Computer Algebra Independent Integration Tests

Summer 2023 edition

1-Algebraic-functions/1.2-Trinomial-products/1.2.2-Quartic/41-1.2.2.4-f-x-^m-d+e-x^2-^q-a+b-x^2+c-x^4-^p

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Contents

CHAPTER 1

INTRODUCTION

This report gives the result of running the computer algebra independent integration test.The download section in on the main webpage contains links to download the problems in plain text format used for all CAS systems. The number of integrals in this report is [413]. This is test number [41].

1.1 Listing of CAS systems tested

The following are the CAS systems tested:

1. Mathematica 13.3.1 (August 16, 2023) on windows 10.
2. Rubi 4.16.1 (Dec 19, 2018) on Mathematica 13.3 on windows 10
3. Maple 2023.1 (July, 12, 2023) on windows 10.
4. Maxima 5.47 (June 1, 2023) using Lisp SBCL 2.3.0 on Linux via sagemath 10.1 (Aug 20, 2023).
5. FriCAS 1.3.9 (July 8, 2023) based on sbcl 2.3.0 on Linux via sagemath 10.1 (Aug 20, 2023).
6. Giac/Xcas 1.9.0-57 (June 26, 2023) on Linux via sagemath 10.1 (Aug 20, 2023).
7. Sympy 1.12 (May 10, 2023) Using Python 3.11.3 on Linux.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 on windows 10.

Maxima and Fricas and Giac are called using Sagemath. This was done using Sagemath `integrate` command by changing the name of the algorithm to use the different CAS systems.

Sympy was run directly in Python not via sagemath.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed. If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	% solved	% Failed
Rubi	100.00 (413)	0.00 (0)
Mathematica	98.31 (406)	1.69 (7)
Maple	91.04 (376)	8.96 (37)
Fricas	79.66 (329)	20.34 (84)
Giac	62.71 (259)	37.29 (154)
Mupad	52.78 (218)	47.22 (195)
Maxima	34.87 (144)	65.13 (269)
Sympy	31.72 (131)	68.28 (282)

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

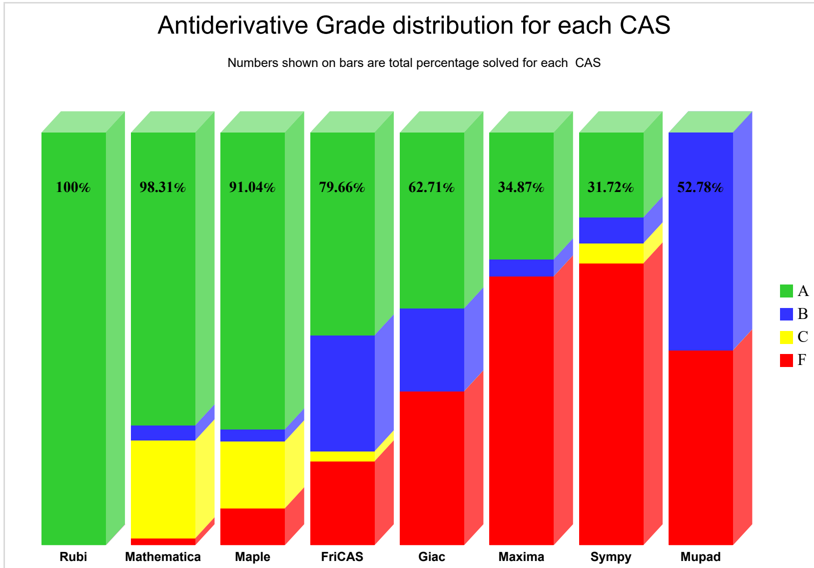
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

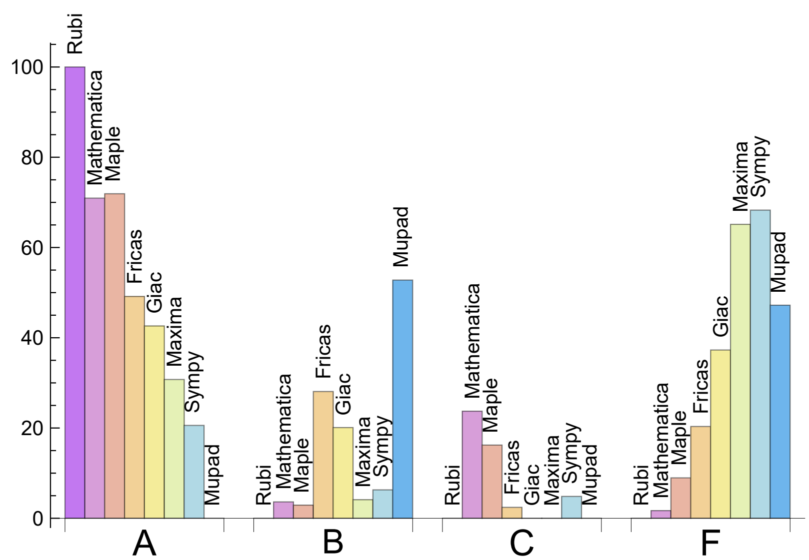
System	% A grade	% B grade	% C grade	% F grade
Rubi	100.000	0.000	0.000	0.000
Maple	71.913	2.906	16.223	8.959
Mathematica	70.944	3.632	23.729	1.695
Fricas	49.153	28.087	2.421	20.339
Giac	42.615	20.097	0.000	37.288
Maxima	30.751	4.116	0.000	65.133
Sympy	20.581	6.295	4.843	68.281
Mupad	0.000	52.785	0.000	47.215

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the grades of the CAS systems.



The following table shows the distribution of the different types of failures for each CAS. There are 3 types failures. The first is when CAS returns the input within the time limit, which means it could not solve it. This is the typical failure and given as **F**.

The second failure is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned. This is assigned **F(-1)**.

The third is due to an exception generated, indicated as **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and Giac) or it could be an indication of an internal error in the CAS itself. This type of error requires more investigation to determine the cause.

System	Number failed	Percentage normal failure	Percentage timeout failure	Percentage exception failure
Rubi	0	0.00	0.00	0.00
Mathematica	7	100.00	0.00	0.00
Maple	37	100.00	0.00	0.00
Fricas	84	80.95	19.05	0.00
Giac	154	73.38	11.04	15.58
Mupad	195	0.00	100.00	0.00
Maxima	269	71.75	0.00	28.25
Sympy	282	67.73	32.27	0.00

Table 1.4: Failure statistics for each CAS

1.3 Time and leaf size Performance

The table below summarizes the performance of each CAS system in terms of time used and leaf size of results.

Mean size is the average leaf size produced by the CAS (before any normalization). The Normalized mean is relative to the mean size of the optimal anti-derivative given in the input files.

For example, if CAS has **Normalized mean** of 3, then the mean size of its leaf size is 3 times as large as the mean size of the optimal leaf size.

Median size is value of leaf size where half the values are larger than this and half are smaller (before any normalization). i.e. The Middle value.

Similarly the **Normalized median** is relative to the median leaf size of the optimal.

For example, if a CAS has Normalized median of 1.2, then its median is 1.2 as large as the median leaf size of the optimal.

System	Mean time (sec)
Maxima	0.24
Rubi	0.35
Giac	0.51
Maple	0.79
Mathematica	3.01
Sympy	3.14
Mupad	8.36
Fricas	13.06

Table 1.5: Time performance for each CAS

System	Mean size	Normalized mean	Median size	Normalized median
Maxima	109.45	1.14	94.00	1.00
Maple	197.24	0.94	146.50	0.84
Rubi	229.98	1.01	189.00	1.00
Mathematica	349.61	1.36	160.50	0.96
Sympy	504.31	3.06	121.00	1.09
Giac	856.24	3.32	145.00	1.12
Fricas	2055.09	6.49	205.00	1.80
Mupad	4535.03	13.49	169.00	1.95

Table 1.6: Leaf size performance for each CAS

1.4 Performance based on number of rules Rubi used

This section shows how each CAS performed based on the number of rules Rubi needed to solve the same integral. One diagram is given for each CAS.

On the y axis is the percentage solved which Rubi itself needed the number of rules given the x axis. These plots show that as more rules are needed then most CAS system percentage of solving decreases which indicates the integral is becoming more complicated to solve.

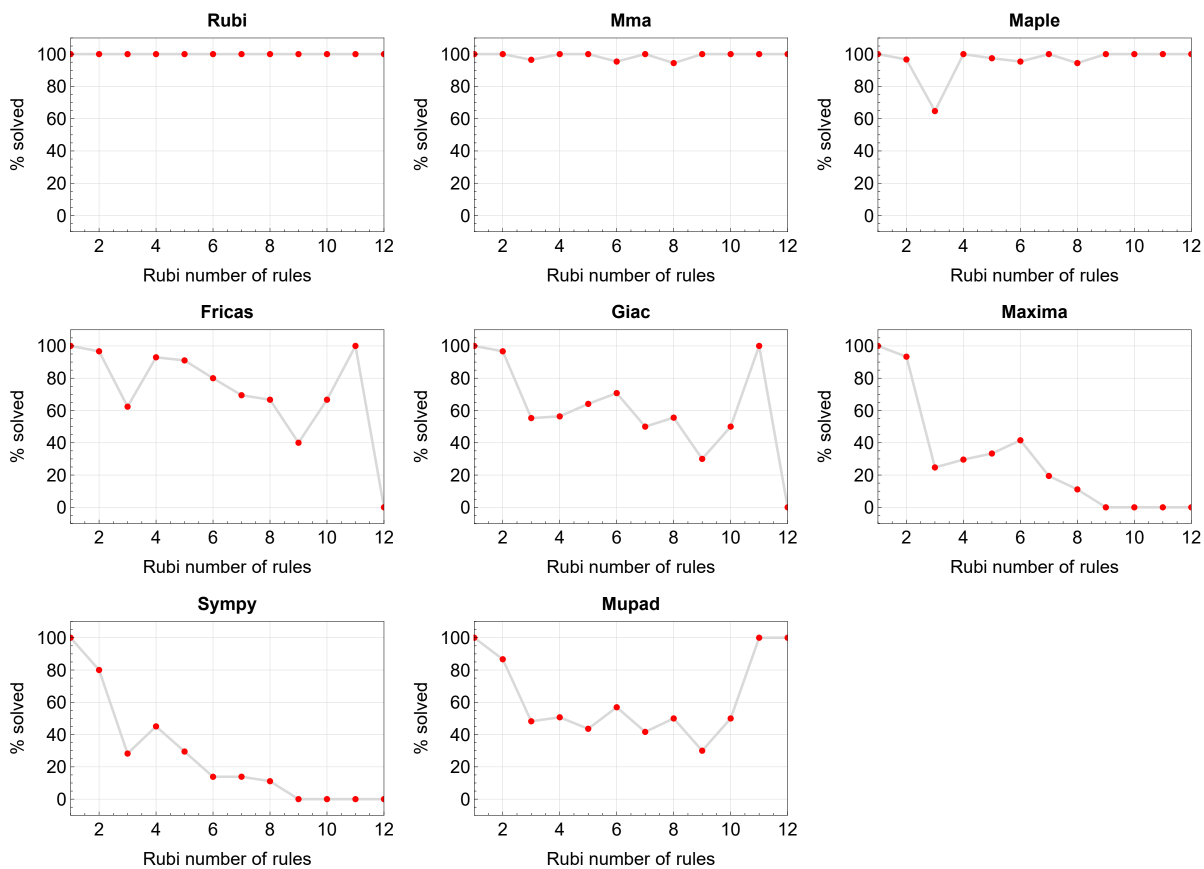


Figure 1.1: Solving statistics per number of Rubi rules used

1.5 Performance based on number of steps Rubi used

This section shows how each CAS performed based on the number of steps Rubi needed to solve the same integral. Note that the number of steps Rubi needed can be much higher than the number of rules, as the same rule could be used more than once.

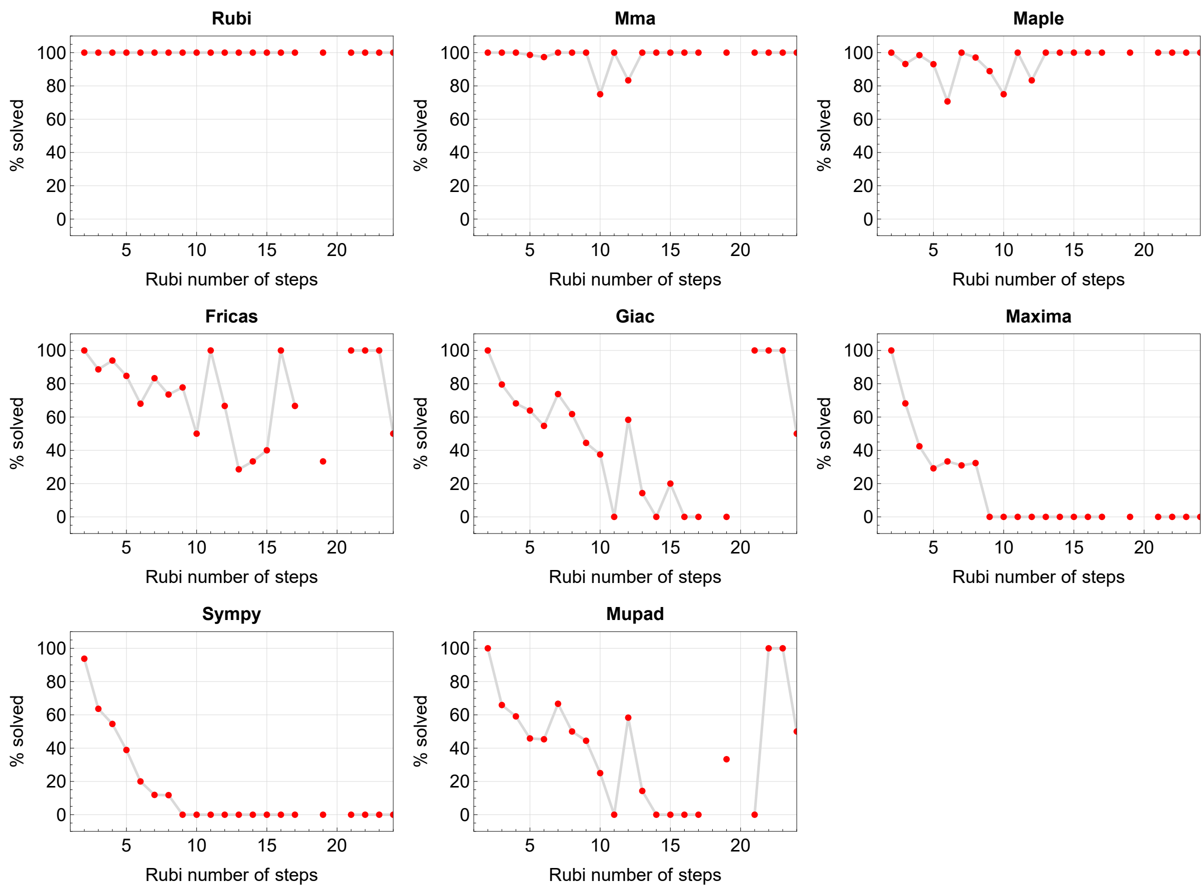


Figure 1.2: Solving statistics per number of Rubi steps used

The above diagram show that the precentage of solved intergals decreases for most CAS systems as the number of steps increases. As expected, for integrals that required less steps by Rubi, CAS systems had more success which indicates the integral was not as hard to solve. As Rubi needed more steps to solve the integral, the solved percentage decreased for most CAS systems which indicates the integral is becoming harder to solve.

1.6 Solved integrals histogram based on leaf size of result

The following shows the distribution of solved integrals for each CAS system based on leaf size of the antiderivatives produced by each CAS. It shows that most integrals solved produced leaf size less than about 100 to 150. The bin size used is 40.

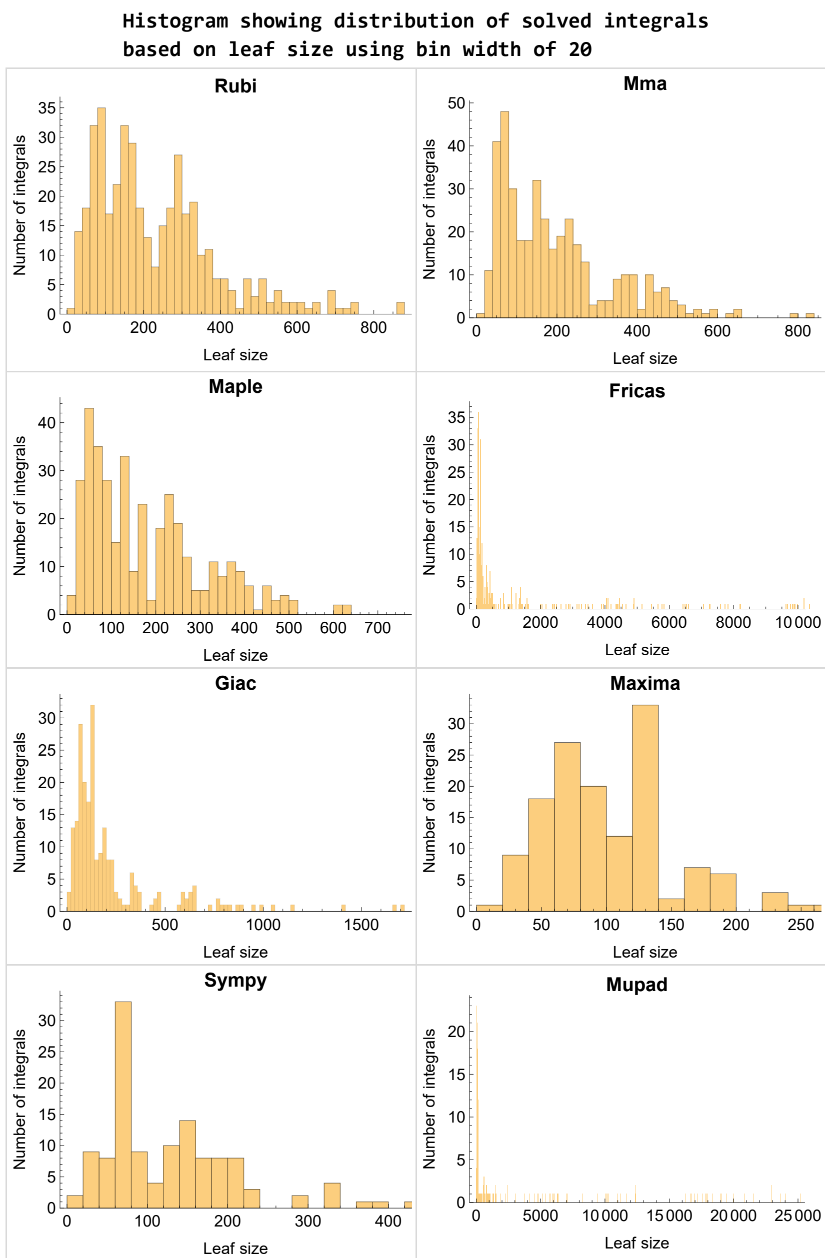


Figure 1.3: Solved integrals based on leaf size distribution

1.7 Solved integrals histogram based on CPU time used

The following shows the distribution of solved integrals for each CAS system based on CPU time used in seconds. The bin size used is 0.1 second.

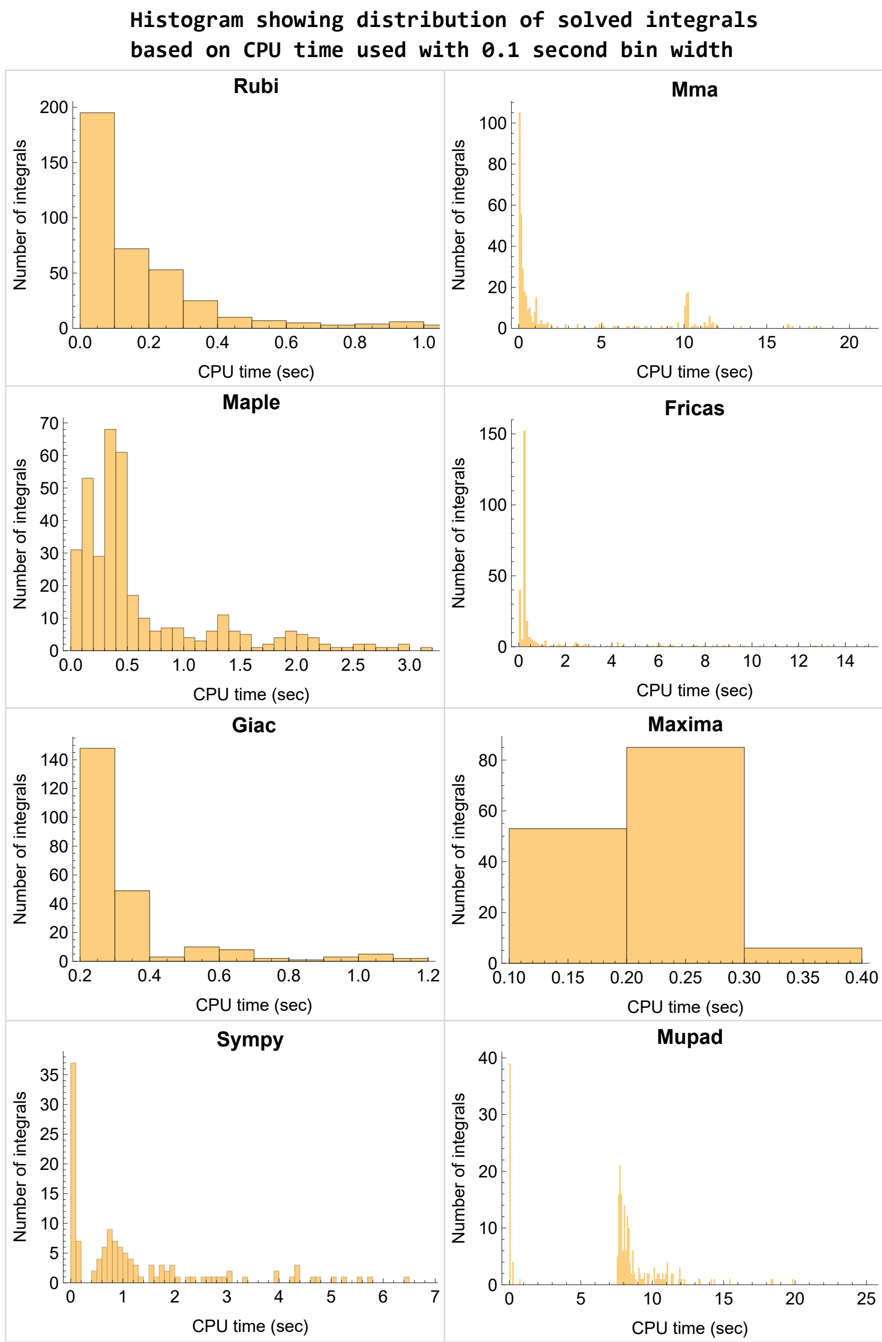


Figure 1.4: Solved integrals histogram based on CPU time used

1.8 Leaf size vs. CPU time used

The following shows the relation between the CPU time used to solve an integral and the leaf size of the antiderivative.

The result for Fricas, Maxima and Giac is shifted more to the right than the other CAS system due to the use of sagemath to call them, which causes an initial slight delay in the timing to start the integration due to overhead of starting a new process each time. This should also be taken into account when looking at the timing of these three CAS systems. Direct calls not using sagemath would result in faster timings, but current implementation uses sagemath as this makes testing much easier to do.

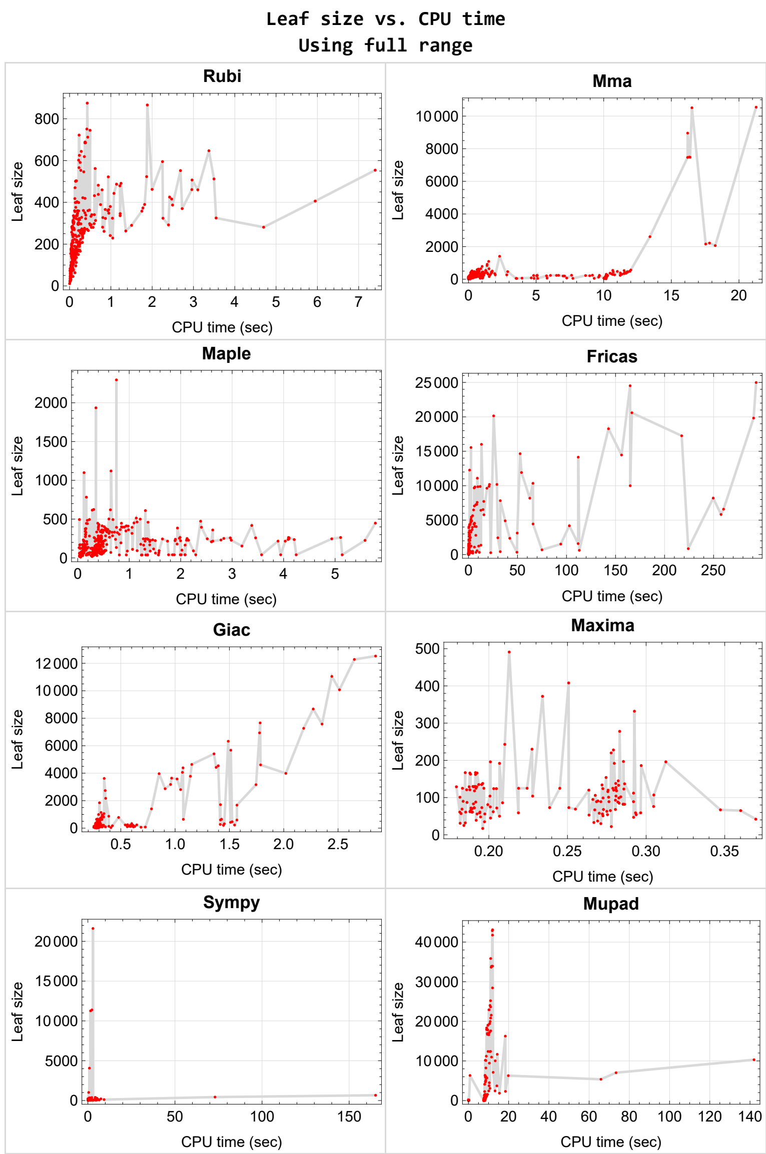


Figure 1.5: Leaf size vs. CPU time. Full range

1.9 list of integrals with no known antiderivative

{}

1.10 List of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

Mupad {}

1.11 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not necessarily mean that the anti-derivative is wrong as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it possible to do further investigation to determine why the result could not be verified.

Rubi {}

Mathematica {151, 152, 153, 154, 155, 163, 164, 165, 166, 167, 168, 189, 190, 192, 193, 199, 200, 201, 202, 203, 225, 397, 398, 399}

Maple {76, 78, 80, 82, 84, 86}

Maxima Verification phase not currently implemented.

Fricas Verification phase not currently implemented.

Sympy Verification phase not currently implemented.

Giac Verification phase not currently implemented.

Mupad Verification phase not currently implemented.

1.12 Timing

The command `AbsoluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of_int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 CPU minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out was not counted in the final statistics.

1.13 Verification

A verification phase was applied on the result of integration for `Rubi` and `Mathematica`.

Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative was correct.

Verification phase also had 3 minutes time out. An integral whose result was not verified could still be correct, but further investigation is needed on those integrals. These integrals were marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.14 Important notes about some of the results

Important note about Maxima results

Since tests were run in a batch mode, and using an automated script, then any integral where Maxima needed an interactive response from the user to answer a question during the evaluation of the integral will fail.

The exception raised is `ValueError`. Therefore Maxima results is lower than what would result if Maxima was run directly and each question was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the `Timofeev` test file, there were about 14 such integrals out of total 705, or about 2 percent. This percentage can be higher or lower depending on the specific input test file.

Such integrals can be identified by looking at the output of the integration in each section for Maxima. The exception message will indicate the cause of error.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath automatic loading of Maxima `abs_integrate` was found to cause some problems. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See `https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/` for reference.

Important note about FriCAS result

There were few integrals which failed due to SageMath interface and not because FriCAS system could not do the integration.

These will fail With error `Exception raised: NotImplementedError`.

The number of such cases seems to be very small. About 1 or 2 percent of all integrals. These can be identified by looking at the exception message given in the result.

Important note about finding leaf size of antiderivative

For Mathematica, Rubi, and Maple, the builtin system function `LeafSize` was used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special builtin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy antiderivative was determined using the following function, thanks to user `slelievre` at `https://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/`

```
def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
```

```
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which was called directly from Python, the following code was used to obtain the leafsize of its result

```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
        leafCount =1
```

Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative. Maple was used to determine the leaf size of Mupad output by post processing Mupad result.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

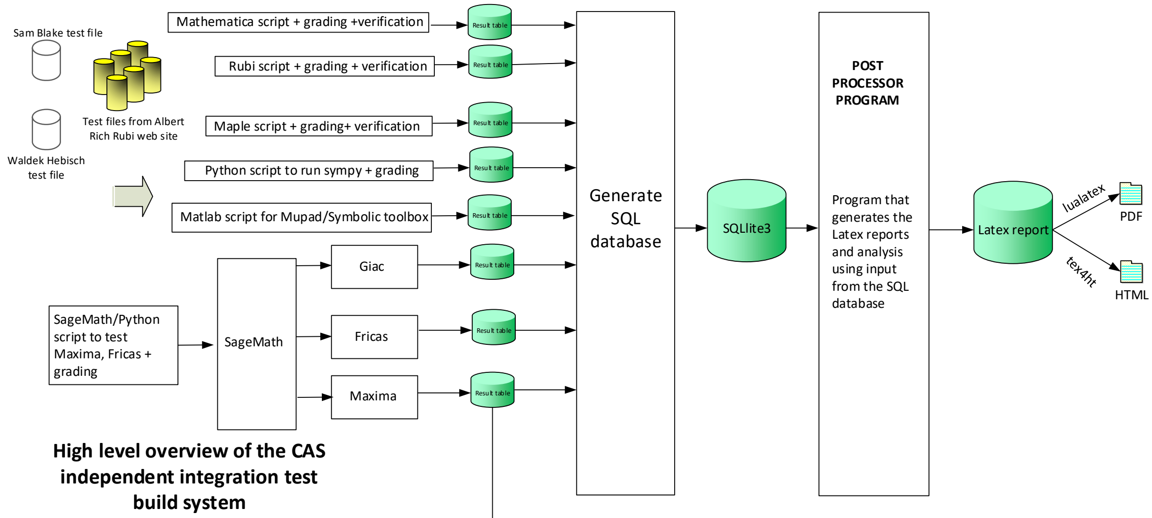
The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine,'cos(x)*sin(x)')
the_variable = evalin(symengine,'x')
anti = int(integrand,the_variable)
```

Which gives `sin(x)^2/2`

1.15 Design of the test system

The following diagram gives a high level view of the current test build system.



One record (line) per one integral result. The line is CSV comma separated. This is description of each record

1. integer, the problem number.
2. integer. 0 for failed, 1 for passed, -1 for timeout, -2 for CAS specific exception. (this is not the grade field)
3. integer. Leaf size of result.
4. integer. Leaf size of the optimal antiderivative.
5. number. CPU time used to solve this integral. 0 if failed.
6. string. The integral in Latex format
7. string. The input used in CAS own syntax.
8. string. The result (antiderivative) produced by CAS in Latex format
9. string. The optimal antiderivative in Latex format.
10. integer. 0 or 1. Indicates if problem has known antiderivative or not
11. String. The result (antiderivative) in CAS own syntax.
12. String. The grade of the antiderivative. Can be “A”, “B”, “C”, or “F”
13. String. Small string description of why the grade was given.
14. integer. 1 if result was verified or 0 if not verified. (For mma, rubi and maple only)

*The following fields are present only in **Rubi** Table file*

15. integer. Number of steps used.
16. integer. Number of rules used.
17. integer. Integrand leaf size.
18. real number. Ratio. Field 16 over field 17
19. String of form “{n,n,..}” which is list of the rules used by Rubi
20. String. The optimal antiderivative in Mathematica syntx

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June 27, 2023
Design.vsdx

CHAPTER 2

DETAILED SUMMARY TABLES OF RESULTS

2.1 List of integrals sorted by grade for each CAS

Rubi

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413 }

B grade { }

C grade { }

F normal fail { }

F(-1) timedout fail { }

F(-2) exception fail { }

Mma

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 20, 21, 22, 23, 24, 25, 26, 32, 33, 34, 35, 36, 37, 38, 44, 45, 46, 47, 48, 49, 55, 57, 59, 61, 62, 63, 64, 65, 67, 69, 70, 71, 72, 73, 74, 75, 77, 78, 79, 80, 81, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 156, 157, 158, 159, 160, 161, 162, 169, 170, 171, 172, 173, 174, 175, 181, 182, 183, 184, 185, 186, 187, 188, 194, 195, 196, 197, 198, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 261, 262, 263, 264, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 311, 312, 313, 314, 315, 322, 323, 324, 325, 326, 332, 333, 334, 335, 336, 342, 343, 344, 345, 346, 347, 354, 355, 358, 359, 360, 376, 377, 378, 379, 380, 388, 391, 392, 393, 401, 402, 403, 404, 405, 406, 413 }

B grade { 56, 58, 60, 66, 68, 76, 82, 362, 363, 365, 366, 373, 374, 385, 394 }

C grade { 15, 16, 17, 18, 19, 27, 28, 29, 30, 31, 39, 40, 41, 42, 43, 50, 51, 52, 53, 54, 151, 152, 153, 154, 155, 163, 164, 165, 166, 167, 168, 176, 177, 178, 179, 180, 189, 190, 191, 192, 193, 199, 200, 201, 202, 203, 224, 259, 260, 265, 266, 310, 316, 317, 318, 319, 320, 321, 327, 328, 329, 330, 331, 337, 338, 339, 340, 341, 348, 349, 350, 351, 352, 353, 356, 357, 361, 364, 367, 368, 369, 370, 371, 372, 375, 381, 382, 383, 384, 386, 387, 389, 390, 395, 396, 397, 398, 399 }

F normal fail { 400, 407, 408, 409, 410, 411, 412 }

F(-1) timedout fail { }

F(-2) exception fail { }

Maple

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 20, 21, 22, 23, 24, 25, 26, 32, 33, 34, 35, 36, 37, 38, 44, 45, 46, 47, 48, 49, 57, 59, 61, 62, 63, 64, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 77, 79, 81, 83, 89, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 110, 111, 112, 113, 114, 115, 116, 117, 122, 123, 124, 127, 128, 129, 130, 131, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 222, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 261, 262, 263, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 311, 312, 313, 314, 315, 322, 323, 324, 325, 326, 332, 333, 334, 335, 336, 342, 343, 344, 345, 346, 347, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399 }

B grade { 55, 56, 58, 60, 65, 87, 88, 125, 126, 220, 221, 264 }

C grade { 15, 16, 17, 18, 19, 27, 28, 29, 30, 31, 39, 40, 41, 42, 43, 50, 51, 52, 53, 54, 76, 78, 80, 82, 84, 85, 86, 107, 108, 109, 118, 119, 120, 121, 132, 133, 134, 135, 136, 259, 260, 265, 266, 310, 316, 317, 318, 319, 320, 321, 327, 328, 329, 330, 331, 337, 338, 339, 340, 341, 348, 349, 350, 351, 352, 353, 413 }

F normal fail { 90, 91, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 223, 224, 225, 226, 227, 228, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412 }

F(-1) timedout fail { }

F(-2) exception fail { }

Fricas

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 44, 45, 46, 47, 48, 49, 50, 57, 59, 61, 62, 63, 64, 67, 69, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 88, 89, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 198, 199, 200, 201, 202, 203, 229, 230, 231, 232, 233, 234, 235, 236, 244, 245, 246, 247, 248, 249, 250, 261, 262, 263, 264, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 296, 297, 298, 299, 311, 312, 313, 314, 315, 324, 336 }

B grade { 55, 56, 58, 60, 65, 66, 68, 70, 87, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 196, 197, 220, 221, 222, 237, 238, 239, 240, 241, 242, 243, 252, 253, 254, 255, 256, 257, 258, 303, 304, 305, 306, 307, 308, 309, 332, 333, 334, 335, 342, 343, 344, 345, 346, 347, 354, 355, 356, 357, 358, 359, 361, 362, 363, 364, 365, 366, 367, 368, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 395, 396, 397, 413 }

C grade { 42, 43, 51, 52, 53, 54, 259, 260, 265, 266 }

F normal fail { 18, 19, 30, 31, 90, 91, 154, 155, 166, 167, 168, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 223, 224, 225, 226, 227, 228, 316, 317, 318, 319, 320, 321, 327, 328, 329, 330, 331, 337, 338, 339, 340, 341, 348, 349, 350, 351, 352, 353, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412 }

F(-1) timedout fail { 251, 295, 300, 301, 302, 310, 322, 323, 325, 326, 360, 369, 370, 394, 398, 399 }

F(-2) exception fail { }

Maxima

A grade { 1, 2, 3, 4, 5, 6, 7, 12, 13, 14, 20, 24, 25, 26, 32, 33, 37, 38, 44, 45, 46, 47, 48, 49, 55, 57, 59, 61, 62, 63, 64, 65, 67, 69, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 93, 94, 95, 96, 97, 98, 99, 100, 101, 137, 138, 139, 140, 142, 143, 144, 145, 146, 147, 148, 149, 150, 156, 157, 158, 159, 160, 161, 162, 181, 182, 183, 184, 185, 186, 187, 188, 194, 195, 196, 197, 198, 220, 221, 222, 229, 230, 231, 232, 233, 234, 235, 236, 244, 245, 246, 247, 248, 249, 250, 251, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279 }

B grade { 8, 9, 10, 11, 21, 22, 23, 34, 35, 36, 56, 58, 60, 66, 68, 70, 92 }

C grade { }

F normal fail { 15, 16, 17, 18, 19, 27, 28, 29, 30, 31, 39, 40, 41, 42, 43, 50, 51, 52, 53, 54, 90, 91, 107, 108, 109, 110, 111, 118, 119, 120, 121, 122, 123, 132, 133, 134, 135, 136, 141, 151, 152, 153, 154, 155, 163, 164, 165, 166, 167, 168, 176, 177, 178, 179, 180, 189, 190, 191, 192, 193, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 223, 224, 225, 226, 227, 228, 259, 260, 261, 262, 263, 264, 265, 266, 310, 314, 315, 316, 317, 318, 319, 320, 321, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413 }

F(-1) timedout fail { }

F(-2) exception fail { 102, 103, 104, 105, 106, 112, 113, 114, 115, 116, 117, 124, 125, 126, 127, 128, 129, 130, 131, 169, 170, 171, 172, 173, 174, 175, 237, 238, 239, 240, 241, 242, 243, 252, 253, 254, 255, 256, 257, 258, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 311, 312, 313, 322, 323, 324 }

Giac

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 20, 21, 22, 23, 24, 32, 33, 34, 35, 44, 45, 46, 47, 48, 49, 57, 59, 61, 62, 63, 64, 67, 69, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 92, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 112, 113, 114, 115, 116, 117, 124, 125, 127, 128, 129, 130, 131, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 156, 158, 159, 160, 161, 169, 170, 171, 173, 181, 182, 183, 184, 185, 194, 195, 196, 197, 198, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 315, 334, 336, 413 }

B grade { 13, 14, 25, 26, 36, 37, 38, 55, 56, 58, 60, 65, 66, 68, 70, 87, 88, 89, 93, 94, 107, 108, 109, 110, 111, 118, 119, 120, 121, 122, 123, 126, 132, 133, 134, 135, 136, 147, 148, 149, 150, 157, 162, 174, 175, 186, 187, 188, 220, 221, 222, 303, 304, 305, 306, 307, 308, 309, 343, 344, 345, 347, 354, 355, 356, 357, 358, 359, 360, 367, 368, 369, 370, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385 }

C grade { }

F normal fail { 15, 16, 17, 18, 19, 27, 28, 29, 30, 31, 39, 40, 41, 42, 43, 50, 51, 52, 53, 54, 90, 91, 151, 152, 153, 154, 155, 163, 164, 165, 166, 167, 168, 176, 177, 178, 179, 180, 189, 190, 191, 192, 193, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 223, 224, 225, 226, 227, 228, 259, 260, 261, 262, 263, 264, 265, 266, 316, 317, 318, 319, 320, 321, 327, 328, 329, 330, 331, 337, 338, 339, 340, 341, 348, 349, 350, 351, 352, 353, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412 }

F(-1) timedout fail { 310, 363, 364, 365, 366, 374, 375, 389, 390, 391, 392, 393, 395, 396, 397, 398, 399 }

F(-2) exception fail { 172, 311, 312, 313, 314, 322, 323, 324, 325, 326, 332, 333, 335, 342, 346, 361, 362, 371, 372, 373, 386, 387, 388, 394 }

Mupad

A grade { }

B grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 18, 20, 21, 22, 23, 24, 25, 26, 30, 32, 33, 34, 35, 36, 37, 38, 42, 44, 45, 46, 47, 48, 49, 53, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 76, 78, 80, 82, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 158, 171, 172, 173, 184, 185, 186, 195, 196, 220, 221, 222, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 354, 355, 356, 357, 358, 359, 360, 367, 368, 369, 370, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385 }

C grade { }

F normal fail { }

F(-1) timedout fail { 15, 16, 17, 19, 27, 28, 29, 31, 39, 40, 41, 43, 50, 51, 52, 54, 75, 77, 79, 81, 83, 84, 85, 86, 87, 88, 89, 90, 91, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 187, 188, 189, 190, 191, 192, 193, 194, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 223, 224, 225, 226, 227, 228, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 361, 362, 363, 364, 365, 366, 371, 372, 373, 374, 375, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413 }

F(-2) exception fail { }

Sympy

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 20, 21, 22, 23, 24, 25, 26, 32, 33, 34, 35, 36, 37, 38, 44, 45, 46, 57, 59, 61, 62, 63, 64, 67, 69, 71, 72, 73, 74, 76, 95, 96, 97, 98, 99, 100, 101, 137, 138, 139, 140, 141, 142, 143, 144, 156, 158, 169, 170, 171, 181, 182, 183, 184, 273, 274, 275, 276, 277, 278, 279, 281, 282, 284, 285, 288, 289, 290, 291, 292, 293 }

B grade { 47, 48, 49, 55, 56, 58, 60, 65, 66, 68, 70, 92, 103, 104, 114, 115, 129, 157, 220, 221, 222, 280, 283, 286, 287, 294 }

C grade { 15, 16, 17, 18, 19, 27, 28, 29, 30, 31, 39, 40, 41, 42, 43, 50, 51, 52, 53, 54 }

F normal fail { 75, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 93, 94, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 172, 173, 174, 175, 176, 177, 178, 179, 180, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 223, 225, 226, 227, 228, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 368, 369, 370, 371, 372, 373, 374, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 405, 413 }

F(-1) timedout fail { 102, 105, 106, 107, 108, 109, 110, 111, 112, 113, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 130, 131, 132, 133, 134, 135, 136, 219, 224, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 367, 375, 400, 401, 402, 403, 404, 406, 407, 408, 409, 410, 411, 412 }

F(-2) exception fail { }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by the table below. The elapsed time is in seconds. For failed result it is given as **F(-1)** if the failure was due to timeout. It is given as **F(-2)** if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given as **F**.

In this table, the column `N.S.` means **normalized size** and is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$. To make the table fit the page, the name `Mathematica` was abbreviated to `MMA`.

Problem 1	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	149	149	149	126	125	125	151	125	125
N.S.	1	1.00	1.00	0.85	0.84	0.84	1.01	0.84	0.84
time (sec)	N/A	0.144	0.006	0.282	0.192	0.248	0.024	0.265	0.063

Problem 2	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	149	149	149	126	125	125	155	125	125
N.S.	1	1.00	1.00	0.85	0.84	0.84	1.04	0.84	0.84
time (sec)	N/A	0.062	0.005	0.288	0.197	0.262	0.024	0.266	0.063

Problem 3	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	89	89	146	125	124	124	150	124	124
N.S.	1	1.00	1.64	1.40	1.39	1.39	1.69	1.39	1.39
time (sec)	N/A	0.054	0.005	0.283	0.203	0.245	0.027	0.268	0.062

Problem 4	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	141	141	141	122	121	121	148	121	121
N.S.	1	1.00	1.00	0.87	0.86	0.86	1.05	0.86	0.86
time (sec)	N/A	0.051	0.004	0.313	0.186	0.255	0.024	0.267	0.060

Problem 5	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	142	142	142	123	125	122	150	125	122
N.S.	1	1.00	1.00	0.87	0.88	0.86	1.06	0.88	0.86
time (sec)	N/A	0.070	0.008	0.254	0.224	0.254	0.079	0.267	0.063

Problem 6	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	139	139	139	122	121	127	143	121	121
N.S.	1	1.00	1.00	0.88	0.87	0.91	1.03	0.87	0.87
time (sec)	N/A	0.068	0.007	0.273	0.188	0.252	0.078	0.286	0.059

Problem 7	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	142	142	142	123	125	129	150	135	122
N.S.	1	1.00	1.00	0.87	0.88	0.91	1.06	0.95	0.86
time (sec)	N/A	0.080	0.007	0.288	0.245	0.237	0.096	0.262	0.060

Problem 8	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	67	67	56	44	102	48	60	54	42
N.S.	1	1.00	0.84	0.66	1.52	0.72	0.90	0.81	0.63
time (sec)	N/A	0.033	0.084	0.355	0.284	0.238	0.606	0.268	7.729

Problem 9	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	51	51	51	39	93	43	54	45	37
N.S.	1	1.00	1.00	0.76	1.82	0.84	1.06	0.88	0.73
time (sec)	N/A	0.034	0.070	0.278	0.280	0.238	0.576	0.262	7.817

Problem 10	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	44	44	42	30	67	34	48	38	32
N.S.	1	1.00	0.95	0.68	1.52	0.77	1.09	0.86	0.73
time (sec)	N/A	0.016	0.070	0.258	0.267	0.237	0.515	0.278	7.564

Problem 11	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	58	58	69	49	99	56	75	76	45
N.S.	1	1.00	1.19	0.84	1.71	0.97	1.29	1.31	0.78
time (sec)	N/A	0.039	0.125	0.464	0.283	0.252	4.779	0.294	7.607

Problem 12	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	59	59	71	49	88	72	83	91	51
N.S.	1	1.00	1.20	0.83	1.49	1.22	1.41	1.54	0.86
time (sec)	N/A	0.036	0.125	0.467	0.282	0.245	3.307	0.283	7.887

Problem 13	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	63	63	72	54	91	72	76	129	56
N.S.	1	1.00	1.14	0.86	1.44	1.14	1.21	2.05	0.89
time (sec)	N/A	0.037	0.198	0.438	0.269	0.247	2.850	0.276	7.739

Problem 14	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	58	58	60	48	59	59	63	116	43
N.S.	1	1.00	1.03	0.83	1.02	1.02	1.09	2.00	0.74
time (sec)	N/A	0.030	0.175	0.356	0.293	0.243	2.569	0.270	7.866

Problem 15	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	A	C	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	208	208	82	40	0	69	78	0	0
N.S.	1	1.00	0.39	0.19	0.00	0.33	0.38	0.00	0.00
time (sec)	N/A	0.086	4.641	5.138	0.000	0.084	1.029	0.000	0.000

Problem 16	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	A	C	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	192	192	68	40	0	63	78	0	0
N.S.	1	1.00	0.35	0.21	0.00	0.33	0.41	0.00	0.00
time (sec)	N/A	0.057	3.957	1.841	0.000	0.080	0.912	0.000	0.000

Problem 17	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	A	C	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	176	176	48	37	0	58	76	0	0
N.S.	1	1.00	0.27	0.21	0.00	0.33	0.43	0.00	0.00
time (sec)	N/A	0.034	3.569	1.237	0.000	0.082	0.820	0.000	0.000

Problem 18	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F	C	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	171	171	53	38	0	0	78	0	61
N.S.	1	1.00	0.31	0.22	0.00	0.00	0.46	0.00	0.36
time (sec)	N/A	0.040	3.520	1.356	0.000	0.000	0.994	0.000	7.773

Problem 19	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F	C	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	192	192	54	40	0	0	83	0	0
N.S.	1	1.00	0.28	0.21	0.00	0.00	0.43	0.00	0.00
time (sec)	N/A	0.053	7.719	2.010	0.000	0.000	1.104	0.000	0.000

Problem 20	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	83	83	66	54	127	58	121	80	52
N.S.	1	1.00	0.80	0.65	1.53	0.70	1.46	0.96	0.63
time (sec)	N/A	0.035	0.103	0.327	0.281	0.242	0.938	0.270	7.642

Problem 21	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	67	67	61	49	118	53	109	71	47
N.S.	1	1.00	0.91	0.73	1.76	0.79	1.63	1.06	0.70
time (sec)	N/A	0.024	0.097	0.296	0.281	0.237	0.848	0.275	7.694

Problem 22	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	60	60	56	44	95	48	95	57	42
N.S.	1	1.00	0.93	0.73	1.58	0.80	1.58	0.95	0.70
time (sec)	N/A	0.018	0.088	0.282	0.266	0.238	0.794	0.268	7.539

Problem 23	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	78	78	79	56	138	67	117	90	55
N.S.	1	1.00	1.01	0.72	1.77	0.86	1.50	1.15	0.71
time (sec)	N/A	0.044	0.142	0.441	0.281	0.256	9.318	0.286	7.579

Problem 24	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	81	81	78	62	122	78	114	102	64
N.S.	1	1.00	0.96	0.77	1.51	0.96	1.41	1.26	0.79
time (sec)	N/A	0.043	0.161	0.497	0.281	0.248	4.354	0.286	8.020

Problem 25	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	86	86	83	65	123	82	133	146	71
N.S.	1	1.00	0.97	0.76	1.43	0.95	1.55	1.70	0.83
time (sec)	N/A	0.046	0.209	0.493	0.286	0.260	5.507	0.283	7.929

Problem 26	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	82	82	83	65	112	82	148	158	82
N.S.	1	1.00	1.01	0.79	1.37	1.00	1.80	1.93	1.00
time (sec)	N/A	0.047	0.229	0.479	0.292	0.262	5.793	0.310	8.079

Problem 27	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	A	C	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	235	235	74	40	0	79	160	0	0
N.S.	1	1.00	0.31	0.17	0.00	0.34	0.68	0.00	0.00
time (sec)	N/A	0.081	7.264	4.244	0.000	0.083	1.800	0.000	0.000

Problem 28	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	A	C	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	219	219	68	40	0	73	160	0	0
N.S.	1	1.00	0.31	0.18	0.00	0.33	0.73	0.00	0.00
time (sec)	N/A	0.078	5.812	1.875	0.000	0.082	1.645	0.000	0.000

Problem 29	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	A	C	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	197	197	49	38	0	68	158	0	0
N.S.	1	1.00	0.25	0.19	0.00	0.35	0.80	0.00	0.00
time (sec)	N/A	0.050	5.115	1.250	0.000	0.085	1.510	0.000	0.000

Problem 30	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F	C	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	199	199	53	38	0	0	160	0	48
N.S.	1	1.00	0.27	0.19	0.00	0.00	0.80	0.00	0.24
time (sec)	N/A	0.050	4.853	1.355	0.000	0.000	1.840	0.000	7.765

Problem 31	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F	C	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	201	201	54	40	0	0	163	0	0
N.S.	1	1.00	0.27	0.20	0.00	0.00	0.81	0.00	0.00
time (sec)	N/A	0.057	9.297	2.087	0.000	0.000	1.728	0.000	0.000

Problem 32	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	67	67	51	39	104	43	48	46	38
N.S.	1	1.00	0.76	0.58	1.55	0.64	0.72	0.69	0.57
time (sec)	N/A	0.041	0.073	0.309	0.281	0.243	0.742	0.293	7.874

Problem 33	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	51	51	42	30	76	34	32	37	32
N.S.	1	1.00	0.82	0.59	1.49	0.67	0.63	0.73	0.63
time (sec)	N/A	0.025	0.094	0.285	0.305	0.244	0.679	0.279	7.835

Problem 34	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	35	35	41	29	65	33	32	33	27
N.S.	1	1.00	1.17	0.83	1.86	0.94	0.91	0.94	0.77
time (sec)	N/A	0.016	0.076	0.279	0.360	0.244	0.634	0.266	7.852

Problem 35	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	24	24	32	20	42	26	22	26	19
N.S.	1	1.00	1.33	0.83	1.75	1.08	0.92	1.08	0.79
time (sec)	N/A	0.010	0.073	0.274	0.370	0.243	0.614	0.272	7.858

Problem 36	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	38	38	50	30	67	41	31	61	30
N.S.	1	1.00	1.32	0.79	1.76	1.08	0.82	1.61	0.79
time (sec)	N/A	0.023	0.090	0.375	0.347	0.243	2.769	0.285	0.298

Problem 37	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	42	42	46	31	47	47	31	66	31
N.S.	1	1.00	1.10	0.74	1.12	1.12	0.74	1.57	0.74
time (sec)	N/A	0.023	0.101	0.297	0.292	0.249	1.599	0.280	7.675

Problem 38	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	58	58	55	43	59	50	88	114	43
N.S.	1	1.00	0.95	0.74	1.02	0.86	1.52	1.97	0.74
time (sec)	N/A	0.032	0.158	0.326	0.297	0.255	3.948	0.280	7.783

Problem 39	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	A	C	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	185	185	74	40	0	59	75	0	0
N.S.	1	1.00	0.40	0.22	0.00	0.32	0.41	0.00	0.00
time (sec)	N/A	0.050	10.026	3.943	0.000	0.082	1.149	0.000	0.000

Problem 40	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	A	C	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	166	166	66	40	0	50	75	0	0
N.S.	1	1.00	0.40	0.24	0.00	0.30	0.45	0.00	0.00
time (sec)	N/A	0.040	10.038	1.765	0.000	0.079	1.050	0.000	0.000

Problem 41	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	A	C	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	155	155	48	38	0	47	73	0	0
N.S.	1	1.00	0.31	0.25	0.00	0.30	0.47	0.00	0.00
time (sec)	N/A	0.026	10.026	0.808	0.000	0.075	0.778	0.000	0.000

Problem 42	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	C	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	173	173	53	38	0	73	75	0	48
N.S.	1	1.00	0.31	0.22	0.00	0.42	0.43	0.00	0.28
time (sec)	N/A	0.040	10.038	1.323	0.000	0.080	0.813	0.000	8.229

Problem 43	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	C	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	189	189	54	40	0	84	80	0	0
N.S.	1	1.00	0.29	0.21	0.00	0.44	0.42	0.00	0.00
time (sec)	N/A	0.056	10.026	1.993	0.000	0.081	0.993	0.000	0.000

Problem 44	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	58	58	51	39	89	62	66	45	97
N.S.	1	1.00	0.88	0.67	1.53	1.07	1.14	0.78	1.67
time (sec)	N/A	0.035	0.111	0.304	0.271	0.240	6.472	0.264	8.231

Problem 45	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	45	45	44	32	63	58	48	39	89
N.S.	1	1.00	0.98	0.71	1.40	1.29	1.07	0.87	1.98
time (sec)	N/A	0.025	0.124	0.296	0.275	0.240	5.013	0.271	8.116

Problem 46	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	35	35	41	29	54	52	39	33	82
N.S.	1	1.00	1.17	0.83	1.54	1.49	1.11	0.94	2.34
time (sec)	N/A	0.017	0.111	0.290	0.294	0.250	4.341	0.263	8.066

Problem 47	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	20	20	20	17	22	31	31	16	16
N.S.	1	1.00	1.00	0.85	1.10	1.55	1.55	0.80	0.80
time (sec)	N/A	0.010	0.103	0.280	0.278	0.257	3.018	0.271	0.032

Problem 48	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	46	52	35	56	61	212	61	40
N.S.	1	1.00	1.13	0.76	1.22	1.33	4.61	1.33	0.87
time (sec)	N/A	0.029	0.143	0.320	0.272	0.257	7.463	0.279	7.974

Problem 49	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	65	65	59	43	68	77	228	82	47
N.S.	1	1.00	0.91	0.66	1.05	1.18	3.51	1.26	0.72
time (sec)	N/A	0.035	0.148	0.326	0.280	0.258	4.622	0.290	7.980

Problem 50	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	A	C	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	196	196	70	40	0	77	75	0	0
N.S.	1	1.00	0.36	0.20	0.00	0.39	0.38	0.00	0.00
time (sec)	N/A	0.051	10.040	3.572	0.000	0.080	2.097	0.000	0.000

Problem 51	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	C	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	177	177	68	40	0	98	75	0	0
N.S.	1	1.00	0.38	0.23	0.00	0.55	0.42	0.00	0.00
time (sec)	N/A	0.048	10.031	2.293	0.000	0.083	1.988	0.000	0.000

Problem 52	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	C	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	180	180	66	38	0	98	73	0	0
N.S.	1	1.00	0.37	0.21	0.00	0.54	0.41	0.00	0.00
time (sec)	N/A	0.040	10.032	1.568	0.000	0.083	1.974	0.000	0.000

Problem 53	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	C	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	196	196	71	38	0	107	75	0	48
N.S.	1	1.00	0.36	0.19	0.00	0.55	0.38	0.00	0.24
time (sec)	N/A	0.049	10.049	0.944	0.000	0.085	2.688	0.000	7.951

Problem 54	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	C	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	214	214	54	40	0	118	80	0	0
N.S.	1	1.00	0.25	0.19	0.00	0.55	0.37	0.00	0.00
time (sec)	N/A	0.070	10.025	2.083	0.000	0.087	3.003	0.000	0.000

Problem 55	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	269	269	189	2295	372	1571	21612	3620	1539
N.S.	1	1.00	0.70	8.53	1.38	5.84	80.34	13.46	5.72
time (sec)	N/A	0.111	0.429	0.752	0.234	0.264	2.913	0.347	8.616

Problem 56	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	63	63	153	122	129	129	134	132	121
N.S.	1	1.00	2.43	1.94	2.05	2.05	2.13	2.10	1.92
time (sec)	N/A	0.130	0.019	0.140	0.179	0.231	0.035	0.275	0.071

Problem 57	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	153	153	153	124	129	129	141	133	123
N.S.	1	1.00	1.00	0.81	0.84	0.84	0.92	0.87	0.80
time (sec)	N/A	0.080	0.018	0.132	0.192	0.231	0.025	0.261	7.625

Problem 58	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	45	45	151	124	129	129	136	133	123
N.S.	1	1.00	3.36	2.76	2.87	2.87	3.02	2.96	2.73
time (sec)	N/A	0.082	0.016	0.139	0.190	0.229	0.044	0.257	0.063

Problem 59	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	153	153	153	124	129	129	139	133	123
N.S.	1	1.00	1.00	0.81	0.84	0.84	0.91	0.87	0.80
time (sec)	N/A	0.061	0.015	0.128	0.187	0.239	0.028	0.283	0.064

Problem 60	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	29	29	149	124	129	129	133	133	123
N.S.	1	1.00	5.14	4.28	4.45	4.45	4.59	4.59	4.24
time (sec)	N/A	0.032	0.011	0.124	0.192	0.256	0.036	0.256	0.063

Problem 61	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	143	143	143	121	125	125	134	130	120
N.S.	1	1.00	1.00	0.85	0.87	0.87	0.94	0.91	0.84
time (sec)	N/A	0.043	0.013	0.145	0.183	0.238	0.032	0.253	0.064

Problem 62	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	93	93	149	122	130	127	131	134	121
N.S.	1	1.00	1.60	1.31	1.40	1.37	1.41	1.44	1.30
time (sec)	N/A	0.032	0.020	0.050	0.191	0.235	0.129	0.263	7.722

Problem 63	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	141	141	141	123	125	131	124	128	119
N.S.	1	1.00	1.00	0.87	0.89	0.93	0.88	0.91	0.84
time (sec)	N/A	0.051	0.023	0.051	0.219	0.244	0.125	0.254	0.064

Problem 64	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	147	147	147	123	130	133	131	144	120
N.S.	1	1.00	1.00	0.84	0.88	0.90	0.89	0.98	0.82
time (sec)	N/A	0.090	0.027	0.051	0.190	0.245	0.151	0.261	7.722

Problem 65	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	203	203	122	1121	192	759	11387	1848	1483
N.S.	1	1.00	0.60	5.52	0.95	3.74	56.09	9.10	7.31
time (sec)	N/A	0.052	0.090	0.647	0.207	0.256	2.325	0.304	8.341

Problem 66	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	34	34	85	29	61	61	76	61	61
N.S.	1	1.00	2.50	0.85	1.79	1.79	2.24	1.79	1.79
time (sec)	N/A	0.028	0.003	0.098	0.190	0.240	0.023	0.251	0.054

Problem 67	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	83	83	83	62	61	61	75	61	61
N.S.	1	1.00	1.00	0.75	0.73	0.73	0.90	0.73	0.73
time (sec)	N/A	0.018	0.007	0.107	0.194	0.229	0.022	0.252	0.053

Problem 68	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	23	23	83	20	61	61	75	61	61
N.S.	1	1.00	3.61	0.87	2.65	2.65	3.26	2.65	2.65
time (sec)	N/A	0.013	0.002	0.063	0.189	0.251	0.022	0.255	0.052

Problem 69	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	83	83	83	62	61	61	75	61	61
N.S.	1	1.00	1.00	0.75	0.73	0.73	0.90	0.73	0.73
time (sec)	N/A	0.015	0.003	0.102	0.185	0.247	0.020	0.263	0.051

Problem 70	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	11	11	11	10	61	61	71	76	61
N.S.	1	1.00	1.00	0.91	5.55	5.55	6.45	6.91	5.55
time (sec)	N/A	0.002	0.002	0.057	0.182	0.253	0.023	0.273	0.052

Problem 71	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	73	73	73	58	57	57	68	57	57
N.S.	1	1.00	1.00	0.79	0.78	0.78	0.93	0.78	0.78
time (sec)	N/A	0.012	0.003	0.106	0.182	0.238	0.028	0.271	0.051

Problem 72	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	80	80	80	59	62	58	75	62	58
N.S.	1	1.00	1.00	0.74	0.78	0.72	0.94	0.78	0.72
time (sec)	N/A	0.019	0.004	0.038	0.203	0.232	0.041	0.283	0.053

Problem 73	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	73	73	73	60	59	62	66	59	59
N.S.	1	1.00	1.00	0.82	0.81	0.85	0.90	0.81	0.81
time (sec)	N/A	0.014	0.003	0.034	0.219	0.235	0.038	0.268	0.052

Problem 74	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	80	80	80	61	62	64	75	69	60
N.S.	1	1.00	1.00	0.76	0.78	0.80	0.94	0.86	0.75
time (sec)	N/A	0.025	0.003	0.041	0.188	0.265	0.048	0.257	0.053

Problem 75	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	145	145	80	91	54	129	0	98	0
N.S.	1	1.00	0.55	0.63	0.37	0.89	0.00	0.68	0.00
time (sec)	N/A	0.059	1.049	2.247	0.275	0.257	0.000	0.270	0.000

Problem 76	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	C	A	A	A	A	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	83	83	276	45	31	29	150	40	103
N.S.	1	1.00	3.33	0.54	0.37	0.35	1.81	0.48	1.24
time (sec)	N/A	0.046	0.547	0.312	0.182	0.259	3.907	0.281	8.083

Problem 77	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	97	97	69	62	33	98	0	57	0
N.S.	1	1.00	0.71	0.64	0.34	1.01	0.00	0.59	0.00
time (sec)	N/A	0.031	1.027	1.510	0.267	0.273	0.000	0.270	0.000

Problem 78	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	A	F	A	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	92	92	125	48	35	33	0	60	83
N.S.	1	1.00	1.36	0.52	0.38	0.36	0.00	0.65	0.90
time (sec)	N/A	0.047	0.378	0.345	0.197	0.252	0.000	0.268	8.171

Problem 79	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	101	101	72	67	37	105	0	61	0
N.S.	1	1.00	0.71	0.66	0.37	1.04	0.00	0.60	0.00
time (sec)	N/A	0.040	1.029	1.577	0.269	0.260	0.000	0.273	0.000

Problem 80	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	A	F	A	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	137	137	165	58	48	48	0	128	125
N.S.	1	1.00	1.20	0.42	0.35	0.35	0.00	0.93	0.91
time (sec)	N/A	0.064	0.357	0.345	0.195	0.254	0.000	0.259	8.320

Problem 81	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	153	153	108	165	125	300	0	98	0
N.S.	1	1.00	0.71	1.08	0.82	1.96	0.00	0.64	0.00
time (sec)	N/A	0.075	1.048	2.086	0.279	0.276	0.000	0.264	0.000

Problem 82	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	C	A	A	F	A	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	77	77	181	37	65	42	0	38	48
N.S.	1	1.00	2.35	0.48	0.84	0.55	0.00	0.49	0.62
time (sec)	N/A	0.043	0.653	0.067	0.188	0.242	0.000	0.275	7.728

Problem 83	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	156	156	108	164	124	301	0	98	0
N.S.	1	1.00	0.69	1.05	0.79	1.93	0.00	0.63	0.00
time (sec)	N/A	0.056	1.041	1.281	0.273	0.275	0.000	0.264	0.000

Problem 84	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	161	161	92	87	88	119	0	106	0
N.S.	1	1.00	0.57	0.54	0.55	0.74	0.00	0.66	0.00
time (sec)	N/A	0.079	1.031	0.104	0.192	0.246	0.000	0.272	0.000

Problem 85	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	190	190	124	176	134	334	0	113	0
N.S.	1	1.00	0.65	0.93	0.71	1.76	0.00	0.59	0.00
time (sec)	N/A	0.123	1.051	1.224	0.272	0.268	0.000	0.271	0.000

Problem 86	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	223	223	130	120	138	205	0	182	0
N.S.	1	1.00	0.58	0.54	0.62	0.92	0.00	0.82	0.00
time (sec)	N/A	0.122	1.056	0.136	0.196	0.250	0.000	0.270	0.000

Problem 87	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	B	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	400	400	160	1099	491	853	0	2171	0
N.S.	1	1.00	0.40	2.75	1.23	2.13	0.00	5.43	0.00
time (sec)	N/A	0.164	0.458	0.123	0.213	0.277	0.000	0.361	0.000

Problem 88	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	276	276	112	495	243	381	0	993	0
N.S.	1	1.00	0.41	1.79	0.88	1.38	0.00	3.60	0.00
time (sec)	N/A	0.106	0.143	0.033	0.210	0.256	0.000	0.296	0.000

Problem 89	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	153	153	86	131	75	94	0	263	0
N.S.	1	1.00	0.56	0.86	0.49	0.61	0.00	1.72	0.00
time (sec)	N/A	0.049	0.064	0.027	0.201	0.249	0.000	0.267	0.000

Problem 90	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	134	134	78	0	0	0	0	0	0
N.S.	1	1.00	0.58	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.053	0.098	0.000	0.000	0.000	0.000	0.000	0.000

Problem 91	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	154	154	101	0	0	0	0	0	0
N.S.	1	1.00	0.66	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.078	0.109	0.000	0.000	0.000	0.000	0.000	0.000

Problem 92	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	34	34	25	40	86	47	155	32	59
N.S.	1	1.00	0.74	1.18	2.53	1.38	4.56	0.94	1.74
time (sec)	N/A	0.021	0.008	0.083	0.209	0.265	4.387	0.269	7.813

Problem 93	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	86	86	45	62	135	92	0	196	108
N.S.	1	1.00	0.52	0.72	1.57	1.07	0.00	2.28	1.26
time (sec)	N/A	0.060	0.059	0.121	0.195	0.267	0.000	0.301	7.926

Problem 94	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	128	128	68	99	196	140	0	331	169
N.S.	1	1.00	0.53	0.77	1.53	1.09	0.00	2.59	1.32
time (sec)	N/A	0.083	0.073	0.161	0.201	0.265	0.000	0.280	7.973

Problem 95	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	166	166	166	170	166	166	202	193	169
N.S.	1	1.00	1.00	1.02	1.00	1.00	1.22	1.16	1.02
time (sec)	N/A	0.257	0.039	0.147	0.188	0.271	0.038	0.273	0.045

Problem 96	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	166	166	166	170	166	166	204	193	169
N.S.	1	1.00	1.00	1.02	1.00	1.00	1.23	1.16	1.02
time (sec)	N/A	0.100	0.037	0.138	0.192	0.247	0.032	0.285	7.899

Problem 97	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	166	166	154	170	166	166	199	193	169
N.S.	1	1.00	0.93	1.02	1.00	1.00	1.20	1.16	1.02
time (sec)	N/A	0.189	0.047	0.128	0.193	0.259	0.036	0.320	0.028

Problem 98	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	161	161	161	166	163	163	199	189	165
N.S.	1	1.00	1.00	1.03	1.01	1.01	1.24	1.17	1.02
time (sec)	N/A	0.074	0.040	0.146	0.188	0.239	0.035	0.325	0.027

Problem 99	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	162	162	162	167	167	164	199	193	166
N.S.	1	1.00	1.00	1.03	1.03	1.01	1.23	1.19	1.02
time (sec)	N/A	0.144	0.077	0.040	0.192	0.244	0.138	0.271	7.774

Problem 100	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	156	156	156	167	162	168	185	185	163
N.S.	1	1.00	1.00	1.07	1.04	1.08	1.19	1.19	1.04
time (sec)	N/A	0.067	0.070	0.043	0.192	0.261	0.142	0.279	0.029

Problem 101	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	162	162	162	169	167	170	197	212	166
N.S.	1	1.00	1.00	1.04	1.03	1.05	1.22	1.31	1.02
time (sec)	N/A	0.140	0.054	0.049	0.185	0.276	0.196	0.296	0.032

Problem 102	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	133	133	126	136	0	421	0	126	1343
N.S.	1	1.00	0.95	1.02	0.00	3.17	0.00	0.95	10.10
time (sec)	N/A	0.138	0.045	0.203	0.000	0.290	0.000	0.616	7.875

Problem 103	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	97	97	93	98	0	312	434	91	979
N.S.	1	1.00	0.96	1.01	0.00	3.22	4.47	0.94	10.09
time (sec)	N/A	0.076	0.046	0.142	0.000	0.284	72.994	0.616	8.008

Problem 104	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	71	71	71	65	0	219	287	67	606
N.S.	1	1.00	1.00	0.92	0.00	3.08	4.04	0.94	8.54
time (sec)	N/A	0.051	0.033	0.096	0.000	0.276	5.220	0.686	7.711

Problem 105	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	78	78	128	76	0	249	0	78	2424
N.S.	1	1.00	1.64	0.97	0.00	3.19	0.00	1.00	31.08
time (sec)	N/A	0.090	0.074	0.067	0.000	0.279	0.000	0.726	10.104

Problem 106	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	112	112	186	126	0	385	0	124	3729
N.S.	1	1.00	1.66	1.12	0.00	3.44	0.00	1.11	33.29
time (sec)	N/A	0.161	0.146	0.090	0.000	0.392	0.000	0.629	10.265

Problem 107	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	B	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	261	261	327	95	0	5140	0	4391	10177
N.S.	1	1.00	1.25	0.36	0.00	19.69	0.00	16.82	38.99
time (sec)	N/A	0.835	0.255	0.101	0.000	2.570	0.000	1.073	8.400

Problem 108	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	B	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	208	208	251	65	0	2632	0	3179	6366
N.S.	1	1.00	1.21	0.31	0.00	12.65	0.00	15.28	30.61
time (sec)	N/A	0.310	0.105	0.070	0.000	0.618	0.000	0.958	8.233

Problem 109	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	B	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	172	172	173	45	0	1569	0	1402	4109
N.S.	1	1.00	1.01	0.26	0.00	9.12	0.00	8.15	23.89
time (sec)	N/A	0.124	0.062	0.059	0.000	0.438	0.000	0.783	8.071

Problem 110	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	189	189	206	173	0	2914	0	2805	6335
N.S.	1	1.00	1.09	0.92	0.00	15.42	0.00	14.84	33.52
time (sec)	N/A	0.240	0.187	0.093	0.000	0.786	0.000	1.051	0.779

Problem 111	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	271	271	267	232	0	5442	0	2870	10101
N.S.	1	1.00	0.99	0.86	0.00	20.08	0.00	10.59	37.27
time (sec)	N/A	0.424	0.206	0.108	0.000	2.886	0.000	0.908	8.694

Problem 112	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	212	212	208	282	0	1323	0	239	2282
N.S.	1	1.00	0.98	1.33	0.00	6.24	0.00	1.13	10.76
time (sec)	N/A	0.237	0.193	0.227	0.000	0.347	0.000	0.647	8.065

Problem 113	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	147	147	160	211	0	849	0	194	1527
N.S.	1	1.00	1.09	1.44	0.00	5.78	0.00	1.32	10.39
time (sec)	N/A	0.123	0.122	0.175	0.000	0.296	0.000	0.614	8.163

Problem 114	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	107	107	111	126	0	538	394	120	283
N.S.	1	1.00	1.04	1.18	0.00	5.03	3.68	1.12	2.64
time (sec)	N/A	0.073	0.057	0.118	0.000	0.258	4.227	0.586	7.612

Problem 115	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	94	94	101	95	0	474	374	102	264
N.S.	1	1.00	1.07	1.01	0.00	5.04	3.98	1.09	2.81
time (sec)	N/A	0.062	0.056	0.102	0.000	0.258	2.279	0.560	7.675

Problem 116	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	150	150	243	212	0	1014	0	201	7119
N.S.	1	1.00	1.62	1.41	0.00	6.76	0.00	1.34	47.46
time (sec)	N/A	0.212	0.226	0.118	0.000	0.596	0.000	0.613	12.299

Problem 117	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	223	223	379	300	0	1635	0	250	10034
N.S.	1	1.00	1.70	1.35	0.00	7.33	0.00	1.12	45.00
time (sec)	N/A	0.269	0.340	0.155	0.000	1.149	0.000	0.562	13.332

Problem 118	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	B	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	425	425	455	217	0	7252	0	5675	16604
N.S.	1	1.00	1.07	0.51	0.00	17.06	0.00	13.35	39.07
time (sec)	N/A	2.422	0.761	0.131	0.000	7.531	0.000	1.513	10.139

Problem 119	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	B	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	336	336	362	178	0	4658	0	4538	12396
N.S.	1	1.00	1.08	0.53	0.00	13.86	0.00	13.51	36.89
time (sec)	N/A	1.224	0.586	0.130	0.000	2.308	0.000	1.397	10.982

Problem 120	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	B	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	276	276	298	152	0	3467	0	3776	9444
N.S.	1	1.00	1.08	0.55	0.00	12.56	0.00	13.68	34.22
time (sec)	N/A	0.363	0.422	0.106	0.000	1.158	0.000	1.140	10.412

Problem 121	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	B	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	293	293	304	178	0	4885	0	4426	12349
N.S.	1	1.00	1.04	0.61	0.00	16.67	0.00	15.11	42.15
time (sec)	N/A	0.521	0.510	0.179	0.000	3.281	0.000	1.378	10.668

Problem 122	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	389	389	382	379	0	7583	0	5408	17591
N.S.	1	1.00	0.98	0.97	0.00	19.49	0.00	13.90	45.22
time (sec)	N/A	0.750	0.698	0.160	0.000	8.891	0.000	1.358	11.087

Problem 123	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	522	522	487	484	0	10190	0	6327	21554
N.S.	1	1.00	0.93	0.93	0.00	19.52	0.00	12.12	41.29
time (sec)	N/A	0.937	0.795	0.187	0.000	21.405	0.000	1.489	11.357

Problem 124	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	365	365	435	624	0	3196	0	598	4501
N.S.	1	1.00	1.19	1.71	0.00	8.76	0.00	1.64	12.33
time (sec)	N/A	0.947	0.428	0.311	0.000	0.535	0.000	1.418	10.379

Problem 125	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F(-2)	B	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	254	254	354	495	0	2167	0	466	3062
N.S.	1	1.00	1.39	1.95	0.00	8.53	0.00	1.83	12.06
time (sec)	N/A	0.260	0.301	0.228	0.000	0.395	0.000	1.525	10.737

Problem 126	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F(-2)	B	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	146	146	261	343	0	1378	0	318	593
N.S.	1	1.00	1.79	2.35	0.00	9.44	0.00	2.18	4.06
time (sec)	N/A	0.091	0.200	0.146	0.000	0.304	0.000	1.453	7.846

Problem 127	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	185	185	233	303	0	1369	0	268	625
N.S.	1	1.00	1.26	1.64	0.00	7.40	0.00	1.45	3.38
time (sec)	N/A	0.163	0.165	0.150	0.000	0.334	0.000	1.416	7.852

Problem 128	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	170	170	172	273	0	1226	0	228	587
N.S.	1	1.00	1.01	1.61	0.00	7.21	0.00	1.34	3.45
time (sec)	N/A	0.104	0.148	0.149	0.000	0.314	0.000	1.548	7.818

Problem 129	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	139	139	142	147	0	1109	661	208	517
N.S.	1	1.00	1.02	1.06	0.00	7.98	4.76	1.50	3.72
time (sec)	N/A	0.083	0.089	0.138	0.000	0.296	165.164	1.445	7.741

Problem 130	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	252	252	396	442	0	2494	0	421	11674
N.S.	1	1.00	1.57	1.75	0.00	9.90	0.00	1.67	46.33
time (sec)	N/A	0.346	0.417	0.187	0.000	1.950	0.000	1.510	14.337

Problem 131	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	363	363	642	616	0	3956	0	648	16265
N.S.	1	1.00	1.77	1.70	0.00	10.90	0.00	1.79	44.81
time (sec)	N/A	0.480	0.903	0.282	0.000	3.981	0.000	1.427	18.347

Problem 132	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	B	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	554	554	644	445	0	9636	0	3987	22911
N.S.	1	1.00	1.16	0.80	0.00	17.39	0.00	7.20	41.36
time (sec)	N/A	7.400	1.520	0.169	0.000	18.804	0.000	2.019	10.325

Problem 133	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	B	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	461	461	543	375	0	7060	0	7578	19041
N.S.	1	1.00	1.18	0.81	0.00	15.31	0.00	16.44	41.30
time (sec)	N/A	2.961	1.269	0.155	0.000	6.011	0.000	2.353	9.742

Problem 134	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	B	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	380	380	447	328	0	5650	0	3164	16688
N.S.	1	1.00	1.18	0.86	0.00	14.87	0.00	8.33	43.92
time (sec)	N/A	0.985	1.080	0.160	0.000	4.207	0.000	1.745	9.432

Problem 135	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	B	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	438	438	436	374	0	7270	0	7267	18992
N.S.	1	1.00	1.00	0.85	0.00	16.60	0.00	16.59	43.36
time (sec)	N/A	0.694	1.013	0.147	0.000	8.330	0.000	2.183	9.694

Problem 136	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	B	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	460	460	516	447	0	9909	0	4605	22914
N.S.	1	1.00	1.12	0.97	0.00	21.54	0.00	10.01	49.81
time (sec)	N/A	0.787	1.297	0.389	0.000	21.044	0.000	1.787	10.184

Problem 137	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	25	25	18	17	17	17	19	17
N.S.	1	1.00	1.00	0.72	0.68	0.68	0.68	0.76	0.68
time (sec)	N/A	0.016	0.008	0.042	0.196	0.256	0.052	0.273	0.033

Problem 138	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	25	25	18	25	17	17	19	17
N.S.	1	1.00	1.00	0.72	1.00	0.68	0.68	0.76	0.68
time (sec)	N/A	0.021	0.004	0.035	0.184	0.254	0.049	0.277	0.017

Problem 139	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	37	37	37	31	30	30	37	30	32
N.S.	1	1.00	1.00	0.84	0.81	0.81	1.00	0.81	0.86
time (sec)	N/A	0.021	0.010	0.047	0.271	0.262	0.056	0.263	7.558

Problem 140	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	37	37	37	31	53	30	37	30	32
N.S.	1	1.00	1.00	0.84	1.43	0.81	1.00	0.81	0.86
time (sec)	N/A	0.026	0.005	0.034	0.264	0.257	0.051	0.261	0.020

Problem 141	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	45	45	45	38	0	47	44	38	41
N.S.	1	1.00	1.00	0.84	0.00	1.04	0.98	0.84	0.91
time (sec)	N/A	0.033	0.023	0.053	0.000	0.255	0.062	0.409	0.031

Problem 142	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	102	102	69	58	104	61	105	102	102
N.S.	1	1.00	0.68	0.57	1.02	0.60	1.03	1.00	1.00
time (sec)	N/A	0.046	0.107	0.334	0.201	0.252	0.776	0.307	0.272

Problem 143	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	81	81	64	53	87	56	94	88	85
N.S.	1	1.00	0.79	0.65	1.07	0.69	1.16	1.09	1.05
time (sec)	N/A	0.035	0.083	0.203	0.186	0.254	0.735	0.287	7.680

Problem 144	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	74	74	59	48	70	51	80	74	67
N.S.	1	1.00	0.80	0.65	0.95	0.69	1.08	1.00	0.91
time (sec)	N/A	0.028	0.081	0.179	0.189	0.263	0.703	0.271	7.666

Problem 145	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	94	94	88	83	89	95	0	98	86
N.S.	1	1.00	0.94	0.88	0.95	1.01	0.00	1.04	0.91
time (sec)	N/A	0.055	0.132	0.521	0.276	0.272	0.000	0.294	0.245

Problem 146	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	97	97	91	83	89	112	0	138	84
N.S.	1	1.00	0.94	0.86	0.92	1.15	0.00	1.42	0.87
time (sec)	N/A	0.056	0.153	0.503	0.268	0.268	0.000	0.305	8.006

Problem 147	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	99	99	91	86	106	112	0	169	0
N.S.	1	1.00	0.92	0.87	1.07	1.13	0.00	1.71	0.00
time (sec)	N/A	0.054	0.182	0.480	0.268	0.260	0.000	0.322	0.000

Problem 148	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	90	90	70	66	99	90	0	189	0
N.S.	1	1.00	0.78	0.73	1.10	1.00	0.00	2.10	0.00
time (sec)	N/A	0.043	0.219	0.409	0.273	0.249	0.000	0.309	0.000

Problem 149	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	111	111	80	71	116	95	0	233	0
N.S.	1	1.00	0.72	0.64	1.05	0.86	0.00	2.10	0.00
time (sec)	N/A	0.059	0.245	0.398	0.276	0.266	0.000	0.290	0.000

Problem 150	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	132	132	80	76	133	100	0	255	0
N.S.	1	1.00	0.61	0.58	1.01	0.76	0.00	1.93	0.00
time (sec)	N/A	0.071	0.307	0.435	0.283	0.260	0.000	0.314	0.000

Problem 151	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	A	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	322	322	237	226	0	139	0	0	0
N.S.	1	1.00	0.74	0.70	0.00	0.43	0.00	0.00	0.00
time (sec)	N/A	0.162	6.063	5.582	0.000	0.085	0.000	0.000	0.000

Problem 152	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	A	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	305	305	234	221	0	134	0	0	0
N.S.	1	1.00	0.77	0.72	0.00	0.44	0.00	0.00	0.00
time (sec)	N/A	0.105	5.090	1.998	0.000	0.097	0.000	0.000	0.000

Problem 153	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	A	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	279	279	229	216	0	129	0	0	0
N.S.	1	1.00	0.82	0.77	0.00	0.46	0.00	0.00	0.00
time (sec)	N/A	0.058	5.058	1.745	0.000	0.093	0.000	0.000	0.000

Problem 154	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	F	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	284	284	231	225	0	0	0	0	0
N.S.	1	1.00	0.81	0.79	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.069	4.815	1.407	0.000	0.000	0.000	0.000	0.000

Problem 155	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	F	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	305	305	237	228	0	0	0	0	0
N.S.	1	1.00	0.78	0.75	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.084	9.679	2.151	0.000	0.000	0.000	0.000	0.000

Problem 156	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	127	127	79	68	135	71	218	207	0
N.S.	1	1.00	0.62	0.54	1.06	0.56	1.72	1.63	0.00
time (sec)	N/A	0.067	0.184	0.237	0.194	0.267	1.206	0.298	0.000

Problem 157	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	106	106	74	63	118	66	190	179	0
N.S.	1	1.00	0.70	0.59	1.11	0.62	1.79	1.69	0.00
time (sec)	N/A	0.045	0.143	0.214	0.191	0.267	1.160	0.324	0.000

Problem 158	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	99	99	69	58	101	61	165	151	127
N.S.	1	1.00	0.70	0.59	1.02	0.62	1.67	1.53	1.28
time (sec)	N/A	0.037	0.132	0.192	0.182	0.275	1.100	0.293	7.785

Problem 159	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	119	119	99	87	120	106	0	113	0
N.S.	1	1.00	0.83	0.73	1.01	0.89	0.00	0.95	0.00
time (sec)	N/A	0.066	0.201	0.475	0.276	0.270	0.000	0.303	0.000

Problem 160	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	122	122	102	96	120	122	0	153	0
N.S.	1	1.00	0.84	0.79	0.98	1.00	0.00	1.25	0.00
time (sec)	N/A	0.070	0.227	0.527	0.264	0.292	0.000	0.313	0.000

Problem 161	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	127	127	101	96	137	122	0	190	0
N.S.	1	1.00	0.80	0.76	1.08	0.96	0.00	1.50	0.00
time (sec)	N/A	0.069	0.255	0.532	0.286	0.280	0.000	0.309	0.000

Problem 162	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	127	127	101	96	154	122	0	227	0
N.S.	1	1.00	0.80	0.76	1.21	0.96	0.00	1.79	0.00
time (sec)	N/A	0.069	0.344	0.548	0.275	0.268	0.000	0.319	0.000

Problem 163	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	A	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	356	356	249	236	0	149	0	0	0
N.S.	1	1.00	0.70	0.66	0.00	0.42	0.00	0.00	0.00
time (sec)	N/A	0.132	9.168	4.206	0.000	0.083	0.000	0.000	0.000

Problem 164	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	A	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	331	331	244	231	0	144	0	0	0
N.S.	1	1.00	0.74	0.70	0.00	0.44	0.00	0.00	0.00
time (sec)	N/A	0.113	7.622	2.783	0.000	0.085	0.000	0.000	0.000

Problem 165	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	A	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	308	308	239	226	0	139	0	0	0
N.S.	1	1.00	0.78	0.73	0.00	0.45	0.00	0.00	0.00
time (sec)	N/A	0.082	6.662	1.405	0.000	0.104	0.000	0.000	0.000

Problem 166	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	F	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	312	312	235	233	0	0	0	0	0
N.S.	1	1.00	0.75	0.75	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.087	6.508	1.456	0.000	0.000	0.000	0.000	0.000

Problem 167	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	F	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	314	314	247	238	0	0	0	0	0
N.S.	1	1.00	0.79	0.76	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.084	10.248	2.127	0.000	0.000	0.000	0.000	0.000

Problem 168	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	F	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	331	331	244	238	0	0	0	0	0
N.S.	1	1.00	0.74	0.72	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.114	10.270	4.088	0.000	0.000	0.000	0.000	0.000

Problem 169	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	A	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	153	153	135	125	0	315	325	136	0
N.S.	1	1.00	0.88	0.82	0.00	2.06	2.12	0.89	0.00
time (sec)	N/A	0.137	0.524	0.186	0.000	0.273	1.052	0.334	0.000

Problem 170	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	A	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	100	100	104	88	0	233	233	96	0
N.S.	1	1.00	1.04	0.88	0.00	2.33	2.33	0.96	0.00
time (sec)	N/A	0.059	0.329	0.121	0.000	0.274	0.920	0.309	0.000

Problem 171	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	76	76	77	65	0	178	165	67	92
N.S.	1	1.00	1.01	0.86	0.00	2.34	2.17	0.88	1.21
time (sec)	N/A	0.043	0.290	0.087	0.000	0.264	0.834	0.294	8.168

Problem 172	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	F	F(-2)	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	90	90	85	76	0	517	0	0	81
N.S.	1	1.00	0.94	0.84	0.00	5.74	0.00	0.00	0.90
time (sec)	N/A	0.061	0.245	0.097	0.000	0.319	0.000	0.000	8.000

Problem 173	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	80	80	81	71	0	197	0	124	103
N.S.	1	1.00	1.01	0.89	0.00	2.46	0.00	1.55	1.29
time (sec)	N/A	0.052	0.334	0.123	0.000	0.305	0.000	0.300	8.149

Problem 174	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	124	124	145	98	0	255	0	339	0
N.S.	1	1.00	1.17	0.79	0.00	2.06	0.00	2.73	0.00
time (sec)	N/A	0.098	0.572	0.135	0.000	0.348	0.000	0.306	0.000

Problem 175	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	177	177	186	138	0	339	0	571	0
N.S.	1	1.00	1.05	0.78	0.00	1.92	0.00	3.23	0.00
time (sec)	N/A	0.153	0.808	0.163	0.000	0.407	0.000	0.310	0.000

Problem 176	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	403	403	532	448	0	431	0	0	0
N.S.	1	1.00	1.32	1.11	0.00	1.07	0.00	0.00	0.00
time (sec)	N/A	0.181	11.508	5.782	0.000	0.094	0.000	0.000	0.000

Problem 177	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	336	336	479	395	0	346	0	0	0
N.S.	1	1.00	1.43	1.18	0.00	1.03	0.00	0.00	0.00
time (sec)	N/A	0.093	11.006	2.403	0.000	0.097	0.000	0.000	0.000

Problem 178	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	283	283	302	362	0	300	0	0	0
N.S.	1	1.00	1.07	1.28	0.00	1.06	0.00	0.00	0.00
time (sec)	N/A	0.054	10.184	0.987	0.000	0.097	0.000	0.000	0.000

Problem 179	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	312	312	448	385	0	294	0	0	0
N.S.	1	1.00	1.44	1.23	0.00	0.94	0.00	0.00	0.00
time (sec)	N/A	0.089	10.707	1.938	0.000	0.086	0.000	0.000	0.000

Problem 180	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	376	376	373	418	0	347	0	0	0
N.S.	1	1.00	0.99	1.11	0.00	0.92	0.00	0.00	0.00
time (sec)	N/A	0.150	10.467	3.381	0.000	0.094	0.000	0.000	0.000

Problem 181	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	98	98	64	53	90	56	65	60	0
N.S.	1	1.00	0.65	0.54	0.92	0.57	0.66	0.61	0.00
time (sec)	N/A	0.059	0.115	0.206	0.184	0.248	0.912	0.291	0.000

Problem 182	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	77	77	59	48	73	51	54	53	0
N.S.	1	1.00	0.77	0.62	0.95	0.66	0.70	0.69	0.00
time (sec)	N/A	0.040	0.102	0.195	0.239	0.241	0.871	0.270	0.000

Problem 183	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	56	56	54	43	56	46	51	46	0
N.S.	1	1.00	0.96	0.77	1.00	0.82	0.91	0.82	0.00
time (sec)	N/A	0.026	0.097	0.181	0.193	0.240	0.816	0.273	0.000

Problem 184	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	49	49	47	36	39	39	42	39	35
N.S.	1	1.00	0.96	0.73	0.80	0.80	0.86	0.80	0.71
time (sec)	N/A	0.020	0.091	0.165	0.192	0.252	0.796	0.275	8.098

Problem 185	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	69	69	63	52	58	75	0	78	56
N.S.	1	1.00	0.91	0.75	0.84	1.09	0.00	1.13	0.81
time (sec)	N/A	0.037	0.121	0.448	0.270	0.264	0.000	0.283	8.203

Problem 186	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	62	62	58	49	51	78	0	101	83
N.S.	1	1.00	0.94	0.79	0.82	1.26	0.00	1.63	1.34
time (sec)	N/A	0.030	0.141	0.332	0.273	0.257	0.000	0.292	0.277

Problem 187	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	83	83	63	64	68	83	0	145	0
N.S.	1	1.00	0.76	0.77	0.82	1.00	0.00	1.75	0.00
time (sec)	N/A	0.043	0.177	0.355	0.273	0.269	0.000	0.294	0.000

Problem 188	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	104	104	75	66	85	90	0	167	0
N.S.	1	1.00	0.72	0.63	0.82	0.87	0.00	1.61	0.00
time (sec)	N/A	0.055	0.240	0.375	0.283	0.264	0.000	0.301	0.000

Problem 189	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	A	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	298	298	229	216	0	129	0	0	0
N.S.	1	1.00	0.77	0.72	0.00	0.43	0.00	0.00	0.00
time (sec)	N/A	0.099	10.226	3.891	0.000	0.099	0.000	0.000	0.000

Problem 190	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	A	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	270	270	222	208	0	122	0	0	0
N.S.	1	1.00	0.82	0.77	0.00	0.45	0.00	0.00	0.00
time (sec)	N/A	0.072	10.228	2.597	0.000	0.093	0.000	0.000	0.000

Problem 191	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	257	257	159	194	0	117	0	0	0
N.S.	1	1.00	0.62	0.75	0.00	0.46	0.00	0.00	0.00
time (sec)	N/A	0.051	10.125	0.840	0.000	0.086	0.000	0.000	0.000

Problem 192	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	A	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	278	278	224	211	0	124	0	0	0
N.S.	1	1.00	0.81	0.76	0.00	0.45	0.00	0.00	0.00
time (sec)	N/A	0.059	10.233	1.400	0.000	0.082	0.000	0.000	0.000

Problem 193	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	A	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	302	302	237	228	0	140	0	0	0
N.S.	1	1.00	0.78	0.75	0.00	0.46	0.00	0.00	0.00
time (sec)	N/A	0.086	10.241	2.114	0.000	0.081	0.000	0.000	0.000

Problem 194	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	77	77	59	48	73	86	0	52	0
N.S.	1	1.00	0.77	0.62	0.95	1.12	0.00	0.68	0.00
time (sec)	N/A	0.040	0.179	0.216	0.196	0.246	0.000	0.273	0.000

Problem 195	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	56	56	54	43	56	81	0	46	52
N.S.	1	1.00	0.96	0.77	1.00	1.45	0.00	0.82	0.93
time (sec)	N/A	0.029	0.144	0.199	0.197	0.259	0.000	0.292	7.849

Problem 196	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	25	25	22	32	46	0	21	21
N.S.	1	1.00	1.00	0.88	1.28	1.84	0.00	0.84	0.84
time (sec)	N/A	0.013	0.144	0.065	0.185	0.260	0.000	0.292	7.578

Problem 197	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	66	66	62	53	65	107	0	78	0
N.S.	1	1.00	0.94	0.80	0.98	1.62	0.00	1.18	0.00
time (sec)	N/A	0.038	0.206	0.339	0.277	0.266	0.000	0.295	0.000

Problem 198	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	90	90	70	61	82	124	0	122	0
N.S.	1	1.00	0.78	0.68	0.91	1.38	0.00	1.36	0.00
time (sec)	N/A	0.049	0.229	0.370	0.286	0.247	0.000	0.311	0.000

Problem 199	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	A	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	307	307	219	216	0	184	0	0	0
N.S.	1	1.00	0.71	0.70	0.00	0.60	0.00	0.00	0.00
time (sec)	N/A	0.082	10.230	4.031	0.000	0.082	0.000	0.000	0.000

Problem 200	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	A	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	286	286	219	216	0	179	0	0	0
N.S.	1	1.00	0.77	0.76	0.00	0.63	0.00	0.00	0.00
time (sec)	N/A	0.070	10.251	2.625	0.000	0.080	0.000	0.000	0.000

Problem 201	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	A	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	282	282	219	216	0	179	0	0	0
N.S.	1	1.00	0.78	0.77	0.00	0.63	0.00	0.00	0.00
time (sec)	N/A	0.059	10.252	1.308	0.000	0.081	0.000	0.000	0.000

Problem 202	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	A	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	309	309	228	223	0	192	0	0	0
N.S.	1	1.00	0.74	0.72	0.00	0.62	0.00	0.00	0.00
time (sec)	N/A	0.079	10.258	1.506	0.000	0.082	0.000	0.000	0.000

Problem 203	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	A	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	326	326	234	228	0	207	0	0	0
N.S.	1	1.00	0.72	0.70	0.00	0.63	0.00	0.00	0.00
time (sec)	N/A	0.109	10.261	2.983	0.000	0.082	0.000	0.000	0.000

Problem 204	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	297	297	430	0	0	0	0	0	0
N.S.	1	1.00	1.45	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.273	11.763	0.000	0.000	0.000	0.000	0.000	0.000

Problem 205	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	297	297	386	0	0	0	0	0	0
N.S.	1	1.00	1.30	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.204	11.613	0.000	0.000	0.000	0.000	0.000	0.000

Problem 206	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	295	295	386	0	0	0	0	0	0
N.S.	1	1.00	1.31	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.205	11.561	0.000	0.000	0.000	0.000	0.000	0.000

Problem 207	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	295	295	370	0	0	0	0	0	0
N.S.	1	1.00	1.25	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.216	11.654	0.000	0.000	0.000	0.000	0.000	0.000

Problem 208	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	299	299	567	0	0	0	0	0	0
N.S.	1	1.00	1.90	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.211	12.005	0.000	0.000	0.000	0.000	0.000	0.000

Problem 209	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	299	299	490	0	0	0	0	0	0
N.S.	1	1.00	1.64	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.209	11.936	0.000	0.000	0.000	0.000	0.000	0.000

Problem 210	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	297	297	487	0	0	0	0	0	0
N.S.	1	1.00	1.64	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.216	11.910	0.000	0.000	0.000	0.000	0.000	0.000

Problem 211	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	297	297	447	0	0	0	0	0	0
N.S.	1	1.00	1.51	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.204	11.745	0.000	0.000	0.000	0.000	0.000	0.000

Problem 212	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	297	297	354	0	0	0	0	0	0
N.S.	1	1.00	1.19	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.207	11.500	0.000	0.000	0.000	0.000	0.000	0.000

Problem 213	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	297	297	242	0	0	0	0	0	0
N.S.	1	1.00	0.81	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.203	11.226	0.000	0.000	0.000	0.000	0.000	0.000

Problem 214	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	295	295	241	0	0	0	0	0	0
N.S.	1	1.00	0.82	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.207	11.221	0.000	0.000	0.000	0.000	0.000	0.000

Problem 215	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	295	295	356	0	0	0	0	0	0
N.S.	1	1.00	1.21	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.206	11.537	0.000	0.000	0.000	0.000	0.000	0.000

Problem 216	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	303	303	375	0	0	0	0	0	0
N.S.	1	1.00	1.24	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.216	11.600	0.000	0.000	0.000	0.000	0.000	0.000

Problem 217	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	303	303	397	0	0	0	0	0	0
N.S.	1	1.00	1.31	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.222	11.597	0.000	0.000	0.000	0.000	0.000	0.000

Problem 218	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	301	301	395	0	0	0	0	0	0
N.S.	1	1.00	1.31	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.221	11.573	0.000	0.000	0.000	0.000	0.000	0.000

Problem 219	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	301	301	460	0	0	0	0	0	0
N.S.	1	1.00	1.53	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.219	11.741	0.000	0.000	0.000	0.000	0.000	0.000

Problem 220	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	243	243	191	1935	408	1357	11266	2736	769
N.S.	1	1.00	0.79	7.96	1.68	5.58	46.36	11.26	3.16
time (sec)	N/A	0.113	0.815	0.354	0.251	0.261	1.529	0.357	8.067

Problem 221	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	155	155	117	783	230	573	4068	1142	429
N.S.	1	1.00	0.75	5.05	1.48	3.70	26.25	7.37	2.77
time (sec)	N/A	0.075	0.449	0.168	0.227	0.255	0.876	0.318	7.867

Problem 222	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	83	83	59	82	104	171	1015	338	171
N.S.	1	1.00	0.71	0.99	1.25	2.06	12.23	4.07	2.06
time (sec)	N/A	0.032	0.080	0.079	0.228	0.255	0.467	0.300	7.677

Problem 223	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	194	194	156	0	0	0	0	0	0
N.S.	1	1.00	0.80	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.208	0.346	0.000	0.000	0.000	0.000	0.000	0.000

Problem 224	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	392	358	160	0	0	0	0	0	0
N.S.	1	0.91	0.41	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.750	1.534	0.000	0.000	0.000	0.000	0.000	0.000

Problem 225	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	319	319	466	0	0	0	0	0	0
N.S.	1	1.00	1.46	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.240	2.900	0.000	0.000	0.000	0.000	0.000	0.000

Problem 226	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	317	317	267	0	0	0	0	0	0
N.S.	1	1.00	0.84	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.213	1.983	0.000	0.000	0.000	0.000	0.000	0.000

Problem 227	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	317	317	267	0	0	0	0	0	0
N.S.	1	1.00	0.84	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.236	2.831	0.000	0.000	0.000	0.000	0.000	0.000

Problem 228	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	323	323	307	0	0	0	0	0	0
N.S.	1	1.00	0.95	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.235	11.343	0.000	0.000	0.000	0.000	0.000	0.000

Problem 229	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	134	134	134	103	120	277	0	123	181
N.S.	1	1.00	1.00	0.77	0.90	2.07	0.00	0.92	1.35
time (sec)	N/A	0.123	0.070	0.412	0.281	2.891	0.000	0.292	8.010

Problem 230	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	118	118	99	92	107	212	0	108	166
N.S.	1	1.00	0.84	0.78	0.91	1.80	0.00	0.92	1.41
time (sec)	N/A	0.098	0.074	0.410	0.305	1.185	0.000	0.281	7.882

Problem 231	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	105	105	77	80	89	170	0	90	138
N.S.	1	1.00	0.73	0.76	0.85	1.62	0.00	0.86	1.31
time (sec)	N/A	0.093	0.031	0.398	0.292	0.730	0.000	0.280	8.061

Problem 232	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	96	96	66	72	82	145	0	85	944
N.S.	1	1.00	0.69	0.75	0.85	1.51	0.00	0.89	9.83
time (sec)	N/A	0.061	0.033	0.389	0.285	0.365	0.000	0.274	8.743

Problem 233	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	96	96	67	75	82	146	0	86	328
N.S.	1	1.00	0.70	0.78	0.85	1.52	0.00	0.90	3.42
time (sec)	N/A	0.042	0.031	0.381	0.285	0.428	0.000	0.284	8.309

Problem 234	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	114	114	134	90	101	201	0	103	527
N.S.	1	1.00	1.18	0.79	0.89	1.76	0.00	0.90	4.62
time (sec)	N/A	0.082	0.061	0.399	0.276	4.431	0.000	0.279	8.043

Problem 235	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	129	129	169	107	120	265	0	131	820
N.S.	1	1.00	1.31	0.83	0.93	2.05	0.00	1.02	6.36
time (sec)	N/A	0.094	0.075	0.386	0.279	22.700	0.000	0.280	8.290

Problem 236	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	156	156	209	127	145	338	0	170	1017
N.S.	1	1.00	1.34	0.81	0.93	2.17	0.00	1.09	6.52
time (sec)	N/A	0.115	0.069	0.425	0.282	49.460	0.000	0.269	8.597

Problem 237	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	359	359	344	279	0	4414	0	370	6097
N.S.	1	1.00	0.96	0.78	0.00	12.30	0.00	1.03	16.98
time (sec)	N/A	0.220	0.261	0.408	0.000	5.530	0.000	0.288	8.484

Problem 238	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	345	345	373	263	0	4354	0	338	5908
N.S.	1	1.00	1.08	0.76	0.00	12.62	0.00	0.98	17.12
time (sec)	N/A	0.190	0.166	0.409	0.000	1.084	0.000	0.282	8.439

Problem 239	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	336	336	233	254	0	4040	0	332	5111
N.S.	1	1.00	0.69	0.76	0.00	12.02	0.00	0.99	15.21
time (sec)	N/A	0.181	0.108	0.406	0.000	0.474	0.000	0.290	8.647

Problem 240	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	337	337	232	246	0	3892	0	340	4720
N.S.	1	1.00	0.69	0.73	0.00	11.55	0.00	1.01	14.01
time (sec)	N/A	0.170	0.097	0.390	0.000	0.364	0.000	0.283	8.459

Problem 241	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	336	336	234	253	0	4084	0	344	4802
N.S.	1	1.00	0.70	0.75	0.00	12.15	0.00	1.02	14.29
time (sec)	N/A	0.176	0.102	0.332	0.000	0.683	0.000	0.313	8.516

Problem 242	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	348	348	389	266	0	4362	0	353	5761
N.S.	1	1.00	1.12	0.76	0.00	12.53	0.00	1.01	16.55
time (sec)	N/A	0.194	0.176	0.404	0.000	1.807	0.000	0.284	8.682

Problem 243	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	360	360	367	284	0	4442	0	368	5972
N.S.	1	1.00	1.02	0.79	0.00	12.34	0.00	1.02	16.59
time (sec)	N/A	0.191	0.304	0.398	0.000	5.978	0.000	0.299	8.707

Problem 244	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	169	169	159	160	220	555	0	258	305
N.S.	1	1.00	0.94	0.95	1.30	3.28	0.00	1.53	1.80
time (sec)	N/A	0.240	0.088	0.431	0.278	8.772	0.000	0.275	8.247

Problem 245	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	150	150	142	146	197	457	0	229	647
N.S.	1	1.00	0.95	0.97	1.31	3.05	0.00	1.53	4.31
time (sec)	N/A	0.161	0.076	0.495	0.286	4.234	0.000	0.286	8.263

Problem 246	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	155	153	149	136	192	487	0	228	528
N.S.	1	0.99	0.96	0.88	1.24	3.14	0.00	1.47	3.41
time (sec)	N/A	0.162	0.072	0.415	0.280	1.787	0.000	0.279	8.276

Problem 247	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	149	148	114	132	186	492	0	196	527
N.S.	1	0.99	0.77	0.89	1.25	3.30	0.00	1.32	3.54
time (sec)	N/A	0.118	0.100	0.423	0.297	1.745	0.000	0.274	8.241

Problem 248	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	151	151	144	146	196	458	0	208	649
N.S.	1	1.00	0.95	0.97	1.30	3.03	0.00	1.38	4.30
time (sec)	N/A	0.117	0.068	0.421	0.313	4.215	0.000	0.291	8.410

Problem 249	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	209	209	241	171	228	686	0	289	1082
N.S.	1	1.00	1.15	0.82	1.09	3.28	0.00	1.38	5.18
time (sec)	N/A	0.162	0.128	0.457	0.279	74.982	0.000	0.283	9.055

Problem 250	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	236	236	248	181	278	852	0	357	1337
N.S.	1	1.00	1.05	0.77	1.18	3.61	0.00	1.51	5.67
time (sec)	N/A	0.173	0.312	0.494	0.283	224.195	0.000	0.268	9.143

Problem 251	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	F(-1)	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	265	265	278	209	332	0	0	361	1545
N.S.	1	1.00	1.05	0.79	1.25	0.00	0.00	1.36	5.83
time (sec)	N/A	0.218	0.303	0.481	0.293	0.000	0.000	0.295	9.492

Problem 252	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	712	712	431	334	0	9856	0	599	18343
N.S.	1	1.00	0.61	0.47	0.00	13.84	0.00	0.84	25.76
time (sec)	N/A	0.440	0.212	0.457	0.000	9.624	0.000	0.293	9.057

Problem 253	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	687	687	428	339	0	9822	0	614	17909
N.S.	1	1.00	0.62	0.49	0.00	14.30	0.00	0.89	26.07
time (sec)	N/A	0.373	0.257	0.460	0.000	6.610	0.000	0.294	9.022

Problem 254	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	685	685	423	327	0	9678	0	607	17180
N.S.	1	1.00	0.62	0.48	0.00	14.13	0.00	0.89	25.08
time (sec)	N/A	0.387	0.183	0.477	0.000	5.778	0.000	0.292	10.458

Problem 255	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	685	685	428	339	0	9774	0	622	17812
N.S.	1	1.00	0.62	0.49	0.00	14.27	0.00	0.91	26.00
time (sec)	N/A	0.370	0.182	0.438	0.000	6.449	0.000	0.297	9.657

Problem 256	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	689	689	429	334	0	9892	0	621	17945
N.S.	1	1.00	0.62	0.48	0.00	14.36	0.00	0.90	26.04
time (sec)	N/A	0.375	0.198	0.368	0.000	11.957	0.000	0.299	9.257

Problem 257	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	745	745	499	355	0	10188	0	659	24015
N.S.	1	1.00	0.67	0.48	0.00	13.68	0.00	0.88	32.23
time (sec)	N/A	0.495	0.242	0.502	0.000	29.014	0.000	0.285	10.732

Problem 258	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	751	751	513	355	0	10352	0	645	20828
N.S.	1	1.00	0.68	0.47	0.00	13.78	0.00	0.86	27.73
time (sec)	N/A	0.419	0.266	0.488	0.000	65.773	0.000	0.277	11.043

Problem 259	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	70	70	40	110	0	32	0	0	0
N.S.	1	1.00	0.57	1.57	0.00	0.46	0.00	0.00	0.00
time (sec)	N/A	0.040	10.186	1.065	0.000	0.120	0.000	0.000	0.000

Problem 260	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	70	70	36	112	0	56	0	0	0
N.S.	1	1.00	0.51	1.60	0.00	0.80	0.00	0.00	0.00
time (sec)	N/A	0.039	10.173	1.303	0.000	0.113	0.000	0.000	0.000

Problem 261	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	99	99	46	88	0	42	0	0	0
N.S.	1	1.00	0.46	0.89	0.00	0.42	0.00	0.00	0.00
time (sec)	N/A	0.036	10.125	1.408	0.000	0.082	0.000	0.000	0.000

Problem 262	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	61	61	37	88	0	31	0	0	0
N.S.	1	1.00	0.61	1.44	0.00	0.51	0.00	0.00	0.00
time (sec)	N/A	0.026	10.152	0.963	0.000	0.085	0.000	0.000	0.000

Problem 263	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	113	113	54	93	0	44	0	0	0
N.S.	1	1.00	0.48	0.82	0.00	0.39	0.00	0.00	0.00
time (sec)	N/A	0.039	10.100	1.581	0.000	0.086	0.000	0.000	0.000

Problem 264	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	57	57	35	93	0	31	0	0	0
N.S.	1	1.00	0.61	1.63	0.00	0.54	0.00	0.00	0.00
time (sec)	N/A	0.032	10.142	1.154	0.000	0.084	0.000	0.000	0.000

Problem 265	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	74	74	60	168	0	75	0	0	0
N.S.	1	1.00	0.81	2.27	0.00	1.01	0.00	0.00	0.00
time (sec)	N/A	0.040	10.118	1.399	0.000	0.103	0.000	0.000	0.000

Problem 266	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	74	74	56	115	0	74	0	0	0
N.S.	1	1.00	0.76	1.55	0.00	1.00	0.00	0.00	0.00
time (sec)	N/A	0.042	10.166	1.474	0.000	0.116	0.000	0.000	0.000

Problem 267	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	243	243	128	128	124	206	0	156	0
N.S.	1	1.00	0.53	0.53	0.51	0.85	0.00	0.64	0.00
time (sec)	N/A	0.082	0.477	0.345	0.205	0.318	0.000	0.276	0.000

Problem 268	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	108	108	56	51	50	50	0	68	0
N.S.	1	1.00	0.52	0.47	0.46	0.46	0.00	0.63	0.00
time (sec)	N/A	0.068	0.037	0.305	0.207	0.280	0.000	0.272	0.000

Problem 269	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	178	178	96	103	81	155	0	109	0
N.S.	1	1.00	0.54	0.58	0.46	0.87	0.00	0.61	0.00
time (sec)	N/A	0.054	0.123	0.324	0.199	0.271	0.000	0.277	0.000

Problem 270	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	152	152	83	80	45	123	0	84	0
N.S.	1	1.00	0.55	0.53	0.30	0.81	0.00	0.55	0.00
time (sec)	N/A	0.061	0.089	0.327	0.201	0.293	0.000	0.272	0.000

Problem 271	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	177	177	101	94	59	134	0	111	0
N.S.	1	1.00	0.57	0.53	0.33	0.76	0.00	0.63	0.00
time (sec)	N/A	0.062	0.173	0.330	0.194	0.269	0.000	0.282	0.000

Problem 272	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	177	177	90	106	83	141	0	100	0
N.S.	1	1.00	0.51	0.60	0.47	0.80	0.00	0.56	0.00
time (sec)	N/A	0.081	0.129	0.340	0.190	0.277	0.000	0.280	0.000

Problem 273	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	78	78	72	73	72	72	76	79	73
N.S.	1	1.00	0.92	0.94	0.92	0.92	0.97	1.01	0.94
time (sec)	N/A	0.087	0.024	0.362	0.205	0.263	0.024	0.259	0.023

Problem 274	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	78	78	78	73	72	72	82	79	73
N.S.	1	1.00	1.00	0.94	0.92	0.92	1.05	1.01	0.94
time (sec)	N/A	0.042	0.016	0.351	0.191	0.258	0.024	0.260	0.017

Problem 275	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	75	75	72	73	72	72	76	79	73
N.S.	1	1.00	0.96	0.97	0.96	0.96	1.01	1.05	0.97
time (sec)	N/A	0.091	0.019	0.350	0.189	0.251	0.025	0.288	0.016

Problem 276	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	73	73	73	70	69	69	78	76	70
N.S.	1	1.00	1.00	0.96	0.95	0.95	1.07	1.04	0.96
time (sec)	N/A	0.030	0.014	0.289	0.183	0.269	0.023	0.285	0.016

Problem 277	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	74	74	74	71	73	70	73	79	70
N.S.	1	1.00	1.00	0.96	0.99	0.95	0.99	1.07	0.95
time (sec)	N/A	0.061	0.018	0.311	0.251	0.242	0.085	0.266	0.019

Problem 278	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	71	71	71	74	69	74	73	74	70
N.S.	1	1.00	1.00	1.04	0.97	1.04	1.03	1.04	0.99
time (sec)	N/A	0.031	0.026	0.302	0.255	0.250	0.080	0.269	0.019

Problem 279	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	74	74	71	73	73	76	71	96	70
N.S.	1	1.00	0.96	0.99	0.99	1.03	0.96	1.30	0.95
time (sec)	N/A	0.063	0.032	0.284	0.191	0.247	0.130	0.269	0.021

Problem 280	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	168	168	165	159	0	426	320	174	251
N.S.	1	1.00	0.98	0.95	0.00	2.54	1.90	1.04	1.49
time (sec)	N/A	0.150	0.105	0.346	0.000	0.335	0.650	0.422	7.700

Problem 281	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	135	135	133	122	0	350	189	136	179
N.S.	1	1.00	0.99	0.90	0.00	2.59	1.40	1.01	1.33
time (sec)	N/A	0.103	0.056	0.346	0.000	0.269	0.621	0.390	7.680

Problem 282	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	106	106	102	90	0	302	162	99	95
N.S.	1	1.00	0.96	0.85	0.00	2.85	1.53	0.93	0.90
time (sec)	N/A	0.073	0.056	0.371	0.000	0.272	0.526	0.359	7.788

Problem 283	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	83	87	88	79	0	268	153	83	77
N.S.	1	1.05	1.06	0.95	0.00	3.23	1.84	1.00	0.93
time (sec)	N/A	0.061	0.048	0.304	0.000	0.267	0.418	0.329	7.720

Problem 284	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	89	86	89	85	0	267	155	89	81
N.S.	1	0.97	1.00	0.96	0.00	3.00	1.74	1.00	0.91
time (sec)	N/A	0.077	0.048	0.329	0.000	0.271	0.595	0.359	7.630

Problem 285	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	106	106	105	94	0	316	167	96	98
N.S.	1	1.00	0.99	0.89	0.00	2.98	1.58	0.91	0.92
time (sec)	N/A	0.093	0.048	0.343	0.000	0.354	0.771	0.325	7.638

Problem 286	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	136	136	135	122	0	360	284	135	128
N.S.	1	1.00	0.99	0.90	0.00	2.65	2.09	0.99	0.94
time (sec)	N/A	0.166	0.066	0.340	0.000	0.266	1.045	0.294	7.696

Problem 287	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	167	167	166	149	0	436	328	173	156
N.S.	1	1.00	0.99	0.89	0.00	2.61	1.96	1.04	0.93
time (sec)	N/A	0.217	0.078	0.347	0.000	0.340	1.248	0.287	7.797

Problem 288	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	173	173	170	151	0	504	235	172	223
N.S.	1	1.00	0.98	0.87	0.00	2.91	1.36	0.99	1.29
time (sec)	N/A	0.209	0.083	0.333	0.000	0.276	1.919	0.307	7.751

Problem 289	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	143	143	141	120	0	462	212	134	137
N.S.	1	1.00	0.99	0.84	0.00	3.23	1.48	0.94	0.96
time (sec)	N/A	0.139	0.063	0.327	0.000	0.256	1.719	0.283	7.710

Problem 290	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	124	124	122	106	0	421	201	116	118
N.S.	1	1.00	0.98	0.85	0.00	3.40	1.62	0.94	0.95
time (sec)	N/A	0.090	0.073	0.319	0.000	0.279	1.294	0.285	7.759

Problem 291	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	115	119	110	107	0	391	196	109	112
N.S.	1	1.03	0.96	0.93	0.00	3.40	1.70	0.95	0.97
time (sec)	N/A	0.079	0.068	0.290	0.000	0.319	0.770	0.286	7.713

Problem 292	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	127	124	124	111	0	421	202	118	118
N.S.	1	0.98	0.98	0.87	0.00	3.31	1.59	0.93	0.93
time (sec)	N/A	0.131	0.099	0.365	0.000	0.268	1.026	0.276	7.700

Problem 293	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	142	142	141	124	0	476	214	131	138
N.S.	1	1.00	0.99	0.87	0.00	3.35	1.51	0.92	0.97
time (sec)	N/A	0.138	0.062	0.365	0.000	0.266	1.351	0.266	7.740

Problem 294	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	171	171	173	151	0	514	330	171	168
N.S.	1	1.00	1.01	0.88	0.00	3.01	1.93	1.00	0.98
time (sec)	N/A	0.243	0.080	0.378	0.000	0.260	1.788	0.272	7.728

Problem 295	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	F(-1)	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	230	230	228	216	0	0	0	233	7024
N.S.	1	1.00	0.99	0.94	0.00	0.00	0.00	1.01	30.54
time (sec)	N/A	0.318	0.180	0.507	0.000	0.000	0.000	0.596	73.398

Problem 296	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	189	189	186	172	0	616	0	194	2304
N.S.	1	1.00	0.98	0.91	0.00	3.26	0.00	1.03	12.19
time (sec)	N/A	0.214	0.121	0.565	0.000	113.048	0.000	0.587	18.413

Problem 297	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	158	158	139	137	0	421	0	155	1853
N.S.	1	1.00	0.88	0.87	0.00	2.66	0.00	0.98	11.73
time (sec)	N/A	0.170	0.068	0.439	0.000	32.438	0.000	0.577	15.421

Problem 298	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	132	132	114	112	0	321	0	131	3704
N.S.	1	1.00	0.86	0.85	0.00	2.43	0.00	0.99	28.06
time (sec)	N/A	0.106	0.049	0.459	0.000	11.072	0.000	0.563	14.133

Problem 299	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	133	133	112	113	0	321	0	133	2434
N.S.	1	1.00	0.84	0.85	0.00	2.41	0.00	1.00	18.30
time (sec)	N/A	0.083	0.048	0.362	0.000	7.675	0.000	0.593	13.230

Problem 300	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	F(-1)	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	167	167	242	166	0	0	0	170	6285
N.S.	1	1.00	1.45	0.99	0.00	0.00	0.00	1.02	37.63
time (sec)	N/A	0.189	0.214	0.419	0.000	0.000	0.000	0.559	19.843

Problem 301	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	F(-1)	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	205	205	331	215	0	0	0	232	5368
N.S.	1	1.00	1.61	1.05	0.00	0.00	0.00	1.13	26.19
time (sec)	N/A	0.263	0.215	0.497	0.000	0.000	0.000	0.541	65.887

Problem 302	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	F(-1)	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	268	268	426	284	0	0	0	327	10300
N.S.	1	1.00	1.59	1.06	0.00	0.00	0.00	1.22	38.43
time (sec)	N/A	0.358	0.277	0.541	0.000	0.000	0.000	0.601	141.998

Problem 303	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	387	387	463	413	0	24520	0	12525	41755
N.S.	1	1.00	1.20	1.07	0.00	63.36	0.00	32.36	107.89
time (sec)	N/A	2.493	0.407	0.474	0.000	164.883	0.000	2.845	11.970

Problem 304	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	323	323	385	331	0	20147	0	11046	33892
N.S.	1	1.00	1.19	1.02	0.00	62.37	0.00	34.20	104.93
time (sec)	N/A	0.869	0.337	0.487	0.000	25.695	0.000	2.443	11.481

Problem 305	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	280	280	323	264	0	15553	0	8670	25202
N.S.	1	1.00	1.15	0.94	0.00	55.55	0.00	30.96	90.01
time (sec)	N/A	0.568	0.222	0.431	0.000	2.574	0.000	2.271	10.978

Problem 306	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	251	251	277	213	0	12269	0	6936	19401
N.S.	1	1.00	1.10	0.85	0.00	48.88	0.00	27.63	77.29
time (sec)	N/A	0.282	0.320	0.436	0.000	1.116	0.000	1.777	10.548

Problem 307	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	254	254	274	215	0	16013	0	7664	23640
N.S.	1	1.00	1.08	0.85	0.00	63.04	0.00	30.17	93.07
time (sec)	N/A	0.307	0.173	0.337	0.000	13.244	0.000	1.783	11.030

Problem 308	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	298	298	340	276	0	20595	0	10072	33644
N.S.	1	1.00	1.14	0.93	0.00	69.11	0.00	33.80	112.90
time (sec)	N/A	0.595	0.262	0.462	0.000	166.674	0.000	2.513	11.326

Problem 309	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	348	348	410	349	0	24988	0	12281	42882
N.S.	1	1.00	1.18	1.00	0.00	71.80	0.00	35.29	123.22
time (sec)	N/A	0.951	0.357	0.488	0.000	293.422	0.000	2.650	11.881

Problem 310	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F(-1)	F	F(-1)	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	866	866	215	264	0	0	0	0	43112
N.S.	1	1.00	0.25	0.30	0.00	0.00	0.00	0.00	49.78
time (sec)	N/A	1.882	0.304	1.988	0.000	0.000	0.000	0.000	12.008

Problem 311	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	272	272	259	369	0	1525	0	0	0
N.S.	1	1.00	0.95	1.36	0.00	5.61	0.00	0.00	0.00
time (sec)	N/A	0.366	0.970	0.571	0.000	94.121	0.000	0.000	0.000

Problem 312	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	208	208	199	286	0	1231	0	0	0
N.S.	1	1.00	0.96	1.38	0.00	5.92	0.00	0.00	0.00
time (sec)	N/A	0.204	0.646	0.456	0.000	10.363	0.000	0.000	0.000

Problem 313	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	168	168	160	190	0	1050	0	0	0
N.S.	1	1.00	0.95	1.13	0.00	6.25	0.00	0.00	0.00
time (sec)	N/A	0.133	0.941	0.404	0.000	0.871	0.000	0.000	0.000

Problem 314	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	186	186	174	240	0	2367	0	0	0
N.S.	1	1.00	0.94	1.29	0.00	12.73	0.00	0.00	0.00
time (sec)	N/A	0.160	0.457	0.446	0.000	42.124	0.000	0.000	0.000

Problem 315	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	361	361	158	225	0	1094	0	214	0
N.S.	1	1.00	0.44	0.62	0.00	3.03	0.00	0.59	0.00
time (sec)	N/A	0.335	0.638	0.453	0.000	0.476	0.000	0.341	0.000

Problem 316	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	424	619	209	246	0	0	0	0	0
N.S.	1	1.46	0.49	0.58	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.357	5.769	4.103	0.000	0.000	0.000	0.000	0.000

Problem 317	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	417	591	204	239	0	0	0	0	0
N.S.	1	1.42	0.49	0.57	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.255	5.058	1.366	0.000	0.000	0.000	0.000	0.000

Problem 318	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	381	470	127	341	0	0	0	0	0
N.S.	1	1.23	0.33	0.90	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.132	9.622	0.621	0.000	0.000	0.000	0.000	0.000

Problem 319	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	399	399	208	240	0	0	0	0	0
N.S.	1	1.00	0.52	0.60	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.139	10.182	1.897	0.000	0.000	0.000	0.000	0.000

Problem 320	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	360	360	154	153	0	0	0	0	0
N.S.	1	1.00	0.43	0.42	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.121	10.173	3.190	0.000	0.000	0.000	0.000	0.000

Problem 321	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	546	546	224	263	0	0	0	0	0
N.S.	1	1.00	0.41	0.48	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.343	10.222	4.095	0.000	0.000	0.000	0.000	0.000

Problem 322	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	F(-1)	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	482	482	545	620	0	0	0	0	0
N.S.	1	1.00	1.13	1.29	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.694	10.668	0.635	0.000	0.000	0.000	0.000	0.000

Problem 323	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	F(-1)	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	360	360	345	494	0	0	0	0	0
N.S.	1	1.00	0.96	1.37	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.445	1.942	0.685	0.000	0.000	0.000	0.000	0.000

Problem 324	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	269	269	266	361	0	1589	0	0	0
N.S.	1	1.00	0.99	1.34	0.00	5.91	0.00	0.00	0.00
time (sec)	N/A	0.302	1.602	0.535	0.000	111.846	0.000	0.000	0.000

Problem 325	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F(-1)	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	350	350	258	360	0	0	0	0	0
N.S.	1	1.00	0.74	1.03	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.371	0.955	0.474	0.000	0.000	0.000	0.000	0.000

Problem 326	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F(-1)	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	562	562	230	344	0	0	0	0	0
N.S.	1	1.00	0.41	0.61	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.619	1.099	0.559	0.000	0.000	0.000	0.000	0.000

Problem 327	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	463	875	214	251	0	0	0	0	0
N.S.	1	1.89	0.46	0.54	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.427	8.963	2.839	0.000	0.000	0.000	0.000	0.000

Problem 328	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	428	602	209	246	0	0	0	0	0
N.S.	1	1.41	0.49	0.57	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.219	10.196	1.351	0.000	0.000	0.000	0.000	0.000

Problem 329	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	722	722	213	258	0	0	0	0	0
N.S.	1	1.00	0.30	0.36	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.230	10.236	2.964	0.000	0.000	0.000	0.000	0.000

Problem 330	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	625	625	219	258	0	0	0	0	0
N.S.	1	1.00	0.35	0.41	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.237	10.246	4.107	0.000	0.000	0.000	0.000	0.000

Problem 331	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	553	553	224	263	0	0	0	0	0
N.S.	1	1.00	0.41	0.48	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.272	10.272	5.107	0.000	0.000	0.000	0.000	0.000

Problem 332	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	173	173	166	211	0	1364	0	0	0
N.S.	1	1.00	0.96	1.22	0.00	7.88	0.00	0.00	0.00
time (sec)	N/A	0.197	0.781	0.442	0.000	12.718	0.000	0.000	0.000

Problem 333	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	137	137	146	171	0	1084	0	0	0
N.S.	1	1.00	1.07	1.25	0.00	7.91	0.00	0.00	0.00
time (sec)	N/A	0.098	0.416	0.362	0.000	0.964	0.000	0.000	0.000

Problem 334	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	86	86	96	101	0	357	0	74	0
N.S.	1	1.00	1.12	1.17	0.00	4.15	0.00	0.86	0.00
time (sec)	N/A	0.055	0.280	0.349	0.000	0.337	0.000	0.276	0.000

Problem 335	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	138	138	144	166	0	1097	0	0	0
N.S.	1	1.00	1.04	1.20	0.00	7.95	0.00	0.00	0.00
time (sec)	N/A	0.129	0.476	0.402	0.000	0.458	0.000	0.000	0.000

Problem 336	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	218	218	209	206	0	1414	0	207	0
N.S.	1	1.00	0.96	0.94	0.00	6.49	0.00	0.95	0.00
time (sec)	N/A	0.175	1.005	0.456	0.000	0.679	0.000	0.296	0.000

Problem 337	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	418	418	127	222	0	0	0	0	0
N.S.	1	1.00	0.30	0.53	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.127	10.208	1.890	0.000	0.000	0.000	0.000	0.000

Problem 338	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	247	247	99	134	0	0	0	0	0
N.S.	1	1.00	0.40	0.54	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.083	10.122	0.750	0.000	0.000	0.000	0.000	0.000

Problem 339	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	245	245	80	70	0	0	0	0	0
N.S.	1	1.00	0.33	0.29	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.067	10.072	0.643	0.000	0.000	0.000	0.000	0.000

Problem 340	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	399	399	147	178	0	0	0	0	0
N.S.	1	1.00	0.37	0.45	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.221	10.185	1.931	0.000	0.000	0.000	0.000	0.000

Problem 341	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	422	422	219	258	0	0	0	0	0
N.S.	1	1.00	0.52	0.61	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.320	10.181	3.456	0.000	0.000	0.000	0.000	0.000

Problem 342	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	236	236	251	365	0	4901	0	0	0
N.S.	1	1.00	1.06	1.55	0.00	20.77	0.00	0.00	0.00
time (sec)	N/A	0.315	0.938	0.586	0.000	37.451	0.000	0.000	0.000

Problem 343	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	167	167	180	229	0	1381	0	468	0
N.S.	1	1.00	1.08	1.37	0.00	8.27	0.00	2.80	0.00
time (sec)	N/A	0.217	0.643	0.437	0.000	0.543	0.000	0.306	0.000

Problem 344	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	159	159	178	216	0	1349	0	451	0
N.S.	1	1.00	1.12	1.36	0.00	8.48	0.00	2.84	0.00
time (sec)	N/A	0.133	0.699	0.432	0.000	0.582	0.000	0.293	0.000

Problem 345	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	166	166	182	223	0	1379	0	468	0
N.S.	1	1.00	1.10	1.34	0.00	8.31	0.00	2.82	0.00
time (sec)	N/A	0.124	0.904	0.395	0.000	0.558	0.000	0.282	0.000

Problem 346	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	266	266	247	285	0	4909	0	0	0
N.S.	1	1.00	0.93	1.07	0.00	18.45	0.00	0.00	0.00
time (sec)	N/A	0.262	1.139	0.677	0.000	2.444	0.000	0.000	0.000

Problem 347	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	419	419	363	413	0	6486	0	777	0
N.S.	1	1.00	0.87	0.99	0.00	15.48	0.00	1.85	0.00
time (sec)	N/A	0.374	1.681	0.945	0.000	5.173	0.000	0.480	0.000

Problem 348	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	449	566	199	246	0	0	0	0	0
N.S.	1	1.26	0.44	0.55	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.228	10.268	4.935	0.000	0.000	0.000	0.000	0.000

Problem 349	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	423	503	199	246	0	0	0	0	0
N.S.	1	1.19	0.47	0.58	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.167	9.675	2.516	0.000	0.000	0.000	0.000	0.000

Problem 350	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	422	501	199	244	0	0	0	0	0
N.S.	1	1.19	0.47	0.58	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.145	7.104	1.646	0.000	0.000	0.000	0.000	0.000

Problem 351	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	423	503	199	246	0	0	0	0	0
N.S.	1	1.19	0.47	0.58	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.155	6.996	1.274	0.000	0.000	0.000	0.000	0.000

Problem 352	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	422	501	199	246	0	0	0	0	0
N.S.	1	1.19	0.47	0.58	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.149	10.150	1.319	0.000	0.000	0.000	0.000	0.000

Problem 353	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	468	644	211	253	0	0	0	0	0
N.S.	1	1.38	0.45	0.54	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.283	10.208	1.975	0.000	0.000	0.000	0.000	0.000

Problem 354	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	406	406	475	473	0	5829	0	959	11195
N.S.	1	1.00	1.17	1.17	0.00	14.36	0.00	2.36	27.57
time (sec)	N/A	5.948	1.304	2.388	0.000	257.626	0.000	0.329	8.899

Problem 355	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	324	324	383	396	0	4182	0	776	8222
N.S.	1	1.00	1.18	1.22	0.00	12.91	0.00	2.40	25.38
time (sec)	N/A	2.261	0.979	0.870	0.000	102.891	0.000	0.322	8.681

Problem 356	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	292	292	350	316	0	2435	0	626	5705
N.S.	1	1.00	1.20	1.08	0.00	8.34	0.00	2.14	19.54
time (sec)	N/A	2.395	0.954	0.607	0.000	29.968	0.000	0.327	9.191

Problem 357	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	202	202	256	227	0	1085	0	449	717
N.S.	1	1.00	1.27	1.12	0.00	5.37	0.00	2.22	3.55
time (sec)	N/A	0.209	0.586	0.387	0.000	6.081	0.000	0.306	8.603

Problem 358	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	281	281	274	358	0	3126	0	724	10964
N.S.	1	1.00	0.98	1.27	0.00	11.12	0.00	2.58	39.02
time (sec)	N/A	0.800	0.636	0.465	0.000	49.740	0.000	0.313	11.487

Problem 359	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	382	370	348	364	0	6592	0	784	19959
N.S.	1	0.97	0.91	0.95	0.00	17.26	0.00	2.05	52.25
time (sec)	N/A	2.726	1.054	0.825	0.000	260.181	0.000	0.325	10.828

Problem 360	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F(-1)	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	552	552	445	463	0	0	0	1057	33925
N.S.	1	1.00	0.81	0.84	0.00	0.00	0.00	1.91	61.46
time (sec)	N/A	2.685	1.700	1.153	0.000	0.000	0.000	0.346	11.971

Problem 361	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	B	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	390	390	786	360	0	6534	0	0	0
N.S.	1	1.00	2.02	0.92	0.00	16.75	0.00	0.00	0.00
time (sec)	N/A	1.818	1.010	2.623	0.000	14.868	0.000	0.000	0.000

Problem 362	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	F	B	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	324	324	7468	368	0	3260	0	0	0
N.S.	1	1.00	23.05	1.14	0.00	10.06	0.00	0.00	0.00
time (sec)	N/A	1.038	16.200	0.400	0.000	2.498	0.000	0.000	0.000

Problem 363	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	F	B	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	240	240	2607	233	0	985	0	0	0
N.S.	1	1.00	10.86	0.97	0.00	4.10	0.00	0.00	0.00
time (sec)	N/A	0.224	13.416	0.365	0.000	0.832	0.000	0.000	0.000

Problem 364	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	B	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	291	291	626	326	0	2402	0	0	0
N.S.	1	1.00	2.15	1.12	0.00	8.25	0.00	0.00	0.00
time (sec)	N/A	0.500	0.734	0.771	0.000	1.634	0.000	0.000	0.000

Problem 365	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	F	B	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	373	373	7477	398	0	4095	0	0	0
N.S.	1	1.00	20.05	1.07	0.00	10.98	0.00	0.00	0.00
time (sec)	N/A	1.776	16.384	0.906	0.000	6.508	0.000	0.000	0.000

Problem 366	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	F	B	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	512	512	10511	500	0	5773	0	0	0
N.S.	1	1.00	20.53	0.98	0.00	11.28	0.00	0.00	0.00
time (sec)	N/A	3.497	16.520	1.214	0.000	15.781	0.000	0.000	0.000

Problem 367	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	B	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	460	460	501	447	0	8200	0	867	16951
N.S.	1	1.00	1.09	0.97	0.00	17.83	0.00	1.88	36.85
time (sec)	N/A	3.105	1.763	0.849	0.000	249.643	0.000	0.389	9.376

Problem 368	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	327	327	373	335	0	4444	0	802	12392
N.S.	1	1.00	1.14	1.02	0.00	13.59	0.00	2.45	37.90
time (sec)	N/A	0.807	1.199	0.569	0.000	65.872	0.000	0.341	9.762

Problem 369	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	F(-1)	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	346	346	379	402	0	0	0	838	28434
N.S.	1	1.00	1.10	1.16	0.00	0.00	0.00	2.42	82.18
time (sec)	N/A	1.226	1.393	0.515	0.000	0.000	0.000	0.341	11.986

Problem 370	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	F(-1)	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	417	416	427	399	0	0	0	898	35855
N.S.	1	1.00	1.02	0.96	0.00	0.00	0.00	2.15	85.98
time (sec)	N/A	2.472	1.735	0.851	0.000	0.000	0.000	0.331	11.061

Problem 371	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	B	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	595	595	1407	460	0	19825	0	0	0
N.S.	1	1.00	2.36	0.77	0.00	33.32	0.00	0.00	0.00
time (sec)	N/A	2.250	2.301	1.366	0.000	290.822	0.000	0.000	0.000

Problem 372	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	B	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	491	491	916	394	0	11917	0	0	0
N.S.	1	1.00	1.87	0.80	0.00	24.27	0.00	0.00	0.00
time (sec)	N/A	1.245	1.380	0.753	0.000	54.072	0.000	0.000	0.000

Problem 373	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	F	B	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	487	487	8958	411	0	7721	0	0	0
N.S.	1	1.00	18.39	0.84	0.00	15.85	0.00	0.00	0.00
time (sec)	N/A	1.137	16.204	0.406	0.000	15.604	0.000	0.000	0.000

Problem 374	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	F	B	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	260	432	7491	347	0	4059	0	0	0
N.S.	1	1.66	28.81	1.33	0.00	15.61	0.00	0.00	0.00
time (sec)	N/A	0.602	16.330	0.936	0.000	7.918	0.000	0.000	0.000

Problem 375	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	B	F(-1)	F(-1)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	523	523	1095	451	0	7830	0	0	0
N.S.	1	1.00	2.09	0.86	0.00	14.97	0.00	0.00	0.00
time (sec)	N/A	1.866	1.494	1.093	0.000	32.456	0.000	0.000	0.000

Problem 376	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	281	281	346	316	0	3615	0	4637	917
N.S.	1	1.00	1.23	1.12	0.00	12.86	0.00	16.50	3.26
time (sec)	N/A	4.699	0.793	1.055	0.000	3.733	0.000	1.153	8.337

Problem 377	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	229	229	285	228	0	2053	0	4060	776
N.S.	1	1.00	1.24	1.00	0.00	8.97	0.00	17.73	3.39
time (sec)	N/A	1.045	0.543	0.536	0.000	1.470	0.000	1.067	8.350

Problem 378	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	182	182	169	162	0	871	0	591	649
N.S.	1	1.00	0.93	0.89	0.00	4.79	0.00	3.25	3.57
time (sec)	N/A	0.158	0.233	0.452	0.000	0.755	0.000	1.569	8.300

Problem 379	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	241	241	264	271	0	1232	0	3639	669
N.S.	1	1.00	1.10	1.12	0.00	5.11	0.00	15.10	2.78
time (sec)	N/A	0.984	0.523	0.461	0.000	2.718	0.000	0.968	8.398

Problem 380	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	290	290	307	330	0	2799	0	1675	825
N.S.	1	1.00	1.06	1.14	0.00	9.65	0.00	5.78	2.84
time (sec)	N/A	1.501	1.454	0.669	0.000	7.499	0.000	1.569	8.241

Problem 381	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	325	325	588	250	0	2860	0	1710	1024
N.S.	1	1.00	1.81	0.77	0.00	8.80	0.00	5.26	3.15
time (sec)	N/A	3.548	0.639	2.124	0.000	1.303	0.000	1.418	8.238

Problem 382	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	263	263	412	262	0	1430	0	3580	870
N.S.	1	1.00	1.57	1.00	0.00	5.44	0.00	13.61	3.31
time (sec)	N/A	1.361	0.411	0.319	0.000	0.672	0.000	1.018	8.340

Problem 383	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	220	220	237	168	0	759	0	641	989
N.S.	1	1.00	1.08	0.76	0.00	3.45	0.00	2.91	4.50
time (sec)	N/A	0.161	0.299	0.392	0.000	0.352	0.000	1.076	8.272

Problem 384	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	265	265	471	239	0	1998	0	3965	1234
N.S.	1	1.00	1.78	0.90	0.00	7.54	0.00	14.96	4.66
time (sec)	N/A	0.453	0.451	0.735	0.000	0.492	0.000	0.853	8.169

Problem 385	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	F	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	96	96	199	100	0	318	0	209	383
N.S.	1	1.00	2.07	1.04	0.00	3.31	0.00	2.18	3.99
time (sec)	N/A	0.150	0.439	2.025	0.000	0.286	0.000	0.330	8.390

Problem 386	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	B	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	479	479	838	394	0	18271	0	0	0
N.S.	1	1.00	1.75	0.82	0.00	38.14	0.00	0.00	0.00
time (sec)	N/A	1.219	1.372	1.056	0.000	142.828	0.000	0.000	0.000

Problem 387	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	B	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	366	366	594	305	0	14654	0	0	0
N.S.	1	1.00	1.62	0.83	0.00	40.04	0.00	0.00	0.00
time (sec)	N/A	0.864	0.775	0.805	0.000	52.679	0.000	0.000	0.000

Problem 388	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	298	298	292	252	0	11094	0	0	0
N.S.	1	1.00	0.98	0.85	0.00	37.23	0.00	0.00	0.00
time (sec)	N/A	0.485	10.674	0.465	0.000	9.052	0.000	0.000	0.000

Problem 389	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	B	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	240	240	227	233	0	3395	0	0	0
N.S.	1	1.00	0.95	0.97	0.00	14.15	0.00	0.00	0.00
time (sec)	N/A	0.226	8.657	0.404	0.000	2.462	0.000	0.000	0.000

Problem 390	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	B	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	243	243	179	224	0	4557	0	0	0
N.S.	1	1.00	0.74	0.92	0.00	18.75	0.00	0.00	0.00
time (sec)	N/A	0.141	0.137	0.379	0.000	6.168	0.000	0.000	0.000

Problem 391	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	280	280	271	266	0	6431	0	0	0
N.S.	1	1.00	0.97	0.95	0.00	22.97	0.00	0.00	0.00
time (sec)	N/A	0.528	10.804	0.617	0.000	12.662	0.000	0.000	0.000

Problem 392	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	341	341	320	332	0	8187	0	0	0
N.S.	1	1.00	0.94	0.97	0.00	24.01	0.00	0.00	0.00
time (sec)	N/A	0.538	10.590	0.714	0.000	62.573	0.000	0.000	0.000

Problem 393	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	443	443	383	410	0	9998	0	0	0
N.S.	1	1.00	0.86	0.93	0.00	22.57	0.00	0.00	0.00
time (sec)	N/A	1.075	11.235	0.979	0.000	164.999	0.000	0.000	0.000

Problem 394	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	F	F(-1)	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	350	507	10546	502	0	0	0	0	0
N.S.	1	1.45	30.13	1.43	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	2.965	21.270	0.606	0.000	0.000	0.000	0.000	0.000

Problem 395	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	B	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	360	360	449	382	0	14462	0	0	0
N.S.	1	1.00	1.25	1.06	0.00	40.17	0.00	0.00	0.00
time (sec)	N/A	0.928	0.686	0.540	0.000	156.125	0.000	0.000	0.000

Problem 396	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	B	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	333	333	463	393	0	14146	0	0	0
N.S.	1	1.00	1.39	1.18	0.00	42.48	0.00	0.00	0.00
time (sec)	N/A	0.518	0.636	0.400	0.000	111.971	0.000	0.000	0.000

Problem 397	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	B	F	F(-1)	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	341	341	2061	429	0	17249	0	0	0
N.S.	1	1.00	6.04	1.26	0.00	50.58	0.00	0.00	0.00
time (sec)	N/A	0.600	18.246	0.413	0.000	217.475	0.000	0.000	0.000

Problem 398	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	F(-1)	F	F(-1)	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	339	462	2158	513	0	0	0	0	0
N.S.	1	1.36	6.37	1.51	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.998	17.538	1.137	0.000	0.000	0.000	0.000	0.000

Problem 399	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	F(-1)	F	F(-1)	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	419	647	2218	610	0	0	0	0	0
N.S.	1	1.54	5.29	1.46	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	3.374	17.822	1.315	0.000	0.000	0.000	0.000	0.000

Problem 400	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	243	243	0	0	0	0	0	0	0
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.311	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Problem 401	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	313	313	272	0	0	0	0	0	0
N.S.	1	1.00	0.87	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.628	1.096	0.000	0.000	0.000	0.000	0.000	0.000

Problem 402	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	256	256	211	0	0	0	0	0	0
N.S.	1	1.00	0.82	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.334	0.534	0.000	0.000	0.000	0.000	0.000	0.000

Problem 403	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	210	210	183	0	0	0	0	0	0
N.S.	1	1.00	0.87	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.183	0.355	0.000	0.000	0.000	0.000	0.000	0.000

Problem 404	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	198	198	168	0	0	0	0	0	0
N.S.	1	1.00	0.85	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.244	0.319	0.000	0.000	0.000	0.000	0.000	0.000

Problem 405	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	262	262	218	0	0	0	0	0	0
N.S.	1	1.00	0.83	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.361	0.423	0.000	0.000	0.000	0.000	0.000	0.000

Problem 406	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	322	322	259	0	0	0	0	0	0
N.S.	1	1.00	0.80	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.389	0.454	0.000	0.000	0.000	0.000	0.000	0.000

Problem 407	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	339	339	0	0	0	0	0	0	0
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.394	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Problem 408	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	273	273	0	0	0	0	0	0	0
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.297	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Problem 409	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	162	162	0	0	0	0	0	0	0
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.143	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Problem 410	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	190	190	0	0	0	0	0	0	0
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.127	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Problem 411	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	264	264	0	0	0	0	0	0	0
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.310	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Problem 412	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	328	328	0	0	0	0	0	0	0
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.291	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Problem 413	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	B	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	40	44	58	101	0	120	0	42	0
N.S.	1	1.10	1.45	2.52	0.00	3.00	0.00	1.05	0.00
time (sec)	N/A	0.044	0.869	0.382	0.000	0.247	0.000	0.301	0.000

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi only. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is also given. The larger this ratio is, the harder the integral is to solve. In this test file, problem number [252] had the largest ratio of [.5000000000000000000]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	3	2	1.00	20	0.100
2	A	2	1	1.00	20	0.050
3	A	4	3	1.00	18	0.167
4	A	2	1	1.00	17	0.059
5	A	3	2	1.00	20	0.100
6	A	2	1	1.00	20	0.050
7	A	3	2	1.00	20	0.100
8	A	5	5	1.00	20	0.250
9	A	4	4	1.00	20	0.200
10	A	4	4	1.00	18	0.222
11	A	7	7	1.00	20	0.350
12	A	7	7	1.00	20	0.350
13	A	7	7	1.00	20	0.350
14	A	6	6	1.00	20	0.300
15	A	6	5	1.00	20	0.250
16	A	5	5	1.00	20	0.250
17	A	4	4	1.00	17	0.235
18	A	4	4	1.00	20	0.200
19	A	5	5	1.00	20	0.250
20	A	6	5	1.00	20	0.250
21	A	5	4	1.00	20	0.200
22	A	5	4	1.00	18	0.222
23	A	8	7	1.00	20	0.350
24	A	8	8	1.00	20	0.400

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
25	A	8	7	1.00	20	0.350
26	A	8	8	1.00	20	0.400
27	A	7	5	1.00	20	0.250
28	A	6	5	1.00	20	0.250
29	A	5	4	1.00	17	0.235
30	A	5	5	1.00	20	0.250
31	A	5	4	1.00	20	0.200
32	A	5	4	1.00	20	0.200
33	A	4	4	1.00	20	0.200
34	A	3	3	1.00	20	0.150
35	A	3	3	1.00	18	0.167
36	A	6	6	1.00	20	0.300
37	A	5	5	1.00	20	0.250
38	A	6	6	1.00	20	0.300
39	A	5	4	1.00	20	0.200
40	A	4	4	1.00	20	0.200
41	A	3	3	1.00	17	0.176
42	A	4	4	1.00	20	0.200
43	A	5	4	1.00	20	0.200
44	A	4	4	1.00	20	0.200
45	A	4	4	1.00	20	0.200
46	A	3	3	1.00	20	0.150
47	A	2	2	1.00	18	0.111
48	A	6	6	1.00	20	0.300
49	A	6	6	1.00	20	0.300
50	A	5	5	1.00	20	0.250
51	A	4	4	1.00	20	0.200
52	A	4	4	1.00	17	0.235
53	A	5	5	1.00	20	0.250
54	A	6	5	1.00	20	0.250
55	A	3	2	1.00	25	0.080
56	A	4	3	1.00	23	0.130
57	A	3	2	1.00	23	0.087
58	A	4	3	1.00	23	0.130
59	A	3	2	1.00	23	0.087

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
60	A	4	3	1.00	21	0.143
61	A	3	2	1.00	20	0.100
62	A	5	4	1.00	23	0.174
63	A	3	2	1.00	23	0.087
64	A	4	3	1.00	23	0.130
65	A	3	2	1.00	23	0.087
66	A	4	3	1.00	21	0.143
67	A	3	2	1.00	21	0.095
68	A	4	3	1.00	21	0.143
69	A	3	2	1.00	21	0.095
70	A	2	2	1.00	19	0.105
71	A	3	2	1.00	18	0.111
72	A	4	3	1.00	21	0.143
73	A	3	2	1.00	21	0.095
74	A	4	3	1.00	21	0.143
75	A	4	4	1.00	33	0.121
76	A	4	4	1.00	31	0.129
77	A	3	3	1.00	30	0.100
78	A	4	3	1.00	33	0.091
79	A	3	3	1.00	33	0.091
80	A	4	3	1.00	33	0.091
81	A	4	4	1.00	33	0.121
82	A	3	3	1.00	31	0.097
83	A	4	4	1.00	30	0.133
84	A	4	3	1.00	33	0.091
85	A	5	4	1.00	33	0.121
86	A	4	3	1.00	33	0.091
87	A	3	2	1.00	35	0.057
88	A	3	2	1.00	35	0.057
89	A	3	2	1.00	35	0.057
90	A	3	3	1.00	35	0.086
91	A	3	3	1.00	35	0.086
92	A	2	2	1.00	29	0.069
93	A	5	4	1.00	31	0.129
94	A	5	4	1.00	31	0.129

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
95	A	3	2	1.00	25	0.080
96	A	2	1	1.00	25	0.040
97	A	3	2	1.00	23	0.087
98	A	2	1	1.00	22	0.045
99	A	3	2	1.00	25	0.080
100	A	2	1	1.00	25	0.040
101	A	3	2	1.00	25	0.080
102	A	7	6	1.00	25	0.240
103	A	6	6	1.00	25	0.240
104	A	5	5	1.00	23	0.217
105	A	7	6	1.00	25	0.240
106	A	7	6	1.00	25	0.240
107	A	5	3	1.00	25	0.120
108	A	4	3	1.00	25	0.120
109	A	3	2	1.00	22	0.091
110	A	4	3	1.00	25	0.120
111	A	5	3	1.00	25	0.120
112	A	7	7	1.00	25	0.280
113	A	6	6	1.00	25	0.240
114	A	4	4	1.00	25	0.160
115	A	4	4	1.00	23	0.174
116	A	8	7	1.00	25	0.280
117	A	8	7	1.00	25	0.280
118	A	6	4	1.00	25	0.160
119	A	5	4	1.00	25	0.160
120	A	4	3	1.00	25	0.120
121	A	4	3	1.00	22	0.136
122	A	5	4	1.00	25	0.160
123	A	6	4	1.00	25	0.160
124	A	8	7	1.00	25	0.280
125	A	7	6	1.00	25	0.240
126	A	5	5	1.00	25	0.200
127	A	5	5	1.00	25	0.200
128	A	5	5	1.00	25	0.200
129	A	5	5	1.00	23	0.217

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
130	A	9	7	1.00	25	0.280
131	A	9	7	1.00	25	0.280
132	A	7	4	1.00	25	0.160
133	A	6	4	1.00	25	0.160
134	A	5	3	1.00	25	0.120
135	A	5	4	1.00	25	0.160
136	A	5	3	1.00	22	0.136
137	A	4	3	1.00	21	0.143
138	A	5	4	1.00	22	0.182
139	A	5	5	1.00	17	0.294
140	A	6	6	1.00	18	0.333
141	A	5	5	1.00	22	0.227
142	A	6	6	1.00	25	0.240
143	A	5	5	1.00	25	0.200
144	A	5	5	1.00	23	0.217
145	A	7	6	1.00	25	0.240
146	A	7	6	1.00	25	0.240
147	A	7	6	1.00	25	0.240
148	A	5	5	1.00	25	0.200
149	A	6	6	1.00	25	0.240
150	A	7	6	1.00	25	0.240
151	A	6	5	1.00	25	0.200
152	A	5	5	1.00	25	0.200
153	A	4	4	1.00	22	0.182
154	A	4	4	1.00	25	0.160
155	A	5	5	1.00	25	0.200
156	A	7	6	1.00	25	0.240
157	A	6	5	1.00	25	0.200
158	A	6	5	1.00	23	0.217
159	A	8	6	1.00	25	0.240
160	A	8	7	1.00	25	0.280
161	A	8	6	1.00	25	0.240
162	A	8	7	1.00	25	0.280
163	A	7	5	1.00	25	0.200
164	A	6	5	1.00	25	0.200

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
165	A	5	4	1.00	22	0.182
166	A	5	5	1.00	25	0.200
167	A	5	4	1.00	25	0.160
168	A	6	5	1.00	25	0.200
169	A	5	5	1.00	27	0.185
170	A	4	4	1.00	27	0.148
171	A	4	4	1.00	25	0.160
172	A	6	5	1.00	27	0.185
173	A	4	4	1.00	27	0.148
174	A	5	5	1.00	27	0.185
175	A	6	5	1.00	27	0.185
176	A	5	4	1.00	27	0.148
177	A	4	4	1.00	27	0.148
178	A	3	3	1.00	24	0.125
179	A	4	4	1.00	27	0.148
180	A	5	4	1.00	27	0.148
181	A	6	5	1.00	25	0.200
182	A	5	5	1.00	25	0.200
183	A	4	4	1.00	25	0.160
184	A	4	4	1.00	23	0.174
185	A	6	5	1.00	25	0.200
186	A	4	4	1.00	25	0.160
187	A	5	5	1.00	25	0.200
188	A	6	5	1.00	25	0.200
189	A	5	4	1.00	25	0.160
190	A	4	4	1.00	25	0.160
191	A	3	3	1.00	22	0.136
192	A	4	4	1.00	25	0.160
193	A	5	4	1.00	25	0.160
194	A	5	5	1.00	25	0.200
195	A	4	4	1.00	25	0.160
196	A	2	2	1.00	23	0.087
197	A	5	5	1.00	25	0.200
198	A	5	5	1.00	25	0.200
199	A	5	5	1.00	25	0.200

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
200	A	4	4	1.00	25	0.160
201	A	4	4	1.00	22	0.182
202	A	5	5	1.00	25	0.200
203	A	6	5	1.00	25	0.200
204	A	6	3	1.00	31	0.097
205	A	6	3	1.00	31	0.097
206	A	6	3	1.00	31	0.097
207	A	6	3	1.00	31	0.097
208	A	6	3	1.00	31	0.097
209	A	6	3	1.00	31	0.097
210	A	6	3	1.00	31	0.097
211	A	6	3	1.00	31	0.097
212	A	6	3	1.00	31	0.097
213	A	6	3	1.00	31	0.097
214	A	6	3	1.00	31	0.097
215	A	6	3	1.00	31	0.097
216	A	6	3	1.00	31	0.097
217	A	6	3	1.00	31	0.097
218	A	6	3	1.00	31	0.097
219	A	6	3	1.00	31	0.097
220	A	2	1	1.00	27	0.037
221	A	2	1	1.00	27	0.037
222	A	2	1	1.00	25	0.040
223	A	3	2	1.00	27	0.074
224	A	4	3	0.91	27	0.111
225	A	6	3	1.00	29	0.103
226	A	6	3	1.00	29	0.103
227	A	6	3	1.00	29	0.103
228	A	6	3	1.00	29	0.103
229	A	6	5	1.00	22	0.227
230	A	6	5	1.00	22	0.227
231	A	6	5	1.00	22	0.227
232	A	6	5	1.00	22	0.227
233	A	6	6	1.00	20	0.300
234	A	6	5	1.00	22	0.227

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
235	A	6	5	1.00	22	0.227
236	A	6	5	1.00	22	0.227
237	A	12	8	1.00	22	0.364
238	A	12	8	1.00	22	0.364
239	A	12	8	1.00	22	0.364
240	A	12	8	1.00	22	0.364
241	A	12	8	1.00	19	0.421
242	A	12	8	1.00	22	0.364
243	A	12	8	1.00	22	0.364
244	A	7	6	1.00	22	0.273
245	A	7	6	1.00	22	0.273
246	A	7	6	0.99	22	0.273
247	A	7	6	0.99	22	0.273
248	A	7	6	1.00	20	0.300
249	A	8	6	1.00	22	0.273
250	A	8	6	1.00	22	0.273
251	A	8	6	1.00	22	0.273
252	A	24	11	1.00	22	0.500
253	A	23	10	1.00	22	0.454
254	A	23	10	1.00	22	0.454
255	A	23	10	1.00	22	0.454
256	A	22	9	1.00	19	0.474
257	A	22	9	1.00	22	0.409
258	A	22	9	1.00	22	0.409
259	A	4	4	1.00	20	0.200
260	A	4	4	1.00	22	0.182
261	A	6	6	1.00	22	0.273
262	A	3	3	1.00	24	0.125
263	A	7	7	1.00	20	0.350
264	A	4	4	1.00	22	0.182
265	A	4	4	1.00	22	0.182
266	A	4	4	1.00	24	0.167
267	A	6	6	1.00	37	0.162
268	A	4	3	1.00	35	0.086
269	A	5	5	1.00	34	0.147

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
270	A	6	6	1.00	37	0.162
271	A	5	5	1.00	37	0.135
272	A	6	6	1.00	37	0.162
273	A	3	2	1.00	25	0.080
274	A	2	1	1.00	25	0.040
275	A	3	2	1.00	23	0.087
276	A	2	1	1.00	22	0.045
277	A	3	2	1.00	25	0.080
278	A	2	1	1.00	25	0.040
279	A	3	2	1.00	25	0.080
280	A	4	3	1.00	25	0.120
281	A	4	3	1.00	25	0.120
282	A	4	3	1.00	25	0.120
283	A	3	3	1.05	22	0.136
284	A	3	3	0.97	25	0.120
285	A	4	3	1.00	25	0.120
286	A	4	3	1.00	25	0.120
287	A	4	3	1.00	25	0.120
288	A	5	4	1.00	25	0.160
289	A	5	4	1.00	25	0.160
290	A	4	4	1.00	25	0.160
291	A	3	3	1.03	22	0.136
292	A	4	4	0.98	25	0.160
293	A	5	3	1.00	25	0.120
294	A	5	4	1.00	25	0.160
295	A	7	6	1.00	27	0.222
296	A	7	6	1.00	27	0.222
297	A	7	6	1.00	27	0.222
298	A	7	6	1.00	27	0.222
299	A	7	7	1.00	25	0.280
300	A	7	6	1.00	27	0.222
301	A	7	6	1.00	27	0.222
302	A	7	6	1.00	27	0.222
303	A	6	3	1.00	27	0.111
304	A	6	3	1.00	27	0.111

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
305	A	6	3	1.00	27	0.111
306	A	6	3	1.00	27	0.111
307	A	6	3	1.00	24	0.125
308	A	6	3	1.00	27	0.111
309	A	6	3	1.00	27	0.111
310	A	19	12	1.00	31	0.387
311	A	8	7	1.00	29	0.241
312	A	7	6	1.00	29	0.207
313	A	7	6	1.00	27	0.222
314	A	9	6	1.00	29	0.207
315	A	21	8	1.00	29	0.276
316	A	17	9	1.46	29	0.310
317	A	13	8	1.42	29	0.276
318	A	7	6	1.23	26	0.231
319	A	8	7	1.00	29	0.241
320	A	7	6	1.00	29	0.207
321	A	13	9	1.00	29	0.310
322	A	9	7	1.00	29	0.241
323	A	8	6	1.00	29	0.207
324	A	8	7	1.00	27	0.259
325	A	14	8	1.00	29	0.276
326	A	24	9	1.00	29	0.310
327	A	19	9	1.89	29	0.310
328	A	12	7	1.41	26	0.269
329	A	13	10	1.00	29	0.345
330	A	13	9	1.00	29	0.310
331	A	15	9	1.00	29	0.310
332	A	7	6	1.00	29	0.207
333	A	6	5	1.00	29	0.172
334	A	3	3	1.00	27	0.111
335	A	7	4	1.00	29	0.138
336	A	10	5	1.00	29	0.172
337	A	4	4	1.00	29	0.138
338	A	3	3	1.00	29	0.103
339	A	3	3	1.00	26	0.115

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
340	A	6	6	1.00	29	0.207
341	A	7	7	1.00	29	0.241
342	A	7	6	1.00	29	0.207
343	A	5	5	1.00	29	0.172
344	A	5	5	1.00	29	0.172
345	A	5	5	1.00	27	0.185
346	A	11	6	1.00	29	0.207
347	A	15	7	1.00	29	0.241
348	A	10	8	1.26	29	0.276
349	A	8	7	1.19	29	0.241
350	A	8	7	1.19	29	0.241
351	A	8	7	1.19	29	0.241
352	A	8	7	1.19	26	0.269
353	A	15	10	1.38	29	0.345
354	A	7	5	1.00	29	0.172
355	A	7	5	1.00	29	0.172
356	A	6	5	1.00	29	0.172
357	A	5	4	1.00	27	0.148
358	A	8	6	1.00	29	0.207
359	A	10	7	0.97	29	0.241
360	A	13	7	1.00	29	0.241
361	A	10	7	1.00	29	0.241
362	A	9	6	1.00	29	0.207
363	A	11	6	1.00	26	0.231
364	A	8	5	1.00	29	0.172
365	A	12	7	1.00	29	0.241
366	A	15	7	1.00	29	0.241
367	A	7	5	1.00	29	0.172
368	A	6	5	1.00	27	0.185
369	A	8	6	1.00	29	0.207
370	A	10	7	1.00	29	0.241
371	A	17	9	1.00	29	0.310
372	A	16	8	1.00	29	0.276
373	A	13	7	1.00	26	0.269
374	A	16	8	1.66	29	0.276

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
375	A	19	10	1.00	29	0.345
376	A	7	5	1.00	29	0.172
377	A	6	5	1.00	29	0.172
378	A	5	4	1.00	27	0.148
379	A	8	6	1.00	29	0.207
380	A	8	6	1.00	29	0.207
381	A	9	6	1.00	29	0.207
382	A	8	5	1.00	29	0.172
383	A	9	5	1.00	26	0.192
384	A	8	5	1.00	29	0.172
385	A	8	6	1.00	25	0.240
386	A	17	7	1.00	29	0.241
387	A	13	7	1.00	29	0.241
388	A	10	6	1.00	29	0.207
389	A	6	3	1.00	29	0.103
390	A	5	3	1.00	26	0.115
391	A	9	5	1.00	29	0.172
392	A	11	6	1.00	29	0.207
393	A	14	6	1.00	29	0.207
394	A	14	7	1.45	29	0.241
395	A	8	5	1.00	29	0.172
396	A	8	5	1.00	29	0.172
397	A	8	5	1.00	26	0.192
398	A	12	8	1.36	29	0.276
399	A	15	8	1.54	29	0.276
400	A	6	3	1.00	29	0.103
401	A	5	3	1.00	27	0.111
402	A	5	3	1.00	27	0.111
403	A	5	3	1.00	27	0.111
404	A	5	3	1.00	25	0.120
405	A	8	5	1.00	27	0.185
406	A	9	5	1.00	27	0.185
407	A	12	8	1.00	27	0.296
408	A	10	6	1.00	27	0.222
409	A	6	3	1.00	27	0.111

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
410	A	5	3	1.00	24	0.125
411	A	10	6	1.00	27	0.222
412	A	12	6	1.00	27	0.222
413	A	5	5	1.10	28	0.179

CHAPTER 3

LISTING OF INTEGRALS

3.1 $\int x^3(d+ex^2)\left(a+cx^4\right)^5\,dx$

Optimal result

Integrand size = 20, antiderivative size = 149

$$\int x^3(d+ex^2)\left(a+cx^4\right)^5\,dx=\frac{1}{4}a^5dx^4+\frac{1}{6}a^5ex^6+\frac{5}{8}a^4cdx^8+\frac{1}{2}a^4cex^{10}+\frac{5}{6}a^3c^2dx^{12}+\frac{5}{7}a^3c^2ex^{14}+\frac{5}{8}a^2c^3dx^{16}+\frac{5}{9}a^2c^3ex^{18}+\frac{1}{4}ac^4dx^{20}+\frac{5}{22}ac^4ex^{22}+\frac{1}{24}c^5dx^{24}+\frac{1}{26}c^5ex^{26}$$

```
[Out] 1/4*a^5*d*x^4+1/6*a^5*e*x^6+5/8*a^4*c*d*x^8+1/2*a^4*c*e*x^10+5/6*a^3*c^2*d*
x^12+5/7*a^3*c^2*e*x^14+5/8*a^2*c^3*d*x^16+5/9*a^2*c^3*e*x^18+1/4*a*c^4*d*x
^20+5/22*a*c^4*e*x^22+1/24*c^5*d*x^24+1/26*c^5*e*x^26
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {1266, 780}

$$\int x^3(d+ex^2)\left(a+cx^4\right)^5\,dx=\frac{1}{4}a^5dx^4+\frac{1}{6}a^5ex^6+\frac{5}{8}a^4cdx^8+\frac{1}{2}a^4cex^{10}+\frac{5}{6}a^3c^2dx^{12}+\frac{5}{7}a^3c^2ex^{14}+\frac{5}{8}a^2c^3dx^{16}+\frac{5}{9}a^2c^3ex^{18}+\frac{1}{4}ac^4dx^{20}+\frac{5}{22}ac^4ex^{22}+\frac{1}{24}c^5dx^{24}+\frac{1}{26}c^5ex^{26}$$

```
[In] Int[x^3*(d + e*x^2)*(a + c*x^4)^5,x]
```

```
[Out] (a^5*d*x^4)/4 + (a^5*e*x^6)/6 + (5*a^4*c*d*x^8)/8 + (a^4*c*e*x^10)/2 + (5*a
^3*c^2*d*x^12)/6 + (5*a^3*c^2*e*x^14)/7 + (5*a^2*c^3*d*x^16)/8 + (5*a^2*c^3
```

```
*e*x^18)/9 + (a*c^4*d*x^20)/4 + (5*a*c^4*e*x^22)/22 + (c^5*d*x^24)/24 + (c^
5*e*x^26)/26
```

Rule 780

```
Int[((e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(e*x)^m*(f + g*x)*(a + c*x^2)^p, x], x] /; F
reeQ[{a, c, e, f, g, m}, x] && IGtQ[p, 0]
```

Rule 1266

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_S
ymbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + c*x^2)^p, x],
x, x^2], x] /; FreeQ[{a, c, d, e, p, q}, x] && IntegerQ[(m + 1)/2]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{1}{2}\text{Subst}\left(\int x(d+ex)\left(a+cx^2\right)^5\,dx, x, x^2\right)\\
&= \frac{1}{2}\text{Subst}\left(\int \left(a^5dx + a^5ex^2 + 5a^4cdx^3 + 5a^4cex^4 + 10a^3c^2dx^5 + 10a^3c^2ex^6\right.\right.\\
&\qquad \left.\left. + 10a^2c^3dx^7 + 10a^2c^3ex^8 + 5ac^4dx^9 + 5ac^4ex^{10} + c^5dx^{11} + c^5ex^{12}\right)\,dx, x, x^2\right)\\
&= \frac{1}{4}a^5dx^4 + \frac{1}{6}a^5ex^6 + \frac{5}{8}a^4cdx^8 + \frac{1}{2}a^4cex^{10} + \frac{5}{6}a^3c^2dx^{12} + \frac{5}{7}a^3c^2ex^{14}\\
&\quad + \frac{5}{8}a^2c^3dx^{16} + \frac{5}{9}a^2c^3ex^{18} + \frac{1}{4}ac^4dx^{20} + \frac{5}{22}ac^4ex^{22} + \frac{1}{24}c^5dx^{24} + \frac{1}{26}c^5ex^{26}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.00

$$\begin{aligned}\int x^3\left(d+ex^2\right)\left(a+cx^4\right)^5\,dx &= \frac{1}{4}a^5dx^4 + \frac{1}{6}a^5ex^6 + \frac{5}{8}a^4cdx^8 + \frac{1}{2}a^4cex^{10}\\
&\quad + \frac{5}{6}a^3c^2dx^{12} + \frac{5}{7}a^3c^2ex^{14} + \frac{5}{8}a^2c^3dx^{16} + \frac{5}{9}a^2c^3ex^{18}\\
&\quad + \frac{1}{4}ac^4dx^{20} + \frac{5}{22}ac^4ex^{22} + \frac{1}{24}c^5dx^{24} + \frac{1}{26}c^5ex^{26}\end{aligned}$$

```
[In] Integrate[x^3*(d + e*x^2)*(a + c*x^4)^5,x]
```

```
[Out] (a^5*d*x^4)/4 + (a^5*e*x^6)/6 + (5*a^4*c*d*x^8)/8 + (a^4*c*e*x^10)/2 + (5*a
^3*c^2*d*x^12)/6 + (5*a^3*c^2*e*x^14)/7 + (5*a^2*c^3*d*x^16)/8 + (5*a^2*c^3
*e*x^18)/9 + (a*c^4*d*x^20)/4 + (5*a*c^4*e*x^22)/22 + (c^5*d*x^24)/24 + (c^
5*e*x^26)/26
```

Maple [A] (verified)

Time = 0.28 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.85

method	result
gosper	$\frac{1}{4}a^5d\,x^4+\frac{1}{6}a^5e\,x^6+\frac{5}{8}a^4cd\,x^8+\frac{1}{2}a^4ce\,x^{10}+\frac{5}{6}a^3c^2d\,x^{12}+\frac{5}{7}a^3c^2e\,x^{14}+\frac{5}{8}a^2c^3d\,x^{16}+\frac{5}{9}a^2c^3e\,x^1$
default	$\frac{1}{4}a^5d\,x^4+\frac{1}{6}a^5e\,x^6+\frac{5}{8}a^4cd\,x^8+\frac{1}{2}a^4ce\,x^{10}+\frac{5}{6}a^3c^2d\,x^{12}+\frac{5}{7}a^3c^2e\,x^{14}+\frac{5}{8}a^2c^3d\,x^{16}+\frac{5}{9}a^2c^3e\,x^1$
norman	$\frac{1}{4}a^5d\,x^4+\frac{1}{6}a^5e\,x^6+\frac{5}{8}a^4cd\,x^8+\frac{1}{2}a^4ce\,x^{10}+\frac{5}{6}a^3c^2d\,x^{12}+\frac{5}{7}a^3c^2e\,x^{14}+\frac{5}{8}a^2c^3d\,x^{16}+\frac{5}{9}a^2c^3e\,x^1$
risch	$\frac{1}{4}a^5d\,x^4+\frac{1}{6}a^5e\,x^6+\frac{5}{8}a^4cd\,x^8+\frac{1}{2}a^4ce\,x^{10}+\frac{5}{6}a^3c^2d\,x^{12}+\frac{5}{7}a^3c^2e\,x^{14}+\frac{5}{8}a^2c^3d\,x^{16}+\frac{5}{9}a^2c^3e\,x^1$
parallelrisch	$\frac{1}{4}a^5d\,x^4+\frac{1}{6}a^5e\,x^6+\frac{5}{8}a^4cd\,x^8+\frac{1}{2}a^4ce\,x^{10}+\frac{5}{6}a^3c^2d\,x^{12}+\frac{5}{7}a^3c^2e\,x^{14}+\frac{5}{8}a^2c^3d\,x^{16}+\frac{5}{9}a^2c^3e\,x^1$

```
[In] int(x^3*(e*x^2+d)*(c*x^4+a)^5,x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*a^5*d*x^4+1/6*a^5*e*x^6+5/8*a^4*c*d*x^8+1/2*a^4*c*e*x^10+5/6*a^3*c^2*d*
x^12+5/7*a^3*c^2*e*x^14+5/8*a^2*c^3*d*x^16+5/9*a^2*c^3*e*x^18+1/4*a*c^4*d*x
^20+5/22*a*c^4*e*x^22+1/24*c^5*d*x^24+1/26*c^5*e*x^26
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.84

$$\begin{aligned}\int x^3\left(d+ex^2\right)\left(a+cx^4\right)^5\,dx=&\ \frac{1}{26}\,c^5ex^{26}+\frac{1}{24}\,c^5dx^{24}+\frac{5}{22}\,ac^4ex^{22}+\frac{1}{4}\,ac^4dx^{20}\\&+\frac{5}{9}\,a^2c^3ex^{18}+\frac{5}{8}\,a^2c^3dx^{16}+\frac{5}{7}\,a^3c^2ex^{14}+\frac{5}{6}\,a^3c^2dx^{12}\\&+\frac{1}{2}\,a^4cex^{10}+\frac{5}{8}\,a^4cdx^8+\frac{1}{6}\,a^5ex^6+\frac{1}{4}\,a^5dx^4\end{aligned}$$

```
[In] integrate(x^3*(e*x^2+d)*(c*x^4+a)^5,x, algorithm="fricas")
```

```
[Out] 1/26*c^5*e*x^26 + 1/24*c^5*d*x^24 + 5/22*a*c^4*e*x^22 + 1/4*a*c^4*d*x^20 +
5/9*a^2*c^3*e*x^18 + 5/8*a^2*c^3*d*x^16 + 5/7*a^3*c^2*e*x^14 + 5/6*a^3*c^2*
d*x^12 + 1/2*a^4*c*e*x^10 + 5/8*a^4*c*d*x^8 + 1/6*a^5*e*x^6 + 1/4*a^5*d*x^4
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.01

$$\int x^3(d+ex^2)\left(a+cx^4\right)^5\,dx=\frac{a^5dx^4}{4}+\frac{a^5ex^6}{6}+\frac{5a^4cdx^8}{8}+\frac{a^4cex^{10}}{2}+\frac{5a^3c^2dx^{12}}{6}+\frac{5a^3c^2ex^{14}}{7}+\frac{5a^2c^3dx^{16}}{8}+\frac{5a^2c^3ex^{18}}{9}+\frac{ac^4dx^{20}}{4}+\frac{5ac^4ex^{22}}{22}+\frac{c^5dx^{24}}{24}+\frac{c^5ex^{26}}{26}$$

```
[In] integrate(x**3*(e*x**2+d)*(c*x**4+a)**5,x)
[Out] a**5*d*x**4/4 + a**5*e*x**6/6 + 5*a**4*c*d*x**8/8 + a**4*c*e*x**10/2 + 5*a*
*3*c**2*d*x**12/6 + 5*a**3*c**2*e*x**14/7 + 5*a**2*c**3*d*x**16/8 + 5*a**2*
c**3*e*x**18/9 + a*c**4*d*x**20/4 + 5*a*c**4*e*x**22/22 + c**5*d*x**24/24 +
 c**5*e*x**26/26
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.84

$$\int x^3(d+ex^2)\left(a+cx^4\right)^5\,dx=\frac{1}{26}\,c^5ex^{26}+\frac{1}{24}\,c^5dx^{24}+\frac{5}{22}\,ac^4ex^{22}+\frac{1}{4}\,ac^4dx^{20}+\frac{5}{9}\,a^2c^3ex^{18}+\frac{5}{8}\,a^2c^3dx^{16}+\frac{5}{7}\,a^3c^2ex^{14}+\frac{5}{6}\,a^3c^2dx^{12}+\frac{1}{2}\,a^4cex^{10}+\frac{5}{8}\,a^4cdx^8+\frac{1}{6}\,a^5ex^6+\frac{1}{4}\,a^5dx^4$$

```
[In] integrate(x^3*(e*x^2+d)*(c*x^4+a)^5,x, algorithm="maxima")
[Out] 1/26*c^5*e*x^26 + 1/24*c^5*d*x^24 + 5/22*a*c^4*e*x^22 + 1/4*a*c^4*d*x^20 +
5/9*a^2*c^3*e*x^18 + 5/8*a^2*c^3*d*x^16 + 5/7*a^3*c^2*e*x^14 + 5/6*a^3*c^2*
d*x^12 + 1/2*a^4*c*e*x^10 + 5/8*a^4*c*d*x^8 + 1/6*a^5*e*x^6 + 1/4*a^5*d*x^4
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.84

$$\int x^3(d+ex^2)\left(a+cx^4\right)^5\,dx=\frac{1}{26}\,c^5ex^{26}+\frac{1}{24}\,c^5dx^{24}+\frac{5}{22}\,ac^4ex^{22}+\frac{1}{4}\,ac^4dx^{20}+\frac{5}{9}\,a^2c^3ex^{18}+\frac{5}{8}\,a^2c^3dx^{16}+\frac{5}{7}\,a^3c^2ex^{14}+\frac{5}{6}\,a^3c^2dx^{12}+\frac{1}{2}\,a^4cex^{10}+\frac{5}{8}\,a^4cdx^8+\frac{1}{6}\,a^5ex^6+\frac{1}{4}\,a^5dx^4$$

```
[In] integrate(x^3*(e*x^2+d)*(c*x^4+a)^5,x, algorithm="giac")
```

```
[Out] 1/26*c^5*e*x^26 + 1/24*c^5*d*x^24 + 5/22*a*c^4*e*x^22 + 1/4*a*c^4*d*x^20 +
5/9*a^2*c^3*e*x^18 + 5/8*a^2*c^3*d*x^16 + 5/7*a^3*c^2*e*x^14 + 5/6*a^3*c^2*
d*x^12 + 1/2*a^4*c*e*x^10 + 5/8*a^4*c*d*x^8 + 1/6*a^5*e*x^6 + 1/4*a^5*d*x^4
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.84

$$\int x^3 (d+ex^2) \left(a+cx^4\right)^5 \, dx = \frac{e\,a^5\,x^6}{6}+\frac{d\,a^5\,x^4}{4}+\frac{e\,a^4\,c\,x^{10}}{2}+\frac{5\,d\,a^4\,c\,x^8}{8}+\frac{5\,e\,a^3\,c^2\,x^{14}}{7}+\frac{5\,d\,a^3\,c^2\,x^{12}}{6}+\frac{5\,e\,a^2\,c^3\,x^{18}}{9}+\frac{5\,d\,a^2\,c^3\,x^{16}}{8}+\frac{5\,e\,a\,c^4\,x^{22}}{22}+\frac{d\,a\,c^4\,x^{20}}{4}+\frac{e\,c^5\,x^{26}}{26}+\frac{d\,c^5\,x^{24}}{24}$$

```
[In] int(x^3*(a + c*x^4)^5*(d + e*x^2),x)
```

```
[Out] (a^5*d*x^4)/4 + (a^5*e*x^6)/6 + (c^5*d*x^24)/24 + (c^5*e*x^26)/26 + (5*a^3*
c^2*d*x^12)/6 + (5*a^2*c^3*d*x^16)/8 + (5*a^3*c^2*e*x^14)/7 + (5*a^2*c^3*e*
x^18)/9 + (5*a^4*c*d*x^8)/8 + (a*c^4*d*x^20)/4 + (a^4*c*e*x^10)/2 + (5*a*c^
4*e*x^22)/22
```

3.2 $\int x^2(d+ex^2)\left(a+cx^4\right)^5 dx$

Optimal result

Integrand size = 20, antiderivative size = 149

$$\int x^2(d+ex^2)\left(a+cx^4\right)^5 dx = \frac{1}{3}a^5dx^3+\frac{1}{5}a^5ex^5+\frac{5}{7}a^4cdx^7+\frac{5}{9}a^4cex^9+\frac{10}{11}a^3c^2dx^{11} + \frac{10}{13}a^3c^2ex^{13}+\frac{2}{3}a^2c^3dx^{15}+\frac{10}{17}a^2c^3ex^{17} + \frac{5}{19}ac^4dx^{19}+\frac{5}{21}ac^4ex^{21}+\frac{1}{23}c^5dx^{23}+\frac{1}{25}c^5ex^{25}$$

```
[Out] 1/3*a^5*d*x^3+1/5*a^5*e*x^5+5/7*a^4*c*d*x^7+5/9*a^4*c*e*x^9+10/11*a^3*c^2*d
*x^11+10/13*a^3*c^2*e*x^13+2/3*a^2*c^3*d*x^15+10/17*a^2*c^3*e*x^17+5/19*a*c
^4*d*x^19+5/21*a*c^4*e*x^21+1/23*c^5*d*x^23+1/25*c^5*e*x^25
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.050, Rules used = {1276}

$$\int x^2(d+ex^2)\left(a+cx^4\right)^5 dx = \frac{1}{3}a^5dx^3+\frac{1}{5}a^5ex^5+\frac{5}{7}a^4cdx^7+\frac{5}{9}a^4cex^9+\frac{10}{11}a^3c^2dx^{11} + \frac{10}{13}a^3c^2ex^{13}+\frac{2}{3}a^2c^3dx^{15}+\frac{10}{17}a^2c^3ex^{17} + \frac{5}{19}ac^4dx^{19}+\frac{5}{21}ac^4ex^{21}+\frac{1}{23}c^5dx^{23}+\frac{1}{25}c^5ex^{25}$$

```
[In] Int[x^2*(d + e*x^2)*(a + c*x^4)^5,x]
```

```
[Out] (a^5*d*x^3)/3 + (a^5*e*x^5)/5 + (5*a^4*c*d*x^7)/7 + (5*a^4*c*e*x^9)/9 + (10
*a^3*c^2*d*x^11)/11 + (10*a^3*c^2*e*x^13)/13 + (2*a^2*c^3*d*x^15)/3 + (10*a
```

```
^2*c^3*e*x^17)/17 + (5*a*c^4*d*x^19)/19 + (5*a*c^4*e*x^21)/21 + (c^5*d*x^23
)/23 + (c^5*e*x^25)/25
```

Rule 1276

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*(a + c*x^4)^p,
x], x] /; FreeQ[{a, c, d, e, f, m, q}, x] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \left(a^5dx^2 + a^5ex^4 + 5a^4cdx^6 + 5a^4cex^8 + 10a^3c^2dx^{10} + 10a^3c^2ex^{12} + 10a^2c^3dx^{14} + 10a^2c^3ex^{16} + 5ac^4dx^{18} + 5ac^4ex^{20} + c^5dx^{22} + c^5ex^{24}\right) dx \\ &= \frac{1}{3}a^5dx^3 + \frac{1}{5}a^5ex^5 + \frac{5}{7}a^4cdx^7 + \frac{5}{9}a^4cex^9 + \frac{10}{11}a^3c^2dx^{11} + \frac{10}{13}a^3c^2ex^{13} + \frac{2}{3}a^2c^3dx^{15} + \frac{10}{17}a^2c^3ex^{17} + \frac{5}{19}ac^4dx^{19} + \frac{5}{21}ac^4ex^{21} + \frac{1}{23}c^5dx^{23} + \frac{1}{25}c^5ex^{25}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.00

$$\int x^2(d+ex^2)\left(a+cx^4\right)^5 dx = \frac{1}{3}a^5dx^3 + \frac{1}{5}a^5ex^5 + \frac{5}{7}a^4cdx^7 + \frac{5}{9}a^4cex^9 + \frac{10}{11}a^3c^2dx^{11} + \frac{10}{13}a^3c^2ex^{13} + \frac{2}{3}a^2c^3dx^{15} + \frac{10}{17}a^2c^3ex^{17} + \frac{5}{19}ac^4dx^{19} + \frac{5}{21}ac^4ex^{21} + \frac{1}{23}c^5dx^{23} + \frac{1}{25}c^5ex^{25}$$

```
[In] Integrate[x^2*(d + e*x^2)*(a + c*x^4)^5,x]
```

```
[Out] (a^5*d*x^3)/3 + (a^5*e*x^5)/5 + (5*a^4*c*d*x^7)/7 + (5*a^4*c*e*x^9)/9 + (10
*a^3*c^2*d*x^11)/11 + (10*a^3*c^2*e*x^13)/13 + (2*a^2*c^3*d*x^15)/3 + (10*a
^2*c^3*e*x^17)/17 + (5*a*c^4*d*x^19)/19 + (5*a*c^4*e*x^21)/21 + (c^5*d*x^23
)/23 + (c^5*e*x^25)/25
```

Maple [A] (verified)

Time = 0.29 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.85

method	result
gosper	$\frac{1}{3}a^5d\,x^3+\frac{1}{5}a^5e\,x^5+\frac{5}{7}a^4cd\,x^7+\frac{5}{9}a^4ce\,x^9+\frac{10}{11}a^3c^2d\,x^{11}+\frac{10}{13}a^3c^2e\,x^{13}+\frac{2}{3}a^2c^3d\,x^{15}+\frac{10}{17}a^2c^3e$
default	$\frac{1}{3}a^5d\,x^3+\frac{1}{5}a^5e\,x^5+\frac{5}{7}a^4cd\,x^7+\frac{5}{9}a^4ce\,x^9+\frac{10}{11}a^3c^2d\,x^{11}+\frac{10}{13}a^3c^2e\,x^{13}+\frac{2}{3}a^2c^3d\,x^{15}+\frac{10}{17}a^2c^3e$
norman	$\frac{1}{3}a^5d\,x^3+\frac{1}{5}a^5e\,x^5+\frac{5}{7}a^4cd\,x^7+\frac{5}{9}a^4ce\,x^9+\frac{10}{11}a^3c^2d\,x^{11}+\frac{10}{13}a^3c^2e\,x^{13}+\frac{2}{3}a^2c^3d\,x^{15}+\frac{10}{17}a^2c^3e$
risch	$\frac{1}{3}a^5d\,x^3+\frac{1}{5}a^5e\,x^5+\frac{5}{7}a^4cd\,x^7+\frac{5}{9}a^4ce\,x^9+\frac{10}{11}a^3c^2d\,x^{11}+\frac{10}{13}a^3c^2e\,x^{13}+\frac{2}{3}a^2c^3d\,x^{15}+\frac{10}{17}a^2c^3e$
parallelrisch	$\frac{1}{3}a^5d\,x^3+\frac{1}{5}a^5e\,x^5+\frac{5}{7}a^4cd\,x^7+\frac{5}{9}a^4ce\,x^9+\frac{10}{11}a^3c^2d\,x^{11}+\frac{10}{13}a^3c^2e\,x^{13}+\frac{2}{3}a^2c^3d\,x^{15}+\frac{10}{17}a^2c^3e$

```
[In] int(x^2*(e*x^2+d)*(c*x^4+a)^5,x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*a^5*d*x^3+1/5*a^5*e*x^5+5/7*a^4*c*d*x^7+5/9*a^4*c*e*x^9+10/11*a^3*c^2*d
*x^11+10/13*a^3*c^2*e*x^13+2/3*a^2*c^3*d*x^15+10/17*a^2*c^3*e*x^17+5/19*a*c
^4*d*x^19+5/21*a*c^4*e*x^21+1/23*c^5*d*x^23+1/25*c^5*e*x^25
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.84

$$\int x^2\left(d+ex^2\right)\left(a+cx^4\right)^5\,dx=\frac{1}{25}\,c^5ex^{25}+\frac{1}{23}\,c^5dx^{23}+\frac{5}{21}\,ac^4ex^{21}+\frac{5}{19}\,ac^4dx^{19}+\frac{10}{17}\,a^2c^3ex^{17}+\frac{2}{3}\,a^2c^3dx^{15}+\frac{10}{13}\,a^3c^2ex^{13}+\frac{10}{11}\,a^3c^2dx^{11}+\frac{5}{9}\,a^4cex^9+\frac{5}{7}\,a^4cdx^7+\frac{1}{5}\,a^5ex^5+\frac{1}{3}\,a^5dx^3$$

```
[In] integrate(x^2*(e*x^2+d)*(c*x^4+a)^5,x, algorithm="fricas")
```

```
[Out] 1/25*c^5*e*x^25 + 1/23*c^5*d*x^23 + 5/21*a*c^4*e*x^21 + 5/19*a*c^4*d*x^19 +
 10/17*a^2*c^3*e*x^17 + 2/3*a^2*c^3*d*x^15 + 10/13*a^3*c^2*e*x^13 + 10/11*a
^3*c^2*d*x^11 + 5/9*a^4*c*e*x^9 + 5/7*a^4*c*d*x^7 + 1/5*a^5*e*x^5 + 1/3*a^5
*d*x^3
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.04

$$\int x^2\left(d+ex^2\right)\left(a+cx^4\right)^5\,dx=\frac{a^5dx^3}{3}+\frac{a^5ex^5}{5}+\frac{5a^4cdx^7}{7}+\frac{5a^4cex^9}{9}+\frac{10a^3c^2dx^{11}}{11}+\frac{10a^3c^2ex^{13}}{13}+\frac{2a^2c^3dx^{15}}{3}+\frac{10a^2c^3ex^{17}}{17}+\frac{5ac^4dx^{19}}{19}+\frac{5ac^4ex^{21}}{21}+\frac{c^5dx^{23}}{23}+\frac{c^5ex^{25}}{25}$$

```
[In] integrate(x**2*(e*x**2+d)*(c*x**4+a)**5,x)
```

```
[Out] a**5*d*x**3/3 + a**5*e*x**5/5 + 5*a**4*c*d*x**7/7 + 5*a**4*c*e*x**9/9 + 10*
a**3*c**2*d*x**11/11 + 10*a**3*c**2*e*x**13/13 + 2*a**2*c**3*d*x**15/3 + 10
*a**2*c**3*e*x**17/17 + 5*a*c**4*d*x**19/19 + 5*a*c**4*e*x**21/21 + c**5*d*
x**23/23 + c**5*e*x**25/25
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.84

$$\int x^2(d+ex^2)\left(a+cx^4\right)^5\,dx = \frac{1}{25}\,c^5ex^{25} + \frac{1}{23}\,c^5dx^{23} + \frac{5}{21}\,ac^4ex^{21} + \frac{5}{19}\,ac^4dx^{19} + \frac{10}{17}\,a^2c^3ex^{17} + \frac{2}{3}\,a^2c^3dx^{15} + \frac{10}{13}\,a^3c^2ex^{13} + \frac{10}{11}\,a^3c^2dx^{11} + \frac{5}{9}\,a^4cex^9 + \frac{5}{7}\,a^4cdx^7 + \frac{1}{5}\,a^5ex^5 + \frac{1}{3}\,a^5dx^3$$

```
[In] integrate(x^2*(e*x^2+d)*(c*x^4+a)^5,x, algorithm="maxima")
```

```
[Out] 1/25*c^5*e*x^25 + 1/23*c^5*d*x^23 + 5/21*a*c^4*e*x^21 + 5/19*a*c^4*d*x^19 +
 10/17*a^2*c^3*e*x^17 + 2/3*a^2*c^3*d*x^15 + 10/13*a^3*c^2*e*x^13 + 10/11*a
^3*c^2*d*x^11 + 5/9*a^4*c*e*x^9 + 5/7*a^4*c*d*x^7 + 1/5*a^5*e*x^5 + 1/3*a^5
*d*x^3
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.84

$$\int x^2(d+ex^2)\left(a+cx^4\right)^5\,dx = \frac{1}{25}\,c^5ex^{25} + \frac{1}{23}\,c^5dx^{23} + \frac{5}{21}\,ac^4ex^{21} + \frac{5}{19}\,ac^4dx^{19} + \frac{10}{17}\,a^2c^3ex^{17} + \frac{2}{3}\,a^2c^3dx^{15} + \frac{10}{13}\,a^3c^2ex^{13} + \frac{10}{11}\,a^3c^2dx^{11} + \frac{5}{9}\,a^4cex^9 + \frac{5}{7}\,a^4cdx^7 + \frac{1}{5}\,a^5ex^5 + \frac{1}{3}\,a^5dx^3$$

```
[In] integrate(x^2*(e*x^2+d)*(c*x^4+a)^5,x, algorithm="giac")
```

```
[Out] 1/25*c^5*e*x^25 + 1/23*c^5*d*x^23 + 5/21*a*c^4*e*x^21 + 5/19*a*c^4*d*x^19 +
 10/17*a^2*c^3*e*x^17 + 2/3*a^2*c^3*d*x^15 + 10/13*a^3*c^2*e*x^13 + 10/11*a
^3*c^2*d*x^11 + 5/9*a^4*c*e*x^9 + 5/7*a^4*c*d*x^7 + 1/5*a^5*e*x^5 + 1/3*a^5
*d*x^3
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.84

$$\int x^2(d+ex^2)\left(a+cx^4\right)^5\,dx=\frac{e\,a^5\,x^5}{5}+\frac{d\,a^5\,x^3}{3}+\frac{5\,e\,a^4\,c\,x^9}{9}+\frac{5\,d\,a^4\,c\,x^7}{7}+\frac{10\,e\,a^3\,c^2\,x^{13}}{13}+\frac{10\,d\,a^3\,c^2\,x^{11}}{11}+\frac{10\,e\,a^2\,c^3\,x^{17}}{17}+\frac{2\,d\,a^2\,c^3\,x^{15}}{3}+\frac{5\,e\,a\,c^4\,x^{21}}{21}+\frac{5\,d\,a\,c^4\,x^{19}}{19}+\frac{e\,c^5\,x^{25}}{25}+\frac{d\,c^5\,x^{23}}{23}$$

```
[In] int(x^2*(a + c*x^4)^5*(d + e*x^2),x)
```

```
[Out] (a^5*d*x^3)/3 + (a^5*e*x^5)/5 + (c^5*d*x^23)/23 + (c^5*e*x^25)/25 + (10*a^3
*c^2*d*x^11)/11 + (2*a^2*c^3*d*x^15)/3 + (10*a^3*c^2*e*x^13)/13 + (10*a^2*c
^3*e*x^17)/17 + (5*a^4*c*d*x^7)/7 + (5*a*c^4*d*x^19)/19 + (5*a^4*c*e*x^9)/9
 + (5*a*c^4*e*x^21)/21
```

3.3 $\int x(d+ex^2)\left(a+cx^4\right)^5\,dx$

Optimal result

Integrand size = 18, antiderivative size = 89

$$\int x(d+ex^2)\left(a+cx^4\right)^5\,dx=\frac{1}{2}a^5dx^2+\frac{5}{6}a^4cdx^6+a^3c^2dx^{10}+\frac{5}{7}a^2c^3dx^{14}+\frac{5}{18}ac^4dx^{18}+\frac{1}{22}c^5dx^{22}+\frac{e(a+cx^4)^6}{24c}$$

```
[Out] 1/2*a^5*d*x^2+5/6*a^4*c*d*x^6+a^3*c^2*d*x^10+5/7*a^2*c^3*d*x^14+5/18*a*c^4*
d*x^18+1/22*c^5*d*x^22+1/24*e*(c*x^4+a)^6/c
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1262, 655, 200}

$$\int x(d+ex^2)\left(a+cx^4\right)^5\,dx=\frac{1}{2}a^5dx^2+\frac{5}{6}a^4cdx^6+a^3c^2dx^{10}+\frac{5}{7}a^2c^3dx^{14}+\frac{5}{18}ac^4dx^{18}+\frac{e(a+cx^4)^6}{24c}+\frac{1}{22}c^5dx^{22}$$

```
[In] Int[x*(d + e*x^2)*(a + c*x^4)^5,x]
```

```
[Out] (a^5*d*x^2)/2 + (5*a^4*c*d*x^6)/6 + a^3*c^2*d*x^10 + (5*a^2*c^3*d*x^14)/7 +
 (5*a*c^4*d*x^18)/18 + (c^5*d*x^22)/22 + (e*(a + c*x^4)^6)/(24*c)
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 655

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[e*((
a + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 1262

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol]
 :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + c*x^2)^p, x], x, x^2], x] /; FreeQ
[{a, c, d, e, p, q}, x]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{1}{2}\text{Subst}\left(\int (d+ex)\left(a+cx^2\right)^5\,dx,x,x^2\right)\\
&= \frac{e(a+cx^4)^6}{24c}+\frac{1}{2}d\text{Subst}\left(\int \left(a+cx^2\right)^5\,dx,x,x^2\right)\\
&= \frac{e(a+cx^4)^6}{24c}+\frac{1}{2}d\text{Subst}\left(\int \left(a^5+5a^4cx^2+10a^3c^2x^4+10a^2c^3x^6+5ac^4x^8+c^5x^{10}\right)\,dx,x,x^2\right)\\
&= \frac{1}{2}a^5dx^2+\frac{5}{6}a^4cdx^6+a^3c^2dx^{10}+\frac{5}{7}a^2c^3dx^{14}+\frac{5}{18}ac^4dx^{18}+\frac{1}{22}c^5dx^{22}+\frac{e(a+cx^4)^6}{24c}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.64

$$
\begin{aligned}
\int x\left(d+ex^2\right)\left(a+cx^4\right)^5\,dx &= \frac{1}{2}a^5dx^2+\frac{1}{4}a^5ex^4+\frac{5}{6}a^4cdx^6+\frac{5}{8}a^4cex^8\\
&\quad+a^3c^2dx^{10}+\frac{5}{6}a^3c^2ex^{12}+\frac{5}{7}a^2c^3dx^{14}+\frac{5}{8}a^2c^3ex^{16}\\
&\quad+\frac{5}{18}ac^4dx^{18}+\frac{1}{4}ac^4ex^{20}+\frac{1}{22}c^5dx^{22}+\frac{1}{24}c^5ex^{24}
\end{aligned}
$$

```
[In] Integrate[x*(d + e*x^2)*(a + c*x^4)^5,x]
```

```
[Out] (a^5*d*x^2)/2 + (a^5*e*x^4)/4 + (5*a^4*c*d*x^6)/6 + (5*a^4*c*e*x^8)/8 + a^3
*c^2*d*x^10 + (5*a^3*c^2*e*x^12)/6 + (5*a^2*c^3*d*x^14)/7 + (5*a^2*c^3*e*x^
16)/8 + (5*a*c^4*d*x^18)/18 + (a*c^4*e*x^20)/4 + (c^5*d*x^22)/22 + (c^5*e*x
^24)/24
```

Maple [A] (verified)

Time = 0.28 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.40

method	result
gosper	$\frac{5}{8}e\,a^2c^3x^{16}+\frac{5}{18}a\,c^4d\,x^{18}+\frac{1}{4}ea\,c^4x^{20}+\frac{1}{22}c^5d\,x^{22}+\frac{1}{24}e\,c^5x^{24}+\frac{1}{2}a^5d\,x^2+\frac{1}{4}e\,a^5x^4+\frac{5}{6}a^4cd\,x^6+$
default	$\frac{5}{8}e\,a^2c^3x^{16}+\frac{5}{18}a\,c^4d\,x^{18}+\frac{1}{4}ea\,c^4x^{20}+\frac{1}{22}c^5d\,x^{22}+\frac{1}{24}e\,c^5x^{24}+\frac{1}{2}a^5d\,x^2+\frac{1}{4}e\,a^5x^4+\frac{5}{6}a^4cd\,x^6+$
norman	$\frac{5}{8}e\,a^2c^3x^{16}+\frac{5}{18}a\,c^4d\,x^{18}+\frac{1}{4}ea\,c^4x^{20}+\frac{1}{22}c^5d\,x^{22}+\frac{1}{24}e\,c^5x^{24}+\frac{1}{2}a^5d\,x^2+\frac{1}{4}e\,a^5x^4+\frac{5}{6}a^4cd\,x^6+$
risch	$\frac{5}{8}e\,a^2c^3x^{16}+\frac{5}{18}a\,c^4d\,x^{18}+\frac{1}{4}ea\,c^4x^{20}+\frac{1}{22}c^5d\,x^{22}+\frac{1}{24}e\,c^5x^{24}+\frac{1}{2}a^5d\,x^2+\frac{1}{4}e\,a^5x^4+\frac{5}{6}a^4cd\,x^6+$
parallelrisch	$\frac{5}{8}e\,a^2c^3x^{16}+\frac{5}{18}a\,c^4d\,x^{18}+\frac{1}{4}ea\,c^4x^{20}+\frac{1}{22}c^5d\,x^{22}+\frac{1}{24}e\,c^5x^{24}+\frac{1}{2}a^5d\,x^2+\frac{1}{4}e\,a^5x^4+\frac{5}{6}a^4cd\,x^6+$

```
[In] int(x*(e*x^2+d)*(c*x^4+a)^5,x,method=_RETURNVERBOSE)
```

```
[Out] 5/8*e*a^2*c^3*x^16+5/18*a*c^4*d*x^18+1/4*e*a*c^4*x^20+1/22*c^5*d*x^22+1/24*
e*c^5*x^24+1/2*a^5*d*x^2+1/4*e*a^5*x^4+5/6*a^4*c*d*x^6+5/8*e*c*a^4*x^8+a^3*
c^2*d*x^10+5/6*e*a^3*c^2*x^12+5/7*a^2*c^3*d*x^14
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.39

$$\int x(d+ex^2)\left(a+cx^4\right)^5\,dx=\frac{1}{24}\,c^5ex^{24}+\frac{1}{22}\,c^5dx^{22}+\frac{1}{4}\,ac^4ex^{20}+\frac{5}{18}\,ac^4dx^{18}+\frac{5}{8}\,a^2c^3ex^{16}+\frac{5}{7}\,a^2c^3dx^{14}+\frac{5}{6}\,a^3c^2ex^{12}+a^3c^2dx^{10}+\frac{5}{8}\,a^4cex^8+\frac{5}{6}\,a^4cdx^6+\frac{1}{4}\,a^5ex^4+\frac{1}{2}\,a^5dx^2$$

```
[In] integrate(x*(e*x^2+d)*(c*x^4+a)^5,x, algorithm="fricas")
```

```
[Out] 1/24*c^5*e*x^24 + 1/22*c^5*d*x^22 + 1/4*a*c^4*e*x^20 + 5/18*a*c^4*d*x^18 +
5/8*a^2*c^3*e*x^16 + 5/7*a^2*c^3*d*x^14 + 5/6*a^3*c^2*e*x^12 + a^3*c^2*d*x^
10 + 5/8*a^4*c*e*x^8 + 5/6*a^4*c*d*x^6 + 1/4*a^5*e*x^4 + 1/2*a^5*d*x^2
```

Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.69

$$\int x(d+ex^2)\left(a+cx^4\right)^5\,dx=\frac{a^5dx^2}{2}+\frac{a^5ex^4}{4}+\frac{5a^4cdx^6}{6}+\frac{5a^4cex^8}{8}+a^3c^2dx^{10}+\frac{5a^3c^2ex^{12}}{6}+\frac{5a^2c^3dx^{14}}{7}+\frac{5a^2c^3ex^{16}}{8}+\frac{5ac^4dx^{18}}{18}+\frac{ac^4ex^{20}}{4}+\frac{c^5dx^{22}}{22}+\frac{c^5ex^{24}}{24}$$

```
[In] integrate(x*(e*x**2+d)*(c*x**4+a)**5,x)
```

```
[Out] a**5*d*x**2/2 + a**5*e*x**4/4 + 5*a**4*c*d*x**6/6 + 5*a**4*c*e*x**8/8 + a**
3*c**2*d*x**10 + 5*a**3*c**2*e*x**12/6 + 5*a**2*c**3*d*x**14/7 + 5*a**2*c**
3*e*x**16/8 + 5*a*c**4*d*x**18/18 + a*c**4*e*x**20/4 + c**5*d*x**22/22 + c*
*5*e*x**24/24
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.39

$$\int x(d+ex^2)\left(a+cx^4\right)^5\,dx=\frac{1}{24}\,c^5ex^{24}+\frac{1}{22}\,c^5dx^{22}+\frac{1}{4}\,ac^4ex^{20}+\frac{5}{18}\,ac^4dx^{18}+\frac{5}{8}\,a^2c^3ex^{16}+\frac{5}{7}\,a^2c^3dx^{14}+\frac{5}{6}\,a^3c^2ex^{12}+a^3c^2dx^{10}+\frac{5}{8}\,a^4cex^8+\frac{5}{6}\,a^4cdx^6+\frac{1}{4}\,a^5ex^4+\frac{1}{2}\,a^5dx^2$$

```
[In] integrate(x*(e*x^2+d)*(c*x^4+a)^5,x, algorithm="maxima")
```

```
[Out] 1/24*c^5*e*x^24 + 1/22*c^5*d*x^22 + 1/4*a*c^4*e*x^20 + 5/18*a*c^4*d*x^18 +
5/8*a^2*c^3*e*x^16 + 5/7*a^2*c^3*d*x^14 + 5/6*a^3*c^2*e*x^12 + a^3*c^2*d*x^
10 + 5/8*a^4*c*e*x^8 + 5/6*a^4*c*d*x^6 + 1/4*a^5*e*x^4 + 1/2*a^5*d*x^2
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.39

$$\int x(d+ex^2)\left(a+cx^4\right)^5\,dx=\frac{1}{24}\,c^5ex^{24}+\frac{1}{22}\,c^5dx^{22}+\frac{1}{4}\,ac^4ex^{20}+\frac{5}{18}\,ac^4dx^{18}+\frac{5}{8}\,a^2c^3ex^{16}+\frac{5}{7}\,a^2c^3dx^{14}+\frac{5}{6}\,a^3c^2ex^{12}+a^3c^2dx^{10}+\frac{5}{8}\,a^4cex^8+\frac{5}{6}\,a^4cdx^6+\frac{1}{4}\,a^5ex^4+\frac{1}{2}\,a^5dx^2$$

```
[In] integrate(x*(e*x^2+d)*(c*x^4+a)^5,x, algorithm="giac")
```

```
[Out] 1/24*c^5*e*x^24 + 1/22*c^5*d*x^22 + 1/4*a*c^4*e*x^20 + 5/18*a*c^4*d*x^18 +
5/8*a^2*c^3*e*x^16 + 5/7*a^2*c^3*d*x^14 + 5/6*a^3*c^2*e*x^12 + a^3*c^2*d*x^
10 + 5/8*a^4*c*e*x^8 + 5/6*a^4*c*d*x^6 + 1/4*a^5*e*x^4 + 1/2*a^5*d*x^2
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.39

$$\int x(d+ex^2)\left(a+cx^4\right)^5\,dx=\frac{e\,a^5\,x^4}{4}+\frac{d\,a^5\,x^2}{2}+\frac{5\,e\,a^4\,c\,x^8}{8}+\frac{5\,d\,a^4\,c\,x^6}{6}+\frac{5\,e\,a^3\,c^2\,x^{12}}{6}+d\,a^3\,c^2\,x^{10}+\frac{5\,e\,a^2\,c^3\,x^{16}}{8}+\frac{5\,d\,a^2\,c^3\,x^{14}}{7}+\frac{e\,a\,c^4\,x^{20}}{4}+\frac{5\,d\,a\,c^4\,x^{18}}{18}+\frac{e\,c^5\,x^{24}}{24}+\frac{d\,c^5\,x^{22}}{22}$$

```
[In] int(x*(a + c*x^4)^5*(d + e*x^2),x)
```

```
[Out] (a^5*d*x^2)/2 + (a^5*e*x^4)/4 + (c^5*d*x^22)/22 + (c^5*e*x^24)/24 + a^3*c^2
*d*x^10 + (5*a^2*c^3*d*x^14)/7 + (5*a^3*c^2*e*x^12)/6 + (5*a^2*c^3*e*x^16)/
8 + (5*a^4*c*d*x^6)/6 + (5*a*c^4*d*x^18)/18 + (5*a^4*c*e*x^8)/8 + (a*c^4*e*
x^20)/4
```

3.4 $\int (d+ex^2)(a+cx^4)^5\,dx$

Optimal result

Integrand size = 17, antiderivative size = 141

$$\int (d+ex^2)(a+cx^4)^5\,dx = a^5dx+\frac{1}{3}a^5ex^3+a^4cdx^5+\frac{5}{7}a^4cex^7+\frac{10}{9}a^3c^2dx^9+\frac{10}{11}a^3c^2ex^{11}+\frac{10}{13}a^2c^3dx^{13}+\frac{2}{3}a^2c^3ex^{15}+\frac{5}{17}ac^4dx^{17}+\frac{5}{19}ac^4ex^{19}+\frac{1}{21}c^5dx^{21}+\frac{1}{23}c^5ex^{23}$$

```
[Out] a^5*d*x+1/3*a^5*e*x^3+a^4*c*d*x^5+5/7*a^4*c*e*x^7+10/9*a^3*c^2*d*x^9+10/11*
a^3*c^2*e*x^11+10/13*a^2*c^3*d*x^13+2/3*a^2*c^3*e*x^15+5/17*a*c^4*d*x^17+5/
19*a*c^4*e*x^19+1/21*c^5*d*x^21+1/23*c^5*e*x^23
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {1168}

$$\int (d+ex^2)(a+cx^4)^5\,dx = a^5dx+\frac{1}{3}a^5ex^3+a^4cdx^5+\frac{5}{7}a^4cex^7+\frac{10}{9}a^3c^2dx^9+\frac{10}{11}a^3c^2ex^{11}+\frac{10}{13}a^2c^3dx^{13}+\frac{2}{3}a^2c^3ex^{15}+\frac{5}{17}ac^4dx^{17}+\frac{5}{19}ac^4ex^{19}+\frac{1}{21}c^5dx^{21}+\frac{1}{23}c^5ex^{23}$$

```
[In] Int[(d + e*x^2)*(a + c*x^4)^5,x]
```

```
[Out] a^5*d*x + (a^5*e*x^3)/3 + a^4*c*d*x^5 + (5*a^4*c*e*x^7)/7 + (10*a^3*c^2*d*x
^9)/9 + (10*a^3*c^2*e*x^11)/11 + (10*a^2*c^3*d*x^13)/13 + (2*a^2*c^3*e*x^15
```

```
)/3 + (5*a*c^4*d*x^17)/17 + (5*a*c^4*e*x^19)/19 + (c^5*d*x^21)/21 + (c^5*e*
x^23)/23
```

Rule 1168

```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol] :> I
nt[ExpandIntegrand[(d + e*x^2)^q*(a + c*x^4)^p, x], x] /; FreeQ[{a, c, d, e
}, x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \left(a^5d + a^5ex^2 + 5a^4cdx^4 + 5a^4cex^6 + 10a^3c^2dx^8 + 10a^3c^2ex^{10} + 10a^2c^3dx^{12} + 10a^2c^3ex^{14} + 5ac^4dx^{16} + 5ac^4ex^{18} + c^5dx^{20} + c^5ex^{22}\right) dx \\ &= a^5dx + \frac{1}{3}a^5ex^3 + a^4cdx^5 + \frac{5}{7}a^4cex^7 + \frac{10}{9}a^3c^2dx^9 + \frac{10}{11}a^3c^2ex^{11} + \frac{10}{13}a^2c^3dx^{13} + \frac{2}{3}a^2c^3ex^{15} + \frac{5}{17}ac^4dx^{17} + \frac{5}{19}ac^4ex^{19} + \frac{1}{21}c^5dx^{21} + \frac{1}{23}c^5ex^{23}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.00

$$\int \left(d+ex^2\right)\left(a+cx^4\right)^5 dx = a^5dx + \frac{1}{3}a^5ex^3 + a^4cdx^5 + \frac{5}{7}a^4cex^7 + \frac{10}{9}a^3c^2dx^9 + \frac{10}{11}a^3c^2ex^{11} + \frac{10}{13}a^2c^3dx^{13} + \frac{2}{3}a^2c^3ex^{15} + \frac{5}{17}ac^4dx^{17} + \frac{5}{19}ac^4ex^{19} + \frac{1}{21}c^5dx^{21} + \frac{1}{23}c^5ex^{23}$$

```
[In] Integrate[(d + e*x^2)*(a + c*x^4)^5,x]
```

```
[Out] a^5*d*x + (a^5*e*x^3)/3 + a^4*c*d*x^5 + (5*a^4*c*e*x^7)/7 + (10*a^3*c^2*d*x
^9)/9 + (10*a^3*c^2*e*x^11)/11 + (10*a^2*c^3*d*x^13)/13 + (2*a^2*c^3*e*x^15
)/3 + (5*a*c^4*d*x^17)/17 + (5*a*c^4*e*x^19)/19 + (c^5*d*x^21)/21 + (c^5*e*
x^23)/23
```

Maple [A] (verified)

Time = 0.31 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.87

method	result
gosper	$a^5dx+\frac{1}{3}a^5e\,x^3+a^4cd\,x^5+\frac{5}{7}a^4ce\,x^7+\frac{10}{9}a^3c^2d\,x^9+\frac{10}{11}a^3c^2e\,x^{11}+\frac{10}{13}a^2c^3d\,x^{13}+\frac{2}{3}a^2c^3e\,x^{15}$
default	$a^5dx+\frac{1}{3}a^5e\,x^3+a^4cd\,x^5+\frac{5}{7}a^4ce\,x^7+\frac{10}{9}a^3c^2d\,x^9+\frac{10}{11}a^3c^2e\,x^{11}+\frac{10}{13}a^2c^3d\,x^{13}+\frac{2}{3}a^2c^3e\,x^{15}$
norman	$a^5dx+\frac{1}{3}a^5e\,x^3+a^4cd\,x^5+\frac{5}{7}a^4ce\,x^7+\frac{10}{9}a^3c^2d\,x^9+\frac{10}{11}a^3c^2e\,x^{11}+\frac{10}{13}a^2c^3d\,x^{13}+\frac{2}{3}a^2c^3e\,x^{15}$
risch	$a^5dx+\frac{1}{3}a^5e\,x^3+a^4cd\,x^5+\frac{5}{7}a^4ce\,x^7+\frac{10}{9}a^3c^2d\,x^9+\frac{10}{11}a^3c^2e\,x^{11}+\frac{10}{13}a^2c^3d\,x^{13}+\frac{2}{3}a^2c^3e\,x^{15}$
parallelrisch	$a^5dx+\frac{1}{3}a^5e\,x^3+a^4cd\,x^5+\frac{5}{7}a^4ce\,x^7+\frac{10}{9}a^3c^2d\,x^9+\frac{10}{11}a^3c^2e\,x^{11}+\frac{10}{13}a^2c^3d\,x^{13}+\frac{2}{3}a^2c^3e\,x^{15}$

```
[In] int((e*x^2+d)*(c*x^4+a)^5,x,method=_RETURNVERBOSE)
```

```
[Out] a^5*d*x+1/3*a^5*e*x^3+a^4*c*d*x^5+5/7*a^4*c*e*x^7+10/9*a^3*c^2*d*x^9+10/11*
a^3*c^2*e*x^11+10/13*a^2*c^3*d*x^13+2/3*a^2*c^3*e*x^15+5/17*a*c^4*d*x^17+5/
19*a*c^4*e*x^19+1/21*c^5*d*x^21+1/23*c^5*e*x^23
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.86

$$\int \left(d+ex^2\right)\left(a+cx^4\right)^5\,dx=\frac{1}{23}\,c^5ex^{23}+\frac{1}{21}\,c^5dx^{21}+\frac{5}{19}\,ac^4ex^{19}+\frac{5}{17}\,ac^4dx^{17}+\frac{2}{3}\,a^2c^3ex^{15}+\frac{10}{13}\,a^2c^3dx^{13}+\frac{10}{11}\,a^3c^2ex^{11}+\frac{10}{9}\,a^3c^2dx^9+\frac{5}{7}\,a^4cex^7+a^4cdx^5+\frac{1}{3}\,a^5ex^3+a^5dx$$

```
[In] integrate((e*x^2+d)*(c*x^4+a)^5,x, algorithm="fricas")
```

```
[Out] 1/23*c^5*e*x^23 + 1/21*c^5*d*x^21 + 5/19*a*c^4*e*x^19 + 5/17*a*c^4*d*x^17 +
 2/3*a^2*c^3*e*x^15 + 10/13*a^2*c^3*d*x^13 + 10/11*a^3*c^2*e*x^11 + 10/9*a^
3*c^2*d*x^9 + 5/7*a^4*c*e*x^7 + a^4*c*d*x^5 + 1/3*a^5*e*x^3 + a^5*d*x
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.05

$$\int \left(d+ex^2\right)\left(a+cx^4\right)^5\,dx=a^5dx+\frac{a^5ex^3}{3}+a^4cdx^5+\frac{5a^4cex^7}{7}+\frac{10a^3c^2dx^9}{9}+\frac{10a^3c^2ex^{11}}{11}+\frac{10a^2c^3dx^{13}}{13}+\frac{2a^2c^3ex^{15}}{3}+\frac{5ac^4dx^{17}}{17}+\frac{5ac^4ex^{19}}{19}+\frac{c^5dx^{21}}{21}+\frac{c^5ex^{23}}{23}$$

```
[In] integrate((e*x**2+d)*(c*x**4+a)**5,x)
```

```
[Out] a**5*d*x + a**5*e*x**3/3 + a**4*c*d*x**5 + 5*a**4*c*e*x**7/7 + 10*a**3*c**2
*d*x**9/9 + 10*a**3*c**2*e*x**11/11 + 10*a**2*c**3*d*x**13/13 + 2*a**2*c**3
*e*x**15/3 + 5*a*c**4*d*x**17/17 + 5*a*c**4*e*x**19/19 + c**5*d*x**21/21 +
c**5*e*x**23/23
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.86

$$\int \left(d+ex^2\right)\left(a+cx^4\right)^5\,dx = \frac{1}{23}\,c^5ex^{23} + \frac{1}{21}\,c^5dx^{21} + \frac{5}{19}\,ac^4ex^{19} + \frac{5}{17}\,ac^4dx^{17} + \frac{2}{3}\,a^2c^3ex^{15} + \frac{10}{13}\,a^2c^3dx^{13} + \frac{10}{11}\,a^3c^2ex^{11} + \frac{10}{9}\,a^3c^2dx^9 + \frac{5}{7}\,a^4cex^7 + a^4cdx^5 + \frac{1}{3}\,a^5ex^3 + a^5dx$$

```
[In] integrate((e*x^2+d)*(c*x^4+a)^5,x, algorithm="maxima")
```

```
[Out] 1/23*c^5*e*x^23 + 1/21*c^5*d*x^21 + 5/19*a*c^4*e*x^19 + 5/17*a*c^4*d*x^17 +
 2/3*a^2*c^3*e*x^15 + 10/13*a^2*c^3*d*x^13 + 10/11*a^3*c^2*e*x^11 + 10/9*a^
3*c^2*d*x^9 + 5/7*a^4*c*e*x^7 + a^4*c*d*x^5 + 1/3*a^5*e*x^3 + a^5*d*x
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.86

$$\int \left(d+ex^2\right)\left(a+cx^4\right)^5\,dx = \frac{1}{23}\,c^5ex^{23} + \frac{1}{21}\,c^5dx^{21} + \frac{5}{19}\,ac^4ex^{19} + \frac{5}{17}\,ac^4dx^{17} + \frac{2}{3}\,a^2c^3ex^{15} + \frac{10}{13}\,a^2c^3dx^{13} + \frac{10}{11}\,a^3c^2ex^{11} + \frac{10}{9}\,a^3c^2dx^9 + \frac{5}{7}\,a^4cex^7 + a^4cdx^5 + \frac{1}{3}\,a^5ex^3 + a^5dx$$

```
[In] integrate((e*x^2+d)*(c*x^4+a)^5,x, algorithm="giac")
```

```
[Out] 1/23*c^5*e*x^23 + 1/21*c^5*d*x^21 + 5/19*a*c^4*e*x^19 + 5/17*a*c^4*d*x^17 +
 2/3*a^2*c^3*e*x^15 + 10/13*a^2*c^3*d*x^13 + 10/11*a^3*c^2*e*x^11 + 10/9*a^
3*c^2*d*x^9 + 5/7*a^4*c*e*x^7 + a^4*c*d*x^5 + 1/3*a^5*e*x^3 + a^5*d*x
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.86

$$\int (d+ex^2) (a+cx^4)^5 \, dx = \frac{e\,a^5\,x^3}{3} + d\,a^5\,x + \frac{5\,e\,a^4\,c\,x^7}{7} + d\,a^4\,c\,x^5 + \frac{10\,e\,a^3\,c^2\,x^{11}}{11} + \frac{10\,d\,a^3\,c^2\,x^9}{9} + \frac{2\,e\,a^2\,c^3\,x^{15}}{3} + \frac{10\,d\,a^2\,c^3\,x^{13}}{13} + \frac{5\,e\,a\,c^4\,x^{19}}{19} + \frac{5\,d\,a\,c^4\,x^{17}}{17} + \frac{e\,c^5\,x^{23}}{23} + \frac{d\,c^5\,x^{21}}{21}$$

```
[In] int((a + c*x^4)^5*(d + e*x^2),x)
```

```
[Out] (a^5*e*x^3)/3 + (c^5*d*x^21)/21 + (c^5*e*x^23)/23 + a^5*d*x + (10*a^3*c^2*d
*x^9)/9 + (10*a^2*c^3*d*x^13)/13 + (10*a^3*c^2*e*x^11)/11 + (2*a^2*c^3*e*x^
15)/3 + a^4*c*d*x^5 + (5*a*c^4*d*x^17)/17 + (5*a^4*c*e*x^7)/7 + (5*a*c^4*e*
x^19)/19
```

3.5 $\int \frac{(d+ex^2)(a+cx^4)^5}{x}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 142

$$\int \frac{(d+ex^2)(a+cx^4)^5}{x}\,dx = \frac{1}{2}a^5ex^2 + \frac{5}{4}a^4cdx^4 + \frac{5}{6}a^4cex^6 + \frac{5}{4}a^3c^2dx^8 + a^3c^2ex^{10} + \frac{5}{6}a^2c^3dx^{12} + \frac{5}{7}a^2c^3ex^{14} + \frac{5}{16}ac^4dx^{16} + \frac{5}{18}ac^4ex^{18} + \frac{1}{20}c^5dx^{20} + \frac{1}{22}c^5ex^{22} + a^5d\log(x)$$

```
[Out] 1/2*a^5*e*x^2+5/4*a^4*c*d*x^4+5/6*a^4*c*e*x^6+5/4*a^3*c^2*d*x^8+a^3*c^2*e*x
^10+5/6*a^2*c^3*d*x^12+5/7*a^2*c^3*e*x^14+5/16*a*c^4*d*x^16+5/18*a*c^4*e*x^
18+1/20*c^5*d*x^20+1/22*c^5*e*x^22+a^5*d*ln(x)
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {1266, 780}

$$\int \frac{(d+ex^2)(a+cx^4)^5}{x}\,dx = a^5d\log(x) + \frac{1}{2}a^5ex^2 + \frac{5}{4}a^4cdx^4 + \frac{5}{6}a^4cex^6 + \frac{5}{4}a^3c^2dx^8 + a^3c^2ex^{10} + \frac{5}{6}a^2c^3dx^{12} + \frac{5}{7}a^2c^3ex^{14} + \frac{5}{16}ac^4dx^{16} + \frac{5}{18}ac^4ex^{18} + \frac{1}{20}c^5dx^{20} + \frac{1}{22}c^5ex^{22}$$

```
[In] Int[((d + e*x^2)*(a + c*x^4)^5)/x,x]
```

```
[Out] (a^5*e*x^2)/2 + (5*a^4*c*d*x^4)/4 + (5*a^4*c*e*x^6)/6 + (5*a^3*c^2*d*x^8)/4
 + a^3*c^2*e*x^10 + (5*a^2*c^3*d*x^12)/6 + (5*a^2*c^3*e*x^14)/7 + (5*a*c^4*
```

```
d*x^16)/16 + (5*a*c^4*e*x^18)/18 + (c^5*d*x^20)/20 + (c^5*e*x^22)/22 + a^5*
d*Log[x]
```

Rule 780

```
Int[((e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(e*x)^m*(f + g*x)*(a + c*x^2)^p, x], x] /; F
reeQ[{a, c, e, f, g, m}, x] && IGtQ[p, 0]
```

Rule 1266

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_S
ymbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + c*x^2)^p, x],
x, x^2], x] /; FreeQ[{a, c, d, e, p, q}, x] && IntegerQ[(m + 1)/2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{2}\text{Subst}\left(\int \frac{(d+ex)\left(a+cx^2\right)^5}{x}\,dx, x, x^2\right)\\
&= \frac{1}{2}\text{Subst}\left(\int \left(a^5e+\frac{a^5d}{x}+5a^4cdx+5a^4cex^2+10a^3c^2dx^3+10a^3c^2ex^4+10a^2c^3dx^5\right.\right.\\
&\qquad\left.\left.+10a^2c^3ex^6+5ac^4dx^7+5ac^4ex^8+c^5dx^9+c^5ex^{10}\right)\,dx, x, x^2\right)\\
&= \frac{1}{2}a^5ex^2+\frac{5}{4}a^4cdx^4+\frac{5}{6}a^4cex^6+\frac{5}{4}a^3c^2dx^8+a^3c^2ex^{10}+\frac{5}{6}a^2c^3dx^{12}\\
&\quad+\frac{5}{7}a^2c^3ex^{14}+\frac{5}{16}ac^4dx^{16}+\frac{5}{18}ac^4ex^{18}+\frac{1}{20}c^5dx^{20}+\frac{1}{22}c^5ex^{22}+a^5d\log(x)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.00

$$\begin{aligned}
\int \frac{(d+ex^2)\left(a+cx^4\right)^5}{x}\,dx &= \frac{1}{2}a^5ex^2+\frac{5}{4}a^4cdx^4+\frac{5}{6}a^4cex^6+\frac{5}{4}a^3c^2dx^8\\
&\quad+a^3c^2ex^{10}+\frac{5}{6}a^2c^3dx^{12}+\frac{5}{7}a^2c^3ex^{14}+\frac{5}{16}ac^4dx^{16}\\
&\quad+\frac{5}{18}ac^4ex^{18}+\frac{1}{20}c^5dx^{20}+\frac{1}{22}c^5ex^{22}+a^5d\log(x)
\end{aligned}$$

```
[In] Integrate[((d + e*x^2)*(a + c*x^4)^5)/x,x]
```

```
[Out] (a^5*e*x^2)/2 + (5*a^4*c*d*x^4)/4 + (5*a^4*c*e*x^6)/6 + (5*a^3*c^2*d*x^8)/4
 + a^3*c^2*e*x^10 + (5*a^2*c^3*d*x^12)/6 + (5*a^2*c^3*e*x^14)/7 + (5*a*c^4*
d*x^16)/16 + (5*a*c^4*e*x^18)/18 + (c^5*d*x^20)/20 + (c^5*e*x^22)/22 + a^5*
d*Log[x]
```

Maple [A] (verified)

Time = 0.25 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.87

method	result
default	$\frac{a^{5} e \,x^{2}}{2}+\frac{5 a^{4} c d \,x^{4}}{4}+\frac{5 a^{4} c e \,x^{6}}{6}+\frac{5 a^{3} c^{2} d \,x^{8}}{4}+a^{3} c^{2} e \,x^{10}+\frac{5 a^{2} c^{3} d \,x^{12}}{6}+\frac{5 a^{2} c^{3} e \,x^{14}}{7}+\frac{5 a \,c^{4} d \,x^{16}}{16}+\frac{5 a \,c^{4} e \,x^{18}}{18}+$
norman	$\frac{a^{5} e \,x^{2}}{2}+\frac{5 a^{4} c d \,x^{4}}{4}+\frac{5 a^{4} c e \,x^{6}}{6}+\frac{5 a^{3} c^{2} d \,x^{8}}{4}+a^{3} c^{2} e \,x^{10}+\frac{5 a^{2} c^{3} d \,x^{12}}{6}+\frac{5 a^{2} c^{3} e \,x^{14}}{7}+\frac{5 a \,c^{4} d \,x^{16}}{16}+\frac{5 a \,c^{4} e \,x^{18}}{18}+$
risch	$\frac{a^{5} e \,x^{2}}{2}+\frac{5 a^{4} c d \,x^{4}}{4}+\frac{5 a^{4} c e \,x^{6}}{6}+\frac{5 a^{3} c^{2} d \,x^{8}}{4}+a^{3} c^{2} e \,x^{10}+\frac{5 a^{2} c^{3} d \,x^{12}}{6}+\frac{5 a^{2} c^{3} e \,x^{14}}{7}+\frac{5 a \,c^{4} d \,x^{16}}{16}+\frac{5 a \,c^{4} e \,x^{18}}{18}+$
parallelrisch	$\frac{a^{5} e \,x^{2}}{2}+\frac{5 a^{4} c d \,x^{4}}{4}+\frac{5 a^{4} c e \,x^{6}}{6}+\frac{5 a^{3} c^{2} d \,x^{8}}{4}+a^{3} c^{2} e \,x^{10}+\frac{5 a^{2} c^{3} d \,x^{12}}{6}+\frac{5 a^{2} c^{3} e \,x^{14}}{7}+\frac{5 a \,c^{4} d \,x^{16}}{16}+\frac{5 a \,c^{4} e \,x^{18}}{18}+$

```
[In] int((e*x^2+d)*(c*x^4+a)^5/x,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*a^5*e*x^2+5/4*a^4*c*d*x^4+5/6*a^4*c*e*x^6+5/4*a^3*c^2*d*x^8+a^3*c^2*e*x
^10+5/6*a^2*c^3*d*x^12+5/7*a^2*c^3*e*x^14+5/16*a*c^4*d*x^16+5/18*a*c^4*e*x^
18+1/20*c^5*d*x^20+1/22*c^5*e*x^22+a^5*d*ln(x)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.86

$$\int \frac{(d+ex^2)(a+cx^4)^5}{x}\,dx = \frac{1}{22}\,c^5ex^{22}+\frac{1}{20}\,c^5dx^{20}+\frac{5}{18}\,ac^4ex^{18}+\frac{5}{16}\,ac^4dx^{16}+\frac{5}{7}\,a^2c^3ex^{14}+\frac{5}{6}\,a^2c^3dx^{12}+a^3c^2ex^{10}+\frac{5}{4}\,a^3c^2dx^8+\frac{5}{6}\,a^4cex^6+\frac{5}{4}\,a^4cdx^4+\frac{1}{2}\,a^5ex^2+a^5d\log(x)$$

```
[In] integrate((e*x^2+d)*(c*x^4+a)^5/x,x, algorithm="fricas")
```

```
[Out] 1/22*c^5*e*x^22 + 1/20*c^5*d*x^20 + 5/18*a*c^4*e*x^18 + 5/16*a*c^4*d*x^16 +
 5/7*a^2*c^3*e*x^14 + 5/6*a^2*c^3*d*x^12 + a^3*c^2*e*x^10 + 5/4*a^3*c^2*d*x
^8 + 5/6*a^4*c*e*x^6 + 5/4*a^4*c*d*x^4 + 1/2*a^5*e*x^2 + a^5*d*log(x)
```

Sympy [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.06

$$\int \frac{(d+ex^2)(a+cx^4)^5}{x}\,dx = a^5d\log(x)+\frac{a^5ex^2}{2}+\frac{5a^4cdx^4}{4}+\frac{5a^4cex^6}{6}+\frac{5a^3c^2dx^8}{4}+a^3c^2ex^{10}+\frac{5a^2c^3dx^{12}}{6}+\frac{5a^2c^3ex^{14}}{7}+\frac{5ac^4dx^{16}}{16}+\frac{5ac^4ex^{18}}{18}+\frac{c^5dx^{20}}{20}+\frac{c^5ex^{22}}{22}$$

```
[In] integrate((e*x**2+d)*(c*x**4+a)**5/x,x)
```

```
[Out] a**5*d*log(x) + a**5*e*x**2/2 + 5*a**4*c*d*x**4/4 + 5*a**4*c*e*x**6/6 + 5*a
**3*c**2*d*x**8/4 + a**3*c**2*e*x**10 + 5*a**2*c**3*d*x**12/6 + 5*a**2*c**3
*e*x**14/7 + 5*a*c**4*d*x**16/16 + 5*a*c**4*e*x**18/18 + c**5*d*x**20/20 +
c**5*e*x**22/22
```

Maxima [A] (verification not implemented)

none

Time = 0.22 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.88

$$\int \frac{(d+ex^2)(a+cx^4)^5}{x}\,dx = \frac{1}{22}\,c^5ex^{22} + \frac{1}{20}\,c^5dx^{20} + \frac{5}{18}\,ac^4ex^{18} + \frac{5}{16}\,ac^4dx^{16} + \frac{5}{7}\,a^2c^3ex^{14} + \frac{5}{6}\,a^2c^3dx^{12} + a^3c^2ex^{10} + \frac{5}{4}\,a^3c^2dx^8 + \frac{5}{6}\,a^4cex^6 + \frac{5}{4}\,a^4cdx^4 + \frac{1}{2}\,a^5ex^2 + \frac{1}{2}\,a^5d\log\left(x^2\right)$$

```
[In] integrate((e*x^2+d)*(c*x^4+a)^5/x,x, algorithm="maxima")
```

```
[Out] 1/22*c^5*e*x^22 + 1/20*c^5*d*x^20 + 5/18*a*c^4*e*x^18 + 5/16*a*c^4*d*x^16 +
 5/7*a^2*c^3*e*x^14 + 5/6*a^2*c^3*d*x^12 + a^3*c^2*e*x^10 + 5/4*a^3*c^2*d*x
^8 + 5/6*a^4*c*e*x^6 + 5/4*a^4*c*d*x^4 + 1/2*a^5*e*x^2 + 1/2*a^5*d*log(x^2)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.88

$$\int \frac{(d+ex^2)(a+cx^4)^5}{x}\,dx = \frac{1}{22}\,c^5ex^{22} + \frac{1}{20}\,c^5dx^{20} + \frac{5}{18}\,ac^4ex^{18} + \frac{5}{16}\,ac^4dx^{16} + \frac{5}{7}\,a^2c^3ex^{14} + \frac{5}{6}\,a^2c^3dx^{12} + a^3c^2ex^{10} + \frac{5}{4}\,a^3c^2dx^8 + \frac{5}{6}\,a^4cex^6 + \frac{5}{4}\,a^4cdx^4 + \frac{1}{2}\,a^5ex^2 + \frac{1}{2}\,a^5d\log\left(x^2\right)$$

```
[In] integrate((e*x^2+d)*(c*x^4+a)^5/x,x, algorithm="giac")
```

```
[Out] 1/22*c^5*e*x^22 + 1/20*c^5*d*x^20 + 5/18*a*c^4*e*x^18 + 5/16*a*c^4*d*x^16 +
 5/7*a^2*c^3*e*x^14 + 5/6*a^2*c^3*d*x^12 + a^3*c^2*e*x^10 + 5/4*a^3*c^2*d*x
^8 + 5/6*a^4*c*e*x^6 + 5/4*a^4*c*d*x^4 + 1/2*a^5*e*x^2 + 1/2*a^5*d*log(x^2)
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.86

$$\int \frac{(d+ex^2)(a+cx^4)^5}{x}\,dx = \frac{a^5\,e\,x^2}{2} + \frac{c^5\,d\,x^{20}}{20} + \frac{c^5\,e\,x^{22}}{22} + a^5\,d\,\ln(x) + \frac{5\,a^3\,c^2\,d\,x^8}{4} + \frac{5\,a^2\,c^3\,d\,x^{12}}{6} + a^3\,c^2\,e\,x^{10} + \frac{5\,a^2\,c^3\,e\,x^{14}}{7} + \frac{5\,a^4\,c\,d\,x^4}{4} + \frac{5\,a\,c^4\,d\,x^{16}}{16} + \frac{5\,a^4\,c\,e\,x^6}{6} + \frac{5\,a\,c^4\,e\,x^{18}}{18}$$

```
[In] int(((a + c*x^4)^5*(d + e*x^2))/x,x)
```

```
[Out] (a^5*e*x^2)/2 + (c^5*d*x^20)/20 + (c^5*e*x^22)/22 + a^5*d*log(x) + (5*a^3*c
^2*d*x^8)/4 + (5*a^2*c^3*d*x^12)/6 + a^3*c^2*e*x^10 + (5*a^2*c^3*e*x^14)/7
+ (5*a^4*c*d*x^4)/4 + (5*a*c^4*d*x^16)/16 + (5*a^4*c*e*x^6)/6 + (5*a*c^4*e*
x^18)/18
```

3.6 $\int \frac{(d+ex^2)(a+cx^4)^5}{x^2} dx$

Optimal result

Integrand size = 20, antiderivative size = 139

$$\int \frac{(d+ex^2)(a+cx^4)^5}{x^2} dx = -\frac{a^5 d}{x} + a^5 ex + \frac{5}{3}a^4 cdx^3 + a^4 cex^5 + \frac{10}{7}a^3 c^2 dx^7 + \frac{10}{9}a^3 c^2 ex^9 + \frac{10}{11}a^2 c^3 dx^{11} + \frac{10}{13}a^2 c^3 ex^{13} + \frac{1}{3}ac^4 dx^{15} + \frac{5}{17}ac^4 ex^{17} + \frac{1}{19}c^5 dx^{19} + \frac{1}{21}c^5 ex^{21}$$

```
[Out] -a^5*d/x+a^5*e*x+5/3*a^4*c*d*x^3+a^4*c*e*x^5+10/7*a^3*c^2*d*x^7+10/9*a^3*c^
2*e*x^9+10/11*a^2*c^3*d*x^11+10/13*a^2*c^3*e*x^13+1/3*a*c^4*d*x^15+5/17*a*c
^4*e*x^17+1/19*c^5*d*x^19+1/21*c^5*e*x^21
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.050, Rules used = {1276}

$$\int \frac{(d+ex^2)(a+cx^4)^5}{x^2} dx = -\frac{a^5 d}{x} + a^5 ex + \frac{5}{3}a^4 cdx^3 + a^4 cex^5 + \frac{10}{7}a^3 c^2 dx^7 + \frac{10}{9}a^3 c^2 ex^9 + \frac{10}{11}a^2 c^3 dx^{11} + \frac{10}{13}a^2 c^3 ex^{13} + \frac{1}{3}ac^4 dx^{15} + \frac{5}{17}ac^4 ex^{17} + \frac{1}{19}c^5 dx^{19} + \frac{1}{21}c^5 ex^{21}$$

```
[In] Int[((d + e*x^2)*(a + c*x^4)^5)/x^2,x]
```

```
[Out] -((a^5*d)/x) + a^5*e*x + (5*a^4*c*d*x^3)/3 + a^4*c*e*x^5 + (10*a^3*c^2*d*x^
7)/7 + (10*a^3*c^2*e*x^9)/9 + (10*a^2*c^3*d*x^11)/11 + (10*a^2*c^3*e*x^13)/
```

```
13 + (a*c^4*d*x^15)/3 + (5*a*c^4*e*x^17)/17 + (c^5*d*x^19)/19 + (c^5*e*x^21
)/21
```

Rule 1276

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*(a + c*x^4)^p,
x], x] /; FreeQ[{a, c, d, e, f, m, q}, x] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rubi steps

$$\text{integral} = \int \left(a^5 e + \frac{a^5 d}{x^2} + 5a^4 c d x^2 + 5a^4 c e x^4 + 10a^3 c^2 d x^6 + 10a^3 c^2 e x^8 + 10a^2 c^3 d x^{10} + 10a^2 c^3 e x^{12} + 5a c^4 d x^{14} + 5a c^4 e x^{16} + c^5 d x^{18} + c^5 e x^{20}\right) dx$$

$$= -\frac{a^5 d}{x} + a^5 e x + \frac{5}{3}a^4 c d x^3 + a^4 c e x^5 + \frac{10}{7}a^3 c^2 d x^7 + \frac{10}{9}a^3 c^2 e x^9 + \frac{10}{11}a^2 c^3 d x^{11} + \frac{10}{13}a^2 c^3 e x^{13} + \frac{1}{3}a c^4 d x^{15} + \frac{5}{17}a c^4 e x^{17} + \frac{1}{19}c^5 d x^{19} + \frac{1}{21}c^5 e x^{21}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.00

$$\int \frac{(d+ex^2)(a+cx^4)^5}{x^2} dx = -\frac{a^5 d}{x} + a^5 e x + \frac{5}{3}a^4 c d x^3 + a^4 c e x^5 + \frac{10}{7}a^3 c^2 d x^7 + \frac{10}{9}a^3 c^2 e x^9 + \frac{10}{11}a^2 c^3 d x^{11} + \frac{10}{13}a^2 c^3 e x^{13} + \frac{1}{3}a c^4 d x^{15} + \frac{5}{17}a c^4 e x^{17} + \frac{1}{19}c^5 d x^{19} + \frac{1}{21}c^5 e x^{21}$$

```
[In] Integrate[((d + e*x^2)*(a + c*x^4)^5)/x^2,x]
```

```
[Out] -((a^5*d)/x) + a^5*e*x + (5*a^4*c*d*x^3)/3 + a^4*c*e*x^5 + (10*a^3*c^2*d*x^
7)/7 + (10*a^3*c^2*e*x^9)/9 + (10*a^2*c^3*d*x^11)/11 + (10*a^2*c^3*e*x^13)/
13 + (a*c^4*d*x^15)/3 + (5*a*c^4*e*x^17)/17 + (c^5*d*x^19)/19 + (c^5*e*x^21
)/21
```

Maple [A] (verified)

Time = 0.27 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.88

method	result
default	$-\frac{a^5 d}{x}+a^5 e x+\frac{5 a^4 c d x^3}{3}+a^4 c e x^5+\frac{10 a^3 c^2 d x^7}{7}+\frac{10 a^3 c^2 e x^9}{9}+\frac{10 a^2 c^3 d x^{11}}{11}+\frac{10 a^2 c^3 e x^{13}}{13}+\frac{a c^4 d x^{15}}{3}+$
risch	$-\frac{a^5 d}{x}+a^5 e x+\frac{5 a^4 c d x^3}{3}+a^4 c e x^5+\frac{10 a^3 c^2 d x^7}{7}+\frac{10 a^3 c^2 e x^9}{9}+\frac{10 a^2 c^3 d x^{11}}{11}+\frac{10 a^2 c^3 e x^{13}}{13}+\frac{a c^4 d x^{15}}{3}+$
norman	$\frac{-d a^5+a^5 e x^2+\frac{5}{3} a^4 c d x^4+a^4 c e x^6+\frac{10}{7} a^3 c^2 d x^8+\frac{10}{9} a^3 c^2 e x^{10}+\frac{10}{11} a^2 c^3 d x^{12}+\frac{10}{13} a^2 c^3 e x^{14}+\frac{1}{3} a c^4 d x^{16}+\frac{5}{17} a c^4 e x^{18}+\frac{1}{19} c^5 d x}{x}$
gosper	$-\frac{-138567 c^5 e x^{22}-153153 c^5 d x^{20}-855855 a c^4 e x^{18}-969969 a c^4 d x^{16}-2238390 a^2 c^3 e x^{14}-2645370 a^2 c^3 d x^{12}-3233230 a^3 c^2 e}{2909907 x}$
parallelrisch	$\frac{138567 c^5 e x^{22}+153153 c^5 d x^{20}+855855 a c^4 e x^{18}+969969 a c^4 d x^{16}+2238390 a^2 c^3 e x^{14}+2645370 a^2 c^3 d x^{12}+3233230 a^3 c^2 e x^{10}}{2909907 x}$

```
[In] int((e*x^2+d)*(c*x^4+a)^5/x^2,x,method=_RETURNVERBOSE)
```

```
[Out] -a^5*d/x+a^5*e*x+5/3*a^4*c*d*x^3+a^4*c*e*x^5+10/7*a^3*c^2*d*x^7+10/9*a^3*c^
2*e*x^9+10/11*a^2*c^3*d*x^11+10/13*a^2*c^3*e*x^13+1/3*a*c^4*d*x^15+5/17*a*c
^4*e*x^17+1/19*c^5*d*x^19+1/21*c^5*e*x^21
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 127, normalized size of antiderivative = 0.91

$$\int \frac{(d+ex^2)(a+cx^4)^5}{x^2}\,dx = \frac{138567\,c^5ex^{22}+153153\,c^5dx^{20}+855855\,ac^4ex^{18}+969969\,ac^4dx^{16}+2238390\,a^2c^3ex^{14}+2645370\,a^2c^3dx^{1}}{290}$$

```
[In] integrate((e*x^2+d)*(c*x^4+a)^5/x^2,x, algorithm="fricas")
```

```
[Out] 1/2909907*(138567*c^5*e*x^22 + 153153*c^5*d*x^20 + 855855*a*c^4*e*x^18 + 96
9969*a*c^4*d*x^16 + 2238390*a^2*c^3*e*x^14 + 2645370*a^2*c^3*d*x^12 + 32332
30*a^3*c^2*e*x^10 + 4157010*a^3*c^2*d*x^8 + 2909907*a^4*c*e*x^6 + 4849845*a
^4*c*d*x^4 + 2909907*a^5*e*x^2 - 2909907*a^5*d)/x
```

Sympy [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.03

$$\int \frac{(d+ex^2)(a+cx^4)^5}{x^2}\,dx = -\frac{a^5d}{x} + a^5ex + \frac{5a^4cdx^3}{3} + a^4cex^5 + \frac{10a^3c^2dx^7}{7} + \frac{10a^3c^2ex^9}{9} + \frac{10a^2c^3dx^{11}}{11} + \frac{10a^2c^3ex^{13}}{13} + \frac{ac^4dx^{15}}{3} + \frac{5ac^4ex^{17}}{17} + \frac{c^5dx^{19}}{19} + \frac{c^5ex^{21}}{21}$$

```
[In] integrate((e*x**2+d)*(c*x**4+a)**5/x**2,x)
```

```
[Out] -a**5*d/x + a**5*e*x + 5*a**4*c*d*x**3/3 + a**4*c*e*x**5 + 10*a**3*c**2*d*x
**7/7 + 10*a**3*c**2*e*x**9/9 + 10*a**2*c**3*d*x**11/11 + 10*a**2*c**3*e*x*
*13/13 + a*c**4*d*x**15/3 + 5*a*c**4*e*x**17/17 + c**5*d*x**19/19 + c**5*e*
x**21/21
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.87

$$\int \frac{(d+ex^2)(a+cx^4)^5}{x^2}\,dx = \frac{1}{21}\,c^5ex^{21} + \frac{1}{19}\,c^5dx^{19} + \frac{5}{17}\,ac^4ex^{17} + \frac{1}{3}\,ac^4dx^{15} + \frac{10}{13}\,a^2c^3ex^{13} + \frac{10}{11}\,a^2c^3dx^{11} + \frac{10}{9}\,a^3c^2ex^9 + \frac{10}{7}\,a^3c^2dx^7 + a^4cex^5 + \frac{5}{3}\,a^4cdx^3 + a^5ex - \frac{a^5d}{x}$$

```
[In] integrate((e*x^2+d)*(c*x^4+a)^5/x^2,x, algorithm="maxima")
```

```
[Out] 1/21*c^5*e*x^21 + 1/19*c^5*d*x^19 + 5/17*a*c^4*e*x^17 + 1/3*a*c^4*d*x^15 +
10/13*a^2*c^3*e*x^13 + 10/11*a^2*c^3*d*x^11 + 10/9*a^3*c^2*e*x^9 + 10/7*a^3
*c^2*d*x^7 + a^4*c*e*x^5 + 5/3*a^4*c*d*x^3 + a^5*e*x - a^5*d/x
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.87

$$\int \frac{(d+ex^2)\,(a+cx^4)^5}{x^2}\,dx = \frac{1}{21}\,c^5ex^{21} + \frac{1}{19}\,c^5dx^{19} + \frac{5}{17}\,ac^4ex^{17} + \frac{1}{3}\,ac^4dx^{15} + \frac{10}{13}\,a^2c^3ex^{13} + \frac{10}{11}\,a^2c^3dx^{11} + \frac{10}{9}\,a^3c^2ex^9 + \frac{10}{7}\,a^3c^2dx^7 + a^4cex^5 + \frac{5}{3}\,a^4cdx^3 + a^5ex - \frac{a^5d}{x}$$

```
[In] integrate((e*x^2+d)*(c*x^4+a)^5/x^2,x, algorithm="giac")
```

```
[Out] 1/21*c^5*e*x^21 + 1/19*c^5*d*x^19 + 5/17*a*c^4*e*x^17 + 1/3*a*c^4*d*x^15 +
10/13*a^2*c^3*e*x^13 + 10/11*a^2*c^3*d*x^11 + 10/9*a^3*c^2*e*x^9 + 10/7*a^3
*c^2*d*x^7 + a^4*c*e*x^5 + 5/3*a^4*c*d*x^3 + a^5*e*x - a^5*d/x
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.87

$$\int \frac{(d+ex^2)\,(a+cx^4)^5}{x^2}\,dx = \frac{c^5\,d\,x^{19}}{19} - \frac{a^5\,d}{x} + \frac{c^5\,e\,x^{21}}{21} + a^5\,e\,x + \frac{10\,a^3\,c^2\,d\,x^7}{7} + \frac{10\,a^2\,c^3\,d\,x^{11}}{11} + \frac{10\,a^3\,c^2\,e\,x^9}{9} + \frac{10\,a^2\,c^3\,e\,x^{13}}{13} + \frac{5\,a^4\,c\,d\,x^3}{3} + \frac{a\,c^4\,d\,x^{15}}{3} + a^4\,c\,e\,x^5 + \frac{5\,a\,c^4\,e\,x^{17}}{17}$$

```
[In] int(((a + c*x^4)^5*(d + e*x^2))/x^2,x)
```

```
[Out] (c^5*d*x^19)/19 - (a^5*d)/x + (c^5*e*x^21)/21 + a^5*e*x + (10*a^3*c^2*d*x^7
)/7 + (10*a^2*c^3*d*x^11)/11 + (10*a^3*c^2*e*x^9)/9 + (10*a^2*c^3*e*x^13)/1
3 + (5*a^4*c*d*x^3)/3 + (a*c^4*d*x^15)/3 + a^4*c*e*x^5 + (5*a*c^4*e*x^17)/1
7
```

3.7 $\int \frac{(d+ex^2)(a+cx^4)^5}{x^3}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 142

$$\int \frac{(d+ex^2)(a+cx^4)^5}{x^3}\,dx = -\frac{a^5d}{2x^2}+\frac{5}{2}a^4cdx^2+\frac{5}{4}a^4cex^4+\frac{5}{3}a^3c^2dx^6+\frac{5}{4}a^3c^2ex^8+a^2c^3dx^{10}+\frac{5}{6}a^2c^3ex^{12}+\frac{5}{14}ac^4dx^{14}+\frac{5}{16}ac^4ex^{16}+\frac{1}{18}c^5dx^{18}+\frac{1}{20}c^5ex^{20}+a^5e\log(x)$$

```
[Out] -1/2*a^5*d/x^2+5/2*a^4*c*d*x^2+5/4*a^4*c*e*x^4+5/3*a^3*c^2*d*x^6+5/4*a^3*c^
2*e*x^8+a^2*c^3*d*x^10+5/6*a^2*c^3*e*x^12+5/14*a*c^4*d*x^14+5/16*a*c^4*e*x^
16+1/18*c^5*d*x^18+1/20*c^5*e*x^20+a^5*e*ln(x)
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {1266, 780}

$$\int \frac{(d+ex^2)(a+cx^4)^5}{x^3}\,dx = -\frac{a^5d}{2x^2}+a^5e\log(x)+\frac{5}{2}a^4cdx^2+\frac{5}{4}a^4cex^4+\frac{5}{3}a^3c^2dx^6+\frac{5}{4}a^3c^2ex^8+a^2c^3dx^{10}+\frac{5}{6}a^2c^3ex^{12}+\frac{5}{14}ac^4dx^{14}+\frac{5}{16}ac^4ex^{16}+\frac{1}{18}c^5dx^{18}+\frac{1}{20}c^5ex^{20}$$

```
[In] Int[((d + e*x^2)*(a + c*x^4)^5)/x^3,x]
```

```
[Out] -1/2*(a^5*d)/x^2 + (5*a^4*c*d*x^2)/2 + (5*a^4*c*e*x^4)/4 + (5*a^3*c^2*d*x^6
)/3 + (5*a^3*c^2*e*x^8)/4 + a^2*c^3*d*x^10 + (5*a^2*c^3*e*x^12)/6 + (5*a*c^
```

```
4*d*x^14)/14 + (5*a*c^4*e*x^16)/16 + (c^5*d*x^18)/18 + (c^5*e*x^20)/20 + a^
5*e*Log[x]
```

Rule 780

```
Int[((e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(e*x)^m*(f + g*x)*(a + c*x^2)^p, x], x] /; F
reeQ[{a, c, e, f, g, m}, x] && IGtQ[p, 0]
```

Rule 1266

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_S
ymbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + c*x^2)^p, x],
x, x^2], x] /; FreeQ[{a, c, d, e, p, q}, x] && IntegerQ[(m + 1)/2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{2}\text{Subst}\left(\int \frac{(d+ex)\left(a+cx^2\right)^5}{x^2}\,dx, x, x^2\right) \\
&= \frac{1}{2}\text{Subst}\left(\int \left(5a^4cd+\frac{a^5d}{x^2}+\frac{a^5e}{x}+5a^4cex+10a^3c^2dx^2+10a^3c^2ex^3+10a^2c^3dx^4 \right.\right. \\
&\qquad \left.\left. +10a^2c^3ex^5+5ac^4dx^6+5ac^4ex^7+c^5dx^8+c^5ex^9\right)\,dx, x, x^2\right) \\
&= -\frac{a^5d}{2x^2}+\frac{5}{2}a^4cdx^2+\frac{5}{4}a^4cex^4+\frac{5}{3}a^3c^2dx^6+\frac{5}{4}a^3c^2ex^8+a^2c^3dx^{10} \\
&\quad +\frac{5}{6}a^2c^3ex^{12}+\frac{5}{14}ac^4dx^{14}+\frac{5}{16}ac^4ex^{16}+\frac{1}{18}c^5dx^{18}+\frac{1}{20}c^5ex^{20}+a^5e\log(x)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.00

$$\begin{aligned}
\int \frac{(d+ex^2)\left(a+cx^4\right)^5}{x^3}\,dx &= -\frac{a^5d}{2x^2}+\frac{5}{2}a^4cdx^2+\frac{5}{4}a^4cex^4+\frac{5}{3}a^3c^2dx^6 \\
&\quad +\frac{5}{4}a^3c^2ex^8+a^2c^3dx^{10}+\frac{5}{6}a^2c^3ex^{12}+\frac{5}{14}ac^4dx^{14} \\
&\quad +\frac{5}{16}ac^4ex^{16}+\frac{1}{18}c^5dx^{18}+\frac{1}{20}c^5ex^{20}+a^5e\log(x)
\end{aligned}$$

```
[In] Integrate[((d + e*x^2)*(a + c*x^4)^5)/x^3,x]
```

```
[Out] -1/2*(a^5*d)/x^2 + (5*a^4*c*d*x^2)/2 + (5*a^4*c*e*x^4)/4 + (5*a^3*c^2*d*x^6
)/3 + (5*a^3*c^2*e*x^8)/4 + a^2*c^3*d*x^10 + (5*a^2*c^3*e*x^12)/6 + (5*a*c^
4*d*x^14)/14 + (5*a*c^4*e*x^16)/16 + (c^5*d*x^18)/18 + (c^5*e*x^20)/20 + a^
5*e*Log[x]
```

Maple [A] (verified)

Time = 0.29 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.87

method	result
default	$-\frac{a^5 d}{2x^2}+\frac{5a^4cd\,x^2}{2}+\frac{5a^4ce\,x^4}{4}+\frac{5a^3c^2d\,x^6}{3}+\frac{5a^3c^2e\,x^8}{4}+a^2c^3d\,x^{10}+\frac{5a^2c^3e\,x^{12}}{6}+\frac{5a\,c^4d\,x^{14}}{14}+\frac{5a\,c^4e\,x^{16}}{16}+$
risch	$-\frac{a^5 d}{2x^2}+\frac{5a^4cd\,x^2}{2}+\frac{5a^4ce\,x^4}{4}+\frac{5a^3c^2d\,x^6}{3}+\frac{5a^3c^2e\,x^8}{4}+a^2c^3d\,x^{10}+\frac{5a^2c^3e\,x^{12}}{6}+\frac{5a\,c^4d\,x^{14}}{14}+\frac{5a\,c^4e\,x^{16}}{16}+$
norman	$\frac{a^2c^3d\,x^{12}-\frac{1}{2}d\,a^5+\frac{1}{18}c^5d\,x^{20}+\frac{1}{20}c^5e\,x^{22}+\frac{5}{14}a\,c^4d\,x^{16}+\frac{5}{16}a\,c^4e\,x^{18}+\frac{5}{6}a^2c^3e\,x^{14}+\frac{5}{3}a^3c^2d\,x^8+\frac{5}{4}a^3c^2e\,x^{10}+\frac{5}{2}a^4cd\,x^4+\frac{5}{4}a^4ce\,x^6}{x^2}$
parallelrisch	$\frac{252c^5e\,x^{22}+280c^5d\,x^{20}+1575a\,c^4e\,x^{18}+1800a\,c^4d\,x^{16}+4200a^2c^3e\,x^{14}+5040a^2c^3d\,x^{12}+6300a^3c^2e\,x^{10}+8400a^3c^2d\,x^8+6300a^4c}{5040x^2}$

```
[In] int((e*x^2+d)*(c*x^4+a)^5/x^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*a^5*d/x^2+5/2*a^4*c*d*x^2+5/4*a^4*c*e*x^4+5/3*a^3*c^2*d*x^6+5/4*a^3*c^
2*e*x^8+a^2*c^3*d*x^10+5/6*a^2*c^3*e*x^12+5/14*a*c^4*d*x^14+5/16*a*c^4*e*x^
16+1/18*c^5*d*x^18+1/20*c^5*e*x^20+a^5*e*ln(x)
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 129, normalized size of antiderivative = 0.91

$$\int \frac{(d+ex^2)\,(a+cx^4)^5}{x^3}\,dx = \frac{252\,c^5ex^{22}+280\,c^5dx^{20}+1575\,ac^4ex^{18}+1800\,ac^4dx^{16}+4200\,a^2c^3ex^{14}+5040\,a^2c^3dx^{12}+6300\,a^3c^2ex^{10}+}{5040\,x^2}$$

```
[In] integrate((e*x^2+d)*(c*x^4+a)^5/x^3,x, algorithm="fricas")
```

```
[Out] 1/5040*(252*c^5*e*x^22 + 280*c^5*d*x^20 + 1575*a*c^4*e*x^18 + 1800*a*c^4*d*
x^16 + 4200*a^2*c^3*e*x^14 + 5040*a^2*c^3*d*x^12 + 6300*a^3*c^2*e*x^10 + 84
00*a^3*c^2*d*x^8 + 6300*a^4*c*e*x^6 + 12600*a^4*c*d*x^4 + 5040*a^5*e*x^2*lo
g(x) - 2520*a^5*d)/x^2
```

Sympy [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.06

$$\int \frac{(d+ex^2)\,(a+cx^4)^5}{x^3}\,dx = -\frac{a^5d}{2x^2}+a^5e\log{(x)}+\frac{5a^4cdx^2}{2}+\frac{5a^4cex^4}{4}+\frac{5a^3c^2dx^6}{3}+\frac{5a^3c^2ex^8}{4}+a^2c^3dx^{10}+\frac{5a^2c^3ex^{12}}{6}+\frac{5ac^4dx^{14}}{14}+\frac{5ac^4ex^{16}}{16}+\frac{c^5dx^{18}}{18}+\frac{c^5ex^{20}}{20}$$

```
[In] integrate((e*x**2+d)*(c*x**4+a)**5/x**3,x)
```

```
[Out] -a**5*d/(2*x**2) + a**5*e*log(x) + 5*a**4*c*d*x**2/2 + 5*a**4*c*e*x**4/4 +
5*a**3*c**2*d*x**6/3 + 5*a**3*c**2*e*x**8/4 + a**2*c**3*d*x**10 + 5*a**2*c*
*3*e*x**12/6 + 5*a*c**4*d*x**14/14 + 5*a*c**4*e*x**16/16 + c**5*d*x**18/18
+ c**5*e*x**20/20
```

Maxima [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.88

$$\int \frac{(d+ex^2)(a+cx^4)^5}{x^3}\,dx = \frac{1}{20}\,c^5ex^{20} + \frac{1}{18}\,c^5dx^{18} + \frac{5}{16}\,ac^4ex^{16} + \frac{5}{14}\,ac^4dx^{14} + \frac{5}{6}\,a^2c^3ex^{12} + a^2c^3dx^{10} + \frac{5}{4}\,a^3c^2ex^8 + \frac{5}{3}\,a^3c^2dx^6 + \frac{5}{4}\,a^4cex^4 + \frac{5}{2}\,a^4cdx^2 + \frac{1}{2}\,a^5e\log\left(x^2\right) - \frac{a^5d}{2\,x^2}$$

```
[In] integrate((e*x^2+d)*(c*x^4+a)^5/x^3,x, algorithm="maxima")
```

```
[Out] 1/20*c^5*e*x^20 + 1/18*c^5*d*x^18 + 5/16*a*c^4*e*x^16 + 5/14*a*c^4*d*x^14 +
 5/6*a^2*c^3*e*x^12 + a^2*c^3*d*x^10 + 5/4*a^3*c^2*e*x^8 + 5/3*a^3*c^2*d*x^
6 + 5/4*a^4*c*e*x^4 + 5/2*a^4*c*d*x^2 + 1/2*a^5*e*log(x^2) - 1/2*a^5*d/x^2
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 135, normalized size of antiderivative = 0.95

$$\int \frac{(d+ex^2)(a+cx^4)^5}{x^3}\,dx = \frac{1}{20}\,c^5ex^{20} + \frac{1}{18}\,c^5dx^{18} + \frac{5}{16}\,ac^4ex^{16} + \frac{5}{14}\,ac^4dx^{14} + \frac{5}{6}\,a^2c^3ex^{12} + a^2c^3dx^{10} + \frac{5}{4}\,a^3c^2ex^8 + \frac{5}{3}\,a^3c^2dx^6 + \frac{5}{4}\,a^4cex^4 + \frac{5}{2}\,a^4cdx^2 + \frac{1}{2}\,a^5e\log\left(x^2\right) - \frac{a^5ex^2 + a^5d}{2\,x^2}$$

```
[In] integrate((e*x^2+d)*(c*x^4+a)^5/x^3,x, algorithm="giac")
```

```
[Out] 1/20*c^5*e*x^20 + 1/18*c^5*d*x^18 + 5/16*a*c^4*e*x^16 + 5/14*a*c^4*d*x^14 +
 5/6*a^2*c^3*e*x^12 + a^2*c^3*d*x^10 + 5/4*a^3*c^2*e*x^8 + 5/3*a^3*c^2*d*x^
6 + 5/4*a^4*c*e*x^4 + 5/2*a^4*c*d*x^2 + 1/2*a^5*e*log(x^2) - 1/2*(a^5*e*x^2
 + a^5*d)/x^2
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.86

$$\int \frac{(d+ex^2)\,(a+cx^4)^5}{x^3}\,dx = \frac{c^5\,d\,x^{18}}{18} - \frac{a^5\,d}{2\,x^2} + \frac{c^5\,e\,x^{20}}{20} + a^5\,e\,\ln(x) + \frac{5\,a^3\,c^2\,d\,x^6}{3} + a^2\,c^3\,d\,x^{10} + \frac{5\,a^3\,c^2\,e\,x^8}{4} + \frac{5\,a^2\,c^3\,e\,x^{12}}{6} + \frac{5\,a^4\,c\,d\,x^2}{2} + \frac{5\,a\,c^4\,d\,x^{14}}{14} + \frac{5\,a^4\,c\,e\,x^4}{4} + \frac{5\,a\,c^4\,e\,x^{16}}{16}$$

```
[In] int(((a + c*x^4)^5*(d + e*x^2))/x^3,x)
```

```
[Out] (c^5*d*x^18)/18 - (a^5*d)/(2*x^2) + (c^5*e*x^20)/20 + a^5*e*log(x) + (5*a^3
*c^2*d*x^6)/3 + a^2*c^3*d*x^10 + (5*a^3*c^2*e*x^8)/4 + (5*a^2*c^3*e*x^12)/6
 + (5*a^4*c*d*x^2)/2 + (5*a*c^4*d*x^14)/14 + (5*a^4*c*e*x^4)/4 + (5*a*c^4*e
*x^16)/16
```

3.8 $\int x^5(2+3x^2)\sqrt{5+x^4}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 67

$$\int x^5(2+3x^2)\sqrt{5+x^4}\,dx = -\frac{5}{8}x^2\sqrt{5+x^4}+\frac{3}{10}x^4(5+x^4)^{3/2} - \frac{1}{4}(4-x^2)\,(5+x^4)^{3/2}-\frac{25}{8}\operatorname{arcsinh}\left(\frac{x^2}{\sqrt{5}}\right)$$

```
[Out] 3/10*x^4*(x^4+5)^(3/2)-1/4*(-x^2+4)*(x^4+5)^(3/2)-25/8*arcsinh(1/5*x^2*5^(1
/2))-5/8*x^2*(x^4+5)^(1/2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {1266, 847, 794, 201, 221}

$$\int x^5(2+3x^2)\sqrt{5+x^4}\,dx = -\frac{25}{8}\operatorname{arcsinh}\left(\frac{x^2}{\sqrt{5}}\right)+\frac{3}{10}(x^4+5)^{3/2}x^4 - \frac{5}{8}\sqrt{x^4+5}x^2-\frac{1}{4}(4-x^2)\,(x^4+5)^{3/2}$$

```
[In] Int[x^5*(2 + 3*x^2)*Sqrt[5 + x^4],x]
```

```
[Out] (-5*x^2*Sqrt[5 + x^4])/8 + (3*x^4*(5 + x^4)^(3/2))/10 - ((4 - x^2)*(5 + x^4
)^(3/2))/4 - (25*ArcSinh[x^2/Sqrt[5]])/8
```

Rule 201

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Dist[a*n*(p/(n*p + 1)), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
```

```
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 794

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*((a + c*x^2)^(p
+ 1)/(2*c*(p + 1)*(2*p + 3))), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rule 847

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[g*(d + e*x)^m*((a + c*x^2)^(p + 1)/(c*(m + 2*p + 2))
), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p*Simp[
c*d*f*(m + 2*p + 2) - a*e*g*m + c*(e*f*(m + 2*p + 2) + d*g*m)*x, x], x], x]
 /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[m, 0] &
& NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]
) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

Rule 1266

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_S
ymbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + c*x^2)^p, x],
x, x^2], x] /; FreeQ[{a, c, d, e, p, q}, x] && IntegerQ[(m + 1)/2]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{1}{2}\text{Subst}\left(\int x^2(2+3x)\sqrt{5+x^2}\,dx, x, x^2\right) \\
&= \frac{3}{10}x^4\left(5+x^4\right)^{3/2} + \frac{1}{10}\text{Subst}\left(\int x(-30+10x)\sqrt{5+x^2}\,dx, x, x^2\right) \\
&= \frac{3}{10}x^4\left(5+x^4\right)^{3/2} - \frac{1}{4}\left(4-x^2\right)\left(5+x^4\right)^{3/2} - \frac{5}{4}\text{Subst}\left(\int \sqrt{5+x^2}\,dx, x, x^2\right) \\
&= -\frac{5}{8}x^2\sqrt{5+x^4} + \frac{3}{10}x^4\left(5+x^4\right)^{3/2} - \frac{1}{4}\left(4-x^2\right)\left(5+x^4\right)^{3/2} - \frac{25}{8}\text{Subst}\left(\int \frac{1}{\sqrt{5+x^2}}\,dx, x, x^2\right) \\
&= -\frac{5}{8}x^2\sqrt{5+x^4} + \frac{3}{10}x^4\left(5+x^4\right)^{3/2} - \frac{1}{4}\left(4-x^2\right)\left(5+x^4\right)^{3/2} - \frac{25}{8}\sinh^{-1}\left(\frac{x^2}{\sqrt{5}}\right)
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.84

$$\int x^5 \left(2+3 x^2\right) \sqrt{5+x^4} \, dx = \frac{1}{40} \sqrt{5+x^4} \left(-200+25 x^2+20 x^4+10 x^6+12 x^8\right) + \frac{25}{8} \log \left(-x^2+\sqrt{5+x^4}\right)$$

```
[In] Integrate[x^5*(2 + 3*x^2)*Sqrt[5 + x^4],x]
[Out] (Sqrt[5 + x^4]*(-200 + 25*x^2 + 20*x^4 + 10*x^6 + 12*x^8))/40 + (25*Log[-x^
2 + Sqrt[5 + x^4]])/8
```

Maple [A] (verified)

Time = 0.36 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.66

method	result	size
risch	$\frac{\left(12 x^{8}+10 x^{6}+20 x^{4}+25 x^{2}-200\right) \sqrt{x^{4}+5}}{40}-\frac{25 \,\operatorname{arcsinh}\left(\frac{x^{2} \sqrt{5}}{5}\right)}{8}$	44
pseudoelliptic	$\frac{\left(12 x^{8}+10 x^{6}+20 x^{4}+25 x^{2}-200\right) \sqrt{x^{4}+5}}{40}-\frac{25 \,\operatorname{arcsinh}\left(\frac{x^{2} \sqrt{5}}{5}\right)}{8}$	44
trager	$\left(\frac{3}{10} x^{8}+\frac{1}{4} x^{6}+\frac{1}{2} x^{4}+\frac{5}{8} x^{2}-5\right) \sqrt{x^{4}+5}-\frac{25 \ln \left(x^{2}+\sqrt{x^{4}+5}\right)}{8}$	46
default	$\frac{\left(x^{4}+5\right)^{\frac{3}{2}} \left(3 x^{4}-10\right)}{10}+\frac{x^{2} \left(x^{4}+5\right)^{\frac{3}{2}}}{4}-\frac{5 x^{2} \sqrt{x^{4}+5}}{8}-\frac{25 \,\operatorname{arcsinh}\left(\frac{x^{2} \sqrt{5}}{5}\right)}{8}$	53
elliptic	$\frac{3 x^{8} \sqrt{x^{4}+5}}{10}+\frac{x^{4} \sqrt{x^{4}+5}}{2}-5 \sqrt{x^{4}+5}+\frac{x^{6} \sqrt{x^{4}+5}}{4}+\frac{5 x^{2} \sqrt{x^{4}+5}}{8}-\frac{25 \,\operatorname{arcsinh}\left(\frac{x^{2} \sqrt{5}}{5}\right)}{8}$	70
meijerg	$-\frac{75 \sqrt{5}\, \left(-\frac{8 \sqrt{\pi}}{15}+\frac{4 \sqrt{\pi}\, \left(1+\frac{x^{4}}{5}\right)^{\frac{3}{2}} \left(-\frac{3 x^{4}}{5}+2\right)}{15}\right)}{8 \sqrt{\pi}}-\frac{25 \left(-\frac{\sqrt{\pi}\, x^{2} \sqrt{5}\, \left(\frac{6 x^{4}}{5}+3\right) \sqrt{1+\frac{x^{4}}{5}}}{30}+\frac{\sqrt{\pi}\, \operatorname{arcsinh}\left(\frac{x^{2} \sqrt{5}}{5}\right)}{2}\right)}{4 \sqrt{\pi}}$	84

```
[In] int(x^5*(3*x^2+2)*(x^4+5)^(1/2),x,method=_RETURNVERBOSE)
[Out] 1/40*(12*x^8+10*x^6+20*x^4+25*x^2-200)*(x^4+5)^(1/2)-25/8*arcsinh(1/5*x^2*5
^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.72

$$\int x^5(2+3x^2)\sqrt{5+x^4}\,dx = \frac{1}{40}\left(12\,x^8+10\,x^6+20\,x^4+25\,x^2-200\right)\sqrt{x^4+5} + \frac{25}{8}\log\left(-x^2+\sqrt{x^4+5}\right)$$

```
[In] integrate(x^5*(3*x^2+2)*(x^4+5)^(1/2),x, algorithm="fricas")
[Out] 1/40*(12*x^8 + 10*x^6 + 20*x^4 + 25*x^2 - 200)*sqrt(x^4 + 5) + 25/8*log(-x^
2 + sqrt(x^4 + 5))
```

Sympy [A] (verification not implemented)

Time = 0.61 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.90

$$\int x^5(2+3x^2)\sqrt{5+x^4}\,dx = \sqrt{x^4+5}\left(\frac{x^6}{4}+\frac{5x^2}{8}\right) + \frac{3\sqrt{x^4+5}\left(\frac{x^8}{5}+\frac{x^4}{3}-\frac{10}{3}\right)}{2} - \frac{25\operatorname{asinh}\left(\frac{\sqrt{5}x^2}{5}\right)}{8}$$

```
[In] integrate(x**5*(3*x**2+2)*(x**4+5)**(1/2),x)
[Out] sqrt(x**4 + 5)*(x**6/4 + 5*x**2/8) + 3*sqrt(x**4 + 5)*(x**8/5 + x**4/3 - 10
/3)/2 - 25*asinh(sqrt(5)*x**2/5)/8
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 102 vs. 2(50) = 100.

Time = 0.28 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.52

$$\int x^5(2+3x^2)\sqrt{5+x^4}\,dx = \frac{3}{10}\left(x^4+5\right)^{\frac{5}{2}} - \frac{5}{2}\left(x^4+5\right)^{\frac{3}{2}} - \frac{25\left(\frac{\sqrt{x^4+5}}{x^2}+\frac{(x^4+5)^{\frac{3}{2}}}{x^6}\right)}{8\left(\frac{2\,(x^4+5)}{x^4}-\frac{(x^4+5)^2}{x^8}-1\right)} - \frac{25}{16}\log\left(\frac{\sqrt{x^4+5}}{x^2}+1\right) + \frac{25}{16}\log\left(\frac{\sqrt{x^4+5}}{x^2}-1\right)$$

```
[In] integrate(x^5*(3*x^2+2)*(x^4+5)^(1/2),x, algorithm="maxima")
[Out] 3/10*(x^4 + 5)^(5/2) - 5/2*(x^4 + 5)^(3/2) - 25/8*(sqrt(x^4 + 5)/x^2 + (x^4
 + 5)^(3/2)/x^6)/(2*(x^4 + 5)/x^4 - (x^4 + 5)^2/x^8 - 1) - 25/16*log(sqrt(x
^4 + 5)/x^2 + 1) + 25/16*log(sqrt(x^4 + 5)/x^2 - 1)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.81

$$\int x^5(2+3x^2)\sqrt{5+x^4}\,dx = \frac{1}{8}\left(2\,x^4+5\right)\sqrt{x^4+5}x^2 + \frac{3}{10}\left(x^4+5\right)^{\frac{5}{2}} - \frac{5}{2}\left(x^4+5\right)^{\frac{3}{2}} + \frac{25}{8}\log\left(-x^2+\sqrt{x^4+5}\right)$$

```
[In] integrate(x^5*(3*x^2+2)*(x^4+5)^(1/2),x, algorithm="giac")
```

```
[Out] 1/8*(2*x^4 + 5)*sqrt(x^4 + 5)*x^2 + 3/10*(x^4 + 5)^(5/2) - 5/2*(x^4 + 5)^(3
/2) + 25/8*log(-x^2 + sqrt(x^4 + 5))
```

Mupad [B] (verification not implemented)

Time = 7.73 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.63

$$\int x^5(2+3x^2)\sqrt{5+x^4}\,dx = \sqrt{x^4+5}\left(\frac{3\,x^8}{10}+\frac{x^6}{4}+\frac{x^4}{2}+\frac{5\,x^2}{8}-5\right) - \frac{25\,\mathrm{asinh}\left(\frac{\sqrt{5}\,x^2}{5}\right)}{8}$$

```
[In] int(x^5*(x^4 + 5)^(1/2)*(3*x^2 + 2),x)
```

```
[Out] (x^4 + 5)^(1/2)*((5*x^2)/8 + x^4/2 + x^6/4 + (3*x^8)/10 - 5) - (25*asinh((5
^(1/2)*x^2)/5))/8
```

3.9 $\int x^3(2+3x^2)\sqrt{5+x^4}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 51

$$\int x^3(2+3x^2)\sqrt{5+x^4}\,dx = -\frac{15}{16}x^2\sqrt{5+x^4}+\frac{1}{24}(8+9x^2)\left(5+x^4\right)^{3/2}-\frac{75}{16}\text{arcsinh}\left(\frac{x^2}{\sqrt{5}}\right)$$

```
[Out] 1/24*(9*x^2+8)*(x^4+5)^(3/2)-75/16*arcsinh(1/5*x^2*5^(1/2))-15/16*x^2*(x^4+
5)^(1/2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {1266, 794, 201, 221}

$$\int x^3(2+3x^2)\sqrt{5+x^4}\,dx = -\frac{75}{16}\text{arcsinh}\left(\frac{x^2}{\sqrt{5}}\right)-\frac{15}{16}\sqrt{x^4+5}x^2+\frac{1}{24}(9x^2+8)\left(x^4+5\right)^{3/2}$$

```
[In] Int[x^3*(2 + 3*x^2)*Sqrt[5 + x^4],x]
```

```
[Out] (-15*x^2*Sqrt[5 + x^4])/16 + ((8 + 9*x^2)*(5 + x^4)^(3/2))/24 - (75*ArcSinh
[x^2/Sqrt[5]])/16
```

Rule 201

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Dist[a*n*(p/(n*p + 1)), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 794

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*((a + c*x^2)^(p
+ 1)/(2*c*(p + 1)*(2*p + 3))), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rule 1266

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_S
ymbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + c*x^2)^p, x],
x, x^2], x] /; FreeQ[{a, c, d, e, p, q}, x] && IntegerQ[(m + 1)/2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{2}\text{Subst}\left(\int x(2+3x)\sqrt{5+x^2}\,dx, x, x^2\right)\\
&= \frac{1}{24}(8+9x^2)\left(5+x^4\right)^{3/2} - \frac{15}{8}\text{Subst}\left(\int \sqrt{5+x^2}\,dx, x, x^2\right)\\
&= -\frac{15}{16}x^2\sqrt{5+x^4} + \frac{1}{24}(8+9x^2)\left(5+x^4\right)^{3/2} - \frac{75}{16}\text{Subst}\left(\int \frac{1}{\sqrt{5+x^2}}\,dx, x, x^2\right)\\
&= -\frac{15}{16}x^2\sqrt{5+x^4} + \frac{1}{24}(8+9x^2)\left(5+x^4\right)^{3/2} - \frac{75}{16}\sinh^{-1}\left(\frac{x^2}{\sqrt{5}}\right)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.00

$$\int x^3(2+3x^2)\sqrt{5+x^4}\,dx = \frac{1}{48}\sqrt{5+x^4}(80+45x^2+16x^4+18x^6) + \frac{75}{16}\log\left(-x^2+\sqrt{5+x^4}\right)$$

```
[In] Integrate[x^3*(2 + 3*x^2)*Sqrt[5 + x^4],x]
[Out] (Sqrt[5 + x^4]*(80 + 45*x^2 + 16*x^4 + 18*x^6))/48 + (75*Log[-x^2 + Sqrt[5
+ x^4]])/16
```

Maple [A] (verified)

Time = 0.28 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.76

method	result	size
risch	$\frac{(18x^6+16x^4+45x^2+80)\sqrt{x^4+5}}{48}-\frac{75\,\operatorname{arcsinh}\left(\frac{x^2\sqrt{5}}{5}\right)}{16}$	39
pseudoelliptic	$\frac{(18x^6+16x^4+45x^2+80)\sqrt{x^4+5}}{48}-\frac{75\,\operatorname{arcsinh}\left(\frac{x^2\sqrt{5}}{5}\right)}{16}$	39
trager	$\left(\frac{3}{8}x^6+\frac{1}{3}x^4+\frac{15}{16}x^2+\frac{5}{3}\right)\sqrt{x^4+5}+\frac{75\ln\left(x^2-\sqrt{x^4+5}\right)}{16}$	43
default	$\frac{3x^2(x^4+5)^{\frac{3}{2}}}{8}-\frac{15x^2\sqrt{x^4+5}}{16}-\frac{75\,\operatorname{arcsinh}\left(\frac{x^2\sqrt{5}}{5}\right)}{16}+\frac{(x^4+5)^{\frac{3}{2}}}{3}$	46
elliptic	$\frac{5\sqrt{x^4+5}}{3}+\frac{3x^6\sqrt{x^4+5}}{8}+\frac{15x^2\sqrt{x^4+5}}{16}-\frac{75\,\operatorname{arcsinh}\left(\frac{x^2\sqrt{5}}{5}\right)}{16}+\frac{x^4\sqrt{x^4+5}}{3}$	58
meijerg	$-\frac{75\left(-\frac{\sqrt{\pi}\,x^2\sqrt{5}\left(\frac{6x^4}{5}+3\right)\sqrt{1+\frac{x^4}{5}}}{30}+\frac{\sqrt{\pi}\,\operatorname{arcsinh}\left(\frac{x^2\sqrt{5}}{5}\right)}{2}\right)}{8\sqrt{\pi}}-\frac{5\sqrt{5}\left(\frac{4\sqrt{\pi}}{3}-\frac{2\sqrt{\pi}\left(2+\frac{2x^4}{5}\right)\sqrt{1+\frac{x^4}{5}}}{3}\right)}{4\sqrt{\pi}}$	84

```
[In] int(x^3*(3*x^2+2)*(x^4+5)^(1/2),x,method=_RETURNVERBOSE)
[Out] 1/48*(18*x^6+16*x^4+45*x^2+80)*(x^4+5)^(1/2)-75/16*arcsinh(1/5*x^2*5^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.84

$$\int x^3(2+3x^2)\sqrt{5+x^4}\,dx = \frac{1}{48}\left(18\,x^6+16\,x^4+45\,x^2+80\right)\sqrt{x^4+5} + \frac{75}{16}\log\left(-x^2+\sqrt{x^4+5}\right)$$

```
[In] integrate(x^3*(3*x^2+2)*(x^4+5)^(1/2),x, algorithm="fricas")
[Out] 1/48*(18*x^6 + 16*x^4 + 45*x^2 + 80)*sqrt(x^4 + 5) + 75/16*log(-x^2 + sqrt(
x^4 + 5))
```

Sympy [A] (verification not implemented)

Time = 0.58 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.06

$$\int x^3(2+3x^2)\sqrt{5+x^4}\,dx = \left(\frac{x^4}{3}+\frac{5}{3}\right)\sqrt{x^4+5}+\frac{3\sqrt{x^4+5}\left(\frac{x^6}{4}+\frac{5x^2}{8}\right)}{2}-\frac{75\operatorname{asinh}\left(\frac{\sqrt{5}x^2}{5}\right)}{16}$$

```
[In] integrate(x**3*(3*x**2+2)*(x**4+5)**(1/2),x)
[Out] (x**4/3 + 5/3)*sqrt(x**4 + 5) + 3*sqrt(x**4 + 5)*(x**6/4 + 5*x**2/8)/2 - 75
*asinh(sqrt(5)*x**2/5)/16
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 93 vs. 2(40) = 80.

Time = 0.28 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.82

$$\int x^3(2+3x^2)\sqrt{5+x^4}\,dx = \frac{1}{3}\left(x^4+5\right)^{\frac{3}{2}}-\frac{75\left(\frac{\sqrt{x^4+5}}{x^2}+\frac{(x^4+5)^{\frac{3}{2}}}{x^6}\right)}{16\left(\frac{2(x^4+5)}{x^4}-\frac{(x^4+5)^2}{x^8}-1\right)} - \frac{75}{32}\log\left(\frac{\sqrt{x^4+5}}{x^2}+1\right)+\frac{75}{32}\log\left(\frac{\sqrt{x^4+5}}{x^2}-1\right)$$

```
[In] integrate(x^3*(3*x^2+2)*(x^4+5)^(1/2),x, algorithm="maxima")
[Out] 1/3*(x^4 + 5)^(3/2) - 75/16*(sqrt(x^4 + 5)/x^2 + (x^4 + 5)^(3/2)/x^6)/(2*(x
^4 + 5)/x^4 - (x^4 + 5)^2/x^8 - 1) - 75/32*log(sqrt(x^4 + 5)/x^2 + 1) + 75/
32*log(sqrt(x^4 + 5)/x^2 - 1)
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.88

$$\int x^3(2+3x^2)\sqrt{5+x^4}\,dx = \frac{3}{16}\left(2x^4+5\right)\sqrt{x^4+5}x^2+\frac{1}{3}\left(x^4+5\right)^{\frac{3}{2}}+\frac{75}{16}\log\left(-x^2+\sqrt{x^4+5}\right)$$

```
[In] integrate(x^3*(3*x^2+2)*(x^4+5)^(1/2),x, algorithm="giac")
[Out] 3/16*(2*x^4 + 5)*sqrt(x^4 + 5)*x^2 + 1/3*(x^4 + 5)^(3/2) + 75/16*log(-x^2 +
 sqrt(x^4 + 5))
```

Mupad [B] (verification not implemented)

Time = 7.82 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.73

$$\int x^3 (2+3x^2) \sqrt{5+x^4} \, dx = \sqrt{x^4+5} \left(\frac{3\,x^6}{8} + \frac{x^4}{3} + \frac{15\,x^2}{16} + \frac{5}{3}\right) - \frac{75\,\mathrm{asinh}\left(\frac{\sqrt{5}\,x^2}{5}\right)}{16}$$

```
[In] int(x^3*(x^4 + 5)^(1/2)*(3*x^2 + 2),x)
[Out] (x^4 + 5)^(1/2)*((15*x^2)/16 + x^4/3 + (3*x^6)/8 + 5/3) - (75*asinh((5^(1/2
)*x^2)/5))/16
```

3.10 $\int x(2+3x^2)\sqrt{5+x^4}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 44

$$\int x(2+3x^2)\sqrt{5+x^4}\,dx=\frac{1}{2}x^2\sqrt{5+x^4}+\frac{1}{2}(5+x^4)^{3/2}+\frac{5}{2}\operatorname{arcsinh}\left(\frac{x^2}{\sqrt{5}}\right)$$

```
[Out] 1/2*(x^4+5)^(3/2)+5/2*arcsinh(1/5*x^2*5^(1/2))+1/2*x^2*(x^4+5)^(1/2)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {1262, 655, 201, 221}

$$\int x(2+3x^2)\sqrt{5+x^4}\,dx=\frac{5}{2}\operatorname{arcsinh}\left(\frac{x^2}{\sqrt{5}}\right)+\frac{1}{2}(x^4+5)^{3/2}+\frac{1}{2}\sqrt{x^4+5}x^2$$

```
[In] Int[x*(2 + 3*x^2)*Sqrt[5 + x^4],x]
```

```
[Out] (x^2*Sqrt[5 + x^4])/2 + (5 + x^4)^(3/2)/2 + (5*ArcSinh[x^2/Sqrt[5]])/2
```

Rule 201

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Dist[a*n*(p/(n*p + 1)), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 655

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[e*((
a + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 1262

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol]
 :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + c*x^2)^p, x], x, x^2], x] /; FreeQ
[{a, c, d, e, p, q}, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{2}\text{Subst}\left(\int (2+3x)\sqrt{5+x^2}\,dx, x, x^2\right)\\
&= \frac{1}{2}\left(5+x^4\right)^{3/2} + \text{Subst}\left(\int \sqrt{5+x^2}\,dx, x, x^2\right)\\
&= \frac{1}{2}x^2\sqrt{5+x^4} + \frac{1}{2}\left(5+x^4\right)^{3/2} + \frac{5}{2}\text{Subst}\left(\int \frac{1}{\sqrt{5+x^2}}\,dx, x, x^2\right)\\
&= \frac{1}{2}x^2\sqrt{5+x^4} + \frac{1}{2}\left(5+x^4\right)^{3/2} + \frac{5}{2}\sinh^{-1}\left(\frac{x^2}{\sqrt{5}}\right)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.95

$$\int x\left(2+3x^2\right)\sqrt{5+x^4}\,dx = \frac{1}{2}\sqrt{5+x^4}\left(5+x^2+x^4\right) - \frac{5}{2}\log\left(-x^2+\sqrt{5+x^4}\right)$$

```
[In] Integrate[x*(2 + 3*x^2)*Sqrt[5 + x^4],x]
[Out] (Sqrt[5 + x^4]*(5 + x^2 + x^4))/2 - (5*Log[-x^2 + Sqrt[5 + x^4]])/2
```

Maple [A] (verified)

Time = 0.26 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.68

method	result	size
risch	$\frac{(x^4+x^2+5)\sqrt{x^4+5}}{2}+\frac{5\,\mathrm{arcsinh}\left(\frac{x^2\sqrt{5}}{5}\right)}{2}$	30
pseudoelliptic	$\frac{(x^4+x^2+5)\sqrt{x^4+5}}{2}+\frac{5\,\mathrm{arcsinh}\left(\frac{x^2\sqrt{5}}{5}\right)}{2}$	30
default	$\frac{(x^4+5)^{\frac{3}{2}}}{2}+\frac{5\,\mathrm{arcsinh}\left(\frac{x^2\sqrt{5}}{5}\right)}{2}+\frac{x^2\sqrt{x^4+5}}{2}$	34
trager	$\left(\frac{1}{2}x^4+\frac{1}{2}x^2+\frac{5}{2}\right)\sqrt{x^4+5}+\frac{5\ln\left(x^2+\sqrt{x^4+5}\right)}{2}$	36
elliptic	$\frac{5\,\mathrm{arcsinh}\left(\frac{x^2\sqrt{5}}{5}\right)}{2}+\frac{5\sqrt{x^4+5}}{2}+\frac{x^4\sqrt{x^4+5}}{2}+\frac{x^2\sqrt{x^4+5}}{2}$	46
meijerg	$-\frac{15\sqrt{5}\left(\frac{4\sqrt{\pi}}{3}-\frac{2\sqrt{\pi}\left(2+\frac{2x^4}{5}\right)\sqrt{1+\frac{x^4}{5}}}{3}\right)}{8\sqrt{\pi}}-\frac{5\left(-\frac{2\sqrt{\pi}\,x^2\sqrt{5}\,\sqrt{1+\frac{x^4}{5}}}{5}-2\sqrt{\pi}\,\mathrm{arcsinh}\left(\frac{x^2\sqrt{5}}{5}\right)\right)}{4\sqrt{\pi}}$	77

```
[In] int(x*(3*x^2+2)*(x^4+5)^(1/2),x,method=_RETURNVERBOSE)
[Out] 1/2*(x^4+x^2+5)*(x^4+5)^(1/2)+5/2*arcsinh(1/5*x^2*5^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.77

$$\int x(2+3x^2)\sqrt{5+x^4}\,dx=\frac{1}{2}\left(x^4+x^2+5\right)\sqrt{x^4+5}-\frac{5}{2}\log\left(-x^2+\sqrt{x^4+5}\right)$$

```
[In] integrate(x*(3*x^2+2)*(x^4+5)^(1/2),x, algorithm="fricas")
[Out] 1/2*(x^4 + x^2 + 5)*sqrt(x^4 + 5) - 5/2*log(-x^2 + sqrt(x^4 + 5))
```

Sympy [A] (verification not implemented)

Time = 0.52 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.09

$$\int x(2+3x^2)\sqrt{5+x^4}\,dx=\frac{x^2\sqrt{x^4+5}}{2}+\frac{3\left(\frac{x^4}{3}+\frac{5}{3}\right)\sqrt{x^4+5}}{2}+\frac{5\,\mathrm{asinh}\left(\frac{\sqrt{5}x^2}{5}\right)}{2}$$

```
[In] integrate(x*(3*x**2+2)*(x**4+5)**(1/2),x)
[Out] x**2*sqrt(x**4 + 5)/2 + 3*(x**4/3 + 5/3)*sqrt(x**4 + 5)/2 + 5*asinh(sqrt(5)
*x**2/5)/2
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 67 vs. $2(33) = 66$.

Time = 0.27 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.52

$$\int x(2+3x^2)\sqrt{5+x^4}\,dx = \frac{1}{2}\left(x^4+5\right)^{\frac{3}{2}} + \frac{5\sqrt{x^4+5}}{2\,x^2\left(\frac{x^4+5}{x^4}-1\right)} + \frac{5}{4}\log\left(\frac{\sqrt{x^4+5}}{x^2}+1\right) - \frac{5}{4}\log\left(\frac{\sqrt{x^4+5}}{x^2}-1\right)$$

```
[In] integrate(x*(3*x^2+2)*(x^4+5)^(1/2),x, algorithm="maxima")
[Out] 1/2*(x^4 + 5)^(3/2) + 5/2*sqrt(x^4 + 5)/(x^2*((x^4 + 5)/x^4 - 1)) + 5/4*log
(sqrt(x^4 + 5)/x^2 + 1) - 5/4*log(sqrt(x^4 + 5)/x^2 - 1)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.86

$$\int x(2+3x^2)\sqrt{5+x^4}\,dx = \frac{1}{2}\sqrt{x^4+5}x^2 + \frac{1}{2}\left(x^4+5\right)^{\frac{3}{2}} - \frac{5}{2}\log\left(-x^2+\sqrt{x^4+5}\right)$$

```
[In] integrate(x*(3*x^2+2)*(x^4+5)^(1/2),x, algorithm="giac")
[Out] 1/2*sqrt(x^4 + 5)*x^2 + 1/2*(x^4 + 5)^(3/2) - 5/2*log(-x^2 + sqrt(x^4 + 5))
```

Mupad [B] (verification not implemented)

Time = 7.56 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.73

$$\int x(2+3x^2)\sqrt{5+x^4}\,dx = \frac{5\,\mathrm{asinh}\left(\frac{\sqrt{5}\,x^2}{5}\right)}{2} + \sqrt{x^4+5}\left(\frac{x^4}{2}+\frac{x^2}{2}+\frac{5}{2}\right)$$

```
[In] int(x*(x^4 + 5)^(1/2)*(3*x^2 + 2),x)
[Out] (5*asinh((5^(1/2)*x^2)/5))/2 + (x^4 + 5)^(1/2)*(x^2/2 + x^4/2 + 5/2)
```

3.11 $\int \frac{(2+3x^2)\sqrt{5+x^4}}{x}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 58

$$\int \frac{(2+3x^2)\sqrt{5+x^4}}{x}\,dx = \frac{1}{4}(4+3x^2)\sqrt{5+x^4}+\frac{15}{4}\text{arcsinh}\left(\frac{x^2}{\sqrt{5}}\right)-\sqrt{5}\text{arctanh}\left(\frac{\sqrt{5+x^4}}{\sqrt{5}}\right)$$

```
[Out] 15/4*arcsinh(1/5*x^2*5^(1/2))-arctanh(1/5*(x^4+5)^(1/2)*5^(1/2))*5^(1/2)+1/
4*(3*x^2+4)*(x^4+5)^(1/2)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.350, Rules used = {1266, 829, 858, 221, 272, 65, 213}

$$\int \frac{(2+3x^2)\sqrt{5+x^4}}{x}\,dx = \frac{15}{4}\text{arcsinh}\left(\frac{x^2}{\sqrt{5}}\right)-\sqrt{5}\text{arctanh}\left(\frac{\sqrt{x^4+5}}{\sqrt{5}}\right)+\frac{1}{4}\sqrt{x^4+5}(3x^2+4)$$

```
[In] Int[((2 + 3*x^2)*Sqrt[5 + x^4])/x,x]
```

```
[Out] ((4 + 3*x^2)*Sqrt[5 + x^4])/4 + (15*ArcSinh[x^2/Sqrt[5]])/4 - Sqrt[5]*ArcTa
nh[Sqrt[5 + x^4]/Sqrt[5]]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 829

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2) - g*c*d*(2*p
+ 1) + g*c*e*(m + 2*p + 1)*x)*((a + c*x^2)^p/(c*e^2*(m + 2*p + 1)*(m + 2*p
+ 2))), x] + Dist[2*(p/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2))), Int[(d + e*x)^
m*(a + c*x^2)^(p - 1)*Simp[f*a*c*e^2*(m + 2*p + 2) + a*c*d*e*g*m - (c^2*f*d
*e*(m + 2*p + 2) - g*(c^2*d^2*(2*p + 1) + a*c*e^2*(m + 2*p + 1)))*x, x], x]
, x] /; FreeQ[{a, c, d, e, f, g, m}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p,
0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 0])) &&  !ILt
Q[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 858

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1266

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_S
ymbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + c*x^2)^p, x],
x, x^2], x] /; FreeQ[{a, c, d, e, p, q}, x] && IntegerQ[(m + 1)/2]
```

Rubi steps

$$\text{integral} = \frac{1}{2}\text{Subst}\left(\int \frac{(2+3x)\sqrt{5+x^2}}{x}\,dx, x, x^2\right)$$

$$
\begin{aligned}
&= \frac{1}{4}(4+3x^2)\sqrt{5+x^4} + \frac{1}{4}\text{Subst}\left(\int \frac{20+15x}{x\sqrt{5+x^2}}\,dx, x, x^2\right) \\
&= \frac{1}{4}(4+3x^2)\sqrt{5+x^4} + \frac{15}{4}\text{Subst}\left(\int \frac{1}{\sqrt{5+x^2}}\,dx, x, x^2\right) + 5\text{Subst}\left(\int \frac{1}{x\sqrt{5+x^2}}\,dx, x, x^2\right) \\
&= \frac{1}{4}(4+3x^2)\sqrt{5+x^4} + \frac{15}{4}\sinh^{-1}\left(\frac{x^2}{\sqrt{5}}\right) + \frac{5}{2}\text{Subst}\left(\int \frac{1}{x\sqrt{5+x}}\,dx, x, x^4\right) \\
&= \frac{1}{4}(4+3x^2)\sqrt{5+x^4} + \frac{15}{4}\sinh^{-1}\left(\frac{x^2}{\sqrt{5}}\right) + 5\text{Subst}\left(\int \frac{1}{-5+x^2}\,dx, x, \sqrt{5+x^4}\right) \\
&= \frac{1}{4}(4+3x^2)\sqrt{5+x^4} + \frac{15}{4}\sinh^{-1}\left(\frac{x^2}{\sqrt{5}}\right) - \sqrt{5}\tanh^{-1}\left(\frac{\sqrt{5+x^4}}{\sqrt{5}}\right)
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.19

$$
\int \frac{(2+3x^2)\sqrt{5+x^4}}{x}\,dx = \frac{1}{4}\left((4+3x^2)\sqrt{5+x^4} + 8\sqrt{5}\text{arctanh}\left(\frac{x^2-\sqrt{5+x^4}}{\sqrt{5}}\right) - 15\log\left(-x^2+\sqrt{5+x^4}\right)\right)
$$

```
[In] Integrate[((2 + 3*x^2)*Sqrt[5 + x^4])/x,x]
[Out] ((4 + 3*x^2)*Sqrt[5 + x^4] + 8*Sqrt[5]*ArcTanh[(x^2 - Sqrt[5 + x^4])/Sqrt[5
]] - 15*Log[-x^2 + Sqrt[5 + x^4]])/4
```

Maple [A] (verified)

Time = 0.46 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.84

method	result
default	$\frac{3x^2\sqrt{x^4+5}}{4}+\frac{15\,\mathrm{arcsinh}\left(\frac{x^2\sqrt{5}}{5}\right)}{4}+\sqrt{x^4+5}-\sqrt{5}\,\mathrm{arctanh}\left(\frac{\sqrt{5}}{\sqrt{x^4+5}}\right)$
elliptic	$\frac{3x^2\sqrt{x^4+5}}{4}+\frac{15\,\mathrm{arcsinh}\left(\frac{x^2\sqrt{5}}{5}\right)}{4}+\sqrt{x^4+5}-\sqrt{5}\,\mathrm{arctanh}\left(\frac{\sqrt{5}}{\sqrt{x^4+5}}\right)$
pseudoelliptic	$\frac{3x^2\sqrt{x^4+5}}{4}+\frac{15\,\mathrm{arcsinh}\left(\frac{x^2\sqrt{5}}{5}\right)}{4}+\sqrt{x^4+5}-\sqrt{5}\,\mathrm{arctanh}\left(\frac{\sqrt{5}}{\sqrt{x^4+5}}\right)$
trager	$\left(\frac{3x^2}{4}+1\right)\sqrt{x^4+5}-\frac{15\ln\left(x^2-\sqrt{x^4+5}\right)}{4}+\mathrm{RootOf}\left(\_Z^2-5\right)\ln\left(\frac{\sqrt{x^4+5}-\mathrm{RootOf}\left(\_Z^2-5\right)}{x^2}\right)$
meijerg	$-\frac{\sqrt{5}\left(4\sqrt{\pi}-4\sqrt{\pi}\,\sqrt{1+\frac{x^4}{5}}+4\sqrt{\pi}\,\ln\left(\frac{1}{2}+\frac{\sqrt{1+\frac{x^4}{5}}}{2}\right)-2(2-2\ln(2)+4\ln(x)-\ln(5))\sqrt{\pi}\right)}{4\sqrt{\pi}}-\frac{15\left(-\frac{2\sqrt{\pi}\,x^2\sqrt{5}\,\sqrt{1+\frac{x^4}{5}}}{5}-2\sqrt{\pi}\,a\right.}{8\sqrt{\pi}}$

```
[In] int((3*x^2+2)*(x^4+5)^(1/2)/x,x,method=_RETURNVERBOSE)
```

```
[Out] 3/4*x^2*(x^4+5)^(1/2)+15/4*arcsinh(1/5*x^2*5^(1/2))+(x^4+5)^(1/2)-5^(1/2)*a
rctanh(5^(1/2)/(x^4+5)^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.97

$$\int \frac{(2+3x^2)\sqrt{5+x^4}}{x}\,dx = \frac{1}{4}\sqrt{x^4+5}(3\,x^2+4)+\sqrt{5}\log\left(-\frac{\sqrt{5}-\sqrt{x^4+5}}{x^2}\right) - \frac{15}{4}\log\left(-x^2+\sqrt{x^4+5}\right)$$

```
[In] integrate((3*x^2+2)*(x^4+5)^(1/2)/x,x, algorithm="fricas")
```

```
[Out] 1/4*sqrt(x^4 + 5)*(3*x^2 + 4) + sqrt(5)*log(-(sqrt(5) - sqrt(x^4 + 5))/x^2)
 - 15/4*log(-x^2 + sqrt(x^4 + 5))
```

Sympy [A] (verification not implemented)

Time = 4.78 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.29

$$\int \frac{(2+3x^2)\sqrt{5+x^4}}{x}\,dx = \frac{3x^2\sqrt{x^4+5}}{4} + \sqrt{5}\left(\sqrt{\frac{x^4}{5}+1}+\frac{\log\left(\sqrt{\frac{x^4}{5}+1}-1\right)}{2}-\frac{\log\left(\sqrt{\frac{x^4}{5}+1}+1\right)}{2}\right) + \frac{15\operatorname{asinh}\left(\frac{\sqrt{5}x^2}{5}\right)}{4}$$

```
[In] integrate((3*x**2+2)*(x**4+5)**(1/2)/x,x)
```

```
[Out] 3*x**2*sqrt(x**4 + 5)/4 + sqrt(5)*(sqrt(x**4/5 + 1) + log(sqrt(x**4/5 + 1)
- 1)/2 - log(sqrt(x**4/5 + 1) + 1)/2) + 15*asinh(sqrt(5)*x**2/5)/4
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 99 vs. 2(46) = 92.

Time = 0.28 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.71

$$\int \frac{(2+3x^2)\sqrt{5+x^4}}{x}\,dx = \frac{1}{2}\sqrt{5}\log\left(-\frac{\sqrt{5}-\sqrt{x^4+5}}{\sqrt{5}+\sqrt{x^4+5}}\right)+\sqrt{x^4+5}+\frac{15\sqrt{x^4+5}}{4x^2\left(\frac{x^4+5}{x^4}-1\right)} + \frac{15}{8}\log\left(\frac{\sqrt{x^4+5}}{x^2}+1\right)-\frac{15}{8}\log\left(\frac{\sqrt{x^4+5}}{x^2}-1\right)$$

```
[In] integrate((3*x^2+2)*(x^4+5)^(1/2)/x,x, algorithm="maxima")
```

```
[Out] 1/2*sqrt(5)*log(-(sqrt(5) - sqrt(x^4 + 5))/(sqrt(5) + sqrt(x^4 + 5))) + sqr
t(x^4 + 5) + 15/4*sqrt(x^4 + 5)/(x^2*((x^4 + 5)/x^4 - 1)) + 15/8*log(sqrt(x
^4 + 5)/x^2 + 1) - 15/8*log(sqrt(x^4 + 5)/x^2 - 1)
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.31

$$\int \frac{(2+3x^2)\sqrt{5+x^4}}{x}\,dx = \frac{1}{4}\sqrt{x^4+5}\left(3\,x^2+4\right)+\sqrt{5}\log\left(-\frac{x^2+\sqrt{5}-\sqrt{x^4+5}}{x^2-\sqrt{5}-\sqrt{x^4+5}}\right) - \frac{15}{4}\log\left(-x^2+\sqrt{x^4+5}\right)$$

```
[In] integrate((3*x^2+2)*(x^4+5)^(1/2)/x,x, algorithm="giac")
[Out] 1/4*sqrt(x^4 + 5)*(3*x^2 + 4) + sqrt(5)*log(-(x^2 + sqrt(5) - sqrt(x^4 + 5)
)/(x^2 - sqrt(5) - sqrt(x^4 + 5))) - 15/4*log(-x^2 + sqrt(x^4 + 5))
```

Mupad [B] (verification not implemented)

Time = 7.61 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.78

$$\int \frac{(2+3x^2)\sqrt{5+x^4}}{x}\,dx = \frac{15\,\mathrm{asinh}\left(\frac{\sqrt{5}\,x^2}{5}\right)}{4}-\sqrt{5}\,\mathrm{atanh}\left(\frac{\sqrt{5}\,\sqrt{x^4+5}}{5}\right)+\sqrt{x^4+5}\,\left(\frac{3\,x^2}{4}+1\right)$$

```
[In] int(((x^4 + 5)^(1/2)*(3*x^2 + 2))/x,x)
[Out] (15*asinh((5^(1/2)*x^2)/5))/4 - 5^(1/2)*atanh((5^(1/2)*(x^4 + 5)^(1/2))/5)
+ (x^4 + 5)^(1/2)*((3*x^2)/4 + 1)
```

3.12 $\int \frac{(2+3x^2)\sqrt{5+x^4}}{x^3}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 59

$$\int \frac{(2+3x^2)\sqrt{5+x^4}}{x^3}\,dx = -\frac{(2-3x^2)\sqrt{5+x^4}}{2x^2} + \operatorname{arcsinh}\left(\frac{x^2}{\sqrt{5}}\right) - \frac{3}{2}\sqrt{5}\operatorname{arctanh}\left(\frac{\sqrt{5+x^4}}{\sqrt{5}}\right)$$

```
[Out] arcsinh(1/5*x^2*5^(1/2))-3/2*arctanh(1/5*(x^4+5)^(1/2)*5^(1/2))*5^(1/2)-1/2
*(-3*x^2+2)*(x^4+5)^(1/2)/x^2
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.350$, Rules used = {1266, 827, 858, 221, 272, 65, 213}

$$\int \frac{(2+3x^2)\sqrt{5+x^4}}{x^3}\,dx = \operatorname{arcsinh}\left(\frac{x^2}{\sqrt{5}}\right) - \frac{3}{2}\sqrt{5}\operatorname{arctanh}\left(\frac{\sqrt{x^4+5}}{\sqrt{5}}\right) - \frac{\sqrt{x^4+5}(2-3x^2)}{2x^2}$$

```
[In] Int[((2 + 3*x^2)*Sqrt[5 + x^4])/x^3,x]
```

```
[Out] -1/2*((2 - 3*x^2)*Sqrt[5 + x^4])/x^2 + ArcSinh[x^2/Sqrt[5]] - (3*Sqrt[5]*Ar
cTanh[Sqrt[5 + x^4]/Sqrt[5]])/2
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 827

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(e*f*(m + 2*p + 2) - d*g*(2*p + 1)
 + e*g*(m + 1)*x)*((a + c*x^2)^p/(e^2*(m + 1)*(m + 2*p + 2))), x] + Dist[p/
(e^2*(m + 1)*(m + 2*p + 2)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^(p - 1)*Simp
[g*(2*a*e + 2*a*e*m) + (g*(2*c*d + 4*c*d*p) - 2*c*e*f*(m + 2*p + 2))*x, x],
 x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && NeQ[c*d^2 + a*e^2, 0] && Rati
onalQ[p] && p > 0 && (LtQ[m, -1] || EqQ[p, 1] || (IntegerQ[p] &&  !Rational
Q[m])) && NeQ[m, -1] &&  !ILtQ[m + 2*p + 1, 0] && (IntegerQ[m] || IntegerQ[
p] || IntegersQ[2*m, 2*p])
```

Rule 858

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1266

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_S
ymbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + c*x^2)^p, x],
x, x^2], x] /; FreeQ[{a, c, d, e, p, q}, x] && IntegerQ[(m + 1)/2]
```

Rubi steps

$$\text{integral} = \frac{1}{2}\text{Subst}\left(\int \frac{(2+3x)\sqrt{5+x^2}}{x^2}\,dx, x, x^2\right)$$

$$= -\frac{(2-3x^2)\sqrt{5+x^4}}{2x^2} - \frac{1}{4}\text{Subst}\left(\int \frac{-30-4x}{x\sqrt{5+x^2}}\,dx, x, x^2\right)$$

$$= -\frac{(2-3x^2)\sqrt{5+x^4}}{2x^2} + \frac{15}{2}\text{Subst}\left(\int \frac{1}{x\sqrt{5+x^2}}\,dx, x, x^2\right) + \text{Subst}\left(\int \frac{1}{\sqrt{5+x^2}}\,dx, x, x^2\right)$$

$$= -\frac{(2-3x^2)\sqrt{5+x^4}}{2x^2} + \sinh^{-1}\left(\frac{x^2}{\sqrt{5}}\right) + \frac{15}{4}\text{Subst}\left(\int \frac{1}{x\sqrt{5+x}}\,dx, x, x^4\right)$$

$$= -\frac{(2-3x^2)\sqrt{5+x^4}}{2x^2} + \sinh^{-1}\left(\frac{x^2}{\sqrt{5}}\right) + \frac{15}{2}\text{Subst}\left(\int \frac{1}{-5+x^2}\,dx, x, \sqrt{5+x^4}\right)$$

$$= -\frac{(2-3x^2)\sqrt{5+x^4}}{2x^2} + \sinh^{-1}\left(\frac{x^2}{\sqrt{5}}\right) - \frac{3}{2}\sqrt{5}\tanh^{-1}\left(\frac{\sqrt{5+x^4}}{\sqrt{5}}\right)$$

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.20

$$\int \frac{(2+3x^2)\sqrt{5+x^4}}{x^3}\,dx = \frac{(-2+3x^2)\sqrt{5+x^4}}{2x^2} + 3\sqrt{5}\text{arctanh}\left(\frac{x^2-\sqrt{5+x^4}}{\sqrt{5}}\right) - \log\left(-x^2+\sqrt{5+x^4}\right)$$

```
[In] Integrate[((2 + 3*x^2)*Sqrt[5 + x^4])/x^3,x]
```

```
[Out] ((-2 + 3*x^2)*Sqrt[5 + x^4])/(2*x^2) + 3*Sqrt[5]*ArcTanh[(x^2 - Sqrt[5 + x^
4])/Sqrt[5]] - Log[-x^2 + Sqrt[5 + x^4]]
```

Maple [A] (verified)

Time = 0.47 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.83

method	result
risch	$-\frac{\sqrt{x^4+5}}{x^2}+\operatorname{arcsinh}\left(\frac{x^2\sqrt{5}}{5}\right)+\frac{3\sqrt{x^4+5}}{2}-\frac{3\sqrt{5}\,\operatorname{arctanh}\left(\frac{\sqrt{5}}{\sqrt{x^4+5}}\right)}{2}$
elliptic	$-\frac{\sqrt{x^4+5}}{x^2}+\operatorname{arcsinh}\left(\frac{x^2\sqrt{5}}{5}\right)+\frac{3\sqrt{x^4+5}}{2}-\frac{3\sqrt{5}\,\operatorname{arctanh}\left(\frac{\sqrt{5}}{\sqrt{x^4+5}}\right)}{2}$
default	$\frac{3\sqrt{x^4+5}}{2}-\frac{3\sqrt{5}\,\operatorname{arctanh}\left(\frac{\sqrt{5}}{\sqrt{x^4+5}}\right)}{2}-\frac{(x^4+5)^{\frac{3}{2}}}{5x^2}+\frac{x^2\sqrt{x^4+5}}{5}+\operatorname{arcsinh}\left(\frac{x^2\sqrt{5}}{5}\right)$
pseudoelliptic	$\frac{-3\sqrt{5}\,\operatorname{arctanh}\left(\frac{\sqrt{5}}{\sqrt{x^4+5}}\right)x^2+3x^2\sqrt{x^4+5}+2\,\operatorname{arcsinh}\left(\frac{x^2\sqrt{5}}{5}\right)x^2-2\sqrt{x^4+5}}{2x^2}$
trager	$\frac{(3x^2-2)\sqrt{x^4+5}}{2x^2}+\ln\left(-x^2-\sqrt{x^4+5}\right)+\frac{3\,\operatorname{RootOf}\left(\_Z^2-5\right)\ln\left(\frac{\sqrt{x^4+5}-\operatorname{RootOf}\left(\_Z^2-5\right)}{x^2}\right)}{2}$
meijerg	$-\frac{\frac{4\sqrt{\pi}\sqrt{5}\sqrt{1+\frac{x^4}{5}}}{x^2}-4\sqrt{\pi}\,\operatorname{arcsinh}\left(\frac{x^2\sqrt{5}}{5}\right)}{4\sqrt{\pi}}-\frac{3\sqrt{5}\left(4\sqrt{\pi}-4\sqrt{\pi}\sqrt{1+\frac{x^4}{5}}+4\sqrt{\pi}\ln\left(\frac{1}{2}+\frac{\sqrt{1+\frac{x^4}{5}}}{2}\right)-2(2-2\ln(2)+4\ln(x)-\ln(5) \ldots\right.}{8\sqrt{\pi}}$

```
[In] int((3*x^2+2)*(x^4+5)^(1/2)/x^3,x,method=_RETURNVERBOSE)
[Out] -(x^4+5)^(1/2)/x^2+arcsinh(1/5*x^2*5^(1/2))+3/2*(x^4+5)^(1/2)-3/2*5^(1/2)*a
rctanh(5^(1/2)/(x^4+5)^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.22

$$\int \frac{(2+3x^2)\sqrt{5+x^4}}{x^3}\,dx = \frac{3\sqrt{5}x^2\log\left(-\frac{\sqrt{5}-\sqrt{x^4+5}}{x^2}\right)-2\,x^2\log\left(-x^2+\sqrt{x^4+5}\right)-2\,x^2+\sqrt{x^4+5}(3\,x^2-2)}{2\,x^2}$$

```
[In] integrate((3*x^2+2)*(x^4+5)^(1/2)/x^3,x, algorithm="fricas")
[Out] 1/2*(3*sqrt(5)*x^2*log(-(sqrt(5) - sqrt(x^4 + 5))/x^2) - 2*x^2*log(-x^2 + s
qrt(x^4 + 5)) - 2*x^2 + sqrt(x^4 + 5)*(3*x^2 - 2))/x^2
```

Sympy [A] (verification not implemented)

Time = 3.31 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.41

$$\int \frac{(2+3x^2)\sqrt{5+x^4}}{x^3}\,dx = -\frac{x^2}{\sqrt{x^4+5}} + \frac{3\sqrt{x^4+5}}{2} + \frac{3\sqrt{5}\log(x^4)}{4} - \frac{3\sqrt{5}\log\left(\sqrt{\frac{x^4}{5}+1}+1\right)}{2} + \operatorname{asinh}\left(\frac{\sqrt{5}x^2}{5}\right) - \frac{5}{x^2\sqrt{x^4+5}}$$

```
[In] integrate((3*x**2+2)*(x**4+5)**(1/2)/x**3,x)
[Out] -x**2/sqrt(x**4 + 5) + 3*sqrt(x**4 + 5)/2 + 3*sqrt(5)*log(x**4)/4 - 3*sqrt(
5)*log(sqrt(x**4/5 + 1) + 1)/2 + asinh(sqrt(5)*x**2/5) - 5/(x**2*sqrt(x**4
+ 5))
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.49

$$\int \frac{(2+3x^2)\sqrt{5+x^4}}{x^3}\,dx = \frac{3}{4}\sqrt{5}\log\left(-\frac{\sqrt{5}-\sqrt{x^4+5}}{\sqrt{5}+\sqrt{x^4+5}}\right) + \frac{3}{2}\sqrt{x^4+5} - \frac{\sqrt{x^4+5}}{x^2} + \frac{1}{2}\log\left(\frac{\sqrt{x^4+5}}{x^2}+1\right) - \frac{1}{2}\log\left(\frac{\sqrt{x^4+5}}{x^2}-1\right)$$

```
[In] integrate((3*x^2+2)*(x^4+5)^(1/2)/x^3,x, algorithm="maxima")
[Out] 3/4*sqrt(5)*log(-(sqrt(5) - sqrt(x^4 + 5))/(sqrt(5) + sqrt(x^4 + 5))) + 3/2
*sqrt(x^4 + 5) - sqrt(x^4 + 5)/x^2 + 1/2*log(sqrt(x^4 + 5)/x^2 + 1) - 1/2*l
og(sqrt(x^4 + 5)/x^2 - 1)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.54

$$\int \frac{(2+3x^2)\sqrt{5+x^4}}{x^3}\,dx = \frac{3}{2}\sqrt{5}\log\left(-\frac{x^2+\sqrt{5}-\sqrt{x^4+5}}{x^2-\sqrt{5}-\sqrt{x^4+5}}\right) + \frac{3}{2}\sqrt{x^4+5} + \frac{10}{\left(x^2-\sqrt{x^4+5}\right)^2-5} - \log\left(-x^2+\sqrt{x^4+5}\right)$$

```
[In] integrate((3*x^2+2)*(x^4+5)^(1/2)/x^3,x, algorithm="giac")
```

```
[Out] 3/2*sqrt(5)*log(-(x^2 + sqrt(5) - sqrt(x^4 + 5))/(x^2 - sqrt(5) - sqrt(x^4
+ 5))) + 3/2*sqrt(x^4 + 5) + 10/((x^2 - sqrt(x^4 + 5))^2 - 5) - log(-x^2 +
sqrt(x^4 + 5))
```

Mupad [B] (verification not implemented)

Time = 7.89 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.86

$$\int \frac{(2+3x^2)\sqrt{5+x^4}}{x^3}\,dx = \operatorname{asinh}\left(\frac{\sqrt{5}\,x^2}{5}\right)+\frac{3\sqrt{x^4+5}}{2}-\frac{\sqrt{x^4+5}}{x^2}+\frac{\sqrt{5}\,\operatorname{atan}\left(\frac{\sqrt{5}\,\sqrt{x^4+5}\,1\mathrm{i}}{5}\right)\,3\mathrm{i}}{2}$$

```
[In] int(((x^4 + 5)^(1/2)*(3*x^2 + 2))/x^3,x)
```

```
[Out] asinh((5^(1/2)*x^2)/5) + (5^(1/2)*atan((5^(1/2)*(x^4 + 5)^(1/2)*1i)/5)*3i)/
2 + (3*(x^4 + 5)^(1/2))/2 - (x^4 + 5)^(1/2)/x^2
```

3.13 $\int \frac{(2+3x^2)\sqrt{5+x^4}}{x^5} dx$

Optimal result

Integrand size = 20, antiderivative size = 63

$$\int \frac{(2+3x^2)\sqrt{5+x^4}}{x^5} dx = -\frac{(1+3x^2)\sqrt{5+x^4}}{2x^4} + \frac{3}{2}\text{arcsinh}\left(\frac{x^2}{\sqrt{5}}\right) - \frac{\text{arctanh}\left(\frac{\sqrt{5+x^4}}{\sqrt{5}}\right)}{2\sqrt{5}}$$

```
[Out] 3/2*arcsinh(1/5*x^2*5^(1/2))-1/10*arctanh(1/5*(x^4+5)^(1/2)*5^(1/2))*5^(1/2
)-1/2*(3*x^2+1)*(x^4+5)^(1/2)/x^4
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.350, Rules used = {1266, 825, 858, 221, 272, 65, 213}

$$\int \frac{(2+3x^2)\sqrt{5+x^4}}{x^5} dx = \frac{3}{2}\text{arcsinh}\left(\frac{x^2}{\sqrt{5}}\right) - \frac{\text{arctanh}\left(\frac{\sqrt{x^4+5}}{\sqrt{5}}\right)}{2\sqrt{5}} - \frac{\sqrt{x^4+5}(3x^2+1)}{2x^4}$$

```
[In] Int[((2 + 3*x^2)*Sqrt[5 + x^4])/x^5,x]
```

```
[Out] -1/2*((1 + 3*x^2)*Sqrt[5 + x^4])/x^4 + (3*ArcSinh[x^2/Sqrt[5]])/2 - ArcTanh
[Sqrt[5 + x^4]/Sqrt[5]]/(2*Sqrt[5])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
```

```
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 825

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(-(d + e*x)^(m + 1))*((a + c*x^2)^p/(e^2*(m + 1)*(m
+ 2)*(c*d^2 + a*e^2)))*((d*g - e*f*(m + 2))*(c*d^2 + a*e^2) - 2*c*d^2*p*(e*
f - d*g) - e*(g*(m + 1)*(c*d^2 + a*e^2) + 2*c*d*p*(e*f - d*g))*x), x] - Dis
t[p/(e^2*(m + 1)*(m + 2)*(c*d^2 + a*e^2)), Int[(d + e*x)^(m + 2)*(a + c*x^2
)^(p - 1)*Simp[2*a*c*e*(e*f - d*g)*(m + 2) - c*(2*c*d*(d*g*(2*p + 1) - e*f*
(m + 2*p + 2)) - 2*a*e^2*g*(m + 1))*x, x], x], x] /; FreeQ[{a, c, d, e, f,
g}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p, 0] && LtQ[m, -2] && LtQ[m + 2*p,
0] &&  !ILtQ[m + 2*p + 3, 0]
```

Rule 858

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1266

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_S
ymbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + c*x^2)^p, x],
x, x^2], x] /; FreeQ[{a, c, d, e, p, q}, x] && IntegerQ[(m + 1)/2]
```

Rubi steps

$$\text{integral} = \frac{1}{2}\text{Subst}\left(\int \frac{(2+3x)\sqrt{5+x^2}}{x^3}\,dx, x, x^2\right)$$
$$= -\frac{(1+3x^2)\sqrt{5+x^4}}{2x^4} - \frac{1}{40}\text{Subst}\left(\int \frac{-20-60x}{x\sqrt{5+x^2}}\,dx, x, x^2\right)$$
$$= -\frac{(1+3x^2)\sqrt{5+x^4}}{2x^4} + \frac{1}{2}\text{Subst}\left(\int \frac{1}{x\sqrt{5+x^2}}\,dx, x, x^2\right) + \frac{3}{2}\text{Subst}\left(\int \frac{1}{\sqrt{5+x^2}}\,dx, x, x^2\right)$$
$$= -\frac{(1+3x^2)\sqrt{5+x^4}}{2x^4} + \frac{3}{2}\sinh^{-1}\left(\frac{x^2}{\sqrt{5}}\right) + \frac{1}{4}\text{Subst}\left(\int \frac{1}{x\sqrt{5+x}}\,dx, x, x^4\right)$$
$$= -\frac{(1+3x^2)\sqrt{5+x^4}}{2x^4} + \frac{3}{2}\sinh^{-1}\left(\frac{x^2}{\sqrt{5}}\right) + \frac{1}{2}\text{Subst}\left(\int \frac{1}{-5+x^2}\,dx, x, \sqrt{5+x^4}\right)$$
$$= -\frac{(1+3x^2)\sqrt{5+x^4}}{2x^4} + \frac{3}{2}\sinh^{-1}\left(\frac{x^2}{\sqrt{5}}\right) - \frac{\tanh^{-1}\left(\frac{\sqrt{5+x^4}}{\sqrt{5}}\right)}{2\sqrt{5}}$$

Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.14

$$\int \frac{(2+3x^2)\sqrt{5+x^4}}{x^5}\,dx = -\frac{(1+3x^2)\sqrt{5+x^4}}{2x^4} + \frac{\text{arctanh}\left(\frac{x^2-\sqrt{5+x^4}}{\sqrt{5}}\right)}{\sqrt{5}} - \frac{3}{2}\log\left(-x^2+\sqrt{5+x^4}\right)$$

```
[In] Integrate[((2 + 3*x^2)*Sqrt[5 + x^4])/x^5,x]
[Out] -1/2*((1 + 3*x^2)*Sqrt[5 + x^4])/x^4 + ArcTanh[(x^2 - Sqrt[5 + x^4])/Sqrt[5
]]/Sqrt[5] - (3*Log[-x^2 + Sqrt[5 + x^4]])/2
```

Maple [A] (verified)

Time = 0.44 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.86

method	result
elliptic	$\frac{3\,\mathrm{arcsinh}\left(\frac{x^{2}\sqrt{5}}{5}\right)}{2}-\frac{\sqrt{5}\,\mathrm{arctanh}\left(\frac{\sqrt{5}}{\sqrt{x^{4}+5}}\right)}{10}-\frac{3\sqrt{x^{4}+5}}{2x^{2}}-\frac{\sqrt{x^{4}+5}}{2x^{4}}$
risch	$-\frac{3x^{6}+x^{4}+15x^{2}+5}{2x^{4}\sqrt{x^{4}+5}}+\frac{3\,\mathrm{arcsinh}\left(\frac{x^{2}\sqrt{5}}{5}\right)}{2}-\frac{\sqrt{5}\,\mathrm{arctanh}\left(\frac{\sqrt{5}}{\sqrt{x^{4}+5}}\right)}{10}$
pseudoelliptic	$\frac{-\sqrt{5}\,\mathrm{arctanh}\left(\frac{\sqrt{5}}{\sqrt{x^{4}+5}}\right)x^{4}+15\,\mathrm{arcsinh}\left(\frac{x^{2}\sqrt{5}}{5}\right)x^{4}-15x^{2}\sqrt{x^{4}+5}-5\sqrt{x^{4}+5}}{10x^{4}}$
trager	$-\frac{(3x^{2}+1)\sqrt{x^{4}+5}}{2x^{4}}+\frac{3\ln\left(-x^{2}-\sqrt{x^{4}+5}\right)}{2}+\frac{\mathrm{RootOf}\left(\_Z^{2}-5\right)\ln\left(\frac{\sqrt{x^{4}+5}-\mathrm{RootOf}\left(\_Z^{2}-5\right)}{x^{2}}\right)}{10}$
default	$-\frac{3(x^{4}+5)^{\frac{3}{2}}}{10x^{2}}+\frac{3x^{2}\sqrt{x^{4}+5}}{10}+\frac{3\,\mathrm{arcsinh}\left(\frac{x^{2}\sqrt{5}}{5}\right)}{2}-\frac{(x^{4}+5)^{\frac{3}{2}}}{10x^{4}}+\frac{\sqrt{x^{4}+5}}{10}-\frac{\sqrt{5}\,\mathrm{arctanh}\left(\frac{\sqrt{5}}{\sqrt{x^{4}+5}}\right)}{10}$
meijerg	$-\frac{\sqrt{5}\left(-\frac{5\sqrt{\pi}\left(8+\frac{4x^{4}}{5}\right)}{4x^{4}}+\frac{10\sqrt{\pi}\sqrt{1+\frac{x^{4}}{5}}}{x^{4}}+2\sqrt{\pi}\,\ln\left(\frac{1}{2}+\frac{\sqrt{1+\frac{x^{4}}{5}}}{2}\right)-(-2\ln(2)-1+4\ln(x)-\ln(5))\sqrt{\pi}+\frac{10\sqrt{\pi}}{x^{4}}\right)}{20\sqrt{\pi}}-\frac{3\left(\frac{4\sqrt{\pi}\,\sqrt{5}\,\sqrt{\cdots}}{x^{2}}\cdots\right.}{\cdots}$

```
[In] int((3*x^2+2)*(x^4+5)^(1/2)/x^5,x,method=_RETURNVERBOSE)
```

```
[Out] 3/2*arcsinh(1/5*x^2*5^(1/2))-1/10*5^(1/2)*arctanh(5^(1/2)/(x^4+5)^(1/2))-3/
2*(x^4+5)^(1/2)/x^2-1/2*(x^4+5)^(1/2)/x^4
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.14

$$\int \frac{(2+3x^{2})\sqrt{5+x^{4}}}{x^{5}}\,dx = \frac{\sqrt{5}x^{4}\log\left(-\frac{\sqrt{5}-\sqrt{x^{4}+5}}{x^{2}}\right)-15\,x^{4}\log\left(-x^{2}+\sqrt{x^{4}+5}\right)-15\,x^{4}-5\,\sqrt{x^{4}+5}(3\,x^{2}+1)}{10\,x^{4}}$$

```
[In] integrate((3*x^2+2)*(x^4+5)^(1/2)/x^5,x, algorithm="fricas")
```

```
[Out] 1/10*(sqrt(5)*x^4*log(-(sqrt(5) - sqrt(x^4 + 5))/x^2) - 15*x^4*log(-x^2 + s
qrt(x^4 + 5)) - 15*x^4 - 5*sqrt(x^4 + 5)*(3*x^2 + 1))/x^4
```

Sympy [A] (verification not implemented)

Time = 2.85 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.21

$$\int \frac{(2+3x^2)\sqrt{5+x^4}}{x^5}\,dx = -\frac{3x^2}{2\sqrt{x^4+5}} - \frac{\sqrt{5}\operatorname{asinh}\left(\frac{\sqrt{5}}{x^2}\right)}{10} + \frac{3\operatorname{asinh}\left(\frac{\sqrt{5}x^2}{5}\right)}{2} - \frac{\sqrt{1+\frac{5}{x^4}}}{2x^2} - \frac{15}{2x^2\sqrt{x^4+5}}$$

```
[In] integrate((3*x**2+2)*(x**4+5)**(1/2)/x**5,x)
[Out] -3*x**2/(2*sqrt(x**4 + 5)) - sqrt(5)*asinh(sqrt(5)/x**2)/10 + 3*asinh(sqrt(
5)*x**2/5)/2 - sqrt(1 + 5/x**4)/(2*x**2) - 15/(2*x**2*sqrt(x**4 + 5))
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.44

$$\int \frac{(2+3x^2)\sqrt{5+x^4}}{x^5}\,dx = \frac{1}{20}\sqrt{5}\log\left(-\frac{\sqrt{5}-\sqrt{x^4+5}}{\sqrt{5}+\sqrt{x^4+5}}\right) - \frac{3\sqrt{x^4+5}}{2x^2} - \frac{\sqrt{x^4+5}}{2x^4} + \frac{3}{4}\log\left(\frac{\sqrt{x^4+5}}{x^2}+1\right) - \frac{3}{4}\log\left(\frac{\sqrt{x^4+5}}{x^2}-1\right)$$

```
[In] integrate((3*x^2+2)*(x^4+5)^(1/2)/x^5,x, algorithm="maxima")
[Out] 1/20*sqrt(5)*log(-(sqrt(5) - sqrt(x^4 + 5))/(sqrt(5) + sqrt(x^4 + 5))) - 3/
2*sqrt(x^4 + 5)/x^2 - 1/2*sqrt(x^4 + 5)/x^4 + 3/4*log(sqrt(x^4 + 5)/x^2 + 1
) - 3/4*log(sqrt(x^4 + 5)/x^2 - 1)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 129 vs. 2(49) = 98.

Time = 0.28 (sec) , antiderivative size = 129, normalized size of antiderivative = 2.05

$$\int \frac{(2+3x^2)\sqrt{5+x^4}}{x^5}\,dx = \frac{1}{10}\sqrt{5}\log\left(-\frac{x^2+\sqrt{5}-\sqrt{x^4+5}}{x^2-\sqrt{5}-\sqrt{x^4+5}}\right) + \frac{\left(x^2-\sqrt{x^4+5}\right)^3+15\left(x^2-\sqrt{x^4+5}\right)^2+5x^2-5\sqrt{x^4+5}-75}{\left(\left(x^2-\sqrt{x^4+5}\right)^2-5\right)^2} - \frac{3}{2}\log\left(-x^2+\sqrt{x^4+5}\right)$$

```
[In] integrate((3*x^2+2)*(x^4+5)^(1/2)/x^5,x, algorithm="giac")
```

```
[Out] 1/10*sqrt(5)*log(-(x^2 + sqrt(5) - sqrt(x^4 + 5))/(x^2 - sqrt(5) - sqrt(x^4
 + 5))) + ((x^2 - sqrt(x^4 + 5))^3 + 15*(x^2 - sqrt(x^4 + 5))^2 + 5*x^2 - 5
*sqrt(x^4 + 5) - 75)/((x^2 - sqrt(x^4 + 5))^2 - 5)^2 - 3/2*log(-x^2 + sqrt(
x^4 + 5))
```

Mupad [B] (verification not implemented)

Time = 7.74 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.89

$$\int \frac{(2+3x^2)\sqrt{5+x^4}}{x^5}\,dx = \frac{3\,\mathrm{asinh}\left(\frac{\sqrt{5}\,x^2}{5}\right)}{2} - \frac{3\,\sqrt{x^4+5}}{2\,x^2} - \frac{\sqrt{x^4+5}}{2\,x^4} + \frac{\sqrt{5}\,\mathrm{atan}\left(\frac{\sqrt{5}\,\sqrt{x^4+5}\,1\mathrm{i}}{5}\right)\,1\mathrm{i}}{10}$$

```
[In] int(((x^4 + 5)^(1/2)*(3*x^2 + 2))/x^5,x)
```

```
[Out] (3*asinh((5^(1/2)*x^2)/5))/2 + (5^(1/2)*atan((5^(1/2)*(x^4 + 5)^(1/2)*1i)/5
)*1i)/10 - (3*(x^4 + 5)^(1/2))/(2*x^2) - (x^4 + 5)^(1/2)/(2*x^4)
```

3.14 $\int \frac{(2+3x^2)\sqrt{5+x^4}}{x^7}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 58

$$\int \frac{(2+3x^2)\sqrt{5+x^4}}{x^7}\,dx = -\frac{3\sqrt{5+x^4}}{4x^4} - \frac{(5+x^4)^{3/2}}{15x^6} - \frac{3\operatorname{arctanh}\left(\frac{\sqrt{5+x^4}}{\sqrt{5}}\right)}{4\sqrt{5}}$$

```
[Out] -1/15*(x^4+5)^(3/2)/x^6-3/20*arctanh(1/5*(x^4+5)^(1/2)*5^(1/2))*5^(1/2)-3/4
*(x^4+5)^(1/2)/x^4
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {1266, 821, 272, 43, 65, 213}

$$\int \frac{(2+3x^2)\sqrt{5+x^4}}{x^7}\,dx = -\frac{3\operatorname{arctanh}\left(\frac{\sqrt{x^4+5}}{\sqrt{5}}\right)}{4\sqrt{5}} - \frac{3\sqrt{x^4+5}}{4x^4} - \frac{(x^4+5)^{3/2}}{15x^6}$$

```
[In] Int[((2 + 3*x^2)*Sqrt[5 + x^4])/x^7,x]
```

```
[Out] (-3*Sqrt[5 + x^4])/(4*x^4) - (5 + x^4)^(3/2)/(15*x^6) - (3*ArcTanh[Sqrt[5 +
 x^4]/Sqrt[5]])/(4*Sqrt[5])
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !IntegerQ[n] && GtQ[n, 0]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 821

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1
)/(2*(p + 1)*(c*d^2 + a*e^2))), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2),
Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m,
p}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 1266

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_S
ymbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + c*x^2)^p, x],
x, x^2], x] /; FreeQ[{a, c, d, e, p, q}, x] && IntegerQ[(m + 1)/2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{2}\text{Subst}\left(\int \frac{(2+3x)\sqrt{5+x^2}}{x^4}\,dx, x, x^2\right) \\
&= -\frac{(5+x^4)^{3/2}}{15x^6} + \frac{3}{2}\text{Subst}\left(\int \frac{\sqrt{5+x^2}}{x^3}\,dx, x, x^2\right) \\
&= -\frac{(5+x^4)^{3/2}}{15x^6} + \frac{3}{4}\text{Subst}\left(\int \frac{\sqrt{5+x}}{x^2}\,dx, x, x^4\right)
\end{aligned}$$

$$= -\frac{3\sqrt{5+x^4}}{4x^4} - \frac{(5+x^4)^{3/2}}{15x^6} + \frac{3}{8}\text{Subst}\left(\int \frac{1}{x\sqrt{5+x}}\,dx, x, x^4\right)$$

$$= -\frac{3\sqrt{5+x^4}}{4x^4} - \frac{(5+x^4)^{3/2}}{15x^6} + \frac{3}{4}\text{Subst}\left(\int \frac{1}{-5+x^2}\,dx, x, \sqrt{5+x^4}\right)$$

$$= -\frac{3\sqrt{5+x^4}}{4x^4} - \frac{(5+x^4)^{3/2}}{15x^6} - \frac{3\tanh^{-1}\left(\frac{\sqrt{5+x^4}}{\sqrt{5}}\right)}{4\sqrt{5}}$$

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.03

$$\int \frac{(2+3x^2)\sqrt{5+x^4}}{x^7}\,dx = -\frac{\sqrt{5+x^4}(20+45x^2+4x^4)}{60x^6} + \frac{3\text{arctanh}\left(\frac{x^2-\sqrt{5+x^4}}{\sqrt{5}}\right)}{2\sqrt{5}}$$

```
[In] Integrate[((2 + 3*x^2)*Sqrt[5 + x^4])/x^7,x]
[Out] -1/60*(Sqrt[5 + x^4]*(20 + 45*x^2 + 4*x^4))/x^6 + (3*ArcTanh[(x^2 - Sqrt[5
+ x^4])/Sqrt[5]])/(2*Sqrt[5])
```

Maple [A] (verified)

Time = 0.36 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.83

method	result
pseudoelliptic	$\frac{-9\sqrt{5}\,\text{arctanh}\left(\frac{\sqrt{5}}{\sqrt{x^4+5}}\right)x^6-(4x^4+45x^2+20)\sqrt{x^4+5}}{60x^6}$
default	$-\frac{(x^4+5)^{\frac{3}{2}}}{15x^6}-\frac{3(x^4+5)^{\frac{3}{2}}}{20x^4}+\frac{3\sqrt{x^4+5}}{20}-\frac{3\sqrt{5}\,\text{arctanh}\left(\frac{\sqrt{5}}{\sqrt{x^4+5}}\right)}{20}$
trager	$-\frac{(4x^4+45x^2+20)\sqrt{x^4+5}}{60x^6}-\frac{3\,\text{RootOf}\left(\_Z^2-5\right)\ln\left(\frac{\text{RootOf}\left(\_Z^2-5\right)+\sqrt{x^4+5}}{x^2}\right)}{20}$
risch	$-\frac{4x^8+45x^6+40x^4+225x^2+100}{60x^6\sqrt{x^4+5}}-\frac{3\sqrt{5}\,\text{arctanh}\left(\frac{\sqrt{5}}{\sqrt{x^4+5}}\right)}{20}$
elliptic	$-\frac{\sqrt{x^4+5}}{15x^2}-\frac{3\sqrt{5}\,\text{arctanh}\left(\frac{\sqrt{5}}{\sqrt{x^4+5}}\right)}{20}-\frac{3\sqrt{x^4+5}}{4x^4}-\frac{\sqrt{x^4+5}}{3x^6}$
meijerg	$-\frac{\sqrt{5}\left(1+\frac{x^4}{5}\right)^{\frac{3}{2}}}{3x^6}-\frac{3\sqrt{5}\left(-\frac{5\sqrt{\pi}\left(8+\frac{4x^4}{5}\right)}{4x^4}+\frac{10\sqrt{\pi}\sqrt{1+\frac{x^4}{5}}}{x^4}+2\sqrt{\pi}\ln\left(\frac{1}{2}+\frac{\sqrt{1+\frac{x^4}{5}}}{2}\right)-(-2\ln(2)-1+4\ln(x)-\ln(5))\sqrt{\pi}+\frac{10\sqrt{}}{x}\right.}{40\sqrt{\pi}}$

```
[In] int((3*x^2+2)*(x^4+5)^(1/2)/x^7,x,method=_RETURNVERBOSE)
```

[Out] 1/60*(-9*5^(1/2)*arctanh(5^(1/2)/(x^4+5)^(1/2))*x^6-(4*x^4+45*x^2+20)*(x^4+5)^(1/2))/x^6

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.02

$$\int \frac{(2+3x^2)\sqrt{5+x^4}}{x^7}\,dx = \frac{9\sqrt{5}x^6\log\left(-\frac{\sqrt{5}-\sqrt{x^4+5}}{x^2}\right)-4\,x^6-(4\,x^4+45\,x^2+20)\sqrt{x^4+5}}{60\,x^6}$$

[In] integrate((3*x^2+2)*(x^4+5)^(1/2)/x^7,x, algorithm="fricas")

[Out] 1/60*(9*sqrt(5)*x^6*log(-(sqrt(5) - sqrt(x^4 + 5))/x^2) - 4*x^6 - (4*x^4 + 45*x^2 + 20)*sqrt(x^4 + 5))/x^6

Sympy [A] (verification not implemented)

Time = 2.57 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.09

$$\int \frac{(2+3x^2)\sqrt{5+x^4}}{x^7}\,dx = -\frac{\sqrt{1+\frac{5}{x^4}}}{15}-\frac{3\sqrt{5}\operatorname{asinh}\left(\frac{\sqrt{5}}{x^2}\right)}{20}-\frac{3\sqrt{1+\frac{5}{x^4}}}{4x^2}-\frac{\sqrt{1+\frac{5}{x^4}}}{3x^4}$$

[In] integrate((3*x**2+2)*(x**4+5)**(1/2)/x**7,x)

[Out] -sqrt(1 + 5/x**4)/15 - 3*sqrt(5)*asinh(sqrt(5)/x**2)/20 - 3*sqrt(1 + 5/x**4)/(4*x**2) - sqrt(1 + 5/x**4)/(3*x**4)

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.02

$$\int \frac{(2+3x^2)\sqrt{5+x^4}}{x^7}\,dx = \frac{3}{40}\sqrt{5}\log\left(-\frac{\sqrt{5}-\sqrt{x^4+5}}{\sqrt{5}+\sqrt{x^4+5}}\right)-\frac{3\sqrt{x^4+5}}{4\,x^4}-\frac{(x^4+5)^{\frac{3}{2}}}{15\,x^6}$$

[In] integrate((3*x^2+2)*(x^4+5)^(1/2)/x^7,x, algorithm="maxima")

[Out] 3/40*sqrt(5)*log(-(sqrt(5) - sqrt(x^4 + 5))/(sqrt(5) + sqrt(x^4 + 5))) - 3/4*sqrt(x^4 + 5)/x^4 - 1/15*(x^4 + 5)^(3/2)/x^6

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 116 vs. $2(43) = 86$.

Time = 0.27 (sec) , antiderivative size = 116, normalized size of antiderivative = 2.00

$$\int \frac{(2+3x^2)\sqrt{5+x^4}}{x^7}\,dx$$
$$= \frac{3}{20}\sqrt{5}\log\left(-\frac{x^2+\sqrt{5}-\sqrt{x^4+5}}{x^2-\sqrt{5}-\sqrt{x^4+5}}\right)$$
$$+ \frac{9\left(x^2-\sqrt{x^4+5}\right)^5+12\left(x^2-\sqrt{x^4+5}\right)^4-225\,x^2+225\sqrt{x^4+5}+100}{6\left(\left(x^2-\sqrt{x^4+5}\right)^2-5\right)^3}$$

```
[In] integrate((3*x^2+2)*(x^4+5)^(1/2)/x^7,x, algorithm="giac")
```

```
[Out] 3/20*sqrt(5)*log(-(x^2 + sqrt(5) - sqrt(x^4 + 5))/(x^2 - sqrt(5) - sqrt(x^4
 + 5))) + 1/6*(9*(x^2 - sqrt(x^4 + 5))^5 + 12*(x^2 - sqrt(x^4 + 5))^4 - 225
*x^2 + 225*sqrt(x^4 + 5) + 100)/((x^2 - sqrt(x^4 + 5))^2 - 5)^3
```

Mupad [B] (verification not implemented)

Time = 7.87 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.74

$$\int \frac{(2+3x^2)\sqrt{5+x^4}}{x^7}\,dx = -\frac{3\sqrt{5}\,\mathrm{atanh}\left(\frac{\sqrt{5}\sqrt{x^4+5}}{5}\right)}{20} - \frac{3\sqrt{x^4+5}}{4\,x^4} - \frac{\left(x^4+5\right)^{3/2}}{15\,x^6}$$

```
[In] int(((x^4 + 5)^(1/2)*(3*x^2 + 2))/x^7,x)
```

```
[Out] - (3*5^(1/2)*atanh((5^(1/2)*(x^4 + 5)^(1/2))/5))/20 - (3*(x^4 + 5)^(1/2))/(
4*x^4) - (x^4 + 5)^(3/2)/(15*x^6)
```

3.15 $\int x^4(2+3x^2)\sqrt{5+x^4}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 208

$$\begin{aligned}\int x^4(2+3x^2)\sqrt{5+x^4}\,dx &= \frac{20}{21}x\sqrt{5+x^4}+\frac{2}{3}x^3\sqrt{5+x^4}-\frac{10x\sqrt{5+x^4}}{\sqrt{5}+x^2}+\frac{1}{21}x^5(6+7x^2)\sqrt{5+x^4}\\ &+\frac{10\sqrt[4]{5}(\sqrt{5}+x^2)\sqrt{\frac{5+x^4}{(\sqrt{5}+x^2)^2}}E\left(2\arctan\left(\frac{x}{\sqrt[4]{5}}\right)|\frac{1}{2}\right)}{\sqrt{5+x^4}}\\ &-\frac{5\sqrt[4]{5}(21+2\sqrt{5})(\sqrt{5}+x^2)\sqrt{\frac{5+x^4}{(\sqrt{5}+x^2)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{x}{\sqrt[4]{5}}\right),\frac{1}{2}\right)}{21\sqrt{5+x^4}}\end{aligned}$$

```
[Out] 20/21*x*(x^4+5)^(1/2)+2/3*x^3*(x^4+5)^(1/2)+1/21*x^5*(7*x^2+6)*(x^4+5)^(1/2
)-10*x*(x^4+5)^(1/2)/(x^2+5^(1/2))+10*5^(1/4)*(cos(2*arctan(1/5*x*5^(3/4)))
^2)^(1/2)/cos(2*arctan(1/5*x*5^(3/4)))*EllipticE(sin(2*arctan(1/5*x*5^(3/4)
)),1/2*2^(1/2))*(x^2+5^(1/2))*((x^4+5)/(x^2+5^(1/2))^2)^(1/2)/(x^4+5)^(1/2)
-5/21*5^(1/4)*(cos(2*arctan(1/5*x*5^(3/4)))^2)^(1/2)/cos(2*arctan(1/5*x*5^(
3/4)))*EllipticF(sin(2*arctan(1/5*x*5^(3/4))),1/2*2^(1/2))*(x^2+5^(1/2))*(2
1+2*5^(1/2))*((x^4+5)/(x^2+5^(1/2))^2)^(1/2)/(x^4+5)^(1/2)
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 208, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {1288, 1294, 1212, 226, 1210}

$$\int x^4(2+3x^2)\sqrt{5+x^4}\,dx = -\frac{5\sqrt[4]{5}(21+2\sqrt{5})(x^2+\sqrt{5})\sqrt{\frac{x^4+5}{(x^2+\sqrt{5})^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{x}{\sqrt[4]{5}}\right),\frac{1}{2}\right)}{21\sqrt{x^4+5}} + \frac{10\sqrt[4]{5}(x^2+\sqrt{5})\sqrt{\frac{x^4+5}{(x^2+\sqrt{5})^2}}E\left(2\arctan\left(\frac{x}{\sqrt[4]{5}}\right)|\frac{1}{2}\right)}{\sqrt{x^4+5}} + \frac{20}{21}\sqrt{x^4+5}x + \frac{2}{3}\sqrt{x^4+5}x^3 - \frac{10\sqrt{x^4+5}x}{x^2+\sqrt{5}} + \frac{1}{21}(7x^2+6)\sqrt{x^4+5}x^5$$

```
[In] Int[x^4*(2 + 3*x^2)*Sqrt[5 + x^4],x]
```

```
[Out] (20*x*Sqrt[5 + x^4])/21 + (2*x^3*Sqrt[5 + x^4])/3 - (10*x*Sqrt[5 + x^4])/(S
qrt[5] + x^2) + (x^5*(6 + 7*x^2)*Sqrt[5 + x^4])/21 + (10*5^(1/4)*(Sqrt[5] +
 x^2)*Sqrt[(5 + x^4)/(Sqrt[5] + x^2)^2]*EllipticE[2*ArcTan[x/5^(1/4)], 1/2]
)/Sqrt[5 + x^4] - (5*5^(1/4)*(21 + 2*Sqrt[5])*(Sqrt[5] + x^2)*Sqrt[(5 + x^4
)/(Sqrt[5] + x^2)^2]*EllipticF[2*ArcTan[x/5^(1/4)], 1/2])/(21*Sqrt[5 + x^4]
)
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 1210

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Rule 1212

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + c*x^4], x], x] - Dist[e/q, I
nt[(1 - q*x^2)/Sqrt[a + c*x^4], x], x] /; NeQ[e + d*q, 0]] /; FreeQ[{a, c,
```

```
d, e}, x] && PosQ[c/a]
```

Rule 1288

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (c_.)*(x_)^4)^(p_.), x
_Symbol] :> Simp[(f*x)^(m + 1)*(a + c*x^4)^p*((c*d*(m + 4*p + 3) + c*e*(4*p
 + m + 1)*x^2)/(c*f*(4*p + m + 1)*(m + 4*p + 3))), x] + Dist[4*a*(p/((4*p +
 m + 1)*(m + 4*p + 3))), Int[(f*x)^m*(a + c*x^4)^(p - 1)*Simp[d*(m + 4*p +
3) + e*(4*p + m + 1)*x^2, x], x], x] /; FreeQ[{a, c, d, e, f, m}, x] && GtQ
[p, 0] && NeQ[4*p + m + 1, 0] && NeQ[m + 4*p + 3, 0] && IntegerQ[2*p] && (I
ntegerQ[p] || IntegerQ[m])
```

Rule 1294

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (c_.)*(x_)^4)^(p_), x_
Symbol] :> Simp[e*f*(f*x)^(m - 1)*((a + c*x^4)^(p + 1)/(c*(m + 4*p + 3))),
x] - Dist[f^2/(c*(m + 4*p + 3)), Int[(f*x)^(m - 2)*(a + c*x^4)^p*(a*e*(m -
1) - c*d*(m + 4*p + 3)*x^2), x], x] /; FreeQ[{a, c, d, e, f, p}, x] && GtQ[
m, 1] && NeQ[m + 4*p + 3, 0] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m
])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{21}x^5(6+7x^2)\sqrt{5+x^4}+\frac{10}{63}\int\frac{x^4(18+21x^2)}{\sqrt{5+x^4}}\,dx\\
&= \frac{2}{3}x^3\sqrt{5+x^4}+\frac{1}{21}x^5(6+7x^2)\sqrt{5+x^4}-\frac{2}{63}\int\frac{x^2(315-90x^2)}{\sqrt{5+x^4}}\,dx\\
&= \frac{20}{21}x\sqrt{5+x^4}+\frac{2}{3}x^3\sqrt{5+x^4}+\frac{1}{21}x^5(6+7x^2)\sqrt{5+x^4}+\frac{2}{189}\int\frac{-450-945x^2}{\sqrt{5+x^4}}\,dx\\
&= \frac{20}{21}x\sqrt{5+x^4}+\frac{2}{3}x^3\sqrt{5+x^4}+\frac{1}{21}x^5(6+7x^2)\sqrt{5+x^4}\\
&\quad+\left(10\sqrt{5}\right)\int\frac{1-\frac{x^2}{\sqrt{5}}}{\sqrt{5+x^4}}\,dx-\frac{1}{21}\left(10\left(10+21\sqrt{5}\right)\right)\int\frac{1}{\sqrt{5+x^4}}\,dx\\
&= \frac{20}{21}x\sqrt{5+x^4}+\frac{2}{3}x^3\sqrt{5+x^4}-\frac{10x\sqrt{5+x^4}}{\sqrt{5}+x^2}+\frac{1}{21}x^5(6+7x^2)\sqrt{5+x^4}\\
&\quad+\frac{10\sqrt[4]{5}(\sqrt{5}+x^2)\sqrt{\frac{5+x^4}{(\sqrt{5}+x^2)^2}}E\left(2\tan^{-1}\left(\frac{x}{\sqrt[4]{5}}\right)|\frac{1}{2}\right)}{\sqrt{5+x^4}}\\
&\quad-\frac{5\sqrt[4]{5}(21+2\sqrt{5})\,(\sqrt{5}+x^2)\sqrt{\frac{5+x^4}{(\sqrt{5}+x^2)^2}}F\left(2\tan^{-1}\left(\frac{x}{\sqrt[4]{5}}\right)|\frac{1}{2}\right)}{21\sqrt{5+x^4}}
\end{aligned}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 4.64 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.39

$$\int x^4(2+3x^2)\sqrt{5+x^4}\,dx = \frac{1}{21}x\left(6(5+x^4)^{3/2}+7x^2(5+x^4)^{3/2}\right.$$
$$\left.-30\sqrt{5}\,\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{1}{4},\frac{5}{4},-\frac{x^4}{5}\right)-35\sqrt{5}x^2\,\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{3}{4},\frac{7}{4},-\frac{x^4}{5}\right)\right)$$

```
[In] Integrate[x^4*(2 + 3*x^2)*Sqrt[5 + x^4],x]
```

```
[Out] (x*(6*(5 + x^4)^(3/2) + 7*x^2*(5 + x^4)^(3/2) - 30*Sqrt[5]*Hypergeometric2F
1[-1/2, 1/4, 5/4, -1/5*x^4] - 35*Sqrt[5]*x^2*Hypergeometric2F1[-1/2, 3/4, 7
/4, -1/5*x^4]))/21
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4.

Time = 5.14 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.19

method	result
meijerg	$\frac{3\sqrt{5}\,x^7\,{}_2F_1\left(-\frac{1}{2},\frac{7}{4};\frac{11}{4};-\frac{x^4}{5}\right)}{7}+\frac{2\sqrt{5}\,x^5\,{}_2F_1\left(-\frac{1}{2},\frac{5}{4};\frac{9}{4};-\frac{x^4}{5}\right)}{5}$
risch	$\frac{x(7x^6+6x^4+14x^2+20)\sqrt{x^4+5}}{21}-\frac{4\sqrt{5}\,\sqrt{25-5i\sqrt{5}\,x^2}\,\sqrt{25+5i\sqrt{5}\,x^2}\,F\left(\frac{x\sqrt{5}\,\sqrt{i\sqrt{5}}}{5},i\right)}{21\sqrt{i\sqrt{5}}\,\sqrt{x^4+5}}-\frac{2i\sqrt{25-5i\sqrt{5}\,x^2}\,\sqrt{25+5i\sqrt{5}\,x^2}\,\left(F\left(\frac{x\sqrt{}}{}\right.\right.}{\sqrt{i\sqrt{5}}\,\sqrt{x^4}}$
default	$\frac{x^7\sqrt{x^4+5}}{3}+\frac{2x^3\sqrt{x^4+5}}{3}-\frac{2i\sqrt{25-5i\sqrt{5}\,x^2}\,\sqrt{25+5i\sqrt{5}\,x^2}\,\left(F\left(\frac{x\sqrt{5}\,\sqrt{i\sqrt{5}}}{5},i\right)-E\left(\frac{x\sqrt{5}\,\sqrt{i\sqrt{5}}}{5},i\right)\right)}{\sqrt{i\sqrt{5}}\,\sqrt{x^4+5}}+\frac{2x^5\sqrt{x^4+5}}{7}+\frac{20x\sqrt{x^4+}}{21}$
elliptic	$\frac{x^7\sqrt{x^4+5}}{3}+\frac{2x^3\sqrt{x^4+5}}{3}-\frac{2i\sqrt{25-5i\sqrt{5}\,x^2}\,\sqrt{25+5i\sqrt{5}\,x^2}\,\left(F\left(\frac{x\sqrt{5}\,\sqrt{i\sqrt{5}}}{5},i\right)-E\left(\frac{x\sqrt{5}\,\sqrt{i\sqrt{5}}}{5},i\right)\right)}{\sqrt{i\sqrt{5}}\,\sqrt{x^4+5}}+\frac{2x^5\sqrt{x^4+5}}{7}+\frac{20x\sqrt{x^4+}}{21}$

```
[In] int(x^4*(3*x^2+2)*(x^4+5)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 3/7*5^(1/2)*x^7*hypergeom([-1/2,7/4],[11/4],-1/5*x^4)+2/5*5^(1/2)*x^5*hyper
geom([-1/2,5/4],[9/4],-1/5*x^4)
```

Fricas [A] (verification not implemented)

none

Time = 0.08 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.33

$$\int x^4(2+3x^2)\sqrt{5+x^4}\,dx = -\frac{210\,(-5)^{\frac{3}{4}}\,xE(\arcsin\left(\frac{(-5)^{\frac{1}{4}}}{x}\right)|-1) - 190\,(-5)^{\frac{3}{4}}\,xF(\arcsin\left(\frac{(-5)^{\frac{1}{4}}}{x}\right)|-1) - (7\,x^8+6\,x^6+14\,x^4+20\,x}{21\,x}$$

```
[In] integrate(x^4*(3*x^2+2)*(x^4+5)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/21*(210*(-5)^(3/4)*x*elliptic_e(arcsin((-5)^(1/4)/x), -1) - 190*(-5)^(3/
4)*x*elliptic_f(arcsin((-5)^(1/4)/x), -1) - (7*x^8 + 6*x^6 + 14*x^4 + 20*x^
2 - 210)*sqrt(x^4 + 5))/x
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.03 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.38

$$\int x^4(2+3x^2)\sqrt{5+x^4}\,dx = \frac{3\sqrt{5}x^7\Gamma\left(\frac{7}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{7}{4}\\ \frac{11}{4}\end{matrix}\middle|\frac{x^4e^{i\pi}}{5}\right)}{4\Gamma\left(\frac{11}{4}\right)} + \frac{\sqrt{5}x^5\Gamma\left(\frac{5}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{5}{4}\\ \frac{9}{4}\end{matrix}\middle|\frac{x^4e^{i\pi}}{5}\right)}{2\Gamma\left(\frac{9}{4}\right)}$$

```
[In] integrate(x**4*(3*x**2+2)*(x**4+5)**(1/2),x)
```

```
[Out] 3*sqrt(5)*x**7*gamma(7/4)*hyper((-1/2, 7/4), (11/4,), x**4*exp_polar(I*pi)/
5)/(4*gamma(11/4)) + sqrt(5)*x**5*gamma(5/4)*hyper((-1/2, 5/4), (9/4,), x**
4*exp_polar(I*pi)/5)/(2*gamma(9/4))
```

Maxima [F]

$$\int x^4(2+3x^2)\sqrt{5+x^4}\,dx = \int \sqrt{x^4+5}\,(3\,x^2+2)\,x^4\,dx$$

```
[In] integrate(x^4*(3*x^2+2)*(x^4+5)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(x^4 + 5)*(3*x^2 + 2)*x^4, x)
```

Giac [F]

$$\int x^4 (2+3x^2) \sqrt{5+x^4} \, dx = \int \sqrt{x^4+5}(3 x^2+2) x^4 \, dx$$

```
[In] integrate(x^4*(3*x^2+2)*(x^4+5)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(x^4 + 5)*(3*x^2 + 2)*x^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^4 (2+3x^2) \sqrt{5+x^4} \, dx = \int x^4 \sqrt{x^4+5} \left(3 x^2+2\right) \, dx$$

```
[In] int(x^4*(x^4 + 5)^(1/2)*(3*x^2 + 2),x)
[Out] int(x^4*(x^4 + 5)^(1/2)*(3*x^2 + 2), x)
```

3.16 $\int x^2(2+3x^2)\sqrt{5+x^4}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 192

$$\begin{aligned}\int x^2(2+3x^2)\sqrt{5+x^4}\,dx &= \frac{10}{7}x\sqrt{5+x^4}+\frac{4x\sqrt{5+x^4}}{\sqrt{5}+x^2}+\frac{1}{35}x^3(14+15x^2)\sqrt{5+x^4}\\ &\quad-\frac{4\sqrt[4]{5}(\sqrt{5}+x^2)\sqrt{\frac{5+x^4}{(\sqrt{5}+x^2)^2}}E\left(2\arctan\left(\frac{x}{\sqrt[4]{5}}\right)|\frac{1}{2}\right)}{\sqrt{5+x^4}}\\ &\quad+\frac{\sqrt[4]{5}(14-5\sqrt{5})(\sqrt{5}+x^2)\sqrt{\frac{5+x^4}{(\sqrt{5}+x^2)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{x}{\sqrt[4]{5}}\right),\frac{1}{2}\right)}{7\sqrt{5+x^4}}\end{aligned}$$

```
[Out] 10/7*x*(x^4+5)^(1/2)+1/35*x^3*(15*x^2+14)*(x^4+5)^(1/2)+4*x*(x^4+5)^(1/2)/(
x^2+5^(1/2))-4*5^(1/4)*(cos(2*arctan(1/5*x*5^(3/4)))^2)^(1/2)/cos(2*arctan(
1/5*x*5^(3/4)))*EllipticE(sin(2*arctan(1/5*x*5^(3/4))),1/2*2^(1/2))*(x^2+5^
(1/2))*((x^4+5)/(x^2+5^(1/2))^2)^(1/2)/(x^4+5)^(1/2)+1/7*5^(1/4)*(cos(2*arc
tan(1/5*x*5^(3/4)))^2)^(1/2)/cos(2*arctan(1/5*x*5^(3/4)))*EllipticF(sin(2*a
rctan(1/5*x*5^(3/4))),1/2*2^(1/2))*(14-5*5^(1/2))*(x^2+5^(1/2))*((x^4+5)/(x
^2+5^(1/2))^2)^(1/2)/(x^4+5)^(1/2)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 192, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {1288, 1294, 1212, 226, 1210}

$$
\begin{aligned}
&\int x^2(2+3x^2)\sqrt{5+x^4}\,dx \\
&= \frac{\sqrt[4]{5}(14-5\sqrt{5})\,(x^2+\sqrt{5})\sqrt{\frac{x^4+5}{(x^2+\sqrt{5})^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{x}{\sqrt[4]{5}}\right),\frac{1}{2}\right)}{7\sqrt{x^4+5}} \\
&\quad - \frac{4\sqrt[4]{5}(x^2+\sqrt{5})\sqrt{\frac{x^4+5}{(x^2+\sqrt{5})^2}}E\left(2\arctan\left(\frac{x}{\sqrt[4]{5}}\right)|\frac{1}{2}\right)}{\sqrt{x^4+5}} \\
&\quad + \frac{10}{7}\sqrt{x^4+5}x + \frac{4\sqrt{x^4+5}x}{x^2+\sqrt{5}} + \frac{1}{35}(15x^2+14)\sqrt{x^4+5}x^3
\end{aligned}
$$

```
[In] Int[x^2*(2 + 3*x^2)*Sqrt[5 + x^4],x]
```

```
[Out] (10*x*Sqrt[5 + x^4])/7 + (4*x*Sqrt[5 + x^4])/(Sqrt[5] + x^2) + (x^3*(14 + 1
5*x^2)*Sqrt[5 + x^4])/35 - (4*5^(1/4)*(Sqrt[5] + x^2)*Sqrt[(5 + x^4)/(Sqrt[
5] + x^2)^2]*EllipticE[2*ArcTan[x/5^(1/4)], 1/2])/Sqrt[5 + x^4] + (5^(1/4)*
(14 - 5*Sqrt[5])*(Sqrt[5] + x^2)*Sqrt[(5 + x^4)/(Sqrt[5] + x^2)^2]*Elliptic
F[2*ArcTan[x/5^(1/4)], 1/2])/(7*Sqrt[5 + x^4])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 1210

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Rule 1212

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + c*x^4], x], x] - Dist[e/q, I
nt[(1 - q*x^2)/Sqrt[a + c*x^4], x], x] /; NeQ[e + d*q, 0]] /; FreeQ[{a, c,
d, e}, x] && PosQ[c/a]
```

Rule 1288

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (c_.)*(x_)^4)^(p_.), x
_Symbol] :> Simp[(f*x)^(m + 1)*(a + c*x^4)^p*((c*d*(m + 4*p + 3) + c*e*(4*p
 + m + 1)*x^2)/(c*f*(4*p + m + 1)*(m + 4*p + 3))), x] + Dist[4*a*(p/((4*p +
 m + 1)*(m + 4*p + 3))), Int[(f*x)^m*(a + c*x^4)^(p - 1)*Simp[d*(m + 4*p +
3) + e*(4*p + m + 1)*x^2, x], x], x] /; FreeQ[{a, c, d, e, f, m}, x] && GtQ
[p, 0] && NeQ[4*p + m + 1, 0] && NeQ[m + 4*p + 3, 0] && IntegerQ[2*p] && (I
ntegerQ[p] || IntegerQ[m])
```

Rule 1294

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (c_.)*(x_)^4)^(p_), x_
Symbol] :> Simp[e*f*(f*x)^(m - 1)*((a + c*x^4)^(p + 1)/(c*(m + 4*p + 3))),
x] - Dist[f^2/(c*(m + 4*p + 3)), Int[(f*x)^(m - 2)*(a + c*x^4)^p*(a*e*(m -
1) - c*d*(m + 4*p + 3)*x^2), x], x] /; FreeQ[{a, c, d, e, f, p}, x] && GtQ[
m, 1] && NeQ[m + 4*p + 3, 0] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m
])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{35}x^3\left(14+15x^2\right)\sqrt{5+x^4}+\frac{2}{7}\int\frac{x^2\left(14+15x^2\right)}{\sqrt{5+x^4}}\,dx \\
&= \frac{10}{7}x\sqrt{5+x^4}+\frac{1}{35}x^3\left(14+15x^2\right)\sqrt{5+x^4}-\frac{2}{21}\int\frac{75-42x^2}{\sqrt{5+x^4}}\,dx \\
&= \frac{10}{7}x\sqrt{5+x^4}+\frac{1}{35}x^3\left(14+15x^2\right)\sqrt{5+x^4} \\
&\quad -\left(4\sqrt{5}\right)\int\frac{1-\frac{x^2}{\sqrt{5}}}{\sqrt{5+x^4}}\,dx-\frac{1}{7}\left(2\left(25-14\sqrt{5}\right)\right)\int\frac{1}{\sqrt{5+x^4}}\,dx \\
&= \frac{10}{7}x\sqrt{5+x^4}+\frac{4x\sqrt{5+x^4}}{\sqrt{5}+x^2}+\frac{1}{35}x^3\left(14+15x^2\right)\sqrt{5+x^4} \\
&\quad -\frac{4\sqrt[4]{5}\left(\sqrt{5}+x^2\right)\sqrt{\frac{5+x^4}{\left(\sqrt{5}+x^2\right)^2}}E\left(2\tan^{-1}\left(\frac{x}{\sqrt[4]{5}}\right)|\frac{1}{2}\right)}{\sqrt{5+x^4}} \\
&\quad +\frac{\sqrt[4]{5}\left(14-5\sqrt{5}\right)\left(\sqrt{5}+x^2\right)\sqrt{\frac{5+x^4}{\left(\sqrt{5}+x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{x}{\sqrt[4]{5}}\right)|\frac{1}{2}\right)}{7\sqrt{5+x^4}}
\end{aligned}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 3.96 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.35

$$\int x^2\left(2+3x^2\right)\sqrt{5+x^4}\,dx = \frac{1}{21}x\left(9\left(5+x^4\right)^{3/2} - 45\sqrt{5}\,\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{1}{4},\frac{5}{4},-\frac{x^4}{5}\right) + 14\sqrt{5}x^2\,\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{3}{4},\frac{7}{4},-\frac{x^4}{5}\right)\right)$$

```
[In] Integrate[x^2*(2 + 3*x^2)*Sqrt[5 + x^4],x]
```

```
[Out] (x*(9*(5 + x^4)^(3/2) - 45*Sqrt[5]*Hypergeometric2F1[-1/2, 1/4, 5/4, -1/5*x
^4] + 14*Sqrt[5]*x^2*Hypergeometric2F1[-1/2, 3/4, 7/4, -1/5*x^4]))/21
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4.

Time = 1.84 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.21

method	result
meijerg	$\frac{3\sqrt{5}\,x^5\,{}_2F_1\left(-\frac{1}{2},\frac{5}{4};\frac{9}{4};-\frac{x^4}{5}\right)}{5}+\frac{2\sqrt{5}\,x^3\,{}_2F_1\left(-\frac{1}{2},\frac{3}{4};\frac{7}{4};-\frac{x^4}{5}\right)}{3}$
risch	$\frac{x\left(15x^4+14x^2+50\right)\sqrt{x^4+5}}{35}-\frac{2\sqrt{5}\,\sqrt{25-5i\sqrt{5}\,x^2}\,\sqrt{25+5i\sqrt{5}\,x^2}\,F\left(\frac{x\sqrt{5}\,\sqrt{i\sqrt{5}}}{5},i\right)}{7\sqrt{i\sqrt{5}}\,\sqrt{x^4+5}}+\frac{4i\sqrt{25-5i\sqrt{5}\,x^2}\,\sqrt{25+5i\sqrt{5}\,x^2}\,\left(F\left(\frac{x\sqrt{5}\,\sqrt{i}}{5}\right.\right.}{5\sqrt{i\sqrt{5}}\,\sqrt{x^4+5}}$
default	$\frac{3x^5\sqrt{x^4+5}}{7}+\frac{10x\sqrt{x^4+5}}{7}-\frac{2\sqrt{5}\,\sqrt{25-5i\sqrt{5}\,x^2}\,\sqrt{25+5i\sqrt{5}\,x^2}\,F\left(\frac{x\sqrt{5}\,\sqrt{i\sqrt{5}}}{5},i\right)}{7\sqrt{i\sqrt{5}}\,\sqrt{x^4+5}}+\frac{2x^3\sqrt{x^4+5}}{5}+\frac{4i\sqrt{25-5i\sqrt{5}\,x^2}\,\sqrt{25+5i\sqrt{5}}}{5}$
elliptic	$\frac{3x^5\sqrt{x^4+5}}{7}+\frac{10x\sqrt{x^4+5}}{7}-\frac{2\sqrt{5}\,\sqrt{25-5i\sqrt{5}\,x^2}\,\sqrt{25+5i\sqrt{5}\,x^2}\,F\left(\frac{x\sqrt{5}\,\sqrt{i\sqrt{5}}}{5},i\right)}{7\sqrt{i\sqrt{5}}\,\sqrt{x^4+5}}+\frac{2x^3\sqrt{x^4+5}}{5}+\frac{4i\sqrt{25-5i\sqrt{5}\,x^2}\,\sqrt{25+5i\sqrt{5}}}{5}$

```
[In] int(x^2*(3*x^2+2)*(x^4+5)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 3/5*5^(1/2)*x^5*hypergeom([-1/2,5/4],[9/4],-1/5*x^4)+2/3*5^(1/2)*x^3*hyperg
eom([-1/2,3/4],[7/4],-1/5*x^4)
```

Fricas [A] (verification not implemented)

none

Time = 0.08 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.33

$$\int x^2\left(2+3x^2\right)\sqrt{5+x^4}\,dx = \frac{140\,(-5)^{\frac{3}{4}}\,xE\left(\arcsin\left(\frac{(-5)^{\frac{1}{4}}}{x}\right)\,|\,-1\right) - 190\,(-5)^{\frac{3}{4}}\,xF\left(\arcsin\left(\frac{(-5)^{\frac{1}{4}}}{x}\right)\,|\,-1\right) + \left(15\,x^6 + 14\,x^4 + 50\,x^2 + 140\right)}{35\,x}$$

```
[In] integrate(x^2*(3*x^2+2)*(x^4+5)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/35*(140*(-5)^(3/4)*x*elliptic_e(arcsin((-5)^(1/4)/x), -1) - 190*(-5)^(3/4
)*x*elliptic_f(arcsin((-5)^(1/4)/x), -1) + (15*x^6 + 14*x^4 + 50*x^2 + 140)
*sqrt(x^4 + 5))/x
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.91 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.41

$$\int x^2\left(2+3x^2\right)\sqrt{5+x^4}\,dx = \frac{3\sqrt{5}x^5\Gamma\left(\frac{5}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{5}{4}\\ \frac{9}{4}\end{matrix}\middle|\frac{x^4e^{i\pi}}{5}\right)}{4\Gamma\left(\frac{9}{4}\right)} + \frac{\sqrt{5}x^3\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{3}{4}\\ \frac{7}{4}\end{matrix}\middle|\frac{x^4e^{i\pi}}{5}\right)}{2\Gamma\left(\frac{7}{4}\right)}$$

```
[In] integrate(x**2*(3*x**2+2)*(x**4+5)**(1/2),x)
```

```
[Out] 3*sqrt(5)*x**5*gamma(5/4)*hyper((-1/2, 5/4), (9/4,), x**4*exp_polar(I*pi)/5
)/(4*gamma(9/4)) + sqrt(5)*x**3*gamma(3/4)*hyper((-1/2, 3/4), (7/4,), x**4*
exp_polar(I*pi)/5)/(2*gamma(7/4))
```

Maxima [F]

$$\int x^2\left(2+3x^2\right)\sqrt{5+x^4}\,dx = \int \sqrt{x^4+5}\left(3\,x^2+2\right)x^2\,dx$$

```
[In] integrate(x^2*(3*x^2+2)*(x^4+5)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(x^4 + 5)*(3*x^2 + 2)*x^2, x)
```

Giac [F]

$$\int x^2(2+3x^2)\sqrt{5+x^4}\,dx=\int\sqrt{x^4+5}(3\,x^2+2)x^2\,dx$$

```
[In] integrate(x^2*(3*x^2+2)*(x^4+5)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(x^4 + 5)*(3*x^2 + 2)*x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^2(2+3x^2)\sqrt{5+x^4}\,dx=\int x^2\sqrt{x^4+5}\,\left(3\,x^2+2\right)\,dx$$

```
[In] int(x^2*(x^4 + 5)^(1/2)*(3*x^2 + 2),x)
[Out] int(x^2*(x^4 + 5)^(1/2)*(3*x^2 + 2), x)
```

3.17 $\int (2+3x^2)\sqrt{5+x^4}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 176

$$\begin{aligned}\int (2+3x^2)\sqrt{5+x^4}\,dx &= \frac{6x\sqrt{5+x^4}}{\sqrt{5}+x^2}+\frac{1}{15}x(10+9x^2)\sqrt{5+x^4} \\ &\quad -\frac{6\sqrt[4]{5}(\sqrt{5}+x^2)\sqrt{\frac{5+x^4}{(\sqrt{5}+x^2)^2}}E\left(2\arctan\left(\frac{x}{\sqrt[4]{5}}\right)|\frac{1}{2}\right)}{\sqrt{5+x^4}} \\ &\quad +\frac{\sqrt[4]{5}(9+2\sqrt{5})(\sqrt{5}+x^2)\sqrt{\frac{5+x^4}{(\sqrt{5}+x^2)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{x}{\sqrt[4]{5}}\right),\frac{1}{2}\right)}{3\sqrt{5+x^4}}\end{aligned}$$

```
[Out] 1/15*x*(9*x^2+10)*(x^4+5)^(1/2)+6*x*(x^4+5)^(1/2)/(x^2+5^(1/2))-6*5^(1/4)*(
cos(2*arctan(1/5*x*5^(3/4)))^2)^(1/2)/cos(2*arctan(1/5*x*5^(3/4)))*Elliptic
E(sin(2*arctan(1/5*x*5^(3/4))),1/2*2^(1/2))*(x^2+5^(1/2))*((x^4+5)/(x^2+5^(
1/2))^2)^(1/2)/(x^4+5)^(1/2)+1/3*5^(1/4)*(cos(2*arctan(1/5*x*5^(3/4)))^2)^(
1/2)/cos(2*arctan(1/5*x*5^(3/4)))*EllipticF(sin(2*arctan(1/5*x*5^(3/4))),1/
2*2^(1/2))*(x^2+5^(1/2))*(9+2*5^(1/2))*((x^4+5)/(x^2+5^(1/2))^2)^(1/2)/(x^4
+5)^(1/2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 176, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.235$, Rules used = {1191, 1212, 226, 1210}

$$\begin{aligned}&\int \left(2+3x^2\right)\sqrt{5+x^4}\,dx\\&=\frac{\sqrt[4]{5}(9+2\sqrt{5})\left(x^2+\sqrt{5}\right)\sqrt{\frac{x^4+5}{\left(x^2+\sqrt{5}\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{x}{\sqrt[4]{5}}\right),\frac{1}{2}\right)}{3\sqrt{x^4+5}}\\&\quad-\frac{6\sqrt[4]{5}(x^2+\sqrt{5})\sqrt{\frac{x^4+5}{\left(x^2+\sqrt{5}\right)^2}}E\left(2\arctan\left(\frac{x}{\sqrt[4]{5}}\right)|\frac{1}{2}\right)}{\sqrt{x^4+5}}\\&\quad+\frac{1}{15}\left(9x^2+10\right)\sqrt{x^4+5}x+\frac{6\sqrt{x^4+5}x}{x^2+\sqrt{5}}\end{aligned}$$

```
[In] Int[(2 + 3*x^2)*Sqrt[5 + x^4],x]
```

```
[Out] (6*x*Sqrt[5 + x^4])/(Sqrt[5] + x^2) + (x*(10 + 9*x^2)*Sqrt[5 + x^4])/15 - (
6*5^(1/4)*(Sqrt[5] + x^2)*Sqrt[(5 + x^4)/(Sqrt[5] + x^2)^2]*EllipticE[2*Arc
Tan[x/5^(1/4)], 1/2])/Sqrt[5 + x^4] + (5^(1/4)*(9 + 2*Sqrt[5])*(Sqrt[5] + x
^2)*Sqrt[(5 + x^4)/(Sqrt[5] + x^2)^2]*EllipticF[2*ArcTan[x/5^(1/4)], 1/2])/
(3*Sqrt[5 + x^4])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 1191

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[x*(
d*(4*p + 3) + e*(4*p + 1)*x^2)*((a + c*x^4)^p/((4*p + 1)*(4*p + 3))), x] +
Dist[2*(p/((4*p + 1)*(4*p + 3))), Int[Simp[2*a*d*(4*p + 3) + (2*a*e*(4*p +
1))*x^2, x]*(a + c*x^4)^(p - 1), x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*
d^2 + a*e^2, 0] && GtQ[p, 0] && FractionQ[p] && IntegerQ[2*p]
```

Rule 1210

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
```

```
}, x] && PosQ[c/a]
```

Rule 1212

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + c*x^4], x], x] - Dist[e/q, I
nt[(1 - q*x^2)/Sqrt[a + c*x^4], x], x] /; NeQ[e + d*q, 0]] /; FreeQ[{a, c,
d, e}, x] && PosQ[c/a]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{15}x\left(10+9x^2\right)\sqrt{5+x^4}+\frac{1}{15}\int\frac{100+90x^2}{\sqrt{5+x^4}}\,dx\\
&= \frac{1}{15}x\left(10+9x^2\right)\sqrt{5+x^4}-\left(6\sqrt{5}\right)\int\frac{1-\frac{x^2}{\sqrt{5}}}{\sqrt{5+x^4}}\,dx+\frac{1}{3}\left(2\left(10+9\sqrt{5}\right)\right)\int\frac{1}{\sqrt{5+x^4}}\,dx\\
&= \frac{6x\sqrt{5+x^4}}{\sqrt{5}+x^2}+\frac{1}{15}x\left(10+9x^2\right)\sqrt{5+x^4}\\
&\quad-\frac{6\sqrt[4]{5}\left(\sqrt{5}+x^2\right)\sqrt{\frac{5+x^4}{\left(\sqrt{5}+x^2\right)^2}}E\left(2\tan^{-1}\left(\frac{x}{\sqrt[4]{5}}\right)|\frac{1}{2}\right)}{\sqrt{5+x^4}}\\
&\quad+\frac{\sqrt[4]{5}\left(9+2\sqrt{5}\right)\left(\sqrt{5}+x^2\right)\sqrt{\frac{5+x^4}{\left(\sqrt{5}+x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{x}{\sqrt[4]{5}}\right)|\frac{1}{2}\right)}{3\sqrt{5+x^4}}
\end{aligned}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 3.57 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.27

$$\int\left(2+3x^2\right)\sqrt{5+x^4}\,dx=\sqrt{5}x\left(2\,\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{1}{4},\frac{5}{4},-\frac{x^4}{5}\right)+x^2\,\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{3}{4},\frac{7}{4},-\frac{x^4}{5}\right)\right)$$

```
[In] Integrate[(2 + 3*x^2)*Sqrt[5 + x^4],x]
```

```
[Out] Sqrt[5]*x*(2*Hypergeometric2F1[-1/2, 1/4, 5/4, -1/5*x^4] + x^2*Hypergeometr
ic2F1[-1/2, 3/4, 7/4, -1/5*x^4])
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4.

Time = 1.24 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.21

method	result
meijerg	$2\sqrt{5}\, x\, {}_2F_1\left(-\frac{1}{2},\frac{1}{4};\frac{5}{4};-\frac{x^4}{5}\right)+\sqrt{5}\, x^3\, {}_2F_1\left(-\frac{1}{2},\frac{3}{4};\frac{7}{4};-\frac{x^4}{5}\right)$
risch	$\frac{x(9x^2+10)\sqrt{x^4+5}}{15}+\frac{4\sqrt{5}\,\sqrt{25-5i\sqrt{5}\,x^2}\,\sqrt{25+5i\sqrt{5}\,x^2}\,F\left(\frac{x\sqrt{5}\,\sqrt{i\sqrt{5}}}{5},i\right)}{15\sqrt{i\sqrt{5}}\,\sqrt{x^4+5}}+\frac{6i\sqrt{25-5i\sqrt{5}\,x^2}\,\sqrt{25+5i\sqrt{5}\,x^2}\left(F\left(\frac{x\sqrt{5}\,\sqrt{i\sqrt{5}}}{5},i\right)-\right.}{5\sqrt{i\sqrt{5}}\,\sqrt{x^4+5}}$
default	$\frac{2x\sqrt{x^4+5}}{3}+\frac{4\sqrt{5}\,\sqrt{25-5i\sqrt{5}\,x^2}\,\sqrt{25+5i\sqrt{5}\,x^2}\,F\left(\frac{x\sqrt{5}\,\sqrt{i\sqrt{5}}}{5},i\right)}{15\sqrt{i\sqrt{5}}\,\sqrt{x^4+5}}+\frac{3x^3\sqrt{x^4+5}}{5}+\frac{6i\sqrt{25-5i\sqrt{5}\,x^2}\,\sqrt{25+5i\sqrt{5}\,x^2}\left(F\left(\frac{x\sqrt{5}\,\sqrt{i\sqrt{}}}{5}\right.\right.}{5\sqrt{i\sqrt{5}}\,\sqrt{x^4+5}}$
elliptic	$\frac{2x\sqrt{x^4+5}}{3}+\frac{4\sqrt{5}\,\sqrt{25-5i\sqrt{5}\,x^2}\,\sqrt{25+5i\sqrt{5}\,x^2}\,F\left(\frac{x\sqrt{5}\,\sqrt{i\sqrt{5}}}{5},i\right)}{15\sqrt{i\sqrt{5}}\,\sqrt{x^4+5}}+\frac{3x^3\sqrt{x^4+5}}{5}+\frac{6i\sqrt{25-5i\sqrt{5}\,x^2}\,\sqrt{25+5i\sqrt{5}\,x^2}\left(F\left(\frac{x\sqrt{5}\,\sqrt{i\sqrt{}}}{5}\right.\right.}{5\sqrt{i\sqrt{5}}\,\sqrt{x^4+5}}$

```
[In] int((3*x^2+2)*(x^4+5)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2*5^(1/2)*x*hypergeom([-1/2,1/4],[5/4],-1/5*x^4)+5^(1/2)*x^3*hypergeom([-1/
2,3/4],[7/4],-1/5*x^4)
```

Fricas [A] (verification not implemented)

none

Time = 0.08 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.33

$$\int (2+3x^2)\sqrt{5+x^4}\,dx$$
$$=\frac{90\,(-5)^{\frac{3}{4}}\,xE(\arcsin\left(\frac{(-5)^{\frac{1}{4}}}{x}\right)\,|-1)-70\,(-5)^{\frac{3}{4}}\,xF(\arcsin\left(\frac{(-5)^{\frac{1}{4}}}{x}\right)\,|-1)+(9\,x^4+10\,x^2+90)\sqrt{x^4+5}}{15\,x}$$

```
[In] integrate((3*x^2+2)*(x^4+5)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/15*(90*(-5)^(3/4)*x*elliptic_e(arcsin((-5)^(1/4)/x), -1) - 70*(-5)^(3/4)*
x*elliptic_f(arcsin((-5)^(1/4)/x), -1) + (9*x^4 + 10*x^2 + 90)*sqrt(x^4 + 5
))/x
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.82 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.43

$$\int \left(2+3x^2\right)\sqrt{5+x^4}\,dx=\frac{3\sqrt{5}x^3\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{3}{4}\\ \frac{7}{4}\end{matrix}\middle|\frac{x^4e^{i\pi}}{5}\right)}{4\Gamma\left(\frac{7}{4}\right)}+\frac{\sqrt{5}x\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{1}{4}\\ \frac{5}{4}\end{matrix}\middle|\frac{x^4e^{i\pi}}{5}\right)}{2\Gamma\left(\frac{5}{4}\right)}$$

```
[In] integrate((3*x**2+2)*(x**4+5)**(1/2),x)
[Out] 3*sqrt(5)*x**3*gamma(3/4)*hyper((-1/2, 3/4), (7/4,), x**4*exp_polar(I*pi)/5
)/(4*gamma(7/4)) + sqrt(5)*x*gamma(1/4)*hyper((-1/2, 1/4), (5/4,), x**4*exp
_polar(I*pi)/5)/(2*gamma(5/4))
```

Maxima [F]

$$\int \left(2+3x^2\right)\sqrt{5+x^4}\,dx=\int \sqrt{x^4+5}\left(3\,x^2+2\right)\,dx$$

```
[In] integrate((3*x^2+2)*(x^4+5)^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(x^4 + 5)*(3*x^2 + 2), x)
```

Giac [F]

$$\int \left(2+3x^2\right)\sqrt{5+x^4}\,dx=\int \sqrt{x^4+5}\left(3\,x^2+2\right)\,dx$$

```
[In] integrate((3*x^2+2)*(x^4+5)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(x^4 + 5)*(3*x^2 + 2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(2+3x^2\right)\sqrt{5+x^4}\,dx=\int \sqrt{x^4+5}\,\left(3\,x^2+2\right)\,dx$$

```
[In] int((x^4 + 5)^(1/2)*(3*x^2 + 2),x)
[Out] int((x^4 + 5)^(1/2)*(3*x^2 + 2), x)
```

3.18 $\int \frac{(2+3x^2)\sqrt{5+x^4}}{x^2}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 171

$$\int \frac{(2+3x^2)\sqrt{5+x^4}}{x^2}\,dx = -\frac{(2-x^2)\sqrt{5+x^4}}{x}+\frac{4x\sqrt{5+x^4}}{\sqrt{5}+x^2}-\frac{4\sqrt[4]{5}(\sqrt{5}+x^2)\sqrt{\frac{5+x^4}{\left(\sqrt{5}+x^2\right)^2}}E\left(2\arctan\left(\frac{x}{\sqrt[4]{5}}\right)|\frac{1}{2}\right)}{\sqrt{5+x^4}}+\frac{\sqrt[4]{5}(2+\sqrt{5})\left(\sqrt{5}+x^2\right)\sqrt{\frac{5+x^4}{\left(\sqrt{5}+x^2\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{x}{\sqrt[4]{5}}\right),\frac{1}{2}\right)}{\sqrt{5+x^4}}$$

```
[Out] -(-x^2+2)*(x^4+5)^(1/2)/x+4*x*(x^4+5)^(1/2)/(x^2+5^(1/2))-4*5^(1/4)*(cos(2*
arctan(1/5*x*5^(3/4)))^2)^(1/2)/cos(2*arctan(1/5*x*5^(3/4)))*EllipticE(sin(
2*arctan(1/5*x*5^(3/4))),1/2*2^(1/2))*(x^2+5^(1/2))*((x^4+5)/(x^2+5^(1/2))^
2)^(1/2)/(x^4+5)^(1/2)+5^(1/4)*(cos(2*arctan(1/5*x*5^(3/4)))^2)^(1/2)/cos(2
*arctan(1/5*x*5^(3/4)))*EllipticF(sin(2*arctan(1/5*x*5^(3/4))),1/2*2^(1/2))
*(2+5^(1/2))*(x^2+5^(1/2))*((x^4+5)/(x^2+5^(1/2))^2)^(1/2)/(x^4+5)^(1/2)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 171, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used

$= \{1286, 1212, 226, 1210\}$

$$\int \frac{(2+3x^2)\sqrt{5+x^4}}{x^2}\,dx$$
$$= \frac{\sqrt[4]{5}(2+\sqrt{5})\left(x^2+\sqrt{5}\right)\sqrt{\frac{x^4+5}{\left(x^2+\sqrt{5}\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{x}{\sqrt[4]{5}}\right),\frac{1}{2}\right)}{\sqrt{x^4+5}} - \frac{4\sqrt[4]{5}(x^2+\sqrt{5})\sqrt{\frac{x^4+5}{\left(x^2+\sqrt{5}\right)^2}}E\left(2\arctan\left(\frac{x}{\sqrt[4]{5}}\right)|\frac{1}{2}\right)}{\sqrt{x^4+5}} + \frac{4\sqrt{x^4+5}x}{x^2+\sqrt{5}} - \frac{(2-x^2)\sqrt{x^4+5}}{x}$$

```
[In] Int[((2 + 3*x^2)*Sqrt[5 + x^4])/x^2,x]
```

```
[Out] -(((2 - x^2)*Sqrt[5 + x^4])/x) + (4*x*Sqrt[5 + x^4])/(Sqrt[5] + x^2) - (4*5
^(1/4)*(Sqrt[5] + x^2)*Sqrt[(5 + x^4)/(Sqrt[5] + x^2)^2]*EllipticE[2*ArcTan
[x/5^(1/4)], 1/2])/Sqrt[5 + x^4] + (5^(1/4)*(2 + Sqrt[5])*(Sqrt[5] + x^2)*S
qrt[(5 + x^4)/(Sqrt[5] + x^2)^2]*EllipticF[2*ArcTan[x/5^(1/4)], 1/2])/Sqrt[
5 + x^4]
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 1210

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Rule 1212

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + c*x^4], x], x] - Dist[e/q, I
nt[(1 - q*x^2)/Sqrt[a + c*x^4], x], x] /; NeQ[e + d*q, 0]] /; FreeQ[{a, c,
d, e}, x] && PosQ[c/a]
```

Rule 1286

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (c_.)*(x_)^4)^(p_.), x
_Symbol] :> Simp[(f*x)^(m + 1)*(a + c*x^4)^p*((d*(m + 4*p + 3) + e*(m + 1)*
x^2)/(f*(m + 1)*(m + 4*p + 3))), x] + Dist[4*(p/(f^2*(m + 1)*(m + 4*p + 3))
), Int[(f*x)^(m + 2)*(a + c*x^4)^(p - 1)*(a*e*(m + 1) - c*d*(m + 4*p + 3)*x
```

```
^2), x], x] /; FreeQ[{a, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1] && m +
4*p + 3 != 0 && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{(2-x^2)\sqrt{5+x^4}}{x} - \frac{2}{3}\int \frac{-15-6x^2}{\sqrt{5+x^4}}\,dx \\
&= -\frac{(2-x^2)\sqrt{5+x^4}}{x} - \left(4\sqrt{5}\right)\int \frac{1-\frac{x^2}{\sqrt{5}}}{\sqrt{5+x^4}}\,dx + \left(2\left(5+2\sqrt{5}\right)\right)\int \frac{1}{\sqrt{5+x^4}}\,dx \\
&= -\frac{(2-x^2)\sqrt{5+x^4}}{x} + \frac{4x\sqrt{5+x^4}}{\sqrt{5}+x^2} \\
&\quad - \frac{4\sqrt[4]{5}\left(\sqrt{5}+x^2\right)\sqrt{\frac{5+x^4}{\left(\sqrt{5}+x^2\right)^2}}E\left(2\tan^{-1}\left(\frac{x}{\sqrt[4]{5}}\right)|\frac{1}{2}\right)}{\sqrt{5+x^4}} \\
&\quad + \frac{\sqrt[4]{5}\left(2+\sqrt{5}\right)\left(\sqrt{5}+x^2\right)\sqrt{\frac{5+x^4}{\left(\sqrt{5}+x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{x}{\sqrt[4]{5}}\right)|\frac{1}{2}\right)}{\sqrt{5+x^4}}
\end{aligned}
$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 3.52 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.31

$$
\int \frac{(2+3x^2)\sqrt{5+x^4}}{x^2}\,dx = -\frac{2\sqrt{5}\,\text{Hypergeometric2F1}\left(-\frac{1}{2},-\frac{1}{4},\frac{3}{4},-\frac{x^4}{5}\right)}{x} + 3\sqrt{5}x\,\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{1}{4},\frac{5}{4},-\frac{x^4}{5}\right)
$$

```
[In] Integrate[((2 + 3*x^2)*Sqrt[5 + x^4])/x^2,x]
[Out] (-2*Sqrt[5]*Hypergeometric2F1[-1/2, -1/4, 3/4, -1/5*x^4])/x + 3*Sqrt[5]*x*H
ypergeometric2F1[-1/2, 1/4, 5/4, -1/5*x^4]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4.

Time = 1.36 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.22

method	result
meijerg	$-\frac{2\sqrt{5}\,{}_2F_1\left(-\frac{1}{2},-\frac{1}{4};\frac{3}{4};-\frac{x^4}{5}\right)}{x}+3\sqrt{5}\,x\,{}_2F_1\left(-\frac{1}{2},\frac{1}{4};\frac{5}{4};-\frac{x^4}{5}\right)$
default	$x\sqrt{x^4+5}+\frac{2\sqrt{5}\sqrt{25-5i\sqrt{5}\,x^2}\sqrt{25+5i\sqrt{5}\,x^2}\,F\left(\frac{x\sqrt{5}\sqrt{i\sqrt{5}}}{5},i\right)}{5\sqrt{i\sqrt{5}}\sqrt{x^4+5}}-\frac{2\sqrt{x^4+5}}{x}+\frac{4i\sqrt{25-5i\sqrt{5}\,x^2}\sqrt{25+5i\sqrt{5}\,x^2}\left(F\left(\frac{x\sqrt{5}\sqrt{i\sqrt{5}}}{5}\right.\right.}{5\sqrt{i\sqrt{5}}\sqrt{x^4+5}}$
elliptic	$x\sqrt{x^4+5}+\frac{2\sqrt{5}\sqrt{25-5i\sqrt{5}\,x^2}\sqrt{25+5i\sqrt{5}\,x^2}\,F\left(\frac{x\sqrt{5}\sqrt{i\sqrt{5}}}{5},i\right)}{5\sqrt{i\sqrt{5}}\sqrt{x^4+5}}-\frac{2\sqrt{x^4+5}}{x}+\frac{4i\sqrt{25-5i\sqrt{5}\,x^2}\sqrt{25+5i\sqrt{5}\,x^2}\left(F\left(\frac{x\sqrt{5}\sqrt{i\sqrt{5}}}{5}\right.\right.}{5\sqrt{i\sqrt{5}}\sqrt{x^4+5}}$
risch	$\frac{x^6-2x^4+5x^2-10}{x\sqrt{x^4+5}}+\frac{2\sqrt{5}\sqrt{25-5i\sqrt{5}\,x^2}\sqrt{25+5i\sqrt{5}\,x^2}\,F\left(\frac{x\sqrt{5}\sqrt{i\sqrt{5}}}{5},i\right)}{5\sqrt{i\sqrt{5}}\sqrt{x^4+5}}+\frac{4i\sqrt{25-5i\sqrt{5}\,x^2}\sqrt{25+5i\sqrt{5}\,x^2}\left(F\left(\frac{x\sqrt{5}\sqrt{i\sqrt{5}}}{5},i\right)-E\left(\right.\right.}{5\sqrt{i\sqrt{5}}\sqrt{x^4+5}}$

```
[In] int((3*x^2+2)*(x^4+5)^(1/2)/x^2,x,method=_RETURNVERBOSE)
[Out] -2*5^(1/2)/x*hypergeom([-1/2,-1/4],[3/4],-1/5*x^4)+3*5^(1/2)*x*hypergeom([-
1/2,1/4],[5/4],-1/5*x^4)
```

Fricas [F]

$$\int \frac{(2+3x^2)\sqrt{5+x^4}}{x^2}\,dx=\int \frac{\sqrt{x^4+5}(3\,x^2+2)}{x^2}\,dx$$

```
[In] integrate((3*x^2+2)*(x^4+5)^(1/2)/x^2,x, algorithm="fricas")
[Out] integral(sqrt(x^4 + 5)*(3*x^2 + 2)/x^2, x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.99 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.46

$$\int \frac{(2+3x^2)\sqrt{5+x^4}}{x^2}\,dx=\frac{3\sqrt{5}x\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\left.\begin{matrix}-\frac{1}{2},\frac{1}{4}\\ \frac{5}{4}\end{matrix}\right|\frac{x^4e^{i\pi}}{5}\right)}{4\Gamma\left(\frac{5}{4}\right)}+\frac{\sqrt{5}\Gamma\left(-\frac{1}{4}\right){}_2F_1\left(\left.\begin{matrix}-\frac{1}{2},-\frac{1}{4}\\ \frac{3}{4}\end{matrix}\right|\frac{x^4e^{i\pi}}{5}\right)}{2x\Gamma\left(\frac{3}{4}\right)}$$

```
[In] integrate((3*x**2+2)*(x**4+5)**(1/2)/x**2,x)
[Out] 3*sqrt(5)*x*gamma(1/4)*hyper((-1/2, 1/4), (5/4,), x**4*exp_polar(I*pi)/5)/(
4*gamma(5/4)) + sqrt(5)*gamma(-1/4)*hyper((-1/2, -1/4), (3/4,), x**4*exp_po
lar(I*pi)/5)/(2*x*gamma(3/4))
```

Maxima [F]

$$\int \frac{(2+3x^2)\sqrt{5+x^4}}{x^2}\,dx = \int \frac{\sqrt{x^4+5}(3\,x^2+2)}{x^2}\,dx$$

```
[In] integrate((3*x^2+2)*(x^4+5)^(1/2)/x^2,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(x^4 + 5)*(3*x^2 + 2)/x^2, x)
```

Giac [F]

$$\int \frac{(2+3x^2)\sqrt{5+x^4}}{x^2}\,dx = \int \frac{\sqrt{x^4+5}(3\,x^2+2)}{x^2}\,dx$$

```
[In] integrate((3*x^2+2)*(x^4+5)^(1/2)/x^2,x, algorithm="giac")
```

```
[Out] integrate(sqrt(x^4 + 5)*(3*x^2 + 2)/x^2, x)
```

Mupad [B] (verification not implemented)

Time = 7.77 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.36

$$\int \frac{(2+3x^2)\sqrt{5+x^4}}{x^2}\,dx = \frac{3\,x\,\sqrt{x^4+5}\,{}_2\mathrm{F}_1\left(-\frac{1}{2},\frac{1}{4};\ \frac{5}{4};\ -\frac{x^4}{5}\right)}{\sqrt{\frac{x^4}{5}+1}} + \frac{2\,\sqrt{x^4+5}\,{}_2\mathrm{F}_1\left(-\frac{1}{2},-\frac{1}{4};\ \frac{3}{4};\ -\frac{5}{x^4}\right)}{x\,\sqrt{\frac{5}{x^4}+1}}$$

```
[In] int(((x^4 + 5)^(1/2)*(3*x^2 + 2))/x^2,x)
```

```
[Out] (3*x*(x^4 + 5)^(1/2)*hypergeom([-1/2, 1/4], 5/4, -x^4/5))/(x^4/5 + 1)^(1/2)
 + (2*(x^4 + 5)^(1/2)*hypergeom([-1/2, -1/4], 3/4, -5/x^4))/(x*(5/x^4 + 1)^
(1/2))
```

3.19 $\int \frac{(2+3x^2)\sqrt{5+x^4}}{x^4}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 192

$$\begin{aligned}&\int \frac{(2+3x^2)\sqrt{5+x^4}}{x^4}\,dx\\&=-\frac{6\sqrt{5+x^4}}{x}-\frac{(2-9x^2)\sqrt{5+x^4}}{3x^3}+\frac{6x\sqrt{5+x^4}}{\sqrt{5}+x^2}\\&\quad-\frac{6\sqrt[4]{5}(\sqrt{5}+x^2)\sqrt{\frac{5+x^4}{(\sqrt{5}+x^2)^2}}E\left(2\arctan\left(\frac{x}{\sqrt[4]{5}}\right)|\frac{1}{2}\right)}{\sqrt{5+x^4}}\\&\quad+\frac{(2+9\sqrt{5})(\sqrt{5}+x^2)\sqrt{\frac{5+x^4}{(\sqrt{5}+x^2)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{x}{\sqrt[4]{5}}\right),\frac{1}{2}\right)}{3\sqrt[4]{5}\sqrt{5+x^4}}\end{aligned}$$

```
[Out] -6*(x^4+5)^(1/2)/x-1/3*(-9*x^2+2)*(x^4+5)^(1/2)/x^3+6*x*(x^4+5)^(1/2)/(x^2+
5^(1/2))-6*5^(1/4)*(cos(2*arctan(1/5*x*5^(3/4)))^2)^(1/2)/cos(2*arctan(1/5*
x*5^(3/4)))*EllipticE(sin(2*arctan(1/5*x*5^(3/4))),1/2*2^(1/2))*(x^2+5^(1/2
))*((x^4+5)/(x^2+5^(1/2))^2)^(1/2)/(x^4+5)^(1/2)+1/15*(cos(2*arctan(1/5*x*5
^(3/4)))^2)^(1/2)/cos(2*arctan(1/5*x*5^(3/4)))*EllipticF(sin(2*arctan(1/5*x
*5^(3/4))),1/2*2^(1/2))*(x^2+5^(1/2))*(2+9*5^(1/2))*((x^4+5)/(x^2+5^(1/2))^
2)^(1/2)*5^(3/4)/(x^4+5)^(1/2)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 192, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {1286, 1296, 1212, 226, 1210}

$$\int \frac{(2+3x^2)\sqrt{5+x^4}}{x^4}\,dx$$

$$= \frac{(2+9\sqrt{5})\,(x^2+\sqrt{5})\,\sqrt{\frac{x^4+5}{(x^2+\sqrt{5})^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{x}{\sqrt[4]{5}}\right),\frac{1}{2}\right)}{3\sqrt[4]{5}\sqrt{x^4+5}} - \frac{6\sqrt[4]{5}(x^2+\sqrt{5})\,\sqrt{\frac{x^4+5}{(x^2+\sqrt{5})^2}}E\left(2\arctan\left(\frac{x}{\sqrt[4]{5}}\right)|\frac{1}{2}\right)}{\sqrt{x^4+5}} - \frac{6\sqrt{x^4+5}}{x} + \frac{6\sqrt{x^4+5}x}{x^2+\sqrt{5}} - \frac{(2-9x^2)\sqrt{x^4+5}}{3x^3}$$

```
[In] Int[((2 + 3*x^2)*Sqrt[5 + x^4])/x^4,x]
```

```
[Out] (-6*Sqrt[5 + x^4])/x - ((2 - 9*x^2)*Sqrt[5 + x^4])/(3*x^3) + (6*x*Sqrt[5 +
x^4])/(Sqrt[5] + x^2) - (6*5^(1/4)*(Sqrt[5] + x^2)*Sqrt[(5 + x^4)/(Sqrt[5]
+ x^2)^2]*EllipticE[2*ArcTan[x/5^(1/4)], 1/2])/Sqrt[5 + x^4] + ((2 + 9*Sqrt
[5])*(Sqrt[5] + x^2)*Sqrt[(5 + x^4)/(Sqrt[5] + x^2)^2]*EllipticF[2*ArcTan[x
/5^(1/4)], 1/2])/(3*5^(1/4)*Sqrt[5 + x^4])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 1210

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Rule 1212

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + c*x^4], x], x] - Dist[e/q, I
nt[(1 - q*x^2)/Sqrt[a + c*x^4], x], x] /; NeQ[e + d*q, 0]] /; FreeQ[{a, c,
d, e}, x] && PosQ[c/a]
```

Rule 1286

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (c_.)*(x_)^4)^(p_.), x
_Symbol] :> Simp[(f*x)^(m + 1)*(a + c*x^4)^p*((d*(m + 4*p + 3) + e*(m + 1)*
x^2)/(f*(m + 1)*(m + 4*p + 3))), x] + Dist[4*(p/(f^2*(m + 1)*(m + 4*p + 3))
), Int[(f*x)^(m + 2)*(a + c*x^4)^(p - 1)*(a*e*(m + 1) - c*d*(m + 4*p + 3)*x
^2), x], x] /; FreeQ[{a, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1] && m +
4*p + 3 != 0 && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1296

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (c_.)*(x_)^4)^(p_), x_
Symbol] :> Simp[d*(f*x)^(m + 1)*((a + c*x^4)^(p + 1)/(a*f*(m + 1))), x] + D
ist[1/(a*f^2*(m + 1)), Int[(f*x)^(m + 2)*(a + c*x^4)^p*(a*e*(m + 1) - c*d*(
m + 4*p + 5)*x^2), x], x] /; FreeQ[{a, c, d, e, f, p}, x] && LtQ[m, -1] &&
IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rubi steps

$$\text{integral} = -\frac{(2-9x^2)\sqrt{5+x^4}}{3x^3} - \frac{2}{3}\int \frac{-45-2x^2}{x^2\sqrt{5+x^4}}\,dx$$

$$= -\frac{6\sqrt{5+x^4}}{x} - \frac{(2-9x^2)\sqrt{5+x^4}}{3x^3} + \frac{2}{15}\int \frac{10+45x^2}{\sqrt{5+x^4}}\,dx$$

$$= -\frac{6\sqrt{5+x^4}}{x} - \frac{(2-9x^2)\sqrt{5+x^4}}{3x^3} - \left(6\sqrt{5}\right)\int \frac{1-\frac{x^2}{\sqrt{5}}}{\sqrt{5+x^4}}\,dx + \frac{1}{3}\left(2\left(2+9\sqrt{5}\right)\right)\int \frac{1}{\sqrt{5+x^4}}\,dx$$

$$= -\frac{6\sqrt{5+x^4}}{x} - \frac{(2-9x^2)\sqrt{5+x^4}}{3x^3} + \frac{6x\sqrt{5+x^4}}{\sqrt{5}+x^2} - \frac{6\sqrt[4]{5}(\sqrt{5}+x^2)\sqrt{\frac{5+x^4}{\left(\sqrt{5}+x^2\right)^2}}E\left(2\tan^{-1}\left(\frac{x}{\sqrt[4]{5}}\right)|\frac{1}{2}\right)}{\sqrt{5+x^4}} + \frac{(2+9\sqrt{5})\left(\sqrt{5}+x^2\right)\sqrt{\frac{5+x^4}{\left(\sqrt{5}+x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{x}{\sqrt[4]{5}}\right)|\frac{1}{2}\right)}{3\sqrt[4]{5}\sqrt{5+x^4}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 7.72 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.28

$$\int \frac{(2+3x^2)\sqrt{5+x^4}}{x^4}\,dx = -\frac{\sqrt{5}\left(2\,\text{Hypergeometric2F1}\left(-\frac{3}{4},-\frac{1}{2},\frac{1}{4},-\frac{x^4}{5}\right)+9x^2\,\text{Hypergeometric2F1}\left(-\frac{1}{2},-\frac{1}{4},\frac{3}{4},-\frac{x^4}{5}\right)\right)}{3x^3}$$

```
[In] Integrate[((2 + 3*x^2)*Sqrt[5 + x^4])/x^4,x]
```

```
[Out] -1/3*(Sqrt[5]*(2*Hypergeometric2F1[-3/4, -1/2, 1/4, -1/5*x^4] + 9*x^2*Hyper
geometric2F1[-1/2, -1/4, 3/4, -1/5*x^4]))/x^3
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4.

Time = 2.01 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.21

method	result
meijerg	$-\frac{2\sqrt{5}\,{}_2F_1\left(-\frac{3}{4},-\frac{1}{2};\frac{1}{4};-\frac{x^4}{5}\right)}{3x^3}-\frac{3\sqrt{5}\,{}_2F_1\left(-\frac{1}{2},-\frac{1}{4};\frac{3}{4};-\frac{x^4}{5}\right)}{x}$
default	$-\frac{3\sqrt{x^4+5}}{x}+\frac{6i\sqrt{25-5i\sqrt{5}\,x^2}\,\sqrt{25+5i\sqrt{5}\,x^2}\left(F\left(\frac{x\sqrt{5}\,\sqrt{i\sqrt{5}}}{5},i\right)-E\left(\frac{x\sqrt{5}\,\sqrt{i\sqrt{5}}}{5},i\right)\right)}{5\sqrt{i\sqrt{5}}\,\sqrt{x^4+5}}-\frac{2\sqrt{x^4+5}}{3x^3}+\frac{4\sqrt{5}\,\sqrt{25-5i\sqrt{5}\,x^2}\,\sqrt{25+5}}{75\sqrt{i\sqrt{5}}}$
elliptic	$-\frac{3\sqrt{x^4+5}}{x}+\frac{6i\sqrt{25-5i\sqrt{5}\,x^2}\,\sqrt{25+5i\sqrt{5}\,x^2}\left(F\left(\frac{x\sqrt{5}\,\sqrt{i\sqrt{5}}}{5},i\right)-E\left(\frac{x\sqrt{5}\,\sqrt{i\sqrt{5}}}{5},i\right)\right)}{5\sqrt{i\sqrt{5}}\,\sqrt{x^4+5}}-\frac{2\sqrt{x^4+5}}{3x^3}+\frac{4\sqrt{5}\,\sqrt{25-5i\sqrt{5}\,x^2}\,\sqrt{25+5}}{75\sqrt{i\sqrt{5}}}$
risch	$-\frac{9x^6+2x^4+45x^2+10}{3x^3\sqrt{x^4+5}}+\frac{4\sqrt{5}\,\sqrt{25-5i\sqrt{5}\,x^2}\,\sqrt{25+5i\sqrt{5}\,x^2}\,F\left(\frac{x\sqrt{5}\,\sqrt{i\sqrt{5}}}{5},i\right)}{75\sqrt{i\sqrt{5}}\,\sqrt{x^4+5}}+\frac{6i\sqrt{25-5i\sqrt{5}\,x^2}\,\sqrt{25+5i\sqrt{5}\,x^2}\left(F\left(\frac{x\sqrt{5}\,\sqrt{i\sqrt{5}}}{5},i\right.\right.}{5\sqrt{i\sqrt{5}}\,\sqrt{x^4+5}}$

```
[In] int((3*x^2+2)*(x^4+5)^(1/2)/x^4,x,method=_RETURNVERBOSE)
```

```
[Out] -2/3*5^(1/2)/x^3*hypergeom([-3/4,-1/2],[1/4],-1/5*x^4)-3*5^(1/2)/x*hypergeo
m([-1/2,-1/4],[3/4],-1/5*x^4)
```

Fricas [F]

$$\int \frac{(2+3x^2)\sqrt{5+x^4}}{x^4}\,dx=\int \frac{\sqrt{x^4+5}(3\,x^2+2)}{x^4}\,dx$$

```
[In] integrate((3*x^2+2)*(x^4+5)^(1/2)/x^4,x, algorithm="fricas")
[Out] integral(sqrt(x^4 + 5)*(3*x^2 + 2)/x^4, x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.10 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.43

$$\int \frac{(2+3x^2)\sqrt{5+x^4}}{x^4}\,dx=\frac{3\sqrt{5}\Gamma\left(-\frac{1}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},-\frac{1}{4}\\ \frac{3}{4}\end{matrix}\middle|\frac{x^4e^{i\pi}}{5}\right)}{4x\Gamma\left(\frac{3}{4}\right)}+\frac{\sqrt{5}\Gamma\left(-\frac{3}{4}\right){}_2F_1\left(\begin{matrix}-\frac{3}{4},-\frac{1}{2}\\ \frac{1}{4}\end{matrix}\middle|\frac{x^4e^{i\pi}}{5}\right)}{2x^3\Gamma\left(\frac{1}{4}\right)}$$

```
[In] integrate((3*x**2+2)*(x**4+5)**(1/2)/x**4,x)
[Out] 3*sqrt(5)*gamma(-1/4)*hyper((-1/2, -1/4), (3/4,), x**4*exp_polar(I*pi)/5)/(
4*x*gamma(3/4)) + sqrt(5)*gamma(-3/4)*hyper((-3/4, -1/2), (1/4,), x**4*exp_
polar(I*pi)/5)/(2*x**3*gamma(1/4))
```

Maxima [F]

$$\int \frac{(2+3x^2)\sqrt{5+x^4}}{x^4}\,dx=\int \frac{\sqrt{x^4+5}(3\,x^2+2)}{x^4}\,dx$$

```
[In] integrate((3*x^2+2)*(x^4+5)^(1/2)/x^4,x, algorithm="maxima")
[Out] integrate(sqrt(x^4 + 5)*(3*x^2 + 2)/x^4, x)
```

Giac [F]

$$\int \frac{(2+3x^2)\sqrt{5+x^4}}{x^4}\,dx = \int \frac{\sqrt{x^4+5}(3\,x^2+2)}{x^4}\,dx$$

```
[In] integrate((3*x^2+2)*(x^4+5)^(1/2)/x^4,x, algorithm="giac")
[Out] integrate(sqrt(x^4 + 5)*(3*x^2 + 2)/x^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(2+3x^2)\sqrt{5+x^4}}{x^4}\,dx = \int \frac{\sqrt{x^4+5}\,(3\,x^2+2)}{x^4}\,dx$$

```
[In] int(((x^4 + 5)^(1/2)*(3*x^2 + 2))/x^4,x)
[Out] int(((x^4 + 5)^(1/2)*(3*x^2 + 2))/x^4, x)
```

3.20 $\int x^5(2+3x^2)\left(5+x^4\right)^{3/2}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 83

$$\int x^5(2+3x^2)\left(5+x^4\right)^{3/2}\,dx=-\frac{25}{16}x^2\sqrt{5+x^4}-\frac{5}{24}x^2\left(5+x^4\right)^{3/2}+\frac{3}{14}x^4\left(5+x^4\right)^{5/2}-\frac{1}{42}(18-7x^2)\left(5+x^4\right)^{5/2}-\frac{125}{16}\text{arcsinh}\left(\frac{x^2}{\sqrt{5}}\right)$$

```
[Out] -5/24*x^2*(x^4+5)^(3/2)+3/14*x^4*(x^4+5)^(5/2)-1/42*(-7*x^2+18)*(x^4+5)^(5/
2)-125/16*arcsinh(1/5*x^2*5^(1/2))-25/16*x^2*(x^4+5)^(1/2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {1266, 847, 794, 201, 221}

$$\int x^5(2+3x^2)\left(5+x^4\right)^{3/2}\,dx=-\frac{125}{16}\text{arcsinh}\left(\frac{x^2}{\sqrt{5}}\right)+\frac{3}{14}\left(x^4+5\right)^{5/2}x^4-\frac{5}{24}\left(x^4+5\right)^{3/2}x^2-\frac{25}{16}\sqrt{x^4+5}x^2-\frac{1}{42}(18-7x^2)\left(x^4+5\right)^{5/2}$$

```
[In] Int[x^5*(2 + 3*x^2)*(5 + x^4)^(3/2),x]
```

```
[Out] (-25*x^2*Sqrt[5 + x^4])/16 - (5*x^2*(5 + x^4)^(3/2))/24 + (3*x^4*(5 + x^4)^
(5/2))/14 - ((18 - 7*x^2)*(5 + x^4)^(5/2))/42 - (125*ArcSinh[x^2/Sqrt[5]])/
16
```

Rule 201

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Dist[a*n*(p/(n*p + 1)), Int[(a + b*x^n)^(p - 1), x], x] /; Free
```

```
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 794

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*((a + c*x^2)^(p
+ 1)/(2*c*(p + 1)*(2*p + 3))), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rule 847

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[g*(d + e*x)^m*((a + c*x^2)^(p + 1)/(c*(m + 2*p + 2))
), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p*Simp[
c*d*f*(m + 2*p + 2) - a*e*g*m + c*(e*f*(m + 2*p + 2) + d*g*m)*x, x], x], x]
 /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[m, 0] &
& NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]
) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

Rule 1266

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_S
ymbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + c*x^2)^p, x],
x, x^2], x] /; FreeQ[{a, c, d, e, p, q}, x] && IntegerQ[(m + 1)/2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{2}\text{Subst}\left(\int x^2(2+3x)\left(5+x^2\right)^{3/2}\,dx,x,x^2\right)\\
&= \frac{3}{14}x^4\left(5+x^4\right)^{5/2}+\frac{1}{14}\text{Subst}\left(\int x(-30+14x)\left(5+x^2\right)^{3/2}\,dx,x,x^2\right)\\
&= \frac{3}{14}x^4\left(5+x^4\right)^{5/2}-\frac{1}{42}\left(18-7x^2\right)\left(5+x^4\right)^{5/2}-\frac{5}{6}\text{Subst}\left(\int \left(5+x^2\right)^{3/2}\,dx,x,x^2\right)\\
&= -\frac{5}{24}x^2\left(5+x^4\right)^{3/2}\\
&\quad +\frac{3}{14}x^4\left(5+x^4\right)^{5/2}-\frac{1}{42}\left(18-7x^2\right)\left(5+x^4\right)^{5/2}-\frac{25}{8}\text{Subst}\left(\int \sqrt{5+x^2}\,dx,x,x^2\right)
\end{aligned}$$

$$= -\frac{25}{16}x^2\sqrt{5+x^4} - \frac{5}{24}x^2(5+x^4)^{3/2} + \frac{3}{14}x^4(5+x^4)^{5/2} - \frac{1}{42}(18-7x^2)(5+x^4)^{5/2} - \frac{125}{16}\text{Subst}\left(\int \frac{1}{\sqrt{5+x^2}}\,dx, x, x^2\right)$$

$$= -\frac{25}{16}x^2\sqrt{5+x^4} - \frac{5}{24}x^2(5+x^4)^{3/2} + \frac{3}{14}x^4(5+x^4)^{5/2} - \frac{1}{42}(18-7x^2)(5+x^4)^{5/2} - \frac{125}{16}\sinh^{-1}\left(\frac{x^2}{\sqrt{5}}\right)$$

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.80

$$\int x^5(2+3x^2)(5+x^4)^{3/2}\,dx = \frac{1}{336}\sqrt{5+x^4}(-3600+525x^2+360x^4+490x^6+576x^8+56x^{10}+72x^{12}) + \frac{125}{16}\log\left(-x^2+\sqrt{5+x^4}\right)$$

```
[In] Integrate[x^5*(2 + 3*x^2)*(5 + x^4)^(3/2),x]
[Out] (Sqrt[5 + x^4]*(-3600 + 525*x^2 + 360*x^4 + 490*x^6 + 576*x^8 + 56*x^10 + 7
2*x^12))/336 + (125*Log[-x^2 + Sqrt[5 + x^4]])/16
```

Maple [A] (verified)

Time = 0.33 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.65

method	result
risch	$\frac{(72x^{12}+56x^{10}+576x^8+490x^6+360x^4+525x^2-3600)\sqrt{x^4+5}}{336} - \frac{125\,\text{arcsinh}\left(\frac{x^2\sqrt{5}}{5}\right)}{16}$
pseudoelliptic	$\frac{(72x^{12}+56x^{10}+576x^8+490x^6+360x^4+525x^2-3600)\sqrt{x^4+5}}{336} - \frac{125\,\text{arcsinh}\left(\frac{x^2\sqrt{5}}{5}\right)}{16}$
trager	$\left(\frac{3}{14}x^{12}+\frac{1}{6}x^{10}+\frac{12}{7}x^8+\frac{35}{24}x^6+\frac{15}{14}x^4+\frac{25}{16}x^2-\frac{75}{7}\right)\sqrt{x^4+5} - \frac{125\ln\left(x^2+\sqrt{x^4+5}\right)}{16}$
default	$\frac{3\sqrt{x^4+5}\,(x^4-2)(x^8+10x^4+25)}{14} + \frac{x^{10}\sqrt{x^4+5}}{6} + \frac{35x^6\sqrt{x^4+5}}{24} + \frac{25x^2\sqrt{x^4+5}}{16} - \frac{125\,\text{arcsinh}\left(\frac{x^2\sqrt{5}}{5}\right)}{16}$
elliptic	$\frac{3x^{12}\sqrt{x^4+5}}{14} + \frac{12x^8\sqrt{x^4+5}}{7} + \frac{15x^4\sqrt{x^4+5}}{14} - \frac{75\sqrt{x^4+5}}{7} + \frac{x^{10}\sqrt{x^4+5}}{6} + \frac{35x^6\sqrt{x^4+5}}{24} + \frac{25x^2\sqrt{x^4+5}}{16} - \frac{125\,\text{arcsin}}{1}$
meijerg	$\frac{1125\sqrt{5}\left(\frac{16\sqrt{\pi}}{105}-\frac{2\sqrt{\pi}\left(-\frac{4}{25}x^{12}-\frac{32}{25}x^8-\frac{4}{5}x^4+8\right)\sqrt{1+\frac{x^4}{5}}}{105}\right)}{16\sqrt{\pi}} + \frac{\frac{25\sqrt{\pi}\,x^2\sqrt{5}\left(\frac{8}{25}x^8+\frac{14}{5}x^4+3\right)\sqrt{1+\frac{x^4}{5}}}{48}-\frac{125\sqrt{\pi}\,\text{arcsinh}\left(\frac{x^2\sqrt{5}}{5}\right)}{16}}{\sqrt{\pi}}$

```
[In] int(x^5*(3*x^2+2)*(x^4+5)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/336*(72*x^12+56*x^10+576*x^8+490*x^6+360*x^4+525*x^2-3600)*(x^4+5)^(1/2)-
125/16*arcsinh(1/5*x^2*5^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.70

$$\int x^5(2+3x^2)\left(5+x^4\right)^{3/2}\,dx=\frac{1}{336}\left(72\,x^{12}+56\,x^{10}+576\,x^8+490\,x^6+360\,x^4+525\,x^2-3600\right)\sqrt{x^4+5}+\frac{125}{16}\log\left(-x^2+\sqrt{x^4+5}\right)$$

```
[In] integrate(x^5*(3*x^2+2)*(x^4+5)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/336*(72*x^12 + 56*x^10 + 576*x^8 + 490*x^6 + 360*x^4 + 525*x^2 - 3600)*sq
rt(x^4 + 5) + 125/16*log(-x^2 + sqrt(x^4 + 5))
```

Sympy [A] (verification not implemented)

Time = 0.94 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.46

$$\int x^5(2+3x^2)\left(5+x^4\right)^{3/2}\,dx=5\sqrt{x^4+5}\left(\frac{x^6}{4}+\frac{5x^2}{8}\right)+\frac{15\sqrt{x^4+5}\left(\frac{x^8}{5}+\frac{x^4}{3}-\frac{10}{3}\right)}{2}+\sqrt{x^4+5}\left(\frac{x^{10}}{6}+\frac{5x^6}{24}-\frac{25x^2}{16}\right)+\frac{3\sqrt{x^4+5}\left(\frac{x^{12}}{7}+\frac{x^8}{7}-\frac{20x^4}{21}+\frac{200}{21}\right)}{2}-\frac{125\,\mathrm{asinh}\left(\frac{\sqrt{5}x^2}{5}\right)}{16}$$

```
[In] integrate(x**5*(3*x**2+2)*(x**4+5)**(3/2),x)
```

```
[Out] 5*sqrt(x**4 + 5)*(x**6/4 + 5*x**2/8) + 15*sqrt(x**4 + 5)*(x**8/5 + x**4/3 -
 10/3)/2 + sqrt(x**4 + 5)*(x**10/6 + 5*x**6/24 - 25*x**2/16) + 3*sqrt(x**4
+ 5)*(x**12/7 + x**8/7 - 20*x**4/21 + 200/21)/2 - 125*asinh(sqrt(5)*x**2/5)
/16
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.53

$$\int x^5(2+3x^2)\,(5+x^4)^{3/2}\,dx = \frac{3}{14}\left(x^4+5\right)^{\frac{7}{2}} - \frac{3}{2}\left(x^4+5\right)^{\frac{5}{2}} - \frac{125\left(\frac{3\sqrt{x^4+5}}{x^2} - \frac{8\left(x^4+5\right)^{\frac{3}{2}}}{x^6} - \frac{3\left(x^4+5\right)^{\frac{5}{2}}}{x^{10}}\right)}{48\left(\frac{3\left(x^4+5\right)}{x^4} - \frac{3\left(x^4+5\right)^2}{x^8} + \frac{\left(x^4+5\right)^3}{x^{12}} - 1\right)} - \frac{125}{32}\log\left(\frac{\sqrt{x^4+5}}{x^2}+1\right) + \frac{125}{32}\log\left(\frac{\sqrt{x^4+5}}{x^2}-1\right)$$

```
[In] integrate(x^5*(3*x^2+2)*(x^4+5)^(3/2),x, algorithm="maxima")
```

```
[Out] 3/14*(x^4 + 5)^(7/2) - 3/2*(x^4 + 5)^(5/2) - 125/48*(3*sqrt(x^4 + 5)/x^2 -
8*(x^4 + 5)^(3/2)/x^6 - 3*(x^4 + 5)^(5/2)/x^10)/(3*(x^4 + 5)/x^4 - 3*(x^4 +
 5)^2/x^8 + (x^4 + 5)^3/x^12 - 1) - 125/32*log(sqrt(x^4 + 5)/x^2 + 1) + 125
/32*log(sqrt(x^4 + 5)/x^2 - 1)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.96

$$\int x^5(2+3x^2)\,(5+x^4)^{3/2}\,dx = \frac{3}{14}\left(x^4+5\right)^{\frac{7}{2}} + \frac{1}{48}\left(2\left(4\,x^4+5\right)x^4-75\right)\sqrt{x^4+5}x^2 + \frac{5}{8}\left(2\,x^4+5\right)\sqrt{x^4+5}x^2 - \frac{3}{2}\left(x^4+5\right)^{\frac{5}{2}} + \frac{125}{16}\log\left(-x^2+\sqrt{x^4+5}\right)$$

```
[In] integrate(x^5*(3*x^2+2)*(x^4+5)^(3/2),x, algorithm="giac")
```

```
[Out] 3/14*(x^4 + 5)^(7/2) + 1/48*(2*(4*x^4 + 5)*x^4 - 75)*sqrt(x^4 + 5)*x^2 + 5/
8*(2*x^4 + 5)*sqrt(x^4 + 5)*x^2 - 3/2*(x^4 + 5)^(5/2) + 125/16*log(-x^2 + s
qrt(x^4 + 5))
```

Mupad [B] (verification not implemented)

Time = 7.64 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.63

$$\int x^5 (2+3x^2) (5+x^4)^{3/2} \, dx = \sqrt{x^4+5}\left(\frac{3\,x^{12}}{14}+\frac{x^{10}}{6}+\frac{12\,x^8}{7}+\frac{35\,x^6}{24}+\frac{15\,x^4}{14}+\frac{25\,x^2}{16}-\frac{75}{7}\right)-\frac{125\,\mathrm{asinh}\left(\frac{\sqrt{5}\,x^2}{5}\right)}{16}$$

```
[In] int(x^5*(x^4 + 5)^(3/2)*(3*x^2 + 2),x)
[Out] (x^4 + 5)^(1/2)*((25*x^2)/16 + (15*x^4)/14 + (35*x^6)/24 + (12*x^8)/7 + x^1
0/6 + (3*x^12)/14 - 75/7) - (125*asinh((5^(1/2)*x^2)/5))/16
```

3.21 $\int x^3(2+3x^2)\left(5+x^4\right)^{3/2}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 67

$$\int x^3(2+3x^2)\left(5+x^4\right)^{3/2}\,dx=-\frac{75}{32}x^2\sqrt{5+x^4}-\frac{5}{16}x^2\left(5+x^4\right)^{3/2}+\frac{1}{20}(4+5x^2)\left(5+x^4\right)^{5/2}-\frac{375}{32}\text{arcsinh}\left(\frac{x^2}{\sqrt{5}}\right)$$

```
[Out] -5/16*x^2*(x^4+5)^(3/2)+1/20*(5*x^2+4)*(x^4+5)^(5/2)-375/32*arcsinh(1/5*x^2
*5^(1/2))-75/32*x^2*(x^4+5)^(1/2)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {1266, 794, 201, 221}

$$\int x^3(2+3x^2)\left(5+x^4\right)^{3/2}\,dx=-\frac{375}{32}\text{arcsinh}\left(\frac{x^2}{\sqrt{5}}\right)+\frac{1}{20}(5x^2+4)\left(x^4+5\right)^{5/2}-\frac{5}{16}x^2\left(x^4+5\right)^{3/2}-\frac{75}{32}x^2\sqrt{x^4+5}$$

```
[In] Int[x^3*(2 + 3*x^2)*(5 + x^4)^(3/2),x]
```

```
[Out] (-75*x^2*Sqrt[5 + x^4])/32 - (5*x^2*(5 + x^4)^(3/2))/16 + ((4 + 5*x^2)*(5 +
 x^4)^(5/2))/20 - (375*ArcSinh[x^2/Sqrt[5]])/32
```

Rule 201

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Dist[a*n*(p/(n*p + 1)), Int[(a + b*x^n)^(p - 1), x], x] /; Free
```

```
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 794

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*((a + c*x^2)^(p
+ 1)/(2*c*(p + 1)*(2*p + 3))), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rule 1266

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_S
ymbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + c*x^2)^p, x],
x, x^2], x] /; FreeQ[{a, c, d, e, p, q}, x] && IntegerQ[(m + 1)/2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{2}\text{Subst}\left(\int x(2+3x)\left(5+x^2\right)^{3/2}\,dx,x,x^2\right)\\
&= \frac{1}{20}\left(4+5x^2\right)\left(5+x^4\right)^{5/2}-\frac{5}{4}\text{Subst}\left(\int \left(5+x^2\right)^{3/2}\,dx,x,x^2\right)\\
&= -\frac{5}{16}x^2\left(5+x^4\right)^{3/2}+\frac{1}{20}\left(4+5x^2\right)\left(5+x^4\right)^{5/2}-\frac{75}{16}\text{Subst}\left(\int \sqrt{5+x^2}\,dx,x,x^2\right)\\
&= -\frac{75}{32}x^2\sqrt{5+x^4}-\frac{5}{16}x^2\left(5+x^4\right)^{3/2}\\
&\quad+\frac{1}{20}\left(4+5x^2\right)\left(5+x^4\right)^{5/2}-\frac{375}{32}\text{Subst}\left(\int \frac{1}{\sqrt{5+x^2}}\,dx,x,x^2\right)\\
&= -\frac{75}{32}x^2\sqrt{5+x^4}-\frac{5}{16}x^2\left(5+x^4\right)^{3/2}+\frac{1}{20}\left(4+5x^2\right)\left(5+x^4\right)^{5/2}-\frac{375}{32}\sinh^{-1}\left(\frac{x^2}{\sqrt{5}}\right)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.91

$$\int x^3(2+3x^2)\left(5+x^4\right)^{3/2}\,dx=\frac{1}{160}\sqrt{5+x^4}\left(800+375x^2+320x^4+350x^6+32x^8+40x^{10}\right)+\frac{375}{32}\log\left(-x^2+\sqrt{5+x^4}\right)$$

```
[In] Integrate[x^3*(2 + 3*x^2)*(5 + x^4)^(3/2),x]
```

```
[Out] (Sqrt[5 + x^4]*(800 + 375*x^2 + 320*x^4 + 350*x^6 + 32*x^8 + 40*x^10))/160
+ (375*Log[-x^2 + Sqrt[5 + x^4]])/32
```

Maple [A] (verified)

Time = 0.30 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.73

method	result
risch	$\frac{(40x^{10}+32x^8+350x^6+320x^4+375x^2+800)\sqrt{x^4+5}}{160}-\frac{375\,\operatorname{arcsinh}\left(\frac{x^2\sqrt{5}}{5}\right)}{32}$
pseudoelliptic	$\frac{(40x^{10}+32x^8+350x^6+320x^4+375x^2+800)\sqrt{x^4+5}}{160}-\frac{375\,\operatorname{arcsinh}\left(\frac{x^2\sqrt{5}}{5}\right)}{32}$
trager	$\left(\frac{1}{4}x^{10}+\frac{1}{5}x^8+\frac{35}{16}x^6+2x^4+\frac{75}{32}x^2+5\right)\sqrt{x^4+5}-\frac{375\ln\left(x^2+\sqrt{x^4+5}\right)}{32}$
default	$\frac{x^{10}\sqrt{x^4+5}}{4}+\frac{35x^6\sqrt{x^4+5}}{16}+\frac{75x^2\sqrt{x^4+5}}{32}-\frac{375\,\operatorname{arcsinh}\left(\frac{x^2\sqrt{5}}{5}\right)}{32}+\frac{(x^4+5)^{\frac{5}{2}}}{5}$
elliptic	$5\sqrt{x^4+5}+\frac{x^{10}\sqrt{x^4+5}}{4}+\frac{35x^6\sqrt{x^4+5}}{16}+\frac{75x^2\sqrt{x^4+5}}{32}-\frac{375\,\operatorname{arcsinh}\left(\frac{x^2\sqrt{5}}{5}\right)}{32}+\frac{x^8\sqrt{x^4+5}}{5}+2x^4\sqrt{x^4+5}$
meijerg	$\frac{\frac{25\sqrt{\pi}\,x^2\sqrt{5}\left(\frac{8}{25}x^8+\frac{14}{5}x^4+3\right)\sqrt{1+\frac{x^4}{5}}}{32}-\frac{375\sqrt{\pi}\,\operatorname{arcsinh}\left(\frac{x^2\sqrt{5}}{5}\right)}{32}}{\sqrt{\pi}}+\frac{75\sqrt{5}\left(-\frac{8\sqrt{\pi}}{15}+\frac{4\sqrt{\pi}\left(\frac{2}{25}x^8+\frac{4}{5}x^4+2\right)\sqrt{1+\frac{x^4}{5}}}{15}\right)}{8\sqrt{\pi}}$

```
[In] int(x^3*(3*x^2+2)*(x^4+5)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/160*(40*x^10+32*x^8+350*x^6+320*x^4+375*x^2+800)*(x^4+5)^(1/2)-375/32*arc
sinh(1/5*x^2*5^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.79

$$\int x^3\left(2+3x^2\right)\left(5+x^4\right)^{3/2}\,dx=\frac{1}{160}\left(40\,x^{10}+32\,x^8+350\,x^6+320\,x^4+375\,x^2+800\right)\sqrt{x^4+5}+\frac{375}{32}\,\log\left(-x^2+\sqrt{x^4+5}\right)$$

```
[In] integrate(x^3*(3*x^2+2)*(x^4+5)^(3/2),x, algorithm="fricas")
[Out] 1/160*(40*x^10 + 32*x^8 + 350*x^6 + 320*x^4 + 375*x^2 + 800)*sqrt(x^4 + 5)
+ 375/32*log(-x^2 + sqrt(x^4 + 5))
```

Sympy [A] (verification not implemented)

Time = 0.85 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.63

$$\int x^3\left(2+3x^2\right)\left(5+x^4\right)^{3/2}\,dx=5\left(\frac{x^4}{3}+\frac{5}{3}\right)\sqrt{x^4+5}+\frac{15\sqrt{x^4+5}\left(\frac{x^6}{4}+\frac{5x^2}{8}\right)}{2}+\sqrt{x^4+5}\left(\frac{x^8}{5}+\frac{x^4}{3}-\frac{10}{3}\right)+\frac{3\sqrt{x^4+5}\left(\frac{x^{10}}{6}+\frac{5x^6}{24}-\frac{25x^2}{16}\right)}{2}-\frac{375\,\mathrm{asinh}\left(\frac{\sqrt{5}x^2}{5}\right)}{32}$$

```
[In] integrate(x**3*(3*x**2+2)*(x**4+5)**(3/2),x)
[Out] 5*(x**4/3 + 5/3)*sqrt(x**4 + 5) + 15*sqrt(x**4 + 5)*(x**6/4 + 5*x**2/8)/2 +
 sqrt(x**4 + 5)*(x**8/5 + x**4/3 - 10/3) + 3*sqrt(x**4 + 5)*(x**10/6 + 5*x*
*6/24 - 25*x**2/16)/2 - 375*asinh(sqrt(5)*x**2/5)/32
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 118 vs. 2(52) = 104.

Time = 0.28 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.76

$$\int x^3\left(2+3x^2\right)\left(5+x^4\right)^{3/2}\,dx=\frac{1}{5}\left(x^4+5\right)^{\frac{5}{2}}-\frac{125\left(\frac{3\sqrt{x^4+5}}{x^2}-\frac{8\left(x^4+5\right)^{\frac{3}{2}}}{x^6}-\frac{3\left(x^4+5\right)^{\frac{5}{2}}}{x^{10}}\right)}{32\left(\frac{3\left(x^4+5\right)}{x^4}-\frac{3\left(x^4+5\right)^2}{x^8}+\frac{\left(x^4+5\right)^3}{x^{12}}-1\right)}-\frac{375}{64}\,\log\left(\frac{\sqrt{x^4+5}}{x^2}+1\right)+\frac{375}{64}\,\log\left(\frac{\sqrt{x^4+5}}{x^2}-1\right)$$

```
[In] integrate(x^3*(3*x^2+2)*(x^4+5)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/5*(x^4 + 5)^(5/2) - 125/32*(3*sqrt(x^4 + 5)/x^2 - 8*(x^4 + 5)^(3/2)/x^6 -
 3*(x^4 + 5)^(5/2)/x^10)/(3*(x^4 + 5)/x^4 - 3*(x^4 + 5)^2/x^8 + (x^4 + 5)^3
/x^12 - 1) - 375/64*log(sqrt(x^4 + 5)/x^2 + 1) + 375/64*log(sqrt(x^4 + 5)/x
^2 - 1)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.06

$$\int x^3(2+3x^2)\left(5+x^4\right)^{3/2}\,dx = \frac{1}{32}\left(2\left(4\,x^4+5\right)x^4-75\right)\sqrt{x^4+5}x^2 + \frac{15}{16}\left(2\,x^4+5\right)\sqrt{x^4+5}x^2 + \frac{1}{5}\left(x^4+5\right)^{\frac{5}{2}} + \frac{375}{32}\,\log\left(-x^2+\sqrt{x^4+5}\right)$$

```
[In] integrate(x^3*(3*x^2+2)*(x^4+5)^(3/2),x, algorithm="giac")
```

```
[Out] 1/32*(2*(4*x^4 + 5)*x^4 - 75)*sqrt(x^4 + 5)*x^2 + 15/16*(2*x^4 + 5)*sqrt(x^
4 + 5)*x^2 + 1/5*(x^4 + 5)^(5/2) + 375/32*log(-x^2 + sqrt(x^4 + 5))
```

Mupad [B] (verification not implemented)

Time = 7.69 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.70

$$\int x^3(2+3x^2)\left(5+x^4\right)^{3/2}\,dx = \sqrt{x^4+5}\left(\frac{x^{10}}{4}+\frac{x^8}{5}+\frac{35\,x^6}{16}+2\,x^4+\frac{75\,x^2}{32}+5\right) - \frac{375\,\mathrm{asinh}\left(\frac{\sqrt{5}\,x^2}{5}\right)}{32}$$

```
[In] int(x^3*(x^4 + 5)^(3/2)*(3*x^2 + 2),x)
```

```
[Out] (x^4 + 5)^(1/2)*((75*x^2)/32 + 2*x^4 + (35*x^6)/16 + x^8/5 + x^10/4 + 5) -
(375*asinh((5^(1/2)*x^2)/5))/32
```

3.22 $\int x(2+3x^2)\left(5+x^4\right)^{3/2} dx$

Optimal result

Integrand size = 18, antiderivative size = 60

$$\int x(2+3x^2)\left(5+x^4\right)^{3/2} dx = \frac{15}{8}x^2\sqrt{5+x^4} + \frac{1}{4}x^2\left(5+x^4\right)^{3/2} + \frac{3}{10}\left(5+x^4\right)^{5/2} + \frac{75}{8}\operatorname{arcsinh}\left(\frac{x^2}{\sqrt{5}}\right)$$

```
[Out] 1/4*x^2*(x^4+5)^(3/2)+3/10*(x^4+5)^(5/2)+75/8*arcsinh(1/5*x^2*5^(1/2))+15/8
*x^2*(x^4+5)^(1/2)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {1262, 655, 201, 221}

$$\int x(2+3x^2)\left(5+x^4\right)^{3/2} dx = \frac{75}{8}\operatorname{arcsinh}\left(\frac{x^2}{\sqrt{5}}\right) + \frac{3}{10}\left(x^4+5\right)^{5/2} + \frac{1}{4}x^2\left(x^4+5\right)^{3/2} + \frac{15}{8}x^2\sqrt{x^4+5}$$

```
[In] Int[x*(2 + 3*x^2)*(5 + x^4)^(3/2),x]
```

```
[Out] (15*x^2*Sqrt[5 + x^4])/8 + (x^2*(5 + x^4)^(3/2))/4 + (3*(5 + x^4)^(5/2))/10
 + (75*ArcSinh[x^2/Sqrt[5]])/8
```

Rule 201

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Dist[a*n*(p/(n*p + 1)), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
```

```
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 655

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[e*((
a + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 1262

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol]
 :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + c*x^2)^p, x], x, x^2], x] /; FreeQ
[{a, c, d, e, p, q}, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{2}\text{Subst}\left(\int (2+3x)\left(5+x^2\right)^{3/2}\,dx, x, x^2\right)\\
&= \frac{3}{10}\left(5+x^4\right)^{5/2}+\text{Subst}\left(\int \left(5+x^2\right)^{3/2}\,dx, x, x^2\right)\\
&= \frac{1}{4}x^2\left(5+x^4\right)^{3/2}+\frac{3}{10}\left(5+x^4\right)^{5/2}+\frac{15}{4}\text{Subst}\left(\int \sqrt{5+x^2}\,dx, x, x^2\right)\\
&= \frac{15}{8}x^2\sqrt{5+x^4}+\frac{1}{4}x^2\left(5+x^4\right)^{3/2}+\frac{3}{10}\left(5+x^4\right)^{5/2}+\frac{75}{8}\text{Subst}\left(\int \frac{1}{\sqrt{5+x^2}}\,dx, x, x^2\right)\\
&= \frac{15}{8}x^2\sqrt{5+x^4}+\frac{1}{4}x^2\left(5+x^4\right)^{3/2}+\frac{3}{10}\left(5+x^4\right)^{5/2}+\frac{75}{8}\sinh^{-1}\left(\frac{x^2}{\sqrt{5}}\right)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.93

$$\int x\left(2+3x^2\right)\left(5+x^4\right)^{3/2}\,dx = \frac{1}{40}\sqrt{5+x^4}\left(300+125x^2+120x^4+10x^6+12x^8\right) - \frac{75}{8}\log\left(-x^2+\sqrt{5+x^4}\right)$$

```
[In] Integrate[x*(2 + 3*x^2)*(5 + x^4)^(3/2),x]
[Out] (Sqrt[5 + x^4]*(300 + 125*x^2 + 120*x^4 + 10*x^6 + 12*x^8))/40 - (75*Log[-x
^2 + Sqrt[5 + x^4]])/8
```

Maple [A] (verified)

Time = 0.28 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.73

method	result	size
risch	$\frac{(12x^8+10x^6+120x^4+125x^2+300)\sqrt{x^4+5}}{40}+\frac{75\,\operatorname{arcsinh}\left(\frac{x^2\sqrt{5}}{5}\right)}{8}$	44
pseudoelliptic	$\frac{(12x^8+10x^6+120x^4+125x^2+300)\sqrt{x^4+5}}{40}+\frac{75\,\operatorname{arcsinh}\left(\frac{x^2\sqrt{5}}{5}\right)}{8}$	44
default	$\frac{75\,\operatorname{arcsinh}\left(\frac{x^2\sqrt{5}}{5}\right)}{8}+\frac{x^6\sqrt{x^4+5}}{4}+\frac{25x^2\sqrt{x^4+5}}{8}+\frac{3(x^4+5)^{\frac{5}{2}}}{10}$	46
trager	$\left(\frac{3}{10}x^8+\frac{1}{4}x^6+3x^4+\frac{25}{8}x^2+\frac{15}{2}\right)\sqrt{x^4+5}-\frac{75\ln\left(x^2-\sqrt{x^4+5}\right)}{8}$	48
elliptic	$\frac{75\,\operatorname{arcsinh}\left(\frac{x^2\sqrt{5}}{5}\right)}{8}+\frac{15\sqrt{x^4+5}}{2}+\frac{3x^8\sqrt{x^4+5}}{10}+3x^4\sqrt{x^4+5}+\frac{x^6\sqrt{x^4+5}}{4}+\frac{25x^2\sqrt{x^4+5}}{8}$	70
meijerg	$\frac{225\sqrt{5}\left(-\frac{8\sqrt{\pi}}{15}+\frac{4\sqrt{\pi}\left(\frac{2}{25}x^8+\frac{4}{5}x^4+2\right)\sqrt{1+\frac{x^4}{5}}}{15}\right)}{16\sqrt{\pi}}+\frac{5\sqrt{\pi}\,x^2\sqrt{5}\left(\frac{x^4}{20}+\frac{5}{8}\right)\sqrt{1+\frac{x^4}{5}}+\frac{75\sqrt{\pi}\,\operatorname{arcsinh}\left(\frac{x^2\sqrt{5}}{5}\right)}{8}}{\sqrt{\pi}}$	88

```
[In] int(x*(3*x^2+2)*(x^4+5)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/40*(12*x^8+10*x^6+120*x^4+125*x^2+300)*(x^4+5)^(1/2)+75/8*arcsinh(1/5*x^2
*5^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.80

$$\int x(2+3x^2)\left(5+x^4\right)^{3/2}\,dx=\frac{1}{40}\left(12\,x^8+10\,x^6+120\,x^4+125\,x^2+300\right)\sqrt{x^4+5}-\frac{75}{8}\log\left(-x^2+\sqrt{x^4+5}\right)$$

```
[In] integrate(x*(3*x^2+2)*(x^4+5)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/40*(12*x^8 + 10*x^6 + 120*x^4 + 125*x^2 + 300)*sqrt(x^4 + 5) - 75/8*log(-
x^2 + sqrt(x^4 + 5))
```

Sympy [A] (verification not implemented)

Time = 0.79 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.58

$$\int x(2+3x^2)\left(5+x^4\right)^{3/2}\,dx=\frac{5x^2\sqrt{x^4+5}}{2}+\frac{15\left(\frac{x^4}{3}+\frac{5}{3}\right)\sqrt{x^4+5}}{2}+\sqrt{x^4+5}\left(\frac{x^6}{4}+\frac{5x^2}{8}\right)+\frac{3\sqrt{x^4+5}\left(\frac{x^8}{5}+\frac{x^4}{3}-\frac{10}{3}\right)}{2}+\frac{75\operatorname{asinh}\left(\frac{\sqrt{5}x^2}{5}\right)}{8}$$

```
[In] integrate(x*(3*x**2+2)*(x**4+5)**(3/2),x)
[Out] 5*x**2*sqrt(x**4 + 5)/2 + 15*(x**4/3 + 5/3)*sqrt(x**4 + 5)/2 + sqrt(x**4 +
5)*(x**6/4 + 5*x**2/8) + 3*sqrt(x**4 + 5)*(x**8/5 + x**4/3 - 10/3)/2 + 75*a
sinh(sqrt(5)*x**2/5)/8
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 95 vs. 2(45) = 90.

Time = 0.27 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.58

$$\int x(2+3x^2)\left(5+x^4\right)^{3/2}\,dx=\frac{3}{10}\left(x^4+5\right)^{\frac{5}{2}}+\frac{25\left(\frac{3\sqrt{x^4+5}}{x^2}-\frac{5\left(x^4+5\right)^{\frac{3}{2}}}{x^6}\right)}{8\left(\frac{2\left(x^4+5\right)}{x^4}-\frac{\left(x^4+5\right)^2}{x^8}-1\right)}+\frac{75}{16}\log\left(\frac{\sqrt{x^4+5}}{x^2}+1\right)-\frac{75}{16}\log\left(\frac{\sqrt{x^4+5}}{x^2}-1\right)$$

```
[In] integrate(x*(3*x^2+2)*(x^4+5)^(3/2),x, algorithm="maxima")
[Out] 3/10*(x^4 + 5)^(5/2) + 25/8*(3*sqrt(x^4 + 5)/x^2 - 5*(x^4 + 5)^(3/2)/x^6)/(
2*(x^4 + 5)/x^4 - (x^4 + 5)^2/x^8 - 1) + 75/16*log(sqrt(x^4 + 5)/x^2 + 1) -
 75/16*log(sqrt(x^4 + 5)/x^2 - 1)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.95

$$\int x(2+3x^2)\left(5+x^4\right)^{3/2}\,dx=\frac{1}{8}\left(2\,x^4+5\right)\sqrt{x^4+5}x^2+\frac{3}{10}\left(x^4+5\right)^{\frac{5}{2}}+\frac{5}{2}\sqrt{x^4+5}x^2-\frac{75}{8}\log\left(-x^2+\sqrt{x^4+5}\right)$$

```
[In] integrate(x*(3*x^2+2)*(x^4+5)^(3/2),x, algorithm="giac")
[Out] 1/8*(2*x^4 + 5)*sqrt(x^4 + 5)*x^2 + 3/10*(x^4 + 5)^(5/2) + 5/2*sqrt(x^4 + 5
)*x^2 - 75/8*log(-x^2 + sqrt(x^4 + 5))
```

Mupad [B] (verification not implemented)

Time = 7.54 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.70

$$\int x(2+3x^2)\left(5+x^4\right)^{3/2}\,dx=\frac{75\,\mathrm{asinh}\left(\frac{\sqrt{5}\,x^2}{5}\right)}{8}+\sqrt{x^4+5}\left(\frac{3\,x^8}{10}+\frac{x^6}{4}+3\,x^4+\frac{25\,x^2}{8}+\frac{15}{2}\right)$$

```
[In] int(x*(x^4 + 5)^(3/2)*(3*x^2 + 2),x)
[Out] (75*asinh((5^(1/2)*x^2)/5))/8 + (x^4 + 5)^(1/2)*((25*x^2)/8 + 3*x^4 + x^6/4
 + (3*x^8)/10 + 15/2)
```

3.23 $\int \frac{(2+3x^2)(5+x^4)^{3/2}}{x} \, dx$

Optimal result

Integrand size = 20, antiderivative size = 78

$$\int \frac{(2+3x^2)(5+x^4)^{3/2}}{x} \, dx = \frac{5}{16}(16+9x^2)\sqrt{5+x^4} + \frac{1}{24}(8+9x^2)(5+x^4)^{3/2} + \frac{225}{16}\text{arcsinh}\left(\frac{x^2}{\sqrt{5}}\right) - 5\sqrt{5}\text{arctanh}\left(\frac{\sqrt{5+x^4}}{\sqrt{5}}\right)$$

```
[Out] 1/24*(9*x^2+8)*(x^4+5)^(3/2)+225/16*arcsinh(1/5*x^2*5^(1/2))-5*arctanh(1/5*
(x^4+5)^(1/2)*5^(1/2))*5^(1/2)+5/16*(9*x^2+16)*(x^4+5)^(1/2)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.350, Rules used = {1266, 829, 858, 221, 272, 65, 213}

$$\int \frac{(2+3x^2)(5+x^4)^{3/2}}{x} \, dx = \frac{225}{16}\text{arcsinh}\left(\frac{x^2}{\sqrt{5}}\right) - 5\sqrt{5}\text{arctanh}\left(\frac{\sqrt{x^4+5}}{\sqrt{5}}\right) + \frac{1}{24}(9x^2+8)(x^4+5)^{3/2} + \frac{5}{16}(9x^2+16)\sqrt{x^4+5}$$

```
[In] Int[((2 + 3*x^2)*(5 + x^4)^(3/2))/x,x]
```

```
[Out] (5*(16 + 9*x^2)*Sqrt[5 + x^4])/16 + ((8 + 9*x^2)*(5 + x^4)^(3/2))/24 + (225
*ArcSinh[x^2/Sqrt[5]])/16 - 5*Sqrt[5]*ArcTanh[Sqrt[5 + x^4]/Sqrt[5]]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 829

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2) - g*c*d*(2*p
+ 1) + g*c*e*(m + 2*p + 1)*x)*((a + c*x^2)^p/(c*e^2*(m + 2*p + 1)*(m + 2*p
+ 2))), x] + Dist[2*(p/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2))), Int[(d + e*x)^
m*(a + c*x^2)^(p - 1)*Simp[f*a*c*e^2*(m + 2*p + 2) + a*c*d*e*g*m - (c^2*f*d
*e*(m + 2*p + 2) - g*(c^2*d^2*(2*p + 1) + a*c*e^2*(m + 2*p + 1)))*x, x], x]
, x] /; FreeQ[{a, c, d, e, f, g, m}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p,
0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 0])) &&  !ILt
Q[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 858

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1266

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_S
ymbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + c*x^2)^p, x],
```

```
x, x^2], x] /; FreeQ[{a, c, d, e, p, q}, x] && IntegerQ[(m + 1)/2]
```

Rubi steps

$$\text{integral} = \frac{1}{2}\text{Subst}\left(\int \frac{(2+3x)\,(5+x^2)^{3/2}}{x}\,dx, x, x^2\right)$$

$$= \frac{1}{24}(8+9x^2)\,(5+x^4)^{3/2} + \frac{1}{8}\text{Subst}\left(\int \frac{(40+45x)\sqrt{5+x^2}}{x}\,dx, x, x^2\right)$$

$$= \frac{5}{16}(16+9x^2)\,\sqrt{5+x^4} + \frac{1}{24}(8+9x^2)\,(5+x^4)^{3/2} + \frac{1}{16}\text{Subst}\left(\int \frac{400+225x}{x\sqrt{5+x^2}}\,dx, x, x^2\right)$$

$$= \frac{5}{16}(16+9x^2)\,\sqrt{5+x^4} + \frac{1}{24}(8+9x^2)\,(5+x^4)^{3/2} + \frac{225}{16}\text{Subst}\left(\int \frac{1}{\sqrt{5+x^2}}\,dx, x, x^2\right) + 25\text{Subst}\left(\int \frac{1}{x\sqrt{5+x^2}}\,dx, x, x^2\right)$$

$$= \frac{5}{16}(16+9x^2)\,\sqrt{5+x^4} + \frac{1}{24}(8+9x^2)\,(5+x^4)^{3/2} + \frac{225}{16}\sinh^{-1}\left(\frac{x^2}{\sqrt{5}}\right) + \frac{25}{2}\text{Subst}\left(\int \frac{1}{x\sqrt{5+x}}\,dx, x, x^4\right)$$

$$= \frac{5}{16}(16+9x^2)\,\sqrt{5+x^4} + \frac{1}{24}(8+9x^2)\,(5+x^4)^{3/2} + \frac{225}{16}\sinh^{-1}\left(\frac{x^2}{\sqrt{5}}\right) + 25\text{Subst}\left(\int \frac{1}{-5+x^2}\,dx, x, \sqrt{5+x^4}\right)$$

$$= \frac{5}{16}(16+9x^2)\,\sqrt{5+x^4} + \frac{1}{24}(8+9x^2)\,(5+x^4)^{3/2} + \frac{225}{16}\sinh^{-1}\left(\frac{x^2}{\sqrt{5}}\right) - 5\sqrt{5}\tanh^{-1}\left(\frac{\sqrt{5+x^4}}{\sqrt{5}}\right)$$

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.01

$$\int \frac{(2+3x^2)\,(5+x^4)^{3/2}}{x}\,dx = \frac{1}{48}\left(\sqrt{5+x^4}(320+225x^2+16x^4+18x^6) + 480\sqrt{5}\text{arctanh}\left(\frac{x^2-\sqrt{5+x^4}}{\sqrt{5}}\right) - 675\log\left(-x^2+\sqrt{5+x^4}\right)\right)$$

```
[In] Integrate[((2 + 3*x^2)*(5 + x^4)^(3/2))/x,x]
[Out] (Sqrt[5 + x^4]*(320 + 225*x^2 + 16*x^4 + 18*x^6) + 480*Sqrt[5]*ArcTanh[(x^2
 - Sqrt[5 + x^4])/Sqrt[5]] - 675*Log[-x^2 + Sqrt[5 + x^4]])/48
```

Maple [A] (verified)

Time = 0.44 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.72

method	result
pseudoelliptic	$-5\sqrt{5}\,\operatorname{arctanh}\left(\frac{\sqrt{5}}{\sqrt{x^4+5}}\right)+\frac{225\,\operatorname{arcsinh}\left(\frac{x^2\sqrt{5}}{5}\right)}{16}+\frac{(18x^6+16x^4+225x^2+320)\sqrt{x^4+5}}{48}$
trager	$\left(\frac{3}{8}x^6+\frac{1}{3}x^4+\frac{75}{16}x^2+\frac{20}{3}\right)\sqrt{x^4+5}+\frac{225\ln\left(-x^2-\sqrt{x^4+5}\right)}{16}-5\,\mathrm{RootOf}\left(\_Z^2-5\right)\ln\left(\frac{\mathrm{RootOf}\left(\_\right.}{}\right.$
default	$\frac{225\,\operatorname{arcsinh}\left(\frac{x^2\sqrt{5}}{5}\right)}{16}+\frac{3x^6\sqrt{x^4+5}}{8}+\frac{75x^2\sqrt{x^4+5}}{16}+\frac{x^4\sqrt{x^4+5}}{3}+\frac{20\sqrt{x^4+5}}{3}-5\sqrt{5}\,\operatorname{arctanh}\left(\frac{\sqrt{5}}{\sqrt{x^4+5}}\right)$
elliptic	$\frac{225\,\operatorname{arcsinh}\left(\frac{x^2\sqrt{5}}{5}\right)}{16}+\frac{3x^6\sqrt{x^4+5}}{8}+\frac{75x^2\sqrt{x^4+5}}{16}+\frac{x^4\sqrt{x^4+5}}{3}+\frac{20\sqrt{x^4+5}}{3}-5\sqrt{5}\,\operatorname{arctanh}\left(\frac{\sqrt{5}}{\sqrt{x^4+5}}\right)$
meijerg	$\frac{15\sqrt{5}\left(-\frac{32\sqrt{\pi}}{9}+\frac{2\sqrt{\pi}\left(\frac{4x^4}{5}+16\right)\sqrt{1+\frac{x^4}{5}}}{9}-\frac{8\sqrt{\pi}\,\ln\left(\frac{1}{2}+\frac{\sqrt{1+\frac{x^4}{5}}}{2}\right)}{3}+\frac{4\left(\frac{8}{3}-2\ln(2)+4\ln(x)-\ln(5)\right)\sqrt{\pi}}{3}\right)}{8\sqrt{\pi}}+\frac{15\sqrt{\pi}\,x^2\sqrt{5}\left(\frac{x^4}{20}+\frac{5}{8}\right)\sqrt{}}{2}$

[In] int((3*x^2+2)*(x^4+5)^(3/2)/x,x,method=_RETURNVERBOSE)

[Out] -5*5^(1/2)*arctanh(5^(1/2)/(x^4+5)^(1/2))+225/16*arcsinh(1/5*x^2*5^(1/2))+1/48*(18*x^6+16*x^4+225*x^2+320)*(x^4+5)^(1/2)

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.86

$$\int \frac{(2+3x^2)\,(5+x^4)^{3/2}}{x}\,dx = \frac{1}{48}\left(18\,x^6+16\,x^4+225\,x^2+320\right)\sqrt{x^4+5} + 5\,\sqrt{5}\log\left(-\frac{\sqrt{5}-\sqrt{x^4+5}}{x^2}\right) - \frac{225}{16}\,\log\left(-x^2+\sqrt{x^4+5}\right)$$

[In] integrate((3*x^2+2)*(x^4+5)^(3/2)/x,x, algorithm="fricas")

[Out] 1/48*(18*x^6 + 16*x^4 + 225*x^2 + 320)*sqrt(x^4 + 5) + 5*sqrt(5)*log(-(sqrt(5) - sqrt(x^4 + 5))/x^2) - 225/16*log(-x^2 + sqrt(x^4 + 5))

Sympy [A] (verification not implemented)

Time = 9.32 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.50

$$\int \frac{(2+3x^2)(5+x^4)^{3/2}}{x}\,dx = \frac{15x^2\sqrt{x^4+5}}{4} + \left(\frac{x^4}{3}+\frac{5}{3}\right)\sqrt{x^4+5} + \frac{3\sqrt{x^4+5}\left(\frac{x^6}{4}+\frac{5x^2}{8}\right)}{2} + 5\sqrt{5}\left(\sqrt{\frac{x^4}{5}+1} + \frac{\log\left(\sqrt{\frac{x^4}{5}+1}-1\right)}{2} - \frac{\log\left(\sqrt{\frac{x^4}{5}+1}+1\right)}{2}\right) + \frac{225\operatorname{asinh}\left(\frac{\sqrt{5}x^2}{5}\right)}{16}$$

```
[In] integrate((3*x**2+2)*(x**4+5)**(3/2)/x,x)
```

```
[Out] 15*x**2*sqrt(x**4 + 5)/4 + (x**4/3 + 5/3)*sqrt(x**4 + 5) + 3*sqrt(x**4 + 5)
*(x**6/4 + 5*x**2/8)/2 + 5*sqrt(5)*(sqrt(x**4/5 + 1) + log(sqrt(x**4/5 + 1)
 - 1)/2 - log(sqrt(x**4/5 + 1) + 1)/2) + 225*asinh(sqrt(5)*x**2/5)/16
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 138 vs. $2(62) = 124$.

Time = 0.28 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.77

$$\int \frac{(2+3x^2)(5+x^4)^{3/2}}{x}\,dx = \frac{1}{3}\left(x^4+5\right)^{\frac{3}{2}} + \frac{5}{2}\sqrt{5}\log\left(-\frac{\sqrt{5}-\sqrt{x^4+5}}{\sqrt{5}+\sqrt{x^4+5}}\right) + 5\sqrt{x^4+5} + \frac{75\left(\frac{3\sqrt{x^4+5}}{x^2} - \frac{5\left(x^4+5\right)^{\frac{3}{2}}}{x^6}\right)}{16\left(\frac{2\left(x^4+5\right)}{x^4} - \frac{\left(x^4+5\right)^2}{x^8} - 1\right)} + \frac{225}{32}\log\left(\frac{\sqrt{x^4+5}}{x^2}+1\right) - \frac{225}{32}\log\left(\frac{\sqrt{x^4+5}}{x^2}-1\right)$$

```
[In] integrate((3*x^2+2)*(x^4+5)^(3/2)/x,x, algorithm="maxima")
```

```
[Out] 1/3*(x^4 + 5)^(3/2) + 5/2*sqrt(5)*log(-(sqrt(5) - sqrt(x^4 + 5))/(sqrt(5) +
 sqrt(x^4 + 5))) + 5*sqrt(x^4 + 5) + 75/16*(3*sqrt(x^4 + 5)/x^2 - 5*(x^4 +
5)^(3/2)/x^6)/(2*(x^4 + 5)/x^4 - (x^4 + 5)^2/x^8 - 1) + 225/32*log(sqrt(x^4
 + 5)/x^2 + 1) - 225/32*log(sqrt(x^4 + 5)/x^2 - 1)
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.15

$$\int \frac{(2+3x^2)(5+x^4)^{3/2}}{x}\,dx = \frac{1}{48}\sqrt{x^4+5}\big((2(9x^2+8)x^2+225)x^2+320\big) + 5\sqrt{5}\log\left(-\frac{x^2+\sqrt{5}-\sqrt{x^4+5}}{x^2-\sqrt{5}-\sqrt{x^4+5}}\right) - \frac{225}{16}\log\left(-x^2+\sqrt{x^4+5}\right)$$

```
[In] integrate((3*x^2+2)*(x^4+5)^(3/2)/x,x, algorithm="giac")
```

```
[Out] 1/48*sqrt(x^4 + 5)*((2*(9*x^2 + 8)*x^2 + 225)*x^2 + 320) + 5*sqrt(5)*log(-(
x^2 + sqrt(5) - sqrt(x^4 + 5))/(x^2 - sqrt(5) - sqrt(x^4 + 5))) - 225/16*lo
g(-x^2 + sqrt(x^4 + 5))
```

Mupad [B] (verification not implemented)

Time = 7.58 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.71

$$\int \frac{(2+3x^2)(5+x^4)^{3/2}}{x}\,dx = \frac{225\,\mathrm{asinh}\left(\frac{\sqrt{5}\,x^2}{5}\right)}{16} - 5\sqrt{5}\,\mathrm{atanh}\left(\frac{\sqrt{5}\sqrt{x^4+5}}{5}\right) + \sqrt{x^4+5}\left(\frac{3x^6}{8}+\frac{x^4}{3}+\frac{75x^2}{16}+\frac{20}{3}\right)$$

```
[In] int(((x^4 + 5)^(3/2)*(3*x^2 + 2))/x,x)
```

```
[Out] (225*asinh((5^(1/2)*x^2)/5))/16 - 5*5^(1/2)*atanh((5^(1/2)*(x^4 + 5)^(1/2))
/5) + (x^4 + 5)^(1/2)*((75*x^2)/16 + x^4/3 + (3*x^6)/8 + 20/3)
```

3.24 $\int \frac{(2+3x^2)(5+x^4)^{3/2}}{x^3} dx$

Optimal result

Integrand size = 20, antiderivative size = 81

$$\int \frac{(2+3x^2)(5+x^4)^{3/2}}{x^3} dx = \frac{3}{2}(5+x^2)\sqrt{5+x^4} - \frac{(2-x^2)(5+x^4)^{3/2}}{2x^2} + \frac{15}{2}\operatorname{arcsinh}\left(\frac{x^2}{\sqrt{5}}\right) - \frac{15}{2}\sqrt{5}\operatorname{arctanh}\left(\frac{\sqrt{5+x^4}}{\sqrt{5}}\right)$$

```
[Out] -1/2*(-x^2+2)*(x^4+5)^(3/2)/x^2+15/2*arcsinh(1/5*x^2*5^(1/2))-15/2*arctanh(
1/5*(x^4+5)^(1/2)*5^(1/2))*5^(1/2)+3/2*(x^2+5)*(x^4+5)^(1/2)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {1266, 827, 829, 858, 221, 272, 65, 213}

$$\int \frac{(2+3x^2)(5+x^4)^{3/2}}{x^3} dx = \frac{15}{2}\operatorname{arcsinh}\left(\frac{x^2}{\sqrt{5}}\right) - \frac{15}{2}\sqrt{5}\operatorname{arctanh}\left(\frac{\sqrt{x^4+5}}{\sqrt{5}}\right) - \frac{(2-x^2)(x^4+5)^{3/2}}{2x^2} + \frac{3}{2}(x^2+5)\sqrt{x^4+5}$$

```
[In] Int[((2 + 3*x^2)*(5 + x^4)^(3/2))/x^3,x]
```

```
[Out] (3*(5 + x^2)*Sqrt[5 + x^4])/2 - ((2 - x^2)*(5 + x^4)^(3/2))/(2*x^2) + (15*A
rcSinh[x^2/Sqrt[5]])/2 - (15*Sqrt[5]*ArcTanh[Sqrt[5 + x^4]/Sqrt[5]])/2
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 827

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(e*f*(m + 2*p + 2) - d*g*(2*p + 1)
 + e*g*(m + 1)*x)*((a + c*x^2)^p/(e^2*(m + 1)*(m + 2*p + 2))), x] + Dist[p/
(e^2*(m + 1)*(m + 2*p + 2)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^(p - 1)*Simp
[g*(2*a*e + 2*a*e*m) + (g*(2*c*d + 4*c*d*p) - 2*c*e*f*(m + 2*p + 2))*x, x],
 x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && NeQ[c*d^2 + a*e^2, 0] && Rati
onalQ[p] && p > 0 && (LtQ[m, -1] || EqQ[p, 1] || (IntegerQ[p] &&  !Rational
Q[m])) && NeQ[m, -1] &&  !ILtQ[m + 2*p + 1, 0] && (IntegerQ[m] || IntegerQ[
p] || IntegersQ[2*m, 2*p])
```

Rule 829

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2) - g*c*d*(2*p
+ 1) + g*c*e*(m + 2*p + 1)*x)*((a + c*x^2)^p/(c*e^2*(m + 2*p + 1)*(m + 2*p
+ 2))), x] + Dist[2*(p/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2))), Int[(d + e*x)^
m*(a + c*x^2)^(p - 1)*Simp[f*a*c*e^2*(m + 2*p + 2) + a*c*d*e*g*m - (c^2*f*d
*e*(m + 2*p + 2) - g*(c^2*d^2*(2*p + 1) + a*c*e^2*(m + 2*p + 1)))*x, x], x]
, x] /; FreeQ[{a, c, d, e, f, g, m}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p,
0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 0])) &&  !ILt
```

```
Q[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 858

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1266

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_S
ymbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + c*x^2)^p, x],
x, x^2], x] /; FreeQ[{a, c, d, e, p, q}, x] && IntegerQ[(m + 1)/2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{2}\text{Subst}\left(\int \frac{(2+3x)\left(5+x^2\right)^{3/2}}{x^2}\,dx,x,x^2\right)\\
&= -\frac{\left(2-x^2\right)\left(5+x^4\right)^{3/2}}{2x^2}-\frac{1}{4}\text{Subst}\left(\int \frac{(-30-12x)\sqrt{5+x^2}}{x}\,dx,x,x^2\right)\\
&= \frac{3}{2}\left(5+x^2\right)\sqrt{5+x^4}-\frac{\left(2-x^2\right)\left(5+x^4\right)^{3/2}}{2x^2}-\frac{1}{8}\text{Subst}\left(\int \frac{-300-60x}{x\sqrt{5+x^2}}\,dx,x,x^2\right)\\
&= \frac{3}{2}\left(5+x^2\right)\sqrt{5+x^4}-\frac{\left(2-x^2\right)\left(5+x^4\right)^{3/2}}{2x^2}\\
&\quad+\frac{15}{2}\text{Subst}\left(\int \frac{1}{\sqrt{5+x^2}}\,dx,x,x^2\right)+\frac{75}{2}\text{Subst}\left(\int \frac{1}{x\sqrt{5+x^2}}\,dx,x,x^2\right)\\
&= \frac{3}{2}\left(5+x^2\right)\sqrt{5+x^4}-\frac{\left(2-x^2\right)\left(5+x^4\right)^{3/2}}{2x^2}\\
&\quad+\frac{15}{2}\sinh^{-1}\left(\frac{x^2}{\sqrt{5}}\right)+\frac{75}{4}\text{Subst}\left(\int \frac{1}{x\sqrt{5+x}}\,dx,x,x^4\right)\\
&= \frac{3}{2}\left(5+x^2\right)\sqrt{5+x^4}-\frac{\left(2-x^2\right)\left(5+x^4\right)^{3/2}}{2x^2}\\
&\quad+\frac{15}{2}\sinh^{-1}\left(\frac{x^2}{\sqrt{5}}\right)+\frac{75}{2}\text{Subst}\left(\int \frac{1}{-5+x^2}\,dx,x,\sqrt{5+x^4}\right)\\
&= \frac{3}{2}\left(5+x^2\right)\sqrt{5+x^4}-\frac{\left(2-x^2\right)\left(5+x^4\right)^{3/2}}{2x^2}+\frac{15}{2}\sinh^{-1}\left(\frac{x^2}{\sqrt{5}}\right)-\frac{15}{2}\sqrt{5}\tanh^{-1}\left(\frac{\sqrt{5+x^4}}{\sqrt{5}}\right)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.96

$$\int \frac{(2+3x^2)(5+x^4)^{3/2}}{x^3}\,dx = \frac{1}{2}\left(\frac{\sqrt{5+x^4}(-10+20x^2+x^4+x^6)}{x^2} + 30\sqrt{5}\operatorname{arctanh}\left(\frac{x^2-\sqrt{5+x^4}}{\sqrt{5}}\right) - 15\log\left(-x^2+\sqrt{5+x^4}\right)\right)$$

```
[In] Integrate[((2 + 3*x^2)*(5 + x^4)^(3/2))/x^3,x]
[Out] ((Sqrt[5 + x^4]*(-10 + 20*x^2 + x^4 + x^6))/x^2 + 30*Sqrt[5]*ArcTanh[(x^2 -
 Sqrt[5 + x^4])/Sqrt[5]] - 15*Log[-x^2 + Sqrt[5 + x^4]])/2
```

Maple [A] (verified)

Time = 0.50 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.77

method	result
pseudoelliptic	$\frac{-15\sqrt{5}\,\operatorname{arctanh}\left(\frac{\sqrt{5}}{\sqrt{x^4+5}}\right)x^2+15\,\operatorname{arcsinh}\left(\frac{x^2\sqrt{5}}{5}\right)x^2+(x^6+x^4+20x^2-10)\sqrt{x^4+5}}{2x^2}$
trager	$\frac{(x^6+x^4+20x^2-10)\sqrt{x^4+5}}{2x^2}+\frac{15\ln\left(-x^2-\sqrt{x^4+5}\right)}{2}+\frac{15\,\operatorname{RootOf}\left(\_Z^2-5\right)\ln\left(\frac{\sqrt{x^4+5}-\operatorname{RootOf}\left(\_Z^2-5\right)}{x^2}\right)}{2}$
default	$\frac{x^4\sqrt{x^4+5}}{2}+10\sqrt{x^4+5}-\frac{15\sqrt{5}\,\operatorname{arctanh}\left(\frac{\sqrt{5}}{\sqrt{x^4+5}}\right)}{2}+\frac{15\,\operatorname{arcsinh}\left(\frac{x^2\sqrt{5}}{5}\right)}{2}+\frac{x^2\sqrt{x^4+5}}{2}-\frac{5\sqrt{x^4+5}}{x^2}$
elliptic	$\frac{x^4\sqrt{x^4+5}}{2}+10\sqrt{x^4+5}-\frac{15\sqrt{5}\,\operatorname{arctanh}\left(\frac{\sqrt{5}}{\sqrt{x^4+5}}\right)}{2}+\frac{15\,\operatorname{arcsinh}\left(\frac{x^2\sqrt{5}}{5}\right)}{2}+\frac{x^2\sqrt{x^4+5}}{2}-\frac{5\sqrt{x^4+5}}{x^2}$
risch	$-\frac{5\sqrt{x^4+5}}{x^2}+\frac{15\,\operatorname{arcsinh}\left(\frac{x^2\sqrt{5}}{5}\right)}{2}+\frac{\sqrt{x^4+5}\,(x^4-10)}{2}+\frac{x^2\sqrt{x^4+5}}{2}+15\sqrt{x^4+5}-\frac{15\sqrt{5}\,\operatorname{arctanh}\left(\frac{\sqrt{5}}{\sqrt{x^4+5}}\right)}{2}$
meijerg	$\frac{-\frac{5\sqrt{\pi}\,\sqrt{5}\left(-\frac{x^4}{10}+1\right)\sqrt{1+\frac{x^4}{5}}}{x^2}+\frac{15\sqrt{\pi}\,\operatorname{arcsinh}\left(\frac{x^2\sqrt{5}}{5}\right)}{2}}{\sqrt{\pi}}+\frac{45\sqrt{5}\left(-\frac{32\sqrt{\pi}}{9}+\frac{2\sqrt{\pi}\left(\frac{4x^4}{5}+16\right)\sqrt{1+\frac{x^4}{5}}}{9}-\frac{8\sqrt{\pi}\,\ln\left(\frac{1}{2}+\frac{\sqrt{1+\frac{x^4}{5}}}{2}\right)}{3}+ \cdots\right)}{16\sqrt{\pi}}$

```
[In] int((3*x^2+2)*(x^4+5)^(3/2)/x^3,x,method=_RETURNVERBOSE)
[Out] 1/2*(-15*5^(1/2)*arctanh(5^(1/2)/(x^4+5)^(1/2))*x^2+15*arcsinh(1/5*x^2*5^(1
/2))*x^2+(x^6+x^4+20*x^2-10)*(x^4+5)^(1/2))/x^2
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.96

$$\int \frac{(2+3x^2)(5+x^4)^{3/2}}{x^3}\,dx = \frac{15\sqrt{5}x^2\log\left(-\frac{\sqrt{5}-\sqrt{x^4+5}}{x^2}\right) - 15\,x^2\log\left(-x^2+\sqrt{x^4+5}\right) - 10\,x^2 + (x^6+x^4+2}{2\,x^2}$$

```
[In] integrate((3*x^2+2)*(x^4+5)^(3/2)/x^3,x, algorithm="fricas")
[Out] 1/2*(15*sqrt(5)*x^2*log(-(sqrt(5) - sqrt(x^4 + 5))/x^2) - 15*x^2*log(-x^2 +
 sqrt(x^4 + 5)) - 10*x^2 + (x^6 + x^4 + 20*x^2 - 10)*sqrt(x^4 + 5))/x^2
```

Sympy [A] (verification not implemented)

Time = 4.35 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.41

$$\int \frac{(2+3x^2)(5+x^4)^{3/2}}{x^3}\,dx = \frac{x^6}{2\sqrt{x^4+5}} - \frac{5x^2}{2\sqrt{x^4+5}} + \frac{(x^4+5)^{\frac{3}{2}}}{2} + \frac{15\sqrt{x^4+5}}{2} + \frac{15\sqrt{5}\log(x^4)}{4} - \frac{15\sqrt{5}\log\left(\sqrt{\frac{x^4}{5}+1}+1\right)}{2} + \frac{15\operatorname{asinh}\left(\frac{\sqrt{5}x^2}{5}\right)}{2} - \frac{25}{x^2\sqrt{x^4+5}}$$

```
[In] integrate((3*x**2+2)*(x**4+5)**(3/2)/x**3,x)
[Out] x**6/(2*sqrt(x**4 + 5)) - 5*x**2/(2*sqrt(x**4 + 5)) + (x**4 + 5)**(3/2)/2 +
 15*sqrt(x**4 + 5)/2 + 15*sqrt(5)*log(x**4)/4 - 15*sqrt(5)*log(sqrt(x**4/5
+ 1) + 1)/2 + 15*asinh(sqrt(5)*x**2/5)/2 - 25/(x**2*sqrt(x**4 + 5))
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.51

$$\int \frac{(2+3x^2)(5+x^4)^{3/2}}{x^3}\,dx = \frac{1}{2}\left(x^4+5\right)^{\frac{3}{2}} + \frac{15}{4}\sqrt{5}\log\left(-\frac{\sqrt{5}-\sqrt{x^4+5}}{\sqrt{5}+\sqrt{x^4+5}}\right) + \frac{15}{2}\sqrt{x^4+5} - \frac{5\sqrt{x^4+5}}{x^2} + \frac{5\sqrt{x^4+5}}{2\,x^2\left(\frac{x^4+5}{x^4}-1\right)} + \frac{15}{4}\log\left(\frac{\sqrt{x^4+5}}{x^2}+1\right) - \frac{15}{4}\log\left(\frac{\sqrt{x^4+5}}{x^2}-1\right)$$

```
[In] integrate((3*x^2+2)*(x^4+5)^(3/2)/x^3,x, algorithm="maxima")
```

```
[Out] 1/2*(x^4 + 5)^(3/2) + 15/4*sqrt(5)*log(-(sqrt(5) - sqrt(x^4 + 5))/(sqrt(5)
+ sqrt(x^4 + 5))) + 15/2*sqrt(x^4 + 5) - 5*sqrt(x^4 + 5)/x^2 + 5/2*sqrt(x^4
 + 5)/(x^2*((x^4 + 5)/x^4 - 1)) + 15/4*log(sqrt(x^4 + 5)/x^2 + 1) - 15/4*lo
g(sqrt(x^4 + 5)/x^2 - 1)
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.26

$$\int \frac{(2+3x^2)(5+x^4)^{3/2}}{x^3}\,dx = \frac{1}{2}\sqrt{x^4+5}\big((x^2+1)x^2+20\big) + \frac{15}{2}\sqrt{5}\log\left(-\frac{x^2+\sqrt{5}-\sqrt{x^4+5}}{x^2-\sqrt{5}-\sqrt{x^4+5}}\right) + \frac{50}{\left(x^2-\sqrt{x^4+5}\right)^2-5} - \frac{15}{2}\log\left(-x^2+\sqrt{x^4+5}\right)$$

```
[In] integrate((3*x^2+2)*(x^4+5)^(3/2)/x^3,x, algorithm="giac")
```

```
[Out] 1/2*sqrt(x^4 + 5)*((x^2 + 1)*x^2 + 20) + 15/2*sqrt(5)*log(-(x^2 + sqrt(5) -
 sqrt(x^4 + 5))/(x^2 - sqrt(5) - sqrt(x^4 + 5))) + 50/((x^2 - sqrt(x^4 + 5)
)^2 - 5) - 15/2*log(-x^2 + sqrt(x^4 + 5))
```

Mupad [B] (verification not implemented)

Time = 8.02 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.79

$$\int \frac{(2+3x^2)(5+x^4)^{3/2}}{x^3}\,dx = \frac{15\,\mathrm{asinh}\left(\frac{\sqrt{5}\,x^2}{5}\right)}{2} + \sqrt{x^4+5}\left(\frac{x^4}{2}+\frac{x^2}{2}+10\right) - \frac{5\sqrt{x^4+5}}{x^2} + \frac{\sqrt{5}\,\mathrm{atan}\left(\frac{\sqrt{5}\sqrt{x^4+5}\,1\mathrm{i}}{5}\right)\,15\mathrm{i}}{2}$$

```
[In] int(((x^4 + 5)^(3/2)*(3*x^2 + 2))/x^3,x)
```

```
[Out] (15*asinh((5^(1/2)*x^2)/5))/2 + (5^(1/2)*atan((5^(1/2)*(x^4 + 5)^(1/2)*1i)/
5)*15i)/2 + (x^4 + 5)^(1/2)*(x^2/2 + x^4/2 + 10) - (5*(x^4 + 5)^(1/2))/x^2
```

3.25 $\int \frac{(2+3x^2)(5+x^4)^{3/2}}{x^5} dx$

Optimal result

Integrand size = 20, antiderivative size = 86

$$\int \frac{(2+3x^2)(5+x^4)^{3/2}}{x^5} dx = -\frac{3(15-2x^2)\sqrt{5+x^4}}{4x^2} - \frac{(2-3x^2)(5+x^4)^{3/2}}{4x^4} + \frac{45}{4}\text{arcsinh}\left(\frac{x^2}{\sqrt{5}}\right) - \frac{3}{2}\sqrt{5}\text{arctanh}\left(\frac{\sqrt{5+x^4}}{\sqrt{5}}\right)$$

```
[Out] -1/4*(-3*x^2+2)*(x^4+5)^(3/2)/x^4+45/4*arcsinh(1/5*x^2*5^(1/2))-3/2*arctanh
(1/5*(x^4+5)^(1/2)*5^(1/2))*5^(1/2)-3/4*(-2*x^2+15)*(x^4+5)^(1/2)/x^2
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.350, Rules used = {1266, 827, 858, 221, 272, 65, 213}

$$\int \frac{(2+3x^2)(5+x^4)^{3/2}}{x^5} dx = \frac{45}{4}\text{arcsinh}\left(\frac{x^2}{\sqrt{5}}\right) - \frac{3}{2}\sqrt{5}\text{arctanh}\left(\frac{\sqrt{x^4+5}}{\sqrt{5}}\right) - \frac{(2-3x^2)(x^4+5)^{3/2}}{4x^4} - \frac{3(15-2x^2)\sqrt{x^4+5}}{4x^2}$$

```
[In] Int[((2 + 3*x^2)*(5 + x^4)^(3/2))/x^5,x]
```

```
[Out] (-3*(15 - 2*x^2)*Sqrt[5 + x^4])/(4*x^2) - ((2 - 3*x^2)*(5 + x^4)^(3/2))/(4*
x^4) + (45*ArcSinh[x^2/Sqrt[5]])/4 - (3*Sqrt[5]*ArcTanh[Sqrt[5 + x^4]/Sqrt[
5]])/2
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 827

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(e*f*(m + 2*p + 2) - d*g*(2*p + 1)
 + e*g*(m + 1)*x)*((a + c*x^2)^p/(e^2*(m + 1)*(m + 2*p + 2))), x] + Dist[p/
(e^2*(m + 1)*(m + 2*p + 2)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^(p - 1)*Simp
[g*(2*a*e + 2*a*e*m) + (g*(2*c*d + 4*c*d*p) - 2*c*e*f*(m + 2*p + 2))*x, x],
 x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && NeQ[c*d^2 + a*e^2, 0] && Rati
onalQ[p] && p > 0 && (LtQ[m, -1] || EqQ[p, 1] || (IntegerQ[p] &&  !Rational
Q[m])) && NeQ[m, -1] &&  !ILtQ[m + 2*p + 1, 0] && (IntegerQ[m] || IntegerQ[
p] || IntegersQ[2*m, 2*p])
```

Rule 858

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1266

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_S
ymbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + c*x^2)^p, x],
```

```
x, x^2], x] /; FreeQ[{a, c, d, e, p, q}, x] && IntegerQ[(m + 1)/2]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{1}{2}\text{Subst}\left(\int \frac{(2+3x)\left(5+x^2\right)^{3/2}}{x^3}\,dx, x, x^2\right) \\
&= -\frac{(2-3x^2)\left(5+x^4\right)^{3/2}}{4x^4} - \frac{3}{16}\text{Subst}\left(\int \frac{(-60-8x)\sqrt{5+x^2}}{x^2}\,dx, x, x^2\right) \\
&= -\frac{3(15-2x^2)\sqrt{5+x^4}}{4x^2} - \frac{(2-3x^2)\left(5+x^4\right)^{3/2}}{4x^4} + \frac{3}{32}\text{Subst}\left(\int \frac{80+120x}{x\sqrt{5+x^2}}\,dx, x, x^2\right) \\
&= -\frac{3(15-2x^2)\sqrt{5+x^4}}{4x^2} - \frac{(2-3x^2)\left(5+x^4\right)^{3/2}}{4x^4} \\
&\quad + \frac{15}{2}\text{Subst}\left(\int \frac{1}{x\sqrt{5+x^2}}\,dx, x, x^2\right) + \frac{45}{4}\text{Subst}\left(\int \frac{1}{\sqrt{5+x^2}}\,dx, x, x^2\right) \\
&= -\frac{3(15-2x^2)\sqrt{5+x^4}}{4x^2} - \frac{(2-3x^2)\left(5+x^4\right)^{3/2}}{4x^4} \\
&\quad + \frac{45}{4}\sinh^{-1}\left(\frac{x^2}{\sqrt{5}}\right) + \frac{15}{4}\text{Subst}\left(\int \frac{1}{x\sqrt{5+x}}\,dx, x, x^4\right) \\
&= -\frac{3(15-2x^2)\sqrt{5+x^4}}{4x^2} - \frac{(2-3x^2)\left(5+x^4\right)^{3/2}}{4x^4} \\
&\quad + \frac{45}{4}\sinh^{-1}\left(\frac{x^2}{\sqrt{5}}\right) + \frac{15}{2}\text{Subst}\left(\int \frac{1}{-5+x^2}\,dx, x, \sqrt{5+x^4}\right) \\
&= -\frac{3(15-2x^2)\sqrt{5+x^4}}{4x^2} - \frac{(2-3x^2)\left(5+x^4\right)^{3/2}}{4x^4} \\
&\quad + \frac{45}{4}\sinh^{-1}\left(\frac{x^2}{\sqrt{5}}\right) - \frac{3}{2}\sqrt{5}\tanh^{-1}\left(\frac{\sqrt{5+x^4}}{\sqrt{5}}\right)
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.97

$$
\begin{aligned}
\int \frac{(2+3x^2)\left(5+x^4\right)^{3/2}}{x^5}\,dx &= 3\sqrt{5}\operatorname{arctanh}\left(\frac{x^2-\sqrt{5+x^4}}{\sqrt{5}}\right) \\
&\quad + \frac{1}{4}\left(\frac{\sqrt{5+x^4}(-10-30x^2+4x^4+3x^6)}{x^4} - 45\log\left(-x^2+\sqrt{5+x^4}\right)\right)
\end{aligned}
$$

```
[In] Integrate[((2 + 3*x^2)*(5 + x^4)^(3/2))/x^5,x]
[Out] 3*Sqrt[5]*ArcTanh[(x^2 - Sqrt[5 + x^4])/Sqrt[5]] + ((Sqrt[5 + x^4]*(-10 - 3
0*x^2 + 4*x^4 + 3*x^6))/x^4 - 45*Log[-x^2 + Sqrt[5 + x^4]])/4
```

Maple [A] (verified)

Time = 0.49 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.76

method	result
pseudoelliptic	$\frac{-6\sqrt{5}\,\operatorname{arctanh}\left(\frac{\sqrt{5}}{\sqrt{x^4+5}}\right)x^4+45\,\operatorname{arcsinh}\left(\frac{x^2\sqrt{5}}{5}\right)x^4+3\sqrt{x^4+5}\,\left(x^6+\frac{4}{3}x^4-10x^2-\frac{10}{3}\right)}{4x^4}$
default	$\frac{45\,\operatorname{arcsinh}\left(\frac{x^2\sqrt{5}}{5}\right)}{4}+\frac{3x^2\sqrt{x^4+5}}{4}-\frac{15\sqrt{x^4+5}}{2x^2}+\sqrt{x^4+5}-\frac{5\sqrt{x^4+5}}{2x^4}-\frac{3\sqrt{5}\,\operatorname{arctanh}\left(\frac{\sqrt{5}}{\sqrt{x^4+5}}\right)}{2}$
elliptic	$\frac{45\,\operatorname{arcsinh}\left(\frac{x^2\sqrt{5}}{5}\right)}{4}+\frac{3x^2\sqrt{x^4+5}}{4}-\frac{15\sqrt{x^4+5}}{2x^2}+\sqrt{x^4+5}-\frac{5\sqrt{x^4+5}}{2x^4}-\frac{3\sqrt{5}\,\operatorname{arctanh}\left(\frac{\sqrt{5}}{\sqrt{x^4+5}}\right)}{2}$
trager	$\frac{(3x^6+4x^4-30x^2-10)\sqrt{x^4+5}}{4x^4}+\frac{3\,\mathrm{RootOf}\left(\_Z^2-5\right)\ln\left(\frac{\sqrt{x^4+5}-\mathrm{RootOf}\left(\_Z^2-5\right)}{x^2}\right)}{2}-\frac{45\ln\left(x^2-\sqrt{x^4+5}\right)}{4}$
risch	$-\frac{5(3x^6+x^4+15x^2+5)}{2x^4\sqrt{x^4+5}}+\frac{45\,\operatorname{arcsinh}\left(\frac{x^2\sqrt{5}}{5}\right)}{4}+\frac{3x^2\sqrt{x^4+5}}{4}+\sqrt{x^4+5}-\frac{3\sqrt{5}\,\operatorname{arctanh}\left(\frac{\sqrt{5}}{\sqrt{x^4+5}}\right)}{2}$
meijerg	$\frac{3\sqrt{5}\left(\frac{5\sqrt{\pi}\left(-\frac{12x^4}{5}+8\right)}{6x^4}-\frac{5\sqrt{\pi}\left(8-\frac{16x^4}{5}\right)\sqrt{1+\frac{x^4}{5}}}{6x^4}-4\sqrt{\pi}\,\ln\left(\frac{1}{2}+\frac{\sqrt{1+\frac{x^4}{5}}}{2}\right)+2(1-2\ln(2)+4\ln(x)-\ln(5))\sqrt{\pi}-\frac{20\sqrt{\pi}}{3x^4}\right)}{8\sqrt{\pi}}+$

[In] int((3*x^2+2)*(x^4+5)^(3/2)/x^5,x,method=_RETURNVERBOSE)

[Out] 1/4*(-6*5^(1/2)*arctanh(5^(1/2)/(x^4+5)^(1/2))*x^4+45*arcsinh(1/5*x^2*5^(1/2))*x^4+3*(x^4+5)^(1/2)*(x^6+4/3*x^4-10*x^2-10/3))/x^4

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.95

$$\int \frac{(2+3x^2)\,(5+x^4)^{3/2}}{x^5}\,dx=\frac{6\,\sqrt{5}x^4\log\left(-\frac{\sqrt{5}-\sqrt{x^4+5}}{x^2}\right)-45\,x^4\log\left(-x^2+\sqrt{x^4+5}\right)-30\,x^4+\left(3\,x^6+4\,x^4\right.}{4\,x^4}$$

[In] integrate((3*x^2+2)*(x^4+5)^(3/2)/x^5,x, algorithm="fricas")

[Out] 1/4*(6*sqrt(5)*x^4*log(-(sqrt(5) - sqrt(x^4 + 5))/x^2) - 45*x^4*log(-x^2 + sqrt(x^4 + 5)) - 30*x^4 + (3*x^6 + 4*x^4 - 30*x^2 - 10)*sqrt(x^4 + 5))/x^4

Sympy [A] (verification not implemented)

Time = 5.51 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.55

$$\int \frac{(2+3x^2)(5+x^4)^{3/2}}{x^5}\,dx = \frac{3x^6}{4\sqrt{x^4+5}} - \frac{15x^2}{4\sqrt{x^4+5}} + \sqrt{x^4+5} + \frac{\sqrt{5}\log\left(x^4\right)}{2} - \sqrt{5}\log\left(\sqrt{\frac{x^4}{5}+1}+1\right) - \frac{\sqrt{5}\operatorname{asinh}\left(\frac{\sqrt{5}}{x^2}\right)}{2} + \frac{45\operatorname{asinh}\left(\frac{\sqrt{5}x^2}{5}\right)}{4} - \frac{5\sqrt{1+\frac{5}{x^4}}}{2x^2} - \frac{75}{2x^2\sqrt{x^4+5}}$$

```
[In] integrate((3*x**2+2)*(x**4+5)**(3/2)/x**5,x)
```

```
[Out] 3*x**6/(4*sqrt(x**4 + 5)) - 15*x**2/(4*sqrt(x**4 + 5)) + sqrt(x**4 + 5) + s
qrt(5)*log(x**4)/2 - sqrt(5)*log(sqrt(x**4/5 + 1) + 1) - sqrt(5)*asinh(sqrt
(5)/x**2)/2 + 45*asinh(sqrt(5)*x**2/5)/4 - 5*sqrt(1 + 5/x**4)/(2*x**2) - 75
/(2*x**2*sqrt(x**4 + 5))
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.43

$$\int \frac{(2+3x^2)(5+x^4)^{3/2}}{x^5}\,dx = \frac{3}{4}\sqrt{5}\log\left(-\frac{\sqrt{5}-\sqrt{x^4+5}}{\sqrt{5}+\sqrt{x^4+5}}\right) + \sqrt{x^4+5} - \frac{15\sqrt{x^4+5}}{2\,x^2} + \frac{15\sqrt{x^4+5}}{4\,x^2\left(\frac{x^4+5}{x^4}-1\right)} - \frac{5\sqrt{x^4+5}}{2\,x^4} + \frac{45}{8}\log\left(\frac{\sqrt{x^4+5}}{x^2}+1\right) - \frac{45}{8}\log\left(\frac{\sqrt{x^4+5}}{x^2}-1\right)$$

```
[In] integrate((3*x^2+2)*(x^4+5)^(3/2)/x^5,x, algorithm="maxima")
```

```
[Out] 3/4*sqrt(5)*log(-(sqrt(5) - sqrt(x^4 + 5))/(sqrt(5) + sqrt(x^4 + 5))) + sqr
t(x^4 + 5) - 15/2*sqrt(x^4 + 5)/x^2 + 15/4*sqrt(x^4 + 5)/(x^2*((x^4 + 5)/x^
4 - 1)) - 5/2*sqrt(x^4 + 5)/x^4 + 45/8*log(sqrt(x^4 + 5)/x^2 + 1) - 45/8*lo
g(sqrt(x^4 + 5)/x^2 - 1)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 146 vs. 2(68) = 136.

Time = 0.28 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.70

$$\int \frac{(2+3x^2)(5+x^4)^{3/2}}{x^5}\,dx = \frac{1}{4}\sqrt{x^4+5}(3x^2+4) + \frac{3}{2}\sqrt{5}\log\left(-\frac{x^2+\sqrt{5}-\sqrt{x^4+5}}{x^2-\sqrt{5}-\sqrt{x^4+5}}\right) + \frac{5\left(\left(x^2-\sqrt{x^4+5}\right)^3+15\left(x^2-\sqrt{x^4+5}\right)^2+5x^2-5\sqrt{x^4+5}-75\right)}{\left(\left(x^2-\sqrt{x^4+5}\right)^2-5\right)^2} - \frac{45}{4}\log\left(-x^2+\sqrt{x^4+5}\right)$$

```
[In] integrate((3*x^2+2)*(x^4+5)^(3/2)/x^5,x, algorithm="giac")
```

```
[Out] 1/4*sqrt(x^4 + 5)*(3*x^2 + 4) + 3/2*sqrt(5)*log(-(x^2 + sqrt(5) - sqrt(x^4
+ 5))/(x^2 - sqrt(5) - sqrt(x^4 + 5))) + 5*((x^2 - sqrt(x^4 + 5))^3 + 15*(x
^2 - sqrt(x^4 + 5))^2 + 5*x^2 - 5*sqrt(x^4 + 5) - 75)/((x^2 - sqrt(x^4 + 5)
)^2 - 5)^2 - 45/4*log(-x^2 + sqrt(x^4 + 5))
```

Mupad [B] (verification not implemented)

Time = 7.93 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.83

$$\int \frac{(2+3x^2)(5+x^4)^{3/2}}{x^5}\,dx = \frac{45\,\mathrm{asinh}\left(\frac{\sqrt{5}\,x^2}{5}\right)}{4} + \sqrt{x^4+5}\left(\frac{3x^2}{4}+1\right) - \frac{15\sqrt{x^4+5}}{2x^2} - \frac{5\sqrt{x^4+5}}{2x^4} + \frac{\sqrt{5}\,\mathrm{atan}\left(\frac{\sqrt{5}\sqrt{x^4+5}\,1\mathrm{i}}{5}\right)3\mathrm{i}}{2}$$

```
[In] int(((x^4 + 5)^(3/2)*(3*x^2 + 2))/x^5,x)
```

```
[Out] (45*asinh((5^(1/2)*x^2)/5))/4 + (5^(1/2)*atan((5^(1/2)*(x^4 + 5)^(1/2)*1i)/
5)*3i)/2 + (x^4 + 5)^(1/2)*((3*x^2)/4 + 1) - (15*(x^4 + 5)^(1/2))/(2*x^2) -
 (5*(x^4 + 5)^(1/2))/(2*x^4)
```

3.26 $\int \frac{(2+3x^2)(5+x^4)^{3/2}}{x^7}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 82

$$\int \frac{(2+3x^2)(5+x^4)^{3/2}}{x^7}\,dx = -\frac{(4-9x^2)\sqrt{5+x^4}}{4x^2} - \frac{(4+9x^2)(5+x^4)^{3/2}}{12x^6} + \operatorname{arcsinh}\left(\frac{x^2}{\sqrt{5}}\right) - \frac{9}{4}\sqrt{5}\operatorname{arctanh}\left(\frac{\sqrt{5+x^4}}{\sqrt{5}}\right)$$

```
[Out] -1/12*(9*x^2+4)*(x^4+5)^(3/2)/x^6+arcsinh(1/5*x^2*5^(1/2))-9/4*arctanh(1/5*
(x^4+5)^(1/2)*5^(1/2))*5^(1/2)-1/4*(-9*x^2+4)*(x^4+5)^(1/2)/x^2
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {1266, 825, 827, 858, 221, 272, 65, 213}

$$\int \frac{(2+3x^2)(5+x^4)^{3/2}}{x^7}\,dx = \operatorname{arcsinh}\left(\frac{x^2}{\sqrt{5}}\right) - \frac{9}{4}\sqrt{5}\operatorname{arctanh}\left(\frac{\sqrt{x^4+5}}{\sqrt{5}}\right) - \frac{(4-9x^2)\sqrt{x^4+5}}{4x^2} - \frac{(9x^2+4)(x^4+5)^{3/2}}{12x^6}$$

```
[In] Int[((2 + 3*x^2)*(5 + x^4)^(3/2))/x^7,x]
```

```
[Out] -1/4*((4 - 9*x^2)*Sqrt[5 + x^4])/x^2 - ((4 + 9*x^2)*(5 + x^4)^(3/2))/(12*x^
6) + ArcSinh[x^2/Sqrt[5]] - (9*Sqrt[5]*ArcTanh[Sqrt[5 + x^4]/Sqrt[5]])/4
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 825

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(-(d + e*x)^(m + 1))*((a + c*x^2)^p/(e^2*(m + 1)*(m
+ 2)*(c*d^2 + a*e^2)))*((d*g - e*f*(m + 2))*(c*d^2 + a*e^2) - 2*c*d^2*p*(e*
f - d*g) - e*(g*(m + 1)*(c*d^2 + a*e^2) + 2*c*d*p*(e*f - d*g))*x), x] - Dis
t[p/(e^2*(m + 1)*(m + 2)*(c*d^2 + a*e^2)), Int[(d + e*x)^(m + 2)*(a + c*x^2
)^(p - 1)*Simp[2*a*c*e*(e*f - d*g)*(m + 2) - c*(2*c*d*(d*g*(2*p + 1) - e*f*
(m + 2*p + 2)) - 2*a*e^2*g*(m + 1))*x, x], x], x] /; FreeQ[{a, c, d, e, f,
g}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p, 0] && LtQ[m, -2] && LtQ[m + 2*p,
0] &&  !ILtQ[m + 2*p + 3, 0]
```

Rule 827

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(e*f*(m + 2*p + 2) - d*g*(2*p + 1)
 + e*g*(m + 1)*x)*((a + c*x^2)^p/(e^2*(m + 1)*(m + 2*p + 2))), x] + Dist[p/
(e^2*(m + 1)*(m + 2*p + 2)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^(p - 1)*Simp
[g*(2*a*e + 2*a*e*m) + (g*(2*c*d + 4*c*d*p) - 2*c*e*f*(m + 2*p + 2))*x, x],
 x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && NeQ[c*d^2 + a*e^2, 0] && Rati
onalQ[p] && p > 0 && (LtQ[m, -1] || EqQ[p, 1] || (IntegerQ[p] &&  !Rational
Q[m])) && NeQ[m, -1] &&  !ILtQ[m + 2*p + 1, 0] && (IntegerQ[m] || IntegerQ[
```

```
p] || IntegersQ[2*m, 2*p])
```

Rule 858

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1266

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_S
ymbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + c*x^2)^p, x],
x, x^2], x] /; FreeQ[{a, c, d, e, p, q}, x] && IntegerQ[(m + 1)/2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{2}\text{Subst}\left(\int \frac{(2+3x)\left(5+x^2\right)^{3/2}}{x^4}\,dx, x, x^2\right)\\
&= -\frac{\left(4+9x^2\right)\left(5+x^4\right)^{3/2}}{12x^6} - \frac{1}{40}\text{Subst}\left(\int \frac{(-40-90x)\sqrt{5+x^2}}{x^2}\,dx, x, x^2\right)\\
&= -\frac{\left(4-9x^2\right)\sqrt{5+x^4}}{4x^2} - \frac{\left(4+9x^2\right)\left(5+x^4\right)^{3/2}}{12x^6} + \frac{1}{80}\text{Subst}\left(\int \frac{900+80x}{x\sqrt{5+x^2}}\,dx, x, x^2\right)\\
&= -\frac{\left(4-9x^2\right)\sqrt{5+x^4}}{4x^2} - \frac{\left(4+9x^2\right)\left(5+x^4\right)^{3/2}}{12x^6}\\
&\quad + \frac{45}{4}\text{Subst}\left(\int \frac{1}{x\sqrt{5+x^2}}\,dx, x, x^2\right) + \text{Subst}\left(\int \frac{1}{\sqrt{5+x^2}}\,dx, x, x^2\right)\\
&= -\frac{\left(4-9x^2\right)\sqrt{5+x^4}}{4x^2} - \frac{\left(4+9x^2\right)\left(5+x^4\right)^{3/2}}{12x^6}\\
&\quad + \sinh^{-1}\left(\frac{x^2}{\sqrt{5}}\right) + \frac{45}{8}\text{Subst}\left(\int \frac{1}{x\sqrt{5+x}}\,dx, x, x^4\right)\\
&= -\frac{\left(4-9x^2\right)\sqrt{5+x^4}}{4x^2} - \frac{\left(4+9x^2\right)\left(5+x^4\right)^{3/2}}{12x^6}\\
&\quad + \sinh^{-1}\left(\frac{x^2}{\sqrt{5}}\right) + \frac{45}{4}\text{Subst}\left(\int \frac{1}{-5+x^2}\,dx, x, \sqrt{5+x^4}\right)\\
&= -\frac{\left(4-9x^2\right)\sqrt{5+x^4}}{4x^2} - \frac{\left(4+9x^2\right)\left(5+x^4\right)^{3/2}}{12x^6} + \sinh^{-1}\left(\frac{x^2}{\sqrt{5}}\right) - \frac{9}{4}\sqrt{5}\tanh^{-1}\left(\frac{\sqrt{5+x^4}}{\sqrt{5}}\right)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.01

$$\int \frac{(2+3x^2)(5+x^4)^{3/2}}{x^7}\,dx = \frac{\sqrt{5+x^4}(-20-45x^2-16x^4+18x^6)}{12x^6} + \frac{9}{2}\sqrt{5}\operatorname{arctanh}\left(\frac{x^2-\sqrt{5+x^4}}{\sqrt{5}}\right) - \log\left(-x^2+\sqrt{5+x^4}\right)$$

```
[In] Integrate[((2 + 3*x^2)*(5 + x^4)^(3/2))/x^7,x]
```

```
[Out] (Sqrt[5 + x^4]*(-20 - 45*x^2 - 16*x^4 + 18*x^6))/(12*x^6) + (9*Sqrt[5]*ArcT
anh[(x^2 - Sqrt[5 + x^4])/Sqrt[5]])/2 - Log[-x^2 + Sqrt[5 + x^4]]
```

Maple [A] (verified)

Time = 0.48 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.79

method	result
pseudoelliptic	$\frac{-9\sqrt{5}\,\operatorname{arctanh}\left(\frac{\sqrt{5}}{\sqrt{x^4+5}}\right)x^6+4\,\operatorname{arcsinh}\left(\frac{x^2\sqrt{5}}{5}\right)x^6+6\left(x^6-\frac{8}{9}x^4-\frac{5}{2}x^2-\frac{10}{9}\right)\sqrt{x^4+5}}{4x^6}$
risch	$-\frac{16x^8+45x^6+100x^4+225x^2+100}{12x^6\sqrt{x^4+5}}+\operatorname{arcsinh}\left(\frac{x^2\sqrt{5}}{5}\right)+\frac{3\sqrt{x^4+5}}{2}-\frac{9\sqrt{5}\,\operatorname{arctanh}\left(\frac{\sqrt{5}}{\sqrt{x^4+5}}\right)}{4}$
default	$\operatorname{arcsinh}\left(\frac{x^2\sqrt{5}}{5}\right)-\frac{4\sqrt{x^4+5}}{3x^2}-\frac{5\sqrt{x^4+5}}{3x^6}+\frac{3\sqrt{x^4+5}}{2}-\frac{15\sqrt{x^4+5}}{4x^4}-\frac{9\sqrt{5}\,\operatorname{arctanh}\left(\frac{\sqrt{5}}{\sqrt{x^4+5}}\right)}{4}$
elliptic	$\operatorname{arcsinh}\left(\frac{x^2\sqrt{5}}{5}\right)-\frac{4\sqrt{x^4+5}}{3x^2}-\frac{5\sqrt{x^4+5}}{3x^6}+\frac{3\sqrt{x^4+5}}{2}-\frac{15\sqrt{x^4+5}}{4x^4}-\frac{9\sqrt{5}\,\operatorname{arctanh}\left(\frac{\sqrt{5}}{\sqrt{x^4+5}}\right)}{4}$
trager	$\frac{(18x^6-16x^4-45x^2-20)\sqrt{x^4+5}}{12x^6}-\ln\left(x^2-\sqrt{x^4+5}\right)+\frac{9\,\operatorname{RootOf}\left(\_Z^2-5\right)\ln\left(\frac{\sqrt{x^4+5}-\operatorname{RootOf}\left(\_Z^2-5\right)}{x^2}\right)}{4}$
meijerg	$\frac{-\frac{5\sqrt{\pi}\sqrt{5}\left(\frac{4x^4}{5}+1\right)\sqrt{1+\frac{x^4}{5}}}{3x^6}+\sqrt{\pi}\,\operatorname{arcsinh}\left(\frac{x^2\sqrt{5}}{5}\right)}{\sqrt{\pi}}+\frac{9\sqrt{5}\left(\frac{5\sqrt{\pi}\left(-\frac{12x^4}{5}+8\right)}{6x^4}-\frac{5\sqrt{\pi}\left(8-\frac{16x^4}{5}\right)\sqrt{1+\frac{x^4}{5}}}{6x^4}-4\sqrt{\pi}\,\ln\left(\frac{1}{2}+\frac{\sqrt{1+\frac{x^4}{5}}}{2}\right.\right.}{16\sqrt{\pi}}$

```
[In] int((3*x^2+2)*(x^4+5)^(3/2)/x^7,x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*(-9*5^(1/2)*arctanh(5^(1/2)/(x^4+5)^(1/2))*x^6+4*arcsinh(1/5*x^2*5^(1/2
))*x^6+6*(x^6-8/9*x^4-5/2*x^2-10/9)*(x^4+5)^(1/2))/x^6
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.00

$$\int \frac{(2+3x^2)(5+x^4)^{3/2}}{x^7}\,dx = \frac{27\sqrt{5}x^6\log\left(-\frac{\sqrt{5}-\sqrt{x^4+5}}{x^2}\right) - 12\,x^6\log\left(-x^2+\sqrt{x^4+5}\right) - 16\,x^6 + (18\,x^6 - 16\,x}{12\,x^6}$$

```
[In] integrate((3*x^2+2)*(x^4+5)^(3/2)/x^7,x, algorithm="fricas")
```

```
[Out] 1/12*(27*sqrt(5)*x^6*log(-(sqrt(5) - sqrt(x^4 + 5))/x^2) - 12*x^6*log(-x^2
+ sqrt(x^4 + 5)) - 16*x^6 + (18*x^6 - 16*x^4 - 45*x^2 - 20)*sqrt(x^4 + 5))/
x^6
```

Sympy [A] (verification not implemented)

Time = 5.79 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.80

$$\begin{aligned}\int \frac{(2+3x^2)(5+x^4)^{3/2}}{x^7}\,dx &= -\frac{x^2}{\sqrt{x^4+5}} - \frac{\sqrt{1+\frac{5}{x^4}}}{3} + \frac{3\sqrt{x^4+5}}{2} \\ &+ \frac{3\sqrt{5}\log(x^4)}{4} - \frac{3\sqrt{5}\log\left(\sqrt{\frac{x^4}{5}+1}+1\right)}{2} - \frac{3\sqrt{5}\operatorname{asinh}\left(\frac{\sqrt{5}}{x^2}\right)}{4} \\ &+ \operatorname{asinh}\left(\frac{\sqrt{5}x^2}{5}\right) - \frac{15\sqrt{1+\frac{5}{x^4}}}{4x^2} - \frac{5}{x^2\sqrt{x^4+5}} - \frac{5\sqrt{1+\frac{5}{x^4}}}{3x^4}\end{aligned}$$

```
[In] integrate((3*x**2+2)*(x**4+5)**(3/2)/x**7,x)
```

```
[Out] -x**2/sqrt(x**4 + 5) - sqrt(1 + 5/x**4)/3 + 3*sqrt(x**4 + 5)/2 + 3*sqrt(5)*
log(x**4)/4 - 3*sqrt(5)*log(sqrt(x**4/5 + 1) + 1)/2 - 3*sqrt(5)*asinh(sqrt(
5)/x**2)/4 + asinh(sqrt(5)*x**2/5) - 15*sqrt(1 + 5/x**4)/(4*x**2) - 5/(x**2
*sqrt(x**4 + 5)) - 5*sqrt(1 + 5/x**4)/(3*x**4)
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.37

$$\begin{aligned}\int \frac{(2+3x^2)(5+x^4)^{3/2}}{x^7}\,dx &= \frac{9}{8}\sqrt{5}\log\left(-\frac{\sqrt{5}-\sqrt{x^4+5}}{\sqrt{5}+\sqrt{x^4+5}}\right) + \frac{3}{2}\sqrt{x^4+5} - \frac{\sqrt{x^4+5}}{x^2} \\ &- \frac{15\sqrt{x^4+5}}{4\,x^4} - \frac{(x^4+5)^{\frac{3}{2}}}{3\,x^6} + \frac{1}{2}\log\left(\frac{\sqrt{x^4+5}}{x^2}+1\right) - \frac{1}{2}\log\left(\frac{\sqrt{x^4+5}}{x^2}-1\right)\end{aligned}$$

```
[In] integrate((3*x^2+2)*(x^4+5)^(3/2)/x^7,x, algorithm="maxima")
```

```
[Out] 9/8*sqrt(5)*log(-(sqrt(5) - sqrt(x^4 + 5))/(sqrt(5) + sqrt(x^4 + 5))) + 3/2
*sqrt(x^4 + 5) - sqrt(x^4 + 5)/x^2 - 15/4*sqrt(x^4 + 5)/x^4 - 1/3*(x^4 + 5)
^(3/2)/x^6 + 1/2*log(sqrt(x^4 + 5)/x^2 + 1) - 1/2*log(sqrt(x^4 + 5)/x^2 - 1
)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 158 vs. $2(66) = 132$.

Time = 0.31 (sec) , antiderivative size = 158, normalized size of antiderivative = 1.93

$$\int \frac{(2+3x^2)(5+x^4)^{3/2}}{x^7}\,dx = \frac{9}{4}\sqrt{5}\log\left(-\frac{x^2+\sqrt{5}-\sqrt{x^4+5}}{x^2-\sqrt{5}-\sqrt{x^4+5}}\right)+\frac{3}{2}\sqrt{x^4+5}+\frac{5\left(9\left(x^2-\sqrt{x^4+5}\right)^5+24\left(x^2-\sqrt{x^4+5}\right)^4-120\left(x^2-\sqrt{x^4+5}\right)^2-225\,x^2+225\sqrt{x^4+5}+400\right)}{6\left(\left(x^2-\sqrt{x^4+5}\right)^2-5\right)^3}-\log\left(-x^2+\sqrt{x^4+5}\right)$$

```
[In] integrate((3*x^2+2)*(x^4+5)^(3/2)/x^7,x, algorithm="giac")
```

```
[Out] 9/4*sqrt(5)*log(-(x^2 + sqrt(5) - sqrt(x^4 + 5))/(x^2 - sqrt(5) - sqrt(x^4
+ 5))) + 3/2*sqrt(x^4 + 5) + 5/6*(9*(x^2 - sqrt(x^4 + 5))^5 + 24*(x^2 - sqr
t(x^4 + 5))^4 - 120*(x^2 - sqrt(x^4 + 5))^2 - 225*x^2 + 225*sqrt(x^4 + 5) +
 400)/((x^2 - sqrt(x^4 + 5))^2 - 5)^3 - log(-x^2 + sqrt(x^4 + 5))
```

Mupad [B] (verification not implemented)

Time = 8.08 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.00

$$\int \frac{(2+3x^2)(5+x^4)^{3/2}}{x^7}\,dx = \operatorname{asinh}\left(\frac{\sqrt{5}\,x^2}{5}\right)+\frac{3\sqrt{x^4+5}}{2}+\sqrt{x^4+5}\left(\frac{2}{3\,x^2}-\frac{5}{3\,x^6}\right)-\frac{2\sqrt{x^4+5}}{x^2}-\frac{15\sqrt{x^4+5}}{4\,x^4}+\frac{\sqrt{5}\,\operatorname{atan}\left(\frac{\sqrt{5}\,\sqrt{x^4+5}\,1\mathrm{i}}{5}\right)\,9\mathrm{i}}{4}$$

```
[In] int(((x^4 + 5)^(3/2)*(3*x^2 + 2))/x^7,x)
```

```
[Out] asinh((5^(1/2)*x^2)/5) + (5^(1/2)*atan((5^(1/2)*(x^4 + 5)^(1/2)*1i)/5)*9i)/
4 + (3*(x^4 + 5)^(1/2))/2 + (x^4 + 5)^(1/2)*(2/(3*x^2) - 5/(3*x^6)) - (2*(x
^4 + 5)^(1/2))/x^2 - (15*(x^4 + 5)^(1/2))/(4*x^4)
```

3.27 $\int x^4(2+3x^2)\left(5+x^4\right)^{3/2}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 235

$$\begin{aligned}\int x^4(2+3x^2)\left(5+x^4\right)^{3/2}\,dx &= \frac{200}{77}x\sqrt{5+x^4}+\frac{20}{13}x^3\sqrt{5+x^4} \\ &- \frac{300x\sqrt{5+x^4}}{13\left(\sqrt{5}+x^2\right)}+\frac{10x^5(78+77x^2)\sqrt{5+x^4}}{1001}+\frac{1}{143}x^5(26+33x^2)\left(5+x^4\right)^{3/2} \\ &+ \frac{300\sqrt[4]{5}\left(\sqrt{5}+x^2\right)\sqrt{\frac{5+x^4}{\left(\sqrt{5}+x^2\right)^2}}E\left(2\arctan\left(\frac{x}{\sqrt[4]{5}}\right)|\frac{1}{2}\right)}{13\sqrt{5+x^4}} \\ &- \frac{50\sqrt[4]{5}(231+26\sqrt{5})\left(\sqrt{5}+x^2\right)\sqrt{\frac{5+x^4}{\left(\sqrt{5}+x^2\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{x}{\sqrt[4]{5}}\right),\frac{1}{2}\right)}{1001\sqrt{5+x^4}}\end{aligned}$$

```
[Out] 1/143*x^5*(33*x^2+26)*(x^4+5)^(3/2)+200/77*x*(x^4+5)^(1/2)+20/13*x^3*(x^4+5
)^(1/2)+10/1001*x^5*(77*x^2+78)*(x^4+5)^(1/2)-300/13*x*(x^4+5)^(1/2)/(x^2+5
^(1/2))+300/13*5^(1/4)*(cos(2*arctan(1/5*x*5^(3/4)))^2)^(1/2)/cos(2*arctan(
1/5*x*5^(3/4)))*EllipticE(sin(2*arctan(1/5*x*5^(3/4))),1/2*2^(1/2))*(x^2+5^
(1/2))*((x^4+5)/(x^2+5^(1/2))^2)^(1/2)/(x^4+5)^(1/2)-50/1001*5^(1/4)*(cos(2
*arctan(1/5*x*5^(3/4)))^2)^(1/2)/cos(2*arctan(1/5*x*5^(3/4)))*EllipticF(sin
(2*arctan(1/5*x*5^(3/4))),1/2*2^(1/2))*(x^2+5^(1/2))*(231+26*5^(1/2))*((x^4
+5)/(x^2+5^(1/2))^2)^(1/2)/(x^4+5)^(1/2)
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 235, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {1288, 1294, 1212, 226, 1210}

$$\int x^4(2+3x^2)\left(5+x^4\right)^{3/2}\,dx =$$

$$-\frac{50\sqrt[4]{5}(231+26\sqrt{5})\left(x^2+\sqrt{5}\right)\sqrt{\frac{x^4+5}{\left(x^2+\sqrt{5}\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{x}{\sqrt[4]{5}}\right),\frac{1}{2}\right)}{1001\sqrt{x^4+5}}$$
$$+\frac{300\sqrt[4]{5}(x^2+\sqrt{5})\sqrt{\frac{x^4+5}{\left(x^2+\sqrt{5}\right)^2}}E\left(2\arctan\left(\frac{x}{\sqrt[4]{5}}\right)|\frac{1}{2}\right)}{13\sqrt{x^4+5}}$$
$$+\frac{200}{77}\sqrt{x^4+5}x+\frac{20}{13}\sqrt{x^4+5}x^3-\frac{300\sqrt{x^4+5}x}{13\left(x^2+\sqrt{5}\right)}$$
$$+\frac{1}{143}\left(33x^2+26\right)\left(x^4+5\right)^{3/2}x^5+\frac{10(77x^2+78)\sqrt{x^4+5}x^5}{1001}$$

```
[In] Int[x^4*(2 + 3*x^2)*(5 + x^4)^(3/2),x]
```

```
[Out] (200*x*Sqrt[5 + x^4])/77 + (20*x^3*Sqrt[5 + x^4])/13 - (300*x*Sqrt[5 + x^4]
)/(13*(Sqrt[5] + x^2)) + (10*x^5*(78 + 77*x^2)*Sqrt[5 + x^4])/1001 + (x^5*(
26 + 33*x^2)*(5 + x^4)^(3/2))/143 + (300*5^(1/4)*(Sqrt[5] + x^2)*Sqrt[(5 +
x^4)/(Sqrt[5] + x^2)^2]*EllipticE[2*ArcTan[x/5^(1/4)], 1/2])/(13*Sqrt[5 + x
^4]) - (50*5^(1/4)*(231 + 26*Sqrt[5])*(Sqrt[5] + x^2)*Sqrt[(5 + x^4)/(Sqrt[
5] + x^2)^2]*EllipticF[2*ArcTan[x/5^(1/4)], 1/2])/(1001*Sqrt[5 + x^4])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 1210

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Rule 1212

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + c*x^4], x], x] - Dist[e/q, I
```

```
nt[(1 - q*x^2)/Sqrt[a + c*x^4], x], x] /; NeQ[e + d*q, 0]] /; FreeQ[{a, c,
d, e}, x] && PosQ[c/a]
```

Rule 1288

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (c_.)*(x_)^4)^(p_.), x
_Symbol] :> Simp[(f*x)^(m + 1)*(a + c*x^4)^p*((c*d*(m + 4*p + 3) + c*e*(4*p
 + m + 1)*x^2)/(c*f*(4*p + m + 1)*(m + 4*p + 3))), x] + Dist[4*a*(p/((4*p +
 m + 1)*(m + 4*p + 3))), Int[(f*x)^m*(a + c*x^4)^(p - 1)*Simp[d*(m + 4*p +
3) + e*(4*p + m + 1)*x^2, x], x], x] /; FreeQ[{a, c, d, e, f, m}, x] && GtQ
[p, 0] && NeQ[4*p + m + 1, 0] && NeQ[m + 4*p + 3, 0] && IntegerQ[2*p] && (I
ntegerQ[p] || IntegerQ[m])
```

Rule 1294

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (c_.)*(x_)^4)^(p_), x_
Symbol] :> Simp[e*f*(f*x)^(m - 1)*((a + c*x^4)^(p + 1)/(c*(m + 4*p + 3))),
x] - Dist[f^2/(c*(m + 4*p + 3)), Int[(f*x)^(m - 2)*(a + c*x^4)^p*(a*e*(m -
1) - c*d*(m + 4*p + 3)*x^2), x], x] /; FreeQ[{a, c, d, e, f, p}, x] && GtQ[
m, 1] && NeQ[m + 4*p + 3, 0] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m
])
```

Rubi steps

$$\text{integral} = \frac{1}{143}x^5(26+33x^2)(5+x^4)^{3/2} + \frac{30}{143}\int x^4(26+33x^2)\sqrt{5+x^4}\,dx$$

$$= \frac{10x^5(78+77x^2)\sqrt{5+x^4}}{1001} + \frac{1}{143}x^5(26+33x^2)(5+x^4)^{3/2} + \frac{100\int \frac{x^4(234+231x^2)}{\sqrt{5+x^4}}\,dx}{3003}$$

$$= \frac{20}{13}x^3\sqrt{5+x^4} + \frac{10x^5(78+77x^2)\sqrt{5+x^4}}{1001} + \frac{1}{143}x^5(26+33x^2)(5+x^4)^{3/2} - \frac{20\int \frac{x^2(3465-1170x^2)}{\sqrt{5+x^4}}\,dx}{3003}$$

$$= \frac{200}{77}x\sqrt{5+x^4} + \frac{20}{13}x^3\sqrt{5+x^4} + \frac{10x^5(78+77x^2)\sqrt{5+x^4}}{1001} + \frac{1}{143}x^5(26+33x^2)(5+x^4)^{3/2} + \frac{20\int \frac{-5850-10395x^2}{\sqrt{5+x^4}}\,dx}{9009}$$

$$= \frac{200}{77}x\sqrt{5+x^4} + \frac{20}{13}x^3\sqrt{5+x^4} + \frac{10x^5(78+77x^2)\sqrt{5+x^4}}{1001} + \frac{1}{143}x^5(26+33x^2)(5+x^4)^{3/2} + \frac{1}{13}\left(300\sqrt{5}\right)\int \frac{1-\frac{x^2}{\sqrt{5}}}{\sqrt{5+x^4}}\,dx - \frac{\left(100\left(130+231\sqrt{5}\right)\right)\int \frac{1}{\sqrt{5+x^4}}\,dx}{1001}$$

$$
\begin{aligned}
&= \frac{200}{77}x\sqrt{5+x^4} + \frac{20}{13}x^3\sqrt{5+x^4} - \frac{300x\sqrt{5+x^4}}{13\left(\sqrt{5}+x^2\right)} \\
&+ \frac{10x^5(78+77x^2)\sqrt{5+x^4}}{1001} + \frac{1}{143}x^5\left(26+33x^2\right)\left(5+x^4\right)^{3/2} \\
&+ \frac{300\sqrt[4]{5}\left(\sqrt{5}+x^2\right)\sqrt{\frac{5+x^4}{\left(\sqrt{5}+x^2\right)^2}}E\left(2\tan^{-1}\left(\frac{x}{\sqrt[4]{5}}\right)|\frac{1}{2}\right)}{13\sqrt{5+x^4}} \\
&- \frac{50\sqrt[4]{5}\left(231+26\sqrt{5}\right)\left(\sqrt{5}+x^2\right)\sqrt{\frac{5+x^4}{\left(\sqrt{5}+x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{x}{\sqrt[4]{5}}\right)|\frac{1}{2}\right)}{1001\sqrt{5+x^4}}
\end{aligned}
$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 7.26 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.31

$$
\int x^4\left(2+3x^2\right)\left(5+x^4\right)^{3/2}\,dx = \frac{1}{143}x\left(\left(26+33x^2\right)\left(5+x^4\right)^{5/2} - 650\sqrt{5}\,\text{Hypergeometric2F1}\left(-\frac{3}{2},\frac{1}{4},\frac{5}{4},-\frac{x^4}{5}\right) - 825\sqrt{5}x^2\,\text{Hypergeometric2F1}\left(-\frac{3}{2},\frac{3}{4},\frac{7}{4},-\frac{x^4}{5}\right)\right)
$$

```
[In] Integrate[x^4*(2 + 3*x^2)*(5 + x^4)^(3/2),x]
[Out] (x*((26 + 33*x^2)*(5 + x^4)^(5/2) - 650*Sqrt[5]*Hypergeometric2F1[-3/2, 1/4
, 5/4, -1/5*x^4] - 825*Sqrt[5]*x^2*Hypergeometric2F1[-3/2, 3/4, 7/4, -1/5*x
^4]))/143
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4.

Time = 4.24 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.17

method	result
meijerg	$\frac{15\sqrt{5}\,x^{7}{}_{2}F_{1}\left(-\frac{3}{2},\frac{7}{4};\frac{11}{4};-\frac{x^{4}}{5}\right)}{7}+2\sqrt{5}\,x^{5}{}_{2}F_{1}\left(-\frac{3}{2},\frac{5}{4};\frac{9}{4};-\frac{x^{4}}{5}\right)$
risch	$\frac{x\left(231x^{10}+182x^{8}+1925x^{6}+1690x^{4}+1540x^{2}+2600\right)\sqrt{x^{4}+5}}{1001}-\frac{40\sqrt{5}\,\sqrt{25-5i\sqrt{5}\,x^{2}}\,\sqrt{25+5i\sqrt{5}\,x^{2}}\,F\left(\frac{x\sqrt{5}\,\sqrt{i\sqrt{5}}}{5},i\right)}{77\sqrt{i\sqrt{5}}\,\sqrt{x^{4}+5}}-\frac{60i\sqrt{25-5i}}{}$
default	$\frac{3x^{11}\sqrt{x^{4}+5}}{13}+\frac{25x^{7}\sqrt{x^{4}+5}}{13}+\frac{20x^{3}\sqrt{x^{4}+5}}{13}-\frac{60i\sqrt{25-5i\sqrt{5}\,x^{2}}\,\sqrt{25+5i\sqrt{5}\,x^{2}}\left(F\left(\frac{x\sqrt{5}\,\sqrt{i\sqrt{5}}}{5},i\right)-E\left(\frac{x\sqrt{5}\,\sqrt{i\sqrt{5}}}{5},i\right)\right)}{13\sqrt{i\sqrt{5}}\,\sqrt{x^{4}+5}}+\frac{2x^{9}\sqrt{}}{11}$
elliptic	$\frac{3x^{11}\sqrt{x^{4}+5}}{13}+\frac{25x^{7}\sqrt{x^{4}+5}}{13}+\frac{20x^{3}\sqrt{x^{4}+5}}{13}-\frac{60i\sqrt{25-5i\sqrt{5}\,x^{2}}\,\sqrt{25+5i\sqrt{5}\,x^{2}}\left(F\left(\frac{x\sqrt{5}\,\sqrt{i\sqrt{5}}}{5},i\right)-E\left(\frac{x\sqrt{5}\,\sqrt{i\sqrt{5}}}{5},i\right)\right)}{13\sqrt{i\sqrt{5}}\,\sqrt{x^{4}+5}}+\frac{2x^{9}\sqrt{}}{11}$

```
[In] int(x^4*(3*x^2+2)*(x^4+5)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 15/7*5^(1/2)*x^7*hypergeom([-3/2,7/4],[11/4],-1/5*x^4)+2*5^(1/2)*x^5*hyperg
eom([-3/2,5/4],[9/4],-1/5*x^4)
```

Fricas [A] (verification not implemented)

none

Time = 0.08 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.34

$$\int x^4(2+3x^2)\left(5+x^4\right)^{3/2}\,dx =$$

$$-\frac{23100\,(-5)^{\frac{3}{4}}\,xE(\arcsin\left(\frac{(-5)^{\frac{1}{4}}}{x}\right)\,|-1)-20500\,(-5)^{\frac{3}{4}}\,xF(\arcsin\left(\frac{(-5)^{\frac{1}{4}}}{x}\right)\,|-1)-(231\,x^{12}+182\,x^{10}+192}{1001\,x}$$

```
[In] integrate(x^4*(3*x^2+2)*(x^4+5)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/1001*(23100*(-5)^(3/4)*x*elliptic_e(arcsin((-5)^(1/4)/x), -1) - 20500*(-
5)^(3/4)*x*elliptic_f(arcsin((-5)^(1/4)/x), -1) - (231*x^12 + 182*x^10 + 19
25*x^8 + 1690*x^6 + 1540*x^4 + 2600*x^2 - 23100)*sqrt(x^4 + 5))/x
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.80 (sec) , antiderivative size = 160, normalized size of antiderivative = 0.68

$$\int x^4(2+3x^2)\left(5+x^4\right)^{3/2}\,dx = \frac{3\sqrt{5}x^{11}\Gamma\left(\frac{11}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{11}{4}\\ \frac{15}{4}\end{matrix}\middle|\frac{x^4e^{i\pi}}{5}\right)}{4\Gamma\left(\frac{15}{4}\right)} + \frac{\sqrt{5}x^9\Gamma\left(\frac{9}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{9}{4}\\ \frac{13}{4}\end{matrix}\middle|\frac{x^4e^{i\pi}}{5}\right)}{2\Gamma\left(\frac{13}{4}\right)} + \frac{15\sqrt{5}x^7\Gamma\left(\frac{7}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{7}{4}\\ \frac{11}{4}\end{matrix}\middle|\frac{x^4e^{i\pi}}{5}\right)}{4\Gamma\left(\frac{11}{4}\right)} + \frac{5\sqrt{5}x^5\Gamma\left(\frac{5}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{5}{4}\\ \frac{9}{4}\end{matrix}\middle|\frac{x^4e^{i\pi}}{5}\right)}{2\Gamma\left(\frac{9}{4}\right)}$$

```
[In] integrate(x**4*(3*x**2+2)*(x**4+5)**(3/2),x)
```

```
[Out] 3*sqrt(5)*x**11*gamma(11/4)*hyper((-1/2, 11/4), (15/4,), x**4*exp_polar(I*p
i)/5)/(4*gamma(15/4)) + sqrt(5)*x**9*gamma(9/4)*hyper((-1/2, 9/4), (13/4,),
 x**4*exp_polar(I*pi)/5)/(2*gamma(13/4)) + 15*sqrt(5)*x**7*gamma(7/4)*hyper
((-1/2, 7/4), (11/4,), x**4*exp_polar(I*pi)/5)/(4*gamma(11/4)) + 5*sqrt(5)*
x**5*gamma(5/4)*hyper((-1/2, 5/4), (9/4,), x**4*exp_polar(I*pi)/5)/(2*gamma
(9/4))
```

Maxima [F]

$$\int x^4(2+3x^2)\left(5+x^4\right)^{3/2}\,dx = \int (x^4+5)^{\frac{3}{2}}(3\,x^2+2)x^4\,dx$$

```
[In] integrate(x^4*(3*x^2+2)*(x^4+5)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((x^4 + 5)^(3/2)*(3*x^2 + 2)*x^4, x)
```

Giac [F]

$$\int x^4(2+3x^2)\left(5+x^4\right)^{3/2}\,dx=\int \left(x^4+5\right)^{\frac{3}{2}}\left(3\,x^2+2\right)x^4\,dx$$

```
[In] integrate(x^4*(3*x^2+2)*(x^4+5)^(3/2),x, algorithm="giac")
[Out] integrate((x^4 + 5)^(3/2)*(3*x^2 + 2)*x^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^4(2+3x^2)\left(5+x^4\right)^{3/2}\,dx=\int x^4\left(x^4+5\right)^{3/2}\left(3\,x^2+2\right)\,dx$$

```
[In] int(x^4*(x^4 + 5)^(3/2)*(3*x^2 + 2),x)
[Out] int(x^4*(x^4 + 5)^(3/2)*(3*x^2 + 2), x)
```

3.28 $\int x^2(2+3x^2)\left(5+x^4\right)^{3/2}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 219

$$\int x^2\left(2+3x^2\right)\left(5+x^4\right)^{3/2}\,dx=\frac{300}{77}x\sqrt{5+x^4}+\frac{40x\sqrt{5+x^4}}{3\left(\sqrt{5}+x^2\right)}+\frac{2}{231}x^3\left(154+135x^2\right)\sqrt{5+x^4}+\frac{1}{99}x^3\left(22+27x^2\right)\left(5+x^4\right)^{3/2}-\frac{40\sqrt[4]{5}\left(\sqrt{5}+x^2\right)\sqrt{\frac{5+x^4}{\left(\sqrt{5}+x^2\right)^2}}E\left(2\arctan\left(\frac{x}{\sqrt[4]{5}}\right)|\frac{1}{2}\right)}{3\sqrt{5+x^4}}+\frac{10\sqrt[4]{5}\left(154-45\sqrt{5}\right)\left(\sqrt{5}+x^2\right)\sqrt{\frac{5+x^4}{\left(\sqrt{5}+x^2\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{x}{\sqrt[4]{5}}\right),\frac{1}{2}\right)}{231\sqrt{5+x^4}}$$

```
[Out] 1/99*x^3*(27*x^2+22)*(x^4+5)^(3/2)+300/77*x*(x^4+5)^(1/2)+2/231*x^3*(135*x^
2+154)*(x^4+5)^(1/2)+40/3*x*(x^4+5)^(1/2)/(x^2+5^(1/2))-40/3*5^(1/4)*(cos(2
*arctan(1/5*x*5^(3/4)))^2)^(1/2)/cos(2*arctan(1/5*x*5^(3/4)))*EllipticE(sin
(2*arctan(1/5*x*5^(3/4))),1/2*2^(1/2))*(x^2+5^(1/2))*((x^4+5)/(x^2+5^(1/2))
^2)^(1/2)/(x^4+5)^(1/2)+10/231*5^(1/4)*(cos(2*arctan(1/5*x*5^(3/4)))^2)^(1/
2)/cos(2*arctan(1/5*x*5^(3/4)))*EllipticF(sin(2*arctan(1/5*x*5^(3/4))),1/2*
2^(1/2))*(154-45*5^(1/2))*(x^2+5^(1/2))*((x^4+5)/(x^2+5^(1/2))^2)^(1/2)/(x^
4+5)^(1/2)
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 219, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {1288, 1294, 1212, 226, 1210}

$$\int x^2(2+3x^2)(5+x^4)^{3/2}\,dx = \frac{10\sqrt[4]{5}(154-45\sqrt{5})(x^2+\sqrt{5})\sqrt{\frac{x^4+5}{(x^2+\sqrt{5})^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{x}{\sqrt[4]{5}}\right),\frac{1}{2}\right)}{231\sqrt{x^4+5}} - \frac{40\sqrt[4]{5}(x^2+\sqrt{5})\sqrt{\frac{x^4+5}{(x^2+\sqrt{5})^2}}E\left(2\arctan\left(\frac{x}{\sqrt[4]{5}}\right)|\frac{1}{2}\right)}{3\sqrt{x^4+5}} + \frac{300}{77}\sqrt{x^4+5}x + \frac{40\sqrt{x^4+5}x}{3(x^2+\sqrt{5})} + \frac{1}{99}(27x^2+22)(x^4+5)^{3/2}x^3 + \frac{2}{231}(135x^2+154)\sqrt{x^4+5}x^3$$

```
[In] Int[x^2*(2 + 3*x^2)*(5 + x^4)^(3/2),x]
```

```
[Out] (300*x*Sqrt[5 + x^4])/77 + (40*x*Sqrt[5 + x^4])/(3*(Sqrt[5] + x^2)) + (2*x^
3*(154 + 135*x^2)*Sqrt[5 + x^4])/231 + (x^3*(22 + 27*x^2)*(5 + x^4)^(3/2))/
99 - (40*5^(1/4)*(Sqrt[5] + x^2)*Sqrt[(5 + x^4)/(Sqrt[5] + x^2)^2]*Elliptic
E[2*ArcTan[x/5^(1/4)], 1/2])/(3*Sqrt[5 + x^4]) + (10*5^(1/4)*(154 - 45*Sqrt
[5])*(Sqrt[5] + x^2)*Sqrt[(5 + x^4)/(Sqrt[5] + x^2)^2]*EllipticF[2*ArcTan[x
/5^(1/4)], 1/2])/(231*Sqrt[5 + x^4])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 1210

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Rule 1212

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + c*x^4], x], x] - Dist[e/q, I
nt[(1 - q*x^2)/Sqrt[a + c*x^4], x], x] /; NeQ[e + d*q, 0]] /; FreeQ[{a, c,
```

```
d, e}, x] && PosQ[c/a]
```

Rule 1288

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (c_.)*(x_)^4)^(p_.), x
_Symbol] :> Simp[(f*x)^(m + 1)*(a + c*x^4)^p*((c*d*(m + 4*p + 3) + c*e*(4*p
 + m + 1)*x^2)/(c*f*(4*p + m + 1)*(m + 4*p + 3))), x] + Dist[4*a*(p/((4*p +
 m + 1)*(m + 4*p + 3))), Int[(f*x)^m*(a + c*x^4)^(p - 1)*Simp[d*(m + 4*p +
3) + e*(4*p + m + 1)*x^2, x], x], x] /; FreeQ[{a, c, d, e, f, m}, x] && GtQ
[p, 0] && NeQ[4*p + m + 1, 0] && NeQ[m + 4*p + 3, 0] && IntegerQ[2*p] && (I
ntegerQ[p] || IntegerQ[m])
```

Rule 1294

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (c_.)*(x_)^4)^(p_), x_
Symbol] :> Simp[e*f*(f*x)^(m - 1)*((a + c*x^4)^(p + 1)/(c*(m + 4*p + 3))),
x] - Dist[f^2/(c*(m + 4*p + 3)), Int[(f*x)^(m - 2)*(a + c*x^4)^p*(a*e*(m -
1) - c*d*(m + 4*p + 3)*x^2), x], x] /; FreeQ[{a, c, d, e, f, p}, x] && GtQ[
m, 1] && NeQ[m + 4*p + 3, 0] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m
])
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{1}{99}x^3\left(22+27x^2\right)\left(5+x^4\right)^{3/2}+\frac{10}{33}\int x^2\left(22+27x^2\right)\sqrt{5+x^4}\,dx \\
&= \frac{2}{231}x^3\left(154+135x^2\right)\sqrt{5+x^4}+\frac{1}{99}x^3\left(22+27x^2\right)\left(5+x^4\right)^{3/2}+\frac{20}{231}\int \frac{x^2\left(154+135x^2\right)}{\sqrt{5+x^4}}\,dx \\
&= \frac{300}{77}x\sqrt{5+x^4}+\frac{2}{231}x^3\left(154+135x^2\right)\sqrt{5+x^4} \\
&\quad +\frac{1}{99}x^3\left(22+27x^2\right)\left(5+x^4\right)^{3/2}-\frac{20}{693}\int \frac{675-462x^2}{\sqrt{5+x^4}}\,dx \\
&= \frac{300}{77}x\sqrt{5+x^4}+\frac{2}{231}x^3\left(154+135x^2\right)\sqrt{5+x^4}+\frac{1}{99}x^3\left(22+27x^2\right)\left(5+x^4\right)^{3/2} \\
&\quad -\frac{1}{3}\left(40\sqrt{5}\right)\int \frac{1-\frac{x^2}{\sqrt{5}}}{\sqrt{5+x^4}}\,dx-\frac{1}{231}\left(20\left(225-154\sqrt{5}\right)\right)\int \frac{1}{\sqrt{5+x^4}}\,dx
\end{aligned}
$$

$$= \frac{300}{77}x\sqrt{5+x^4} + \frac{40x\sqrt{5+x^4}}{3\left(\sqrt{5}+x^2\right)} + \frac{2}{231}x^3\left(154+135x^2\right)\sqrt{5+x^4}$$
$$+ \frac{1}{99}x^3\left(22+27x^2\right)\left(5+x^4\right)^{3/2} - \frac{40\sqrt[4]{5}\left(\sqrt{5}+x^2\right)\sqrt{\frac{5+x^4}{\left(\sqrt{5}+x^2\right)^2}}E\left(2\tan^{-1}\left(\frac{x}{\sqrt[4]{5}}\right)|\frac{1}{2}\right)}{3\sqrt{5+x^4}}$$
$$+ \frac{10\sqrt[4]{5}\left(154-45\sqrt{5}\right)\left(\sqrt{5}+x^2\right)\sqrt{\frac{5+x^4}{\left(\sqrt{5}+x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{x}{\sqrt[4]{5}}\right)|\frac{1}{2}\right)}{231\sqrt{5+x^4}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 5.81 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.31

$$\int x^2\left(2+3x^2\right)\left(5+x^4\right)^{3/2}\,dx = \frac{1}{33}x\left(9\left(5+x^4\right)^{5/2}\right.$$
$$\left.-225\sqrt{5}\,\text{Hypergeometric2F1}\left(-\frac{3}{2},\frac{1}{4},\frac{5}{4},-\frac{x^4}{5}\right)+110\sqrt{5}x^2\,\text{Hypergeometric2F1}\left(-\frac{3}{2},\frac{3}{4},\frac{7}{4},-\frac{x^4}{5}\right)\right)$$

```
[In] Integrate[x^2*(2 + 3*x^2)*(5 + x^4)^(3/2),x]
```

```
[Out] (x*(9*(5 + x^4)^(5/2) - 225*Sqrt[5]*Hypergeometric2F1[-3/2, 1/4, 5/4, -1/5*
x^4] + 110*Sqrt[5]*x^2*Hypergeometric2F1[-3/2, 3/4, 7/4, -1/5*x^4]))/33
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4.

Time = 1.88 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.18

method	result
meijerg	$3\sqrt{5}\,x^5{}_2F_1\left(-\frac{3}{2},\frac{5}{4};\frac{9}{4};-\frac{x^4}{5}\right)+\frac{10\sqrt{5}\,x^3{}_2F_1\left(-\frac{3}{2},\frac{3}{4};\frac{7}{4};-\frac{x^4}{5}\right)}{3}$
risch	$\frac{x\left(189x^8+154x^6+1755x^4+1694x^2+2700\right)\sqrt{x^4+5}}{693}-\frac{60\sqrt{5}\,\sqrt{25-5i\sqrt{5}\,x^2}\,\sqrt{25+5i\sqrt{5}\,x^2}\,F\left(\frac{x\sqrt{5}\,\sqrt{i\sqrt{5}}}{5},i\right)}{77\sqrt{i\sqrt{5}}\,\sqrt{x^4+5}}+\frac{8i\sqrt{25-5i\sqrt{5}\,x^2}\,\sqrt{25-}}{}$
default	$\frac{3x^9\sqrt{x^4+5}}{11}+\frac{195x^5\sqrt{x^4+5}}{77}+\frac{300x\sqrt{x^4+5}}{77}-\frac{60\sqrt{5}\,\sqrt{25-5i\sqrt{5}\,x^2}\,\sqrt{25+5i\sqrt{5}\,x^2}\,F\left(\frac{x\sqrt{5}\,\sqrt{i\sqrt{5}}}{5},i\right)}{77\sqrt{i\sqrt{5}}\,\sqrt{x^4+5}}+\frac{2x^7\sqrt{x^4+5}}{9}+\frac{22x^3\sqrt{x^4-}}{9}$
elliptic	$\frac{3x^9\sqrt{x^4+5}}{11}+\frac{195x^5\sqrt{x^4+5}}{77}+\frac{300x\sqrt{x^4+5}}{77}-\frac{60\sqrt{5}\,\sqrt{25-5i\sqrt{5}\,x^2}\,\sqrt{25+5i\sqrt{5}\,x^2}\,F\left(\frac{x\sqrt{5}\,\sqrt{i\sqrt{5}}}{5},i\right)}{77\sqrt{i\sqrt{5}}\,\sqrt{x^4+5}}+\frac{2x^7\sqrt{x^4+5}}{9}+\frac{22x^3\sqrt{x^4-}}{9}$

```
[In] int(x^2*(3*x^2+2)*(x^4+5)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 3*5^(1/2)*x^5*hypergeom([-3/2,5/4],[9/4],-1/5*x^4)+10/3*5^(1/2)*x^3*hyperge
om([-3/2,3/4],[7/4],-1/5*x^4)
```

Fricas [A] (verification not implemented)

none

Time = 0.08 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.33

$$\int x^2(2+3x^2)(5+x^4)^{3/2}\,dx = \frac{9240\,(-5)^{\frac{3}{4}}\,xE(\arcsin\left(\frac{(-5)^{\frac{1}{4}}}{x}\right)|-1) - 11940\,(-5)^{\frac{3}{4}}\,xF(\arcsin\left(\frac{(-5)^{\frac{1}{4}}}{x}\right)|-1) + (189\,x^{10} + 15}{693\,x}$$

```
[In] integrate(x^2*(3*x^2+2)*(x^4+5)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/693*(9240*(-5)^(3/4)*x*elliptic_e(arcsin((-5)^(1/4)/x), -1) - 11940*(-5)^
(3/4)*x*elliptic_f(arcsin((-5)^(1/4)/x), -1) + (189*x^10 + 154*x^8 + 1755*x
^6 + 1694*x^4 + 2700*x^2 + 9240)*sqrt(x^4 + 5))/x
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.64 (sec) , antiderivative size = 160, normalized size of antiderivative = 0.73

$$\int x^2(2+3x^2)\left(5+x^4\right)^{3/2}\,dx = \frac{3\sqrt{5}x^9\Gamma\left(\frac{9}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{9}{4}\\ \frac{13}{4}\end{matrix}\middle|\frac{x^4e^{i\pi}}{5}\right)}{4\Gamma\left(\frac{13}{4}\right)} + \frac{\sqrt{5}x^7\Gamma\left(\frac{7}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{7}{4}\\ \frac{11}{4}\end{matrix}\middle|\frac{x^4e^{i\pi}}{5}\right)}{2\Gamma\left(\frac{11}{4}\right)} + \frac{15\sqrt{5}x^5\Gamma\left(\frac{5}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{5}{4}\\ \frac{9}{4}\end{matrix}\middle|\frac{x^4e^{i\pi}}{5}\right)}{4\Gamma\left(\frac{9}{4}\right)} + \frac{5\sqrt{5}x^3\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{3}{4}\\ \frac{7}{4}\end{matrix}\middle|\frac{x^4e^{i\pi}}{5}\right)}{2\Gamma\left(\frac{7}{4}\right)}$$

```
[In] integrate(x**2*(3*x**2+2)*(x**4+5)**(3/2),x)
```

```
[Out] 3*sqrt(5)*x**9*gamma(9/4)*hyper((-1/2, 9/4), (13/4,), x**4*exp_polar(I*pi)/
5)/(4*gamma(13/4)) + sqrt(5)*x**7*gamma(7/4)*hyper((-1/2, 7/4), (11/4,), x*
*4*exp_polar(I*pi)/5)/(2*gamma(11/4)) + 15*sqrt(5)*x**5*gamma(5/4)*hyper((-
1/2, 5/4), (9/4,), x**4*exp_polar(I*pi)/5)/(4*gamma(9/4)) + 5*sqrt(5)*x**3*
gamma(3/4)*hyper((-1/2, 3/4), (7/4,), x**4*exp_polar(I*pi)/5)/(2*gamma(7/4)
)
```

Maxima [F]

$$\int x^2(2+3x^2)\left(5+x^4\right)^{3/2}\,dx=\int (x^4+5)^{\frac{3}{2}}(3\,x^2+2)x^2\,dx$$

```
[In] integrate(x^2*(3*x^2+2)*(x^4+5)^(3/2),x, algorithm="maxima")
[Out] integrate((x^4 + 5)^(3/2)*(3*x^2 + 2)*x^2, x)
```

Giac [F]

$$\int x^2(2+3x^2)\left(5+x^4\right)^{3/2}\,dx=\int (x^4+5)^{\frac{3}{2}}(3\,x^2+2)x^2\,dx$$

```
[In] integrate(x^2*(3*x^2+2)*(x^4+5)^(3/2),x, algorithm="giac")
[Out] integrate((x^4 + 5)^(3/2)*(3*x^2 + 2)*x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^2(2+3x^2)\left(5+x^4\right)^{3/2}\,dx=\int x^2\,\left(x^4+5\right)^{3/2}\,\left(3\,x^2+2\right)\,dx$$

```
[In] int(x^2*(x^4 + 5)^(3/2)*(3*x^2 + 2),x)
[Out] int(x^2*(x^4 + 5)^(3/2)*(3*x^2 + 2), x)
```

3.29 $\int (2+3x^2)(5+x^4)^{3/2}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 197

$$\int (2+3x^2)(5+x^4)^{3/2}\,dx = \frac{20x\sqrt{5+x^4}}{\sqrt{5}+x^2}+\frac{2}{7}x(10+7x^2)\sqrt{5+x^4} + \frac{1}{21}x(6+7x^2)(5+x^4)^{3/2} - \frac{20\sqrt[4]{5}(\sqrt{5}+x^2)\sqrt{\frac{5+x^4}{(\sqrt{5}+x^2)^2}}E\left(2\arctan\left(\frac{x}{\sqrt[4]{5}}\right)\middle|\frac{1}{2}\right)}{\sqrt{5+x^4}} + \frac{10\sqrt[4]{5}(7+2\sqrt{5})(\sqrt{5}+x^2)\sqrt{\frac{5+x^4}{(\sqrt{5}+x^2)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{x}{\sqrt[4]{5}}\right),\frac{1}{2}\right)}{7\sqrt{5+x^4}}$$

```
[Out] 1/21*x*(7*x^2+6)*(x^4+5)^(3/2)+2/7*x*(7*x^2+10)*(x^4+5)^(1/2)+20*x*(x^4+5)^
(1/2)/(x^2+5^(1/2))-20*5^(1/4)*(cos(2*arctan(1/5*x*5^(3/4)))^2)^(1/2)/cos(2
*arctan(1/5*x*5^(3/4)))*EllipticE(sin(2*arctan(1/5*x*5^(3/4))),1/2*2^(1/2))
*(x^2+5^(1/2))*((x^4+5)/(x^2+5^(1/2))^2)^(1/2)/(x^4+5)^(1/2)+10/7*5^(1/4)*(
cos(2*arctan(1/5*x*5^(3/4)))^2)^(1/2)/cos(2*arctan(1/5*x*5^(3/4)))*Elliptic
F(sin(2*arctan(1/5*x*5^(3/4))),1/2*2^(1/2))*(x^2+5^(1/2))*(7+2*5^(1/2))*((x
^4+5)/(x^2+5^(1/2))^2)^(1/2)/(x^4+5)^(1/2)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 197, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {1191, 1212, 226, 1210}

$$\int (2+3x^2)\left(5+x^4\right)^{3/2}\,dx = \frac{10\sqrt[4]{5}(7+2\sqrt{5})\left(x^2+\sqrt{5}\right)\sqrt{\frac{x^4+5}{\left(x^2+\sqrt{5}\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{x}{\sqrt[4]{5}}\right),\frac{1}{2}\right)}{7\sqrt{x^4+5}} - \frac{20\sqrt[4]{5}\left(x^2+\sqrt{5}\right)\sqrt{\frac{x^4+5}{\left(x^2+\sqrt{5}\right)^2}}E\left(2\arctan\left(\frac{x}{\sqrt[4]{5}}\right)|\frac{1}{2}\right)}{\sqrt{x^4+5}} + \frac{1}{21}x(7x^2+6)\left(x^4+5\right)^{3/2} + \frac{2}{7}x(7x^2+10)\sqrt{x^4+5} + \frac{20x\sqrt{x^4+5}}{x^2+\sqrt{5}}$$

```
[In] Int[(2 + 3*x^2)*(5 + x^4)^(3/2),x]
```

```
[Out] (20*x*Sqrt[5 + x^4])/(Sqrt[5] + x^2) + (2*x*(10 + 7*x^2)*Sqrt[5 + x^4])/7 +
 (x*(6 + 7*x^2)*(5 + x^4)^(3/2))/21 - (20*5^(1/4)*(Sqrt[5] + x^2)*Sqrt[(5 +
 x^4)/(Sqrt[5] + x^2)^2]*EllipticE[2*ArcTan[x/5^(1/4)], 1/2])/Sqrt[5 + x^4]
 + (10*5^(1/4)*(7 + 2*Sqrt[5])*(Sqrt[5] + x^2)*Sqrt[(5 + x^4)/(Sqrt[5] + x^
2)^2]*EllipticF[2*ArcTan[x/5^(1/4)], 1/2])/(7*Sqrt[5 + x^4])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 1191

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[x*(
d*(4*p + 3) + e*(4*p + 1)*x^2)*((a + c*x^4)^p/((4*p + 1)*(4*p + 3))), x] +
Dist[2*(p/((4*p + 1)*(4*p + 3))), Int[Simp[2*a*d*(4*p + 3) + (2*a*e*(4*p +
1))*x^2, x]*(a + c*x^4)^(p - 1), x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*
d^2 + a*e^2, 0] && GtQ[p, 0] && FractionQ[p] && IntegerQ[2*p]
```

Rule 1210

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
```

```
}, x] && PosQ[c/a]
```

Rule 1212

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + c*x^4], x], x] - Dist[e/q, I
nt[(1 - q*x^2)/Sqrt[a + c*x^4], x], x] /; NeQ[e + d*q, 0]] /; FreeQ[{a, c,
d, e}, x] && PosQ[c/a]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{1}{21}x(6+7x^2)\left(5+x^4\right)^{3/2}+\frac{1}{21}\int \left(180+210x^2\right)\sqrt{5+x^4}\,dx \\
&= \frac{2}{7}x(10+7x^2)\sqrt{5+x^4}+\frac{1}{21}x(6+7x^2)\left(5+x^4\right)^{3/2}+\frac{1}{315}\int \frac{9000+6300x^2}{\sqrt{5+x^4}}\,dx \\
&= \frac{2}{7}x(10+7x^2)\sqrt{5+x^4}+\frac{1}{21}x(6+7x^2)\left(5+x^4\right)^{3/2} \\
&\quad -\left(20\sqrt{5}\right)\int \frac{1-\frac{x^2}{\sqrt{5}}}{\sqrt{5+x^4}}\,dx+\frac{1}{7}\left(20\left(10+7\sqrt{5}\right)\right)\int \frac{1}{\sqrt{5+x^4}}\,dx \\
&= \frac{20x\sqrt{5+x^4}}{\sqrt{5}+x^2}+\frac{2}{7}x(10+7x^2)\sqrt{5+x^4}+\frac{1}{21}x(6+7x^2)\left(5+x^4\right)^{3/2} \\
&\quad -\frac{20\sqrt[4]{5}(\sqrt{5}+x^2)\sqrt{\frac{5+x^4}{\left(\sqrt{5}+x^2\right)^2}}E\left(2\tan^{-1}\left(\frac{x}{\sqrt[4]{5}}\right)|\frac{1}{2}\right)}{\sqrt{5+x^4}} \\
&\quad +\frac{10\sqrt[4]{5}(7+2\sqrt{5})\left(\sqrt{5}+x^2\right)\sqrt{\frac{5+x^4}{\left(\sqrt{5}+x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{x}{\sqrt[4]{5}}\right)|\frac{1}{2}\right)}{7\sqrt{5+x^4}}
\end{aligned}
$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 5.11 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.25

$$
\int \left(2+3x^2\right)\left(5+x^4\right)^{3/2}\,dx = 5\sqrt{5}x\left(2\,\text{Hypergeometric2F1}\left(-\frac{3}{2},\frac{1}{4},\frac{5}{4},-\frac{x^4}{5}\right)+x^2\,\text{Hypergeometric2F1}\left(-\frac{3}{2},\frac{3}{4},\frac{7}{4},-\frac{x^4}{5}\right)\right)
$$

```
[In] Integrate[(2 + 3*x^2)*(5 + x^4)^(3/2),x]
[Out] 5*Sqrt[5]*x*(2*Hypergeometric2F1[-3/2, 1/4, 5/4, -1/5*x^4] + x^2*Hypergeome
tric2F1[-3/2, 3/4, 7/4, -1/5*x^4])
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4.

Time = 1.25 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.19

method	result
meijerg	$10\sqrt{5}\, x\,{}_2F_1\left(-\frac{3}{2},\frac{1}{4};\frac{5}{4};-\frac{x^4}{5}\right)+5\sqrt{5}\, x^3\,{}_2F_1\left(-\frac{3}{2},\frac{3}{4};\frac{7}{4};-\frac{x^4}{5}\right)$
risch	$\frac{x(7x^6+6x^4+77x^2+90)\sqrt{x^4+5}}{21}+\frac{8\sqrt{5}\,\sqrt{25-5i\sqrt{5}\,x^2}\,\sqrt{25+5i\sqrt{5}\,x^2}\,F\left(\frac{x\sqrt{5}\,\sqrt{i\sqrt{5}}}{5},i\right)}{7\sqrt{i\sqrt{5}}\,\sqrt{x^4+5}}+\frac{4i\sqrt{25-5i\sqrt{5}\,x^2}\,\sqrt{25+5i\sqrt{5}\,x^2}\left(F\left(\frac{x\sqrt{5}}{5}\right.\right.}{\sqrt{i\sqrt{5}}\,\sqrt{x^4+5}}$
default	$\frac{2x^5\sqrt{x^4+5}}{7}+\frac{30x\sqrt{x^4+5}}{7}+\frac{8\sqrt{5}\,\sqrt{25-5i\sqrt{5}\,x^2}\,\sqrt{25+5i\sqrt{5}\,x^2}\,F\left(\frac{x\sqrt{5}\,\sqrt{i\sqrt{5}}}{5},i\right)}{7\sqrt{i\sqrt{5}}\,\sqrt{x^4+5}}+\frac{x^7\sqrt{x^4+5}}{3}+\frac{11x^3\sqrt{x^4+5}}{3}+\frac{4i\sqrt{25-5i\sqrt{5}\,x}}{}$
elliptic	$\frac{2x^5\sqrt{x^4+5}}{7}+\frac{30x\sqrt{x^4+5}}{7}+\frac{8\sqrt{5}\,\sqrt{25-5i\sqrt{5}\,x^2}\,\sqrt{25+5i\sqrt{5}\,x^2}\,F\left(\frac{x\sqrt{5}\,\sqrt{i\sqrt{5}}}{5},i\right)}{7\sqrt{i\sqrt{5}}\,\sqrt{x^4+5}}+\frac{x^7\sqrt{x^4+5}}{3}+\frac{11x^3\sqrt{x^4+5}}{3}+\frac{4i\sqrt{25-5i\sqrt{5}\,x}}{}$

```
[In] int((3*x^2+2)*(x^4+5)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 10*5^(1/2)*x*hypergeom([-3/2,1/4],[5/4],-1/5*x^4)+5*5^(1/2)*x^3*hypergeom([
-3/2,3/4],[7/4],-1/5*x^4)
```

Fricas [A] (verification not implemented)

none

Time = 0.08 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.35

$$\int (2+3x^2)\,(5+x^4)^{3/2}\,dx=\frac{420\,(-5)^{\frac{3}{4}}\,xE(\arcsin\left(\frac{(-5)^{\frac{1}{4}}}{x}\right)|-1)-300\,(-5)^{\frac{3}{4}}\,xF(\arcsin\left(\frac{(-5)^{\frac{1}{4}}}{x}\right)|-1)+(7\,x^8+6\,x^6+77\,x^4}{21\,x}$$

```
[In] integrate((3*x^2+2)*(x^4+5)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/21*(420*(-5)^(3/4)*x*elliptic_e(arcsin((-5)^(1/4)/x), -1) - 300*(-5)^(3/4
)*x*elliptic_f(arcsin((-5)^(1/4)/x), -1) + (7*x^8 + 6*x^6 + 77*x^4 + 90*x^2
 + 420)*sqrt(x^4 + 5))/x
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.51 (sec) , antiderivative size = 158, normalized size of antiderivative = 0.80

$$\int (2+3x^2)\left(5+x^4\right)^{3/2}\,dx = \frac{3\sqrt{5}x^7\Gamma\left(\frac{7}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{7}{4}\\ \frac{11}{4}\end{matrix}\middle|\frac{x^4e^{i\pi}}{5}\right)}{4\Gamma\left(\frac{11}{4}\right)} + \frac{\sqrt{5}x^5\Gamma\left(\frac{5}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{5}{4}\\ \frac{9}{4}\end{matrix}\middle|\frac{x^4e^{i\pi}}{5}\right)}{2\Gamma\left(\frac{9}{4}\right)} + \frac{15\sqrt{5}x^3\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{3}{4}\\ \frac{7}{4}\end{matrix}\middle|\frac{x^4e^{i\pi}}{5}\right)}{4\Gamma\left(\frac{7}{4}\right)} + \frac{5\sqrt{5}x\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{1}{4}\\ \frac{5}{4}\end{matrix}\middle|\frac{x^4e^{i\pi}}{5}\right)}{2\Gamma\left(\frac{5}{4}\right)}$$

```
[In] integrate((3*x**2+2)*(x**4+5)**(3/2),x)
[Out] 3*sqrt(5)*x**7*gamma(7/4)*hyper((-1/2, 7/4), (11/4,), x**4*exp_polar(I*pi)/
5)/(4*gamma(11/4)) + sqrt(5)*x**5*gamma(5/4)*hyper((-1/2, 5/4), (9/4,), x**
4*exp_polar(I*pi)/5)/(2*gamma(9/4)) + 15*sqrt(5)*x**3*gamma(3/4)*hyper((-1/
2, 3/4), (7/4,), x**4*exp_polar(I*pi)/5)/(4*gamma(7/4)) + 5*sqrt(5)*x*gamma
(1/4)*hyper((-1/2, 1/4), (5/4,), x**4*exp_polar(I*pi)/5)/(2*gamma(5/4))
```

Maxima [F]

$$\int (2+3x^2)\left(5+x^4\right)^{3/2}\,dx = \int \left(x^4+5\right)^{\frac{3}{2}}\left(3\,x^2+2\right)dx$$

```
[In] integrate((3*x^2+2)*(x^4+5)^(3/2),x, algorithm="maxima")
[Out] integrate((x^4 + 5)^(3/2)*(3*x^2 + 2), x)
```

Giac [F]

$$\int (2+3x^2) (5+x^4)^{3/2} \, dx = \int (x^4+5)^{\frac{3}{2}} (3 x^2+2) \, dx$$

```
[In] integrate((3*x^2+2)*(x^4+5)^(3/2),x, algorithm="giac")
[Out] integrate((x^4 + 5)^(3/2)*(3*x^2 + 2), x)
```

Mupad [F(-1)]

Timed out.

$$\int (2+3x^2) (5+x^4)^{3/2} \, dx = \int (x^4+5)^{3/2} (3 x^2+2) \, dx$$

```
[In] int((x^4 + 5)^(3/2)*(3*x^2 + 2),x)
[Out] int((x^4 + 5)^(3/2)*(3*x^2 + 2), x)
```

3.30 $\int \frac{(2+3x^2)(5+x^4)^{3/2}}{x^2} \, dx$

Optimal result

Integrand size = 20, antiderivative size = 199

$$\int \frac{(2+3x^2)(5+x^4)^{3/2}}{x^2} \, dx = \frac{24x\sqrt{5+x^4}}{\sqrt{5}+x^2} + \frac{6}{35}x(25+14x^2)\sqrt{5+x^4} - \frac{(14-3x^2)(5+x^4)^{3/2}}{7x} - \frac{24\sqrt[4]{5}(\sqrt{5}+x^2)\sqrt{\frac{5+x^4}{(\sqrt{5}+x^2)^2}}E\left(2\arctan\left(\frac{x}{\sqrt[4]{5}}\right)|\frac{1}{2}\right)}{\sqrt{5+x^4}} + \frac{6\sqrt[4]{5}(14+5\sqrt{5})(\sqrt{5}+x^2)\sqrt{\frac{5+x^4}{(\sqrt{5}+x^2)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{x}{\sqrt[4]{5}}\right),\frac{1}{2}\right)}{7\sqrt{5+x^4}}$$

```
[Out] -1/7*(-3*x^2+14)*(x^4+5)^(3/2)/x+6/35*x*(14*x^2+25)*(x^4+5)^(1/2)+24*x*(x^4
+5)^(1/2)/(x^2+5^(1/2))-24*5^(1/4)*(cos(2*arctan(1/5*x*5^(3/4)))^2)^(1/2)/c
os(2*arctan(1/5*x*5^(3/4)))*EllipticE(sin(2*arctan(1/5*x*5^(3/4))),1/2*2^(1
/2))*(x^2+5^(1/2))*((x^4+5)/(x^2+5^(1/2))^2)^(1/2)/(x^4+5)^(1/2)+6/7*5^(1/4
)*(cos(2*arctan(1/5*x*5^(3/4)))^2)^(1/2)/cos(2*arctan(1/5*x*5^(3/4)))*Ellip
ticF(sin(2*arctan(1/5*x*5^(3/4))),1/2*2^(1/2))*(x^2+5^(1/2))*(14+5*5^(1/2))
*((x^4+5)/(x^2+5^(1/2))^2)^(1/2)/(x^4+5)^(1/2)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 199, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {1286, 1191, 1212, 226, 1210}

$$\int \frac{(2+3x^2)(5+x^4)^{3/2}}{x^2}\,dx = \frac{6\sqrt[4]{5}(14+5\sqrt{5})(x^2+\sqrt{5})\sqrt{\frac{x^4+5}{(x^2+\sqrt{5})^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{x}{\sqrt[4]{5}}\right),\frac{1}{2}\right)}{7\sqrt{x^4+5}} - \frac{24\sqrt[4]{5}(x^2+\sqrt{5})\sqrt{\frac{x^4+5}{(x^2+\sqrt{5})^2}}E\left(2\arctan\left(\frac{x}{\sqrt[4]{5}}\right)|\frac{1}{2}\right)}{\sqrt{x^4+5}} - \frac{(14-3x^2)(x^4+5)^{3/2}}{7x} + \frac{6}{35}x(14x^2+25)\sqrt{x^4+5} + \frac{24x\sqrt{x^4+5}}{x^2+\sqrt{5}}$$

```
[In] Int[((2 + 3*x^2)*(5 + x^4)^(3/2))/x^2,x]
```

```
[Out] (24*x*Sqrt[5 + x^4])/(Sqrt[5] + x^2) + (6*x*(25 + 14*x^2)*Sqrt[5 + x^4])/35
 - ((14 - 3*x^2)*(5 + x^4)^(3/2))/(7*x) - (24*5^(1/4)*(Sqrt[5] + x^2)*Sqrt[
(5 + x^4)/(Sqrt[5] + x^2)^2]*EllipticE[2*ArcTan[x/5^(1/4)], 1/2])/Sqrt[5 +
x^4] + (6*5^(1/4)*(14 + 5*Sqrt[5])*(Sqrt[5] + x^2)*Sqrt[(5 + x^4)/(Sqrt[5]
+ x^2)^2]*EllipticF[2*ArcTan[x/5^(1/4)], 1/2])/(7*Sqrt[5 + x^4])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 1191

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[x*(
d*(4*p + 3) + e*(4*p + 1)*x^2)*((a + c*x^4)^p/((4*p + 1)*(4*p + 3))), x] +
Dist[2*(p/((4*p + 1)*(4*p + 3))), Int[Simp[2*a*d*(4*p + 3) + (2*a*e*(4*p +
1))*x^2, x]*(a + c*x^4)^(p - 1), x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*
d^2 + a*e^2, 0] && GtQ[p, 0] && FractionQ[p] && IntegerQ[2*p]
```

Rule 1210

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Rule 1212

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + c*x^4], x], x] - Dist[e/q, I
nt[(1 - q*x^2)/Sqrt[a + c*x^4], x], x] /; NeQ[e + d*q, 0]] /; FreeQ[{a, c,
d, e}, x] && PosQ[c/a]
```

Rule 1286

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (c_.)*(x_)^4)^(p_.), x
_Symbol] :> Simp[(f*x)^(m + 1)*(a + c*x^4)^p*((d*(m + 4*p + 3) + e*(m + 1)*
x^2)/(f*(m + 1)*(m + 4*p + 3))), x] + Dist[4*(p/(f^2*(m + 1)*(m + 4*p + 3))
), Int[(f*x)^(m + 2)*(a + c*x^4)^(p - 1)*(a*e*(m + 1) - c*d*(m + 4*p + 3)*x
^2), x], x] /; FreeQ[{a, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1] && m +
4*p + 3 != 0 && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{(14-3x^2)(5+x^4)^{3/2}}{7x} - \frac{6}{7}\int \left(-15-14x^2\right)\sqrt{5+x^4}\,dx \\
&= \frac{6}{35}x\left(25+14x^2\right)\sqrt{5+x^4} - \frac{(14-3x^2)(5+x^4)^{3/2}}{7x} - \frac{2}{35}\int \frac{-750-420x^2}{\sqrt{5+x^4}}\,dx \\
&= \frac{6}{35}x\left(25+14x^2\right)\sqrt{5+x^4} - \frac{(14-3x^2)(5+x^4)^{3/2}}{7x} \\
&\quad - \left(24\sqrt{5}\right)\int \frac{1-\frac{x^2}{\sqrt{5}}}{\sqrt{5+x^4}}\,dx + \frac{1}{7}\left(12\left(25+14\sqrt{5}\right)\right)\int \frac{1}{\sqrt{5+x^4}}\,dx \\
&= \frac{24x\sqrt{5+x^4}}{\sqrt{5}+x^2} + \frac{6}{35}x\left(25+14x^2\right)\sqrt{5+x^4} - \frac{(14-3x^2)(5+x^4)^{3/2}}{7x} \\
&\quad - \frac{24\sqrt[4]{5}\left(\sqrt{5}+x^2\right)\sqrt{\frac{5+x^4}{\left(\sqrt{5}+x^2\right)^2}}\,E\left(2\tan^{-1}\left(\frac{x}{\sqrt[4]{5}}\right)|\frac{1}{2}\right)}{\sqrt{5+x^4}} \\
&\quad + \frac{6\sqrt[4]{5}\left(14+5\sqrt{5}\right)\left(\sqrt{5}+x^2\right)\sqrt{\frac{5+x^4}{\left(\sqrt{5}+x^2\right)^2}}\,F\left(2\tan^{-1}\left(\frac{x}{\sqrt[4]{5}}\right)|\frac{1}{2}\right)}{7\sqrt{5+x^4}}
\end{aligned}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 4.85 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.27

$$\int \frac{(2+3x^2)(5+x^4)^{3/2}}{x^2}\,dx = -\frac{10\sqrt{5}\,\text{Hypergeometric2F1}\left(-\frac{3}{2},-\frac{1}{4},\frac{3}{4},-\frac{x^4}{5}\right)}{x} + 15\sqrt{5}x\,\text{Hypergeometric2F1}\left(-\frac{3}{2},\frac{1}{4},\frac{5}{4},-\frac{x^4}{5}\right)$$

```
[In] Integrate[((2 + 3*x^2)*(5 + x^4)^(3/2))/x^2,x]
[Out] (-10*Sqrt[5]*Hypergeometric2F1[-3/2, -1/4, 3/4, -1/5*x^4])/x + 15*Sqrt[5]*x
*Hypergeometric2F1[-3/2, 1/4, 5/4, -1/5*x^4]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4.

Time = 1.36 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.19

method	result
meijerg	$-\frac{10\sqrt{5}\,{}_2F_1\left(-\frac{3}{2},-\frac{1}{4};\frac{3}{4};-\frac{x^4}{5}\right)}{x}+15\sqrt{5}\,x\,{}_2F_1\left(-\frac{3}{2},\frac{1}{4};\frac{5}{4};-\frac{x^4}{5}\right)$
risch	$\frac{15x^{10}+14x^8+300x^6-280x^4+1125x^2-1750}{35x\sqrt{x^4+5}}+\frac{12\sqrt{5}\,\sqrt{25-5i\sqrt{5}\,x^2}\,\sqrt{25+5i\sqrt{5}\,x^2}\,F\left(\frac{x\sqrt{5}\,\sqrt{i\sqrt{5}}}{5},i\right)}{7\sqrt{i\sqrt{5}}\,\sqrt{x^4+5}}+\frac{24i\sqrt{25-5i\sqrt{5}\,x^2}\,\sqrt{25+5i\sqrt{}}}{}$
default	$\frac{3x^5\sqrt{x^4+5}}{7}+\frac{45x\sqrt{x^4+5}}{7}+\frac{12\sqrt{5}\,\sqrt{25-5i\sqrt{5}\,x^2}\,\sqrt{25+5i\sqrt{5}\,x^2}\,F\left(\frac{x\sqrt{5}\,\sqrt{i\sqrt{5}}}{5},i\right)}{7\sqrt{i\sqrt{5}}\,\sqrt{x^4+5}}-\frac{10\sqrt{x^4+5}}{x}+\frac{2x^3\sqrt{x^4+5}}{5}+\frac{24i\sqrt{25-5i\sqrt{5}}}{}$
elliptic	$\frac{3x^5\sqrt{x^4+5}}{7}+\frac{45x\sqrt{x^4+5}}{7}+\frac{12\sqrt{5}\,\sqrt{25-5i\sqrt{5}\,x^2}\,\sqrt{25+5i\sqrt{5}\,x^2}\,F\left(\frac{x\sqrt{5}\,\sqrt{i\sqrt{5}}}{5},i\right)}{7\sqrt{i\sqrt{5}}\,\sqrt{x^4+5}}-\frac{10\sqrt{x^4+5}}{x}+\frac{2x^3\sqrt{x^4+5}}{5}+\frac{24i\sqrt{25-5i\sqrt{5}}}{}$

```
[In] int((3*x^2+2)*(x^4+5)^(3/2)/x^2,x,method=_RETURNVERBOSE)
[Out] -10*5^(1/2)/x*hypergeom([-3/2,-1/4],[3/4],-1/5*x^4)+15*5^(1/2)*x*hypergeom(
[-3/2,1/4],[5/4],-1/5*x^4)
```

Fricas [F]

$$\int \frac{(2+3x^2)\left(5+x^4\right)^{3/2}}{x^2}\,dx = \int \frac{\left(x^4+5\right)^{\frac{3}{2}}\left(3\,x^2+2\right)}{x^2}\,dx$$

```
[In] integrate((3*x^2+2)*(x^4+5)^(3/2)/x^2,x, algorithm="fricas")
```

```
[Out] integral((3*x^6 + 2*x^4 + 15*x^2 + 10)*sqrt(x^4 + 5)/x^2, x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.84 (sec) , antiderivative size = 160, normalized size of antiderivative = 0.80

$$\int \frac{(2+3x^2)\left(5+x^4\right)^{3/2}}{x^2}\,dx = \frac{3\sqrt{5}x^5\Gamma\left(\frac{5}{4}\right)\,{}_2F_1\left(\left.\begin{matrix}-\frac{1}{2},\frac{5}{4}\\ \frac{9}{4}\end{matrix}\right|\frac{x^4e^{i\pi}}{5}\right)}{4\Gamma\left(\frac{9}{4}\right)} + \frac{\sqrt{5}x^3\Gamma\left(\frac{3}{4}\right)\,{}_2F_1\left(\left.\begin{matrix}-\frac{1}{2},\frac{3}{4}\\ \frac{7}{4}\end{matrix}\right|\frac{x^4e^{i\pi}}{5}\right)}{2\Gamma\left(\frac{7}{4}\right)} + \frac{15\sqrt{5}x\Gamma\left(\frac{1}{4}\right)\,{}_2F_1\left(\left.\begin{matrix}-\frac{1}{2},\frac{1}{4}\\ \frac{5}{4}\end{matrix}\right|\frac{x^4e^{i\pi}}{5}\right)}{4\Gamma\left(\frac{5}{4}\right)} + \frac{5\sqrt{5}\Gamma\left(-\frac{1}{4}\right)\,{}_2F_1\left(\left.\begin{matrix}-\frac{1}{2},-\frac{1}{4}\\ \frac{3}{4}\end{matrix}\right|\frac{x^4e^{i\pi}}{5}\right)}{2x\Gamma\left(\frac{3}{4}\right)}$$

```
[In] integrate((3*x**2+2)*(x**4+5)**(3/2)/x**2,x)
```

```
[Out] 3*sqrt(5)*x**5*gamma(5/4)*hyper((-1/2, 5/4), (9/4,), x**4*exp_polar(I*pi)/5
)/(4*gamma(9/4)) + sqrt(5)*x**3*gamma(3/4)*hyper((-1/2, 3/4), (7/4,), x**4*
exp_polar(I*pi)/5)/(2*gamma(7/4)) + 15*sqrt(5)*x*gamma(1/4)*hyper((-1/2, 1/
4), (5/4,), x**4*exp_polar(I*pi)/5)/(4*gamma(5/4)) + 5*sqrt(5)*gamma(-1/4)*
hyper((-1/2, -1/4), (3/4,), x**4*exp_polar(I*pi)/5)/(2*x*gamma(3/4))
```

Maxima [F]

$$\int \frac{(2+3x^2)\left(5+x^4\right)^{3/2}}{x^2}\,dx = \int \frac{\left(x^4+5\right)^{\frac{3}{2}}(3\,x^2+2)}{x^2}\,dx$$

```
[In] integrate((3*x^2+2)*(x^4+5)^(3/2)/x^2,x, algorithm="maxima")
[Out] integrate((x^4 + 5)^(3/2)*(3*x^2 + 2)/x^2, x)
```

Giac [F]

$$\int \frac{(2+3x^2)\left(5+x^4\right)^{3/2}}{x^2}\,dx = \int \frac{\left(x^4+5\right)^{\frac{3}{2}}(3\,x^2+2)}{x^2}\,dx$$

```
[In] integrate((3*x^2+2)*(x^4+5)^(3/2)/x^2,x, algorithm="giac")
[Out] integrate((x^4 + 5)^(3/2)*(3*x^2 + 2)/x^2, x)
```

Mupad [B] (verification not implemented)

Time = 7.77 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.24

$$\int \frac{(2+3x^2)\left(5+x^4\right)^{3/2}}{x^2}\,dx = 15\,\sqrt{5}\,x\,{}_2\mathrm{F}_1\left(-\frac{3}{2},\frac{1}{4};\ \frac{5}{4};\ -\frac{x^4}{5}\right) + \frac{2\left(x^4+5\right)^{3/2}{}_2\mathrm{F}_1\left(-\frac{3}{2},-\frac{5}{4};\ -\frac{1}{4};\ -\frac{5}{x^4}\right)}{5\,x\left(\frac{5}{x^4}+1\right)^{3/2}}$$

```
[In] int(((x^4 + 5)^(3/2)*(3*x^2 + 2))/x^2,x)
[Out] 15*5^(1/2)*x*hypergeom([-3/2, 1/4], 5/4, -x^4/5) + (2*(x^4 + 5)^(3/2)*hyper
geom([-3/2, -5/4], -1/4, -5/x^4))/(5*x*(5/x^4 + 1)^(3/2))
```

3.31 $\int \frac{(2+3x^2)(5+x^4)^{3/2}}{x^4}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 201

$$\int \frac{(2+3x^2)(5+x^4)^{3/2}}{x^4}\,dx = -\frac{2(27-2x^2)\sqrt{5+x^4}}{3x}+\frac{36x\sqrt{5+x^4}}{\sqrt{5}+x^2} - \frac{(10-9x^2)(5+x^4)^{3/2}}{15x^3} - \frac{36\sqrt[4]{5}(\sqrt{5}+x^2)\sqrt{\frac{5+x^4}{(\sqrt{5}+x^2)^2}}E\left(2\arctan\left(\frac{x}{\sqrt[4]{5}}\right)|\frac{1}{2}\right)}{\sqrt{5+x^4}} + \frac{2\sqrt[4]{5}(27+2\sqrt{5})(\sqrt{5}+x^2)\sqrt{\frac{5+x^4}{(\sqrt{5}+x^2)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{x}{\sqrt[4]{5}}\right),\frac{1}{2}\right)}{3\sqrt{5+x^4}}$$

```
[Out] -1/15*(-9*x^2+10)*(x^4+5)^(3/2)/x^3-2/3*(-2*x^2+27)*(x^4+5)^(1/2)/x+36*x*(x
^4+5)^(1/2)/(x^2+5^(1/2))-36*5^(1/4)*(cos(2*arctan(1/5*x*5^(3/4)))^2)^(1/2)
/cos(2*arctan(1/5*x*5^(3/4)))*EllipticE(sin(2*arctan(1/5*x*5^(3/4))),1/2*2^
(1/2))*(x^2+5^(1/2))*((x^4+5)/(x^2+5^(1/2))^2)^(1/2)/(x^4+5)^(1/2)+2/3*5^(1
/4)*(cos(2*arctan(1/5*x*5^(3/4)))^2)^(1/2)/cos(2*arctan(1/5*x*5^(3/4)))*Ell
ipticF(sin(2*arctan(1/5*x*5^(3/4))),1/2*2^(1/2))*(x^2+5^(1/2))*(27+2*5^(1/2
))*((x^4+5)/(x^2+5^(1/2))^2)^(1/2)/(x^4+5)^(1/2)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 201, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {1286, 1212, 226, 1210}

$$\int \frac{(2+3x^2)(5+x^4)^{3/2}}{x^4}\,dx = \frac{2\sqrt[4]{5}(27+2\sqrt{5})(x^2+\sqrt{5})\sqrt{\frac{x^4+5}{(x^2+\sqrt{5})^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{x}{\sqrt[4]{5}}\right),\frac{1}{2}\right)}{3\sqrt{x^4+5}} - \frac{36\sqrt[4]{5}(x^2+\sqrt{5})\sqrt{\frac{x^4+5}{(x^2+\sqrt{5})^2}}E\left(2\arctan\left(\frac{x}{\sqrt[4]{5}}\right)|\frac{1}{2}\right)}{\sqrt{x^4+5}} - \frac{2(27-2x^2)\sqrt{x^4+5}}{3x} + \frac{36x\sqrt{x^4+5}}{x^2+\sqrt{5}} - \frac{(10-9x^2)(x^4+5)^{3/2}}{15x^3}$$

```
[In] Int[((2 + 3*x^2)*(5 + x^4)^(3/2))/x^4,x]
```

```
[Out] (-2*(27 - 2*x^2)*Sqrt[5 + x^4])/(3*x) + (36*x*Sqrt[5 + x^4])/(Sqrt[5] + x^2
) - ((10 - 9*x^2)*(5 + x^4)^(3/2))/(15*x^3) - (36*5^(1/4)*(Sqrt[5] + x^2)*S
qrt[(5 + x^4)/(Sqrt[5] + x^2)^2]*EllipticE[2*ArcTan[x/5^(1/4)], 1/2])/Sqrt[
5 + x^4] + (2*5^(1/4)*(27 + 2*Sqrt[5])*(Sqrt[5] + x^2)*Sqrt[(5 + x^4)/(Sqrt
[5] + x^2)^2]*EllipticF[2*ArcTan[x/5^(1/4)], 1/2])/(3*Sqrt[5 + x^4])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 1210

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Rule 1212

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + c*x^4], x], x] - Dist[e/q, I
nt[(1 - q*x^2)/Sqrt[a + c*x^4], x], x] /; NeQ[e + d*q, 0]] /; FreeQ[{a, c,
d, e}, x] && PosQ[c/a]
```

Rule 1286

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (c_.)*(x_)^4)^(p_.), x
_Symbol] :> Simp[(f*x)^(m + 1)*(a + c*x^4)^p*((d*(m + 4*p + 3) + e*(m + 1)*
x^2)/(f*(m + 1)*(m + 4*p + 3))), x] + Dist[4*(p/(f^2*(m + 1)*(m + 4*p + 3))
), Int[(f*x)^(m + 2)*(a + c*x^4)^(p - 1)*(a*e*(m + 1) - c*d*(m + 4*p + 3)*x
^2), x], x] /; FreeQ[{a, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1] && m +
4*p + 3 != 0 && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rubi steps

$$\text{integral} = -\frac{(10-9x^2)\,(5+x^4)^{3/2}}{15x^3} - \frac{2}{5}\int \frac{(-45-10x^2)\sqrt{5+x^4}}{x^2}\,dx$$

$$= -\frac{2(27-2x^2)\sqrt{5+x^4}}{3x} - \frac{(10-9x^2)\,(5+x^4)^{3/2}}{15x^3} + \frac{4}{15}\int \frac{50+135x^2}{\sqrt{5+x^4}}\,dx$$

$$= -\frac{2(27-2x^2)\sqrt{5+x^4}}{3x} - \frac{(10-9x^2)\,(5+x^4)^{3/2}}{15x^3} - \left(36\sqrt{5}\right)\int \frac{1-\frac{x^2}{\sqrt{5}}}{\sqrt{5+x^4}}\,dx + \frac{1}{3}\left(4\left(10+27\sqrt{5}\right)\right)\int \frac{1}{\sqrt{5+x^4}}\,dx$$

$$= -\frac{2(27-2x^2)\sqrt{5+x^4}}{3x} + \frac{36x\sqrt{5+x^4}}{\sqrt{5}+x^2} - \frac{(10-9x^2)\,(5+x^4)^{3/2}}{15x^3} - \frac{36\sqrt[4]{5}(\sqrt{5}+x^2)\sqrt{\frac{5+x^4}{\left(\sqrt{5}+x^2\right)^2}}E\left(2\tan^{-1}\left(\frac{x}{\sqrt[4]{5}}\right)|\frac{1}{2}\right)}{\sqrt{5+x^4}} + \frac{2\sqrt[4]{5}(27+2\sqrt{5})\,(\sqrt{5}+x^2)\sqrt{\frac{5+x^4}{\left(\sqrt{5}+x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{x}{\sqrt[4]{5}}\right)|\frac{1}{2}\right)}{3\sqrt{5+x^4}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 9.30 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.27

$$\int \frac{(2+3x^2)\,(5+x^4)^{3/2}}{x^4}\,dx = -\frac{5\sqrt{5}\left(2\,\text{Hypergeometric2F1}\left(-\frac{3}{2},-\frac{3}{4},\frac{1}{4},-\frac{x^4}{5}\right)+9x^2\,\text{Hypergeometric2F1}\left(-\frac{3}{2},-\frac{1}{4},\frac{3}{4},-\frac{x^4}{5}\right)\right)}{3x^3}$$

```
[In] Integrate[((2 + 3*x^2)*(5 + x^4)^(3/2))/x^4,x]
[Out] (-5*Sqrt[5]*(2*Hypergeometric2F1[-3/2, -3/4, 1/4, -1/5*x^4] + 9*x^2*Hyperge
ometric2F1[-3/2, -1/4, 3/4, -1/5*x^4]))/(3*x^3)
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4.

Time = 2.09 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.20

method	result
meijerg	$-\frac{10\sqrt{5}\,{}_2F_1\left(-\frac{3}{2},-\frac{3}{4};\frac{1}{4};-\frac{x^4}{5}\right)}{3x^3}-\frac{15\sqrt{5}\,{}_2F_1\left(-\frac{3}{2},-\frac{1}{4};\frac{3}{4};-\frac{x^4}{5}\right)}{x}$
risch	$\frac{9x^{10}+10x^8-180x^6-1125x^2-250}{15x^3\sqrt{x^4+5}}+\frac{8\sqrt{5}\,\sqrt{25-5i\sqrt{5}\,x^2}\,\sqrt{25+5i\sqrt{5}\,x^2}\,F\left(\frac{x\sqrt{5}\,\sqrt{i\sqrt{5}}}{5},i\right)}{15\sqrt{i\sqrt{5}}\,\sqrt{x^4+5}}+\frac{36i\sqrt{25-5i\sqrt{5}\,x^2}\,\sqrt{25+5i\sqrt{5}\,x^2}\,\left(F\left(\frac{x}{}\right.\right.}{5\sqrt{i\sqrt{5}}\,\sqrt{x}}$
default	$-\frac{15\sqrt{x^4+5}}{x}+\frac{3x^3\sqrt{x^4+5}}{5}+\frac{36i\sqrt{25-5i\sqrt{5}\,x^2}\,\sqrt{25+5i\sqrt{5}\,x^2}\,\left(F\left(\frac{x\sqrt{5}\,\sqrt{i\sqrt{5}}}{5},i\right)-E\left(\frac{x\sqrt{5}\,\sqrt{i\sqrt{5}}}{5},i\right)\right)}{5\sqrt{i\sqrt{5}}\,\sqrt{x^4+5}}-\frac{10\sqrt{x^4+5}}{3x^3}+\frac{2x\sqrt{x^4+5}}{3}$
elliptic	$-\frac{15\sqrt{x^4+5}}{x}+\frac{3x^3\sqrt{x^4+5}}{5}+\frac{36i\sqrt{25-5i\sqrt{5}\,x^2}\,\sqrt{25+5i\sqrt{5}\,x^2}\,\left(F\left(\frac{x\sqrt{5}\,\sqrt{i\sqrt{5}}}{5},i\right)-E\left(\frac{x\sqrt{5}\,\sqrt{i\sqrt{5}}}{5},i\right)\right)}{5\sqrt{i\sqrt{5}}\,\sqrt{x^4+5}}-\frac{10\sqrt{x^4+5}}{3x^3}+\frac{2x\sqrt{x^4+5}}{3}$

```
[In] int((3*x^2+2)*(x^4+5)^(3/2)/x^4,x,method=_RETURNVERBOSE)
```

```
[Out] -10/3*5^(1/2)/x^3*hypergeom([-3/2,-3/4],[1/4],-1/5*x^4)-15*5^(1/2)/x*hyperg
eom([-3/2,-1/4],[3/4],-1/5*x^4)
```

Fricas [F]

$$\int \frac{(2+3x^2)\,(5+x^4)^{3/2}}{x^4}\,dx=\int \frac{(x^4+5)^{\frac{3}{2}}\,(3\,x^2+2)}{x^4}\,dx$$

```
[In] integrate((3*x^2+2)*(x^4+5)^(3/2)/x^4,x, algorithm="fricas")
```

```
[Out] integral((3*x^6 + 2*x^4 + 15*x^2 + 10)*sqrt(x^4 + 5)/x^4, x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.73 (sec) , antiderivative size = 163, normalized size of antiderivative = 0.81

$$\int \frac{(2+3x^2)(5+x^4)^{3/2}}{x^4}\,dx = \frac{3\sqrt{5}x^3\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{3}{4}\\ \frac{7}{4}\end{matrix}\middle|\frac{x^4e^{i\pi}}{5}\right)}{4\Gamma\left(\frac{7}{4}\right)} + \frac{\sqrt{5}x\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{1}{4}\\ \frac{5}{4}\end{matrix}\middle|\frac{x^4e^{i\pi}}{5}\right)}{2\Gamma\left(\frac{5}{4}\right)} + \frac{15\sqrt{5}\Gamma\left(-\frac{1}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},-\frac{1}{4}\\ \frac{3}{4}\end{matrix}\middle|\frac{x^4e^{i\pi}}{5}\right)}{4x\Gamma\left(\frac{3}{4}\right)} + \frac{5\sqrt{5}\Gamma\left(-\frac{3}{4}\right){}_2F_1\left(\begin{matrix}-\frac{3}{4},-\frac{1}{2}\\ \frac{1}{4}\end{matrix}\middle|\frac{x^4e^{i\pi}}{5}\right)}{2x^3\Gamma\left(\frac{1}{4}\right)}$$

```
[In] integrate((3*x**2+2)*(x**4+5)**(3/2)/x**4,x)
```

```
[Out] 3*sqrt(5)*x**3*gamma(3/4)*hyper((-1/2, 3/4), (7/4,), x**4*exp_polar(I*pi)/5
)/(4*gamma(7/4)) + sqrt(5)*x*gamma(1/4)*hyper((-1/2, 1/4), (5/4,), x**4*exp
_polar(I*pi)/5)/(2*gamma(5/4)) + 15*sqrt(5)*gamma(-1/4)*hyper((-1/2, -1/4),
 (3/4,), x**4*exp_polar(I*pi)/5)/(4*x*gamma(3/4)) + 5*sqrt(5)*gamma(-3/4)*h
yper((-3/4, -1/2), (1/4,), x**4*exp_polar(I*pi)/5)/(2*x**3*gamma(1/4))
```

Maxima [F]

$$\int \frac{(2+3x^2)(5+x^4)^{3/2}}{x^4}\,dx = \int \frac{(x^4+5)^{\frac{3}{2}}(3\,x^2+2)}{x^4}\,dx$$

```
[In] integrate((3*x^2+2)*(x^4+5)^(3/2)/x^4,x, algorithm="maxima")
```

```
[Out] integrate((x^4 + 5)^(3/2)*(3*x^2 + 2)/x^4, x)
```

Giac [F]

$$\int \frac{(2+3x^2)(5+x^4)^{3/2}}{x^4}\,dx = \int \frac{(x^4+5)^{\frac{3}{2}}(3\,x^2+2)}{x^4}\,dx$$

```
[In] integrate((3*x^2+2)*(x^4+5)^(3/2)/x^4,x, algorithm="giac")
[Out] integrate((x^4 + 5)^(3/2)*(3*x^2 + 2)/x^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(2+3x^2)(5+x^4)^{3/2}}{x^4}\,dx = \int \frac{(x^4+5)^{3/2}(3\,x^2+2)}{x^4}\,dx$$

```
[In] int(((x^4 + 5)^(3/2)*(3*x^2 + 2))/x^4,x)
[Out] int(((x^4 + 5)^(3/2)*(3*x^2 + 2))/x^4, x)
```

3.32 $\int \frac{x^7(2+3x^2)}{\sqrt{5+x^4}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 67

$$\int \frac{x^7(2+3x^2)}{\sqrt{5+x^4}}\,dx = \frac{1}{3}x^4\sqrt{5+x^4}+\frac{3}{8}x^6\sqrt{5+x^4}-\frac{5}{48}\left(32+27x^2\right)\sqrt{5+x^4}+\frac{225}{16}\operatorname{arcsinh}\left(\frac{x^2}{\sqrt{5}}\right)$$

```
[Out] 225/16*arcsinh(1/5*x^2*5^(1/2))+1/3*x^4*(x^4+5)^(1/2)+3/8*x^6*(x^4+5)^(1/2)
-5/48*(27*x^2+32)*(x^4+5)^(1/2)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {1266, 847, 794, 221}

$$\int \frac{x^7(2+3x^2)}{\sqrt{5+x^4}}\,dx = \frac{225}{16}\operatorname{arcsinh}\left(\frac{x^2}{\sqrt{5}}\right)+\frac{1}{3}\sqrt{x^4+5}x^4+\frac{3}{8}\sqrt{x^4+5}x^6-\frac{5}{48}\left(27x^2+32\right)\sqrt{x^4+5}$$

```
[In] Int[(x^7*(2 + 3*x^2))/Sqrt[5 + x^4],x]
```

```
[Out] (x^4*Sqrt[5 + x^4])/3 + (3*x^6*Sqrt[5 + x^4])/8 - (5*(32 + 27*x^2)*Sqrt[5 +
 x^4])/48 + (225*ArcSinh[x^2/Sqrt[5]])/16
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 794

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*((a + c*x^2)^(p
```

```
+ 1)/(2*c*(p + 1)*(2*p + 3))), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rule 847

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[g*(d + e*x)^m*((a + c*x^2)^(p + 1)/(c*(m + 2*p + 2))
), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p*Simp[
c*d*f*(m + 2*p + 2) - a*e*g*m + c*(e*f*(m + 2*p + 2) + d*g*m)*x, x], x], x]
 /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[m, 0] &
& NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]
) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

Rule 1266

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_S
ymbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + c*x^2)^p, x],
x, x^2], x] /; FreeQ[{a, c, d, e, p, q}, x] && IntegerQ[(m + 1)/2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{2}\text{Subst}\left(\int \frac{x^3(2+3x)}{\sqrt{5+x^2}}\,dx, x, x^2\right)\\
&= \frac{3}{8}x^6\sqrt{5+x^4} + \frac{1}{8}\text{Subst}\left(\int \frac{x^2(-45+8x)}{\sqrt{5+x^2}}\,dx, x, x^2\right)\\
&= \frac{1}{3}x^4\sqrt{5+x^4} + \frac{3}{8}x^6\sqrt{5+x^4} + \frac{1}{24}\text{Subst}\left(\int \frac{(-80-135x)x}{\sqrt{5+x^2}}\,dx, x, x^2\right)\\
&= \frac{1}{3}x^4\sqrt{5+x^4} + \frac{3}{8}x^6\sqrt{5+x^4} - \frac{5}{48}\left(32+27x^2\right)\sqrt{5+x^4} + \frac{225}{16}\text{Subst}\left(\int \frac{1}{\sqrt{5+x^2}}\,dx, x, x^2\right)\\
&= \frac{1}{3}x^4\sqrt{5+x^4} + \frac{3}{8}x^6\sqrt{5+x^4} - \frac{5}{48}\left(32+27x^2\right)\sqrt{5+x^4} + \frac{225}{16}\sinh^{-1}\left(\frac{x^2}{\sqrt{5}}\right)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.76

$$\int \frac{x^7(2+3x^2)}{\sqrt{5+x^4}}\,dx = \frac{1}{48}\sqrt{5+x^4}\left(-160-135x^2+16x^4+18x^6\right) - \frac{225}{16}\log\left(-x^2+\sqrt{5+x^4}\right)$$

```
[In] Integrate[(x^7*(2 + 3*x^2))/Sqrt[5 + x^4],x]
[Out] (Sqrt[5 + x^4]*(-160 - 135*x^2 + 16*x^4 + 18*x^6))/48 - (225*Log[-x^2 + Sqr
t[5 + x^4]])/16
```

Maple [A] (verified)

Time = 0.31 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.58

method	result	size
risch	$\frac{(18x^6+16x^4-135x^2-160)\sqrt{x^4+5}}{48}+\frac{225 \operatorname{arcsinh}\left(\frac{x^2\sqrt{5}}{5}\right)}{16}$	39
pseudoelliptic	$\frac{(18x^6+16x^4-135x^2-160)\sqrt{x^4+5}}{48}+\frac{225 \operatorname{arcsinh}\left(\frac{x^2\sqrt{5}}{5}\right)}{16}$	39
trager	$\left(\frac{3}{8}x^6+\frac{1}{3}x^4-\frac{45}{16}x^2-\frac{10}{3}\right)\sqrt{x^4+5}-\frac{225\ln\left(x^2-\sqrt{x^4+5}\right)}{16}$	43
default	$\frac{3x^6\sqrt{x^4+5}}{8}-\frac{45x^2\sqrt{x^4+5}}{16}+\frac{225 \operatorname{arcsinh}\left(\frac{x^2\sqrt{5}}{5}\right)}{16}+\frac{\sqrt{x^4+5}\,(x^4-10)}{3}$	51
elliptic	$\frac{3x^6\sqrt{x^4+5}}{8}-\frac{45x^2\sqrt{x^4+5}}{16}+\frac{225 \operatorname{arcsinh}\left(\frac{x^2\sqrt{5}}{5}\right)}{16}+\frac{x^4\sqrt{x^4+5}}{3}-\frac{10\sqrt{x^4+5}}{3}$	58
meijerg	$\frac{-\frac{3\sqrt{\pi}\,x^2\sqrt{5}\,\left(-2x^4+15\right)\sqrt{1+\frac{x^4}{5}}}{16}+\frac{225\sqrt{\pi}\,\operatorname{arcsinh}\left(\frac{x^2\sqrt{5}}{5}\right)}{16}}{\sqrt{\pi}}+\frac{5\sqrt{5}\left(\frac{4\sqrt{\pi}}{3}-\frac{\sqrt{\pi}\,\left(-\frac{4x^4}{5}+8\right)\sqrt{1+\frac{x^4}{5}}}{6}\right)}{2\sqrt{\pi}}$	84

```
[In] int(x^7*(3*x^2+2)/(x^4+5)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/48*(18*x^6+16*x^4-135*x^2-160)*(x^4+5)^(1/2)+225/16*arcsinh(1/5*x^2*5^(1/
2))
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.64

$$\int \frac{x^7(2+3x^2)}{\sqrt{5+x^4}}\,dx=\frac{1}{48}\left(18\,x^6+16\,x^4-135\,x^2-160\right)\sqrt{x^4+5}-\frac{225}{16}\,\log\left(-x^2+\sqrt{x^4+5}\right)$$

```
[In] integrate(x^7*(3*x^2+2)/(x^4+5)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/48*(18*x^6 + 16*x^4 - 135*x^2 - 160)*sqrt(x^4 + 5) - 225/16*log(-x^2 + sq
rt(x^4 + 5))
```

Sympy [A] (verification not implemented)

Time = 0.74 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.72

$$\int \frac{x^7(2+3x^2)}{\sqrt{5+x^4}}\,dx = \frac{\sqrt{x^4+5}\cdot\left(\frac{3x^6}{4}+\frac{2x^4}{3}-\frac{45x^2}{8}-\frac{20}{3}\right)}{2}+\frac{225\operatorname{asinh}\left(\frac{\sqrt{5}x^2}{5}\right)}{16}$$

```
[In] integrate(x**7*(3*x**2+2)/(x**4+5)**(1/2),x)
```

```
[Out] sqrt(x**4 + 5)*(3*x**6/4 + 2*x**4/3 - 45*x**2/8 - 20/3)/2 + 225*asinh(sqrt(
5)*x**2/5)/16
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.55

$$\int \frac{x^7(2+3x^2)}{\sqrt{5+x^4}}\,dx = \frac{1}{3}\left(x^4+5\right)^{\frac{3}{2}}-5\sqrt{x^4+5}-\frac{75\left(\frac{5\sqrt{x^4+5}}{x^2}-\frac{3\left(x^4+5\right)^{\frac{3}{2}}}{x^6}\right)}{16\left(\frac{2\left(x^4+5\right)}{x^4}-\frac{\left(x^4+5\right)^2}{x^8}-1\right)}+\frac{225}{32}\log\left(\frac{\sqrt{x^4+5}}{x^2}+1\right)-\frac{225}{32}\log\left(\frac{\sqrt{x^4+5}}{x^2}-1\right)$$

```
[In] integrate(x^7*(3*x^2+2)/(x^4+5)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/3*(x^4 + 5)^(3/2) - 5*sqrt(x^4 + 5) - 75/16*(5*sqrt(x^4 + 5)/x^2 - 3*(x^4
 + 5)^(3/2)/x^6)/(2*(x^4 + 5)/x^4 - (x^4 + 5)^2/x^8 - 1) + 225/32*log(sqrt(
x^4 + 5)/x^2 + 1) - 225/32*log(sqrt(x^4 + 5)/x^2 - 1)
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.69

$$\int \frac{x^7(2+3x^2)}{\sqrt{5+x^4}}\,dx = \frac{1}{48}\sqrt{x^4+5}\left(\left(2\left(9x^2+8\right)x^2-135\right)x^2-160\right)-\frac{225}{16}\log\left(-x^2+\sqrt{x^4+5}\right)$$

```
[In] integrate(x^7*(3*x^2+2)/(x^4+5)^(1/2),x, algorithm="giac")
```

```
[Out] 1/48*sqrt(x^4 + 5)*((2*(9*x^2 + 8)*x^2 - 135)*x^2 - 160) - 225/16*log(-x^2
+ sqrt(x^4 + 5))
```

Mupad [B] (verification not implemented)

Time = 7.87 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.57

$$\int \frac{x^7(2+3x^2)}{\sqrt{5+x^4}}\,dx = \frac{225\,\mathrm{asinh}\left(\frac{\sqrt{5}\,x^2}{5}\right)}{16} - \sqrt{x^4+5}\left(-\frac{3\,x^6}{8} - \frac{x^4}{3} + \frac{45\,x^2}{16} + \frac{10}{3}\right)$$

```
[In] int((x^7*(3*x^2 + 2))/(x^4 + 5)^(1/2),x)
```

```
[Out] (225*asinh((5^(1/2)*x^2)/5))/16 - (x^4 + 5)^(1/2)*((45*x^2)/16 - x^4/3 - (3
*x^6)/8 + 10/3)
```

3.33 $\int \frac{x^5(2+3x^2)}{\sqrt{5+x^4}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 51

$$\int \frac{x^5(2+3x^2)}{\sqrt{5+x^4}}\,dx = \frac{1}{2}x^4\sqrt{5+x^4}-\frac{1}{2}\left(10-x^2\right)\sqrt{5+x^4}-\frac{5}{2}\text{arcsinh}\left(\frac{x^2}{\sqrt{5}}\right)$$

```
[Out] -5/2*arcsinh(1/5*x^2*5^(1/2))+1/2*x^4*(x^4+5)^(1/2)-1/2*(-x^2+10)*(x^4+5)^(
1/2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {1266, 847, 794, 221}

$$\int \frac{x^5(2+3x^2)}{\sqrt{5+x^4}}\,dx = -\frac{5}{2}\text{arcsinh}\left(\frac{x^2}{\sqrt{5}}\right)+\frac{1}{2}\sqrt{x^4+5}x^4-\frac{1}{2}\left(10-x^2\right)\sqrt{x^4+5}$$

```
[In] Int[(x^5*(2 + 3*x^2))/Sqrt[5 + x^4],x]
```

```
[Out] (x^4*Sqrt[5 + x^4])/2 - ((10 - x^2)*Sqrt[5 + x^4])/2 - (5*ArcSinh[x^2/Sqrt[
5]])/2
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 794

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*((a + c*x^2)^(p
```

```
+ 1)/(2*c*(p + 1)*(2*p + 3))), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rule 847

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[g*(d + e*x)^m*((a + c*x^2)^(p + 1)/(c*(m + 2*p + 2))
), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p*Simp[
c*d*f*(m + 2*p + 2) - a*e*g*m + c*(e*f*(m + 2*p + 2) + d*g*m)*x, x], x], x]
 /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[m, 0] &
& NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]
) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

Rule 1266

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_S
ymbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + c*x^2)^p, x],
x, x^2], x] /; FreeQ[{a, c, d, e, p, q}, x] && IntegerQ[(m + 1)/2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{2}\text{Subst}\left(\int \frac{x^2(2+3x)}{\sqrt{5+x^2}}\,dx, x, x^2\right) \\
&= \frac{1}{2}x^4\sqrt{5+x^4} + \frac{1}{6}\text{Subst}\left(\int \frac{x(-30+6x)}{\sqrt{5+x^2}}\,dx, x, x^2\right) \\
&= \frac{1}{2}x^4\sqrt{5+x^4} - \frac{1}{2}\left(10-x^2\right)\sqrt{5+x^4} - \frac{5}{2}\text{Subst}\left(\int \frac{1}{\sqrt{5+x^2}}\,dx, x, x^2\right) \\
&= \frac{1}{2}x^4\sqrt{5+x^4} - \frac{1}{2}\left(10-x^2\right)\sqrt{5+x^4} - \frac{5}{2}\sinh^{-1}\left(\frac{x^2}{\sqrt{5}}\right)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.82

$$\int \frac{x^5(2+3x^2)}{\sqrt{5+x^4}}\,dx = \frac{1}{2}\sqrt{5+x^4}\left(-10+x^2+x^4\right) + \frac{5}{2}\log\left(-x^2+\sqrt{5+x^4}\right)$$

```
[In] Integrate[(x^5*(2 + 3*x^2))/Sqrt[5 + x^4],x]
[Out] (Sqrt[5 + x^4]*(-10 + x^2 + x^4))/2 + (5*Log[-x^2 + Sqrt[5 + x^4]])/2
```

Maple [A] (verified)

Time = 0.28 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.59

method	result	size
risch	$\frac{(x^4+x^2-10)\sqrt{x^4+5}}{2}-\frac{5\,\operatorname{arcsinh}\left(\frac{x^2\sqrt{5}}{5}\right)}{2}$	30
pseudoelliptic	$\frac{(x^4+x^2-10)\sqrt{x^4+5}}{2}-\frac{5\,\operatorname{arcsinh}\left(\frac{x^2\sqrt{5}}{5}\right)}{2}$	30
trager	$\left(\frac{1}{2}x^4+\frac{1}{2}x^2-5\right)\sqrt{x^4+5}-\frac{5\ln\left(x^2+\sqrt{x^4+5}\right)}{2}$	36
default	$\frac{\sqrt{x^4+5}\,(x^4-10)}{2}+\frac{x^2\sqrt{x^4+5}}{2}-\frac{5\,\operatorname{arcsinh}\left(\frac{x^2\sqrt{5}}{5}\right)}{2}$	39
elliptic	$\frac{x^4\sqrt{x^4+5}}{2}-5\sqrt{x^4+5}+\frac{x^2\sqrt{x^4+5}}{2}-\frac{5\,\operatorname{arcsinh}\left(\frac{x^2\sqrt{5}}{5}\right)}{2}$	46
meijerg	$\frac{15\sqrt{5}\left(\frac{4\sqrt{\pi}}{3}-\frac{\sqrt{\pi}\left(-\frac{4x^4}{5}+8\right)\sqrt{1+\frac{x^4}{5}}}{6}\right)}{4\sqrt{\pi}}+\frac{\frac{\sqrt{\pi}\,x^2\sqrt{5}\,\sqrt{1+\frac{x^4}{5}}}{2}-\frac{5\sqrt{\pi}\,\operatorname{arcsinh}\left(\frac{x^2\sqrt{5}}{5}\right)}{2}}{\sqrt{\pi}}$	77

```
[In] int(x^5*(3*x^2+2)/(x^4+5)^(1/2),x,method=_RETURNVERBOSE)
[Out] 1/2*(x^4+x^2-10)*(x^4+5)^(1/2)-5/2*arcsinh(1/5*x^2*5^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.67

$$\int \frac{x^5(2+3x^2)}{\sqrt{5+x^4}}\,dx = \frac{1}{2}\left(x^4+x^2-10\right)\sqrt{x^4+5}+\frac{5}{2}\log\left(-x^2+\sqrt{x^4+5}\right)$$

```
[In] integrate(x^5*(3*x^2+2)/(x^4+5)^(1/2),x, algorithm="fricas")
[Out] 1/2*(x^4 + x^2 - 10)*sqrt(x^4 + 5) + 5/2*log(-x^2 + sqrt(x^4 + 5))
```

Sympy [A] (verification not implemented)

Time = 0.68 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.63

$$\int \frac{x^5(2+3x^2)}{\sqrt{5+x^4}}\,dx = \frac{\sqrt{x^4+5}(x^4+x^2-10)}{2}-\frac{5\operatorname{asinh}\left(\frac{\sqrt{5}x^2}{5}\right)}{2}$$

```
[In] integrate(x**5*(3*x**2+2)/(x**4+5)**(1/2),x)
[Out] sqrt(x**4 + 5)*(x**4 + x**2 - 10)/2 - 5*asinh(sqrt(5)*x**2/5)/2
```

Maxima [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.49

$$\int \frac{x^5(2+3x^2)}{\sqrt{5+x^4}}\,dx = \frac{1}{2}\left(x^4+5\right)^{\frac{3}{2}} - \frac{15}{2}\sqrt{x^4+5} + \frac{5\sqrt{x^4+5}}{2\,x^2\left(\frac{x^4+5}{x^4}-1\right)} - \frac{5}{4}\log\left(\frac{\sqrt{x^4+5}}{x^2}+1\right) + \frac{5}{4}\log\left(\frac{\sqrt{x^4+5}}{x^2}-1\right)$$

```
[In] integrate(x^5*(3*x^2+2)/(x^4+5)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/2*(x^4 + 5)^(3/2) - 15/2*sqrt(x^4 + 5) + 5/2*sqrt(x^4 + 5)/(x^2*((x^4 + 5
)/x^4 - 1)) - 5/4*log(sqrt(x^4 + 5)/x^2 + 1) + 5/4*log(sqrt(x^4 + 5)/x^2 -
1)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.73

$$\int \frac{x^5(2+3x^2)}{\sqrt{5+x^4}}\,dx = \frac{1}{2}\sqrt{x^4+5}\left(\left(x^2+1\right)x^2-10\right) + \frac{5}{2}\log\left(-x^2+\sqrt{x^4+5}\right)$$

```
[In] integrate(x^5*(3*x^2+2)/(x^4+5)^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(x^4 + 5)*((x^2 + 1)*x^2 - 10) + 5/2*log(-x^2 + sqrt(x^4 + 5))
```

Mupad [B] (verification not implemented)

Time = 7.84 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.63

$$\int \frac{x^5(2+3x^2)}{\sqrt{5+x^4}}\,dx = \sqrt{x^4+5}\left(\frac{x^4}{2}+\frac{x^2}{2}-5\right) - \frac{5\,\mathrm{asinh}\left(\frac{\sqrt{5}\,x^2}{5}\right)}{2}$$

```
[In] int((x^5*(3*x^2 + 2))/(x^4 + 5)^(1/2),x)
```

```
[Out] (x^4 + 5)^(1/2)*(x^2/2 + x^4/2 - 5) - (5*asinh((5^(1/2)*x^2)/5))/2
```

3.34 $\int \frac{x^3(2+3x^2)}{\sqrt{5+x^4}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 35

$$\int \frac{x^3(2+3x^2)}{\sqrt{5+x^4}}\,dx = \frac{1}{4}(4+3x^2)\sqrt{5+x^4} - \frac{15}{4}\text{arcsinh}\left(\frac{x^2}{\sqrt{5}}\right)$$

```
[Out] -15/4*arcsinh(1/5*x^2*5^(1/2))+1/4*(3*x^2+4)*(x^4+5)^(1/2)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {1266, 794, 221}

$$\int \frac{x^3(2+3x^2)}{\sqrt{5+x^4}}\,dx = \frac{1}{4}(3x^2+4)\sqrt{x^4+5} - \frac{15}{4}\text{arcsinh}\left(\frac{x^2}{\sqrt{5}}\right)$$

```
[In] Int[(x^3*(2 + 3*x^2))/Sqrt[5 + x^4],x]
```

```
[Out] ((4 + 3*x^2)*Sqrt[5 + x^4])/4 - (15*ArcSinh[x^2/Sqrt[5]])/4
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 794

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*((a + c*x^2)^(p
+ 1)/(2*c*(p + 1)*(2*p + 3))), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
```

```
Q[p, -1]
```

Rule 1266

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_S
ymbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + c*x^2)^p, x],
x, x^2], x] /; FreeQ[{a, c, d, e, p, q}, x] && IntegerQ[(m + 1)/2]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{1}{2}\text{Subst}\left(\int \frac{x(2+3x)}{\sqrt{5+x^2}}\,dx, x, x^2\right)\\ &= \frac{1}{4}(4+3x^2)\sqrt{5+x^4} - \frac{15}{4}\text{Subst}\left(\int \frac{1}{\sqrt{5+x^2}}\,dx, x, x^2\right)\\ &= \frac{1}{4}(4+3x^2)\sqrt{5+x^4} - \frac{15}{4}\sinh^{-1}\left(\frac{x^2}{\sqrt{5}}\right)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.17

$$\int \frac{x^3(2+3x^2)}{\sqrt{5+x^4}}\,dx = \frac{1}{4}(4+3x^2)\sqrt{5+x^4} + \frac{15}{4}\log\left(-x^2+\sqrt{5+x^4}\right)$$

```
[In] Integrate[(x^3*(2 + 3*x^2))/Sqrt[5 + x^4],x]
[Out] ((4 + 3*x^2)*Sqrt[5 + x^4])/4 + (15*Log[-x^2 + Sqrt[5 + x^4]])/4
```

Maple [A] (verified)

Time = 0.28 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.83

method	result	size
risch	$-\frac{15\,\mathrm{arcsinh}\left(\frac{x^2\sqrt{5}}{5}\right)}{4}+\frac{(3x^2+4)\sqrt{x^4+5}}{4}$	29
default	$\frac{3x^2\sqrt{x^4+5}}{4}-\frac{15\,\mathrm{arcsinh}\left(\frac{x^2\sqrt{5}}{5}\right)}{4}+\sqrt{x^4+5}$	32
elliptic	$\frac{3x^2\sqrt{x^4+5}}{4}-\frac{15\,\mathrm{arcsinh}\left(\frac{x^2\sqrt{5}}{5}\right)}{4}+\sqrt{x^4+5}$	32
pseudoelliptic	$\frac{3x^2\sqrt{x^4+5}}{4}-\frac{15\,\mathrm{arcsinh}\left(\frac{x^2\sqrt{5}}{5}\right)}{4}+\sqrt{x^4+5}$	32
trager	$\left(\frac{3x^2}{4}+1\right)\sqrt{x^4+5}+\frac{15\ln\left(x^2-\sqrt{x^4+5}\right)}{4}$	33
meijerg	$\frac{\frac{3\sqrt{\pi}\,x^2\sqrt{5}\,\sqrt{1+\frac{x^4}{5}}}{4}-\frac{15\sqrt{\pi}\,\mathrm{arcsinh}\left(\frac{x^2\sqrt{5}}{5}\right)}{4}}{\sqrt{\pi}}+\frac{\sqrt{5}\left(-2\sqrt{\pi}+2\sqrt{\pi}\,\sqrt{1+\frac{x^4}{5}}\right)}{2\sqrt{\pi}}$	70

```
[In] int(x^3*(3*x^2+2)/(x^4+5)^(1/2),x,method=_RETURNVERBOSE)
[Out] -15/4*arcsinh(1/5*x^2*5^(1/2))+1/4*(3*x^2+4)*(x^4+5)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.94

$$\int \frac{x^3(2+3x^2)}{\sqrt{5+x^4}}\,dx = \frac{1}{4}\sqrt{x^4+5}(3\,x^2+4)+\frac{15}{4}\,\log\left(-x^2+\sqrt{x^4+5}\right)$$

```
[In] integrate(x^3*(3*x^2+2)/(x^4+5)^(1/2),x, algorithm="fricas")
[Out] 1/4*sqrt(x^4 + 5)*(3*x^2 + 4) + 15/4*log(-x^2 + sqrt(x^4 + 5))
```

Sympy [A] (verification not implemented)

Time = 0.63 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.91

$$\int \frac{x^3(2+3x^2)}{\sqrt{5+x^4}}\,dx = \frac{\left(\frac{3x^2}{2}+2\right)\sqrt{x^4+5}}{2}-\frac{15\,\mathrm{asinh}\left(\frac{\sqrt{5}x^2}{5}\right)}{4}$$

```
[In] integrate(x**3*(3*x**2+2)/(x**4+5)**(1/2),x)
[Out] (3*x**2/2 + 2)*sqrt(x**4 + 5)/2 - 15*asinh(sqrt(5)*x**2/5)/4
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 65 vs. 2(28) = 56.

Time = 0.36 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.86

$$\int \frac{x^3(2+3x^2)}{\sqrt{5+x^4}}\,dx = \sqrt{x^4+5} + \frac{15\sqrt{x^4+5}}{4\,x^2\left(\frac{x^4+5}{x^4}-1\right)} - \frac{15}{8}\log\left(\frac{\sqrt{x^4+5}}{x^2}+1\right) + \frac{15}{8}\log\left(\frac{\sqrt{x^4+5}}{x^2}-1\right)$$

```
[In] integrate(x^3*(3*x^2+2)/(x^4+5)^(1/2),x, algorithm="maxima")
[Out] sqrt(x^4 + 5) + 15/4*sqrt(x^4 + 5)/(x^2*((x^4 + 5)/x^4 - 1)) - 15/8*log(sqr
t(x^4 + 5)/x^2 + 1) + 15/8*log(sqrt(x^4 + 5)/x^2 - 1)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.94

$$\int \frac{x^3(2+3x^2)}{\sqrt{5+x^4}}\,dx = \frac{1}{4}\sqrt{x^4+5}\left(3\,x^2+4\right) + \frac{15}{4}\log\left(-x^2+\sqrt{x^4+5}\right)$$

```
[In] integrate(x^3*(3*x^2+2)/(x^4+5)^(1/2),x, algorithm="giac")
[Out] 1/4*sqrt(x^4 + 5)*(3*x^2 + 4) + 15/4*log(-x^2 + sqrt(x^4 + 5))
```

Mupad [B] (verification not implemented)

Time = 7.85 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.77

$$\int \frac{x^3(2+3x^2)}{\sqrt{5+x^4}}\,dx = \sqrt{x^4+5}\left(\frac{3\,x^2}{4}+1\right) - \frac{15\,\mathrm{asinh}\left(\frac{\sqrt{5}\,x^2}{5}\right)}{4}$$

```
[In] int((x^3*(3*x^2 + 2))/(x^4 + 5)^(1/2),x)
[Out] (x^4 + 5)^(1/2)*((3*x^2)/4 + 1) - (15*asinh((5^(1/2)*x^2)/5))/4
```

3.35 $\int \frac{x(2+3x^2)}{\sqrt{5+x^4}}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 24

$$\int \frac{x(2+3x^2)}{\sqrt{5+x^4}}\,dx = \frac{3\sqrt{5+x^4}}{2} + \operatorname{arcsinh}\left(\frac{x^2}{\sqrt{5}}\right)$$

```
[Out] arcsinh(1/5*x^2*5^(1/2))+3/2*(x^4+5)^(1/2)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1262, 655, 221}

$$\int \frac{x(2+3x^2)}{\sqrt{5+x^4}}\,dx = \operatorname{arcsinh}\left(\frac{x^2}{\sqrt{5}}\right) + \frac{3\sqrt{x^4+5}}{2}$$

```
[In] Int[(x*(2 + 3*x^2))/Sqrt[5 + x^4],x]
```

```
[Out] (3*Sqrt[5 + x^4])/2 + ArcSinh[x^2/Sqrt[5]]
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 655

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[e*((
a + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 1262

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol]
 :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + c*x^2)^p, x], x, x^2], x] /; FreeQ
[{a, c, d, e, p, q}, x]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{1}{2}\text{Subst}\left(\int \frac{2+3x}{\sqrt{5+x^2}}\,dx, x, x^2\right)\\ &= \frac{3\sqrt{5+x^4}}{2} + \text{Subst}\left(\int \frac{1}{\sqrt{5+x^2}}\,dx, x, x^2\right)\\ &= \frac{3\sqrt{5+x^4}}{2} + \sinh^{-1}\left(\frac{x^2}{\sqrt{5}}\right)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.33

$$\int \frac{x(2+3x^2)}{\sqrt{5+x^4}}\,dx = \frac{3\sqrt{5+x^4}}{2} - \log\left(-x^2+\sqrt{5+x^4}\right)$$

```
[In] Integrate[(x*(2 + 3*x^2))/Sqrt[5 + x^4],x]
[Out] (3*Sqrt[5 + x^4])/2 - Log[-x^2 + Sqrt[5 + x^4]]
```

Maple [A] (verified)

Time = 0.27 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.83

method	result	size
default	$\operatorname{arcsinh}\left(\frac{x^2\sqrt{5}}{5}\right)+\frac{3\sqrt{x^4+5}}{2}$	20
risch	$\operatorname{arcsinh}\left(\frac{x^2\sqrt{5}}{5}\right)+\frac{3\sqrt{x^4+5}}{2}$	20
elliptic	$\operatorname{arcsinh}\left(\frac{x^2\sqrt{5}}{5}\right)+\frac{3\sqrt{x^4+5}}{2}$	20
pseudoelliptic	$\operatorname{arcsinh}\left(\frac{x^2\sqrt{5}}{5}\right)+\frac{3\sqrt{x^4+5}}{2}$	20
trager	$\frac{3\sqrt{x^4+5}}{2}+\ln\left(x^2+\sqrt{x^4+5}\right)$	23
meijerg	$\frac{3\sqrt{5}\left(-2\sqrt{\pi}+2\sqrt{\pi}\sqrt{1+\frac{x^4}{5}}\right)}{4\sqrt{\pi}}+\operatorname{arcsinh}\left(\frac{x^2\sqrt{5}}{5}\right)$	39

```
[In] int(x*(3*x^2+2)/(x^4+5)^(1/2),x,method=_RETURNVERBOSE)
[Out] arcsinh(1/5*x^2*5^(1/2))+3/2*(x^4+5)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{x(2+3x^2)}{\sqrt{5+x^4}}\,dx = \frac{3}{2}\sqrt{x^4+5} - \log\left(-x^2+\sqrt{x^4+5}\right)$$

```
[In] integrate(x*(3*x^2+2)/(x^4+5)^(1/2),x, algorithm="fricas")
[Out] 3/2*sqrt(x^4 + 5) - log(-x^2 + sqrt(x^4 + 5))
```

Sympy [A] (verification not implemented)

Time = 0.61 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{x(2+3x^2)}{\sqrt{5+x^4}}\,dx = \frac{3\sqrt{x^4+5}}{2} + \operatorname{asinh}\left(\frac{\sqrt{5}x^2}{5}\right)$$

```
[In] integrate(x*(3*x**2+2)/(x**4+5)**(1/2),x)
[Out] 3*sqrt(x**4 + 5)/2 + asinh(sqrt(5)*x**2/5)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 42 vs. $2(19) = 38$.

Time = 0.37 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.75

$$\int \frac{x(2+3x^2)}{\sqrt{5+x^4}}\,dx = \frac{3}{2}\sqrt{x^4+5} + \frac{1}{2}\log\left(\frac{\sqrt{x^4+5}}{x^2}+1\right) - \frac{1}{2}\log\left(\frac{\sqrt{x^4+5}}{x^2}-1\right)$$

```
[In] integrate(x*(3*x^2+2)/(x^4+5)^(1/2),x, algorithm="maxima")
[Out] 3/2*sqrt(x^4 + 5) + 1/2*log(sqrt(x^4 + 5)/x^2 + 1) - 1/2*log(sqrt(x^4 + 5)/
x^2 - 1)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{x(2+3x^2)}{\sqrt{5+x^4}}\,dx = \frac{3}{2}\sqrt{x^4+5} - \log\left(-x^2+\sqrt{x^4+5}\right)$$

```
[In] integrate(x*(3*x^2+2)/(x^4+5)^(1/2),x, algorithm="giac")
[Out] 3/2*sqrt(x^4 + 5) - log(-x^2 + sqrt(x^4 + 5))
```

Mupad [B] (verification not implemented)

Time = 7.86 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.79

$$\int \frac{x(2+3x^2)}{\sqrt{5+x^4}}\,dx = \operatorname{asinh}\left(\frac{\sqrt{5}\,x^2}{5}\right) + \frac{3\sqrt{x^4+5}}{2}$$

```
[In] int((x*(3*x^2 + 2))/(x^4 + 5)^(1/2),x)
[Out] asinh((5^(1/2)*x^2)/5) + (3*(x^4 + 5)^(1/2))/2
```

3.36 $\int \frac{2+3x^2}{x\sqrt{5+x^4}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 38

$$\int \frac{2+3x^2}{x\sqrt{5+x^4}}\,dx=\frac{3}{2}\text{arcsinh}\left(\frac{x^2}{\sqrt{5}}\right)-\frac{\text{arctanh}\left(\frac{\sqrt{5+x^4}}{\sqrt{5}}\right)}{\sqrt{5}}$$

```
[Out] 3/2*arcsinh(1/5*x^2*5^(1/2))-1/5*arctanh(1/5*(x^4+5)^(1/2)*5^(1/2))*5^(1/2)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {1266, 858, 221, 272, 65, 213}

$$\int \frac{2+3x^2}{x\sqrt{5+x^4}}\,dx=\frac{3}{2}\text{arcsinh}\left(\frac{x^2}{\sqrt{5}}\right)-\frac{\text{arctanh}\left(\frac{\sqrt{x^4+5}}{\sqrt{5}}\right)}{\sqrt{5}}$$

```
[In] Int[(2 + 3*x^2)/(x*Sqrt[5 + x^4]),x]
```

```
[Out] (3*ArcSinh[x^2/Sqrt[5]])/2 - ArcTanh[Sqrt[5 + x^4]/Sqrt[5]]/Sqrt[5]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 858

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1266

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_S
ymbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + c*x^2)^p, x],
x, x^2], x] /; FreeQ[{a, c, d, e, p, q}, x] && IntegerQ[(m + 1)/2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{2}\text{Subst}\left(\int \frac{2+3x}{x\sqrt{5+x^2}}\,dx, x, x^2\right) \\
&= \frac{3}{2}\text{Subst}\left(\int \frac{1}{\sqrt{5+x^2}}\,dx, x, x^2\right) + \text{Subst}\left(\int \frac{1}{x\sqrt{5+x^2}}\,dx, x, x^2\right) \\
&= \frac{3}{2}\sinh^{-1}\left(\frac{x^2}{\sqrt{5}}\right) + \frac{1}{2}\text{Subst}\left(\int \frac{1}{x\sqrt{5+x}}\,dx, x, x^4\right) \\
&= \frac{3}{2}\sinh^{-1}\left(\frac{x^2}{\sqrt{5}}\right) + \text{Subst}\left(\int \frac{1}{-5+x^2}\,dx, x, \sqrt{5+x^4}\right) \\
&= \frac{3}{2}\sinh^{-1}\left(\frac{x^2}{\sqrt{5}}\right) - \frac{\tanh^{-1}\left(\frac{\sqrt{5+x^4}}{\sqrt{5}}\right)}{\sqrt{5}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.32

$$\int \frac{2+3x^2}{x\sqrt{5+x^4}}\,dx=\frac{2\text{arctanh}\left(\frac{x^2-\sqrt{5+x^4}}{\sqrt{5}}\right)}{\sqrt{5}}-\frac{3}{2}\log\left(-x^2+\sqrt{5+x^4}\right)$$

```
[In] Integrate[(2 + 3*x^2)/(x*Sqrt[5 + x^4]),x]
[Out] (2*ArcTanh[(x^2 - Sqrt[5 + x^4])/Sqrt[5]])/Sqrt[5] - (3*Log[-x^2 + Sqrt[5 +
 x^4]])/2
```

Maple [A] (verified)

Time = 0.38 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.79

method	result	size
default	$\frac{3\,\text{arcsinh}\left(\frac{x^2\sqrt{5}}{5}\right)}{2}-\frac{\sqrt{5}\,\text{arctanh}\left(\frac{\sqrt{5}}{\sqrt{x^4+5}}\right)}{5}$	30
elliptic	$\frac{3\,\text{arcsinh}\left(\frac{x^2\sqrt{5}}{5}\right)}{2}-\frac{\sqrt{5}\,\text{arctanh}\left(\frac{\sqrt{5}}{\sqrt{x^4+5}}\right)}{5}$	30
pseudoelliptic	$\frac{3\,\text{arcsinh}\left(\frac{x^2\sqrt{5}}{5}\right)}{2}-\frac{\sqrt{5}\,\text{arctanh}\left(\frac{\sqrt{5}}{\sqrt{x^4+5}}\right)}{5}$	30
trager	$-\frac{\text{RootOf}\left(\_Z^2-5\right)\ln\left(\frac{\text{RootOf}\left(\_Z^2-5\right)+\sqrt{x^4+5}}{x^2}\right)}{5}-\frac{3\ln\left(x^2-\sqrt{x^4+5}\right)}{2}$	45
meijerg	$\frac{\sqrt{5}\left(-2\sqrt{\pi}\,\ln\left(\frac{1}{2}+\frac{\sqrt{1+\frac{x^4}{5}}}{2}\right)+(-2\ln(2)+4\ln(x)-\ln(5))\sqrt{\pi}\right)}{10\sqrt{\pi}}+\frac{3\,\text{arcsinh}\left(\frac{x^2\sqrt{5}}{5}\right)}{2}$	58

```
[In] int((3*x^2+2)/x/(x^4+5)^(1/2),x,method=_RETURNVERBOSE)
[Out] 3/2*arcsinh(1/5*x^2*5^(1/2))-1/5*5^(1/2)*arctanh(5^(1/2)/(x^4+5)^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.08

$$\int \frac{2+3x^2}{x\sqrt{5+x^4}}\,dx = \frac{1}{5}\sqrt{5}\log\left(-\frac{\sqrt{5}-\sqrt{x^4+5}}{x^2}\right) - \frac{3}{2}\log\left(-x^2+\sqrt{x^4+5}\right)$$

```
[In] integrate((3*x^2+2)/x/(x^4+5)^(1/2),x, algorithm="fricas")
[Out] 1/5*sqrt(5)*log(-(sqrt(5) - sqrt(x^4 + 5))/x^2) - 3/2*log(-x^2 + sqrt(x^4 +
 5))
```

Sympy [A] (verification not implemented)

Time = 2.77 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.82

$$\int \frac{2+3x^2}{x\sqrt{5+x^4}}\,dx = -\frac{\sqrt{5}\,\text{asinh}\left(\frac{\sqrt{5}}{x^2}\right)}{5} + \frac{3\,\text{asinh}\left(\frac{\sqrt{5}x^2}{5}\right)}{2}$$

```
[In] integrate((3*x**2+2)/x/(x**4+5)**(1/2),x)
[Out] -sqrt(5)*asinh(sqrt(5)/x**2)/5 + 3*asinh(sqrt(5)*x**2/5)/2
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 67 vs. 2(30) = 60.

Time = 0.35 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.76

$$\int \frac{2+3x^2}{x\sqrt{5+x^4}}\,dx = \frac{1}{10}\sqrt{5}\log\left(-\frac{\sqrt{5}-\sqrt{x^4+5}}{\sqrt{5}+\sqrt{x^4+5}}\right) + \frac{3}{4}\log\left(\frac{\sqrt{x^4+5}}{x^2}+1\right) - \frac{3}{4}\log\left(\frac{\sqrt{x^4+5}}{x^2}-1\right)$$

```
[In] integrate((3*x^2+2)/x/(x^4+5)^(1/2),x, algorithm="maxima")
[Out] 1/10*sqrt(5)*log(-(sqrt(5) - sqrt(x^4 + 5))/(sqrt(5) + sqrt(x^4 + 5))) + 3/
4*log(sqrt(x^4 + 5)/x^2 + 1) - 3/4*log(sqrt(x^4 + 5)/x^2 - 1)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 61 vs. $2(30) = 60$.

Time = 0.28 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.61

$$\int \frac{2+3x^2}{x\sqrt{5+x^4}}\,dx = \frac{1}{5}\sqrt{5}\log\left(-\frac{x^2+\sqrt{5}-\sqrt{x^4+5}}{x^2-\sqrt{5}-\sqrt{x^4+5}}\right) - \frac{3}{2}\log\left(-x^2+\sqrt{x^4+5}\right)$$

```
[In] integrate((3*x^2+2)/x/(x^4+5)^(1/2),x, algorithm="giac")
```

```
[Out] 1/5*sqrt(5)*log(-(x^2 + sqrt(5) - sqrt(x^4 + 5))/(x^2 - sqrt(5) - sqrt(x^4
+ 5))) - 3/2*log(-x^2 + sqrt(x^4 + 5))
```

Mupad [B] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.79

$$\int \frac{2+3x^2}{x\sqrt{5+x^4}}\,dx = \frac{3\operatorname{asinh}\left(\frac{\sqrt{5}\,x^2}{5}\right)}{2} - \frac{\sqrt{5}\operatorname{atanh}\left(\frac{\sqrt{5}\,\sqrt{x^4+5}}{5}\right)}{5}$$

```
[In] int((3*x^2 + 2)/(x*(x^4 + 5)^(1/2)),x)
```

```
[Out] (3*asinh((5^(1/2)*x^2)/5))/2 - (5^(1/2)*atanh((5^(1/2)*(x^4 + 5)^(1/2))/5))
/5
```

3.37 $\int \frac{2+3x^2}{x^3\sqrt{5+x^4}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 42

$$\int \frac{2+3x^2}{x^3\sqrt{5+x^4}}\,dx=-\frac{\sqrt{5+x^4}}{5x^2}-\frac{3\text{arctanh}\left(\frac{\sqrt{5+x^4}}{\sqrt{5}}\right)}{2\sqrt{5}}$$

```
[Out] -3/10*arctanh(1/5*(x^4+5)^(1/2)*5^(1/2))*5^(1/2)-1/5*(x^4+5)^(1/2)/x^2
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {1266, 821, 272, 65, 213}

$$\int \frac{2+3x^2}{x^3\sqrt{5+x^4}}\,dx=-\frac{3\text{arctanh}\left(\frac{\sqrt{x^4+5}}{\sqrt{5}}\right)}{2\sqrt{5}}-\frac{\sqrt{x^4+5}}{5x^2}$$

```
[In] Int[(2 + 3*x^2)/(x^3*Sqrt[5 + x^4]),x]
```

```
[Out] -1/5*Sqrt[5 + x^4]/x^2 - (3*ArcTanh[Sqrt[5 + x^4]/Sqrt[5]])/(2*Sqrt[5])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 821

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1
)/(2*(p + 1)*(c*d^2 + a*e^2))), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2),
Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m,
p}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 1266

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_S
ymbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + c*x^2)^p, x],
x, x^2], x] /; FreeQ[{a, c, d, e, p, q}, x] && IntegerQ[(m + 1)/2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{2}\text{Subst}\left(\int \frac{2+3x}{x^2\sqrt{5+x^2}}\,dx, x, x^2\right)\\
&= -\frac{\sqrt{5+x^4}}{5x^2}+\frac{3}{2}\text{Subst}\left(\int \frac{1}{x\sqrt{5+x^2}}\,dx, x, x^2\right)\\
&= -\frac{\sqrt{5+x^4}}{5x^2}+\frac{3}{4}\text{Subst}\left(\int \frac{1}{x\sqrt{5+x}}\,dx, x, x^4\right)\\
&= -\frac{\sqrt{5+x^4}}{5x^2}+\frac{3}{2}\text{Subst}\left(\int \frac{1}{-5+x^2}\,dx, x, \sqrt{5+x^4}\right)\\
&= -\frac{\sqrt{5+x^4}}{5x^2}-\frac{3\tanh^{-1}\left(\frac{\sqrt{5+x^4}}{\sqrt{5}}\right)}{2\sqrt{5}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.10

$$\int \frac{2+3x^2}{x^3\sqrt{5+x^4}}\,dx = -\frac{\sqrt{5+x^4}}{5x^2}+\frac{3\text{arctanh}\left(\frac{x^2-\sqrt{5+x^4}}{\sqrt{5}}\right)}{\sqrt{5}}$$

```
[In] Integrate[(2 + 3*x^2)/(x^3*Sqrt[5 + x^4]),x]
[Out] -1/5*Sqrt[5 + x^4]/x^2 + (3*ArcTanh[(x^2 - Sqrt[5 + x^4])/Sqrt[5]])/Sqrt[5]
```

Maple [A] (verified)

Time = 0.30 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.74

method	result	size
default	$-\frac{3\sqrt{5}\,\operatorname{arctanh}\left(\frac{\sqrt{5}}{\sqrt{x^4+5}}\right)}{10}-\frac{\sqrt{x^4+5}}{5x^2}$	31
risch	$-\frac{3\sqrt{5}\,\operatorname{arctanh}\left(\frac{\sqrt{5}}{\sqrt{x^4+5}}\right)}{10}-\frac{\sqrt{x^4+5}}{5x^2}$	31
elliptic	$-\frac{3\sqrt{5}\,\operatorname{arctanh}\left(\frac{\sqrt{5}}{\sqrt{x^4+5}}\right)}{10}-\frac{\sqrt{x^4+5}}{5x^2}$	31
pseudoelliptic	$\frac{-3\sqrt{5}\,\operatorname{arctanh}\left(\frac{\sqrt{5}}{\sqrt{x^4+5}}\right)x^2-2\sqrt{x^4+5}}{10x^2}$	36
trager	$-\frac{\sqrt{x^4+5}}{5x^2}-\frac{3\,\operatorname{RootOf}\left(\_Z^2-5\right)\ln\left(\frac{\operatorname{RootOf}\left(\_Z^2-5\right)+\sqrt{x^4+5}}{x^2}\right)}{10}$	41
meijerg	$-\frac{\sqrt{5}\,\sqrt{1+\frac{x^4}{5}}}{5x^2}+\frac{3\sqrt{5}\left(-2\sqrt{\pi}\,\ln\left(\frac{1}{2}+\frac{\sqrt{1+\frac{x^4}{5}}}{2}\right)+\left(-2\ln\left(2\right)+4\ln\left(x\right)-\ln\left(5\right)\right)\sqrt{\pi}\right)}{20\sqrt{\pi}}$	64

```
[In] int((3*x^2+2)/x^3/(x^4+5)^(1/2),x,method=_RETURNVERBOSE)
[Out] -3/10*5^(1/2)*arctanh(5^(1/2)/(x^4+5)^(1/2))-1/5*(x^4+5)^(1/2)/x^2
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.12

$$\int \frac{2+3x^2}{x^3\sqrt{5+x^4}}\,dx = \frac{3\sqrt{5}x^2\log\left(-\frac{\sqrt{5}-\sqrt{x^4+5}}{x^2}\right) - 2\,x^2 - 2\sqrt{x^4+5}}{10\,x^2}$$

```
[In] integrate((3*x^2+2)/x^3/(x^4+5)^(1/2),x, algorithm="fricas")
[Out] 1/10*(3*sqrt(5)*x^2*log(-(sqrt(5) - sqrt(x^4 + 5))/x^2) - 2*x^2 - 2*sqrt(x^
4 + 5))/x^2
```

Sympy [A] (verification not implemented)

Time = 1.60 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.74

$$\int \frac{2+3x^2}{x^3\sqrt{5+x^4}}\,dx = -\frac{\sqrt{1+\frac{5}{x^4}}}{5} - \frac{3\sqrt{5}\operatorname{asinh}\left(\frac{\sqrt{5}}{x^2}\right)}{10}$$

```
[In] integrate((3*x**2+2)/x**3/(x**4+5)**(1/2),x)
[Out] -sqrt(1 + 5/x**4)/5 - 3*sqrt(5)*asinh(sqrt(5)/x**2)/10
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.12

$$\int \frac{2+3x^2}{x^3\sqrt{5+x^4}}\,dx = \frac{3}{20}\sqrt{5}\log\left(-\frac{\sqrt{5}-\sqrt{x^4+5}}{\sqrt{5}+\sqrt{x^4+5}}\right) - \frac{\sqrt{x^4+5}}{5\,x^2}$$

```
[In] integrate((3*x^2+2)/x^3/(x^4+5)^(1/2),x, algorithm="maxima")
[Out] 3/20*sqrt(5)*log(-(sqrt(5) - sqrt(x^4 + 5))/(sqrt(5) + sqrt(x^4 + 5))) - 1/
5*sqrt(x^4 + 5)/x^2
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 66 vs. $2(31) = 62$.

Time = 0.28 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.57

$$\int \frac{2+3x^2}{x^3\sqrt{5+x^4}}\,dx = \frac{3}{10}\sqrt{5}\log\left(-\frac{x^2+\sqrt{5}-\sqrt{x^4+5}}{x^2-\sqrt{5}-\sqrt{x^4+5}}\right) + \frac{2}{\left(x^2-\sqrt{x^4+5}\right)^2-5}$$

```
[In] integrate((3*x^2+2)/x^3/(x^4+5)^(1/2),x, algorithm="giac")
```

```
[Out] 3/10*sqrt(5)*log(-(x^2 + sqrt(5) - sqrt(x^4 + 5))/(x^2 - sqrt(5) - sqrt(x^4
 + 5))) + 2/((x^2 - sqrt(x^4 + 5))^2 - 5)
```

Mupad [B] (verification not implemented)

Time = 7.68 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.74

$$\int \frac{2+3x^2}{x^3\sqrt{5+x^4}}\,dx = -\frac{3\sqrt{5}\,\mathrm{atanh}\left(\frac{\sqrt{5}\,\sqrt{x^4+5}}{5}\right)}{10} - \frac{\sqrt{x^4+5}}{5\,x^2}$$

```
[In] int((3*x^2 + 2)/(x^3*(x^4 + 5)^(1/2)),x)
```

```
[Out] - (3*5^(1/2)*atanh((5^(1/2)*(x^4 + 5)^(1/2))/5))/10 - (x^4 + 5)^(1/2)/(5*x^
2)
```

3.38 $\int \frac{2+3x^2}{x^5\sqrt{5+x^4}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 58

$$\int \frac{2+3x^2}{x^5\sqrt{5+x^4}}\,dx = -\frac{\sqrt{5+x^4}}{10x^4}-\frac{3\sqrt{5+x^4}}{10x^2}+\frac{\operatorname{arctanh}\left(\frac{\sqrt{5+x^4}}{\sqrt{5}}\right)}{10\sqrt{5}}$$

```
[Out] 1/50*arctanh(1/5*(x^4+5)^(1/2)*5^(1/2))*5^(1/2)-1/10*(x^4+5)^(1/2)/x^4-3/10
*(x^4+5)^(1/2)/x^2
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {1266, 849, 821, 272, 65, 213}

$$\int \frac{2+3x^2}{x^5\sqrt{5+x^4}}\,dx = \frac{\operatorname{arctanh}\left(\frac{\sqrt{x^4+5}}{\sqrt{5}}\right)}{10\sqrt{5}}-\frac{\sqrt{x^4+5}}{10x^4}-\frac{3\sqrt{x^4+5}}{10x^2}$$

```
[In] Int[(2 + 3*x^2)/(x^5*Sqrt[5 + x^4]),x]
```

```
[Out] -1/10*Sqrt[5 + x^4]/x^4 - (3*Sqrt[5 + x^4])/(10*x^2) + ArcTanh[Sqrt[5 + x^4
]/Sqrt[5]]/(10*Sqrt[5])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
```

```
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 821

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1
)/(2*(p + 1)*(c*d^2 + a*e^2))), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2),
Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m,
p}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 849

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(e*f - d*g)*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1)/(
(m + 1)*(c*d^2 + a*e^2))), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d +
e*x)^(m + 1)*(a + c*x^2)^p*Simp[(c*d*f + a*e*g)*(m + 1) - c*(e*f - d*g)*(m
+ 2*p + 3)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 +
a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*
p])
```

Rule 1266

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_S
ymbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + c*x^2)^p, x],
x, x^2], x] /; FreeQ[{a, c, d, e, p, q}, x] && IntegerQ[(m + 1)/2]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{1}{2}\text{Subst}\left(\int \frac{2+3x}{x^3\sqrt{5+x^2}}\,dx, x, x^2\right)\\ &= -\frac{\sqrt{5+x^4}}{10x^4} - \frac{1}{20}\text{Subst}\left(\int \frac{-30+2x}{x^2\sqrt{5+x^2}}\,dx, x, x^2\right)\end{aligned}$$

$$= -\frac{\sqrt{5+x^4}}{10x^4} - \frac{3\sqrt{5+x^4}}{10x^2} - \frac{1}{10}\text{Subst}\left(\int \frac{1}{x\sqrt{5+x^2}}\,dx, x, x^2\right)$$

$$= -\frac{\sqrt{5+x^4}}{10x^4} - \frac{3\sqrt{5+x^4}}{10x^2} - \frac{1}{20}\text{Subst}\left(\int \frac{1}{x\sqrt{5+x}}\,dx, x, x^4\right)$$

$$= -\frac{\sqrt{5+x^4}}{10x^4} - \frac{3\sqrt{5+x^4}}{10x^2} - \frac{1}{10}\text{Subst}\left(\int \frac{1}{-5+x^2}\,dx, x, \sqrt{5+x^4}\right)$$

$$= -\frac{\sqrt{5+x^4}}{10x^4} - \frac{3\sqrt{5+x^4}}{10x^2} + \frac{\tanh^{-1}\left(\frac{\sqrt{5+x^4}}{\sqrt{5}}\right)}{10\sqrt{5}}$$

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.95

$$\int \frac{2+3x^2}{x^5\sqrt{5+x^4}}\,dx = \frac{1}{50}\left(-\frac{5(1+3x^2)\sqrt{5+x^4}}{x^4} - 2\sqrt{5}\text{arctanh}\left(\frac{x^2-\sqrt{5+x^4}}{\sqrt{5}}\right)\right)$$

```
[In] Integrate[(2 + 3*x^2)/(x^5*Sqrt[5 + x^4]),x]
[Out] ((-5*(1 + 3*x^2)*Sqrt[5 + x^4])/x^4 - 2*Sqrt[5]*ArcTanh[(x^2 - Sqrt[5 + x^4
])/Sqrt[5]])/50
```

Maple [A] (verified)

Time = 0.33 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.74

method	result	size
default	$-\frac{3\sqrt{x^4+5}}{10x^2}-\frac{\sqrt{x^4+5}}{10x^4}+\frac{\sqrt{5}\,\operatorname{arctanh}\left(\frac{\sqrt{5}}{\sqrt{x^4+5}}\right)}{50}$	43
elliptic	$-\frac{3\sqrt{x^4+5}}{10x^2}-\frac{\sqrt{x^4+5}}{10x^4}+\frac{\sqrt{5}\,\operatorname{arctanh}\left(\frac{\sqrt{5}}{\sqrt{x^4+5}}\right)}{50}$	43
risch	$-\frac{3x^6+x^4+15x^2+5}{10x^4\sqrt{x^4+5}}+\frac{\sqrt{5}\,\operatorname{arctanh}\left(\frac{\sqrt{5}}{\sqrt{x^4+5}}\right)}{50}$	46
pseudoelliptic	$\frac{\sqrt{5}\,\operatorname{arctanh}\left(\frac{\sqrt{5}}{\sqrt{x^4+5}}\right)x^4-15x^2\sqrt{x^4+5}-5\sqrt{x^4+5}}{50x^4}$	47
trager	$-\frac{(3x^2+1)\sqrt{x^4+5}}{10x^4}-\frac{\operatorname{RootOf}\left(\_Z^2-5\right)\ln\left(\frac{\sqrt{x^4+5}-\operatorname{RootOf}\left(\_Z^2-5\right)}{x^2}\right)}{50}$	50
meijerg	$\frac{\sqrt{5}\left(\frac{5\sqrt{\pi}\left(8+\frac{4x^4}{5}\right)}{8x^4}-\frac{5\sqrt{\pi}\sqrt{1+\frac{x^4}{5}}}{x^4}+\sqrt{\pi}\ln\left(\frac{1}{2}+\frac{\sqrt{1+\frac{x^4}{5}}}{2}\right)-\frac{(1-2\ln(2)+4\ln(x)-\ln(5))\sqrt{\pi}}{2}-\frac{5\sqrt{\pi}}{x^4}\right)}{50\sqrt{\pi}}-\frac{3\sqrt{5}\sqrt{1+\frac{x^4}{5}}}{10x^2}$	105

```
[In] int((3*x^2+2)/x^5/(x^4+5)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -3/10*(x^4+5)^(1/2)/x^2-1/10*(x^4+5)^(1/2)/x^4+1/50*5^(1/2)*arctanh(5^(1/2)
/(x^4+5)^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.86

$$\int \frac{2+3x^2}{x^5\sqrt{5+x^4}}\,dx=\frac{\sqrt{5}x^4\log\left(\frac{\sqrt{5}+\sqrt{x^4+5}}{x^2}\right)-15\,x^4-5\,\sqrt{x^4+5}(3\,x^2+1)}{50\,x^4}$$

```
[In] integrate((3*x^2+2)/x^5/(x^4+5)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/50*(sqrt(5)*x^4*log((sqrt(5) + sqrt(x^4 + 5))/x^2) - 15*x^4 - 5*sqrt(x^4
+ 5)*(3*x^2 + 1))/x^4
```

Sympy [A] (verification not implemented)

Time = 3.95 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.52

$$\int \frac{2+3x^2}{x^5\sqrt{5+x^4}}\,dx = \frac{\sqrt{5}\left(-\frac{\log\left(\sqrt{\frac{x^4}{5}+1}-1\right)}{4}+\frac{\log\left(\sqrt{\frac{x^4}{5}+1}+1\right)}{4}-\frac{1}{4\left(\sqrt{\frac{x^4}{5}+1}+1\right)}-\frac{1}{4\left(\sqrt{\frac{x^4}{5}+1}-1\right)}\right)}{25} - \frac{3\sqrt{5}\sqrt{5x^4+25}}{50x^2}$$

```
[In] integrate((3*x**2+2)/x**5/(x**4+5)**(1/2),x)
```

```
[Out] sqrt(5)*(-log(sqrt(x**4/5 + 1) - 1)/4 + log(sqrt(x**4/5 + 1) + 1)/4 - 1/(4*
(sqrt(x**4/5 + 1) + 1)) - 1/(4*(sqrt(x**4/5 + 1) - 1)))/25 - 3*sqrt(5)*sqrt
(5*x**4 + 25)/(50*x**2)
```

Maxima [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.02

$$\int \frac{2+3x^2}{x^5\sqrt{5+x^4}}\,dx = -\frac{1}{100}\sqrt{5}\log\left(-\frac{\sqrt{5}-\sqrt{x^4+5}}{\sqrt{5}+\sqrt{x^4+5}}\right) - \frac{3\sqrt{x^4+5}}{10\,x^2} - \frac{\sqrt{x^4+5}}{10\,x^4}$$

```
[In] integrate((3*x^2+2)/x^5/(x^4+5)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/100*sqrt(5)*log(-(sqrt(5) - sqrt(x^4 + 5))/(sqrt(5) + sqrt(x^4 + 5))) -
3/10*sqrt(x^4 + 5)/x^2 - 1/10*sqrt(x^4 + 5)/x^4
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 114 vs. $2(43) = 86$.

Time = 0.28 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.97

$$\int \frac{2+3x^2}{x^5\sqrt{5+x^4}}\,dx = -\frac{1}{50}\sqrt{5}\log\left(-\frac{x^2+\sqrt{5}-\sqrt{x^4+5}}{x^2-\sqrt{5}-\sqrt{x^4+5}}\right) + \frac{\left(x^2-\sqrt{x^4+5}\right)^3+15\left(x^2-\sqrt{x^4+5}\right)^2+5\,x^2-5\sqrt{x^4+5}-75}{5\left(\left(x^2-\sqrt{x^4+5}\right)^2-5\right)^2}$$

```
[In] integrate((3*x^2+2)/x^5/(x^4+5)^(1/2),x, algorithm="giac")
```

```
[Out] -1/50*sqrt(5)*log(-(x^2 + sqrt(5) - sqrt(x^4 + 5))/(x^2 - sqrt(5) - sqrt(x^
4 + 5))) + 1/5*((x^2 - sqrt(x^4 + 5))^3 + 15*(x^2 - sqrt(x^4 + 5))^2 + 5*x^
2 - 5*sqrt(x^4 + 5) - 75)/((x^2 - sqrt(x^4 + 5))^2 - 5)^2
```

Mupad [B] (verification not implemented)

Time = 7.78 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.74

$$\int \frac{2+3x^2}{x^5\sqrt{5+x^4}}\,dx=\frac{\sqrt{5}\,\mathrm{atanh}\left(\frac{\sqrt{5}\,\sqrt{x^4+5}}{5}\right)}{50}-\frac{3\,\sqrt{x^4+5}}{10\,x^2}-\frac{\sqrt{x^4+5}}{10\,x^4}$$

```
[In] int((3*x^2 + 2)/(x^5*(x^4 + 5)^(1/2)),x)
```

```
[Out] (5^(1/2)*atanh((5^(1/2)*(x^4 + 5)^(1/2))/5))/50 - (3*(x^4 + 5)^(1/2))/(10*x
^2) - (x^4 + 5)^(1/2)/(10*x^4)
```

3.39 $\int \frac{x^4(2+3x^2)}{\sqrt{5+x^4}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 185

$$\int \frac{x^4(2+3x^2)}{\sqrt{5+x^4}}\,dx = \frac{2}{3}x\sqrt{5+x^4}+\frac{3}{5}x^3\sqrt{5+x^4}-\frac{9x\sqrt{5+x^4}}{\sqrt{5}+x^2} + \frac{9\sqrt[4]{5}(\sqrt{5}+x^2)\sqrt{\frac{5+x^4}{(\sqrt{5}+x^2)^2}}E\left(2\arctan\left(\frac{x}{\sqrt[4]{5}}\right)|\frac{1}{2}\right)}{\sqrt{5+x^4}} - \frac{\sqrt[4]{5}(27+2\sqrt{5})\,(\sqrt{5}+x^2)\sqrt{\frac{5+x^4}{(\sqrt{5}+x^2)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{x}{\sqrt[4]{5}}\right),\frac{1}{2}\right)}{6\sqrt{5+x^4}}$$

```
[Out] 2/3*x*(x^4+5)^(1/2)+3/5*x^3*(x^4+5)^(1/2)-9*x*(x^4+5)^(1/2)/(x^2+5^(1/2))+9
*5^(1/4)*(cos(2*arctan(1/5*x*5^(3/4)))^2)^(1/2)/cos(2*arctan(1/5*x*5^(3/4))
)*EllipticE(sin(2*arctan(1/5*x*5^(3/4))),1/2*2^(1/2))*(x^2+5^(1/2))*((x^4+5
)/(x^2+5^(1/2))^2)^(1/2)/(x^4+5)^(1/2)-1/6*5^(1/4)*(cos(2*arctan(1/5*x*5^(3
/4)))^2)^(1/2)/cos(2*arctan(1/5*x*5^(3/4)))*EllipticF(sin(2*arctan(1/5*x*5^
(3/4))),1/2*2^(1/2))*(x^2+5^(1/2))*(27+2*5^(1/2))*((x^4+5)/(x^2+5^(1/2))^2)
^(1/2)/(x^4+5)^(1/2)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 185, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {1294, 1212, 226, 1210}

$$\int \frac{x^4(2+3x^2)}{\sqrt{5+x^4}}\,dx = -\frac{\sqrt[4]{5}(27+2\sqrt{5})\left(x^2+\sqrt{5}\right)\sqrt{\frac{x^4+5}{\left(x^2+\sqrt{5}\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{x}{\sqrt[4]{5}}\right),\frac{1}{2}\right)}{6\sqrt{x^4+5}} + \frac{9\sqrt[4]{5}(x^2+\sqrt{5})\sqrt{\frac{x^4+5}{\left(x^2+\sqrt{5}\right)^2}}E\left(2\arctan\left(\frac{x}{\sqrt[4]{5}}\right)|\frac{1}{2}\right)}{\sqrt{x^4+5}} + \frac{2}{3}\sqrt{x^4+5}x + \frac{3}{5}\sqrt{x^4+5}x^3 - \frac{9\sqrt{x^4+5}x}{x^2+\sqrt{5}}$$

```
[In] Int[(x^4*(2 + 3*x^2))/Sqrt[5 + x^4],x]
```

```
[Out] (2*x*Sqrt[5 + x^4])/3 + (3*x^3*Sqrt[5 + x^4])/5 - (9*x*Sqrt[5 + x^4])/(Sqrt
[5] + x^2) + (9*5^(1/4)*(Sqrt[5] + x^2)*Sqrt[(5 + x^4)/(Sqrt[5] + x^2)^2]*E
llipticE[2*ArcTan[x/5^(1/4)], 1/2])/Sqrt[5 + x^4] - (5^(1/4)*(27 + 2*Sqrt[5
])*(Sqrt[5] + x^2)*Sqrt[(5 + x^4)/(Sqrt[5] + x^2)^2]*EllipticF[2*ArcTan[x/5
^(1/4)], 1/2])/(6*Sqrt[5 + x^4])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 1210

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Rule 1212

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + c*x^4], x], x] - Dist[e/q, I
nt[(1 - q*x^2)/Sqrt[a + c*x^4], x], x] /; NeQ[e + d*q, 0]] /; FreeQ[{a, c,
d, e}, x] && PosQ[c/a]
```

Rule 1294

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (c_.)*(x_)^4)^(p_), x_
Symbol] :> Simp[e*f*(f*x)^(m - 1)*((a + c*x^4)^(p + 1)/(c*(m + 4*p + 3))),
x] - Dist[f^2/(c*(m + 4*p + 3)), Int[(f*x)^(m - 2)*(a + c*x^4)^p*(a*e*(m -
1) - c*d*(m + 4*p + 3)*x^2), x], x] /; FreeQ[{a, c, d, e, f, p}, x] && GtQ[
m, 1] && NeQ[m + 4*p + 3, 0] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m
])
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{3}{5}x^3\sqrt{5+x^4} - \frac{1}{5}\int \frac{x^2(45-10x^2)}{\sqrt{5+x^4}}\,dx \\
&= \frac{2}{3}x\sqrt{5+x^4} + \frac{3}{5}x^3\sqrt{5+x^4} + \frac{1}{15}\int \frac{-50-135x^2}{\sqrt{5+x^4}}\,dx \\
&= \frac{2}{3}x\sqrt{5+x^4} + \frac{3}{5}x^3\sqrt{5+x^4} + \left(9\sqrt{5}\right)\int \frac{1-\frac{x^2}{\sqrt{5}}}{\sqrt{5+x^4}}\,dx - \frac{1}{3}\left(10+27\sqrt{5}\right)\int \frac{1}{\sqrt{5+x^4}}\,dx \\
&= \frac{2}{3}x\sqrt{5+x^4} + \frac{3}{5}x^3\sqrt{5+x^4} - \frac{9x\sqrt{5+x^4}}{\sqrt{5}+x^2} \\
&\quad + \frac{9\sqrt[4]{5}\left(\sqrt{5}+x^2\right)\sqrt{\frac{5+x^4}{\left(\sqrt{5}+x^2\right)^2}}\,E\left(2\tan^{-1}\left(\frac{x}{\sqrt[4]{5}}\right)|\frac{1}{2}\right)}{\sqrt{5+x^4}} \\
&\quad - \frac{\sqrt[4]{5}\left(27+2\sqrt{5}\right)\left(\sqrt{5}+x^2\right)\sqrt{\frac{5+x^4}{\left(\sqrt{5}+x^2\right)^2}}\,F\left(2\tan^{-1}\left(\frac{x}{\sqrt[4]{5}}\right)|\frac{1}{2}\right)}{6\sqrt{5+x^4}}
\end{aligned}
$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.03 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.40

$$
\int \frac{x^4(2+3x^2)}{\sqrt{5+x^4}}\,dx = \frac{1}{15}x\left(\left(10+9x^2\right)\sqrt{5+x^4} - 10\sqrt{5}\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{5}{4},-\frac{x^4}{5}\right) - 9\sqrt{5}x^2\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3}{4},\frac{7}{4},-\frac{x^4}{5}\right)\right)
$$

```
[In] Integrate[(x^4*(2 + 3*x^2))/Sqrt[5 + x^4],x]
[Out] (x*((10 + 9*x^2)*Sqrt[5 + x^4] - 10*Sqrt[5]*Hypergeometric2F1[1/4, 1/2, 5/4
, -1/5*x^4] - 9*Sqrt[5]*x^2*Hypergeometric2F1[1/2, 3/4, 7/4, -1/5*x^4]))/15
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4.

Time = 3.94 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.22

method	result
meijerg	$\frac{3\sqrt{5}\, x^{7}{}_{2}F_{1}\left(\frac{1}{2},\frac{7}{4};\frac{11}{4};-\frac{x^{4}}{5}\right)}{35}+\frac{2\sqrt{5}\, x^{5}{}_{2}F_{1}\left(\frac{1}{2},\frac{5}{4};\frac{9}{4};-\frac{x^{4}}{5}\right)}{25}$
risch	$\frac{x(9x^{2}+10)\sqrt{x^{4}+5}}{15}-\frac{2\sqrt{5}\,\sqrt{25-5i\sqrt{5}\,x^{2}}\,\sqrt{25+5i\sqrt{5}\,x^{2}}\,F\left(\frac{x\sqrt{5}\,\sqrt{i\sqrt{5}}}{5},i\right)}{15\sqrt{i\sqrt{5}}\,\sqrt{x^{4}+5}}-\frac{9i\sqrt{25-5i\sqrt{5}\,x^{2}}\,\sqrt{25+5i\sqrt{5}\,x^{2}}\left(F\left(\frac{x\sqrt{5}\,\sqrt{i\sqrt{5}}}{5},i\right)-\right.}{5\sqrt{i\sqrt{5}}\,\sqrt{x^{4}+5}}$
default	$\frac{3x^{3}\sqrt{x^{4}+5}}{5}-\frac{9i\sqrt{25-5i\sqrt{5}\,x^{2}}\,\sqrt{25+5i\sqrt{5}\,x^{2}}\left(F\left(\frac{x\sqrt{5}\,\sqrt{i\sqrt{5}}}{5},i\right)-E\left(\frac{x\sqrt{5}\,\sqrt{i\sqrt{5}}}{5},i\right)\right)}{5\sqrt{i\sqrt{5}}\,\sqrt{x^{4}+5}}+\frac{2x\sqrt{x^{4}+5}}{3}-\frac{2\sqrt{5}\,\sqrt{25-5i\sqrt{5}\,x^{2}}\,\sqrt{25+}}{15\sqrt{i\sqrt{5}}}$
elliptic	$\frac{3x^{3}\sqrt{x^{4}+5}}{5}-\frac{9i\sqrt{25-5i\sqrt{5}\,x^{2}}\,\sqrt{25+5i\sqrt{5}\,x^{2}}\left(F\left(\frac{x\sqrt{5}\,\sqrt{i\sqrt{5}}}{5},i\right)-E\left(\frac{x\sqrt{5}\,\sqrt{i\sqrt{5}}}{5},i\right)\right)}{5\sqrt{i\sqrt{5}}\,\sqrt{x^{4}+5}}+\frac{2x\sqrt{x^{4}+5}}{3}-\frac{2\sqrt{5}\,\sqrt{25-5i\sqrt{5}\,x^{2}}\,\sqrt{25+}}{15\sqrt{i\sqrt{5}}}$

```
[In] int(x^4*(3*x^2+2)/(x^4+5)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 3/35*5^(1/2)*x^7*hypergeom([1/2,7/4],[11/4],-1/5*x^4)+2/25*5^(1/2)*x^5*hype
rgeom([1/2,5/4],[9/4],-1/5*x^4)
```

Fricas [A] (verification not implemented)

none

Time = 0.08 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.32

$$\int \frac{x^4(2+3x^2)}{\sqrt{5+x^4}}\,dx = -\frac{135\,(-5)^{\frac{3}{4}}\,xE(\arcsin\left(\frac{(-5)^{\frac{1}{4}}}{x}\right)\,|-1)-125\,(-5)^{\frac{3}{4}}\,xF(\arcsin\left(\frac{(-5)^{\frac{1}{4}}}{x}\right)\,|-1)-(9\,x^4+10\,x^2-135)\sqrt{x^4}}{15\,x}$$

```
[In] integrate(x^4*(3*x^2+2)/(x^4+5)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/15*(135*(-5)^(3/4)*x*elliptic_e(arcsin((-5)^(1/4)/x), -1) - 125*(-5)^(3/
4)*x*elliptic_f(arcsin((-5)^(1/4)/x), -1) - (9*x^4 + 10*x^2 - 135)*sqrt(x^4
 + 5))/x
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.15 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.41

$$\int \frac{x^4(2+3x^2)}{\sqrt{5+x^4}}\,dx = \frac{3\sqrt{5}x^7\Gamma\left(\frac{7}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{7}{4}\\ \frac{11}{4}\end{matrix}\middle|\frac{x^4e^{i\pi}}{5}\right)}{20\Gamma\left(\frac{11}{4}\right)} + \frac{\sqrt{5}x^5\Gamma\left(\frac{5}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{5}{4}\\ \frac{9}{4}\end{matrix}\middle|\frac{x^4e^{i\pi}}{5}\right)}{10\Gamma\left(\frac{9}{4}\right)}$$

```
[In] integrate(x**4*(3*x**2+2)/(x**4+5)**(1/2),x)
[Out] 3*sqrt(5)*x**7*gamma(7/4)*hyper((1/2, 7/4), (11/4,), x**4*exp_polar(I*pi)/5
)/(20*gamma(11/4)) + sqrt(5)*x**5*gamma(5/4)*hyper((1/2, 5/4), (9/4,), x**4
*exp_polar(I*pi)/5)/(10*gamma(9/4))
```

Maxima [F]

$$\int \frac{x^4(2+3x^2)}{\sqrt{5+x^4}}\,dx = \int \frac{(3\,x^2+2)x^4}{\sqrt{x^4+5}}\,dx$$

```
[In] integrate(x^4*(3*x^2+2)/(x^4+5)^(1/2),x, algorithm="maxima")
[Out] integrate((3*x^2 + 2)*x^4/sqrt(x^4 + 5), x)
```

Giac [F]

$$\int \frac{x^4(2+3x^2)}{\sqrt{5+x^4}}\,dx = \int \frac{(3\,x^2+2)x^4}{\sqrt{x^4+5}}\,dx$$

```
[In] integrate(x^4*(3*x^2+2)/(x^4+5)^(1/2),x, algorithm="giac")
[Out] integrate((3*x^2 + 2)*x^4/sqrt(x^4 + 5), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^4(2+3x^2)}{\sqrt{5+x^4}}\,dx = \int \frac{x^4\,(3\,x^2+2)}{\sqrt{x^4+5}}\,dx$$

```
[In] int((x^4*(3*x^2 + 2))/(x^4 + 5)^(1/2),x)
[Out] int((x^4*(3*x^2 + 2))/(x^4 + 5)^(1/2), x)
```

3.40 $\int \frac{x^2(2+3x^2)}{\sqrt{5+x^4}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 166

$$\int \frac{x^2(2+3x^2)}{\sqrt{5+x^4}}\,dx = x\sqrt{5+x^4}+\frac{2x\sqrt{5+x^4}}{\sqrt{5}+x^2} - \frac{2\sqrt[4]{5}\left(\sqrt{5}+x^2\right)\sqrt{\frac{5+x^4}{\left(\sqrt{5}+x^2\right)^2}}E\left(2\arctan\left(\frac{x}{\sqrt[4]{5}}\right)|\frac{1}{2}\right)}{\sqrt{5+x^4}} + \frac{\sqrt[4]{5}\left(2-\sqrt{5}\right)\left(\sqrt{5}+x^2\right)\sqrt{\frac{5+x^4}{\left(\sqrt{5}+x^2\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{x}{\sqrt[4]{5}}\right),\frac{1}{2}\right)}{2\sqrt{5+x^4}}$$

```
[Out] x*(x^4+5)^(1/2)+2*x*(x^4+5)^(1/2)/(x^2+5^(1/2))-2*5^(1/4)*(cos(2*arctan(1/5
*x*5^(3/4)))^2)^(1/2)/cos(2*arctan(1/5*x*5^(3/4)))*EllipticE(sin(2*arctan(1
/5*x*5^(3/4))),1/2*2^(1/2))*(x^2+5^(1/2))*((x^4+5)/(x^2+5^(1/2))^2)^(1/2)/(
x^4+5)^(1/2)+1/2*5^(1/4)*(cos(2*arctan(1/5*x*5^(3/4)))^2)^(1/2)/cos(2*arcta
n(1/5*x*5^(3/4)))*EllipticF(sin(2*arctan(1/5*x*5^(3/4))),1/2*2^(1/2))*(2-5^
(1/2))*(x^2+5^(1/2))*((x^4+5)/(x^2+5^(1/2))^2)^(1/2)/(x^4+5)^(1/2)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 166, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used

$= \{1294, 1212, 226, 1210\}$

$$\int \frac{x^2(2+3x^2)}{\sqrt{5+x^4}}\,dx = \frac{\sqrt[4]{5}(2-\sqrt{5})\left(x^2+\sqrt{5}\right)\sqrt{\frac{x^4+5}{\left(x^2+\sqrt{5}\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{x}{\sqrt[4]{5}}\right),\frac{1}{2}\right)}{2\sqrt{x^4+5}} - \frac{2\sqrt[4]{5}(x^2+\sqrt{5})\sqrt{\frac{x^4+5}{\left(x^2+\sqrt{5}\right)^2}}E\left(2\arctan\left(\frac{x}{\sqrt[4]{5}}\right)|\frac{1}{2}\right)}{\sqrt{x^4+5}} + \sqrt{x^4+5}x + \frac{2\sqrt{x^4+5}x}{x^2+\sqrt{5}}$$

```
[In] Int[(x^2*(2 + 3*x^2))/Sqrt[5 + x^4],x]
```

```
[Out] x*Sqrt[5 + x^4] + (2*x*Sqrt[5 + x^4])/(Sqrt[5] + x^2) - (2*5^(1/4)*(Sqrt[5]
 + x^2)*Sqrt[(5 + x^4)/(Sqrt[5] + x^2)^2]*EllipticE[2*ArcTan[x/5^(1/4)], 1/
2])/Sqrt[5 + x^4] + (5^(1/4)*(2 - Sqrt[5])*(Sqrt[5] + x^2)*Sqrt[(5 + x^4)/(
Sqrt[5] + x^2)^2]*EllipticF[2*ArcTan[x/5^(1/4)], 1/2])/(2*Sqrt[5 + x^4])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 1210

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Rule 1212

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + c*x^4], x], x] - Dist[e/q, I
nt[(1 - q*x^2)/Sqrt[a + c*x^4], x], x] /; NeQ[e + d*q, 0]] /; FreeQ[{a, c,
d, e}, x] && PosQ[c/a]
```

Rule 1294

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (c_.)*(x_)^4)^(p_), x_
Symbol] :> Simp[e*f*(f*x)^(m - 1)*((a + c*x^4)^(p + 1)/(c*(m + 4*p + 3))),
x] - Dist[f^2/(c*(m + 4*p + 3)), Int[(f*x)^(m - 2)*(a + c*x^4)^p*(a*e*(m -
1) - c*d*(m + 4*p + 3)*x^2), x], x] /; FreeQ[{a, c, d, e, f, p}, x] && GtQ[
m, 1] && NeQ[m + 4*p + 3, 0] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m
```

```
])
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= x\sqrt{5+x^4}-\frac{1}{3}\int \frac{15-6x^2}{\sqrt{5+x^4}}\,dx\\
&= x\sqrt{5+x^4}-\left(2\sqrt{5}\right)\int \frac{1-\frac{x^2}{\sqrt{5}}}{\sqrt{5+x^4}}\,dx-\left(5-2\sqrt{5}\right)\int \frac{1}{\sqrt{5+x^4}}\,dx\\
&= x\sqrt{5+x^4}+\frac{2x\sqrt{5+x^4}}{\sqrt{5}+x^2}-\frac{2\sqrt[4]{5}\left(\sqrt{5}+x^2\right)\sqrt{\frac{5+x^4}{\left(\sqrt{5}+x^2\right)^2}}E\left(2\tan^{-1}\left(\frac{x}{\sqrt[4]{5}}\right)|\frac{1}{2}\right)}{\sqrt{5+x^4}}\\
&\quad+\frac{\sqrt[4]{5}\left(2-\sqrt{5}\right)\left(\sqrt{5}+x^2\right)\sqrt{\frac{5+x^4}{\left(\sqrt{5}+x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{x}{\sqrt[4]{5}}\right)|\frac{1}{2}\right)}{2\sqrt{5+x^4}}
\end{aligned}
$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.04 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.40

$$
\int \frac{x^2(2+3x^2)}{\sqrt{5+x^4}}\,dx = x\sqrt{5+x^4}-\sqrt{5}x\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{5}{4},-\frac{x^4}{5}\right)+\frac{2x^3\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3}{4},\frac{7}{4},-\frac{x^4}{5}\right)}{3\sqrt{5}}
$$

```
[In] Integrate[(x^2*(2 + 3*x^2))/Sqrt[5 + x^4],x]
[Out] x*Sqrt[5 + x^4] - Sqrt[5]*x*Hypergeometric2F1[1/4, 1/2, 5/4, -1/5*x^4] + (2
*x^3*Hypergeometric2F1[1/2, 3/4, 7/4, -1/5*x^4])/(3*Sqrt[5])
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4.

Time = 1.76 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.24

method	result
meijerg	$\frac{3\sqrt{5}\,x^{5}\,{}_{2}F_{1}\left(\frac{1}{2},\frac{5}{4};\frac{9}{4};-\frac{x^{4}}{5}\right)}{25}+\frac{2\sqrt{5}\,x^{3}\,{}_{2}F_{1}\left(\frac{1}{2},\frac{3}{4};\frac{7}{4};-\frac{x^{4}}{5}\right)}{15}$
default	$x\sqrt{x^{4}+5}-\frac{\sqrt{5}\,\sqrt{25-5i\sqrt{5}\,x^{2}}\,\sqrt{25+5i\sqrt{5}\,x^{2}}\,F\left(\frac{x\sqrt{5}\,\sqrt{i\sqrt{5}}}{5},i\right)}{5\sqrt{i\sqrt{5}}\,\sqrt{x^{4}+5}}+\frac{2i\sqrt{25-5i\sqrt{5}\,x^{2}}\,\sqrt{25+5i\sqrt{5}\,x^{2}}\,\left(F\left(\frac{x\sqrt{5}\,\sqrt{i\sqrt{5}}}{5},i\right)-E\left(\frac{x\sqrt{5}}{}\right.\right.}{5\sqrt{i\sqrt{5}}\,\sqrt{x^{4}+5}}$
risch	$x\sqrt{x^{4}+5}-\frac{\sqrt{5}\,\sqrt{25-5i\sqrt{5}\,x^{2}}\,\sqrt{25+5i\sqrt{5}\,x^{2}}\,F\left(\frac{x\sqrt{5}\,\sqrt{i\sqrt{5}}}{5},i\right)}{5\sqrt{i\sqrt{5}}\,\sqrt{x^{4}+5}}+\frac{2i\sqrt{25-5i\sqrt{5}\,x^{2}}\,\sqrt{25+5i\sqrt{5}\,x^{2}}\,\left(F\left(\frac{x\sqrt{5}\,\sqrt{i\sqrt{5}}}{5},i\right)-E\left(\frac{x\sqrt{5}}{}\right.\right.}{5\sqrt{i\sqrt{5}}\,\sqrt{x^{4}+5}}$
elliptic	$x\sqrt{x^{4}+5}-\frac{\sqrt{5}\,\sqrt{25-5i\sqrt{5}\,x^{2}}\,\sqrt{25+5i\sqrt{5}\,x^{2}}\,F\left(\frac{x\sqrt{5}\,\sqrt{i\sqrt{5}}}{5},i\right)}{5\sqrt{i\sqrt{5}}\,\sqrt{x^{4}+5}}+\frac{2i\sqrt{25-5i\sqrt{5}\,x^{2}}\,\sqrt{25+5i\sqrt{5}\,x^{2}}\,\left(F\left(\frac{x\sqrt{5}\,\sqrt{i\sqrt{5}}}{5},i\right)-E\left(\frac{x\sqrt{5}}{}\right.\right.}{5\sqrt{i\sqrt{5}}\,\sqrt{x^{4}+5}}$

```
[In] int(x^2*(3*x^2+2)/(x^4+5)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 3/25*5^(1/2)*x^5*hypergeom([1/2,5/4],[9/4],-1/5*x^4)+2/15*5^(1/2)*x^3*hyper
geom([1/2,3/4],[7/4],-1/5*x^4)
```

Fricas [A] (verification not implemented)

none

Time = 0.08 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.30

$$\int \frac{x^2(2+3x^2)}{\sqrt{5+x^4}}\,dx = \frac{2\,(-5)^{\frac{3}{4}}\,xE(\arcsin\left(\frac{(-5)^{\frac{1}{4}}}{x}\right)\,|-1)-3\,(-5)^{\frac{3}{4}}\,xF(\arcsin\left(\frac{(-5)^{\frac{1}{4}}}{x}\right)\,|-1)+\sqrt{x^4+5}(x^2+2)}{x}$$

```
[In] integrate(x^2*(3*x^2+2)/(x^4+5)^(1/2),x, algorithm="fricas")
```

```
[Out] (2*(-5)^(3/4)*x*elliptic_e(arcsin((-5)^(1/4)/x), -1) - 3*(-5)^(3/4)*x*ellip
tic_f(arcsin((-5)^(1/4)/x), -1) + sqrt(x^4 + 5)*(x^2 + 2))/x
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.05 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.45

$$\int \frac{x^2(2+3x^2)}{\sqrt{5+x^4}}\,dx = \frac{3\sqrt{5}x^5\Gamma\left(\frac{5}{4}\right)\,{}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{5}{4}\\ \frac{9}{4}\end{matrix}\middle|\frac{x^4e^{i\pi}}{5}\right)}{20\Gamma\left(\frac{9}{4}\right)}+\frac{\sqrt{5}x^3\Gamma\left(\frac{3}{4}\right)\,{}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{3}{4}\\ \frac{7}{4}\end{matrix}\middle|\frac{x^4e^{i\pi}}{5}\right)}{10\Gamma\left(\frac{7}{4}\right)}$$

```
[In] integrate(x**2*(3*x**2+2)/(x**4+5)**(1/2),x)
```

```
[Out] 3*sqrt(5)*x**5*gamma(5/4)*hyper((1/2, 5/4), (9/4,), x**4*exp_polar(I*pi)/5)
/(20*gamma(9/4)) + sqrt(5)*x**3*gamma(3/4)*hyper((1/2, 3/4), (7/4,), x**4*e
xp_polar(I*pi)/5)/(10*gamma(7/4))
```

Maxima [F]

$$\int \frac{x^2(2+3x^2)}{\sqrt{5+x^4}}\,dx = \int \frac{(3\,x^2+2)x^2}{\sqrt{x^4+5}}\,dx$$

```
[In] integrate(x^2*(3*x^2+2)/(x^4+5)^(1/2),x, algorithm="maxima")
[Out] integrate((3*x^2 + 2)*x^2/sqrt(x^4 + 5), x)
```

Giac [F]

$$\int \frac{x^2(2+3x^2)}{\sqrt{5+x^4}}\,dx = \int \frac{(3\,x^2+2)x^2}{\sqrt{x^4+5}}\,dx$$

```
[In] integrate(x^2*(3*x^2+2)/(x^4+5)^(1/2),x, algorithm="giac")
[Out] integrate((3*x^2 + 2)*x^2/sqrt(x^4 + 5), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2(2+3x^2)}{\sqrt{5+x^4}}\,dx = \int \frac{x^2\,(3\,x^2+2)}{\sqrt{x^4+5}}\,dx$$

```
[In] int((x^2*(3*x^2 + 2))/(x^4 + 5)^(1/2),x)
[Out] int((x^2*(3*x^2 + 2))/(x^4 + 5)^(1/2), x)
```

3.41 $\int \frac{2+3x^2}{\sqrt{5+x^4}}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 155

$$\int \frac{2+3x^2}{\sqrt{5+x^4}}\,dx=\frac{3x\sqrt{5+x^4}}{\sqrt{5}+x^2}-\frac{3\sqrt[4]{5}(\sqrt{5}+x^2)\sqrt{\frac{5+x^4}{\left(\sqrt{5}+x^2\right)^2}}E\left(2\arctan\left(\frac{x}{\sqrt[4]{5}}\right)|\frac{1}{2}\right)}{\sqrt{5+x^4}}+\frac{\left(2+3\sqrt{5}\right)\left(\sqrt{5}+x^2\right)\sqrt{\frac{5+x^4}{\left(\sqrt{5}+x^2\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{x}{\sqrt[4]{5}}\right),\frac{1}{2}\right)}{2\sqrt[4]{5}\sqrt{5+x^4}}$$

```
[Out] 3*x*(x^4+5)^(1/2)/(x^2+5^(1/2))-3*5^(1/4)*(cos(2*arctan(1/5*x*5^(3/4)))^2)^
(1/2)/cos(2*arctan(1/5*x*5^(3/4)))*EllipticE(sin(2*arctan(1/5*x*5^(3/4))),1
/2*2^(1/2))*(x^2+5^(1/2))*((x^4+5)/(x^2+5^(1/2))^2)^(1/2)/(x^4+5)^(1/2)+1/1
0*(cos(2*arctan(1/5*x*5^(3/4)))^2)^(1/2)/cos(2*arctan(1/5*x*5^(3/4)))*Ellip
ticF(sin(2*arctan(1/5*x*5^(3/4))),1/2*2^(1/2))*(x^2+5^(1/2))*(2+3*5^(1/2))*
((x^4+5)/(x^2+5^(1/2))^2)^(1/2)*5^(3/4)/(x^4+5)^(1/2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {1212, 226, 1210}

$$\int \frac{2+3x^2}{\sqrt{5+x^4}}\,dx=\frac{\left(2+3\sqrt{5}\right)\left(x^2+\sqrt{5}\right)\sqrt{\frac{x^4+5}{\left(x^2+\sqrt{5}\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{x}{\sqrt[4]{5}}\right),\frac{1}{2}\right)}{2\sqrt[4]{5}\sqrt{x^4+5}}-\frac{3\sqrt[4]{5}(x^2+\sqrt{5})\sqrt{\frac{x^4+5}{\left(x^2+\sqrt{5}\right)^2}}E\left(2\arctan\left(\frac{x}{\sqrt[4]{5}}\right)|\frac{1}{2}\right)}{\sqrt{x^4+5}}+\frac{3\sqrt{x^4+5}x}{x^2+\sqrt{5}}$$

```
[In] Int[(2 + 3*x^2)/Sqrt[5 + x^4],x]
```

```
[Out] (3*x*Sqrt[5 + x^4])/(Sqrt[5] + x^2) - (3*5^(1/4)*(Sqrt[5] + x^2)*Sqrt[(5 +
x^4)/(Sqrt[5] + x^2)^2]*EllipticE[2*ArcTan[x/5^(1/4)], 1/2])/Sqrt[5 + x^4]
+ ((2 + 3*Sqrt[5])*(Sqrt[5] + x^2)*Sqrt[(5 + x^4)/(Sqrt[5] + x^2)^2]*Ellipt
icF[2*ArcTan[x/5^(1/4)], 1/2])/(2*5^(1/4)*Sqrt[5 + x^4])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 1210

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Rule 1212

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + c*x^4], x], x] - Dist[e/q, I
nt[(1 - q*x^2)/Sqrt[a + c*x^4], x], x] /; NeQ[e + d*q, 0]] /; FreeQ[{a, c,
d, e}, x] && PosQ[c/a]
```

Rubi steps

$$\text{integral} = -\left(\left(3\sqrt{5}\right)\int \frac{1-\frac{x^2}{\sqrt{5}}}{\sqrt{5+x^4}}\,dx\right) + \left(2+3\sqrt{5}\right)\int \frac{1}{\sqrt{5+x^4}}\,dx$$

$$= \frac{3x\sqrt{5+x^4}}{\sqrt{5}+x^2} - \frac{3\sqrt[4]{5}\left(\sqrt{5}+x^2\right)\sqrt{\frac{5+x^4}{\left(\sqrt{5}+x^2\right)^2}}E\left(2\tan^{-1}\left(\frac{x}{\sqrt[4]{5}}\right)|\frac{1}{2}\right)}{\sqrt{5+x^4}} + \frac{\left(2+3\sqrt{5}\right)\left(\sqrt{5}+x^2\right)\sqrt{\frac{5+x^4}{\left(\sqrt{5}+x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{x}{\sqrt[4]{5}}\right)|\frac{1}{2}\right)}{2\sqrt[4]{5}\sqrt{5+x^4}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.03 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.31

$$\int \frac{2+3x^2}{\sqrt{5+x^4}}\,dx = \frac{x\left(2\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{5}{4},-\frac{x^4}{5}\right)+x^2\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3}{4},\frac{7}{4},-\frac{x^4}{5}\right)\right)}{\sqrt{5}}$$

```
[In] Integrate[(2 + 3*x^2)/Sqrt[5 + x^4],x]
[Out] (x*(2*Hypergeometric2F1[1/4, 1/2, 5/4, -1/5*x^4] + x^2*Hypergeometric2F1[1/
2, 3/4, 7/4, -1/5*x^4]))/Sqrt[5]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4.

Time = 0.81 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.25

method	result	s
meijerg	$\frac{2\sqrt{5}\,x\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};-\frac{x^4}{5}\right)}{5}+\frac{\sqrt{5}\,x^3\,{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{7}{4};-\frac{x^4}{5}\right)}{5}$	3
default	$\frac{2\sqrt{5}\,\sqrt{25-5i\sqrt{5}\,x^2}\,\sqrt{25+5i\sqrt{5}\,x^2}\,F\left(\frac{x\sqrt{5}\,\sqrt{i\sqrt{5}}}{5},i\right)}{25\sqrt{i\sqrt{5}}\,\sqrt{x^4+5}}+\frac{3i\sqrt{25-5i\sqrt{5}\,x^2}\,\sqrt{25+5i\sqrt{5}\,x^2}\,\left(F\left(\frac{x\sqrt{5}\,\sqrt{i\sqrt{5}}}{5},i\right)-E\left(\frac{x\sqrt{5}\,\sqrt{i\sqrt{5}}}{5},i\right)\right)}{5\sqrt{i\sqrt{5}}\,\sqrt{x^4+5}}$	1
elliptic	$\frac{2\sqrt{5}\,\sqrt{25-5i\sqrt{5}\,x^2}\,\sqrt{25+5i\sqrt{5}\,x^2}\,F\left(\frac{x\sqrt{5}\,\sqrt{i\sqrt{5}}}{5},i\right)}{25\sqrt{i\sqrt{5}}\,\sqrt{x^4+5}}+\frac{3i\sqrt{25-5i\sqrt{5}\,x^2}\,\sqrt{25+5i\sqrt{5}\,x^2}\,\left(F\left(\frac{x\sqrt{5}\,\sqrt{i\sqrt{5}}}{5},i\right)-E\left(\frac{x\sqrt{5}\,\sqrt{i\sqrt{5}}}{5},i\right)\right)}{5\sqrt{i\sqrt{5}}\,\sqrt{x^4+5}}$	1

```
[In] int((3*x^2+2)/(x^4+5)^(1/2),x,method=_RETURNVERBOSE)
[Out] 2/5*5^(1/2)*x*hypergeom([1/4,1/2],[5/4],-1/5*x^4)+1/5*5^(1/2)*x^3*hypergeom
([1/2,3/4],[7/4],-1/5*x^4)
```

Fricas [A] (verification not implemented)

none

Time = 0.07 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.30

$$\int \frac{2+3x^2}{\sqrt{5+x^4}}\,dx = \frac{15\,(-5)^{\frac{3}{4}}\,xE(\arcsin\left(\frac{(-5)^{\frac{1}{4}}}{x}\right)\,|-1) - 13\,(-5)^{\frac{3}{4}}\,xF(\arcsin\left(\frac{(-5)^{\frac{1}{4}}}{x}\right)\,|-1) + 15\,\sqrt{x^4+5}}{5\,x}$$

```
[In] integrate((3*x^2+2)/(x^4+5)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/5*(15*(-5)^(3/4)*x*elliptic_e(arcsin((-5)^(1/4)/x), -1) - 13*(-5)^(3/4)*x
*elliptic_f(arcsin((-5)^(1/4)/x), -1) + 15*sqrt(x^4 + 5))/x
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.78 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.47

$$\int \frac{2+3x^2}{\sqrt{5+x^4}}\,dx = \frac{3\sqrt{5}x^3\Gamma\left(\frac{3}{4}\right)\,{}_2F_1\left(\left.\begin{matrix}\frac{1}{2},\frac{3}{4}\\\frac{7}{4}\end{matrix}\right|\frac{x^4e^{i\pi}}{5}\right)}{20\Gamma\left(\frac{7}{4}\right)} + \frac{\sqrt{5}x\Gamma\left(\frac{1}{4}\right)\,{}_2F_1\left(\left.\begin{matrix}\frac{1}{4},\frac{1}{2}\\\frac{5}{4}\end{matrix}\right|\frac{x^4e^{i\pi}}{5}\right)}{10\Gamma\left(\frac{5}{4}\right)}$$

```
[In] integrate((3*x**2+2)/(x**4+5)**(1/2),x)
```

```
[Out] 3*sqrt(5)*x**3*gamma(3/4)*hyper((1/2, 3/4), (7/4,), x**4*exp_polar(I*pi)/5)
/(20*gamma(7/4)) + sqrt(5)*x*gamma(1/4)*hyper((1/4, 1/2), (5/4,), x**4*exp_
polar(I*pi)/5)/(10*gamma(5/4))
```

Maxima [F]

$$\int \frac{2+3x^2}{\sqrt{5+x^4}}\,dx = \int \frac{3\,x^2+2}{\sqrt{x^4+5}}\,dx$$

```
[In] integrate((3*x^2+2)/(x^4+5)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((3*x^2 + 2)/sqrt(x^4 + 5), x)
```

Giac [F]

$$\int \frac{2+3x^2}{\sqrt{5+x^4}}\,dx = \int \frac{3\,x^2+2}{\sqrt{x^4+5}}\,dx$$

```
[In] integrate((3*x^2+2)/(x^4+5)^(1/2),x, algorithm="giac")
[Out] integrate((3*x^2 + 2)/sqrt(x^4 + 5), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{2+3x^2}{\sqrt{5+x^4}}\,dx = \int \frac{3\,x^2+2}{\sqrt{x^4+5}}\,dx$$

```
[In] int((3*x^2 + 2)/(x^4 + 5)^(1/2),x)
[Out] int((3*x^2 + 2)/(x^4 + 5)^(1/2), x)
```

3.42 $\int \frac{2+3x^2}{x^2\sqrt{5+x^4}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 173

$$\int \frac{2+3x^2}{x^2\sqrt{5+x^4}}\,dx = -\frac{2\sqrt{5+x^4}}{5x}+\frac{2x\sqrt{5+x^4}}{5\left(\sqrt{5}+x^2\right)} - \frac{2\left(\sqrt{5}+x^2\right)\sqrt{\frac{5+x^4}{\left(\sqrt{5}+x^2\right)^2}}E\left(2\arctan\left(\frac{x}{\sqrt[4]{5}}\right)|\frac{1}{2}\right)}{5^{3/4}\sqrt{5+x^4}} + \frac{\left(2+3\sqrt{5}\right)\left(\sqrt{5}+x^2\right)\sqrt{\frac{5+x^4}{\left(\sqrt{5}+x^2\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{x}{\sqrt[4]{5}}\right),\frac{1}{2}\right)}{2\ 5^{3/4}\sqrt{5+x^4}}$$

```
[Out] -2/5*(x^4+5)^(1/2)/x+2/5*x*(x^4+5)^(1/2)/(x^2+5^(1/2))-2/5*5^(1/4)*(cos(2*a
rctan(1/5*x*5^(3/4)))^2)^(1/2)/cos(2*arctan(1/5*x*5^(3/4)))*EllipticE(sin(2
*arctan(1/5*x*5^(3/4))),1/2*2^(1/2))*(x^2+5^(1/2))*((x^4+5)/(x^2+5^(1/2))^2
)^(1/2)/(x^4+5)^(1/2)+1/10*(cos(2*arctan(1/5*x*5^(3/4)))^2)^(1/2)/cos(2*arc
tan(1/5*x*5^(3/4)))*EllipticF(sin(2*arctan(1/5*x*5^(3/4))),1/2*2^(1/2))*(x^
2+5^(1/2))*(2+3*5^(1/2))*((x^4+5)/(x^2+5^(1/2))^2)^(1/2)*5^(1/4)/(x^4+5)^(1
/2)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 173, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {1296, 1212, 226, 1210}

$$\int \frac{2+3x^2}{x^2\sqrt{5+x^4}}\,dx = \frac{\left(2+3\sqrt{5}\right)\left(x^2+\sqrt{5}\right)\sqrt{\frac{x^4+5}{\left(x^2+\sqrt{5}\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{x}{\sqrt[4]{5}}\right),\frac{1}{2}\right)}{2\ 5^{3/4}\sqrt{x^4+5}} - \frac{2\left(x^2+\sqrt{5}\right)\sqrt{\frac{x^4+5}{\left(x^2+\sqrt{5}\right)^2}}E\left(2\arctan\left(\frac{x}{\sqrt[4]{5}}\right)|\frac{1}{2}\right)}{5^{3/4}\sqrt{x^4+5}} - \frac{2\sqrt{x^4+5}}{5x} + \frac{2\sqrt{x^4+5}x}{5\left(x^2+\sqrt{5}\right)}$$

```
[In] Int[(2 + 3*x^2)/(x^2*Sqrt[5 + x^4]),x]
```

```
[Out] (-2*Sqrt[5 + x^4])/(5*x) + (2*x*Sqrt[5 + x^4])/(5*(Sqrt[5] + x^2)) - (2*(Sq
rt[5] + x^2)*Sqrt[(5 + x^4)/(Sqrt[5] + x^2)^2]*EllipticE[2*ArcTan[x/5^(1/4)
], 1/2])/(5^(3/4)*Sqrt[5 + x^4]) + ((2 + 3*Sqrt[5])*(Sqrt[5] + x^2)*Sqrt[(5
 + x^4)/(Sqrt[5] + x^2)^2]*EllipticF[2*ArcTan[x/5^(1/4)], 1/2])/(2*5^(3/4)*
Sqrt[5 + x^4])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 1210

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Rule 1212

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + c*x^4], x], x] - Dist[e/q, I
nt[(1 - q*x^2)/Sqrt[a + c*x^4], x], x] /; NeQ[e + d*q, 0]] /; FreeQ[{a, c,
d, e}, x] && PosQ[c/a]
```

Rule 1296

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (c_.)*(x_)^4)^(p_), x_
Symbol] :> Simp[d*(f*x)^(m + 1)*((a + c*x^4)^(p + 1)/(a*f*(m + 1))), x] + D
ist[1/(a*f^2*(m + 1)), Int[(f*x)^(m + 2)*(a + c*x^4)^p*(a*e*(m + 1) - c*d*(
m + 4*p + 5)*x^2), x], x] /; FreeQ[{a, c, d, e, f, p}, x] && LtQ[m, -1] &&
IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{2\sqrt{5+x^4}}{5x} - \frac{1}{5}\int \frac{-15-2x^2}{\sqrt{5+x^4}}\,dx\\ &= -\frac{2\sqrt{5+x^4}}{5x} - \frac{2\int \frac{1-\frac{x^2}{\sqrt{5}}}{\sqrt{5+x^4}}\,dx}{\sqrt{5}} + \frac{1}{5}\left(15+2\sqrt{5}\right)\int \frac{1}{\sqrt{5+x^4}}\,dx\\ &= -\frac{2\sqrt{5+x^4}}{5x} + \frac{2x\sqrt{5+x^4}}{5\left(\sqrt{5}+x^2\right)} - \frac{2\left(\sqrt{5}+x^2\right)\sqrt{\frac{5+x^4}{\left(\sqrt{5}+x^2\right)^2}}E\left(2\tan^{-1}\left(\frac{x}{\sqrt[4]{5}}\right)|\frac{1}{2}\right)}{5^{3/4}\sqrt{5+x^4}}\\ &\quad + \frac{\left(2+3\sqrt{5}\right)\left(\sqrt{5}+x^2\right)\sqrt{\frac{5+x^4}{\left(\sqrt{5}+x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{x}{\sqrt[4]{5}}\right)|\frac{1}{2}\right)}{2\ 5^{3/4}\sqrt{5+x^4}}\end{aligned}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.04 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.31

$$\int \frac{2+3x^2}{x^2\sqrt{5+x^4}}\,dx = -\frac{2\,\text{Hypergeometric2F1}\left(-\frac{1}{4},\frac{1}{2},\frac{3}{4},-\frac{x^4}{5}\right)}{\sqrt{5}x} + \frac{3x\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{5}{4},-\frac{x^4}{5}\right)}{\sqrt{5}}$$

```
[In] Integrate[(2 + 3*x^2)/(x^2*Sqrt[5 + x^4]),x]
```

```
[Out] (-2*Hypergeometric2F1[-1/4, 1/2, 3/4, -1/5*x^4])/(Sqrt[5]*x) + (3*x*Hyperge
ometric2F1[1/4, 1/2, 5/4, -1/5*x^4])/Sqrt[5]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4.

Time = 1.32 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.22

method	result
meijerg	$-\frac{2\sqrt{5}\,{}_2F_1\left(-\frac{1}{4},\frac{1}{2};\frac{3}{4};-\frac{x^4}{5}\right)}{5x}+\frac{3\sqrt{5}\,x\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};-\frac{x^4}{5}\right)}{5}$
default	$\frac{3\sqrt{5}\,\sqrt{25-5i\sqrt{5}\,x^2}\,\sqrt{25+5i\sqrt{5}\,x^2}\,F\left(\frac{x\sqrt{5}\,\sqrt{i\sqrt{5}}}{5},i\right)}{25\sqrt{i\sqrt{5}}\,\sqrt{x^4+5}}-\frac{2\sqrt{x^4+5}}{5x}+\frac{2i\sqrt{25-5i\sqrt{5}\,x^2}\,\sqrt{25+5i\sqrt{5}\,x^2}\,\left(F\left(\frac{x\sqrt{5}\,\sqrt{i\sqrt{5}}}{5},i\right)-E\left(\frac{x\sqrt{5}\,\sqrt{}}{5}\right.\right.}{25\sqrt{i\sqrt{5}}\,\sqrt{x^4+5}}$
risch	$\frac{3\sqrt{5}\,\sqrt{25-5i\sqrt{5}\,x^2}\,\sqrt{25+5i\sqrt{5}\,x^2}\,F\left(\frac{x\sqrt{5}\,\sqrt{i\sqrt{5}}}{5},i\right)}{25\sqrt{i\sqrt{5}}\,\sqrt{x^4+5}}-\frac{2\sqrt{x^4+5}}{5x}+\frac{2i\sqrt{25-5i\sqrt{5}\,x^2}\,\sqrt{25+5i\sqrt{5}\,x^2}\,\left(F\left(\frac{x\sqrt{5}\,\sqrt{i\sqrt{5}}}{5},i\right)-E\left(\frac{x\sqrt{5}\,\sqrt{}}{5}\right.\right.}{25\sqrt{i\sqrt{5}}\,\sqrt{x^4+5}}$
elliptic	$\frac{3\sqrt{5}\,\sqrt{25-5i\sqrt{5}\,x^2}\,\sqrt{25+5i\sqrt{5}\,x^2}\,F\left(\frac{x\sqrt{5}\,\sqrt{i\sqrt{5}}}{5},i\right)}{25\sqrt{i\sqrt{5}}\,\sqrt{x^4+5}}-\frac{2\sqrt{x^4+5}}{5x}+\frac{2i\sqrt{25-5i\sqrt{5}\,x^2}\,\sqrt{25+5i\sqrt{5}\,x^2}\,\left(F\left(\frac{x\sqrt{5}\,\sqrt{i\sqrt{5}}}{5},i\right)-E\left(\frac{x\sqrt{5}\,\sqrt{}}{5}\right.\right.}{25\sqrt{i\sqrt{5}}\,\sqrt{x^4+5}}$

```
[In] int((3*x^2+2)/x^2/(x^4+5)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/5*5^(1/2)/x*hypergeom([-1/4,1/2],[3/4],-1/5*x^4)+3/5*5^(1/2)*x*hypergeom
([1/4,1/2],[5/4],-1/5*x^4)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.08 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.42

$$\int \frac{2+3x^2}{x^2\sqrt{5+x^4}}\,dx = \frac{-2i\sqrt{5}x\sqrt{i\sqrt{5}}E(\arcsin\left(\frac{1}{5}\sqrt{5}x\sqrt{i\sqrt{5}}\right)|-1)-13i\sqrt{5}x\sqrt{i\sqrt{5}}F(\arcsin\left(\frac{1}{5}\sqrt{5}x\sqrt{i\sqrt{5}}\right)|-1)-10\sqrt{x^4}}{25x}$$

```
[In] integrate((3*x^2+2)/x^2/(x^4+5)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/25*(-2*I*sqrt(5)*x*sqrt(I*sqrt(5))*elliptic_e(arcsin(1/5*sqrt(5)*x*sqrt(I
*sqrt(5))), -1) - 13*I*sqrt(5)*x*sqrt(I*sqrt(5))*elliptic_f(arcsin(1/5*sqrt
(5)*x*sqrt(I*sqrt(5))), -1) - 10*sqrt(x^4 + 5))/x
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.81 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.43

$$\int \frac{2+3x^2}{x^2\sqrt{5+x^4}}\,dx = \frac{3\sqrt{5}x\Gamma\left(\frac{1}{4}\right)\,_2F_1\left(\begin{matrix}\frac{1}{4},\frac{1}{2}\\ \frac{5}{4}\end{matrix}\middle|\frac{x^4e^{i\pi}}{5}\right)}{20\Gamma\left(\frac{5}{4}\right)} + \frac{\sqrt{5}\Gamma\left(-\frac{1}{4}\right)\,_2F_1\left(\begin{matrix}-\frac{1}{4},\frac{1}{2}\\ \frac{3}{4}\end{matrix}\middle|\frac{x^4e^{i\pi}}{5}\right)}{10x\Gamma\left(\frac{3}{4}\right)}$$

```
[In] integrate((3*x**2+2)/x**2/(x**4+5)**(1/2),x)
[Out] 3*sqrt(5)*x*gamma(1/4)*hyper((1/4, 1/2), (5/4,), x**4*exp_polar(I*pi)/5)/(2
0*gamma(5/4)) + sqrt(5)*gamma(-1/4)*hyper((-1/4, 1/2), (3/4,), x**4*exp_pol
ar(I*pi)/5)/(10*x*gamma(3/4))
```

Maxima [F]

$$\int \frac{2+3x^2}{x^2\sqrt{5+x^4}}\,dx = \int \frac{3\,x^2+2}{\sqrt{x^4+5}x^2}\,dx$$

```
[In] integrate((3*x^2+2)/x^2/(x^4+5)^(1/2),x, algorithm="maxima")
[Out] integrate((3*x^2 + 2)/(sqrt(x^4 + 5)*x^2), x)
```

Giac [F]

$$\int \frac{2+3x^2}{x^2\sqrt{5+x^4}}\,dx = \int \frac{3\,x^2+2}{\sqrt{x^4+5}x^2}\,dx$$

```
[In] integrate((3*x^2+2)/x^2/(x^4+5)^(1/2),x, algorithm="giac")
[Out] integrate((3*x^2 + 2)/(sqrt(x^4 + 5)*x^2), x)
```

Mupad [B] (verification not implemented)

Time = 8.23 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.28

$$\int \frac{2+3x^2}{x^2\sqrt{5+x^4}}\,dx = \frac{3\sqrt{5}\,x\,{}_2\mathrm{F}_1\left(\frac{1}{4},\frac{1}{2};\ \frac{5}{4};\ -\frac{x^4}{5}\right)}{5} - \frac{2\sqrt{\frac{5}{x^4}+1}\,{}_2\mathrm{F}_1\left(\frac{1}{2},\frac{3}{4};\ \frac{7}{4};\ -\frac{5}{x^4}\right)}{3\,x\,\sqrt{x^4+5}}$$

```
[In] int((3*x^2 + 2)/(x^2*(x^4 + 5)^(1/2)),x)
[Out] (3*5^(1/2)*x*hypergeom([1/4, 1/2], 5/4, -x^4/5))/5 - (2*(5/x^4 + 1)^(1/2)*h
ypergeom([1/2, 3/4], 7/4, -5/x^4))/(3*x*(x^4 + 5)^(1/2))
```

3.43 $\int \frac{2+3x^2}{x^4\sqrt{5+x^4}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 189

$$\int \frac{2+3x^2}{x^4\sqrt{5+x^4}}\,dx = -\frac{2\sqrt{5+x^4}}{15x^3}-\frac{3\sqrt{5+x^4}}{5x}+\frac{3x\sqrt{5+x^4}}{5\left(\sqrt{5}+x^2\right)} - \frac{3\left(\sqrt{5}+x^2\right)\sqrt{\frac{5+x^4}{\left(\sqrt{5}+x^2\right)^2}}E\left(2\arctan\left(\frac{x}{\sqrt[4]{5}}\right)|\frac{1}{2}\right)}{5^{3/4}\sqrt{5+x^4}} - \frac{\left(2-9\sqrt{5}\right)\left(\sqrt{5}+x^2\right)\sqrt{\frac{5+x^4}{\left(\sqrt{5}+x^2\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{x}{\sqrt[4]{5}}\right),\frac{1}{2}\right)}{30\sqrt[4]{5}\sqrt{5+x^4}}$$

```
[Out] -2/15*(x^4+5)^(1/2)/x^3-3/5*(x^4+5)^(1/2)/x+3/5*x*(x^4+5)^(1/2)/(x^2+5^(1/2
))-3/5*5^(1/4)*(cos(2*arctan(1/5*x*5^(3/4)))^2)^(1/2)/cos(2*arctan(1/5*x*5^
(3/4)))*EllipticE(sin(2*arctan(1/5*x*5^(3/4))),1/2*2^(1/2))*(x^2+5^(1/2))*(
(x^4+5)/(x^2+5^(1/2))^2)^(1/2)/(x^4+5)^(1/2)-1/150*(cos(2*arctan(1/5*x*5^(3
/4)))^2)^(1/2)/cos(2*arctan(1/5*x*5^(3/4)))*EllipticF(sin(2*arctan(1/5*x*5^
(3/4))),1/2*2^(1/2))*(2-9*5^(1/2))*(x^2+5^(1/2))*((x^4+5)/(x^2+5^(1/2))^2)^
(1/2)*5^(3/4)/(x^4+5)^(1/2)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 189, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {1296, 1212, 226, 1210}

$$\int \frac{2+3x^2}{x^4\sqrt{5+x^4}}\,dx = -\frac{\left(2-9\sqrt{5}\right)\left(x^2+\sqrt{5}\right)\sqrt{\frac{x^4+5}{\left(x^2+\sqrt{5}\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{x}{\sqrt[4]{5}}\right),\frac{1}{2}\right)}{30\sqrt[4]{5}\sqrt{x^4+5}} - \frac{3\left(x^2+\sqrt{5}\right)\sqrt{\frac{x^4+5}{\left(x^2+\sqrt{5}\right)^2}}E\left(2\arctan\left(\frac{x}{\sqrt[4]{5}}\right)|\frac{1}{2}\right)}{5^{3/4}\sqrt{x^4+5}} - \frac{3\sqrt{x^4+5}}{5x} - \frac{2\sqrt{x^4+5}}{15x^3} + \frac{3\sqrt{x^4+5}x}{5\left(x^2+\sqrt{5}\right)}$$

```
[In] Int[(2 + 3*x^2)/(x^4*Sqrt[5 + x^4]),x]
```

```
[Out] (-2*Sqrt[5 + x^4])/(15*x^3) - (3*Sqrt[5 + x^4])/(5*x) + (3*x*Sqrt[5 + x^4])
/(5*(Sqrt[5] + x^2)) - (3*(Sqrt[5] + x^2)*Sqrt[(5 + x^4)/(Sqrt[5] + x^2)^2]
*EllipticE[2*ArcTan[x/5^(1/4)], 1/2])/(5^(3/4)*Sqrt[5 + x^4]) - ((2 - 9*Sqr
t[5])*(Sqrt[5] + x^2)*Sqrt[(5 + x^4)/(Sqrt[5] + x^2)^2]*EllipticF[2*ArcTan[
x/5^(1/4)], 1/2])/(30*5^(1/4)*Sqrt[5 + x^4])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 1210

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Rule 1212

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + c*x^4], x], x] - Dist[e/q, I
nt[(1 - q*x^2)/Sqrt[a + c*x^4], x], x] /; NeQ[e + d*q, 0]] /; FreeQ[{a, c,
d, e}, x] && PosQ[c/a]
```

Rule 1296

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (c_.)*(x_)^4)^(p_), x_
Symbol] :> Simp[d*(f*x)^(m + 1)*((a + c*x^4)^(p + 1)/(a*f*(m + 1))), x] + D
ist[1/(a*f^2*(m + 1)), Int[(f*x)^(m + 2)*(a + c*x^4)^p*(a*e*(m + 1) - c*d*(
m + 4*p + 5)*x^2), x], x] /; FreeQ[{a, c, d, e, f, p}, x] && LtQ[m, -1] &&
IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{2\sqrt{5+x^4}}{15x^3}-\frac{1}{15}\int \frac{-45+2x^2}{x^2\sqrt{5+x^4}}\,dx \\
&= -\frac{2\sqrt{5+x^4}}{15x^3}-\frac{3\sqrt{5+x^4}}{5x}+\frac{1}{75}\int \frac{-10+45x^2}{\sqrt{5+x^4}}\,dx \\
&= -\frac{2\sqrt{5+x^4}}{15x^3}-\frac{3\sqrt{5+x^4}}{5x}-\frac{3\int \frac{1-\frac{x^2}{\sqrt{5}}}{\sqrt{5+x^4}}\,dx}{\sqrt{5}}+\frac{1}{15}\left(-2+9\sqrt{5}\right)\int \frac{1}{\sqrt{5+x^4}}\,dx \\
&= -\frac{2\sqrt{5+x^4}}{15x^3}-\frac{3\sqrt{5+x^4}}{5x}+\frac{3x\sqrt{5+x^4}}{5\left(\sqrt{5}+x^2\right)} \\
&\quad -\frac{3\left(\sqrt{5}+x^2\right)\sqrt{\frac{5+x^4}{\left(\sqrt{5}+x^2\right)^2}}E\left(2\tan^{-1}\left(\frac{x}{\sqrt[4]{5}}\right)|\frac{1}{2}\right)}{5^{3/4}\sqrt{5+x^4}} \\
&\quad -\frac{\left(2-9\sqrt{5}\right)\left(\sqrt{5}+x^2\right)\sqrt{\frac{5+x^4}{\left(\sqrt{5}+x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{x}{\sqrt[4]{5}}\right)|\frac{1}{2}\right)}{30\sqrt[4]{5}\sqrt{5+x^4}}
\end{aligned}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.03 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.29

$$\int \frac{2+3x^2}{x^4\sqrt{5+x^4}}\,dx = -\frac{2\,\text{Hypergeometric2F1}\left(-\frac{3}{4},\frac{1}{2},\frac{1}{4},-\frac{x^4}{5}\right)+9x^2\,\text{Hypergeometric2F1}\left(-\frac{1}{4},\frac{1}{2},\frac{3}{4},-\frac{x^4}{5}\right)}{3\sqrt{5}x^3}$$

```
[In] Integrate[(2 + 3*x^2)/(x^4*Sqrt[5 + x^4]),x]
[Out] -1/3*(2*Hypergeometric2F1[-3/4, 1/2, 1/4, -1/5*x^4] + 9*x^2*Hypergeometric2
F1[-1/4, 1/2, 3/4, -1/5*x^4])/(Sqrt[5]*x^3)
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4.

Time = 1.99 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.21

method	result
meijerg	$-\frac{2\sqrt{5}\,{}_2F_1\left(-\frac{3}{4},\frac{1}{2};\frac{1}{4};-\frac{x^4}{5}\right)}{15x^3}-\frac{3\sqrt{5}\,{}_2F_1\left(-\frac{1}{4},\frac{1}{2};\frac{3}{4};-\frac{x^4}{5}\right)}{5x}$
default	$-\frac{3\sqrt{x^4+5}}{5x}+\frac{3i\sqrt{25-5i\sqrt{5}\,x^2}\,\sqrt{25+5i\sqrt{5}\,x^2}\left(F\left(\frac{x\sqrt{5}\,\sqrt{i\sqrt{5}}}{5},i\right)-E\left(\frac{x\sqrt{5}\,\sqrt{i\sqrt{5}}}{5},i\right)\right)}{25\sqrt{i\sqrt{5}}\,\sqrt{x^4+5}}-\frac{2\sqrt{x^4+5}}{15x^3}-\frac{2\sqrt{5}\,\sqrt{25-5i\sqrt{5}\,x^2}\,\sqrt{25+5}}{375\sqrt{i\sqrt{5}}}$
elliptic	$-\frac{3\sqrt{x^4+5}}{5x}+\frac{3i\sqrt{25-5i\sqrt{5}\,x^2}\,\sqrt{25+5i\sqrt{5}\,x^2}\left(F\left(\frac{x\sqrt{5}\,\sqrt{i\sqrt{5}}}{5},i\right)-E\left(\frac{x\sqrt{5}\,\sqrt{i\sqrt{5}}}{5},i\right)\right)}{25\sqrt{i\sqrt{5}}\,\sqrt{x^4+5}}-\frac{2\sqrt{x^4+5}}{15x^3}-\frac{2\sqrt{5}\,\sqrt{25-5i\sqrt{5}\,x^2}\,\sqrt{25+5}}{375\sqrt{i\sqrt{5}}}$
risch	$-\frac{9x^6+2x^4+45x^2+10}{15x^3\sqrt{x^4+5}}-\frac{2\sqrt{5}\,\sqrt{25-5i\sqrt{5}\,x^2}\,\sqrt{25+5i\sqrt{5}\,x^2}\,F\left(\frac{x\sqrt{5}\,\sqrt{i\sqrt{5}}}{5},i\right)}{375\sqrt{i\sqrt{5}}\,\sqrt{x^4+5}}+\frac{3i\sqrt{25-5i\sqrt{5}\,x^2}\,\sqrt{25+5i\sqrt{5}\,x^2}\left(F\left(\frac{x\sqrt{5}\,\sqrt{i\sqrt{5}}}{5},i\right.\right.}{25\sqrt{i\sqrt{5}}\,\sqrt{x^4+5}}$

```
[In] int((3*x^2+2)/x^4/(x^4+5)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/15*5^(1/2)/x^3*hypergeom([-3/4,1/2],[1/4],-1/5*x^4)-3/5*5^(1/2)/x*hyperg
eom([-1/4,1/2],[3/4],-1/5*x^4)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.08 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.44

$$\int \frac{2+3x^2}{x^4\sqrt{5+x^4}}\,dx = \frac{-9i\,\sqrt{5}x^3\sqrt{i\,\sqrt{5}}E(\arcsin\left(\frac{1}{5}\,\sqrt{5}x\sqrt{i\,\sqrt{5}}\right)|-1)+11i\,\sqrt{5}x^3\sqrt{i\,\sqrt{5}}F(\arcsin\left(\frac{1}{5}\,\sqrt{5}x\sqrt{i\,\sqrt{5}}\right)|-1)-5\,\sqrt{x}}{75\,x^3}$$

```
[In] integrate((3*x^2+2)/x^4/(x^4+5)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/75*(-9*I*sqrt(5)*x^3*sqrt(I*sqrt(5))*elliptic_e(arcsin(1/5*sqrt(5)*x*sqrt
(I*sqrt(5))), -1) + 11*I*sqrt(5)*x^3*sqrt(I*sqrt(5))*elliptic_f(arcsin(1/5*
sqrt(5)*x*sqrt(I*sqrt(5))), -1) - 5*sqrt(x^4 + 5)*(9*x^2 + 2))/x^3
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.99 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.42

$$\int \frac{2+3x^2}{x^4\sqrt{5+x^4}}\,dx = \frac{3\sqrt{5}\Gamma\left(-\frac{1}{4}\right){}_2F_1\left(\begin{matrix} -\frac{1}{4},\frac{1}{2} \\ \frac{3}{4} \end{matrix}\middle| \frac{x^4e^{i\pi}}{5}\right)}{20x\Gamma\left(\frac{3}{4}\right)} + \frac{\sqrt{5}\Gamma\left(-\frac{3}{4}\right){}_2F_1\left(\begin{matrix} -\frac{3}{4},\frac{1}{2} \\ \frac{1}{4} \end{matrix}\middle| \frac{x^4e^{i\pi}}{5}\right)}{10x^3\Gamma\left(\frac{1}{4}\right)}$$

```
[In] integrate((3*x**2+2)/x**4/(x**4+5)**(1/2),x)
[Out] 3*sqrt(5)*gamma(-1/4)*hyper((-1/4, 1/2), (3/4,), x**4*exp_polar(I*pi)/5)/(2
0*x*gamma(3/4)) + sqrt(5)*gamma(-3/4)*hyper((-3/4, 1/2), (1/4,), x**4*exp_p
olar(I*pi)/5)/(10*x**3*gamma(1/4))
```

Maxima [F]

$$\int \frac{2+3x^2}{x^4\sqrt{5+x^4}}\,dx = \int \frac{3\,x^2+2}{\sqrt{x^4+5}x^4}\,dx$$

```
[In] integrate((3*x^2+2)/x^4/(x^4+5)^(1/2),x, algorithm="maxima")
[Out] integrate((3*x^2 + 2)/(sqrt(x^4 + 5)*x^4), x)
```

Giac [F]

$$\int \frac{2+3x^2}{x^4\sqrt{5+x^4}}\,dx = \int \frac{3\,x^2+2}{\sqrt{x^4+5}x^4}\,dx$$

```
[In] integrate((3*x^2+2)/x^4/(x^4+5)^(1/2),x, algorithm="giac")
[Out] integrate((3*x^2 + 2)/(sqrt(x^4 + 5)*x^4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{2+3x^2}{x^4\sqrt{5+x^4}}\,dx = \int \frac{3\,x^2+2}{x^4\,\sqrt{x^4+5}}\,dx$$

```
[In] int((3*x^2 + 2)/(x^4*(x^4 + 5)^(1/2)),x)
[Out] int((3*x^2 + 2)/(x^4*(x^4 + 5)^(1/2)), x)
```

3.44 $\int \frac{x^7(2+3x^2)}{(5+x^4)^{3/2}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 58

$$\int \frac{x^7(2+3x^2)}{(5+x^4)^{3/2}}\,dx = -\frac{x^4(2+3x^2)}{2\sqrt{5+x^4}} + \frac{1}{4}(8+9x^2)\sqrt{5+x^4} - \frac{45}{4}\operatorname{arcsinh}\left(\frac{x^2}{\sqrt{5}}\right)$$

```
[Out] -45/4*arcsinh(1/5*x^2*5^(1/2))-1/2*x^4*(3*x^2+2)/(x^4+5)^(1/2)+1/4*(9*x^2+8
)*(x^4+5)^(1/2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {1266, 833, 794, 221}

$$\int \frac{x^7(2+3x^2)}{(5+x^4)^{3/2}}\,dx = -\frac{45}{4}\operatorname{arcsinh}\left(\frac{x^2}{\sqrt{5}}\right) - \frac{(3x^2+2)\,x^4}{2\sqrt{x^4+5}} + \frac{1}{4}(9x^2+8)\sqrt{x^4+5}$$

```
[In] Int[(x^7*(2 + 3*x^2))/(5 + x^4)^(3/2),x]
```

```
[Out] -1/2*(x^4*(2 + 3*x^2))/Sqrt[5 + x^4] + ((8 + 9*x^2)*Sqrt[5 + x^4])/4 - (45*
ArcSinh[x^2/Sqrt[5]])/4
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 794

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*((a + c*x^2)^(p
+ 1)/(2*c*(p + 1)*(2*p + 3))), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rule 833

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1)*((a*(e*f + d*g
) - (c*d*f - a*e*g)*x)/(2*a*c*(p + 1))), x] - Dist[1/(2*a*c*(p + 1)), Int[(
d + e*x)^(m - 2)*(a + c*x^2)^(p + 1)*Simp[a*e*(e*f*(m - 1) + d*g*m) - c*d^2
*f*(2*p + 3) + e*(a*e*g*m - c*d*f*(m + 2*p + 2))*x, x], x], x] /; FreeQ[{a,
 c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 1] &&
(EqQ[d, 0] || (EqQ[m, 2] && EqQ[p, -3] && RationalQ[a, c, d, e, f, g]) ||
!ILtQ[m + 2*p + 3, 0])
```

Rule 1266

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_S
ymbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + c*x^2)^p, x],
x, x^2], x] /; FreeQ[{a, c, d, e, p, q}, x] && IntegerQ[(m + 1)/2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{2}\text{Subst}\left(\int \frac{x^3(2+3x)}{\left(5+x^2\right)^{3/2}}\,dx, x, x^2\right)\\
&= -\frac{x^4(2+3x^2)}{2\sqrt{5+x^4}} + \frac{1}{10}\text{Subst}\left(\int \frac{x(20+45x)}{\sqrt{5+x^2}}\,dx, x, x^2\right)\\
&= -\frac{x^4(2+3x^2)}{2\sqrt{5+x^4}} + \frac{1}{4}\left(8+9x^2\right)\sqrt{5+x^4} - \frac{45}{4}\text{Subst}\left(\int \frac{1}{\sqrt{5+x^2}}\,dx, x, x^2\right)\\
&= -\frac{x^4(2+3x^2)}{2\sqrt{5+x^4}} + \frac{1}{4}\left(8+9x^2\right)\sqrt{5+x^4} - \frac{45}{4}\sinh^{-1}\left(\frac{x^2}{\sqrt{5}}\right)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.88

$$\int \frac{x^7(2+3x^2)}{(5+x^4)^{3/2}}\,dx = \frac{40+45x^2+4x^4+3x^6}{4\sqrt{5+x^4}} + \frac{45}{4}\log\left(-x^2+\sqrt{5+x^4}\right)$$

```
[In] Integrate[(x^7*(2 + 3*x^2))/(5 + x^4)^(3/2),x]
[Out] (40 + 45*x^2 + 4*x^4 + 3*x^6)/(4*Sqrt[5 + x^4]) + (45*Log[-x^2 + Sqrt[5 + x
^4]])/4
```

Maple [A] (verified)

Time = 0.30 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.67

method	result	size
risch	$\frac{3x^6+4x^4+45x^2+40}{4\sqrt{x^4+5}}-\frac{45\,\mathrm{arcsinh}\left(\frac{x^2\sqrt{5}}{5}\right)}{4}$	39
trager	$\frac{3x^6+4x^4+45x^2+40}{4\sqrt{x^4+5}}+\frac{45\ln\left(x^2-\sqrt{x^4+5}\right)}{4}$	44
pseudoelliptic	$\frac{3x^6+4x^4-45\,\mathrm{arcsinh}\left(\frac{x^2\sqrt{5}}{5}\right)\sqrt{x^4+5}+45x^2+40}{4\sqrt{x^4+5}}$	45
default	$\frac{3x^6}{4\sqrt{x^4+5}}+\frac{45x^2}{4\sqrt{x^4+5}}-\frac{45\,\mathrm{arcsinh}\left(\frac{x^2\sqrt{5}}{5}\right)}{4}+\frac{x^4+10}{\sqrt{x^4+5}}$	50
elliptic	$\frac{3x^6}{4\sqrt{x^4+5}}+\frac{45x^2}{4\sqrt{x^4+5}}-\frac{45\,\mathrm{arcsinh}\left(\frac{x^2\sqrt{5}}{5}\right)}{4}+\frac{x^4}{\sqrt{x^4+5}}+\frac{10}{\sqrt{x^4+5}}$	57
meijerg	$\frac{\frac{3\sqrt{\pi}\,x^2\sqrt{5}\left(x^4+15\right)}{20\sqrt{1+\frac{x^4}{5}}}-\frac{45\sqrt{\pi}\,\mathrm{arcsinh}\left(\frac{x^2\sqrt{5}}{5}\right)}{4}}{\sqrt{\pi}}+\frac{\sqrt{5}\left(-2\sqrt{\pi}+\frac{\sqrt{\pi}\left(8+\frac{4x^4}{5}\right)}{4\sqrt{1+\frac{x^4}{5}}}\right)}{\sqrt{\pi}}$	81

```
[In] int(x^7*(3*x^2+2)/(x^4+5)^(3/2),x,method=_RETURNVERBOSE)
[Out] 1/4*(3*x^6+4*x^4+45*x^2+40)/(x^4+5)^(1/2)-45/4*arcsinh(1/5*x^2*5^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.07

$$\int \frac{x^7(2+3x^2)}{(5+x^4)^{3/2}}\,dx = \frac{30\,x^4+45\,(x^4+5)\log\left(-x^2+\sqrt{x^4+5}\right)+(3\,x^6+4\,x^4+45\,x^2+40)\sqrt{x^4+5}+150}{4\,(x^4+5)}$$

```
[In] integrate(x^7*(3*x^2+2)/(x^4+5)^(3/2),x, algorithm="fricas")
[Out] 1/4*(30*x^4 + 45*(x^4 + 5)*log(-x^2 + sqrt(x^4 + 5)) + (3*x^6 + 4*x^4 + 45*
x^2 + 40)*sqrt(x^4 + 5) + 150)/(x^4 + 5)
```

Sympy [A] (verification not implemented)

Time = 6.47 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.14

$$\int \frac{x^7(2+3x^2)}{(5+x^4)^{3/2}}\,dx = \frac{3x^6}{4\sqrt{x^4+5}}+\frac{x^4}{\sqrt{x^4+5}}+\frac{45x^2}{4\sqrt{x^4+5}}-\frac{45\operatorname{asinh}\left(\frac{\sqrt{5}x^2}{5}\right)}{4}+\frac{10}{\sqrt{x^4+5}}$$

```
[In] integrate(x**7*(3*x**2+2)/(x**4+5)**(3/2),x)
[Out] 3*x**6/(4*sqrt(x**4 + 5)) + x**4/sqrt(x**4 + 5) + 45*x**2/(4*sqrt(x**4 + 5)
) - 45*asinh(sqrt(5)*x**2/5)/4 + 10/sqrt(x**4 + 5)
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.53

$$\int \frac{x^7(2+3x^2)}{(5+x^4)^{3/2}}\,dx = \sqrt{x^4+5}-\frac{15\left(\frac{3\,(x^4+5)}{x^4}-2\right)}{4\left(\frac{\sqrt{x^4+5}}{x^2}-\frac{(x^4+5)^{\frac{3}{2}}}{x^6}\right)}+\frac{5}{\sqrt{x^4+5}} -\frac{45}{8}\log\left(\frac{\sqrt{x^4+5}}{x^2}+1\right)+\frac{45}{8}\log\left(\frac{\sqrt{x^4+5}}{x^2}-1\right)$$

```
[In] integrate(x^7*(3*x^2+2)/(x^4+5)^(3/2),x, algorithm="maxima")
[Out] sqrt(x^4 + 5) - 15/4*(3*(x^4 + 5)/x^4 - 2)/(sqrt(x^4 + 5)/x^2 - (x^4 + 5)^(
3/2)/x^6) + 5/sqrt(x^4 + 5) - 45/8*log(sqrt(x^4 + 5)/x^2 + 1) + 45/8*log(sq
rt(x^4 + 5)/x^2 - 1)
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.78

$$\int \frac{x^7(2+3x^2)}{(5+x^4)^{3/2}}\,dx = \frac{((3\,x^2+4)x^2+45)x^2+40}{4\,\sqrt{x^4+5}} + \frac{45}{4}\,\log\left(-x^2+\sqrt{x^4+5}\right)$$

```
[In] integrate(x^7*(3*x^2+2)/(x^4+5)^(3/2),x, algorithm="giac")
```

```
[Out] 1/4*(((3*x^2 + 4)*x^2 + 45)*x^2 + 40)/sqrt(x^4 + 5) + 45/4*log(-x^2 + sqrt(
x^4 + 5))
```

Mupad [B] (verification not implemented)

Time = 8.23 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.67

$$\int \frac{x^7(2+3x^2)}{(5+x^4)^{3/2}}\,dx = \sqrt{x^4+5}\left(\frac{3\,x^2}{4}+1\right) - \frac{45\,\mathrm{asinh}\left(\frac{\sqrt{5}\,x^2}{5}\right)}{4} + \frac{\sqrt{5}\,\left(10+\sqrt{5}\,15\mathrm{i}\right)\,\sqrt{x^4+5}\,1\mathrm{i}}{20\,\left(-x^2+\sqrt{5}\,1\mathrm{i}\right)} - \frac{\sqrt{5}\,\left(-10+\sqrt{5}\,15\mathrm{i}\right)\,\sqrt{x^4+5}\,1\mathrm{i}}{20\,\left(x^2+\sqrt{5}\,1\mathrm{i}\right)}$$

```
[In] int((x^7*(3*x^2 + 2))/(x^4 + 5)^(3/2),x)
```

```
[Out] (x^4 + 5)^(1/2)*((3*x^2)/4 + 1) - (45*asinh((5^(1/2)*x^2)/5))/4 + (5^(1/2)*
(5^(1/2)*15i + 10)*(x^4 + 5)^(1/2)*1i)/(20*(5^(1/2)*1i - x^2)) - (5^(1/2)*(
5^(1/2)*15i - 10)*(x^4 + 5)^(1/2)*1i)/(20*(5^(1/2)*1i + x^2))
```

3.45 $\int \frac{x^5(2+3x^2)}{(5+x^4)^{3/2}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 45

$$\int \frac{x^5(2+3x^2)}{(5+x^4)^{3/2}}\,dx=-\frac{x^2(2+3x^2)}{2\sqrt{5+x^4}}+3\sqrt{5+x^4}+\operatorname{arcsinh}\left(\frac{x^2}{\sqrt{5}}\right)$$

```
[Out] arcsinh(1/5*x^2*5^(1/2))-1/2*x^2*(3*x^2+2)/(x^4+5)^(1/2)+3*(x^4+5)^(1/2)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {1266, 833, 655, 221}

$$\int \frac{x^5(2+3x^2)}{(5+x^4)^{3/2}}\,dx=\operatorname{arcsinh}\left(\frac{x^2}{\sqrt{5}}\right)+3\sqrt{x^4+5}-\frac{(3x^2+2)\,x^2}{2\sqrt{x^4+5}}$$

```
[In] Int[(x^5*(2 + 3*x^2))/(5 + x^4)^(3/2),x]
```

```
[Out] -1/2*(x^2*(2 + 3*x^2))/Sqrt[5 + x^4] + 3*Sqrt[5 + x^4] + ArcSinh[x^2/Sqrt[5
]]
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 655

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[e*((
a + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
```

```
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 833

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1)*((a*(e*f + d*g
) - (c*d*f - a*e*g)*x)/(2*a*c*(p + 1))), x] - Dist[1/(2*a*c*(p + 1)), Int[(
d + e*x)^(m - 2)*(a + c*x^2)^(p + 1)*Simp[a*e*(e*f*(m - 1) + d*g*m) - c*d^2
*f*(2*p + 3) + e*(a*e*g*m - c*d*f*(m + 2*p + 2))*x, x], x], x] /; FreeQ[{a,
 c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 1] &&
(EqQ[d, 0] || (EqQ[m, 2] && EqQ[p, -3] && RationalQ[a, c, d, e, f, g]) ||
!ILtQ[m + 2*p + 3, 0])
```

Rule 1266

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_S
ymbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + c*x^2)^p, x],
x, x^2], x] /; FreeQ[{a, c, d, e, p, q}, x] && IntegerQ[(m + 1)/2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{2}\text{Subst}\left(\int \frac{x^2(2+3x)}{(5+x^2)^{3/2}}\,dx, x, x^2\right) \\
&= -\frac{x^2(2+3x^2)}{2\sqrt{5+x^4}} + \frac{1}{10}\text{Subst}\left(\int \frac{10+30x}{\sqrt{5+x^2}}\,dx, x, x^2\right) \\
&= -\frac{x^2(2+3x^2)}{2\sqrt{5+x^4}} + 3\sqrt{5+x^4} + \text{Subst}\left(\int \frac{1}{\sqrt{5+x^2}}\,dx, x, x^2\right) \\
&= -\frac{x^2(2+3x^2)}{2\sqrt{5+x^4}} + 3\sqrt{5+x^4} + \sinh^{-1}\left(\frac{x^2}{\sqrt{5}}\right)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.98

$$\int \frac{x^5(2+3x^2)}{(5+x^4)^{3/2}}\,dx = \frac{30-2x^2+3x^4}{2\sqrt{5+x^4}} - \log\left(-x^2+\sqrt{5+x^4}\right)$$

```
[In] Integrate[(x^5*(2 + 3*x^2))/(5 + x^4)^(3/2),x]
[Out] (30 - 2*x^2 + 3*x^4)/(2*Sqrt[5 + x^4]) - Log[-x^2 + Sqrt[5 + x^4]]
```

Maple [A] (verified)

Time = 0.30 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.71

method	result	size
risch	$\frac{3x^4-2x^2+30}{2\sqrt{x^4+5}}+\operatorname{arcsinh}\left(\frac{x^2\sqrt{5}}{5}\right)$	32
trager	$\frac{3x^4-2x^2+30}{2\sqrt{x^4+5}}+\ln\left(x^2+\sqrt{x^4+5}\right)$	35
default	$\frac{\frac{3x^4}{2}+15}{\sqrt{x^4+5}}-\frac{x^2}{\sqrt{x^4+5}}+\operatorname{arcsinh}\left(\frac{x^2\sqrt{5}}{5}\right)$	37
pseudoelliptic	$\frac{3x^4+2\operatorname{arcsinh}\left(\frac{x^2\sqrt{5}}{5}\right)\sqrt{x^4+5}-2x^2+30}{2\sqrt{x^4+5}}$	40
elliptic	$\frac{3x^4}{2\sqrt{x^4+5}}+\frac{15}{\sqrt{x^4+5}}-\frac{x^2}{\sqrt{x^4+5}}+\operatorname{arcsinh}\left(\frac{x^2\sqrt{5}}{5}\right)$	44
meijerg	$\frac{3\sqrt{5}\left(-2\sqrt{\pi}+\frac{\sqrt{\pi}\left(8+\frac{4x^4}{5}\right)}{4\sqrt{1+\frac{x^4}{5}}}\right)}{2\sqrt{\pi}}+\frac{-\frac{\sqrt{\pi}\,x^2\sqrt{5}}{5\sqrt{1+\frac{x^4}{5}}}+\sqrt{\pi}\operatorname{arcsinh}\left(\frac{x^2\sqrt{5}}{5}\right)}{\sqrt{\pi}}$	75

```
[In] int(x^5*(3*x^2+2)/(x^4+5)^(3/2),x,method=_RETURNVERBOSE)
[Out] 1/2*(3*x^4-2*x^2+30)/(x^4+5)^(1/2)+arcsinh(1/5*x^2*5^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.29

$$\int \frac{x^5(2+3x^2)}{(5+x^4)^{3/2}}\,dx=-\frac{2\,x^4+2\,(x^4+5)\log\left(-x^2+\sqrt{x^4+5}\right)-(3\,x^4-2\,x^2+30)\sqrt{x^4+5}+10}{2\,(x^4+5)}$$

```
[In] integrate(x^5*(3*x^2+2)/(x^4+5)^(3/2),x, algorithm="fricas")
[Out] -1/2*(2*x^4 + 2*(x^4 + 5)*log(-x^2 + sqrt(x^4 + 5)) - (3*x^4 - 2*x^2 + 30)*
sqrt(x^4 + 5) + 10)/(x^4 + 5)
```

Sympy [A] (verification not implemented)

Time = 5.01 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.07

$$\int \frac{x^5(2+3x^2)}{(5+x^4)^{3/2}}\,dx=\frac{3x^4}{2\sqrt{x^4+5}}-\frac{x^2}{\sqrt{x^4+5}}+\operatorname{asinh}\left(\frac{\sqrt{5}x^2}{5}\right)+\frac{15}{\sqrt{x^4+5}}$$

```
[In] integrate(x**5*(3*x**2+2)/(x**4+5)**(3/2),x)
[Out] 3*x**4/(2*sqrt(x**4 + 5)) - x**2/sqrt(x**4 + 5) + asinh(sqrt(5)*x**2/5) + 1
5/sqrt(x**4 + 5)
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.40

$$\int \frac{x^5(2+3x^2)}{(5+x^4)^{3/2}}\,dx = -\frac{x^2}{\sqrt{x^4+5}} + \frac{3}{2}\sqrt{x^4+5} + \frac{15}{2\sqrt{x^4+5}} + \frac{1}{2}\log\left(\frac{\sqrt{x^4+5}}{x^2}+1\right) - \frac{1}{2}\log\left(\frac{\sqrt{x^4+5}}{x^2}-1\right)$$

```
[In] integrate(x^5*(3*x^2+2)/(x^4+5)^(3/2),x, algorithm="maxima")
```

```
[Out] -x^2/sqrt(x^4 + 5) + 3/2*sqrt(x^4 + 5) + 15/2/sqrt(x^4 + 5) + 1/2*log(sqrt(
x^4 + 5)/x^2 + 1) - 1/2*log(sqrt(x^4 + 5)/x^2 - 1)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.87

$$\int \frac{x^5(2+3x^2)}{(5+x^4)^{3/2}}\,dx = \frac{(3x^2-2)x^2+30}{2\sqrt{x^4+5}} - \log\left(-x^2+\sqrt{x^4+5}\right)$$

```
[In] integrate(x^5*(3*x^2+2)/(x^4+5)^(3/2),x, algorithm="giac")
```

```
[Out] 1/2*((3*x^2 - 2)*x^2 + 30)/sqrt(x^4 + 5) - log(-x^2 + sqrt(x^4 + 5))
```

Mupad [B] (verification not implemented)

Time = 8.12 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.98

$$\int \frac{x^5(2+3x^2)}{(5+x^4)^{3/2}}\,dx = \operatorname{asinh}\left(\frac{\sqrt{5}\,x^2}{5}\right) + \frac{3\sqrt{x^4+5}}{2} - \frac{\sqrt{5}\left(-15+\sqrt{5}\,2\mathrm{i}\right)\sqrt{x^4+5}\,1\mathrm{i}}{20\left(-x^2+\sqrt{5}\,1\mathrm{i}\right)} + \frac{\sqrt{5}\left(15+\sqrt{5}\,2\mathrm{i}\right)\sqrt{x^4+5}\,1\mathrm{i}}{20\left(x^2+\sqrt{5}\,1\mathrm{i}\right)}$$

```
[In] int((x^5*(3*x^2 + 2))/(x^4 + 5)^(3/2),x)
```

```
[Out] asinh((5^(1/2)*x^2)/5) + (3*(x^4 + 5)^(1/2))/2 - (5^(1/2)*(5^(1/2)*2i - 15)
*(x^4 + 5)^(1/2)*1i)/(20*(5^(1/2)*1i - x^2)) + (5^(1/2)*(5^(1/2)*2i + 15)*(
x^4 + 5)^(1/2)*1i)/(20*(5^(1/2)*1i + x^2))
```

3.46 $\int \frac{x^3(2+3x^2)}{(5+x^4)^{3/2}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 35

$$\int \frac{x^3(2+3x^2)}{(5+x^4)^{3/2}}\,dx = \frac{-2-3x^2}{2\sqrt{5+x^4}} + \frac{3}{2}\operatorname{arcsinh}\left(\frac{x^2}{\sqrt{5}}\right)$$

```
[Out] 3/2*arcsinh(1/5*x^2*5^(1/2))+1/2*(-3*x^2-2)/(x^4+5)^(1/2)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {1266, 792, 221}

$$\int \frac{x^3(2+3x^2)}{(5+x^4)^{3/2}}\,dx = \frac{3}{2}\operatorname{arcsinh}\left(\frac{x^2}{\sqrt{5}}\right) - \frac{3x^2+2}{2\sqrt{x^4+5}}$$

```
[In] Int[(x^3*(2 + 3*x^2))/(5 + x^4)^(3/2),x]
```

```
[Out] -1/2*(2 + 3*x^2)/Sqrt[5 + x^4] + (3*ArcSinh[x^2/Sqrt[5]])/2
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 792

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(a*(e*f + d*g) - (c*d*f - a*e*g)*x)*((a + c*x^2)^(p + 1)/(
2*a*c*(p + 1))), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(2*a*c*(p + 1)), Int[(
```

```
a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && LtQ[p, -1]
```

Rule 1266

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_S
ymbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + c*x^2)^p, x],
x, x^2], x] /; FreeQ[{a, c, d, e, p, q}, x] && IntegerQ[(m + 1)/2]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{1}{2}\text{Subst}\left(\int \frac{x(2+3x)}{(5+x^2)^{3/2}}\,dx, x, x^2\right)\\ &= -\frac{2+3x^2}{2\sqrt{5+x^4}} + \frac{3}{2}\text{Subst}\left(\int \frac{1}{\sqrt{5+x^2}}\,dx, x, x^2\right)\\ &= -\frac{2+3x^2}{2\sqrt{5+x^4}} + \frac{3}{2}\sinh^{-1}\left(\frac{x^2}{\sqrt{5}}\right)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.17

$$\int \frac{x^3(2+3x^2)}{(5+x^4)^{3/2}}\,dx = \frac{-2-3x^2}{2\sqrt{5+x^4}} - \frac{3}{2}\log\left(-x^2+\sqrt{5+x^4}\right)$$

```
[In] Integrate[(x^3*(2 + 3*x^2))/(5 + x^4)^(3/2),x]
[Out] (-2 - 3*x^2)/(2*Sqrt[5 + x^4]) - (3*Log[-x^2 + Sqrt[5 + x^4]])/2
```

Maple [A] (verified)

Time = 0.29 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.83

method	result	size
risch	$-\frac{3x^2+2}{2\sqrt{x^4+5}}+\frac{3\,\mathrm{arcsinh}\left(\frac{x^2\sqrt{5}}{5}\right)}{2}$	29
trager	$-\frac{3x^2+2}{2\sqrt{x^4+5}}+\frac{3\ln\left(x^2+\sqrt{x^4+5}\right)}{2}$	32
pseudoelliptic	$-\frac{3\left(x^2-\mathrm{arcsinh}\left(\frac{x^2\sqrt{5}}{5}\right)\sqrt{x^4+5}+\frac{2}{3}\right)}{2\sqrt{x^4+5}}$	33
default	$-\frac{3x^2}{2\sqrt{x^4+5}}+\frac{3\,\mathrm{arcsinh}\left(\frac{x^2\sqrt{5}}{5}\right)}{2}-\frac{1}{\sqrt{x^4+5}}$	34
elliptic	$-\frac{3x^2}{2\sqrt{x^4+5}}+\frac{3\,\mathrm{arcsinh}\left(\frac{x^2\sqrt{5}}{5}\right)}{2}-\frac{1}{\sqrt{x^4+5}}$	34
meijerg	$\frac{-\frac{3\sqrt{\pi}\,x^2\sqrt{5}}{10\sqrt{1+\frac{x^4}{5}}}+\frac{3\sqrt{\pi}\,\mathrm{arcsinh}\left(\frac{x^2\sqrt{5}}{5}\right)}{2}}{\sqrt{\pi}}+\frac{\sqrt{5}\left(\sqrt{\pi}-\frac{\sqrt{\pi}}{\sqrt{1+\frac{x^4}{5}}}\right)}{5\sqrt{\pi}}$	67

```
[In] int(x^3*(3*x^2+2)/(x^4+5)^(3/2),x,method=_RETURNVERBOSE)
[Out] -1/2*(3*x^2+2)/(x^4+5)^(1/2)+3/2*arcsinh(1/5*x^2*5^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.49

$$\int \frac{x^3(2+3x^2)}{(5+x^4)^{3/2}}\,dx = -\frac{3\,x^4+3\,(x^4+5)\log\left(-x^2+\sqrt{x^4+5}\right)+\sqrt{x^4+5}(3\,x^2+2)+15}{2\,(x^4+5)}$$

```
[In] integrate(x^3*(3*x^2+2)/(x^4+5)^(3/2),x, algorithm="fricas")
[Out] -1/2*(3*x^4 + 3*(x^4 + 5)*log(-x^2 + sqrt(x^4 + 5)) + sqrt(x^4 + 5)*(3*x^2
+ 2) + 15)/(x^4 + 5)
```

Sympy [A] (verification not implemented)

Time = 4.34 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.11

$$\int \frac{x^3(2+3x^2)}{(5+x^4)^{3/2}}\,dx = -\frac{3x^2}{2\sqrt{x^4+5}}+\frac{3\,\mathrm{asinh}\left(\frac{\sqrt{5}x^2}{5}\right)}{2}-\frac{1}{\sqrt{x^4+5}}$$

```
[In] integrate(x**3*(3*x**2+2)/(x**4+5)**(3/2),x)
[Out] -3*x**2/(2*sqrt(x**4 + 5)) + 3*asinh(sqrt(5)*x**2/5)/2 - 1/sqrt(x**4 + 5)
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.54

$$\int \frac{x^3(2+3x^2)}{(5+x^4)^{3/2}}\,dx = -\frac{3\,x^2}{2\sqrt{x^4+5}} - \frac{1}{\sqrt{x^4+5}} + \frac{3}{4}\,\log\left(\frac{\sqrt{x^4+5}}{x^2}+1\right) - \frac{3}{4}\,\log\left(\frac{\sqrt{x^4+5}}{x^2}-1\right)$$

```
[In] integrate(x^3*(3*x^2+2)/(x^4+5)^(3/2),x, algorithm="maxima")
```

```
[Out] -3/2*x^2/sqrt(x^4 + 5) - 1/sqrt(x^4 + 5) + 3/4*log(sqrt(x^4 + 5)/x^2 + 1) -
 3/4*log(sqrt(x^4 + 5)/x^2 - 1)
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.94

$$\int \frac{x^3(2+3x^2)}{(5+x^4)^{3/2}}\,dx = -\frac{3\,x^2+2}{2\sqrt{x^4+5}} - \frac{3}{2}\,\log\left(-x^2+\sqrt{x^4+5}\right)$$

```
[In] integrate(x^3*(3*x^2+2)/(x^4+5)^(3/2),x, algorithm="giac")
```

```
[Out] -1/2*(3*x^2 + 2)/sqrt(x^4 + 5) - 3/2*log(-x^2 + sqrt(x^4 + 5))
```

Mupad [B] (verification not implemented)

Time = 8.07 (sec) , antiderivative size = 82, normalized size of antiderivative = 2.34

$$\int \frac{x^3(2+3x^2)}{(5+x^4)^{3/2}}\,dx = \frac{3\,\mathrm{asinh}\left(\frac{\sqrt{5}\,x^2}{5}\right)}{2} - \frac{\sqrt{5}\,\left(2+\sqrt{5}\,3\mathrm{i}\right)\,\sqrt{x^4+5}\,1\mathrm{i}}{20\,\left(-x^2+\sqrt{5}\,1\mathrm{i}\right)} + \frac{\sqrt{5}\,\left(-2+\sqrt{5}\,3\mathrm{i}\right)\,\sqrt{x^4+5}\,1\mathrm{i}}{20\,\left(x^2+\sqrt{5}\,1\mathrm{i}\right)}$$

```
[In] int((x^3*(3*x^2 + 2))/(x^4 + 5)^(3/2),x)
```

```
[Out] (3*asinh((5^(1/2)*x^2)/5))/2 - (5^(1/2)*(5^(1/2)*3i + 2)*(x^4 + 5)^(1/2)*1i
)/(20*(5^(1/2)*1i - x^2)) + (5^(1/2)*(5^(1/2)*3i - 2)*(x^4 + 5)^(1/2)*1i)/(
20*(5^(1/2)*1i + x^2))
```

3.47 $\int \frac{x(2+3x^2)}{(5+x^4)^{3/2}}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 20

$$\int \frac{x(2+3x^2)}{(5+x^4)^{3/2}}\,dx = \frac{-15+2x^2}{10\sqrt{5+x^4}}$$

```
[Out] 1/10*(2*x^2-15)/(x^4+5)^(1/2)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {1262, 651}

$$\int \frac{x(2+3x^2)}{(5+x^4)^{3/2}}\,dx = -\frac{15-2x^2}{10\sqrt{x^4+5}}$$

```
[In] Int[(x*(2 + 3*x^2))/(5 + x^4)^(3/2),x]
```

```
[Out] -1/10*(15 - 2*x^2)/Sqrt[5 + x^4]
```

Rule 651

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2)^(3/2), x_Symbol] :> Simp[((-a
)*e + c*d*x)/(a*c*Sqrt[a + c*x^2]), x] /; FreeQ[{a, c, d, e}, x]
```

Rule 1262

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol]
 :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + c*x^2)^p, x], x, x^2], x] /; FreeQ
[{a, c, d, e, p, q}, x]
```

Rubi steps

$$\text{integral} = \frac{1}{2}\text{Subst}\left(\int \frac{2+3x}{(5+x^2)^{3/2}}\,dx, x, x^2\right)$$
$$= -\frac{15-2x^2}{10\sqrt{5+x^4}}$$

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{x(2+3x^2)}{(5+x^4)^{3/2}}\,dx = \frac{-15+2x^2}{10\sqrt{5+x^4}}$$

```
[In] Integrate[(x*(2 + 3*x^2))/(5 + x^4)^(3/2),x]
[Out] (-15 + 2*x^2)/(10*Sqrt[5 + x^4])
```

Maple [A] (verified)

Time = 0.28 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.85

method	result	size
gosper	$\frac{2x^2-15}{10\sqrt{x^4+5}}$	17
trager	$\frac{2x^2-15}{10\sqrt{x^4+5}}$	17
risch	$\frac{2x^2-15}{10\sqrt{x^4+5}}$	17
elliptic	$\frac{2x^2-15}{10\sqrt{x^4+5}}$	17
pseudoelliptic	$\frac{2x^2-15}{10\sqrt{x^4+5}}$	17
default	$\frac{x^2}{5\sqrt{x^4+5}}-\frac{3}{2\sqrt{x^4+5}}$	23
meijerg	$\frac{3\sqrt{5}\left(\sqrt{\pi}-\frac{\sqrt{\pi}}{\sqrt{1+\frac{x^4}{5}}}\right)}{10\sqrt{\pi}}+\frac{\sqrt{5}\,x^2}{25\sqrt{1+\frac{x^4}{5}}}$	45

```
[In] int(x*(3*x^2+2)/(x^4+5)^(3/2),x,method=_RETURNVERBOSE)
[Out] 1/10*(2*x^2-15)/(x^4+5)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.55

$$\int \frac{x(2+3x^2)}{(5+x^4)^{3/2}}\,dx = \frac{2\,x^4+\sqrt{x^4+5}(2\,x^2-15)+10}{10\,(x^4+5)}$$

```
[In] integrate(x*(3*x^2+2)/(x^4+5)^(3/2),x, algorithm="fricas")
[Out] 1/10*(2*x^4 + sqrt(x^4 + 5)*(2*x^2 - 15) + 10)/(x^4 + 5)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 31 vs. 2(15) = 30.

Time = 3.02 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.55

$$\int \frac{x(2+3x^2)}{(5+x^4)^{3/2}}\,dx = \frac{\sqrt{5}x^2}{5\sqrt{5x^4+25}} - \frac{3}{2\sqrt{x^4+5}}$$

```
[In] integrate(x*(3*x**2+2)/(x**4+5)**(3/2),x)
[Out] sqrt(5)*x**2/(5*sqrt(5*x**4 + 25)) - 3/(2*sqrt(x**4 + 5))
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{x(2+3x^2)}{(5+x^4)^{3/2}}\,dx = \frac{x^2}{5\,\sqrt{x^4+5}} - \frac{3}{2\,\sqrt{x^4+5}}$$

```
[In] integrate(x*(3*x^2+2)/(x^4+5)^(3/2),x, algorithm="maxima")
[Out] 1/5*x^2/sqrt(x^4 + 5) - 3/2/sqrt(x^4 + 5)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.80

$$\int \frac{x(2+3x^2)}{(5+x^4)^{3/2}}\,dx = \frac{2\,x^2-15}{10\,\sqrt{x^4+5}}$$

```
[In] integrate(x*(3*x^2+2)/(x^4+5)^(3/2),x, algorithm="giac")
[Out] 1/10*(2*x^2 - 15)/sqrt(x^4 + 5)
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.80

$$\int \frac{x(2+3x^2)}{(5+x^4)^{3/2}}\,dx = \frac{2\,x^2-15}{10\,\sqrt{x^4+5}}$$

```
[In] int((x*(3*x^2 + 2))/(x^4 + 5)^(3/2),x)
[Out] (2*x^2 - 15)/(10*(x^4 + 5)^(1/2))
```

3.48 $\int \frac{2+3x^2}{x(5+x^4)^{3/2}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 46

$$\int \frac{2+3x^2}{x(5+x^4)^{3/2}}\,dx = \frac{2+3x^2}{10\sqrt{5+x^4}} - \frac{\operatorname{arctanh}\left(\frac{\sqrt{5+x^4}}{\sqrt{5}}\right)}{5\sqrt{5}}$$

```
[Out] -1/25*arctanh(1/5*(x^4+5)^(1/2)*5^(1/2))*5^(1/2)+1/10*(3*x^2+2)/(x^4+5)^(1/
2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {1266, 837, 12, 272, 65, 213}

$$\int \frac{2+3x^2}{x(5+x^4)^{3/2}}\,dx = \frac{3x^2+2}{10\sqrt{x^4+5}} - \frac{\operatorname{arctanh}\left(\frac{\sqrt{x^4+5}}{\sqrt{5}}\right)}{5\sqrt{5}}$$

```
[In] Int[(2 + 3*x^2)/(x*(5 + x^4)^(3/2)),x]
```

```
[Out] (2 + 3*x^2)/(10*Sqrt[5 + x^4]) - ArcTanh[Sqrt[5 + x^4]/Sqrt[5]]/(5*Sqrt[5])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 837

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> Simp[(-(d + e*x)^(m + 1))*(f*a*c*e - a*g*c*d + c*(c*d*f +
a*e*g)*x)*((a + c*x^2)^(p + 1)/(2*a*c*(p + 1)*(c*d^2 + a*e^2))), x] + Dist[
1/(2*a*c*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*(a + c*x^2)^(p + 1)*Simp
[f*(c^2*d^2*(2*p + 3) + a*c*e^2*(m + 2*p + 3)) - a*c*d*e*g*m + c*e*(c*d*f +
 a*e*g)*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[
c*d^2 + a*e^2, 0] && LtQ[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ
[2*m, 2*p])
```

Rule 1266

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_S
ymbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + c*x^2)^p, x],
x, x^2], x] /; FreeQ[{a, c, d, e, p, q}, x] && IntegerQ[(m + 1)/2]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{1}{2}\text{Subst}\left(\int \frac{2+3x}{x\left(5+x^2\right)^{3/2}}\,dx, x, x^2\right)\\
&= \frac{2+3x^2}{10\sqrt{5+x^4}} - \frac{1}{50}\text{Subst}\left(\int -\frac{10}{x\sqrt{5+x^2}}\,dx, x, x^2\right)\\
&= \frac{2+3x^2}{10\sqrt{5+x^4}} + \frac{1}{5}\text{Subst}\left(\int \frac{1}{x\sqrt{5+x^2}}\,dx, x, x^2\right)
\end{aligned}
$$

$$= \frac{2+3x^2}{10\sqrt{5+x^4}} + \frac{1}{10}\text{Subst}\left(\int \frac{1}{x\sqrt{5+x}}\,dx, x, x^4\right)$$

$$= \frac{2+3x^2}{10\sqrt{5+x^4}} + \frac{1}{5}\text{Subst}\left(\int \frac{1}{-5+x^2}\,dx, x, \sqrt{5+x^4}\right)$$

$$= \frac{2+3x^2}{10\sqrt{5+x^4}} - \frac{\tanh^{-1}\left(\frac{\sqrt{5+x^4}}{\sqrt{5}}\right)}{5\sqrt{5}}$$

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.13

$$\int \frac{2+3x^2}{x\,(5+x^4)^{3/2}}\,dx = \frac{1}{50}\left(\frac{5(2+3x^2)}{\sqrt{5+x^4}} + 4\sqrt{5}\text{arctanh}\left(\frac{x^2-\sqrt{5+x^4}}{\sqrt{5}}\right)\right)$$

```
[In] Integrate[(2 + 3*x^2)/(x*(5 + x^4)^(3/2)),x]
[Out] ((5*(2 + 3*x^2))/Sqrt[5 + x^4] + 4*Sqrt[5]*ArcTanh[(x^2 - Sqrt[5 + x^4])/Sq
rt[5]])/50
```

Maple [A] (verified)

Time = 0.32 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.76

method	result	size
risch	$\frac{3x^2+2}{10\sqrt{x^4+5}} - \frac{\sqrt{5}\,\text{arctanh}\left(\frac{\sqrt{5}}{\sqrt{x^4+5}}\right)}{25}$	35
default	$\frac{3x^2}{10\sqrt{x^4+5}} + \frac{1}{5\sqrt{x^4+5}} - \frac{\sqrt{5}\,\text{arctanh}\left(\frac{\sqrt{5}}{\sqrt{x^4+5}}\right)}{25}$	40
elliptic	$\frac{3x^2}{10\sqrt{x^4+5}} + \frac{1}{5\sqrt{x^4+5}} - \frac{\sqrt{5}\,\text{arctanh}\left(\frac{\sqrt{5}}{\sqrt{x^4+5}}\right)}{25}$	40
pseudoelliptic	$\frac{-2\sqrt{5}\,\text{arctanh}\left(\frac{\sqrt{5}}{\sqrt{x^4+5}}\right)\sqrt{x^4+5}+15x^2+10}{50\sqrt{x^4+5}}$	41
trager	$\frac{3x^2+2}{10\sqrt{x^4+5}} + \frac{\text{RootOf}\left(\_Z^2-5\right)\ln\left(\frac{\sqrt{x^4+5}-\text{RootOf}\left(\_Z^2-5\right)}{x^2}\right)}{25}$	47
meijerg	$\frac{\sqrt{5}\left(-\sqrt{\pi}+\frac{\sqrt{\pi}}{\sqrt{1+\frac{x^4}{5}}}-\sqrt{\pi}\,\ln\left(\frac{1}{2}+\frac{\sqrt{1+\frac{x^4}{5}}}{2}\right)+\frac{(2-2\ln(2)+4\ln(x)-\ln(5))\sqrt{\pi}}{2}\right)}{25\sqrt{\pi}} + \frac{3\sqrt{5}\,x^2}{50\sqrt{1+\frac{x^4}{5}}}$	84

```
[In] int((3*x^2+2)/x/(x^4+5)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/10*(3*x^2+2)/(x^4+5)^(1/2)-1/25*5^(1/2)*arctanh(5^(1/2)/(x^4+5)^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.33

$$\int \frac{2+3x^2}{x\,(5+x^4)^{3/2}}\,dx = \frac{15\,x^4+2\,\sqrt{5}(x^4+5)\log\left(-\frac{\sqrt{5}-\sqrt{x^4+5}}{x^2}\right)+5\,\sqrt{x^4+5}(3\,x^2+2)+75}{50\,(x^4+5)}$$

```
[In] integrate((3*x^2+2)/x/(x^4+5)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/50*(15*x^4 + 2*sqrt(5)*(x^4 + 5)*log(-(sqrt(5) - sqrt(x^4 + 5))/x^2) + 5*
sqrt(x^4 + 5)*(3*x^2 + 2) + 75)/(x^4 + 5)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 212 vs. $2(41) = 82$.

Time = 7.46 (sec) , antiderivative size = 212, normalized size of antiderivative = 4.61

$$\begin{aligned}\int \frac{2+3x^2}{x\,(5+x^4)^{3/2}}\,dx &= \frac{2x^4\log(x^4)}{20\sqrt{5}x^4+100\sqrt{5}} - \frac{4x^4\log\left(\sqrt{\frac{x^4}{5}+1}+1\right)}{20\sqrt{5}x^4+100\sqrt{5}} \\ &\quad - \frac{2x^4\log(5)}{20\sqrt{5}x^4+100\sqrt{5}} + \frac{3x^2}{10\sqrt{x^4+5}} + \frac{4\sqrt{5}\sqrt{x^4+5}}{20\sqrt{5}x^4+100\sqrt{5}} \\ &\quad + \frac{10\log(x^4)}{20\sqrt{5}x^4+100\sqrt{5}} - \frac{20\log\left(\sqrt{\frac{x^4}{5}+1}+1\right)}{20\sqrt{5}x^4+100\sqrt{5}} - \frac{10\log(5)}{20\sqrt{5}x^4+100\sqrt{5}}\end{aligned}$$

```
[In] integrate((3*x**2+2)/x/(x**4+5)**(3/2),x)
```

```
[Out] 2*x**4*log(x**4)/(20*sqrt(5)*x**4 + 100*sqrt(5)) - 4*x**4*log(sqrt(x**4/5 +
 1) + 1)/(20*sqrt(5)*x**4 + 100*sqrt(5)) - 2*x**4*log(5)/(20*sqrt(5)*x**4 +
 100*sqrt(5)) + 3*x**2/(10*sqrt(x**4 + 5)) + 4*sqrt(5)*sqrt(x**4 + 5)/(20*s
qrt(5)*x**4 + 100*sqrt(5)) + 10*log(x**4)/(20*sqrt(5)*x**4 + 100*sqrt(5)) -
 20*log(sqrt(x**4/5 + 1) + 1)/(20*sqrt(5)*x**4 + 100*sqrt(5)) - 10*log(5)/(
20*sqrt(5)*x**4 + 100*sqrt(5))
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.22

$$\int \frac{2+3x^2}{x\left(5+x^4\right)^{3/2}}\,dx = \frac{3\,x^2}{10\,\sqrt{x^4+5}} + \frac{1}{50}\,\sqrt{5}\log\left(-\frac{\sqrt{5}-\sqrt{x^4+5}}{\sqrt{5}+\sqrt{x^4+5}}\right) + \frac{1}{5\,\sqrt{x^4+5}}$$

```
[In] integrate((3*x^2+2)/x/(x^4+5)^(3/2),x, algorithm="maxima")
```

```
[Out] 3/10*x^2/sqrt(x^4 + 5) + 1/50*sqrt(5)*log(-(sqrt(5) - sqrt(x^4 + 5))/(sqrt(
5) + sqrt(x^4 + 5))) + 1/5/sqrt(x^4 + 5)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.33

$$\int \frac{2+3x^2}{x\left(5+x^4\right)^{3/2}}\,dx = \frac{1}{25}\,\sqrt{5}\log\left(x^2+\sqrt{5}-\sqrt{x^4+5}\right) - \frac{1}{25}\,\sqrt{5}\log\left(-x^2+\sqrt{5}+\sqrt{x^4+5}\right) + \frac{3\,x^2+2}{10\,\sqrt{x^4+5}}$$

```
[In] integrate((3*x^2+2)/x/(x^4+5)^(3/2),x, algorithm="giac")
```

```
[Out] 1/25*sqrt(5)*log(x^2 + sqrt(5) - sqrt(x^4 + 5)) - 1/25*sqrt(5)*log(-x^2 + s
qrt(5) + sqrt(x^4 + 5)) + 1/10*(3*x^2 + 2)/sqrt(x^4 + 5)
```

Mupad [B] (verification not implemented)

Time = 7.97 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.87

$$\int \frac{2+3x^2}{x\left(5+x^4\right)^{3/2}}\,dx = \frac{1}{5\,\sqrt{x^4+5}} - \frac{\sqrt{5}\,\mathrm{atanh}\left(\frac{\sqrt{5}\,\sqrt{x^4+5}}{5}\right)}{25} + \frac{3\,x^2}{10\,\sqrt{x^4+5}}$$

```
[In] int((3*x^2 + 2)/(x*(x^4 + 5)^(3/2)),x)
```

```
[Out] 1/(5*(x^4 + 5)^(1/2)) - (5^(1/2)*atanh((5^(1/2)*(x^4 + 5)^(1/2))/5))/25 + (
3*x^2)/(10*(x^4 + 5)^(1/2))
```

3.49 $\int \frac{2+3x^2}{x^3\left(5+x^4\right)^{3/2}} dx$

Optimal result

Integrand size = 20, antiderivative size = 65

$$\int \frac{2+3x^2}{x^3\left(5+x^4\right)^{3/2}}\,dx=\frac{2+3x^2}{10x^2\sqrt{5+x^4}}-\frac{2\sqrt{5+x^4}}{25x^2}-\frac{3\text{arctanh}\left(\frac{\sqrt{5+x^4}}{\sqrt{5}}\right)}{10\sqrt{5}}$$

```
[Out] -3/50*arctanh(1/5*(x^4+5)^(1/2)*5^(1/2))*5^(1/2)+1/10*(3*x^2+2)/x^2/(x^4+5)
^(1/2)-2/25*(x^4+5)^(1/2)/x^2
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {1266, 837, 821, 272, 65, 213}

$$\int \frac{2+3x^2}{x^3\left(5+x^4\right)^{3/2}}\,dx=-\frac{3\text{arctanh}\left(\frac{\sqrt{x^4+5}}{\sqrt{5}}\right)}{10\sqrt{5}}+\frac{3x^2+2}{10x^2\sqrt{x^4+5}}-\frac{2\sqrt{x^4+5}}{25x^2}$$

```
[In] Int[(2 + 3*x^2)/(x^3*(5 + x^4)^(3/2)),x]
```

```
[Out] (2 + 3*x^2)/(10*x^2*Sqrt[5 + x^4]) - (2*Sqrt[5 + x^4])/(25*x^2) - (3*ArcTan
h[Sqrt[5 + x^4]/Sqrt[5]])/(10*Sqrt[5])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
```

```
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 821

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1
)/(2*(p + 1)*(c*d^2 + a*e^2))), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2),
Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m,
p}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 837

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> Simp[(-(d + e*x)^(m + 1))*(f*a*c*e - a*g*c*d + c*(c*d*f +
a*e*g)*x)*((a + c*x^2)^(p + 1)/(2*a*c*(p + 1)*(c*d^2 + a*e^2))), x] + Dist[
1/(2*a*c*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*(a + c*x^2)^(p + 1)*Simp
[f*(c^2*d^2*(2*p + 3) + a*c*e^2*(m + 2*p + 3)) - a*c*d*e*g*m + c*e*(c*d*f +
 a*e*g)*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[
c*d^2 + a*e^2, 0] && LtQ[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ
[2*m, 2*p])
```

Rule 1266

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_S
ymbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + c*x^2)^p, x],
x, x^2], x] /; FreeQ[{a, c, d, e, p, q}, x] && IntegerQ[(m + 1)/2]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{1}{2}\text{Subst}\left(\int \frac{2+3x}{x^2\left(5+x^2\right)^{3/2}}\,dx, x, x^2\right)\\ &= \frac{2+3x^2}{10x^2\sqrt{5+x^4}} - \frac{1}{50}\text{Subst}\left(\int \frac{-20-15x}{x^2\sqrt{5+x^2}}\,dx, x, x^2\right)\end{aligned}$$

$$= \frac{2+3x^2}{10x^2\sqrt{5+x^4}} - \frac{2\sqrt{5+x^4}}{25x^2} + \frac{3}{10}\text{Subst}\left(\int \frac{1}{x\sqrt{5+x^2}}\,dx, x, x^2\right)$$

$$= \frac{2+3x^2}{10x^2\sqrt{5+x^4}} - \frac{2\sqrt{5+x^4}}{25x^2} + \frac{3}{20}\text{Subst}\left(\int \frac{1}{x\sqrt{5+x}}\,dx, x, x^4\right)$$

$$= \frac{2+3x^2}{10x^2\sqrt{5+x^4}} - \frac{2\sqrt{5+x^4}}{25x^2} + \frac{3}{10}\text{Subst}\left(\int \frac{1}{-5+x^2}\,dx, x, \sqrt{5+x^4}\right)$$

$$= \frac{2+3x^2}{10x^2\sqrt{5+x^4}} - \frac{2\sqrt{5+x^4}}{25x^2} - \frac{3\tanh^{-1}\left(\frac{\sqrt{5+x^4}}{\sqrt{5}}\right)}{10\sqrt{5}}$$

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.91

$$\int \frac{2+3x^2}{x^3\left(5+x^4\right)^{3/2}}\,dx = \frac{1}{50}\left(\frac{-10+15x^2-4x^4}{x^2\sqrt{5+x^4}} + 6\sqrt{5}\text{arctanh}\left(\frac{x^2-\sqrt{5+x^4}}{\sqrt{5}}\right)\right)$$

```
[In] Integrate[(2 + 3*x^2)/(x^3*(5 + x^4)^(3/2)),x]
```

```
[Out] ((-10 + 15*x^2 - 4*x^4)/(x^2*Sqrt[5 + x^4]) + 6*Sqrt[5]*ArcTanh[(x^2 - Sqrt
[5 + x^4])/Sqrt[5]])/50
```

Maple [A] (verified)

Time = 0.33 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.66

method	result	size
risch	$-\frac{4x^{4}-15x^{2}+10}{50x^{2}\sqrt{x^{4}+5}}-\frac{3\sqrt{5}\,\operatorname{arctanh}\left(\frac{\sqrt{5}}{\sqrt{x^{4}+5}}\right)}{50}$	43
default	$\frac{3}{10\sqrt{x^{4}+5}}-\frac{3\sqrt{5}\,\operatorname{arctanh}\left(\frac{\sqrt{5}}{\sqrt{x^{4}+5}}\right)}{50}-\frac{2x^{4}+5}{25x^{2}\sqrt{x^{4}+5}}$	47
elliptic	$-\frac{1}{5x^{2}\sqrt{x^{4}+5}}-\frac{2x^{2}}{25\sqrt{x^{4}+5}}+\frac{3}{10\sqrt{x^{4}+5}}-\frac{3\sqrt{5}\,\operatorname{arctanh}\left(\frac{\sqrt{5}}{\sqrt{x^{4}+5}}\right)}{50}$	52
pseudoelliptic	$\frac{-3\sqrt{5}\,\operatorname{arctanh}\left(\frac{\sqrt{5}}{\sqrt{x^{4}+5}}\right)x^{2}\sqrt{x^{4}+5}-4x^{4}+15x^{2}-10}{50x^{2}\sqrt{x^{4}+5}}$	52
trager	$-\frac{4x^{4}-15x^{2}+10}{50x^{2}\sqrt{x^{4}+5}}+\frac{3\operatorname{RootOf}\left(\_Z^{2}-5\right)\ln\left(\frac{\sqrt{x^{4}+5}-\operatorname{RootOf}\left(\_Z^{2}-5\right)}{x^{2}}\right)}{50}$	55
meijerg	$-\frac{\sqrt{5}\left(1+\frac{2x^{4}}{5}\right)}{25x^{2}\sqrt{1+\frac{x^{4}}{5}}}+\frac{3\sqrt{5}\left(-\sqrt{\pi}+\frac{\sqrt{\pi}}{\sqrt{1+\frac{x^{4}}{5}}}-\sqrt{\pi}\,\ln\left(\frac{1}{2}+\frac{\sqrt{1+\frac{x^{4}}{5}}}{2}\right)+\frac{(2-2\ln(2)+4\ln(x)-\ln(5))\sqrt{\pi}}{2}\right)}{50\sqrt{\pi}}$	91

```
[In] int((3*x^2+2)/x^3/(x^4+5)^(3/2),x,method=_RETURNVERBOSE)
[Out] -1/50*(4*x^4-15*x^2+10)/x^2/(x^4+5)^(1/2)-3/50*5^(1/2)*arctanh(5^(1/2)/(x^4
+5)^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.18

$$\int \frac{2+3x^2}{x^3\left(5+x^4\right)^{3/2}}\,dx = -\frac{4\,x^{6}-3\,\sqrt{5}(x^{6}+5\,x^{2})\log\left(-\frac{\sqrt{5}-\sqrt{x^{4}+5}}{x^{2}}\right)+20\,x^{2}+(4\,x^{4}-15\,x^{2}+10)\sqrt{x^{4}+5}}{50\,(x^{6}+5\,x^{2})}$$

```
[In] integrate((3*x^2+2)/x^3/(x^4+5)^(3/2),x, algorithm="fricas")
[Out] -1/50*(4*x^6 - 3*sqrt(5)*(x^6 + 5*x^2)*log(-(sqrt(5) - sqrt(x^4 + 5))/x^2)
+ 20*x^2 + (4*x^4 - 15*x^2 + 10)*sqrt(x^4 + 5))/(x^6 + 5*x^2)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 228 vs. $2(61) = 122$.

Time = 4.62 (sec) , antiderivative size = 228, normalized size of antiderivative = 3.51

$$\int \frac{2+3x^2}{x^3\left(5+x^4\right)^{3/2}}\,dx = \frac{3x^4\log\left(x^4\right)}{20\sqrt{5}x^4+100\sqrt{5}} - \frac{6x^4\log\left(\sqrt{\frac{x^4}{5}+1}+1\right)}{20\sqrt{5}x^4+100\sqrt{5}} - \frac{3x^4\log\left(5\right)}{20\sqrt{5}x^4+100\sqrt{5}} + \frac{6\sqrt{5}\sqrt{x^4+5}}{20\sqrt{5}x^4+100\sqrt{5}} + \frac{15\log\left(x^4\right)}{20\sqrt{5}x^4+100\sqrt{5}} - \frac{30\log\left(\sqrt{\frac{x^4}{5}+1}+1\right)}{20\sqrt{5}x^4+100\sqrt{5}} - \frac{15\log\left(5\right)}{20\sqrt{5}x^4+100\sqrt{5}} - \frac{2}{25\sqrt{1+\frac{5}{x^4}}} - \frac{1}{5x^4\sqrt{1+\frac{5}{x^4}}}$$

```
[In] integrate((3*x**2+2)/x**3/(x**4+5)**(3/2),x)
```

```
[Out] 3*x**4*log(x**4)/(20*sqrt(5)*x**4 + 100*sqrt(5)) - 6*x**4*log(sqrt(x**4/5 +
 1) + 1)/(20*sqrt(5)*x**4 + 100*sqrt(5)) - 3*x**4*log(5)/(20*sqrt(5)*x**4 +
 100*sqrt(5)) + 6*sqrt(5)*sqrt(x**4 + 5)/(20*sqrt(5)*x**4 + 100*sqrt(5)) +
15*log(x**4)/(20*sqrt(5)*x**4 + 100*sqrt(5)) - 30*log(sqrt(x**4/5 + 1) + 1)
/(20*sqrt(5)*x**4 + 100*sqrt(5)) - 15*log(5)/(20*sqrt(5)*x**4 + 100*sqrt(5)
) - 2/(25*sqrt(1 + 5/x**4)) - 1/(5*x**4*sqrt(1 + 5/x**4))
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.05

$$\int \frac{2+3x^2}{x^3\left(5+x^4\right)^{3/2}}\,dx = -\frac{x^2}{25\sqrt{x^4+5}} + \frac{3}{100}\sqrt{5}\log\left(-\frac{\sqrt{5}-\sqrt{x^4+5}}{\sqrt{5}+\sqrt{x^4+5}}\right) + \frac{3}{10\sqrt{x^4+5}} - \frac{\sqrt{x^4+5}}{25\,x^2}$$

```
[In] integrate((3*x^2+2)/x^3/(x^4+5)^(3/2),x, algorithm="maxima")
```

```
[Out] -1/25*x^2/sqrt(x^4 + 5) + 3/100*sqrt(5)*log(-(sqrt(5) - sqrt(x^4 + 5))/(sqr
t(5) + sqrt(x^4 + 5))) + 3/10/sqrt(x^4 + 5) - 1/25*sqrt(x^4 + 5)/x^2
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.26

$$\int \frac{2+3x^2}{x^3\left(5+x^4\right)^{3/2}}\,dx = \frac{3}{50}\sqrt{5}\log\left(-\frac{x^2+\sqrt{5}-\sqrt{x^4+5}}{x^2-\sqrt{5}-\sqrt{x^4+5}}\right) - \frac{2\,x^2-15}{50\,\sqrt{x^4+5}} + \frac{2}{5\left(\left(x^2-\sqrt{x^4+5}\right)^2-5\right)}$$

```
[In] integrate((3*x^2+2)/x^3/(x^4+5)^(3/2),x, algorithm="giac")
```

```
[Out] 3/50*sqrt(5)*log(-(x^2 + sqrt(5) - sqrt(x^4 + 5))/(x^2 - sqrt(5) - sqrt(x^4
 + 5))) - 1/50*(2*x^2 - 15)/sqrt(x^4 + 5) + 2/5/((x^2 - sqrt(x^4 + 5))^2 -
5)
```

Mupad [B] (verification not implemented)

Time = 7.98 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.72

$$\int \frac{2+3x^2}{x^3\left(5+x^4\right)^{3/2}}\,dx = \frac{3}{10\,\sqrt{x^4+5}} - \frac{3\,\sqrt{5}\,\mathrm{atanh}\left(\frac{\sqrt{5}\,\sqrt{x^4+5}}{5}\right)}{50} - \frac{2\,x^4+5}{25\,x^2\,\sqrt{x^4+5}}$$

```
[In] int((3*x^2 + 2)/(x^3*(x^4 + 5)^(3/2)),x)
```

```
[Out] 3/(10*(x^4 + 5)^(1/2)) - (3*5^(1/2)*atanh((5^(1/2)*(x^4 + 5)^(1/2))/5))/50
- (2*x^4 + 5)/(25*x^2*(x^4 + 5)^(1/2))
```

3.50 $\int \frac{x^4(2+3x^2)}{(5+x^4)^{3/2}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 196

$$\int \frac{x^4(2+3x^2)}{(5+x^4)^{3/2}}\,dx = -\frac{x^3(15-2x^2)}{10\sqrt{5+x^4}} - \frac{1}{5}x\sqrt{5+x^4} + \frac{9x\sqrt{5+x^4}}{2\left(\sqrt{5}+x^2\right)} - \frac{9\sqrt[4]{5}(\sqrt{5}+x^2)\sqrt{\frac{5+x^4}{\left(\sqrt{5}+x^2\right)^2}}E\left(2\arctan\left(\frac{x}{\sqrt[4]{5}}\right)|\frac{1}{2}\right)}{2\sqrt{5+x^4}} + \frac{\left(2+9\sqrt{5}\right)\left(\sqrt{5}+x^2\right)\sqrt{\frac{5+x^4}{\left(\sqrt{5}+x^2\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{x}{\sqrt[4]{5}}\right),\frac{1}{2}\right)}{4\sqrt[4]{5}\sqrt{5+x^4}}$$

```
[Out] -1/10*x^3*(-2*x^2+15)/(x^4+5)^(1/2)-1/5*x*(x^4+5)^(1/2)+9/2*x*(x^4+5)^(1/2)
/(x^2+5^(1/2))-9/2*5^(1/4)*(cos(2*arctan(1/5*x*5^(3/4)))^2)^(1/2)/cos(2*arc
tan(1/5*x*5^(3/4)))*EllipticE(sin(2*arctan(1/5*x*5^(3/4))),1/2*2^(1/2))*(x^
2+5^(1/2))*((x^4+5)/(x^2+5^(1/2))^2)^(1/2)/(x^4+5)^(1/2)+1/20*(cos(2*arctan
(1/5*x*5^(3/4)))^2)^(1/2)/cos(2*arctan(1/5*x*5^(3/4)))*EllipticF(sin(2*arct
an(1/5*x*5^(3/4))),1/2*2^(1/2))*(x^2+5^(1/2))*(2+9*5^(1/2))*((x^4+5)/(x^2+5
^(1/2))^2)^(1/2)*5^(3/4)/(x^4+5)^(1/2)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 196, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {1290, 1294, 1212, 226, 1210}

$$\int \frac{x^4(2+3x^2)}{(5+x^4)^{3/2}}\,dx = \frac{\left(2+9\sqrt{5}\right)\left(x^2+\sqrt{5}\right)\sqrt{\frac{x^4+5}{\left(x^2+\sqrt{5}\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{x}{\sqrt[4]{5}}\right),\frac{1}{2}\right)}{4\sqrt[4]{5}\sqrt{x^4+5}} - \frac{9\sqrt[4]{5}(x^2+\sqrt{5})\sqrt{\frac{x^4+5}{\left(x^2+\sqrt{5}\right)^2}}E\left(2\arctan\left(\frac{x}{\sqrt[4]{5}}\right)|\frac{1}{2}\right)}{2\sqrt{x^4+5}} - \frac{1}{5}\sqrt{x^4+5}x + \frac{9\sqrt{x^4+5}x}{2\left(x^2+\sqrt{5}\right)} - \frac{\left(15-2x^2\right)x^3}{10\sqrt{x^4+5}}$$

```
[In] Int[(x^4*(2 + 3*x^2))/(5 + x^4)^(3/2),x]
```

```
[Out] -1/10*(x^3*(15 - 2*x^2))/Sqrt[5 + x^4] - (x*Sqrt[5 + x^4])/5 + (9*x*Sqrt[5
+ x^4])/(2*(Sqrt[5] + x^2)) - (9*5^(1/4)*(Sqrt[5] + x^2)*Sqrt[(5 + x^4)/(Sq
rt[5] + x^2)^2]*EllipticE[2*ArcTan[x/5^(1/4)], 1/2])/(2*Sqrt[5 + x^4]) + ((
2 + 9*Sqrt[5])*(Sqrt[5] + x^2)*Sqrt[(5 + x^4)/(Sqrt[5] + x^2)^2]*EllipticF[
2*ArcTan[x/5^(1/4)], 1/2])/(4*5^(1/4)*Sqrt[5 + x^4])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 1210

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Rule 1212

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + c*x^4], x], x] - Dist[e/q, I
nt[(1 - q*x^2)/Sqrt[a + c*x^4], x], x] /; NeQ[e + d*q, 0]] /; FreeQ[{a, c,
d, e}, x] && PosQ[c/a]
```

Rule 1290

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (c_.)*(x_)^4)^(p_.), x
_Symbol] :> Simp[f*(f*x)^(m - 1)*(a + c*x^4)^(p + 1)*((a*e - c*d*x^2)/(4*a*
c*(p + 1))), x] - Dist[f^2/(4*a*c*(p + 1)), Int[(f*x)^(m - 2)*(a + c*x^4)^(
p + 1)*(a*e*(m - 1) - c*d*(4*p + 4 + m + 1)*x^2), x], x] /; FreeQ[{a, c, d,
 e, f}, x] && LtQ[p, -1] && GtQ[m, 1] && IntegerQ[2*p] && (IntegerQ[p] || I
ntegerQ[m])
```

Rule 1294

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (c_.)*(x_)^4)^(p_), x_
Symbol] :> Simp[e*f*(f*x)^(m - 1)*((a + c*x^4)^(p + 1)/(c*(m + 4*p + 3))),
x] - Dist[f^2/(c*(m + 4*p + 3)), Int[(f*x)^(m - 2)*(a + c*x^4)^p*(a*e*(m -
1) - c*d*(m + 4*p + 3)*x^2), x], x] /; FreeQ[{a, c, d, e, f, p}, x] && GtQ[
m, 1] && NeQ[m + 4*p + 3, 0] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m
])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{x^3(15-2x^2)}{10\sqrt{5+x^4}}+\frac{1}{10}\int\frac{x^2(45-6x^2)}{\sqrt{5+x^4}}\,dx\\
&= -\frac{x^3(15-2x^2)}{10\sqrt{5+x^4}}-\frac{1}{5}x\sqrt{5+x^4}-\frac{1}{30}\int\frac{-30-135x^2}{\sqrt{5+x^4}}\,dx\\
&= -\frac{x^3(15-2x^2)}{10\sqrt{5+x^4}}-\frac{1}{5}x\sqrt{5+x^4}-\frac{1}{2}\left(9\sqrt{5}\right)\int\frac{1-\frac{x^2}{\sqrt{5}}}{\sqrt{5+x^4}}\,dx-\frac{1}{2}\left(-2-9\sqrt{5}\right)\int\frac{1}{\sqrt{5+x^4}}\,dx\\
&= -\frac{x^3(15-2x^2)}{10\sqrt{5+x^4}}-\frac{1}{5}x\sqrt{5+x^4}+\frac{9x\sqrt{5+x^4}}{2\left(\sqrt{5}+x^2\right)}\\
&\quad-\frac{9\sqrt[4]{5}\left(\sqrt{5}+x^2\right)\sqrt{\frac{5+x^4}{\left(\sqrt{5}+x^2\right)^2}}E\left(2\tan^{-1}\left(\frac{x}{\sqrt[4]{5}}\right)|\frac{1}{2}\right)}{2\sqrt{5+x^4}}\\
&\quad+\frac{\left(2+9\sqrt{5}\right)\left(\sqrt{5}+x^2\right)\sqrt{\frac{5+x^4}{\left(\sqrt{5}+x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{x}{\sqrt[4]{5}}\right)|\frac{1}{2}\right)}{4\sqrt[4]{5}\sqrt{5+x^4}}
\end{aligned}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.04 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.36

$$\int \frac{x^4(2+3x^2)}{(5+x^4)^{3/2}}\,dx = \frac{x(-1+3x^2)}{\sqrt{5+x^4}} + \frac{x\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{5}{4},-\frac{x^4}{5}\right)}{\sqrt{5}} - \frac{3x^3\,\text{Hypergeometric2F1}\left(\frac{3}{4},\frac{3}{2},\frac{7}{4},-\frac{x^4}{5}\right)}{\sqrt{5}}$$

```
[In] Integrate[(x^4*(2 + 3*x^2))/(5 + x^4)^(3/2),x]
```

```
[Out] (x*(-1 + 3*x^2))/Sqrt[5 + x^4] + (x*Hypergeometric2F1[1/4, 1/2, 5/4, -1/5*x
^4])/Sqrt[5] - (3*x^3*Hypergeometric2F1[3/4, 3/2, 7/4, -1/5*x^4])/Sqrt[5]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4.

Time = 3.57 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.20

method	result
meijerg	$\frac{3\sqrt{5}\,x^7\,{}_2F_1\left(\frac{3}{2},\frac{7}{4};\frac{11}{4};-\frac{x^4}{5}\right)}{175}+\frac{2\sqrt{5}\,x^5\,{}_2F_1\left(\frac{5}{4},\frac{3}{2};\frac{9}{4};-\frac{x^4}{5}\right)}{125}$
risch	$-\frac{x(3x^2+2)}{2\sqrt{x^4+5}}+\frac{\sqrt{5}\,\sqrt{25-5i\sqrt{5}\,x^2}\,\sqrt{25+5i\sqrt{5}\,x^2}\,F\left(\frac{x\sqrt{5}\,\sqrt{i\sqrt{5}}}{5},i\right)}{25\sqrt{i\sqrt{5}}\,\sqrt{x^4+5}}+\frac{9i\sqrt{25-5i\sqrt{5}\,x^2}\,\sqrt{25+5i\sqrt{5}\,x^2}\left(F\left(\frac{x\sqrt{5}\,\sqrt{i\sqrt{5}}}{5},i\right)-E\left(\cdots\right.\right.}{10\sqrt{i\sqrt{5}}\,\sqrt{x^4+5}}\cdots$
elliptic	$-\frac{2\left(\frac{3}{4}x^3+\frac{1}{2}x\right)}{\sqrt{x^4+5}}+\frac{\sqrt{5}\,\sqrt{25-5i\sqrt{5}\,x^2}\,\sqrt{25+5i\sqrt{5}\,x^2}\,F\left(\frac{x\sqrt{5}\,\sqrt{i\sqrt{5}}}{5},i\right)}{25\sqrt{i\sqrt{5}}\,\sqrt{x^4+5}}+\frac{9i\sqrt{25-5i\sqrt{5}\,x^2}\,\sqrt{25+5i\sqrt{5}\,x^2}\left(F\left(\frac{x\sqrt{5}\,\sqrt{i\sqrt{5}}}{5},i\right)-E\left(\cdots\right.\right.}{10\sqrt{i\sqrt{5}}\,\sqrt{x^4+5}}\cdots$
default	$-\frac{3x^3}{2\sqrt{x^4+5}}+\frac{9i\sqrt{25-5i\sqrt{5}\,x^2}\,\sqrt{25+5i\sqrt{5}\,x^2}\left(F\left(\frac{x\sqrt{5}\,\sqrt{i\sqrt{5}}}{5},i\right)-E\left(\frac{x\sqrt{5}\,\sqrt{i\sqrt{5}}}{5},i\right)\right)}{10\sqrt{i\sqrt{5}}\,\sqrt{x^4+5}}-\frac{x}{\sqrt{x^4+5}}+\frac{\sqrt{5}\,\sqrt{25-5i\sqrt{5}\,x^2}\,\sqrt{25+5i\cdots}}{25\sqrt{i\sqrt{5}}\,\sqrt{\cdots}}\cdots$

```
[In] int(x^4*(3*x^2+2)/(x^4+5)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 3/175*5^(1/2)*x^7*hypergeom([3/2,7/4],[11/4],-1/5*x^4)+2/125*5^(1/2)*x^5*hy
pergeom([5/4,3/2],[9/4],-1/5*x^4)
```

Fricas [A] (verification not implemented)

none

Time = 0.08 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.39

$$\int \frac{x^4(2+3x^2)}{(5+x^4)^{3/2}}\,dx = \frac{45\,(-5)^{\frac{3}{4}}\,(x^5+5\,x)E(\arcsin\left(\frac{(-5)^{\frac{1}{4}}}{x}\right)\,|\,-1) - 43\,(-5)^{\frac{3}{4}}\,(x^5+5\,x)F(\arcsin\left(\frac{(-5)^{\frac{1}{4}}}{x}\right)\,|\,-1}{10\,(x^5+5\,x)}$$

```
[In] integrate(x^4*(3*x^2+2)/(x^4+5)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/10*(45*(-5)^(3/4)*(x^5 + 5*x)*elliptic_e(arcsin((-5)^(1/4)/x), -1) - 43*(
-5)^(3/4)*(x^5 + 5*x)*elliptic_f(arcsin((-5)^(1/4)/x), -1) + 5*(6*x^4 - 2*x
^2 + 45)*sqrt(x^4 + 5))/(x^5 + 5*x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 2.10 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.38

$$\int \frac{x^4(2+3x^2)}{(5+x^4)^{3/2}}\,dx = \frac{3\sqrt{5}x^7\Gamma\left(\frac{7}{4}\right)\,{}_2F_1\left(\begin{matrix}\frac{3}{2},\frac{7}{4}\\\frac{11}{4}\end{matrix}\middle|\frac{x^4e^{i\pi}}{5}\right)}{100\Gamma\left(\frac{11}{4}\right)} + \frac{\sqrt{5}x^5\Gamma\left(\frac{5}{4}\right)\,{}_2F_1\left(\begin{matrix}\frac{5}{4},\frac{3}{2}\\\frac{9}{4}\end{matrix}\middle|\frac{x^4e^{i\pi}}{5}\right)}{50\Gamma\left(\frac{9}{4}\right)}$$

```
[In] integrate(x**4*(3*x**2+2)/(x**4+5)**(3/2),x)
```

```
[Out] 3*sqrt(5)*x**7*gamma(7/4)*hyper((3/2, 7/4), (11/4,), x**4*exp_polar(I*pi)/5
)/(100*gamma(11/4)) + sqrt(5)*x**5*gamma(5/4)*hyper((5/4, 3/2), (9/4,), x**
4*exp_polar(I*pi)/5)/(50*gamma(9/4))
```

Maxima [F]

$$\int \frac{x^4(2+3x^2)}{(5+x^4)^{3/2}}\,dx = \int \frac{(3\,x^2+2)x^4}{(x^4+5)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(x^4*(3*x^2+2)/(x^4+5)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((3*x^2 + 2)*x^4/(x^4 + 5)^(3/2), x)
```

Giac [F]

$$\int \frac{x^4(2+3x^2)}{(5+x^4)^{3/2}}\,dx = \int \frac{(3\,x^2+2)x^4}{(x^4+5)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(x^4*(3*x^2+2)/(x^4+5)^(3/2),x, algorithm="giac")
[Out] integrate((3*x^2 + 2)*x^4/(x^4 + 5)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^4(2+3x^2)}{(5+x^4)^{3/2}}\,dx = \int \frac{x^4\,(3\,x^2+2)}{(x^4+5)^{3/2}}\,dx$$

```
[In] int((x^4*(3*x^2 + 2))/(x^4 + 5)^(3/2),x)
[Out] int((x^4*(3*x^2 + 2))/(x^4 + 5)^(3/2), x)
```

3.51 $\int \frac{x^2(2+3x^2)}{(5+x^4)^{3/2}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 177

$$\int \frac{x^2(2+3x^2)}{(5+x^4)^{3/2}}\,dx = -\frac{x(15-2x^2)}{10\sqrt{5+x^4}} - \frac{x\sqrt{5+x^4}}{5\left(\sqrt{5}+x^2\right)} + \frac{\left(\sqrt{5}+x^2\right)\sqrt{\frac{5+x^4}{\left(\sqrt{5}+x^2\right)^2}}E\left(2\arctan\left(\frac{x}{\sqrt[4]{5}}\right)|\frac{1}{2}\right)}{5^{3/4}\sqrt{5+x^4}} - \frac{\left(2-3\sqrt{5}\right)\left(\sqrt{5}+x^2\right)\sqrt{\frac{5+x^4}{\left(\sqrt{5}+x^2\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{x}{\sqrt[4]{5}}\right),\frac{1}{2}\right)}{4\ 5^{3/4}\sqrt{5+x^4}}$$

```
[Out] -1/10*x*(-2*x^2+15)/(x^4+5)^(1/2)-1/5*x*(x^4+5)^(1/2)/(x^2+5^(1/2))+1/5*5^(
1/4)*(cos(2*arctan(1/5*x*5^(3/4)))^2)^(1/2)/cos(2*arctan(1/5*x*5^(3/4)))*El
lipticE(sin(2*arctan(1/5*x*5^(3/4))),1/2*2^(1/2))*(x^2+5^(1/2))*((x^4+5)/(x
^2+5^(1/2))^2)^(1/2)/(x^4+5)^(1/2)-1/20*(cos(2*arctan(1/5*x*5^(3/4)))^2)^(1
/2)/cos(2*arctan(1/5*x*5^(3/4)))*EllipticF(sin(2*arctan(1/5*x*5^(3/4))),1/2
*2^(1/2))*(2-3*5^(1/2))*(x^2+5^(1/2))*((x^4+5)/(x^2+5^(1/2))^2)^(1/2)*5^(1/
4)/(x^4+5)^(1/2)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 177, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {1290, 1212, 226, 1210}

$$\int \frac{x^2(2+3x^2)}{(5+x^4)^{3/2}}\,dx = -\frac{\left(2-3\sqrt{5}\right)\left(x^2+\sqrt{5}\right)\sqrt{\frac{x^4+5}{\left(x^2+\sqrt{5}\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{x}{\sqrt[4]{5}}\right),\frac{1}{2}\right)}{4\,5^{3/4}\sqrt{x^4+5}} + \frac{\left(x^2+\sqrt{5}\right)\sqrt{\frac{x^4+5}{\left(x^2+\sqrt{5}\right)^2}}E\left(2\arctan\left(\frac{x}{\sqrt[4]{5}}\right)|\frac{1}{2}\right)}{5^{3/4}\sqrt{x^4+5}} - \frac{\sqrt{x^4+5}\,x}{5\left(x^2+\sqrt{5}\right)} - \frac{\left(15-2x^2\right)x}{10\sqrt{x^4+5}}$$

```
[In] Int[(x^2*(2 + 3*x^2))/(5 + x^4)^(3/2),x]
```

```
[Out] -1/10*(x*(15 - 2*x^2))/Sqrt[5 + x^4] - (x*Sqrt[5 + x^4])/(5*(Sqrt[5] + x^2)
) + ((Sqrt[5] + x^2)*Sqrt[(5 + x^4)/(Sqrt[5] + x^2)^2]*EllipticE[2*ArcTan[x
/5^(1/4)], 1/2])/(5^(3/4)*Sqrt[5 + x^4]) - ((2 - 3*Sqrt[5])*(Sqrt[5] + x^2)
*Sqrt[(5 + x^4)/(Sqrt[5] + x^2)^2]*EllipticF[2*ArcTan[x/5^(1/4)], 1/2])/(4*
5^(3/4)*Sqrt[5 + x^4])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 1210

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Rule 1212

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + c*x^4], x], x] - Dist[e/q, I
nt[(1 - q*x^2)/Sqrt[a + c*x^4], x], x] /; NeQ[e + d*q, 0]] /; FreeQ[{a, c,
d, e}, x] && PosQ[c/a]
```

Rule 1290

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (c_.)*(x_)^4)^(p_.), x
_Symbol] :> Simp[f*(f*x)^(m - 1)*(a + c*x^4)^(p + 1)*((a*e - c*d*x^2)/(4*a*
```

```
c*(p + 1))), x] - Dist[f^2/(4*a*c*(p + 1)), Int[(f*x)^(m - 2)*(a + c*x^4)^(
p + 1)*(a*e*(m - 1) - c*d*(4*p + 4 + m + 1)*x^2), x], x] /; FreeQ[{a, c, d,
 e, f}, x] && LtQ[p, -1] && GtQ[m, 1] && IntegerQ[2*p] && (IntegerQ[p] || I
ntegerQ[m])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{x(15-2x^2)}{10\sqrt{5+x^4}}+\frac{1}{10}\int\frac{15-2x^2}{\sqrt{5+x^4}}\,dx\\
&= -\frac{x(15-2x^2)}{10\sqrt{5+x^4}}+\frac{\int\frac{1-\frac{x^2}{\sqrt{5}}}{\sqrt{5+x^4}}\,dx}{\sqrt{5}}+\frac{1}{10}\left(15-2\sqrt{5}\right)\int\frac{1}{\sqrt{5+x^4}}\,dx\\
&= -\frac{x(15-2x^2)}{10\sqrt{5+x^4}}-\frac{x\sqrt{5+x^4}}{5\left(\sqrt{5}+x^2\right)}+\frac{\left(\sqrt{5}+x^2\right)\sqrt{\frac{5+x^4}{\left(\sqrt{5}+x^2\right)^2}}E\left(2\tan^{-1}\left(\frac{x}{\sqrt[4]{5}}\right)|\frac{1}{2}\right)}{5^{3/4}\sqrt{5+x^4}}\\
&\quad-\frac{\left(2-3\sqrt{5}\right)\left(\sqrt{5}+x^2\right)\sqrt{\frac{5+x^4}{\left(\sqrt{5}+x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{x}{\sqrt[4]{5}}\right)|\frac{1}{2}\right)}{4\ 5^{3/4}\sqrt{5+x^4}}
\end{aligned}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.03 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.38

$$\int\frac{x^2(2+3x^2)}{(5+x^4)^{3/2}}\,dx=\frac{1}{150}x\left(-\frac{225}{\sqrt{5+x^4}}+45\sqrt{5}\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{5}{4},-\frac{x^4}{5}\right)+4\sqrt{5}x^2\,\text{Hypergeometric2F1}\left(\frac{3}{4},\frac{3}{2},\frac{7}{4},-\frac{x^4}{5}\right)\right)$$

```
[In] Integrate[(x^2*(2 + 3*x^2))/(5 + x^4)^(3/2),x]
```

```
[Out] (x*(-225/Sqrt[5 + x^4] + 45*Sqrt[5]*Hypergeometric2F1[1/4, 1/2, 5/4, -1/5*x
^4] + 4*Sqrt[5]*x^2*Hypergeometric2F1[3/4, 3/2, 7/4, -1/5*x^4]))/150
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4.

Time = 2.29 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.23

method	result
meijerg	$\frac{3\sqrt{5}\,x^{5}{}_{2}F_{1}\left(\frac{5}{4},\frac{3}{2};\frac{9}{4};-\frac{x^{4}}{5}\right)}{125}+\frac{2\sqrt{5}\,x^{3}{}_{2}F_{1}\left(\frac{3}{4},\frac{3}{2};\frac{7}{4};-\frac{x^{4}}{5}\right)}{75}$
risch	$\frac{x(2x^{2}-15)}{10\sqrt{x^{4}+5}}+\frac{3\sqrt{5}\,\sqrt{25-5i\sqrt{5}\,x^{2}}\,\sqrt{25+5i\sqrt{5}\,x^{2}}\,F\left(\frac{x\sqrt{5}\,\sqrt{i\sqrt{5}}}{5},i\right)}{50\sqrt{i\sqrt{5}}\,\sqrt{x^{4}+5}}-\frac{i\sqrt{25-5i\sqrt{5}\,x^{2}}\,\sqrt{25+5i\sqrt{5}\,x^{2}}\left(F\left(\frac{x\sqrt{5}\,\sqrt{i\sqrt{5}}}{5},i\right)-E\left(\frac{x\sqrt{5}}{5}\right.\right.}{25\sqrt{i\sqrt{5}}\,\sqrt{x^{4}+5}}$
elliptic	$-\frac{2\left(-\frac{1}{10}x^{3}+\frac{3}{4}x\right)}{\sqrt{x^{4}+5}}+\frac{3\sqrt{5}\,\sqrt{25-5i\sqrt{5}\,x^{2}}\,\sqrt{25+5i\sqrt{5}\,x^{2}}\,F\left(\frac{x\sqrt{5}\,\sqrt{i\sqrt{5}}}{5},i\right)}{50\sqrt{i\sqrt{5}}\,\sqrt{x^{4}+5}}-\frac{i\sqrt{25-5i\sqrt{5}\,x^{2}}\,\sqrt{25+5i\sqrt{5}\,x^{2}}\left(F\left(\frac{x\sqrt{5}\,\sqrt{i\sqrt{5}}}{5},i\right)-E\right.}{25\sqrt{i\sqrt{5}}\,\sqrt{x^{4}+5}}$
default	$-\frac{3x}{2\sqrt{x^{4}+5}}+\frac{3\sqrt{5}\,\sqrt{25-5i\sqrt{5}\,x^{2}}\,\sqrt{25+5i\sqrt{5}\,x^{2}}\,F\left(\frac{x\sqrt{5}\,\sqrt{i\sqrt{5}}}{5},i\right)}{50\sqrt{i\sqrt{5}}\,\sqrt{x^{4}+5}}+\frac{x^{3}}{5\sqrt{x^{4}+5}}-\frac{i\sqrt{25-5i\sqrt{5}\,x^{2}}\,\sqrt{25+5i\sqrt{5}\,x^{2}}\left(F\left(\frac{x\sqrt{5}\,\sqrt{i\sqrt{5}}}{5},\right.\right.}{25\sqrt{i\sqrt{5}}\,\sqrt{x^{4}+5}}$

```
[In] int(x^2*(3*x^2+2)/(x^4+5)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 3/125*5^(1/2)*x^5*hypergeom([5/4,3/2],[9/4],-1/5*x^4)+2/75*5^(1/2)*x^3*hype
rgeom([3/4,3/2],[7/4],-1/5*x^4)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.08 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.55

$$\int \frac{x^2(2+3x^2)}{(5+x^4)^{3/2}}\,dx =$$
$$-\frac{2\sqrt{5}(-i\,x^4-5i)\sqrt{i\sqrt{5}}E(\arcsin\left(\frac{1}{5}\sqrt{5}x\sqrt{i\sqrt{5}}\right)|-1)+17\sqrt{5}(i\,x^4+5i)\sqrt{i\sqrt{5}}F(\arcsin\left(\frac{1}{5}\sqrt{5}x\sqrt{i\sqrt{5}}\right)}{50\,(x^4+5)}$$

```
[In] integrate(x^2*(3*x^2+2)/(x^4+5)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/50*(2*sqrt(5)*(-I*x^4 - 5*I)*sqrt(I*sqrt(5))*elliptic_e(arcsin(1/5*sqrt(
5)*x*sqrt(I*sqrt(5))), -1) + 17*sqrt(5)*(I*x^4 + 5*I)*sqrt(I*sqrt(5))*ellip
tic_f(arcsin(1/5*sqrt(5)*x*sqrt(I*sqrt(5))), -1) - 5*sqrt(x^4 + 5)*(2*x^3 -
 15*x))/(x^4 + 5)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.99 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.42

$$\int \frac{x^2(2+3x^2)}{(5+x^4)^{3/2}}\,dx = \frac{3\sqrt{5}x^5\Gamma\left(\frac{5}{4}\right){}_2F_1\left(\begin{matrix}\frac{5}{4},\frac{3}{2}\\\frac{9}{4}\end{matrix}\middle|\frac{x^4e^{i\pi}}{5}\right)}{100\Gamma\left(\frac{9}{4}\right)} + \frac{\sqrt{5}x^3\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\begin{matrix}\frac{3}{4},\frac{3}{2}\\\frac{7}{4}\end{matrix}\middle|\frac{x^4e^{i\pi}}{5}\right)}{50\Gamma\left(\frac{7}{4}\right)}$$

```
[In] integrate(x**2*(3*x**2+2)/(x**4+5)**(3/2),x)
[Out] 3*sqrt(5)*x**5*gamma(5/4)*hyper((5/4, 3/2), (9/4,), x**4*exp_polar(I*pi)/5)
/(100*gamma(9/4)) + sqrt(5)*x**3*gamma(3/4)*hyper((3/4, 3/2), (7/4,), x**4*
exp_polar(I*pi)/5)/(50*gamma(7/4))
```

Maxima [F]

$$\int \frac{x^2(2+3x^2)}{(5+x^4)^{3/2}}\,dx = \int \frac{(3\,x^2+2)x^2}{(x^4+5)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(x^2*(3*x^2+2)/(x^4+5)^(3/2),x, algorithm="maxima")
[Out] integrate((3*x^2 + 2)*x^2/(x^4 + 5)^(3/2), x)
```

Giac [F]

$$\int \frac{x^2(2+3x^2)}{(5+x^4)^{3/2}}\,dx = \int \frac{(3\,x^2+2)x^2}{(x^4+5)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(x^2*(3*x^2+2)/(x^4+5)^(3/2),x, algorithm="giac")
[Out] integrate((3*x^2 + 2)*x^2/(x^4 + 5)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2(2+3x^2)}{(5+x^4)^{3/2}}\,dx = \int \frac{x^2\,(3\,x^2+2)}{(x^4+5)^{3/2}}\,dx$$

```
[In] int((x^2*(3*x^2 + 2))/(x^4 + 5)^(3/2),x)
[Out] int((x^2*(3*x^2 + 2))/(x^4 + 5)^(3/2), x)
```

3.52 $\int \frac{2+3x^2}{\left(5+x^4\right)^{3/2}}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 180

$$\int \frac{2+3x^2}{\left(5+x^4\right)^{3/2}}\,dx=\frac{x(2+3x^2)}{10\sqrt{5+x^4}}-\frac{3x\sqrt{5+x^4}}{10\left(\sqrt{5}+x^2\right)}+\frac{3\left(\sqrt{5}+x^2\right)\sqrt{\frac{5+x^4}{\left(\sqrt{5}+x^2\right)^2}}E\left(2\arctan\left(\frac{x}{\sqrt[4]{5}}\right)|\frac{1}{2}\right)}{2\,5^{3/4}\sqrt{5+x^4}}+\frac{\left(2-3\sqrt{5}\right)\left(\sqrt{5}+x^2\right)\sqrt{\frac{5+x^4}{\left(\sqrt{5}+x^2\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{x}{\sqrt[4]{5}}\right),\frac{1}{2}\right)}{20\sqrt[4]{5}\sqrt{5+x^4}}$$

```
[Out] 1/10*x*(3*x^2+2)/(x^4+5)^(1/2)-3/10*x*(x^4+5)^(1/2)/(x^2+5^(1/2))+3/10*5^(1
/4)*(cos(2*arctan(1/5*x*5^(3/4)))^2)^(1/2)/cos(2*arctan(1/5*x*5^(3/4)))*Ell
ipticE(sin(2*arctan(1/5*x*5^(3/4))),1/2*2^(1/2))*(x^2+5^(1/2))*((x^4+5)/(x^
2+5^(1/2))^2)^(1/2)/(x^4+5)^(1/2)+1/100*(cos(2*arctan(1/5*x*5^(3/4)))^2)^(1
/2)/cos(2*arctan(1/5*x*5^(3/4)))*EllipticF(sin(2*arctan(1/5*x*5^(3/4))),1/2
*2^(1/2))*(2-3*5^(1/2))*(x^2+5^(1/2))*((x^4+5)/(x^2+5^(1/2))^2)^(1/2)*5^(3/
4)/(x^4+5)^(1/2)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 180, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.235$, Rules used = {1193, 1212, 226, 1210}

$$\int \frac{2+3x^2}{(5+x^4)^{3/2}}\,dx = \frac{\left(2-3\sqrt{5}\right)\left(x^2+\sqrt{5}\right)\sqrt{\frac{x^4+5}{\left(x^2+\sqrt{5}\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{x}{\sqrt[4]{5}}\right),\frac{1}{2}\right)}{20\sqrt[4]{5}\sqrt{x^4+5}} + \frac{3\left(x^2+\sqrt{5}\right)\sqrt{\frac{x^4+5}{\left(x^2+\sqrt{5}\right)^2}}E\left(2\arctan\left(\frac{x}{\sqrt[4]{5}}\right)|\frac{1}{2}\right)}{2\ 5^{3/4}\sqrt{x^4+5}} - \frac{3\sqrt{x^4+5}x}{10\left(x^2+\sqrt{5}\right)} + \frac{\left(3x^2+2\right)x}{10\sqrt{x^4+5}}$$

```
[In] Int[(2 + 3*x^2)/(5 + x^4)^(3/2),x]
```

```
[Out] (x*(2 + 3*x^2))/(10*Sqrt[5 + x^4]) - (3*x*Sqrt[5 + x^4])/(10*(Sqrt[5] + x^2
)) + (3*(Sqrt[5] + x^2)*Sqrt[(5 + x^4)/(Sqrt[5] + x^2)^2]*EllipticE[2*ArcTa
n[x/5^(1/4)], 1/2])/(2*5^(3/4)*Sqrt[5 + x^4]) + ((2 - 3*Sqrt[5])*(Sqrt[5] +
 x^2)*Sqrt[(5 + x^4)/(Sqrt[5] + x^2)^2]*EllipticF[2*ArcTan[x/5^(1/4)], 1/2]
)/(20*5^(1/4)*Sqrt[5 + x^4])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 1193

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[(-x
)*(d + e*x^2)*((a + c*x^4)^(p + 1)/(4*a*(p + 1))), x] + Dist[1/(4*a*(p + 1)
), Int[Simp[d*(4*p + 5) + e*(4*p + 7)*x^2, x]*(a + c*x^4)^(p + 1), x], x] /
; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && IntegerQ
[2*p]
```

Rule 1210

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Rule 1212

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + c*x^4], x], x] - Dist[e/q, I
```

```
nt[(1 - q*x^2)/Sqrt[a + c*x^4], x], x] /; NeQ[e + d*q, 0]] /; FreeQ[{a, c,
d, e}, x] && PosQ[c/a]
```

Rubi steps

$$\text{integral} = \frac{x(2+3x^2)}{10\sqrt{5+x^4}} - \frac{1}{10}\int \frac{-2+3x^2}{\sqrt{5+x^4}}\,dx$$

$$= \frac{x(2+3x^2)}{10\sqrt{5+x^4}} + \frac{3\int \frac{1-\frac{x^2}{\sqrt{5}}}{\sqrt{5+x^4}}\,dx}{2\sqrt{5}} - \frac{1}{10}\left(-2+3\sqrt{5}\right)\int \frac{1}{\sqrt{5+x^4}}\,dx$$

$$= \frac{x(2+3x^2)}{10\sqrt{5+x^4}} - \frac{3x\sqrt{5+x^4}}{10\left(\sqrt{5}+x^2\right)} + \frac{3\left(\sqrt{5}+x^2\right)\sqrt{\frac{5+x^4}{\left(\sqrt{5}+x^2\right)^2}}E\left(2\tan^{-1}\left(\frac{x}{\sqrt[4]{5}}\right)|\frac{1}{2}\right)}{2\ 5^{3/4}\sqrt{5+x^4}}$$
$$+ \frac{\left(2-3\sqrt{5}\right)\left(\sqrt{5}+x^2\right)\sqrt{\frac{5+x^4}{\left(\sqrt{5}+x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{x}{\sqrt[4]{5}}\right)|\frac{1}{2}\right)}{20\sqrt[4]{5}\sqrt{5+x^4}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.03 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.37

$$\int \frac{2+3x^2}{(5+x^4)^{3/2}}\,dx = \frac{1}{25}x\left(\frac{5}{\sqrt{5+x^4}} + \sqrt{5}\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{5}{4},-\frac{x^4}{5}\right) + \sqrt{5}x^2\,\text{Hypergeometric2F1}\left(\frac{3}{4},\frac{3}{2},\frac{7}{4},-\frac{x^4}{5}\right)\right)$$

```
[In] Integrate[(2 + 3*x^2)/(5 + x^4)^(3/2),x]
[Out] (x*(5/Sqrt[5 + x^4] + Sqrt[5]*Hypergeometric2F1[1/4, 1/2, 5/4, -1/5*x^4] +
Sqrt[5]*x^2*Hypergeometric2F1[3/4, 3/2, 7/4, -1/5*x^4]))/25
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4.

Time = 1.57 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.21

method	result
meijerg	$\frac{2\sqrt{5}\,x\,{}_2F_1\left(\frac{1}{4},\frac{3}{2};\frac{5}{4};-\frac{x^4}{5}\right)}{25}+\frac{\sqrt{5}\,x^3\,{}_2F_1\left(\frac{3}{4},\frac{3}{2};\frac{7}{4};-\frac{x^4}{5}\right)}{25}$
risch	$\frac{x(3x^2+2)}{10\sqrt{x^4+5}}+\frac{\sqrt{5}\,\sqrt{25-5i\sqrt{5}\,x^2}\,\sqrt{25+5i\sqrt{5}\,x^2}\,F\left(\frac{x\sqrt{5}\,\sqrt{i\sqrt{5}}}{5},i\right)}{125\sqrt{i\sqrt{5}}\,\sqrt{x^4+5}}-\frac{3i\sqrt{25-5i\sqrt{5}\,x^2}\,\sqrt{25+5i\sqrt{5}\,x^2}\left(F\left(\frac{x\sqrt{5}\,\sqrt{i\sqrt{5}}}{5},i\right)-E\left(\frac{x\sqrt{5}\,\sqrt{}}{5}\right.\right.}{50\sqrt{i\sqrt{5}}\,\sqrt{x^4+5}}$
elliptic	$-\frac{2\left(-\frac{3}{20}x^3-\frac{1}{10}x\right)}{\sqrt{x^4+5}}+\frac{\sqrt{5}\,\sqrt{25-5i\sqrt{5}\,x^2}\,\sqrt{25+5i\sqrt{5}\,x^2}\,F\left(\frac{x\sqrt{5}\,\sqrt{i\sqrt{5}}}{5},i\right)}{125\sqrt{i\sqrt{5}}\,\sqrt{x^4+5}}-\frac{3i\sqrt{25-5i\sqrt{5}\,x^2}\,\sqrt{25+5i\sqrt{5}\,x^2}\left(F\left(\frac{x\sqrt{5}\,\sqrt{i\sqrt{5}}}{5},i\right)-E\right.}{50\sqrt{i\sqrt{5}}\,\sqrt{x^4+5}}$
default	$\frac{x}{5\sqrt{x^4+5}}+\frac{\sqrt{5}\,\sqrt{25-5i\sqrt{5}\,x^2}\,\sqrt{25+5i\sqrt{5}\,x^2}\,F\left(\frac{x\sqrt{5}\,\sqrt{i\sqrt{5}}}{5},i\right)}{125\sqrt{i\sqrt{5}}\,\sqrt{x^4+5}}+\frac{3x^3}{10\sqrt{x^4+5}}-\frac{3i\sqrt{25-5i\sqrt{5}\,x^2}\,\sqrt{25+5i\sqrt{5}\,x^2}\left(F\left(\frac{x\sqrt{5}\,\sqrt{i\sqrt{5}}}{5},i\right.\right.}{50\sqrt{i\sqrt{5}}\,\sqrt{x^4+5}}$

```
[In] int((3*x^2+2)/(x^4+5)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/25*5^(1/2)*x*hypergeom([1/4,3/2],[5/4],-1/5*x^4)+1/25*5^(1/2)*x^3*hyperge
om([3/4,3/2],[7/4],-1/5*x^4)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.08 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.54

$$\int \frac{2+3x^2}{(5+x^4)^{3/2}}\,dx = -\frac{3\sqrt{5}(-i\,x^4-5i)\sqrt{i\sqrt{5}}E(\arcsin\left(\frac{1}{5}\sqrt{5}x\sqrt{i\sqrt{5}}\right)|-1)+5\sqrt{5}(i\,x^4+5i)\sqrt{i\sqrt{5}}F(\arcsin\left(\frac{1}{5}\sqrt{5}x\sqrt{i\sqrt{5}}\right)}{50\,(x^4+5)}$$

```
[In] integrate((3*x^2+2)/(x^4+5)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/50*(3*sqrt(5)*(-I*x^4 - 5*I)*sqrt(I*sqrt(5))*elliptic_e(arcsin(1/5*sqrt(
5)*x*sqrt(I*sqrt(5))), -1) + 5*sqrt(5)*(I*x^4 + 5*I)*sqrt(I*sqrt(5))*ellipt
ic_f(arcsin(1/5*sqrt(5)*x*sqrt(I*sqrt(5))), -1) - 5*sqrt(x^4 + 5)*(3*x^3 +
2*x))/(x^4 + 5)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.97 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.41

$$\int \frac{2+3x^2}{(5+x^4)^{3/2}}\,dx = \frac{3\sqrt{5}x^3\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\begin{matrix}\frac{3}{4},\frac{3}{2}\\ \frac{7}{4}\end{matrix}\middle|\frac{x^4e^{i\pi}}{5}\right)}{100\Gamma\left(\frac{7}{4}\right)}+\frac{\sqrt{5}x\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{4},\frac{3}{2}\\ \frac{5}{4}\end{matrix}\middle|\frac{x^4e^{i\pi}}{5}\right)}{50\Gamma\left(\frac{5}{4}\right)}$$

```
[In] integrate((3*x**2+2)/(x**4+5)**(3/2),x)
```

```
[Out] 3*sqrt(5)*x**3*gamma(3/4)*hyper((3/4, 3/2), (7/4,), x**4*exp_polar(I*pi)/5)
/(100*gamma(7/4)) + sqrt(5)*x*gamma(1/4)*hyper((1/4, 3/2), (5/4,), x**4*exp
_polar(I*pi)/5)/(50*gamma(5/4))
```

Maxima [F]

$$\int \frac{2+3x^2}{(5+x^4)^{3/2}}\,dx = \int \frac{3\,x^2+2}{(x^4+5)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((3*x^2+2)/(x^4+5)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((3*x^2 + 2)/(x^4 + 5)^(3/2), x)
```

Giac [F]

$$\int \frac{2+3x^2}{(5+x^4)^{3/2}}\,dx = \int \frac{3\,x^2+2}{(x^4+5)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((3*x^2+2)/(x^4+5)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((3*x^2 + 2)/(x^4 + 5)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{2+3x^2}{(5+x^4)^{3/2}}\,dx = \int \frac{3\,x^2+2}{(x^4+5)^{3/2}}\,dx$$

```
[In] int((3*x^2 + 2)/(x^4 + 5)^(3/2),x)
```

```
[Out] int((3*x^2 + 2)/(x^4 + 5)^(3/2), x)
```

3.53 $\int \frac{2+3x^2}{x^2(5+x^4)^{3/2}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 196

$$\int \frac{2+3x^2}{x^2(5+x^4)^{3/2}}\,dx = \frac{2+3x^2}{10x\sqrt{5+x^4}} - \frac{3\sqrt{5+x^4}}{25x} + \frac{3x\sqrt{5+x^4}}{25\left(\sqrt{5}+x^2\right)} - \frac{3\left(\sqrt{5}+x^2\right)\sqrt{\frac{5+x^4}{\left(\sqrt{5}+x^2\right)^2}}E\left(2\arctan\left(\frac{x}{\sqrt[4]{5}}\right)|\frac{1}{2}\right)}{5\ 5^{3/4}\sqrt{5+x^4}} + \frac{3\left(2+\sqrt{5}\right)\left(\sqrt{5}+x^2\right)\sqrt{\frac{5+x^4}{\left(\sqrt{5}+x^2\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{x}{\sqrt[4]{5}}\right),\frac{1}{2}\right)}{20\ 5^{3/4}\sqrt{5+x^4}}$$

```
[Out] 1/10*(3*x^2+2)/x/(x^4+5)^(1/2)-3/25*(x^4+5)^(1/2)/x+3/25*x*(x^4+5)^(1/2)/(x
^2+5^(1/2))-3/25*5^(1/4)*(cos(2*arctan(1/5*x*5^(3/4)))^2)^(1/2)/cos(2*arcta
n(1/5*x*5^(3/4)))*EllipticE(sin(2*arctan(1/5*x*5^(3/4))),1/2*2^(1/2))*(x^2+
5^(1/2))*((x^4+5)/(x^2+5^(1/2))^2)^(1/2)/(x^4+5)^(1/2)+3/100*5^(1/4)*(cos(2
*arctan(1/5*x*5^(3/4)))^2)^(1/2)/cos(2*arctan(1/5*x*5^(3/4)))*EllipticF(sin
(2*arctan(1/5*x*5^(3/4))),1/2*2^(1/2))*(2+5^(1/2))*(x^2+5^(1/2))*((x^4+5)/(
x^2+5^(1/2))^2)^(1/2)/(x^4+5)^(1/2)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 196, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {1292, 1296, 1212, 226, 1210}

$$\int \frac{2+3x^2}{x^2\left(5+x^4\right)^{3/2}}\,dx = \frac{3\left(2+\sqrt{5}\right)\left(x^2+\sqrt{5}\right)\sqrt{\frac{x^4+5}{\left(x^2+\sqrt{5}\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{x}{\sqrt[4]{5}}\right),\frac{1}{2}\right)}{20\ 5^{3/4}\sqrt{x^4+5}} - \frac{3\left(x^2+\sqrt{5}\right)\sqrt{\frac{x^4+5}{\left(x^2+\sqrt{5}\right)^2}}E\left(2\arctan\left(\frac{x}{\sqrt[4]{5}}\right)|\frac{1}{2}\right)}{5\ 5^{3/4}\sqrt{x^4+5}} - \frac{3\sqrt{x^4+5}}{25x} + \frac{3\sqrt{x^4+5}x}{25\left(x^2+\sqrt{5}\right)} + \frac{3x^2+2}{10\sqrt{x^4+5}x}$$

```
[In] Int[(2 + 3*x^2)/(x^2*(5 + x^4)^(3/2)),x]
```

```
[Out] (2 + 3*x^2)/(10*x*Sqrt[5 + x^4]) - (3*Sqrt[5 + x^4])/(25*x) + (3*x*Sqrt[5 +
 x^4])/(25*(Sqrt[5] + x^2)) - (3*(Sqrt[5] + x^2)*Sqrt[(5 + x^4)/(Sqrt[5] +
x^2)^2]*EllipticE[2*ArcTan[x/5^(1/4)], 1/2])/(5*5^(3/4)*Sqrt[5 + x^4]) + (3
*(2 + Sqrt[5])*(Sqrt[5] + x^2)*Sqrt[(5 + x^4)/(Sqrt[5] + x^2)^2]*EllipticF[
2*ArcTan[x/5^(1/4)], 1/2])/(20*5^(3/4)*Sqrt[5 + x^4])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 1210

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Rule 1212

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + c*x^4], x], x] - Dist[e/q, I
nt[(1 - q*x^2)/Sqrt[a + c*x^4], x], x] /; NeQ[e + d*q, 0]] /; FreeQ[{a, c,
d, e}, x] && PosQ[c/a]
```

Rule 1292

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (c_.)*(x_)^4)^(p_), x_
Symbol] :> Simp[(-(f*x)^(m + 1))*(a + c*x^4)^(p + 1)*((d + e*x^2)/(4*a*f*(p
```

```
 + 1))), x] + Dist[1/(4*a*(p + 1)), Int[(f*x)^m*(a + c*x^4)^(p + 1)*Simp[d*
(m + 4*(p + 1) + 1) + e*(m + 2*(2*p + 3) + 1)*x^2, x], x], x] /; FreeQ[{a,
c, d, e, f, m}, x] && LtQ[p, -1] && IntegerQ[2*p] && (IntegerQ[p] || Intege
rQ[m])
```

Rule 1296

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (c_.)*(x_)^4)^(p_), x_
Symbol] :> Simp[d*(f*x)^(m + 1)*((a + c*x^4)^(p + 1)/(a*f*(m + 1))), x] + D
ist[1/(a*f^2*(m + 1)), Int[(f*x)^(m + 2)*(a + c*x^4)^p*(a*e*(m + 1) - c*d*(
m + 4*p + 5)*x^2), x], x] /; FreeQ[{a, c, d, e, f, p}, x] && LtQ[m, -1] &&
IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{2+3x^2}{10x\sqrt{5+x^4}}-\frac{1}{10}\int\frac{-6-3x^2}{x^2\sqrt{5+x^4}}\,dx\\
&= \frac{2+3x^2}{10x\sqrt{5+x^4}}-\frac{3\sqrt{5+x^4}}{25x}+\frac{1}{50}\int\frac{15+6x^2}{\sqrt{5+x^4}}\,dx\\
&= \frac{2+3x^2}{10x\sqrt{5+x^4}}-\frac{3\sqrt{5+x^4}}{25x}-\frac{3\int\frac{1-\frac{x^2}{\sqrt{5}}}{\sqrt{5+x^4}}\,dx}{5\sqrt{5}}+\frac{1}{50}\left(3\left(5+2\sqrt{5}\right)\right)\int\frac{1}{\sqrt{5+x^4}}\,dx\\
&= \frac{2+3x^2}{10x\sqrt{5+x^4}}-\frac{3\sqrt{5+x^4}}{25x}+\frac{3x\sqrt{5+x^4}}{25\left(\sqrt{5}+x^2\right)}\\
&\quad-\frac{3\left(\sqrt{5}+x^2\right)\sqrt{\frac{5+x^4}{\left(\sqrt{5}+x^2\right)^2}}E\left(2\tan^{-1}\left(\frac{x}{\sqrt[4]{5}}\right)|\frac{1}{2}\right)}{5\ 5^{3/4}\sqrt{5+x^4}}\\
&\quad+\frac{3\left(2+\sqrt{5}\right)\left(\sqrt{5}+x^2\right)\sqrt{\frac{5+x^4}{\left(\sqrt{5}+x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{x}{\sqrt[4]{5}}\right)|\frac{1}{2}\right)}{20\ 5^{3/4}\sqrt{5+x^4}}
\end{aligned}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.05 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.36

$$\int\frac{2+3x^2}{x^2\left(5+x^4\right)^{3/2}}\,dx=\frac{3x}{10\sqrt{5+x^4}}-\frac{2\,\text{Hypergeometric2F1}\left(-\frac{1}{4},\frac{3}{2},\frac{3}{4},-\frac{x^4}{5}\right)}{5\sqrt{5}x}+\frac{3x\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{5}{4},-\frac{x^4}{5}\right)}{10\sqrt{5}}$$

```
[In] Integrate[(2 + 3*x^2)/(x^2*(5 + x^4)^(3/2)),x]
[Out] (3*x)/(10*Sqrt[5 + x^4]) - (2*Hypergeometric2F1[-1/4, 3/2, 3/4, -1/5*x^4])/
(5*Sqrt[5]*x) + (3*x*Hypergeometric2F1[1/4, 1/2, 5/4, -1/5*x^4])/(10*Sqrt[5
])
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4.

Time = 0.94 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.19

method	result
meijerg	$-\frac{2\sqrt{5}\,{}_2F_1\left(-\frac{1}{4},\frac{3}{2};\frac{3}{4};-\frac{x^4}{5}\right)}{25x}+\frac{3\sqrt{5}\,x\,{}_2F_1\left(\frac{1}{4},\frac{3}{2};\frac{5}{4};-\frac{x^4}{5}\right)}{25}$
risch	$-\frac{6x^4-15x^2+20}{50x\sqrt{x^4+5}}+\frac{3\sqrt{5}\,\sqrt{25-5i\sqrt{5}\,x^2}\,\sqrt{25+5i\sqrt{5}\,x^2}\,F\left(\frac{x\sqrt{5}\,\sqrt{i\sqrt{5}}}{5},i\right)}{250\sqrt{i\sqrt{5}}\,\sqrt{x^4+5}}+\frac{3i\sqrt{25-5i\sqrt{5}\,x^2}\,\sqrt{25+5i\sqrt{5}\,x^2}\,\left(F\left(\frac{x\sqrt{5}\,\sqrt{i\sqrt{5}}}{5},i\right)-E\left(\right.\right.}{125\sqrt{i\sqrt{5}}\,\sqrt{x^4+5}}$
elliptic	$-\frac{2\sqrt{x^4+5}}{25x}-\frac{2\left(\frac{1}{50}x^3-\frac{3}{20}x\right)}{\sqrt{x^4+5}}+\frac{3\sqrt{5}\,\sqrt{25-5i\sqrt{5}\,x^2}\,\sqrt{25+5i\sqrt{5}\,x^2}\,F\left(\frac{x\sqrt{5}\,\sqrt{i\sqrt{5}}}{5},i\right)}{250\sqrt{i\sqrt{5}}\,\sqrt{x^4+5}}+\frac{3i\sqrt{25-5i\sqrt{5}\,x^2}\,\sqrt{25+5i\sqrt{5}\,x^2}\,\left(F\left(\frac{x\sqrt{5}\,\sqrt{}}{5}\right.\right.}{125\sqrt{i\sqrt{5}}\,\sqrt{x^4+}}$
default	$\frac{3x}{10\sqrt{x^4+5}}+\frac{3\sqrt{5}\,\sqrt{25-5i\sqrt{5}\,x^2}\,\sqrt{25+5i\sqrt{5}\,x^2}\,F\left(\frac{x\sqrt{5}\,\sqrt{i\sqrt{5}}}{5},i\right)}{250\sqrt{i\sqrt{5}}\,\sqrt{x^4+5}}-\frac{2\sqrt{x^4+5}}{25x}-\frac{x^3}{25\sqrt{x^4+5}}+\frac{3i\sqrt{25-5i\sqrt{5}\,x^2}\,\sqrt{25+5i\sqrt{5}\,x^2}\,\left(F\right.}{125\sqrt{i\sqrt{}}}$

```
[In] int((3*x^2+2)/x^2/(x^4+5)^(3/2),x,method=_RETURNVERBOSE)
[Out] -2/25*5^(1/2)/x*hypergeom([-1/4,3/2],[3/4],-1/5*x^4)+3/25*5^(1/2)*x*hyperge
om([1/4,3/2],[5/4],-1/5*x^4)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.09 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.55

$$\int \frac{2+3x^2}{x^2\left(5+x^4\right)^{3/2}}\,dx =$$
$$-\frac{6\sqrt{5}(i\,x^5+5i\,x)\sqrt{i\sqrt{5}}E(\arcsin\left(\frac{1}{5}\sqrt{5}x\sqrt{i\sqrt{5}}\right)\mid -1)+9\sqrt{5}(i\,x^5+5i\,x)\sqrt{i\sqrt{5}}F(\arcsin\left(\frac{1}{5}\sqrt{5}x\sqrt{i\sqrt{5}}\right)}{250\left(x^5+5\,x\right)}$$

```
[In] integrate((3*x^2+2)/x^2/(x^4+5)^(3/2),x, algorithm="fricas")
[Out] -1/250*(6*sqrt(5)*(I*x^5 + 5*I*x)*sqrt(I*sqrt(5))*elliptic_e(arcsin(1/5*sqr
t(5)*x*sqrt(I*sqrt(5))), -1) + 9*sqrt(5)*(I*x^5 + 5*I*x)*sqrt(I*sqrt(5))*el
liptic_f(arcsin(1/5*sqrt(5)*x*sqrt(I*sqrt(5))), -1) + 5*(6*x^4 - 15*x^2 + 2
0)*sqrt(x^4 + 5))/(x^5 + 5*x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 2.69 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.38

$$\int \frac{2+3x^2}{x^2\left(5+x^4\right)^{3/2}}\,dx = \frac{3\sqrt{5}x\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{4},\frac{3}{2}\\ \frac{5}{4}\end{matrix}\middle|\frac{x^4e^{i\pi}}{5}\right)}{100\Gamma\left(\frac{5}{4}\right)}+\frac{\sqrt{5}\Gamma\left(-\frac{1}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{4},\frac{3}{2}\\ \frac{3}{4}\end{matrix}\middle|\frac{x^4e^{i\pi}}{5}\right)}{50x\Gamma\left(\frac{3}{4}\right)}$$

```
[In] integrate((3*x**2+2)/x**2/(x**4+5)**(3/2),x)
[Out] 3*sqrt(5)*x*gamma(1/4)*hyper((1/4, 3/2), (5/4,), x**4*exp_polar(I*pi)/5)/(1
00*gamma(5/4)) + sqrt(5)*gamma(-1/4)*hyper((-1/4, 3/2), (3/4,), x**4*exp_po
lar(I*pi)/5)/(50*x*gamma(3/4))
```

Maxima [F]

$$\int \frac{2+3x^2}{x^2\left(5+x^4\right)^{3/2}}\,dx = \int \frac{3\,x^2+2}{\left(x^4+5\right)^{\frac{3}{2}}x^2}\,dx$$

```
[In] integrate((3*x^2+2)/x^2/(x^4+5)^(3/2),x, algorithm="maxima")
[Out] integrate((3*x^2 + 2)/((x^4 + 5)^(3/2)*x^2), x)
```

Giac [F]

$$\int \frac{2+3x^2}{x^2\left(5+x^4\right)^{3/2}}\,dx = \int \frac{3\,x^2+2}{\left(x^4+5\right)^{\frac{3}{2}}x^2}\,dx$$

```
[In] integrate((3*x^2+2)/x^2/(x^4+5)^(3/2),x, algorithm="giac")
[Out] integrate((3*x^2 + 2)/((x^4 + 5)^(3/2)*x^2), x)
```

Mupad [B] (verification not implemented)

Time = 7.95 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.24

$$\int \frac{2+3x^2}{x^2 \left(5+x^4\right)^{3/2}}\,dx = \frac{3\sqrt{5}\,x\,{}_2\mathrm{F}_1\left(\frac{1}{4},\frac{3}{2};\ \frac{5}{4};\ -\frac{x^4}{5}\right)}{25} - \frac{2\left(\frac{5}{x^4}+1\right)^{3/2}{}_2\mathrm{F}_1\left(\frac{3}{2},\frac{7}{4};\ \frac{11}{4};\ -\frac{5}{x^4}\right)}{7\,x\left(x^4+5\right)^{3/2}}$$

```
[In] int((3*x^2 + 2)/(x^2*(x^4 + 5)^(3/2)),x)
```

```
[Out] (3*5^(1/2)*x*hypergeom([1/4, 3/2], 5/4, -x^4/5))/25 - (2*(5/x^4 + 1)^(3/2)*
hypergeom([3/2, 7/4], 11/4, -5/x^4))/(7*x*(x^4 + 5)^(3/2))
```

3.54 $\int \frac{2+3x^2}{x^4\left(5+x^4\right)^{3/2}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 214

$$\int \frac{2+3x^2}{x^4\left(5+x^4\right)^{3/2}}\,dx = \frac{2+3x^2}{10x^3\sqrt{5+x^4}} - \frac{\sqrt{5+x^4}}{15x^3} - \frac{9\sqrt{5+x^4}}{50x} + \frac{9x\sqrt{5+x^4}}{50\left(\sqrt{5}+x^2\right)} - \frac{9\left(\sqrt{5}+x^2\right)\sqrt{\frac{5+x^4}{\left(\sqrt{5}+x^2\right)^2}}\,E\left(2\arctan\left(\frac{x}{\sqrt[4]{5}}\right)|\frac{1}{2}\right)}{10\ 5^{3/4}\sqrt{5+x^4}} + \frac{\left(27-2\sqrt{5}\right)\left(\sqrt{5}+x^2\right)\sqrt{\frac{5+x^4}{\left(\sqrt{5}+x^2\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{x}{\sqrt[4]{5}}\right),\frac{1}{2}\right)}{60\ 5^{3/4}\sqrt{5+x^4}}$$

```
[Out] 1/10*(3*x^2+2)/x^3/(x^4+5)^(1/2)-1/15*(x^4+5)^(1/2)/x^3-9/50*(x^4+5)^(1/2)/
x+9/50*x*(x^4+5)^(1/2)/(x^2+5^(1/2))-9/50*5^(1/4)*(cos(2*arctan(1/5*x*5^(3/
4)))^2)^(1/2)/cos(2*arctan(1/5*x*5^(3/4)))*EllipticE(sin(2*arctan(1/5*x*5^(
3/4))),1/2*2^(1/2))*(x^2+5^(1/2))*((x^4+5)/(x^2+5^(1/2))^2)^(1/2)/(x^4+5)^(
1/2)+1/300*(cos(2*arctan(1/5*x*5^(3/4)))^2)^(1/2)/cos(2*arctan(1/5*x*5^(3/4
)))*EllipticF(sin(2*arctan(1/5*x*5^(3/4))),1/2*2^(1/2))*(27-2*5^(1/2))*(x^2
+5^(1/2))*((x^4+5)/(x^2+5^(1/2))^2)^(1/2)*5^(1/4)/(x^4+5)^(1/2)
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 214, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {1292, 1296, 1212, 226, 1210}

$$\int \frac{2+3x^2}{x^4\left(5+x^4\right)^{3/2}}\,dx = \frac{\left(27-2\sqrt{5}\right)\left(x^2+\sqrt{5}\right)\sqrt{\frac{x^4+5}{\left(x^2+\sqrt{5}\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{x}{\sqrt[4]{5}}\right),\frac{1}{2}\right)}{60\ 5^{3/4}\sqrt{x^4+5}} - \frac{9\left(x^2+\sqrt{5}\right)\sqrt{\frac{x^4+5}{\left(x^2+\sqrt{5}\right)^2}}E\left(2\arctan\left(\frac{x}{\sqrt[4]{5}}\right)|\frac{1}{2}\right)}{10\ 5^{3/4}\sqrt{x^4+5}} - \frac{9\sqrt{x^4+5}}{50x} - \frac{\sqrt{x^4+5}}{15x^3} + \frac{9\sqrt{x^4+5}x}{50\left(x^2+\sqrt{5}\right)} + \frac{3x^2+2}{10\sqrt{x^4+5}x^3}$$

```
[In] Int[(2 + 3*x^2)/(x^4*(5 + x^4)^(3/2)),x]
```

```
[Out] (2 + 3*x^2)/(10*x^3*Sqrt[5 + x^4]) - Sqrt[5 + x^4]/(15*x^3) - (9*Sqrt[5 + x
^4])/(50*x) + (9*x*Sqrt[5 + x^4])/(50*(Sqrt[5] + x^2)) - (9*(Sqrt[5] + x^2)
*Sqrt[(5 + x^4)/(Sqrt[5] + x^2)^2]*EllipticE[2*ArcTan[x/5^(1/4)], 1/2])/(10
*5^(3/4)*Sqrt[5 + x^4]) + ((27 - 2*Sqrt[5])*(Sqrt[5] + x^2)*Sqrt[(5 + x^4)/
(Sqrt[5] + x^2)^2]*EllipticF[2*ArcTan[x/5^(1/4)], 1/2])/(60*5^(3/4)*Sqrt[5
+ x^4])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 1210

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Rule 1212

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + c*x^4], x], x] - Dist[e/q, I
nt[(1 - q*x^2)/Sqrt[a + c*x^4], x], x] /; NeQ[e + d*q, 0]] /; FreeQ[{a, c,
d, e}, x] && PosQ[c/a]
```

Rule 1292

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (c_.)*(x_)^4)^(p_), x_
Symbol] :> Simp[(-(f*x)^(m + 1))*(a + c*x^4)^(p + 1)*((d + e*x^2)/(4*a*f*(p
 + 1))), x] + Dist[1/(4*a*(p + 1)), Int[(f*x)^m*(a + c*x^4)^(p + 1)*Simp[d*
(m + 4*(p + 1) + 1) + e*(m + 2*(2*p + 3) + 1)*x^2, x], x], x] /; FreeQ[{a,
c, d, e, f, m}, x] && LtQ[p, -1] && IntegerQ[2*p] && (IntegerQ[p] || Intege
rQ[m])
```

Rule 1296

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (c_.)*(x_)^4)^(p_), x_
Symbol] :> Simp[d*(f*x)^(m + 1)*((a + c*x^4)^(p + 1)/(a*f*(m + 1))), x] + D
ist[1/(a*f^2*(m + 1)), Int[(f*x)^(m + 2)*(a + c*x^4)^p*(a*e*(m + 1) - c*d*(
m + 4*p + 5)*x^2), x], x] /; FreeQ[{a, c, d, e, f, p}, x] && LtQ[m, -1] &&
IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{2+3x^2}{10x^3\sqrt{5+x^4}} - \frac{1}{10}\int \frac{-10-9x^2}{x^4\sqrt{5+x^4}}\,dx \\
&= \frac{2+3x^2}{10x^3\sqrt{5+x^4}} - \frac{\sqrt{5+x^4}}{15x^3} + \frac{1}{150}\int \frac{135-10x^2}{x^2\sqrt{5+x^4}}\,dx \\
&= \frac{2+3x^2}{10x^3\sqrt{5+x^4}} - \frac{\sqrt{5+x^4}}{15x^3} - \frac{9\sqrt{5+x^4}}{50x} - \frac{1}{750}\int \frac{50-135x^2}{\sqrt{5+x^4}}\,dx \\
&= \frac{2+3x^2}{10x^3\sqrt{5+x^4}} - \frac{\sqrt{5+x^4}}{15x^3} - \frac{9\sqrt{5+x^4}}{50x} - \frac{9\int \frac{1-\frac{x^2}{\sqrt{5}}}{\sqrt{5+x^4}}\,dx}{10\sqrt{5}} - \frac{1}{150}\left(10-27\sqrt{5}\right)\int \frac{1}{\sqrt{5+x^4}}\,dx \\
&= \frac{2+3x^2}{10x^3\sqrt{5+x^4}} - \frac{\sqrt{5+x^4}}{15x^3} - \frac{9\sqrt{5+x^4}}{50x} + \frac{9x\sqrt{5+x^4}}{50\left(\sqrt{5}+x^2\right)} \\
&\quad - \frac{9\left(\sqrt{5}+x^2\right)\sqrt{\frac{5+x^4}{\left(\sqrt{5}+x^2\right)^2}}\,E\left(2\tan^{-1}\left(\frac{x}{\sqrt[4]{5}}\right)|\frac{1}{2}\right)}{10\ 5^{3/4}\sqrt{5+x^4}} \\
&\quad + \frac{\left(27-2\sqrt{5}\right)\left(\sqrt{5}+x^2\right)\sqrt{\frac{5+x^4}{\left(\sqrt{5}+x^2\right)^2}}\,F\left(2\tan^{-1}\left(\frac{x}{\sqrt[4]{5}}\right)|\frac{1}{2}\right)}{60\ 5^{3/4}\sqrt{5+x^4}}
\end{aligned}
$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.03 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.25

$$\int \frac{2+3x^2}{x^4\left(5+x^4\right)^{3/2}}\,dx = -\frac{2\,\text{Hypergeometric2F1}\left(-\frac{3}{4},\frac{3}{2},\frac{1}{4},-\frac{x^4}{5}\right)+9x^2\,\text{Hypergeometric2F1}\left(-\frac{1}{4},\frac{3}{2},\frac{3}{4},-\frac{x^4}{5}\right)}{15\sqrt{5}x^3}$$

```
[In] Integrate[(2 + 3*x^2)/(x^4*(5 + x^4)^(3/2)),x]
```

```
[Out] -1/15*(2*Hypergeometric2F1[-3/4, 3/2, 1/4, -1/5*x^4] + 9*x^2*Hypergeometric
2F1[-1/4, 3/2, 3/4, -1/5*x^4])/(Sqrt[5]*x^3)
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4.

Time = 2.08 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.19

method	result
meijerg	$-\frac{2\sqrt{5}\,{}_2F_1\left(-\frac{3}{4},\frac{3}{2};\frac{1}{4};-\frac{x^4}{5}\right)}{75x^3}-\frac{3\sqrt{5}\,{}_2F_1\left(-\frac{1}{4},\frac{3}{2};\frac{3}{4};-\frac{x^4}{5}\right)}{25x}$
risch	$-\frac{27x^6+10x^4+90x^2+20}{150x^3\sqrt{x^4+5}}-\frac{\sqrt{5}\,\sqrt{25-5i\sqrt{5}\,x^2}\,\sqrt{25+5i\sqrt{5}\,x^2}\,F\left(\frac{x\sqrt{5}\,\sqrt{i\sqrt{5}}}{5},i\right)}{375\sqrt{i\sqrt{5}}\,\sqrt{x^4+5}}+\frac{9i\sqrt{25-5i\sqrt{5}\,x^2}\,\sqrt{25+5i\sqrt{5}\,x^2}\,\left(F\left(\frac{x\sqrt{5}\,\sqrt{i\sqrt{5}}}{5},i\right.\right.}{250\sqrt{i\sqrt{5}}\,\sqrt{x^4+5}}$
elliptic	$-\frac{2\sqrt{x^4+5}}{75x^3}-\frac{3\sqrt{x^4+5}}{25x}-\frac{2\left(\frac{3}{100}x^3+\frac{1}{50}x\right)}{\sqrt{x^4+5}}-\frac{\sqrt{5}\,\sqrt{25-5i\sqrt{5}\,x^2}\,\sqrt{25+5i\sqrt{5}\,x^2}\,F\left(\frac{x\sqrt{5}\,\sqrt{i\sqrt{5}}}{5},i\right)}{375\sqrt{i\sqrt{5}}\,\sqrt{x^4+5}}+\frac{9i\sqrt{25-5i\sqrt{5}\,x^2}\,\sqrt{25+5i\sqrt{5}\,x}}{250}$
default	$-\frac{3\sqrt{x^4+5}}{25x}-\frac{3x^3}{50\sqrt{x^4+5}}+\frac{9i\sqrt{25-5i\sqrt{5}\,x^2}\,\sqrt{25+5i\sqrt{5}\,x^2}\,\left(F\left(\frac{x\sqrt{5}\,\sqrt{i\sqrt{5}}}{5},i\right)-E\left(\frac{x\sqrt{5}\,\sqrt{i\sqrt{5}}}{5},i\right)\right)}{250\sqrt{i\sqrt{5}}\,\sqrt{x^4+5}}-\frac{x}{25\sqrt{x^4+5}}-\frac{2\sqrt{x^4+5}}{75x^3}-$

```
[In] int((3*x^2+2)/x^4/(x^4+5)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/75*5^(1/2)/x^3*hypergeom([-3/4,3/2],[1/4],-1/5*x^4)-3/25*5^(1/2)/x*hyper
geom([-1/4,3/2],[3/4],-1/5*x^4)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.09 (sec) , antiderivative size = 118, normalized size of antiderivative = 0.55

$$\int \frac{2+3x^2}{x^4 (5+x^4)^{3/2}} dx = -\frac{27\sqrt{5}(i x^7 + 5i x^3)\sqrt{i\sqrt{5}}E(\arcsin\left(\frac{1}{5}\sqrt{5}x\sqrt{i\sqrt{5}}\right) | -1) + 37\sqrt{5}(-i x^7 - 5i x^3)\sqrt{i\sqrt{5}}F(\arcsin\left(\frac{1}{5}\sqrt{5}x\sqrt{\cdots}\right.}{750 (x^7 + 5x^3)}$$

```
[In] integrate((3*x^2+2)/x^4/(x^4+5)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/750*(27*sqrt(5)*(I*x^7 + 5*I*x^3)*sqrt(I*sqrt(5))*elliptic_e(arcsin(1/5*
sqrt(5)*x*sqrt(I*sqrt(5))), -1) + 37*sqrt(5)*(-I*x^7 - 5*I*x^3)*sqrt(I*sqrt
(5))*elliptic_f(arcsin(1/5*sqrt(5)*x*sqrt(I*sqrt(5))), -1) + 5*(27*x^6 + 10
*x^4 + 90*x^2 + 20)*sqrt(x^4 + 5))/(x^7 + 5*x^3)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 3.00 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.37

$$\int \frac{2+3x^2}{x^4 (5+x^4)^{3/2}} dx = \frac{3\sqrt{5}\Gamma\left(-\frac{1}{4}\right) {}_2F_1\left(\begin{matrix} -\frac{1}{4}, \frac{3}{2} \\ \frac{3}{4} \end{matrix}\middle| \frac{x^4 e^{i\pi}}{5}\right)}{100x\Gamma\left(\frac{3}{4}\right)} + \frac{\sqrt{5}\Gamma\left(-\frac{3}{4}\right) {}_2F_1\left(\begin{matrix} -\frac{3}{4}, \frac{3}{2} \\ \frac{1}{4} \end{matrix}\middle| \frac{x^4 e^{i\pi}}{5}\right)}{50x^3\Gamma\left(\frac{1}{4}\right)}$$

```
[In] integrate((3*x**2+2)/x**4/(x**4+5)**(3/2),x)
```

```
[Out] 3*sqrt(5)*gamma(-1/4)*hyper((-1/4, 3/2), (3/4,), x**4*exp_polar(I*pi)/5)/(1
00*x*gamma(3/4)) + sqrt(5)*gamma(-3/4)*hyper((-3/4, 3/2), (1/4,), x**4*exp_
polar(I*pi)/5)/(50*x**3*gamma(1/4))
```

Maxima [F]

$$\int \frac{2+3x^2}{x^4 (5+x^4)^{3/2}} dx = \int \frac{3x^2+2}{(x^4+5)^{\frac{3}{2}}x^4} dx$$

```
[In] integrate((3*x^2+2)/x^4/(x^4+5)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((3*x^2 + 2)/((x^4 + 5)^(3/2)*x^4), x)
```

Giac [F]

$$\int \frac{2+3x^2}{x^4 (5+x^4)^{3/2}}\,dx = \int \frac{3\,x^2+2}{(x^4+5)^{\frac{3}{2}}x^4}\,dx$$

```
[In] integrate((3*x^2+2)/x^4/(x^4+5)^(3/2),x, algorithm="giac")
[Out] integrate((3*x^2 + 2)/((x^4 + 5)^(3/2)*x^4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{2+3x^2}{x^4 (5+x^4)^{3/2}}\,dx = \int \frac{3\,x^2+2}{x^4\,(x^4+5)^{3/2}}\,dx$$

```
[In] int((3*x^2 + 2)/(x^4*(x^4 + 5)^(3/2)),x)
[Out] int((3*x^2 + 2)/(x^4*(x^4 + 5)^(3/2)), x)
```

3.55 $\int (fx)^m (d+ex^2) (1+2x^2+x^4)^5 \, dx$

Optimal result

Integrand size = 25, antiderivative size = 269

$$\int (fx)^m (d+ex^2) (1+2x^2+x^4)^5 \, dx = \frac{d(fx)^{1+m}}{f(1+m)} + \frac{(10d+e)(fx)^{3+m}}{f^3(3+m)} + \frac{5(9d+2e)(fx)^{5+m}}{f^5(5+m)} + \frac{15(8d+3e)(fx)^{7+m}}{f^7(7+m)} + \frac{30(7d+4e)(fx)^{9+m}}{f^9(9+m)} + \frac{42(6d+5e)(fx)^{11+m}}{f^{11}(11+m)} + \frac{42(5d+6e)(fx)^{13+m}}{f^{13}(13+m)} + \frac{30(4d+7e)(fx)^{15+m}}{f^{15}(15+m)} + \frac{15(3d+8e)(fx)^{17+m}}{f^{17}(17+m)} + \frac{5(2d+9e)(fx)^{19+m}}{f^{19}(19+m)} + \frac{(d+10e)(fx)^{21+m}}{f^{21}(21+m)} + \frac{e(fx)^{23+m}}{f^{23}(23+m)}$$

```
[Out] d*(f*x)^(1+m)/f/(1+m)+(10*d+e)*(f*x)^(3+m)/f^3/(3+m)+5*(9*d+2*e)*(f*x)^(5+m
)/f^5/(5+m)+15*(8*d+3*e)*(f*x)^(7+m)/f^7/(7+m)+30*(7*d+4*e)*(f*x)^(9+m)/f^9
/(9+m)+42*(6*d+5*e)*(f*x)^(11+m)/f^11/(11+m)+42*(5*d+6*e)*(f*x)^(13+m)/f^13
/(13+m)+30*(4*d+7*e)*(f*x)^(15+m)/f^15/(15+m)+15*(3*d+8*e)*(f*x)^(17+m)/f^1
7/(17+m)+5*(2*d+9*e)*(f*x)^(19+m)/f^19/(19+m)+(d+10*e)*(f*x)^(21+m)/f^21/(2
1+m)+e*(f*x)^(23+m)/f^23/(23+m)
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 269, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.080, Rules used = {28, 459}

$$\int (fx)^m (d+ex^2) (1+2x^2+x^4)^5 \, dx = \frac{(d+10e)(fx)^{m+21}}{f^{21}(m+21)} + \frac{5(2d+9e)(fx)^{m+19}}{f^{19}(m+19)} + \frac{15(3d+8e)(fx)^{m+17}}{f^{17}(m+17)} + \frac{30(4d+7e)(fx)^{m+15}}{f^{15}(m+15)} + \frac{42(5d+6e)(fx)^{m+13}}{f^{13}(m+13)} + \frac{42(6d+5e)(fx)^{m+11}}{f^{11}(m+11)} + \frac{30(7d+4e)(fx)^{m+9}}{f^9(m+9)} + \frac{15(8d+3e)(fx)^{m+7}}{f^7(m+7)} + \frac{5(9d+2e)(fx)^{m+5}}{f^5(m+5)} + \frac{(10d+e)(fx)^{m+3}}{f^3(m+3)} + \frac{d(fx)^{m+1}}{f(m+1)} + \frac{e(fx)^{m+23}}{f^{23}(m+23)}$$

```
[In] Int[(f*x)^m*(d + e*x^2)*(1 + 2*x^2 + x^4)^5,x]
```

```
[Out] (d*(f*x)^(1 + m))/(f*(1 + m)) + ((10*d + e)*(f*x)^(3 + m))/(f^3*(3 + m)) +
(5*(9*d + 2*e)*(f*x)^(5 + m))/(f^5*(5 + m)) + (15*(8*d + 3*e)*(f*x)^(7 + m)
)/(f^7*(7 + m)) + (30*(7*d + 4*e)*(f*x)^(9 + m))/(f^9*(9 + m)) + (42*(6*d +
 5*e)*(f*x)^(11 + m))/(f^11*(11 + m)) + (42*(5*d + 6*e)*(f*x)^(13 + m))/(f^
13*(13 + m)) + (30*(4*d + 7*e)*(f*x)^(15 + m))/(f^15*(15 + m)) + (15*(3*d +
 8*e)*(f*x)^(17 + m))/(f^17*(17 + m)) + (5*(2*d + 9*e)*(f*x)^(19 + m))/(f^1
9*(19 + m)) + ((d + 10*e)*(f*x)^(21 + m))/(f^21*(21 + m)) + (e*(f*x)^(23 +
m))/(f^23*(23 + m))
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int (fx)^m \left(1+x^2\right)^{10} \left(d+ex^2\right) \, dx \\
&= \int \left(d(fx)^m + \frac{(10d+e)(fx)^{2+m}}{f^2} + \frac{5(9d+2e)(fx)^{4+m}}{f^4} + \frac{15(8d+3e)(fx)^{6+m}}{f^6} \right. \\
&\quad + \frac{30(7d+4e)(fx)^{8+m}}{f^8} + \frac{42(6d+5e)(fx)^{10+m}}{f^{10}} + \frac{42(5d+6e)(fx)^{12+m}}{f^{12}} \\
&\quad + \frac{30(4d+7e)(fx)^{14+m}}{f^{14}} + \frac{15(3d+8e)(fx)^{16+m}}{f^{16}} + \frac{5(2d+9e)(fx)^{18+m}}{f^{18}} \\
&\quad \left. + \frac{(d+10e)(fx)^{20+m}}{f^{20}} + \frac{e(fx)^{22+m}}{f^{22}} \right) dx \\
&= \frac{d(fx)^{1+m}}{f(1+m)} + \frac{(10d+e)(fx)^{3+m}}{f^3(3+m)} + \frac{5(9d+2e)(fx)^{5+m}}{f^5(5+m)} \\
&\quad + \frac{15(8d+3e)(fx)^{7+m}}{f^7(7+m)} + \frac{30(7d+4e)(fx)^{9+m}}{f^9(9+m)} + \frac{42(6d+5e)(fx)^{11+m}}{f^{11}(11+m)} \\
&\quad + \frac{42(5d+6e)(fx)^{13+m}}{f^{13}(13+m)} + \frac{30(4d+7e)(fx)^{15+m}}{f^{15}(15+m)} + \frac{15(3d+8e)(fx)^{17+m}}{f^{17}(17+m)} \\
&\quad + \frac{5(2d+9e)(fx)^{19+m}}{f^{19}(19+m)} + \frac{(d+10e)(fx)^{21+m}}{f^{21}(21+m)} + \frac{e(fx)^{23+m}}{f^{23}(23+m)}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.43 (sec) , antiderivative size = 189, normalized size of antiderivative = 0.70

$$
\begin{aligned}
\int (fx)^m \left(d+ex^2\right) \left(1+2x^2+x^4\right)^5 \, dx = x(fx)^m \left(\frac{d}{1+m} + \frac{(10d+e)x^2}{3+m} + \frac{5(9d+2e)x^4}{5+m} \right. \\
+ \frac{15(8d+3e)x^6}{7+m} + \frac{30(7d+4e)x^8}{9+m} \\
+ \frac{42(6d+5e)x^{10}}{11+m} + \frac{42(5d+6e)x^{12}}{13+m} \\
+ \frac{30(4d+7e)x^{14}}{15+m} + \frac{15(3d+8e)x^{16}}{17+m} \\
\left. + \frac{5(2d+9e)x^{18}}{19+m} + \frac{(d+10e)x^{20}}{21+m} + \frac{ex^{22}}{23+m} \right)
\end{aligned}
$$

```
[In] Integrate[(f*x)^m*(d + e*x^2)*(1 + 2*x^2 + x^4)^5,x]
```

```
[Out] x*(f*x)^m*(d/(1 + m) + ((10*d + e)*x^2)/(3 + m) + (5*(9*d + 2*e)*x^4)/(5 +
m) + (15*(8*d + 3*e)*x^6)/(7 + m) + (30*(7*d + 4*e)*x^8)/(9 + m) + (42*(6*d
 + 5*e)*x^10)/(11 + m) + (42*(5*d + 6*e)*x^12)/(13 + m) + (30*(4*d + 7*e)*x
^14)/(15 + m) + (15*(3*d + 8*e)*x^16)/(17 + m) + (5*(2*d + 9*e)*x^18)/(19 +
 m) + ((d + 10*e)*x^20)/(21 + m) + (e*x^22)/(23 + m))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 2294 vs. 2(269) = 538.

Time = 0.75 (sec) , antiderivative size = 2295, normalized size of antiderivative = 8.53

method	result	size
gosper	Expression too large to display	2295
risch	Expression too large to display	2295
parallelrisch	Expression too large to display	3621

```
[In] int((f*x)^m*(e*x^2+d)*(x^4+2*x^2+1)^5,x,method=_RETURNVERBOSE)
```

```
[Out] (f*x)^m*(e*m^11*x^22+121*e*m^10*x^22+d*m^11*x^20+10*e*m^11*x^20+6435*e*m^9*
x^22+123*d*m^10*x^20+1230*e*m^10*x^20+197835*e*m^8*x^22+10*d*m^11*x^18+6635
*d*m^9*x^20+45*e*m^11*x^18+66350*e*m^9*x^20+3889578*e*m^7*x^22+1250*d*m^10*
x^18+206505*d*m^8*x^20+5625*e*m^10*x^18+2065050*e*m^8*x^20+51069018*e*m^6*x
^22+45*d*m^11*x^16+68430*d*m^9*x^18+4103178*d*m^7*x^20+120*e*m^11*x^16+3079
35*e*m^9*x^18+41031780*e*m^7*x^20+453714470*e*m^5*x^22+5715*d*m^10*x^16+215
8230*d*m^8*x^18+54362574*d*m^6*x^20+15240*e*m^10*x^16+9712035*e*m^8*x^18+54
3625740*e*m^6*x^20+2702025590*e*m^4*x^22+120*d*m^11*x^14+317655*d*m^9*x^16+
43391460*d*m^7*x^18+486687830*d*m^5*x^20+210*e*m^11*x^14+847080*e*m^9*x^16+
195261570*e*m^7*x^18+4866878300*e*m^5*x^20+10431670821*e*m^3*x^22+15480*d*m
^10*x^14+10162665*d*m^8*x^16+580855380*d*m^6*x^18+2917013970*d*m^4*x^20+270
90*e*m^10*x^14+27100440*e*m^8*x^16+2613849210*e*m^6*x^18+29170139700*e*m^4*
x^20+24372200061*e*m^2*x^22+210*d*m^11*x^12+873960*d*m^9*x^14+207024930*d*m
^7*x^16+5246766620*d*m^5*x^18+11320966021*d*m^3*x^20+252*e*m^11*x^12+152943
0*e*m^9*x^14+552066480*e*m^7*x^16+23610449790*e*m^5*x^18+113209660210*e*m^3
*x^20+29985521895*e*m*x^22+27510*d*m^10*x^12+28391400*d*m^8*x^14+2804395230
*d*m^6*x^16+31686018220*d*m^4*x^18+26560342503*d*m^2*x^20+33012*e*m^10*x^12
+49684950*e*m^8*x^14+7478387280*e*m^6*x^16+142587081990*e*m^4*x^18+26560342
5030*e*m^2*x^20+13749310575*e*x^22+252*d*m^11*x^10+1578150*d*m^9*x^12+58690
2960*d*m^7*x^14+25598865870*d*m^5*x^16+123748247730*d*m^3*x^18+32778930735*
d*m*x^20+210*e*m^11*x^10+1893780*e*m^9*x^12+1027080180*e*m^7*x^14+682636423
20*e*m^5*x^16+556867114785*e*m^3*x^18+327789307350*e*m*x^20+33516*d*m^10*x^
10+52110450*d*m^8*x^12+8059973040*d*m^6*x^14+156004908210*d*m^4*x^16+291789
582570*d*m^2*x^18+15058768725*d*x^20+27930*e*m^10*x^10+62532540*e*m^8*x^12+
14104952820*e*m^6*x^14+416013088560*e*m^4*x^16+1313053121565*e*m^2*x^18+150
587687250*e*x^20+210*d*m^11*x^8+1954260*d*m^9*x^10+1094918580*d*m^7*x^12+74
496630480*d*m^5*x^14+613938233025*d*m^3*x^16+361459164150*d*m*x^18+120*e*m^
11*x^8+1628550*e*m^9*x^10+1313902296*e*m^7*x^12+130369103340*e*m^5*x^14+163
7168621400*e*m^3*x^16+1626566238675*e*m*x^18+28350*d*m^10*x^8+65654820*d*m^
8*x^10+15277213980*d*m^6*x^12+459045550800*d*m^4*x^14+1456578341055*d*m^2*x
^16+166439022750*d*x^18+16200*e*m^10*x^8+54712350*e*m^8*x^10+18332656776*e*
m^6*x^12+803329713900*e*m^4*x^14+3884208909480*e*m^2*x^16+748975602375*e*x^
18+120*d*m^11*x^6+1680630*d*m^9*x^8+1404622296*d*m^7*x^10+143339613900*d*m^
```

5*x^12+1823707864920*d*m^3*x^14+1812743750475*d*m*x^16+45*e*m^11*x^6+960360
*e*m^9*x^8+1170518580*e*m^7*x^10+172007536680*e*m^5*x^12+3191488763610*e*m^
3*x^14+4833983334600*e*m*x^16+16440*d*m^10*x^6+57500730*d*m^8*x^8+199625413
68*d*m^6*x^10+895451283300*d*m^4*x^12+4360457499480*d*m^2*x^14+837090379125
*d*x^16+6165*e*m^10*x^6+32857560*e*m^8*x^8+16635451140*e*m^6*x^10+107454153
9960*e*m^4*x^12+7630800624090*e*m^2*x^14+2232241011000*e*x^16+45*d*m^11*x^4
+991080*d*m^9*x^6+1254847860*d*m^7*x^8+190744119720*d*m^5*x^10+360056778921
0*d*m^3*x^12+5458672303560*d*m*x^14+10*e*m^11*x^4+371655*e*m^9*x^6+71705592
0*e*m^7*x^8+158953433100*e*m^5*x^10+4320681347052*e*m^3*x^12+9552676531230*
e*m*x^14+6255*d*m^10*x^4+34563240*d*m^8*x^6+18217524780*d*m^6*x^8+121245419
9880*d*m^4*x^10+8695750818510*d*m^2*x^12+2529873145800*d*x^14+1390*e*m^10*x
^4+12961215*e*m^8*x^6+10410014160*e*m^6*x^8+1010378499900*e*m^4*x^10+104349
00982212*e*m^2*x^12+4427278005150*e*x^14+10*d*m^11*x^2+383535*d*m^9*x^4+770
831280*d*m^7*x^6+177985672620*d*m^5*x^8+4952725167852*d*m^3*x^10+1096992525
1950*d*m*x^12+e*m^11*x^2+85230*e*m^9*x^4+289061730*e*m^7*x^6+101706098640*e
*m^5*x^8+4127270973210*e*m^3*x^10+13163910302340*e*m*x^12+1410*d*m^10*x^2+1
3645125*d*m^8*x^4+11467698480*d*m^6*x^6+1156995210420*d*m^4*x^8+12123781647
516*d*m^2*x^10+5108397698250*d*x^12+141*e*m^10*x^2+3032250*e*m^8*x^4+430038
6930*e*m^6*x^6+661140120240*e*m^4*x^8+10103151372930*e*m^2*x^10+61300772379
00*e*x^12+d*m^11+87950*d*m^9*x^2+311564610*d*m^7*x^4+115122336720*d*m^5*x^6
+4828477578330*d*m^3*x^8+15456024948420*d*m*x^10+8795*e*m^9*x^2+69236580*e*
m^7*x^4+43170876270*e*m^5*x^6+2759130044760*e*m^3*x^8+12880020790350*e*m*x^
10+143*d*m^10+3194550*d*m^8*x^2+4765995990*d*m^6*x^4+770638650960*d*m^4*x^6
+12046833873270*d*m^2*x^8+7244636735700*d*x^10+319455*e*m^8*x^2+1059110220*
e*m^6*x^4+288989494110*e*m^4*x^6+6883905070440*e*m^2*x^8+6037197279750*e*x^
10+9075*d*m^9+74814180*d*m^7*x^2+49443604830*d*m^5*x^4+3314920570200*d*m^3*
x^6+15593181033150*d*m*x^8+7481418*e*m^7*x^2+10987467740*e*m^5*x^4+12430952
13825*e*m^3*x^6+8910389161800*e*m*x^8+336765*d*m^8+1180850580*d*m^6*x^2+343
967603850*d*m^4*x^4+8511631481880*d*m^2*x^6+7378796675250*d*x^8+118085058*e
*m^6*x^2+76437245300*e*m^4*x^4+3191861805705*e*m^2*x^6+4216455243000*e*x^8+
8103018*d*m^7+12740467100*d*m^5*x^2+1546183653345*d*m^3*x^4+11284114422600*
d*m*x^6+1274046710*e*m^5*x^2+343596367410*e*m^3*x^4+4231542908475*e*m*x^6+1
32426294*d*m^6+93153182700*d*m^4*x^2+4162610035755*d*m^2*x^4+5421156741000*
d*x^6+9315318270*e*m^4*x^2+925024452390*e*m^2*x^4+2032933777875*e*x^6+14958
75590*d*m^5+446323045810*d*m^3*x^2+5761525369635*d*m*x^4+44632304581*e*m^3*
x^2+1280338971030*e*m*x^4+11641582810*d*m^4+1304037152010*d*m^2*x^2+2846107
289025*d*x^4+130403715201*e*m^2*x^2+632468286450*e*x^4+60936676581*d*m^3+19
93349776950*d*m*x^2+199334977695*e*m*x^2+203363952363*d*m^2+1054113810750*d
*x^2+105411381075*e*x^2+387182170935*d*m+316234143225*d)*x/(1+m)/(3+m)/(5+m
)/(7+m)/(9+m)/(11+m)/(13+m)/(15+m)/(17+m)/(19+m)/(21+m)/(23+m)

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1571 vs. $2(269) = 538$.

Time = 0.26 (sec) , antiderivative size = 1571, normalized size of antiderivative = 5.84

$$\int (fx)^m \left(d + ex^2\right) \left(1 + 2x^2 + x^4\right)^5 \, dx = \text{Too large to display}$$

```
[In] integrate((f*x)^m*(e*x^2+d)*(x^4+2*x^2+1)^5,x, algorithm="fricas")
```

```
[Out] ((e*m^11 + 121*e*m^10 + 6435*e*m^9 + 197835*e*m^8 + 3889578*e*m^7 + 5106901
8*e*m^6 + 453714470*e*m^5 + 2702025590*e*m^4 + 10431670821*e*m^3 + 24372200
061*e*m^2 + 29985521895*e*m + 13749310575*e)*x^23 + ((d + 10*e)*m^11 + 123*
(d + 10*e)*m^10 + 6635*(d + 10*e)*m^9 + 206505*(d + 10*e)*m^8 + 4103178*(d
+ 10*e)*m^7 + 54362574*(d + 10*e)*m^6 + 486687830*(d + 10*e)*m^5 + 29170139
70*(d + 10*e)*m^4 + 11320966021*(d + 10*e)*m^3 + 26560342503*(d + 10*e)*m^2
 + 32778930735*(d + 10*e)*m + 15058768725*d + 150587687250*e)*x^21 + 5*((2*
d + 9*e)*m^11 + 125*(2*d + 9*e)*m^10 + 6843*(2*d + 9*e)*m^9 + 215823*(2*d +
 9*e)*m^8 + 4339146*(2*d + 9*e)*m^7 + 58085538*(2*d + 9*e)*m^6 + 524676662*
(2*d + 9*e)*m^5 + 3168601822*(2*d + 9*e)*m^4 + 12374824773*(2*d + 9*e)*m^3
+ 29178958257*(2*d + 9*e)*m^2 + 36145916415*(2*d + 9*e)*m + 33287804550*d +
 149795120475*e)*x^19 + 15*((3*d + 8*e)*m^11 + 127*(3*d + 8*e)*m^10 + 7059*
(3*d + 8*e)*m^9 + 225837*(3*d + 8*e)*m^8 + 4600554*(3*d + 8*e)*m^7 + 623198
94*(3*d + 8*e)*m^6 + 568863686*(3*d + 8*e)*m^5 + 3466775738*(3*d + 8*e)*m^4
 + 13643071845*(3*d + 8*e)*m^3 + 32368407579*(3*d + 8*e)*m^2 + 40283194455*
(3*d + 8*e)*m + 55806025275*d + 148816067400*e)*x^17 + 30*((4*d + 7*e)*m^11
 + 129*(4*d + 7*e)*m^10 + 7283*(4*d + 7*e)*m^9 + 236595*(4*d + 7*e)*m^8 + 4
890858*(4*d + 7*e)*m^7 + 67166442*(4*d + 7*e)*m^6 + 620805254*(4*d + 7*e)*m
^5 + 3825379590*(4*d + 7*e)*m^4 + 15197565541*(4*d + 7*e)*m^3 + 36337145829
*(4*d + 7*e)*m^2 + 45488935863*(4*d + 7*e)*m + 84329104860*d + 147575933505
*e)*x^15 + 42*((5*d + 6*e)*m^11 + 131*(5*d + 6*e)*m^10 + 7515*(5*d + 6*e)*m
^9 + 248145*(5*d + 6*e)*m^8 + 5213898*(5*d + 6*e)*m^7 + 72748638*(5*d + 6*e
)*m^6 + 682569590*(5*d + 6*e)*m^5 + 4264053730*(5*d + 6*e)*m^4 + 1714556090
1*(5*d + 6*e)*m^3 + 41408337231*(5*d + 6*e)*m^2 + 52237739295*(5*d + 6*e)*m
 + 121628516625*d + 145954219950*e)*x^13 + 42*((6*d + 5*e)*m^11 + 133*(6*d
+ 5*e)*m^10 + 7755*(6*d + 5*e)*m^9 + 260535*(6*d + 5*e)*m^8 + 5573898*(6*d
+ 5*e)*m^7 + 79216434*(6*d + 5*e)*m^6 + 756921110*(6*d + 5*e)*m^5 + 4811326
190*(6*d + 5*e)*m^4 + 19653671301*(6*d + 5*e)*m^3 + 48110244633*(6*d + 5*e)
*m^2 + 61333432335*(6*d + 5*e)*m + 172491350850*d + 143742792375*e)*x^11 +
30*((7*d + 4*e)*m^11 + 135*(7*d + 4*e)*m^10 + 8003*(7*d + 4*e)*m^9 + 273813
*(7*d + 4*e)*m^8 + 5975466*(7*d + 4*e)*m^7 + 86750118*(7*d + 4*e)*m^6 + 847
550822*(7*d + 4*e)*m^5 + 5509501002*(7*d + 4*e)*m^4 + 22992750373*(7*d + 4*
e)*m^3 + 57365875587*(7*d + 4*e)*m^2 + 74253243015*(7*d + 4*e)*m + 24595988
9175*d + 140548508100*e)*x^9 + 15*((8*d + 3*e)*m^11 + 137*(8*d + 3*e)*m^10
+ 8259*(8*d + 3*e)*m^9 + 288027*(8*d + 3*e)*m^8 + 6423594*(8*d + 3*e)*m^7 +
```

```
 95564154*(8*d + 3*e)*m^6 + 959352806*(8*d + 3*e)*m^5 + 6421988758*(8*d + 3
*e)*m^4 + 27624338085*(8*d + 3*e)*m^3 + 70930262349*(8*d + 3*e)*m^2 + 94034
286855*(8*d + 3*e)*m + 361410449400*d + 135528918525*e)*x^7 + 5*((9*d + 2*e
)*m^11 + 139*(9*d + 2*e)*m^10 + 8523*(9*d + 2*e)*m^9 + 303225*(9*d + 2*e)*m
^8 + 6923658*(9*d + 2*e)*m^7 + 105911022*(9*d + 2*e)*m^6 + 1098746774*(9*d
+ 2*e)*m^5 + 7643724530*(9*d + 2*e)*m^4 + 34359636741*(9*d + 2*e)*m^3 + 925
02445239*(9*d + 2*e)*m^2 + 128033897103*(9*d + 2*e)*m + 569221457805*d + 12
6493657290*e)*x^5 + ((10*d + e)*m^11 + 141*(10*d + e)*m^10 + 8795*(10*d + e
)*m^9 + 319455*(10*d + e)*m^8 + 7481418*(10*d + e)*m^7 + 118085058*(10*d +
e)*m^6 + 1274046710*(10*d + e)*m^5 + 9315318270*(10*d + e)*m^4 + 4463230458
1*(10*d + e)*m^3 + 130403715201*(10*d + e)*m^2 + 199334977695*(10*d + e)*m
+ 1054113810750*d + 105411381075*e)*x^3 + (d*m^11 + 143*d*m^10 + 9075*d*m^9
 + 336765*d*m^8 + 8103018*d*m^7 + 132426294*d*m^6 + 1495875590*d*m^5 + 1164
1582810*d*m^4 + 60936676581*d*m^3 + 203363952363*d*m^2 + 387182170935*d*m +
 316234143225*d)*x)*(f*x)^m/(m^12 + 144*m^11 + 9218*m^10 + 345840*m^9 + 843
9783*m^8 + 140529312*m^7 + 1628301884*m^6 + 13137458400*m^5 + 72578259391*m
^4 + 264300628944*m^3 + 590546123298*m^2 + 703416314160*m + 316234143225)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 21612 vs. $2(228) = 456$.

Time = 2.91 (sec) , antiderivative size = 21612, normalized size of antiderivative = 80.34

$$\int (fx)^m\left(d+ex^2\right)\left(1+2x^2+x^4\right)^5\,dx=\text{Too large to display}$$

```
[In] integrate((f*x)**m*(e*x**2+d)*(x**4+2*x**2+1)**5,x)
```

```
[Out] Piecewise(((-d/(2*x**2) - 5*d/(2*x**4) - 15*d/(2*x**6) - 15*d/x**8 - 21*d/x
**10 - 21*d/x**12 - 15*d/x**14 - 15*d/(2*x**16) - 5*d/(2*x**18) - d/(2*x**2
0) - d/(22*x**22) + e*log(x) - 5*e/x**2 - 45*e/(4*x**4) - 20*e/x**6 - 105*e
/(4*x**8) - 126*e/(5*x**10) - 35*e/(2*x**12) - 60*e/(7*x**14) - 45*e/(16*x*
*16) - 5*e/(9*x**18) - e/(20*x**20))/f**23, Eq(m, -23)), ((d*log(x) - 5*d/x
**2 - 45*d/(4*x**4) - 20*d/x**6 - 105*d/(4*x**8) - 126*d/(5*x**10) - 35*d/(
2*x**12) - 60*d/(7*x**14) - 45*d/(16*x**16) - 5*d/(9*x**18) - d/(20*x**20)
+ e*x**2/2 + 10*e*log(x) - 45*e/(2*x**2) - 30*e/x**4 - 35*e/x**6 - 63*e/(2*
x**8) - 21*e/x**10 - 10*e/x**12 - 45*e/(14*x**14) - 5*e/(8*x**16) - e/(18*x
**18))/f**21, Eq(m, -21)), ((d*x**2/2 + 10*d*log(x) - 45*d/(2*x**2) - 30*d/
x**4 - 35*d/x**6 - 63*d/(2*x**8) - 21*d/x**10 - 10*d/x**12 - 45*d/(14*x**14
) - 5*d/(8*x**16) - d/(18*x**18) + e*x**4/4 + 5*e*x**2 + 45*e*log(x) - 60*e
/x**2 - 105*e/(2*x**4) - 42*e/x**6 - 105*e/(4*x**8) - 12*e/x**10 - 15*e/(4*
x**12) - 5*e/(7*x**14) - e/(16*x**16))/f**19, Eq(m, -19)), ((d*x**4/4 + 5*d
*x**2 + 45*d*log(x) - 60*d/x**2 - 105*d/(2*x**4) - 42*d/x**6 - 105*d/(4*x**
8) - 12*d/x**10 - 15*d/(4*x**12) - 5*d/(7*x**14) - d/(16*x**16) + e*x**6/6
+ 5*e*x**4/2 + 45*e*x**2/2 + 120*e*log(x) - 105*e/x**2 - 63*e/x**4 - 35*e/x
```

```
**6 - 15*e/x**8 - 9*e/(2*x**10) - 5*e/(6*x**12) - e/(14*x**14))/f**17, Eq(m
, -17)), ((d*x**6/6 + 5*d*x**4/2 + 45*d*x**2/2 + 120*d*log(x) - 105*d/x**2
- 63*d/x**4 - 35*d/x**6 - 15*d/x**8 - 9*d/(2*x**10) - 5*d/(6*x**12) - d/(14
*x**14) + e*x**8/8 + 5*e*x**6/3 + 45*e*x**4/4 + 60*e*x**2 + 210*e*log(x) -
126*e/x**2 - 105*e/(2*x**4) - 20*e/x**6 - 45*e/(8*x**8) - e/x**10 - e/(12*x
**12))/f**15, Eq(m, -15)), ((d*x**8/8 + 5*d*x**6/3 + 45*d*x**4/4 + 60*d*x**
2 + 210*d*log(x) - 126*d/x**2 - 105*d/(2*x**4) - 20*d/x**6 - 45*d/(8*x**8)
- d/x**10 - d/(12*x**12) + e*x**10/10 + 5*e*x**8/4 + 15*e*x**6/2 + 30*e*x**
4 + 105*e*x**2 + 252*e*log(x) - 105*e/x**2 - 30*e/x**4 - 15*e/(2*x**6) - 5*
e/(4*x**8) - e/(10*x**10))/f**13, Eq(m, -13)), ((d*x**10/10 + 5*d*x**8/4 +
15*d*x**6/2 + 30*d*x**4 + 105*d*x**2 + 252*d*log(x) - 105*d/x**2 - 30*d/x**
4 - 15*d/(2*x**6) - 5*d/(4*x**8) - d/(10*x**10) + e*x**12/12 + e*x**10 + 45
*e*x**8/8 + 20*e*x**6 + 105*e*x**4/2 + 126*e*x**2 + 210*e*log(x) - 60*e/x**
2 - 45*e/(4*x**4) - 5*e/(3*x**6) - e/(8*x**8))/f**11, Eq(m, -11)), ((d*x**1
2/12 + d*x**10 + 45*d*x**8/8 + 20*d*x**6 + 105*d*x**4/2 + 126*d*x**2 + 210*
d*log(x) - 60*d/x**2 - 45*d/(4*x**4) - 5*d/(3*x**6) - d/(8*x**8) + e*x**14/
14 + 5*e*x**12/6 + 9*e*x**10/2 + 15*e*x**8 + 35*e*x**6 + 63*e*x**4 + 105*e*
x**2 + 120*e*log(x) - 45*e/(2*x**2) - 5*e/(2*x**4) - e/(6*x**6))/f**9, Eq(m
, -9)), ((d*x**14/14 + 5*d*x**12/6 + 9*d*x**10/2 + 15*d*x**8 + 35*d*x**6 +
63*d*x**4 + 105*d*x**2 + 120*d*log(x) - 45*d/(2*x**2) - 5*d/(2*x**4) - d/(6
*x**6) + e*x**16/16 + 5*e*x**14/7 + 15*e*x**12/4 + 12*e*x**10 + 105*e*x**8/
4 + 42*e*x**6 + 105*e*x**4/2 + 60*e*x**2 + 45*e*log(x) - 5*e/x**2 - e/(4*x*
*4))/f**7, Eq(m, -7)), ((d*x**16/16 + 5*d*x**14/7 + 15*d*x**12/4 + 12*d*x**
10 + 105*d*x**8/4 + 42*d*x**6 + 105*d*x**4/2 + 60*d*x**2 + 45*d*log(x) - 5*
d/x**2 - d/(4*x**4) + e*x**18/18 + 5*e*x**16/8 + 45*e*x**14/14 + 10*e*x**12
 + 21*e*x**10 + 63*e*x**8/2 + 35*e*x**6 + 30*e*x**4 + 45*e*x**2/2 + 10*e*lo
g(x) - e/(2*x**2))/f**5, Eq(m, -5)), ((d*x**18/18 + 5*d*x**16/8 + 45*d*x**1
4/14 + 10*d*x**12 + 21*d*x**10 + 63*d*x**8/2 + 35*d*x**6 + 30*d*x**4 + 45*d
*x**2/2 + 10*d*log(x) - d/(2*x**2) + e*x**20/20 + 5*e*x**18/9 + 45*e*x**16/
16 + 60*e*x**14/7 + 35*e*x**12/2 + 126*e*x**10/5 + 105*e*x**8/4 + 20*e*x**6
 + 45*e*x**4/4 + 5*e*x**2 + e*log(x))/f**3, Eq(m, -3)), ((d*x**20/20 + 5*d*
x**18/9 + 45*d*x**16/16 + 60*d*x**14/7 + 35*d*x**12/2 + 126*d*x**10/5 + 105
*d*x**8/4 + 20*d*x**6 + 45*d*x**4/4 + 5*d*x**2 + d*log(x) + e*x**22/22 + e*
x**20/2 + 5*e*x**18/2 + 15*e*x**16/2 + 15*e*x**14 + 21*e*x**12 + 21*e*x**10
 + 15*e*x**8 + 15*e*x**6/2 + 5*e*x**4/2 + e*x**2/2)/f, Eq(m, -1)), (d*m**11
*x**21*(f*x)**m/(m**12 + 144*m**11 + 9218*m**10 + 345840*m**9 + 8439783*m**
8 + 140529312*m**7 + 1628301884*m**6 + 13137458400*m**5 + 72578259391*m**4
+ 264300628944*m**3 + 590546123298*m**2 + 703416314160*m + 316234143225) +
10*d*m**11*x**19*(f*x)**m/(m**12 + 144*m**11 + 9218*m**10 + 345840*m**9 + 8
439783*m**8 + 140529312*m**7 + 1628301884*m**6 + 13137458400*m**5 + 7257825
9391*m**4 + 264300628944*m**3 + 590546123298*m**2 + 703416314160*m + 316234
143225) + 45*d*m**11*x**17*(f*x)**m/(m**12 + 144*m**11 + 9218*m**10 + 34584
0*m**9 + 8439783*m**8 + 140529312*m**7 + 1628301884*m**6 + 13137458400*m**5
 + 72578259391*m**4 + 264300628944*m**3 + 590546123298*m**2 + 703416314160*
m + 316234143225) + 120*d*m**11*x**15*(f*x)**m/(m**12 + 144*m**11 + 9218*m*
```

```
*10 + 345840*m**9 + 8439783*m**8 + 140529312*m**7 + 1628301884*m**6 + 13137
458400*m**5 + 72578259391*m**4 + 264300628944*m**3 + 590546123298*m**2 + 70
3416314160*m + 316234143225) + 210*d*m**11*x**13*(f*x)**m/(m**12 + 144*m**1
1 + 9218*m**10 + 345840*m**9 + 8439783*m**8 + 140529312*m**7 + 1628301884*m
**6 + 13137458400*m**5 + 72578259391*m**4 + 264300628944*m**3 + 59054612329
8*m**2 + 703416314160*m + 316234143225) + 252*d*m**11*x**11*(f*x)**m/(m**12
 + 144*m**11 + 9218*m**10 + 345840*m**9 + 8439783*m**8 + 140529312*m**7 + 1
628301884*m**6 + 13137458400*m**5 + 72578259391*m**4 + 264300628944*m**3 +
590546123298*m**2 + 703416314160*m + 316234143225) + 210*d*m**11*x**9*(f*x)
**m/(m**12 + 144*m**11 + 9218*m**10 + 345840*m**9 + 8439783*m**8 + 14052931
2*m**7 + 1628301884*m**6 + 13137458400*m**5 + 72578259391*m**4 + 2643006289
44*m**3 + 590546123298*m**2 + 703416314160*m + 316234143225) + 120*d*m**11*
x**7*(f*x)**m/(m**12 + 144*m**11 + 9218*m**10 + 345840*m**9 + 8439783*m**8
+ 140529312*m**7 + 1628301884*m**6 + 13137458400*m**5 + 72578259391*m**4 +
264300628944*m**3 + 590546123298*m**2 + 703416314160*m + 316234143225) + 45
*d*m**11*x**5*(f*x)**m/(m**12 + 144*m**11 + 9218*m**10 + 345840*m**9 + 8439
783*m**8 + 140529312*m**7 + 1628301884*m**6 + 13137458400*m**5 + 7257825939
1*m**4 + 264300628944*m**3 + 590546123298*m**2 + 703416314160*m + 316234143
225) + 10*d*m**11*x**3*(f*x)**m/(m**12 + 144*m**11 + 9218*m**10 + 345840*m*
*9 + 8439783*m**8 + 140529312*m**7 + 1628301884*m**6 + 13137458400*m**5 + 7
2578259391*m**4 + 264300628944*m**3 + 590546123298*m**2 + 703416314160*m +
316234143225) + d*m**11*x*(f*x)**m/(m**12 + 144*m**11 + 9218*m**10 + 345840
*m**9 + 8439783*m**8 + 140529312*m**7 + 1628301884*m**6 + 13137458400*m**5
+ 72578259391*m**4 + 264300628944*m**3 + 590546123298*m**2 + 703416314160*m
 + 316234143225) + 123*d*m**10*x**21*(f*x)**m/(m**12 + 144*m**11 + 9218*m**
10 + 345840*m**9 + 8439783*m**8 + 140529312*m**7 + 1628301884*m**6 + 131374
58400*m**5 + 72578259391*m**4 + 264300628944*m**3 + 590546123298*m**2 + 703
416314160*m + 316234143225) + 1250*d*m**10*x**19*(f*x)**m/(m**12 + 144*m**1
1 + 9218*m**10 + 345840*m**9 + 8439783*m**8 + 140529312*m**7 + 1628301884*m
**6 + 13137458400*m**5 + 72578259391*m**4 + 264300628944*m**3 + 59054612329
8*m**2 + 703416314160*m + 316234143225) + 5715*d*m**10*x**17*(f*x)**m/(m**1
2 + 144*m**11 + 9218*m**10 + 345840*m**9 + 8439783*m**8 + 140529312*m**7 +
1628301884*m**6 + 13137458400*m**5 + 72578259391*m**4 + 264300628944*m**3 +
 590546123298*m**2 + 703416314160*m + 316234143225) + 15480*d*m**10*x**15*(
f*x)**m/(m**12 + 144*m**11 + 9218*m**10 + 345840*m**9 + 8439783*m**8 + 1405
29312*m**7 + 1628301884*m**6 + 13137458400*m**5 + 72578259391*m**4 + 264300
628944*m**3 + 590546123298*m**2 + 703416314160*m + 316234143225) + 27510*d*
m**10*x**13*(f*x)**m/(m**12 + 144*m**11 + 9218*m**10 + 345840*m**9 + 843978
3*m**8 + 140529312*m**7 + 1628301884*m**6 + 13137458400*m**5 + 72578259391*
m**4 + 264300628944*m**3 + 590546123298*m**2 + 703416314160*m + 31623414322
5) + 33516*d*m**10*x**11*(f*x)**m/(m**12 + 144*m**11 + 9218*m**10 + 345840*
m**9 + 8439783*m**8 + 140529312*m**7 + 1628301884*m**6 + 13137458400*m**5 +
 72578259391*m**4 + 264300628944*m**3 + 590546123298*m**2 + 703416314160*m
+ 316234143225) + 28350*d*m**10*x**9*(f*x)**m/(m**12 + 144*m**11 + 9218*m**
10 + 345840*m**9 + 8439783*m**8 + 140529312*m**7 + 1628301884*m**6 + 131374
```

58400*m**5 + 72578259391*m**4 + 264300628944*m**3 + 590546123298*m**2 + 703
416314160*m + 316234143225) + 16440*d*m**10*x**7*(f*x)**m/(m**12 + 144*m**1
1 + 9218*m**10 + 345840*m**9 + 8439783*m**8 + 140529312*m**7 + 1628301884*m
6 + 13137458400*m5 + 72578259391*m**4 + 264300628944*m**3 + 59054612329
8*m**2 + 703416314160*m + 316234143225) + 6255*d*m**10*x**5*(f*x)**m/(m**12
 + 144*m**11 + 9218*m**10 + 345840*m**9 + 8439783*m**8 + 140529312*m**7 + 1
628301884*m**6 + 13137458400*m**5 + 72578259391*m**4 + 264300628944*m**3 +
590546123298*m**2 + 703416314160*m + 316234143225) + 1410*d*m**10*x**3*(f*x
)**m/(m**12 + 144*m**11 + 9218*m**10 + 345840*m**9 + 8439783*m**8 + 1405293
12*m**7 + 1628301884*m**6 + 13137458400*m**5 + 72578259391*m**4 + 264300628
944*m**3 + 590546123298*m**2 + 703416314160*m + 316234143225) + 143*d*m**10
x(f*x)**m/(m**12 + 144*m**11 + 9218*m**10 + 345840*m**9 + 8439783*m**8 +
140529312*m**7 + 1628301884*m**6 + 13137458400*m**5 + 72578259391*m**4 + 26
4300628944*m**3 + 590546123298*m**2 + 703416314160*m + 316234143225) + 6635
*d*m**9*x**21*(f*x)**m/(m**12 + 144*m**11 + 9218*m**10 + 345840*m**9 + 8439
783*m**8 + 140529312*m**7 + 1628301884*m**6 + 13137458400*m**5 + 7257825939
1*m**4 + 264300628944*m**3 + 590546123298*m**2 + 703416314160*m + 316234143
225) + 68430*d*m**9*x**19*(f*x)**m/(m**12 + 144*m**11 + 9218*m**10 + 345840
*m**9 + 8439783*m**8 + 140529312*m**7 + 1628301884*m**6 + 13137458400*m**5
+ 72578259391*m**4 + 264300628944*m**3 + 590546123298*m**2 + 703416314160*m
 + 316234143225) + 317655*d*m**9*x**17*(f*x)**m/(m**12 + 144*m**11 + 9218*m
10 + 345840*m9 + 8439783*m**8 + 140529312*m**7 + 1628301884*m**6 + 1313
7458400*m**5 + 72578259391*m**4 + 264300628944*m**3 + 590546123298*m**2 + 7
03416314160*m + 316234143225) + 873960*d*m**9*x**15*(f*x)**m/(m**12 + 144*m
11 + 9218*m10 + 345840*m**9 + 8439783*m**8 + 140529312*m**7 + 162830188
4*m**6 + 13137458400*m**5 + 72578259391*m**4 + 264300628944*m**3 + 59054612
3298*m**2 + 703416314160*m + 316234143225) + 1578150*d*m**9*x**13*(f*x)**m/
(m**12 + 144*m**11 + 9218*m**10 + 345840*m**9 + 8439783*m**8 + 140529312*m*
*7 + 1628301884*m**6 + 13137458400*m**5 + 72578259391*m**4 + 264300628944*m
3 + 590546123298*m2 + 703416314160*m + 316234143225) + 1954260*d*m**9*x
11*(f*x)m/(m**12 + 144*m**11 + 9218*m**10 + 345840*m**9 + 8439783*m**8
+ 140529312*m**7 + 1628301884*m**6 + 13137458400*m**5 + 72578259391*m**4 +
264300628944*m**3 + 590546123298*m**2 + 703416314160*m + 316234143225) + 16
80630*d*m**9*x**9*(f*x)**m/(m**12 + 144*m**11 + 9218*m**10 + 345840*m**9 +
8439783*m**8 + 140529312*m**7 + 1628301884*m**6 + 13137458400*m**5 + 725782
59391*m**4 + 264300628944*m**3 + 590546123298*m**2 + 703416314160*m + 31623
4143225) + 991080*d*m**9*x**7*(f*x)**m/(m**12 + 144*m**11 + 9218*m**10 + 34
5840*m**9 + 8439783*m**8 + 140529312*m**7 + 1628301884*m**6 + 13137458400*m
5 + 72578259391*m4 + 264300628944*m**3 + 590546123298*m**2 + 7034163141
60*m + 316234143225) + 383535*d*m**9*x**5*(f*x)**m/(m**12 + 144*m**11 + 921
8*m**10 + 345840*m**9 + 8439783*m**8 + 140529312*m**7 + 1628301884*m**6 + 1
3137458400*m**5 + 72578259391*m**4 + 264300628944*m**3 + 590546123298*m**2
+ 703416314160*m + 316234143225) + 87950*d*m**9*x**3*(f*x)**m/(m**12 + 144*
m**11 + 9218*m**10 + 345840*m**9 + 8439783*m**8 + 140529312*m**7 + 16283018
84*m**6 + 13137458400*m**5 + 72578259391*m**4 + 264300628944*m**3 + 5905461

23298*m**2 + 703416314160*m + 316234143225) + 9075*d*m**9*x*(f*x)**m/(m**12
 + 144*m**11 + 9218*m**10 + 345840*m**9 + 8439783*m**8 + 140529312*m**7 + 1
628301884*m**6 + 13137458400*m**5 + 72578259391*m**4 + 264300628944*m**3 +
590546123298*m**2 + 703416314160*m + 316234143225) + 206505*d*m**8*x**21*(f
*x)**m/(m**12 + 144*m**11 + 9218*m**10 + 345840*m**9 + 8439783*m**8 + 14052
9312*m**7 + 1628301884*m**6 + 13137458400*m**5 + 72578259391*m**4 + 2643006
28944*m**3 + 590546123298*m**2 + 703416314160*m + 316234143225) + 2158230*d
*m**8*x**19*(f*x)**m/(m**12 + 144*m**11 + 9218*m**10 + 345840*m**9 + 843978
3*m**8 + 140529312*m**7 + 1628301884*m**6 + 13137458400*m**5 + 72578259391*
m**4 + 264300628944*m**3 + 590546123298*m**2 + 703416314160*m + 31623414322
5) + 10162665*d*m**8*x**17*(f*x)**m/(m**12 + 144*m**11 + 9218*m**10 + 34584
0*m**9 + 8439783*m**8 + 140529312*m**7 + 1628301884*m**6 + 13137458400*m**5
 + 72578259391*m**4 + 264300628944*m**3 + 590546123298*m**2 + 703416314160*
m + 316234143225) + 28391400*d*m**8*x**15*(f*x)**m/(m**12 + 144*m**11 + 921
8*m**10 + 345840*m**9 + 8439783*m**8 + 140529312*m**7 + 1628301884*m**6 + 1
3137458400*m**5 + 72578259391*m**4 + 264300628944*m**3 + 590546123298*m**2
 + 703416314160*m + 316234143225) + 52110450*d*m**8*x**13*(f*x)**m/(m**12 +
144*m**11 + 9218*m**10 + 345840*m**9 + 8439783*m**8 + 140529312*m**7 + 1628
301884*m**6 + 13137458400*m**5 + 72578259391*m**4 + 264300628944*m**3 + 590
546123298*m**2 + 703416314160*m + 316234143225) + 65654820*d*m**8*x**11*(f*
x)**m/(m**12 + 144*m**11 + 9218*m**10 + 345840*m**9 + 8439783*m**8 + 140529
312*m**7 + 1628301884*m**6 + 13137458400*m**5 + 72578259391*m**4 + 26430062
8944*m**3 + 590546123298*m**2 + 703416314160*m + 316234143225) + 57500730*d
*m**8*x**9*(f*x)**m/(m**12 + 144*m**11 + 9218*m**10 + 345840*m**9 + 8439783
*m**8 + 140529312*m**7 + 1628301884*m**6 + 13137458400*m**5 + 72578259391*m
4 + 264300628944*m3 + 590546123298*m**2 + 703416314160*m + 316234143225
) + 34563240*d*m**8*x**7*(f*x)**m/(m**12 + 144*m**11 + 9218*m**10 + 345840*
m**9 + 8439783*m**8 + 140529312*m**7 + 1628301884*m**6 + 13137458400*m**5 +
 72578259391*m**4 + 264300628944*m**3 + 590546123298*m**2 + 703416314160*m
 + 316234143225) + 13645125*d*m**8*x**5*(f*x)**m/(m**12 + 144*m**11 + 9218*m
10 + 345840*m9 + 8439783*m**8 + 140529312*m**7 + 1628301884*m**6 + 1313
7458400*m**5 + 72578259391*m**4 + 264300628944*m**3 + 590546123298*m**2 + 7
03416314160*m + 316234143225) + 3194550*d*m**8*x**3*(f*x)**m/(m**12 + 144*m
11 + 9218*m10 + 345840*m**9 + 8439783*m**8 + 140529312*m**7 + 162830188
4*m**6 + 13137458400*m**5 + 72578259391*m**4 + 264300628944*m**3 + 59054612
3298*m**2 + 703416314160*m + 316234143225) + 336765*d*m**8*x*(f*x)**m/(m**1
2 + 144*m**11 + 9218*m**10 + 345840*m**9 + 8439783*m**8 + 140529312*m**7 +
1628301884*m**6 + 13137458400*m**5 + 72578259391*m**4 + 264300628944*m**3 +
 590546123298*m**2 + 703416314160*m + 316234143225) + 4103178*d*m**7*x**21*
(f*x)**m/(m**12 + 144*m**11 + 9218*m**10 + 345840*m**9 + 8439783*m**8 + 140
529312*m**7 + 1628301884*m**6 + 13137458400*m**5 + 72578259391*m**4 + 26430
0628944*m**3 + 590546123298*m**2 + 703416314160*m + 316234143225) + 4339146
0*d*m**7*x**19*(f*x)**m/(m**12 + 144*m**11 + 9218*m**10 + 345840*m**9 + 843
9783*m**8 + 140529312*m**7 + 1628301884*m**6 + 13137458400*m**5 + 725782593
91*m**4 + 264300628944*m**3 + 590546123298*m**2 + 703416314160*m + 31623414

3225) + 207024930*d*m**7*x**17*(f*x)**m/(m**12 + 144*m**11 + 9218*m**10 + 3
45840*m**9 + 8439783*m**8 + 140529312*m**7 + 1628301884*m**6 + 13137458400*
m**5 + 72578259391*m**4 + 264300628944*m**3 + 590546123298*m**2 + 703416314
160*m + 316234143225) + 586902960*d*m**7*x**15*(f*x)**m/(m**12 + 144*m**11
+ 9218*m**10 + 345840*m**9 + 8439783*m**8 + 140529312*m**7 + 1628301884*m**
6 + 13137458400*m**5 + 72578259391*m**4 + 264300628944*m**3 + 590546123298*
m**2 + 703416314160*m + 316234143225) + 1094918580*d*m**7*x**13*(f*x)**m/(m
12 + 144*m11 + 9218*m**10 + 345840*m**9 + 8439783*m**8 + 140529312*m**7
 + 1628301884*m**6 + 13137458400*m**5 + 72578259391*m**4 + 264300628944*m**
3 + 590546123298*m**2 + 703416314160*m + 316234143225) + 1404622296*d*m**7*
x**11*(f*x)**m/(m**12 + 144*m**11 + 9218*m**10 + 345840*m**9 + 8439783*m**8
 + 140529312*m**7 + 1628301884*m**6 + 13137458400*m**5 + 72578259391*m**4 +
 264300628944*m**3 + 590546123298*m**2 + 703416314160*m + 316234143225) + 1
254847860*d*m**7*x**9*(f*x)**m/(m**12 + 144*m**11 + 9218*m**10 + 345840*m**
9 + 8439783*m**8 + 140529312*m**7 + 1628301884*m**6 + 13137458400*m**5 + 72
578259391*m**4 + 264300628944*m**3 + 590546123298*m**2 + 703416314160*m + 3
16234143225) + 770831280*d*m**7*x**7*(f*x)**m/(m**12 + 144*m**11 + 9218*m**
10 + 345840*m**9 + 8439783*m**8 + 140529312*m**7 + 1628301884*m**6 + 131374
58400*m**5 + 72578259391*m**4 + 264300628944*m**3 + 590546123298*m**2 + 703
416314160*m + 316234143225) + 311564610*d*m**7*x**5*(f*x)**m/(m**12 + 144*m
11 + 9218*m10 + 345840*m**9 + 8439783*m**8 + 140529312*m**7 + 162830188
4*m**6 + 13137458400*m**5 + 72578259391*m**4 + 264300628944*m**3 + 59054612
3298*m**2 + 703416314160*m + 316234143225) + 74814180*d*m**7*x**3*(f*x)**m/
(m**12 + 144*m**11 + 9218*m**10 + 345840*m**9 + 8439783*m**8 + 140529312*m*
*7 + 1628301884*m**6 + 13137458400*m**5 + 72578259391*m**4 + 264300628944*m
3 + 590546123298*m2 + 703416314160*m + 316234143225) + 8103018*d*m**7*x
*(f*x)**m/(m**12 + 144*m**11 + 9218*m**10 + 345840*m**9 + 8439783*m**8 + 14
0529312*m**7 + 1628301884*m**6 + 13137458400*m**5 + 72578259391*m**4 + 2643
00628944*m**3 + 590546123298*m**2 + 703416314160*m + 316234143225) + 543625
74*d*m**6*x**21*(f*x)**m/(m**12 + 144*m**11 + 9218*m**10 + 345840*m**9 + 84
39783*m**8 + 140529312*m**7 + 1628301884*m**6 + 13137458400*m**5 + 72578259
391*m**4 + 264300628944*m**3 + 590546123298*m**2 + 703416314160*m + 3162341
43225) + 580855380*d*m**6*x**19*(f*x)**m/(m**12 + 144*m**11 + 9218*m**10 +
345840*m**9 + 8439783*m**8 + 140529312*m**7 + 1628301884*m**6 + 13137458400
*m**5 + 72578259391*m**4 + 264300628944*m**3 + 590546123298*m**2 + 70341631
4160*m + 316234143225) + 2804395230*d*m**6*x**17*(f*x)**m/(m**12 + 144*m**1
1 + 9218*m**10 + 345840*m**9 + 8439783*m**8 + 140529312*m**7 + 1628301884*m
6 + 13137458400*m5 + 72578259391*m**4 + 264300628944*m**3 + 59054612329
8*m**2 + 703416314160*m + 316234143225) + 8059973040*d*m**6*x**15*(f*x)**m/
(m**12 + 144*m**11 + 9218*m**10 + 345840*m**9 + 8439783*m**8 + 140529312*m*
*7 + 1628301884*m**6 + 13137458400*m**5 + 72578259391*m**4 + 264300628944*m
3 + 590546123298*m2 + 703416314160*m + 316234143225) + 15277213980*d*m*
*6*x**13*(f*x)**m/(m**12 + 144*m**11 + 9218*m**10 + 345840*m**9 + 8439783*m
8 + 140529312*m7 + 1628301884*m**6 + 13137458400*m**5 + 72578259391*m**
4 + 264300628944*m**3 + 590546123298*m**2 + 703416314160*m + 316234143225)

+ 19962541368*d*m**6*x**11*(f*x)**m/(m**12 + 144*m**11 + 9218*m**10 + 34584
0*m**9 + 8439783*m**8 + 140529312*m**7 + 1628301884*m**6 + 13137458400*m**5
 + 72578259391*m**4 + 264300628944*m**3 + 590546123298*m**2 + 703416314160*
m + 316234143225) + 18217524780*d*m**6*x**9*(f*x)**m/(m**12 + 144*m**11 + 9
218*m**10 + 345840*m**9 + 8439783*m**8 + 140529312*m**7 + 1628301884*m**6 +
 13137458400*m**5 + 72578259391*m**4 + 264300628944*m**3 + 590546123298*m**
2 + 703416314160*m + 316234143225) + 11467698480*d*m**6*x**7*(f*x)**m/(m**1
2 + 144*m**11 + 9218*m**10 + 345840*m**9 + 8439783*m**8 + 140529312*m**7 +
1628301884*m**6 + 13137458400*m**5 + 72578259391*m**4 + 264300628944*m**3 +
 590546123298*m**2 + 703416314160*m + 316234143225) + 4765995990*d*m**6*x**
5*(f*x)**m/(m**12 + 144*m**11 + 9218*m**10 + 345840*m**9 + 8439783*m**8 + 1
40529312*m**7 + 1628301884*m**6 + 13137458400*m**5 + 72578259391*m**4 + 264
300628944*m**3 + 590546123298*m**2 + 703416314160*m + 316234143225) + 11808
50580*d*m**6*x**3*(f*x)**m/(m**12 + 144*m**11 + 9218*m**10 + 345840*m**9 +
8439783*m**8 + 140529312*m**7 + 1628301884*m**6 + 13137458400*m**5 + 725782
59391*m**4 + 264300628944*m**3 + 590546123298*m**2 + 703416314160*m + 31623
4143225) + 132426294*d*m**6*x*(f*x)**m/(m**12 + 144*m**11 + 9218*m**10 + 34
5840*m**9 + 8439783*m**8 + 140529312*m**7 + 1628301884*m**6 + 13137458400*m
5 + 72578259391*m4 + 264300628944*m**3 + 590546123298*m**2 + 7034163141
60*m + 316234143225) + 486687830*d*m**5*x**21*(f*x)**m/(m**12 + 144*m**11 +
 9218*m**10 + 345840*m**9 + 8439783*m**8 + 140529312*m**7 + 1628301884*m**6
 + 13137458400*m**5 + 72578259391*m**4 + 264300628944*m**3 + 590546123298*m
2 + 703416314160*m + 316234143225) + 5246766620*d*m5*x**19*(f*x)**m/(m*
*12 + 144*m**11 + 9218*m**10 + 345840*m**9 + 8439783*m**8 + 140529312*m**7
+ 1628301884*m**6 + 13137458400*m**5 + 72578259391*m**4 + 264300628944*m**3
 + 590546123298*m**2 + 703416314160*m + 316234143225) + 25598865870*d*m**5*
x**17*(f*x)**m/(m**12 + 144*m**11 + 9218*m**10 + 345840*m**9 + 8439783*m**8
 + 140529312*m**7 + 1628301884*m**6 + 13137458400*m**5 + 72578259391*m**4 +
 264300628944*m**3 + 590546123298*m**2 + 703416314160*m + 316234143225) + 7
4496630480*d*m**5*x**15*(f*x)**m/(m**12 + 144*m**11 + 9218*m**10 + 345840*m
9 + 8439783*m8 + 140529312*m**7 + 1628301884*m**6 + 13137458400*m**5 +
72578259391*m**4 + 264300628944*m**3 + 590546123298*m**2 + 703416314160*m +
 316234143225) + 143339613900*d*m**5*x**13*(f*x)**m/(m**12 + 144*m**11 + 92
18*m**10 + 345840*m**9 + 8439783*m**8 + 140529312*m**7 + 1628301884*m**6 +
13137458400*m**5 + 72578259391*m**4 + 264300628944*m**3 + 590546123298*m**2
 + 703416314160*m + 316234143225) + 190744119720*d*m**5*x**11*(f*x)**m/(m**
12 + 144*m**11 + 9218*m**10 + 345840*m**9 + 8439783*m**8 + 140529312*m**7 +
 1628301884*m**6 + 13137458400*m**5 + 72578259391*m**4 + 264300628944*m**3
+ 590546123298*m**2 + 703416314160*m + 316234143225) + 177985672620*d*m**5*
x**9*(f*x)**m/(m**12 + 144*m**11 + 9218*m**10 + 345840*m**9 + 8439783*m**8
+ 140529312*m**7 + 1628301884*m**6 + 13137458400*m**5 + 72578259391*m**4 +
264300628944*m**3 + 590546123298*m**2 + 703416314160*m + 316234143225) + 11
5122336720*d*m**5*x**7*(f*x)**m/(m**12 + 144*m**11 + 9218*m**10 + 345840*m*
*9 + 8439783*m**8 + 140529312*m**7 + 1628301884*m**6 + 13137458400*m**5 + 7
2578259391*m**4 + 264300628944*m**3 + 590546123298*m**2 + 703416314160*m +

316234143225) + 49443604830*d*m**5*x**5*(f*x)**m/(m**12 + 144*m**11 + 9218*
m**10 + 345840*m**9 + 8439783*m**8 + 140529312*m**7 + 1628301884*m**6 + 131
37458400*m**5 + 72578259391*m**4 + 264300628944*m**3 + 590546123298*m**2 +
703416314160*m + 316234143225) + 12740467100*d*m**5*x**3*(f*x)**m/(m**12 +
144*m**11 + 9218*m**10 + 345840*m**9 + 8439783*m**8 + 140529312*m**7 + 1628
301884*m**6 + 13137458400*m**5 + 72578259391*m**4 + 264300628944*m**3 + 590
546123298*m**2 + 703416314160*m + 316234143225) + 1495875590*d*m**5*x*(f*x)
m/(m12 + 144*m**11 + 9218*m**10 + 345840*m**9 + 8439783*m**8 + 14052931
2*m**7 + 1628301884*m**6 + 13137458400*m**5 + 72578259391*m**4 + 2643006289
44*m**3 + 590546123298*m**2 + 703416314160*m + 316234143225) + 2917013970*d
*m**4*x**21*(f*x)**m/(m**12 + 144*m**11 + 9218*m**10 + 345840*m**9 + 843978
3*m**8 + 140529312*m**7 + 1628301884*m**6 + 13137458400*m**5 + 72578259391*
m**4 + 264300628944*m**3 + 590546123298*m**2 + 703416314160*m + 31623414322
5) + 31686018220*d*m**4*x**19*(f*x)**m/(m**12 + 144*m**11 + 9218*m**10 + 34
5840*m**9 + 8439783*m**8 + 140529312*m**7 + 1628301884*m**6 + 13137458400*m
5 + 72578259391*m4 + 264300628944*m**3 + 590546123298*m**2 + 7034163141
60*m + 316234143225) + 156004908210*d*m**4*x**17*(f*x)**m/(m**12 + 144*m**1
1 + 9218*m**10 + 345840*m**9 + 8439783*m**8 + 140529312*m**7 + 1628301884*m
6 + 13137458400*m5 + 72578259391*m**4 + 264300628944*m**3 + 59054612329
8*m**2 + 703416314160*m + 316234143225) + 459045550800*d*m**4*x**15*(f*x)**
m/(m**12 + 144*m**11 + 9218*m**10 + 345840*m**9 + 8439783*m**8 + 140529312*
m**7 + 1628301884*m**6 + 13137458400*m**5 + 72578259391*m**4 + 264300628944
*m**3 + 590546123298*m**2 + 703416314160*m + 316234143225) + 895451283300*d
*m**4*x**13*(f*x)**m/(m**12 + 144*m**11 + 9218*m**10 + 345840*m**9 + 843978
3*m**8 + 140529312*m**7 + 1628301884*m**6 + 13137458400*m**5 + 72578259391*
m**4 + 264300628944*m**3 + 590546123298*m**2 + 703416314160*m + 31623414322
5) + 1212454199880*d*m**4*x**11*(f*x)**m/(m**12 + 144*m**11 + 9218*m**10 +
345840*m**9 + 8439783*m**8 + 140529312*m**7 + 1628301884*m**6 + 13137458400
*m**5 + 72578259391*m**4 + 264300628944*m**3 + 590546123298*m**2 + 70341631
4160*m + 316234143225) + 1156995210420*d*m**4*x**9*(f*x)**m/(m**12 + 144*m*
*11 + 9218*m**10 + 345840*m**9 + 8439783*m**8 + 140529312*m**7 + 1628301884
*m**6 + 13137458400*m**5 + 72578259391*m**4 + 264300628944*m**3 + 590546123
298*m**2 + 703416314160*m + 316234143225) + 770638650960*d*m**4*x**7*(f*x)*
*m/(m**12 + 144*m**11 + 9218*m**10 + 345840*m**9 + 8439783*m**8 + 140529312
*m**7 + 1628301884*m**6 + 13137458400*m**5 + 72578259391*m**4 + 26430062894
4*m**3 + 590546123298*m**2 + 703416314160*m + 316234143225) + 343967603850*
d*m**4*x**5*(f*x)**m/(m**12 + 144*m**11 + 9218*m**10 + 345840*m**9 + 843978
3*m**8 + 140529312*m**7 + 1628301884*m**6 + 13137458400*m**5 + 72578259391*
m**4 + 264300628944*m**3 + 590546123298*m**2 + 703416314160*m + 31623414322
5) + 93153182700*d*m**4*x**3*(f*x)**m/(m**12 + 144*m**11 + 9218*m**10 + 345
840*m**9 + 8439783*m**8 + 140529312*m**7 + 1628301884*m**6 + 13137458400*m*
*5 + 72578259391*m**4 + 264300628944*m**3 + 590546123298*m**2 + 70341631416
0*m + 316234143225) + 11641582810*d*m**4*x*(f*x)**m/(m**12 + 144*m**11 + 92
18*m**10 + 345840*m**9 + 8439783*m**8 + 140529312*m**7 + 1628301884*m**6 +
13137458400*m**5 + 72578259391*m**4 + 264300628944*m**3 + 590546123298*m**2

+ 703416314160*m + 316234143225) + 11320966021*d*m**3*x**21*(f*x)**m/(m**1
2 + 144*m**11 + 9218*m**10 + 345840*m**9 + 8439783*m**8 + 140529312*m**7 +
1628301884*m**6 + 13137458400*m**5 + 72578259391*m**4 + 264300628944*m**3 +
590546123298*m**2 + 703416314160*m + 316234143225) + 123748247730*d*m**3*x
19*(f*x)m/(m**12 + 144*m**11 + 9218*m**10 + 345840*m**9 + 8439783*m**8
+ 140529312*m**7 + 1628301884*m**6 + 13137458400*m**5 + 72578259391*m**4 +
264300628944*m**3 + 590546123298*m**2 + 703416314160*m + 316234143225) + 61
3938233025*d*m**3*x**17*(f*x)**m/(m**12 + 144*m**11 + 9218*m**10 + 345840*m
9 + 8439783*m8 + 140529312*m**7 + 1628301884*m**6 + 13137458400*m**5 +
72578259391*m**4 + 264300628944*m**3 + 590546123298*m**2 + 703416314160*m +
316234143225) + 1823707864920*d*m**3*x**15*(f*x)**m/(m**12 + 144*m**11 + 9
218*m**10 + 345840*m**9 + 8439783*m**8 + 140529312*m**7 + 1628301884*m**6 +
13137458400*m**5 + 72578259391*m**4 + 264300628944*m**3 + 590546123298*m**
2 + 703416314160*m + 316234143225) + 3600567789210*d*m**3*x**13*(f*x)**m/(m
12 + 144*m11 + 9218*m**10 + 345840*m**9 + 8439783*m**8 + 140529312*m**7
+ 1628301884*m**6 + 13137458400*m**5 + 72578259391*m**4 + 264300628944*m**
3 + 590546123298*m**2 + 703416314160*m + 316234143225) + 4952725167852*d*m*
*3*x**11*(f*x)**m/(m**12 + 144*m**11 + 9218*m**10 + 345840*m**9 + 8439783*m
8 + 140529312*m7 + 1628301884*m**6 + 13137458400*m**5 + 72578259391*m**
4 + 264300628944*m**3 + 590546123298*m**2 + 703416314160*m + 316234143225)
+ 4828477578330*d*m**3*x**9*(f*x)**m/(m**12 + 144*m**11 + 9218*m**10 + 3458
40*m**9 + 8439783*m**8 + 140529312*m**7 + 1628301884*m**6 + 13137458400*m**
5 + 72578259391*m**4 + 264300628944*m**3 + 590546123298*m**2 + 703416314160
*m + 316234143225) + 3314920570200*d*m**3*x**7*(f*x)**m/(m**12 + 144*m**11
+ 9218*m**10 + 345840*m**9 + 8439783*m**8 + 140529312*m**7 + 1628301884*m**
6 + 13137458400*m**5 + 72578259391*m**4 + 264300628944*m**3 + 590546123298*
m**2 + 703416314160*m + 316234143225) + 1546183653345*d*m**3*x**5*(f*x)**m/
(m**12 + 144*m**11 + 9218*m**10 + 345840*m**9 + 8439783*m**8 + 140529312*m*
*7 + 1628301884*m**6 + 13137458400*m**5 + 72578259391*m**4 + 264300628944*m
3 + 590546123298*m2 + 703416314160*m + 316234143225) + 446323045810*d*m
3*x3*(f*x)**m/(m**12 + 144*m**11 + 9218*m**10 + 345840*m**9 + 8439783*m
8 + 140529312*m7 + 1628301884*m**6 + 13137458400*m**5 + 72578259391*m**
4 + 264300628944*m**3 + 590546123298*m**2 + 703416314160*m + 316234143225)
+ 60936676581*d*m**3*x*(f*x)**m/(m**12 + 144*m**11 + 9218*m**10 + 345840*m*
*9 + 8439783*m**8 + 140529312*m**7 + 1628301884*m**6 + 13137458400*m**5 + 7
2578259391*m**4 + 264300628944*m**3 + 590546123298*m**2 + 703416314160*m +
316234143225) + 26560342503*d*m**2*x**21*(f*x)**m/(m**12 + 144*m**11 + 9218
*m**10 + 345840*m**9 + 8439783*m**8 + 140529312*m**7 + 1628301884*m**6 + 13
137458400*m**5 + 72578259391*m**4 + 264300628944*m**3 + 590546123298*m**2 +
703416314160*m + 316234143225) + 291789582570*d*m**2*x**19*(f*x)**m/(m**12
+ 144*m**11 + 9218*m**10 + 345840*m**9 + 8439783*m**8 + 140529312*m**7 + 1
628301884*m**6 + 13137458400*m**5 + 72578259391*m**4 + 264300628944*m**3 +
590546123298*m**2 + 703416314160*m + 316234143225) + 1456578341055*d*m**2*x
17*(f*x)m/(m**12 + 144*m**11 + 9218*m**10 + 345840*m**9 + 8439783*m**8
+ 140529312*m**7 + 1628301884*m**6 + 13137458400*m**5 + 72578259391*m**4 +

264300628944*m**3 + 590546123298*m**2 + 703416314160*m + 316234143225) + 43
60457499480*d*m**2*x**15*(f*x)**m/(m**12 + 144*m**11 + 9218*m**10 + 345840*
m**9 + 8439783*m**8 + 140529312*m**7 + 1628301884*m**6 + 13137458400*m**5 +
 72578259391*m**4 + 264300628944*m**3 + 590546123298*m**2 + 703416314160*m
+ 316234143225) + 8695750818510*d*m**2*x**13*(f*x)**m/(m**12 + 144*m**11 +
9218*m**10 + 345840*m**9 + 8439783*m**8 + 140529312*m**7 + 1628301884*m**6
+ 13137458400*m**5 + 72578259391*m**4 + 264300628944*m**3 + 590546123298*m*
*2 + 703416314160*m + 316234143225) + 12123781647516*d*m**2*x**11*(f*x)**m/
(m**12 + 144*m**11 + 9218*m**10 + 345840*m**9 + 8439783*m**8 + 140529312*m*
*7 + 1628301884*m**6 + 13137458400*m**5 + 72578259391*m**4 + 264300628944*m
3 + 590546123298*m2 + 703416314160*m + 316234143225) + 12046833873270*d
*m**2*x**9*(f*x)**m/(m**12 + 144*m**11 + 9218*m**10 + 345840*m**9 + 8439783
*m**8 + 140529312*m**7 + 1628301884*m**6 + 13137458400*m**5 + 72578259391*m
4 + 264300628944*m3 + 590546123298*m**2 + 703416314160*m + 316234143225
) + 8511631481880*d*m**2*x**7*(f*x)**m/(m**12 + 144*m**11 + 9218*m**10 + 34
5840*m**9 + 8439783*m**8 + 140529312*m**7 + 1628301884*m**6 + 13137458400*m
5 + 72578259391*m4 + 264300628944*m**3 + 590546123298*m**2 + 7034163141
60*m + 316234143225) + 4162610035755*d*m**2*x**5*(f*x)**m/(m**12 + 144*m**1
1 + 9218*m**10 + 345840*m**9 + 8439783*m**8 + 140529312*m**7 + 1628301884*m
6 + 13137458400*m5 + 72578259391*m**4 + 264300628944*m**3 + 59054612329
8*m**2 + 703416314160*m + 316234143225) + 1304037152010*d*m**2*x**3*(f*x)**
m/(m**12 + 144*m**11 + 9218*m**10 + 345840*m**9 + 8439783*m**8 + 140529312*
m**7 + 1628301884*m**6 + 13137458400*m**5 + 72578259391*m**4 + 264300628944
*m**3 + 590546123298*m**2 + 703416314160*m + 316234143225) + 203363952363*d
*m**2*x*(f*x)**m/(m**12 + 144*m**11 + 9218*m**10 + 345840*m**9 + 8439783*m*
*8 + 140529312*m**7 + 1628301884*m**6 + 13137458400*m**5 + 72578259391*m**4
 + 264300628944*m**3 + 590546123298*m**2 + 703416314160*m + 316234143225) +
 32778930735*d*m*x**21*(f*x)**m/(m**12 + 144*m**11 + 9218*m**10 + 345840*m*
*9 + 8439783*m**8 + 140529312*m**7 + 1628301884*m**6 + 13137458400*m**5 + 7
2578259391*m**4 + 264300628944*m**3 + 590546123298*m**2 + 703416314160*m +
316234143225) + 361459164150*d*m*x**19*(f*x)**m/(m**12 + 144*m**11 + 9218*m
10 + 345840*m9 + 8439783*m**8 + 140529312*m**7 + 1628301884*m**6 + 1313
7458400*m**5 + 72578259391*m**4 + 264300628944*m**3 + 590546123298*m**2 + 7
03416314160*m + 316234143225) + 1812743750475*d*m*x**17*(f*x)**m/(m**12 + 1
44*m**11 + 9218*m**10 + 345840*m**9 + 8439783*m**8 + 140529312*m**7 + 16283
01884*m**6 + 13137458400*m**5 + 72578259391*m**4 + 264300628944*m**3 + 5905
46123298*m**2 + 703416314160*m + 316234143225) + 5458672303560*d*m*x**15*(f
*x)**m/(m**12 + 144*m**11 + 9218*m**10 + 345840*m**9 + 8439783*m**8 + 14052
9312*m**7 + 1628301884*m**6 + 13137458400*m**5 + 72578259391*m**4 + 2643006
28944*m**3 + 590546123298*m**2 + 703416314160*m + 316234143225) + 109699252
51950*d*m*x**13*(f*x)**m/(m**12 + 144*m**11 + 9218*m**10 + 345840*m**9 + 84
39783*m**8 + 140529312*m**7 + 1628301884*m**6 + 13137458400*m**5 + 72578259
391*m**4 + 264300628944*m**3 + 590546123298*m**2 + 703416314160*m + 3162341
43225) + 15456024948420*d*m*x**11*(f*x)**m/(m**12 + 144*m**11 + 9218*m**10
+ 345840*m**9 + 8439783*m**8 + 140529312*m**7 + 1628301884*m**6 + 131374584

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00*m**5 + 72578259391*m**4 + 264300628944*m**3 + 590546123298*m**2 + 703416
314160*m + 316234143225) + 15593181033150*d*m*x**9*(f*x)**m/(m**12 + 144*m*
*11 + 9218*m**10 + 345840*m**9 + 8439783*m**8 + 140529312*m**7 + 1628301884
*m**6 + 13137458400*m**5 + 72578259391*m**4 + 264300628944*m**3 + 590546123
298*m**2 + 703416314160*m + 316234143225) + 11284114422600*d*m*x**7*(f*x)**
m/(m**12 + 144*m**11 + 9218*m**10 + 345840*m**9 + 8439783*m**8 + 140529312*
m**7 + 1628301884*m**6 + 13137458400*m**5 + 72578259391*m**4 + 264300628944
*m**3 + 590546123298*m**2 + 703416314160*m + 316234143225) + 5761525369635*
d*m*x**5*(f*x)**m/(m**12 + 144*m**11 + 9218*m**10 + 345840*m**9 + 8439783*m
**8 + 140529312*m**7 + 1628301884*m**6 + 13137458400*m**5 + 72578259391*m**
4 + 264300628944*m**3 + 590546123298*m**2 + 703416314160*m + 316234143225)
+ 1993349776950*d*m*x**3*(f*x)**m/(m**12 + 144*m**11 + 9218*m**10 + 345840*
m**9 + 8439783*m**8 + 140529312*m**7 + 1628301884*m**6 + 13137458400*m**5 +
 72578259391*m**4 + 264300628944*m**3 + 590546123298*m**2 + 703416314160*m
+ 316234143225) + 387182170935*d*m*x*(f*x)**m/(m**12 + 144*m**11 + 9218*m**
10 + 345840*m**9 + 8439783*m**8 + 140529312*m**7 + 1628301884*m**6 + 131374
58400*m**5 + 72578259391*m**4 + 264300628944*m**3 + 590546123298*m**2 + 703
416314160*m + 316234143225) + 15058768725*d*x**21*(f*x)**m/(m**12 + 144*m**
11 + 9218*m**10 + 345840*m**9 + 8439783*m**8 + 140529312*m**7 + 1628301884*
m**6 + 13137458400*m**5 + 72578259391*m**4 + 264300628944*m**3 + 5905461232
98*m**2 + 703416314160*m + 316234143225) + 166439022750*d*x**19*(f*x)**m/(m
**12 + 144*m**11 + 9218*m**10 + 345840*m**9 + 8439783*m**8 + 140529312*m**7
 + 1628301884*m**6 + 13137458400*m**5 + 72578259391*m**4 + 264300628944*m**
3 + 590546123298*m**2 + 703416314160*m + 316234143225) + 837090379125*d*x**
17*(f*x)**m/(m**12 + 144*m**11 + 9218*m**10 + 345840*m**9 + 8439783*m**8 +
140529312*m**7 + 1628301884*m**6 + 13137458400*m**5 + 72578259391*m**4 + 26
4300628944*m**3 + 590546123298*m**2 + 703416314160*m + 316234143225) + 2529
873145800*d*x**15*(f*x)**m/(m**12 + 144*m**11 + 9218*m**10 + 345840*m**9 +
8439783*m**8 + 140529312*m**7 + 1628301884*m**6 + 13137458400*m**5 + 725782
59391*m**4 + 264300628944*m**3 + 590546123298*m**2 + 703416314160*m + 31623
4143225) + 5108397698250*d*x**13*(f*x)**m/(m**12 + 144*m**11 + 9218*m**10 +
 345840*m**9 + 8439783*m**8 + 140529312*m**7 + 1628301884*m**6 + 1313745840
0*m**5 + 72578259391*m**4 + 264300628944*m**3 + 590546123298*m**2 + 7034163
14160*m + 316234143225) + 7244636735700*d*x**11*(f*x)**m/(m**12 + 144*m**11
 + 9218*m**10 + 345840*m**9 + 8439783*m**8 + 140529312*m**7 + 1628301884*m*
*6 + 13137458400*m**5 + 72578259391*m**4 + 264300628944*m**3 + 590546123298
*m**2 + 703416314160*m + 316234143225) + 7378796675250*d*x**9*(f*x)**m/(m**
12 + 144*m**11 + 9218*m**10 + 345840*m**9 + 8439783*m**8 + 140529312*m**7 +
 1628301884*m**6 + 13137458400*m**5 + 72578259391*m**4 + 264300628944*m**3
+ 590546123298*m**2 + 703416314160*m + 316234143225) + 5421156741000*d*x**7
*(f*x)**m/(m**12 + 144*m**11 + 9218*m**10 + 345840*m**9 + 8439783*m**8 + 14
0529312*m**7 + 1628301884*m**6 + 13137458400*m**5 + 72578259391*m**4 + 2643
00628944*m**3 + 590546123298*m**2 + 703416314160*m + 316234143225) + 284610
7289025*d*x**5*(f*x)**m/(m**12 + 144*m**11 + 9218*m**10 + 345840*m**9 + 843
9783*m**8 + 140529312*m**7 + 1628301884*m**6 + 13137458400*m**5 + 725782593
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91*m**4 + 264300628944*m**3 + 590546123298*m**2 + 703416314160*m + 31623414
3225) + 1054113810750*d*x**3*(f*x)**m/(m**12 + 144*m**11 + 9218*m**10 + 345
840*m**9 + 8439783*m**8 + 140529312*m**7 + 1628301884*m**6 + 13137458400*m*
*5 + 72578259391*m**4 + 264300628944*m**3 + 590546123298*m**2 + 70341631416
0*m + 316234143225) + 316234143225*d*x*(f*x)**m/(m**12 + 144*m**11 + 9218*m
**10 + 345840*m**9 + 8439783*m**8 + 140529312*m**7 + 1628301884*m**6 + 1313
7458400*m**5 + 72578259391*m**4 + 264300628944*m**3 + 590546123298*m**2 + 7
03416314160*m + 316234143225) + e*m**11*x**23*(f*x)**m/(m**12 + 144*m**11 +
 9218*m**10 + 345840*m**9 + 8439783*m**8 + 140529312*m**7 + 1628301884*m**6
 + 13137458400*m**5 + 72578259391*m**4 + 264300628944*m**3 + 590546123298*m
**2 + 703416314160*m + 316234143225) + 10*e*m**11*x**21*(f*x)**m/(m**12 + 1
44*m**11 + 9218*m**10 + 345840*m**9 + 8439783*m**8 + 140529312*m**7 + 16283
01884*m**6 + 13137458400*m**5 + 72578259391*m**4 + 264300628944*m**3 + 5905
46123298*m**2 + 703416314160*m + 316234143225) + 45*e*m**11*x**19*(f*x)**m/
(m**12 + 144*m**11 + 9218*m**10 + 345840*m**9 + 8439783*m**8 + 140529312*m*
*7 + 1628301884*m**6 + 13137458400*m**5 + 72578259391*m**4 + 264300628944*m
**3 + 590546123298*m**2 + 703416314160*m + 316234143225) + 120*e*m**11*x**1
7*(f*x)**m/(m**12 + 144*m**11 + 9218*m**10 + 345840*m**9 + 8439783*m**8 + 1
40529312*m**7 + 1628301884*m**6 + 13137458400*m**5 + 72578259391*m**4 + 264
300628944*m**3 + 590546123298*m**2 + 703416314160*m + 316234143225) + 210*e
*m**11*x**15*(f*x)**m/(m**12 + 144*m**11 + 9218*m**10 + 345840*m**9 + 84397
83*m**8 + 140529312*m**7 + 1628301884*m**6 + 13137458400*m**5 + 72578259391
*m**4 + 264300628944*m**3 + 590546123298*m**2 + 703416314160*m + 3162341432
25) + 252*e*m**11*x**13*(f*x)**m/(m**12 + 144*m**11 + 9218*m**10 + 345840*m
**9 + 8439783*m**8 + 140529312*m**7 + 1628301884*m**6 + 13137458400*m**5 +
72578259391*m**4 + 264300628944*m**3 + 590546123298*m**2 + 703416314160*m +
 316234143225) + 210*e*m**11*x**11*(f*x)**m/(m**12 + 144*m**11 + 9218*m**10
 + 345840*m**9 + 8439783*m**8 + 140529312*m**7 + 1628301884*m**6 + 13137458
400*m**5 + 72578259391*m**4 + 264300628944*m**3 + 590546123298*m**2 + 70341
6314160*m + 316234143225) + 120*e*m**11*x**9*(f*x)**m/(m**12 + 144*m**11 +
9218*m**10 + 345840*m**9 + 8439783*m**8 + 140529312*m**7 + 1628301884*m**6
+ 13137458400*m**5 + 72578259391*m**4 + 264300628944*m**3 + 590546123298*m*
*2 + 703416314160*m + 316234143225) + 45*e*m**11*x**7*(f*x)**m/(m**12 + 144
*m**11 + 9218*m**10 + 345840*m**9 + 8439783*m**8 + 140529312*m**7 + 1628301
884*m**6 + 13137458400*m**5 + 72578259391*m**4 + 264300628944*m**3 + 590546
123298*m**2 + 703416314160*m + 316234143225) + 10*e*m**11*x**5*(f*x)**m/(m*
*12 + 144*m**11 + 9218*m**10 + 345840*m**9 + 8439783*m**8 + 140529312*m**7
+ 1628301884*m**6 + 13137458400*m**5 + 72578259391*m**4 + 264300628944*m**3
 + 590546123298*m**2 + 703416314160*m + 316234143225) + e*m**11*x**3*(f*x)*
*m/(m**12 + 144*m**11 + 9218*m**10 + 345840*m**9 + 8439783*m**8 + 140529312
*m**7 + 1628301884*m**6 + 13137458400*m**5 + 72578259391*m**4 + 26430062894
4*m**3 + 590546123298*m**2 + 703416314160*m + 316234143225) + 121*e*m**10*x
**23*(f*x)**m/(m**12 + 144*m**11 + 9218*m**10 + 345840*m**9 + 8439783*m**8
+ 140529312*m**7 + 1628301884*m**6 + 13137458400*m**5 + 72578259391*m**4 +
264300628944*m**3 + 590546123298*m**2 + 703416314160*m + 316234143225) + 12
```

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30*e*m**10*x**21*(f*x)**m/(m**12 + 144*m**11 + 9218*m**10 + 345840*m**9 + 8
439783*m**8 + 140529312*m**7 + 1628301884*m**6 + 13137458400*m**5 + 7257825
9391*m**4 + 264300628944*m**3 + 590546123298*m**2 + 703416314160*m + 316234
143225) + 5625*e*m**10*x**19*(f*x)**m/(m**12 + 144*m**11 + 9218*m**10 + 345
840*m**9 + 8439783*m**8 + 140529312*m**7 + 1628301884*m**6 + 13137458400*m*
*5 + 72578259391*m**4 + 264300628944*m**3 + 590546123298*m**2 + 70341631416
0*m + 316234143225) + 15240*e*m**10*x**17*(f*x)**m/(m**12 + 144*m**11 + 921
8*m**10 + 345840*m**9 + 8439783*m**8 + 140529312*m**7 + 1628301884*m**6 + 1
3137458400*m**5 + 72578259391*m**4 + 264300628944*m**3 + 590546123298*m**2
+ 703416314160*m + 316234143225) + 27090*e*m**10*x**15*(f*x)**m/(m**12 + 14
4*m**11 + 9218*m**10 + 345840*m**9 + 8439783*m**8 + 140529312*m**7 + 162830
1884*m**6 + 13137458400*m**5 + 72578259391*m**4 + 264300628944*m**3 + 59054
6123298*m**2 + 703416314160*m + 316234143225) + 33012*e*m**10*x**13*(f*x)**
m/(m**12 + 144*m**11 + 9218*m**10 + 345840*m**9 + 8439783*m**8 + 140529312*
m**7 + 1628301884*m**6 + 13137458400*m**5 + 72578259391*m**4 + 264300628944
*m**3 + 590546123298*m**2 + 703416314160*m + 316234143225) + 27930*e*m**10*
x**11*(f*x)**m/(m**12 + 144*m**11 + 9218*m**10 + 345840*m**9 + 8439783*m**8
 + 140529312*m**7 + 1628301884*m**6 + 13137458400*m**5 + 72578259391*m**4 +
 264300628944*m**3 + 590546123298*m**2 + 703416314160*m + 316234143225) + 1
6200*e*m**10*x**9*(f*x)**m/(m**12 + 144*m**11 + 9218*m**10 + 345840*m**9 +
8439783*m**8 + 140529312*m**7 + 1628301884*m**6 + 13137458400*m**5 + 725782
59391*m**4 + 264300628944*m**3 + 590546123298*m**2 + 703416314160*m + 31623
4143225) + 6165*e*m**10*x**7*(f*x)**m/(m**12 + 144*m**11 + 9218*m**10 + 345
840*m**9 + 8439783*m**8 + 140529312*m**7 + 1628301884*m**6 + 13137458400*m*
*5 + 72578259391*m**4 + 264300628944*m**3 + 590546123298*m**2 + 70341631416
0*m + 316234143225) + 1390*e*m**10*x**5*(f*x)**m/(m**12 + 144*m**11 + 9218*
m**10 + 345840*m**9 + 8439783*m**8 + 140529312*m**7 + 1628301884*m**6 + 131
37458400*m**5 + 72578259391*m**4 + 264300628944*m**3 + 590546123298*m**2 +
703416314160*m + 316234143225) + 141*e*m**10*x**3*(f*x)**m/(m**12 + 144*m**
11 + 9218*m**10 + 345840*m**9 + 8439783*m**8 + 140529312*m**7 + 1628301884*
m**6 + 13137458400*m**5 + 72578259391*m**4 + 264300628944*m**3 + 5905461232
98*m**2 + 703416314160*m + 316234143225) + 6435*e*m**9*x**23*(f*x)**m/(m**1
2 + 144*m**11 + 9218*m**10 + 345840*m**9 + 8439783*m**8 + 140529312*m**7 +
1628301884*m**6 + 13137458400*m**5 + 72578259391*m**4 + 264300628944*m**3 +
 590546123298*m**2 + 703416314160*m + 316234143225) + 66350*e*m**9*x**21*(f
*x)**m/(m**12 + 144*m**11 + 9218*m**10 + 345840*m**9 + 8439783*m**8 + 14052
9312*m**7 + 1628301884*m**6 + 13137458400*m**5 + 72578259391*m**4 + 2643006
28944*m**3 + 590546123298*m**2 + 703416314160*m + 316234143225) + 307935*e*
m**9*x**19*(f*x)**m/(m**12 + 144*m**11 + 9218*m**10 + 345840*m**9 + 8439783
*m**8 + 140529312*m**7 + 1628301884*m**6 + 13137458400*m**5 + 72578259391*m
**4 + 264300628944*m**3 + 590546123298*m**2 + 703416314160*m + 316234143225
) + 847080*e*m**9*x**17*(f*x)**m/(m**12 + 144*m**11 + 9218*m**10 + 345840*m
**9 + 8439783*m**8 + 140529312*m**7 + 1628301884*m**6 + 13137458400*m**5 +
72578259391*m**4 + 264300628944*m**3 + 590546123298*m**2 + 703416314160*m +
 316234143225) + 1529430*e*m**9*x**15*(f*x)**m/(m**12 + 144*m**11 + 9218*m*
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*10 + 345840*m**9 + 8439783*m**8 + 140529312*m**7 + 1628301884*m**6 + 13137
458400*m**5 + 72578259391*m**4 + 264300628944*m**3 + 590546123298*m**2 + 70
3416314160*m + 316234143225) + 1893780*e*m**9*x**13*(f*x)**m/(m**12 + 144*m
**11 + 9218*m**10 + 345840*m**9 + 8439783*m**8 + 140529312*m**7 + 162830188
4*m**6 + 13137458400*m**5 + 72578259391*m**4 + 264300628944*m**3 + 59054612
3298*m**2 + 703416314160*m + 316234143225) + 1628550*e*m**9*x**11*(f*x)**m/
(m**12 + 144*m**11 + 9218*m**10 + 345840*m**9 + 8439783*m**8 + 140529312*m*
*7 + 1628301884*m**6 + 13137458400*m**5 + 72578259391*m**4 + 264300628944*m
**3 + 590546123298*m**2 + 703416314160*m + 316234143225) + 960360*e*m**9*x*
*9*(f*x)**m/(m**12 + 144*m**11 + 9218*m**10 + 345840*m**9 + 8439783*m**8 +
140529312*m**7 + 1628301884*m**6 + 13137458400*m**5 + 72578259391*m**4 + 26
4300628944*m**3 + 590546123298*m**2 + 703416314160*m + 316234143225) + 3716
55*e*m**9*x**7*(f*x)**m/(m**12 + 144*m**11 + 9218*m**10 + 345840*m**9 + 843
9783*m**8 + 140529312*m**7 + 1628301884*m**6 + 13137458400*m**5 + 725782593
91*m**4 + 264300628944*m**3 + 590546123298*m**2 + 703416314160*m + 31623414
3225) + 85230*e*m**9*x**5*(f*x)**m/(m**12 + 144*m**11 + 9218*m**10 + 345840
*m**9 + 8439783*m**8 + 140529312*m**7 + 1628301884*m**6 + 13137458400*m**5
+ 72578259391*m**4 + 264300628944*m**3 + 590546123298*m**2 + 703416314160*m
 + 316234143225) + 8795*e*m**9*x**3*(f*x)**m/(m**12 + 144*m**11 + 9218*m**1
0 + 345840*m**9 + 8439783*m**8 + 140529312*m**7 + 1628301884*m**6 + 1313745
8400*m**5 + 72578259391*m**4 + 264300628944*m**3 + 590546123298*m**2 + 7034
16314160*m + 316234143225) + 197835*e*m**8*x**23*(f*x)**m/(m**12 + 144*m**1
1 + 9218*m**10 + 345840*m**9 + 8439783*m**8 + 140529312*m**7 + 1628301884*m
**6 + 13137458400*m**5 + 72578259391*m**4 + 264300628944*m**3 + 59054612329
8*m**2 + 703416314160*m + 316234143225) + 2065050*e*m**8*x**21*(f*x)**m/(m*
*12 + 144*m**11 + 9218*m**10 + 345840*m**9 + 8439783*m**8 + 140529312*m**7
+ 1628301884*m**6 + 13137458400*m**5 + 72578259391*m**4 + 264300628944*m**3
 + 590546123298*m**2 + 703416314160*m + 316234143225) + 9712035*e*m**8*x**1
9*(f*x)**m/(m**12 + 144*m**11 + 9218*m**10 + 345840*m**9 + 8439783*m**8 + 1
40529312*m**7 + 1628301884*m**6 + 13137458400*m**5 + 72578259391*m**4 + 264
300628944*m**3 + 590546123298*m**2 + 703416314160*m + 316234143225) + 27100
440*e*m**8*x**17*(f*x)**m/(m**12 + 144*m**11 + 9218*m**10 + 345840*m**9 + 8
439783*m**8 + 140529312*m**7 + 1628301884*m**6 + 13137458400*m**5 + 7257825
9391*m**4 + 264300628944*m**3 + 590546123298*m**2 + 703416314160*m + 316234
143225) + 49684950*e*m**8*x**15*(f*x)**m/(m**12 + 144*m**11 + 9218*m**10 +
345840*m**9 + 8439783*m**8 + 140529312*m**7 + 1628301884*m**6 + 13137458400
*m**5 + 72578259391*m**4 + 264300628944*m**3 + 590546123298*m**2 + 70341631
4160*m + 316234143225) + 62532540*e*m**8*x**13*(f*x)**m/(m**12 + 144*m**11
+ 9218*m**10 + 345840*m**9 + 8439783*m**8 + 140529312*m**7 + 1628301884*m**
6 + 13137458400*m**5 + 72578259391*m**4 + 264300628944*m**3 + 590546123298*
m**2 + 703416314160*m + 316234143225) + 54712350*e*m**8*x**11*(f*x)**m/(m**
12 + 144*m**11 + 9218*m**10 + 345840*m**9 + 8439783*m**8 + 140529312*m**7 +
 1628301884*m**6 + 13137458400*m**5 + 72578259391*m**4 + 264300628944*m**3
+ 590546123298*m**2 + 703416314160*m + 316234143225) + 32857560*e*m**8*x**9
*(f*x)**m/(m**12 + 144*m**11 + 9218*m**10 + 345840*m**9 + 8439783*m**8 + 14
```

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0529312*m**7 + 1628301884*m**6 + 13137458400*m**5 + 72578259391*m**4 + 2643
00628944*m**3 + 590546123298*m**2 + 703416314160*m + 316234143225) + 129612
15*e*m**8*x**7*(f*x)**m/(m**12 + 144*m**11 + 9218*m**10 + 345840*m**9 + 843
9783*m**8 + 140529312*m**7 + 1628301884*m**6 + 13137458400*m**5 + 725782593
91*m**4 + 264300628944*m**3 + 590546123298*m**2 + 703416314160*m + 31623414
3225) + 3032250*e*m**8*x**5*(f*x)**m/(m**12 + 144*m**11 + 9218*m**10 + 3458
40*m**9 + 8439783*m**8 + 140529312*m**7 + 1628301884*m**6 + 13137458400*m**
5 + 72578259391*m**4 + 264300628944*m**3 + 590546123298*m**2 + 703416314160
*m + 316234143225) + 319455*e*m**8*x**3*(f*x)**m/(m**12 + 144*m**11 + 9218*
m**10 + 345840*m**9 + 8439783*m**8 + 140529312*m**7 + 1628301884*m**6 + 131
37458400*m**5 + 72578259391*m**4 + 264300628944*m**3 + 590546123298*m**2 +
703416314160*m + 316234143225) + 3889578*e*m**7*x**23*(f*x)**m/(m**12 + 144
*m**11 + 9218*m**10 + 345840*m**9 + 8439783*m**8 + 140529312*m**7 + 1628301
884*m**6 + 13137458400*m**5 + 72578259391*m**4 + 264300628944*m**3 + 590546
123298*m**2 + 703416314160*m + 316234143225) + 41031780*e*m**7*x**21*(f*x)*
*m/(m**12 + 144*m**11 + 9218*m**10 + 345840*m**9 + 8439783*m**8 + 140529312
*m**7 + 1628301884*m**6 + 13137458400*m**5 + 72578259391*m**4 + 26430062894
4*m**3 + 590546123298*m**2 + 703416314160*m + 316234143225) + 195261570*e*m
**7*x**19*(f*x)**m/(m**12 + 144*m**11 + 9218*m**10 + 345840*m**9 + 8439783*
m**8 + 140529312*m**7 + 1628301884*m**6 + 13137458400*m**5 + 72578259391*m*
*4 + 264300628944*m**3 + 590546123298*m**2 + 703416314160*m + 316234143225)
 + 552066480*e*m**7*x**17*(f*x)**m/(m**12 + 144*m**11 + 9218*m**10 + 345840
*m**9 + 8439783*m**8 + 140529312*m**7 + 1628301884*m**6 + 13137458400*m**5
+ 72578259391*m**4 + 264300628944*m**3 + 590546123298*m**2 + 703416314160*m
 + 316234143225) + 1027080180*e*m**7*x**15*(f*x)**m/(m**12 + 144*m**11 + 92
18*m**10 + 345840*m**9 + 8439783*m**8 + 140529312*m**7 + 1628301884*m**6 +
13137458400*m**5 + 72578259391*m**4 + 264300628944*m**3 + 590546123298*m**2
 + 703416314160*m + 316234143225) + 1313902296*e*m**7*x**13*(f*x)**m/(m**12
 + 144*m**11 + 9218*m**10 + 345840*m**9 + 8439783*m**8 + 140529312*m**7 + 1
628301884*m**6 + 13137458400*m**5 + 72578259391*m**4 + 264300628944*m**3 +
590546123298*m**2 + 703416314160*m + 316234143225) + 1170518580*e*m**7*x**1
1*(f*x)**m/(m**12 + 144*m**11 + 9218*m**10 + 345840*m**9 + 8439783*m**8 + 1
40529312*m**7 + 1628301884*m**6 + 13137458400*m**5 + 72578259391*m**4 + 264
300628944*m**3 + 590546123298*m**2 + 703416314160*m + 316234143225) + 71705
5920*e*m**7*x**9*(f*x)**m/(m**12 + 144*m**11 + 9218*m**10 + 345840*m**9 + 8
439783*m**8 + 140529312*m**7 + 1628301884*m**6 + 13137458400*m**5 + 7257825
9391*m**4 + 264300628944*m**3 + 590546123298*m**2 + 703416314160*m + 316234
143225) + 289061730*e*m**7*x**7*(f*x)**m/(m**12 + 144*m**11 + 9218*m**10 +
345840*m**9 + 8439783*m**8 + 140529312*m**7 + 1628301884*m**6 + 13137458400
*m**5 + 72578259391*m**4 + 264300628944*m**3 + 590546123298*m**2 + 70341631
4160*m + 316234143225) + 69236580*e*m**7*x**5*(f*x)**m/(m**12 + 144*m**11 +
 9218*m**10 + 345840*m**9 + 8439783*m**8 + 140529312*m**7 + 1628301884*m**6
 + 13137458400*m**5 + 72578259391*m**4 + 264300628944*m**3 + 590546123298*m
**2 + 703416314160*m + 316234143225) + 7481418*e*m**7*x**3*(f*x)**m/(m**12
+ 144*m**11 + 9218*m**10 + 345840*m**9 + 8439783*m**8 + 140529312*m**7 + 16
```

```
28301884*m**6 + 13137458400*m**5 + 72578259391*m**4 + 264300628944*m**3 + 5
90546123298*m**2 + 703416314160*m + 316234143225) + 51069018*e*m**6*x**23*(
f*x)**m/(m**12 + 144*m**11 + 9218*m**10 + 345840*m**9 + 8439783*m**8 + 1405
29312*m**7 + 1628301884*m**6 + 13137458400*m**5 + 72578259391*m**4 + 264300
628944*m**3 + 590546123298*m**2 + 703416314160*m + 316234143225) + 54362574
0*e*m**6*x**21*(f*x)**m/(m**12 + 144*m**11 + 9218*m**10 + 345840*m**9 + 843
9783*m**8 + 140529312*m**7 + 1628301884*m**6 + 13137458400*m**5 + 725782593
91*m**4 + 264300628944*m**3 + 590546123298*m**2 + 703416314160*m + 31623414
3225) + 2613849210*e*m**6*x**19*(f*x)**m/(m**12 + 144*m**11 + 9218*m**10 +
345840*m**9 + 8439783*m**8 + 140529312*m**7 + 1628301884*m**6 + 13137458400
*m**5 + 72578259391*m**4 + 264300628944*m**3 + 590546123298*m**2 + 70341631
4160*m + 316234143225) + 7478387280*e*m**6*x**17*(f*x)**m/(m**12 + 144*m**1
1 + 9218*m**10 + 345840*m**9 + 8439783*m**8 + 140529312*m**7 + 1628301884*m
**6 + 13137458400*m**5 + 72578259391*m**4 + 264300628944*m**3 + 59054612329
8*m**2 + 703416314160*m + 316234143225) + 14104952820*e*m**6*x**15*(f*x)**m
/(m**12 + 144*m**11 + 9218*m**10 + 345840*m**9 + 8439783*m**8 + 140529312*m
**7 + 1628301884*m**6 + 13137458400*m**5 + 72578259391*m**4 + 264300628944*
m**3 + 590546123298*m**2 + 703416314160*m + 316234143225) + 18332656776*e*m
**6*x**13*(f*x)**m/(m**12 + 144*m**11 + 9218*m**10 + 345840*m**9 + 8439783*
m**8 + 140529312*m**7 + 1628301884*m**6 + 13137458400*m**5 + 72578259391*m*
*4 + 264300628944*m**3 + 590546123298*m**2 + 703416314160*m + 316234143225)
 + 16635451140*e*m**6*x**11*(f*x)**m/(m**12 + 144*m**11 + 9218*m**10 + 3458
40*m**9 + 8439783*m**8 + 140529312*m**7 + 1628301884*m**6 + 13137458400*m**
5 + 72578259391*m**4 + 264300628944*m**3 + 590546123298*m**2 + 703416314160
*m + 316234143225) + 10410014160*e*m**6*x**9*(f*x)**m/(m**12 + 144*m**11 +
9218*m**10 + 345840*m**9 + 8439783*m**8 + 140529312*m**7 + 1628301884*m**6
+ 13137458400*m**5 + 72578259391*m**4 + 264300628944*m**3 + 590546123298*m*
*2 + 703416314160*m + 316234143225) + 4300386930*e*m**6*x**7*(f*x)**m/(m**1
2 + 144*m**11 + 9218*m**10 + 345840*m**9 + 8439783*m**8 + 140529312*m**7 +
1628301884*m**6 + 13137458400*m**5 + 72578259391*m**4 + 264300628944*m**3 +
 590546123298*m**2 + 703416314160*m + 316234143225) + 1059110220*e*m**6*x**
5*(f*x)**m/(m**12 + 144*m**11 + 9218*m**10 + 345840*m**9 + 8439783*m**8 + 1
40529312*m**7 + 1628301884*m**6 + 13137458400*m**5 + 72578259391*m**4 + 264
300628944*m**3 + 590546123298*m**2 + 703416314160*m + 316234143225) + 11808
5058*e*m**6*x**3*(f*x)**m/(m**12 + 144*m**11 + 9218*m**10 + 345840*m**9 + 8
439783*m**8 + 140529312*m**7 + 1628301884*m**6 + 13137458400*m**5 + 7257825
9391*m**4 + 264300628944*m**3 + 590546123298*m**2 + 703416314160*m + 316234
143225) + 453714470*e*m**5*x**23*(f*x)**m/(m**12 + 144*m**11 + 9218*m**10 +
 345840*m**9 + 8439783*m**8 + 140529312*m**7 + 1628301884*m**6 + 1313745840
0*m**5 + 72578259391*m**4 + 264300628944*m**3 + 590546123298*m**2 + 7034163
14160*m + 316234143225) + 4866878300*e*m**5*x**21*(f*x)**m/(m**12 + 144*m**
11 + 9218*m**10 + 345840*m**9 + 8439783*m**8 + 140529312*m**7 + 1628301884*
m**6 + 13137458400*m**5 + 72578259391*m**4 + 264300628944*m**3 + 5905461232
98*m**2 + 703416314160*m + 316234143225) + 23610449790*e*m**5*x**19*(f*x)**
m/(m**12 + 144*m**11 + 9218*m**10 + 345840*m**9 + 8439783*m**8 + 140529312*
```

m**7 + 1628301884*m**6 + 13137458400*m**5 + 72578259391*m**4 + 264300628944
*m**3 + 590546123298*m**2 + 703416314160*m + 316234143225) + 68263642320*e*
m**5*x**17*(f*x)**m/(m**12 + 144*m**11 + 9218*m**10 + 345840*m**9 + 8439783
*m**8 + 140529312*m**7 + 1628301884*m**6 + 13137458400*m**5 + 72578259391*m
4 + 264300628944*m3 + 590546123298*m**2 + 703416314160*m + 316234143225
) + 130369103340*e*m**5*x**15*(f*x)**m/(m**12 + 144*m**11 + 9218*m**10 + 34
5840*m**9 + 8439783*m**8 + 140529312*m**7 + 1628301884*m**6 + 13137458400*m
5 + 72578259391*m4 + 264300628944*m**3 + 590546123298*m**2 + 7034163141
60*m + 316234143225) + 172007536680*e*m**5*x**13*(f*x)**m/(m**12 + 144*m**1
1 + 9218*m**10 + 345840*m**9 + 8439783*m**8 + 140529312*m**7 + 1628301884*m
6 + 13137458400*m5 + 72578259391*m**4 + 264300628944*m**3 + 59054612329
8*m**2 + 703416314160*m + 316234143225) + 158953433100*e*m**5*x**11*(f*x)**
m/(m**12 + 144*m**11 + 9218*m**10 + 345840*m**9 + 8439783*m**8 + 140529312*
m**7 + 1628301884*m**6 + 13137458400*m**5 + 72578259391*m**4 + 264300628944
*m**3 + 590546123298*m**2 + 703416314160*m + 316234143225) + 101706098640*e
*m**5*x**9*(f*x)**m/(m**12 + 144*m**11 + 9218*m**10 + 345840*m**9 + 8439783
*m**8 + 140529312*m**7 + 1628301884*m**6 + 13137458400*m**5 + 72578259391*m
4 + 264300628944*m3 + 590546123298*m**2 + 703416314160*m + 316234143225
) + 43170876270*e*m**5*x**7*(f*x)**m/(m**12 + 144*m**11 + 9218*m**10 + 3458
40*m**9 + 8439783*m**8 + 140529312*m**7 + 1628301884*m**6 + 13137458400*m**
5 + 72578259391*m**4 + 264300628944*m**3 + 590546123298*m**2 + 703416314160
*m + 316234143225) + 10987467740*e*m**5*x**5*(f*x)**m/(m**12 + 144*m**11 +
9218*m**10 + 345840*m**9 + 8439783*m**8 + 140529312*m**7 + 1628301884*m**6
+ 13137458400*m**5 + 72578259391*m**4 + 264300628944*m**3 + 590546123298*m*
*2 + 703416314160*m + 316234143225) + 1274046710*e*m**5*x**3*(f*x)**m/(m**1
2 + 144*m**11 + 9218*m**10 + 345840*m**9 + 8439783*m**8 + 140529312*m**7 +
1628301884*m**6 + 13137458400*m**5 + 72578259391*m**4 + 264300628944*m**3 +
590546123298*m**2 + 703416314160*m + 316234143225) + 2702025590*e*m**4*x**
23*(f*x)**m/(m**12 + 144*m**11 + 9218*m**10 + 345840*m**9 + 8439783*m**8 +
140529312*m**7 + 1628301884*m**6 + 13137458400*m**5 + 72578259391*m**4 + 26
4300628944*m**3 + 590546123298*m**2 + 703416314160*m + 316234143225) + 2917
0139700*e*m**4*x**21*(f*x)**m/(m**12 + 144*m**11 + 9218*m**10 + 345840*m**9
+ 8439783*m**8 + 140529312*m**7 + 1628301884*m**6 + 13137458400*m**5 + 725
78259391*m**4 + 264300628944*m**3 + 590546123298*m**2 + 703416314160*m + 31
6234143225) + 142587081990*e*m**4*x**19*(f*x)**m/(m**12 + 144*m**11 + 9218*
m**10 + 345840*m**9 + 8439783*m**8 + 140529312*m**7 + 1628301884*m**6 + 131
37458400*m**5 + 72578259391*m**4 + 264300628944*m**3 + 590546123298*m**2 +
703416314160*m + 316234143225) + 416013088560*e*m**4*x**17*(f*x)**m/(m**12
+ 144*m**11 + 9218*m**10 + 345840*m**9 + 8439783*m**8 + 140529312*m**7 + 16
28301884*m**6 + 13137458400*m**5 + 72578259391*m**4 + 264300628944*m**3 + 5
90546123298*m**2 + 703416314160*m + 316234143225) + 803329713900*e*m**4*x**
15*(f*x)**m/(m**12 + 144*m**11 + 9218*m**10 + 345840*m**9 + 8439783*m**8 +
140529312*m**7 + 1628301884*m**6 + 13137458400*m**5 + 72578259391*m**4 + 26
4300628944*m**3 + 590546123298*m**2 + 703416314160*m + 316234143225) + 1074
541539960*e*m**4*x**13*(f*x)**m/(m**12 + 144*m**11 + 9218*m**10 + 345840*m*

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*9 + 8439783*m**8 + 140529312*m**7 + 1628301884*m**6 + 13137458400*m**5 + 7
2578259391*m**4 + 264300628944*m**3 + 590546123298*m**2 + 703416314160*m +
316234143225) + 1010378499900*e*m**4*x**11*(f*x)**m/(m**12 + 144*m**11 + 92
18*m**10 + 345840*m**9 + 8439783*m**8 + 140529312*m**7 + 1628301884*m**6 +
13137458400*m**5 + 72578259391*m**4 + 264300628944*m**3 + 590546123298*m**2
 + 703416314160*m + 316234143225) + 661140120240*e*m**4*x**9*(f*x)**m/(m**1
2 + 144*m**11 + 9218*m**10 + 345840*m**9 + 8439783*m**8 + 140529312*m**7 +
1628301884*m**6 + 13137458400*m**5 + 72578259391*m**4 + 264300628944*m**3 +
 590546123298*m**2 + 703416314160*m + 316234143225) + 288989494110*e*m**4*x
**7*(f*x)**m/(m**12 + 144*m**11 + 9218*m**10 + 345840*m**9 + 8439783*m**8 +
 140529312*m**7 + 1628301884*m**6 + 13137458400*m**5 + 72578259391*m**4 + 2
64300628944*m**3 + 590546123298*m**2 + 703416314160*m + 316234143225) + 764
37245300*e*m**4*x**5*(f*x)**m/(m**12 + 144*m**11 + 9218*m**10 + 345840*m**9
 + 8439783*m**8 + 140529312*m**7 + 1628301884*m**6 + 13137458400*m**5 + 725
78259391*m**4 + 264300628944*m**3 + 590546123298*m**2 + 703416314160*m + 31
6234143225) + 9315318270*e*m**4*x**3*(f*x)**m/(m**12 + 144*m**11 + 9218*m**
10 + 345840*m**9 + 8439783*m**8 + 140529312*m**7 + 1628301884*m**6 + 131374
58400*m**5 + 72578259391*m**4 + 264300628944*m**3 + 590546123298*m**2 + 703
416314160*m + 316234143225) + 10431670821*e*m**3*x**23*(f*x)**m/(m**12 + 14
4*m**11 + 9218*m**10 + 345840*m**9 + 8439783*m**8 + 140529312*m**7 + 162830
1884*m**6 + 13137458400*m**5 + 72578259391*m**4 + 264300628944*m**3 + 59054
6123298*m**2 + 703416314160*m + 316234143225) + 113209660210*e*m**3*x**21*(
f*x)**m/(m**12 + 144*m**11 + 9218*m**10 + 345840*m**9 + 8439783*m**8 + 1405
29312*m**7 + 1628301884*m**6 + 13137458400*m**5 + 72578259391*m**4 + 264300
628944*m**3 + 590546123298*m**2 + 703416314160*m + 316234143225) + 55686711
4785*e*m**3*x**19*(f*x)**m/(m**12 + 144*m**11 + 9218*m**10 + 345840*m**9 +
8439783*m**8 + 140529312*m**7 + 1628301884*m**6 + 13137458400*m**5 + 725782
59391*m**4 + 264300628944*m**3 + 590546123298*m**2 + 703416314160*m + 31623
4143225) + 1637168621400*e*m**3*x**17*(f*x)**m/(m**12 + 144*m**11 + 9218*m*
*10 + 345840*m**9 + 8439783*m**8 + 140529312*m**7 + 1628301884*m**6 + 13137
458400*m**5 + 72578259391*m**4 + 264300628944*m**3 + 590546123298*m**2 + 70
3416314160*m + 316234143225) + 3191488763610*e*m**3*x**15*(f*x)**m/(m**12 +
 144*m**11 + 9218*m**10 + 345840*m**9 + 8439783*m**8 + 140529312*m**7 + 162
8301884*m**6 + 13137458400*m**5 + 72578259391*m**4 + 264300628944*m**3 + 59
0546123298*m**2 + 703416314160*m + 316234143225) + 4320681347052*e*m**3*x**
13*(f*x)**m/(m**12 + 144*m**11 + 9218*m**10 + 345840*m**9 + 8439783*m**8 +
140529312*m**7 + 1628301884*m**6 + 13137458400*m**5 + 72578259391*m**4 + 26
4300628944*m**3 + 590546123298*m**2 + 703416314160*m + 316234143225) + 4127
270973210*e*m**3*x**11*(f*x)**m/(m**12 + 144*m**11 + 9218*m**10 + 345840*m*
*9 + 8439783*m**8 + 140529312*m**7 + 1628301884*m**6 + 13137458400*m**5 + 7
2578259391*m**4 + 264300628944*m**3 + 590546123298*m**2 + 703416314160*m +
316234143225) + 2759130044760*e*m**3*x**9*(f*x)**m/(m**12 + 144*m**11 + 921
8*m**10 + 345840*m**9 + 8439783*m**8 + 140529312*m**7 + 1628301884*m**6 + 1
3137458400*m**5 + 72578259391*m**4 + 264300628944*m**3 + 590546123298*m**2
+ 703416314160*m + 316234143225) + 1243095213825*e*m**3*x**7*(f*x)**m/(m**1
```

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2 + 144*m**11 + 9218*m**10 + 345840*m**9 + 8439783*m**8 + 140529312*m**7 +
1628301884*m**6 + 13137458400*m**5 + 72578259391*m**4 + 264300628944*m**3 +
 590546123298*m**2 + 703416314160*m + 316234143225) + 343596367410*e*m**3*x
**5*(f*x)**m/(m**12 + 144*m**11 + 9218*m**10 + 345840*m**9 + 8439783*m**8 +
 140529312*m**7 + 1628301884*m**6 + 13137458400*m**5 + 72578259391*m**4 + 2
64300628944*m**3 + 590546123298*m**2 + 703416314160*m + 316234143225) + 446
32304581*e*m**3*x**3*(f*x)**m/(m**12 + 144*m**11 + 9218*m**10 + 345840*m**9
 + 8439783*m**8 + 140529312*m**7 + 1628301884*m**6 + 13137458400*m**5 + 725
78259391*m**4 + 264300628944*m**3 + 590546123298*m**2 + 703416314160*m + 31
6234143225) + 24372200061*e*m**2*x**23*(f*x)**m/(m**12 + 144*m**11 + 9218*m
**10 + 345840*m**9 + 8439783*m**8 + 140529312*m**7 + 1628301884*m**6 + 1313
7458400*m**5 + 72578259391*m**4 + 264300628944*m**3 + 590546123298*m**2 + 7
03416314160*m + 316234143225) + 265603425030*e*m**2*x**21*(f*x)**m/(m**12 +
 144*m**11 + 9218*m**10 + 345840*m**9 + 8439783*m**8 + 140529312*m**7 + 162
8301884*m**6 + 13137458400*m**5 + 72578259391*m**4 + 264300628944*m**3 + 59
0546123298*m**2 + 703416314160*m + 316234143225) + 1313053121565*e*m**2*x**
19*(f*x)**m/(m**12 + 144*m**11 + 9218*m**10 + 345840*m**9 + 8439783*m**8 +
140529312*m**7 + 1628301884*m**6 + 13137458400*m**5 + 72578259391*m**4 + 26
4300628944*m**3 + 590546123298*m**2 + 703416314160*m + 316234143225) + 3884
208909480*e*m**2*x**17*(f*x)**m/(m**12 + 144*m**11 + 9218*m**10 + 345840*m*
*9 + 8439783*m**8 + 140529312*m**7 + 1628301884*m**6 + 13137458400*m**5 + 7
2578259391*m**4 + 264300628944*m**3 + 590546123298*m**2 + 703416314160*m +
316234143225) + 7630800624090*e*m**2*x**15*(f*x)**m/(m**12 + 144*m**11 + 92
18*m**10 + 345840*m**9 + 8439783*m**8 + 140529312*m**7 + 1628301884*m**6 +
13137458400*m**5 + 72578259391*m**4 + 264300628944*m**3 + 590546123298*m**2
 + 703416314160*m + 316234143225) + 10434900982212*e*m**2*x**13*(f*x)**m/(m
**12 + 144*m**11 + 9218*m**10 + 345840*m**9 + 8439783*m**8 + 140529312*m**7
 + 1628301884*m**6 + 13137458400*m**5 + 72578259391*m**4 + 264300628944*m**
3 + 590546123298*m**2 + 703416314160*m + 316234143225) + 10103151372930*e*m
**2*x**11*(f*x)**m/(m**12 + 144*m**11 + 9218*m**10 + 345840*m**9 + 8439783*
m**8 + 140529312*m**7 + 1628301884*m**6 + 13137458400*m**5 + 72578259391*m*
*4 + 264300628944*m**3 + 590546123298*m**2 + 703416314160*m + 316234143225)
 + 6883905070440*e*m**2*x**9*(f*x)**m/(m**12 + 144*m**11 + 9218*m**10 + 345
840*m**9 + 8439783*m**8 + 140529312*m**7 + 1628301884*m**6 + 13137458400*m*
*5 + 72578259391*m**4 + 264300628944*m**3 + 590546123298*m**2 + 70341631416
0*m + 316234143225) + 3191861805705*e*m**2*x**7*(f*x)**m/(m**12 + 144*m**11
 + 9218*m**10 + 345840*m**9 + 8439783*m**8 + 140529312*m**7 + 1628301884*m*
*6 + 13137458400*m**5 + 72578259391*m**4 + 264300628944*m**3 + 590546123298
*m**2 + 703416314160*m + 316234143225) + 925024452390*e*m**2*x**5*(f*x)**m/
(m**12 + 144*m**11 + 9218*m**10 + 345840*m**9 + 8439783*m**8 + 140529312*m*
*7 + 1628301884*m**6 + 13137458400*m**5 + 72578259391*m**4 + 264300628944*m
**3 + 590546123298*m**2 + 703416314160*m + 316234143225) + 130403715201*e*m
**2*x**3*(f*x)**m/(m**12 + 144*m**11 + 9218*m**10 + 345840*m**9 + 8439783*m
**8 + 140529312*m**7 + 1628301884*m**6 + 13137458400*m**5 + 72578259391*m**
4 + 264300628944*m**3 + 590546123298*m**2 + 703416314160*m + 316234143225)
```

+ 29985521895*e*m*x**23*(f*x)**m/(m**12 + 144*m**11 + 9218*m**10 + 345840*m
9 + 8439783*m8 + 140529312*m**7 + 1628301884*m**6 + 13137458400*m**5 +
72578259391*m**4 + 264300628944*m**3 + 590546123298*m**2 + 703416314160*m +
 316234143225) + 327789307350*e*m*x**21*(f*x)**m/(m**12 + 144*m**11 + 9218*
m**10 + 345840*m**9 + 8439783*m**8 + 140529312*m**7 + 1628301884*m**6 + 131
37458400*m**5 + 72578259391*m**4 + 264300628944*m**3 + 590546123298*m**2 +
703416314160*m + 316234143225) + 1626566238675*e*m*x**19*(f*x)**m/(m**12 +
144*m**11 + 9218*m**10 + 345840*m**9 + 8439783*m**8 + 140529312*m**7 + 1628
301884*m**6 + 13137458400*m**5 + 72578259391*m**4 + 264300628944*m**3 + 590
546123298*m**2 + 703416314160*m + 316234143225) + 4833983334600*e*m*x**17*(
f*x)**m/(m**12 + 144*m**11 + 9218*m**10 + 345840*m**9 + 8439783*m**8 + 1405
29312*m**7 + 1628301884*m**6 + 13137458400*m**5 + 72578259391*m**4 + 264300
628944*m**3 + 590546123298*m**2 + 703416314160*m + 316234143225) + 95526765
31230*e*m*x**15*(f*x)**m/(m**12 + 144*m**11 + 9218*m**10 + 345840*m**9 + 84
39783*m**8 + 140529312*m**7 + 1628301884*m**6 + 13137458400*m**5 + 72578259
391*m**4 + 264300628944*m**3 + 590546123298*m**2 + 703416314160*m + 3162341
43225) + 13163910302340*e*m*x**13*(f*x)**m/(m**12 + 144*m**11 + 9218*m**10
+ 345840*m**9 + 8439783*m**8 + 140529312*m**7 + 1628301884*m**6 + 131374584
00*m**5 + 72578259391*m**4 + 264300628944*m**3 + 590546123298*m**2 + 703416
314160*m + 316234143225) + 12880020790350*e*m*x**11*(f*x)**m/(m**12 + 144*m
11 + 9218*m10 + 345840*m**9 + 8439783*m**8 + 140529312*m**7 + 162830188
4*m**6 + 13137458400*m**5 + 72578259391*m**4 + 264300628944*m**3 + 59054612
3298*m**2 + 703416314160*m + 316234143225) + 8910389161800*e*m*x**9*(f*x)**
m/(m**12 + 144*m**11 + 9218*m**10 + 345840*m**9 + 8439783*m**8 + 140529312*
m**7 + 1628301884*m**6 + 13137458400*m**5 + 72578259391*m**4 + 264300628944
*m**3 + 590546123298*m**2 + 703416314160*m + 316234143225) + 4231542908475*
e*m*x**7*(f*x)**m/(m**12 + 144*m**11 + 9218*m**10 + 345840*m**9 + 8439783*m
8 + 140529312*m7 + 1628301884*m**6 + 13137458400*m**5 + 72578259391*m**
4 + 264300628944*m**3 + 590546123298*m**2 + 703416314160*m + 316234143225)
+ 1280338971030*e*m*x**5*(f*x)**m/(m**12 + 144*m**11 + 9218*m**10 + 345840*
m**9 + 8439783*m**8 + 140529312*m**7 + 1628301884*m**6 + 13137458400*m**5 +
 72578259391*m**4 + 264300628944*m**3 + 590546123298*m**2 + 703416314160*m
+ 316234143225) + 199334977695*e*m*x**3*(f*x)**m/(m**12 + 144*m**11 + 9218*
m**10 + 345840*m**9 + 8439783*m**8 + 140529312*m**7 + 1628301884*m**6 + 131
37458400*m**5 + 72578259391*m**4 + 264300628944*m**3 + 590546123298*m**2 +
703416314160*m + 316234143225) + 13749310575*e*x**23*(f*x)**m/(m**12 + 144*
m**11 + 9218*m**10 + 345840*m**9 + 8439783*m**8 + 140529312*m**7 + 16283018
84*m**6 + 13137458400*m**5 + 72578259391*m**4 + 264300628944*m**3 + 5905461
23298*m**2 + 703416314160*m + 316234143225) + 150587687250*e*x**21*(f*x)**m
/(m**12 + 144*m**11 + 9218*m**10 + 345840*m**9 + 8439783*m**8 + 140529312*m
7 + 1628301884*m6 + 13137458400*m**5 + 72578259391*m**4 + 264300628944*
m**3 + 590546123298*m**2 + 703416314160*m + 316234143225) + 748975602375*e*
x**19*(f*x)**m/(m**12 + 144*m**11 + 9218*m**10 + 345840*m**9 + 8439783*m**8
 + 140529312*m**7 + 1628301884*m**6 + 13137458400*m**5 + 72578259391*m**4 +
 264300628944*m**3 + 590546123298*m**2 + 703416314160*m + 316234143225) + 2

```
232241011000*e*x**17*(f*x)**m/(m**12 + 144*m**11 + 9218*m**10 + 345840*m**9
 + 8439783*m**8 + 140529312*m**7 + 1628301884*m**6 + 13137458400*m**5 + 725
78259391*m**4 + 264300628944*m**3 + 590546123298*m**2 + 703416314160*m + 31
6234143225) + 4427278005150*e*x**15*(f*x)**m/(m**12 + 144*m**11 + 9218*m**1
0 + 345840*m**9 + 8439783*m**8 + 140529312*m**7 + 1628301884*m**6 + 1313745
8400*m**5 + 72578259391*m**4 + 264300628944*m**3 + 590546123298*m**2 + 7034
16314160*m + 316234143225) + 6130077237900*e*x**13*(f*x)**m/(m**12 + 144*m*
*11 + 9218*m**10 + 345840*m**9 + 8439783*m**8 + 140529312*m**7 + 1628301884
*m**6 + 13137458400*m**5 + 72578259391*m**4 + 264300628944*m**3 + 590546123
298*m**2 + 703416314160*m + 316234143225) + 6037197279750*e*x**11*(f*x)**m/
(m**12 + 144*m**11 + 9218*m**10 + 345840*m**9 + 8439783*m**8 + 140529312*m*
*7 + 1628301884*m**6 + 13137458400*m**5 + 72578259391*m**4 + 264300628944*m
**3 + 590546123298*m**2 + 703416314160*m + 316234143225) + 4216455243000*e*
x**9*(f*x)**m/(m**12 + 144*m**11 + 9218*m**10 + 345840*m**9 + 8439783*m**8
+ 140529312*m**7 + 1628301884*m**6 + 13137458400*m**5 + 72578259391*m**4 +
264300628944*m**3 + 590546123298*m**2 + 703416314160*m + 316234143225) + 20
32933777875*e*x**7*(f*x)**m/(m**12 + 144*m**11 + 9218*m**10 + 345840*m**9 +
 8439783*m**8 + 140529312*m**7 + 1628301884*m**6 + 13137458400*m**5 + 72578
259391*m**4 + 264300628944*m**3 + 590546123298*m**2 + 703416314160*m + 3162
34143225) + 632468286450*e*x**5*(f*x)**m/(m**12 + 144*m**11 + 9218*m**10 +
345840*m**9 + 8439783*m**8 + 140529312*m**7 + 1628301884*m**6 + 13137458400
*m**5 + 72578259391*m**4 + 264300628944*m**3 + 590546123298*m**2 + 70341631
4160*m + 316234143225) + 105411381075*e*x**3*(f*x)**m/(m**12 + 144*m**11 +
9218*m**10 + 345840*m**9 + 8439783*m**8 + 140529312*m**7 + 1628301884*m**6
+ 13137458400*m**5 + 72578259391*m**4 + 264300628944*m**3 + 590546123298*m*
*2 + 703416314160*m + 316234143225), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 372, normalized size of antiderivative = 1.38

$$\int (fx)^m (d+ex^2) (1+2x^2+x^4)^5 dx = \frac{ef^m x^{23} x^m}{m+23} + \frac{df^m x^{21} x^m}{m+21} + \frac{10 ef^m x^{21} x^m}{m+21} + \frac{10 df^m x^{19} x^m}{m+19} + \frac{45 ef^m x^{19} x^m}{m+19} + \frac{45 df^m x^{17} x^m}{m+17} + \frac{120 ef^m x^{17} x^m}{m+17} + \frac{120 df^m x^{15} x^m}{m+15} + \frac{210 ef^m x^{15} x^m}{m+15} + \frac{210 df^m x^{13} x^m}{m+13} + \frac{252 ef^m x^{13} x^m}{m+13} + \frac{252 df^m x^{11} x^m}{m+11} + \frac{210 ef^m x^{11} x^m}{m+11} + \frac{210 df^m x^9 x^m}{m+9} + \frac{120 ef^m x^9 x^m}{m+9} + \frac{120 df^m x^7 x^m}{m+7} + \frac{45 ef^m x^7 x^m}{m+7} + \frac{45 df^m x^5 x^m}{m+5} + \frac{10 ef^m x^5 x^m}{m+5} + \frac{10 df^m x^3 x^m}{m+3} + \frac{ef^m x^3 x^m}{m+3} + \frac{(fx)^{m+1} d}{f(m+1)}$$

```
[In] integrate((f*x)^m*(e*x^2+d)*(x^4+2*x^2+1)^5,x, algorithm="maxima")
```

```
[Out] e*f^m*x^23*x^m/(m + 23) + d*f^m*x^21*x^m/(m + 21) + 10*e*f^m*x^21*x^m/(m +
21) + 10*d*f^m*x^19*x^m/(m + 19) + 45*e*f^m*x^19*x^m/(m + 19) + 45*d*f^m*x^
17*x^m/(m + 17) + 120*e*f^m*x^17*x^m/(m + 17) + 120*d*f^m*x^15*x^m/(m + 15)
 + 210*e*f^m*x^15*x^m/(m + 15) + 210*d*f^m*x^13*x^m/(m + 13) + 252*e*f^m*x^
13*x^m/(m + 13) + 252*d*f^m*x^11*x^m/(m + 11) + 210*e*f^m*x^11*x^m/(m + 11)
 + 210*d*f^m*x^9*x^m/(m + 9) + 120*e*f^m*x^9*x^m/(m + 9) + 120*d*f^m*x^7*x^
m/(m + 7) + 45*e*f^m*x^7*x^m/(m + 7) + 45*d*f^m*x^5*x^m/(m + 5) + 10*e*f^m*
x^5*x^m/(m + 5) + 10*d*f^m*x^3*x^m/(m + 3) + e*f^m*x^3*x^m/(m + 3) + (f*x)^
(m + 1)*d/(f*(m + 1))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3620 vs. $2(269) = 538$.

Time = 0.35 (sec) , antiderivative size = 3620, normalized size of antiderivative = 13.46

$$\int (fx)^m (d+ex^2) (1+2x^2+x^4)^5 dx = \text{Too large to display}$$

```
[In] integrate((f*x)^m*(e*x^2+d)*(x^4+2*x^2+1)^5,x, algorithm="giac")
```

```
[Out] ((f*x)^m*e*m^11*x^23 + 121*(f*x)^m*e*m^10*x^23 + (f*x)^m*d*m^11*x^21 + 10*(
f*x)^m*e*m^11*x^21 + 6435*(f*x)^m*e*m^9*x^23 + 123*(f*x)^m*d*m^10*x^21 + 12
30*(f*x)^m*e*m^10*x^21 + 197835*(f*x)^m*e*m^8*x^23 + 10*(f*x)^m*d*m^11*x^19
 + 45*(f*x)^m*e*m^11*x^19 + 6635*(f*x)^m*d*m^9*x^21 + 66350*(f*x)^m*e*m^9*x
^21 + 3889578*(f*x)^m*e*m^7*x^23 + 1250*(f*x)^m*d*m^10*x^19 + 5625*(f*x)^m*
```

e*m^10*x^19 + 206505*(f*x)^m*d*m^8*x^21 + 2065050*(f*x)^m*e*m^8*x^21 + 5106
9018*(f*x)^m*e*m^6*x^23 + 45*(f*x)^m*d*m^11*x^17 + 120*(f*x)^m*e*m^11*x^17
+ 68430*(f*x)^m*d*m^9*x^19 + 307935*(f*x)^m*e*m^9*x^19 + 4103178*(f*x)^m*d*
m^7*x^21 + 41031780*(f*x)^m*e*m^7*x^21 + 453714470*(f*x)^m*e*m^5*x^23 + 571
5*(f*x)^m*d*m^10*x^17 + 15240*(f*x)^m*e*m^10*x^17 + 2158230*(f*x)^m*d*m^8*x
^19 + 9712035*(f*x)^m*e*m^8*x^19 + 54362574*(f*x)^m*d*m^6*x^21 + 543625740*
(f*x)^m*e*m^6*x^21 + 2702025590*(f*x)^m*e*m^4*x^23 + 120*(f*x)^m*d*m^11*x^1
5 + 210*(f*x)^m*e*m^11*x^15 + 317655*(f*x)^m*d*m^9*x^17 + 847080*(f*x)^m*e*
m^9*x^17 + 43391460*(f*x)^m*d*m^7*x^19 + 195261570*(f*x)^m*e*m^7*x^19 + 486
687830*(f*x)^m*d*m^5*x^21 + 4866878300*(f*x)^m*e*m^5*x^21 + 10431670821*(f*
x)^m*e*m^3*x^23 + 15480*(f*x)^m*d*m^10*x^15 + 27090*(f*x)^m*e*m^10*x^15 + 1
0162665*(f*x)^m*d*m^8*x^17 + 27100440*(f*x)^m*e*m^8*x^17 + 580855380*(f*x)^
m*d*m^6*x^19 + 2613849210*(f*x)^m*e*m^6*x^19 + 2917013970*(f*x)^m*d*m^4*x^2
1 + 29170139700*(f*x)^m*e*m^4*x^21 + 24372200061*(f*x)^m*e*m^2*x^23 + 210*(
f*x)^m*d*m^11*x^13 + 252*(f*x)^m*e*m^11*x^13 + 873960*(f*x)^m*d*m^9*x^15 +
1529430*(f*x)^m*e*m^9*x^15 + 207024930*(f*x)^m*d*m^7*x^17 + 552066480*(f*x)
^m*e*m^7*x^17 + 5246766620*(f*x)^m*d*m^5*x^19 + 23610449790*(f*x)^m*e*m^5*x
^19 + 11320966021*(f*x)^m*d*m^3*x^21 + 113209660210*(f*x)^m*e*m^3*x^21 + 29
985521895*(f*x)^m*e*m*x^23 + 27510*(f*x)^m*d*m^10*x^13 + 33012*(f*x)^m*e*m^
10*x^13 + 28391400*(f*x)^m*d*m^8*x^15 + 49684950*(f*x)^m*e*m^8*x^15 + 28043
95230*(f*x)^m*d*m^6*x^17 + 7478387280*(f*x)^m*e*m^6*x^17 + 31686018220*(f*x
)^m*d*m^4*x^19 + 142587081990*(f*x)^m*e*m^4*x^19 + 26560342503*(f*x)^m*d*m^
2*x^21 + 265603425030*(f*x)^m*e*m^2*x^21 + 13749310575*(f*x)^m*e*x^23 + 252
*(f*x)^m*d*m^11*x^11 + 210*(f*x)^m*e*m^11*x^11 + 1578150*(f*x)^m*d*m^9*x^13
 + 1893780*(f*x)^m*e*m^9*x^13 + 586902960*(f*x)^m*d*m^7*x^15 + 1027080180*(
f*x)^m*e*m^7*x^15 + 25598865870*(f*x)^m*d*m^5*x^17 + 68263642320*(f*x)^m*e*
m^5*x^17 + 123748247730*(f*x)^m*d*m^3*x^19 + 556867114785*(f*x)^m*e*m^3*x^1
9 + 32778930735*(f*x)^m*d*m*x^21 + 327789307350*(f*x)^m*e*m*x^21 + 33516*(f
*x)^m*d*m^10*x^11 + 27930*(f*x)^m*e*m^10*x^11 + 52110450*(f*x)^m*d*m^8*x^13
 + 62532540*(f*x)^m*e*m^8*x^13 + 8059973040*(f*x)^m*d*m^6*x^15 + 1410495282
0*(f*x)^m*e*m^6*x^15 + 156004908210*(f*x)^m*d*m^4*x^17 + 416013088560*(f*x)
^m*e*m^4*x^17 + 291789582570*(f*x)^m*d*m^2*x^19 + 1313053121565*(f*x)^m*e*m
^2*x^19 + 15058768725*(f*x)^m*d*x^21 + 150587687250*(f*x)^m*e*x^21 + 210*(f
*x)^m*d*m^11*x^9 + 120*(f*x)^m*e*m^11*x^9 + 1954260*(f*x)^m*d*m^9*x^11 + 16
28550*(f*x)^m*e*m^9*x^11 + 1094918580*(f*x)^m*d*m^7*x^13 + 1313902296*(f*x)
^m*e*m^7*x^13 + 74496630480*(f*x)^m*d*m^5*x^15 + 130369103340*(f*x)^m*e*m^5
x^15 + 613938233025(f*x)^m*d*m^3*x^17 + 1637168621400*(f*x)^m*e*m^3*x^17
+ 361459164150*(f*x)^m*d*m*x^19 + 1626566238675*(f*x)^m*e*m*x^19 + 28350*(f
*x)^m*d*m^10*x^9 + 16200*(f*x)^m*e*m^10*x^9 + 65654820*(f*x)^m*d*m^8*x^11 +
 54712350*(f*x)^m*e*m^8*x^11 + 15277213980*(f*x)^m*d*m^6*x^13 + 18332656776
*(f*x)^m*e*m^6*x^13 + 459045550800*(f*x)^m*d*m^4*x^15 + 803329713900*(f*x)^
m*e*m^4*x^15 + 1456578341055*(f*x)^m*d*m^2*x^17 + 3884208909480*(f*x)^m*e*m
^2*x^17 + 166439022750*(f*x)^m*d*x^19 + 748975602375*(f*x)^m*e*x^19 + 120*(
f*x)^m*d*m^11*x^7 + 45*(f*x)^m*e*m^11*x^7 + 1680630*(f*x)^m*d*m^9*x^9 + 960
360*(f*x)^m*e*m^9*x^9 + 1404622296*(f*x)^m*d*m^7*x^11 + 1170518580*(f*x)^m*

e*m^7*x^11 + 143339613900*(f*x)^m*d*m^5*x^13 + 172007536680*(f*x)^m*e*m^5*x^13 + 1823707864920*(f*x)^m*d*m^3*x^15 + 3191488763610*(f*x)^m*e*m^3*x^15 + 1812743750475*(f*x)^m*d*m*x^17 + 4833983334600*(f*x)^m*e*m*x^17 + 16440*(f*x)^m*d*m^10*x^7 + 6165*(f*x)^m*e*m^10*x^7 + 57500730*(f*x)^m*d*m^8*x^9 + 32857560*(f*x)^m*e*m^8*x^9 + 19962541368*(f*x)^m*d*m^6*x^11 + 16635451140*(f*x)^m*e*m^6*x^11 + 895451283300*(f*x)^m*d*m^4*x^13 + 1074541539960*(f*x)^m*e*m^4*x^13 + 4360457499480*(f*x)^m*d*m^2*x^15 + 7630800624090*(f*x)^m*e*m^2*x^15 + 837090379125*(f*x)^m*d*x^17 + 2232241011000*(f*x)^m*e*x^17 + 45*(f*x)^m*d*m^11*x^5 + 10*(f*x)^m*e*m^11*x^5 + 991080*(f*x)^m*d*m^9*x^7 + 371655*(f*x)^m*e*m^9*x^7 + 1254847860*(f*x)^m*d*m^7*x^9 + 717055920*(f*x)^m*e*m^7*x^9 + 190744119720*(f*x)^m*d*m^5*x^11 + 158953433100*(f*x)^m*e*m^5*x^11 + 3600567789210*(f*x)^m*d*m^3*x^13 + 4320681347052*(f*x)^m*e*m^3*x^13 + 5458672303560*(f*x)^m*d*m*x^15 + 9552676531230*(f*x)^m*e*m*x^15 + 6255*(f*x)^m*d*m^10*x^5 + 1390*(f*x)^m*e*m^10*x^5 + 34563240*(f*x)^m*d*m^8*x^7 + 12961215*(f*x)^m*e*m^8*x^7 + 18217524780*(f*x)^m*d*m^6*x^9 + 10410014160*(f*x)^m*e*m^6*x^9 + 1212454199880*(f*x)^m*d*m^4*x^11 + 1010378499900*(f*x)^m*e*m^4*x^11 + 8695750818510*(f*x)^m*d*m^2*x^13 + 10434900982212*(f*x)^m*e*m^2*x^13 + 2529873145800*(f*x)^m*d*x^15 + 4427278005150*(f*x)^m*e*x^15 + 10*(f*x)^m*d*m^11*x^3 + (f*x)^m*e*m^11*x^3 + 383535*(f*x)^m*d*m^9*x^5 + 85230*(f*x)^m*e*m^9*x^5 + 770831280*(f*x)^m*d*m^7*x^7 + 289061730*(f*x)^m*e*m^7*x^7 + 177985672620*(f*x)^m*d*m^5*x^9 + 101706098640*(f*x)^m*e*m^5*x^9 + 4952725167852*(f*x)^m*d*m^3*x^11 + 4127270973210*(f*x)^m*e*m^3*x^11 + 10969925251950*(f*x)^m*d*m*x^13 + 13163910302340*(f*x)^m*e*m*x^13 + 1410*(f*x)^m*d*m^10*x^3 + 141*(f*x)^m*e*m^10*x^3 + 13645125*(f*x)^m*d*m^8*x^5 + 3032250*(f*x)^m*e*m^8*x^5 + 11467698480*(f*x)^m*d*m^6*x^7 + 4300386930*(f*x)^m*e*m^6*x^7 + 1156995210420*(f*x)^m*d*m^4*x^9 + 661140120240*(f*x)^m*e*m^4*x^9 + 12123781647516*(f*x)^m*d*m^2*x^11 + 10103151372930*(f*x)^m*e*m^2*x^11 + 5108397698250*(f*x)^m*d*x^13 + 6130077237900*(f*x)^m*e*x^13 + (f*x)^m*d*m^11*x + 87950*(f*x)^m*d*m^9*x^3 + 8795*(f*x)^m*e*m^9*x^3 + 311564610*(f*x)^m*d*m^7*x^5 + 69236580*(f*x)^m*e*m^7*x^5 + 115122336720*(f*x)^m*d*m^5*x^7 + 43170876270*(f*x)^m*e*m^5*x^7 + 4828477578330*(f*x)^m*d*m^3*x^9 + 2759130044760*(f*x)^m*e*m^3*x^9 + 15456024948420*(f*x)^m*d*m*x^11 + 12880020790350*(f*x)^m*e*m*x^11 + 143*(f*x)^m*d*m^10*x + 3194550*(f*x)^m*d*m^8*x^3 + 319455*(f*x)^m*e*m^8*x^3 + 4765995990*(f*x)^m*d*m^6*x^5 + 1059110220*(f*x)^m*e*m^6*x^5 + 770638650960*(f*x)^m*d*m^4*x^7 + 288989494110*(f*x)^m*e*m^4*x^7 + 12046833873270*(f*x)^m*d*m^2*x^9 + 6883905070440*(f*x)^m*e*m^2*x^9 + 7244636735700*(f*x)^m*d*x^11 + 6037197279750*(f*x)^m*e*x^11 + 9075*(f*x)^m*d*m^9*x + 74814180*(f*x)^m*d*m^7*x^3 + 7481418*(f*x)^m*e*m^7*x^3 + 49443604830*(f*x)^m*d*m^5*x^5 + 10987467740*(f*x)^m*e*m^5*x^5 + 3314920570200*(f*x)^m*d*m^3*x^7 + 1243095213825*(f*x)^m*e*m^3*x^7 + 15593181033150*(f*x)^m*d*m*x^9 + 8910389161800*(f*x)^m*e*m*x^9 + 336765*(f*x)^m*d*m^8*x + 1180850580*(f*x)^m*d*m^6*x^3 + 118085058*(f*x)^m*e*m^6*x^3 + 343967603850*(f*x)^m*d*m^4*x^5 + 76437245300*(f*x)^m*e*m^4*x^5 + 8511631481880*(f*x)^m*d*m^2*x^7 + 3191861805705*(f*x)^m*e*m^2*x^7 + 7378796675250*(f*x)^m*d*x^9 + 4216455243000*(f*x)^m*e*x^9 + 8103018*(f*x)^m*d*m^7*x + 12740467100*(f*x)^m*d*m^5*x^3 + 1274046710*(f*x)^m*e*m

```
^5*x^3 + 1546183653345*(f*x)^m*d*m^3*x^5 + 343596367410*(f*x)^m*e*m^3*x^5 +
 11284114422600*(f*x)^m*d*m*x^7 + 4231542908475*(f*x)^m*e*m*x^7 + 132426294
*(f*x)^m*d*m^6*x + 93153182700*(f*x)^m*d*m^4*x^3 + 9315318270*(f*x)^m*e*m^4
*x^3 + 4162610035755*(f*x)^m*d*m^2*x^5 + 925024452390*(f*x)^m*e*m^2*x^5 + 5
421156741000*(f*x)^m*d*x^7 + 2032933777875*(f*x)^m*e*x^7 + 1495875590*(f*x)
^m*d*m^5*x + 446323045810*(f*x)^m*d*m^3*x^3 + 44632304581*(f*x)^m*e*m^3*x^3
 + 5761525369635*(f*x)^m*d*m*x^5 + 1280338971030*(f*x)^m*e*m*x^5 + 11641582
810*(f*x)^m*d*m^4*x + 1304037152010*(f*x)^m*d*m^2*x^3 + 130403715201*(f*x)^
m*e*m^2*x^3 + 2846107289025*(f*x)^m*d*x^5 + 632468286450*(f*x)^m*e*x^5 + 60
936676581*(f*x)^m*d*m^3*x + 1993349776950*(f*x)^m*d*m*x^3 + 199334977695*(f
*x)^m*e*m*x^3 + 203363952363*(f*x)^m*d*m^2*x + 1054113810750*(f*x)^m*d*x^3
+ 105411381075*(f*x)^m*e*x^3 + 387182170935*(f*x)^m*d*m*x + 316234143225*(f
*x)^m*d*x)/(m^12 + 144*m^11 + 9218*m^10 + 345840*m^9 + 8439783*m^8 + 140529
312*m^7 + 1628301884*m^6 + 13137458400*m^5 + 72578259391*m^4 + 264300628944
*m^3 + 590546123298*m^2 + 703416314160*m + 316234143225)
```

Mupad [B] (verification not implemented)

Time = 8.62 (sec) , antiderivative size = 1539, normalized size of antiderivative = 5.72

$$\int (fx)^m \left(d+ex^2\right) \left(1+2x^2+x^4\right)^5 \, dx = \text{Too large to display}$$

```
[In] int((f*x)^m*(d + e*x^2)*(2*x^2 + x^4 + 1)^5,x)
```

```
[Out] (d*x*(f*x)^m*(387182170935*m + 203363952363*m^2 + 60936676581*m^3 + 1164158
2810*m^4 + 1495875590*m^5 + 132426294*m^6 + 8103018*m^7 + 336765*m^8 + 9075
*m^9 + 143*m^10 + m^11 + 316234143225))/(703416314160*m + 590546123298*m^2
+ 264300628944*m^3 + 72578259391*m^4 + 13137458400*m^5 + 1628301884*m^6 + 1
40529312*m^7 + 8439783*m^8 + 345840*m^9 + 9218*m^10 + 144*m^11 + m^12 + 316
234143225) + (e*x^23*(f*x)^m*(29985521895*m + 24372200061*m^2 + 10431670821
*m^3 + 2702025590*m^4 + 453714470*m^5 + 51069018*m^6 + 3889578*m^7 + 197835
*m^8 + 6435*m^9 + 121*m^10 + m^11 + 13749310575))/(703416314160*m + 5905461
23298*m^2 + 264300628944*m^3 + 72578259391*m^4 + 13137458400*m^5 + 16283018
84*m^6 + 140529312*m^7 + 8439783*m^8 + 345840*m^9 + 9218*m^10 + 144*m^11 +
m^12 + 316234143225) + (30*x^15*(f*x)^m*(4*d + 7*e)*(45488935863*m + 363371
45829*m^2 + 15197565541*m^3 + 3825379590*m^4 + 620805254*m^5 + 67166442*m^6
 + 4890858*m^7 + 236595*m^8 + 7283*m^9 + 129*m^10 + m^11 + 21082276215))/(7
03416314160*m + 590546123298*m^2 + 264300628944*m^3 + 72578259391*m^4 + 131
37458400*m^5 + 1628301884*m^6 + 140529312*m^7 + 8439783*m^8 + 345840*m^9 +
9218*m^10 + 144*m^11 + m^12 + 316234143225) + (42*x^13*(f*x)^m*(5*d + 6*e)*
(52237739295*m + 41408337231*m^2 + 17145560901*m^3 + 4264053730*m^4 + 68256
9590*m^5 + 72748638*m^6 + 5213898*m^7 + 248145*m^8 + 7515*m^9 + 131*m^10 +
m^11 + 24325703325))/(703416314160*m + 590546123298*m^2 + 264300628944*m^3
+ 72578259391*m^4 + 13137458400*m^5 + 1628301884*m^6 + 140529312*m^7 + 8439
```

783*m^8 + 345840*m^9 + 9218*m^10 + 144*m^11 + m^12 + 316234143225) + (30*x^
9*(f*x)^m*(7*d + 4*e)*(74253243015*m + 57365875587*m^2 + 22992750373*m^3 +
5509501002*m^4 + 847550822*m^5 + 86750118*m^6 + 5975466*m^7 + 273813*m^8 +
8003*m^9 + 135*m^10 + m^11 + 35137127025))/(703416314160*m + 590546123298*m
^2 + 264300628944*m^3 + 72578259391*m^4 + 13137458400*m^5 + 1628301884*m^6
+ 140529312*m^7 + 8439783*m^8 + 345840*m^9 + 9218*m^10 + 144*m^11 + m^12 +
316234143225) + (x^3*(f*x)^m*(10*d + e)*(199334977695*m + 130403715201*m^2
+ 44632304581*m^3 + 9315318270*m^4 + 1274046710*m^5 + 118085058*m^6 + 74814
18*m^7 + 319455*m^8 + 8795*m^9 + 141*m^10 + m^11 + 105411381075))/(70341631
4160*m + 590546123298*m^2 + 264300628944*m^3 + 72578259391*m^4 + 1313745840
0*m^5 + 1628301884*m^6 + 140529312*m^7 + 8439783*m^8 + 345840*m^9 + 9218*m^
10 + 144*m^11 + m^12 + 316234143225) + (5*x^19*(f*x)^m*(2*d + 9*e)*(3614591
6415*m + 29178958257*m^2 + 12374824773*m^3 + 3168601822*m^4 + 524676662*m^5
 + 58085538*m^6 + 4339146*m^7 + 215823*m^8 + 6843*m^9 + 125*m^10 + m^11 + 1
6643902275))/(703416314160*m + 590546123298*m^2 + 264300628944*m^3 + 725782
59391*m^4 + 13137458400*m^5 + 1628301884*m^6 + 140529312*m^7 + 8439783*m^8
+ 345840*m^9 + 9218*m^10 + 144*m^11 + m^12 + 316234143225) + (42*x^11*(f*x)
^m*(6*d + 5*e)*(61333432335*m + 48110244633*m^2 + 19653671301*m^3 + 4811326
190*m^4 + 756921110*m^5 + 79216434*m^6 + 5573898*m^7 + 260535*m^8 + 7755*m^
9 + 133*m^10 + m^11 + 28748558475))/(703416314160*m + 590546123298*m^2 + 26
4300628944*m^3 + 72578259391*m^4 + 13137458400*m^5 + 1628301884*m^6 + 14052
9312*m^7 + 8439783*m^8 + 345840*m^9 + 9218*m^10 + 144*m^11 + m^12 + 3162341
43225) + (15*x^7*(f*x)^m*(8*d + 3*e)*(94034286855*m + 70930262349*m^2 + 276
24338085*m^3 + 6421988758*m^4 + 959352806*m^5 + 95564154*m^6 + 6423594*m^7
+ 288027*m^8 + 8259*m^9 + 137*m^10 + m^11 + 45176306175))/(703416314160*m +
 590546123298*m^2 + 264300628944*m^3 + 72578259391*m^4 + 13137458400*m^5 +
1628301884*m^6 + 140529312*m^7 + 8439783*m^8 + 345840*m^9 + 9218*m^10 + 144
*m^11 + m^12 + 316234143225) + (5*x^5*(f*x)^m*(9*d + 2*e)*(128033897103*m +
 92502445239*m^2 + 34359636741*m^3 + 7643724530*m^4 + 1098746774*m^5 + 1059
11022*m^6 + 6923658*m^7 + 303225*m^8 + 8523*m^9 + 139*m^10 + m^11 + 6324682
8645))/(703416314160*m + 590546123298*m^2 + 264300628944*m^3 + 72578259391*
m^4 + 13137458400*m^5 + 1628301884*m^6 + 140529312*m^7 + 8439783*m^8 + 3458
40*m^9 + 9218*m^10 + 144*m^11 + m^12 + 316234143225) + (15*x^17*(f*x)^m*(3*
d + 8*e)*(40283194455*m + 32368407579*m^2 + 13643071845*m^3 + 3466775738*m^
4 + 568863686*m^5 + 62319894*m^6 + 4600554*m^7 + 225837*m^8 + 7059*m^9 + 12
7*m^10 + m^11 + 18602008425))/(703416314160*m + 590546123298*m^2 + 26430062
8944*m^3 + 72578259391*m^4 + 13137458400*m^5 + 1628301884*m^6 + 140529312*m
^7 + 8439783*m^8 + 345840*m^9 + 9218*m^10 + 144*m^11 + m^12 + 316234143225)
 + (x^21*(f*x)^m*(d + 10*e)*(32778930735*m + 26560342503*m^2 + 11320966021*
m^3 + 2917013970*m^4 + 486687830*m^5 + 54362574*m^6 + 4103178*m^7 + 206505*
m^8 + 6635*m^9 + 123*m^10 + m^11 + 15058768725))/(703416314160*m + 59054612
3298*m^2 + 264300628944*m^3 + 72578259391*m^4 + 13137458400*m^5 + 162830188
4*m^6 + 140529312*m^7 + 8439783*m^8 + 345840*m^9 + 9218*m^10 + 144*m^11 + m
^12 + 316234143225)

3.56 $\int x^5(d+ex^2)\left(1+2x^2+x^4\right)^5 \, dx$

Optimal result

Integrand size = 23, antiderivative size = 63

$$\int x^5(d+ex^2)\left(1+2x^2+x^4\right)^5 \, dx = \frac{1}{22}(d-e)\left(1+x^2\right)^{11} - \frac{1}{24}(2d-3e)\left(1+x^2\right)^{12} + \frac{1}{26}(d-3e)\left(1+x^2\right)^{13} + \frac{1}{28}e\left(1+x^2\right)^{14}$$

```
[Out] 1/22*(d-e)*(x^2+1)^11-1/24*(2*d-3*e)*(x^2+1)^12+1/26*(d-3*e)*(x^2+1)^13+1/2
8*e*(x^2+1)^14
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {28, 457, 77}

$$\int x^5(d+ex^2)\left(1+2x^2+x^4\right)^5 \, dx = \frac{1}{26}\left(x^2+1\right)^{13}(d-3e) - \frac{1}{24}\left(x^2+1\right)^{12}(2d-3e) + \frac{1}{22}\left(x^2+1\right)^{11}(d-e) + \frac{1}{28}e\left(x^2+1\right)^{14}$$

```
[In] Int[x^5*(d + e*x^2)*(1 + 2*x^2 + x^4)^5,x]
```

```
[Out] ((d - e)*(1 + x^2)^11)/22 - ((2*d - 3*e)*(1 + x^2)^12)/24 + ((d - 3*e)*(1 +
 x^2)^13)/26 + (e*(1 + x^2)^14)/28
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
```

```
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 77

```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_Sym
bol] :> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[
{a, b, d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*
e + a*f, 0] && ( !IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] ||
(GeQ[n + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || E
qQ[p, 1])
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int x^5 (1+x^2)^{10} (d+ex^2)\, dx \\
&= \frac{1}{2}\text{Subst}\left(\int x^2(1+x)^{10}(d+ex)\, dx, x, x^2\right) \\
&= \frac{1}{2}\text{Subst}\left(\int \left((d-e)(1+x)^{10}+(-2d+3e)(1+x)^{11}+(d-3e)(1+x)^{12}+e(1+x)^{13}\right)\, dx, x, x^2\right) \\
&= \frac{1}{22}(d-e)\left(1+x^2\right)^{11}-\frac{1}{24}(2d-3e)\left(1+x^2\right)^{12}+\frac{1}{26}(d-3e)\left(1+x^2\right)^{13}+\frac{1}{28}e\left(1+x^2\right)^{14}
\end{aligned}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 153 vs. $2(63) = 126$.

Time = 0.02 (sec) , antiderivative size = 153, normalized size of antiderivative = 2.43

$$\begin{aligned}
\int x^5 (d+ex^2) (1+2x^2+x^4)^5\, dx = {} & \frac{dx^6}{6}+\frac{1}{8}(10d+e)x^8+\frac{1}{2}(9d+2e)x^{10} \\
& +\frac{5}{4}(8d+3e)x^{12}+\frac{15}{7}(7d+4e)x^{14}+\frac{21}{8}(6d+5e)x^{16} \\
& +\frac{7}{3}(5d+6e)x^{18}+\frac{3}{2}(4d+7e)x^{20}+\frac{15}{22}(3d+8e)x^{22} \\
& +\frac{5}{24}(2d+9e)x^{24}+\frac{1}{26}(d+10e)x^{26}+\frac{ex^{28}}{28}
\end{aligned}$$

```
[In] Integrate[x^5*(d + e*x^2)*(1 + 2*x^2 + x^4)^5,x]
```

```
[Out] (d*x^6)/6 + ((10*d + e)*x^8)/8 + ((9*d + 2*e)*x^10)/2 + (5*(8*d + 3*e)*x^12
)/4 + (15*(7*d + 4*e)*x^14)/7 + (21*(6*d + 5*e)*x^16)/8 + (7*(5*d + 6*e)*x^
18)/3 + (3*(4*d + 7*e)*x^20)/2 + (15*(3*d + 8*e)*x^22)/22 + (5*(2*d + 9*e)*
x^24)/24 + ((d + 10*e)*x^26)/26 + (e*x^28)/28
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 121 vs. $2(55) = 110$.

Time = 0.14 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.94

method	result
norman	$\left(\frac{5d}{12}+\frac{15e}{8}\right)x^{24}+\left(\frac{d}{26}+\frac{5e}{13}\right)x^{26}+\frac{e\,x^{28}}{28}+\left(\frac{35d}{3}+14e\right)x^{18}+\left(6d+\frac{21e}{2}\right)x^{20}+\left(\frac{45d}{22}+\frac{60e}{11}\right)x^{22}+$
default	$\frac{e\,x^{28}}{28}+\frac{(d+10e)x^{26}}{26}+\frac{(10d+45e)x^{24}}{24}+\frac{(45d+120e)x^{22}}{22}+\frac{(120d+210e)x^{20}}{20}+\frac{(210d+252e)x^{18}}{18}+\frac{(252d+210e)x^{16}}{16}$
risch	$\frac{1}{28}e\,x^{28}+\frac{1}{26}x^{26}d+\frac{5}{13}x^{26}e+\frac{5}{12}x^{24}d+\frac{15}{8}x^{24}e+\frac{45}{22}x^{22}d+\frac{60}{11}e\,x^{22}+6d\,x^{20}+\frac{21}{2}e\,x^{20}+\frac{35}{3}d\,x^{1}$
parallelrisch	$\frac{1}{28}e\,x^{28}+\frac{1}{26}x^{26}d+\frac{5}{13}x^{26}e+\frac{5}{12}x^{24}d+\frac{15}{8}x^{24}e+\frac{45}{22}x^{22}d+\frac{60}{11}e\,x^{22}+6d\,x^{20}+\frac{21}{2}e\,x^{20}+\frac{35}{3}d\,x^{1}$
gosper	$x^{6}\left(858e\,x^{22}+924d\,x^{20}+9240e\,x^{20}+10010d\,x^{18}+45045e\,x^{18}+49140d\,x^{16}+131040e\,x^{16}+144144d\,x^{14}+252252e\,x^{14}+280280d\,x\right.$

```
[In] int(x^5*(e*x^2+d)*(x^4+2*x^2+1)^5,x,method=_RETURNVERBOSE)
```

```
[Out] (5/12*d+15/8*e)*x^24+(1/26*d+5/13*e)*x^26+1/28*e*x^28+(35/3*d+14*e)*x^18+(6
*d+21/2*e)*x^20+(45/22*d+60/11*e)*x^22+(63/4*d+105/8*e)*x^16+(9/2*d+e)*x^10
+(10*d+15/4*e)*x^12+(15*d+60/7*e)*x^14+1/6*d*x^6+(5/4*d+1/8*e)*x^8
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 129 vs. $2(55) = 110$.

Time = 0.23 (sec) , antiderivative size = 129, normalized size of antiderivative = 2.05

$$\int x^5(d+ex^2)\left(1+2x^2+x^4\right)^5\,dx=\frac{1}{28}\,ex^{28}+\frac{1}{26}\,(d+10\,e)x^{26}+\frac{5}{24}\,(2\,d+9\,e)x^{24}+\frac{15}{22}\,(3\,d+8\,e)x^{22}+\frac{3}{2}\,(4\,d+7\,e)x^{20}+\frac{7}{3}\,(5\,d+6\,e)x^{18}+\frac{21}{8}\,(6\,d+5\,e)x^{16}+\frac{15}{7}\,(7\,d+4\,e)x^{14}+\frac{5}{4}\,(8\,d+3\,e)x^{12}+\frac{1}{2}\,(9\,d+2\,e)x^{10}+\frac{1}{8}\,(10\,d+e)x^{8}+\frac{1}{6}\,dx^{6}$$

```
[In] integrate(x^5*(e*x^2+d)*(x^4+2*x^2+1)^5,x, algorithm="fricas")
```

```
[Out] 1/28*e*x^28 + 1/26*(d + 10*e)*x^26 + 5/24*(2*d + 9*e)*x^24 + 15/22*(3*d + 8
*e)*x^22 + 3/2*(4*d + 7*e)*x^20 + 7/3*(5*d + 6*e)*x^18 + 21/8*(6*d + 5*e)*x
^16 + 15/7*(7*d + 4*e)*x^14 + 5/4*(8*d + 3*e)*x^12 + 1/2*(9*d + 2*e)*x^10 +
 1/8*(10*d + e)*x^8 + 1/6*d*x^6
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 134 vs. $2(51) = 102$.

Time = 0.03 (sec) , antiderivative size = 134, normalized size of antiderivative = 2.13

$$\int x^5 (d+ex^2)\left(1+2x^2+x^4\right)^5\,dx = \frac{dx^6}{6}+\frac{ex^{28}}{28}+x^{26}\left(\frac{d}{26}+\frac{5e}{13}\right)+x^{24}\cdot\left(\frac{5d}{12}+\frac{15e}{8}\right)+x^{22}\cdot\left(\frac{45d}{22}+\frac{60e}{11}\right)+x^{20}\cdot\left(6d+\frac{21e}{2}\right)+x^{18}\cdot\left(\frac{35d}{3}+14e\right)+x^{16}\cdot\left(\frac{63d}{4}+\frac{105e}{8}\right)+x^{14}\cdot\left(15d+\frac{60e}{7}\right)+x^{12}\cdot\left(10d+\frac{15e}{4}\right)+x^{10}\cdot\left(\frac{9d}{2}+e\right)+x^{8}\cdot\left(\frac{5d}{4}+\frac{e}{8}\right)$$

```
[In] integrate(x**5*(e*x**2+d)*(x**4+2*x**2+1)**5,x)
```

```
[Out] d*x**6/6 + e*x**28/28 + x**26*(d/26 + 5*e/13) + x**24*(5*d/12 + 15*e/8) + x
**22*(45*d/22 + 60*e/11) + x**20*(6*d + 21*e/2) + x**18*(35*d/3 + 14*e) + x
**16*(63*d/4 + 105*e/8) + x**14*(15*d + 60*e/7) + x**12*(10*d + 15*e/4) + x
**10*(9*d/2 + e) + x**8*(5*d/4 + e/8)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 129 vs. $2(55) = 110$.

Time = 0.18 (sec) , antiderivative size = 129, normalized size of antiderivative = 2.05

$$\int x^5 (d+ex^2)\left(1+2x^2+x^4\right)^5\,dx = \frac{1}{28}ex^{28}+\frac{1}{26}(d+10e)x^{26}+\frac{5}{24}(2d+9e)x^{24}+\frac{15}{22}(3d+8e)x^{22}+\frac{3}{2}(4d+7e)x^{20}+\frac{7}{3}(5d+6e)x^{18}+\frac{21}{8}(6d+5e)x^{16}+\frac{15}{7}(7d+4e)x^{14}+\frac{5}{4}(8d+3e)x^{12}+\frac{1}{2}(9d+2e)x^{10}+\frac{1}{8}(10d+e)x^{8}+\frac{1}{6}dx^{6}$$

```
[In] integrate(x^5*(e*x^2+d)*(x^4+2*x^2+1)^5,x, algorithm="maxima")
```

```
[Out] 1/28*e*x^28 + 1/26*(d + 10*e)*x^26 + 5/24*(2*d + 9*e)*x^24 + 15/22*(3*d + 8
*e)*x^22 + 3/2*(4*d + 7*e)*x^20 + 7/3*(5*d + 6*e)*x^18 + 21/8*(6*d + 5*e)*x
^16 + 15/7*(7*d + 4*e)*x^14 + 5/4*(8*d + 3*e)*x^12 + 1/2*(9*d + 2*e)*x^10 +
 1/8*(10*d + e)*x^8 + 1/6*d*x^6
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 132 vs. $2(55) = 110$.

Time = 0.27 (sec) , antiderivative size = 132, normalized size of antiderivative = 2.10

$$\int x^5(d+ex^2)\left(1+2x^2+x^4\right)^5\,dx = \frac{1}{28}\,ex^{28}+\frac{1}{26}\,dx^{26}+\frac{5}{13}\,ex^{26}+\frac{5}{12}\,dx^{24}+\frac{15}{8}\,ex^{24}+\frac{45}{22}\,dx^{22} + \frac{60}{11}\,ex^{22}+6\,dx^{20}+\frac{21}{2}\,ex^{20}+\frac{35}{3}\,dx^{18}+14\,ex^{18} + \frac{63}{4}\,dx^{16}+\frac{105}{8}\,ex^{16}+15\,dx^{14}+\frac{60}{7}\,ex^{14}+10\,dx^{12} + \frac{15}{4}\,ex^{12}+\frac{9}{2}\,dx^{10}+ex^{10}+\frac{5}{4}\,dx^{8}+\frac{1}{8}\,ex^{8}+\frac{1}{6}\,dx^{6}$$

```
[In] integrate(x^5*(e*x^2+d)*(x^4+2*x^2+1)^5,x, algorithm="giac")
```

```
[Out] 1/28*e*x^28 + 1/26*d*x^26 + 5/13*e*x^26 + 5/12*d*x^24 + 15/8*e*x^24 + 45/22
*d*x^22 + 60/11*e*x^22 + 6*d*x^20 + 21/2*e*x^20 + 35/3*d*x^18 + 14*e*x^18 +
 63/4*d*x^16 + 105/8*e*x^16 + 15*d*x^14 + 60/7*e*x^14 + 10*d*x^12 + 15/4*e*
x^12 + 9/2*d*x^10 + e*x^10 + 5/4*d*x^8 + 1/8*e*x^8 + 1/6*d*x^6
```

Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.92

$$\int x^5\left(d+ex^2\right)\left(1+2x^2+x^4\right)^5\,dx = \frac{e\,x^{28}}{28}+\left(\frac{d}{26}+\frac{5\,e}{13}\right)\,x^{26}+\left(\frac{5\,d}{12}+\frac{15\,e}{8}\right)\,x^{24} + \left(\frac{45\,d}{22}+\frac{60\,e}{11}\right)\,x^{22}+\left(6\,d+\frac{21\,e}{2}\right)\,x^{20} + \left(\frac{35\,d}{3}+14\,e\right)\,x^{18}+\left(\frac{63\,d}{4}+\frac{105\,e}{8}\right)\,x^{16} + \left(15\,d+\frac{60\,e}{7}\right)\,x^{14}+\left(10\,d+\frac{15\,e}{4}\right)\,x^{12} + \left(\frac{9\,d}{2}+e\right)\,x^{10}+\left(\frac{5\,d}{4}+\frac{e}{8}\right)\,x^{8}+\frac{d\,x^6}{6}$$

```
[In] int(x^5*(d + e*x^2)*(2*x^2 + x^4 + 1)^5,x)
```

```
[Out] x^8*((5*d)/4 + e/8) + x^12*(10*d + (15*e)/4) + x^20*(6*d + (21*e)/2) + x^24
*((5*d)/12 + (15*e)/8) + x^18*((35*d)/3 + 14*e) + x^26*(d/26 + (5*e)/13) +
x^14*(15*d + (60*e)/7) + x^22*((45*d)/22 + (60*e)/11) + x^16*((63*d)/4 + (1
05*e)/8) + (d*x^6)/6 + (e*x^28)/28 + x^10*((9*d)/2 + e)
```

3.57 $\int x^4(d+ex^2)\left(1+2x^2+x^4\right)^5\,dx$

Optimal result

Integrand size = 23, antiderivative size = 153

$$\int x^4\left(d+ex^2\right)\left(1+2x^2+x^4\right)^5\,dx=\frac{dx^5}{5}+\frac{1}{7}(10d+e)x^7+\frac{5}{9}(9d+2e)x^9+\frac{15}{11}(8d+3e)x^{11}+\frac{30}{13}(7d+4e)x^{13}+\frac{14}{5}(6d+5e)x^{15}+\frac{42}{17}(5d+6e)x^{17}+\frac{30}{19}(4d+7e)x^{19}+\frac{5}{7}(3d+8e)x^{21}+\frac{5}{23}(2d+9e)x^{23}+\frac{1}{25}(d+10e)x^{25}+\frac{ex^{27}}{27}$$

```
[Out] 1/5*d*x^5+1/7*(10*d+e)*x^7+5/9*(9*d+2*e)*x^9+15/11*(8*d+3*e)*x^11+30/13*(7*
d+4*e)*x^13+14/5*(6*d+5*e)*x^15+42/17*(5*d+6*e)*x^17+30/19*(4*d+7*e)*x^19+5
/7*(3*d+8*e)*x^21+5/23*(2*d+9*e)*x^23+1/25*(d+10*e)*x^25+1/27*e*x^27
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {28, 459}

$$\int x^4\left(d+ex^2\right)\left(1+2x^2+x^4\right)^5\,dx=\frac{1}{25}x^{25}(d+10e)+\frac{5}{23}x^{23}(2d+9e)+\frac{5}{7}x^{21}(3d+8e)+\frac{30}{19}x^{19}(4d+7e)+\frac{42}{17}x^{17}(5d+6e)+\frac{14}{5}x^{15}(6d+5e)+\frac{30}{13}x^{13}(7d+4e)+\frac{15}{11}x^{11}(8d+3e)+\frac{5}{9}x^9(9d+2e)+\frac{1}{7}x^7(10d+e)+\frac{dx^5}{5}+\frac{ex^{27}}{27}$$

```
[In] Int[x^4*(d + e*x^2)*(1 + 2*x^2 + x^4)^5,x]
```

```
[Out] (d*x^5)/5 + ((10*d + e)*x^7)/7 + (5*(9*d + 2*e)*x^9)/9 + (15*(8*d + 3*e)*x^
11)/11 + (30*(7*d + 4*e)*x^13)/13 + (14*(6*d + 5*e)*x^15)/5 + (42*(5*d + 6*
e)*x^17)/17 + (30*(4*d + 7*e)*x^19)/19 + (5*(3*d + 8*e)*x^21)/7 + (5*(2*d +
 9*e)*x^23)/23 + ((d + 10*e)*x^25)/25 + (e*x^27)/27
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int x^4\left(1+x^2\right)^{10}\left(d+ex^2\right)\,dx\\
&= \int \left(dx^4+(10d+e)x^6+5(9d+2e)x^8+15(8d+3e)x^{10}+30(7d+4e)x^{12}+42(6d+5e)x^{14}\right.\\
&\qquad +42(5d+6e)x^{16}+30(4d+7e)x^{18}+15(3d+8e)x^{20}+5(2d+9e)x^{22}\\
&\qquad \left.+(d+10e)x^{24}+ex^{26}\right)\,dx\\
&= \frac{dx^5}{5}+\frac{1}{7}(10d+e)x^7+\frac{5}{9}(9d+2e)x^9+\frac{15}{11}(8d+3e)x^{11}+\frac{30}{13}(7d+4e)x^{13}+\frac{14}{5}(6d+5e)x^{15}\\
&\quad +\frac{42}{17}(5d+6e)x^{17}+\frac{30}{19}(4d+7e)x^{19}+\frac{5}{7}(3d+8e)x^{21}+\frac{5}{23}(2d+9e)x^{23}+\frac{1}{25}(d+10e)x^{25}\\
&\quad +\frac{ex^{27}}{27}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.00

$$\begin{aligned}
\int x^4\left(d+ex^2\right)\left(1+2x^2+x^4\right)^5\,dx &= \frac{dx^5}{5}+\frac{1}{7}(10d+e)x^7+\frac{5}{9}(9d+2e)x^9\\
&\quad +\frac{15}{11}(8d+3e)x^{11}+\frac{30}{13}(7d+4e)x^{13}+\frac{14}{5}(6d+5e)x^{15}\\
&\quad +\frac{42}{17}(5d+6e)x^{17}+\frac{30}{19}(4d+7e)x^{19}+\frac{5}{7}(3d+8e)x^{21}\\
&\quad +\frac{5}{23}(2d+9e)x^{23}+\frac{1}{25}(d+10e)x^{25}+\frac{ex^{27}}{27}
\end{aligned}$$

```
[In] Integrate[x^4*(d + e*x^2)*(1 + 2*x^2 + x^4)^5,x]
```

```
[Out] (d*x^5)/5 + ((10*d + e)*x^7)/7 + (5*(9*d + 2*e)*x^9)/9 + (15*(8*d + 3*e)*x^
11)/11 + (30*(7*d + 4*e)*x^13)/13 + (14*(6*d + 5*e)*x^15)/5 + (42*(5*d + 6*
e)*x^17)/17 + (30*(4*d + 7*e)*x^19)/19 + (5*(3*d + 8*e)*x^21)/7 + (5*(2*d +
 9*e)*x^23)/23 + ((d + 10*e)*x^25)/25 + (e*x^27)/27
```

Maple [A] (verified)

Time = 0.13 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.81

method	result
norman	$\left(\frac{10d}{23}+\frac{45e}{23}\right)x^{23}+\left(\frac{d}{25}+\frac{2e}{5}\right)x^{25}+\frac{e\,x^{27}}{27}+\left(\frac{15d}{7}+\frac{40e}{7}\right)x^{21}+\left(\frac{210d}{17}+\frac{252e}{17}\right)x^{17}+\left(\frac{120d}{19}+\frac{210e}{19}\right)x^{1}$
default	$\frac{e\,x^{27}}{27}+\frac{(d+10e)x^{25}}{25}+\frac{(10d+45e)x^{23}}{23}+\frac{(45d+120e)x^{21}}{21}+\frac{(120d+210e)x^{19}}{19}+\frac{(210d+252e)x^{17}}{17}+\frac{(252d+210e)x^{15}}{15}+$
risch	$\frac{1}{27}e\,x^{27}+\frac{1}{25}x^{25}d+\frac{2}{5}e\,x^{25}+\frac{10}{23}x^{23}d+\frac{45}{23}e\,x^{23}+\frac{15}{7}x^{21}d+\frac{40}{7}e\,x^{21}+\frac{120}{19}x^{19}d+\frac{210}{19}x^{19}e+\frac{210}{17}x^{17}$
parallelrisch	$\frac{1}{27}e\,x^{27}+\frac{1}{25}x^{25}d+\frac{2}{5}e\,x^{25}+\frac{10}{23}x^{23}d+\frac{45}{23}e\,x^{23}+\frac{15}{7}x^{21}d+\frac{40}{7}e\,x^{21}+\frac{120}{19}x^{19}d+\frac{210}{19}x^{19}e+\frac{210}{17}x^{17}$
gosper	$\frac{x^5(185910725e\,x^{22}+200783583d\,x^{20}+2007835830e\,x^{20}+2182430250d\,x^{18}+9820936125e\,x^{18}+10756263375d\,x^{16}+28683369000e}{}$

```
[In] int(x^4*(e*x^2+d)*(x^4+2*x^2+1)^5,x,method=_RETURNVERBOSE)
```

```
[Out] (10/23*d+45/23*e)*x^23+(1/25*d+2/5*e)*x^25+1/27*e*x^27+(15/7*d+40/7*e)*x^21
+(210/17*d+252/17*e)*x^17+(120/19*d+210/19*e)*x^19+(5*d+10/9*e)*x^9+(120/11
*d+45/11*e)*x^11+(210/13*d+120/13*e)*x^13+(84/5*d+14*e)*x^15+1/5*x^5*d+(10/
7*d+1/7*e)*x^7
```

Fricas [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 129, normalized size of antiderivative = 0.84

$$\begin{aligned}\int x^4(d+ex^2)\left(1+2x^2+x^4\right)^5\,dx &= \frac{1}{27}\,ex^{27}+\frac{1}{25}\,(d+10\,e)x^{25}+\frac{5}{23}\,(2\,d+9\,e)x^{23}\\ &+\frac{5}{7}\,(3\,d+8\,e)x^{21}+\frac{30}{19}\,(4\,d+7\,e)x^{19}\\ &+\frac{42}{17}\,(5\,d+6\,e)x^{17}+\frac{14}{5}\,(6\,d+5\,e)x^{15}\\ &+\frac{30}{13}\,(7\,d+4\,e)x^{13}+\frac{15}{11}\,(8\,d+3\,e)x^{11}\\ &+\frac{5}{9}\,(9\,d+2\,e)x^9+\frac{1}{7}\,(10\,d+e)x^7+\frac{1}{5}\,dx^5\end{aligned}$$

```
[In] integrate(x^4*(e*x^2+d)*(x^4+2*x^2+1)^5,x, algorithm="fricas")
```

```
[Out] 1/27*e*x^27 + 1/25*(d + 10*e)*x^25 + 5/23*(2*d + 9*e)*x^23 + 5/7*(3*d + 8*e
)*x^21 + 30/19*(4*d + 7*e)*x^19 + 42/17*(5*d + 6*e)*x^17 + 14/5*(6*d + 5*e)
*x^15 + 30/13*(7*d + 4*e)*x^13 + 15/11*(8*d + 3*e)*x^11 + 5/9*(9*d + 2*e)*x
^9 + 1/7*(10*d + e)*x^7 + 1/5*d*x^5
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 141, normalized size of antiderivative = 0.92

$$\int x^4(d+ex^2)\left(1+2x^2+x^4\right)^5\,dx = \frac{dx^5}{5}+\frac{ex^{27}}{27}+x^{25}\left(\frac{d}{25}+\frac{2e}{5}\right)+x^{23}\cdot\left(\frac{10d}{23}+\frac{45e}{23}\right)+x^{21}\cdot\left(\frac{15d}{7}+\frac{40e}{7}\right)+x^{19}\cdot\left(\frac{120d}{19}+\frac{210e}{19}\right)+x^{17}\cdot\left(\frac{210d}{17}+\frac{252e}{17}\right)+x^{15}\cdot\left(\frac{84d}{5}+14e\right)+x^{13}\cdot\left(\frac{210d}{13}+\frac{120e}{13}\right)+x^{11}\cdot\left(\frac{120d}{11}+\frac{45e}{11}\right)+x^9\cdot\left(5d+\frac{10e}{9}\right)+x^7\cdot\left(\frac{10d}{7}+\frac{e}{7}\right)$$

```
[In] integrate(x**4*(e*x**2+d)*(x**4+2*x**2+1)**5,x)
```

```
[Out] d*x**5/5 + e*x**27/27 + x**25*(d/25 + 2*e/5) + x**23*(10*d/23 + 45*e/23) +
x**21*(15*d/7 + 40*e/7) + x**19*(120*d/19 + 210*e/19) + x**17*(210*d/17 + 2
52*e/17) + x**15*(84*d/5 + 14*e) + x**13*(210*d/13 + 120*e/13) + x**11*(120
*d/11 + 45*e/11) + x**9*(5*d + 10*e/9) + x**7*(10*d/7 + e/7)
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 129, normalized size of antiderivative = 0.84

$$\int x^4(d+ex^2)\left(1+2x^2+x^4\right)^5\,dx = \frac{1}{27}\,ex^{27}+\frac{1}{25}\,(d+10\,e)x^{25}+\frac{5}{23}\,(2\,d+9\,e)x^{23}+\frac{5}{7}\,(3\,d+8\,e)x^{21}+\frac{30}{19}\,(4\,d+7\,e)x^{19}+\frac{42}{17}\,(5\,d+6\,e)x^{17}+\frac{14}{5}\,(6\,d+5\,e)x^{15}+\frac{30}{13}\,(7\,d+4\,e)x^{13}+\frac{15}{11}\,(8\,d+3\,e)x^{11}+\frac{5}{9}\,(9\,d+2\,e)x^9+\frac{1}{7}\,(10\,d+e)x^7+\frac{1}{5}\,dx^5$$

```
[In] integrate(x^4*(e*x^2+d)*(x^4+2*x^2+1)^5,x, algorithm="maxima")
```

```
[Out] 1/27*e*x^27 + 1/25*(d + 10*e)*x^25 + 5/23*(2*d + 9*e)*x^23 + 5/7*(3*d + 8*e
)*x^21 + 30/19*(4*d + 7*e)*x^19 + 42/17*(5*d + 6*e)*x^17 + 14/5*(6*d + 5*e)
*x^15 + 30/13*(7*d + 4*e)*x^13 + 15/11*(8*d + 3*e)*x^11 + 5/9*(9*d + 2*e)*x
^9 + 1/7*(10*d + e)*x^7 + 1/5*d*x^5
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 133, normalized size of antiderivative = 0.87

$$\int x^4(d+ex^2)\left(1+2x^2+x^4\right)^5\,dx = \frac{1}{27}\,ex^{27}+\frac{1}{25}\,dx^{25}+\frac{2}{5}\,ex^{25}+\frac{10}{23}\,dx^{23}+\frac{45}{23}\,ex^{23}+\frac{15}{7}\,dx^{21} + \frac{40}{7}\,ex^{21}+\frac{120}{19}\,dx^{19}+\frac{210}{19}\,ex^{19}+\frac{210}{17}\,dx^{17}+\frac{252}{17}\,ex^{17} + \frac{84}{5}\,dx^{15}+14\,ex^{15}+\frac{210}{13}\,dx^{13}+\frac{120}{13}\,ex^{13}+\frac{120}{11}\,dx^{11} + \frac{45}{11}\,ex^{11}+5\,dx^9+\frac{10}{9}\,ex^9+\frac{10}{7}\,dx^7+\frac{1}{7}\,ex^7+\frac{1}{5}\,dx^5$$

```
[In] integrate(x^4*(e*x^2+d)*(x^4+2*x^2+1)^5,x, algorithm="giac")
```

```
[Out] 1/27*e*x^27 + 1/25*d*x^25 + 2/5*e*x^25 + 10/23*d*x^23 + 45/23*e*x^23 + 15/7
*d*x^21 + 40/7*e*x^21 + 120/19*d*x^19 + 210/19*e*x^19 + 210/17*d*x^17 + 252
/17*e*x^17 + 84/5*d*x^15 + 14*e*x^15 + 210/13*d*x^13 + 120/13*e*x^13 + 120/
11*d*x^11 + 45/11*e*x^11 + 5*d*x^9 + 10/9*e*x^9 + 10/7*d*x^7 + 1/7*e*x^7 +
1/5*d*x^5
```

Mupad [B] (verification not implemented)

Time = 7.62 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.80

$$\int x^4(d+ex^2)\left(1+2x^2+x^4\right)^5\,dx = \frac{e\,x^{27}}{27}+\left(\frac{d}{25}+\frac{2\,e}{5}\right)\,x^{25}+\left(\frac{10\,d}{23}+\frac{45\,e}{23}\right)\,x^{23} + \left(\frac{15\,d}{7}+\frac{40\,e}{7}\right)\,x^{21}+\left(\frac{120\,d}{19}+\frac{210\,e}{19}\right)\,x^{19} + \left(\frac{210\,d}{17}+\frac{252\,e}{17}\right)\,x^{17}+\left(\frac{84\,d}{5}+14\,e\right)\,x^{15} + \left(\frac{210\,d}{13}+\frac{120\,e}{13}\right)\,x^{13}+\left(\frac{120\,d}{11}+\frac{45\,e}{11}\right)\,x^{11} + \left(5\,d+\frac{10\,e}{9}\right)\,x^9+\left(\frac{10\,d}{7}+\frac{e}{7}\right)\,x^7+\frac{d\,x^5}{5}$$

```
[In] int(x^4*(d + e*x^2)*(2*x^2 + x^4 + 1)^5,x)
```

```
[Out] x^7*((10*d)/7 + e/7) + x^9*(5*d + (10*e)/9) + x^25*(d/25 + (2*e)/5) + x^21*
((15*d)/7 + (40*e)/7) + x^15*((84*d)/5 + 14*e) + x^23*((10*d)/23 + (45*e)/2
3) + x^11*((120*d)/11 + (45*e)/11) + x^13*((210*d)/13 + (120*e)/13) + x^19*
((120*d)/19 + (210*e)/19) + x^17*((210*d)/17 + (252*e)/17) + (d*x^5)/5 + (e
*x^27)/27
```

3.58 $\int x^3(d+ex^2)\left(1+2x^2+x^4\right)^5\,dx$

Optimal result

Integrand size = 23, antiderivative size = 45

$$\int x^3\left(d+ex^2\right)\left(1+2x^2+x^4\right)^5\,dx=-\frac{1}{22}(d-e)\left(1+x^2\right)^{11}+\frac{1}{24}(d-2e)\left(1+x^2\right)^{12}+\frac{1}{26}e\left(1+x^2\right)^{13}$$

```
[Out] -1/22*(d-e)*(x^2+1)^11+1/24*(d-2*e)*(x^2+1)^12+1/26*e*(x^2+1)^13
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {28, 457, 77}

$$\int x^3\left(d+ex^2\right)\left(1+2x^2+x^4\right)^5\,dx=\frac{1}{24}\left(x^2+1\right)^{12}(d-2e)-\frac{1}{22}\left(x^2+1\right)^{11}(d-e)+\frac{1}{26}e\left(x^2+1\right)^{13}$$

```
[In] Int[x^3*(d + e*x^2)*(1 + 2*x^2 + x^4)^5,x]
```

```
[Out] -1/22*((d - e)*(1 + x^2)^11) + ((d - 2*e)*(1 + x^2)^12)/24 + (e*(1 + x^2)^1
3)/26
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 77

```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_Sym
bol] :> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[
{a, b, d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*
e + a*f, 0] && ( !IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] ||
(GeQ[n + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || E
qQ[p, 1])
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int x^3\left(1+x^2\right)^{10}\left(d+ex^2\right)\,dx \\
&= \frac{1}{2}\text{Subst}\left(\int x(1+x)^{10}(d+ex)\,dx,x,x^2\right) \\
&= \frac{1}{2}\text{Subst}\left(\int \left((-d+e)(1+x)^{10}+(d-2e)(1+x)^{11}+e(1+x)^{12}\right)\,dx,x,x^2\right) \\
&= -\frac{1}{22}(d-e)\left(1+x^2\right)^{11}+\frac{1}{24}(d-2e)\left(1+x^2\right)^{12}+\frac{1}{26}e\left(1+x^2\right)^{13}
\end{aligned}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 151 vs. $2(45) = 90$.

Time = 0.02 (sec) , antiderivative size = 151, normalized size of antiderivative = 3.36

$$\begin{aligned}
\int x^3\left(d+ex^2\right)\left(1+2x^2+x^4\right)^5\,dx &= \frac{dx^4}{4}+\frac{1}{6}(10d+e)x^6+\frac{5}{8}(9d+2e)x^8 \\
&+\frac{3}{2}(8d+3e)x^{10}+\frac{5}{2}(7d+4e)x^{12}+3(6d+5e)x^{14} \\
&+\frac{21}{8}(5d+6e)x^{16}+\frac{5}{3}(4d+7e)x^{18}+\frac{3}{4}(3d+8e)x^{20} \\
&+\frac{5}{22}(2d+9e)x^{22}+\frac{1}{24}(d+10e)x^{24}+\frac{ex^{26}}{26}
\end{aligned}$$

```
[In] Integrate[x^3*(d + e*x^2)*(1 + 2*x^2 + x^4)^5,x]
```

```
[Out] (d*x^4)/4 + ((10*d + e)*x^6)/6 + (5*(9*d + 2*e)*x^8)/8 + (3*(8*d + 3*e)*x^1
0)/2 + (5*(7*d + 4*e)*x^12)/2 + 3*(6*d + 5*e)*x^14 + (21*(5*d + 6*e)*x^16)/
8 + (5*(4*d + 7*e)*x^18)/3 + (3*(3*d + 8*e)*x^20)/4 + (5*(2*d + 9*e)*x^22)/
22 + ((d + 10*e)*x^24)/24 + (e*x^26)/26
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 123 vs. $2(39) = 78$.

Time = 0.14 (sec) , antiderivative size = 124, normalized size of antiderivative = 2.76

method	result
norman	$\frac{x^{26}e}{26}+\left(\frac{5d}{11}+\frac{45e}{22}\right)x^{22}+\left(\frac{d}{24}+\frac{5e}{12}\right)x^{24}+\left(\frac{105d}{8}+\frac{63e}{4}\right)x^{16}+\left(\frac{20d}{3}+\frac{35e}{3}\right)x^{18}+\left(\frac{9d}{4}+6e\right)x^{20}+($
default	$\frac{x^{26}e}{26}+\frac{(d+10e)x^{24}}{24}+\frac{(10d+45e)x^{22}}{22}+\frac{(45d+120e)x^{20}}{20}+\frac{(120d+210e)x^{18}}{18}+\frac{(210d+252e)x^{16}}{16}+\frac{(252d+210e)x^{14}}{14}+$
risch	$\frac{1}{26}x^{26}e+\frac{1}{24}x^{24}d+\frac{5}{12}x^{24}e+\frac{5}{11}x^{22}d+\frac{45}{22}e\,x^{22}+\frac{9}{4}d\,x^{20}+6e\,x^{20}+\frac{20}{3}d\,x^{18}+\frac{35}{3}e\,x^{18}+\frac{105}{8}d\,x^{16}$
parallelrisch	$\frac{1}{26}x^{26}e+\frac{1}{24}x^{24}d+\frac{5}{12}x^{24}e+\frac{5}{11}x^{22}d+\frac{45}{22}e\,x^{22}+\frac{9}{4}d\,x^{20}+6e\,x^{20}+\frac{20}{3}d\,x^{18}+\frac{35}{3}e\,x^{18}+\frac{105}{8}d\,x^{16}$
gosper	$\frac{x^4(132e\,x^{22}+143d\,x^{20}+1430e\,x^{20}+1560d\,x^{18}+7020e\,x^{18}+7722d\,x^{16}+20592e\,x^{16}+22880d\,x^{14}+40040e\,x^{14}+45045d\,x^{12}+54054}{34}$

```
[In] int(x^3*(e*x^2+d)*(x^4+2*x^2+1)^5,x,method=_RETURNVERBOSE)
```

```
[Out] 1/26*x^26*e+(5/11*d+45/22*e)*x^22+(1/24*d+5/12*e)*x^24+(105/8*d+63/4*e)*x^1
6+(20/3*d+35/3*e)*x^18+(9/4*d+6*e)*x^20+(35/2*d+10*e)*x^12+(18*d+15*e)*x^14
+(45/8*d+5/4*e)*x^8+(12*d+9/2*e)*x^10+1/4*d*x^4+(5/3*d+1/6*e)*x^6
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 129 vs. $2(39) = 78$.

Time = 0.23 (sec) , antiderivative size = 129, normalized size of antiderivative = 2.87

$$\begin{aligned}\int x^3(d+ex^2)\left(1+2x^2+x^4\right)^5\,dx &= \frac{1}{26}\,ex^{26}+\frac{1}{24}\,(d+10\,e)x^{24}+\frac{5}{22}\,(2\,d+9\,e)x^{22}\\ &\quad+\frac{3}{4}\,(3\,d+8\,e)x^{20}+\frac{5}{3}\,(4\,d+7\,e)x^{18}+\frac{21}{8}\,(5\,d+6\,e)x^{16}\\ &\quad+3\,(6\,d+5\,e)x^{14}+\frac{5}{2}\,(7\,d+4\,e)x^{12}+\frac{3}{2}\,(8\,d+3\,e)x^{10}\\ &\quad+\frac{5}{8}\,(9\,d+2\,e)x^{8}+\frac{1}{6}\,(10\,d+e)x^{6}+\frac{1}{4}\,dx^{4}\end{aligned}$$

```
[In] integrate(x^3*(e*x^2+d)*(x^4+2*x^2+1)^5,x, algorithm="fricas")
```

```
[Out] 1/26*e*x^26 + 1/24*(d + 10*e)*x^24 + 5/22*(2*d + 9*e)*x^22 + 3/4*(3*d + 8*e
)*x^20 + 5/3*(4*d + 7*e)*x^18 + 21/8*(5*d + 6*e)*x^16 + 3*(6*d + 5*e)*x^14
+ 5/2*(7*d + 4*e)*x^12 + 3/2*(8*d + 3*e)*x^10 + 5/8*(9*d + 2*e)*x^8 + 1/6*(
10*d + e)*x^6 + 1/4*d*x^4
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 136 vs. $2(36) = 72$.

Time = 0.04 (sec) , antiderivative size = 136, normalized size of antiderivative = 3.02

$$\int x^3(d+ex^2)\left(1+2x^2+x^4\right)^5\,dx = \frac{dx^4}{4} + \frac{ex^{26}}{26} + x^{24}\left(\frac{d}{24}+\frac{5e}{12}\right) + x^{22}\cdot\left(\frac{5d}{11}+\frac{45e}{22}\right) + x^{20}\cdot\left(\frac{9d}{4}+6e\right) + x^{18}\cdot\left(\frac{20d}{3}+\frac{35e}{3}\right) + x^{16}\cdot\left(\frac{105d}{8}+\frac{63e}{4}\right) + x^{14}\cdot(18d+15e) + x^{12}\cdot\left(\frac{35d}{2}+10e\right) + x^{10}\cdot\left(12d+\frac{9e}{2}\right) + x^8\cdot\left(\frac{45d}{8}+\frac{5e}{4}\right) + x^6\cdot\left(\frac{5d}{3}+\frac{e}{6}\right)$$

```
[In] integrate(x**3*(e*x**2+d)*(x**4+2*x**2+1)**5,x)
```

```
[Out] d*x**4/4 + e*x**26/26 + x**24*(d/24 + 5*e/12) + x**22*(5*d/11 + 45*e/22) +
x**20*(9*d/4 + 6*e) + x**18*(20*d/3 + 35*e/3) + x**16*(105*d/8 + 63*e/4) +
x**14*(18*d + 15*e) + x**12*(35*d/2 + 10*e) + x**10*(12*d + 9*e/2) + x**8*(
45*d/8 + 5*e/4) + x**6*(5*d/3 + e/6)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 129 vs. $2(39) = 78$.

Time = 0.19 (sec) , antiderivative size = 129, normalized size of antiderivative = 2.87

$$\int x^3(d+ex^2)\left(1+2x^2+x^4\right)^5\,dx = \frac{1}{26}\,ex^{26} + \frac{1}{24}\,(d+10\,e)x^{24} + \frac{5}{22}\,(2\,d+9\,e)x^{22} + \frac{3}{4}\,(3\,d+8\,e)x^{20} + \frac{5}{3}\,(4\,d+7\,e)x^{18} + \frac{21}{8}\,(5\,d+6\,e)x^{16} + 3\,(6\,d+5\,e)x^{14} + \frac{5}{2}\,(7\,d+4\,e)x^{12} + \frac{3}{2}\,(8\,d+3\,e)x^{10} + \frac{5}{8}\,(9\,d+2\,e)x^8 + \frac{1}{6}\,(10\,d+e)x^6 + \frac{1}{4}\,dx^4$$

```
[In] integrate(x^3*(e*x^2+d)*(x^4+2*x^2+1)^5,x, algorithm="maxima")
```

```
[Out] 1/26*e*x^26 + 1/24*(d + 10*e)*x^24 + 5/22*(2*d + 9*e)*x^22 + 3/4*(3*d + 8*e
)*x^20 + 5/3*(4*d + 7*e)*x^18 + 21/8*(5*d + 6*e)*x^16 + 3*(6*d + 5*e)*x^14
+ 5/2*(7*d + 4*e)*x^12 + 3/2*(8*d + 3*e)*x^10 + 5/8*(9*d + 2*e)*x^8 + 1/6*(
10*d + e)*x^6 + 1/4*d*x^4
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 133 vs. $2(39) = 78$.

Time = 0.26 (sec) , antiderivative size = 133, normalized size of antiderivative = 2.96

$$\int x^3(d+ex^2)\left(1+2x^2+x^4\right)^5\,dx = \frac{1}{26}\,ex^{26}+\frac{1}{24}\,dx^{24}+\frac{5}{12}\,ex^{24}+\frac{5}{11}\,dx^{22}+\frac{45}{22}\,ex^{22}+\frac{9}{4}\,dx^{20} + 6\,ex^{20}+\frac{20}{3}\,dx^{18}+\frac{35}{3}\,ex^{18}+\frac{105}{8}\,dx^{16}+\frac{63}{4}\,ex^{16} + 18\,dx^{14}+15\,ex^{14}+\frac{35}{2}\,dx^{12}+10\,ex^{12}+12\,dx^{10} + \frac{9}{2}\,ex^{10}+\frac{45}{8}\,dx^{8}+\frac{5}{4}\,ex^{8}+\frac{5}{3}\,dx^{6}+\frac{1}{6}\,ex^{6}+\frac{1}{4}\,dx^{4}$$

```
[In] integrate(x^3*(e*x^2+d)*(x^4+2*x^2+1)^5,x, algorithm="giac")
```

```
[Out] 1/26*e*x^26 + 1/24*d*x^24 + 5/12*e*x^24 + 5/11*d*x^22 + 45/22*e*x^22 + 9/4*
d*x^20 + 6*e*x^20 + 20/3*d*x^18 + 35/3*e*x^18 + 105/8*d*x^16 + 63/4*e*x^16
+ 18*d*x^14 + 15*e*x^14 + 35/2*d*x^12 + 10*e*x^12 + 12*d*x^10 + 9/2*e*x^10
+ 45/8*d*x^8 + 5/4*e*x^8 + 5/3*d*x^6 + 1/6*e*x^6 + 1/4*d*x^4
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 123, normalized size of antiderivative = 2.73

$$\int x^3\left(d+ex^2\right)\left(1+2x^2+x^4\right)^5\,dx = \frac{e\,x^{26}}{26}+\left(\frac{d}{24}+\frac{5\,e}{12}\right)\,x^{24}+\left(\frac{5\,d}{11}+\frac{45\,e}{22}\right)\,x^{22} + \left(\frac{9\,d}{4}+6\,e\right)\,x^{20}+\left(\frac{20\,d}{3}+\frac{35\,e}{3}\right)\,x^{18} + \left(\frac{105\,d}{8}+\frac{63\,e}{4}\right)\,x^{16}+(18\,d+15\,e)\,x^{14} + \left(\frac{35\,d}{2}+10\,e\right)\,x^{12}+\left(12\,d+\frac{9\,e}{2}\right)\,x^{10} + \left(\frac{45\,d}{8}+\frac{5\,e}{4}\right)\,x^{8}+\left(\frac{5\,d}{3}+\frac{e}{6}\right)\,x^{6}+\frac{d\,x^4}{4}$$

```
[In] int(x^3*(d + e*x^2)*(2*x^2 + x^4 + 1)^5,x)
```

```
[Out] x^6*((5*d)/3 + e/6) + x^10*(12*d + (9*e)/2) + x^20*((9*d)/4 + 6*e) + x^14*(
18*d + 15*e) + x^12*((35*d)/2 + 10*e) + x^24*(d/24 + (5*e)/12) + x^8*((45*d
)/8 + (5*e)/4) + x^18*((20*d)/3 + (35*e)/3) + x^22*((5*d)/11 + (45*e)/22) +
 x^16*((105*d)/8 + (63*e)/4) + (d*x^4)/4 + (e*x^26)/26
```

3.59 $\int x^2(d+ex^2)\left(1+2x^2+x^4\right)^5\,dx$

Optimal result

Integrand size = 23, antiderivative size = 153

$$\int x^2(d+ex^2)\left(1+2x^2+x^4\right)^5\,dx=\frac{dx^3}{3}+\frac{1}{5}(10d+e)x^5+\frac{5}{7}(9d+2e)x^7+\frac{5}{3}(8d+3e)x^9+\frac{30}{11}(7d+4e)x^{11}+\frac{42}{13}(6d+5e)x^{13}+\frac{14}{5}(5d+6e)x^{15}+\frac{30}{17}(4d+7e)x^{17}+\frac{15}{19}(3d+8e)x^{19}+\frac{5}{21}(2d+9e)x^{21}+\frac{1}{23}(d+10e)x^{23}+\frac{ex^{25}}{25}$$

```
[Out] 1/3*d*x^3+1/5*(10*d+e)*x^5+5/7*(9*d+2*e)*x^7+5/3*(8*d+3*e)*x^9+30/11*(7*d+4
*e)*x^11+42/13*(6*d+5*e)*x^13+14/5*(5*d+6*e)*x^15+30/17*(4*d+7*e)*x^17+15/1
9*(3*d+8*e)*x^19+5/21*(2*d+9*e)*x^21+1/23*(d+10*e)*x^23+1/25*e*x^25
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {28, 459}

$$\int x^2(d+ex^2)\left(1+2x^2+x^4\right)^5\,dx=\frac{1}{23}x^{23}(d+10e)+\frac{5}{21}x^{21}(2d+9e)+\frac{15}{19}x^{19}(3d+8e)+\frac{30}{17}x^{17}(4d+7e)+\frac{14}{5}x^{15}(5d+6e)+\frac{42}{13}x^{13}(6d+5e)+\frac{30}{11}x^{11}(7d+4e)+\frac{5}{3}x^9(8d+3e)+\frac{5}{7}x^7(9d+2e)+\frac{1}{5}x^5(10d+e)+\frac{dx^3}{3}+\frac{ex^{25}}{25}$$

```
[In] Int[x^2*(d + e*x^2)*(1 + 2*x^2 + x^4)^5,x]
```

```
[Out] (d*x^3)/3 + ((10*d + e)*x^5)/5 + (5*(9*d + 2*e)*x^7)/7 + (5*(8*d + 3*e)*x^9
)/3 + (30*(7*d + 4*e)*x^11)/11 + (42*(6*d + 5*e)*x^13)/13 + (14*(5*d + 6*e)
*x^15)/5 + (30*(4*d + 7*e)*x^17)/17 + (15*(3*d + 8*e)*x^19)/19 + (5*(2*d +
9*e)*x^21)/21 + ((d + 10*e)*x^23)/23 + (e*x^25)/25
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int x^2\left(1+x^2\right)^{10}\left(d+ex^2\right)\,dx\\
&= \int \left(dx^2+(10d+e)x^4+5(9d+2e)x^6+15(8d+3e)x^8+30(7d+4e)x^{10}+42(6d+5e)x^{12}\right.\\
&\qquad +42(5d+6e)x^{14}+30(4d+7e)x^{16}+15(3d+8e)x^{18}+5(2d+9e)x^{20}\\
&\qquad \left.+(d+10e)x^{22}+ex^{24}\right)\,dx\\
&= \frac{dx^3}{3}+\frac{1}{5}(10d+e)x^5+\frac{5}{7}(9d+2e)x^7+\frac{5}{3}(8d+3e)x^9+\frac{30}{11}(7d+4e)x^{11}+\frac{42}{13}(6d+5e)x^{13}\\
&\quad +\frac{14}{5}(5d+6e)x^{15}+\frac{30}{17}(4d+7e)x^{17}+\frac{15}{19}(3d+8e)x^{19}+\frac{5}{21}(2d+9e)x^{21}+\frac{1}{23}(d+10e)x^{23}\\
&\quad +\frac{ex^{25}}{25}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.00

$$\begin{aligned}
\int x^2\left(d+ex^2\right)\left(1+2x^2+x^4\right)^5\,dx &= \frac{dx^3}{3}+\frac{1}{5}(10d+e)x^5+\frac{5}{7}(9d+2e)x^7\\
&\quad +\frac{5}{3}(8d+3e)x^9+\frac{30}{11}(7d+4e)x^{11}+\frac{42}{13}(6d+5e)x^{13}\\
&\quad +\frac{14}{5}(5d+6e)x^{15}+\frac{30}{17}(4d+7e)x^{17}+\frac{15}{19}(3d+8e)x^{19}\\
&\quad +\frac{5}{21}(2d+9e)x^{21}+\frac{1}{23}(d+10e)x^{23}+\frac{ex^{25}}{25}
\end{aligned}$$

```
[In] Integrate[x^2*(d + e*x^2)*(1 + 2*x^2 + x^4)^5,x]
```

```
[Out] (d*x^3)/3 + ((10*d + e)*x^5)/5 + (5*(9*d + 2*e)*x^7)/7 + (5*(8*d + 3*e)*x^9
)/3 + (30*(7*d + 4*e)*x^11)/11 + (42*(6*d + 5*e)*x^13)/13 + (14*(5*d + 6*e)
*x^15)/5 + (30*(4*d + 7*e)*x^17)/17 + (15*(3*d + 8*e)*x^19)/19 + (5*(2*d +
9*e)*x^21)/21 + ((d + 10*e)*x^23)/23 + (e*x^25)/25
```

Maple [A] (verified)

Time = 0.13 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.81

method	result
norman	$\left(\frac{d}{23}+\frac{10e}{23}\right)x^{23}+\frac{e\,x^{25}}{25}+\left(\frac{10d}{21}+\frac{15e}{7}\right)x^{21}+\left(14d+\frac{84e}{5}\right)x^{15}+\left(\frac{120d}{17}+\frac{210e}{17}\right)x^{17}+\left(\frac{45d}{19}+\frac{120e}{19}\right)$
default	$\frac{e\,x^{25}}{25}+\frac{(d+10e)x^{23}}{23}+\frac{(10d+45e)x^{21}}{21}+\frac{(45d+120e)x^{19}}{19}+\frac{(120d+210e)x^{17}}{17}+\frac{(210d+252e)x^{15}}{15}+\frac{(252d+210e)x^{13}}{13}$
risch	$\frac{1}{25}e\,x^{25}+\frac{1}{23}x^{23}d+\frac{10}{23}e\,x^{23}+\frac{10}{21}x^{21}d+\frac{15}{7}e\,x^{21}+\frac{45}{19}x^{19}d+\frac{120}{19}x^{19}e+\frac{120}{17}x^{17}d+\frac{210}{17}x^{17}e+14x$
parallelrisch	$\frac{1}{25}e\,x^{25}+\frac{1}{23}x^{23}d+\frac{10}{23}e\,x^{23}+\frac{10}{21}x^{21}d+\frac{15}{7}e\,x^{21}+\frac{45}{19}x^{19}d+\frac{120}{19}x^{19}e+\frac{120}{17}x^{17}d+\frac{210}{17}x^{17}e+14x$
gosper	$x^{3}\left(22309287e\,x^{22}+24249225d\,x^{20}+242492250e\,x^{20}+265586750d\,x^{18}+1195140375e\,x^{18}+1320944625d\,x^{16}+3522519000e\,x^{16}\right.$

```
[In] int(x^2*(e*x^2+d)*(x^4+2*x^2+1)^5,x,method=_RETURNVERBOSE)
```

```
[Out] (1/23*d+10/23*e)*x^23+1/25*e*x^25+(10/21*d+15/7*e)*x^21+(14*d+84/5*e)*x^15+
(120/17*d+210/17*e)*x^17+(45/19*d+120/19*e)*x^19+(45/7*d+10/7*e)*x^7+(40/3*
d+5*e)*x^9+(210/11*d+120/11*e)*x^11+(252/13*d+210/13*e)*x^13+1/3*x^3*d+(2*d
+1/5*e)*x^5
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 129, normalized size of antiderivative = 0.84

$$\begin{aligned}\int x^2\left(d+ex^2\right)\left(1+2x^2+x^4\right)^5\,dx=&\ \frac{1}{25}\,ex^{25}+\frac{1}{23}\,(d+10\,e)x^{23}+\frac{5}{21}\,(2\,d+9\,e)x^{21}\\&+\frac{15}{19}\,(3\,d+8\,e)x^{19}+\frac{30}{17}\,(4\,d+7\,e)x^{17}\\&+\frac{14}{5}\,(5\,d+6\,e)x^{15}+\frac{42}{13}\,(6\,d+5\,e)x^{13}\\&+\frac{30}{11}\,(7\,d+4\,e)x^{11}+\frac{5}{3}\,(8\,d+3\,e)x^{9}\\&+\frac{5}{7}\,(9\,d+2\,e)x^{7}+\frac{1}{5}\,(10\,d+e)x^{5}+\frac{1}{3}\,dx^{3}\end{aligned}$$

```
[In] integrate(x^2*(e*x^2+d)*(x^4+2*x^2+1)^5,x, algorithm="fricas")
```

```
[Out] 1/25*e*x^25 + 1/23*(d + 10*e)*x^23 + 5/21*(2*d + 9*e)*x^21 + 15/19*(3*d + 8
*e)*x^19 + 30/17*(4*d + 7*e)*x^17 + 14/5*(5*d + 6*e)*x^15 + 42/13*(6*d + 5*
e)*x^13 + 30/11*(7*d + 4*e)*x^11 + 5/3*(8*d + 3*e)*x^9 + 5/7*(9*d + 2*e)*x^
7 + 1/5*(10*d + e)*x^5 + 1/3*d*x^3
```

Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 139, normalized size of antiderivative = 0.91

$$\int x^2(d+ex^2)\left(1+2x^2+x^4\right)^5\,dx = \frac{dx^3}{3}+\frac{ex^{25}}{25}+x^{23}\left(\frac{d}{23}+\frac{10e}{23}\right)+x^{21}\cdot\left(\frac{10d}{21}+\frac{15e}{7}\right)+x^{19}\cdot\left(\frac{45d}{19}+\frac{120e}{19}\right)+x^{17}\cdot\left(\frac{120d}{17}+\frac{210e}{17}\right)+x^{15}\cdot\left(14d+\frac{84e}{5}\right)+x^{13}\cdot\left(\frac{252d}{13}+\frac{210e}{13}\right)+x^{11}\cdot\left(\frac{210d}{11}+\frac{120e}{11}\right)+x^{9}\cdot\left(\frac{40d}{3}+5e\right)+x^{7}\cdot\left(\frac{45d}{7}+\frac{10e}{7}\right)+x^{5}\cdot\left(2d+\frac{e}{5}\right)$$

```
[In] integrate(x**2*(e*x**2+d)*(x**4+2*x**2+1)**5,x)
```

```
[Out] d*x**3/3 + e*x**25/25 + x**23*(d/23 + 10*e/23) + x**21*(10*d/21 + 15*e/7) +
 x**19*(45*d/19 + 120*e/19) + x**17*(120*d/17 + 210*e/17) + x**15*(14*d + 8
4*e/5) + x**13*(252*d/13 + 210*e/13) + x**11*(210*d/11 + 120*e/11) + x**9*(
40*d/3 + 5*e) + x**7*(45*d/7 + 10*e/7) + x**5*(2*d + e/5)
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 129, normalized size of antiderivative = 0.84

$$\int x^2(d+ex^2)\left(1+2x^2+x^4\right)^5\,dx = \frac{1}{25}\,ex^{25}+\frac{1}{23}\,(d+10\,e)x^{23}+\frac{5}{21}\,(2\,d+9\,e)x^{21}+\frac{15}{19}\,(3\,d+8\,e)x^{19}+\frac{30}{17}\,(4\,d+7\,e)x^{17}+\frac{14}{5}\,(5\,d+6\,e)x^{15}+\frac{42}{13}\,(6\,d+5\,e)x^{13}+\frac{30}{11}\,(7\,d+4\,e)x^{11}+\frac{5}{3}\,(8\,d+3\,e)x^{9}+\frac{5}{7}\,(9\,d+2\,e)x^{7}+\frac{1}{5}\,(10\,d+e)x^{5}+\frac{1}{3}\,dx^{3}$$

```
[In] integrate(x^2*(e*x^2+d)*(x^4+2*x^2+1)^5,x, algorithm="maxima")
```

```
[Out] 1/25*e*x^25 + 1/23*(d + 10*e)*x^23 + 5/21*(2*d + 9*e)*x^21 + 15/19*(3*d + 8
*e)*x^19 + 30/17*(4*d + 7*e)*x^17 + 14/5*(5*d + 6*e)*x^15 + 42/13*(6*d + 5*
e)*x^13 + 30/11*(7*d + 4*e)*x^11 + 5/3*(8*d + 3*e)*x^9 + 5/7*(9*d + 2*e)*x^
7 + 1/5*(10*d + e)*x^5 + 1/3*d*x^3
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 133, normalized size of antiderivative = 0.87

$$\int x^2(d+ex^2)\left(1+2x^2+x^4\right)^5\,dx=\frac{1}{25}\,ex^{25}+\frac{1}{23}\,dx^{23}+\frac{10}{23}\,ex^{23}+\frac{10}{21}\,dx^{21}+\frac{15}{7}\,ex^{21}+\frac{45}{19}\,dx^{19}+\frac{120}{19}\,ex^{19}+\frac{120}{17}\,dx^{17}+\frac{210}{17}\,ex^{17}+14\,dx^{15}+\frac{84}{5}\,ex^{15}+\frac{252}{13}\,dx^{13}+\frac{210}{13}\,ex^{13}+\frac{210}{11}\,dx^{11}+\frac{120}{11}\,ex^{11}+\frac{40}{3}\,dx^{9}+5\,ex^{9}+\frac{45}{7}\,dx^{7}+\frac{10}{7}\,ex^{7}+2\,dx^{5}+\frac{1}{5}\,ex^{5}+\frac{1}{3}\,dx^{3}$$

```
[In] integrate(x^2*(e*x^2+d)*(x^4+2*x^2+1)^5,x, algorithm="giac")
```

```
[Out] 1/25*e*x^25 + 1/23*d*x^23 + 10/23*e*x^23 + 10/21*d*x^21 + 15/7*e*x^21 + 45/
19*d*x^19 + 120/19*e*x^19 + 120/17*d*x^17 + 210/17*e*x^17 + 14*d*x^15 + 84/
5*e*x^15 + 252/13*d*x^13 + 210/13*e*x^13 + 210/11*d*x^11 + 120/11*e*x^11 +
40/3*d*x^9 + 5*e*x^9 + 45/7*d*x^7 + 10/7*e*x^7 + 2*d*x^5 + 1/5*e*x^5 + 1/3*
d*x^3
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.80

$$\int x^2(d+ex^2)\left(1+2x^2+x^4\right)^5\,dx=\frac{e\,x^{25}}{25}+\left(\frac{d}{23}+\frac{10\,e}{23}\right)\,x^{23}+\left(\frac{10\,d}{21}+\frac{15\,e}{7}\right)\,x^{21}+\left(\frac{45\,d}{19}+\frac{120\,e}{19}\right)\,x^{19}+\left(\frac{120\,d}{17}+\frac{210\,e}{17}\right)\,x^{17}+\left(14\,d+\frac{84\,e}{5}\right)\,x^{15}+\left(\frac{252\,d}{13}+\frac{210\,e}{13}\right)\,x^{13}+\left(\frac{210\,d}{11}+\frac{120\,e}{11}\right)\,x^{11}+\left(\frac{40\,d}{3}+5\,e\right)\,x^{9}+\left(\frac{45\,d}{7}+\frac{10\,e}{7}\right)\,x^{7}+\left(2\,d+\frac{e}{5}\right)\,x^{5}+\frac{d\,x^3}{3}$$

```
[In] int(x^2*(d + e*x^2)*(2*x^2 + x^4 + 1)^5,x)
```

```
[Out] x^5*(2*d + e/5) + x^9*((40*d)/3 + 5*e) + x^21*((10*d)/21 + (15*e)/7) + x^7*
((45*d)/7 + (10*e)/7) + x^23*(d/23 + (10*e)/23) + x^15*(14*d + (84*e)/5) +
x^19*((45*d)/19 + (120*e)/19) + x^11*((210*d)/11 + (120*e)/11) + x^17*((120
*d)/17 + (210*e)/17) + x^13*((252*d)/13 + (210*e)/13) + (d*x^3)/3 + (e*x^25
)/25
```

3.60 $\int x(d+ex^2)\left(1+2x^2+x^4\right)^5 \, dx$

Optimal result

Integrand size = 21, antiderivative size = 29

$$\int x\left(d+ex^2\right)\left(1+2x^2+x^4\right)^5 \, dx = \frac{1}{22}(d-e)\left(1+x^2\right)^{11}+\frac{1}{24}e\left(1+x^2\right)^{12}$$

```
[Out] 1/22*(d-e)*(x^2+1)^11+1/24*e*(x^2+1)^12
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {28, 455, 45}

$$\int x\left(d+ex^2\right)\left(1+2x^2+x^4\right)^5 \, dx = \frac{1}{22}\left(x^2+1\right)^{11}(d-e)+\frac{1}{24}e\left(x^2+1\right)^{12}$$

```
[In] Int[x*(d + e*x^2)*(1 + 2*x^2 + x^4)^5,x]
```

```
[Out] ((d - e)*(1 + x^2)^11)/22 + (e*(1 + x^2)^12)/24
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
```

```
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int x\left(1+x^2\right)^{10}\left(d+ex^2\right)\,dx \\
&= \frac{1}{2}\text{Subst}\left(\int(1+x)^{10}(d+ex)\,dx,x,x^2\right) \\
&= \frac{1}{2}\text{Subst}\left(\int\left((d-e)(1+x)^{10}+e(1+x)^{11}\right)\,dx,x,x^2\right) \\
&= \frac{1}{22}(d-e)\left(1+x^2\right)^{11}+\frac{1}{24}e\left(1+x^2\right)^{12}
\end{aligned}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 149 vs. $2(29) = 58$.

Time = 0.01 (sec) , antiderivative size = 149, normalized size of antiderivative = 5.14

$$\begin{aligned}
\int x\left(d+ex^2\right)\left(1+2x^2+x^4\right)^5\,dx &= \frac{dx^2}{2}+\frac{1}{4}(10d+e)x^4+\frac{5}{6}(9d+2e)x^6 \\
&\quad+\frac{15}{8}(8d+3e)x^8+3(7d+4e)x^{10}+\frac{7}{2}(6d+5e)x^{12} \\
&\quad+3(5d+6e)x^{14}+\frac{15}{8}(4d+7e)x^{16}+\frac{5}{6}(3d+8e)x^{18} \\
&\quad+\frac{1}{4}(2d+9e)x^{20}+\frac{1}{22}(d+10e)x^{22}+\frac{ex^{24}}{24}
\end{aligned}$$

```
[In] Integrate[x*(d + e*x^2)*(1 + 2*x^2 + x^4)^5,x]
```

```
[Out] (d*x^2)/2 + ((10*d + e)*x^4)/4 + (5*(9*d + 2*e)*x^6)/6 + (15*(8*d + 3*e)*x^
8)/8 + 3*(7*d + 4*e)*x^10 + (7*(6*d + 5*e)*x^12)/2 + 3*(5*d + 6*e)*x^14 + (
15*(4*d + 7*e)*x^16)/8 + (5*(3*d + 8*e)*x^18)/6 + ((2*d + 9*e)*x^20)/4 + ((
d + 10*e)*x^22)/22 + (e*x^24)/24
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 123 vs. 2(25) = 50.

Time = 0.12 (sec) , antiderivative size = 124, normalized size of antiderivative = 4.28

method	result
norman	$\frac{x^{24}e}{24}+\left(\frac{d}{2}+\frac{9e}{4}\right)x^{20}+\left(\frac{d}{22}+\frac{5e}{11}\right)x^{22}+\left(\frac{5d}{2}+\frac{20e}{3}\right)x^{18}+(15d+18e)\,x^{14}+\left(\frac{15d}{2}+\frac{105e}{8}\right)x^{16}+$
default	$\frac{x^{24}e}{24}+\frac{(d+10e)x^{22}}{22}+\frac{(10d+45e)x^{20}}{20}+\frac{(45d+120e)x^{18}}{18}+\frac{(120d+210e)x^{16}}{16}+\frac{(210d+252e)x^{14}}{14}+\frac{(252d+210e)x^{12}}{12}$
risch	$\frac{1}{24}x^{24}e+\frac{1}{22}x^{22}d+\frac{5}{11}e\,x^{22}+\frac{1}{2}d\,x^{20}+\frac{9}{4}e\,x^{20}+\frac{5}{2}d\,x^{18}+\frac{20}{3}e\,x^{18}+\frac{15}{2}d\,x^{16}+\frac{105}{8}e\,x^{16}+15d\,x$
parallelrisch	$\frac{1}{24}x^{24}e+\frac{1}{22}x^{22}d+\frac{5}{11}e\,x^{22}+\frac{1}{2}d\,x^{20}+\frac{9}{4}e\,x^{20}+\frac{5}{2}d\,x^{18}+\frac{20}{3}e\,x^{18}+\frac{15}{2}d\,x^{16}+\frac{105}{8}e\,x^{16}+15d\,x$
gosper	$\frac{x^{2}(11e\,x^{22}+12d\,x^{20}+120e\,x^{20}+132d\,x^{18}+594e\,x^{18}+660d\,x^{16}+1760e\,x^{16}+1980d\,x^{14}+3465e\,x^{14}+3960d\,x^{12}+4752e\,x^{12}+5544}{264}$

[In] int(x*(e*x^2+d)*(x^4+2*x^2+1)^5,x,method=_RETURNVERBOSE)

[Out] 1/24*x^24*e+(1/2*d+9/4*e)*x^20+(1/22*d+5/11*e)*x^22+(5/2*d+20/3*e)*x^18+(15*d+18*e)*x^14+(15/2*d+105/8*e)*x^16+(15/2*d+5/3*e)*x^6+(15*d+45/8*e)*x^8+(21*d+12*e)*x^10+(21*d+35/2*e)*x^12+1/2*d*x^2+(5/2*d+1/4*e)*x^4

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 129 vs. 2(25) = 50.

Time = 0.26 (sec) , antiderivative size = 129, normalized size of antiderivative = 4.45

$$\int x(d+ex^2)\left(1+2x^2+x^4\right)^5\,dx=\frac{1}{24}\,ex^{24}+\frac{1}{22}\,(d+10\,e)x^{22}+\frac{1}{4}\,(2\,d+9\,e)x^{20}+\frac{5}{6}\,(3\,d+8\,e)x^{18}+\frac{15}{8}\,(4\,d+7\,e)x^{16}+3\,(5\,d+6\,e)x^{14}+\frac{7}{2}\,(6\,d+5\,e)x^{12}+3\,(7\,d+4\,e)x^{10}+\frac{15}{8}\,(8\,d+3\,e)x^{8}+\frac{5}{6}\,(9\,d+2\,e)x^{6}+\frac{1}{4}\,(10\,d+e)x^{4}+\frac{1}{2}\,dx^{2}$$

[In] integrate(x*(e*x^2+d)*(x^4+2*x^2+1)^5,x, algorithm="fricas")

[Out] 1/24*e*x^24 + 1/22*(d + 10*e)*x^22 + 1/4*(2*d + 9*e)*x^20 + 5/6*(3*d + 8*e)*x^18 + 15/8*(4*d + 7*e)*x^16 + 3*(5*d + 6*e)*x^14 + 7/2*(6*d + 5*e)*x^12 + 3*(7*d + 4*e)*x^10 + 15/8*(8*d + 3*e)*x^8 + 5/6*(9*d + 2*e)*x^6 + 1/4*(10*d + e)*x^4 + 1/2*d*x^2

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 133 vs. $2(22) = 44$.

Time = 0.04 (sec) , antiderivative size = 133, normalized size of antiderivative = 4.59

$$\int x(d+ex^2)\left(1+2x^2+x^4\right)^5\,dx = \frac{dx^2}{2}+\frac{ex^{24}}{24}+x^{22}\left(\frac{d}{22}+\frac{5e}{11}\right)+x^{20}\left(\frac{d}{2}+\frac{9e}{4}\right)+x^{18}\cdot\left(\frac{5d}{2}+\frac{20e}{3}\right)+x^{16}\cdot\left(\frac{15d}{2}+\frac{105e}{8}\right)+x^{14}\cdot(15d+18e)+x^{12}\cdot\left(21d+\frac{35e}{2}\right)+x^{10}\cdot(21d+12e)+x^8\cdot\left(15d+\frac{45e}{8}\right)+x^6\cdot\left(\frac{15d}{2}+\frac{5e}{3}\right)+x^4\cdot\left(\frac{5d}{2}+\frac{e}{4}\right)$$

```
[In] integrate(x*(e*x**2+d)*(x**4+2*x**2+1)**5,x)
```

```
[Out] d*x**2/2 + e*x**24/24 + x**22*(d/22 + 5*e/11) + x**20*(d/2 + 9*e/4) + x**18
*(5*d/2 + 20*e/3) + x**16*(15*d/2 + 105*e/8) + x**14*(15*d + 18*e) + x**12*
(21*d + 35*e/2) + x**10*(21*d + 12*e) + x**8*(15*d + 45*e/8) + x**6*(15*d/2
 + 5*e/3) + x**4*(5*d/2 + e/4)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 129 vs. $2(25) = 50$.

Time = 0.19 (sec) , antiderivative size = 129, normalized size of antiderivative = 4.45

$$\int x(d+ex^2)\left(1+2x^2+x^4\right)^5\,dx = \frac{1}{24}ex^{24}+\frac{1}{22}(d+10e)x^{22}+\frac{1}{4}(2d+9e)x^{20}+\frac{5}{6}(3d+8e)x^{18}+\frac{15}{8}(4d+7e)x^{16}+3(5d+6e)x^{14}+\frac{7}{2}(6d+5e)x^{12}+3(7d+4e)x^{10}+\frac{15}{8}(8d+3e)x^8+\frac{5}{6}(9d+2e)x^6+\frac{1}{4}(10d+e)x^4+\frac{1}{2}dx^2$$

```
[In] integrate(x*(e*x^2+d)*(x^4+2*x^2+1)^5,x, algorithm="maxima")
```

```
[Out] 1/24*e*x^24 + 1/22*(d + 10*e)*x^22 + 1/4*(2*d + 9*e)*x^20 + 5/6*(3*d + 8*e)
*x^18 + 15/8*(4*d + 7*e)*x^16 + 3*(5*d + 6*e)*x^14 + 7/2*(6*d + 5*e)*x^12 +
 3*(7*d + 4*e)*x^10 + 15/8*(8*d + 3*e)*x^8 + 5/6*(9*d + 2*e)*x^6 + 1/4*(10*
d + e)*x^4 + 1/2*d*x^2
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 133 vs. $2(25) = 50$.

Time = 0.26 (sec) , antiderivative size = 133, normalized size of antiderivative = 4.59

$$\int x(d+ex^2)\left(1+2x^2+x^4\right)^5\,dx = \frac{1}{24}\,ex^{24}+\frac{1}{22}\,dx^{22}+\frac{5}{11}\,ex^{22}+\frac{1}{2}\,dx^{20}+\frac{9}{4}\,ex^{20}+\frac{5}{2}\,dx^{18}+\frac{20}{3}\,ex^{18}+\frac{15}{2}\,dx^{16}+\frac{105}{8}\,ex^{16}+15\,dx^{14}+18\,ex^{14}+21\,dx^{12}+\frac{35}{2}\,ex^{12}+21\,dx^{10}+12\,ex^{10}+15\,dx^{8}+\frac{45}{8}\,ex^{8}+\frac{15}{2}\,dx^{6}+\frac{5}{3}\,ex^{6}+\frac{5}{2}\,dx^{4}+\frac{1}{4}\,ex^{4}+\frac{1}{2}\,dx^{2}$$

```
[In] integrate(x*(e*x^2+d)*(x^4+2*x^2+1)^5,x, algorithm="giac")
```

```
[Out] 1/24*e*x^24 + 1/22*d*x^22 + 5/11*e*x^22 + 1/2*d*x^20 + 9/4*e*x^20 + 5/2*d*x
^18 + 20/3*e*x^18 + 15/2*d*x^16 + 105/8*e*x^16 + 15*d*x^14 + 18*e*x^14 + 21
*d*x^12 + 35/2*e*x^12 + 21*d*x^10 + 12*e*x^10 + 15*d*x^8 + 45/8*e*x^8 + 15/
2*d*x^6 + 5/3*e*x^6 + 5/2*d*x^4 + 1/4*e*x^4 + 1/2*d*x^2
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 123, normalized size of antiderivative = 4.24

$$\int x(d+ex^2)\left(1+2x^2+x^4\right)^5\,dx = \frac{e\,x^{24}}{24}+\left(\frac{d}{22}+\frac{5\,e}{11}\right)x^{22}+\left(\frac{d}{2}+\frac{9\,e}{4}\right)x^{20}+\left(\frac{5\,d}{2}+\frac{20\,e}{3}\right)x^{18}+\left(\frac{15\,d}{2}+\frac{105\,e}{8}\right)x^{16}+(15\,d+18\,e)\,x^{14}+\left(21\,d+\frac{35\,e}{2}\right)x^{12}+(21\,d+12\,e)\,x^{10}+\left(15\,d+\frac{45\,e}{8}\right)x^{8}+\left(\frac{15\,d}{2}+\frac{5\,e}{3}\right)x^{6}+\left(\frac{5\,d}{2}+\frac{e}{4}\right)x^{4}+\frac{d\,x^2}{2}$$

```
[In] int(x*(d + e*x^2)*(2*x^2 + x^4 + 1)^5,x)
```

```
[Out] x^4*((5*d)/2 + e/4) + x^6*((15*d)/2 + (5*e)/3) + x^20*(d/2 + (9*e)/4) + x^1
0*(21*d + 12*e) + x^14*(15*d + 18*e) + x^18*((5*d)/2 + (20*e)/3) + x^22*(d/
22 + (5*e)/11) + x^12*(21*d + (35*e)/2) + x^8*(15*d + (45*e)/8) + x^16*((15
*d)/2 + (105*e)/8) + (d*x^2)/2 + (e*x^24)/24
```

3.61 $\int (d+ex^2)(1+2x^2+x^4)^5 \, dx$

Optimal result

Integrand size = 20, antiderivative size = 143

$$\int (d+ex^2)\left(1+2x^2+x^4\right)^5 \, dx = dx+\frac{1}{3}(10d+e)x^3+(9d+2e)x^5+\frac{15}{7}(8d+3e)x^7 +\frac{10}{3}(7d+4e)x^9+\frac{42}{11}(6d+5e)x^{11} +\frac{42}{13}(5d+6e)x^{13}+2(4d+7e)x^{15}+\frac{15}{17}(3d+8e)x^{17} +\frac{5}{19}(2d+9e)x^{19}+\frac{1}{21}(d+10e)x^{21}+\frac{ex^{23}}{23}$$

```
[Out] d*x+1/3*(10*d+e)*x^3+(9*d+2*e)*x^5+15/7*(8*d+3*e)*x^7+10/3*(7*d+4*e)*x^9+42
/11*(6*d+5*e)*x^11+42/13*(5*d+6*e)*x^13+2*(4*d+7*e)*x^15+15/17*(3*d+8*e)*x^
17+5/19*(2*d+9*e)*x^19+1/21*(d+10*e)*x^21+1/23*e*x^23
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {28, 380}

$$\int (d+ex^2)\left(1+2x^2+x^4\right)^5 \, dx = \frac{1}{21}x^{21}(d+10e)+\frac{5}{19}x^{19}(2d+9e)+\frac{15}{17}x^{17}(3d+8e) +2x^{15}(4d+7e)+\frac{42}{13}x^{13}(5d+6e)+\frac{42}{11}x^{11}(6d+5e) +\frac{10}{3}x^9(7d+4e)+\frac{15}{7}x^7(8d+3e) +x^5(9d+2e)+\frac{1}{3}x^3(10d+e)+dx+\frac{ex^{23}}{23}$$

```
[In] Int[(d + e*x^2)*(1 + 2*x^2 + x^4)^5,x]
```

```
[Out] d*x + ((10*d + e)*x^3)/3 + (9*d + 2*e)*x^5 + (15*(8*d + 3*e)*x^7)/7 + (10*(
7*d + 4*e)*x^9)/3 + (42*(6*d + 5*e)*x^11)/11 + (42*(5*d + 6*e)*x^13)/13 + 2
*(4*d + 7*e)*x^15 + (15*(3*d + 8*e)*x^17)/17 + (5*(2*d + 9*e)*x^19)/19 + ((
d + 10*e)*x^21)/21 + (e*x^23)/23
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 380

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> Int[ExpandIntegrand[(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b
, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(1+x^2\right)^{10} \left(d+ex^2\right)\,dx \\
&= \int \left(d+(10d+e)x^2+5(9d+2e)x^4+15(8d+3e)x^6+30(7d+4e)x^8+42(6d+5e)x^{10}\right. \\
&\qquad +42(5d+6e)x^{12}+30(4d+7e)x^{14}+15(3d+8e)x^{16}+5(2d+9e)x^{18} \\
&\qquad \left.+(d+10e)x^{20}+ex^{22}\right)\,dx \\
&= dx+\frac{1}{3}(10d+e)x^3+(9d+2e)x^5+\frac{15}{7}(8d+3e)x^7+\frac{10}{3}(7d+4e)x^9+\frac{42}{11}(6d+5e)x^{11} \\
&\quad +\frac{42}{13}(5d+6e)x^{13}+2(4d+7e)x^{15}+\frac{15}{17}(3d+8e)x^{17}+\frac{5}{19}(2d+9e)x^{19}+\frac{1}{21}(d+10e)x^{21} \\
&\quad +\frac{ex^{23}}{23}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.00

$$\begin{aligned}
\int \left(d+ex^2\right)\left(1+2x^2+x^4\right)^5\,dx &= dx+\frac{1}{3}(10d+e)x^3+(9d+2e)x^5+\frac{15}{7}(8d+3e)x^7 \\
&\quad +\frac{10}{3}(7d+4e)x^9+\frac{42}{11}(6d+5e)x^{11} \\
&\quad +\frac{42}{13}(5d+6e)x^{13}+2(4d+7e)x^{15}+\frac{15}{17}(3d+8e)x^{17} \\
&\quad +\frac{5}{19}(2d+9e)x^{19}+\frac{1}{21}(d+10e)x^{21}+\frac{ex^{23}}{23}
\end{aligned}$$

```
[In] Integrate[(d + e*x^2)*(1 + 2*x^2 + x^4)^5,x]
```

```
[Out] d*x + ((10*d + e)*x^3)/3 + (9*d + 2*e)*x^5 + (15*(8*d + 3*e)*x^7)/7 + (10*(
7*d + 4*e)*x^9)/3 + (42*(6*d + 5*e)*x^11)/11 + (42*(5*d + 6*e)*x^13)/13 + 2
*(4*d + 7*e)*x^15 + (15*(3*d + 8*e)*x^17)/17 + (5*(2*d + 9*e)*x^19)/19 + ((
d + 10*e)*x^21)/21 + (e*x^23)/23
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.85

method	result
norman	$dx+\left(\frac{10d}{3}+\frac{e}{3}\right)x^{3}+\left(9d+2e\right)x^{5}+\left(\frac{120d}{7}+\frac{45e}{7}\right)x^{7}+\left(\frac{70d}{3}+\frac{40e}{3}\right)x^{9}+\left(\frac{252d}{11}+\frac{210e}{11}\right)x^{11}+\left(\frac{210}{13}\right.$
default	$\frac{e\,x^{23}}{23}+\frac{(d+10e)x^{21}}{21}+\frac{(10d+45e)x^{19}}{19}+\frac{(45d+120e)x^{17}}{17}+\frac{(120d+210e)x^{15}}{15}+\frac{(210d+252e)x^{13}}{13}+\frac{(252d+210e)x^{11}}{11}+$
risch	$\frac{1}{23}e\,x^{23}+\frac{1}{21}x^{21}d+\frac{10}{21}e\,x^{21}+\frac{10}{19}x^{19}d+\frac{45}{19}x^{19}e+\frac{45}{17}x^{17}d+\frac{120}{17}x^{17}e+8x^{15}d+14x^{15}e+\frac{210}{13}x^{13}d$
parallelrisch	$\frac{1}{23}e\,x^{23}+\frac{1}{21}x^{21}d+\frac{10}{21}e\,x^{21}+\frac{10}{19}x^{19}d+\frac{45}{19}x^{19}e+\frac{45}{17}x^{17}d+\frac{120}{17}x^{17}e+8x^{15}d+14x^{15}e+\frac{210}{13}x^{13}d$
gosper	$x(969969e\,x^{22}+1062347d\,x^{20}+10623470e\,x^{20}+11741730d\,x^{18}+52837785e\,x^{18}+59053995d\,x^{16}+157477320e\,x^{16}+178474296d\,x$

```
[In] int((e*x^2+d)*(x^4+2*x^2+1)^5,x,method=_RETURNVERBOSE)
```

```
[Out] d*x+(10/3*d+1/3*e)*x^3+(9*d+2*e)*x^5+(120/7*d+45/7*e)*x^7+(70/3*d+40/3*e)*x
^9+(252/11*d+210/11*e)*x^11+(210/13*d+252/13*e)*x^13+(8*d+14*e)*x^15+(45/17
*d+120/17*e)*x^17+(10/19*d+45/19*e)*x^19+(1/21*d+10/21*e)*x^21+1/23*e*x^23
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.87

$$\begin{aligned}\int \left(d+ex^2\right)\left(1+2x^2+x^4\right)^5\,dx =& \frac{1}{23}\,ex^{23}+\frac{1}{21}\,(d+10\,e)x^{21}+\frac{5}{19}\,(2\,d+9\,e)x^{19}\\&+\frac{15}{17}\,(3\,d+8\,e)x^{17}+2\,(4\,d+7\,e)x^{15}+\frac{42}{13}\,(5\,d+6\,e)x^{13}\\&+\frac{42}{11}\,(6\,d+5\,e)x^{11}+\frac{10}{3}\,(7\,d+4\,e)x^{9}\\&+\frac{15}{7}\,(8\,d+3\,e)x^{7}+(9\,d+2\,e)x^{5}+\frac{1}{3}\,(10\,d+e)x^{3}+dx\end{aligned}$$

```
[In] integrate((e*x^2+d)*(x^4+2*x^2+1)^5,x, algorithm="fricas")
```

```
[Out] 1/23*e*x^23 + 1/21*(d + 10*e)*x^21 + 5/19*(2*d + 9*e)*x^19 + 15/17*(3*d + 8
*e)*x^17 + 2*(4*d + 7*e)*x^15 + 42/13*(5*d + 6*e)*x^13 + 42/11*(6*d + 5*e)*
x^11 + 10/3*(7*d + 4*e)*x^9 + 15/7*(8*d + 3*e)*x^7 + (9*d + 2*e)*x^5 + 1/3*
(10*d + e)*x^3 + d*x
```

Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 134, normalized size of antiderivative = 0.94

$$\int \left(d+ex^2\right)\left(1+2x^2+x^4\right)^5\,dx = dx+\frac{ex^{23}}{23}+x^{21}\left(\frac{d}{21}+\frac{10e}{21}\right)+x^{19}\cdot\left(\frac{10d}{19}+\frac{45e}{19}\right)+x^{17}\cdot\left(\frac{45d}{17}+\frac{120e}{17}\right)+x^{15}\cdot(8d+14e)+x^{13}\cdot\left(\frac{210d}{13}+\frac{252e}{13}\right)+x^{11}\cdot\left(\frac{252d}{11}+\frac{210e}{11}\right)+x^{9}\cdot\left(\frac{70d}{3}+\frac{40e}{3}\right)+x^{7}\cdot\left(\frac{120d}{7}+\frac{45e}{7}\right)+x^{5}\cdot(9d+2e)+x^{3}\cdot\left(\frac{10d}{3}+\frac{e}{3}\right)$$

```
[In] integrate((e*x**2+d)*(x**4+2*x**2+1)**5,x)
```

```
[Out] d*x + e*x**23/23 + x**21*(d/21 + 10*e/21) + x**19*(10*d/19 + 45*e/19) + x**
17*(45*d/17 + 120*e/17) + x**15*(8*d + 14*e) + x**13*(210*d/13 + 252*e/13)
+ x**11*(252*d/11 + 210*e/11) + x**9*(70*d/3 + 40*e/3) + x**7*(120*d/7 + 45
*e/7) + x**5*(9*d + 2*e) + x**3*(10*d/3 + e/3)
```

Maxima [A] (verification not implemented)

none

Time = 0.18 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.87

$$\int \left(d+ex^2\right)\left(1+2x^2+x^4\right)^5\,dx = \frac{1}{23}\,ex^{23}+\frac{1}{21}\,(d+10\,e)x^{21}+\frac{5}{19}\,(2\,d+9\,e)x^{19}+\frac{15}{17}\,(3\,d+8\,e)x^{17}+2\,(4\,d+7\,e)x^{15}+\frac{42}{13}\,(5\,d+6\,e)x^{13}+\frac{42}{11}\,(6\,d+5\,e)x^{11}+\frac{10}{3}\,(7\,d+4\,e)x^{9}+\frac{15}{7}\,(8\,d+3\,e)x^{7}+(9\,d+2\,e)x^{5}+\frac{1}{3}\,(10\,d+e)x^{3}+dx$$

```
[In] integrate((e*x^2+d)*(x^4+2*x^2+1)^5,x, algorithm="maxima")
```

```
[Out] 1/23*e*x^23 + 1/21*(d + 10*e)*x^21 + 5/19*(2*d + 9*e)*x^19 + 15/17*(3*d + 8
*e)*x^17 + 2*(4*d + 7*e)*x^15 + 42/13*(5*d + 6*e)*x^13 + 42/11*(6*d + 5*e)*
x^11 + 10/3*(7*d + 4*e)*x^9 + 15/7*(8*d + 3*e)*x^7 + (9*d + 2*e)*x^5 + 1/3*
(10*d + e)*x^3 + d*x
```

Giac [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 130, normalized size of antiderivative = 0.91

$$\int (d+ex^2)\left(1+2x^2+x^4\right)^5\,dx = \frac{1}{23}\,ex^{23} + \frac{1}{21}\,dx^{21} + \frac{10}{21}\,ex^{21} + \frac{10}{19}\,dx^{19} + \frac{45}{19}\,ex^{19} + \frac{45}{17}\,dx^{17} + \frac{120}{17}\,ex^{17} + 8\,dx^{15} + 14\,ex^{15} + \frac{210}{13}\,dx^{13} + \frac{252}{13}\,ex^{13} + \frac{252}{11}\,dx^{11} + \frac{210}{11}\,ex^{11} + \frac{70}{3}\,dx^9 + \frac{40}{3}\,ex^9 + \frac{120}{7}\,dx^7 + \frac{45}{7}\,ex^7 + 9\,dx^5 + 2\,ex^5 + \frac{10}{3}\,dx^3 + \frac{1}{3}\,ex^3 + dx$$

```
[In] integrate((e*x^2+d)*(x^4+2*x^2+1)^5,x, algorithm="giac")
```

```
[Out] 1/23*e*x^23 + 1/21*d*x^21 + 10/21*e*x^21 + 10/19*d*x^19 + 45/19*e*x^19 + 45
/17*d*x^17 + 120/17*e*x^17 + 8*d*x^15 + 14*e*x^15 + 210/13*d*x^13 + 252/13*
e*x^13 + 252/11*d*x^11 + 210/11*e*x^11 + 70/3*d*x^9 + 40/3*e*x^9 + 120/7*d*
x^7 + 45/7*e*x^7 + 9*d*x^5 + 2*e*x^5 + 10/3*d*x^3 + 1/3*e*x^3 + d*x
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 120, normalized size of antiderivative = 0.84

$$\int (d+ex^2)\left(1+2x^2+x^4\right)^5\,dx = \frac{e\,x^{23}}{23} + \left(\frac{d}{21}+\frac{10\,e}{21}\right)x^{21} + \left(\frac{10\,d}{19}+\frac{45\,e}{19}\right)x^{19} + \left(\frac{45\,d}{17}+\frac{120\,e}{17}\right)x^{17} + (8\,d+14\,e)\,x^{15} + \left(\frac{210\,d}{13}+\frac{252\,e}{13}\right)x^{13} + \left(\frac{252\,d}{11}+\frac{210\,e}{11}\right)x^{11} + \left(\frac{70\,d}{3}+\frac{40\,e}{3}\right)x^9 + \left(\frac{120\,d}{7}+\frac{45\,e}{7}\right)x^7 + (9\,d+2\,e)\,x^5 + \left(\frac{10\,d}{3}+\frac{e}{3}\right)x^3 + d\,x$$

```
[In] int((d + e*x^2)*(2*x^2 + x^4 + 1)^5,x)
```

```
[Out] x^5*(9*d + 2*e) + x^3*((10*d)/3 + e/3) + x^15*(8*d + 14*e) + x^21*(d/21 + (
10*e)/21) + x^19*((10*d)/19 + (45*e)/19) + x^9*((70*d)/3 + (40*e)/3) + x^7*
((120*d)/7 + (45*e)/7) + x^17*((45*d)/17 + (120*e)/17) + x^11*((252*d)/11 +
 (210*e)/11) + x^13*((210*d)/13 + (252*e)/13) + d*x + (e*x^23)/23
```

3.62 $\int \frac{(d+ex^2)(1+2x^2+x^4)^5}{x} \, dx$

Optimal result

Integrand size = 23, antiderivative size = 93

$$\int \frac{(d+ex^2)(1+2x^2+x^4)^5}{x} \, dx = 5dx^2 + \frac{45dx^4}{4} + 20dx^6 + \frac{105dx^8}{4} + \frac{126dx^{10}}{5} + \frac{35dx^{12}}{2} + \frac{60dx^{14}}{7} + \frac{45dx^{16}}{16} + \frac{5dx^{18}}{9} + \frac{dx^{20}}{20} + \frac{1}{22}e(1+x^2)^{11} + d\log(x)$$

```
[Out] 5*d*x^2+45/4*d*x^4+20*d*x^6+105/4*d*x^8+126/5*d*x^10+35/2*d*x^12+60/7*d*x^1
4+45/16*d*x^16+5/9*d*x^18+1/20*d*x^20+1/22*e*(x^2+1)^11+d*ln(x)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {28, 457, 81, 45}

$$\int \frac{(d+ex^2)(1+2x^2+x^4)^5}{x} \, dx = \frac{dx^{20}}{20} + \frac{5dx^{18}}{9} + \frac{45dx^{16}}{16} + \frac{60dx^{14}}{7} + \frac{35dx^{12}}{2} + \frac{126dx^{10}}{5} + \frac{105dx^8}{4} + 20dx^6 + \frac{45dx^4}{4} + 5dx^2 + d\log(x) + \frac{1}{22}e(x^2+1)^{11}$$

```
[In] Int[((d + e*x^2)*(1 + 2*x^2 + x^4)^5)/x,x]
```

```
[Out] 5*d*x^2 + (45*d*x^4)/4 + 20*d*x^6 + (105*d*x^8)/4 + (126*d*x^10)/5 + (35*d*
x^12)/2 + (60*d*x^14)/7 + (45*d*x^16)/16 + (5*d*x^18)/9 + (d*x^20)/20 + (e*
(1 + x^2)^11)/22 + d*Log[x]
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 81

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p +
 2))), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \frac{(1+x^2)^{10}\,(d+ex^2)}{x}\,dx \\
&= \frac{1}{2}\text{Subst}\left(\int \frac{(1+x)^{10}(d+ex)}{x}\,dx, x, x^2\right) \\
&= \frac{1}{22}e\left(1+x^2\right)^{11} + \frac{1}{2}d\text{Subst}\left(\int \frac{(1+x)^{10}}{x}\,dx, x, x^2\right) \\
&= \frac{1}{22}e\left(1+x^2\right)^{11} + \frac{1}{2}d\text{Subst}\left(\int \left(10+\frac{1}{x}+45x+120x^2+210x^3+252x^4+210x^5 \right.\right. \\
&\qquad \left.\left. +120x^6+45x^7+10x^8+x^9\right)\,dx, x, x^2\right) \\
&= 5dx^2+\frac{45dx^4}{4}+20dx^6+\frac{105dx^8}{4}+\frac{126dx^{10}}{5}+\frac{35dx^{12}}{2}+\frac{60dx^{14}}{7} \\
&\quad +\frac{45dx^{16}}{16}+\frac{5dx^{18}}{9}+\frac{dx^{20}}{20}+\frac{1}{22}e\left(1+x^2\right)^{11}+d\log(x)
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.60

$$\int \frac{(d+ex^2)(1+2x^2+x^4)^5}{x}\,dx = \frac{1}{2}(10d+e)x^2+\frac{5}{4}(9d+2e)x^4+\frac{5}{2}(8d+3e)x^6+\frac{15}{4}(7d+4e)x^8+\frac{21}{5}(6d+5e)x^{10}+\frac{7}{2}(5d+6e)x^{12}+\frac{15}{7}(4d+7e)x^{14}+\frac{15}{16}(3d+8e)x^{16}+\frac{5}{18}(2d+9e)x^{18}+\frac{1}{20}(d+10e)x^{20}+\frac{ex^{22}}{22}+d\log(x)$$

```
[In] Integrate[((d + e*x^2)*(1 + 2*x^2 + x^4)^5)/x,x]
```

```
[Out] ((10*d + e)*x^2)/2 + (5*(9*d + 2*e)*x^4)/4 + (5*(8*d + 3*e)*x^6)/2 + (15*(7
*d + 4*e)*x^8)/4 + (21*(6*d + 5*e)*x^10)/5 + (7*(5*d + 6*e)*x^12)/2 + (15*(
4*d + 7*e)*x^14)/7 + (15*(3*d + 8*e)*x^16)/16 + (5*(2*d + 9*e)*x^18)/18 + (
(d + 10*e)*x^20)/20 + (e*x^22)/22 + d*Log[x]
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.31

method	result
norman	$\left(5d+\frac{e}{2}\right)x^2+\left(20d+\frac{15e}{2}\right)x^6+\left(\frac{d}{20}+\frac{e}{2}\right)x^{20}+\left(\frac{5d}{9}+\frac{5e}{2}\right)x^{18}+\left(\frac{35d}{2}+21e\right)x^{12}+\left(\frac{45d}{4}+\frac{5e}{2}\right)$
default	$\frac{e\,x^{22}}{22}+\frac{d\,x^{20}}{20}+\frac{e\,x^{20}}{2}+\frac{5d\,x^{18}}{9}+\frac{5e\,x^{18}}{2}+\frac{45d\,x^{16}}{16}+\frac{15e\,x^{16}}{2}+\frac{60d\,x^{14}}{7}+15e\,x^{14}+\frac{35d\,x^{12}}{2}+21e\,x^{12}+$
risch	$\frac{e\,x^{22}}{22}+\frac{d\,x^{20}}{20}+\frac{e\,x^{20}}{2}+\frac{5d\,x^{18}}{9}+\frac{5e\,x^{18}}{2}+\frac{45d\,x^{16}}{16}+\frac{15e\,x^{16}}{2}+\frac{60d\,x^{14}}{7}+15e\,x^{14}+\frac{35d\,x^{12}}{2}+21e\,x^{12}+$
parallelrisch	$\frac{e\,x^{22}}{22}+\frac{d\,x^{20}}{20}+\frac{e\,x^{20}}{2}+\frac{5d\,x^{18}}{9}+\frac{5e\,x^{18}}{2}+\frac{45d\,x^{16}}{16}+\frac{15e\,x^{16}}{2}+\frac{60d\,x^{14}}{7}+15e\,x^{14}+\frac{35d\,x^{12}}{2}+21e\,x^{12}+$

```
[In] int((e*x^2+d)*(x^4+2*x^2+1)^5/x,x,method=_RETURNVERBOSE)
```

```
[Out] (5*d+1/2*e)*x^2+(20*d+15/2*e)*x^6+(1/20*d+1/2*e)*x^20+(5/9*d+5/2*e)*x^18+(3
5/2*d+21*e)*x^12+(45/4*d+5/2*e)*x^4+(45/16*d+15/2*e)*x^16+(60/7*d+15*e)*x^1
4+(105/4*d+15*e)*x^8+(126/5*d+21*e)*x^10+1/22*e*x^22+d*ln(x)
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.37

$$\int \frac{(d+ex^2)(1+2x^2+x^4)^5}{x}\,dx = \frac{1}{22}\,ex^{22} + \frac{1}{20}\,(d+10\,e)x^{20} + \frac{5}{18}\,(2\,d+9\,e)x^{18} + \frac{15}{16}\,(3\,d+8\,e)x^{16} + \frac{15}{7}\,(4\,d+7\,e)x^{14} + \frac{7}{2}\,(5\,d+6\,e)x^{12} + \frac{21}{5}\,(6\,d+5\,e)x^{10} + \frac{15}{4}\,(7\,d+4\,e)x^{8} + \frac{5}{2}\,(8\,d+3\,e)x^{6} + \frac{5}{4}\,(9\,d+2\,e)x^{4} + \frac{1}{2}\,(10\,d+e)x^{2} + d\log(x)$$

```
[In] integrate((e*x^2+d)*(x^4+2*x^2+1)^5/x,x, algorithm="fricas")
```

```
[Out] 1/22*e*x^22 + 1/20*(d + 10*e)*x^20 + 5/18*(2*d + 9*e)*x^18 + 15/16*(3*d + 8
*e)*x^16 + 15/7*(4*d + 7*e)*x^14 + 7/2*(5*d + 6*e)*x^12 + 21/5*(6*d + 5*e)*
x^10 + 15/4*(7*d + 4*e)*x^8 + 5/2*(8*d + 3*e)*x^6 + 5/4*(9*d + 2*e)*x^4 + 1
/2*(10*d + e)*x^2 + d*log(x)
```

Sympy [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.41

$$\int \frac{(d+ex^2)(1+2x^2+x^4)^5}{x}\,dx = d\log(x) + \frac{ex^{22}}{22} + x^{20}\left(\frac{d}{20}+\frac{e}{2}\right) + x^{18}\cdot\left(\frac{5d}{9}+\frac{5e}{2}\right) + x^{16}\cdot\left(\frac{45d}{16}+\frac{15e}{2}\right) + x^{14}\cdot\left(\frac{60d}{7}+15e\right) + x^{12}\cdot\left(\frac{35d}{2}+21e\right) + x^{10}\cdot\left(\frac{126d}{5}+21e\right) + x^{8}\cdot\left(\frac{105d}{4}+15e\right) + x^{6}\cdot\left(20d+\frac{15e}{2}\right) + x^{4}\cdot\left(\frac{45d}{4}+\frac{5e}{2}\right) + x^{2}\cdot\left(5d+\frac{e}{2}\right)$$

```
[In] integrate((e*x**2+d)*(x**4+2*x**2+1)**5/x,x)
```

```
[Out] d*log(x) + e*x**22/22 + x**20*(d/20 + e/2) + x**18*(5*d/9 + 5*e/2) + x**16*
(45*d/16 + 15*e/2) + x**14*(60*d/7 + 15*e) + x**12*(35*d/2 + 21*e) + x**10*
(126*d/5 + 21*e) + x**8*(105*d/4 + 15*e) + x**6*(20*d + 15*e/2) + x**4*(45*
d/4 + 5*e/2) + x**2*(5*d + e/2)
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.40

$$\int \frac{(d+ex^2)(1+2x^2+x^4)^5}{x}\,dx = \frac{1}{22}\,ex^{22} + \frac{1}{20}\,(d+10\,e)x^{20} + \frac{5}{18}\,(2\,d+9\,e)x^{18} + \frac{15}{16}\,(3\,d+8\,e)x^{16} + \frac{15}{7}\,(4\,d+7\,e)x^{14} + \frac{7}{2}\,(5\,d+6\,e)x^{12} + \frac{21}{5}\,(6\,d+5\,e)x^{10} + \frac{15}{4}\,(7\,d+4\,e)x^{8} + \frac{5}{2}\,(8\,d+3\,e)x^{6} + \frac{5}{4}\,(9\,d+2\,e)x^{4} + \frac{1}{2}\,(10\,d+e)x^{2} + \frac{1}{2}\,d\log\left(x^2\right)$$

```
[In] integrate((e*x^2+d)*(x^4+2*x^2+1)^5/x,x, algorithm="maxima")
```

```
[Out] 1/22*e*x^22 + 1/20*(d + 10*e)*x^20 + 5/18*(2*d + 9*e)*x^18 + 15/16*(3*d + 8
*e)*x^16 + 15/7*(4*d + 7*e)*x^14 + 7/2*(5*d + 6*e)*x^12 + 21/5*(6*d + 5*e)*
x^10 + 15/4*(7*d + 4*e)*x^8 + 5/2*(8*d + 3*e)*x^6 + 5/4*(9*d + 2*e)*x^4 + 1
/2*(10*d + e)*x^2 + 1/2*d*log(x^2)
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.44

$$\int \frac{(d+ex^2)(1+2x^2+x^4)^5}{x}\,dx = \frac{1}{22}\,ex^{22} + \frac{1}{20}\,dx^{20} + \frac{1}{2}\,ex^{20} + \frac{5}{9}\,dx^{18} + \frac{5}{2}\,ex^{18} + \frac{45}{16}\,dx^{16} + \frac{15}{2}\,ex^{16} + \frac{60}{7}\,dx^{14} + 15\,ex^{14} + \frac{35}{2}\,dx^{12} + 21\,ex^{12} + \frac{126}{5}\,dx^{10} + 21\,ex^{10} + \frac{105}{4}\,dx^{8} + 15\,ex^{8} + 20\,dx^{6} + \frac{15}{2}\,ex^{6} + \frac{45}{4}\,dx^{4} + \frac{5}{2}\,ex^{4} + 5\,dx^{2} + \frac{1}{2}\,ex^{2} + \frac{1}{2}\,d\log\left(x^2\right)$$

```
[In] integrate((e*x^2+d)*(x^4+2*x^2+1)^5/x,x, algorithm="giac")
```

```
[Out] 1/22*e*x^22 + 1/20*d*x^20 + 1/2*e*x^20 + 5/9*d*x^18 + 5/2*e*x^18 + 45/16*d*
x^16 + 15/2*e*x^16 + 60/7*d*x^14 + 15*e*x^14 + 35/2*d*x^12 + 21*e*x^12 + 12
6/5*d*x^10 + 21*e*x^10 + 105/4*d*x^8 + 15*e*x^8 + 20*d*x^6 + 15/2*e*x^6 + 4
5/4*d*x^4 + 5/2*e*x^4 + 5*d*x^2 + 1/2*e*x^2 + 1/2*d*log(x^2)
```

Mupad [B] (verification not implemented)

Time = 7.72 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.30

$$\int \frac{(d+ex^2)(1+2x^2+x^4)^5}{x}\,dx = x^2\left(5\,d+\frac{e}{2}\right)+x^{18}\left(\frac{5\,d}{9}+\frac{5\,e}{2}\right)+x^6\left(20\,d+\frac{15\,e}{2}\right)+x^{20}\left(\frac{d}{20}+\frac{e}{2}\right)+x^4\left(\frac{45\,d}{4}+\frac{5\,e}{2}\right)+x^{12}\left(\frac{35\,d}{2}+21\,e\right)+x^{16}\left(\frac{45\,d}{16}+\frac{15\,e}{2}\right)+x^{14}\left(\frac{60\,d}{7}+15\,e\right)+x^8\left(\frac{105\,d}{4}+15\,e\right)+x^{10}\left(\frac{126\,d}{5}+21\,e\right)+\frac{e\,x^{22}}{22}+d\,\ln(x)$$

```
[In] int(((d + e*x^2)*(2*x^2 + x^4 + 1)^5)/x,x)
```

```
[Out] x^2*(5*d + e/2) + x^18*((5*d)/9 + (5*e)/2) + x^6*(20*d + (15*e)/2) + x^20*(
d/20 + e/2) + x^4*((45*d)/4 + (5*e)/2) + x^12*((35*d)/2 + 21*e) + x^16*((45
*d)/16 + (15*e)/2) + x^14*((60*d)/7 + 15*e) + x^8*((105*d)/4 + 15*e) + x^10
*((126*d)/5 + 21*e) + (e*x^22)/22 + d*log(x)
```

3.63 $\int \frac{(d+ex^2)(1+2x^2+x^4)^5}{x^2} dx$

Optimal result

Integrand size = 23, antiderivative size = 141

$$\int \frac{(d+ex^2)(1+2x^2+x^4)^5}{x^2} dx = -\frac{d}{x}+(10d+e)x+\frac{5}{3}(9d+2e)x^3+3(8d+3e)x^5 + \frac{30}{7}(7d+4e)x^7+\frac{14}{3}(6d+5e)x^9 + \frac{42}{11}(5d+6e)x^{11}+\frac{30}{13}(4d+7e)x^{13}+(3d+8e)x^{15} + \frac{5}{17}(2d+9e)x^{17}+\frac{1}{19}(d+10e)x^{19}+\frac{ex^{21}}{21}$$

```
[Out] -d/x+(10*d+e)*x+5/3*(9*d+2*e)*x^3+3*(8*d+3*e)*x^5+30/7*(7*d+4*e)*x^7+14/3*(
6*d+5*e)*x^9+42/11*(5*d+6*e)*x^11+30/13*(4*d+7*e)*x^13+(3*d+8*e)*x^15+5/17*
(2*d+9*e)*x^17+1/19*(d+10*e)*x^19+1/21*e*x^21
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {28, 459}

$$\int \frac{(d+ex^2)(1+2x^2+x^4)^5}{x^2} dx = \frac{1}{19}x^{19}(d+10e)+\frac{5}{17}x^{17}(2d+9e)+x^{15}(3d+8e) + \frac{30}{13}x^{13}(4d+7e)+\frac{42}{11}x^{11}(5d+6e) + \frac{14}{3}x^9(6d+5e)+\frac{30}{7}x^7(7d+4e)+3x^5(8d+3e) + \frac{5}{3}x^3(9d+2e)+x(10d+e)-\frac{d}{x}+\frac{ex^{21}}{21}$$

```
[In] Int[((d + e*x^2)*(1 + 2*x^2 + x^4)^5)/x^2,x]
```

```
[Out] -(d/x) + (10*d + e)*x + (5*(9*d + 2*e)*x^3)/3 + 3*(8*d + 3*e)*x^5 + (30*(7*
d + 4*e)*x^7)/7 + (14*(6*d + 5*e)*x^9)/3 + (42*(5*d + 6*e)*x^11)/11 + (30*(
4*d + 7*e)*x^13)/13 + (3*d + 8*e)*x^15 + (5*(2*d + 9*e)*x^17)/17 + ((d + 10
*e)*x^19)/19 + (e*x^21)/21
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \frac{(1+x^2)^{10}\,(d+ex^2)}{x^2}\,dx \\
&= \int \left(10d\left(1+\frac{e}{10d}\right)+\frac{d}{x^2}+5(9d+2e)x^2+15(8d+3e)x^4+30(7d+4e)x^6+42(6d+5e)x^8 \right. \\
&\qquad + 42(5d+6e)x^{10}+30(4d+7e)x^{12}+15(3d+8e)x^{14}+5(2d+9e)x^{16} \\
&\qquad \left. + (d+10e)x^{18}+ex^{20}\right)\,dx \\
&= -\frac{d}{x}+(10d+e)x+\frac{5}{3}(9d+2e)x^3+3(8d+3e)x^5+\frac{30}{7}(7d+4e)x^7+\frac{14}{3}(6d+5e)x^9 \\
&\quad +\frac{42}{11}(5d+6e)x^{11}+\frac{30}{13}(4d+7e)x^{13}+(3d+8e)x^{15}+\frac{5}{17}(2d+9e)x^{17}+\frac{1}{19}(d+10e)x^{19} \\
&\quad +\frac{ex^{21}}{21}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.00

$$\int \frac{(d+ex^2)\,(1+2x^2+x^4)^5}{x^2}\,dx = -\frac{d}{x}+(10d+e)x+\frac{5}{3}(9d+2e)x^3+3(8d+3e)x^5+\frac{30}{7}(7d+4e)x^7+\frac{14}{3}(6d+5e)x^9+\frac{42}{11}(5d+6e)x^{11}+\frac{30}{13}(4d+7e)x^{13}+(3d+8e)x^{15}+\frac{5}{17}(2d+9e)x^{17}+\frac{1}{19}(d+10e)x^{19}+\frac{ex^{21}}{21}$$

```
[In] Integrate[((d + e*x^2)*(1 + 2*x^2 + x^4)^5)/x^2,x]
```

```
[Out] -(d/x) + (10*d + e)*x + (5*(9*d + 2*e)*x^3)/3 + 3*(8*d + 3*e)*x^5 + (30*(7*
d + 4*e)*x^7)/7 + (14*(6*d + 5*e)*x^9)/3 + (42*(5*d + 6*e)*x^11)/11 + (30*(
4*d + 7*e)*x^13)/13 + (3*d + 8*e)*x^15 + (5*(2*d + 9*e)*x^17)/17 + ((d + 10
*e)*x^19)/19 + (e*x^21)/21
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.87

method	result
norman	$\frac{-d+(10d+e)x^2+\left(15d+\frac{10e}{3}\right)x^4+(24d+9e)x^6+\left(30d+\frac{120e}{7}\right)x^8+\left(28d+\frac{70e}{3}\right)x^{10}+\left(\frac{210d}{11}+\frac{252e}{11}\right)x^{12}+\left(\frac{120d}{13}+\frac{210e}{13}\right)x^{14}+(3d+8\ldots}{x}$
default	$\frac{e\,x^{21}}{21}+\frac{x^{19}d}{19}+\frac{10x^{19}e}{19}+\frac{10x^{17}d}{17}+\frac{45x^{17}e}{17}+3x^{15}d+8x^{15}e+\frac{120x^{13}d}{13}+\frac{210x^{13}e}{13}+\frac{210x^{11}d}{11}+\frac{252e\,x^{11}}{11}$
risch	$\frac{e\,x^{21}}{21}+\frac{x^{19}d}{19}+\frac{10x^{19}e}{19}+\frac{10x^{17}d}{17}+\frac{45x^{17}e}{17}+3x^{15}d+8x^{15}e+\frac{120x^{13}d}{13}+\frac{210x^{13}e}{13}+\frac{210x^{11}d}{11}+\frac{252e\,x^{11}}{11}$
gosper	$46189e\,x^{22}+51051d\,x^{20}+510510e\,x^{20}+570570d\,x^{18}+2567565e\,x^{18}+2909907d\,x^{16}+7759752e\,x^{16}+8953560d\,x^{14}+15668730e\,x\ldots$
parallelrisch	$46189e\,x^{22}+51051d\,x^{20}+510510e\,x^{20}+570570d\,x^{18}+2567565e\,x^{18}+2909907d\,x^{16}+7759752e\,x^{16}+8953560d\,x^{14}+15668730e\,x\ldots$

```
[In] int((e*x^2+d)*(x^4+2*x^2+1)^5/x^2,x,method=_RETURNVERBOSE)
```

```
[Out] (-d+(10*d+e)*x^2+(15*d+10/3*e)*x^4+(24*d+9*e)*x^6+(30*d+120/7*e)*x^8+(28*d+
70/3*e)*x^10+(210/11*d+252/11*e)*x^12+(120/13*d+210/13*e)*x^14+(3*d+8*e)*x^
16+(10/17*d+45/17*e)*x^18+(1/19*d+10/19*e)*x^20+1/21*e*x^22)/x
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.93

$$\int \frac{(d+ex^2)(1+2x^2+x^4)^5}{x^2}\,dx = \frac{46189\,ex^{22} + 51051\,(d+10\,e)x^{20} + 285285\,(2\,d+9\,e)x^{18} + 969969\,(3\,d+8\,e)x^{16} + 2238390\,(4\,d+7\,e)x^{14} \cdots}{\cdots}$$

```
[In] integrate((e*x^2+d)*(x^4+2*x^2+1)^5/x^2,x, algorithm="fricas")
```

```
[Out] 1/969969*(46189*e*x^22 + 51051*(d + 10*e)*x^20 + 285285*(2*d + 9*e)*x^18 +
969969*(3*d + 8*e)*x^16 + 2238390*(4*d + 7*e)*x^14 + 3703518*(5*d + 6*e)*x^
12 + 4526522*(6*d + 5*e)*x^10 + 4157010*(7*d + 4*e)*x^8 + 2909907*(8*d + 3*
e)*x^6 + 1616615*(9*d + 2*e)*x^4 + 969969*(10*d + e)*x^2 - 969969*d)/x
```

Sympy [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.88

$$\int \frac{(d+ex^2)(1+2x^2+x^4)^5}{x^2}\,dx = -\frac{d}{x} + \frac{ex^{21}}{21} + x^{19}\left(\frac{d}{19}+\frac{10e}{19}\right) + x^{17}\cdot\left(\frac{10d}{17}+\frac{45e}{17}\right) + x^{15}\cdot(3d+8e) + x^{13}\cdot\left(\frac{120d}{13}+\frac{210e}{13}\right) + x^{11}\cdot\left(\frac{210d}{11}+\frac{252e}{11}\right) + x^9\cdot\left(28d+\frac{70e}{3}\right) + x^7\cdot\left(30d+\frac{120e}{7}\right) + x^5\cdot(24d+9e) + x^3\cdot\left(15d+\frac{10e}{3}\right) + x(10d+e)$$

```
[In] integrate((e*x**2+d)*(x**4+2*x**2+1)**5/x**2,x)
```

```
[Out] -d/x + e*x**21/21 + x**19*(d/19 + 10*e/19) + x**17*(10*d/17 + 45*e/17) + x*
*15*(3*d + 8*e) + x**13*(120*d/13 + 210*e/13) + x**11*(210*d/11 + 252*e/11)
 + x**9*(28*d + 70*e/3) + x**7*(30*d + 120*e/7) + x**5*(24*d + 9*e) + x**3*
(15*d + 10*e/3) + x*(10*d + e)
```

Maxima [A] (verification not implemented)

none

Time = 0.22 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.89

$$\int \frac{(d+ex^2)(1+2x^2+x^4)^5}{x^2}\,dx = \frac{1}{21}\,ex^{21} + \frac{1}{19}\,(d+10\,e)x^{19} + \frac{5}{17}\,(2\,d+9\,e)x^{17} + (3\,d+8\,e)x^{15} + \frac{30}{13}\,(4\,d+7\,e)x^{13} + \frac{42}{11}\,(5\,d+6\,e)x^{11} + \frac{14}{3}\,(6\,d+5\,e)x^9 + \frac{30}{7}\,(7\,d+4\,e)x^7 + 3\,(8\,d+3\,e)x^5 + \frac{5}{3}\,(9\,d+2\,e)x^3 + (10\,d+e)x - \frac{d}{x}$$

```
[In] integrate((e*x^2+d)*(x^4+2*x^2+1)^5/x^2,x, algorithm="maxima")
```

```
[Out] 1/21*e*x^21 + 1/19*(d + 10*e)*x^19 + 5/17*(2*d + 9*e)*x^17 + (3*d + 8*e)*x^
15 + 30/13*(4*d + 7*e)*x^13 + 42/11*(5*d + 6*e)*x^11 + 14/3*(6*d + 5*e)*x^9
 + 30/7*(7*d + 4*e)*x^7 + 3*(8*d + 3*e)*x^5 + 5/3*(9*d + 2*e)*x^3 + (10*d +
 e)*x - d/x
```

Giac [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.91

$$\int \frac{(d+ex^2)(1+2x^2+x^4)^5}{x^2}\,dx = \frac{1}{21}\,ex^{21} + \frac{1}{19}\,dx^{19} + \frac{10}{19}\,ex^{19} + \frac{10}{17}\,dx^{17} + \frac{45}{17}\,ex^{17} + 3\,dx^{15} + 8\,ex^{15} + \frac{120}{13}\,dx^{13} + \frac{210}{13}\,ex^{13} + \frac{210}{11}\,dx^{11} + \frac{252}{11}\,ex^{11} + 28\,dx^9 + \frac{70}{3}\,ex^9 + 30\,dx^7 + \frac{120}{7}\,ex^7 + 24\,dx^5 + 9\,ex^5 + 15\,dx^3 + \frac{10}{3}\,ex^3 + 10\,dx + ex - \frac{d}{x}$$

```
[In] integrate((e*x^2+d)*(x^4+2*x^2+1)^5/x^2,x, algorithm="giac")
```

```
[Out] 1/21*e*x^21 + 1/19*d*x^19 + 10/19*e*x^19 + 10/17*d*x^17 + 45/17*e*x^17 + 3*
d*x^15 + 8*e*x^15 + 120/13*d*x^13 + 210/13*e*x^13 + 210/11*d*x^11 + 252/11*
e*x^11 + 28*d*x^9 + 70/3*e*x^9 + 30*d*x^7 + 120/7*e*x^7 + 24*d*x^5 + 9*e*x^
5 + 15*d*x^3 + 10/3*e*x^3 + 10*d*x + e*x - d/x
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.84

$$\int \frac{(d+ex^2)(1+2x^2+x^4)^5}{x^2}\,dx = x^{15}\,(3\,d+8\,e)+x^3\,\left(15\,d+\frac{10\,e}{3}\right)+x^5\,(24\,d+9\,e)+x^{19}\,\left(\frac{d}{19}+\frac{10\,e}{19}\right)+x^{17}\,\left(\frac{10\,d}{17}+\frac{45\,e}{17}\right)+x^9\,\left(28\,d+\frac{70\,e}{3}\right)+x^7\,\left(30\,d+\frac{120\,e}{7}\right)+x^{13}\,\left(\frac{120\,d}{13}+\frac{210\,e}{13}\right)+x^{11}\,\left(\frac{210\,d}{11}+\frac{252\,e}{11}\right)+x\,(10\,d+e)-\frac{d}{x}+\frac{e\,x^{21}}{21}$$

```
[In] int(((d + e*x^2)*(2*x^2 + x^4 + 1)^5)/x^2,x)
```

```
[Out] x^15*(3*d + 8*e) + x^3*(15*d + (10*e)/3) + x^5*(24*d + 9*e) + x^19*(d/19 +
(10*e)/19) + x^17*((10*d)/17 + (45*e)/17) + x^9*(28*d + (70*e)/3) + x^7*(30
*d + (120*e)/7) + x^13*((120*d)/13 + (210*e)/13) + x^11*((210*d)/11 + (252*
e)/11) + x*(10*d + e) - d/x + (e*x^21)/21
```

3.64 $\int \frac{(d+ex^2)(1+2x^2+x^4)^5}{x^3} \, dx$

Optimal result

Integrand size = 23, antiderivative size = 147

$$\int \frac{(d+ex^2)(1+2x^2+x^4)^5}{x^3} \, dx = -\frac{d}{2x^2} + \frac{5}{2}(9d+2e)x^2 + \frac{15}{4}(8d+3e)x^4 + 5(7d+4e)x^6 + \frac{21}{4}(6d+5e)x^8 + \frac{21}{5}(5d+6e)x^{10} + \frac{5}{2}(4d+7e)x^{12} + \frac{15}{14}(3d+8e)x^{14} + \frac{5}{16}(2d+9e)x^{16} + \frac{1}{18}(d+10e)x^{18} + \frac{ex^{20}}{20} + (10d+e)\log(x)$$

```
[Out] -1/2*d/x^2+5/2*(9*d+2*e)*x^2+15/4*(8*d+3*e)*x^4+5*(7*d+4*e)*x^6+21/4*(6*d+5
*e)*x^8+21/5*(5*d+6*e)*x^10+5/2*(4*d+7*e)*x^12+15/14*(3*d+8*e)*x^14+5/16*(2
*d+9*e)*x^16+1/18*(d+10*e)*x^18+1/20*e*x^20+(10*d+e)*ln(x)
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {28, 457, 77}

$$\int \frac{(d+ex^2)(1+2x^2+x^4)^5}{x^3} \, dx = \frac{1}{18}x^{18}(d+10e) + \frac{5}{16}x^{16}(2d+9e) + \frac{15}{14}x^{14}(3d+8e) + \frac{5}{2}x^{12}(4d+7e) + \frac{21}{5}x^{10}(5d+6e) + \frac{21}{4}x^8(6d+5e) + 5x^6(7d+4e) + \frac{15}{4}x^4(8d+3e) + \frac{5}{2}x^2(9d+2e) + (10d+e)\log(x) - \frac{d}{2x^2} + \frac{ex^{20}}{20}$$

```
[In] Int[((d + e*x^2)*(1 + 2*x^2 + x^4)^5)/x^3,x]
```

```
[Out] -1/2*d/x^2 + (5*(9*d + 2*e)*x^2)/2 + (15*(8*d + 3*e)*x^4)/4 + 5*(7*d + 4*e)
*x^6 + (21*(6*d + 5*e)*x^8)/4 + (21*(5*d + 6*e)*x^10)/5 + (5*(4*d + 7*e)*x^
12)/2 + (15*(3*d + 8*e)*x^14)/14 + (5*(2*d + 9*e)*x^16)/16 + ((d + 10*e)*x^
18)/18 + (e*x^20)/20 + (10*d + e)*Log[x]
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 77

```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_Sym
bol] :> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[
{a, b, d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*
e + a*f, 0] && ( !IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] ||
(GeQ[n + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || E
qQ[p, 1])
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \frac{\left(1+x^2\right)^{10}\left(d+ex^2\right)}{x^3}\,dx\\
&= \frac{1}{2}\text{Subst}\left(\int \frac{(1+x)^{10}(d+ex)}{x^2}\,dx, x, x^2\right)\\
&= \frac{1}{2}\text{Subst}\left(\int \left(5(9d+2e)+\frac{d}{x^2}+\frac{10d+e}{x}+15(8d+3e)x+30(7d+4e)x^2 + 42(6d+5e)x^3+42(5d+6e)x^4+30(4d+7e)x^5+15(3d+8e)x^6 + 5(2d+9e)x^7+(d+10e)x^8+ex^9\right)dx, x, x^2\right)\\
&= -\frac{d}{2x^2}+\frac{5}{2}(9d+2e)x^2+\frac{15}{4}(8d+3e)x^4+5(7d+4e)x^6+\frac{21}{4}(6d+5e)x^8+\frac{21}{5}(5d+6e)x^{10} + \frac{5}{2}(4d+7e)x^{12}+\frac{15}{14}(3d+8e)x^{14}+\frac{5}{16}(2d+9e)x^{16}+\frac{1}{18}(d+10e)x^{18}+\frac{ex^{20}}{20}+(10d + e)\log(x)
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.00

$$\int \frac{(d+ex^2)(1+2x^2+x^4)^5}{x^3}\,dx = -\frac{d}{2x^2}+\frac{5}{2}(9d+2e)x^2+\frac{15}{4}(8d+3e)x^4 +5(7d+4e)x^6+\frac{21}{4}(6d+5e)x^8+\frac{21}{5}(5d+6e)x^{10} +\frac{5}{2}(4d+7e)x^{12}+\frac{15}{14}(3d+8e)x^{14}+\frac{5}{16}(2d+9e)x^{16} +\frac{1}{18}(d+10e)x^{18}+\frac{ex^{20}}{20}+(10d+e)\log(x)$$

```
[In] Integrate[((d + e*x^2)*(1 + 2*x^2 + x^4)^5)/x^3,x]
```

```
[Out] -1/2*d/x^2 + (5*(9*d + 2*e)*x^2)/2 + (15*(8*d + 3*e)*x^4)/4 + 5*(7*d + 4*e)
*x^6 + (21*(6*d + 5*e)*x^8)/4 + (21*(5*d + 6*e)*x^10)/5 + (5*(4*d + 7*e)*x^
12)/2 + (15*(3*d + 8*e)*x^14)/14 + (5*(2*d + 9*e)*x^16)/16 + ((d + 10*e)*x^
18)/18 + (e*x^20)/20 + (10*d + e)*Log[x]
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.84

method	result
norman	$\frac{\left(10d+\frac{35e}{2}\right)x^{14}+\left(21d+\frac{126e}{5}\right)x^{12}+\left(30d+\frac{45e}{4}\right)x^{6}+(35d+20e)x^{8}+\left(\frac{d}{18}+\frac{5e}{9}\right)x^{20}+\left(\frac{5d}{8}+\frac{45e}{16}\right)x^{18}+\left(\frac{45d}{2}+5e\right)x^{4}+\left(\frac{45d}{14}+\frac{60e}{7}\right)x^{1}}{x^2}$
default	$\frac{e\,x^{20}}{20}+\frac{d\,x^{18}}{18}+\frac{5e\,x^{18}}{9}+\frac{5d\,x^{16}}{8}+\frac{45e\,x^{16}}{16}+\frac{45d\,x^{14}}{14}+\frac{60e\,x^{14}}{7}+10d\,x^{12}+\frac{35e\,x^{12}}{2}+21d\,x^{10}+\frac{126e\,x^{10}}{5}$
risch	$\frac{e\,x^{20}}{20}+\frac{d\,x^{18}}{18}+\frac{5e\,x^{18}}{9}+\frac{5d\,x^{16}}{8}+\frac{45e\,x^{16}}{16}+\frac{45d\,x^{14}}{14}+\frac{60e\,x^{14}}{7}+10d\,x^{12}+\frac{35e\,x^{12}}{2}+21d\,x^{10}+\frac{126e\,x^{10}}{5}$
parallelrisch	$252e\,x^{22}+280d\,x^{20}+2800e\,x^{20}+3150d\,x^{18}+14175e\,x^{18}+16200d\,x^{16}+43200e\,x^{16}+50400d\,x^{14}+88200e\,x^{14}+105840d\,x^{12}+127$

```
[In] int((e*x^2+d)*(x^4+2*x^2+1)^5/x^3,x,method=_RETURNVERBOSE)
```

```
[Out] ((10*d+35/2*e)*x^14+(21*d+126/5*e)*x^12+(30*d+45/4*e)*x^6+(35*d+20*e)*x^8+(
1/18*d+5/9*e)*x^20+(5/8*d+45/16*e)*x^18+(45/2*d+5*e)*x^4+(45/14*d+60/7*e)*x
^16+(63/2*d+105/4*e)*x^10-1/2*d+1/20*e*x^22)/x^2+(10*d+e)*ln(x)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 133, normalized size of antiderivative = 0.90

$$\int \frac{(d+ex^2)(1+2x^2+x^4)^5}{x^3}\,dx$$
$$= \frac{252\,ex^{22}+280\,(d+10\,e)x^{20}+1575\,(2\,d+9\,e)x^{18}+5400\,(3\,d+8\,e)x^{16}+12600\,(4\,d+7\,e)x^{14}+21168\,(5\,d}{}$$

```
[In] integrate((e*x^2+d)*(x^4+2*x^2+1)^5/x^3,x, algorithm="fricas")
```

```
[Out] 1/5040*(252*e*x^22 + 280*(d + 10*e)*x^20 + 1575*(2*d + 9*e)*x^18 + 5400*(3*
d + 8*e)*x^16 + 12600*(4*d + 7*e)*x^14 + 21168*(5*d + 6*e)*x^12 + 26460*(6*
d + 5*e)*x^10 + 25200*(7*d + 4*e)*x^8 + 18900*(8*d + 3*e)*x^6 + 12600*(9*d
+ 2*e)*x^4 + 5040*(10*d + e)*x^2*log(x) - 2520*d)/x^2
```

Sympy [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.89

$$\int \frac{(d+ex^2)(1+2x^2+x^4)^5}{x^3}\,dx = -\frac{d}{2x^2}+\frac{ex^{20}}{20}+x^{18}\left(\frac{d}{18}+\frac{5e}{9}\right)+x^{16}\cdot\left(\frac{5d}{8}+\frac{45e}{16}\right)+x^{14}\cdot\left(\frac{45d}{14}+\frac{60e}{7}\right)+x^{12}\cdot\left(10d+\frac{35e}{2}\right)+x^{10}\cdot\left(21d+\frac{126e}{5}\right)+x^{8}\cdot\left(\frac{63d}{2}+\frac{105e}{4}\right)+x^{6}\cdot(35d+20e)+x^{4}\cdot\left(30d+\frac{45e}{4}\right)+x^{2}\cdot\left(\frac{45d}{2}+5e\right)+(10d+e)\log(x)$$

```
[In] integrate((e*x**2+d)*(x**4+2*x**2+1)**5/x**3,x)
```

```
[Out] -d/(2*x**2) + e*x**20/20 + x**18*(d/18 + 5*e/9) + x**16*(5*d/8 + 45*e/16) +
 x**14*(45*d/14 + 60*e/7) + x**12*(10*d + 35*e/2) + x**10*(21*d + 126*e/5)
+ x**8*(63*d/2 + 105*e/4) + x**6*(35*d + 20*e) + x**4*(30*d + 45*e/4) + x**
2*(45*d/2 + 5*e) + (10*d + e)*log(x)
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 130, normalized size of antiderivative = 0.88

$$\int \frac{(d+ex^2)\,(1+2x^2+x^4)^5}{x^3}\,dx = \frac{1}{20}\,ex^{20} + \frac{1}{18}\,(d+10\,e)x^{18} + \frac{5}{16}\,(2\,d+9\,e)x^{16} + \frac{15}{14}\,(3\,d+8\,e)x^{14} + \frac{5}{2}\,(4\,d+7\,e)x^{12} + \frac{21}{5}\,(5\,d+6\,e)x^{10} + \frac{21}{4}\,(6\,d+5\,e)x^{8} + 5\,(7\,d+4\,e)x^{6} + \frac{15}{4}\,(8\,d+3\,e)x^{4} + \frac{5}{2}\,(9\,d+2\,e)x^{2} + \frac{1}{2}\,(10\,d+e)\log\left(x^2\right) - \frac{d}{2\,x^2}$$

```
[In] integrate((e*x^2+d)*(x^4+2*x^2+1)^5/x^3,x, algorithm="maxima")
```

```
[Out] 1/20*e*x^20 + 1/18*(d + 10*e)*x^18 + 5/16*(2*d + 9*e)*x^16 + 15/14*(3*d + 8
*e)*x^14 + 5/2*(4*d + 7*e)*x^12 + 21/5*(5*d + 6*e)*x^10 + 21/4*(6*d + 5*e)*
x^8 + 5*(7*d + 4*e)*x^6 + 15/4*(8*d + 3*e)*x^4 + 5/2*(9*d + 2*e)*x^2 + 1/2*
(10*d + e)*log(x^2) - 1/2*d/x^2
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 144, normalized size of antiderivative = 0.98

$$\int \frac{(d+ex^2)\,(1+2x^2+x^4)^5}{x^3}\,dx = \frac{1}{20}\,ex^{20} + \frac{1}{18}\,dx^{18} + \frac{5}{9}\,ex^{18} + \frac{5}{8}\,dx^{16} + \frac{45}{16}\,ex^{16} + \frac{45}{14}\,dx^{14} + \frac{60}{7}\,ex^{14} + 10\,dx^{12} + \frac{35}{2}\,ex^{12} + 21\,dx^{10} + \frac{126}{5}\,ex^{10} + \frac{63}{2}\,dx^{8} + \frac{105}{4}\,ex^{8} + 35\,dx^{6} + 20\,ex^{6} + 30\,dx^{4} + \frac{45}{4}\,ex^{4} + \frac{45}{2}\,dx^{2} + 5\,ex^{2} + \frac{1}{2}\,(10\,d+e)\log\left(x^2\right) - \frac{10\,dx^2+ex^2+d}{2\,x^2}$$

```
[In] integrate((e*x^2+d)*(x^4+2*x^2+1)^5/x^3,x, algorithm="giac")
```

```
[Out] 1/20*e*x^20 + 1/18*d*x^18 + 5/9*e*x^18 + 5/8*d*x^16 + 45/16*e*x^16 + 45/14*
d*x^14 + 60/7*e*x^14 + 10*d*x^12 + 35/2*e*x^12 + 21*d*x^10 + 126/5*e*x^10 +
 63/2*d*x^8 + 105/4*e*x^8 + 35*d*x^6 + 20*e*x^6 + 30*d*x^4 + 45/4*e*x^4 + 4
5/2*d*x^2 + 5*e*x^2 + 1/2*(10*d + e)*log(x^2) - 1/2*(10*d*x^2 + e*x^2 + d)/
x^2
```

Mupad [B] (verification not implemented)

Time = 7.72 (sec) , antiderivative size = 120, normalized size of antiderivative = 0.82

$$\int \frac{(d+ex^2)\,(1+2x^2+x^4)^5}{x^3}\,dx = x^{18}\left(\frac{d}{18}+\frac{5\,e}{9}\right)+x^2\left(\frac{45\,d}{2}+5\,e\right)+x^{12}\left(10\,d+\frac{35\,e}{2}\right)+x^6\,(35\,d+20\,e)+x^4\left(30\,d+\frac{45\,e}{4}\right)+x^{16}\left(\frac{5\,d}{8}+\frac{45\,e}{16}\right)+x^{14}\left(\frac{45\,d}{14}+\frac{60\,e}{7}\right)+x^{10}\left(21\,d+\frac{126\,e}{5}\right)+x^8\left(\frac{63\,d}{2}+\frac{105\,e}{4}\right)-\frac{d}{2\,x^2}+\frac{e\,x^{20}}{20}+\ln(x)\,(10\,d+e)$$

```
[In] int(((d + e*x^2)*(2*x^2 + x^4 + 1)^5)/x^3,x)
```

```
[Out] x^18*(d/18 + (5*e)/9) + x^2*((45*d)/2 + 5*e) + x^12*(10*d + (35*e)/2) + x^6
*(35*d + 20*e) + x^4*(30*d + (45*e)/4) + x^16*((5*d)/8 + (45*e)/16) + x^14*
((45*d)/14 + (60*e)/7) + x^10*(21*d + (126*e)/5) + x^8*((63*d)/2 + (105*e)/
4) - d/(2*x^2) + (e*x^20)/20 + log(x)*(10*d + e)
```

3.65 $\int (fx)^m (1+x^2) (1+2x^2+x^4)^5 \, dx$

Optimal result

Integrand size = 23, antiderivative size = 203

$$\begin{aligned}\int (fx)^m \left(1+x^2\right) \left(1+2x^2+x^4\right)^5 \, dx = {} & \frac{(fx)^{1+m}}{f(1+m)}+\frac{11(fx)^{3+m}}{f^3(3+m)}+\frac{55(fx)^{5+m}}{f^5(5+m)} \\ & +\frac{165(fx)^{7+m}}{f^7(7+m)}+\frac{330(fx)^{9+m}}{f^9(9+m)}+\frac{462(fx)^{11+m}}{f^{11}(11+m)} \\ & +\frac{462(fx)^{13+m}}{f^{13}(13+m)}+\frac{330(fx)^{15+m}}{f^{15}(15+m)}+\frac{165(fx)^{17+m}}{f^{17}(17+m)} \\ & +\frac{55(fx)^{19+m}}{f^{19}(19+m)}+\frac{11(fx)^{21+m}}{f^{21}(21+m)}+\frac{(fx)^{23+m}}{f^{23}(23+m)}\end{aligned}$$

```
[Out] (f*x)^(1+m)/f/(1+m)+11*(f*x)^(3+m)/f^3/(3+m)+55*(f*x)^(5+m)/f^5/(5+m)+165*(
f*x)^(7+m)/f^7/(7+m)+330*(f*x)^(9+m)/f^9/(9+m)+462*(f*x)^(11+m)/f^11/(11+m)
+462*(f*x)^(13+m)/f^13/(13+m)+330*(f*x)^(15+m)/f^15/(15+m)+165*(f*x)^(17+m)
/f^17/(17+m)+55*(f*x)^(19+m)/f^19/(19+m)+11*(f*x)^(21+m)/f^21/(21+m)+(f*x)^
(23+m)/f^23/(23+m)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 203, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used

$= \{28, 276\}$

$$\int (fx)^m \left(1+x^2\right) \left(1+2x^2+x^4\right)^5 \, dx = \frac{(fx)^{m+23}}{f^{23}(m+23)} + \frac{11(fx)^{m+21}}{f^{21}(m+21)} + \frac{55(fx)^{m+19}}{f^{19}(m+19)} + \frac{165(fx)^{m+17}}{f^{17}(m+17)} + \frac{330(fx)^{m+15}}{f^{15}(m+15)} + \frac{462(fx)^{m+13}}{f^{13}(m+13)} + \frac{462(fx)^{m+11}}{f^{11}(m+11)} + \frac{330(fx)^{m+9}}{f^{9}(m+9)} + \frac{165(fx)^{m+7}}{f^{7}(m+7)} + \frac{55(fx)^{m+5}}{f^{5}(m+5)} + \frac{11(fx)^{m+3}}{f^{3}(m+3)} + \frac{(fx)^{m+1}}{f(m+1)}$$

```
[In] Int[(f*x)^m*(1 + x^2)*(1 + 2*x^2 + x^4)^5,x]
```

```
[Out] (f*x)^(1 + m)/(f*(1 + m)) + (11*(f*x)^(3 + m))/(f^3*(3 + m)) + (55*(f*x)^(5
 + m))/(f^5*(5 + m)) + (165*(f*x)^(7 + m))/(f^7*(7 + m)) + (330*(f*x)^(9 +
m))/(f^9*(9 + m)) + (462*(f*x)^(11 + m))/(f^11*(11 + m)) + (462*(f*x)^(13 +
 m))/(f^13*(13 + m)) + (330*(f*x)^(15 + m))/(f^15*(15 + m)) + (165*(f*x)^(1
7 + m))/(f^17*(17 + m)) + (55*(f*x)^(19 + m))/(f^19*(19 + m)) + (11*(f*x)^(
21 + m))/(f^21*(21 + m)) + (f*x)^(23 + m)/(f^23*(23 + m))
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\text{integral} = \int (fx)^m \left(1+x^2\right)^{11} \, dx$$

$$= \int \left((fx)^m + \frac{11(fx)^{2+m}}{f^2} + \frac{55(fx)^{4+m}}{f^4} + \frac{165(fx)^{6+m}}{f^6} + \frac{330(fx)^{8+m}}{f^8} + \frac{462(fx)^{10+m}}{f^{10}} + \frac{462(fx)^{12+m}}{f^{12}} + \frac{330(fx)^{14+m}}{f^{14}} + \frac{165(fx)^{16+m}}{f^{16}} + \frac{55(fx)^{18+m}}{f^{18}} + \frac{11(fx)^{20+m}}{f^{20}} + \frac{(fx)^{22+m}}{f^{22}}\right) dx$$

$$= \frac{(fx)^{1+m}}{f(1+m)} + \frac{11(fx)^{3+m}}{f^3(3+m)} + \frac{55(fx)^{5+m}}{f^5(5+m)} + \frac{165(fx)^{7+m}}{f^7(7+m)} + \frac{330(fx)^{9+m}}{f^9(9+m)} + \frac{462(fx)^{11+m}}{f^{11}(11+m)} + \frac{462(fx)^{13+m}}{f^{13}(13+m)} + \frac{330(fx)^{15+m}}{f^{15}(15+m)} + \frac{165(fx)^{17+m}}{f^{17}(17+m)} + \frac{55(fx)^{19+m}}{f^{19}(19+m)} + \frac{11(fx)^{21+m}}{f^{21}(21+m)} + \frac{(fx)^{23+m}}{f^{23}(23+m)}$$

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.60

$$\int (fx)^m \left(1+x^2\right) \left(1+2x^2+x^4\right)^5 \, dx = x(fx)^m \left(\frac{1}{1+m} + \frac{11x^2}{3+m} + \frac{55x^4}{5+m} + \frac{165x^6}{7+m} + \frac{330x^8}{9+m} + \frac{462x^{10}}{11+m} + \frac{462x^{12}}{13+m} + \frac{330x^{14}}{15+m} + \frac{165x^{16}}{17+m} + \frac{55x^{18}}{19+m} + \frac{11x^{20}}{21+m} + \frac{x^{22}}{23+m}\right)$$

```
[In] Integrate[(f*x)^m*(1 + x^2)*(1 + 2*x^2 + x^4)^5,x]
```

```
[Out] x*(f*x)^m*((1 + m)^(-1) + (11*x^2)/(3 + m) + (55*x^4)/(5 + m) + (165*x^6)/(
7 + m) + (330*x^8)/(9 + m) + (462*x^10)/(11 + m) + (462*x^12)/(13 + m) + (3
30*x^14)/(15 + m) + (165*x^16)/(17 + m) + (55*x^18)/(19 + m) + (11*x^20)/(2
1 + m) + x^22/(23 + m))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1120 vs. 2(203) = 406.

Time = 0.65 (sec) , antiderivative size = 1121, normalized size of antiderivative = 5.52

method	result	size
gosper	Expression too large to display	1121
risch	Expression too large to display	1121
parallelrisch	Expression too large to display	1849

```
[In] int((f*x)^m*(x^2+1)*(x^4+2*x^2+1)^5,x,method=_RETURNVERBOSE)
```

```
[Out] (f*x)^m*(m^11*x^22+121*m^10*x^22+11*m^11*x^20+6435*m^9*x^22+1353*m^10*x^20+
197835*m^8*x^22+55*m^11*x^18+72985*m^9*x^20+3889578*m^7*x^22+6875*m^10*x^18
+2271555*m^8*x^20+51069018*m^6*x^22+165*m^11*x^16+376365*m^9*x^18+45134958*
m^7*x^20+453714470*m^5*x^22+20955*m^10*x^16+11870265*m^8*x^18+597988314*m^6
*x^20+2702025590*m^4*x^22+330*m^11*x^14+1164735*m^9*x^16+238653030*m^7*x^18
+5353566130*m^5*x^20+10431670821*m^3*x^22+42570*m^10*x^14+37263105*m^8*x^16
```

```
+3194704590*m^6*x^18+32087153670*m^4*x^20+24372200061*m^2*x^22+462*m^11*x^1
2+2403390*m^9*x^14+759091410*m^7*x^16+28857216410*m^5*x^18+124530626231*m^3
*x^20+29985521895*m*x^22+60522*m^10*x^12+78076350*m^8*x^14+10282782510*m^6*
x^16+174273100210*m^4*x^18+292163767533*m^2*x^20+13749310575*x^22+462*m^11*
x^10+3471930*m^9*x^12+1613983140*m^7*x^14+93862508190*m^5*x^16+680615362515
*m^3*x^18+360568238085*m*x^20+61446*m^10*x^10+114642990*m^8*x^12+2216492586
0*m^6*x^14+572017996770*m^4*x^16+1604842704135*m^2*x^18+165646455975*x^20+3
30*m^11*x^8+3582810*m^9*x^10+2408820876*m^7*x^12+204865733820*m^5*x^14+2251
106854425*m^3*x^16+1988025402825*m*x^18+44550*m^10*x^8+120367170*m^8*x^10+3
3609870756*m^6*x^12+1262375264700*m^4*x^14+5340787250535*m^2*x^16+915414625
125*x^18+165*m^11*x^6+2640990*m^9*x^8+2575140876*m^7*x^10+315347150580*m^5*
x^12+5015196628530*m^3*x^14+6646727085075*m*x^16+22605*m^10*x^6+90358290*m^
8*x^8+36597992508*m^6*x^10+1969992823260*m^4*x^12+11991258123570*m^2*x^14+3
069331390125*x^16+55*m^11*x^4+1362735*m^9*x^6+1971903780*m^7*x^8+3496975528
20*m^5*x^10+7921249136262*m^3*x^12+15011348834790*m*x^14+7645*m^10*x^4+4752
4455*m^8*x^6+28627538940*m^6*x^8+2222832699780*m^4*x^10+19130651800722*m^2*
x^12+6957151150950*x^14+11*m^11*x^2+468765*m^9*x^4+1059893010*m^7*x^6+27969
1771260*m^5*x^8+9079996141062*m^3*x^10+24133835554290*m*x^12+1551*m^10*x^2+
16677375*m^8*x^4+15768085410*m^6*x^6+1818135330660*m^4*x^8+22226933020446*m
^2*x^10+11238474936150*x^12+m^11+96745*m^9*x^2+380801190*m^7*x^4+1582932129
90*m^5*x^6+7587607623090*m^3*x^8+28336045738770*m*x^10+143*m^10+3514005*m^8
*x^2+5825106210*m^6*x^4+1059628145070*m^4*x^6+18930738943710*m^2*x^8+132818
34015450*x^10+9075*m^9+82295598*m^7*x^2+60431072570*m^5*x^4+4558015784025*m
^3*x^6+24503570194950*m*x^8+336765*m^8+1298935638*m^6*x^2+420404849150*m^4*
x^4+11703493287585*m^2*x^6+11595251918250*x^8+8103018*m^7+14014513810*m^5*x
^2+1889780020755*m^3*x^4+15515657331075*m*x^6+132426294*m^6+102468500970*m^
4*x^2+5087634488145*m^2*x^4+7454090518875*x^6+1495875590*m^5+490955350391*m
^3*x^2+7041864340665*m*x^4+11641582810*m^4+1434440867211*m^2*x^2+3478575575
475*x^4+60936676581*m^3+2192684754645*m*x^2+203363952363*m^2+1159525191825*
x^2+387182170935*m+316234143225)*x/(1+m)/(3+m)/(5+m)/(7+m)/(9+m)/(11+m)/(13
+m)/(15+m)/(17+m)/(19+m)/(21+m)/(23+m)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 759 vs. $2(203) = 406$.

Time = 0.26 (sec) , antiderivative size = 759, normalized size of antiderivative = 3.74

$$\int (fx)^m \left(1+x^2\right) \left(1+2x^2+x^4\right)^5 dx = \text{Too large to display}$$

```
[In] integrate((f*x)^m*(x^2+1)*(x^4+2*x^2+1)^5,x, algorithm="fricas")
```

```
[Out] ((m^11 + 121*m^10 + 6435*m^9 + 197835*m^8 + 3889578*m^7 + 51069018*m^6 + 45
3714470*m^5 + 2702025590*m^4 + 10431670821*m^3 + 24372200061*m^2 + 29985521
895*m + 13749310575)*x^23 + 11*(m^11 + 123*m^10 + 6635*m^9 + 206505*m^8 + 4
```

```
103178*m^7 + 54362574*m^6 + 486687830*m^5 + 2917013970*m^4 + 11320966021*m^
3 + 26560342503*m^2 + 32778930735*m + 15058768725)*x^21 + 55*(m^11 + 125*m^
10 + 6843*m^9 + 215823*m^8 + 4339146*m^7 + 58085538*m^6 + 524676662*m^5 + 3
168601822*m^4 + 12374824773*m^3 + 29178958257*m^2 + 36145916415*m + 1664390
2275)*x^19 + 165*(m^11 + 127*m^10 + 7059*m^9 + 225837*m^8 + 4600554*m^7 + 6
2319894*m^6 + 568863686*m^5 + 3466775738*m^4 + 13643071845*m^3 + 3236840757
9*m^2 + 40283194455*m + 18602008425)*x^17 + 330*(m^11 + 129*m^10 + 7283*m^9
 + 236595*m^8 + 4890858*m^7 + 67166442*m^6 + 620805254*m^5 + 3825379590*m^4
 + 15197565541*m^3 + 36337145829*m^2 + 45488935863*m + 21082276215)*x^15 +
462*(m^11 + 131*m^10 + 7515*m^9 + 248145*m^8 + 5213898*m^7 + 72748638*m^6 +
 682569590*m^5 + 4264053730*m^4 + 17145560901*m^3 + 41408337231*m^2 + 52237
739295*m + 24325703325)*x^13 + 462*(m^11 + 133*m^10 + 7755*m^9 + 260535*m^8
 + 5573898*m^7 + 79216434*m^6 + 756921110*m^5 + 4811326190*m^4 + 1965367130
1*m^3 + 48110244633*m^2 + 61333432335*m + 28748558475)*x^11 + 330*(m^11 + 1
35*m^10 + 8003*m^9 + 273813*m^8 + 5975466*m^7 + 86750118*m^6 + 847550822*m^
5 + 5509501002*m^4 + 22992750373*m^3 + 57365875587*m^2 + 74253243015*m + 35
137127025)*x^9 + 165*(m^11 + 137*m^10 + 8259*m^9 + 288027*m^8 + 6423594*m^7
 + 95564154*m^6 + 959352806*m^5 + 6421988758*m^4 + 27624338085*m^3 + 709302
62349*m^2 + 94034286855*m + 45176306175)*x^7 + 55*(m^11 + 139*m^10 + 8523*m
^9 + 303225*m^8 + 6923658*m^7 + 105911022*m^6 + 1098746774*m^5 + 7643724530
*m^4 + 34359636741*m^3 + 92502445239*m^2 + 128033897103*m + 63246828645)*x^
5 + 11*(m^11 + 141*m^10 + 8795*m^9 + 319455*m^8 + 7481418*m^7 + 118085058*m
^6 + 1274046710*m^5 + 9315318270*m^4 + 44632304581*m^3 + 130403715201*m^2 +
 199334977695*m + 105411381075)*x^3 + (m^11 + 143*m^10 + 9075*m^9 + 336765*
m^8 + 8103018*m^7 + 132426294*m^6 + 1495875590*m^5 + 11641582810*m^4 + 6093
6676581*m^3 + 203363952363*m^2 + 387182170935*m + 316234143225)*x)*(f*x)^m/
(m^12 + 144*m^11 + 9218*m^10 + 345840*m^9 + 8439783*m^8 + 140529312*m^7 + 1
628301884*m^6 + 13137458400*m^5 + 72578259391*m^4 + 264300628944*m^3 + 5905
46123298*m^2 + 703416314160*m + 316234143225)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 11387 vs. $2(177) = 354$.

Time = 2.32 (sec) , antiderivative size = 11387, normalized size of antiderivative = 56.09

$$\int (fx)^m \left(1+x^2\right) \left(1+2x^2+x^4\right)^5 \, dx = \text{Too large to display}$$

```
[In] integrate((f*x)**m*(x**2+1)*(x**4+2*x**2+1)**5,x)
```

```
[Out] Piecewise(((log(x) - 11/(2*x**2) - 55/(4*x**4) - 55/(2*x**6) - 165/(4*x**8)
 - 231/(5*x**10) - 77/(2*x**12) - 165/(7*x**14) - 165/(16*x**16) - 55/(18*x
**18) - 11/(20*x**20) - 1/(22*x**22))/f**23, Eq(m, -23)), ((x**2/2 + 11*log
(x) - 55/(2*x**2) - 165/(4*x**4) - 55/x**6 - 231/(4*x**8) - 231/(5*x**10) -
 55/(2*x**12) - 165/(14*x**14) - 55/(16*x**16) - 11/(18*x**18) - 1/(20*x**2
```

0))/f**21, Eq(m, -21)), ((x**4/4 + 11*x**2/2 + 55*log(x) - 165/(2*x**2) - 165/(2*x**4) - 77/x**6 - 231/(4*x**8) - 33/x**10 - 55/(4*x**12) - 55/(14*x**14) - 11/(16*x**16) - 1/(18*x**18))/f**19, Eq(m, -19)), ((x**6/6 + 11*x**4/4 + 55*x**2/2 + 165*log(x) - 165/x**2 - 231/(2*x**4) - 77/x**6 - 165/(4*x**8) - 33/(2*x**10) - 55/(12*x**12) - 11/(14*x**14) - 1/(16*x**16))/f**17, Eq(m, -17)), ((x**8/8 + 11*x**6/6 + 55*x**4/4 + 165*x**2/2 + 330*log(x) - 231/x**2 - 231/(2*x**4) - 55/x**6 - 165/(8*x**8) - 11/(2*x**10) - 11/(12*x**12) - 1/(14*x**14))/f**15, Eq(m, -15)), ((x**10/10 + 11*x**8/8 + 55*x**6/6 + 165*x**4/4 + 165*x**2 + 462*log(x) - 231/x**2 - 165/(2*x**4) - 55/(2*x**6) - 55/(8*x**8) - 11/(10*x**10) - 1/(12*x**12))/f**13, Eq(m, -13)), ((x**12/12 + 11*x**10/10 + 55*x**8/8 + 55*x**6/2 + 165*x**4/2 + 231*x**2 + 462*log(x) - 165/x**2 - 165/(4*x**4) - 55/(6*x**6) - 11/(8*x**8) - 1/(10*x**10))/f**11, Eq(m, -11)), ((x**14/14 + 11*x**12/12 + 11*x**10/2 + 165*x**8/8 + 55*x**6 + 231*x**4/2 + 231*x**2 + 330*log(x) - 165/(2*x**2) - 55/(4*x**4) - 11/(6*x**6) - 1/(8*x**8))/f**9, Eq(m, -9)), ((x**16/16 + 11*x**14/14 + 55*x**12/12 + 33*x**10/2 + 165*x**8/4 + 77*x**6 + 231*x**4/2 + 165*x**2 + 165*log(x) - 55/(2*x**2) - 11/(4*x**4) - 1/(6*x**6))/f**7, Eq(m, -7)), ((x**18/18 + 11*x**16/16 + 55*x**14/14 + 55*x**12/4 + 33*x**10 + 231*x**8/4 + 77*x**6 + 165*x**4/2 + 165*x**2/2 + 55*log(x) - 11/(2*x**2) - 1/(4*x**4))/f**5, Eq(m, -5)), ((x**20/20 + 11*x**18/18 + 55*x**16/16 + 165*x**14/14 + 55*x**12/2 + 231*x**10/5 + 231*x**8/4 + 55*x**6 + 165*x**4/4 + 55*x**2/2 + 11*log(x) - 1/(2*x**2))/f**3, Eq(m, -3)), ((x**22/22 + 11*x**20/20 + 55*x**18/18 + 165*x**16/16 + 165*x**14/7 + 77*x**12/2 + 231*x**10/5 + 165*x**8/4 + 55*x**6/2 + 55*x**4/4 + 11*x**2/2 + log(x))/f, Eq(m, -1)), (m**11*x**23*(f*x)**m/(m**12 + 144*m**11 + 9218*m**10 + 345840*m**9 + 8439783*m**8 + 140529312*m**7 + 1628301884*m**6 + 13137458400*m**5 + 72578259391*m**4 + 264300628944*m**3 + 590546123298*m**2 + 703416314160*m + 316234143225) + 11*m**11*x**21*(f*x)**m/(m**12 + 144*m**11 + 9218*m**10 + 345840*m**9 + 8439783*m**8 + 140529312*m**7 + 1628301884*m**6 + 13137458400*m**5 + 72578259391*m**4 + 264300628944*m**3 + 590546123298*m**2 + 703416314160*m + 316234143225) + 55*m**11*x**19*(f*x)**m/(m**12 + 144*m**11 + 9218*m**10 + 345840*m**9 + 8439783*m**8 + 140529312*m**7 + 1628301884*m**6 + 13137458400*m**5 + 72578259391*m**4 + 264300628944*m**3 + 590546123298*m**2 + 703416314160*m + 316234143225) + 165*m**11*x**17*(f*x)**m/(m**12 + 144*m**11 + 9218*m**10 + 345840*m**9 + 8439783*m**8 + 140529312*m**7 + 1628301884*m**6 + 13137458400*m**5 + 72578259391*m**4 + 264300628944*m**3 + 590546123298*m**2 + 703416314160*m + 316234143225) + 330*m**11*x**15*(f*x)**m/(m**12 + 144*m**11 + 9218*m**10 + 345840*m**9 + 8439783*m**8 + 140529312*m**7 + 1628301884*m**6 + 13137458400*m**5 + 72578259391*m**4 + 264300628944*m**3 + 590546123298*m**2 + 703416314160*m + 316234143225) + 462*m**11*x**13*(f*x)**m/(m**12 + 144*m**11 + 9218*m**10 + 345840*m**9 + 8439783*m**8 + 140529312*m**7 + 1628301884*m**6 + 13137458400*m**5 + 72578259391*m**4 + 264300628944*m**3 + 590546123298*m**2 + 703416314160*m + 316234143225) + 462*m**11*x**11*(f*x)**m/(m**12 + 144*m**11 + 9218*m**10 + 345840*m**9 + 8439783*m**8 + 140529312*m**7 + 1628301884*m**6 + 13137458400*m**5 + 72578259391*m**4 + 264300628944*m**3 + 590546123298*m**2 + 7

03416314160*m + 316234143225) + 330*m**11*x**9*(f*x)**m/(m**12 + 144*m**11 + 9218*m**10 + 345840*m**9 + 8439783*m**8 + 140529312*m**7 + 1628301884*m**6 + 13137458400*m**5 + 72578259391*m**4 + 264300628944*m**3 + 590546123298*m**2 + 703416314160*m + 316234143225) + 165*m**11*x**7*(f*x)**m/(m**12 + 144*m**11 + 9218*m**10 + 345840*m**9 + 8439783*m**8 + 140529312*m**7 + 1628301884*m**6 + 13137458400*m**5 + 72578259391*m**4 + 264300628944*m**3 + 590546123298*m**2 + 703416314160*m + 316234143225) + 55*m**11*x**5*(f*x)**m/(m**12 + 144*m**11 + 9218*m**10 + 345840*m**9 + 8439783*m**8 + 140529312*m**7 + 1628301884*m**6 + 13137458400*m**5 + 72578259391*m**4 + 264300628944*m**3 + 590546123298*m**2 + 703416314160*m + 316234143225) + 11*m**11*x**3*(f*x)**m/(m**12 + 144*m**11 + 9218*m**10 + 345840*m**9 + 8439783*m**8 + 140529312*m**7 + 1628301884*m**6 + 13137458400*m**5 + 72578259391*m**4 + 264300628944*m**3 + 590546123298*m**2 + 703416314160*m + 316234143225) + m**11*x*(f*x)**m/(m**12 + 144*m**11 + 9218*m**10 + 345840*m**9 + 8439783*m**8 + 140529312*m**7 + 1628301884*m**6 + 13137458400*m**5 + 72578259391*m**4 + 264300628944*m**3 + 590546123298*m**2 + 703416314160*m + 316234143225) + 121*m**10*x**23*(f*x)**m/(m**12 + 144*m**11 + 9218*m**10 + 345840*m**9 + 8439783*m**8 + 140529312*m**7 + 1628301884*m**6 + 13137458400*m**5 + 72578259391*m**4 + 264300628944*m**3 + 590546123298*m**2 + 703416314160*m + 316234143225) + 1353*m**10*x**21*(f*x)**m/(m**12 + 144*m**11 + 9218*m**10 + 345840*m**9 + 8439783*m**8 + 140529312*m**7 + 1628301884*m**6 + 13137458400*m**5 + 72578259391*m**4 + 264300628944*m**3 + 590546123298*m**2 + 703416314160*m + 316234143225) + 6875*m**10*x**19*(f*x)**m/(m**12 + 144*m**11 + 9218*m**10 + 345840*m**9 + 8439783*m**8 + 140529312*m**7 + 1628301884*m**6 + 13137458400*m**5 + 72578259391*m**4 + 264300628944*m**3 + 590546123298*m**2 + 703416314160*m + 316234143225) + 20955*m**10*x**17*(f*x)**m/(m**12 + 144*m**11 + 9218*m**10 + 345840*m**9 + 8439783*m**8 + 140529312*m**7 + 1628301884*m**6 + 13137458400*m**5 + 72578259391*m**4 + 264300628944*m**3 + 590546123298*m**2 + 703416314160*m + 316234143225) + 42570*m**10*x**15*(f*x)**m/(m**12 + 144*m**11 + 9218*m**10 + 345840*m**9 + 8439783*m**8 + 140529312*m**7 + 1628301884*m**6 + 13137458400*m**5 + 72578259391*m**4 + 264300628944*m**3 + 590546123298*m**2 + 703416314160*m + 316234143225) + 60522*m**10*x**13*(f*x)**m/(m**12 + 144*m**11 + 9218*m**10 + 345840*m**9 + 8439783*m**8 + 140529312*m**7 + 1628301884*m**6 + 13137458400*m**5 + 72578259391*m**4 + 264300628944*m**3 + 590546123298*m**2 + 703416314160*m + 316234143225) + 61446*m**10*x**11*(f*x)**m/(m**12 + 144*m**11 + 9218*m**10 + 345840*m**9 + 8439783*m**8 + 140529312*m**7 + 1628301884*m**6 + 13137458400*m**5 + 72578259391*m**4 + 264300628944*m**3 + 590546123298*m**2 + 703416314160*m + 316234143225) + 44550*m**10*x**9*(f*x)**m/(m**12 + 144*m**11 + 9218*m**10 + 345840*m**9 + 8439783*m**8 + 140529312*m**7 + 1628301884*m**6 + 13137458400*m**5 + 72578259391*m**4 + 264300628944*m**3 + 590546123298*m**2 + 703416314160*m + 316234143225) + 22605*m**10*x**7*(f*x)**m/(m**12 + 144*m**11 + 9218*m**10 + 345840*m**9 + 8439783*m**8 + 140529312*m**7 + 1628301884*m**6 + 13137458400*m**5 + 72578259391*m**4 + 264300628944*m**3 + 590546123298*m**2 + 703416314160*m + 316234143225) + 7645*m**10*x**5*(f*x)**m/(m**12 + 144*m**11 + 9218*m**10 + 345840*m**

9 + 8439783*m**8 + 140529312*m**7 + 1628301884*m**6 + 13137458400*m**5 + 72
578259391*m**4 + 264300628944*m**3 + 590546123298*m**2 + 703416314160*m + 3
16234143225) + 1551*m**10*x**3*(f*x)**m/(m**12 + 144*m**11 + 9218*m**10 + 3
45840*m**9 + 8439783*m**8 + 140529312*m**7 + 1628301884*m**6 + 13137458400*
m**5 + 72578259391*m**4 + 264300628944*m**3 + 590546123298*m**2 + 703416314
160*m + 316234143225) + 143*m**10*x*(f*x)**m/(m**12 + 144*m**11 + 9218*m**1
0 + 345840*m**9 + 8439783*m**8 + 140529312*m**7 + 1628301884*m**6 + 1313745
8400*m**5 + 72578259391*m**4 + 264300628944*m**3 + 590546123298*m**2 + 7034
16314160*m + 316234143225) + 6435*m**9*x**23*(f*x)**m/(m**12 + 144*m**11 +
9218*m**10 + 345840*m**9 + 8439783*m**8 + 140529312*m**7 + 1628301884*m**6
+ 13137458400*m**5 + 72578259391*m**4 + 264300628944*m**3 + 590546123298*m*
*2 + 703416314160*m + 316234143225) + 72985*m**9*x**21*(f*x)**m/(m**12 + 14
4*m**11 + 9218*m**10 + 345840*m**9 + 8439783*m**8 + 140529312*m**7 + 162830
1884*m**6 + 13137458400*m**5 + 72578259391*m**4 + 264300628944*m**3 + 59054
6123298*m**2 + 703416314160*m + 316234143225) + 376365*m**9*x**19*(f*x)**m/
(m**12 + 144*m**11 + 9218*m**10 + 345840*m**9 + 8439783*m**8 + 140529312*m*
*7 + 1628301884*m**6 + 13137458400*m**5 + 72578259391*m**4 + 264300628944*m
3 + 590546123298*m2 + 703416314160*m + 316234143225) + 1164735*m**9*x**
17*(f*x)**m/(m**12 + 144*m**11 + 9218*m**10 + 345840*m**9 + 8439783*m**8 +
140529312*m**7 + 1628301884*m**6 + 13137458400*m**5 + 72578259391*m**4 + 26
4300628944*m**3 + 590546123298*m**2 + 703416314160*m + 316234143225) + 2403
390*m**9*x**15*(f*x)**m/(m**12 + 144*m**11 + 9218*m**10 + 345840*m**9 + 843
9783*m**8 + 140529312*m**7 + 1628301884*m**6 + 13137458400*m**5 + 725782593
91*m**4 + 264300628944*m**3 + 590546123298*m**2 + 703416314160*m + 31623414
3225) + 3471930*m**9*x**13*(f*x)**m/(m**12 + 144*m**11 + 9218*m**10 + 34584
0*m**9 + 8439783*m**8 + 140529312*m**7 + 1628301884*m**6 + 13137458400*m**5
 + 72578259391*m**4 + 264300628944*m**3 + 590546123298*m**2 + 703416314160*
m + 316234143225) + 3582810*m**9*x**11*(f*x)**m/(m**12 + 144*m**11 + 9218*m
10 + 345840*m9 + 8439783*m**8 + 140529312*m**7 + 1628301884*m**6 + 1313
7458400*m**5 + 72578259391*m**4 + 264300628944*m**3 + 590546123298*m**2 + 7
03416314160*m + 316234143225) + 2640990*m**9*x**9*(f*x)**m/(m**12 + 144*m**
11 + 9218*m**10 + 345840*m**9 + 8439783*m**8 + 140529312*m**7 + 1628301884*
m**6 + 13137458400*m**5 + 72578259391*m**4 + 264300628944*m**3 + 5905461232
98*m**2 + 703416314160*m + 316234143225) + 1362735*m**9*x**7*(f*x)**m/(m**1
2 + 144*m**11 + 9218*m**10 + 345840*m**9 + 8439783*m**8 + 140529312*m**7 +
1628301884*m**6 + 13137458400*m**5 + 72578259391*m**4 + 264300628944*m**3 +
 590546123298*m**2 + 703416314160*m + 316234143225) + 468765*m**9*x**5*(f*x
)**m/(m**12 + 144*m**11 + 9218*m**10 + 345840*m**9 + 8439783*m**8 + 1405293
12*m**7 + 1628301884*m**6 + 13137458400*m**5 + 72578259391*m**4 + 264300628
944*m**3 + 590546123298*m**2 + 703416314160*m + 316234143225) + 96745*m**9*
x**3*(f*x)**m/(m**12 + 144*m**11 + 9218*m**10 + 345840*m**9 + 8439783*m**8
+ 140529312*m**7 + 1628301884*m**6 + 13137458400*m**5 + 72578259391*m**4 +
264300628944*m**3 + 590546123298*m**2 + 703416314160*m + 316234143225) + 90
75*m**9*x*(f*x)**m/(m**12 + 144*m**11 + 9218*m**10 + 345840*m**9 + 8439783*
m**8 + 140529312*m**7 + 1628301884*m**6 + 13137458400*m**5 + 72578259391*m*

*4 + 264300628944*m**3 + 590546123298*m**2 + 703416314160*m + 316234143225)
 + 197835*m**8*x**23*(f*x)**m/(m**12 + 144*m**11 + 9218*m**10 + 345840*m**9
 + 8439783*m**8 + 140529312*m**7 + 1628301884*m**6 + 13137458400*m**5 + 725
78259391*m**4 + 264300628944*m**3 + 590546123298*m**2 + 703416314160*m + 31
6234143225) + 2271555*m**8*x**21*(f*x)**m/(m**12 + 144*m**11 + 9218*m**10 +
 345840*m**9 + 8439783*m**8 + 140529312*m**7 + 1628301884*m**6 + 1313745840
0*m**5 + 72578259391*m**4 + 264300628944*m**3 + 590546123298*m**2 + 7034163
14160*m + 316234143225) + 11870265*m**8*x**19*(f*x)**m/(m**12 + 144*m**11 +
 9218*m**10 + 345840*m**9 + 8439783*m**8 + 140529312*m**7 + 1628301884*m**6
 + 13137458400*m**5 + 72578259391*m**4 + 264300628944*m**3 + 590546123298*m
2 + 703416314160*m + 316234143225) + 37263105*m8*x**17*(f*x)**m/(m**12
+ 144*m**11 + 9218*m**10 + 345840*m**9 + 8439783*m**8 + 140529312*m**7 + 16
28301884*m**6 + 13137458400*m**5 + 72578259391*m**4 + 264300628944*m**3 + 5
90546123298*m**2 + 703416314160*m + 316234143225) + 78076350*m**8*x**15*(f*
x)**m/(m**12 + 144*m**11 + 9218*m**10 + 345840*m**9 + 8439783*m**8 + 140529
312*m**7 + 1628301884*m**6 + 13137458400*m**5 + 72578259391*m**4 + 26430062
8944*m**3 + 590546123298*m**2 + 703416314160*m + 316234143225) + 114642990*
m**8*x**13*(f*x)**m/(m**12 + 144*m**11 + 9218*m**10 + 345840*m**9 + 8439783
*m**8 + 140529312*m**7 + 1628301884*m**6 + 13137458400*m**5 + 72578259391*m
4 + 264300628944*m3 + 590546123298*m**2 + 703416314160*m + 316234143225
) + 120367170*m**8*x**11*(f*x)**m/(m**12 + 144*m**11 + 9218*m**10 + 345840*
m**9 + 8439783*m**8 + 140529312*m**7 + 1628301884*m**6 + 13137458400*m**5 +
 72578259391*m**4 + 264300628944*m**3 + 590546123298*m**2 + 703416314160*m
+ 316234143225) + 90358290*m**8*x**9*(f*x)**m/(m**12 + 144*m**11 + 9218*m**
10 + 345840*m**9 + 8439783*m**8 + 140529312*m**7 + 1628301884*m**6 + 131374
58400*m**5 + 72578259391*m**4 + 264300628944*m**3 + 590546123298*m**2 + 703
416314160*m + 316234143225) + 47524455*m**8*x**7*(f*x)**m/(m**12 + 144*m**1
1 + 9218*m**10 + 345840*m**9 + 8439783*m**8 + 140529312*m**7 + 1628301884*m
6 + 13137458400*m5 + 72578259391*m**4 + 264300628944*m**3 + 59054612329
8*m**2 + 703416314160*m + 316234143225) + 16677375*m**8*x**5*(f*x)**m/(m**1
2 + 144*m**11 + 9218*m**10 + 345840*m**9 + 8439783*m**8 + 140529312*m**7 +
1628301884*m**6 + 13137458400*m**5 + 72578259391*m**4 + 264300628944*m**3 +
 590546123298*m**2 + 703416314160*m + 316234143225) + 3514005*m**8*x**3*(f*
x)**m/(m**12 + 144*m**11 + 9218*m**10 + 345840*m**9 + 8439783*m**8 + 140529
312*m**7 + 1628301884*m**6 + 13137458400*m**5 + 72578259391*m**4 + 26430062
8944*m**3 + 590546123298*m**2 + 703416314160*m + 316234143225) + 336765*m**
8*x*(f*x)**m/(m**12 + 144*m**11 + 9218*m**10 + 345840*m**9 + 8439783*m**8 +
 140529312*m**7 + 1628301884*m**6 + 13137458400*m**5 + 72578259391*m**4 + 2
64300628944*m**3 + 590546123298*m**2 + 703416314160*m + 316234143225) + 388
9578*m**7*x**23*(f*x)**m/(m**12 + 144*m**11 + 9218*m**10 + 345840*m**9 + 84
39783*m**8 + 140529312*m**7 + 1628301884*m**6 + 13137458400*m**5 + 72578259
391*m**4 + 264300628944*m**3 + 590546123298*m**2 + 703416314160*m + 3162341
43225) + 45134958*m**7*x**21*(f*x)**m/(m**12 + 144*m**11 + 9218*m**10 + 345
840*m**9 + 8439783*m**8 + 140529312*m**7 + 1628301884*m**6 + 13137458400*m*
*5 + 72578259391*m**4 + 264300628944*m**3 + 590546123298*m**2 + 70341631416

0*m + 316234143225) + 238653030*m**7*x**19*(f*x)**m/(m**12 + 144*m**11 + 92
18*m**10 + 345840*m**9 + 8439783*m**8 + 140529312*m**7 + 1628301884*m**6 +
13137458400*m**5 + 72578259391*m**4 + 264300628944*m**3 + 590546123298*m**2
 + 703416314160*m + 316234143225) + 759091410*m**7*x**17*(f*x)**m/(m**12 +
144*m**11 + 9218*m**10 + 345840*m**9 + 8439783*m**8 + 140529312*m**7 + 1628
301884*m**6 + 13137458400*m**5 + 72578259391*m**4 + 264300628944*m**3 + 590
546123298*m**2 + 703416314160*m + 316234143225) + 1613983140*m**7*x**15*(f*
x)**m/(m**12 + 144*m**11 + 9218*m**10 + 345840*m**9 + 8439783*m**8 + 140529
312*m**7 + 1628301884*m**6 + 13137458400*m**5 + 72578259391*m**4 + 26430062
8944*m**3 + 590546123298*m**2 + 703416314160*m + 316234143225) + 2408820876
*m**7*x**13*(f*x)**m/(m**12 + 144*m**11 + 9218*m**10 + 345840*m**9 + 843978
3*m**8 + 140529312*m**7 + 1628301884*m**6 + 13137458400*m**5 + 72578259391*
m**4 + 264300628944*m**3 + 590546123298*m**2 + 703416314160*m + 31623414322
5) + 2575140876*m**7*x**11*(f*x)**m/(m**12 + 144*m**11 + 9218*m**10 + 34584
0*m**9 + 8439783*m**8 + 140529312*m**7 + 1628301884*m**6 + 13137458400*m**5
 + 72578259391*m**4 + 264300628944*m**3 + 590546123298*m**2 + 703416314160*
m + 316234143225) + 1971903780*m**7*x**9*(f*x)**m/(m**12 + 144*m**11 + 9218
*m**10 + 345840*m**9 + 8439783*m**8 + 140529312*m**7 + 1628301884*m**6 + 13
137458400*m**5 + 72578259391*m**4 + 264300628944*m**3 + 590546123298*m**2 +
 703416314160*m + 316234143225) + 1059893010*m**7*x**7*(f*x)**m/(m**12 + 14
4*m**11 + 9218*m**10 + 345840*m**9 + 8439783*m**8 + 140529312*m**7 + 162830
1884*m**6 + 13137458400*m**5 + 72578259391*m**4 + 264300628944*m**3 + 59054
6123298*m**2 + 703416314160*m + 316234143225) + 380801190*m**7*x**5*(f*x)**
m/(m**12 + 144*m**11 + 9218*m**10 + 345840*m**9 + 8439783*m**8 + 140529312*
m**7 + 1628301884*m**6 + 13137458400*m**5 + 72578259391*m**4 + 264300628944
*m**3 + 590546123298*m**2 + 703416314160*m + 316234143225) + 82295598*m**7*
x**3*(f*x)**m/(m**12 + 144*m**11 + 9218*m**10 + 345840*m**9 + 8439783*m**8
+ 140529312*m**7 + 1628301884*m**6 + 13137458400*m**5 + 72578259391*m**4 +
264300628944*m**3 + 590546123298*m**2 + 703416314160*m + 316234143225) + 81
03018*m**7*x*(f*x)**m/(m**12 + 144*m**11 + 9218*m**10 + 345840*m**9 + 84397
83*m**8 + 140529312*m**7 + 1628301884*m**6 + 13137458400*m**5 + 72578259391
*m**4 + 264300628944*m**3 + 590546123298*m**2 + 703416314160*m + 3162341432
25) + 51069018*m**6*x**23*(f*x)**m/(m**12 + 144*m**11 + 9218*m**10 + 345840
*m**9 + 8439783*m**8 + 140529312*m**7 + 1628301884*m**6 + 13137458400*m**5
+ 72578259391*m**4 + 264300628944*m**3 + 590546123298*m**2 + 703416314160*m
 + 316234143225) + 597988314*m**6*x**21*(f*x)**m/(m**12 + 144*m**11 + 9218*
m**10 + 345840*m**9 + 8439783*m**8 + 140529312*m**7 + 1628301884*m**6 + 131
37458400*m**5 + 72578259391*m**4 + 264300628944*m**3 + 590546123298*m**2 +
703416314160*m + 316234143225) + 3194704590*m**6*x**19*(f*x)**m/(m**12 + 14
4*m**11 + 9218*m**10 + 345840*m**9 + 8439783*m**8 + 140529312*m**7 + 162830
1884*m**6 + 13137458400*m**5 + 72578259391*m**4 + 264300628944*m**3 + 59054
6123298*m**2 + 703416314160*m + 316234143225) + 10282782510*m**6*x**17*(f*x
)**m/(m**12 + 144*m**11 + 9218*m**10 + 345840*m**9 + 8439783*m**8 + 1405293
12*m**7 + 1628301884*m**6 + 13137458400*m**5 + 72578259391*m**4 + 264300628
944*m**3 + 590546123298*m**2 + 703416314160*m + 316234143225) + 22164925860

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*m**6*x**15*(f*x)**m/(m**12 + 144*m**11 + 9218*m**10 + 345840*m**9 + 843978
3*m**8 + 140529312*m**7 + 1628301884*m**6 + 13137458400*m**5 + 72578259391*
m**4 + 264300628944*m**3 + 590546123298*m**2 + 703416314160*m + 31623414322
5) + 33609870756*m**6*x**13*(f*x)**m/(m**12 + 144*m**11 + 9218*m**10 + 3458
40*m**9 + 8439783*m**8 + 140529312*m**7 + 1628301884*m**6 + 13137458400*m**
5 + 72578259391*m**4 + 264300628944*m**3 + 590546123298*m**2 + 703416314160
*m + 316234143225) + 36597992508*m**6*x**11*(f*x)**m/(m**12 + 144*m**11 + 9
218*m**10 + 345840*m**9 + 8439783*m**8 + 140529312*m**7 + 1628301884*m**6 +
 13137458400*m**5 + 72578259391*m**4 + 264300628944*m**3 + 590546123298*m**
2 + 703416314160*m + 316234143225) + 28627538940*m**6*x**9*(f*x)**m/(m**12
+ 144*m**11 + 9218*m**10 + 345840*m**9 + 8439783*m**8 + 140529312*m**7 + 16
28301884*m**6 + 13137458400*m**5 + 72578259391*m**4 + 264300628944*m**3 + 5
90546123298*m**2 + 703416314160*m + 316234143225) + 15768085410*m**6*x**7*(
f*x)**m/(m**12 + 144*m**11 + 9218*m**10 + 345840*m**9 + 8439783*m**8 + 1405
29312*m**7 + 1628301884*m**6 + 13137458400*m**5 + 72578259391*m**4 + 264300
628944*m**3 + 590546123298*m**2 + 703416314160*m + 316234143225) + 58251062
10*m**6*x**5*(f*x)**m/(m**12 + 144*m**11 + 9218*m**10 + 345840*m**9 + 84397
83*m**8 + 140529312*m**7 + 1628301884*m**6 + 13137458400*m**5 + 72578259391
*m**4 + 264300628944*m**3 + 590546123298*m**2 + 703416314160*m + 3162341432
25) + 1298935638*m**6*x**3*(f*x)**m/(m**12 + 144*m**11 + 9218*m**10 + 34584
0*m**9 + 8439783*m**8 + 140529312*m**7 + 1628301884*m**6 + 13137458400*m**5
 + 72578259391*m**4 + 264300628944*m**3 + 590546123298*m**2 + 703416314160*
m + 316234143225) + 132426294*m**6*x*(f*x)**m/(m**12 + 144*m**11 + 9218*m**
10 + 345840*m**9 + 8439783*m**8 + 140529312*m**7 + 1628301884*m**6 + 131374
58400*m**5 + 72578259391*m**4 + 264300628944*m**3 + 590546123298*m**2 + 703
416314160*m + 316234143225) + 453714470*m**5*x**23*(f*x)**m/(m**12 + 144*m*
*11 + 9218*m**10 + 345840*m**9 + 8439783*m**8 + 140529312*m**7 + 1628301884
*m**6 + 13137458400*m**5 + 72578259391*m**4 + 264300628944*m**3 + 590546123
298*m**2 + 703416314160*m + 316234143225) + 5353566130*m**5*x**21*(f*x)**m/
(m**12 + 144*m**11 + 9218*m**10 + 345840*m**9 + 8439783*m**8 + 140529312*m*
*7 + 1628301884*m**6 + 13137458400*m**5 + 72578259391*m**4 + 264300628944*m
**3 + 590546123298*m**2 + 703416314160*m + 316234143225) + 28857216410*m**5
*x**19*(f*x)**m/(m**12 + 144*m**11 + 9218*m**10 + 345840*m**9 + 8439783*m**
8 + 140529312*m**7 + 1628301884*m**6 + 13137458400*m**5 + 72578259391*m**4
+ 264300628944*m**3 + 590546123298*m**2 + 703416314160*m + 316234143225) +
93862508190*m**5*x**17*(f*x)**m/(m**12 + 144*m**11 + 9218*m**10 + 345840*m*
*9 + 8439783*m**8 + 140529312*m**7 + 1628301884*m**6 + 13137458400*m**5 + 7
2578259391*m**4 + 264300628944*m**3 + 590546123298*m**2 + 703416314160*m +
316234143225) + 204865733820*m**5*x**15*(f*x)**m/(m**12 + 144*m**11 + 9218*
m**10 + 345840*m**9 + 8439783*m**8 + 140529312*m**7 + 1628301884*m**6 + 131
37458400*m**5 + 72578259391*m**4 + 264300628944*m**3 + 590546123298*m**2 +
703416314160*m + 316234143225) + 315347150580*m**5*x**13*(f*x)**m/(m**12 +
144*m**11 + 9218*m**10 + 345840*m**9 + 8439783*m**8 + 140529312*m**7 + 1628
301884*m**6 + 13137458400*m**5 + 72578259391*m**4 + 264300628944*m**3 + 590
546123298*m**2 + 703416314160*m + 316234143225) + 349697552820*m**5*x**11*(
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f*x)**m/(m**12 + 144*m**11 + 9218*m**10 + 345840*m**9 + 8439783*m**8 + 1405
29312*m**7 + 1628301884*m**6 + 13137458400*m**5 + 72578259391*m**4 + 264300
628944*m**3 + 590546123298*m**2 + 703416314160*m + 316234143225) + 27969177
1260*m**5*x**9*(f*x)**m/(m**12 + 144*m**11 + 9218*m**10 + 345840*m**9 + 843
9783*m**8 + 140529312*m**7 + 1628301884*m**6 + 13137458400*m**5 + 725782593
91*m**4 + 264300628944*m**3 + 590546123298*m**2 + 703416314160*m + 31623414
3225) + 158293212990*m**5*x**7*(f*x)**m/(m**12 + 144*m**11 + 9218*m**10 + 3
45840*m**9 + 8439783*m**8 + 140529312*m**7 + 1628301884*m**6 + 13137458400*
m**5 + 72578259391*m**4 + 264300628944*m**3 + 590546123298*m**2 + 703416314
160*m + 316234143225) + 60431072570*m**5*x**5*(f*x)**m/(m**12 + 144*m**11 +
 9218*m**10 + 345840*m**9 + 8439783*m**8 + 140529312*m**7 + 1628301884*m**6
 + 13137458400*m**5 + 72578259391*m**4 + 264300628944*m**3 + 590546123298*m
2 + 703416314160*m + 316234143225) + 14014513810*m5*x**3*(f*x)**m/(m**1
2 + 144*m**11 + 9218*m**10 + 345840*m**9 + 8439783*m**8 + 140529312*m**7 +
1628301884*m**6 + 13137458400*m**5 + 72578259391*m**4 + 264300628944*m**3 +
 590546123298*m**2 + 703416314160*m + 316234143225) + 1495875590*m**5*x*(f*
x)**m/(m**12 + 144*m**11 + 9218*m**10 + 345840*m**9 + 8439783*m**8 + 140529
312*m**7 + 1628301884*m**6 + 13137458400*m**5 + 72578259391*m**4 + 26430062
8944*m**3 + 590546123298*m**2 + 703416314160*m + 316234143225) + 2702025590
*m**4*x**23*(f*x)**m/(m**12 + 144*m**11 + 9218*m**10 + 345840*m**9 + 843978
3*m**8 + 140529312*m**7 + 1628301884*m**6 + 13137458400*m**5 + 72578259391*
m**4 + 264300628944*m**3 + 590546123298*m**2 + 703416314160*m + 31623414322
5) + 32087153670*m**4*x**21*(f*x)**m/(m**12 + 144*m**11 + 9218*m**10 + 3458
40*m**9 + 8439783*m**8 + 140529312*m**7 + 1628301884*m**6 + 13137458400*m**
5 + 72578259391*m**4 + 264300628944*m**3 + 590546123298*m**2 + 703416314160
*m + 316234143225) + 174273100210*m**4*x**19*(f*x)**m/(m**12 + 144*m**11 +
9218*m**10 + 345840*m**9 + 8439783*m**8 + 140529312*m**7 + 1628301884*m**6
+ 13137458400*m**5 + 72578259391*m**4 + 264300628944*m**3 + 590546123298*m*
*2 + 703416314160*m + 316234143225) + 572017996770*m**4*x**17*(f*x)**m/(m**
12 + 144*m**11 + 9218*m**10 + 345840*m**9 + 8439783*m**8 + 140529312*m**7 +
 1628301884*m**6 + 13137458400*m**5 + 72578259391*m**4 + 264300628944*m**3
+ 590546123298*m**2 + 703416314160*m + 316234143225) + 1262375264700*m**4*x
15*(f*x)m/(m**12 + 144*m**11 + 9218*m**10 + 345840*m**9 + 8439783*m**8
+ 140529312*m**7 + 1628301884*m**6 + 13137458400*m**5 + 72578259391*m**4 +
264300628944*m**3 + 590546123298*m**2 + 703416314160*m + 316234143225) + 19
69992823260*m**4*x**13*(f*x)**m/(m**12 + 144*m**11 + 9218*m**10 + 345840*m*
*9 + 8439783*m**8 + 140529312*m**7 + 1628301884*m**6 + 13137458400*m**5 + 7
2578259391*m**4 + 264300628944*m**3 + 590546123298*m**2 + 703416314160*m +
316234143225) + 2222832699780*m**4*x**11*(f*x)**m/(m**12 + 144*m**11 + 9218
*m**10 + 345840*m**9 + 8439783*m**8 + 140529312*m**7 + 1628301884*m**6 + 13
137458400*m**5 + 72578259391*m**4 + 264300628944*m**3 + 590546123298*m**2 +
 703416314160*m + 316234143225) + 1818135330660*m**4*x**9*(f*x)**m/(m**12 +
 144*m**11 + 9218*m**10 + 345840*m**9 + 8439783*m**8 + 140529312*m**7 + 162
8301884*m**6 + 13137458400*m**5 + 72578259391*m**4 + 264300628944*m**3 + 59
0546123298*m**2 + 703416314160*m + 316234143225) + 1059628145070*m**4*x**7*

(f*x)**m/(m**12 + 144*m**11 + 9218*m**10 + 345840*m**9 + 8439783*m**8 + 140
529312*m**7 + 1628301884*m**6 + 13137458400*m**5 + 72578259391*m**4 + 26430
0628944*m**3 + 590546123298*m**2 + 703416314160*m + 316234143225) + 4204048
49150*m**4*x**5*(f*x)**m/(m**12 + 144*m**11 + 9218*m**10 + 345840*m**9 + 84
39783*m**8 + 140529312*m**7 + 1628301884*m**6 + 13137458400*m**5 + 72578259
391*m**4 + 264300628944*m**3 + 590546123298*m**2 + 703416314160*m + 3162341
43225) + 102468500970*m**4*x**3*(f*x)**m/(m**12 + 144*m**11 + 9218*m**10 +
345840*m**9 + 8439783*m**8 + 140529312*m**7 + 1628301884*m**6 + 13137458400
*m**5 + 72578259391*m**4 + 264300628944*m**3 + 590546123298*m**2 + 70341631
4160*m + 316234143225) + 11641582810*m**4*x*(f*x)**m/(m**12 + 144*m**11 + 9
218*m**10 + 345840*m**9 + 8439783*m**8 + 140529312*m**7 + 1628301884*m**6 +
13137458400*m**5 + 72578259391*m**4 + 264300628944*m**3 + 590546123298*m**
2 + 703416314160*m + 316234143225) + 10431670821*m**3*x**23*(f*x)**m/(m**12
+ 144*m**11 + 9218*m**10 + 345840*m**9 + 8439783*m**8 + 140529312*m**7 + 1
628301884*m**6 + 13137458400*m**5 + 72578259391*m**4 + 264300628944*m**3 +
590546123298*m**2 + 703416314160*m + 316234143225) + 124530626231*m**3*x**2
1*(f*x)**m/(m**12 + 144*m**11 + 9218*m**10 + 345840*m**9 + 8439783*m**8 + 1
40529312*m**7 + 1628301884*m**6 + 13137458400*m**5 + 72578259391*m**4 + 264
300628944*m**3 + 590546123298*m**2 + 703416314160*m + 316234143225) + 68061
5362515*m**3*x**19*(f*x)**m/(m**12 + 144*m**11 + 9218*m**10 + 345840*m**9 +
8439783*m**8 + 140529312*m**7 + 1628301884*m**6 + 13137458400*m**5 + 72578
259391*m**4 + 264300628944*m**3 + 590546123298*m**2 + 703416314160*m + 3162
34143225) + 2251106854425*m**3*x**17*(f*x)**m/(m**12 + 144*m**11 + 9218*m**
10 + 345840*m**9 + 8439783*m**8 + 140529312*m**7 + 1628301884*m**6 + 131374
58400*m**5 + 72578259391*m**4 + 264300628944*m**3 + 590546123298*m**2 + 703
416314160*m + 316234143225) + 5015196628530*m**3*x**15*(f*x)**m/(m**12 + 14
4*m**11 + 9218*m**10 + 345840*m**9 + 8439783*m**8 + 140529312*m**7 + 162830
1884*m**6 + 13137458400*m**5 + 72578259391*m**4 + 264300628944*m**3 + 59054
6123298*m**2 + 703416314160*m + 316234143225) + 7921249136262*m**3*x**13*(f
*x)**m/(m**12 + 144*m**11 + 9218*m**10 + 345840*m**9 + 8439783*m**8 + 14052
9312*m**7 + 1628301884*m**6 + 13137458400*m**5 + 72578259391*m**4 + 2643006
28944*m**3 + 590546123298*m**2 + 703416314160*m + 316234143225) + 907999614
1062*m**3*x**11*(f*x)**m/(m**12 + 144*m**11 + 9218*m**10 + 345840*m**9 + 84
39783*m**8 + 140529312*m**7 + 1628301884*m**6 + 13137458400*m**5 + 72578259
391*m**4 + 264300628944*m**3 + 590546123298*m**2 + 703416314160*m + 3162341
43225) + 7587607623090*m**3*x**9*(f*x)**m/(m**12 + 144*m**11 + 9218*m**10 +
345840*m**9 + 8439783*m**8 + 140529312*m**7 + 1628301884*m**6 + 1313745840
0*m**5 + 72578259391*m**4 + 264300628944*m**3 + 590546123298*m**2 + 7034163
14160*m + 316234143225) + 4558015784025*m**3*x**7*(f*x)**m/(m**12 + 144*m**
11 + 9218*m**10 + 345840*m**9 + 8439783*m**8 + 140529312*m**7 + 1628301884*
m**6 + 13137458400*m**5 + 72578259391*m**4 + 264300628944*m**3 + 5905461232
98*m**2 + 703416314160*m + 316234143225) + 1889780020755*m**3*x**5*(f*x)**m
/(m**12 + 144*m**11 + 9218*m**10 + 345840*m**9 + 8439783*m**8 + 140529312*m
7 + 1628301884*m6 + 13137458400*m**5 + 72578259391*m**4 + 264300628944*
m**3 + 590546123298*m**2 + 703416314160*m + 316234143225) + 490955350391*m*

```
*3*x**3*(f*x)**m/(m**12 + 144*m**11 + 9218*m**10 + 345840*m**9 + 8439783*m*
*8 + 140529312*m**7 + 1628301884*m**6 + 13137458400*m**5 + 72578259391*m**4
 + 264300628944*m**3 + 590546123298*m**2 + 703416314160*m + 316234143225) +
 60936676581*m**3*x*(f*x)**m/(m**12 + 144*m**11 + 9218*m**10 + 345840*m**9
+ 8439783*m**8 + 140529312*m**7 + 1628301884*m**6 + 13137458400*m**5 + 7257
8259391*m**4 + 264300628944*m**3 + 590546123298*m**2 + 703416314160*m + 316
234143225) + 24372200061*m**2*x**23*(f*x)**m/(m**12 + 144*m**11 + 9218*m**1
0 + 345840*m**9 + 8439783*m**8 + 140529312*m**7 + 1628301884*m**6 + 1313745
8400*m**5 + 72578259391*m**4 + 264300628944*m**3 + 590546123298*m**2 + 7034
16314160*m + 316234143225) + 292163767533*m**2*x**21*(f*x)**m/(m**12 + 144*
m**11 + 9218*m**10 + 345840*m**9 + 8439783*m**8 + 140529312*m**7 + 16283018
84*m**6 + 13137458400*m**5 + 72578259391*m**4 + 264300628944*m**3 + 5905461
23298*m**2 + 703416314160*m + 316234143225) + 1604842704135*m**2*x**19*(f*x
)**m/(m**12 + 144*m**11 + 9218*m**10 + 345840*m**9 + 8439783*m**8 + 1405293
12*m**7 + 1628301884*m**6 + 13137458400*m**5 + 72578259391*m**4 + 264300628
944*m**3 + 590546123298*m**2 + 703416314160*m + 316234143225) + 53407872505
35*m**2*x**17*(f*x)**m/(m**12 + 144*m**11 + 9218*m**10 + 345840*m**9 + 8439
783*m**8 + 140529312*m**7 + 1628301884*m**6 + 13137458400*m**5 + 7257825939
1*m**4 + 264300628944*m**3 + 590546123298*m**2 + 703416314160*m + 316234143
225) + 11991258123570*m**2*x**15*(f*x)**m/(m**12 + 144*m**11 + 9218*m**10 +
 345840*m**9 + 8439783*m**8 + 140529312*m**7 + 1628301884*m**6 + 1313745840
0*m**5 + 72578259391*m**4 + 264300628944*m**3 + 590546123298*m**2 + 7034163
14160*m + 316234143225) + 19130651800722*m**2*x**13*(f*x)**m/(m**12 + 144*m
**11 + 9218*m**10 + 345840*m**9 + 8439783*m**8 + 140529312*m**7 + 162830188
4*m**6 + 13137458400*m**5 + 72578259391*m**4 + 264300628944*m**3 + 59054612
3298*m**2 + 703416314160*m + 316234143225) + 22226933020446*m**2*x**11*(f*x
)**m/(m**12 + 144*m**11 + 9218*m**10 + 345840*m**9 + 8439783*m**8 + 1405293
12*m**7 + 1628301884*m**6 + 13137458400*m**5 + 72578259391*m**4 + 264300628
944*m**3 + 590546123298*m**2 + 703416314160*m + 316234143225) + 18930738943
710*m**2*x**9*(f*x)**m/(m**12 + 144*m**11 + 9218*m**10 + 345840*m**9 + 8439
783*m**8 + 140529312*m**7 + 1628301884*m**6 + 13137458400*m**5 + 7257825939
1*m**4 + 264300628944*m**3 + 590546123298*m**2 + 703416314160*m + 316234143
225) + 11703493287585*m**2*x**7*(f*x)**m/(m**12 + 144*m**11 + 9218*m**10 +
345840*m**9 + 8439783*m**8 + 140529312*m**7 + 1628301884*m**6 + 13137458400
*m**5 + 72578259391*m**4 + 264300628944*m**3 + 590546123298*m**2 + 70341631
4160*m + 316234143225) + 5087634488145*m**2*x**5*(f*x)**m/(m**12 + 144*m**1
1 + 9218*m**10 + 345840*m**9 + 8439783*m**8 + 140529312*m**7 + 1628301884*m
**6 + 13137458400*m**5 + 72578259391*m**4 + 264300628944*m**3 + 59054612329
8*m**2 + 703416314160*m + 316234143225) + 1434440867211*m**2*x**3*(f*x)**m/
(m**12 + 144*m**11 + 9218*m**10 + 345840*m**9 + 8439783*m**8 + 140529312*m*
*7 + 1628301884*m**6 + 13137458400*m**5 + 72578259391*m**4 + 264300628944*m
**3 + 590546123298*m**2 + 703416314160*m + 316234143225) + 203363952363*m**
2*x*(f*x)**m/(m**12 + 144*m**11 + 9218*m**10 + 345840*m**9 + 8439783*m**8 +
 140529312*m**7 + 1628301884*m**6 + 13137458400*m**5 + 72578259391*m**4 + 2
64300628944*m**3 + 590546123298*m**2 + 703416314160*m + 316234143225) + 299
```

85521895*m*x**23*(f*x)**m/(m**12 + 144*m**11 + 9218*m**10 + 345840*m**9 + 8439783*m**8 + 140529312*m**7 + 1628301884*m**6 + 13137458400*m**5 + 72578259391*m**4 + 264300628944*m**3 + 590546123298*m**2 + 703416314160*m + 316234143225) + 360568238085*m*x**21*(f*x)**m/(m**12 + 144*m**11 + 9218*m**10 + 345840*m**9 + 8439783*m**8 + 140529312*m**7 + 1628301884*m**6 + 13137458400*m**5 + 72578259391*m**4 + 264300628944*m**3 + 590546123298*m**2 + 703416314160*m + 316234143225) + 1988025402825*m*x**19*(f*x)**m/(m**12 + 144*m**11 + 9218*m**10 + 345840*m**9 + 8439783*m**8 + 140529312*m**7 + 1628301884*m**6 + 13137458400*m**5 + 72578259391*m**4 + 264300628944*m**3 + 590546123298*m**2 + 703416314160*m + 316234143225) + 6646727085075*m*x**17*(f*x)**m/(m**12 + 144*m**11 + 9218*m**10 + 345840*m**9 + 8439783*m**8 + 140529312*m**7 + 1628301884*m**6 + 13137458400*m**5 + 72578259391*m**4 + 264300628944*m**3 + 590546123298*m**2 + 703416314160*m + 316234143225) + 15011348834790*m*x**15*(f*x)**m/(m**12 + 144*m**11 + 9218*m**10 + 345840*m**9 + 8439783*m**8 + 140529312*m**7 + 1628301884*m**6 + 13137458400*m**5 + 72578259391*m**4 + 264300628944*m**3 + 590546123298*m**2 + 703416314160*m + 316234143225) + 24133835554290*m*x**13*(f*x)**m/(m**12 + 144*m**11 + 9218*m**10 + 345840*m**9 + 8439783*m**8 + 140529312*m**7 + 1628301884*m**6 + 13137458400*m**5 + 72578259391*m**4 + 264300628944*m**3 + 590546123298*m**2 + 703416314160*m + 316234143225) + 28336045738770*m*x**11*(f*x)**m/(m**12 + 144*m**11 + 9218*m**10 + 345840*m**9 + 8439783*m**8 + 140529312*m**7 + 1628301884*m**6 + 13137458400*m**5 + 72578259391*m**4 + 264300628944*m**3 + 590546123298*m**2 + 703416314160*m + 316234143225) + 24503570194950*m*x**9*(f*x)**m/(m**12 + 144*m**11 + 9218*m**10 + 345840*m**9 + 8439783*m**8 + 140529312*m**7 + 1628301884*m**6 + 13137458400*m**5 + 72578259391*m**4 + 264300628944*m**3 + 590546123298*m**2 + 703416314160*m + 316234143225) + 15515657331075*m*x**7*(f*x)**m/(m**12 + 144*m**11 + 9218*m**10 + 345840*m**9 + 8439783*m**8 + 140529312*m**7 + 1628301884*m**6 + 13137458400*m**5 + 72578259391*m**4 + 264300628944*m**3 + 590546123298*m**2 + 703416314160*m + 316234143225) + 7041864340665*m*x**5*(f*x)**m/(m**12 + 144*m**11 + 9218*m**10 + 345840*m**9 + 8439783*m**8 + 140529312*m**7 + 1628301884*m**6 + 13137458400*m**5 + 72578259391*m**4 + 264300628944*m**3 + 590546123298*m**2 + 703416314160*m + 316234143225) + 2192684754645*m*x**3*(f*x)**m/(m**12 + 144*m**11 + 9218*m**10 + 345840*m**9 + 8439783*m**8 + 140529312*m**7 + 1628301884*m**6 + 13137458400*m**5 + 72578259391*m**4 + 264300628944*m**3 + 590546123298*m**2 + 703416314160*m + 316234143225) + 387182170935*m*x*(f*x)**m/(m**12 + 144*m**11 + 9218*m**10 + 345840*m**9 + 8439783*m**8 + 140529312*m**7 + 1628301884*m**6 + 13137458400*m**5 + 72578259391*m**4 + 264300628944*m**3 + 590546123298*m**2 + 703416314160*m + 316234143225) + 13749310575*x**23*(f*x)**m/(m**12 + 144*m**11 + 9218*m**10 + 345840*m**9 + 8439783*m**8 + 140529312*m**7 + 1628301884*m**6 + 13137458400*m**5 + 72578259391*m**4 + 264300628944*m**3 + 590546123298*m**2 + 703416314160*m + 316234143225) + 165646455975*x**21*(f*x)**m/(m**12 + 144*m**11 + 9218*m**10 + 345840*m**9 + 8439783*m**8 + 140529312*m**7 + 1628301884*m**6 + 13137458400*m**5 + 72578259391*m**4 + 264300628944*m**3 + 590546123298*m**2 + 703416314160*m + 316234143225) + 915414625125*x**19*(f*x)**m/(m**

```
12 + 144*m**11 + 9218*m**10 + 345840*m**9 + 8439783*m**8 + 140529312*m**7 +
 1628301884*m**6 + 13137458400*m**5 + 72578259391*m**4 + 264300628944*m**3
+ 590546123298*m**2 + 703416314160*m + 316234143225) + 3069331390125*x**17*
(f*x)**m/(m**12 + 144*m**11 + 9218*m**10 + 345840*m**9 + 8439783*m**8 + 140
529312*m**7 + 1628301884*m**6 + 13137458400*m**5 + 72578259391*m**4 + 26430
0628944*m**3 + 590546123298*m**2 + 703416314160*m + 316234143225) + 6957151
150950*x**15*(f*x)**m/(m**12 + 144*m**11 + 9218*m**10 + 345840*m**9 + 84397
83*m**8 + 140529312*m**7 + 1628301884*m**6 + 13137458400*m**5 + 72578259391
*m**4 + 264300628944*m**3 + 590546123298*m**2 + 703416314160*m + 3162341432
25) + 11238474936150*x**13*(f*x)**m/(m**12 + 144*m**11 + 9218*m**10 + 34584
0*m**9 + 8439783*m**8 + 140529312*m**7 + 1628301884*m**6 + 13137458400*m**5
 + 72578259391*m**4 + 264300628944*m**3 + 590546123298*m**2 + 703416314160*
m + 316234143225) + 13281834015450*x**11*(f*x)**m/(m**12 + 144*m**11 + 9218
*m**10 + 345840*m**9 + 8439783*m**8 + 140529312*m**7 + 1628301884*m**6 + 13
137458400*m**5 + 72578259391*m**4 + 264300628944*m**3 + 590546123298*m**2 +
 703416314160*m + 316234143225) + 11595251918250*x**9*(f*x)**m/(m**12 + 144
*m**11 + 9218*m**10 + 345840*m**9 + 8439783*m**8 + 140529312*m**7 + 1628301
884*m**6 + 13137458400*m**5 + 72578259391*m**4 + 264300628944*m**3 + 590546
123298*m**2 + 703416314160*m + 316234143225) + 7454090518875*x**7*(f*x)**m/
(m**12 + 144*m**11 + 9218*m**10 + 345840*m**9 + 8439783*m**8 + 140529312*m*
*7 + 1628301884*m**6 + 13137458400*m**5 + 72578259391*m**4 + 264300628944*m
**3 + 590546123298*m**2 + 703416314160*m + 316234143225) + 3478575575475*x*
*5*(f*x)**m/(m**12 + 144*m**11 + 9218*m**10 + 345840*m**9 + 8439783*m**8 +
140529312*m**7 + 1628301884*m**6 + 13137458400*m**5 + 72578259391*m**4 + 26
4300628944*m**3 + 590546123298*m**2 + 703416314160*m + 316234143225) + 1159
525191825*x**3*(f*x)**m/(m**12 + 144*m**11 + 9218*m**10 + 345840*m**9 + 843
9783*m**8 + 140529312*m**7 + 1628301884*m**6 + 13137458400*m**5 + 725782593
91*m**4 + 264300628944*m**3 + 590546123298*m**2 + 703416314160*m + 31623414
3225) + 316234143225*x*(f*x)**m/(m**12 + 144*m**11 + 9218*m**10 + 345840*m*
*9 + 8439783*m**8 + 140529312*m**7 + 1628301884*m**6 + 13137458400*m**5 + 7
2578259391*m**4 + 264300628944*m**3 + 590546123298*m**2 + 703416314160*m +
316234143225), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 192, normalized size of antiderivative = 0.95

$$\int (fx)^m \left(1+x^2\right) \left(1+2x^2+x^4\right)^5 \, dx = \frac{f^m x^{23} x^m}{m+23} + \frac{11 f^m x^{21} x^m}{m+21} + \frac{55 f^m x^{19} x^m}{m+19} + \frac{165 f^m x^{17} x^m}{m+17} + \frac{330 f^m x^{15} x^m}{m+15} + \frac{462 f^m x^{13} x^m}{m+13} + \frac{462 f^m x^{11} x^m}{m+11} + \frac{330 f^m x^9 x^m}{m+9} + \frac{165 f^m x^7 x^m}{m+7} + \frac{55 f^m x^5 x^m}{m+5} + \frac{11 f^m x^3 x^m}{m+3} + \frac{(fx)^{m+1}}{f(m+1)}$$

```
[In] integrate((f*x)^m*(x^2+1)*(x^4+2*x^2+1)^5,x, algorithm="maxima")
```

```
[Out] f^m*x^23*x^m/(m + 23) + 11*f^m*x^21*x^m/(m + 21) + 55*f^m*x^19*x^m/(m + 19)
 + 165*f^m*x^17*x^m/(m + 17) + 330*f^m*x^15*x^m/(m + 15) + 462*f^m*x^13*x^m
/(m + 13) + 462*f^m*x^11*x^m/(m + 11) + 330*f^m*x^9*x^m/(m + 9) + 165*f^m*x
^7*x^m/(m + 7) + 55*f^m*x^5*x^m/(m + 5) + 11*f^m*x^3*x^m/(m + 3) + (f*x)^(m
 + 1)/(f*(m + 1))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1848 vs. 2(203) = 406.

Time = 0.30 (sec) , antiderivative size = 1848, normalized size of antiderivative = 9.10

$$\int (fx)^m \left(1+x^2\right) \left(1+2x^2+x^4\right)^5 \, dx = \text{Too large to display}$$

```
[In] integrate((f*x)^m*(x^2+1)*(x^4+2*x^2+1)^5,x, algorithm="giac")
```

```
[Out] ((f*x)^m*m^11*x^23 + 121*(f*x)^m*m^10*x^23 + 11*(f*x)^m*m^11*x^21 + 6435*(f
*x)^m*m^9*x^23 + 1353*(f*x)^m*m^10*x^21 + 197835*(f*x)^m*m^8*x^23 + 55*(f*x
)^m*m^11*x^19 + 72985*(f*x)^m*m^9*x^21 + 3889578*(f*x)^m*m^7*x^23 + 6875*(f
*x)^m*m^10*x^19 + 2271555*(f*x)^m*m^8*x^21 + 51069018*(f*x)^m*m^6*x^23 + 16
5*(f*x)^m*m^11*x^17 + 376365*(f*x)^m*m^9*x^19 + 45134958*(f*x)^m*m^7*x^21 +
 453714470*(f*x)^m*m^5*x^23 + 20955*(f*x)^m*m^10*x^17 + 11870265*(f*x)^m*m^
8*x^19 + 597988314*(f*x)^m*m^6*x^21 + 2702025590*(f*x)^m*m^4*x^23 + 330*(f*
x)^m*m^11*x^15 + 1164735*(f*x)^m*m^9*x^17 + 238653030*(f*x)^m*m^7*x^19 + 53
53566130*(f*x)^m*m^5*x^21 + 10431670821*(f*x)^m*m^3*x^23 + 42570*(f*x)^m*m^
10*x^15 + 37263105*(f*x)^m*m^8*x^17 + 3194704590*(f*x)^m*m^6*x^19 + 3208715
3670*(f*x)^m*m^4*x^21 + 24372200061*(f*x)^m*m^2*x^23 + 462*(f*x)^m*m^11*x^1
3 + 2403390*(f*x)^m*m^9*x^15 + 759091410*(f*x)^m*m^7*x^17 + 28857216410*(f*
x)^m*m^5*x^19 + 124530626231*(f*x)^m*m^3*x^21 + 29985521895*(f*x)^m*m*x^23
+ 60522*(f*x)^m*m^10*x^13 + 78076350*(f*x)^m*m^8*x^15 + 10282782510*(f*x)^m
*m^6*x^17 + 174273100210*(f*x)^m*m^4*x^19 + 292163767533*(f*x)^m*m^2*x^21 +
```

```
13749310575*(f*x)^m*x^23 + 462*(f*x)^m*m^11*x^11 + 3471930*(f*x)^m*m^9*x^1
3 + 1613983140*(f*x)^m*m^7*x^15 + 93862508190*(f*x)^m*m^5*x^17 + 6806153625
15*(f*x)^m*m^3*x^19 + 360568238085*(f*x)^m*m*x^21 + 61446*(f*x)^m*m^10*x^11
 + 114642990*(f*x)^m*m^8*x^13 + 22164925860*(f*x)^m*m^6*x^15 + 572017996770
*(f*x)^m*m^4*x^17 + 1604842704135*(f*x)^m*m^2*x^19 + 165646455975*(f*x)^m*x
^21 + 330*(f*x)^m*m^11*x^9 + 3582810*(f*x)^m*m^9*x^11 + 2408820876*(f*x)^m*
m^7*x^13 + 204865733820*(f*x)^m*m^5*x^15 + 2251106854425*(f*x)^m*m^3*x^17 +
 1988025402825*(f*x)^m*m*x^19 + 44550*(f*x)^m*m^10*x^9 + 120367170*(f*x)^m*
m^8*x^11 + 33609870756*(f*x)^m*m^6*x^13 + 1262375264700*(f*x)^m*m^4*x^15 +
5340787250535*(f*x)^m*m^2*x^17 + 915414625125*(f*x)^m*x^19 + 165*(f*x)^m*m^
11*x^7 + 2640990*(f*x)^m*m^9*x^9 + 2575140876*(f*x)^m*m^7*x^11 + 3153471505
80*(f*x)^m*m^5*x^13 + 5015196628530*(f*x)^m*m^3*x^15 + 6646727085075*(f*x)^
m*m*x^17 + 22605*(f*x)^m*m^10*x^7 + 90358290*(f*x)^m*m^8*x^9 + 36597992508*
(f*x)^m*m^6*x^11 + 1969992823260*(f*x)^m*m^4*x^13 + 11991258123570*(f*x)^m*
m^2*x^15 + 3069331390125*(f*x)^m*x^17 + 55*(f*x)^m*m^11*x^5 + 1362735*(f*x)
^m*m^9*x^7 + 1971903780*(f*x)^m*m^7*x^9 + 349697552820*(f*x)^m*m^5*x^11 + 7
921249136262*(f*x)^m*m^3*x^13 + 15011348834790*(f*x)^m*m*x^15 + 7645*(f*x)^
m*m^10*x^5 + 47524455*(f*x)^m*m^8*x^7 + 28627538940*(f*x)^m*m^6*x^9 + 22228
32699780*(f*x)^m*m^4*x^11 + 19130651800722*(f*x)^m*m^2*x^13 + 6957151150950
*(f*x)^m*x^15 + 11*(f*x)^m*m^11*x^3 + 468765*(f*x)^m*m^9*x^5 + 1059893010*(
f*x)^m*m^7*x^7 + 279691771260*(f*x)^m*m^5*x^9 + 9079996141062*(f*x)^m*m^3*x
^11 + 24133835554290*(f*x)^m*m*x^13 + 1551*(f*x)^m*m^10*x^3 + 16677375*(f*x
)^m*m^8*x^5 + 15768085410*(f*x)^m*m^6*x^7 + 1818135330660*(f*x)^m*m^4*x^9 +
 22226933020446*(f*x)^m*m^2*x^11 + 11238474936150*(f*x)^m*x^13 + (f*x)^m*m^
11*x + 96745*(f*x)^m*m^9*x^3 + 380801190*(f*x)^m*m^7*x^5 + 158293212990*(f*
x)^m*m^5*x^7 + 7587607623090*(f*x)^m*m^3*x^9 + 28336045738770*(f*x)^m*m*x^1
1 + 143*(f*x)^m*m^10*x + 3514005*(f*x)^m*m^8*x^3 + 5825106210*(f*x)^m*m^6*x
^5 + 1059628145070*(f*x)^m*m^4*x^7 + 18930738943710*(f*x)^m*m^2*x^9 + 13281
834015450*(f*x)^m*x^11 + 9075*(f*x)^m*m^9*x + 82295598*(f*x)^m*m^7*x^3 + 60
431072570*(f*x)^m*m^5*x^5 + 4558015784025*(f*x)^m*m^3*x^7 + 24503570194950*
(f*x)^m*m*x^9 + 336765*(f*x)^m*m^8*x + 1298935638*(f*x)^m*m^6*x^3 + 4204048
49150*(f*x)^m*m^4*x^5 + 11703493287585*(f*x)^m*m^2*x^7 + 11595251918250*(f*
x)^m*x^9 + 8103018*(f*x)^m*m^7*x + 14014513810*(f*x)^m*m^5*x^3 + 1889780020
755*(f*x)^m*m^3*x^5 + 15515657331075*(f*x)^m*m*x^7 + 132426294*(f*x)^m*m^6*
x + 102468500970*(f*x)^m*m^4*x^3 + 5087634488145*(f*x)^m*m^2*x^5 + 74540905
18875*(f*x)^m*x^7 + 1495875590*(f*x)^m*m^5*x + 490955350391*(f*x)^m*m^3*x^3
 + 7041864340665*(f*x)^m*m*x^5 + 11641582810*(f*x)^m*m^4*x + 1434440867211*
(f*x)^m*m^2*x^3 + 3478575575475*(f*x)^m*x^5 + 60936676581*(f*x)^m*m^3*x + 2
192684754645*(f*x)^m*m*x^3 + 203363952363*(f*x)^m*m^2*x + 1159525191825*(f*
x)^m*x^3 + 387182170935*(f*x)^m*m*x + 316234143225*(f*x)^m*x)/(m^12 + 144*m
^11 + 9218*m^10 + 345840*m^9 + 8439783*m^8 + 140529312*m^7 + 1628301884*m^6
 + 13137458400*m^5 + 72578259391*m^4 + 264300628944*m^3 + 590546123298*m^2
+ 703416314160*m + 316234143225)
```

Mupad [B] (verification not implemented)

Time = 8.34 (sec) , antiderivative size = 1483, normalized size of antiderivative = 7.31

$$\int (fx)^m \left(1+x^2\right) \left(1+2x^2+x^4\right)^5 \, dx = \text{Too large to display}$$

```
[In] int((x^2 + 1)*(f*x)^m*(2*x^2 + x^4 + 1)^5,x)
```

```
[Out] (x^3*(f*x)^m*(2192684754645*m + 1434440867211*m^2 + 490955350391*m^3 + 1024
68500970*m^4 + 14014513810*m^5 + 1298935638*m^6 + 82295598*m^7 + 3514005*m^
8 + 96745*m^9 + 1551*m^10 + 11*m^11 + 1159525191825))/(703416314160*m + 590
546123298*m^2 + 264300628944*m^3 + 72578259391*m^4 + 13137458400*m^5 + 1628
301884*m^6 + 140529312*m^7 + 8439783*m^8 + 345840*m^9 + 9218*m^10 + 144*m^1
1 + m^12 + 316234143225) + (x^19*(f*x)^m*(1988025402825*m + 1604842704135*m
^2 + 680615362515*m^3 + 174273100210*m^4 + 28857216410*m^5 + 3194704590*m^6
 + 238653030*m^7 + 11870265*m^8 + 376365*m^9 + 6875*m^10 + 55*m^11 + 915414
625125))/(703416314160*m + 590546123298*m^2 + 264300628944*m^3 + 7257825939
1*m^4 + 13137458400*m^5 + 1628301884*m^6 + 140529312*m^7 + 8439783*m^8 + 34
5840*m^9 + 9218*m^10 + 144*m^11 + m^12 + 316234143225) + (x^11*(f*x)^m*(283
36045738770*m + 22226933020446*m^2 + 9079996141062*m^3 + 2222832699780*m^4
+ 349697552820*m^5 + 36597992508*m^6 + 2575140876*m^7 + 120367170*m^8 + 358
2810*m^9 + 61446*m^10 + 462*m^11 + 13281834015450))/(703416314160*m + 59054
6123298*m^2 + 264300628944*m^3 + 72578259391*m^4 + 13137458400*m^5 + 162830
1884*m^6 + 140529312*m^7 + 8439783*m^8 + 345840*m^9 + 9218*m^10 + 144*m^11
+ m^12 + 316234143225) + (x^21*(f*x)^m*(360568238085*m + 292163767533*m^2 +
 124530626231*m^3 + 32087153670*m^4 + 5353566130*m^5 + 597988314*m^6 + 4513
4958*m^7 + 2271555*m^8 + 72985*m^9 + 1353*m^10 + 11*m^11 + 165646455975))/(
703416314160*m + 590546123298*m^2 + 264300628944*m^3 + 72578259391*m^4 + 13
137458400*m^5 + 1628301884*m^6 + 140529312*m^7 + 8439783*m^8 + 345840*m^9 +
 9218*m^10 + 144*m^11 + m^12 + 316234143225) + (x^5*(f*x)^m*(7041864340665*
m + 5087634488145*m^2 + 1889780020755*m^3 + 420404849150*m^4 + 60431072570*
m^5 + 5825106210*m^6 + 380801190*m^7 + 16677375*m^8 + 468765*m^9 + 7645*m^1
0 + 55*m^11 + 3478575575475))/(703416314160*m + 590546123298*m^2 + 26430062
8944*m^3 + 72578259391*m^4 + 13137458400*m^5 + 1628301884*m^6 + 140529312*m
^7 + 8439783*m^8 + 345840*m^9 + 9218*m^10 + 144*m^11 + m^12 + 316234143225)
 + (x^17*(f*x)^m*(6646727085075*m + 5340787250535*m^2 + 2251106854425*m^3 +
 572017996770*m^4 + 93862508190*m^5 + 10282782510*m^6 + 759091410*m^7 + 372
63105*m^8 + 1164735*m^9 + 20955*m^10 + 165*m^11 + 3069331390125))/(70341631
4160*m + 590546123298*m^2 + 264300628944*m^3 + 72578259391*m^4 + 1313745840
0*m^5 + 1628301884*m^6 + 140529312*m^7 + 8439783*m^8 + 345840*m^9 + 9218*m^
10 + 144*m^11 + m^12 + 316234143225) + (x*(f*x)^m*(387182170935*m + 2033639
52363*m^2 + 60936676581*m^3 + 11641582810*m^4 + 1495875590*m^5 + 132426294*
m^6 + 8103018*m^7 + 336765*m^8 + 9075*m^9 + 143*m^10 + m^11 + 316234143225)
)/(703416314160*m + 590546123298*m^2 + 264300628944*m^3 + 72578259391*m^4 +
```

```
 13137458400*m^5 + 1628301884*m^6 + 140529312*m^7 + 8439783*m^8 + 345840*m^
9 + 9218*m^10 + 144*m^11 + m^12 + 316234143225) + (x^23*(f*x)^m*(2998552189
5*m + 24372200061*m^2 + 10431670821*m^3 + 2702025590*m^4 + 453714470*m^5 +
51069018*m^6 + 3889578*m^7 + 197835*m^8 + 6435*m^9 + 121*m^10 + m^11 + 1374
9310575))/(703416314160*m + 590546123298*m^2 + 264300628944*m^3 + 725782593
91*m^4 + 13137458400*m^5 + 1628301884*m^6 + 140529312*m^7 + 8439783*m^8 + 3
45840*m^9 + 9218*m^10 + 144*m^11 + m^12 + 316234143225) + (x^7*(f*x)^m*(155
15657331075*m + 11703493287585*m^2 + 4558015784025*m^3 + 1059628145070*m^4
+ 158293212990*m^5 + 15768085410*m^6 + 1059893010*m^7 + 47524455*m^8 + 1362
735*m^9 + 22605*m^10 + 165*m^11 + 7454090518875))/(703416314160*m + 5905461
23298*m^2 + 264300628944*m^3 + 72578259391*m^4 + 13137458400*m^5 + 16283018
84*m^6 + 140529312*m^7 + 8439783*m^8 + 345840*m^9 + 9218*m^10 + 144*m^11 +
m^12 + 316234143225) + (x^15*(f*x)^m*(15011348834790*m + 11991258123570*m^2
 + 5015196628530*m^3 + 1262375264700*m^4 + 204865733820*m^5 + 22164925860*m
^6 + 1613983140*m^7 + 78076350*m^8 + 2403390*m^9 + 42570*m^10 + 330*m^11 +
6957151150950))/(703416314160*m + 590546123298*m^2 + 264300628944*m^3 + 725
78259391*m^4 + 13137458400*m^5 + 1628301884*m^6 + 140529312*m^7 + 8439783*m
^8 + 345840*m^9 + 9218*m^10 + 144*m^11 + m^12 + 316234143225) + (x^9*(f*x)^
m*(24503570194950*m + 18930738943710*m^2 + 7587607623090*m^3 + 181813533066
0*m^4 + 279691771260*m^5 + 28627538940*m^6 + 1971903780*m^7 + 90358290*m^8
+ 2640990*m^9 + 44550*m^10 + 330*m^11 + 11595251918250))/(703416314160*m +
590546123298*m^2 + 264300628944*m^3 + 72578259391*m^4 + 13137458400*m^5 + 1
628301884*m^6 + 140529312*m^7 + 8439783*m^8 + 345840*m^9 + 9218*m^10 + 144*
m^11 + m^12 + 316234143225) + (x^13*(f*x)^m*(24133835554290*m + 19130651800
722*m^2 + 7921249136262*m^3 + 1969992823260*m^4 + 315347150580*m^5 + 336098
70756*m^6 + 2408820876*m^7 + 114642990*m^8 + 3471930*m^9 + 60522*m^10 + 462
*m^11 + 11238474936150))/(703416314160*m + 590546123298*m^2 + 264300628944*
m^3 + 72578259391*m^4 + 13137458400*m^5 + 1628301884*m^6 + 140529312*m^7 +
8439783*m^8 + 345840*m^9 + 9218*m^10 + 144*m^11 + m^12 + 316234143225)
```

3.66 $\int x^5(1+x^2)\left(1+2x^2+x^4\right)^5\,dx$

Optimal result

Integrand size = 21, antiderivative size = 34

$$\int x^5\left(1+x^2\right)\left(1+2x^2+x^4\right)^5\,dx=\frac{1}{24}\left(1+x^2\right)^{12}-\frac{1}{13}\left(1+x^2\right)^{13}+\frac{1}{28}\left(1+x^2\right)^{14}$$

```
[Out] 1/24*(x^2+1)^12-1/13*(x^2+1)^13+1/28*(x^2+1)^14
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {28, 272, 45}

$$\int x^5\left(1+x^2\right)\left(1+2x^2+x^4\right)^5\,dx=\frac{1}{28}\left(x^2+1\right)^{14}-\frac{1}{13}\left(x^2+1\right)^{13}+\frac{1}{24}\left(x^2+1\right)^{12}$$

```
[In] Int[x^5*(1 + x^2)*(1 + 2*x^2 + x^4)^5,x]
```

```
[Out] (1 + x^2)^12/24 - (1 + x^2)^13/13 + (1 + x^2)^14/28
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
```

```
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int x^5 \left(1+x^2\right)^{11} \, dx \\
&= \frac{1}{2}\text{Subst}\left(\int x^2(1+x)^{11} \, dx, x, x^2\right) \\
&= \frac{1}{2}\text{Subst}\left(\int \left((1+x)^{11}-2(1+x)^{12}+(1+x)^{13}\right) \, dx, x, x^2\right) \\
&= \frac{1}{24}\left(1+x^2\right)^{12}-\frac{1}{13}\left(1+x^2\right)^{13}+\frac{1}{28}\left(1+x^2\right)^{14}
\end{aligned}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 85 vs. $2(34) = 68$.

Time = 0.00 (sec) , antiderivative size = 85, normalized size of antiderivative = 2.50

$$\int x^5 \left(1+x^2\right) \left(1+2x^2+x^4\right)^5 \, dx = \frac{x^6}{6}+\frac{11x^8}{8}+\frac{11x^{10}}{2}+\frac{55x^{12}}{4}+\frac{165x^{14}}{7}+\frac{231x^{16}}{8}+\frac{77x^{18}}{3}+\frac{33x^{20}}{2}+\frac{15x^{22}}{2}+\frac{55x^{24}}{24}+\frac{11x^{26}}{26}+\frac{x^{28}}{28}$$

```
[In] Integrate[x^5*(1 + x^2)*(1 + 2*x^2 + x^4)^5,x]
```

```
[Out] x^6/6 + (11*x^8)/8 + (11*x^10)/2 + (55*x^12)/4 + (165*x^14)/7 + (231*x^16)/
8 + (77*x^18)/3 + (33*x^20)/2 + (15*x^22)/2 + (55*x^24)/24 + (11*x^26)/26 +
 x^28/28
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.85

method	result
default	$\frac{(x^2+1)^{12}}{24}-\frac{(x^2+1)^{13}}{13}+\frac{(x^2+1)^{14}}{28}$
norman	$\frac{15}{2}x^{22}+\frac{55}{24}x^{24}+\frac{11}{26}x^{26}+\frac{1}{28}x^{28}+\frac{1}{6}x^6+\frac{11}{8}x^8+\frac{11}{2}x^{10}+\frac{55}{4}x^{12}+\frac{165}{7}x^{14}+\frac{231}{8}x^{16}+\frac{77}{3}x^{18}+$
parallelrisch	$\frac{15}{2}x^{22}+\frac{55}{24}x^{24}+\frac{11}{26}x^{26}+\frac{1}{28}x^{28}+\frac{1}{6}x^6+\frac{11}{8}x^8+\frac{11}{2}x^{10}+\frac{55}{4}x^{12}+\frac{165}{7}x^{14}+\frac{231}{8}x^{16}+\frac{77}{3}x^{18}+$
gosper	$\frac{x^6(78x^{22}+924x^{20}+5005x^{18}+16380x^{16}+36036x^{14}+56056x^{12}+63063x^{10}+51480x^8+30030x^6+12012x^4+3003x^2+364)}{2184}$
risch	$\frac{11}{2}x^{10}+\frac{55}{4}x^{12}+\frac{165}{7}x^{14}+\frac{11}{8}x^8+\frac{1}{6}x^6+\frac{1}{2184}+\frac{55}{24}x^{24}+\frac{11}{26}x^{26}+\frac{1}{28}x^{28}+\frac{33}{2}x^{20}+\frac{77}{3}x^{18}+\frac{231}{8}$

```
[In] int(x^5*(x^2+1)*(x^4+2*x^2+1)^5,x,method=_RETURNVERBOSE)
[Out] 1/24*(x^2+1)^12-1/13*(x^2+1)^13+1/28*(x^2+1)^14
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 61 vs. 2(28) = 56.

Time = 0.24 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.79

$$\int x^5(1+x^2)(1+2x^2+x^4)^5\,dx=\frac{1}{28}x^{28}+\frac{11}{26}x^{26}+\frac{55}{24}x^{24}+\frac{15}{2}x^{22}+\frac{33}{2}x^{20}+\frac{77}{3}x^{18}+\frac{231}{8}x^{16}+\frac{165}{7}x^{14}+\frac{55}{4}x^{12}+\frac{11}{2}x^{10}+\frac{11}{8}x^8+\frac{1}{6}x^6$$

```
[In] integrate(x^5*(x^2+1)*(x^4+2*x^2+1)^5,x, algorithm="fricas")
[Out] 1/28*x^28 + 11/26*x^26 + 55/24*x^24 + 15/2*x^22 + 33/2*x^20 + 77/3*x^18 + 2
31/8*x^16 + 165/7*x^14 + 55/4*x^12 + 11/2*x^10 + 11/8*x^8 + 1/6*x^6
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 76 vs. 2(24) = 48.

Time = 0.02 (sec) , antiderivative size = 76, normalized size of antiderivative = 2.24

$$\int x^5(1+x^2)(1+2x^2+x^4)^5\,dx=\frac{x^{28}}{28}+\frac{11x^{26}}{26}+\frac{55x^{24}}{24}+\frac{15x^{22}}{2}+\frac{33x^{20}}{2}+\frac{77x^{18}}{3}+\frac{231x^{16}}{8}+\frac{165x^{14}}{7}+\frac{55x^{12}}{4}+\frac{11x^{10}}{2}+\frac{11x^8}{8}+\frac{x^6}{6}$$

```
[In] integrate(x**5*(x**2+1)*(x**4+2*x**2+1)**5,x)
[Out] x**28/28 + 11*x**26/26 + 55*x**24/24 + 15*x**22/2 + 33*x**20/2 + 77*x**18/3
 + 231*x**16/8 + 165*x**14/7 + 55*x**12/4 + 11*x**10/2 + 11*x**8/8 + x**6/6
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 61 vs. $2(28) = 56$.

Time = 0.19 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.79

$$\int x^5(1+x^2)\left(1+2x^2+x^4\right)^5\,dx = \frac{1}{28}x^{28}+\frac{11}{26}x^{26}+\frac{55}{24}x^{24}+\frac{15}{2}x^{22}+\frac{33}{2}x^{20}+\frac{77}{3}x^{18}+\frac{231}{8}x^{16}+\frac{165}{7}x^{14}+\frac{55}{4}x^{12}+\frac{11}{2}x^{10}+\frac{11}{8}x^{8}+\frac{1}{6}x^{6}$$

```
[In] integrate(x^5*(x^2+1)*(x^4+2*x^2+1)^5,x, algorithm="maxima")
```

```
[Out] 1/28*x^28 + 11/26*x^26 + 55/24*x^24 + 15/2*x^22 + 33/2*x^20 + 77/3*x^18 + 2
31/8*x^16 + 165/7*x^14 + 55/4*x^12 + 11/2*x^10 + 11/8*x^8 + 1/6*x^6
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 61 vs. $2(28) = 56$.

Time = 0.25 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.79

$$\int x^5(1+x^2)\left(1+2x^2+x^4\right)^5\,dx = \frac{1}{28}x^{28}+\frac{11}{26}x^{26}+\frac{55}{24}x^{24}+\frac{15}{2}x^{22}+\frac{33}{2}x^{20}+\frac{77}{3}x^{18}+\frac{231}{8}x^{16}+\frac{165}{7}x^{14}+\frac{55}{4}x^{12}+\frac{11}{2}x^{10}+\frac{11}{8}x^{8}+\frac{1}{6}x^{6}$$

```
[In] integrate(x^5*(x^2+1)*(x^4+2*x^2+1)^5,x, algorithm="giac")
```

```
[Out] 1/28*x^28 + 11/26*x^26 + 55/24*x^24 + 15/2*x^22 + 33/2*x^20 + 77/3*x^18 + 2
31/8*x^16 + 165/7*x^14 + 55/4*x^12 + 11/2*x^10 + 11/8*x^8 + 1/6*x^6
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.79

$$\int x^5(1+x^2)\left(1+2x^2+x^4\right)^5\,dx = \frac{x^{28}}{28}+\frac{11\,x^{26}}{26}+\frac{55\,x^{24}}{24}+\frac{15\,x^{22}}{2}+\frac{33\,x^{20}}{2}+\frac{77\,x^{18}}{3}+\frac{231\,x^{16}}{8}+\frac{165\,x^{14}}{7}+\frac{55\,x^{12}}{4}+\frac{11\,x^{10}}{2}+\frac{11\,x^{8}}{8}+\frac{x^{6}}{6}$$

```
[In] int(x^5*(x^2 + 1)*(2*x^2 + x^4 + 1)^5,x)
```

```
[Out] x^6/6 + (11*x^8)/8 + (11*x^10)/2 + (55*x^12)/4 + (165*x^14)/7 + (231*x^16)/
8 + (77*x^18)/3 + (33*x^20)/2 + (15*x^22)/2 + (55*x^24)/24 + (11*x^26)/26 +
 x^28/28
```

3.67 $\int x^4(1+x^2)\left(1+2x^2+x^4\right)^5\,dx$

Optimal result

Integrand size = 21, antiderivative size = 83

$$\int x^4(1+x^2)\left(1+2x^2+x^4\right)^5\,dx=\frac{x^5}{5}+\frac{11x^7}{7}+\frac{55x^9}{9}+15x^{11}+\frac{330x^{13}}{13}+\frac{154x^{15}}{5}+\frac{462x^{17}}{17}+\frac{330x^{19}}{19}+\frac{55x^{21}}{7}+\frac{55x^{23}}{23}+\frac{11x^{25}}{25}+\frac{x^{27}}{27}$$

```
[Out] 1/5*x^5+11/7*x^7+55/9*x^9+15*x^11+330/13*x^13+154/5*x^15+462/17*x^17+330/19
*x^19+55/7*x^21+55/23*x^23+11/25*x^25+1/27*x^27
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {28, 276}

$$\int x^4(1+x^2)\left(1+2x^2+x^4\right)^5\,dx=\frac{x^{27}}{27}+\frac{11x^{25}}{25}+\frac{55x^{23}}{23}+\frac{55x^{21}}{7}+\frac{330x^{19}}{19}+\frac{462x^{17}}{17}+\frac{154x^{15}}{5}+\frac{330x^{13}}{13}+15x^{11}+\frac{55x^9}{9}+\frac{11x^7}{7}+\frac{x^5}{5}$$

```
[In] Int[x^4*(1 + x^2)*(1 + 2*x^2 + x^4)^5,x]
```

```
[Out] x^5/5 + (11*x^7)/7 + (55*x^9)/9 + 15*x^11 + (330*x^13)/13 + (154*x^15)/5 +
(462*x^17)/17 + (330*x^19)/19 + (55*x^21)/7 + (55*x^23)/23 + (11*x^25)/25 +
 x^27/27
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
```

```
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int x^4\left(1+x^2\right)^{11}\,dx\\ &= \int \left(x^4+11x^6+55x^8+165x^{10}+330x^{12}+462x^{14}+462x^{16}+330x^{18}+165x^{20}\right.\\ &\qquad \left.+55x^{22}+11x^{24}+x^{26}\right)\,dx\\ &= \frac{x^5}{5}+\frac{11x^7}{7}+\frac{55x^9}{9}+15x^{11}+\frac{330x^{13}}{13}+\frac{154x^{15}}{5}\\ &\quad +\frac{462x^{17}}{17}+\frac{330x^{19}}{19}+\frac{55x^{21}}{7}+\frac{55x^{23}}{23}+\frac{11x^{25}}{25}+\frac{x^{27}}{27}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.00

$$\int x^4\left(1+x^2\right)\left(1+2x^2+x^4\right)^5\,dx = \frac{x^5}{5}+\frac{11x^7}{7}+\frac{55x^9}{9}+15x^{11}+\frac{330x^{13}}{13}+\frac{154x^{15}}{5}+\frac{462x^{17}}{17}+\frac{330x^{19}}{19}+\frac{55x^{21}}{7}+\frac{55x^{23}}{23}+\frac{11x^{25}}{25}+\frac{x^{27}}{27}$$

```
[In] Integrate[x^4*(1 + x^2)*(1 + 2*x^2 + x^4)^5,x]
```

```
[Out] x^5/5 + (11*x^7)/7 + (55*x^9)/9 + 15*x^11 + (330*x^13)/13 + (154*x^15)/5 +
(462*x^17)/17 + (330*x^19)/19 + (55*x^21)/7 + (55*x^23)/23 + (11*x^25)/25 +
 x^27/27
```

Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.75

method	result
default	$\frac{1}{5}x^5+\frac{11}{7}x^7+\frac{55}{9}x^9+15x^{11}+\frac{330}{13}x^{13}+\frac{154}{5}x^{15}+\frac{462}{17}x^{17}+\frac{330}{19}x^{19}+\frac{55}{7}x^{21}+\frac{55}{23}x^{23}+\frac{11}{25}x^{25}+$
norman	$\frac{1}{5}x^5+\frac{11}{7}x^7+\frac{55}{9}x^9+15x^{11}+\frac{330}{13}x^{13}+\frac{154}{5}x^{15}+\frac{462}{17}x^{17}+\frac{330}{19}x^{19}+\frac{55}{7}x^{21}+\frac{55}{23}x^{23}+\frac{11}{25}x^{25}+$
risch	$\frac{1}{5}x^5+\frac{11}{7}x^7+\frac{55}{9}x^9+15x^{11}+\frac{330}{13}x^{13}+\frac{154}{5}x^{15}+\frac{462}{17}x^{17}+\frac{330}{19}x^{19}+\frac{55}{7}x^{21}+\frac{55}{23}x^{23}+\frac{11}{25}x^{25}+$
parallelrisch	$\frac{1}{5}x^5+\frac{11}{7}x^7+\frac{55}{9}x^9+15x^{11}+\frac{330}{13}x^{13}+\frac{154}{5}x^{15}+\frac{462}{17}x^{17}+\frac{330}{19}x^{19}+\frac{55}{7}x^{21}+\frac{55}{23}x^{23}+\frac{11}{25}x^{25}+$
gosper	$\frac{x^5(16900975x^{22}+200783583x^{20}+1091215125x^{18}+3585421125x^{16}+7925667750x^{14}+12401338950x^{12}+14054850810x^{10}+115}{456326325}$

[In] int(x^4*(x^2+1)*(x^4+2*x^2+1)^5,x,method=_RETURNVERBOSE)

[Out] 1/5*x^5+11/7*x^7+55/9*x^9+15*x^11+330/13*x^13+154/5*x^15+462/17*x^17+330/19*x^19+55/7*x^21+55/23*x^23+11/25*x^25+1/27*x^27

Fricas [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.73

$$\int x^4(1+x^2)\left(1+2x^2+x^4\right)^5\,dx=\frac{1}{27}\,x^{27}+\frac{11}{25}\,x^{25}+\frac{55}{23}\,x^{23}+\frac{55}{7}\,x^{21}+\frac{330}{19}\,x^{19}+\frac{462}{17}\,x^{17}+\frac{154}{5}\,x^{15}+\frac{330}{13}\,x^{13}+15\,x^{11}+\frac{55}{9}\,x^9+\frac{11}{7}\,x^7+\frac{1}{5}\,x^5$$

[In] integrate(x^4*(x^2+1)*(x^4+2*x^2+1)^5,x, algorithm="fricas")

[Out] 1/27*x^27 + 11/25*x^25 + 55/23*x^23 + 55/7*x^21 + 330/19*x^19 + 462/17*x^17 + 154/5*x^15 + 330/13*x^13 + 15*x^11 + 55/9*x^9 + 11/7*x^7 + 1/5*x^5

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.90

$$\int x^4(1+x^2)\left(1+2x^2+x^4\right)^5\,dx=\frac{x^{27}}{27}+\frac{11x^{25}}{25}+\frac{55x^{23}}{23}+\frac{55x^{21}}{7}+\frac{330x^{19}}{19}+\frac{462x^{17}}{17}+\frac{154x^{15}}{5}+\frac{330x^{13}}{13}+15x^{11}+\frac{55x^9}{9}+\frac{11x^7}{7}+\frac{x^5}{5}$$

[In] integrate(x**4*(x**2+1)*(x**4+2*x**2+1)**5,x)

[Out] x**27/27 + 11*x**25/25 + 55*x**23/23 + 55*x**21/7 + 330*x**19/19 + 462*x**17/17 + 154*x**15/5 + 330*x**13/13 + 15*x**11 + 55*x**9/9 + 11*x**7/7 + x**5/5

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.73

$$\int x^4(1+x^2)\left(1+2x^2+x^4\right)^5\,dx = \frac{1}{27}\,x^{27}+\frac{11}{25}\,x^{25}+\frac{55}{23}\,x^{23}+\frac{55}{7}\,x^{21}+\frac{330}{19}\,x^{19}+\frac{462}{17}\,x^{17}+\frac{154}{5}\,x^{15}+\frac{330}{13}\,x^{13}+15\,x^{11}+\frac{55}{9}\,x^{9}+\frac{11}{7}\,x^{7}+\frac{1}{5}\,x^{5}$$

```
[In] integrate(x^4*(x^2+1)*(x^4+2*x^2+1)^5,x, algorithm="maxima")
[Out] 1/27*x^27 + 11/25*x^25 + 55/23*x^23 + 55/7*x^21 + 330/19*x^19 + 462/17*x^17
 + 154/5*x^15 + 330/13*x^13 + 15*x^11 + 55/9*x^9 + 11/7*x^7 + 1/5*x^5
```

Giac [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.73

$$\int x^4(1+x^2)\left(1+2x^2+x^4\right)^5\,dx = \frac{1}{27}\,x^{27}+\frac{11}{25}\,x^{25}+\frac{55}{23}\,x^{23}+\frac{55}{7}\,x^{21}+\frac{330}{19}\,x^{19}+\frac{462}{17}\,x^{17}+\frac{154}{5}\,x^{15}+\frac{330}{13}\,x^{13}+15\,x^{11}+\frac{55}{9}\,x^{9}+\frac{11}{7}\,x^{7}+\frac{1}{5}\,x^{5}$$

```
[In] integrate(x^4*(x^2+1)*(x^4+2*x^2+1)^5,x, algorithm="giac")
[Out] 1/27*x^27 + 11/25*x^25 + 55/23*x^23 + 55/7*x^21 + 330/19*x^19 + 462/17*x^17
 + 154/5*x^15 + 330/13*x^13 + 15*x^11 + 55/9*x^9 + 11/7*x^7 + 1/5*x^5
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.73

$$\int x^4(1+x^2)\left(1+2x^2+x^4\right)^5\,dx = \frac{x^{27}}{27}+\frac{11\,x^{25}}{25}+\frac{55\,x^{23}}{23}+\frac{55\,x^{21}}{7}+\frac{330\,x^{19}}{19}+\frac{462\,x^{17}}{17}+\frac{154\,x^{15}}{5}+\frac{330\,x^{13}}{13}+15\,x^{11}+\frac{55\,x^{9}}{9}+\frac{11\,x^{7}}{7}+\frac{x^{5}}{5}$$

```
[In] int(x^4*(x^2 + 1)*(2*x^2 + x^4 + 1)^5,x)
[Out] x^5/5 + (11*x^7)/7 + (55*x^9)/9 + 15*x^11 + (330*x^13)/13 + (154*x^15)/5 +
(462*x^17)/17 + (330*x^19)/19 + (55*x^21)/7 + (55*x^23)/23 + (11*x^25)/25 +
 x^27/27
```

3.68 $\int x^3(1+x^2)\left(1+2x^2+x^4\right)^5\,dx$

Optimal result

Integrand size = 21, antiderivative size = 23

$$\int x^3\left(1+x^2\right)\left(1+2x^2+x^4\right)^5\,dx=-\frac{1}{24}\left(1+x^2\right)^{12}+\frac{1}{26}\left(1+x^2\right)^{13}$$

```
[Out] -1/24*(x^2+1)^12+1/26*(x^2+1)^13
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {28, 272, 45}

$$\int x^3\left(1+x^2\right)\left(1+2x^2+x^4\right)^5\,dx=\frac{1}{26}\left(x^2+1\right)^{13}-\frac{1}{24}\left(x^2+1\right)^{12}$$

```
[In] Int[x^3*(1 + x^2)*(1 + 2*x^2 + x^4)^5,x]
```

```
[Out] -1/24*(1 + x^2)^12 + (1 + x^2)^13/26
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
```

```
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int x^3 \left(1+x^2\right)^{11} dx \\
&= \frac{1}{2}\text{Subst}\left(\int x(1+x)^{11} dx, x, x^2\right) \\
&= \frac{1}{2}\text{Subst}\left(\int \left(-(1+x)^{11}+(1+x)^{12}\right) dx, x, x^2\right) \\
&= -\frac{1}{24}\left(1+x^2\right)^{12}+\frac{1}{26}\left(1+x^2\right)^{13}
\end{aligned}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 83 vs. 2(23) = 46.

Time = 0.00 (sec) , antiderivative size = 83, normalized size of antiderivative = 3.61

$$\int x^3 \left(1+x^2\right) \left(1+2x^2+x^4\right)^5 dx = \frac{x^4}{4}+\frac{11x^6}{6}+\frac{55x^8}{8}+\frac{33x^{10}}{2}+\frac{55x^{12}}{2}+33x^{14} +\frac{231x^{16}}{8}+\frac{55x^{18}}{3}+\frac{33x^{20}}{4}+\frac{5x^{22}}{2}+\frac{11x^{24}}{24}+\frac{x^{26}}{26}$$

```
[In] Integrate[x^3*(1 + x^2)*(1 + 2*x^2 + x^4)^5,x]
[Out] x^4/4 + (11*x^6)/6 + (55*x^8)/8 + (33*x^10)/2 + (55*x^12)/2 + 33*x^14 + (23
1*x^16)/8 + (55*x^18)/3 + (33*x^20)/4 + (5*x^22)/2 + (11*x^24)/24 + x^26/26
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.87

method	result
default	$-\frac{(x^2+1)^{12}}{24}+\frac{(x^2+1)^{13}}{26}$
norman	$\frac{5}{2}x^{22}+\frac{11}{24}x^{24}+\frac{1}{26}x^{26}+\frac{1}{4}x^4+\frac{11}{6}x^6+\frac{55}{8}x^8+\frac{33}{2}x^{10}+\frac{55}{2}x^{12}+33x^{14}+\frac{231}{8}x^{16}+\frac{55}{3}x^{18}+\frac{33}{4}x$
parallelrisch	$\frac{5}{2}x^{22}+\frac{11}{24}x^{24}+\frac{1}{26}x^{26}+\frac{1}{4}x^4+\frac{11}{6}x^6+\frac{55}{8}x^8+\frac{33}{2}x^{10}+\frac{55}{2}x^{12}+33x^{14}+\frac{231}{8}x^{16}+\frac{55}{3}x^{18}+\frac{33}{4}x$
gosper	$\frac{x^4(12x^{22}+143x^{20}+780x^{18}+2574x^{16}+5720x^{14}+9009x^{12}+10296x^{10}+8580x^8+5148x^6+2145x^4+572x^2+78)}{312}$
risch	$\frac{33}{2}x^{10}+\frac{55}{2}x^{12}+\frac{1}{4}x^4+33x^{14}+\frac{55}{8}x^8+\frac{11}{6}x^6+\frac{11}{24}x^{24}+\frac{1}{26}x^{26}+\frac{33}{4}x^{20}+\frac{55}{3}x^{18}+\frac{231}{8}x^{16}+\frac{5}{2}x$

```
[In] int(x^3*(x^2+1)*(x^4+2*x^2+1)^5,x,method=_RETURNVERBOSE)
```

```
[Out] -1/24*(x^2+1)^12+1/26*(x^2+1)^13
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 61 vs. 2(19) = 38.

Time = 0.25 (sec) , antiderivative size = 61, normalized size of antiderivative = 2.65

$$\int x^3(1+x^2)\left(1+2x^2+x^4\right)^5\,dx=\frac{1}{26}\,x^{26}+\frac{11}{24}\,x^{24}+\frac{5}{2}\,x^{22}+\frac{33}{4}\,x^{20}+\frac{55}{3}\,x^{18}+\frac{231}{8}\,x^{16}+33\,x^{14}+\frac{55}{2}\,x^{12}+\frac{33}{2}\,x^{10}+\frac{55}{8}\,x^8+\frac{11}{6}\,x^6+\frac{1}{4}\,x^4$$

```
[In] integrate(x^3*(x^2+1)*(x^4+2*x^2+1)^5,x, algorithm="fricas")
```

```
[Out] 1/26*x^26 + 11/24*x^24 + 5/2*x^22 + 33/4*x^20 + 55/3*x^18 + 231/8*x^16 + 33
*x^14 + 55/2*x^12 + 33/2*x^10 + 55/8*x^8 + 11/6*x^6 + 1/4*x^4
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 75 vs. 2(15) = 30.

Time = 0.02 (sec) , antiderivative size = 75, normalized size of antiderivative = 3.26

$$\int x^3(1+x^2)\left(1+2x^2+x^4\right)^5\,dx=\frac{x^{26}}{26}+\frac{11x^{24}}{24}+\frac{5x^{22}}{2}+\frac{33x^{20}}{4}+\frac{55x^{18}}{3}+\frac{231x^{16}}{8}+33x^{14}+\frac{55x^{12}}{2}+\frac{33x^{10}}{2}+\frac{55x^8}{8}+\frac{11x^6}{6}+\frac{x^4}{4}$$

```
[In] integrate(x**3*(x**2+1)*(x**4+2*x**2+1)**5,x)
```

```
[Out] x**26/26 + 11*x**24/24 + 5*x**22/2 + 33*x**20/4 + 55*x**18/3 + 231*x**16/8
+ 33*x**14 + 55*x**12/2 + 33*x**10/2 + 55*x**8/8 + 11*x**6/6 + x**4/4
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 61 vs. $2(19) = 38$.

Time = 0.19 (sec) , antiderivative size = 61, normalized size of antiderivative = 2.65

$$\int x^3(1+x^2)\left(1+2x^2+x^4\right)^5\,dx = \frac{1}{26}\,x^{26} + \frac{11}{24}\,x^{24} + \frac{5}{2}\,x^{22} + \frac{33}{4}\,x^{20} + \frac{55}{3}\,x^{18} + \frac{231}{8}\,x^{16} + 33\,x^{14} + \frac{55}{2}\,x^{12} + \frac{33}{2}\,x^{10} + \frac{55}{8}\,x^{8} + \frac{11}{6}\,x^{6} + \frac{1}{4}\,x^{4}$$

```
[In] integrate(x^3*(x^2+1)*(x^4+2*x^2+1)^5,x, algorithm="maxima")
[Out] 1/26*x^26 + 11/24*x^24 + 5/2*x^22 + 33/4*x^20 + 55/3*x^18 + 231/8*x^16 + 33
*x^14 + 55/2*x^12 + 33/2*x^10 + 55/8*x^8 + 11/6*x^6 + 1/4*x^4
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 61 vs. $2(19) = 38$.

Time = 0.26 (sec) , antiderivative size = 61, normalized size of antiderivative = 2.65

$$\int x^3(1+x^2)\left(1+2x^2+x^4\right)^5\,dx = \frac{1}{26}\,x^{26} + \frac{11}{24}\,x^{24} + \frac{5}{2}\,x^{22} + \frac{33}{4}\,x^{20} + \frac{55}{3}\,x^{18} + \frac{231}{8}\,x^{16} + 33\,x^{14} + \frac{55}{2}\,x^{12} + \frac{33}{2}\,x^{10} + \frac{55}{8}\,x^{8} + \frac{11}{6}\,x^{6} + \frac{1}{4}\,x^{4}$$

```
[In] integrate(x^3*(x^2+1)*(x^4+2*x^2+1)^5,x, algorithm="giac")
[Out] 1/26*x^26 + 11/24*x^24 + 5/2*x^22 + 33/4*x^20 + 55/3*x^18 + 231/8*x^16 + 33
*x^14 + 55/2*x^12 + 33/2*x^10 + 55/8*x^8 + 11/6*x^6 + 1/4*x^4
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 61, normalized size of antiderivative = 2.65

$$\int x^3(1+x^2)\left(1+2x^2+x^4\right)^5\,dx = \frac{x^{26}}{26} + \frac{11\,x^{24}}{24} + \frac{5\,x^{22}}{2} + \frac{33\,x^{20}}{4} + \frac{55\,x^{18}}{3} + \frac{231\,x^{16}}{8} + 33\,x^{14} + \frac{55\,x^{12}}{2} + \frac{33\,x^{10}}{2} + \frac{55\,x^{8}}{8} + \frac{11\,x^{6}}{6} + \frac{x^{4}}{4}$$

```
[In] int(x^3*(x^2 + 1)*(2*x^2 + x^4 + 1)^5,x)
[Out] x^4/4 + (11*x^6)/6 + (55*x^8)/8 + (33*x^10)/2 + (55*x^12)/2 + 33*x^14 + (23
1*x^16)/8 + (55*x^18)/3 + (33*x^20)/4 + (5*x^22)/2 + (11*x^24)/24 + x^26/26
```

3.69 $\int x^2(1+x^2)\left(1+2x^2+x^4\right)^5\,dx$

Optimal result

Integrand size = 21, antiderivative size = 83

$$\int x^2(1+x^2)\left(1+2x^2+x^4\right)^5\,dx=\frac{x^3}{3}+\frac{11x^5}{5}+\frac{55x^7}{7}+\frac{55x^9}{3}+30x^{11}+\frac{462x^{13}}{13}+\frac{154x^{15}}{5}+\frac{330x^{17}}{17}+\frac{165x^{19}}{19}+\frac{55x^{21}}{21}+\frac{11x^{23}}{23}+\frac{x^{25}}{25}$$

```
[Out] 1/3*x^3+11/5*x^5+55/7*x^7+55/3*x^9+30*x^11+462/13*x^13+154/5*x^15+330/17*x^
17+165/19*x^19+55/21*x^21+11/23*x^23+1/25*x^25
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {28, 276}

$$\int x^2(1+x^2)\left(1+2x^2+x^4\right)^5\,dx=\frac{x^{25}}{25}+\frac{11x^{23}}{23}+\frac{55x^{21}}{21}+\frac{165x^{19}}{19}+\frac{330x^{17}}{17}+\frac{154x^{15}}{5}+\frac{462x^{13}}{13}+30x^{11}+\frac{55x^9}{3}+\frac{55x^7}{7}+\frac{11x^5}{5}+\frac{x^3}{3}$$

```
[In] Int[x^2*(1 + x^2)*(1 + 2*x^2 + x^4)^5,x]
```

```
[Out] x^3/3 + (11*x^5)/5 + (55*x^7)/7 + (55*x^9)/3 + 30*x^11 + (462*x^13)/13 + (1
54*x^15)/5 + (330*x^17)/17 + (165*x^19)/19 + (55*x^21)/21 + (11*x^23)/23 +
x^25/25
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
```

```
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int x^2\left(1+x^2\right)^{11}\,dx\\ &= \int \left(x^2+11x^4+55x^6+165x^8+330x^{10}+462x^{12}+462x^{14}+330x^{16}+165x^{18}+55x^{20}+11x^{22}+x^{24}\right)dx\\ &= \frac{x^3}{3}+\frac{11x^5}{5}+\frac{55x^7}{7}+\frac{55x^9}{3}+30x^{11}+\frac{462x^{13}}{13}+\frac{154x^{15}}{5}+\frac{330x^{17}}{17}+\frac{165x^{19}}{19}+\frac{55x^{21}}{21}+\frac{11x^{23}}{23}+\frac{x^{25}}{25}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.00

$$\int x^2\left(1+x^2\right)\left(1+2x^2+x^4\right)^5\,dx = \frac{x^3}{3}+\frac{11x^5}{5}+\frac{55x^7}{7}+\frac{55x^9}{3}+30x^{11}+\frac{462x^{13}}{13}+\frac{154x^{15}}{5}+\frac{330x^{17}}{17}+\frac{165x^{19}}{19}+\frac{55x^{21}}{21}+\frac{11x^{23}}{23}+\frac{x^{25}}{25}$$

```
[In] Integrate[x^2*(1 + x^2)*(1 + 2*x^2 + x^4)^5,x]
```

```
[Out] x^3/3 + (11*x^5)/5 + (55*x^7)/7 + (55*x^9)/3 + 30*x^11 + (462*x^13)/13 + (1
54*x^15)/5 + (330*x^17)/17 + (165*x^19)/19 + (55*x^21)/21 + (11*x^23)/23 +
x^25/25
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.75

method	result
default	$\frac{1}{3}x^3+\frac{11}{5}x^5+\frac{55}{7}x^7+\frac{55}{3}x^9+30x^{11}+\frac{462}{13}x^{13}+\frac{154}{5}x^{15}+\frac{330}{17}x^{17}+\frac{165}{19}x^{19}+\frac{55}{21}x^{21}+\frac{11}{23}x^{23}+$
norman	$\frac{1}{3}x^3+\frac{11}{5}x^5+\frac{55}{7}x^7+\frac{55}{3}x^9+30x^{11}+\frac{462}{13}x^{13}+\frac{154}{5}x^{15}+\frac{330}{17}x^{17}+\frac{165}{19}x^{19}+\frac{55}{21}x^{21}+\frac{11}{23}x^{23}+$
risch	$\frac{1}{3}x^3+\frac{11}{5}x^5+\frac{55}{7}x^7+\frac{55}{3}x^9+30x^{11}+\frac{462}{13}x^{13}+\frac{154}{5}x^{15}+\frac{330}{17}x^{17}+\frac{165}{19}x^{19}+\frac{55}{21}x^{21}+\frac{11}{23}x^{23}+$
parallelrisch	$\frac{1}{3}x^3+\frac{11}{5}x^5+\frac{55}{7}x^7+\frac{55}{3}x^9+30x^{11}+\frac{462}{13}x^{13}+\frac{154}{5}x^{15}+\frac{330}{17}x^{17}+\frac{165}{19}x^{19}+\frac{55}{21}x^{21}+\frac{11}{23}x^{23}+$
gosper	$\frac{x^3(2028117x^{22}+24249225x^{20}+132793375x^{18}+440314875x^{16}+984233250x^{14}+1561650090x^{12}+1801903950x^{10}+1521087750}{50702925}$

[In] int(x^2*(x^2+1)*(x^4+2*x^2+1)^5,x,method=_RETURNVERBOSE)

[Out] 1/3*x^3+11/5*x^5+55/7*x^7+55/3*x^9+30*x^11+462/13*x^13+154/5*x^15+330/17*x^17+165/19*x^19+55/21*x^21+11/23*x^23+1/25*x^25

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.73

$$\int x^2(1+x^2)\left(1+2x^2+x^4\right)^5\,dx=\frac{1}{25}\,x^{25}+\frac{11}{23}\,x^{23}+\frac{55}{21}\,x^{21}+\frac{165}{19}\,x^{19}+\frac{330}{17}\,x^{17}+\frac{154}{5}\,x^{15}+\frac{462}{13}\,x^{13}+30\,x^{11}+\frac{55}{3}\,x^9+\frac{55}{7}\,x^7+\frac{11}{5}\,x^5+\frac{1}{3}\,x^3$$

[In] integrate(x^2*(x^2+1)*(x^4+2*x^2+1)^5,x, algorithm="fricas")

[Out] 1/25*x^25 + 11/23*x^23 + 55/21*x^21 + 165/19*x^19 + 330/17*x^17 + 154/5*x^15 + 462/13*x^13 + 30*x^11 + 55/3*x^9 + 55/7*x^7 + 11/5*x^5 + 1/3*x^3

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.90

$$\int x^2(1+x^2)\left(1+2x^2+x^4\right)^5\,dx=\frac{x^{25}}{25}+\frac{11x^{23}}{23}+\frac{55x^{21}}{21}+\frac{165x^{19}}{19}+\frac{330x^{17}}{17}+\frac{154x^{15}}{5}+\frac{462x^{13}}{13}+30x^{11}+\frac{55x^9}{3}+\frac{55x^7}{7}+\frac{11x^5}{5}+\frac{x^3}{3}$$

[In] integrate(x**2*(x**2+1)*(x**4+2*x**2+1)**5,x)

[Out] x**25/25 + 11*x**23/23 + 55*x**21/21 + 165*x**19/19 + 330*x**17/17 + 154*x**15/5 + 462*x**13/13 + 30*x**11 + 55*x**9/3 + 55*x**7/7 + 11*x**5/5 + x**3/3

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.73

$$\int x^2(1+x^2)\left(1+2x^2+x^4\right)^5\,dx = \frac{1}{25}\,x^{25} + \frac{11}{23}\,x^{23} + \frac{55}{21}\,x^{21} + \frac{165}{19}\,x^{19} + \frac{330}{17}\,x^{17} + \frac{154}{5}\,x^{15} + \frac{462}{13}\,x^{13} + 30\,x^{11} + \frac{55}{3}\,x^9 + \frac{55}{7}\,x^7 + \frac{11}{5}\,x^5 + \frac{1}{3}\,x^3$$

```
[In] integrate(x^2*(x^2+1)*(x^4+2*x^2+1)^5,x, algorithm="maxima")
```

```
[Out] 1/25*x^25 + 11/23*x^23 + 55/21*x^21 + 165/19*x^19 + 330/17*x^17 + 154/5*x^1
5 + 462/13*x^13 + 30*x^11 + 55/3*x^9 + 55/7*x^7 + 11/5*x^5 + 1/3*x^3
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.73

$$\int x^2(1+x^2)\left(1+2x^2+x^4\right)^5\,dx = \frac{1}{25}\,x^{25} + \frac{11}{23}\,x^{23} + \frac{55}{21}\,x^{21} + \frac{165}{19}\,x^{19} + \frac{330}{17}\,x^{17} + \frac{154}{5}\,x^{15} + \frac{462}{13}\,x^{13} + 30\,x^{11} + \frac{55}{3}\,x^9 + \frac{55}{7}\,x^7 + \frac{11}{5}\,x^5 + \frac{1}{3}\,x^3$$

```
[In] integrate(x^2*(x^2+1)*(x^4+2*x^2+1)^5,x, algorithm="giac")
```

```
[Out] 1/25*x^25 + 11/23*x^23 + 55/21*x^21 + 165/19*x^19 + 330/17*x^17 + 154/5*x^1
5 + 462/13*x^13 + 30*x^11 + 55/3*x^9 + 55/7*x^7 + 11/5*x^5 + 1/3*x^3
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.73

$$\int x^2(1+x^2)\left(1+2x^2+x^4\right)^5\,dx = \frac{x^{25}}{25} + \frac{11\,x^{23}}{23} + \frac{55\,x^{21}}{21} + \frac{165\,x^{19}}{19} + \frac{330\,x^{17}}{17} + \frac{154\,x^{15}}{5} + \frac{462\,x^{13}}{13} + 30\,x^{11} + \frac{55\,x^9}{3} + \frac{55\,x^7}{7} + \frac{11\,x^5}{5} + \frac{x^3}{3}$$

```
[In] int(x^2*(x^2 + 1)*(2*x^2 + x^4 + 1)^5,x)
```

```
[Out] x^3/3 + (11*x^5)/5 + (55*x^7)/7 + (55*x^9)/3 + 30*x^11 + (462*x^13)/13 + (1
54*x^15)/5 + (330*x^17)/17 + (165*x^19)/19 + (55*x^21)/21 + (11*x^23)/23 +
x^25/25
```

3.70 $\int x(1+x^2)\left(1+2x^2+x^4\right)^5 dx$

Optimal result

Integrand size = 19, antiderivative size = 11

$$\int x\left(1+x^2\right)\left(1+2x^2+x^4\right)^5 dx=\frac{1}{24}\left(1+x^2\right)^{12}$$

```
[Out] 1/24*(x^2+1)^12
```

Rubi [A] (verified)

Time = 0.00 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {28, 267}

$$\int x\left(1+x^2\right)\left(1+2x^2+x^4\right)^5 dx=\frac{1}{24}\left(x^2+1\right)^{12}$$

```
[In] Int[x*(1 + x^2)*(1 + 2*x^2 + x^4)^5,x]
```

```
[Out] (1 + x^2)^12/24
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\begin{aligned} \text{integral} &= \int x\left(1+x^2\right)^{11}\,dx \\ &= \frac{1}{24}\left(1+x^2\right)^{12} \end{aligned}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

$$\int x\left(1+x^2\right)\left(1+2x^2+x^4\right)^5\,dx=\frac{1}{24}\left(1+x^2\right)^{12}$$

```
[In] Integrate[x*(1 + x^2)*(1 + 2*x^2 + x^4)^5,x]
[Out] (1 + x^2)^12/24
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.91

method	result
default	$\frac{\left(x^2+1\right)^{12}}{24}$
gosper	$\frac{x^2\left(x^{22}+12x^{20}+66x^{18}+220x^{16}+495x^{14}+792x^{12}+924x^{10}+792x^8+495x^6+220x^4+66x^2+12\right)}{24}$
norman	$\frac{1}{24}x^{24}+\frac{1}{2}x^{22}+\frac{11}{4}x^{20}+\frac{55}{6}x^{18}+33x^{14}+\frac{165}{8}x^{16}+\frac{165}{8}x^8+33x^{10}+\frac{77}{2}x^{12}+\frac{1}{2}x^2+\frac{11}{4}x^4+\frac{55}{6}x^6$
parallelrisch	$\frac{1}{24}x^{24}+\frac{1}{2}x^{22}+\frac{11}{4}x^{20}+\frac{55}{6}x^{18}+33x^{14}+\frac{165}{8}x^{16}+\frac{165}{8}x^8+33x^{10}+\frac{77}{2}x^{12}+\frac{1}{2}x^2+\frac{11}{4}x^4+\frac{55}{6}x^6$
risch	$\frac{1}{24}x^{24}+\frac{1}{2}x^{22}+\frac{11}{4}x^{20}+\frac{55}{6}x^{18}+\frac{165}{8}x^{16}+33x^{14}+\frac{77}{2}x^{12}+33x^{10}+\frac{165}{8}x^8+\frac{55}{6}x^6+\frac{11}{4}x^4+\frac{1}{2}x^2$

```
[In] int(x*(x^2+1)*(x^4+2*x^2+1)^5,x,method=_RETURNVERBOSE)
[Out] 1/24*(x^2+1)^12
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 61 vs. 2(9) = 18.

Time = 0.25 (sec) , antiderivative size = 61, normalized size of antiderivative = 5.55

$$\int x(1+x^2)\left(1+2x^2+x^4\right)^5\,dx = \frac{1}{24}\,x^{24}+\frac{1}{2}\,x^{22}+\frac{11}{4}\,x^{20}+\frac{55}{6}\,x^{18}+\frac{165}{8}\,x^{16}+33\,x^{14}+\frac{77}{2}\,x^{12}+33\,x^{10}+\frac{165}{8}\,x^{8}+\frac{55}{6}\,x^{6}+\frac{11}{4}\,x^{4}+\frac{1}{2}\,x^{2}$$

```
[In] integrate(x*(x^2+1)*(x^4+2*x^2+1)^5,x, algorithm="fricas")
```

```
[Out] 1/24*x^24 + 1/2*x^22 + 11/4*x^20 + 55/6*x^18 + 165/8*x^16 + 33*x^14 + 77/2*
x^12 + 33*x^10 + 165/8*x^8 + 55/6*x^6 + 11/4*x^4 + 1/2*x^2
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 71 vs. 2(7) = 14.

Time = 0.02 (sec) , antiderivative size = 71, normalized size of antiderivative = 6.45

$$\int x(1+x^2)\left(1+2x^2+x^4\right)^5\,dx = \frac{x^{24}}{24}+\frac{x^{22}}{2}+\frac{11x^{20}}{4}+\frac{55x^{18}}{6}+\frac{165x^{16}}{8}+33x^{14}+\frac{77x^{12}}{2}+33x^{10}+\frac{165x^{8}}{8}+\frac{55x^{6}}{6}+\frac{11x^{4}}{4}+\frac{x^{2}}{2}$$

```
[In] integrate(x*(x**2+1)*(x**4+2*x**2+1)**5,x)
```

```
[Out] x**24/24 + x**22/2 + 11*x**20/4 + 55*x**18/6 + 165*x**16/8 + 33*x**14 + 77*
x**12/2 + 33*x**10 + 165*x**8/8 + 55*x**6/6 + 11*x**4/4 + x**2/2
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 61 vs. 2(9) = 18.

Time = 0.18 (sec) , antiderivative size = 61, normalized size of antiderivative = 5.55

$$\int x(1+x^2)\left(1+2x^2+x^4\right)^5\,dx = \frac{1}{24}\,x^{24}+\frac{1}{2}\,x^{22}+\frac{11}{4}\,x^{20}+\frac{55}{6}\,x^{18}+\frac{165}{8}\,x^{16}+33\,x^{14}+\frac{77}{2}\,x^{12}+33\,x^{10}+\frac{165}{8}\,x^{8}+\frac{55}{6}\,x^{6}+\frac{11}{4}\,x^{4}+\frac{1}{2}\,x^{2}$$

```
[In] integrate(x*(x^2+1)*(x^4+2*x^2+1)^5,x, algorithm="maxima")
```

```
[Out] 1/24*x^24 + 1/2*x^22 + 11/4*x^20 + 55/6*x^18 + 165/8*x^16 + 33*x^14 + 77/2*
x^12 + 33*x^10 + 165/8*x^8 + 55/6*x^6 + 11/4*x^4 + 1/2*x^2
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 76 vs. $2(9) = 18$.

Time = 0.27 (sec) , antiderivative size = 76, normalized size of antiderivative = 6.91

$$\int x(1+x^2)\left(1+2x^2+x^4\right)^5\,dx = \frac{1}{24}\left(x^4+2\,x^2\right)^6+\frac{1}{4}\left(x^4+2\,x^2\right)^5+\frac{5}{8}\left(x^4+2\,x^2\right)^4 + \frac{1}{4}\,x^4+\frac{5}{6}\left(x^4+2\,x^2\right)^3+\frac{5}{8}\left(x^4+2\,x^2\right)^2+\frac{1}{2}\,x^2$$

```
[In] integrate(x*(x^2+1)*(x^4+2*x^2+1)^5,x, algorithm="giac")
[Out] 1/24*(x^4 + 2*x^2)^6 + 1/4*(x^4 + 2*x^2)^5 + 5/8*(x^4 + 2*x^2)^4 + 1/4*x^4
+ 5/6*(x^4 + 2*x^2)^3 + 5/8*(x^4 + 2*x^2)^2 + 1/2*x^2
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 61, normalized size of antiderivative = 5.55

$$\int x(1+x^2)\left(1+2x^2+x^4\right)^5\,dx = \frac{x^{24}}{24}+\frac{x^{22}}{2}+\frac{11\,x^{20}}{4}+\frac{55\,x^{18}}{6}+\frac{165\,x^{16}}{8}+33\,x^{14} + \frac{77\,x^{12}}{2}+33\,x^{10}+\frac{165\,x^8}{8}+\frac{55\,x^6}{6}+\frac{11\,x^4}{4}+\frac{x^2}{2}$$

```
[In] int(x*(x^2 + 1)*(2*x^2 + x^4 + 1)^5,x)
[Out] x^2/2 + (11*x^4)/4 + (55*x^6)/6 + (165*x^8)/8 + 33*x^10 + (77*x^12)/2 + 33*
x^14 + (165*x^16)/8 + (55*x^18)/6 + (11*x^20)/4 + x^22/2 + x^24/24
```

3.71 $\int (1+x^2)(1+2x^2+x^4)^5\,dx$

Optimal result

Integrand size = 18, antiderivative size = 73

$$\int (1+x^2)(1+2x^2+x^4)^5\,dx = x+\frac{11x^3}{3}+11x^5+\frac{165x^7}{7}+\frac{110x^9}{3}+42x^{11}+\frac{462x^{13}}{13}+22x^{15}+\frac{165x^{17}}{17}+\frac{55x^{19}}{19}+\frac{11x^{21}}{21}+\frac{x^{23}}{23}$$

```
[Out] x+11/3*x^3+11*x^5+165/7*x^7+110/3*x^9+42*x^11+462/13*x^13+22*x^15+165/17*x^
17+55/19*x^19+11/21*x^21+1/23*x^23
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {28, 200}

$$\int (1+x^2)(1+2x^2+x^4)^5\,dx = \frac{x^{23}}{23}+\frac{11x^{21}}{21}+\frac{55x^{19}}{19}+\frac{165x^{17}}{17}+22x^{15}+\frac{462x^{13}}{13}+42x^{11}+\frac{110x^9}{3}+\frac{165x^7}{7}+11x^5+\frac{11x^3}{3}+x$$

```
[In] Int[(1 + x^2)*(1 + 2*x^2 + x^4)^5,x]
```

```
[Out] x + (11*x^3)/3 + 11*x^5 + (165*x^7)/7 + (110*x^9)/3 + 42*x^11 + (462*x^13)/
13 + 22*x^15 + (165*x^17)/17 + (55*x^19)/19 + (11*x^21)/21 + x^23/23
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
```

```
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \left(1+x^2\right)^{11} \, dx \\ &= \int \left(1+11x^2+55x^4+165x^6+330x^8+462x^{10}+462x^{12}+330x^{14}+165x^{16} \right. \\ &\qquad \left. +55x^{18}+11x^{20}+x^{22}\right) \, dx \\ &= x+\frac{11x^3}{3}+11x^5+\frac{165x^7}{7}+\frac{110x^9}{3}+42x^{11}+\frac{462x^{13}}{13}+22x^{15}+\frac{165x^{17}}{17}+\frac{55x^{19}}{19}+\frac{11x^{21}}{21} \\ &\quad +\frac{x^{23}}{23}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.00

$$\int \left(1+x^2\right) \left(1+2x^2+x^4\right)^5 \, dx = x+\frac{11x^3}{3}+11x^5+\frac{165x^7}{7}+\frac{110x^9}{3}+42x^{11} +\frac{462x^{13}}{13}+22x^{15}+\frac{165x^{17}}{17}+\frac{55x^{19}}{19}+\frac{11x^{21}}{21}+\frac{x^{23}}{23}$$

```
[In] Integrate[(1 + x^2)*(1 + 2*x^2 + x^4)^5,x]
[Out] x + (11*x^3)/3 + 11*x^5 + (165*x^7)/7 + (110*x^9)/3 + 42*x^11 + (462*x^13)/
13 + 22*x^15 + (165*x^17)/17 + (55*x^19)/19 + (11*x^21)/21 + x^23/23
```

Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.79

method	result
default	$x+\frac{11}{3}x^3+11x^5+\frac{165}{7}x^7+\frac{110}{3}x^9+42x^{11}+\frac{462}{13}x^{13}+22x^{15}+\frac{165}{17}x^{17}+\frac{55}{19}x^{19}+\frac{11}{21}x^{21}+\frac{1}{23}x^{2}$
norman	$x+\frac{11}{3}x^3+11x^5+\frac{165}{7}x^7+\frac{110}{3}x^9+42x^{11}+\frac{462}{13}x^{13}+22x^{15}+\frac{165}{17}x^{17}+\frac{55}{19}x^{19}+\frac{11}{21}x^{21}+\frac{1}{23}x^{2}$
risch	$x+\frac{11}{3}x^3+11x^5+\frac{165}{7}x^7+\frac{110}{3}x^9+42x^{11}+\frac{462}{13}x^{13}+22x^{15}+\frac{165}{17}x^{17}+\frac{55}{19}x^{19}+\frac{11}{21}x^{21}+\frac{1}{23}x^{2}$
parallelrisch	$x+\frac{11}{3}x^3+11x^5+\frac{165}{7}x^7+\frac{110}{3}x^9+42x^{11}+\frac{462}{13}x^{13}+22x^{15}+\frac{165}{17}x^{17}+\frac{55}{19}x^{19}+\frac{11}{21}x^{21}+\frac{1}{23}x^{2}$
gosper	$\frac{x\left(88179x^{22}+1062347x^{20}+5870865x^{18}+19684665x^{16}+44618574x^{14}+72076158x^{12}+85180914x^{10}+74364290x^8+47805615x^6}{2028117}$

```
[In] int((x^2+1)*(x^4+2*x^2+1)^5,x,method=_RETURNVERBOSE)
```

```
[Out] x+11/3*x^3+11*x^5+165/7*x^7+110/3*x^9+42*x^11+462/13*x^13+22*x^15+165/17*x^
17+55/19*x^19+11/21*x^21+1/23*x^23
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.78

$$\int \left(1+x^2\right)\left(1+2x^2+x^4\right)^5\,dx=\frac{1}{23}\,x^{23}+\frac{11}{21}\,x^{21}+\frac{55}{19}\,x^{19}+\frac{165}{17}\,x^{17}+22\,x^{15}+\frac{462}{13}\,x^{13}+42\,x^{11}+\frac{110}{3}\,x^{9}+\frac{165}{7}\,x^{7}+11\,x^{5}+\frac{11}{3}\,x^{3}+x$$

```
[In] integrate((x^2+1)*(x^4+2*x^2+1)^5,x, algorithm="fricas")
```

```
[Out] 1/23*x^23 + 11/21*x^21 + 55/19*x^19 + 165/17*x^17 + 22*x^15 + 462/13*x^13 +
 42*x^11 + 110/3*x^9 + 165/7*x^7 + 11*x^5 + 11/3*x^3 + x
```

Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.93

$$\int \left(1+x^2\right)\left(1+2x^2+x^4\right)^5\,dx=\frac{x^{23}}{23}+\frac{11x^{21}}{21}+\frac{55x^{19}}{19}+\frac{165x^{17}}{17}+22x^{15}+\frac{462x^{13}}{13}+42x^{11}+\frac{110x^{9}}{3}+\frac{165x^{7}}{7}+11x^{5}+\frac{11x^{3}}{3}+x$$

```
[In] integrate((x**2+1)*(x**4+2*x**2+1)**5,x)
```

```
[Out] x**23/23 + 11*x**21/21 + 55*x**19/19 + 165*x**17/17 + 22*x**15 + 462*x**13/
13 + 42*x**11 + 110*x**9/3 + 165*x**7/7 + 11*x**5 + 11*x**3/3 + x
```

Maxima [A] (verification not implemented)

none

Time = 0.18 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.78

$$\int (1+x^2)\left(1+2x^2+x^4\right)^5\,dx = \frac{1}{23}\,x^{23} + \frac{11}{21}\,x^{21} + \frac{55}{19}\,x^{19} + \frac{165}{17}\,x^{17} + 22\,x^{15} + \frac{462}{13}\,x^{13} + 42\,x^{11} + \frac{110}{3}\,x^{9} + \frac{165}{7}\,x^{7} + 11\,x^{5} + \frac{11}{3}\,x^{3} + x$$

```
[In] integrate((x^2+1)*(x^4+2*x^2+1)^5,x, algorithm="maxima")
```

```
[Out] 1/23*x^23 + 11/21*x^21 + 55/19*x^19 + 165/17*x^17 + 22*x^15 + 462/13*x^13 +
 42*x^11 + 110/3*x^9 + 165/7*x^7 + 11*x^5 + 11/3*x^3 + x
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.78

$$\int (1+x^2)\left(1+2x^2+x^4\right)^5\,dx = \frac{1}{23}\,x^{23} + \frac{11}{21}\,x^{21} + \frac{55}{19}\,x^{19} + \frac{165}{17}\,x^{17} + 22\,x^{15} + \frac{462}{13}\,x^{13} + 42\,x^{11} + \frac{110}{3}\,x^{9} + \frac{165}{7}\,x^{7} + 11\,x^{5} + \frac{11}{3}\,x^{3} + x$$

```
[In] integrate((x^2+1)*(x^4+2*x^2+1)^5,x, algorithm="giac")
```

```
[Out] 1/23*x^23 + 11/21*x^21 + 55/19*x^19 + 165/17*x^17 + 22*x^15 + 462/13*x^13 +
 42*x^11 + 110/3*x^9 + 165/7*x^7 + 11*x^5 + 11/3*x^3 + x
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.78

$$\int (1+x^2)\left(1+2x^2+x^4\right)^5\,dx = \frac{x^{23}}{23} + \frac{11\,x^{21}}{21} + \frac{55\,x^{19}}{19} + \frac{165\,x^{17}}{17} + 22\,x^{15} + \frac{462\,x^{13}}{13} + 42\,x^{11} + \frac{110\,x^{9}}{3} + \frac{165\,x^{7}}{7} + 11\,x^{5} + \frac{11\,x^{3}}{3} + x$$

```
[In] int((x^2 + 1)*(2*x^2 + x^4 + 1)^5,x)
```

```
[Out] x + (11*x^3)/3 + 11*x^5 + (165*x^7)/7 + (110*x^9)/3 + 42*x^11 + (462*x^13)/
13 + 22*x^15 + (165*x^17)/17 + (55*x^19)/19 + (11*x^21)/21 + x^23/23
```

3.72 $\int \frac{(1+x^2)(1+2x^2+x^4)^5}{x} \, dx$

Optimal result

Integrand size = 21, antiderivative size = 80

$$\int \frac{(1+x^2)(1+2x^2+x^4)^5}{x} \, dx = \frac{11x^2}{2}+\frac{55x^4}{4}+\frac{55x^6}{2}+\frac{165x^8}{4}+\frac{231x^{10}}{5}+\frac{77x^{12}}{2}+\frac{165x^{14}}{7}+\frac{165x^{16}}{16}+\frac{55x^{18}}{18}+\frac{11x^{20}}{20}+\frac{x^{22}}{22}+\log(x)$$

```
[Out] 11/2*x^2+55/4*x^4+55/2*x^6+165/4*x^8+231/5*x^10+77/2*x^12+165/7*x^14+165/16
*x^16+55/18*x^18+11/20*x^20+1/22*x^22+ln(x)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {28, 272, 45}

$$\int \frac{(1+x^2)(1+2x^2+x^4)^5}{x} \, dx = \frac{x^{22}}{22}+\frac{11x^{20}}{20}+\frac{55x^{18}}{18}+\frac{165x^{16}}{16}+\frac{165x^{14}}{7}+\frac{77x^{12}}{2}+\frac{231x^{10}}{5}+\frac{165x^8}{4}+\frac{55x^6}{2}+\frac{55x^4}{4}+\frac{11x^2}{2}+\log(x)$$

```
[In] Int[((1 + x^2)*(1 + 2*x^2 + x^4)^5)/x,x]
```

```
[Out] (11*x^2)/2 + (55*x^4)/4 + (55*x^6)/2 + (165*x^8)/4 + (231*x^10)/5 + (77*x^1
2)/2 + (165*x^14)/7 + (165*x^16)/16 + (55*x^18)/18 + (11*x^20)/20 + x^22/22
 + Log[x]
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
```

```
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \frac{(1+x^2)^{11}}{x}\,dx\\ &= \frac{1}{2}\text{Subst}\left(\int \frac{(1+x)^{11}}{x}\,dx, x, x^2\right)\\ &= \frac{1}{2}\text{Subst}\left(\int \left(11+\frac{1}{x}+55x+165x^2+330x^3+462x^4+462x^5+330x^6+165x^7\right.\right.\\ &\qquad \left.\left.+55x^8+11x^9+x^{10}\right)\,dx, x, x^2\right)\\ &= \frac{11x^2}{2}+\frac{55x^4}{4}+\frac{55x^6}{2}+\frac{165x^8}{4}+\frac{231x^{10}}{5}+\frac{77x^{12}}{2}\\ &\quad +\frac{165x^{14}}{7}+\frac{165x^{16}}{16}+\frac{55x^{18}}{18}+\frac{11x^{20}}{20}+\frac{x^{22}}{22}+\log(x)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.00

$$\int \frac{(1+x^2)(1+2x^2+x^4)^5}{x}\,dx = \frac{11x^2}{2}+\frac{55x^4}{4}+\frac{55x^6}{2}+\frac{165x^8}{4}+\frac{231x^{10}}{5}+\frac{77x^{12}}{2} +\frac{165x^{14}}{7}+\frac{165x^{16}}{16}+\frac{55x^{18}}{18}+\frac{11x^{20}}{20}+\frac{x^{22}}{22}+\log(x)$$

```
[In] Integrate[((1 + x^2)*(1 + 2*x^2 + x^4)^5)/x,x]
```

```
[Out] (11*x^2)/2 + (55*x^4)/4 + (55*x^6)/2 + (165*x^8)/4 + (231*x^10)/5 + (77*x^1
2)/2 + (165*x^14)/7 + (165*x^16)/16 + (55*x^18)/18 + (11*x^20)/20 + x^22/22
 + Log[x]
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.74

method	result	s
default	$\frac{11x^2}{2}+\frac{55x^4}{4}+\frac{55x^6}{2}+\frac{165x^8}{4}+\frac{231x^{10}}{5}+\frac{77x^{12}}{2}+\frac{165x^{14}}{7}+\frac{165x^{16}}{16}+\frac{55x^{18}}{18}+\frac{11x^{20}}{20}+\frac{x^{22}}{22}+\ln(x)$	5
norman	$\frac{11x^2}{2}+\frac{55x^4}{4}+\frac{55x^6}{2}+\frac{165x^8}{4}+\frac{231x^{10}}{5}+\frac{77x^{12}}{2}+\frac{165x^{14}}{7}+\frac{165x^{16}}{16}+\frac{55x^{18}}{18}+\frac{11x^{20}}{20}+\frac{x^{22}}{22}+\ln(x)$	5
risch	$\frac{11x^2}{2}+\frac{55x^4}{4}+\frac{55x^6}{2}+\frac{165x^8}{4}+\frac{231x^{10}}{5}+\frac{77x^{12}}{2}+\frac{165x^{14}}{7}+\frac{165x^{16}}{16}+\frac{55x^{18}}{18}+\frac{11x^{20}}{20}+\frac{x^{22}}{22}+\ln(x)$	5
parallelrisch	$\frac{11x^2}{2}+\frac{55x^4}{4}+\frac{55x^6}{2}+\frac{165x^8}{4}+\frac{231x^{10}}{5}+\frac{77x^{12}}{2}+\frac{165x^{14}}{7}+\frac{165x^{16}}{16}+\frac{55x^{18}}{18}+\frac{11x^{20}}{20}+\frac{x^{22}}{22}+\ln(x)$	5

```
[In] int((x^2+1)*(x^4+2*x^2+1)^5/x,x,method=_RETURNVERBOSE)
```

```
[Out] 11/2*x^2+55/4*x^4+55/2*x^6+165/4*x^8+231/5*x^10+77/2*x^12+165/7*x^14+165/16
*x^16+55/18*x^18+11/20*x^20+1/22*x^22+ln(x)
```

Fricas [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.72

$$\int \frac{(1+x^2)(1+2x^2+x^4)^5}{x}\,dx = \frac{1}{22}x^{22}+\frac{11}{20}x^{20}+\frac{55}{18}x^{18}+\frac{165}{16}x^{16}+\frac{165}{7}x^{14}+\frac{77}{2}x^{12} + \frac{231}{5}x^{10}+\frac{165}{4}x^8+\frac{55}{2}x^6+\frac{55}{4}x^4+\frac{11}{2}x^2+\log(x)$$

```
[In] integrate((x^2+1)*(x^4+2*x^2+1)^5/x,x, algorithm="fricas")
```

```
[Out] 1/22*x^22 + 11/20*x^20 + 55/18*x^18 + 165/16*x^16 + 165/7*x^14 + 77/2*x^12
+ 231/5*x^10 + 165/4*x^8 + 55/2*x^6 + 55/4*x^4 + 11/2*x^2 + log(x)
```

Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.94

$$\int \frac{(1+x^2)(1+2x^2+x^4)^5}{x}\,dx = \frac{x^{22}}{22}+\frac{11x^{20}}{20}+\frac{55x^{18}}{18}+\frac{165x^{16}}{16}+\frac{165x^{14}}{7}+\frac{77x^{12}}{2} + \frac{231x^{10}}{5}+\frac{165x^8}{4}+\frac{55x^6}{2}+\frac{55x^4}{4}+\frac{11x^2}{2}+\log(x)$$

```
[In] integrate((x**2+1)*(x**4+2*x**2+1)**5/x,x)
```

```
[Out] x**22/22 + 11*x**20/20 + 55*x**18/18 + 165*x**16/16 + 165*x**14/7 + 77*x**1
2/2 + 231*x**10/5 + 165*x**8/4 + 55*x**6/2 + 55*x**4/4 + 11*x**2/2 + log(x)
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.78

$$\int \frac{(1+x^2)(1+2x^2+x^4)^5}{x}\,dx = \frac{1}{22}\,x^{22}+\frac{11}{20}\,x^{20}+\frac{55}{18}\,x^{18}+\frac{165}{16}\,x^{16}+\frac{165}{7}\,x^{14}+\frac{77}{2}\,x^{12}+\frac{231}{5}\,x^{10}+\frac{165}{4}\,x^{8}+\frac{55}{2}\,x^{6}+\frac{55}{4}\,x^{4}+\frac{11}{2}\,x^{2}+\frac{1}{2}\,\log\left(x^2\right)$$

```
[In] integrate((x^2+1)*(x^4+2*x^2+1)^5/x,x, algorithm="maxima")
```

```
[Out] 1/22*x^22 + 11/20*x^20 + 55/18*x^18 + 165/16*x^16 + 165/7*x^14 + 77/2*x^12
+ 231/5*x^10 + 165/4*x^8 + 55/2*x^6 + 55/4*x^4 + 11/2*x^2 + 1/2*log(x^2)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.78

$$\int \frac{(1+x^2)(1+2x^2+x^4)^5}{x}\,dx = \frac{1}{22}\,x^{22}+\frac{11}{20}\,x^{20}+\frac{55}{18}\,x^{18}+\frac{165}{16}\,x^{16}+\frac{165}{7}\,x^{14}+\frac{77}{2}\,x^{12}+\frac{231}{5}\,x^{10}+\frac{165}{4}\,x^{8}+\frac{55}{2}\,x^{6}+\frac{55}{4}\,x^{4}+\frac{11}{2}\,x^{2}+\frac{1}{2}\,\log\left(x^2\right)$$

```
[In] integrate((x^2+1)*(x^4+2*x^2+1)^5/x,x, algorithm="giac")
```

```
[Out] 1/22*x^22 + 11/20*x^20 + 55/18*x^18 + 165/16*x^16 + 165/7*x^14 + 77/2*x^12
+ 231/5*x^10 + 165/4*x^8 + 55/2*x^6 + 55/4*x^4 + 11/2*x^2 + 1/2*log(x^2)
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.72

$$\int \frac{(1+x^2)(1+2x^2+x^4)^5}{x}\,dx = \ln(x)+\frac{11\,x^2}{2}+\frac{55\,x^4}{4}+\frac{55\,x^6}{2}+\frac{165\,x^8}{4}+\frac{231\,x^{10}}{5}+\frac{77\,x^{12}}{2}+\frac{165\,x^{14}}{7}+\frac{165\,x^{16}}{16}+\frac{55\,x^{18}}{18}+\frac{11\,x^{20}}{20}+\frac{x^{22}}{22}$$

```
[In] int(((x^2 + 1)*(2*x^2 + x^4 + 1)^5)/x,x)
```

```
[Out] log(x) + (11*x^2)/2 + (55*x^4)/4 + (55*x^6)/2 + (165*x^8)/4 + (231*x^10)/5
+ (77*x^12)/2 + (165*x^14)/7 + (165*x^16)/16 + (55*x^18)/18 + (11*x^20)/20
+ x^22/22
```

3.73 $\int \frac{(1+x^2)(1+2x^2+x^4)^5}{x^2} dx$

Optimal result

Integrand size = 21, antiderivative size = 73

$$\int \frac{(1+x^2)(1+2x^2+x^4)^5}{x^2} dx = -\frac{1}{x}+11x+\frac{55x^3}{3}+33x^5+\frac{330x^7}{7}+\frac{154x^9}{3}+42x^{11}+\frac{330x^{13}}{13}+11x^{15}+\frac{55x^{17}}{17}+\frac{11x^{19}}{19}+\frac{x^{21}}{21}$$

```
[Out] -1/x+11*x+55/3*x^3+33*x^5+330/7*x^7+154/3*x^9+42*x^11+330/13*x^13+11*x^15+5
5/17*x^17+11/19*x^19+1/21*x^21
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {28, 276}

$$\int \frac{(1+x^2)(1+2x^2+x^4)^5}{x^2} dx = \frac{x^{21}}{21}+\frac{11x^{19}}{19}+\frac{55x^{17}}{17}+11x^{15}+\frac{330x^{13}}{13}+42x^{11}+\frac{154x^9}{3}+\frac{330x^7}{7}+33x^5+\frac{55x^3}{3}+11x-\frac{1}{x}$$

```
[In] Int[((1 + x^2)*(1 + 2*x^2 + x^4)^5)/x^2,x]
```

```
[Out] -x^(-1) + 11*x + (55*x^3)/3 + 33*x^5 + (330*x^7)/7 + (154*x^9)/3 + 42*x^11
+ (330*x^13)/13 + 11*x^15 + (55*x^17)/17 + (11*x^19)/19 + x^21/21
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
```

```
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \frac{(1+x^2)^{11}}{x^2}\,dx \\
&= \int \left(11+\frac{1}{x^2}+55x^2+165x^4+330x^6+462x^8+462x^{10}+330x^{12}+165x^{14}+55x^{16}+11x^{18}+x^{20}\right) dx \\
&= -\frac{1}{x}+11x+\frac{55x^3}{3}+33x^5+\frac{330x^7}{7}+\frac{154x^9}{3}+42x^{11}+\frac{330x^{13}}{13}+11x^{15}+\frac{55x^{17}}{17}+\frac{11x^{19}}{19}+\frac{x^{21}}{21}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.00

$$\int \frac{(1+x^2)(1+2x^2+x^4)^5}{x^2}\,dx = -\frac{1}{x}+11x+\frac{55x^3}{3}+33x^5+\frac{330x^7}{7}+\frac{154x^9}{3}+42x^{11}+\frac{330x^{13}}{13}+11x^{15}+\frac{55x^{17}}{17}+\frac{11x^{19}}{19}+\frac{x^{21}}{21}$$

```
[In] Integrate[((1 + x^2)*(1 + 2*x^2 + x^4)^5)/x^2,x]
```

```
[Out] -x^(-1) + 11*x + (55*x^3)/3 + 33*x^5 + (330*x^7)/7 + (154*x^9)/3 + 42*x^11
+ (330*x^13)/13 + 11*x^15 + (55*x^17)/17 + (11*x^19)/19 + x^21/21
```

Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.82

method	result
default	$-\frac{1}{x}+11x+\frac{55x^{3}}{3}+33x^{5}+\frac{330x^{7}}{7}+\frac{154x^{9}}{3}+42x^{11}+\frac{330x^{13}}{13}+11x^{15}+\frac{55x^{17}}{17}+\frac{11x^{19}}{19}+\frac{x^{21}}{21}$
risch	$-\frac{1}{x}+11x+\frac{55x^{3}}{3}+33x^{5}+\frac{330x^{7}}{7}+\frac{154x^{9}}{3}+42x^{11}+\frac{330x^{13}}{13}+11x^{15}+\frac{55x^{17}}{17}+\frac{11x^{19}}{19}+\frac{x^{21}}{21}$
norman	$\frac{-1+11x^{2}+\frac{55}{3}x^{4}+33x^{6}+\frac{330}{7}x^{8}+\frac{154}{3}x^{10}+42x^{12}+\frac{330}{13}x^{14}+11x^{16}+\frac{55}{17}x^{18}+\frac{11}{19}x^{20}+\frac{1}{21}x^{22}}{x}$
gosper	$\frac{4199x^{22}+51051x^{20}+285285x^{18}+969969x^{16}+2238390x^{14}+3703518x^{12}+4526522x^{10}+4157010x^{8}+2909907x^{6}+1616615x^{4}+9}{88179x}$
parallelrisch	$\frac{4199x^{22}+51051x^{20}+285285x^{18}+969969x^{16}+2238390x^{14}+3703518x^{12}+4526522x^{10}+4157010x^{8}+2909907x^{6}+1616615x^{4}+9}{88179x}$

```
[In] int((x^2+1)*(x^4+2*x^2+1)^5/x^2,x,method=_RETURNVERBOSE)
[Out] -1/x+11*x+55/3*x^3+33*x^5+330/7*x^7+154/3*x^9+42*x^11+330/13*x^13+11*x^15+5
5/17*x^17+11/19*x^19+1/21*x^21
```

Fricas [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.85

$$\int \frac{(1+x^2)(1+2x^2+x^4)^5}{x^2}\,dx = \frac{4199\,x^{22}+51051\,x^{20}+285285\,x^{18}+969969\,x^{16}+2238390\,x^{14}+3703518\,x^{12}+4526522\,x^{10}+4157010\,x}{88179\,x}$$

```
[In] integrate((x^2+1)*(x^4+2*x^2+1)^5/x^2,x, algorithm="fricas")
[Out] 1/88179*(4199*x^22 + 51051*x^20 + 285285*x^18 + 969969*x^16 + 2238390*x^14
+ 3703518*x^12 + 4526522*x^10 + 4157010*x^8 + 2909907*x^6 + 1616615*x^4 + 9
69969*x^2 - 88179)/x
```

Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.90

$$\int \frac{(1+x^2)(1+2x^2+x^4)^5}{x^2}\,dx = \frac{x^{21}}{21}+\frac{11x^{19}}{19}+\frac{55x^{17}}{17}+11x^{15}+\frac{330x^{13}}{13}+42x^{11}+\frac{154x^{9}}{3}+\frac{330x^{7}}{7}+33x^{5}+\frac{55x^{3}}{3}+11x-\frac{1}{x}$$

```
[In] integrate((x**2+1)*(x**4+2*x**2+1)**5/x**2,x)
[Out] x**21/21 + 11*x**19/19 + 55*x**17/17 + 11*x**15 + 330*x**13/13 + 42*x**11 +
 154*x**9/3 + 330*x**7/7 + 33*x**5 + 55*x**3/3 + 11*x - 1/x
```

Maxima [A] (verification not implemented)

none

Time = 0.22 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.81

$$\int \frac{(1+x^2)(1+2x^2+x^4)^5}{x^2}\,dx = \frac{1}{21}\,x^{21} + \frac{11}{19}\,x^{19} + \frac{55}{17}\,x^{17} + 11\,x^{15} + \frac{330}{13}\,x^{13} + 42\,x^{11} + \frac{154}{3}\,x^9 + \frac{330}{7}\,x^7 + 33\,x^5 + \frac{55}{3}\,x^3 + 11\,x - \frac{1}{x}$$

```
[In] integrate((x^2+1)*(x^4+2*x^2+1)^5/x^2,x, algorithm="maxima")
```

```
[Out] 1/21*x^21 + 11/19*x^19 + 55/17*x^17 + 11*x^15 + 330/13*x^13 + 42*x^11 + 154
/3*x^9 + 330/7*x^7 + 33*x^5 + 55/3*x^3 + 11*x - 1/x
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.81

$$\int \frac{(1+x^2)(1+2x^2+x^4)^5}{x^2}\,dx = \frac{1}{21}\,x^{21} + \frac{11}{19}\,x^{19} + \frac{55}{17}\,x^{17} + 11\,x^{15} + \frac{330}{13}\,x^{13} + 42\,x^{11} + \frac{154}{3}\,x^9 + \frac{330}{7}\,x^7 + 33\,x^5 + \frac{55}{3}\,x^3 + 11\,x - \frac{1}{x}$$

```
[In] integrate((x^2+1)*(x^4+2*x^2+1)^5/x^2,x, algorithm="giac")
```

```
[Out] 1/21*x^21 + 11/19*x^19 + 55/17*x^17 + 11*x^15 + 330/13*x^13 + 42*x^11 + 154
/3*x^9 + 330/7*x^7 + 33*x^5 + 55/3*x^3 + 11*x - 1/x
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.81

$$\int \frac{(1+x^2)(1+2x^2+x^4)^5}{x^2}\,dx = 11\,x - \frac{1}{x} + \frac{55\,x^3}{3} + 33\,x^5 + \frac{330\,x^7}{7} + \frac{154\,x^9}{3} + 42\,x^{11} + \frac{330\,x^{13}}{13} + 11\,x^{15} + \frac{55\,x^{17}}{17} + \frac{11\,x^{19}}{19} + \frac{x^{21}}{21}$$

```
[In] int(((x^2 + 1)*(2*x^2 + x^4 + 1)^5)/x^2,x)
```

```
[Out] 11*x - 1/x + (55*x^3)/3 + 33*x^5 + (330*x^7)/7 + (154*x^9)/3 + 42*x^11 + (3
30*x^13)/13 + 11*x^15 + (55*x^17)/17 + (11*x^19)/19 + x^21/21
```

3.74 $\int \frac{(1+x^2)(1+2x^2+x^4)^5}{x^3} \, dx$

Optimal result

Integrand size = 21, antiderivative size = 80

$$\int \frac{(1+x^2)(1+2x^2+x^4)^5}{x^3} \, dx = -\frac{1}{2x^2}+\frac{55x^2}{2}+\frac{165x^4}{4}+55x^6+\frac{231x^8}{4}+\frac{231x^{10}}{5}+\frac{55x^{12}}{2}+\frac{165x^{14}}{14}+\frac{55x^{16}}{16}+\frac{11x^{18}}{18}+\frac{x^{20}}{20}+11\log(x)$$

```
[Out] -1/2/x^2+55/2*x^2+165/4*x^4+55*x^6+231/4*x^8+231/5*x^10+55/2*x^12+165/14*x^
14+55/16*x^16+11/18*x^18+1/20*x^20+11*ln(x)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {28, 272, 45}

$$\int \frac{(1+x^2)(1+2x^2+x^4)^5}{x^3} \, dx = \frac{x^{20}}{20}+\frac{11x^{18}}{18}+\frac{55x^{16}}{16}+\frac{165x^{14}}{14}+\frac{55x^{12}}{2}+\frac{231x^{10}}{5}+\frac{231x^8}{4}+55x^6+\frac{165x^4}{4}+\frac{55x^2}{2}-\frac{1}{2x^2}+11\log(x)$$

```
[In] Int[((1 + x^2)*(1 + 2*x^2 + x^4)^5)/x^3,x]
```

```
[Out] -1/2*1/x^2 + (55*x^2)/2 + (165*x^4)/4 + 55*x^6 + (231*x^8)/4 + (231*x^10)/5
 + (55*x^12)/2 + (165*x^14)/14 + (55*x^16)/16 + (11*x^18)/18 + x^20/20 + 11
*Log[x]
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \frac{(1+x^2)^{11}}{x^3}\,dx \\
&= \frac{1}{2}\text{Subst}\left(\int \frac{(1+x)^{11}}{x^2}\,dx, x, x^2\right) \\
&= \frac{1}{2}\text{Subst}\left(\int \left(55+\frac{1}{x^2}+\frac{11}{x}+165x+330x^2+462x^3+462x^4+330x^5+165x^6 \right.\right. \\
&\qquad \left.\left. +55x^7+11x^8+x^9\right)\,dx, x, x^2\right) \\
&= -\frac{1}{2x^2}+\frac{55x^2}{2}+\frac{165x^4}{4}+55x^6+\frac{231x^8}{4}+\frac{231x^{10}}{5} \\
&\quad +\frac{55x^{12}}{2}+\frac{165x^{14}}{14}+\frac{55x^{16}}{16}+\frac{11x^{18}}{18}+\frac{x^{20}}{20}+11\log(x)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.00

$$\int \frac{(1+x^2)(1+2x^2+x^4)^5}{x^3}\,dx = -\frac{1}{2x^2}+\frac{55x^2}{2}+\frac{165x^4}{4}+55x^6+\frac{231x^8}{4}+\frac{231x^{10}}{5} +\frac{55x^{12}}{2}+\frac{165x^{14}}{14}+\frac{55x^{16}}{16}+\frac{11x^{18}}{18}+\frac{x^{20}}{20}+11\log(x)$$

```
[In] Integrate[((1 + x^2)*(1 + 2*x^2 + x^4)^5)/x^3,x]
[Out] -1/2*1/x^2 + (55*x^2)/2 + (165*x^4)/4 + 55*x^6 + (231*x^8)/4 + (231*x^10)/5
 + (55*x^12)/2 + (165*x^14)/14 + (55*x^16)/16 + (11*x^18)/18 + x^20/20 + 11
*Log[x]
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.76

method	result
default	$-\frac{1}{2x^{2}}+\frac{55x^{2}}{2}+\frac{165x^{4}}{4}+55x^{6}+\frac{231x^{8}}{4}+\frac{231x^{10}}{5}+\frac{55x^{12}}{2}+\frac{165x^{14}}{14}+\frac{55x^{16}}{16}+\frac{11x^{18}}{18}+\frac{x^{20}}{20}+11\ln (x$
risch	$-\frac{1}{2x^{2}}+\frac{55x^{2}}{2}+\frac{165x^{4}}{4}+55x^{6}+\frac{231x^{8}}{4}+\frac{231x^{10}}{5}+\frac{55x^{12}}{2}+\frac{165x^{14}}{14}+\frac{55x^{16}}{16}+\frac{11x^{18}}{18}+\frac{x^{20}}{20}+11\ln (x$
norman	$\frac{-\frac{1}{2}+\frac{55}{2}x^{4}+\frac{165}{4}x^{6}+55x^{8}+\frac{231}{4}x^{10}+\frac{231}{5}x^{12}+\frac{55}{2}x^{14}+\frac{165}{14}x^{16}+\frac{55}{16}x^{18}+\frac{11}{18}x^{20}+\frac{1}{20}x^{22}}{x^{2}}+11\ln \left(x\right)$
parallelrisch	$\frac{252x^{22}+3080x^{20}+17325x^{18}+59400x^{16}+138600x^{14}+232848x^{12}+291060x^{10}+277200x^{8}+207900x^{6}+138600x^{4}+55440\ln(x)x}{5040x^{2}}$

```
[In] int((x^2+1)*(x^4+2*x^2+1)^5/x^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/2/x^2+55/2*x^2+165/4*x^4+55*x^6+231/4*x^8+231/5*x^10+55/2*x^12+165/14*x^
14+55/16*x^16+11/18*x^18+1/20*x^20+11*ln(x)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.80

$$\int \frac{(1+x^2)\left(1+2x^2+x^4\right)^5}{x^3}\,dx = \frac{252\,x^{22}+3080\,x^{20}+17325\,x^{18}+59400\,x^{16}+138600\,x^{14}+232848\,x^{12}+291060\,x^{10}+277200\,x^{8}+20790}{5040\,x^{2}}$$

```
[In] integrate((x^2+1)*(x^4+2*x^2+1)^5/x^3,x, algorithm="fricas")
```

```
[Out] 1/5040*(252*x^22 + 3080*x^20 + 17325*x^18 + 59400*x^16 + 138600*x^14 + 2328
48*x^12 + 291060*x^10 + 277200*x^8 + 207900*x^6 + 138600*x^4 + 55440*x^2*lo
g(x) - 2520)/x^2
```

Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.94

$$\int \frac{(1+x^2)\left(1+2x^2+x^4\right)^5}{x^3}\,dx = \frac{x^{20}}{20}+\frac{11x^{18}}{18}+\frac{55x^{16}}{16}+\frac{165x^{14}}{14}+\frac{55x^{12}}{2}+\frac{231x^{10}}{5} + \frac{231x^{8}}{4}+55x^{6}+\frac{165x^{4}}{4}+\frac{55x^{2}}{2}+11\log\left(x\right)-\frac{1}{2x^{2}}$$

```
[In] integrate((x**2+1)*(x**4+2*x**2+1)**5/x**3,x)
```

```
[Out] x**20/20 + 11*x**18/18 + 55*x**16/16 + 165*x**14/14 + 55*x**12/2 + 231*x**1
0/5 + 231*x**8/4 + 55*x**6 + 165*x**4/4 + 55*x**2/2 + 11*log(x) - 1/(2*x**2
)
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.78

$$\int \frac{(1+x^2)(1+2x^2+x^4)^5}{x^3}\,dx = \frac{1}{20}x^{20} + \frac{11}{18}x^{18} + \frac{55}{16}x^{16} + \frac{165}{14}x^{14} + \frac{55}{2}x^{12} + \frac{231}{5}x^{10} + \frac{231}{4}x^8 + 55\,x^6 + \frac{165}{4}x^4 + \frac{55}{2}x^2 - \frac{1}{2\,x^2} + \frac{11}{2}\log\left(x^2\right)$$

```
[In] integrate((x^2+1)*(x^4+2*x^2+1)^5/x^3,x, algorithm="maxima")
```

```
[Out] 1/20*x^20 + 11/18*x^18 + 55/16*x^16 + 165/14*x^14 + 55/2*x^12 + 231/5*x^10
+ 231/4*x^8 + 55*x^6 + 165/4*x^4 + 55/2*x^2 - 1/2/x^2 + 11/2*log(x^2)
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.86

$$\int \frac{(1+x^2)(1+2x^2+x^4)^5}{x^3}\,dx = \frac{1}{20}x^{20} + \frac{11}{18}x^{18} + \frac{55}{16}x^{16} + \frac{165}{14}x^{14} + \frac{55}{2}x^{12} + \frac{231}{5}x^{10} + \frac{231}{4}x^8 + 55\,x^6 + \frac{165}{4}x^4 + \frac{55}{2}x^2 - \frac{11\,x^2+1}{2\,x^2} + \frac{11}{2}\log\left(x^2\right)$$

```
[In] integrate((x^2+1)*(x^4+2*x^2+1)^5/x^3,x, algorithm="giac")
```

```
[Out] 1/20*x^20 + 11/18*x^18 + 55/16*x^16 + 165/14*x^14 + 55/2*x^12 + 231/5*x^10
+ 231/4*x^8 + 55*x^6 + 165/4*x^4 + 55/2*x^2 - 1/2*(11*x^2 + 1)/x^2 + 11/2*l
og(x^2)
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.75

$$\int \frac{(1+x^2)(1+2x^2+x^4)^5}{x^3}\,dx = 11\,\ln(x) - \frac{1}{2\,x^2} + \frac{55\,x^2}{2} + \frac{165\,x^4}{4} + 55\,x^6 + \frac{231\,x^8}{4} + \frac{231\,x^{10}}{5} + \frac{55\,x^{12}}{2} + \frac{165\,x^{14}}{14} + \frac{55\,x^{16}}{16} + \frac{11\,x^{18}}{18} + \frac{x^{20}}{20}$$

```
[In] int(((x^2 + 1)*(2*x^2 + x^4 + 1)^5)/x^3,x)
```

```
[Out] 11*log(x) - 1/(2*x^2) + (55*x^2)/2 + (165*x^4)/4 + 55*x^6 + (231*x^8)/4 + (
231*x^10)/5 + (55*x^12)/2 + (165*x^14)/14 + (55*x^16)/16 + (11*x^18)/18 + x
^20/20
```

3.75 $\int \frac{x^2(d+ex^2)}{\sqrt{a^2+2abx^2+b^2x^4}}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 145

$$\int \frac{x^2(d+ex^2)}{\sqrt{a^2+2abx^2+b^2x^4}}\,dx = \frac{(bd-ae)x(a+bx^2)}{b^2\sqrt{a^2+2abx^2+b^2x^4}}+\frac{ex^3(a+bx^2)}{3b\sqrt{a^2+2abx^2+b^2x^4}} - \frac{\sqrt{a}(bd-ae)\,(a+bx^2)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{b^{5/2}\sqrt{a^2+2abx^2+b^2x^4}}$$

```
[Out] (-a*e+b*d)*x*(b*x^2+a)/b^2/((b*x^2+a)^2)^(1/2)+1/3*e*x^3*(b*x^2+a)/b/((b*x^
2+a)^2)^(1/2)-(-a*e+b*d)*(b*x^2+a)*arctan(x*b^(1/2)/a^(1/2))*a^(1/2)/b^(5/2
)/((b*x^2+a)^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.121, Rules used = {1264, 470, 327, 211}

$$\int \frac{x^2(d+ex^2)}{\sqrt{a^2+2abx^2+b^2x^4}}\,dx = -\frac{\sqrt{a}(a+bx^2)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)(bd-ae)}{b^{5/2}\sqrt{a^2+2abx^2+b^2x^4}} + \frac{x(a+bx^2)\,(bd-ae)}{b^2\sqrt{a^2+2abx^2+b^2x^4}}+\frac{ex^3(a+bx^2)}{3b\sqrt{a^2+2abx^2+b^2x^4}}$$

```
[In] Int[(x^2*(d + e*x^2))/Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4],x]
```

```
[Out] ((b*d - a*e)*x*(a + b*x^2))/(b^2*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (e*x^3*
(a + b*x^2))/(3*b*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (Sqrt[a]*(b*d - a*e)*(
a + b*x^2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(b^(5/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2
*x^4])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 1264

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_), x_Symbol] :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^Int
Part[p]*(b/2 + c*x^2)^(2*FracPart[p])), Int[(f*x)^m*(d + e*x^2)^q*(b/2 + c*
x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, f, m, p, q}, x] && EqQ[b^2 - 4*
a*c, 0] &&  !IntegerQ[p]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{(ab+b^2x^2)\int \frac{x^2(d+ex^2)}{ab+b^2x^2}\,dx}{\sqrt{a^2+2abx^2+b^2x^4}}\\
&= \frac{ex^3(a+bx^2)}{3b\sqrt{a^2+2abx^2+b^2x^4}} - \frac{((-3b^2d+3abe)\,(ab+b^2x^2))\int \frac{x^2}{ab+b^2x^2}\,dx}{3b^2\sqrt{a^2+2abx^2+b^2x^4}}\\
&= \frac{(bd-ae)x(a+bx^2)}{b^2\sqrt{a^2+2abx^2+b^2x^4}} + \frac{ex^3(a+bx^2)}{3b\sqrt{a^2+2abx^2+b^2x^4}} + \frac{(a(-3b^2d+3abe)\,(ab+b^2x^2))\int \frac{1}{ab+b^2x^2}\,dx}{3b^3\sqrt{a^2+2abx^2+b^2x^4}}\\
&= \frac{(bd-ae)x(a+bx^2)}{b^2\sqrt{a^2+2abx^2+b^2x^4}} + \frac{ex^3(a+bx^2)}{3b\sqrt{a^2+2abx^2+b^2x^4}} - \frac{\sqrt{a}(bd-ae)\,(a+bx^2)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{b^{5/2}\sqrt{a^2+2abx^2+b^2x^4}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 1.05 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.55

$$\int \frac{x^2(d+ex^2)}{\sqrt{a^2+2abx^2+b^2x^4}}\,dx = \frac{(a+bx^2)\left(\sqrt{b}x(3bd-3ae+bex^2)+3\sqrt{a}(-bd+ae)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\right)}{3b^{5/2}\sqrt{(a+bx^2)^2}}$$

```
[In] Integrate[(x^2*(d + e*x^2))/Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4],x]
[Out] ((a + b*x^2)*(Sqrt[b]*x*(3*b*d - 3*a*e + b*e*x^2) + 3*Sqrt[a]*(-(b*d) + a*e
)*ArcTan[(Sqrt[b]*x)/Sqrt[a]]))/(3*b^(5/2)*Sqrt[(a + b*x^2)^2])
```

Maple [A] (verified)

Time = 2.25 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.63

method	result
default	$-\frac{(b\,x^2+a)\left(-\sqrt{ab}\,be\,x^3+3\sqrt{ab}\,aex-3\sqrt{ab}\,bdx-3\arctan\left(\frac{bx}{\sqrt{ab}}\right)a^2e+3\arctan\left(\frac{bx}{\sqrt{ab}}\right)abd\right)}{3\sqrt{(b\,x^2+a)^2}\,b^2\sqrt{ab}}$
risch	$\frac{\sqrt{(b\,x^2+a)^2}\,\left(\frac{1}{3}e\,x^3b-aex+bdx\right)}{(b\,x^2+a)b^2}+\frac{\sqrt{(b\,x^2+a)^2}\,\sqrt{-ab}\,(ae-bd)\ln\left(-\sqrt{-ab}\,x+a\right)}{2(b\,x^2+a)b^3}-\frac{\sqrt{(b\,x^2+a)^2}\,\sqrt{-ab}\,(ae-bd)\ln\left(\sqrt{-ab}\,x+a\right)}{2(b\,x^2+a)b^3}$

```
[In] int(x^2*(e*x^2+d)/((b*x^2+a)^2)^(1/2),x,method=_RETURNVERBOSE)
[Out] -1/3*(b*x^2+a)*(-(a*b)^(1/2)*b*e*x^3+3*(a*b)^(1/2)*a*e*x-3*(a*b)^(1/2)*b*d*
x-3*arctan(b*x/(a*b)^(1/2))*a^2*e+3*arctan(b*x/(a*b)^(1/2))*a*b*d)/((b*x^2+
a)^2)^(1/2)/b^2/(a*b)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 129, normalized size of antiderivative = 0.89

$$\int \frac{x^2(d+ex^2)}{\sqrt{a^2+2abx^2+b^2x^4}}\,dx = \left[\frac{2\,bex^3-3\,(bd-ae)\sqrt{-\frac{a}{b}}\log\left(\frac{bx^2+2\,bx\sqrt{-\frac{a}{b}}-a}{bx^2+a}\right)+6\,(bd-ae)x}{6\,b^2}, \frac{bex^3-3\,(bd-ae)\sqrt{\frac{a}{b}}\arctan\left(\frac{bx\sqrt{\frac{a}{b}}}{a}\right)+}{3\,b^2}\right.$$

```
[In] integrate(x^2*(e*x^2+d)/((b*x^2+a)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/6*(2*b*e*x^3 - 3*(b*d - a*e)*sqrt(-a/b)*log((b*x^2 + 2*b*x*sqrt(-a/b) -
a)/(b*x^2 + a)) + 6*(b*d - a*e)*x)/b^2, 1/3*(b*e*x^3 - 3*(b*d - a*e)*sqrt(a
/b)*arctan(b*x*sqrt(a/b)/a) + 3*(b*d - a*e)*x)/b^2]
```

Sympy [F]

$$\int \frac{x^2(d+ex^2)}{\sqrt{a^2+2abx^2+b^2x^4}}\,dx = \int \frac{x^2(d+ex^2)}{\sqrt{(a+bx^2)^2}}\,dx$$

```
[In] integrate(x**2*(e*x**2+d)/((b*x**2+a)**2)**(1/2),x)
```

```
[Out] Integral(x**2*(d + e*x**2)/sqrt((a + b*x**2)**2), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.37

$$\int \frac{x^2(d+ex^2)}{\sqrt{a^2+2abx^2+b^2x^4}}\,dx = -\frac{(abd-a^2e)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}b^2} + \frac{bex^3+3\,(bd-ae)x}{3\,b^2}$$

```
[In] integrate(x^2*(e*x^2+d)/((b*x^2+a)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] -(a*b*d - a^2*e)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b^2) + 1/3*(b*e*x^3 + 3*(
b*d - a*e)*x)/b^2
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.68

$$\int \frac{x^2(d+ex^2)}{\sqrt{a^2+2abx^2+b^2x^4}}\,dx = -\frac{(abd\text{sgn}(bx^2+a)-a^2e\text{sgn}(bx^2+a))\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}b^2} + \frac{b^2ex^3\text{sgn}(bx^2+a)+3\,b^2dx\text{sgn}(bx^2+a)-3\,abex\text{sgn}(bx^2+a)}{3\,b^3}$$

```
[In] integrate(x^2*(e*x^2+d)/((b*x^2+a)^2)^(1/2),x, algorithm="giac")
```

```
[Out] -(a*b*d*sgn(b*x^2 + a) - a^2*e*sgn(b*x^2 + a))*arctan(b*x/sqrt(a*b))/(sqrt(
a*b)*b^2) + 1/3*(b^2*e*x^3*sgn(b*x^2 + a) + 3*b^2*d*x*sgn(b*x^2 + a) - 3*a*
b*e*x*sgn(b*x^2 + a))/b^3
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2(d+ex^2)}{\sqrt{a^2+2abx^2+b^2x^4}}\,dx = \int \frac{x^2\,(e\,x^2+d)}{\sqrt{(b\,x^2+a)^2}}\,dx$$

```
[In] int((x^2*(d + e*x^2))/((a + b*x^2)^2)^(1/2),x)
[Out] int((x^2*(d + e*x^2))/((a + b*x^2)^2)^(1/2), x)
```

3.76 $\int \frac{x(d+ex^2)}{\sqrt{a^2+2abx^2+b^2x^4}}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 83

$$\int \frac{x(d+ex^2)}{\sqrt{a^2+2abx^2+b^2x^4}}\,dx = \frac{e\sqrt{a^2+2abx^2+b^2x^4}}{2b^2} + \frac{(bd-ae)\,(a+bx^2)\log(a+bx^2)}{2b^2\sqrt{a^2+2abx^2+b^2x^4}}$$

```
[Out] 1/2*(-a*e+b*d)*(b*x^2+a)*ln(b*x^2+a)/b^2/((b*x^2+a)^2)^(1/2)+1/2*e*((b*x^2+
a)^2)^(1/2)/b^2
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.129, Rules used = {1261, 654, 622, 31}

$$\int \frac{x(d+ex^2)}{\sqrt{a^2+2abx^2+b^2x^4}}\,dx = \frac{(a+bx^2)\,(bd-ae)\log(a+bx^2)}{2b^2\sqrt{a^2+2abx^2+b^2x^4}} + \frac{e\sqrt{a^2+2abx^2+b^2x^4}}{2b^2}$$

```
[In] Int[(x*(d + e*x^2))/Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4],x]
```

```
[Out] (e*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(2*b^2) + ((b*d - a*e)*(a + b*x^2)*Log[
a + b*x^2])/(2*b^2*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 622

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[(b/2 + c*x)
/Sqrt[a + b*x + c*x^2], Int[1/(b/2 + c*x), x], x] /; FreeQ[{a, b, c}, x] &&
```

```
 EqQ[b^2 - 4*a*c, 0]
```

Rule 654

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 1261

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(
p_.), x_Symbol] :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x],
 x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{1}{2}\text{Subst}\left(\int \frac{d+ex}{\sqrt{a^2+2abx+b^2x^2}}\,dx, x, x^2\right)\\
&= \frac{e\sqrt{a^2+2abx^2+b^2x^4}}{2b^2} + \frac{(bd-ae)\text{Subst}\left(\int \frac{1}{\sqrt{a^2+2abx+b^2x^2}}\,dx, x, x^2\right)}{2b}\\
&= \frac{e\sqrt{a^2+2abx^2+b^2x^4}}{2b^2} + \frac{\left((bd-ae)\left(ab+b^2x^2\right)\right)\text{Subst}\left(\int \frac{1}{ab+b^2x}\,dx, x, x^2\right)}{2b\sqrt{a^2+2abx^2+b^2x^4}}\\
&= \frac{e\sqrt{a^2+2abx^2+b^2x^4}}{2b^2} + \frac{(bd-ae)\left(a+bx^2\right)\log\left(a+bx^2\right)}{2b^2\sqrt{a^2+2abx^2+b^2x^4}}
\end{aligned}
$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 276 vs. 2(83) = 166.

Time = 0.55 (sec) , antiderivative size = 276, normalized size of antiderivative = 3.33

$$
\int \frac{x(d+ex^2)}{\sqrt{a^2+2abx^2+b^2x^4}}\,dx = -\frac{(2a+bx^2)\left(bex^2\left(\sqrt{a^2}bx^2+a\left(\sqrt{a^2}-\sqrt{(a+bx^2)^2}\right)\right)-(-bd+ae)\left(-a^2-abx^2+\sqrt{a^2}\sqrt{(a+bx^2)^2}\right.\right.}{2b^2\left(\sqrt{a^2}-\sqrt{(a+b\right.}}
$$

```
[In] Integrate[(x*(d + e*x^2))/Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4],x]
```

```
[Out] -1/2*((2*a + b*x^2)*(b*e*x^2*(Sqrt[a^2]*b*x^2 + a*(Sqrt[a^2] - Sqrt[(a + b*
x^2)^2])) - (-(b*d) + a*e)*(-a^2 - a*b*x^2 + Sqrt[a^2]*Sqrt[(a + b*x^2)^2])
```

```
*Log[Sqrt[a^2] - b*x^2 - Sqrt[(a + b*x^2)^2]] + (-(b*d) + a*e)*(-a^2 - a*b*
x^2 + Sqrt[a^2]*Sqrt[(a + b*x^2)^2])*Log[b^2*(Sqrt[a^2] + b*x^2 - Sqrt[(a +
 b*x^2)^2])]))/(b^2*(Sqrt[a^2] - Sqrt[(a + b*x^2)^2])*(Sqrt[a^2]*b*x^2 + a*
(Sqrt[a^2] - Sqrt[(a + b*x^2)^2])))
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.31 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.54

method	result	size
pseudoelliptic	$-\frac{\left(-e\,x^{2} b+\ln\left(b\,x^{2}+a\right) a e-\ln\left(b\,x^{2}+a\right) b d\right) \operatorname{csgn}\left(b\,x^{2}+a\right)}{2 b^{2}}$	45
default	$-\frac{\left(b\,x^{2}+a\right)\left(-e\,x^{2} b+\ln\left(b\,x^{2}+a\right) a e-\ln\left(b\,x^{2}+a\right) b d\right)}{2\sqrt{\left(b\,x^{2}+a\right)^{2}}\, b^{2}}$	55
risch	$\frac{\sqrt{\left(b\,x^{2}+a\right)^{2}}\, e\,x^{2}}{2\left(b\,x^{2}+a\right) b}-\frac{\sqrt{\left(b\,x^{2}+a\right)^{2}}\,\left(a e-b d\right)\ln\left(b\,x^{2}+a\right)}{2\left(b\,x^{2}+a\right) b^{2}}$	72

```
[In] int(x*(e*x^2+d)/((b*x^2+a)^2)^(1/2),x,method=_RETURNVERBOSE)
[Out] -1/2*(-e*x^2*b+ln(b*x^2+a)*a*e-ln(b*x^2+a)*b*d)*csgn(b*x^2+a)/b^2
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.35

$$\int \frac{x(d+ex^2)}{\sqrt{a^2+2abx^2+b^2x^4}}\,dx=\frac{bex^2+(bd-ae)\log\left(bx^2+a\right)}{2\,b^2}$$

```
[In] integrate(x*(e*x^2+d)/((b*x^2+a)^2)^(1/2),x, algorithm="fricas")
[Out] 1/2*(b*e*x^2 + (b*d - a*e)*log(b*x^2 + a))/b^2
```

Sympy [A] (verification not implemented)

Time = 3.91 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.81

$$\int \frac{x(d+ex^2)}{\sqrt{a^2+2abx^2+b^2x^4}}\,dx=\frac{\begin{cases} \frac{\left(\frac{a}{b}+x^2\right)\left(-\frac{ae}{b}+d\right)\log\left(\frac{a}{b}+x^2\right)}{\sqrt{b^2\left(\frac{a}{b}+x^2\right)^2}}+\frac{e\sqrt{a^2+2abx^2+b^2x^4}}{b^2} & \text{for}\ b^2\neq 0 \\ \frac{2d\sqrt{a^2+2abx^2}+\frac{e\left(-a^2\sqrt{a^2+2abx^2}+\frac{\left(a^2+2abx^2\right)^{\frac{3}{2}}}{3}\right)}{ab}}{2ab} & \text{for}\ ab\neq 0 \\ \frac{dx^2+\frac{ex^4}{2}}{\sqrt{a^2}} & \text{otherwise} \end{cases}}{2}$$

```
[In] integrate(x*(e*x**2+d)/((b*x**2+a)**2)**(1/2),x)
```

```
[Out] Piecewise(((a/b + x**2)*(-a*e/b + d)*log(a/b + x**2)/sqrt(b**2*(a/b + x**2)
**2) + e*sqrt(a**2 + 2*a*b*x**2 + b**2*x**4)/b**2, Ne(b**2, 0)), ((2*d*sqrt
(a**2 + 2*a*b*x**2) + e*(-a**2*sqrt(a**2 + 2*a*b*x**2) + (a**2 + 2*a*b*x**2
)**(3/2)/3)/(a*b))/(2*a*b), Ne(a*b, 0)), ((d*x**2 + e*x**4/2)/sqrt(a**2), T
rue))/2
```

Maxima [A] (verification not implemented)

none

Time = 0.18 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.37

$$\int \frac{x(d+ex^2)}{\sqrt{a^2+2abx^2+b^2x^4}}\,dx = \frac{ex^2}{2\,b} + \frac{(bd-ae)\log\left(bx^2+a\right)}{2\,b^2}$$

```
[In] integrate(x*(e*x^2+d)/((b*x^2+a)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/2*e*x^2/b + 1/2*(b*d - a*e)*log(b*x^2 + a)/b^2
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.48

$$\int \frac{x(d+ex^2)}{\sqrt{a^2+2abx^2+b^2x^4}}\,dx = \frac{1}{2}\left(\frac{ex^2}{b} + \frac{(bd-ae)\log\left(|bx^2+a|\right)}{b^2}\right)\operatorname{sgn}\left(bx^2+a\right)$$

```
[In] integrate(x*(e*x^2+d)/((b*x^2+a)^2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*(e*x^2/b + (b*d - a*e)*log(abs(b*x^2 + a))/b^2)*sgn(b*x^2 + a)
```

Mupad [B] (verification not implemented)

Time = 8.08 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.24

$$\int \frac{x(d+ex^2)}{\sqrt{a^2+2abx^2+b^2x^4}}\,dx = \frac{e\sqrt{a^2+2\,a\,b\,x^2+b^2\,x^4}}{2\,b^2} - \frac{a\,b\,e\,\ln\left(a\,b+\sqrt{{\left(b\,x^2+a\right)}^2}\,\sqrt{b^2}+b^2\,x^2\right)}{2\,{\left(b^2\right)}^{3/2}} + \frac{b^2\,d\,\ln\left(b^2\,x^2+a\,b\right)\,\mathrm{sign}(2\,b^2\,x^2+2\,a\,b)}{2\,{\left(b^2\right)}^{3/2}}$$

```
[In] int((x*(d + e*x^2))/((a + b*x^2)^2)^(1/2),x)
```

```
[Out] (e*(a^2 + b^2*x^4 + 2*a*b*x^2)^(1/2))/(2*b^2) - (a*b*e*log(a*b + ((a + b*x^
2)^2)^(1/2)*(b^2)^(1/2) + b^2*x^2))/(2*(b^2)^(3/2)) + (b^2*d*log(a*b + b^2*
x^2)*sign(2*a*b + 2*b^2*x^2))/(2*(b^2)^(3/2))
```

3.77 $\int \frac{d+ex^2}{\sqrt{a^2+2abx^2+b^2x^4}}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 97

$$\int \frac{d+ex^2}{\sqrt{a^2+2abx^2+b^2x^4}}\,dx=\frac{ex(a+bx^2)}{b\sqrt{a^2+2abx^2+b^2x^4}}+\frac{(bd-ae)\,(a+bx^2)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}b^{3/2}\sqrt{a^2+2abx^2+b^2x^4}}$$

```
[Out] e*x*(b*x^2+a)/b/((b*x^2+a)^2)^(1/2)+(-a*e+b*d)*(b*x^2+a)*arctan(x*b^(1/2)/a
^(1/2))/b^(3/2)/a^(1/2)/((b*x^2+a)^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {1162, 396, 211}

$$\int \frac{d+ex^2}{\sqrt{a^2+2abx^2+b^2x^4}}\,dx=\frac{(a+bx^2)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)(bd-ae)}{\sqrt{a}b^{3/2}\sqrt{a^2+2abx^2+b^2x^4}}+\frac{ex(a+bx^2)}{b\sqrt{a^2+2abx^2+b^2x^4}}$$

```
[In] Int[(d + e*x^2)/Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4],x]
```

```
[Out] (e*x*(a + b*x^2))/(b*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + ((b*d - a*e)*(a + b
*x^2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(Sqrt[a]*b^(3/2)*Sqrt[a^2 + 2*a*b*x^2 +
b^2*x^4])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 396

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_),
x_Symbol] :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^
2)^(2*FracPart[p])), Int[(d + e*x^2)^q*(b/2 + c*x^2)^(2*p), x], x] /; FreeQ
[{a, b, c, d, e, p, q}, x] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[p]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{(ab+b^2x^2)\int \frac{d+ex^2}{ab+b^2x^2}\,dx}{\sqrt{a^2+2abx^2+b^2x^4}}\\
&= \frac{ex(a+bx^2)}{b\sqrt{a^2+2abx^2+b^2x^4}} - \frac{((-b^2d+abe)(ab+b^2x^2))\int \frac{1}{ab+b^2x^2}\,dx}{b^2\sqrt{a^2+2abx^2+b^2x^4}}\\
&= \frac{ex(a+bx^2)}{b\sqrt{a^2+2abx^2+b^2x^4}} + \frac{(bd-ae)(a+bx^2)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}b^{3/2}\sqrt{a^2+2abx^2+b^2x^4}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 1.03 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.71

$$\int \frac{d+ex^2}{\sqrt{a^2+2abx^2+b^2x^4}}\,dx = -\frac{(a+bx^2)\left(-\sqrt{a}\sqrt{b}ex+(-bd+ae)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\right)}{\sqrt{a}b^{3/2}\sqrt{(a+bx^2)^2}}$$

```
[In] Integrate[(d + e*x^2)/Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4],x]
[Out] -(((a + b*x^2)*(-(Sqrt[a]*Sqrt[b]*e*x) + (-(b*d) + a*e)*ArcTan[(Sqrt[b]*x)/
Sqrt[a]]))/(Sqrt[a]*b^(3/2)*Sqrt[(a + b*x^2)^2]))
```

Maple [A] (verified)

Time = 1.51 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.64

method	result	size
default	$\frac{(b\,x^2+a)\left(ex\sqrt{ab}-\arctan\left(\frac{bx}{\sqrt{ab}}\right)ae+\arctan\left(\frac{bx}{\sqrt{ab}}\right)bd\right)}{\sqrt{(b\,x^2+a)^2}\,b\sqrt{ab}}$	62
risch	$\frac{\sqrt{(b\,x^2+a)^2}\,ex}{(b\,x^2+a)b}-\frac{\sqrt{(b\,x^2+a)^2}\,(ae-bd)\ln\left(bx-\sqrt{-ab}\right)}{2(b\,x^2+a)b\sqrt{-ab}}+\frac{\sqrt{(b\,x^2+a)^2}\,(ae-bd)\ln\left(-bx-\sqrt{-ab}\right)}{2(b\,x^2+a)b\sqrt{-ab}}$	133

```
[In] int((e*x^2+d)/((b*x^2+a)^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] (b*x^2+a)*(e*x*(a*b)^(1/2)-arctan(b*x/(a*b)^(1/2))*a*e+arctan(b*x/(a*b)^(1/
2))*b*d)/((b*x^2+a)^2)^(1/2)/b/(a*b)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.01

$$\int \frac{d+ex^2}{\sqrt{a^2+2abx^2+b^2x^4}}\,dx = \left[\frac{2\,abex+\sqrt{-ab}(bd-ae)\log\left(\frac{bx^2+2\sqrt{-ab}x-a}{bx^2+a}\right)}{2\,ab^2}, \frac{abex+\sqrt{ab}(bd-ae)\arctan\left(\frac{\sqrt{ab}x}{a}\right)}{ab^2}\right]$$

```
[In] integrate((e*x^2+d)/((b*x^2+a)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*(2*a*b*e*x + sqrt(-a*b)*(b*d - a*e)*log((b*x^2 + 2*sqrt(-a*b)*x - a)/(
b*x^2 + a)))/(a*b^2), (a*b*e*x + sqrt(a*b)*(b*d - a*e)*arctan(sqrt(a*b)*x/a
))/(a*b^2)]
```

Sympy [F]

$$\int \frac{d+ex^2}{\sqrt{a^2+2abx^2+b^2x^4}}\,dx = \int \frac{d+ex^2}{\sqrt{(a+bx^2)^2}}\,dx$$

```
[In] integrate((e*x**2+d)/((b*x**2+a)**2)**(1/2),x)
```

```
[Out] Integral((d + e*x**2)/sqrt((a + b*x**2)**2), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.34

$$\int \frac{d+ex^2}{\sqrt{a^2+2abx^2+b^2x^4}}\,dx = \frac{ex}{b} + \frac{(bd-ae)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}b}$$

```
[In] integrate((e*x^2+d)/((b*x^2+a)^2)^(1/2),x, algorithm="maxima")
[Out] e*x/b + (b*d - a*e)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.59

$$\int \frac{d+ex^2}{\sqrt{a^2+2abx^2+b^2x^4}}\,dx = \frac{ex\text{sgn}(bx^2+a)}{b} + \frac{(bd\text{sgn}(bx^2+a)-ae\text{sgn}(bx^2+a))\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}b}$$

```
[In] integrate((e*x^2+d)/((b*x^2+a)^2)^(1/2),x, algorithm="giac")
[Out] e*x*sgn(b*x^2 + a)/b + (b*d*sgn(b*x^2 + a) - a*e*sgn(b*x^2 + a))*arctan(b*x
/sqrt(a*b))/(sqrt(a*b)*b)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{d+ex^2}{\sqrt{a^2+2abx^2+b^2x^4}}\,dx = \int \frac{e\,x^2+d}{\sqrt{{\left(b\,x^2+a\right)}^2}}\,dx$$

```
[In] int((d + e*x^2)/((a + b*x^2)^2)^(1/2),x)
[Out] int((d + e*x^2)/((a + b*x^2)^2)^(1/2), x)
```

3.78 $\int \frac{d+ex^2}{x\sqrt{a^2+2abx^2+b^2x^4}}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 92

$$\int \frac{d+ex^2}{x\sqrt{a^2+2abx^2+b^2x^4}}\,dx = \frac{d(a+bx^2)\log(x)}{a\sqrt{a^2+2abx^2+b^2x^4}} - \frac{(bd-ae)\,(a+bx^2)\log\left(a+bx^2\right)}{2ab\sqrt{a^2+2abx^2+b^2x^4}}$$

```
[Out] d*(b*x^2+a)*ln(x)/a/((b*x^2+a)^2)^(1/2)-1/2*(-a*e+b*d)*(b*x^2+a)*ln(b*x^2+a
)/a/b/((b*x^2+a)^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {1264, 457, 78}

$$\int \frac{d+ex^2}{x\sqrt{a^2+2abx^2+b^2x^4}}\,dx = \frac{d\log(x)\,(a+bx^2)}{a\sqrt{a^2+2abx^2+b^2x^4}} - \frac{(a+bx^2)\,(bd-ae)\log\left(a+bx^2\right)}{2ab\sqrt{a^2+2abx^2+b^2x^4}}$$

```
[In] Int[(d + e*x^2)/(x*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]),x]
```

```
[Out] (d*(a + b*x^2)*Log[x])/(a*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - ((b*d - a*e)*(
a + b*x^2)*Log[a + b*x^2])/(2*a*b*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1264

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_), x_Symbol] :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^Int
Part[p]*(b/2 + c*x^2)^(2*FracPart[p])), Int[(f*x)^m*(d + e*x^2)^q*(b/2 + c*
x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, f, m, p, q}, x] && EqQ[b^2 - 4*
a*c, 0] &&  !IntegerQ[p]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{(ab+b^2x^2)\int \frac{d+ex^2}{x(ab+b^2x^2)}\,dx}{\sqrt{a^2+2abx^2+b^2x^4}} \\
&= \frac{(ab+b^2x^2)\operatorname{Subst}\left(\int \frac{d+ex}{x(ab+b^2x)}\,dx,x,x^2\right)}{2\sqrt{a^2+2abx^2+b^2x^4}} \\
&= \frac{(ab+b^2x^2)\operatorname{Subst}\left(\int \left(\frac{d}{abx}+\frac{-bd+ae}{ab(a+bx)}\right)\,dx,x,x^2\right)}{2\sqrt{a^2+2abx^2+b^2x^4}} \\
&= \frac{d(a+bx^2)\log(x)}{a\sqrt{a^2+2abx^2+b^2x^4}}-\frac{(bd-ae)\,(a+bx^2)\log\left(a+bx^2\right)}{2ab\sqrt{a^2+2abx^2+b^2x^4}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.38 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.36

$$\int \frac{d+ex^2}{x\sqrt{a^2+2abx^2+b^2x^4}}\,dx = \frac{1}{4}\left(\left(\frac{2d}{a}-\frac{4e}{b}\right)\operatorname{arctanh}\left(\frac{\sqrt{a^2}-\sqrt{(a+bx^2)^2}}{bx^2}\right) + \frac{d\left(-2\log\left(x^2\right)+\log\left(\sqrt{a^2}-bx^2-\sqrt{(a+bx^2)^2}\right)+\log\left(\sqrt{a^2}+bx^2-\sqrt{(a+bx^2)^2}\right)\right)}{\sqrt{a^2}}\right)$$

```
[In] Integrate[(d + e*x^2)/(x*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]),x]
[Out] (((2*d)/a - (4*e)/b)*ArcTanh[(Sqrt[a^2] - Sqrt[(a + b*x^2)^2])/(b*x^2)] + (
d*(-2*Log[x^2] + Log[Sqrt[a^2] - b*x^2 - Sqrt[(a + b*x^2)^2]] + Log[Sqrt[a^
2] + b*x^2 - Sqrt[(a + b*x^2)^2]]))/Sqrt[a^2])/4
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.34 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.52

method	result	size
pseudoelliptic	$\frac{\left(d \ln \left(x^{2}\right) b+\ln \left(b \,x^{2}+a \right) a e-\ln \left(b \,x^{2}+a \right) b d \right) \operatorname{csgn}\left(b \,x^{2}+a \right)}{2 a b}$	48
default	$\frac{\left(b \,x^{2}+a \right) \left(2 d \ln \left(x \right) b+\ln \left(b \,x^{2}+a \right) a e-\ln \left(b \,x^{2}+a \right) b d \right)}{2 \sqrt{\left(b \,x^{2}+a \right)^{2}}\, a b}$	57
risch	$\frac{\sqrt{\left(b \,x^{2}+a \right)^{2}}\, d \ln \left(x \right)}{\left(b \,x^{2}+a \right) a}+\frac{\sqrt{\left(b \,x^{2}+a \right)^{2}}\, \left(a e-b d \right) \ln \left(-b \,x^{2}-a \right)}{2 \left(b \,x^{2}+a \right) a b}$	76

```
[In] int((e*x^2+d)/x/((b*x^2+a)^2)^(1/2),x,method=_RETURNVERBOSE)
[Out] 1/2*(d*ln(x^2)*b+ln(b*x^2+a)*a*e-ln(b*x^2+a)*b*d)*csgn(b*x^2+a)/a/b
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.36

$$\int \frac{d+ex^2}{x\sqrt{a^2+2abx^2+b^2x^4}}\,dx = \frac{2\,bd\log\left(x\right)-\left(bd-ae\right)\log\left(bx^2+a\right)}{2\,ab}$$

```
[In] integrate((e*x^2+d)/x/((b*x^2+a)^2)^(1/2),x, algorithm="fricas")
[Out] 1/2*(2*b*d*log(x) - (b*d - a*e)*log(b*x^2 + a))/(a*b)
```

Sympy [F]

$$\int \frac{d+ex^2}{x\sqrt{a^2+2abx^2+b^2x^4}}\,dx = \int \frac{d+ex^2}{x\sqrt{\left(a+bx^2\right)^2}}\,dx$$

```
[In] integrate((e*x**2+d)/x/((b*x**2+a)**2)**(1/2),x)
[Out] Integral((d + e*x**2)/(x*sqrt((a + b*x**2)**2)), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.38

$$\int \frac{d+ex^2}{x\sqrt{a^2+2abx^2+b^2x^4}}\,dx = \frac{d\log\left(x^2\right)}{2\,a} - \frac{(bd-ae)\log\left(bx^2+a\right)}{2\,ab}$$

```
[In] integrate((e*x^2+d)/x/((b*x^2+a)^2)^(1/2),x, algorithm="maxima")
[Out] 1/2*d*log(x^2)/a - 1/2*(b*d - a*e)*log(b*x^2 + a)/(a*b)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.65

$$\int \frac{d+ex^2}{x\sqrt{a^2+2abx^2+b^2x^4}}\,dx = \frac{d\log\left(x^2\right)\mathrm{sgn}(bx^2+a)}{2\,a} - \frac{(bd\mathrm{sgn}(bx^2+a)-ae\mathrm{sgn}(bx^2+a))\log\left(|bx^2+a|\right)}{2\,ab}$$

```
[In] integrate((e*x^2+d)/x/((b*x^2+a)^2)^(1/2),x, algorithm="giac")
[Out] 1/2*d*log(x^2)*sgn(b*x^2 + a)/a - 1/2*(b*d*sgn(b*x^2 + a) - a*e*sgn(b*x^2 +
 a))*log(abs(b*x^2 + a))/(a*b)
```

Mupad [B] (verification not implemented)

Time = 8.17 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.90

$$\int \frac{d+ex^2}{x\sqrt{a^2+2abx^2+b^2x^4}}\,dx = \frac{e\,\ln\left(b^2\,x^2+a\,b\right)\,\mathrm{sign}(2\,b^2\,x^2+2\,a\,b)}{2\,\sqrt{b^2}} - \frac{d\,\ln\left(\frac{1}{x^2}\right)}{2\,\sqrt{a^2}} - \frac{d\,\ln\left(\sqrt{{\left(b\,x^2+a\right)}^2}\,\sqrt{a^2}+a^2+a\,b\,x^2\right)}{2\,\sqrt{a^2}}$$

```
[In] int((d + e*x^2)/(x*((a + b*x^2)^2)^(1/2)),x)
[Out] (e*log(a*b + b^2*x^2)*sign(2*a*b + 2*b^2*x^2))/(2*(b^2)^(1/2)) - (d*log(1/x
^2))/(2*(a^2)^(1/2)) - (d*log(((a + b*x^2)^2)^(1/2)*(a^2)^(1/2) + a^2 + a*b
*x^2))/(2*(a^2)^(1/2))
```

3.79 $\int \frac{d+ex^2}{x^2\sqrt{a^2+2abx^2+b^2x^4}}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 101

$$\int \frac{d+ex^2}{x^2\sqrt{a^2+2abx^2+b^2x^4}}\,dx = -\frac{d(a+bx^2)}{ax\sqrt{a^2+2abx^2+b^2x^4}}-\frac{(bd-ae)\,(a+bx^2)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{a^{3/2}\sqrt{b}\sqrt{a^2+2abx^2+b^2x^4}}$$

```
[Out] -d*(b*x^2+a)/a/x/((b*x^2+a)^2)^(1/2)-(-a*e+b*d)*(b*x^2+a)*arctan(x*b^(1/2)/
a^(1/2))/a^(3/2)/b^(1/2)/((b*x^2+a)^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {1264, 464, 211}

$$\int \frac{d+ex^2}{x^2\sqrt{a^2+2abx^2+b^2x^4}}\,dx = -\frac{d(a+bx^2)}{ax\sqrt{a^2+2abx^2+b^2x^4}}-\frac{(a+bx^2)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)(bd-ae)}{a^{3/2}\sqrt{b}\sqrt{a^2+2abx^2+b^2x^4}}$$

```
[In] Int[(d + e*x^2)/(x^2*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]),x]
```

```
[Out] -((d*(a + b*x^2))/(a*x*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])) - ((b*d - a*e)*(a
+ b*x^2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(a^(3/2)*Sqrt[b]*Sqrt[a^2 + 2*a*b*x^2
 + b^2*x^4])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 464

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 1264

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_), x_Symbol] :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^Int
Part[p]*(b/2 + c*x^2)^(2*FracPart[p])), Int[(f*x)^m*(d + e*x^2)^q*(b/2 + c*
x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, f, m, p, q}, x] && EqQ[b^2 - 4*
a*c, 0] &&  !IntegerQ[p]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{(ab+b^2x^2)\int\frac{d+ex^2}{x^2(ab+b^2x^2)}\,dx}{\sqrt{a^2+2abx^2+b^2x^4}}\\
&= -\frac{d(a+bx^2)}{ax\sqrt{a^2+2abx^2+b^2x^4}}-\frac{((b^2d-abe)(ab+b^2x^2))\int\frac{1}{ab+b^2x^2}\,dx}{ab\sqrt{a^2+2abx^2+b^2x^4}}\\
&= -\frac{d(a+bx^2)}{ax\sqrt{a^2+2abx^2+b^2x^4}}-\frac{(bd-ae)(a+bx^2)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{a^{3/2}\sqrt{b}\sqrt{a^2+2abx^2+b^2x^4}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 1.03 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.71

$$\int\frac{d+ex^2}{x^2\sqrt{a^2+2abx^2+b^2x^4}}\,dx=\frac{(a+bx^2)\left(-\sqrt{a}\sqrt{b}d+(-bdx+aex)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\right)}{a^{3/2}\sqrt{b}x\sqrt{(a+bx^2)^2}}$$

```
[In] Integrate[(d + e*x^2)/(x^2*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]),x]
[Out] ((a + b*x^2)*(-(Sqrt[a]*Sqrt[b]*d) + (-(b*d*x) + a*e*x)*ArcTan[(Sqrt[b]*x)/
Sqrt[a]]))/(a^(3/2)*Sqrt[b]*x*Sqrt[(a + b*x^2)^2])
```

Maple [A] (verified)

Time = 1.58 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.66

method	result	size
default	$-\frac{\left(b\,x^{2}+a\right)\left(-\arctan\left(\frac{bx}{\sqrt{ab}}\right)aex+\arctan\left(\frac{bx}{\sqrt{ab}}\right)bdx+d\sqrt{ab}\right)}{\sqrt{\left(b\,x^{2}+a\right)^{2}}\,a\sqrt{ab}\,x}$	67
risch	$-\frac{\sqrt{\left(b\,x^{2}+a\right)^{2}}\,d}{\left(b\,x^{2}+a\right)ax}-\frac{\sqrt{\left(b\,x^{2}+a\right)^{2}}\,\left(ae-bd\right)\ln\left(-\sqrt{-ab}\,x+a\right)}{2\left(b\,x^{2}+a\right)\sqrt{-ab}\,a}+\frac{\sqrt{\left(b\,x^{2}+a\right)^{2}}\,\left(ae-bd\right)\ln\left(-\sqrt{-ab}\,x-a\right)}{2\left(b\,x^{2}+a\right)\sqrt{-ab}\,a}$	135

```
[In] int((e*x^2+d)/x^2/((b*x^2+a)^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -(b*x^2+a)*(-arctan(b*x/(a*b)^(1/2))*a*e*x+arctan(b*x/(a*b)^(1/2))*b*d*x+d*
(a*b)^(1/2))/((b*x^2+a)^2)^(1/2)/a/(a*b)^(1/2)/x
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.04

$$\int \frac{d+ex^2}{x^2\sqrt{a^2+2abx^2+b^2x^4}}\,dx = \left[\frac{\sqrt{-ab}(bd-ae)x\log\left(\frac{bx^2-2\sqrt{-ab}x-a}{bx^2+a}\right)-2\,abd}{2\,a^2bx}, -\frac{\sqrt{ab}(bd-ae)x\arctan\left(\frac{\sqrt{ab}x}{a}\right)+abd}{a^2bx}\right]$$

```
[In] integrate((e*x^2+d)/x^2/((b*x^2+a)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*(sqrt(-a*b)*(b*d - a*e)*x*log((b*x^2 - 2*sqrt(-a*b)*x - a)/(b*x^2 + a)
) - 2*a*b*d)/(a^2*b*x), -(sqrt(a*b)*(b*d - a*e)*x*arctan(sqrt(a*b)*x/a) + a
*b*d)/(a^2*b*x)]
```

Sympy [F]

$$\int \frac{d+ex^2}{x^2\sqrt{a^2+2abx^2+b^2x^4}}\,dx = \int \frac{d+ex^2}{x^2\sqrt{(a+bx^2)^2}}\,dx$$

```
[In] integrate((e*x**2+d)/x**2/((b*x**2+a)**2)**(1/2),x)
```

```
[Out] Integral((d + e*x**2)/(x**2*sqrt((a + b*x**2)**2)), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.37

$$\int \frac{d+ex^2}{x^2\sqrt{a^2+2abx^2+b^2x^4}}\,dx = -\frac{(bd-ae)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}a} - \frac{d}{ax}$$

```
[In] integrate((e*x^2+d)/x^2/((b*x^2+a)^2)^(1/2),x, algorithm="maxima")
[Out] -(b*d - a*e)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a) - d/(a*x)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.60

$$\int \frac{d+ex^2}{x^2\sqrt{a^2+2abx^2+b^2x^4}}\,dx = -\frac{(bd\text{sgn}(bx^2+a) - ae\text{sgn}(bx^2+a))\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}a} - \frac{d\text{sgn}(bx^2+a)}{ax}$$

```
[In] integrate((e*x^2+d)/x^2/((b*x^2+a)^2)^(1/2),x, algorithm="giac")
[Out] -(b*d*sgn(b*x^2 + a) - a*e*sgn(b*x^2 + a))*arctan(b*x/sqrt(a*b))/(sqrt(a*b)
*a) - d*sgn(b*x^2 + a)/(a*x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{d+ex^2}{x^2\sqrt{a^2+2abx^2+b^2x^4}}\,dx = \int \frac{e\,x^2+d}{x^2\,\sqrt{{\left(b\,x^2+a\right)}^2}}\,dx$$

```
[In] int((d + e*x^2)/(x^2*((a + b*x^2)^2)^(1/2)),x)
[Out] int((d + e*x^2)/(x^2*((a + b*x^2)^2)^(1/2)), x)
```

3.80 $\int \frac{d+ex^2}{x^3\sqrt{a^2+2abx^2+b^2x^4}}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 137

$$\int \frac{d+ex^2}{x^3\sqrt{a^2+2abx^2+b^2x^4}}\,dx = -\frac{d(a+bx^2)}{2ax^2\sqrt{a^2+2abx^2+b^2x^4}} - \frac{(bd-ae)\,(a+bx^2)\log(x)}{a^2\sqrt{a^2+2abx^2+b^2x^4}} + \frac{(bd-ae)\,(a+bx^2)\log\left(a+bx^2\right)}{2a^2\sqrt{a^2+2abx^2+b^2x^4}}$$

```
[Out] -1/2*d*(b*x^2+a)/a/x^2/((b*x^2+a)^2)^(1/2)-(-a*e+b*d)*(b*x^2+a)*ln(x)/a^2/(
(b*x^2+a)^2)^(1/2)+1/2*(-a*e+b*d)*(b*x^2+a)*ln(b*x^2+a)/a^2/((b*x^2+a)^2)^(
1/2)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {1264, 457, 78}

$$\int \frac{d+ex^2}{x^3\sqrt{a^2+2abx^2+b^2x^4}}\,dx = -\frac{\log(x)\,(a+bx^2)\,(bd-ae)}{a^2\sqrt{a^2+2abx^2+b^2x^4}} + \frac{(a+bx^2)\,(bd-ae)\log\left(a+bx^2\right)}{2a^2\sqrt{a^2+2abx^2+b^2x^4}} - \frac{d(a+bx^2)}{2ax^2\sqrt{a^2+2abx^2+b^2x^4}}$$

```
[In] Int[(d + e*x^2)/(x^3*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]),x]
```

```
[Out] -1/2*(d*(a + b*x^2))/(a*x^2*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - ((b*d - a*e)
*(a + b*x^2)*Log[x])/(a^2*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + ((b*d - a*e)*(
a + b*x^2)*Log[a + b*x^2])/(2*a^2*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1264

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_), x_Symbol] :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^Int
Part[p]*(b/2 + c*x^2)^(2*FracPart[p])), Int[(f*x)^m*(d + e*x^2)^q*(b/2 + c*
x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, f, m, p, q}, x] && EqQ[b^2 - 4*
a*c, 0] &&  !IntegerQ[p]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{(ab+b^2x^2)\int \frac{d+ex^2}{x^3(ab+b^2x^2)}\,dx}{\sqrt{a^2+2abx^2+b^2x^4}}\\
&= \frac{(ab+b^2x^2)\,\text{Subst}\left(\int \frac{d+ex}{x^2(ab+b^2x)}\,dx,x,x^2\right)}{2\sqrt{a^2+2abx^2+b^2x^4}}\\
&= \frac{(ab+b^2x^2)\,\text{Subst}\left(\int \left(\frac{d}{abx^2}+\frac{-bd+ae}{a^2bx}+\frac{bd-ae}{a^2(a+bx)}\right)\,dx,x,x^2\right)}{2\sqrt{a^2+2abx^2+b^2x^4}}\\
&= -\frac{d(a+bx^2)}{2ax^2\sqrt{a^2+2abx^2+b^2x^4}}-\frac{(bd-ae)\,(a+bx^2)\log(x)}{a^2\sqrt{a^2+2abx^2+b^2x^4}}+\frac{(bd-ae)\,(a+bx^2)\log\left(a+bx^2\right)}{2a^2\sqrt{a^2+2abx^2+b^2x^4}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.36 (sec) , antiderivative size = 165, normalized size of antiderivative = 1.20

$$\int \frac{d+ex^2}{x^3\sqrt{a^2+2abx^2+b^2x^4}}\,dx$$
$$= \frac{a^2d-\sqrt{a^2}d\sqrt{(a+bx^2)^2}+2a(bd-ae)x^2\log(x^2)-\left(-a+\sqrt{a^2}\right)(-bd+ae)x^2\log\left(\sqrt{a^2}-bx^2-\sqrt{(a+\right.}{4\,(a^2)^{3/2}\,x^2}$$

```
[In] Integrate[(d + e*x^2)/(x^3*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]),x]
```

```
[Out] (a^2*d - Sqrt[a^2]*d*Sqrt[(a + b*x^2)^2] + 2*a*(b*d - a*e)*x^2*Log[x^2] - (
-a + Sqrt[a^2])*(-(b*d) + a*e)*x^2*Log[Sqrt[a^2] - b*x^2 - Sqrt[(a + b*x^2)
^2]] + (a + Sqrt[a^2])*(-(b*d) + a*e)*x^2*Log[Sqrt[a^2] + b*x^2 - Sqrt[(a +
 b*x^2)^2]])/(4*(a^2)^(3/2)*x^2)
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.34 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.42

method	result	size
pseudoelliptic	$-\frac{\left(x^{2}\left(ae-bd\right)\ln\left(b\,x^{2}+a\right)-x^{2}\left(ae-bd\right)\ln\left(x^{2}\right)+da\right)\operatorname{csgn}\left(b\,x^{2}+a\right)}{2a^{2}x^{2}}$	58
default	$\frac{\left(b\,x^{2}+a\right)\left(2\ln\left(x\right)ae\,x^{2}-2\ln\left(x\right)bd\,x^{2}-\ln\left(b\,x^{2}+a\right)ae\,x^{2}+\ln\left(b\,x^{2}+a\right)bd\,x^{2}-da\right)}{2\sqrt{\left(b\,x^{2}+a\right)^{2}}\,a^{2}x^{2}}$	79
risch	$-\frac{\sqrt{\left(b\,x^{2}+a\right)^{2}}\,d}{2\left(b\,x^{2}+a\right)a\,x^{2}}+\frac{\sqrt{\left(b\,x^{2}+a\right)^{2}}\,\left(ae-bd\right)\ln\left(x\right)}{\left(b\,x^{2}+a\right)a^{2}}-\frac{\sqrt{\left(b\,x^{2}+a\right)^{2}}\,\left(ae-bd\right)\ln\left(b\,x^{2}+a\right)}{2\left(b\,x^{2}+a\right)a^{2}}$	106

```
[In] int((e*x^2+d)/x^3/((b*x^2+a)^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*(x^2*(a*e-b*d)*ln(b*x^2+a)-x^2*(a*e-b*d)*ln(x^2)+d*a)*csgn(b*x^2+a)/a^
2/x^2
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.35

$$\int \frac{d+ex^2}{x^3\sqrt{a^2+2abx^2+b^2x^4}}\,dx = \frac{(bd-ae)x^2\log\left(bx^2+a\right)-2\,(bd-ae)x^2\log\left(x\right)-ad}{2\,a^2x^2}$$

```
[In] integrate((e*x^2+d)/x^3/((b*x^2+a)^2)^(1/2),x, algorithm="fricas")
[Out] 1/2*((b*d - a*e)*x^2*log(b*x^2 + a) - 2*(b*d - a*e)*x^2*log(x) - a*d)/(a^2*
x^2)
```

Sympy [F]

$$\int \frac{d+ex^2}{x^3\sqrt{a^2+2abx^2+b^2x^4}}\,dx = \int \frac{d+ex^2}{x^3\sqrt{\left(a+bx^2\right)^2}}\,dx$$

```
[In] integrate((e*x**2+d)/x**3/((b*x**2+a)**2)**(1/2),x)
[Out] Integral((d + e*x**2)/(x**3*sqrt((a + b*x**2)**2)), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.35

$$\int \frac{d+ex^2}{x^3\sqrt{a^2+2abx^2+b^2x^4}}\,dx = \frac{(bd-ae)\log\left(bx^2+a\right)}{2\,a^2} - \frac{(bd-ae)\log\left(x^2\right)}{2\,a^2} - \frac{d}{2\,ax^2}$$

```
[In] integrate((e*x^2+d)/x^3/((b*x^2+a)^2)^(1/2),x, algorithm="maxima")
[Out] 1/2*(b*d - a*e)*log(b*x^2 + a)/a^2 - 1/2*(b*d - a*e)*log(x^2)/a^2 - 1/2*d/(
a*x^2)
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.93

$$\int \frac{d+ex^2}{x^3\sqrt{a^2+2abx^2+b^2x^4}}\,dx = -\frac{(bd\text{sgn}(bx^2+a)-ae\text{sgn}(bx^2+a))\log(x^2)}{2\,a^2} + \frac{(b^2d\text{sgn}(bx^2+a)-abe\text{sgn}(bx^2+a))\log(|bx^2+a|)}{2\,a^2b} + \frac{bdx^2\text{sgn}(bx^2+a)-aex^2\text{sgn}(bx^2+a)-ad\text{sgn}(bx^2+a)}{2\,a^2x^2}$$

```
[In] integrate((e*x^2+d)/x^3/((b*x^2+a)^2)^(1/2),x, algorithm="giac")
```

```
[Out] -1/2*(b*d*sgn(b*x^2 + a) - a*e*sgn(b*x^2 + a))*log(x^2)/a^2 + 1/2*(b^2*d*sg
n(b*x^2 + a) - a*b*e*sgn(b*x^2 + a))*log(abs(b*x^2 + a))/(a^2*b) + 1/2*(b*d
*x^2*sgn(b*x^2 + a) - a*e*x^2*sgn(b*x^2 + a) - a*d*sgn(b*x^2 + a))/(a^2*x^2
)
```

Mupad [B] (verification not implemented)

Time = 8.32 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.91

$$\int \frac{d+ex^2}{x^3\sqrt{a^2+2abx^2+b^2x^4}}\,dx = \frac{a\,b\,d\,\text{atanh}\left(\frac{a^2+b\,a\,x^2}{\sqrt{a^2}\,\sqrt{a^2+2\,a\,b\,x^2+b^2\,x^4}}\right)}{2\,{\left(a^2\right)}^{3/2}} - \frac{e\,\ln\left(\frac{1}{x^2}\right)}{2\,\sqrt{a^2}} - \frac{d\,\sqrt{a^2+2\,a\,b\,x^2+b^2\,x^4}}{2\,a^2\,x^2} - \frac{e\,\ln\left(\sqrt{{\left(b\,x^2+a\right)}^2}\,\sqrt{a^2}+a^2+a\,b\,x^2\right)}{2\,\sqrt{a^2}}$$

```
[In] int((d + e*x^2)/(x^3*((a + b*x^2)^2)^(1/2)),x)
```

```
[Out] (a*b*d*atanh((a^2 + a*b*x^2)/((a^2)^(1/2)*(a^2 + b^2*x^4 + 2*a*b*x^2)^(1/2)
)))/(2*(a^2)^(3/2)) - (e*log(1/x^2))/(2*(a^2)^(1/2)) - (d*(a^2 + b^2*x^4 +
2*a*b*x^2)^(1/2))/(2*a^2*x^2) - (e*log(((a + b*x^2)^2)^(1/2)*(a^2)^(1/2) +
a^2 + a*b*x^2))/(2*(a^2)^(1/2))
```

3.81 $\int \frac{x^2(d+ex^2)}{(a^2+2abx^2+b^2x^4)^{3/2}} dx$

Optimal result

Integrand size = 33, antiderivative size = 153

$$\int \frac{x^2(d+ex^2)}{(a^2+2abx^2+b^2x^4)^{3/2}} dx = \frac{(bd-5ae)x}{8ab^2\sqrt{a^2+2abx^2+b^2x^4}} - \frac{(bd-ae)x}{4b^2(a+bx^2)\sqrt{a^2+2abx^2+b^2x^4}} + \frac{(bd+3ae)(a+bx^2)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8a^{3/2}b^{5/2}\sqrt{a^2+2abx^2+b^2x^4}}$$

```
[Out] 1/8*(-5*a*e+b*d)*x/a/b^2/((b*x^2+a)^2)^(1/2)-1/4*(-a*e+b*d)*x/b^2/(b*x^2+a)
/((b*x^2+a)^2)^(1/2)+1/8*(3*a*e+b*d)*(b*x^2+a)*arctan(x*b^(1/2)/a^(1/2))/a^
(3/2)/b^(5/2)/((b*x^2+a)^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.121, Rules used = {1264, 466, 393, 211}

$$\int \frac{x^2(d+ex^2)}{(a^2+2abx^2+b^2x^4)^{3/2}} dx = \frac{x(bd-5ae)}{8ab^2\sqrt{a^2+2abx^2+b^2x^4}} - \frac{x(bd-ae)}{4b^2(a+bx^2)\sqrt{a^2+2abx^2+b^2x^4}} + \frac{(a+bx^2)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)(3ae+bd)}{8a^{3/2}b^{5/2}\sqrt{a^2+2abx^2+b^2x^4}}$$

```
[In] Int[(x^2*(d + e*x^2))/(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2),x]
```

```
[Out] ((b*d - 5*a*e)*x)/(8*a*b^2*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - ((b*d - a*e)*
x)/(4*b^2*(a + b*x^2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + ((b*d + 3*a*e)*(a
```

```
+ b*x^2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(8*a^(3/2)*b^(5/2)*Sqrt[a^2 + 2*a*b*x
^2 + b^2*x^4])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 393

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Dist[(a*d -
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; F
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n
 + p, 0])
```

Rule 466

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[(-a)^(m/2 - 1)*(b*c - a*d)*x*((a + b*x^2)^(p + 1)/(2*b^(m/2 + 1)*(p
+ 1))), x] + Dist[1/(2*b^(m/2 + 1)*(p + 1)), Int[(a + b*x^2)^(p + 1)*Expand
ToSum[2*b*(p + 1)*x^2*Together[(b^(m/2)*x^(m - 2)*(c + d*x^2) - (-a)^(m/2 -
 1)*(b*c - a*d))/(a + b*x^2)] - (-a)^(m/2 - 1)*(b*c - a*d), x], x], x] /; F
reeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && IGtQ[m/2, 0] &&
 (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

Rule 1264

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_), x_Symbol] :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^Int
Part[p]*(b/2 + c*x^2)^(2*FracPart[p])), Int[(f*x)^m*(d + e*x^2)^q*(b/2 + c*
x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, f, m, p, q}, x] && EqQ[b^2 - 4*
a*c, 0] &&  !IntegerQ[p]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\left(b^2(ab+b^2x^2)\right)\int \frac{x^2(d+ex^2)}{(ab+b^2x^2)^3}\,dx}{\sqrt{a^2+2abx^2+b^2x^4}} \\
&= -\frac{(bd-ae)x}{4b^2\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}} - \frac{(ab+b^2x^2)\int \frac{-b(bd-ae)-4b^2ex^2}{(ab+b^2x^2)^2}\,dx}{4b^2\sqrt{a^2+2abx^2+b^2x^4}} \\
&= \frac{(bd-5ae)x}{8ab^2\sqrt{a^2+2abx^2+b^2x^4}} - \frac{(bd-ae)x}{4b^2\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}} \\
&\quad + \frac{\left((bd+3ae)\left(ab+b^2x^2\right)\right)\int \frac{1}{ab+b^2x^2}\,dx}{8ab^2\sqrt{a^2+2abx^2+b^2x^4}}
\end{aligned}$$

$$= \frac{(bd-5ae)x}{8ab^2\sqrt{a^2+2abx^2+b^2x^4}} - \frac{(bd-ae)x}{4b^2(a+bx^2)\sqrt{a^2+2abx^2+b^2x^4}} + \frac{(bd+3ae)(a+bx^2)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8a^{3/2}b^{5/2}\sqrt{a^2+2abx^2+b^2x^4}}$$

Mathematica [A] (verified)

Time = 1.05 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.71

$$\int \frac{x^2(d+ex^2)}{(a^2+2abx^2+b^2x^4)^{3/2}}\,dx = \frac{-\sqrt{a}\sqrt{b}x(3a^2e-b^2dx^2+ab(d+5ex^2))+(bd+3ae)(a+bx^2)^2\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8a^{3/2}b^{5/2}(a+bx^2)\sqrt{(a+bx^2)^2}}$$

```
[In] Integrate[(x^2*(d + e*x^2))/(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2),x]
```

```
[Out] (-(Sqrt[a]*Sqrt[b]*x*(3*a^2*e - b^2*d*x^2 + a*b*(d + 5*e*x^2))) + (b*d + 3*
a*e)*(a + b*x^2)^2*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(8*a^(3/2)*b^(5/2)*(a + b*x
^2)*Sqrt[(a + b*x^2)^2])
```

Maple [A] (verified)

Time = 2.09 (sec) , antiderivative size = 165, normalized size of antiderivative = 1.08

method	result
risch	$\frac{\sqrt{(b\,x^2+a)^2}\left(-\frac{(5ae-bd)x^3}{8ab}-\frac{(3ae+bd)x}{8b^2}\right)}{(b\,x^2+a)^3}-\frac{\sqrt{(b\,x^2+a)^2}\,(3ae+bd)\ln\left(bx+\sqrt{-ab}\right)}{16(b\,x^2+a)\sqrt{-ab}\,b^2a}+\frac{\sqrt{(b\,x^2+a)^2}\,(3ae+bd)\ln\left(-bx+\sqrt{-ab}\right)}{16(b\,x^2+a)\sqrt{-ab}\,b^2a}$
default	$-\frac{\left(-3\arctan\left(\frac{bx}{\sqrt{ab}}\right)a\,b^2e\,x^4-\arctan\left(\frac{bx}{\sqrt{ab}}\right)b^3d\,x^4+5\sqrt{ab}\,abe\,x^3-\sqrt{ab}\,b^2d\,x^3-6\arctan\left(\frac{bx}{\sqrt{ab}}\right)a^2be\,x^2-2\arctan\left(\frac{bx}{\sqrt{ab}}\right)a\,b^2d\,x^2+3\sqrt{ab}\right.}{8\sqrt{ab}\,a\,b^2\left((b\,x^2+a)^2\right)^{\frac{3}{2}}}$

```
[In] int(x^2*(e*x^2+d)/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] ((b*x^2+a)^2)^(1/2)/(b*x^2+a)^3*(-1/8*(5*a*e-b*d)/a/b*x^3-1/8*(3*a*e+b*d)/b
^2*x)-1/16*((b*x^2+a)^2)^(1/2)/(b*x^2+a)/(-a*b)^(1/2)*(3*a*e+b*d)/b^2/a*ln(
b*x+(-a*b)^(1/2))+1/16*((b*x^2+a)^2)^(1/2)/(b*x^2+a)/(-a*b)^(1/2)*(3*a*e+b*
d)/b^2/a*ln(-b*x+(-a*b)^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 300, normalized size of antiderivative = 1.96

$$\int \frac{x^2(d+ex^2)}{(a^2+2abx^2+b^2x^4)^{3/2}}\,dx = \left[\frac{2\,(ab^3d-5\,a^2b^2e)x^3-((b^3d+3\,ab^2e)x^4+a^2bd+3\,a^3e+2\,(ab^2d+3\,a^2be)}{16\,(a^2b^5x^4+2\,a^3b^4x^2+a^4}\right.$$

```
[In] integrate(x^2*(e*x^2+d)/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="fricas"
)
```

```
[Out] [1/16*(2*(a*b^3*d - 5*a^2*b^2*e)*x^3 - ((b^3*d + 3*a*b^2*e)*x^4 + a^2*b*d +
 3*a^3*e + 2*(a*b^2*d + 3*a^2*b*e)*x^2)*sqrt(-a*b)*log((b*x^2 - 2*sqrt(-a*b
)*x - a)/(b*x^2 + a)) - 2*(a^2*b^2*d + 3*a^3*b*e)*x)/(a^2*b^5*x^4 + 2*a^3*b
^4*x^2 + a^4*b^3), 1/8*((a*b^3*d - 5*a^2*b^2*e)*x^3 + ((b^3*d + 3*a*b^2*e)*
x^4 + a^2*b*d + 3*a^3*e + 2*(a*b^2*d + 3*a^2*b*e)*x^2)*sqrt(a*b)*arctan(sqr
t(a*b)*x/a) - (a^2*b^2*d + 3*a^3*b*e)*x)/(a^2*b^5*x^4 + 2*a^3*b^4*x^2 + a^4
*b^3)]
```

Sympy [F]

$$\int \frac{x^2(d+ex^2)}{(a^2+2abx^2+b^2x^4)^{3/2}}\,dx = \int \frac{x^2(d+ex^2)}{\left((a+bx^2)^2\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(x**2*(e*x**2+d)/(b**2*x**4+2*a*b*x**2+a**2)**(3/2),x)
```

```
[Out] Integral(x**2*(d + e*x**2)/((a + b*x**2)**2)**(3/2), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.82

$$\int \frac{x^2(d+ex^2)}{(a^2+2abx^2+b^2x^4)^{3/2}}\,dx = -\frac{1}{8}\,e\left(\frac{5\,bx^3+3\,ax}{b^4x^4+2\,ab^3x^2+a^2b^2}-\frac{3\,\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}b^2}\right) + \frac{1}{8}\,d\left(\frac{bx^3-ax}{ab^3x^4+2\,a^2b^2x^2+a^3b}+\frac{\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}ab}\right)$$

```
[In] integrate(x^2*(e*x^2+d)/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="maxima"
)
```

```
[Out] -1/8*e*((5*b*x^3 + 3*a*x)/(b^4*x^4 + 2*a*b^3*x^2 + a^2*b^2) - 3*arctan(b*x/
sqrt(a*b))/(sqrt(a*b)*b^2)) + 1/8*d*((b*x^3 - a*x)/(a*b^3*x^4 + 2*a^2*b^2*x
^2 + a^3*b) + arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a*b))
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.64

$$\int \frac{x^2(d+ex^2)}{(a^2+2abx^2+b^2x^4)^{3/2}}\,dx = \frac{(bd+3\,ae)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\,\sqrt{ab}ab^2\mathrm{sgn}\left(bx^2+a\right)} + \frac{b^2dx^3-5\,abex^3-abdx-3\,a^2ex}{8\left(bx^2+a\right)^2ab^2\mathrm{sgn}\left(bx^2+a\right)}$$

```
[In] integrate(x^2*(e*x^2+d)/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="giac")
```

```
[Out] 1/8*(b*d + 3*a*e)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a*b^2*sgn(b*x^2 + a)) +
1/8*(b^2*d*x^3 - 5*a*b*e*x^3 - a*b*d*x - 3*a^2*e*x)/((b*x^2 + a)^2*a*b^2*sg
n(b*x^2 + a))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2(d+ex^2)}{(a^2+2abx^2+b^2x^4)^{3/2}}\,dx = \int \frac{x^2\left(e\,x^2+d\right)}{\left(a^2+2\,a\,b\,x^2+b^2\,x^4\right)^{3/2}}\,dx$$

```
[In] int((x^2*(d + e*x^2))/(a^2 + b^2*x^4 + 2*a*b*x^2)^(3/2),x)
```

```
[Out] int((x^2*(d + e*x^2))/(a^2 + b^2*x^4 + 2*a*b*x^2)^(3/2), x)
```

3.82 $\int \frac{x(d+ex^2)}{(a^2+2abx^2+b^2x^4)^{3/2}} dx$

Optimal result

Integrand size = 31, antiderivative size = 77

$$\int \frac{x(d+ex^2)}{(a^2+2abx^2+b^2x^4)^{3/2}} dx = -\frac{e}{2b^2\sqrt{a^2+2abx^2+b^2x^4}} - \frac{bd-ae}{4b^2(a+bx^2)\sqrt{a^2+2abx^2+b^2x^4}}$$

```
[Out] -1/2*e/b^2/((b*x^2+a)^2)^(1/2)+1/4*(a*e-b*d)/b^2/(b*x^2+a)/((b*x^2+a)^2)^(1
/2)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.097, Rules used = {1261, 654, 621}

$$\int \frac{x(d+ex^2)}{(a^2+2abx^2+b^2x^4)^{3/2}} dx = -\frac{bd-ae}{4b^2(a+bx^2)\sqrt{a^2+2abx^2+b^2x^4}} - \frac{e}{2b^2\sqrt{a^2+2abx^2+b^2x^4}}$$

```
[In] Int[(x*(d + e*x^2))/(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2),x]
```

```
[Out] -1/2*e/(b^2*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (b*d - a*e)/(4*b^2*(a + b*x^
2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])
```

Rule 621

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[2*((a + b*x
+ c*x^2)^(p + 1)/((2*p + 1)*(b + 2*c*x))), x] /; FreeQ[{a, b, c, p}, x] &&
EqQ[b^2 - 4*a*c, 0] && LtQ[p, -1]
```

Rule 654

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 1261

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(
p_.), x_Symbol] :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x],
 x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{2}\text{Subst}\left(\int \frac{d+ex}{\left(a^2+2abx+b^2x^2\right)^{3/2}}\,dx, x, x^2\right)\\
&= -\frac{e}{2b^2\sqrt{a^2+2abx^2+b^2x^4}} + \frac{(bd-ae)\text{Subst}\left(\int \frac{1}{\left(a^2+2abx+b^2x^2\right)^{3/2}}\,dx, x, x^2\right)}{2b}\\
&= -\frac{e}{2b^2\sqrt{a^2+2abx^2+b^2x^4}} - \frac{bd-ae}{4b^2\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}}
\end{aligned}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 181 vs. 2(77) = 154.

Time = 0.65 (sec) , antiderivative size = 181, normalized size of antiderivative = 2.35

$$\int \frac{x(d+ex^2)}{\left(a^2+2abx^2+b^2x^4\right)^{3/2}}\,dx =$$
$$-\frac{x^2\left(a^3bdx^2+a^2b^2ex^6+a^4(2d+ex^2)+a\left(-b^3dx^6+\sqrt{a^2}bx^2\sqrt{(a+bx^2)^2}(d+ex^2)\right)-\sqrt{a^2}\sqrt{(a+bx^2)^2}(b^2\right.}{4a^4\left(a+bx^2\right)\left(\sqrt{a^2}bx^2+a\left(\sqrt{a^2}-\sqrt{(a+bx^2)^2}\right)\right)}$$

```
[In] Integrate[(x*(d + e*x^2))/(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2),x]
```

```
[Out] -1/4*(x^2*(a^3*b*d*x^2 + a^2*b^2*e*x^6 + a^4*(2*d + e*x^2) + a*(-(b^3*d*x^6
) + Sqrt[a^2]*b*x^2*Sqrt[(a + b*x^2)^2]*(d + e*x^2)) - Sqrt[a^2]*Sqrt[(a +
b*x^2)^2]*(b^2*d*x^4 + a^2*(2*d + e*x^2))))/(a^4*(a + b*x^2)*(Sqrt[a^2]*b*x
^2 + a*(Sqrt[a^2] - Sqrt[(a + b*x^2)^2])))
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 2.

Time = 0.07 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.48

method	result	size
pseudoelliptic	$-\frac{\left(\left(2 e \,x^{2}+d \right) b +a e \right) \operatorname{csgn}\left(b \,x^{2}+a \right)}{4 \left(b \,x^{2}+a \right)^{2} b^{2}}$	37
gosper	$-\frac{\left(b \,x^{2}+a \right) \left(2 e \,x^{2} b +a e +b d \right)}{4 b^{2} \left(\left(b \,x^{2}+a \right)^{2}\right)^{\frac{3}{2}}}$	38
default	$-\frac{\left(b \,x^{2}+a \right) \left(2 e \,x^{2} b +a e +b d \right)}{4 b^{2} \left(\left(b \,x^{2}+a \right)^{2}\right)^{\frac{3}{2}}}$	38
risch	$\frac{\sqrt{\left(b \,x^{2}+a \right)^{2}}\, \left(-\frac{e \,x^{2}}{2 b}-\frac{a e +b d}{4 b^{2}}\right)}{\left(b \,x^{2}+a \right)^{3}}$	44

```
[In] int(x*(e*x^2+d)/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x,method=_RETURNVERBOSE)
[Out] -1/4*((2*e*x^2+d)*b+a*e)*csgn(b*x^2+a)/(b*x^2+a)^2/b^2
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.55

$$\int \frac{x(d+ex^2)}{(a^2+2abx^2+b^2x^4)^{3/2}}\,dx = -\frac{2\,bex^2+bd+ae}{4\,(b^4x^4+2\,ab^3x^2+a^2b^2)}$$

```
[In] integrate(x*(e*x^2+d)/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="fricas")
[Out] -1/4*(2*b*e*x^2 + b*d + a*e)/(b^4*x^4 + 2*a*b^3*x^2 + a^2*b^2)
```

Sympy [F]

$$\int \frac{x(d+ex^2)}{(a^2+2abx^2+b^2x^4)^{3/2}}\,dx = \int \frac{x(d+ex^2)}{((a+bx^2)^2)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(x*(e*x**2+d)/(b**2*x**4+2*a*b*x**2+a**2)**(3/2),x)
[Out] Integral(x*(d + e*x**2)/((a + b*x**2)**2)**(3/2), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.84

$$\int \frac{x(d+ex^2)}{(a^2+2abx^2+b^2x^4)^{3/2}}\,dx = -\frac{(2\,bx^2+a)e}{4\,(b^4x^4+2\,ab^3x^2+a^2b^2)} - \frac{d}{4\,(b^3x^4+2\,ab^2x^2+a^2b)}$$

```
[In] integrate(x*(e*x^2+d)/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="maxima")
[Out] -1/4*(2*b*x^2 + a)*e/(b^4*x^4 + 2*a*b^3*x^2 + a^2*b^2) - 1/4*d/(b^3*x^4 + 2
*a*b^2*x^2 + a^2*b)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.49

$$\int \frac{x(d+ex^2)}{(a^2+2abx^2+b^2x^4)^{3/2}}\,dx = -\frac{2\,bex^2+bd+ae}{4\,(bx^2+a)^2b^2\mathrm{sgn}\,(bx^2+a)}$$

```
[In] integrate(x*(e*x^2+d)/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="giac")
[Out] -1/4*(2*b*e*x^2 + b*d + a*e)/((b*x^2 + a)^2*b^2*sgn(b*x^2 + a))
```

Mupad [B] (verification not implemented)

Time = 7.73 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.62

$$\int \frac{x(d+ex^2)}{(a^2+2abx^2+b^2x^4)^{3/2}}\,dx = -\frac{\sqrt{a^2+2\,a\,b\,x^2+b^2\,x^4}\,\left(2\,b\,e\,x^2+a\,e+b\,d\right)}{4\,b^2\,\left(b\,x^2+a\right)^3}$$

```
[In] int((x*(d + e*x^2))/(a^2 + b^2*x^4 + 2*a*b*x^2)^(3/2),x)
[Out] -((a^2 + b^2*x^4 + 2*a*b*x^2)^(1/2)*(a*e + b*d + 2*b*e*x^2))/(4*b^2*(a + b*
x^2)^3)
```

3.83 $\int \frac{d+ex^2}{(a^2+2abx^2+b^2x^4)^{3/2}}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 156

$$\int \frac{d+ex^2}{(a^2+2abx^2+b^2x^4)^{3/2}}\,dx = \frac{(3bd+ae)x}{8a^2b\sqrt{a^2+2abx^2+b^2x^4}} + \frac{(bd-ae)x}{4ab(a+bx^2)\sqrt{a^2+2abx^2+b^2x^4}} + \frac{(3bd+ae)(a+bx^2)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8a^{5/2}b^{3/2}\sqrt{a^2+2abx^2+b^2x^4}}$$

```
[Out] 1/8*(a*e+3*b*d)*x/a^2/b/((b*x^2+a)^2)^(1/2)+1/4*(-a*e+b*d)*x/a/b/(b*x^2+a)/
((b*x^2+a)^2)^(1/2)+1/8*(a*e+3*b*d)*(b*x^2+a)*arctan(x*b^(1/2)/a^(1/2))/a^(
5/2)/b^(3/2)/((b*x^2+a)^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {1162, 393, 205, 211}

$$\int \frac{d+ex^2}{(a^2+2abx^2+b^2x^4)^{3/2}}\,dx = \frac{x(ae+3bd)}{8a^2b\sqrt{a^2+2abx^2+b^2x^4}} + \frac{x(bd-ae)}{4ab(a+bx^2)\sqrt{a^2+2abx^2+b^2x^4}} + \frac{(a+bx^2)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)(ae+3bd)}{8a^{5/2}b^{3/2}\sqrt{a^2+2abx^2+b^2x^4}}$$

```
[In] Int[(d + e*x^2)/(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2),x]
```

```
[Out] ((3*b*d + a*e)*x)/(8*a^2*b*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + ((b*d - a*e)*
x)/(4*a*b*(a + b*x^2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + ((3*b*d + a*e)*(a
```

```
+ b*x^2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(8*a^(5/2)*b^(3/2)*Sqrt[a^2 + 2*a*b*x
^2 + b^2*x^4])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integ
erQ[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denom
inator[p + 1/n] < Denominator[p])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 393

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Dist[(a*d -
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; F
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n
 + p, 0])
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_),
x_Symbol] :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^
2)^(2*FracPart[p])), Int[(d + e*x^2)^q*(b/2 + c*x^2)^(2*p), x], x] /; FreeQ
[{a, b, c, d, e, p, q}, x] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[p]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\left(b^2(ab+b^2x^2)\right)\int\frac{d+ex^2}{(ab+b^2x^2)^3}\,dx}{\sqrt{a^2+2abx^2+b^2x^4}}\\
&= \frac{(bd-ae)x}{4ab\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}}+\frac{\left((3bd+ae)\left(ab+b^2x^2\right)\right)\int\frac{1}{(ab+b^2x^2)^2}\,dx}{4a\sqrt{a^2+2abx^2+b^2x^4}}\\
&= \frac{(3bd+ae)x}{8a^2b\sqrt{a^2+2abx^2+b^2x^4}}+\frac{(bd-ae)x}{4ab\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}}\\
&\quad+\frac{\left((3bd+ae)\left(ab+b^2x^2\right)\right)\int\frac{1}{ab+b^2x^2}\,dx}{8a^2b\sqrt{a^2+2abx^2+b^2x^4}}
\end{aligned}
$$

$$= \frac{(3bd+ae)x}{8a^2b\sqrt{a^2+2abx^2+b^2x^4}} + \frac{(bd-ae)x}{4ab\,(a+bx^2)\sqrt{a^2+2abx^2+b^2x^4}} + \frac{(3bd+ae)\,(a+bx^2)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8a^{5/2}b^{3/2}\sqrt{a^2+2abx^2+b^2x^4}}$$

Mathematica [A] (verified)

Time = 1.04 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.69

$$\int \frac{d+ex^2}{(a^2+2abx^2+b^2x^4)^{3/2}}\,dx = \frac{\sqrt{a}\sqrt{b}x(-a^2e+3b^2dx^2+ab(5d+ex^2)) + (3bd+ae)\,(a+bx^2)^2\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right.}{8a^{5/2}b^{3/2}\,(a+bx^2)\sqrt{(a+bx^2)^2}}$$

```
[In] Integrate[(d + e*x^2)/(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2),x]
```

```
[Out] (Sqrt[a]*Sqrt[b]*x*(-(a^2*e) + 3*b^2*d*x^2 + a*b*(5*d + e*x^2)) + (3*b*d +
a*e)*(a + b*x^2)^2*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(8*a^(5/2)*b^(3/2)*(a + b*x
^2)*Sqrt[(a + b*x^2)^2])
```

Maple [A] (verified)

Time = 1.28 (sec) , antiderivative size = 164, normalized size of antiderivative = 1.05

method	result
risch	$\frac{\sqrt{(b\,x^2+a)^2}\left(\frac{(ae+3bd)x^3}{8a^2}-\frac{(ae-5bd)x}{8ab}\right)}{(b\,x^2+a)^3}-\frac{\sqrt{(b\,x^2+a)^2}\,(ae+3bd)\ln\left(bx+\sqrt{-ab}\right)}{16(b\,x^2+a)\sqrt{-ab}\,b\,a^2}+\frac{\sqrt{(b\,x^2+a)^2}\,(ae+3bd)\ln\left(-bx+\sqrt{-ab}\right)}{16(b\,x^2+a)\sqrt{-ab}\,b\,a^2}$
default	$-\frac{\left(-\arctan\left(\frac{bx}{\sqrt{ab}}\right)a\,b^2e\,x^4-3\arctan\left(\frac{bx}{\sqrt{ab}}\right)b^3d\,x^4-\sqrt{ab}\,abe\,x^3-3\sqrt{ab}\,b^2d\,x^3-2\arctan\left(\frac{bx}{\sqrt{ab}}\right)a^2be\,x^2-6\arctan\left(\frac{bx}{\sqrt{ab}}\right)a\,b^2d\,x^2+\sqrt{a}\right.}{8\sqrt{ab}\,b\,a^2\left((b\,x^2+a)^2\right)^{\frac{3}{2}}}$

```
[In] int((e*x^2+d)/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] ((b*x^2+a)^2)^(1/2)/(b*x^2+a)^3*(1/8*(a*e+3*b*d)/a^2*x^3-1/8*(a*e-5*b*d)/a/
b*x)-1/16*((b*x^2+a)^2)^(1/2)/(b*x^2+a)/(-a*b)^(1/2)*(a*e+3*b*d)/b/a^2*ln(b
*x+(-a*b)^(1/2))+1/16*((b*x^2+a)^2)^(1/2)/(b*x^2+a)/(-a*b)^(1/2)*(a*e+3*b*d
)/b/a^2*ln(-b*x+(-a*b)^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 301, normalized size of antiderivative = 1.93

$$\int \frac{d+ex^2}{(a^2+2abx^2+b^2x^4)^{3/2}}\,dx = \left[\frac{2\,(3\,ab^3d+a^2b^2e)x^3-((3\,b^3d+ab^2e)x^4+3\,a^2bd+a^3e+2\,(3\,ab^2d+a^2be)x}{16\,(a^3b^4x^4+2\,a^4b^3x^2+a^5b^2}\right.$$

```
[In] integrate((e*x^2+d)/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/16*(2*(3*a*b^3*d + a^2*b^2*e)*x^3 - ((3*b^3*d + a*b^2*e)*x^4 + 3*a^2*b*d
 + a^3*e + 2*(3*a*b^2*d + a^2*b*e)*x^2)*sqrt(-a*b)*log((b*x^2 - 2*sqrt(-a*b
)*x - a)/(b*x^2 + a)) + 2*(5*a^2*b^2*d - a^3*b*e)*x)/(a^3*b^4*x^4 + 2*a^4*b
^3*x^2 + a^5*b^2), 1/8*((3*a*b^3*d + a^2*b^2*e)*x^3 + ((3*b^3*d + a*b^2*e)*
x^4 + 3*a^2*b*d + a^3*e + 2*(3*a*b^2*d + a^2*b*e)*x^2)*sqrt(a*b)*arctan(sqr
t(a*b)*x/a) + (5*a^2*b^2*d - a^3*b*e)*x)/(a^3*b^4*x^4 + 2*a^4*b^3*x^2 + a^5
*b^2)]
```

Sympy [F]

$$\int \frac{d+ex^2}{(a^2+2abx^2+b^2x^4)^{3/2}}\,dx = \int \frac{d+ex^2}{\left((a+bx^2)^2\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((e*x**2+d)/(b**2*x**4+2*a*b*x**2+a**2)**(3/2),x)
```

```
[Out] Integral((d + e*x**2)/((a + b*x**2)**2)**(3/2), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.79

$$\int \frac{d+ex^2}{(a^2+2abx^2+b^2x^4)^{3/2}}\,dx = \frac{1}{8}\,d\left(\frac{3\,bx^3+5\,ax}{a^2b^2x^4+2\,a^3bx^2+a^4}+\frac{3\,\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}a^2}\right) + \frac{1}{8}\,e\left(\frac{bx^3-ax}{ab^3x^4+2\,a^2b^2x^2+a^3b}+\frac{\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}ab}\right)$$

```
[In] integrate((e*x^2+d)/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/8*d*((3*b*x^3 + 5*a*x)/(a^2*b^2*x^4 + 2*a^3*b*x^2 + a^4) + 3*arctan(b*x/s
qrt(a*b))/(sqrt(a*b)*a^2)) + 1/8*e*((b*x^3 - a*x)/(a*b^3*x^4 + 2*a^2*b^2*x^
2 + a^3*b) + arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a*b))
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.63

$$\int \frac{d+ex^2}{(a^2+2abx^2+b^2x^4)^{3/2}}\,dx=\frac{(3\,bd+ae)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\,\sqrt{ab}a^2b\mathrm{sgn}\,(bx^2+a)}+\frac{3\,b^2dx^3+abex^3+5\,abdx-a^2ex}{8\,(bx^2+a)^2a^2b\mathrm{sgn}\,(bx^2+a)}$$

```
[In] integrate((e*x^2+d)/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="giac")
[Out] 1/8*(3*b*d + a*e)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^2*b*sgn(b*x^2 + a)) +
1/8*(3*b^2*d*x^3 + a*b*e*x^3 + 5*a*b*d*x - a^2*e*x)/((b*x^2 + a)^2*a^2*b*sg
n(b*x^2 + a))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{d+ex^2}{(a^2+2abx^2+b^2x^4)^{3/2}}\,dx=\int \frac{e\,x^2+d}{(a^2+2\,a\,b\,x^2+b^2\,x^4)^{3/2}}\,dx$$

```
[In] int((d + e*x^2)/(a^2 + b^2*x^4 + 2*a*b*x^2)^(3/2),x)
[Out] int((d + e*x^2)/(a^2 + b^2*x^4 + 2*a*b*x^2)^(3/2), x)
```

3.84 $\int \frac{d+ex^2}{x\left(a^2+2abx^2+b^2x^4\right)^{3/2}}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 161

$$\int \frac{d+ex^2}{x\left(a^2+2abx^2+b^2x^4\right)^{3/2}}\,dx = \frac{d}{2a^2\sqrt{a^2+2abx^2+b^2x^4}} + \frac{bd-ae}{4ab\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}} + \frac{d(a+bx^2)\log(x)}{a^3\sqrt{a^2+2abx^2+b^2x^4}} - \frac{d(a+bx^2)\log\left(a+bx^2\right)}{2a^3\sqrt{a^2+2abx^2+b^2x^4}}$$

```
[Out] 1/2*d/a^2/((b*x^2+a)^2)^(1/2)+1/4*(-a*e+b*d)/a/b/(b*x^2+a)/((b*x^2+a)^2)^(1
/2)+d*(b*x^2+a)*ln(x)/a^3/((b*x^2+a)^2)^(1/2)-1/2*d*(b*x^2+a)*ln(b*x^2+a)/a
^3/((b*x^2+a)^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 161, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {1264, 457, 78}

$$\int \frac{d+ex^2}{x\left(a^2+2abx^2+b^2x^4\right)^{3/2}}\,dx = \frac{bd-ae}{4ab\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}} + \frac{d}{2a^2\sqrt{a^2+2abx^2+b^2x^4}} + \frac{d\log(x)\left(a+bx^2\right)}{a^3\sqrt{a^2+2abx^2+b^2x^4}} - \frac{d(a+bx^2)\log\left(a+bx^2\right)}{2a^3\sqrt{a^2+2abx^2+b^2x^4}}$$

```
[In] Int[(d + e*x^2)/(x*(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2)),x]
```

```
[Out] d/(2*a^2*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (b*d - a*e)/(4*a*b*(a + b*x^2)*
Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (d*(a + b*x^2)*Log[x])/(a^3*Sqrt[a^2 + 2
*a*b*x^2 + b^2*x^4]) - (d*(a + b*x^2)*Log[a + b*x^2])/(2*a^3*Sqrt[a^2 + 2*a
*b*x^2 + b^2*x^4])
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1264

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_), x_Symbol] :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^Int
Part[p]*(b/2 + c*x^2)^(2*FracPart[p])), Int[(f*x)^m*(d + e*x^2)^q*(b/2 + c*
x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, f, m, p, q}, x] && EqQ[b^2 - 4*
a*c, 0] &&  !IntegerQ[p]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{(b^2(ab+b^2x^2))\int \frac{d+ex^2}{x(ab+b^2x^2)^3}\,dx}{\sqrt{a^2+2abx^2+b^2x^4}}\\
&= \frac{(b^2(ab+b^2x^2))\,\text{Subst}\left(\int \frac{d+ex}{x(ab+b^2x)^3}\,dx,x,x^2\right)}{2\sqrt{a^2+2abx^2+b^2x^4}}\\
&= \frac{(b^2(ab+b^2x^2))\,\text{Subst}\left(\int \left(\frac{d}{a^3b^3x}+\frac{-bd+ae}{ab^3(a+bx)^3}-\frac{d}{a^2b^2(a+bx)^2}-\frac{d}{a^3b^2(a+bx)}\right)dx,x,x^2\right)}{2\sqrt{a^2+2abx^2+b^2x^4}}\\
&= \frac{d}{2a^2\sqrt{a^2+2abx^2+b^2x^4}}+\frac{bd-ae}{4ab\,(a+bx^2)\sqrt{a^2+2abx^2+b^2x^4}}\\
&\quad+\frac{d(a+bx^2)\log(x)}{a^3\sqrt{a^2+2abx^2+b^2x^4}}-\frac{d(a+bx^2)\log\left(a+bx^2\right)}{2a^3\sqrt{a^2+2abx^2+b^2x^4}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 1.03 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.57

$$\int \frac{d+ex^2}{x\left(a^2+2abx^2+b^2x^4\right)^{3/2}}\,dx = \frac{a(3abd-a^2e+2b^2dx^2)+4bd(a+bx^2)^2\log(x)-2bd(a+bx^2)^2\log\left(a+bx^2\right.}{4a^3b\left(a+bx^2\right)\sqrt{\left(a+bx^2\right)^2}}$$

```
[In] Integrate[(d + e*x^2)/(x*(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2)),x]
[Out] (a*(3*a*b*d - a^2*e + 2*b^2*d*x^2) + 4*b*d*(a + b*x^2)^2*Log[x] - 2*b*d*(a
+ b*x^2)^2*Log[a + b*x^2])/(4*a^3*b*(a + b*x^2)*Sqrt[(a + b*x^2)^2])
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.10 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.54

method	result
pseudoelliptic	$-\frac{\operatorname{csgn}\left(b\,x^{2}+a\right)\left(2db\left(b\,x^{2}+a\right)^{2}\ln\left(b\,x^{2}+a\right)-2db\left(b\,x^{2}+a\right)^{2}\ln\left(x^{2}\right)+a\left(-2b^{2}d\,x^{2}+e\,a^{2}-3dab\right)\right)}{4a^{3}b\left(b\,x^{2}+a\right)^{2}}$
risch	$\frac{\sqrt{\left(b\,x^{2}+a\right)^{2}}\,\left(\frac{bd\,x^{2}}{2a^{2}}-\frac{ae-3bd}{4ab}\right)}{\left(b\,x^{2}+a\right)^{3}}+\frac{\sqrt{\left(b\,x^{2}+a\right)^{2}}\,d\ln\left(x\right)}{\left(b\,x^{2}+a\right)a^{3}}-\frac{\sqrt{\left(b\,x^{2}+a\right)^{2}}\,d\ln\left(b\,x^{2}+a\right)}{2\left(b\,x^{2}+a\right)a^{3}}$
default	$\frac{\left(4\ln\left(x\right)b^{3}d\,x^{4}-2\ln\left(b\,x^{2}+a\right)b^{3}d\,x^{4}+8\ln\left(x\right)a\,b^{2}d\,x^{2}-4\ln\left(b\,x^{2}+a\right)a\,b^{2}d\,x^{2}+2b^{2}d\,x^{2}a+4\ln\left(x\right)a^{2}bd-2\ln\left(b\,x^{2}+a\right)a^{2}bd-a^{3}e+3a\right.}{4b\,a^{3}\left(\left(b\,x^{2}+a\right)^{2}\right)^{\frac{3}{2}}}$

```
[In] int((e*x^2+d)/x/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x,method=_RETURNVERBOSE)
[Out] -1/4*csgn(b*x^2+a)*(2*d*b*(b*x^2+a)^2*ln(b*x^2+a)-2*d*b*(b*x^2+a)^2*ln(x^2)
+a*(-2*b^2*d*x^2+a^2*e-3*a*b*d))/a^3/b/(b*x^2+a)^2
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.74

$$\int \frac{d+ex^2}{x\left(a^2+2abx^2+b^2x^4\right)^{3/2}}\,dx = \frac{2\,ab^2dx^2+3\,a^2bd-a^3e-2\left(b^3dx^4+2\,ab^2dx^2+a^2bd\right)\log\left(bx^2+a\right)+4\left(b^3d\right.}{4\left(a^3b^3x^4+2\,a^4b^2x^2+a^5b\right)}$$

```
[In] integrate((e*x^2+d)/x/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="fricas")
[Out] 1/4*(2*a*b^2*d*x^2 + 3*a^2*b*d - a^3*e - 2*(b^3*d*x^4 + 2*a*b^2*d*x^2 + a^2
*b*d)*log(b*x^2 + a) + 4*(b^3*d*x^4 + 2*a*b^2*d*x^2 + a^2*b*d)*log(x))/(a^3
*b^3*x^4 + 2*a^4*b^2*x^2 + a^5*b)
```

Sympy [F]

$$\int \frac{d+ex^2}{x\left(a^2+2abx^2+b^2x^4\right)^{3/2}}\,dx = \int \frac{d+ex^2}{x\left(\left(a+bx^2\right)^2\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((e*x**2+d)/x/(b**2*x**4+2*a*b*x**2+a**2)**(3/2),x)
[Out] Integral((d + e*x**2)/(x*((a + b*x**2)**2)**(3/2)), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.55

$$\int \frac{d+ex^2}{x\left(a^2+2abx^2+b^2x^4\right)^{3/2}}\,dx = \frac{1}{4}\,d\left(\frac{2\,bx^2+3\,a}{a^2b^2x^4+2\,a^3bx^2+a^4}-\frac{2\,\log\left(bx^2+a\right)}{a^3}+\frac{4\,\log\left(x\right)}{a^3}\right) - \frac{e}{4\left(b^3x^4+2\,ab^2x^2+a^2b\right)}$$

```
[In] integrate((e*x^2+d)/x/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="maxima")
[Out] 1/4*d*((2*b*x^2 + 3*a)/(a^2*b^2*x^4 + 2*a^3*b*x^2 + a^4) - 2*log(b*x^2 + a)
/a^3 + 4*log(x)/a^3) - 1/4*e/(b^3*x^4 + 2*a*b^2*x^2 + a^2*b)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.66

$$\int \frac{d+ex^2}{x\left(a^2+2abx^2+b^2x^4\right)^{3/2}}\,dx = \frac{d\log\left(x^2\right)}{2\,a^3\mathrm{sgn}\left(bx^2+a\right)} - \frac{d\log\left(\left|bx^2+a\right|\right)}{2\,a^3\mathrm{sgn}\left(bx^2+a\right)} + \frac{3\,b^3dx^4+8\,ab^2dx^2+6\,a^2bd-a^3e}{4\left(bx^2+a\right)^2a^3b\mathrm{sgn}\left(bx^2+a\right)}$$

```
[In] integrate((e*x^2+d)/x/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="giac")
[Out] 1/2*d*log(x^2)/(a^3*sgn(b*x^2 + a)) - 1/2*d*log(abs(b*x^2 + a))/(a^3*sgn(b*
x^2 + a)) + 1/4*(3*b^3*d*x^4 + 8*a*b^2*d*x^2 + 6*a^2*b*d - a^3*e)/((b*x^2 +
 a)^2*a^3*b*sgn(b*x^2 + a))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{d+ex^2}{x\left(a^2+2abx^2+b^2x^4\right)^{3/2}}\,dx=\int \frac{e\,x^2+d}{x\,\left(a^2+2\,a\,b\,x^2+b^2\,x^4\right)^{3/2}}\,dx$$

```
[In] int((d + e*x^2)/(x*(a^2 + b^2*x^4 + 2*a*b*x^2)^(3/2)),x)
[Out] int((d + e*x^2)/(x*(a^2 + b^2*x^4 + 2*a*b*x^2)^(3/2)), x)
```

3.85 $\int \frac{d+ex^2}{x^2\left(a^2+2abx^2+b^2x^4\right)^{3/2}}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 190

$$\int \frac{d+ex^2}{x^2\left(a^2+2abx^2+b^2x^4\right)^{3/2}}\,dx = -\frac{(7bd-3ae)x}{8a^3\sqrt{a^2+2abx^2+b^2x^4}} - \frac{(bd-ae)x}{4a^2\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}} - \frac{d\left(a+bx^2\right)}{a^3x\sqrt{a^2+2abx^2+b^2x^4}} - \frac{3(5bd-ae)\left(a+bx^2\right)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8a^{7/2}\sqrt{b}\sqrt{a^2+2abx^2+b^2x^4}}$$

```
[Out] -1/8*(-3*a*e+7*b*d)*x/a^3/((b*x^2+a)^2)^(1/2)-1/4*(-a*e+b*d)*x/a^2/(b*x^2+a
)/((b*x^2+a)^2)^(1/2)-d*(b*x^2+a)/a^3/x/((b*x^2+a)^2)^(1/2)-3/8*(-a*e+5*b*d
)*(b*x^2+a)*arctan(x*b^(1/2)/a^(1/2))/a^(7/2)/b^(1/2)/((b*x^2+a)^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 190, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.121, Rules used = {1264, 467, 464, 211}

$$\int \frac{d+ex^2}{x^2\left(a^2+2abx^2+b^2x^4\right)^{3/2}}\,dx = -\frac{x(bd-ae)}{4a^2\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}} - \frac{3\left(a+bx^2\right)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)(5bd-ae)}{8a^{7/2}\sqrt{b}\sqrt{a^2+2abx^2+b^2x^4}} - \frac{x(7bd-3ae)}{8a^3\sqrt{a^2+2abx^2+b^2x^4}} - \frac{d\left(a+bx^2\right)}{a^3x\sqrt{a^2+2abx^2+b^2x^4}}$$

```
[In] Int[(d + e*x^2)/(x^2*(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2)),x]
```

```
[Out] -1/8*((7*b*d - 3*a*e)*x)/(a^3*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - ((b*d - a*
e)*x)/(4*a^2*(a + b*x^2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (d*(a + b*x^2))
/(a^3*x*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (3*(5*b*d - a*e)*(a + b*x^2)*Arc
Tan[(Sqrt[b]*x)/Sqrt[a]])/(8*a^(7/2)*Sqrt[b]*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4
])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 464

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 467

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[(-a)^(m/2 - 1)*(b*c - a*d)*x*((a + b*x^2)^(p + 1)/(2*b^(m/2 + 1)*(p
+ 1))), x] + Dist[1/(2*b^(m/2 + 1)*(p + 1)), Int[x^m*(a + b*x^2)^(p + 1)*Ex
pandToSum[2*b*(p + 1)*Together[(b^(m/2)*(c + d*x^2) - (-a)^(m/2 - 1)*(b*c -
 a*d)*x^(-m + 2))/(a + b*x^2)] - ((-a)^(m/2 - 1)*(b*c - a*d))/x^m, x], x],
x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && ILtQ[m/2
, 0] && (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

Rule 1264

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_), x_Symbol] :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^Int
Part[p]*(b/2 + c*x^2)^(2*FracPart[p])), Int[(f*x)^m*(d + e*x^2)^q*(b/2 + c*
x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, f, m, p, q}, x] && EqQ[b^2 - 4*
a*c, 0] &&  !IntegerQ[p]
```

Rubi steps

$$\text{integral} = \frac{(b^2(ab+b^2x^2))\int \frac{d+ex^2}{x^2(ab+b^2x^2)^3}\,dx}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$$= -\frac{(bd-ae)x}{4a^2\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}} - \frac{(b^2(ab+b^2x^2))\int \frac{-\frac{4d}{ab}+\frac{3(bd-ae)x^2}{a^2b}}{x^2(ab+b^2x^2)^2}\,dx}{4\sqrt{a^2+2abx^2+b^2x^4}}$$

$$= -\frac{(7bd-3ae)x}{8a^3\sqrt{a^2+2abx^2+b^2x^4}} - \frac{(bd-ae)x}{4a^2(a+bx^2)\sqrt{a^2+2abx^2+b^2x^4}} + \frac{(b^2(ab+b^2x^2))\int \frac{\frac{8d}{a^2b^2}-\frac{(7bd-3ae)x^2}{a^3b^2}}{x^2(ab+b^2x^2)}dx}{8\sqrt{a^2+2abx^2+b^2x^4}}$$

$$= -\frac{(7bd-3ae)x}{8a^3\sqrt{a^2+2abx^2+b^2x^4}} - \frac{(bd-ae)x}{4a^2(a+bx^2)\sqrt{a^2+2abx^2+b^2x^4}} - \frac{d(a+bx^2)}{a^3x\sqrt{a^2+2abx^2+b^2x^4}} - \frac{(3(5bd-ae)(ab+b^2x^2))\int \frac{1}{ab+b^2x^2}dx}{8a^3\sqrt{a^2+2abx^2+b^2x^4}}$$

$$= -\frac{(7bd-3ae)x}{8a^3\sqrt{a^2+2abx^2+b^2x^4}} - \frac{(bd-ae)x}{4a^2(a+bx^2)\sqrt{a^2+2abx^2+b^2x^4}} - \frac{d(a+bx^2)}{a^3x\sqrt{a^2+2abx^2+b^2x^4}} - \frac{3(5bd-ae)(a+bx^2)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8a^{7/2}\sqrt{b}\sqrt{a^2+2abx^2+b^2x^4}}$$

Mathematica [A] (verified)

Time = 1.05 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.65

$$\int \frac{d+ex^2}{x^2(a^2+2abx^2+b^2x^4)^{3/2}}dx = \frac{\sqrt{a}\sqrt{b}(-15b^2dx^4+a^2(-8d+5ex^2)+ab(-25dx^2+3ex^4))+3(-5bd+a}{8a^{7/2}\sqrt{b}x(a+bx^2)\sqrt{(a+bx^2)^2}}$$

```
[In] Integrate[(d + e*x^2)/(x^2*(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2)),x]
```

```
[Out] (Sqrt[a]*Sqrt[b]*(-15*b^2*d*x^4 + a^2*(-8*d + 5*e*x^2) + a*b*(-25*d*x^2 + 3
*e*x^4)) + 3*(-5*b*d + a*e)*x*(a + b*x^2)^2*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(8
*a^(7/2)*Sqrt[b]*x*(a + b*x^2)*Sqrt[(a + b*x^2)^2])
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.22 (sec) , antiderivative size = 176, normalized size of antiderivative = 0.93

method	result
risch	$\frac{\sqrt{(bx^2+a)^2}\left(\frac{3b(ae-5bd)x^4}{8a^3}+\frac{5(ae-5bd)x^2}{8a^2}-\frac{d}{a}\right)}{(bx^2+a)^3x} + \frac{3\sqrt{(bx^2+a)^2}\left(\sum_{\_R=\mathrm{RootOf}\left(a^7\_Z^2b+e^2a^2-10abde+25b^2d^2\right)}\_R\ln\left(\left(3\_R^2a^7b\right.\right.\right.}{16(bx^2+a)}$
default	$-\frac{\left(-3\arctan\left(\frac{bx}{\sqrt{ab}}\right)ab^2ex^5+15\arctan\left(\frac{bx}{\sqrt{ab}}\right)b^3dx^5-3\sqrt{ab}\,abex^4+15\sqrt{ab}\,b^2dx^4-6\arctan\left(\frac{bx}{\sqrt{ab}}\right)a^2bex^3+30\arctan\left(\frac{bx}{\sqrt{ab}}\right)ab^2dx\right.}{8\sqrt{ab}\,xa^3\left((bx^2+a)^2\right)^{\frac{3}{2}}}$

```
[In] int((e*x^2+d)/x^2/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] ((b*x^2+a)^2)^(1/2)/(b*x^2+a)^3*(3/8*b*(a*e-5*b*d)/a^3*x^4+5/8/a^2*(a*e-5*b
*d)*x^2-d/a)/x+3/16*((b*x^2+a)^2)^(1/2)/(b*x^2+a)*sum(_R*ln((3*_R^2*a^7*b+2
*a^2*e^2-20*a*b*d*e+50*b^2*d^2)*x+(-a^5*e+5*a^4*b*d)*_R),_R=RootOf(_Z^2*a^7
*b+a^2*e^2-10*a*b*d*e+25*b^2*d^2))
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 334, normalized size of antiderivative = 1.76

$$\int \frac{d+ex^2}{x^2\left(a^2+2abx^2+b^2x^4\right)^{3/2}}\,dx = \left[-\frac{16\,a^3bd+6\,(5\,ab^3d-a^2b^2e)x^4+10\,(5\,a^2b^2d-a^3be)x^2-3\,((5\,b^3d-ab^2\right.}{16\,(a^4b^3x^5+2}$$

$$-\frac{8\,a^3bd+3\,(5\,ab^3d-a^2b^2e)x^4+5\,(5\,a^2b^2d-a^3be)x^2+3\,((5\,b^3d-ab^2e)x^5+2\,(5\,ab^2d-a^2be)x^3+(5\,a^2bd-}{8\,(a^4b^3x^5+2\,a^5b^2x^3+a^6bx)}$$

```
[In] integrate((e*x^2+d)/x^2/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="fricas"
)
```

```
[Out] [-1/16*(16*a^3*b*d + 6*(5*a*b^3*d - a^2*b^2*e)*x^4 + 10*(5*a^2*b^2*d - a^3*
b*e)*x^2 - 3*((5*b^3*d - a*b^2*e)*x^5 + 2*(5*a*b^2*d - a^2*b*e)*x^3 + (5*a^
2*b*d - a^3*e)*x)*sqrt(-a*b)*log((b*x^2 - 2*sqrt(-a*b)*x - a)/(b*x^2 + a)))
/(a^4*b^3*x^5 + 2*a^5*b^2*x^3 + a^6*b*x), -1/8*(8*a^3*b*d + 3*(5*a*b^3*d -
a^2*b^2*e)*x^4 + 5*(5*a^2*b^2*d - a^3*b*e)*x^2 + 3*((5*b^3*d - a*b^2*e)*x^5
 + 2*(5*a*b^2*d - a^2*b*e)*x^3 + (5*a^2*b*d - a^3*e)*x)*sqrt(a*b)*arctan(sq
rt(a*b)*x/a))/(a^4*b^3*x^5 + 2*a^5*b^2*x^3 + a^6*b*x)]
```

Sympy [F]

$$\int \frac{d+ex^2}{x^2\left(a^2+2abx^2+b^2x^4\right)^{3/2}}\,dx = \int \frac{d+ex^2}{x^2\left(\left(a+bx^2\right)^2\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((e*x**2+d)/x**2/(b**2*x**4+2*a*b*x**2+a**2)**(3/2),x)
```

```
[Out] Integral((d + e*x**2)/(x**2*((a + b*x**2)**2)**(3/2)), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 134, normalized size of antiderivative = 0.71

$$\int \frac{d+ex^2}{x^2\left(a^2+2abx^2+b^2x^4\right)^{3/2}}\,dx = -\frac{1}{8}\,d\left(\frac{15\,b^2x^4+25\,abx^2+8\,a^2}{a^3b^2x^5+2\,a^4bx^3+a^5x}+\frac{15\,b\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}a^3}\right) + \frac{1}{8}\,e\left(\frac{3\,bx^3+5\,ax}{a^2b^2x^4+2\,a^3bx^2+a^4}+\frac{3\,\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}a^2}\right)$$

```
[In] integrate((e*x^2+d)/x^2/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="maxima"
)
[Out] -1/8*d*((15*b^2*x^4 + 25*a*b*x^2 + 8*a^2)/(a^3*b^2*x^5 + 2*a^4*b*x^3 + a^5*
x) + 15*b*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^3)) + 1/8*e*((3*b*x^3 + 5*a*x)
/(a^2*b^2*x^4 + 2*a^3*b*x^2 + a^4) + 3*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^2
))
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.59

$$\int \frac{d+ex^2}{x^2\left(a^2+2abx^2+b^2x^4\right)^{3/2}}\,dx = -\frac{3\,(5\,bd-ae)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\,\sqrt{ab}a^3\mathrm{sgn}\left(bx^2+a\right)} - \frac{d}{a^3x\mathrm{sgn}\left(bx^2+a\right)} - \frac{7\,b^2dx^3-3\,abex^3+9\,abdx-5\,a^2ex}{8\,\left(bx^2+a\right)^2a^3\mathrm{sgn}\left(bx^2+a\right)}$$

```
[In] integrate((e*x^2+d)/x^2/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="giac")
[Out] -3/8*(5*b*d - a*e)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^3*sgn(b*x^2 + a)) - d
/(a^3*x*sgn(b*x^2 + a)) - 1/8*(7*b^2*d*x^3 - 3*a*b*e*x^3 + 9*a*b*d*x - 5*a^
2*e*x)/((b*x^2 + a)^2*a^3*sgn(b*x^2 + a))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{d+ex^2}{x^2\left(a^2+2abx^2+b^2x^4\right)^{3/2}}\,dx=\int \frac{e\,x^2+d}{x^2\,\left(a^2+2\,a\,b\,x^2+b^2\,x^4\right)^{3/2}}\,dx$$

```
[In] int((d + e*x^2)/(x^2*(a^2 + b^2*x^4 + 2*a*b*x^2)^(3/2)),x)
[Out] int((d + e*x^2)/(x^2*(a^2 + b^2*x^4 + 2*a*b*x^2)^(3/2)), x)
```

3.86 $\int \frac{d+ex^2}{x^3(a^2+2abx^2+b^2x^4)^{3/2}}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 223

$$\int \frac{d+ex^2}{x^3(a^2+2abx^2+b^2x^4)^{3/2}}\,dx = -\frac{2bd-ae}{2a^3\sqrt{a^2+2abx^2+b^2x^4}} - \frac{bd-ae}{4a^2(a+bx^2)\sqrt{a^2+2abx^2+b^2x^4}} - \frac{d(a+bx^2)}{2a^3x^2\sqrt{a^2+2abx^2+b^2x^4}} - \frac{(3bd-ae)(a+bx^2)\log(x)}{a^4\sqrt{a^2+2abx^2+b^2x^4}} + \frac{(3bd-ae)(a+bx^2)\log(a+bx^2)}{2a^4\sqrt{a^2+2abx^2+b^2x^4}}$$

```
[Out] 1/2*(a*e-2*b*d)/a^3/((b*x^2+a)^2)^(1/2)+1/4*(a*e-b*d)/a^2/(b*x^2+a)/((b*x^2
+a)^2)^(1/2)-1/2*d*(b*x^2+a)/a^3/x^2/((b*x^2+a)^2)^(1/2)-(-a*e+3*b*d)*(b*x^
2+a)*ln(x)/a^4/((b*x^2+a)^2)^(1/2)+1/2*(-a*e+3*b*d)*(b*x^2+a)*ln(b*x^2+a)/a
^4/((b*x^2+a)^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 223, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {1264, 457, 78}

$$\int \frac{d+ex^2}{x^3(a^2+2abx^2+b^2x^4)^{3/2}}\,dx = -\frac{bd-ae}{4a^2(a+bx^2)\sqrt{a^2+2abx^2+b^2x^4}} - \frac{\log(x)(a+bx^2)(3bd-ae)}{a^4\sqrt{a^2+2abx^2+b^2x^4}} + \frac{(a+bx^2)(3bd-ae)\log(a+bx^2)}{2a^4\sqrt{a^2+2abx^2+b^2x^4}} - \frac{2bd-ae}{2a^3\sqrt{a^2+2abx^2+b^2x^4}} - \frac{d(a+bx^2)}{2a^3x^2\sqrt{a^2+2abx^2+b^2x^4}}$$

```
[In] Int[(d + e*x^2)/(x^3*(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2)),x]
```

```
[Out] -1/2*(2*b*d - a*e)/(a^3*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (b*d - a*e)/(4*a
^2*(a + b*x^2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (d*(a + b*x^2))/(2*a^3*x^
2*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - ((3*b*d - a*e)*(a + b*x^2)*Log[x])/(a^
4*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + ((3*b*d - a*e)*(a + b*x^2)*Log[a + b*x
^2])/(2*a^4*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1264

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_), x_Symbol] :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^Int
Part[p]*(b/2 + c*x^2)^(2*FracPart[p])), Int[(f*x)^m*(d + e*x^2)^q*(b/2 + c*
x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, f, m, p, q}, x] && EqQ[b^2 - 4*
a*c, 0] &&  !IntegerQ[p]
```

Rubi steps

$$\text{integral} = \frac{(b^2(ab+b^2x^2))\int \frac{d+ex^2}{x^3(ab+b^2x^2)^3}\,dx}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$$= \frac{(b^2(ab+b^2x^2))\,\text{Subst}\left(\int \frac{d+ex}{x^2(ab+b^2x)^3}\,dx, x, x^2\right)}{2\sqrt{a^2+2abx^2+b^2x^4}}$$

$$= \frac{(b^2(ab+b^2x^2))\,\text{Subst}\left(\int \left(\frac{d}{a^3b^3x^2}+\frac{-3bd+ae}{a^4b^3x}+\frac{bd-ae}{a^2b^2(a+bx)^3}+\frac{2bd-ae}{a^3b^2(a+bx)^2}+\frac{3bd-ae}{a^4b^2(a+bx)}\right)dx, x, x^2\right)}{2\sqrt{a^2+2abx^2+b^2x^4}}$$

$$= -\frac{2bd - ae}{2a^3\sqrt{a^2 + 2abx^2 + b^2x^4}} - \frac{bd - ae}{4a^2\left(a + bx^2\right)\sqrt{a^2 + 2abx^2 + b^2x^4}} - \frac{d(a + bx^2)}{2a^3x^2\sqrt{a^2 + 2abx^2 + b^2x^4}} - \frac{(3bd - ae)\left(a + bx^2\right)\log(x)}{a^4\sqrt{a^2 + 2abx^2 + b^2x^4}} + \frac{(3bd - ae)\left(a + bx^2\right)\log\left(a + bx^2\right)}{2a^4\sqrt{a^2 + 2abx^2 + b^2x^4}}$$

Mathematica [A] (verified)

Time = 1.06 (sec) , antiderivative size = 130, normalized size of antiderivative = 0.58

$$\int \frac{d + ex^2}{x^3\left(a^2 + 2abx^2 + b^2x^4\right)^{3/2}}\,dx = \frac{a(-6b^2dx^4 + a^2(-2d + 3ex^2) + ab(-9dx^2 + 2ex^4)) + 4(-3bd + ae)x^2(a + \ldots}{4a^4x^2\left(a + bx^2\right)\sqrt{(a + b \ldots}}$$

```
[In] Integrate[(d + e*x^2)/(x^3*(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2)),x]
```

```
[Out] (a*(-6*b^2*d*x^4 + a^2*(-2*d + 3*e*x^2) + a*b*(-9*d*x^2 + 2*e*x^4)) + 4*(-3
*b*d + a*e)*x^2*(a + b*x^2)^2*Log[x] + 2*(3*b*d - a*e)*x^2*(a + b*x^2)^2*Lo
g[a + b*x^2])/(4*a^4*x^2*(a + b*x^2)*Sqrt[(a + b*x^2)^2])
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.14 (sec) , antiderivative size = 120, normalized size of antiderivative = 0.54

method	result
pseudoelliptic	$-\frac{\operatorname{csgn}(b\,x^2+a)\left(x^2\left(b\,x^2+a\right)^2(ae-3bd)\ln\left(b\,x^2+a\right)-x^2\left(b\,x^2+a\right)^2(ae-3bd)\ln\left(x^2\right)+a\left(\left(-abe+3b^2d\right)x^4-\frac{3a(ae-3bd)x^2}{2}+d\,a \ldots\right.\right.}{2\left(b\,x^2+a\right)^2x^2a^4}$
risch	$\frac{\sqrt{\left(b\,x^2+a\right)^2}\left(\frac{b(ae-3bd)x^4}{2a^3}+\frac{3(ae-3bd)x^2}{4a^2}-\frac{d}{2a}\right)}{\left(b\,x^2+a\right)^3x^2}+\frac{\sqrt{\left(b\,x^2+a\right)^2}\,(ae-3bd)\ln(x)}{\left(b\,x^2+a\right)a^4}-\frac{\sqrt{\left(b\,x^2+a\right)^2}\,(ae-3bd)\ln\left(b\,x^2+a\right)}{2\left(b\,x^2+a\right)a^4}$
default	$\frac{(4\ln(x)a\,b^2e\,x^6-12\ln(x)b^3d\,x^6-2\ln\left(b\,x^2+a\right)a\,b^2e\,x^6+6\ln\left(b\,x^2+a\right)b^3d\,x^6+8\ln(x)a^2be\,x^4-24\ln(x)a\,b^2d\,x^4-4\ln\left(b\,x^2+a\right)a \ldots}{}$

```
[In] int((e*x^2+d)/x^3/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*csgn(b*x^2+a)*(x^2*(b*x^2+a)^2*(a*e-3*b*d)*ln(b*x^2+a)-x^2*(b*x^2+a)^2
*(a*e-3*b*d)*ln(x^2)+a*((-a*b*e+3*b^2*d)*x^4-3/2*a*(a*e-3*b*d)*x^2+d*a^2))/
(b*x^2+a)^2/x^2/a^4
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 205, normalized size of antiderivative = 0.92

$$\int \frac{d+ex^2}{x^3 \left(a^2+2abx^2+b^2x^4\right)^{3/2}}\,dx =$$

$$-\frac{2\,(3\,ab^2d-a^2be)x^4+2\,a^3d+3\,(3\,a^2bd-a^3e)x^2-2\,((3\,b^3d-ab^2e)x^6+2\,(3\,ab^2d-a^2be)x^4+(3\,a^2bd-a^3e}{4\,(a^4b^2x^6+2\,a^5bx^4+a}$$

```
[In] integrate((e*x^2+d)/x^3/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="fricas"
)
[Out] -1/4*(2*(3*a*b^2*d - a^2*b*e)*x^4 + 2*a^3*d + 3*(3*a^2*b*d - a^3*e)*x^2 - 2
*((3*b^3*d - a*b^2*e)*x^6 + 2*(3*a*b^2*d - a^2*b*e)*x^4 + (3*a^2*b*d - a^3*
e)*x^2)*log(b*x^2 + a) + 4*((3*b^3*d - a*b^2*e)*x^6 + 2*(3*a*b^2*d - a^2*b*
e)*x^4 + (3*a^2*b*d - a^3*e)*x^2)*log(x))/(a^4*b^2*x^6 + 2*a^5*b*x^4 + a^6*
x^2)
```

Sympy [F]

$$\int \frac{d+ex^2}{x^3 \left(a^2+2abx^2+b^2x^4\right)^{3/2}}\,dx = \int \frac{d+ex^2}{x^3 \left(\left(a+bx^2\right)^2\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((e*x**2+d)/x**3/(b**2*x**4+2*a*b*x**2+a**2)**(3/2),x)
[Out] Integral((d + e*x**2)/(x**3*((a + b*x**2)**2)**(3/2)), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.62

$$\int \frac{d+ex^2}{x^3 \left(a^2+2abx^2+b^2x^4\right)^{3/2}}\,dx = -\frac{1}{4}\,d\left(\frac{6\,b^2x^4+9\,abx^2+2\,a^2}{a^3b^2x^6+2\,a^4bx^4+a^5x^2}-\frac{6\,b\log\left(bx^2+a\right)}{a^4}+\frac{12\,b\log\left(x\right)}{a^4}\right) + \frac{1}{4}\,e\left(\frac{2\,bx^2+3\,a}{a^2b^2x^4+2\,a^3bx^2+a^4}-\frac{2\,\log\left(bx^2+a\right)}{a^3}+\frac{4\,\log\left(x\right)}{a^3}\right)$$

```
[In] integrate((e*x^2+d)/x^3/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="maxima"
)
[Out] -1/4*d*((6*b^2*x^4 + 9*a*b*x^2 + 2*a^2)/(a^3*b^2*x^6 + 2*a^4*b*x^4 + a^5*x^
2) - 6*b*log(b*x^2 + a)/a^4 + 12*b*log(x)/a^4) + 1/4*e*((2*b*x^2 + 3*a)/(a^
2*b^2*x^4 + 2*a^3*b*x^2 + a^4) - 2*log(b*x^2 + a)/a^3 + 4*log(x)/a^3)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 182, normalized size of antiderivative = 0.82

$$\int \frac{d+ex^2}{x^3 (a^2+2abx^2+b^2x^4)^{3/2}}\,dx = -\frac{(3\,bd-ae)\log\left(x^2\right)}{2\,a^4\mathrm{sgn}\left(bx^2+a\right)} + \frac{(3\,b^2d-abe)\log\left(|bx^2+a|\right)}{2\,a^4b\mathrm{sgn}\left(bx^2+a\right)} - \frac{9\,b^3dx^4-3\,ab^2ex^4+22\,ab^2dx^2-8\,a^2bex^2+14\,a^2bd-6\,a^3e}{4\,(bx^2+a)^2a^4\mathrm{sgn}\left(bx^2+a\right)} + \frac{3\,bdx^2-aex^2-ad}{2\,a^4x^2\mathrm{sgn}\left(bx^2+a\right)}$$

```
[In] integrate((e*x^2+d)/x^3/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="giac")
```

```
[Out] -1/2*(3*b*d - a*e)*log(x^2)/(a^4*sgn(b*x^2 + a)) + 1/2*(3*b^2*d - a*b*e)*lo
g(abs(b*x^2 + a))/(a^4*b*sgn(b*x^2 + a)) - 1/4*(9*b^3*d*x^4 - 3*a*b^2*e*x^4
 + 22*a*b^2*d*x^2 - 8*a^2*b*e*x^2 + 14*a^2*b*d - 6*a^3*e)/((b*x^2 + a)^2*a^
4*sgn(b*x^2 + a)) + 1/2*(3*b*d*x^2 - a*e*x^2 - a*d)/(a^4*x^2*sgn(b*x^2 + a)
)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{d+ex^2}{x^3 (a^2+2abx^2+b^2x^4)^{3/2}}\,dx = \int \frac{e\,x^2+d}{x^3\,{\left(a^2+2\,a\,b\,x^2+b^2\,x^4\right)}^{3/2}} \,d x$$

```
[In] int((d + e*x^2)/(x^3*(a^2 + b^2*x^4 + 2*a*b*x^2)^(3/2)),x)
```

```
[Out] int((d + e*x^2)/(x^3*(a^2 + b^2*x^4 + 2*a*b*x^2)^(3/2)), x)
```

3.87 $\int (fx)^m (d+ex^2) (a^2+2abx^2+b^2x^4)^{5/2} dx$

Optimal result

Integrand size = 35, antiderivative size = 400

$$\int (fx)^m (d+ex^2) (a^2+2abx^2+b^2x^4)^{5/2} dx = \frac{a^5d(fx)^{1+m}\sqrt{a^2+2abx^2+b^2x^4}}{f(1+m)(a+bx^2)} + \frac{a^4(5bd+ae)(fx)^{3+m}\sqrt{a^2+2abx^2+b^2x^4}}{f^3(3+m)(a+bx^2)} + \frac{5a^3b(2bd+ae)(fx)^{5+m}\sqrt{a^2+2abx^2+b^2x^4}}{f^5(5+m)(a+bx^2)} + \frac{10a^2b^2(bd+ae)(fx)^{7+m}\sqrt{a^2+2abx^2+b^2x^4}}{f^7(7+m)(a+bx^2)} + \frac{5ab^3(bd+2ae)(fx)^{9+m}\sqrt{a^2+2abx^2+b^2x^4}}{f^9(9+m)(a+bx^2)} + \frac{b^4(bd+5ae)(fx)^{11+m}\sqrt{a^2+2abx^2+b^2x^4}}{f^{11}(11+m)(a+bx^2)} + \frac{b^5e(fx)^{13+m}\sqrt{a^2+2abx^2+b^2x^4}}{f^{13}(13+m)(a+bx^2)}$$

```
[Out] a^5*d*(f*x)^(1+m)*((b*x^2+a)^2)^(1/2)/f/(1+m)/(b*x^2+a)+a^4*(a*e+5*b*d)*(f*
x)^(3+m)*((b*x^2+a)^2)^(1/2)/f^3/(3+m)/(b*x^2+a)+5*a^3*b*(a*e+2*b*d)*(f*x)^
(5+m)*((b*x^2+a)^2)^(1/2)/f^5/(5+m)/(b*x^2+a)+10*a^2*b^2*(a*e+b*d)*(f*x)^(7
+m)*((b*x^2+a)^2)^(1/2)/f^7/(7+m)/(b*x^2+a)+5*a*b^3*(2*a*e+b*d)*(f*x)^(9+m)
*((b*x^2+a)^2)^(1/2)/f^9/(9+m)/(b*x^2+a)+b^4*(5*a*e+b*d)*(f*x)^(11+m)*((b*x
^2+a)^2)^(1/2)/f^11/(11+m)/(b*x^2+a)+b^5*e*(f*x)^(13+m)*((b*x^2+a)^2)^(1/2)
/f^13/(13+m)/(b*x^2+a)
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 400, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.057, Rules used = {1264, 459}

$$\int (fx)^m \left(d+ex^2\right) \left(a^2+2abx^2+b^2x^4\right)^{5/2} dx = \frac{10a^2b^2\sqrt{a^2+2abx^2+b^2x^4}(fx)^{m+7}(ae+bd)}{f^7(m+7)\left(a+bx^2\right)} + \frac{b^5e\sqrt{a^2+2abx^2+b^2x^4}(fx)^{m+13}}{f^{13}(m+13)\left(a+bx^2\right)} + \frac{b^4\sqrt{a^2+2abx^2+b^2x^4}(fx)^{m+11}(5ae+bd)}{f^{11}(m+11)\left(a+bx^2\right)} + \frac{5ab^3\sqrt{a^2+2abx^2+b^2x^4}(fx)^{m+9}(2ae+bd)}{f^9(m+9)\left(a+bx^2\right)} + \frac{a^5d\sqrt{a^2+2abx^2+b^2x^4}(fx)^{m+1}}{f(m+1)\left(a+bx^2\right)} + \frac{a^4\sqrt{a^2+2abx^2+b^2x^4}(fx)^{m+3}(ae+5bd)}{f^3(m+3)\left(a+bx^2\right)} + \frac{5a^3b\sqrt{a^2+2abx^2+b^2x^4}(fx)^{m+5}(ae+2bd)}{f^5(m+5)\left(a+bx^2\right)}$$

```
[In] Int[(f*x)^m*(d + e*x^2)*(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2),x]
```

```
[Out] (a^5*d*(f*x)^(1 + m)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(f*(1 + m)*(a + b*x^2
)) + (a^4*(5*b*d + a*e)*(f*x)^(3 + m)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(f^3
*(3 + m)*(a + b*x^2)) + (5*a^3*b*(2*b*d + a*e)*(f*x)^(5 + m)*Sqrt[a^2 + 2*a
*b*x^2 + b^2*x^4])/(f^5*(5 + m)*(a + b*x^2)) + (10*a^2*b^2*(b*d + a*e)*(f*x
)^(7 + m)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(f^7*(7 + m)*(a + b*x^2)) + (5*a
*b^3*(b*d + 2*a*e)*(f*x)^(9 + m)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(f^9*(9 +
 m)*(a + b*x^2)) + (b^4*(b*d + 5*a*e)*(f*x)^(11 + m)*Sqrt[a^2 + 2*a*b*x^2 +
 b^2*x^4])/(f^11*(11 + m)*(a + b*x^2)) + (b^5*e*(f*x)^(13 + m)*Sqrt[a^2 + 2
*a*b*x^2 + b^2*x^4])/(f^13*(13 + m)*(a + b*x^2))
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rule 1264

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_), x_Symbol] :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^Int
Part[p]*(b/2 + c*x^2)^(2*FracPart[p])), Int[(f*x)^m*(d + e*x^2)^q*(b/2 + c*
x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, f, m, p, q}, x] && EqQ[b^2 - 4*
a*c, 0] &&  !IntegerQ[p]
```

Rubi steps

$$\text{integral} = \frac{\sqrt{a^2+2abx^2+b^2x^4}\int (fx)^m\left(ab+b^2x^2\right)^5\left(d+ex^2\right)\,dx}{b^4\left(ab+b^2x^2\right)}$$

$$= \frac{\sqrt{a^2+2abx^2+b^2x^4}\int\left(a^5b^5d(fx)^m+\frac{a^4b^5(5bd+ae)(fx)^{2+m}}{f^2}+\frac{5a^3b^6(2bd+ae)(fx)^{4+m}}{f^4}+\frac{10a^2b^7(bd+ae)(fx)^{6+m}}{f^6}\right.}{b^4\left(ab+b^2x^2\right)}$$

$$= \frac{a^5d(fx)^{1+m}\sqrt{a^2+2abx^2+b^2x^4}}{f(1+m)\left(a+bx^2\right)}+\frac{a^4(5bd+ae)(fx)^{3+m}\sqrt{a^2+2abx^2+b^2x^4}}{f^3(3+m)\left(a+bx^2\right)}$$
$$+\frac{5a^3b(2bd+ae)(fx)^{5+m}\sqrt{a^2+2abx^2+b^2x^4}}{f^5(5+m)\left(a+bx^2\right)}$$
$$+\frac{10a^2b^2(bd+ae)(fx)^{7+m}\sqrt{a^2+2abx^2+b^2x^4}}{f^7(7+m)\left(a+bx^2\right)}$$
$$+\frac{5ab^3(bd+2ae)(fx)^{9+m}\sqrt{a^2+2abx^2+b^2x^4}}{f^9(9+m)\left(a+bx^2\right)}$$
$$+\frac{b^4(bd+5ae)(fx)^{11+m}\sqrt{a^2+2abx^2+b^2x^4}}{f^{11}(11+m)\left(a+bx^2\right)}+\frac{b^5e(fx)^{13+m}\sqrt{a^2+2abx^2+b^2x^4}}{f^{13}(13+m)\left(a+bx^2\right)}$$

Mathematica [A] (verified)

Time = 0.46 (sec) , antiderivative size = 160, normalized size of antiderivative = 0.40

$$\int (fx)^m\left(d+ex^2\right)\left(a^2+2abx^2+b^2x^4\right)^{5/2}\,dx = \frac{x(fx)^m\sqrt{\left(a+bx^2\right)^2}\left(\frac{a^5d}{1+m}+\frac{a^4(5bd+ae)x^2}{3+m}+\frac{5a^3b(2bd+ae)x^4}{5+m}+\frac{10a^2b^2(bd+ae)x^6}{7+m}+\frac{5ab^3(bd+2ae)x^8}{9+m}+\frac{b^4(bd\right.}{a+bx^2}$$

```
[In] Integrate[(f*x)^m*(d + e*x^2)*(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2),x]
```

```
[Out] (x*(f*x)^m*Sqrt[(a + b*x^2)^2]*((a^5*d)/(1 + m) + (a^4*(5*b*d + a*e)*x^2)/(
3 + m) + (5*a^3*b*(2*b*d + a*e)*x^4)/(5 + m) + (10*a^2*b^2*(b*d + a*e)*x^6)
/(7 + m) + (5*a*b^3*(b*d + 2*a*e)*x^8)/(9 + m) + (b^4*(b*d + 5*a*e)*x^10)/(
11 + m) + (b^5*e*x^12)/(13 + m)))/(a + b*x^2)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1098 vs. 2(323) = 646.

Time = 0.12 (sec) , antiderivative size = 1099, normalized size of antiderivative = 2.75

method	result	size
gosper	Expression too large to display	1099
risch	Expression too large to display	1099

```
[In] int((f*x)^m*(e*x^2+d)*(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x,method=_RETURNVERBOSE
)
```

```
[Out] x*(b^5*e*m^6*x^12+36*b^5*e*m^5*x^12+5*a*b^4*e*m^6*x^10+b^5*d*m^6*x^10+505*b
^5*e*m^4*x^12+190*a*b^4*e*m^5*x^10+38*b^5*d*m^5*x^10+3480*b^5*e*m^3*x^12+10
*a^2*b^3*e*m^6*x^8+5*a*b^4*d*m^6*x^8+2775*a*b^4*e*m^4*x^10+555*b^5*d*m^4*x^
10+12139*b^5*e*m^2*x^12+400*a^2*b^3*e*m^5*x^8+200*a*b^4*d*m^5*x^8+19700*a*b
^4*e*m^3*x^10+3940*b^5*d*m^3*x^10+19524*b^5*e*m*x^12+10*a^3*b^2*e*m^6*x^6+1
0*a^2*b^3*d*m^6*x^6+6130*a^2*b^3*e*m^4*x^8+3065*a*b^4*d*m^4*x^8+70195*a*b^4
*e*m^2*x^10+14039*b^5*d*m^2*x^10+10395*b^5*e*x^12+420*a^3*b^2*e*m^5*x^6+420
*a^2*b^3*d*m^5*x^6+45280*a^2*b^3*e*m^3*x^8+22640*a*b^4*d*m^3*x^8+114510*a*b
^4*e*m*x^10+22902*b^5*d*m*x^10+5*a^4*b*e*m^6*x^4+10*a^3*b^2*d*m^6*x^4+6790*
a^3*b^2*e*m^4*x^6+6790*a^2*b^3*d*m^4*x^6+166270*a^2*b^3*e*m^2*x^8+83135*a*b
^4*d*m^2*x^8+61425*a*b^4*e*x^10+12285*b^5*d*x^10+220*a^4*b*e*m^5*x^4+440*a^
3*b^2*d*m^5*x^4+52920*a^3*b^2*e*m^3*x^6+52920*a^2*b^3*d*m^3*x^6+276880*a^2*
b^3*e*m*x^8+138440*a*b^4*d*m*x^8+a^5*e*m^6*x^2+5*a^4*b*d*m^6*x^2+3765*a^4*b
*e*m^4*x^4+7530*a^3*b^2*d*m^4*x^4+203350*a^3*b^2*e*m^2*x^6+203350*a^2*b^3*d
*m^2*x^6+150150*a^2*b^3*e*x^8+75075*a*b^4*d*x^8+46*a^5*e*m^5*x^2+230*a^4*b*
d*m^5*x^2+31400*a^4*b*e*m^3*x^4+62800*a^3*b^2*d*m^3*x^4+349860*a^3*b^2*e*m*
x^6+349860*a^2*b^3*d*m*x^6+a^5*d*m^6+835*a^5*e*m^4*x^2+4175*a^4*b*d*m^4*x^2
+129895*a^4*b*e*m^2*x^4+259790*a^3*b^2*d*m^2*x^4+193050*a^3*b^2*e*x^6+19305
0*a^2*b^3*d*x^6+48*a^5*d*m^5+7540*a^5*e*m^3*x^2+37700*a^4*b*d*m^3*x^2+23718
0*a^4*b*e*m*x^4+474360*a^3*b^2*d*m*x^4+925*a^5*d*m^4+34759*a^5*e*m^2*x^2+17
3795*a^4*b*d*m^2*x^2+135135*a^4*b*e*x^4+270270*a^3*b^2*d*x^4+9120*a^5*d*m^3
+73054*a^5*e*m*x^2+365270*a^4*b*d*m*x^2+48259*a^5*d*m^2+45045*a^5*e*x^2+225
225*a^4*b*d*x^2+129072*a^5*d*m+135135*a^5*d)*(f*x)^m*((b*x^2+a)^2)^(5/2)/(1
3+m)/(11+m)/(9+m)/(7+m)/(5+m)/(3+m)/(1+m)/(b*x^2+a)^5
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 853 vs. 2(323) = 646.

Time = 0.28 (sec) , antiderivative size = 853, normalized size of antiderivative = 2.13

$$\int (fx)^m (d+ex^2) (a^2+2abx^2+b^2x^4)^{5/2}\,dx = \frac{((b^5em^6+36\,b^5em^5+505\,b^5em^4+3480\,b^5em^3+12139\,b^5em^2+19524\,b^5em+10395\,b^5e)x^{13}}{}$$

```
[In] integrate((f*x)^m*(e*x^2+d)*(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="fri
cas")
```

```
[Out] ((b^5*e*m^6 + 36*b^5*e*m^5 + 505*b^5*e*m^4 + 3480*b^5*e*m^3 + 12139*b^5*e*m
^2 + 19524*b^5*e*m + 10395*b^5*e)*x^13 + ((b^5*d + 5*a*b^4*e)*m^6 + 12285*b
^5*d + 61425*a*b^4*e + 38*(b^5*d + 5*a*b^4*e)*m^5 + 555*(b^5*d + 5*a*b^4*e)
*m^4 + 3940*(b^5*d + 5*a*b^4*e)*m^3 + 14039*(b^5*d + 5*a*b^4*e)*m^2 + 22902
*(b^5*d + 5*a*b^4*e)*m)*x^11 + 5*((a*b^4*d + 2*a^2*b^3*e)*m^6 + 15015*a*b^4
*d + 30030*a^2*b^3*e + 40*(a*b^4*d + 2*a^2*b^3*e)*m^5 + 613*(a*b^4*d + 2*a^
2*b^3*e)*m^4 + 4528*(a*b^4*d + 2*a^2*b^3*e)*m^3 + 16627*(a*b^4*d + 2*a^2*b^
3*e)*m^2 + 27688*(a*b^4*d + 2*a^2*b^3*e)*m)*x^9 + 10*((a^2*b^3*d + a^3*b^2*
e)*m^6 + 19305*a^2*b^3*d + 19305*a^3*b^2*e + 42*(a^2*b^3*d + a^3*b^2*e)*m^5
 + 679*(a^2*b^3*d + a^3*b^2*e)*m^4 + 5292*(a^2*b^3*d + a^3*b^2*e)*m^3 + 203
35*(a^2*b^3*d + a^3*b^2*e)*m^2 + 34986*(a^2*b^3*d + a^3*b^2*e)*m)*x^7 + 5*(
(2*a^3*b^2*d + a^4*b*e)*m^6 + 54054*a^3*b^2*d + 27027*a^4*b*e + 44*(2*a^3*b
^2*d + a^4*b*e)*m^5 + 753*(2*a^3*b^2*d + a^4*b*e)*m^4 + 6280*(2*a^3*b^2*d +
 a^4*b*e)*m^3 + 25979*(2*a^3*b^2*d + a^4*b*e)*m^2 + 47436*(2*a^3*b^2*d + a^
4*b*e)*m)*x^5 + ((5*a^4*b*d + a^5*e)*m^6 + 225225*a^4*b*d + 45045*a^5*e + 4
6*(5*a^4*b*d + a^5*e)*m^5 + 835*(5*a^4*b*d + a^5*e)*m^4 + 7540*(5*a^4*b*d +
 a^5*e)*m^3 + 34759*(5*a^4*b*d + a^5*e)*m^2 + 73054*(5*a^4*b*d + a^5*e)*m)*
x^3 + (a^5*d*m^6 + 48*a^5*d*m^5 + 925*a^5*d*m^4 + 9120*a^5*d*m^3 + 48259*a^
5*d*m^2 + 129072*a^5*d*m + 135135*a^5*d)*x)*(f*x)^m/(m^7 + 49*m^6 + 973*m^5
 + 10045*m^4 + 57379*m^3 + 177331*m^2 + 264207*m + 135135)
```

Sympy [F]

$$\int (fx)^m (d+ex^2) (a^2+2abx^2+b^2x^4)^{5/2}\,dx = \int (fx)^m (d+ex^2) \left(\left(a+bx^2\right)^2\right)^{\frac{5}{2}}\,dx$$

```
[In] integrate((f*x)**m*(e*x**2+d)*(b**2*x**4+2*a*b*x**2+a**2)**(5/2),x)
```

```
[Out] Integral((f*x)**m*(d + e*x**2)*((a + b*x**2)**2)**(5/2), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 491, normalized size of antiderivative = 1.23

$$\int (fx)^m (d+ex^2) (a^2+2abx^2 +b^2x^4)^{5/2} dx = \frac{((m^5+25m^4+230m^3+950m^2+1689m+945)b^5f^mx^{11}+5(m^5+27m^4+262m^3+112}{} + \frac{((m^5+35m^4+470m^3+3010m^2+9129m+10395)b^5f^mx^{13}+5(m^5+37m^4+518m^3+3422m^2+10}{}$$

```
[In] integrate((f*x)^m*(e*x^2+d)*(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="max
ima")
```

```
[Out] ((m^5 + 25*m^4 + 230*m^3 + 950*m^2 + 1689*m + 945)*b^5*f^m*x^11 + 5*(m^5 +
27*m^4 + 262*m^3 + 1122*m^2 + 2041*m + 1155)*a*b^4*f^m*x^9 + 10*(m^5 + 29*m
^4 + 302*m^3 + 1366*m^2 + 2577*m + 1485)*a^2*b^3*f^m*x^7 + 10*(m^5 + 31*m^4
 + 350*m^3 + 1730*m^2 + 3489*m + 2079)*a^3*b^2*f^m*x^5 + 5*(m^5 + 33*m^4 +
406*m^3 + 2262*m^2 + 5353*m + 3465)*a^4*b*f^m*x^3 + (m^5 + 35*m^4 + 470*m^3
 + 3010*m^2 + 9129*m + 10395)*a^5*f^m*x)*d*x^m/(m^6 + 36*m^5 + 505*m^4 + 34
80*m^3 + 12139*m^2 + 19524*m + 10395) + ((m^5 + 35*m^4 + 470*m^3 + 3010*m^2
 + 9129*m + 10395)*b^5*f^m*x^13 + 5*(m^5 + 37*m^4 + 518*m^3 + 3422*m^2 + 10
617*m + 12285)*a*b^4*f^m*x^11 + 10*(m^5 + 39*m^4 + 574*m^3 + 3954*m^2 + 126
73*m + 15015)*a^2*b^3*f^m*x^9 + 10*(m^5 + 41*m^4 + 638*m^3 + 4654*m^2 + 156
81*m + 19305)*a^3*b^2*f^m*x^7 + 5*(m^5 + 43*m^4 + 710*m^3 + 5570*m^2 + 2040
9*m + 27027)*a^4*b*f^m*x^5 + (m^5 + 45*m^4 + 790*m^3 + 6750*m^2 + 28009*m +
 45045)*a^5*f^m*x^3)*e*x^m/(m^6 + 48*m^5 + 925*m^4 + 9120*m^3 + 48259*m^2 +
 129072*m + 135135)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2171 vs. 2(323) = 646.

Time = 0.36 (sec) , antiderivative size = 2171, normalized size of antiderivative = 5.43

$$\int (fx)^m (d+ex^2) (a^2+2abx^2+b^2x^4)^{5/2} dx = \text{Too large to display}$$

```
[In] integrate((f*x)^m*(e*x^2+d)*(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="gia
c")
```

```
[Out] ((f*x)^m*b^5*e*m^6*x^13*sgn(b*x^2 + a) + 36*(f*x)^m*b^5*e*m^5*x^13*sgn(b*x^
2 + a) + (f*x)^m*b^5*d*m^6*x^11*sgn(b*x^2 + a) + 5*(f*x)^m*a*b^4*e*m^6*x^11
*sgn(b*x^2 + a) + 505*(f*x)^m*b^5*e*m^4*x^13*sgn(b*x^2 + a) + 38*(f*x)^m*b^
```

5*d*m^5*x^11*sgn(b*x^2 + a) + 190*(f*x)^m*a*b^4*e*m^5*x^11*sgn(b*x^2 + a) + 3480*(f*x)^m*b^5*e*m^3*x^13*sgn(b*x^2 + a) + 5*(f*x)^m*a*b^4*d*m^6*x^9*sgn(b*x^2 + a) + 10*(f*x)^m*a^2*b^3*e*m^6*x^9*sgn(b*x^2 + a) + 555*(f*x)^m*b^5*d*m^4*x^11*sgn(b*x^2 + a) + 2775*(f*x)^m*a*b^4*e*m^4*x^11*sgn(b*x^2 + a) + 12139*(f*x)^m*b^5*e*m^2*x^13*sgn(b*x^2 + a) + 200*(f*x)^m*a*b^4*d*m^5*x^9*sgn(b*x^2 + a) + 400*(f*x)^m*a^2*b^3*e*m^5*x^9*sgn(b*x^2 + a) + 3940*(f*x)^m*b^5*d*m^3*x^11*sgn(b*x^2 + a) + 19700*(f*x)^m*a*b^4*e*m^3*x^11*sgn(b*x^2 + a) + 19524*(f*x)^m*b^5*e*m*x^13*sgn(b*x^2 + a) + 10*(f*x)^m*a^2*b^3*d*m^6*x^7*sgn(b*x^2 + a) + 10*(f*x)^m*a^3*b^2*e*m^6*x^7*sgn(b*x^2 + a) + 3065*(f*x)^m*a*b^4*d*m^4*x^9*sgn(b*x^2 + a) + 6130*(f*x)^m*a^2*b^3*e*m^4*x^9*sgn(b*x^2 + a) + 14039*(f*x)^m*b^5*d*m^2*x^11*sgn(b*x^2 + a) + 70195*(f*x)^m*a*b^4*e*m^2*x^11*sgn(b*x^2 + a) + 10395*(f*x)^m*b^5*e*x^13*sgn(b*x^2 + a) + 420*(f*x)^m*a^2*b^3*d*m^5*x^7*sgn(b*x^2 + a) + 420*(f*x)^m*a^3*b^2*e*m^5*x^7*sgn(b*x^2 + a) + 22640*(f*x)^m*a*b^4*d*m^3*x^9*sgn(b*x^2 + a) + 45280*(f*x)^m*a^2*b^3*e*m^3*x^9*sgn(b*x^2 + a) + 22902*(f*x)^m*b^5*d*m*x^11*sgn(b*x^2 + a) + 114510*(f*x)^m*a*b^4*e*m*x^11*sgn(b*x^2 + a) + 10*(f*x)^m*a^3*b^2*d*m^6*x^5*sgn(b*x^2 + a) + 5*(f*x)^m*a^4*b*e*m^6*x^5*sgn(b*x^2 + a) + 6790*(f*x)^m*a^2*b^3*d*m^4*x^7*sgn(b*x^2 + a) + 6790*(f*x)^m*a^3*b^2*e*m^4*x^7*sgn(b*x^2 + a) + 83135*(f*x)^m*a*b^4*d*m^2*x^9*sgn(b*x^2 + a) + 166270*(f*x)^m*a^2*b^3*e*m^2*x^9*sgn(b*x^2 + a) + 12285*(f*x)^m*b^5*d*x^11*sgn(b*x^2 + a) + 61425*(f*x)^m*a*b^4*e*x^11*sgn(b*x^2 + a) + 440*(f*x)^m*a^3*b^2*d*m^5*x^5*sgn(b*x^2 + a) + 220*(f*x)^m*a^4*b*e*m^5*x^5*sgn(b*x^2 + a) + 52920*(f*x)^m*a^2*b^3*d*m^3*x^7*sgn(b*x^2 + a) + 52920*(f*x)^m*a^3*b^2*e*m^3*x^7*sgn(b*x^2 + a) + 138440*(f*x)^m*a*b^4*d*m*x^9*sgn(b*x^2 + a) + 276880*(f*x)^m*a^2*b^3*e*m*x^9*sgn(b*x^2 + a) + 5*(f*x)^m*a^4*b*d*m^6*x^3*sgn(b*x^2 + a) + (f*x)^m*a^5*e*m^6*x^3*sgn(b*x^2 + a) + 7530*(f*x)^m*a^3*b^2*d*m^4*x^5*sgn(b*x^2 + a) + 3765*(f*x)^m*a^4*b*e*m^4*x^5*sgn(b*x^2 + a) + 203350*(f*x)^m*a^2*b^3*d*m^2*x^7*sgn(b*x^2 + a) + 203350*(f*x)^m*a^3*b^2*e*m^2*x^7*sgn(b*x^2 + a) + 75075*(f*x)^m*a*b^4*d*x^9*sgn(b*x^2 + a) + 150150*(f*x)^m*a^2*b^3*e*x^9*sgn(b*x^2 + a) + 230*(f*x)^m*a^4*b*d*m^5*x^3*sgn(b*x^2 + a) + 46*(f*x)^m*a^5*e*m^5*x^3*sgn(b*x^2 + a) + 62800*(f*x)^m*a^3*b^2*d*m^3*x^5*sgn(b*x^2 + a) + 31400*(f*x)^m*a^4*b*e*m^3*x^5*sgn(b*x^2 + a) + 349860*(f*x)^m*a^2*b^3*d*m*x^7*sgn(b*x^2 + a) + 349860*(f*x)^m*a^3*b^2*e*m*x^7*sgn(b*x^2 + a) + (f*x)^m*a^5*d*m^6*x*sgn(b*x^2 + a) + 4175*(f*x)^m*a^4*b*d*m^4*x^3*sgn(b*x^2 + a) + 835*(f*x)^m*a^5*e*m^4*x^3*sgn(b*x^2 + a) + 259790*(f*x)^m*a^3*b^2*d*m^2*x^5*sgn(b*x^2 + a) + 129895*(f*x)^m*a^4*b*e*m^2*x^5*sgn(b*x^2 + a) + 193050*(f*x)^m*a^2*b^3*d*x^7*sgn(b*x^2 + a) + 193050*(f*x)^m*a^3*b^2*e*x^7*sgn(b*x^2 + a) + 48*(f*x)^m*a^5*d*m^5*x*sgn(b*x^2 + a) + 37700*(f*x)^m*a^4*b*d*m^3*x^3*sgn(b*x^2 + a) + 7540*(f*x)^m*a^5*e*m^3*x^3*sgn(b*x^2 + a) + 474360*(f*x)^m*a^3*b^2*d*m*x^5*sgn(b*x^2 + a) + 237180*(f*x)^m*a^4*b*e*m*x^5*sgn(b*x^2 + a) + 925*(f*x)^m*a^5*d*m^4*x*sgn(b*x^2 + a) + 173795*(f*x)^m*a^4*b*d*m^2*x^3*sgn(b*x^2 + a) + 34759*(f*x)^m*a^5*e*m^2*x^3*sgn(b*x^2 + a) + 270270*(f*x)^m*a^3*b^2*d*x^5*sgn(b*x^2 + a) + 135135*(f*x)^m*a^4*b*e*x^5*sgn(b*x^2 + a) + 9120*(f*x)^m*a^5*d*m^3*x*sgn(b*x^2 + a) + 365270*(f*x)^m*a^4*b*d*m*x^3*sgn(b*x^2 + a) + 73054*(f*x)^m*a^5*e*m*x^3*sgn(b*x^2 + a) + 4825

```
9*(f*x)^m*a^5*d*m^2*x*sgn(b*x^2 + a) + 225225*(f*x)^m*a^4*b*d*x^3*sgn(b*x^2
 + a) + 45045*(f*x)^m*a^5*e*x^3*sgn(b*x^2 + a) + 129072*(f*x)^m*a^5*d*m*x*s
gn(b*x^2 + a) + 135135*(f*x)^m*a^5*d*x*sgn(b*x^2 + a))/(m^7 + 49*m^6 + 973*
m^5 + 10045*m^4 + 57379*m^3 + 177331*m^2 + 264207*m + 135135)
```

Mupad [F(-1)]

Timed out.

$$\int (fx)^m (d+ex^2) (a^2+2abx^2+b^2x^4)^{5/2} dx = \int (f x)^m (e x^2+d) (a^2+2 a b x^2+b^2 x^4)^{5/2} dx$$

```
[In] int((f*x)^m*(d + e*x^2)*(a^2 + b^2*x^4 + 2*a*b*x^2)^(5/2),x)
[Out] int((f*x)^m*(d + e*x^2)*(a^2 + b^2*x^4 + 2*a*b*x^2)^(5/2), x)
```

3.88 $\int (fx)^m (d+ex^2) (a^2+2abx^2+b^2x^4)^{3/2} dx$

Optimal result

Integrand size = 35, antiderivative size = 276

$$\int (fx)^m (d+ex^2) \left(a^2+2abx^2+b^2x^4\right)^{3/2} dx = \frac{a^3d(fx)^{1+m}\sqrt{a^2+2abx^2+b^2x^4}}{f(1+m)(a+bx^2)} + \frac{a^2(3bd+ae)(fx)^{3+m}\sqrt{a^2+2abx^2+b^2x^4}}{f^3(3+m)(a+bx^2)} + \frac{3ab(bd+ae)(fx)^{5+m}\sqrt{a^2+2abx^2+b^2x^4}}{f^5(5+m)(a+bx^2)} + \frac{b^2(bd+3ae)(fx)^{7+m}\sqrt{a^2+2abx^2+b^2x^4}}{f^7(7+m)(a+bx^2)} + \frac{b^3e(fx)^{9+m}\sqrt{a^2+2abx^2+b^2x^4}}{f^9(9+m)(a+bx^2)}$$

```
[Out] a^3*d*(f*x)^(1+m)*((b*x^2+a)^2)^(1/2)/f/(1+m)/(b*x^2+a)+a^2*(a*e+3*b*d)*(f*
x)^(3+m)*((b*x^2+a)^2)^(1/2)/f^3/(3+m)/(b*x^2+a)+3*a*b*(a*e+b*d)*(f*x)^(5+m
)*((b*x^2+a)^2)^(1/2)/f^5/(5+m)/(b*x^2+a)+b^2*(3*a*e+b*d)*(f*x)^(7+m)*((b*x
^2+a)^2)^(1/2)/f^7/(7+m)/(b*x^2+a)+b^3*e*(f*x)^(9+m)*((b*x^2+a)^2)^(1/2)/f^
9/(9+m)/(b*x^2+a)
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 276, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.057, Rules used

= {1264, 459}

$$\int (fx)^m \left(d+ex^2\right) \left(a^2+2abx^2+b^2x^4\right)^{3/2} dx = \frac{b^2\sqrt{a^2+2abx^2+b^2x^4}(fx)^{m+7}(3ae+bd)}{f^7(m+7)\left(a+bx^2\right)} + \frac{3ab\sqrt{a^2+2abx^2+b^2x^4}(fx)^{m+5}(ae+bd)}{f^5(m+5)\left(a+bx^2\right)} + \frac{a^2\sqrt{a^2+2abx^2+b^2x^4}(fx)^{m+3}(ae+3bd)}{f^3(m+3)\left(a+bx^2\right)} + \frac{b^3e\sqrt{a^2+2abx^2+b^2x^4}(fx)^{m+9}}{f^9(m+9)\left(a+bx^2\right)} + \frac{a^3d\sqrt{a^2+2abx^2+b^2x^4}(fx)^{m+1}}{f(m+1)\left(a+bx^2\right)}$$

```
[In] Int[(f*x)^m*(d + e*x^2)*(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2),x]
```

```
[Out] (a^3*d*(f*x)^(1 + m)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(f*(1 + m)*(a + b*x^2
)) + (a^2*(3*b*d + a*e)*(f*x)^(3 + m)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(f^3
*(3 + m)*(a + b*x^2)) + (3*a*b*(b*d + a*e)*(f*x)^(5 + m)*Sqrt[a^2 + 2*a*b*x
^2 + b^2*x^4])/(f^5*(5 + m)*(a + b*x^2)) + (b^2*(b*d + 3*a*e)*(f*x)^(7 + m)
*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(f^7*(7 + m)*(a + b*x^2)) + (b^3*e*(f*x)^
(9 + m)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(f^9*(9 + m)*(a + b*x^2))
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rule 1264

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_), x_Symbol] :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^Int
Part[p]*(b/2 + c*x^2)^(2*FracPart[p])), Int[(f*x)^m*(d + e*x^2)^q*(b/2 + c*
x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, f, m, p, q}, x] && EqQ[b^2 - 4*
a*c, 0] &&  !IntegerQ[p]
```

Rubi steps

$$\text{integral} = \frac{\sqrt{a^2+2abx^2+b^2x^4}\int (fx)^m \left(ab+b^2x^2\right)^3 \left(d+ex^2\right) dx}{b^2\left(ab+b^2x^2\right)}$$

$$= \frac{\sqrt{a^2+2abx^2+b^2x^4}\int \left(a^3b^3d(fx)^m + \frac{a^2b^3(3bd+ae)(fx)^{2+m}}{f^2} + \frac{3ab^4(bd+ae)(fx)^{4+m}}{f^4} + \frac{b^5(bd+3ae)(fx)^{6+m}}{f^6} + \dots\right.}{b^2\left(ab+b^2x^2\right)}$$

$$
\begin{aligned}
&= \frac{a^3d(fx)^{1+m}\sqrt{a^2+2abx^2+b^2x^4}}{f(1+m)\,(a+bx^2)} + \frac{a^2(3bd+ae)(fx)^{3+m}\sqrt{a^2+2abx^2+b^2x^4}}{f^3(3+m)\,(a+bx^2)} \\
&\quad + \frac{3ab(bd+ae)(fx)^{5+m}\sqrt{a^2+2abx^2+b^2x^4}}{f^5(5+m)\,(a+bx^2)} \\
&\quad + \frac{b^2(bd+3ae)(fx)^{7+m}\sqrt{a^2+2abx^2+b^2x^4}}{f^7(7+m)\,(a+bx^2)} + \frac{b^3e(fx)^{9+m}\sqrt{a^2+2abx^2+b^2x^4}}{f^9(9+m)\,(a+bx^2)}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.41

$$
\int (fx)^m\,(d+ex^2)\,(a^2+2abx^2+b^2x^4)^{3/2}\,dx = \frac{x(fx)^m\left((a+bx^2)^2\right)^{3/2}\left(\frac{a^3d}{1+m}+\frac{a^2(3bd+ae)x^2}{3+m}+\frac{3ab(bd+ae)x^4}{5+m}+\frac{b^2(bd+3ae)x^6}{7+m}+\frac{b^3ex^8}{9+m}\right)}{(a+bx^2)^3}
$$

```
[In] Integrate[(f*x)^m*(d + e*x^2)*(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2),x]
```

```
[Out] (x*(f*x)^m*((a + b*x^2)^2)^(3/2)*((a^3*d)/(1 + m) + (a^2*(3*b*d + a*e)*x^2)
/(3 + m) + (3*a*b*(b*d + a*e)*x^4)/(5 + m) + (b^2*(b*d + 3*a*e)*x^6)/(7 + m
) + (b^3*e*x^8)/(9 + m)))/(a + b*x^2)^3
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 494 vs. 2(221) = 442.

Time = 0.03 (sec) , antiderivative size = 495, normalized size of antiderivative = 1.79

method	result
gosper	$\frac{x\left(b^3e\,m^4x^8+16b^3e\,m^3x^8+3a\,b^2e\,m^4x^6+b^3d\,m^4x^6+86b^3e\,m^2x^8+54a\,b^2e\,m^3x^6+18b^3d\,m^3x^6+176m\,x^8b^3e+3a^2be\,m^4x^4+3a\,b^2d\,m^4x^4\right.}{}$
risch	$\frac{\sqrt{(b\,x^2+a)^2}\,\left(b^3e\,m^4x^8+16b^3e\,m^3x^8+3a\,b^2e\,m^4x^6+b^3d\,m^4x^6+86b^3e\,m^2x^8+54a\,b^2e\,m^3x^6+18b^3d\,m^3x^6+176m\,x^8b^3e+3a^2be\,m^4x^4+\right.}{}$

```
[In] int((f*x)^m*(e*x^2+d)*(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x,method=_RETURNVERBOSE
)
```

```
[Out] x*(b^3*e*m^4*x^8+16*b^3*e*m^3*x^8+3*a*b^2*e*m^4*x^6+b^3*d*m^4*x^6+86*b^3*e*
m^2*x^8+54*a*b^2*e*m^3*x^6+18*b^3*d*m^3*x^6+176*b^3*e*m*x^8+3*a^2*b*e*m^4*x
^4+3*a*b^2*d*m^4*x^4+312*a*b^2*e*m^2*x^6+104*b^3*d*m^2*x^6+105*b^3*e*x^8+60
*a^2*b*e*m^3*x^4+60*a*b^2*d*m^3*x^4+666*a*b^2*e*m*x^6+222*b^3*d*m*x^6+a^3*e
*m^4*x^2+3*a^2*b*d*m^4*x^2+390*a^2*b*e*m^2*x^4+390*a*b^2*d*m^2*x^4+405*a*b^
2*e*x^6+135*b^3*d*x^6+22*a^3*e*m^3*x^2+66*a^2*b*d*m^3*x^2+900*a^2*b*e*m*x^4
```

```
+900*a*b^2*d*m*x^4+a^3*d*m^4+164*a^3*e*m^2*x^2+492*a^2*b*d*m^2*x^2+567*a^2*
b*e*x^4+567*a*b^2*d*x^4+24*a^3*d*m^3+458*a^3*e*m*x^2+1374*a^2*b*d*m*x^2+206
*a^3*d*m^2+315*a^3*e*x^2+945*a^2*b*d*x^2+744*a^3*d*m+945*a^3*d)*(f*x)^m*((b
*x^2+a)^2)^(3/2)/(9+m)/(7+m)/(5+m)/(3+m)/(1+m)/(b*x^2+a)^3
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 381, normalized size of antiderivative = 1.38

$$\int (fx)^m \left(d+ex^2\right) \left(a^2+2abx^2+b^2x^4\right)^{3/2} dx = \frac{((b^3em^4+16\,b^3em^3+86\,b^3em^2+176\,b^3em+105\,b^3e)x^9+((b^3d+3\,ab^2e)m^4+135\,b^3d+40$$

```
[In] integrate((f*x)^m*(e*x^2+d)*(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="fri
cas")
```

```
[Out] ((b^3*e*m^4 + 16*b^3*e*m^3 + 86*b^3*e*m^2 + 176*b^3*e*m + 105*b^3*e)*x^9 +
((b^3*d + 3*a*b^2*e)*m^4 + 135*b^3*d + 405*a*b^2*e + 18*(b^3*d + 3*a*b^2*e)
*m^3 + 104*(b^3*d + 3*a*b^2*e)*m^2 + 222*(b^3*d + 3*a*b^2*e)*m)*x^7 + 3*((a
*b^2*d + a^2*b*e)*m^4 + 189*a*b^2*d + 189*a^2*b*e + 20*(a*b^2*d + a^2*b*e)*
m^3 + 130*(a*b^2*d + a^2*b*e)*m^2 + 300*(a*b^2*d + a^2*b*e)*m)*x^5 + ((3*a^
2*b*d + a^3*e)*m^4 + 945*a^2*b*d + 315*a^3*e + 22*(3*a^2*b*d + a^3*e)*m^3 +
 164*(3*a^2*b*d + a^3*e)*m^2 + 458*(3*a^2*b*d + a^3*e)*m)*x^3 + (a^3*d*m^4
+ 24*a^3*d*m^3 + 206*a^3*d*m^2 + 744*a^3*d*m + 945*a^3*d)*x)*(f*x)^m/(m^5 +
 25*m^4 + 230*m^3 + 950*m^2 + 1689*m + 945)
```

Sympy [F]

$$\int (fx)^m \left(d+ex^2\right) \left(a^2+2abx^2+b^2x^4\right)^{3/2} dx = \int (fx)^m \left(d+ex^2\right) \left(\left(a+bx^2\right)^2\right)^{\frac{3}{2}} dx$$

```
[In] integrate((f*x)**m*(e*x**2+d)*(b**2*x**4+2*a*b*x**2+a**2)**(3/2),x)
```

```
[Out] Integral((f*x)**m*(d + e*x**2)*((a + b*x**2)**2)**(3/2), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 243, normalized size of antiderivative = 0.88

$$\int (fx)^m \left(d+ex^2\right) \left(a^2+2abx^2+b^2x^4\right)^{3/2} \, dx = \frac{((m^3+9\,m^2+23\,m+15)b^3 f^m x^7+3\,(m^3+11\,m^2+31\,m+21)ab^2 f^m x^5+3\,(m^3+13\,m^2+\cdots}{m^4+16\,m^3+86\,m^2+176\,m+1\cdots} + \frac{((m^3+15\,m^2+71\,m+105)b^3 f^m x^9+3\,(m^3+17\,m^2+87\,m+135)ab^2 f^m x^7+3\,(m^3+19\,m^2+111\,m+1\cdots}{m^4+24\,m^3+206\,m^2+744\,m+945}$$

```
[In] integrate((f*x)^m*(e*x^2+d)*(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="max
ima")
```

```
[Out] ((m^3 + 9*m^2 + 23*m + 15)*b^3*f^m*x^7 + 3*(m^3 + 11*m^2 + 31*m + 21)*a*b^2
*f^m*x^5 + 3*(m^3 + 13*m^2 + 47*m + 35)*a^2*b*f^m*x^3 + (m^3 + 15*m^2 + 71*
m + 105)*a^3*f^m*x)*d*x^m/(m^4 + 16*m^3 + 86*m^2 + 176*m + 105) + ((m^3 + 1
5*m^2 + 71*m + 105)*b^3*f^m*x^9 + 3*(m^3 + 17*m^2 + 87*m + 135)*a*b^2*f^m*x
^7 + 3*(m^3 + 19*m^2 + 111*m + 189)*a^2*b*f^m*x^5 + (m^3 + 21*m^2 + 143*m +
 315)*a^3*f^m*x^3)*e*x^m/(m^4 + 24*m^3 + 206*m^2 + 744*m + 945)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 993 vs. 2(221) = 442.

Time = 0.30 (sec) , antiderivative size = 993, normalized size of antiderivative = 3.60

$$\int (fx)^m \left(d+ex^2\right) \left(a^2+2abx^2+b^2x^4\right)^{3/2} \, dx = \text{Too large to display}$$

```
[In] integrate((f*x)^m*(e*x^2+d)*(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="gia
c")
```

```
[Out] ((f*x)^m*b^3*e*m^4*x^9*sgn(b*x^2 + a) + 16*(f*x)^m*b^3*e*m^3*x^9*sgn(b*x^2
+ a) + (f*x)^m*b^3*d*m^4*x^7*sgn(b*x^2 + a) + 3*(f*x)^m*a*b^2*e*m^4*x^7*sgn
(b*x^2 + a) + 86*(f*x)^m*b^3*e*m^2*x^9*sgn(b*x^2 + a) + 18*(f*x)^m*b^3*d*m^
3*x^7*sgn(b*x^2 + a) + 54*(f*x)^m*a*b^2*e*m^3*x^7*sgn(b*x^2 + a) + 176*(f*x
)^m*b^3*e*m*x^9*sgn(b*x^2 + a) + 3*(f*x)^m*a*b^2*d*m^4*x^5*sgn(b*x^2 + a) +
 3*(f*x)^m*a^2*b*e*m^4*x^5*sgn(b*x^2 + a) + 104*(f*x)^m*b^3*d*m^2*x^7*sgn(b
*x^2 + a) + 312*(f*x)^m*a*b^2*e*m^2*x^7*sgn(b*x^2 + a) + 105*(f*x)^m*b^3*e*
x^9*sgn(b*x^2 + a) + 60*(f*x)^m*a*b^2*d*m^3*x^5*sgn(b*x^2 + a) + 60*(f*x)^m
*a^2*b*e*m^3*x^5*sgn(b*x^2 + a) + 222*(f*x)^m*b^3*d*m*x^7*sgn(b*x^2 + a) +
666*(f*x)^m*a*b^2*e*m*x^7*sgn(b*x^2 + a) + 3*(f*x)^m*a^2*b*d*m^4*x^3*sgn(b*
x^2 + a) + (f*x)^m*a^3*e*m^4*x^3*sgn(b*x^2 + a) + 390*(f*x)^m*a*b^2*d*m^2*x
```

```
^5*sgn(b*x^2 + a) + 390*(f*x)^m*a^2*b*e*m^2*x^5*sgn(b*x^2 + a) + 135*(f*x)^
m*b^3*d*x^7*sgn(b*x^2 + a) + 405*(f*x)^m*a*b^2*e*x^7*sgn(b*x^2 + a) + 66*(f
*x)^m*a^2*b*d*m^3*x^3*sgn(b*x^2 + a) + 22*(f*x)^m*a^3*e*m^3*x^3*sgn(b*x^2 +
 a) + 900*(f*x)^m*a*b^2*d*m*x^5*sgn(b*x^2 + a) + 900*(f*x)^m*a^2*b*e*m*x^5*
sgn(b*x^2 + a) + (f*x)^m*a^3*d*m^4*x*sgn(b*x^2 + a) + 492*(f*x)^m*a^2*b*d*m
^2*x^3*sgn(b*x^2 + a) + 164*(f*x)^m*a^3*e*m^2*x^3*sgn(b*x^2 + a) + 567*(f*x
)^m*a*b^2*d*x^5*sgn(b*x^2 + a) + 567*(f*x)^m*a^2*b*e*x^5*sgn(b*x^2 + a) + 2
4*(f*x)^m*a^3*d*m^3*x*sgn(b*x^2 + a) + 1374*(f*x)^m*a^2*b*d*m*x^3*sgn(b*x^2
 + a) + 458*(f*x)^m*a^3*e*m*x^3*sgn(b*x^2 + a) + 206*(f*x)^m*a^3*d*m^2*x*sg
n(b*x^2 + a) + 945*(f*x)^m*a^2*b*d*x^3*sgn(b*x^2 + a) + 315*(f*x)^m*a^3*e*x
^3*sgn(b*x^2 + a) + 744*(f*x)^m*a^3*d*m*x*sgn(b*x^2 + a) + 945*(f*x)^m*a^3*
d*x*sgn(b*x^2 + a))/(m^5 + 25*m^4 + 230*m^3 + 950*m^2 + 1689*m + 945)
```

Mupad [F(-1)]

Timed out.

$$\int (fx)^m \left(d+ex^2\right) \left(a^2+2abx^2+b^2x^4\right)^{3/2} dx = \int (f\,x)^m \left(e\,x^2+d\right) \left(a^2+2\,a\,b\,x^2+b^2\,x^4\right)^{3/2} dx$$

```
[In] int((f*x)^m*(d + e*x^2)*(a^2 + b^2*x^4 + 2*a*b*x^2)^(3/2),x)
[Out] int((f*x)^m*(d + e*x^2)*(a^2 + b^2*x^4 + 2*a*b*x^2)^(3/2), x)
```

3.89 $\int (fx)^m (d+ex^2) \sqrt{a^2+2abx^2+b^2x^4} \, dx$

Optimal result

Integrand size = 35, antiderivative size = 153

$$\int (fx)^m (d+ex^2) \sqrt{a^2+2abx^2+b^2x^4} \, dx = \frac{ad(fx)^{1+m}\sqrt{a^2+2abx^2+b^2x^4}}{f(1+m)(a+bx^2)} + \frac{(bd+ae)(fx)^{3+m}\sqrt{a^2+2abx^2+b^2x^4}}{f^3(3+m)(a+bx^2)} + \frac{be(fx)^{5+m}\sqrt{a^2+2abx^2+b^2x^4}}{f^5(5+m)(a+bx^2)}$$

```
[Out] a*d*(f*x)^(1+m)*((b*x^2+a)^2)^(1/2)/f/(1+m)/(b*x^2+a)+(a*e+b*d)*(f*x)^(3+m)
*((b*x^2+a)^2)^(1/2)/f^3/(3+m)/(b*x^2+a)+b*e*(f*x)^(5+m)*((b*x^2+a)^2)^(1/2
)/f^5/(5+m)/(b*x^2+a)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.057, Rules used = {1264, 459}

$$\int (fx)^m (d+ex^2) \sqrt{a^2+2abx^2+b^2x^4} \, dx = \frac{\sqrt{a^2+2abx^2+b^2x^4}(fx)^{m+3}(ae+bd)}{f^3(m+3)(a+bx^2)} + \frac{ad\sqrt{a^2+2abx^2+b^2x^4}(fx)^{m+1}}{f(m+1)(a+bx^2)} + \frac{be\sqrt{a^2+2abx^2+b^2x^4}(fx)^{m+5}}{f^5(m+5)(a+bx^2)}$$

```
[In] Int[(f*x)^m*(d + e*x^2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4],x]
```

```
[Out] (a*d*(f*x)^(1 + m)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(f*(1 + m)*(a + b*x^2))
 + ((b*d + a*e)*(f*x)^(3 + m)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(f^3*(3 + m)
*(a + b*x^2)) + (b*e*(f*x)^(5 + m)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(f^5*(5
 + m)*(a + b*x^2))
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rule 1264

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_), x_Symbol] :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^Int
Part[p]*(b/2 + c*x^2)^(2*FracPart[p])), Int[(f*x)^m*(d + e*x^2)^q*(b/2 + c*
x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, f, m, p, q}, x] && EqQ[b^2 - 4*
a*c, 0] &&  !IntegerQ[p]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\sqrt{a^2+2abx^2+b^2x^4}\int (fx)^m\left(ab+b^2x^2\right)\left(d+ex^2\right)\,dx}{ab+b^2x^2}\\
&= \frac{\sqrt{a^2+2abx^2+b^2x^4}\int \left(abd(fx)^m+\frac{b(bd+ae)(fx)^{2+m}}{f^2}+\frac{b^2e(fx)^{4+m}}{f^4}\right)\,dx}{ab+b^2x^2}\\
&= \frac{ad(fx)^{1+m}\sqrt{a^2+2abx^2+b^2x^4}}{f(1+m)\left(a+bx^2\right)}+\frac{(bd+ae)(fx)^{3+m}\sqrt{a^2+2abx^2+b^2x^4}}{f^3(3+m)\left(a+bx^2\right)}\\
&\quad+\frac{be(fx)^{5+m}\sqrt{a^2+2abx^2+b^2x^4}}{f^5(5+m)\left(a+bx^2\right)}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.56

$$
\int (fx)^m\left(d+ex^2\right)\sqrt{a^2+2abx^2+b^2x^4}\,dx
$$

$$
=\frac{x(fx)^m\sqrt{\left(a+bx^2\right)^2}\left(a(5+m)\left(d(3+m)+e(1+m)x^2\right)+b(1+m)x^2\left(d(5+m)+e(3+m)x^2\right)\right)}{(1+m)(3+m)(5+m)\left(a+bx^2\right)}
$$

```
[In] Integrate[(f*x)^m*(d + e*x^2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4],x]
```

```
[Out] (x*(f*x)^m*Sqrt[(a + b*x^2)^2]*(a*(5 + m)*(d*(3 + m) + e*(1 + m)*x^2) + b*(
1 + m)*x^2*(d*(5 + m) + e*(3 + m)*x^2)))/((1 + m)*(3 + m)*(5 + m)*(a + b*x^
2))
```

Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.86

method	result	si
gosper	$\frac{x\left(b e\,m^{2} x^{4}+4 b e m\,x^{4}+a e\,m^{2} x^{2}+b d\,m^{2} x^{2}+3 b e\,x^{4}+6 a e m\,x^{2}+6 b d m\,x^{2}+a d\,m^{2}+5 a e\,x^{2}+5 b d\,x^{2}+8 a d m+15 d a\right)\left(f x\right)^{m} \sqrt{\left(b\,x^{2}+a\right)^{2}}}{\left(5+m\right)\left(3+m\right)\left(1+m\right)\left(b\,x^{2}+a\right)}$	13
risch	$\frac{x\left(b e\,m^{2} x^{4}+4 b e m\,x^{4}+a e\,m^{2} x^{2}+b d\,m^{2} x^{2}+3 b e\,x^{4}+6 a e m\,x^{2}+6 b d m\,x^{2}+a d\,m^{2}+5 a e\,x^{2}+5 b d\,x^{2}+8 a d m+15 d a\right)\left(f x\right)^{m} \sqrt{\left(b\,x^{2}+a\right)^{2}}}{\left(5+m\right)\left(3+m\right)\left(1+m\right)\left(b\,x^{2}+a\right)}$	13

```
[In] int((f*x)^m*(e*x^2+d)*(b^2*x^4+2*a*b*x^2+a^2)^(1/2),x,method=_RETURNVERBOSE
)
```

```
[Out] x*(b*e*m^2*x^4+4*b*e*m*x^4+a*e*m^2*x^2+b*d*m^2*x^2+3*b*e*x^4+6*a*e*m*x^2+6*
b*d*m*x^2+a*d*m^2+5*a*e*x^2+5*b*d*x^2+8*a*d*m+15*a*d)*(f*x)^m*((b*x^2+a)^2)
^(1/2)/(5+m)/(3+m)/(1+m)/(b*x^2+a)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.61

$$\int (fx)^m \left(d+ex^2\right) \sqrt{a^2+2abx^2+b^2x^4}\,dx = \frac{\left((bem^2+4\,bem+3\,be)x^5+((bd+ae)m^2+5\,bd+5\,ae+6\,(bd+ae)m)x^3+(adm^2+8\,adm+15\,ad)x\right)(f}{m^3+9\,m^2+23\,m+15}$$

```
[In] integrate((f*x)^m*(e*x^2+d)*(b^2*x^4+2*a*b*x^2+a^2)^(1/2),x, algorithm="fri
cas")
```

```
[Out] ((b*e*m^2 + 4*b*e*m + 3*b*e)*x^5 + ((b*d + a*e)*m^2 + 5*b*d + 5*a*e + 6*(b*
d + a*e)*m)*x^3 + (a*d*m^2 + 8*a*d*m + 15*a*d)*x)*(f*x)^m/(m^3 + 9*m^2 + 23
*m + 15)
```

Sympy [F]

$$\int (fx)^m \left(d+ex^2\right) \sqrt{a^2+2abx^2+b^2x^4}\,dx = \int (fx)^m \left(d+ex^2\right) \sqrt{\left(a+bx^2\right)^2}\,dx$$

```
[In] integrate((f*x)**m*(e*x**2+d)*(b**2*x**4+2*a*b*x**2+a**2)**(1/2),x)
```

```
[Out] Integral((f*x)**m*(d + e*x**2)*sqrt((a + b*x**2)**2), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.49

$$\int (fx)^m \left(d+ex^2\right)\sqrt{a^2+2abx^2+b^2x^4}\,dx = \frac{(bf^m(m+1)x^3+af^m(m+3)x)dx^m}{m^2+4m+3} + \frac{(bf^m(m+3)x^5+af^m(m+5)x^3)ex^m}{m^2+8m+15}$$

```
[In] integrate((f*x)^m*(e*x^2+d)*(b^2*x^4+2*a*b*x^2+a^2)^(1/2),x, algorithm="max
ima")
```

```
[Out] (b*f^m*(m + 1)*x^3 + a*f^m*(m + 3)*x)*d*x^m/(m^2 + 4*m + 3) + (b*f^m*(m + 3
)*x^5 + a*f^m*(m + 5)*x^3)*e*x^m/(m^2 + 8*m + 15)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 263 vs. 2(120) = 240.

Time = 0.27 (sec) , antiderivative size = 263, normalized size of antiderivative = 1.72

$$\int (fx)^m \left(d+ex^2\right)\sqrt{a^2+2abx^2+b^2x^4}\,dx$$
$$= \frac{(fx)^m bem^2x^5\mathrm{sgn}(bx^2+a) + 4(fx)^m bemx^5\mathrm{sgn}(bx^2+a) + (fx)^m bdm^2x^3\mathrm{sgn}(bx^2+a) + (fx)^m aem^2x^3\mathrm{s}}{}$$

```
[In] integrate((f*x)^m*(e*x^2+d)*(b^2*x^4+2*a*b*x^2+a^2)^(1/2),x, algorithm="gia
c")
```

```
[Out] ((f*x)^m*b*e*m^2*x^5*sgn(b*x^2 + a) + 4*(f*x)^m*b*e*m*x^5*sgn(b*x^2 + a) +
(f*x)^m*b*d*m^2*x^3*sgn(b*x^2 + a) + (f*x)^m*a*e*m^2*x^3*sgn(b*x^2 + a) + 3
*(f*x)^m*b*e*x^5*sgn(b*x^2 + a) + 6*(f*x)^m*b*d*m*x^3*sgn(b*x^2 + a) + 6*(f
*x)^m*a*e*m*x^3*sgn(b*x^2 + a) + (f*x)^m*a*d*m^2*x*sgn(b*x^2 + a) + 5*(f*x)
^m*b*d*x^3*sgn(b*x^2 + a) + 5*(f*x)^m*a*e*x^3*sgn(b*x^2 + a) + 8*(f*x)^m*a*
d*m*x*sgn(b*x^2 + a) + 15*(f*x)^m*a*d*x*sgn(b*x^2 + a))/(m^3 + 9*m^2 + 23*m
 + 15)
```

Mupad [F(-1)]

Timed out.

$$\int (fx)^m \left(d+ex^2\right) \sqrt{a^2+2abx^2+b^2x^4}\,dx = \int (f\,x)^m \left(e\,x^2+d\right) \sqrt{a^2+2\,a\,b\,x^2+b^2\,x^4}\,dx$$

```
[In] int((f*x)^m*(d + e*x^2)*(a^2 + b^2*x^4 + 2*a*b*x^2)^(1/2),x)
[Out] int((f*x)^m*(d + e*x^2)*(a^2 + b^2*x^4 + 2*a*b*x^2)^(1/2), x)
```

3.90 $\int \frac{(fx)^m(d+ex^2)}{\sqrt{a^2+2abx^2+b^2x^4}}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 134

$$\int \frac{(fx)^m (d+ex^2)}{\sqrt{a^2+2abx^2+b^2x^4}}\,dx = \frac{e(fx)^{1+m}(a+bx^2)}{bf(1+m)\sqrt{a^2+2abx^2+b^2x^4}} + \frac{(bd-ae)(fx)^{1+m}(a+bx^2)\,\text{Hypergeometric2F1}\left(1,\frac{1+m}{2},\frac{3+m}{2},-\frac{bx^2}{a}\right)}{abf(1+m)\sqrt{a^2+2abx^2+b^2x^4}}$$

```
[Out] e*(f*x)^(1+m)*(b*x^2+a)/b/f/(1+m)/((b*x^2+a)^2)^(1/2)+(-a*e+b*d)*(f*x)^(1+m
)*(b*x^2+a)*hypergeom([1, 1/2+1/2*m],[3/2+1/2*m],-b*x^2/a)/a/b/f/(1+m)/((b*
x^2+a)^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.086, Rules used = {1264, 470, 371}

$$\int \frac{(fx)^m (d+ex^2)}{\sqrt{a^2+2abx^2+b^2x^4}}\,dx = \frac{(a+bx^2)(fx)^{m+1}(bd-ae)\,\text{Hypergeometric2F1}\left(1,\frac{m+1}{2},\frac{m+3}{2},-\frac{bx^2}{a}\right)}{abf(m+1)\sqrt{a^2+2abx^2+b^2x^4}} + \frac{e(a+bx^2)(fx)^{m+1}}{bf(m+1)\sqrt{a^2+2abx^2+b^2x^4}}$$

```
[In] Int[((f*x)^m*(d + e*x^2))/Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4],x]
```

```
[Out] (e*(f*x)^(1 + m)*(a + b*x^2))/(b*f*(1 + m)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])
 + ((b*d - a*e)*(f*x)^(1 + m)*(a + b*x^2)*Hypergeometric2F1[1, (1 + m)/2, (
3 + m)/2, -((b*x^2)/a)])/(a*b*f*(1 + m)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 1264

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_), x_Symbol] :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^Int
Part[p]*(b/2 + c*x^2)^(2*FracPart[p])), Int[(f*x)^m*(d + e*x^2)^q*(b/2 + c*
x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, f, m, p, q}, x] && EqQ[b^2 - 4*
a*c, 0] &&  !IntegerQ[p]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{(ab+b^2x^2)\int \frac{(fx)^m(d+ex^2)}{ab+b^2x^2}\,dx}{\sqrt{a^2+2abx^2+b^2x^4}}\\
&= \frac{e(fx)^{1+m}(a+bx^2)}{bf(1+m)\sqrt{a^2+2abx^2+b^2x^4}} - \frac{((-b^2d(1+m)+abe(1+m))(ab+b^2x^2))\int \frac{(fx)^m}{ab+b^2x^2}\,dx}{b^2(1+m)\sqrt{a^2+2abx^2+b^2x^4}}\\
&= \frac{e(fx)^{1+m}(a+bx^2)}{bf(1+m)\sqrt{a^2+2abx^2+b^2x^4}} + \frac{(bd-ae)(fx)^{1+m}(a+bx^2)\,{}_2F_1\left(1,\frac{1+m}{2};\frac{3+m}{2};-\frac{bx^2}{a}\right)}{abf(1+m)\sqrt{a^2+2abx^2+b^2x^4}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.58

$$\int \frac{(fx)^m (d+ex^2)}{\sqrt{a^2+2abx^2+b^2x^4}}\,dx = -\frac{x(fx)^m (a+bx^2)\left(-ae+(-bd+ae)\,\text{Hypergeometric2F1}\left(1,\frac{1+m}{2},\frac{3+m}{2},-\frac{bx^2}{a}\right)\right)}{ab(1+m)\sqrt{(a+bx^2)^2}}$$

```
[In] Integrate[((f*x)^m*(d + e*x^2))/Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4],x]
[Out] -((x*(f*x)^m*(a + b*x^2)*(-(a*e) + (-(b*d) + a*e)*Hypergeometric2F1[1, (1 +
 m)/2, (3 + m)/2, -((b*x^2)/a)]))/(a*b*(1 + m)*Sqrt[(a + b*x^2)^2]))
```

Maple [F]

$$\int \frac{(fx)^m (e\,x^2+d)}{\sqrt{b^2x^4+2ab\,x^2+a^2}}dx$$

```
[In] int((f*x)^m*(e*x^2+d)/(b^2*x^4+2*a*b*x^2+a^2)^(1/2),x)
[Out] int((f*x)^m*(e*x^2+d)/(b^2*x^4+2*a*b*x^2+a^2)^(1/2),x)
```

Fricas [F]

$$\int \frac{(fx)^m (d+ex^2)}{\sqrt{a^2+2abx^2+b^2x^4}}\,dx = \int \frac{(ex^2+d)(fx)^m}{\sqrt{b^2x^4+2\,abx^2+a^2}}\,dx$$

```
[In] integrate((f*x)^m*(e*x^2+d)/(b^2*x^4+2*a*b*x^2+a^2)^(1/2),x, algorithm="fri
cas")
[Out] integral((e*x^2 + d)*(f*x)^m/sqrt(b^2*x^4 + 2*a*b*x^2 + a^2), x)
```

Sympy [F]

$$\int \frac{(fx)^m (d+ex^2)}{\sqrt{a^2+2abx^2+b^2x^4}}\,dx = \int \frac{(fx)^m (d+ex^2)}{\sqrt{(a+bx^2)^2}}\,dx$$

```
[In] integrate((f*x)**m*(e*x**2+d)/(b**2*x**4+2*a*b*x**2+a**2)**(1/2),x)
[Out] Integral((f*x)**m*(d + e*x**2)/sqrt((a + b*x**2)**2), x)
```

Maxima [F]

$$\int \frac{(fx)^m (d+ex^2)}{\sqrt{a^2+2abx^2+b^2x^4}}\,dx = \int \frac{(ex^2+d)(fx)^m}{\sqrt{b^2x^4+2\,abx^2+a^2}}\,dx$$

```
[In] integrate((f*x)^m*(e*x^2+d)/(b^2*x^4+2*a*b*x^2+a^2)^(1/2),x, algorithm="max
ima")
[Out] integrate((e*x^2 + d)*(f*x)^m/sqrt(b^2*x^4 + 2*a*b*x^2 + a^2), x)
```

Giac [F]

$$\int \frac{(fx)^m (d+ex^2)}{\sqrt{a^2+2abx^2+b^2x^4}}\,dx = \int \frac{(ex^2+d)(fx)^m}{\sqrt{b^2x^4+2\,abx^2+a^2}}\,dx$$

```
[In] integrate((f*x)^m*(e*x^2+d)/(b^2*x^4+2*a*b*x^2+a^2)^(1/2),x, algorithm="gia
c")
[Out] integrate((e*x^2 + d)*(f*x)^m/sqrt(b^2*x^4 + 2*a*b*x^2 + a^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(fx)^m (d+ex^2)}{\sqrt{a^2+2abx^2+b^2x^4}}\,dx = \int \frac{(f\,x)^m\,(e\,x^2+d)}{\sqrt{a^2+2\,a\,b\,x^2+b^2\,x^4}}\,dx$$

```
[In] int(((f*x)^m*(d + e*x^2))/(a^2 + b^2*x^4 + 2*a*b*x^2)^(1/2),x)
[Out] int(((f*x)^m*(d + e*x^2))/(a^2 + b^2*x^4 + 2*a*b*x^2)^(1/2), x)
```

3.91 $\int \frac{(fx)^m(d+ex^2)}{(a^2+2abx^2+b^2x^4)^{3/2}}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 154

$$\int \frac{(fx)^m(d+ex^2)}{(a^2+2abx^2+b^2x^4)^{3/2}}\,dx = \frac{(bd-ae)(fx)^{1+m}}{4abf(a+bx^2)\sqrt{a^2+2abx^2+b^2x^4}} + \frac{(bd(3-m)+ae(1+m))(fx)^{1+m}(a+bx^2)\,\text{Hypergeometric2F1}\left(2,\frac{1+m}{2},\frac{3+m}{2},-\frac{bx^2}{a}\right)}{4a^3bf(1+m)\sqrt{a^2+2abx^2+b^2x^4}}$$

```
[Out] 1/4*(-a*e+b*d)*(f*x)^(1+m)/a/b/f/(b*x^2+a)/((b*x^2+a)^2)^(1/2)+1/4*(b*d*(3-
m)+a*e*(1+m))*(f*x)^(1+m)*(b*x^2+a)*hypergeom([2, 1/2+1/2*m],[3/2+1/2*m],-b
*x^2/a)/a^3/b/f/(1+m)/((b*x^2+a)^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.086, Rules used = {1264, 468, 371}

$$\int \frac{(fx)^m(d+ex^2)}{(a^2+2abx^2+b^2x^4)^{3/2}}\,dx = \frac{(fx)^{m+1}(bd-ae)}{4abf(a+bx^2)\sqrt{a^2+2abx^2+b^2x^4}} + \frac{(a+bx^2)(fx)^{m+1}(ae(m+1)+bd(3-m))\,\text{Hypergeometric2F1}\left(2,\frac{m+1}{2},\frac{m+3}{2},-\frac{bx^2}{a}\right)}{4a^3bf(m+1)\sqrt{a^2+2abx^2+b^2x^4}}$$

```
[In] Int[((f*x)^m*(d + e*x^2))/(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2),x]
```

```
[Out] ((b*d - a*e)*(f*x)^(1 + m))/(4*a*b*f*(a + b*x^2)*Sqrt[a^2 + 2*a*b*x^2 + b^2
*x^4]) + ((b*d*(3 - m) + a*e*(1 + m))*(f*x)^(1 + m)*(a + b*x^2)*Hypergeomet
```

```
ric2F1[2, (1 + m)/2, (3 + m)/2, -((b*x^2)/a)])/(4*a^3*b*f*(1 + m)*Sqrt[a^2
+ 2*a*b*x^2 + b^2*x^4])
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 468

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a
*b*e*n*(p + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*
(p + 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e,
m, n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && Ne
Q[p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1,
 m, (-n)*(p + 1)]))
```

Rule 1264

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_), x_Symbol] :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^Int
Part[p]*(b/2 + c*x^2)^(2*FracPart[p])), Int[(f*x)^m*(d + e*x^2)^q*(b/2 + c*
x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, f, m, p, q}, x] && EqQ[b^2 - 4*
a*c, 0] &&  !IntegerQ[p]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\left(b^2\left(ab+b^2x^2\right)\right)\int \frac{(fx)^m\left(d+ex^2\right)}{\left(ab+b^2x^2\right)^3}\,dx}{\sqrt{a^2+2abx^2+b^2x^4}} \\
&= \frac{(bd-ae)(fx)^{1+m}}{4abf\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}}+\frac{\left((bd(3-m)+ae(1+m))\left(ab+b^2x^2\right)\right)\int \frac{(fx)^m}{\left(ab+b^2x^2\right)^2}\,dx}{4a\sqrt{a^2+2abx^2+b^2x^4}} \\
&= \frac{(bd-ae)(fx)^{1+m}}{4abf\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}} \\
&\quad+\frac{(bd(3-m)+ae(1+m))(fx)^{1+m}\left(a+bx^2\right)\,{}_2F_1\left(2,\frac{1+m}{2};\frac{3+m}{2};-\frac{bx^2}{a}\right)}{4a^3bf(1+m)\sqrt{a^2+2abx^2+b^2x^4}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.66

$$\int \frac{(fx)^m (d+ex^2)}{(a^2+2abx^2+b^2x^4)^{3/2}}\,dx = \frac{x(fx)^m (a+bx^2) \left(ae\,\text{Hypergeometric2F1}\left(2,\frac{1+m}{2},\frac{3+m}{2},-\frac{bx^2}{a}\right)+(bd-ae)\right.}{a^3b(1+m)\sqrt{(a+bx^2)^2}}$$

```
[In] Integrate[((f*x)^m*(d + e*x^2))/(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2),x]
```

```
[Out] (x*(f*x)^m*(a + b*x^2)*(a*e*Hypergeometric2F1[2, (1 + m)/2, (3 + m)/2, -((b
*x^2)/a)] + (b*d - a*e)*Hypergeometric2F1[3, (1 + m)/2, (3 + m)/2, -((b*x^2
)/a)]))/(a^3*b*(1 + m)*Sqrt[(a + b*x^2)^2])
```

Maple [F]

$$\int \frac{(fx)^m (e\,x^2+d)}{(b^2x^4+2ab\,x^2+a^2)^{\frac{3}{2}}}dx$$

```
[In] int((f*x)^m*(e*x^2+d)/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x)
```

```
[Out] int((f*x)^m*(e*x^2+d)/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x)
```

Fricas [F]

$$\int \frac{(fx)^m (d+ex^2)}{(a^2+2abx^2+b^2x^4)^{3/2}}\,dx = \int \frac{(ex^2+d)(fx)^m}{(b^2x^4+2\,abx^2+a^2)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((f*x)^m*(e*x^2+d)/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="fri
cas")
```

```
[Out] integral(sqrt(b^2*x^4 + 2*a*b*x^2 + a^2)*(e*x^2 + d)*(f*x)^m/(b^4*x^8 + 4*a
*b^3*x^6 + 6*a^2*b^2*x^4 + 4*a^3*b*x^2 + a^4), x)
```

Sympy [F]

$$\int \frac{(fx)^m (d+ex^2)}{(a^2+2abx^2+b^2x^4)^{3/2}}\,dx = \int \frac{(fx)^m (d+ex^2)}{((a+bx^2)^2)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((f*x)**m*(e*x**2+d)/(b**2*x**4+2*a*b*x**2+a**2)**(3/2),x)
```

```
[Out] Integral((f*x)**m*(d + e*x**2)/((a + b*x**2)**2)**(3/2), x)
```

Maxima [F]

$$\int \frac{(fx)^m (d+ex^2)}{(a^2+2abx^2+b^2x^4)^{3/2}}\,dx = \int \frac{(ex^2+d)(fx)^m}{(b^2x^4+2\,abx^2+a^2)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((f*x)^m*(e*x^2+d)/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="max
ima")
[Out] integrate((e*x^2 + d)*(f*x)^m/(b^2*x^4 + 2*a*b*x^2 + a^2)^(3/2), x)
```

Giac [F]

$$\int \frac{(fx)^m (d+ex^2)}{(a^2+2abx^2+b^2x^4)^{3/2}}\,dx = \int \frac{(ex^2+d)(fx)^m}{(b^2x^4+2\,abx^2+a^2)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((f*x)^m*(e*x^2+d)/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="gia
c")
[Out] integrate((e*x^2 + d)*(f*x)^m/(b^2*x^4 + 2*a*b*x^2 + a^2)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(fx)^m (d+ex^2)}{(a^2+2abx^2+b^2x^4)^{3/2}}\,dx = \int \frac{(f\,x)^m\,(e\,x^2+d)}{(a^2+2\,a\,b\,x^2+b^2\,x^4)^{3/2}}\,dx$$

```
[In] int(((f*x)^m*(d + e*x^2))/(a^2 + b^2*x^4 + 2*a*b*x^2)^(3/2),x)
[Out] int(((f*x)^m*(d + e*x^2))/(a^2 + b^2*x^4 + 2*a*b*x^2)^(3/2), x)
```

3.92 $\int x(a+bx^2)\left(a^2+2abx^2+b^2x^4\right)^p\,dx$

Optimal result

Integrand size = 29, antiderivative size = 34

$$\int x\left(a+bx^2\right)\left(a^2+2abx^2+b^2x^4\right)^p\,dx=\frac{\left(a^2+2abx^2+b^2x^4\right)^{1+p}}{4b(1+p)}$$

```
[Out] 1/4*(b^2*x^4+2*a*b*x^2+a^2)^(p+1)/b/(p+1)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.069, Rules used = {1261, 643}

$$\int x\left(a+bx^2\right)\left(a^2+2abx^2+b^2x^4\right)^p\,dx=\frac{\left(a^2+2abx^2+b^2x^4\right)^{p+1}}{4b(p+1)}$$

```
[In] Int[x*(a + b*x^2)*(a^2 + 2*a*b*x^2 + b^2*x^4)^p,x]
```

```
[Out] (a^2 + 2*a*b*x^2 + b^2*x^4)^(1 + p)/(4*b*(1 + p))
```

Rule 643

```
Int[((d_) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol
] :> Simp[d*((a + b*x + c*x^2)^(p + 1)/(b*(p + 1))), x] /; FreeQ[{a, b, c,
d, e, p}, x] && EqQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 1261

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(
p_.), x_Symbol] :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x],
 x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]
```

Rubi steps

$$\text{integral} = \frac{1}{2}\text{Subst}\left(\int (a+bx)\left(a^2+2abx+b^2x^2\right)^p\,dx,x,x^2\right)$$
$$= \frac{\left(a^2+2abx^2+b^2x^4\right)^{1+p}}{4b(1+p)}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.74

$$\int x\left(a+bx^2\right)\left(a^2+2abx^2+b^2x^4\right)^p\,dx = \frac{\left(\left(a+bx^2\right)^2\right)^{1+p}}{4b(1+p)}$$

```
[In] Integrate[x*(a + b*x^2)*(a^2 + 2*a*b*x^2 + b^2*x^4)^p,x]
[Out] ((a + b*x^2)^2)^(1 + p)/(4*b*(1 + p))
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.18

method	result	size
gosper	$\frac{\left(b\,x^2+a\right)^2\left(b^2x^4+2ab\,x^2+a^2\right)^p}{4b(1+p)}$	40
risch	$\frac{\left(b^2x^4+2ab\,x^2+a^2\right)\left(\left(b\,x^2+a\right)^2\right)^p}{4b(1+p)}$	40
parallelrisch	$\frac{x^4\left(b^2x^4+2ab\,x^2+a^2\right)^p a\,b^2+2x^2\left(b^2x^4+2ab\,x^2+a^2\right)^p a^2b+a^3\left(b^2x^4+2ab\,x^2+a^2\right)^p}{4ab(1+p)}$	96
norman	$\frac{a\,x^2 e^{p\ln\left(b^2x^4+2ab\,x^2+a^2\right)}}{2p+2}+\frac{a^2 e^{p\ln\left(b^2x^4+2ab\,x^2+a^2\right)}}{4b(1+p)}+\frac{b\,x^4 e^{p\ln\left(b^2x^4+2ab\,x^2+a^2\right)}}{4+4p}$	103

```
[In] int(x*(b*x^2+a)*(b^2*x^4+2*a*b*x^2+a^2)^p,x,method=_RETURNVERBOSE)
[Out] 1/4*(b*x^2+a)^2/b/(1+p)*(b^2*x^4+2*a*b*x^2+a^2)^p
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.38

$$\int x(a+bx^2)\left(a^2+2abx^2+b^2x^4\right)^p\,dx=\frac{(b^2x^4+2\,abx^2+a^2)(b^2x^4+2\,abx^2+a^2)^p}{4\,(bp+b)}$$

```
[In] integrate(x*(b*x^2+a)*(b^2*x^4+2*a*b*x^2+a^2)^p,x, algorithm="fricas")
[Out] 1/4*(b^2*x^4 + 2*a*b*x^2 + a^2)*(b^2*x^4 + 2*a*b*x^2 + a^2)^p/(b*p + b)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 155 vs. 2(27) = 54.

Time = 4.39 (sec) , antiderivative size = 155, normalized size of antiderivative = 4.56

$$\int x(a+bx^2)\left(a^2+2abx^2+b^2x^4\right)^p\,dx$$
$$=\begin{cases}\frac{x^2}{2a} & \text{for } b=0\wedge p=-1\\ \frac{ax^2(a^2)^p}{2} & \text{for } b=0\\ \frac{\log\left(x-\sqrt{-\frac{a}{b}}\right)}{2b}+\frac{\log\left(x+\sqrt{-\frac{a}{b}}\right)}{2b} & \text{for } p=-1\\ \frac{a^2(a^2+2abx^2+b^2x^4)^p}{4bp+4b}+\frac{2abx^2(a^2+2abx^2+b^2x^4)^p}{4bp+4b}+\frac{b^2x^4(a^2+2abx^2+b^2x^4)^p}{4bp+4b} & \text{otherwise}\end{cases}$$

```
[In] integrate(x*(b*x**2+a)*(b**2*x**4+2*a*b*x**2+a**2)**p,x)
[Out] Piecewise((x**2/(2*a), Eq(b, 0) & Eq(p, -1)), (a*x**2*(a**2)**p/2, Eq(b, 0)
), (log(x - sqrt(-a/b))/(2*b) + log(x + sqrt(-a/b))/(2*b), Eq(p, -1)), (a**
2*(a**2 + 2*a*b*x**2 + b**2*x**4)**p/(4*b*p + 4*b) + 2*a*b*x**2*(a**2 + 2*a
*b*x**2 + b**2*x**4)**p/(4*b*p + 4*b) + b**2*x**4*(a**2 + 2*a*b*x**2 + b**2
*x**4)**p/(4*b*p + 4*b), True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 86 vs. 2(32) = 64.

Time = 0.21 (sec) , antiderivative size = 86, normalized size of antiderivative = 2.53

$$\int x(a+bx^2)\left(a^2+2abx^2+b^2x^4\right)^p\,dx=\frac{(bx^2+a)(bx^2+a)^{2\,p}a}{2\,b(2\,p+1)}+\frac{(b^2(2\,p+1)x^4+2\,abpx^2-a^2)(bx^2+a)^{2\,p}}{4\,(2\,p^2+3\,p+1)b}$$

```
[In] integrate(x*(b*x^2+a)*(b^2*x^4+2*a*b*x^2+a^2)^p,x, algorithm="maxima")
```

```
[Out] 1/2*(b*x^2 + a)*(b*x^2 + a)^(2*p)*a/(b*(2*p + 1)) + 1/4*(b^2*(2*p + 1)*x^4
+ 2*a*b*p*x^2 - a^2)*(b*x^2 + a)^(2*p)/((2*p^2 + 3*p + 1)*b)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.94

$$\int x(a+bx^2)\left(a^2+2abx^2+b^2x^4\right)^p\,dx=\frac{\left(b^2x^4+2\,abx^2+a^2\right)^{p+1}}{4\,b(p+1)}$$

```
[In] integrate(x*(b*x^2+a)*(b^2*x^4+2*a*b*x^2+a^2)^p,x, algorithm="giac")
```

```
[Out] 1/4*(b^2*x^4 + 2*a*b*x^2 + a^2)^(p + 1)/(b*(p + 1))
```

Mupad [B] (verification not implemented)

Time = 7.81 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.74

$$\int x(a+bx^2)\left(a^2+2abx^2+b^2x^4\right)^p\,dx=\left(a^2+2\,a\,b\,x^2+b^2\,x^4\right)^p\left(\frac{a^2}{4\,b\,\left(p+1\right)}+\frac{a\,x^2}{2\,\left(p+1\right)}+\frac{b\,x^4}{4\,\left(p+1\right)}\right)$$

```
[In] int(x*(a + b*x^2)*(a^2 + b^2*x^4 + 2*a*b*x^2)^p,x)
```

```
[Out] (a^2 + b^2*x^4 + 2*a*b*x^2)^p*(a^2/(4*b*(p + 1)) + (a*x^2)/(2*(p + 1)) + (b
*x^4)/(4*(p + 1)))
```

3.93 $\int x^3(a+bx^2)\left(a^2+2abx^2+b^2x^4\right)^p\,dx$

Optimal result

Integrand size = 31, antiderivative size = 86

$$\int x^3(a+bx^2)\left(a^2+2abx^2+b^2x^4\right)^p\,dx=-\frac{a(a+bx^2)^2\left(a^2+2abx^2+b^2x^4\right)^p}{4b^2(1+p)}+\frac{(a+bx^2)^3\left(a^2+2abx^2+b^2x^4\right)^p}{2b^2(3+2p)}$$

```
[Out] -1/4*a*(b*x^2+a)^2*(b^2*x^4+2*a*b*x^2+a^2)^p/b^2/(p+1)+1/2*(b*x^2+a)^3*(b^2
*x^4+2*a*b*x^2+a^2)^p/b^2/(3+2*p)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.129, Rules used = {1263, 784, 21, 45}

$$\int x^3(a+bx^2)\left(a^2+2abx^2+b^2x^4\right)^p\,dx=\frac{(a+bx^2)^3\left(a^2+2abx^2+b^2x^4\right)^p}{2b^2(2p+3)}-\frac{a(a+bx^2)^2\left(a^2+2abx^2+b^2x^4\right)^p}{4b^2(p+1)}$$

```
[In] Int[x^3*(a + b*x^2)*(a^2 + 2*a*b*x^2 + b^2*x^4)^p,x]
```

```
[Out] -1/4*(a*(a + b*x^2)^2*(a^2 + 2*a*b*x^2 + b^2*x^4)^p)/(b^2*(1 + p)) + ((a +
b*x^2)^3*(a^2 + 2*a*b*x^2 + b^2*x^4)^p)/(2*b^2*(3 + 2*p))
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
```

```
&& EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
a + b*x])
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 784

```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (
c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/(c^IntPa
rt[p]*(b/2 + c*x)^(2*FracPart[p])), Int[(d + e*x)^m*(f + g*x)*(b/2 + c*x)^(
2*p), x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && EqQ[b^2 - 4*a*c, 0]
```

Rule 1263

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && EqQ[b
^2 - 4*a*c, 0] &&  !IntegerQ[p] && IGtQ[(m + 1)/2, 0]
```

Rubi steps

$$\text{integral} = \frac{1}{2}\text{Subst}\left(\int x(a+bx)\left(a^2+2abx+b^2x^2\right)^p\,dx, x, x^2\right)$$

$$= \frac{1}{2}\left(\left(b\left(a+bx^2\right)\right)^{-2p}\left(a^2+2abx^2+b^2x^4\right)^p\right)\text{Subst}\left(\int x(a+bx)\left(ab+b^2x\right)^{2p}\,dx, x, x^2\right)$$

$$= \frac{\left(\left(b\left(a+bx^2\right)\right)^{-2p}\left(a^2+2abx^2+b^2x^4\right)^p\right)\text{Subst}\left(\int x\left(ab+b^2x\right)^{1+2p}\,dx, x, x^2\right)}{2b}$$

$$= \frac{\left(\left(b\left(a+bx^2\right)\right)^{-2p}\left(a^2+2abx^2+b^2x^4\right)^p\right)\text{Subst}\left(\int\left(-\frac{a\left(ab+b^2x\right)^{1+2p}}{b}+\frac{\left(ab+b^2x\right)^{2+2p}}{b^2}\right)dx, x, x^2\right)}{2b}$$

$$= -\frac{a\left(a+bx^2\right)^2\left(a^2+2abx^2+b^2x^4\right)^p}{4b^2(1+p)}+\frac{\left(a+bx^2\right)^3\left(a^2+2abx^2+b^2x^4\right)^p}{2b^2(3+2p)}$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.52

$$\int x^3\left(a+bx^2\right)\left(a^2+2abx^2+b^2x^4\right)^p\,dx=\frac{\left(\left(a+bx^2\right)^2\right)^{1+p}\left(-a+2b(1+p)x^2\right)}{4b^2(1+p)(3+2p)}$$

```
[In] Integrate[x^3*(a + b*x^2)*(a^2 + 2*a*b*x^2 + b^2*x^4)^p,x]
[Out] (((a + b*x^2)^2)^(1 + p)*(-a + 2*b*(1 + p)*x^2))/(4*b^2*(1 + p)*(3 + 2*p))
```

Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.72

method	result
gosper	$-\frac{\left(b^{2} x^{4}+2 a b \,x^{2}+a^{2}\right)^{p} \left(-2 x^{2} p b -2 b \,x^{2}+a \right) \left(b \,x^{2}+a \right)^{2}}{4 b^{2} \left(2 p^{2}+5 p +3\right)}$
risch	$-\frac{\left(-2 b^{3} p \,x^{6}-2 b^{3} x^{6}-4 a \,b^{2} p \,x^{4}-3 b^{2} x^{4} a -2 a^{2} b p \,x^{2}+a^{3}\right) \left(\left(b \,x^{2}+a \right)^{2}\right)^{p}}{4 \left(1+p \right) \left(3+2 p \right) b^{2}}$
norman	$-\frac{a^{3} {\mathrm e}^{p \ln \left(b^{2} x^{4}+2 a b \,x^{2}+a^{2}\right)}}{4 b^{2} \left(2 p^{2}+5 p +3\right)}+\frac{b \,x^{6} {\mathrm e}^{p \ln \left(b^{2} x^{4}+2 a b \,x^{2}+a^{2}\right)}}{6+4 p}+\frac{a \left(4 p +3\right) x^{4} {\mathrm e}^{p \ln \left(b^{2} x^{4}+2 a b \,x^{2}+a^{2}\right)}}{8 p^{2}+20 p +12}+\frac{p \,a^{2} x^{2} {\mathrm e}^{p \ln \left(b^{2} x^{4}+2 a b \,x^{2}+a^{2}\right)}}{2 b \left(2 p^{2}+5 p +3\right)}$
parallelrisch	$\frac{2 x^{6} \left(b^{2} x^{4}+2 a b \,x^{2}+a^{2}\right)^{p} b^{3} p +2 x^{6} \left(b^{2} x^{4}+2 a b \,x^{2}+a^{2}\right)^{p} b^{3}+4 x^{4} \left(b^{2} x^{4}+2 a b \,x^{2}+a^{2}\right)^{p} a \,b^{2} p +3 x^{4} \left(b^{2} x^{4}+2 a b \,x^{2}+a^{2}\right)^{p} a \,b^{2}+2 x^{2} \left(b^{2} x\right.}{4 b^{2} \left(2 p^{2}+5 p +3\right)}$

```
[In] int(x^3*(b*x^2+a)*(b^2*x^4+2*a*b*x^2+a^2)^p,x,method=_RETURNVERBOSE)
[Out] -1/4*(b^2*x^4+2*a*b*x^2+a^2)^p*(-2*b*p*x^2-2*b*x^2+a)*(b*x^2+a)^2/b^2/(2*p^
2+5*p+3)
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.07

$$\int x^3\left(a+bx^2\right)\left(a^2+2abx^2+b^2x^4\right)^p\,dx$$
$$=\frac{(2\,(b^3p+b^3)x^6+2\,a^2bpx^2+(4\,ab^2p+3\,ab^2)x^4-a^3)(b^2x^4+2\,abx^2+a^2)^p}{4\,(2\,b^2p^2+5\,b^2p+3\,b^2)}$$

```
[In] integrate(x^3*(b*x^2+a)*(b^2*x^4+2*a*b*x^2+a^2)^p,x, algorithm="fricas")
[Out] 1/4*(2*(b^3*p + b^3)*x^6 + 2*a^2*b*p*x^2 + (4*a*b^2*p + 3*a*b^2)*x^4 - a^3)
*(b^2*x^4 + 2*a*b*x^2 + a^2)^p/(2*b^2*p^2 + 5*b^2*p + 3*b^2)
```

Sympy [F]

$$\int x^3(a+bx^2)\left(a^2+2abx^2+b^2x^4\right)^p\,dx$$
$$=\begin{cases}\frac{ax^4(a^2)^p}{4}\\ \int\frac{x^3(a+bx^2)}{\left((a+bx^2)^2\right)^{\frac{3}{2}}}\,dx\\ -\frac{a\log\left(x-\sqrt{-\frac{a}{b}}\right)}{2b^2}-\frac{a\log\left(x+\sqrt{-\frac{a}{b}}\right)}{2b^2}+\frac{x^2}{2b}\\ -\frac{a^3(a^2+2abx^2+b^2x^4)^p}{8b^2p^2+20b^2p+12b^2}+\frac{2a^2bpx^2(a^2+2abx^2+b^2x^4)^p}{8b^2p^2+20b^2p+12b^2}+\frac{4ab^2px^4(a^2+2abx^2+b^2x^4)^p}{8b^2p^2+20b^2p+12b^2}+\frac{3ab^2x^4(a^2+2abx^2+b^2x^4)^p}{8b^2p^2+20b^2p+12b^2}+\frac{2b^3px^6(a^2+2abx^2+b^2x^4)^p}{8b^2p^2+20b^2p+12b^2}\end{cases}$$

```
[In] integrate(x**3*(b*x**2+a)*(b**2*x**4+2*a*b*x**2+a**2)**p,x)
```

```
[Out] Piecewise((a*x**4*(a**2)**p/4, Eq(b, 0)), (Integral(x**3*(a + b*x**2)/((a +
 b*x**2)**2)**(3/2), x), Eq(p, -3/2)), (-a*log(x - sqrt(-a/b))/(2*b**2) - a
*log(x + sqrt(-a/b))/(2*b**2) + x**2/(2*b), Eq(p, -1)), (-a**3*(a**2 + 2*a*
b*x**2 + b**2*x**4)**p/(8*b**2*p**2 + 20*b**2*p + 12*b**2) + 2*a**2*b*p*x**
2*(a**2 + 2*a*b*x**2 + b**2*x**4)**p/(8*b**2*p**2 + 20*b**2*p + 12*b**2) +
4*a*b**2*p*x**4*(a**2 + 2*a*b*x**2 + b**2*x**4)**p/(8*b**2*p**2 + 20*b**2*p
 + 12*b**2) + 3*a*b**2*x**4*(a**2 + 2*a*b*x**2 + b**2*x**4)**p/(8*b**2*p**2
 + 20*b**2*p + 12*b**2) + 2*b**3*p*x**6*(a**2 + 2*a*b*x**2 + b**2*x**4)**p/
(8*b**2*p**2 + 20*b**2*p + 12*b**2) + 2*b**3*x**6*(a**2 + 2*a*b*x**2 + b**2
*x**4)**p/(8*b**2*p**2 + 20*b**2*p + 12*b**2), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.57

$$\int x^3(a+bx^2)\left(a^2+2abx^2+b^2x^4\right)^p\,dx$$
$$=\frac{(b^2(2p+1)x^4+2abpx^2-a^2)(bx^2+a)^{2p}a}{4(2p^2+3p+1)b^2}+\frac{((2p^2+3p+1)b^3x^6+(2p^2+p)ab^2x^4-2a^2bpx^2+a^3)(bx^2+a)^{2p}}{2(4p^3+12p^2+11p+3)b^2}$$

```
[In] integrate(x^3*(b*x^2+a)*(b^2*x^4+2*a*b*x^2+a^2)^p,x, algorithm="maxima")
```

```
[Out] 1/4*(b^2*(2*p + 1)*x^4 + 2*a*b*p*x^2 - a^2)*(b*x^2 + a)^(2*p)*a/((2*p^2 + 3
*p + 1)*b^2) + 1/2*((2*p^2 + 3*p + 1)*b^3*x^6 + (2*p^2 + p)*a*b^2*x^4 - 2*a
^2*b*p*x^2 + a^3)*(b*x^2 + a)^(2*p)/((4*p^3 + 12*p^2 + 11*p + 3)*b^2)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 196 vs. 2(82) = 164.

Time = 0.30 (sec) , antiderivative size = 196, normalized size of antiderivative = 2.28

$$\int x^3(a+bx^2)\left(a^2+2abx^2+b^2x^4\right)^p\,dx$$
$$=\frac{2\,(b^2x^4+2\,abx^2+a^2)^pb^3px^6+2\,(b^2x^4+2\,abx^2+a^2)^pb^3x^6+4\,(b^2x^4+2\,abx^2+a^2)^pab^2px^4+3\,(b^2x^4+2\,a}{4\,(2\,b^2p^2+5\,b^2p+3\,b^2)}$$

```
[In] integrate(x^3*(b*x^2+a)*(b^2*x^4+2*a*b*x^2+a^2)^p,x, algorithm="giac")
```

```
[Out] 1/4*(2*(b^2*x^4 + 2*a*b*x^2 + a^2)^p*b^3*p*x^6 + 2*(b^2*x^4 + 2*a*b*x^2 + a
^2)^p*b^3*x^6 + 4*(b^2*x^4 + 2*a*b*x^2 + a^2)^p*a*b^2*p*x^4 + 3*(b^2*x^4 +
2*a*b*x^2 + a^2)^p*a*b^2*x^4 + 2*(b^2*x^4 + 2*a*b*x^2 + a^2)^p*a^2*b*p*x^2
- (b^2*x^4 + 2*a*b*x^2 + a^2)^p*a^3)/(2*b^2*p^2 + 5*b^2*p + 3*b^2)
```

Mupad [B] (verification not implemented)

Time = 7.93 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.26

$$\int x^3(a+bx^2)\left(a^2+2abx^2+b^2x^4\right)^p\,dx=\left(a^2+2\,a\,b\,x^2+b^2\,x^4\right)^p\left(\frac{b\,x^6\,(p+1)}{2\,\left(2\,p^2+5\,p+3\right)}-\frac{a^3}{4\,b^2\,\left(2\,p^2+5\,p+3\right)}+\frac{a\,x^4\,(4\,p+3)}{4\,\left(2\,p^2+5\,p+3\right)}+\frac{a^2\,p\,x^2}{2\,b\,\left(2\,p^2+5\,p+3\right)}\right)$$

```
[In] int(x^3*(a + b*x^2)*(a^2 + b^2*x^4 + 2*a*b*x^2)^p,x)
```

```
[Out] (a^2 + b^2*x^4 + 2*a*b*x^2)^p*((b*x^6*(p + 1))/(2*(5*p + 2*p^2 + 3)) - a^3/
(4*b^2*(5*p + 2*p^2 + 3)) + (a*x^4*(4*p + 3))/(4*(5*p + 2*p^2 + 3)) + (a^2*
p*x^2)/(2*b*(5*p + 2*p^2 + 3)))
```

3.94 $\int x^5(a+bx^2)\left(a^2+2abx^2+b^2x^4\right)^p\,dx$

Optimal result

Integrand size = 31, antiderivative size = 128

$$\int x^5(a+bx^2)\left(a^2+2abx^2+b^2x^4\right)^p\,dx=\frac{a^2(a+bx^2)^2\left(a^2+2abx^2+b^2x^4\right)^p}{4b^3(1+p)}-\frac{a(a+bx^2)^3\left(a^2+2abx^2+b^2x^4\right)^p}{b^3(3+2p)}+\frac{(a+bx^2)^4\left(a^2+2abx^2+b^2x^4\right)^p}{4b^3(2+p)}$$

```
[Out] 1/4*a^2*(b*x^2+a)^2*(b^2*x^4+2*a*b*x^2+a^2)^p/b^3/(p+1)-a*(b*x^2+a)^3*(b^2*
x^4+2*a*b*x^2+a^2)^p/b^3/(3+2*p)+1/4*(b*x^2+a)^4*(b^2*x^4+2*a*b*x^2+a^2)^p/
b^3/(2+p)
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.129, Rules used = {1263, 784, 21, 45}

$$\int x^5(a+bx^2)\left(a^2+2abx^2+b^2x^4\right)^p\,dx=\frac{(a+bx^2)^4\left(a^2+2abx^2+b^2x^4\right)^p}{4b^3(p+2)}-\frac{a(a+bx^2)^3\left(a^2+2abx^2+b^2x^4\right)^p}{b^3(2p+3)}+\frac{a^2(a+bx^2)^2\left(a^2+2abx^2+b^2x^4\right)^p}{4b^3(p+1)}$$

```
[In] Int[x^5*(a + b*x^2)*(a^2 + 2*a*b*x^2 + b^2*x^4)^p,x]
```

```
[Out] (a^2*(a + b*x^2)^2*(a^2 + 2*a*b*x^2 + b^2*x^4)^p)/(4*b^3*(1 + p)) - (a*(a +
 b*x^2)^3*(a^2 + 2*a*b*x^2 + b^2*x^4)^p)/(b^3*(3 + 2*p)) + ((a + b*x^2)^4*(
a^2 + 2*a*b*x^2 + b^2*x^4)^p)/(4*b^3*(2 + p))
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 784

```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (
c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/(c^IntPa
rt[p]*(b/2 + c*x)^(2*FracPart[p])), Int[(d + e*x)^m*(f + g*x)*(b/2 + c*x)^(
2*p), x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && EqQ[b^2 - 4*a*c, 0]
```

Rule 1263

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && EqQ[b
^2 - 4*a*c, 0] &&  !IntegerQ[p] && IGtQ[(m + 1)/2, 0]
```

Rubi steps

$$\text{integral} = \frac{1}{2}\text{Subst}\left(\int x^2(a+bx)\left(a^2+2abx+b^2x^2\right)^p\,dx,x,x^2\right)$$

$$= \frac{1}{2}\left(\left(b\left(a+bx^2\right)\right)^{-2p}\left(a^2+2abx^2+b^2x^4\right)^p\right)\text{Subst}\left(\int x^2(a+bx)\left(ab+b^2x\right)^{2p}\,dx,x,x^2\right)$$

$$= \frac{\left(\left(b(a+bx^2)\right)^{-2p}\left(a^2+2abx^2+b^2x^4\right)^p\right)\text{Subst}\left(\int x^2\left(ab+b^2x\right)^{1+2p}\,dx,x,x^2\right)}{2b}$$

$$= \frac{\left(\left(b(a+bx^2)\right)^{-2p}\left(a^2+2abx^2+b^2x^4\right)^p\right)\text{Subst}\left(\int\left(\frac{a^2\left(ab+b^2x\right)^{1+2p}}{b^2}-\frac{2a\left(ab+b^2x\right)^{2+2p}}{b^3}+\frac{\left(ab+b^2x\right)^{3+2p}}{b^4}\right)dx\right.}{2b}$$

$$= \frac{a^2(a+bx^2)^2 (a^2+2abx^2+b^2x^4)^p}{4b^3(1+p)} - \frac{a(a+bx^2)^3 (a^2+2abx^2+b^2x^4)^p}{b^3(3+2p)} + \frac{(a+bx^2)^4 (a^2+2abx^2+b^2x^4)^p}{4b^3(2+p)}$$

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.53

$$\int x^5 (a+bx^2) (a^2+2abx^2+b^2x^4)^p \, dx = \frac{\left((a+bx^2)^2\right)^{1+p} (a^2-2ab(1+p)x^2+b^2(3+5p+2p^2)\, x^4)}{4b^3(1+p)(2+p)(3+2p)}$$

```
[In] Integrate[x^5*(a + b*x^2)*(a^2 + 2*a*b*x^2 + b^2*x^4)^p,x]
```

```
[Out] (((a + b*x^2)^2)^(1 + p)*(a^2 - 2*a*b*(1 + p)*x^2 + b^2*(3 + 5*p + 2*p^2)*x
^4))/(4*b^3*(1 + p)*(2 + p)*(3 + 2*p))
```

Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.77

method	result
gosper	$\frac{(b\,x^2+a)^2 (2b^2p^2x^4+5b^2p\,x^4+3b^2x^4-2abp\,x^2-2ab\,x^2+a^2)(b^2x^4+2ab\,x^2+a^2)^p}{4b^3(2p^3+9p^2+13p+6)}$
risch	$\frac{(2b^4p^2x^8+5b^4p\,x^8+4a\,b^3p^2x^6+3b^4x^8+8a\,b^3p\,x^6+2a^2b^2p^2x^4+4a\,b^3x^6+a^2b^2p\,x^4-2a^3p\,x^2b+a^4)\left((b\,x^2+a)^2\right)^p}{4(3+2p)(2+p)(1+p)b^3}$
norman	$\frac{a(1+p)x^6 e^{p\ln\left(b^2x^4+2ab\,x^2+a^2\right)}}{2p^2+7p+6} + \frac{a^4 e^{p\ln\left(b^2x^4+2ab\,x^2+a^2\right)}}{4b^3(2p^3+9p^2+13p+6)} + \frac{b\,x^8 e^{p\ln\left(b^2x^4+2ab\,x^2+a^2\right)}}{8+4p} - \frac{p\,a^3x^2 e^{p\ln\left(b^2x^4+2ab\,x^2+a^2\right)}}{2b^2(2p^3+9p^2+13p+6)} +$
parallelrisch	$\frac{2x^8(b^2x^4+2ab\,x^2+a^2)^p a\,b^4p^2+5x^8(b^2x^4+2ab\,x^2+a^2)^p a\,b^4p+3x^8(b^2x^4+2ab\,x^2+a^2)^p a\,b^4+4x^6(b^2x^4+2ab\,x^2+a^2)^p a^2b^3p^2+8a}{}$

```
[In] int(x^5*(b*x^2+a)*(b^2*x^4+2*a*b*x^2+a^2)^p,x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*(b*x^2+a)^2*(2*b^2*p^2*x^4+5*b^2*p*x^4+3*b^2*x^4-2*a*b*p*x^2-2*a*b*x^2+
a^2)*(b^2*x^4+2*a*b*x^2+a^2)^p/b^3/(2*p^3+9*p^2+13*p+6)
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.09

$$\int x^5(a+bx^2)\left(a^2+2abx^2+b^2x^4\right)^p\,dx = \frac{((2\,b^4p^2+5\,b^4p+3\,b^4)x^8-2\,a^3bpx^2+4\,(ab^3p^2+2\,ab^3p+ab^3)x^6+(2\,a^2b^2p^2+a^2b^2p)x^4+a^4)(b^2x^4+2\,a}{4\,(2\,b^3p^3+9\,b^3p^2+13\,b^3p+6\,b^3)}$$

```
[In] integrate(x^5*(b*x^2+a)*(b^2*x^4+2*a*b*x^2+a^2)^p,x, algorithm="fricas")
```

```
[Out] 1/4*((2*b^4*p^2 + 5*b^4*p + 3*b^4)*x^8 - 2*a^3*b*p*x^2 + 4*(a*b^3*p^2 + 2*a
*b^3*p + a*b^3)*x^6 + (2*a^2*b^2*p^2 + a^2*b^2*p)*x^4 + a^4)*(b^2*x^4 + 2*a
*b*x^2 + a^2)^p/(2*b^3*p^3 + 9*b^3*p^2 + 13*b^3*p + 6*b^3)
```

Sympy [F]

$$\int x^5(a+bx^2)\left(a^2+2abx^2+b^2x^4\right)^p\,dx = \begin{cases} \frac{ax^6(a^2)^p}{6} \\ \frac{2a^2\log\left(x-\sqrt{-\frac{a}{b}}\right)}{4a^2b^3+8ab^4x^2+4b^5x^4}+\frac{2a^2\log\left(x+\sqrt{-\frac{a}{b}}\right)}{4a^2b^3+8ab^4x^2+4b^5x^4}+\frac{3a^2}{4a^2b^3+8ab^4x^2+4b^5x^4}+\frac{4abx^2\log\left(x-\sqrt{-\frac{a}{b}}\right)}{4a^2b^3+8ab^4x^2+4b^5x^4}+\frac{4abx^2\log\left(x+\sqrt{-\frac{a}{b}}\right)}{4a^2b^3+8ab^4x^2+4b^5x^4}+\frac{}{4a^2b^3+} \\ \int\frac{x^5(a+bx^2)}{\left((a+bx^2)^2\right)^{\frac{3}{2}}}\,dx \\ \frac{a^2\log\left(x-\sqrt{-\frac{a}{b}}\right)}{2b^3}+\frac{a^2\log\left(x+\sqrt{-\frac{a}{b}}\right)}{2b^3}-\frac{ax^2}{2b^2}+\frac{x^4}{4b} \\ \frac{a^4(a^2+2abx^2+b^2x^4)^p}{8b^3p^3+36b^3p^2+52b^3p+24b^3}-\frac{2a^3bpx^2(a^2+2abx^2+b^2x^4)^p}{8b^3p^3+36b^3p^2+52b^3p+24b^3}+\frac{2a^2b^2p^2x^4(a^2+2abx^2+b^2x^4)^p}{8b^3p^3+36b^3p^2+52b^3p+24b^3}+\frac{a^2b^2px^4(a^2+2abx^2+b^2x^4)^p}{8b^3p^3+36b^3p^2+52b^3p+24b^3}+\frac{4ab^3p^2a}{8b^3p^3+} \end{cases}$$

```
[In] integrate(x**5*(b*x**2+a)*(b**2*x**4+2*a*b*x**2+a**2)**p,x)
```

```
[Out] Piecewise((a*x**6*(a**2)**p/6, Eq(b, 0)), (2*a**2*log(x - sqrt(-a/b))/(4*a*
*2*b**3 + 8*a*b**4*x**2 + 4*b**5*x**4) + 2*a**2*log(x + sqrt(-a/b))/(4*a**2
*b**3 + 8*a*b**4*x**2 + 4*b**5*x**4) + 3*a**2/(4*a**2*b**3 + 8*a*b**4*x**2
+ 4*b**5*x**4) + 4*a*b*x**2*log(x - sqrt(-a/b))/(4*a**2*b**3 + 8*a*b**4*x**
2 + 4*b**5*x**4) + 4*a*b*x**2*log(x + sqrt(-a/b))/(4*a**2*b**3 + 8*a*b**4*x
**2 + 4*b**5*x**4) + 4*a*b*x**2/(4*a**2*b**3 + 8*a*b**4*x**2 + 4*b**5*x**4)
 + 2*b**2*x**4*log(x - sqrt(-a/b))/(4*a**2*b**3 + 8*a*b**4*x**2 + 4*b**5*x*
*4) + 2*b**2*x**4*log(x + sqrt(-a/b))/(4*a**2*b**3 + 8*a*b**4*x**2 + 4*b**5
*x**4), Eq(p, -2)), (Integral(x**5*(a + b*x**2)/((a + b*x**2)**2)**(3/2), x
), Eq(p, -3/2)), (a**2*log(x - sqrt(-a/b))/(2*b**3) + a**2*log(x + sqrt(-a/
b))/(2*b**3) - a*x**2/(2*b**2) + x**4/(4*b), Eq(p, -1)), (a**4*(a**2 + 2*a*
```

```
b*x**2 + b**2*x**4)**p/(8*b**3*p**3 + 36*b**3*p**2 + 52*b**3*p + 24*b**3) -
 2*a**3*b*p*x**2*(a**2 + 2*a*b*x**2 + b**2*x**4)**p/(8*b**3*p**3 + 36*b**3*
p**2 + 52*b**3*p + 24*b**3) + 2*a**2*b**2*p**2*x**4*(a**2 + 2*a*b*x**2 + b*
*2*x**4)**p/(8*b**3*p**3 + 36*b**3*p**2 + 52*b**3*p + 24*b**3) + a**2*b**2*
p*x**4*(a**2 + 2*a*b*x**2 + b**2*x**4)**p/(8*b**3*p**3 + 36*b**3*p**2 + 52*
b**3*p + 24*b**3) + 4*a*b**3*p**2*x**6*(a**2 + 2*a*b*x**2 + b**2*x**4)**p/(
8*b**3*p**3 + 36*b**3*p**2 + 52*b**3*p + 24*b**3) + 8*a*b**3*p*x**6*(a**2 +
 2*a*b*x**2 + b**2*x**4)**p/(8*b**3*p**3 + 36*b**3*p**2 + 52*b**3*p + 24*b*
*3) + 4*a*b**3*x**6*(a**2 + 2*a*b*x**2 + b**2*x**4)**p/(8*b**3*p**3 + 36*b*
*3*p**2 + 52*b**3*p + 24*b**3) + 2*b**4*p**2*x**8*(a**2 + 2*a*b*x**2 + b**2
*x**4)**p/(8*b**3*p**3 + 36*b**3*p**2 + 52*b**3*p + 24*b**3) + 5*b**4*p*x**
8*(a**2 + 2*a*b*x**2 + b**2*x**4)**p/(8*b**3*p**3 + 36*b**3*p**2 + 52*b**3*
p + 24*b**3) + 3*b**4*x**8*(a**2 + 2*a*b*x**2 + b**2*x**4)**p/(8*b**3*p**3
+ 36*b**3*p**2 + 52*b**3*p + 24*b**3), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 196, normalized size of antiderivative = 1.53

$$\int x^5 (a+bx^2) (a^2+2abx^2+b^2x^4)^p \, dx$$
$$= \frac{((2\,p^2+3\,p+1)b^3x^6+(2\,p^2+p)ab^2x^4-2\,a^2bpx^2+a^3)(bx^2+a)^{2\,p}a}{2\,(4\,p^3+12\,p^2+11\,p+3)b^3}$$
$$+ \frac{((4\,p^3+12\,p^2+11\,p+3)b^4x^8+2\,(2\,p^3+3\,p^2+p)ab^3x^6-3\,(2\,p^2+p)a^2b^2x^4+6\,a^3bpx^2-3\,a^4)(bx^2+a)}{4\,(4\,p^4+20\,p^3+35\,p^2+25\,p+6)b^3}$$

```
[In] integrate(x^5*(b*x^2+a)*(b^2*x^4+2*a*b*x^2+a^2)^p,x, algorithm="maxima")
```

```
[Out] 1/2*((2*p^2 + 3*p + 1)*b^3*x^6 + (2*p^2 + p)*a*b^2*x^4 - 2*a^2*b*p*x^2 + a^
3)*(b*x^2 + a)^(2*p)*a/((4*p^3 + 12*p^2 + 11*p + 3)*b^3) + 1/4*((4*p^3 + 12
*p^2 + 11*p + 3)*b^4*x^8 + 2*(2*p^3 + 3*p^2 + p)*a*b^3*x^6 - 3*(2*p^2 + p)*
a^2*b^2*x^4 + 6*a^3*b*p*x^2 - 3*a^4)*(b*x^2 + a)^(2*p)/((4*p^4 + 20*p^3 + 3
5*p^2 + 25*p + 6)*b^3)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 331 vs. $2(124) = 248$.

Time = 0.28 (sec) , antiderivative size = 331, normalized size of antiderivative = 2.59

$$\int x^5(a+bx^2)\left(a^2+2abx^2+b^2x^4\right)^p\,dx$$
$$= \frac{2\left(b^2x^4+2\,abx^2+a^2\right)^pb^4p^2x^8+5\left(b^2x^4+2\,abx^2+a^2\right)^pb^4px^8+4\left(b^2x^4+2\,abx^2+a^2\right)^pab^3p^2x^6+3\left(b^2x^4+\right.}{}$$

```
[In] integrate(x^5*(b*x^2+a)*(b^2*x^4+2*a*b*x^2+a^2)^p,x, algorithm="giac")
```

```
[Out] 1/4*(2*(b^2*x^4 + 2*a*b*x^2 + a^2)^p*b^4*p^2*x^8 + 5*(b^2*x^4 + 2*a*b*x^2 +
 a^2)^p*b^4*p*x^8 + 4*(b^2*x^4 + 2*a*b*x^2 + a^2)^p*a*b^3*p^2*x^6 + 3*(b^2*
x^4 + 2*a*b*x^2 + a^2)^p*b^4*x^8 + 8*(b^2*x^4 + 2*a*b*x^2 + a^2)^p*a*b^3*p*
x^6 + 2*(b^2*x^4 + 2*a*b*x^2 + a^2)^p*a^2*b^2*p^2*x^4 + 4*(b^2*x^4 + 2*a*b*
x^2 + a^2)^p*a*b^3*x^6 + (b^2*x^4 + 2*a*b*x^2 + a^2)^p*a^2*b^2*p*x^4 - 2*(b
^2*x^4 + 2*a*b*x^2 + a^2)^p*a^3*b*p*x^2 + (b^2*x^4 + 2*a*b*x^2 + a^2)^p*a^4
)/(2*b^3*p^3 + 9*b^3*p^2 + 13*b^3*p + 6*b^3)
```

Mupad [B] (verification not implemented)

Time = 7.97 (sec) , antiderivative size = 169, normalized size of antiderivative = 1.32

$$\int x^5(a+bx^2)\left(a^2+2abx^2+b^2x^4\right)^p\,dx$$
$$= \left(a^2+2\,a\,b\,x^2+b^2\,x^4\right)^p\left(\frac{a^4}{4\,b^3\,\left(2\,p^3+9\,p^2+13\,p+6\right)}+\frac{a\,x^6\,{\left(p+1\right)}^2}{2\,p^3+9\,p^2+13\,p+6}+\frac{b\,x^8\,\left(2\,p^2+5\,p+3\right)}{4\,\left(2\,p^3+9\,p^2+13\,p+6\right)}-\frac{a^3\,p\,x^2}{2\,b^2\,\left(2\,p^3+9\,p^2+13\,p+6\right)}+\frac{a^2\,p\,x^4\,\left(2\,p+1\right)}{4\,b\,\left(2\,p^3+9\,p^2+13\,p+6\right)}\right)$$

```
[In] int(x^5*(a + b*x^2)*(a^2 + b^2*x^4 + 2*a*b*x^2)^p,x)
```

```
[Out] (a^2 + b^2*x^4 + 2*a*b*x^2)^p*(a^4/(4*b^3*(13*p + 9*p^2 + 2*p^3 + 6)) + (a*
x^6*(p + 1)^2)/(13*p + 9*p^2 + 2*p^3 + 6) + (b*x^8*(5*p + 2*p^2 + 3))/(4*(1
3*p + 9*p^2 + 2*p^3 + 6)) - (a^3*p*x^2)/(2*b^2*(13*p + 9*p^2 + 2*p^3 + 6))
+ (a^2*p*x^4*(2*p + 1))/(4*b*(13*p + 9*p^2 + 2*p^3 + 6)))
```

3.95 $\int x^3(A+Bx^2)\left(a+bx^2+cx^4\right)^3\,dx$

Optimal result

Integrand size = 25, antiderivative size = 166

$$\int x^3\left(A+Bx^2\right)\left(a+bx^2+cx^4\right)^3\,dx=\frac{1}{4}a^3Ax^4+\frac{1}{6}a^2(3Ab+aB)x^6+\frac{3}{8}a\left(abB+A\left(b^2+ac\right)\right)x^8+\frac{1}{10}\left(3aB\left(b^2+ac\right)+A\left(b^3+6abc\right)\right)x^{10}+\frac{1}{12}\left(b^3B+3Ab^2c+6abBc+3aAc^2\right)x^{12}+\frac{3}{14}c\left(b^2B+Abc+aBc\right)x^{14}+\frac{1}{16}c^2(3bB+Ac)x^{16}+\frac{1}{18}Bc^3x^{18}$$

```
[Out] 1/4*a^3*A*x^4+1/6*a^2*(3*A*b+B*a)*x^6+3/8*a*(a*b*B+A*(a*c+b^2))*x^8+1/10*(3
*a*B*(a*c+b^2)+A*(6*a*b*c+b^3))*x^10+1/12*(3*A*a*c^2+3*A*b^2*c+6*B*a*b*c+B*
b^3)*x^12+3/14*c*(A*b*c+B*a*c+B*b^2)*x^14+1/16*c^2*(A*c+3*B*b)*x^16+1/18*B*
c^3*x^18
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 166, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.080, Rules used

$= \{1265, 779\}$

$$\int x^3 (A+Bx^2) \left(a+bx^2+cx^4\right)^3 \, dx = \frac{1}{4}a^3Ax^4+\frac{1}{6}a^2x^6(aB+3Ab)+\frac{3}{14}cx^{14}\left(aBc+Abc+b^2B\right) + \frac{3}{8}ax^8\left(A\left(ac+b^2\right)+abB\right) + \frac{1}{12}x^{12}\left(3aAc^2+6abBc+3Ab^2c+b^3B\right) + \frac{1}{10}x^{10}\left(A\left(6abc+b^3\right)+3aB\left(ac+b^2\right)\right) + \frac{1}{16}c^2x^{16}(Ac+3bB)+\frac{1}{18}Bc^3x^{18}$$

```
[In] Int[x^3*(A + B*x^2)*(a + b*x^2 + c*x^4)^3,x]
```

```
[Out] (a^3*A*x^4)/4 + (a^2*(3*A*b + a*B)*x^6)/6 + (3*a*(a*b*B + A*(b^2 + a*c))*x^
8)/8 + ((3*a*B*(b^2 + a*c) + A*(b^3 + 6*a*b*c))*x^10)/10 + ((b^3*B + 3*A*b^
2*c + 6*a*b*B*c + 3*a*A*c^2)*x^12)/12 + (3*c*(b^2*B + A*b*c + a*B*c)*x^14)/
14 + (c^2*(3*b*B + A*c)*x^16)/16 + (B*c^3*x^18)/18
```

Rule 779

```
Int[((e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_
)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(f + g*x)*(a + b*x + c
*x^2)^p, x], x] /; FreeQ[{a, b, c, e, f, g, m}, x] && IntegerQ[p] && (GtQ[p
, 0] || (EqQ[a, 0] && IntegerQ[m]))
```

Rule 1265

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rubi steps

$$\text{integral} = \frac{1}{2}\text{Subst}\left(\int x(A+Bx)\left(a+bx+cx^2\right)^3 \, dx, x, x^2\right)$$

$$= \frac{1}{2}\text{Subst}\left(\int \left(a^3Ax+a^2(3Ab+aB)x^2+3a\left(abB+A\left(b^2+ac\right)\right)x^3 + \left(3aB\left(b^2+ac\right)+A\left(b^3+6abc\right)\right)x^4+\left(b^3B+3Ab^2c+6abBc+3aAc^2\right)x^5 + 3c\left(b^2B+Abc+aBc\right)x^6+c^2(3bB+Ac)x^7+Bc^3x^8\right) dx, x, x^2\right)$$

$$= \frac{1}{4}a^3Ax^4 + \frac{1}{6}a^2(3Ab + aB)x^6 + \frac{3}{8}a\left(abB + A\left(b^2 + ac\right)\right)x^8 + \frac{1}{10}\left(3aB\left(b^2 + ac\right) + A\left(b^3 + 6abc\right)\right)x^{10} + \frac{1}{12}\left(b^3B + 3Ab^2c + 6abBc + 3aAc^2\right)x^{12} + \frac{3}{14}c\left(b^2B + Abc + aBc\right)x^{14} + \frac{1}{16}c^2(3bB + Ac)x^{16} + \frac{1}{18}Bc^3x^{18}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 166, normalized size of antiderivative = 1.00

$$\int x^3\left(A+Bx^2\right)\left(a+bx^2+cx^4\right)^3\,dx = \frac{1}{4}a^3Ax^4 + \frac{1}{6}a^2(3Ab+aB)x^6 + \frac{3}{8}a\left(abB+A\left(b^2+ac\right)\right)x^8 + \frac{1}{10}\left(3aB\left(b^2+ac\right)+A\left(b^3+6abc\right)\right)x^{10} + \frac{1}{12}\left(b^3B+3Ab^2c+6abBc+3aAc^2\right)x^{12} + \frac{3}{14}c\left(b^2B+Abc+aBc\right)x^{14} + \frac{1}{16}c^2(3bB+Ac)x^{16} + \frac{1}{18}Bc^3x^{18}$$

```
[In] Integrate[x^3*(A + B*x^2)*(a + b*x^2 + c*x^4)^3,x]
```

```
[Out] (a^3*A*x^4)/4 + (a^2*(3*A*b + a*B)*x^6)/6 + (3*a*(a*b*B + A*(b^2 + a*c))*x^
8)/8 + ((3*a*B*(b^2 + a*c) + A*(b^3 + 6*a*b*c))*x^10)/10 + ((b^3*B + 3*A*b^
2*c + 6*a*b*B*c + 3*a*A*c^2)*x^12)/12 + (3*c*(b^2*B + A*b*c + a*B*c)*x^14)/
14 + (c^2*(3*b*B + A*c)*x^16)/16 + (B*c^3*x^18)/18
```

Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.02

method	result
norman	$\frac{a^3Ax^4}{4} + \left(\frac{1}{2}Aa^2b + \frac{1}{6}Ba^3\right)x^6 + \left(\frac{3}{8}Aca^2 + \frac{3}{8}Aab^2 + \frac{3}{8}Ba^2b\right)x^8 + \left(\frac{3}{5}Aabc + \frac{1}{10}Ab^3 + \frac{3}{10}a^2Bc\right.$
gosper	$\frac{1}{4}a^3Ax^4 + \frac{1}{2}x^6Aa^2b + \frac{1}{6}x^6Ba^3 + \frac{3}{8}x^8Aca^2 + \frac{3}{8}x^8Aab^2 + \frac{3}{8}x^8Ba^2b + \frac{3}{5}x^{10}Aabc + \frac{1}{10}x^{10}Ab^3$
risch	$\frac{1}{4}a^3Ax^4 + \frac{1}{2}x^6Aa^2b + \frac{1}{6}x^6Ba^3 + \frac{3}{8}x^8Aca^2 + \frac{3}{8}x^8Aab^2 + \frac{3}{8}x^8Ba^2b + \frac{3}{5}x^{10}Aabc + \frac{1}{10}x^{10}Ab^3$
parallelrisch	$\frac{1}{4}a^3Ax^4 + \frac{1}{2}x^6Aa^2b + \frac{1}{6}x^6Ba^3 + \frac{3}{8}x^8Aca^2 + \frac{3}{8}x^8Aab^2 + \frac{3}{8}x^8Ba^2b + \frac{3}{5}x^{10}Aabc + \frac{1}{10}x^{10}Ab^3$
default	$\frac{Bc^3x^{18}}{18} + \frac{\left(Ac^3+3Bbc^2\right)x^{16}}{16} + \frac{\left(3Abc^2+B\left(ac^2+2b^2c+c\left(2ac+b^2\right)\right)\right)x^{14}}{14} + \frac{\left(A\left(ac^2+2b^2c+c\left(2ac+b^2\right)\right)+B\left(4abc+b\left(2ac+b\right.\right.\right.}{12}$

```
[In] int(x^3*(B*x^2+A)*(c*x^4+b*x^2+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*a^3*A*x^4+(1/2*A*a^2*b+1/6*B*a^3)*x^6+(3/8*A*c*a^2+3/8*A*a*b^2+3/8*B*a^
2*b)*x^8+(3/5*A*a*b*c+1/10*A*b^3+3/10*a^2*B*c+3/10*B*a*b^2)*x^10+(1/4*A*a*c
```

```
^2+1/4*A*b^2*c+1/2*B*a*b*c+1/12*B*b^3)*x^12+(3/14*A*b*c^2+3/14*B*a*c^2+3/14
*B*b^2*c)*x^14+(1/16*A*c^3+3/16*B*b*c^2)*x^16+1/18*B*c^3*x^18
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 166, normalized size of antiderivative = 1.00

$$\int x^3(A+Bx^2)\left(a+bx^2+cx^4\right)^3\,dx = \frac{1}{18}\,Bc^3x^{18} + \frac{1}{16}\,(3\,Bbc^2+Ac^3)x^{16} + \frac{3}{14}\,(Bb^2c+(Ba+Ab)c^2)x^{14} + \frac{1}{12}\,(Bb^3+3\,Aac^2+3\,(2\,Bab+Ab^2)c)x^{12} + \frac{1}{10}\,(3\,Bab^2+Ab^3+3\,(Ba^2+2\,Aab)c)x^{10} + \frac{3}{8}\,(Ba^2b+Aab^2+Aa^2c)x^8 + \frac{1}{4}\,Aa^3x^4 + \frac{1}{6}\,(Ba^3+3\,Aa^2b)x^6$$

```
[In] integrate(x^3*(B*x^2+A)*(c*x^4+b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] 1/18*B*c^3*x^18 + 1/16*(3*B*b*c^2 + A*c^3)*x^16 + 3/14*(B*b^2*c + (B*a + A*
b)*c^2)*x^14 + 1/12*(B*b^3 + 3*A*a*c^2 + 3*(2*B*a*b + A*b^2)*c)*x^12 + 1/10
*(3*B*a*b^2 + A*b^3 + 3*(B*a^2 + 2*A*a*b)*c)*x^10 + 3/8*(B*a^2*b + A*a*b^2
+ A*a^2*c)*x^8 + 1/4*A*a^3*x^4 + 1/6*(B*a^3 + 3*A*a^2*b)*x^6
```

Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 202, normalized size of antiderivative = 1.22

$$\int x^3(A+Bx^2)\left(a+bx^2+cx^4\right)^3\,dx = \frac{Aa^3x^4}{4} + \frac{Bc^3x^{18}}{18} + x^{16}\left(\frac{Ac^3}{16}+\frac{3Bbc^2}{16}\right) + x^{14}\cdot\left(\frac{3Abc^2}{14}+\frac{3Bac^2}{14}+\frac{3Bb^2c}{14}\right) + x^{12}\left(\frac{Aac^2}{4}+\frac{Ab^2c}{4}+\frac{Babc}{2}+\frac{Bb^3}{12}\right) + x^{10}\cdot\left(\frac{3Aabc}{5}+\frac{Ab^3}{10}+\frac{3Ba^2c}{10}+\frac{3Bab^2}{10}\right) + x^8\cdot\left(\frac{3Aa^2c}{8}+\frac{3Aab^2}{8}+\frac{3Ba^2b}{8}\right) + x^6\left(\frac{Aa^2b}{2}+\frac{Ba^3}{6}\right)$$

```
[In] integrate(x**3*(B*x**2+A)*(c*x**4+b*x**2+a)**3,x)
```

```
[Out] A*a**3*x**4/4 + B*c**3*x**18/18 + x**16*(A*c**3/16 + 3*B*b*c**2/16) + x**14
*(3*A*b*c**2/14 + 3*B*a*c**2/14 + 3*B*b**2*c/14) + x**12*(A*a*c**2/4 + A*b*
*2*c/4 + B*a*b*c/2 + B*b**3/12) + x**10*(3*A*a*b*c/5 + A*b**3/10 + 3*B*a**2
*c/10 + 3*B*a*b**2/10) + x**8*(3*A*a**2*c/8 + 3*A*a*b**2/8 + 3*B*a**2*b/8)
+ x**6*(A*a**2*b/2 + B*a**3/6)
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 166, normalized size of antiderivative = 1.00

$$\int x^3(A+Bx^2)\left(a+bx^2+cx^4\right)^3\,dx = \frac{1}{18}\,Bc^3x^{18} + \frac{1}{16}\,(3\,Bbc^2+Ac^3)x^{16} + \frac{3}{14}\,(Bb^2c+(Ba+Ab)c^2)x^{14} + \frac{1}{12}\,\left(Bb^3+3\,Aac^2+3\,\left(2\,Bab+Ab^2\right)c\right)x^{12} + \frac{1}{10}\,\left(3\,Bab^2+Ab^3+3\,\left(Ba^2+2\,Aab\right)c\right)x^{10} + \frac{3}{8}\,(Ba^2b+Aab^2+Aa^2c)x^8 + \frac{1}{4}\,Aa^3x^4 + \frac{1}{6}\,(Ba^3+3\,Aa^2b)x^6$$

```
[In] integrate(x^3*(B*x^2+A)*(c*x^4+b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] 1/18*B*c^3*x^18 + 1/16*(3*B*b*c^2 + A*c^3)*x^16 + 3/14*(B*b^2*c + (B*a + A*
b)*c^2)*x^14 + 1/12*(B*b^3 + 3*A*a*c^2 + 3*(2*B*a*b + A*b^2)*c)*x^12 + 1/10
*(3*B*a*b^2 + A*b^3 + 3*(B*a^2 + 2*A*a*b)*c)*x^10 + 3/8*(B*a^2*b + A*a*b^2
+ A*a^2*c)*x^8 + 1/4*A*a^3*x^4 + 1/6*(B*a^3 + 3*A*a^2*b)*x^6
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 193, normalized size of antiderivative = 1.16

$$\int x^3(A+Bx^2)\left(a+bx^2+cx^4\right)^3\,dx = \frac{1}{18}\,Bc^3x^{18} + \frac{3}{16}\,Bbc^2x^{16} + \frac{1}{16}\,Ac^3x^{16} + \frac{3}{14}\,Bb^2cx^{14} + \frac{3}{14}\,Bac^2x^{14} + \frac{3}{14}\,Abc^2x^{14} + \frac{1}{12}\,Bb^3x^{12} + \frac{1}{2}\,Babcx^{12} + \frac{1}{4}\,Ab^2cx^{12} + \frac{1}{4}\,Aac^2x^{12} + \frac{3}{10}\,Bab^2x^{10} + \frac{1}{10}\,Ab^3x^{10} + \frac{3}{10}\,Ba^2cx^{10} + \frac{3}{5}\,Aabcx^{10} + \frac{3}{8}\,Ba^2bx^8 + \frac{3}{8}\,Aab^2x^8 + \frac{3}{8}\,Aa^2cx^8 + \frac{1}{6}\,Ba^3x^6 + \frac{1}{2}\,Aa^2bx^6 + \frac{1}{4}\,Aa^3x^4$$

```
[In] integrate(x^3*(B*x^2+A)*(c*x^4+b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] 1/18*B*c^3*x^18 + 3/16*B*b*c^2*x^16 + 1/16*A*c^3*x^16 + 3/14*B*b^2*c*x^14 +
 3/14*B*a*c^2*x^14 + 3/14*A*b*c^2*x^14 + 1/12*B*b^3*x^12 + 1/2*B*a*b*c*x^12
 + 1/4*A*b^2*c*x^12 + 1/4*A*a*c^2*x^12 + 3/10*B*a*b^2*x^10 + 1/10*A*b^3*x^1
0 + 3/10*B*a^2*c*x^10 + 3/5*A*a*b*c*x^10 + 3/8*B*a^2*b*x^8 + 3/8*A*a*b^2*x^
8 + 3/8*A*a^2*c*x^8 + 1/6*B*a^3*x^6 + 1/2*A*a^2*b*x^6 + 1/4*A*a^3*x^4
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 169, normalized size of antiderivative = 1.02

$$\int x^3(A+Bx^2)\left(a+bx^2+cx^4\right)^3\,dx = x^{10}\left(\frac{3\,B\,c\,a^2}{10}+\frac{3\,B\,a\,b^2}{10}+\frac{3\,A\,c\,a\,b}{5}+\frac{A\,b^3}{10}\right) + x^{12}\left(\frac{B\,b^3}{12}+\frac{A\,b^2\,c}{4}+\frac{B\,a\,b\,c}{2}+\frac{A\,a\,c^2}{4}\right) + x^6\left(\frac{B\,a^3}{6}+\frac{A\,b\,a^2}{2}\right)+x^{16}\left(\frac{A\,c^3}{16}+\frac{3\,B\,b\,c^2}{16}\right) + x^8\left(\frac{3\,B\,a^2\,b}{8}+\frac{3\,A\,c\,a^2}{8}+\frac{3\,A\,a\,b^2}{8}\right) + x^{14}\left(\frac{3\,B\,b^2\,c}{14}+\frac{3\,A\,b\,c^2}{14}+\frac{3\,B\,a\,c^2}{14}\right) + \frac{A\,a^3\,x^4}{4}+\frac{B\,c^3\,x^{18}}{18}$$

```
[In] int(x^3*(A + B*x^2)*(a + b*x^2 + c*x^4)^3,x)
```

```
[Out] x^10*((A*b^3)/10 + (3*B*a*b^2)/10 + (3*B*a^2*c)/10 + (3*A*a*b*c)/5) + x^12*
((B*b^3)/12 + (A*a*c^2)/4 + (A*b^2*c)/4 + (B*a*b*c)/2) + x^6*((B*a^3)/6 + (
A*a^2*b)/2) + x^16*((A*c^3)/16 + (3*B*b*c^2)/16) + x^8*((3*A*a*b^2)/8 + (3*
A*a^2*c)/8 + (3*B*a^2*b)/8) + x^14*((3*A*b*c^2)/14 + (3*B*a*c^2)/14 + (3*B*
b^2*c)/14) + (A*a^3*x^4)/4 + (B*c^3*x^18)/18
```

3.96 $\int x^2(A+Bx^2)\left(a+bx^2+cx^4\right)^3\,dx$

Optimal result

Integrand size = 25, antiderivative size = 166

$$\int x^2\left(A+Bx^2\right)\left(a+bx^2+cx^4\right)^3\,dx=\frac{1}{3}a^3Ax^3+\frac{1}{5}a^2(3Ab+aB)x^5+\frac{3}{7}a\left(abB+A\left(b^2+ac\right)\right)x^7+\frac{1}{9}\left(3aB\left(b^2+ac\right)+A\left(b^3+6abc\right)\right)x^9+\frac{1}{11}\left(b^3B+3Ab^2c+6abBc+3aAc^2\right)x^{11}+\frac{3}{13}c\left(b^2B+Abc+aBc\right)x^{13}+\frac{1}{15}c^2(3bB+Ac)x^{15}+\frac{1}{17}Bc^3x^{17}$$

```
[Out] 1/3*a^3*A*x^3+1/5*a^2*(3*A*b+B*a)*x^5+3/7*a*(a*b*B+A*(a*c+b^2))*x^7+1/9*(3*
a*B*(a*c+b^2)+A*(6*a*b*c+b^3))*x^9+1/11*(3*A*a*c^2+3*A*b^2*c+6*B*a*b*c+B*b^
3)*x^11+3/13*c*(A*b*c+B*a*c+B*b^2)*x^13+1/15*c^2*(A*c+3*B*b)*x^15+1/17*B*c^
3*x^17
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 166, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.040, Rules used

$= \{1275\}$

$$\int x^2\left(A+Bx^2\right)\left(a+bx^2+cx^4\right)^3\,dx=\frac{1}{3}a^3Ax^3+\frac{1}{5}a^2x^5(aB+3Ab)+\frac{3}{13}cx^{13}\left(aBc+Abc+b^2B\right)+\frac{3}{7}ax^7\left(A\left(ac+b^2\right)+abB\right)+\frac{1}{11}x^{11}\left(3aAc^2+6abBc+3Ab^2c+b^3B\right)+\frac{1}{9}x^9\left(A\left(6abc+b^3\right)+3aB\left(ac+b^2\right)\right)+\frac{1}{15}c^2x^{15}(Ac+3bB)+\frac{1}{17}Bc^3x^{17}$$

```
[In] Int[x^2*(A + B*x^2)*(a + b*x^2 + c*x^4)^3,x]
```

```
[Out] (a^3*A*x^3)/3 + (a^2*(3*A*b + a*B)*x^5)/5 + (3*a*(a*b*B + A*(b^2 + a*c))*x^
7)/7 + ((3*a*B*(b^2 + a*c) + A*(b^3 + 6*a*b*c))*x^9)/9 + ((b^3*B + 3*A*b^2*
c + 6*a*b*B*c + 3*a*A*c^2)*x^11)/11 + (3*c*(b^2*B + A*b*c + a*B*c)*x^13)/13
 + (c^2*(3*b*B + A*c)*x^15)/15 + (B*c^3*x^17)/17
```

Rule 1275

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*
(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && NeQ[
b^2 - 4*a*c, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rubi steps

$$\text{integral}=\int\left(a^3Ax^2+a^2(3Ab+aB)x^4+3a\left(abB+A\left(b^2+ac\right)\right)x^6+\left(3aB\left(b^2+ac\right)+A\left(b^3+6abc\right)\right)x^8+\left(b^3B+3Ab^2c+6abBc+3aAc^2\right)x^{10}+3c\left(b^2B+Abc+aBc\right)x^{12}+c^2(3bB+Ac)x^{14}+Bc^3x^{16}\right)dx$$

$$=\frac{1}{3}a^3Ax^3+\frac{1}{5}a^2(3Ab+aB)x^5+\frac{3}{7}a\left(abB+A\left(b^2+ac\right)\right)x^7+\frac{1}{9}\left(3aB\left(b^2+ac\right)+A\left(b^3+6abc\right)\right)x^9+\frac{1}{11}\left(b^3B+3Ab^2c+6abBc+3aAc^2\right)x^{11}+\frac{3}{13}c\left(b^2B+Abc+aBc\right)x^{13}+\frac{1}{15}c^2(3bB+Ac)x^{15}+\frac{1}{17}Bc^3x^{17}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 166, normalized size of antiderivative = 1.00

$$\int x^2(A+Bx^2)\left(a+bx^2+cx^4\right)^3\,dx=\frac{1}{3}a^3Ax^3+\frac{1}{5}a^2(3Ab+aB)x^5+\frac{3}{7}a\left(abB+A\left(b^2+ac\right)\right)x^7+\frac{1}{9}\left(3aB\left(b^2+ac\right)+A\left(b^3+6abc\right)\right)x^9+\frac{1}{11}\left(b^3B+3Ab^2c+6abBc+3aAc^2\right)x^{11}+\frac{3}{13}c\left(b^2B+Abc+aBc\right)x^{13}+\frac{1}{15}c^2(3bB+Ac)x^{15}+\frac{1}{17}Bc^3x^{17}$$

[In] Integrate[x^2*(A + B*x^2)*(a + b*x^2 + c*x^4)^3,x]

[Out] (a^3*A*x^3)/3 + (a^2*(3*A*b + a*B)*x^5)/5 + (3*a*(a*b*B + A*(b^2 + a*c))*x^7)/7 + ((3*a*B*(b^2 + a*c) + A*(b^3 + 6*a*b*c))*x^9)/9 + ((b^3*B + 3*A*b^2*c + 6*a*b*B*c + 3*a*A*c^2)*x^11)/11 + (3*c*(b^2*B + A*b*c + a*B*c)*x^13)/13 + (c^2*(3*b*B + A*c)*x^15)/15 + (B*c^3*x^17)/17

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.02

method	result
norman	$\frac{a^3Ax^3}{3}+\left(\frac{3}{5}Aa^2b+\frac{1}{5}Ba^3\right)x^5+\left(\frac{3}{7}Aca^2+\frac{3}{7}Aab^2+\frac{3}{7}Ba^2b\right)x^7+\left(\frac{2}{3}Aabc+\frac{1}{9}Ab^3+\frac{1}{3}a^2Bc+\right.$
gosper	$\frac{1}{3}a^3Ax^3+\frac{3}{5}x^5Aa^2b+\frac{1}{5}x^5Ba^3+\frac{3}{7}x^7Aca^2+\frac{3}{7}x^7Aab^2+\frac{3}{7}x^7Ba^2b+\frac{2}{3}x^9Aabc+\frac{1}{9}x^9Ab^3+$
risch	$\frac{1}{3}a^3Ax^3+\frac{3}{5}x^5Aa^2b+\frac{1}{5}x^5Ba^3+\frac{3}{7}x^7Aca^2+\frac{3}{7}x^7Aab^2+\frac{3}{7}x^7Ba^2b+\frac{2}{3}x^9Aabc+\frac{1}{9}x^9Ab^3+$
parallelrisch	$\frac{1}{3}a^3Ax^3+\frac{3}{5}x^5Aa^2b+\frac{1}{5}x^5Ba^3+\frac{3}{7}x^7Aca^2+\frac{3}{7}x^7Aab^2+\frac{3}{7}x^7Ba^2b+\frac{2}{3}x^9Aabc+\frac{1}{9}x^9Ab^3+$
default	$\frac{Bc^3x^{17}}{17}+\frac{(Ac^3+3Bbc^2)x^{15}}{15}+\frac{\left(3Abc^2+B\left(ac^2+2b^2c+c\left(2ac+b^2\right)\right)\right)x^{13}}{13}+\frac{\left(A\left(ac^2+2b^2c+c\left(2ac+b^2\right)\right)+B\left(4abc+b\left(2ac+b\right.\right.\right.}{11}$

[In] int(x^2*(B*x^2+A)*(c*x^4+b*x^2+a)^3,x,method=_RETURNVERBOSE)

[Out] 1/3*a^3*A*x^3+(3/5*A*a^2*b+1/5*B*a^3)*x^5+(3/7*A*c*a^2+3/7*A*a*b^2+3/7*B*a^2*b)*x^7+(2/3*A*a*b*c+1/9*A*b^3+1/3*a^2*B*c+1/3*B*a*b^2)*x^9+(3/11*A*a*c^2+3/11*A*b^2*c+6/11*B*a*b*c+1/11*B*b^3)*x^11+(3/13*A*b*c^2+3/13*B*a*c^2+3/13*B*b^2*c)*x^13+(1/15*A*c^3+1/5*B*b*c^2)*x^15+1/17*B*c^3*x^17

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 166, normalized size of antiderivative = 1.00

$$\int x^2(A+Bx^2)\left(a+bx^2+cx^4\right)^3\,dx = \frac{1}{17}\,Bc^3x^{17}+\frac{1}{15}\left(3\,Bbc^2+Ac^3\right)x^{15}+\frac{3}{13}\left(Bb^2c+(Ba+Ab)c^2\right)x^{13}+\frac{1}{11}\left(Bb^3+3\,Aac^2+3\left(2\,Bab+Ab^2\right)c\right)x^{11}+\frac{1}{9}\left(3\,Bab^2+Ab^3+3\left(Ba^2+2\,Aab\right)c\right)x^9+\frac{3}{7}\left(Ba^2b+Aab^2+Aa^2c\right)x^7+\frac{1}{3}\,Aa^3x^3+\frac{1}{5}\left(Ba^3+3\,Aa^2b\right)x^5$$

```
[In] integrate(x^2*(B*x^2+A)*(c*x^4+b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] 1/17*B*c^3*x^17 + 1/15*(3*B*b*c^2 + A*c^3)*x^15 + 3/13*(B*b^2*c + (B*a + A*
b)*c^2)*x^13 + 1/11*(B*b^3 + 3*A*a*c^2 + 3*(2*B*a*b + A*b^2)*c)*x^11 + 1/9*
(3*B*a*b^2 + A*b^3 + 3*(B*a^2 + 2*A*a*b)*c)*x^9 + 3/7*(B*a^2*b + A*a*b^2 +
A*a^2*c)*x^7 + 1/3*A*a^3*x^3 + 1/5*(B*a^3 + 3*A*a^2*b)*x^5
```

Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 204, normalized size of antiderivative = 1.23

$$\int x^2(A+Bx^2)\left(a+bx^2+cx^4\right)^3\,dx = \frac{Aa^3x^3}{3}+\frac{Bc^3x^{17}}{17}+x^{15}\left(\frac{Ac^3}{15}+\frac{Bbc^2}{5}\right)+x^{13}\cdot\left(\frac{3Abc^2}{13}+\frac{3Bac^2}{13}+\frac{3Bb^2c}{13}\right)+x^{11}\cdot\left(\frac{3Aac^2}{11}+\frac{3Ab^2c}{11}+\frac{6Babc}{11}+\frac{Bb^3}{11}\right)+x^9\cdot\left(\frac{2Aabc}{3}+\frac{Ab^3}{9}+\frac{Ba^2c}{3}+\frac{Bab^2}{3}\right)+x^7\cdot\left(\frac{3Aa^2c}{7}+\frac{3Aab^2}{7}+\frac{3Ba^2b}{7}\right)+x^5\cdot\left(\frac{3Aa^2b}{5}+\frac{Ba^3}{5}\right)$$

```
[In] integrate(x**2*(B*x**2+A)*(c*x**4+b*x**2+a)**3,x)
```

```
[Out] A*a**3*x**3/3 + B*c**3*x**17/17 + x**15*(A*c**3/15 + B*b*c**2/5) + x**13*(3
*A*b*c**2/13 + 3*B*a*c**2/13 + 3*B*b**2*c/13) + x**11*(3*A*a*c**2/11 + 3*A*
b**2*c/11 + 6*B*a*b*c/11 + B*b**3/11) + x**9*(2*A*a*b*c/3 + A*b**3/9 + B*a*
*2*c/3 + B*a*b**2/3) + x**7*(3*A*a**2*c/7 + 3*A*a*b**2/7 + 3*B*a**2*b/7) +
x**5*(3*A*a**2*b/5 + B*a**3/5)
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 166, normalized size of antiderivative = 1.00

$$\int x^2(A+Bx^2)\left(a+bx^2+cx^4\right)^3\,dx = \frac{1}{17}Bc^3x^{17}+\frac{1}{15}\left(3Bbc^2+Ac^3\right)x^{15} + \frac{3}{13}\left(Bb^2c+(Ba+Ab)c^2\right)x^{13} + \frac{1}{11}\left(Bb^3+3Aac^2+3\left(2Bab+Ab^2\right)c\right)x^{11} + \frac{1}{9}\left(3Bab^2+Ab^3+3\left(Ba^2+2Aab\right)c\right)x^9 + \frac{3}{7}\left(Ba^2b+Aab^2+Aa^2c\right)x^7 + \frac{1}{3}Aa^3x^3+\frac{1}{5}\left(Ba^3+3Aa^2b\right)x^5$$

```
[In] integrate(x^2*(B*x^2+A)*(c*x^4+b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] 1/17*B*c^3*x^17 + 1/15*(3*B*b*c^2 + A*c^3)*x^15 + 3/13*(B*b^2*c + (B*a + A*
b)*c^2)*x^13 + 1/11*(B*b^3 + 3*A*a*c^2 + 3*(2*B*a*b + A*b^2)*c)*x^11 + 1/9*
(3*B*a*b^2 + A*b^3 + 3*(B*a^2 + 2*A*a*b)*c)*x^9 + 3/7*(B*a^2*b + A*a*b^2 +
A*a^2*c)*x^7 + 1/3*A*a^3*x^3 + 1/5*(B*a^3 + 3*A*a^2*b)*x^5
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 193, normalized size of antiderivative = 1.16

$$\int x^2(A+Bx^2)\left(a+bx^2+cx^4\right)^3\,dx = \frac{1}{17}Bc^3x^{17}+\frac{1}{5}Bbc^2x^{15}+\frac{1}{15}Ac^3x^{15}+\frac{3}{13}Bb^2cx^{13} + \frac{3}{13}Bac^2x^{13}+\frac{3}{13}Abc^2x^{13}+\frac{1}{11}Bb^3x^{11}+\frac{6}{11}Babcx^{11} + \frac{3}{11}Ab^2cx^{11}+\frac{3}{11}Aac^2x^{11}+\frac{1}{3}Bab^2x^9+\frac{1}{9}Ab^3x^9 + \frac{1}{3}Ba^2cx^9+\frac{2}{3}Aabcx^9+\frac{3}{7}Ba^2bx^7+\frac{3}{7}Aab^2x^7 + \frac{3}{7}Aa^2cx^7+\frac{1}{5}Ba^3x^5+\frac{3}{5}Aa^2bx^5+\frac{1}{3}Aa^3x^3$$

```
[In] integrate(x^2*(B*x^2+A)*(c*x^4+b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] 1/17*B*c^3*x^17 + 1/5*B*b*c^2*x^15 + 1/15*A*c^3*x^15 + 3/13*B*b^2*c*x^13 +
3/13*B*a*c^2*x^13 + 3/13*A*b*c^2*x^13 + 1/11*B*b^3*x^11 + 6/11*B*a*b*c*x^11
 + 3/11*A*b^2*c*x^11 + 3/11*A*a*c^2*x^11 + 1/3*B*a*b^2*x^9 + 1/9*A*b^3*x^9
+ 1/3*B*a^2*c*x^9 + 2/3*A*a*b*c*x^9 + 3/7*B*a^2*b*x^7 + 3/7*A*a*b^2*x^7 + 3
/7*A*a^2*c*x^7 + 1/5*B*a^3*x^5 + 3/5*A*a^2*b*x^5 + 1/3*A*a^3*x^3
```

Mupad [B] (verification not implemented)

Time = 7.90 (sec) , antiderivative size = 169, normalized size of antiderivative = 1.02

$$\int x^2(A+Bx^2)\left(a+bx^2+cx^4\right)^3\,dx = x^9\left(\frac{B\,c\,a^2}{3}+\frac{B\,a\,b^2}{3}+\frac{2\,A\,c\,a\,b}{3}+\frac{A\,b^3}{9}\right) + x^{11}\left(\frac{B\,b^3}{11}+\frac{3\,A\,b^2\,c}{11}+\frac{6\,B\,a\,b\,c}{11}+\frac{3\,A\,a\,c^2}{11}\right) + x^5\left(\frac{B\,a^3}{5}+\frac{3\,A\,b\,a^2}{5}\right)+x^{15}\left(\frac{A\,c^3}{15}+\frac{B\,b\,c^2}{5}\right) + x^7\left(\frac{3\,B\,a^2\,b}{7}+\frac{3\,A\,c\,a^2}{7}+\frac{3\,A\,a\,b^2}{7}\right) + x^{13}\left(\frac{3\,B\,b^2\,c}{13}+\frac{3\,A\,b\,c^2}{13}+\frac{3\,B\,a\,c^2}{13}\right) + \frac{A\,a^3\,x^3}{3}+\frac{B\,c^3\,x^{17}}{17}$$

```
[In] int(x^2*(A + B*x^2)*(a + b*x^2 + c*x^4)^3,x)
```

```
[Out] x^9*((A*b^3)/9 + (B*a*b^2)/3 + (B*a^2*c)/3 + (2*A*a*b*c)/3) + x^11*((B*b^3)
/11 + (3*A*a*c^2)/11 + (3*A*b^2*c)/11 + (6*B*a*b*c)/11) + x^5*((B*a^3)/5 +
(3*A*a^2*b)/5) + x^15*((A*c^3)/15 + (B*b*c^2)/5) + x^7*((3*A*a*b^2)/7 + (3*
A*a^2*c)/7 + (3*B*a^2*b)/7) + x^13*((3*A*b*c^2)/13 + (3*B*a*c^2)/13 + (3*B*
b^2*c)/13) + (A*a^3*x^3)/3 + (B*c^3*x^17)/17
```

3.97 $\int x(A+Bx^2)\left(a+bx^2+cx^4\right)^3\,dx$

Optimal result

Integrand size = 23, antiderivative size = 166

$$\begin{aligned}\int x(A+Bx^2)\left(a+bx^2+cx^4\right)^3\,dx &= \frac{1}{2}a^3Ax^2+\frac{1}{4}a^2(3Ab+aB)x^4+\frac{1}{2}a\left(abB+A\left(b^2+ac\right)\right)x^6\\ &\quad+\frac{1}{8}\left(3aB\left(b^2+ac\right)+A\left(b^3+6abc\right)\right)x^8\\ &\quad+\frac{1}{10}\left(b^3B+3Ab^2c+6abBc+3aAc^2\right)x^{10}\\ &\quad+\frac{1}{4}c\left(b^2B+Abc+aBc\right)x^{12}\\ &\quad+\frac{1}{14}c^2(3bB+Ac)x^{14}+\frac{1}{16}Bc^3x^{16}\end{aligned}$$

```
[Out] 1/2*a^3*A*x^2+1/4*a^2*(3*A*b+B*a)*x^4+1/2*a*(a*b*B+A*(a*c+b^2))*x^6+1/8*(3*
a*B*(a*c+b^2)+A*(6*a*b*c+b^3))*x^8+1/10*(3*A*a*c^2+3*A*b^2*c+6*B*a*b*c+B*b^
3)*x^10+1/4*c*(A*b*c+B*a*c+B*b^2)*x^12+1/14*c^2*(A*c+3*B*b)*x^14+1/16*B*c^3
*x^16
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 166, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used

$= \{1261, 645\}$

$$\int x\left(A+Bx^2\right)\left(a+bx^2+cx^4\right)^3\,dx = \frac{1}{2}a^3Ax^2+\frac{1}{4}a^2x^4(aB+3Ab) + \frac{1}{4}cx^{12}\left(aBc+Abc+b^2B\right)+\frac{1}{2}ax^6\left(A\left(ac+b^2\right)+abB\right) + \frac{1}{10}x^{10}\left(3aAc^2+6abBc+3Ab^2c+b^3B\right) + \frac{1}{8}x^8\left(A\left(6abc+b^3\right)+3aB\left(ac+b^2\right)\right) + \frac{1}{14}c^2x^{14}(Ac+3bB)+\frac{1}{16}Bc^3x^{16}$$

```
[In] Int[x*(A + B*x^2)*(a + b*x^2 + c*x^4)^3,x]
```

```
[Out] (a^3*A*x^2)/2 + (a^2*(3*A*b + a*B)*x^4)/4 + (a*(a*b*B + A*(b^2 + a*c))*x^6)
/2 + ((3*a*B*(b^2 + a*c) + A*(b^3 + 6*a*b*c))*x^8)/8 + ((b^3*B + 3*A*b^2*c
+ 6*a*b*B*c + 3*a*A*c^2)*x^10)/10 + (c*(b^2*B + A*b*c + a*B*c)*x^12)/4 + (c
^2*(3*b*B + A*c)*x^14)/14 + (B*c^3*x^16)/16
```

Rule 645

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbo
l] :> Int[ExpandIntegrand[(d + e*x)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a
, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && IntegerQ[p] && (GtQ[p, 0] || Eq
Q[a, 0])
```

Rule 1261

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(
p_.), x_Symbol] :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x],
 x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]
```

Rubi steps

$$\text{integral} = \frac{1}{2}\text{Subst}\left(\int (A+Bx)\left(a+bx+cx^2\right)^3\,dx, x, x^2\right)$$

$$= \frac{1}{2}\text{Subst}\left(\int \left(a^3A+a^2(3Ab+aB)x+3a\left(abB+A\left(b^2+ac\right)\right)x^2 + \left(3aB\left(b^2+ac\right)+A\left(b^3+6abc\right)\right)x^3+\left(b^3B+3Ab^2c+6abBc+3aAc^2\right)x^4 + 3c\left(b^2B+Abc+aBc\right)x^5+c^2(3bB+Ac)x^6+Bc^3x^7\right)dx, x, x^2\right)$$

$$= \frac{1}{2}a^3Ax^2+\frac{1}{4}a^2(3Ab+aB)x^4+\frac{1}{2}a\left(abB+A\left(b^2+ac\right)\right)x^6 + \frac{1}{8}\left(3aB\left(b^2+ac\right)+A\left(b^3+6abc\right)\right)x^8+\frac{1}{10}\left(b^3B+3Ab^2c+6abBc+3aAc^2\right)x^{10} + \frac{1}{4}c\left(b^2B+Abc+aBc\right)x^{12}+\frac{1}{14}c^2(3bB+Ac)x^{14}+\frac{1}{16}Bc^3x^{16}$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 154, normalized size of antiderivative = 0.93

$$\int x(A+Bx^2)\left(a+bx^2+cx^4\right)^3\,dx=\frac{1}{560}x^2\big(280a^3A+140a^2(3Ab+aB)x^2+280a\big(abB+A\big(b^2+ac\big)\big)\,x^4+70\big(3aB\big(b^2+ac\big)+A\big(b^3+6abc\big)\big)\,x^6+56\big(b^3B+3Ab^2c+6abBc+3aAc^2\big)\,x^8+140c\big(b^2B+Abc+aBc\big)\,x^{10}+40c^2(3bB+Ac)x^{12}+35Bc^3x^{14}\big)$$

```
[In] Integrate[x*(A + B*x^2)*(a + b*x^2 + c*x^4)^3,x]
```

```
[Out] (x^2*(280*a^3*A + 140*a^2*(3*A*b + a*B)*x^2 + 280*a*(a*b*B + A*(b^2 + a*c))
*x^4 + 70*(3*a*B*(b^2 + a*c) + A*(b^3 + 6*a*b*c))*x^6 + 56*(b^3*B + 3*A*b^2
*c + 6*a*b*B*c + 3*a*A*c^2)*x^8 + 140*c*(b^2*B + A*b*c + a*B*c)*x^10 + 40*c
^2*(3*b*B + A*c)*x^12 + 35*B*c^3*x^14))/560
```

Maple [A] (verified)

Time = 0.13 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.02

method	result
norman	$\frac{a^3Ax^2}{2}+\left(\frac{3}{4}Aa^2b+\frac{1}{4}Ba^3\right)x^4+\left(\frac{1}{2}Aca^2+\frac{1}{2}Aab^2+\frac{1}{2}Ba^2b\right)x^6+\left(\frac{3}{4}Aabc+\frac{1}{8}Ab^3+\frac{3}{8}a^2Bc+\right.$
gosper	$\frac{1}{2}a^3Ax^2+\frac{3}{4}x^4Aa^2b+\frac{1}{4}x^4Ba^3+\frac{1}{2}x^6Aca^2+\frac{1}{2}x^6Aab^2+\frac{1}{2}x^6Ba^2b+\frac{3}{4}x^8Aabc+\frac{1}{8}x^8Ab^3+$
risch	$\frac{1}{2}a^3Ax^2+\frac{3}{4}x^4Aa^2b+\frac{1}{4}x^4Ba^3+\frac{1}{2}x^6Aca^2+\frac{1}{2}x^6Aab^2+\frac{1}{2}x^6Ba^2b+\frac{3}{4}x^8Aabc+\frac{1}{8}x^8Ab^3+$
parallelrisch	$\frac{1}{2}a^3Ax^2+\frac{3}{4}x^4Aa^2b+\frac{1}{4}x^4Ba^3+\frac{1}{2}x^6Aca^2+\frac{1}{2}x^6Aab^2+\frac{1}{2}x^6Ba^2b+\frac{3}{4}x^8Aabc+\frac{1}{8}x^8Ab^3+$
default	$\frac{Bc^3x^{16}}{16}+\frac{(Ac^3+3Bbc^2)x^{14}}{14}+\frac{(3Abc^2+B(ac^2+2b^2c+c(2ac+b^2)))x^{12}}{12}+\frac{(A(ac^2+2b^2c+c(2ac+b^2))+B(4abc+b(2ac+b$

```
[In] int(x*(B*x^2+A)*(c*x^4+b*x^2+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*a^3*A*x^2+(3/4*A*a^2*b+1/4*B*a^3)*x^4+(1/2*A*c*a^2+1/2*A*a*b^2+1/2*B*a^
2*b)*x^6+(3/4*A*a*b*c+1/8*A*b^3+3/8*a^2*B*c+3/8*B*a*b^2)*x^8+(3/10*A*a*c^2+
3/10*A*b^2*c+3/5*B*a*b*c+1/10*B*b^3)*x^10+(1/4*A*b*c^2+1/4*B*a*c^2+1/4*B*b^
2*c)*x^12+(1/14*A*c^3+3/14*B*b*c^2)*x^14+1/16*B*c^3*x^16
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 166, normalized size of antiderivative = 1.00

$$\int x(A+Bx^2)\left(a+bx^2+cx^4\right)^3\,dx = \frac{1}{16}\,Bc^3x^{16}+\frac{1}{14}\left(3\,Bbc^2+Ac^3\right)x^{14}+\frac{1}{4}\left(Bb^2c+(Ba+Ab)c^2\right)x^{12}+\frac{1}{10}\left(Bb^3+3\,Aac^2+3\left(2\,Bab+Ab^2\right)c\right)x^{10}+\frac{1}{8}\left(3\,Bab^2+Ab^3+3\left(Ba^2+2\,Aab\right)c\right)x^8+\frac{1}{2}\left(Ba^2b+Aab^2+Aa^2c\right)x^6+\frac{1}{2}\,Aa^3x^2+\frac{1}{4}\left(Ba^3+3\,Aa^2b\right)x^4$$

```
[In] integrate(x*(B*x^2+A)*(c*x^4+b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] 1/16*B*c^3*x^16 + 1/14*(3*B*b*c^2 + A*c^3)*x^14 + 1/4*(B*b^2*c + (B*a + A*b
)*c^2)*x^12 + 1/10*(B*b^3 + 3*A*a*c^2 + 3*(2*B*a*b + A*b^2)*c)*x^10 + 1/8*(
3*B*a*b^2 + A*b^3 + 3*(B*a^2 + 2*A*a*b)*c)*x^8 + 1/2*(B*a^2*b + A*a*b^2 + A
*a^2*c)*x^6 + 1/2*A*a^3*x^2 + 1/4*(B*a^3 + 3*A*a^2*b)*x^4
```

Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 199, normalized size of antiderivative = 1.20

$$\int x(A+Bx^2)\left(a+bx^2+cx^4\right)^3\,dx = \frac{Aa^3x^2}{2}+\frac{Bc^3x^{16}}{16}+x^{14}\left(\frac{Ac^3}{14}+\frac{3Bbc^2}{14}\right)+x^{12}\left(\frac{Abc^2}{4}+\frac{Bac^2}{4}+\frac{Bb^2c}{4}\right)+x^{10}\cdot\left(\frac{3Aac^2}{10}+\frac{3Ab^2c}{10}+\frac{3Babc}{5}+\frac{Bb^3}{10}\right)+x^8\cdot\left(\frac{3Aabc}{4}+\frac{Ab^3}{8}+\frac{3Ba^2c}{8}+\frac{3Bab^2}{8}\right)+x^6\left(\frac{Aa^2c}{2}+\frac{Aab^2}{2}+\frac{Ba^2b}{2}\right)+x^4\cdot\left(\frac{3Aa^2b}{4}+\frac{Ba^3}{4}\right)$$

```
[In] integrate(x*(B*x**2+A)*(c*x**4+b*x**2+a)**3,x)
```

```
[Out] A*a**3*x**2/2 + B*c**3*x**16/16 + x**14*(A*c**3/14 + 3*B*b*c**2/14) + x**12
*(A*b*c**2/4 + B*a*c**2/4 + B*b**2*c/4) + x**10*(3*A*a*c**2/10 + 3*A*b**2*c
/10 + 3*B*a*b*c/5 + B*b**3/10) + x**8*(3*A*a*b*c/4 + A*b**3/8 + 3*B*a**2*c/
8 + 3*B*a*b**2/8) + x**6*(A*a**2*c/2 + A*a*b**2/2 + B*a**2*b/2) + x**4*(3*A
*a**2*b/4 + B*a**3/4)
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 166, normalized size of antiderivative = 1.00

$$\int x(A+Bx^2)\left(a+bx^2+cx^4\right)^3\,dx=\frac{1}{16}\,Bc^3x^{16}+\frac{1}{14}\left(3\,Bbc^2+Ac^3\right)x^{14}+\frac{1}{4}\left(Bb^2c+(Ba+Ab)c^2\right)x^{12}+\frac{1}{10}\left(Bb^3+3\,Aac^2+3\left(2\,Bab+Ab^2\right)c\right)x^{10}+\frac{1}{8}\left(3\,Bab^2+Ab^3+3\left(Ba^2+2\,Aab\right)c\right)x^8+\frac{1}{2}\left(Ba^2b+Aab^2+Aa^2c\right)x^6+\frac{1}{2}\,Aa^3x^2+\frac{1}{4}\left(Ba^3+3\,Aa^2b\right)x^4$$

```
[In] integrate(x*(B*x^2+A)*(c*x^4+b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] 1/16*B*c^3*x^16 + 1/14*(3*B*b*c^2 + A*c^3)*x^14 + 1/4*(B*b^2*c + (B*a + A*b
)*c^2)*x^12 + 1/10*(B*b^3 + 3*A*a*c^2 + 3*(2*B*a*b + A*b^2)*c)*x^10 + 1/8*(
3*B*a*b^2 + A*b^3 + 3*(B*a^2 + 2*A*a*b)*c)*x^8 + 1/2*(B*a^2*b + A*a*b^2 + A
*a^2*c)*x^6 + 1/2*A*a^3*x^2 + 1/4*(B*a^3 + 3*A*a^2*b)*x^4
```

Giac [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 193, normalized size of antiderivative = 1.16

$$\int x(A+Bx^2)\left(a+bx^2+cx^4\right)^3\,dx=\frac{1}{16}\,Bc^3x^{16}+\frac{3}{14}\,Bbc^2x^{14}+\frac{1}{14}\,Ac^3x^{14}+\frac{1}{4}\,Bb^2cx^{12}+\frac{1}{4}\,Bac^2x^{12}+\frac{1}{4}\,Abc^2x^{12}+\frac{1}{10}\,Bb^3x^{10}+\frac{3}{5}\,Babcx^{10}+\frac{3}{10}\,Ab^2cx^{10}+\frac{3}{10}\,Aac^2x^{10}+\frac{3}{8}\,Bab^2x^8+\frac{1}{8}\,Ab^3x^8+\frac{3}{8}\,Ba^2cx^8+\frac{3}{4}\,Aabcx^8+\frac{1}{2}\,Ba^2bx^6+\frac{1}{2}\,Aab^2x^6+\frac{1}{2}\,Aa^2cx^6+\frac{1}{4}\,Ba^3x^4+\frac{3}{4}\,Aa^2bx^4+\frac{1}{2}\,Aa^3x^2$$

```
[In] integrate(x*(B*x^2+A)*(c*x^4+b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] 1/16*B*c^3*x^16 + 3/14*B*b*c^2*x^14 + 1/14*A*c^3*x^14 + 1/4*B*b^2*c*x^12 +
1/4*B*a*c^2*x^12 + 1/4*A*b*c^2*x^12 + 1/10*B*b^3*x^10 + 3/5*B*a*b*c*x^10 +
3/10*A*b^2*c*x^10 + 3/10*A*a*c^2*x^10 + 3/8*B*a*b^2*x^8 + 1/8*A*b^3*x^8 + 3
/8*B*a^2*c*x^8 + 3/4*A*a*b*c*x^8 + 1/2*B*a^2*b*x^6 + 1/2*A*a*b^2*x^6 + 1/2*
A*a^2*c*x^6 + 1/4*B*a^3*x^4 + 3/4*A*a^2*b*x^4 + 1/2*A*a^3*x^2
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 169, normalized size of antiderivative = 1.02

$$\int x(A+Bx^2)\left(a+bx^2+cx^4\right)^3\,dx = x^8\left(\frac{3\,B\,c\,a^2}{8}+\frac{3\,B\,a\,b^2}{8}+\frac{3\,A\,c\,a\,b}{4}+\frac{A\,b^3}{8}\right)+x^{10}\left(\frac{B\,b^3}{10}+\frac{3\,A\,b^2\,c}{10}+\frac{3\,B\,a\,b\,c}{5}+\frac{3\,A\,a\,c^2}{10}\right)+x^4\left(\frac{B\,a^3}{4}+\frac{3\,A\,b\,a^2}{4}\right)+x^{14}\left(\frac{A\,c^3}{14}+\frac{3\,B\,b\,c^2}{14}\right)+x^6\left(\frac{B\,a^2\,b}{2}+\frac{A\,c\,a^2}{2}+\frac{A\,a\,b^2}{2}\right)+x^{12}\left(\frac{B\,b^2\,c}{4}+\frac{A\,b\,c^2}{4}+\frac{B\,a\,c^2}{4}\right)+\frac{A\,a^3\,x^2}{2}+\frac{B\,c^3\,x^{16}}{16}$$

```
[In] int(x*(A + B*x^2)*(a + b*x^2 + c*x^4)^3,x)
```

```
[Out] x^8*((A*b^3)/8 + (3*B*a*b^2)/8 + (3*B*a^2*c)/8 + (3*A*a*b*c)/4) + x^10*((B*
b^3)/10 + (3*A*a*c^2)/10 + (3*A*b^2*c)/10 + (3*B*a*b*c)/5) + x^4*((B*a^3)/4
 + (3*A*a^2*b)/4) + x^14*((A*c^3)/14 + (3*B*b*c^2)/14) + x^6*((A*a*b^2)/2 +
 (A*a^2*c)/2 + (B*a^2*b)/2) + x^12*((A*b*c^2)/4 + (B*a*c^2)/4 + (B*b^2*c)/4
) + (A*a^3*x^2)/2 + (B*c^3*x^16)/16
```

3.98 $\int (A+Bx^2)(a+bx^2+cx^4)^3\,dx$

Optimal result

Integrand size = 22, antiderivative size = 161

$$\int (A+Bx^2)(a+bx^2+cx^4)^3\,dx = a^3Ax+\frac{1}{3}a^2(3Ab+aB)x^3+\frac{3}{5}a\big(abB+A\big(b^2+ac\big)\big)x^5 +\frac{1}{7}\big(3aB\big(b^2+ac\big)+A\big(b^3+6abc\big)\big)x^7 +\frac{1}{9}\big(b^3B+3Ab^2c+6abBc+3aAc^2\big)x^9 +\frac{3}{11}c\big(b^2B+Abc+aBc\big)x^{11} +\frac{1}{13}c^2(3bB+Ac)x^{13}+\frac{1}{15}Bc^3x^{15}$$

```
[Out] a^3*A*x+1/3*a^2*(3*A*b+B*a)*x^3+3/5*a*(a*b*B+A*(a*c+b^2))*x^5+1/7*(3*a*B*(a
*c+b^2)+A*(6*a*b*c+b^3))*x^7+1/9*(3*A*a*c^2+3*A*b^2*c+6*B*a*b*c+B*b^3)*x^9+
3/11*c*(A*b*c+B*a*c+B*b^2)*x^11+1/13*c^2*(A*c+3*B*b)*x^13+1/15*B*c^3*x^15
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 161, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used

$= \{1167\}$

$$\int \left(A+Bx^2\right)\left(a+bx^2+cx^4\right)^3\,dx = a^3Ax+\frac{1}{3}a^2x^3(aB+3Ab)+\frac{3}{11}cx^{11}\left(aBc+Abc+b^2B\right)+\frac{3}{5}ax^5\left(A\left(ac+b^2\right)+abB\right)+\frac{1}{9}x^9\left(3aAc^2+6abBc+3Ab^2c+b^3B\right)+\frac{1}{7}x^7\left(A\left(6abc+b^3\right)+3aB\left(ac+b^2\right)\right)+\frac{1}{13}c^2x^{13}(Ac+3bB)+\frac{1}{15}Bc^3x^{15}$$

```
[In] Int[(A + B*x^2)*(a + b*x^2 + c*x^4)^3,x]
```

```
[Out] a^3*A*x + (a^2*(3*A*b + a*B)*x^3)/3 + (3*a*(a*b*B + A*(b^2 + a*c))*x^5)/5 +
 ((3*a*B*(b^2 + a*c) + A*(b^3 + 6*a*b*c))*x^7)/7 + ((b^3*B + 3*A*b^2*c + 6*
a*b*B*c + 3*a*A*c^2)*x^9)/9 + (3*c*(b^2*B + A*b*c + a*B*c)*x^11)/11 + (c^2*
(3*b*B + A*c)*x^13)/13 + (B*c^3*x^15)/15
```

Rule 1167

```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^q*(a + b*x^2 + c*x^4)^p, x],
x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e
 + a*e^2, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \left(a^3A+a^2(3Ab+aB)x^2+3a\left(abB+A\left(b^2+ac\right)\right)x^4+\left(3aB\left(b^2+ac\right)+A\left(b^3+6abc\right)\right)x^6+\left(b^3B+3Ab^2c+6abBc+3aAc^2\right)x^8+3c\left(b^2B+Abc+aBc\right)x^{10}+c^2(3bB+Ac)x^{12}+Bc^3x^{14}\right)dx\\ &= a^3Ax+\frac{1}{3}a^2(3Ab+aB)x^3+\frac{3}{5}a\left(abB+A\left(b^2+ac\right)\right)x^5+\frac{1}{7}\left(3aB\left(b^2+ac\right)+A\left(b^3+6abc\right)\right)x^7+\frac{1}{9}\left(b^3B+3Ab^2c+6abBc+3aAc^2\right)x^9+\frac{3}{11}c\left(b^2B+Abc+aBc\right)x^{11}+\frac{1}{13}c^2(3bB+Ac)x^{13}+\frac{1}{15}Bc^3x^{15}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 161, normalized size of antiderivative = 1.00

$$\int \left(A+Bx^2\right)\left(a+bx^2+cx^4\right)^3\,dx = a^3Ax+\frac{1}{3}a^2(3Ab+aB)x^3+\frac{3}{5}a\left(abB+A\left(b^2+ac\right)\right)x^5 + \frac{1}{7}\left(3aB\left(b^2+ac\right)+A\left(b^3+6abc\right)\right)x^7 + \frac{1}{9}\left(b^3B+3Ab^2c+6abBc+3aAc^2\right)x^9 + \frac{3}{11}c\left(b^2B+Abc+aBc\right)x^{11} + \frac{1}{13}c^2(3bB+Ac)x^{13}+\frac{1}{15}Bc^3x^{15}$$

```
[In] Integrate[(A + B*x^2)*(a + b*x^2 + c*x^4)^3,x]
```

```
[Out] a^3*A*x + (a^2*(3*A*b + a*B)*x^3)/3 + (3*a*(a*b*B + A*(b^2 + a*c))*x^5)/5 +
 ((3*a*B*(b^2 + a*c) + A*(b^3 + 6*a*b*c))*x^7)/7 + ((b^3*B + 3*A*b^2*c + 6*
a*b*B*c + 3*a*A*c^2)*x^9)/9 + (3*c*(b^2*B + A*b*c + a*B*c)*x^11)/11 + (c^2*
(3*b*B + A*c)*x^13)/13 + (B*c^3*x^15)/15
```

Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 166, normalized size of antiderivative = 1.03

method	result
norman	$a^3Ax+\left(Aa^2b+\frac{1}{3}Ba^3\right)x^3+\left(\frac{3}{5}Aca^2+\frac{3}{5}Aab^2+\frac{3}{5}Ba^2b\right)x^5+\left(\frac{6}{7}Aabc+\frac{1}{7}Ab^3+\frac{3}{7}a^2Bc+\right.$
gosper	$a^3Ax+x^3Aa^2b+\frac{1}{3}x^3Ba^3+\frac{3}{5}x^5Aca^2+\frac{3}{5}x^5Aab^2+\frac{3}{5}x^5Ba^2b+\frac{6}{7}x^7Aabc+\frac{1}{7}x^7Ab^3+\frac{3}{7}x^7a$
risch	$a^3Ax+x^3Aa^2b+\frac{1}{3}x^3Ba^3+\frac{3}{5}x^5Aca^2+\frac{3}{5}x^5Aab^2+\frac{3}{5}x^5Ba^2b+\frac{6}{7}x^7Aabc+\frac{1}{7}x^7Ab^3+\frac{3}{7}x^7a$
parallelrisch	$a^3Ax+x^3Aa^2b+\frac{1}{3}x^3Ba^3+\frac{3}{5}x^5Aca^2+\frac{3}{5}x^5Aab^2+\frac{3}{5}x^5Ba^2b+\frac{6}{7}x^7Aabc+\frac{1}{7}x^7Ab^3+\frac{3}{7}x^7a$
default	$\frac{Bc^3x^{15}}{15}+\frac{(Ac^3+3Bbc^2)x^{13}}{13}+\frac{(3Abc^2+B(ac^2+2b^2c+c(2ac+b^2)))x^{11}}{11}+\frac{(A(ac^2+2b^2c+c(2ac+b^2))+B(4abc+b(2ac+b}{9}$

```
[In] int((B*x^2+A)*(c*x^4+b*x^2+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] a^3*A*x+(A*a^2*b+1/3*B*a^3)*x^3+(3/5*A*c*a^2+3/5*A*a*b^2+3/5*B*a^2*b)*x^5+(
6/7*A*a*b*c+1/7*A*b^3+3/7*a^2*B*c+3/7*B*a*b^2)*x^7+(1/3*A*a*c^2+1/3*A*b^2*c
+2/3*B*a*b*c+1/9*B*b^3)*x^9+(3/11*A*b*c^2+3/11*B*a*c^2+3/11*B*b^2*c)*x^11+(
1/13*A*c^3+3/13*B*b*c^2)*x^13+1/15*B*c^3*x^15
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.01

$$\int (A+Bx^2)\left(a+bx^2+cx^4\right)^3\,dx = \frac{1}{15}\,Bc^3x^{15} + \frac{1}{13}\,(3\,Bbc^2+Ac^3)x^{13} + \frac{3}{11}\,(Bb^2c+(Ba+Ab)c^2)x^{11} + \frac{1}{9}\,\left(Bb^3+3\,Aac^2+3\,\left(2\,Bab+Ab^2\right)c\right)x^9 + \frac{1}{7}\,\left(3\,Bab^2+Ab^3+3\,\left(Ba^2+2\,Aab\right)c\right)x^7 + \frac{3}{5}\,\left(Ba^2b+Aab^2+Aa^2c\right)x^5 + Aa^3x + \frac{1}{3}\,\left(Ba^3+3\,Aa^2b\right)x^3$$

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] 1/15*B*c^3*x^15 + 1/13*(3*B*b*c^2 + A*c^3)*x^13 + 3/11*(B*b^2*c + (B*a + A*
b)*c^2)*x^11 + 1/9*(B*b^3 + 3*A*a*c^2 + 3*(2*B*a*b + A*b^2)*c)*x^9 + 1/7*(3
*B*a*b^2 + A*b^3 + 3*(B*a^2 + 2*A*a*b)*c)*x^7 + 3/5*(B*a^2*b + A*a*b^2 + A*
a^2*c)*x^5 + A*a^3*x + 1/3*(B*a^3 + 3*A*a^2*b)*x^3
```

Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 199, normalized size of antiderivative = 1.24

$$\int (A+Bx^2)\left(a+bx^2+cx^4\right)^3\,dx = Aa^3x + \frac{Bc^3x^{15}}{15} + x^{13}\left(\frac{Ac^3}{13}+\frac{3Bbc^2}{13}\right) + x^{11}\cdot\left(\frac{3Abc^2}{11}+\frac{3Bac^2}{11}+\frac{3Bb^2c}{11}\right) + x^9\left(\frac{Aac^2}{3}+\frac{Ab^2c}{3}+\frac{2Babc}{3}+\frac{Bb^3}{9}\right) + x^7\cdot\left(\frac{6Aabc}{7}+\frac{Ab^3}{7}+\frac{3Ba^2c}{7}+\frac{3Bab^2}{7}\right) + x^5\cdot\left(\frac{3Aa^2c}{5}+\frac{3Aab^2}{5}+\frac{3Ba^2b}{5}\right) + x^3\left(Aa^2b+\frac{Ba^3}{3}\right)$$

```
[In] integrate((B*x**2+A)*(c*x**4+b*x**2+a)**3,x)
```

```
[Out] A*a**3*x + B*c**3*x**15/15 + x**13*(A*c**3/13 + 3*B*b*c**2/13) + x**11*(3*A
*b*c**2/11 + 3*B*a*c**2/11 + 3*B*b**2*c/11) + x**9*(A*a*c**2/3 + A*b**2*c/3
 + 2*B*a*b*c/3 + B*b**3/9) + x**7*(6*A*a*b*c/7 + A*b**3/7 + 3*B*a**2*c/7 +
3*B*a*b**2/7) + x**5*(3*A*a**2*c/5 + 3*A*a*b**2/5 + 3*B*a**2*b/5) + x**3*(A
*a**2*b + B*a**3/3)
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.01

$$\int (A+Bx^2)(a+bx^2+cx^4)^3\,dx = \frac{1}{15}Bc^3x^{15} + \frac{1}{13}(3Bbc^2+Ac^3)x^{13} + \frac{3}{11}(Bb^2c+(Ba+Ab)c^2)x^{11} + \frac{1}{9}(Bb^3+3Aac^2+3(2Bab+Ab^2)c)x^9 + \frac{1}{7}(3Bab^2+Ab^3+3(Ba^2+2Aab)c)x^7 + \frac{3}{5}(Ba^2b+Aab^2+Aa^2c)x^5 + Aa^3x + \frac{1}{3}(Ba^3+3Aa^2b)x^3$$

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] 1/15*B*c^3*x^15 + 1/13*(3*B*b*c^2 + A*c^3)*x^13 + 3/11*(B*b^2*c + (B*a + A*
b)*c^2)*x^11 + 1/9*(B*b^3 + 3*A*a*c^2 + 3*(2*B*a*b + A*b^2)*c)*x^9 + 1/7*(3
*B*a*b^2 + A*b^3 + 3*(B*a^2 + 2*A*a*b)*c)*x^7 + 3/5*(B*a^2*b + A*a*b^2 + A*
a^2*c)*x^5 + A*a^3*x + 1/3*(B*a^3 + 3*A*a^2*b)*x^3
```

Giac [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 189, normalized size of antiderivative = 1.17

$$\int (A+Bx^2)(a+bx^2+cx^4)^3\,dx = \frac{1}{15}Bc^3x^{15} + \frac{3}{13}Bbc^2x^{13} + \frac{1}{13}Ac^3x^{13} + \frac{3}{11}Bb^2cx^{11} + \frac{3}{11}Bac^2x^{11} + \frac{3}{11}Abc^2x^{11} + \frac{1}{9}Bb^3x^9 + \frac{2}{3}Babcx^9 + \frac{1}{3}Ab^2cx^9 + \frac{1}{3}Aac^2x^9 + \frac{3}{7}Bab^2x^7 + \frac{1}{7}Ab^3x^7 + \frac{3}{7}Ba^2cx^7 + \frac{6}{7}Aabcx^7 + \frac{3}{5}Ba^2bx^5 + \frac{3}{5}Aab^2x^5 + \frac{3}{5}Aa^2cx^5 + \frac{1}{3}Ba^3x^3 + Aa^2bx^3 + Aa^3x$$

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] 1/15*B*c^3*x^15 + 3/13*B*b*c^2*x^13 + 1/13*A*c^3*x^13 + 3/11*B*b^2*c*x^11 +
 3/11*B*a*c^2*x^11 + 3/11*A*b*c^2*x^11 + 1/9*B*b^3*x^9 + 2/3*B*a*b*c*x^9 +
1/3*A*b^2*c*x^9 + 1/3*A*a*c^2*x^9 + 3/7*B*a*b^2*x^7 + 1/7*A*b^3*x^7 + 3/7*B
*a^2*c*x^7 + 6/7*A*a*b*c*x^7 + 3/5*B*a^2*b*x^5 + 3/5*A*a*b^2*x^5 + 3/5*A*a^
2*c*x^5 + 1/3*B*a^3*x^3 + A*a^2*b*x^3 + A*a^3*x
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 165, normalized size of antiderivative = 1.02

$$\int \left(A+Bx^2\right)\left(a+bx^2+cx^4\right)^3\,dx = x^7\left(\frac{3\,B\,c\,a^2}{7}+\frac{3\,B\,a\,b^2}{7}+\frac{6\,A\,c\,a\,b}{7}+\frac{A\,b^3}{7}\right) + x^9\left(\frac{B\,b^3}{9}+\frac{A\,b^2\,c}{3}+\frac{2\,B\,a\,b\,c}{3}+\frac{A\,a\,c^2}{3}\right) + x^3\left(\frac{B\,a^3}{3}+A\,b\,a^2\right)+x^{13}\left(\frac{A\,c^3}{13}+\frac{3\,B\,b\,c^2}{13}\right) + x^5\left(\frac{3\,B\,a^2\,b}{5}+\frac{3\,A\,c\,a^2}{5}+\frac{3\,A\,a\,b^2}{5}\right) + x^{11}\left(\frac{3\,B\,b^2\,c}{11}+\frac{3\,A\,b\,c^2}{11}+\frac{3\,B\,a\,c^2}{11}\right) + \frac{B\,c^3\,x^{15}}{15}+A\,a^3\,x$$

```
[In] int((A + B*x^2)*(a + b*x^2 + c*x^4)^3,x)
```

```
[Out] x^7*((A*b^3)/7 + (3*B*a*b^2)/7 + (3*B*a^2*c)/7 + (6*A*a*b*c)/7) + x^9*((B*b
^3)/9 + (A*a*c^2)/3 + (A*b^2*c)/3 + (2*B*a*b*c)/3) + x^3*((B*a^3)/3 + A*a^2
*b) + x^13*((A*c^3)/13 + (3*B*b*c^2)/13) + x^5*((3*A*a*b^2)/5 + (3*A*a^2*c)
/5 + (3*B*a^2*b)/5) + x^11*((3*A*b*c^2)/11 + (3*B*a*c^2)/11 + (3*B*b^2*c)/1
1) + (B*c^3*x^15)/15 + A*a^3*x
```

3.99 $\int \frac{(A+Bx^2)(a+bx^2+cx^4)^3}{x} \, dx$

Optimal result

Integrand size = 25, antiderivative size = 162

$$\int \frac{(A+Bx^2)(a+bx^2+cx^4)^3}{x} \, dx = \frac{1}{2}a^2(3Ab+aB)x^2 + \frac{3}{4}a\left(abB+A\left(b^2+ac\right)\right)x^4 + \frac{1}{6}\left(3aB\left(b^2+ac\right)+A\left(b^3+6abc\right)\right)x^6 + \frac{1}{8}\left(b^3B+3Ab^2c+6abBc+3aAc^2\right)x^8 + \frac{3}{10}c\left(b^2B+Abc+aBc\right)x^{10} + \frac{1}{12}c^2(3bB+Ac)x^{12} + \frac{1}{14}Bc^3x^{14} + a^3A\log(x)$$

```
[Out] 1/2*a^2*(3*A*b+B*a)*x^2+3/4*a*(a*b*B+A*(a*c+b^2))*x^4+1/6*(3*a*B*(a*c+b^2)+
A*(6*a*b*c+b^3))*x^6+1/8*(3*A*a*c^2+3*A*b^2*c+6*B*a*b*c+B*b^3)*x^8+3/10*c*(
A*b*c+B*a*c+B*b^2)*x^10+1/12*c^2*(A*c+3*B*b)*x^12+1/14*B*c^3*x^14+a^3*A*ln(
x)
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 162, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.080$, Rules used

$= \{1265, 779\}$

$$\int \frac{(A+Bx^2)(a+bx^2+cx^4)^3}{x}\,dx = a^3A\log(x)+\frac{1}{2}a^2x^2(aB+3Ab)+\frac{3}{10}cx^{10}\left(aBc+Abc+b^2B\right)+\frac{3}{4}ax^4\left(A\left(ac+b^2\right)+abB\right)+\frac{1}{8}x^8\left(3aAc^2+6abBc+3Ab^2c+b^3B\right)+\frac{1}{6}x^6\left(A\left(6abc+b^3\right)+3aB\left(ac+b^2\right)\right)+\frac{1}{12}c^2x^{12}(Ac+3bB)+\frac{1}{14}Bc^3x^{14}$$

```
[In] Int[((A + B*x^2)*(a + b*x^2 + c*x^4)^3)/x,x]
```

```
[Out] (a^2*(3*A*b + a*B)*x^2)/2 + (3*a*(a*b*B + A*(b^2 + a*c))*x^4)/4 + ((3*a*B*(
b^2 + a*c) + A*(b^3 + 6*a*b*c))*x^6)/6 + ((b^3*B + 3*A*b^2*c + 6*a*b*B*c +
3*a*A*c^2)*x^8)/8 + (3*c*(b^2*B + A*b*c + a*B*c)*x^10)/10 + (c^2*(3*b*B + A
*c)*x^12)/12 + (B*c^3*x^14)/14 + a^3*A*Log[x]
```

Rule 779

```
Int[((e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_
)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(f + g*x)*(a + b*x + c
*x^2)^p, x], x] /; FreeQ[{a, b, c, e, f, g, m}, x] && IntegerQ[p] && (GtQ[p
, 0] || (EqQ[a, 0] && IntegerQ[m]))
```

Rule 1265

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rubi steps

$$\text{integral} = \frac{1}{2}\text{Subst}\left(\int \frac{(A+Bx)(a+bx+cx^2)^3}{x}\,dx, x, x^2\right)$$

$$= \frac{1}{2}\text{Subst}\left(\int \left(a^2(3Ab+aB)+\frac{a^3A}{x}+3a\left(abB+A\left(b^2+ac\right)\right)x+\left(3aB\left(b^2+ac\right)+A\left(b^3+6abc\right)\right)x^2+\left(b^3B+3Ab^2c+6abBc+3aAc^2\right)x^3+3c\left(b^2B+Abc+aBc\right)x^4+c^2(3bB+Ac)x^5+Bc^3x^6\right)dx, x, x^2\right)$$

$$= \frac{1}{2}a^2(3Ab+aB)x^2 + \frac{3}{4}a\left(abB + A\left(b^2+ac\right)\right)x^4 + \frac{1}{6}\left(3aB\left(b^2+ac\right) + A\left(b^3+6abc\right)\right)x^6 + \frac{1}{8}\left(b^3B + 3Ab^2c + 6abBc + 3aAc^2\right)x^8 + \frac{3}{10}c\left(b^2B + Abc + aBc\right)x^{10} + \frac{1}{12}c^2(3bB+Ac)x^{12} + \frac{1}{14}Bc^3x^{14} + a^3A\log(x)$$

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 162, normalized size of antiderivative = 1.00

$$\int \frac{\left(A+Bx^2\right)\left(a+bx^2+cx^4\right)^3}{x}\,dx = \frac{1}{2}a^2(3Ab+aB)x^2 + \frac{3}{4}a\left(abB + A\left(b^2+ac\right)\right)x^4 + \frac{1}{6}\left(3aB\left(b^2+ac\right) + A\left(b^3+6abc\right)\right)x^6 + \frac{1}{8}\left(b^3B + 3Ab^2c + 6abBc + 3aAc^2\right)x^8 + \frac{3}{10}c\left(b^2B + Abc + aBc\right)x^{10} + \frac{1}{12}c^2(3bB+Ac)x^{12} + \frac{1}{14}Bc^3x^{14} + a^3A\log(x)$$

```
[In] Integrate[((A + B*x^2)*(a + b*x^2 + c*x^4)^3)/x,x]
```

```
[Out] (a^2*(3*A*b + a*B)*x^2)/2 + (3*a*(a*b*B + A*(b^2 + a*c))*x^4)/4 + ((3*a*B*(
b^2 + a*c) + A*(b^3 + 6*a*b*c))*x^6)/6 + ((b^3*B + 3*A*b^2*c + 6*a*b*B*c +
3*a*A*c^2)*x^8)/8 + (3*c*(b^2*B + A*b*c + a*B*c)*x^10)/10 + (c^2*(3*b*B + A
*c)*x^12)/12 + (B*c^3*x^14)/14 + a^3*A*Log[x]
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 167, normalized size of antiderivative = 1.03

method	result
norman	$\left(\frac{1}{12}A\,c^3+\frac{1}{4}Bb\,c^2\right)x^{12}+\left(\frac{3}{2}A\,a^2b+\frac{1}{2}B\,a^3\right)x^2+\left(\frac{3}{10}Ab\,c^2+\frac{3}{10}Ba\,c^2+\frac{3}{10}B\,b^2c\right)x^{10}+\left(\frac{3}{4}Ac\,a^2\right.$
default	$\frac{B\,c^3x^{14}}{14}+\frac{A\,c^3x^{12}}{12}+\frac{Bb\,c^2x^{12}}{4}+\frac{3Ab\,c^2x^{10}}{10}+\frac{3Ba\,c^2x^{10}}{10}+\frac{3B\,b^2c\,x^{10}}{10}+\frac{3Aa\,c^2x^8}{8}+\frac{3A\,b^2c\,x^8}{8}+\frac{3Babc\,x^8}{4}+B$
risch	$\frac{B\,c^3x^{14}}{14}+\frac{A\,c^3x^{12}}{12}+\frac{Bb\,c^2x^{12}}{4}+\frac{3Ab\,c^2x^{10}}{10}+\frac{3Ba\,c^2x^{10}}{10}+\frac{3B\,b^2c\,x^{10}}{10}+\frac{3Aa\,c^2x^8}{8}+\frac{3A\,b^2c\,x^8}{8}+\frac{3Babc\,x^8}{4}+B$
parallelrisch	$\frac{B\,c^3x^{14}}{14}+\frac{A\,c^3x^{12}}{12}+\frac{Bb\,c^2x^{12}}{4}+\frac{3Ab\,c^2x^{10}}{10}+\frac{3Ba\,c^2x^{10}}{10}+\frac{3B\,b^2c\,x^{10}}{10}+\frac{3Aa\,c^2x^8}{8}+\frac{3A\,b^2c\,x^8}{8}+\frac{3Babc\,x^8}{4}+B$

```
[In] int((B*x^2+A)*(c*x^4+b*x^2+a)^3/x,x,method=_RETURNVERBOSE)
```

```
[Out] (1/12*A*c^3+1/4*B*b*c^2)*x^12+(3/2*A*a^2*b+1/2*B*a^3)*x^2+(3/10*A*b*c^2+3/1
0*B*a*c^2+3/10*B*b^2*c)*x^10+(3/4*A*c*a^2+3/4*A*a*b^2+3/4*B*a^2*b)*x^4+(3/8
```

```
*A*a*c^2+3/8*A*b^2*c+3/4*B*a*b*c+1/8*B*b^3)*x^8+(A*a*b*c+1/6*A*b^3+1/2*a^2*
B*c+1/2*B*a*b^2)*x^6+1/14*B*c^3*x^14+a^3*A*ln(x)
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 164, normalized size of antiderivative = 1.01

$$\int \frac{(A+Bx^2)(a+bx^2+cx^4)^3}{x}\,dx = \frac{1}{14}Bc^3x^{14} + \frac{1}{12}(3Bbc^2+Ac^3)x^{12} + \frac{3}{10}(Bb^2c+(Ba+Ab)c^2)x^{10} + \frac{1}{8}(Bb^3+3Aac^2+3(2Bab+Ab^2)c)x^8 + \frac{1}{6}(3Bab^2+Ab^3+3(Ba^2+2Aab)c)x^6 + \frac{3}{4}(Ba^2b+Aab^2+Aa^2c)x^4 + Aa^3\log(x) + \frac{1}{2}(Ba^3+3Aa^2b)x^2$$

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2+a)^3/x,x, algorithm="fricas")
```

```
[Out] 1/14*B*c^3*x^14 + 1/12*(3*B*b*c^2 + A*c^3)*x^12 + 3/10*(B*b^2*c + (B*a + A*
b)*c^2)*x^10 + 1/8*(B*b^3 + 3*A*a*c^2 + 3*(2*B*a*b + A*b^2)*c)*x^8 + 1/6*(3
*B*a*b^2 + A*b^3 + 3*(B*a^2 + 2*A*a*b)*c)*x^6 + 3/4*(B*a^2*b + A*a*b^2 + A*
a^2*c)*x^4 + A*a^3*log(x) + 1/2*(B*a^3 + 3*A*a^2*b)*x^2
```

Sympy [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 199, normalized size of antiderivative = 1.23

$$\int \frac{(A+Bx^2)(a+bx^2+cx^4)^3}{x}\,dx = Aa^3\log(x) + \frac{Bc^3x^{14}}{14} + x^{12}\left(\frac{Ac^3}{12}+\frac{Bbc^2}{4}\right) + x^{10}\cdot\left(\frac{3Abc^2}{10}+\frac{3Bac^2}{10}+\frac{3Bb^2c}{10}\right) + x^8\cdot\left(\frac{3Aac^2}{8}+\frac{3Ab^2c}{8}+\frac{3Babc}{4}+\frac{Bb^3}{8}\right) + x^6\left(Aabc+\frac{Ab^3}{6}+\frac{Ba^2c}{2}+\frac{Bab^2}{2}\right) + x^4\cdot\left(\frac{3Aa^2c}{4}+\frac{3Aab^2}{4}+\frac{3Ba^2b}{4}\right) + x^2\cdot\left(\frac{3Aa^2b}{2}+\frac{Ba^3}{2}\right)$$

```
[In] integrate((B*x**2+A)*(c*x**4+b*x**2+a)**3/x,x)
```

```
[Out] A*a**3*log(x) + B*c**3*x**14/14 + x**12*(A*c**3/12 + B*b*c**2/4) + x**10*(3
*A*b*c**2/10 + 3*B*a*c**2/10 + 3*B*b**2*c/10) + x**8*(3*A*a*c**2/8 + 3*A*b*
*2*c/8 + 3*B*a*b*c/4 + B*b**3/8) + x**6*(A*a*b*c + A*b**3/6 + B*a**2*c/2 +
B*a*b**2/2) + x**4*(3*A*a**2*c/4 + 3*A*a*b**2/4 + 3*B*a**2*b/4) + x**2*(3*A
*a**2*b/2 + B*a**3/2)
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 167, normalized size of antiderivative = 1.03

$$\int \frac{(A+Bx^2)\,(a+bx^2+cx^4)^3}{x}\,dx = \frac{1}{14}\,Bc^3x^{14} + \frac{1}{12}\,(3\,Bbc^2 + Ac^3)x^{12} + \frac{3}{10}\,(Bb^2c + (Ba + Ab)c^2)x^{10} + \frac{1}{8}\,(Bb^3 + 3\,Aac^2 + 3\,(2\,Bab + Ab^2)c)x^8 + \frac{1}{6}\,(3\,Bab^2 + Ab^3 + 3\,(Ba^2 + 2\,Aab)c)x^6 + \frac{3}{4}\,(Ba^2b + Aab^2 + Aa^2c)x^4 + \frac{1}{2}\,Aa^3\log\left(x^2\right) + \frac{1}{2}\,(Ba^3 + 3\,Aa^2b)x^2$$

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2+a)^3/x,x, algorithm="maxima")
```

```
[Out] 1/14*B*c^3*x^14 + 1/12*(3*B*b*c^2 + A*c^3)*x^12 + 3/10*(B*b^2*c + (B*a + A*
b)*c^2)*x^10 + 1/8*(B*b^3 + 3*A*a*c^2 + 3*(2*B*a*b + A*b^2)*c)*x^8 + 1/6*(3
*B*a*b^2 + A*b^3 + 3*(B*a^2 + 2*A*a*b)*c)*x^6 + 3/4*(B*a^2*b + A*a*b^2 + A*
a^2*c)*x^4 + 1/2*A*a^3*log(x^2) + 1/2*(B*a^3 + 3*A*a^2*b)*x^2
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 193, normalized size of antiderivative = 1.19

$$\int \frac{(A+Bx^2)\,(a+bx^2+cx^4)^3}{x}\,dx = \frac{1}{14}\,Bc^3x^{14} + \frac{1}{4}\,Bbc^2x^{12} + \frac{1}{12}\,Ac^3x^{12} + \frac{3}{10}\,Bb^2cx^{10} + \frac{3}{10}\,Bac^2x^{10} + \frac{3}{10}\,Abc^2x^{10} + \frac{1}{8}\,Bb^3x^8 + \frac{3}{4}\,Babcx^8 + \frac{3}{8}\,Ab^2cx^8 + \frac{3}{8}\,Aac^2x^8 + \frac{1}{2}\,Bab^2x^6 + \frac{1}{6}\,Ab^3x^6 + \frac{1}{2}\,Ba^2cx^6 + Aabcx^6 + \frac{3}{4}\,Ba^2bx^4 + \frac{3}{4}\,Aab^2x^4 + \frac{3}{4}\,Aa^2cx^4 + \frac{1}{2}\,Ba^3x^2 + \frac{3}{2}\,Aa^2bx^2 + \frac{1}{2}\,Aa^3\log\left(x^2\right)$$

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2+a)^3/x,x, algorithm="giac")
```

```
[Out] 1/14*B*c^3*x^14 + 1/4*B*b*c^2*x^12 + 1/12*A*c^3*x^12 + 3/10*B*b^2*c*x^10 +
3/10*B*a*c^2*x^10 + 3/10*A*b*c^2*x^10 + 1/8*B*b^3*x^8 + 3/4*B*a*b*c*x^8 + 3
/8*A*b^2*c*x^8 + 3/8*A*a*c^2*x^8 + 1/2*B*a*b^2*x^6 + 1/6*A*b^3*x^6 + 1/2*B*
a^2*c*x^6 + A*a*b*c*x^6 + 3/4*B*a^2*b*x^4 + 3/4*A*a*b^2*x^4 + 3/4*A*a^2*c*x
^4 + 1/2*B*a^3*x^2 + 3/2*A*a^2*b*x^2 + 1/2*A*a^3*log(x^2)
```

Mupad [B] (verification not implemented)

Time = 7.77 (sec) , antiderivative size = 166, normalized size of antiderivative = 1.02

$$\int \frac{(A+Bx^2)(a+bx^2+cx^4)^3}{x}\,dx = x^6\left(\frac{B\,c\,a^2}{2}+\frac{B\,a\,b^2}{2}+A\,c\,a\,b+\frac{A\,b^3}{6}\right) + x^8\left(\frac{B\,b^3}{8}+\frac{3\,A\,b^2\,c}{8}+\frac{3\,B\,a\,b\,c}{4}+\frac{3\,A\,a\,c^2}{8}\right) + x^2\left(\frac{B\,a^3}{2}+\frac{3\,A\,b\,a^2}{2}\right) + x^{12}\left(\frac{A\,c^3}{12}+\frac{B\,b\,c^2}{4}\right) + x^4\left(\frac{3\,B\,a^2\,b}{4}+\frac{3\,A\,c\,a^2}{4}+\frac{3\,A\,a\,b^2}{4}\right) + x^{10}\left(\frac{3\,B\,b^2\,c}{10}+\frac{3\,A\,b\,c^2}{10}+\frac{3\,B\,a\,c^2}{10}\right) + \frac{B\,c^3\,x^{14}}{14} + A\,a^3\,\ln(x)$$

```
[In] int(((A + B*x^2)*(a + b*x^2 + c*x^4)^3)/x,x)
```

```
[Out] x^6*((A*b^3)/6 + (B*a*b^2)/2 + (B*a^2*c)/2 + A*a*b*c) + x^8*((B*b^3)/8 + (3
*A*a*c^2)/8 + (3*A*b^2*c)/8 + (3*B*a*b*c)/4) + x^2*((B*a^3)/2 + (3*A*a^2*b)
/2) + x^12*((A*c^3)/12 + (B*b*c^2)/4) + x^4*((3*A*a*b^2)/4 + (3*A*a^2*c)/4
+ (3*B*a^2*b)/4) + x^10*((3*A*b*c^2)/10 + (3*B*a*c^2)/10 + (3*B*b^2*c)/10)
+ (B*c^3*x^14)/14 + A*a^3*log(x)
```

3.100 $\int \frac{(A+Bx^2)(a+bx^2+cx^4)^3}{x^2}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 156

$$\int \frac{(A+Bx^2)(a+bx^2+cx^4)^3}{x^2}\,dx = -\frac{a^3A}{x}+a^2(3Ab+aB)x+a\big(abB+A\big(b^2+ac\big)\big)\,x^3+\frac{1}{5}\big(3aB\big(b^2+ac\big)+A\big(b^3+6abc\big)\big)\,x^5+\frac{1}{7}\big(b^3B+3Ab^2c+6abBc+3aAc^2\big)\,x^7+\frac{1}{3}c\big(b^2B+Abc+aBc\big)\,x^9+\frac{1}{11}c^2(3bB+Ac)x^{11}+\frac{1}{13}Bc^3x^{13}$$

```
[Out] -a^3*A/x+a^2*(3*A*b+B*a)*x+a*(a*b*B+A*(a*c+b^2))*x^3+1/5*(3*a*B*(a*c+b^2)+A
*(6*a*b*c+b^3))*x^5+1/7*(3*A*a*c^2+3*A*b^2*c+6*B*a*b*c+B*b^3)*x^7+1/3*c*(A*
b*c+B*a*c+B*b^2)*x^9+1/11*c^2*(A*c+3*B*b)*x^11+1/13*B*c^3*x^13
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.040, Rules used

$= \{1275\}$

$$\int \frac{(A+Bx^2)\,(a+bx^2+cx^4)^3}{x^2}\,dx = -\frac{a^3A}{x} + a^2x(aB+3Ab) + \frac{1}{3}cx^9\left(aBc+Abc+b^2B\right) + ax^3\left(A\left(ac+b^2\right)+abB\right) + \frac{1}{7}x^7\left(3aAc^2+6abBc+3Ab^2c+b^3B\right) + \frac{1}{5}x^5\left(A\left(6abc+b^3\right)+3aB\left(ac+b^2\right)\right) + \frac{1}{11}c^2x^{11}(Ac+3bB) + \frac{1}{13}Bc^3x^{13}$$

```
[In] Int[((A + B*x^2)*(a + b*x^2 + c*x^4)^3)/x^2,x]
```

```
[Out] -((a^3*A)/x) + a^2*(3*A*b + a*B)*x + a*(a*b*B + A*(b^2 + a*c))*x^3 + ((3*a*
B*(b^2 + a*c) + A*(b^3 + 6*a*b*c))*x^5)/5 + ((b^3*B + 3*A*b^2*c + 6*a*b*B*c
 + 3*a*A*c^2)*x^7)/7 + (c*(b^2*B + A*b*c + a*B*c)*x^9)/3 + (c^2*(3*b*B + A*
c)*x^11)/11 + (B*c^3*x^13)/13
```

Rule 1275

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*
(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && NeQ[
b^2 - 4*a*c, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rubi steps

$$\text{integral} = \int \left(a^2(3Ab+aB) + \frac{a^3A}{x^2} + 3a\left(abB+A\left(b^2+ac\right)\right)x^2 + \left(3aB\left(b^2+ac\right)+A\left(b^3+6abc\right)\right)x^4 + \left(b^3B+3Ab^2c+6abBc+3aAc^2\right)x^6 + 3c\left(b^2B+Abc+aBc\right)x^8 + c^2(3bB+Ac)x^{10} + Bc^3x^{12}\right)dx$$

$$= -\frac{a^3A}{x} + a^2(3Ab+aB)x + a\left(abB+A\left(b^2+ac\right)\right)x^3 + \frac{1}{5}\left(3aB\left(b^2+ac\right)+A\left(b^3+6abc\right)\right)x^5 + \frac{1}{7}\left(b^3B+3Ab^2c+6abBc+3aAc^2\right)x^7 + \frac{1}{3}c\left(b^2B+Abc+aBc\right)x^9 + \frac{1}{11}c^2(3bB+Ac)x^{11} + \frac{1}{13}Bc^3x^{13}$$

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.00

$$\int \frac{(A+Bx^2)(a+bx^2+cx^4)^3}{x^2}\,dx = -\frac{a^3A}{x}+a^2(3Ab+aB)x+a\left(abB+A\left(b^2+ac\right)\right)x^3 + \frac{1}{5}\left(3aB\left(b^2+ac\right)+A\left(b^3+6abc\right)\right)x^5 + \frac{1}{7}\left(b^3B+3Ab^2c+6abBc+3aAc^2\right)x^7 + \frac{1}{3}c\left(b^2B+Abc+aBc\right)x^9 + \frac{1}{11}c^2(3bB+Ac)x^{11}+\frac{1}{13}Bc^3x^{13}$$

```
[In] Integrate[((A + B*x^2)*(a + b*x^2 + c*x^4)^3)/x^2,x]
```

```
[Out] -((a^3*A)/x) + a^2*(3*A*b + a*B)*x + a*(a*b*B + A*(b^2 + a*c))*x^3 + ((3*a*
B*(b^2 + a*c) + A*(b^3 + 6*a*b*c))*x^5)/5 + ((b^3*B + 3*A*b^2*c + 6*a*b*B*c
 + 3*a*A*c^2)*x^7)/7 + (c*(b^2*B + A*b*c + a*B*c)*x^9)/3 + (c^2*(3*b*B + A*
c)*x^11)/11 + (B*c^3*x^13)/13
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 167, normalized size of antiderivative = 1.07

method	result
norman	$\frac{-Aa^3+(3Aa^2b+Ba^3)x^2+(Aca^2+Aab^2+Ba^2b)x^4+(\frac{6}{5}Aabc+\frac{1}{5}Ab^3+\frac{3}{5}a^2Bc+\frac{3}{5}Bab^2)x^6+(\frac{3}{7}Aac^2+\frac{3}{7}Ab^2c+\frac{6}{7}Babc+\frac{1}{7}Bb^3}{x}$
default	$\frac{Bc^3x^{13}}{13}+\frac{Ac^3x^{11}}{11}+\frac{3Bbc^2x^{11}}{11}+\frac{Abc^2x^9}{3}+\frac{Bac^2x^9}{3}+\frac{Bb^2cx^9}{3}+\frac{3Aac^2x^7}{7}+\frac{3Ab^2cx^7}{7}+\frac{6Babcx^7}{7}+\frac{Bb^3x^7}{7}$
risch	$\frac{Bc^3x^{13}}{13}+\frac{Ac^3x^{11}}{11}+\frac{3Bbc^2x^{11}}{11}+\frac{Abc^2x^9}{3}+\frac{Bac^2x^9}{3}+\frac{Bb^2cx^9}{3}+\frac{3Aac^2x^7}{7}+\frac{3Ab^2cx^7}{7}+\frac{6Babcx^7}{7}+\frac{Bb^3x^7}{7}$
gosper	$-\frac{-1155Bc^3x^{14}-1365Ac^3x^{12}-4095Bbc^2x^{12}-5005Abc^2x^{10}-5005Bac^2x^{10}-5005Bb^2cx^{10}-6435Aac^2x^8-6435Ab^2cx^8-12}{}$
parallelrisch	$\frac{1155Bc^3x^{14}+1365Ac^3x^{12}+4095Bbc^2x^{12}+5005Abc^2x^{10}+5005Bac^2x^{10}+5005Bb^2cx^{10}+6435Aac^2x^8+6435Ab^2cx^8+12870}{}$

```
[In] int((B*x^2+A)*(c*x^4+b*x^2+a)^3/x^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/x*(-A*a^3+(3*A*a^2*b+B*a^3)*x^2+(A*a^2*c+A*a*b^2+B*a^2*b)*x^4+(6/5*A*a*b*
c+1/5*A*b^3+3/5*a^2*B*c+3/5*B*a*b^2)*x^6+(3/7*A*a*c^2+3/7*A*b^2*c+6/7*B*a*b
*c+1/7*B*b^3)*x^8+(1/3*A*b*c^2+1/3*B*a*c^2+1/3*B*b^2*c)*x^10+(1/11*A*c^3+3/
11*B*b*c^2)*x^12+1/13*B*c^3*x^14)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 168, normalized size of antiderivative = 1.08

$$\int \frac{(A+Bx^2)(a+bx^2+cx^4)^3}{x^2}\,dx$$
$$= \frac{1155\,Bc^3x^{14}+1365\,(3\,Bbc^2+Ac^3)x^{12}+5005\,(Bb^2c+(Ba+Ab)c^2)x^{10}+2145\,(Bb^3+3\,Aac^2+3\,(2\,Bab}{}$$

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2+a)^3/x^2,x, algorithm="fricas")
```

```
[Out] 1/15015*(1155*B*c^3*x^14 + 1365*(3*B*b*c^2 + A*c^3)*x^12 + 5005*(B*b^2*c +
(B*a + A*b)*c^2)*x^10 + 2145*(B*b^3 + 3*A*a*c^2 + 3*(2*B*a*b + A*b^2)*c)*x^
8 + 3003*(3*B*a*b^2 + A*b^3 + 3*(B*a^2 + 2*A*a*b)*c)*x^6 + 15015*(B*a^2*b +
 A*a*b^2 + A*a^2*c)*x^4 - 15015*A*a^3 + 15015*(B*a^3 + 3*A*a^2*b)*x^2)/x
```

Sympy [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 185, normalized size of antiderivative = 1.19

$$\int \frac{(A+Bx^2)(a+bx^2+cx^4)^3}{x^2}\,dx = -\frac{Aa^3}{x}+\frac{Bc^3x^{13}}{13}+x^{11}\left(\frac{Ac^3}{11}+\frac{3Bbc^2}{11}\right)+x^9\left(\frac{Abc^2}{3}+\frac{Bac^2}{3}+\frac{Bb^2c}{3}\right)+x^7\cdot\left(\frac{3Aac^2}{7}+\frac{3Ab^2c}{7}+\frac{6Babc}{7}+\frac{Bb^3}{7}\right)+x^5\cdot\left(\frac{6Aabc}{5}+\frac{Ab^3}{5}+\frac{3Ba^2c}{5}+\frac{3Bab^2}{5}\right)+x^3\left(Aa^2c+Aab^2+Ba^2b\right)+x\left(3Aa^2b+Ba^3\right)$$

```
[In] integrate((B*x**2+A)*(c*x**4+b*x**2+a)**3/x**2,x)
```

```
[Out] -A*a**3/x + B*c**3*x**13/13 + x**11*(A*c**3/11 + 3*B*b*c**2/11) + x**9*(A*b
*c**2/3 + B*a*c**2/3 + B*b**2*c/3) + x**7*(3*A*a*c**2/7 + 3*A*b**2*c/7 + 6*
B*a*b*c/7 + B*b**3/7) + x**5*(6*A*a*b*c/5 + A*b**3/5 + 3*B*a**2*c/5 + 3*B*a
*b**2/5) + x**3*(A*a**2*c + A*a*b**2 + B*a**2*b) + x*(3*A*a**2*b + B*a**3)
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 162, normalized size of antiderivative = 1.04

$$\int \frac{(A+Bx^2)(a+bx^2+cx^4)^3}{x^2}\,dx = \frac{1}{13}Bc^3x^{13} + \frac{1}{11}\left(3Bbc^2 + Ac^3\right)x^{11} + \frac{1}{3}\left(Bb^2c + (Ba+Ab)c^2\right)x^9 + \frac{1}{7}\left(Bb^3 + 3Aac^2 + 3\left(2Bab + Ab^2\right)c\right)x^7 + \frac{1}{5}\left(3Bab^2 + Ab^3 + 3\left(Ba^2 + 2Aab\right)c\right)x^5 + \left(Ba^2b + Aab^2 + Aa^2c\right)x^3 - \frac{Aa^3}{x} + \left(Ba^3 + 3Aa^2b\right)x$$

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2+a)^3/x^2,x, algorithm="maxima")
```

```
[Out] 1/13*B*c^3*x^13 + 1/11*(3*B*b*c^2 + A*c^3)*x^11 + 1/3*(B*b^2*c + (B*a + A*b
)*c^2)*x^9 + 1/7*(B*b^3 + 3*A*a*c^2 + 3*(2*B*a*b + A*b^2)*c)*x^7 + 1/5*(3*B
*a*b^2 + A*b^3 + 3*(B*a^2 + 2*A*a*b)*c)*x^5 + (B*a^2*b + A*a*b^2 + A*a^2*c)
*x^3 - A*a^3/x + (B*a^3 + 3*A*a^2*b)*x
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 185, normalized size of antiderivative = 1.19

$$\int \frac{(A+Bx^2)(a+bx^2+cx^4)^3}{x^2}\,dx = \frac{1}{13}Bc^3x^{13} + \frac{3}{11}Bbc^2x^{11} + \frac{1}{11}Ac^3x^{11} + \frac{1}{3}Bb^2cx^9 + \frac{1}{3}Bac^2x^9 + \frac{1}{3}Abc^2x^9 + \frac{1}{7}Bb^3x^7 + \frac{6}{7}Babcx^7 + \frac{3}{7}Ab^2cx^7 + \frac{3}{7}Aac^2x^7 + \frac{3}{5}Bab^2x^5 + \frac{1}{5}Ab^3x^5 + \frac{3}{5}Ba^2cx^5 + \frac{6}{5}Aabcx^5 + Ba^2bx^3 + Aab^2x^3 + Aa^2cx^3 + Ba^3x + 3Aa^2bx - \frac{Aa^3}{x}$$

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2+a)^3/x^2,x, algorithm="giac")
```

```
[Out] 1/13*B*c^3*x^13 + 3/11*B*b*c^2*x^11 + 1/11*A*c^3*x^11 + 1/3*B*b^2*c*x^9 + 1
/3*B*a*c^2*x^9 + 1/3*A*b*c^2*x^9 + 1/7*B*b^3*x^7 + 6/7*B*a*b*c*x^7 + 3/7*A*
b^2*c*x^7 + 3/7*A*a*c^2*x^7 + 3/5*B*a*b^2*x^5 + 1/5*A*b^3*x^5 + 3/5*B*a^2*c
*x^5 + 6/5*A*a*b*c*x^5 + B*a^2*b*x^3 + A*a*b^2*x^3 + A*a^2*c*x^3 + B*a^3*x
+ 3*A*a^2*b*x - A*a^3/x
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.04

$$\int \frac{(A+Bx^2)(a+bx^2+cx^4)^3}{x^2}\,dx = x^5\left(\frac{3\,B\,c\,a^2}{5}+\frac{3\,B\,a\,b^2}{5}+\frac{6\,A\,c\,a\,b}{5}+\frac{A\,b^3}{5}\right) + x^7\left(\frac{B\,b^3}{7}+\frac{3\,A\,b^2\,c}{7}+\frac{6\,B\,a\,b\,c}{7}+\frac{3\,A\,a\,c^2}{7}\right) + x\left(B\,a^3+3\,A\,b\,a^2\right)+x^{11}\left(\frac{A\,c^3}{11}+\frac{3\,B\,b\,c^2}{11}\right) + x^3\left(B\,a^2\,b+A\,c\,a^2+A\,a\,b^2\right) + x^9\left(\frac{B\,b^2\,c}{3}+\frac{A\,b\,c^2}{3}+\frac{B\,a\,c^2}{3}\right)-\frac{A\,a^3}{x}+\frac{B\,c^3\,x^{13}}{13}$$

```
[In] int(((A + B*x^2)*(a + b*x^2 + c*x^4)^3)/x^2,x)
```

```
[Out] x^5*((A*b^3)/5 + (3*B*a*b^2)/5 + (3*B*a^2*c)/5 + (6*A*a*b*c)/5) + x^7*((B*b
^3)/7 + (3*A*a*c^2)/7 + (3*A*b^2*c)/7 + (6*B*a*b*c)/7) + x*(B*a^3 + 3*A*a^2
*b) + x^11*((A*c^3)/11 + (3*B*b*c^2)/11) + x^3*(A*a*b^2 + A*a^2*c + B*a^2*b
) + x^9*((A*b*c^2)/3 + (B*a*c^2)/3 + (B*b^2*c)/3) - (A*a^3)/x + (B*c^3*x^13
)/13
```

3.101 $\int \frac{(A+Bx^2)(a+bx^2+cx^4)^3}{x^3}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 162

$$\begin{aligned}\int \frac{(A+Bx^2)(a+bx^2+cx^4)^3}{x^3}\,dx = {} & -\frac{a^3A}{2x^2}+\frac{3}{2}a\big(abB+A\big(b^2+ac\big)\big)x^2 \\ & +\frac{1}{4}\big(3aB\big(b^2+ac\big)+A\big(b^3+6abc\big)\big)x^4 \\ & +\frac{1}{6}\big(b^3B+3Ab^2c+6abBc+3aAc^2\big)x^6 \\ & +\frac{3}{8}c\big(b^2B+Abc+aBc\big)x^8+\frac{1}{10}c^2(3bB+Ac)x^{10} \\ & +\frac{1}{12}Bc^3x^{12}+a^2(3Ab+aB)\log(x)\end{aligned}$$

```
[Out] -1/2*a^3*A/x^2+3/2*a*(a*b*B+A*(a*c+b^2))*x^2+1/4*(3*a*B*(a*c+b^2)+A*(6*a*b*
c+b^3))*x^4+1/6*(3*A*a*c^2+3*A*b^2*c+6*B*a*b*c+B*b^3)*x^6+3/8*c*(A*b*c+B*a*
c+B*b^2)*x^8+1/10*c^2*(A*c+3*B*b)*x^10+1/12*B*c^3*x^12+a^2*(3*A*b+B*a)*ln(x
)
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 162, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.080$, Rules used

$= \{1265, 779\}$

$$\int \frac{(A+Bx^2)(a+bx^2+cx^4)^3}{x^3}\,dx = -\frac{a^3A}{2x^2} + a^2\log(x)(aB+3Ab) + \frac{3}{8}cx^8\left(aBc+Abc+b^2B\right) + \frac{3}{2}ax^2\left(A\left(ac+b^2\right)+abB\right) + \frac{1}{6}x^6\left(3aAc^2+6abBc+3Ab^2c+b^3B\right) + \frac{1}{4}x^4\left(A\left(6abc+b^3\right)+3aB\left(ac+b^2\right)\right) + \frac{1}{10}c^2x^{10}(Ac+3bB) + \frac{1}{12}Bc^3x^{12}$$

```
[In] Int[((A + B*x^2)*(a + b*x^2 + c*x^4)^3)/x^3,x]
```

```
[Out] -1/2*(a^3*A)/x^2 + (3*a*(a*b*B + A*(b^2 + a*c))*x^2)/2 + ((3*a*B*(b^2 + a*c
) + A*(b^3 + 6*a*b*c))*x^4)/4 + ((b^3*B + 3*A*b^2*c + 6*a*b*B*c + 3*a*A*c^2
)*x^6)/6 + (3*c*(b^2*B + A*b*c + a*B*c)*x^8)/8 + (c^2*(3*b*B + A*c)*x^10)/1
0 + (B*c^3*x^12)/12 + a^2*(3*A*b + a*B)*Log[x]
```

Rule 779

```
Int[((e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_
)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(f + g*x)*(a + b*x + c
*x^2)^p, x], x] /; FreeQ[{a, b, c, e, f, g, m}, x] && IntegerQ[p] && (GtQ[p
, 0] || (EqQ[a, 0] && IntegerQ[m]))
```

Rule 1265

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rubi steps

$$\text{integral} = \frac{1}{2}\text{Subst}\left(\int \frac{(A+Bx)(a+bx+cx^2)^3}{x^2}\,dx, x, x^2\right)$$

$$= \frac{1}{2}\text{Subst}\left(\int \left(3a\left(abB+A\left(b^2+ac\right)\right) + \frac{a^3A}{x^2} + \frac{a^2(3Ab+aB)}{x} + \left(3aB\left(b^2+ac\right)+A\left(b^3+6abc\right)\right)x + \left(b^3B+3Ab^2c+6abBc+3aAc^2\right)x^2 + 3c\left(b^2B+Abc+aBc\right)x^3 + c^2(3bB+Ac)x^4 + Bc^3x^5\right)dx, x, x^2\right)$$

$$= -\frac{a^3A}{2x^2} + \frac{3}{2}a\left(abB + A\left(b^2 + ac\right)\right)x^2 + \frac{1}{4}\left(3aB\left(b^2 + ac\right) + A\left(b^3 + 6abc\right)\right)x^4 + \frac{1}{6}\left(b^3B + 3Ab^2c + 6abBc + 3aAc^2\right)x^6 + \frac{3}{8}c\left(b^2B + Abc + aBc\right)x^8 + \frac{1}{10}c^2(3bB + Ac)x^{10} + \frac{1}{12}Bc^3x^{12} + a^2(3Ab + aB)\log(x)$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 162, normalized size of antiderivative = 1.00

$$\int \frac{(A + Bx^2)(a + bx^2 + cx^4)^3}{x^3}\,dx = -\frac{a^3A}{2x^2} + \frac{3}{2}a\left(abB + A\left(b^2 + ac\right)\right)x^2 + \frac{1}{4}\left(3aB\left(b^2 + ac\right) + A\left(b^3 + 6abc\right)\right)x^4 + \frac{1}{6}\left(b^3B + 3Ab^2c + 6abBc + 3aAc^2\right)x^6 + \frac{3}{8}c\left(b^2B + Abc + aBc\right)x^8 + \frac{1}{10}c^2(3bB + Ac)x^{10} + \frac{1}{12}Bc^3x^{12} + a^2(3Ab + aB)\log(x)$$

```
[In] Integrate[((A + B*x^2)*(a + b*x^2 + c*x^4)^3)/x^3,x]
```

```
[Out] -1/2*(a^3*A)/x^2 + (3*a*(a*b*B + A*(b^2 + a*c))*x^2)/2 + ((3*a*B*(b^2 + a*c
) + A*(b^3 + 6*a*b*c))*x^4)/4 + ((b^3*B + 3*A*b^2*c + 6*a*b*B*c + 3*a*A*c^2
)*x^6)/6 + (3*c*(b^2*B + A*b*c + a*B*c)*x^8)/8 + (c^2*(3*b*B + A*c)*x^10)/1
0 + (B*c^3*x^12)/12 + a^2*(3*A*b + a*B)*Log[x]
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 169, normalized size of antiderivative = 1.04

method	result
norman	$\frac{\left(\frac{1}{10}A\,c^3+\frac{3}{10}Bb\,c^2\right)x^{12}+\left(\frac{3}{8}Ab\,c^2+\frac{3}{8}Ba\,c^2+\frac{3}{8}B\,b^2c\right)x^{10}+\left(\frac{3}{2}Ac\,a^2+\frac{3}{2}Aa\,b^2+\frac{3}{2}B\,a^2b\right)x^4+\left(\frac{1}{2}Aa\,c^2+\frac{1}{2}A\,b^2c+Babc+\frac{1}{6}B\,b^3\right)x^8+(}{x^2}$
default	$\frac{B\,c^3x^{12}}{12}+\frac{A\,c^3x^{10}}{10}+\frac{3Bb\,c^2x^{10}}{10}+\frac{3Ab\,c^2x^8}{8}+\frac{3Ba\,c^2x^8}{8}+\frac{3B\,b^2c\,x^8}{8}+\frac{Aa\,c^2x^6}{2}+\frac{A\,b^2c\,x^6}{2}+Babc\,x^6+\frac{B\,b^3x}{6}$
risch	$\frac{B\,c^3x^{12}}{12}+\frac{A\,c^3x^{10}}{10}+\frac{3Bb\,c^2x^{10}}{10}+\frac{3Ab\,c^2x^8}{8}+\frac{3Ba\,c^2x^8}{8}+\frac{3B\,b^2c\,x^8}{8}+\frac{Aa\,c^2x^6}{2}+\frac{A\,b^2c\,x^6}{2}+Babc\,x^6+\frac{B\,b^3x}{6}$
parallelrisch	$\frac{10B\,c^3x^{14}+12A\,c^3x^{12}+36Bb\,c^2x^{12}+45Ab\,c^2x^{10}+45Ba\,c^2x^{10}+45B\,b^2c\,x^{10}+60Aa\,c^2x^8+60A\,b^2c\,x^8+120Babc\,x^8+20B\,b^3x^8+1}{1}$

```
[In] int((B*x^2+A)*(c*x^4+b*x^2+a)^3/x^3,x,method=_RETURNVERBOSE)
```

```
[Out] ((1/10*A*c^3+3/10*B*b*c^2)*x^12+(3/8*A*b*c^2+3/8*B*a*c^2+3/8*B*b^2*c)*x^10+
(3/2*A*c*a^2+3/2*A*a*b^2+3/2*B*a^2*b)*x^4+(1/2*A*a*c^2+1/2*A*b^2*c+B*a*b*c+
```

```
1/6*B*b^3)*x^8+(3/2*A*a*b*c+1/4*A*b^3+3/4*a^2*B*c+3/4*B*a*b^2)*x^6-1/2*A*a^
3+1/12*B*c^3*x^14)/x^2+(3*A*a^2*b+B*a^3)*ln(x)
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.05

$$\int \frac{(A+Bx^2)(a+bx^2+cx^4)^3}{x^3}\,dx$$
$$= \frac{10\,Bc^3x^{14}+12\,(3\,Bbc^2+Ac^3)x^{12}+45\,(Bb^2c+(Ba+Ab)c^2)x^{10}+20\,(Bb^3+3\,Aac^2+3\,(2\,Bab+Ab^2)c)}{}$$

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2+a)^3/x^3,x, algorithm="fricas")
```

```
[Out] 1/120*(10*B*c^3*x^14 + 12*(3*B*b*c^2 + A*c^3)*x^12 + 45*(B*b^2*c + (B*a + A
*b)*c^2)*x^10 + 20*(B*b^3 + 3*A*a*c^2 + 3*(2*B*a*b + A*b^2)*c)*x^8 + 30*(3*
B*a*b^2 + A*b^3 + 3*(B*a^2 + 2*A*a*b)*c)*x^6 + 180*(B*a^2*b + A*a*b^2 + A*a
^2*c)*x^4 - 60*A*a^3 + 120*(B*a^3 + 3*A*a^2*b)*x^2*log(x))/x^2
```

Sympy [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 197, normalized size of antiderivative = 1.22

$$\int \frac{(A+Bx^2)(a+bx^2+cx^4)^3}{x^3}\,dx = -\frac{Aa^3}{2x^2}+\frac{Bc^3x^{12}}{12}+a^2\cdot(3Ab+Ba)\log(x)$$
$$+x^{10}\left(\frac{Ac^3}{10}+\frac{3Bbc^2}{10}\right)+x^8\cdot\left(\frac{3Abc^2}{8}+\frac{3Bac^2}{8}+\frac{3Bb^2c}{8}\right)$$
$$+x^6\left(\frac{Aac^2}{2}+\frac{Ab^2c}{2}+Babc+\frac{Bb^3}{6}\right)+x^4$$
$$\cdot\left(\frac{3Aabc}{2}+\frac{Ab^3}{4}+\frac{3Ba^2c}{4}+\frac{3Bab^2}{4}\right)$$
$$+x^2\cdot\left(\frac{3Aa^2c}{2}+\frac{3Aab^2}{2}+\frac{3Ba^2b}{2}\right)$$

```
[In] integrate((B*x**2+A)*(c*x**4+b*x**2+a)**3/x**3,x)
```

```
[Out] -A*a**3/(2*x**2) + B*c**3*x**12/12 + a**2*(3*A*b + B*a)*log(x) + x**10*(A*c
**3/10 + 3*B*b*c**2/10) + x**8*(3*A*b*c**2/8 + 3*B*a*c**2/8 + 3*B*b**2*c/8)
 + x**6*(A*a*c**2/2 + A*b**2*c/2 + B*a*b*c + B*b**3/6) + x**4*(3*A*a*b*c/2
+ A*b**3/4 + 3*B*a**2*c/4 + 3*B*a*b**2/4) + x**2*(3*A*a**2*c/2 + 3*A*a*b**2
/2 + 3*B*a**2*b/2)
```

Maxima [A] (verification not implemented)

none

Time = 0.18 (sec) , antiderivative size = 167, normalized size of antiderivative = 1.03

$$\int \frac{(A+Bx^2)\,(a+bx^2+cx^4)^3}{x^3}\,dx = \frac{1}{12}\,Bc^3x^{12} + \frac{1}{10}\,\left(3\,Bbc^2 + Ac^3\right)x^{10} + \frac{3}{8}\,\left(Bb^2c + (Ba+Ab)c^2\right)x^8 + \frac{1}{6}\,\left(Bb^3 + 3\,Aac^2 + 3\,\left(2\,Bab + Ab^2\right)c\right)x^6 + \frac{1}{4}\,\left(3\,Bab^2 + Ab^3 + 3\,\left(Ba^2 + 2\,Aab\right)c\right)x^4 + \frac{3}{2}\,\left(Ba^2b + Aab^2 + Aa^2c\right)x^2 - \frac{Aa^3}{2\,x^2} + \frac{1}{2}\,\left(Ba^3 + 3\,Aa^2b\right)\log\left(x^2\right)$$

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2+a)^3/x^3,x, algorithm="maxima")
```

```
[Out] 1/12*B*c^3*x^12 + 1/10*(3*B*b*c^2 + A*c^3)*x^10 + 3/8*(B*b^2*c + (B*a + A*b
)*c^2)*x^8 + 1/6*(B*b^3 + 3*A*a*c^2 + 3*(2*B*a*b + A*b^2)*c)*x^6 + 1/4*(3*B
*a*b^2 + A*b^3 + 3*(B*a^2 + 2*A*a*b)*c)*x^4 + 3/2*(B*a^2*b + A*a*b^2 + A*a^
2*c)*x^2 - 1/2*A*a^3/x^2 + 1/2*(B*a^3 + 3*A*a^2*b)*log(x^2)
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 212, normalized size of antiderivative = 1.31

$$\int \frac{(A+Bx^2)\,(a+bx^2+cx^4)^3}{x^3}\,dx = \frac{1}{12}\,Bc^3x^{12} + \frac{3}{10}\,Bbc^2x^{10} + \frac{1}{10}\,Ac^3x^{10} + \frac{3}{8}\,Bb^2cx^8 + \frac{3}{8}\,Bac^2x^8 + \frac{3}{8}\,Abc^2x^8 + \frac{1}{6}\,Bb^3x^6 + Babcx^6 + \frac{1}{2}\,Ab^2cx^6 + \frac{1}{2}\,Aac^2x^6 + \frac{3}{4}\,Bab^2x^4 + \frac{1}{4}\,Ab^3x^4 + \frac{3}{4}\,Ba^2cx^4 + \frac{3}{2}\,Aabcx^4 + \frac{3}{2}\,Ba^2bx^2 + \frac{3}{2}\,Aab^2x^2 + \frac{3}{2}\,Aa^2cx^2 + \frac{1}{2}\,\left(Ba^3 + 3\,Aa^2b\right)\log\left(x^2\right) - \frac{Ba^3x^2 + 3\,Aa^2bx^2 + Aa^3}{2\,x^2}$$

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2+a)^3/x^3,x, algorithm="giac")
```

```
[Out] 1/12*B*c^3*x^12 + 3/10*B*b*c^2*x^10 + 1/10*A*c^3*x^10 + 3/8*B*b^2*c*x^8 + 3
/8*B*a*c^2*x^8 + 3/8*A*b*c^2*x^8 + 1/6*B*b^3*x^6 + B*a*b*c*x^6 + 1/2*A*b^2*
```

```
c*x^6 + 1/2*A*a*c^2*x^6 + 3/4*B*a*b^2*x^4 + 1/4*A*b^3*x^4 + 3/4*B*a^2*c*x^4
 + 3/2*A*a*b*c*x^4 + 3/2*B*a^2*b*x^2 + 3/2*A*a*b^2*x^2 + 3/2*A*a^2*c*x^2 +
1/2*(B*a^3 + 3*A*a^2*b)*log(x^2) - 1/2*(B*a^3*x^2 + 3*A*a^2*b*x^2 + A*a^3)/
x^2
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 166, normalized size of antiderivative = 1.02

$$\int \frac{(A+Bx^2)(a+bx^2+cx^4)^3}{x^3}\,dx = x^4\left(\frac{3\,B\,c\,a^2}{4}+\frac{3\,B\,a\,b^2}{4}+\frac{3\,A\,c\,a\,b}{2}+\frac{A\,b^3}{4}\right) + x^6\left(\frac{B\,b^3}{6}+\frac{A\,b^2\,c}{2}+B\,a\,b\,c+\frac{A\,a\,c^2}{2}\right) + x^{10}\left(\frac{A\,c^3}{10}+\frac{3\,B\,b\,c^2}{10}\right)+\ln(x)\,\left(B\,a^3+3\,A\,b\,a^2\right) + x^2\left(\frac{3\,B\,a^2\,b}{2}+\frac{3\,A\,c\,a^2}{2}+\frac{3\,A\,a\,b^2}{2}\right) + x^8\left(\frac{3\,B\,b^2\,c}{8}+\frac{3\,A\,b\,c^2}{8}+\frac{3\,B\,a\,c^2}{8}\right)-\frac{A\,a^3}{2\,x^2}+\frac{B\,c^3\,x^{12}}{12}$$

```
[In] int(((A + B*x^2)*(a + b*x^2 + c*x^4)^3)/x^3,x)
```

```
[Out] x^4*((A*b^3)/4 + (3*B*a*b^2)/4 + (3*B*a^2*c)/4 + (3*A*a*b*c)/2) + x^6*((B*b
^3)/6 + (A*a*c^2)/2 + (A*b^2*c)/2 + B*a*b*c) + x^10*((A*c^3)/10 + (3*B*b*c^
2)/10) + log(x)*(B*a^3 + 3*A*a^2*b) + x^2*((3*A*a*b^2)/2 + (3*A*a^2*c)/2 +
(3*B*a^2*b)/2) + x^8*((3*A*b*c^2)/8 + (3*B*a*c^2)/8 + (3*B*b^2*c)/8) - (A*a
^3)/(2*x^2) + (B*c^3*x^12)/12
```

3.102 $\int \frac{x^5(A+Bx^2)}{a+bx^2+cx^4} dx$

Optimal result

Integrand size = 25, antiderivative size = 133

$$\int \frac{x^5(A+Bx^2)}{a+bx^2+cx^4} dx = -\frac{(bB-Ac)x^2}{2c^2} + \frac{Bx^4}{4c} + \frac{(b^3B-Ab^2c-3abBc+2aAc^2)\operatorname{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2c^3\sqrt{b^2-4ac}} + \frac{(b^2B-Abc-aBc)\log(a+bx^2+cx^4)}{4c^3}$$

```
[Out] -1/2*(-A*c+B*b)*x^2/c^2+1/4*B*x^4/c+1/4*(-A*b*c-B*a*c+B*b^2)*ln(c*x^4+b*x^2
+a)/c^3+1/2*(2*A*a*c^2-A*b^2*c-3*B*a*b*c+B*b^3)*arctanh((2*c*x^2+b)/(-4*a*c
+b^2)^(1/2))/c^3/(-4*a*c+b^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {1265, 814, 648, 632, 212, 642}

$$\int \frac{x^5(A+Bx^2)}{a+bx^2+cx^4} dx = \frac{(2aAc^2-3abBc-Ab^2c+b^3B)\operatorname{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2c^3\sqrt{b^2-4ac}} + \frac{(-aBc-Abc+b^2B)\log(a+bx^2+cx^4)}{4c^3} - \frac{x^2(bB-Ac)}{2c^2} + \frac{Bx^4}{4c}$$

```
[In] Int[(x^5*(A + B*x^2))/(a + b*x^2 + c*x^4),x]
```

```
[Out] -1/2*((b*B - A*c)*x^2)/c^2 + (B*x^4)/(4*c) + ((b^3*B - A*b^2*c - 3*a*b*B*c
+ 2*a*A*c^2)*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(2*c^3*Sqrt[b^2 - 4*
a*c]) + ((b^2*B - A*b*c - a*B*c)*Log[a + b*x^2 + c*x^4])/(4*c^3)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 814

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)/(a +
 b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 1265

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{1}{2}\text{Subst}\left(\int \frac{x^2(A+Bx)}{a+bx+cx^2}\,dx, x, x^2\right)\\ &= \frac{1}{2}\text{Subst}\left(\int \left(-\frac{bB-Ac}{c^2}+\frac{Bx}{c}+\frac{a(bB-Ac)+(b^2B-Abc-aBc)\,x}{c^2\,(a+bx+cx^2)}\right)\,dx, x, x^2\right)\end{aligned}$$

$$= -\frac{(bB-Ac)x^2}{2c^2}+\frac{Bx^4}{4c}+\frac{\text{Subst}\left(\int \frac{a(bB-Ac)+(b^2B-Abc-aBc)x}{a+bx+cx^2}\,dx,x,x^2\right)}{2c^2}$$

$$= -\frac{(bB-Ac)x^2}{2c^2}+\frac{Bx^4}{4c}+\frac{(b^2B-Abc-aBc)\,\text{Subst}\left(\int \frac{b+2cx}{a+bx+cx^2}\,dx,x,x^2\right)}{4c^3} - \frac{(b^3B-Ab^2c-3abBc+2aAc^2)\,\text{Subst}\left(\int \frac{1}{a+bx+cx^2}\,dx,x,x^2\right)}{4c^3}$$

$$= -\frac{(bB-Ac)x^2}{2c^2}+\frac{Bx^4}{4c}+\frac{(b^2B-Abc-aBc)\log(a+bx^2+cx^4)}{4c^3} + \frac{(b^3B-Ab^2c-3abBc+2aAc^2)\,\text{Subst}\left(\int \frac{1}{b^2-4ac-x^2}\,dx,x,b+2cx^2\right)}{2c^3}$$

$$= -\frac{(bB-Ac)x^2}{2c^2}+\frac{Bx^4}{4c}+\frac{(b^3B-Ab^2c-3abBc+2aAc^2)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2c^3\sqrt{b^2-4ac}} + \frac{(b^2B-Abc-aBc)\log(a+bx^2+cx^4)}{4c^3}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.95

$$\int \frac{x^5(A+Bx^2)}{a+bx^2+cx^4}\,dx$$

$$= \frac{2c(-bB+Ac)x^2+Bc^2x^4+\frac{2(-b^3B+Ab^2c+3abBc-2aAc^2)\arctan\left(\frac{b+2cx^2}{\sqrt{-b^2+4ac}}\right)}{\sqrt{-b^2+4ac}}+(b^2B-Abc-aBc)\log(a+bx^2+c}{4c^3}$$

```
[In] Integrate[(x^5*(A + B*x^2))/(a + b*x^2 + c*x^4),x]
```

```
[Out] (2*c*(-(b*B) + A*c)*x^2 + B*c^2*x^4 + (2*(-(b^3*B) + A*b^2*c + 3*a*b*B*c -
2*a*A*c^2)*ArcTan[(b + 2*c*x^2)/Sqrt[-b^2 + 4*a*c]])/Sqrt[-b^2 + 4*a*c] + (
b^2*B - A*b*c - a*B*c)*Log[a + b*x^2 + c*x^4])/(4*c^3)
```

Maple [A] (verified)

Time = 0.20 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.02

method	result	size
default	$\frac{\frac{1}{2}Bx^4c+Acx^2-Bbx^2}{2c^2}+\frac{\frac{\left(-Abc-Bac+Bb^2\right)\ln\left(cx^4+bx^2+a\right)}{2c}+\frac{2\left(-Aac+abB-\frac{\left(-Abc-Bac+Bb^2\right)b}{2c}\right)\arctan\left(\frac{2cx^2+b}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}}}{2c^2}$	136
risch	Expression too large to display	2131

```
[In] int(x^5*(B*x^2+A)/(c*x^4+b*x^2+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2/c^2*(1/2*B*x^4*c+A*c*x^2-B*b*x^2)+1/2/c^2*(1/2*(-A*b*c-B*a*c+B*b^2)/c*l
n(c*x^4+b*x^2+a)+2*(-A*a*c+a*b*B-1/2*(-A*b*c-B*a*c+B*b^2)*b/c)/(4*a*c-b^2)^
(1/2)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2)))
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 421, normalized size of antiderivative = 3.17

$$\int \frac{x^5(A+Bx^2)}{a+bx^2+cx^4}\,dx$$
$$= \left[\frac{(Bb^2c^2-4\,Bac^3)x^4-2\,(Bb^3c+4\,Aac^3-(4\,Bab+Ab^2)c^2)x^2+(Bb^3+2\,Aac^2-(3\,Bab+Ab^2)c)\sqrt{b^2-}}{4\,(b}\right.$$

```
[In] integrate(x^5*(B*x^2+A)/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] [1/4*((B*b^2*c^2 - 4*B*a*c^3)*x^4 - 2*(B*b^3*c + 4*A*a*c^3 - (4*B*a*b + A*b
^2)*c^2)*x^2 + (B*b^3 + 2*A*a*c^2 - (3*B*a*b + A*b^2)*c)*sqrt(b^2 - 4*a*c)*
log((2*c^2*x^4 + 2*b*c*x^2 + b^2 - 2*a*c + (2*c*x^2 + b)*sqrt(b^2 - 4*a*c))
/(c*x^4 + b*x^2 + a)) + (B*b^4 + 4*(B*a^2 + A*a*b)*c^2 - (5*B*a*b^2 + A*b^3
)*c)*log(c*x^4 + b*x^2 + a))/(b^2*c^3 - 4*a*c^4), 1/4*((B*b^2*c^2 - 4*B*a*c
^3)*x^4 - 2*(B*b^3*c + 4*A*a*c^3 - (4*B*a*b + A*b^2)*c^2)*x^2 + 2*(B*b^3 +
2*A*a*c^2 - (3*B*a*b + A*b^2)*c)*sqrt(-b^2 + 4*a*c)*arctan(-(2*c*x^2 + b)*s
qrt(-b^2 + 4*a*c)/(b^2 - 4*a*c)) + (B*b^4 + 4*(B*a^2 + A*a*b)*c^2 - (5*B*a*
b^2 + A*b^3)*c)*log(c*x^4 + b*x^2 + a))/(b^2*c^3 - 4*a*c^4)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^5(A+Bx^2)}{a+bx^2+cx^4}\,dx = \text{Timed out}$$

```
[In] integrate(x**5*(B*x**2+A)/(c*x**4+b*x**2+a),x)
```

```
[Out] Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^5(A+Bx^2)}{a+bx^2+cx^4}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(x^5*(B*x^2+A)/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

Giac [A] (verification not implemented)

none

Time = 0.62 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.95

$$\int \frac{x^5(A+Bx^2)}{a+bx^2+cx^4}\,dx = \frac{Bcx^4-2\,Bbx^2+2\,Acx^2}{4\,c^2}+\frac{(Bb^2-Bac-Abc)\log\left(cx^4+bx^2+a\right)}{4\,c^3} - \frac{(Bb^3-3\,Babc-Ab^2c+2\,Aac^2)\arctan\left(\frac{2\,cx^2+b}{\sqrt{-b^2+4\,ac}}\right)}{2\,\sqrt{-b^2+4\,ac}c^3}$$

```
[In] integrate(x^5*(B*x^2+A)/(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] 1/4*(B*c*x^4 - 2*B*b*x^2 + 2*A*c*x^2)/c^2 + 1/4*(B*b^2 - B*a*c - A*b*c)*log
(c*x^4 + b*x^2 + a)/c^3 - 1/2*(B*b^3 - 3*B*a*b*c - A*b^2*c + 2*A*a*c^2)*arc
tan((2*c*x^2 + b)/sqrt(-b^2 + 4*a*c))/(sqrt(-b^2 + 4*a*c)*c^3)
```

Mupad [B] (verification not implemented)

Time = 7.88 (sec) , antiderivative size = 1343, normalized size of antiderivative = 10.10

$$\int \frac{x^5(A+Bx^2)}{a+bx^2+cx^4}\,dx = x^2\left(\frac{A}{2\,c}-\frac{B\,b}{2\,c^2}\right)+\frac{B\,x^4}{4\,c}$$

$$-\frac{\ln\left(c\,x^4+b\,x^2+a\right)\,\left(8\,B\,a^2\,c^2-10\,B\,a\,b^2\,c+8\,A\,a\,b\,c^2+2\,B\,b^4-2\,A\,b^3\,c\right)}{2\,\left(16\,a\,c^4-4\,b^2\,c^3\right)}$$

$$+\frac{\operatorname{atan}\left(\frac{2\,c^4\,\left(4\,a\,c-b^2\right)\,\left(x^2\,\left(\frac{\frac{\left(\frac{-6\,B\,b^3\,c^3+6\,A\,b^2\,c^4+10\,B\,a\,b\,c^4-4\,A\,a\,c^5}{c^4}-\frac{4\,b\,c^2\,\left(8\,B\,a^2\,c^2-10\,B\,a\,b^2\,c+8\,A\,a\,b\,c^2+2\,B\,b^4-2\,A\,b^3\,c\right)}{16\,a\,c^4-4\,b^2\,c^3}\right)\,\left(B\,b^3-A\,b^2\,c-\cdots\right.}{8\,c^3\,\sqrt{4\,a\,c-b^2}}\cdots}{a}\cdots\right.\right.}{\cdots}\right)}{\cdots}$$

```
[In] int((x^5*(A + B*x^2))/(a + b*x^2 + c*x^4),x)
```

```
[Out] x^2*(A/(2*c) - (B*b)/(2*c^2)) + (B*x^4)/(4*c) - (log(a + b*x^2 + c*x^4)*(2*
B*b^4 + 8*B*a^2*c^2 - 2*A*b^3*c + 8*A*a*b*c^2 - 10*B*a*b^2*c))/(2*(16*a*c^4
 - 4*b^2*c^3)) + (atan((2*c^4*(4*a*c - b^2)*(x^2*(((((6*A*b^2*c^4 - 6*B*b^3
*c^3 - 4*A*a*c^5 + 10*B*a*b*c^4)/c^4 - (4*b*c^2*(2*B*b^4 + 8*B*a^2*c^2 - 2*
A*b^3*c + 8*A*a*b*c^2 - 10*B*a*b^2*c))/(16*a*c^4 - 4*b^2*c^3))*(B*b^3 + 2*A
*a*c^2 - A*b^2*c - 3*B*a*b*c))/(8*c^3*(4*a*c - b^2)^(1/2)) - (b*(B*b^3 + 2*
A*a*c^2 - A*b^2*c - 3*B*a*b*c)*(2*B*b^4 + 8*B*a^2*c^2 - 2*A*b^3*c + 8*A*a*b
*c^2 - 10*B*a*b^2*c))/(2*c*(4*a*c - b^2)^(1/2)*(16*a*c^4 - 4*b^2*c^3)))/a -
 (b*((((6*A*b^2*c^4 - 6*B*b^3*c^3 - 4*A*a*c^5 + 10*B*a*b*c^4)/c^4 - (4*b*c^
2*(2*B*b^4 + 8*B*a^2*c^2 - 2*A*b^3*c + 8*A*a*b*c^2 - 10*B*a*b^2*c))/(16*a*c
^4 - 4*b^2*c^3))*(2*B*b^4 + 8*B*a^2*c^2 - 2*A*b^3*c + 8*A*a*b*c^2 - 10*B*a*
b^2*c))/(2*(16*a*c^4 - 4*b^2*c^3)) - (B^2*b^5 + A^2*b^3*c^2 - 2*A*B*b^4*c -
 A*B*a^2*c^3 - A^2*a*b*c^3 - 3*B^2*a*b^3*c + 2*B^2*a^2*b*c^2 + 4*A*B*a*b^2*
c^2)/c^4 + (b*(B*b^3 + 2*A*a*c^2 - A*b^2*c - 3*B*a*b*c)^2)/(2*c^4*(4*a*c -
b^2))))/(2*a*(4*a*c - b^2)^(1/2))) + ((((8*B*a^2*c^4 + 8*A*a*b*c^4 - 8*B*a*
b^2*c^3)/c^4 - (8*a*c^2*(2*B*b^4 + 8*B*a^2*c^2 - 2*A*b^3*c + 8*A*a*b*c^2 -
10*B*a*b^2*c))/(16*a*c^4 - 4*b^2*c^3))*(B*b^3 + 2*A*a*c^2 - A*b^2*c - 3*B*a
*b*c))/(8*c^3*(4*a*c - b^2)^(1/2)) - (a*(B*b^3 + 2*A*a*c^2 - A*b^2*c - 3*B*
a*b*c)*(2*B*b^4 + 8*B*a^2*c^2 - 2*A*b^3*c + 8*A*a*b*c^2 - 10*B*a*b^2*c))/(c
*(4*a*c - b^2)^(1/2)*(16*a*c^4 - 4*b^2*c^3)))/a - (b*((((8*B*a^2*c^4 + 8*A*
a*b*c^4 - 8*B*a*b^2*c^3)/c^4 - (8*a*c^2*(2*B*b^4 + 8*B*a^2*c^2 - 2*A*b^3*c
```

```
+ 8*A*a*b*c^2 - 10*B*a*b^2*c))/(16*a*c^4 - 4*b^2*c^3))*(2*B*b^4 + 8*B*a^2*c
^2 - 2*A*b^3*c + 8*A*a*b*c^2 - 10*B*a*b^2*c))/(2*(16*a*c^4 - 4*b^2*c^3)) -
(B^2*a*b^4 + B^2*a^3*c^2 + A^2*a*b^2*c^2 - 2*B^2*a^2*b^2*c - 2*A*B*a*b^3*c
+ 2*A*B*a^2*b*c^2)/c^4 + (a*(B*b^3 + 2*A*a*c^2 - A*b^2*c - 3*B*a*b*c)^2)/(c
^4*(4*a*c - b^2)))))/(2*a*(4*a*c - b^2)^(1/2))))/(B^2*b^6 + 4*A^2*a^2*c^4 +
A^2*b^4*c^2 - 2*A*B*b^5*c + 9*B^2*a^2*b^2*c^2 - 6*B^2*a*b^4*c - 4*A^2*a*b^2
*c^3 + 10*A*B*a*b^3*c^2 - 12*A*B*a^2*b*c^3))*(B*b^3 + 2*A*a*c^2 - A*b^2*c -
 3*B*a*b*c))/(2*c^3*(4*a*c - b^2)^(1/2))
```

3.103 $\int \frac{x^3(A+Bx^2)}{a+bx^2+cx^4}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 97

$$\int \frac{x^3(A+Bx^2)}{a+bx^2+cx^4}\,dx = \frac{Bx^2}{2c} - \frac{(b^2B-Abc-2aBc)\,\text{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2c^2\sqrt{b^2-4ac}} - \frac{(bB-Ac)\log\left(a+bx^2+cx^4\right)}{4c^2}$$

```
[Out] 1/2*B*x^2/c-1/4*(-A*c+B*b)*ln(c*x^4+b*x^2+a)/c^2-1/2*(-A*b*c-2*B*a*c+B*b^2)
*arctanh((2*c*x^2+b)/(-4*a*c+b^2)^(1/2))/c^2/(-4*a*c+b^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {1265, 787, 648, 632, 212, 642}

$$\int \frac{x^3(A+Bx^2)}{a+bx^2+cx^4}\,dx = -\frac{(-2aBc-Abc+b^2B)\,\text{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2c^2\sqrt{b^2-4ac}} - \frac{(bB-Ac)\log\left(a+bx^2+cx^4\right)}{4c^2} + \frac{Bx^2}{2c}$$

```
[In] Int[(x^3*(A + B*x^2))/(a + b*x^2 + c*x^4),x]
```

```
[Out] (B*x^2)/(2*c) - ((b^2*B - A*b*c - 2*a*B*c)*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 -
 4*a*c]])/(2*c^2*Sqrt[b^2 - 4*a*c]) - ((b*B - A*c)*Log[a + b*x^2 + c*x^4])/
(4*c^2)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 787

```
Int[(((d_.) + (e_.)*(x_))*((f_) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) + (c_.)*
(x_)^2), x_Symbol] :> Simp[e*g*(x/c), x] + Dist[1/c, Int[(c*d*f - a*e*g + (
c*e*f + c*d*g - b*e*g)*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e
, f, g}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1265

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rubi steps

$$\begin{aligned} \text{integral} &= \frac{1}{2}\text{Subst}\left(\int \frac{x(A+Bx)}{a+bx+cx^2}\,dx, x, x^2\right) \\ &= \frac{Bx^2}{2c} + \frac{\text{Subst}\left(\int \frac{-aB+(-bB+Ac)x}{a+bx+cx^2}\,dx, x, x^2\right)}{2c} \end{aligned}$$

$$=\frac{Bx^2}{2c}-\frac{(bB-Ac)\text{Subst}\left(\int\frac{b+2cx}{a+bx+cx^2}\,dx,x,x^2\right)}{4c^2}+\frac{(b^2B-Abc-2aBc)\,\text{Subst}\left(\int\frac{1}{a+bx+cx^2}\,dx,x,x^2\right)}{4c^2}$$

$$=\frac{Bx^2}{2c}-\frac{(bB-Ac)\log\left(a+bx^2+cx^4\right)}{4c^2}-\frac{(b^2B-Abc-2aBc)\,\text{Subst}\left(\int\frac{1}{b^2-4ac-x^2}\,dx,x,b+2cx^2\right)}{2c^2}$$

$$=\frac{Bx^2}{2c}-\frac{(b^2B-Abc-2aBc)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2c^2\sqrt{b^2-4ac}}-\frac{(bB-Ac)\log\left(a+bx^2+cx^4\right)}{4c^2}$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.96

$$\int\frac{x^3(A+Bx^2)}{a+bx^2+cx^4}\,dx=\frac{2Bcx^2+\frac{2(b^2B-Abc-2aBc)\arctan\left(\frac{b+2cx^2}{\sqrt{-b^2+4ac}}\right)}{\sqrt{-b^2+4ac}}+(-bB+Ac)\log\left(a+bx^2+cx^4\right)}{4c^2}$$

```
[In] Integrate[(x^3*(A + B*x^2))/(a + b*x^2 + c*x^4),x]
```

```
[Out] (2*B*c*x^2 + (2*(b^2*B - A*b*c - 2*a*B*c)*ArcTan[(b + 2*c*x^2)/Sqrt[-b^2 +
4*a*c]])/Sqrt[-b^2 + 4*a*c] + (-(b*B) + A*c)*Log[a + b*x^2 + c*x^4])/(4*c^2
)
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.01

method	result	size
default	$\frac{B\,x^2}{2c}+\frac{\frac{(Ac-Bb)\ln\left(c\,x^4+b\,x^2+a\right)}{2c}+\frac{2\left(-Ba-\frac{(Ac-Bb)b}{2c}\right)\arctan\left(\frac{2c\,x^2+b}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}}}{2c}$	98
risch	Expression too large to display	1398

```
[In] int(x^3*(B*x^2+A)/(c*x^4+b*x^2+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*B*x^2/c+1/2/c*(1/2*(A*c-B*b)/c*ln(c*x^4+b*x^2+a)+2*(-B*a-1/2*(A*c-B*b)*
b/c)/(4*a*c-b^2)^(1/2)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2)))
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 312, normalized size of antiderivative = 3.22

$$\int \frac{x^3(A+Bx^2)}{a+bx^2+cx^4}\,dx$$
$$= \left[\frac{2\,(Bb^2c-4\,Bac^2)x^2-(Bb^2-(2\,Ba+Ab)c)\sqrt{b^2-4\,ac}\log\left(\frac{2\,c^2x^4+2\,bcx^2+b^2-2\,ac+(2\,cx^2+b)\sqrt{b^2-4\,ac}}{cx^4+bx^2+a}\right)-(Bb^3\ldots}{4\,(b^2c^2-4\,ac^3)}\right.$$

```
[In] integrate(x^3*(B*x^2+A)/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] [1/4*(2*(B*b^2*c - 4*B*a*c^2)*x^2 - (B*b^2 - (2*B*a + A*b)*c)*sqrt(b^2 - 4*
a*c)*log((2*c^2*x^4 + 2*b*c*x^2 + b^2 - 2*a*c + (2*c*x^2 + b)*sqrt(b^2 - 4*
a*c))/(c*x^4 + b*x^2 + a)) - (B*b^3 + 4*A*a*c^2 - (4*B*a*b + A*b^2)*c)*log(
c*x^4 + b*x^2 + a))/(b^2*c^2 - 4*a*c^3), 1/4*(2*(B*b^2*c - 4*B*a*c^2)*x^2 -
 2*(B*b^2 - (2*B*a + A*b)*c)*sqrt(-b^2 + 4*a*c)*arctan(-(2*c*x^2 + b)*sqrt(
-b^2 + 4*a*c)/(b^2 - 4*a*c)) - (B*b^3 + 4*A*a*c^2 - (4*B*a*b + A*b^2)*c)*lo
g(c*x^4 + b*x^2 + a))/(b^2*c^2 - 4*a*c^3)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 434 vs. 2(90) = 180.

Time = 72.99 (sec) , antiderivative size = 434, normalized size of antiderivative = 4.47

$$\int \frac{x^3(A+Bx^2)}{a+bx^2+cx^4}\,dx = \frac{Bx^2}{2c} + \left(-\frac{-Ac+Bb}{4c^2}\right.$$
$$\left.-\frac{\sqrt{-4ac+b^2}(Abc+2Bac-Bb^2)}{4c^2\cdot(4ac-b^2)}\right)\log\left(x^2+\frac{2Aac-Bab-8ac^2\left(-\frac{-Ac+Bb}{4c^2}-\frac{\sqrt{-4ac+b^2}(Abc+2Bac-Bb^2)}{4c^2\cdot(4ac-b^2)}\right)\ldots}{Abc+2Bac-B\ldots}\right.$$
$$+\left(-\frac{-Ac+Bb}{4c^2}\right.$$
$$\left.+\frac{\sqrt{-4ac+b^2}(Abc+2Bac-Bb^2)}{4c^2\cdot(4ac-b^2)}\right)\log\left(x^2+\frac{2Aac-Bab-8ac^2\left(-\frac{-Ac+Bb}{4c^2}+\frac{\sqrt{-4ac+b^2}(Abc+2Bac-Bb^2)}{4c^2\cdot(4ac-b^2)}\right)\ldots}{Abc+2Bac-B\ldots}\right.$$

```
[In] integrate(x**3*(B*x**2+A)/(c*x**4+b*x**2+a),x)
```

```
[Out] B*x**2/(2*c) + (-(-A*c + B*b)/(4*c**2) - sqrt(-4*a*c + b**2)*(A*b*c + 2*B*a
*c - B*b**2)/(4*c**2*(4*a*c - b**2)))*log(x**2 + (2*A*a*c - B*a*b - 8*a*c**
2*(-(-A*c + B*b)/(4*c**2) - sqrt(-4*a*c + b**2)*(A*b*c + 2*B*a*c - B*b**2)/
```

```
(4*c**2*(4*a*c - b**2))) + 2*b**2*c*(-(-A*c + B*b)/(4*c**2) - sqrt(-4*a*c +
 b**2)*(A*b*c + 2*B*a*c - B*b**2)/(4*c**2*(4*a*c - b**2))))/(A*b*c + 2*B*a*
c - B*b**2)) + (-(-A*c + B*b)/(4*c**2) + sqrt(-4*a*c + b**2)*(A*b*c + 2*B*a
*c - B*b**2)/(4*c**2*(4*a*c - b**2)))*log(x**2 + (2*A*a*c - B*a*b - 8*a*c**
2*(-(-A*c + B*b)/(4*c**2) + sqrt(-4*a*c + b**2)*(A*b*c + 2*B*a*c - B*b**2)/
(4*c**2*(4*a*c - b**2))) + 2*b**2*c*(-(-A*c + B*b)/(4*c**2) + sqrt(-4*a*c +
 b**2)*(A*b*c + 2*B*a*c - B*b**2)/(4*c**2*(4*a*c - b**2))))/(A*b*c + 2*B*a*
c - B*b**2))
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^3(A+Bx^2)}{a+bx^2+cx^4}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(x^3*(B*x^2+A)/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

Giac [A] (verification not implemented)

none

Time = 0.62 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.94

$$\int \frac{x^3(A+Bx^2)}{a+bx^2+cx^4}\,dx = \frac{Bx^2}{2\,c} - \frac{(Bb-Ac)\log\left(cx^4+bx^2+a\right)}{4\,c^2} + \frac{(Bb^2-2\,Bac-Abc)\arctan\left(\frac{2\,cx^2+b}{\sqrt{-b^2+4\,ac}}\right)}{2\,\sqrt{-b^2+4\,ac}c^2}$$

```
[In] integrate(x^3*(B*x^2+A)/(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] 1/2*B*x^2/c - 1/4*(B*b - A*c)*log(c*x^4 + b*x^2 + a)/c^2 + 1/2*(B*b^2 - 2*B
*a*c - A*b*c)*arctan((2*c*x^2 + b)/sqrt(-b^2 + 4*a*c))/(sqrt(-b^2 + 4*a*c)*
c^2)
```

Mupad [B] (verification not implemented)

Time = 8.01 (sec) , antiderivative size = 979, normalized size of antiderivative = 10.09

$$\int \frac{x^3(A+Bx^2)}{a+bx^2+cx^4}\,dx = \frac{B\,x^2}{2\,c}+\frac{\ln\left(c\,x^4+b\,x^2+a\right)\,\left(2\,B\,b^3-2\,A\,b^2\,c-8\,B\,a\,b\,c+8\,A\,a\,c^2\right)}{2\,\left(16\,a\,c^3-4\,b^2\,c^2\right)}$$

$$-\frac{\mathrm{atan}\left(\frac{2\,c^2\,\left(4\,a\,c-b^2\right)\left(\frac{\frac{\left(\frac{8\,A\,a\,c^3-8\,B\,a\,b\,c^2}{c^2}-\frac{8\,a\,c^2\,\left(2\,B\,b^3-2\,A\,b^2\,c-8\,B\,a\,b\,c+8\,A\,a\,c^2\right)}{16\,a\,c^3-4\,b^2\,c^2}\right)\left(-B\,b^2+A\,c\,b+2\,B\,a\,c\right)}{8\,c^2\,\sqrt{4\,a\,c-b^2}}-\frac{a\,\left(-B\,b^2+A\,c\,b+2\,B\,a\,c\right)\left(2\,B\,b^3-2\cdots\right.}{\sqrt{4\,a\,c-b^2}\,\left(16\,a\,c^3\cdots\right.}}{a}\cdots\right.}{\cdots}\right.}{\cdots}$$

```
[In] int((x^3*(A + B*x^2))/(a + b*x^2 + c*x^4),x)
```

```
[Out] (B*x^2)/(2*c) + (log(a + b*x^2 + c*x^4)*(2*B*b^3 + 8*A*a*c^2 - 2*A*b^2*c -
8*B*a*b*c))/(2*(16*a*c^3 - 4*b^2*c^2)) - (atan((2*c^2*(4*a*c - b^2)*(((((8*
A*a*c^3 - 8*B*a*b*c^2)/c^2 - (8*a*c^2*(2*B*b^3 + 8*A*a*c^2 - 2*A*b^2*c - 8*
B*a*b*c))/(16*a*c^3 - 4*b^2*c^2))*(A*b*c - B*b^2 + 2*B*a*c))/(8*c^2*(4*a*c
- b^2)^(1/2)) - (a*(A*b*c - B*b^2 + 2*B*a*c)*(2*B*b^3 + 8*A*a*c^2 - 2*A*b^2
*c - 8*B*a*b*c))/((4*a*c - b^2)^(1/2)*(16*a*c^3 - 4*b^2*c^2)))/a + x^2*((((
(6*A*b*c^3 - 6*B*b^2*c^2 + 4*B*a*c^3)/c^2 - (4*b*c^2*(2*B*b^3 + 8*A*a*c^2 -
 2*A*b^2*c - 8*B*a*b*c))/(16*a*c^3 - 4*b^2*c^2))*(A*b*c - B*b^2 + 2*B*a*c))
/(8*c^2*(4*a*c - b^2)^(1/2)) - (b*(A*b*c - B*b^2 + 2*B*a*c)*(2*B*b^3 + 8*A*
a*c^2 - 2*A*b^2*c - 8*B*a*b*c))/(2*(4*a*c - b^2)^(1/2)*(16*a*c^3 - 4*b^2*c^
2)))/a + (b*((((6*A*b*c^3 - 6*B*b^2*c^2 + 4*B*a*c^3)/c^2 - (4*b*c^2*(2*B*b^
3 + 8*A*a*c^2 - 2*A*b^2*c - 8*B*a*b*c))/(16*a*c^3 - 4*b^2*c^2))*(2*B*b^3 +
8*A*a*c^2 - 2*A*b^2*c - 8*B*a*b*c))/(2*(16*a*c^3 - 4*b^2*c^2)) - (B^2*b^3 +
 A^2*b*c^2 + A*B*a*c^2 - 2*A*B*b^2*c - B^2*a*b*c)/c^2 + (b*(A*b*c - B*b^2 +
 2*B*a*c)^2)/(2*c^2*(4*a*c - b^2))))/(2*a*(4*a*c - b^2)^(1/2))) + (b*((((8*
A*a*c^3 - 8*B*a*b*c^2)/c^2 - (8*a*c^2*(2*B*b^3 + 8*A*a*c^2 - 2*A*b^2*c - 8*
B*a*b*c))/(16*a*c^3 - 4*b^2*c^2))*(2*B*b^3 + 8*A*a*c^2 - 2*A*b^2*c - 8*B*a*
b*c))/(2*(16*a*c^3 - 4*b^2*c^2)) - (A^2*a*c^2 + B^2*a*b^2 - 2*A*B*a*b*c)/c^
2 + (a*(A*b*c - B*b^2 + 2*B*a*c)^2)/(c^2*(4*a*c - b^2))))/(2*a*(4*a*c - b^2
)^(1/2))))/(B^2*b^4 + A^2*b^2*c^2 + 4*B^2*a^2*c^2 - 2*A*B*b^3*c - 4*B^2*a*b
^2*c + 4*A*B*a*b*c^2))*(A*b*c - B*b^2 + 2*B*a*c))/(2*c^2*(4*a*c - b^2)^(1/2
))
```

3.104 $\int \frac{x(A+Bx^2)}{a+bx^2+cx^4} \, dx$

Optimal result

Integrand size = 23, antiderivative size = 71

$$\int \frac{x(A+Bx^2)}{a+bx^2+cx^4} \, dx = \frac{(bB-2Ac)\text{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2c\sqrt{b^2-4ac}} + \frac{B\log(a+bx^2+cx^4)}{4c}$$

```
[Out] 1/4*B*ln(c*x^4+b*x^2+a)/c+1/2*(-2*A*c+B*b)*arctanh((2*c*x^2+b)/(-4*a*c+b^2)
^(1/2))/c/(-4*a*c+b^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {1261, 648, 632, 212, 642}

$$\int \frac{x(A+Bx^2)}{a+bx^2+cx^4} \, dx = \frac{(bB-2Ac)\text{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2c\sqrt{b^2-4ac}} + \frac{B\log(a+bx^2+cx^4)}{4c}$$

```
[In] Int[(x*(A + B*x^2))/(a + b*x^2 + c*x^4),x]
```

```
[Out] ((b*B - 2*A*c)*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(2*c*Sqrt[b^2 - 4*
a*c]) + (B*Log[a + b*x^2 + c*x^4])/(4*c)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1261

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(
p_.), x_Symbol] :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x],
 x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{1}{2}\text{Subst}\left(\int \frac{A+Bx}{a+bx+cx^2}\,dx,x,x^2\right)\\
&= \frac{B\text{Subst}\left(\int \frac{b+2cx}{a+bx+cx^2}\,dx,x,x^2\right)}{4c}+\frac{(-bB+2Ac)\text{Subst}\left(\int \frac{1}{a+bx+cx^2}\,dx,x,x^2\right)}{4c}\\
&= \frac{B\log\left(a+bx^2+cx^4\right)}{4c}-\frac{(-bB+2Ac)\text{Subst}\left(\int \frac{1}{b^2-4ac-x^2}\,dx,x,b+2cx^2\right)}{2c}\\
&= \frac{(bB-2Ac)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2c\sqrt{b^2-4ac}}+\frac{B\log\left(a+bx^2+cx^4\right)}{4c}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.00

$$\int \frac{x(A+Bx^2)}{a+bx^2+cx^4}\,dx = \frac{-\frac{2(bB-2Ac)\arctan\left(\frac{b+2cx^2}{\sqrt{-b^2+4ac}}\right)}{\sqrt{-b^2+4ac}}+B\log\left(a+bx^2+cx^4\right)}{4c}$$

```
[In] Integrate[(x*(A + B*x^2))/(a + b*x^2 + c*x^4),x]
[Out] ((-2*(b*B - 2*A*c)*ArcTan[(b + 2*c*x^2)/Sqrt[-b^2 + 4*a*c]])/Sqrt[-b^2 + 4*
a*c] + B*Log[a + b*x^2 + c*x^4])/(4*c)
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.92

method	result
default	$\frac{B\ln(cx^4+bx^2+a)}{4c}+\frac{\left(A-\frac{Bb}{2c}\right)\arctan\left(\frac{2cx^2+b}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}}$
risch	$\frac{\ln\left(\left(-8Aac^2+2Ab^2c+4Babc-Bb^3-\sqrt{-(4ac-b^2)(2Ac-Bb)^2}\,b\right)x^2-2\sqrt{-(4ac-b^2)(2Ac-Bb)^2}\,a\right)Ba}{4ac-b^2}-\frac{\ln\left(\left(-8Aac^2+2Ab^2c-\right.\right.}{}$

```
[In] int(x*(B*x^2+A)/(c*x^4+b*x^2+a),x,method=_RETURNVERBOSE)
[Out] 1/4*B*ln(c*x^4+b*x^2+a)/c+(A-1/2*B*b/c)/(4*a*c-b^2)^(1/2)*arctan((2*c*x^2+b
)/(4*a*c-b^2)^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 219, normalized size of antiderivative = 3.08

$$\int \frac{x(A+Bx^2)}{a+bx^2+cx^4}\,dx$$
$$=\left[-\frac{(Bb-2\,Ac)\sqrt{b^2-4\,ac}\log\left(\frac{2\,c^2x^4+2\,bcx^2+b^2-2\,ac-(2\,cx^2+b)\sqrt{b^2-4\,ac}}{cx^4+bx^2+a}\right)-(Bb^2-4\,Bac)\log\left(cx^4+bx^2+a\right)}{4\,(b^2c-4\,ac^2)},\right.$$

```
[In] integrate(x*(B*x^2+A)/(c*x^4+b*x^2+a),x, algorithm="fricas")
[Out] [-1/4*((B*b - 2*A*c)*sqrt(b^2 - 4*a*c)*log((2*c^2*x^4 + 2*b*c*x^2 + b^2 - 2
*a*c - (2*c*x^2 + b)*sqrt(b^2 - 4*a*c))/(c*x^4 + b*x^2 + a)) - (B*b^2 - 4*B
*a*c)*log(c*x^4 + b*x^2 + a))/(b^2*c - 4*a*c^2), 1/4*(2*(B*b - 2*A*c)*sqrt(
```

```
-b^2 + 4*a*c)*arctan(-(2*c*x^2 + b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4*a*c)) + (B*
b^2 - 4*B*a*c)*log(c*x^4 + b*x^2 + a))/(b^2*c - 4*a*c^2)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 287 vs. $2(61) = 122$.

Time = 5.22 (sec) , antiderivative size = 287, normalized size of antiderivative = 4.04

$$\int \frac{x(A+Bx^2)}{a+bx^2+cx^4}\,dx = \left(\frac{B}{4c} - \frac{(-2Ac+Bb)\sqrt{-4ac+b^2}}{4c(4ac-b^2)}\right)\log\left(x^2 + \frac{-Ab+2Ba-8ac\left(\frac{B}{4c}-\frac{(-2Ac+Bb)\sqrt{-4ac+b^2}}{4c(4ac-b^2)}\right)+2b^2\left(\frac{B}{4c}-\frac{(-2Ac+B}{4c(4}\right.}{-2Ac+Bb}\right. + \left(\frac{B}{4c} + \frac{(-2Ac+Bb)\sqrt{-4ac+b^2}}{4c(4ac-b^2)}\right)\log\left(x^2 + \frac{-Ab+2Ba-8ac\left(\frac{B}{4c}+\frac{(-2Ac+Bb)\sqrt{-4ac+b^2}}{4c(4ac-b^2)}\right)+2b^2\left(\frac{B}{4c}+\frac{(-2Ac+Bb}{4c(4}\right.}{-2Ac+Bb}\right.$$

```
[In] integrate(x*(B*x**2+A)/(c*x**4+b*x**2+a),x)
```

```
[Out] (B/(4*c) - (-2*A*c + B*b)*sqrt(-4*a*c + b**2)/(4*c*(4*a*c - b**2)))*log(x**
2 + (-A*b + 2*B*a - 8*a*c*(B/(4*c) - (-2*A*c + B*b)*sqrt(-4*a*c + b**2)/(4*
c*(4*a*c - b**2))) + 2*b**2*(B/(4*c) - (-2*A*c + B*b)*sqrt(-4*a*c + b**2)/(
4*c*(4*a*c - b**2))))/(-2*A*c + B*b)) + (B/(4*c) + (-2*A*c + B*b)*sqrt(-4*a
*c + b**2)/(4*c*(4*a*c - b**2)))*log(x**2 + (-A*b + 2*B*a - 8*a*c*(B/(4*c)
+ (-2*A*c + B*b)*sqrt(-4*a*c + b**2)/(4*c*(4*a*c - b**2))) + 2*b**2*(B/(4*c
) + (-2*A*c + B*b)*sqrt(-4*a*c + b**2)/(4*c*(4*a*c - b**2))))/(-2*A*c + B*b
))
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x(A+Bx^2)}{a+bx^2+cx^4}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(x*(B*x^2+A)/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

Giac [A] (verification not implemented)

none

Time = 0.69 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.94

$$\int \frac{x(A+Bx^2)}{a+bx^2+cx^4}\,dx = \frac{B\log\left(cx^4+bx^2+a\right)}{4\,c} - \frac{(Bb-2\,Ac)\arctan\left(\frac{2\,cx^2+b}{\sqrt{-b^2+4\,ac}}\right)}{2\,\sqrt{-b^2+4\,ac}c}$$

```
[In] integrate(x*(B*x^2+A)/(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] 1/4*B*log(c*x^4 + b*x^2 + a)/c - 1/2*(B*b - 2*A*c)*arctan((2*c*x^2 + b)/sqr
t(-b^2 + 4*a*c))/(sqrt(-b^2 + 4*a*c)*c)
```

Mupad [B] (verification not implemented)

Time = 7.71 (sec) , antiderivative size = 606, normalized size of antiderivative = 8.54

$$\int \frac{x(A+Bx^2)}{a+bx^2+cx^4}\,dx = -\frac{\ln\left(c\,x^4+b\,x^2+a\right)\,\left(2\,B\,b^2-8\,B\,a\,c\right)}{2\,\left(16\,a\,c^2-4\,b^2\,c\right)}$$

$$-\frac{\mathrm{atan}\left(\frac{2\,\left(4\,a\,c-b^2\right)\,\left(x^2\,\left(\frac{\frac{(2\,A\,c-B\,b)\,\left(6\,B\,b\,c-4\,A\,c^2+\frac{4\,b\,c^2\,\left(2\,B\,b^2-8\,B\,a\,c\right)}{16\,a\,c^2-4\,b^2\,c}\right)}{8\,c\,\sqrt{4\,a\,c-b^2}}+\frac{b\,c\,\left(2\,B\,b^2-8\,B\,a\,c\right)\,(2\,A\,c-B\,b)}{2\,\left(16\,a\,c^2-4\,b^2\,c\right)\,\sqrt{4\,a\,c-b^2}}}{a}+\frac{b\,\left(B^2\,b-A\,B\,c-\frac{b\,(2\,A\,c-B\,b)^2}{2\,\left(4\,a\,c-b^2\right)}+\cdots\right.}{\cdots}\right.\right.}{\cdots}\right.}{\cdots}$$

```
[In] int((x*(A + B*x^2))/(a + b*x^2 + c*x^4),x)
```

```
[Out] - (log(a + b*x^2 + c*x^4)*(2*B*b^2 - 8*B*a*c))/(2*(16*a*c^2 - 4*b^2*c)) - (
atan((2*(4*a*c - b^2)*(x^2*((((2*A*c - B*b)*(6*B*b*c - 4*A*c^2 + (4*b*c^2*(
2*B*b^2 - 8*B*a*c))/(16*a*c^2 - 4*b^2*c)))/(8*c*(4*a*c - b^2)^(1/2)) + (b*c
*(2*B*b^2 - 8*B*a*c)*(2*A*c - B*b))/(2*(16*a*c^2 - 4*b^2*c)*(4*a*c - b^2)^(
1/2)))/a + (b*(B^2*b - A*B*c - (b*(2*A*c - B*b)^2)/(2*(4*a*c - b^2)) + ((2*
B*b^2 - 8*B*a*c)*(6*B*b*c - 4*A*c^2 + (4*b*c^2*(2*B*b^2 - 8*B*a*c))/(16*a*c
^2 - 4*b^2*c)))/(2*(16*a*c^2 - 4*b^2*c))))/(2*a*(4*a*c - b^2)^(1/2))) + (((
8*B*a*c + (8*a*c^2*(2*B*b^2 - 8*B*a*c))/(16*a*c^2 - 4*b^2*c))*(2*A*c - B*b)
)/(8*c*(4*a*c - b^2)^(1/2)) + (a*c*(2*B*b^2 - 8*B*a*c)*(2*A*c - B*b))/((16*
```

```
a*c^2 - 4*b^2*c)*(4*a*c - b^2)^(1/2)))/a + (b*(B^2*a + ((2*B*b^2 - 8*B*a*c)
*(8*B*a*c + (8*a*c^2*(2*B*b^2 - 8*B*a*c))/(16*a*c^2 - 4*b^2*c)))/(2*(16*a*c
^2 - 4*b^2*c)) - (a*(2*A*c - B*b)^2)/(4*a*c - b^2)))/(2*a*(4*a*c - b^2)^(1/
2))))/(4*A^2*c^2 + B^2*b^2 - 4*A*B*b*c))*(2*A*c - B*b))/(2*c*(4*a*c - b^2)^
(1/2))
```

3.105 $\int \frac{A+Bx^2}{x(a+bx^2+cx^4)}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 78

$$\int \frac{A+Bx^2}{x(a+bx^2+cx^4)}\,dx=\frac{(Ab-2aB)\operatorname{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2a\sqrt{b^2-4ac}}+\frac{A\log(x)}{a}-\frac{A\log\left(a+bx^2+cx^4\right)}{4a}$$

```
[Out] A*ln(x)/a-1/4*A*ln(c*x^4+b*x^2+a)/a+1/2*(A*b-2*B*a)*arctanh((2*c*x^2+b)/(-4
*a*c+b^2)^(1/2))/a/(-4*a*c+b^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {1265, 814, 648, 632, 212, 642}

$$\int \frac{A+Bx^2}{x(a+bx^2+cx^4)}\,dx=\frac{(Ab-2aB)\operatorname{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2a\sqrt{b^2-4ac}}-\frac{A\log\left(a+bx^2+cx^4\right)}{4a}+\frac{A\log(x)}{a}$$

```
[In] Int[(A + B*x^2)/(x*(a + b*x^2 + c*x^4)),x]
```

```
[Out] ((A*b - 2*a*B)*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(2*a*Sqrt[b^2 - 4*
a*c]) + (A*Log[x])/a - (A*Log[a + b*x^2 + c*x^4])/(4*a)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 814

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)/(a +
 b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 1265

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{2}\text{Subst}\left(\int \frac{A+Bx}{x\,(a+bx+cx^2)}\,dx,x,x^2\right)\\
&= \frac{1}{2}\text{Subst}\left(\int \left(\frac{A}{ax}+\frac{-Ab+aB-Acx}{a\,(a+bx+cx^2)}\right)\,dx,x,x^2\right)\\
&= \frac{A\log(x)}{a}+\frac{\text{Subst}\left(\int \frac{-Ab+aB-Acx}{a+bx+cx^2}\,dx,x,x^2\right)}{2a}\\
&= \frac{A\log(x)}{a}-\frac{A\text{Subst}\left(\int \frac{b+2cx}{a+bx+cx^2}\,dx,x,x^2\right)}{4a}+\frac{(-Ab+2aB)\text{Subst}\left(\int \frac{1}{a+bx+cx^2}\,dx,x,x^2\right)}{4a}\\
&= \frac{A\log(x)}{a}-\frac{A\log\left(a+bx^2+cx^4\right)}{4a}-\frac{(-Ab+2aB)\text{Subst}\left(\int \frac{1}{b^2-4ac-x^2}\,dx,x,b+2cx^2\right)}{2a}
\end{aligned}$$

$$= \frac{(Ab-2aB)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2a\sqrt{b^2-4ac}} + \frac{A\log(x)}{a} - \frac{A\log(a+bx^2+cx^4)}{4a}$$

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.64

$$\int \frac{A+Bx^2}{x(a+bx^2+cx^4)}\,dx$$
$$= \frac{4A\sqrt{b^2-4ac}\log(x) - \left(-2aB + A\left(b+\sqrt{b^2-4ac}\right)\right)\log\left(b-\sqrt{b^2-4ac}+2cx^2\right) + \left(-2aB + A\left(b-\sqrt{b^2\cdot\right.\right.}{4a\sqrt{b^2-4ac}}$$

```
[In] Integrate[(A + B*x^2)/(x*(a + b*x^2 + c*x^4)),x]
[Out] (4*A*Sqrt[b^2 - 4*a*c]*Log[x] - (-2*a*B + A*(b + Sqrt[b^2 - 4*a*c]))*Log[b
- Sqrt[b^2 - 4*a*c] + 2*c*x^2] + (-2*a*B + A*(b - Sqrt[b^2 - 4*a*c]))*Log[b
 + Sqrt[b^2 - 4*a*c] + 2*c*x^2])/(4*a*Sqrt[b^2 - 4*a*c])
```

Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.97

method	result
default	$\frac{A\ln(x)}{a} - \frac{\frac{A\ln(c\,x^4+b\,x^2+a)}{2} + \frac{2\left(\frac{Ab}{2}-Ba\right)\arctan\left(\frac{2c\,x^2+b}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}}}{2a}$
risch	$\frac{A\ln(x)}{a} + \frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\left(4c\,a^2-b^2a\right)\_Z^2+\left(4Aac-A\,b^2\right)\_Z+A^2c-bBA+B^2a\right)} \_R\ln\left(\left(\left(10ac-3b^2\right)\_R^2+\left(5Ac-Bb\right)\_R+2B^2\right)x\right.\right.}{2}$

```
[In] int((B*x^2+A)/x/(c*x^4+b*x^2+a),x,method=_RETURNVERBOSE)
[Out] A*ln(x)/a-1/2/a*(1/2*A*ln(c*x^4+b*x^2+a)+2*(1/2*A*b-B*a)/(4*a*c-b^2)^(1/2)*
arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2)))
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 249, normalized size of antiderivative = 3.19

$$\int \frac{A+Bx^2}{x\,(a+bx^2+cx^4)}\,dx$$
$$= \left[-\frac{(2\,Ba-Ab)\sqrt{b^2-4\,ac}\log\left(\frac{2\,c^2x^4+2\,bcx^2+b^2-2\,ac+(2\,cx^2+b)\sqrt{b^2-4\,ac}}{cx^4+bx^2+a}\right)+(Ab^2-4\,Aac)\log\left(cx^4+bx^2+a\right)-4}{4\,(ab^2-4\,a^2c)} \right.$$
$$-\frac{2\,(2\,Ba-Ab)\sqrt{-b^2+4\,ac}\arctan\left(-\frac{(2\,cx^2+b)\sqrt{-b^2+4\,ac}}{b^2-4\,ac}\right)+(Ab^2-4\,Aac)\log\left(cx^4+bx^2+a\right)-4\,(Ab^2-4}{4\,(ab^2-4\,a^2c)}$$

```
[In] integrate((B*x^2+A)/x/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] [-1/4*((2*B*a - A*b)*sqrt(b^2 - 4*a*c)*log((2*c^2*x^4 + 2*b*c*x^2 + b^2 - 2
*a*c + (2*c*x^2 + b)*sqrt(b^2 - 4*a*c))/(c*x^4 + b*x^2 + a)) + (A*b^2 - 4*A
*a*c)*log(c*x^4 + b*x^2 + a) - 4*(A*b^2 - 4*A*a*c)*log(x))/(a*b^2 - 4*a^2*c
), -1/4*(2*(2*B*a - A*b)*sqrt(-b^2 + 4*a*c)*arctan(-(2*c*x^2 + b)*sqrt(-b^2
 + 4*a*c)/(b^2 - 4*a*c)) + (A*b^2 - 4*A*a*c)*log(c*x^4 + b*x^2 + a) - 4*(A*
b^2 - 4*A*a*c)*log(x))/(a*b^2 - 4*a^2*c)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A+Bx^2}{x\,(a+bx^2+cx^4)}\,dx = \text{Timed out}$$

```
[In] integrate((B*x**2+A)/x/(c*x**4+b*x**2+a),x)
```

```
[Out] Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{A+Bx^2}{x\,(a+bx^2+cx^4)}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((B*x^2+A)/x/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

Giac [A] (verification not implemented)

none

Time = 0.73 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.00

$$\int \frac{A+Bx^2}{x(a+bx^2+cx^4)}\,dx = -\frac{A\log\left(cx^4+bx^2+a\right)}{4\,a}+\frac{A\log\left(x^2\right)}{2\,a}+\frac{(2\,Ba-Ab)\arctan\left(\frac{2\,cx^2+b}{\sqrt{-b^2+4\,ac}}\right)}{2\,\sqrt{-b^2+4\,ac}a}$$

```
[In] integrate((B*x^2+A)/x/(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] -1/4*A*log(c*x^4 + b*x^2 + a)/a + 1/2*A*log(x^2)/a + 1/2*(2*B*a - A*b)*arct
an((2*c*x^2 + b)/sqrt(-b^2 + 4*a*c))/(sqrt(-b^2 + 4*a*c)*a)
```

Mupad [B] (verification not implemented)

Time = 10.10 (sec) , antiderivative size = 2424, normalized size of antiderivative = 31.08

$$\int \frac{A+Bx^2}{x(a+bx^2+cx^4)}\,dx = \text{Too large to display}$$

```
[In] int((A + B*x^2)/(x*(a + b*x^2 + c*x^4)),x)
```

```
[Out] (A*log(x))/a - (log((A*B^2*c^2 + ((A + a*(-(A*b - 2*B*a)^2/(a^2*(4*a*c - b^
2)))^(1/2))*(B^2*a*c^2 + ((A + a*(-(A*b - 2*B*a)^2/(a^2*(4*a*c - b^2)))^(1/
2))*(4*A*b^2*c^2 + 2*c^2*x^2*(5*A*b*c - 4*B*b^2 + 10*B*a*c) - 4*B*a*b*c^2 +
 (b*c^2*(A + a*(-(A*b - 2*B*a)^2/(a^2*(4*a*c - b^2)))^(1/2))*(a*b + 3*b^2*x
^2 - 10*a*c*x^2))/a))/(4*a) - 4*A*B*b*c^2 - B*c^2*x^2*(5*A*c + B*b)))/(4*a)
 + B^3*c^2*x^2)*(A*B^2*c^2 + ((A - a*(-(A*b - 2*B*a)^2/(a^2*(4*a*c - b^2)))
^(1/2))*(B^2*a*c^2 + ((A - a*(-(A*b - 2*B*a)^2/(a^2*(4*a*c - b^2)))^(1/2))*
(4*A*b^2*c^2 + 2*c^2*x^2*(5*A*b*c - 4*B*b^2 + 10*B*a*c) - 4*B*a*b*c^2 + (b*
c^2*(A - a*(-(A*b - 2*B*a)^2/(a^2*(4*a*c - b^2)))^(1/2))*(a*b + 3*b^2*x^2 -
 10*a*c*x^2))/a))/(4*a) - 4*A*B*b*c^2 - B*c^2*x^2*(5*A*c + B*b)))/(4*a) + B
^3*c^2*x^2))*(2*A*b^2 - 8*A*a*c))/(2*(4*a*b^2 - 16*a^2*c)) - (atan((2*(4*a*
c - b^2)^(3/2)*(3*A*b^3 - B*a*b^2 + B*a^2*c - 8*A*a*b*c)*(A*B^2*c^2 + ((2*A
*b^2 - 8*A*a*c)*(((2*A*b^2 - 8*A*a*c)*(4*A*b^2*c^2 - 4*B*a*b*c^2 + (2*a*b^2
*c^2*(2*A*b^2 - 8*A*a*c))/(4*a*b^2 - 16*a^2*c)))/(2*(4*a*b^2 - 16*a^2*c)) +
 B^2*a*c^2 - 4*A*B*b*c^2))/(2*(4*a*b^2 - 16*a^2*c)) - ((A*b - 2*B*a)*(((A*b
 - 2*B*a)*(4*A*b^2*c^2 - 4*B*a*b*c^2 + (2*a*b^2*c^2*(2*A*b^2 - 8*A*a*c))/(4
*a*b^2 - 16*a^2*c)))/(4*a*(4*a*c - b^2)^(1/2)) + (b^2*c^2*(2*A*b^2 - 8*A*a*
c)*(A*b - 2*B*a))/(2*(4*a*b^2 - 16*a^2*c)*(4*a*c - b^2)^(1/2))))/(4*a*(4*a*
c - b^2)^(1/2)) - (b^2*c^2*(2*A*b^2 - 8*A*a*c)*(A*b - 2*B*a)^2)/(8*a*(4*a*b
^2 - 16*a^2*c)*(4*a*c - b^2))))/(c^2*(A^2*b^2*c^2 + 4*B^2*a^2*c^2 - 4*A*B*a
```

```
*b*c^2)*(6*A^2*b^2 - B^2*a^2 - 25*A^2*a*c + A*B*a*b)) - (16*a^3*x^2*(((3*A*
b^3 - B*a*b^2 + B*a^2*c - 8*A*a*b*c)*(((2*A*b^2 - 8*A*a*c)*(B^2*b*c^2 + 5*A
*B*c^3 - ((2*A*b^2 - 8*A*a*c)*(((2*A*b^2 - 8*A*a*c)*(12*b^3*c^2 - 40*a*b*c^
3))/(2*(4*a*b^2 - 16*a^2*c)) - 8*B*b^2*c^2 + 10*A*b*c^3 + 20*B*a*c^3))/(2*(
4*a*b^2 - 16*a^2*c))))/(2*(4*a*b^2 - 16*a^2*c)) - B^3*c^2 + ((A*b - 2*B*a)*
(((A*b - 2*B*a)*(((2*A*b^2 - 8*A*a*c)*(12*b^3*c^2 - 40*a*b*c^3))/(2*(4*a*b^
2 - 16*a^2*c)) - 8*B*b^2*c^2 + 10*A*b*c^3 + 20*B*a*c^3))/(4*a*(4*a*c - b^2)
^(1/2)) + ((2*A*b^2 - 8*A*a*c)*(12*b^3*c^2 - 40*a*b*c^3)*(A*b - 2*B*a))/(8*
a*(4*a*b^2 - 16*a^2*c)*(4*a*c - b^2)^(1/2))))/(4*a*(4*a*c - b^2)^(1/2)) + (
(2*A*b^2 - 8*A*a*c)*(12*b^3*c^2 - 40*a*b*c^3)*(A*b - 2*B*a)^2)/(32*a^2*(4*a
*b^2 - 16*a^2*c)*(4*a*c - b^2))))/(8*a^3*c^2*(6*A^2*b^2 - B^2*a^2 - 25*A^2*
a*c + A*B*a*b)) + ((((12*b^3*c^2 - 40*a*b*c^3)*(A*b - 2*B*a)^3)/(64*a^3*(4*
a*c - b^2)^(3/2)) - ((2*A*b^2 - 8*A*a*c)*(((A*b - 2*B*a)*(((2*A*b^2 - 8*A*a
*c)*(12*b^3*c^2 - 40*a*b*c^3))/(2*(4*a*b^2 - 16*a^2*c)) - 8*B*b^2*c^2 + 10*
A*b*c^3 + 20*B*a*c^3))/(4*a*(4*a*c - b^2)^(1/2)) + ((2*A*b^2 - 8*A*a*c)*(12
*b^3*c^2 - 40*a*b*c^3)*(A*b - 2*B*a))/(8*a*(4*a*b^2 - 16*a^2*c)*(4*a*c - b^
2)^(1/2))))/(2*(4*a*b^2 - 16*a^2*c)) + ((A*b - 2*B*a)*(B^2*b*c^2 + 5*A*B*c^
3 - ((2*A*b^2 - 8*A*a*c)*(((2*A*b^2 - 8*A*a*c)*(12*b^3*c^2 - 40*a*b*c^3))/(
2*(4*a*b^2 - 16*a^2*c)) - 8*B*b^2*c^2 + 10*A*b*c^3 + 20*B*a*c^3))/(2*(4*a*b
^2 - 16*a^2*c))))/(4*a*(4*a*c - b^2)^(1/2)))*(3*A*b^4 + 10*A*a^2*c^2 - B*a*
b^3 - 14*A*a*b^2*c + 3*B*a^2*b*c))/(8*a^3*c^2*(4*a*c - b^2)^(1/2)*(6*A^2*b^
2 - B^2*a^2 - 25*A^2*a*c + A*B*a*b)))*(4*a*c - b^2)^(3/2))/(A^2*b^2*c^2 + 4
*B^2*a^2*c^2 - 4*A*B*a*b*c^2) + (2*(4*a*c - b^2)*(((2*A*b^2 - 8*A*a*c)*(((A
*b - 2*B*a)*(4*A*b^2*c^2 - 4*B*a*b*c^2 + (2*a*b^2*c^2*(2*A*b^2 - 8*A*a*c))/
(4*a*b^2 - 16*a^2*c)))/(4*a*(4*a*c - b^2)^(1/2)) + (b^2*c^2*(2*A*b^2 - 8*A*
a*c)*(A*b - 2*B*a))/(2*(4*a*b^2 - 16*a^2*c)*(4*a*c - b^2)^(1/2))))/(2*(4*a*
b^2 - 16*a^2*c)) + ((A*b - 2*B*a)*(((2*A*b^2 - 8*A*a*c)*(4*A*b^2*c^2 - 4*B*
a*b*c^2 + (2*a*b^2*c^2*(2*A*b^2 - 8*A*a*c))/(4*a*b^2 - 16*a^2*c)))/(2*(4*a*
b^2 - 16*a^2*c)) + B^2*a*c^2 - 4*A*B*b*c^2))/(4*a*(4*a*c - b^2)^(1/2)) - (b
^2*c^2*(A*b - 2*B*a)^3)/(16*a^2*(4*a*c - b^2)^(3/2)))*(3*A*b^4 + 10*A*a^2*c
^2 - B*a*b^3 - 14*A*a*b^2*c + 3*B*a^2*b*c))/(c^2*(A^2*b^2*c^2 + 4*B^2*a^2*c
^2 - 4*A*B*a*b*c^2)*(6*A^2*b^2 - B^2*a^2 - 25*A^2*a*c + A*B*a*b)))*(A*b - 2
*B*a))/(2*a*(4*a*c - b^2)^(1/2))
```

3.106 $\int \frac{A+Bx^2}{x^3(a+bx^2+cx^4)}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 112

$$\int \frac{A+Bx^2}{x^3\left(a+bx^2+cx^4\right)}\,dx=-\frac{A}{2ax^2}-\frac{\left(Ab^2-abB-2aAc\right)\operatorname{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2a^2\sqrt{b^2-4ac}}-\frac{(Ab-aB)\log(x)}{a^2}+\frac{(Ab-aB)\log\left(a+bx^2+cx^4\right)}{4a^2}$$

```
[Out] -1/2*A/a/x^2-(A*b-B*a)*ln(x)/a^2+1/4*(A*b-B*a)*ln(c*x^4+b*x^2+a)/a^2-1/2*(-
2*A*a*c+A*b^2-B*a*b)*arctanh((2*c*x^2+b)/(-4*a*c+b^2)^(1/2))/a^2/(-4*a*c+b^
2)^(1/2)
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {1265, 814, 648, 632, 212, 642}

$$\int \frac{A+Bx^2}{x^3\left(a+bx^2+cx^4\right)}\,dx=-\frac{\left(-2aAc-abB+Ab^2\right)\operatorname{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2a^2\sqrt{b^2-4ac}}+\frac{(Ab-aB)\log\left(a+bx^2+cx^4\right)}{4a^2}-\frac{\log(x)(Ab-aB)}{a^2}-\frac{A}{2ax^2}$$

```
[In] Int[(A + B*x^2)/(x^3*(a + b*x^2 + c*x^4)),x]
```

```
[Out] -1/2*A/(a*x^2) - ((A*b^2 - a*b*B - 2*a*A*c)*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2
- 4*a*c]])/(2*a^2*Sqrt[b^2 - 4*a*c]) - ((A*b - a*B)*Log[x])/a^2 + ((A*b - a
*B)*Log[a + b*x^2 + c*x^4])/(4*a^2)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 814

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)/(a +
 b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 1265

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{2}\text{Subst}\left(\int \frac{A+Bx}{x^2\left(a+bx+cx^2\right)}\,dx, x, x^2\right)\\
&= \frac{1}{2}\text{Subst}\left(\int \left(\frac{A}{ax^2}+\frac{-Ab+aB}{a^2x}+\frac{-abB+A(b^2-ac)+(Ab-aB)cx}{a^2\left(a+bx+cx^2\right)}\right)\,dx, x, x^2\right)
\end{aligned}$$

$$= -\frac{A}{2ax^2} - \frac{(Ab-aB)\log(x)}{a^2} + \frac{\text{Subst}\left(\int \frac{-abB+A(b^2-ac)+(Ab-aB)cx}{a+bx+cx^2}\,dx, x, x^2\right)}{2a^2}$$

$$= -\frac{A}{2ax^2} - \frac{(Ab-aB)\log(x)}{a^2} + \frac{(Ab-aB)\text{Subst}\left(\int \frac{b+2cx}{a+bx+cx^2}\,dx, x, x^2\right)}{4a^2} + \frac{(-abB+A(b^2-2ac))\,\text{Subst}\left(\int \frac{1}{a+bx+cx^2}\,dx, x, x^2\right)}{4a^2}$$

$$= -\frac{A}{2ax^2} - \frac{(Ab-aB)\log(x)}{a^2} + \frac{(Ab-aB)\log\left(a+bx^2+cx^4\right)}{4a^2} - \frac{(-abB+A(b^2-2ac))\,\text{Subst}\left(\int \frac{1}{b^2-4ac-x^2}\,dx, x, b+2cx^2\right)}{2a^2}$$

$$= -\frac{A}{2ax^2} + \frac{(abB-A(b^2-2ac))\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2a^2\sqrt{b^2-4ac}} - \frac{(Ab-aB)\log(x)}{a^2} + \frac{(Ab-aB)\log\left(a+bx^2+cx^4\right)}{4a^2}$$

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 186, normalized size of antiderivative = 1.66

$$\int \frac{A+Bx^2}{x^3\,(a+bx^2+cx^4)}\,dx = \frac{-\frac{2aA}{x^2} + 4(-Ab+aB)\log(x) + \frac{\left(-aB\left(b+\sqrt{b^2-4ac}\right)+A\left(b^2-2ac+b\sqrt{b^2-4ac}\right)\right)\log\left(b-\sqrt{b^2-4ac}+2cx^2\right)}{\sqrt{b^2-4ac}} + \frac{\left(aB\left(b-\sqrt{b^2-4ac}\right)+A\right.}{}}{4a^2}$$

```
[In] Integrate[(A + B*x^2)/(x^3*(a + b*x^2 + c*x^4)),x]
```

```
[Out] ((-2*a*A)/x^2 + 4*(-(A*b) + a*B)*Log[x] + ((-(a*B*(b + Sqrt[b^2 - 4*a*c]))
+ A*(b^2 - 2*a*c + b*Sqrt[b^2 - 4*a*c]))*Log[b - Sqrt[b^2 - 4*a*c] + 2*c*x^
2])/Sqrt[b^2 - 4*a*c] + ((a*B*(b - Sqrt[b^2 - 4*a*c]) + A*(-b^2 + 2*a*c + b
*Sqrt[b^2 - 4*a*c]))*Log[b + Sqrt[b^2 - 4*a*c] + 2*c*x^2])/Sqrt[b^2 - 4*a*c
])/(4*a^2)
```

Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.12

method	result
default	$-\frac{A}{2a\,x^{2}}+\frac{(-Ab+Ba)\ln(x)}{a^{2}}-\frac{\frac{(-Abc+Bac)\ln\left(c\,x^{4}+b\,x^{2}+a\right)}{2c}+\frac{2\left(Aac-A\,b^{2}+abB-\frac{(-Abc+Bac)b}{2c}\right)\arctan\left(\frac{2c\,x^{2}+b}{\sqrt{4ac-b^{2}}}\right)}{\sqrt{4ac-b^{2}}}}{2a^{2}}$
risch	$-\frac{A}{2a\,x^{2}}-\frac{\ln(x)Ab}{a^{2}}+\frac{\ln(x)B}{a}+\frac{\left(\sum_{\_R=\text{RootOf}\left(\left(4a^{3}c-a^{2}b^{2}\right)\_Z^{2}+\left(-4Aabc+A\,b^{3}+4a^{2}Bc-Ba\,b^{2}\right)\_Z+A^{2}c^{2}-bBAc+B^{2}ac\right)}\_R\ln\left(\left(\right.\right.\right.}{}$

```
[In] int((B*x^2+A)/x^3/(c*x^4+b*x^2+a),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*A/a/x^2+1/a^2*(-A*b+B*a)*ln(x)-1/2/a^2*(1/2*(-A*b*c+B*a*c)/c*ln(c*x^4+
b*x^2+a)+2*(A*a*c-A*b^2+a*b*B-1/2*(-A*b*c+B*a*c)*b/c)/(4*a*c-b^2)^(1/2)*arc
tan((2*c*x^2+b)/(4*a*c-b^2)^(1/2)))
```

Fricas [A] (verification not implemented)

none

Time = 0.39 (sec) , antiderivative size = 385, normalized size of antiderivative = 3.44

$$\int \frac{A+Bx^2}{x^3\left(a+bx^2+cx^4\right)}\,dx = \left[\frac{(Bab-Ab^2+2\,Aac)\sqrt{b^2-4\,ac}x^2\log\left(\frac{2\,c^2x^4+2\,bcx^2+b^2-2\,ac+(2\,cx^2+b)\sqrt{b^2-4\,ac}}{cx^4+bx^2+a}\right)-2\,Aab^2+8\,Aa^2c-(Bab^2-}{4\,(a^2b^2-4\,a^3c)}\right.$$

```
[In] integrate((B*x^2+A)/x^3/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] [1/4*((B*a*b - A*b^2 + 2*A*a*c)*sqrt(b^2 - 4*a*c)*x^2*log((2*c^2*x^4 + 2*b*
c*x^2 + b^2 - 2*a*c + (2*c*x^2 + b)*sqrt(b^2 - 4*a*c))/(c*x^4 + b*x^2 + a))
 - 2*A*a*b^2 + 8*A*a^2*c - (B*a*b^2 - A*b^3 - 4*(B*a^2 - A*a*b)*c)*x^2*log(
c*x^4 + b*x^2 + a) + 4*(B*a*b^2 - A*b^3 - 4*(B*a^2 - A*a*b)*c)*x^2*log(x))/
((a^2*b^2 - 4*a^3*c)*x^2), 1/4*(2*(B*a*b - A*b^2 + 2*A*a*c)*sqrt(-b^2 + 4*a
*c)*x^2*arctan(-(2*c*x^2 + b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4*a*c)) - 2*A*a*b^2
 + 8*A*a^2*c - (B*a*b^2 - A*b^3 - 4*(B*a^2 - A*a*b)*c)*x^2*log(c*x^4 + b*x^
2 + a) + 4*(B*a*b^2 - A*b^3 - 4*(B*a^2 - A*a*b)*c)*x^2*log(x))/((a^2*b^2 -
4*a^3*c)*x^2)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A+Bx^2}{x^3\left(a+bx^2+cx^4\right)}\,dx=\text{Timed out}$$

```
[In] integrate((B*x**2+A)/x**3/(c*x**4+b*x**2+a),x)
```

```
[Out] Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{A+Bx^2}{x^3\left(a+bx^2+cx^4\right)}\,dx=\text{Exception raised: ValueError}$$

```
[In] integrate((B*x^2+A)/x^3/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

Giac [A] (verification not implemented)

none

Time = 0.63 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.11

$$\int \frac{A+Bx^2}{x^3\left(a+bx^2+cx^4\right)}\,dx=-\frac{(Ba-Ab)\log\left(cx^4+bx^2+a\right)}{4\,a^2}+\frac{(Ba-Ab)\log\left(x^2\right)}{2\,a^2}-\frac{\left(Bab-Ab^2+2\,Aac\right)\arctan\left(\frac{2\,cx^2+b}{\sqrt{-b^2+4\,ac}}\right)}{2\,\sqrt{-b^2+4\,ac}a^2}-\frac{Bax^2-Abx^2+Aa}{2\,a^2x^2}$$

```
[In] integrate((B*x^2+A)/x^3/(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] -1/4*(B*a - A*b)*log(c*x^4 + b*x^2 + a)/a^2 + 1/2*(B*a - A*b)*log(x^2)/a^2
- 1/2*(B*a*b - A*b^2 + 2*A*a*c)*arctan((2*c*x^2 + b)/sqrt(-b^2 + 4*a*c))/(s
qrt(-b^2 + 4*a*c)*a^2) - 1/2*(B*a*x^2 - A*b*x^2 + A*a)/(a^2*x^2)
```

Mupad [B] (verification not implemented)

Time = 10.27 (sec) , antiderivative size = 3729, normalized size of antiderivative = 33.29

$$\int \frac{A+Bx^2}{x^3\left(a+bx^2+cx^4\right)}\,dx = \text{Too large to display}$$

```
[In] int((A + B*x^2)/(x^3*(a + b*x^2 + c*x^4)),x)
```

```
[Out] - A/(2*a*x^2) - (log(x)*(A*b - B*a))/a^2 - (log(((A^3*c^5*x^2)/a^3 - (((((4
*b*c^2*(A*a*c - A*b^2 + B*a*b))/a - (2*c^3*x^2*(A*b^2 + 10*A*a*c - 5*B*a*b)
)/a + (b*c^2*(a*b + 3*b^2*x^2 - 10*a*c*x^2)*(B*a - A*b + a^2*(-(2*A*a*c - A
*b^2 + B*a*b)^2/(a^4*(4*a*c - b^2)))^(1/2)))/a^2)*(B*a - A*b + a^2*(-(2*A*a
*c - A*b^2 + B*a*b)^2/(a^4*(4*a*c - b^2)))^(1/2)))/(4*a^2) + (A*c^3*(A*a*c
- 4*A*b^2 + 4*B*a*b))/a^2 - (A*c^4*x^2*(6*A*b - 5*B*a))/a^2)*(B*a - A*b + a
^2*(-(2*A*a*c - A*b^2 + B*a*b)^2/(a^4*(4*a*c - b^2)))^(1/2)))/(4*a^2) + (A^
2*c^4*(A*b - B*a))/a^3)*((((((2*c^3*x^2*(A*b^2 + 10*A*a*c - 5*B*a*b))/a - (
4*b*c^2*(A*a*c - A*b^2 + B*a*b))/a + (b*c^2*(a*b + 3*b^2*x^2 - 10*a*c*x^2)*
(A*b - B*a + a^2*(-(2*A*a*c - A*b^2 + B*a*b)^2/(a^4*(4*a*c - b^2)))^(1/2)))
/a^2)*(A*b - B*a + a^2*(-(2*A*a*c - A*b^2 + B*a*b)^2/(a^4*(4*a*c - b^2)))^(
1/2)))/(4*a^2) + (A*c^3*(A*a*c - 4*A*b^2 + 4*B*a*b))/a^2 - (A*c^4*x^2*(6*A*
b - 5*B*a))/a^2)*(A*b - B*a + a^2*(-(2*A*a*c - A*b^2 + B*a*b)^2/(a^4*(4*a*c
 - b^2)))^(1/2)))/(4*a^2) + (A^3*c^5*x^2)/a^3 + (A^2*c^4*(A*b - B*a))/a^3))
*(2*A*b^3 - 2*B*a*b^2 + 8*B*a^2*c - 8*A*a*b*c))/(2*(16*a^3*c - 4*a^2*b^2))
- (atan((16*a^6*x^2*((((((5*A*B*a^2*c^4 - 6*A^2*a*b*c^4)/a^3 - (((20*A*a^3*
c^4 - 10*B*a^3*b*c^3 + 2*A*a^2*b^2*c^3)/a^3 + ((40*a^4*b*c^3 - 12*a^3*b^3*c
^2)*(2*A*b^3 - 2*B*a*b^2 + 8*B*a^2*c - 8*A*a*b*c))/(2*a^3*(16*a^3*c - 4*a^2
*b^2)))*(2*A*b^3 - 2*B*a*b^2 + 8*B*a^2*c - 8*A*a*b*c))/(2*(16*a^3*c - 4*a^2
*b^2)))*(2*A*b^3 - 2*B*a*b^2 + 8*B*a^2*c - 8*A*a*b*c))/(2*(16*a^3*c - 4*a^2
*b^2)) - (A^3*c^5)/a^3 + (((((20*A*a^3*c^4 - 10*B*a^3*b*c^3 + 2*A*a^2*b^2*c
^3)/a^3 + ((40*a^4*b*c^3 - 12*a^3*b^3*c^2)*(2*A*b^3 - 2*B*a*b^2 + 8*B*a^2*c
 - 8*A*a*b*c))/(2*a^3*(16*a^3*c - 4*a^2*b^2)))*(2*A*a*c - A*b^2 + B*a*b))/(
4*a^2*(4*a*c - b^2)^(1/2)) + ((40*a^4*b*c^3 - 12*a^3*b^3*c^2)*(2*A*a*c - A*
b^2 + B*a*b)*(2*A*b^3 - 2*B*a*b^2 + 8*B*a^2*c - 8*A*a*b*c))/(8*a^5*(4*a*c -
 b^2)^(1/2)*(16*a^3*c - 4*a^2*b^2)))*(2*A*a*c - A*b^2 + B*a*b))/(4*a^2*(4*a
*c - b^2)^(1/2)) + ((40*a^4*b*c^3 - 12*a^3*b^3*c^2)*(2*A*a*c - A*b^2 + B*a*
b)^2*(2*A*b^3 - 2*B*a*b^2 + 8*B*a^2*c - 8*A*a*b*c))/(32*a^7*(4*a*c - b^2)*(
16*a^3*c - 4*a^2*b^2)))*(3*A*b^4 + A*a^2*c^2 - 3*B*a*b^3 - 9*A*a*b^2*c + 8*
B*a^2*b*c))/(8*a^3*c^2*(25*B^2*a^3*c - 6*A^2*b^4 + A^2*a^2*c^2 - 6*B^2*a^2*
b^2 + 12*A*B*a*b^3 + 24*A^2*a*b^2*c - 49*A*B*a^2*b*c)) + (((((5*A*B*a^2*c^4
 - 6*A^2*a*b*c^4)/a^3 - (((20*A*a^3*c^4 - 10*B*a^3*b*c^3 + 2*A*a^2*b^2*c^3)
/a^3 + ((40*a^4*b*c^3 - 12*a^3*b^3*c^2)*(2*A*b^3 - 2*B*a*b^2 + 8*B*a^2*c -
8*A*a*b*c))/(2*a^3*(16*a^3*c - 4*a^2*b^2)))*(2*A*b^3 - 2*B*a*b^2 + 8*B*a^2*
c - 8*A*a*b*c))/(2*(16*a^3*c - 4*a^2*b^2)))*(2*A*a*c - A*b^2 + B*a*b))/(4*a
^2*(4*a*c - b^2)^(1/2)) - (((((20*A*a^3*c^4 - 10*B*a^3*b*c^3 + 2*A*a^2*b^2*
```

c^3)/a^3 + ((40*a^4*b*c^3 - 12*a^3*b^3*c^2)*(2*A*b^3 - 2*B*a*b^2 + 8*B*a^2*c - 8*A*a*b*c))/(2*a^3*(16*a^3*c - 4*a^2*b^2)))*(2*A*a*c - A*b^2 + B*a*b))/(4*a^2*(4*a*c - b^2)^(1/2)) + ((40*a^4*b*c^3 - 12*a^3*b^3*c^2)*(2*A*a*c - A*b^2 + B*a*b)*(2*A*b^3 - 2*B*a*b^2 + 8*B*a^2*c - 8*A*a*b*c))/(8*a^5*(4*a*c - b^2)^(1/2)*(16*a^3*c - 4*a^2*b^2)))*(2*A*b^3 - 2*B*a*b^2 + 8*B*a^2*c - 8*A*a*b*c))/(2*(16*a^3*c - 4*a^2*b^2)) + ((40*a^4*b*c^3 - 12*a^3*b^3*c^2)*(2*A*a*c - A*b^2 + B*a*b)^3)/(64*a^9*(4*a*c - b^2)^(3/2)))*(6*A*b^5 - 20*B*a^3*c^2 - 6*B*a*b^4 - 30*A*a*b^3*c + 26*A*a^2*b*c^2 + 28*B*a^2*b^2*c))/(16*a^3*c^2*(4*a*c - b^2)^(1/2)*(25*B^2*a^3*c - 6*A^2*b^4 + A^2*a^2*c^2 - 6*B^2*a^2*b^2 + 12*A*B*a*b^3 + 24*A^2*a*b^2*c - 49*A*B*a^2*b*c)))*(4*a*c - b^2)^(3/2))/(4*A^2*a^2*c^4 + A^2*b^4*c^2 + B^2*a^2*b^2*c^2 - 4*A^2*a*b^2*c^3 - 2*A*B*a*b^3*c^2 + 4*A*B*a^2*b*c^3) + (a^3*(4*a*c - b^2)*((((((4*A*a^3*b*c^3 - 4*A*a^2*b^3*c^2 + 4*B*a^3*b^2*c^2)/a^3 + (2*a*b^2*c^2*(2*A*b^3 - 2*B*a*b^2 + 8*B*a^2*c - 8*A*a*b*c))/(16*a^3*c - 4*a^2*b^2))*(2*A*a*c - A*b^2 + B*a*b))/(4*a^2*(4*a*c - b^2)^(1/2)) + (b^2*c^2*(2*A*a*c - A*b^2 + B*a*b)*(2*A*b^3 - 2*B*a*b^2 + 8*B*a^2*c - 8*A*a*b*c))/(2*a*(4*a*c - b^2)^(1/2)*(16*a^3*c - 4*a^2*b^2)))*(2*A*b^3 - 2*B*a*b^2 + 8*B*a^2*c - 8*A*a*b*c))/(2*(16*a^3*c - 4*a^2*b^2)) + (((A^2*a^2*c^4 - 4*A^2*a*b^2*c^3 + 4*A*B*a^2*b*c^3)/a^3 + (((4*A*a^3*b*c^3 - 4*A*a^2*b^3*c^2 + 4*B*a^3*b^2*c^2)/a^3 + (2*a*b^2*c^2*(2*A*b^3 - 2*B*a*b^2 + 8*B*a^2*c - 8*A*a*b*c))/(16*a^3*c - 4*a^2*b^2))*(2*A*b^3 - 2*B*a*b^2 + 8*B*a^2*c - 8*A*a*b*c))/(2*(16*a^3*c - 4*a^2*b^2)))*(2*A*a*c - A*b^2 + B*a*b))/(4*a^2*(4*a*c - b^2)^(1/2)) - (b^2*c^2*(2*A*a*c - A*b^2 + B*a*b)^3)/(16*a^5*(4*a*c - b^2)^(3/2)))*(6*A*b^5 - 20*B*a^3*c^2 - 6*B*a*b^4 - 30*A*a*b^3*c + 26*A*a^2*b*c^2 + 28*B*a^2*b^2*c))/(c^2*(4*A^2*a^2*c^4 + A^2*b^4*c^2 + B^2*a^2*b^2*c^2 - 4*A^2*a*b^2*c^3 - 2*A*B*a*b^3*c^2 + 4*A*B*a^2*b*c^3)*(25*B^2*a^3*c - 6*A^2*b^4 + A^2*a^2*c^2 - 6*B^2*a^2*b^2 + 12*A*B*a*b^3 + 24*A^2*a*b^2*c - 49*A*B*a^2*b*c)) - (2*a^3*(4*a*c - b^2)^(3/2)*((A^3*b*c^4 - A^2*B*a*c^4)/a^3 - (((A^2*a^2*c^4 - 4*A^2*a*b^2*c^3 + 4*A*B*a^2*b*c^3)/a^3 + (((4*A*a^3*b*c^3 - 4*A*a^2*b^3*c^2 + 4*B*a^3*b^2*c^2)/a^3 + (2*a*b^2*c^2*(2*A*b^3 - 2*B*a*b^2 + 8*B*a^2*c - 8*A*a*b*c))/(16*a^3*c - 4*a^2*b^2))*(2*A*b^3 - 2*B*a*b^2 + 8*B*a^2*c - 8*A*a*b*c))/(2*(16*a^3*c - 4*a^2*b^2)))*(2*A*b^3 - 2*B*a*b^2 + 8*B*a^2*c - 8*A*a*b*c))/(2*(16*a^3*c - 4*a^2*b^2)) + (((((4*A*a^3*b*c^3 - 4*A*a^2*b^3*c^2 + 4*B*a^3*b^2*c^2)/a^3 + (2*a*b^2*c^2*(2*A*b^3 - 2*B*a*b^2 + 8*B*a^2*c - 8*A*a*b*c))/(16*a^3*c - 4*a^2*b^2))*(2*A*a*c - A*b^2 + B*a*b))/(4*a^2*(4*a*c - b^2)^(1/2)) + (b^2*c^2*(2*A*a*c - A*b^2 + B*a*b)*(2*A*b^3 - 2*B*a*b^2 + 8*B*a^2*c - 8*A*a*b*c))/(2*a*(4*a*c - b^2)^(1/2)*(16*a^3*c - 4*a^2*b^2)))*(2*A*a*c - A*b^2 + B*a*b))/(4*a^2*(4*a*c - b^2)^(1/2)) + (b^2*c^2*(2*A*a*c - A*b^2 + B*a*b)^2*(2*A*b^3 - 2*B*a*b^2 + 8*B*a^2*c - 8*A*a*b*c))/(8*a^3*(4*a*c - b^2)*(16*a^3*c - 4*a^2*b^2)))*(3*A*b^4 + A*a^2*c^2 - 3*B*a*b^3 - 9*A*a*b^2*c + 8*B*a^2*b*c))/(c^2*(4*A^2*a^2*c^4 + A^2*b^4*c^2 + B^2*a^2*b^2*c^2 - 4*A^2*a*b^2*c^3 - 2*A*B*a*b^3*c^2 + 4*A*B*a^2*b*c^3)*(25*B^2*a^3*c - 6*A^2*b^4 + A^2*a^2*c^2 - 6*B^2*a^2*b^2 + 12*A*B*a*b^3 + 24*A^2*a*b^2*c - 49*A*B*a^2*b*c)))*(2*A*a*c - A*b^2 + B*a*b))/(2*a^2*(4*a*c - b^2)^(1/2))

3.107 $\int \frac{x^4(A+Bx^2)}{a+bx^2+cx^4}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 261

$$\int \frac{x^4(A+Bx^2)}{a+bx^2+cx^4}\,dx = -\frac{(bB-Ac)x}{c^2}+\frac{Bx^3}{3c} + \frac{\left(b^2B-Abc-aBc-\frac{b^3B-Ab^2c-3abBc+2aAc^2}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}c^{5/2}\sqrt{b-\sqrt{b^2-4ac}}} + \frac{\left(b^2B-Abc-aBc+\frac{b^3B-Ab^2c-3abBc+2aAc^2}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{2}c^{5/2}\sqrt{b+\sqrt{b^2-4ac}}}$$

```
[Out] -(-A*c+B*b)*x/c^2+1/3*B*x^3/c+1/2*arctan(x*2^(1/2)*c^(1/2)/(b-(-4*a*c+b^2)^
(1/2))^(1/2))*(B*b^2-A*b*c-B*a*c+(-2*A*a*c^2+A*b^2*c+3*B*a*b*c-B*b^3)/(-4*a
*c+b^2)^(1/2))/c^(5/2)*2^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2)+1/2*arctan(x*2^
(1/2)*c^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2))*(B*b^2-A*b*c-B*a*c+(2*A*a*c^2-A
*b^2*c-3*B*a*b*c+B*b^3)/(-4*a*c+b^2)^(1/2))/c^(5/2)*2^(1/2)/(b+(-4*a*c+b^2)
^(1/2))^(1/2)
```

Rubi [A] (verified)

Time = 0.83 (sec) , antiderivative size = 261, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.120$, Rules used

$= \{1293, 1180, 211\}$

$$\int \frac{x^4(A+Bx^2)}{a+bx^2+cx^4}\,dx = \frac{\left(-\frac{2aAc^2-3abBc-Ab^2c+b^3B}{\sqrt{b^2-4ac}}-aBc-Abc+b^2B\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}c^{5/2}\sqrt{b-\sqrt{b^2-4ac}}} + \frac{\left(\frac{2aAc^2-3abBc-Ab^2c+b^3B}{\sqrt{b^2-4ac}}-aBc-Abc+b^2B\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}c^{5/2}\sqrt{\sqrt{b^2-4ac}+b}} - \frac{x(bB-Ac)}{c^2}+\frac{Bx^3}{3c}$$

```
[In] Int[(x^4*(A + B*x^2))/(a + b*x^2 + c*x^4),x]
```

```
[Out] -(((b*B - A*c)*x)/c^2) + (B*x^3)/(3*c) + ((b^2*B - A*b*c - a*B*c - (b^3*B -
 A*b^2*c - 3*a*b*B*c + 2*a*A*c^2)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c
]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*c^(5/2)*Sqrt[b - Sqrt[b^2 - 4*a
*c]]) + ((b^2*B - A*b*c - a*B*c + (b^3*B - A*b^2*c - 3*a*b*B*c + 2*a*A*c^2)
/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]]
)/(Sqrt[2]*c^(5/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1293

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[e*f*(f*x)^(m - 1)*((a + b*x^2 + c*x^4)^(p +
1)/(c*(m + 4*p + 3))), x] - Dist[f^2/(c*(m + 4*p + 3)), Int[(f*x)^(m - 2)*(
a + b*x^2 + c*x^4)^p*Simp[a*e*(m - 1) + (b*e*(m + 2*p + 1) - c*d*(m + 4*p +
 3))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c,
 0] && GtQ[m, 1] && NeQ[m + 4*p + 3, 0] && IntegerQ[2*p] && (IntegerQ[p] ||
 IntegerQ[m])
```

Rubi steps

$$\text{integral} = \frac{Bx^3}{3c} - \frac{\int \frac{x^2(3aB+3(bB-Ac)x^2)}{a+bx^2+cx^4}\,dx}{3c}$$

$$= -\frac{(bB-Ac)x}{c^2} + \frac{Bx^3}{3c} + \frac{\int \frac{3a(bB-Ac)+3(b^2B-Abc-aBc)x^2}{a+bx^2+cx^4}\,dx}{3c^2}$$

$$= -\frac{(bB-Ac)x}{c^2} + \frac{Bx^3}{3c} + \frac{\left(b^2B-Abc-aBc-\frac{b^3B-Ab^2c-3abBc+2aAc^2}{\sqrt{b^2-4ac}}\right)\int \frac{1}{\frac{b}{2}-\frac{1}{2}\sqrt{b^2-4ac}+cx^2}\,dx}{2c^2} + \frac{\left(b^2B-Abc-aBc+\frac{b^3B-Ab^2c-3abBc+2aAc^2}{\sqrt{b^2-4ac}}\right)\int \frac{1}{\frac{b}{2}+\frac{1}{2}\sqrt{b^2-4ac}+cx^2}\,dx}{2c^2}$$

$$= -\frac{(bB-Ac)x}{c^2} + \frac{Bx^3}{3c} + \frac{\left(b^2B-Abc-aBc-\frac{b^3B-Ab^2c-3abBc+2aAc^2}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}c^{5/2}\sqrt{b-\sqrt{b^2-4ac}}} + \frac{\left(b^2B-Abc-aBc+\frac{b^3B-Ab^2c-3abBc+2aAc^2}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{2}c^{5/2}\sqrt{b+\sqrt{b^2-4ac}}}$$

Mathematica [A] (verified)

Time = 0.25 (sec) , antiderivative size = 327, normalized size of antiderivative = 1.25

$$\int \frac{x^4(A+Bx^2)}{a+bx^2+cx^4}\,dx = \frac{(-bB+Ac)x}{c^2} + \frac{Bx^3}{3c} + \frac{\left(-b^3B+Ab^2c+3abBc-2aAc^2+b^2B\sqrt{b^2-4ac}-Abc\sqrt{b^2-4ac}-aBc\sqrt{b^2-4ac}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}c^{5/2}\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}} + \frac{\left(b^3B-Ab^2c-3abBc+2aAc^2+b^2B\sqrt{b^2-4ac}-Abc\sqrt{b^2-4ac}-aBc\sqrt{b^2-4ac}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{2}c^{5/2}\sqrt{b^2-4ac}\sqrt{b+\sqrt{b^2-4ac}}}$$

```
[In] Integrate[(x^4*(A + B*x^2))/(a + b*x^2 + c*x^4),x]
[Out] ((-(b*B) + A*c)*x)/c^2 + (B*x^3)/(3*c) + ((-(b^3*B) + A*b^2*c + 3*a*b*B*c -
 2*a*A*c^2 + b^2*B*Sqrt[b^2 - 4*a*c] - A*b*c*Sqrt[b^2 - 4*a*c] - a*B*c*Sqrt
[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sq
rt[2]*c^(5/2)*Sqrt[b^2 - 4*a*c]*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + ((b^3*B - A*
b^2*c - 3*a*b*B*c + 2*a*A*c^2 + b^2*B*Sqrt[b^2 - 4*a*c] - A*b*c*Sqrt[b^2 -
4*a*c] - a*B*c*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[
b^2 - 4*a*c]]])/(Sqrt[2]*c^(5/2)*Sqrt[b^2 - 4*a*c]*Sqrt[b + Sqrt[b^2 - 4*a*
c]])
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.10 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.36

method	result
risch	$\frac{B\,x^{3}}{3c}+\frac{Ax}{c}-\frac{Bbx}{c^{2}}+\frac{\sum_{\_R=\mathrm{RootOf}\left(c\_Z^{4}+\_Z^{2}b+a\right)}\frac{\left(\left(-Abc-Bac+B\,b^{2}\right)\_R^{2}-Aac+abB\right)\ln\left(x-\_R\right)}{2c\_R^{3}+\_Rb}}{2c^{2}}$
default	$\frac{\frac{1}{3}Bc\,x^{3}+Acx-Bbx}{c^{2}}+\frac{\frac{\left(-Abc\sqrt{-4ac+b^{2}}+2Aa\,c^{2}-A\,b^{2}c-Bac\sqrt{-4ac+b^{2}}+B\,b^{2}\sqrt{-4ac+b^{2}}-3Babc+B\,b^{3}\right)\sqrt{2}\,\arctan\left(\frac{cx\sqrt{2}}{\sqrt{\left(b+\sqrt{-4ac+b^{2}}\right)c}}\right)}{2c\sqrt{-4ac+b^{2}}\,\sqrt{\left(b+\sqrt{-4ac+b^{2}}\right)c}}-}{}$

[In] int(x^4*(B*x^2+A)/(c*x^4+b*x^2+a),x,method=_RETURNVERBOSE)

[Out] 1/3*B*x^3/c+1/c*A*x-1/c^2*B*b*x+1/2/c^2*sum(((-A*b*c-B*a*c+B*b^2)*_R^2-A*a*c+a*b*B)/(2*_R^3*c+_R*b)*ln(x-_R),_R=RootOf(_Z^4*c+_Z^2*b+a))

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 5140 vs. 2(225) = 450.

Time = 2.57 (sec) , antiderivative size = 5140, normalized size of antiderivative = 19.69

$$\int \frac{x^4(A+Bx^2)}{a+bx^2+cx^4}\,dx=\text{Too large to display}$$

[In] integrate(x^4*(B*x^2+A)/(c*x^4+b*x^2+a),x, algorithm="fricas")

[Out] Too large to include

Sympy [F(-1)]

Timed out.

$$\int \frac{x^4(A+Bx^2)}{a+bx^2+cx^4}\,dx=\text{Timed out}$$

[In] integrate(x**4*(B*x**2+A)/(c*x**4+b*x**2+a),x)

[Out] Timed out

Maxima [F]

$$\int \frac{x^4(A+Bx^2)}{a+bx^2+cx^4}\,dx=\int \frac{(Bx^2+A)x^4}{cx^4+bx^2+a}\,dx$$

```
[In] integrate(x^4*(B*x^2+A)/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

```
[Out] 1/3*(B*c*x^3 - 3*(B*b - A*c)*x)/c^2 - integrate(-(B*a*b - A*a*c + (B*b^2 -
(B*a + A*b)*c)*x^2)/(c*x^4 + b*x^2 + a), x)/c^2
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 4391 vs. $2(225) = 450$.

Time = 1.07 (sec) , antiderivative size = 4391, normalized size of antiderivative = 16.82

$$\int \frac{x^4(A+Bx^2)}{a+bx^2+cx^4}\,dx=\text{Too large to display}$$

```
[In] integrate(x^4*(B*x^2+A)/(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] 1/8*((2*b^5*c^3 - 16*a*b^3*c^4 + 32*a^2*b*c^5 - sqrt(2)*sqrt(b^2 - 4*a*c)*s
qrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^5*c + 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c
 + sqrt(b^2 - 4*a*c)*c)*a*b^3*c^2 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c +
sqrt(b^2 - 4*a*c)*c)*b^4*c^2 - 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt
(b^2 - 4*a*c)*c)*a^2*b*c^3 - 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^
2 - 4*a*c)*c)*a*b^2*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4
*a*c)*c)*b^3*c^3 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)
*c)*a*b*c^4 - 2*(b^2 - 4*a*c)*b^3*c^3 + 8*(b^2 - 4*a*c)*a*b*c^4)*A*c^2 - (2
*b^6*c^2 - 18*a*b^4*c^3 + 48*a^2*b^2*c^4 - 32*a^3*c^5 - sqrt(2)*sqrt(b^2 -
4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^6 + 9*sqrt(2)*sqrt(b^2 - 4*a*c)*sq
rt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^4*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*
c + sqrt(b^2 - 4*a*c)*c)*b^5*c - 24*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sq
rt(b^2 - 4*a*c)*c)*a^2*b^2*c^2 - 10*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sq
rt(b^2 - 4*a*c)*c)*a*b^3*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^
2 - 4*a*c)*c)*b^4*c^2 + 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 -
4*a*c)*c)*a^3*c^3 + 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c
)*c)*a^2*b*c^3 + 5*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c
)*a*b^2*c^3 - 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a
^2*c^4 - 2*(b^2 - 4*a*c)*b^4*c^2 + 10*(b^2 - 4*a*c)*a*b^2*c^3 - 8*(b^2 - 4*
a*c)*a^2*c^4)*B*c^2 - 2*(sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^4*c^3
- 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^4 - 2*sqrt(2)*sqrt(b*
c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c^4 - 2*a*b^4*c^4 + 16*sqrt(2)*sqrt(b*c + sq
rt(b^2 - 4*a*c)*c)*a^3*c^5 + 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*
b*c^5 + sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^5 + 16*a^2*b^2*c^5
```

- 4*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*c^6 - 32*a^3*c^6 + 2*(b^2 - 4*a*c)*a*b^2*c^4 - 8*(b^2 - 4*a*c)*a^2*c^5)*A*abs(c) + 2*(sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^5*c^2 - 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c^3 - 2*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^4*c^3 - 2*a*b^5*c^3 + 16*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b*c^4 + 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^4 + sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c^4 + 16*a^2*b^3*c^4 - 4*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b*c^5 - 32*a^3*b*c^5 + 2*(b^2 - 4*a*c)*a*b^3*c^3 - 8*(b^2 - 4*a*c)*a^2*b*c^4)*B*abs(c) - (2*b^5*c^5 - 12*a*b^3*c^6 + 16*a^2*b*c^7 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^5*c^3 + 6*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c^4 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^4*c^4 - 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b*c^5 - 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^5 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^3*c^5 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c^6 - 2*(b^2 - 4*a*c)*b^3*c^5 + 4*(b^2 - 4*a*c)*a*b*c^6)*A + (2*b^6*c^4 - 14*a*b^4*c^5 + 24*a^2*b^2*c^6 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^6*c^2 + 7*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^4*c^3 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^5*c^3 - 12*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^4 - 6*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^4*c^4 + 3*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^5 - 2*(b^2 - 4*a*c)*b^4*c^4 + 6*(b^2 - 4*a*c)*a*b^2*c^5)*B)*arctan(2*sqrt(1/2)*x/sqrt((b*c^3 + sqrt(b^2*c^6 - 4*a*c^7))/c^4))/((a*b^4*c^4 - 8*a^2*b^2*c^5 - 2*a*b^3*c^5 + 16*a^3*c^6 + 8*a^2*b*c^6 + a*b^2*c^6 - 4*a^2*c^7)*c^2) + 1/8*((2*b^5*c^3 - 16*a*b^3*c^4 + 32*a^2*b*c^5 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^5*c + 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^3*c^2 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^4*c^2 - 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b*c^3 - 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^2*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^3*c^3 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b*c^4 - 2*(b^2 - 4*a*c)*b^3*c^3 + 8*(b^2 - 4*a*c)*a*b*c^4)*A*c^2 - (2*b^6*c^2 - 18*a*b^4*c^3 + 48*a^2*b^2*c^4 - 32*a^3*c^5 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^6 + 9*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^4*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^5*c - 24*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^2 - 10*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^3*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^4*c^2 + 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*c^3 + 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b*c^3 + 5*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^2*c^3 - 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*c^4 - 2*(b^2 - 4*a*c)*b^4*c^2 + 10*(b^2 -

```
4*a*c)*a*b^2*c^3 - 8*(b^2 - 4*a*c)*a^2*c^4)*B*c^2 - 2*(sqrt(2)*sqrt(b*c - s
qrt(b^2 - 4*a*c)*c)*a*b^4*c^3 - 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a
^2*b^2*c^4 - 2*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^3*c^4 + 2*a*b^4*
c^4 + 16*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*c^5 + 8*sqrt(2)*sqrt(b
*c - sqrt(b^2 - 4*a*c)*c)*a^2*b*c^5 + sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*
c)*a*b^2*c^5 - 16*a^2*b^2*c^5 - 4*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a
^2*c^6 + 32*a^3*c^6 - 2*(b^2 - 4*a*c)*a*b^2*c^4 + 8*(b^2 - 4*a*c)*a^2*c^5)*
A*abs(c) + 2*(sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^5*c^2 - 8*sqrt(2)
*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c^3 - 2*sqrt(2)*sqrt(b*c - sqrt(b^
2 - 4*a*c)*c)*a*b^4*c^3 + 2*a*b^5*c^3 + 16*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*
a*c)*c)*a^3*b*c^4 + 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^4 +
 sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^3*c^4 - 16*a^2*b^3*c^4 - 4*sqr
t(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b*c^5 + 32*a^3*b*c^5 - 2*(b^2 - 4*
a*c)*a*b^3*c^3 + 8*(b^2 - 4*a*c)*a^2*b*c^4)*B*abs(c) - (2*b^5*c^5 - 12*a*b^
3*c^6 + 16*a^2*b*c^7 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*
c)*c)*b^5*c^3 + 6*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)
*a*b^3*c^4 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^
4*c^4 - 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b*c
^5 - 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^2*c^5
- sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^3*c^5 + 2*sqr
t(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b*c^6 - 2*(b^2 - 4
*a*c)*b^3*c^5 + 4*(b^2 - 4*a*c)*a*b*c^6)*A + (2*b^6*c^4 - 14*a*b^4*c^5 + 24
*a^2*b^2*c^6 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^
6*c^2 + 7*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^4*c
^3 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^5*c^3 -
12*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^4 -
6*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^3*c^4 - sqr
t(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^4*c^4 + 3*sqrt(2)*
sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^2*c^5 - 2*(b^2 - 4*a*
c)*b^4*c^4 + 6*(b^2 - 4*a*c)*a*b^2*c^5)*B)*arctan(2*sqrt(1/2)*x/sqrt((b*c^3
 - sqrt(b^2*c^6 - 4*a*c^7))/c^4))/((a*b^4*c^4 - 8*a^2*b^2*c^5 - 2*a*b^3*c^5
 + 16*a^3*c^6 + 8*a^2*b*c^6 + a*b^2*c^6 - 4*a^2*c^7)*c^2) + 1/3*(B*c^2*x^3
- 3*B*b*c*x + 3*A*c^2*x)/c^3
```

Mupad [B] (verification not implemented)

Time = 8.40 (sec) , antiderivative size = 10177, normalized size of antiderivative = 38.99

$$\int \frac{x^4(A+Bx^2)}{a+bx^2+cx^4}\,dx = \text{Too large to display}$$

```
[In] int((x^4*(A + B*x^2))/(a + b*x^2 + c*x^4),x)
```

```
[Out] x*(A/c - (B*b)/c^2) - atan(((((16*A*a^2*c^5 - 4*A*a*b^2*c^4 + 4*B*a*b^3*c^3
 - 16*B*a^2*b*c^4)/c^3 - (2*x*(4*b^3*c^5 - 16*a*b*c^6)*(-(B^2*b^7 + A^2*b^5
```

*c^2 + B^2*b^4*(-(4*a*c - b^2)^3)^(1/2) - 2*A*B*b^6*c + 25*B^2*a^2*b^3*c^2 + A^2*b^2*c^2*(-(4*a*c - b^2)^3)^(1/2) + B^2*a^2*c^2*(-(4*a*c - b^2)^3)^(1/2) + 16*A*B*a^3*c^4 - 9*B^2*a*b^5*c - 7*A^2*a*b^3*c^3 + 12*A^2*a^2*b*c^4 - A^2*a*c^3*(-(4*a*c - b^2)^3)^(1/2) - 20*B^2*a^3*b*c^3 - 36*A*B*a^2*b^2*c^3 - 3*B^2*a*b^2*c*(-(4*a*c - b^2)^3)^(1/2) + 16*A*B*a*b^4*c^2 - 2*A*B*b^3*c*(-(4*a*c - b^2)^3)^(1/2) + 4*A*B*a*b*c^2*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^7 + b^4*c^5 - 8*a*b^2*c^6)))^(1/2))/c^3)*(-(B^2*b^7 + A^2*b^5*c^2 + B^2*b^4*(-(4*a*c - b^2)^3)^(1/2) - 2*A*B*b^6*c + 25*B^2*a^2*b^3*c^2 + A^2*b^2*c^2*(-(4*a*c - b^2)^3)^(1/2) + B^2*a^2*c^2*(-(4*a*c - b^2)^3)^(1/2) + 16*A*B*a^3*c^4 - 9*B^2*a*b^5*c - 7*A^2*a*b^3*c^3 + 12*A^2*a^2*b*c^4 - A^2*a*c^3*(-(4*a*c - b^2)^3)^(1/2) - 20*B^2*a^3*b*c^3 - 36*A*B*a^2*b^2*c^3 - 3*B^2*a*b^2*c*(-(4*a*c - b^2)^3)^(1/2) + 16*A*B*a*b^4*c^2 - 2*A*B*b^3*c*(-(4*a*c - b^2)^3)^(1/2) + 4*A*B*a*b*c^2*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^7 + b^4*c^5 - 8*a*b^2*c^6)))^(1/2) - (2*x*(B^2*b^6 + 2*A^2*a^2*c^4 + A^2*b^4*c^2 - 2*B^2*a^3*c^3 - 2*A*B*b^5*c + 9*B^2*a^2*b^2*c^2 - 6*B^2*a*b^4*c - 4*A^2*a*b^2*c^3 + 10*A*B*a*b^3*c^2 - 10*A*B*a^2*b*c^3))/c^3)*(-(B^2*b^7 + A^2*b^5*c^2 + B^2*b^4*(-(4*a*c - b^2)^3)^(1/2) - 2*A*B*b^6*c + 25*B^2*a^2*b^3*c^2 + A^2*b^2*c^2*(-(4*a*c - b^2)^3)^(1/2) + B^2*a^2*c^2*(-(4*a*c - b^2)^3)^(1/2) + 16*A*B*a^3*c^4 - 9*B^2*a*b^5*c - 7*A^2*a*b^3*c^3 + 12*A^2*a^2*b*c^4 - A^2*a*c^3*(-(4*a*c - b^2)^3)^(1/2) - 20*B^2*a^3*b*c^3 - 36*A*B*a^2*b^2*c^3 - 3*B^2*a*b^2*c*(-(4*a*c - b^2)^3)^(1/2) + 16*A*B*a*b^4*c^2 - 2*A*B*b^3*c*(-(4*a*c - b^2)^3)^(1/2) + 4*A*B*a*b*c^2*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^7 + b^4*c^5 - 8*a*b^2*c^6)))^(1/2)*1i - (((16*A*a^2*c^5 - 4*A*a*b^2*c^4 + 4*B*a*b^3*c^3 - 16*B*a^2*b*c^4)/c^3 + (2*x*(4*b^3*c^5 - 16*a*b*c^6)*(-(B^2*b^7 + A^2*b^5*c^2 + B^2*b^4*(-(4*a*c - b^2)^3)^(1/2) - 2*A*B*b^6*c + 25*B^2*a^2*b^3*c^2 + A^2*b^2*c^2*(-(4*a*c - b^2)^3)^(1/2) + B^2*a^2*c^2*(-(4*a*c - b^2)^3)^(1/2) + 16*A*B*a^3*c^4 - 9*B^2*a*b^5*c - 7*A^2*a*b^3*c^3 + 12*A^2*a^2*b*c^4 - A^2*a*c^3*(-(4*a*c - b^2)^3)^(1/2) - 20*B^2*a^3*b*c^3 - 36*A*B*a^2*b^2*c^3 - 3*B^2*a*b^2*c*(-(4*a*c - b^2)^3)^(1/2) + 16*A*B*a*b^4*c^2 - 2*A*B*b^3*c*(-(4*a*c - b^2)^3)^(1/2) + 4*A*B*a*b*c^2*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^7 + b^4*c^5 - 8*a*b^2*c^6)))^(1/2))/c^3)*(-(B^2*b^7 + A^2*b^5*c^2 + B^2*b^4*(-(4*a*c - b^2)^3)^(1/2) - 2*A*B*b^6*c + 25*B^2*a^2*b^3*c^2 + A^2*b^2*c^2*(-(4*a*c - b^2)^3)^(1/2) + B^2*a^2*c^2*(-(4*a*c - b^2)^3)^(1/2) + 16*A*B*a^3*c^4 - 9*B^2*a*b^5*c - 7*A^2*a*b^3*c^3 + 12*A^2*a^2*b*c^4 - A^2*a*c^3*(-(4*a*c - b^2)^3)^(1/2) - 20*B^2*a^3*b*c^3 - 36*A*B*a^2*b^2*c^3 - 3*B^2*a*b^2*c*(-(4*a*c - b^2)^3)^(1/2) + 16*A*B*a*b^4*c^2 - 2*A*B*b^3*c*(-(4*a*c - b^2)^3)^(1/2) + 4*A*B*a*b*c^2*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^7 + b^4*c^5 - 8*a*b^2*c^6)))^(1/2) + (2*x*(B^2*b^6 + 2*A^2*a^2*c^4 + A^2*b^4*c^2 - 2*B^2*a^3*c^3 - 2*A*B*b^5*c + 9*B^2*a^2*b^2*c^2 - 6*B^2*a*b^4*c - 4*A^2*a*b^2*c^3 + 10*A*B*a*b^3*c^2 - 10*A*B*a^2*b*c^3))/c^3)*(-(B^2*b^7 + A^2*b^5*c^2 + B^2*b^4*(-(4*a*c - b^2)^3)^(1/2) - 2*A*B*b^6*c + 25*B^2*a^2*b^3*c^2 + A^2*b^2*c^2*(-(4*a*c - b^2)^3)^(1/2) + B^2*a^2*c^2*(-(4*a*c - b^2)^3)^(1/2) + 16*A*B*a^3*c^4 - 9*B^2*a*b^5*c - 7*A^2*a*b^3*c^3 + 12*A^2*a^2*b*c^4 - A^2*a*c^3*(-(4*a*c - b^2)^3)^(1/2) - 20*B^2*a^3*b*c^3 - 36*A*B*a^2*b^2*c^3 - 3*B^2*a*b^2*c*(-(4*a*c - b^2)^3)^(1/2) + 16*A*B*a*b^4*c^2 -

2*A*B*b^3*c*(-(4*a*c - b^2)^3)^(1/2) + 4*A*B*a*b*c^2*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^7 + b^4*c^5 - 8*a*b^2*c^6)))^(1/2)*1i)/((((16*A*a^2*c^5 - 4*A*a*b^2*c^4 + 4*B*a*b^3*c^3 - 16*B*a^2*b*c^4)/c^3 - (2*x*(4*b^3*c^5 - 16*a*b*c^6)*(-(B^2*b^7 + A^2*b^5*c^2 + B^2*b^4*(-(4*a*c - b^2)^3)^(1/2) - 2*A*B*b^6*c + 25*B^2*a^2*b^3*c^2 + A^2*b^2*c^2*(-(4*a*c - b^2)^3)^(1/2) + B^2*a^2*c^2*(-(4*a*c - b^2)^3)^(1/2) + 16*A*B*a^3*c^4 - 9*B^2*a*b^5*c - 7*A^2*a*b^3*c^3 + 12*A^2*a^2*b*c^4 - A^2*a*c^3*(-(4*a*c - b^2)^3)^(1/2) - 20*B^2*a^3*b*c^3 - 36*A*B*a^2*b^2*c^3 - 3*B^2*a*b^2*c*(-(4*a*c - b^2)^3)^(1/2) + 16*A*B*a*b^4*c^2 - 2*A*B*b^3*c*(-(4*a*c - b^2)^3)^(1/2) + 4*A*B*a*b*c^2*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^7 + b^4*c^5 - 8*a*b^2*c^6)))^(1/2))/c^3)*(-(B^2*b^7 + A^2*b^5*c^2 + B^2*b^4*(-(4*a*c - b^2)^3)^(1/2) - 2*A*B*b^6*c + 25*B^2*a^2*b^3*c^2 + A^2*b^2*c^2*(-(4*a*c - b^2)^3)^(1/2) + B^2*a^2*c^2*(-(4*a*c - b^2)^3)^(1/2) + 16*A*B*a^3*c^4 - 9*B^2*a*b^5*c - 7*A^2*a*b^3*c^3 + 12*A^2*a^2*b*c^4 - A^2*a*c^3*(-(4*a*c - b^2)^3)^(1/2) - 20*B^2*a^3*b*c^3 - 36*A*B*a^2*b^2*c^3 - 3*B^2*a*b^2*c*(-(4*a*c - b^2)^3)^(1/2) + 16*A*B*a*b^4*c^2 - 2*A*B*b^3*c*(-(4*a*c - b^2)^3)^(1/2) + 4*A*B*a*b*c^2*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^7 + b^4*c^5 - 8*a*b^2*c^6)))^(1/2) - (2*x*(B^2*b^6 + 2*A^2*a^2*c^4 + A^2*b^4*c^2 - 2*B^2*a^3*c^3 - 2*A*B*b^5*c + 9*B^2*a^2*b^2*c^2 - 6*B^2*a*b^4*c - 4*A^2*a*b^2*c^3 + 10*A*B*a*b^3*c^2 - 10*A*B*a^2*b*c^3))/c^3)*(-(B^2*b^7 + A^2*b^5*c^2 + B^2*b^4*(-(4*a*c - b^2)^3)^(1/2) - 2*A*B*b^6*c + 25*B^2*a^2*b^3*c^2 + A^2*b^2*c^2*(-(4*a*c - b^2)^3)^(1/2) + B^2*a^2*c^2*(-(4*a*c - b^2)^3)^(1/2) + 16*A*B*a^3*c^4 - 9*B^2*a*b^5*c - 7*A^2*a*b^3*c^3 + 12*A^2*a^2*b*c^4 - A^2*a*c^3*(-(4*a*c - b^2)^3)^(1/2) - 20*B^2*a^3*b*c^3 - 36*A*B*a^2*b^2*c^3 - 3*B^2*a*b^2*c*(-(4*a*c - b^2)^3)^(1/2) + 16*A*B*a*b^4*c^2 - 2*A*B*b^3*c*(-(4*a*c - b^2)^3)^(1/2) + 4*A*B*a*b*c^2*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^7 + b^4*c^5 - 8*a*b^2*c^6)))^(1/2) + (((16*A*a^2*c^5 - 4*A*a*b^2*c^4 + 4*B*a*b^3*c^3 - 16*B*a^2*b*c^4)/c^3 + (2*x*(4*b^3*c^5 - 16*a*b*c^6)*(-(B^2*b^7 + A^2*b^5*c^2 + B^2*b^4*(-(4*a*c - b^2)^3)^(1/2) - 2*A*B*b^6*c + 25*B^2*a^2*b^3*c^2 + A^2*b^2*c^2*(-(4*a*c - b^2)^3)^(1/2) + B^2*a^2*c^2*(-(4*a*c - b^2)^3)^(1/2) + 16*A*B*a^3*c^4 - 9*B^2*a*b^5*c - 7*A^2*a*b^3*c^3 + 12*A^2*a^2*b*c^4 - A^2*a*c^3*(-(4*a*c - b^2)^3)^(1/2) - 20*B^2*a^3*b*c^3 - 36*A*B*a^2*b^2*c^3 - 3*B^2*a*b^2*c*(-(4*a*c - b^2)^3)^(1/2) + 16*A*B*a*b^4*c^2 - 2*A*B*b^3*c*(-(4*a*c - b^2)^3)^(1/2) + 4*A*B*a*b*c^2*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^7 + b^4*c^5 - 8*a*b^2*c^6)))^(1/2))/c^3)*(-(B^2*b^7 + A^2*b^5*c^2 + B^2*b^4*(-(4*a*c - b^2)^3)^(1/2) - 2*A*B*b^6*c + 25*B^2*a^2*b^3*c^2 + A^2*b^2*c^2*(-(4*a*c - b^2)^3)^(1/2) + B^2*a^2*c^2*(-(4*a*c - b^2)^3)^(1/2) + 16*A*B*a^3*c^4 - 9*B^2*a*b^5*c - 7*A^2*a*b^3*c^3 + 12*A^2*a^2*b*c^4 - A^2*a*c^3*(-(4*a*c - b^2)^3)^(1/2) - 20*B^2*a^3*b*c^3 - 36*A*B*a^2*b^2*c^3 - 3*B^2*a*b^2*c*(-(4*a*c - b^2)^3)^(1/2) + 16*A*B*a*b^4*c^2 - 2*A*B*b^3*c*(-(4*a*c - b^2)^3)^(1/2) + 4*A*B*a*b*c^2*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^7 + b^4*c^5 - 8*a*b^2*c^6)))^(1/2) + (2*x*(B^2*b^6 + 2*A^2*a^2*c^4 + A^2*b^4*c^2 - 2*B^2*a^3*c^3 - 2*A*B*b^5*c + 9*B^2*a^2*b^2*c^2 - 6*B^2*a*b^4*c - 4*A^2*a*b^2*c^3 + 10*A*B*a*b^3*c^2 - 10*A*B*a^2*b*c^3))/c^3)*(-(B^2*b^7 + A^2*b^5*c^2 + B^2*b^4*(-(4*a*c - b^2)^3)^(1/2) - 2*A*B*b^6*c + 25*B^2*a^2*b^3*c^2 + A^2*b^2*c^2*(-(4*a*c - b^2)^3)^(

1/2) + B^2*a^2*c^2*(-(4*a*c - b^2)^3)^(1/2) + 16*A*B*a^3*c^4 - 9*B^2*a*b^5*
c - 7*A^2*a*b^3*c^3 + 12*A^2*a^2*b*c^4 - A^2*a*c^3*(-(4*a*c - b^2)^3)^(1/2)
 - 20*B^2*a^3*b*c^3 - 36*A*B*a^2*b^2*c^3 - 3*B^2*a*b^2*c*(-(4*a*c - b^2)^3)
^(1/2) + 16*A*B*a*b^4*c^2 - 2*A*B*b^3*c*(-(4*a*c - b^2)^3)^(1/2) + 4*A*B*a*
b*c^2*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^7 + b^4*c^5 - 8*a*b^2*c^6)))^(
1/2) + (2*(B^3*a^4*c - B^3*a^3*b^2 + A*B^2*a^2*b^3 + A^2*B*a^3*c^2 + A^3*a^
2*b*c^2 - 2*A^2*B*a^2*b^2*c))/c^3))*(-(B^2*b^7 + A^2*b^5*c^2 + B^2*b^4*(-(4
*a*c - b^2)^3)^(1/2) - 2*A*B*b^6*c + 25*B^2*a^2*b^3*c^2 + A^2*b^2*c^2*(-(4*
a*c - b^2)^3)^(1/2) + B^2*a^2*c^2*(-(4*a*c - b^2)^3)^(1/2) + 16*A*B*a^3*c^4
 - 9*B^2*a*b^5*c - 7*A^2*a*b^3*c^3 + 12*A^2*a^2*b*c^4 - A^2*a*c^3*(-(4*a*c
- b^2)^3)^(1/2) - 20*B^2*a^3*b*c^3 - 36*A*B*a^2*b^2*c^3 - 3*B^2*a*b^2*c*(-(
4*a*c - b^2)^3)^(1/2) + 16*A*B*a*b^4*c^2 - 2*A*B*b^3*c*(-(4*a*c - b^2)^3)^(
1/2) + 4*A*B*a*b*c^2*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^7 + b^4*c^5 - 8
*a*b^2*c^6)))^(1/2)*2i - atan(((((16*A*a^2*c^5 - 4*A*a*b^2*c^4 + 4*B*a*b^3*
c^3 - 16*B*a^2*b*c^4)/c^3 - (2*x*(4*b^3*c^5 - 16*a*b*c^6)*(-(B^2*b^7 + A^2*
b^5*c^2 - B^2*b^4*(-(4*a*c - b^2)^3)^(1/2) - 2*A*B*b^6*c + 25*B^2*a^2*b^3*c
^2 - A^2*b^2*c^2*(-(4*a*c - b^2)^3)^(1/2) - B^2*a^2*c^2*(-(4*a*c - b^2)^3)^
(1/2) + 16*A*B*a^3*c^4 - 9*B^2*a*b^5*c - 7*A^2*a*b^3*c^3 + 12*A^2*a^2*b*c^4
 + A^2*a*c^3*(-(4*a*c - b^2)^3)^(1/2) - 20*B^2*a^3*b*c^3 - 36*A*B*a^2*b^2*c
^3 + 3*B^2*a*b^2*c*(-(4*a*c - b^2)^3)^(1/2) + 16*A*B*a*b^4*c^2 + 2*A*B*b^3*
c*(-(4*a*c - b^2)^3)^(1/2) - 4*A*B*a*b*c^2*(-(4*a*c - b^2)^3)^(1/2))/(8*(16
*a^2*c^7 + b^4*c^5 - 8*a*b^2*c^6)))^(1/2))/c^3)*(-(B^2*b^7 + A^2*b^5*c^2 -
B^2*b^4*(-(4*a*c - b^2)^3)^(1/2) - 2*A*B*b^6*c + 25*B^2*a^2*b^3*c^2 - A^2*b
^2*c^2*(-(4*a*c - b^2)^3)^(1/2) - B^2*a^2*c^2*(-(4*a*c - b^2)^3)^(1/2) + 16
*A*B*a^3*c^4 - 9*B^2*a*b^5*c - 7*A^2*a*b^3*c^3 + 12*A^2*a^2*b*c^4 + A^2*a*c
^3*(-(4*a*c - b^2)^3)^(1/2) - 20*B^2*a^3*b*c^3 - 36*A*B*a^2*b^2*c^3 + 3*B^2
*a*b^2*c*(-(4*a*c - b^2)^3)^(1/2) + 16*A*B*a*b^4*c^2 + 2*A*B*b^3*c*(-(4*a*c
 - b^2)^3)^(1/2) - 4*A*B*a*b*c^2*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^7 +
 b^4*c^5 - 8*a*b^2*c^6)))^(1/2) - (2*x*(B^2*b^6 + 2*A^2*a^2*c^4 + A^2*b^4*c
^2 - 2*B^2*a^3*c^3 - 2*A*B*b^5*c + 9*B^2*a^2*b^2*c^2 - 6*B^2*a*b^4*c - 4*A^
2*a*b^2*c^3 + 10*A*B*a*b^3*c^2 - 10*A*B*a^2*b*c^3))/c^3)*(-(B^2*b^7 + A^2*b
^5*c^2 - B^2*b^4*(-(4*a*c - b^2)^3)^(1/2) - 2*A*B*b^6*c + 25*B^2*a^2*b^3*c^
2 - A^2*b^2*c^2*(-(4*a*c - b^2)^3)^(1/2) - B^2*a^2*c^2*(-(4*a*c - b^2)^3)^(
1/2) + 16*A*B*a^3*c^4 - 9*B^2*a*b^5*c - 7*A^2*a*b^3*c^3 + 12*A^2*a^2*b*c^4
+ A^2*a*c^3*(-(4*a*c - b^2)^3)^(1/2) - 20*B^2*a^3*b*c^3 - 36*A*B*a^2*b^2*c^
3 + 3*B^2*a*b^2*c*(-(4*a*c - b^2)^3)^(1/2) + 16*A*B*a*b^4*c^2 + 2*A*B*b^3*c
*(-(4*a*c - b^2)^3)^(1/2) - 4*A*B*a*b*c^2*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*
a^2*c^7 + b^4*c^5 - 8*a*b^2*c^6)))^(1/2)*1i - (((16*A*a^2*c^5 - 4*A*a*b^2*c
^4 + 4*B*a*b^3*c^3 - 16*B*a^2*b*c^4)/c^3 + (2*x*(4*b^3*c^5 - 16*a*b*c^6)*(-
(B^2*b^7 + A^2*b^5*c^2 - B^2*b^4*(-(4*a*c - b^2)^3)^(1/2) - 2*A*B*b^6*c + 2
5*B^2*a^2*b^3*c^2 - A^2*b^2*c^2*(-(4*a*c - b^2)^3)^(1/2) - B^2*a^2*c^2*(-(4
*a*c - b^2)^3)^(1/2) + 16*A*B*a^3*c^4 - 9*B^2*a*b^5*c - 7*A^2*a*b^3*c^3 + 1
2*A^2*a^2*b*c^4 + A^2*a*c^3*(-(4*a*c - b^2)^3)^(1/2) - 20*B^2*a^3*b*c^3 - 3
6*A*B*a^2*b^2*c^3 + 3*B^2*a*b^2*c*(-(4*a*c - b^2)^3)^(1/2) + 16*A*B*a*b^4*c
^2 + 2*A*B*b^3*c*(-(4*a*c - b^2)^3)^(1/2) - 4*A*B*a*b*c^2*(-(4*a*c - b^2)^3

)^(1/2))/(8*(16*a^2*c^7 + b^4*c^5 - 8*a*b^2*c^6)))^(1/2))/c^3)*(-(B^2*b^7 + A^2*b^5*c^2 - B^2*b^4*(-(4*a*c - b^2)^3)^(1/2) - 2*A*B*b^6*c + 25*B^2*a^2*b^3*c^2 - A^2*b^2*c^2*(-(4*a*c - b^2)^3)^(1/2) - B^2*a^2*c^2*(-(4*a*c - b^2)^3)^(1/2) + 16*A*B*a^3*c^4 - 9*B^2*a*b^5*c - 7*A^2*a*b^3*c^3 + 12*A^2*a^2*b*c^4 + A^2*a*c^3*(-(4*a*c - b^2)^3)^(1/2) - 20*B^2*a^3*b*c^3 - 36*A*B*a^2*b^2*c^3 + 3*B^2*a*b^2*c*(-(4*a*c - b^2)^3)^(1/2) + 16*A*B*a*b^4*c^2 + 2*A*B*b^3*c*(-(4*a*c - b^2)^3)^(1/2) - 4*A*B*a*b*c^2*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^7 + b^4*c^5 - 8*a*b^2*c^6)))^(1/2) + (2*x*(B^2*b^6 + 2*A^2*a^2*c^4 + A^2*b^4*c^2 - 2*B^2*a^3*c^3 - 2*A*B*b^5*c + 9*B^2*a^2*b^2*c^2 - 6*B^2*a*b^4*c - 4*A^2*a*b^2*c^3 + 10*A*B*a*b^3*c^2 - 10*A*B*a^2*b*c^3))/c^3)*(-(B^2*b^7 + A^2*b^5*c^2 - B^2*b^4*(-(4*a*c - b^2)^3)^(1/2) - 2*A*B*b^6*c + 25*B^2*a^2*b^3*c^2 - A^2*b^2*c^2*(-(4*a*c - b^2)^3)^(1/2) - B^2*a^2*c^2*(-(4*a*c - b^2)^3)^(1/2) + 16*A*B*a^3*c^4 - 9*B^2*a*b^5*c - 7*A^2*a*b^3*c^3 + 12*A^2*a^2*b*c^4 + A^2*a*c^3*(-(4*a*c - b^2)^3)^(1/2) - 20*B^2*a^3*b*c^3 - 36*A*B*a^2*b^2*c^3 + 3*B^2*a*b^2*c*(-(4*a*c - b^2)^3)^(1/2) + 16*A*B*a*b^4*c^2 + 2*A*B*b^3*c*(-(4*a*c - b^2)^3)^(1/2) - 4*A*B*a*b*c^2*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^7 + b^4*c^5 - 8*a*b^2*c^6)))^(1/2)*1i)/((((16*A*a^2*c^5 - 4*A*a*b^2*c^4 + 4*B*a*b^3*c^3 - 16*B*a^2*b*c^4)/c^3 - (2*x*(4*b^3*c^5 - 16*a*b*c^6)*(-(B^2*b^7 + A^2*b^5*c^2 - B^2*b^4*(-(4*a*c - b^2)^3)^(1/2) - 2*A*B*b^6*c + 25*B^2*a^2*b^3*c^2 - A^2*b^2*c^2*(-(4*a*c - b^2)^3)^(1/2) - B^2*a^2*c^2*(-(4*a*c - b^2)^3)^(1/2) + 16*A*B*a^3*c^4 - 9*B^2*a*b^5*c - 7*A^2*a*b^3*c^3 + 12*A^2*a^2*b*c^4 + A^2*a*c^3*(-(4*a*c - b^2)^3)^(1/2) - 20*B^2*a^3*b*c^3 - 36*A*B*a^2*b^2*c^3 + 3*B^2*a*b^2*c*(-(4*a*c - b^2)^3)^(1/2) + 16*A*B*a*b^4*c^2 + 2*A*B*b^3*c*(-(4*a*c - b^2)^3)^(1/2) - 4*A*B*a*b*c^2*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^7 + b^4*c^5 - 8*a*b^2*c^6)))^(1/2))/c^3)*(-(B^2*b^7 + A^2*b^5*c^2 - B^2*b^4*(-(4*a*c - b^2)^3)^(1/2) - 2*A*B*b^6*c + 25*B^2*a^2*b^3*c^2 - A^2*b^2*c^2*(-(4*a*c - b^2)^3)^(1/2) - B^2*a^2*c^2*(-(4*a*c - b^2)^3)^(1/2) + 16*A*B*a^3*c^4 - 9*B^2*a*b^5*c - 7*A^2*a*b^3*c^3 + 12*A^2*a^2*b*c^4 + A^2*a*c^3*(-(4*a*c - b^2)^3)^(1/2) - 20*B^2*a^3*b*c^3 - 36*A*B*a^2*b^2*c^3 + 3*B^2*a*b^2*c*(-(4*a*c - b^2)^3)^(1/2) + 16*A*B*a*b^4*c^2 + 2*A*B*b^3*c*(-(4*a*c - b^2)^3)^(1/2) - 4*A*B*a*b*c^2*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^7 + b^4*c^5 - 8*a*b^2*c^6)))^(1/2) - (2*x*(B^2*b^6 + 2*A^2*a^2*c^4 + A^2*b^4*c^2 - 2*B^2*a^3*c^3 - 2*A*B*b^5*c + 9*B^2*a^2*b^2*c^2 - 6*B^2*a*b^4*c - 4*A^2*a*b^2*c^3 + 10*A*B*a*b^3*c^2 - 10*A*B*a^2*b*c^3))/c^3)*(-(B^2*b^7 + A^2*b^5*c^2 - B^2*b^4*(-(4*a*c - b^2)^3)^(1/2) - 2*A*B*b^6*c + 25*B^2*a^2*b^3*c^2 - A^2*b^2*c^2*(-(4*a*c - b^2)^3)^(1/2) - B^2*a^2*c^2*(-(4*a*c - b^2)^3)^(1/2) + 16*A*B*a^3*c^4 - 9*B^2*a*b^5*c - 7*A^2*a*b^3*c^3 + 12*A^2*a^2*b*c^4 + A^2*a*c^3*(-(4*a*c - b^2)^3)^(1/2) - 20*B^2*a^3*b*c^3 - 36*A*B*a^2*b^2*c^3 + 3*B^2*a*b^2*c*(-(4*a*c - b^2)^3)^(1/2) + 16*A*B*a*b^4*c^2 + 2*A*B*b^3*c*(-(4*a*c - b^2)^3)^(1/2) - 4*A*B*a*b*c^2*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^7 + b^4*c^5 - 8*a*b^2*c^6)))^(1/2) + (((16*A*a^2*c^5 - 4*A*a*b^2*c^4 + 4*B*a*b^3*c^3 - 16*B*a^2*b*c^4)/c^3 + (2*x*(4*b^3*c^5 - 16*a*b*c^6)*(-(B^2*b^7 + A^2*b^5*c^2 - B^2*b^4*(-(4*a*c - b^2)^3)^(1/2) - 2*A*B*b^6*c + 25*B^2*a^2*b^3*c^2 - A^2*b^2*c^2*(-(4*a*c - b^2)^3)^(1/2) - B^2*a^2*c^2*(-(4*a*c - b^2)^3)^(1/2) + 16*A*B*a^3*c^4 - 9*B^2*a*

```
b^5*c - 7*A^2*a*b^3*c^3 + 12*A^2*a^2*b*c^4 + A^2*a*c^3*(-(4*a*c - b^2)^3)^(
1/2) - 20*B^2*a^3*b*c^3 - 36*A*B*a^2*b^2*c^3 + 3*B^2*a*b^2*c*(-(4*a*c - b^2
)^3)^(1/2) + 16*A*B*a*b^4*c^2 + 2*A*B*b^3*c*(-(4*a*c - b^2)^3)^(1/2) - 4*A*
B*a*b*c^2*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^7 + b^4*c^5 - 8*a*b^2*c^6)
))^(1/2))/c^3)*(-(B^2*b^7 + A^2*b^5*c^2 - B^2*b^4*(-(4*a*c - b^2)^3)^(1/2)
- 2*A*B*b^6*c + 25*B^2*a^2*b^3*c^2 - A^2*b^2*c^2*(-(4*a*c - b^2)^3)^(1/2) -
 B^2*a^2*c^2*(-(4*a*c - b^2)^3)^(1/2) + 16*A*B*a^3*c^4 - 9*B^2*a*b^5*c - 7*
A^2*a*b^3*c^3 + 12*A^2*a^2*b*c^4 + A^2*a*c^3*(-(4*a*c - b^2)^3)^(1/2) - 20*
B^2*a^3*b*c^3 - 36*A*B*a^2*b^2*c^3 + 3*B^2*a*b^2*c*(-(4*a*c - b^2)^3)^(1/2)
 + 16*A*B*a*b^4*c^2 + 2*A*B*b^3*c*(-(4*a*c - b^2)^3)^(1/2) - 4*A*B*a*b*c^2*
(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^7 + b^4*c^5 - 8*a*b^2*c^6)))^(1/2) +
 (2*x*(B^2*b^6 + 2*A^2*a^2*c^4 + A^2*b^4*c^2 - 2*B^2*a^3*c^3 - 2*A*B*b^5*c
+ 9*B^2*a^2*b^2*c^2 - 6*B^2*a*b^4*c - 4*A^2*a*b^2*c^3 + 10*A*B*a*b^3*c^2 -
10*A*B*a^2*b*c^3))/c^3)*(-(B^2*b^7 + A^2*b^5*c^2 - B^2*b^4*(-(4*a*c - b^2)^
3)^(1/2) - 2*A*B*b^6*c + 25*B^2*a^2*b^3*c^2 - A^2*b^2*c^2*(-(4*a*c - b^2)^3
)^(1/2) - B^2*a^2*c^2*(-(4*a*c - b^2)^3)^(1/2) + 16*A*B*a^3*c^4 - 9*B^2*a*b
^5*c - 7*A^2*a*b^3*c^3 + 12*A^2*a^2*b*c^4 + A^2*a*c^3*(-(4*a*c - b^2)^3)^(1
/2) - 20*B^2*a^3*b*c^3 - 36*A*B*a^2*b^2*c^3 + 3*B^2*a*b^2*c*(-(4*a*c - b^2)
^3)^(1/2) + 16*A*B*a*b^4*c^2 + 2*A*B*b^3*c*(-(4*a*c - b^2)^3)^(1/2) - 4*A*B
*a*b*c^2*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^7 + b^4*c^5 - 8*a*b^2*c^6))
)^(1/2) + (2*(B^3*a^4*c - B^3*a^3*b^2 + A*B^2*a^2*b^3 + A^2*B*a^3*c^2 + A^3
*a^2*b*c^2 - 2*A^2*B*a^2*b^2*c))/c^3))*(-(B^2*b^7 + A^2*b^5*c^2 - B^2*b^4*(
-(4*a*c - b^2)^3)^(1/2) - 2*A*B*b^6*c + 25*B^2*a^2*b^3*c^2 - A^2*b^2*c^2*(-
(4*a*c - b^2)^3)^(1/2) - B^2*a^2*c^2*(-(4*a*c - b^2)^3)^(1/2) + 16*A*B*a^3*
c^4 - 9*B^2*a*b^5*c - 7*A^2*a*b^3*c^3 + 12*A^2*a^2*b*c^4 + A^2*a*c^3*(-(4*a
*c - b^2)^3)^(1/2) - 20*B^2*a^3*b*c^3 - 36*A*B*a^2*b^2*c^3 + 3*B^2*a*b^2*c*
(-(4*a*c - b^2)^3)^(1/2) + 16*A*B*a*b^4*c^2 + 2*A*B*b^3*c*(-(4*a*c - b^2)^3
)^(1/2) - 4*A*B*a*b*c^2*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^7 + b^4*c^5
- 8*a*b^2*c^6)))^(1/2)*2i + (B*x^3)/(3*c)
```

3.108 $\int \frac{x^2(A+Bx^2)}{a+bx^2+cx^4}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 208

$$\int \frac{x^2(A+Bx^2)}{a+bx^2+cx^4}\,dx=\frac{Bx}{c}-\frac{\left(bB-Ac-\frac{b^2B-Abc-2aBc}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}c^{3/2}\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\left(bB-Ac+\frac{b^2B-Abc-2aBc}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{2}c^{3/2}\sqrt{b+\sqrt{b^2-4ac}}}$$

```
[Out] B*x/c-1/2*arctan(x*2^(1/2)*c^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2))*(B*b-A*c+(
A*b*c+2*B*a*c-B*b^2)/(-4*a*c+b^2)^(1/2))/c^(3/2)*2^(1/2)/(b-(-4*a*c+b^2)^(1
/2))^(1/2)-1/2*arctan(x*2^(1/2)*c^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2))*(B*b-
A*c+(-A*b*c-2*B*a*c+B*b^2)/(-4*a*c+b^2)^(1/2))/c^(3/2)*2^(1/2)/(b+(-4*a*c+b
^2)^(1/2))^(1/2)
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 208, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {1293, 1180, 211}

$$\int \frac{x^2(A+Bx^2)}{a+bx^2+cx^4}\,dx=-\frac{\left(-\frac{-2aBc-Abc+b^2B}{\sqrt{b^2-4ac}}-Ac+bB\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}c^{3/2}\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\left(\frac{-2aBc-Abc+b^2B}{\sqrt{b^2-4ac}}-Ac+bB\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}c^{3/2}\sqrt{\sqrt{b^2-4ac}+b}}+\frac{Bx}{c}$$

```
[In] Int[(x^2*(A + B*x^2))/(a + b*x^2 + c*x^4),x]
```

```
[Out] (B*x)/c - ((b*B - A*c - (b^2*B - A*b*c - 2*a*B*c)/Sqrt[b^2 - 4*a*c])*ArcTan
[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*c^(3/2)*Sqrt[b
- Sqrt[b^2 - 4*a*c]]) - ((b*B - A*c + (b^2*B - A*b*c - 2*a*B*c)/Sqrt[b^2 -
4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*c
^(3/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1293

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[e*f*(f*x)^(m - 1)*((a + b*x^2 + c*x^4)^(p +
1)/(c*(m + 4*p + 3))), x] - Dist[f^2/(c*(m + 4*p + 3)), Int[(f*x)^(m - 2)*(
a + b*x^2 + c*x^4)^p*Simp[a*e*(m - 1) + (b*e*(m + 2*p + 1) - c*d*(m + 4*p +
 3))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c,
 0] && GtQ[m, 1] && NeQ[m + 4*p + 3, 0] && IntegerQ[2*p] && (IntegerQ[p] ||
 IntegerQ[m])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{Bx}{c} - \frac{\int \frac{aB+(bB-Ac)x^2}{a+bx^2+cx^4}\,dx}{c} \\
&= \frac{Bx}{c} - \frac{\left(bB-Ac-\frac{b^2B-Abc-2aBc}{\sqrt{b^2-4ac}}\right)\int \frac{1}{\frac{b}{2}-\frac{1}{2}\sqrt{b^2-4ac}+cx^2}\,dx}{2c} \\
&\quad - \frac{\left(bB-Ac+\frac{b^2B-Abc-2aBc}{\sqrt{b^2-4ac}}\right)\int \frac{1}{\frac{b}{2}+\frac{1}{2}\sqrt{b^2-4ac}+cx^2}\,dx}{2c} \\
&= \frac{Bx}{c} - \frac{\left(bB-Ac-\frac{b^2B-Abc-2aBc}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}c^{3/2}\sqrt{b-\sqrt{b^2-4ac}}} \\
&\quad - \frac{\left(bB-Ac+\frac{b^2B-Abc-2aBc}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{2}c^{3/2}\sqrt{b+\sqrt{b^2-4ac}}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 251, normalized size of antiderivative = 1.21

$$\int \frac{x^2(A+Bx^2)}{a+bx^2+cx^4}\,dx$$
$$=\frac{Bx}{c}-\frac{\left(-b^2B+Abc+2aBc+bB\sqrt{b^2-4ac}-Ac\sqrt{b^2-4ac}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}c^{3/2}\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}}$$
$$-\frac{\left(b^2B-Abc-2aBc+bB\sqrt{b^2-4ac}-Ac\sqrt{b^2-4ac}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{2}c^{3/2}\sqrt{b^2-4ac}\sqrt{b+\sqrt{b^2-4ac}}}$$

```
[In] Integrate[(x^2*(A + B*x^2))/(a + b*x^2 + c*x^4),x]
```

```
[Out] (B*x)/c - ((-(b^2*B) + A*b*c + 2*a*B*c + b*B*Sqrt[b^2 - 4*a*c] - A*c*Sqrt[b
^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt
[2]*c^(3/2)*Sqrt[b^2 - 4*a*c]*Sqrt[b - Sqrt[b^2 - 4*a*c]]) - ((b^2*B - A*b*
c - 2*a*B*c + b*B*Sqrt[b^2 - 4*a*c] - A*c*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2
]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*c^(3/2)*Sqrt[b^2 - 4*a*
c]*Sqrt[b + Sqrt[b^2 - 4*a*c]])
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.07 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.31

method	result
risch	$\frac{Bx}{c}+\frac{\sum_{\_R=\operatorname{RootOf}\left(c\_Z^4+\_Z^2b+a\right)}\frac{\left(\_R^2(Ac-Bb)-Ba\right)\ln\left(x-\_R\right)}{2c\_R^3+\_Rb}}{2c}$
default	$\frac{Bx}{c}+\frac{\left(Ac\sqrt{-4ac+b^2}+Abc-Bb\sqrt{-4ac+b^2}+2Bac-B\,b^2\right)\sqrt{2}\,\arctan\left(\frac{cx\sqrt{2}}{\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}}\right)}{2c\sqrt{-4ac+b^2}\,\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}}-\frac{\left(Ac\sqrt{-4ac+b^2}-Abc-Bb\sqrt{-4ac+b^2}\right.}{2c\sqrt{-4ac+}}$

```
[In] int(x^2*(B*x^2+A)/(c*x^4+b*x^2+a),x,method=_RETURNVERBOSE)
```

```
[Out] B*x/c+1/2/c*sum((_R^2*(A*c-B*b)-B*a)/(2*_R^3*c+_R*b)*ln(x-_R),_R=RootOf(_Z^
4*c+_Z^2*b+a))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2632 vs. $2(172) = 344$.

Time = 0.62 (sec) , antiderivative size = 2632, normalized size of antiderivative = 12.65

$$\int \frac{x^2(A+Bx^2)}{a+bx^2+cx^4}\,dx = \text{Too large to display}$$

```
[In] integrate(x^2*(B*x^2+A)/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] 1/2*(sqrt(1/2)*c*sqrt(-(B^2*b^3 + (4*A*B*a + A^2*b)*c^2 - (3*B^2*a*b + 2*A*
B*b^2)*c + (b^2*c^3 - 4*a*c^4)*sqrt((B^4*b^4 + A^4*c^4 - 2*(A^2*B^2*a + 2*A
^3*B*b)*c^3 + (B^4*a^2 + 4*A*B^3*a*b + 6*A^2*B^2*b^2)*c^2 - 2*(B^4*a*b^2 +
2*A*B^3*b^3)*c)/(b^2*c^6 - 4*a*c^7)))/(b^2*c^3 - 4*a*c^4))*log(2*(B^4*a*b^2
 - A*B^3*b^3 - 3*A^3*B*b*c^2 + A^4*c^3 - (B^4*a^2 + A*B^3*a*b - 3*A^2*B^2*b
^2)*c)*x + sqrt(1/2)*(B^3*b^4 - 4*A^2*B*a*c^3 + (4*B^3*a^2 + 8*A*B^2*a*b +
A^2*B*b^2)*c^2 - (5*B^3*a*b^2 + 2*A*B^2*b^3)*c - (B*b^3*c^3 + 8*A*a*c^5 - 2
*(2*B*a*b + A*b^2)*c^4)*sqrt((B^4*b^4 + A^4*c^4 - 2*(A^2*B^2*a + 2*A^3*B*b)
*c^3 + (B^4*a^2 + 4*A*B^3*a*b + 6*A^2*B^2*b^2)*c^2 - 2*(B^4*a*b^2 + 2*A*B^3
*b^3)*c)/(b^2*c^6 - 4*a*c^7)))*sqrt(-(B^2*b^3 + (4*A*B*a + A^2*b)*c^2 - (3*
B^2*a*b + 2*A*B*b^2)*c + (b^2*c^3 - 4*a*c^4)*sqrt((B^4*b^4 + A^4*c^4 - 2*(A
^2*B^2*a + 2*A^3*B*b)*c^3 + (B^4*a^2 + 4*A*B^3*a*b + 6*A^2*B^2*b^2)*c^2 - 2
*(B^4*a*b^2 + 2*A*B^3*b^3)*c)/(b^2*c^6 - 4*a*c^7)))/(b^2*c^3 - 4*a*c^4))) -
 sqrt(1/2)*c*sqrt(-(B^2*b^3 + (4*A*B*a + A^2*b)*c^2 - (3*B^2*a*b + 2*A*B*b^
2)*c + (b^2*c^3 - 4*a*c^4)*sqrt((B^4*b^4 + A^4*c^4 - 2*(A^2*B^2*a + 2*A^3*B
*b)*c^3 + (B^4*a^2 + 4*A*B^3*a*b + 6*A^2*B^2*b^2)*c^2 - 2*(B^4*a*b^2 + 2*A*
B^3*b^3)*c)/(b^2*c^6 - 4*a*c^7)))/(b^2*c^3 - 4*a*c^4))*log(2*(B^4*a*b^2 - A
*B^3*b^3 - 3*A^3*B*b*c^2 + A^4*c^3 - (B^4*a^2 + A*B^3*a*b - 3*A^2*B^2*b^2)*
c)*x - sqrt(1/2)*(B^3*b^4 - 4*A^2*B*a*c^3 + (4*B^3*a^2 + 8*A*B^2*a*b + A^2*
B*b^2)*c^2 - (5*B^3*a*b^2 + 2*A*B^2*b^3)*c - (B*b^3*c^3 + 8*A*a*c^5 - 2*(2*
B*a*b + A*b^2)*c^4)*sqrt((B^4*b^4 + A^4*c^4 - 2*(A^2*B^2*a + 2*A^3*B*b)*c^3
 + (B^4*a^2 + 4*A*B^3*a*b + 6*A^2*B^2*b^2)*c^2 - 2*(B^4*a*b^2 + 2*A*B^3*b^3
)*c)/(b^2*c^6 - 4*a*c^7)))*sqrt(-(B^2*b^3 + (4*A*B*a + A^2*b)*c^2 - (3*B^2*
a*b + 2*A*B*b^2)*c + (b^2*c^3 - 4*a*c^4)*sqrt((B^4*b^4 + A^4*c^4 - 2*(A^2*B
^2*a + 2*A^3*B*b)*c^3 + (B^4*a^2 + 4*A*B^3*a*b + 6*A^2*B^2*b^2)*c^2 - 2*(B^
4*a*b^2 + 2*A*B^3*b^3)*c)/(b^2*c^6 - 4*a*c^7)))/(b^2*c^3 - 4*a*c^4))) + sqr
t(1/2)*c*sqrt(-(B^2*b^3 + (4*A*B*a + A^2*b)*c^2 - (3*B^2*a*b + 2*A*B*b^2)*c
 - (b^2*c^3 - 4*a*c^4)*sqrt((B^4*b^4 + A^4*c^4 - 2*(A^2*B^2*a + 2*A^3*B*b)*
c^3 + (B^4*a^2 + 4*A*B^3*a*b + 6*A^2*B^2*b^2)*c^2 - 2*(B^4*a*b^2 + 2*A*B^3*
b^3)*c)/(b^2*c^6 - 4*a*c^7)))/(b^2*c^3 - 4*a*c^4))*log(2*(B^4*a*b^2 - A*B^3
*b^3 - 3*A^3*B*b*c^2 + A^4*c^3 - (B^4*a^2 + A*B^3*a*b - 3*A^2*B^2*b^2)*c)*x
 + sqrt(1/2)*(B^3*b^4 - 4*A^2*B*a*c^3 + (4*B^3*a^2 + 8*A*B^2*a*b + A^2*B*b^
2)*c^2 - (5*B^3*a*b^2 + 2*A*B^2*b^3)*c + (B*b^3*c^3 + 8*A*a*c^5 - 2*(2*B*a*
b + A*b^2)*c^4)*sqrt((B^4*b^4 + A^4*c^4 - 2*(A^2*B^2*a + 2*A^3*B*b)*c^3 + (
B^4*a^2 + 4*A*B^3*a*b + 6*A^2*B^2*b^2)*c^2 - 2*(B^4*a*b^2 + 2*A*B^3*b^3)*c)
```

```
/(b^2*c^6 - 4*a*c^7)))*sqrt(-(B^2*b^3 + (4*A*B*a + A^2*b)*c^2 - (3*B^2*a*b
+ 2*A*B*b^2)*c - (b^2*c^3 - 4*a*c^4)*sqrt((B^4*b^4 + A^4*c^4 - 2*(A^2*B^2*a
 + 2*A^3*B*b)*c^3 + (B^4*a^2 + 4*A*B^3*a*b + 6*A^2*B^2*b^2)*c^2 - 2*(B^4*a*
b^2 + 2*A*B^3*b^3)*c)/(b^2*c^6 - 4*a*c^7)))/(b^2*c^3 - 4*a*c^4))) - sqrt(1/
2)*c*sqrt(-(B^2*b^3 + (4*A*B*a + A^2*b)*c^2 - (3*B^2*a*b + 2*A*B*b^2)*c - (
b^2*c^3 - 4*a*c^4)*sqrt((B^4*b^4 + A^4*c^4 - 2*(A^2*B^2*a + 2*A^3*B*b)*c^3
+ (B^4*a^2 + 4*A*B^3*a*b + 6*A^2*B^2*b^2)*c^2 - 2*(B^4*a*b^2 + 2*A*B^3*b^3)
*c)/(b^2*c^6 - 4*a*c^7)))/(b^2*c^3 - 4*a*c^4))*log(2*(B^4*a*b^2 - A*B^3*b^3
 - 3*A^3*B*b*c^2 + A^4*c^3 - (B^4*a^2 + A*B^3*a*b - 3*A^2*B^2*b^2)*c)*x - s
qrt(1/2)*(B^3*b^4 - 4*A^2*B*a*c^3 + (4*B^3*a^2 + 8*A*B^2*a*b + A^2*B*b^2)*c
^2 - (5*B^3*a*b^2 + 2*A*B^2*b^3)*c + (B*b^3*c^3 + 8*A*a*c^5 - 2*(2*B*a*b +
A*b^2)*c^4)*sqrt((B^4*b^4 + A^4*c^4 - 2*(A^2*B^2*a + 2*A^3*B*b)*c^3 + (B^4*
a^2 + 4*A*B^3*a*b + 6*A^2*B^2*b^2)*c^2 - 2*(B^4*a*b^2 + 2*A*B^3*b^3)*c)/(b^
2*c^6 - 4*a*c^7)))*sqrt(-(B^2*b^3 + (4*A*B*a + A^2*b)*c^2 - (3*B^2*a*b + 2*
A*B*b^2)*c - (b^2*c^3 - 4*a*c^4)*sqrt((B^4*b^4 + A^4*c^4 - 2*(A^2*B^2*a + 2
*A^3*B*b)*c^3 + (B^4*a^2 + 4*A*B^3*a*b + 6*A^2*B^2*b^2)*c^2 - 2*(B^4*a*b^2
+ 2*A*B^3*b^3)*c)/(b^2*c^6 - 4*a*c^7)))/(b^2*c^3 - 4*a*c^4))) + 2*B*x)/c
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^2(A+Bx^2)}{a+bx^2+cx^4}\,dx=\text{Timed out}$$

```
[In] integrate(x**2*(B*x**2+A)/(c*x**4+b*x**2+a),x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{x^2(A+Bx^2)}{a+bx^2+cx^4}\,dx=\int \frac{(Bx^2+A)x^2}{cx^4+bx^2+a}\,dx$$

```
[In] integrate(x^2*(B*x^2+A)/(c*x^4+b*x^2+a),x, algorithm="maxima")
[Out] B*x/c + integrate(-((B*b - A*c)*x^2 + B*a)/(c*x^4 + b*x^2 + a), x)/c
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3179 vs. 2(172) = 344.

Time = 0.96 (sec) , antiderivative size = 3179, normalized size of antiderivative = 15.28

$$\int \frac{x^2(A+Bx^2)}{a+bx^2+cx^4}\,dx = \text{Too large to display}$$

```
[In] integrate(x^2*(B*x^2+A)/(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] B*x/c + 1/8*((2*b^4*c^3 - 16*a*b^2*c^4 + 32*a^2*c^5 - sqrt(2)*sqrt(b^2 - 4*
a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^4*c + 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sq
rt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^2 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(
b*c + sqrt(b^2 - 4*a*c)*c)*b^3*c^2 - 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c
+ sqrt(b^2 - 4*a*c)*c)*a^2*c^3 - 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqr
t(b^2 - 4*a*c)*c)*a*b*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 -
 4*a*c)*c)*b^2*c^3 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*
c)*c)*a*c^4 - 2*(b^2 - 4*a*c)*b^2*c^3 + 8*(b^2 - 4*a*c)*a*c^4)*A*c^2 - (2*b
^5*c^2 - 16*a*b^3*c^3 + 32*a^2*b*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c +
 sqrt(b^2 - 4*a*c)*c)*b^5 + 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2
 - 4*a*c)*c)*a*b^3*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*
a*c)*c)*b^4*c - 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c
)*a^2*b*c^2 - 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a
*b^2*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^3*c^
2 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c^3 - 2
*(b^2 - 4*a*c)*b^3*c^2 + 8*(b^2 - 4*a*c)*a*b*c^3)*B*c^2 - 2*(sqrt(2)*sqrt(b
*c + sqrt(b^2 - 4*a*c)*c)*a*b^4*c^2 - 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c
)*c)*a^2*b^2*c^3 - 2*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c^3 - 2*
a*b^4*c^3 + 16*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*c^4 + 8*sqrt(2)*
sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b*c^4 + sqrt(2)*sqrt(b*c + sqrt(b^2 - 4
*a*c)*c)*a*b^2*c^4 + 16*a^2*b^2*c^4 - 4*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c
)*c)*a^2*c^5 - 32*a^3*c^5 + 2*(b^2 - 4*a*c)*a*b^2*c^3 - 8*(b^2 - 4*a*c)*a^2
*c^4)*B*abs(c) - (2*b^4*c^5 - 8*a*b^2*c^6 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(
b*c + sqrt(b^2 - 4*a*c)*c)*b^4*c^3 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c +
 sqrt(b^2 - 4*a*c)*c)*a*b^2*c^4 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sq
rt(b^2 - 4*a*c)*c)*b^3*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2
- 4*a*c)*c)*b^2*c^5 - 2*(b^2 - 4*a*c)*b^2*c^5)*A + (2*b^5*c^4 - 12*a*b^3*c^
5 + 16*a^2*b*c^6 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c
)*b^5*c^2 + 6*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b
^3*c^3 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^4*c^
3 - 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b*c^4 -
 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^4 - sq
rt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^3*c^4 + 2*sqrt(2)
*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c^5 - 2*(b^2 - 4*a*c
)*b^3*c^4 + 4*(b^2 - 4*a*c)*a*b*c^5)*B)*arctan(2*sqrt(1/2)*x/sqrt((b*c + sq
```

rt(b^2*c^2 - 4*a*c^3))/c^2))/((a*b^4*c^3 - 8*a^2*b^2*c^4 - 2*a*b^3*c^4 + 16*a^3*c^5 + 8*a^2*b*c^5 + a*b^2*c^5 - 4*a^2*c^6)*c^2) - 1/8*((2*b^4*c^3 - 16*a*b^2*c^4 + 32*a^2*c^5 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^4*c + 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^2*c^2 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^3*c^2 - 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*c^3 - 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^2*c^3 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*c^4 - 2*(b^2 - 4*a*c)*b^2*c^3 + 8*(b^2 - 4*a*c)*a*c^4)*A*c^2 - (2*b^5*c^2 - 16*a*b^3*c^3 + 32*a^2*b*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^5 + 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^3*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^4*c - 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b*c^2 - 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^2*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^3*c^2 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b*c^3 - 2*(b^2 - 4*a*c)*b^3*c^2 + 8*(b^2 - 4*a*c)*a*b*c^3)*B*c^2 + 2*(sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^4*c^2 - 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^3 - 2*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^3*c^3 + 2*a*b^4*c^3 + 16*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*c^4 + 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b*c^4 + sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^2*c^4 - 16*a^2*b^2*c^4 - 4*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*c^5 + 32*a^3*c^5 - 2*(b^2 - 4*a*c)*a*b^2*c^3 + 8*(b^2 - 4*a*c)*a^2*c^4)*B*abs(c) - (2*b^4*c^5 - 8*a*b^2*c^6 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^4*c^3 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^2*c^4 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^3*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^2*c^5 - 2*(b^2 - 4*a*c)*b^2*c^5)*A + (2*b^5*c^4 - 12*a*b^3*c^5 + 16*a^2*b*c^6 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^5*c^2 + 6*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^3*c^3 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^4*c^3 - 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b*c^4 - 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^2*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^3*c^4 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b*c^5 - 2*(b^2 - 4*a*c)*b^3*c^4 + 4*(b^2 - 4*a*c)*a*b*c^5)*B)*arctan(2*sqrt(1/2)*x/sqrt((b*c - sqrt(b^2*c^2 - 4*a*c^3))/c^2))/((a*b^4*c^3 - 8*a^2*b^2*c^4 - 2*a*b^3*c^4 + 16*a^3*c^5 + 8*a^2*b*c^5 + a*b^2*c^5 - 4*a^2*c^6)*c^2)

Mupad [B] (verification not implemented)

Time = 8.23 (sec) , antiderivative size = 6366, normalized size of antiderivative = 30.61

$$\int \frac{x^2(A+Bx^2)}{a+bx^2+cx^4}\,dx = \text{Too large to display}$$

```
[In] int((x^2*(A + B*x^2))/(a + b*x^2 + c*x^4),x)
```

```
[Out] (B*x)/c - atan(((((16*B*a^2*c^3 - 4*B*a*b^2*c^2)/c - (2*x*(4*b^3*c^3 - 16*a
*b*c^4)*(-(B^2*b^5 + A^2*b^3*c^2 - A^2*c^2*(-(4*a*c - b^2)^3)^(1/2) - B^2*b
^2*(-(4*a*c - b^2)^3)^(1/2) - 2*A*B*b^4*c - 16*A*B*a^2*c^3 - 4*A^2*a*b*c^3
- 7*B^2*a*b^3*c + B^2*a*c*(-(4*a*c - b^2)^3)^(1/2) + 12*B^2*a^2*b*c^2 + 2*A
*B*b*c*(-(4*a*c - b^2)^3)^(1/2) + 12*A*B*a*b^2*c^2)/(8*(16*a^2*c^5 + b^4*c^
3 - 8*a*b^2*c^4)))^(1/2))/c)*(-(B^2*b^5 + A^2*b^3*c^2 - A^2*c^2*(-(4*a*c -
b^2)^3)^(1/2) - B^2*b^2*(-(4*a*c - b^2)^3)^(1/2) - 2*A*B*b^4*c - 16*A*B*a^2
*c^3 - 4*A^2*a*b*c^3 - 7*B^2*a*b^3*c + B^2*a*c*(-(4*a*c - b^2)^3)^(1/2) + 1
2*B^2*a^2*b*c^2 + 2*A*B*b*c*(-(4*a*c - b^2)^3)^(1/2) + 12*A*B*a*b^2*c^2)/(8
*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2) - (2*x*(B^2*b^4 - 2*A^2*a*c^3
 + A^2*b^2*c^2 + 2*B^2*a^2*c^2 - 2*A*B*b^3*c - 4*B^2*a*b^2*c + 6*A*B*a*b*c^
2))/c)*(-(B^2*b^5 + A^2*b^3*c^2 - A^2*c^2*(-(4*a*c - b^2)^3)^(1/2) - B^2*b^
2*(-(4*a*c - b^2)^3)^(1/2) - 2*A*B*b^4*c - 16*A*B*a^2*c^3 - 4*A^2*a*b*c^3 -
 7*B^2*a*b^3*c + B^2*a*c*(-(4*a*c - b^2)^3)^(1/2) + 12*B^2*a^2*b*c^2 + 2*A*
B*b*c*(-(4*a*c - b^2)^3)^(1/2) + 12*A*B*a*b^2*c^2)/(8*(16*a^2*c^5 + b^4*c^3
 - 8*a*b^2*c^4)))^(1/2)*1i - (((16*B*a^2*c^3 - 4*B*a*b^2*c^2)/c + (2*x*(4*b
^3*c^3 - 16*a*b*c^4)*(-(B^2*b^5 + A^2*b^3*c^2 - A^2*c^2*(-(4*a*c - b^2)^3)^
(1/2) - B^2*b^2*(-(4*a*c - b^2)^3)^(1/2) - 2*A*B*b^4*c - 16*A*B*a^2*c^3 - 4
*A^2*a*b*c^3 - 7*B^2*a*b^3*c + B^2*a*c*(-(4*a*c - b^2)^3)^(1/2) + 12*B^2*a^
2*b*c^2 + 2*A*B*b*c*(-(4*a*c - b^2)^3)^(1/2) + 12*A*B*a*b^2*c^2)/(8*(16*a^2
*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2))/c)*(-(B^2*b^5 + A^2*b^3*c^2 - A^2*c^
2*(-(4*a*c - b^2)^3)^(1/2) - B^2*b^2*(-(4*a*c - b^2)^3)^(1/2) - 2*A*B*b^4*c
 - 16*A*B*a^2*c^3 - 4*A^2*a*b*c^3 - 7*B^2*a*b^3*c + B^2*a*c*(-(4*a*c - b^2)
^3)^(1/2) + 12*B^2*a^2*b*c^2 + 2*A*B*b*c*(-(4*a*c - b^2)^3)^(1/2) + 12*A*B*
a*b^2*c^2)/(8*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2) + (2*x*(B^2*b^4
- 2*A^2*a*c^3 + A^2*b^2*c^2 + 2*B^2*a^2*c^2 - 2*A*B*b^3*c - 4*B^2*a*b^2*c +
 6*A*B*a*b*c^2))/c)*(-(B^2*b^5 + A^2*b^3*c^2 - A^2*c^2*(-(4*a*c - b^2)^3)^(
1/2) - B^2*b^2*(-(4*a*c - b^2)^3)^(1/2) - 2*A*B*b^4*c - 16*A*B*a^2*c^3 - 4*
A^2*a*b*c^3 - 7*B^2*a*b^3*c + B^2*a*c*(-(4*a*c - b^2)^3)^(1/2) + 12*B^2*a^2
*b*c^2 + 2*A*B*b*c*(-(4*a*c - b^2)^3)^(1/2) + 12*A*B*a*b^2*c^2)/(8*(16*a^2*
c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2)*1i)/((((16*B*a^2*c^3 - 4*B*a*b^2*c^2)/
c - (2*x*(4*b^3*c^3 - 16*a*b*c^4)*(-(B^2*b^5 + A^2*b^3*c^2 - A^2*c^2*(-(4*a
*c - b^2)^3)^(1/2) - B^2*b^2*(-(4*a*c - b^2)^3)^(1/2) - 2*A*B*b^4*c - 16*A*
B*a^2*c^3 - 4*A^2*a*b*c^3 - 7*B^2*a*b^3*c + B^2*a*c*(-(4*a*c - b^2)^3)^(1/2
) + 12*B^2*a^2*b*c^2 + 2*A*B*b*c*(-(4*a*c - b^2)^3)^(1/2) + 12*A*B*a*b^2*c^
2)/(8*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2))/c)*(-(B^2*b^5 + A^2*b^3
```

```
*c^2 - A^2*c^2*(-(4*a*c - b^2)^3)^(1/2) - B^2*b^2*(-(4*a*c - b^2)^3)^(1/2)
- 2*A*B*b^4*c - 16*A*B*a^2*c^3 - 4*A^2*a*b*c^3 - 7*B^2*a*b^3*c + B^2*a*c*(-
(4*a*c - b^2)^3)^(1/2) + 12*B^2*a^2*b*c^2 + 2*A*B*b*c*(-(4*a*c - b^2)^3)^(1
/2) + 12*A*B*a*b^2*c^2)/(8*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2) - (
2*x*(B^2*b^4 - 2*A^2*a*c^3 + A^2*b^2*c^2 + 2*B^2*a^2*c^2 - 2*A*B*b^3*c - 4*
B^2*a*b^2*c + 6*A*B*a*b*c^2))/c)*(-(B^2*b^5 + A^2*b^3*c^2 - A^2*c^2*(-(4*a*
c - b^2)^3)^(1/2) - B^2*b^2*(-(4*a*c - b^2)^3)^(1/2) - 2*A*B*b^4*c - 16*A*B
*a^2*c^3 - 4*A^2*a*b*c^3 - 7*B^2*a*b^3*c + B^2*a*c*(-(4*a*c - b^2)^3)^(1/2)
 + 12*B^2*a^2*b*c^2 + 2*A*B*b*c*(-(4*a*c - b^2)^3)^(1/2) + 12*A*B*a*b^2*c^2
)/(8*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2) + (((16*B*a^2*c^3 - 4*B*a
*b^2*c^2)/c + (2*x*(4*b^3*c^3 - 16*a*b*c^4)*(-(B^2*b^5 + A^2*b^3*c^2 - A^2*
c^2*(-(4*a*c - b^2)^3)^(1/2) - B^2*b^2*(-(4*a*c - b^2)^3)^(1/2) - 2*A*B*b^4
*c - 16*A*B*a^2*c^3 - 4*A^2*a*b*c^3 - 7*B^2*a*b^3*c + B^2*a*c*(-(4*a*c - b^
2)^3)^(1/2) + 12*B^2*a^2*b*c^2 + 2*A*B*b*c*(-(4*a*c - b^2)^3)^(1/2) + 12*A*
B*a*b^2*c^2)/(8*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2))/c)*(-(B^2*b^5
 + A^2*b^3*c^2 - A^2*c^2*(-(4*a*c - b^2)^3)^(1/2) - B^2*b^2*(-(4*a*c - b^2)
^3)^(1/2) - 2*A*B*b^4*c - 16*A*B*a^2*c^3 - 4*A^2*a*b*c^3 - 7*B^2*a*b^3*c +
B^2*a*c*(-(4*a*c - b^2)^3)^(1/2) + 12*B^2*a^2*b*c^2 + 2*A*B*b*c*(-(4*a*c -
b^2)^3)^(1/2) + 12*A*B*a*b^2*c^2)/(8*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))
^(1/2) + (2*x*(B^2*b^4 - 2*A^2*a*c^3 + A^2*b^2*c^2 + 2*B^2*a^2*c^2 - 2*A*B*
b^3*c - 4*B^2*a*b^2*c + 6*A*B*a*b*c^2))/c)*(-(B^2*b^5 + A^2*b^3*c^2 - A^2*c
^2*(-(4*a*c - b^2)^3)^(1/2) - B^2*b^2*(-(4*a*c - b^2)^3)^(1/2) - 2*A*B*b^4*
c - 16*A*B*a^2*c^3 - 4*A^2*a*b*c^3 - 7*B^2*a*b^3*c + B^2*a*c*(-(4*a*c - b^2
)^3)^(1/2) + 12*B^2*a^2*b*c^2 + 2*A*B*b*c*(-(4*a*c - b^2)^3)^(1/2) + 12*A*B
*a*b^2*c^2)/(8*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2) - (2*(A^3*a*c^2
 - B^3*a^2*b + A*B^2*a*b^2 + A*B^2*a^2*c - 2*A^2*B*a*b*c))/c))*(-(B^2*b^5 +
 A^2*b^3*c^2 - A^2*c^2*(-(4*a*c - b^2)^3)^(1/2) - B^2*b^2*(-(4*a*c - b^2)^3
)^(1/2) - 2*A*B*b^4*c - 16*A*B*a^2*c^3 - 4*A^2*a*b*c^3 - 7*B^2*a*b^3*c + B^
2*a*c*(-(4*a*c - b^2)^3)^(1/2) + 12*B^2*a^2*b*c^2 + 2*A*B*b*c*(-(4*a*c - b^
2)^3)^(1/2) + 12*A*B*a*b^2*c^2)/(8*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(
1/2)*2i - atan(((((16*B*a^2*c^3 - 4*B*a*b^2*c^2)/c - (2*x*(4*b^3*c^3 - 16*a
*b*c^4)*(-(B^2*b^5 + A^2*b^3*c^2 + A^2*c^2*(-(4*a*c - b^2)^3)^(1/2) + B^2*b
^2*(-(4*a*c - b^2)^3)^(1/2) - 2*A*B*b^4*c - 16*A*B*a^2*c^3 - 4*A^2*a*b*c^3
- 7*B^2*a*b^3*c - B^2*a*c*(-(4*a*c - b^2)^3)^(1/2) + 12*B^2*a^2*b*c^2 - 2*A
*B*b*c*(-(4*a*c - b^2)^3)^(1/2) + 12*A*B*a*b^2*c^2)/(8*(16*a^2*c^5 + b^4*c^
3 - 8*a*b^2*c^4)))^(1/2))/c)*(-(B^2*b^5 + A^2*b^3*c^2 + A^2*c^2*(-(4*a*c -
b^2)^3)^(1/2) + B^2*b^2*(-(4*a*c - b^2)^3)^(1/2) - 2*A*B*b^4*c - 16*A*B*a^2
*c^3 - 4*A^2*a*b*c^3 - 7*B^2*a*b^3*c - B^2*a*c*(-(4*a*c - b^2)^3)^(1/2) + 1
2*B^2*a^2*b*c^2 - 2*A*B*b*c*(-(4*a*c - b^2)^3)^(1/2) + 12*A*B*a*b^2*c^2)/(8
*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2) - (2*x*(B^2*b^4 - 2*A^2*a*c^3
 + A^2*b^2*c^2 + 2*B^2*a^2*c^2 - 2*A*B*b^3*c - 4*B^2*a*b^2*c + 6*A*B*a*b*c^
2))/c)*(-(B^2*b^5 + A^2*b^3*c^2 + A^2*c^2*(-(4*a*c - b^2)^3)^(1/2) + B^2*b^
2*(-(4*a*c - b^2)^3)^(1/2) - 2*A*B*b^4*c - 16*A*B*a^2*c^3 - 4*A^2*a*b*c^3 -
 7*B^2*a*b^3*c - B^2*a*c*(-(4*a*c - b^2)^3)^(1/2) + 12*B^2*a^2*b*c^2 - 2*A*
B*b*c*(-(4*a*c - b^2)^3)^(1/2) + 12*A*B*a*b^2*c^2)/(8*(16*a^2*c^5 + b^4*c^3
```

```
 - 8*a*b^2*c^4)))^(1/2)*1i - (((16*B*a^2*c^3 - 4*B*a*b^2*c^2)/c + (2*x*(4*b
^3*c^3 - 16*a*b*c^4)*(-(B^2*b^5 + A^2*b^3*c^2 + A^2*c^2*(-(4*a*c - b^2)^3)^
(1/2) + B^2*b^2*(-(4*a*c - b^2)^3)^(1/2) - 2*A*B*b^4*c - 16*A*B*a^2*c^3 - 4
*A^2*a*b*c^3 - 7*B^2*a*b^3*c - B^2*a*c*(-(4*a*c - b^2)^3)^(1/2) + 12*B^2*a^
2*b*c^2 - 2*A*B*b*c*(-(4*a*c - b^2)^3)^(1/2) + 12*A*B*a*b^2*c^2)/(8*(16*a^2
*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2))/c)*(-(B^2*b^5 + A^2*b^3*c^2 + A^2*c^
2*(-(4*a*c - b^2)^3)^(1/2) + B^2*b^2*(-(4*a*c - b^2)^3)^(1/2) - 2*A*B*b^4*c
 - 16*A*B*a^2*c^3 - 4*A^2*a*b*c^3 - 7*B^2*a*b^3*c - B^2*a*c*(-(4*a*c - b^2)
^3)^(1/2) + 12*B^2*a^2*b*c^2 - 2*A*B*b*c*(-(4*a*c - b^2)^3)^(1/2) + 12*A*B*
a*b^2*c^2)/(8*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2) + (2*x*(B^2*b^4
- 2*A^2*a*c^3 + A^2*b^2*c^2 + 2*B^2*a^2*c^2 - 2*A*B*b^3*c - 4*B^2*a*b^2*c +
 6*A*B*a*b*c^2))/c)*(-(B^2*b^5 + A^2*b^3*c^2 + A^2*c^2*(-(4*a*c - b^2)^3)^(
1/2) + B^2*b^2*(-(4*a*c - b^2)^3)^(1/2) - 2*A*B*b^4*c - 16*A*B*a^2*c^3 - 4*
A^2*a*b*c^3 - 7*B^2*a*b^3*c - B^2*a*c*(-(4*a*c - b^2)^3)^(1/2) + 12*B^2*a^2
*b*c^2 - 2*A*B*b*c*(-(4*a*c - b^2)^3)^(1/2) + 12*A*B*a*b^2*c^2)/(8*(16*a^2*
c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2)*1i)/((((16*B*a^2*c^3 - 4*B*a*b^2*c^2)/
c - (2*x*(4*b^3*c^3 - 16*a*b*c^4)*(-(B^2*b^5 + A^2*b^3*c^2 + A^2*c^2*(-(4*a
*c - b^2)^3)^(1/2) + B^2*b^2*(-(4*a*c - b^2)^3)^(1/2) - 2*A*B*b^4*c - 16*A*
B*a^2*c^3 - 4*A^2*a*b*c^3 - 7*B^2*a*b^3*c - B^2*a*c*(-(4*a*c - b^2)^3)^(1/2
) + 12*B^2*a^2*b*c^2 - 2*A*B*b*c*(-(4*a*c - b^2)^3)^(1/2) + 12*A*B*a*b^2*c^
2)/(8*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2))/c)*(-(B^2*b^5 + A^2*b^3
*c^2 + A^2*c^2*(-(4*a*c - b^2)^3)^(1/2) + B^2*b^2*(-(4*a*c - b^2)^3)^(1/2)
- 2*A*B*b^4*c - 16*A*B*a^2*c^3 - 4*A^2*a*b*c^3 - 7*B^2*a*b^3*c - B^2*a*c*(-
(4*a*c - b^2)^3)^(1/2) + 12*B^2*a^2*b*c^2 - 2*A*B*b*c*(-(4*a*c - b^2)^3)^(1
/2) + 12*A*B*a*b^2*c^2)/(8*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2) - (
2*x*(B^2*b^4 - 2*A^2*a*c^3 + A^2*b^2*c^2 + 2*B^2*a^2*c^2 - 2*A*B*b^3*c - 4*
B^2*a*b^2*c + 6*A*B*a*b*c^2))/c)*(-(B^2*b^5 + A^2*b^3*c^2 + A^2*c^2*(-(4*a*
c - b^2)^3)^(1/2) + B^2*b^2*(-(4*a*c - b^2)^3)^(1/2) - 2*A*B*b^4*c - 16*A*B
*a^2*c^3 - 4*A^2*a*b*c^3 - 7*B^2*a*b^3*c - B^2*a*c*(-(4*a*c - b^2)^3)^(1/2)
 + 12*B^2*a^2*b*c^2 - 2*A*B*b*c*(-(4*a*c - b^2)^3)^(1/2) + 12*A*B*a*b^2*c^2
)/(8*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2) + (((16*B*a^2*c^3 - 4*B*a
*b^2*c^2)/c + (2*x*(4*b^3*c^3 - 16*a*b*c^4)*(-(B^2*b^5 + A^2*b^3*c^2 + A^2*
c^2*(-(4*a*c - b^2)^3)^(1/2) + B^2*b^2*(-(4*a*c - b^2)^3)^(1/2) - 2*A*B*b^4
*c - 16*A*B*a^2*c^3 - 4*A^2*a*b*c^3 - 7*B^2*a*b^3*c - B^2*a*c*(-(4*a*c - b^
2)^3)^(1/2) + 12*B^2*a^2*b*c^2 - 2*A*B*b*c*(-(4*a*c - b^2)^3)^(1/2) + 12*A*
B*a*b^2*c^2)/(8*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2))/c)*(-(B^2*b^5
 + A^2*b^3*c^2 + A^2*c^2*(-(4*a*c - b^2)^3)^(1/2) + B^2*b^2*(-(4*a*c - b^2)
^3)^(1/2) - 2*A*B*b^4*c - 16*A*B*a^2*c^3 - 4*A^2*a*b*c^3 - 7*B^2*a*b^3*c -
B^2*a*c*(-(4*a*c - b^2)^3)^(1/2) + 12*B^2*a^2*b*c^2 - 2*A*B*b*c*(-(4*a*c -
b^2)^3)^(1/2) + 12*A*B*a*b^2*c^2)/(8*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))
^(1/2) + (2*x*(B^2*b^4 - 2*A^2*a*c^3 + A^2*b^2*c^2 + 2*B^2*a^2*c^2 - 2*A*B*
b^3*c - 4*B^2*a*b^2*c + 6*A*B*a*b*c^2))/c)*(-(B^2*b^5 + A^2*b^3*c^2 + A^2*c
^2*(-(4*a*c - b^2)^3)^(1/2) + B^2*b^2*(-(4*a*c - b^2)^3)^(1/2) - 2*A*B*b^4*
c - 16*A*B*a^2*c^3 - 4*A^2*a*b*c^3 - 7*B^2*a*b^3*c - B^2*a*c*(-(4*a*c - b^2
)^3)^(1/2) + 12*B^2*a^2*b*c^2 - 2*A*B*b*c*(-(4*a*c - b^2)^3)^(1/2) + 12*A*B
```

```
*a*b^2*c^2)/(8*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2) - (2*(A^3*a*c^2
 - B^3*a^2*b + A*B^2*a*b^2 + A*B^2*a^2*c - 2*A^2*B*a*b*c))/c))*(-(B^2*b^5 +
 A^2*b^3*c^2 + A^2*c^2*(-(4*a*c - b^2)^3)^(1/2) + B^2*b^2*(-(4*a*c - b^2)^3
)^(1/2) - 2*A*B*b^4*c - 16*A*B*a^2*c^3 - 4*A^2*a*b*c^3 - 7*B^2*a*b^3*c - B^
2*a*c*(-(4*a*c - b^2)^3)^(1/2) + 12*B^2*a^2*b*c^2 - 2*A*B*b*c*(-(4*a*c - b^
2)^3)^(1/2) + 12*A*B*a*b^2*c^2)/(8*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(
1/2)*2i
```

3.109 $\int \frac{A+Bx^2}{a+bx^2+cx^4} dx$

Optimal result

Integrand size = 22, antiderivative size = 172

$$\int \frac{A+Bx^2}{a+bx^2+cx^4}\,dx=\frac{\left(B-\frac{bB-2Ac}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\left(B+\frac{bB-2Ac}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b+\sqrt{b^2-4ac}}}$$

```
[Out] 1/2*arctan(x*2^(1/2)*c^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2))*(B+(2*A*c-B*b)/(
-4*a*c+b^2)^(1/2))*2^(1/2)/c^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2)+1/2*arctan(
x*2^(1/2)*c^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2))*(B+(-2*A*c+B*b)/(-4*a*c+b^2
)^(1/2))*2^(1/2)/c^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2)
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 172, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {1180, 211}

$$\int \frac{A+Bx^2}{a+bx^2+cx^4}\,dx=\frac{\left(B-\frac{bB-2Ac}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\left(\frac{bB-2Ac}{\sqrt{b^2-4ac}}+B\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}\sqrt{c}\sqrt{\sqrt{b^2-4ac}+b}}$$

```
[In] Int[(A + B*x^2)/(a + b*x^2 + c*x^4),x]
```

```
[Out] ((B - (b*B - 2*A*c)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b -
Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*Sqrt[c]*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + ((B +
(b*B - 2*A*c)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b
^2 - 4*a*c]]])/(Sqrt[2]*Sqrt[c]*Sqrt[b + Sqrt[b^2 - 4*a*c]])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{2}\left(B-\frac{bB-2Ac}{\sqrt{b^2-4ac}}\right)\int\frac{1}{\frac{b}{2}-\frac{1}{2}\sqrt{b^2-4ac}+cx^2}\,dx\\
&\quad+\frac{1}{2}\left(B+\frac{bB-2Ac}{\sqrt{b^2-4ac}}\right)\int\frac{1}{\frac{b}{2}+\frac{1}{2}\sqrt{b^2-4ac}+cx^2}\,dx\\
&=\frac{\left(B-\frac{bB-2Ac}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\left(B+\frac{bB-2Ac}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b+\sqrt{b^2-4ac}}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 173, normalized size of antiderivative = 1.01

$$\int\frac{A+Bx^2}{a+bx^2+cx^4}\,dx=\frac{\frac{\left(-bB+2Ac+B\sqrt{b^2-4ac}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\left(bB-2Ac+B\sqrt{b^2-4ac}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{b+\sqrt{b^2-4ac}}}}{\sqrt{2}\sqrt{c}\sqrt{b^2-4ac}}$$

```
[In] Integrate[(A + B*x^2)/(a + b*x^2 + c*x^4),x]
```

```
[Out] (((-(b*B) + 2*A*c + B*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b
- Sqrt[b^2 - 4*a*c]]])/Sqrt[b - Sqrt[b^2 - 4*a*c]] + ((b*B - 2*A*c + B*Sqrt
[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/Sqr
t[b + Sqrt[b^2 - 4*a*c]])/(Sqrt[2]*Sqrt[c]*Sqrt[b^2 - 4*a*c])
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.06 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.26

method	result
risch	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(c\_Z^4+\_Z^2b+a\right)}\frac{\left(B\_R^2+A\right)\ln\left(x-\_R\right)}{2c\_R^3+\_Rb}\right)}{2}$
default	$4c\left(\frac{\left(-2Ac+B\sqrt{-4ac+b^2}+Bb\right)\sqrt{2}\arctan\left(\frac{cx\sqrt{2}}{\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}}\right)}{8\sqrt{-4ac+b^2}\,c\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}}-\frac{\left(2Ac+B\sqrt{-4ac+b^2}-Bb\right)\sqrt{2}\,\mathrm{arctanh}\left(\frac{cx\sqrt{2}}{\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}}\right.}{8\sqrt{-4ac+b^2}\,c\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}}\right.$

```
[In] int((B*x^2+A)/(c*x^4+b*x^2+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*sum((B*_R^2+A)/(2*_R^3*c+_R*b)*ln(x-_R),_R=RootOf(_Z^4*c+_Z^2*b+a))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1569 vs. 2(138) = 276.

Time = 0.44 (sec) , antiderivative size = 1569, normalized size of antiderivative = 9.12

$$\int \frac{A+Bx^2}{a+bx^2+cx^4}\,dx = \text{Too large to display}$$

```
[In] integrate((B*x^2+A)/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] 1/2*sqrt(1/2)*sqrt(-(B^2*a*b - (4*A*B*a - A^2*b)*c + (a*b^2*c - 4*a^2*c^2)*
sqrt((B^4*a^2 - 2*A^2*B^2*a*c + A^4*c^2)/(a^2*b^2*c^2 - 4*a^3*c^3)))/(a*b^2
*c - 4*a^2*c^2))*log(-2*(B^4*a^2 - A*B^3*a*b + A^3*B*b*c - A^4*c^2)*x + sqr
t(1/2)*(A*B^2*a*b^2 + 4*A^3*a*c^2 - (4*A*B^2*a^2 + A^3*b^2)*c + (4*(2*B*a^3
 - A*a^2*b)*c^2 - (2*B*a^2*b^2 - A*a*b^3)*c)*sqrt((B^4*a^2 - 2*A^2*B^2*a*c
+ A^4*c^2)/(a^2*b^2*c^2 - 4*a^3*c^3)))*sqrt(-(B^2*a*b - (4*A*B*a - A^2*b)*c
 + (a*b^2*c - 4*a^2*c^2)*sqrt((B^4*a^2 - 2*A^2*B^2*a*c + A^4*c^2)/(a^2*b^2*
c^2 - 4*a^3*c^3)))/(a*b^2*c - 4*a^2*c^2))) - 1/2*sqrt(1/2)*sqrt(-(B^2*a*b -
 (4*A*B*a - A^2*b)*c + (a*b^2*c - 4*a^2*c^2)*sqrt((B^4*a^2 - 2*A^2*B^2*a*c
+ A^4*c^2)/(a^2*b^2*c^2 - 4*a^3*c^3)))/(a*b^2*c - 4*a^2*c^2))*log(-2*(B^4*a
^2 - A*B^3*a*b + A^3*B*b*c - A^4*c^2)*x - sqrt(1/2)*(A*B^2*a*b^2 + 4*A^3*a*
c^2 - (4*A*B^2*a^2 + A^3*b^2)*c + (4*(2*B*a^3 - A*a^2*b)*c^2 - (2*B*a^2*b^2
 - A*a*b^3)*c)*sqrt((B^4*a^2 - 2*A^2*B^2*a*c + A^4*c^2)/(a^2*b^2*c^2 - 4*a^
3*c^3)))*sqrt(-(B^2*a*b - (4*A*B*a - A^2*b)*c + (a*b^2*c - 4*a^2*c^2)*sqrt(
(B^4*a^2 - 2*A^2*B^2*a*c + A^4*c^2)/(a^2*b^2*c^2 - 4*a^3*c^3)))/(a*b^2*c -
4*a^2*c^2))) + 1/2*sqrt(1/2)*sqrt(-(B^2*a*b - (4*A*B*a - A^2*b)*c - (a*b^2*
```

```
c - 4*a^2*c^2)*sqrt((B^4*a^2 - 2*A^2*B^2*a*c + A^4*c^2)/(a^2*b^2*c^2 - 4*a^
3*c^3)))/(a*b^2*c - 4*a^2*c^2))*log(-2*(B^4*a^2 - A*B^3*a*b + A^3*B*b*c - A
^4*c^2)*x + sqrt(1/2)*(A*B^2*a*b^2 + 4*A^3*a*c^2 - (4*A*B^2*a^2 + A^3*b^2)*
c - (4*(2*B*a^3 - A*a^2*b)*c^2 - (2*B*a^2*b^2 - A*a*b^3)*c)*sqrt((B^4*a^2 -
 2*A^2*B^2*a*c + A^4*c^2)/(a^2*b^2*c^2 - 4*a^3*c^3)))*sqrt(-(B^2*a*b - (4*A
*B*a - A^2*b)*c - (a*b^2*c - 4*a^2*c^2)*sqrt((B^4*a^2 - 2*A^2*B^2*a*c + A^4
*c^2)/(a^2*b^2*c^2 - 4*a^3*c^3)))/(a*b^2*c - 4*a^2*c^2))) - 1/2*sqrt(1/2)*s
qrt(-(B^2*a*b - (4*A*B*a - A^2*b)*c - (a*b^2*c - 4*a^2*c^2)*sqrt((B^4*a^2 -
 2*A^2*B^2*a*c + A^4*c^2)/(a^2*b^2*c^2 - 4*a^3*c^3)))/(a*b^2*c - 4*a^2*c^2)
)*log(-2*(B^4*a^2 - A*B^3*a*b + A^3*B*b*c - A^4*c^2)*x - sqrt(1/2)*(A*B^2*a
*b^2 + 4*A^3*a*c^2 - (4*A*B^2*a^2 + A^3*b^2)*c - (4*(2*B*a^3 - A*a^2*b)*c^2
 - (2*B*a^2*b^2 - A*a*b^3)*c)*sqrt((B^4*a^2 - 2*A^2*B^2*a*c + A^4*c^2)/(a^2
*b^2*c^2 - 4*a^3*c^3)))*sqrt(-(B^2*a*b - (4*A*B*a - A^2*b)*c - (a*b^2*c - 4
*a^2*c^2)*sqrt((B^4*a^2 - 2*A^2*B^2*a*c + A^4*c^2)/(a^2*b^2*c^2 - 4*a^3*c^3
)))/(a*b^2*c - 4*a^2*c^2)))
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A + Bx^2}{a + bx^2 + cx^4}\,dx = \text{Timed out}$$

```
[In] integrate((B*x**2+A)/(c*x**4+b*x**2+a),x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{A + Bx^2}{a + bx^2 + cx^4}\,dx = \int \frac{Bx^2 + A}{cx^4 + bx^2 + a}\,dx$$

```
[In] integrate((B*x^2+A)/(c*x^4+b*x^2+a),x, algorithm="maxima")
[Out] integrate((B*x^2 + A)/(c*x^4 + b*x^2 + a), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1402 vs. 2(138) = 276.

Time = 0.78 (sec) , antiderivative size = 1402, normalized size of antiderivative = 8.15

$$\int \frac{A + Bx^2}{a + bx^2 + cx^4} \, dx = \text{Too large to display}$$

```
[In] integrate((B*x^2+A)/(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] 1/4*((sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^4 - 8*sqrt(2)*sqrt(b*c + sq
rt(b^2 - 4*a*c)*c)*a*b^2*c - 2*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^3*
c - 2*b^4*c + 16*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*c^2 + 8*sqrt(2
)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c^2 + sqrt(2)*sqrt(b*c + sqrt(b^2 - 4
*a*c)*c)*b^2*c^2 + 16*a*b^2*c^2 + 2*b^3*c^2 - 4*sqrt(2)*sqrt(b*c + sqrt(b^2
 - 4*a*c)*c)*a*c^3 - 32*a^2*c^3 - 8*a*b*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqr
t(b*c + sqrt(b^2 - 4*a*c)*c)*b^3 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + s
qrt(b^2 - 4*a*c)*c)*a*b*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2
 - 4*a*c)*c)*b^2*c - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)
*c)*b*c^2 + 2*(b^2 - 4*a*c)*b^2*c - 8*(b^2 - 4*a*c)*a*c^2 - 2*(b^2 - 4*a*c)
*b*c^2)*A - 2*(2*a*b^2*c^2 - 8*a^2*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c
 + sqrt(b^2 - 4*a*c)*c)*a*b^2 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt
(b^2 - 4*a*c)*c)*a^2*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 -
4*a*c)*c)*a*b*c - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)
*a*c^2 - 2*(b^2 - 4*a*c)*a*c^2)*B)*arctan(2*sqrt(1/2)*x/sqrt((b + sqrt(b^2
- 4*a*c))/c))/((a*b^4 - 8*a^2*b^2*c - 2*a*b^3*c + 16*a^3*c^2 + 8*a^2*b*c^2
+ a*b^2*c^2 - 4*a^2*c^3)*abs(c)) + 1/4*((sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*
c)*c)*b^4 - 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^2*c - 2*sqrt(2)*s
qrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^3*c + 2*b^4*c + 16*sqrt(2)*sqrt(b*c - sqrt
(b^2 - 4*a*c)*c)*a^2*c^2 + 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b*c^
2 + sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^2*c^2 - 16*a*b^2*c^2 + 2*b^3*
c^2 - 4*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*c^3 + 32*a^2*c^3 - 8*a*b*
c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^3 + 4*sqr
t(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b*c + 2*sqrt(2)*sq
rt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^2*c - sqrt(2)*sqrt(b^2 -
4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b*c^2 - 2*(b^2 - 4*a*c)*b^2*c + 8*(b
^2 - 4*a*c)*a*c^2 - 2*(b^2 - 4*a*c)*b*c^2)*A - 2*(2*a*b^2*c^2 - 8*a^2*c^3 -
 sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^2 + 4*sqrt(2
)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*c + 2*sqrt(2)*sqrt(
b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b*c - sqrt(2)*sqrt(b^2 - 4*a
*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*c^2 - 2*(b^2 - 4*a*c)*a*c^2)*B)*arcta
n(2*sqrt(1/2)*x/sqrt((b - sqrt(b^2 - 4*a*c))/c))/((a*b^4 - 8*a^2*b^2*c - 2*
a*b^3*c + 16*a^3*c^2 + 8*a^2*b*c^2 + a*b^2*c^2 - 4*a^2*c^3)*abs(c))
```

Mupad [B] (verification not implemented)

Time = 8.07 (sec) , antiderivative size = 4109, normalized size of antiderivative = 23.89

$$\int \frac{A+Bx^2}{a+bx^2+cx^4} \, dx = \text{Too large to display}$$

```
[In] int((A + B*x^2)/(a + b*x^2 + c*x^4),x)
```

```
[Out] - atan((((-(B^2*a*b^3 + B^2*a*(-(4*a*c - b^2)^3)^(1/2) + A^2*b^3*c - A^2*c*
(-(4*a*c - b^2)^3)^(1/2) + 16*A*B*a^2*c^2 - 4*A^2*a*b*c^2 - 4*B^2*a^2*b*c -
 4*A*B*a*b^2*c)/(8*(16*a^3*c^3 - 8*a^2*b^2*c^2 + a*b^4*c)))^(1/2)*(x*(8*b^3
*c^2 - 32*a*b*c^3)*(-(B^2*a*b^3 + B^2*a*(-(4*a*c - b^2)^3)^(1/2) + A^2*b^3*
c - A^2*c*(-(4*a*c - b^2)^3)^(1/2) + 16*A*B*a^2*c^2 - 4*A^2*a*b*c^2 - 4*B^2
*a^2*b*c - 4*A*B*a*b^2*c)/(8*(16*a^3*c^3 - 8*a^2*b^2*c^2 + a*b^4*c)))^(1/2)
 - 4*A*b^2*c^2 + 16*A*a*c^3) + x*(4*A^2*c^3 - 4*B^2*a*c^2 + 2*B^2*b^2*c - 4
*A*B*b*c^2))*(-(B^2*a*b^3 + B^2*a*(-(4*a*c - b^2)^3)^(1/2) + A^2*b^3*c - A^
2*c*(-(4*a*c - b^2)^3)^(1/2) + 16*A*B*a^2*c^2 - 4*A^2*a*b*c^2 - 4*B^2*a^2*b
*c - 4*A*B*a*b^2*c)/(8*(16*a^3*c^3 - 8*a^2*b^2*c^2 + a*b^4*c)))^(1/2)*1i +
((-(B^2*a*b^3 + B^2*a*(-(4*a*c - b^2)^3)^(1/2) + A^2*b^3*c - A^2*c*(-(4*a*c
 - b^2)^3)^(1/2) + 16*A*B*a^2*c^2 - 4*A^2*a*b*c^2 - 4*B^2*a^2*b*c - 4*A*B*a
*b^2*c)/(8*(16*a^3*c^3 - 8*a^2*b^2*c^2 + a*b^4*c)))^(1/2)*(4*A*b^2*c^2 + x*
(8*b^3*c^2 - 32*a*b*c^3)*(-(B^2*a*b^3 + B^2*a*(-(4*a*c - b^2)^3)^(1/2) + A^
2*b^3*c - A^2*c*(-(4*a*c - b^2)^3)^(1/2) + 16*A*B*a^2*c^2 - 4*A^2*a*b*c^2 -
 4*B^2*a^2*b*c - 4*A*B*a*b^2*c)/(8*(16*a^3*c^3 - 8*a^2*b^2*c^2 + a*b^4*c)))
^(1/2) - 16*A*a*c^3) + x*(4*A^2*c^3 - 4*B^2*a*c^2 + 2*B^2*b^2*c - 4*A*B*b*c
^2))*(-(B^2*a*b^3 + B^2*a*(-(4*a*c - b^2)^3)^(1/2) + A^2*b^3*c - A^2*c*(-(4
*a*c - b^2)^3)^(1/2) + 16*A*B*a^2*c^2 - 4*A^2*a*b*c^2 - 4*B^2*a^2*b*c - 4*A
*B*a*b^2*c)/(8*(16*a^3*c^3 - 8*a^2*b^2*c^2 + a*b^4*c)))^(1/2)*1i)/(((-(B^2*
a*b^3 + B^2*a*(-(4*a*c - b^2)^3)^(1/2) + A^2*b^3*c - A^2*c*(-(4*a*c - b^2)^
3)^(1/2) + 16*A*B*a^2*c^2 - 4*A^2*a*b*c^2 - 4*B^2*a^2*b*c - 4*A*B*a*b^2*c)/
(8*(16*a^3*c^3 - 8*a^2*b^2*c^2 + a*b^4*c)))^(1/2)*(x*(8*b^3*c^2 - 32*a*b*c^
3)*(-(B^2*a*b^3 + B^2*a*(-(4*a*c - b^2)^3)^(1/2) + A^2*b^3*c - A^2*c*(-(4*a
*c - b^2)^3)^(1/2) + 16*A*B*a^2*c^2 - 4*A^2*a*b*c^2 - 4*B^2*a^2*b*c - 4*A*B
*a*b^2*c)/(8*(16*a^3*c^3 - 8*a^2*b^2*c^2 + a*b^4*c)))^(1/2) - 4*A*b^2*c^2 +
 16*A*a*c^3) + x*(4*A^2*c^3 - 4*B^2*a*c^2 + 2*B^2*b^2*c - 4*A*B*b*c^2))*(-(
B^2*a*b^3 + B^2*a*(-(4*a*c - b^2)^3)^(1/2) + A^2*b^3*c - A^2*c*(-(4*a*c - b
^2)^3)^(1/2) + 16*A*B*a^2*c^2 - 4*A^2*a*b*c^2 - 4*B^2*a^2*b*c - 4*A*B*a*b^2
*c)/(8*(16*a^3*c^3 - 8*a^2*b^2*c^2 + a*b^4*c)))^(1/2) - ((-(B^2*a*b^3 + B^2
*a*(-(4*a*c - b^2)^3)^(1/2) + A^2*b^3*c - A^2*c*(-(4*a*c - b^2)^3)^(1/2) +
16*A*B*a^2*c^2 - 4*A^2*a*b*c^2 - 4*B^2*a^2*b*c - 4*A*B*a*b^2*c)/(8*(16*a^3*
c^3 - 8*a^2*b^2*c^2 + a*b^4*c)))^(1/2)*(4*A*b^2*c^2 + x*(8*b^3*c^2 - 32*a*b
*c^3)*(-(B^2*a*b^3 + B^2*a*(-(4*a*c - b^2)^3)^(1/2) + A^2*b^3*c - A^2*c*(-(
4*a*c - b^2)^3)^(1/2) + 16*A*B*a^2*c^2 - 4*A^2*a*b*c^2 - 4*B^2*a^2*b*c - 4*
A*B*a*b^2*c)/(8*(16*a^3*c^3 - 8*a^2*b^2*c^2 + a*b^4*c)))^(1/2) - 16*A*a*c^3
```

```
) + x*(4*A^2*c^3 - 4*B^2*a*c^2 + 2*B^2*b^2*c - 4*A*B*b*c^2))*(-(B^2*a*b^3 +
 B^2*a*(-(4*a*c - b^2)^3)^(1/2) + A^2*b^3*c - A^2*c*(-(4*a*c - b^2)^3)^(1/2
) + 16*A*B*a^2*c^2 - 4*A^2*a*b*c^2 - 4*B^2*a^2*b*c - 4*A*B*a*b^2*c)/(8*(16*
a^3*c^3 - 8*a^2*b^2*c^2 + a*b^4*c)))^(1/2) + 2*A^2*B*c^2 + 2*B^3*a*c - 2*A*
B^2*b*c))*(-(B^2*a*b^3 + B^2*a*(-(4*a*c - b^2)^3)^(1/2) + A^2*b^3*c - A^2*c
*(-(4*a*c - b^2)^3)^(1/2) + 16*A*B*a^2*c^2 - 4*A^2*a*b*c^2 - 4*B^2*a^2*b*c
- 4*A*B*a*b^2*c)/(8*(16*a^3*c^3 - 8*a^2*b^2*c^2 + a*b^4*c)))^(1/2)*2i - ata
n((((-(B^2*a*b^3 - B^2*a*(-(4*a*c - b^2)^3)^(1/2) + A^2*b^3*c + A^2*c*(-(4*
a*c - b^2)^3)^(1/2) + 16*A*B*a^2*c^2 - 4*A^2*a*b*c^2 - 4*B^2*a^2*b*c - 4*A*
B*a*b^2*c)/(8*(16*a^3*c^3 - 8*a^2*b^2*c^2 + a*b^4*c)))^(1/2)*(x*(8*b^3*c^2
- 32*a*b*c^3)*(-(B^2*a*b^3 - B^2*a*(-(4*a*c - b^2)^3)^(1/2) + A^2*b^3*c + A
^2*c*(-(4*a*c - b^2)^3)^(1/2) + 16*A*B*a^2*c^2 - 4*A^2*a*b*c^2 - 4*B^2*a^2*
b*c - 4*A*B*a*b^2*c)/(8*(16*a^3*c^3 - 8*a^2*b^2*c^2 + a*b^4*c)))^(1/2) - 4*
A*b^2*c^2 + 16*A*a*c^3) + x*(4*A^2*c^3 - 4*B^2*a*c^2 + 2*B^2*b^2*c - 4*A*B*
b*c^2))*(-(B^2*a*b^3 - B^2*a*(-(4*a*c - b^2)^3)^(1/2) + A^2*b^3*c + A^2*c*(
-(4*a*c - b^2)^3)^(1/2) + 16*A*B*a^2*c^2 - 4*A^2*a*b*c^2 - 4*B^2*a^2*b*c -
4*A*B*a*b^2*c)/(8*(16*a^3*c^3 - 8*a^2*b^2*c^2 + a*b^4*c)))^(1/2)*1i + ((-(B
^2*a*b^3 - B^2*a*(-(4*a*c - b^2)^3)^(1/2) + A^2*b^3*c + A^2*c*(-(4*a*c - b^
2)^3)^(1/2) + 16*A*B*a^2*c^2 - 4*A^2*a*b*c^2 - 4*B^2*a^2*b*c - 4*A*B*a*b^2*
c)/(8*(16*a^3*c^3 - 8*a^2*b^2*c^2 + a*b^4*c)))^(1/2)*(4*A*b^2*c^2 + x*(8*b^
3*c^2 - 32*a*b*c^3)*(-(B^2*a*b^3 - B^2*a*(-(4*a*c - b^2)^3)^(1/2) + A^2*b^3
*c + A^2*c*(-(4*a*c - b^2)^3)^(1/2) + 16*A*B*a^2*c^2 - 4*A^2*a*b*c^2 - 4*B^
2*a^2*b*c - 4*A*B*a*b^2*c)/(8*(16*a^3*c^3 - 8*a^2*b^2*c^2 + a*b^4*c)))^(1/2
) - 16*A*a*c^3) + x*(4*A^2*c^3 - 4*B^2*a*c^2 + 2*B^2*b^2*c - 4*A*B*b*c^2))*
(-(B^2*a*b^3 - B^2*a*(-(4*a*c - b^2)^3)^(1/2) + A^2*b^3*c + A^2*c*(-(4*a*c
- b^2)^3)^(1/2) + 16*A*B*a^2*c^2 - 4*A^2*a*b*c^2 - 4*B^2*a^2*b*c - 4*A*B*a*
b^2*c)/(8*(16*a^3*c^3 - 8*a^2*b^2*c^2 + a*b^4*c)))^(1/2)*1i)/(((-(B^2*a*b^3
 - B^2*a*(-(4*a*c - b^2)^3)^(1/2) + A^2*b^3*c + A^2*c*(-(4*a*c - b^2)^3)^(1
/2) + 16*A*B*a^2*c^2 - 4*A^2*a*b*c^2 - 4*B^2*a^2*b*c - 4*A*B*a*b^2*c)/(8*(1
6*a^3*c^3 - 8*a^2*b^2*c^2 + a*b^4*c)))^(1/2)*(x*(8*b^3*c^2 - 32*a*b*c^3)*(-
(B^2*a*b^3 - B^2*a*(-(4*a*c - b^2)^3)^(1/2) + A^2*b^3*c + A^2*c*(-(4*a*c -
b^2)^3)^(1/2) + 16*A*B*a^2*c^2 - 4*A^2*a*b*c^2 - 4*B^2*a^2*b*c - 4*A*B*a*b^
2*c)/(8*(16*a^3*c^3 - 8*a^2*b^2*c^2 + a*b^4*c)))^(1/2) - 4*A*b^2*c^2 + 16*A
*a*c^3) + x*(4*A^2*c^3 - 4*B^2*a*c^2 + 2*B^2*b^2*c - 4*A*B*b*c^2))*(-(B^2*a
*b^3 - B^2*a*(-(4*a*c - b^2)^3)^(1/2) + A^2*b^3*c + A^2*c*(-(4*a*c - b^2)^3
)^(1/2) + 16*A*B*a^2*c^2 - 4*A^2*a*b*c^2 - 4*B^2*a^2*b*c - 4*A*B*a*b^2*c)/(
8*(16*a^3*c^3 - 8*a^2*b^2*c^2 + a*b^4*c)))^(1/2) - ((-(B^2*a*b^3 - B^2*a*(-
(4*a*c - b^2)^3)^(1/2) + A^2*b^3*c + A^2*c*(-(4*a*c - b^2)^3)^(1/2) + 16*A*
B*a^2*c^2 - 4*A^2*a*b*c^2 - 4*B^2*a^2*b*c - 4*A*B*a*b^2*c)/(8*(16*a^3*c^3 -
 8*a^2*b^2*c^2 + a*b^4*c)))^(1/2)*(4*A*b^2*c^2 + x*(8*b^3*c^2 - 32*a*b*c^3)
*(-(B^2*a*b^3 - B^2*a*(-(4*a*c - b^2)^3)^(1/2) + A^2*b^3*c + A^2*c*(-(4*a*c
 - b^2)^3)^(1/2) + 16*A*B*a^2*c^2 - 4*A^2*a*b*c^2 - 4*B^2*a^2*b*c - 4*A*B*a
*b^2*c)/(8*(16*a^3*c^3 - 8*a^2*b^2*c^2 + a*b^4*c)))^(1/2) - 16*A*a*c^3) + x
*(4*A^2*c^3 - 4*B^2*a*c^2 + 2*B^2*b^2*c - 4*A*B*b*c^2))*(-(B^2*a*b^3 - B^2*
a*(-(4*a*c - b^2)^3)^(1/2) + A^2*b^3*c + A^2*c*(-(4*a*c - b^2)^3)^(1/2) + 1
```

```
6*A*B*a^2*c^2 - 4*A^2*a*b*c^2 - 4*B^2*a^2*b*c - 4*A*B*a*b^2*c)/(8*(16*a^3*c
^3 - 8*a^2*b^2*c^2 + a*b^4*c)))^(1/2) + 2*A^2*B*c^2 + 2*B^3*a*c - 2*A*B^2*b
*c))*(-(B^2*a*b^3 - B^2*a*(-(4*a*c - b^2)^3)^(1/2) + A^2*b^3*c + A^2*c*(-(4
*a*c - b^2)^3)^(1/2) + 16*A*B*a^2*c^2 - 4*A^2*a*b*c^2 - 4*B^2*a^2*b*c - 4*A
*B*a*b^2*c)/(8*(16*a^3*c^3 - 8*a^2*b^2*c^2 + a*b^4*c)))^(1/2)*2i
```

3.110 $\int \frac{A+Bx^2}{x^2(a+bx^2+cx^4)}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 189

$$\int \frac{A+Bx^2}{x^2(a+bx^2+cx^4)}\,dx = -\frac{A}{ax} - \frac{\sqrt{c}\left(A+\frac{Ab-2aB}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}a\sqrt{b-\sqrt{b^2-4ac}}} - \frac{\sqrt{c}\left(A-\frac{Ab-2aB}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{2}a\sqrt{b+\sqrt{b^2-4ac}}}$$

```
[Out] -A/a/x-1/2*arctan(x*2^(1/2)*c^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2))*c^(1/2)*(
A+(A*b-2*B*a)/(-4*a*c+b^2)^(1/2))/a*2^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2)-1/
2*arctan(x*2^(1/2)*c^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2))*c^(1/2)*(A+(-A*b+2
*B*a)/(-4*a*c+b^2)^(1/2))/a*2^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2)
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 189, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {1295, 1180, 211}

$$\int \frac{A+Bx^2}{x^2(a+bx^2+cx^4)}\,dx = -\frac{\sqrt{c}\left(\frac{Ab-2aB}{\sqrt{b^2-4ac}}+A\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}a\sqrt{b-\sqrt{b^2-4ac}}} - \frac{\sqrt{c}\left(A-\frac{Ab-2aB}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}a\sqrt{\sqrt{b^2-4ac}+b}} - \frac{A}{ax}$$

```
[In] Int[(A + B*x^2)/(x^2*(a + b*x^2 + c*x^4)),x]
```

```
[Out] -(A/(a*x)) - (Sqrt[c]*(A + (A*b - 2*a*B)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]
*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*a*Sqrt[b - Sqrt[b^2 - 4*
a*c]]) - (Sqrt[c]*(A - (A*b - 2*a*B)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqr
t[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*a*Sqrt[b + Sqrt[b^2 - 4*a*c]
])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1295

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[d*(f*x)^(m + 1)*((a + b*x^2 + c*x^4)^(p + 1)
/(a*f*(m + 1))), x] + Dist[1/(a*f^2*(m + 1)), Int[(f*x)^(m + 2)*(a + b*x^2
+ c*x^4)^p*Simp[a*e*(m + 1) - b*d*(m + 2*p + 3) - c*d*(m + 4*p + 5)*x^2, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[m
, -1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{A}{ax} - \frac{\int \frac{Ab-aB+Acx^2}{a+bx^2+cx^4}\,dx}{a} \\
&= -\frac{A}{ax} - \frac{\left(c\left(A-\frac{Ab-2aB}{\sqrt{b^2-4ac}}\right)\right)\int \frac{1}{\frac{b}{2}+\frac{1}{2}\sqrt{b^2-4ac}+cx^2}\,dx}{2a} - \frac{\left(c\left(A+\frac{Ab-2aB}{\sqrt{b^2-4ac}}\right)\right)\int \frac{1}{\frac{b}{2}-\frac{1}{2}\sqrt{b^2-4ac}+cx^2}\,dx}{2a} \\
&= -\frac{A}{ax} - \frac{\sqrt{c}\left(A+\frac{Ab-2aB}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}a\sqrt{b-\sqrt{b^2-4ac}}} - \frac{\sqrt{c}\left(A-\frac{Ab-2aB}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{2}a\sqrt{b+\sqrt{b^2-4ac}}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 206, normalized size of antiderivative = 1.09

$$\int \frac{A+Bx^2}{x^2\left(a+bx^2+cx^4\right)}\,dx = -\frac{\frac{2A}{x}+\frac{\sqrt{2}\sqrt{c}\left(-2aB+A\left(b+\sqrt{b^2-4ac}\right)\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\sqrt{2}\sqrt{c}\left(2aB+A\left(-b+\sqrt{b^2-4ac}\right)\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}\sqrt{b+\sqrt{b^2-4ac}}}}{2a}$$

```
[In] Integrate[(A + B*x^2)/(x^2*(a + b*x^2 + c*x^4)),x]
```

```
[Out] -1/2*((2*A)/x + (Sqrt[2]*Sqrt[c]*(-2*a*B + A*(b + Sqrt[b^2 - 4*a*c]))*ArcTa
n[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[b^2 - 4*a*c]*Sqrt
[b - Sqrt[b^2 - 4*a*c]]) + (Sqrt[2]*Sqrt[c]*(2*a*B + A*(-b + Sqrt[b^2 - 4*a
*c]))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[b^2 -
4*a*c]*Sqrt[b + Sqrt[b^2 - 4*a*c]]))/a
```

Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 173, normalized size of antiderivative = 0.92

method	result
default	$\frac{4c\left(\frac{\left(-A\sqrt{-4ac+b^2}+Ab-2Ba\right)\sqrt{2}\,\arctan\left(\frac{cx\sqrt{2}}{\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}}\right)}{8\sqrt{-4ac+b^2}\,\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}}-\frac{\left(-A\sqrt{-4ac+b^2}-Ab+2Ba\right)\sqrt{2}\,\operatorname{arctanh}\left(\frac{cx\sqrt{2}}{\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}}\right)}{8\sqrt{-4ac+b^2}\,\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}}\right)}{a}-\frac{A}{ax}$
risch	$-\frac{A}{ax}+\frac{\left(\sum_{\_R=\operatorname{RootOf}\left(\left(16a^5c^2-8a^4b^2c+b^4a^3\right)\_Z^4+\left(12A^2a^2bc^2-7A^2ab^3c+A^2b^5-16ABa^3c^2+12ABa^2b^2c-2ABab^4-4B^2a^3bc+B^2a^2b^3\right.\right.}\right.}{}$

```
[In] int((B*x^2+A)/x^2/(c*x^4+b*x^2+a),x,method=_RETURNVERBOSE)
```

```
[Out] 4/a*c*(1/8*(-A*(-4*a*c+b^2)^(1/2)+A*b-2*B*a)/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b
+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)
^(1/2))-1/8*(-A*(-4*a*c+b^2)^(1/2)-A*b+2*B*a)/(-4*a*c+b^2)^(1/2)*2^(1/2)/((
-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(c*x*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2)
)*c)^(1/2)))-A/a/x
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2914 vs. $2(155) = 310$.

Time = 0.79 (sec) , antiderivative size = 2914, normalized size of antiderivative = 15.42

$$\int \frac{A + Bx^2}{x^2 (a + bx^2 + cx^4)} \, dx = \text{Too large to display}$$

```
[In] integrate((B*x^2+A)/x^2/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] 1/2*(sqrt(1/2)*a*x*sqrt(-(B^2*a^2*b - 2*A*B*a*b^2 + A^2*b^3 + (4*A*B*a^2 -
3*A^2*a*b)*c + (a^3*b^2 - 4*a^4*c)*sqrt((B^4*a^4 - 4*A*B^3*a^3*b + 6*A^2*B^
2*a^2*b^2 - 4*A^3*B*a*b^3 + A^4*b^4 + A^4*a^2*c^2 - 2*(A^2*B^2*a^3 - 2*A^3*
B*a^2*b + A^4*a*b^2)*c)/(a^6*b^2 - 4*a^7*c)))/(a^3*b^2 - 4*a^4*c))*log(2*(A
^4*a*c^3 + (A^3*B*a*b - A^4*b^2)*c^2 - (B^4*a^3 - 3*A*B^3*a^2*b + 3*A^2*B^2
*a*b^2 - A^3*B*b^3)*c)*x + sqrt(1/2)*(B^3*a^3*b^2 - 3*A*B^2*a^2*b^3 + 3*A^2
*B*a*b^4 - A^3*b^5 + 4*(A^2*B*a^3 - A^3*a^2*b)*c^2 - (4*B^3*a^4 - 12*A*B^2*
a^3*b + 13*A^2*B*a^2*b^2 - 5*A^3*a*b^3)*c - (B*a^4*b^3 - A*a^3*b^4 - 8*A*a^
5*c^2 - 2*(2*B*a^5*b - 3*A*a^4*b^2)*c)*sqrt((B^4*a^4 - 4*A*B^3*a^3*b + 6*A^
2*B^2*a^2*b^2 - 4*A^3*B*a*b^3 + A^4*b^4 + A^4*a^2*c^2 - 2*(A^2*B^2*a^3 - 2*
A^3*B*a^2*b + A^4*a*b^2)*c)/(a^6*b^2 - 4*a^7*c)))*sqrt(-(B^2*a^2*b - 2*A*B*
a*b^2 + A^2*b^3 + (4*A*B*a^2 - 3*A^2*a*b)*c + (a^3*b^2 - 4*a^4*c)*sqrt((B^4
*a^4 - 4*A*B^3*a^3*b + 6*A^2*B^2*a^2*b^2 - 4*A^3*B*a*b^3 + A^4*b^4 + A^4*a^
2*c^2 - 2*(A^2*B^2*a^3 - 2*A^3*B*a^2*b + A^4*a*b^2)*c)/(a^6*b^2 - 4*a^7*c))
)/(a^3*b^2 - 4*a^4*c))) - sqrt(1/2)*a*x*sqrt(-(B^2*a^2*b - 2*A*B*a*b^2 + A^
2*b^3 + (4*A*B*a^2 - 3*A^2*a*b)*c + (a^3*b^2 - 4*a^4*c)*sqrt((B^4*a^4 - 4*A
*B^3*a^3*b + 6*A^2*B^2*a^2*b^2 - 4*A^3*B*a*b^3 + A^4*b^4 + A^4*a^2*c^2 - 2*
(A^2*B^2*a^3 - 2*A^3*B*a^2*b + A^4*a*b^2)*c)/(a^6*b^2 - 4*a^7*c)))/(a^3*b^2
 - 4*a^4*c))*log(2*(A^4*a*c^3 + (A^3*B*a*b - A^4*b^2)*c^2 - (B^4*a^3 - 3*A*
B^3*a^2*b + 3*A^2*B^2*a*b^2 - A^3*B*b^3)*c)*x - sqrt(1/2)*(B^3*a^3*b^2 - 3*
A*B^2*a^2*b^3 + 3*A^2*B*a*b^4 - A^3*b^5 + 4*(A^2*B*a^3 - A^3*a^2*b)*c^2 - (
4*B^3*a^4 - 12*A*B^2*a^3*b + 13*A^2*B*a^2*b^2 - 5*A^3*a*b^3)*c - (B*a^4*b^3
 - A*a^3*b^4 - 8*A*a^5*c^2 - 2*(2*B*a^5*b - 3*A*a^4*b^2)*c)*sqrt((B^4*a^4 -
 4*A*B^3*a^3*b + 6*A^2*B^2*a^2*b^2 - 4*A^3*B*a*b^3 + A^4*b^4 + A^4*a^2*c^2
- 2*(A^2*B^2*a^3 - 2*A^3*B*a^2*b + A^4*a*b^2)*c)/(a^6*b^2 - 4*a^7*c)))*sqrt
(-(B^2*a^2*b - 2*A*B*a*b^2 + A^2*b^3 + (4*A*B*a^2 - 3*A^2*a*b)*c + (a^3*b^2
 - 4*a^4*c)*sqrt((B^4*a^4 - 4*A*B^3*a^3*b + 6*A^2*B^2*a^2*b^2 - 4*A^3*B*a*b
^3 + A^4*b^4 + A^4*a^2*c^2 - 2*(A^2*B^2*a^3 - 2*A^3*B*a^2*b + A^4*a*b^2)*c)
/(a^6*b^2 - 4*a^7*c)))/(a^3*b^2 - 4*a^4*c))) + sqrt(1/2)*a*x*sqrt(-(B^2*a^2
*b - 2*A*B*a*b^2 + A^2*b^3 + (4*A*B*a^2 - 3*A^2*a*b)*c - (a^3*b^2 - 4*a^4*c
)*sqrt((B^4*a^4 - 4*A*B^3*a^3*b + 6*A^2*B^2*a^2*b^2 - 4*A^3*B*a*b^3 + A^4*b
^4 + A^4*a^2*c^2 - 2*(A^2*B^2*a^3 - 2*A^3*B*a^2*b + A^4*a*b^2)*c)/(a^6*b^2
- 4*a^7*c)))/(a^3*b^2 - 4*a^4*c))*log(2*(A^4*a*c^3 + (A^3*B*a*b - A^4*b^2)*
c^2 - (B^4*a^3 - 3*A*B^3*a^2*b + 3*A^2*B^2*a*b^2 - A^3*B*b^3)*c)*x + sqrt(1
/2)*(B^3*a^3*b^2 - 3*A*B^2*a^2*b^3 + 3*A^2*B*a*b^4 - A^3*b^5 + 4*(A^2*B*a^3
```

```
 - A^3*a^2*b)*c^2 - (4*B^3*a^4 - 12*A*B^2*a^3*b + 13*A^2*B*a^2*b^2 - 5*A^3*
a*b^3)*c + (B*a^4*b^3 - A*a^3*b^4 - 8*A*a^5*c^2 - 2*(2*B*a^5*b - 3*A*a^4*b^
2)*c)*sqrt((B^4*a^4 - 4*A*B^3*a^3*b + 6*A^2*B^2*a^2*b^2 - 4*A^3*B*a*b^3 + A
^4*b^4 + A^4*a^2*c^2 - 2*(A^2*B^2*a^3 - 2*A^3*B*a^2*b + A^4*a*b^2)*c)/(a^6*
b^2 - 4*a^7*c)))*sqrt(-(B^2*a^2*b - 2*A*B*a*b^2 + A^2*b^3 + (4*A*B*a^2 - 3*
A^2*a*b)*c - (a^3*b^2 - 4*a^4*c)*sqrt((B^4*a^4 - 4*A*B^3*a^3*b + 6*A^2*B^2*
a^2*b^2 - 4*A^3*B*a*b^3 + A^4*b^4 + A^4*a^2*c^2 - 2*(A^2*B^2*a^3 - 2*A^3*B*
a^2*b + A^4*a*b^2)*c)/(a^6*b^2 - 4*a^7*c)))/(a^3*b^2 - 4*a^4*c))) - sqrt(1/
2)*a*x*sqrt(-(B^2*a^2*b - 2*A*B*a*b^2 + A^2*b^3 + (4*A*B*a^2 - 3*A^2*a*b)*c
 - (a^3*b^2 - 4*a^4*c)*sqrt((B^4*a^4 - 4*A*B^3*a^3*b + 6*A^2*B^2*a^2*b^2 -
4*A^3*B*a*b^3 + A^4*b^4 + A^4*a^2*c^2 - 2*(A^2*B^2*a^3 - 2*A^3*B*a^2*b + A^
4*a*b^2)*c)/(a^6*b^2 - 4*a^7*c)))/(a^3*b^2 - 4*a^4*c))*log(2*(A^4*a*c^3 + (
A^3*B*a*b - A^4*b^2)*c^2 - (B^4*a^3 - 3*A*B^3*a^2*b + 3*A^2*B^2*a*b^2 - A^3
*B*b^3)*c)*x - sqrt(1/2)*(B^3*a^3*b^2 - 3*A*B^2*a^2*b^3 + 3*A^2*B*a*b^4 - A
^3*b^5 + 4*(A^2*B*a^3 - A^3*a^2*b)*c^2 - (4*B^3*a^4 - 12*A*B^2*a^3*b + 13*A
^2*B*a^2*b^2 - 5*A^3*a*b^3)*c + (B*a^4*b^3 - A*a^3*b^4 - 8*A*a^5*c^2 - 2*(2
*B*a^5*b - 3*A*a^4*b^2)*c)*sqrt((B^4*a^4 - 4*A*B^3*a^3*b + 6*A^2*B^2*a^2*b^
2 - 4*A^3*B*a*b^3 + A^4*b^4 + A^4*a^2*c^2 - 2*(A^2*B^2*a^3 - 2*A^3*B*a^2*b
+ A^4*a*b^2)*c)/(a^6*b^2 - 4*a^7*c)))*sqrt(-(B^2*a^2*b - 2*A*B*a*b^2 + A^2*
b^3 + (4*A*B*a^2 - 3*A^2*a*b)*c - (a^3*b^2 - 4*a^4*c)*sqrt((B^4*a^4 - 4*A*B
^3*a^3*b + 6*A^2*B^2*a^2*b^2 - 4*A^3*B*a*b^3 + A^4*b^4 + A^4*a^2*c^2 - 2*(A
^2*B^2*a^3 - 2*A^3*B*a^2*b + A^4*a*b^2)*c)/(a^6*b^2 - 4*a^7*c)))/(a^3*b^2 -
 4*a^4*c))) - 2*A)/(a*x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A + Bx^2}{x^2 (a + bx^2 + cx^4)} \, dx = \text{Timed out}$$

```
[In] integrate((B*x**2+A)/x**2/(c*x**4+b*x**2+a),x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{A + Bx^2}{x^2 (a + bx^2 + cx^4)} \, dx = \int \frac{Bx^2 + A}{(cx^4 + bx^2 + a)x^2} \, dx$$

```
[In] integrate((B*x^2+A)/x^2/(c*x^4+b*x^2+a),x, algorithm="maxima")
[Out] -integrate((A*c*x^2 - B*a + A*b)/(c*x^4 + b*x^2 + a), x)/a - A/(a*x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2805 vs. $2(155) = 310$.

Time = 1.05 (sec) , antiderivative size = 2805, normalized size of antiderivative = 14.84

$$\int \frac{A + Bx^2}{x^2 (a + bx^2 + cx^4)} \, dx = \text{Too large to display}$$

```
[In] integrate((B*x^2+A)/x^2/(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] -A/(a*x) + 1/8*((2*b^4*c^2 - 16*a*b^2*c^3 + 32*a^2*c^4 - sqrt(2)*sqrt(b^2 -
 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^4 + 8*sqrt(2)*sqrt(b^2 - 4*a*c)*s
qrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b
*c + sqrt(b^2 - 4*a*c)*c)*b^3*c - 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + s
qrt(b^2 - 4*a*c)*c)*a^2*c^2 - 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b
^2 - 4*a*c)*c)*a*b*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*
a*c)*c)*b^2*c^2 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*
c)*a*c^3 - 2*(b^2 - 4*a*c)*b^2*c^2 + 8*(b^2 - 4*a*c)*a*c^3)*A*a^2 - 2*(sqrt
(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^5 - 8*sqrt(2)*sqrt(b*c + sqrt(b^2 -
 4*a*c)*c)*a^2*b^3*c - 2*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^4*c -
2*a*b^5*c + 16*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b*c^2 + 8*sqrt(2
)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^2 + sqrt(2)*sqrt(b*c + sqrt(b^2
 - 4*a*c)*c)*a*b^3*c^2 + 16*a^2*b^3*c^2 - 4*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4
*a*c)*c)*a^2*b*c^3 - 32*a^3*b*c^3 + 2*(b^2 - 4*a*c)*a*b^3*c - 8*(b^2 - 4*a*
c)*a^2*b*c^2)*A*abs(a) + 2*(sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^4
 - 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^2*c - 2*sqrt(2)*sqrt(b*c
 + sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c - 2*a^2*b^4*c + 16*sqrt(2)*sqrt(b*c + sqr
t(b^2 - 4*a*c)*c)*a^4*c^2 + 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b
*c^2 + sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^2 + 16*a^3*b^2*c^2
 - 4*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*c^3 - 32*a^4*c^3 + 2*(b^2
- 4*a*c)*a^2*b^2*c - 8*(b^2 - 4*a*c)*a^3*c^2)*B*abs(a) + (2*a^2*b^4*c^2 - 8
*a^3*b^2*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^
2*b^4 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^2
*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c
- sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^2 - 2
*(b^2 - 4*a*c)*a^2*b^2*c^2)*A - 2*(2*a^3*b^3*c^2 - 8*a^4*b*c^3 - sqrt(2)*sq
rt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^3 + 4*sqrt(2)*sqrt(b^
2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b*c + 2*sqrt(2)*sqrt(b^2 - 4
*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^2*c - sqrt(2)*sqrt(b^2 - 4*a*c)
*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b*c^2 - 2*(b^2 - 4*a*c)*a^3*b*c^2)*B)*
arctan(2*sqrt(1/2)*x/sqrt((a*b + sqrt(a^2*b^2 - 4*a^3*c))/(a*c)))/((a^3*b^4
 - 8*a^4*b^2*c - 2*a^3*b^3*c + 16*a^5*c^2 + 8*a^4*b*c^2 + a^3*b^2*c^2 - 4*a
^4*c^3)*abs(a)*abs(c)) + 1/8*((2*b^4*c^2 - 16*a*b^2*c^3 + 32*a^2*c^4 - sqrt
(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^4 + 8*sqrt(2)*sqrt(
b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^2*c + 2*sqrt(2)*sqrt(b^2 -
```

```
4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^3*c - 16*sqrt(2)*sqrt(b^2 - 4*a*c
)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*c^2 - 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqr
t(b*c - sqrt(b^2 - 4*a*c)*c)*a*b*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c -
 sqrt(b^2 - 4*a*c)*c)*b^2*c^2 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt
(b^2 - 4*a*c)*c)*a*c^3 - 2*(b^2 - 4*a*c)*b^2*c^2 + 8*(b^2 - 4*a*c)*a*c^3)*A
*a^2 - 2*(sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^5 - 8*sqrt(2)*sqrt(b*
c - sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c - 2*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)
*c)*a*b^4*c + 2*a*b^5*c + 16*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b*
c^2 + 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^2 + sqrt(2)*sqrt(
b*c - sqrt(b^2 - 4*a*c)*c)*a*b^3*c^2 - 16*a^2*b^3*c^2 - 4*sqrt(2)*sqrt(b*c
- sqrt(b^2 - 4*a*c)*c)*a^2*b*c^3 + 32*a^3*b*c^3 - 2*(b^2 - 4*a*c)*a*b^3*c +
 8*(b^2 - 4*a*c)*a^2*b*c^2)*A*abs(a) + 2*(sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a
*c)*c)*a^2*b^4 - 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^2*c - 2*sq
rt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c + 2*a^2*b^4*c + 16*sqrt(2)*
sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*c^2 + 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4
*a*c)*c)*a^3*b*c^2 + sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^2 -
16*a^3*b^2*c^2 - 4*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*c^3 + 32*a^4
*c^3 - 2*(b^2 - 4*a*c)*a^2*b^2*c + 8*(b^2 - 4*a*c)*a^3*c^2)*B*abs(a) + (2*a
^2*b^4*c^2 - 8*a^3*b^2*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2
- 4*a*c)*c)*a^2*b^4 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a
*c)*c)*a^3*b^2*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)
*c)*a^2*b^3*c - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a
^2*b^2*c^2 - 2*(b^2 - 4*a*c)*a^2*b^2*c^2)*A - 2*(2*a^3*b^3*c^2 - 8*a^4*b*c^
3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^3 + 4*s
qrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*b*c + 2*sqrt(2
)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^2*c - sqrt(2)*sqr
t(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b*c^2 - 2*(b^2 - 4*a*c)*
a^3*b*c^2)*B)*arctan(2*sqrt(1/2)*x/sqrt((a*b - sqrt(a^2*b^2 - 4*a^3*c))/(a*
c)))/((a^3*b^4 - 8*a^4*b^2*c - 2*a^3*b^3*c + 16*a^5*c^2 + 8*a^4*b*c^2 + a^3
*b^2*c^2 - 4*a^4*c^3)*abs(a)*abs(c))
```

Mupad [B] (verification not implemented)

Time = 0.78 (sec) , antiderivative size = 6335, normalized size of antiderivative = 33.52

$$\int \frac{A+Bx^2}{x^2\left(a+bx^2+cx^4\right)}\,dx = \text{Too large to display}$$

```
[In] int((A + B*x^2)/(x^2*(a + b*x^2 + c*x^4)),x)
```

```
[Out] - atan(((x*(4*A^2*a^4*c^4 - 4*B^2*a^5*c^3 - 2*A^2*a^3*b^2*c^3 + 4*A*B*a^4*b
*c^3) + (-(A^2*b^5 + B^2*a^2*b^3 + A^2*b^2*(-(4*a*c - b^2)^3)^(1/2) + B^2*a
^2*(-(4*a*c - b^2)^3)^(1/2) - 2*A*B*a*b^4 - 16*A*B*a^3*c^2 - 7*A^2*a*b^3*c
- A^2*a*c*(-(4*a*c - b^2)^3)^(1/2) - 4*B^2*a^3*b*c + 12*A^2*a^2*b*c^2 - 2*A
*B*a*b*(-(4*a*c - b^2)^3)^(1/2) + 12*A*B*a^2*b^2*c)/(8*(a^3*b^4 + 16*a^5*c^
```

```
2 - 8*a^4*b^2*c)))^(1/2)*(x*(32*a^6*b*c^3 - 8*a^5*b^3*c^2)*(-(A^2*b^5 + B^2
*a^2*b^3 + A^2*b^2*(-(4*a*c - b^2)^3)^(1/2) + B^2*a^2*(-(4*a*c - b^2)^3)^(1
/2) - 2*A*B*a*b^4 - 16*A*B*a^3*c^2 - 7*A^2*a*b^3*c - A^2*a*c*(-(4*a*c - b^2
)^3)^(1/2) - 4*B^2*a^3*b*c + 12*A^2*a^2*b*c^2 - 2*A*B*a*b*(-(4*a*c - b^2)^3
)^(1/2) + 12*A*B*a^2*b^2*c)/(8*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c)))^(1/2)
 - 16*B*a^6*c^3 + 16*A*a^5*b*c^3 - 4*A*a^4*b^3*c^2 + 4*B*a^5*b^2*c^2))*(-(A
^2*b^5 + B^2*a^2*b^3 + A^2*b^2*(-(4*a*c - b^2)^3)^(1/2) + B^2*a^2*(-(4*a*c
- b^2)^3)^(1/2) - 2*A*B*a*b^4 - 16*A*B*a^3*c^2 - 7*A^2*a*b^3*c - A^2*a*c*(-
(4*a*c - b^2)^3)^(1/2) - 4*B^2*a^3*b*c + 12*A^2*a^2*b*c^2 - 2*A*B*a*b*(-(4*
a*c - b^2)^3)^(1/2) + 12*A*B*a^2*b^2*c)/(8*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^
2*c)))^(1/2)*1i + (x*(4*A^2*a^4*c^4 - 4*B^2*a^5*c^3 - 2*A^2*a^3*b^2*c^3 + 4
*A*B*a^4*b*c^3) + (-(A^2*b^5 + B^2*a^2*b^3 + A^2*b^2*(-(4*a*c - b^2)^3)^(1/
2) + B^2*a^2*(-(4*a*c - b^2)^3)^(1/2) - 2*A*B*a*b^4 - 16*A*B*a^3*c^2 - 7*A^
2*a*b^3*c - A^2*a*c*(-(4*a*c - b^2)^3)^(1/2) - 4*B^2*a^3*b*c + 12*A^2*a^2*b
*c^2 - 2*A*B*a*b*(-(4*a*c - b^2)^3)^(1/2) + 12*A*B*a^2*b^2*c)/(8*(a^3*b^4 +
 16*a^5*c^2 - 8*a^4*b^2*c)))^(1/2)*(16*B*a^6*c^3 + x*(32*a^6*b*c^3 - 8*a^5*
b^3*c^2)*(-(A^2*b^5 + B^2*a^2*b^3 + A^2*b^2*(-(4*a*c - b^2)^3)^(1/2) + B^2*
a^2*(-(4*a*c - b^2)^3)^(1/2) - 2*A*B*a*b^4 - 16*A*B*a^3*c^2 - 7*A^2*a*b^3*c
 - A^2*a*c*(-(4*a*c - b^2)^3)^(1/2) - 4*B^2*a^3*b*c + 12*A^2*a^2*b*c^2 - 2*
A*B*a*b*(-(4*a*c - b^2)^3)^(1/2) + 12*A*B*a^2*b^2*c)/(8*(a^3*b^4 + 16*a^5*c
^2 - 8*a^4*b^2*c)))^(1/2) - 16*A*a^5*b*c^3 + 4*A*a^4*b^3*c^2 - 4*B*a^5*b^2*
c^2))*(-(A^2*b^5 + B^2*a^2*b^3 + A^2*b^2*(-(4*a*c - b^2)^3)^(1/2) + B^2*a^2
*(-(4*a*c - b^2)^3)^(1/2) - 2*A*B*a*b^4 - 16*A*B*a^3*c^2 - 7*A^2*a*b^3*c -
A^2*a*c*(-(4*a*c - b^2)^3)^(1/2) - 4*B^2*a^3*b*c + 12*A^2*a^2*b*c^2 - 2*A*B
*a*b*(-(4*a*c - b^2)^3)^(1/2) + 12*A*B*a^2*b^2*c)/(8*(a^3*b^4 + 16*a^5*c^2
- 8*a^4*b^2*c)))^(1/2)*1i)/((x*(4*A^2*a^4*c^4 - 4*B^2*a^5*c^3 - 2*A^2*a^3*b
^2*c^3 + 4*A*B*a^4*b*c^3) + (-(A^2*b^5 + B^2*a^2*b^3 + A^2*b^2*(-(4*a*c - b
^2)^3)^(1/2) + B^2*a^2*(-(4*a*c - b^2)^3)^(1/2) - 2*A*B*a*b^4 - 16*A*B*a^3*
c^2 - 7*A^2*a*b^3*c - A^2*a*c*(-(4*a*c - b^2)^3)^(1/2) - 4*B^2*a^3*b*c + 12
*A^2*a^2*b*c^2 - 2*A*B*a*b*(-(4*a*c - b^2)^3)^(1/2) + 12*A*B*a^2*b^2*c)/(8*
(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c)))^(1/2)*(x*(32*a^6*b*c^3 - 8*a^5*b^3*c
^2)*(-(A^2*b^5 + B^2*a^2*b^3 + A^2*b^2*(-(4*a*c - b^2)^3)^(1/2) + B^2*a^2*(
-(4*a*c - b^2)^3)^(1/2) - 2*A*B*a*b^4 - 16*A*B*a^3*c^2 - 7*A^2*a*b^3*c - A^
2*a*c*(-(4*a*c - b^2)^3)^(1/2) - 4*B^2*a^3*b*c + 12*A^2*a^2*b*c^2 - 2*A*B*a
*b*(-(4*a*c - b^2)^3)^(1/2) + 12*A*B*a^2*b^2*c)/(8*(a^3*b^4 + 16*a^5*c^2 -
8*a^4*b^2*c)))^(1/2) - 16*B*a^6*c^3 + 16*A*a^5*b*c^3 - 4*A*a^4*b^3*c^2 + 4*
B*a^5*b^2*c^2))*(-(A^2*b^5 + B^2*a^2*b^3 + A^2*b^2*(-(4*a*c - b^2)^3)^(1/2)
 + B^2*a^2*(-(4*a*c - b^2)^3)^(1/2) - 2*A*B*a*b^4 - 16*A*B*a^3*c^2 - 7*A^2*
a*b^3*c - A^2*a*c*(-(4*a*c - b^2)^3)^(1/2) - 4*B^2*a^3*b*c + 12*A^2*a^2*b*c
^2 - 2*A*B*a*b*(-(4*a*c - b^2)^3)^(1/2) + 12*A*B*a^2*b^2*c)/(8*(a^3*b^4 + 1
6*a^5*c^2 - 8*a^4*b^2*c)))^(1/2) - (x*(4*A^2*a^4*c^4 - 4*B^2*a^5*c^3 - 2*A^
2*a^3*b^2*c^3 + 4*A*B*a^4*b*c^3) + (-(A^2*b^5 + B^2*a^2*b^3 + A^2*b^2*(-(4*
a*c - b^2)^3)^(1/2) + B^2*a^2*(-(4*a*c - b^2)^3)^(1/2) - 2*A*B*a*b^4 - 16*A
*B*a^3*c^2 - 7*A^2*a*b^3*c - A^2*a*c*(-(4*a*c - b^2)^3)^(1/2) - 4*B^2*a^3*b
*c + 12*A^2*a^2*b*c^2 - 2*A*B*a*b*(-(4*a*c - b^2)^3)^(1/2) + 12*A*B*a^2*b^2
```

```
*c)/(8*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c)))^(1/2)*(16*B*a^6*c^3 + x*(32*a
^6*b*c^3 - 8*a^5*b^3*c^2)*(-(A^2*b^5 + B^2*a^2*b^3 + A^2*b^2*(-(4*a*c - b^2
)^3)^(1/2) + B^2*a^2*(-(4*a*c - b^2)^3)^(1/2) - 2*A*B*a*b^4 - 16*A*B*a^3*c^
2 - 7*A^2*a*b^3*c - A^2*a*c*(-(4*a*c - b^2)^3)^(1/2) - 4*B^2*a^3*b*c + 12*A
^2*a^2*b*c^2 - 2*A*B*a*b*(-(4*a*c - b^2)^3)^(1/2) + 12*A*B*a^2*b^2*c)/(8*(a
^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c)))^(1/2) - 16*A*a^5*b*c^3 + 4*A*a^4*b^3*c
^2 - 4*B*a^5*b^2*c^2))*(-(A^2*b^5 + B^2*a^2*b^3 + A^2*b^2*(-(4*a*c - b^2)^3
)^(1/2) + B^2*a^2*(-(4*a*c - b^2)^3)^(1/2) - 2*A*B*a*b^4 - 16*A*B*a^3*c^2 -
 7*A^2*a*b^3*c - A^2*a*c*(-(4*a*c - b^2)^3)^(1/2) - 4*B^2*a^3*b*c + 12*A^2*
a^2*b*c^2 - 2*A*B*a*b*(-(4*a*c - b^2)^3)^(1/2) + 12*A*B*a^2*b^2*c)/(8*(a^3*
b^4 + 16*a^5*c^2 - 8*a^4*b^2*c)))^(1/2) + 2*A^3*a^3*c^4 + 2*A*B^2*a^4*c^3 -
 2*A^2*B*a^3*b*c^3))*(-(A^2*b^5 + B^2*a^2*b^3 + A^2*b^2*(-(4*a*c - b^2)^3)^
(1/2) + B^2*a^2*(-(4*a*c - b^2)^3)^(1/2) - 2*A*B*a*b^4 - 16*A*B*a^3*c^2 - 7
*A^2*a*b^3*c - A^2*a*c*(-(4*a*c - b^2)^3)^(1/2) - 4*B^2*a^3*b*c + 12*A^2*a^
2*b*c^2 - 2*A*B*a*b*(-(4*a*c - b^2)^3)^(1/2) + 12*A*B*a^2*b^2*c)/(8*(a^3*b^
4 + 16*a^5*c^2 - 8*a^4*b^2*c)))^(1/2)*2i - atan(((x*(4*A^2*a^4*c^4 - 4*B^2*
a^5*c^3 - 2*A^2*a^3*b^2*c^3 + 4*A*B*a^4*b*c^3) + (-(A^2*b^5 + B^2*a^2*b^3 -
 A^2*b^2*(-(4*a*c - b^2)^3)^(1/2) - B^2*a^2*(-(4*a*c - b^2)^3)^(1/2) - 2*A*
B*a*b^4 - 16*A*B*a^3*c^2 - 7*A^2*a*b^3*c + A^2*a*c*(-(4*a*c - b^2)^3)^(1/2)
 - 4*B^2*a^3*b*c + 12*A^2*a^2*b*c^2 + 2*A*B*a*b*(-(4*a*c - b^2)^3)^(1/2) +
12*A*B*a^2*b^2*c)/(8*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c)))^(1/2)*(x*(32*a^
6*b*c^3 - 8*a^5*b^3*c^2)*(-(A^2*b^5 + B^2*a^2*b^3 - A^2*b^2*(-(4*a*c - b^2)
^3)^(1/2) - B^2*a^2*(-(4*a*c - b^2)^3)^(1/2) - 2*A*B*a*b^4 - 16*A*B*a^3*c^2
 - 7*A^2*a*b^3*c + A^2*a*c*(-(4*a*c - b^2)^3)^(1/2) - 4*B^2*a^3*b*c + 12*A^
2*a^2*b*c^2 + 2*A*B*a*b*(-(4*a*c - b^2)^3)^(1/2) + 12*A*B*a^2*b^2*c)/(8*(a^
3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c)))^(1/2) - 16*B*a^6*c^3 + 16*A*a^5*b*c^3 -
 4*A*a^4*b^3*c^2 + 4*B*a^5*b^2*c^2))*(-(A^2*b^5 + B^2*a^2*b^3 - A^2*b^2*(-(
4*a*c - b^2)^3)^(1/2) - B^2*a^2*(-(4*a*c - b^2)^3)^(1/2) - 2*A*B*a*b^4 - 16
*A*B*a^3*c^2 - 7*A^2*a*b^3*c + A^2*a*c*(-(4*a*c - b^2)^3)^(1/2) - 4*B^2*a^3
*b*c + 12*A^2*a^2*b*c^2 + 2*A*B*a*b*(-(4*a*c - b^2)^3)^(1/2) + 12*A*B*a^2*b
^2*c)/(8*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c)))^(1/2)*1i + (x*(4*A^2*a^4*c^
4 - 4*B^2*a^5*c^3 - 2*A^2*a^3*b^2*c^3 + 4*A*B*a^4*b*c^3) + (-(A^2*b^5 + B^2
*a^2*b^3 - A^2*b^2*(-(4*a*c - b^2)^3)^(1/2) - B^2*a^2*(-(4*a*c - b^2)^3)^(1
/2) - 2*A*B*a*b^4 - 16*A*B*a^3*c^2 - 7*A^2*a*b^3*c + A^2*a*c*(-(4*a*c - b^2
)^3)^(1/2) - 4*B^2*a^3*b*c + 12*A^2*a^2*b*c^2 + 2*A*B*a*b*(-(4*a*c - b^2)^3
)^(1/2) + 12*A*B*a^2*b^2*c)/(8*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c)))^(1/2)
*(16*B*a^6*c^3 + x*(32*a^6*b*c^3 - 8*a^5*b^3*c^2)*(-(A^2*b^5 + B^2*a^2*b^3
- A^2*b^2*(-(4*a*c - b^2)^3)^(1/2) - B^2*a^2*(-(4*a*c - b^2)^3)^(1/2) - 2*A
*B*a*b^4 - 16*A*B*a^3*c^2 - 7*A^2*a*b^3*c + A^2*a*c*(-(4*a*c - b^2)^3)^(1/2
) - 4*B^2*a^3*b*c + 12*A^2*a^2*b*c^2 + 2*A*B*a*b*(-(4*a*c - b^2)^3)^(1/2) +
 12*A*B*a^2*b^2*c)/(8*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c)))^(1/2) - 16*A*a
^5*b*c^3 + 4*A*a^4*b^3*c^2 - 4*B*a^5*b^2*c^2))*(-(A^2*b^5 + B^2*a^2*b^3 - A
^2*b^2*(-(4*a*c - b^2)^3)^(1/2) - B^2*a^2*(-(4*a*c - b^2)^3)^(1/2) - 2*A*B*
a*b^4 - 16*A*B*a^3*c^2 - 7*A^2*a*b^3*c + A^2*a*c*(-(4*a*c - b^2)^3)^(1/2) -
 4*B^2*a^3*b*c + 12*A^2*a^2*b*c^2 + 2*A*B*a*b*(-(4*a*c - b^2)^3)^(1/2) + 12
```

```
*A*B*a^2*b^2*c)/(8*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c)))^(1/2)*1i)/((x*(4*
A^2*a^4*c^4 - 4*B^2*a^5*c^3 - 2*A^2*a^3*b^2*c^3 + 4*A*B*a^4*b*c^3) + (-(A^2
*b^5 + B^2*a^2*b^3 - A^2*b^2*(-(4*a*c - b^2)^3)^(1/2) - B^2*a^2*(-(4*a*c -
b^2)^3)^(1/2) - 2*A*B*a*b^4 - 16*A*B*a^3*c^2 - 7*A^2*a*b^3*c + A^2*a*c*(-(4
*a*c - b^2)^3)^(1/2) - 4*B^2*a^3*b*c + 12*A^2*a^2*b*c^2 + 2*A*B*a*b*(-(4*a*
c - b^2)^3)^(1/2) + 12*A*B*a^2*b^2*c)/(8*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*
c)))^(1/2)*(x*(32*a^6*b*c^3 - 8*a^5*b^3*c^2)*(-(A^2*b^5 + B^2*a^2*b^3 - A^2
*b^2*(-(4*a*c - b^2)^3)^(1/2) - B^2*a^2*(-(4*a*c - b^2)^3)^(1/2) - 2*A*B*a*
b^4 - 16*A*B*a^3*c^2 - 7*A^2*a*b^3*c + A^2*a*c*(-(4*a*c - b^2)^3)^(1/2) - 4
*B^2*a^3*b*c + 12*A^2*a^2*b*c^2 + 2*A*B*a*b*(-(4*a*c - b^2)^3)^(1/2) + 12*A
*B*a^2*b^2*c)/(8*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c)))^(1/2) - 16*B*a^6*c^
3 + 16*A*a^5*b*c^3 - 4*A*a^4*b^3*c^2 + 4*B*a^5*b^2*c^2))*(-(A^2*b^5 + B^2*a
^2*b^3 - A^2*b^2*(-(4*a*c - b^2)^3)^(1/2) - B^2*a^2*(-(4*a*c - b^2)^3)^(1/2
) - 2*A*B*a*b^4 - 16*A*B*a^3*c^2 - 7*A^2*a*b^3*c + A^2*a*c*(-(4*a*c - b^2)^
3)^(1/2) - 4*B^2*a^3*b*c + 12*A^2*a^2*b*c^2 + 2*A*B*a*b*(-(4*a*c - b^2)^3)^
(1/2) + 12*A*B*a^2*b^2*c)/(8*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c)))^(1/2) -
 (x*(4*A^2*a^4*c^4 - 4*B^2*a^5*c^3 - 2*A^2*a^3*b^2*c^3 + 4*A*B*a^4*b*c^3) +
 (-(A^2*b^5 + B^2*a^2*b^3 - A^2*b^2*(-(4*a*c - b^2)^3)^(1/2) - B^2*a^2*(-(4
*a*c - b^2)^3)^(1/2) - 2*A*B*a*b^4 - 16*A*B*a^3*c^2 - 7*A^2*a*b^3*c + A^2*a
*c*(-(4*a*c - b^2)^3)^(1/2) - 4*B^2*a^3*b*c + 12*A^2*a^2*b*c^2 + 2*A*B*a*b*
(-(4*a*c - b^2)^3)^(1/2) + 12*A*B*a^2*b^2*c)/(8*(a^3*b^4 + 16*a^5*c^2 - 8*a
^4*b^2*c)))^(1/2)*(16*B*a^6*c^3 + x*(32*a^6*b*c^3 - 8*a^5*b^3*c^2)*(-(A^2*b
^5 + B^2*a^2*b^3 - A^2*b^2*(-(4*a*c - b^2)^3)^(1/2) - B^2*a^2*(-(4*a*c - b^
2)^3)^(1/2) - 2*A*B*a*b^4 - 16*A*B*a^3*c^2 - 7*A^2*a*b^3*c + A^2*a*c*(-(4*a
*c - b^2)^3)^(1/2) - 4*B^2*a^3*b*c + 12*A^2*a^2*b*c^2 + 2*A*B*a*b*(-(4*a*c
- b^2)^3)^(1/2) + 12*A*B*a^2*b^2*c)/(8*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c)
))^(1/2) - 16*A*a^5*b*c^3 + 4*A*a^4*b^3*c^2 - 4*B*a^5*b^2*c^2))*(-(A^2*b^5
+ B^2*a^2*b^3 - A^2*b^2*(-(4*a*c - b^2)^3)^(1/2) - B^2*a^2*(-(4*a*c - b^2)^
3)^(1/2) - 2*A*B*a*b^4 - 16*A*B*a^3*c^2 - 7*A^2*a*b^3*c + A^2*a*c*(-(4*a*c
- b^2)^3)^(1/2) - 4*B^2*a^3*b*c + 12*A^2*a^2*b*c^2 + 2*A*B*a*b*(-(4*a*c - b
^2)^3)^(1/2) + 12*A*B*a^2*b^2*c)/(8*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c)))^
(1/2) + 2*A^3*a^3*c^4 + 2*A*B^2*a^4*c^3 - 2*A^2*B*a^3*b*c^3))*(-(A^2*b^5 +
B^2*a^2*b^3 - A^2*b^2*(-(4*a*c - b^2)^3)^(1/2) - B^2*a^2*(-(4*a*c - b^2)^3)
^(1/2) - 2*A*B*a*b^4 - 16*A*B*a^3*c^2 - 7*A^2*a*b^3*c + A^2*a*c*(-(4*a*c -
b^2)^3)^(1/2) - 4*B^2*a^3*b*c + 12*A^2*a^2*b*c^2 + 2*A*B*a*b*(-(4*a*c - b^2
)^3)^(1/2) + 12*A*B*a^2*b^2*c)/(8*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c)))^(1
/2)*2i - A/(a*x)
```

3.111 $\int \frac{A+Bx^2}{x^4(a+bx^2+cx^4)}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 271

$$\begin{aligned}
&\int \frac{A+Bx^2}{x^4\left(a+bx^2+cx^4\right)}\,dx \\
&= -\frac{A}{3ax^3}+\frac{Ab-aB}{a^2x} \\
&\quad -\frac{\sqrt{c}\left(aB\left(b+\sqrt{b^2-4ac}\right)-A\left(b^2-2ac+b\sqrt{b^2-4ac}\right)\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}a^2\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}} \\
&\quad +\frac{\sqrt{c}\left(aB\left(b-\sqrt{b^2-4ac}\right)-A\left(b^2-2ac-b\sqrt{b^2-4ac}\right)\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{2}a^2\sqrt{b^2-4ac}\sqrt{b+\sqrt{b^2-4ac}}}
\end{aligned}$$

```
[Out] -1/3*A/a/x^3+(A*b-B*a)/a^2/x-1/2*arctan(x*2^(1/2)*c^(1/2)/(b-(-4*a*c+b^2)^(
1/2))^(1/2))*c^(1/2)*(a*B*(b+(-4*a*c+b^2)^(1/2))-A*(b^2-2*a*c+b*(-4*a*c+b^2
)^(1/2)))/a^2*2^(1/2)/(-4*a*c+b^2)^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2)+1/2*a
rctan(x*2^(1/2)*c^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2))*c^(1/2)*(a*B*(b-(-4*a
*c+b^2)^(1/2))-A*(b^2-2*a*c-b*(-4*a*c+b^2)^(1/2)))/a^2*2^(1/2)/(-4*a*c+b^2)
^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2)
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 271, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {1295, 1180, 211}

$$\begin{aligned}\int \frac{A+Bx^2}{x^4\left(a+bx^2+cx^4\right)}\,dx &= -\frac{\sqrt{c}\left(aB\left(\sqrt{b^2-4ac}+b\right)-A\left(b\sqrt{b^2-4ac}-2ac+b^2\right)\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}a^2\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}}\\&\quad+\frac{\sqrt{c}\left(aB\left(b-\sqrt{b^2-4ac}\right)-A\left(-b\sqrt{b^2-4ac}-2ac+b^2\right)\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}a^2\sqrt{b^2-4ac}\sqrt{\sqrt{b^2-4ac}+b}}\\&\quad+\frac{Ab-aB}{a^2x}-\frac{A}{3ax^3}\end{aligned}$$

```
[In] Int[(A + B*x^2)/(x^4*(a + b*x^2 + c*x^4)),x]
```

```
[Out] -1/3*A/(a*x^3) + (A*b - a*B)/(a^2*x) - (Sqrt[c]*(a*B*(b + Sqrt[b^2 - 4*a*c]
) - A*(b^2 - 2*a*c + b*Sqrt[b^2 - 4*a*c]))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[
b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*a^2*Sqrt[b^2 - 4*a*c]*Sqrt[b - Sqrt[b^2 -
 4*a*c]]) + (Sqrt[c]*(a*B*(b - Sqrt[b^2 - 4*a*c]) - A*(b^2 - 2*a*c - b*Sqrt
[b^2 - 4*a*c]))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(S
qrt[2]*a^2*Sqrt[b^2 - 4*a*c]*Sqrt[b + Sqrt[b^2 - 4*a*c]])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1295

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[d*(f*x)^(m + 1)*((a + b*x^2 + c*x^4)^(p + 1)
/(a*f*(m + 1))), x] + Dist[1/(a*f^2*(m + 1)), Int[(f*x)^(m + 2)*(a + b*x^2
+ c*x^4)^p*Simp[a*e*(m + 1) - b*d*(m + 2*p + 3) - c*d*(m + 4*p + 5)*x^2, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[m
, -1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{A}{3ax^3} - \frac{\int \frac{3(Ab-aB)+3Acx^2}{x^2(a+bx^2+cx^4)}\,dx}{3a} \\
&= -\frac{A}{3ax^3} + \frac{Ab-aB}{a^2x} + \frac{\int \frac{3(Ab^2-abB-aAc)+3(Ab-aB)cx^2}{a+bx^2+cx^4}\,dx}{3a^2} \\
&= -\frac{A}{3ax^3} + \frac{Ab-aB}{a^2x} \\
&\quad + \frac{\left(c\left(aB\left(b-\sqrt{b^2-4ac}\right)-A\left(b^2-2ac-b\sqrt{b^2-4ac}\right)\right)\right)\int \frac{1}{\frac{b}{2}+\frac{1}{2}\sqrt{b^2-4ac}+cx^2}\,dx}{2a^2\sqrt{b^2-4ac}} \\
&\quad - \frac{\left(c\left(aB\left(b+\sqrt{b^2-4ac}\right)-A\left(b^2-2ac+b\sqrt{b^2-4ac}\right)\right)\right)\int \frac{1}{\frac{b}{2}-\frac{1}{2}\sqrt{b^2-4ac}+cx^2}\,dx}{2a^2\sqrt{b^2-4ac}} \\
&= -\frac{A}{3ax^3} + \frac{Ab-aB}{a^2x} \\
&\quad - \frac{\sqrt{c}\left(aB\left(b+\sqrt{b^2-4ac}\right)-A\left(b^2-2ac+b\sqrt{b^2-4ac}\right)\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}a^2\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}} \\
&\quad + \frac{\sqrt{c}\left(aB\left(b-\sqrt{b^2-4ac}\right)-A\left(b^2-2ac-b\sqrt{b^2-4ac}\right)\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{2}a^2\sqrt{b^2-4ac}\sqrt{b+\sqrt{b^2-4ac}}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 267, normalized size of antiderivative = 0.99

$$
\int \frac{A+Bx^2}{x^4\left(a+bx^2+cx^4\right)}\,dx
= \frac{-\frac{2aA}{x^3}+\frac{6Ab-6aB}{x}-\frac{3\sqrt{2}\sqrt{c}\left(aB\left(b+\sqrt{b^2-4ac}\right)-A\left(b^2-2ac+b\sqrt{b^2-4ac}\right)\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{3\sqrt{2}\sqrt{c}\left(aB\left(b-\sqrt{b^2-4ac}\right)+A\left(-b\right.\right.}{\sqrt{b^2-4}}}{6a^2}
$$

```
[In] Integrate[(A + B*x^2)/(x^4*(a + b*x^2 + c*x^4)),x]
```

```
[Out] ((-2*a*A)/x^3 + (6*A*b - 6*a*B)/x - (3*Sqrt[2]*Sqrt[c]*(a*B*(b + Sqrt[b^2 -
 4*a*c]) - A*(b^2 - 2*a*c + b*Sqrt[b^2 - 4*a*c]))*ArcTan[(Sqrt[2]*Sqrt[c]*x
)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[b^2 - 4*a*c]*Sqrt[b - Sqrt[b^2 - 4*a*
c]]) + (3*Sqrt[2]*Sqrt[c]*(a*B*(b - Sqrt[b^2 - 4*a*c]) + A*(-b^2 + 2*a*c +
b*Sqrt[b^2 - 4*a*c]))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]
]])/(Sqrt[b^2 - 4*a*c]*Sqrt[b + Sqrt[b^2 - 4*a*c]]))/(6*a^2)
```

Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 232, normalized size of antiderivative = 0.86

method	result
default	$-\frac{A}{3a\,x^{3}}-\frac{-Ab+Ba}{x\,a^{2}}+\frac{4c\left(\frac{\left(Ab\sqrt{-4ac+b^{2}}+2Aac-A\,b^{2}-Ba\sqrt{-4ac+b^{2}}+abB\right)\sqrt{2}\,\arctan\left(\frac{cx\sqrt{2}}{\sqrt{\left(b+\sqrt{-4ac+b^{2}}\right)c}}\right)}{8\sqrt{-4ac+b^{2}}\,\sqrt{\left(b+\sqrt{-4ac+b^{2}}\right)c}}-\right.}{a^{2}}$ $\left(Ab\sqrt{-4ac+b^{2}}-2Aac+A\right.$
risch	$\frac{\frac{(Ab-Ba)x^{2}}{a^{2}}-\frac{A}{3a}}{x^{3}}+\left(\_R=\mathrm{RootOf}\left(\left(16a^{7}c^{2}-8b^{2}c\,a^{6}+b^{4}a^{5}\right)\_Z^{4}+\left(-20A^{2}a^{3}b\,c^{3}+25A^{2}a^{2}b^{3}c^{2}-9A^{2}a\,b^{5}c+A^{2}b^{7}+16AB\,a^{4}c^{3}-36AB\,a^{3}b^{2}c^{2}+1\ldots\right.\right.\right.$

```
[In] int((B*x^2+A)/x^4/(c*x^4+b*x^2+a),x,method=_RETURNVERBOSE)
```

```
[Out] -1/3*A/a/x^3-(-A*b+B*a)/x/a^2+4/a^2*c*(1/8*(A*b*(-4*a*c+b^2)^(1/2)+2*A*a*c-
A*b^2-B*a*(-4*a*c+b^2)^(1/2)+a*b*B)/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+
b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))-1
/8*(A*b*(-4*a*c+b^2)^(1/2)-2*A*a*c+A*b^2-B*a*(-4*a*c+b^2)^(1/2)-a*b*B)/(-4*
a*c+b^2)^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(c*x*2^(1/2
)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 5442 vs. $2(227) = 454$.

Time = 2.89 (sec) , antiderivative size = 5442, normalized size of antiderivative = 20.08

$$\int \frac{A+Bx^2}{x^4\left(a+bx^2+cx^4\right)}\,dx = \text{Too large to display}$$

```
[In] integrate((B*x^2+A)/x^4/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A+Bx^2}{x^4\left(a+bx^2+cx^4\right)}\,dx = \text{Timed out}$$

```
[In] integrate((B*x**2+A)/x**4/(c*x**4+b*x**2+a),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{A+Bx^2}{x^4\left(a+bx^2+cx^4\right)}\,dx=\int \frac{Bx^2+A}{(cx^4+bx^2+a)x^4}\,dx$$

```
[In] integrate((B*x^2+A)/x^4/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

```
[Out] integrate(-((B*a - A*b)*c*x^2 + B*a*b - A*b^2 + A*a*c)/(c*x^4 + b*x^2 + a),
 x)/a^2 - 1/3*(3*(B*a - A*b)*x^2 + A*a)/(a^2*x^3)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2870 vs. $2(227) = 454$.

Time = 0.91 (sec) , antiderivative size = 2870, normalized size of antiderivative = 10.59

$$\int \frac{A+Bx^2}{x^4\left(a+bx^2+cx^4\right)}\,dx=\text{Too large to display}$$

```
[In] integrate((B*x^2+A)/x^4/(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] 1/4*((sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^6 - 9*sqrt(2)*sqrt(b*c + sq
rt(b^2 - 4*a*c)*c)*a*b^4*c - 2*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^5*
c - 2*b^6*c + 24*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^2 + 10*s
qrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c^2 + sqrt(2)*sqrt(b*c + sqrt(
b^2 - 4*a*c)*c)*b^4*c^2 + 18*a*b^4*c^2 + 2*b^5*c^2 - 16*sqrt(2)*sqrt(b*c +
sqrt(b^2 - 4*a*c)*c)*a^3*c^3 - 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^
2*b*c^3 - 5*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^3 - 48*a^2*b^2*
c^3 - 14*a*b^3*c^3 + 4*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*c^4 + 32
*a^3*c^4 + 24*a^2*b*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4
*a*c)*c)*b^5 + 7*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*
a*b^3*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^4*c
 - 12*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b*c^2 -
 6*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^2 - sq
rt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^3*c^2 + 3*sqrt(2)
*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c^3 + 2*(b^2 - 4*a*c
)*b^4*c - 10*(b^2 - 4*a*c)*a*b^2*c^2 - 2*(b^2 - 4*a*c)*b^3*c^2 + 8*(b^2 - 4
*a*c)*a^2*c^3 + 6*(b^2 - 4*a*c)*a*b*c^3)*A - (sqrt(2)*sqrt(b*c + sqrt(b^2 -
 4*a*c)*c)*a*b^5 - 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c - 2*
sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^4*c - 2*a*b^5*c + 16*sqrt(2)*sq
rt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b*c^2 + 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4
*a*c)*c)*a^2*b^2*c^2 + sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c^2 +
16*a^2*b^3*c^2 + 2*a*b^4*c^2 - 4*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^
2*b*c^3 - 32*a^3*b*c^3 - 12*a^2*b^2*c^3 + 16*a^3*c^4 - sqrt(2)*sqrt(b^2 - 4
*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^4 + 6*sqrt(2)*sqrt(b^2 - 4*a*c)*s
```

qrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt
(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c - 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c
+ sqrt(b^2 - 4*a*c)*c)*a^3*c^2 - 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqr
t(b^2 - 4*a*c)*c)*a^2*b*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2
 - 4*a*c)*c)*a*b^2*c^2 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 -
4*a*c)*c)*a^2*c^3 + 2*(b^2 - 4*a*c)*a*b^3*c - 8*(b^2 - 4*a*c)*a^2*b*c^2 - 2
*(b^2 - 4*a*c)*a*b^2*c^2 + 4*(b^2 - 4*a*c)*a^2*c^3)*B)*arctan(2*sqrt(1/2)*x
/sqrt((a^2*b + sqrt(a^4*b^2 - 4*a^5*c))/(a^2*c)))/((a^3*b^4 - 8*a^4*b^2*c -
 2*a^3*b^3*c + 16*a^5*c^2 + 8*a^4*b*c^2 + a^3*b^2*c^2 - 4*a^4*c^3)*abs(c))
+ 1/4*((sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^6 - 9*sqrt(2)*sqrt(b*c -
sqrt(b^2 - 4*a*c)*c)*a*b^4*c - 2*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^
5*c + 2*b^6*c + 24*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^2 + 10
*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^3*c^2 + sqrt(2)*sqrt(b*c - sqr
t(b^2 - 4*a*c)*c)*b^4*c^2 - 18*a*b^4*c^2 - 2*b^5*c^2 - 16*sqrt(2)*sqrt(b*c
- sqrt(b^2 - 4*a*c)*c)*a^3*c^3 - 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*
a^2*b*c^3 - 5*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^2*c^3 + 48*a^2*b^
2*c^3 + 14*a*b^3*c^3 + 4*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*c^4 -
32*a^3*c^4 - 24*a^2*b*c^4 + sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 -
 4*a*c)*c)*b^5 - 7*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c
)*a*b^3*c - 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^4
*c + 12*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b*c^2
 + 6*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^2*c^2 +
sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^3*c^2 - 3*sqrt(
2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b*c^3 - 2*(b^2 - 4*a
*c)*b^4*c + 10*(b^2 - 4*a*c)*a*b^2*c^2 + 2*(b^2 - 4*a*c)*b^3*c^2 - 8*(b^2 -
 4*a*c)*a^2*c^3 - 6*(b^2 - 4*a*c)*a*b*c^3)*A - (sqrt(2)*sqrt(b*c - sqrt(b^2
 - 4*a*c)*c)*a*b^5 - 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c -
2*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^4*c + 2*a*b^5*c + 16*sqrt(2)*
sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b*c^2 + 8*sqrt(2)*sqrt(b*c - sqrt(b^2 -
 4*a*c)*c)*a^2*b^2*c^2 + sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^3*c^2
- 16*a^2*b^3*c^2 - 2*a*b^4*c^2 - 4*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*
a^2*b*c^3 + 32*a^3*b*c^3 + 12*a^2*b^2*c^3 - 16*a^3*c^4 + sqrt(2)*sqrt(b^2 -
 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^4 - 6*sqrt(2)*sqrt(b^2 - 4*a*c)
*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c - 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sq
rt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^3*c + 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*
c - sqrt(b^2 - 4*a*c)*c)*a^3*c^2 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - s
qrt(b^2 - 4*a*c)*c)*a^2*b*c^2 + sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b
^2 - 4*a*c)*c)*a*b^2*c^2 - 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2
- 4*a*c)*c)*a^2*c^3 - 2*(b^2 - 4*a*c)*a*b^3*c + 8*(b^2 - 4*a*c)*a^2*b*c^2 +
 2*(b^2 - 4*a*c)*a*b^2*c^2 - 4*(b^2 - 4*a*c)*a^2*c^3)*B)*arctan(2*sqrt(1/2)
*x/sqrt((a^2*b - sqrt(a^4*b^2 - 4*a^5*c))/(a^2*c)))/((a^3*b^4 - 8*a^4*b^2*c
 - 2*a^3*b^3*c + 16*a^5*c^2 + 8*a^4*b*c^2 + a^3*b^2*c^2 - 4*a^4*c^3)*abs(c)
) - 1/3*(3*B*a*x^2 - 3*A*b*x^2 + A*a)/(a^2*x^3)

Mupad [B] (verification not implemented)

Time = 8.69 (sec) , antiderivative size = 10101, normalized size of antiderivative = 37.27

$$\int \frac{A+Bx^2}{x^4\left(a+bx^2+cx^4\right)}\,dx=\text{Too large to display}$$

```
[In] int((A + B*x^2)/(x^4*(a + b*x^2 + c*x^4)),x)
```

```
[Out] - (A/(3*a) - (x^2*(A*b - B*a))/a^2)/x^3 - atan((((-(A^2*b^7 + B^2*a^2*b^5 +
 A^2*b^4*(-(4*a*c - b^2)^3)^(1/2) - 2*A*B*a*b^6 + 25*A^2*a^2*b^3*c^2 + A^2*
a^2*c^2*(-(4*a*c - b^2)^3)^(1/2) + B^2*a^2*b^2*(-(4*a*c - b^2)^3)^(1/2) + 1
6*A*B*a^4*c^3 - 9*A^2*a*b^5*c - 20*A^2*a^3*b*c^3 - 7*B^2*a^3*b^3*c + 12*B^2
*a^4*b*c^2 - B^2*a^3*c*(-(4*a*c - b^2)^3)^(1/2) - 36*A*B*a^3*b^2*c^2 - 3*A^
2*a*b^2*c*(-(4*a*c - b^2)^3)^(1/2) - 2*A*B*a*b^3*(-(4*a*c - b^2)^3)^(1/2) +
 16*A*B*a^2*b^4*c + 4*A*B*a^2*b*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^5*b^4 + 1
6*a^7*c^2 - 8*a^6*b^2*c)))^(1/2)*(16*A*a^10*c^4 + x*(32*a^11*b*c^3 - 8*a^10
*b^3*c^2)*(-(A^2*b^7 + B^2*a^2*b^5 + A^2*b^4*(-(4*a*c - b^2)^3)^(1/2) - 2*A
*B*a*b^6 + 25*A^2*a^2*b^3*c^2 + A^2*a^2*c^2*(-(4*a*c - b^2)^3)^(1/2) + B^2*
a^2*b^2*(-(4*a*c - b^2)^3)^(1/2) + 16*A*B*a^4*c^3 - 9*A^2*a*b^5*c - 20*A^2*
a^3*b*c^3 - 7*B^2*a^3*b^3*c + 12*B^2*a^4*b*c^2 - B^2*a^3*c*(-(4*a*c - b^2)^
3)^(1/2) - 36*A*B*a^3*b^2*c^2 - 3*A^2*a*b^2*c*(-(4*a*c - b^2)^3)^(1/2) - 2*
A*B*a*b^3*(-(4*a*c - b^2)^3)^(1/2) + 16*A*B*a^2*b^4*c + 4*A*B*a^2*b*c*(-(4*
a*c - b^2)^3)^(1/2))/(8*(a^5*b^4 + 16*a^7*c^2 - 8*a^6*b^2*c)))^(1/2) + 16*B
*a^10*b*c^3 + 4*A*a^8*b^4*c^2 - 20*A*a^9*b^2*c^3 - 4*B*a^9*b^3*c^2) - x*(4*
A^2*a^8*c^5 - 4*B^2*a^9*c^4 + 2*A^2*a^6*b^4*c^3 - 8*A^2*a^7*b^2*c^4 + 2*B^2
*a^8*b^2*c^3 - 4*A*B*a^7*b^3*c^3 + 12*A*B*a^8*b*c^4))*(-(A^2*b^7 + B^2*a^2*
b^5 + A^2*b^4*(-(4*a*c - b^2)^3)^(1/2) - 2*A*B*a*b^6 + 25*A^2*a^2*b^3*c^2 +
 A^2*a^2*c^2*(-(4*a*c - b^2)^3)^(1/2) + B^2*a^2*b^2*(-(4*a*c - b^2)^3)^(1/2
) + 16*A*B*a^4*c^3 - 9*A^2*a*b^5*c - 20*A^2*a^3*b*c^3 - 7*B^2*a^3*b^3*c + 1
2*B^2*a^4*b*c^2 - B^2*a^3*c*(-(4*a*c - b^2)^3)^(1/2) - 36*A*B*a^3*b^2*c^2 -
 3*A^2*a*b^2*c*(-(4*a*c - b^2)^3)^(1/2) - 2*A*B*a*b^3*(-(4*a*c - b^2)^3)^(1
/2) + 16*A*B*a^2*b^4*c + 4*A*B*a^2*b*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^5*b^
4 + 16*a^7*c^2 - 8*a^6*b^2*c)))^(1/2)*1i - ((-(A^2*b^7 + B^2*a^2*b^5 + A^2*
b^4*(-(4*a*c - b^2)^3)^(1/2) - 2*A*B*a*b^6 + 25*A^2*a^2*b^3*c^2 + A^2*a^2*c
^2*(-(4*a*c - b^2)^3)^(1/2) + B^2*a^2*b^2*(-(4*a*c - b^2)^3)^(1/2) + 16*A*B
*a^4*c^3 - 9*A^2*a*b^5*c - 20*A^2*a^3*b*c^3 - 7*B^2*a^3*b^3*c + 12*B^2*a^4*
b*c^2 - B^2*a^3*c*(-(4*a*c - b^2)^3)^(1/2) - 36*A*B*a^3*b^2*c^2 - 3*A^2*a*b
^2*c*(-(4*a*c - b^2)^3)^(1/2) - 2*A*B*a*b^3*(-(4*a*c - b^2)^3)^(1/2) + 16*A
*B*a^2*b^4*c + 4*A*B*a^2*b*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^5*b^4 + 16*a^7
*c^2 - 8*a^6*b^2*c)))^(1/2)*(16*A*a^10*c^4 - x*(32*a^11*b*c^3 - 8*a^10*b^3*
c^2)*(-(A^2*b^7 + B^2*a^2*b^5 + A^2*b^4*(-(4*a*c - b^2)^3)^(1/2) - 2*A*B*a*
b^6 + 25*A^2*a^2*b^3*c^2 + A^2*a^2*c^2*(-(4*a*c - b^2)^3)^(1/2) + B^2*a^2*b
^2*(-(4*a*c - b^2)^3)^(1/2) + 16*A*B*a^4*c^3 - 9*A^2*a*b^5*c - 20*A^2*a^3*b
*c^3 - 7*B^2*a^3*b^3*c + 12*B^2*a^4*b*c^2 - B^2*a^3*c*(-(4*a*c - b^2)^3)^(1
```

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/2) - 36*A*B*a^3*b^2*c^2 - 3*A^2*a*b^2*c*(-(4*a*c - b^2)^3)^(1/2) - 2*A*B*a
*b^3*(-(4*a*c - b^2)^3)^(1/2) + 16*A*B*a^2*b^4*c + 4*A*B*a^2*b*c*(-(4*a*c -
 b^2)^3)^(1/2))/(8*(a^5*b^4 + 16*a^7*c^2 - 8*a^6*b^2*c)))^(1/2) + 16*B*a^10
*b*c^3 + 4*A*a^8*b^4*c^2 - 20*A*a^9*b^2*c^3 - 4*B*a^9*b^3*c^2) + x*(4*A^2*a
^8*c^5 - 4*B^2*a^9*c^4 + 2*A^2*a^6*b^4*c^3 - 8*A^2*a^7*b^2*c^4 + 2*B^2*a^8*
b^2*c^3 - 4*A*B*a^7*b^3*c^3 + 12*A*B*a^8*b*c^4))*(-(A^2*b^7 + B^2*a^2*b^5 +
 A^2*b^4*(-(4*a*c - b^2)^3)^(1/2) - 2*A*B*a*b^6 + 25*A^2*a^2*b^3*c^2 + A^2*
a^2*c^2*(-(4*a*c - b^2)^3)^(1/2) + B^2*a^2*b^2*(-(4*a*c - b^2)^3)^(1/2) + 1
6*A*B*a^4*c^3 - 9*A^2*a*b^5*c - 20*A^2*a^3*b*c^3 - 7*B^2*a^3*b^3*c + 12*B^2
*a^4*b*c^2 - B^2*a^3*c*(-(4*a*c - b^2)^3)^(1/2) - 36*A*B*a^3*b^2*c^2 - 3*A^
2*a*b^2*c*(-(4*a*c - b^2)^3)^(1/2) - 2*A*B*a*b^3*(-(4*a*c - b^2)^3)^(1/2) +
 16*A*B*a^2*b^4*c + 4*A*B*a^2*b*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^5*b^4 + 1
6*a^7*c^2 - 8*a^6*b^2*c)))^(1/2)*1i)/(((-(A^2*b^7 + B^2*a^2*b^5 + A^2*b^4*(
-(4*a*c - b^2)^3)^(1/2) - 2*A*B*a*b^6 + 25*A^2*a^2*b^3*c^2 + A^2*a^2*c^2*(-
(4*a*c - b^2)^3)^(1/2) + B^2*a^2*b^2*(-(4*a*c - b^2)^3)^(1/2) + 16*A*B*a^4*
c^3 - 9*A^2*a*b^5*c - 20*A^2*a^3*b*c^3 - 7*B^2*a^3*b^3*c + 12*B^2*a^4*b*c^2
 - B^2*a^3*c*(-(4*a*c - b^2)^3)^(1/2) - 36*A*B*a^3*b^2*c^2 - 3*A^2*a*b^2*c*
(-(4*a*c - b^2)^3)^(1/2) - 2*A*B*a*b^3*(-(4*a*c - b^2)^3)^(1/2) + 16*A*B*a^
2*b^4*c + 4*A*B*a^2*b*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^5*b^4 + 16*a^7*c^2
 - 8*a^6*b^2*c)))^(1/2)*(16*A*a^10*c^4 + x*(32*a^11*b*c^3 - 8*a^10*b^3*c^2)*
(-(A^2*b^7 + B^2*a^2*b^5 + A^2*b^4*(-(4*a*c - b^2)^3)^(1/2) - 2*A*B*a*b^6 +
 25*A^2*a^2*b^3*c^2 + A^2*a^2*c^2*(-(4*a*c - b^2)^3)^(1/2) + B^2*a^2*b^2*(-
(4*a*c - b^2)^3)^(1/2) + 16*A*B*a^4*c^3 - 9*A^2*a*b^5*c - 20*A^2*a^3*b*c^3
 - 7*B^2*a^3*b^3*c + 12*B^2*a^4*b*c^2 - B^2*a^3*c*(-(4*a*c - b^2)^3)^(1/2) -
 36*A*B*a^3*b^2*c^2 - 3*A^2*a*b^2*c*(-(4*a*c - b^2)^3)^(1/2) - 2*A*B*a*b^3*
(-(4*a*c - b^2)^3)^(1/2) + 16*A*B*a^2*b^4*c + 4*A*B*a^2*b*c*(-(4*a*c - b^2)
^3)^(1/2))/(8*(a^5*b^4 + 16*a^7*c^2 - 8*a^6*b^2*c)))^(1/2) + 16*B*a^10*b*c^
3 + 4*A*a^8*b^4*c^2 - 20*A*a^9*b^2*c^3 - 4*B*a^9*b^3*c^2) - x*(4*A^2*a^8*c^
5 - 4*B^2*a^9*c^4 + 2*A^2*a^6*b^4*c^3 - 8*A^2*a^7*b^2*c^4 + 2*B^2*a^8*b^2*c
^3 - 4*A*B*a^7*b^3*c^3 + 12*A*B*a^8*b*c^4))*(-(A^2*b^7 + B^2*a^2*b^5 + A^2*
b^4*(-(4*a*c - b^2)^3)^(1/2) - 2*A*B*a*b^6 + 25*A^2*a^2*b^3*c^2 + A^2*a^2*c
^2*(-(4*a*c - b^2)^3)^(1/2) + B^2*a^2*b^2*(-(4*a*c - b^2)^3)^(1/2) + 16*A*B
*a^4*c^3 - 9*A^2*a*b^5*c - 20*A^2*a^3*b*c^3 - 7*B^2*a^3*b^3*c + 12*B^2*a^4*
b*c^2 - B^2*a^3*c*(-(4*a*c - b^2)^3)^(1/2) - 36*A*B*a^3*b^2*c^2 - 3*A^2*a*b
^2*c*(-(4*a*c - b^2)^3)^(1/2) - 2*A*B*a*b^3*(-(4*a*c - b^2)^3)^(1/2) + 16*A
*B*a^2*b^4*c + 4*A*B*a^2*b*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^5*b^4 + 16*a^7
*c^2 - 8*a^6*b^2*c)))^(1/2) + ((-(A^2*b^7 + B^2*a^2*b^5 + A^2*b^4*(-(4*a*c
 - b^2)^3)^(1/2) - 2*A*B*a*b^6 + 25*A^2*a^2*b^3*c^2 + A^2*a^2*c^2*(-(4*a*c -
 b^2)^3)^(1/2) + B^2*a^2*b^2*(-(4*a*c - b^2)^3)^(1/2) + 16*A*B*a^4*c^3 - 9*
A^2*a*b^5*c - 20*A^2*a^3*b*c^3 - 7*B^2*a^3*b^3*c + 12*B^2*a^4*b*c^2 - B^2*a
^3*c*(-(4*a*c - b^2)^3)^(1/2) - 36*A*B*a^3*b^2*c^2 - 3*A^2*a*b^2*c*(-(4*a*c
 - b^2)^3)^(1/2) - 2*A*B*a*b^3*(-(4*a*c - b^2)^3)^(1/2) + 16*A*B*a^2*b^4*c
+ 4*A*B*a^2*b*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^5*b^4 + 16*a^7*c^2 - 8*a^6*
b^2*c)))^(1/2)*(16*A*a^10*c^4 - x*(32*a^11*b*c^3 - 8*a^10*b^3*c^2)*(-(A^2*b
^7 + B^2*a^2*b^5 + A^2*b^4*(-(4*a*c - b^2)^3)^(1/2) - 2*A*B*a*b^6 + 25*A^2*
```

a^2*b^3*c^2 + A^2*a^2*c^2*(-(4*a*c - b^2)^3)^(1/2) + B^2*a^2*b^2*(-(4*a*c - b^2)^3)^(1/2) + 16*A*B*a^4*c^3 - 9*A^2*a*b^5*c - 20*A^2*a^3*b*c^3 - 7*B^2*a^3*b^3*c + 12*B^2*a^4*b*c^2 - B^2*a^3*c*(-(4*a*c - b^2)^3)^(1/2) - 36*A*B*a^3*b^2*c^2 - 3*A^2*a*b^2*c*(-(4*a*c - b^2)^3)^(1/2) - 2*A*B*a*b^3*(-(4*a*c - b^2)^3)^(1/2) + 16*A*B*a^2*b^4*c + 4*A*B*a^2*b*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^5*b^4 + 16*a^7*c^2 - 8*a^6*b^2*c)))^(1/2) + 16*B*a^10*b*c^3 + 4*A*a^8*b^4*c^2 - 20*A*a^9*b^2*c^3 - 4*B*a^9*b^3*c^2) + x*(4*A^2*a^8*c^5 - 4*B^2*a^9*c^4 + 2*A^2*a^6*b^4*c^3 - 8*A^2*a^7*b^2*c^4 + 2*B^2*a^8*b^2*c^3 - 4*A*B*a^7*b^3*c^3 + 12*A*B*a^8*b*c^4))*(-(A^2*b^7 + B^2*a^2*b^5 + A^2*b^4*(-(4*a*c - b^2)^3)^(1/2) - 2*A*B*a*b^6 + 25*A^2*a^2*b^3*c^2 + A^2*a^2*c^2*(-(4*a*c - b^2)^3)^(1/2) + B^2*a^2*b^2*(-(4*a*c - b^2)^3)^(1/2) + 16*A*B*a^4*c^3 - 9*A^2*a*b^5*c - 20*A^2*a^3*b*c^3 - 7*B^2*a^3*b^3*c + 12*B^2*a^4*b*c^2 - B^2*a^3*c*(-(4*a*c - b^2)^3)^(1/2) - 36*A*B*a^3*b^2*c^2 - 3*A^2*a*b^2*c*(-(4*a*c - b^2)^3)^(1/2) - 2*A*B*a*b^3*(-(4*a*c - b^2)^3)^(1/2) + 16*A*B*a^2*b^4*c + 4*A*B*a^2*b*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^5*b^4 + 16*a^7*c^2 - 8*a^6*b^2*c)))^(1/2) + 2*B^3*a^8*c^4 + 2*A^2*B*a^7*c^5 - 2*A^3*a^6*b*c^5 - 4*A*B^2*a^7*b*c^4 + 2*A^2*B*a^6*b^2*c^4))*(-(A^2*b^7 + B^2*a^2*b^5 + A^2*b^4*(-(4*a*c - b^2)^3)^(1/2) - 2*A*B*a*b^6 + 25*A^2*a^2*b^3*c^2 + A^2*a^2*c^2*(-(4*a*c - b^2)^3)^(1/2) + B^2*a^2*b^2*(-(4*a*c - b^2)^3)^(1/2) + 16*A*B*a^4*c^3 - 9*A^2*a*b^5*c - 20*A^2*a^3*b*c^3 - 7*B^2*a^3*b^3*c + 12*B^2*a^4*b*c^2 - B^2*a^3*c*(-(4*a*c - b^2)^3)^(1/2) - 36*A*B*a^3*b^2*c^2 - 3*A^2*a*b^2*c*(-(4*a*c - b^2)^3)^(1/2) - 2*A*B*a*b^3*(-(4*a*c - b^2)^3)^(1/2) + 16*A*B*a^2*b^4*c + 4*A*B*a^2*b*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^5*b^4 + 16*a^7*c^2 - 8*a^6*b^2*c)))^(1/2)*2i - atan((((-(A^2*b^7 + B^2*a^2*b^5 - A^2*b^4*(-(4*a*c - b^2)^3)^(1/2) - 2*A*B*a*b^6 + 25*A^2*a^2*b^3*c^2 - A^2*a^2*c^2*(-(4*a*c - b^2)^3)^(1/2) - B^2*a^2*b^2*(-(4*a*c - b^2)^3)^(1/2) + 16*A*B*a^4*c^3 - 9*A^2*a*b^5*c - 20*A^2*a^3*b*c^3 - 7*B^2*a^3*b^3*c + 12*B^2*a^4*b*c^2 + B^2*a^3*c*(-(4*a*c - b^2)^3)^(1/2) - 36*A*B*a^3*b^2*c^2 + 3*A^2*a*b^2*c*(-(4*a*c - b^2)^3)^(1/2) + 2*A*B*a*b^3*(-(4*a*c - b^2)^3)^(1/2) + 16*A*B*a^2*b^4*c - 4*A*B*a^2*b*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^5*b^4 + 16*a^7*c^2 - 8*a^6*b^2*c)))^(1/2)*(16*A*a^10*c^4 + x*(32*a^11*b*c^3 - 8*a^10*b^3*c^2)*(-(A^2*b^7 + B^2*a^2*b^5 - A^2*b^4*(-(4*a*c - b^2)^3)^(1/2) - 2*A*B*a*b^6 + 25*A^2*a^2*b^3*c^2 - A^2*a^2*c^2*(-(4*a*c - b^2)^3)^(1/2) - B^2*a^2*b^2*(-(4*a*c - b^2)^3)^(1/2) + 16*A*B*a^4*c^3 - 9*A^2*a*b^5*c - 20*A^2*a^3*b*c^3 - 7*B^2*a^3*b^3*c + 12*B^2*a^4*b*c^2 + B^2*a^3*c*(-(4*a*c - b^2)^3)^(1/2) - 36*A*B*a^3*b^2*c^2 + 3*A^2*a*b^2*c*(-(4*a*c - b^2)^3)^(1/2) + 2*A*B*a*b^3*(-(4*a*c - b^2)^3)^(1/2) + 16*A*B*a^2*b^4*c - 4*A*B*a^2*b*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^5*b^4 + 16*a^7*c^2 - 8*a^6*b^2*c)))^(1/2) + 16*B*a^10*b*c^3 + 4*A*a^8*b^4*c^2 - 20*A*a^9*b^2*c^3 - 4*B*a^9*b^3*c^2) - x*(4*A^2*a^8*c^5 - 4*B^2*a^9*c^4 + 2*A^2*a^6*b^4*c^3 - 8*A^2*a^7*b^2*c^4 + 2*B^2*a^8*b^2*c^3 - 4*A*B*a^7*b^3*c^3 + 12*A*B*a^8*b*c^4))*(-(A^2*b^7 + B^2*a^2*b^5 - A^2*b^4*(-(4*a*c - b^2)^3)^(1/2) - 2*A*B*a*b^6 + 25*A^2*a^2*b^3*c^2 - A^2*a^2*c^2*(-(4*a*c - b^2)^3)^(1/2) - B^2*a^2*b^2*(-(4*a*c - b^2)^3)^(1/2) + 16*A*B*a^4*c^3 - 9*A^2*a*b^5*c - 20*A^2*a^3*b*c^3 - 7*B^2*a^3*b^3*c + 12*B^2*a^4*b*c^2 + B^2*a^3*c*(-(4*a*c - b^2)^3)^(1/2) - 36*A*B*a^3*b^2*c^2 + 3*A^2*a*b^2

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*c*(-(4*a*c - b^2)^3)^(1/2) + 2*A*B*a*b^3*(-(4*a*c - b^2)^3)^(1/2) + 16*A*B
*a^2*b^4*c - 4*A*B*a^2*b*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^5*b^4 + 16*a^7*c
^2 - 8*a^6*b^2*c)))^(1/2)*1i - ((-(A^2*b^7 + B^2*a^2*b^5 - A^2*b^4*(-(4*a*c
 - b^2)^3)^(1/2) - 2*A*B*a*b^6 + 25*A^2*a^2*b^3*c^2 - A^2*a^2*c^2*(-(4*a*c
- b^2)^3)^(1/2) - B^2*a^2*b^2*(-(4*a*c - b^2)^3)^(1/2) + 16*A*B*a^4*c^3 - 9
*A^2*a*b^5*c - 20*A^2*a^3*b*c^3 - 7*B^2*a^3*b^3*c + 12*B^2*a^4*b*c^2 + B^2*
a^3*c*(-(4*a*c - b^2)^3)^(1/2) - 36*A*B*a^3*b^2*c^2 + 3*A^2*a*b^2*c*(-(4*a*
c - b^2)^3)^(1/2) + 2*A*B*a*b^3*(-(4*a*c - b^2)^3)^(1/2) + 16*A*B*a^2*b^4*c
 - 4*A*B*a^2*b*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^5*b^4 + 16*a^7*c^2 - 8*a^6
*b^2*c)))^(1/2)*(16*A*a^10*c^4 - x*(32*a^11*b*c^3 - 8*a^10*b^3*c^2)*(-(A^2*
b^7 + B^2*a^2*b^5 - A^2*b^4*(-(4*a*c - b^2)^3)^(1/2) - 2*A*B*a*b^6 + 25*A^2
*a^2*b^3*c^2 - A^2*a^2*c^2*(-(4*a*c - b^2)^3)^(1/2) - B^2*a^2*b^2*(-(4*a*c
- b^2)^3)^(1/2) + 16*A*B*a^4*c^3 - 9*A^2*a*b^5*c - 20*A^2*a^3*b*c^3 - 7*B^2
*a^3*b^3*c + 12*B^2*a^4*b*c^2 + B^2*a^3*c*(-(4*a*c - b^2)^3)^(1/2) - 36*A*B
*a^3*b^2*c^2 + 3*A^2*a*b^2*c*(-(4*a*c - b^2)^3)^(1/2) + 2*A*B*a*b^3*(-(4*a*
c - b^2)^3)^(1/2) + 16*A*B*a^2*b^4*c - 4*A*B*a^2*b*c*(-(4*a*c - b^2)^3)^(1/
2))/(8*(a^5*b^4 + 16*a^7*c^2 - 8*a^6*b^2*c)))^(1/2) + 16*B*a^10*b*c^3 + 4*A
*a^8*b^4*c^2 - 20*A*a^9*b^2*c^3 - 4*B*a^9*b^3*c^2) + x*(4*A^2*a^8*c^5 - 4*B
^2*a^9*c^4 + 2*A^2*a^6*b^4*c^3 - 8*A^2*a^7*b^2*c^4 + 2*B^2*a^8*b^2*c^3 - 4*
A*B*a^7*b^3*c^3 + 12*A*B*a^8*b*c^4))*(-(A^2*b^7 + B^2*a^2*b^5 - A^2*b^4*(-(
4*a*c - b^2)^3)^(1/2) - 2*A*B*a*b^6 + 25*A^2*a^2*b^3*c^2 - A^2*a^2*c^2*(-(4
*a*c - b^2)^3)^(1/2) - B^2*a^2*b^2*(-(4*a*c - b^2)^3)^(1/2) + 16*A*B*a^4*c^
3 - 9*A^2*a*b^5*c - 20*A^2*a^3*b*c^3 - 7*B^2*a^3*b^3*c + 12*B^2*a^4*b*c^2 +
 B^2*a^3*c*(-(4*a*c - b^2)^3)^(1/2) - 36*A*B*a^3*b^2*c^2 + 3*A^2*a*b^2*c*(-
(4*a*c - b^2)^3)^(1/2) + 2*A*B*a*b^3*(-(4*a*c - b^2)^3)^(1/2) + 16*A*B*a^2*
b^4*c - 4*A*B*a^2*b*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^5*b^4 + 16*a^7*c^2 -
8*a^6*b^2*c)))^(1/2)*1i)/(((-(A^2*b^7 + B^2*a^2*b^5 - A^2*b^4*(-(4*a*c - b^
2)^3)^(1/2) - 2*A*B*a*b^6 + 25*A^2*a^2*b^3*c^2 - A^2*a^2*c^2*(-(4*a*c - b^2
)^3)^(1/2) - B^2*a^2*b^2*(-(4*a*c - b^2)^3)^(1/2) + 16*A*B*a^4*c^3 - 9*A^2*
a*b^5*c - 20*A^2*a^3*b*c^3 - 7*B^2*a^3*b^3*c + 12*B^2*a^4*b*c^2 + B^2*a^3*c
*(-(4*a*c - b^2)^3)^(1/2) - 36*A*B*a^3*b^2*c^2 + 3*A^2*a*b^2*c*(-(4*a*c - b
^2)^3)^(1/2) + 2*A*B*a*b^3*(-(4*a*c - b^2)^3)^(1/2) + 16*A*B*a^2*b^4*c - 4*
A*B*a^2*b*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^5*b^4 + 16*a^7*c^2 - 8*a^6*b^2*
c)))^(1/2)*(16*A*a^10*c^4 + x*(32*a^11*b*c^3 - 8*a^10*b^3*c^2)*(-(A^2*b^7 +
 B^2*a^2*b^5 - A^2*b^4*(-(4*a*c - b^2)^3)^(1/2) - 2*A*B*a*b^6 + 25*A^2*a^2*
b^3*c^2 - A^2*a^2*c^2*(-(4*a*c - b^2)^3)^(1/2) - B^2*a^2*b^2*(-(4*a*c - b^2
)^3)^(1/2) + 16*A*B*a^4*c^3 - 9*A^2*a*b^5*c - 20*A^2*a^3*b*c^3 - 7*B^2*a^3*
b^3*c + 12*B^2*a^4*b*c^2 + B^2*a^3*c*(-(4*a*c - b^2)^3)^(1/2) - 36*A*B*a^3*
b^2*c^2 + 3*A^2*a*b^2*c*(-(4*a*c - b^2)^3)^(1/2) + 2*A*B*a*b^3*(-(4*a*c - b
^2)^3)^(1/2) + 16*A*B*a^2*b^4*c - 4*A*B*a^2*b*c*(-(4*a*c - b^2)^3)^(1/2))/(
8*(a^5*b^4 + 16*a^7*c^2 - 8*a^6*b^2*c)))^(1/2) + 16*B*a^10*b*c^3 + 4*A*a^8*
b^4*c^2 - 20*A*a^9*b^2*c^3 - 4*B*a^9*b^3*c^2) - x*(4*A^2*a^8*c^5 - 4*B^2*a^
9*c^4 + 2*A^2*a^6*b^4*c^3 - 8*A^2*a^7*b^2*c^4 + 2*B^2*a^8*b^2*c^3 - 4*A*B*a
^7*b^3*c^3 + 12*A*B*a^8*b*c^4))*(-(A^2*b^7 + B^2*a^2*b^5 - A^2*b^4*(-(4*a*c
 - b^2)^3)^(1/2) - 2*A*B*a*b^6 + 25*A^2*a^2*b^3*c^2 - A^2*a^2*c^2*(-(4*a*c
```

```
- b^2)^3)^(1/2) - B^2*a^2*b^2*(-(4*a*c - b^2)^3)^(1/2) + 16*A*B*a^4*c^3 - 9
*A^2*a*b^5*c - 20*A^2*a^3*b*c^3 - 7*B^2*a^3*b^3*c + 12*B^2*a^4*b*c^2 + B^2*
a^3*c*(-(4*a*c - b^2)^3)^(1/2) - 36*A*B*a^3*b^2*c^2 + 3*A^2*a*b^2*c*(-(4*a*
c - b^2)^3)^(1/2) + 2*A*B*a*b^3*(-(4*a*c - b^2)^3)^(1/2) + 16*A*B*a^2*b^4*c
 - 4*A*B*a^2*b*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^5*b^4 + 16*a^7*c^2 - 8*a^6
*b^2*c)))^(1/2) + ((-(A^2*b^7 + B^2*a^2*b^5 - A^2*b^4*(-(4*a*c - b^2)^3)^(1
/2) - 2*A*B*a*b^6 + 25*A^2*a^2*b^3*c^2 - A^2*a^2*c^2*(-(4*a*c - b^2)^3)^(1/
2) - B^2*a^2*b^2*(-(4*a*c - b^2)^3)^(1/2) + 16*A*B*a^4*c^3 - 9*A^2*a*b^5*c
- 20*A^2*a^3*b*c^3 - 7*B^2*a^3*b^3*c + 12*B^2*a^4*b*c^2 + B^2*a^3*c*(-(4*a*
c - b^2)^3)^(1/2) - 36*A*B*a^3*b^2*c^2 + 3*A^2*a*b^2*c*(-(4*a*c - b^2)^3)^(
1/2) + 2*A*B*a*b^3*(-(4*a*c - b^2)^3)^(1/2) + 16*A*B*a^2*b^4*c - 4*A*B*a^2*
b*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^5*b^4 + 16*a^7*c^2 - 8*a^6*b^2*c)))^(1/
2)*(16*A*a^10*c^4 - x*(32*a^11*b*c^3 - 8*a^10*b^3*c^2)*(-(A^2*b^7 + B^2*a^2
*b^5 - A^2*b^4*(-(4*a*c - b^2)^3)^(1/2) - 2*A*B*a*b^6 + 25*A^2*a^2*b^3*c^2
- A^2*a^2*c^2*(-(4*a*c - b^2)^3)^(1/2) - B^2*a^2*b^2*(-(4*a*c - b^2)^3)^(1/
2) + 16*A*B*a^4*c^3 - 9*A^2*a*b^5*c - 20*A^2*a^3*b*c^3 - 7*B^2*a^3*b^3*c +
12*B^2*a^4*b*c^2 + B^2*a^3*c*(-(4*a*c - b^2)^3)^(1/2) - 36*A*B*a^3*b^2*c^2
+ 3*A^2*a*b^2*c*(-(4*a*c - b^2)^3)^(1/2) + 2*A*B*a*b^3*(-(4*a*c - b^2)^3)^(
1/2) + 16*A*B*a^2*b^4*c - 4*A*B*a^2*b*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^5*b
^4 + 16*a^7*c^2 - 8*a^6*b^2*c)))^(1/2) + 16*B*a^10*b*c^3 + 4*A*a^8*b^4*c^2
- 20*A*a^9*b^2*c^3 - 4*B*a^9*b^3*c^2) + x*(4*A^2*a^8*c^5 - 4*B^2*a^9*c^4 +
2*A^2*a^6*b^4*c^3 - 8*A^2*a^7*b^2*c^4 + 2*B^2*a^8*b^2*c^3 - 4*A*B*a^7*b^3*c
^3 + 12*A*B*a^8*b*c^4))*(-(A^2*b^7 + B^2*a^2*b^5 - A^2*b^4*(-(4*a*c - b^2)^
3)^(1/2) - 2*A*B*a*b^6 + 25*A^2*a^2*b^3*c^2 - A^2*a^2*c^2*(-(4*a*c - b^2)^3
)^(1/2) - B^2*a^2*b^2*(-(4*a*c - b^2)^3)^(1/2) + 16*A*B*a^4*c^3 - 9*A^2*a*b
^5*c - 20*A^2*a^3*b*c^3 - 7*B^2*a^3*b^3*c + 12*B^2*a^4*b*c^2 + B^2*a^3*c*(-
(4*a*c - b^2)^3)^(1/2) - 36*A*B*a^3*b^2*c^2 + 3*A^2*a*b^2*c*(-(4*a*c - b^2)
^3)^(1/2) + 2*A*B*a*b^3*(-(4*a*c - b^2)^3)^(1/2) + 16*A*B*a^2*b^4*c - 4*A*B
*a^2*b*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^5*b^4 + 16*a^7*c^2 - 8*a^6*b^2*c))
)^(1/2) + 2*B^3*a^8*c^4 + 2*A^2*B*a^7*c^5 - 2*A^3*a^6*b*c^5 - 4*A*B^2*a^7*b
*c^4 + 2*A^2*B*a^6*b^2*c^4))*(-(A^2*b^7 + B^2*a^2*b^5 - A^2*b^4*(-(4*a*c -
b^2)^3)^(1/2) - 2*A*B*a*b^6 + 25*A^2*a^2*b^3*c^2 - A^2*a^2*c^2*(-(4*a*c - b
^2)^3)^(1/2) - B^2*a^2*b^2*(-(4*a*c - b^2)^3)^(1/2) + 16*A*B*a^4*c^3 - 9*A^
2*a*b^5*c - 20*A^2*a^3*b*c^3 - 7*B^2*a^3*b^3*c + 12*B^2*a^4*b*c^2 + B^2*a^3
*c*(-(4*a*c - b^2)^3)^(1/2) - 36*A*B*a^3*b^2*c^2 + 3*A^2*a*b^2*c*(-(4*a*c -
 b^2)^3)^(1/2) + 2*A*B*a*b^3*(-(4*a*c - b^2)^3)^(1/2) + 16*A*B*a^2*b^4*c -
4*A*B*a^2*b*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^5*b^4 + 16*a^7*c^2 - 8*a^6*b^
2*c)))^(1/2)*2i
```

3.112 $\int \frac{x^7(A+Bx^2)}{(a+bx^2+cx^4)^2} dx$

Optimal result

Integrand size = 25, antiderivative size = 212

$$\begin{aligned}\int \frac{x^7(A+Bx^2)}{(a+bx^2+cx^4)^2} dx &= \frac{(2b^2B-Abc-6aBc)\,x^2}{2c^2\,(b^2-4ac)} - \frac{x^4(a(bB-2Ac)+(b^2B-Abc-2aBc)\,x^2)}{2c\,(b^2-4ac)\,(a+bx^2+cx^4)} \\ &\quad - \frac{(2b^4B-Ab^3c-12ab^2Bc+6aAbc^2+12a^2Bc^2)\operatorname{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2c^3\,(b^2-4ac)^{3/2}} \\ &\quad - \frac{(2bB-Ac)\log(a+bx^2+cx^4)}{4c^3}\end{aligned}$$

```
[Out] 1/2*(-A*b*c-6*B*a*c+2*B*b^2)*x^2/c^2/(-4*a*c+b^2)-1/2*x^4*(a*(-2*A*c+B*b)+(
-A*b*c-2*B*a*c+B*b^2)*x^2)/c/(-4*a*c+b^2)/(c*x^4+b*x^2+a)-1/2*(6*A*a*b*c^2-
A*b^3*c+12*B*a^2*c^2-12*B*a*b^2*c+2*B*b^4)*arctanh((2*c*x^2+b)/(-4*a*c+b^2)
^(1/2))/c^3/(-4*a*c+b^2)^(3/2)-1/4*(-A*c+2*B*b)*ln(c*x^4+b*x^2+a)/c^3
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 212, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.280$, Rules used

$= \{1265, 832, 787, 648, 632, 212, 642\}$

$$\int \frac{x^7(A+Bx^2)}{(a+bx^2+cx^4)^2}\,dx = -\frac{(12a^2Bc^2+6aAbc^2-12ab^2Bc-Ab^3c+2b^4B)\operatorname{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2c^3(b^2-4ac)^{3/2}} + \frac{x^2(-6aBc-Abc+2b^2B)}{2c^2(b^2-4ac)} - \frac{x^4(x^2(-2aBc-Abc+b^2B)+a(bB-2Ac))}{2c(b^2-4ac)(a+bx^2+cx^4)} - \frac{(2bB-Ac)\log(a+bx^2+cx^4)}{4c^3}$$

```
[In] Int[(x^7*(A + B*x^2))/(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] ((2*b^2*B - A*b*c - 6*a*B*c)*x^2)/(2*c^2*(b^2 - 4*a*c)) - (x^4*(a*(b*B - 2*
A*c) + (b^2*B - A*b*c - 2*a*B*c)*x^2))/(2*c*(b^2 - 4*a*c)*(a + b*x^2 + c*x^
4)) - ((2*b^4*B - A*b^3*c - 12*a*b^2*B*c + 6*a*A*b*c^2 + 12*a^2*B*c^2)*ArcT
anh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(2*c^3*(b^2 - 4*a*c)^(3/2)) - ((2*b*B
 - A*c)*Log[a + b*x^2 + c*x^4])/(4*c^3)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 787

```
Int[(((d_.) + (e_.)*(x_))*((f_) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) + (c_.)*
(x_)^2), x_Symbol] :> Simp[e*g*(x/c), x] + Dist[1/c, Int[(c*d*f - a*e*g + (
c*e*f + c*d*g - b*e*g)*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e
, f, g}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 832

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(-(d + e*x)^(m - 1))*(a + b*x + c*x^2)
^(p + 1)*((2*a*c*(e*f + d*g) - b*(c*d*f + a*e*g) - (2*c^2*d*f + b^2*e*g - c
*(b*e*f + b*d*g + 2*a*e*g))*x)/(c*(p + 1)*(b^2 - 4*a*c))), x] - Dist[1/(c*(
p + 1)*(b^2 - 4*a*c)), Int[(d + e*x)^(m - 2)*(a + b*x + c*x^2)^(p + 1)*Simp
[2*c^2*d^2*f*(2*p + 3) + b*e*g*(a*e*(m - 1) + b*d*(p + 2)) - c*(2*a*e*(e*f*
(m - 1) + d*g*m) + b*d*(d*g*(2*p + 3) - e*f*(m - 2*p - 4))) + e*(b^2*e*g*(m
 + p + 1) + 2*c^2*d*f*(m + 2*p + 2) - c*(2*a*e*g*m + b*(e*f + d*g)*(m + 2*p
 + 2)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*c
, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 1] && ((EqQ[m
, 2] && EqQ[p, -3] && RationalQ[a, b, c, d, e, f, g]) ||  !ILtQ[m + 2*p + 3
, 0])
```

Rule 1265

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{2}\text{Subst}\left(\int \frac{x^3(A+Bx)}{(a+bx+cx^2)^2}\,dx, x, x^2\right)\\
&= -\frac{x^4\left(a(bB-2Ac)+\left(b^2B-Abc-2aBc\right)x^2\right)}{2c\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}\\
&\quad+\frac{\text{Subst}\left(\int \frac{x\left(2a(bB-2Ac)+\left(2b^2B-Abc-6aBc\right)x\right)}{a+bx+cx^2}\,dx, x, x^2\right)}{2c\left(b^2-4ac\right)}\\
&= \frac{\left(2b^2B-Abc-6aBc\right)x^2}{2c^2\left(b^2-4ac\right)}-\frac{x^4\left(a(bB-2Ac)+\left(b^2B-Abc-2aBc\right)x^2\right)}{2c\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}\\
&\quad+\frac{\text{Subst}\left(\int \frac{-a\left(2b^2B-Abc-6aBc\right)+\left(2ac(bB-2Ac)-b\left(2b^2B-Abc-6aBc\right)\right)x}{a+bx+cx^2}\,dx, x, x^2\right)}{2c^2\left(b^2-4ac\right)}
\end{aligned}$$

$$= \frac{(2b^2B - Abc - 6aBc)\,x^2}{2c^2\,(b^2 - 4ac)} - \frac{x^4(a(bB - 2Ac) + (b^2B - Abc - 2aBc)\,x^2)}{2c\,(b^2 - 4ac)\,(a + bx^2 + cx^4)} - \frac{(2bB - Ac)\text{Subst}\left(\int \frac{b+2cx}{a+bx+cx^2}\,dx, x, x^2\right)}{4c^3} + \frac{(2b^4B - Ab^3c - 12ab^2Bc + 6aAbc^2 + 12a^2Bc^2)\,\text{Subst}\left(\int \frac{1}{a+bx+cx^2}\,dx, x, x^2\right)}{4c^3\,(b^2 - 4ac)}$$

$$= \frac{(2b^2B - Abc - 6aBc)\,x^2}{2c^2\,(b^2 - 4ac)} - \frac{x^4(a(bB - 2Ac) + (b^2B - Abc - 2aBc)\,x^2)}{2c\,(b^2 - 4ac)\,(a + bx^2 + cx^4)} - \frac{(2bB - Ac)\log(a + bx^2 + cx^4)}{4c^3} - \frac{(2b^4B - Ab^3c - 12ab^2Bc + 6aAbc^2 + 12a^2Bc^2)\,\text{Subst}\left(\int \frac{1}{b^2-4ac-x^2}\,dx, x, b + 2cx^2\right)}{2c^3\,(b^2 - 4ac)}$$

$$= \frac{(2b^2B - Abc - 6aBc)\,x^2}{2c^2\,(b^2 - 4ac)} - \frac{x^4(a(bB - 2Ac) + (b^2B - Abc - 2aBc)\,x^2)}{2c\,(b^2 - 4ac)\,(a + bx^2 + cx^4)} - \frac{(2b^4B - Ab^3c - 12ab^2Bc + 6aAbc^2 + 12a^2Bc^2)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2c^3\,(b^2 - 4ac)^{3/2}} - \frac{(2bB - Ac)\log(a + bx^2 + cx^4)}{4c^3}$$

Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 208, normalized size of antiderivative = 0.98

$$\int \frac{x^7(A + Bx^2)}{(a + bx^2 + cx^4)^2}\,dx$$

$$= \frac{2Bcx^2 - \frac{2\left(b^3(bB-Ac)x^2+a^2c(-3bB+2c(A+Bx^2))+ab(b^2B+3Ac^2x^2-bc(A+4Bx^2))\right)}{(b^2-4ac)(a+bx^2+cx^4)} - \frac{2(2b^4B-Ab^3c-12ab^2Bc+6aAbc^2+12a^2Bc^2)\text{ a}}{(-b^2+4ac)^{3/2}}}{4c^3}$$

```
[In] Integrate[(x^7*(A + B*x^2))/(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] (2*B*c*x^2 - (2*(b^3*(b*B - A*c)*x^2 + a^2*c*(-3*b*B + 2*c*(A + B*x^2)) + a
*b*(b^2*B + 3*A*c^2*x^2 - b*c*(A + 4*B*x^2))))/((b^2 - 4*a*c)*(a + b*x^2 +
c*x^4)) - (2*(2*b^4*B - A*b^3*c - 12*a*b^2*B*c + 6*a*A*b*c^2 + 12*a^2*B*c^2
)*ArcTan[(b + 2*c*x^2)/Sqrt[-b^2 + 4*a*c]])/(-b^2 + 4*a*c)^(3/2) + (-2*b*B
+ A*c)*Log[a + b*x^2 + c*x^4])/(4*c^3)
```

Maple [A] (verified)

Time = 0.23 (sec) , antiderivative size = 282, normalized size of antiderivative = 1.33

method	result
default	$\frac{B\,x^{2}}{2c^{2}}+\frac{\frac{\left(3Aab\,c^{2}-A\,b^{3}c+2B\,a^{2}c^{2}-4Ba\,b^{2}c+B\,b^{4}\right)x^{2}}{c\left(4ac-b^{2}\right)}+\frac{a\left(2Aa\,c^{2}-A\,b^{2}c-3Babc+B\,b^{3}\right)}{c\left(4ac-b^{2}\right)}}{c\,x^{4}+b\,x^{2}+a}+\frac{\frac{\left(4Aa\,c^{2}-A\,b^{2}c-8Babc+2B\,b^{3}\right)\ln\left(c\,x^{4}+b\,x^{2}+a\right)}{2c}+\frac{2}{}}{2c^{2}}$
risch	Expression too large to display

```
[In] int(x^7*(B*x^2+A)/(c*x^4+b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*B*x^2/c^2+1/2/c^2*(((3*A*a*b*c^2-A*b^3*c+2*B*a^2*c^2-4*B*a*b^2*c+B*b^4)
/c/(4*a*c-b^2)*x^2+a*(2*A*a*c^2-A*b^2*c-3*B*a*b*c+B*b^3)/c/(4*a*c-b^2))/(c*
x^4+b*x^2+a)+1/(4*a*c-b^2)*(1/2*(4*A*a*c^2-A*b^2*c-8*B*a*b*c+2*B*b^3)/c*ln(
c*x^4+b*x^2+a)+2*(-A*a*b*c-6*a^2*B*c+2*B*a*b^2-1/2*(4*A*a*c^2-A*b^2*c-8*B*a
*b*c+2*B*b^3)*b/c)/(4*a*c-b^2)^(1/2)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2)))
)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 650 vs. 2(200) = 400.

Time = 0.35 (sec) , antiderivative size = 1323, normalized size of antiderivative = 6.24

$$\int \frac{x^7(A+Bx^2)}{(a+bx^2+cx^4)^2}\,dx = \text{Too large to display}$$

```
[In] integrate(x^7*(B*x^2+A)/(c*x^4+b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] [-1/4*(2*B*a*b^5 - 16*A*a^3*c^3 - 2*(B*b^4*c^2 - 8*B*a*b^2*c^3 + 16*B*a^2*c
^4)*x^6 - 2*(B*b^5*c - 8*B*a*b^3*c^2 + 16*B*a^2*b*c^3)*x^4 + 12*(2*B*a^3*b
+ A*a^2*b^2)*c^2 + 2*(B*b^6 - 12*(2*B*a^3 + A*a^2*b)*c^3 + (26*B*a^2*b^2 +
7*A*a*b^3)*c^2 - (9*B*a*b^4 + A*b^5)*c)*x^2 + (2*B*a*b^4 + (2*B*b^4*c + 6*(
2*B*a^2 + A*a*b)*c^3 - (12*B*a*b^2 + A*b^3)*c^2)*x^4 + 6*(2*B*a^3 + A*a^2*b
)*c^2 + (2*B*b^5 + 6*(2*B*a^2*b + A*a*b^2)*c^2 - (12*B*a*b^3 + A*b^4)*c)*x^
2 - (12*B*a^2*b^2 + A*a*b^3)*c)*sqrt(b^2 - 4*a*c)*log((2*c^2*x^4 + 2*b*c*x^
2 + b^2 - 2*a*c + (2*c*x^2 + b)*sqrt(b^2 - 4*a*c))/(c*x^4 + b*x^2 + a)) - 2
*(7*B*a^2*b^3 + A*a*b^4)*c + (2*B*a*b^5 - 16*A*a^3*c^3 + (2*B*b^5*c - 16*A*
a^2*c^4 + 8*(4*B*a^2*b + A*a*b^2)*c^3 - (16*B*a*b^3 + A*b^4)*c^2)*x^4 + 8*(
4*B*a^3*b + A*a^2*b^2)*c^2 + (2*B*b^6 - 16*A*a^2*b*c^3 + 8*(4*B*a^2*b^2 + A
*a*b^3)*c^2 - (16*B*a*b^4 + A*b^5)*c)*x^2 - (16*B*a^2*b^3 + A*a*b^4)*c)*log
(c*x^4 + b*x^2 + a))/(a*b^4*c^3 - 8*a^2*b^2*c^4 + 16*a^3*c^5 + (b^4*c^4 - 8
*a*b^2*c^5 + 16*a^2*c^6)*x^4 + (b^5*c^3 - 8*a*b^3*c^4 + 16*a^2*b*c^5)*x^2),
 -1/4*(2*B*a*b^5 - 16*A*a^3*c^3 - 2*(B*b^4*c^2 - 8*B*a*b^2*c^3 + 16*B*a^2*c
```

```
^4)*x^6 - 2*(B*b^5*c - 8*B*a*b^3*c^2 + 16*B*a^2*b*c^3)*x^4 + 12*(2*B*a^3*b
+ A*a^2*b^2)*c^2 + 2*(B*b^6 - 12*(2*B*a^3 + A*a^2*b)*c^3 + (26*B*a^2*b^2 +
7*A*a*b^3)*c^2 - (9*B*a*b^4 + A*b^5)*c)*x^2 + 2*(2*B*a*b^4 + (2*B*b^4*c + 6
*(2*B*a^2 + A*a*b)*c^3 - (12*B*a*b^2 + A*b^3)*c^2)*x^4 + 6*(2*B*a^3 + A*a^2
*b)*c^2 + (2*B*b^5 + 6*(2*B*a^2*b + A*a*b^2)*c^2 - (12*B*a*b^3 + A*b^4)*c)*
x^2 - (12*B*a^2*b^2 + A*a*b^3)*c)*sqrt(-b^2 + 4*a*c)*arctan(-(2*c*x^2 + b)*
sqrt(-b^2 + 4*a*c)/(b^2 - 4*a*c)) - 2*(7*B*a^2*b^3 + A*a*b^4)*c + (2*B*a*b^
5 - 16*A*a^3*c^3 + (2*B*b^5*c - 16*A*a^2*c^4 + 8*(4*B*a^2*b + A*a*b^2)*c^3
- (16*B*a*b^3 + A*b^4)*c^2)*x^4 + 8*(4*B*a^3*b + A*a^2*b^2)*c^2 + (2*B*b^6
- 16*A*a^2*b*c^3 + 8*(4*B*a^2*b^2 + A*a*b^3)*c^2 - (16*B*a*b^4 + A*b^5)*c)*
x^2 - (16*B*a^2*b^3 + A*a*b^4)*c)*log(c*x^4 + b*x^2 + a))/(a*b^4*c^3 - 8*a^
2*b^2*c^4 + 16*a^3*c^5 + (b^4*c^4 - 8*a*b^2*c^5 + 16*a^2*c^6)*x^4 + (b^5*c^
3 - 8*a*b^3*c^4 + 16*a^2*b*c^5)*x^2)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^7(A+Bx^2)}{(a+bx^2+cx^4)^2}\,dx = \text{Timed out}$$

```
[In] integrate(x**7*(B*x**2+A)/(c*x**4+b*x**2+a)**2,x)
```

```
[Out] Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^7(A+Bx^2)}{(a+bx^2+cx^4)^2}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(x^7*(B*x^2+A)/(c*x^4+b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

Giac [A] (verification not implemented)

none

Time = 0.65 (sec) , antiderivative size = 239, normalized size of antiderivative = 1.13

$$\int \frac{x^7(A+Bx^2)}{(a+bx^2+cx^4)^2}\,dx$$
$$=\frac{Bx^2}{2\,c^2}+\frac{(2\,Bb^4-12\,Bab^2c-Ab^3c+12\,Ba^2c^2+6\,Aabc^2)\arctan\left(\frac{2\,cx^2+b}{\sqrt{-b^2+4\,ac}}\right)}{2\,(b^2c^3-4\,ac^4)\sqrt{-b^2+4\,ac}}$$
$$+\frac{2\,Bb^3x^4-8\,Babcx^4-Ab^2cx^4+4\,Aac^2x^4+Ab^3x^2-4\,Ba^2cx^2-2\,Aabcx^2-2\,Ba^2b+Aab^2}{4\,(cx^4+bx^2+a)(b^2c^2-4\,ac^3)}$$
$$-\frac{(2\,Bb-Ac)\log\left(cx^4+bx^2+a\right)}{4\,c^3}$$

```
[In] integrate(x^7*(B*x^2+A)/(c*x^4+b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] 1/2*B*x^2/c^2 + 1/2*(2*B*b^4 - 12*B*a*b^2*c - A*b^3*c + 12*B*a^2*c^2 + 6*A*
a*b*c^2)*arctan((2*c*x^2 + b)/sqrt(-b^2 + 4*a*c))/((b^2*c^3 - 4*a*c^4)*sqrt
(-b^2 + 4*a*c)) + 1/4*(2*B*b^3*x^4 - 8*B*a*b*c*x^4 - A*b^2*c*x^4 + 4*A*a*c^
2*x^4 + A*b^3*x^2 - 4*B*a^2*c*x^2 - 2*A*a*b*c*x^2 - 2*B*a^2*b + A*a*b^2)/((
c*x^4 + b*x^2 + a)*(b^2*c^2 - 4*a*c^3)) - 1/4*(2*B*b - A*c)*log(c*x^4 + b*x
^2 + a)/c^3
```

Mupad [B] (verification not implemented)

Time = 8.06 (sec) , antiderivative size = 2282, normalized size of antiderivative = 10.76

$$\int \frac{x^7(A+Bx^2)}{(a+bx^2+cx^4)^2}\,dx=\text{Too large to display}$$

```
[In] int((x^7*(A + B*x^2))/(a + b*x^2 + c*x^4)^2,x)
```

```
[Out] ((a*(B*b^3 + 2*A*a*c^2 - A*b^2*c - 3*B*a*b*c))/(2*c*(4*a*c - b^2)) + (x^2*(
B*b^4 + 2*B*a^2*c^2 - A*b^3*c + 3*A*a*b*c^2 - 4*B*a*b^2*c))/(2*c*(4*a*c - b
^2)))/(a*c^2 + c^3*x^4 + b*c^2*x^2) + (B*x^2)/(2*c^2) + (log(a + b*x^2 + c*
x^4)*(4*B*b^7 + 128*A*a^3*c^4 - 2*A*b^6*c - 48*B*a*b^5*c + 24*A*a*b^4*c^2 -
 256*B*a^3*b*c^3 - 96*A*a^2*b^2*c^3 + 192*B*a^2*b^3*c^2))/(2*(256*a^3*c^6 -
 4*b^6*c^3 + 48*a*b^4*c^4 - 192*a^2*b^2*c^5)) - (atan(((8*a*c^5*(4*a*c - b^
2)^3 - 2*b^2*c^4*(4*a*c - b^2)^3)*(x^2*(((((24*B*a^2*c^5 - 6*A*b^3*c^4 + 12
*B*b^4*c^3 + 28*A*a*b*c^5 - 56*B*a*b^2*c^4)/(4*a*c^5 - b^2*c^4) + ((8*b^3*c
^6 - 32*a*b*c^7)*(4*B*b^7 + 128*A*a^3*c^4 - 2*A*b^6*c - 48*B*a*b^5*c + 24*A
*a*b^4*c^2 - 256*B*a^3*b*c^3 - 96*A*a^2*b^2*c^3 + 192*B*a^2*b^3*c^2))/(2*(4
*a*c^5 - b^2*c^4)*(256*a^3*c^6 - 4*b^6*c^3 + 48*a*b^4*c^4 - 192*a^2*b^2*c^5
```

```
)))*(2*B*b^4 + 12*B*a^2*c^2 - A*b^3*c + 6*A*a*b*c^2 - 12*B*a*b^2*c))/(8*c^3
*(4*a*c - b^2)^(3/2)) + ((8*b^3*c^6 - 32*a*b*c^7)*(2*B*b^4 + 12*B*a^2*c^2 -
 A*b^3*c + 6*A*a*b*c^2 - 12*B*a*b^2*c)*(4*B*b^7 + 128*A*a^3*c^4 - 2*A*b^6*c
 - 48*B*a*b^5*c + 24*A*a*b^4*c^2 - 256*B*a^3*b*c^3 - 96*A*a^2*b^2*c^3 + 192
*B*a^2*b^3*c^2))/(16*c^3*(4*a*c - b^2)^(3/2)*(4*a*c^5 - b^2*c^4)*(256*a^3*c
^6 - 4*b^6*c^3 + 48*a*b^4*c^4 - 192*a^2*b^2*c^5)))/(a*(4*a*c - b^2)) + (b*(
(4*B^2*b^5 + A^2*b^3*c^2 - 4*A*B*b^4*c - 6*A*B*a^2*c^3 - 5*A^2*a*b*c^3 - 20
*B^2*a*b^3*c + 12*B^2*a^2*b*c^2 + 20*A*B*a*b^2*c^2)/(4*a*c^5 - b^2*c^4) + (
((24*B*a^2*c^5 - 6*A*b^3*c^4 + 12*B*b^4*c^3 + 28*A*a*b*c^5 - 56*B*a*b^2*c^4
)/(4*a*c^5 - b^2*c^4) + ((8*b^3*c^6 - 32*a*b*c^7)*(4*B*b^7 + 128*A*a^3*c^4
- 2*A*b^6*c - 48*B*a*b^5*c + 24*A*a*b^4*c^2 - 256*B*a^3*b*c^3 - 96*A*a^2*b^
2*c^3 + 192*B*a^2*b^3*c^2))/(2*(4*a*c^5 - b^2*c^4)*(256*a^3*c^6 - 4*b^6*c^3
 + 48*a*b^4*c^4 - 192*a^2*b^2*c^5)))*(4*B*b^7 + 128*A*a^3*c^4 - 2*A*b^6*c -
 48*B*a*b^5*c + 24*A*a*b^4*c^2 - 256*B*a^3*b*c^3 - 96*A*a^2*b^2*c^3 + 192*B
*a^2*b^3*c^2))/(2*(256*a^3*c^6 - 4*b^6*c^3 + 48*a*b^4*c^4 - 192*a^2*b^2*c^5
)) - (((b^3*c^6)/2 - 2*a*b*c^7)*(2*B*b^4 + 12*B*a^2*c^2 - A*b^3*c + 6*A*a*b
*c^2 - 12*B*a*b^2*c)^2)/(c^6*(4*a*c - b^2)^3*(4*a*c^5 - b^2*c^4))))/(2*a*(4
*a*c - b^2)^(3/2))) + ((((8*A*a*c^4 - 16*B*a*b*c^3)/c^4 - (8*a*c^2*(4*B*b^7
 + 128*A*a^3*c^4 - 2*A*b^6*c - 48*B*a*b^5*c + 24*A*a*b^4*c^2 - 256*B*a^3*b*
c^3 - 96*A*a^2*b^2*c^3 + 192*B*a^2*b^3*c^2))/(256*a^3*c^6 - 4*b^6*c^3 + 48*
a*b^4*c^4 - 192*a^2*b^2*c^5))*(2*B*b^4 + 12*B*a^2*c^2 - A*b^3*c + 6*A*a*b*c
^2 - 12*B*a*b^2*c))/(8*c^3*(4*a*c - b^2)^(3/2)) - (a*(2*B*b^4 + 12*B*a^2*c^
2 - A*b^3*c + 6*A*a*b*c^2 - 12*B*a*b^2*c)*(4*B*b^7 + 128*A*a^3*c^4 - 2*A*b^
6*c - 48*B*a*b^5*c + 24*A*a*b^4*c^2 - 256*B*a^3*b*c^3 - 96*A*a^2*b^2*c^3 +
192*B*a^2*b^3*c^2))/(c*(4*a*c - b^2)^(3/2)*(256*a^3*c^6 - 4*b^6*c^3 + 48*a*
b^4*c^4 - 192*a^2*b^2*c^5)))/(a*(4*a*c - b^2)) + (b*((((8*A*a*c^4 - 16*B*a*
b*c^3)/c^4 - (8*a*c^2*(4*B*b^7 + 128*A*a^3*c^4 - 2*A*b^6*c - 48*B*a*b^5*c +
 24*A*a*b^4*c^2 - 256*B*a^3*b*c^3 - 96*A*a^2*b^2*c^3 + 192*B*a^2*b^3*c^2))/
(256*a^3*c^6 - 4*b^6*c^3 + 48*a*b^4*c^4 - 192*a^2*b^2*c^5))*(4*B*b^7 + 128*
A*a^3*c^4 - 2*A*b^6*c - 48*B*a*b^5*c + 24*A*a*b^4*c^2 - 256*B*a^3*b*c^3 - 9
6*A*a^2*b^2*c^3 + 192*B*a^2*b^3*c^2))/(2*(256*a^3*c^6 - 4*b^6*c^3 + 48*a*b^
4*c^4 - 192*a^2*b^2*c^5)) - (A^2*a*c^2 + 4*B^2*a*b^2 - 4*A*B*a*b*c)/c^4 + (
a*(2*B*b^4 + 12*B*a^2*c^2 - A*b^3*c + 6*A*a*b*c^2 - 12*B*a*b^2*c)^2)/(c^4*(
4*a*c - b^2)^3)))/(2*a*(4*a*c - b^2)^(3/2))))/(4*B^2*b^8 + A^2*b^6*c^2 + 14
4*B^2*a^4*c^4 - 4*A*B*b^7*c + 36*A^2*a^2*b^2*c^4 + 192*B^2*a^2*b^4*c^2 - 28
8*B^2*a^3*b^2*c^3 - 48*B^2*a*b^6*c - 12*A^2*a*b^4*c^3 - 168*A*B*a^2*b^3*c^3
 + 48*A*B*a*b^5*c^2 + 144*A*B*a^3*b*c^4))*(2*B*b^4 + 12*B*a^2*c^2 - A*b^3*c
 + 6*A*a*b*c^2 - 12*B*a*b^2*c))/(2*c^3*(4*a*c - b^2)^(3/2))
```

3.113 $\int \frac{x^5(A+Bx^2)}{(a+bx^2+cx^4)^2}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 147

$$\int \frac{x^5(A+Bx^2)}{(a+bx^2+cx^4)^2}\,dx = -\frac{x^2(a(bB-2Ac)+(b^2B-Abc-2aBc)\,x^2)}{2c\,(b^2-4ac)\,(a+bx^2+cx^4)} + \frac{(b^3B-6abBc+4aAc^2)\,\text{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2c^2\,(b^2-4ac)^{3/2}} + \frac{B\log\left(a+bx^2+cx^4\right)}{4c^2}$$

```
[Out] -1/2*x^2*(a*(-2*A*c+B*b)+(-A*b*c-2*B*a*c+B*b^2)*x^2)/c/(-4*a*c+b^2)/(c*x^4+
b*x^2+a)+1/2*(4*A*a*c^2-6*B*a*b*c+B*b^3)*arctanh((2*c*x^2+b)/(-4*a*c+b^2)^(
1/2))/c^2/(-4*a*c+b^2)^(3/2)+1/4*B*ln(c*x^4+b*x^2+a)/c^2
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {1265, 832, 648, 632, 212, 642}

$$\int \frac{x^5(A+Bx^2)}{(a+bx^2+cx^4)^2}\,dx = \frac{(4aAc^2-6abBc+b^3B)\,\text{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2c^2\,(b^2-4ac)^{3/2}} - \frac{x^2(x^2(-2aBc-Abc+b^2B)+a(bB-2Ac))}{2c\,(b^2-4ac)\,(a+bx^2+cx^4)} + \frac{B\log\left(a+bx^2+cx^4\right)}{4c^2}$$

```
[In] Int[(x^5*(A + B*x^2))/(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] -1/2*(x^2*(a*(b*B - 2*A*c) + (b^2*B - A*b*c - 2*a*B*c)*x^2))/(c*(b^2 - 4*a*
c)*(a + b*x^2 + c*x^4)) + ((b^3*B - 6*a*b*B*c + 4*a*A*c^2)*ArcTanh[(b + 2*c
*x^2)/Sqrt[b^2 - 4*a*c]])/(2*c^2*(b^2 - 4*a*c)^(3/2)) + (B*Log[a + b*x^2 +
c*x^4])/(4*c^2)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 832

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(-(d + e*x)^(m - 1))*(a + b*x + c*x^2)
^(p + 1)*((2*a*c*(e*f + d*g) - b*(c*d*f + a*e*g) - (2*c^2*d*f + b^2*e*g - c
*(b*e*f + b*d*g + 2*a*e*g))*x)/(c*(p + 1)*(b^2 - 4*a*c))), x] - Dist[1/(c*(
p + 1)*(b^2 - 4*a*c)), Int[(d + e*x)^(m - 2)*(a + b*x + c*x^2)^(p + 1)*Simp
[2*c^2*d^2*f*(2*p + 3) + b*e*g*(a*e*(m - 1) + b*d*(p + 2)) - c*(2*a*e*(e*f*
(m - 1) + d*g*m) + b*d*(d*g*(2*p + 3) - e*f*(m - 2*p - 4))) + e*(b^2*e*g*(m
 + p + 1) + 2*c^2*d*f*(m + 2*p + 2) - c*(2*a*e*g*m + b*(e*f + d*g)*(m + 2*p
 + 2)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*c
, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 1] && ((EqQ[m
, 2] && EqQ[p, -3] && RationalQ[a, b, c, d, e, f, g]) ||  !ILtQ[m + 2*p + 3
, 0])
```

Rule 1265

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rubi steps

$$\text{integral} = \frac{1}{2}\text{Subst}\left(\int \frac{x^2(A+Bx)}{(a+bx+cx^2)^2}\,dx, x, x^2\right)$$

$$= -\frac{x^2(a(bB-2Ac)+(b^2B-Abc-2aBc)\,x^2)}{2c\,(b^2-4ac)\,(a+bx^2+cx^4)} + \frac{\text{Subst}\left(\int \frac{a(bB-2Ac)+B(b^2-4ac)x}{a+bx+cx^2}\,dx, x, x^2\right)}{2c\,(b^2-4ac)}$$

$$= -\frac{x^2(a(bB-2Ac)+(b^2B-Abc-2aBc)\,x^2)}{2c\,(b^2-4ac)\,(a+bx^2+cx^4)} + \frac{B\text{Subst}\left(\int \frac{b+2cx}{a+bx+cx^2}\,dx, x, x^2\right)}{4c^2} - \frac{(b^3B-6abBc+4aAc^2)\,\text{Subst}\left(\int \frac{1}{a+bx+cx^2}\,dx, x, x^2\right)}{4c^2\,(b^2-4ac)}$$

$$= -\frac{x^2(a(bB-2Ac)+(b^2B-Abc-2aBc)\,x^2)}{2c\,(b^2-4ac)\,(a+bx^2+cx^4)} + \frac{B\log\left(a+bx^2+cx^4\right)}{4c^2} + \frac{(b^3B-6abBc+4aAc^2)\,\text{Subst}\left(\int \frac{1}{b^2-4ac-x^2}\,dx, x, b+2cx^2\right)}{2c^2\,(b^2-4ac)}$$

$$= -\frac{x^2(a(bB-2Ac)+(b^2B-Abc-2aBc)\,x^2)}{2c\,(b^2-4ac)\,(a+bx^2+cx^4)} + \frac{(b^3B-6abBc+4aAc^2)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2c^2\,(b^2-4ac)^{3/2}} + \frac{B\log\left(a+bx^2+cx^4\right)}{4c^2}$$

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 160, normalized size of antiderivative = 1.09

$$\int \frac{x^5(A+Bx^2)}{(a+bx^2+cx^4)^2}\,dx$$

$$= \frac{-\frac{2\left(2a^2Bc+b^2(-bB+Ac)x^2+a\left(-b^2B-2Ac^2x^2+bc(A+3Bx^2)\right)\right)}{(b^2-4ac)(a+bx^2+cx^4)} + \frac{2(b^3B-6abBc+4aAc^2)\arctan\left(\frac{b+2cx^2}{\sqrt{-b^2+4ac}}\right)}{(-b^2+4ac)^{3/2}} + B\log\left(a+bx^2+cx\right.}{4c^2}$$

```
[In] Integrate[(x^5*(A + B*x^2))/(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] ((-2*(2*a^2*B*c + b^2*(-(b*B) + A*c)*x^2 + a*(-(b^2*B) - 2*A*c^2*x^2 + b*c*
(A + 3*B*x^2))))/((b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) + (2*(b^3*B - 6*a*b*B*
c + 4*a*A*c^2)*ArcTan[(b + 2*c*x^2)/Sqrt[-b^2 + 4*a*c]])/(-b^2 + 4*a*c)^(3/
2) + B*Log[a + b*x^2 + c*x^4])/(4*c^2)
```

Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 211, normalized size of antiderivative = 1.44

method	result
default	$\frac{-\frac{(2Aac^2-Ab^2c-3Babc+Bb^3)x^2}{c^2(4ac-b^2)}+\frac{a(Abc+2Bac-Bb^2)}{c^2(4ac-b^2)}}{2cx^4+2bx^2+2a}+\frac{\frac{(4Bac-Bb^2)\ln(cx^4+bx^2+a)}{2c}+\frac{2\left(2Aac-abB-\frac{(4Bac-Bb^2)b}{2c}\right)\arctan\left(\frac{2cx^2}{\sqrt{4ac}}\right)}{\sqrt{4ac-b^2}}}{2(4ac-b^2)c}$
risch	Expression too large to display

```
[In] int(x^5*(B*x^2+A)/(c*x^4+b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*(-1/c^2*(2*A*a*c^2-A*b^2*c-3*B*a*b*c+B*b^3)/(4*a*c-b^2)*x^2+a*(A*b*c+2*
B*a*c-B*b^2)/c^2/(4*a*c-b^2))/(c*x^4+b*x^2+a)+1/2/(4*a*c-b^2)/c*(1/2*(4*B*a
*c-B*b^2)/c*ln(c*x^4+b*x^2+a)+2*(2*A*a*c-a*b*B-1/2*(4*B*a*c-B*b^2)*b/c)/(4*
a*c-b^2)^(1/2)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 412 vs. $2(137) = 274$.

Time = 0.30 (sec) , antiderivative size = 849, normalized size of antiderivative = 5.78

$$\int \frac{x^5(A+Bx^2)}{(a+bx^2+cx^4)^2}\,dx$$
$$= \left[\frac{2\,Bab^4+8\,(2\,Ba^3+Aa^2b)c^2+2\,(Bb^5-8\,Aa^2c^3+6\,(2\,Ba^2b+Aab^2)c^2-(7\,Bab^3+Ab^4)c)x^2-(Bab^3}{}\right.$$

```
[In] integrate(x^5*(B*x^2+A)/(c*x^4+b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] [1/4*(2*B*a*b^4 + 8*(2*B*a^3 + A*a^2*b)*c^2 + 2*(B*b^5 - 8*A*a^2*c^3 + 6*(2
*B*a^2*b + A*a*b^2)*c^2 - (7*B*a*b^3 + A*b^4)*c)*x^2 - (B*a*b^3 - 6*B*a^2*b
*c + 4*A*a^2*c^2 + (B*b^3*c - 6*B*a*b*c^2 + 4*A*a*c^3)*x^4 + (B*b^4 - 6*B*a
*b^2*c + 4*A*a*b*c^2)*x^2)*sqrt(b^2 - 4*a*c)*log((2*c^2*x^4 + 2*b*c*x^2 + b
^2 - 2*a*c - (2*c*x^2 + b)*sqrt(b^2 - 4*a*c))/(c*x^4 + b*x^2 + a)) - 2*(6*B
*a^2*b^2 + A*a*b^3)*c + (B*a*b^4 - 8*B*a^2*b^2*c + 16*B*a^3*c^2 + (B*b^4*c
- 8*B*a*b^2*c^2 + 16*B*a^2*c^3)*x^4 + (B*b^5 - 8*B*a*b^3*c + 16*B*a^2*b*c^2
)*x^2)*log(c*x^4 + b*x^2 + a))/(a*b^4*c^2 - 8*a^2*b^2*c^3 + 16*a^3*c^4 + (b
^4*c^3 - 8*a*b^2*c^4 + 16*a^2*c^5)*x^4 + (b^5*c^2 - 8*a*b^3*c^3 + 16*a^2*b*
c^4)*x^2), 1/4*(2*B*a*b^4 + 8*(2*B*a^3 + A*a^2*b)*c^2 + 2*(B*b^5 - 8*A*a^2*
c^3 + 6*(2*B*a^2*b + A*a*b^2)*c^2 - (7*B*a*b^3 + A*b^4)*c)*x^2 + 2*(B*a*b^3
 - 6*B*a^2*b*c + 4*A*a^2*c^2 + (B*b^3*c - 6*B*a*b*c^2 + 4*A*a*c^3)*x^4 + (B
*b^4 - 6*B*a*b^2*c + 4*A*a*b*c^2)*x^2)*sqrt(-b^2 + 4*a*c)*arctan(-(2*c*x^2
+ b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4*a*c)) - 2*(6*B*a^2*b^2 + A*a*b^3)*c + (B*a
*b^4 - 8*B*a^2*b^2*c + 16*B*a^3*c^2 + (B*b^4*c - 8*B*a*b^2*c^2 + 16*B*a^2*c
```

```
^3)*x^4 + (B*b^5 - 8*B*a*b^3*c + 16*B*a^2*b*c^2)*x^2)*log(c*x^4 + b*x^2 + a
))/(a*b^4*c^2 - 8*a^2*b^2*c^3 + 16*a^3*c^4 + (b^4*c^3 - 8*a*b^2*c^4 + 16*a^
2*c^5)*x^4 + (b^5*c^2 - 8*a*b^3*c^3 + 16*a^2*b*c^4)*x^2)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^5(A+Bx^2)}{(a+bx^2+cx^4)^2}\,dx = \text{Timed out}$$

```
[In] integrate(x**5*(B*x**2+A)/(c*x**4+b*x**2+a)**2,x)
```

```
[Out] Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^5(A+Bx^2)}{(a+bx^2+cx^4)^2}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(x^5*(B*x^2+A)/(c*x^4+b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

Giac [A] (verification not implemented)

none

Time = 0.61 (sec) , antiderivative size = 194, normalized size of antiderivative = 1.32

$$\int \frac{x^5(A+Bx^2)}{(a+bx^2+cx^4)^2}\,dx = -\frac{(Bb^3-6\,Babc+4\,Aac^2)\arctan\left(\frac{2\,cx^2+b}{\sqrt{-b^2+4\,ac}}\right)}{2\,(b^2c^2-4\,ac^3)\sqrt{-b^2+4\,ac}} + \frac{B\log\left(cx^4+bx^2+a\right)}{4\,c^2} - \frac{Bb^2cx^4-4\,Bac^2x^4-Bb^3x^2+2\,Babcx^2+2\,Ab^2cx^2-4\,Aac^2x^2-Bab^2+2\,Aabc}{4\,(cx^4+bx^2+a)(b^2c^2-4\,ac^3)}$$

```
[In] integrate(x^5*(B*x^2+A)/(c*x^4+b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] -1/2*(B*b^3 - 6*B*a*b*c + 4*A*a*c^2)*arctan((2*c*x^2 + b)/sqrt(-b^2 + 4*a*c
))/((b^2*c^2 - 4*a*c^3)*sqrt(-b^2 + 4*a*c)) + 1/4*B*log(c*x^4 + b*x^2 + a)/
```

```
c^2 - 1/4*(B*b^2*c*x^4 - 4*B*a*c^2*x^4 - B*b^3*x^2 + 2*B*a*b*c*x^2 + 2*A*b^
2*c*x^2 - 4*A*a*c^2*x^2 - B*a*b^2 + 2*A*a*b*c)/((c*x^4 + b*x^2 + a)*(b^2*c^
2 - 4*a*c^3))
```

Mupad [B] (verification not implemented)

Time = 8.16 (sec) , antiderivative size = 1527, normalized size of antiderivative = 10.39

$$\int \frac{x^5(A+Bx^2)}{(a+bx^2+cx^4)^2}\,dx = \text{Too large to display}$$

```
[In] int((x^5*(A + B*x^2))/(a + b*x^2 + c*x^4)^2,x)
[Out] - ((x^2*(B*b^3 + 2*A*a*c^2 - A*b^2*c - 3*B*a*b*c))/(2*c^2*(4*a*c - b^2)) -
(a*(A*b*c - B*b^2 + 2*B*a*c))/(2*c^2*(4*a*c - b^2)))/(a + b*x^2 + c*x^4) -
(log(a + b*x^2 + c*x^4)*(2*B*b^6 - 128*B*a^3*c^3 - 24*B*a*b^4*c + 96*B*a^2*
b^2*c^2))/(2*(256*a^3*c^5 - 4*b^6*c^2 + 48*a*b^4*c^3 - 192*a^2*b^2*c^4)) -
(atan(((8*a*c^3*(4*a*c - b^2)^3 - 2*b^2*c^2*(4*a*c - b^2)^3)*((((8*B*a + (8
*a*c^2*(2*B*b^6 - 128*B*a^3*c^3 - 24*B*a*b^4*c + 96*B*a^2*b^2*c^2))/(256*a^
3*c^5 - 4*b^6*c^2 + 48*a*b^4*c^3 - 192*a^2*b^2*c^4))*(B*b^3 + 4*A*a*c^2 - 6
*B*a*b*c))/(8*c^2*(4*a*c - b^2)^(3/2)) + (a*(B*b^3 + 4*A*a*c^2 - 6*B*a*b*c)
*(2*B*b^6 - 128*B*a^3*c^3 - 24*B*a*b^4*c + 96*B*a^2*b^2*c^2))/((4*a*c - b^2
)^(3/2)*(256*a^3*c^5 - 4*b^6*c^2 + 48*a*b^4*c^3 - 192*a^2*b^2*c^4)))/(a*(4*
a*c - b^2)) - x^2*(((((6*B*b^3*c^2 + 8*A*a*c^4 - 28*B*a*b*c^3)/(4*a*c^3 - b
^2*c^2) + ((8*b^3*c^4 - 32*a*b*c^5)*(2*B*b^6 - 128*B*a^3*c^3 - 24*B*a*b^4*c
 + 96*B*a^2*b^2*c^2))/(2*(4*a*c^3 - b^2*c^2)*(256*a^3*c^5 - 4*b^6*c^2 + 48*
a*b^4*c^3 - 192*a^2*b^2*c^4)))*(B*b^3 + 4*A*a*c^2 - 6*B*a*b*c))/(8*c^2*(4*a
*c - b^2)^(3/2)) + ((8*b^3*c^4 - 32*a*b*c^5)*(B*b^3 + 4*A*a*c^2 - 6*B*a*b*c
)*(2*B*b^6 - 128*B*a^3*c^3 - 24*B*a*b^4*c + 96*B*a^2*b^2*c^2))/(16*c^2*(4*a
*c - b^2)^(3/2)*(4*a*c^3 - b^2*c^2)*(256*a^3*c^5 - 4*b^6*c^2 + 48*a*b^4*c^3
 - 192*a^2*b^2*c^4)))/(a*(4*a*c - b^2)) + (b*((B^2*b^3 + 2*A*B*a*c^2 - 5*B^
2*a*b*c)/(4*a*c^3 - b^2*c^2) + (((6*B*b^3*c^2 + 8*A*a*c^4 - 28*B*a*b*c^3)/(
4*a*c^3 - b^2*c^2) + ((8*b^3*c^4 - 32*a*b*c^5)*(2*B*b^6 - 128*B*a^3*c^3 - 2
4*B*a*b^4*c + 96*B*a^2*b^2*c^2))/(2*(4*a*c^3 - b^2*c^2)*(256*a^3*c^5 - 4*b^
6*c^2 + 48*a*b^4*c^3 - 192*a^2*b^2*c^4)))*(2*B*b^6 - 128*B*a^3*c^3 - 24*B*a
*b^4*c + 96*B*a^2*b^2*c^2))/(2*(256*a^3*c^5 - 4*b^6*c^2 + 48*a*b^4*c^3 - 19
2*a^2*b^2*c^4)) - (((b^3*c^4)/2 - 2*a*b*c^5)*(B*b^3 + 4*A*a*c^2 - 6*B*a*b*c
)^2)/(c^4*(4*a*c - b^2)^3*(4*a*c^3 - b^2*c^2))))/(2*a*(4*a*c - b^2)^(3/2)))
 + (b*(((8*B*a + (8*a*c^2*(2*B*b^6 - 128*B*a^3*c^3 - 24*B*a*b^4*c + 96*B*a^
2*b^2*c^2))/(256*a^3*c^5 - 4*b^6*c^2 + 48*a*b^4*c^3 - 192*a^2*b^2*c^4))*(2*
B*b^6 - 128*B*a^3*c^3 - 24*B*a*b^4*c + 96*B*a^2*b^2*c^2))/(2*(256*a^3*c^5 -
 4*b^6*c^2 + 48*a*b^4*c^3 - 192*a^2*b^2*c^4)) + (B^2*a)/c^2 - (a*(B*b^3 + 4
*A*a*c^2 - 6*B*a*b*c)^2)/(c^2*(4*a*c - b^2)^3)))/(2*a*(4*a*c - b^2)^(3/2)))
)/(B^2*b^6 + 16*A^2*a^2*c^4 + 36*B^2*a^2*b^2*c^2 - 12*B^2*a*b^4*c + 8*A*B*a
*b^3*c^2 - 48*A*B*a^2*b*c^3))*(B*b^3 + 4*A*a*c^2 - 6*B*a*b*c))/(2*c^2*(4*a*
c - b^2)^(3/2))
```

3.114 $\int \frac{x^3(A+Bx^2)}{(a+bx^2+cx^4)^2} dx$

Optimal result

Integrand size = 25, antiderivative size = 107

$$\int \frac{x^3(A+Bx^2)}{(a+bx^2+cx^4)^2} dx = -\frac{a(bB-2Ac)+(b^2B-Abc-2aBc)x^2}{2c(b^2-4ac)(a+bx^2+cx^4)} - \frac{(Ab-2aB)\operatorname{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{(b^2-4ac)^{3/2}}$$

```
[Out] 1/2*(-a*(-2*A*c+B*b)-(-A*b*c-2*B*a*c+B*b^2)*x^2)/c/(-4*a*c+b^2)/(c*x^4+b*x^
2+a)-(A*b-2*B*a)*arctanh((2*c*x^2+b)/(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(3/2)
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {1265, 791, 632, 212}

$$\int \frac{x^3(A+Bx^2)}{(a+bx^2+cx^4)^2} dx = -\frac{(Ab-2aB)\operatorname{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{(b^2-4ac)^{3/2}} - \frac{x^2(-2aBc-Abc+b^2B)+a(bB-2Ac)}{2c(b^2-4ac)(a+bx^2+cx^4)}$$

```
[In] Int[(x^3*(A + B*x^2))/(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] -1/2*(a*(b*B - 2*A*c) + (b^2*B - A*b*c - 2*a*B*c)*x^2)/(c*(b^2 - 4*a*c)*(a
+ b*x^2 + c*x^4)) - ((A*b - 2*a*B)*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]]
)/(b^2 - 4*a*c)^(3/2)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 791

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> Simp[(-(2*a*c*(e*f + d*g) - b*(c*d*f + a*e*g) - (
b^2*e*g - b*c*(e*f + d*g) + 2*c*(c*d*f - a*e*g))*x))*((a + b*x + c*x^2)^(p
+ 1)/(c*(p + 1)*(b^2 - 4*a*c))), x] - Dist[(b^2*e*g*(p + 2) - 2*a*c*e*g + c
*(2*c*d*f - b*(e*f + d*g))*(2*p + 3))/(c*(p + 1)*(b^2 - 4*a*c)), Int[(a + b
*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 -
 4*a*c, 0] && LtQ[p, -1]
```

Rule 1265

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rubi steps

$$\text{integral} = \frac{1}{2}\text{Subst}\left(\int \frac{x(A+Bx)}{(a+bx+cx^2)^2}\,dx, x, x^2\right)$$

$$= -\frac{a(bB-2Ac)+(b^2B-Abc-2aBc)\,x^2}{2c\,(b^2-4ac)\,(a+bx^2+cx^4)} + \frac{(Ab-2aB)\text{Subst}\left(\int \frac{1}{a+bx+cx^2}\,dx, x, x^2\right)}{2\,(b^2-4ac)}$$

$$= -\frac{a(bB-2Ac)+(b^2B-Abc-2aBc)\,x^2}{2c\,(b^2-4ac)\,(a+bx^2+cx^4)} - \frac{(Ab-2aB)\text{Subst}\left(\int \frac{1}{b^2-4ac-x^2}\,dx, x, b+2cx^2\right)}{b^2-4ac}$$

$$= -\frac{a(bB-2Ac)+(b^2B-Abc-2aBc)\,x^2}{2c\,(b^2-4ac)\,(a+bx^2+cx^4)} - \frac{(Ab-2aB)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{(b^2-4ac)^{3/2}}$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.04

$$\int \frac{x^3(A+Bx^2)}{(a+bx^2+cx^4)^2}\,dx = \frac{abB+b(bB-Ac)x^2-2ac(A+Bx^2)}{2c\,(-b^2+4ac)\,(a+bx^2+cx^4)} - \frac{(Ab-2aB)\arctan\left(\frac{b+2cx^2}{\sqrt{-b^2+4ac}}\right)}{(-b^2+4ac)^{3/2}}$$

```
[In] Integrate[(x^3*(A + B*x^2))/(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] (a*b*B + b*(b*B - A*c)*x^2 - 2*a*c*(A + B*x^2))/(2*c*(-b^2 + 4*a*c)*(a + b*
x^2 + c*x^4)) - ((A*b - 2*a*B)*ArcTan[(b + 2*c*x^2)/Sqrt[-b^2 + 4*a*c]])/(-
b^2 + 4*a*c)^(3/2)
```

Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.18

method	result
default	$\frac{-\frac{\left(Abc+2Bac-B\,b^2\right)x^2}{c\left(4ac-b^2\right)}-\frac{a(2Ac-Bb)}{\left(4ac-b^2\right)c}}{2c\,x^4+2b\,x^2+2a}-\frac{(Ab-2Ba)\arctan\left(\frac{2c\,x^2+b}{\sqrt{4ac-b^2}}\right)}{(4ac-b^2)^{\frac{3}{2}}}$
risch	$\frac{-\frac{\left(Abc+2Bac-B\,b^2\right)x^2}{2c\left(4ac-b^2\right)}-\frac{a(2Ac-Bb)}{2\left(4ac-b^2\right)c}}{c\,x^4+b\,x^2+a}+\frac{\ln\left(\left(-(-4ac+b^2)^{\frac{3}{2}}+4abc-b^3\right)x^2+8c\,a^2-2b^2a\right)Ab}{2(-4ac+b^2)^{\frac{3}{2}}}-\frac{\ln\left(\left(-(-4ac+b^2)^{\frac{3}{2}}+4abc-b^3\right)x^2+\right.}{(-4ac+b^2)^{\frac{3}{2}}}$

```
[In] int(x^3*(B*x^2+A)/(c*x^4+b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*(-(A*b*c+2*B*a*c-B*b^2)/c/(4*a*c-b^2)*x^2-a*(2*A*c-B*b)/(4*a*c-b^2)/c)/
(c*x^4+b*x^2+a)-(A*b-2*B*a)/(4*a*c-b^2)^(3/2)*arctan((2*c*x^2+b)/(4*a*c-b^2
)^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 257 vs. 2(101) = 202.

Time = 0.26 (sec) , antiderivative size = 538, normalized size of antiderivative = 5.03

$$\int \frac{x^3(A+Bx^2)}{(a+bx^2+cx^4)^2}\,dx$$
$$= \left[-\frac{Bab^3+8\,Aa^2c^2+(Bb^4+4\,(2\,Ba^2+Aab)c^2-(6\,Bab^2+Ab^3)c)x^2-((2\,Ba-Ab)c^2x^4+(2\,Bab-A}{2\,(ab^4c-8\,a^2b^2c^2+16\,a^3c^3+(b^4c^2-8\,ab^2c^3+}\right.$$
$$-\frac{Bab^3+8\,Aa^2c^2+(Bb^4+4\,(2\,Ba^2+Aab)c^2-(6\,Bab^2+Ab^3)c)x^2-2\,((2\,Ba-Ab)c^2x^4+(2\,Bab-A}{2\,(ab^4c-8\,a^2b^2c^2+16\,a^3c^3+(b^4c^2-8\,ab^2c^3+16\,a}$$

```
[In] integrate(x^3*(B*x^2+A)/(c*x^4+b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] [-1/2*(B*a*b^3 + 8*A*a^2*c^2 + (B*b^4 + 4*(2*B*a^2 + A*a*b)*c^2 - (6*B*a*b^
2 + A*b^3)*c)*x^2 - ((2*B*a - A*b)*c^2*x^4 + (2*B*a*b - A*b^2)*c*x^2 + (2*B
*a^2 - A*a*b)*c)*sqrt(b^2 - 4*a*c)*log((2*c^2*x^4 + 2*b*c*x^2 + b^2 - 2*a*c
 + (2*c*x^2 + b)*sqrt(b^2 - 4*a*c))/(c*x^4 + b*x^2 + a)) - 2*(2*B*a^2*b + A
*a*b^2)*c)/(a*b^4*c - 8*a^2*b^2*c^2 + 16*a^3*c^3 + (b^4*c^2 - 8*a*b^2*c^3 +
 16*a^2*c^4)*x^4 + (b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*x^2), -1/2*(B*a*b^3
 + 8*A*a^2*c^2 + (B*b^4 + 4*(2*B*a^2 + A*a*b)*c^2 - (6*B*a*b^2 + A*b^3)*c)*
x^2 - 2*((2*B*a - A*b)*c^2*x^4 + (2*B*a*b - A*b^2)*c*x^2 + (2*B*a^2 - A*a*b
)*c)*sqrt(-b^2 + 4*a*c)*arctan(-(2*c*x^2 + b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4*a
*c)) - 2*(2*B*a^2*b + A*a*b^2)*c)/(a*b^4*c - 8*a^2*b^2*c^2 + 16*a^3*c^3 + (
b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*x^4 + (b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c
^3)*x^2)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 394 vs. $2(99) = 198$.

Time = 4.23 (sec) , antiderivative size = 394, normalized size of antiderivative = 3.68

$$\int \frac{x^3(A+Bx^2)}{(a+bx^2+cx^4)^2}\,dx =$$
$$-\frac{\sqrt{-\frac{1}{(4ac-b^2)^3}}(-Ab+2Ba)\log\left(x^2+\frac{-Ab^2+2Bab-16a^2c^2\sqrt{-\frac{1}{\left(4ac-b^2\right)^3}}(-Ab+2Ba)+8ab^2c\sqrt{-\frac{1}{\left(4ac-b^2\right)^3}}(-Ab+2Ba)-b^4\sqrt{}}{-2Abc+4Bac}\right.}{2}$$
$$+\frac{\sqrt{-\frac{1}{(4ac-b^2)^3}}(-Ab+2Ba)\log\left(x^2+\frac{-Ab^2+2Bab+16a^2c^2\sqrt{-\frac{1}{\left(4ac-b^2\right)^3}}(-Ab+2Ba)-8ab^2c\sqrt{-\frac{1}{\left(4ac-b^2\right)^3}}(-Ab+2Ba)+b^4\sqrt{}}{-2Abc+4Bac}\right.}{2}$$
$$+\frac{-2Aac+Bab+x^2(-Abc-2Bac+Bb^2)}{8a^2c^2-2ab^2c+x^4\cdot(8ac^3-2b^2c^2)+x^2\cdot(8abc^2-2b^3c)}$$

```
[In] integrate(x**3*(B*x**2+A)/(c*x**4+b*x**2+a)**2,x)
```

```
[Out] -sqrt(-1/(4*a*c - b**2)**3)*(-A*b + 2*B*a)*log(x**2 + (-A*b**2 + 2*B*a*b -
16*a**2*c**2*sqrt(-1/(4*a*c - b**2)**3)*(-A*b + 2*B*a) + 8*a*b**2*c*sqrt(-1
/(4*a*c - b**2)**3)*(-A*b + 2*B*a) - b**4*sqrt(-1/(4*a*c - b**2)**3)*(-A*b
+ 2*B*a))/(-2*A*b*c + 4*B*a*c))/2 + sqrt(-1/(4*a*c - b**2)**3)*(-A*b + 2*B*
a)*log(x**2 + (-A*b**2 + 2*B*a*b + 16*a**2*c**2*sqrt(-1/(4*a*c - b**2)**3)*
(-A*b + 2*B*a) - 8*a*b**2*c*sqrt(-1/(4*a*c - b**2)**3)*(-A*b + 2*B*a) + b**
4*sqrt(-1/(4*a*c - b**2)**3)*(-A*b + 2*B*a))/(-2*A*b*c + 4*B*a*c))/2 + (-2*
A*a*c + B*a*b + x**2*(-A*b*c - 2*B*a*c + B*b**2))/(8*a**2*c**2 - 2*a*b**2*c
 + x**4*(8*a*c**3 - 2*b**2*c**2) + x**2*(8*a*b*c**2 - 2*b**3*c))
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^3(A+Bx^2)}{(a+bx^2+cx^4)^2}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(x^3*(B*x^2+A)/(c*x^4+b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

Giac [A] (verification not implemented)

none

Time = 0.59 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.12

$$\int \frac{x^3(A+Bx^2)}{(a+bx^2+cx^4)^2}\,dx = -\frac{(2\,Ba-Ab)\arctan\left(\frac{2\,cx^2+b}{\sqrt{-b^2+4\,ac}}\right)}{(b^2-4\,ac)\sqrt{-b^2+4\,ac}} - \frac{Bb^2x^2-2\,Bacx^2-Abcx^2+Bab-2\,Aac}{2\,(cx^4+bx^2+a)(b^2c-4\,ac^2)}$$

```
[In] integrate(x^3*(B*x^2+A)/(c*x^4+b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] -(2*B*a - A*b)*arctan((2*c*x^2 + b)/sqrt(-b^2 + 4*a*c))/((b^2 - 4*a*c)*sqrt
(-b^2 + 4*a*c)) - 1/2*(B*b^2*x^2 - 2*B*a*c*x^2 - A*b*c*x^2 + B*a*b - 2*A*a*
c)/((c*x^4 + b*x^2 + a)*(b^2*c - 4*a*c^2))
```

Mupad [B] (verification not implemented)

Time = 7.61 (sec) , antiderivative size = 283, normalized size of antiderivative = 2.64

$$\int \frac{x^3(A+Bx^2)}{(a+bx^2+cx^4)^2}\,dx = -\frac{\frac{x^2\,(-B\,b^2+A\,c\,b+2\,B\,a\,c)}{2\,c\,(4\,a\,c-b^2)}+\frac{a\,(2\,A\,c-B\,b)}{2\,c\,(4\,a\,c-b^2)}}{c\,x^4+b\,x^2+a} - \frac{\operatorname{atan}\left(\frac{(4\,a\,c-b^2)^4\,\left(x^2\,\left(\frac{(A\,b-2\,B\,a)\,\left(A\,b\,c^2-2\,B\,a\,c^2\right)}{a\,\left(4\,a\,c-b^2\right)^{7/2}}+\frac{\left(2\,b^3\,c^2-8\,a\,b\,c^3\right)\,(A\,b-2\,B\,a)^2\,\left(b^3-4\,a\,b\,c\right)}{2\,a\,\left(4\,a\,c-b^2\right)^{13/2}}\right)-\frac{2\,c^2\,(A\,b-2\,B\,a)^2\,\left(b^3-4\,a\,b\,c\right)}{\left(4\,a\,c-b^2\right)^{11/2}}\right)}{2\,A^2\,b^2\,c^2-8\,A\,B\,a\,b\,c^2+8\,B^2\,a^2\,c^2}\right)\,(A\,b}{(4\,a\,c-b^2)^{3/2}}$$

```
[In] int((x^3*(A + B*x^2))/(a + b*x^2 + c*x^4)^2,x)
```

```
[Out] - ((x^2*(A*b*c - B*b^2 + 2*B*a*c))/(2*c*(4*a*c - b^2)) + (a*(2*A*c - B*b))/
(2*c*(4*a*c - b^2)))/(a + b*x^2 + c*x^4) - (atan(((4*a*c - b^2)^4*(x^2*(((A
*b - 2*B*a)*(A*b*c^2 - 2*B*a*c^2))/(a*(4*a*c - b^2)^(7/2)) + ((2*b^3*c^2 -
8*a*b*c^3)*(A*b - 2*B*a)^2*(b^3 - 4*a*b*c))/(2*a*(4*a*c - b^2)^(13/2))) - (
2*c^2*(A*b - 2*B*a)^2*(b^3 - 4*a*b*c))/(4*a*c - b^2)^(11/2)))/(2*A^2*b^2*c^
2 + 8*B^2*a^2*c^2 - 8*A*B*a*b*c^2))*(A*b - 2*B*a))/(4*a*c - b^2)^(3/2)
```

3.115 $\int \frac{x(A+Bx^2)}{(a+bx^2+cx^4)^2}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 94

$$\int \frac{x(A+Bx^2)}{(a+bx^2+cx^4)^2}\,dx = -\frac{Ab-2aB-(bB-2Ac)x^2}{2(b^2-4ac)(a+bx^2+cx^4)} - \frac{(bB-2Ac)\operatorname{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{(b^2-4ac)^{3/2}}$$

```
[Out] 1/2*(-A*b+2*B*a+(-2*A*c+B*b)*x^2)/(-4*a*c+b^2)/(c*x^4+b*x^2+a)-(-2*A*c+B*b)
*arctanh((2*c*x^2+b)/(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(3/2)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {1261, 652, 632, 212}

$$\int \frac{x(A+Bx^2)}{(a+bx^2+cx^4)^2}\,dx = -\frac{(bB-2Ac)\operatorname{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{(b^2-4ac)^{3/2}} - \frac{-2aB-(x^2(bB-2Ac))+Ab}{2(b^2-4ac)(a+bx^2+cx^4)}$$

```
[In] Int[(x*(A + B*x^2))/(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] -1/2*(A*b - 2*a*B - (b*B - 2*A*c)*x^2)/((b^2 - 4*a*c)*(a + b*x^2 + c*x^4))
- ((b*B - 2*A*c)*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(b^2 - 4*a*c)^(3
/2)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 652

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[((b*d - 2*a*e + (2*c*d - b*e)*x)/((p + 1)*(b^2 - 4*a*c)))*(a + b*
x + c*x^2)^(p + 1), x] - Dist[(2*p + 3)*((2*c*d - b*e)/((p + 1)*(b^2 - 4*a*
c))), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] &&
 NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2]
```

Rule 1261

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(
p_.), x_Symbol] :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x],
 x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{2}\text{Subst}\left(\int \frac{A+Bx}{(a+bx+cx^2)^2}\,dx, x, x^2\right)\\
&= -\frac{Ab-2aB-(bB-2Ac)x^2}{2(b^2-4ac)(a+bx^2+cx^4)} + \frac{(bB-2Ac)\text{Subst}\left(\int \frac{1}{a+bx+cx^2}\,dx, x, x^2\right)}{2(b^2-4ac)}\\
&= -\frac{Ab-2aB-(bB-2Ac)x^2}{2(b^2-4ac)(a+bx^2+cx^4)} - \frac{(bB-2Ac)\text{Subst}\left(\int \frac{1}{b^2-4ac-x^2}\,dx, x, b+2cx^2\right)}{b^2-4ac}\\
&= -\frac{Ab-2aB-(bB-2Ac)x^2}{2(b^2-4ac)(a+bx^2+cx^4)} - \frac{(bB-2Ac)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{(b^2-4ac)^{3/2}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.07

$$\int \frac{x(A+Bx^2)}{(a+bx^2+cx^4)^2}\,dx = \frac{\frac{B(2a+bx^2)-A(b+2cx^2)}{a+bx^2+cx^4} + \frac{2(bB-2Ac)\arctan\left(\frac{b+2cx^2}{\sqrt{-b^2+4ac}}\right)}{\sqrt{-b^2+4ac}}}{2(b^2-4ac)}$$

```
[In] Integrate[(x*(A + B*x^2))/(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] ((B*(2*a + b*x^2) - A*(b + 2*c*x^2))/(a + b*x^2 + c*x^4) + (2*(b*B - 2*A*c)
*ArcTan[(b + 2*c*x^2)/Sqrt[-b^2 + 4*a*c]])/Sqrt[-b^2 + 4*a*c])/(2*(b^2 - 4*
a*c))
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.01

method	result
default	$\frac{(2Ac-Bb)x^2+Ab-2Ba}{2(4ac-b^2)(c\,x^4+b\,x^2+a)}+\frac{(2Ac-Bb)\arctan\left(\frac{2c\,x^2+b}{\sqrt{4ac-b^2}}\right)}{(4ac-b^2)^{\frac{3}{2}}}$
risch	$\frac{\frac{(2Ac-Bb)x^2}{8ac-2b^2}+\frac{Ab-2Ba}{8ac-2b^2}}{c\,x^4+b\,x^2+a}+\frac{\ln\left(\left(\left(-4ac+b^2\right)^{\frac{3}{2}}+4abc-b^3\right)x^2+8c\,a^2-2b^2a\right)Ac}{(-4ac+b^2)^{\frac{3}{2}}}-\frac{\ln\left(\left(\left(-4ac+b^2\right)^{\frac{3}{2}}+4abc-b^3\right)x^2+8c\,a^2-2b^2a\right)Bb}{2(-4ac+b^2)^{\frac{3}{2}}}$

```
[In] int(x*(B*x^2+A)/(c*x^4+b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*((2*A*c-B*b)*x^2+A*b-2*B*a)/(4*a*c-b^2)/(c*x^4+b*x^2+a)+(2*A*c-B*b)/(4*
a*c-b^2)^(3/2)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 225 vs. $2(88) = 176$.

Time = 0.26 (sec) , antiderivative size = 474, normalized size of antiderivative = 5.04

$$\int \frac{x(A+Bx^2)}{(a+bx^2+cx^4)^2}\,dx$$
$$= \left[\frac{2\,Bab^2 - Ab^3 + (Bb^3 + 8\,Aac^2 - 2\,(2\,Bab + Ab^2)c)x^2 + ((Bbc - 2\,Ac^2)x^4 + Bab - 2\,Aac + (Bb^2 - 2\,Abc}{2\,(ab^4 - 8\,a^2b^2c + 16\,a^3c^2 + (b^4c - 8\,ab^2c^2 + 16\,a^2c^3)x^4 +}\right.$$

```
[In] integrate(x*(B*x^2+A)/(c*x^4+b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] [1/2*(2*B*a*b^2 - A*b^3 + (B*b^3 + 8*A*a*c^2 - 2*(2*B*a*b + A*b^2)*c)*x^2 +
 ((B*b*c - 2*A*c^2)*x^4 + B*a*b - 2*A*a*c + (B*b^2 - 2*A*b*c)*x^2)*sqrt(b^2
 - 4*a*c)*log((2*c^2*x^4 + 2*b*c*x^2 + b^2 - 2*a*c - (2*c*x^2 + b)*sqrt(b^2
 - 4*a*c))/(c*x^4 + b*x^2 + a)) - 4*(2*B*a^2 - A*a*b)*c)/(a*b^4 - 8*a^2*b^2
*c + 16*a^3*c^2 + (b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*x^4 + (b^5 - 8*a*b^3*c
 + 16*a^2*b*c^2)*x^2), 1/2*(2*B*a*b^2 - A*b^3 + (B*b^3 + 8*A*a*c^2 - 2*(2*B
*a*b + A*b^2)*c)*x^2 - 2*((B*b*c - 2*A*c^2)*x^4 + B*a*b - 2*A*a*c + (B*b^2
- 2*A*b*c)*x^2)*sqrt(-b^2 + 4*a*c)*arctan(-(2*c*x^2 + b)*sqrt(-b^2 + 4*a*c)
/(b^2 - 4*a*c)) - 4*(2*B*a^2 - A*a*b)*c)/(a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2
+ (b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*x^4 + (b^5 - 8*a*b^3*c + 16*a^2*b*c^2)
*x^2)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 374 vs. 2(83) = 166.

Time = 2.28 (sec) , antiderivative size = 374, normalized size of antiderivative = 3.98

$$\int \frac{x(A+Bx^2)}{(a+bx^2+cx^4)^2}\,dx = \frac{\sqrt{-\frac{1}{(4ac-b^2)^3}}(-2Ac+Bb)\log\left(x^2+\frac{-2Abc+Bb^2-16a^2c^2\sqrt{-\frac{1}{(4ac-b^2)^3}}(-2Ac+Bb)+8ab^2c\sqrt{-\frac{1}{(4ac-b^2)^3}}(-2Ac+Bb)-b^4\sqrt{-\frac{1}{(}}}{-4Ac^2+2Bbc}\right.}{2} - \frac{\sqrt{-\frac{1}{(4ac-b^2)^3}}(-2Ac+Bb)\log\left(x^2+\frac{-2Abc+Bb^2+16a^2c^2\sqrt{-\frac{1}{(4ac-b^2)^3}}(-2Ac+Bb)-8ab^2c\sqrt{-\frac{1}{(4ac-b^2)^3}}(-2Ac+Bb)+b^4\sqrt{}}{-4Ac^2+2Bbc}\right.}{2} + \frac{Ab-2Ba+x^2\cdot(2Ac-Bb)}{8a^2c-2ab^2+x^4\cdot(8ac^2-2b^2c)+x^2\cdot(8abc-2b^3)}$$

```
[In] integrate(x*(B*x**2+A)/(c*x**4+b*x**2+a)**2,x)
```

```
[Out] sqrt(-1/(4*a*c - b**2)**3)*(-2*A*c + B*b)*log(x**2 + (-2*A*b*c + B*b**2 - 1
6*a**2*c**2*sqrt(-1/(4*a*c - b**2)**3)*(-2*A*c + B*b) + 8*a*b**2*c*sqrt(-1/
(4*a*c - b**2)**3)*(-2*A*c + B*b) - b**4*sqrt(-1/(4*a*c - b**2)**3)*(-2*A*c
 + B*b))/(-4*A*c**2 + 2*B*b*c))/2 - sqrt(-1/(4*a*c - b**2)**3)*(-2*A*c + B*
b)*log(x**2 + (-2*A*b*c + B*b**2 + 16*a**2*c**2*sqrt(-1/(4*a*c - b**2)**3)*
(-2*A*c + B*b) - 8*a*b**2*c*sqrt(-1/(4*a*c - b**2)**3)*(-2*A*c + B*b) + b**
4*sqrt(-1/(4*a*c - b**2)**3)*(-2*A*c + B*b))/(-4*A*c**2 + 2*B*b*c))/2 + (A*
b - 2*B*a + x**2*(2*A*c - B*b))/(8*a**2*c - 2*a*b**2 + x**4*(8*a*c**2 - 2*b
**2*c) + x**2*(8*a*b*c - 2*b**3))
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x(A+Bx^2)}{(a+bx^2+cx^4)^2}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(x*(B*x^2+A)/(c*x^4+b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

Giac [A] (verification not implemented)

none

Time = 0.56 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.09

$$\int \frac{x(A+Bx^2)}{(a+bx^2+cx^4)^2}\,dx = \frac{(Bb-2\,Ac)\arctan\left(\frac{2\,cx^2+b}{\sqrt{-b^2+4\,ac}}\right)}{(b^2-4\,ac)\sqrt{-b^2+4\,ac}} + \frac{Bbx^2-2\,Acx^2+2\,Ba-Ab}{2\,(cx^4+bx^2+a)(b^2-4\,ac)}$$

```
[In] integrate(x*(B*x^2+A)/(c*x^4+b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] (B*b - 2*A*c)*arctan((2*c*x^2 + b)/sqrt(-b^2 + 4*a*c))/((b^2 - 4*a*c)*sqrt(
-b^2 + 4*a*c)) + 1/2*(B*b*x^2 - 2*A*c*x^2 + 2*B*a - A*b)/((c*x^4 + b*x^2 +
a)*(b^2 - 4*a*c))
```

Mupad [B] (verification not implemented)

Time = 7.67 (sec) , antiderivative size = 264, normalized size of antiderivative = 2.81

$$\int \frac{x(A+Bx^2)}{(a+bx^2+cx^4)^2}\,dx = \frac{\frac{A\,b-2\,B\,a}{2\,(4\,a\,c-b^2)}+\frac{x^2\,(2\,A\,c-B\,b)}{2\,(4\,a\,c-b^2)}}{c\,x^4+b\,x^2+a} + \frac{\mathrm{atan}\left(\frac{\left(x^2\left(\frac{(2\,A\,c-B\,b)\,\left(2\,A\,c^3-B\,b\,c^2\right)}{a\,\left(4\,a\,c-b^2\right)^{7/2}}+\frac{\left(2\,b^3\,c^2-8\,a\,b\,c^3\right)\,(2\,A\,c-B\,b)^2\,\left(b^3-4\,a\,b\,c\right)}{2\,a\,\left(4\,a\,c-b^2\right)^{13/2}}\right)-\frac{2\,c^2\,(2\,A\,c-B\,b)^2\,\left(b^3-4\,a\,b\,c\right)}{\left(4\,a\,c-b^2\right)^{11/2}}\right)\,\left(4\,a\,c-b^2\right)^4}{8\,A^2\,c^4-8\,A\,B\,b\,c^3+2\,B^2\,b^2\,c^2}\right)\,(2\,A\,c-}{(4\,a\,c-b^2)^{3/2}}$$

```
[In] int((x*(A + B*x^2))/(a + b*x^2 + c*x^4)^2,x)
```

```
[Out] ((A*b - 2*B*a)/(2*(4*a*c - b^2)) + (x^2*(2*A*c - B*b))/(2*(4*a*c - b^2)))/(
a + b*x^2 + c*x^4) + (atan(((x^2*(((2*A*c - B*b)*(2*A*c^3 - B*b*c^2))/(a*(4
*a*c - b^2)^(7/2)) + ((2*b^3*c^2 - 8*a*b*c^3)*(2*A*c - B*b)^2*(b^3 - 4*a*b*
c))/(2*a*(4*a*c - b^2)^(13/2))) - (2*c^2*(2*A*c - B*b)^2*(b^3 - 4*a*b*c))/(
4*a*c - b^2)^(11/2))*(4*a*c - b^2)^4)/(8*A^2*c^4 + 2*B^2*b^2*c^2 - 8*A*B*b*
c^3))*(2*A*c - B*b))/(4*a*c - b^2)^(3/2)
```

3.116 $\int \frac{A+Bx^2}{x(a+bx^2+cx^4)^2}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 150

$$\int \frac{A+Bx^2}{x\left(a+bx^2+cx^4\right)^2}\,dx = -\frac{abB-A(b^2-2ac)-(Ab-2aB)cx^2}{2a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} + \frac{\left(4a^2Bc+A(b^3-6abc)\right)\operatorname{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2a^2\left(b^2-4ac\right)^{3/2}} + \frac{A\log(x)}{a^2} - \frac{A\log\left(a+bx^2+cx^4\right)}{4a^2}$$

```
[Out] 1/2*(-a*b*B+A*(-2*a*c+b^2)+(A*b-2*B*a)*c*x^2)/a/(-4*a*c+b^2)/(c*x^4+b*x^2+a
)+1/2*(4*a^2*B*c+A*(-6*a*b*c+b^3))*arctanh((2*c*x^2+b)/(-4*a*c+b^2)^(1/2))/
a^2/(-4*a*c+b^2)^(3/2)+A*ln(x)/a^2-1/4*A*ln(c*x^4+b*x^2+a)/a^2
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.280, Rules used = {1265, 836, 814, 648, 632, 212, 642}

$$\int \frac{A+Bx^2}{x\left(a+bx^2+cx^4\right)^2}\,dx = \frac{\left(4a^2Bc+A(b^3-6abc)\right)\operatorname{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2a^2\left(b^2-4ac\right)^{3/2}} - \frac{A\log\left(a+bx^2+cx^4\right)}{4a^2} + \frac{A\log(x)}{a^2} - \frac{-A(b^2-2ac)-\left(cx^2(Ab-2aB)\right)+abB}{2a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}$$

```
[In] Int[(A + B*x^2)/(x*(a + b*x^2 + c*x^4)^2),x]
```

```
[Out] -1/2*(a*b*B - A*(b^2 - 2*a*c) - (A*b - 2*a*B)*c*x^2)/(a*(b^2 - 4*a*c)*(a +
b*x^2 + c*x^4)) + ((4*a^2*B*c + A*(b^3 - 6*a*b*c))*ArcTanh[(b + 2*c*x^2)/Sq
rt[b^2 - 4*a*c]])/(2*a^2*(b^2 - 4*a*c)^(3/2)) + (A*Log[x])/a^2 - (A*Log[a +
 b*x^2 + c*x^4])/(4*a^2)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 814

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)/(a +
 b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 836

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(f*(b*c*d - b^2*e + 2
*a*c*e) - a*g*(2*c*d - b*e) + c*(f*(2*c*d - b*e) - g*(b*d - 2*a*e))*x)*((a
+ b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2))), x]
 + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^m*
(a + b*x + c*x^2)^(p + 1)*Simp[f*(b*c*d*e*(2*p - m + 2) + b^2*e^2*(p + m +
2) - 2*c^2*d^2*(2*p + 3) - 2*a*c*e^2*(m + 2*p + 3)) - g*(a*e*(b*e - 2*c*d*m
 + b*e*m) - b*d*(3*c*d - b*e + 2*c*d*p - b*e*p)) + c*e*(g*(b*d - 2*a*e) - f
```

```
*(2*c*d - b*e))*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1
] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 1265

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rubi steps

$$\text{integral} = \frac{1}{2}\text{Subst}\left(\int \frac{A+Bx}{x\left(a+bx+cx^2\right)^2}\,dx, x, x^2\right)$$

$$= -\frac{abB - A(b^2-2ac) - (Ab-2aB)cx^2}{2a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} - \frac{\text{Subst}\left(\int \frac{-A(b^2-4ac)-(Ab-2aB)cx}{x(a+bx+cx^2)}\,dx, x, x^2\right)}{2a\left(b^2-4ac\right)}$$

$$= -\frac{abB - A(b^2-2ac) - (Ab-2aB)cx^2}{2a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} - \frac{\text{Subst}\left(\int \left(\frac{A(-b^2+4ac)}{ax} + \frac{2a^2Bc+A(b^3-5abc)+Ac(b^2-4ac)x}{a(a+bx+cx^2)}\right)dx, x, x^2\right)}{2a\left(b^2-4ac\right)}$$

$$= -\frac{abB - A(b^2-2ac) - (Ab-2aB)cx^2}{2a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} + \frac{A\log(x)}{a^2} - \frac{\text{Subst}\left(\int \frac{2a^2Bc+A(b^3-5abc)+Ac(b^2-4ac)x}{a+bx+cx^2}\,dx, x, x^2\right)}{2a^2\left(b^2-4ac\right)}$$

$$= -\frac{abB - A(b^2-2ac) - (Ab-2aB)cx^2}{2a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} + \frac{A\log(x)}{a^2} - \frac{A\text{Subst}\left(\int \frac{b+2cx}{a+bx+cx^2}\,dx, x, x^2\right)}{4a^2} - \frac{\left(4a^2Bc+A(b^3-6abc)\right)\text{Subst}\left(\int \frac{1}{a+bx+cx^2}\,dx, x, x^2\right)}{4a^2\left(b^2-4ac\right)}$$

$$= -\frac{abB - A(b^2-2ac) - (Ab-2aB)cx^2}{2a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} + \frac{A\log(x)}{a^2} - \frac{A\log\left(a+bx^2+cx^4\right)}{4a^2} + \frac{\left(4a^2Bc+A(b^3-6abc)\right)\text{Subst}\left(\int \frac{1}{b^2-4ac-x^2}\,dx, x, b+2cx^2\right)}{2a^2\left(b^2-4ac\right)}$$

$$= -\frac{abB - A(b^2-2ac) - (Ab-2aB)cx^2}{2a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} + \frac{\left(4a^2Bc+A(b^3-6abc)\right)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2a^2\left(b^2-4ac\right)^{3/2}} + \frac{A\log(x)}{a^2} - \frac{A\log\left(a+bx^2+cx^4\right)}{4a^2}$$

Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 243, normalized size of antiderivative = 1.62

$$\int \frac{A+Bx^2}{x\left(a+bx^2+cx^4\right)^2}\,dx$$
$$= \frac{-\frac{2a\left(aB\left(b+2cx^2\right)-A\left(b^2-2ac+bcx^2\right)\right)}{\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}+4A\log(x)-\frac{\left(4a^2Bc+A\left(b^3-6abc+b^2\sqrt{b^2-4ac}-4ac\sqrt{b^2-4ac}\right)\right)\log\left(b-\sqrt{b^2-4ac}+2cx^2\right)}{\left(b^2-4ac\right)^{3/2}}+\frac{(4a\cdots}{\cdots}}{4a^2}$$

```
[In] Integrate[(A + B*x^2)/(x*(a + b*x^2 + c*x^4)^2),x]
```

```
[Out] ((-2*a*(a*B*(b + 2*c*x^2) - A*(b^2 - 2*a*c + b*c*x^2)))/((b^2 - 4*a*c)*(a +
 b*x^2 + c*x^4)) + 4*A*Log[x] - ((4*a^2*B*c + A*(b^3 - 6*a*b*c + b^2*Sqrt[b
^2 - 4*a*c] - 4*a*c*Sqrt[b^2 - 4*a*c]))*Log[b - Sqrt[b^2 - 4*a*c] + 2*c*x^2
])/(b^2 - 4*a*c)^(3/2) + ((4*a^2*B*c + A*(b^3 - 6*a*b*c - b^2*Sqrt[b^2 - 4*
a*c] + 4*a*c*Sqrt[b^2 - 4*a*c]))*Log[b + Sqrt[b^2 - 4*a*c] + 2*c*x^2])/(b^2
 - 4*a*c)^(3/2))/(4*a^2)
```

Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 212, normalized size of antiderivative = 1.41

method	result
default	$\frac{A\ln(x)}{a^2}-\frac{\frac{\frac{ac(Ab-2Ba)x^2}{4ac-b^2}-\frac{a\left(2Aac-Ab^2+abB\right)}{4ac-b^2}}{cx^4+bx^2+a}+\frac{\frac{\left(4Aac^2-Ab^2c\right)\ln\left(cx^4+bx^2+a\right)}{2c}+\frac{2\left(5Aabc-Ab^3-2a^2Bc-\frac{\left(4Aac^2-Ab^2c\right)b}{2c}\right)\arctan\left(\frac{2\cdots}{\sqrt{\cdots}}\right)}{\sqrt{4ac-b^2}}}{4ac-b^2}}{2a^2}$
risch	$\frac{-\frac{c(Ab-2Ba)x^2}{2a\left(4ac-b^2\right)}+\frac{2Aac-Ab^2+abB}{2\left(4ac-b^2\right)a}}{cx^4+bx^2+a}+\frac{A\ln(x)}{a^2}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\left(64a^5c^3-48b^2a^4c^2+12b^4a^3c-a^2b^6\right)\_Z^2+\left(64c^3a^3A-48a^2b^2c^2A+12ab^4cA\cdots\right.\right.}\cdots\right.}{\cdots}$

```
[In] int((B*x^2+A)/x/(c*x^4+b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] A*ln(x)/a^2-1/2/a^2*((a*c*(A*b-2*B*a)/(4*a*c-b^2)*x^2-a*(2*A*a*c-A*b^2+B*a*
b)/(4*a*c-b^2))/(c*x^4+b*x^2+a)+1/(4*a*c-b^2)*(1/2*(4*A*a*c^2-A*b^2*c)/c*ln
(c*x^4+b*x^2+a)+2*(5*A*a*b*c-A*b^3-2*a^2*B*c-1/2*(4*A*a*c^2-A*b^2*c)*b/c)/(
4*a*c-b^2)^(1/2)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 495 vs. $2(140) = 280$.

Time = 0.60 (sec) , antiderivative size = 1014, normalized size of antiderivative = 6.76

$$\int \frac{A + Bx^2}{x(a + bx^2 + cx^4)^2}\,dx$$
$$= \Bigg[-\frac{2\,Ba^2b^3 - 2\,Aab^4 - 16\,Aa^3c^2 - 2\,(4\,(2\,Ba^3 - Aa^2b)c^2 - (2\,Ba^2b^2 - Aab^3)c)x^2 - (Aab^3 + (Ab^3c + 2\,(2}{}$$

$$-\frac{2\,Ba^2b^3 - 2\,Aab^4 - 16\,Aa^3c^2 - 2\,(4\,(2\,Ba^3 - Aa^2b)c^2 - (2\,Ba^2b^2 - Aab^3)c)x^2 - 2\,(Aab^3 + (Ab^3c + 2\,(}{}$$

```
[In] integrate((B*x^2+A)/x/(c*x^4+b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] [-1/4*(2*B*a^2*b^3 - 2*A*a*b^4 - 16*A*a^3*c^2 - 2*(4*(2*B*a^3 - A*a^2*b)*c^
2 - (2*B*a^2*b^2 - A*a*b^3)*c)*x^2 - (A*a*b^3 + (A*b^3*c + 2*(2*B*a^2 - 3*A
*a*b)*c^2)*x^4 + (A*b^4 + 2*(2*B*a^2*b - 3*A*a*b^2)*c)*x^2 + 2*(2*B*a^3 - 3
*A*a^2*b)*c)*sqrt(b^2 - 4*a*c)*log((2*c^2*x^4 + 2*b*c*x^2 + b^2 - 2*a*c + (
2*c*x^2 + b)*sqrt(b^2 - 4*a*c))/(c*x^4 + b*x^2 + a)) - 4*(2*B*a^3*b - 3*A*a
^2*b^2)*c + (A*a*b^4 - 8*A*a^2*b^2*c + 16*A*a^3*c^2 + (A*b^4*c - 8*A*a*b^2*
c^2 + 16*A*a^2*c^3)*x^4 + (A*b^5 - 8*A*a*b^3*c + 16*A*a^2*b*c^2)*x^2)*log(c
*x^4 + b*x^2 + a) - 4*(A*a*b^4 - 8*A*a^2*b^2*c + 16*A*a^3*c^2 + (A*b^4*c -
8*A*a*b^2*c^2 + 16*A*a^2*c^3)*x^4 + (A*b^5 - 8*A*a*b^3*c + 16*A*a^2*b*c^2)*
x^2)*log(x))/(a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2 + (a^2*b^4*c - 8*a^3*b^2*c
^2 + 16*a^4*c^3)*x^4 + (a^2*b^5 - 8*a^3*b^3*c + 16*a^4*b*c^2)*x^2), -1/4*(2
*B*a^2*b^3 - 2*A*a*b^4 - 16*A*a^3*c^2 - 2*(4*(2*B*a^3 - A*a^2*b)*c^2 - (2*B
*a^2*b^2 - A*a*b^3)*c)*x^2 - 2*(A*a*b^3 + (A*b^3*c + 2*(2*B*a^2 - 3*A*a*b)*
c^2)*x^4 + (A*b^4 + 2*(2*B*a^2*b - 3*A*a*b^2)*c)*x^2 + 2*(2*B*a^3 - 3*A*a^2
*b)*c)*sqrt(-b^2 + 4*a*c)*arctan(-(2*c*x^2 + b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4
*a*c)) - 4*(2*B*a^3*b - 3*A*a^2*b^2)*c + (A*a*b^4 - 8*A*a^2*b^2*c + 16*A*a^
3*c^2 + (A*b^4*c - 8*A*a*b^2*c^2 + 16*A*a^2*c^3)*x^4 + (A*b^5 - 8*A*a*b^3*c
 + 16*A*a^2*b*c^2)*x^2)*log(c*x^4 + b*x^2 + a) - 4*(A*a*b^4 - 8*A*a^2*b^2*c
 + 16*A*a^3*c^2 + (A*b^4*c - 8*A*a*b^2*c^2 + 16*A*a^2*c^3)*x^4 + (A*b^5 - 8
*A*a*b^3*c + 16*A*a^2*b*c^2)*x^2)*log(x))/(a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c
^2 + (a^2*b^4*c - 8*a^3*b^2*c^2 + 16*a^4*c^3)*x^4 + (a^2*b^5 - 8*a^3*b^3*c
+ 16*a^4*b*c^2)*x^2)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A+Bx^2}{x\left(a+bx^2+cx^4\right)^2}\,dx=\text{Timed out}$$

```
[In] integrate((B*x**2+A)/x/(c*x**4+b*x**2+a)**2,x)
```

```
[Out] Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{A+Bx^2}{x\left(a+bx^2+cx^4\right)^2}\,dx=\text{Exception raised: ValueError}$$

```
[In] integrate((B*x^2+A)/x/(c*x^4+b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

Giac [A] (verification not implemented)

none

Time = 0.61 (sec) , antiderivative size = 201, normalized size of antiderivative = 1.34

$$\begin{aligned}\int \frac{A+Bx^2}{x\left(a+bx^2+cx^4\right)^2}\,dx&=-\frac{\left(Ab^3+4\,Ba^2c-6\,Aabc\right)\arctan\left(\frac{2\,cx^2+b}{\sqrt{-b^2+4\,ac}}\right)}{2\,\left(a^2b^2-4\,a^3c\right)\sqrt{-b^2+4\,ac}}-\frac{A\log\left(cx^4+bx^2+a\right)}{4\,a^2}+\frac{A\log\left(x^2\right)}{2\,a^2}\\&\quad+\frac{Ab^2cx^4-4\,Aac^2x^4+Ab^3x^2-4\,Ba^2cx^2-2\,Aabcx^2-2\,Ba^2b+3\,Aab^2-8\,Aa^2c}{4\,\left(cx^4+bx^2+a\right)\left(a^2b^2-4\,a^3c\right)}\end{aligned}$$

```
[In] integrate((B*x^2+A)/x/(c*x^4+b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] -1/2*(A*b^3 + 4*B*a^2*c - 6*A*a*b*c)*arctan((2*c*x^2 + b)/sqrt(-b^2 + 4*a*c
))/((a^2*b^2 - 4*a^3*c)*sqrt(-b^2 + 4*a*c)) - 1/4*A*log(c*x^4 + b*x^2 + a)/
a^2 + 1/2*A*log(x^2)/a^2 + 1/4*(A*b^2*c*x^4 - 4*A*a*c^2*x^4 + A*b^3*x^2 - 4
*B*a^2*c*x^2 - 2*A*a*b*c*x^2 - 2*B*a^2*b + 3*A*a*b^2 - 8*A*a^2*c)/((c*x^4 +
 b*x^2 + a)*(a^2*b^2 - 4*a^3*c))
```

Mupad [B] (verification not implemented)

Time = 12.30 (sec) , antiderivative size = 7119, normalized size of antiderivative = 47.46

$$\int \frac{A + Bx^2}{x\left(a + bx^2 + cx^4\right)^2}\,dx = \text{Too large to display}$$

[In] int((A + B*x^2)/(x*(a + b*x^2 + c*x^4)^2),x)

[Out] ((2*A*a*c - A*b^2 + B*a*b)/(2*a*(4*a*c - b^2)) - (c*x^2*(A*b - 2*B*a))/(2*a *(4*a*c - b^2)))/(a + b*x^2 + c*x^4) + (A*log(x))/a^2 - (log((((A + a^2*(-(A*b^3 + 4*B*a^2*c - 6*A*a*b*c)^2/(a^4*(4*a*c - b^2)^3))^(1/2))*(((A + a^2*(-(A*b^3 + 4*B*a^2*c - 6*A*a*b*c)^2/(a^4*(4*a*c - b^2)^3))^(1/2))*((4*b*c^2* (A*b^3 + 2*B*a^2*c - 5*A*a*b*c))/(a*(4*a*c - b^2)) - (b*c^2*(A + a^2*(-(A*b ^3 + 4*B*a^2*c - 6*A*a*b*c)^2/(a^4*(4*a*c - b^2)^3))^(1/2))*(a*b + 3*b^2*x^ 2 - 10*a*c*x^2))/a^2 + (2*c^3*x^2*(A*b^3 + 8*B*a*b^2 - 20*B*a^2*c - 10*A*a* b*c))/(a*(4*a*c - b^2))))/(4*a^2) + (c^3*(A*b - 2*B*a)*(4*A*b^3 + 2*B*a^2*c - 17*A*a*b*c))/(a^2*(4*a*c - b^2)^2) - (2*c^4*x^2*(A*b - 2*B*a)*(10*A*a*c - 3*A*b^2 + B*a*b))/(a^2*(4*a*c - b^2)^2)))/(4*a^2) + (c^5*x^2*(A*b - 2*B*a)^3)/(a^3*(4*a*c - b^2)^3) - (A*c^4*(A*b - 2*B*a)^2)/(a^3*(4*a*c - b^2)^2)) *(((A - a^2*(-(A*b^3 + 4*B*a^2*c - 6*A*a*b*c)^2/(a^4*(4*a*c - b^2)^3))^(1/2))*(((A - a^2*(-(A*b^3 + 4*B*a^2*c - 6*A*a*b*c)^2/(a^4*(4*a*c - b^2)^3))^(1 /2))*((4*b*c^2*(A*b^3 + 2*B*a^2*c - 5*A*a*b*c))/(a*(4*a*c - b^2)) - (b*c^2* (A - a^2*(-(A*b^3 + 4*B*a^2*c - 6*A*a*b*c)^2/(a^4*(4*a*c - b^2)^3))^(1/2))* (a*b + 3*b^2*x^2 - 10*a*c*x^2))/a^2 + (2*c^3*x^2*(A*b^3 + 8*B*a*b^2 - 20*B* a^2*c - 10*A*a*b*c))/(a*(4*a*c - b^2))))/(4*a^2) + (c^3*(A*b - 2*B*a)*(4*A* b^3 + 2*B*a^2*c - 17*A*a*b*c))/(a^2*(4*a*c - b^2)^2) - (2*c^4*x^2*(A*b - 2* B*a)*(10*A*a*c - 3*A*b^2 + B*a*b))/(a^2*(4*a*c - b^2)^2)))/(4*a^2) + (c^5*x ^2*(A*b - 2*B*a)^3)/(a^3*(4*a*c - b^2)^3) - (A*c^4*(A*b - 2*B*a)^2)/(a^3*(4 *a*c - b^2)^2)))*(2*A*b^6 - 128*A*a^3*c^3 - 24*A*a*b^4*c + 96*A*a^2*b^2*c^2))/(2*(4*a^2*b^6 - 256*a^5*c^3 - 48*a^3*b^4*c + 192*a^4*b^2*c^2)) - (atan((x^2*((((A^3*b^3*c^5 - 8*B^3*a^3*c^5 + 12*A*B^2*a^2*b*c^5 - 6*A^2*B*a*b^2*c^ 5)/(a^3*b^6 - 64*a^6*c^3 - 12*a^4*b^4*c + 48*a^5*b^2*c^2) + (((44*A^2*a^2*b ^3*c^5 - 4*B^2*a^3*b^3*c^4 + 160*A*B*a^4*c^6 - 6*A^2*a*b^5*c^4 - 80*A^2*a^3 *b*c^6 + 16*B^2*a^4*b*c^5 + 14*A*B*a^2*b^4*c^4 - 96*A*B*a^3*b^2*c^5)/(a^3*b ^6 - 64*a^6*c^3 - 12*a^4*b^4*c + 48*a^5*b^2*c^2) - (((640*B*a^6*c^6 + 320*A *a^5*b*c^6 - 2*A*a^2*b^7*c^3 + 36*A*a^3*b^5*c^4 - 192*A*a^4*b^3*c^5 - 16*B* a^3*b^6*c^3 + 168*B*a^4*b^4*c^4 - 576*B*a^5*b^2*c^5)/(a^3*b^6 - 64*a^6*c^3 - 12*a^4*b^4*c + 48*a^5*b^2*c^2) - ((2*A*b^6 - 128*A*a^3*c^3 - 24*A*a*b^4*c + 96*A*a^2*b^2*c^2)*(2560*a^7*b*c^6 + 12*a^3*b^9*c^2 - 184*a^4*b^7*c^3 + 1 056*a^5*b^5*c^4 - 2688*a^6*b^3*c^5))/(2*(a^3*b^6 - 64*a^6*c^3 - 12*a^4*b^4* c + 48*a^5*b^2*c^2)*(4*a^2*b^6 - 256*a^5*c^3 - 48*a^3*b^4*c + 192*a^4*b^2*c ^2)))*(2*A*b^6 - 128*A*a^3*c^3 - 24*A*a*b^4*c + 96*A*a^2*b^2*c^2))/(2*(4*a^ 2*b^6 - 256*a^5*c^3 - 48*a^3*b^4*c + 192*a^4*b^2*c^2)))*(2*A*b^6 - 128*A*a^ 3*c^3 - 24*A*a*b^4*c + 96*A*a^2*b^2*c^2))/(2*(4*a^2*b^6 - 256*a^5*c^3 - 48*

a^3*b^4*c + 192*a^4*b^2*c^2)) + (((((640*B*a^6*c^6 + 320*A*a^5*b*c^6 - 2*A*
a^2*b^7*c^3 + 36*A*a^3*b^5*c^4 - 192*A*a^4*b^3*c^5 - 16*B*a^3*b^6*c^3 + 168
*B*a^4*b^4*c^4 - 576*B*a^5*b^2*c^5)/(a^3*b^6 - 64*a^6*c^3 - 12*a^4*b^4*c +
48*a^5*b^2*c^2) - ((2*A*b^6 - 128*A*a^3*c^3 - 24*A*a*b^4*c + 96*A*a^2*b^2*c
^2)*(2560*a^7*b*c^6 + 12*a^3*b^9*c^2 - 184*a^4*b^7*c^3 + 1056*a^5*b^5*c^4 -
 2688*a^6*b^3*c^5))/(2*(a^3*b^6 - 64*a^6*c^3 - 12*a^4*b^4*c + 48*a^5*b^2*c^
2)*(4*a^2*b^6 - 256*a^5*c^3 - 48*a^3*b^4*c + 192*a^4*b^2*c^2)))*(A*b^3 + 4*
B*a^2*c - 6*A*a*b*c))/(4*a^2*(4*a*c - b^2)^(3/2)) - ((A*b^3 + 4*B*a^2*c - 6
*A*a*b*c)*(2*A*b^6 - 128*A*a^3*c^3 - 24*A*a*b^4*c + 96*A*a^2*b^2*c^2)*(2560
*a^7*b*c^6 + 12*a^3*b^9*c^2 - 184*a^4*b^7*c^3 + 1056*a^5*b^5*c^4 - 2688*a^6
*b^3*c^5))/(8*a^2*(4*a*c - b^2)^(3/2)*(a^3*b^6 - 64*a^6*c^3 - 12*a^4*b^4*c
+ 48*a^5*b^2*c^2)*(4*a^2*b^6 - 256*a^5*c^3 - 48*a^3*b^4*c + 192*a^4*b^2*c^2
)))*(A*b^3 + 4*B*a^2*c - 6*A*a*b*c))/(4*a^2*(4*a*c - b^2)^(3/2)) - ((A*b^3
+ 4*B*a^2*c - 6*A*a*b*c)^2*(2*A*b^6 - 128*A*a^3*c^3 - 24*A*a*b^4*c + 96*A*a
^2*b^2*c^2)*(2560*a^7*b*c^6 + 12*a^3*b^9*c^2 - 184*a^4*b^7*c^3 + 1056*a^5*b
^5*c^4 - 2688*a^6*b^3*c^5))/(32*a^4*(4*a*c - b^2)^3*(a^3*b^6 - 64*a^6*c^3 -
 12*a^4*b^4*c + 48*a^5*b^2*c^2)*(4*a^2*b^6 - 256*a^5*c^3 - 48*a^3*b^4*c + 1
92*a^4*b^2*c^2)))*(3*A*b^5 - 2*B*a^3*c^2 - 21*A*a*b^3*c + 33*A*a^2*b*c^2 +
2*B*a^2*b^2*c))/(8*a^3*c^2*(4*a*c - b^2)^3*(400*A^2*a^3*c^3 - 6*A^2*b^6 + 4
*B^2*a^4*c^2 - 291*A^2*a^2*b^2*c^2 + 72*A^2*a*b^4*c + 2*A*B*a^2*b^3*c - 12*
A*B*a^3*b*c^2)) + (((((((640*B*a^6*c^6 + 320*A*a^5*b*c^6 - 2*A*a^2*b^7*c^3
+ 36*A*a^3*b^5*c^4 - 192*A*a^4*b^3*c^5 - 16*B*a^3*b^6*c^3 + 168*B*a^4*b^4*c
^4 - 576*B*a^5*b^2*c^5)/(a^3*b^6 - 64*a^6*c^3 - 12*a^4*b^4*c + 48*a^5*b^2*c
^2) - ((2*A*b^6 - 128*A*a^3*c^3 - 24*A*a*b^4*c + 96*A*a^2*b^2*c^2)*(2560*a^
7*b*c^6 + 12*a^3*b^9*c^2 - 184*a^4*b^7*c^3 + 1056*a^5*b^5*c^4 - 2688*a^6*b^
3*c^5))/(2*(a^3*b^6 - 64*a^6*c^3 - 12*a^4*b^4*c + 48*a^5*b^2*c^2)*(4*a^2*b^
6 - 256*a^5*c^3 - 48*a^3*b^4*c + 192*a^4*b^2*c^2)))*(A*b^3 + 4*B*a^2*c - 6*
A*a*b*c))/(4*a^2*(4*a*c - b^2)^(3/2)) - ((A*b^3 + 4*B*a^2*c - 6*A*a*b*c)*(2
*A*b^6 - 128*A*a^3*c^3 - 24*A*a*b^4*c + 96*A*a^2*b^2*c^2)*(2560*a^7*b*c^6 +
 12*a^3*b^9*c^2 - 184*a^4*b^7*c^3 + 1056*a^5*b^5*c^4 - 2688*a^6*b^3*c^5))/(
8*a^2*(4*a*c - b^2)^(3/2)*(a^3*b^6 - 64*a^6*c^3 - 12*a^4*b^4*c + 48*a^5*b^2
c^2)(4*a^2*b^6 - 256*a^5*c^3 - 48*a^3*b^4*c + 192*a^4*b^2*c^2)))*(2*A*b^6
 - 128*A*a^3*c^3 - 24*A*a*b^4*c + 96*A*a^2*b^2*c^2))/(2*(4*a^2*b^6 - 256*a^
5*c^3 - 48*a^3*b^4*c + 192*a^4*b^2*c^2)) - (((44*A^2*a^2*b^3*c^5 - 4*B^2*a^
3*b^3*c^4 + 160*A*B*a^4*c^6 - 6*A^2*a*b^5*c^4 - 80*A^2*a^3*b*c^6 + 16*B^2*a
^4*b*c^5 + 14*A*B*a^2*b^4*c^4 - 96*A*B*a^3*b^2*c^5)/(a^3*b^6 - 64*a^6*c^3 -
 12*a^4*b^4*c + 48*a^5*b^2*c^2) - (((640*B*a^6*c^6 + 320*A*a^5*b*c^6 - 2*A*
a^2*b^7*c^3 + 36*A*a^3*b^5*c^4 - 192*A*a^4*b^3*c^5 - 16*B*a^3*b^6*c^3 + 168
*B*a^4*b^4*c^4 - 576*B*a^5*b^2*c^5)/(a^3*b^6 - 64*a^6*c^3 - 12*a^4*b^4*c +
48*a^5*b^2*c^2) - ((2*A*b^6 - 128*A*a^3*c^3 - 24*A*a*b^4*c + 96*A*a^2*b^2*c
^2)*(2560*a^7*b*c^6 + 12*a^3*b^9*c^2 - 184*a^4*b^7*c^3 + 1056*a^5*b^5*c^4 -
 2688*a^6*b^3*c^5))/(2*(a^3*b^6 - 64*a^6*c^3 - 12*a^4*b^4*c + 48*a^5*b^2*c^
2)*(4*a^2*b^6 - 256*a^5*c^3 - 48*a^3*b^4*c + 192*a^4*b^2*c^2)))*(2*A*b^6 -
128*A*a^3*c^3 - 24*A*a*b^4*c + 96*A*a^2*b^2*c^2))/(2*(4*a^2*b^6 - 256*a^5*c
^3 - 48*a^3*b^4*c + 192*a^4*b^2*c^2)))*(A*b^3 + 4*B*a^2*c - 6*A*a*b*c))/(4*

a^2*(4*a*c - b^2)^(3/2)) + ((A*b^3 + 4*B*a^2*c - 6*A*a*b*c)^3*(2560*a^7*b*c ^6 + 12*a^3*b^9*c^2 - 184*a^4*b^7*c^3 + 1056*a^5*b^5*c^4 - 2688*a^6*b^3*c^5))/(64*a^6*(4*a*c - b^2)^(9/2)*(a^3*b^6 - 64*a^6*c^3 - 12*a^4*b^4*c + 48*a^ 5*b^2*c^2)))*(3*A*b^6 - 40*A*a^3*c^3 - 27*A*a*b^4*c + 2*B*a^2*b^3*c - 6*B*a ^3*b*c^2 + 69*A*a^2*b^2*c^2))/(8*a^3*c^2*(4*a*c - b^2)^(7/2)*(400*A^2*a^3*c ^3 - 6*A^2*b^6 + 4*B^2*a^4*c^2 - 291*A^2*a^2*b^2*c^2 + 72*A^2*a*b^4*c + 2*A *B*a^2*b^3*c - 12*A*B*a^3*b*c^2)))*(16*a^6*b^6*(4*a*c - b^2)^(9/2) - 1024*a ^9*c^3*(4*a*c - b^2)^(9/2) - 192*a^7*b^4*c*(4*a*c - b^2)^(9/2) + 768*a^8*b^ 2*c^2*(4*a*c - b^2)^(9/2)))/(A^2*b^6*c^2 + 16*B^2*a^4*c^4 + 36*A^2*a^2*b^2* c^4 - 12*A^2*a*b^4*c^3 + 8*A*B*a^2*b^3*c^3 - 48*A*B*a^3*b*c^4) - (((((((4*A *a^2*b^6*c^2 - 32*B*a^5*b*c^4 - 36*A*a^3*b^4*c^3 + 80*A*a^4*b^2*c^4 + 8*B*a ^4*b^3*c^3)/(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c) + ((4*a^4*b^6*c^2 - 32*a^5 *b^4*c^3 + 64*a^6*b^2*c^4)*(2*A*b^6 - 128*A*a^3*c^3 - 24*A*a*b^4*c + 96*A*a ^2*b^2*c^2))/(2*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c)*(4*a^2*b^6 - 256*a^5*c ^3 - 48*a^3*b^4*c + 192*a^4*b^2*c^2)))*(A*b^3 + 4*B*a^2*c - 6*A*a*b*c))/(4* a^2*(4*a*c - b^2)^(3/2)) + ((4*a^4*b^6*c^2 - 32*a^5*b^4*c^3 + 64*a^6*b^2*c^ 4)*(A*b^3 + 4*B*a^2*c - 6*A*a*b*c)*(2*A*b^6 - 128*A*a^3*c^3 - 24*A*a*b^4*c + 96*A*a^2*b^2*c^2))/(8*a^2*(4*a*c - b^2)^(3/2)*(a^3*b^4 + 16*a^5*c^2 - 8*a ^4*b^2*c)*(4*a^2*b^6 - 256*a^5*c^3 - 48*a^3*b^4*c + 192*a^4*b^2*c^2)))*(2*A *b^6 - 128*A*a^3*c^3 - 24*A*a*b^4*c + 96*A*a^2*b^2*c^2))/(2*(4*a^2*b^6 - 25 6*a^5*c^3 - 48*a^3*b^4*c + 192*a^4*b^2*c^2)) + (((4*B^2*a^4*c^4 + 17*A^2*a^ 2*b^2*c^4 - 4*A^2*a*b^4*c^3 + 8*A*B*a^2*b^3*c^3 - 36*A*B*a^3*b*c^4)/(a^3*b^ 4 + 16*a^5*c^2 - 8*a^4*b^2*c) + (((4*A*a^2*b^6*c^2 - 32*B*a^5*b*c^4 - 36*A* a^3*b^4*c^3 + 80*A*a^4*b^2*c^4 + 8*B*a^4*b^3*c^3)/(a^3*b^4 + 16*a^5*c^2 - 8 *a^4*b^2*c) + ((4*a^4*b^6*c^2 - 32*a^5*b^4*c^3 + 64*a^6*b^2*c^4)*(2*A*b^6 - 128*A*a^3*c^3 - 24*A*a*b^4*c + 96*A*a^2*b^2*c^2))/(2*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c)*(4*a^2*b^6 - 256*a^5*c^3 - 48*a^3*b^4*c + 192*a^4*b^2*c^2)))*(2*A*b^6 - 128*A*a^3*c^3 - 24*A*a*b^4*c + 96*A*a^2*b^2*c^2))/(2*(4*a^2*b^ 6 - 256*a^5*c^3 - 48*a^3*b^4*c + 192*a^4*b^2*c^2)))*(A*b^3 + 4*B*a^2*c - 6* A*a*b*c))/(4*a^2*(4*a*c - b^2)^(3/2)) - ((4*a^4*b^6*c^2 - 32*a^5*b^4*c^3 + 64*a^6*b^2*c^4)*(A*b^3 + 4*B*a^2*c - 6*A*a*b*c)^3)/(64*a^6*(4*a*c - b^2)^(9 /2)*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c)))*(16*a^6*b^6*(4*a*c - b^2)^(9/2) - 1024*a^9*c^3*(4*a*c - b^2)^(9/2) - 192*a^7*b^4*c*(4*a*c - b^2)^(9/2) + 76 8*a^8*b^2*c^2*(4*a*c - b^2)^(9/2))*(3*A*b^6 - 40*A*a^3*c^3 - 27*A*a*b^4*c + 2*B*a^2*b^3*c - 6*B*a^3*b*c^2 + 69*A*a^2*b^2*c^2))/(8*a^3*c^2*(4*a*c - b^2)^(7/2)*(A^2*b^6*c^2 + 16*B^2*a^4*c^4 + 36*A^2*a^2*b^2*c^4 - 12*A^2*a*b^4*c ^3 + 8*A*B*a^2*b^3*c^3 - 48*A*B*a^3*b*c^4)*(400*A^2*a^3*c^3 - 6*A^2*b^6 + 4 *B^2*a^4*c^2 - 291*A^2*a^2*b^2*c^2 + 72*A^2*a*b^4*c + 2*A*B*a^2*b^3*c - 12* A*B*a^3*b*c^2)) + ((16*a^6*b^6*(4*a*c - b^2)^(9/2) - 1024*a^9*c^3*(4*a*c - b^2)^(9/2) - 192*a^7*b^4*c*(4*a*c - b^2)^(9/2) + 768*a^8*b^2*c^2*(4*a*c - b ^2)^(9/2))*((A^3*b^2*c^4 + 4*A*B^2*a^2*c^4 - 4*A^2*B*a*b*c^4)/(a^3*b^4 + 16 *a^5*c^2 - 8*a^4*b^2*c) + (((4*B^2*a^4*c^4 + 17*A^2*a^2*b^2*c^4 - 4*A^2*a*b ^4*c^3 + 8*A*B*a^2*b^3*c^3 - 36*A*B*a^3*b*c^4)/(a^3*b^4 + 16*a^5*c^2 - 8*a^ 4*b^2*c) + (((4*A*a^2*b^6*c^2 - 32*B*a^5*b*c^4 - 36*A*a^3*b^4*c^3 + 80*A*a^ 4*b^2*c^4 + 8*B*a^4*b^3*c^3)/(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c) + ((4*a^4

```
*b^6*c^2 - 32*a^5*b^4*c^3 + 64*a^6*b^2*c^4)*(2*A*b^6 - 128*A*a^3*c^3 - 24*A
*a*b^4*c + 96*A*a^2*b^2*c^2))/(2*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c)*(4*a^
2*b^6 - 256*a^5*c^3 - 48*a^3*b^4*c + 192*a^4*b^2*c^2)))*(2*A*b^6 - 128*A*a^
3*c^3 - 24*A*a*b^4*c + 96*A*a^2*b^2*c^2))/(2*(4*a^2*b^6 - 256*a^5*c^3 - 48*
a^3*b^4*c + 192*a^4*b^2*c^2)))*(2*A*b^6 - 128*A*a^3*c^3 - 24*A*a*b^4*c + 96
*A*a^2*b^2*c^2))/(2*(4*a^2*b^6 - 256*a^5*c^3 - 48*a^3*b^4*c + 192*a^4*b^2*c
^2)) - (((((4*A*a^2*b^6*c^2 - 32*B*a^5*b*c^4 - 36*A*a^3*b^4*c^3 + 80*A*a^4*
b^2*c^4 + 8*B*a^4*b^3*c^3)/(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c) + ((4*a^4*b
^6*c^2 - 32*a^5*b^4*c^3 + 64*a^6*b^2*c^4)*(2*A*b^6 - 128*A*a^3*c^3 - 24*A*a
*b^4*c + 96*A*a^2*b^2*c^2))/(2*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c)*(4*a^2*
b^6 - 256*a^5*c^3 - 48*a^3*b^4*c + 192*a^4*b^2*c^2)))*(A*b^3 + 4*B*a^2*c -
6*A*a*b*c))/(4*a^2*(4*a*c - b^2)^(3/2)) + ((4*a^4*b^6*c^2 - 32*a^5*b^4*c^3
+ 64*a^6*b^2*c^4)*(A*b^3 + 4*B*a^2*c - 6*A*a*b*c)*(2*A*b^6 - 128*A*a^3*c^3
- 24*A*a*b^4*c + 96*A*a^2*b^2*c^2))/(8*a^2*(4*a*c - b^2)^(3/2)*(a^3*b^4 + 1
6*a^5*c^2 - 8*a^4*b^2*c)*(4*a^2*b^6 - 256*a^5*c^3 - 48*a^3*b^4*c + 192*a^4*
b^2*c^2)))*(A*b^3 + 4*B*a^2*c - 6*A*a*b*c))/(4*a^2*(4*a*c - b^2)^(3/2)) - (
(4*a^4*b^6*c^2 - 32*a^5*b^4*c^3 + 64*a^6*b^2*c^4)*(A*b^3 + 4*B*a^2*c - 6*A*
a*b*c)^2*(2*A*b^6 - 128*A*a^3*c^3 - 24*A*a*b^4*c + 96*A*a^2*b^2*c^2))/(32*a
^4*(4*a*c - b^2)^3*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c)*(4*a^2*b^6 - 256*a^
5*c^3 - 48*a^3*b^4*c + 192*a^4*b^2*c^2)))*(3*A*b^5 - 2*B*a^3*c^2 - 21*A*a*b
^3*c + 33*A*a^2*b*c^2 + 2*B*a^2*b^2*c))/(8*a^3*c^2*(4*a*c - b^2)^3*(A^2*b^6
*c^2 + 16*B^2*a^4*c^4 + 36*A^2*a^2*b^2*c^4 - 12*A^2*a*b^4*c^3 + 8*A*B*a^2*b
^3*c^3 - 48*A*B*a^3*b*c^4)*(400*A^2*a^3*c^3 - 6*A^2*b^6 + 4*B^2*a^4*c^2 - 2
91*A^2*a^2*b^2*c^2 + 72*A^2*a*b^4*c + 2*A*B*a^2*b^3*c - 12*A*B*a^3*b*c^2)))
*(A*b^3 + 4*B*a^2*c - 6*A*a*b*c))/(2*a^2*(4*a*c - b^2)^(3/2))
```

3.117 $\int \frac{A+Bx^2}{x^3\left(a+bx^2+cx^4\right)^2}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 223

$$\int \frac{A+Bx^2}{x^3\left(a+bx^2+cx^4\right)^2}\,dx = -\frac{2Ab^2-abB-6aAc}{2a^2\left(b^2-4ac\right)x^2}-\frac{abB-A(b^2-2ac)-(Ab-2aB)cx^2}{2a\left(b^2-4ac\right)x^2\left(a+bx^2+cx^4\right)} + \frac{(abB(b^2-6ac)-2A(b^4-6ab^2c+6a^2c^2))\operatorname{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2a^3\left(b^2-4ac\right)^{3/2}} - \frac{(2Ab-aB)\log(x)}{a^3}+\frac{(2Ab-aB)\log\left(a+bx^2+cx^4\right)}{4a^3}$$

```
[Out] 1/2*(6*A*a*c-2*A*b^2+B*a*b)/a^2/(-4*a*c+b^2)/x^2+1/2*(-a*b*B+A*(-2*a*c+b^2)
+(A*b-2*B*a)*c*x^2)/a/(-4*a*c+b^2)/x^2/(c*x^4+b*x^2+a)+1/2*(a*b*B*(-6*a*c+b
^2)-2*A*(6*a^2*c^2-6*a*b^2*c+b^4))*arctanh((2*c*x^2+b)/(-4*a*c+b^2)^(1/2))/
a^3/(-4*a*c+b^2)^(3/2)-(2*A*b-B*a)*ln(x)/a^3+1/4*(2*A*b-B*a)*ln(c*x^4+b*x^2
+a)/a^3
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 223, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.280, Rules used = {1265, 836, 814, 648, 632, 212, 642}

$$\int \frac{A+Bx^2}{x^3\left(a+bx^2+cx^4\right)^2}\,dx = \frac{(2Ab-aB)\log\left(a+bx^2+cx^4\right)}{4a^3} - \frac{\log(x)(2Ab-aB)}{a^3} - \frac{-6aAc-abB+2Ab^2}{2a^2x^2\left(b^2-4ac\right)} + \frac{\left(abB(b^2-6ac)-2A(6a^2c^2-6ab^2c+b^4)\right)\operatorname{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2a^3\left(b^2-4ac\right)^{3/2}} - \frac{-A(b^2-2ac)-\left(cx^2(Ab-2aB)\right)+abB}{2ax^2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}$$

```
[In] Int[(A + B*x^2)/(x^3*(a + b*x^2 + c*x^4)^2),x]
```

```
[Out] -1/2*(2*A*b^2 - a*b*B - 6*a*A*c)/(a^2*(b^2 - 4*a*c)*x^2) - (a*b*B - A*(b^2
- 2*a*c) - (A*b - 2*a*B)*c*x^2)/(2*a*(b^2 - 4*a*c)*x^2*(a + b*x^2 + c*x^4))
 + ((a*b*B*(b^2 - 6*a*c) - 2*A*(b^4 - 6*a*b^2*c + 6*a^2*c^2))*ArcTanh[(b +
2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(2*a^3*(b^2 - 4*a*c)^(3/2)) - ((2*A*b - a*B)*L
og[x])/a^3 + ((2*A*b - a*B)*Log[a + b*x^2 + c*x^4])/(4*a^3)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
```

```
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 814

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)/(a +
 b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 836

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(f*(b*c*d - b^2*e + 2
*a*c*e) - a*g*(2*c*d - b*e) + c*(f*(2*c*d - b*e) - g*(b*d - 2*a*e))*x)*((a
+ b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2))), x]
 + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^m*
(a + b*x + c*x^2)^(p + 1)*Simp[f*(b*c*d*e*(2*p - m + 2) + b^2*e^2*(p + m +
2) - 2*c^2*d^2*(2*p + 3) - 2*a*c*e^2*(m + 2*p + 3)) - g*(a*e*(b*e - 2*c*d*m
 + b*e*m) - b*d*(3*c*d - b*e + 2*c*d*p - b*e*p)) + c*e*(g*(b*d - 2*a*e) - f
*(2*c*d - b*e))*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1
] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 1265

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rubi steps

$$\text{integral} = \frac{1}{2}\text{Subst}\left(\int \frac{A+Bx}{x^2\left(a+bx+cx^2\right)^2}\,dx, x, x^2\right)$$

$$= -\frac{abB-A(b^2-2ac)-(Ab-2aB)cx^2}{2a\left(b^2-4ac\right)x^2\left(a+bx^2+cx^4\right)} - \frac{\text{Subst}\left(\int \frac{-2Ab^2+abB+6aAc-2(Ab-2aB)cx}{x^2(a+bx+cx^2)}\,dx, x, x^2\right)}{2a\left(b^2-4ac\right)}$$

$$= -\frac{abB-A(b^2-2ac)-(Ab-2aB)cx^2}{2a\left(b^2-4ac\right)x^2\left(a+bx^2+cx^4\right)}$$

$$- \frac{\text{Subst}\Big(\int \Big(\frac{-2Ab^2+abB+6aAc}{ax^2} + \frac{(-2Ab+aB)(-b^2+4ac)}{a^2x} + \frac{abB(b^2-5ac)-2A(b^4-5ab^2c+3a^2c^2)-(2Ab-aB)c(b^2-4ac)x}{a^2(a+bx+cx^2)}}{2a\left(b^2-4ac\right)}$$

$$= -\frac{2Ab^2-abB-6aAc}{2a^2(b^2-4ac)x^2} - \frac{abB-A(b^2-2ac)-(Ab-2aB)cx^2}{2a(b^2-4ac)x^2(a+bx^2+cx^4)} - \frac{(2Ab-aB)\log(x)}{a^3} - \frac{\text{Subst}\left(\int \frac{abB(b^2-5ac)-2A(b^4-5ab^2c+3a^2c^2)-(2Ab-aB)c(b^2-4ac)x}{a+bx+cx^2}\,dx, x, x^2\right)}{2a^3(b^2-4ac)}$$

$$= -\frac{2Ab^2-abB-6aAc}{2a^2(b^2-4ac)x^2} - \frac{abB-A(b^2-2ac)-(Ab-2aB)cx^2}{2a(b^2-4ac)x^2(a+bx^2+cx^4)} - \frac{(2Ab-aB)\log(x)}{a^3} + \frac{(2Ab-aB)\text{Subst}\left(\int \frac{b+2cx}{a+bx+cx^2}\,dx, x, x^2\right)}{4a^3} - \frac{(abB(b^2-6ac)-2A(b^4-6ab^2c+6a^2c^2))\,\text{Subst}\left(\int \frac{1}{a+bx+cx^2}\,dx, x, x^2\right)}{4a^3(b^2-4ac)}$$

$$= -\frac{2Ab^2-abB-6aAc}{2a^2(b^2-4ac)x^2} - \frac{abB-A(b^2-2ac)-(Ab-2aB)cx^2}{2a(b^2-4ac)x^2(a+bx^2+cx^4)} - \frac{(2Ab-aB)\log(x)}{a^3} + \frac{(2Ab-aB)\log(a+bx^2+cx^4)}{4a^3} + \frac{(abB(b^2-6ac)-2A(b^4-6ab^2c+6a^2c^2))\,\text{Subst}\left(\int \frac{1}{b^2-4ac-x^2}\,dx, x, b+2cx^2\right)}{2a^3(b^2-4ac)}$$

$$= -\frac{2Ab^2-abB-6aAc}{2a^2(b^2-4ac)x^2} - \frac{abB-A(b^2-2ac)-(Ab-2aB)cx^2}{2a(b^2-4ac)x^2(a+bx^2+cx^4)} + \frac{(abB(b^2-6ac)-2A(b^4-6ab^2c+6a^2c^2))\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2a^3(b^2-4ac)^{3/2}} - \frac{(2Ab-aB)\log(x)}{a^3} + \frac{(2Ab-aB)\log(a+bx^2+cx^4)}{4a^3}$$

Mathematica [A] (verified)

Time = 0.34 (sec) , antiderivative size = 379, normalized size of antiderivative = 1.70

$$\int \frac{A+Bx^2}{x^3(a+bx^2+cx^4)^2}\,dx$$

$$= \frac{-\frac{2aA}{x^2} - \frac{2a(aB(-b^2+2ac-bcx^2)+A(b^3-3abc+b^2cx^2-2ac^2x^2))}{(b^2-4ac)(a+bx^2+cx^4)} + 4(-2Ab+aB)\log(x) + \frac{\left(aB\left(-b^3+6abc-b^2\sqrt{b^2-4ac}+4ac\sqrt{b^2-\right.\right.}{}}{}$$

```
[In] Integrate[(A + B*x^2)/(x^3*(a + b*x^2 + c*x^4)^2),x]
```

```
[Out] ((-2*a*A)/x^2 - (2*a*(a*B*(-b^2 + 2*a*c - b*c*x^2) + A*(b^3 - 3*a*b*c + b^2
*c*x^2 - 2*a*c^2*x^2)))/((b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) + 4*(-2*A*b + a
*B)*Log[x] + ((a*B*(-b^3 + 6*a*b*c - b^2*Sqrt[b^2 - 4*a*c] + 4*a*c*Sqrt[b^2
 - 4*a*c]) + 2*A*(b^4 - 6*a*b^2*c + 6*a^2*c^2 + b^3*Sqrt[b^2 - 4*a*c] - 4*a
*b*c*Sqrt[b^2 - 4*a*c]))*Log[b - Sqrt[b^2 - 4*a*c] + 2*c*x^2])/(b^2 - 4*a*c
)^(3/2) + ((a*B*(b^3 - 6*a*b*c - b^2*Sqrt[b^2 - 4*a*c] + 4*a*c*Sqrt[b^2 - 4
```

```
*a*c]) + 2*A*(-b^4 + 6*a*b^2*c - 6*a^2*c^2 + b^3*Sqrt[b^2 - 4*a*c] - 4*a*b*
c*Sqrt[b^2 - 4*a*c]))*Log[b + Sqrt[b^2 - 4*a*c] + 2*c*x^2])/(b^2 - 4*a*c)^(
3/2))/(4*a^3)
```

Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 300, normalized size of antiderivative = 1.35

method	result
default	$-\frac{A}{2a^2x^2}+\frac{(-2Ab+Ba)\ln(x)}{a^3}-\frac{\frac{ac\left(2Aac-A\,b^2+abB\right)x^2}{4ac-b^2}+\frac{a\left(3Aabc-A\,b^3-2a^2Bc+Ba\,b^2\right)}{4ac-b^2}}{c\,x^4+b\,x^2+a}+\frac{\frac{\left(-8Aab\,c^2+2A\,b^3c+4B\,a^2c^2-Ba\,b^2c\right)\ln\left(c\right.}{2c}}{}$
risch	$\frac{-\frac{c\left(6Aac-2A\,b^2+abB\right)x^4}{2a^2\left(4ac-b^2\right)}-\frac{\left(7Aabc-2A\,b^3-2a^2Bc+Ba\,b^2\right)x^2}{2\left(4ac-b^2\right)a^2}-\frac{A}{2a}}{x^2(c\,x^4+b\,x^2+a)}-\frac{2\ln(x)Ab}{a^3}+\frac{\ln(x)B}{a^2}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\left(64a^6c^3-48b^2a^5c^2+12a^4b\right.\right.}\right.}{}$

```
[In] int((B*x^2+A)/x^3/(c*x^4+b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*A/a^2/x^2+(-2*A*b+B*a)/a^3*ln(x)-1/2/a^3*((a*c*(2*A*a*c-A*b^2+B*a*b)/(
4*a*c-b^2)*x^2+a*(3*A*a*b*c-A*b^3-2*B*a^2*c+B*a*b^2)/(4*a*c-b^2))/(c*x^4+b*
x^2+a)+1/(4*a*c-b^2)*(1/2*(-8*A*a*b*c^2+2*A*b^3*c+4*B*a^2*c^2-B*a*b^2*c)/c*
ln(c*x^4+b*x^2+a)+2*(6*A*a^2*c^2-10*A*a*b^2*c+2*A*b^4+5*a^2*b*B*c-B*a*b^3-1
/2*(-8*A*a*b*c^2+2*A*b^3*c+4*B*a^2*c^2-B*a*b^2*c)*b/c)/(4*a*c-b^2)^(1/2)*ar
ctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 806 vs. $2(207) = 414$.

Time = 1.15 (sec) , antiderivative size = 1635, normalized size of antiderivative = 7.33

$$\int \frac{A+Bx^2}{x^3\left(a+bx^2+cx^4\right)^2}\,dx=\text{Too large to display}$$

```
[In] integrate((B*x^2+A)/x^3/(c*x^4+b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] [-1/4*(2*A*a^2*b^4 - 16*A*a^3*b^2*c + 32*A*a^4*c^2 + 2*(24*A*a^3*c^3 + 2*(2
*B*a^3*b - 7*A*a^2*b^2)*c^2 - (B*a^2*b^3 - 2*A*a*b^4)*c)*x^4 - 2*(B*a^2*b^4
 - 2*A*a*b^5 + 4*(2*B*a^4 - 7*A*a^3*b)*c^2 - 3*(2*B*a^3*b^2 - 5*A*a^2*b^3)*
c)*x^2 + ((12*A*a^2*c^3 + 6*(B*a^2*b - 2*A*a*b^2)*c^2 - (B*a*b^3 - 2*A*b^4)
*c)*x^6 - (B*a*b^4 - 2*A*b^5 - 12*A*a^2*b*c^2 - 6*(B*a^2*b^2 - 2*A*a*b^3)*c
)*x^4 - (B*a^2*b^3 - 2*A*a*b^4 - 12*A*a^3*c^2 - 6*(B*a^3*b - 2*A*a^2*b^2)*c
)*x^2)*sqrt(b^2 - 4*a*c)*log((2*c^2*x^4 + 2*b*c*x^2 + b^2 - 2*a*c + (2*c*x^
2 + b)*sqrt(b^2 - 4*a*c))/(c*x^4 + b*x^2 + a)) + ((16*(B*a^3 - 2*A*a^2*b)*c
^3 - 8*(B*a^2*b^2 - 2*A*a*b^3)*c^2 + (B*a*b^4 - 2*A*b^5)*c)*x^6 + (B*a*b^5
```

```
- 2*A*b^6 + 16*(B*a^3*b - 2*A*a^2*b^2)*c^2 - 8*(B*a^2*b^3 - 2*A*a*b^4)*c)*x
^4 + (B*a^2*b^4 - 2*A*a*b^5 + 16*(B*a^4 - 2*A*a^3*b)*c^2 - 8*(B*a^3*b^2 - 2
*A*a^2*b^3)*c)*x^2)*log(c*x^4 + b*x^2 + a) - 4*((16*(B*a^3 - 2*A*a^2*b)*c^3
 - 8*(B*a^2*b^2 - 2*A*a*b^3)*c^2 + (B*a*b^4 - 2*A*b^5)*c)*x^6 + (B*a*b^5 -
2*A*b^6 + 16*(B*a^3*b - 2*A*a^2*b^2)*c^2 - 8*(B*a^2*b^3 - 2*A*a*b^4)*c)*x^4
 + (B*a^2*b^4 - 2*A*a*b^5 + 16*(B*a^4 - 2*A*a^3*b)*c^2 - 8*(B*a^3*b^2 - 2*A
*a^2*b^3)*c)*x^2)*log(x))/((a^3*b^4*c - 8*a^4*b^2*c^2 + 16*a^5*c^3)*x^6 + (
a^3*b^5 - 8*a^4*b^3*c + 16*a^5*b*c^2)*x^4 + (a^4*b^4 - 8*a^5*b^2*c + 16*a^6
*c^2)*x^2), -1/4*(2*A*a^2*b^4 - 16*A*a^3*b^2*c + 32*A*a^4*c^2 + 2*(24*A*a^3
*c^3 + 2*(2*B*a^3*b - 7*A*a^2*b^2)*c^2 - (B*a^2*b^3 - 2*A*a*b^4)*c)*x^4 - 2
*(B*a^2*b^4 - 2*A*a*b^5 + 4*(2*B*a^4 - 7*A*a^3*b)*c^2 - 3*(2*B*a^3*b^2 - 5*
A*a^2*b^3)*c)*x^2 + 2*((12*A*a^2*c^3 + 6*(B*a^2*b - 2*A*a*b^2)*c^2 - (B*a*b
^3 - 2*A*b^4)*c)*x^6 - (B*a*b^4 - 2*A*b^5 - 12*A*a^2*b*c^2 - 6*(B*a^2*b^2 -
 2*A*a*b^3)*c)*x^4 - (B*a^2*b^3 - 2*A*a*b^4 - 12*A*a^3*c^2 - 6*(B*a^3*b - 2
*A*a^2*b^2)*c)*x^2)*sqrt(-b^2 + 4*a*c)*arctan(-(2*c*x^2 + b)*sqrt(-b^2 + 4*
a*c)/(b^2 - 4*a*c)) + ((16*(B*a^3 - 2*A*a^2*b)*c^3 - 8*(B*a^2*b^2 - 2*A*a*b
^3)*c^2 + (B*a*b^4 - 2*A*b^5)*c)*x^6 + (B*a*b^5 - 2*A*b^6 + 16*(B*a^3*b - 2
*A*a^2*b^2)*c^2 - 8*(B*a^2*b^3 - 2*A*a*b^4)*c)*x^4 + (B*a^2*b^4 - 2*A*a*b^5
 + 16*(B*a^4 - 2*A*a^3*b)*c^2 - 8*(B*a^3*b^2 - 2*A*a^2*b^3)*c)*x^2)*log(c*x
^4 + b*x^2 + a) - 4*((16*(B*a^3 - 2*A*a^2*b)*c^3 - 8*(B*a^2*b^2 - 2*A*a*b^3
)*c^2 + (B*a*b^4 - 2*A*b^5)*c)*x^6 + (B*a*b^5 - 2*A*b^6 + 16*(B*a^3*b - 2*A
*a^2*b^2)*c^2 - 8*(B*a^2*b^3 - 2*A*a*b^4)*c)*x^4 + (B*a^2*b^4 - 2*A*a*b^5 +
 16*(B*a^4 - 2*A*a^3*b)*c^2 - 8*(B*a^3*b^2 - 2*A*a^2*b^3)*c)*x^2)*log(x))/(
(a^3*b^4*c - 8*a^4*b^2*c^2 + 16*a^5*c^3)*x^6 + (a^3*b^5 - 8*a^4*b^3*c + 16*
a^5*b*c^2)*x^4 + (a^4*b^4 - 8*a^5*b^2*c + 16*a^6*c^2)*x^2)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A + Bx^2}{x^3 \left(a + bx^2 + cx^4\right)^2}\, dx = \text{Timed out}$$

```
[In] integrate((B*x**2+A)/x**3/(c*x**4+b*x**2+a)**2,x)
[Out] Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{A+Bx^2}{x^3\left(a+bx^2+cx^4\right)^2}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((B*x^2+A)/x^3/(c*x^4+b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

Giac [A] (verification not implemented)

none

Time = 0.56 (sec) , antiderivative size = 250, normalized size of antiderivative = 1.12

$$\int \frac{A+Bx^2}{x^3\left(a+bx^2+cx^4\right)^2}\,dx = -\frac{\left(Bab^3-2\,Ab^4-6\,Ba^2bc+12\,Aab^2c-12\,Aa^2c^2\right)\arctan\left(\frac{2\,cx^2+b}{\sqrt{-b^2+4\,ac}}\right)}{2\,\left(a^3b^2-4\,a^4c\right)\sqrt{-b^2+4\,ac}} + \frac{Babcx^4-2\,Ab^2cx^4+6\,Aac^2x^4+Bab^2x^2-2\,Ab^3x^2-2\,Ba^2cx^2+7\,Aabcx^2-Aab^2+4\,Aa^2c}{2\,(cx^6+bx^4+ax^2)(a^2b^2-4\,a^3c)} - \frac{(Ba-2\,Ab)\log\left(cx^4+bx^2+a\right)}{4\,a^3} + \frac{(Ba-2\,Ab)\log\left(x^2\right)}{2\,a^3}$$

```
[In] integrate((B*x^2+A)/x^3/(c*x^4+b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] -1/2*(B*a*b^3 - 2*A*b^4 - 6*B*a^2*b*c + 12*A*a*b^2*c - 12*A*a^2*c^2)*arctan
((2*c*x^2 + b)/sqrt(-b^2 + 4*a*c))/((a^3*b^2 - 4*a^4*c)*sqrt(-b^2 + 4*a*c))
 + 1/2*(B*a*b*c*x^4 - 2*A*b^2*c*x^4 + 6*A*a*c^2*x^4 + B*a*b^2*x^2 - 2*A*b^3
*x^2 - 2*B*a^2*c*x^2 + 7*A*a*b*c*x^2 - A*a*b^2 + 4*A*a^2*c)/((c*x^6 + b*x^4
 + a*x^2)*(a^2*b^2 - 4*a^3*c)) - 1/4*(B*a - 2*A*b)*log(c*x^4 + b*x^2 + a)/a
^3 + 1/2*(B*a - 2*A*b)*log(x^2)/a^3
```

Mupad [B] (verification not implemented)

Time = 13.33 (sec) , antiderivative size = 10034, normalized size of antiderivative = 45.00

$$\int \frac{A+Bx^2}{x^3\left(a+bx^2+cx^4\right)^2}\,dx = \text{Too large to display}$$

```
[In] int((A + B*x^2)/(x^3*(a + b*x^2 + c*x^4)^2),x)
```

```
[Out] (log(((c^4*(2*A*b - B*a)*(6*A*a*c - 2*A*b^2 + B*a*b)^2)/(a^6*(4*a*c - b^2)^
2) - ((((B*a - 2*A*b + a^3*(-(2*A*b^4 + 12*A*a^2*c^2 - B*a*b^3 - 12*A*a*b^2
*c + 6*B*a^2*b*c)^2/(a^6*(4*a*c - b^2)^3))^(1/2))*((b*c^2*(B*a - 2*A*b + a^
3*(-(2*A*b^4 + 12*A*a^2*c^2 - B*a*b^3 - 12*A*a*b^2*c + 6*B*a^2*b*c)^2/(a^6*
(4*a*c - b^2)^3))^(1/2))*(a*b + 3*b^2*x^2 - 10*a*c*x^2))/a^3 + (4*b*c^2*(2*
A*b^4 + 6*A*a^2*c^2 - B*a*b^3 - 10*A*a*b^2*c + 5*B*a^2*b*c))/(a^2*(4*a*c -
b^2)) + (2*c^3*x^2*(2*A*b^4 - 60*A*a^2*c^2 - B*a*b^3 + 4*A*a*b^2*c + 10*B*a
^2*b*c))/(a^2*(4*a*c - b^2))))/(4*a^3) + (c^3*(36*A^2*a^3*c^3 - 16*A^2*b^6
- 4*B^2*a^2*b^4 + 16*A*B*a*b^5 - 216*A^2*a^2*b^2*c^2 + 116*A^2*a*b^4*c + 17
*B^2*a^3*b^2*c - 92*A*B*a^2*b^3*c + 108*A*B*a^3*b*c^2))/(a^4*(4*a*c - b^2)^
2) - (2*c^4*x^2*(12*A^2*b^5 + 3*B^2*a^2*b^3 - 12*A*B*a*b^4 - 60*A*B*a^3*c^2
 - 82*A^2*a*b^3*c - 10*B^2*a^3*b*c + 138*A^2*a^2*b*c^2 + 61*A*B*a^2*b^2*c))
/(a^4*(4*a*c - b^2)^2))*(B*a - 2*A*b + a^3*(-(2*A*b^4 + 12*A*a^2*c^2 - B*a*
b^3 - 12*A*a*b^2*c + 6*B*a^2*b*c)^2/(a^6*(4*a*c - b^2)^3))^(1/2)))/(4*a^3)
+ (c^5*x^2*(6*A*a*c - 2*A*b^2 + B*a*b)^3)/(a^6*(4*a*c - b^2)^3))*((c^4*(2*A
*b - B*a)*(6*A*a*c - 2*A*b^2 + B*a*b)^2)/(a^6*(4*a*c - b^2)^2) - ((((2*A*b
- B*a + a^3*(-(2*A*b^4 + 12*A*a^2*c^2 - B*a*b^3 - 12*A*a*b^2*c + 6*B*a^2*b*
c)^2/(a^6*(4*a*c - b^2)^3))^(1/2))*((4*b*c^2*(2*A*b^4 + 6*A*a^2*c^2 - B*a*b
^3 - 10*A*a*b^2*c + 5*B*a^2*b*c))/(a^2*(4*a*c - b^2)) - (b*c^2*(2*A*b - B*a
 + a^3*(-(2*A*b^4 + 12*A*a^2*c^2 - B*a*b^3 - 12*A*a*b^2*c + 6*B*a^2*b*c)^2/
(a^6*(4*a*c - b^2)^3))^(1/2))*(a*b + 3*b^2*x^2 - 10*a*c*x^2))/a^3 + (2*c^3*
x^2*(2*A*b^4 - 60*A*a^2*c^2 - B*a*b^3 + 4*A*a*b^2*c + 10*B*a^2*b*c))/(a^2*(
4*a*c - b^2))))/(4*a^3) - (c^3*(36*A^2*a^3*c^3 - 16*A^2*b^6 - 4*B^2*a^2*b^4
 + 16*A*B*a*b^5 - 216*A^2*a^2*b^2*c^2 + 116*A^2*a*b^4*c + 17*B^2*a^3*b^2*c
- 92*A*B*a^2*b^3*c + 108*A*B*a^3*b*c^2))/(a^4*(4*a*c - b^2)^2) + (2*c^4*x^2
*(12*A^2*b^5 + 3*B^2*a^2*b^3 - 12*A*B*a*b^4 - 60*A*B*a^3*c^2 - 82*A^2*a*b^3
*c - 10*B^2*a^3*b*c + 138*A^2*a^2*b*c^2 + 61*A*B*a^2*b^2*c))/(a^4*(4*a*c -
b^2)^2))*(2*A*b - B*a + a^3*(-(2*A*b^4 + 12*A*a^2*c^2 - B*a*b^3 - 12*A*a*b^
2*c + 6*B*a^2*b*c)^2/(a^6*(4*a*c - b^2)^3))^(1/2)))/(4*a^3) + (c^5*x^2*(6*A
*a*c - 2*A*b^2 + B*a*b)^3)/(a^6*(4*a*c - b^2)^3)))*(4*A*b^7 + 128*B*a^4*c^3
 - 2*B*a*b^6 - 48*A*a*b^5*c - 256*A*a^3*b*c^3 + 24*B*a^2*b^4*c + 192*A*a^2*
b^3*c^2 - 96*B*a^3*b^2*c^2))/(2*(4*a^3*b^6 - 256*a^6*c^3 - 48*a^4*b^4*c + 1
92*a^5*b^2*c^2)) - (log(x)*(2*A*b - B*a))/a^3 - (A/(2*a) - (x^2*(2*A*b^3 -
B*a*b^2 + 2*B*a^2*c - 7*A*a*b*c))/(2*a^2*(4*a*c - b^2)) + (c*x^4*(6*A*a*c -
 2*A*b^2 + B*a*b))/(2*a^2*(4*a*c - b^2)))/(a*x^2 + b*x^4 + c*x^6) + (atan((
x^2*((((216*A^3*a^3*c^8 - 8*A^3*b^6*c^5 - 216*A^3*a^2*b^2*c^7 + B^3*a^3*b^3
```

*c^5 + 72*A^3*a*b^4*c^6 + 12*A^2*B*a*b^5*c^5 + 108*A^2*B*a^3*b*c^7 - 6*A*B^
2*a^2*b^4*c^5 + 18*A*B^2*a^3*b^2*c^6 - 72*A^2*B*a^2*b^3*c^6)/(a^6*b^6 - 64*
a^9*c^3 - 12*a^7*b^4*c + 48*a^8*b^2*c^2) + (((24*A^2*a^2*b^7*c^4 - 260*A^2*
a^3*b^5*c^5 + 932*A^2*a^4*b^3*c^6 + 6*B^2*a^4*b^5*c^4 - 44*B^2*a^5*b^3*c^5
+ 480*A*B*a^6*c^7 - 1104*A^2*a^5*b*c^7 + 80*B^2*a^6*b*c^6 - 24*A*B*a^3*b^6*
c^4 + 218*A*B*a^4*b^4*c^5 - 608*A*B*a^5*b^2*c^6)/(a^6*b^6 - 64*a^9*c^3 - 12
*a^7*b^4*c + 48*a^8*b^2*c^2) + (((1920*A*a^8*c^7 - 320*B*a^8*b*c^6 - 4*A*a^
4*b^8*c^3 + 24*A*a^5*b^6*c^4 + 120*A*a^6*b^4*c^5 - 1088*A*a^7*b^2*c^6 + 2*B
*a^5*b^7*c^3 - 36*B*a^6*b^5*c^4 + 192*B*a^7*b^3*c^5)/(a^6*b^6 - 64*a^9*c^3
- 12*a^7*b^4*c + 48*a^8*b^2*c^2) - ((2560*a^10*b*c^6 + 12*a^6*b^9*c^2 - 184
*a^7*b^7*c^3 + 1056*a^8*b^5*c^4 - 2688*a^9*b^3*c^5)*(4*A*b^7 + 128*B*a^4*c^
3 - 2*B*a*b^6 - 48*A*a*b^5*c - 256*A*a^3*b*c^3 + 24*B*a^2*b^4*c + 192*A*a^2
*b^3*c^2 - 96*B*a^3*b^2*c^2))/(2*(a^6*b^6 - 64*a^9*c^3 - 12*a^7*b^4*c + 48*
a^8*b^2*c^2)*(4*a^3*b^6 - 256*a^6*c^3 - 48*a^4*b^4*c + 192*a^5*b^2*c^2)))*(
4*A*b^7 + 128*B*a^4*c^3 - 2*B*a*b^6 - 48*A*a*b^5*c - 256*A*a^3*b*c^3 + 24*B
*a^2*b^4*c + 192*A*a^2*b^3*c^2 - 96*B*a^3*b^2*c^2))/(2*(4*a^3*b^6 - 256*a^6
*c^3 - 48*a^4*b^4*c + 192*a^5*b^2*c^2)))*(4*A*b^7 + 128*B*a^4*c^3 - 2*B*a*b
^6 - 48*A*a*b^5*c - 256*A*a^3*b*c^3 + 24*B*a^2*b^4*c + 192*A*a^2*b^3*c^2 -
96*B*a^3*b^2*c^2))/(2*(4*a^3*b^6 - 256*a^6*c^3 - 48*a^4*b^4*c + 192*a^5*b^2
*c^2)) - (((((1920*A*a^8*c^7 - 320*B*a^8*b*c^6 - 4*A*a^4*b^8*c^3 + 24*A*a^5
*b^6*c^4 + 120*A*a^6*b^4*c^5 - 1088*A*a^7*b^2*c^6 + 2*B*a^5*b^7*c^3 - 36*B*
a^6*b^5*c^4 + 192*B*a^7*b^3*c^5)/(a^6*b^6 - 64*a^9*c^3 - 12*a^7*b^4*c + 48*
a^8*b^2*c^2) - ((2560*a^10*b*c^6 + 12*a^6*b^9*c^2 - 184*a^7*b^7*c^3 + 1056*
a^8*b^5*c^4 - 2688*a^9*b^3*c^5)*(4*A*b^7 + 128*B*a^4*c^3 - 2*B*a*b^6 - 48*A
*a*b^5*c - 256*A*a^3*b*c^3 + 24*B*a^2*b^4*c + 192*A*a^2*b^3*c^2 - 96*B*a^3*
b^2*c^2))/(2*(a^6*b^6 - 64*a^9*c^3 - 12*a^7*b^4*c + 48*a^8*b^2*c^2)*(4*a^3*
b^6 - 256*a^6*c^3 - 48*a^4*b^4*c + 192*a^5*b^2*c^2)))*(2*A*b^4 + 12*A*a^2*c
^2 - B*a*b^3 - 12*A*a*b^2*c + 6*B*a^2*b*c))/(4*a^3*(4*a*c - b^2)^(3/2)) - (
(2560*a^10*b*c^6 + 12*a^6*b^9*c^2 - 184*a^7*b^7*c^3 + 1056*a^8*b^5*c^4 - 26
88*a^9*b^3*c^5)*(2*A*b^4 + 12*A*a^2*c^2 - B*a*b^3 - 12*A*a*b^2*c + 6*B*a^2*
b*c)*(4*A*b^7 + 128*B*a^4*c^3 - 2*B*a*b^6 - 48*A*a*b^5*c - 256*A*a^3*b*c^3
+ 24*B*a^2*b^4*c + 192*A*a^2*b^3*c^2 - 96*B*a^3*b^2*c^2))/(8*a^3*(4*a*c - b
^2)^(3/2)*(a^6*b^6 - 64*a^9*c^3 - 12*a^7*b^4*c + 48*a^8*b^2*c^2)*(4*a^3*b^6
- 256*a^6*c^3 - 48*a^4*b^4*c + 192*a^5*b^2*c^2)))*(2*A*b^4 + 12*A*a^2*c^2
- B*a*b^3 - 12*A*a*b^2*c + 6*B*a^2*b*c))/(4*a^3*(4*a*c - b^2)^(3/2)) + ((25
60*a^10*b*c^6 + 12*a^6*b^9*c^2 - 184*a^7*b^7*c^3 + 1056*a^8*b^5*c^4 - 2688*
a^9*b^3*c^5)*(2*A*b^4 + 12*A*a^2*c^2 - B*a*b^3 - 12*A*a*b^2*c + 6*B*a^2*b*c
)^2*(4*A*b^7 + 128*B*a^4*c^3 - 2*B*a*b^6 - 48*A*a*b^5*c - 256*A*a^3*b*c^3 +
24*B*a^2*b^4*c + 192*A*a^2*b^3*c^2 - 96*B*a^3*b^2*c^2))/(32*a^6*(4*a*c - b
^2)^3*(a^6*b^6 - 64*a^9*c^3 - 12*a^7*b^4*c + 48*a^8*b^2*c^2)*(4*a^3*b^6 - 2
56*a^6*c^3 - 48*a^4*b^4*c + 192*a^5*b^2*c^2)))*(6*A*a^3*c^3 - 6*A*b^6 + 3*B
*a*b^5 + 42*A*a*b^4*c - 21*B*a^2*b^3*c + 33*B*a^3*b*c^2 - 72*A*a^2*b^2*c^2)
)/(8*a^3*c^2*(4*a*c - b^2)^3*(36*A^2*a^4*c^4 - 24*A^2*b^8 - 6*B^2*a^2*b^6 +
400*B^2*a^5*c^3 + 24*A*B*a*b^7 - 1152*A^2*a^2*b^4*c^2 + 1528*A^2*a^3*b^2*c
^3 - 291*B^2*a^4*b^2*c^2 + 288*A^2*a*b^6*c + 72*B^2*a^3*b^4*c + 1158*A*B*a^

3*b^3*c^2 - 288*A*B*a^2*b^5*c - 1564*A*B*a^4*b*c^3)) + (((((((1920*A*a^8*c^7 - 320*B*a^8*b*c^6 - 4*A*a^4*b^8*c^3 + 24*A*a^5*b^6*c^4 + 120*A*a^6*b^4*c^5 - 1088*A*a^7*b^2*c^6 + 2*B*a^5*b^7*c^3 - 36*B*a^6*b^5*c^4 + 192*B*a^7*b^3*c^5)/(a^6*b^6 - 64*a^9*c^3 - 12*a^7*b^4*c + 48*a^8*b^2*c^2) - ((2560*a^10*b*c^6 + 12*a^6*b^9*c^2 - 184*a^7*b^7*c^3 + 1056*a^8*b^5*c^4 - 2688*a^9*b^3*c^5)*(4*A*b^7 + 128*B*a^4*c^3 - 2*B*a*b^6 - 48*A*a*b^5*c - 256*A*a^3*b*c^3 + 24*B*a^2*b^4*c + 192*A*a^2*b^3*c^2 - 96*B*a^3*b^2*c^2))/(2*(a^6*b^6 - 64*a^9*c^3 - 12*a^7*b^4*c + 48*a^8*b^2*c^2)*(4*a^3*b^6 - 256*a^6*c^3 - 48*a^4*b^4*c + 192*a^5*b^2*c^2)))*(2*A*b^4 + 12*A*a^2*c^2 - B*a*b^3 - 12*A*a*b^2*c + 6*B*a^2*b*c))/(4*a^3*(4*a*c - b^2)^(3/2)) - ((2560*a^10*b*c^6 + 12*a^6*b^9*c^2 - 184*a^7*b^7*c^3 + 1056*a^8*b^5*c^4 - 2688*a^9*b^3*c^5)*(2*A*b^4 + 12*A*a^2*c^2 - B*a*b^3 - 12*A*a*b^2*c + 6*B*a^2*b*c)*(4*A*b^7 + 128*B*a^4*c^3 - 2*B*a*b^6 - 48*A*a*b^5*c - 256*A*a^3*b*c^3 + 24*B*a^2*b^4*c + 192*A*a^2*b^3*c^2 - 96*B*a^3*b^2*c^2))/(8*a^3*(4*a*c - b^2)^(3/2)*(a^6*b^6 - 64*a^9*c^3 - 12*a^7*b^4*c + 48*a^8*b^2*c^2)*(4*a^3*b^6 - 256*a^6*c^3 - 48*a^4*b^4*c + 192*a^5*b^2*c^2)))*(4*A*b^7 + 128*B*a^4*c^3 - 2*B*a*b^6 - 48*A*a*b^5*c - 256*A*a^3*b*c^3 + 24*B*a^2*b^4*c + 192*A*a^2*b^3*c^2 - 96*B*a^3*b^2*c^2))/(2*(4*a^3*b^6 - 256*a^6*c^3 - 48*a^4*b^4*c + 192*a^5*b^2*c^2)) + (((24*A^2*a^2*b^7*c^4 - 260*A^2*a^3*b^5*c^5 + 932*A^2*a^4*b^3*c^6 + 6*B^2*a^4*b^5*c^4 - 44*B^2*a^5*b^3*c^5 + 480*A*B*a^6*c^7 - 1104*A^2*a^5*b*c^7 + 80*B^2*a^6*b*c^6 - 24*A*B*a^3*b^6*c^4 + 218*A*B*a^4*b^4*c^5 - 608*A*B*a^5*b^2*c^6)/(a^6*b^6 - 64*a^9*c^3 - 12*a^7*b^4*c + 48*a^8*b^2*c^2) + (((1920*A*a^8*c^7 - 320*B*a^8*b*c^6 - 4*A*a^4*b^8*c^3 + 24*A*a^5*b^6*c^4 + 120*A*a^6*b^4*c^5 - 1088*A*a^7*b^2*c^6 + 2*B*a^5*b^7*c^3 - 36*B*a^6*b^5*c^4 + 192*B*a^7*b^3*c^5)/(a^6*b^6 - 64*a^9*c^3 - 12*a^7*b^4*c + 48*a^8*b^2*c^2) - ((2560*a^10*b*c^6 + 12*a^6*b^9*c^2 - 184*a^7*b^7*c^3 + 1056*a^8*b^5*c^4 - 2688*a^9*b^3*c^5)*(4*A*b^7 + 128*B*a^4*c^3 - 2*B*a*b^6 - 48*A*a*b^5*c - 256*A*a^3*b*c^3 + 24*B*a^2*b^4*c + 192*A*a^2*b^3*c^2 - 96*B*a^3*b^2*c^2))/(2*(a^6*b^6 - 64*a^9*c^3 - 12*a^7*b^4*c + 48*a^8*b^2*c^2)*(4*a^3*b^6 - 256*a^6*c^3 - 48*a^4*b^4*c + 192*a^5*b^2*c^2)))*(4*A*b^7 + 128*B*a^4*c^3 - 2*B*a*b^6 - 48*A*a*b^5*c - 256*A*a^3*b*c^3 + 24*B*a^2*b^4*c + 192*A*a^2*b^3*c^2 - 96*B*a^3*b^2*c^2))/(2*(4*a^3*b^6 - 256*a^6*c^3 - 48*a^4*b^4*c + 192*a^5*b^2*c^2)))*(2*A*b^4 + 12*A*a^2*c^2 - B*a*b^3 - 12*A*a*b^2*c + 6*B*a^2*b*c))/(4*a^3*(4*a*c - b^2)^(3/2)) + ((2560*a^10*b*c^6 + 12*a^6*b^9*c^2 - 184*a^7*b^7*c^3 + 1056*a^8*b^5*c^4 - 2688*a^9*b^3*c^5)*(2*A*b^4 + 12*A*a^2*c^2 - B*a*b^3 - 12*A*a*b^2*c + 6*B*a^2*b*c)^3)/(64*a^9*(4*a*c - b^2)^(9/2)*(a^6*b^6 - 64*a^9*c^3 - 12*a^7*b^4*c + 48*a^8*b^2*c^2)))*(768*A*b^7 + 5120*B*a^4*c^3 - 384*B*a*b^6 - 6912*A*a*b^5*c - 12544*A*a^3*b*c^3 + 3456*B*a^2*b^4*c + 18432*A*a^2*b^3*c^2 - 8832*B*a^3*b^2*c^2))/(1024*a^3*c^2*(4*a*c - b^2)^(7/2)*(36*A^2*a^4*c^4 - 24*A^2*b^8 - 6*B^2*a^2*b^6 + 400*B^2*a^5*c^3 + 24*A*B*a*b^7 - 1152*A^2*a^2*b^4*c^2 + 1528*A^2*a^3*b^2*c^3 - 291*B^2*a^4*b^2*c^2 + 288*A^2*a*b^6*c + 72*B^2*a^3*b^4*c + 1158*A*B*a^3*b^3*c^2 - 288*A*B*a^2*b^5*c - 1564*A*B*a^4*b*c^3)))*(16*a^9*b^6*(4*a*c - b^2)^(9/2) - 1024*a^12*c^3*(4*a*c - b^2)^(9/2) - 192*a^10*b^4*c*(4*a*c - b^2)^(9/2) + 768*a^11*b^2*c^2*(4*a*c - b^2)^(9/2)))/(144*A^2*a^4*c^6 + 4*A^2*b^8*c^2 + 192*A^2*a^2*b^4*c^4 - 288*A^2*a^3*b^2* c^5 + B^2*a^2*b^6*c^2 - 12*B^2*a^3*b^4*c^3 + 36*B^2*a^4*b^2*c^4 - 48*A^2*a*b^6*c^3 + 48*A*B*a^2*b^5*c^3 - 168*A*B*a^3*b^3*c^4 - 4*A*B*a*b^7*c^2 + 144*A*B*a^4*b*c^5) + (((((((96*A*a^7*b*c^5 - 8*A*a^4*b^7*c^2 + 72*A*a^5*b^5*c^3 - 184*A*a^6*b^3*c^4 + 4*B*a^5*b^6*c^2 - 36*B*a^6*b^4*c^3 + 80*B*a^7*b^2*c^4)/(a^6*b^4 + 16*a^8*c^2 - 8*a^7*b^2*c) - ((4*a^7*b^6*c^2 - 32*a^8*b^4*c^3 + 64*a^9*b^2*c^4)*(4*A*b^7 + 128*B*a^4*c^3 - 2*B*a*b^6 - 48*A*a*b^5*c - 256*A*a^3*b*c^3 + 24*B*a^2*b^4*c + 192*A*a^2*b^3*c^2 - 96*B*a^3*b^2*c^2))/(2*(a^6*b^4 + 16*a^8*c^2 - 8*a^7*b^2*c)*(4*a^3*b^6 - 256*a^6*c^3 - 48*a^4*b^4*c + 192*a^5*b^2*c^2)))*(2*A*b^4 + 12*A*a^2*c^2 - B*a*b^3 - 12*A*a*b^2*c + 6*B*a^2*b*c))/(4*a^3*(4*a*c - b^2)^(3/2)) - ((4*a^7*b^6*c^2 - 32*a^8*b^4*c^3 + 64*a^9*b^2*c^4)*(2*A*b^4 + 12*A*a^2*c^2 - B*a*b^3 - 12*A*a*b^2*c + 6*B*a^2*b*c)*(4*A*b^7 + 128*B*a^4*c^3 - 2*B*a*b^6 - 48*A*a*b^5*c - 256*A*a^3*b*c^3 + 24*B*a^2*b^4*c + 192*A*a^2*b^3*c^2 - 96*B*a^3*b^2*c^2))/(8*a^3*(4*a*c - b^2)^(3/2)*(a^6*b^4 + 16*a^8*c^2 - 8*a^7*b^2*c)*(4*a^3*b^6 - 256*a^6*c^3 - 48*a^4*b^4*c + 192*a^5*b^2*c^2)))*(4*A*b^7 + 128*B*a^4*c^3 - 2*B*a*b^6 - 48*A*a*b^5*c - 256*A*a^3*b*c^3 + 24*B*a^2*b^4*c + 192*A*a^2*b^3*c^2 - 96*B*a^3*b^2*c^2))/(2*(4*a^3*b^6 - 256*a^6*c^3 - 48*a^4*b^4*c + 192*a^5*b^2*c^2)) - (((36*A^2*a^5*c^6 - 16*A^2*a^2*b^6*c^3 + 116*A^2*a^3*b^4*c^4 - 216*A^2*a^4*b^2*c^5 - 4*B^2*a^4*b^4*c^3 + 17*B^2*a^5*b^2*c^4 + 16*A*B*a^3*b^5*c^3 - 92*A*B*a^4*b^3*c^4 + 108*A*B*a^5*b*c^5)/(a^6*b^4 + 16*a^8*c^2 - 8*a^7*b^2*c) - (((96*A*a^7*b*c^5 - 8*A*a^4*b^7*c^2 + 72*A*a^5*b^5*c^3 - 184*A*a^6*b^3*c^4 + 4*B*a^5*b^6*c^2 - 36*B*a^6*b^4*c^3 + 80*B*a^7*b^2*c^4)/(a^6*b^4 + 16*a^8*c^2 - 8*a^7*b^2*c) - ((4*a^7*b^6*c^2 - 32*a^8*b^4*c^3 + 64*a^9*b^2*c^4)*(4*A*b^7 + 128*B*a^4*c^3 - 2*B*a*b^6 - 48*A*a*b^5*c - 256*A*a^3*b*c^3 + 24*B*a^2*b^4*c + 192*A*a^2*b^3*c^2 - 96*B*a^3*b^2*c^2))/(2*(a^6*b^4 + 16*a^8*c^2 - 8*a^7*b^2*c)*(4*a^3*b^6 - 256*a^6*c^3 - 48*a^4*b^4*c + 192*a^5*b^2*c^2)))*(4*A*b^7 + 128*B*a^4*c^3 - 2*B*a*b^6 - 48*A*a*b^5*c - 256*A*a^3*b*c^3 + 24*B*a^2*b^4*c + 192*A*a^2*b^3*c^2 - 96*B*a^3*b^2*c^2))/(2*(4*a^3*b^6 - 256*a^6*c^3 - 48*a^4*b^4*c + 192*a^5*b^2*c^2)))*(2*A*b^4 + 12*A*a^2*c^2 - B*a*b^3 - 12*A*a*b^2*c + 6*B*a^2*b*c))/(4*a^3*(4*a*c - b^2)^(3/2)) + ((4*a^7*b^6*c^2 - 32*a^8*b^4*c^3 + 64*a^9*b^2*c^4)*(2*A*b^4 + 12*A*a^2*c^2 - B*a*b^3 - 12*A*a*b^2*c + 6*B*a^2*b*c)^3)/(64*a^9*(4*a*c - b^2)^(9/2)*(a^6*b^4 + 16*a^8*c^2 - 8*a^7*b^2*c)))*(16*a^9*b^6*(4*a*c - b^2)^(9/2) - 1024*a^12*c^3*(4*a*c - b^2)^(9/2) - 192*a^10*b^4*c*(4*a*c - b^2)^(9/2) + 768*a^11*b^2*c^2*(4*a*c - b^2)^(9/2))*(768*A*b^7 + 5120*B*a^4*c^3 - 384*B*a*b^6 - 6912*A*a*b^5*c - 12544*A*a^3*b*c^3 + 3456*B*a^2*b^4*c + 18432*A*a^2*b^3*c^2 - 8832*B*a^3*b^2*c^2))/(1024*a^3*c^2*(4*a*c - b^2)^(7/2)*(144*A^2*a^4*c^6 + 4*A^2*b^8*c^2 + 192*A^2*a^2*b^4*c^4 - 288*A^2*a^3*b^2*c^5 + B^2*a^2*b^6*c^2 - 12*B^2*a^3*b^4*c^3 + 36*B^2*a^4*b^2*c^4 - 48*A^2*a*b^6*c^3 + 48*A*B*a^2*b^5*c^3 - 168*A*B*a^3*b^3*c^4 - 4*A*B*a*b^7*c^2 + 144*A*B*a^4*b*c^5)*(36*A^2*a^4*c^4 - 24*A^2*b^8 - 6*B^2*a^2*b^6 + 400*B^2*a^5*c^3 + 24*A*B*a*b^7 - 1152*A^2*a^2*b^4*c^2 + 1528*A^2*a^3*b^2*c^3 - 291*B^2*a^4*b^2*c^2 + 288*A^2*a*b^6*c + 72*B^2*a^3*b^4*c + 1158*A*B*a^3*b^3*c^2 - 288*A*B*a^2*b^5*c - 1564*A*B*a^4*b*c^3)) + ((16*a^9*b^6*(4*a*c - b^2)^(9/2) - 1024*a^12*c^3*(4*a*c - b^2)^(9/2) - 192*a^10*b^4*c*(4*a*c - b^2)^(9/2) + 768*a^11*b^2*c^2*(4*a*c - b^2

```
)^(9/2))*((B^3*a^3*b^2*c^4 - 8*A^3*b^5*c^4 + 36*A^2*B*a^3*c^6 + 48*A^3*a*b^
3*c^5 - 72*A^3*a^2*b*c^6 + 12*A*B^2*a^3*b*c^5 + 12*A^2*B*a*b^4*c^4 - 6*A*B^
2*a^2*b^3*c^4 - 48*A^2*B*a^2*b^2*c^5)/(a^6*b^4 + 16*a^8*c^2 - 8*a^7*b^2*c)
- (((36*A^2*a^5*c^6 - 16*A^2*a^2*b^6*c^3 + 116*A^2*a^3*b^4*c^4 - 216*A^2*a^
4*b^2*c^5 - 4*B^2*a^4*b^4*c^3 + 17*B^2*a^5*b^2*c^4 + 16*A*B*a^3*b^5*c^3 - 9
2*A*B*a^4*b^3*c^4 + 108*A*B*a^5*b*c^5)/(a^6*b^4 + 16*a^8*c^2 - 8*a^7*b^2*c)
 - (((96*A*a^7*b*c^5 - 8*A*a^4*b^7*c^2 + 72*A*a^5*b^5*c^3 - 184*A*a^6*b^3*c
^4 + 4*B*a^5*b^6*c^2 - 36*B*a^6*b^4*c^3 + 80*B*a^7*b^2*c^4)/(a^6*b^4 + 16*a
^8*c^2 - 8*a^7*b^2*c) - ((4*a^7*b^6*c^2 - 32*a^8*b^4*c^3 + 64*a^9*b^2*c^4)*
(4*A*b^7 + 128*B*a^4*c^3 - 2*B*a*b^6 - 48*A*a*b^5*c - 256*A*a^3*b*c^3 + 24*
B*a^2*b^4*c + 192*A*a^2*b^3*c^2 - 96*B*a^3*b^2*c^2))/(2*(a^6*b^4 + 16*a^8*c
^2 - 8*a^7*b^2*c)*(4*a^3*b^6 - 256*a^6*c^3 - 48*a^4*b^4*c + 192*a^5*b^2*c^2
)))*(4*A*b^7 + 128*B*a^4*c^3 - 2*B*a*b^6 - 48*A*a*b^5*c - 256*A*a^3*b*c^3 +
 24*B*a^2*b^4*c + 192*A*a^2*b^3*c^2 - 96*B*a^3*b^2*c^2))/(2*(4*a^3*b^6 - 25
6*a^6*c^3 - 48*a^4*b^4*c + 192*a^5*b^2*c^2)))*(4*A*b^7 + 128*B*a^4*c^3 - 2*
B*a*b^6 - 48*A*a*b^5*c - 256*A*a^3*b*c^3 + 24*B*a^2*b^4*c + 192*A*a^2*b^3*c
^2 - 96*B*a^3*b^2*c^2))/(2*(4*a^3*b^6 - 256*a^6*c^3 - 48*a^4*b^4*c + 192*a^
5*b^2*c^2)) - (((((96*A*a^7*b*c^5 - 8*A*a^4*b^7*c^2 + 72*A*a^5*b^5*c^3 - 18
4*A*a^6*b^3*c^4 + 4*B*a^5*b^6*c^2 - 36*B*a^6*b^4*c^3 + 80*B*a^7*b^2*c^4)/(a
^6*b^4 + 16*a^8*c^2 - 8*a^7*b^2*c) - ((4*a^7*b^6*c^2 - 32*a^8*b^4*c^3 + 64*
a^9*b^2*c^4)*(4*A*b^7 + 128*B*a^4*c^3 - 2*B*a*b^6 - 48*A*a*b^5*c - 256*A*a^
3*b*c^3 + 24*B*a^2*b^4*c + 192*A*a^2*b^3*c^2 - 96*B*a^3*b^2*c^2))/(2*(a^6*b
^4 + 16*a^8*c^2 - 8*a^7*b^2*c)*(4*a^3*b^6 - 256*a^6*c^3 - 48*a^4*b^4*c + 19
2*a^5*b^2*c^2)))*(2*A*b^4 + 12*A*a^2*c^2 - B*a*b^3 - 12*A*a*b^2*c + 6*B*a^2
*b*c))/(4*a^3*(4*a*c - b^2)^(3/2)) - ((4*a^7*b^6*c^2 - 32*a^8*b^4*c^3 + 64*
a^9*b^2*c^4)*(2*A*b^4 + 12*A*a^2*c^2 - B*a*b^3 - 12*A*a*b^2*c + 6*B*a^2*b*c
)*(4*A*b^7 + 128*B*a^4*c^3 - 2*B*a*b^6 - 48*A*a*b^5*c - 256*A*a^3*b*c^3 + 2
4*B*a^2*b^4*c + 192*A*a^2*b^3*c^2 - 96*B*a^3*b^2*c^2))/(8*a^3*(4*a*c - b^2)
^(3/2)*(a^6*b^4 + 16*a^8*c^2 - 8*a^7*b^2*c)*(4*a^3*b^6 - 256*a^6*c^3 - 48*a
^4*b^4*c + 192*a^5*b^2*c^2)))*(2*A*b^4 + 12*A*a^2*c^2 - B*a*b^3 - 12*A*a*b^
2*c + 6*B*a^2*b*c))/(4*a^3*(4*a*c - b^2)^(3/2)) + ((4*a^7*b^6*c^2 - 32*a^8*
b^4*c^3 + 64*a^9*b^2*c^4)*(2*A*b^4 + 12*A*a^2*c^2 - B*a*b^3 - 12*A*a*b^2*c
+ 6*B*a^2*b*c)^2*(4*A*b^7 + 128*B*a^4*c^3 - 2*B*a*b^6 - 48*A*a*b^5*c - 256*
A*a^3*b*c^3 + 24*B*a^2*b^4*c + 192*A*a^2*b^3*c^2 - 96*B*a^3*b^2*c^2))/(32*a
^6*(4*a*c - b^2)^3*(a^6*b^4 + 16*a^8*c^2 - 8*a^7*b^2*c)*(4*a^3*b^6 - 256*a^
6*c^3 - 48*a^4*b^4*c + 192*a^5*b^2*c^2)))*(6*A*a^3*c^3 - 6*A*b^6 + 3*B*a*b^
5 + 42*A*a*b^4*c - 21*B*a^2*b^3*c + 33*B*a^3*b*c^2 - 72*A*a^2*b^2*c^2))/(8*
a^3*c^2*(4*a*c - b^2)^3*(144*A^2*a^4*c^6 + 4*A^2*b^8*c^2 + 192*A^2*a^2*b^4*
c^4 - 288*A^2*a^3*b^2*c^5 + B^2*a^2*b^6*c^2 - 12*B^2*a^3*b^4*c^3 + 36*B^2*a
^4*b^2*c^4 - 48*A^2*a*b^6*c^3 + 48*A*B*a^2*b^5*c^3 - 168*A*B*a^3*b^3*c^4 -
4*A*B*a*b^7*c^2 + 144*A*B*a^4*b*c^5)*(36*A^2*a^4*c^4 - 24*A^2*b^8 - 6*B^2*a
^2*b^6 + 400*B^2*a^5*c^3 + 24*A*B*a*b^7 - 1152*A^2*a^2*b^4*c^2 + 1528*A^2*a
^3*b^2*c^3 - 291*B^2*a^4*b^2*c^2 + 288*A^2*a*b^6*c + 72*B^2*a^3*b^4*c + 115
8*A*B*a^3*b^3*c^2 - 288*A*B*a^2*b^5*c - 1564*A*B*a^4*b*c^3)))*(2*A*b^4 + 12
*A*a^2*c^2 - B*a*b^3 - 12*A*a*b^2*c + 6*B*a^2*b*c))/(2*a^3*(4*a*c - b^2)^(3
```

```
/2))
```

3.118 $\int \frac{x^6(A+Bx^2)}{(a+bx^2+cx^4)^2}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 425

$$\int \frac{x^6(A+Bx^2)}{(a+bx^2+cx^4)^2}\,dx = \frac{(3b^2B-Abc-10aBc)\,x}{2c^2\,(b^2-4ac)} - \frac{(bB-2Ac)x^3}{2c\,(b^2-4ac)} - \frac{x^5(Ab-2aB-(bB-2Ac)x^2)}{2\,(b^2-4ac)\,(a+bx^2+cx^4)} - \frac{\left(3b^3B-Ab^2c-13abBc+6aAc^2-\frac{3b^4B-Ab^3c-19ab^2Bc+8aAbc^2+20a^2Bc^2}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}c^{5/2}\,(b^2-4ac)\,\sqrt{b-\sqrt{b^2-4ac}}} - \frac{\left(3b^3B-Ab^2c-13abBc+6aAc^2+\frac{3b^4B-Ab^3c-19ab^2Bc+8aAbc^2+20a^2Bc^2}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}c^{5/2}\,(b^2-4ac)\,\sqrt{b+\sqrt{b^2-4ac}}}$$

```
[Out] 1/2*(-A*b*c-10*B*a*c+3*B*b^2)*x/c^2/(-4*a*c+b^2)-1/2*(-2*A*c+B*b)*x^3/c/(-4
*a*c+b^2)-1/2*x^5*(A*b-2*B*a-(-2*A*c+B*b)*x^2)/(-4*a*c+b^2)/(c*x^4+b*x^2+a)
-1/4*arctan(x*2^(1/2)*c^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2))*(3*B*b^3-A*b^2*
c-13*B*a*b*c+6*A*a*c^2+(-8*A*a*b*c^2+A*b^3*c-20*B*a^2*c^2+19*B*a*b^2*c-3*B*
b^4)/(-4*a*c+b^2)^(1/2))/c^(5/2)/(-4*a*c+b^2)*2^(1/2)/(b-(-4*a*c+b^2)^(1/2)
)^(1/2)-1/4*arctan(x*2^(1/2)*c^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2))*(3*B*b^3
-A*b^2*c-13*B*a*b*c+6*A*a*c^2+(8*A*a*b*c^2-A*b^3*c+20*B*a^2*c^2-19*B*a*b^2*
c+3*B*b^4)/(-4*a*c+b^2)^(1/2))/c^(5/2)/(-4*a*c+b^2)*2^(1/2)/(b+(-4*a*c+b^2)
^(1/2))^(1/2)
```

Rubi [A] (verified)

Time = 2.42 (sec) , antiderivative size = 425, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {1289, 1293, 1180, 211}

$$\int \frac{x^6(A+Bx^2)}{(a+bx^2+cx^4)^2}\,dx = -\frac{\left(-\frac{20a^2Bc^2+8aAbc^2-19ab^2Bc-Ab^3c+3b^4B}{\sqrt{b^2-4ac}}+6aAc^2-13abBc-Ab^2c+3b^3B\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}c^{5/2}\left(b^2-4ac\right)\sqrt{b-\sqrt{b^2-4ac}}} - \frac{\left(\frac{20a^2Bc^2+8aAbc^2-19ab^2Bc-Ab^3c+3b^4B}{\sqrt{b^2-4ac}}+6aAc^2-13abBc-Ab^2c+3b^3B\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{2\sqrt{2}c^{5/2}\left(b^2-4ac\right)\sqrt{\sqrt{b^2-4ac}+b}} + \frac{x(-10aBc-Abc+3b^2B)}{2c^2\left(b^2-4ac\right)} - \frac{x^3(bB-2Ac)}{2c\left(b^2-4ac\right)} - \frac{x^5(-2aB-(x^2(bB-2Ac))+Ab)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}$$

```
[In] Int[(x^6*(A + B*x^2))/(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] ((3*b^2*B - A*b*c - 10*a*B*c)*x)/(2*c^2*(b^2 - 4*a*c)) - ((b*B - 2*A*c)*x^3
)/(2*c*(b^2 - 4*a*c)) - (x^5*(A*b - 2*a*B - (b*B - 2*A*c)*x^2))/(2*(b^2 - 4
*a*c)*(a + b*x^2 + c*x^4)) - ((3*b^3*B - A*b^2*c - 13*a*b*B*c + 6*a*A*c^2 -
 (3*b^4*B - A*b^3*c - 19*a*b^2*B*c + 8*a*A*b*c^2 + 20*a^2*B*c^2)/Sqrt[b^2 -
 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2
]*c^(5/2)*(b^2 - 4*a*c)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) - ((3*b^3*B - A*b^2*c
- 13*a*b*B*c + 6*a*A*c^2 + (3*b^4*B - A*b^3*c - 19*a*b^2*B*c + 8*a*A*b*c^2
+ 20*a^2*B*c^2)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt
[b^2 - 4*a*c]]])/(2*Sqrt[2]*c^(5/2)*(b^2 - 4*a*c)*Sqrt[b + Sqrt[b^2 - 4*a*c
]])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1289

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_.), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(a + b*x^2 + c*x^4)^(p + 1)
```

```
*((b*d - 2*a*e - (b*e - 2*c*d)*x^2)/(2*(p + 1)*(b^2 - 4*a*c))), x] - Dist[f
^2/(2*(p + 1)*(b^2 - 4*a*c)), Int[(f*x)^(m - 2)*(a + b*x^2 + c*x^4)^(p + 1)
*Simp[(m - 1)*(b*d - 2*a*e) - (4*p + 4 + m + 1)*(b*e - 2*c*d)*x^2, x], x],
x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] &&
GtQ[m, 1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1293

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[e*f*(f*x)^(m - 1)*((a + b*x^2 + c*x^4)^(p +
1)/(c*(m + 4*p + 3))), x] - Dist[f^2/(c*(m + 4*p + 3)), Int[(f*x)^(m - 2)*(
a + b*x^2 + c*x^4)^p*Simp[a*e*(m - 1) + (b*e*(m + 2*p + 1) - c*d*(m + 4*p +
 3))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c,
 0] && GtQ[m, 1] && NeQ[m + 4*p + 3, 0] && IntegerQ[2*p] && (IntegerQ[p] ||
 IntegerQ[m])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{x^5(Ab-2aB-(bB-2Ac)x^2)}{2(b^2-4ac)(a+bx^2+cx^4)}+\frac{\int \frac{x^4(5(Ab-2aB)-3(bB-2Ac)x^2)}{a+bx^2+cx^4}\,dx}{2(b^2-4ac)}\\
&= -\frac{(bB-2Ac)x^3}{2c(b^2-4ac)}-\frac{x^5(Ab-2aB-(bB-2Ac)x^2)}{2(b^2-4ac)(a+bx^2+cx^4)}-\frac{\int \frac{x^2(-9a(bB-2Ac)-3(3b^2B-Abc-10aBc)x^2)}{a+bx^2+cx^4}\,dx}{6c(b^2-4ac)}\\
&= \frac{(3b^2B-Abc-10aBc)\,x}{2c^2(b^2-4ac)}-\frac{(bB-2Ac)x^3}{2c(b^2-4ac)}-\frac{x^5(Ab-2aB-(bB-2Ac)x^2)}{2(b^2-4ac)(a+bx^2+cx^4)}\\
&\quad+\frac{\int \frac{-3a(3b^2B-Abc-10aBc)-3(3b^3B-Ab^2c-13abBc+6aAc^2)x^2}{a+bx^2+cx^4}\,dx}{6c^2(b^2-4ac)}\\
&= \frac{(3b^2B-Abc-10aBc)\,x}{2c^2(b^2-4ac)}-\frac{(bB-2Ac)x^3}{2c(b^2-4ac)}-\frac{x^5(Ab-2aB-(bB-2Ac)x^2)}{2(b^2-4ac)(a+bx^2+cx^4)}\\
&\quad-\frac{\left(3b^3B-Ab^2c-13abBc+6aAc^2-\frac{3b^4B-Ab^3c-19ab^2Bc+8aAbc^2+20a^2Bc^2}{\sqrt{b^2-4ac}}\right)\int \frac{1}{\frac{b}{2}-\frac{1}{2}\sqrt{b^2-4ac}+cx^2}\,dx}{4c^2(b^2-4ac)}\\
&\quad-\frac{\left(3b^3B-Ab^2c-13abBc+6aAc^2+\frac{3b^4B-Ab^3c-19ab^2Bc+8aAbc^2+20a^2Bc^2}{\sqrt{b^2-4ac}}\right)\int \frac{1}{\frac{b}{2}+\frac{1}{2}\sqrt{b^2-4ac}+cx^2}\,dx}{4c^2(b^2-4ac)}\\
&= \frac{(3b^2B-Abc-10aBc)\,x}{2c^2(b^2-4ac)}-\frac{(bB-2Ac)x^3}{2c(b^2-4ac)}-\frac{x^5(Ab-2aB-(bB-2Ac)x^2)}{2(b^2-4ac)(a+bx^2+cx^4)}\\
&\quad-\frac{\left(3b^3B-Ab^2c-13abBc+6aAc^2-\frac{3b^4B-Ab^3c-19ab^2Bc+8aAbc^2+20a^2Bc^2}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}c^{5/2}(b^2-4ac)\sqrt{b-\sqrt{b^2-4ac}}}\\
&\quad-\frac{\left(3b^3B-Ab^2c-13abBc+6aAc^2+\frac{3b^4B-Ab^3c-19ab^2Bc+8aAbc^2+20a^2Bc^2}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}c^{5/2}(b^2-4ac)\sqrt{b+\sqrt{b^2-4ac}}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.76 (sec) , antiderivative size = 455, normalized size of antiderivative = 1.07

$$\int \frac{x^6(A+Bx^2)}{(a+bx^2+cx^4)^2}\,dx$$

$$= \frac{4B\sqrt{c}x + \frac{2\sqrt{c}x(-2a^2Bc+b^2(bB-Ac)x^2+a(b^2B+2Ac^2x^2-bc(A+3Bx^2)))}{(b^2-4ac)(a+bx^2+cx^4)} - \frac{\sqrt{2}\left(-3b^4B+b^2c\left(19aB-A\sqrt{b^2-4ac}\right)+2ac^2\left(-10aB+3A\sqrt{\ldots}\right.\right.}{\ldots}}{\ldots}$$

```
[In] Integrate[(x^6*(A + B*x^2))/(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] (4*B*Sqrt[c]*x + (2*Sqrt[c]*x*(-2*a^2*B*c + b^2*(b*B - A*c)*x^2 + a*(b^2*B
+ 2*A*c^2*x^2 - b*c*(A + 3*B*x^2))))/((b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) -
(Sqrt[2]*(-3*b^4*B + b^2*c*(19*a*B - A*Sqrt[b^2 - 4*a*c]) + 2*a*c^2*(-10*a*
B + 3*A*Sqrt[b^2 - 4*a*c]) + b^3*(A*c + 3*B*Sqrt[b^2 - 4*a*c]) - a*b*c*(8*A
*c + 13*B*Sqrt[b^2 - 4*a*c]))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2
- 4*a*c]]])/((b^2 - 4*a*c)^(3/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) - (Sqrt[2]*(3
*b^4*B - b^2*c*(19*a*B + A*Sqrt[b^2 - 4*a*c]) + 2*a*c^2*(10*a*B + 3*A*Sqrt[
b^2 - 4*a*c]) + a*b*c*(8*A*c - 13*B*Sqrt[b^2 - 4*a*c]) + b^3*(-(A*c) + 3*B*
Sqrt[b^2 - 4*a*c]))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]]
)/((b^2 - 4*a*c)^(3/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]]))/(4*c^(5/2))
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.13 (sec) , antiderivative size = 217, normalized size of antiderivative = 0.51

method	result
risch	$\frac{Bx}{c^2} + \frac{-\frac{\left(2Aa\,c^2-A\,b^2c-3Babc+B\,b^3\right)x^3}{2\left(4ac-b^2\right)}+\frac{a\left(Abc+2Bac-B\,b^2\right)x}{8ac-2b^2}}{c^2\left(c\,x^4+b\,x^2+a\right)} + \frac{\sum_{\_R=\mathrm{RootOf}\left(c\_Z^4+\_Z^2b+a\right)} \frac{\left(\frac{\left(6Aa\,c^2-A\,b^2c-13Babc+3B\,b^3\right)\_\ldots}{4ac-b^2}\ldots\right.}{2c\_\ldots}}{4c^2}$
default	$\frac{Bx}{c^2} + \frac{-\frac{\left(2Aa\,c^2-A\,b^2c-3Babc+B\,b^3\right)x^3}{2\left(4ac-b^2\right)}+\frac{a\left(Abc+2Bac-B\,b^2\right)x}{8ac-2b^2}}{c\,x^4+b\,x^2+a} + \frac{2c\left(\frac{\left(6Aa\,c^2\sqrt{-4ac+b^2}-A\,b^2c\sqrt{-4ac+b^2}+8Aab\,c^2-A\,b^3c-13Babc\sqrt{-4a\ldots}\right.}{8c\sqrt{-4\ldots}}\ldots\right.}{\ldots}$

```
[In] int(x^6*(B*x^2+A)/(c*x^4+b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] B*x/c^2+(-1/2*(2*A*a*c^2-A*b^2*c-3*B*a*b*c+B*b^3)/(4*a*c-b^2)*x^3+1/2*a*(A*
b*c+2*B*a*c-B*b^2)/(4*a*c-b^2)*x)/c^2/(c*x^4+b*x^2+a)+1/4/c^2*sum(((6*A*a*c
```

```
^2-A*b^2*c-13*B*a*b*c+3*B*b^3)/(4*a*c-b^2)*_R^2-a*(A*b*c+10*B*a*c-3*B*b^2)/
(4*a*c-b^2))/(2*_R^3*c+_R*b)*ln(x-_R),_R=RootOf(_Z^4*c+_Z^2*b+a))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 7252 vs. $2(379) = 758$.

Time = 7.53 (sec) , antiderivative size = 7252, normalized size of antiderivative = 17.06

$$\int \frac{x^6(A+Bx^2)}{(a+bx^2+cx^4)^2}\,dx = \text{Too large to display}$$

```
[In] integrate(x^6*(B*x^2+A)/(c*x^4+b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^6(A+Bx^2)}{(a+bx^2+cx^4)^2}\,dx = \text{Timed out}$$

```
[In] integrate(x**6*(B*x**2+A)/(c*x**4+b*x**2+a)**2,x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{x^6(A+Bx^2)}{(a+bx^2+cx^4)^2}\,dx = \int \frac{(Bx^2+A)x^6}{(cx^4+bx^2+a)^2}\,dx$$

```
[In] integrate(x^6*(B*x^2+A)/(c*x^4+b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] 1/2*((B*b^3 + 2*A*a*c^2 - (3*B*a*b + A*b^2)*c)*x^3 + (B*a*b^2 - (2*B*a^2 +
A*a*b)*c)*x)/(a*b^2*c^2 - 4*a^2*c^3 + (b^2*c^3 - 4*a*c^4)*x^4 + (b^3*c^2 -
4*a*b*c^3)*x^2) + B*x/c^2 - 1/2*integrate((3*B*a*b^2 + (3*B*b^3 + 6*A*a*c^2
 - (13*B*a*b + A*b^2)*c)*x^2 - (10*B*a^2 + A*a*b)*c)/(c*x^4 + b*x^2 + a), x
)/(b^2*c^2 - 4*a*c^3)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 5675 vs. 2(379) = 758.

Time = 1.51 (sec) , antiderivative size = 5675, normalized size of antiderivative = 13.35

$$\int \frac{x^6(A+Bx^2)}{(a+bx^2+cx^4)^2}\,dx = \text{Too large to display}$$

```
[In] integrate(x^6*(B*x^2+A)/(c*x^4+b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] B*x/c^2 + 1/2*(B*b^3*x^3 - 3*B*a*b*c*x^3 - A*b^2*c*x^3 + 2*A*a*c^2*x^3 + B*
a*b^2*x - 2*B*a^2*c*x - A*a*b*c*x)/((c*x^4 + b*x^2 + a)*(b^2*c^2 - 4*a*c^3)
) + 1/16*((2*b^4*c^3 - 20*a*b^2*c^4 + 48*a^2*c^5 - sqrt(2)*sqrt(b^2 - 4*a*c
)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^4*c + 10*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt
(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^2 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*
c + sqrt(b^2 - 4*a*c)*c)*b^3*c^2 - 24*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c +
sqrt(b^2 - 4*a*c)*c)*a^2*c^3 - 12*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt
(b^2 - 4*a*c)*c)*a*b*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 -
4*a*c)*c)*b^2*c^3 + 6*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c
)*c)*a*c^4 - 2*(b^2 - 4*a*c)*b^2*c^3 + 12*(b^2 - 4*a*c)*a*c^4)*(b^2*c^2 - 4
*a*c^3)^2*A - (6*b^5*c^2 - 50*a*b^3*c^3 + 104*a^2*b*c^4 - 3*sqrt(2)*sqrt(b^
2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^5 + 25*sqrt(2)*sqrt(b^2 - 4*a*
c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c + 6*sqrt(2)*sqrt(b^2 - 4*a*c)*sq
rt(b*c + sqrt(b^2 - 4*a*c)*c)*b^4*c - 52*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c
 + sqrt(b^2 - 4*a*c)*c)*a^2*b*c^2 - 26*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c +
 sqrt(b^2 - 4*a*c)*c)*a*b^2*c^2 - 3*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sq
rt(b^2 - 4*a*c)*c)*b^3*c^2 + 13*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b
^2 - 4*a*c)*c)*a*b*c^3 - 6*(b^2 - 4*a*c)*b^3*c^2 + 26*(b^2 - 4*a*c)*a*b*c^3
)*(b^2*c^2 - 4*a*c^3)^2*B + 2*(sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^
5*c^4 - 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c^5 - 2*sqrt(2)*s
qrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^4*c^5 - 2*a*b^5*c^5 + 16*sqrt(2)*sqrt(b*
c + sqrt(b^2 - 4*a*c)*c)*a^3*b*c^6 + 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)
*c)*a^2*b^2*c^6 + sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c^6 + 16*a^
2*b^3*c^6 - 4*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b*c^7 - 32*a^3*b*
c^7 + 2*(b^2 - 4*a*c)*a*b^3*c^5 - 8*(b^2 - 4*a*c)*a^2*b*c^6)*A*abs(b^2*c^2
- 4*a*c^3) - 2*(3*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^6*c^3 - 34*sq
rt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^4*c^4 - 6*sqrt(2)*sqrt(b*c + sq
rt(b^2 - 4*a*c)*c)*a*b^5*c^4 - 6*a*b^6*c^4 + 128*sqrt(2)*sqrt(b*c + sqrt(b^
2 - 4*a*c)*c)*a^3*b^2*c^5 + 44*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*
b^3*c^5 + 3*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^4*c^5 + 68*a^2*b^4*
c^5 - 160*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*c^6 - 80*sqrt(2)*sqrt
(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b*c^6 - 22*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*
a*c)*c)*a^2*b^2*c^6 - 256*a^3*b^2*c^6 + 40*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*
a*c)*c)*a^3*c^7 + 320*a^4*c^7 + 6*(b^2 - 4*a*c)*a*b^4*c^4 - 44*(b^2 - 4*a*c
)*a^2*b^2*c^5 + 80*(b^2 - 4*a*c)*a^3*c^6)*B*abs(b^2*c^2 - 4*a*c^3) - (2*b^8
```

```
*c^7 - 32*a*b^6*c^8 + 160*a^2*b^4*c^9 - 256*a^3*b^2*c^10 - sqrt(2)*sqrt(b^2
 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^8*c^5 + 16*sqrt(2)*sqrt(b^2 - 4
*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^6*c^6 + 2*sqrt(2)*sqrt(b^2 - 4*a*
c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^7*c^6 - 80*sqrt(2)*sqrt(b^2 - 4*a*c)*s
qrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^4*c^7 - 24*sqrt(2)*sqrt(b^2 - 4*a*c)*s
qrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^5*c^7 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b
*c + sqrt(b^2 - 4*a*c)*c)*b^6*c^7 + 128*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c
+ sqrt(b^2 - 4*a*c)*c)*a^3*b^2*c^8 + 64*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c
+ sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c^8 + 12*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c
+ sqrt(b^2 - 4*a*c)*c)*a*b^4*c^8 - 32*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c +
sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^9 - 2*(b^2 - 4*a*c)*b^6*c^7 + 24*(b^2 - 4*a*
c)*a*b^4*c^8 - 64*(b^2 - 4*a*c)*a^2*b^2*c^9)*A + (6*b^9*c^6 - 86*a*b^7*c^7
+ 440*a^2*b^5*c^8 - 928*a^3*b^3*c^9 + 640*a^4*b*c^10 - 3*sqrt(2)*sqrt(b^2 -
 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^9*c^4 + 43*sqrt(2)*sqrt(b^2 - 4*a
*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^7*c^5 + 6*sqrt(2)*sqrt(b^2 - 4*a*c)
*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^8*c^5 - 220*sqrt(2)*sqrt(b^2 - 4*a*c)*sq
rt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^5*c^6 - 62*sqrt(2)*sqrt(b^2 - 4*a*c)*sq
rt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^6*c^6 - 3*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(
b*c + sqrt(b^2 - 4*a*c)*c)*b^7*c^6 + 464*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c
 + sqrt(b^2 - 4*a*c)*c)*a^3*b^3*c^7 + 192*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*
c + sqrt(b^2 - 4*a*c)*c)*a^2*b^4*c^7 + 31*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*
c + sqrt(b^2 - 4*a*c)*c)*a*b^5*c^7 - 320*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c
 + sqrt(b^2 - 4*a*c)*c)*a^4*b*c^8 - 160*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c
+ sqrt(b^2 - 4*a*c)*c)*a^3*b^2*c^8 - 96*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c
+ sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c^8 + 80*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c
+ sqrt(b^2 - 4*a*c)*c)*a^3*b*c^9 - 6*(b^2 - 4*a*c)*b^7*c^6 + 62*(b^2 - 4*a*
c)*a*b^5*c^7 - 192*(b^2 - 4*a*c)*a^2*b^3*c^8 + 160*(b^2 - 4*a*c)*a^3*b*c^9)
*B)*arctan(2*sqrt(1/2)*x/sqrt((b^3*c^2 - 4*a*b*c^3 + sqrt((b^3*c^2 - 4*a*b*
c^3)^2 - 4*(a*b^2*c^2 - 4*a^2*c^3)*(b^2*c^3 - 4*a*c^4)))/(b^2*c^3 - 4*a*c^4
)))/((a*b^6*c^5 - 12*a^2*b^4*c^6 - 2*a*b^5*c^6 + 48*a^3*b^2*c^7 + 16*a^2*b^
3*c^7 + a*b^4*c^7 - 64*a^4*c^8 - 32*a^3*b*c^8 - 8*a^2*b^2*c^8 + 16*a^3*c^9)
*abs(b^2*c^2 - 4*a*c^3)*abs(c)) - 1/16*((2*b^4*c^3 - 20*a*b^2*c^4 + 48*a^2*
c^5 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^4*c + 10*
sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^2*c^2 + 2*sqr
t(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^3*c^2 - 24*sqrt(2)
*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*c^3 - 12*sqrt(2)*sqr
t(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b*c^3 - sqrt(2)*sqrt(b^2 -
 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^2*c^3 + 6*sqrt(2)*sqrt(b^2 - 4*a*
c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*c^4 - 2*(b^2 - 4*a*c)*b^2*c^3 + 12*(b^
2 - 4*a*c)*a*c^4)*(b^2*c^2 - 4*a*c^3)^2*A - (6*b^5*c^2 - 50*a*b^3*c^3 + 104
*a^2*b*c^4 - 3*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^
5 + 25*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^3*c +
6*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^4*c - 52*sqrt
(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b*c^2 - 26*sqrt(2
)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^2*c^2 - 3*sqrt(2)*s
```

qrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^3*c^2 + 13*sqrt(2)*sqrt(
b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b*c^3 - 6*(b^2 - 4*a*c)*b^3*
c^2 + 26*(b^2 - 4*a*c)*a*b*c^3)*(b^2*c^2 - 4*a*c^3)^2*B - 2*(sqrt(2)*sqrt(b
*c - sqrt(b^2 - 4*a*c)*c)*a*b^5*c^4 - 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c
)*c)*a^2*b^3*c^5 - 2*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^4*c^5 + 2*
a*b^5*c^5 + 16*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b*c^6 + 8*sqrt(2
)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^6 + sqrt(2)*sqrt(b*c - sqrt(b^2
 - 4*a*c)*c)*a*b^3*c^6 - 16*a^2*b^3*c^6 - 4*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4
*a*c)*c)*a^2*b*c^7 + 32*a^3*b*c^7 - 2*(b^2 - 4*a*c)*a*b^3*c^5 + 8*(b^2 - 4*
a*c)*a^2*b*c^6)*A*abs(b^2*c^2 - 4*a*c^3) + 2*(3*sqrt(2)*sqrt(b*c - sqrt(b^2
 - 4*a*c)*c)*a*b^6*c^3 - 34*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^4
*c^4 - 6*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^5*c^4 + 6*a*b^6*c^4 +
128*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^2*c^5 + 44*sqrt(2)*sqrt(b
*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c^5 + 3*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a
*c)*c)*a*b^4*c^5 - 68*a^2*b^4*c^5 - 160*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c
)*c)*a^4*c^6 - 80*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b*c^6 - 22*sq
rt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^6 + 256*a^3*b^2*c^6 + 40*sq
rt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*c^7 - 320*a^4*c^7 - 6*(b^2 - 4*a*
c)*a*b^4*c^4 + 44*(b^2 - 4*a*c)*a^2*b^2*c^5 - 80*(b^2 - 4*a*c)*a^3*c^6)*B*a
bs(b^2*c^2 - 4*a*c^3) - (2*b^8*c^7 - 32*a*b^6*c^8 + 160*a^2*b^4*c^9 - 256*a
^3*b^2*c^10 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^8
*c^5 + 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^6*c
^6 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^7*c^6 -
80*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^4*c^7 -
24*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^5*c^7 - sq
rt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^6*c^7 + 128*sqrt(
2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^2*c^8 + 64*sqrt(
2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c^8 + 12*sqrt(
2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^4*c^8 - 32*sqrt(2)
*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^9 - 2*(b^2 - 4
*a*c)*b^6*c^7 + 24*(b^2 - 4*a*c)*a*b^4*c^8 - 64*(b^2 - 4*a*c)*a^2*b^2*c^9)*
A + (6*b^9*c^6 - 86*a*b^7*c^7 + 440*a^2*b^5*c^8 - 928*a^3*b^3*c^9 + 640*a^4
*b*c^10 - 3*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^9*c
^4 + 43*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^7*c^5
 + 6*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^8*c^5 - 22
0*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^5*c^6 - 6
2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^6*c^6 - 3*s
qrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^7*c^6 + 464*sqrt
(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^3*c^7 + 192*sqr
t(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^4*c^7 + 31*sqr
t(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^5*c^7 - 320*sqrt
(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*b*c^8 - 160*sqrt(
2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^2*c^8 - 96*sqrt(
2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c^8 + 80*sqrt(
2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b*c^9 - 6*(b^2 - 4

```
*a*c)*b^7*c^6 + 62*(b^2 - 4*a*c)*a*b^5*c^7 - 192*(b^2 - 4*a*c)*a^2*b^3*c^8
+ 160*(b^2 - 4*a*c)*a^3*b*c^9)*B)*arctan(2*sqrt(1/2)*x/sqrt((b^3*c^2 - 4*a*
b*c^3 - sqrt((b^3*c^2 - 4*a*b*c^3)^2 - 4*(a*b^2*c^2 - 4*a^2*c^3)*(b^2*c^3 -
 4*a*c^4)))/(b^2*c^3 - 4*a*c^4)))/((a*b^6*c^5 - 12*a^2*b^4*c^6 - 2*a*b^5*c^
6 + 48*a^3*b^2*c^7 + 16*a^2*b^3*c^7 + a*b^4*c^7 - 64*a^4*c^8 - 32*a^3*b*c^8
 - 8*a^2*b^2*c^8 + 16*a^3*c^9)*abs(b^2*c^2 - 4*a*c^3)*abs(c))
```

Mupad [B] (verification not implemented)

Time = 10.14 (sec) , antiderivative size = 16604, normalized size of antiderivative = 39.07

$$\int \frac{x^6(A+Bx^2)}{(a+bx^2+cx^4)^2}\,dx = \text{Too large to display}$$

```
[In] int((x^6*(A + B*x^2))/(a + b*x^2 + c*x^4)^2,x)
```

```
[Out] (B*x)/c^2 - atan(((((10240*B*a^5*c^7 - 16*A*a*b^7*c^4 + 1024*A*a^4*b*c^7 +
48*B*a*b^8*c^3 + 192*A*a^2*b^5*c^5 - 768*A*a^3*b^3*c^6 - 736*B*a^2*b^6*c^4
+ 4224*B*a^3*b^4*c^5 - 10752*B*a^4*b^2*c^6)/(8*(64*a^3*c^6 - b^6*c^3 + 12*a
*b^4*c^4 - 48*a^2*b^2*c^5)) - (x*((9*B^2*b^4*(-(4*a*c - b^2)^9)^(1/2) - A^2
*b^11*c^2 - 9*B^2*b^13 + 6*A*B*b^12*c - 288*A^2*a^2*b^7*c^4 + 1504*A^2*a^3*
b^5*c^5 - 3840*A^2*a^4*b^3*c^6 - 2077*B^2*a^2*b^9*c^2 + 10656*B^2*a^3*b^7*c
^3 - 30240*B^2*a^4*b^5*c^4 + 44800*B^2*a^5*b^3*c^5 + A^2*b^2*c^2*(-(4*a*c -
 b^2)^9)^(1/2) + 25*B^2*a^2*c^2*(-(4*a*c - b^2)^9)^(1/2) + 15360*A*B*a^6*c^
7 + 213*B^2*a*b^11*c + 27*A^2*a*b^9*c^3 + 3840*A^2*a^5*b*c^7 - 9*A^2*a*c^3*
(-(4*a*c - b^2)^9)^(1/2) - 26880*B^2*a^6*b*c^6 + 1548*A*B*a^2*b^8*c^3 - 806
4*A*B*a^3*b^6*c^4 + 22400*A*B*a^4*b^4*c^5 - 30720*A*B*a^5*b^2*c^6 - 51*B^2*
a*b^2*c*(-(4*a*c - b^2)^9)^(1/2) - 152*A*B*a*b^10*c^2 - 6*A*B*b^3*c*(-(4*a*
c - b^2)^9)^(1/2) + 44*A*B*a*b*c^2*(-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a^6*
c^11 + b^12*c^5 - 24*a*b^10*c^6 + 240*a^2*b^8*c^7 - 1280*a^3*b^6*c^8 + 3840
*a^4*b^4*c^9 - 6144*a^5*b^2*c^10)))^(1/2)*(16*b^7*c^5 - 192*a*b^5*c^6 - 102
4*a^3*b*c^8 + 768*a^2*b^3*c^7))/(2*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))*(
(9*B^2*b^4*(-(4*a*c - b^2)^9)^(1/2) - A^2*b^11*c^2 - 9*B^2*b^13 + 6*A*B*b^1
2*c - 288*A^2*a^2*b^7*c^4 + 1504*A^2*a^3*b^5*c^5 - 3840*A^2*a^4*b^3*c^6 - 2
077*B^2*a^2*b^9*c^2 + 10656*B^2*a^3*b^7*c^3 - 30240*B^2*a^4*b^5*c^4 + 44800
*B^2*a^5*b^3*c^5 + A^2*b^2*c^2*(-(4*a*c - b^2)^9)^(1/2) + 25*B^2*a^2*c^2*(-
(4*a*c - b^2)^9)^(1/2) + 15360*A*B*a^6*c^7 + 213*B^2*a*b^11*c + 27*A^2*a*b^
9*c^3 + 3840*A^2*a^5*b*c^7 - 9*A^2*a*c^3*(-(4*a*c - b^2)^9)^(1/2) - 26880*B
^2*a^6*b*c^6 + 1548*A*B*a^2*b^8*c^3 - 8064*A*B*a^3*b^6*c^4 + 22400*A*B*a^4*
b^4*c^5 - 30720*A*B*a^5*b^2*c^6 - 51*B^2*a*b^2*c*(-(4*a*c - b^2)^9)^(1/2) -
 152*A*B*a*b^10*c^2 - 6*A*B*b^3*c*(-(4*a*c - b^2)^9)^(1/2) + 44*A*B*a*b*c^2
*(-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a^6*c^11 + b^12*c^5 - 24*a*b^10*c^6 +
240*a^2*b^8*c^7 - 1280*a^3*b^6*c^8 + 3840*a^4*b^4*c^9 - 6144*a^5*b^2*c^10))
)^(1/2) - (x*(9*B^2*b^8 - 72*A^2*a^3*c^5 + A^2*b^6*c^2 + 200*B^2*a^4*c^4 -
6*A*B*b^7*c + 74*A^2*a^2*b^2*c^4 + 481*B^2*a^2*b^4*c^2 - 718*B^2*a^3*b^2*c^
```

3 - 114*B^2*a*b^6*c - 16*A^2*a*b^4*c^3 - 374*A*B*a^2*b^3*c^3 + 86*A*B*a*b^5
*c^2 + 472*A*B*a^3*b*c^4))/(2*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))*((9*B^
2*b^4*(-(4*a*c - b^2)^9)^(1/2) - A^2*b^11*c^2 - 9*B^2*b^13 + 6*A*B*b^12*c -
288*A^2*a^2*b^7*c^4 + 1504*A^2*a^3*b^5*c^5 - 3840*A^2*a^4*b^3*c^6 - 2077*B
^2*a^2*b^9*c^2 + 10656*B^2*a^3*b^7*c^3 - 30240*B^2*a^4*b^5*c^4 + 44800*B^2*
a^5*b^3*c^5 + A^2*b^2*c^2*(-(4*a*c - b^2)^9)^(1/2) + 25*B^2*a^2*c^2*(-(4*a*
c - b^2)^9)^(1/2) + 15360*A*B*a^6*c^7 + 213*B^2*a*b^11*c + 27*A^2*a*b^9*c^3
+ 3840*A^2*a^5*b*c^7 - 9*A^2*a*c^3*(-(4*a*c - b^2)^9)^(1/2) - 26880*B^2*a^
6*b*c^6 + 1548*A*B*a^2*b^8*c^3 - 8064*A*B*a^3*b^6*c^4 + 22400*A*B*a^4*b^4*c
^5 - 30720*A*B*a^5*b^2*c^6 - 51*B^2*a*b^2*c*(-(4*a*c - b^2)^9)^(1/2) - 152*
A*B*a*b^10*c^2 - 6*A*B*b^3*c*(-(4*a*c - b^2)^9)^(1/2) + 44*A*B*a*b*c^2*(-(4
*a*c - b^2)^9)^(1/2))/(32*(4096*a^6*c^11 + b^12*c^5 - 24*a*b^10*c^6 + 240*a
^2*b^8*c^7 - 1280*a^3*b^6*c^8 + 3840*a^4*b^4*c^9 - 6144*a^5*b^2*c^10)))^(1/
2)*1i - (((10240*B*a^5*c^7 - 16*A*a*b^7*c^4 + 1024*A*a^4*b*c^7 + 48*B*a*b^8
*c^3 + 192*A*a^2*b^5*c^5 - 768*A*a^3*b^3*c^6 - 736*B*a^2*b^6*c^4 + 4224*B*a
^3*b^4*c^5 - 10752*B*a^4*b^2*c^6)/(8*(64*a^3*c^6 - b^6*c^3 + 12*a*b^4*c^4 -
48*a^2*b^2*c^5)) + (x*((9*B^2*b^4*(-(4*a*c - b^2)^9)^(1/2) - A^2*b^11*c^2
- 9*B^2*b^13 + 6*A*B*b^12*c - 288*A^2*a^2*b^7*c^4 + 1504*A^2*a^3*b^5*c^5 -
3840*A^2*a^4*b^3*c^6 - 2077*B^2*a^2*b^9*c^2 + 10656*B^2*a^3*b^7*c^3 - 30240
*B^2*a^4*b^5*c^4 + 44800*B^2*a^5*b^3*c^5 + A^2*b^2*c^2*(-(4*a*c - b^2)^9)^(
1/2) + 25*B^2*a^2*c^2*(-(4*a*c - b^2)^9)^(1/2) + 15360*A*B*a^6*c^7 + 213*B^
2*a*b^11*c + 27*A^2*a*b^9*c^3 + 3840*A^2*a^5*b*c^7 - 9*A^2*a*c^3*(-(4*a*c -
b^2)^9)^(1/2) - 26880*B^2*a^6*b*c^6 + 1548*A*B*a^2*b^8*c^3 - 8064*A*B*a^3*
b^6*c^4 + 22400*A*B*a^4*b^4*c^5 - 30720*A*B*a^5*b^2*c^6 - 51*B^2*a*b^2*c*(-
(4*a*c - b^2)^9)^(1/2) - 152*A*B*a*b^10*c^2 - 6*A*B*b^3*c*(-(4*a*c - b^2)^9
)^(1/2) + 44*A*B*a*b*c^2*(-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a^6*c^11 + b^1
2*c^5 - 24*a*b^10*c^6 + 240*a^2*b^8*c^7 - 1280*a^3*b^6*c^8 + 3840*a^4*b^4*c
^9 - 6144*a^5*b^2*c^10)))^(1/2)*(16*b^7*c^5 - 192*a*b^5*c^6 - 1024*a^3*b*c^
8 + 768*a^2*b^3*c^7))/(2*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))*((9*B^2*b^4
*(-(4*a*c - b^2)^9)^(1/2) - A^2*b^11*c^2 - 9*B^2*b^13 + 6*A*B*b^12*c - 288*
A^2*a^2*b^7*c^4 + 1504*A^2*a^3*b^5*c^5 - 3840*A^2*a^4*b^3*c^6 - 2077*B^2*a^
2*b^9*c^2 + 10656*B^2*a^3*b^7*c^3 - 30240*B^2*a^4*b^5*c^4 + 44800*B^2*a^5*b
^3*c^5 + A^2*b^2*c^2*(-(4*a*c - b^2)^9)^(1/2) + 25*B^2*a^2*c^2*(-(4*a*c - b
^2)^9)^(1/2) + 15360*A*B*a^6*c^7 + 213*B^2*a*b^11*c + 27*A^2*a*b^9*c^3 + 38
40*A^2*a^5*b*c^7 - 9*A^2*a*c^3*(-(4*a*c - b^2)^9)^(1/2) - 26880*B^2*a^6*b*c
^6 + 1548*A*B*a^2*b^8*c^3 - 8064*A*B*a^3*b^6*c^4 + 22400*A*B*a^4*b^4*c^5 -
30720*A*B*a^5*b^2*c^6 - 51*B^2*a*b^2*c*(-(4*a*c - b^2)^9)^(1/2) - 152*A*B*a
*b^10*c^2 - 6*A*B*b^3*c*(-(4*a*c - b^2)^9)^(1/2) + 44*A*B*a*b*c^2*(-(4*a*c
- b^2)^9)^(1/2))/(32*(4096*a^6*c^11 + b^12*c^5 - 24*a*b^10*c^6 + 240*a^2*b^
8*c^7 - 1280*a^3*b^6*c^8 + 3840*a^4*b^4*c^9 - 6144*a^5*b^2*c^10)))^(1/2) +
(x*(9*B^2*b^8 - 72*A^2*a^3*c^5 + A^2*b^6*c^2 + 200*B^2*a^4*c^4 - 6*A*B*b^7*
c + 74*A^2*a^2*b^2*c^4 + 481*B^2*a^2*b^4*c^2 - 718*B^2*a^3*b^2*c^3 - 114*B^
2*a*b^6*c - 16*A^2*a*b^4*c^3 - 374*A*B*a^2*b^3*c^3 + 86*A*B*a*b^5*c^2 + 472
*A*B*a^3*b*c^4))/(2*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))*((9*B^2*b^4*(-(4
*a*c - b^2)^9)^(1/2) - A^2*b^11*c^2 - 9*B^2*b^13 + 6*A*B*b^12*c - 288*A^2*a

```
^2*b^7*c^4 + 1504*A^2*a^3*b^5*c^5 - 3840*A^2*a^4*b^3*c^6 - 2077*B^2*a^2*b^9
*c^2 + 10656*B^2*a^3*b^7*c^3 - 30240*B^2*a^4*b^5*c^4 + 44800*B^2*a^5*b^3*c^
5 + A^2*b^2*c^2*(-(4*a*c - b^2)^9)^(1/2) + 25*B^2*a^2*c^2*(-(4*a*c - b^2)^9
)^(1/2) + 15360*A*B*a^6*c^7 + 213*B^2*a*b^11*c + 27*A^2*a*b^9*c^3 + 3840*A^
2*a^5*b*c^7 - 9*A^2*a*c^3*(-(4*a*c - b^2)^9)^(1/2) - 26880*B^2*a^6*b*c^6 +
1548*A*B*a^2*b^8*c^3 - 8064*A*B*a^3*b^6*c^4 + 22400*A*B*a^4*b^4*c^5 - 30720
*A*B*a^5*b^2*c^6 - 51*B^2*a*b^2*c*(-(4*a*c - b^2)^9)^(1/2) - 152*A*B*a*b^10
*c^2 - 6*A*B*b^3*c*(-(4*a*c - b^2)^9)^(1/2) + 44*A*B*a*b*c^2*(-(4*a*c - b^2
)^9)^(1/2))/(32*(4096*a^6*c^11 + b^12*c^5 - 24*a*b^10*c^6 + 240*a^2*b^8*c^7
 - 1280*a^3*b^6*c^8 + 3840*a^4*b^4*c^9 - 6144*a^5*b^2*c^10)))^(1/2)*1i)/(((
(10240*B*a^5*c^7 - 16*A*a*b^7*c^4 + 1024*A*a^4*b*c^7 + 48*B*a*b^8*c^3 + 192
*A*a^2*b^5*c^5 - 768*A*a^3*b^3*c^6 - 736*B*a^2*b^6*c^4 + 4224*B*a^3*b^4*c^5
 - 10752*B*a^4*b^2*c^6)/(8*(64*a^3*c^6 - b^6*c^3 + 12*a*b^4*c^4 - 48*a^2*b^
2*c^5)) - (x*((9*B^2*b^4*(-(4*a*c - b^2)^9)^(1/2) - A^2*b^11*c^2 - 9*B^2*b^
13 + 6*A*B*b^12*c - 288*A^2*a^2*b^7*c^4 + 1504*A^2*a^3*b^5*c^5 - 3840*A^2*a
^4*b^3*c^6 - 2077*B^2*a^2*b^9*c^2 + 10656*B^2*a^3*b^7*c^3 - 30240*B^2*a^4*b
^5*c^4 + 44800*B^2*a^5*b^3*c^5 + A^2*b^2*c^2*(-(4*a*c - b^2)^9)^(1/2) + 25*
B^2*a^2*c^2*(-(4*a*c - b^2)^9)^(1/2) + 15360*A*B*a^6*c^7 + 213*B^2*a*b^11*c
 + 27*A^2*a*b^9*c^3 + 3840*A^2*a^5*b*c^7 - 9*A^2*a*c^3*(-(4*a*c - b^2)^9)^(
1/2) - 26880*B^2*a^6*b*c^6 + 1548*A*B*a^2*b^8*c^3 - 8064*A*B*a^3*b^6*c^4 +
22400*A*B*a^4*b^4*c^5 - 30720*A*B*a^5*b^2*c^6 - 51*B^2*a*b^2*c*(-(4*a*c - b
^2)^9)^(1/2) - 152*A*B*a*b^10*c^2 - 6*A*B*b^3*c*(-(4*a*c - b^2)^9)^(1/2) +
44*A*B*a*b*c^2*(-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a^6*c^11 + b^12*c^5 - 24
*a*b^10*c^6 + 240*a^2*b^8*c^7 - 1280*a^3*b^6*c^8 + 3840*a^4*b^4*c^9 - 6144*
a^5*b^2*c^10)))^(1/2)*(16*b^7*c^5 - 192*a*b^5*c^6 - 1024*a^3*b*c^8 + 768*a^
2*b^3*c^7))/(2*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))*((9*B^2*b^4*(-(4*a*c
 - b^2)^9)^(1/2) - A^2*b^11*c^2 - 9*B^2*b^13 + 6*A*B*b^12*c - 288*A^2*a^2*b^
7*c^4 + 1504*A^2*a^3*b^5*c^5 - 3840*A^2*a^4*b^3*c^6 - 2077*B^2*a^2*b^9*c^2
+ 10656*B^2*a^3*b^7*c^3 - 30240*B^2*a^4*b^5*c^4 + 44800*B^2*a^5*b^3*c^5 + A
^2*b^2*c^2*(-(4*a*c - b^2)^9)^(1/2) + 25*B^2*a^2*c^2*(-(4*a*c - b^2)^9)^(1/
2) + 15360*A*B*a^6*c^7 + 213*B^2*a*b^11*c + 27*A^2*a*b^9*c^3 + 3840*A^2*a^5
*b*c^7 - 9*A^2*a*c^3*(-(4*a*c - b^2)^9)^(1/2) - 26880*B^2*a^6*b*c^6 + 1548*
A*B*a^2*b^8*c^3 - 8064*A*B*a^3*b^6*c^4 + 22400*A*B*a^4*b^4*c^5 - 30720*A*B*
a^5*b^2*c^6 - 51*B^2*a*b^2*c*(-(4*a*c - b^2)^9)^(1/2) - 152*A*B*a*b^10*c^2
 - 6*A*B*b^3*c*(-(4*a*c - b^2)^9)^(1/2) + 44*A*B*a*b*c^2*(-(4*a*c - b^2)^9)^
(1/2))/(32*(4096*a^6*c^11 + b^12*c^5 - 24*a*b^10*c^6 + 240*a^2*b^8*c^7 - 12
80*a^3*b^6*c^8 + 3840*a^4*b^4*c^9 - 6144*a^5*b^2*c^10)))^(1/2) - (x*(9*B^2*
b^8 - 72*A^2*a^3*c^5 + A^2*b^6*c^2 + 200*B^2*a^4*c^4 - 6*A*B*b^7*c + 74*A^2
*a^2*b^2*c^4 + 481*B^2*a^2*b^4*c^2 - 718*B^2*a^3*b^2*c^3 - 114*B^2*a*b^6*c
 - 16*A^2*a*b^4*c^3 - 374*A*B*a^2*b^3*c^3 + 86*A*B*a*b^5*c^2 + 472*A*B*a^3*b
*c^4))/(2*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))*((9*B^2*b^4*(-(4*a*c - b^2
)^9)^(1/2) - A^2*b^11*c^2 - 9*B^2*b^13 + 6*A*B*b^12*c - 288*A^2*a^2*b^7*c^4
 + 1504*A^2*a^3*b^5*c^5 - 3840*A^2*a^4*b^3*c^6 - 2077*B^2*a^2*b^9*c^2 + 106
56*B^2*a^3*b^7*c^3 - 30240*B^2*a^4*b^5*c^4 + 44800*B^2*a^5*b^3*c^5 + A^2*b^
2*c^2*(-(4*a*c - b^2)^9)^(1/2) + 25*B^2*a^2*c^2*(-(4*a*c - b^2)^9)^(1/2) +
```

15360*A*B*a^6*c^7 + 213*B^2*a*b^11*c + 27*A^2*a*b^9*c^3 + 3840*A^2*a^5*b*c^
7 - 9*A^2*a*c^3*(-(4*a*c - b^2)^9)^(1/2) - 26880*B^2*a^6*b*c^6 + 1548*A*B*a
^2*b^8*c^3 - 8064*A*B*a^3*b^6*c^4 + 22400*A*B*a^4*b^4*c^5 - 30720*A*B*a^5*b
^2*c^6 - 51*B^2*a*b^2*c*(-(4*a*c - b^2)^9)^(1/2) - 152*A*B*a*b^10*c^2 - 6*A
*B*b^3*c*(-(4*a*c - b^2)^9)^(1/2) + 44*A*B*a*b*c^2*(-(4*a*c - b^2)^9)^(1/2)
)/(32*(4096*a^6*c^11 + b^12*c^5 - 24*a*b^10*c^6 + 240*a^2*b^8*c^7 - 1280*a^
3*b^6*c^8 + 3840*a^4*b^4*c^9 - 6144*a^5*b^2*c^10)))^(1/2) - (216*A^3*a^4*c^
4 - 63*B^3*a^3*b^5 + 5*A^3*a^2*b^4*c^2 - 66*A^3*a^3*b^2*c^3 + 45*A*B^2*a^2*
b^6 + 600*A*B^2*a^5*c^3 + 573*B^3*a^4*b^3*c - 1300*B^3*a^5*b*c^2 - 402*A*B^
2*a^3*b^4*c - 30*A^2*B*a^2*b^5*c - 924*A^2*B*a^4*b*c^3 + 762*A*B^2*a^4*b^2*
c^2 + 339*A^2*B*a^3*b^3*c^2)/(4*(64*a^3*c^6 - b^6*c^3 + 12*a*b^4*c^4 - 48*a
^2*b^2*c^5)) + (((10240*B*a^5*c^7 - 16*A*a*b^7*c^4 + 1024*A*a^4*b*c^7 + 48*
B*a*b^8*c^3 + 192*A*a^2*b^5*c^5 - 768*A*a^3*b^3*c^6 - 736*B*a^2*b^6*c^4 + 4
224*B*a^3*b^4*c^5 - 10752*B*a^4*b^2*c^6)/(8*(64*a^3*c^6 - b^6*c^3 + 12*a*b^
4*c^4 - 48*a^2*b^2*c^5)) + (x*((9*B^2*b^4*(-(4*a*c - b^2)^9)^(1/2) - A^2*b^
11*c^2 - 9*B^2*b^13 + 6*A*B*b^12*c - 288*A^2*a^2*b^7*c^4 + 1504*A^2*a^3*b^5
*c^5 - 3840*A^2*a^4*b^3*c^6 - 2077*B^2*a^2*b^9*c^2 + 10656*B^2*a^3*b^7*c^3
- 30240*B^2*a^4*b^5*c^4 + 44800*B^2*a^5*b^3*c^5 + A^2*b^2*c^2*(-(4*a*c - b^
2)^9)^(1/2) + 25*B^2*a^2*c^2*(-(4*a*c - b^2)^9)^(1/2) + 15360*A*B*a^6*c^7 +
213*B^2*a*b^11*c + 27*A^2*a*b^9*c^3 + 3840*A^2*a^5*b*c^7 - 9*A^2*a*c^3*(-(
4*a*c - b^2)^9)^(1/2) - 26880*B^2*a^6*b*c^6 + 1548*A*B*a^2*b^8*c^3 - 8064*A
*B*a^3*b^6*c^4 + 22400*A*B*a^4*b^4*c^5 - 30720*A*B*a^5*b^2*c^6 - 51*B^2*a*b
^2*c*(-(4*a*c - b^2)^9)^(1/2) - 152*A*B*a*b^10*c^2 - 6*A*B*b^3*c*(-(4*a*c -
b^2)^9)^(1/2) + 44*A*B*a*b*c^2*(-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a^6*c^1
1 + b^12*c^5 - 24*a*b^10*c^6 + 240*a^2*b^8*c^7 - 1280*a^3*b^6*c^8 + 3840*a^
4*b^4*c^9 - 6144*a^5*b^2*c^10)))^(1/2)*(16*b^7*c^5 - 192*a*b^5*c^6 - 1024*a
^3*b*c^8 + 768*a^2*b^3*c^7))/(2*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))*((9*
B^2*b^4*(-(4*a*c - b^2)^9)^(1/2) - A^2*b^11*c^2 - 9*B^2*b^13 + 6*A*B*b^12*c
- 288*A^2*a^2*b^7*c^4 + 1504*A^2*a^3*b^5*c^5 - 3840*A^2*a^4*b^3*c^6 - 2077
*B^2*a^2*b^9*c^2 + 10656*B^2*a^3*b^7*c^3 - 30240*B^2*a^4*b^5*c^4 + 44800*B^
2*a^5*b^3*c^5 + A^2*b^2*c^2*(-(4*a*c - b^2)^9)^(1/2) + 25*B^2*a^2*c^2*(-(4*
a*c - b^2)^9)^(1/2) + 15360*A*B*a^6*c^7 + 213*B^2*a*b^11*c + 27*A^2*a*b^9*c
^3 + 3840*A^2*a^5*b*c^7 - 9*A^2*a*c^3*(-(4*a*c - b^2)^9)^(1/2) - 26880*B^2*
a^6*b*c^6 + 1548*A*B*a^2*b^8*c^3 - 8064*A*B*a^3*b^6*c^4 + 22400*A*B*a^4*b^4
*c^5 - 30720*A*B*a^5*b^2*c^6 - 51*B^2*a*b^2*c*(-(4*a*c - b^2)^9)^(1/2) - 15
2*A*B*a*b^10*c^2 - 6*A*B*b^3*c*(-(4*a*c - b^2)^9)^(1/2) + 44*A*B*a*b*c^2*(-
(4*a*c - b^2)^9)^(1/2))/(32*(4096*a^6*c^11 + b^12*c^5 - 24*a*b^10*c^6 + 240
*a^2*b^8*c^7 - 1280*a^3*b^6*c^8 + 3840*a^4*b^4*c^9 - 6144*a^5*b^2*c^10)))^(
1/2) + (x*(9*B^2*b^8 - 72*A^2*a^3*c^5 + A^2*b^6*c^2 + 200*B^2*a^4*c^4 - 6*A
*B*b^7*c + 74*A^2*a^2*b^2*c^4 + 481*B^2*a^2*b^4*c^2 - 718*B^2*a^3*b^2*c^3 -
114*B^2*a*b^6*c - 16*A^2*a*b^4*c^3 - 374*A*B*a^2*b^3*c^3 + 86*A*B*a*b^5*c^
2 + 472*A*B*a^3*b*c^4))/(2*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))*((9*B^2*b
^4*(-(4*a*c - b^2)^9)^(1/2) - A^2*b^11*c^2 - 9*B^2*b^13 + 6*A*B*b^12*c - 28
8*A^2*a^2*b^7*c^4 + 1504*A^2*a^3*b^5*c^5 - 3840*A^2*a^4*b^3*c^6 - 2077*B^2*
a^2*b^9*c^2 + 10656*B^2*a^3*b^7*c^3 - 30240*B^2*a^4*b^5*c^4 + 44800*B^2*a^5

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*b^3*c^5 + A^2*b^2*c^2*(-(4*a*c - b^2)^9)^(1/2) + 25*B^2*a^2*c^2*(-(4*a*c -
 b^2)^9)^(1/2) + 15360*A*B*a^6*c^7 + 213*B^2*a*b^11*c + 27*A^2*a*b^9*c^3 +
3840*A^2*a^5*b*c^7 - 9*A^2*a*c^3*(-(4*a*c - b^2)^9)^(1/2) - 26880*B^2*a^6*b
*c^6 + 1548*A*B*a^2*b^8*c^3 - 8064*A*B*a^3*b^6*c^4 + 22400*A*B*a^4*b^4*c^5
- 30720*A*B*a^5*b^2*c^6 - 51*B^2*a*b^2*c*(-(4*a*c - b^2)^9)^(1/2) - 152*A*B
*a*b^10*c^2 - 6*A*B*b^3*c*(-(4*a*c - b^2)^9)^(1/2) + 44*A*B*a*b*c^2*(-(4*a*
c - b^2)^9)^(1/2))/(32*(4096*a^6*c^11 + b^12*c^5 - 24*a*b^10*c^6 + 240*a^2*
b^8*c^7 - 1280*a^3*b^6*c^8 + 3840*a^4*b^4*c^9 - 6144*a^5*b^2*c^10)))^(1/2))
)*((9*B^2*b^4*(-(4*a*c - b^2)^9)^(1/2) - A^2*b^11*c^2 - 9*B^2*b^13 + 6*A*B*
b^12*c - 288*A^2*a^2*b^7*c^4 + 1504*A^2*a^3*b^5*c^5 - 3840*A^2*a^4*b^3*c^6
- 2077*B^2*a^2*b^9*c^2 + 10656*B^2*a^3*b^7*c^3 - 30240*B^2*a^4*b^5*c^4 + 44
800*B^2*a^5*b^3*c^5 + A^2*b^2*c^2*(-(4*a*c - b^2)^9)^(1/2) + 25*B^2*a^2*c^2
*(-(4*a*c - b^2)^9)^(1/2) + 15360*A*B*a^6*c^7 + 213*B^2*a*b^11*c + 27*A^2*a
*b^9*c^3 + 3840*A^2*a^5*b*c^7 - 9*A^2*a*c^3*(-(4*a*c - b^2)^9)^(1/2) - 2688
0*B^2*a^6*b*c^6 + 1548*A*B*a^2*b^8*c^3 - 8064*A*B*a^3*b^6*c^4 + 22400*A*B*a
^4*b^4*c^5 - 30720*A*B*a^5*b^2*c^6 - 51*B^2*a*b^2*c*(-(4*a*c - b^2)^9)^(1/2
) - 152*A*B*a*b^10*c^2 - 6*A*B*b^3*c*(-(4*a*c - b^2)^9)^(1/2) + 44*A*B*a*b*
c^2*(-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a^6*c^11 + b^12*c^5 - 24*a*b^10*c^6
 + 240*a^2*b^8*c^7 - 1280*a^3*b^6*c^8 + 3840*a^4*b^4*c^9 - 6144*a^5*b^2*c^1
0)))^(1/2)*2i - ((x^3*(B*b^3 + 2*A*a*c^2 - A*b^2*c - 3*B*a*b*c))/(2*(4*a*c
- b^2)) - (x*(2*B*a^2*c - B*a*b^2 + A*a*b*c))/(2*(4*a*c - b^2)))/(a*c^2 + c
^3*x^4 + b*c^2*x^2) - atan(((((10240*B*a^5*c^7 - 16*A*a*b^7*c^4 + 1024*A*a^
4*b*c^7 + 48*B*a*b^8*c^3 + 192*A*a^2*b^5*c^5 - 768*A*a^3*b^3*c^6 - 736*B*a^
2*b^6*c^4 + 4224*B*a^3*b^4*c^5 - 10752*B*a^4*b^2*c^6)/(8*(64*a^3*c^6 - b^6*
c^3 + 12*a*b^4*c^4 - 48*a^2*b^2*c^5)) - (x*(-(9*B^2*b^13 + A^2*b^11*c^2 + 9
*B^2*b^4*(-(4*a*c - b^2)^9)^(1/2) - 6*A*B*b^12*c + 288*A^2*a^2*b^7*c^4 - 15
04*A^2*a^3*b^5*c^5 + 3840*A^2*a^4*b^3*c^6 + 2077*B^2*a^2*b^9*c^2 - 10656*B^
2*a^3*b^7*c^3 + 30240*B^2*a^4*b^5*c^4 - 44800*B^2*a^5*b^3*c^5 + A^2*b^2*c^2
*(-(4*a*c - b^2)^9)^(1/2) + 25*B^2*a^2*c^2*(-(4*a*c - b^2)^9)^(1/2) - 15360
*A*B*a^6*c^7 - 213*B^2*a*b^11*c - 27*A^2*a*b^9*c^3 - 3840*A^2*a^5*b*c^7 - 9
*A^2*a*c^3*(-(4*a*c - b^2)^9)^(1/2) + 26880*B^2*a^6*b*c^6 - 1548*A*B*a^2*b^
8*c^3 + 8064*A*B*a^3*b^6*c^4 - 22400*A*B*a^4*b^4*c^5 + 30720*A*B*a^5*b^2*c^
6 - 51*B^2*a*b^2*c*(-(4*a*c - b^2)^9)^(1/2) + 152*A*B*a*b^10*c^2 - 6*A*B*b^
3*c*(-(4*a*c - b^2)^9)^(1/2) + 44*A*B*a*b*c^2*(-(4*a*c - b^2)^9)^(1/2))/(32
*(4096*a^6*c^11 + b^12*c^5 - 24*a*b^10*c^6 + 240*a^2*b^8*c^7 - 1280*a^3*b^6
*c^8 + 3840*a^4*b^4*c^9 - 6144*a^5*b^2*c^10)))^(1/2)*(16*b^7*c^5 - 192*a*b^
5*c^6 - 1024*a^3*b*c^8 + 768*a^2*b^3*c^7))/(2*(16*a^2*c^5 + b^4*c^3 - 8*a*b
^2*c^4)))*(-(9*B^2*b^13 + A^2*b^11*c^2 + 9*B^2*b^4*(-(4*a*c - b^2)^9)^(1/2)
 - 6*A*B*b^12*c + 288*A^2*a^2*b^7*c^4 - 1504*A^2*a^3*b^5*c^5 + 3840*A^2*a^4
*b^3*c^6 + 2077*B^2*a^2*b^9*c^2 - 10656*B^2*a^3*b^7*c^3 + 30240*B^2*a^4*b^5
*c^4 - 44800*B^2*a^5*b^3*c^5 + A^2*b^2*c^2*(-(4*a*c - b^2)^9)^(1/2) + 25*B^
2*a^2*c^2*(-(4*a*c - b^2)^9)^(1/2) - 15360*A*B*a^6*c^7 - 213*B^2*a*b^11*c -
 27*A^2*a*b^9*c^3 - 3840*A^2*a^5*b*c^7 - 9*A^2*a*c^3*(-(4*a*c - b^2)^9)^(1/
2) + 26880*B^2*a^6*b*c^6 - 1548*A*B*a^2*b^8*c^3 + 8064*A*B*a^3*b^6*c^4 - 22
400*A*B*a^4*b^4*c^5 + 30720*A*B*a^5*b^2*c^6 - 51*B^2*a*b^2*c*(-(4*a*c - b^2
```

)^9)^(1/2) + 152*A*B*a*b^10*c^2 - 6*A*B*b^3*c*(-(4*a*c - b^2)^9)^(1/2) + 44
*A*B*a*b*c^2*(-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a^6*c^11 + b^12*c^5 - 24*a
*b^10*c^6 + 240*a^2*b^8*c^7 - 1280*a^3*b^6*c^8 + 3840*a^4*b^4*c^9 - 6144*a^
5*b^2*c^10)))^(1/2) - (x*(9*B^2*b^8 - 72*A^2*a^3*c^5 + A^2*b^6*c^2 + 200*B^
2*a^4*c^4 - 6*A*B*b^7*c + 74*A^2*a^2*b^2*c^4 + 481*B^2*a^2*b^4*c^2 - 718*B^
2*a^3*b^2*c^3 - 114*B^2*a*b^6*c - 16*A^2*a*b^4*c^3 - 374*A*B*a^2*b^3*c^3 +
86*A*B*a*b^5*c^2 + 472*A*B*a^3*b*c^4))/(2*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c
^4)))*(-(9*B^2*b^13 + A^2*b^11*c^2 + 9*B^2*b^4*(-(4*a*c - b^2)^9)^(1/2) - 6
*A*B*b^12*c + 288*A^2*a^2*b^7*c^4 - 1504*A^2*a^3*b^5*c^5 + 3840*A^2*a^4*b^3
*c^6 + 2077*B^2*a^2*b^9*c^2 - 10656*B^2*a^3*b^7*c^3 + 30240*B^2*a^4*b^5*c^4
 - 44800*B^2*a^5*b^3*c^5 + A^2*b^2*c^2*(-(4*a*c - b^2)^9)^(1/2) + 25*B^2*a^
2*c^2*(-(4*a*c - b^2)^9)^(1/2) - 15360*A*B*a^6*c^7 - 213*B^2*a*b^11*c - 27*
A^2*a*b^9*c^3 - 3840*A^2*a^5*b*c^7 - 9*A^2*a*c^3*(-(4*a*c - b^2)^9)^(1/2) +
 26880*B^2*a^6*b*c^6 - 1548*A*B*a^2*b^8*c^3 + 8064*A*B*a^3*b^6*c^4 - 22400*
A*B*a^4*b^4*c^5 + 30720*A*B*a^5*b^2*c^6 - 51*B^2*a*b^2*c*(-(4*a*c - b^2)^9)
^(1/2) + 152*A*B*a*b^10*c^2 - 6*A*B*b^3*c*(-(4*a*c - b^2)^9)^(1/2) + 44*A*B
*a*b*c^2*(-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a^6*c^11 + b^12*c^5 - 24*a*b^1
0*c^6 + 240*a^2*b^8*c^7 - 1280*a^3*b^6*c^8 + 3840*a^4*b^4*c^9 - 6144*a^5*b^
2*c^10)))^(1/2)*1i - (((10240*B*a^5*c^7 - 16*A*a*b^7*c^4 + 1024*A*a^4*b*c^7
 + 48*B*a*b^8*c^3 + 192*A*a^2*b^5*c^5 - 768*A*a^3*b^3*c^6 - 736*B*a^2*b^6*c
^4 + 4224*B*a^3*b^4*c^5 - 10752*B*a^4*b^2*c^6)/(8*(64*a^3*c^6 - b^6*c^3 + 1
2*a*b^4*c^4 - 48*a^2*b^2*c^5)) + (x*(-(9*B^2*b^13 + A^2*b^11*c^2 + 9*B^2*b^
4*(-(4*a*c - b^2)^9)^(1/2) - 6*A*B*b^12*c + 288*A^2*a^2*b^7*c^4 - 1504*A^2*
a^3*b^5*c^5 + 3840*A^2*a^4*b^3*c^6 + 2077*B^2*a^2*b^9*c^2 - 10656*B^2*a^3*b
^7*c^3 + 30240*B^2*a^4*b^5*c^4 - 44800*B^2*a^5*b^3*c^5 + A^2*b^2*c^2*(-(4*a
*c - b^2)^9)^(1/2) + 25*B^2*a^2*c^2*(-(4*a*c - b^2)^9)^(1/2) - 15360*A*B*a^
6*c^7 - 213*B^2*a*b^11*c - 27*A^2*a*b^9*c^3 - 3840*A^2*a^5*b*c^7 - 9*A^2*a*
c^3*(-(4*a*c - b^2)^9)^(1/2) + 26880*B^2*a^6*b*c^6 - 1548*A*B*a^2*b^8*c^3 +
 8064*A*B*a^3*b^6*c^4 - 22400*A*B*a^4*b^4*c^5 + 30720*A*B*a^5*b^2*c^6 - 51*
B^2*a*b^2*c*(-(4*a*c - b^2)^9)^(1/2) + 152*A*B*a*b^10*c^2 - 6*A*B*b^3*c*(-(
4*a*c - b^2)^9)^(1/2) + 44*A*B*a*b*c^2*(-(4*a*c - b^2)^9)^(1/2))/(32*(4096*
a^6*c^11 + b^12*c^5 - 24*a*b^10*c^6 + 240*a^2*b^8*c^7 - 1280*a^3*b^6*c^8 +
3840*a^4*b^4*c^9 - 6144*a^5*b^2*c^10)))^(1/2)*(16*b^7*c^5 - 192*a*b^5*c^6 -
 1024*a^3*b*c^8 + 768*a^2*b^3*c^7))/(2*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)
))*(-(9*B^2*b^13 + A^2*b^11*c^2 + 9*B^2*b^4*(-(4*a*c - b^2)^9)^(1/2) - 6*A*
B*b^12*c + 288*A^2*a^2*b^7*c^4 - 1504*A^2*a^3*b^5*c^5 + 3840*A^2*a^4*b^3*c^
6 + 2077*B^2*a^2*b^9*c^2 - 10656*B^2*a^3*b^7*c^3 + 30240*B^2*a^4*b^5*c^4 -
44800*B^2*a^5*b^3*c^5 + A^2*b^2*c^2*(-(4*a*c - b^2)^9)^(1/2) + 25*B^2*a^2*c
^2*(-(4*a*c - b^2)^9)^(1/2) - 15360*A*B*a^6*c^7 - 213*B^2*a*b^11*c - 27*A^2
*a*b^9*c^3 - 3840*A^2*a^5*b*c^7 - 9*A^2*a*c^3*(-(4*a*c - b^2)^9)^(1/2) + 26
880*B^2*a^6*b*c^6 - 1548*A*B*a^2*b^8*c^3 + 8064*A*B*a^3*b^6*c^4 - 22400*A*B
*a^4*b^4*c^5 + 30720*A*B*a^5*b^2*c^6 - 51*B^2*a*b^2*c*(-(4*a*c - b^2)^9)^(1
/2) + 152*A*B*a*b^10*c^2 - 6*A*B*b^3*c*(-(4*a*c - b^2)^9)^(1/2) + 44*A*B*a*
b*c^2*(-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a^6*c^11 + b^12*c^5 - 24*a*b^10*c
^6 + 240*a^2*b^8*c^7 - 1280*a^3*b^6*c^8 + 3840*a^4*b^4*c^9 - 6144*a^5*b^2*c

^10)))^(1/2) + (x*(9*B^2*b^8 - 72*A^2*a^3*c^5 + A^2*b^6*c^2 + 200*B^2*a^4*c^4 - 6*A*B*b^7*c + 74*A^2*a^2*b^2*c^4 + 481*B^2*a^2*b^4*c^2 - 718*B^2*a^3*b^2*c^3 - 114*B^2*a*b^6*c - 16*A^2*a*b^4*c^3 - 374*A*B*a^2*b^3*c^3 + 86*A*B*a*b^5*c^2 + 472*A*B*a^3*b*c^4))/(2*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))*(-(9*B^2*b^13 + A^2*b^11*c^2 + 9*B^2*b^4*(-(4*a*c - b^2)^9)^(1/2) - 6*A*B*b^12*c + 288*A^2*a^2*b^7*c^4 - 1504*A^2*a^3*b^5*c^5 + 3840*A^2*a^4*b^3*c^6 + 2077*B^2*a^2*b^9*c^2 - 10656*B^2*a^3*b^7*c^3 + 30240*B^2*a^4*b^5*c^4 - 44800*B^2*a^5*b^3*c^5 + A^2*b^2*c^2*(-(4*a*c - b^2)^9)^(1/2) + 25*B^2*a^2*c^2*(-(4*a*c - b^2)^9)^(1/2) - 15360*A*B*a^6*c^7 - 213*B^2*a*b^11*c - 27*A^2*a*b^9*c^3 - 3840*A^2*a^5*b*c^7 - 9*A^2*a*c^3*(-(4*a*c - b^2)^9)^(1/2) + 26880*B^2*a^6*b*c^6 - 1548*A*B*a^2*b^8*c^3 + 8064*A*B*a^3*b^6*c^4 - 22400*A*B*a^4*b^4*c^5 + 30720*A*B*a^5*b^2*c^6 - 51*B^2*a*b^2*c*(-(4*a*c - b^2)^9)^(1/2) + 152*A*B*a*b^10*c^2 - 6*A*B*b^3*c*(-(4*a*c - b^2)^9)^(1/2) + 44*A*B*a*b*c^2*(-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a^6*c^11 + b^12*c^5 - 24*a*b^10*c^6 + 240*a^2*b^8*c^7 - 1280*a^3*b^6*c^8 + 3840*a^4*b^4*c^9 - 6144*a^5*b^2*c^10)))^(1/2)*1i)/((((10240*B*a^5*c^7 - 16*A*a*b^7*c^4 + 1024*A*a^4*b*c^7 + 48*B*a*b^8*c^3 + 192*A*a^2*b^5*c^5 - 768*A*a^3*b^3*c^6 - 736*B*a^2*b^6*c^4 + 4224*B*a^3*b^4*c^5 - 10752*B*a^4*b^2*c^6)/(8*(64*a^3*c^6 - b^6*c^3 + 12*a*b^4*c^4 - 48*a^2*b^2*c^5)) - (x*(-(9*B^2*b^13 + A^2*b^11*c^2 + 9*B^2*b^4*(-(4*a*c - b^2)^9)^(1/2) - 6*A*B*b^12*c + 288*A^2*a^2*b^7*c^4 - 1504*A^2*a^3*b^5*c^5 + 3840*A^2*a^4*b^3*c^6 + 2077*B^2*a^2*b^9*c^2 - 10656*B^2*a^3*b^7*c^3 + 30240*B^2*a^4*b^5*c^4 - 44800*B^2*a^5*b^3*c^5 + A^2*b^2*c^2*(-(4*a*c - b^2)^9)^(1/2) + 25*B^2*a^2*c^2*(-(4*a*c - b^2)^9)^(1/2) - 15360*A*B*a^6*c^7 - 213*B^2*a*b^11*c - 27*A^2*a*b^9*c^3 - 3840*A^2*a^5*b*c^7 - 9*A^2*a*c^3*(-(4*a*c - b^2)^9)^(1/2) + 26880*B^2*a^6*b*c^6 - 1548*A*B*a^2*b^8*c^3 + 8064*A*B*a^3*b^6*c^4 - 22400*A*B*a^4*b^4*c^5 + 30720*A*B*a^5*b^2*c^6 - 51*B^2*a*b^2*c*(-(4*a*c - b^2)^9)^(1/2) + 152*A*B*a*b^10*c^2 - 6*A*B*b^3*c*(-(4*a*c - b^2)^9)^(1/2) + 44*A*B*a*b*c^2*(-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a^6*c^11 + b^12*c^5 - 24*a*b^10*c^6 + 240*a^2*b^8*c^7 - 1280*a^3*b^6*c^8 + 3840*a^4*b^4*c^9 - 6144*a^5*b^2*c^10)))^(1/2)*(16*b^7*c^5 - 192*a*b^5*c^6 - 1024*a^3*b*c^8 + 768*a^2*b^3*c^7))/(2*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))*(-(9*B^2*b^13 + A^2*b^11*c^2 + 9*B^2*b^4*(-(4*a*c - b^2)^9)^(1/2) - 6*A*B*b^12*c + 288*A^2*a^2*b^7*c^4 - 1504*A^2*a^3*b^5*c^5 + 3840*A^2*a^4*b^3*c^6 + 2077*B^2*a^2*b^9*c^2 - 10656*B^2*a^3*b^7*c^3 + 30240*B^2*a^4*b^5*c^4 - 44800*B^2*a^5*b^3*c^5 + A^2*b^2*c^2*(-(4*a*c - b^2)^9)^(1/2) + 25*B^2*a^2*c^2*(-(4*a*c - b^2)^9)^(1/2) - 15360*A*B*a^6*c^7 - 213*B^2*a*b^11*c - 27*A^2*a*b^9*c^3 - 3840*A^2*a^5*b*c^7 - 9*A^2*a*c^3*(-(4*a*c - b^2)^9)^(1/2) + 26880*B^2*a^6*b*c^6 - 1548*A*B*a^2*b^8*c^3 + 8064*A*B*a^3*b^6*c^4 - 22400*A*B*a^4*b^4*c^5 + 30720*A*B*a^5*b^2*c^6 - 51*B^2*a*b^2*c*(-(4*a*c - b^2)^9)^(1/2) + 152*A*B*a*b^10*c^2 - 6*A*B*b^3*c*(-(4*a*c - b^2)^9)^(1/2) + 44*A*B*a*b*c^2*(-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a^6*c^11 + b^12*c^5 - 24*a*b^10*c^6 + 240*a^2*b^8*c^7 - 1280*a^3*b^6*c^8 + 3840*a^4*b^4*c^9 - 6144*a^5*b^2*c^10)))^(1/2) - (x*(9*B^2*b^8 - 72*A^2*a^3*c^5 + A^2*b^6*c^2 + 200*B^2*a^4*c^4 - 6*A*B*b^7*c + 74*A^2*a^2*b^2*c^4 + 481*B^2*a^2*b^4*c^2 - 718*B^2*a^3*b^2*c^3 - 114*B^2*a*b^6*c - 16*A^2*a*b^4*c^3 - 374*A*B*a^2*b^3*c^3 + 86*A*B*a*b^5*c

^2 + 472*A*B*a^3*b*c^4))/(2*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))*(-(9*B^2
*b^13 + A^2*b^11*c^2 + 9*B^2*b^4*(-(4*a*c - b^2)^9)^(1/2) - 6*A*B*b^12*c +
288*A^2*a^2*b^7*c^4 - 1504*A^2*a^3*b^5*c^5 + 3840*A^2*a^4*b^3*c^6 + 2077*B^
2*a^2*b^9*c^2 - 10656*B^2*a^3*b^7*c^3 + 30240*B^2*a^4*b^5*c^4 - 44800*B^2*a
^5*b^3*c^5 + A^2*b^2*c^2*(-(4*a*c - b^2)^9)^(1/2) + 25*B^2*a^2*c^2*(-(4*a*c
 - b^2)^9)^(1/2) - 15360*A*B*a^6*c^7 - 213*B^2*a*b^11*c - 27*A^2*a*b^9*c^3
- 3840*A^2*a^5*b*c^7 - 9*A^2*a*c^3*(-(4*a*c - b^2)^9)^(1/2) + 26880*B^2*a^6
*b*c^6 - 1548*A*B*a^2*b^8*c^3 + 8064*A*B*a^3*b^6*c^4 - 22400*A*B*a^4*b^4*c^
5 + 30720*A*B*a^5*b^2*c^6 - 51*B^2*a*b^2*c*(-(4*a*c - b^2)^9)^(1/2) + 152*A
*B*a*b^10*c^2 - 6*A*B*b^3*c*(-(4*a*c - b^2)^9)^(1/2) + 44*A*B*a*b*c^2*(-(4*
a*c - b^2)^9)^(1/2))/(32*(4096*a^6*c^11 + b^12*c^5 - 24*a*b^10*c^6 + 240*a^
2*b^8*c^7 - 1280*a^3*b^6*c^8 + 3840*a^4*b^4*c^9 - 6144*a^5*b^2*c^10)))^(1/2
) - (216*A^3*a^4*c^4 - 63*B^3*a^3*b^5 + 5*A^3*a^2*b^4*c^2 - 66*A^3*a^3*b^2*
c^3 + 45*A*B^2*a^2*b^6 + 600*A*B^2*a^5*c^3 + 573*B^3*a^4*b^3*c - 1300*B^3*a
^5*b*c^2 - 402*A*B^2*a^3*b^4*c - 30*A^2*B*a^2*b^5*c - 924*A^2*B*a^4*b*c^3 +
 762*A*B^2*a^4*b^2*c^2 + 339*A^2*B*a^3*b^3*c^2)/(4*(64*a^3*c^6 - b^6*c^3 +
12*a*b^4*c^4 - 48*a^2*b^2*c^5)) + (((10240*B*a^5*c^7 - 16*A*a*b^7*c^4 + 102
4*A*a^4*b*c^7 + 48*B*a*b^8*c^3 + 192*A*a^2*b^5*c^5 - 768*A*a^3*b^3*c^6 - 73
6*B*a^2*b^6*c^4 + 4224*B*a^3*b^4*c^5 - 10752*B*a^4*b^2*c^6)/(8*(64*a^3*c^6
- b^6*c^3 + 12*a*b^4*c^4 - 48*a^2*b^2*c^5)) + (x*(-(9*B^2*b^13 + A^2*b^11*c
^2 + 9*B^2*b^4*(-(4*a*c - b^2)^9)^(1/2) - 6*A*B*b^12*c + 288*A^2*a^2*b^7*c^
4 - 1504*A^2*a^3*b^5*c^5 + 3840*A^2*a^4*b^3*c^6 + 2077*B^2*a^2*b^9*c^2 - 10
656*B^2*a^3*b^7*c^3 + 30240*B^2*a^4*b^5*c^4 - 44800*B^2*a^5*b^3*c^5 + A^2*b
^2*c^2*(-(4*a*c - b^2)^9)^(1/2) + 25*B^2*a^2*c^2*(-(4*a*c - b^2)^9)^(1/2) -
 15360*A*B*a^6*c^7 - 213*B^2*a*b^11*c - 27*A^2*a*b^9*c^3 - 3840*A^2*a^5*b*c
^7 - 9*A^2*a*c^3*(-(4*a*c - b^2)^9)^(1/2) + 26880*B^2*a^6*b*c^6 - 1548*A*B*
a^2*b^8*c^3 + 8064*A*B*a^3*b^6*c^4 - 22400*A*B*a^4*b^4*c^5 + 30720*A*B*a^5*
b^2*c^6 - 51*B^2*a*b^2*c*(-(4*a*c - b^2)^9)^(1/2) + 152*A*B*a*b^10*c^2 - 6*
A*B*b^3*c*(-(4*a*c - b^2)^9)^(1/2) + 44*A*B*a*b*c^2*(-(4*a*c - b^2)^9)^(1/2
))/(32*(4096*a^6*c^11 + b^12*c^5 - 24*a*b^10*c^6 + 240*a^2*b^8*c^7 - 1280*a
^3*b^6*c^8 + 3840*a^4*b^4*c^9 - 6144*a^5*b^2*c^10)))^(1/2)*(16*b^7*c^5 - 19
2*a*b^5*c^6 - 1024*a^3*b*c^8 + 768*a^2*b^3*c^7))/(2*(16*a^2*c^5 + b^4*c^3 -
 8*a*b^2*c^4)))*(-(9*B^2*b^13 + A^2*b^11*c^2 + 9*B^2*b^4*(-(4*a*c - b^2)^9)
^(1/2) - 6*A*B*b^12*c + 288*A^2*a^2*b^7*c^4 - 1504*A^2*a^3*b^5*c^5 + 3840*A
^2*a^4*b^3*c^6 + 2077*B^2*a^2*b^9*c^2 - 10656*B^2*a^3*b^7*c^3 + 30240*B^2*a
^4*b^5*c^4 - 44800*B^2*a^5*b^3*c^5 + A^2*b^2*c^2*(-(4*a*c - b^2)^9)^(1/2) +
 25*B^2*a^2*c^2*(-(4*a*c - b^2)^9)^(1/2) - 15360*A*B*a^6*c^7 - 213*B^2*a*b^
11*c - 27*A^2*a*b^9*c^3 - 3840*A^2*a^5*b*c^7 - 9*A^2*a*c^3*(-(4*a*c - b^2)^
9)^(1/2) + 26880*B^2*a^6*b*c^6 - 1548*A*B*a^2*b^8*c^3 + 8064*A*B*a^3*b^6*c^
4 - 22400*A*B*a^4*b^4*c^5 + 30720*A*B*a^5*b^2*c^6 - 51*B^2*a*b^2*c*(-(4*a*c
 - b^2)^9)^(1/2) + 152*A*B*a*b^10*c^2 - 6*A*B*b^3*c*(-(4*a*c - b^2)^9)^(1/2
) + 44*A*B*a*b*c^2*(-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a^6*c^11 + b^12*c^5
- 24*a*b^10*c^6 + 240*a^2*b^8*c^7 - 1280*a^3*b^6*c^8 + 3840*a^4*b^4*c^9 - 6
144*a^5*b^2*c^10)))^(1/2) + (x*(9*B^2*b^8 - 72*A^2*a^3*c^5 + A^2*b^6*c^2 +
200*B^2*a^4*c^4 - 6*A*B*b^7*c + 74*A^2*a^2*b^2*c^4 + 481*B^2*a^2*b^4*c^2 -

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718*B^2*a^3*b^2*c^3 - 114*B^2*a*b^6*c - 16*A^2*a*b^4*c^3 - 374*A*B*a^2*b^3*
c^3 + 86*A*B*a*b^5*c^2 + 472*A*B*a^3*b*c^4))/(2*(16*a^2*c^5 + b^4*c^3 - 8*a
*b^2*c^4)))*(-(9*B^2*b^13 + A^2*b^11*c^2 + 9*B^2*b^4*(-(4*a*c - b^2)^9)^(1/
2) - 6*A*B*b^12*c + 288*A^2*a^2*b^7*c^4 - 1504*A^2*a^3*b^5*c^5 + 3840*A^2*a
^4*b^3*c^6 + 2077*B^2*a^2*b^9*c^2 - 10656*B^2*a^3*b^7*c^3 + 30240*B^2*a^4*b
^5*c^4 - 44800*B^2*a^5*b^3*c^5 + A^2*b^2*c^2*(-(4*a*c - b^2)^9)^(1/2) + 25*
B^2*a^2*c^2*(-(4*a*c - b^2)^9)^(1/2) - 15360*A*B*a^6*c^7 - 213*B^2*a*b^11*c
 - 27*A^2*a*b^9*c^3 - 3840*A^2*a^5*b*c^7 - 9*A^2*a*c^3*(-(4*a*c - b^2)^9)^(
1/2) + 26880*B^2*a^6*b*c^6 - 1548*A*B*a^2*b^8*c^3 + 8064*A*B*a^3*b^6*c^4 -
22400*A*B*a^4*b^4*c^5 + 30720*A*B*a^5*b^2*c^6 - 51*B^2*a*b^2*c*(-(4*a*c - b
^2)^9)^(1/2) + 152*A*B*a*b^10*c^2 - 6*A*B*b^3*c*(-(4*a*c - b^2)^9)^(1/2) +
44*A*B*a*b*c^2*(-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a^6*c^11 + b^12*c^5 - 24
*a*b^10*c^6 + 240*a^2*b^8*c^7 - 1280*a^3*b^6*c^8 + 3840*a^4*b^4*c^9 - 6144*
a^5*b^2*c^10)))^(1/2)))*(-(9*B^2*b^13 + A^2*b^11*c^2 + 9*B^2*b^4*(-(4*a*c -
 b^2)^9)^(1/2) - 6*A*B*b^12*c + 288*A^2*a^2*b^7*c^4 - 1504*A^2*a^3*b^5*c^5
+ 3840*A^2*a^4*b^3*c^6 + 2077*B^2*a^2*b^9*c^2 - 10656*B^2*a^3*b^7*c^3 + 302
40*B^2*a^4*b^5*c^4 - 44800*B^2*a^5*b^3*c^5 + A^2*b^2*c^2*(-(4*a*c - b^2)^9)
^(1/2) + 25*B^2*a^2*c^2*(-(4*a*c - b^2)^9)^(1/2) - 15360*A*B*a^6*c^7 - 213*
B^2*a*b^11*c - 27*A^2*a*b^9*c^3 - 3840*A^2*a^5*b*c^7 - 9*A^2*a*c^3*(-(4*a*c
 - b^2)^9)^(1/2) + 26880*B^2*a^6*b*c^6 - 1548*A*B*a^2*b^8*c^3 + 8064*A*B*a^
3*b^6*c^4 - 22400*A*B*a^4*b^4*c^5 + 30720*A*B*a^5*b^2*c^6 - 51*B^2*a*b^2*c*
(-(4*a*c - b^2)^9)^(1/2) + 152*A*B*a*b^10*c^2 - 6*A*B*b^3*c*(-(4*a*c - b^2)
^9)^(1/2) + 44*A*B*a*b*c^2*(-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a^6*c^11 + b
^12*c^5 - 24*a*b^10*c^6 + 240*a^2*b^8*c^7 - 1280*a^3*b^6*c^8 + 3840*a^4*b^4
*c^9 - 6144*a^5*b^2*c^10)))^(1/2)*2i
```

3.119 $\int \frac{x^4(A+Bx^2)}{(a+bx^2+cx^4)^2}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 336

$$\int \frac{x^4(A+Bx^2)}{(a+bx^2+cx^4)^2}\,dx = -\frac{(bB-2Ac)x}{2c(b^2-4ac)} - \frac{x^3(Ab-2aB-(bB-2Ac)x^2)}{2(b^2-4ac)(a+bx^2+cx^4)} + \frac{\left(b^2B+Abc-6aBc-\frac{b^3B+Ab^2c-8abBc+4aAc^2}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}c^{3/2}(b^2-4ac)\sqrt{b-\sqrt{b^2-4ac}}} + \frac{\left(b^2B+Abc-6aBc+\frac{b^3B+Ab^2c-8abBc+4aAc^2}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}c^{3/2}(b^2-4ac)\sqrt{b+\sqrt{b^2-4ac}}}$$

```
[Out] -1/2*(-2*A*c+B*b)*x/c/(-4*a*c+b^2)-1/2*x^3*(A*b-2*B*a-(-2*A*c+B*b)*x^2)/(-4
*a*c+b^2)/(c*x^4+b*x^2+a)+1/4*arctan(x*2^(1/2)*c^(1/2)/(b-(-4*a*c+b^2)^(1/2
))^(1/2))*(B*b^2+A*b*c-6*B*a*c+(-4*A*a*c^2-A*b^2*c+8*B*a*b*c-B*b^3)/(-4*a*c
+b^2)^(1/2))/c^(3/2)/(-4*a*c+b^2)*2^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2)+1/4*
arctan(x*2^(1/2)*c^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2))*(B*b^2+A*b*c-6*B*a*c
+(4*A*a*c^2+A*b^2*c-8*B*a*b*c+B*b^3)/(-4*a*c+b^2)^(1/2))/c^(3/2)/(-4*a*c+b^
2)*2^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2)
```

Rubi [A] (verified)

Time = 1.22 (sec) , antiderivative size = 336, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {1289, 1293, 1180, 211}

$$\int \frac{x^4(A+Bx^2)}{(a+bx^2+cx^4)^2}\,dx = \frac{\left(-\frac{4aAc^2-8abBc+Ab^2c+b^3B}{\sqrt{b^2-4ac}}-6aBc+Abc+b^2B\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}c^{3/2}\left(b^2-4ac\right)\sqrt{b-\sqrt{b^2-4ac}}} + \frac{\left(\frac{4aAc^2-8abBc+Ab^2c+b^3B}{\sqrt{b^2-4ac}}-6aBc+Abc+b^2B\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{2\sqrt{2}c^{3/2}\left(b^2-4ac\right)\sqrt{\sqrt{b^2-4ac}+b}} - \frac{x^3(-2aB-(x^2(bB-2Ac))+Ab)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} - \frac{x(bB-2Ac)}{2c\left(b^2-4ac\right)}$$

```
[In] Int[(x^4*(A + B*x^2))/(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] -1/2*((b*B - 2*A*c)*x)/(c*(b^2 - 4*a*c)) - (x^3*(A*b - 2*a*B - (b*B - 2*A*c
)*x^2))/(2*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) + ((b^2*B + A*b*c - 6*a*B*c -
 (b^3*B + A*b^2*c - 8*a*b*B*c + 4*a*A*c^2)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[
2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]*c^(3/2)*(b^2 - 4*a*c
)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + ((b^2*B + A*b*c - 6*a*B*c + (b^3*B + A*b^2
*c - 8*a*b*B*c + 4*a*A*c^2)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/S
qrt[b + Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]*c^(3/2)*(b^2 - 4*a*c)*Sqrt[b + Sqrt
[b^2 - 4*a*c]])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1289

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_.), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(a + b*x^2 + c*x^4)^(p + 1)
*((b*d - 2*a*e - (b*e - 2*c*d)*x^2)/(2*(p + 1)*(b^2 - 4*a*c))), x] - Dist[f
^2/(2*(p + 1)*(b^2 - 4*a*c)), Int[(f*x)^(m - 2)*(a + b*x^2 + c*x^4)^(p + 1)
```

```
*Simp[(m - 1)*(b*d - 2*a*e) - (4*p + 4 + m + 1)*(b*e - 2*c*d)*x^2, x], x],
x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] &&
GtQ[m, 1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1293

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[e*f*(f*x)^(m - 1)*((a + b*x^2 + c*x^4)^(p +
1)/(c*(m + 4*p + 3))), x] - Dist[f^2/(c*(m + 4*p + 3)), Int[(f*x)^(m - 2)*(
a + b*x^2 + c*x^4)^p*Simp[a*e*(m - 1) + (b*e*(m + 2*p + 1) - c*d*(m + 4*p +
 3))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c,
 0] && GtQ[m, 1] && NeQ[m + 4*p + 3, 0] && IntegerQ[2*p] && (IntegerQ[p] ||
 IntegerQ[m])
```

Rubi steps

$$\text{integral} = -\frac{x^3(Ab-2aB-(bB-2Ac)x^2)}{2(b^2-4ac)(a+bx^2+cx^4)} + \frac{\int \frac{x^2(3(Ab-2aB)+(-bB+2Ac)x^2)}{a+bx^2+cx^4}\,dx}{2(b^2-4ac)}$$

$$= -\frac{(bB-2Ac)x}{2c(b^2-4ac)} - \frac{x^3(Ab-2aB-(bB-2Ac)x^2)}{2(b^2-4ac)(a+bx^2+cx^4)} - \frac{\int \frac{-a(bB-2Ac)+(-b^2B-Abc+6aBc)x^2}{a+bx^2+cx^4}\,dx}{2c(b^2-4ac)}$$

$$= -\frac{(bB-2Ac)x}{2c(b^2-4ac)} - \frac{x^3(Ab-2aB-(bB-2Ac)x^2)}{2(b^2-4ac)(a+bx^2+cx^4)} + \frac{\left(b^2B+Abc-6aBc-\frac{b^3B+Ab^2c-8abBc+4aAc^2}{\sqrt{b^2-4ac}}\right)\int \frac{1}{\frac{b}{2}-\frac{1}{2}\sqrt{b^2-4ac}+cx^2}\,dx}{4c(b^2-4ac)} + \frac{\left(b^2B+Abc-6aBc+\frac{b^3B+Ab^2c-8abBc+4aAc^2}{\sqrt{b^2-4ac}}\right)\int \frac{1}{\frac{b}{2}+\frac{1}{2}\sqrt{b^2-4ac}+cx^2}\,dx}{4c(b^2-4ac)}$$

$$= -\frac{(bB-2Ac)x}{2c(b^2-4ac)} - \frac{x^3(Ab-2aB-(bB-2Ac)x^2)}{2(b^2-4ac)(a+bx^2+cx^4)} + \frac{\left(b^2B+Abc-6aBc-\frac{b^3B+Ab^2c-8abBc+4aAc^2}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}c^{3/2}(b^2-4ac)\sqrt{b-\sqrt{b^2-4ac}}} + \frac{\left(b^2B+Abc-6aBc+\frac{b^3B+Ab^2c-8abBc+4aAc^2}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}c^{3/2}(b^2-4ac)\sqrt{b+\sqrt{b^2-4ac}}}$$

Mathematica [A] (verified)

Time = 0.59 (sec) , antiderivative size = 362, normalized size of antiderivative = 1.08

$$\int \frac{x^4(A+Bx^2)}{(a+bx^2+cx^4)^2}\,dx$$
$$= \frac{\frac{2\sqrt{c}(-abBx+b(-bB+Ac)x^3+2acx(A+Bx^2))}{(b^2-4ac)(a+bx^2+cx^4)}+\frac{\sqrt{2}\left(-b^3B+bc\left(8aB+A\sqrt{b^2-4ac}\right)+b^2\left(-Ac+B\sqrt{b^2-4ac}\right)-2ac\left(2Ac+3B\sqrt{b^2-4ac}\right)\right)\arctan\left(\right.}{(b^2-4ac)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}}}{4c^{3/2}}$$

```
[In] Integrate[(x^4*(A + B*x^2))/(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] ((2*Sqrt[c]*(-(a*b*B*x) + b*(-(b*B) + A*c)*x^3 + 2*a*c*x*(A + B*x^2)))/((b^
2 - 4*a*c)*(a + b*x^2 + c*x^4)) + (Sqrt[2]*(-(b^3*B) + b*c*(8*a*B + A*Sqrt[
b^2 - 4*a*c]) + b^2*(-(A*c) + B*Sqrt[b^2 - 4*a*c]) - 2*a*c*(2*A*c + 3*B*Sqr
t[b^2 - 4*a*c]))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(
(b^2 - 4*a*c)^(3/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (Sqrt[2]*(b^3*B + 2*a*c*
(2*A*c - 3*B*Sqrt[b^2 - 4*a*c]) + b^2*(A*c + B*Sqrt[b^2 - 4*a*c]) + b*(-8*a
*B*c + A*c*Sqrt[b^2 - 4*a*c]))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2
 - 4*a*c]]])/((b^2 - 4*a*c)^(3/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]]))/(4*c^(3/2))
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.13 (sec) , antiderivative size = 178, normalized size of antiderivative = 0.53

method	result
risch	$\frac{-\frac{\left(Abc+2Bac-B\,b^2\right)x^3}{2c\left(4ac-b^2\right)}-\frac{a(2Ac-Bb)x}{2\left(4ac-b^2\right)c}}{c\,x^4+b\,x^2+a}+\frac{\sum_{\_R=\mathrm{RootOf}\left(c\_Z^4+\_Z^2b+a\right)}\frac{\left(-\frac{\left(Abc-6Bac+B\,b^2\right)\_R^2}{4ac-b^2}+\frac{a(2Ac-Bb)}{4ac-b^2}\right)\ln\left(x-\_R\right)}{2c\_R^3+\_Rb}}{4c}$
default	$\frac{-\frac{\left(Abc+2Bac-B\,b^2\right)x^3}{2c\left(4ac-b^2\right)}-\frac{a(2Ac-Bb)x}{2\left(4ac-b^2\right)c}}{c\,x^4+b\,x^2+a}+\frac{\frac{\left(-Abc\sqrt{-4ac+b^2}-4Aa\,c^2-A\,b^2c+6Bac\sqrt{-4ac+b^2}-B\,b^2\sqrt{-4ac+b^2}+8Babc-B\,b^3\right)\sqrt{2}\,\arctan\left(\frac{}{\sqrt{(b+}}\right.}{4c\sqrt{-4ac+b^2}\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}}}{}$

```
[In] int(x^4*(B*x^2+A)/(c*x^4+b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] (-1/2*(A*b*c+2*B*a*c-B*b^2)/c/(4*a*c-b^2)*x^3-1/2*a*(2*A*c-B*b)/(4*a*c-b^2)
/c*x)/(c*x^4+b*x^2+a)+1/4/c*sum((-(A*b*c-6*B*a*c+B*b^2)/(4*a*c-b^2)*_R^2+a*
(2*A*c-B*b)/(4*a*c-b^2))/(2*_R^3*c+_R*b)*ln(x-_R),_R=RootOf(_Z^4*c+_Z^2*b+a
))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 4658 vs. $2(292) = 584$.

Time = 2.31 (sec) , antiderivative size = 4658, normalized size of antiderivative = 13.86

$$\int \frac{x^4(A+Bx^2)}{(a+bx^2+cx^4)^2}\,dx = \text{Too large to display}$$

```
[In] integrate(x^4*(B*x^2+A)/(c*x^4+b*x^2+a)^2,x, algorithm="fricas")
[Out] -1/4*(2*(B*b^2 - (2*B*a + A*b)*c)*x^3 + sqrt(1/2)*((b^2*c^2 - 4*a*c^3)*x^4
+ a*b^2*c - 4*a^2*c^2 + (b^3*c - 4*a*b*c^2)*x^2)*sqrt(-(B^2*b^5 - 12*(4*A*B
*a^2 - A^2*a*b)*c^3 + (60*B^2*a^2*b - 12*A*B*a*b^2 + A^2*b^3)*c^2 - (15*B^2
*a*b^3 - 2*A*B*b^4)*c + (b^6*c^3 - 12*a*b^4*c^4 + 48*a^2*b^2*c^5 - 64*a^3*c
^6)*sqrt((B^4*b^4 + A^4*c^4 - 2*(9*A^2*B^2*a - 2*A^3*B*b)*c^3 + 3*(27*B^4*a
^2 - 12*A*B^3*a*b + 2*A^2*B^2*b^2)*c^2 - 2*(9*B^4*a*b^2 - 2*A*B^3*b^3)*c)/(
b^6*c^6 - 12*a*b^4*c^7 + 48*a^2*b^2*c^8 - 64*a^3*c^9)))/(b^6*c^3 - 12*a*b^4
*c^4 + 48*a^2*b^2*c^5 - 64*a^3*c^6))*log(-(5*B^4*a*b^4 - 3*A*B^3*b^5 - 4*A^
4*a*c^4 + (20*A^3*B*a*b - 3*A^4*b^2)*c^3 + 3*(108*B^4*a^3 - 108*A*B^3*a^2*b
 + 28*A^2*B^2*a*b^2 - 3*A^3*B*b^3)*c^2 - (81*B^4*a^2*b^2 - 65*A*B^3*a*b^3 +
 9*A^2*B^2*b^4)*c)*x + 1/2*sqrt(1/2)*(B^3*b^7 - 17*B^3*a*b^5*c - 32*A^3*a^2
*c^5 + 16*(18*A*B^2*a^3 - 3*A^2*B*a^2*b + A^3*a*b^2)*c^4 - 2*(72*B^3*a^3*b
+ 72*A*B^2*a^2*b^2 - 12*A^2*B*a*b^3 + A^3*b^4)*c^3 + (88*B^3*a^2*b^3 + 18*A
*B^2*a*b^4 - 3*A^2*B*b^5)*c^2 - (B*b^8*c^3 + 256*(3*B*a^4 - A*a^3*b)*c^7 -
64*(10*B*a^3*b^2 - 3*A*a^2*b^3)*c^6 + 48*(4*B*a^2*b^4 - A*a*b^5)*c^5 - 4*(6
*B*a*b^6 - A*b^7)*c^4)*sqrt((B^4*b^4 + A^4*c^4 - 2*(9*A^2*B^2*a - 2*A^3*B*b
)*c^3 + 3*(27*B^4*a^2 - 12*A*B^3*a*b + 2*A^2*B^2*b^2)*c^2 - 2*(9*B^4*a*b^2
- 2*A*B^3*b^3)*c)/(b^6*c^6 - 12*a*b^4*c^7 + 48*a^2*b^2*c^8 - 64*a^3*c^9)))*
sqrt(-(B^2*b^5 - 12*(4*A*B*a^2 - A^2*a*b)*c^3 + (60*B^2*a^2*b - 12*A*B*a*b^
2 + A^2*b^3)*c^2 - (15*B^2*a*b^3 - 2*A*B*b^4)*c + (b^6*c^3 - 12*a*b^4*c^4 +
 48*a^2*b^2*c^5 - 64*a^3*c^6)*sqrt((B^4*b^4 + A^4*c^4 - 2*(9*A^2*B^2*a - 2*
A^3*B*b)*c^3 + 3*(27*B^4*a^2 - 12*A*B^3*a*b + 2*A^2*B^2*b^2)*c^2 - 2*(9*B^4
*a*b^2 - 2*A*B^3*b^3)*c)/(b^6*c^6 - 12*a*b^4*c^7 + 48*a^2*b^2*c^8 - 64*a^3*
c^9)))/(b^6*c^3 - 12*a*b^4*c^4 + 48*a^2*b^2*c^5 - 64*a^3*c^6))) - sqrt(1/2)
*((b^2*c^2 - 4*a*c^3)*x^4 + a*b^2*c - 4*a^2*c^2 + (b^3*c - 4*a*b*c^2)*x^2)*
sqrt(-(B^2*b^5 - 12*(4*A*B*a^2 - A^2*a*b)*c^3 + (60*B^2*a^2*b - 12*A*B*a*b^
2 + A^2*b^3)*c^2 - (15*B^2*a*b^3 - 2*A*B*b^4)*c + (b^6*c^3 - 12*a*b^4*c^4 +
 48*a^2*b^2*c^5 - 64*a^3*c^6)*sqrt((B^4*b^4 + A^4*c^4 - 2*(9*A^2*B^2*a - 2*
A^3*B*b)*c^3 + 3*(27*B^4*a^2 - 12*A*B^3*a*b + 2*A^2*B^2*b^2)*c^2 - 2*(9*B^4
*a*b^2 - 2*A*B^3*b^3)*c)/(b^6*c^6 - 12*a*b^4*c^7 + 48*a^2*b^2*c^8 - 64*a^3*
c^9)))/(b^6*c^3 - 12*a*b^4*c^4 + 48*a^2*b^2*c^5 - 64*a^3*c^6))*log(-(5*B^4*
a*b^4 - 3*A*B^3*b^5 - 4*A^4*a*c^4 + (20*A^3*B*a*b - 3*A^4*b^2)*c^3 + 3*(108
*B^4*a^3 - 108*A*B^3*a^2*b + 28*A^2*B^2*a*b^2 - 3*A^3*B*b^3)*c^2 - (81*B^4*
a^2*b^2 - 65*A*B^3*a*b^3 + 9*A^2*B^2*b^4)*c)*x - 1/2*sqrt(1/2)*(B^3*b^7 - 1
7*B^3*a*b^5*c - 32*A^3*a^2*c^5 + 16*(18*A*B^2*a^3 - 3*A^2*B*a^2*b + A^3*a*b
```

```
^2)*c^4 - 2*(72*B^3*a^3*b + 72*A*B^2*a^2*b^2 - 12*A^2*B*a*b^3 + A^3*b^4)*c^
3 + (88*B^3*a^2*b^3 + 18*A*B^2*a*b^4 - 3*A^2*B*b^5)*c^2 - (B*b^8*c^3 + 256*
(3*B*a^4 - A*a^3*b)*c^7 - 64*(10*B*a^3*b^2 - 3*A*a^2*b^3)*c^6 + 48*(4*B*a^2
*b^4 - A*a*b^5)*c^5 - 4*(6*B*a*b^6 - A*b^7)*c^4)*sqrt((B^4*b^4 + A^4*c^4 -
2*(9*A^2*B^2*a - 2*A^3*B*b)*c^3 + 3*(27*B^4*a^2 - 12*A*B^3*a*b + 2*A^2*B^2*
b^2)*c^2 - 2*(9*B^4*a*b^2 - 2*A*B^3*b^3)*c)/(b^6*c^6 - 12*a*b^4*c^7 + 48*a^
2*b^2*c^8 - 64*a^3*c^9)))*sqrt(-(B^2*b^5 - 12*(4*A*B*a^2 - A^2*a*b)*c^3 + (
60*B^2*a^2*b - 12*A*B*a*b^2 + A^2*b^3)*c^2 - (15*B^2*a*b^3 - 2*A*B*b^4)*c +
 (b^6*c^3 - 12*a*b^4*c^4 + 48*a^2*b^2*c^5 - 64*a^3*c^6)*sqrt((B^4*b^4 + A^4
*c^4 - 2*(9*A^2*B^2*a - 2*A^3*B*b)*c^3 + 3*(27*B^4*a^2 - 12*A*B^3*a*b + 2*A
^2*B^2*b^2)*c^2 - 2*(9*B^4*a*b^2 - 2*A*B^3*b^3)*c)/(b^6*c^6 - 12*a*b^4*c^7
+ 48*a^2*b^2*c^8 - 64*a^3*c^9)))/(b^6*c^3 - 12*a*b^4*c^4 + 48*a^2*b^2*c^5 -
 64*a^3*c^6))) + sqrt(1/2)*((b^2*c^2 - 4*a*c^3)*x^4 + a*b^2*c - 4*a^2*c^2 +
 (b^3*c - 4*a*b*c^2)*x^2)*sqrt(-(B^2*b^5 - 12*(4*A*B*a^2 - A^2*a*b)*c^3 + (
60*B^2*a^2*b - 12*A*B*a*b^2 + A^2*b^3)*c^2 - (15*B^2*a*b^3 - 2*A*B*b^4)*c -
 (b^6*c^3 - 12*a*b^4*c^4 + 48*a^2*b^2*c^5 - 64*a^3*c^6)*sqrt((B^4*b^4 + A^4
*c^4 - 2*(9*A^2*B^2*a - 2*A^3*B*b)*c^3 + 3*(27*B^4*a^2 - 12*A*B^3*a*b + 2*A
^2*B^2*b^2)*c^2 - 2*(9*B^4*a*b^2 - 2*A*B^3*b^3)*c)/(b^6*c^6 - 12*a*b^4*c^7
+ 48*a^2*b^2*c^8 - 64*a^3*c^9)))/(b^6*c^3 - 12*a*b^4*c^4 + 48*a^2*b^2*c^5 -
 64*a^3*c^6))*log(-(5*B^4*a*b^4 - 3*A*B^3*b^5 - 4*A^4*a*c^4 + (20*A^3*B*a*b
 - 3*A^4*b^2)*c^3 + 3*(108*B^4*a^3 - 108*A*B^3*a^2*b + 28*A^2*B^2*a*b^2 - 3
*A^3*B*b^3)*c^2 - (81*B^4*a^2*b^2 - 65*A*B^3*a*b^3 + 9*A^2*B^2*b^4)*c)*x +
1/2*sqrt(1/2)*(B^3*b^7 - 17*B^3*a*b^5*c - 32*A^3*a^2*c^5 + 16*(18*A*B^2*a^3
 - 3*A^2*B*a^2*b + A^3*a*b^2)*c^4 - 2*(72*B^3*a^3*b + 72*A*B^2*a^2*b^2 - 12
*A^2*B*a*b^3 + A^3*b^4)*c^3 + (88*B^3*a^2*b^3 + 18*A*B^2*a*b^4 - 3*A^2*B*b^
5)*c^2 + (B*b^8*c^3 + 256*(3*B*a^4 - A*a^3*b)*c^7 - 64*(10*B*a^3*b^2 - 3*A*
a^2*b^3)*c^6 + 48*(4*B*a^2*b^4 - A*a*b^5)*c^5 - 4*(6*B*a*b^6 - A*b^7)*c^4)*
sqrt((B^4*b^4 + A^4*c^4 - 2*(9*A^2*B^2*a - 2*A^3*B*b)*c^3 + 3*(27*B^4*a^2 -
 12*A*B^3*a*b + 2*A^2*B^2*b^2)*c^2 - 2*(9*B^4*a*b^2 - 2*A*B^3*b^3)*c)/(b^6*
c^6 - 12*a*b^4*c^7 + 48*a^2*b^2*c^8 - 64*a^3*c^9)))*sqrt(-(B^2*b^5 - 12*(4*
A*B*a^2 - A^2*a*b)*c^3 + (60*B^2*a^2*b - 12*A*B*a*b^2 + A^2*b^3)*c^2 - (15*
B^2*a*b^3 - 2*A*B*b^4)*c - (b^6*c^3 - 12*a*b^4*c^4 + 48*a^2*b^2*c^5 - 64*a^
3*c^6)*sqrt((B^4*b^4 + A^4*c^4 - 2*(9*A^2*B^2*a - 2*A^3*B*b)*c^3 + 3*(27*B^
4*a^2 - 12*A*B^3*a*b + 2*A^2*B^2*b^2)*c^2 - 2*(9*B^4*a*b^2 - 2*A*B^3*b^3)*c
)/(b^6*c^6 - 12*a*b^4*c^7 + 48*a^2*b^2*c^8 - 64*a^3*c^9)))/(b^6*c^3 - 12*a*
b^4*c^4 + 48*a^2*b^2*c^5 - 64*a^3*c^6))) - sqrt(1/2)*((b^2*c^2 - 4*a*c^3)*x
^4 + a*b^2*c - 4*a^2*c^2 + (b^3*c - 4*a*b*c^2)*x^2)*sqrt(-(B^2*b^5 - 12*(4*
A*B*a^2 - A^2*a*b)*c^3 + (60*B^2*a^2*b - 12*A*B*a*b^2 + A^2*b^3)*c^2 - (15*
B^2*a*b^3 - 2*A*B*b^4)*c - (b^6*c^3 - 12*a*b^4*c^4 + 48*a^2*b^2*c^5 - 64*a^
3*c^6)*sqrt((B^4*b^4 + A^4*c^4 - 2*(9*A^2*B^2*a - 2*A^3*B*b)*c^3 + 3*(27*B^
4*a^2 - 12*A*B^3*a*b + 2*A^2*B^2*b^2)*c^2 - 2*(9*B^4*a*b^2 - 2*A*B^3*b^3)*c
)/(b^6*c^6 - 12*a*b^4*c^7 + 48*a^2*b^2*c^8 - 64*a^3*c^9)))/(b^6*c^3 - 12*a*
b^4*c^4 + 48*a^2*b^2*c^5 - 64*a^3*c^6))*log(-(5*B^4*a*b^4 - 3*A*B^3*b^5 - 4
*A^4*a*c^4 + (20*A^3*B*a*b - 3*A^4*b^2)*c^3 + 3*(108*B^4*a^3 - 108*A*B^3*a^
2*b + 28*A^2*B^2*a*b^2 - 3*A^3*B*b^3)*c^2 - (81*B^4*a^2*b^2 - 65*A*B^3*a*b^
```

```
3 + 9*A^2*B^2*b^4)*c)*x - 1/2*sqrt(1/2)*(B^3*b^7 - 17*B^3*a*b^5*c - 32*A^3*
a^2*c^5 + 16*(18*A*B^2*a^3 - 3*A^2*B*a^2*b + A^3*a*b^2)*c^4 - 2*(72*B^3*a^3
*b + 72*A*B^2*a^2*b^2 - 12*A^2*B*a*b^3 + A^3*b^4)*c^3 + (88*B^3*a^2*b^3 + 1
8*A*B^2*a*b^4 - 3*A^2*B*b^5)*c^2 + (B*b^8*c^3 + 256*(3*B*a^4 - A*a^3*b)*c^7
 - 64*(10*B*a^3*b^2 - 3*A*a^2*b^3)*c^6 + 48*(4*B*a^2*b^4 - A*a*b^5)*c^5 - 4
*(6*B*a*b^6 - A*b^7)*c^4)*sqrt((B^4*b^4 + A^4*c^4 - 2*(9*A^2*B^2*a - 2*A^3*
B*b)*c^3 + 3*(27*B^4*a^2 - 12*A*B^3*a*b + 2*A^2*B^2*b^2)*c^2 - 2*(9*B^4*a*b
^2 - 2*A*B^3*b^3)*c)/(b^6*c^6 - 12*a*b^4*c^7 + 48*a^2*b^2*c^8 - 64*a^3*c^9)
))*sqrt(-(B^2*b^5 - 12*(4*A*B*a^2 - A^2*a*b)*c^3 + (60*B^2*a^2*b - 12*A*B*a
*b^2 + A^2*b^3)*c^2 - (15*B^2*a*b^3 - 2*A*B*b^4)*c - (b^6*c^3 - 12*a*b^4*c^
4 + 48*a^2*b^2*c^5 - 64*a^3*c^6)*sqrt((B^4*b^4 + A^4*c^4 - 2*(9*A^2*B^2*a -
 2*A^3*B*b)*c^3 + 3*(27*B^4*a^2 - 12*A*B^3*a*b + 2*A^2*B^2*b^2)*c^2 - 2*(9*
B^4*a*b^2 - 2*A*B^3*b^3)*c)/(b^6*c^6 - 12*a*b^4*c^7 + 48*a^2*b^2*c^8 - 64*a
^3*c^9)))/(b^6*c^3 - 12*a*b^4*c^4 + 48*a^2*b^2*c^5 - 64*a^3*c^6)) + 2*(B*a
*b - 2*A*a*c)*x)/((b^2*c^2 - 4*a*c^3)*x^4 + a*b^2*c - 4*a^2*c^2 + (b^3*c -
4*a*b*c^2)*x^2)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^4(A+Bx^2)}{(a+bx^2+cx^4)^2}\,dx = \text{Timed out}$$

```
[In] integrate(x**4*(B*x**2+A)/(c*x**4+b*x**2+a)**2,x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{x^4(A+Bx^2)}{(a+bx^2+cx^4)^2}\,dx = \int \frac{(Bx^2+A)x^4}{(cx^4+bx^2+a)^2}\,dx$$

```
[In] integrate(x^4*(B*x^2+A)/(c*x^4+b*x^2+a)^2,x, algorithm="maxima")
[Out] -1/2*((B*b^2 - (2*B*a + A*b)*c)*x^3 + (B*a*b - 2*A*a*c)*x)/((b^2*c^2 - 4*a*
c^3)*x^4 + a*b^2*c - 4*a^2*c^2 + (b^3*c - 4*a*b*c^2)*x^2) + 1/2*integrate((
B*a*b - 2*A*a*c + (B*b^2 - (6*B*a - A*b)*c)*x^2)/(c*x^4 + b*x^2 + a), x)/(b
^2*c - 4*a*c^2)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 4538 vs. 2(292) = 584.

Time = 1.40 (sec) , antiderivative size = 4538, normalized size of antiderivative = 13.51

$$\int \frac{x^4(A+Bx^2)}{(a+bx^2+cx^4)^2}\,dx = \text{Too large to display}$$

```
[In] integrate(x^4*(B*x^2+A)/(c*x^4+b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] -1/2*(B*b^2*x^3 - 2*B*a*c*x^3 - A*b*c*x^3 + B*a*b*x - 2*A*a*c*x)/((c*x^4 +
b*x^2 + a)*(b^2*c - 4*a*c^2)) + 1/16*((2*b^3*c^3 - 8*a*b*c^4 - sqrt(2)*sqrt
(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^3*c + 4*sqrt(2)*sqrt(b^2 -
4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c^2 + 2*sqrt(2)*sqrt(b^2 - 4*a*c
)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^2*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(
b*c + sqrt(b^2 - 4*a*c)*c)*b*c^3 - 2*(b^2 - 4*a*c)*b*c^3)*(b^2*c - 4*a*c^2)
^2*A + (2*b^4*c^2 - 20*a*b^2*c^3 + 48*a^2*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*s
qrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^4 + 10*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c
+ sqrt(b^2 - 4*a*c)*c)*a*b^2*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqr
t(b^2 - 4*a*c)*c)*b^3*c - 24*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2
- 4*a*c)*c)*a^2*c^2 - 12*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*
a*c)*c)*a*b*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)
*b^2*c^2 + 6*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*c^
3 - 2*(b^2 - 4*a*c)*b^2*c^2 + 12*(b^2 - 4*a*c)*a*c^3)*(b^2*c - 4*a*c^2)^2*B
 - 4*(sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^4*c^3 - 8*sqrt(2)*sqrt(b*
c + sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^4 - 2*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*
c)*c)*a*b^3*c^4 - 2*a*b^4*c^4 + 16*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*
a^3*c^5 + 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b*c^5 + sqrt(2)*sqr
t(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^5 + 16*a^2*b^2*c^5 - 4*sqrt(2)*sqrt(b*
c + sqrt(b^2 - 4*a*c)*c)*a^2*c^6 - 32*a^3*c^6 + 2*(b^2 - 4*a*c)*a*b^2*c^4 -
 8*(b^2 - 4*a*c)*a^2*c^5)*A*abs(b^2*c - 4*a*c^2) + 2*(sqrt(2)*sqrt(b*c + sq
rt(b^2 - 4*a*c)*c)*a*b^5*c^2 - 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^
2*b^3*c^3 - 2*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^4*c^3 - 2*a*b^5*c
^3 + 16*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b*c^4 + 8*sqrt(2)*sqrt(
b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^4 + sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*
c)*c)*a*b^3*c^4 + 16*a^2*b^3*c^4 - 4*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c
)*a^2*b*c^5 - 32*a^3*b*c^5 + 2*(b^2 - 4*a*c)*a*b^3*c^3 - 8*(b^2 - 4*a*c)*a^
2*b*c^4)*B*abs(b^2*c - 4*a*c^2) - (2*b^7*c^5 - 8*a*b^5*c^6 - 32*a^2*b^3*c^7
 + 128*a^3*b*c^8 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c
)*b^7*c^3 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b
^5*c^4 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^6*c^
4 + 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c^
5 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^5*c^5 - 64*
sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b*c^6 - 32*sq
rt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^6 + 16*sq
```

rt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b*c^7 - 2*(b^2
- 4*a*c)*b^5*c^5 + 32*(b^2 - 4*a*c)*a^2*b*c^7)*A - (2*b^8*c^4 - 32*a*b^6*c^
5 + 160*a^2*b^4*c^6 - 256*a^3*b^2*c^7 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c
+ sqrt(b^2 - 4*a*c)*c)*b^8*c^2 + 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sq
rt(b^2 - 4*a*c)*c)*a*b^6*c^3 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(
b^2 - 4*a*c)*c)*b^7*c^3 - 80*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2
- 4*a*c)*c)*a^2*b^4*c^4 - 24*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2
- 4*a*c)*c)*a*b^5*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a
*c)*c)*b^6*c^4 + 128*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)
*c)*a^3*b^2*c^5 + 64*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)
*c)*a^2*b^3*c^5 + 12*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)
*c)*a*b^4*c^5 - 32*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c
)*a^2*b^2*c^6 - 2*(b^2 - 4*a*c)*b^6*c^4 + 24*(b^2 - 4*a*c)*a*b^4*c^5 - 64*(
b^2 - 4*a*c)*a^2*b^2*c^6)*B)*arctan(2*sqrt(1/2)*x/sqrt((b^3*c - 4*a*b*c^2 +
sqrt((b^3*c - 4*a*b*c^2)^2 - 4*(a*b^2*c - 4*a^2*c^2)*(b^2*c^2 - 4*a*c^3)))
/(b^2*c^2 - 4*a*c^3)))/((a*b^6*c^3 - 12*a^2*b^4*c^4 - 2*a*b^5*c^4 + 48*a^3*
b^2*c^5 + 16*a^2*b^3*c^5 + a*b^4*c^5 - 64*a^4*c^6 - 32*a^3*b*c^6 - 8*a^2*b^
2*c^6 + 16*a^3*c^7)*abs(b^2*c - 4*a*c^2)*abs(c)) - 1/16*((2*b^3*c^3 - 8*a*b
*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^3*c + 4*
sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b*c^2 + 2*sqrt(
2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^2*c^2 - sqrt(2)*sqrt
(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b*c^3 - 2*(b^2 - 4*a*c)*b*c^3
)*(b^2*c - 4*a*c^2)^2*A + (2*b^4*c^2 - 20*a*b^2*c^3 + 48*a^2*c^4 - sqrt(2)*
sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^4 + 10*sqrt(2)*sqrt(b^2
- 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^2*c + 2*sqrt(2)*sqrt(b^2 - 4*
a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^3*c - 24*sqrt(2)*sqrt(b^2 - 4*a*c)*s
qrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*c^2 - 12*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(
b*c - sqrt(b^2 - 4*a*c)*c)*a*b*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - s
qrt(b^2 - 4*a*c)*c)*b^2*c^2 + 6*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b
^2 - 4*a*c)*c)*a*c^3 - 2*(b^2 - 4*a*c)*b^2*c^2 + 12*(b^2 - 4*a*c)*a*c^3)*(b
^2*c - 4*a*c^2)^2*B + 4*(sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^4*c^3
- 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^4 - 2*sqrt(2)*sqrt(b*
c - sqrt(b^2 - 4*a*c)*c)*a*b^3*c^4 + 2*a*b^4*c^4 + 16*sqrt(2)*sqrt(b*c - sq
rt(b^2 - 4*a*c)*c)*a^3*c^5 + 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*
b*c^5 + sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^2*c^5 - 16*a^2*b^2*c^5
- 4*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*c^6 + 32*a^3*c^6 - 2*(b^2 -
4*a*c)*a*b^2*c^4 + 8*(b^2 - 4*a*c)*a^2*c^5)*A*abs(b^2*c - 4*a*c^2) - 2*(sq
rt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^5*c^2 - 8*sqrt(2)*sqrt(b*c - sqrt
(b^2 - 4*a*c)*c)*a^2*b^3*c^3 - 2*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*
b^4*c^3 + 2*a*b^5*c^3 + 16*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b*c^
4 + 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^4 + sqrt(2)*sqrt(b*
c - sqrt(b^2 - 4*a*c)*c)*a*b^3*c^4 - 16*a^2*b^3*c^4 - 4*sqrt(2)*sqrt(b*c -
sqrt(b^2 - 4*a*c)*c)*a^2*b*c^5 + 32*a^3*b*c^5 - 2*(b^2 - 4*a*c)*a*b^3*c^3 +
8*(b^2 - 4*a*c)*a^2*b*c^4)*B*abs(b^2*c - 4*a*c^2) - (2*b^7*c^5 - 8*a*b^5*c
^6 - 32*a^2*b^3*c^7 + 128*a^3*b*c^8 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c -

```
sqrt(b^2 - 4*a*c)*c)*b^7*c^3 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(
b^2 - 4*a*c)*c)*a*b^5*c^4 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2
 - 4*a*c)*c)*b^6*c^4 + 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4
*a*c)*c)*a^2*b^3*c^5 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*
c)*c)*b^5*c^5 - 64*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c
)*a^3*b*c^6 - 32*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*
a^2*b^2*c^6 + 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*
a^2*b*c^7 - 2*(b^2 - 4*a*c)*b^5*c^5 + 32*(b^2 - 4*a*c)*a^2*b*c^7)*A - (2*b^
8*c^4 - 32*a*b^6*c^5 + 160*a^2*b^4*c^6 - 256*a^3*b^2*c^7 - sqrt(2)*sqrt(b^2
 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^8*c^2 + 16*sqrt(2)*sqrt(b^2 - 4
*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^6*c^3 + 2*sqrt(2)*sqrt(b^2 - 4*a*
c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^7*c^3 - 80*sqrt(2)*sqrt(b^2 - 4*a*c)*s
qrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^4*c^4 - 24*sqrt(2)*sqrt(b^2 - 4*a*c)*s
qrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^5*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b
*c - sqrt(b^2 - 4*a*c)*c)*b^6*c^4 + 128*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c
- sqrt(b^2 - 4*a*c)*c)*a^3*b^2*c^5 + 64*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c
- sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c^5 + 12*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c
- sqrt(b^2 - 4*a*c)*c)*a*b^4*c^5 - 32*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c -
sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^6 - 2*(b^2 - 4*a*c)*b^6*c^4 + 24*(b^2 - 4*a*
c)*a*b^4*c^5 - 64*(b^2 - 4*a*c)*a^2*b^2*c^6)*B)*arctan(2*sqrt(1/2)*x/sqrt((
b^3*c - 4*a*b*c^2 - sqrt((b^3*c - 4*a*b*c^2)^2 - 4*(a*b^2*c - 4*a^2*c^2)*(b
^2*c^2 - 4*a*c^3)))/(b^2*c^2 - 4*a*c^3)))/((a*b^6*c^3 - 12*a^2*b^4*c^4 - 2*
a*b^5*c^4 + 48*a^3*b^2*c^5 + 16*a^2*b^3*c^5 + a*b^4*c^5 - 64*a^4*c^6 - 32*a
^3*b*c^6 - 8*a^2*b^2*c^6 + 16*a^3*c^7)*abs(b^2*c - 4*a*c^2)*abs(c))
```

Mupad [B] (verification not implemented)

Time = 10.98 (sec) , antiderivative size = 12396, normalized size of antiderivative = 36.89

$$\int \frac{x^4(A+Bx^2)}{(a+bx^2+cx^4)^2}\,dx = \text{Too large to display}$$

```
[In] int((x^4*(A + B*x^2))/(a + b*x^2 + c*x^4)^2,x)
```

```
[Out] - ((x^3*(A*b*c - B*b^2 + 2*B*a*c))/(2*c*(4*a*c - b^2)) + (x*(2*A*a*c - B*a*
b))/(2*c*(4*a*c - b^2)))/(a + b*x^2 + c*x^4) - atan(((((2048*A*a^4*c^6 - 32
*A*a*b^6*c^3 + 16*B*a*b^7*c^2 - 1024*B*a^4*b*c^5 + 384*A*a^2*b^4*c^4 - 1536
*A*a^3*b^2*c^5 - 192*B*a^2*b^5*c^3 + 768*B*a^3*b^3*c^4)/(8*(b^6*c - 64*a^3*
c^4 - 12*a*b^4*c^2 + 48*a^2*b^2*c^3)) - (x*(-(B^2*b^11 + A^2*b^9*c^2 + A^2*
c^2*(-(4*a*c - b^2)^9)^(1/2) + B^2*b^2*(-(4*a*c - b^2)^9)^(1/2) + 2*A*B*b^1
0*c - 96*A^2*a^2*b^5*c^4 + 512*A^2*a^3*b^3*c^5 + 288*B^2*a^2*b^7*c^2 - 1504
*B^2*a^3*b^5*c^3 + 3840*B^2*a^4*b^3*c^4 + 3072*A*B*a^5*c^6 - 27*B^2*a*b^9*c
 - 9*B^2*a*c*(-(4*a*c - b^2)^9)^(1/2) - 768*A^2*a^4*b*c^6 - 3840*B^2*a^5*b*
c^5 + 192*A*B*a^2*b^6*c^3 - 128*A*B*a^3*b^4*c^4 - 1536*A*B*a^4*b^2*c^5 + 2*
A*B*b*c*(-(4*a*c - b^2)^9)^(1/2) - 36*A*B*a*b^8*c^2)/(32*(4096*a^6*c^9 + b^
```

```
12*c^3 - 24*a*b^10*c^4 + 240*a^2*b^8*c^5 - 1280*a^3*b^6*c^6 + 3840*a^4*b^4*
c^7 - 6144*a^5*b^2*c^8)))^(1/2)*(16*b^7*c^3 - 192*a*b^5*c^4 - 1024*a^3*b*c^
6 + 768*a^2*b^3*c^5))/(2*(b^4*c + 16*a^2*c^3 - 8*a*b^2*c^2)))*(-(B^2*b^11 +
 A^2*b^9*c^2 + A^2*c^2*(-(4*a*c - b^2)^9)^(1/2) + B^2*b^2*(-(4*a*c - b^2)^9
)^(1/2) + 2*A*B*b^10*c - 96*A^2*a^2*b^5*c^4 + 512*A^2*a^3*b^3*c^5 + 288*B^2
*a^2*b^7*c^2 - 1504*B^2*a^3*b^5*c^3 + 3840*B^2*a^4*b^3*c^4 + 3072*A*B*a^5*c
^6 - 27*B^2*a*b^9*c - 9*B^2*a*c*(-(4*a*c - b^2)^9)^(1/2) - 768*A^2*a^4*b*c^
6 - 3840*B^2*a^5*b*c^5 + 192*A*B*a^2*b^6*c^3 - 128*A*B*a^3*b^4*c^4 - 1536*A
*B*a^4*b^2*c^5 + 2*A*B*b*c*(-(4*a*c - b^2)^9)^(1/2) - 36*A*B*a*b^8*c^2)/(32
*(4096*a^6*c^9 + b^12*c^3 - 24*a*b^10*c^4 + 240*a^2*b^8*c^5 - 1280*a^3*b^6*
c^6 + 3840*a^4*b^4*c^7 - 6144*a^5*b^2*c^8)))^(1/2) - (x*(B^2*b^6 + 8*A^2*a^
2*c^4 + A^2*b^4*c^2 - 72*B^2*a^3*c^3 + 2*A*B*b^5*c + 74*B^2*a^2*b^2*c^2 - 1
6*B^2*a*b^4*c + 2*A^2*a*b^2*c^3 - 14*A*B*a*b^3*c^2 - 8*A*B*a^2*b*c^3))/(2*(
b^4*c + 16*a^2*c^3 - 8*a*b^2*c^2)))*(-(B^2*b^11 + A^2*b^9*c^2 + A^2*c^2*(-(
4*a*c - b^2)^9)^(1/2) + B^2*b^2*(-(4*a*c - b^2)^9)^(1/2) + 2*A*B*b^10*c - 9
6*A^2*a^2*b^5*c^4 + 512*A^2*a^3*b^3*c^5 + 288*B^2*a^2*b^7*c^2 - 1504*B^2*a^
3*b^5*c^3 + 3840*B^2*a^4*b^3*c^4 + 3072*A*B*a^5*c^6 - 27*B^2*a*b^9*c - 9*B^
2*a*c*(-(4*a*c - b^2)^9)^(1/2) - 768*A^2*a^4*b*c^6 - 3840*B^2*a^5*b*c^5 + 1
92*A*B*a^2*b^6*c^3 - 128*A*B*a^3*b^4*c^4 - 1536*A*B*a^4*b^2*c^5 + 2*A*B*b*c
*(-(4*a*c - b^2)^9)^(1/2) - 36*A*B*a*b^8*c^2)/(32*(4096*a^6*c^9 + b^12*c^3
- 24*a*b^10*c^4 + 240*a^2*b^8*c^5 - 1280*a^3*b^6*c^6 + 3840*a^4*b^4*c^7 - 6
144*a^5*b^2*c^8)))^(1/2)*1i - (((2048*A*a^4*c^6 - 32*A*a*b^6*c^3 + 16*B*a*b
^7*c^2 - 1024*B*a^4*b*c^5 + 384*A*a^2*b^4*c^4 - 1536*A*a^3*b^2*c^5 - 192*B*
a^2*b^5*c^3 + 768*B*a^3*b^3*c^4)/(8*(b^6*c - 64*a^3*c^4 - 12*a*b^4*c^2 + 48
*a^2*b^2*c^3)) + (x*(-(B^2*b^11 + A^2*b^9*c^2 + A^2*c^2*(-(4*a*c - b^2)^9)^
(1/2) + B^2*b^2*(-(4*a*c - b^2)^9)^(1/2) + 2*A*B*b^10*c - 96*A^2*a^2*b^5*c^
4 + 512*A^2*a^3*b^3*c^5 + 288*B^2*a^2*b^7*c^2 - 1504*B^2*a^3*b^5*c^3 + 3840
*B^2*a^4*b^3*c^4 + 3072*A*B*a^5*c^6 - 27*B^2*a*b^9*c - 9*B^2*a*c*(-(4*a*c -
 b^2)^9)^(1/2) - 768*A^2*a^4*b*c^6 - 3840*B^2*a^5*b*c^5 + 192*A*B*a^2*b^6*c
^3 - 128*A*B*a^3*b^4*c^4 - 1536*A*B*a^4*b^2*c^5 + 2*A*B*b*c*(-(4*a*c - b^2)
^9)^(1/2) - 36*A*B*a*b^8*c^2)/(32*(4096*a^6*c^9 + b^12*c^3 - 24*a*b^10*c^4
+ 240*a^2*b^8*c^5 - 1280*a^3*b^6*c^6 + 3840*a^4*b^4*c^7 - 6144*a^5*b^2*c^8)
))^(1/2)*(16*b^7*c^3 - 192*a*b^5*c^4 - 1024*a^3*b*c^6 + 768*a^2*b^3*c^5))/(
2*(b^4*c + 16*a^2*c^3 - 8*a*b^2*c^2)))*(-(B^2*b^11 + A^2*b^9*c^2 + A^2*c^2*
(-(4*a*c - b^2)^9)^(1/2) + B^2*b^2*(-(4*a*c - b^2)^9)^(1/2) + 2*A*B*b^10*c
- 96*A^2*a^2*b^5*c^4 + 512*A^2*a^3*b^3*c^5 + 288*B^2*a^2*b^7*c^2 - 1504*B^2
*a^3*b^5*c^3 + 3840*B^2*a^4*b^3*c^4 + 3072*A*B*a^5*c^6 - 27*B^2*a*b^9*c - 9
*B^2*a*c*(-(4*a*c - b^2)^9)^(1/2) - 768*A^2*a^4*b*c^6 - 3840*B^2*a^5*b*c^5
+ 192*A*B*a^2*b^6*c^3 - 128*A*B*a^3*b^4*c^4 - 1536*A*B*a^4*b^2*c^5 + 2*A*B*
b*c*(-(4*a*c - b^2)^9)^(1/2) - 36*A*B*a*b^8*c^2)/(32*(4096*a^6*c^9 + b^12*c
^3 - 24*a*b^10*c^4 + 240*a^2*b^8*c^5 - 1280*a^3*b^6*c^6 + 3840*a^4*b^4*c^7
- 6144*a^5*b^2*c^8)))^(1/2) + (x*(B^2*b^6 + 8*A^2*a^2*c^4 + A^2*b^4*c^2 - 7
2*B^2*a^3*c^3 + 2*A*B*b^5*c + 74*B^2*a^2*b^2*c^2 - 16*B^2*a*b^4*c + 2*A^2*a
*b^2*c^3 - 14*A*B*a*b^3*c^2 - 8*A*B*a^2*b*c^3))/(2*(b^4*c + 16*a^2*c^3 - 8*
a*b^2*c^2)))*(-(B^2*b^11 + A^2*b^9*c^2 + A^2*c^2*(-(4*a*c - b^2)^9)^(1/2) +
```

B^2*b^2*(-(4*a*c - b^2)^9)^(1/2) + 2*A*B*b^10*c - 96*A^2*a^2*b^5*c^4 + 512
*A^2*a^3*b^3*c^5 + 288*B^2*a^2*b^7*c^2 - 1504*B^2*a^3*b^5*c^3 + 3840*B^2*a^
4*b^3*c^4 + 3072*A*B*a^5*c^6 - 27*B^2*a*b^9*c - 9*B^2*a*c*(-(4*a*c - b^2)^9
)^(1/2) - 768*A^2*a^4*b*c^6 - 3840*B^2*a^5*b*c^5 + 192*A*B*a^2*b^6*c^3 - 12
8*A*B*a^3*b^4*c^4 - 1536*A*B*a^4*b^2*c^5 + 2*A*B*b*c*(-(4*a*c - b^2)^9)^(1/
2) - 36*A*B*a*b^8*c^2)/(32*(4096*a^6*c^9 + b^12*c^3 - 24*a*b^10*c^4 + 240*a
^2*b^8*c^5 - 1280*a^3*b^6*c^6 + 3840*a^4*b^4*c^7 - 6144*a^5*b^2*c^8)))^(1/2
)*1i)/((((2048*A*a^4*c^6 - 32*A*a*b^6*c^3 + 16*B*a*b^7*c^2 - 1024*B*a^4*b*c
^5 + 384*A*a^2*b^4*c^4 - 1536*A*a^3*b^2*c^5 - 192*B*a^2*b^5*c^3 + 768*B*a^3
*b^3*c^4)/(8*(b^6*c - 64*a^3*c^4 - 12*a*b^4*c^2 + 48*a^2*b^2*c^3)) - (x*(-(
B^2*b^11 + A^2*b^9*c^2 + A^2*c^2*(-(4*a*c - b^2)^9)^(1/2) + B^2*b^2*(-(4*a*
c - b^2)^9)^(1/2) + 2*A*B*b^10*c - 96*A^2*a^2*b^5*c^4 + 512*A^2*a^3*b^3*c^5
 + 288*B^2*a^2*b^7*c^2 - 1504*B^2*a^3*b^5*c^3 + 3840*B^2*a^4*b^3*c^4 + 3072
*A*B*a^5*c^6 - 27*B^2*a*b^9*c - 9*B^2*a*c*(-(4*a*c - b^2)^9)^(1/2) - 768*A^
2*a^4*b*c^6 - 3840*B^2*a^5*b*c^5 + 192*A*B*a^2*b^6*c^3 - 128*A*B*a^3*b^4*c^
4 - 1536*A*B*a^4*b^2*c^5 + 2*A*B*b*c*(-(4*a*c - b^2)^9)^(1/2) - 36*A*B*a*b^
8*c^2)/(32*(4096*a^6*c^9 + b^12*c^3 - 24*a*b^10*c^4 + 240*a^2*b^8*c^5 - 128
0*a^3*b^6*c^6 + 3840*a^4*b^4*c^7 - 6144*a^5*b^2*c^8)))^(1/2)*(16*b^7*c^3 -
192*a*b^5*c^4 - 1024*a^3*b*c^6 + 768*a^2*b^3*c^5))/(2*(b^4*c + 16*a^2*c^3 -
 8*a*b^2*c^2)))*(-(B^2*b^11 + A^2*b^9*c^2 + A^2*c^2*(-(4*a*c - b^2)^9)^(1/2
) + B^2*b^2*(-(4*a*c - b^2)^9)^(1/2) + 2*A*B*b^10*c - 96*A^2*a^2*b^5*c^4 +
512*A^2*a^3*b^3*c^5 + 288*B^2*a^2*b^7*c^2 - 1504*B^2*a^3*b^5*c^3 + 3840*B^2
*a^4*b^3*c^4 + 3072*A*B*a^5*c^6 - 27*B^2*a*b^9*c - 9*B^2*a*c*(-(4*a*c - b^2
)^9)^(1/2) - 768*A^2*a^4*b*c^6 - 3840*B^2*a^5*b*c^5 + 192*A*B*a^2*b^6*c^3 -
 128*A*B*a^3*b^4*c^4 - 1536*A*B*a^4*b^2*c^5 + 2*A*B*b*c*(-(4*a*c - b^2)^9)^
(1/2) - 36*A*B*a*b^8*c^2)/(32*(4096*a^6*c^9 + b^12*c^3 - 24*a*b^10*c^4 + 24
0*a^2*b^8*c^5 - 1280*a^3*b^6*c^6 + 3840*a^4*b^4*c^7 - 6144*a^5*b^2*c^8)))^(
1/2) - (x*(B^2*b^6 + 8*A^2*a^2*c^4 + A^2*b^4*c^2 - 72*B^2*a^3*c^3 + 2*A*B*b
^5*c + 74*B^2*a^2*b^2*c^2 - 16*B^2*a*b^4*c + 2*A^2*a*b^2*c^3 - 14*A*B*a*b^3
*c^2 - 8*A*B*a^2*b*c^3))/(2*(b^4*c + 16*a^2*c^3 - 8*a*b^2*c^2)))*(-(B^2*b^1
1 + A^2*b^9*c^2 + A^2*c^2*(-(4*a*c - b^2)^9)^(1/2) + B^2*b^2*(-(4*a*c - b^2
)^9)^(1/2) + 2*A*B*b^10*c - 96*A^2*a^2*b^5*c^4 + 512*A^2*a^3*b^3*c^5 + 288*
B^2*a^2*b^7*c^2 - 1504*B^2*a^3*b^5*c^3 + 3840*B^2*a^4*b^3*c^4 + 3072*A*B*a^
5*c^6 - 27*B^2*a*b^9*c - 9*B^2*a*c*(-(4*a*c - b^2)^9)^(1/2) - 768*A^2*a^4*b
*c^6 - 3840*B^2*a^5*b*c^5 + 192*A*B*a^2*b^6*c^3 - 128*A*B*a^3*b^4*c^4 - 153
6*A*B*a^4*b^2*c^5 + 2*A*B*b*c*(-(4*a*c - b^2)^9)^(1/2) - 36*A*B*a*b^8*c^2)/
(32*(4096*a^6*c^9 + b^12*c^3 - 24*a*b^10*c^4 + 240*a^2*b^8*c^5 - 1280*a^3*b
^6*c^6 + 3840*a^4*b^4*c^7 - 6144*a^5*b^2*c^8)))^(1/2) - (3*A*B^2*a*b^5 - 21
6*B^3*a^4*c^2 - 5*B^3*a^2*b^4 - 24*A^2*B*a^3*c^3 + 3*A^3*a*b^3*c^2 + 4*A^3*
a^2*b*c^3 + 66*B^3*a^3*b^2*c - 51*A*B^2*a^2*b^3*c + 204*A*B^2*a^3*b*c^2 - 4
2*A^2*B*a^2*b^2*c^2 + 6*A^2*B*a*b^4*c)/(4*(b^6*c - 64*a^3*c^4 - 12*a*b^4*c^
2 + 48*a^2*b^2*c^3)) + (((2048*A*a^4*c^6 - 32*A*a*b^6*c^3 + 16*B*a*b^7*c^2
- 1024*B*a^4*b*c^5 + 384*A*a^2*b^4*c^4 - 1536*A*a^3*b^2*c^5 - 192*B*a^2*b^5
*c^3 + 768*B*a^3*b^3*c^4)/(8*(b^6*c - 64*a^3*c^4 - 12*a*b^4*c^2 + 48*a^2*b^
2*c^3)) + (x*(-(B^2*b^11 + A^2*b^9*c^2 + A^2*c^2*(-(4*a*c - b^2)^9)^(1/2) +

B^2*b^2*(-(4*a*c - b^2)^9)^(1/2) + 2*A*B*b^10*c - 96*A^2*a^2*b^5*c^4 + 512
*A^2*a^3*b^3*c^5 + 288*B^2*a^2*b^7*c^2 - 1504*B^2*a^3*b^5*c^3 + 3840*B^2*a^
4*b^3*c^4 + 3072*A*B*a^5*c^6 - 27*B^2*a*b^9*c - 9*B^2*a*c*(-(4*a*c - b^2)^9
)^(1/2) - 768*A^2*a^4*b*c^6 - 3840*B^2*a^5*b*c^5 + 192*A*B*a^2*b^6*c^3 - 12
8*A*B*a^3*b^4*c^4 - 1536*A*B*a^4*b^2*c^5 + 2*A*B*b*c*(-(4*a*c - b^2)^9)^(1/
2) - 36*A*B*a*b^8*c^2)/(32*(4096*a^6*c^9 + b^12*c^3 - 24*a*b^10*c^4 + 240*a
^2*b^8*c^5 - 1280*a^3*b^6*c^6 + 3840*a^4*b^4*c^7 - 6144*a^5*b^2*c^8)))^(1/2
)*(16*b^7*c^3 - 192*a*b^5*c^4 - 1024*a^3*b*c^6 + 768*a^2*b^3*c^5))/(2*(b^4*
c + 16*a^2*c^3 - 8*a*b^2*c^2)))*(-(B^2*b^11 + A^2*b^9*c^2 + A^2*c^2*(-(4*a*
c - b^2)^9)^(1/2) + B^2*b^2*(-(4*a*c - b^2)^9)^(1/2) + 2*A*B*b^10*c - 96*A^
2*a^2*b^5*c^4 + 512*A^2*a^3*b^3*c^5 + 288*B^2*a^2*b^7*c^2 - 1504*B^2*a^3*b^
5*c^3 + 3840*B^2*a^4*b^3*c^4 + 3072*A*B*a^5*c^6 - 27*B^2*a*b^9*c - 9*B^2*a*
c*(-(4*a*c - b^2)^9)^(1/2) - 768*A^2*a^4*b*c^6 - 3840*B^2*a^5*b*c^5 + 192*A
*B*a^2*b^6*c^3 - 128*A*B*a^3*b^4*c^4 - 1536*A*B*a^4*b^2*c^5 + 2*A*B*b*c*(-(
4*a*c - b^2)^9)^(1/2) - 36*A*B*a*b^8*c^2)/(32*(4096*a^6*c^9 + b^12*c^3 - 24
*a*b^10*c^4 + 240*a^2*b^8*c^5 - 1280*a^3*b^6*c^6 + 3840*a^4*b^4*c^7 - 6144*
a^5*b^2*c^8)))^(1/2) + (x*(B^2*b^6 + 8*A^2*a^2*c^4 + A^2*b^4*c^2 - 72*B^2*a
^3*c^3 + 2*A*B*b^5*c + 74*B^2*a^2*b^2*c^2 - 16*B^2*a*b^4*c + 2*A^2*a*b^2*c^
3 - 14*A*B*a*b^3*c^2 - 8*A*B*a^2*b*c^3))/(2*(b^4*c + 16*a^2*c^3 - 8*a*b^2*c
^2)))*(-(B^2*b^11 + A^2*b^9*c^2 + A^2*c^2*(-(4*a*c - b^2)^9)^(1/2) + B^2*b^
2*(-(4*a*c - b^2)^9)^(1/2) + 2*A*B*b^10*c - 96*A^2*a^2*b^5*c^4 + 512*A^2*a^
3*b^3*c^5 + 288*B^2*a^2*b^7*c^2 - 1504*B^2*a^3*b^5*c^3 + 3840*B^2*a^4*b^3*c
^4 + 3072*A*B*a^5*c^6 - 27*B^2*a*b^9*c - 9*B^2*a*c*(-(4*a*c - b^2)^9)^(1/2)
 - 768*A^2*a^4*b*c^6 - 3840*B^2*a^5*b*c^5 + 192*A*B*a^2*b^6*c^3 - 128*A*B*a
^3*b^4*c^4 - 1536*A*B*a^4*b^2*c^5 + 2*A*B*b*c*(-(4*a*c - b^2)^9)^(1/2) - 36
*A*B*a*b^8*c^2)/(32*(4096*a^6*c^9 + b^12*c^3 - 24*a*b^10*c^4 + 240*a^2*b^8*
c^5 - 1280*a^3*b^6*c^6 + 3840*a^4*b^4*c^7 - 6144*a^5*b^2*c^8)))^(1/2)))*(-(
B^2*b^11 + A^2*b^9*c^2 + A^2*c^2*(-(4*a*c - b^2)^9)^(1/2) + B^2*b^2*(-(4*a*
c - b^2)^9)^(1/2) + 2*A*B*b^10*c - 96*A^2*a^2*b^5*c^4 + 512*A^2*a^3*b^3*c^5
 + 288*B^2*a^2*b^7*c^2 - 1504*B^2*a^3*b^5*c^3 + 3840*B^2*a^4*b^3*c^4 + 3072
*A*B*a^5*c^6 - 27*B^2*a*b^9*c - 9*B^2*a*c*(-(4*a*c - b^2)^9)^(1/2) - 768*A^
2*a^4*b*c^6 - 3840*B^2*a^5*b*c^5 + 192*A*B*a^2*b^6*c^3 - 128*A*B*a^3*b^4*c^
4 - 1536*A*B*a^4*b^2*c^5 + 2*A*B*b*c*(-(4*a*c - b^2)^9)^(1/2) - 36*A*B*a*b^
8*c^2)/(32*(4096*a^6*c^9 + b^12*c^3 - 24*a*b^10*c^4 + 240*a^2*b^8*c^5 - 128
0*a^3*b^6*c^6 + 3840*a^4*b^4*c^7 - 6144*a^5*b^2*c^8)))^(1/2)*2i - atan(((((
2048*A*a^4*c^6 - 32*A*a*b^6*c^3 + 16*B*a*b^7*c^2 - 1024*B*a^4*b*c^5 + 384*A
*a^2*b^4*c^4 - 1536*A*a^3*b^2*c^5 - 192*B*a^2*b^5*c^3 + 768*B*a^3*b^3*c^4)/
(8*(b^6*c - 64*a^3*c^4 - 12*a*b^4*c^2 + 48*a^2*b^2*c^3)) - (x*((A^2*c^2*(-(
4*a*c - b^2)^9)^(1/2) - A^2*b^9*c^2 - B^2*b^11 + B^2*b^2*(-(4*a*c - b^2)^9)
^(1/2) - 2*A*B*b^10*c + 96*A^2*a^2*b^5*c^4 - 512*A^2*a^3*b^3*c^5 - 288*B^2*
a^2*b^7*c^2 + 1504*B^2*a^3*b^5*c^3 - 3840*B^2*a^4*b^3*c^4 - 3072*A*B*a^5*c^
6 + 27*B^2*a*b^9*c - 9*B^2*a*c*(-(4*a*c - b^2)^9)^(1/2) + 768*A^2*a^4*b*c^6
 + 3840*B^2*a^5*b*c^5 - 192*A*B*a^2*b^6*c^3 + 128*A*B*a^3*b^4*c^4 + 1536*A*
B*a^4*b^2*c^5 + 2*A*B*b*c*(-(4*a*c - b^2)^9)^(1/2) + 36*A*B*a*b^8*c^2)/(32*
(4096*a^6*c^9 + b^12*c^3 - 24*a*b^10*c^4 + 240*a^2*b^8*c^5 - 1280*a^3*b^6*c

```
^6 + 3840*a^4*b^4*c^7 - 6144*a^5*b^2*c^8)))^(1/2)*(16*b^7*c^3 - 192*a*b^5*c
^4 - 1024*a^3*b*c^6 + 768*a^2*b^3*c^5))/(2*(b^4*c + 16*a^2*c^3 - 8*a*b^2*c^
2)))*((A^2*c^2*(-(4*a*c - b^2)^9)^(1/2) - A^2*b^9*c^2 - B^2*b^11 + B^2*b^2*
(-(4*a*c - b^2)^9)^(1/2) - 2*A*B*b^10*c + 96*A^2*a^2*b^5*c^4 - 512*A^2*a^3*
b^3*c^5 - 288*B^2*a^2*b^7*c^2 + 1504*B^2*a^3*b^5*c^3 - 3840*B^2*a^4*b^3*c^4
 - 3072*A*B*a^5*c^6 + 27*B^2*a*b^9*c - 9*B^2*a*c*(-(4*a*c - b^2)^9)^(1/2) +
 768*A^2*a^4*b*c^6 + 3840*B^2*a^5*b*c^5 - 192*A*B*a^2*b^6*c^3 + 128*A*B*a^3
*b^4*c^4 + 1536*A*B*a^4*b^2*c^5 + 2*A*B*b*c*(-(4*a*c - b^2)^9)^(1/2) + 36*A
*B*a*b^8*c^2)/(32*(4096*a^6*c^9 + b^12*c^3 - 24*a*b^10*c^4 + 240*a^2*b^8*c^
5 - 1280*a^3*b^6*c^6 + 3840*a^4*b^4*c^7 - 6144*a^5*b^2*c^8)))^(1/2) - (x*(B
^2*b^6 + 8*A^2*a^2*c^4 + A^2*b^4*c^2 - 72*B^2*a^3*c^3 + 2*A*B*b^5*c + 74*B^
2*a^2*b^2*c^2 - 16*B^2*a*b^4*c + 2*A^2*a*b^2*c^3 - 14*A*B*a*b^3*c^2 - 8*A*B
*a^2*b*c^3))/(2*(b^4*c + 16*a^2*c^3 - 8*a*b^2*c^2)))*((A^2*c^2*(-(4*a*c - b
^2)^9)^(1/2) - A^2*b^9*c^2 - B^2*b^11 + B^2*b^2*(-(4*a*c - b^2)^9)^(1/2) -
2*A*B*b^10*c + 96*A^2*a^2*b^5*c^4 - 512*A^2*a^3*b^3*c^5 - 288*B^2*a^2*b^7*c
^2 + 1504*B^2*a^3*b^5*c^3 - 3840*B^2*a^4*b^3*c^4 - 3072*A*B*a^5*c^6 + 27*B^
2*a*b^9*c - 9*B^2*a*c*(-(4*a*c - b^2)^9)^(1/2) + 768*A^2*a^4*b*c^6 + 3840*B
^2*a^5*b*c^5 - 192*A*B*a^2*b^6*c^3 + 128*A*B*a^3*b^4*c^4 + 1536*A*B*a^4*b^2
*c^5 + 2*A*B*b*c*(-(4*a*c - b^2)^9)^(1/2) + 36*A*B*a*b^8*c^2)/(32*(4096*a^6
*c^9 + b^12*c^3 - 24*a*b^10*c^4 + 240*a^2*b^8*c^5 - 1280*a^3*b^6*c^6 + 3840
*a^4*b^4*c^7 - 6144*a^5*b^2*c^8)))^(1/2)*1i - (((2048*A*a^4*c^6 - 32*A*a*b^
6*c^3 + 16*B*a*b^7*c^2 - 1024*B*a^4*b*c^5 + 384*A*a^2*b^4*c^4 - 1536*A*a^3*
b^2*c^5 - 192*B*a^2*b^5*c^3 + 768*B*a^3*b^3*c^4)/(8*(b^6*c - 64*a^3*c^4 - 1
2*a*b^4*c^2 + 48*a^2*b^2*c^3)) + (x*((A^2*c^2*(-(4*a*c - b^2)^9)^(1/2) - A^
2*b^9*c^2 - B^2*b^11 + B^2*b^2*(-(4*a*c - b^2)^9)^(1/2) - 2*A*B*b^10*c + 96
*A^2*a^2*b^5*c^4 - 512*A^2*a^3*b^3*c^5 - 288*B^2*a^2*b^7*c^2 + 1504*B^2*a^3
*b^5*c^3 - 3840*B^2*a^4*b^3*c^4 - 3072*A*B*a^5*c^6 + 27*B^2*a*b^9*c - 9*B^2
*a*c*(-(4*a*c - b^2)^9)^(1/2) + 768*A^2*a^4*b*c^6 + 3840*B^2*a^5*b*c^5 - 19
2*A*B*a^2*b^6*c^3 + 128*A*B*a^3*b^4*c^4 + 1536*A*B*a^4*b^2*c^5 + 2*A*B*b*c*
(-(4*a*c - b^2)^9)^(1/2) + 36*A*B*a*b^8*c^2)/(32*(4096*a^6*c^9 + b^12*c^3 -
 24*a*b^10*c^4 + 240*a^2*b^8*c^5 - 1280*a^3*b^6*c^6 + 3840*a^4*b^4*c^7 - 61
44*a^5*b^2*c^8)))^(1/2)*(16*b^7*c^3 - 192*a*b^5*c^4 - 1024*a^3*b*c^6 + 768*
a^2*b^3*c^5))/(2*(b^4*c + 16*a^2*c^3 - 8*a*b^2*c^2)))*((A^2*c^2*(-(4*a*c -
b^2)^9)^(1/2) - A^2*b^9*c^2 - B^2*b^11 + B^2*b^2*(-(4*a*c - b^2)^9)^(1/2) -
 2*A*B*b^10*c + 96*A^2*a^2*b^5*c^4 - 512*A^2*a^3*b^3*c^5 - 288*B^2*a^2*b^7*
c^2 + 1504*B^2*a^3*b^5*c^3 - 3840*B^2*a^4*b^3*c^4 - 3072*A*B*a^5*c^6 + 27*B
^2*a*b^9*c - 9*B^2*a*c*(-(4*a*c - b^2)^9)^(1/2) + 768*A^2*a^4*b*c^6 + 3840*
B^2*a^5*b*c^5 - 192*A*B*a^2*b^6*c^3 + 128*A*B*a^3*b^4*c^4 + 1536*A*B*a^4*b^
2*c^5 + 2*A*B*b*c*(-(4*a*c - b^2)^9)^(1/2) + 36*A*B*a*b^8*c^2)/(32*(4096*a^
6*c^9 + b^12*c^3 - 24*a*b^10*c^4 + 240*a^2*b^8*c^5 - 1280*a^3*b^6*c^6 + 384
0*a^4*b^4*c^7 - 6144*a^5*b^2*c^8)))^(1/2) + (x*(B^2*b^6 + 8*A^2*a^2*c^4 + A
^2*b^4*c^2 - 72*B^2*a^3*c^3 + 2*A*B*b^5*c + 74*B^2*a^2*b^2*c^2 - 16*B^2*a*b
^4*c + 2*A^2*a*b^2*c^3 - 14*A*B*a*b^3*c^2 - 8*A*B*a^2*b*c^3))/(2*(b^4*c + 1
6*a^2*c^3 - 8*a*b^2*c^2)))*((A^2*c^2*(-(4*a*c - b^2)^9)^(1/2) - A^2*b^9*c^2
 - B^2*b^11 + B^2*b^2*(-(4*a*c - b^2)^9)^(1/2) - 2*A*B*b^10*c + 96*A^2*a^2*
```

b^5*c^4 - 512*A^2*a^3*b^3*c^5 - 288*B^2*a^2*b^7*c^2 + 1504*B^2*a^3*b^5*c^3
- 3840*B^2*a^4*b^3*c^4 - 3072*A*B*a^5*c^6 + 27*B^2*a*b^9*c - 9*B^2*a*c*(-(4
*a*c - b^2)^9)^(1/2) + 768*A^2*a^4*b*c^6 + 3840*B^2*a^5*b*c^5 - 192*A*B*a^2
*b^6*c^3 + 128*A*B*a^3*b^4*c^4 + 1536*A*B*a^4*b^2*c^5 + 2*A*B*b*c*(-(4*a*c
- b^2)^9)^(1/2) + 36*A*B*a*b^8*c^2)/(32*(4096*a^6*c^9 + b^12*c^3 - 24*a*b^1
0*c^4 + 240*a^2*b^8*c^5 - 1280*a^3*b^6*c^6 + 3840*a^4*b^4*c^7 - 6144*a^5*b^
2*c^8)))^(1/2)*1i)/((((2048*A*a^4*c^6 - 32*A*a*b^6*c^3 + 16*B*a*b^7*c^2 - 1
024*B*a^4*b*c^5 + 384*A*a^2*b^4*c^4 - 1536*A*a^3*b^2*c^5 - 192*B*a^2*b^5*c^
3 + 768*B*a^3*b^3*c^4)/(8*(b^6*c - 64*a^3*c^4 - 12*a*b^4*c^2 + 48*a^2*b^2*c
^3)) - (x*((A^2*c^2*(-(4*a*c - b^2)^9)^(1/2) - A^2*b^9*c^2 - B^2*b^11 + B^2
b^2(-(4*a*c - b^2)^9)^(1/2) - 2*A*B*b^10*c + 96*A^2*a^2*b^5*c^4 - 512*A^2
*a^3*b^3*c^5 - 288*B^2*a^2*b^7*c^2 + 1504*B^2*a^3*b^5*c^3 - 3840*B^2*a^4*b^
3*c^4 - 3072*A*B*a^5*c^6 + 27*B^2*a*b^9*c - 9*B^2*a*c*(-(4*a*c - b^2)^9)^(1
/2) + 768*A^2*a^4*b*c^6 + 3840*B^2*a^5*b*c^5 - 192*A*B*a^2*b^6*c^3 + 128*A*
B*a^3*b^4*c^4 + 1536*A*B*a^4*b^2*c^5 + 2*A*B*b*c*(-(4*a*c - b^2)^9)^(1/2) +
 36*A*B*a*b^8*c^2)/(32*(4096*a^6*c^9 + b^12*c^3 - 24*a*b^10*c^4 + 240*a^2*b
^8*c^5 - 1280*a^3*b^6*c^6 + 3840*a^4*b^4*c^7 - 6144*a^5*b^2*c^8)))^(1/2)*(1
6*b^7*c^3 - 192*a*b^5*c^4 - 1024*a^3*b*c^6 + 768*a^2*b^3*c^5))/(2*(b^4*c +
16*a^2*c^3 - 8*a*b^2*c^2)))*((A^2*c^2*(-(4*a*c - b^2)^9)^(1/2) - A^2*b^9*c^
2 - B^2*b^11 + B^2*b^2*(-(4*a*c - b^2)^9)^(1/2) - 2*A*B*b^10*c + 96*A^2*a^2
*b^5*c^4 - 512*A^2*a^3*b^3*c^5 - 288*B^2*a^2*b^7*c^2 + 1504*B^2*a^3*b^5*c^3
 - 3840*B^2*a^4*b^3*c^4 - 3072*A*B*a^5*c^6 + 27*B^2*a*b^9*c - 9*B^2*a*c*(-(
4*a*c - b^2)^9)^(1/2) + 768*A^2*a^4*b*c^6 + 3840*B^2*a^5*b*c^5 - 192*A*B*a^
2*b^6*c^3 + 128*A*B*a^3*b^4*c^4 + 1536*A*B*a^4*b^2*c^5 + 2*A*B*b*c*(-(4*a*c
 - b^2)^9)^(1/2) + 36*A*B*a*b^8*c^2)/(32*(4096*a^6*c^9 + b^12*c^3 - 24*a*b^
10*c^4 + 240*a^2*b^8*c^5 - 1280*a^3*b^6*c^6 + 3840*a^4*b^4*c^7 - 6144*a^5*b
^2*c^8)))^(1/2) - (x*(B^2*b^6 + 8*A^2*a^2*c^4 + A^2*b^4*c^2 - 72*B^2*a^3*c^
3 + 2*A*B*b^5*c + 74*B^2*a^2*b^2*c^2 - 16*B^2*a*b^4*c + 2*A^2*a*b^2*c^3 - 1
4*A*B*a*b^3*c^2 - 8*A*B*a^2*b*c^3))/(2*(b^4*c + 16*a^2*c^3 - 8*a*b^2*c^2)))
*((A^2*c^2*(-(4*a*c - b^2)^9)^(1/2) - A^2*b^9*c^2 - B^2*b^11 + B^2*b^2*(-(4
*a*c - b^2)^9)^(1/2) - 2*A*B*b^10*c + 96*A^2*a^2*b^5*c^4 - 512*A^2*a^3*b^3*
c^5 - 288*B^2*a^2*b^7*c^2 + 1504*B^2*a^3*b^5*c^3 - 3840*B^2*a^4*b^3*c^4 - 3
072*A*B*a^5*c^6 + 27*B^2*a*b^9*c - 9*B^2*a*c*(-(4*a*c - b^2)^9)^(1/2) + 768
*A^2*a^4*b*c^6 + 3840*B^2*a^5*b*c^5 - 192*A*B*a^2*b^6*c^3 + 128*A*B*a^3*b^4
*c^4 + 1536*A*B*a^4*b^2*c^5 + 2*A*B*b*c*(-(4*a*c - b^2)^9)^(1/2) + 36*A*B*a
*b^8*c^2)/(32*(4096*a^6*c^9 + b^12*c^3 - 24*a*b^10*c^4 + 240*a^2*b^8*c^5 -
1280*a^3*b^6*c^6 + 3840*a^4*b^4*c^7 - 6144*a^5*b^2*c^8)))^(1/2) - (3*A*B^2*
a*b^5 - 216*B^3*a^4*c^2 - 5*B^3*a^2*b^4 - 24*A^2*B*a^3*c^3 + 3*A^3*a*b^3*c^
2 + 4*A^3*a^2*b*c^3 + 66*B^3*a^3*b^2*c - 51*A*B^2*a^2*b^3*c + 204*A*B^2*a^3
*b*c^2 - 42*A^2*B*a^2*b^2*c^2 + 6*A^2*B*a*b^4*c)/(4*(b^6*c - 64*a^3*c^4 - 1
2*a*b^4*c^2 + 48*a^2*b^2*c^3)) + (((2048*A*a^4*c^6 - 32*A*a*b^6*c^3 + 16*B*
a*b^7*c^2 - 1024*B*a^4*b*c^5 + 384*A*a^2*b^4*c^4 - 1536*A*a^3*b^2*c^5 - 192
*B*a^2*b^5*c^3 + 768*B*a^3*b^3*c^4)/(8*(b^6*c - 64*a^3*c^4 - 12*a*b^4*c^2 +
 48*a^2*b^2*c^3)) + (x*((A^2*c^2*(-(4*a*c - b^2)^9)^(1/2) - A^2*b^9*c^2 - B
^2*b^11 + B^2*b^2*(-(4*a*c - b^2)^9)^(1/2) - 2*A*B*b^10*c + 96*A^2*a^2*b^5*

c^4 - 512*A^2*a^3*b^3*c^5 - 288*B^2*a^2*b^7*c^2 + 1504*B^2*a^3*b^5*c^3 - 3840*B^2*a^4*b^3*c^4 - 3072*A*B*a^5*c^6 + 27*B^2*a*b^9*c - 9*B^2*a*c*(-(4*a*c - b^2)^9)^(1/2) + 768*A^2*a^4*b*c^6 + 3840*B^2*a^5*b*c^5 - 192*A*B*a^2*b^6*c^3 + 128*A*B*a^3*b^4*c^4 + 1536*A*B*a^4*b^2*c^5 + 2*A*B*b*c*(-(4*a*c - b^2)^9)^(1/2) + 36*A*B*a*b^8*c^2)/(32*(4096*a^6*c^9 + b^12*c^3 - 24*a*b^10*c^4 + 240*a^2*b^8*c^5 - 1280*a^3*b^6*c^6 + 3840*a^4*b^4*c^7 - 6144*a^5*b^2*c^8)))^(1/2)*(16*b^7*c^3 - 192*a*b^5*c^4 - 1024*a^3*b*c^6 + 768*a^2*b^3*c^5))/(2*(b^4*c + 16*a^2*c^3 - 8*a*b^2*c^2)))*((A^2*c^2*(-(4*a*c - b^2)^9)^(1/2) - A^2*b^9*c^2 - B^2*b^11 + B^2*b^2*(-(4*a*c - b^2)^9)^(1/2) - 2*A*B*b^10*c + 96*A^2*a^2*b^5*c^4 - 512*A^2*a^3*b^3*c^5 - 288*B^2*a^2*b^7*c^2 + 1504*B^2*a^3*b^5*c^3 - 3840*B^2*a^4*b^3*c^4 - 3072*A*B*a^5*c^6 + 27*B^2*a*b^9*c - 9*B^2*a*c*(-(4*a*c - b^2)^9)^(1/2) + 768*A^2*a^4*b*c^6 + 3840*B^2*a^5*b*c^5 - 192*A*B*a^2*b^6*c^3 + 128*A*B*a^3*b^4*c^4 + 1536*A*B*a^4*b^2*c^5 + 2*A*B*b*c*(-(4*a*c - b^2)^9)^(1/2) + 36*A*B*a*b^8*c^2)/(32*(4096*a^6*c^9 + b^12*c^3 - 24*a*b^10*c^4 + 240*a^2*b^8*c^5 - 1280*a^3*b^6*c^6 + 3840*a^4*b^4*c^7 - 6144*a^5*b^2*c^8)))^(1/2) + (x*(B^2*b^6 + 8*A^2*a^2*c^4 + A^2*b^4*c^2 - 72*B^2*a^3*c^3 + 2*A*B*b^5*c + 74*B^2*a^2*b^2*c^2 - 16*B^2*a*b^4*c + 2*A^2*a*b^2*c^3 - 14*A*B*a*b^3*c^2 - 8*A*B*a^2*b*c^3))/(2*(b^4*c + 16*a^2*c^3 - 8*a*b^2*c^2)))*((A^2*c^2*(-(4*a*c - b^2)^9)^(1/2) - A^2*b^9*c^2 - B^2*b^11 + B^2*b^2*(-(4*a*c - b^2)^9)^(1/2) - 2*A*B*b^10*c + 96*A^2*a^2*b^5*c^4 - 512*A^2*a^3*b^3*c^5 - 288*B^2*a^2*b^7*c^2 + 1504*B^2*a^3*b^5*c^3 - 3840*B^2*a^4*b^3*c^4 - 3072*A*B*a^5*c^6 + 27*B^2*a*b^9*c - 9*B^2*a*c*(-(4*a*c - b^2)^9)^(1/2) + 768*A^2*a^4*b*c^6 + 3840*B^2*a^5*b*c^5 - 192*A*B*a^2*b^6*c^3 + 128*A*B*a^3*b^4*c^4 + 1536*A*B*a^4*b^2*c^5 + 2*A*B*b*c*(-(4*a*c - b^2)^9)^(1/2) + 36*A*B*a*b^8*c^2)/(32*(4096*a^6*c^9 + b^12*c^3 - 24*a*b^10*c^4 + 240*a^2*b^8*c^5 - 1280*a^3*b^6*c^6 + 3840*a^4*b^4*c^7 - 6144*a^5*b^2*c^8)))^(1/2)))*((A^2*c^2*(-(4*a*c - b^2)^9)^(1/2) - A^2*b^9*c^2 - B^2*b^11 + B^2*b^2*(-(4*a*c - b^2)^9)^(1/2) - 2*A*B*b^10*c + 96*A^2*a^2*b^5*c^4 - 512*A^2*a^3*b^3*c^5 - 288*B^2*a^2*b^7*c^2 + 1504*B^2*a^3*b^5*c^3 - 3840*B^2*a^4*b^3*c^4 - 3072*A*B*a^5*c^6 + 27*B^2*a*b^9*c - 9*B^2*a*c*(-(4*a*c - b^2)^9)^(1/2) + 768*A^2*a^4*b*c^6 + 3840*B^2*a^5*b*c^5 - 192*A*B*a^2*b^6*c^3 + 128*A*B*a^3*b^4*c^4 + 1536*A*B*a^4*b^2*c^5 + 2*A*B*b*c*(-(4*a*c - b^2)^9)^(1/2) + 36*A*B*a*b^8*c^2)/(32*(4096*a^6*c^9 + b^12*c^3 - 24*a*b^10*c^4 + 240*a^2*b^8*c^5 - 1280*a^3*b^6*c^6 + 3840*a^4*b^4*c^7 - 6144*a^5*b^2*c^8)))^(1/2)*2i

3.120 $\int \frac{x^2(A+Bx^2)}{(a+bx^2+cx^4)^2} dx$

Optimal result

Integrand size = 25, antiderivative size = 276

$$\int \frac{x^2(A+Bx^2)}{(a+bx^2+cx^4)^2} dx = -\frac{x(Ab-2aB-(bB-2Ac)x^2)}{2(b^2-4ac)(a+bx^2+cx^4)} + \frac{\left(bB-2Ac-\frac{b^2B-4Abc+4aBc}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}\sqrt{c}(b^2-4ac)\sqrt{b-\sqrt{b^2-4ac}}} + \frac{\left(bB-2Ac+\frac{b^2B-4Abc+4aBc}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}\sqrt{c}(b^2-4ac)\sqrt{b+\sqrt{b^2-4ac}}}$$

```
[Out] -1/2*x*(A*b-2*B*a-(-2*A*c+B*b)*x^2)/(-4*a*c+b^2)/(c*x^4+b*x^2+a)+1/4*arctan
(x*2^(1/2)*c^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2))*(B*b-2*A*c+(4*A*b*c-4*B*a*
c-B*b^2)/(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)*2^(1/2)/c^(1/2)/(b-(-4*a*c+b^2)^(
1/2))^(1/2)+1/4*arctan(x*2^(1/2)*c^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2))*(B*b
-2*A*c+(-4*A*b*c+4*B*a*c+B*b^2)/(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)*2^(1/2)/c^
(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2)
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 276, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used

$= \{1289, 1180, 211\}$

$$\int \frac{x^2(A+Bx^2)}{(a+bx^2+cx^4)^2}\,dx = \frac{\left(-\frac{4aBc-4Abc+b^2B}{\sqrt{b^2-4ac}}-2Ac+bB\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}\sqrt{c}\,(b^2-4ac)\sqrt{b-\sqrt{b^2-4ac}}} + \frac{\left(\frac{4aBc-4Abc+b^2B}{\sqrt{b^2-4ac}}-2Ac+bB\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{2\sqrt{2}\sqrt{c}\,(b^2-4ac)\sqrt{\sqrt{b^2-4ac}+b}} - \frac{x(-2aB-(x^2(bB-2Ac))+Ab)}{2\,(b^2-4ac)\,(a+bx^2+cx^4)}$$

```
[In] Int[(x^2*(A + B*x^2))/(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] -1/2*(x*(A*b - 2*a*B - (b*B - 2*A*c)*x^2))/((b^2 - 4*a*c)*(a + b*x^2 + c*x^
4)) + ((b*B - 2*A*c - (b^2*B - 4*A*b*c + 4*a*B*c)/Sqrt[b^2 - 4*a*c])*ArcTan
[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]*Sqrt[c]*(b^2
- 4*a*c)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + ((b*B - 2*A*c + (b^2*B - 4*A*b*c +
4*a*B*c)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 -
4*a*c]]])/(2*Sqrt[2]*Sqrt[c]*(b^2 - 4*a*c)*Sqrt[b + Sqrt[b^2 - 4*a*c]])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1289

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_.), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(a + b*x^2 + c*x^4)^(p + 1)
*((b*d - 2*a*e - (b*e - 2*c*d)*x^2)/(2*(p + 1)*(b^2 - 4*a*c))), x] - Dist[f
^2/(2*(p + 1)*(b^2 - 4*a*c)), Int[(f*x)^(m - 2)*(a + b*x^2 + c*x^4)^(p + 1)
*Simp[(m - 1)*(b*d - 2*a*e) - (4*p + 4 + m + 1)*(b*e - 2*c*d)*x^2, x], x],
x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] &&
GtQ[m, 1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rubi steps

$$\text{integral} = -\frac{x(Ab-2aB-(bB-2Ac)x^2)}{2(b^2-4ac)(a+bx^2+cx^4)} + \frac{\int \frac{Ab-2aB+(bB-2Ac)x^2}{a+bx^2+cx^4}\,dx}{2(b^2-4ac)}$$

$$= -\frac{x(Ab-2aB-(bB-2Ac)x^2)}{2(b^2-4ac)(a+bx^2+cx^4)} + \frac{\left(bB-2Ac-\frac{b^2B-4Abc+4aBc}{\sqrt{b^2-4ac}}\right)\int \frac{1}{\frac{b}{2}-\frac{1}{2}\sqrt{b^2-4ac}+cx^2}\,dx}{4(b^2-4ac)} + \frac{\left(bB-2Ac+\frac{b^2B-4Abc+4aBc}{\sqrt{b^2-4ac}}\right)\int \frac{1}{\frac{b}{2}+\frac{1}{2}\sqrt{b^2-4ac}+cx^2}\,dx}{4(b^2-4ac)}$$

$$= -\frac{x(Ab-2aB-(bB-2Ac)x^2)}{2(b^2-4ac)(a+bx^2+cx^4)} + \frac{\left(bB-2Ac-\frac{b^2B-4Abc+4aBc}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}\sqrt{c}(b^2-4ac)\sqrt{b-\sqrt{b^2-4ac}}} + \frac{\left(bB-2Ac+\frac{b^2B-4Abc+4aBc}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}\sqrt{c}(b^2-4ac)\sqrt{b+\sqrt{b^2-4ac}}}$$

Mathematica [A] (verified)

Time = 0.42 (sec) , antiderivative size = 298, normalized size of antiderivative = 1.08

$$\int \frac{x^2(A+Bx^2)}{(a+bx^2+cx^4)^2}\,dx = \frac{1}{4}\left(\frac{2x(B(2a+bx^2)-A(b+2cx^2))}{(b^2-4ac)(a+bx^2+cx^4)} + \frac{\sqrt{2}\left(-b^2B+4Abc-4aBc+bB\sqrt{b^2-4ac}-2Ac\sqrt{b^2-4ac}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{c}(b^2-4ac)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}} + \frac{\sqrt{2}\left(b^2B-4Abc+4aBc+bB\sqrt{b^2-4ac}-2Ac\sqrt{b^2-4ac}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{c}(b^2-4ac)^{3/2}\sqrt{b+\sqrt{b^2-4ac}}}\right)$$

```
[In] Integrate[(x^2*(A + B*x^2))/(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] ((2*x*(B*(2*a + b*x^2) - A*(b + 2*c*x^2)))/((b^2 - 4*a*c)*(a + b*x^2 + c*x^
4)) + (Sqrt[2]*(-(b^2*B) + 4*A*b*c - 4*a*B*c + b*B*Sqrt[b^2 - 4*a*c] - 2*A*
c*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]
])/(Sqrt[c]*(b^2 - 4*a*c)^(3/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (Sqrt[2]*(b^
```

```
2*B - 4*A*b*c + 4*a*B*c + b*B*Sqrt[b^2 - 4*a*c] - 2*A*c*Sqrt[b^2 - 4*a*c])*
ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[c]*(b^2 - 4*
a*c)^(3/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]]))/4
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.11 (sec) , antiderivative size = 152, normalized size of antiderivative = 0.55

method	result
risch	$\frac{\frac{(2Ac-Bb)x^3}{8ac-2b^2}+\frac{(Ab-2Ba)x}{8ac-2b^2}}{c\,x^4+b\,x^2+a}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(c\_Z^4+\_Z^2b+a\right)}\frac{\left(\frac{(2Ac-Bb)\_R^2}{4ac-b^2}-\frac{Ab-2Ba}{4ac-b^2}\right)\ln\left(x-\_R\right)}{2c\_R^3+\_Rb}\right)}{4}$
default	$\frac{\frac{(2Ac-Bb)x^3}{8ac-2b^2}+\frac{(Ab-2Ba)x}{8ac-2b^2}}{c\,x^4+b\,x^2+a}+\frac{2c\left(\frac{\left(2Ac\sqrt{-4ac+b^2}+4Abc-Bb\sqrt{-4ac+b^2}-4Bac-B\,b^2\right)\sqrt{2}\,\arctan\left(\frac{cx\sqrt{2}}{\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}}\right)}{8c\sqrt{-4ac+b^2}\,\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}}-\frac{\left(2Ac\sqrt{-4ac+b^2}-4A\cdots\right.}{\cdots}\right.}{4ac-b^2\cdots}$

```
[In] int(x^2*(B*x^2+A)/(c*x^4+b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] (1/2*(2*A*c-B*b)/(4*a*c-b^2)*x^3+1/2*(A*b-2*B*a)/(4*a*c-b^2)*x)/(c*x^4+b*x^
2+a)+1/4*sum(((2*A*c-B*b)/(4*a*c-b^2)*_R^2-(A*b-2*B*a)/(4*a*c-b^2))/(2*_R^3
*c+_R*b)*ln(x-_R),_R=RootOf(_Z^4*c+_Z^2*b+a))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3467 vs. 2(234) = 468.

Time = 1.16 (sec) , antiderivative size = 3467, normalized size of antiderivative = 12.56

$$\int \frac{x^2(A+Bx^2)}{(a+bx^2+cx^4)^2}\,dx = \text{Too large to display}$$

```
[In] integrate(x^2*(B*x^2+A)/(c*x^4+b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] 1/4*(2*(B*b - 2*A*c)*x^3 - sqrt(1/2)*((b^2*c - 4*a*c^2)*x^4 + a*b^2 - 4*a^2
*c + (b^3 - 4*a*b*c)*x^2)*sqrt(-(B^2*a*b^3 - 4*(4*A*B*a^2 - 3*A^2*a*b)*c^2
+ (12*B^2*a^2*b - 12*A*B*a*b^2 + A^2*b^3)*c + (a*b^6*c - 12*a^2*b^4*c^2 + 4
8*a^3*b^2*c^3 - 64*a^4*c^4)*sqrt((B^4*a^2 - 2*A^2*B^2*a*c + A^4*c^2)/(a^2*b
^6*c^2 - 12*a^3*b^4*c^3 + 48*a^4*b^2*c^4 - 64*a^5*c^5)))/(a*b^6*c - 12*a^2*
b^4*c^2 + 48*a^3*b^2*c^3 - 64*a^4*c^4))*log(-(3*B^4*a^2*b^2 - A*B^3*a*b^3 -
 4*A^4*a*c^3 + 3*(4*A^3*B*a*b - A^4*b^2)*c^2 + (4*B^4*a^3 - 12*A*B^3*a^2*b
+ A^3*B*b^3)*c)*x + 1/2*sqrt(1/2)*(2*B^3*a^2*b^4 - A*B^2*a*b^5 - 16*(2*A^2*
```

```
B*a^3 - A^3*a^2*b)*c^3 + 8*(4*B^3*a^4 - 2*A*B^2*a^3*b + 2*A^2*B*a^2*b^2 - A
^3*a*b^3)*c^2 - (16*B^3*a^3*b^2 - 8*A*B^2*a^2*b^3 + 2*A^2*B*a*b^4 - A^3*b^5
)*c + (192*B*a^4*b^3*c^3 + 256*A*a^5*c^5 - 128*(2*B*a^5*b + A*a^4*b^2)*c^4
- 8*(6*B*a^3*b^5 - A*a^2*b^6)*c^2 + (4*B*a^2*b^7 - A*a*b^8)*c)*sqrt((B^4*a^
2 - 2*A^2*B^2*a*c + A^4*c^2)/(a^2*b^6*c^2 - 12*a^3*b^4*c^3 + 48*a^4*b^2*c^4
 - 64*a^5*c^5)))*sqrt(-(B^2*a*b^3 - 4*(4*A*B*a^2 - 3*A^2*a*b)*c^2 + (12*B^2
*a^2*b - 12*A*B*a*b^2 + A^2*b^3)*c + (a*b^6*c - 12*a^2*b^4*c^2 + 48*a^3*b^2
*c^3 - 64*a^4*c^4)*sqrt((B^4*a^2 - 2*A^2*B^2*a*c + A^4*c^2)/(a^2*b^6*c^2 -
12*a^3*b^4*c^3 + 48*a^4*b^2*c^4 - 64*a^5*c^5)))/(a*b^6*c - 12*a^2*b^4*c^2 +
 48*a^3*b^2*c^3 - 64*a^4*c^4))) + sqrt(1/2)*((b^2*c - 4*a*c^2)*x^4 + a*b^2
- 4*a^2*c + (b^3 - 4*a*b*c)*x^2)*sqrt(-(B^2*a*b^3 - 4*(4*A*B*a^2 - 3*A^2*a*
b)*c^2 + (12*B^2*a^2*b - 12*A*B*a*b^2 + A^2*b^3)*c + (a*b^6*c - 12*a^2*b^4*
c^2 + 48*a^3*b^2*c^3 - 64*a^4*c^4)*sqrt((B^4*a^2 - 2*A^2*B^2*a*c + A^4*c^2)
/(a^2*b^6*c^2 - 12*a^3*b^4*c^3 + 48*a^4*b^2*c^4 - 64*a^5*c^5)))/(a*b^6*c -
12*a^2*b^4*c^2 + 48*a^3*b^2*c^3 - 64*a^4*c^4))*log(-(3*B^4*a^2*b^2 - A*B^3*
a*b^3 - 4*A^4*a*c^3 + 3*(4*A^3*B*a*b - A^4*b^2)*c^2 + (4*B^4*a^3 - 12*A*B^3
*a^2*b + A^3*B*b^3)*c)*x - 1/2*sqrt(1/2)*(2*B^3*a^2*b^4 - A*B^2*a*b^5 - 16*
(2*A^2*B*a^3 - A^3*a^2*b)*c^3 + 8*(4*B^3*a^4 - 2*A*B^2*a^3*b + 2*A^2*B*a^2*
b^2 - A^3*a*b^3)*c^2 - (16*B^3*a^3*b^2 - 8*A*B^2*a^2*b^3 + 2*A^2*B*a*b^4 -
A^3*b^5)*c + (192*B*a^4*b^3*c^3 + 256*A*a^5*c^5 - 128*(2*B*a^5*b + A*a^4*b^
2)*c^4 - 8*(6*B*a^3*b^5 - A*a^2*b^6)*c^2 + (4*B*a^2*b^7 - A*a*b^8)*c)*sqrt(
(B^4*a^2 - 2*A^2*B^2*a*c + A^4*c^2)/(a^2*b^6*c^2 - 12*a^3*b^4*c^3 + 48*a^4*
b^2*c^4 - 64*a^5*c^5)))*sqrt(-(B^2*a*b^3 - 4*(4*A*B*a^2 - 3*A^2*a*b)*c^2 +
(12*B^2*a^2*b - 12*A*B*a*b^2 + A^2*b^3)*c + (a*b^6*c - 12*a^2*b^4*c^2 + 48*
a^3*b^2*c^3 - 64*a^4*c^4)*sqrt((B^4*a^2 - 2*A^2*B^2*a*c + A^4*c^2)/(a^2*b^6
*c^2 - 12*a^3*b^4*c^3 + 48*a^4*b^2*c^4 - 64*a^5*c^5)))/(a*b^6*c - 12*a^2*b^
4*c^2 + 48*a^3*b^2*c^3 - 64*a^4*c^4))) - sqrt(1/2)*((b^2*c - 4*a*c^2)*x^4 +
 a*b^2 - 4*a^2*c + (b^3 - 4*a*b*c)*x^2)*sqrt(-(B^2*a*b^3 - 4*(4*A*B*a^2 - 3
*A^2*a*b)*c^2 + (12*B^2*a^2*b - 12*A*B*a*b^2 + A^2*b^3)*c - (a*b^6*c - 12*a
^2*b^4*c^2 + 48*a^3*b^2*c^3 - 64*a^4*c^4)*sqrt((B^4*a^2 - 2*A^2*B^2*a*c + A
^4*c^2)/(a^2*b^6*c^2 - 12*a^3*b^4*c^3 + 48*a^4*b^2*c^4 - 64*a^5*c^5)))/(a*b
^6*c - 12*a^2*b^4*c^2 + 48*a^3*b^2*c^3 - 64*a^4*c^4))*log(-(3*B^4*a^2*b^2 -
 A*B^3*a*b^3 - 4*A^4*a*c^3 + 3*(4*A^3*B*a*b - A^4*b^2)*c^2 + (4*B^4*a^3 - 1
2*A*B^3*a^2*b + A^3*B*b^3)*c)*x + 1/2*sqrt(1/2)*(2*B^3*a^2*b^4 - A*B^2*a*b^
5 - 16*(2*A^2*B*a^3 - A^3*a^2*b)*c^3 + 8*(4*B^3*a^4 - 2*A*B^2*a^3*b + 2*A^2
*B*a^2*b^2 - A^3*a*b^3)*c^2 - (16*B^3*a^3*b^2 - 8*A*B^2*a^2*b^3 + 2*A^2*B*a
*b^4 - A^3*b^5)*c - (192*B*a^4*b^3*c^3 + 256*A*a^5*c^5 - 128*(2*B*a^5*b + A
*a^4*b^2)*c^4 - 8*(6*B*a^3*b^5 - A*a^2*b^6)*c^2 + (4*B*a^2*b^7 - A*a*b^8)*c
)*sqrt((B^4*a^2 - 2*A^2*B^2*a*c + A^4*c^2)/(a^2*b^6*c^2 - 12*a^3*b^4*c^3 +
48*a^4*b^2*c^4 - 64*a^5*c^5)))*sqrt(-(B^2*a*b^3 - 4*(4*A*B*a^2 - 3*A^2*a*b)
*c^2 + (12*B^2*a^2*b - 12*A*B*a*b^2 + A^2*b^3)*c - (a*b^6*c - 12*a^2*b^4*c^
2 + 48*a^3*b^2*c^3 - 64*a^4*c^4)*sqrt((B^4*a^2 - 2*A^2*B^2*a*c + A^4*c^2)/(
a^2*b^6*c^2 - 12*a^3*b^4*c^3 + 48*a^4*b^2*c^4 - 64*a^5*c^5)))/(a*b^6*c - 12
*a^2*b^4*c^2 + 48*a^3*b^2*c^3 - 64*a^4*c^4))) + sqrt(1/2)*((b^2*c - 4*a*c^2
)*x^4 + a*b^2 - 4*a^2*c + (b^3 - 4*a*b*c)*x^2)*sqrt(-(B^2*a*b^3 - 4*(4*A*B*
```

```
a^2 - 3*A^2*a*b)*c^2 + (12*B^2*a^2*b - 12*A*B*a*b^2 + A^2*b^3)*c - (a*b^6*c
 - 12*a^2*b^4*c^2 + 48*a^3*b^2*c^3 - 64*a^4*c^4)*sqrt((B^4*a^2 - 2*A^2*B^2*
a*c + A^4*c^2)/(a^2*b^6*c^2 - 12*a^3*b^4*c^3 + 48*a^4*b^2*c^4 - 64*a^5*c^5)
))/(a*b^6*c - 12*a^2*b^4*c^2 + 48*a^3*b^2*c^3 - 64*a^4*c^4))*log(-(3*B^4*a^
2*b^2 - A*B^3*a*b^3 - 4*A^4*a*c^3 + 3*(4*A^3*B*a*b - A^4*b^2)*c^2 + (4*B^4*
a^3 - 12*A*B^3*a^2*b + A^3*B*b^3)*c)*x - 1/2*sqrt(1/2)*(2*B^3*a^2*b^4 - A*B
^2*a*b^5 - 16*(2*A^2*B*a^3 - A^3*a^2*b)*c^3 + 8*(4*B^3*a^4 - 2*A*B^2*a^3*b
 + 2*A^2*B*a^2*b^2 - A^3*a*b^3)*c^2 - (16*B^3*a^3*b^2 - 8*A*B^2*a^2*b^3 + 2*
A^2*B*a*b^4 - A^3*b^5)*c - (192*B*a^4*b^3*c^3 + 256*A*a^5*c^5 - 128*(2*B*a^
5*b + A*a^4*b^2)*c^4 - 8*(6*B*a^3*b^5 - A*a^2*b^6)*c^2 + (4*B*a^2*b^7 - A*a
*b^8)*c)*sqrt((B^4*a^2 - 2*A^2*B^2*a*c + A^4*c^2)/(a^2*b^6*c^2 - 12*a^3*b^4
*c^3 + 48*a^4*b^2*c^4 - 64*a^5*c^5)))*sqrt(-(B^2*a*b^3 - 4*(4*A*B*a^2 - 3*A
^2*a*b)*c^2 + (12*B^2*a^2*b - 12*A*B*a*b^2 + A^2*b^3)*c - (a*b^6*c - 12*a^2
*b^4*c^2 + 48*a^3*b^2*c^3 - 64*a^4*c^4)*sqrt((B^4*a^2 - 2*A^2*B^2*a*c + A^4
*c^2)/(a^2*b^6*c^2 - 12*a^3*b^4*c^3 + 48*a^4*b^2*c^4 - 64*a^5*c^5)))/(a*b^6
*c - 12*a^2*b^4*c^2 + 48*a^3*b^2*c^3 - 64*a^4*c^4))) + 2*(2*B*a - A*b)*x)/(
(b^2*c - 4*a*c^2)*x^4 + a*b^2 - 4*a^2*c + (b^3 - 4*a*b*c)*x^2)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^2(A+Bx^2)}{(a+bx^2+cx^4)^2}\,dx = \text{Timed out}$$

```
[In] integrate(x**2*(B*x**2+A)/(c*x**4+b*x**2+a)**2,x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{x^2(A+Bx^2)}{(a+bx^2+cx^4)^2}\,dx = \int \frac{(Bx^2+A)x^2}{(cx^4+bx^2+a)^2}\,dx$$

```
[In] integrate(x^2*(B*x^2+A)/(c*x^4+b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] 1/2*((B*b - 2*A*c)*x^3 + (2*B*a - A*b)*x)/((b^2*c - 4*a*c^2)*x^4 + a*b^2 -
4*a^2*c + (b^3 - 4*a*b*c)*x^2) - 1/2*integrate(-((B*b - 2*A*c)*x^2 - 2*B*a
+ A*b)/(c*x^4 + b*x^2 + a), x)/(b^2 - 4*a*c)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3776 vs. 2(234) = 468.

Time = 1.14 (sec) , antiderivative size = 3776, normalized size of antiderivative = 13.68

$$\int \frac{x^2(A+Bx^2)}{(a+bx^2+cx^4)^2}\,dx = \text{Too large to display}$$

```
[In] integrate(x^2*(B*x^2+A)/(c*x^4+b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] 1/2*(B*b*x^3 - 2*A*c*x^3 + 2*B*a*x - A*b*x)/((c*x^4 + b*x^2 + a)*(b^2 - 4*a
*c)) + 1/16*(2*(2*b^2*c^3 - 8*a*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c +
sqrt(b^2 - 4*a*c)*c)*b^2*c + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^
2 - 4*a*c)*c)*a*c^2 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a
*c)*c)*b*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*c^
3 - 2*(b^2 - 4*a*c)*c^3)*(b^2 - 4*a*c)^2*A - (2*b^3*c^2 - 8*a*b*c^3 - sqrt(
2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^3 + 4*sqrt(2)*sqrt(b
^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c + 2*sqrt(2)*sqrt(b^2 - 4*
a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^2*c - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt
(b*c + sqrt(b^2 - 4*a*c)*c)*b*c^2 - 2*(b^2 - 4*a*c)*b*c^2)*(b^2 - 4*a*c)^2*
B + 2*(sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^5*c - 8*sqrt(2)*sqrt(b*c +
 sqrt(b^2 - 4*a*c)*c)*a*b^3*c^2 - 2*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)
*b^4*c^2 - 2*b^5*c^2 + 16*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b*c^3
 + 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^3 + sqrt(2)*sqrt(b*c +
 sqrt(b^2 - 4*a*c)*c)*b^3*c^3 + 16*a*b^3*c^3 - 4*sqrt(2)*sqrt(b*c + sqrt(b^
2 - 4*a*c)*c)*a*b*c^4 - 32*a^2*b*c^4 + 2*(b^2 - 4*a*c)*b^3*c^2 - 8*(b^2 - 4
*a*c)*a*b*c^3)*A*abs(b^2 - 4*a*c) - 4*(sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)
*c)*a*b^4*c - 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^2 - 2*sqr
t(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c^2 - 2*a*b^4*c^2 + 16*sqrt(2)*s
qrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*c^3 + 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*
a*c)*c)*a^2*b*c^3 + sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^3 + 16*
a^2*b^2*c^3 - 4*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*c^4 - 32*a^3*c^
4 + 2*(b^2 - 4*a*c)*a*b^2*c^2 - 8*(b^2 - 4*a*c)*a^2*c^3)*B*abs(b^2 - 4*a*c)
 - 4*(2*b^6*c^3 - 16*a*b^4*c^4 + 32*a^2*b^2*c^5 - sqrt(2)*sqrt(b^2 - 4*a*c)
*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^6*c + 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b
*c + sqrt(b^2 - 4*a*c)*c)*a*b^4*c^2 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c
+ sqrt(b^2 - 4*a*c)*c)*b^5*c^2 - 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sq
rt(b^2 - 4*a*c)*c)*a^2*b^2*c^3 - 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqr
t(b^2 - 4*a*c)*c)*a*b^3*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2
 - 4*a*c)*c)*b^4*c^3 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*
a*c)*c)*a*b^2*c^4 - 2*(b^2 - 4*a*c)*b^4*c^3 + 8*(b^2 - 4*a*c)*a*b^2*c^4)*A
+ (2*b^7*c^2 - 8*a*b^5*c^3 - 32*a^2*b^3*c^4 + 128*a^3*b*c^5 - sqrt(2)*sqrt(
b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^7 + 4*sqrt(2)*sqrt(b^2 - 4*a
*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^5*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*s
qrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^6*c + 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*
```

c + sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c +
 sqrt(b^2 - 4*a*c)*c)*b^5*c^2 - 64*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqr
t(b^2 - 4*a*c)*c)*a^3*b*c^3 - 32*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(
b^2 - 4*a*c)*c)*a^2*b^2*c^3 + 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(
b^2 - 4*a*c)*c)*a^2*b*c^4 - 2*(b^2 - 4*a*c)*b^5*c^2 + 32*(b^2 - 4*a*c)*a^2*
b*c^4)*B)*arctan(2*sqrt(1/2)*x/sqrt((b^3 - 4*a*b*c + sqrt((b^3 - 4*a*b*c)^2
 - 4*(a*b^2 - 4*a^2*c)*(b^2*c - 4*a*c^2)))/(b^2*c - 4*a*c^2)))/((a*b^6*c -
12*a^2*b^4*c^2 - 2*a*b^5*c^2 + 48*a^3*b^2*c^3 + 16*a^2*b^3*c^3 + a*b^4*c^3
- 64*a^4*c^4 - 32*a^3*b*c^4 - 8*a^2*b^2*c^4 + 16*a^3*c^5)*abs(b^2 - 4*a*c)*
abs(c)) + 1/16*(2*(2*b^2*c^3 - 8*a*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c
 - sqrt(b^2 - 4*a*c)*c)*b^2*c + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt
(b^2 - 4*a*c)*c)*a*c^2 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 -
4*a*c)*c)*b*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)
c^3 - 2(b^2 - 4*a*c)*c^3)*(b^2 - 4*a*c)^2*A - (2*b^3*c^2 - 8*a*b*c^3 - sq
rt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^3 + 4*sqrt(2)*sqr
t(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b*c + 2*sqrt(2)*sqrt(b^2 -
 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^2*c - sqrt(2)*sqrt(b^2 - 4*a*c)*s
qrt(b*c - sqrt(b^2 - 4*a*c)*c)*b*c^2 - 2*(b^2 - 4*a*c)*b*c^2)*(b^2 - 4*a*c)
^2*B + 2*(sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^5*c - 8*sqrt(2)*sqrt(b*
c - sqrt(b^2 - 4*a*c)*c)*a*b^3*c^2 - 2*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)
*c)*b^4*c^2 + 2*b^5*c^2 + 16*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b*
c^3 + 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^2*c^3 + sqrt(2)*sqrt(b*
c - sqrt(b^2 - 4*a*c)*c)*b^3*c^3 - 16*a*b^3*c^3 - 4*sqrt(2)*sqrt(b*c - sqrt
(b^2 - 4*a*c)*c)*a*b*c^4 + 32*a^2*b*c^4 - 2*(b^2 - 4*a*c)*b^3*c^2 + 8*(b^2
- 4*a*c)*a*b*c^3)*A*abs(b^2 - 4*a*c) - 4*(sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a
*c)*c)*a*b^4*c - 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^2 - 2*
sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^3*c^2 + 2*a*b^4*c^2 + 16*sqrt(2
)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*c^3 + 8*sqrt(2)*sqrt(b*c - sqrt(b^2 -
 4*a*c)*c)*a^2*b*c^3 + sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^2*c^3 -
16*a^2*b^2*c^3 - 4*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*c^4 + 32*a^3
c^4 - 2(b^2 - 4*a*c)*a*b^2*c^2 + 8*(b^2 - 4*a*c)*a^2*c^3)*B*abs(b^2 - 4*a
c) - 4(2*b^6*c^3 - 16*a*b^4*c^4 + 32*a^2*b^2*c^5 - sqrt(2)*sqrt(b^2 - 4*a
*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^6*c + 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqr
t(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^4*c^2 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b
*c - sqrt(b^2 - 4*a*c)*c)*b^5*c^2 - 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c -
 sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^3 - 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c -
sqrt(b^2 - 4*a*c)*c)*a*b^3*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(
b^2 - 4*a*c)*c)*b^4*c^3 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 -
 4*a*c)*c)*a*b^2*c^4 - 2*(b^2 - 4*a*c)*b^4*c^3 + 8*(b^2 - 4*a*c)*a*b^2*c^4)
*A + (2*b^7*c^2 - 8*a*b^5*c^3 - 32*a^2*b^3*c^4 + 128*a^3*b*c^5 - sqrt(2)*sq
rt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^7 + 4*sqrt(2)*sqrt(b^2 -
4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^5*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c
)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^6*c + 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt
(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*
c - sqrt(b^2 - 4*a*c)*c)*b^5*c^2 - 64*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c -

```
sqrt(b^2 - 4*a*c)*c)*a^3*b*c^3 - 32*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sq
rt(b^2 - 4*a*c)*c)*a^2*b^2*c^3 + 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sq
rt(b^2 - 4*a*c)*c)*a^2*b*c^4 - 2*(b^2 - 4*a*c)*b^5*c^2 + 32*(b^2 - 4*a*c)*a
^2*b*c^4)*B)*arctan(2*sqrt(1/2)*x/sqrt((b^3 - 4*a*b*c - sqrt((b^3 - 4*a*b*c
)^2 - 4*(a*b^2 - 4*a^2*c)*(b^2*c - 4*a*c^2)))/(b^2*c - 4*a*c^2)))/((a*b^6*c
 - 12*a^2*b^4*c^2 - 2*a*b^5*c^2 + 48*a^3*b^2*c^3 + 16*a^2*b^3*c^3 + a*b^4*c
^3 - 64*a^4*c^4 - 32*a^3*b*c^4 - 8*a^2*b^2*c^4 + 16*a^3*c^5)*abs(b^2 - 4*a*
c)*abs(c))
```

Mupad [B] (verification not implemented)

Time = 10.41 (sec) , antiderivative size = 9444, normalized size of antiderivative = 34.22

$$\int \frac{x^2(A+Bx^2)}{(a+bx^2+cx^4)^2}\,dx = \text{Too large to display}$$

```
[In] int((x^2*(A + B*x^2))/(a + b*x^2 + c*x^4)^2,x)
```

```
[Out] ((x*(A*b - 2*B*a))/(2*(4*a*c - b^2)) + (x^3*(2*A*c - B*b))/(2*(4*a*c - b^2)
))/(a + b*x^2 + c*x^4) - atan(((((16*A*b^7*c^2 + 2048*B*a^4*c^5 - 192*A*a*b
^5*c^3 - 1024*A*a^3*b*c^5 - 32*B*a*b^6*c^2 + 768*A*a^2*b^3*c^4 + 384*B*a^2*
b^4*c^3 - 1536*B*a^3*b^2*c^4)/(8*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*
b^4*c)) - (x*(-(B^2*a*b^9 - B^2*a*(-(4*a*c - b^2)^9)^(1/2) + A^2*b^9*c + A^
2*c*(-(4*a*c - b^2)^9)^(1/2) - 96*A^2*a^2*b^5*c^3 + 512*A^2*a^3*b^3*c^4 - 9
6*B^2*a^3*b^5*c^2 + 512*B^2*a^4*b^3*c^3 + 1024*A*B*a^5*c^5 - 768*A^2*a^4*b*
c^5 - 768*B^2*a^5*b*c^4 + 128*A*B*a^2*b^6*c^2 - 384*A*B*a^3*b^4*c^3 - 12*A*
B*a*b^8*c)/(32*(4096*a^7*c^7 - 24*a^2*b^10*c^2 + 240*a^3*b^8*c^3 - 1280*a^4
*b^6*c^4 + 3840*a^5*b^4*c^5 - 6144*a^6*b^2*c^6 + a*b^12*c)))^(1/2)*(16*b^7*
c^2 - 192*a*b^5*c^3 - 1024*a^3*b*c^5 + 768*a^2*b^3*c^4))/(2*(b^4 + 16*a^2*c
^2 - 8*a*b^2*c)))*(-(B^2*a*b^9 - B^2*a*(-(4*a*c - b^2)^9)^(1/2) + A^2*b^9*c
 + A^2*c*(-(4*a*c - b^2)^9)^(1/2) - 96*A^2*a^2*b^5*c^3 + 512*A^2*a^3*b^3*c^
4 - 96*B^2*a^3*b^5*c^2 + 512*B^2*a^4*b^3*c^3 + 1024*A*B*a^5*c^5 - 768*A^2*a
^4*b*c^5 - 768*B^2*a^5*b*c^4 + 128*A*B*a^2*b^6*c^2 - 384*A*B*a^3*b^4*c^3 -
12*A*B*a*b^8*c)/(32*(4096*a^7*c^7 - 24*a^2*b^10*c^2 + 240*a^3*b^8*c^3 - 128
0*a^4*b^6*c^4 + 3840*a^5*b^4*c^5 - 6144*a^6*b^2*c^6 + a*b^12*c)))^(1/2) - (
x*(B^2*b^4*c - 8*A^2*a*c^4 + 10*A^2*b^2*c^3 + 8*B^2*a^2*c^3 - 6*A*B*b^3*c^2
 + 2*B^2*a*b^2*c^2 - 8*A*B*a*b*c^3))/(2*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)))*(-
(B^2*a*b^9 - B^2*a*(-(4*a*c - b^2)^9)^(1/2) + A^2*b^9*c + A^2*c*(-(4*a*c -
b^2)^9)^(1/2) - 96*A^2*a^2*b^5*c^3 + 512*A^2*a^3*b^3*c^4 - 96*B^2*a^3*b^5*c
^2 + 512*B^2*a^4*b^3*c^3 + 1024*A*B*a^5*c^5 - 768*A^2*a^4*b*c^5 - 768*B^2*a
^5*b*c^4 + 128*A*B*a^2*b^6*c^2 - 384*A*B*a^3*b^4*c^3 - 12*A*B*a*b^8*c)/(32*
(4096*a^7*c^7 - 24*a^2*b^10*c^2 + 240*a^3*b^8*c^3 - 1280*a^4*b^6*c^4 + 3840
*a^5*b^4*c^5 - 6144*a^6*b^2*c^6 + a*b^12*c)))^(1/2)*1i - (((16*A*b^7*c^2 +
2048*B*a^4*c^5 - 192*A*a*b^5*c^3 - 1024*A*a^3*b*c^5 - 32*B*a*b^6*c^2 + 768*
A*a^2*b^3*c^4 + 384*B*a^2*b^4*c^3 - 1536*B*a^3*b^2*c^4)/(8*(b^6 - 64*a^3*c^
```

3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)) + (x*(-(B^2*a*b^9 - B^2*a*(-(4*a*c - b^2)^9)^(1/2) + A^2*b^9*c + A^2*c*(-(4*a*c - b^2)^9)^(1/2) - 96*A^2*a^2*b^5*c^3 + 512*A^2*a^3*b^3*c^4 - 96*B^2*a^3*b^5*c^2 + 512*B^2*a^4*b^3*c^3 + 1024*A*B*a^5*c^5 - 768*A^2*a^4*b*c^5 - 768*B^2*a^5*b*c^4 + 128*A*B*a^2*b^6*c^2 - 384*A*B*a^3*b^4*c^3 - 12*A*B*a*b^8*c)/(32*(4096*a^7*c^7 - 24*a^2*b^10*c^2 + 240*a^3*b^8*c^3 - 1280*a^4*b^6*c^4 + 3840*a^5*b^4*c^5 - 6144*a^6*b^2*c^6 + a*b^12*c)))^(1/2)*(16*b^7*c^2 - 192*a*b^5*c^3 - 1024*a^3*b*c^5 + 768*a^2*b^3*c^4))/(2*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)))*(-(B^2*a*b^9 - B^2*a*(-(4*a*c - b^2)^9)^(1/2) + A^2*b^9*c + A^2*c*(-(4*a*c - b^2)^9)^(1/2) - 96*A^2*a^2*b^5*c^3 + 512*A^2*a^3*b^3*c^4 - 96*B^2*a^3*b^5*c^2 + 512*B^2*a^4*b^3*c^3 + 1024*A*B*a^5*c^5 - 768*A^2*a^4*b*c^5 - 768*B^2*a^5*b*c^4 + 128*A*B*a^2*b^6*c^2 - 384*A*B*a^3*b^4*c^3 - 12*A*B*a*b^8*c)/(32*(4096*a^7*c^7 - 24*a^2*b^10*c^2 + 240*a^3*b^8*c^3 - 1280*a^4*b^6*c^4 + 3840*a^5*b^4*c^5 - 6144*a^6*b^2*c^6 + a*b^12*c)))^(1/2) + (x*(B^2*b^4*c - 8*A^2*a*c^4 + 10*A^2*b^2*c^3 + 8*B^2*a^2*c^3 - 6*A*B*b^3*c^2 + 2*B^2*a*b^2*c^2 - 8*A*B*a*b*c^3))/(2*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)))*(-(B^2*a*b^9 - B^2*a*(-(4*a*c - b^2)^9)^(1/2) + A^2*b^9*c + A^2*c*(-(4*a*c - b^2)^9)^(1/2) - 96*A^2*a^2*b^5*c^3 + 512*A^2*a^3*b^3*c^4 - 96*B^2*a^3*b^5*c^2 + 512*B^2*a^4*b^3*c^3 + 1024*A*B*a^5*c^5 - 768*A^2*a^4*b*c^5 - 768*B^2*a^5*b*c^4 + 128*A*B*a^2*b^6*c^2 - 384*A*B*a^3*b^4*c^3 - 12*A*B*a*b^8*c)/(32*(4096*a^7*c^7 - 24*a^2*b^10*c^2 + 240*a^3*b^8*c^3 - 1280*a^4*b^6*c^4 + 3840*a^5*b^4*c^5 - 6144*a^6*b^2*c^6 + a*b^12*c)))^(1/2)*1i)/((8*A^3*a*c^4 + 6*A^3*b^2*c^3 + A*B^2*b^4*c - 3*B^3*a*b^3*c + 8*A*B^2*a^2*c^3 - 5*A^2*B*b^3*c^2 - 4*B^3*a^2*b*c^2 + 18*A*B^2*a*b^2*c^2 - 28*A^2*B*a*b*c^3)/(4*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)) + (((16*A*b^7*c^2 + 2048*B*a^4*c^5 - 192*A*a*b^5*c^3 - 1024*A*a^3*b*c^5 - 32*B*a*b^6*c^2 + 768*A*a^2*b^3*c^4 + 384*B*a^2*b^4*c^3 - 1536*B*a^3*b^2*c^4)/(8*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)) - (x*(-(B^2*a*b^9 - B^2*a*(-(4*a*c - b^2)^9)^(1/2) + A^2*b^9*c + A^2*c*(-(4*a*c - b^2)^9)^(1/2) - 96*A^2*a^2*b^5*c^3 + 512*A^2*a^3*b^3*c^4 - 96*B^2*a^3*b^5*c^2 + 512*B^2*a^4*b^3*c^3 + 1024*A*B*a^5*c^5 - 768*A^2*a^4*b*c^5 - 768*B^2*a^5*b*c^4 + 128*A*B*a^2*b^6*c^2 - 384*A*B*a^3*b^4*c^3 - 12*A*B*a*b^8*c)/(32*(4096*a^7*c^7 - 24*a^2*b^10*c^2 + 240*a^3*b^8*c^3 - 1280*a^4*b^6*c^4 + 3840*a^5*b^4*c^5 - 6144*a^6*b^2*c^6 + a*b^12*c)))^(1/2)*(16*b^7*c^2 - 192*a*b^5*c^3 - 1024*a^3*b*c^5 + 768*a^2*b^3*c^4))/(2*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)))*(-(B^2*a*b^9 - B^2*a*(-(4*a*c - b^2)^9)^(1/2) + A^2*b^9*c + A^2*c*(-(4*a*c - b^2)^9)^(1/2) - 96*A^2*a^2*b^5*c^3 + 512*A^2*a^3*b^3*c^4 - 96*B^2*a^3*b^5*c^2 + 512*B^2*a^4*b^3*c^3 + 1024*A*B*a^5*c^5 - 768*A^2*a^4*b*c^5 - 768*B^2*a^5*b*c^4 + 128*A*B*a^2*b^6*c^2 - 384*A*B*a^3*b^4*c^3 - 12*A*B*a*b^8*c)/(32*(4096*a^7*c^7 - 24*a^2*b^10*c^2 + 240*a^3*b^8*c^3 - 1280*a^4*b^6*c^4 + 3840*a^5*b^4*c^5 - 6144*a^6*b^2*c^6 + a*b^12*c)))^(1/2) - (x*(B^2*b^4*c - 8*A^2*a*c^4 + 10*A^2*b^2*c^3 + 8*B^2*a^2*c^3 - 6*A*B*b^3*c^2 + 2*B^2*a*b^2*c^2 - 8*A*B*a*b*c^3))/(2*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)))*(-(B^2*a*b^9 - B^2*a*(-(4*a*c - b^2)^9)^(1/2) + A^2*b^9*c + A^2*c*(-(4*a*c - b^2)^9)^(1/2) - 96*A^2*a^2*b^5*c^3 + 512*A^2*a^3*b^3*c^4 - 96*B^2*a^3*b^5*c^2 + 512*B^2*a^4*b^3*c^3 + 1024*A*B*a^5*c^5 - 768*A^2*a^4*b*c^5 - 768*B^2*a^5*b*c^4 + 128*A*B*a^2*b^6*c^2 - 384*A*

B*a^3*b^4*c^3 - 12*A*B*a*b^8*c)/(32*(4096*a^7*c^7 - 24*a^2*b^10*c^2 + 240*a
^3*b^8*c^3 - 1280*a^4*b^6*c^4 + 3840*a^5*b^4*c^5 - 6144*a^6*b^2*c^6 + a*b^1
2*c)))^(1/2) + (((16*A*b^7*c^2 + 2048*B*a^4*c^5 - 192*A*a*b^5*c^3 - 1024*A*
a^3*b*c^5 - 32*B*a*b^6*c^2 + 768*A*a^2*b^3*c^4 + 384*B*a^2*b^4*c^3 - 1536*B
*a^3*b^2*c^4)/(8*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)) + (x*(-(
B^2*a*b^9 - B^2*a*(-(4*a*c - b^2)^9)^(1/2) + A^2*b^9*c + A^2*c*(-(4*a*c - b
^2)^9)^(1/2) - 96*A^2*a^2*b^5*c^3 + 512*A^2*a^3*b^3*c^4 - 96*B^2*a^3*b^5*c^
2 + 512*B^2*a^4*b^3*c^3 + 1024*A*B*a^5*c^5 - 768*A^2*a^4*b*c^5 - 768*B^2*a^
5*b*c^4 + 128*A*B*a^2*b^6*c^2 - 384*A*B*a^3*b^4*c^3 - 12*A*B*a*b^8*c)/(32*(
4096*a^7*c^7 - 24*a^2*b^10*c^2 + 240*a^3*b^8*c^3 - 1280*a^4*b^6*c^4 + 3840*
a^5*b^4*c^5 - 6144*a^6*b^2*c^6 + a*b^12*c)))^(1/2)*(16*b^7*c^2 - 192*a*b^5*
c^3 - 1024*a^3*b*c^5 + 768*a^2*b^3*c^4))/(2*(b^4 + 16*a^2*c^2 - 8*a*b^2*c))
)*(-(B^2*a*b^9 - B^2*a*(-(4*a*c - b^2)^9)^(1/2) + A^2*b^9*c + A^2*c*(-(4*a*
c - b^2)^9)^(1/2) - 96*A^2*a^2*b^5*c^3 + 512*A^2*a^3*b^3*c^4 - 96*B^2*a^3*b
^5*c^2 + 512*B^2*a^4*b^3*c^3 + 1024*A*B*a^5*c^5 - 768*A^2*a^4*b*c^5 - 768*B
^2*a^5*b*c^4 + 128*A*B*a^2*b^6*c^2 - 384*A*B*a^3*b^4*c^3 - 12*A*B*a*b^8*c)/
(32*(4096*a^7*c^7 - 24*a^2*b^10*c^2 + 240*a^3*b^8*c^3 - 1280*a^4*b^6*c^4 +
3840*a^5*b^4*c^5 - 6144*a^6*b^2*c^6 + a*b^12*c)))^(1/2) + (x*(B^2*b^4*c - 8
*A^2*a*c^4 + 10*A^2*b^2*c^3 + 8*B^2*a^2*c^3 - 6*A*B*b^3*c^2 + 2*B^2*a*b^2*c
^2 - 8*A*B*a*b*c^3))/(2*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)))*(-(B^2*a*b^9 - B^2
a(-(4*a*c - b^2)^9)^(1/2) + A^2*b^9*c + A^2*c*(-(4*a*c - b^2)^9)^(1/2) -
96*A^2*a^2*b^5*c^3 + 512*A^2*a^3*b^3*c^4 - 96*B^2*a^3*b^5*c^2 + 512*B^2*a^4
*b^3*c^3 + 1024*A*B*a^5*c^5 - 768*A^2*a^4*b*c^5 - 768*B^2*a^5*b*c^4 + 128*A
*B*a^2*b^6*c^2 - 384*A*B*a^3*b^4*c^3 - 12*A*B*a*b^8*c)/(32*(4096*a^7*c^7 -
24*a^2*b^10*c^2 + 240*a^3*b^8*c^3 - 1280*a^4*b^6*c^4 + 3840*a^5*b^4*c^5 - 6
144*a^6*b^2*c^6 + a*b^12*c)))^(1/2)))*(-(B^2*a*b^9 - B^2*a*(-(4*a*c - b^2)^
9)^(1/2) + A^2*b^9*c + A^2*c*(-(4*a*c - b^2)^9)^(1/2) - 96*A^2*a^2*b^5*c^3
+ 512*A^2*a^3*b^3*c^4 - 96*B^2*a^3*b^5*c^2 + 512*B^2*a^4*b^3*c^3 + 1024*A*B
*a^5*c^5 - 768*A^2*a^4*b*c^5 - 768*B^2*a^5*b*c^4 + 128*A*B*a^2*b^6*c^2 - 38
4*A*B*a^3*b^4*c^3 - 12*A*B*a*b^8*c)/(32*(4096*a^7*c^7 - 24*a^2*b^10*c^2 + 2
40*a^3*b^8*c^3 - 1280*a^4*b^6*c^4 + 3840*a^5*b^4*c^5 - 6144*a^6*b^2*c^6 + a
*b^12*c)))^(1/2)*2i - atan(((((16*A*b^7*c^2 + 2048*B*a^4*c^5 - 192*A*a*b^5*
c^3 - 1024*A*a^3*b*c^5 - 32*B*a*b^6*c^2 + 768*A*a^2*b^3*c^4 + 384*B*a^2*b^4
*c^3 - 1536*B*a^3*b^2*c^4)/(8*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4
c)) - (x(-(B^2*a*b^9 + B^2*a*(-(4*a*c - b^2)^9)^(1/2) + A^2*b^9*c - A^2*c
*(-(4*a*c - b^2)^9)^(1/2) - 96*A^2*a^2*b^5*c^3 + 512*A^2*a^3*b^3*c^4 - 96*B
^2*a^3*b^5*c^2 + 512*B^2*a^4*b^3*c^3 + 1024*A*B*a^5*c^5 - 768*A^2*a^4*b*c^5
 - 768*B^2*a^5*b*c^4 + 128*A*B*a^2*b^6*c^2 - 384*A*B*a^3*b^4*c^3 - 12*A*B*a
*b^8*c)/(32*(4096*a^7*c^7 - 24*a^2*b^10*c^2 + 240*a^3*b^8*c^3 - 1280*a^4*b^
6*c^4 + 3840*a^5*b^4*c^5 - 6144*a^6*b^2*c^6 + a*b^12*c)))^(1/2)*(16*b^7*c^2
 - 192*a*b^5*c^3 - 1024*a^3*b*c^5 + 768*a^2*b^3*c^4))/(2*(b^4 + 16*a^2*c^2
- 8*a*b^2*c)))*(-(B^2*a*b^9 + B^2*a*(-(4*a*c - b^2)^9)^(1/2) + A^2*b^9*c -
A^2*c*(-(4*a*c - b^2)^9)^(1/2) - 96*A^2*a^2*b^5*c^3 + 512*A^2*a^3*b^3*c^4 -
 96*B^2*a^3*b^5*c^2 + 512*B^2*a^4*b^3*c^3 + 1024*A*B*a^5*c^5 - 768*A^2*a^4*
b*c^5 - 768*B^2*a^5*b*c^4 + 128*A*B*a^2*b^6*c^2 - 384*A*B*a^3*b^4*c^3 - 12*

A*B*a*b^8*c)/(32*(4096*a^7*c^7 - 24*a^2*b^10*c^2 + 240*a^3*b^8*c^3 - 1280*a^4*b^6*c^4 + 3840*a^5*b^4*c^5 - 6144*a^6*b^2*c^6 + a*b^12*c)))^(1/2) - (x*(B^2*b^4*c - 8*A^2*a*c^4 + 10*A^2*b^2*c^3 + 8*B^2*a^2*c^3 - 6*A*B*b^3*c^2 + 2*B^2*a*b^2*c^2 - 8*A*B*a*b*c^3))/(2*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)))*(-(B^2*a*b^9 + B^2*a*(-(4*a*c - b^2)^9)^(1/2) + A^2*b^9*c - A^2*c*(-(4*a*c - b^2)^9)^(1/2) - 96*A^2*a^2*b^5*c^3 + 512*A^2*a^3*b^3*c^4 - 96*B^2*a^3*b^5*c^2 + 512*B^2*a^4*b^3*c^3 + 1024*A*B*a^5*c^5 - 768*A^2*a^4*b*c^5 - 768*B^2*a^5*b*c^4 + 128*A*B*a^2*b^6*c^2 - 384*A*B*a^3*b^4*c^3 - 12*A*B*a*b^8*c)/(32*(4096*a^7*c^7 - 24*a^2*b^10*c^2 + 240*a^3*b^8*c^3 - 1280*a^4*b^6*c^4 + 3840*a^5*b^4*c^5 - 6144*a^6*b^2*c^6 + a*b^12*c)))^(1/2)*1i - (((16*A*b^7*c^2 + 2048*B*a^4*c^5 - 192*A*a*b^5*c^3 - 1024*A*a^3*b*c^5 - 32*B*a*b^6*c^2 + 768*A*a^2*b^3*c^4 + 384*B*a^2*b^4*c^3 - 1536*B*a^3*b^2*c^4)/(8*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)) + (x*(-(B^2*a*b^9 + B^2*a*(-(4*a*c - b^2)^9)^(1/2) + A^2*b^9*c - A^2*c*(-(4*a*c - b^2)^9)^(1/2) - 96*A^2*a^2*b^5*c^3 + 512*A^2*a^3*b^3*c^4 - 96*B^2*a^3*b^5*c^2 + 512*B^2*a^4*b^3*c^3 + 1024*A*B*a^5*c^5 - 768*A^2*a^4*b*c^5 - 768*B^2*a^5*b*c^4 + 128*A*B*a^2*b^6*c^2 - 384*A*B*a^3*b^4*c^3 - 12*A*B*a*b^8*c)/(32*(4096*a^7*c^7 - 24*a^2*b^10*c^2 + 240*a^3*b^8*c^3 - 1280*a^4*b^6*c^4 + 3840*a^5*b^4*c^5 - 6144*a^6*b^2*c^6 + a*b^12*c)))^(1/2)*(16*b^7*c^2 - 192*a*b^5*c^3 - 1024*a^3*b*c^5 + 768*a^2*b^3*c^4))/(2*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)))*(-(B^2*a*b^9 + B^2*a*(-(4*a*c - b^2)^9)^(1/2) + A^2*b^9*c - A^2*c*(-(4*a*c - b^2)^9)^(1/2) - 96*A^2*a^2*b^5*c^3 + 512*A^2*a^3*b^3*c^4 - 96*B^2*a^3*b^5*c^2 + 512*B^2*a^4*b^3*c^3 + 1024*A*B*a^5*c^5 - 768*A^2*a^4*b*c^5 - 768*B^2*a^5*b*c^4 + 128*A*B*a^2*b^6*c^2 - 384*A*B*a^3*b^4*c^3 - 12*A*B*a*b^8*c)/(32*(4096*a^7*c^7 - 24*a^2*b^10*c^2 + 240*a^3*b^8*c^3 - 1280*a^4*b^6*c^4 + 3840*a^5*b^4*c^5 - 6144*a^6*b^2*c^6 + a*b^12*c)))^(1/2) + (x*(B^2*b^4*c - 8*A^2*a*c^4 + 10*A^2*b^2*c^3 + 8*B^2*a^2*c^3 - 6*A*B*b^3*c^2 + 2*B^2*a*b^2*c^2 - 8*A*B*a*b*c^3))/(2*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)))*(-(B^2*a*b^9 + B^2*a*(-(4*a*c - b^2)^9)^(1/2) + A^2*b^9*c - A^2*c*(-(4*a*c - b^2)^9)^(1/2) - 96*A^2*a^2*b^5*c^3 + 512*A^2*a^3*b^3*c^4 - 96*B^2*a^3*b^5*c^2 + 512*B^2*a^4*b^3*c^3 + 1024*A*B*a^5*c^5 - 768*A^2*a^4*b*c^5 - 768*B^2*a^5*b*c^4 + 128*A*B*a^2*b^6*c^2 - 384*A*B*a^3*b^4*c^3 - 12*A*B*a*b^8*c)/(32*(4096*a^7*c^7 - 24*a^2*b^10*c^2 + 240*a^3*b^8*c^3 - 1280*a^4*b^6*c^4 + 3840*a^5*b^4*c^5 - 6144*a^6*b^2*c^6 + a*b^12*c)))^(1/2)*1i)/((8*A^3*a*c^4 + 6*A^3*b^2*c^3 + A*B^2*b^4*c - 3*B^3*a*b^3*c + 8*A*B^2*a^2*c^3 - 5*A^2*B*b^3*c^2 - 4*B^3*a^2*b*c^2 + 18*A*B^2*a*b^2*c^2 - 28*A^2*B*a*b*c^3)/(4*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)) + (((16*A*b^7*c^2 + 2048*B*a^4*c^5 - 192*A*a*b^5*c^3 - 1024*A*a^3*b*c^5 - 32*B*a*b^6*c^2 + 768*A*a^2*b^3*c^4 + 384*B*a^2*b^4*c^3 - 1536*B*a^3*b^2*c^4)/(8*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)) - (x*(-(B^2*a*b^9 + B^2*a*(-(4*a*c - b^2)^9)^(1/2) + A^2*b^9*c - A^2*c*(-(4*a*c - b^2)^9)^(1/2) - 96*A^2*a^2*b^5*c^3 + 512*A^2*a^3*b^3*c^4 - 96*B^2*a^3*b^5*c^2 + 512*B^2*a^4*b^3*c^3 + 1024*A*B*a^5*c^5 - 768*A^2*a^4*b*c^5 - 768*B^2*a^5*b*c^4 + 128*A*B*a^2*b^6*c^2 - 384*A*B*a^3*b^4*c^3 - 12*A*B*a*b^8*c)/(32*(4096*a^7*c^7 - 24*a^2*b^10*c^2 + 240*a^3*b^8*c^3 - 1280*a^4*b^6*c^4 + 3840*a^5*b^4*c^5 - 6144*a^6*b^2*c^6 + a*b^12*c)))^(1/2)*(16*b^7*c^2 - 192*a*b^5*c^3 - 1024*a^3*b*c^5 + 768

*a^2*b^3*c^4))/(2*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)))*(-(B^2*a*b^9 + B^2*a*(-(
4*a*c - b^2)^9)^(1/2) + A^2*b^9*c - A^2*c*(-(4*a*c - b^2)^9)^(1/2) - 96*A^2
*a^2*b^5*c^3 + 512*A^2*a^3*b^3*c^4 - 96*B^2*a^3*b^5*c^2 + 512*B^2*a^4*b^3*c
^3 + 1024*A*B*a^5*c^5 - 768*A^2*a^4*b*c^5 - 768*B^2*a^5*b*c^4 + 128*A*B*a^2
*b^6*c^2 - 384*A*B*a^3*b^4*c^3 - 12*A*B*a*b^8*c)/(32*(4096*a^7*c^7 - 24*a^2
*b^10*c^2 + 240*a^3*b^8*c^3 - 1280*a^4*b^6*c^4 + 3840*a^5*b^4*c^5 - 6144*a^
6*b^2*c^6 + a*b^12*c)))^(1/2) - (x*(B^2*b^4*c - 8*A^2*a*c^4 + 10*A^2*b^2*c^
3 + 8*B^2*a^2*c^3 - 6*A*B*b^3*c^2 + 2*B^2*a*b^2*c^2 - 8*A*B*a*b*c^3))/(2*(b
^4 + 16*a^2*c^2 - 8*a*b^2*c)))*(-(B^2*a*b^9 + B^2*a*(-(4*a*c - b^2)^9)^(1/2
) + A^2*b^9*c - A^2*c*(-(4*a*c - b^2)^9)^(1/2) - 96*A^2*a^2*b^5*c^3 + 512*A
^2*a^3*b^3*c^4 - 96*B^2*a^3*b^5*c^2 + 512*B^2*a^4*b^3*c^3 + 1024*A*B*a^5*c^
5 - 768*A^2*a^4*b*c^5 - 768*B^2*a^5*b*c^4 + 128*A*B*a^2*b^6*c^2 - 384*A*B*a
^3*b^4*c^3 - 12*A*B*a*b^8*c)/(32*(4096*a^7*c^7 - 24*a^2*b^10*c^2 + 240*a^3*
b^8*c^3 - 1280*a^4*b^6*c^4 + 3840*a^5*b^4*c^5 - 6144*a^6*b^2*c^6 + a*b^12*c
)))^(1/2) + (((16*A*b^7*c^2 + 2048*B*a^4*c^5 - 192*A*a*b^5*c^3 - 1024*A*a^3
*b*c^5 - 32*B*a*b^6*c^2 + 768*A*a^2*b^3*c^4 + 384*B*a^2*b^4*c^3 - 1536*B*a^
3*b^2*c^4)/(8*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)) + (x*(-(B^2
*a*b^9 + B^2*a*(-(4*a*c - b^2)^9)^(1/2) + A^2*b^9*c - A^2*c*(-(4*a*c - b^2)
^9)^(1/2) - 96*A^2*a^2*b^5*c^3 + 512*A^2*a^3*b^3*c^4 - 96*B^2*a^3*b^5*c^2 +
 512*B^2*a^4*b^3*c^3 + 1024*A*B*a^5*c^5 - 768*A^2*a^4*b*c^5 - 768*B^2*a^5*b
*c^4 + 128*A*B*a^2*b^6*c^2 - 384*A*B*a^3*b^4*c^3 - 12*A*B*a*b^8*c)/(32*(409
6*a^7*c^7 - 24*a^2*b^10*c^2 + 240*a^3*b^8*c^3 - 1280*a^4*b^6*c^4 + 3840*a^5
*b^4*c^5 - 6144*a^6*b^2*c^6 + a*b^12*c)))^(1/2)*(16*b^7*c^2 - 192*a*b^5*c^3
 - 1024*a^3*b*c^5 + 768*a^2*b^3*c^4))/(2*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)))*(
-(B^2*a*b^9 + B^2*a*(-(4*a*c - b^2)^9)^(1/2) + A^2*b^9*c - A^2*c*(-(4*a*c -
 b^2)^9)^(1/2) - 96*A^2*a^2*b^5*c^3 + 512*A^2*a^3*b^3*c^4 - 96*B^2*a^3*b^5*
c^2 + 512*B^2*a^4*b^3*c^3 + 1024*A*B*a^5*c^5 - 768*A^2*a^4*b*c^5 - 768*B^2*
a^5*b*c^4 + 128*A*B*a^2*b^6*c^2 - 384*A*B*a^3*b^4*c^3 - 12*A*B*a*b^8*c)/(32
*(4096*a^7*c^7 - 24*a^2*b^10*c^2 + 240*a^3*b^8*c^3 - 1280*a^4*b^6*c^4 + 384
0*a^5*b^4*c^5 - 6144*a^6*b^2*c^6 + a*b^12*c)))^(1/2) + (x*(B^2*b^4*c - 8*A^
2*a*c^4 + 10*A^2*b^2*c^3 + 8*B^2*a^2*c^3 - 6*A*B*b^3*c^2 + 2*B^2*a*b^2*c^2
 - 8*A*B*a*b*c^3))/(2*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)))*(-(B^2*a*b^9 + B^2*a*
(-(4*a*c - b^2)^9)^(1/2) + A^2*b^9*c - A^2*c*(-(4*a*c - b^2)^9)^(1/2) - 96*
A^2*a^2*b^5*c^3 + 512*A^2*a^3*b^3*c^4 - 96*B^2*a^3*b^5*c^2 + 512*B^2*a^4*b^
3*c^3 + 1024*A*B*a^5*c^5 - 768*A^2*a^4*b*c^5 - 768*B^2*a^5*b*c^4 + 128*A*B*
a^2*b^6*c^2 - 384*A*B*a^3*b^4*c^3 - 12*A*B*a*b^8*c)/(32*(4096*a^7*c^7 - 24*
a^2*b^10*c^2 + 240*a^3*b^8*c^3 - 1280*a^4*b^6*c^4 + 3840*a^5*b^4*c^5 - 6144
*a^6*b^2*c^6 + a*b^12*c)))^(1/2)))*(-(B^2*a*b^9 + B^2*a*(-(4*a*c - b^2)^9)^
(1/2) + A^2*b^9*c - A^2*c*(-(4*a*c - b^2)^9)^(1/2) - 96*A^2*a^2*b^5*c^3 + 5
12*A^2*a^3*b^3*c^4 - 96*B^2*a^3*b^5*c^2 + 512*B^2*a^4*b^3*c^3 + 1024*A*B*a^
5*c^5 - 768*A^2*a^4*b*c^5 - 768*B^2*a^5*b*c^4 + 128*A*B*a^2*b^6*c^2 - 384*A
*B*a^3*b^4*c^3 - 12*A*B*a*b^8*c)/(32*(4096*a^7*c^7 - 24*a^2*b^10*c^2 + 240*
a^3*b^8*c^3 - 1280*a^4*b^6*c^4 + 3840*a^5*b^4*c^5 - 6144*a^6*b^2*c^6 + a*b^
12*c)))^(1/2)*2i

3.121 $\int \frac{A+Bx^2}{(a+bx^2+cx^4)^2}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 293

$$\int \frac{A+Bx^2}{(a+bx^2+cx^4)^2}\,dx = \frac{x(Ab^2-abB-2aAc+(Ab-2aB)cx^2)}{2a(b^2-4ac)(a+bx^2+cx^4)} + \frac{\sqrt{c}\left(Ab-2aB+\frac{4abB+A(b^2-12ac)}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}a(b^2-4ac)\sqrt{b-\sqrt{b^2-4ac}}} + \frac{\sqrt{c}\left(Ab-2aB-\frac{Ab^2+4abB-12aAc}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}a(b^2-4ac)\sqrt{b+\sqrt{b^2-4ac}}}$$

```
[Out] 1/2*x*(A*b^2-a*b*B-2*A*a*c+(A*b-2*B*a)*c*x^2)/a/(-4*a*c+b^2)/(c*x^4+b*x^2+a
)+1/4*arctan(x*2^(1/2)*c^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2))*c^(1/2)*(A*b-2
*B*a+(4*a*b*B+A*(-12*a*c+b^2))/(-4*a*c+b^2)^(1/2))/a/(-4*a*c+b^2)*2^(1/2)/(
b-(-4*a*c+b^2)^(1/2))^(1/2)+1/4*arctan(x*2^(1/2)*c^(1/2)/(b+(-4*a*c+b^2)^(1
/2))^(1/2))*c^(1/2)*(A*b-2*B*a+(12*A*a*c-A*b^2-4*B*a*b)/(-4*a*c+b^2)^(1/2))
/a/(-4*a*c+b^2)*2^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2)
```

Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 293, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used

$= \{1192, 1180, 211\}$

$$\int \frac{A+Bx^2}{\left(a+bx^2+cx^4\right)^2}\,dx = \frac{\sqrt{c}\left(\frac{A(b^2-12ac)+4abB}{\sqrt{b^2-4ac}}-2aB+Ab\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}a\left(b^2-4ac\right)\sqrt{b-\sqrt{b^2-4ac}}} + \frac{\sqrt{c}\left(-\frac{-12aAc+4abB+Ab^2}{\sqrt{b^2-4ac}}-2aB+Ab\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{2\sqrt{2}a\left(b^2-4ac\right)\sqrt{\sqrt{b^2-4ac}+b}} + \frac{x(cx^2(Ab-2aB)-2aAc-abB+Ab^2)}{2a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}$$

```
[In] Int[(A + B*x^2)/(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] (x*(A*b^2 - a*b*B - 2*a*A*c + (A*b - 2*a*B)*c*x^2))/(2*a*(b^2 - 4*a*c)*(a +
 b*x^2 + c*x^4)) + (Sqrt[c]*(A*b - 2*a*B + (4*a*b*B + A*(b^2 - 12*a*c))/Sqr
t[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(2
*Sqrt[2]*a*(b^2 - 4*a*c)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (Sqrt[c]*(A*b - 2*a
*B - (A*b^2 + 4*a*b*B - 12*a*A*c)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c
]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]*a*(b^2 - 4*a*c)*Sqrt[b + Sqrt
[b^2 - 4*a*c]])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1192

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[x*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)*((a + b*x^2 +
 c*x^4)^(p + 1)/(2*a*(p + 1)*(b^2 - 4*a*c))), x] + Dist[1/(2*a*(p + 1)*(b^2
 - 4*a*c)), Int[Simp[(2*p + 3)*d*b^2 - a*b*e - 2*a*c*d*(4*p + 5) + (4*p + 7
)*(d*b - 2*a*e)*c*x^2, x]*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; FreeQ[{a,
b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
LtQ[p, -1] && IntegerQ[2*p]
```

Rubi steps

$$\text{integral} = \frac{x(Ab^2 - abB - 2aAc + (Ab - 2aB)cx^2)}{2a(b^2-4ac)(a+bx^2+cx^4)} - \frac{\int \frac{-Ab^2-abB+6aAc-(Ab-2aB)cx^2}{a+bx^2+cx^4}\,dx}{2a(b^2-4ac)}$$

$$= \frac{x(Ab^2 - abB - 2aAc + (Ab - 2aB)cx^2)}{2a(b^2-4ac)(a+bx^2+cx^4)} + \frac{\left(c\left(Ab - 2aB - \frac{Ab^2+4abB-12aAc}{\sqrt{b^2-4ac}}\right)\right)\int \frac{1}{\frac{b}{2}+\frac{1}{2}\sqrt{b^2-4ac}+cx^2}\,dx}{4a(b^2-4ac)} + \frac{\left(c\left(2aB\left(2b-\sqrt{b^2-4ac}\right)+A\left(b^2-12ac+b\sqrt{b^2-4ac}\right)\right)\right)\int \frac{1}{\frac{b}{2}-\frac{1}{2}\sqrt{b^2-4ac}+cx^2}\,dx}{4a(b^2-4ac)^{3/2}}$$

$$= \frac{x(Ab^2 - abB - 2aAc + (Ab - 2aB)cx^2)}{2a(b^2-4ac)(a+bx^2+cx^4)} + \frac{\sqrt{c}\left(2aB\left(2b-\sqrt{b^2-4ac}\right)+A\left(b^2-12ac+b\sqrt{b^2-4ac}\right)\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}a(b^2-4ac)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}} + \frac{\sqrt{c}\left(Ab-2aB-\frac{Ab^2+4abB-12aAc}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}a(b^2-4ac)\sqrt{b+\sqrt{b^2-4ac}}}$$

Mathematica [A] (verified)

Time = 0.51 (sec) , antiderivative size = 304, normalized size of antiderivative = 1.04

$$\int \frac{A+Bx^2}{(a+bx^2+cx^4)^2}\,dx$$

$$= \frac{\frac{2x\left(-aB(b+2cx^2)+A(b^2-2ac+bcx^2)\right)}{(b^2-4ac)(a+bx^2+cx^4)} + \frac{\sqrt{2}\sqrt{c}\left(-2aB\left(-2b+\sqrt{b^2-4ac}\right)+A\left(b^2-12ac+b\sqrt{b^2-4ac}\right)\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{(b^2-4ac)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}} + \frac{\sqrt{2}\sqrt{c}\left(-2aB\right.}{}}{4a}$$

```
[In] Integrate[(A + B*x^2)/(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] ((2*x*(-(a*B*(b + 2*c*x^2)) + A*(b^2 - 2*a*c + b*c*x^2)))/((b^2 - 4*a*c)*(a
 + b*x^2 + c*x^4)) + (Sqrt[2]*Sqrt[c]*(-2*a*B*(-2*b + Sqrt[b^2 - 4*a*c]) +
A*(b^2 - 12*a*c + b*Sqrt[b^2 - 4*a*c]))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b -
 Sqrt[b^2 - 4*a*c]]])/((b^2 - 4*a*c)^(3/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (
Sqrt[2]*Sqrt[c]*(-2*a*B*(2*b + Sqrt[b^2 - 4*a*c]) + A*(-b^2 + 12*a*c + b*Sq
rt[b^2 - 4*a*c]))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/
((b^2 - 4*a*c)^(3/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]]))/(4*a)
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.18 (sec) , antiderivative size = 178, normalized size of antiderivative = 0.61

method	result
risch	$\frac{-\frac{c(Ab-2Ba)x^3}{2a(4ac-b^2)}+\frac{(2Aac-Ab^2+abB)x}{2(4ac-b^2)a}}{cx^4+bx^2+a}+\frac{\sum_{\_R=\text{RootOf}(c\_Z^4+\_Z^2b+a)}\frac{\left(-\frac{c(Ab-2Ba)\_R^2}{4ac-b^2}+\frac{6Aac-Ab^2-abB}{4ac-b^2}\right)\ln(x-\_R)}{2c\_R^3+\_Rb}}{4a}$
default	$16c^2\left(-\frac{-\frac{\left(-A\sqrt{-4ac+b^2}-Ab+2Ba\right)\sqrt{-4ac+b^2}\,x}{16ac\left(x^2+\frac{b}{2c}-\frac{\sqrt{-4ac+b^2}}{2c}\right)}-\frac{\left(12A\sqrt{-4ac+b^2}\,ac-3A\sqrt{-4ac+b^2}\,b^2+28Aabc-3Ab^3-8a^2Bc-6Bab^2\right)\left(\sqrt{-4ac+b^2}-2\cdots\right.}{16a\left(4ac+3b^2\right)\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}}}{4c(4ac-b^2)\sqrt{-4ac+b^2}}\right.$ ⋯

```
[In] int((B*x^2+A)/(c*x^4+b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] (-1/2*c*(A*b-2*B*a)/a/(4*a*c-b^2)*x^3+1/2*(2*A*a*c-A*b^2+B*a*b)/(4*a*c-b^2)
/a*x)/(c*x^4+b*x^2+a)+1/4/a*sum((-c*(A*b-2*B*a)/(4*a*c-b^2)*_R^2+(6*A*a*c-A
*b^2-B*a*b)/(4*a*c-b^2))/(2*_R^3*c+_R*b)*ln(x-_R),_R=RootOf(_Z^4*c+_Z^2*b+a
))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 4885 vs. 2(254) = 508.

Time = 3.28 (sec) , antiderivative size = 4885, normalized size of antiderivative = 16.67

$$\int \frac{A+Bx^2}{(a+bx^2+cx^4)^2}\,dx = \text{Too large to display}$$

```
[In] integrate((B*x^2+A)/(c*x^4+b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] -1/4*(2*(2*B*a - A*b)*c*x^3 - sqrt(1/2)*((a*b^2*c - 4*a^2*c^2)*x^4 + a^2*b^
2 - 4*a^3*c + (a*b^3 - 4*a^2*b*c)*x^2)*sqrt(-(B^2*a^2*b^3 + 2*A*B*a*b^4 + A
^2*b^5 - 12*(4*A*B*a^3 - 5*A^2*a^2*b)*c^2 + 3*(4*B^2*a^3*b - 4*A*B*a^2*b^2
- 5*A^2*a*b^3)*c + (a^3*b^6 - 12*a^4*b^4*c + 48*a^5*b^2*c^2 - 64*a^6*c^3)*s
qrt((B^4*a^4 + 4*A*B^3*a^3*b + 6*A^2*B^2*a^2*b^2 + 4*A^3*B*a*b^3 + A^4*b^4
+ 81*A^4*a^2*c^2 - 18*(A^2*B^2*a^3 + 2*A^3*B*a^2*b + A^4*a*b^2)*c)/(a^6*b^6
 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3)))/(a^3*b^6 - 12*a^4*b^4*c +
48*a^5*b^2*c^2 - 64*a^6*c^3))*log((324*A^4*a^2*c^4 - 81*(4*A^3*B*a^2*b + A^
4*a*b^2)*c^3 - (4*B^4*a^4 - 20*A*B^3*a^3*b - 84*A^2*B^2*a^2*b^2 - 65*A^3*B*
a*b^3 - 5*A^4*b^4)*c^2 - 3*(B^4*a^3*b^2 + 3*A*B^3*a^2*b^3 + 3*A^2*B^2*a*b^4
```

```
 + A^3*B*b^5)*c)*x + 1/2*sqrt(1/2)*(B^3*a^3*b^5 + 3*A*B^2*a^2*b^6 + 3*A^2*B
*a*b^7 + A^3*b^8 + 864*A^3*a^4*c^4 - 48*(2*A*B^2*a^5 + 7*A^2*B*a^4*b + 14*A
^3*a^3*b^2)*c^3 + 2*(8*B^3*a^5*b + 48*A*B^2*a^4*b^2 + 108*A^2*B*a^3*b^3 + 9
5*A^3*a^2*b^4)*c^2 - (8*B^3*a^4*b^3 + 30*A*B^2*a^3*b^4 + 45*A^2*B*a^2*b^5 +
 23*A^3*a*b^6)*c - (B*a^4*b^8 + A*a^3*b^9 + 144*A*a^5*b^5*c^2 - 256*(B*a^8
- 2*A*a^7*b)*c^4 + 64*(2*B*a^7*b^2 - 7*A*a^6*b^3)*c^3 - 4*(2*B*a^5*b^6 + 5*
A*a^4*b^7)*c)*sqrt((B^4*a^4 + 4*A*B^3*a^3*b + 6*A^2*B^2*a^2*b^2 + 4*A^3*B*a
*b^3 + A^4*b^4 + 81*A^4*a^2*c^2 - 18*(A^2*B^2*a^3 + 2*A^3*B*a^2*b + A^4*a*b
^2)*c)/(a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3)))*sqrt(-(B^2*
a^2*b^3 + 2*A*B*a*b^4 + A^2*b^5 - 12*(4*A*B*a^3 - 5*A^2*a^2*b)*c^2 + 3*(4*B
^2*a^3*b - 4*A*B*a^2*b^2 - 5*A^2*a*b^3)*c + (a^3*b^6 - 12*a^4*b^4*c + 48*a^
5*b^2*c^2 - 64*a^6*c^3)*sqrt((B^4*a^4 + 4*A*B^3*a^3*b + 6*A^2*B^2*a^2*b^2 +
 4*A^3*B*a*b^3 + A^4*b^4 + 81*A^4*a^2*c^2 - 18*(A^2*B^2*a^3 + 2*A^3*B*a^2*b
 + A^4*a*b^2)*c)/(a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3)))/(
a^3*b^6 - 12*a^4*b^4*c + 48*a^5*b^2*c^2 - 64*a^6*c^3))) + sqrt(1/2)*((a*b^2
*c - 4*a^2*c^2)*x^4 + a^2*b^2 - 4*a^3*c + (a*b^3 - 4*a^2*b*c)*x^2)*sqrt(-(B
^2*a^2*b^3 + 2*A*B*a*b^4 + A^2*b^5 - 12*(4*A*B*a^3 - 5*A^2*a^2*b)*c^2 + 3*(
4*B^2*a^3*b - 4*A*B*a^2*b^2 - 5*A^2*a*b^3)*c + (a^3*b^6 - 12*a^4*b^4*c + 48
*a^5*b^2*c^2 - 64*a^6*c^3)*sqrt((B^4*a^4 + 4*A*B^3*a^3*b + 6*A^2*B^2*a^2*b^
2 + 4*A^3*B*a*b^3 + A^4*b^4 + 81*A^4*a^2*c^2 - 18*(A^2*B^2*a^3 + 2*A^3*B*a^
2*b + A^4*a*b^2)*c)/(a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3))
)/(a^3*b^6 - 12*a^4*b^4*c + 48*a^5*b^2*c^2 - 64*a^6*c^3))*log((324*A^4*a^2*
c^4 - 81*(4*A^3*B*a^2*b + A^4*a*b^2)*c^3 - (4*B^4*a^4 - 20*A*B^3*a^3*b - 84
*A^2*B^2*a^2*b^2 - 65*A^3*B*a*b^3 - 5*A^4*b^4)*c^2 - 3*(B^4*a^3*b^2 + 3*A*B
^3*a^2*b^3 + 3*A^2*B^2*a*b^4 + A^3*B*b^5)*c)*x - 1/2*sqrt(1/2)*(B^3*a^3*b^5
 + 3*A*B^2*a^2*b^6 + 3*A^2*B*a*b^7 + A^3*b^8 + 864*A^3*a^4*c^4 - 48*(2*A*B^
2*a^5 + 7*A^2*B*a^4*b + 14*A^3*a^3*b^2)*c^3 + 2*(8*B^3*a^5*b + 48*A*B^2*a^4
*b^2 + 108*A^2*B*a^3*b^3 + 95*A^3*a^2*b^4)*c^2 - (8*B^3*a^4*b^3 + 30*A*B^2*
a^3*b^4 + 45*A^2*B*a^2*b^5 + 23*A^3*a*b^6)*c - (B*a^4*b^8 + A*a^3*b^9 + 144
*A*a^5*b^5*c^2 - 256*(B*a^8 - 2*A*a^7*b)*c^4 + 64*(2*B*a^7*b^2 - 7*A*a^6*b^
3)*c^3 - 4*(2*B*a^5*b^6 + 5*A*a^4*b^7)*c)*sqrt((B^4*a^4 + 4*A*B^3*a^3*b + 6
*A^2*B^2*a^2*b^2 + 4*A^3*B*a*b^3 + A^4*b^4 + 81*A^4*a^2*c^2 - 18*(A^2*B^2*a
^3 + 2*A^3*B*a^2*b + A^4*a*b^2)*c)/(a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2
 - 64*a^9*c^3)))*sqrt(-(B^2*a^2*b^3 + 2*A*B*a*b^4 + A^2*b^5 - 12*(4*A*B*a^3
 - 5*A^2*a^2*b)*c^2 + 3*(4*B^2*a^3*b - 4*A*B*a^2*b^2 - 5*A^2*a*b^3)*c + (a^
3*b^6 - 12*a^4*b^4*c + 48*a^5*b^2*c^2 - 64*a^6*c^3)*sqrt((B^4*a^4 + 4*A*B^3
*a^3*b + 6*A^2*B^2*a^2*b^2 + 4*A^3*B*a*b^3 + A^4*b^4 + 81*A^4*a^2*c^2 - 18*
(A^2*B^2*a^3 + 2*A^3*B*a^2*b + A^4*a*b^2)*c)/(a^6*b^6 - 12*a^7*b^4*c + 48*a
^8*b^2*c^2 - 64*a^9*c^3)))/(a^3*b^6 - 12*a^4*b^4*c + 48*a^5*b^2*c^2 - 64*a^
6*c^3))) - sqrt(1/2)*((a*b^2*c - 4*a^2*c^2)*x^4 + a^2*b^2 - 4*a^3*c + (a*b^
3 - 4*a^2*b*c)*x^2)*sqrt(-(B^2*a^2*b^3 + 2*A*B*a*b^4 + A^2*b^5 - 12*(4*A*B*
a^3 - 5*A^2*a^2*b)*c^2 + 3*(4*B^2*a^3*b - 4*A*B*a^2*b^2 - 5*A^2*a*b^3)*c -
(a^3*b^6 - 12*a^4*b^4*c + 48*a^5*b^2*c^2 - 64*a^6*c^3)*sqrt((B^4*a^4 + 4*A*
B^3*a^3*b + 6*A^2*B^2*a^2*b^2 + 4*A^3*B*a*b^3 + A^4*b^4 + 81*A^4*a^2*c^2 -
18*(A^2*B^2*a^3 + 2*A^3*B*a^2*b + A^4*a*b^2)*c)/(a^6*b^6 - 12*a^7*b^4*c + 4
```

```
8*a^8*b^2*c^2 - 64*a^9*c^3)))/(a^3*b^6 - 12*a^4*b^4*c + 48*a^5*b^2*c^2 - 64
*a^6*c^3))*log((324*A^4*a^2*c^4 - 81*(4*A^3*B*a^2*b + A^4*a*b^2)*c^3 - (4*B
^4*a^4 - 20*A*B^3*a^3*b - 84*A^2*B^2*a^2*b^2 - 65*A^3*B*a*b^3 - 5*A^4*b^4)*
c^2 - 3*(B^4*a^3*b^2 + 3*A*B^3*a^2*b^3 + 3*A^2*B^2*a*b^4 + A^3*B*b^5)*c)*x
+ 1/2*sqrt(1/2)*(B^3*a^3*b^5 + 3*A*B^2*a^2*b^6 + 3*A^2*B*a*b^7 + A^3*b^8 +
864*A^3*a^4*c^4 - 48*(2*A*B^2*a^5 + 7*A^2*B*a^4*b + 14*A^3*a^3*b^2)*c^3 + 2
*(8*B^3*a^5*b + 48*A*B^2*a^4*b^2 + 108*A^2*B*a^3*b^3 + 95*A^3*a^2*b^4)*c^2
- (8*B^3*a^4*b^3 + 30*A*B^2*a^3*b^4 + 45*A^2*B*a^2*b^5 + 23*A^3*a*b^6)*c +
(B*a^4*b^8 + A*a^3*b^9 + 144*A*a^5*b^5*c^2 - 256*(B*a^8 - 2*A*a^7*b)*c^4 +
64*(2*B*a^7*b^2 - 7*A*a^6*b^3)*c^3 - 4*(2*B*a^5*b^6 + 5*A*a^4*b^7)*c)*sqrt(
(B^4*a^4 + 4*A*B^3*a^3*b + 6*A^2*B^2*a^2*b^2 + 4*A^3*B*a*b^3 + A^4*b^4 + 81
*A^4*a^2*c^2 - 18*(A^2*B^2*a^3 + 2*A^3*B*a^2*b + A^4*a*b^2)*c)/(a^6*b^6 - 1
2*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3)))*sqrt(-(B^2*a^2*b^3 + 2*A*B*a*b
^4 + A^2*b^5 - 12*(4*A*B*a^3 - 5*A^2*a^2*b)*c^2 + 3*(4*B^2*a^3*b - 4*A*B*a^
2*b^2 - 5*A^2*a*b^3)*c - (a^3*b^6 - 12*a^4*b^4*c + 48*a^5*b^2*c^2 - 64*a^6*
c^3)*sqrt((B^4*a^4 + 4*A*B^3*a^3*b + 6*A^2*B^2*a^2*b^2 + 4*A^3*B*a*b^3 + A^
4*b^4 + 81*A^4*a^2*c^2 - 18*(A^2*B^2*a^3 + 2*A^3*B*a^2*b + A^4*a*b^2)*c)/(a
^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3)))/(a^3*b^6 - 12*a^4*b^
4*c + 48*a^5*b^2*c^2 - 64*a^6*c^3))) + sqrt(1/2)*((a*b^2*c - 4*a^2*c^2)*x^4
 + a^2*b^2 - 4*a^3*c + (a*b^3 - 4*a^2*b*c)*x^2)*sqrt(-(B^2*a^2*b^3 + 2*A*B*
a*b^4 + A^2*b^5 - 12*(4*A*B*a^3 - 5*A^2*a^2*b)*c^2 + 3*(4*B^2*a^3*b - 4*A*B
*a^2*b^2 - 5*A^2*a*b^3)*c - (a^3*b^6 - 12*a^4*b^4*c + 48*a^5*b^2*c^2 - 64*a
^6*c^3)*sqrt((B^4*a^4 + 4*A*B^3*a^3*b + 6*A^2*B^2*a^2*b^2 + 4*A^3*B*a*b^3 +
 A^4*b^4 + 81*A^4*a^2*c^2 - 18*(A^2*B^2*a^3 + 2*A^3*B*a^2*b + A^4*a*b^2)*c)
/(a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3)))/(a^3*b^6 - 12*a^4
*b^4*c + 48*a^5*b^2*c^2 - 64*a^6*c^3))*log((324*A^4*a^2*c^4 - 81*(4*A^3*B*a
^2*b + A^4*a*b^2)*c^3 - (4*B^4*a^4 - 20*A*B^3*a^3*b - 84*A^2*B^2*a^2*b^2 -
65*A^3*B*a*b^3 - 5*A^4*b^4)*c^2 - 3*(B^4*a^3*b^2 + 3*A*B^3*a^2*b^3 + 3*A^2*
B^2*a*b^4 + A^3*B*b^5)*c)*x - 1/2*sqrt(1/2)*(B^3*a^3*b^5 + 3*A*B^2*a^2*b^6
+ 3*A^2*B*a*b^7 + A^3*b^8 + 864*A^3*a^4*c^4 - 48*(2*A*B^2*a^5 + 7*A^2*B*a^4
*b + 14*A^3*a^3*b^2)*c^3 + 2*(8*B^3*a^5*b + 48*A*B^2*a^4*b^2 + 108*A^2*B*a^
3*b^3 + 95*A^3*a^2*b^4)*c^2 - (8*B^3*a^4*b^3 + 30*A*B^2*a^3*b^4 + 45*A^2*B*
a^2*b^5 + 23*A^3*a*b^6)*c + (B*a^4*b^8 + A*a^3*b^9 + 144*A*a^5*b^5*c^2 - 25
6*(B*a^8 - 2*A*a^7*b)*c^4 + 64*(2*B*a^7*b^2 - 7*A*a^6*b^3)*c^3 - 4*(2*B*a^5
*b^6 + 5*A*a^4*b^7)*c)*sqrt((B^4*a^4 + 4*A*B^3*a^3*b + 6*A^2*B^2*a^2*b^2 +
4*A^3*B*a*b^3 + A^4*b^4 + 81*A^4*a^2*c^2 - 18*(A^2*B^2*a^3 + 2*A^3*B*a^2*b
+ A^4*a*b^2)*c)/(a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3)))*sq
rt(-(B^2*a^2*b^3 + 2*A*B*a*b^4 + A^2*b^5 - 12*(4*A*B*a^3 - 5*A^2*a^2*b)*c^2
 + 3*(4*B^2*a^3*b - 4*A*B*a^2*b^2 - 5*A^2*a*b^3)*c - (a^3*b^6 - 12*a^4*b^4*
c + 48*a^5*b^2*c^2 - 64*a^6*c^3)*sqrt((B^4*a^4 + 4*A*B^3*a^3*b + 6*A^2*B^2*
a^2*b^2 + 4*A^3*B*a*b^3 + A^4*b^4 + 81*A^4*a^2*c^2 - 18*(A^2*B^2*a^3 + 2*A^
3*B*a^2*b + A^4*a*b^2)*c)/(a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9
*c^3)))/(a^3*b^6 - 12*a^4*b^4*c + 48*a^5*b^2*c^2 - 64*a^6*c^3))) + 2*(B*a*b
 - A*b^2 + 2*A*a*c)*x)/((a*b^2*c - 4*a^2*c^2)*x^4 + a^2*b^2 - 4*a^3*c + (a*
b^3 - 4*a^2*b*c)*x^2)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A+Bx^2}{(a+bx^2+cx^4)^2}\,dx = \text{Timed out}$$

```
[In] integrate((B*x**2+A)/(c*x**4+b*x**2+a)**2,x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{A+Bx^2}{(a+bx^2+cx^4)^2}\,dx = \int \frac{Bx^2+A}{(cx^4+bx^2+a)^2}\,dx$$

```
[In] integrate((B*x^2+A)/(c*x^4+b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] -1/2*((2*B*a - A*b)*c*x^3 + (B*a*b - A*b^2 + 2*A*a*c)*x)/((a*b^2*c - 4*a^2*
c^2)*x^4 + a^2*b^2 - 4*a^3*c + (a*b^3 - 4*a^2*b*c)*x^2) + 1/2*integrate(-((
2*B*a - A*b)*c*x^2 - B*a*b - A*b^2 + 6*A*a*c)/(c*x^4 + b*x^2 + a), x)/(a*b^
2 - 4*a^2*c)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 4426 vs. $2(254) = 508$.

Time = 1.38 (sec) , antiderivative size = 4426, normalized size of antiderivative = 15.11

$$\int \frac{A+Bx^2}{(a+bx^2+cx^4)^2}\,dx = \text{Too large to display}$$

```
[In] integrate((B*x^2+A)/(c*x^4+b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] -1/2*(2*B*a*c*x^3 - A*b*c*x^3 + B*a*b*x - A*b^2*x + 2*A*a*c*x)/((c*x^4 + b*
x^2 + a)*(a*b^2 - 4*a^2*c)) + 1/16*((2*b^3*c^2 - 8*a*b*c^3 - sqrt(2)*sqrt(b
^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^3 + 4*sqrt(2)*sqrt(b^2 - 4*a*
c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt
(b*c + sqrt(b^2 - 4*a*c)*c)*b^2*c - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sq
rt(b^2 - 4*a*c)*c)*b*c^2 - 2*(b^2 - 4*a*c)*b*c^2)*(a*b^2 - 4*a^2*c)^2*A - 2
*(2*a*b^2*c^2 - 8*a^2*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 -
 4*a*c)*c)*a*b^2 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)
*c)*a^2*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b
*c - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*c^2 - 2*(b
^2 - 4*a*c)*a*c^2)*(a*b^2 - 4*a^2*c)^2*B + 2*(sqrt(2)*sqrt(b*c + sqrt(b^2 -
```

4*a*c)*c)*a*b^6 - 14*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^4*c - 2
*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^5*c - 2*a*b^6*c + 64*sqrt(2)*s
qrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^2*c^2 + 20*sqrt(2)*sqrt(b*c + sqrt(b^2
 - 4*a*c)*c)*a^2*b^3*c^2 + sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^4*c^
2 + 28*a^2*b^4*c^2 - 96*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*c^3 - 4
8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b*c^3 - 10*sqrt(2)*sqrt(b*c +
 sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^3 - 128*a^3*b^2*c^3 + 24*sqrt(2)*sqrt(b*c +
 sqrt(b^2 - 4*a*c)*c)*a^3*c^4 + 192*a^4*c^4 + 2*(b^2 - 4*a*c)*a*b^4*c - 20*
(b^2 - 4*a*c)*a^2*b^2*c^2 + 48*(b^2 - 4*a*c)*a^3*c^3)*A*abs(a*b^2 - 4*a^2*c
) + 2*(sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^5 - 8*sqrt(2)*sqrt(b*c
 + sqrt(b^2 - 4*a*c)*c)*a^3*b^3*c - 2*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*
c)*a^2*b^4*c - 2*a^2*b^5*c + 16*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4
*b*c^2 + 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^2*c^2 + sqrt(2)*sq
rt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c^2 + 16*a^3*b^3*c^2 - 4*sqrt(2)*sqrt
(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b*c^3 - 32*a^4*b*c^3 + 2*(b^2 - 4*a*c)*a^2*
b^3*c - 8*(b^2 - 4*a*c)*a^3*b*c^2)*B*abs(a*b^2 - 4*a^2*c) + (2*a^2*b^7*c^2
 - 40*a^3*b^5*c^3 + 224*a^4*b^3*c^4 - 384*a^5*b*c^5 - sqrt(2)*sqrt(b^2 - 4*a
*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^7 + 20*sqrt(2)*sqrt(b^2 - 4*a*c)*
sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^5*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqr
t(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^6*c - 112*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt
(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b^3*c^2 - 32*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt
(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^4*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*
c + sqrt(b^2 - 4*a*c)*c)*a^2*b^5*c^2 + 192*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b
*c + sqrt(b^2 - 4*a*c)*c)*a^5*b*c^3 + 96*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c
 + sqrt(b^2 - 4*a*c)*c)*a^4*b^2*c^3 + 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c
 + sqrt(b^2 - 4*a*c)*c)*a^3*b^3*c^3 - 48*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c
 + sqrt(b^2 - 4*a*c)*c)*a^4*b*c^4 - 2*(b^2 - 4*a*c)*a^2*b^5*c^2 + 32*(b^2 -
 4*a*c)*a^3*b^3*c^3 - 96*(b^2 - 4*a*c)*a^4*b*c^4)*A + 4*(2*a^3*b^6*c^2 - 16
*a^4*b^4*c^3 + 32*a^5*b^2*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b
^2 - 4*a*c)*c)*a^3*b^6 + 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 -
4*a*c)*c)*a^4*b^4*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a
*c)*c)*a^3*b^5*c - 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c
)*c)*a^5*b^2*c^2 - 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)
*c)*a^4*b^3*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)
*a^3*b^4*c^2 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*
a^4*b^2*c^3 - 2*(b^2 - 4*a*c)*a^3*b^4*c^2 + 8*(b^2 - 4*a*c)*a^4*b^2*c^3)*B)
*arctan(2*sqrt(1/2)*x/sqrt((a*b^3 - 4*a^2*b*c + sqrt((a*b^3 - 4*a^2*b*c)^2
 - 4*(a^2*b^2 - 4*a^3*c)*(a*b^2*c - 4*a^2*c^2)))/(a*b^2*c - 4*a^2*c^2)))/((a
^3*b^6 - 12*a^4*b^4*c - 2*a^3*b^5*c + 48*a^5*b^2*c^2 + 16*a^4*b^3*c^2 + a^3
*b^4*c^2 - 64*a^6*c^3 - 32*a^5*b*c^3 - 8*a^4*b^2*c^3 + 16*a^5*c^4)*abs(a*b^
2 - 4*a^2*c)*abs(c)) - 1/16*((2*b^3*c^2 - 8*a*b*c^3 - sqrt(2)*sqrt(b^2 - 4*
a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^3 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt
(b*c - sqrt(b^2 - 4*a*c)*c)*a*b*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c -
sqrt(b^2 - 4*a*c)*c)*b^2*c - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2
 - 4*a*c)*c)*b*c^2 - 2*(b^2 - 4*a*c)*b*c^2)*(a*b^2 - 4*a^2*c)^2*A - 2*(2*a*b

^2*c^2 - 8*a^2*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^2 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b*c - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*c^2 - 2*(b^2 - 4*a*c)*a*c^2)*(a*b^2 - 4*a^2*c)^2*B - 2*(sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^6 - 14*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^4*c - 2*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^5*c + 2*a*b^6*c + 64*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^2*c^2 + 20*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c^2 + sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^4*c^2 - 28*a^2*b^4*c^2 - 96*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*c^3 - 48*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b*c^3 - 10*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^3 + 128*a^3*b^2*c^3 + 24*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*c^4 - 192*a^4*c^4 - 2*(b^2 - 4*a*c)*a*b^4*c + 20*(b^2 - 4*a*c)*a^2*b^2*c^2 - 48*(b^2 - 4*a*c)*a^3*c^3)*A*abs(a*b^2 - 4*a^2*c) - 2*(sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^5 - 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^3*c - 2*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^4*c + 2*a^2*b^5*c + 16*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*b*c^2 + 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^2*c^2 + sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c^2 - 16*a^3*b^3*c^2 - 4*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b*c^3 + 32*a^4*b*c^3 - 2*(b^2 - 4*a*c)*a^2*b^3*c + 8*(b^2 - 4*a*c)*a^3*b*c^2)*B*abs(a*b^2 - 4*a^2*c) + (2*a^2*b^7*c^2 - 40*a^3*b^5*c^3 + 224*a^4*b^3*c^4 - 384*a^5*b*c^5 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^7 + 20*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^5*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^6*c - 112*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*b^3*c^2 - 32*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^4*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^5*c^2 + 192*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^5*b*c^3 + 96*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*b^2*c^3 + 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^3*c^3 - 48*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*b*c^4 - 2*(b^2 - 4*a*c)*a^2*b^5*c^2 + 32*(b^2 - 4*a*c)*a^3*b^3*c^3 - 96*(b^2 - 4*a*c)*a^4*b*c^4)*A + 4*(2*a^3*b^6*c^2 - 16*a^4*b^4*c^3 + 32*a^5*b^2*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^6 + 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*b^4*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^5*c - 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^5*b^2*c^2 - 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*b^3*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^4*c^2 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*b^2*c^3 - 2*(b^2 - 4*a*c)*a^3*b^4*c^2 + 8*(b^2 - 4*a*c)*a^4*b^2*c^3)*B)*arctan(2*sqrt(1/2)*x/sqrt((a*b^3 - 4*a^2*b*c - sqrt((a*b^3 - 4*a^2*b*c)^2 - 4*(a^2*b^2 - 4*a^3*c)*(a*b^2*c - 4*a^2*c^2)))/(a*b^2*c - 4*a^2*c^2)))/((a^3*b^6 - 12*a^4*b^4*c - 2*a^3*b^5*c + 48*a^5*b^2*c^2 + 16*a^4*b^3*c^2 + a^3*b^4*c^2 - 64*a^6*c^3 - 32*a^5*b*c^3 - 8*a^4*b^2*c^3 + 16*a^5*c^4)*abs(a*b^2 - 4*a

```
^2*c)*abs(c))
```

Mupad [B] (verification not implemented)

Time = 10.67 (sec) , antiderivative size = 12349, normalized size of antiderivative = 42.15

$$\int \frac{A+Bx^2}{(a+bx^2+cx^4)^2}\,dx = \text{Too large to display}$$

```
[In] int((A + B*x^2)/(a + b*x^2 + c*x^4)^2,x)
```

```
[Out] atan(((((6144*A*a^5*c^6 + 16*A*a*b^8*c^2 - 1024*B*a^5*b*c^5 - 288*A*a^2*b^6
*c^3 + 1920*A*a^3*b^4*c^4 - 5632*A*a^4*b^2*c^5 + 16*B*a^2*b^7*c^2 - 192*B*a
^3*b^5*c^3 + 768*B*a^4*b^3*c^4)/(8*(a^2*b^6 - 64*a^5*c^3 - 12*a^3*b^4*c + 4
8*a^4*b^2*c^2)) - (x*(-(A^2*b^11 + B^2*a^2*b^9 + A^2*b^2*(-(4*a*c - b^2)^9)
^(1/2) + B^2*a^2*(-(4*a*c - b^2)^9)^(1/2) + 2*A*B*a*b^10 + 288*A^2*a^2*b^7*
c^2 - 1504*A^2*a^3*b^5*c^3 + 3840*A^2*a^4*b^3*c^4 - 96*B^2*a^4*b^5*c^2 + 51
2*B^2*a^5*b^3*c^3 + 3072*A*B*a^6*c^5 - 27*A^2*a*b^9*c - 9*A^2*a*c*(-(4*a*c
- b^2)^9)^(1/2) - 3840*A^2*a^5*b*c^5 - 768*B^2*a^6*b*c^4 + 192*A*B*a^3*b^6*
c^2 - 128*A*B*a^4*b^4*c^3 - 1536*A*B*a^5*b^2*c^4 + 2*A*B*a*b*(-(4*a*c - b^2
)^9)^(1/2) - 36*A*B*a^2*b^8*c)/(32*(a^3*b^12 + 4096*a^9*c^6 - 24*a^4*b^10*c
 + 240*a^5*b^8*c^2 - 1280*a^6*b^6*c^3 + 3840*a^7*b^4*c^4 - 6144*a^8*b^2*c^5
)))^(1/2)*(1024*a^5*b*c^5 - 16*a^2*b^7*c^2 + 192*a^3*b^5*c^3 - 768*a^4*b^3*
c^4))/(2*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)))*(-(A^2*b^11 + B^2*a^2*b^9 +
 A^2*b^2*(-(4*a*c - b^2)^9)^(1/2) + B^2*a^2*(-(4*a*c - b^2)^9)^(1/2) + 2*A*
B*a*b^10 + 288*A^2*a^2*b^7*c^2 - 1504*A^2*a^3*b^5*c^3 + 3840*A^2*a^4*b^3*c^
4 - 96*B^2*a^4*b^5*c^2 + 512*B^2*a^5*b^3*c^3 + 3072*A*B*a^6*c^5 - 27*A^2*a*
b^9*c - 9*A^2*a*c*(-(4*a*c - b^2)^9)^(1/2) - 3840*A^2*a^5*b*c^5 - 768*B^2*a
^6*b*c^4 + 192*A*B*a^3*b^6*c^2 - 128*A*B*a^4*b^4*c^3 - 1536*A*B*a^5*b^2*c^4
 + 2*A*B*a*b*(-(4*a*c - b^2)^9)^(1/2) - 36*A*B*a^2*b^8*c)/(32*(a^3*b^12 + 4
096*a^9*c^6 - 24*a^4*b^10*c + 240*a^5*b^8*c^2 - 1280*a^6*b^6*c^3 + 3840*a^7
*b^4*c^4 - 6144*a^8*b^2*c^5)))^(1/2) + (x*(72*A^2*a^2*c^5 + A^2*b^4*c^3 - 8
*B^2*a^3*c^4 + 10*B^2*a^2*b^2*c^3 - 14*A^2*a*b^2*c^4 + 2*A*B*a*b^3*c^3 - 40
*A*B*a^2*b*c^4))/(2*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)))*(-(A^2*b^11 + B^
2*a^2*b^9 + A^2*b^2*(-(4*a*c - b^2)^9)^(1/2) + B^2*a^2*(-(4*a*c - b^2)^9)^(
1/2) + 2*A*B*a*b^10 + 288*A^2*a^2*b^7*c^2 - 1504*A^2*a^3*b^5*c^3 + 3840*A^2
*a^4*b^3*c^4 - 96*B^2*a^4*b^5*c^2 + 512*B^2*a^5*b^3*c^3 + 3072*A*B*a^6*c^5
- 27*A^2*a*b^9*c - 9*A^2*a*c*(-(4*a*c - b^2)^9)^(1/2) - 3840*A^2*a^5*b*c^5
- 768*B^2*a^6*b*c^4 + 192*A*B*a^3*b^6*c^2 - 128*A*B*a^4*b^4*c^3 - 1536*A*B*
a^5*b^2*c^4 + 2*A*B*a*b*(-(4*a*c - b^2)^9)^(1/2) - 36*A*B*a^2*b^8*c)/(32*(a
^3*b^12 + 4096*a^9*c^6 - 24*a^4*b^10*c + 240*a^5*b^8*c^2 - 1280*a^6*b^6*c^3
 + 3840*a^7*b^4*c^4 - 6144*a^8*b^2*c^5)))^(1/2)*1i - (((6144*A*a^5*c^6 + 16
*A*a*b^8*c^2 - 1024*B*a^5*b*c^5 - 288*A*a^2*b^6*c^3 + 1920*A*a^3*b^4*c^4 -
5632*A*a^4*b^2*c^5 + 16*B*a^2*b^7*c^2 - 192*B*a^3*b^5*c^3 + 768*B*a^4*b^3*c
^4)/(8*(a^2*b^6 - 64*a^5*c^3 - 12*a^3*b^4*c + 48*a^4*b^2*c^2)) + (x*(-(A^2*
```

b^11 + B^2*a^2*b^9 + A^2*b^2*(-(4*a*c - b^2)^9)^(1/2) + B^2*a^2*(-(4*a*c - b^2)^9)^(1/2) + 2*A*B*a*b^10 + 288*A^2*a^2*b^7*c^2 - 1504*A^2*a^3*b^5*c^3 + 3840*A^2*a^4*b^3*c^4 - 96*B^2*a^4*b^5*c^2 + 512*B^2*a^5*b^3*c^3 + 3072*A*B *a^6*c^5 - 27*A^2*a*b^9*c - 9*A^2*a*c*(-(4*a*c - b^2)^9)^(1/2) - 3840*A^2*a ^5*b*c^5 - 768*B^2*a^6*b*c^4 + 192*A*B*a^3*b^6*c^2 - 128*A*B*a^4*b^4*c^3 - 1536*A*B*a^5*b^2*c^4 + 2*A*B*a*b*(-(4*a*c - b^2)^9)^(1/2) - 36*A*B*a^2*b^8* c)/(32*(a^3*b^12 + 4096*a^9*c^6 - 24*a^4*b^10*c + 240*a^5*b^8*c^2 - 1280*a^ 6*b^6*c^3 + 3840*a^7*b^4*c^4 - 6144*a^8*b^2*c^5)))^(1/2)*(1024*a^5*b*c^5 - 16*a^2*b^7*c^2 + 192*a^3*b^5*c^3 - 768*a^4*b^3*c^4))/(2*(a^2*b^4 + 16*a^4*c ^2 - 8*a^3*b^2*c)))*(-(A^2*b^11 + B^2*a^2*b^9 + A^2*b^2*(-(4*a*c - b^2)^9)^ (1/2) + B^2*a^2*(-(4*a*c - b^2)^9)^(1/2) + 2*A*B*a*b^10 + 288*A^2*a^2*b^7*c ^2 - 1504*A^2*a^3*b^5*c^3 + 3840*A^2*a^4*b^3*c^4 - 96*B^2*a^4*b^5*c^2 + 512 *B^2*a^5*b^3*c^3 + 3072*A*B*a^6*c^5 - 27*A^2*a*b^9*c - 9*A^2*a*c*(-(4*a*c - b^2)^9)^(1/2) - 3840*A^2*a^5*b*c^5 - 768*B^2*a^6*b*c^4 + 192*A*B*a^3*b^6*c ^2 - 128*A*B*a^4*b^4*c^3 - 1536*A*B*a^5*b^2*c^4 + 2*A*B*a*b*(-(4*a*c - b^2) ^9)^(1/2) - 36*A*B*a^2*b^8*c)/(32*(a^3*b^12 + 4096*a^9*c^6 - 24*a^4*b^10*c + 240*a^5*b^8*c^2 - 1280*a^6*b^6*c^3 + 3840*a^7*b^4*c^4 - 6144*a^8*b^2*c^5)))^(1/2) - (x*(72*A^2*a^2*c^5 + A^2*b^4*c^3 - 8*B^2*a^3*c^4 + 10*B^2*a^2*b^ 2*c^3 - 14*A^2*a*b^2*c^4 + 2*A*B*a*b^3*c^3 - 40*A*B*a^2*b*c^4))/(2*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)))*(-(A^2*b^11 + B^2*a^2*b^9 + A^2*b^2*(-(4*a*c - b^2)^9)^(1/2) + B^2*a^2*(-(4*a*c - b^2)^9)^(1/2) + 2*A*B*a*b^10 + 288*A^ 2*a^2*b^7*c^2 - 1504*A^2*a^3*b^5*c^3 + 3840*A^2*a^4*b^3*c^4 - 96*B^2*a^4*b^ 5*c^2 + 512*B^2*a^5*b^3*c^3 + 3072*A*B*a^6*c^5 - 27*A^2*a*b^9*c - 9*A^2*a*c *(-(4*a*c - b^2)^9)^(1/2) - 3840*A^2*a^5*b*c^5 - 768*B^2*a^6*b*c^4 + 192*A* B*a^3*b^6*c^2 - 128*A*B*a^4*b^4*c^3 - 1536*A*B*a^5*b^2*c^4 + 2*A*B*a*b*(-(4 *a*c - b^2)^9)^(1/2) - 36*A*B*a^2*b^8*c)/(32*(a^3*b^12 + 4096*a^9*c^6 - 24* a^4*b^10*c + 240*a^5*b^8*c^2 - 1280*a^6*b^6*c^3 + 3840*a^7*b^4*c^4 - 6144*a ^8*b^2*c^5)))^(1/2)*1i)/((((6144*A*a^5*c^6 + 16*A*a*b^8*c^2 - 1024*B*a^5*b* c^5 - 288*A*a^2*b^6*c^3 + 1920*A*a^3*b^4*c^4 - 5632*A*a^4*b^2*c^5 + 16*B*a^ 2*b^7*c^2 - 192*B*a^3*b^5*c^3 + 768*B*a^4*b^3*c^4)/(8*(a^2*b^6 - 64*a^5*c^3 - 12*a^3*b^4*c + 48*a^4*b^2*c^2)) - (x*(-(A^2*b^11 + B^2*a^2*b^9 + A^2*b^2 *(-(4*a*c - b^2)^9)^(1/2) + B^2*a^2*(-(4*a*c - b^2)^9)^(1/2) + 2*A*B*a*b^10 + 288*A^2*a^2*b^7*c^2 - 1504*A^2*a^3*b^5*c^3 + 3840*A^2*a^4*b^3*c^4 - 96*B ^2*a^4*b^5*c^2 + 512*B^2*a^5*b^3*c^3 + 3072*A*B*a^6*c^5 - 27*A^2*a*b^9*c - 9*A^2*a*c*(-(4*a*c - b^2)^9)^(1/2) - 3840*A^2*a^5*b*c^5 - 768*B^2*a^6*b*c^4 + 192*A*B*a^3*b^6*c^2 - 128*A*B*a^4*b^4*c^3 - 1536*A*B*a^5*b^2*c^4 + 2*A*B *a*b*(-(4*a*c - b^2)^9)^(1/2) - 36*A*B*a^2*b^8*c)/(32*(a^3*b^12 + 4096*a^9* c^6 - 24*a^4*b^10*c + 240*a^5*b^8*c^2 - 1280*a^6*b^6*c^3 + 3840*a^7*b^4*c^4 - 6144*a^8*b^2*c^5)))^(1/2)*(1024*a^5*b*c^5 - 16*a^2*b^7*c^2 + 192*a^3*b^5 *c^3 - 768*a^4*b^3*c^4))/(2*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)))*(-(A^2*b ^11 + B^2*a^2*b^9 + A^2*b^2*(-(4*a*c - b^2)^9)^(1/2) + B^2*a^2*(-(4*a*c - b ^2)^9)^(1/2) + 2*A*B*a*b^10 + 288*A^2*a^2*b^7*c^2 - 1504*A^2*a^3*b^5*c^3 + 3840*A^2*a^4*b^3*c^4 - 96*B^2*a^4*b^5*c^2 + 512*B^2*a^5*b^3*c^3 + 3072*A*B* a^6*c^5 - 27*A^2*a*b^9*c - 9*A^2*a*c*(-(4*a*c - b^2)^9)^(1/2) - 3840*A^2*a^ 5*b*c^5 - 768*B^2*a^6*b*c^4 + 192*A*B*a^3*b^6*c^2 - 128*A*B*a^4*b^4*c^3 - 1

536*A*B*a^5*b^2*c^4 + 2*A*B*a*b*(-(4*a*c - b^2)^9)^(1/2) - 36*A*B*a^2*b^8*c
)/(32*(a^3*b^12 + 4096*a^9*c^6 - 24*a^4*b^10*c + 240*a^5*b^8*c^2 - 1280*a^6
*b^6*c^3 + 3840*a^7*b^4*c^4 - 6144*a^8*b^2*c^5)))^(1/2) + (x*(72*A^2*a^2*c^
5 + A^2*b^4*c^3 - 8*B^2*a^3*c^4 + 10*B^2*a^2*b^2*c^3 - 14*A^2*a*b^2*c^4 + 2
*A*B*a*b^3*c^3 - 40*A*B*a^2*b*c^4))/(2*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)
))*(-(A^2*b^11 + B^2*a^2*b^9 + A^2*b^2*(-(4*a*c - b^2)^9)^(1/2) + B^2*a^2*(
-(4*a*c - b^2)^9)^(1/2) + 2*A*B*a*b^10 + 288*A^2*a^2*b^7*c^2 - 1504*A^2*a^3
*b^5*c^3 + 3840*A^2*a^4*b^3*c^4 - 96*B^2*a^4*b^5*c^2 + 512*B^2*a^5*b^3*c^3
+ 3072*A*B*a^6*c^5 - 27*A^2*a*b^9*c - 9*A^2*a*c*(-(4*a*c - b^2)^9)^(1/2) -
3840*A^2*a^5*b*c^5 - 768*B^2*a^6*b*c^4 + 192*A*B*a^3*b^6*c^2 - 128*A*B*a^4*
b^4*c^3 - 1536*A*B*a^5*b^2*c^4 + 2*A*B*a*b*(-(4*a*c - b^2)^9)^(1/2) - 36*A*
B*a^2*b^8*c)/(32*(a^3*b^12 + 4096*a^9*c^6 - 24*a^4*b^10*c + 240*a^5*b^8*c^2
 - 1280*a^6*b^6*c^3 + 3840*a^7*b^4*c^4 - 6144*a^8*b^2*c^5)))^(1/2) + (((614
4*A*a^5*c^6 + 16*A*a*b^8*c^2 - 1024*B*a^5*b*c^5 - 288*A*a^2*b^6*c^3 + 1920*
A*a^3*b^4*c^4 - 5632*A*a^4*b^2*c^5 + 16*B*a^2*b^7*c^2 - 192*B*a^3*b^5*c^3 +
 768*B*a^4*b^3*c^4)/(8*(a^2*b^6 - 64*a^5*c^3 - 12*a^3*b^4*c + 48*a^4*b^2*c^
2)) + (x*(-(A^2*b^11 + B^2*a^2*b^9 + A^2*b^2*(-(4*a*c - b^2)^9)^(1/2) + B^2
a^2(-(4*a*c - b^2)^9)^(1/2) + 2*A*B*a*b^10 + 288*A^2*a^2*b^7*c^2 - 1504*A
^2*a^3*b^5*c^3 + 3840*A^2*a^4*b^3*c^4 - 96*B^2*a^4*b^5*c^2 + 512*B^2*a^5*b^
3*c^3 + 3072*A*B*a^6*c^5 - 27*A^2*a*b^9*c - 9*A^2*a*c*(-(4*a*c - b^2)^9)^(1
/2) - 3840*A^2*a^5*b*c^5 - 768*B^2*a^6*b*c^4 + 192*A*B*a^3*b^6*c^2 - 128*A*
B*a^4*b^4*c^3 - 1536*A*B*a^5*b^2*c^4 + 2*A*B*a*b*(-(4*a*c - b^2)^9)^(1/2) -
 36*A*B*a^2*b^8*c)/(32*(a^3*b^12 + 4096*a^9*c^6 - 24*a^4*b^10*c + 240*a^5*b
^8*c^2 - 1280*a^6*b^6*c^3 + 3840*a^7*b^4*c^4 - 6144*a^8*b^2*c^5)))^(1/2)*(1
024*a^5*b*c^5 - 16*a^2*b^7*c^2 + 192*a^3*b^5*c^3 - 768*a^4*b^3*c^4))/(2*(a^
2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)))*(-(A^2*b^11 + B^2*a^2*b^9 + A^2*b^2*(-(
4*a*c - b^2)^9)^(1/2) + B^2*a^2*(-(4*a*c - b^2)^9)^(1/2) + 2*A*B*a*b^10 + 2
88*A^2*a^2*b^7*c^2 - 1504*A^2*a^3*b^5*c^3 + 3840*A^2*a^4*b^3*c^4 - 96*B^2*a
^4*b^5*c^2 + 512*B^2*a^5*b^3*c^3 + 3072*A*B*a^6*c^5 - 27*A^2*a*b^9*c - 9*A^
2*a*c*(-(4*a*c - b^2)^9)^(1/2) - 3840*A^2*a^5*b*c^5 - 768*B^2*a^6*b*c^4 + 1
92*A*B*a^3*b^6*c^2 - 128*A*B*a^4*b^4*c^3 - 1536*A*B*a^5*b^2*c^4 + 2*A*B*a*b
*(-(4*a*c - b^2)^9)^(1/2) - 36*A*B*a^2*b^8*c)/(32*(a^3*b^12 + 4096*a^9*c^6
- 24*a^4*b^10*c + 240*a^5*b^8*c^2 - 1280*a^6*b^6*c^3 + 3840*a^7*b^4*c^4 - 6
144*a^8*b^2*c^5)))^(1/2) - (x*(72*A^2*a^2*c^5 + A^2*b^4*c^3 - 8*B^2*a^3*c^4
 + 10*B^2*a^2*b^2*c^3 - 14*A^2*a*b^2*c^4 + 2*A*B*a*b^3*c^3 - 40*A*B*a^2*b*c
^4))/(2*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)))*(-(A^2*b^11 + B^2*a^2*b^9 +
A^2*b^2*(-(4*a*c - b^2)^9)^(1/2) + B^2*a^2*(-(4*a*c - b^2)^9)^(1/2) + 2*A*B
*a*b^10 + 288*A^2*a^2*b^7*c^2 - 1504*A^2*a^3*b^5*c^3 + 3840*A^2*a^4*b^3*c^4
 - 96*B^2*a^4*b^5*c^2 + 512*B^2*a^5*b^3*c^3 + 3072*A*B*a^6*c^5 - 27*A^2*a*b
^9*c - 9*A^2*a*c*(-(4*a*c - b^2)^9)^(1/2) - 3840*A^2*a^5*b*c^5 - 768*B^2*a^
6*b*c^4 + 192*A*B*a^3*b^6*c^2 - 128*A*B*a^4*b^4*c^3 - 1536*A*B*a^5*b^2*c^4
+ 2*A*B*a*b*(-(4*a*c - b^2)^9)^(1/2) - 36*A*B*a^2*b^8*c)/(32*(a^3*b^12 + 40
96*a^9*c^6 - 24*a^4*b^10*c + 240*a^5*b^8*c^2 - 1280*a^6*b^6*c^3 + 3840*a^7*
b^4*c^4 - 6144*a^8*b^2*c^5)))^(1/2) + (5*A^3*b^3*c^4 + 8*B^3*a^3*c^4 + 6*B^
3*a^2*b^2*c^3 - 36*A^3*a*b*c^5 + 72*A^2*B*a^2*c^5 - 3*A^2*B*b^4*c^3 + 3*A*B

^2*a*b^3*c^3 - 60*A*B^2*a^2*b*c^4 + 18*A^2*B*a*b^2*c^4)/(4*(a^2*b^6 - 64*a^5*c^3 - 12*a^3*b^4*c + 48*a^4*b^2*c^2))))*(-(A^2*b^11 + B^2*a^2*b^9 + A^2*b^2*(-(4*a*c - b^2)^9)^(1/2) + B^2*a^2*(-(4*a*c - b^2)^9)^(1/2) + 2*A*B*a*b^10 + 288*A^2*a^2*b^7*c^2 - 1504*A^2*a^3*b^5*c^3 + 3840*A^2*a^4*b^3*c^4 - 96*B^2*a^4*b^5*c^2 + 512*B^2*a^5*b^3*c^3 + 3072*A*B*a^6*c^5 - 27*A^2*a*b^9*c - 9*A^2*a*c*(-(4*a*c - b^2)^9)^(1/2) - 3840*A^2*a^5*b*c^5 - 768*B^2*a^6*b*c^4 + 192*A*B*a^3*b^6*c^2 - 128*A*B*a^4*b^4*c^3 - 1536*A*B*a^5*b^2*c^4 + 2*A*B*a*b*(-(4*a*c - b^2)^9)^(1/2) - 36*A*B*a^2*b^8*c)/(32*(a^3*b^12 + 4096*a^9*c^6 - 24*a^4*b^10*c + 240*a^5*b^8*c^2 - 1280*a^6*b^6*c^3 + 3840*a^7*b^4*c^4 - 6144*a^8*b^2*c^5)))^(1/2)*2i + atan(((((6144*A*a^5*c^6 + 16*A*a*b^8*c^2 - 1024*B*a^5*b*c^5 - 288*A*a^2*b^6*c^3 + 1920*A*a^3*b^4*c^4 - 5632*A*a^4*b^2*c^5 + 16*B*a^2*b^7*c^2 - 192*B*a^3*b^5*c^3 + 768*B*a^4*b^3*c^4)/(8*(a^2*b^6 - 64*a^5*c^3 - 12*a^3*b^4*c + 48*a^4*b^2*c^2)) - (x*((A^2*b^2*(-(4*a*c - b^2)^9)^(1/2) - B^2*a^2*b^9 - A^2*b^11 + B^2*a^2*(-(4*a*c - b^2)^9)^(1/2) - 2*A*B*a*b^10 - 288*A^2*a^2*b^7*c^2 + 1504*A^2*a^3*b^5*c^3 - 3840*A^2*a^4*b^3*c^4 + 96*B^2*a^4*b^5*c^2 - 512*B^2*a^5*b^3*c^3 - 3072*A*B*a^6*c^5 + 27*A^2*a*b^9*c - 9*A^2*a*c*(-(4*a*c - b^2)^9)^(1/2) + 3840*A^2*a^5*b*c^5 + 768*B^2*a^6*b*c^4 - 192*A*B*a^3*b^6*c^2 + 128*A*B*a^4*b^4*c^3 + 1536*A*B*a^5*b^2*c^4 + 2*A*B*a*b*(-(4*a*c - b^2)^9)^(1/2) + 36*A*B*a^2*b^8*c)/(32*(a^3*b^12 + 4096*a^9*c^6 - 24*a^4*b^10*c + 240*a^5*b^8*c^2 - 1280*a^6*b^6*c^3 + 3840*a^7*b^4*c^4 - 6144*a^8*b^2*c^5)))^(1/2)*(1024*a^5*b*c^5 - 16*a^2*b^7*c^2 + 192*a^3*b^5*c^3 - 768*a^4*b^3*c^4))/(2*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)))*((A^2*b^2*(-(4*a*c - b^2)^9)^(1/2) - B^2*a^2*b^9 - A^2*b^11 + B^2*a^2*(-(4*a*c - b^2)^9)^(1/2) - 2*A*B*a*b^10 - 288*A^2*a^2*b^7*c^2 + 1504*A^2*a^3*b^5*c^3 - 3840*A^2*a^4*b^3*c^4 + 96*B^2*a^4*b^5*c^2 - 512*B^2*a^5*b^3*c^3 - 3072*A*B*a^6*c^5 + 27*A^2*a*b^9*c - 9*A^2*a*c*(-(4*a*c - b^2)^9)^(1/2) + 3840*A^2*a^5*b*c^5 + 768*B^2*a^6*b*c^4 - 192*A*B*a^3*b^6*c^2 + 128*A*B*a^4*b^4*c^3 + 1536*A*B*a^5*b^2*c^4 + 2*A*B*a*b*(-(4*a*c - b^2)^9)^(1/2) + 36*A*B*a^2*b^8*c)/(32*(a^3*b^12 + 4096*a^9*c^6 - 24*a^4*b^10*c + 240*a^5*b^8*c^2 - 1280*a^6*b^6*c^3 + 3840*a^7*b^4*c^4 - 6144*a^8*b^2*c^5)))^(1/2) + (x*(72*A^2*a^2*c^5 + A^2*b^4*c^3 - 8*B^2*a^3*c^4 + 10*B^2*a^2*b^2*c^3 - 14*A^2*a*b^2*c^4 + 2*A*B*a*b^3*c^3 - 40*A*B*a^2*b*c^4))/(2*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)))*((A^2*b^2*(-(4*a*c - b^2)^9)^(1/2) - B^2*a^2*b^9 - A^2*b^11 + B^2*a^2*(-(4*a*c - b^2)^9)^(1/2) - 2*A*B*a*b^10 - 288*A^2*a^2*b^7*c^2 + 1504*A^2*a^3*b^5*c^3 - 3840*A^2*a^4*b^3*c^4 + 96*B^2*a^4*b^5*c^2 - 512*B^2*a^5*b^3*c^3 - 3072*A*B*a^6*c^5 + 27*A^2*a*b^9*c - 9*A^2*a*c*(-(4*a*c - b^2)^9)^(1/2) + 3840*A^2*a^5*b*c^5 + 768*B^2*a^6*b*c^4 - 192*A*B*a^3*b^6*c^2 + 128*A*B*a^4*b^4*c^3 + 1536*A*B*a^5*b^2*c^4 + 2*A*B*a*b*(-(4*a*c - b^2)^9)^(1/2) + 36*A*B*a^2*b^8*c)/(32*(a^3*b^12 + 4096*a^9*c^6 - 24*a^4*b^10*c + 240*a^5*b^8*c^2 - 1280*a^6*b^6*c^3 + 3840*a^7*b^4*c^4 - 6144*a^8*b^2*c^5)))^(1/2)*1i - (((6144*A*a^5*c^6 + 16*A*a*b^8*c^2 - 1024*B*a^5*b*c^5 - 288*A*a^2*b^6*c^3 + 1920*A*a^3*b^4*c^4 - 5632*A*a^4*b^2*c^5 + 16*B*a^2*b^7*c^2 - 192*B*a^3*b^5*c^3 + 768*B*a^4*b^3*c^4)/(8*(a^2*b^6 - 64*a^5*c^3 - 12*a^3*b^4*c + 48*a^4*b^2*c^2)) + (x*((A^2*b^2*(-(4*a*c - b^2)^9)^(1/2) - B^2*a^2*b^9 - A^2*b^11 + B^2*a^2*(-(4*a*c - b^2)^9)^(1/2) - 2*A*B*a*b^10 - 288*A^2*a^2*

b^7*c^2 + 1504*A^2*a^3*b^5*c^3 - 3840*A^2*a^4*b^3*c^4 + 96*B^2*a^4*b^5*c^2
- 512*B^2*a^5*b^3*c^3 - 3072*A*B*a^6*c^5 + 27*A^2*a*b^9*c - 9*A^2*a*c*(-(4*
a*c - b^2)^9)^(1/2) + 3840*A^2*a^5*b*c^5 + 768*B^2*a^6*b*c^4 - 192*A*B*a^3*
b^6*c^2 + 128*A*B*a^4*b^4*c^3 + 1536*A*B*a^5*b^2*c^4 + 2*A*B*a*b*(-(4*a*c -
b^2)^9)^(1/2) + 36*A*B*a^2*b^8*c)/(32*(a^3*b^12 + 4096*a^9*c^6 - 24*a^4*b^
10*c + 240*a^5*b^8*c^2 - 1280*a^6*b^6*c^3 + 3840*a^7*b^4*c^4 - 6144*a^8*b^2
c^5)))^(1/2)(1024*a^5*b*c^5 - 16*a^2*b^7*c^2 + 192*a^3*b^5*c^3 - 768*a^4*
b^3*c^4))/(2*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)))*((A^2*b^2*(-(4*a*c - b^
2)^9)^(1/2) - B^2*a^2*b^9 - A^2*b^11 + B^2*a^2*(-(4*a*c - b^2)^9)^(1/2) - 2
*A*B*a*b^10 - 288*A^2*a^2*b^7*c^2 + 1504*A^2*a^3*b^5*c^3 - 3840*A^2*a^4*b^3
*c^4 + 96*B^2*a^4*b^5*c^2 - 512*B^2*a^5*b^3*c^3 - 3072*A*B*a^6*c^5 + 27*A^2
*a*b^9*c - 9*A^2*a*c*(-(4*a*c - b^2)^9)^(1/2) + 3840*A^2*a^5*b*c^5 + 768*B^
2*a^6*b*c^4 - 192*A*B*a^3*b^6*c^2 + 128*A*B*a^4*b^4*c^3 + 1536*A*B*a^5*b^2*
c^4 + 2*A*B*a*b*(-(4*a*c - b^2)^9)^(1/2) + 36*A*B*a^2*b^8*c)/(32*(a^3*b^12
+ 4096*a^9*c^6 - 24*a^4*b^10*c + 240*a^5*b^8*c^2 - 1280*a^6*b^6*c^3 + 3840*
a^7*b^4*c^4 - 6144*a^8*b^2*c^5)))^(1/2) - (x*(72*A^2*a^2*c^5 + A^2*b^4*c^3
- 8*B^2*a^3*c^4 + 10*B^2*a^2*b^2*c^3 - 14*A^2*a*b^2*c^4 + 2*A*B*a*b^3*c^3 -
40*A*B*a^2*b*c^4))/(2*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)))*((A^2*b^2*(-(
4*a*c - b^2)^9)^(1/2) - B^2*a^2*b^9 - A^2*b^11 + B^2*a^2*(-(4*a*c - b^2)^9)
^(1/2) - 2*A*B*a*b^10 - 288*A^2*a^2*b^7*c^2 + 1504*A^2*a^3*b^5*c^3 - 3840*A
^2*a^4*b^3*c^4 + 96*B^2*a^4*b^5*c^2 - 512*B^2*a^5*b^3*c^3 - 3072*A*B*a^6*c^
5 + 27*A^2*a*b^9*c - 9*A^2*a*c*(-(4*a*c - b^2)^9)^(1/2) + 3840*A^2*a^5*b*c^
5 + 768*B^2*a^6*b*c^4 - 192*A*B*a^3*b^6*c^2 + 128*A*B*a^4*b^4*c^3 + 1536*A*
B*a^5*b^2*c^4 + 2*A*B*a*b*(-(4*a*c - b^2)^9)^(1/2) + 36*A*B*a^2*b^8*c)/(32*
(a^3*b^12 + 4096*a^9*c^6 - 24*a^4*b^10*c + 240*a^5*b^8*c^2 - 1280*a^6*b^6*c
^3 + 3840*a^7*b^4*c^4 - 6144*a^8*b^2*c^5)))^(1/2)*1i)/((((6144*A*a^5*c^6 +
16*A*a*b^8*c^2 - 1024*B*a^5*b*c^5 - 288*A*a^2*b^6*c^3 + 1920*A*a^3*b^4*c^4
- 5632*A*a^4*b^2*c^5 + 16*B*a^2*b^7*c^2 - 192*B*a^3*b^5*c^3 + 768*B*a^4*b^3
c^4)/(8(a^2*b^6 - 64*a^5*c^3 - 12*a^3*b^4*c + 48*a^4*b^2*c^2)) - (x*((A^2
b^2(-(4*a*c - b^2)^9)^(1/2) - B^2*a^2*b^9 - A^2*b^11 + B^2*a^2*(-(4*a*c -
b^2)^9)^(1/2) - 2*A*B*a*b^10 - 288*A^2*a^2*b^7*c^2 + 1504*A^2*a^3*b^5*c^3
- 3840*A^2*a^4*b^3*c^4 + 96*B^2*a^4*b^5*c^2 - 512*B^2*a^5*b^3*c^3 - 3072*A*
B*a^6*c^5 + 27*A^2*a*b^9*c - 9*A^2*a*c*(-(4*a*c - b^2)^9)^(1/2) + 3840*A^2*
a^5*b*c^5 + 768*B^2*a^6*b*c^4 - 192*A*B*a^3*b^6*c^2 + 128*A*B*a^4*b^4*c^3 +
1536*A*B*a^5*b^2*c^4 + 2*A*B*a*b*(-(4*a*c - b^2)^9)^(1/2) + 36*A*B*a^2*b^8
c)/(32(a^3*b^12 + 4096*a^9*c^6 - 24*a^4*b^10*c + 240*a^5*b^8*c^2 - 1280*a
^6*b^6*c^3 + 3840*a^7*b^4*c^4 - 6144*a^8*b^2*c^5)))^(1/2)*(1024*a^5*b*c^5 -
16*a^2*b^7*c^2 + 192*a^3*b^5*c^3 - 768*a^4*b^3*c^4))/(2*(a^2*b^4 + 16*a^4*
c^2 - 8*a^3*b^2*c)))*((A^2*b^2*(-(4*a*c - b^2)^9)^(1/2) - B^2*a^2*b^9 - A^2
*b^11 + B^2*a^2*(-(4*a*c - b^2)^9)^(1/2) - 2*A*B*a*b^10 - 288*A^2*a^2*b^7*c
^2 + 1504*A^2*a^3*b^5*c^3 - 3840*A^2*a^4*b^3*c^4 + 96*B^2*a^4*b^5*c^2 - 512
*B^2*a^5*b^3*c^3 - 3072*A*B*a^6*c^5 + 27*A^2*a*b^9*c - 9*A^2*a*c*(-(4*a*c -
b^2)^9)^(1/2) + 3840*A^2*a^5*b*c^5 + 768*B^2*a^6*b*c^4 - 192*A*B*a^3*b^6*c
^2 + 128*A*B*a^4*b^4*c^3 + 1536*A*B*a^5*b^2*c^4 + 2*A*B*a*b*(-(4*a*c - b^2)
^9)^(1/2) + 36*A*B*a^2*b^8*c)/(32*(a^3*b^12 + 4096*a^9*c^6 - 24*a^4*b^10*c

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+ 240*a^5*b^8*c^2 - 1280*a^6*b^6*c^3 + 3840*a^7*b^4*c^4 - 6144*a^8*b^2*c^5)
))^(1/2) + (x*(72*A^2*a^2*c^5 + A^2*b^4*c^3 - 8*B^2*a^3*c^4 + 10*B^2*a^2*b^
2*c^3 - 14*A^2*a*b^2*c^4 + 2*A*B*a*b^3*c^3 - 40*A*B*a^2*b*c^4))/(2*(a^2*b^4
 + 16*a^4*c^2 - 8*a^3*b^2*c)))*((A^2*b^2*(-(4*a*c - b^2)^9)^(1/2) - B^2*a^2
*b^9 - A^2*b^11 + B^2*a^2*(-(4*a*c - b^2)^9)^(1/2) - 2*A*B*a*b^10 - 288*A^2
*a^2*b^7*c^2 + 1504*A^2*a^3*b^5*c^3 - 3840*A^2*a^4*b^3*c^4 + 96*B^2*a^4*b^5
*c^2 - 512*B^2*a^5*b^3*c^3 - 3072*A*B*a^6*c^5 + 27*A^2*a*b^9*c - 9*A^2*a*c*
(-(4*a*c - b^2)^9)^(1/2) + 3840*A^2*a^5*b*c^5 + 768*B^2*a^6*b*c^4 - 192*A*B
*a^3*b^6*c^2 + 128*A*B*a^4*b^4*c^3 + 1536*A*B*a^5*b^2*c^4 + 2*A*B*a*b*(-(4*
a*c - b^2)^9)^(1/2) + 36*A*B*a^2*b^8*c)/(32*(a^3*b^12 + 4096*a^9*c^6 - 24*a
^4*b^10*c + 240*a^5*b^8*c^2 - 1280*a^6*b^6*c^3 + 3840*a^7*b^4*c^4 - 6144*a^
8*b^2*c^5)))^(1/2) + (((6144*A*a^5*c^6 + 16*A*a*b^8*c^2 - 1024*B*a^5*b*c^5
- 288*A*a^2*b^6*c^3 + 1920*A*a^3*b^4*c^4 - 5632*A*a^4*b^2*c^5 + 16*B*a^2*b^
7*c^2 - 192*B*a^3*b^5*c^3 + 768*B*a^4*b^3*c^4)/(8*(a^2*b^6 - 64*a^5*c^3 - 1
2*a^3*b^4*c + 48*a^4*b^2*c^2)) + (x*((A^2*b^2*(-(4*a*c - b^2)^9)^(1/2) - B^
2*a^2*b^9 - A^2*b^11 + B^2*a^2*(-(4*a*c - b^2)^9)^(1/2) - 2*A*B*a*b^10 - 28
8*A^2*a^2*b^7*c^2 + 1504*A^2*a^3*b^5*c^3 - 3840*A^2*a^4*b^3*c^4 + 96*B^2*a^
4*b^5*c^2 - 512*B^2*a^5*b^3*c^3 - 3072*A*B*a^6*c^5 + 27*A^2*a*b^9*c - 9*A^2
*a*c*(-(4*a*c - b^2)^9)^(1/2) + 3840*A^2*a^5*b*c^5 + 768*B^2*a^6*b*c^4 - 19
2*A*B*a^3*b^6*c^2 + 128*A*B*a^4*b^4*c^3 + 1536*A*B*a^5*b^2*c^4 + 2*A*B*a*b*
(-(4*a*c - b^2)^9)^(1/2) + 36*A*B*a^2*b^8*c)/(32*(a^3*b^12 + 4096*a^9*c^6 -
 24*a^4*b^10*c + 240*a^5*b^8*c^2 - 1280*a^6*b^6*c^3 + 3840*a^7*b^4*c^4 - 61
44*a^8*b^2*c^5)))^(1/2)*(1024*a^5*b*c^5 - 16*a^2*b^7*c^2 + 192*a^3*b^5*c^3
- 768*a^4*b^3*c^4))/(2*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)))*((A^2*b^2*(-(
4*a*c - b^2)^9)^(1/2) - B^2*a^2*b^9 - A^2*b^11 + B^2*a^2*(-(4*a*c - b^2)^9)
^(1/2) - 2*A*B*a*b^10 - 288*A^2*a^2*b^7*c^2 + 1504*A^2*a^3*b^5*c^3 - 3840*A
^2*a^4*b^3*c^4 + 96*B^2*a^4*b^5*c^2 - 512*B^2*a^5*b^3*c^3 - 3072*A*B*a^6*c^
5 + 27*A^2*a*b^9*c - 9*A^2*a*c*(-(4*a*c - b^2)^9)^(1/2) + 3840*A^2*a^5*b*c^
5 + 768*B^2*a^6*b*c^4 - 192*A*B*a^3*b^6*c^2 + 128*A*B*a^4*b^4*c^3 + 1536*A*
B*a^5*b^2*c^4 + 2*A*B*a*b*(-(4*a*c - b^2)^9)^(1/2) + 36*A*B*a^2*b^8*c)/(32*
(a^3*b^12 + 4096*a^9*c^6 - 24*a^4*b^10*c + 240*a^5*b^8*c^2 - 1280*a^6*b^6*c
^3 + 3840*a^7*b^4*c^4 - 6144*a^8*b^2*c^5)))^(1/2) - (x*(72*A^2*a^2*c^5 + A^
2*b^4*c^3 - 8*B^2*a^3*c^4 + 10*B^2*a^2*b^2*c^3 - 14*A^2*a*b^2*c^4 + 2*A*B*a
*b^3*c^3 - 40*A*B*a^2*b*c^4))/(2*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)))*((A
^2*b^2*(-(4*a*c - b^2)^9)^(1/2) - B^2*a^2*b^9 - A^2*b^11 + B^2*a^2*(-(4*a*c
 - b^2)^9)^(1/2) - 2*A*B*a*b^10 - 288*A^2*a^2*b^7*c^2 + 1504*A^2*a^3*b^5*c^
3 - 3840*A^2*a^4*b^3*c^4 + 96*B^2*a^4*b^5*c^2 - 512*B^2*a^5*b^3*c^3 - 3072*
A*B*a^6*c^5 + 27*A^2*a*b^9*c - 9*A^2*a*c*(-(4*a*c - b^2)^9)^(1/2) + 3840*A^
2*a^5*b*c^5 + 768*B^2*a^6*b*c^4 - 192*A*B*a^3*b^6*c^2 + 128*A*B*a^4*b^4*c^3
 + 1536*A*B*a^5*b^2*c^4 + 2*A*B*a*b*(-(4*a*c - b^2)^9)^(1/2) + 36*A*B*a^2*b
^8*c)/(32*(a^3*b^12 + 4096*a^9*c^6 - 24*a^4*b^10*c + 240*a^5*b^8*c^2 - 1280
*a^6*b^6*c^3 + 3840*a^7*b^4*c^4 - 6144*a^8*b^2*c^5)))^(1/2) + (5*A^3*b^3*c^
4 + 8*B^3*a^3*c^4 + 6*B^3*a^2*b^2*c^3 - 36*A^3*a*b*c^5 + 72*A^2*B*a^2*c^5 -
 3*A^2*B*b^4*c^3 + 3*A*B^2*a*b^3*c^3 - 60*A*B^2*a^2*b*c^4 + 18*A^2*B*a*b^2*
c^4)/(4*(a^2*b^6 - 64*a^5*c^3 - 12*a^3*b^4*c + 48*a^4*b^2*c^2))))*((A^2*b^2
```

```
*(-(4*a*c - b^2)^9)^(1/2) - B^2*a^2*b^9 - A^2*b^11 + B^2*a^2*(-(4*a*c - b^2
)^9)^(1/2) - 2*A*B*a*b^10 - 288*A^2*a^2*b^7*c^2 + 1504*A^2*a^3*b^5*c^3 - 38
40*A^2*a^4*b^3*c^4 + 96*B^2*a^4*b^5*c^2 - 512*B^2*a^5*b^3*c^3 - 3072*A*B*a^
6*c^5 + 27*A^2*a*b^9*c - 9*A^2*a*c*(-(4*a*c - b^2)^9)^(1/2) + 3840*A^2*a^5*
b*c^5 + 768*B^2*a^6*b*c^4 - 192*A*B*a^3*b^6*c^2 + 128*A*B*a^4*b^4*c^3 + 153
6*A*B*a^5*b^2*c^4 + 2*A*B*a*b*(-(4*a*c - b^2)^9)^(1/2) + 36*A*B*a^2*b^8*c)/
(32*(a^3*b^12 + 4096*a^9*c^6 - 24*a^4*b^10*c + 240*a^5*b^8*c^2 - 1280*a^6*b
^6*c^3 + 3840*a^7*b^4*c^4 - 6144*a^8*b^2*c^5)))^(1/2)*2i + ((x*(2*A*a*c - A
*b^2 + B*a*b))/(2*a*(4*a*c - b^2)) - (c*x^3*(A*b - 2*B*a))/(2*a*(4*a*c - b^
2)))/(a + b*x^2 + c*x^4)
```

3.122 $\int \frac{A+Bx^2}{x^2\left(a+bx^2+cx^4\right)^2}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 389

$$\int \frac{A+Bx^2}{x^2\left(a+bx^2+cx^4\right)^2}\,dx = -\frac{3Ab^2-abB-10aAc}{2a^2\left(b^2-4ac\right)x} - \frac{abB-A(b^2-2ac)-(Ab-2aB)cx^2}{2a\left(b^2-4ac\right)x\left(a+bx^2+cx^4\right)}$$
$$+\frac{\sqrt{c}\left(aB\left(b^2-12ac+b\sqrt{b^2-4ac}\right)-A\left(3b^3-16abc+3b^2\sqrt{b^2-4ac}-10ac\sqrt{b^2-4ac}\right)\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-}}}\right.}{2\sqrt{2}a^2\left(b^2-4ac\right)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}}$$
$$-\frac{\sqrt{c}\left(3Ab^2-abB-10aAc+\frac{aB(b^2-12ac)-A(3b^3-16abc)}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}a^2\left(b^2-4ac\right)\sqrt{b+\sqrt{b^2-4ac}}}$$

```
[Out] 1/2*(10*A*a*c-3*A*b^2+B*a*b)/a^2/(-4*a*c+b^2)/x+1/2*(-a*b*B+A*(-2*a*c+b^2)+
(A*b-2*B*a)*c*x^2)/a/(-4*a*c+b^2)/x/(c*x^4+b*x^2+a)+1/4*arctan(x*2^(1/2)*c^
(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2))*c^(1/2)*(a*B*(b^2-12*a*c+b*(-4*a*c+b^2)
^(1/2))-A*(3*b^3-16*a*b*c+3*b^2*(-4*a*c+b^2)^(1/2)-10*a*c*(-4*a*c+b^2)^(1/2
)))/a^2/(-4*a*c+b^2)^(3/2)*2^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2)-1/4*arctan(
x*2^(1/2)*c^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2))*c^(1/2)*(3*A*b^2-a*b*B-10*A
*a*c+(a*B*(-12*a*c+b^2)-A*(-16*a*b*c+3*b^3))/(-4*a*c+b^2)^(1/2))/a^2/(-4*a*
c+b^2)*2^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2)
```

Rubi [A] (verified)

Time = 0.75 (sec) , antiderivative size = 389, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {1291, 1295, 1180, 211}

$$\int \frac{A+Bx^2}{x^2\left(a+bx^2+cx^4\right)^2}\,dx$$
$$=\frac{\sqrt{c}\left(aB\left(b\sqrt{b^2-4ac}-12ac+b^2\right)-A\left(3b^2\sqrt{b^2-4ac}-10ac\sqrt{b^2-4ac}-16abc+3b^3\right)\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}a^2\left(b^2-4ac\right)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\sqrt{c}\left(\frac{aB\left(b^2-12ac\right)-A\left(3b^3-16abc\right)}{\sqrt{b^2-4ac}}-10aAc-abB+3Ab^2\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{2\sqrt{2}a^2\left(b^2-4ac\right)\sqrt{\sqrt{b^2-4ac}+b}}-\frac{-10aAc-abB+3Ab^2}{2a^2x\left(b^2-4ac\right)}-\frac{-A\left(b^2-2ac\right)-\left(cx^2(Ab-2aB)\right)+abB}{2ax\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}$$

```
[In] Int[(A + B*x^2)/(x^2*(a + b*x^2 + c*x^4)^2),x]
```

```
[Out] -1/2*(3*A*b^2 - a*b*B - 10*a*A*c)/(a^2*(b^2 - 4*a*c)*x) - (a*b*B - A*(b^2 -
 2*a*c) - (A*b - 2*a*B)*c*x^2)/(2*a*(b^2 - 4*a*c)*x*(a + b*x^2 + c*x^4)) +
(Sqrt[c]*(a*B*(b^2 - 12*a*c + b*Sqrt[b^2 - 4*a*c]) - A*(3*b^3 - 16*a*b*c +
3*b^2*Sqrt[b^2 - 4*a*c] - 10*a*c*Sqrt[b^2 - 4*a*c]))*ArcTan[(Sqrt[2]*Sqrt[c
]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]*a^2*(b^2 - 4*a*c)^(3/2)*Sqrt[
b - Sqrt[b^2 - 4*a*c]]) - (Sqrt[c]*(3*A*b^2 - a*b*B - 10*a*A*c + (a*B*(b^2
- 12*a*c) - A*(3*b^3 - 16*a*b*c))/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c
]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]*a^2*(b^2 - 4*a*c)*Sqrt[b + Sq
rt[b^2 - 4*a*c]])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1291

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[(-(f*x)^(m + 1))*(a + b*x^2 + c*x^4)^(p + 1)
*((d*(b^2 - 2*a*c) - a*b*e + (b*d - 2*a*e)*c*x^2)/(2*a*f*(p + 1)*(b^2 - 4*a
```

```
*c))), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), Int[(f*x)^m*(a + b*x^2 + c*
x^4)^(p + 1)*Simp[d*(b^2*(m + 2*(p + 1) + 1) - 2*a*c*(m + 4*(p + 1) + 1)) -
 a*b*e*(m + 1) + c*(m + 2*(2*p + 3) + 1)*(b*d - 2*a*e)*x^2, x], x], x] /; F
reeQ[{a, b, c, d, e, f, m}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && Inte
gerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1295

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[d*(f*x)^(m + 1)*((a + b*x^2 + c*x^4)^(p + 1)
/(a*f*(m + 1))), x] + Dist[1/(a*f^2*(m + 1)), Int[(f*x)^(m + 2)*(a + b*x^2
+ c*x^4)^p*Simp[a*e*(m + 1) - b*d*(m + 2*p + 3) - c*d*(m + 4*p + 5)*x^2, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[m
, -1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{abB - A(b^2-2ac) - (Ab-2aB)cx^2}{2a(b^2-4ac)x(a+bx^2+cx^4)} - \frac{\int \frac{-3Ab^2+abB+10aAc-3(Ab-2aB)cx^2}{x^2(a+bx^2+cx^4)}\,dx}{2a(b^2-4ac)} \\
&= -\frac{3Ab^2-abB-10aAc}{2a^2(b^2-4ac)x} - \frac{abB - A(b^2-2ac) - (Ab-2aB)cx^2}{2a(b^2-4ac)x(a+bx^2+cx^4)} \\
&\quad + \frac{\int \frac{aB(b^2-6ac)-A(3b^3-13abc)-c(3Ab^2-abB-10aAc)x^2}{a+bx^2+cx^4}\,dx}{2a^2(b^2-4ac)} \\
&= -\frac{3Ab^2-abB-10aAc}{2a^2(b^2-4ac)x} - \frac{abB - A(b^2-2ac) - (Ab-2aB)cx^2}{2a(b^2-4ac)x(a+bx^2+cx^4)} \\
&\quad + \frac{\left(c\left(aB\left(b^2-12ac+b\sqrt{b^2-4ac}\right)-A\left(3b^3-16abc+3b^2\sqrt{b^2-4ac}-10ac\sqrt{b^2-4ac}\right)\right)\right)\int \frac{}{\frac{b}{2}-\frac{1}{2}\sqrt{b^2}}}{4a^2(b^2-4ac)^{3/2}} \\
&\quad - \frac{\left(c\left(3Ab^2-abB-10aAc+\frac{aB(b^2-12ac)-A(3b^3-16abc)}{\sqrt{b^2-4ac}}\right)\right)\int \frac{1}{\frac{b}{2}+\frac{1}{2}\sqrt{b^2-4ac}+cx^2}\,dx}{4a^2(b^2-4ac)} \\
&= -\frac{3Ab^2-abB-10aAc}{2a^2(b^2-4ac)x} - \frac{abB - A(b^2-2ac) - (Ab-2aB)cx^2}{2a(b^2-4ac)x(a+bx^2+cx^4)} \\
&\quad + \frac{\sqrt{c}\left(aB\left(b^2-12ac+b\sqrt{b^2-4ac}\right)-A\left(3b^3-16abc+3b^2\sqrt{b^2-4ac}-10ac\sqrt{b^2-4ac}\right)\right)\tan^{-1}\left(\frac{}{\sqrt{}}\right.}{2\sqrt{2}a^2(b^2-4ac)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}} \\
&\quad - \frac{\sqrt{c}\left(3Ab^2-abB-10aAc+\frac{aB(b^2-12ac)-A(3b^3-16abc)}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}a^2(b^2-4ac)\sqrt{b+\sqrt{b^2-4ac}}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.70 (sec) , antiderivative size = 382, normalized size of antiderivative = 0.98

$$\int \frac{A+Bx^2}{x^2\left(a+bx^2+cx^4\right)^2}\,dx$$

$$= \frac{-\frac{4A}{x}+\frac{2x\left(aB\left(b^2-2ac+bcx^2\right)-A\left(b^3-3abc+b^2cx^2-2ac^2x^2\right)\right)}{\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}+\frac{\sqrt{2}\sqrt{c}\left(aB\left(b^2-12ac+b\sqrt{b^2-4ac}\right)+A\left(-3b^3+16abc-3b^2\sqrt{b^2-4ac}+10ac\sqrt{\right.\right.}}{\left(b^2-4ac\right)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}}}{}$$

[In] Integrate[(A + B*x^2)/(x^2*(a + b*x^2 + c*x^4)^2),x]

[Out] ((-4*A)/x + (2*x*(a*B*(b^2 - 2*a*c + b*c*x^2) - A*(b^3 - 3*a*b*c + b^2*c*x^2 - 2*a*c^2*x^2)))/((b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) + (Sqrt[2]*Sqrt[c]*(a*B*(b^2 - 12*a*c + b*Sqrt[b^2 - 4*a*c]) + A*(-3*b^3 + 16*a*b*c - 3*b^2*Sqrt[b^2 - 4*a*c] + 10*a*c*Sqrt[b^2 - 4*a*c]))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/((b^2 - 4*a*c)^(3/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (Sqrt[2]*Sqrt[c]*(a*B*(-b^2 + 12*a*c + b*Sqrt[b^2 - 4*a*c]) + A*(3*b^3 - 16*a*b*c - 3*b^2*Sqrt[b^2 - 4*a*c] + 10*a*c*Sqrt[b^2 - 4*a*c]))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/((b^2 - 4*a*c)^(3/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]]))/(4*a^2)

Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 379, normalized size of antiderivative = 0.97

method	result
default	$-\frac{A}{a^2x}-\frac{\frac{c\left(2Aac-A\,b^2+abB\right)x^3}{8ac-2b^2}+\frac{\left(3Aabc-A\,b^3-2a^2Bc+Ba\,b^2\right)x}{8ac-2b^2}}{c\,x^4+b\,x^2+a}+2c\left(\frac{\left(10A\sqrt{-4ac+b^2}\,ac-3A\sqrt{-4ac+b^2}\,b^2-16Aabc+3A\,b^3+abB\sqrt{-4ac+\right.}}{8\sqrt{-4ac+b^2}\,\sqrt{\left(b+\sqrt{-\right.}}}\right.$
risch	Expression too large to display

[In] int((B*x^2+A)/x^2/(c*x^4+b*x^2+a)^2,x,method=_RETURNVERBOSE)

[Out] -A/a^2/x-1/a^2*((1/2*c*(2*A*a*c-A*b^2+B*a*b)/(4*a*c-b^2)*x^3+1/2*(3*A*a*b*c-A*b^3-2*B*a^2*c+B*a*b^2)/(4*a*c-b^2)*x)/(c*x^4+b*x^2+a)+2/(4*a*c-b^2)*c*(1/8*(10*A*(-4*a*c+b^2)^(1/2)*a*c-3*A*(-4*a*c+b^2)^(1/2)*b^2-16*A*a*b*c+3*A*b^3+a*b*B*(-4*a*c+b^2)^(1/2)+12*a^2*B*c-B*a*b^2)/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))-1/8*(10*A*(-4*a*c+b^2)^(1/2)*a*c-3*A*(-4*a*c+b^2)^(1/2)*b^2+16*A*a*b*c-3*A*b^3+a*b*B*(-4*a*c+b^2)^(1/2)-12*a^2*B*c+B*a*b^2)/(-4*a*c+b^2)^(1

```
/2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(c*x*2^(1/2)/((-b+(-4*
a*c+b^2)^(1/2))*c)^(1/2)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 7583 vs. $2(338) = 676$.

Time = 8.89 (sec) , antiderivative size = 7583, normalized size of antiderivative = 19.49

$$\int \frac{A+Bx^2}{x^2\left(a+bx^2+cx^4\right)^2}\,dx = \text{Too large to display}$$

```
[In] integrate((B*x^2+A)/x^2/(c*x^4+b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A+Bx^2}{x^2\left(a+bx^2+cx^4\right)^2}\,dx = \text{Timed out}$$

```
[In] integrate((B*x**2+A)/x**2/(c*x**4+b*x**2+a)**2,x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{A+Bx^2}{x^2\left(a+bx^2+cx^4\right)^2}\,dx = \int \frac{Bx^2+A}{\left(cx^4+bx^2+a\right)^2x^2}\,dx$$

```
[In] integrate((B*x^2+A)/x^2/(c*x^4+b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] 1/2*((10*A*a*c^2 + (B*a*b - 3*A*b^2)*c)*x^4 - 2*A*a*b^2 + 8*A*a^2*c + (B*a*
b^2 - 3*A*b^3 - (2*B*a^2 - 11*A*a*b)*c)*x^2)/((a^2*b^2*c - 4*a^3*c^2)*x^5 +
 (a^2*b^3 - 4*a^3*b*c)*x^3 + (a^3*b^2 - 4*a^4*c)*x) + 1/2*integrate((B*a*b^
2 - 3*A*b^3 + (10*A*a*c^2 + (B*a*b - 3*A*b^2)*c)*x^2 - (6*B*a^2 - 13*A*a*b)
*c)/(c*x^4 + b*x^2 + a), x)/(a^2*b^2 - 4*a^3*c)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 5408 vs. 2(338) = 676.

Time = 1.36 (sec) , antiderivative size = 5408, normalized size of antiderivative = 13.90

$$\int \frac{A+Bx^2}{x^2\left(a+bx^2+cx^4\right)^2}\,dx = \text{Too large to display}$$

```
[In] integrate((B*x^2+A)/x^2/(c*x^4+b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] 1/2*(B*a*b*c*x^4 - 3*A*b^2*c*x^4 + 10*A*a*c^2*x^4 + B*a*b^2*x^2 - 3*A*b^3*x
^2 - 2*B*a^2*c*x^2 + 11*A*a*b*c*x^2 - 2*A*a*b^2 + 8*A*a^2*c)/((c*x^5 + b*x^
3 + a*x)*(a^2*b^2 - 4*a^3*c)) - 1/16*((6*b^4*c^2 - 44*a*b^2*c^3 + 80*a^2*c^
4 - 3*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^4 + 22*sq
rt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c + 6*sqrt(2)
*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^3*c - 40*sqrt(2)*sqrt(
b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*c^2 - 20*sqrt(2)*sqrt(b^2
- 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c^2 - 3*sqrt(2)*sqrt(b^2 - 4*a
*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^2*c^2 + 10*sqrt(2)*sqrt(b^2 - 4*a*c)*
sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*c^3 - 6*(b^2 - 4*a*c)*b^2*c^2 + 20*(b^2 -
 4*a*c)*a*c^3)*(a^2*b^2 - 4*a^3*c)^2*A - (2*a*b^3*c^2 - 8*a^2*b*c^3 - sqrt(
2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3 + 4*sqrt(2)*sqrt
(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b*c + 2*sqrt(2)*sqrt(b^2
- 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c - sqrt(2)*sqrt(b^2 - 4*a*c
)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c^2 - 2*(b^2 - 4*a*c)*a*b*c^2)*(a^2*b
^2 - 4*a^3*c)^2*B + 2*(3*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^7 -
37*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^5*c - 6*sqrt(2)*sqrt(b*c +
 sqrt(b^2 - 4*a*c)*c)*a^2*b^6*c - 6*a^2*b^7*c + 152*sqrt(2)*sqrt(b*c + sqrt
(b^2 - 4*a*c)*c)*a^4*b^3*c^2 + 50*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a
^3*b^4*c^2 + 3*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^5*c^2 + 74*a^3
*b^5*c^2 - 208*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^5*b*c^3 - 104*sqrt
(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b^2*c^3 - 25*sqrt(2)*sqrt(b*c + sqr
t(b^2 - 4*a*c)*c)*a^3*b^3*c^3 - 304*a^4*b^3*c^3 + 52*sqrt(2)*sqrt(b*c + sqr
t(b^2 - 4*a*c)*c)*a^4*b*c^4 + 416*a^5*b*c^4 + 6*(b^2 - 4*a*c)*a^2*b^5*c - 5
0*(b^2 - 4*a*c)*a^3*b^3*c^2 + 104*(b^2 - 4*a*c)*a^4*b*c^3)*A*abs(a^2*b^2 -
4*a^3*c) - 2*(sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^6 - 14*sqrt(2)*
sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b^4*c - 2*sqrt(2)*sqrt(b*c + sqrt(b^2 -
 4*a*c)*c)*a^3*b^5*c - 2*a^3*b^6*c + 64*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c
)*c)*a^5*b^2*c^2 + 20*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b^3*c^2 +
 sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^4*c^2 + 28*a^4*b^4*c^2 - 96*
sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^6*c^3 - 48*sqrt(2)*sqrt(b*c + sqr
t(b^2 - 4*a*c)*c)*a^5*b*c^3 - 10*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^
4*b^2*c^3 - 128*a^5*b^2*c^3 + 24*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^
5*c^4 + 192*a^6*c^4 + 2*(b^2 - 4*a*c)*a^3*b^4*c - 20*(b^2 - 4*a*c)*a^4*b^2*
c^2 + 48*(b^2 - 4*a*c)*a^5*c^3)*B*abs(a^2*b^2 - 4*a^3*c) + (6*a^4*b^8*c^2 -
```

80*a^5*b^6*c^3 + 352*a^6*b^4*c^4 - 512*a^7*b^2*c^5 - 3*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b^8 + 40*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^5*b^6*c + 6*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b^7*c - 176*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^6*b^4*c^2 - 56*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^5*b^5*c^2 - 3*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b^6*c^2 + 256*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^7*b^2*c^3 + 128*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^6*b^3*c^3 + 28*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^5*b^4*c^3 - 64*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^6*b^2*c^4 - 6*(b^2 - 4*a*c)*a^4*b^6*c^2 + 56*(b^2 - 4*a*c)*a^5*b^4*c^3 - 128*(b^2 - 4*a*c)*a^6*b^2*c^4)*A - (2*a^5*b^7*c^2 - 40*a^6*b^5*c^3 + 224*a^7*b^3*c^4 - 384*a^8*b*c^5 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^5*b^7 + 20*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^6*b^5*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^5*b^6*c - 112*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^7*b^3*c^2 - 32*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^6*b^4*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^5*b^5*c^2 + 192*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^8*b*c^3 + 96*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^7*b^2*c^3 + 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^6*b^3*c^3 - 48*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^7*b*c^4 - 2*(b^2 - 4*a*c)*a^5*b^5*c^2 + 32*(b^2 - 4*a*c)*a^6*b^3*c^3 - 96*(b^2 - 4*a*c)*a^7*b*c^4)*B)*arctan(2*sqrt(1/2)*x/sqrt((a^2*b^3 - 4*a^3*b*c + sqrt((a^2*b^3 - 4*a^3*b*c)^2 - 4*(a^3*b^2 - 4*a^4*c)*(a^2*b^2*c - 4*a^3*c^2)))/(a^2*b^2*c - 4*a^3*c^2)))/((a^5*b^6 - 12*a^6*b^4*c - 2*a^5*b^5*c + 48*a^7*b^2*c^2 + 16*a^6*b^3*c^2 + a^5*b^4*c^2 - 64*a^8*c^3 - 32*a^7*b*c^3 - 8*a^6*b^2*c^3 + 16*a^7*c^4)*abs(a^2*b^2 - 4*a^3*c)*abs(c)) + 1/16*((6*b^4*c^2 - 44*a*b^2*c^3 + 80*a^2*c^4 - 3*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^4 + 22*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^2*c + 6*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^3*c - 40*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*c^2 - 20*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b*c^2 - 3*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^2*c^2 + 10*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*c^3 - 6*(b^2 - 4*a*c)*b^2*c^2 + 20*(b^2 - 4*a*c)*a*c^3)*(a^2*b^2 - 4*a^3*c)^2*A - (2*a*b^3*c^2 - 8*a^2*b*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^3 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^2*c - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b*c^2 - 2*(b^2 - 4*a*c)*a*b*c^2)*(a^2*b^2 - 4*a^3*c)^2*B - 2*(3*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^7 - 37*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^5*c - 6*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^6*c + 6*a^2*b^7*c + 152*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*b^3*c^2 + 50*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^4*c^2 + 3*

sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^5*c^2 - 74*a^3*b^5*c^2 - 208*
sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^5*b*c^3 - 104*sqrt(2)*sqrt(b*c -
sqrt(b^2 - 4*a*c)*c)*a^4*b^2*c^3 - 25*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*
c)*a^3*b^3*c^3 + 304*a^4*b^3*c^3 + 52*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*
c)*a^4*b*c^4 - 416*a^5*b*c^4 - 6*(b^2 - 4*a*c)*a^2*b^5*c + 50*(b^2 - 4*a*c)
*a^3*b^3*c^2 - 104*(b^2 - 4*a*c)*a^4*b*c^3)*A*abs(a^2*b^2 - 4*a^3*c) + 2*(s
qrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^6 - 14*sqrt(2)*sqrt(b*c - sqrt
(b^2 - 4*a*c)*c)*a^4*b^4*c - 2*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*
b^5*c + 2*a^3*b^6*c + 64*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^5*b^2*c^
2 + 20*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*b^3*c^2 + sqrt(2)*sqrt(b
*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^4*c^2 - 28*a^4*b^4*c^2 - 96*sqrt(2)*sqrt(b*
c - sqrt(b^2 - 4*a*c)*c)*a^6*c^3 - 48*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*
c)*a^5*b*c^3 - 10*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*b^2*c^3 + 128
*a^5*b^2*c^3 + 24*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^5*c^4 - 192*a^6
c^4 - 2(b^2 - 4*a*c)*a^3*b^4*c + 20*(b^2 - 4*a*c)*a^4*b^2*c^2 - 48*(b^2 -
4*a*c)*a^5*c^3)*B*abs(a^2*b^2 - 4*a^3*c) + (6*a^4*b^8*c^2 - 80*a^5*b^6*c^3
+ 352*a^6*b^4*c^4 - 512*a^7*b^2*c^5 - 3*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c
- sqrt(b^2 - 4*a*c)*c)*a^4*b^8 + 40*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - s
qrt(b^2 - 4*a*c)*c)*a^5*b^6*c + 6*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt
(b^2 - 4*a*c)*c)*a^4*b^7*c - 176*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(
b^2 - 4*a*c)*c)*a^6*b^4*c^2 - 56*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(
b^2 - 4*a*c)*c)*a^5*b^5*c^2 - 3*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b
^2 - 4*a*c)*c)*a^4*b^6*c^2 + 256*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(
b^2 - 4*a*c)*c)*a^7*b^2*c^3 + 128*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt
(b^2 - 4*a*c)*c)*a^6*b^3*c^3 + 28*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt
(b^2 - 4*a*c)*c)*a^5*b^4*c^3 - 64*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt
(b^2 - 4*a*c)*c)*a^6*b^2*c^4 - 6*(b^2 - 4*a*c)*a^4*b^6*c^2 + 56*(b^2 - 4*a*
c)*a^5*b^4*c^3 - 128*(b^2 - 4*a*c)*a^6*b^2*c^4)*A - (2*a^5*b^7*c^2 - 40*a^6
*b^5*c^3 + 224*a^7*b^3*c^4 - 384*a^8*b*c^5 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt
(b*c - sqrt(b^2 - 4*a*c)*c)*a^5*b^7 + 20*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c
- sqrt(b^2 - 4*a*c)*c)*a^6*b^5*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c -
sqrt(b^2 - 4*a*c)*c)*a^5*b^6*c - 112*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - s
qrt(b^2 - 4*a*c)*c)*a^7*b^3*c^2 - 32*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - s
qrt(b^2 - 4*a*c)*c)*a^6*b^4*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt
(b^2 - 4*a*c)*c)*a^5*b^5*c^2 + 192*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqr
t(b^2 - 4*a*c)*c)*a^8*b*c^3 + 96*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(
b^2 - 4*a*c)*c)*a^7*b^2*c^3 + 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(
b^2 - 4*a*c)*c)*a^6*b^3*c^3 - 48*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(
b^2 - 4*a*c)*c)*a^7*b*c^4 - 2*(b^2 - 4*a*c)*a^5*b^5*c^2 + 32*(b^2 - 4*a*c)*
a^6*b^3*c^3 - 96*(b^2 - 4*a*c)*a^7*b*c^4)*B)*arctan(2*sqrt(1/2)*x/sqrt((a^2
*b^3 - 4*a^3*b*c - sqrt((a^2*b^3 - 4*a^3*b*c)^2 - 4*(a^3*b^2 - 4*a^4*c)*(a^
2*b^2*c - 4*a^3*c^2)))/(a^2*b^2*c - 4*a^3*c^2)))/((a^5*b^6 - 12*a^6*b^4*c -
2*a^5*b^5*c + 48*a^7*b^2*c^2 + 16*a^6*b^3*c^2 + a^5*b^4*c^2 - 64*a^8*c^3 -
32*a^7*b*c^3 - 8*a^6*b^2*c^3 + 16*a^7*c^4)*abs(a^2*b^2 - 4*a^3*c)*abs(c))

Mupad [B] (verification not implemented)

Time = 11.09 (sec) , antiderivative size = 17591, normalized size of antiderivative = 45.22

$$\int \frac{A+Bx^2}{x^2\left(a+bx^2+cx^4\right)^2}\,dx = \text{Too large to display}$$

```
[In] int((A + B*x^2)/(x^2*(a + b*x^2 + c*x^4)^2),x)
```

```
[Out] - (A/a - (x^2*(3*A*b^3 - B*a*b^2 + 2*B*a^2*c - 11*A*a*b*c))/(2*a^2*(4*a*c -
 b^2)) + (c*x^4*(10*A*a*c - 3*A*b^2 + B*a*b))/(2*a^2*(4*a*c - b^2)))/(a*x +
 b*x^3 + c*x^5) - atan((((-(9*A^2*b^13 + B^2*a^2*b^11 + 9*A^2*b^4*(-(4*a*c
- b^2)^9)^(1/2) - 6*A*B*a*b^12 + 2077*A^2*a^2*b^9*c^2 - 10656*A^2*a^3*b^7*c
^3 + 30240*A^2*a^4*b^5*c^4 - 44800*A^2*a^5*b^3*c^5 + 25*A^2*a^2*c^2*(-(4*a*
c - b^2)^9)^(1/2) + B^2*a^2*b^2*(-(4*a*c - b^2)^9)^(1/2) + 288*B^2*a^4*b^7*
c^2 - 1504*B^2*a^5*b^5*c^3 + 3840*B^2*a^6*b^3*c^4 - 15360*A*B*a^7*c^6 - 213
*A^2*a*b^11*c + 26880*A^2*a^6*b*c^6 - 27*B^2*a^3*b^9*c - 3840*B^2*a^7*b*c^5
 - 9*B^2*a^3*c*(-(4*a*c - b^2)^9)^(1/2) - 1548*A*B*a^3*b^8*c^2 + 8064*A*B*a
^4*b^6*c^3 - 22400*A*B*a^5*b^4*c^4 + 30720*A*B*a^6*b^2*c^5 - 51*A^2*a*b^2*c
*(-(4*a*c - b^2)^9)^(1/2) - 6*A*B*a*b^3*(-(4*a*c - b^2)^9)^(1/2) + 152*A*B*
a^2*b^10*c + 44*A*B*a^2*b*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^5*b^12 + 4096*
a^11*c^6 - 24*a^6*b^10*c + 240*a^7*b^8*c^2 - 1280*a^8*b^6*c^3 + 3840*a^9*b^
4*c^4 - 6144*a^10*b^2*c^5)))^(1/2)*(x*(-(9*A^2*b^13 + B^2*a^2*b^11 + 9*A^2*
b^4*(-(4*a*c - b^2)^9)^(1/2) - 6*A*B*a*b^12 + 2077*A^2*a^2*b^9*c^2 - 10656*
A^2*a^3*b^7*c^3 + 30240*A^2*a^4*b^5*c^4 - 44800*A^2*a^5*b^3*c^5 + 25*A^2*a^
2*c^2*(-(4*a*c - b^2)^9)^(1/2) + B^2*a^2*b^2*(-(4*a*c - b^2)^9)^(1/2) + 288
*B^2*a^4*b^7*c^2 - 1504*B^2*a^5*b^5*c^3 + 3840*B^2*a^6*b^3*c^4 - 15360*A*B*
a^7*c^6 - 213*A^2*a*b^11*c + 26880*A^2*a^6*b*c^6 - 27*B^2*a^3*b^9*c - 3840*
B^2*a^7*b*c^5 - 9*B^2*a^3*c*(-(4*a*c - b^2)^9)^(1/2) - 1548*A*B*a^3*b^8*c^2
 + 8064*A*B*a^4*b^6*c^3 - 22400*A*B*a^5*b^4*c^4 + 30720*A*B*a^6*b^2*c^5 - 5
1*A^2*a*b^2*c*(-(4*a*c - b^2)^9)^(1/2) - 6*A*B*a*b^3*(-(4*a*c - b^2)^9)^(1/
2) + 152*A*B*a^2*b^10*c + 44*A*B*a^2*b*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^5
*b^12 + 4096*a^11*c^6 - 24*a^6*b^10*c + 240*a^7*b^8*c^2 - 1280*a^8*b^6*c^3
+ 3840*a^9*b^4*c^4 - 6144*a^10*b^2*c^5)))^(1/2)*(1048576*a^16*b*c^8 + 256*a
^10*b^13*c^2 - 6144*a^11*b^11*c^3 + 61440*a^12*b^9*c^4 - 327680*a^13*b^7*c^
5 + 983040*a^14*b^5*c^6 - 1572864*a^15*b^3*c^7) - 393216*B*a^15*c^8 + 85196
8*A*a^14*b*c^8 + 192*A*a^8*b^13*c^2 - 4672*A*a^9*b^11*c^3 + 47360*A*a^10*b^
9*c^4 - 256000*A*a^11*b^7*c^5 + 778240*A*a^12*b^5*c^6 - 1261568*A*a^13*b^3*
c^7 - 64*B*a^9*b^12*c^2 + 1664*B*a^10*b^10*c^3 - 17920*B*a^11*b^8*c^4 + 102
400*B*a^12*b^6*c^5 - 327680*B*a^13*b^4*c^6 + 557056*B*a^14*b^2*c^7) + x*(20
4800*A^2*a^12*c^9 - 73728*B^2*a^13*c^8 + 144*A^2*a^6*b^12*c^3 - 3264*A^2*a^
7*b^10*c^4 + 30112*A^2*a^8*b^8*c^5 - 143360*A^2*a^9*b^6*c^6 + 365568*A^2*a^
10*b^4*c^7 - 458752*A^2*a^11*b^2*c^8 + 16*B^2*a^8*b^10*c^3 - 416*B^2*a^9*b^
8*c^4 + 4608*B^2*a^10*b^6*c^5 - 25600*B^2*a^11*b^4*c^6 + 69632*B^2*a^12*b^2
*c^7 - 96*A*B*a^7*b^11*c^3 + 2336*A*B*a^8*b^9*c^4 - 22528*A*B*a^9*b^7*c^5 +
```

107520*A*B*a^10*b^5*c^6 - 253952*A*B*a^11*b^3*c^7 + 237568*A*B*a^12*b*c^8)
)*(-(9*A^2*b^13 + B^2*a^2*b^11 + 9*A^2*b^4*(-(4*a*c - b^2)^9)^(1/2) - 6*A*B
*a*b^12 + 2077*A^2*a^2*b^9*c^2 - 10656*A^2*a^3*b^7*c^3 + 30240*A^2*a^4*b^5*
c^4 - 44800*A^2*a^5*b^3*c^5 + 25*A^2*a^2*c^2*(-(4*a*c - b^2)^9)^(1/2) + B^2
*a^2*b^2*(-(4*a*c - b^2)^9)^(1/2) + 288*B^2*a^4*b^7*c^2 - 1504*B^2*a^5*b^5*
c^3 + 3840*B^2*a^6*b^3*c^4 - 15360*A*B*a^7*c^6 - 213*A^2*a*b^11*c + 26880*A
^2*a^6*b*c^6 - 27*B^2*a^3*b^9*c - 3840*B^2*a^7*b*c^5 - 9*B^2*a^3*c*(-(4*a*c
 - b^2)^9)^(1/2) - 1548*A*B*a^3*b^8*c^2 + 8064*A*B*a^4*b^6*c^3 - 22400*A*B*
a^5*b^4*c^4 + 30720*A*B*a^6*b^2*c^5 - 51*A^2*a*b^2*c*(-(4*a*c - b^2)^9)^(1/
2) - 6*A*B*a*b^3*(-(4*a*c - b^2)^9)^(1/2) + 152*A*B*a^2*b^10*c + 44*A*B*a^2
*b*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^5*b^12 + 4096*a^11*c^6 - 24*a^6*b^10*
c + 240*a^7*b^8*c^2 - 1280*a^8*b^6*c^3 + 3840*a^9*b^4*c^4 - 6144*a^10*b^2*c
^5)))^(1/2)*1i + ((-(9*A^2*b^13 + B^2*a^2*b^11 + 9*A^2*b^4*(-(4*a*c - b^2)^
9)^(1/2) - 6*A*B*a*b^12 + 2077*A^2*a^2*b^9*c^2 - 10656*A^2*a^3*b^7*c^3 + 30
240*A^2*a^4*b^5*c^4 - 44800*A^2*a^5*b^3*c^5 + 25*A^2*a^2*c^2*(-(4*a*c - b^2
)^9)^(1/2) + B^2*a^2*b^2*(-(4*a*c - b^2)^9)^(1/2) + 288*B^2*a^4*b^7*c^2 - 1
504*B^2*a^5*b^5*c^3 + 3840*B^2*a^6*b^3*c^4 - 15360*A*B*a^7*c^6 - 213*A^2*a*
b^11*c + 26880*A^2*a^6*b*c^6 - 27*B^2*a^3*b^9*c - 3840*B^2*a^7*b*c^5 - 9*B^
2*a^3*c*(-(4*a*c - b^2)^9)^(1/2) - 1548*A*B*a^3*b^8*c^2 + 8064*A*B*a^4*b^6*
c^3 - 22400*A*B*a^5*b^4*c^4 + 30720*A*B*a^6*b^2*c^5 - 51*A^2*a*b^2*c*(-(4*a
*c - b^2)^9)^(1/2) - 6*A*B*a*b^3*(-(4*a*c - b^2)^9)^(1/2) + 152*A*B*a^2*b^1
0*c + 44*A*B*a^2*b*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^5*b^12 + 4096*a^11*c^
6 - 24*a^6*b^10*c + 240*a^7*b^8*c^2 - 1280*a^8*b^6*c^3 + 3840*a^9*b^4*c^4 -
 6144*a^10*b^2*c^5)))^(1/2)*(x*(-(9*A^2*b^13 + B^2*a^2*b^11 + 9*A^2*b^4*(-(
4*a*c - b^2)^9)^(1/2) - 6*A*B*a*b^12 + 2077*A^2*a^2*b^9*c^2 - 10656*A^2*a^3
*b^7*c^3 + 30240*A^2*a^4*b^5*c^4 - 44800*A^2*a^5*b^3*c^5 + 25*A^2*a^2*c^2*(
-(4*a*c - b^2)^9)^(1/2) + B^2*a^2*b^2*(-(4*a*c - b^2)^9)^(1/2) + 288*B^2*a^
4*b^7*c^2 - 1504*B^2*a^5*b^5*c^3 + 3840*B^2*a^6*b^3*c^4 - 15360*A*B*a^7*c^6
 - 213*A^2*a*b^11*c + 26880*A^2*a^6*b*c^6 - 27*B^2*a^3*b^9*c - 3840*B^2*a^7
*b*c^5 - 9*B^2*a^3*c*(-(4*a*c - b^2)^9)^(1/2) - 1548*A*B*a^3*b^8*c^2 + 8064
*A*B*a^4*b^6*c^3 - 22400*A*B*a^5*b^4*c^4 + 30720*A*B*a^6*b^2*c^5 - 51*A^2*a
*b^2*c*(-(4*a*c - b^2)^9)^(1/2) - 6*A*B*a*b^3*(-(4*a*c - b^2)^9)^(1/2) + 15
2*A*B*a^2*b^10*c + 44*A*B*a^2*b*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^5*b^12 +
 4096*a^11*c^6 - 24*a^6*b^10*c + 240*a^7*b^8*c^2 - 1280*a^8*b^6*c^3 + 3840*
a^9*b^4*c^4 - 6144*a^10*b^2*c^5)))^(1/2)*(1048576*a^16*b*c^8 + 256*a^10*b^1
3*c^2 - 6144*a^11*b^11*c^3 + 61440*a^12*b^9*c^4 - 327680*a^13*b^7*c^5 + 983
040*a^14*b^5*c^6 - 1572864*a^15*b^3*c^7) + 393216*B*a^15*c^8 - 851968*A*a^1
4*b*c^8 - 192*A*a^8*b^13*c^2 + 4672*A*a^9*b^11*c^3 - 47360*A*a^10*b^9*c^4 +
 256000*A*a^11*b^7*c^5 - 778240*A*a^12*b^5*c^6 + 1261568*A*a^13*b^3*c^7 + 6
4*B*a^9*b^12*c^2 - 1664*B*a^10*b^10*c^3 + 17920*B*a^11*b^8*c^4 - 102400*B*a
^12*b^6*c^5 + 327680*B*a^13*b^4*c^6 - 557056*B*a^14*b^2*c^7) + x*(204800*A^
2*a^12*c^9 - 73728*B^2*a^13*c^8 + 144*A^2*a^6*b^12*c^3 - 3264*A^2*a^7*b^10*
c^4 + 30112*A^2*a^8*b^8*c^5 - 143360*A^2*a^9*b^6*c^6 + 365568*A^2*a^10*b^4*
c^7 - 458752*A^2*a^11*b^2*c^8 + 16*B^2*a^8*b^10*c^3 - 416*B^2*a^9*b^8*c^4 +
 4608*B^2*a^10*b^6*c^5 - 25600*B^2*a^11*b^4*c^6 + 69632*B^2*a^12*b^2*c^7 -

96*A*B*a^7*b^11*c^3 + 2336*A*B*a^8*b^9*c^4 - 22528*A*B*a^9*b^7*c^5 + 107520
*A*B*a^10*b^5*c^6 - 253952*A*B*a^11*b^3*c^7 + 237568*A*B*a^12*b*c^8))*(-(9*
A^2*b^13 + B^2*a^2*b^11 + 9*A^2*b^4*(-(4*a*c - b^2)^9)^(1/2) - 6*A*B*a*b^12
 + 2077*A^2*a^2*b^9*c^2 - 10656*A^2*a^3*b^7*c^3 + 30240*A^2*a^4*b^5*c^4 - 4
4800*A^2*a^5*b^3*c^5 + 25*A^2*a^2*c^2*(-(4*a*c - b^2)^9)^(1/2) + B^2*a^2*b^
2*(-(4*a*c - b^2)^9)^(1/2) + 288*B^2*a^4*b^7*c^2 - 1504*B^2*a^5*b^5*c^3 + 3
840*B^2*a^6*b^3*c^4 - 15360*A*B*a^7*c^6 - 213*A^2*a*b^11*c + 26880*A^2*a^6*
b*c^6 - 27*B^2*a^3*b^9*c - 3840*B^2*a^7*b*c^5 - 9*B^2*a^3*c*(-(4*a*c - b^2)
^9)^(1/2) - 1548*A*B*a^3*b^8*c^2 + 8064*A*B*a^4*b^6*c^3 - 22400*A*B*a^5*b^4
*c^4 + 30720*A*B*a^6*b^2*c^5 - 51*A^2*a*b^2*c*(-(4*a*c - b^2)^9)^(1/2) - 6*
A*B*a*b^3*(-(4*a*c - b^2)^9)^(1/2) + 152*A*B*a^2*b^10*c + 44*A*B*a^2*b*c*(-
(4*a*c - b^2)^9)^(1/2))/(32*(a^5*b^12 + 4096*a^11*c^6 - 24*a^6*b^10*c + 240
*a^7*b^8*c^2 - 1280*a^8*b^6*c^3 + 3840*a^9*b^4*c^4 - 6144*a^10*b^2*c^5)))^(
1/2)*1i)/(((-(9*A^2*b^13 + B^2*a^2*b^11 + 9*A^2*b^4*(-(4*a*c - b^2)^9)^(1/2
) - 6*A*B*a*b^12 + 2077*A^2*a^2*b^9*c^2 - 10656*A^2*a^3*b^7*c^3 + 30240*A^2
*a^4*b^5*c^4 - 44800*A^2*a^5*b^3*c^5 + 25*A^2*a^2*c^2*(-(4*a*c - b^2)^9)^(1
/2) + B^2*a^2*b^2*(-(4*a*c - b^2)^9)^(1/2) + 288*B^2*a^4*b^7*c^2 - 1504*B^2
*a^5*b^5*c^3 + 3840*B^2*a^6*b^3*c^4 - 15360*A*B*a^7*c^6 - 213*A^2*a*b^11*c
 + 26880*A^2*a^6*b*c^6 - 27*B^2*a^3*b^9*c - 3840*B^2*a^7*b*c^5 - 9*B^2*a^3*c
*(-(4*a*c - b^2)^9)^(1/2) - 1548*A*B*a^3*b^8*c^2 + 8064*A*B*a^4*b^6*c^3 - 2
2400*A*B*a^5*b^4*c^4 + 30720*A*B*a^6*b^2*c^5 - 51*A^2*a*b^2*c*(-(4*a*c - b^
2)^9)^(1/2) - 6*A*B*a*b^3*(-(4*a*c - b^2)^9)^(1/2) + 152*A*B*a^2*b^10*c + 4
4*A*B*a^2*b*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^5*b^12 + 4096*a^11*c^6 - 24*
a^6*b^10*c + 240*a^7*b^8*c^2 - 1280*a^8*b^6*c^3 + 3840*a^9*b^4*c^4 - 6144*a
^10*b^2*c^5)))^(1/2)*(x*(-(9*A^2*b^13 + B^2*a^2*b^11 + 9*A^2*b^4*(-(4*a*c -
 b^2)^9)^(1/2) - 6*A*B*a*b^12 + 2077*A^2*a^2*b^9*c^2 - 10656*A^2*a^3*b^7*c^
3 + 30240*A^2*a^4*b^5*c^4 - 44800*A^2*a^5*b^3*c^5 + 25*A^2*a^2*c^2*(-(4*a*c
 - b^2)^9)^(1/2) + B^2*a^2*b^2*(-(4*a*c - b^2)^9)^(1/2) + 288*B^2*a^4*b^7*c
^2 - 1504*B^2*a^5*b^5*c^3 + 3840*B^2*a^6*b^3*c^4 - 15360*A*B*a^7*c^6 - 213*
A^2*a*b^11*c + 26880*A^2*a^6*b*c^6 - 27*B^2*a^3*b^9*c - 3840*B^2*a^7*b*c^5
 - 9*B^2*a^3*c*(-(4*a*c - b^2)^9)^(1/2) - 1548*A*B*a^3*b^8*c^2 + 8064*A*B*a^
4*b^6*c^3 - 22400*A*B*a^5*b^4*c^4 + 30720*A*B*a^6*b^2*c^5 - 51*A^2*a*b^2*c*
(-(4*a*c - b^2)^9)^(1/2) - 6*A*B*a*b^3*(-(4*a*c - b^2)^9)^(1/2) + 152*A*B*a
^2*b^10*c + 44*A*B*a^2*b*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^5*b^12 + 4096*a
^11*c^6 - 24*a^6*b^10*c + 240*a^7*b^8*c^2 - 1280*a^8*b^6*c^3 + 3840*a^9*b^4
*c^4 - 6144*a^10*b^2*c^5)))^(1/2)*(1048576*a^16*b*c^8 + 256*a^10*b^13*c^2 -
 6144*a^11*b^11*c^3 + 61440*a^12*b^9*c^4 - 327680*a^13*b^7*c^5 + 983040*a^1
4*b^5*c^6 - 1572864*a^15*b^3*c^7) - 393216*B*a^15*c^8 + 851968*A*a^14*b*c^8
 + 192*A*a^8*b^13*c^2 - 4672*A*a^9*b^11*c^3 + 47360*A*a^10*b^9*c^4 - 256000
*A*a^11*b^7*c^5 + 778240*A*a^12*b^5*c^6 - 1261568*A*a^13*b^3*c^7 - 64*B*a^9
*b^12*c^2 + 1664*B*a^10*b^10*c^3 - 17920*B*a^11*b^8*c^4 + 102400*B*a^12*b^6
*c^5 - 327680*B*a^13*b^4*c^6 + 557056*B*a^14*b^2*c^7) + x*(204800*A^2*a^12*
c^9 - 73728*B^2*a^13*c^8 + 144*A^2*a^6*b^12*c^3 - 3264*A^2*a^7*b^10*c^4 + 3
0112*A^2*a^8*b^8*c^5 - 143360*A^2*a^9*b^6*c^6 + 365568*A^2*a^10*b^4*c^7 - 4
58752*A^2*a^11*b^2*c^8 + 16*B^2*a^8*b^10*c^3 - 416*B^2*a^9*b^8*c^4 + 4608*B

^2*a^10*b^6*c^5 - 25600*B^2*a^11*b^4*c^6 + 69632*B^2*a^12*b^2*c^7 - 96*A*B*
a^7*b^11*c^3 + 2336*A*B*a^8*b^9*c^4 - 22528*A*B*a^9*b^7*c^5 + 107520*A*B*a^
10*b^5*c^6 - 253952*A*B*a^11*b^3*c^7 + 237568*A*B*a^12*b*c^8))*(-(9*A^2*b^1
3 + B^2*a^2*b^11 + 9*A^2*b^4*(-(4*a*c - b^2)^9)^(1/2) - 6*A*B*a*b^12 + 2077
*A^2*a^2*b^9*c^2 - 10656*A^2*a^3*b^7*c^3 + 30240*A^2*a^4*b^5*c^4 - 44800*A^
2*a^5*b^3*c^5 + 25*A^2*a^2*c^2*(-(4*a*c - b^2)^9)^(1/2) + B^2*a^2*b^2*(-(4*
a*c - b^2)^9)^(1/2) + 288*B^2*a^4*b^7*c^2 - 1504*B^2*a^5*b^5*c^3 + 3840*B^2
*a^6*b^3*c^4 - 15360*A*B*a^7*c^6 - 213*A^2*a*b^11*c + 26880*A^2*a^6*b*c^6 -
 27*B^2*a^3*b^9*c - 3840*B^2*a^7*b*c^5 - 9*B^2*a^3*c*(-(4*a*c - b^2)^9)^(1/
2) - 1548*A*B*a^3*b^8*c^2 + 8064*A*B*a^4*b^6*c^3 - 22400*A*B*a^5*b^4*c^4 +
30720*A*B*a^6*b^2*c^5 - 51*A^2*a*b^2*c*(-(4*a*c - b^2)^9)^(1/2) - 6*A*B*a*b
^3*(-(4*a*c - b^2)^9)^(1/2) + 152*A*B*a^2*b^10*c + 44*A*B*a^2*b*c*(-(4*a*c
- b^2)^9)^(1/2))/(32*(a^5*b^12 + 4096*a^11*c^6 - 24*a^6*b^10*c + 240*a^7*b^
8*c^2 - 1280*a^8*b^6*c^3 + 3840*a^9*b^4*c^4 - 6144*a^10*b^2*c^5)))^(1/2) -
((-(9*A^2*b^13 + B^2*a^2*b^11 + 9*A^2*b^4*(-(4*a*c - b^2)^9)^(1/2) - 6*A*B*
a*b^12 + 2077*A^2*a^2*b^9*c^2 - 10656*A^2*a^3*b^7*c^3 + 30240*A^2*a^4*b^5*c
^4 - 44800*A^2*a^5*b^3*c^5 + 25*A^2*a^2*c^2*(-(4*a*c - b^2)^9)^(1/2) + B^2*
a^2*b^2*(-(4*a*c - b^2)^9)^(1/2) + 288*B^2*a^4*b^7*c^2 - 1504*B^2*a^5*b^5*c
^3 + 3840*B^2*a^6*b^3*c^4 - 15360*A*B*a^7*c^6 - 213*A^2*a*b^11*c + 26880*A^
2*a^6*b*c^6 - 27*B^2*a^3*b^9*c - 3840*B^2*a^7*b*c^5 - 9*B^2*a^3*c*(-(4*a*c
- b^2)^9)^(1/2) - 1548*A*B*a^3*b^8*c^2 + 8064*A*B*a^4*b^6*c^3 - 22400*A*B*a
^5*b^4*c^4 + 30720*A*B*a^6*b^2*c^5 - 51*A^2*a*b^2*c*(-(4*a*c - b^2)^9)^(1/2
) - 6*A*B*a*b^3*(-(4*a*c - b^2)^9)^(1/2) + 152*A*B*a^2*b^10*c + 44*A*B*a^2*
b*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^5*b^12 + 4096*a^11*c^6 - 24*a^6*b^10*c
 + 240*a^7*b^8*c^2 - 1280*a^8*b^6*c^3 + 3840*a^9*b^4*c^4 - 6144*a^10*b^2*c^
5)))^(1/2)*(x*(-(9*A^2*b^13 + B^2*a^2*b^11 + 9*A^2*b^4*(-(4*a*c - b^2)^9)^(
1/2) - 6*A*B*a*b^12 + 2077*A^2*a^2*b^9*c^2 - 10656*A^2*a^3*b^7*c^3 + 30240*
A^2*a^4*b^5*c^4 - 44800*A^2*a^5*b^3*c^5 + 25*A^2*a^2*c^2*(-(4*a*c - b^2)^9)
^(1/2) + B^2*a^2*b^2*(-(4*a*c - b^2)^9)^(1/2) + 288*B^2*a^4*b^7*c^2 - 1504*
B^2*a^5*b^5*c^3 + 3840*B^2*a^6*b^3*c^4 - 15360*A*B*a^7*c^6 - 213*A^2*a*b^11
*c + 26880*A^2*a^6*b*c^6 - 27*B^2*a^3*b^9*c - 3840*B^2*a^7*b*c^5 - 9*B^2*a^
3*c*(-(4*a*c - b^2)^9)^(1/2) - 1548*A*B*a^3*b^8*c^2 + 8064*A*B*a^4*b^6*c^3
- 22400*A*B*a^5*b^4*c^4 + 30720*A*B*a^6*b^2*c^5 - 51*A^2*a*b^2*c*(-(4*a*c -
 b^2)^9)^(1/2) - 6*A*B*a*b^3*(-(4*a*c - b^2)^9)^(1/2) + 152*A*B*a^2*b^10*c
+ 44*A*B*a^2*b*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^5*b^12 + 4096*a^11*c^6 -
24*a^6*b^10*c + 240*a^7*b^8*c^2 - 1280*a^8*b^6*c^3 + 3840*a^9*b^4*c^4 - 614
4*a^10*b^2*c^5)))^(1/2)*(1048576*a^16*b*c^8 + 256*a^10*b^13*c^2 - 6144*a^11
*b^11*c^3 + 61440*a^12*b^9*c^4 - 327680*a^13*b^7*c^5 + 983040*a^14*b^5*c^6
- 1572864*a^15*b^3*c^7) + 393216*B*a^15*c^8 - 851968*A*a^14*b*c^8 - 192*A*a
^8*b^13*c^2 + 4672*A*a^9*b^11*c^3 - 47360*A*a^10*b^9*c^4 + 256000*A*a^11*b^
7*c^5 - 778240*A*a^12*b^5*c^6 + 1261568*A*a^13*b^3*c^7 + 64*B*a^9*b^12*c^2
- 1664*B*a^10*b^10*c^3 + 17920*B*a^11*b^8*c^4 - 102400*B*a^12*b^6*c^5 + 327
680*B*a^13*b^4*c^6 - 557056*B*a^14*b^2*c^7) + x*(204800*A^2*a^12*c^9 - 7372
8*B^2*a^13*c^8 + 144*A^2*a^6*b^12*c^3 - 3264*A^2*a^7*b^10*c^4 + 30112*A^2*a
^8*b^8*c^5 - 143360*A^2*a^9*b^6*c^6 + 365568*A^2*a^10*b^4*c^7 - 458752*A^2*

a^11*b^2*c^8 + 16*B^2*a^8*b^10*c^3 - 416*B^2*a^9*b^8*c^4 + 4608*B^2*a^10*b^6*c^5 - 25600*B^2*a^11*b^4*c^6 + 69632*B^2*a^12*b^2*c^7 - 96*A*B*a^7*b^11*c^3 + 2336*A*B*a^8*b^9*c^4 - 22528*A*B*a^9*b^7*c^5 + 107520*A*B*a^10*b^5*c^6 - 253952*A*B*a^11*b^3*c^7 + 237568*A*B*a^12*b*c^8))*(-(9*A^2*b^13 + B^2*a^2*b^11 + 9*A^2*b^4*(-(4*a*c - b^2)^9)^(1/2) - 6*A*B*a*b^12 + 2077*A^2*a^2*b^9*c^2 - 10656*A^2*a^3*b^7*c^3 + 30240*A^2*a^4*b^5*c^4 - 44800*A^2*a^5*b^3*c^5 + 25*A^2*a^2*c^2*(-(4*a*c - b^2)^9)^(1/2) + B^2*a^2*b^2*(-(4*a*c - b^2)^9)^(1/2) + 288*B^2*a^4*b^7*c^2 - 1504*B^2*a^5*b^5*c^3 + 3840*B^2*a^6*b^3*c^4 - 15360*A*B*a^7*c^6 - 213*A^2*a*b^11*c + 26880*A^2*a^6*b*c^6 - 27*B^2*a^3*b^9*c - 3840*B^2*a^7*b*c^5 - 9*B^2*a^3*c*(-(4*a*c - b^2)^9)^(1/2) - 1548*A*B*a^3*b^8*c^2 + 8064*A*B*a^4*b^6*c^3 - 22400*A*B*a^5*b^4*c^4 + 30720*A*B*a^6*b^2*c^5 - 51*A^2*a*b^2*c*(-(4*a*c - b^2)^9)^(1/2) - 6*A*B*a*b^3*(-(4*a*c - b^2)^9)^(1/2) + 152*A*B*a^2*b^10*c + 44*A*B*a^2*b*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^5*b^12 + 4096*a^11*c^6 - 24*a^6*b^10*c + 240*a^7*b^8*c^2 - 1280*a^8*b^6*c^3 + 3840*a^9*b^4*c^4 - 6144*a^10*b^2*c^5)))^(1/2) + 128000*A^3*a^10*c^9 + 504*A^3*a^6*b^8*c^5 - 8112*A^3*a^7*b^6*c^6 + 48704*A^3*a^8*b^4*c^7 - 129280*A^3*a^9*b^2*c^8 - 40*B^3*a^8*b^7*c^4 + 608*B^3*a^9*b^5*c^5 - 2944*B^3*a^10*b^3*c^6 + 46080*A*B^2*a^11*c^8 + 4608*B^3*a^11*b*c^7 - 84480*A^2*B*a^10*b*c^8 + 240*A*B^2*a^7*b^8*c^4 - 3792*A*B^2*a^8*b^6*c^5 + 21696*A*B^2*a^9*b^4*c^6 - 52992*A*B^2*a^10*b^2*c^7 - 360*A^2*B*a^6*b^9*c^4 + 5736*A^2*B*a^7*b^7*c^5 - 33888*A^2*B*a^8*b^5*c^6 + 87936*A^2*B*a^9*b^3*c^7))*(-(9*A^2*b^13 + B^2*a^2*b^11 + 9*A^2*b^4*(-(4*a*c - b^2)^9)^(1/2) - 6*A*B*a*b^12 + 2077*A^2*a^2*b^9*c^2 - 10656*A^2*a^3*b^7*c^3 + 30240*A^2*a^4*b^5*c^4 - 44800*A^2*a^5*b^3*c^5 + 25*A^2*a^2*c^2*(-(4*a*c - b^2)^9)^(1/2) + B^2*a^2*b^2*(-(4*a*c - b^2)^9)^(1/2) + 288*B^2*a^4*b^7*c^2 - 1504*B^2*a^5*b^5*c^3 + 3840*B^2*a^6*b^3*c^4 - 15360*A*B*a^7*c^6 - 213*A^2*a*b^11*c + 26880*A^2*a^6*b*c^6 - 27*B^2*a^3*b^9*c - 3840*B^2*a^7*b*c^5 - 9*B^2*a^3*c*(-(4*a*c - b^2)^9)^(1/2) - 1548*A*B*a^3*b^8*c^2 + 8064*A*B*a^4*b^6*c^3 - 22400*A*B*a^5*b^4*c^4 + 30720*A*B*a^6*b^2*c^5 - 51*A^2*a*b^2*c*(-(4*a*c - b^2)^9)^(1/2) - 6*A*B*a*b^3*(-(4*a*c - b^2)^9)^(1/2) + 152*A*B*a^2*b^10*c + 44*A*B*a^2*b*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^5*b^12 + 4096*a^11*c^6 - 24*a^6*b^10*c + 240*a^7*b^8*c^2 - 1280*a^8*b^6*c^3 + 3840*a^9*b^4*c^4 - 6144*a^10*b^2*c^5)))^(1/2)*2i - atan(((((9*A^2*b^4*(-(4*a*c - b^2)^9)^(1/2) - B^2*a^2*b^11 - 9*A^2*b^13 + 6*A*B*a*b^12 - 2077*A^2*a^2*b^9*c^2 + 10656*A^2*a^3*b^7*c^3 - 30240*A^2*a^4*b^5*c^4 + 44800*A^2*a^5*b^3*c^5 + 25*A^2*a^2*c^2*(-(4*a*c - b^2)^9)^(1/2) + B^2*a^2*b^2*(-(4*a*c - b^2)^9)^(1/2) - 288*B^2*a^4*b^7*c^2 + 1504*B^2*a^5*b^5*c^3 - 3840*B^2*a^6*b^3*c^4 + 15360*A*B*a^7*c^6 + 213*A^2*a*b^11*c - 26880*A^2*a^6*b*c^6 + 27*B^2*a^3*b^9*c + 3840*B^2*a^7*b*c^5 - 9*B^2*a^3*c*(-(4*a*c - b^2)^9)^(1/2) + 1548*A*B*a^3*b^8*c^2 - 8064*A*B*a^4*b^6*c^3 + 22400*A*B*a^5*b^4*c^4 - 30720*A*B*a^6*b^2*c^5 - 51*A^2*a*b^2*c*(-(4*a*c - b^2)^9)^(1/2) - 6*A*B*a*b^3*(-(4*a*c - b^2)^9)^(1/2) - 152*A*B*a^2*b^10*c + 44*A*B*a^2*b*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^5*b^12 + 4096*a^11*c^6 - 24*a^6*b^10*c + 240*a^7*b^8*c^2 - 1280*a^8*b^6*c^3 + 3840*a^9*b^4*c^4 - 6144*a^10*b^2*c^5)))^(1/2)*(x*((9*A^2*b^4*(-(4*a*c - b^2)^9)^(1/2) - B^2*a^2*b^11 - 9*A^2*b^13 + 6*A*B*a*b^12 - 2077*A^2*a^2*b^9*c^2 + 10656*A^2*a^3*b

^7*c^3 - 30240*A^2*a^4*b^5*c^4 + 44800*A^2*a^5*b^3*c^5 + 25*A^2*a^2*c^2*(-(
4*a*c - b^2)^9)^(1/2) + B^2*a^2*b^2*(-(4*a*c - b^2)^9)^(1/2) - 288*B^2*a^4*
b^7*c^2 + 1504*B^2*a^5*b^5*c^3 - 3840*B^2*a^6*b^3*c^4 + 15360*A*B*a^7*c^6 +
213*A^2*a*b^11*c - 26880*A^2*a^6*b*c^6 + 27*B^2*a^3*b^9*c + 3840*B^2*a^7*b
*c^5 - 9*B^2*a^3*c*(-(4*a*c - b^2)^9)^(1/2) + 1548*A*B*a^3*b^8*c^2 - 8064*A
*B*a^4*b^6*c^3 + 22400*A*B*a^5*b^4*c^4 - 30720*A*B*a^6*b^2*c^5 - 51*A^2*a*b
^2*c*(-(4*a*c - b^2)^9)^(1/2) - 6*A*B*a*b^3*(-(4*a*c - b^2)^9)^(1/2) - 152*
A*B*a^2*b^10*c + 44*A*B*a^2*b*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^5*b^12 + 4
096*a^11*c^6 - 24*a^6*b^10*c + 240*a^7*b^8*c^2 - 1280*a^8*b^6*c^3 + 3840*a^
9*b^4*c^4 - 6144*a^10*b^2*c^5)))^(1/2)*(1048576*a^16*b*c^8 + 256*a^10*b^13*
c^2 - 6144*a^11*b^11*c^3 + 61440*a^12*b^9*c^4 - 327680*a^13*b^7*c^5 + 98304
0*a^14*b^5*c^6 - 1572864*a^15*b^3*c^7) - 393216*B*a^15*c^8 + 851968*A*a^14*
b*c^8 + 192*A*a^8*b^13*c^2 - 4672*A*a^9*b^11*c^3 + 47360*A*a^10*b^9*c^4 - 2
56000*A*a^11*b^7*c^5 + 778240*A*a^12*b^5*c^6 - 1261568*A*a^13*b^3*c^7 - 64*
B*a^9*b^12*c^2 + 1664*B*a^10*b^10*c^3 - 17920*B*a^11*b^8*c^4 + 102400*B*a^1
2*b^6*c^5 - 327680*B*a^13*b^4*c^6 + 557056*B*a^14*b^2*c^7) + x*(204800*A^2*
a^12*c^9 - 73728*B^2*a^13*c^8 + 144*A^2*a^6*b^12*c^3 - 3264*A^2*a^7*b^10*c^
4 + 30112*A^2*a^8*b^8*c^5 - 143360*A^2*a^9*b^6*c^6 + 365568*A^2*a^10*b^4*c^
7 - 458752*A^2*a^11*b^2*c^8 + 16*B^2*a^8*b^10*c^3 - 416*B^2*a^9*b^8*c^4 + 4
608*B^2*a^10*b^6*c^5 - 25600*B^2*a^11*b^4*c^6 + 69632*B^2*a^12*b^2*c^7 - 96
*A*B*a^7*b^11*c^3 + 2336*A*B*a^8*b^9*c^4 - 22528*A*B*a^9*b^7*c^5 + 107520*A
*B*a^10*b^5*c^6 - 253952*A*B*a^11*b^3*c^7 + 237568*A*B*a^12*b*c^8))*((9*A^2
b^4(-(4*a*c - b^2)^9)^(1/2) - B^2*a^2*b^11 - 9*A^2*b^13 + 6*A*B*a*b^12 -
2077*A^2*a^2*b^9*c^2 + 10656*A^2*a^3*b^7*c^3 - 30240*A^2*a^4*b^5*c^4 + 4480
0*A^2*a^5*b^3*c^5 + 25*A^2*a^2*c^2*(-(4*a*c - b^2)^9)^(1/2) + B^2*a^2*b^2*(
-(4*a*c - b^2)^9)^(1/2) - 288*B^2*a^4*b^7*c^2 + 1504*B^2*a^5*b^5*c^3 - 3840
*B^2*a^6*b^3*c^4 + 15360*A*B*a^7*c^6 + 213*A^2*a*b^11*c - 26880*A^2*a^6*b*c
^6 + 27*B^2*a^3*b^9*c + 3840*B^2*a^7*b*c^5 - 9*B^2*a^3*c*(-(4*a*c - b^2)^9)
^(1/2) + 1548*A*B*a^3*b^8*c^2 - 8064*A*B*a^4*b^6*c^3 + 22400*A*B*a^5*b^4*c^
4 - 30720*A*B*a^6*b^2*c^5 - 51*A^2*a*b^2*c*(-(4*a*c - b^2)^9)^(1/2) - 6*A*B
*a*b^3*(-(4*a*c - b^2)^9)^(1/2) - 152*A*B*a^2*b^10*c + 44*A*B*a^2*b*c*(-(4*
a*c - b^2)^9)^(1/2))/(32*(a^5*b^12 + 4096*a^11*c^6 - 24*a^6*b^10*c + 240*a^
7*b^8*c^2 - 1280*a^8*b^6*c^3 + 3840*a^9*b^4*c^4 - 6144*a^10*b^2*c^5)))^(1/2
)*1i + (((9*A^2*b^4*(-(4*a*c - b^2)^9)^(1/2) - B^2*a^2*b^11 - 9*A^2*b^13 +
6*A*B*a*b^12 - 2077*A^2*a^2*b^9*c^2 + 10656*A^2*a^3*b^7*c^3 - 30240*A^2*a^4
*b^5*c^4 + 44800*A^2*a^5*b^3*c^5 + 25*A^2*a^2*c^2*(-(4*a*c - b^2)^9)^(1/2)
+ B^2*a^2*b^2*(-(4*a*c - b^2)^9)^(1/2) - 288*B^2*a^4*b^7*c^2 + 1504*B^2*a^5
*b^5*c^3 - 3840*B^2*a^6*b^3*c^4 + 15360*A*B*a^7*c^6 + 213*A^2*a*b^11*c - 26
880*A^2*a^6*b*c^6 + 27*B^2*a^3*b^9*c + 3840*B^2*a^7*b*c^5 - 9*B^2*a^3*c*(-(
4*a*c - b^2)^9)^(1/2) + 1548*A*B*a^3*b^8*c^2 - 8064*A*B*a^4*b^6*c^3 + 22400
*A*B*a^5*b^4*c^4 - 30720*A*B*a^6*b^2*c^5 - 51*A^2*a*b^2*c*(-(4*a*c - b^2)^9
)^(1/2) - 6*A*B*a*b^3*(-(4*a*c - b^2)^9)^(1/2) - 152*A*B*a^2*b^10*c + 44*A*
B*a^2*b*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^5*b^12 + 4096*a^11*c^6 - 24*a^6*
b^10*c + 240*a^7*b^8*c^2 - 1280*a^8*b^6*c^3 + 3840*a^9*b^4*c^4 - 6144*a^10*
b^2*c^5)))^(1/2)*(x*((9*A^2*b^4*(-(4*a*c - b^2)^9)^(1/2) - B^2*a^2*b^11 - 9

*A^2*b^13 + 6*A*B*a*b^12 - 2077*A^2*a^2*b^9*c^2 + 10656*A^2*a^3*b^7*c^3 - 3
0240*A^2*a^4*b^5*c^4 + 44800*A^2*a^5*b^3*c^5 + 25*A^2*a^2*c^2*(-(4*a*c - b^
2)^9)^(1/2) + B^2*a^2*b^2*(-(4*a*c - b^2)^9)^(1/2) - 288*B^2*a^4*b^7*c^2 +
1504*B^2*a^5*b^5*c^3 - 3840*B^2*a^6*b^3*c^4 + 15360*A*B*a^7*c^6 + 213*A^2*a
*b^11*c - 26880*A^2*a^6*b*c^6 + 27*B^2*a^3*b^9*c + 3840*B^2*a^7*b*c^5 - 9*B
^2*a^3*c*(-(4*a*c - b^2)^9)^(1/2) + 1548*A*B*a^3*b^8*c^2 - 8064*A*B*a^4*b^6
*c^3 + 22400*A*B*a^5*b^4*c^4 - 30720*A*B*a^6*b^2*c^5 - 51*A^2*a*b^2*c*(-(4*
a*c - b^2)^9)^(1/2) - 6*A*B*a*b^3*(-(4*a*c - b^2)^9)^(1/2) - 152*A*B*a^2*b^
10*c + 44*A*B*a^2*b*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^5*b^12 + 4096*a^11*c
^6 - 24*a^6*b^10*c + 240*a^7*b^8*c^2 - 1280*a^8*b^6*c^3 + 3840*a^9*b^4*c^4
- 6144*a^10*b^2*c^5)))^(1/2)*(1048576*a^16*b*c^8 + 256*a^10*b^13*c^2 - 6144
*a^11*b^11*c^3 + 61440*a^12*b^9*c^4 - 327680*a^13*b^7*c^5 + 983040*a^14*b^5
*c^6 - 1572864*a^15*b^3*c^7) + 393216*B*a^15*c^8 - 851968*A*a^14*b*c^8 - 19
2*A*a^8*b^13*c^2 + 4672*A*a^9*b^11*c^3 - 47360*A*a^10*b^9*c^4 + 256000*A*a^
11*b^7*c^5 - 778240*A*a^12*b^5*c^6 + 1261568*A*a^13*b^3*c^7 + 64*B*a^9*b^12
*c^2 - 1664*B*a^10*b^10*c^3 + 17920*B*a^11*b^8*c^4 - 102400*B*a^12*b^6*c^5
+ 327680*B*a^13*b^4*c^6 - 557056*B*a^14*b^2*c^7) + x*(204800*A^2*a^12*c^9 -
 73728*B^2*a^13*c^8 + 144*A^2*a^6*b^12*c^3 - 3264*A^2*a^7*b^10*c^4 + 30112*
A^2*a^8*b^8*c^5 - 143360*A^2*a^9*b^6*c^6 + 365568*A^2*a^10*b^4*c^7 - 458752
*A^2*a^11*b^2*c^8 + 16*B^2*a^8*b^10*c^3 - 416*B^2*a^9*b^8*c^4 + 4608*B^2*a^
10*b^6*c^5 - 25600*B^2*a^11*b^4*c^6 + 69632*B^2*a^12*b^2*c^7 - 96*A*B*a^7*b
^11*c^3 + 2336*A*B*a^8*b^9*c^4 - 22528*A*B*a^9*b^7*c^5 + 107520*A*B*a^10*b^
5*c^6 - 253952*A*B*a^11*b^3*c^7 + 237568*A*B*a^12*b*c^8))*((9*A^2*b^4*(-(4*
a*c - b^2)^9)^(1/2) - B^2*a^2*b^11 - 9*A^2*b^13 + 6*A*B*a*b^12 - 2077*A^2*a
^2*b^9*c^2 + 10656*A^2*a^3*b^7*c^3 - 30240*A^2*a^4*b^5*c^4 + 44800*A^2*a^5*
b^3*c^5 + 25*A^2*a^2*c^2*(-(4*a*c - b^2)^9)^(1/2) + B^2*a^2*b^2*(-(4*a*c -
b^2)^9)^(1/2) - 288*B^2*a^4*b^7*c^2 + 1504*B^2*a^5*b^5*c^3 - 3840*B^2*a^6*b
^3*c^4 + 15360*A*B*a^7*c^6 + 213*A^2*a*b^11*c - 26880*A^2*a^6*b*c^6 + 27*B^
2*a^3*b^9*c + 3840*B^2*a^7*b*c^5 - 9*B^2*a^3*c*(-(4*a*c - b^2)^9)^(1/2) + 1
548*A*B*a^3*b^8*c^2 - 8064*A*B*a^4*b^6*c^3 + 22400*A*B*a^5*b^4*c^4 - 30720*
A*B*a^6*b^2*c^5 - 51*A^2*a*b^2*c*(-(4*a*c - b^2)^9)^(1/2) - 6*A*B*a*b^3*(-(
4*a*c - b^2)^9)^(1/2) - 152*A*B*a^2*b^10*c + 44*A*B*a^2*b*c*(-(4*a*c - b^2)
^9)^(1/2))/(32*(a^5*b^12 + 4096*a^11*c^6 - 24*a^6*b^10*c + 240*a^7*b^8*c^2
- 1280*a^8*b^6*c^3 + 3840*a^9*b^4*c^4 - 6144*a^10*b^2*c^5)))^(1/2)*1i)/((((
9*A^2*b^4*(-(4*a*c - b^2)^9)^(1/2) - B^2*a^2*b^11 - 9*A^2*b^13 + 6*A*B*a*b^
12 - 2077*A^2*a^2*b^9*c^2 + 10656*A^2*a^3*b^7*c^3 - 30240*A^2*a^4*b^5*c^4 +
 44800*A^2*a^5*b^3*c^5 + 25*A^2*a^2*c^2*(-(4*a*c - b^2)^9)^(1/2) + B^2*a^2*
b^2*(-(4*a*c - b^2)^9)^(1/2) - 288*B^2*a^4*b^7*c^2 + 1504*B^2*a^5*b^5*c^3 -
 3840*B^2*a^6*b^3*c^4 + 15360*A*B*a^7*c^6 + 213*A^2*a*b^11*c - 26880*A^2*a^
6*b*c^6 + 27*B^2*a^3*b^9*c + 3840*B^2*a^7*b*c^5 - 9*B^2*a^3*c*(-(4*a*c - b^
2)^9)^(1/2) + 1548*A*B*a^3*b^8*c^2 - 8064*A*B*a^4*b^6*c^3 + 22400*A*B*a^5*b
^4*c^4 - 30720*A*B*a^6*b^2*c^5 - 51*A^2*a*b^2*c*(-(4*a*c - b^2)^9)^(1/2) -
6*A*B*a*b^3*(-(4*a*c - b^2)^9)^(1/2) - 152*A*B*a^2*b^10*c + 44*A*B*a^2*b*c*
(-(4*a*c - b^2)^9)^(1/2))/(32*(a^5*b^12 + 4096*a^11*c^6 - 24*a^6*b^10*c + 2
40*a^7*b^8*c^2 - 1280*a^8*b^6*c^3 + 3840*a^9*b^4*c^4 - 6144*a^10*b^2*c^5)))

^(1/2)*(x*((9*A^2*b^4*(-(4*a*c - b^2)^9)^(1/2) - B^2*a^2*b^11 - 9*A^2*b^13
+ 6*A*B*a*b^12 - 2077*A^2*a^2*b^9*c^2 + 10656*A^2*a^3*b^7*c^3 - 30240*A^2*a
^4*b^5*c^4 + 44800*A^2*a^5*b^3*c^5 + 25*A^2*a^2*c^2*(-(4*a*c - b^2)^9)^(1/2
) + B^2*a^2*b^2*(-(4*a*c - b^2)^9)^(1/2) - 288*B^2*a^4*b^7*c^2 + 1504*B^2*a
^5*b^5*c^3 - 3840*B^2*a^6*b^3*c^4 + 15360*A*B*a^7*c^6 + 213*A^2*a*b^11*c -
26880*A^2*a^6*b*c^6 + 27*B^2*a^3*b^9*c + 3840*B^2*a^7*b*c^5 - 9*B^2*a^3*c*(
-(4*a*c - b^2)^9)^(1/2) + 1548*A*B*a^3*b^8*c^2 - 8064*A*B*a^4*b^6*c^3 + 224
00*A*B*a^5*b^4*c^4 - 30720*A*B*a^6*b^2*c^5 - 51*A^2*a*b^2*c*(-(4*a*c - b^2)
^9)^(1/2) - 6*A*B*a*b^3*(-(4*a*c - b^2)^9)^(1/2) - 152*A*B*a^2*b^10*c + 44*
A*B*a^2*b*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^5*b^12 + 4096*a^11*c^6 - 24*a^
6*b^10*c + 240*a^7*b^8*c^2 - 1280*a^8*b^6*c^3 + 3840*a^9*b^4*c^4 - 6144*a^1
0*b^2*c^5)))^(1/2)*(1048576*a^16*b*c^8 + 256*a^10*b^13*c^2 - 6144*a^11*b^11
*c^3 + 61440*a^12*b^9*c^4 - 327680*a^13*b^7*c^5 + 983040*a^14*b^5*c^6 - 157
2864*a^15*b^3*c^7) - 393216*B*a^15*c^8 + 851968*A*a^14*b*c^8 + 192*A*a^8*b^
13*c^2 - 4672*A*a^9*b^11*c^3 + 47360*A*a^10*b^9*c^4 - 256000*A*a^11*b^7*c^5
 + 778240*A*a^12*b^5*c^6 - 1261568*A*a^13*b^3*c^7 - 64*B*a^9*b^12*c^2 + 166
4*B*a^10*b^10*c^3 - 17920*B*a^11*b^8*c^4 + 102400*B*a^12*b^6*c^5 - 327680*B
*a^13*b^4*c^6 + 557056*B*a^14*b^2*c^7) + x*(204800*A^2*a^12*c^9 - 73728*B^2
*a^13*c^8 + 144*A^2*a^6*b^12*c^3 - 3264*A^2*a^7*b^10*c^4 + 30112*A^2*a^8*b^
8*c^5 - 143360*A^2*a^9*b^6*c^6 + 365568*A^2*a^10*b^4*c^7 - 458752*A^2*a^11*
b^2*c^8 + 16*B^2*a^8*b^10*c^3 - 416*B^2*a^9*b^8*c^4 + 4608*B^2*a^10*b^6*c^5
 - 25600*B^2*a^11*b^4*c^6 + 69632*B^2*a^12*b^2*c^7 - 96*A*B*a^7*b^11*c^3 +
2336*A*B*a^8*b^9*c^4 - 22528*A*B*a^9*b^7*c^5 + 107520*A*B*a^10*b^5*c^6 - 25
3952*A*B*a^11*b^3*c^7 + 237568*A*B*a^12*b*c^8))*((9*A^2*b^4*(-(4*a*c - b^2)
^9)^(1/2) - B^2*a^2*b^11 - 9*A^2*b^13 + 6*A*B*a*b^12 - 2077*A^2*a^2*b^9*c^2
 + 10656*A^2*a^3*b^7*c^3 - 30240*A^2*a^4*b^5*c^4 + 44800*A^2*a^5*b^3*c^5 +
25*A^2*a^2*c^2*(-(4*a*c - b^2)^9)^(1/2) + B^2*a^2*b^2*(-(4*a*c - b^2)^9)^(1
/2) - 288*B^2*a^4*b^7*c^2 + 1504*B^2*a^5*b^5*c^3 - 3840*B^2*a^6*b^3*c^4 + 1
5360*A*B*a^7*c^6 + 213*A^2*a*b^11*c - 26880*A^2*a^6*b*c^6 + 27*B^2*a^3*b^9*
c + 3840*B^2*a^7*b*c^5 - 9*B^2*a^3*c*(-(4*a*c - b^2)^9)^(1/2) + 1548*A*B*a^
3*b^8*c^2 - 8064*A*B*a^4*b^6*c^3 + 22400*A*B*a^5*b^4*c^4 - 30720*A*B*a^6*b^
2*c^5 - 51*A^2*a*b^2*c*(-(4*a*c - b^2)^9)^(1/2) - 6*A*B*a*b^3*(-(4*a*c - b^
2)^9)^(1/2) - 152*A*B*a^2*b^10*c + 44*A*B*a^2*b*c*(-(4*a*c - b^2)^9)^(1/2))
/(32*(a^5*b^12 + 4096*a^11*c^6 - 24*a^6*b^10*c + 240*a^7*b^8*c^2 - 1280*a^8
*b^6*c^3 + 3840*a^9*b^4*c^4 - 6144*a^10*b^2*c^5)))^(1/2) - (((9*A^2*b^4*(-(
4*a*c - b^2)^9)^(1/2) - B^2*a^2*b^11 - 9*A^2*b^13 + 6*A*B*a*b^12 - 2077*A^2
*a^2*b^9*c^2 + 10656*A^2*a^3*b^7*c^3 - 30240*A^2*a^4*b^5*c^4 + 44800*A^2*a^
5*b^3*c^5 + 25*A^2*a^2*c^2*(-(4*a*c - b^2)^9)^(1/2) + B^2*a^2*b^2*(-(4*a*c
- b^2)^9)^(1/2) - 288*B^2*a^4*b^7*c^2 + 1504*B^2*a^5*b^5*c^3 - 3840*B^2*a^6
*b^3*c^4 + 15360*A*B*a^7*c^6 + 213*A^2*a*b^11*c - 26880*A^2*a^6*b*c^6 + 27*
B^2*a^3*b^9*c + 3840*B^2*a^7*b*c^5 - 9*B^2*a^3*c*(-(4*a*c - b^2)^9)^(1/2) +
 1548*A*B*a^3*b^8*c^2 - 8064*A*B*a^4*b^6*c^3 + 22400*A*B*a^5*b^4*c^4 - 3072
0*A*B*a^6*b^2*c^5 - 51*A^2*a*b^2*c*(-(4*a*c - b^2)^9)^(1/2) - 6*A*B*a*b^3*(
-(4*a*c - b^2)^9)^(1/2) - 152*A*B*a^2*b^10*c + 44*A*B*a^2*b*c*(-(4*a*c - b^
2)^9)^(1/2))/(32*(a^5*b^12 + 4096*a^11*c^6 - 24*a^6*b^10*c + 240*a^7*b^8*c^

2 - 1280*a^8*b^6*c^3 + 3840*a^9*b^4*c^4 - 6144*a^10*b^2*c^5)))^(1/2)*(x*((9
*A^2*b^4*(-(4*a*c - b^2)^9)^(1/2) - B^2*a^2*b^11 - 9*A^2*b^13 + 6*A*B*a*b^1
2 - 2077*A^2*a^2*b^9*c^2 + 10656*A^2*a^3*b^7*c^3 - 30240*A^2*a^4*b^5*c^4 +
44800*A^2*a^5*b^3*c^5 + 25*A^2*a^2*c^2*(-(4*a*c - b^2)^9)^(1/2) + B^2*a^2*b
^2*(-(4*a*c - b^2)^9)^(1/2) - 288*B^2*a^4*b^7*c^2 + 1504*B^2*a^5*b^5*c^3 -
3840*B^2*a^6*b^3*c^4 + 15360*A*B*a^7*c^6 + 213*A^2*a*b^11*c - 26880*A^2*a^6
*b*c^6 + 27*B^2*a^3*b^9*c + 3840*B^2*a^7*b*c^5 - 9*B^2*a^3*c*(-(4*a*c - b^2
)^9)^(1/2) + 1548*A*B*a^3*b^8*c^2 - 8064*A*B*a^4*b^6*c^3 + 22400*A*B*a^5*b^
4*c^4 - 30720*A*B*a^6*b^2*c^5 - 51*A^2*a*b^2*c*(-(4*a*c - b^2)^9)^(1/2) - 6
*A*B*a*b^3*(-(4*a*c - b^2)^9)^(1/2) - 152*A*B*a^2*b^10*c + 44*A*B*a^2*b*c*(
-(4*a*c - b^2)^9)^(1/2))/(32*(a^5*b^12 + 4096*a^11*c^6 - 24*a^6*b^10*c + 24
0*a^7*b^8*c^2 - 1280*a^8*b^6*c^3 + 3840*a^9*b^4*c^4 - 6144*a^10*b^2*c^5)))^
(1/2)*(1048576*a^16*b*c^8 + 256*a^10*b^13*c^2 - 6144*a^11*b^11*c^3 + 61440*
a^12*b^9*c^4 - 327680*a^13*b^7*c^5 + 983040*a^14*b^5*c^6 - 1572864*a^15*b^3
*c^7) + 393216*B*a^15*c^8 - 851968*A*a^14*b*c^8 - 192*A*a^8*b^13*c^2 + 4672
*A*a^9*b^11*c^3 - 47360*A*a^10*b^9*c^4 + 256000*A*a^11*b^7*c^5 - 778240*A*a
^12*b^5*c^6 + 1261568*A*a^13*b^3*c^7 + 64*B*a^9*b^12*c^2 - 1664*B*a^10*b^10
*c^3 + 17920*B*a^11*b^8*c^4 - 102400*B*a^12*b^6*c^5 + 327680*B*a^13*b^4*c^6
 - 557056*B*a^14*b^2*c^7) + x*(204800*A^2*a^12*c^9 - 73728*B^2*a^13*c^8 + 1
44*A^2*a^6*b^12*c^3 - 3264*A^2*a^7*b^10*c^4 + 30112*A^2*a^8*b^8*c^5 - 14336
0*A^2*a^9*b^6*c^6 + 365568*A^2*a^10*b^4*c^7 - 458752*A^2*a^11*b^2*c^8 + 16*
B^2*a^8*b^10*c^3 - 416*B^2*a^9*b^8*c^4 + 4608*B^2*a^10*b^6*c^5 - 25600*B^2*
a^11*b^4*c^6 + 69632*B^2*a^12*b^2*c^7 - 96*A*B*a^7*b^11*c^3 + 2336*A*B*a^8*
b^9*c^4 - 22528*A*B*a^9*b^7*c^5 + 107520*A*B*a^10*b^5*c^6 - 253952*A*B*a^11
*b^3*c^7 + 237568*A*B*a^12*b*c^8))*((9*A^2*b^4*(-(4*a*c - b^2)^9)^(1/2) - B
^2*a^2*b^11 - 9*A^2*b^13 + 6*A*B*a*b^12 - 2077*A^2*a^2*b^9*c^2 + 10656*A^2*
a^3*b^7*c^3 - 30240*A^2*a^4*b^5*c^4 + 44800*A^2*a^5*b^3*c^5 + 25*A^2*a^2*c^
2*(-(4*a*c - b^2)^9)^(1/2) + B^2*a^2*b^2*(-(4*a*c - b^2)^9)^(1/2) - 288*B^2
*a^4*b^7*c^2 + 1504*B^2*a^5*b^5*c^3 - 3840*B^2*a^6*b^3*c^4 + 15360*A*B*a^7*
c^6 + 213*A^2*a*b^11*c - 26880*A^2*a^6*b*c^6 + 27*B^2*a^3*b^9*c + 3840*B^2*
a^7*b*c^5 - 9*B^2*a^3*c*(-(4*a*c - b^2)^9)^(1/2) + 1548*A*B*a^3*b^8*c^2 - 8
064*A*B*a^4*b^6*c^3 + 22400*A*B*a^5*b^4*c^4 - 30720*A*B*a^6*b^2*c^5 - 51*A^
2*a*b^2*c*(-(4*a*c - b^2)^9)^(1/2) - 6*A*B*a*b^3*(-(4*a*c - b^2)^9)^(1/2) -
 152*A*B*a^2*b^10*c + 44*A*B*a^2*b*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^5*b^1
2 + 4096*a^11*c^6 - 24*a^6*b^10*c + 240*a^7*b^8*c^2 - 1280*a^8*b^6*c^3 + 38
40*a^9*b^4*c^4 - 6144*a^10*b^2*c^5)))^(1/2) + 128000*A^3*a^10*c^9 + 504*A^3
*a^6*b^8*c^5 - 8112*A^3*a^7*b^6*c^6 + 48704*A^3*a^8*b^4*c^7 - 129280*A^3*a^
9*b^2*c^8 - 40*B^3*a^8*b^7*c^4 + 608*B^3*a^9*b^5*c^5 - 2944*B^3*a^10*b^3*c^
6 + 46080*A*B^2*a^11*c^8 + 4608*B^3*a^11*b*c^7 - 84480*A^2*B*a^10*b*c^8 + 2
40*A*B^2*a^7*b^8*c^4 - 3792*A*B^2*a^8*b^6*c^5 + 21696*A*B^2*a^9*b^4*c^6 - 5
2992*A*B^2*a^10*b^2*c^7 - 360*A^2*B*a^6*b^9*c^4 + 5736*A^2*B*a^7*b^7*c^5 -
33888*A^2*B*a^8*b^5*c^6 + 87936*A^2*B*a^9*b^3*c^7))*((9*A^2*b^4*(-(4*a*c -
b^2)^9)^(1/2) - B^2*a^2*b^11 - 9*A^2*b^13 + 6*A*B*a*b^12 - 2077*A^2*a^2*b^9
*c^2 + 10656*A^2*a^3*b^7*c^3 - 30240*A^2*a^4*b^5*c^4 + 44800*A^2*a^5*b^3*c^
5 + 25*A^2*a^2*c^2*(-(4*a*c - b^2)^9)^(1/2) + B^2*a^2*b^2*(-(4*a*c - b^2)^9

```
)^(1/2) - 288*B^2*a^4*b^7*c^2 + 1504*B^2*a^5*b^5*c^3 - 3840*B^2*a^6*b^3*c^4
 + 15360*A*B*a^7*c^6 + 213*A^2*a*b^11*c - 26880*A^2*a^6*b*c^6 + 27*B^2*a^3*
b^9*c + 3840*B^2*a^7*b*c^5 - 9*B^2*a^3*c*(-(4*a*c - b^2)^9)^(1/2) + 1548*A*
B*a^3*b^8*c^2 - 8064*A*B*a^4*b^6*c^3 + 22400*A*B*a^5*b^4*c^4 - 30720*A*B*a^
6*b^2*c^5 - 51*A^2*a*b^2*c*(-(4*a*c - b^2)^9)^(1/2) - 6*A*B*a*b^3*(-(4*a*c
- b^2)^9)^(1/2) - 152*A*B*a^2*b^10*c + 44*A*B*a^2*b*c*(-(4*a*c - b^2)^9)^(1
/2))/(32*(a^5*b^12 + 4096*a^11*c^6 - 24*a^6*b^10*c + 240*a^7*b^8*c^2 - 1280
*a^8*b^6*c^3 + 3840*a^9*b^4*c^4 - 6144*a^10*b^2*c^5)))^(1/2)*2i
```

3.123 $\int \frac{A+Bx^2}{x^4\left(a+bx^2+cx^4\right)^2}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 522

$$\int \frac{A+Bx^2}{x^4\left(a+bx^2+cx^4\right)^2}\,dx = -\frac{5Ab^2-3abB-14aAc}{6a^2\left(b^2-4ac\right)x^3}$$

$$-\frac{aB(3b^2-10ac)-A(5b^3-19abc)}{2a^3\left(b^2-4ac\right)x}-\frac{abB-A(b^2-2ac)-(Ab-2aB)cx^2}{2a\left(b^2-4ac\right)x^3\left(a+bx^2+cx^4\right)}$$

$$-\frac{\sqrt{c}\left(aB\left(3b^3-16abc+3b^2\sqrt{b^2-4ac}-10ac\sqrt{b^2-4ac}\right)-A\left(5b^4-29ab^2c+28a^2c^2+5b^3\sqrt{b^2-4ac}-1\right.\right.}{2\sqrt{2}a^3\left(b^2-4ac\right)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}}$$

$$+\frac{\sqrt{c}\left(aB\left(3b^3-16abc-3b^2\sqrt{b^2-4ac}+10ac\sqrt{b^2-4ac}\right)-A\left(5b^4-29ab^2c+28a^2c^2-5b^3\sqrt{b^2-4ac}+1\right.\right.}{2\sqrt{2}a^3\left(b^2-4ac\right)^{3/2}\sqrt{b+\sqrt{b^2-4ac}}}$$

```
[Out] 1/6*(14*A*a*c-5*A*b^2+3*B*a*b)/a^2/(-4*a*c+b^2)/x^3+1/2*(-a*B*(-10*a*c+3*b^
2)+A*(-19*a*b*c+5*b^3))/a^3/(-4*a*c+b^2)/x+1/2*(-a*b*B+A*(-2*a*c+b^2)+(A*b-
2*B*a)*c*x^2)/a/(-4*a*c+b^2)/x^3/(c*x^4+b*x^2+a)-1/4*arctan(x*2^(1/2)*c^(1/
2)/(b-(-4*a*c+b^2)^(1/2))^(1/2))*c^(1/2)*(a*B*(3*b^3-16*a*b*c+3*b^2*(-4*a*c
+b^2)^(1/2)-10*a*c*(-4*a*c+b^2)^(1/2))-A*(5*b^4-29*a*b^2*c+28*a^2*c^2+5*(-4
*a*c+b^2)^(1/2)*b^3-19*(-4*a*c+b^2)^(1/2)*a*b*c))/a^3/(-4*a*c+b^2)^(3/2)*2^
(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2)+1/4*arctan(x*2^(1/2)*c^(1/2)/(b+(-4*a*c+
b^2)^(1/2))^(1/2))*c^(1/2)*(a*B*(3*b^3-16*a*b*c-3*b^2*(-4*a*c+b^2)^(1/2)+10
*a*c*(-4*a*c+b^2)^(1/2))-A*(5*b^4-29*a*b^2*c+28*a^2*c^2-5*(-4*a*c+b^2)^(1/2
)*b^3+19*(-4*a*c+b^2)^(1/2)*a*b*c))/a^3/(-4*a*c+b^2)^(3/2)*2^(1/2)/(b+(-4*a
*c+b^2)^(1/2))^(1/2)
```

Rubi [A] (verified)

Time = 0.94 (sec) , antiderivative size = 522, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {1291, 1295, 1180, 211}

$$\int \frac{A+Bx^2}{x^4\left(a+bx^2+cx^4\right)^2}\,dx = -\frac{aB(3b^2-10ac)-A(5b^3-19abc)}{2a^3x\left(b^2-4ac\right)} - \frac{-14aAc-3abB+5Ab^2}{6a^2x^3\left(b^2-4ac\right)}$$
$$-\frac{\sqrt{c}\left(aB\left(3b^2\sqrt{b^2-4ac}-10ac\sqrt{b^2-4ac}-16abc+3b^3\right)-A\left(28a^2c^2-29ab^2c-19abc\sqrt{b^2-4ac}+5b^3\right.\right.}{2\sqrt{2}a^3\left(b^2-4ac\right)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}}$$
$$+\frac{\sqrt{c}\left(aB\left(-3b^2\sqrt{b^2-4ac}+10ac\sqrt{b^2-4ac}-16abc+3b^3\right)-A\left(28a^2c^2-29ab^2c+19abc\sqrt{b^2-4ac}-5b\right.\right.}{2\sqrt{2}a^3\left(b^2-4ac\right)^{3/2}\sqrt{\sqrt{b^2-4ac}+b}}$$
$$-\frac{-A(b^2-2ac)-\left(cx^2(Ab-2aB)\right)+abB}{2ax^3\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}$$

```
[In] Int[(A + B*x^2)/(x^4*(a + b*x^2 + c*x^4)^2),x]
```

```
[Out] -1/6*(5*A*b^2 - 3*a*b*B - 14*a*A*c)/(a^2*(b^2 - 4*a*c)*x^3) - (a*B*(3*b^2 -
 10*a*c) - A*(5*b^3 - 19*a*b*c))/(2*a^3*(b^2 - 4*a*c)*x) - (a*b*B - A*(b^2
- 2*a*c) - (A*b - 2*a*B)*c*x^2)/(2*a*(b^2 - 4*a*c)*x^3*(a + b*x^2 + c*x^4))
 - (Sqrt[c]*(a*B*(3*b^3 - 16*a*b*c + 3*b^2*Sqrt[b^2 - 4*a*c] - 10*a*c*Sqrt[
b^2 - 4*a*c]) - A*(5*b^4 - 29*a*b^2*c + 28*a^2*c^2 + 5*b^3*Sqrt[b^2 - 4*a*c
] - 19*a*b*c*Sqrt[b^2 - 4*a*c]))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b
^2 - 4*a*c]]])/(2*Sqrt[2]*a^3*(b^2 - 4*a*c)^(3/2)*Sqrt[b - Sqrt[b^2 - 4*a*c
]]) + (Sqrt[c]*(a*B*(3*b^3 - 16*a*b*c - 3*b^2*Sqrt[b^2 - 4*a*c] + 10*a*c*Sq
rt[b^2 - 4*a*c]) - A*(5*b^4 - 29*a*b^2*c + 28*a^2*c^2 - 5*b^3*Sqrt[b^2 - 4*
a*c] + 19*a*b*c*Sqrt[b^2 - 4*a*c]))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqr
t[b^2 - 4*a*c]]])/(2*Sqrt[2]*a^3*(b^2 - 4*a*c)^(3/2)*Sqrt[b + Sqrt[b^2 - 4*
a*c]])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1291

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[(-(f*x)^(m + 1))*(a + b*x^2 + c*x^4)^(p + 1)
*((d*(b^2 - 2*a*c) - a*b*e + (b*d - 2*a*e)*c*x^2)/(2*a*f*(p + 1)*(b^2 - 4*a
*c))), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), Int[(f*x)^m*(a + b*x^2 + c*
x^4)^(p + 1)*Simp[d*(b^2*(m + 2*(p + 1) + 1) - 2*a*c*(m + 4*(p + 1) + 1)) -
 a*b*e*(m + 1) + c*(m + 2*(2*p + 3) + 1)*(b*d - 2*a*e)*x^2, x], x], x] /; F
reeQ[{a, b, c, d, e, f, m}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && Inte
gerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1295

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[d*(f*x)^(m + 1)*((a + b*x^2 + c*x^4)^(p + 1)
/(a*f*(m + 1))), x] + Dist[1/(a*f^2*(m + 1)), Int[(f*x)^(m + 2)*(a + b*x^2
+ c*x^4)^p*Simp[a*e*(m + 1) - b*d*(m + 2*p + 3) - c*d*(m + 4*p + 5)*x^2, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[m
, -1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{abB-A(b^2-2ac)-(Ab-2aB)cx^2}{2a\left(b^2-4ac\right)x^3\left(a+bx^2+cx^4\right)}-\frac{\int \frac{-5Ab^2+3abB+14aAc-5(Ab-2aB)cx^2}{x^4(a+bx^2+cx^4)}\,dx}{2a\left(b^2-4ac\right)}\\
&= -\frac{5Ab^2-3abB-14aAc}{6a^2\left(b^2-4ac\right)x^3}-\frac{abB-A(b^2-2ac)-(Ab-2aB)cx^2}{2a\left(b^2-4ac\right)x^3\left(a+bx^2+cx^4\right)}\\
&\quad+\frac{\int \frac{-3\left(5Ab^3-3ab^2B-19aAbc+10a^2Bc\right)-3c\left(5Ab^2-3abB-14aAc\right)x^2}{x^2(a+bx^2+cx^4)}\,dx}{6a^2\left(b^2-4ac\right)}\\
&= -\frac{5Ab^2-3abB-14aAc}{6a^2\left(b^2-4ac\right)x^3}-\frac{aB(3b^2-10ac)-A(5b^3-19abc)}{2a^3\left(b^2-4ac\right)x}\\
&\quad-\frac{abB-A(b^2-2ac)-(Ab-2aB)cx^2}{2a\left(b^2-4ac\right)x^3\left(a+bx^2+cx^4\right)}\\
&\quad-\frac{\int \frac{3\left(abB\left(3b^2-13ac\right)-A\left(5b^4-24ab^2c+14a^2c^2\right)\right)+3c\left(aB\left(3b^2-10ac\right)-A\left(5b^3-19abc\right)\right)x^2}{a+bx^2+cx^4}\,dx}{6a^3\left(b^2-4ac\right)}\\
&= -\frac{5Ab^2-3abB-14aAc}{6a^2\left(b^2-4ac\right)x^3}-\frac{aB(3b^2-10ac)-A(5b^3-19abc)}{2a^3\left(b^2-4ac\right)x}\\
&\quad-\frac{abB-A(b^2-2ac)-(Ab-2aB)cx^2}{2a\left(b^2-4ac\right)x^3\left(a+bx^2+cx^4\right)}\\
&\quad-\frac{\left(c\left(aB\left(3b^3-16abc+3b^2\sqrt{b^2-4ac}-10ac\sqrt{b^2-4ac}\right)-A\left(5b^4-29ab^2c+28a^2c^2+5b^3\sqrt{b^2-4\right.\right.\right.}{4a^3\left(b^2-4ac\right)^{3/2}}\\
&\quad+\frac{\left(c\left(aB\left(3b^3-16abc-3b^2\sqrt{b^2-4ac}+10ac\sqrt{b^2-4ac}\right)-A\left(5b^4-29ab^2c+28a^2c^2-5b^3\sqrt{b^2-4\right.\right.\right.}{4a^3\left(b^2-4ac\right)^{3/2}}
\end{aligned}$$

$$= -\frac{5Ab^2 - 3abB - 14aAc}{6a^2(b^2-4ac)x^3} - \frac{aB(3b^2-10ac) - A(5b^3-19abc)}{2a^3(b^2-4ac)x}$$
$$- \frac{abB - A(b^2-2ac) - (Ab-2aB)cx^2}{2a(b^2-4ac)x^3(a+bx^2+cx^4)}$$
$$- \frac{\sqrt{c}\big(aB\big(3b^3 - 16abc + 3b^2\sqrt{b^2-4ac} - 10ac\sqrt{b^2-4ac}\big) - A\big(5b^4 - 29ab^2c + 28a^2c^2 + 5b^3\sqrt{b^2 -}}{2\sqrt{2}a^3(b^2-4ac)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}}$$
$$+ \frac{\sqrt{c}\big(aB\big(3b^3 - 16abc - 3b^2\sqrt{b^2-4ac} + 10ac\sqrt{b^2-4ac}\big) - A\big(5b^4 - 29ab^2c + 28a^2c^2 - 5b^3\sqrt{b^2 -}}{2\sqrt{2}a^3(b^2-4ac)^{3/2}\sqrt{b+\sqrt{b^2-4ac}}}$$

Mathematica [A] (verified)

Time = 0.79 (sec) , antiderivative size = 487, normalized size of antiderivative = 0.93

$$\int \frac{A+Bx^2}{x^4(a+bx^2+cx^4)^2}\,dx$$
$$= \frac{-\frac{4aA}{x^3} + \frac{24Ab-12aB}{x} + \frac{6x\left(aB(-b^3+3abc-b^2cx^2+2ac^2x^2)+A(b^4-4ab^2c+2a^2c^2+b^3cx^2-3abc^2x^2)\right)}{(b^2-4ac)(a+bx^2+cx^4)} + \frac{3\sqrt{2}\sqrt{c}\Big(aB\Big(-3b^3+16abc-3b^2\sqrt{b}}{}}{}$$

```
[In] Integrate[(A + B*x^2)/(x^4*(a + b*x^2 + c*x^4)^2),x]
```

```
[Out] ((-4*a*A)/x^3 + (24*A*b - 12*a*B)/x + (6*x*(a*B*(-b^3 + 3*a*b*c - b^2*c*x^2
 + 2*a*c^2*x^2) + A*(b^4 - 4*a*b^2*c + 2*a^2*c^2 + b^3*c*x^2 - 3*a*b*c^2*x^
2)))/((b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) + (3*Sqrt[2]*Sqrt[c]*(a*B*(-3*b^3
+ 16*a*b*c - 3*b^2*Sqrt[b^2 - 4*a*c] + 10*a*c*Sqrt[b^2 - 4*a*c]) + A*(5*b^4
 - 29*a*b^2*c + 28*a^2*c^2 + 5*b^3*Sqrt[b^2 - 4*a*c] - 19*a*b*c*Sqrt[b^2 -
4*a*c]))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/((b^2 - 4
*a*c)^(3/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) - (3*Sqrt[2]*Sqrt[c]*(a*B*(-3*b^3
+ 16*a*b*c + 3*b^2*Sqrt[b^2 - 4*a*c] - 10*a*c*Sqrt[b^2 - 4*a*c]) + A*(5*b^4
 - 29*a*b^2*c + 28*a^2*c^2 - 5*b^3*Sqrt[b^2 - 4*a*c] + 19*a*b*c*Sqrt[b^2 -
4*a*c]))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/((b^2 - 4
*a*c)^(3/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]]))/(12*a^3)
```

Maple [A] (verified)

Time = 0.19 (sec) , antiderivative size = 484, normalized size of antiderivative = 0.93

method	result
default	$-\frac{A}{3a^2x^3}-\frac{-2Ab+Ba}{x\,a^3}-\frac{-\frac{c\left(3Aabc-A\,b^3-2a^2Bc+Ba\,b^2\right)x^3}{2\left(4ac-b^2\right)}+\frac{\left(2A\,a^2c^2-4Aa\,b^2c+A\,b^4+3a^2bBc-Ba\,b^3\right)x}{8ac-2b^2}}{c\,x^4+b\,x^2+a}+\frac{2c\left(\frac{\left(-19Aabc\sqrt{-4ac+b^2}+5A\right.}{}\right.}{}$
risch	Expression too large to display

```
[In] int((B*x^2+A)/x^4/(c*x^4+b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/3*A/a^2/x^3-(-2*A*b+B*a)/x/a^3-1/a^3*((-1/2*c*(3*A*a*b*c-A*b^3-2*B*a^2*c
+B*a*b^2)/(4*a*c-b^2)*x^3+1/2*(2*A*a^2*c^2-4*A*a*b^2*c+A*b^4+3*B*a^2*b*c-B*
a*b^3)/(4*a*c-b^2)*x)/(c*x^4+b*x^2+a)+2/(4*a*c-b^2)*c*(1/8*(-19*A*a*b*c*(-4
*a*c+b^2)^(1/2)+5*A*b^3*(-4*a*c+b^2)^(1/2)-28*A*a^2*c^2+29*A*a*b^2*c-5*A*b^
4+10*a^2*B*c*(-4*a*c+b^2)^(1/2)-3*B*a*b^2*(-4*a*c+b^2)^(1/2)-16*a^2*b*B*c+3
*B*a*b^3)/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arcta
n(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))-1/8*(-19*A*a*b*c*(-4*a*c+b^
2)^(1/2)+5*A*b^3*(-4*a*c+b^2)^(1/2)+28*A*a^2*c^2-29*A*a*b^2*c+5*A*b^4+10*a^
2*B*c*(-4*a*c+b^2)^(1/2)-3*B*a*b^2*(-4*a*c+b^2)^(1/2)+16*a^2*b*B*c-3*B*a*b^
3)/(-4*a*c+b^2)^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(c*x
*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 10190 vs. $2(460) = 920$.

Time = 21.40 (sec) , antiderivative size = 10190, normalized size of antiderivative = 19.52

$$\int \frac{A+Bx^2}{x^4\left(a+bx^2+cx^4\right)^2}\,dx = \text{Too large to display}$$

```
[In] integrate((B*x^2+A)/x^4/(c*x^4+b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A+Bx^2}{x^4\left(a+bx^2+cx^4\right)^2}\,dx = \text{Timed out}$$

```
[In] integrate((B*x**2+A)/x**4/(c*x**4+b*x**2+a)**2,x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{A+Bx^2}{x^4\left(a+bx^2+cx^4\right)^2}\,dx = \int \frac{Bx^2+A}{\left(cx^4+bx^2+a\right)^2x^4}\,dx$$

```
[In] integrate((B*x^2+A)/x^4/(c*x^4+b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] 1/6*(3*((10*B*a^2 - 19*A*a*b)*c^2 - (3*B*a*b^2 - 5*A*b^3)*c)*x^6 - 2*A*a^2*
b^2 + 8*A*a^3*c - (9*B*a*b^3 - 15*A*b^4 - 14*A*a^2*c^2 - (33*B*a^2*b - 62*A
*a*b^2)*c)*x^4 - 2*(3*B*a^2*b^2 - 5*A*a*b^3 - 4*(3*B*a^3 - 5*A*a^2*b)*c)*x^
2)/((a^3*b^2*c - 4*a^4*c^2)*x^7 + (a^3*b^3 - 4*a^4*b*c)*x^5 + (a^4*b^2 - 4*
a^5*c)*x^3) - 1/2*integrate((3*B*a*b^3 - 5*A*b^4 - 14*A*a^2*c^2 - ((10*B*a^
2 - 19*A*a*b)*c^2 - (3*B*a*b^2 - 5*A*b^3)*c)*x^2 - (13*B*a^2*b - 24*A*a*b^2
)*c)/(c*x^4 + b*x^2 + a), x)/(a^3*b^2 - 4*a^4*c)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 6327 vs. $2(460) = 920$.

Time = 1.49 (sec) , antiderivative size = 6327, normalized size of antiderivative = 12.12

$$\int \frac{A+Bx^2}{x^4\left(a+bx^2+cx^4\right)^2}\,dx = \text{Too large to display}$$

```
[In] integrate((B*x^2+A)/x^4/(c*x^4+b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] -1/2*(B*a*b^2*c*x^3 - A*b^3*c*x^3 - 2*B*a^2*c^2*x^3 + 3*A*a*b*c^2*x^3 + B*a
*b^3*x - A*b^4*x - 3*B*a^2*b*c*x + 4*A*a*b^2*c*x - 2*A*a^2*c^2*x)/((a^3*b^2
 - 4*a^4*c)*(c*x^4 + b*x^2 + a)) + 1/16*((10*b^5*c^2 - 78*a*b^3*c^3 + 152*a
^2*b*c^4 - 5*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^5
+ 39*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c + 10
*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^4*c - 76*sqrt(
2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b*c^2 - 38*sqrt(2)
*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^2 - 5*sqrt(2)*sq
rt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^3*c^2 + 19*sqrt(2)*sqrt(b
^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c^3 - 10*(b^2 - 4*a*c)*b^3*
c^2 + 38*(b^2 - 4*a*c)*a*b*c^3)*(a^3*b^2 - 4*a^4*c)^2*A - (6*a*b^4*c^2 - 44
*a^2*b^2*c^3 + 80*a^3*c^4 - 3*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2
 - 4*a*c)*c)*a*b^4 + 22*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a
*c)*c)*a^2*b^2*c + 6*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)
*c)*a*b^3*c - 40*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*
a^3*c^2 - 20*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*
b*c^2 - 3*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c
^2 + 10*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*c^3 -
```

6*(b^2 - 4*a*c)*a*b^2*c^2 + 20*(b^2 - 4*a*c)*a^2*c^3)*(a^3*b^2 - 4*a^4*c)^
2*B + 2*(5*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^8 - 64*sqrt(2)*sqr
t(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b^6*c - 10*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4
*a*c)*c)*a^3*b^7*c - 10*a^3*b^8*c + 286*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c
)*c)*a^5*b^4*c^2 + 88*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b^5*c^2 +
 5*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^6*c^2 + 128*a^4*b^6*c^2 -
496*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^6*b^2*c^3 - 220*sqrt(2)*sqrt(
b*c + sqrt(b^2 - 4*a*c)*c)*a^5*b^3*c^3 - 44*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4
*a*c)*c)*a^4*b^4*c^3 - 572*a^5*b^4*c^3 + 224*sqrt(2)*sqrt(b*c + sqrt(b^2 -
4*a*c)*c)*a^7*c^4 + 112*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^6*b*c^4 +
 110*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^5*b^2*c^4 + 992*a^6*b^2*c^4
- 56*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^6*c^5 - 448*a^7*c^5 + 10*(b^
2 - 4*a*c)*a^3*b^6*c - 88*(b^2 - 4*a*c)*a^4*b^4*c^2 + 220*(b^2 - 4*a*c)*a^5
*b^2*c^3 - 112*(b^2 - 4*a*c)*a^6*c^4)*A*abs(a^3*b^2 - 4*a^4*c) - 2*(3*sqrt(
2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b^7 - 37*sqrt(2)*sqrt(b*c + sqrt(b^2
 - 4*a*c)*c)*a^5*b^5*c - 6*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b^6*
c - 6*a^4*b^7*c + 152*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^6*b^3*c^2 +
 50*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^5*b^4*c^2 + 3*sqrt(2)*sqrt(b*
c + sqrt(b^2 - 4*a*c)*c)*a^4*b^5*c^2 + 74*a^5*b^5*c^2 - 208*sqrt(2)*sqrt(b*
c + sqrt(b^2 - 4*a*c)*c)*a^7*b*c^3 - 104*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*
c)*c)*a^6*b^2*c^3 - 25*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^5*b^3*c^3
- 304*a^6*b^3*c^3 + 52*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^6*b*c^4 +
416*a^7*b*c^4 + 6*(b^2 - 4*a*c)*a^4*b^5*c - 50*(b^2 - 4*a*c)*a^5*b^3*c^2 +
104*(b^2 - 4*a*c)*a^6*b*c^3)*B*abs(a^3*b^2 - 4*a^4*c) + (10*a^6*b^9*c^2 - 1
38*a^7*b^7*c^3 + 680*a^8*b^5*c^4 - 1376*a^9*b^3*c^5 + 896*a^10*b*c^6 - 5*sq
rt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^6*b^9 + 69*sqrt(2
)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^7*b^7*c + 10*sqrt(2)*
sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^6*b^8*c - 340*sqrt(2)*s
qrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^8*b^5*c^2 - 98*sqrt(2)*s
qrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^7*b^6*c^2 - 5*sqrt(2)*sq
rt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^6*b^7*c^2 + 688*sqrt(2)*s
qrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^9*b^3*c^3 + 288*sqrt(2)*
sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^8*b^4*c^3 + 49*sqrt(2)*
sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^7*b^5*c^3 - 448*sqrt(2)
*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^10*b*c^4 - 224*sqrt(2)
*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^9*b^2*c^4 - 144*sqrt(2
)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^8*b^3*c^4 + 112*sqrt(
2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^9*b*c^5 - 10*(b^2 -
4*a*c)*a^6*b^7*c^2 + 98*(b^2 - 4*a*c)*a^7*b^5*c^3 - 288*(b^2 - 4*a*c)*a^8*b
^3*c^4 + 224*(b^2 - 4*a*c)*a^9*b*c^5)*A - (6*a^7*b^8*c^2 - 80*a^8*b^6*c^3 +
 352*a^9*b^4*c^4 - 512*a^10*b^2*c^5 - 3*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c
+ sqrt(b^2 - 4*a*c)*c)*a^7*b^8 + 40*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sq
rt(b^2 - 4*a*c)*c)*a^8*b^6*c + 6*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(
b^2 - 4*a*c)*c)*a^7*b^7*c - 176*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b
^2 - 4*a*c)*c)*a^9*b^4*c^2 - 56*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b

^2 - 4*a*c)*c)*a^8*b^5*c^2 - 3*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^
2 - 4*a*c)*c)*a^7*b^6*c^2 + 256*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b
^2 - 4*a*c)*c)*a^10*b^2*c^3 + 128*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt
(b^2 - 4*a*c)*c)*a^9*b^3*c^3 + 28*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt
(b^2 - 4*a*c)*c)*a^8*b^4*c^3 - 64*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt
(b^2 - 4*a*c)*c)*a^9*b^2*c^4 - 6*(b^2 - 4*a*c)*a^7*b^6*c^2 + 56*(b^2 - 4*a*
c)*a^8*b^4*c^3 - 128*(b^2 - 4*a*c)*a^9*b^2*c^4)*B)*arctan(2*sqrt(1/2)*x/sqr
t((a^3*b^3 - 4*a^4*b*c + sqrt((a^3*b^3 - 4*a^4*b*c)^2 - 4*(a^4*b^2 - 4*a^5*
c)*(a^3*b^2*c - 4*a^4*c^2)))/(a^3*b^2*c - 4*a^4*c^2)))/((a^7*b^6 - 12*a^8*b
^4*c - 2*a^7*b^5*c + 48*a^9*b^2*c^2 + 16*a^8*b^3*c^2 + a^7*b^4*c^2 - 64*a^1
0*c^3 - 32*a^9*b*c^3 - 8*a^8*b^2*c^3 + 16*a^9*c^4)*abs(a^3*b^2 - 4*a^4*c)*a
bs(c)) - 1/16*((10*b^5*c^2 - 78*a*b^3*c^3 + 152*a^2*b*c^4 - 5*sqrt(2)*sqrt(
b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^5 + 39*sqrt(2)*sqrt(b^2 - 4*
a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^3*c + 10*sqrt(2)*sqrt(b^2 - 4*a*c)
*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^4*c - 76*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(
b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b*c^2 - 38*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*
c - sqrt(b^2 - 4*a*c)*c)*a*b^2*c^2 - 5*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c -
 sqrt(b^2 - 4*a*c)*c)*b^3*c^2 + 19*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqr
t(b^2 - 4*a*c)*c)*a*b*c^3 - 10*(b^2 - 4*a*c)*b^3*c^2 + 38*(b^2 - 4*a*c)*a*b
c^3)(a^3*b^2 - 4*a^4*c)^2*A - (6*a*b^4*c^2 - 44*a^2*b^2*c^3 + 80*a^3*c^4
 - 3*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^4 + 22*sq
rt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c + 6*sqrt(
2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^3*c - 40*sqrt(2)*s
qrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*c^2 - 20*sqrt(2)*sqrt(
b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b*c^2 - 3*sqrt(2)*sqrt(b^2
 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^2*c^2 + 10*sqrt(2)*sqrt(b^2 -
 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*c^3 - 6*(b^2 - 4*a*c)*a*b^2*c^2
 + 20*(b^2 - 4*a*c)*a^2*c^3)*(a^3*b^2 - 4*a^4*c)^2*B - 2*(5*sqrt(2)*sqrt(b*
c - sqrt(b^2 - 4*a*c)*c)*a^3*b^8 - 64*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*
c)*a^4*b^6*c - 10*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^7*c + 10*a^
3*b^8*c + 286*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^5*b^4*c^2 + 88*sqrt
(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*b^5*c^2 + 5*sqrt(2)*sqrt(b*c - sqrt
(b^2 - 4*a*c)*c)*a^3*b^6*c^2 - 128*a^4*b^6*c^2 - 496*sqrt(2)*sqrt(b*c - sqr
t(b^2 - 4*a*c)*c)*a^6*b^2*c^3 - 220*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)
*a^5*b^3*c^3 - 44*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*b^4*c^3 + 572
*a^5*b^4*c^3 + 224*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^7*c^4 + 112*sq
rt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^6*b*c^4 + 110*sqrt(2)*sqrt(b*c - sq
rt(b^2 - 4*a*c)*c)*a^5*b^2*c^4 - 992*a^6*b^2*c^4 - 56*sqrt(2)*sqrt(b*c - sq
rt(b^2 - 4*a*c)*c)*a^6*c^5 + 448*a^7*c^5 - 10*(b^2 - 4*a*c)*a^3*b^6*c + 88*
(b^2 - 4*a*c)*a^4*b^4*c^2 - 220*(b^2 - 4*a*c)*a^5*b^2*c^3 + 112*(b^2 - 4*a*
c)*a^6*c^4)*A*abs(a^3*b^2 - 4*a^4*c) + 2*(3*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4
*a*c)*c)*a^4*b^7 - 37*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^5*b^5*c - 6
*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*b^6*c + 6*a^4*b^7*c + 152*sqrt
(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^6*b^3*c^2 + 50*sqrt(2)*sqrt(b*c - sqr
t(b^2 - 4*a*c)*c)*a^5*b^4*c^2 + 3*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a

^4*b^5*c^2 - 74*a^5*b^5*c^2 - 208*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^7*b*c^3 - 104*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^6*b^2*c^3 - 25*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^5*b^3*c^3 + 304*a^6*b^3*c^3 + 52*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^6*b*c^4 - 416*a^7*b*c^4 - 6*(b^2 - 4*a*c)*a^4*b^5*c + 50*(b^2 - 4*a*c)*a^5*b^3*c^2 - 104*(b^2 - 4*a*c)*a^6*b*c^3)*B*abs(a^3*b^2 - 4*a^4*c) + (10*a^6*b^9*c^2 - 138*a^7*b^7*c^3 + 680*a^8*b^5*c^4 - 1376*a^9*b^3*c^5 + 896*a^10*b*c^6 - 5*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^6*b^9 + 69*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^7*b^7*c + 10*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^6*b^8*c - 340*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^8*b^5*c^2 - 98*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^7*b^6*c^2 - 5*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^6*b^7*c^2 + 688*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^9*b^3*c^3 + 288*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^8*b^4*c^3 + 49*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^7*b^5*c^3 - 448*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^10*b*c^4 - 224*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^9*b^2*c^4 - 144*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^8*b^3*c^4 + 112*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^9*b*c^5 - 10*(b^2 - 4*a*c)*a^6*b^7*c^2 + 98*(b^2 - 4*a*c)*a^7*b^5*c^3 - 288*(b^2 - 4*a*c)*a^8*b^3*c^4 + 224*(b^2 - 4*a*c)*a^9*b*c^5)*A - (6*a^7*b^8*c^2 - 80*a^8*b^6*c^3 + 352*a^9*b^4*c^4 - 512*a^10*b^2*c^5 - 3*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^7*b^8 + 40*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^8*b^6*c + 6*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^7*b^7*c - 176*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^9*b^4*c^2 - 56*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^8*b^5*c^2 - 3*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^7*b^6*c^2 + 256*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^10*b^2*c^3 + 128*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^9*b^3*c^3 + 28*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^8*b^4*c^3 - 64*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^9*b^2*c^4 - 6*(b^2 - 4*a*c)*a^7*b^6*c^2 + 56*(b^2 - 4*a*c)*a^8*b^4*c^3 - 128*(b^2 - 4*a*c)*a^9*b^2*c^4)*B)*arctan(2*sqrt(1/2)*x/sqrt((a^3*b^3 - 4*a^4*b*c - sqrt((a^3*b^3 - 4*a^4*b*c)^2 - 4*(a^4*b^2 - 4*a^5*c)*(a^3*b^2*c - 4*a^4*c^2)))/(a^3*b^2*c - 4*a^4*c^2)))/((a^7*b^6 - 12*a^8*b^4*c - 2*a^7*b^5*c + 48*a^9*b^2*c^2 + 16*a^8*b^3*c^2 + a^7*b^4*c^2 - 64*a^10*c^3 - 32*a^9*b*c^3 - 8*a^8*b^2*c^3 + 16*a^9*c^4)*abs(a^3*b^2 - 4*a^4*c)*abs(c)) - 1/3*(3*B*a*x^2 - 6*A*b*x^2 + A*a)/(a^3*x^3)

Mupad [B] (verification not implemented)

Time = 11.36 (sec) , antiderivative size = 21554, normalized size of antiderivative = 41.29

$$\int \frac{A+Bx^2}{x^4\left(a+bx^2+cx^4\right)^2}\,dx = \text{Too large to display}$$

```
[In] int((A + B*x^2)/(x^4*(a + b*x^2 + c*x^4)^2),x)
[Out] - atan((((-(25*A^2*b^15 + 9*B^2*a^2*b^13 - 25*A^2*b^6*(-(4*a*c - b^2)^9)^(1
/2) - 30*A*B*a*b^14 + 6366*A^2*a^2*b^11*c^2 - 35767*A^2*a^3*b^9*c^3 + 11692
8*A^2*a^4*b^7*c^4 - 219744*A^2*a^5*b^5*c^5 + 215040*A^2*a^6*b^3*c^6 + 49*A^
2*a^3*c^3*(-(4*a*c - b^2)^9)^(1/2) - 9*B^2*a^2*b^4*(-(4*a*c - b^2)^9)^(1/2)
 + 2077*B^2*a^4*b^9*c^2 - 10656*B^2*a^5*b^7*c^3 + 30240*B^2*a^6*b^5*c^4 - 4
4800*B^2*a^7*b^3*c^5 - 25*B^2*a^4*c^2*(-(4*a*c - b^2)^9)^(1/2) + 35840*A*B*
a^8*c^7 - 615*A^2*a*b^13*c - 80640*A^2*a^7*b*c^7 - 213*B^2*a^3*b^11*c + 268
80*B^2*a^8*b*c^6 - 246*A^2*a^2*b^2*c^2*(-(4*a*c - b^2)^9)^(1/2) - 7278*A*B*
a^3*b^10*c^2 + 39132*A*B*a^4*b^8*c^3 - 119616*A*B*a^5*b^6*c^4 + 201600*A*B*
a^6*b^4*c^5 - 161280*A*B*a^7*b^2*c^6 + 165*A^2*a*b^4*c*(-(4*a*c - b^2)^9)^(
1/2) + 51*B^2*a^3*b^2*c*(-(4*a*c - b^2)^9)^(1/2) + 30*A*B*a*b^5*(-(4*a*c -
b^2)^9)^(1/2) + 724*A*B*a^2*b^12*c - 184*A*B*a^2*b^3*c*(-(4*a*c - b^2)^9)^(
1/2) + 186*A*B*a^3*b*c^2*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^7*b^12 + 4096*a^1
3*c^6 - 24*a^8*b^10*c + 240*a^9*b^8*c^2 - 1280*a^10*b^6*c^3 + 3840*a^11*b^4
*c^4 - 6144*a^12*b^2*c^5)))^(1/2)*(917504*A*a^19*c^9 + x*(-(25*A^2*b^15 + 9
*B^2*a^2*b^13 - 25*A^2*b^6*(-(4*a*c - b^2)^9)^(1/2) - 30*A*B*a*b^14 + 6366*
A^2*a^2*b^11*c^2 - 35767*A^2*a^3*b^9*c^3 + 116928*A^2*a^4*b^7*c^4 - 219744*
A^2*a^5*b^5*c^5 + 215040*A^2*a^6*b^3*c^6 + 49*A^2*a^3*c^3*(-(4*a*c - b^2)^9
)^(1/2) - 9*B^2*a^2*b^4*(-(4*a*c - b^2)^9)^(1/2) + 2077*B^2*a^4*b^9*c^2 - 1
0656*B^2*a^5*b^7*c^3 + 30240*B^2*a^6*b^5*c^4 - 44800*B^2*a^7*b^3*c^5 - 25*B
^2*a^4*c^2*(-(4*a*c - b^2)^9)^(1/2) + 35840*A*B*a^8*c^7 - 615*A^2*a*b^13*c
- 80640*A^2*a^7*b*c^7 - 213*B^2*a^3*b^11*c + 26880*B^2*a^8*b*c^6 - 246*A^2*
a^2*b^2*c^2*(-(4*a*c - b^2)^9)^(1/2) - 7278*A*B*a^3*b^10*c^2 + 39132*A*B*a^
4*b^8*c^3 - 119616*A*B*a^5*b^6*c^4 + 201600*A*B*a^6*b^4*c^5 - 161280*A*B*a^
7*b^2*c^6 + 165*A^2*a*b^4*c*(-(4*a*c - b^2)^9)^(1/2) + 51*B^2*a^3*b^2*c*(-(
4*a*c - b^2)^9)^(1/2) + 30*A*B*a*b^5*(-(4*a*c - b^2)^9)^(1/2) + 724*A*B*a^2
*b^12*c - 184*A*B*a^2*b^3*c*(-(4*a*c - b^2)^9)^(1/2) + 186*A*B*a^3*b*c^2*(-
(4*a*c - b^2)^9)^(1/2))/(32*(a^7*b^12 + 4096*a^13*c^6 - 24*a^8*b^10*c + 240
*a^9*b^8*c^2 - 1280*a^10*b^6*c^3 + 3840*a^11*b^4*c^4 - 6144*a^12*b^2*c^5)))
^(1/2)*(1048576*a^21*b*c^8 + 256*a^15*b^13*c^2 - 6144*a^16*b^11*c^3 + 61440
*a^17*b^9*c^4 - 327680*a^18*b^7*c^5 + 983040*a^19*b^5*c^6 - 1572864*a^20*b^
3*c^7) + 851968*B*a^19*b*c^8 - 320*A*a^12*b^14*c^2 + 7936*A*a^13*b^12*c^3 -
 82816*A*a^14*b^10*c^4 + 468480*A*a^15*b^8*c^5 - 1536000*A*a^16*b^6*c^6 + 2
867200*A*a^17*b^4*c^7 - 2719744*A*a^18*b^2*c^8 + 192*B*a^13*b^13*c^2 - 4672
*B*a^14*b^11*c^3 + 47360*B*a^15*b^9*c^4 - 256000*B*a^16*b^7*c^5 + 778240*B*
a^17*b^5*c^6 - 1261568*B*a^18*b^3*c^7) - x*(401408*A^2*a^16*c^10 - 204800*B
```

^2*a^17*c^9 - 400*A^2*a^9*b^14*c^3 + 9440*A^2*a^10*b^12*c^4 - 92816*A^2*a^1
1*b^10*c^5 + 488096*A^2*a^12*b^8*c^6 - 1458688*A^2*a^13*b^6*c^7 + 2401280*A
^2*a^14*b^4*c^8 - 1871872*A^2*a^15*b^2*c^9 - 144*B^2*a^11*b^12*c^3 + 3264*B
^2*a^12*b^10*c^4 - 30112*B^2*a^13*b^8*c^5 + 143360*B^2*a^14*b^6*c^6 - 36556
8*B^2*a^15*b^4*c^7 + 458752*B^2*a^16*b^2*c^8 + 480*A*B*a^10*b^13*c^3 - 1110
4*A*B*a^11*b^11*c^4 + 105824*A*B*a^12*b^9*c^5 - 530432*A*B*a^13*b^7*c^6 + 1
469440*A*B*a^14*b^5*c^7 - 2121728*A*B*a^15*b^3*c^8 + 1236992*A*B*a^16*b*c^9
))*(-(25*A^2*b^15 + 9*B^2*a^2*b^13 - 25*A^2*b^6*(-(4*a*c - b^2)^9)^(1/2) -
30*A*B*a*b^14 + 6366*A^2*a^2*b^11*c^2 - 35767*A^2*a^3*b^9*c^3 + 116928*A^2*
a^4*b^7*c^4 - 219744*A^2*a^5*b^5*c^5 + 215040*A^2*a^6*b^3*c^6 + 49*A^2*a^3*
c^3*(-(4*a*c - b^2)^9)^(1/2) - 9*B^2*a^2*b^4*(-(4*a*c - b^2)^9)^(1/2) + 207
7*B^2*a^4*b^9*c^2 - 10656*B^2*a^5*b^7*c^3 + 30240*B^2*a^6*b^5*c^4 - 44800*B
^2*a^7*b^3*c^5 - 25*B^2*a^4*c^2*(-(4*a*c - b^2)^9)^(1/2) + 35840*A*B*a^8*c^
7 - 615*A^2*a*b^13*c - 80640*A^2*a^7*b*c^7 - 213*B^2*a^3*b^11*c + 26880*B^2
*a^8*b*c^6 - 246*A^2*a^2*b^2*c^2*(-(4*a*c - b^2)^9)^(1/2) - 7278*A*B*a^3*b^
10*c^2 + 39132*A*B*a^4*b^8*c^3 - 119616*A*B*a^5*b^6*c^4 + 201600*A*B*a^6*b^
4*c^5 - 161280*A*B*a^7*b^2*c^6 + 165*A^2*a*b^4*c*(-(4*a*c - b^2)^9)^(1/2) +
51*B^2*a^3*b^2*c*(-(4*a*c - b^2)^9)^(1/2) + 30*A*B*a*b^5*(-(4*a*c - b^2)^9
)^(1/2) + 724*A*B*a^2*b^12*c - 184*A*B*a^2*b^3*c*(-(4*a*c - b^2)^9)^(1/2) +
186*A*B*a^3*b*c^2*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^7*b^12 + 4096*a^13*c^6
- 24*a^8*b^10*c + 240*a^9*b^8*c^2 - 1280*a^10*b^6*c^3 + 3840*a^11*b^4*c^4 -
6144*a^12*b^2*c^5)))^(1/2)*1i - ((-(25*A^2*b^15 + 9*B^2*a^2*b^13 - 25*A^2*
b^6*(-(4*a*c - b^2)^9)^(1/2) - 30*A*B*a*b^14 + 6366*A^2*a^2*b^11*c^2 - 3576
7*A^2*a^3*b^9*c^3 + 116928*A^2*a^4*b^7*c^4 - 219744*A^2*a^5*b^5*c^5 + 21504
0*A^2*a^6*b^3*c^6 + 49*A^2*a^3*c^3*(-(4*a*c - b^2)^9)^(1/2) - 9*B^2*a^2*b^4
*(-(4*a*c - b^2)^9)^(1/2) + 2077*B^2*a^4*b^9*c^2 - 10656*B^2*a^5*b^7*c^3 +
30240*B^2*a^6*b^5*c^4 - 44800*B^2*a^7*b^3*c^5 - 25*B^2*a^4*c^2*(-(4*a*c - b
^2)^9)^(1/2) + 35840*A*B*a^8*c^7 - 615*A^2*a*b^13*c - 80640*A^2*a^7*b*c^7 -
213*B^2*a^3*b^11*c + 26880*B^2*a^8*b*c^6 - 246*A^2*a^2*b^2*c^2*(-(4*a*c -
b^2)^9)^(1/2) - 7278*A*B*a^3*b^10*c^2 + 39132*A*B*a^4*b^8*c^3 - 119616*A*B*
a^5*b^6*c^4 + 201600*A*B*a^6*b^4*c^5 - 161280*A*B*a^7*b^2*c^6 + 165*A^2*a*b
^4*c*(-(4*a*c - b^2)^9)^(1/2) + 51*B^2*a^3*b^2*c*(-(4*a*c - b^2)^9)^(1/2) +
30*A*B*a*b^5*(-(4*a*c - b^2)^9)^(1/2) + 724*A*B*a^2*b^12*c - 184*A*B*a^2*b
^3*c*(-(4*a*c - b^2)^9)^(1/2) + 186*A*B*a^3*b*c^2*(-(4*a*c - b^2)^9)^(1/2))
/(32*(a^7*b^12 + 4096*a^13*c^6 - 24*a^8*b^10*c + 240*a^9*b^8*c^2 - 1280*a^1
0*b^6*c^3 + 3840*a^11*b^4*c^4 - 6144*a^12*b^2*c^5)))^(1/2)*(917504*A*a^19*c
^9 - x*(-(25*A^2*b^15 + 9*B^2*a^2*b^13 - 25*A^2*b^6*(-(4*a*c - b^2)^9)^(1/2
) - 30*A*B*a*b^14 + 6366*A^2*a^2*b^11*c^2 - 35767*A^2*a^3*b^9*c^3 + 116928*
A^2*a^4*b^7*c^4 - 219744*A^2*a^5*b^5*c^5 + 215040*A^2*a^6*b^3*c^6 + 49*A^2*
a^3*c^3*(-(4*a*c - b^2)^9)^(1/2) - 9*B^2*a^2*b^4*(-(4*a*c - b^2)^9)^(1/2) +
2077*B^2*a^4*b^9*c^2 - 10656*B^2*a^5*b^7*c^3 + 30240*B^2*a^6*b^5*c^4 - 448
00*B^2*a^7*b^3*c^5 - 25*B^2*a^4*c^2*(-(4*a*c - b^2)^9)^(1/2) + 35840*A*B*a^
8*c^7 - 615*A^2*a*b^13*c - 80640*A^2*a^7*b*c^7 - 213*B^2*a^3*b^11*c + 26880
*B^2*a^8*b*c^6 - 246*A^2*a^2*b^2*c^2*(-(4*a*c - b^2)^9)^(1/2) - 7278*A*B*a^
3*b^10*c^2 + 39132*A*B*a^4*b^8*c^3 - 119616*A*B*a^5*b^6*c^4 + 201600*A*B*a^

6*b^4*c^5 - 161280*A*B*a^7*b^2*c^6 + 165*A^2*a*b^4*c*(-(4*a*c - b^2)^9)^(1/
2) + 51*B^2*a^3*b^2*c*(-(4*a*c - b^2)^9)^(1/2) + 30*A*B*a*b^5*(-(4*a*c - b^
2)^9)^(1/2) + 724*A*B*a^2*b^12*c - 184*A*B*a^2*b^3*c*(-(4*a*c - b^2)^9)^(1/
2) + 186*A*B*a^3*b*c^2*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^7*b^12 + 4096*a^13*
c^6 - 24*a^8*b^10*c + 240*a^9*b^8*c^2 - 1280*a^10*b^6*c^3 + 3840*a^11*b^4*c
^4 - 6144*a^12*b^2*c^5)))^(1/2)*(1048576*a^21*b*c^8 + 256*a^15*b^13*c^2 - 6
144*a^16*b^11*c^3 + 61440*a^17*b^9*c^4 - 327680*a^18*b^7*c^5 + 983040*a^19*
b^5*c^6 - 1572864*a^20*b^3*c^7) + 851968*B*a^19*b*c^8 - 320*A*a^12*b^14*c^2
 + 7936*A*a^13*b^12*c^3 - 82816*A*a^14*b^10*c^4 + 468480*A*a^15*b^8*c^5 - 1
536000*A*a^16*b^6*c^6 + 2867200*A*a^17*b^4*c^7 - 2719744*A*a^18*b^2*c^8 + 1
92*B*a^13*b^13*c^2 - 4672*B*a^14*b^11*c^3 + 47360*B*a^15*b^9*c^4 - 256000*B
*a^16*b^7*c^5 + 778240*B*a^17*b^5*c^6 - 1261568*B*a^18*b^3*c^7) + x*(401408
*A^2*a^16*c^10 - 204800*B^2*a^17*c^9 - 400*A^2*a^9*b^14*c^3 + 9440*A^2*a^10
*b^12*c^4 - 92816*A^2*a^11*b^10*c^5 + 488096*A^2*a^12*b^8*c^6 - 1458688*A^2
*a^13*b^6*c^7 + 2401280*A^2*a^14*b^4*c^8 - 1871872*A^2*a^15*b^2*c^9 - 144*B
^2*a^11*b^12*c^3 + 3264*B^2*a^12*b^10*c^4 - 30112*B^2*a^13*b^8*c^5 + 143360
*B^2*a^14*b^6*c^6 - 365568*B^2*a^15*b^4*c^7 + 458752*B^2*a^16*b^2*c^8 + 480
*A*B*a^10*b^13*c^3 - 11104*A*B*a^11*b^11*c^4 + 105824*A*B*a^12*b^9*c^5 - 53
0432*A*B*a^13*b^7*c^6 + 1469440*A*B*a^14*b^5*c^7 - 2121728*A*B*a^15*b^3*c^8
 + 1236992*A*B*a^16*b*c^9))*(-(25*A^2*b^15 + 9*B^2*a^2*b^13 - 25*A^2*b^6*(-
(4*a*c - b^2)^9)^(1/2) - 30*A*B*a*b^14 + 6366*A^2*a^2*b^11*c^2 - 35767*A^2*
a^3*b^9*c^3 + 116928*A^2*a^4*b^7*c^4 - 219744*A^2*a^5*b^5*c^5 + 215040*A^2*
a^6*b^3*c^6 + 49*A^2*a^3*c^3*(-(4*a*c - b^2)^9)^(1/2) - 9*B^2*a^2*b^4*(-(4*
a*c - b^2)^9)^(1/2) + 2077*B^2*a^4*b^9*c^2 - 10656*B^2*a^5*b^7*c^3 + 30240*
B^2*a^6*b^5*c^4 - 44800*B^2*a^7*b^3*c^5 - 25*B^2*a^4*c^2*(-(4*a*c - b^2)^9)
^(1/2) + 35840*A*B*a^8*c^7 - 615*A^2*a*b^13*c - 80640*A^2*a^7*b*c^7 - 213*B
^2*a^3*b^11*c + 26880*B^2*a^8*b*c^6 - 246*A^2*a^2*b^2*c^2*(-(4*a*c - b^2)^9
)^(1/2) - 7278*A*B*a^3*b^10*c^2 + 39132*A*B*a^4*b^8*c^3 - 119616*A*B*a^5*b^
6*c^4 + 201600*A*B*a^6*b^4*c^5 - 161280*A*B*a^7*b^2*c^6 + 165*A^2*a*b^4*c*(
-(4*a*c - b^2)^9)^(1/2) + 51*B^2*a^3*b^2*c*(-(4*a*c - b^2)^9)^(1/2) + 30*A*
B*a*b^5*(-(4*a*c - b^2)^9)^(1/2) + 724*A*B*a^2*b^12*c - 184*A*B*a^2*b^3*c*(
-(4*a*c - b^2)^9)^(1/2) + 186*A*B*a^3*b*c^2*(-(4*a*c - b^2)^9)^(1/2))/(32*(
a^7*b^12 + 4096*a^13*c^6 - 24*a^8*b^10*c + 240*a^9*b^8*c^2 - 1280*a^10*b^6*
c^3 + 3840*a^11*b^4*c^4 - 6144*a^12*b^2*c^5)))^(1/2)*1i)/(((-(25*A^2*b^15 +
 9*B^2*a^2*b^13 - 25*A^2*b^6*(-(4*a*c - b^2)^9)^(1/2) - 30*A*B*a*b^14 + 636
6*A^2*a^2*b^11*c^2 - 35767*A^2*a^3*b^9*c^3 + 116928*A^2*a^4*b^7*c^4 - 21974
4*A^2*a^5*b^5*c^5 + 215040*A^2*a^6*b^3*c^6 + 49*A^2*a^3*c^3*(-(4*a*c - b^2)
^9)^(1/2) - 9*B^2*a^2*b^4*(-(4*a*c - b^2)^9)^(1/2) + 2077*B^2*a^4*b^9*c^2 -
 10656*B^2*a^5*b^7*c^3 + 30240*B^2*a^6*b^5*c^4 - 44800*B^2*a^7*b^3*c^5 - 25
*B^2*a^4*c^2*(-(4*a*c - b^2)^9)^(1/2) + 35840*A*B*a^8*c^7 - 615*A^2*a*b^13*
c - 80640*A^2*a^7*b*c^7 - 213*B^2*a^3*b^11*c + 26880*B^2*a^8*b*c^6 - 246*A^
2*a^2*b^2*c^2*(-(4*a*c - b^2)^9)^(1/2) - 7278*A*B*a^3*b^10*c^2 + 39132*A*B*
a^4*b^8*c^3 - 119616*A*B*a^5*b^6*c^4 + 201600*A*B*a^6*b^4*c^5 - 161280*A*B*
a^7*b^2*c^6 + 165*A^2*a*b^4*c*(-(4*a*c - b^2)^9)^(1/2) + 51*B^2*a^3*b^2*c*(
-(4*a*c - b^2)^9)^(1/2) + 30*A*B*a*b^5*(-(4*a*c - b^2)^9)^(1/2) + 724*A*B*a

^2*b^12*c - 184*A*B*a^2*b^3*c*(-(4*a*c - b^2)^9)^(1/2) + 186*A*B*a^3*b*c^2*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^7*b^12 + 4096*a^13*c^6 - 24*a^8*b^10*c + 240*a^9*b^8*c^2 - 1280*a^10*b^6*c^3 + 3840*a^11*b^4*c^4 - 6144*a^12*b^2*c^5)))^(1/2)*(917504*A*a^19*c^9 + x*(-(25*A^2*b^15 + 9*B^2*a^2*b^13 - 25*A^2*b^6*(-(4*a*c - b^2)^9)^(1/2) - 30*A*B*a*b^14 + 6366*A^2*a^2*b^11*c^2 - 35767*A^2*a^3*b^9*c^3 + 116928*A^2*a^4*b^7*c^4 - 219744*A^2*a^5*b^5*c^5 + 215040*A^2*a^6*b^3*c^6 + 49*A^2*a^3*c^3*(-(4*a*c - b^2)^9)^(1/2) - 9*B^2*a^2*b^4*(-(4*a*c - b^2)^9)^(1/2) + 2077*B^2*a^4*b^9*c^2 - 10656*B^2*a^5*b^7*c^3 + 30240*B^2*a^6*b^5*c^4 - 44800*B^2*a^7*b^3*c^5 - 25*B^2*a^4*c^2*(-(4*a*c - b^2)^9)^(1/2) + 35840*A*B*a^8*c^7 - 615*A^2*a*b^13*c - 80640*A^2*a^7*b*c^7 - 213*B^2*a^3*b^11*c + 26880*B^2*a^8*b*c^6 - 246*A^2*a^2*b^2*c^2*(-(4*a*c - b^2)^9)^(1/2) - 7278*A*B*a^3*b^10*c^2 + 39132*A*B*a^4*b^8*c^3 - 119616*A*B*a^5*b^6*c^4 + 201600*A*B*a^6*b^4*c^5 - 161280*A*B*a^7*b^2*c^6 + 165*A^2*a*b^4*c*(-(4*a*c - b^2)^9)^(1/2) + 51*B^2*a^3*b^2*c*(-(4*a*c - b^2)^9)^(1/2) + 30*A*B*a*b^5*(-(4*a*c - b^2)^9)^(1/2) + 724*A*B*a^2*b^12*c - 184*A*B*a^2*b^3*c*(-(4*a*c - b^2)^9)^(1/2) + 186*A*B*a^3*b*c^2*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^7*b^12 + 4096*a^13*c^6 - 24*a^8*b^10*c + 240*a^9*b^8*c^2 - 1280*a^10*b^6*c^3 + 3840*a^11*b^4*c^4 - 6144*a^12*b^2*c^5)))^(1/2)*(1048576*a^21*b*c^8 + 256*a^15*b^13*c^2 - 6144*a^16*b^11*c^3 + 61440*a^17*b^9*c^4 - 327680*a^18*b^7*c^5 + 983040*a^19*b^5*c^6 - 1572864*a^20*b^3*c^7) + 851968*B*a^19*b*c^8 - 320*A*a^12*b^14*c^2 + 7936*A*a^13*b^12*c^3 - 82816*A*a^14*b^10*c^4 + 468480*A*a^15*b^8*c^5 - 1536000*A*a^16*b^6*c^6 + 2867200*A*a^17*b^4*c^7 - 2719744*A*a^18*b^2*c^8 + 192*B*a^13*b^13*c^2 - 4672*B*a^14*b^11*c^3 + 47360*B*a^15*b^9*c^4 - 256000*B*a^16*b^7*c^5 + 778240*B*a^17*b^5*c^6 - 1261568*B*a^18*b^3*c^7) - x*(401408*A^2*a^16*c^10 - 204800*B^2*a^17*c^9 - 400*A^2*a^9*b^14*c^3 + 9440*A^2*a^10*b^12*c^4 - 92816*A^2*a^11*b^10*c^5 + 488096*A^2*a^12*b^8*c^6 - 1458688*A^2*a^13*b^6*c^7 + 2401280*A^2*a^14*b^4*c^8 - 1871872*A^2*a^15*b^2*c^9 - 144*B^2*a^11*b^12*c^3 + 3264*B^2*a^12*b^10*c^4 - 30112*B^2*a^13*b^8*c^5 + 143360*B^2*a^14*b^6*c^6 - 365568*B^2*a^15*b^4*c^7 + 458752*B^2*a^16*b^2*c^8 + 480*A*B*a^10*b^13*c^3 - 11104*A*B*a^11*b^11*c^4 + 105824*A*B*a^12*b^9*c^5 - 530432*A*B*a^13*b^7*c^6 + 1469440*A*B*a^14*b^5*c^7 - 2121728*A*B*a^15*b^3*c^8 + 1236992*A*B*a^16*b*c^9))*(-(25*A^2*b^15 + 9*B^2*a^2*b^13 - 25*A^2*b^6*(-(4*a*c - b^2)^9)^(1/2) - 30*A*B*a*b^14 + 6366*A^2*a^2*b^11*c^2 - 35767*A^2*a^3*b^9*c^3 + 116928*A^2*a^4*b^7*c^4 - 219744*A^2*a^5*b^5*c^5 + 215040*A^2*a^6*b^3*c^6 + 49*A^2*a^3*c^3*(-(4*a*c - b^2)^9)^(1/2) - 9*B^2*a^2*b^4*(-(4*a*c - b^2)^9)^(1/2) + 2077*B^2*a^4*b^9*c^2 - 10656*B^2*a^5*b^7*c^3 + 30240*B^2*a^6*b^5*c^4 - 44800*B^2*a^7*b^3*c^5 - 25*B^2*a^4*c^2*(-(4*a*c - b^2)^9)^(1/2) + 35840*A*B*a^8*c^7 - 615*A^2*a*b^13*c - 80640*A^2*a^7*b*c^7 - 213*B^2*a^3*b^11*c + 26880*B^2*a^8*b*c^6 - 246*A^2*a^2*b^2*c^2*(-(4*a*c - b^2)^9)^(1/2) - 7278*A*B*a^3*b^10*c^2 + 39132*A*B*a^4*b^8*c^3 - 119616*A*B*a^5*b^6*c^4 + 201600*A*B*a^6*b^4*c^5 - 161280*A*B*a^7*b^2*c^6 + 165*A^2*a*b^4*c*(-(4*a*c - b^2)^9)^(1/2) + 51*B^2*a^3*b^2*c*(-(4*a*c - b^2)^9)^(1/2) + 30*A*B*a*b^5*(-(4*a*c - b^2)^9)^(1/2) + 724*A*B*a^2*b^12*c - 184*A*B*a^2*b^3*c*(-(4*a*c - b^2)^9)^(1/2) + 186*A*B*a^3*b*c^2*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^7*b^12 + 4096*a^13*c^6 - 24*a^8*b^10*c + 240*a^9

*b^8*c^2 - 1280*a^10*b^6*c^3 + 3840*a^11*b^4*c^4 - 6144*a^12*b^2*c^5)))^(1/
2) + ((-(25*A^2*b^15 + 9*B^2*a^2*b^13 - 25*A^2*b^6*(-(4*a*c - b^2)^9)^(1/2)
 - 30*A*B*a*b^14 + 6366*A^2*a^2*b^11*c^2 - 35767*A^2*a^3*b^9*c^3 + 116928*A
^2*a^4*b^7*c^4 - 219744*A^2*a^5*b^5*c^5 + 215040*A^2*a^6*b^3*c^6 + 49*A^2*a
^3*c^3*(-(4*a*c - b^2)^9)^(1/2) - 9*B^2*a^2*b^4*(-(4*a*c - b^2)^9)^(1/2) +
2077*B^2*a^4*b^9*c^2 - 10656*B^2*a^5*b^7*c^3 + 30240*B^2*a^6*b^5*c^4 - 4480
0*B^2*a^7*b^3*c^5 - 25*B^2*a^4*c^2*(-(4*a*c - b^2)^9)^(1/2) + 35840*A*B*a^8
*c^7 - 615*A^2*a*b^13*c - 80640*A^2*a^7*b*c^7 - 213*B^2*a^3*b^11*c + 26880*
B^2*a^8*b*c^6 - 246*A^2*a^2*b^2*c^2*(-(4*a*c - b^2)^9)^(1/2) - 7278*A*B*a^3
*b^10*c^2 + 39132*A*B*a^4*b^8*c^3 - 119616*A*B*a^5*b^6*c^4 + 201600*A*B*a^6
*b^4*c^5 - 161280*A*B*a^7*b^2*c^6 + 165*A^2*a*b^4*c*(-(4*a*c - b^2)^9)^(1/2
) + 51*B^2*a^3*b^2*c*(-(4*a*c - b^2)^9)^(1/2) + 30*A*B*a*b^5*(-(4*a*c - b^2
)^9)^(1/2) + 724*A*B*a^2*b^12*c - 184*A*B*a^2*b^3*c*(-(4*a*c - b^2)^9)^(1/2
) + 186*A*B*a^3*b*c^2*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^7*b^12 + 4096*a^13*c
^6 - 24*a^8*b^10*c + 240*a^9*b^8*c^2 - 1280*a^10*b^6*c^3 + 3840*a^11*b^4*c^
4 - 6144*a^12*b^2*c^5)))^(1/2)*(917504*A*a^19*c^9 - x*(-(25*A^2*b^15 + 9*B^
2*a^2*b^13 - 25*A^2*b^6*(-(4*a*c - b^2)^9)^(1/2) - 30*A*B*a*b^14 + 6366*A^2
*a^2*b^11*c^2 - 35767*A^2*a^3*b^9*c^3 + 116928*A^2*a^4*b^7*c^4 - 219744*A^2
*a^5*b^5*c^5 + 215040*A^2*a^6*b^3*c^6 + 49*A^2*a^3*c^3*(-(4*a*c - b^2)^9)^(
1/2) - 9*B^2*a^2*b^4*(-(4*a*c - b^2)^9)^(1/2) + 2077*B^2*a^4*b^9*c^2 - 1065
6*B^2*a^5*b^7*c^3 + 30240*B^2*a^6*b^5*c^4 - 44800*B^2*a^7*b^3*c^5 - 25*B^2*
a^4*c^2*(-(4*a*c - b^2)^9)^(1/2) + 35840*A*B*a^8*c^7 - 615*A^2*a*b^13*c - 8
0640*A^2*a^7*b*c^7 - 213*B^2*a^3*b^11*c + 26880*B^2*a^8*b*c^6 - 246*A^2*a^2
*b^2*c^2*(-(4*a*c - b^2)^9)^(1/2) - 7278*A*B*a^3*b^10*c^2 + 39132*A*B*a^4*b
^8*c^3 - 119616*A*B*a^5*b^6*c^4 + 201600*A*B*a^6*b^4*c^5 - 161280*A*B*a^7*b
^2*c^6 + 165*A^2*a*b^4*c*(-(4*a*c - b^2)^9)^(1/2) + 51*B^2*a^3*b^2*c*(-(4*a
*c - b^2)^9)^(1/2) + 30*A*B*a*b^5*(-(4*a*c - b^2)^9)^(1/2) + 724*A*B*a^2*b^
12*c - 184*A*B*a^2*b^3*c*(-(4*a*c - b^2)^9)^(1/2) + 186*A*B*a^3*b*c^2*(-(4*
a*c - b^2)^9)^(1/2))/(32*(a^7*b^12 + 4096*a^13*c^6 - 24*a^8*b^10*c + 240*a^
9*b^8*c^2 - 1280*a^10*b^6*c^3 + 3840*a^11*b^4*c^4 - 6144*a^12*b^2*c^5)))^(1
/2)*(1048576*a^21*b*c^8 + 256*a^15*b^13*c^2 - 6144*a^16*b^11*c^3 + 61440*a^
17*b^9*c^4 - 327680*a^18*b^7*c^5 + 983040*a^19*b^5*c^6 - 1572864*a^20*b^3*c
^7) + 851968*B*a^19*b*c^8 - 320*A*a^12*b^14*c^2 + 7936*A*a^13*b^12*c^3 - 82
816*A*a^14*b^10*c^4 + 468480*A*a^15*b^8*c^5 - 1536000*A*a^16*b^6*c^6 + 2867
200*A*a^17*b^4*c^7 - 2719744*A*a^18*b^2*c^8 + 192*B*a^13*b^13*c^2 - 4672*B*
a^14*b^11*c^3 + 47360*B*a^15*b^9*c^4 - 256000*B*a^16*b^7*c^5 + 778240*B*a^1
7*b^5*c^6 - 1261568*B*a^18*b^3*c^7) + x*(401408*A^2*a^16*c^10 - 204800*B^2*
a^17*c^9 - 400*A^2*a^9*b^14*c^3 + 9440*A^2*a^10*b^12*c^4 - 92816*A^2*a^11*b
^10*c^5 + 488096*A^2*a^12*b^8*c^6 - 1458688*A^2*a^13*b^6*c^7 + 2401280*A^2*
a^14*b^4*c^8 - 1871872*A^2*a^15*b^2*c^9 - 144*B^2*a^11*b^12*c^3 + 3264*B^2*
a^12*b^10*c^4 - 30112*B^2*a^13*b^8*c^5 + 143360*B^2*a^14*b^6*c^6 - 365568*B
^2*a^15*b^4*c^7 + 458752*B^2*a^16*b^2*c^8 + 480*A*B*a^10*b^13*c^3 - 11104*A
*B*a^11*b^11*c^4 + 105824*A*B*a^12*b^9*c^5 - 530432*A*B*a^13*b^7*c^6 + 1469
440*A*B*a^14*b^5*c^7 - 2121728*A*B*a^15*b^3*c^8 + 1236992*A*B*a^16*b*c^9))*
(-(25*A^2*b^15 + 9*B^2*a^2*b^13 - 25*A^2*b^6*(-(4*a*c - b^2)^9)^(1/2) - 30*

A*B*a*b^14 + 6366*A^2*a^2*b^11*c^2 - 35767*A^2*a^3*b^9*c^3 + 116928*A^2*a^4*b^7*c^4 - 219744*A^2*a^5*b^5*c^5 + 215040*A^2*a^6*b^3*c^6 + 49*A^2*a^3*c^3*(-(4*a*c - b^2)^9)^(1/2) - 9*B^2*a^2*b^4*(-(4*a*c - b^2)^9)^(1/2) + 2077*B^2*a^4*b^9*c^2 - 10656*B^2*a^5*b^7*c^3 + 30240*B^2*a^6*b^5*c^4 - 44800*B^2*a^7*b^3*c^5 - 25*B^2*a^4*c^2*(-(4*a*c - b^2)^9)^(1/2) + 35840*A*B*a^8*c^7 - 615*A^2*a*b^13*c - 80640*A^2*a^7*b*c^7 - 213*B^2*a^3*b^11*c + 26880*B^2*a^8*b*c^6 - 246*A^2*a^2*b^2*c^2*(-(4*a*c - b^2)^9)^(1/2) - 7278*A*B*a^3*b^10*c^2 + 39132*A*B*a^4*b^8*c^3 - 119616*A*B*a^5*b^6*c^4 + 201600*A*B*a^6*b^4*c^5 - 161280*A*B*a^7*b^2*c^6 + 165*A^2*a*b^4*c*(-(4*a*c - b^2)^9)^(1/2) + 51*B^2*a^3*b^2*c*(-(4*a*c - b^2)^9)^(1/2) + 30*A*B*a*b^5*(-(4*a*c - b^2)^9)^(1/2) + 724*A*B*a^2*b^12*c - 184*A*B*a^2*b^3*c*(-(4*a*c - b^2)^9)^(1/2) + 186*A*B*a^3*b*c^2*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^7*b^12 + 4096*a^13*c^6 - 24*a^8*b^10*c + 240*a^9*b^8*c^2 - 1280*a^10*b^6*c^3 + 3840*a^11*b^4*c^4 - 6144*a^12*b^2*c^5)))^(1/2) + 128000*B^3*a^15*c^9 - 1800*A^3*a^9*b^9*c^6 + 29080*A^3*a^10*b^7*c^7 - 176032*A^3*a^11*b^5*c^8 + 473216*A^3*a^12*b^3*c^9 + 504*B^3*a^11*b^8*c^5 - 8112*B^3*a^12*b^6*c^6 + 48704*B^3*a^13*b^4*c^7 - 129280*B^3*a^14*b^2*c^8 + 250880*A^2*B*a^14*c^10 - 476672*A^3*a^13*b*c^10 - 442880*A*B^2*a^14*b*c^9 - 1680*A*B^2*a^10*b^9*c^5 + 27176*A*B^2*a^11*b^7*c^6 - 164448*A*B^2*a^12*b^5*c^7 + 441216*A*B^2*a^13*b^3*c^8 + 1400*A^2*B*a^9*b^10*c^5 - 21680*A^2*B*a^10*b^8*c^6 + 121648*A^2*B*a^11*b^6*c^7 - 275264*A^2*B*a^12*b^4*c^8 + 121088*A^2*B*a^13*b^2*c^9))*(-(25*A^2*b^15 + 9*B^2*a^2*b^13 - 25*A^2*b^6*(-(4*a*c - b^2)^9)^(1/2) - 30*A*B*a*b^14 + 6366*A^2*a^2*b^11*c^2 - 35767*A^2*a^3*b^9*c^3 + 116928*A^2*a^4*b^7*c^4 - 219744*A^2*a^5*b^5*c^5 + 215040*A^2*a^6*b^3*c^6 + 49*A^2*a^3*c^3*(-(4*a*c - b^2)^9)^(1/2) - 9*B^2*a^2*b^4*(-(4*a*c - b^2)^9)^(1/2) + 2077*B^2*a^4*b^9*c^2 - 10656*B^2*a^5*b^7*c^3 + 30240*B^2*a^6*b^5*c^4 - 44800*B^2*a^7*b^3*c^5 - 25*B^2*a^4*c^2*(-(4*a*c - b^2)^9)^(1/2) + 35840*A*B*a^8*c^7 - 615*A^2*a*b^13*c - 80640*A^2*a^7*b*c^7 - 213*B^2*a^3*b^11*c + 26880*B^2*a^8*b*c^6 - 246*A^2*a^2*b^2*c^2*(-(4*a*c - b^2)^9)^(1/2) - 7278*A*B*a^3*b^10*c^2 + 39132*A*B*a^4*b^8*c^3 - 119616*A*B*a^5*b^6*c^4 + 201600*A*B*a^6*b^4*c^5 - 161280*A*B*a^7*b^2*c^6 + 165*A^2*a*b^4*c*(-(4*a*c - b^2)^9)^(1/2) + 51*B^2*a^3*b^2*c*(-(4*a*c - b^2)^9)^(1/2) + 30*A*B*a*b^5*(-(4*a*c - b^2)^9)^(1/2) + 724*A*B*a^2*b^12*c - 184*A*B*a^2*b^3*c*(-(4*a*c - b^2)^9)^(1/2) + 186*A*B*a^3*b*c^2*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^7*b^12 + 4096*a^13*c^6 - 24*a^8*b^10*c + 240*a^9*b^8*c^2 - 1280*a^10*b^6*c^3 + 3840*a^11*b^4*c^4 - 6144*a^12*b^2*c^5)))^(1/2)*2i - atan((((-(25*A^2*b^15 + 9*B^2*a^2*b^13 + 25*A^2*b^6*(-(4*a*c - b^2)^9)^(1/2) - 30*A*B*a*b^14 + 6366*A^2*a^2*b^11*c^2 - 35767*A^2*a^3*b^9*c^3 + 116928*A^2*a^4*b^7*c^4 - 219744*A^2*a^5*b^5*c^5 + 215040*A^2*a^6*b^3*c^6 - 49*A^2*a^3*c^3*(-(4*a*c - b^2)^9)^(1/2) + 9*B^2*a^2*b^4*(-(4*a*c - b^2)^9)^(1/2) + 2077*B^2*a^4*b^9*c^2 - 10656*B^2*a^5*b^7*c^3 + 30240*B^2*a^6*b^5*c^4 - 44800*B^2*a^7*b^3*c^5 + 25*B^2*a^4*c^2*(-(4*a*c - b^2)^9)^(1/2) + 35840*A*B*a^8*c^7 - 615*A^2*a*b^13*c - 80640*A^2*a^7*b*c^7 - 213*B^2*a^3*b^11*c + 26880*B^2*a^8*b*c^6 + 246*A^2*a^2*b^2*c^2*(-(4*a*c - b^2)^9)^(1/2) - 7278*A*B*a^3*b^10*c^2 + 39132*A*B*a^4*b^8*c^3 - 119616*A*B*a^5*b^6*c^4 + 201600*A*B*a^6*b^4*c^5 - 161280*A*B*a^7*b^2*c^6 - 165*A^2*a*b^4*c*(-(4*a*c - b^2)^9)^(1/2

) - 51*B^2*a^3*b^2*c*(-(4*a*c - b^2)^9)^(1/2) - 30*A*B*a*b^5*(-(4*a*c - b^2
)^9)^(1/2) + 724*A*B*a^2*b^12*c + 184*A*B*a^2*b^3*c*(-(4*a*c - b^2)^9)^(1/2
) - 186*A*B*a^3*b*c^2*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^7*b^12 + 4096*a^13*c
^6 - 24*a^8*b^10*c + 240*a^9*b^8*c^2 - 1280*a^10*b^6*c^3 + 3840*a^11*b^4*c^
4 - 6144*a^12*b^2*c^5)))^(1/2)*(917504*A*a^19*c^9 + x*(-(25*A^2*b^15 + 9*B^
2*a^2*b^13 + 25*A^2*b^6*(-(4*a*c - b^2)^9)^(1/2) - 30*A*B*a*b^14 + 6366*A^2
*a^2*b^11*c^2 - 35767*A^2*a^3*b^9*c^3 + 116928*A^2*a^4*b^7*c^4 - 219744*A^2
*a^5*b^5*c^5 + 215040*A^2*a^6*b^3*c^6 - 49*A^2*a^3*c^3*(-(4*a*c - b^2)^9)^(
1/2) + 9*B^2*a^2*b^4*(-(4*a*c - b^2)^9)^(1/2) + 2077*B^2*a^4*b^9*c^2 - 1065
6*B^2*a^5*b^7*c^3 + 30240*B^2*a^6*b^5*c^4 - 44800*B^2*a^7*b^3*c^5 + 25*B^2*
a^4*c^2*(-(4*a*c - b^2)^9)^(1/2) + 35840*A*B*a^8*c^7 - 615*A^2*a*b^13*c - 8
0640*A^2*a^7*b*c^7 - 213*B^2*a^3*b^11*c + 26880*B^2*a^8*b*c^6 + 246*A^2*a^2
*b^2*c^2*(-(4*a*c - b^2)^9)^(1/2) - 7278*A*B*a^3*b^10*c^2 + 39132*A*B*a^4*b
^8*c^3 - 119616*A*B*a^5*b^6*c^4 + 201600*A*B*a^6*b^4*c^5 - 161280*A*B*a^7*b
^2*c^6 - 165*A^2*a*b^4*c*(-(4*a*c - b^2)^9)^(1/2) - 51*B^2*a^3*b^2*c*(-(4*a
*c - b^2)^9)^(1/2) - 30*A*B*a*b^5*(-(4*a*c - b^2)^9)^(1/2) + 724*A*B*a^2*b^
12*c + 184*A*B*a^2*b^3*c*(-(4*a*c - b^2)^9)^(1/2) - 186*A*B*a^3*b*c^2*(-(4*
a*c - b^2)^9)^(1/2))/(32*(a^7*b^12 + 4096*a^13*c^6 - 24*a^8*b^10*c + 240*a^
9*b^8*c^2 - 1280*a^10*b^6*c^3 + 3840*a^11*b^4*c^4 - 6144*a^12*b^2*c^5)))^(1
/2)*(1048576*a^21*b*c^8 + 256*a^15*b^13*c^2 - 6144*a^16*b^11*c^3 + 61440*a^
17*b^9*c^4 - 327680*a^18*b^7*c^5 + 983040*a^19*b^5*c^6 - 1572864*a^20*b^3*c
^7) + 851968*B*a^19*b*c^8 - 320*A*a^12*b^14*c^2 + 7936*A*a^13*b^12*c^3 - 82
816*A*a^14*b^10*c^4 + 468480*A*a^15*b^8*c^5 - 1536000*A*a^16*b^6*c^6 + 2867
200*A*a^17*b^4*c^7 - 2719744*A*a^18*b^2*c^8 + 192*B*a^13*b^13*c^2 - 4672*B*
a^14*b^11*c^3 + 47360*B*a^15*b^9*c^4 - 256000*B*a^16*b^7*c^5 + 778240*B*a^1
7*b^5*c^6 - 1261568*B*a^18*b^3*c^7) - x*(401408*A^2*a^16*c^10 - 204800*B^2*
a^17*c^9 - 400*A^2*a^9*b^14*c^3 + 9440*A^2*a^10*b^12*c^4 - 92816*A^2*a^11*b
^10*c^5 + 488096*A^2*a^12*b^8*c^6 - 1458688*A^2*a^13*b^6*c^7 + 2401280*A^2*
a^14*b^4*c^8 - 1871872*A^2*a^15*b^2*c^9 - 144*B^2*a^11*b^12*c^3 + 3264*B^2*
a^12*b^10*c^4 - 30112*B^2*a^13*b^8*c^5 + 143360*B^2*a^14*b^6*c^6 - 365568*B
^2*a^15*b^4*c^7 + 458752*B^2*a^16*b^2*c^8 + 480*A*B*a^10*b^13*c^3 - 11104*A
*B*a^11*b^11*c^4 + 105824*A*B*a^12*b^9*c^5 - 530432*A*B*a^13*b^7*c^6 + 1469
440*A*B*a^14*b^5*c^7 - 2121728*A*B*a^15*b^3*c^8 + 1236992*A*B*a^16*b*c^9))*
(-(25*A^2*b^15 + 9*B^2*a^2*b^13 + 25*A^2*b^6*(-(4*a*c - b^2)^9)^(1/2) - 30*
A*B*a*b^14 + 6366*A^2*a^2*b^11*c^2 - 35767*A^2*a^3*b^9*c^3 + 116928*A^2*a^4
*b^7*c^4 - 219744*A^2*a^5*b^5*c^5 + 215040*A^2*a^6*b^3*c^6 - 49*A^2*a^3*c^3
*(-(4*a*c - b^2)^9)^(1/2) + 9*B^2*a^2*b^4*(-(4*a*c - b^2)^9)^(1/2) + 2077*B
^2*a^4*b^9*c^2 - 10656*B^2*a^5*b^7*c^3 + 30240*B^2*a^6*b^5*c^4 - 44800*B^2*
a^7*b^3*c^5 + 25*B^2*a^4*c^2*(-(4*a*c - b^2)^9)^(1/2) + 35840*A*B*a^8*c^7 -
 615*A^2*a*b^13*c - 80640*A^2*a^7*b*c^7 - 213*B^2*a^3*b^11*c + 26880*B^2*a^
8*b*c^6 + 246*A^2*a^2*b^2*c^2*(-(4*a*c - b^2)^9)^(1/2) - 7278*A*B*a^3*b^10*
c^2 + 39132*A*B*a^4*b^8*c^3 - 119616*A*B*a^5*b^6*c^4 + 201600*A*B*a^6*b^4*c
^5 - 161280*A*B*a^7*b^2*c^6 - 165*A^2*a*b^4*c*(-(4*a*c - b^2)^9)^(1/2) - 51
*B^2*a^3*b^2*c*(-(4*a*c - b^2)^9)^(1/2) - 30*A*B*a*b^5*(-(4*a*c - b^2)^9)^(
1/2) + 724*A*B*a^2*b^12*c + 184*A*B*a^2*b^3*c*(-(4*a*c - b^2)^9)^(1/2) - 18

6*A*B*a^3*b*c^2*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^7*b^12 + 4096*a^13*c^6 - 24*a^8*b^10*c + 240*a^9*b^8*c^2 - 1280*a^10*b^6*c^3 + 3840*a^11*b^4*c^4 - 6144*a^12*b^2*c^5)))^(1/2)*1i - ((-(25*A^2*b^15 + 9*B^2*a^2*b^13 + 25*A^2*b^6*(-(4*a*c - b^2)^9)^(1/2) - 30*A*B*a*b^14 + 6366*A^2*a^2*b^11*c^2 - 35767*A^2*a^3*b^9*c^3 + 116928*A^2*a^4*b^7*c^4 - 219744*A^2*a^5*b^5*c^5 + 215040*A^2*a^6*b^3*c^6 - 49*A^2*a^3*c^3*(-(4*a*c - b^2)^9)^(1/2) + 9*B^2*a^2*b^4*(-(4*a*c - b^2)^9)^(1/2) + 2077*B^2*a^4*b^9*c^2 - 10656*B^2*a^5*b^7*c^3 + 30240*B^2*a^6*b^5*c^4 - 44800*B^2*a^7*b^3*c^5 + 25*B^2*a^4*c^2*(-(4*a*c - b^2)^9)^(1/2) + 35840*A*B*a^8*c^7 - 615*A^2*a*b^13*c - 80640*A^2*a^7*b*c^7 - 213*B^2*a^3*b^11*c + 26880*B^2*a^8*b*c^6 + 246*A^2*a^2*b^2*c^2*(-(4*a*c - b^2)^9)^(1/2) - 7278*A*B*a^3*b^10*c^2 + 39132*A*B*a^4*b^8*c^3 - 119616*A*B*a^5*b^6*c^4 + 201600*A*B*a^6*b^4*c^5 - 161280*A*B*a^7*b^2*c^6 - 165*A^2*a*b^4*c*(-(4*a*c - b^2)^9)^(1/2) - 51*B^2*a^3*b^2*c*(-(4*a*c - b^2)^9)^(1/2) - 30*A*B*a*b^5*(-(4*a*c - b^2)^9)^(1/2) + 724*A*B*a^2*b^12*c + 184*A*B*a^2*b^3*c*(-(4*a*c - b^2)^9)^(1/2) - 186*A*B*a^3*b*c^2*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^7*b^12 + 4096*a^13*c^6 - 24*a^8*b^10*c + 240*a^9*b^8*c^2 - 1280*a^10*b^6*c^3 + 3840*a^11*b^4*c^4 - 6144*a^12*b^2*c^5)))^(1/2)*(917504*A*a^19*c^9 - x*(-(25*A^2*b^15 + 9*B^2*a^2*b^13 + 25*A^2*b^6*(-(4*a*c - b^2)^9)^(1/2) - 30*A*B*a*b^14 + 6366*A^2*a^2*b^11*c^2 - 35767*A^2*a^3*b^9*c^3 + 116928*A^2*a^4*b^7*c^4 - 219744*A^2*a^5*b^5*c^5 + 215040*A^2*a^6*b^3*c^6 - 49*A^2*a^3*c^3*(-(4*a*c - b^2)^9)^(1/2) + 9*B^2*a^2*b^4*(-(4*a*c - b^2)^9)^(1/2) + 2077*B^2*a^4*b^9*c^2 - 10656*B^2*a^5*b^7*c^3 + 30240*B^2*a^6*b^5*c^4 - 44800*B^2*a^7*b^3*c^5 + 25*B^2*a^4*c^2*(-(4*a*c - b^2)^9)^(1/2) + 35840*A*B*a^8*c^7 - 615*A^2*a*b^13*c - 80640*A^2*a^7*b*c^7 - 213*B^2*a^3*b^11*c + 26880*B^2*a^8*b*c^6 + 246*A^2*a^2*b^2*c^2*(-(4*a*c - b^2)^9)^(1/2) - 7278*A*B*a^3*b^10*c^2 + 39132*A*B*a^4*b^8*c^3 - 119616*A*B*a^5*b^6*c^4 + 201600*A*B*a^6*b^4*c^5 - 161280*A*B*a^7*b^2*c^6 - 165*A^2*a*b^4*c*(-(4*a*c - b^2)^9)^(1/2) - 51*B^2*a^3*b^2*c*(-(4*a*c - b^2)^9)^(1/2) - 30*A*B*a*b^5*(-(4*a*c - b^2)^9)^(1/2) + 724*A*B*a^2*b^12*c + 184*A*B*a^2*b^3*c*(-(4*a*c - b^2)^9)^(1/2) - 186*A*B*a^3*b*c^2*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^7*b^12 + 4096*a^13*c^6 - 24*a^8*b^10*c + 240*a^9*b^8*c^2 - 1280*a^10*b^6*c^3 + 3840*a^11*b^4*c^4 - 6144*a^12*b^2*c^5)))^(1/2)*(1048576*a^21*b*c^8 + 256*a^15*b^13*c^2 - 6144*a^16*b^11*c^3 + 61440*a^17*b^9*c^4 - 327680*a^18*b^7*c^5 + 983040*a^19*b^5*c^6 - 1572864*a^20*b^3*c^7) + 851968*B*a^19*b*c^8 - 320*A*a^12*b^14*c^2 + 7936*A*a^13*b^12*c^3 - 82816*A*a^14*b^10*c^4 + 468480*A*a^15*b^8*c^5 - 1536000*A*a^16*b^6*c^6 + 2867200*A*a^17*b^4*c^7 - 2719744*A*a^18*b^2*c^8 + 192*B*a^13*b^13*c^2 - 4672*B*a^14*b^11*c^3 + 47360*B*a^15*b^9*c^4 - 256000*B*a^16*b^7*c^5 + 778240*B*a^17*b^5*c^6 - 1261568*B*a^18*b^3*c^7) + x*(401408*A^2*a^16*c^10 - 204800*B^2*a^17*c^9 - 400*A^2*a^9*b^14*c^3 + 9440*A^2*a^10*b^12*c^4 - 92816*A^2*a^11*b^10*c^5 + 488096*A^2*a^12*b^8*c^6 - 1458688*A^2*a^13*b^6*c^7 + 2401280*A^2*a^14*b^4*c^8 - 1871872*A^2*a^15*b^2*c^9 - 144*B^2*a^11*b^12*c^3 + 3264*B^2*a^12*b^10*c^4 - 30112*B^2*a^13*b^8*c^5 + 143360*B^2*a^14*b^6*c^6 - 365568*B^2*a^15*b^4*c^7 + 458752*B^2*a^16*b^2*c^8 + 480*A*B*a^10*b^13*c^3 - 11104*A*B*a^11*b^11*c^4 + 105824*A*B*a^12*b^9*c^5 - 530432*A*B*a^13*b^7*c^6 + 1469440*A*B*a^14*b^5*c^7 - 2121728*A*B*a^15*b^3*c^8 +

1236992*A*B*a^16*b*c^9))*(-(25*A^2*b^15 + 9*B^2*a^2*b^13 + 25*A^2*b^6*(-(4*a*c - b^2)^9)^(1/2) - 30*A*B*a*b^14 + 6366*A^2*a^2*b^11*c^2 - 35767*A^2*a^3*b^9*c^3 + 116928*A^2*a^4*b^7*c^4 - 219744*A^2*a^5*b^5*c^5 + 215040*A^2*a^6*b^3*c^6 - 49*A^2*a^3*c^3*(-(4*a*c - b^2)^9)^(1/2) + 9*B^2*a^2*b^4*(-(4*a*c - b^2)^9)^(1/2) + 2077*B^2*a^4*b^9*c^2 - 10656*B^2*a^5*b^7*c^3 + 30240*B^2*a^6*b^5*c^4 - 44800*B^2*a^7*b^3*c^5 + 25*B^2*a^4*c^2*(-(4*a*c - b^2)^9)^(1/2) + 35840*A*B*a^8*c^7 - 615*A^2*a*b^13*c - 80640*A^2*a^7*b*c^7 - 213*B^2*a^3*b^11*c + 26880*B^2*a^8*b*c^6 + 246*A^2*a^2*b^2*c^2*(-(4*a*c - b^2)^9)^(1/2) - 7278*A*B*a^3*b^10*c^2 + 39132*A*B*a^4*b^8*c^3 - 119616*A*B*a^5*b^6*c^4 + 201600*A*B*a^6*b^4*c^5 - 161280*A*B*a^7*b^2*c^6 - 165*A^2*a*b^4*c*(-(4*a*c - b^2)^9)^(1/2) - 51*B^2*a^3*b^2*c*(-(4*a*c - b^2)^9)^(1/2) - 30*A*B*a*b^5*(-(4*a*c - b^2)^9)^(1/2) + 724*A*B*a^2*b^12*c + 184*A*B*a^2*b^3*c*(-(4*a*c - b^2)^9)^(1/2) - 186*A*B*a^3*b*c^2*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^7*b^12 + 4096*a^13*c^6 - 24*a^8*b^10*c + 240*a^9*b^8*c^2 - 1280*a^10*b^6*c^3 + 3840*a^11*b^4*c^4 - 6144*a^12*b^2*c^5)))^(1/2)*1i)/(((-(25*A^2*b^15 + 9*B^2*a^2*b^13 + 25*A^2*b^6*(-(4*a*c - b^2)^9)^(1/2) - 30*A*B*a*b^14 + 6366*A^2*a^2*b^11*c^2 - 35767*A^2*a^3*b^9*c^3 + 116928*A^2*a^4*b^7*c^4 - 219744*A^2*a^5*b^5*c^5 + 215040*A^2*a^6*b^3*c^6 - 49*A^2*a^3*c^3*(-(4*a*c - b^2)^9)^(1/2) + 9*B^2*a^2*b^4*(-(4*a*c - b^2)^9)^(1/2) + 2077*B^2*a^4*b^9*c^2 - 10656*B^2*a^5*b^7*c^3 + 30240*B^2*a^6*b^5*c^4 - 44800*B^2*a^7*b^3*c^5 + 25*B^2*a^4*c^2*(-(4*a*c - b^2)^9)^(1/2) + 35840*A*B*a^8*c^7 - 615*A^2*a*b^13*c - 80640*A^2*a^7*b*c^7 - 213*B^2*a^3*b^11*c + 26880*B^2*a^8*b*c^6 + 246*A^2*a^2*b^2*c^2*(-(4*a*c - b^2)^9)^(1/2) - 7278*A*B*a^3*b^10*c^2 + 39132*A*B*a^4*b^8*c^3 - 119616*A*B*a^5*b^6*c^4 + 201600*A*B*a^6*b^4*c^5 - 161280*A*B*a^7*b^2*c^6 - 165*A^2*a*b^4*c*(-(4*a*c - b^2)^9)^(1/2) - 51*B^2*a^3*b^2*c*(-(4*a*c - b^2)^9)^(1/2) - 30*A*B*a*b^5*(-(4*a*c - b^2)^9)^(1/2) + 724*A*B*a^2*b^12*c + 184*A*B*a^2*b^3*c*(-(4*a*c - b^2)^9)^(1/2) - 186*A*B*a^3*b*c^2*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^7*b^12 + 4096*a^13*c^6 - 24*a^8*b^10*c + 240*a^9*b^8*c^2 - 1280*a^10*b^6*c^3 + 3840*a^11*b^4*c^4 - 6144*a^12*b^2*c^5)))^(1/2)*(917504*A*a^19*c^9 + x*(-(25*A^2*b^15 + 9*B^2*a^2*b^13 + 25*A^2*b^6*(-(4*a*c - b^2)^9)^(1/2) - 30*A*B*a*b^14 + 6366*A^2*a^2*b^11*c^2 - 35767*A^2*a^3*b^9*c^3 + 116928*A^2*a^4*b^7*c^4 - 219744*A^2*a^5*b^5*c^5 + 215040*A^2*a^6*b^3*c^6 - 49*A^2*a^3*c^3*(-(4*a*c - b^2)^9)^(1/2) + 9*B^2*a^2*b^4*(-(4*a*c - b^2)^9)^(1/2) + 2077*B^2*a^4*b^9*c^2 - 10656*B^2*a^5*b^7*c^3 + 30240*B^2*a^6*b^5*c^4 - 44800*B^2*a^7*b^3*c^5 + 25*B^2*a^4*c^2*(-(4*a*c - b^2)^9)^(1/2) + 35840*A*B*a^8*c^7 - 615*A^2*a*b^13*c - 80640*A^2*a^7*b*c^7 - 213*B^2*a^3*b^11*c + 26880*B^2*a^8*b*c^6 + 246*A^2*a^2*b^2*c^2*(-(4*a*c - b^2)^9)^(1/2) - 7278*A*B*a^3*b^10*c^2 + 39132*A*B*a^4*b^8*c^3 - 119616*A*B*a^5*b^6*c^4 + 201600*A*B*a^6*b^4*c^5 - 161280*A*B*a^7*b^2*c^6 - 165*A^2*a*b^4*c*(-(4*a*c - b^2)^9)^(1/2) - 51*B^2*a^3*b^2*c*(-(4*a*c - b^2)^9)^(1/2) - 30*A*B*a*b^5*(-(4*a*c - b^2)^9)^(1/2) + 724*A*B*a^2*b^12*c + 184*A*B*a^2*b^3*c*(-(4*a*c - b^2)^9)^(1/2) - 186*A*B*a^3*b*c^2*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^7*b^12 + 4096*a^13*c^6 - 24*a^8*b^10*c + 240*a^9*b^8*c^2 - 1280*a^10*b^6*c^3 + 3840*a^11*b^4*c^4 - 6144*a^12*b^2*c^5)))^(1/2)*(1048576*a^21*b*c^8 + 256*a^15*b^13*c^2 - 6144*a^16*b^11*c^3 + 61440*a^17*b^9*c^4 - 327680*a^18* b^7*c^5 + 983040*a^19*b^5*c^6 - 1572864*a^20*b^3*c^7) + 851968*B*a^19*b*c^8 - 320*A*a^12*b^14*c^2 + 7936*A*a^13*b^12*c^3 - 82816*A*a^14*b^10*c^4 + 468480*A*a^15*b^8*c^5 - 1536000*A*a^16*b^6*c^6 + 2867200*A*a^17*b^4*c^7 - 2719744*A*a^18*b^2*c^8 + 192*B*a^13*b^13*c^2 - 4672*B*a^14*b^11*c^3 + 47360*B*a^15*b^9*c^4 - 256000*B*a^16*b^7*c^5 + 778240*B*a^17*b^5*c^6 - 1261568*B*a^18*b^3*c^7) - x*(401408*A^2*a^16*c^10 - 204800*B^2*a^17*c^9 - 400*A^2*a^9*b^14*c^3 + 9440*A^2*a^10*b^12*c^4 - 92816*A^2*a^11*b^10*c^5 + 488096*A^2*a^12*b^8*c^6 - 1458688*A^2*a^13*b^6*c^7 + 2401280*A^2*a^14*b^4*c^8 - 1871872*A^2*a^15*b^2*c^9 - 144*B^2*a^11*b^12*c^3 + 3264*B^2*a^12*b^10*c^4 - 30112*B^2*a^13*b^8*c^5 + 143360*B^2*a^14*b^6*c^6 - 365568*B^2*a^15*b^4*c^7 + 458752*B^2*a^16*b^2*c^8 + 480*A*B*a^10*b^13*c^3 - 11104*A*B*a^11*b^11*c^4 + 105824*A*B*a^12*b^9*c^5 - 530432*A*B*a^13*b^7*c^6 + 1469440*A*B*a^14*b^5*c^7 - 2121728*A*B*a^15*b^3*c^8 + 1236992*A*B*a^16*b*c^9))*(-(25*A^2*b^15 + 9*B^2*a^2*b^13 + 25*A^2*b^6*(-(4*a*c - b^2)^9)^(1/2) - 30*A*B*a*b^14 + 6366*A^2*a^2*b^11*c^2 - 35767*A^2*a^3*b^9*c^3 + 116928*A^2*a^4*b^7*c^4 - 219744*A^2*a^5*b^5*c^5 + 215040*A^2*a^6*b^3*c^6 - 49*A^2*a^3*c^3*(-(4*a*c - b^2)^9)^(1/2) + 9*B^2*a^2*b^4*(-(4*a*c - b^2)^9)^(1/2) + 2077*B^2*a^4*b^9*c^2 - 10656*B^2*a^5*b^7*c^3 + 30240*B^2*a^6*b^5*c^4 - 44800*B^2*a^7*b^3*c^5 + 25*B^2*a^4*c^2*(-(4*a*c - b^2)^9)^(1/2) + 35840*A*B*a^8*c^7 - 615*A^2*a*b^13*c - 80640*A^2*a^7*b*c^7 - 213*B^2*a^3*b^11*c + 26880*B^2*a^8*b*c^6 + 246*A^2*a^2*b^2*c^2*(-(4*a*c - b^2)^9)^(1/2) - 7278*A*B*a^3*b^10*c^2 + 39132*A*B*a^4*b^8*c^3 - 119616*A*B*a^5*b^6*c^4 + 201600*A*B*a^6*b^4*c^5 - 161280*A*B*a^7*b^2*c^6 - 165*A^2*a*b^4*c*(-(4*a*c - b^2)^9)^(1/2) - 51*B^2*a^3*b^2*c*(-(4*a*c - b^2)^9)^(1/2) - 30*A*B*a*b^5*(-(4*a*c - b^2)^9)^(1/2) + 724*A*B*a^2*b^12*c + 184*A*B*a^2*b^3*c*(-(4*a*c - b^2)^9)^(1/2) - 186*A*B*a^3*b*c^2*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^7*b^12 + 4096*a^13*c^6 - 24*a^8*b^10*c + 240*a^9*b^8*c^2 - 1280*a^10*b^6*c^3 + 3840*a^11*b^4*c^4 - 6144*a^12*b^2*c^5)))^(1/2) + ((-(25*A^2*b^15 + 9*B^2*a^2*b^13 + 25*A^2*b^6*(-(4*a*c - b^2)^9)^(1/2) - 30*A*B*a*b^14 + 6366*A^2*a^2*b^11*c^2 - 35767*A^2*a^3*b^9*c^3 + 116928*A^2*a^4*b^7*c^4 - 219744*A^2*a^5*b^5*c^5 + 215040*A^2*a^6*b^3*c^6 - 49*A^2*a^3*c^3*(-(4*a*c - b^2)^9)^(1/2) + 9*B^2*a^2*b^4*(-(4*a*c - b^2)^9)^(1/2) + 2077*B^2*a^4*b^9*c^2 - 10656*B^2*a^5*b^7*c^3 + 30240*B^2*a^6*b^5*c^4 - 44800*B^2*a^7*b^3*c^5 + 25*B^2*a^4*c^2*(-(4*a*c - b^2)^9)^(1/2) + 35840*A*B*a^8*c^7 - 615*A^2*a*b^13*c - 80640*A^2*a^7*b*c^7 - 213*B^2*a^3*b^11*c + 26880*B^2*a^8*b*c^6 + 246*A^2*a^2*b^2*c^2*(-(4*a*c - b^2)^9)^(1/2) - 7278*A*B*a^3*b^10*c^2 + 39132*A*B*a^4*b^8*c^3 - 119616*A*B*a^5*b^6*c^4 + 201600*A*B*a^6*b^4*c^5 - 161280*A*B*a^7*b^2*c^6 - 165*A^2*a*b^4*c*(-(4*a*c - b^2)^9)^(1/2) - 51*B^2*a^3*b^2*c*(-(4*a*c - b^2)^9)^(1/2) - 30*A*B*a*b^5*(-(4*a*c - b^2)^9)^(1/2) + 724*A*B*a^2*b^12*c + 184*A*B*a^2*b^3*c*(-(4*a*c - b^2)^9)^(1/2) - 186*A*B*a^3*b*c^2*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^7*b^12 + 4096*a^13*c^6 - 24*a^8*b^10*c + 240*a^9*b^8*c^2 - 1280*a^10*b^6*c^3 + 3840*a^11*b^4*c^4 - 6144*a^12*b^2*c^5)))^(1/2)*(917504*A*a^19*c^9 - x*(-(25*A^2*b^15 + 9*B^2*a^2*b^13 + 25*A^2*b^6*(-(4*a*c - b^2)^9)^(1/2) - 30*A*B*a*b^14 + 6366*A^2*a^2*b^11*c^2 - 35767*A^2*a^3*b^9*c^3 + 116928*A^2*a^4*b^7*c^4 - 219744*A^2*a^5*b^5*c^5 + 215040*A^2*a^6*b^3*c^6 - 49*A^2*a^3*c^3*(-(4*a*c - b^2)^9)^(1/2

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) + 9*B^2*a^2*b^4*(-(4*a*c - b^2)^9)^(1/2) + 2077*B^2*a^4*b^9*c^2 - 10656*B
^2*a^5*b^7*c^3 + 30240*B^2*a^6*b^5*c^4 - 44800*B^2*a^7*b^3*c^5 + 25*B^2*a^4
*c^2*(-(4*a*c - b^2)^9)^(1/2) + 35840*A*B*a^8*c^7 - 615*A^2*a*b^13*c - 8064
0*A^2*a^7*b*c^7 - 213*B^2*a^3*b^11*c + 26880*B^2*a^8*b*c^6 + 246*A^2*a^2*b^
2*c^2*(-(4*a*c - b^2)^9)^(1/2) - 7278*A*B*a^3*b^10*c^2 + 39132*A*B*a^4*b^8*
c^3 - 119616*A*B*a^5*b^6*c^4 + 201600*A*B*a^6*b^4*c^5 - 161280*A*B*a^7*b^2*
c^6 - 165*A^2*a*b^4*c*(-(4*a*c - b^2)^9)^(1/2) - 51*B^2*a^3*b^2*c*(-(4*a*c
- b^2)^9)^(1/2) - 30*A*B*a*b^5*(-(4*a*c - b^2)^9)^(1/2) + 724*A*B*a^2*b^12*
c + 184*A*B*a^2*b^3*c*(-(4*a*c - b^2)^9)^(1/2) - 186*A*B*a^3*b*c^2*(-(4*a*c
 - b^2)^9)^(1/2))/(32*(a^7*b^12 + 4096*a^13*c^6 - 24*a^8*b^10*c + 240*a^9*b
^8*c^2 - 1280*a^10*b^6*c^3 + 3840*a^11*b^4*c^4 - 6144*a^12*b^2*c^5)))^(1/2)
*(1048576*a^21*b*c^8 + 256*a^15*b^13*c^2 - 6144*a^16*b^11*c^3 + 61440*a^17*
b^9*c^4 - 327680*a^18*b^7*c^5 + 983040*a^19*b^5*c^6 - 1572864*a^20*b^3*c^7)
 + 851968*B*a^19*b*c^8 - 320*A*a^12*b^14*c^2 + 7936*A*a^13*b^12*c^3 - 82816
*A*a^14*b^10*c^4 + 468480*A*a^15*b^8*c^5 - 1536000*A*a^16*b^6*c^6 + 2867200
*A*a^17*b^4*c^7 - 2719744*A*a^18*b^2*c^8 + 192*B*a^13*b^13*c^2 - 4672*B*a^1
4*b^11*c^3 + 47360*B*a^15*b^9*c^4 - 256000*B*a^16*b^7*c^5 + 778240*B*a^17*b
^5*c^6 - 1261568*B*a^18*b^3*c^7) + x*(401408*A^2*a^16*c^10 - 204800*B^2*a^1
7*c^9 - 400*A^2*a^9*b^14*c^3 + 9440*A^2*a^10*b^12*c^4 - 92816*A^2*a^11*b^10
*c^5 + 488096*A^2*a^12*b^8*c^6 - 1458688*A^2*a^13*b^6*c^7 + 2401280*A^2*a^1
4*b^4*c^8 - 1871872*A^2*a^15*b^2*c^9 - 144*B^2*a^11*b^12*c^3 + 3264*B^2*a^1
2*b^10*c^4 - 30112*B^2*a^13*b^8*c^5 + 143360*B^2*a^14*b^6*c^6 - 365568*B^2*
a^15*b^4*c^7 + 458752*B^2*a^16*b^2*c^8 + 480*A*B*a^10*b^13*c^3 - 11104*A*B*
a^11*b^11*c^4 + 105824*A*B*a^12*b^9*c^5 - 530432*A*B*a^13*b^7*c^6 + 1469440
*A*B*a^14*b^5*c^7 - 2121728*A*B*a^15*b^3*c^8 + 1236992*A*B*a^16*b*c^9))*(-(
25*A^2*b^15 + 9*B^2*a^2*b^13 + 25*A^2*b^6*(-(4*a*c - b^2)^9)^(1/2) - 30*A*B
*a*b^14 + 6366*A^2*a^2*b^11*c^2 - 35767*A^2*a^3*b^9*c^3 + 116928*A^2*a^4*b^
7*c^4 - 219744*A^2*a^5*b^5*c^5 + 215040*A^2*a^6*b^3*c^6 - 49*A^2*a^3*c^3*(-
(4*a*c - b^2)^9)^(1/2) + 9*B^2*a^2*b^4*(-(4*a*c - b^2)^9)^(1/2) + 2077*B^2*
a^4*b^9*c^2 - 10656*B^2*a^5*b^7*c^3 + 30240*B^2*a^6*b^5*c^4 - 44800*B^2*a^7
*b^3*c^5 + 25*B^2*a^4*c^2*(-(4*a*c - b^2)^9)^(1/2) + 35840*A*B*a^8*c^7 - 61
5*A^2*a*b^13*c - 80640*A^2*a^7*b*c^7 - 213*B^2*a^3*b^11*c + 26880*B^2*a^8*b
*c^6 + 246*A^2*a^2*b^2*c^2*(-(4*a*c - b^2)^9)^(1/2) - 7278*A*B*a^3*b^10*c^2
 + 39132*A*B*a^4*b^8*c^3 - 119616*A*B*a^5*b^6*c^4 + 201600*A*B*a^6*b^4*c^5
- 161280*A*B*a^7*b^2*c^6 - 165*A^2*a*b^4*c*(-(4*a*c - b^2)^9)^(1/2) - 51*B^
2*a^3*b^2*c*(-(4*a*c - b^2)^9)^(1/2) - 30*A*B*a*b^5*(-(4*a*c - b^2)^9)^(1/2
) + 724*A*B*a^2*b^12*c + 184*A*B*a^2*b^3*c*(-(4*a*c - b^2)^9)^(1/2) - 186*A
*B*a^3*b*c^2*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^7*b^12 + 4096*a^13*c^6 - 24*a
^8*b^10*c + 240*a^9*b^8*c^2 - 1280*a^10*b^6*c^3 + 3840*a^11*b^4*c^4 - 6144*
a^12*b^2*c^5)))^(1/2) + 128000*B^3*a^15*c^9 - 1800*A^3*a^9*b^9*c^6 + 29080*
A^3*a^10*b^7*c^7 - 176032*A^3*a^11*b^5*c^8 + 473216*A^3*a^12*b^3*c^9 + 504*
B^3*a^11*b^8*c^5 - 8112*B^3*a^12*b^6*c^6 + 48704*B^3*a^13*b^4*c^7 - 129280*
B^3*a^14*b^2*c^8 + 250880*A^2*B*a^14*c^10 - 476672*A^3*a^13*b*c^10 - 442880
*A*B^2*a^14*b*c^9 - 1680*A*B^2*a^10*b^9*c^5 + 27176*A*B^2*a^11*b^7*c^6 - 16
4448*A*B^2*a^12*b^5*c^7 + 441216*A*B^2*a^13*b^3*c^8 + 1400*A^2*B*a^9*b^10*c
```

```
^5 - 21680*A^2*B*a^10*b^8*c^6 + 121648*A^2*B*a^11*b^6*c^7 - 275264*A^2*B*a^
12*b^4*c^8 + 121088*A^2*B*a^13*b^2*c^9))*(-(25*A^2*b^15 + 9*B^2*a^2*b^13 +
25*A^2*b^6*(-(4*a*c - b^2)^9)^(1/2) - 30*A*B*a*b^14 + 6366*A^2*a^2*b^11*c^2
 - 35767*A^2*a^3*b^9*c^3 + 116928*A^2*a^4*b^7*c^4 - 219744*A^2*a^5*b^5*c^5
+ 215040*A^2*a^6*b^3*c^6 - 49*A^2*a^3*c^3*(-(4*a*c - b^2)^9)^(1/2) + 9*B^2*
a^2*b^4*(-(4*a*c - b^2)^9)^(1/2) + 2077*B^2*a^4*b^9*c^2 - 10656*B^2*a^5*b^7
*c^3 + 30240*B^2*a^6*b^5*c^4 - 44800*B^2*a^7*b^3*c^5 + 25*B^2*a^4*c^2*(-(4*
a*c - b^2)^9)^(1/2) + 35840*A*B*a^8*c^7 - 615*A^2*a*b^13*c - 80640*A^2*a^7*
b*c^7 - 213*B^2*a^3*b^11*c + 26880*B^2*a^8*b*c^6 + 246*A^2*a^2*b^2*c^2*(-(4
*a*c - b^2)^9)^(1/2) - 7278*A*B*a^3*b^10*c^2 + 39132*A*B*a^4*b^8*c^3 - 1196
16*A*B*a^5*b^6*c^4 + 201600*A*B*a^6*b^4*c^5 - 161280*A*B*a^7*b^2*c^6 - 165*
A^2*a*b^4*c*(-(4*a*c - b^2)^9)^(1/2) - 51*B^2*a^3*b^2*c*(-(4*a*c - b^2)^9)^
(1/2) - 30*A*B*a*b^5*(-(4*a*c - b^2)^9)^(1/2) + 724*A*B*a^2*b^12*c + 184*A*
B*a^2*b^3*c*(-(4*a*c - b^2)^9)^(1/2) - 186*A*B*a^3*b*c^2*(-(4*a*c - b^2)^9)
^(1/2))/(32*(a^7*b^12 + 4096*a^13*c^6 - 24*a^8*b^10*c + 240*a^9*b^8*c^2 - 1
280*a^10*b^6*c^3 + 3840*a^11*b^4*c^4 - 6144*a^12*b^2*c^5)))^(1/2)*2i - (A/(
3*a) - (x^2*(5*A*b - 3*B*a))/(3*a^2) + (x^4*(15*A*b^4 + 14*A*a^2*c^2 - 9*B*
a*b^3 - 62*A*a*b^2*c + 33*B*a^2*b*c))/(6*a^3*(4*a*c - b^2)) + (c*x^6*(5*A*b
^3 - 3*B*a*b^2 + 10*B*a^2*c - 19*A*a*b*c))/(2*a^3*(4*a*c - b^2)))/(a*x^3 +
b*x^5 + c*x^7)
```

3.124 $\int \frac{x^{11}(A+Bx^2)}{(a+bx^2+cx^4)^3}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 365

$$\int \frac{x^{11}(A+Bx^2)}{(a+bx^2+cx^4)^3}\,dx = \frac{(3b^4B-Ab^3c-21ab^2Bc+7aAbc^2+30a^2Bc^2)\,x^2}{2c^3\,(b^2-4ac)^2} - \frac{x^8(a(bB-2Ac)+(b^2B-Abc-2aBc)\,x^2)}{4c\,(b^2-4ac)\,(a+bx^2+cx^4)^2} - \frac{x^4(a(3b^3B-Ab^2c-18abBc+16aAc^2)+(3b^4B-Ab^3c-20ab^2Bc+10aAbc^2+20a^2Bc^2)\,x^2)}{4c^2\,(b^2-4ac)^2\,(a+bx^2+cx^4)} - \frac{(3b^6B-Ab^5c-30ab^4Bc+10aAb^3c^2+90a^2b^2Bc^2-30a^2Abc^3-60a^3Bc^3)\,\mathrm{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2c^4\,(b^2-4ac)^{5/2}} - \frac{(3bB-Ac)\log(a+bx^2+cx^4)}{4c^4}$$

```
[Out] 1/2*(7*A*a*b*c^2-A*b^3*c+30*B*a^2*c^2-21*B*a*b^2*c+3*B*b^4)*x^2/c^3/(-4*a*c
+b^2)^2-1/4*x^8*(a*(-2*A*c+B*b)+(-A*b*c-2*B*a*c+B*b^2)*x^2)/c/(-4*a*c+b^2)/
(c*x^4+b*x^2+a)^2-1/4*x^4*(a*(16*A*a*c^2-A*b^2*c-18*B*a*b*c+3*B*b^3)+(10*A*
a*b*c^2-A*b^3*c+20*B*a^2*c^2-20*B*a*b^2*c+3*B*b^4)*x^2)/c^2/(-4*a*c+b^2)^2/
(c*x^4+b*x^2+a)-1/2*(-30*A*a^2*b*c^3+10*A*a*b^3*c^2-A*b^5*c-60*B*a^3*c^3+90
*B*a^2*b^2*c^2-30*B*a*b^4*c+3*B*b^6)*arctanh((2*c*x^2+b)/(-4*a*c+b^2)^(1/2)
)/c^4/(-4*a*c+b^2)^(5/2)-1/4*(-A*c+3*B*b)*ln(c*x^4+b*x^2+a)/c^4
```

Rubi [A] (verified)

Time = 0.95 (sec) , antiderivative size = 365, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.280, Rules used = {1265, 832, 787, 648, 632, 212, 642}

$$\int \frac{x^{11}(A+Bx^2)}{(a+bx^2+cx^4)^3}\,dx = -\frac{x^4(x^2(20a^2Bc^2+10aAbc^2-20ab^2Bc-Ab^3c+3b^4B)+a(16aAc^2-18abBc-Ab^2c+3b^3B))}{4c^2(b^2-4ac)^2(a+bx^2+cx^4)} + \frac{x^2(30a^2Bc^2+7aAbc^2-21ab^2Bc-Ab^3c+3b^4B)}{2c^3(b^2-4ac)^2} - \frac{(-60a^3Bc^3-30a^2Abc^3+90a^2b^2Bc^2+10aAb^3c^2-30ab^4Bc-Ab^5c+3b^6B)\,\text{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2c^4(b^2-4ac)^{5/2}} - \frac{x^8(x^2(-2aBc-Abc+b^2B)+a(bB-2Ac))}{4c(b^2-4ac)(a+bx^2+cx^4)^2} - \frac{(3bB-Ac)\log(a+bx^2+cx^4)}{4c^4}$$

```
[In] Int[(x^11*(A + B*x^2))/(a + b*x^2 + c*x^4)^3,x]
```

```
[Out] ((3*b^4*B - A*b^3*c - 21*a*b^2*B*c + 7*a*A*b*c^2 + 30*a^2*B*c^2)*x^2)/(2*c^
3*(b^2 - 4*a*c)^2) - (x^8*(a*(b*B - 2*A*c) + (b^2*B - A*b*c - 2*a*B*c)*x^2)
)/(4*c*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)^2) - (x^4*(a*(3*b^3*B - A*b^2*c -
18*a*b*B*c + 16*a*A*c^2) + (3*b^4*B - A*b^3*c - 20*a*b^2*B*c + 10*a*A*b*c^2
 + 20*a^2*B*c^2)*x^2))/(4*c^2*(b^2 - 4*a*c)^2*(a + b*x^2 + c*x^4)) - ((3*b^
6*B - A*b^5*c - 30*a*b^4*B*c + 10*a*A*b^3*c^2 + 90*a^2*b^2*B*c^2 - 30*a^2*A
*b*c^3 - 60*a^3*B*c^3)*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(2*c^4*(b^
2 - 4*a*c)^(5/2)) - ((3*b*B - A*c)*Log[a + b*x^2 + c*x^4])/(4*c^4)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
```

```
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 787

```
Int[(((d_.) + (e_.)*(x_))*((f_) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) + (c_.)*
(x_)^2), x_Symbol] :> Simp[e*g*(x/c), x] + Dist[1/c, Int[(c*d*f - a*e*g + (
c*e*f + c*d*g - b*e*g)*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e
, f, g}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 832

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(-(d + e*x)^(m - 1))*(a + b*x + c*x^2)
^(p + 1)*((2*a*c*(e*f + d*g) - b*(c*d*f + a*e*g) - (2*c^2*d*f + b^2*e*g - c
*(b*e*f + b*d*g + 2*a*e*g))*x)/(c*(p + 1)*(b^2 - 4*a*c))), x] - Dist[1/(c*(
p + 1)*(b^2 - 4*a*c)), Int[(d + e*x)^(m - 2)*(a + b*x + c*x^2)^(p + 1)*Simp
[2*c^2*d^2*f*(2*p + 3) + b*e*g*(a*e*(m - 1) + b*d*(p + 2)) - c*(2*a*e*(e*f*
(m - 1) + d*g*m) + b*d*(d*g*(2*p + 3) - e*f*(m - 2*p - 4))) + e*(b^2*e*g*(m
 + p + 1) + 2*c^2*d*f*(m + 2*p + 2) - c*(2*a*e*g*m + b*(e*f + d*g)*(m + 2*p
 + 2)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*c
, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 1] && ((EqQ[m
, 2] && EqQ[p, -3] && RationalQ[a, b, c, d, e, f, g]) ||  !ILtQ[m + 2*p + 3
, 0])
```

Rule 1265

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rubi steps

$$\text{integral} = \frac{1}{2}\text{Subst}\left(\int \frac{x^5(A+Bx)}{(a+bx+cx^2)^3}\,dx, x, x^2\right)$$

$$=-\frac{x^8(a(bB-2Ac)+(b^2B-Abc-2aBc)\,x^2)}{4c\,(b^2-4ac)\,(a+bx^2+cx^4)^2}+\frac{\text{Subst}\left(\int \frac{x^3(4a(bB-2Ac)+(3b^2B-Abc-10aBc)x)}{(a+bx+cx^2)^2}\,dx,x,x^2\right)}{4c\,(b^2-4ac)}$$

$$=-\frac{x^8(a(bB-2Ac)+(b^2B-Abc-2aBc)\,x^2)}{4c\,(b^2-4ac)\,(a+bx^2+cx^4)^2}-\frac{x^4(a(3b^3B-Ab^2c-18abBc+16aAc^2)+(3b^4B-Ab^3c-20ab^2Bc+10aAbc^2+20a^2Bc^2)\,x^2)}{4c^2\,(b^2-4ac)^2\,(a+bx^2+cx^4)}+\frac{\text{Subst}\left(\int \frac{x(2a(3b^3B-Ab^2c-18abBc+16aAc^2)+2(3b^4B-Ab^3c-21ab^2Bc+7aAbc^2+30a^2Bc^2)x)}{a+bx+cx^2}\,dx,x,x^2\right)}{4c^2\,(b^2-4ac)^2}$$

$$=\frac{(3b^4B-Ab^3c-21ab^2Bc+7aAbc^2+30a^2Bc^2)\,x^2}{2c^3\,(b^2-4ac)^2}-\frac{x^8(a(bB-2Ac)+(b^2B-Abc-2aBc)\,x^2)}{4c\,(b^2-4ac)\,(a+bx^2+cx^4)^2}-\frac{x^4(a(3b^3B-Ab^2c-18abBc+16aAc^2)+(3b^4B-Ab^3c-20ab^2Bc+10aAbc^2+20a^2Bc^2)\,x^2)}{4c^2\,(b^2-4ac)^2\,(a+bx^2+cx^4)}+\frac{\text{Subst}\left(\int \frac{-2a(3b^4B-Ab^3c-21ab^2Bc+7aAbc^2+30a^2Bc^2)+(2ac(3b^3B-Ab^2c-18abBc+16aAc^2)-2b(3b^4B-Ab^3c-21ab^2Bc+7a}{a+bx+cx^2}\right.}{4c^3\,(b^2-4ac)^2}$$

$$=\frac{(3b^4B-Ab^3c-21ab^2Bc+7aAbc^2+30a^2Bc^2)\,x^2}{2c^3\,(b^2-4ac)^2}-\frac{x^8(a(bB-2Ac)+(b^2B-Abc-2aBc)\,x^2)}{4c\,(b^2-4ac)\,(a+bx^2+cx^4)^2}-\frac{x^4(a(3b^3B-Ab^2c-18abBc+16aAc^2)+(3b^4B-Ab^3c-20ab^2Bc+10aAbc^2+20a^2Bc^2)\,x^2)}{4c^2\,(b^2-4ac)^2\,(a+bx^2+cx^4)}-\frac{(3bB-Ac)\text{Subst}\left(\int \frac{b+2cx}{a+bx+cx^2}\,dx,x,x^2\right)}{4c^4}+\frac{(3b^6B-Ab^5c-30ab^4Bc+10aAb^3c^2+90a^2b^2Bc^2-30a^2Abc^3-60a^3Bc^3)\,\text{Subst}\left(\int \frac{1}{a+bx+cx^2}\,dx,\right.}{4c^4\,(b^2-4ac)^2}$$

$$= \frac{(3b^4B - Ab^3c - 21ab^2Bc + 7aAbc^2 + 30a^2Bc^2)\,x^2}{2c^3\,(b^2-4ac)^2} - \frac{x^8(a(bB-2Ac) + (b^2B - Abc - 2aBc)\,x^2)}{4c\,(b^2-4ac)\,(a+bx^2+cx^4)^2} - \frac{x^4(a(3b^3B - Ab^2c - 18abBc + 16aAc^2) + (3b^4B - Ab^3c - 20ab^2Bc + 10aAbc^2 + 20a^2Bc^2)\,x^2)}{4c^2\,(b^2-4ac)^2\,(a+bx^2+cx^4)} - \frac{(3bB - Ac)\log(a+bx^2+cx^4)}{4c^4} - \frac{(3b^6B - Ab^5c - 30ab^4Bc + 10aAb^3c^2 + 90a^2b^2Bc^2 - 30a^2Abc^3 - 60a^3Bc^3)\,\mathrm{Subst}\left(\int \frac{1}{b^2-4ac-x^2}\,d\right.}{2c^4\,(b^2-4ac)^2}$$

$$= \frac{(3b^4B - Ab^3c - 21ab^2Bc + 7aAbc^2 + 30a^2Bc^2)\,x^2}{2c^3\,(b^2-4ac)^2} - \frac{x^8(a(bB-2Ac) + (b^2B - Abc - 2aBc)\,x^2)}{4c\,(b^2-4ac)\,(a+bx^2+cx^4)^2} - \frac{x^4(a(3b^3B - Ab^2c - 18abBc + 16aAc^2) + (3b^4B - Ab^3c - 20ab^2Bc + 10aAbc^2 + 20a^2Bc^2)\,x^2)}{4c^2\,(b^2-4ac)^2\,(a+bx^2+cx^4)} - \frac{(3b^6B - Ab^5c - 30ab^4Bc + 10aAb^3c^2 + 90a^2b^2Bc^2 - 30a^2Abc^3 - 60a^3Bc^3)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2c^4\,(b^2-4ac)^{5/2}} - \frac{(3bB - Ac)\log(a+bx^2+cx^4)}{4c^4}$$

Mathematica [A] (verified)

Time = 0.43 (sec) , antiderivative size = 435, normalized size of antiderivative = 1.19

$$\int \frac{x^{11}(A+Bx^2)}{(a+bx^2+cx^4)^3}\,dx$$

$$= \frac{2Bc^2x^2 + \frac{b^7B - b^6c(A+6Bx^2) + 4a^3c^4(8A+9Bx^2) - 3a^2b^2c^3(13A+34Bx^2) + ab^4c^2(11A+48Bx^2) + ab^3c^2(61aB-30Acx^2) + 2b^5c(-7aB+2A}{(b^2-4ac)^2(a+bx^2+cx^4)}}{}$$

```
[In] Integrate[(x^11*(A + B*x^2))/(a + b*x^2 + c*x^4)^3,x]
```

```
[Out] (2*B*c^2*x^2 + (b^7*B - b^6*c*(A + 6*B*x^2) + 4*a^3*c^4*(8*A + 9*B*x^2) - 3
*a^2*b^2*c^3*(13*A + 34*B*x^2) + a*b^4*c^2*(11*A + 48*B*x^2) + a*b^3*c^2*(6
1*a*B - 30*A*c*x^2) + 2*b^5*c*(-7*a*B + 2*A*c*x^2) + 2*a^2*b*c^3*(-39*a*B +
 25*A*c*x^2))/((b^2 - 4*a*c)^2*(a + b*x^2 + c*x^4)) + (b^5*(-(b*B) + A*c)*x
^2 + a^3*c^2*(-5*b*B + 2*c*(A + B*x^2)) + a*b^3*(-(b^2*B) - 5*A*c^2*x^2 + b
*c*(A + 6*B*x^2)) + a^2*b*c*(5*b^2*B + 5*A*c^2*x^2 - b*c*(4*A + 9*B*x^2)))/
((b^2 - 4*a*c)*(a + b*x^2 + c*x^4)^2) - (2*c*(-3*b^6*B + A*b^5*c + 30*a*b^4
*B*c - 10*a*A*b^3*c^2 - 90*a^2*b^2*B*c^2 + 30*a^2*A*b*c^3 + 60*a^3*B*c^3)*A
rcTan[(b + 2*c*x^2)/Sqrt[-b^2 + 4*a*c]])/(-b^2 + 4*a*c)^(5/2) + c*(-3*b*B +
 A*c)*Log[a + b*x^2 + c*x^4])/(4*c^5)
```

Maple [A] (verified)

Time = 0.31 (sec) , antiderivative size = 624, normalized size of antiderivative = 1.71

method	result
default	$\frac{B\,x^2}{2c^3}+\frac{\frac{\left(25A\,a^2 b\,c^3-15Aa\,b^3c^2+2A\,b^5c+18B\,a^3c^3-51B\,a^2b^2c^2+24Ba\,b^4c-3B\,b^6\right)x^6}{16a^2c^2-8a\,b^2c+b^4}+\frac{\left(32A\,a^3c^4+11A\,a^2b^2c^3-19Aa\,b^4c^2+3A\,b^6c-42B\,a^3b\,c^3\right.}{2c\left(16a^2c^2-8a\,b^2c+b^4\right.}}{}$
risch	Expression too large to display

```
[In] int(x^11*(B*x^2+A)/(c*x^4+b*x^2+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*B*x^2/c^3+1/2/c^3*(((25*A*a^2*b*c^3-15*A*a*b^3*c^2+2*A*b^5*c+18*B*a^3*c
^3-51*B*a^2*b^2*c^2+24*B*a*b^4*c-3*B*b^6)/(16*a^2*c^2-8*a*b^2*c+b^4)*x^6+1/
2*(32*A*a^3*c^4+11*A*a^2*b^2*c^3-19*A*a*b^4*c^2+3*A*b^6*c-42*B*a^3*b*c^3-41
*B*a^2*b^3*c^2+34*B*a*b^5*c-5*B*b^7)/c/(16*a^2*c^2-8*a*b^2*c+b^4)*x^4+a*(31
*A*a^2*b*c^3-22*A*a*b^3*c^2+3*A*b^5*c+14*B*a^3*c^3-71*B*a^2*b^2*c^2+38*B*a*
b^4*c-5*B*b^6)/c/(16*a^2*c^2-8*a*b^2*c+b^4)*x^2+1/2*a^2*(24*A*a^2*c^3-21*A*
a*b^2*c^2+3*A*b^4*c-58*B*a^2*b*c^2+36*B*a*b^3*c-5*B*b^5)/c/(16*a^2*c^2-8*a*
b^2*c+b^4))/(c*x^4+b*x^2+a)^2+1/(16*a^2*c^2-8*a*b^2*c+b^4)*(1/2*(16*A*a^2*c
^3-8*A*a*b^2*c^2+A*b^4*c-48*B*a^2*b*c^2+24*B*a*b^3*c-3*B*b^5)/c*ln(c*x^4+b*
x^2+a)+2*(-7*A*a^2*b*c^2+A*a*b^3*c-30*a^3*B*c^2+21*B*a^2*b^2*c-3*B*a*b^4-1/
2*(16*A*a^2*c^3-8*A*a*b^2*c^2+A*b^4*c-48*B*a^2*b*c^2+24*B*a*b^3*c-3*B*b^5)*
b/c)/(4*a*c-b^2)^(1/2)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1586 vs. $2(351) = 702$.

Time = 0.53 (sec) , antiderivative size = 3196, normalized size of antiderivative = 8.76

$$\int \frac{x^{11}(A+Bx^2)}{(a+bx^2+cx^4)^3}\,dx=\text{Too large to display}$$

```
[In] integrate(x^11*(B*x^2+A)/(c*x^4+b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] [1/4*(2*(B*b^6*c^3 - 12*B*a*b^4*c^4 + 48*B*a^2*b^2*c^5 - 64*B*a^3*c^6)*x^10
 - 5*B*a^2*b^7 - 96*A*a^5*c^4 + 4*(B*b^7*c^2 - 12*B*a*b^5*c^3 + 48*B*a^2*b^
3*c^4 - 64*B*a^3*b*c^5)*x^8 - 2*(2*B*b^8*c + 100*(2*B*a^4 + A*a^3*b)*c^5 -
(254*B*a^3*b^2 + 85*A*a^2*b^3)*c^4 + (123*B*a^2*b^4 + 23*A*a*b^5)*c^3 - 2*(
13*B*a*b^6 + A*b^7)*c^2)*x^6 - (5*B*b^9 + 128*A*a^4*c^5 + 4*(22*B*a^4*b + 3
*A*a^3*b^2)*c^4 - (314*B*a^3*b^3 + 87*A*a^2*b^4)*c^3 + (225*B*a^2*b^5 + 31*
A*a*b^6)*c^2 - (58*B*a*b^7 + 3*A*b^8)*c)*x^4 + 4*(58*B*a^5*b + 27*A*a^4*b^2
)*c^3 - (202*B*a^4*b^3 + 33*A*a^3*b^4)*c^2 - 2*(5*B*a*b^8 + 4*(30*B*a^5 + 3
1*A*a^4*b)*c^4 - (346*B*a^4*b^2 + 119*A*a^3*b^3)*c^3 + (235*B*a^3*b^4 + 34*
```

```
A*a^2*b^5)*c^2 - (59*B*a^2*b^6 + 3*A*a*b^7)*c)*x^2 - (3*B*a^2*b^6 + (3*B*b^
6*c^2 - 30*(2*B*a^3 + A*a^2*b)*c^5 + 10*(9*B*a^2*b^2 + A*a*b^3)*c^4 - (30*B
*a*b^4 + A*b^5)*c^3)*x^8 + 2*(3*B*b^7*c - 30*(2*B*a^3*b + A*a^2*b^2)*c^4 +
10*(9*B*a^2*b^3 + A*a*b^4)*c^3 - (30*B*a*b^5 + A*b^6)*c^2)*x^6 + (3*B*b^8 -
 60*(2*B*a^4 + A*a^3*b)*c^4 + 10*(12*B*a^3*b^2 - A*a^2*b^3)*c^3 + 2*(15*B*a
^2*b^4 + 4*A*a*b^5)*c^2 - (24*B*a*b^6 + A*b^7)*c)*x^4 - 30*(2*B*a^5 + A*a^4
*b)*c^3 + 10*(9*B*a^4*b^2 + A*a^3*b^3)*c^2 + 2*(3*B*a*b^7 - 30*(2*B*a^4*b +
 A*a^3*b^2)*c^3 + 10*(9*B*a^3*b^3 + A*a^2*b^4)*c^2 - (30*B*a^2*b^5 + A*a*b^
6)*c)*x^2 - (30*B*a^3*b^4 + A*a^2*b^5)*c)*sqrt(b^2 - 4*a*c)*log((2*c^2*x^4
+ 2*b*c*x^2 + b^2 - 2*a*c + (2*c*x^2 + b)*sqrt(b^2 - 4*a*c))/(c*x^4 + b*x^2
 + a)) + (56*B*a^3*b^5 + 3*A*a^2*b^6)*c - (3*B*a^2*b^7 + 64*A*a^5*c^4 + (3*
B*b^7*c^2 + 64*A*a^3*c^6 - 48*(4*B*a^3*b + A*a^2*b^2)*c^5 + 12*(12*B*a^2*b^
3 + A*a*b^4)*c^4 - (36*B*a*b^5 + A*b^6)*c^3)*x^8 + 2*(3*B*b^8*c + 64*A*a^3*
b*c^5 - 48*(4*B*a^3*b^2 + A*a^2*b^3)*c^4 + 12*(12*B*a^2*b^4 + A*a*b^5)*c^3
- (36*B*a*b^6 + A*b^7)*c^2)*x^6 + (3*B*b^9 + 128*A*a^4*c^5 - 32*(12*B*a^4*b
 + A*a^3*b^2)*c^4 + 24*(4*B*a^3*b^3 - A*a^2*b^4)*c^3 + 2*(36*B*a^2*b^5 + 5*
A*a*b^6)*c^2 - (30*B*a*b^7 + A*b^8)*c)*x^4 - 48*(4*B*a^5*b + A*a^4*b^2)*c^3
 + 12*(12*B*a^4*b^3 + A*a^3*b^4)*c^2 + 2*(3*B*a*b^8 + 64*A*a^4*b*c^4 - 48*(
4*B*a^4*b^2 + A*a^3*b^3)*c^3 + 12*(12*B*a^3*b^4 + A*a^2*b^5)*c^2 - (36*B*a^
2*b^6 + A*a*b^7)*c)*x^2 - (36*B*a^3*b^5 + A*a^2*b^6)*c)*log(c*x^4 + b*x^2 +
 a))/(a^2*b^6*c^4 - 12*a^3*b^4*c^5 + 48*a^4*b^2*c^6 - 64*a^5*c^7 + (b^6*c^6
 - 12*a*b^4*c^7 + 48*a^2*b^2*c^8 - 64*a^3*c^9)*x^8 + 2*(b^7*c^5 - 12*a*b^5*
c^6 + 48*a^2*b^3*c^7 - 64*a^3*b*c^8)*x^6 + (b^8*c^4 - 10*a*b^6*c^5 + 24*a^2
*b^4*c^6 + 32*a^3*b^2*c^7 - 128*a^4*c^8)*x^4 + 2*(a*b^7*c^4 - 12*a^2*b^5*c^
5 + 48*a^3*b^3*c^6 - 64*a^4*b*c^7)*x^2), 1/4*(2*(B*b^6*c^3 - 12*B*a*b^4*c^4
 + 48*B*a^2*b^2*c^5 - 64*B*a^3*c^6)*x^10 - 5*B*a^2*b^7 - 96*A*a^5*c^4 + 4*(
B*b^7*c^2 - 12*B*a*b^5*c^3 + 48*B*a^2*b^3*c^4 - 64*B*a^3*b*c^5)*x^8 - 2*(2*
B*b^8*c + 100*(2*B*a^4 + A*a^3*b)*c^5 - (254*B*a^3*b^2 + 85*A*a^2*b^3)*c^4
+ (123*B*a^2*b^4 + 23*A*a*b^5)*c^3 - 2*(13*B*a*b^6 + A*b^7)*c^2)*x^6 - (5*B
*b^9 + 128*A*a^4*c^5 + 4*(22*B*a^4*b + 3*A*a^3*b^2)*c^4 - (314*B*a^3*b^3 +
87*A*a^2*b^4)*c^3 + (225*B*a^2*b^5 + 31*A*a*b^6)*c^2 - (58*B*a*b^7 + 3*A*b^
8)*c)*x^4 + 4*(58*B*a^5*b + 27*A*a^4*b^2)*c^3 - (202*B*a^4*b^3 + 33*A*a^3*b
^4)*c^2 - 2*(5*B*a*b^8 + 4*(30*B*a^5 + 31*A*a^4*b)*c^4 - (346*B*a^4*b^2 + 1
19*A*a^3*b^3)*c^3 + (235*B*a^3*b^4 + 34*A*a^2*b^5)*c^2 - (59*B*a^2*b^6 + 3*
A*a*b^7)*c)*x^2 - 2*(3*B*a^2*b^6 + (3*B*b^6*c^2 - 30*(2*B*a^3 + A*a^2*b)*c^
5 + 10*(9*B*a^2*b^2 + A*a*b^3)*c^4 - (30*B*a*b^4 + A*b^5)*c^3)*x^8 + 2*(3*B
*b^7*c - 30*(2*B*a^3*b + A*a^2*b^2)*c^4 + 10*(9*B*a^2*b^3 + A*a*b^4)*c^3 -
(30*B*a*b^5 + A*b^6)*c^2)*x^6 + (3*B*b^8 - 60*(2*B*a^4 + A*a^3*b)*c^4 + 10*
(12*B*a^3*b^2 - A*a^2*b^3)*c^3 + 2*(15*B*a^2*b^4 + 4*A*a*b^5)*c^2 - (24*B*a
*b^6 + A*b^7)*c)*x^4 - 30*(2*B*a^5 + A*a^4*b)*c^3 + 10*(9*B*a^4*b^2 + A*a^3
*b^3)*c^2 + 2*(3*B*a*b^7 - 30*(2*B*a^4*b + A*a^3*b^2)*c^3 + 10*(9*B*a^3*b^3
 + A*a^2*b^4)*c^2 - (30*B*a^2*b^5 + A*a*b^6)*c)*x^2 - (30*B*a^3*b^4 + A*a^2
*b^5)*c)*sqrt(-b^2 + 4*a*c)*arctan(-(2*c*x^2 + b)*sqrt(-b^2 + 4*a*c)/(b^2 -
 4*a*c)) + (56*B*a^3*b^5 + 3*A*a^2*b^6)*c - (3*B*a^2*b^7 + 64*A*a^5*c^4 + (
3*B*b^7*c^2 + 64*A*a^3*c^6 - 48*(4*B*a^3*b + A*a^2*b^2)*c^5 + 12*(12*B*a^2*
```

```
b^3 + A*a*b^4)*c^4 - (36*B*a*b^5 + A*b^6)*c^3)*x^8 + 2*(3*B*b^8*c + 64*A*a^
3*b*c^5 - 48*(4*B*a^3*b^2 + A*a^2*b^3)*c^4 + 12*(12*B*a^2*b^4 + A*a*b^5)*c^
3 - (36*B*a*b^6 + A*b^7)*c^2)*x^6 + (3*B*b^9 + 128*A*a^4*c^5 - 32*(12*B*a^4
*b + A*a^3*b^2)*c^4 + 24*(4*B*a^3*b^3 - A*a^2*b^4)*c^3 + 2*(36*B*a^2*b^5 +
5*A*a*b^6)*c^2 - (30*B*a*b^7 + A*b^8)*c)*x^4 - 48*(4*B*a^5*b + A*a^4*b^2)*c
^3 + 12*(12*B*a^4*b^3 + A*a^3*b^4)*c^2 + 2*(3*B*a*b^8 + 64*A*a^4*b*c^4 - 48
*(4*B*a^4*b^2 + A*a^3*b^3)*c^3 + 12*(12*B*a^3*b^4 + A*a^2*b^5)*c^2 - (36*B*
a^2*b^6 + A*a*b^7)*c)*x^2 - (36*B*a^3*b^5 + A*a^2*b^6)*c)*log(c*x^4 + b*x^2
 + a))/(a^2*b^6*c^4 - 12*a^3*b^4*c^5 + 48*a^4*b^2*c^6 - 64*a^5*c^7 + (b^6*c
^6 - 12*a*b^4*c^7 + 48*a^2*b^2*c^8 - 64*a^3*c^9)*x^8 + 2*(b^7*c^5 - 12*a*b^
5*c^6 + 48*a^2*b^3*c^7 - 64*a^3*b*c^8)*x^6 + (b^8*c^4 - 10*a*b^6*c^5 + 24*a
^2*b^4*c^6 + 32*a^3*b^2*c^7 - 128*a^4*c^8)*x^4 + 2*(a*b^7*c^4 - 12*a^2*b^5*
c^5 + 48*a^3*b^3*c^6 - 64*a^4*b*c^7)*x^2)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^{11}(A+Bx^2)}{(a+bx^2+cx^4)^3}\,dx = \text{Timed out}$$

```
[In] integrate(x**11*(B*x**2+A)/(c*x**4+b*x**2+a)**3,x)
```

```
[Out] Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^{11}(A+Bx^2)}{(a+bx^2+cx^4)^3}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(x^11*(B*x^2+A)/(c*x^4+b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

Giac [A] (verification not implemented)

none

Time = 1.42 (sec) , antiderivative size = 598, normalized size of antiderivative = 1.64

$$\int \frac{x^{11}(A+Bx^2)}{(a+bx^2+cx^4)^3}\,dx = \frac{(3\,Bb^6-30\,Bab^4c-Ab^5c+90\,Ba^2b^2c^2+10\,Aab^3c^2-60\,Ba^3c^3-30\,Aa^2bc^3)\arctan\left(\frac{2\,cx^2+b}{\sqrt{-b^2+4\,ac}}\right)}{2\,(b^4c^4-8\,ab^2c^5+16\,a^2c^6)\sqrt{-b^2+4\,ac}} + \frac{Bx^2}{2\,c^3} + \frac{9\,Bb^5c^2x^8-72\,Bab^3c^3x^8-3\,Ab^4c^3x^8+144\,Ba^2bc^4x^8+24\,Aab^2c^4x^8-48\,Aa^2c^5x^8+6\,Bb^6cx^6-48\,Ba}{} - \frac{(3\,Bb-Ac)\log(cx^4+bx^2+a)}{4\,c^4}$$

```
[In] integrate(x^11*(B*x^2+A)/(c*x^4+b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] 1/2*(3*B*b^6 - 30*B*a*b^4*c - A*b^5*c + 90*B*a^2*b^2*c^2 + 10*A*a*b^3*c^2 -
 60*B*a^3*c^3 - 30*A*a^2*b*c^3)*arctan((2*c*x^2 + b)/sqrt(-b^2 + 4*a*c))/((
b^4*c^4 - 8*a*b^2*c^5 + 16*a^2*c^6)*sqrt(-b^2 + 4*a*c)) + 1/2*B*x^2/c^3 + 1
/8*(9*B*b^5*c^2*x^8 - 72*B*a*b^3*c^3*x^8 - 3*A*b^4*c^3*x^8 + 144*B*a^2*b*c^
4*x^8 + 24*A*a*b^2*c^4*x^8 - 48*A*a^2*c^5*x^8 + 6*B*b^6*c*x^6 - 48*B*a*b^4*
c^2*x^6 + 2*A*b^5*c^2*x^6 + 84*B*a^2*b^2*c^3*x^6 - 12*A*a*b^3*c^3*x^6 + 72*
B*a^3*c^4*x^6 + 4*A*a^2*b*c^4*x^6 - B*b^7*x^4 + 14*B*a*b^5*c*x^4 + 3*A*b^6*
c*x^4 - 82*B*a^2*b^3*c^2*x^4 - 20*A*a*b^4*c^2*x^4 + 204*B*a^3*b*c^3*x^4 + 2
2*A*a^2*b^2*c^3*x^4 - 32*A*a^3*c^4*x^4 - 2*B*a*b^6*x^2 + 8*B*a^2*b^4*c*x^2
+ 6*A*a*b^5*c*x^2 + 4*B*a^3*b^2*c^2*x^2 - 40*A*a^2*b^3*c^2*x^2 + 56*B*a^4*c
^3*x^2 + 28*A*a^3*b*c^3*x^2 - B*a^2*b^5 + 3*A*a^2*b^4*c + 28*B*a^4*b*c^2 -
18*A*a^3*b^2*c^2)/((b^4*c^4 - 8*a*b^2*c^5 + 16*a^2*c^6)*(c*x^4 + b*x^2 + a)
^2) - 1/4*(3*B*b - A*c)*log(c*x^4 + b*x^2 + a)/c^4
```

Mupad [B] (verification not implemented)

Time = 10.38 (sec) , antiderivative size = 4501, normalized size of antiderivative = 12.33

$$\int \frac{x^{11}(A+Bx^2)}{(a+bx^2+cx^4)^3}\,dx = \text{Too large to display}$$

```
[In] int((x^11*(A + B*x^2))/(a + b*x^2 + c*x^4)^3,x)
```

```
[Out] ((x^6*(18*B*a^3*c^3 - 3*B*b^6 + 2*A*b^5*c + 24*B*a*b^4*c - 15*A*a*b^3*c^2 +
 25*A*a^2*b*c^3 - 51*B*a^2*b^2*c^2))/(2*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (
```

```
a*(24*A*a^3*c^3 - 5*B*a*b^5 + 3*A*a*b^4*c + 36*B*a^2*b^3*c - 58*B*a^3*b*c^2
 - 21*A*a^2*b^2*c^2))/(4*c*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (x^2*(14*B*a^4
*c^3 - 5*B*a*b^6 + 3*A*a*b^5*c + 31*A*a^3*b*c^3 + 38*B*a^2*b^4*c - 22*A*a^2
*b^3*c^2 - 71*B*a^3*b^2*c^2))/(2*c*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) - (x^4*(
5*B*b^7 - 32*A*a^3*c^4 - 3*A*b^6*c - 34*B*a*b^5*c + 19*A*a*b^4*c^2 + 42*B*a
^3*b*c^3 - 11*A*a^2*b^2*c^3 + 41*B*a^2*b^3*c^2))/(4*c*(b^4 + 16*a^2*c^2 - 8
*a*b^2*c)))/(a^2*c^3 + c^5*x^8 + x^4*(2*a*c^4 + b^2*c^3) + 2*b*c^4*x^6 + 2*
a*b*c^3*x^2) + (B*x^2)/(2*c^3) + (log(((a*(A*c - 3*B*b)^2)/c^6 - (((8*a*(A*
c - 3*B*b))/c^2 - (2*(2*a + b*x^2)*(A*c - 3*B*b + c^4*(-(60*B*a^3*c^3 - 3*B
*b^6 + A*b^5*c + 30*B*a*b^4*c - 10*A*a*b^3*c^2 + 30*A*a^2*b*c^3 - 90*B*a^2*
b^2*c^2)^2/(c^8*(4*a*c - b^2)^5))^(1/2)))/c^2 + (2*x^2*(60*B*a^3*c^3 - 9*B*
b^6 + 3*A*b^5*c + 78*B*a*b^4*c - 26*A*a*b^3*c^2 + 62*A*a^2*b*c^3 - 186*B*a^
2*b^2*c^2))/(c^2*(4*a*c - b^2)^2))*(A*c - 3*B*b + c^4*(-(60*B*a^3*c^3 - 3*B
*b^6 + A*b^5*c + 30*B*a*b^4*c - 10*A*a*b^3*c^2 + 30*A*a^2*b*c^3 - 90*B*a^2*
b^2*c^2)^2/(c^8*(4*a*c - b^2)^5))^(1/2)))/(4*c^4) + (x^2*(A*c - 3*B*b)*(30*
B*a^3*c^3 - 3*B*b^6 + A*b^5*c + 27*B*a*b^4*c - 9*A*a*b^3*c^2 + 23*A*a^2*b*c
^3 - 69*B*a^2*b^2*c^2))/(c^6*(4*a*c - b^2)^2))*((a*(A*c - 3*B*b)^2)/c^6 + (
((2*(2*a + b*x^2)*(3*B*b - A*c + c^4*(-(60*B*a^3*c^3 - 3*B*b^6 + A*b^5*c +
30*B*a*b^4*c - 10*A*a*b^3*c^2 + 30*A*a^2*b*c^3 - 90*B*a^2*b^2*c^2)^2/(c^8*(
4*a*c - b^2)^5))^(1/2)))/c^2 + (8*a*(A*c - 3*B*b))/c^2 + (2*x^2*(60*B*a^3*c
^3 - 9*B*b^6 + 3*A*b^5*c + 78*B*a*b^4*c - 26*A*a*b^3*c^2 + 62*A*a^2*b*c^3 -
 186*B*a^2*b^2*c^2))/(c^2*(4*a*c - b^2)^2))*(3*B*b - A*c + c^4*(-(60*B*a^3*
c^3 - 3*B*b^6 + A*b^5*c + 30*B*a*b^4*c - 10*A*a*b^3*c^2 + 30*A*a^2*b*c^3 -
90*B*a^2*b^2*c^2)^2/(c^8*(4*a*c - b^2)^5))^(1/2)))/(4*c^4) + (x^2*(A*c - 3*
B*b)*(30*B*a^3*c^3 - 3*B*b^6 + A*b^5*c + 27*B*a*b^4*c - 9*A*a*b^3*c^2 + 23*
A*a^2*b*c^3 - 69*B*a^2*b^2*c^2))/(c^6*(4*a*c - b^2)^2)))*(6*B*b^11 + 2048*A
*a^5*c^6 - 2*A*b^10*c - 120*B*a*b^9*c + 40*A*a*b^8*c^2 - 6144*B*a^5*b*c^5 -
 320*A*a^2*b^6*c^3 + 1280*A*a^3*b^4*c^4 - 2560*A*a^4*b^2*c^5 + 960*B*a^2*b^
7*c^2 - 3840*B*a^3*b^5*c^3 + 7680*B*a^4*b^3*c^4))/(2*(4096*a^5*c^9 - 4*b^10
*c^4 + 80*a*b^8*c^5 - 640*a^2*b^6*c^6 + 2560*a^3*b^4*c^7 - 5120*a^4*b^2*c^8
)) - (atan(((32*a^2*c^8*(4*a*c - b^2)^5 + 2*b^4*c^6*(4*a*c - b^2)^5 - 16*a*
b^2*c^7*(4*a*c - b^2)^5)*(x^2*(((((6*A*b^5*c^5 + 120*B*a^3*c^7 - 18*B*b^6*c
^4 - 52*A*a*b^3*c^6 + 124*A*a^2*b*c^7 + 156*B*a*b^4*c^5 - 372*B*a^2*b^2*c^6
)/(16*a^2*c^8 + b^4*c^6 - 8*a*b^2*c^7) - ((8*b^5*c^8 - 64*a*b^3*c^9 + 128*a
^2*b*c^10)*(6*B*b^11 + 2048*A*a^5*c^6 - 2*A*b^10*c - 120*B*a*b^9*c + 40*A*a
*b^8*c^2 - 6144*B*a^5*b*c^5 - 320*A*a^2*b^6*c^3 + 1280*A*a^3*b^4*c^4 - 2560
*A*a^4*b^2*c^5 + 960*B*a^2*b^7*c^2 - 3840*B*a^3*b^5*c^3 + 7680*B*a^4*b^3*c^
4))/(2*(16*a^2*c^8 + b^4*c^6 - 8*a*b^2*c^7)*(4096*a^5*c^9 - 4*b^10*c^4 + 80
*a*b^8*c^5 - 640*a^2*b^6*c^6 + 2560*a^3*b^4*c^7 - 5120*a^4*b^2*c^8)))*(60*B
*a^3*c^3 - 3*B*b^6 + A*b^5*c + 30*B*a*b^4*c - 10*A*a*b^3*c^2 + 30*A*a^2*b*c
^3 - 90*B*a^2*b^2*c^2))/(8*c^4*(4*a*c - b^2)^(5/2)) - ((8*b^5*c^8 - 64*a*b^
3*c^9 + 128*a^2*b*c^10)*(60*B*a^3*c^3 - 3*B*b^6 + A*b^5*c + 30*B*a*b^4*c -
10*A*a*b^3*c^2 + 30*A*a^2*b*c^3 - 90*B*a^2*b^2*c^2)*(6*B*b^11 + 2048*A*a^5*
c^6 - 2*A*b^10*c - 120*B*a*b^9*c + 40*A*a*b^8*c^2 - 6144*B*a^5*b*c^5 - 320*
A*a^2*b^6*c^3 + 1280*A*a^3*b^4*c^4 - 2560*A*a^4*b^2*c^5 + 960*B*a^2*b^7*c^2
```

- 3840*B*a^3*b^5*c^3 + 7680*B*a^4*b^3*c^4))/(16*c^4*(4*a*c - b^2)^(5/2)*(16*a^2*c^8 + b^4*c^6 - 8*a*b^2*c^7)*(4096*a^5*c^9 - 4*b^10*c^4 + 80*a*b^8*c^5 - 640*a^2*b^6*c^6 + 2560*a^3*b^4*c^7 - 5120*a^4*b^2*c^8)))/(a*(4*a*c - b^2)^2) + (b*((((6*A*b^5*c^5 + 120*B*a^3*c^7 - 18*B*b^6*c^4 - 52*A*a*b^3*c^6 + 124*A*a^2*b*c^7 + 156*B*a*b^4*c^5 - 372*B*a^2*b^2*c^6)/(16*a^2*c^8 + b^4*c^6 - 8*a*b^2*c^7) - ((8*b^5*c^8 - 64*a*b^3*c^9 + 128*a^2*b*c^10)*(6*B*b^11 + 2048*A*a^5*c^6 - 2*A*b^10*c - 120*B*a*b^9*c + 40*A*a*b^8*c^2 - 6144*B*a^5*b*c^5 - 320*A*a^2*b^6*c^3 + 1280*A*a^3*b^4*c^4 - 2560*A*a^4*b^2*c^5 + 960*B*a^2*b^7*c^2 - 3840*B*a^3*b^5*c^3 + 7680*B*a^4*b^3*c^4))/(2*(16*a^2*c^8 + b^4*c^6 - 8*a*b^2*c^7)*(4096*a^5*c^9 - 4*b^10*c^4 + 80*a*b^8*c^5 - 640*a^2*b^6*c^6 + 2560*a^3*b^4*c^7 - 5120*a^4*b^2*c^8)))*(6*B*b^11 + 2048*A*a^5*c^6 - 2*A*b^10*c - 120*B*a*b^9*c + 40*A*a*b^8*c^2 - 6144*B*a^5*b*c^5 - 320*A*a^2*b^6*c^3 + 1280*A*a^3*b^4*c^4 - 2560*A*a^4*b^2*c^5 + 960*B*a^2*b^7*c^2 - 3840*B*a^3*b^5*c^3 + 7680*B*a^4*b^3*c^4))/(2*(4096*a^5*c^9 - 4*b^10*c^4 + 80*a*b^8*c^5 - 640*a^2*b^6*c^6 + 2560*a^3*b^4*c^7 - 5120*a^4*b^2*c^8)) - (9*B^2*b^7 + A^2*b^5*c^2 - 6*A*B*b^6*c + 207*B^2*a^2*b^3*c^2 + 30*A*B*a^3*c^4 - 81*B^2*a*b^5*c - 9*A^2*a*b^3*c^3 + 23*A^2*a^2*b*c^4 - 90*B^2*a^3*b*c^3 - 138*A*B*a^2*b^2*c^3 + 54*A*B*a*b^4*c^2)/(16*a^2*c^8 + b^4*c^6 - 8*a*b^2*c^7) + (((b^5*c^8)/2 - 4*a*b^3*c^9 + 8*a^2*b*c^10)*(60*B*a^3*c^3 - 3*B*b^6 + A*b^5*c + 30*B*a*b^4*c - 10*A*a*b^3*c^2 + 30*A*a^2*b*c^3 - 90*B*a^2*b^2*c^2)^2)/(c^8*(4*a*c - b^2)^5*(16*a^2*c^8 + b^4*c^6 - 8*a*b^2*c^7))))/(2*a*(4*a*c - b^2)^(5/2))) + ((((8*A*a*c^5 - 24*B*a*b*c^4)/c^6 - (8*a*c^2*(6*B*b^11 + 2048*A*a^5*c^6 - 2*A*b^10*c - 120*B*a*b^9*c + 40*A*a*b^8*c^2 - 6144*B*a^5*b*c^5 - 320*A*a^2*b^6*c^3 + 1280*A*a^3*b^4*c^4 - 2560*A*a^4*b^2*c^5 + 960*B*a^2*b^7*c^2 - 3840*B*a^3*b^5*c^3 + 7680*B*a^4*b^3*c^4))/(4096*a^5*c^9 - 4*b^10*c^4 + 80*a*b^8*c^5 - 640*a^2*b^6*c^6 + 2560*a^3*b^4*c^7 - 5120*a^4*b^2*c^8))*(60*B*a^3*c^3 - 3*B*b^6 + A*b^5*c + 30*B*a*b^4*c - 10*A*a*b^3*c^2 + 30*A*a^2*b*c^3 - 90*B*a^2*b^2*c^2))/(8*c^4*(4*a*c - b^2)^(5/2)) - (a*(60*B*a^3*c^3 - 3*B*b^6 + A*b^5*c + 30*B*a*b^4*c - 10*A*a*b^3*c^2 + 30*A*a^2*b*c^3 - 90*B*a^2*b^2*c^2)*(6*B*b^11 + 2048*A*a^5*c^6 - 2*A*b^10*c - 120*B*a*b^9*c + 40*A*a*b^8*c^2 - 6144*B*a^5*b*c^5 - 320*A*a^2*b^6*c^3 + 1280*A*a^3*b^4*c^4 - 2560*A*a^4*b^2*c^5 + 960*B*a^2*b^7*c^2 - 3840*B*a^3*b^5*c^3 + 7680*B*a^4*b^3*c^4))/(c^2*(4*a*c - b^2)^(5/2)*(4096*a^5*c^9 - 4*b^10*c^4 + 80*a*b^8*c^5 - 640*a^2*b^6*c^6 + 2560*a^3*b^4*c^7 - 5120*a^4*b^2*c^8)))/(a*(4*a*c - b^2)^2) + (b*((((8*A*a*c^5 - 24*B*a*b*c^4)/c^6 - (8*a*c^2*(6*B*b^11 + 2048*A*a^5*c^6 - 2*A*b^10*c - 120*B*a*b^9*c + 40*A*a*b^8*c^2 - 6144*B*a^5*b*c^5 - 320*A*a^2*b^6*c^3 + 1280*A*a^3*b^4*c^4 - 2560*A*a^4*b^2*c^5 + 960*B*a^2*b^7*c^2 - 3840*B*a^3*b^5*c^3 + 7680*B*a^4*b^3*c^4))/(4096*a^5*c^9 - 4*b^10*c^4 + 80*a*b^8*c^5 - 640*a^2*b^6*c^6 + 2560*a^3*b^4*c^7 - 5120*a^4*b^2*c^8))*(6*B*b^11 + 2048*A*a^5*c^6 - 2*A*b^10*c - 120*B*a*b^9*c + 40*A*a*b^8*c^2 - 6144*B*a^5*b*c^5 - 320*A*a^2*b^6*c^3 + 1280*A*a^3*b^4*c^4 - 2560*A*a^4*b^2*c^5 + 960*B*a^2*b^7*c^2 - 3840*B*a^3*b^5*c^3 + 7680*B*a^4*b^3*c^4))/(2*(4096*a^5*c^9 - 4*b^10*c^4 + 80*a*b^8*c^5 - 640*a^2*b^6*c^6 + 2560*a^3*b^4*c^7 - 5120*a^4*b^2*c^8)) - (A^2*a*c^2 + 9*B^2*a*b^2 - 6*A*B*a*b*c)/c^6 + (a*(60*B*a^3*c^3 - 3*B*b^6 + A*b^5*c + 30*B*a*b^4*c - 10*A*a*b^3*c^2 + 30*A*

```
a^2*b*c^3 - 90*B*a^2*b^2*c^2)^2)/(c^6*(4*a*c - b^2)^5)))/(2*a*(4*a*c - b^2)
^(5/2))))/(9*B^2*b^12 + A^2*b^10*c^2 + 3600*B^2*a^6*c^6 - 6*A*B*b^11*c + 16
0*A^2*a^2*b^6*c^4 - 600*A^2*a^3*b^4*c^5 + 900*A^2*a^4*b^2*c^6 + 1440*B^2*a^
2*b^8*c^2 - 5760*B^2*a^3*b^6*c^3 + 11700*B^2*a^4*b^4*c^4 - 10800*B^2*a^5*b^
2*c^5 - 180*B^2*a*b^10*c - 20*A^2*a*b^8*c^3 - 960*A*B*a^2*b^7*c^3 + 3720*A*
B*a^3*b^5*c^4 - 6600*A*B*a^4*b^3*c^5 + 120*A*B*a*b^9*c^2 + 3600*A*B*a^5*b*c
^6))*(60*B*a^3*c^3 - 3*B*b^6 + A*b^5*c + 30*B*a*b^4*c - 10*A*a*b^3*c^2 + 30
*A*a^2*b*c^3 - 90*B*a^2*b^2*c^2))/(2*c^4*(4*a*c - b^2)^(5/2))
```

3.125 $\int \frac{x^9(A+Bx^2)}{(a+bx^2+cx^4)^3} dx$

Optimal result

Integrand size = 25, antiderivative size = 254

$$\int \frac{x^9(A+Bx^2)}{(a+bx^2+cx^4)^3} dx = -\frac{x^6(a(bB-2Ac)+(b^2B-Abc-2aBc)x^2)}{4c(b^2-4ac)(a+bx^2+cx^4)^2} - \frac{x^2(2a(b^3B-7abBc+6aAc^2)+(2b^4B-15ab^2Bc+6aAbc^2+16a^2Bc^2)x^2)}{4c^2(b^2-4ac)^2(a+bx^2+cx^4)} + \frac{(b^5B-10ab^3Bc+30a^2bBc^2-12a^2Ac^3)\operatorname{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2c^3(b^2-4ac)^{5/2}} + \frac{B\log(a+bx^2+cx^4)}{4c^3}$$

```
[Out] -1/4*x^6*(a*(-2*A*c+B*b)+(-A*b*c-2*B*a*c+B*b^2)*x^2)/c/(-4*a*c+b^2)/(c*x^4+
b*x^2+a)^2-1/4*x^2*(2*a*(6*A*a*c^2-7*B*a*b*c+B*b^3)+(6*A*a*b*c^2+16*B*a^2*c
^2-15*B*a*b^2*c+2*B*b^4)*x^2)/c^2/(-4*a*c+b^2)^2/(c*x^4+b*x^2+a)+1/2*(-12*A
*a^2*c^3+30*B*a^2*b*c^2-10*B*a*b^3*c+B*b^5)*arctanh((2*c*x^2+b)/(-4*a*c+b^2
)^(1/2))/c^3/(-4*a*c+b^2)^(5/2)+1/4*B*ln(c*x^4+b*x^2+a)/c^3
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 254, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used

$= \{1265, 832, 648, 632, 212, 642\}$

$$\int \frac{x^9(A+Bx^2)}{(a+bx^2+cx^4)^3}\,dx$$
$$= \frac{(-12a^2Ac^3+30a^2bBc^2-10ab^3Bc+b^5B)\operatorname{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2c^3(b^2-4ac)^{5/2}} - \frac{x^2(x^2(16a^2Bc^2+6aAbc^2-15ab^2Bc+2b^4B)+2a(6aAc^2-7abBc+b^3B))}{4c^2(b^2-4ac)^2(a+bx^2+cx^4)} - \frac{x^6(x^2(-2aBc-Abc+b^2B)+a(bB-2Ac))}{4c(b^2-4ac)(a+bx^2+cx^4)^2} + \frac{B\log(a+bx^2+cx^4)}{4c^3}$$

```
[In] Int[(x^9*(A + B*x^2))/(a + b*x^2 + c*x^4)^3,x]
```

```
[Out] -1/4*(x^6*(a*(b*B - 2*A*c) + (b^2*B - A*b*c - 2*a*B*c)*x^2))/(c*(b^2 - 4*a*
c)*(a + b*x^2 + c*x^4)^2) - (x^2*(2*a*(b^3*B - 7*a*b*B*c + 6*a*A*c^2) + (2*
b^4*B - 15*a*b^2*B*c + 6*a*A*b*c^2 + 16*a^2*B*c^2)*x^2))/(4*c^2*(b^2 - 4*a*
c)^2*(a + b*x^2 + c*x^4)) + ((b^5*B - 10*a*b^3*B*c + 30*a^2*b*B*c^2 - 12*a^
2*A*c^3)*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(2*c^3*(b^2 - 4*a*c)^(5/
2)) + (B*Log[a + b*x^2 + c*x^4])/(4*c^3)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 832

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(-(d + e*x)^(m - 1))*(a + b*x + c*x^2)
^(p + 1)*((2*a*c*(e*f + d*g) - b*(c*d*f + a*e*g) - (2*c^2*d*f + b^2*e*g - c
*(b*e*f + b*d*g + 2*a*e*g))*x)/(c*(p + 1)*(b^2 - 4*a*c))), x] - Dist[1/(c*(
p + 1)*(b^2 - 4*a*c)), Int[(d + e*x)^(m - 2)*(a + b*x + c*x^2)^(p + 1)*Simp
[2*c^2*d^2*f*(2*p + 3) + b*e*g*(a*e*(m - 1) + b*d*(p + 2)) - c*(2*a*e*(e*f*
(m - 1) + d*g*m) + b*d*(d*g*(2*p + 3) - e*f*(m - 2*p - 4))) + e*(b^2*e*g*(m
 + p + 1) + 2*c^2*d*f*(m + 2*p + 2) - c*(2*a*e*g*m + b*(e*f + d*g)*(m + 2*p
 + 2)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*c
, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 1] && ((EqQ[m
, 2] && EqQ[p, -3] && RationalQ[a, b, c, d, e, f, g]) ||  !ILtQ[m + 2*p + 3
, 0])
```

Rule 1265

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rubi steps

$$\text{integral} = \frac{1}{2}\text{Subst}\left(\int \frac{x^4(A+Bx)}{(a+bx+cx^2)^3}\,dx, x, x^2\right)$$

$$= -\frac{x^6(a(bB-2Ac)+(b^2B-Abc-2aBc)\,x^2)}{4c\,(b^2-4ac)\,(a+bx^2+cx^4)^2} + \frac{\text{Subst}\left(\int \frac{x^2(3a(bB-2Ac)+2B(b^2-4ac)x)}{(a+bx+cx^2)^2}\,dx, x, x^2\right)}{4c\,(b^2-4ac)}$$

$$= -\frac{x^6(a(bB-2Ac)+(b^2B-Abc-2aBc)\,x^2)}{4c\,(b^2-4ac)\,(a+bx^2+cx^4)^2} - \frac{x^2(2a(b^3B-7abBc+6aAc^2)+(2b^4B-15ab^2Bc+6aAbc^2+16a^2Bc^2)\,x^2)}{4c^2\,(b^2-4ac)^2\,(a+bx^2+cx^4)} + \frac{\text{Subst}\left(\int \frac{2a(b^3B-7abBc+6aAc^2)+2B(b^2-4ac)^2x}{a+bx+cx^2}\,dx, x, x^2\right)}{4c^2\,(b^2-4ac)^2}$$

$$
\begin{aligned}
&= -\frac{x^6(a(bB-2Ac)+(b^2B-Abc-2aBc)\,x^2)}{4c\,(b^2-4ac)\,(a+bx^2+cx^4)^2} \\
&\quad - \frac{x^2(2a(b^3B-7abBc+6aAc^2)+(2b^4B-15ab^2Bc+6aAbc^2+16a^2Bc^2)\,x^2)}{4c^2\,(b^2-4ac)^2\,(a+bx^2+cx^4)} \\
&\quad + \frac{B\mathrm{Subst}\left(\int \frac{b+2cx}{a+bx+cx^2}\,dx, x, x^2\right)}{4c^3} \\
&\quad - \frac{(b^5B-10ab^3Bc+30a^2bBc^2-12a^2Ac^3)\,\mathrm{Subst}\left(\int \frac{1}{a+bx+cx^2}\,dx, x, x^2\right)}{4c^3\,(b^2-4ac)^2}
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{x^6(a(bB-2Ac)+(b^2B-Abc-2aBc)\,x^2)}{4c\,(b^2-4ac)\,(a+bx^2+cx^4)^2} \\
&\quad - \frac{x^2(2a(b^3B-7abBc+6aAc^2)+(2b^4B-15ab^2Bc+6aAbc^2+16a^2Bc^2)\,x^2)}{4c^2\,(b^2-4ac)^2\,(a+bx^2+cx^4)} \\
&\quad + \frac{B\log\left(a+bx^2+cx^4\right)}{4c^3} \\
&\quad + \frac{(b^5B-10ab^3Bc+30a^2bBc^2-12a^2Ac^3)\,\mathrm{Subst}\left(\int \frac{1}{b^2-4ac-x^2}\,dx, x, b+2cx^2\right)}{2c^3\,(b^2-4ac)^2}
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{x^6(a(bB-2Ac)+(b^2B-Abc-2aBc)\,x^2)}{4c\,(b^2-4ac)\,(a+bx^2+cx^4)^2} \\
&\quad - \frac{x^2(2a(b^3B-7abBc+6aAc^2)+(2b^4B-15ab^2Bc+6aAbc^2+16a^2Bc^2)\,x^2)}{4c^2\,(b^2-4ac)^2\,(a+bx^2+cx^4)} \\
&\quad + \frac{(b^5B-10ab^3Bc+30a^2bBc^2-12a^2Ac^3)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2c^3\,(b^2-4ac)^{5/2}} \\
&\quad + \frac{B\log\left(a+bx^2+cx^4\right)}{4c^3}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.30 (sec) , antiderivative size = 354, normalized size of antiderivative = 1.39

$$
\int \frac{x^9(A+Bx^2)}{(a+bx^2+cx^4)^3}\,dx
$$

$$
= \frac{\frac{-b^6B+b^5c(A+4Bx^2)-2ab^3c^2(4A+15Bx^2)+2a^2bc^3(11A+25Bx^2)+4a^2c^3(8aB-5Acx^2)+b^4c(11aB-2Acx^2)+ab^2c^2(-39aB+16Acx^2)}{(b^2-4ac)^2(a+bx^2+cx^4)} + \frac{2a^3 I
$$

```
[In] Integrate[(x^9*(A + B*x^2))/(a + b*x^2 + c*x^4)^3,x]
```

```
[Out] ((-(b^6*B) + b^5*c*(A + 4*B*x^2) - 2*a*b^3*c^2*(4*A + 15*B*x^2) + 2*a^2*b*c
^3*(11*A + 25*B*x^2) + 4*a^2*c^3*(8*a*B - 5*A*c*x^2) + b^4*c*(11*a*B - 2*A*
c*x^2) + a*b^2*c^2*(-39*a*B + 16*A*c*x^2))/((b^2 - 4*a*c)^2*(a + b*x^2 + c*
```

```
x^4)) + (2*a^3*B*c^2 + b^4*(b*B - A*c)*x^2 + a*b^2*(b^2*B + 4*A*c^2*x^2 - b
*c*(A + 5*B*x^2)) + a^2*c*(-4*b^2*B - 2*A*c^2*x^2 + b*c*(3*A + 5*B*x^2)))/(
(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)^2) - (2*c*(b^5*B - 10*a*b^3*B*c + 30*a^2*
b*B*c^2 - 12*a^2*A*c^3)*ArcTan[(b + 2*c*x^2)/Sqrt[-b^2 + 4*a*c]])/(-b^2 + 4
*a*c)^(5/2) + B*c*Log[a + b*x^2 + c*x^4])/(4*c^4)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 494 vs. $2(242) = 484$.

Time = 0.23 (sec) , antiderivative size = 495, normalized size of antiderivative = 1.95

method	result
default	$\frac{-\frac{\left(10A\,a^{2}c^{3}-8Aa\,b^{2}c^{2}+A\,b^{4}c-25B\,a^{2}b\,c^{2}+15Ba\,b^{3}c-2B\,b^{5}\right)x^{6}}{c^{2}\left(16a^{2}c^{2}-8a\,b^{2}c+b^{4}\right)}+\frac{\left(2A\,a^{2}b\,c^{3}+8Aa\,b^{3}c^{2}-A\,b^{5}c+32B\,a^{3}c^{3}+11B\,a^{2}b^{2}c^{2}-19Ba\,b^{4}c+3B\,b^{6}\right)x^{4}}{2c^{3}\left(16a^{2}c^{2}-8a\,b^{2}c+b^{4}\right)}-}{2\left(c\,x^{4}+b\,x^{2}+a\right)^{2}}$
risch	Expression too large to display

```
[In] int(x^9*(B*x^2+A)/(c*x^4+b*x^2+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*(-1/c^2*(10*A*a^2*c^3-8*A*a*b^2*c^2+A*b^4*c-25*B*a^2*b*c^2+15*B*a*b^3*c
-2*B*b^5)/(16*a^2*c^2-8*a*b^2*c+b^4)*x^6+1/2*(2*A*a^2*b*c^3+8*A*a*b^3*c^2-A
*b^5*c+32*B*a^3*c^3+11*B*a^2*b^2*c^2-19*B*a*b^4*c+3*B*b^6)/c^3/(16*a^2*c^2-
8*a*b^2*c+b^4)*x^4-a*(6*A*a^2*c^3-10*A*a*b^2*c^2+A*b^4*c-31*B*a^2*b*c^2+22*
B*a*b^3*c-3*B*b^5)/c^3/(16*a^2*c^2-8*a*b^2*c+b^4)*x^2+1/2*a^2*(10*A*a*b*c^2
-A*b^3*c+24*B*a^2*c^2-21*B*a*b^2*c+3*B*b^4)/c^3/(16*a^2*c^2-8*a*b^2*c+b^4))
/(c*x^4+b*x^2+a)^2+1/2/c^2/(16*a^2*c^2-8*a*b^2*c+b^4)*(1/2*(16*B*a^2*c^2-8*
B*a*b^2*c+B*b^4)/c*ln(c*x^4+b*x^2+a)+2*(6*A*a^2*c^2-7*a^2*b*B*c+B*a*b^3-1/2
*(16*B*a^2*c^2-8*B*a*b^2*c+B*b^4)*b/c)/(4*a*c-b^2)^(1/2)*arctan((2*c*x^2+b)
/(4*a*c-b^2)^(1/2)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1071 vs. $2(242) = 484$.

Time = 0.40 (sec) , antiderivative size = 2167, normalized size of antiderivative = 8.53

$$\int \frac{x^9(A+Bx^2)}{(a+bx^2+cx^4)^3}\,dx = \text{Too large to display}$$

```
[In] integrate(x^9*(B*x^2+A)/(c*x^4+b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] [1/4*(3*B*a^2*b^6 + 2*(2*B*b^7*c + 40*A*a^3*c^5 - 2*(50*B*a^3*b + 21*A*a^2*
b^2)*c^4 + (85*B*a^2*b^3 + 12*A*a*b^4)*c^3 - (23*B*a*b^5 + A*b^6)*c^2)*x^6
+ (3*B*b^8 - 8*(16*B*a^4 + A*a^3*b)*c^4 - 6*(2*B*a^3*b^2 + 5*A*a^2*b^3)*c^3
 + 3*(29*B*a^2*b^4 + 4*A*a*b^5)*c^2 - (31*B*a*b^6 + A*b^7)*c)*x^4 - 8*(12*B
```

```
*a^5 + 5*A*a^4*b)*c^3 + 2*(54*B*a^4*b^2 + 7*A*a^3*b^3)*c^2 + 2*(3*B*a*b^7 +
 24*A*a^4*c^4 - 2*(62*B*a^4*b + 23*A*a^3*b^2)*c^3 + 7*(17*B*a^3*b^3 + 2*A*a
^2*b^4)*c^2 - (34*B*a^2*b^5 + A*a*b^6)*c)*x^2 - ((B*b^5*c^2 - 10*B*a*b^3*c^
3 + 30*B*a^2*b*c^4 - 12*A*a^2*c^5)*x^8 + B*a^2*b^5 - 10*B*a^3*b^3*c + 30*B*
a^4*b*c^2 - 12*A*a^4*c^3 + 2*(B*b^6*c - 10*B*a*b^4*c^2 + 30*B*a^2*b^2*c^3 -
 12*A*a^2*b*c^4)*x^6 + (B*b^7 - 8*B*a*b^5*c + 10*B*a^2*b^3*c^2 - 24*A*a^3*c
^4 + 12*(5*B*a^3*b - A*a^2*b^2)*c^3)*x^4 + 2*(B*a*b^6 - 10*B*a^2*b^4*c + 30
*B*a^3*b^2*c^2 - 12*A*a^3*b*c^3)*x^2)*sqrt(b^2 - 4*a*c)*log((2*c^2*x^4 + 2*
b*c*x^2 + b^2 - 2*a*c - (2*c*x^2 + b)*sqrt(b^2 - 4*a*c))/(c*x^4 + b*x^2 + a
)) - (33*B*a^3*b^4 + A*a^2*b^5)*c + (B*a^2*b^6 - 12*B*a^3*b^4*c + 48*B*a^4*
b^2*c^2 - 64*B*a^5*c^3 + (B*b^6*c^2 - 12*B*a*b^4*c^3 + 48*B*a^2*b^2*c^4 - 6
4*B*a^3*c^5)*x^8 + 2*(B*b^7*c - 12*B*a*b^5*c^2 + 48*B*a^2*b^3*c^3 - 64*B*a^
3*b*c^4)*x^6 + (B*b^8 - 10*B*a*b^6*c + 24*B*a^2*b^4*c^2 + 32*B*a^3*b^2*c^3
- 128*B*a^4*c^4)*x^4 + 2*(B*a*b^7 - 12*B*a^2*b^5*c + 48*B*a^3*b^3*c^2 - 64*
B*a^4*b*c^3)*x^2)*log(c*x^4 + b*x^2 + a))/(a^2*b^6*c^3 - 12*a^3*b^4*c^4 + 4
8*a^4*b^2*c^5 - 64*a^5*c^6 + (b^6*c^5 - 12*a*b^4*c^6 + 48*a^2*b^2*c^7 - 64*
a^3*c^8)*x^8 + 2*(b^7*c^4 - 12*a*b^5*c^5 + 48*a^2*b^3*c^6 - 64*a^3*b*c^7)*x
^6 + (b^8*c^3 - 10*a*b^6*c^4 + 24*a^2*b^4*c^5 + 32*a^3*b^2*c^6 - 128*a^4*c^
7)*x^4 + 2*(a*b^7*c^3 - 12*a^2*b^5*c^4 + 48*a^3*b^3*c^5 - 64*a^4*b*c^6)*x^2
), 1/4*(3*B*a^2*b^6 + 2*(2*B*b^7*c + 40*A*a^3*c^5 - 2*(50*B*a^3*b + 21*A*a^
2*b^2)*c^4 + (85*B*a^2*b^3 + 12*A*a*b^4)*c^3 - (23*B*a*b^5 + A*b^6)*c^2)*x^
6 + (3*B*b^8 - 8*(16*B*a^4 + A*a^3*b)*c^4 - 6*(2*B*a^3*b^2 + 5*A*a^2*b^3)*c
^3 + 3*(29*B*a^2*b^4 + 4*A*a*b^5)*c^2 - (31*B*a*b^6 + A*b^7)*c)*x^4 - 8*(12
*B*a^5 + 5*A*a^4*b)*c^3 + 2*(54*B*a^4*b^2 + 7*A*a^3*b^3)*c^2 + 2*(3*B*a*b^7
 + 24*A*a^4*c^4 - 2*(62*B*a^4*b + 23*A*a^3*b^2)*c^3 + 7*(17*B*a^3*b^3 + 2*A
*a^2*b^4)*c^2 - (34*B*a^2*b^5 + A*a*b^6)*c)*x^2 + 2*((B*b^5*c^2 - 10*B*a*b^
3*c^3 + 30*B*a^2*b*c^4 - 12*A*a^2*c^5)*x^8 + B*a^2*b^5 - 10*B*a^3*b^3*c + 3
0*B*a^4*b*c^2 - 12*A*a^4*c^3 + 2*(B*b^6*c - 10*B*a*b^4*c^2 + 30*B*a^2*b^2*c
^3 - 12*A*a^2*b*c^4)*x^6 + (B*b^7 - 8*B*a*b^5*c + 10*B*a^2*b^3*c^2 - 24*A*a
^3*c^4 + 12*(5*B*a^3*b - A*a^2*b^2)*c^3)*x^4 + 2*(B*a*b^6 - 10*B*a^2*b^4*c
+ 30*B*a^3*b^2*c^2 - 12*A*a^3*b*c^3)*x^2)*sqrt(-b^2 + 4*a*c)*arctan(-(2*c*x
^2 + b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4*a*c)) - (33*B*a^3*b^4 + A*a^2*b^5)*c +
(B*a^2*b^6 - 12*B*a^3*b^4*c + 48*B*a^4*b^2*c^2 - 64*B*a^5*c^3 + (B*b^6*c^2
- 12*B*a*b^4*c^3 + 48*B*a^2*b^2*c^4 - 64*B*a^3*c^5)*x^8 + 2*(B*b^7*c - 12*B
*a*b^5*c^2 + 48*B*a^2*b^3*c^3 - 64*B*a^3*b*c^4)*x^6 + (B*b^8 - 10*B*a*b^6*c
 + 24*B*a^2*b^4*c^2 + 32*B*a^3*b^2*c^3 - 128*B*a^4*c^4)*x^4 + 2*(B*a*b^7 -
12*B*a^2*b^5*c + 48*B*a^3*b^3*c^2 - 64*B*a^4*b*c^3)*x^2)*log(c*x^4 + b*x^2
+ a))/(a^2*b^6*c^3 - 12*a^3*b^4*c^4 + 48*a^4*b^2*c^5 - 64*a^5*c^6 + (b^6*c^
5 - 12*a*b^4*c^6 + 48*a^2*b^2*c^7 - 64*a^3*c^8)*x^8 + 2*(b^7*c^4 - 12*a*b^5
*c^5 + 48*a^2*b^3*c^6 - 64*a^3*b*c^7)*x^6 + (b^8*c^3 - 10*a*b^6*c^4 + 24*a^
2*b^4*c^5 + 32*a^3*b^2*c^6 - 128*a^4*c^7)*x^4 + 2*(a*b^7*c^3 - 12*a^2*b^5*c
^4 + 48*a^3*b^3*c^5 - 64*a^4*b*c^6)*x^2)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^9(A+Bx^2)}{(a+bx^2+cx^4)^3}\,dx = \text{Timed out}$$

```
[In] integrate(x**9*(B*x**2+A)/(c*x**4+b*x**2+a)**3,x)
[Out] Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^9(A+Bx^2)}{(a+bx^2+cx^4)^3}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(x^9*(B*x^2+A)/(c*x^4+b*x^2+a)^3,x, algorithm="maxima")
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

Giac [A] (verification not implemented)

none

Time = 1.52 (sec) , antiderivative size = 466, normalized size of antiderivative = 1.83

$$\int \frac{x^9(A+Bx^2)}{(a+bx^2+cx^4)^3}\,dx = -\frac{(Bb^5-10\,Bab^3c+30\,Ba^2bc^2-12\,Aa^2c^3)\arctan\left(\frac{2\,cx^2+b}{\sqrt{-b^2+4\,ac}}\right)}{2\,(b^4c^3-8\,ab^2c^4+16\,a^2c^5)\sqrt{-b^2+4\,ac}}+\frac{B\log\left(cx^4+bx^2+a\right)}{4\,c^3} - \frac{3\,Bb^4c^2x^8-24\,Bab^2c^3x^8+48\,Ba^2c^4x^8-2\,Bb^5cx^6+12\,Bab^3c^2x^6+4\,Ab^4c^2x^6-4\,Ba^2bc^3x^6-32\,Aab^2}{}$$

```
[In] integrate(x^9*(B*x^2+A)/(c*x^4+b*x^2+a)^3,x, algorithm="giac")
[Out] -1/2*(B*b^5 - 10*B*a*b^3*c + 30*B*a^2*b*c^2 - 12*A*a^2*c^3)*arctan((2*c*x^2
 + b)/sqrt(-b^2 + 4*a*c))/((b^4*c^3 - 8*a*b^2*c^4 + 16*a^2*c^5)*sqrt(-b^2 +
 4*a*c)) + 1/4*B*log(c*x^4 + b*x^2 + a)/c^3 - 1/8*(3*B*b^4*c^2*x^8 - 24*B*a
*b^2*c^3*x^8 + 48*B*a^2*c^4*x^8 - 2*B*b^5*c*x^6 + 12*B*a*b^3*c^2*x^6 + 4*A*
b^4*c^2*x^6 - 4*B*a^2*b*c^3*x^6 - 32*A*a*b^2*c^3*x^6 + 40*A*a^2*c^4*x^6 - 3
```

```
*B*b^6*x^4 + 20*B*a*b^4*c*x^4 + 2*A*b^5*c*x^4 - 22*B*a^2*b^2*c^2*x^4 - 16*A
*a*b^3*c^2*x^4 + 32*B*a^3*c^3*x^4 - 4*A*a^2*b*c^3*x^4 - 6*B*a*b^5*x^2 + 40*
B*a^2*b^3*c*x^2 + 4*A*a*b^4*c*x^2 - 28*B*a^3*b*c^2*x^2 - 40*A*a^2*b^2*c^2*x
^2 + 24*A*a^3*c^3*x^2 - 3*B*a^2*b^4 + 18*B*a^3*b^2*c + 2*A*a^2*b^3*c - 20*A
*a^3*b*c^2)/((b^4*c^3 - 8*a*b^2*c^4 + 16*a^2*c^5)*(c*x^4 + b*x^2 + a)^2)
```

Mupad [B] (verification not implemented)

Time = 10.74 (sec) , antiderivative size = 3062, normalized size of antiderivative = 12.06

$$\int \frac{x^9(A+Bx^2)}{(a+bx^2+cx^4)^3}\,dx = \text{Too large to display}$$

```
[In] int((x^9*(A + B*x^2))/(a + b*x^2 + c*x^4)^3,x)
```

```
[Out] ((x^4*(3*B*b^6 + 32*B*a^3*c^3 - A*b^5*c - 19*B*a*b^4*c + 8*A*a*b^3*c^2 + 2*
A*a^2*b*c^3 + 11*B*a^2*b^2*c^2))/(4*c^3*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (
x^6*(2*B*b^5 - 10*A*a^2*c^3 - A*b^4*c - 15*B*a*b^3*c + 8*A*a*b^2*c^2 + 25*B
*a^2*b*c^2))/(2*c^2*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (a*(24*B*a^3*c^2 + 3*
B*a*b^4 - A*a*b^3*c + 10*A*a^2*b*c^2 - 21*B*a^2*b^2*c))/(4*c^3*(b^4 + 16*a^
2*c^2 - 8*a*b^2*c)) - (x^2*(6*A*a^3*c^3 - 3*B*a*b^5 + A*a*b^4*c + 22*B*a^2*
b^3*c - 31*B*a^3*b*c^2 - 10*A*a^2*b^2*c^2))/(2*c^3*(b^4 + 16*a^2*c^2 - 8*a*
b^2*c)))/(x^4*(2*a*c + b^2) + a^2 + c^2*x^8 + 2*a*b*x^2 + 2*b*c*x^6) - (log
(((B^2*a)/c^4 - ((B + c^3*(-(B*b^5 - 12*A*a^2*c^3 - 10*B*a*b^3*c + 30*B*a^2
*b*c^2)^2/(c^6*(4*a*c - b^2)^5))^(1/2))*((8*B*a)/c - (2*(B + c^3*(-(B*b^5 -
 12*A*a^2*c^3 - 10*B*a*b^3*c + 30*B*a^2*b*c^2)^2/(c^6*(4*a*c - b^2)^5))^(1/
2))*(2*a + b*x^2))/c + (2*x^2*(3*B*b^5 - 12*A*a^2*c^3 - 26*B*a*b^3*c + 62*B
*a^2*b*c^2))/(c*(4*a*c - b^2)^2)))/(4*c^3) + (B*x^2*(B*b^5 - 6*A*a^2*c^3 -
9*B*a*b^3*c + 23*B*a^2*b*c^2))/(c^4*(4*a*c - b^2)^2))*((B^2*a)/c^4 - ((B -
c^3*(-(B*b^5 - 12*A*a^2*c^3 - 10*B*a*b^3*c + 30*B*a^2*b*c^2)^2/(c^6*(4*a*c
- b^2)^5))^(1/2))*((8*B*a)/c - (2*(B - c^3*(-(B*b^5 - 12*A*a^2*c^3 - 10*B*a
*b^3*c + 30*B*a^2*b*c^2)^2/(c^6*(4*a*c - b^2)^5))^(1/2))*(2*a + b*x^2))/c +
 (2*x^2*(3*B*b^5 - 12*A*a^2*c^3 - 26*B*a*b^3*c + 62*B*a^2*b*c^2))/(c*(4*a*c
 - b^2)^2)))/(4*c^3) + (B*x^2*(B*b^5 - 6*A*a^2*c^3 - 9*B*a*b^3*c + 23*B*a^2
*b*c^2))/(c^4*(4*a*c - b^2)^2)))*(2*B*b^10 - 2048*B*a^5*c^5 - 40*B*a*b^8*c
+ 320*B*a^2*b^6*c^2 - 1280*B*a^3*b^4*c^3 + 2560*B*a^4*b^2*c^4))/(2*(4096*a^
5*c^8 - 4*b^10*c^3 + 80*a*b^8*c^4 - 640*a^2*b^6*c^5 + 2560*a^3*b^4*c^6 - 51
20*a^4*b^2*c^7)) + (atan(((32*a^2*c^6*(4*a*c - b^2)^5 + 2*b^4*c^4*(4*a*c -
b^2)^5 - 16*a*b^2*c^5*(4*a*c - b^2)^5)*(x^2*(((((24*A*a^2*c^6 - 6*B*b^5*c^3
 + 52*B*a*b^3*c^4 - 124*B*a^2*b*c^5)/(16*a^2*c^6 + b^4*c^4 - 8*a*b^2*c^5) -
 ((8*b^5*c^6 - 64*a*b^3*c^7 + 128*a^2*b*c^8)*(2*B*b^10 - 2048*B*a^5*c^5 - 4
0*B*a*b^8*c + 320*B*a^2*b^6*c^2 - 1280*B*a^3*b^4*c^3 + 2560*B*a^4*b^2*c^4))
/(2*(16*a^2*c^6 + b^4*c^4 - 8*a*b^2*c^5)*(4096*a^5*c^8 - 4*b^10*c^3 + 80*a*
b^8*c^4 - 640*a^2*b^6*c^5 + 2560*a^3*b^4*c^6 - 5120*a^4*b^2*c^7)))*(B*b^5 -
 12*A*a^2*c^3 - 10*B*a*b^3*c + 30*B*a^2*b*c^2))/(8*c^3*(4*a*c - b^2)^(5/2))
```

```
 - ((8*b^5*c^6 - 64*a*b^3*c^7 + 128*a^2*b*c^8)*(B*b^5 - 12*A*a^2*c^3 - 10*B
*a*b^3*c + 30*B*a^2*b*c^2)*(2*B*b^10 - 2048*B*a^5*c^5 - 40*B*a*b^8*c + 320*
B*a^2*b^6*c^2 - 1280*B*a^3*b^4*c^3 + 2560*B*a^4*b^2*c^4))/(16*c^3*(4*a*c -
b^2)^(5/2)*(16*a^2*c^6 + b^4*c^4 - 8*a*b^2*c^5)*(4096*a^5*c^8 - 4*b^10*c^3
+ 80*a*b^8*c^4 - 640*a^2*b^6*c^5 + 2560*a^3*b^4*c^6 - 5120*a^4*b^2*c^7)))/(
a*(4*a*c - b^2)^2) - (b*((((24*A*a^2*c^6 - 6*B*b^5*c^3 + 52*B*a*b^3*c^4 - 1
24*B*a^2*b*c^5)/(16*a^2*c^6 + b^4*c^4 - 8*a*b^2*c^5) - ((8*b^5*c^6 - 64*a*b
^3*c^7 + 128*a^2*b*c^8)*(2*B*b^10 - 2048*B*a^5*c^5 - 40*B*a*b^8*c + 320*B*a
^2*b^6*c^2 - 1280*B*a^3*b^4*c^3 + 2560*B*a^4*b^2*c^4))/(2*(16*a^2*c^6 + b^4
*c^4 - 8*a*b^2*c^5)*(4096*a^5*c^8 - 4*b^10*c^3 + 80*a*b^8*c^4 - 640*a^2*b^6
*c^5 + 2560*a^3*b^4*c^6 - 5120*a^4*b^2*c^7)))*(2*B*b^10 - 2048*B*a^5*c^5 -
40*B*a*b^8*c + 320*B*a^2*b^6*c^2 - 1280*B*a^3*b^4*c^3 + 2560*B*a^4*b^2*c^4)
)/(2*(4096*a^5*c^8 - 4*b^10*c^3 + 80*a*b^8*c^4 - 640*a^2*b^6*c^5 + 2560*a^3
*b^4*c^6 - 5120*a^4*b^2*c^7)) - (B^2*b^5 - 6*A*B*a^2*c^3 - 9*B^2*a*b^3*c +
23*B^2*a^2*b*c^2)/(16*a^2*c^6 + b^4*c^4 - 8*a*b^2*c^5) + (((b^5*c^6)/2 - 4*
a*b^3*c^7 + 8*a^2*b*c^8)*(B*b^5 - 12*A*a^2*c^3 - 10*B*a*b^3*c + 30*B*a^2*b*
c^2)^2)/(c^6*(4*a*c - b^2)^5*(16*a^2*c^6 + b^4*c^4 - 8*a*b^2*c^5))))/(2*a*(
4*a*c - b^2)^(5/2))) - ((((8*B*a)/c + (8*a*c^2*(2*B*b^10 - 2048*B*a^5*c^5 -
 40*B*a*b^8*c + 320*B*a^2*b^6*c^2 - 1280*B*a^3*b^4*c^3 + 2560*B*a^4*b^2*c^4
))/(4096*a^5*c^8 - 4*b^10*c^3 + 80*a*b^8*c^4 - 640*a^2*b^6*c^5 + 2560*a^3*b
^4*c^6 - 5120*a^4*b^2*c^7))*(B*b^5 - 12*A*a^2*c^3 - 10*B*a*b^3*c + 30*B*a^2
*b*c^2))/(8*c^3*(4*a*c - b^2)^(5/2)) + (a*(B*b^5 - 12*A*a^2*c^3 - 10*B*a*b^
3*c + 30*B*a^2*b*c^2)*(2*B*b^10 - 2048*B*a^5*c^5 - 40*B*a*b^8*c + 320*B*a^2
*b^6*c^2 - 1280*B*a^3*b^4*c^3 + 2560*B*a^4*b^2*c^4))/(c*(4*a*c - b^2)^(5/2)
*(4096*a^5*c^8 - 4*b^10*c^3 + 80*a*b^8*c^4 - 640*a^2*b^6*c^5 + 2560*a^3*b^4
*c^6 - 5120*a^4*b^2*c^7)))/(a*(4*a*c - b^2)^2) + (b*((B^2*a)/c^4 + (((8*B*a
)/c + (8*a*c^2*(2*B*b^10 - 2048*B*a^5*c^5 - 40*B*a*b^8*c + 320*B*a^2*b^6*c^
2 - 1280*B*a^3*b^4*c^3 + 2560*B*a^4*b^2*c^4))/(4096*a^5*c^8 - 4*b^10*c^3 +
80*a*b^8*c^4 - 640*a^2*b^6*c^5 + 2560*a^3*b^4*c^6 - 5120*a^4*b^2*c^7))*(2*B
*b^10 - 2048*B*a^5*c^5 - 40*B*a*b^8*c + 320*B*a^2*b^6*c^2 - 1280*B*a^3*b^4*
c^3 + 2560*B*a^4*b^2*c^4))/(2*(4096*a^5*c^8 - 4*b^10*c^3 + 80*a*b^8*c^4 - 6
40*a^2*b^6*c^5 + 2560*a^3*b^4*c^6 - 5120*a^4*b^2*c^7)) - (a*(B*b^5 - 12*A*a
^2*c^3 - 10*B*a*b^3*c + 30*B*a^2*b*c^2)^2)/(c^4*(4*a*c - b^2)^5)))/(2*a*(4*
a*c - b^2)^(5/2))))/(B^2*b^10 + 144*A^2*a^4*c^6 + 160*B^2*a^2*b^6*c^2 - 600
*B^2*a^3*b^4*c^3 + 900*B^2*a^4*b^2*c^4 - 20*B^2*a*b^8*c - 24*A*B*a^2*b^5*c^
3 + 240*A*B*a^3*b^3*c^4 - 720*A*B*a^4*b*c^5))*(B*b^5 - 12*A*a^2*c^3 - 10*B*
a*b^3*c + 30*B*a^2*b*c^2))/(2*c^3*(4*a*c - b^2)^(5/2))
```

3.126 $\int \frac{x^7(A+Bx^2)}{(a+bx^2+cx^4)^3}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 146

$$\int \frac{x^7(A+Bx^2)}{(a+bx^2+cx^4)^3}\,dx = -\frac{x^6(Ab-2aB-(bB-2Ac)x^2)}{4(b^2-4ac)(a+bx^2+cx^4)^2}+\frac{3(Ab-2aB)x^2(2a+bx^2)}{4(b^2-4ac)^2(a+bx^2+cx^4)} + \frac{3a(Ab-2aB)\operatorname{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{(b^2-4ac)^{5/2}}$$

```
[Out] -1/4*x^6*(A*b-2*B*a-(-2*A*c+B*b)*x^2)/(-4*a*c+b^2)/(c*x^4+b*x^2+a)^2+3/4*(A
*b-2*B*a)*x^2*(b*x^2+2*a)/(-4*a*c+b^2)^2/(c*x^4+b*x^2+a)+3*a*(A*b-2*B*a)*ar
ctanh((2*c*x^2+b)/(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(5/2)
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {1265, 818, 736, 632, 212}

$$\int \frac{x^7(A+Bx^2)}{(a+bx^2+cx^4)^3}\,dx = \frac{3a(Ab-2aB)\operatorname{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{(b^2-4ac)^{5/2}}+\frac{3x^2(2a+bx^2)(Ab-2aB)}{4(b^2-4ac)^2(a+bx^2+cx^4)} - \frac{x^6(-2aB-(x^2(bB-2Ac))+Ab)}{4(b^2-4ac)(a+bx^2+cx^4)^2}$$

```
[In] Int[(x^7*(A + B*x^2))/(a + b*x^2 + c*x^4)^3,x]
```

```
[Out] -1/4*(x^6*(A*b - 2*a*B - (b*B - 2*A*c)*x^2))/((b^2 - 4*a*c)*(a + b*x^2 + c*
x^4)^2) + (3*(A*b - 2*a*B)*x^2*(2*a + b*x^2))/(4*(b^2 - 4*a*c)^2*(a + b*x^2
```

```
 + c*x^4)) + (3*a*(A*b - 2*a*B)*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(
b^2 - 4*a*c)^(5/2)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 736

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m - 1)*(d*b - 2*a*e + (2*c*d - b*e)*x)*((a + b*x
+ c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] - Dist[2*(2*p + 3)*((c*d^2 -
b*d*e + a*e^2)/((p + 1)*(b^2 - 4*a*c))), Int[(d + e*x)^(m - 2)*(a + b*x + c
*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] &
& NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && EqQ[m + 2*p + 2,
0] && LtQ[p, -1]
```

Rule 818

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d + e*x)^m*(a + b*x + c*x^2)^(p + 1)*(
(b*f - 2*a*g + (2*c*f - b*g)*x)/((p + 1)*(b^2 - 4*a*c))), x] - Dist[m*((b*(
e*f + d*g) - 2*(c*d*f + a*e*g))/((p + 1)*(b^2 - 4*a*c))), Int[(d + e*x)^(m
- 1)*(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] &
& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[Simplify[m +
2*p + 3], 0] && LtQ[p, -1]
```

Rule 1265

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rubi steps

$$\text{integral} = \frac{1}{2}\text{Subst}\left(\int \frac{x^3(A+Bx)}{(a+bx+cx^2)^3}\,dx, x, x^2\right)$$

$$= -\frac{x^6(Ab - 2aB - (bB - 2Ac)x^2)}{4(b^2 - 4ac)(a + bx^2 + cx^4)^2} + \frac{(3(Ab - 2aB))\text{Subst}\left(\int \frac{x^2}{(a+bx+cx^2)^2}\,dx, x, x^2\right)}{4(b^2 - 4ac)}$$

$$= -\frac{x^6(Ab - 2aB - (bB - 2Ac)x^2)}{4(b^2 - 4ac)(a + bx^2 + cx^4)^2} + \frac{3(Ab - 2aB)x^2(2a + bx^2)}{4(b^2 - 4ac)^2(a + bx^2 + cx^4)} - \frac{(3a(Ab - 2aB))\text{Subst}\left(\int \frac{1}{a+bx+cx^2}\,dx, x, x^2\right)}{2(b^2 - 4ac)^2}$$

$$= -\frac{x^6(Ab - 2aB - (bB - 2Ac)x^2)}{4(b^2 - 4ac)(a + bx^2 + cx^4)^2} + \frac{3(Ab - 2aB)x^2(2a + bx^2)}{4(b^2 - 4ac)^2(a + bx^2 + cx^4)} + \frac{(3a(Ab - 2aB))\text{Subst}\left(\int \frac{1}{b^2-4ac-x^2}\,dx, x, b + 2cx^2\right)}{(b^2 - 4ac)^2}$$

$$= -\frac{x^6(Ab - 2aB - (bB - 2Ac)x^2)}{4(b^2 - 4ac)(a + bx^2 + cx^4)^2} + \frac{3(Ab - 2aB)x^2(2a + bx^2)}{4(b^2 - 4ac)^2(a + bx^2 + cx^4)} + \frac{3a(Ab - 2aB)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{(b^2 - 4ac)^{5/2}}$$

Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 261, normalized size of antiderivative = 1.79

$$\int \frac{x^7(A + Bx^2)}{(a + bx^2 + cx^4)^3}\,dx$$

$$= \frac{1}{4}\left(\frac{b^5B - 8ab^3Bc - b^4c(A + 2Bx^2) - 4a^2c^3(4A + 5Bx^2) + ab^2c^2(5A + 16Bx^2) + 2abc^2(11aB - 3Acx^2)}{c^3(b^2 - 4ac)^2(a + bx^2 + cx^4)} + \frac{b^3(bB - Ac)x^2 + a^2c(-3bB + 2c(A + Bx^2)) + ab(b^2B + 3Ac^2x^2 - bc(A + 4Bx^2))}{c^3(-b^2 + 4ac)(a + bx^2 + cx^4)^2} - \frac{12a(Ab - 2aB)\arctan\left(\frac{b+2cx^2}{\sqrt{-b^2+4ac}}\right)}{(-b^2 + 4ac)^{5/2}}\right)$$

```
[In] Integrate[(x^7*(A + B*x^2))/(a + b*x^2 + c*x^4)^3,x]
```

```
[Out] ((b^5*B - 8*a*b^3*B*c - b^4*c*(A + 2*B*x^2) - 4*a^2*c^3*(4*A + 5*B*x^2) + a
*b^2*c^2*(5*A + 16*B*x^2) + 2*a*b*c^2*(11*a*B - 3*A*c*x^2))/(c^3*(b^2 - 4*a
*c)^2*(a + b*x^2 + c*x^4)) + (b^3*(b*B - A*c)*x^2 + a^2*c*(-3*b*B + 2*c*(A
+ B*x^2)) + a*b*(b^2*B + 3*A*c^2*x^2 - b*c*(A + 4*B*x^2)))/(c^3*(-b^2 + 4*a
*c)*(a + b*x^2 + c*x^4)^2) - (12*a*(A*b - 2*a*B)*ArcTan[(b + 2*c*x^2)/Sqrt[
-b^2 + 4*a*c]])/(-b^2 + 4*a*c)^(5/2))/4
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 342 vs. $2(138) = 276$.

Time = 0.15 (sec) , antiderivative size = 343, normalized size of antiderivative = 2.35

method	result
default	$\frac{-\frac{\left(3Aab\,c^{2}+10B\,a^{2}c^{2}-8Ba\,b^{2}c+B\,b^{4}\right)x^{6}}{c\left(16a^{2}c^{2}-8a\,b^{2}c+b^{4}\right)}-\frac{\left(16A\,a^{2}c^{3}+Aa\,b^{2}c^{2}+A\,b^{4}c-2B\,a^{2}b\,c^{2}-8Ba\,b^{3}c+B\,b^{5}\right)x^{4}}{2\left(16a^{2}c^{2}-8a\,b^{2}c+b^{4}\right)c^{2}}-\frac{a\left(5Aab\,c^{2}+A\,b^{3}c+6B\,a^{2}c^{2}-10Ba\,b^{2}c+\ldots\right.}{\left(16a^{2}c^{2}-8a\,b^{2}c+b^{4}\right)c^{2}}}{2\left(c\,x^{4}+b\,x^{2}+a\right)^{2}}$
risch	$\frac{-\frac{\left(3Aab\,c^{2}+10B\,a^{2}c^{2}-8Ba\,b^{2}c+B\,b^{4}\right)x^{6}}{2c\left(16a^{2}c^{2}-8a\,b^{2}c+b^{4}\right)}-\frac{\left(16A\,a^{2}c^{3}+Aa\,b^{2}c^{2}+A\,b^{4}c-2B\,a^{2}b\,c^{2}-8Ba\,b^{3}c+B\,b^{5}\right)x^{4}}{4\left(16a^{2}c^{2}-8a\,b^{2}c+b^{4}\right)c^{2}}-\frac{a\left(5Aab\,c^{2}+A\,b^{3}c+6B\,a^{2}c^{2}-10Ba\,b^{2}c+\ldots\right.}{2\left(16a^{2}c^{2}-8a\,b^{2}c+b^{4}\right)c^{2}}}{\left(c\,x^{4}+b\,x^{2}+a\right)^{2}}$

```
[In] int(x^7*(B*x^2+A)/(c*x^4+b*x^2+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*(-(3*A*a*b*c^2+10*B*a^2*c^2-8*B*a*b^2*c+B*b^4)/c/(16*a^2*c^2-8*a*b^2*c+
b^4)*x^6-1/2*(16*A*a^2*c^3+A*a*b^2*c^2+A*b^4*c-2*B*a^2*b*c^2-8*B*a*b^3*c+B*
b^5)/(16*a^2*c^2-8*a*b^2*c+b^4)/c^2*x^4-a*(5*A*a*b*c^2+A*b^3*c+6*B*a^2*c^2-
10*B*a*b^2*c+B*b^4)/(16*a^2*c^2-8*a*b^2*c+b^4)/c^2*x^2-1/2*a^2/c^2*(8*A*a*c
^2+A*b^2*c-10*B*a*b*c+B*b^3)/(16*a^2*c^2-8*a*b^2*c+b^4))/(c*x^4+b*x^2+a)^2-
3*a*(A*b-2*B*a)/(16*a^2*c^2-8*a*b^2*c+b^4)/(4*a*c-b^2)^(1/2)*arctan((2*c*x^
2+b)/(4*a*c-b^2)^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 677 vs. $2(140) = 280$.

Time = 0.30 (sec) , antiderivative size = 1378, normalized size of antiderivative = 9.44

$$\int \frac{x^7(A+Bx^2)}{\left(a+bx^2+cx^4\right)^3}\,dx = \text{Too large to display}$$

```
[In] integrate(x^7*(B*x^2+A)/(c*x^4+b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] [-1/4*(B*a^2*b^5 - 32*A*a^4*c^3 + 2*(B*b^6*c - 12*B*a*b^4*c^2 - 4*(10*B*a^3
 + 3*A*a^2*b)*c^4 + 3*(14*B*a^2*b^2 + A*a*b^3)*c^3)*x^6 + (B*b^7 - 64*A*a^3
*c^4 + 4*(2*B*a^3*b + 3*A*a^2*b^2)*c^3 + 3*(10*B*a^2*b^3 - A*a*b^4)*c^2 - (
12*B*a*b^5 - A*b^6)*c)*x^4 + 4*(10*B*a^4*b + A*a^3*b^2)*c^2 + 2*(B*a*b^6 -
4*(6*B*a^4 + 5*A*a^3*b)*c^3 + (46*B*a^3*b^2 + A*a^2*b^3)*c^2 - (14*B*a^2*b^
4 - A*a*b^5)*c)*x^2 + 6*((2*B*a^2 - A*a*b)*c^4*x^8 + 2*(2*B*a^2*b - A*a*b^2
)*c^3*x^6 + 2*(2*B*a^3*b - A*a^2*b^2)*c^2*x^2 + (2*(2*B*a^3 - A*a^2*b)*c^3
+ (2*B*a^2*b^2 - A*a*b^3)*c^2)*x^4 + (2*B*a^4 - A*a^3*b)*c^2)*sqrt(b^2 - 4*
a*c)*log((2*c^2*x^4 + 2*b*c*x^2 + b^2 - 2*a*c + (2*c*x^2 + b)*sqrt(b^2 - 4*
a*c))/(c*x^4 + b*x^2 + a)) - (14*B*a^3*b^3 - A*a^2*b^4)*c)/(a^2*b^6*c^2 - 1
2*a^3*b^4*c^3 + 48*a^4*b^2*c^4 - 64*a^5*c^5 + (b^6*c^4 - 12*a*b^4*c^5 + 48*
a^2*b^2*c^6 - 64*a^3*c^7)*x^8 + 2*(b^7*c^3 - 12*a*b^5*c^4 + 48*a^2*b^3*c^5
```

```
- 64*a^3*b*c^6)*x^6 + (b^8*c^2 - 10*a*b^6*c^3 + 24*a^2*b^4*c^4 + 32*a^3*b^2
*c^5 - 128*a^4*c^6)*x^4 + 2*(a*b^7*c^2 - 12*a^2*b^5*c^3 + 48*a^3*b^3*c^4 -
64*a^4*b*c^5)*x^2), -1/4*(B*a^2*b^5 - 32*A*a^4*c^3 + 2*(B*b^6*c - 12*B*a*b^
4*c^2 - 4*(10*B*a^3 + 3*A*a^2*b)*c^4 + 3*(14*B*a^2*b^2 + A*a*b^3)*c^3)*x^6
+ (B*b^7 - 64*A*a^3*c^4 + 4*(2*B*a^3*b + 3*A*a^2*b^2)*c^3 + 3*(10*B*a^2*b^3
 - A*a*b^4)*c^2 - (12*B*a*b^5 - A*b^6)*c)*x^4 + 4*(10*B*a^4*b + A*a^3*b^2)*
c^2 + 2*(B*a*b^6 - 4*(6*B*a^4 + 5*A*a^3*b)*c^3 + (46*B*a^3*b^2 + A*a^2*b^3)
*c^2 - (14*B*a^2*b^4 - A*a*b^5)*c)*x^2 + 12*((2*B*a^2 - A*a*b)*c^4*x^8 + 2*
(2*B*a^2*b - A*a*b^2)*c^3*x^6 + 2*(2*B*a^3*b - A*a^2*b^2)*c^2*x^2 + (2*(2*B
*a^3 - A*a^2*b)*c^3 + (2*B*a^2*b^2 - A*a*b^3)*c^2)*x^4 + (2*B*a^4 - A*a^3*b
)*c^2)*sqrt(-b^2 + 4*a*c)*arctan(-(2*c*x^2 + b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4
*a*c)) - (14*B*a^3*b^3 - A*a^2*b^4)*c)/(a^2*b^6*c^2 - 12*a^3*b^4*c^3 + 48*a
^4*b^2*c^4 - 64*a^5*c^5 + (b^6*c^4 - 12*a*b^4*c^5 + 48*a^2*b^2*c^6 - 64*a^3
*c^7)*x^8 + 2*(b^7*c^3 - 12*a*b^5*c^4 + 48*a^2*b^3*c^5 - 64*a^3*b*c^6)*x^6
+ (b^8*c^2 - 10*a*b^6*c^3 + 24*a^2*b^4*c^4 + 32*a^3*b^2*c^5 - 128*a^4*c^6)*
x^4 + 2*(a*b^7*c^2 - 12*a^2*b^5*c^3 + 48*a^3*b^3*c^4 - 64*a^4*b*c^5)*x^2)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^7(A+Bx^2)}{(a+bx^2+cx^4)^3}\,dx = \text{Timed out}$$

```
[In] integrate(x**7*(B*x**2+A)/(c*x**4+b*x**2+a)**3,x)
```

```
[Out] Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^7(A+Bx^2)}{(a+bx^2+cx^4)^3}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(x^7*(B*x^2+A)/(c*x^4+b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 318 vs. $2(140) = 280$.

Time = 1.45 (sec) , antiderivative size = 318, normalized size of antiderivative = 2.18

$$\int \frac{x^7(A+Bx^2)}{(a+bx^2+cx^4)^3}\,dx = \frac{3\,(2\,Ba^2 - Aab)\arctan\left(\frac{2\,cx^2+b}{\sqrt{-b^2+4\,ac}}\right)}{(b^4-8\,ab^2c+16\,a^2c^2)\sqrt{-b^2+4\,ac}} - \frac{2\,Bb^4cx^6 - 16\,Bab^2c^2x^6 + 20\,Ba^2c^3x^6 + 6\,Aabc^3x^6 + Bb^5x^4 - 8\,Bab^3cx^4 + Ab^4cx^4 - 2\,Ba^2bc^2x^4 + Aa}{4\,(b^4c^2 -}$$

```
[In] integrate(x^7*(B*x^2+A)/(c*x^4+b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] 3*(2*B*a^2 - A*a*b)*arctan((2*c*x^2 + b)/sqrt(-b^2 + 4*a*c))/((b^4 - 8*a*b^
2*c + 16*a^2*c^2)*sqrt(-b^2 + 4*a*c)) - 1/4*(2*B*b^4*c*x^6 - 16*B*a*b^2*c^2
*x^6 + 20*B*a^2*c^3*x^6 + 6*A*a*b*c^3*x^6 + B*b^5*x^4 - 8*B*a*b^3*c*x^4 + A
*b^4*c*x^4 - 2*B*a^2*b*c^2*x^4 + A*a*b^2*c^2*x^4 + 16*A*a^2*c^3*x^4 + 2*B*a
*b^4*x^2 - 20*B*a^2*b^2*c*x^2 + 2*A*a*b^3*c*x^2 + 12*B*a^3*c^2*x^2 + 10*A*a
^2*b*c^2*x^2 + B*a^2*b^3 - 10*B*a^3*b*c + A*a^2*b^2*c + 8*A*a^3*c^2)/((b^4*
c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*(c*x^4 + b*x^2 + a)^2)
```

Mupad [B] (verification not implemented)

Time = 7.85 (sec) , antiderivative size = 593, normalized size of antiderivative = 4.06

$$\int \frac{x^7(A+Bx^2)}{(a+bx^2+cx^4)^3}\,dx$$

$$= \frac{3\,a\,\mathrm{atan}\left(\frac{\left(x^2\left(\frac{3\,(A\,b-2\,B\,a)\,\left(6\,B\,a^2\,c^2-3\,A\,a\,b\,c^2\right)}{\left(4\,a\,c-b^2\right)^{9/2}\,\left(16\,a^2\,c^2-8\,a\,b^2\,c+b^4\right)}-\frac{9\,a\,b\,(A\,b-2\,B\,a)^2\,\left(32\,a^2\,b\,c^4-16\,a\,b^3\,c^3+2\,b^5\,c^2\right)}{2\,\left(4\,a\,c-b^2\right)^{15/2}\,\left(16\,a^2\,c^2-8\,a\,b^2\,c+b^4\right)}\right)-\frac{18\,a^2\,b\,c^2\,(A\,b-2\,B\,a)^2}{\left(4\,a\,c-b^2\right)^{15/2}}\right)\left(b^4\,\left(4\,a\,c-b^2\right)^{5}\right.}{18\,A^2\,a^2\,b^2\,c^2-72\,A\,B\,a^3\,b\,c^2+72\,B^2\,a^4\,c^2}\right.}{\left(4\,a\,c-b^2\right)^{5/2}}$$

$$- \frac{\frac{x^4\,\left(-2\,B\,a^2\,b\,c^2+16\,A\,a^2\,c^3-8\,B\,a\,b^3\,c+A\,a\,b^2\,c^2+B\,b^5+A\,b^4\,c\right)}{4\,c^2\,\left(16\,a^2\,c^2-8\,a\,b^2\,c+b^4\right)}+\frac{a^2\,\left(B\,b^3+A\,b^2\,c-10\,B\,a\,b\,c+8\,A\,a\,c^2\right)}{4\,c^2\,\left(16\,a^2\,c^2-8\,a\,b^2\,c+b^4\right)}+\frac{x^6\,\left(10\,B\,a^2\,c^2-8\,B\,a\,b^2\,c+3\,A\right.}{2\,c\,\left(16\,a^2\,c^2-8\,a\,b^2\,c+\right.}}{x^4\,\left(b^2+2\,a\,c\right)+a^2+c^2\,x^8+2\,a\,b\,x^2+2\,b\,c\,x^6}$$

```
[In] int((x^7*(A + B*x^2))/(a + b*x^2 + c*x^4)^3,x)
```

```
[Out] (3*a*atan(((x^2*((3*(A*b - 2*B*a)*(6*B*a^2*c^2 - 3*A*a*b*c^2))/((4*a*c - b^
2)^(9/2)*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) - (9*a*b*(A*b - 2*B*a)^2*(2*b^5*c^
2 - 16*a*b^3*c^3 + 32*a^2*b*c^4))/(2*(4*a*c - b^2)^(15/2)*(b^4 + 16*a^2*c^2
 - 8*a*b^2*c))) - (18*a^2*b*c^2*(A*b - 2*B*a)^2)/(4*a*c - b^2)^(15/2))*(b^4
*(4*a*c - b^2)^5 + 16*a^2*c^2*(4*a*c - b^2)^5 - 8*a*b^2*c*(4*a*c - b^2)^5))
/(72*B^2*a^4*c^2 + 18*A^2*a^2*b^2*c^2 - 72*A*B*a^3*b*c^2))*(A*b - 2*B*a))/(
```

```
4*a*c - b^2)^(5/2) - ((x^4*(B*b^5 + 16*A*a^2*c^3 + A*b^4*c - 8*B*a*b^3*c +
A*a*b^2*c^2 - 2*B*a^2*b*c^2))/(4*c^2*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (a^2
*(B*b^3 + 8*A*a*c^2 + A*b^2*c - 10*B*a*b*c))/(4*c^2*(b^4 + 16*a^2*c^2 - 8*a
*b^2*c)) + (x^6*(B*b^4 + 10*B*a^2*c^2 + 3*A*a*b*c^2 - 8*B*a*b^2*c))/(2*c*(b
^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (a*x^2*(B*b^4 + 6*B*a^2*c^2 + A*b^3*c + 5*A
*a*b*c^2 - 10*B*a*b^2*c))/(2*c^2*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)))/(x^4*(2*a
*c + b^2) + a^2 + c^2*x^8 + 2*a*b*x^2 + 2*b*c*x^6)
```

3.127 $\int \frac{x^5(A+Bx^2)}{(a+bx^2+cx^4)^3}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 185

$$\int \frac{x^5(A+Bx^2)}{(a+bx^2+cx^4)^3}\,dx = -\frac{x^4(Ab-2aB-(bB-2Ac)x^2)}{4(b^2-4ac)(a+bx^2+cx^4)^2} - \frac{a(b^2B-6Abc+8aBc)+(b^3B-4Ab^2c+2abBc+4aAc^2)x^2}{4c(b^2-4ac)^2(a+bx^2+cx^4)} + \frac{(3abB-A(b^2+2ac))\operatorname{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{(b^2-4ac)^{5/2}}$$

```
[Out] -1/4*x^4*(A*b-2*B*a-(-2*A*c+B*b)*x^2)/(-4*a*c+b^2)/(c*x^4+b*x^2+a)^2+1/4*(-
a*(-6*A*b*c+8*B*a*c+B*b^2)-(4*A*a*c^2-4*A*b^2*c+2*B*a*b*c+B*b^3)*x^2)/c/(-4
*a*c+b^2)^2/(c*x^4+b*x^2+a)+(3*a*b*B-A*(2*a*c+b^2))*arctanh((2*c*x^2+b)/(-4
*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(5/2)
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 185, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {1265, 834, 791, 632, 212}

$$\int \frac{x^5(A+Bx^2)}{(a+bx^2+cx^4)^3}\,dx = \frac{(3abB-A(2ac+b^2))\operatorname{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{(b^2-4ac)^{5/2}} - \frac{x^4(-2aB-(x^2(bB-2Ac))+Ab)}{4(b^2-4ac)(a+bx^2+cx^4)^2} - \frac{a(8aBc-6Abc+b^2B)+x^2(4aAc^2+2abBc-4Ab^2c+b^3B)}{4c(b^2-4ac)^2(a+bx^2+cx^4)}$$

```
[In] Int[(x^5*(A + B*x^2))/(a + b*x^2 + c*x^4)^3,x]
```

```
[Out] -1/4*(x^4*(A*b - 2*a*B - (b*B - 2*A*c)*x^2))/((b^2 - 4*a*c)*(a + b*x^2 + c*
x^4)^2) - (a*(b^2*B - 6*A*b*c + 8*a*B*c) + (b^3*B - 4*A*b^2*c + 2*a*b*B*c +
 4*a*A*c^2)*x^2)/(4*c*(b^2 - 4*a*c)^2*(a + b*x^2 + c*x^4)) + ((3*a*b*B - A*
(b^2 + 2*a*c))*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(b^2 - 4*a*c)^(5/2
)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 791

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> Simp[(-(2*a*c*(e*f + d*g) - b*(c*d*f + a*e*g) - (
b^2*e*g - b*c*(e*f + d*g) + 2*c*(c*d*f - a*e*g))*x))*((a + b*x + c*x^2)^(p
+ 1)/(c*(p + 1)*(b^2 - 4*a*c))), x] - Dist[(b^2*e*g*(p + 2) - 2*a*c*e*g + c
*(2*c*d*f - b*(e*f + d*g))*(2*p + 3))/(c*(p + 1)*(b^2 - 4*a*c)), Int[(a + b
*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 -
 4*a*c, 0] && LtQ[p, -1]
```

Rule 834

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d + e*x)^m*(a + b*x + c*x^2)^(p + 1)*(
(f*b - 2*a*g + (2*c*f - b*g)*x)/((p + 1)*(b^2 - 4*a*c))), x] + Dist[1/((p +
 1)*(b^2 - 4*a*c)), Int[(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1)*Simp[g*
(2*a*e*m + b*d*(2*p + 3)) - f*(b*e*m + 2*c*d*(2*p + 3)) - e*(2*c*f - b*g)*(
m + 2*p + 3)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 0] && (I
ntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 1265

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rubi steps

$$\text{integral} = \frac{1}{2}\text{Subst}\left(\int \frac{x^2(A+Bx)}{(a+bx+cx^2)^3}\,dx, x, x^2\right)$$

$$= -\frac{x^4(Ab-2aB-(bB-2Ac)x^2)}{4(b^2-4ac)(a+bx^2+cx^4)^2} - \frac{\text{Subst}\left(\int \frac{x(-2(Ab-2aB)-(bB-2Ac)x)}{(a+bx+cx^2)^2}\,dx, x, x^2\right)}{4(b^2-4ac)}$$

$$= -\frac{x^4(Ab-2aB-(bB-2Ac)x^2)}{4(b^2-4ac)(a+bx^2+cx^4)^2} - \frac{a(b^2B-6Abc+8aBc)+(b^3B-4Ab^2c+2abBc+4aAc^2)x^2}{4c(b^2-4ac)^2(a+bx^2+cx^4)} - \frac{(3abB-A(b^2+2ac))\,\text{Subst}\left(\int \frac{1}{a+bx+cx^2}\,dx, x, x^2\right)}{2(b^2-4ac)^2}$$

$$= -\frac{x^4(Ab-2aB-(bB-2Ac)x^2)}{4(b^2-4ac)(a+bx^2+cx^4)^2} - \frac{a(b^2B-6Abc+8aBc)+(b^3B-4Ab^2c+2abBc+4aAc^2)x^2}{4c(b^2-4ac)^2(a+bx^2+cx^4)} + \frac{(3abB-A(b^2+2ac))\,\text{Subst}\left(\int \frac{1}{b^2-4ac-x^2}\,dx, x, b+2cx^2\right)}{(b^2-4ac)^2}$$

$$= -\frac{x^4(Ab-2aB-(bB-2Ac)x^2)}{4(b^2-4ac)(a+bx^2+cx^4)^2} - \frac{a(b^2B-6Abc+8aBc)+(b^3B-4Ab^2c+2abBc+4aAc^2)x^2}{4c(b^2-4ac)^2(a+bx^2+cx^4)} + \frac{(3abB-A(b^2+2ac))\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{(b^2-4ac)^{5/2}}$$

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 233, normalized size of antiderivative = 1.26

$$\int \frac{x^5(A+Bx^2)}{(a+bx^2+cx^4)^3}\,dx$$

$$= \frac{1}{4}\left(\frac{-b^4B+Ab^3c+2abc^2(A-3Bx^2)+4ac^2(-4aB+Acx^2)+b^2c(5aB+2Acx^2)}{c^2(b^2-4ac)^2(a+bx^2+cx^4)} + \frac{2a^2Bc+b^2(-bB+Ac)x^2+a(-b^2B-2Ac^2x^2+bc(A+3Bx^2))}{c^2(-b^2+4ac)(a+bx^2+cx^4)^2} + \frac{4(-3abB+A(b^2+2ac))\arctan\left(\frac{b+2cx^2}{\sqrt{-b^2+4ac}}\right)}{(-b^2+4ac)^{5/2}}\right)$$

```
[In] Integrate[(x^5*(A + B*x^2))/(a + b*x^2 + c*x^4)^3,x]
```

```
[Out] ((-(b^4*B) + A*b^3*c + 2*a*b*c^2*(A - 3*B*x^2) + 4*a*c^2*(-4*a*B + A*c*x^2)
 + b^2*c*(5*a*B + 2*A*c*x^2))/(c^2*(b^2 - 4*a*c)^2*(a + b*x^2 + c*x^4)) + (
2*a^2*B*c + b^2*(-(b*B) + A*c)*x^2 + a*(-(b^2*B) - 2*A*c^2*x^2 + b*c*(A + 3
*B*x^2)))/(c^2*(-b^2 + 4*a*c)*(a + b*x^2 + c*x^4)^2) + (4*(-3*a*b*B + A*(b^
2 + 2*a*c))*ArcTan[(b + 2*c*x^2)/Sqrt[-b^2 + 4*a*c]])/(-b^2 + 4*a*c)^(5/2))
/4
```

Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 303, normalized size of antiderivative = 1.64

method	result
default	$\frac{\frac{c\left(2Aac+A\,b^2-3abB\right)x^6}{16a^2c^2-8a\,b^2c+b^4}+\frac{\left(6Aab\,c^2+3A\,b^3c-16B\,a^2c^2-Ba\,b^2c-B\,b^4\right)x^4}{2c\left(16a^2c^2-8a\,b^2c+b^4\right)}-\frac{a\left(2Aa\,c^2-5A\,b^2c+5Babc+B\,b^3\right)x^2}{c\left(16a^2c^2-8a\,b^2c+b^4\right)}+\frac{a^2\left(6Abc-8Bac-B\,b^2\right)}{2c\left(16a^2c^2-8a\,b^2c+b^4\right)}}{2(c\,x^4+b\,x^2+a)^2}+\frac{(2A}{}$
risch	$\frac{\frac{c\left(2Aac+A\,b^2-3abB\right)x^6}{32a^2c^2-16a\,b^2c+2b^4}+\frac{\left(6Aab\,c^2+3A\,b^3c-16B\,a^2c^2-Ba\,b^2c-B\,b^4\right)x^4}{4c\left(16a^2c^2-8a\,b^2c+b^4\right)}-\frac{a\left(2Aa\,c^2-5A\,b^2c+5Babc+B\,b^3\right)x^2}{2c\left(16a^2c^2-8a\,b^2c+b^4\right)}+\frac{a^2\left(6Abc-8Bac-B\,b^2\right)}{4c\left(16a^2c^2-8a\,b^2c+b^4\right)}}{(c\,x^4+b\,x^2+a)^2}-\frac{\ln(}{}$

```
[In] int(x^5*(B*x^2+A)/(c*x^4+b*x^2+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*(c*(2*A*a*c+A*b^2-3*B*a*b)/(16*a^2*c^2-8*a*b^2*c+b^4)*x^6+1/2*(6*A*a*b*
c^2+3*A*b^3*c-16*B*a^2*c^2-B*a*b^2*c-B*b^4)/c/(16*a^2*c^2-8*a*b^2*c+b^4)*x^
4-1/c*a*(2*A*a*c^2-5*A*b^2*c+5*B*a*b*c+B*b^3)/(16*a^2*c^2-8*a*b^2*c+b^4)*x^
2+1/2*a^2*(6*A*b*c-8*B*a*c-B*b^2)/c/(16*a^2*c^2-8*a*b^2*c+b^4))/(c*x^4+b*x^
2+a)^2+(2*A*a*c+A*b^2-3*B*a*b)/(16*a^2*c^2-8*a*b^2*c+b^4)/(4*a*c-b^2)^(1/2)
*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 672 vs. 2(177) = 354.

Time = 0.33 (sec) , antiderivative size = 1369, normalized size of antiderivative = 7.40

$$\int \frac{x^5(A+Bx^2)}{(a+bx^2+cx^4)^3}\,dx = \text{Too large to display}$$

```
[In] integrate(x^5*(B*x^2+A)/(c*x^4+b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] [-1/4*(B*a^2*b^4 + 2*(8*A*a^2*c^4 - 2*(6*B*a^2*b - A*a*b^2)*c^3 + (3*B*a*b^
3 - A*b^4)*c^2)*x^6 + (B*b^6 - 8*(8*B*a^3 - 3*A*a^2*b)*c^3 + 6*(2*B*a^2*b^2
 + A*a*b^3)*c^2 - 3*(B*a*b^4 + A*b^5)*c)*x^4 - 8*(4*B*a^4 - 3*A*a^3*b)*c^2
+ 2*(B*a*b^5 - 8*A*a^3*c^3 - 2*(10*B*a^3*b - 11*A*a^2*b^2)*c^2 + (B*a^2*b^3
 - 5*A*a*b^4)*c)*x^2 - 2*((2*A*a*c^4 - (3*B*a*b - A*b^2)*c^3)*x^8 + 2*(2*A*
a*b*c^3 - (3*B*a*b^2 - A*b^3)*c^2)*x^6 + 2*A*a^3*c^2 + (4*A*a^2*c^3 - 2*(3*
```

```
B*a^2*b - 2*A*a*b^2)*c^2 - (3*B*a*b^3 - A*b^4)*c)*x^4 + 2*(2*A*a^2*b*c^2 -
(3*B*a^2*b^2 - A*a*b^3)*c)*x^2 - (3*B*a^3*b - A*a^2*b^2)*c)*sqrt(b^2 - 4*a*
c)*log((2*c^2*x^4 + 2*b*c*x^2 + b^2 - 2*a*c - (2*c*x^2 + b)*sqrt(b^2 - 4*a*
c))/(c*x^4 + b*x^2 + a)) + 2*(2*B*a^3*b^2 - 3*A*a^2*b^3)*c)/(a^2*b^6*c - 12
*a^3*b^4*c^2 + 48*a^4*b^2*c^3 - 64*a^5*c^4 + (b^6*c^3 - 12*a*b^4*c^4 + 48*a
^2*b^2*c^5 - 64*a^3*c^6)*x^8 + 2*(b^7*c^2 - 12*a*b^5*c^3 + 48*a^2*b^3*c^4 -
 64*a^3*b*c^5)*x^6 + (b^8*c - 10*a*b^6*c^2 + 24*a^2*b^4*c^3 + 32*a^3*b^2*c^
4 - 128*a^4*c^5)*x^4 + 2*(a*b^7*c - 12*a^2*b^5*c^2 + 48*a^3*b^3*c^3 - 64*a^
4*b*c^4)*x^2), -1/4*(B*a^2*b^4 + 2*(8*A*a^2*c^4 - 2*(6*B*a^2*b - A*a*b^2)*c
^3 + (3*B*a*b^3 - A*b^4)*c^2)*x^6 + (B*b^6 - 8*(8*B*a^3 - 3*A*a^2*b)*c^3 +
6*(2*B*a^2*b^2 + A*a*b^3)*c^2 - 3*(B*a*b^4 + A*b^5)*c)*x^4 - 8*(4*B*a^4 - 3
*A*a^3*b)*c^2 + 2*(B*a*b^5 - 8*A*a^3*c^3 - 2*(10*B*a^3*b - 11*A*a^2*b^2)*c^
2 + (B*a^2*b^3 - 5*A*a*b^4)*c)*x^2 + 4*((2*A*a*c^4 - (3*B*a*b - A*b^2)*c^3)
*x^8 + 2*(2*A*a*b*c^3 - (3*B*a*b^2 - A*b^3)*c^2)*x^6 + 2*A*a^3*c^2 + (4*A*a
^2*c^3 - 2*(3*B*a^2*b - 2*A*a*b^2)*c^2 - (3*B*a*b^3 - A*b^4)*c)*x^4 + 2*(2*
A*a^2*b*c^2 - (3*B*a^2*b^2 - A*a*b^3)*c)*x^2 - (3*B*a^3*b - A*a^2*b^2)*c)*s
qrt(-b^2 + 4*a*c)*arctan(-(2*c*x^2 + b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4*a*c)) +
 2*(2*B*a^3*b^2 - 3*A*a^2*b^3)*c)/(a^2*b^6*c - 12*a^3*b^4*c^2 + 48*a^4*b^2*
c^3 - 64*a^5*c^4 + (b^6*c^3 - 12*a*b^4*c^4 + 48*a^2*b^2*c^5 - 64*a^3*c^6)*x
^8 + 2*(b^7*c^2 - 12*a*b^5*c^3 + 48*a^2*b^3*c^4 - 64*a^3*b*c^5)*x^6 + (b^8*
c - 10*a*b^6*c^2 + 24*a^2*b^4*c^3 + 32*a^3*b^2*c^4 - 128*a^4*c^5)*x^4 + 2*(
a*b^7*c - 12*a^2*b^5*c^2 + 48*a^3*b^3*c^3 - 64*a^4*b*c^4)*x^2)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^5(A+Bx^2)}{(a+bx^2+cx^4)^3}\,dx = \text{Timed out}$$

```
[In] integrate(x**5*(B*x**2+A)/(c*x**4+b*x**2+a)**3,x)
[Out] Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^5(A+Bx^2)}{(a+bx^2+cx^4)^3}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(x^5*(B*x^2+A)/(c*x^4+b*x^2+a)^3,x, algorithm="maxima")
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

Giac [A] (verification not implemented)

none

Time = 1.42 (sec) , antiderivative size = 268, normalized size of antiderivative = 1.45

$$\int \frac{x^5(A+Bx^2)}{(a+bx^2+cx^4)^3}\,dx = -\frac{(3\,Bab-Ab^2-2\,Aac)\arctan\left(\frac{2\,cx^2+b}{\sqrt{-b^2+4\,ac}}\right)}{(b^4-8\,ab^2c+16\,a^2c^2)\sqrt{-b^2+4\,ac}} - \frac{6\,Babc^2x^6-2\,Ab^2c^2x^6-4\,Aac^3x^6+Bb^4x^4+Bab^2cx^4-3\,Ab^3cx^4+16\,Ba^2c^2x^4-6\,Aabc^2x^4+2\,Bab^3x}{4\,(b^4c-8\,ab^2c^2+16\,a^2c^3)(cx^4+bx^2+}$$

[In] integrate(x^5*(B*x^2+A)/(c*x^4+b*x^2+a)^3,x, algorithm="giac")

[Out] -(3*B*a*b - A*b^2 - 2*A*a*c)*arctan((2*c*x^2 + b)/sqrt(-b^2 + 4*a*c))/((b^4 - 8*a*b^2*c + 16*a^2*c^2)*sqrt(-b^2 + 4*a*c)) - 1/4*(6*B*a*b*c^2*x^6 - 2*A*b^2*c^2*x^6 - 4*A*a*c^3*x^6 + B*b^4*x^4 + B*a*b^2*c*x^4 - 3*A*b^3*c*x^4 + 16*B*a^2*c^2*x^4 - 6*A*a*b*c^2*x^4 + 2*B*a*b^3*x^2 + 10*B*a^2*b*c*x^2 - 10*A*a*b^2*c*x^2 + 4*A*a^2*c^2*x^2 + B*a^2*b^2 + 8*B*a^3*c - 6*A*a^2*b*c)/((b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*(c*x^4 + b*x^2 + a)^2)

Mupad [B] (verification not implemented)

Time = 7.85 (sec) , antiderivative size = 625, normalized size of antiderivative = 3.38

$$\int \frac{x^5(A+Bx^2)}{(a+bx^2+cx^4)^3}\,dx = \frac{\mathrm{atan}\left(\frac{\left(x^2\left(\frac{\left(A\,b^2\,c^2-3\,B\,a\,b\,c^2+2\,A\,a\,c^3\right)\left(A\,b^2-3\,B\,a\,b+2\,A\,a\,c\right)}{a\left(4\,a\,c-b^2\right)^{9/2}\left(16\,a^2\,c^2-8\,a\,b^2\,c+b^4\right)}+\frac{b\left(A\,b^2-3\,B\,a\,b+2\,A\,a\,c\right)^2\left(32\,a^2\,b\,c^4-16\,a\,b^3\,c^3+2\,b^5\,c^2\right)}{2\,a\left(4\,a\,c-b^2\right)^{15/2}\left(16\,a^2\,c^2-8\,a\,b^2\,c+b^4\right)}\right)+\frac{2\,b\,c^2\left(A\,b^2-3\,B\,a\,b+2\,A\,a\right.}{\left(4\,a\,c-b^2\right)^{15/2}}}{8\,A^2\,a^2\,c^4+8\,A^2\,a\,b^2\,c^3+2\,A^2\,b^4\,c^2-24\,A\,B\,a^2\,b\,c^3-12\,A\,B\,a\,b^3\,c^2+18\,B^2\,a^2}\right.}{\left(4\,a\,c-b^2\right)^{5/2}} - \frac{\frac{x^4\left(16\,B\,a^2\,c^2+B\,a\,b^2\,c-6\,A\,a\,b\,c^2+B\,b^4-3\,A\,b^3\,c\right)}{4\,c\left(16\,a^2\,c^2-8\,a\,b^2\,c+b^4\right)}-\frac{c\,x^6\left(A\,b^2-3\,B\,a\,b+2\,A\,a\,c\right)}{2\left(16\,a^2\,c^2-8\,a\,b^2\,c+b^4\right)}+\frac{a\left(8\,B\,c\,a^2+B\,a\,b^2-6\,A\,c\,a\,b\right)}{4\,c\left(16\,a^2\,c^2-8\,a\,b^2\,c+b^4\right)}+\frac{x^2\left(5\,B\,a^2\,b\,c+2\,A\,a^2\,c\right.}{2\,c\left(16\,a^2\,c^2-\right.}}{x^4\left(b^2+2\,a\,c\right)+a^2+c^2\,x^8+2\,a\,b\,x^2+2\,b\,c\,x^6}$$

[In] int((x^5*(A + B*x^2))/(a + b*x^2 + c*x^4)^3,x)

[Out] (atan(((x^2*(((A*b^2*c^2 + 2*A*a*c^3 - 3*B*a*b*c^2)*(A*b^2 + 2*A*a*c - 3*B*a*b))/(a*(4*a*c - b^2)^(9/2)*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (b*(A*b^2 + 2*A*a*c - 3*B*a*b)^2*(2*b^5*c^2 - 16*a*b^3*c^3 + 32*a^2*b*c^4))/(2*a*(4*a*c - b^2)^(15/2)*(b^4 + 16*a^2*c^2 - 8*a*b^2*c))) + (2*b*c^2*(A*b^2 + 2*A*a*c - 3*B*a*b)^2)/(4*a*c - b^2)^(15/2))*(b^4*(4*a*c - b^2)^5 + 16*a^2*c^2*(4*a*c - b^2)^5 - 8*a*b^2*c*(4*a*c - b^2)^5))/(8*A^2*a^2*c^4 + 2*A^2*b^4*c^2 + 18*B^2*a^2*b^2*c^2 + 8*A^2*a*b^2*c^3 - 12*A*B*a*b^3*c^2 - 24*A*B*a^2*b*c^3)

```
)*(A*b^2 + 2*A*a*c - 3*B*a*b))/(4*a*c - b^2)^(5/2) - ((x^4*(B*b^4 + 16*B*a^
2*c^2 - 3*A*b^3*c - 6*A*a*b*c^2 + B*a*b^2*c))/(4*c*(b^4 + 16*a^2*c^2 - 8*a*
b^2*c)) - (c*x^6*(A*b^2 + 2*A*a*c - 3*B*a*b))/(2*(b^4 + 16*a^2*c^2 - 8*a*b^
2*c)) + (a*(B*a*b^2 + 8*B*a^2*c - 6*A*a*b*c))/(4*c*(b^4 + 16*a^2*c^2 - 8*a*
b^2*c)) + (x^2*(2*A*a^2*c^2 + B*a*b^3 - 5*A*a*b^2*c + 5*B*a^2*b*c))/(2*c*(b
^4 + 16*a^2*c^2 - 8*a*b^2*c)))/(x^4*(2*a*c + b^2) + a^2 + c^2*x^8 + 2*a*b*x
^2 + 2*b*c*x^6)
```

3.128 $\int \frac{x^3(A+Bx^2)}{(a+bx^2+cx^4)^3}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 170

$$\int \frac{x^3(A+Bx^2)}{(a+bx^2+cx^4)^3}\,dx=-\frac{a(bB-2Ac)+(b^2B-Abc-2aBc)\,x^2}{4c\,(b^2-4ac)\,(a+bx^2+cx^4)^2}+\frac{(b^2B-3Abc+2aBc)\,(b+2cx^2)}{4c\,(b^2-4ac)^2\,(a+bx^2+cx^4)}-\frac{(b^2B-3Abc+2aBc)\,\text{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{(b^2-4ac)^{5/2}}$$

```
[Out] 1/4*(-a*(-2*A*c+B*b)-(-A*b*c-2*B*a*c+B*b^2)*x^2)/c/(-4*a*c+b^2)/(c*x^4+b*x^
2+a)^2+1/4*(-3*A*b*c+2*B*a*c+B*b^2)*(2*c*x^2+b)/c/(-4*a*c+b^2)^2/(c*x^4+b*x
^2+a)-(-3*A*b*c+2*B*a*c+B*b^2)*arctanh((2*c*x^2+b)/(-4*a*c+b^2)^(1/2))/(-4*
a*c+b^2)^(5/2)
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {1265, 791, 628, 632, 212}

$$\int \frac{x^3(A+Bx^2)}{(a+bx^2+cx^4)^3}\,dx=-\frac{(2aBc-3Abc+b^2B)\,\text{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{(b^2-4ac)^{5/2}}+\frac{(b+2cx^2)\,(2aBc-3Abc+b^2B)}{4c\,(b^2-4ac)^2\,(a+bx^2+cx^4)}-\frac{x^2(-2aBc-Abc+b^2B)+a(bB-2Ac)}{4c\,(b^2-4ac)\,(a+bx^2+cx^4)^2}$$

```
[In] Int[(x^3*(A + B*x^2))/(a + b*x^2 + c*x^4)^3,x]
```

```
[Out] -1/4*(a*(b*B - 2*A*c) + (b^2*B - A*b*c - 2*a*B*c)*x^2)/(c*(b^2 - 4*a*c)*(a
+ b*x^2 + c*x^4)^2) + ((b^2*B - 3*A*b*c + 2*a*B*c)*(b + 2*c*x^2))/(4*c*(b^2
 - 4*a*c)^2*(a + b*x^2 + c*x^4)) - ((b^2*B - 3*A*b*c + 2*a*B*c)*ArcTanh[(b
+ 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(b^2 - 4*a*c)^(5/2)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] - Dist[2*c*((2*p +
 3)/((p + 1)*(b^2 - 4*a*c))), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2] && Int
egerQ[4*p]
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 791

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> Simp[(-(2*a*c*(e*f + d*g) - b*(c*d*f + a*e*g) - (
b^2*e*g - b*c*(e*f + d*g) + 2*c*(c*d*f - a*e*g))*x))*((a + b*x + c*x^2)^(p
+ 1)/(c*(p + 1)*(b^2 - 4*a*c))), x] - Dist[(b^2*e*g*(p + 2) - 2*a*c*e*g + c
*(2*c*d*f - b*(e*f + d*g))*(2*p + 3))/(c*(p + 1)*(b^2 - 4*a*c)), Int[(a + b
*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 -
 4*a*c, 0] && LtQ[p, -1]
```

Rule 1265

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rubi steps

$$\text{integral} = \frac{1}{2}\text{Subst}\left(\int \frac{x(A+Bx)}{(a+bx+cx^2)^3}\,dx, x, x^2\right)$$

$$= -\frac{a(bB-2Ac)+(b^2B-Abc-2aBc)\,x^2}{4c\,(b^2-4ac)\,(a+bx^2+cx^4)^2} - \frac{(b^2B-3Abc+2aBc)\,\text{Subst}\left(\int \frac{1}{(a+bx+cx^2)^2}\,dx,x,x^2\right)}{4c\,(b^2-4ac)}$$

$$= -\frac{a(bB-2Ac)+(b^2B-Abc-2aBc)\,x^2}{4c\,(b^2-4ac)\,(a+bx^2+cx^4)^2} + \frac{(b^2B-3Abc+2aBc)\,(b+2cx^2)}{4c\,(b^2-4ac)^2\,(a+bx^2+cx^4)} + \frac{(b^2B-3Abc+2aBc)\,\text{Subst}\left(\int \frac{1}{a+bx+cx^2}\,dx,x,x^2\right)}{2\,(b^2-4ac)^2}$$

$$= -\frac{a(bB-2Ac)+(b^2B-Abc-2aBc)\,x^2}{4c\,(b^2-4ac)\,(a+bx^2+cx^4)^2} + \frac{(b^2B-3Abc+2aBc)\,(b+2cx^2)}{4c\,(b^2-4ac)^2\,(a+bx^2+cx^4)} - \frac{(b^2B-3Abc+2aBc)\,\text{Subst}\left(\int \frac{1}{b^2-4ac-x^2}\,dx,x,b+2cx^2\right)}{(b^2-4ac)^2}$$

$$= -\frac{a(bB-2Ac)+(b^2B-Abc-2aBc)\,x^2}{4c\,(b^2-4ac)\,(a+bx^2+cx^4)^2} + \frac{(b^2B-3Abc+2aBc)\,(b+2cx^2)}{4c\,(b^2-4ac)^2\,(a+bx^2+cx^4)} - \frac{(b^2B-3Abc+2aBc)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{(b^2-4ac)^{5/2}}$$

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 172, normalized size of antiderivative = 1.01

$$\int \frac{x^3(A+Bx^2)}{(a+bx^2+cx^4)^3}\,dx = \frac{1}{4}\left(\frac{(b^2B-3Abc+2aBc)\,(b+2cx^2)}{c\,(b^2-4ac)^2\,(a+bx^2+cx^4)} + \frac{abB+b(bB-Ac)x^2-2ac(A+Bx^2)}{c\,(-b^2+4ac)\,(a+bx^2+cx^4)^2} + \frac{4(b^2B-3Abc+2aBc)\arctan\left(\frac{b+2cx^2}{\sqrt{-b^2+4ac}}\right)}{(-b^2+4ac)^{5/2}}\right)$$

```
[In] Integrate[(x^3*(A + B*x^2))/(a + b*x^2 + c*x^4)^3,x]
```

```
[Out] (((b^2*B - 3*A*b*c + 2*a*B*c)*(b + 2*c*x^2))/(c*(b^2 - 4*a*c)^2*(a + b*x^2
+ c*x^4)) + (a*b*B + b*(b*B - A*c)*x^2 - 2*a*c*(A + B*x^2))/(c*(-b^2 + 4*a*
c)*(a + b*x^2 + c*x^4)^2) + (4*(b^2*B - 3*A*b*c + 2*a*B*c)*ArcTan[(b + 2*c*
x^2)/Sqrt[-b^2 + 4*a*c]])/(-b^2 + 4*a*c)^(5/2))/4
```

Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 273, normalized size of antiderivative = 1.61

method	result
default	$\frac{-\frac{c\left(3Abc-2Bac-B\,b^{2}\right)x^{6}}{16a^{2}c^{2}-8a\,b^{2}c+b^{4}}-\frac{3b\left(3Abc-2Bac-B\,b^{2}\right)x^{4}}{2\left(16a^{2}c^{2}-8a\,b^{2}c+b^{4}\right)}-\frac{\left(5Aabc+A\,b^{3}+2a^{2}Bc-5Ba\,b^{2}\right)x^{2}}{16a^{2}c^{2}-8a\,b^{2}c+b^{4}}-\frac{a\left(8Aac+A\,b^{2}-6abB\right)}{2\left(16a^{2}c^{2}-8a\,b^{2}c+b^{4}\right)}}{2\left(c\,x^{4}+b\,x^{2}+a\right)^{2}}-\frac{\left(3Abc-2Bac-B\,b^{2}\right)}{\left(16a^{2}c^{2}-8a\,b^{2}c\right.}$
risch	$\frac{-\frac{c\left(3Abc-2Bac-B\,b^{2}\right)x^{6}}{2\left(16a^{2}c^{2}-8a\,b^{2}c+b^{4}\right)}-\frac{3b\left(3Abc-2Bac-B\,b^{2}\right)x^{4}}{4\left(16a^{2}c^{2}-8a\,b^{2}c+b^{4}\right)}-\frac{\left(5Aabc+A\,b^{3}+2a^{2}Bc-5Ba\,b^{2}\right)x^{2}}{2\left(16a^{2}c^{2}-8a\,b^{2}c+b^{4}\right)}-\frac{a\left(8Aac+A\,b^{2}-6abB\right)}{4\left(16a^{2}c^{2}-8a\,b^{2}c+b^{4}\right)}}{\left(c\,x^{4}+b\,x^{2}+a\right)^{2}}-\frac{3\ln\left(\left(-\left(-4ac+b^{2}\right)^{\frac{5}{2}}\right.\right.}{}$

```
[In] int(x^3*(B*x^2+A)/(c*x^4+b*x^2+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*(-c*(3*A*b*c-2*B*a*c-B*b^2)/(16*a^2*c^2-8*a*b^2*c+b^4)*x^6-3/2*b*(3*A*b
*c-2*B*a*c-B*b^2)/(16*a^2*c^2-8*a*b^2*c+b^4)*x^4-(5*A*a*b*c+A*b^3+2*B*a^2*c
-5*B*a*b^2)/(16*a^2*c^2-8*a*b^2*c+b^4)*x^2-1/2*a*(8*A*a*c+A*b^2-6*B*a*b)/(1
6*a^2*c^2-8*a*b^2*c+b^4))/(c*x^4+b*x^2+a)^2-(3*A*b*c-2*B*a*c-B*b^2)/(16*a^2
*c^2-8*a*b^2*c+b^4)/(4*a*c-b^2)^(1/2)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 601 vs. $2(162) = 324$.

Time = 0.31 (sec) , antiderivative size = 1226, normalized size of antiderivative = 7.21

$$\int \frac{x^3(A+Bx^2)}{(a+bx^2+cx^4)^3}\,dx = \text{Too large to display}$$

```
[In] integrate(x^3*(B*x^2+A)/(c*x^4+b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] [1/4*(2*(B*b^4*c - 4*(2*B*a^2 - 3*A*a*b)*c^3 - (2*B*a*b^2 + 3*A*b^3)*c^2)*x
^6 + 6*B*a^2*b^3 - A*a*b^4 + 32*A*a^3*c^2 + 3*(B*b^5 - 4*(2*B*a^2*b - 3*A*a
*b^2)*c^2 - (2*B*a*b^3 + 3*A*b^4)*c)*x^4 + 2*(5*B*a*b^4 - A*b^5 + 4*(2*B*a^
3 + 5*A*a^2*b)*c^2 - (22*B*a^2*b^2 + A*a*b^3)*c)*x^2 - 2*((B*b^2*c^2 + (2*B
*a - 3*A*b)*c^3)*x^8 + 2*(B*b^3*c + (2*B*a*b - 3*A*b^2)*c^2)*x^6 + B*a^2*b^
2 + (B*b^4 + 2*(2*B*a^2 - 3*A*a*b)*c^2 + (4*B*a*b^2 - 3*A*b^3)*c)*x^4 + 2*(
B*a*b^3 + (2*B*a^2*b - 3*A*a*b^2)*c)*x^2 + (2*B*a^3 - 3*A*a^2*b)*c)*sqrt(b^
2 - 4*a*c)*log((2*c^2*x^4 + 2*b*c*x^2 + b^2 - 2*a*c + (2*c*x^2 + b)*sqrt(b^
2 - 4*a*c))/(c*x^4 + b*x^2 + a)) - 4*(6*B*a^3*b + A*a^2*b^2)*c)/((b^6*c^2 -
 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*x^8 + a^2*b^6 - 12*a^3*b^4*c +
 48*a^4*b^2*c^2 - 64*a^5*c^3 + 2*(b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 6
4*a^3*b*c^4)*x^6 + (b^8 - 10*a*b^6*c + 24*a^2*b^4*c^2 + 32*a^3*b^2*c^3 - 12
8*a^4*c^4)*x^4 + 2*(a*b^7 - 12*a^2*b^5*c + 48*a^3*b^3*c^2 - 64*a^4*b*c^3)*x
^2), 1/4*(2*(B*b^4*c - 4*(2*B*a^2 - 3*A*a*b)*c^3 - (2*B*a*b^2 + 3*A*b^3)*c^
2)*x^6 + 6*B*a^2*b^3 - A*a*b^4 + 32*A*a^3*c^2 + 3*(B*b^5 - 4*(2*B*a^2*b - 3
*A*a*b^2)*c^2 - (2*B*a*b^3 + 3*A*b^4)*c)*x^4 + 2*(5*B*a*b^4 - A*b^5 + 4*(2*
```

```
B*a^3 + 5*A*a^2*b)*c^2 - (22*B*a^2*b^2 + A*a*b^3)*c)*x^2 - 4*((B*b^2*c^2 +
(2*B*a - 3*A*b)*c^3)*x^8 + 2*(B*b^3*c + (2*B*a*b - 3*A*b^2)*c^2)*x^6 + B*a^
2*b^2 + (B*b^4 + 2*(2*B*a^2 - 3*A*a*b)*c^2 + (4*B*a*b^2 - 3*A*b^3)*c)*x^4 +
 2*(B*a*b^3 + (2*B*a^2*b - 3*A*a*b^2)*c)*x^2 + (2*B*a^3 - 3*A*a^2*b)*c)*sqr
t(-b^2 + 4*a*c)*arctan(-(2*c*x^2 + b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4*a*c)) - 4
*(6*B*a^3*b + A*a^2*b^2)*c)/((b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*
a^3*c^5)*x^8 + a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3 + 2*(b^
7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4)*x^6 + (b^8 - 10*a*b^6*c
 + 24*a^2*b^4*c^2 + 32*a^3*b^2*c^3 - 128*a^4*c^4)*x^4 + 2*(a*b^7 - 12*a^2*b
^5*c + 48*a^3*b^3*c^2 - 64*a^4*b*c^3)*x^2)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^3(A+Bx^2)}{(a+bx^2+cx^4)^3}\,dx = \text{Timed out}$$

```
[In] integrate(x**3*(B*x**2+A)/(c*x**4+b*x**2+a)**3,x)
[Out] Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^3(A+Bx^2)}{(a+bx^2+cx^4)^3}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(x^3*(B*x^2+A)/(c*x^4+b*x^2+a)^3,x, algorithm="maxima")
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

Giac [A] (verification not implemented)

none

Time = 1.55 (sec) , antiderivative size = 228, normalized size of antiderivative = 1.34

$$\int \frac{x^3(A+Bx^2)}{(a+bx^2+cx^4)^3}\,dx = \frac{(Bb^2+2\,Bac-3\,Abc)\arctan\left(\frac{2\,cx^2+b}{\sqrt{-b^2+4\,ac}}\right)}{(b^4-8\,ab^2c+16\,a^2c^2)\sqrt{-b^2+4\,ac}} + \frac{2\,Bb^2cx^6+4\,Bac^2x^6-6\,Abc^2x^6+3\,Bb^3x^4+6\,Babcx^4-9\,Ab^2cx^4+10\,Bab^2x^2-2\,Ab^3x^2-4\,Ba^2cx^2-}{4\,(cx^4+bx^2+a)^2(b^4-8\,ab^2c+16\,a^2c^2)}$$

```
[In] integrate(x^3*(B*x^2+A)/(c*x^4+b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] (B*b^2 + 2*B*a*c - 3*A*b*c)*arctan((2*c*x^2 + b)/sqrt(-b^2 + 4*a*c))/((b^4
- 8*a*b^2*c + 16*a^2*c^2)*sqrt(-b^2 + 4*a*c)) + 1/4*(2*B*b^2*c*x^6 + 4*B*a*
c^2*x^6 - 6*A*b*c^2*x^6 + 3*B*b^3*x^4 + 6*B*a*b*c*x^4 - 9*A*b^2*c*x^4 + 10*
B*a*b^2*x^2 - 2*A*b^3*x^2 - 4*B*a^2*c*x^2 - 10*A*a*b*c*x^2 + 6*B*a^2*b - A*
a*b^2 - 8*A*a^2*c)/((c*x^4 + b*x^2 + a)^2*(b^4 - 8*a*b^2*c + 16*a^2*c^2))
```

Mupad [B] (verification not implemented)

Time = 7.82 (sec) , antiderivative size = 587, normalized size of antiderivative = 3.45

$$\int \frac{x^3(A+Bx^2)}{(a+bx^2+cx^4)^3}\,dx$$
$$= \frac{\operatorname{atan}\left(\frac{\left(x^2\left(\frac{(B\,b^2\,c^2-3\,A\,b\,c^3+2\,B\,a\,c^3)\,(B\,b^2-3\,A\,c\,b+2\,B\,a\,c)}{a\,(4\,a\,c-b^2)^{9/2}\,(16\,a^2\,c^2-8\,a\,b^2\,c+b^4)}+\frac{b\,(B\,b^2-3\,A\,c\,b+2\,B\,a\,c)^2\,(32\,a^2\,b\,c^4-16\,a\,b^3\,c^3+2\,b^5\,c^2)}{2\,a\,(4\,a\,c-b^2)^{15/2}\,(16\,a^2\,c^2-8\,a\,b^2\,c+b^4)}\right)+\frac{2\,b\,c^2\,(B\,b^2-3\,A\,c\,b+2\,B\,c\cdots}{(4\,a\,c-b^2)^{15/2}}\cdots}{18\,A^2\,b^2\,c^4-24\,A\,B\,a\,b\,c^4-12\,A\,B\,b^3\,c^3+8\,B^2\,a^2\,c^4+8\,B^2\,a\,b^2\,c^3+2\,B^2\,b^4\cdots}\right.\cdots}{(4\,a\,c-b^2)^{5/2}}$$
$$-\frac{\frac{-6\,B\,a^2\,b+8\,A\,c\,a^2+A\,a\,b^2}{4\,(16\,a^2\,c^2-8\,a\,b^2\,c+b^4)}+\frac{x^2\,(2\,B\,c\,a^2-5\,B\,a\,b^2+5\,A\,c\,a\,b+A\,b^3)}{2\,(16\,a^2\,c^2-8\,a\,b^2\,c+b^4)}-\frac{3\,b\,x^4\,(B\,b^2-3\,A\,c\,b+2\,B\,a\,c)}{4\,(16\,a^2\,c^2-8\,a\,b^2\,c+b^4)}-\frac{c\,x^6\,(B\,b^2-3\,A\,c\,b+2\,B\,a\,c)}{2\,(16\,a^2\,c^2-8\,a\,b^2\,c+b^4)}}{x^4\,(b^2+2\,a\,c)+a^2+c^2\,x^8+2\,a\,b\,x^2+2\,b\,c\,x^6}$$

```
[In] int((x^3*(A + B*x^2))/(a + b*x^2 + c*x^4)^3,x)
```

```
[Out] (atan(((x^2*(((B*b^2*c^2 - 3*A*b*c^3 + 2*B*a*c^3)*(B*b^2 - 3*A*b*c + 2*B*a*
c))/(a*(4*a*c - b^2)^(9/2)*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (b*(B*b^2 - 3*
A*b*c + 2*B*a*c)^2*(2*b^5*c^2 - 16*a*b^3*c^3 + 32*a^2*b*c^4))/(2*a*(4*a*c -
 b^2)^(15/2)*(b^4 + 16*a^2*c^2 - 8*a*b^2*c))) + (2*b*c^2*(B*b^2 - 3*A*b*c +
 2*B*a*c)^2)/(4*a*c - b^2)^(15/2))*(b^4*(4*a*c - b^2)^5 + 16*a^2*c^2*(4*a*c
 - b^2)^5 - 8*a*b^2*c*(4*a*c - b^2)^5))/(18*A^2*b^2*c^4 + 8*B^2*a^2*c^4 + 2
*B^2*b^4*c^2 - 12*A*B*b^3*c^3 + 8*B^2*a*b^2*c^3 - 24*A*B*a*b*c^4))*(B*b^2 -
 3*A*b*c + 2*B*a*c))/(4*a*c - b^2)^(5/2) - ((A*a*b^2 + 8*A*a^2*c - 6*B*a^2*
b)/(4*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (x^2*(A*b^3 - 5*B*a*b^2 + 2*B*a^2*c
 + 5*A*a*b*c))/(2*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) - (3*b*x^4*(B*b^2 - 3*A*b
*c + 2*B*a*c))/(4*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) - (c*x^6*(B*b^2 - 3*A*b*c
 + 2*B*a*c))/(2*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)))/(x^4*(2*a*c + b^2) + a^2 +
 c^2*x^8 + 2*a*b*x^2 + 2*b*c*x^6)
```

3.129 $\int \frac{x(A+Bx^2)}{(a+bx^2+cx^4)^3}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 139

$$\int \frac{x(A+Bx^2)}{(a+bx^2+cx^4)^3}\,dx=-\frac{Ab-2aB-(bB-2Ac)x^2}{4(b^2-4ac)(a+bx^2+cx^4)^2}-\frac{3(bB-2Ac)(b+2cx^2)}{4(b^2-4ac)^2(a+bx^2+cx^4)}+\frac{3c(bB-2Ac)\operatorname{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{(b^2-4ac)^{5/2}}$$

```
[Out] 1/4*(-A*b+2*B*a+(-2*A*c+B*b)*x^2)/(-4*a*c+b^2)/(c*x^4+b*x^2+a)^2-3/4*(-2*A*
c+B*b)*(2*c*x^2+b)/(-4*a*c+b^2)^2/(c*x^4+b*x^2+a)+3*c*(-2*A*c+B*b)*arctanh(
(2*c*x^2+b)/(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(5/2)
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {1261, 652, 628, 632, 212}

$$\int \frac{x(A+Bx^2)}{(a+bx^2+cx^4)^3}\,dx=\frac{3c(bB-2Ac)\operatorname{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{(b^2-4ac)^{5/2}}-\frac{3(b+2cx^2)(bB-2Ac)}{4(b^2-4ac)^2(a+bx^2+cx^4)}-\frac{-2aB-(x^2(bB-2Ac))+Ab}{4(b^2-4ac)(a+bx^2+cx^4)^2}$$

```
[In] Int[(x*(A + B*x^2))/(a + b*x^2 + c*x^4)^3,x]
```

```
[Out] -1/4*(A*b - 2*a*B - (b*B - 2*A*c)*x^2)/((b^2 - 4*a*c)*(a + b*x^2 + c*x^4)^2
) - (3*(b*B - 2*A*c)*(b + 2*c*x^2))/(4*(b^2 - 4*a*c)^2*(a + b*x^2 + c*x^4))
```

```
 + (3*c*(b*B - 2*A*c)*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(b^2 - 4*a*
c)^(5/2)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] - Dist[2*c*((2*p +
 3)/((p + 1)*(b^2 - 4*a*c))), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2] && Int
egerQ[4*p]
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 652

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[((b*d - 2*a*e + (2*c*d - b*e)*x)/((p + 1)*(b^2 - 4*a*c)))*(a + b*
x + c*x^2)^(p + 1), x] - Dist[(2*p + 3)*((2*c*d - b*e)/((p + 1)*(b^2 - 4*a*
c))), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] &&
 NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2]
```

Rule 1261

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(
p_.), x_Symbol] :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x],
 x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{1}{2}\text{Subst}\left(\int \frac{A+Bx}{(a+bx+cx^2)^3}\,dx, x, x^2\right)\\ &= -\frac{Ab-2aB-(bB-2Ac)x^2}{4\,(b^2-4ac)\,(a+bx^2+cx^4)^2}+\frac{(3(bB-2Ac))\text{Subst}\left(\int \frac{1}{(a+bx+cx^2)^2}\,dx, x, x^2\right)}{4\,(b^2-4ac)}\end{aligned}$$

$$= -\frac{Ab - 2aB - (bB - 2Ac)x^2}{4(b^2 - 4ac)(a + bx^2 + cx^4)^2} - \frac{3(bB - 2Ac)(b + 2cx^2)}{4(b^2 - 4ac)^2(a + bx^2 + cx^4)} - \frac{(3c(bB - 2Ac))\text{Subst}\left(\int \frac{1}{a+bx+cx^2}\,dx, x, x^2\right)}{2(b^2 - 4ac)^2}$$

$$= -\frac{Ab - 2aB - (bB - 2Ac)x^2}{4(b^2 - 4ac)(a + bx^2 + cx^4)^2} - \frac{3(bB - 2Ac)(b + 2cx^2)}{4(b^2 - 4ac)^2(a + bx^2 + cx^4)} + \frac{(3c(bB - 2Ac))\text{Subst}\left(\int \frac{1}{b^2-4ac-x^2}\,dx, x, b + 2cx^2\right)}{(b^2 - 4ac)^2}$$

$$= -\frac{Ab - 2aB - (bB - 2Ac)x^2}{4(b^2 - 4ac)(a + bx^2 + cx^4)^2} - \frac{3(bB - 2Ac)(b + 2cx^2)}{4(b^2 - 4ac)^2(a + bx^2 + cx^4)} + \frac{3c(bB - 2Ac)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{(b^2 - 4ac)^{5/2}}$$

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.02

$$\int \frac{x(A + Bx^2)}{(a + bx^2 + cx^4)^3}\,dx = \frac{-\frac{3(bB-2Ac)(b+2cx^2)}{a+bx^2+cx^4} + \frac{(b^2-4ac)(B(2a+bx^2)-A(b+2cx^2))}{(a+bx^2+cx^4)^2} - \frac{12c(bB-2Ac)\arctan\left(\frac{b+2cx^2}{\sqrt{-b^2+4ac}}\right)}{\sqrt{-b^2+4ac}}}{4(b^2 - 4ac)^2}$$

```
[In] Integrate[(x*(A + B*x^2))/(a + b*x^2 + c*x^4)^3,x]
```

```
[Out] ((-3*(b*B - 2*A*c)*(b + 2*c*x^2))/(a + b*x^2 + c*x^4) + ((b^2 - 4*a*c)*(B*(
2*a + b*x^2) - A*(b + 2*c*x^2)))/(a + b*x^2 + c*x^4)^2 - (12*c*(b*B - 2*A*c
)*ArcTan[(b + 2*c*x^2)/Sqrt[-b^2 + 4*a*c]])/Sqrt[-b^2 + 4*a*c])/(4*(b^2 - 4
*a*c)^2)
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.06

method	result
default	$\frac{(2Ac-Bb)x^2+Ab-2Ba}{4(4ac-b^2)(c\,x^4+b\,x^2+a)^2}+\frac{3(2Ac-Bb)\left(\frac{2c\,x^2+b}{\left(4ac-b^2\right)\left(c\,x^4+b\,x^2+a\right)}+\frac{4c\arctan\left(\frac{2c\,x^2+b}{\sqrt{4ac-b^2}}\right)}{\left(4ac-b^2\right)^{\frac{3}{2}}}\right)}{4(4ac-b^2)}$
risch	$\frac{\frac{3c^2(2Ac-Bb)x^6}{2\left(16a^2c^2-8a\,b^2c+b^4\right)}+\frac{9bc(2Ac-Bb)x^4}{4\left(16a^2c^2-8a\,b^2c+b^4\right)}+\frac{\left(5ac+b^2\right)(2Ac-Bb)x^2}{32a^2c^2-16a\,b^2c+2b^4}+\frac{10Aabc-A\,b^3-8a^2Bc-Ba\,b^2}{64a^2c^2-32a\,b^2c+4b^4}}{(c\,x^4+b\,x^2+a)^2}-\frac{3c^2\ln\left(\left(\left(-4ac+b^2\right)^{\frac{5}{2}}-16a^2b\,c\right.\right.}{}$

[In] int(x*(B*x^2+A)/(c*x^4+b*x^2+a)^3,x,method=_RETURNVERBOSE)

[Out] 1/4*((2*A*c-B*b)*x^2+A*b-2*B*a)/(4*a*c-b^2)/(c*x^4+b*x^2+a)^2+3/4*(2*A*c-B*b)/(4*a*c-b^2)*((2*c*x^2+b)/(4*a*c-b^2)/(c*x^4+b*x^2+a)+4*c/(4*a*c-b^2)^(3/2)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2)))

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 542 vs. $2(131) = 262$.

Time = 0.30 (sec) , antiderivative size = 1109, normalized size of antiderivative = 7.98

$$\int \frac{x(A+Bx^2)}{(a+bx^2+cx^4)^3}\,dx = \text{Too large to display}$$

[In] integrate(x*(B*x^2+A)/(c*x^4+b*x^2+a)^3,x, algorithm="fricas")

[Out] [-1/4*(6*(B*b^3*c^2 + 8*A*a*c^4 - 2*(2*B*a*b + A*b^2)*c^3)*x^6 + B*a*b^4 + A*b^5 + 9*(B*b^4*c + 8*A*a*b*c^3 - 2*(2*B*a*b^2 + A*b^3)*c^2)*x^4 - 8*(4*B*a^3 - 5*A*a^2*b)*c^2 + 2*(B*b^5 + 40*A*a^2*c^3 - 2*(10*B*a^2*b + A*a*b^2)*c^2 + (B*a*b^3 - 2*A*b^4)*c)*x^2 + 6*((B*b*c^3 - 2*A*c^4)*x^8 + 2*(B*b^2*c^2 - 2*A*b*c^3)*x^6 + B*a^2*b*c - 2*A*a^2*c^2 + (B*b^3*c - 4*A*a*c^3 + 2*(B*a*b - A*b^2)*c^2)*x^4 + 2*(B*a*b^2*c - 2*A*a*b*c^2)*x^2)*sqrt(b^2 - 4*a*c)*log((2*c^2*x^4 + 2*b*c*x^2 + b^2 - 2*a*c - (2*c*x^2 + b)*sqrt(b^2 - 4*a*c))/(c*x^4 + b*x^2 + a)) + 2*(2*B*a^2*b^2 - 7*A*a*b^3)*c)/((b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*x^8 + a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3 + 2*(b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4)*x^6 + (b^8 - 10*a*b^6*c + 24*a^2*b^4*c^2 + 32*a^3*b^2*c^3 - 128*a^4*c^4)*x^4 + 2*(a*b^7 - 12*a^2*b^5*c + 48*a^3*b^3*c^2 - 64*a^4*b*c^3)*x^2), -1/4*(6*(B*b^3*c^2 + 8*A*a*c^4 - 2*(2*B*a*b + A*b^2)*c^3)*x^6 + B*a*b^4 + A*b^5 + 9*(B*b^4*c + 8*A*a*b*c^3 - 2*(2*B*a*b^2 + A*b^3)*c^2)*x^4 - 8*(4*B*a^3 - 5*A*a^2*b)*c^2 + 2*(B*b^5 + 40*A*a^2*c^3 - 2*(10*B*a^2*b + A*a*b^2)*c^2 + (B*a*b^3 - 2*A*b^4)*c)*x^2 - 12*((B*b*c^3 - 2*A*c^4)*x^8 + 2*(B*b^2*c^2 - 2*A*b*c^3)*x^6 + B*a^2*b*c - 2*A*a^2*c^2 + (B*b^3*c - 4*A*a*c^3 + 2*(B*a*b - A*b^2)*c^2)*x^4 + 2*(B*a*b^2*c - 2*A*a*b*c^2)*x^2)*sqrt(-b^2 + 4*a*c)*arctan(-(2*c*x^2 + b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4*a*c)) + 2*(2*B*a^2*b^2 - 7*A*a*b^3)*c)/((b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*x^8 + a^2*

```
b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3 + 2*(b^7*c - 12*a*b^5*c^2
+ 48*a^2*b^3*c^3 - 64*a^3*b*c^4)*x^6 + (b^8 - 10*a*b^6*c + 24*a^2*b^4*c^2 +
 32*a^3*b^2*c^3 - 128*a^4*c^4)*x^4 + 2*(a*b^7 - 12*a^2*b^5*c + 48*a^3*b^3*c
^2 - 64*a^4*b*c^3)*x^2)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 661 vs. $2(134) = 268$.

Time = 165.16 (sec) , antiderivative size = 661, normalized size of antiderivative = 4.76

$$\int \frac{x(A+Bx^2)}{(a+bx^2+cx^4)^3}\,dx$$

$$= \frac{3c\sqrt{-\frac{1}{(4ac-b^2)^5}}(-2Ac+Bb)\log\left(x^2+\frac{-6Abc^2+3Bb^2c-192a^3c^4\sqrt{-\frac{1}{(4ac-b^2)^5}}(-2Ac+Bb)+144a^2b^2c^3\sqrt{-\frac{1}{(4ac-b^2)^5}}(-2Ac+Bb)}{-12Ac^3+6Bbc^2}\right.}{2}$$

$$-\frac{3c\sqrt{-\frac{1}{(4ac-b^2)^5}}(-2Ac+Bb)\log\left(x^2+\frac{-6Abc^2+3Bb^2c+192a^3c^4\sqrt{-\frac{1}{(4ac-b^2)^5}}(-2Ac+Bb)-144a^2b^2c^3\sqrt{-\frac{1}{(4ac-b^2)^5}}(-2Ac+}{-12Ac^3+6Bbc^2}\right.}{2}$$

$$+\frac{10Aabc-Ab^3-8Ba^2c-Bab^2+x^6\cdot(12Ac^3-6Bbc^2)+x^4\cdot(18Abc^2-9Bb^2c)+x^2}{64a^4c^2-32a^3b^2c+4a^2b^4+x^8\cdot(64a^2c^4-32ab^2c^3+4b^4c^2)+x^6\cdot(128a^2bc^3-64ab^3c^2+8b^5c)+x^4\cdot(12}$$

```
[In] integrate(x*(B*x**2+A)/(c*x**4+b*x**2+a)**3,x)
```

```
[Out] 3*c*sqrt(-1/(4*a*c - b**2)**5)*(-2*A*c + B*b)*log(x**2 + (-6*A*b*c**2 + 3*B
*b**2*c - 192*a**3*c**4*sqrt(-1/(4*a*c - b**2)**5)*(-2*A*c + B*b) + 144*a**
2*b**2*c**3*sqrt(-1/(4*a*c - b**2)**5)*(-2*A*c + B*b) - 36*a*b**4*c**2*sqrt
(-1/(4*a*c - b**2)**5)*(-2*A*c + B*b) + 3*b**6*c*sqrt(-1/(4*a*c - b**2)**5)
*(-2*A*c + B*b))/(-12*A*c**3 + 6*B*b*c**2))/2 - 3*c*sqrt(-1/(4*a*c - b**2)*
*5)*(-2*A*c + B*b)*log(x**2 + (-6*A*b*c**2 + 3*B*b**2*c + 192*a**3*c**4*sqr
t(-1/(4*a*c - b**2)**5)*(-2*A*c + B*b) - 144*a**2*b**2*c**3*sqrt(-1/(4*a*c
- b**2)**5)*(-2*A*c + B*b) + 36*a*b**4*c**2*sqrt(-1/(4*a*c - b**2)**5)*(-2*
A*c + B*b) - 3*b**6*c*sqrt(-1/(4*a*c - b**2)**5)*(-2*A*c + B*b))/(-12*A*c**
3 + 6*B*b*c**2))/2 + (10*A*a*b*c - A*b**3 - 8*B*a**2*c - B*a*b**2 + x**6*(1
2*A*c**3 - 6*B*b*c**2) + x**4*(18*A*b*c**2 - 9*B*b**2*c) + x**2*(20*A*a*c**
2 + 4*A*b**2*c - 10*B*a*b*c - 2*B*b**3))/(64*a**4*c**2 - 32*a**3*b**2*c + 4
*a**2*b**4 + x**8*(64*a**2*c**4 - 32*a*b**2*c**3 + 4*b**4*c**2) + x**6*(128
*a**2*b*c**3 - 64*a*b**3*c**2 + 8*b**5*c) + x**4*(128*a**3*c**3 - 24*a*b**4
*c + 4*b**6) + x**2*(128*a**3*b*c**2 - 64*a**2*b**3*c + 8*a*b**5))
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x(A+Bx^2)}{(a+bx^2+cx^4)^3}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(x*(B*x^2+A)/(c*x^4+b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

Giac [A] (verification not implemented)

none

Time = 1.45 (sec) , antiderivative size = 208, normalized size of antiderivative = 1.50

$$\int \frac{x(A+Bx^2)}{(a+bx^2+cx^4)^3}\,dx = -\frac{3\,(Bbc-2\,Ac^2)\arctan\left(\frac{2\,cx^2+b}{\sqrt{-b^2+4\,ac}}\right)}{(b^4-8\,ab^2c+16\,a^2c^2)\sqrt{-b^2+4\,ac}} - \frac{6\,Bbc^2x^6-12\,Ac^3x^6+9\,Bb^2cx^4-18\,Abc^2x^4+2\,Bb^3x^2+10\,Babcx^2-4\,Ab^2cx^2-20\,Aac^2x^2+Bab^2 \cdots}{4\,(cx^4+bx^2+a)^2(b^4-8\,ab^2c+16\,a^2c^2)}$$

```
[In] integrate(x*(B*x^2+A)/(c*x^4+b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] -3*(B*b*c - 2*A*c^2)*arctan((2*c*x^2 + b)/sqrt(-b^2 + 4*a*c))/((b^4 - 8*a*b
^2*c + 16*a^2*c^2)*sqrt(-b^2 + 4*a*c)) - 1/4*(6*B*b*c^2*x^6 - 12*A*c^3*x^6
+ 9*B*b^2*c*x^4 - 18*A*b*c^2*x^4 + 2*B*b^3*x^2 + 10*B*a*b*c*x^2 - 4*A*b^2*c
*x^2 - 20*A*a*c^2*x^2 + B*a*b^2 + A*b^3 + 8*B*a^2*c - 10*A*a*b*c)/((c*x^4 +
 b*x^2 + a)^2*(b^4 - 8*a*b^2*c + 16*a^2*c^2))
```

Mupad [B] (verification not implemented)

Time = 7.74 (sec) , antiderivative size = 517, normalized size of antiderivative = 3.72

$$\int \frac{x(A+Bx^2)}{(a+bx^2+cx^4)^3}\,dx$$

$$= \frac{3\,c\,\mathrm{atan}\left(\frac{\left(x^2\left(\frac{3\,c\,(2\,A\,c-B\,b)\left(6\,A\,c^4-3\,B\,b\,c^3\right)}{a\left(4\,a\,c-b^2\right)^{9/2}\left(16\,a^2\,c^2-8\,a\,b^2\,c+b^4\right)}+\frac{9\,b\,c^2\,(2\,A\,c-B\,b)^2\left(32\,a^2\,b\,c^4-16\,a\,b^3\,c^3+2\,b^5\,c^2\right)}{2\,a\left(4\,a\,c-b^2\right)^{15/2}\left(16\,a^2\,c^2-8\,a\,b^2\,c+b^4\right)}\right)+\frac{18\,b\,c^4\,(2\,A\,c-B\,b)^2}{\left(4\,a\,c-b^2\right)^{15/2}}\right)\left(b^4\left(4\,a\,c-b^2\right)^{5} \cdots}{72\,A^2\,c^6-72\,A\,B\,b\,c^5+18\,B^2\,b^2\,c^4}\right. \cdots}{\left(4\,a\,c-b^2\right)^{5/2}} - \frac{\frac{8\,B\,c\,a^2+B\,a\,b^2-10\,A\,c\,a\,b+A\,b^3}{4\,(16\,a^2\,c^2-8\,a\,b^2\,c+b^4)}-\frac{9\,x^4\,\left(2\,A\,b\,c^2-B\,b^2\,c\right)}{4\,(16\,a^2\,c^2-8\,a\,b^2\,c+b^4)}+\frac{x^2\,\left(B\,b^3-2\,A\,b^2\,c+5\,B\,a\,b\,c-10\,A\,a\,c^2\right)}{2\,(16\,a^2\,c^2-8\,a\,b^2\,c+b^4)}-\frac{3\,c^2\,x^6\,(2\,A\,c-B\,b)}{2\,(16\,a^2\,c^2-8\,a\,b^2\,c+b^4)}}{x^4\,\left(b^2+2\,a\,c\right)+a^2+c^2\,x^8+2\,a\,b\,x^2+2\,b\,c\,x^6}$$

```
[In] int((x*(A + B*x^2))/(a + b*x^2 + c*x^4)^3,x)
```

```
[Out] (3*c*atan(((x^2*((3*c*(2*A*c - B*b)*(6*A*c^4 - 3*B*b*c^3))/(a*(4*a*c - b^2)
^(9/2)*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (9*b*c^2*(2*A*c - B*b)^2*(2*b^5*c^
2 - 16*a*b^3*c^3 + 32*a^2*b*c^4))/(2*a*(4*a*c - b^2)^(15/2)*(b^4 + 16*a^2*c
^2 - 8*a*b^2*c))) + (18*b*c^4*(2*A*c - B*b)^2)/(4*a*c - b^2)^(15/2))*(b^4*(
4*a*c - b^2)^5 + 16*a^2*c^2*(4*a*c - b^2)^5 - 8*a*b^2*c*(4*a*c - b^2)^5))/(
72*A^2*c^6 + 18*B^2*b^2*c^4 - 72*A*B*b*c^5))*(2*A*c - B*b))/(4*a*c - b^2)^(
5/2) - ((A*b^3 + B*a*b^2 + 8*B*a^2*c - 10*A*a*b*c)/(4*(b^4 + 16*a^2*c^2 - 8
*a*b^2*c)) - (9*x^4*(2*A*b*c^2 - B*b^2*c))/(4*(b^4 + 16*a^2*c^2 - 8*a*b^2*c
)) + (x^2*(B*b^3 - 10*A*a*c^2 - 2*A*b^2*c + 5*B*a*b*c))/(2*(b^4 + 16*a^2*c^
2 - 8*a*b^2*c)) - (3*c^2*x^6*(2*A*c - B*b))/(2*(b^4 + 16*a^2*c^2 - 8*a*b^2*
c)))/(x^4*(2*a*c + b^2) + a^2 + c^2*x^8 + 2*a*b*x^2 + 2*b*c*x^6)
```

3.130 $\int \frac{A+Bx^2}{x(a+bx^2+cx^4)^3} dx$

Optimal result

Integrand size = 25, antiderivative size = 252

$$\begin{aligned}
&\int \frac{A+Bx^2}{x\left(a+bx^2+cx^4\right)^3} dx \\
&= -\frac{abB-A(b^2-2ac)-(Ab-2aB)cx^2}{4a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2} \\
&\quad+\frac{6a^2bBc+A(2b^4-15ab^2c+16a^2c^2)+2c(6a^2Bc+A(b^3-7abc))\,x^2}{4a^2\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)} \\
&\quad-\frac{(12a^3Bc^2-A(b^5-10ab^3c+30a^2bc^2))\operatorname{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2a^3\left(b^2-4ac\right)^{5/2}} \\
&\quad+\frac{A\log(x)}{a^3}-\frac{A\log\left(a+bx^2+cx^4\right)}{4a^3}
\end{aligned}$$

```
[Out] 1/4*(-a*b*B+A*(-2*a*c+b^2)+(A*b-2*B*a)*c*x^2)/a/(-4*a*c+b^2)/(c*x^4+b*x^2+a
)^2+1/4*(6*a^2*b*B*c+A*(16*a^2*c^2-15*a*b^2*c+2*b^4)+2*c*(6*a^2*B*c+A*(-7*a
*b*c+b^3))*x^2)/a^2/(-4*a*c+b^2)^2/(c*x^4+b*x^2+a)-1/2*(12*a^3*B*c^2-A*(30*
a^2*b*c^2-10*a*b^3*c+b^5))*arctanh((2*c*x^2+b)/(-4*a*c+b^2)^(1/2))/a^3/(-4*
a*c+b^2)^(5/2)+A*ln(x)/a^3-1/4*A*ln(c*x^4+b*x^2+a)/a^3
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 252, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.280, Rules used = {1265, 836, 814, 648, 632, 212, 642}

$$\begin{aligned}
&\int \frac{A+Bx^2}{x\left(a+bx^2+cx^4\right)^3}\,dx \\
&= -\frac{A\log\left(a+bx^2+cx^4\right)}{4a^3}+\frac{A\log(x)}{a^3} \\
&\quad+\frac{2cx^2(6a^2Bc+A(b^3-7abc))+A(16a^2c^2-15ab^2c+2b^4)+6a^2bBc}{4a^2\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)} \\
&\quad-\frac{(12a^3Bc^2-A(30a^2bc^2-10ab^3c+b^5))\,\text{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2a^3\left(b^2-4ac\right)^{5/2}} \\
&\quad-\frac{-A(b^2-2ac)-(cx^2(Ab-2aB))+abB}{4a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2}
\end{aligned}$$

```
[In] Int[(A + B*x^2)/(x*(a + b*x^2 + c*x^4)^3),x]
```

```
[Out] -1/4*(a*b*B - A*(b^2 - 2*a*c) - (A*b - 2*a*B)*c*x^2)/(a*(b^2 - 4*a*c)*(a +
b*x^2 + c*x^4)^2) + (6*a^2*b*B*c + A*(2*b^4 - 15*a*b^2*c + 16*a^2*c^2) + 2*
c*(6*a^2*B*c + A*(b^3 - 7*a*b*c))*x^2)/(4*a^2*(b^2 - 4*a*c)^2*(a + b*x^2 +
c*x^4)) - ((12*a^3*B*c^2 - A*(b^5 - 10*a*b^3*c + 30*a^2*b*c^2))*ArcTanh[(b
+ 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(2*a^3*(b^2 - 4*a*c)^(5/2)) + (A*Log[x])/a^3
 - (A*Log[a + b*x^2 + c*x^4])/(4*a^3)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 814

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)/(a +
 b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 836

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(f*(b*c*d - b^2*e + 2
*a*c*e) - a*g*(2*c*d - b*e) + c*(f*(2*c*d - b*e) - g*(b*d - 2*a*e))*x)*((a
+ b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2))), x]
 + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^m*
(a + b*x + c*x^2)^(p + 1)*Simp[f*(b*c*d*e*(2*p - m + 2) + b^2*e^2*(p + m +
2) - 2*c^2*d^2*(2*p + 3) - 2*a*c*e^2*(m + 2*p + 3)) - g*(a*e*(b*e - 2*c*d*m
 + b*e*m) - b*d*(3*c*d - b*e + 2*c*d*p - b*e*p)) + c*e*(g*(b*d - 2*a*e) - f
*(2*c*d - b*e))*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1
] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 1265

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rubi steps

$$\text{integral} = \frac{1}{2}\text{Subst}\left(\int \frac{A+Bx}{x\,(a+bx+cx^2)^3}\,dx, x, x^2\right)$$

$$= -\frac{abB-A(b^2-2ac)-(Ab-2aB)cx^2}{4a\,(b^2-4ac)\,(a+bx^2+cx^4)^2} - \frac{\text{Subst}\left(\int \frac{-2A(b^2-4ac)-3(Ab-2aB)cx}{x(a+bx+cx^2)^2}\,dx, x, x^2\right)}{4a\,(b^2-4ac)}$$

$$= -\frac{abB - A(b^2-2ac) - (Ab-2aB)cx^2}{4a(b^2-4ac)(a+bx^2+cx^4)^2} + \frac{6a^2bBc + A(2b^4-15ab^2c+16a^2c^2) + 2c(6a^2Bc + A(b^3-7abc))x^2}{4a^2(b^2-4ac)^2(a+bx^2+cx^4)} + \frac{\text{Subst}\left(\int \frac{2A(b^2-4ac)^2+2c(Ab^3-7aAbc+6a^2Bc)x}{x(a+bx+cx^2)}dx, x, x^2\right)}{4a^2(b^2-4ac)^2}$$

$$= -\frac{abB - A(b^2-2ac) - (Ab-2aB)cx^2}{4a(b^2-4ac)(a+bx^2+cx^4)^2} + \frac{6a^2bBc + A(2b^4-15ab^2c+16a^2c^2) + 2c(6a^2Bc + A(b^3-7abc))x^2}{4a^2(b^2-4ac)^2(a+bx^2+cx^4)} + \frac{\text{Subst}\left(\int \left(\frac{2A(-b^2+4ac)^2}{ax} + \frac{2\left(6a^3Bc^2 - A(b^5-9ab^3c+23a^2bc^2) - Ac(b^2-4ac)^2x\right)}{a(a+bx+cx^2)}\right)dx, x, x^2\right)}{4a^2(b^2-4ac)^2}$$

$$= -\frac{abB - A(b^2-2ac) - (Ab-2aB)cx^2}{4a(b^2-4ac)(a+bx^2+cx^4)^2} + \frac{6a^2bBc + A(2b^4-15ab^2c+16a^2c^2) + 2c(6a^2Bc + A(b^3-7abc))x^2}{4a^2(b^2-4ac)^2(a+bx^2+cx^4)} + \frac{A\log(x)}{a^3} + \frac{\text{Subst}\left(\int \frac{6a^3Bc^2 - A(b^5-9ab^3c+23a^2bc^2) - Ac(b^2-4ac)^2x}{a+bx+cx^2}dx, x, x^2\right)}{2a^3(b^2-4ac)^2}$$

$$= -\frac{abB - A(b^2-2ac) - (Ab-2aB)cx^2}{4a(b^2-4ac)(a+bx^2+cx^4)^2} + \frac{6a^2bBc + A(2b^4-15ab^2c+16a^2c^2) + 2c(6a^2Bc + A(b^3-7abc))x^2}{4a^2(b^2-4ac)^2(a+bx^2+cx^4)} + \frac{A\log(x)}{a^3} - \frac{A\text{Subst}\left(\int \frac{b+2cx}{a+bx+cx^2}dx, x, x^2\right)}{4a^3} + \frac{(12a^3Bc^2 - A(b^5-10ab^3c+30a^2bc^2))\,\text{Subst}\left(\int \frac{1}{a+bx+cx^2}dx, x, x^2\right)}{4a^3(b^2-4ac)^2}$$

$$= -\frac{abB - A(b^2-2ac) - (Ab-2aB)cx^2}{4a(b^2-4ac)(a+bx^2+cx^4)^2} + \frac{6a^2bBc + A(2b^4-15ab^2c+16a^2c^2) + 2c(6a^2Bc + A(b^3-7abc))x^2}{4a^2(b^2-4ac)^2(a+bx^2+cx^4)} + \frac{A\log(x)}{a^3} - \frac{A\log(a+bx^2+cx^4)}{4a^3} - \frac{(12a^3Bc^2 - A(b^5-10ab^3c+30a^2bc^2))\,\text{Subst}\left(\int \frac{1}{b^2-4ac-x^2}dx, x, b+2cx^2\right)}{2a^3(b^2-4ac)^2}$$

$$= -\frac{abB - A(b^2-2ac) - (Ab-2aB)cx^2}{4a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2} + \frac{6a^2bBc + A(2b^4-15ab^2c+16a^2c^2) + 2c(6a^2Bc + A(b^3-7abc))\,x^2}{4a^2\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)} - \frac{(12a^3Bc^2 - A(b^5-10ab^3c+30a^2bc^2))\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2a^3\left(b^2-4ac\right)^{5/2}} + \frac{A\log(x)}{a^3} - \frac{A\log\left(a+bx^2+cx^4\right)}{4a^3}$$

Mathematica [A] (verified)

Time = 0.42 (sec) , antiderivative size = 396, normalized size of antiderivative = 1.57

$$\int \frac{A+Bx^2}{x\left(a+bx^2+cx^4\right)^3}\,dx$$

$$= \frac{\frac{a^2\left(-aB(b+2cx^2)+A(b^2-2ac+bcx^2)\right)}{(b^2-4ac)(a+bx^2+cx^4)^2} + \frac{a\left(2Ab^3(b+cx^2)-aAbc(15b+14cx^2)+2a^2c(3bB+8Ac+6Bcx^2)\right)}{(b^2-4ac)^2(a+bx^2+cx^4)} + 4A\log(x) - \frac{\left(-12a^3Bc^2+A\right.}{}}{}$$

```
[In] Integrate[(A + B*x^2)/(x*(a + b*x^2 + c*x^4)^3),x]
```

```
[Out] ((a^2*(-(a*B*(b + 2*c*x^2)) + A*(b^2 - 2*a*c + b*c*x^2)))/((b^2 - 4*a*c)*(a
 + b*x^2 + c*x^4)^2) + (a*(2*A*b^3*(b + c*x^2) - a*A*b*c*(15*b + 14*c*x^2)
+ 2*a^2*c*(3*b*B + 8*A*c + 6*B*c*x^2)))/((b^2 - 4*a*c)^2*(a + b*x^2 + c*x^4
)) + 4*A*Log[x] - ((-12*a^3*B*c^2 + A*(b^5 - 10*a*b^3*c + 30*a^2*b*c^2 + b^
4*Sqrt[b^2 - 4*a*c] - 8*a*b^2*c*Sqrt[b^2 - 4*a*c] + 16*a^2*c^2*Sqrt[b^2 - 4
*a*c]))*Log[b - Sqrt[b^2 - 4*a*c] + 2*c*x^2])/(b^2 - 4*a*c)^(5/2) - ((12*a^
3*B*c^2 + A*(-b^5 + 10*a*b^3*c - 30*a^2*b*c^2 + b^4*Sqrt[b^2 - 4*a*c] - 8*a
*b^2*c*Sqrt[b^2 - 4*a*c] + 16*a^2*c^2*Sqrt[b^2 - 4*a*c]))*Log[b + Sqrt[b^2
- 4*a*c] + 2*c*x^2])/(b^2 - 4*a*c)^(5/2))/(4*a^3)
```

Maple [A] (verified)

Time = 0.19 (sec) , antiderivative size = 442, normalized size of antiderivative = 1.75

method	result
default	$\frac{A\ln(x)}{a^3}-\frac{\frac{\frac{a\,c^2\left(7Aabc-A\,b^3-6a^2Bc\right)x^6}{16a^2c^2-8a\,b^2c+b^4}-\frac{ac\left(16A\,a^2c^2-29Aa\,b^2c+4A\,b^4+18a^2bBc\right)x^4}{2\left(16a^2c^2-8a\,b^2c+b^4\right)}+\frac{a\left(A\,a^2b\,c^2+6Aa\,b^3c-A\,b^5-10a^3B\,c^2-2B\,a^2b^2c\right)x^2}{16a^2c^2-8a\,b^2c+b^4}-}{\left(c\,x^4+b\,x^2+a\right)^2}}{}$
risch	$\frac{-\frac{c^2\left(7Aabc-A\,b^3-6a^2Bc\right)x^6}{2a^2\left(16a^2c^2-8a\,b^2c+b^4\right)}+\frac{c\left(16A\,a^2c^2-29Aa\,b^2c+4A\,b^4+18a^2bBc\right)x^4}{4\left(16a^2c^2-8a\,b^2c+b^4\right)a^2}-\frac{\left(A\,a^2b\,c^2+6Aa\,b^3c-A\,b^5-10a^3B\,c^2-2B\,a^2b^2c\right)x^2}{2a^2\left(16a^2c^2-8a\,b^2c+b^4\right)}+\frac{24A\,a^2c^2-21A}{4a(1}}{(c\,x^4+b\,x^2+a)^2}$

```
[In] int((B*x^2+A)/x/(c*x^4+b*x^2+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] A*ln(x)/a^3-1/2/a^3*((a*c^2*(7*A*a*b*c-A*b^3-6*B*a^2*c)/(16*a^2*c^2-8*a*b^2
*c+b^4)*x^6-1/2*a*c*(16*A*a^2*c^2-29*A*a*b^2*c+4*A*b^4+18*B*a^2*b*c)/(16*a^
2*c^2-8*a*b^2*c+b^4)*x^4+a*(A*a^2*b*c^2+6*A*a*b^3*c-A*b^5-10*B*a^3*c^2-2*B*
a^2*b^2*c)/(16*a^2*c^2-8*a*b^2*c+b^4)*x^2-1/2*a^2*(24*A*a^2*c^2-21*A*a*b^2*
c+3*A*b^4+10*B*a^2*b*c-B*a*b^3)/(16*a^2*c^2-8*a*b^2*c+b^4))/(c*x^4+b*x^2+a)
^2+1/(16*a^2*c^2-8*a*b^2*c+b^4)*(1/2*(16*A*a^2*c^3-8*A*a*b^2*c^2+A*b^4*c)/c
*ln(c*x^4+b*x^2+a)+2*(23*A*a^2*b*c^2-9*A*a*b^3*c+A*b^5-6*a^3*B*c^2-1/2*(16*
A*a^2*c^3-8*A*a*b^2*c^2+A*b^4*c)*b/c)/(4*a*c-b^2)^(1/2)*arctan((2*c*x^2+b)/
(4*a*c-b^2)^(1/2))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1235 vs. 2(240) = 480.

Time = 1.95 (sec) , antiderivative size = 2494, normalized size of antiderivative = 9.90

$$\int \frac{A+Bx^2}{x\left(a+bx^2+cx^4\right)^3}\,dx=\text{Too large to display}$$

```
[In] integrate((B*x^2+A)/x/(c*x^4+b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] [-1/4*(B*a^3*b^5 - 3*A*a^2*b^6 + 96*A*a^5*c^3 - 2*(A*a*b^5*c^2 - 4*(6*B*a^4
 - 7*A*a^3*b)*c^4 + (6*B*a^3*b^2 - 11*A*a^2*b^3)*c^3)*x^6 - (4*A*a*b^6*c -
64*A*a^4*c^4 - 12*(6*B*a^4*b - 11*A*a^3*b^2)*c^3 + 9*(2*B*a^3*b^3 - 5*A*a^2
*b^4)*c^2)*x^4 + 4*(10*B*a^5*b - 27*A*a^4*b^2)*c^2 - 2*(A*a*b^7 - 4*(10*B*a
^5 - A*a^4*b)*c^3 + (2*B*a^4*b^2 + 23*A*a^3*b^3)*c^2 + 2*(B*a^3*b^4 - 5*A*a
^2*b^5)*c)*x^2 - ((A*b^5*c^2 - 10*A*a*b^3*c^3 - 6*(2*B*a^3 - 5*A*a^2*b)*c^4
)*x^8 + A*a^2*b^5 - 10*A*a^3*b^3*c + 2*(A*b^6*c - 10*A*a*b^4*c^2 - 6*(2*B*a
^3*b - 5*A*a^2*b^2)*c^3)*x^6 + (A*b^7 - 8*A*a*b^5*c - 12*(2*B*a^4 - 5*A*a^3
*b)*c^3 - 2*(6*B*a^3*b^2 - 5*A*a^2*b^3)*c^2)*x^4 - 6*(2*B*a^5 - 5*A*a^4*b)*
c^2 + 2*(A*a*b^6 - 10*A*a^2*b^4*c - 6*(2*B*a^4*b - 5*A*a^3*b^2)*c^2)*x^2)*s
```

qrt(b^2 - 4*a*c)*log((2*c^2*x^4 + 2*b*c*x^2 + b^2 - 2*a*c + (2*c*x^2 + b)*s
qrt(b^2 - 4*a*c))/(c*x^4 + b*x^2 + a)) - (14*B*a^4*b^3 - 33*A*a^3*b^4)*c +
(A*a^2*b^6 - 12*A*a^3*b^4*c + 48*A*a^4*b^2*c^2 - 64*A*a^5*c^3 + (A*b^6*c^2
- 12*A*a*b^4*c^3 + 48*A*a^2*b^2*c^4 - 64*A*a^3*c^5)*x^8 + 2*(A*b^7*c - 12*A
*a*b^5*c^2 + 48*A*a^2*b^3*c^3 - 64*A*a^3*b*c^4)*x^6 + (A*b^8 - 10*A*a*b^6*c
+ 24*A*a^2*b^4*c^2 + 32*A*a^3*b^2*c^3 - 128*A*a^4*c^4)*x^4 + 2*(A*a*b^7 -
12*A*a^2*b^5*c + 48*A*a^3*b^3*c^2 - 64*A*a^4*b*c^3)*x^2)*log(c*x^4 + b*x^2
+ a) - 4*(A*a^2*b^6 - 12*A*a^3*b^4*c + 48*A*a^4*b^2*c^2 - 64*A*a^5*c^3 + (A
*b^6*c^2 - 12*A*a*b^4*c^3 + 48*A*a^2*b^2*c^4 - 64*A*a^3*c^5)*x^8 + 2*(A*b^7
*c - 12*A*a*b^5*c^2 + 48*A*a^2*b^3*c^3 - 64*A*a^3*b*c^4)*x^6 + (A*b^8 - 10*
A*a*b^6*c + 24*A*a^2*b^4*c^2 + 32*A*a^3*b^2*c^3 - 128*A*a^4*c^4)*x^4 + 2*(A
*a*b^7 - 12*A*a^2*b^5*c + 48*A*a^3*b^3*c^2 - 64*A*a^4*b*c^3)*x^2)*log(x))/(
a^5*b^6 - 12*a^6*b^4*c + 48*a^7*b^2*c^2 - 64*a^8*c^3 + (a^3*b^6*c^2 - 12*a^
4*b^4*c^3 + 48*a^5*b^2*c^4 - 64*a^6*c^5)*x^8 + 2*(a^3*b^7*c - 12*a^4*b^5*c^
2 + 48*a^5*b^3*c^3 - 64*a^6*b*c^4)*x^6 + (a^3*b^8 - 10*a^4*b^6*c + 24*a^5*b
^4*c^2 + 32*a^6*b^2*c^3 - 128*a^7*c^4)*x^4 + 2*(a^4*b^7 - 12*a^5*b^5*c + 48
*a^6*b^3*c^2 - 64*a^7*b*c^3)*x^2), -1/4*(B*a^3*b^5 - 3*A*a^2*b^6 + 96*A*a^5
c^3 - 2(A*a*b^5*c^2 - 4*(6*B*a^4 - 7*A*a^3*b)*c^4 + (6*B*a^3*b^2 - 11*A*a
^2*b^3)*c^3)*x^6 - (4*A*a*b^6*c - 64*A*a^4*c^4 - 12*(6*B*a^4*b - 11*A*a^3*b
^2)*c^3 + 9*(2*B*a^3*b^3 - 5*A*a^2*b^4)*c^2)*x^4 + 4*(10*B*a^5*b - 27*A*a^4
*b^2)*c^2 - 2*(A*a*b^7 - 4*(10*B*a^5 - A*a^4*b)*c^3 + (2*B*a^4*b^2 + 23*A*a
^3*b^3)*c^2 + 2*(B*a^3*b^4 - 5*A*a^2*b^5)*c)*x^2 - 2*((A*b^5*c^2 - 10*A*a*b
^3*c^3 - 6*(2*B*a^3 - 5*A*a^2*b)*c^4)*x^8 + A*a^2*b^5 - 10*A*a^3*b^3*c + 2*
(A*b^6*c - 10*A*a*b^4*c^2 - 6*(2*B*a^3*b - 5*A*a^2*b^2)*c^3)*x^6 + (A*b^7 -
8*A*a*b^5*c - 12*(2*B*a^4 - 5*A*a^3*b)*c^3 - 2*(6*B*a^3*b^2 - 5*A*a^2*b^3)
*c^2)*x^4 - 6*(2*B*a^5 - 5*A*a^4*b)*c^2 + 2*(A*a*b^6 - 10*A*a^2*b^4*c - 6*(
2*B*a^4*b - 5*A*a^3*b^2)*c^2)*x^2)*sqrt(-b^2 + 4*a*c)*arctan(-(2*c*x^2 + b)
*sqrt(-b^2 + 4*a*c)/(b^2 - 4*a*c)) - (14*B*a^4*b^3 - 33*A*a^3*b^4)*c + (A*a
^2*b^6 - 12*A*a^3*b^4*c + 48*A*a^4*b^2*c^2 - 64*A*a^5*c^3 + (A*b^6*c^2 - 12
*A*a*b^4*c^3 + 48*A*a^2*b^2*c^4 - 64*A*a^3*c^5)*x^8 + 2*(A*b^7*c - 12*A*a*b
^5*c^2 + 48*A*a^2*b^3*c^3 - 64*A*a^3*b*c^4)*x^6 + (A*b^8 - 10*A*a*b^6*c + 2
4*A*a^2*b^4*c^2 + 32*A*a^3*b^2*c^3 - 128*A*a^4*c^4)*x^4 + 2*(A*a*b^7 - 12*A
*a^2*b^5*c + 48*A*a^3*b^3*c^2 - 64*A*a^4*b*c^3)*x^2)*log(c*x^4 + b*x^2 + a)
- 4*(A*a^2*b^6 - 12*A*a^3*b^4*c + 48*A*a^4*b^2*c^2 - 64*A*a^5*c^3 + (A*b^6
*c^2 - 12*A*a*b^4*c^3 + 48*A*a^2*b^2*c^4 - 64*A*a^3*c^5)*x^8 + 2*(A*b^7*c -
12*A*a*b^5*c^2 + 48*A*a^2*b^3*c^3 - 64*A*a^3*b*c^4)*x^6 + (A*b^8 - 10*A*a*
b^6*c + 24*A*a^2*b^4*c^2 + 32*A*a^3*b^2*c^3 - 128*A*a^4*c^4)*x^4 + 2*(A*a*b
^7 - 12*A*a^2*b^5*c + 48*A*a^3*b^3*c^2 - 64*A*a^4*b*c^3)*x^2)*log(x))/(a^5*
b^6 - 12*a^6*b^4*c + 48*a^7*b^2*c^2 - 64*a^8*c^3 + (a^3*b^6*c^2 - 12*a^4*b^
4*c^3 + 48*a^5*b^2*c^4 - 64*a^6*c^5)*x^8 + 2*(a^3*b^7*c - 12*a^4*b^5*c^2 +
48*a^5*b^3*c^3 - 64*a^6*b*c^4)*x^6 + (a^3*b^8 - 10*a^4*b^6*c + 24*a^5*b^4*c
^2 + 32*a^6*b^2*c^3 - 128*a^7*c^4)*x^4 + 2*(a^4*b^7 - 12*a^5*b^5*c + 48*a^6
*b^3*c^2 - 64*a^7*b*c^3)*x^2)]

Sympy [F(-1)]

Timed out.

$$\int \frac{A+Bx^2}{x\left(a+bx^2+cx^4\right)^3}\,dx = \text{Timed out}$$

```
[In] integrate((B*x**2+A)/x/(c*x**4+b*x**2+a)**3,x)
```

```
[Out] Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{A+Bx^2}{x\left(a+bx^2+cx^4\right)^3}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((B*x^2+A)/x/(c*x^4+b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

Giac [A] (verification not implemented)

none

Time = 1.51 (sec) , antiderivative size = 421, normalized size of antiderivative = 1.67

$$\int \frac{A+Bx^2}{x\left(a+bx^2+cx^4\right)^3}\,dx = -\frac{\left(Ab^5-10\,Aab^3c-12\,Ba^3c^2+30\,Aa^2bc^2\right)\arctan\left(\frac{2\,cx^2+b}{\sqrt{-b^2+4\,ac}}\right)}{2\,\left(a^3b^4-8\,a^4b^2c+16\,a^5c^2\right)\sqrt{-b^2+4\,ac}} - \frac{A\log\left(cx^4+bx^2+a\right)}{4\,a^3} + \frac{A\log\left(x^2\right)}{2\,a^3} + \frac{3\,Ab^4c^2x^8-24\,Aab^2c^3x^8+48\,Aa^2c^4x^8+6\,Ab^5cx^6-44\,Aab^3c^2x^6+24\,Ba^3c^3x^6+68\,Aa^2bc^3x^6+3\,Ab^6x^4}{}$$

```
[In] integrate((B*x^2+A)/x/(c*x^4+b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] -1/2*(A*b^5 - 10*A*a*b^3*c - 12*B*a^3*c^2 + 30*A*a^2*b*c^2)*arctan((2*c*x^2
 + b)/sqrt(-b^2 + 4*a*c))/((a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2)*sqrt(-b^2 +
 4*a*c)) - 1/4*A*log(c*x^4 + b*x^2 + a)/a^3 + 1/2*A*log(x^2)/a^3 + 1/8*(3*A
*b^4*c^2*x^8 - 24*A*a*b^2*c^3*x^8 + 48*A*a^2*c^4*x^8 + 6*A*b^5*c*x^6 - 44*A
*a*b^3*c^2*x^6 + 24*B*a^3*c^3*x^6 + 68*A*a^2*b*c^3*x^6 + 3*A*b^6*x^4 - 10*A
*a*b^4*c*x^4 + 36*B*a^3*b*c^2*x^4 - 58*A*a^2*b^2*c^2*x^4 + 128*A*a^3*c^3*x^
```

```
4 + 10*A*a*b^5*x^2 + 8*B*a^3*b^2*c*x^2 - 72*A*a^2*b^3*c*x^2 + 40*B*a^4*c^2*
x^2 + 92*A*a^3*b*c^2*x^2 - 2*B*a^3*b^3 + 9*A*a^2*b^4 + 20*B*a^4*b*c - 66*A*
a^3*b^2*c + 96*A*a^4*c^2)/((a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2)*(c*x^4 + b*
x^2 + a)^2)
```

Mupad [B] (verification not implemented)

Time = 14.34 (sec) , antiderivative size = 11674, normalized size of antiderivative = 46.33

$$\int \frac{A+Bx^2}{x\left(a+bx^2+cx^4\right)^3}\,dx = \text{Too large to display}$$

```
[In] int((A + B*x^2)/(x*(a + b*x^2 + c*x^4)^3),x)
```

```
[Out] ((3*A*b^4 + 24*A*a^2*c^2 - B*a*b^3 - 21*A*a*b^2*c + 10*B*a^2*b*c)/(4*a*(b^4
 + 16*a^2*c^2 - 8*a*b^2*c)) + (x^2*(A*b^5 + 10*B*a^3*c^2 - 6*A*a*b^3*c - A*
a^2*b*c^2 + 2*B*a^2*b^2*c))/(2*a^2*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (x^4*(
16*A*a^2*c^3 + 4*A*b^4*c - 29*A*a*b^2*c^2 + 18*B*a^2*b*c^2))/(4*a^2*(b^4 +
16*a^2*c^2 - 8*a*b^2*c)) + (c^2*x^6*(A*b^3 + 6*B*a^2*c - 7*A*a*b*c))/(2*a^2
*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)))/(x^4*(2*a*c + b^2) + a^2 + c^2*x^8 + 2*a*
b*x^2 + 2*b*c*x^6) + (A*log(x))/a^3 - (log(((c^5*x^2*(A*b^3 + 6*B*a^2*c - 7
*A*a*b*c)^3)/(a^6*(4*a*c - b^2)^6) - ((A + a^3*(-(A*b^5 - 12*B*a^3*c^2 - 10
*A*a*b^3*c + 30*A*a^2*b*c^2)^2/(a^6*(4*a*c - b^2)^5))^(1/2))*((c^3*(4*A^2*b
^8 - 36*B^2*a^5*c^3 + 302*A^2*a^2*b^4*c^2 - 497*A^2*a^3*b^2*c^3 - 61*A^2*a*
b^6*c - 204*A*B*a^3*b^3*c^2 + 24*A*B*a^2*b^5*c + 468*A*B*a^4*b*c^3))/(a^4*(
4*a*c - b^2)^4) - ((A + a^3*(-(A*b^5 - 12*B*a^3*c^2 - 10*A*a*b^3*c + 30*A*a
^2*b*c^2)^2/(a^6*(4*a*c - b^2)^5))^(1/2))*((2*c^3*x^2*(A*b^5 + 60*B*a^3*c^2
 - 2*A*a*b^3*c + 10*A*a^2*b*c^2 - 24*B*a^2*b^2*c))/(a^2*(4*a*c - b^2)^2) +
(4*b*c^2*(A*b^5 - 6*B*a^3*c^2 - 9*A*a*b^3*c + 23*A*a^2*b*c^2))/(a^2*(4*a*c
- b^2)^2) + (b*c^2*(A + a^3*(-(A*b^5 - 12*B*a^3*c^2 - 10*A*a*b^3*c + 30*A*a
^2*b*c^2)^2/(a^6*(4*a*c - b^2)^5))^(1/2))*(a*b + 3*b^2*x^2 - 10*a*c*x^2))/a
^3))/(4*a^3) + (c^4*x^2*(6*A^2*b^7 + 409*A^2*a^2*b^3*c^2 + 480*A*B*a^4*c^3
- 89*A^2*a*b^5*c - 560*A^2*a^3*b*c^3 + 36*B^2*a^4*b*c^2 - 324*A*B*a^3*b^2*c
^2 + 42*A*B*a^2*b^4*c))/(a^4*(4*a*c - b^2)^4)))/(4*a^3) + (A*c^4*(A*b^3 + 6
*B*a^2*c - 7*A*a*b*c)^2)/(a^6*(4*a*c - b^2)^4))*((c^5*x^2*(A*b^3 + 6*B*a^2*
c - 7*A*a*b*c)^3)/(a^6*(4*a*c - b^2)^6) - ((A - a^3*(-(A*b^5 - 12*B*a^3*c^2
 - 10*A*a*b^3*c + 30*A*a^2*b*c^2)^2/(a^6*(4*a*c - b^2)^5))^(1/2))*((c^3*(4*
A^2*b^8 - 36*B^2*a^5*c^3 + 302*A^2*a^2*b^4*c^2 - 497*A^2*a^3*b^2*c^3 - 61*A
^2*a*b^6*c - 204*A*B*a^3*b^3*c^2 + 24*A*B*a^2*b^5*c + 468*A*B*a^4*b*c^3))/(
a^4*(4*a*c - b^2)^4) - ((A - a^3*(-(A*b^5 - 12*B*a^3*c^2 - 10*A*a*b^3*c + 3
0*A*a^2*b*c^2)^2/(a^6*(4*a*c - b^2)^5))^(1/2))*((2*c^3*x^2*(A*b^5 + 60*B*a^
3*c^2 - 2*A*a*b^3*c + 10*A*a^2*b*c^2 - 24*B*a^2*b^2*c))/(a^2*(4*a*c - b^2)^
2) + (4*b*c^2*(A*b^5 - 6*B*a^3*c^2 - 9*A*a*b^3*c + 23*A*a^2*b*c^2))/(a^2*(4
*a*c - b^2)^2) + (b*c^2*(A - a^3*(-(A*b^5 - 12*B*a^3*c^2 - 10*A*a*b^3*c + 3
0*A*a^2*b*c^2)^2/(a^6*(4*a*c - b^2)^5))^(1/2))*(a*b + 3*b^2*x^2 - 10*a*c*x^
```

2))/a^3))/(4*a^3) + (c^4*x^2*(6*A^2*b^7 + 409*A^2*a^2*b^3*c^2 + 480*A*B*a^4
*c^3 - 89*A^2*a*b^5*c - 560*A^2*a^3*b*c^3 + 36*B^2*a^4*b*c^2 - 324*A*B*a^3*
b^2*c^2 + 42*A*B*a^2*b^4*c))/(a^4*(4*a*c - b^2)^4)))/(4*a^3) + (A*c^4*(A*b^
3 + 6*B*a^2*c - 7*A*a*b*c)^2)/(a^6*(4*a*c - b^2)^4)))*(2*A*b^10 - 2048*A*a^
5*c^5 - 40*A*a*b^8*c + 320*A*a^2*b^6*c^2 - 1280*A*a^3*b^4*c^3 + 2560*A*a^4*
b^2*c^4))/(2*(4*a^3*b^10 - 4096*a^8*c^5 - 80*a^4*b^8*c + 640*a^5*b^6*c^2 -
2560*a^6*b^4*c^3 + 5120*a^7*b^2*c^4)) - (atan((x^2*((((((((30720*B*a^11*c^9
 + 5120*A*a^10*b*c^9 + 2*A*a^4*b^13*c^3 - 36*A*a^5*b^11*c^4 + 276*A*a^6*b^9
*c^5 - 1216*A*a^7*b^7*c^6 + 3456*A*a^8*b^5*c^7 - 6144*A*a^9*b^3*c^8 - 48*B*
a^6*b^10*c^4 + 888*B*a^7*b^8*c^5 - 6528*B*a^8*b^6*c^6 + 23808*B*a^9*b^4*c^7
 - 43008*B*a^10*b^2*c^8)/(a^6*b^12 + 4096*a^12*c^6 - 24*a^7*b^10*c + 240*a^
8*b^8*c^2 - 1280*a^9*b^6*c^3 + 3840*a^10*b^4*c^4 - 6144*a^11*b^2*c^5) - ((2
*A*b^10 - 2048*A*a^5*c^5 - 40*A*a*b^8*c + 320*A*a^2*b^6*c^2 - 1280*A*a^3*b^
4*c^3 + 2560*A*a^4*b^2*c^4)*(163840*a^13*b*c^9 - 12*a^6*b^15*c^2 + 328*a^7*
b^13*c^3 - 3840*a^8*b^11*c^4 + 24960*a^9*b^9*c^5 - 97280*a^10*b^7*c^6 + 227
328*a^11*b^5*c^7 - 294912*a^12*b^3*c^8))/(2*(4*a^3*b^10 - 4096*a^8*c^5 - 80
*a^4*b^8*c + 640*a^5*b^6*c^2 - 2560*a^6*b^4*c^3 + 5120*a^7*b^2*c^4)*(a^6*b^
12 + 4096*a^12*c^6 - 24*a^7*b^10*c + 240*a^8*b^8*c^2 - 1280*a^9*b^6*c^3 + 3
840*a^10*b^4*c^4 - 6144*a^11*b^2*c^5)))*(A*b^5 - 12*B*a^3*c^2 - 10*A*a*b^3*
c + 30*A*a^2*b*c^2))/(4*a^3*(4*a*c - b^2)^(5/2)) - ((A*b^5 - 12*B*a^3*c^2 -
 10*A*a*b^3*c + 30*A*a^2*b*c^2)*(2*A*b^10 - 2048*A*a^5*c^5 - 40*A*a*b^8*c +
 320*A*a^2*b^6*c^2 - 1280*A*a^3*b^4*c^3 + 2560*A*a^4*b^2*c^4)*(163840*a^13*
b*c^9 - 12*a^6*b^15*c^2 + 328*a^7*b^13*c^3 - 3840*a^8*b^11*c^4 + 24960*a^9*
b^9*c^5 - 97280*a^10*b^7*c^6 + 227328*a^11*b^5*c^7 - 294912*a^12*b^3*c^8))/
(8*a^3*(4*a*c - b^2)^(5/2)*(4*a^3*b^10 - 4096*a^8*c^5 - 80*a^4*b^8*c + 640*
a^5*b^6*c^2 - 2560*a^6*b^4*c^3 + 5120*a^7*b^2*c^4)*(a^6*b^12 + 4096*a^12*c^
6 - 24*a^7*b^10*c + 240*a^8*b^8*c^2 - 1280*a^9*b^6*c^3 + 3840*a^10*b^4*c^4
 - 6144*a^11*b^2*c^5)))*(2*A*b^10 - 2048*A*a^5*c^5 - 40*A*a*b^8*c + 320*A*a^
2*b^6*c^2 - 1280*A*a^3*b^4*c^3 + 2560*A*a^4*b^2*c^4))/(2*(4*a^3*b^10 - 4096
*a^8*c^5 - 80*a^4*b^8*c + 640*a^5*b^6*c^2 - 2560*a^6*b^4*c^3 + 5120*a^7*b^2
*c^4)) - (((6*A^2*a^2*b^11*c^4 - 137*A^2*a^3*b^9*c^5 + 1217*A^2*a^4*b^7*c^6
 - 5256*A^2*a^5*b^5*c^7 + 11024*A^2*a^6*b^3*c^8 + 36*B^2*a^6*b^5*c^6 - 288*
B^2*a^7*b^3*c^7 + 7680*A*B*a^8*c^9 - 8960*A^2*a^7*b*c^9 + 576*B^2*a^8*b*c^8
 + 42*A*B*a^4*b^8*c^5 - 660*A*B*a^5*b^6*c^6 + 3744*A*B*a^6*b^4*c^7 - 9024*A
*B*a^7*b^2*c^8)/(a^6*b^12 + 4096*a^12*c^6 - 24*a^7*b^10*c + 240*a^8*b^8*c^2
 - 1280*a^9*b^6*c^3 + 3840*a^10*b^4*c^4 - 6144*a^11*b^2*c^5) - (((30720*B*a
^11*c^9 + 5120*A*a^10*b*c^9 + 2*A*a^4*b^13*c^3 - 36*A*a^5*b^11*c^4 + 276*A*
a^6*b^9*c^5 - 1216*A*a^7*b^7*c^6 + 3456*A*a^8*b^5*c^7 - 6144*A*a^9*b^3*c^8
 - 48*B*a^6*b^10*c^4 + 888*B*a^7*b^8*c^5 - 6528*B*a^8*b^6*c^6 + 23808*B*a^9*
b^4*c^7 - 43008*B*a^10*b^2*c^8)/(a^6*b^12 + 4096*a^12*c^6 - 24*a^7*b^10*c +
 240*a^8*b^8*c^2 - 1280*a^9*b^6*c^3 + 3840*a^10*b^4*c^4 - 6144*a^11*b^2*c^5
) - ((2*A*b^10 - 2048*A*a^5*c^5 - 40*A*a*b^8*c + 320*A*a^2*b^6*c^2 - 1280*A
*a^3*b^4*c^3 + 2560*A*a^4*b^2*c^4)*(163840*a^13*b*c^9 - 12*a^6*b^15*c^2 + 3
28*a^7*b^13*c^3 - 3840*a^8*b^11*c^4 + 24960*a^9*b^9*c^5 - 97280*a^10*b^7*c^
6 + 227328*a^11*b^5*c^7 - 294912*a^12*b^3*c^8))/(2*(4*a^3*b^10 - 4096*a^8*c

^5 - 80*a^4*b^8*c + 640*a^5*b^6*c^2 - 2560*a^6*b^4*c^3 + 5120*a^7*b^2*c^4)*
(a^6*b^12 + 4096*a^12*c^6 - 24*a^7*b^10*c + 240*a^8*b^8*c^2 - 1280*a^9*b^6*
c^3 + 3840*a^10*b^4*c^4 - 6144*a^11*b^2*c^5)))*(2*A*b^10 - 2048*A*a^5*c^5 -
40*A*a*b^8*c + 320*A*a^2*b^6*c^2 - 1280*A*a^3*b^4*c^3 + 2560*A*a^4*b^2*c^4
))/(2*(4*a^3*b^10 - 4096*a^8*c^5 - 80*a^4*b^8*c + 640*a^5*b^6*c^2 - 2560*a^
6*b^4*c^3 + 5120*a^7*b^2*c^4)))*(A*b^5 - 12*B*a^3*c^2 - 10*A*a*b^3*c + 30*A
*a^2*b*c^2))/(4*a^3*(4*a*c - b^2)^(5/2)) + ((A*b^5 - 12*B*a^3*c^2 - 10*A*a*
b^3*c + 30*A*a^2*b*c^2)^3*(163840*a^13*b*c^9 - 12*a^6*b^15*c^2 + 328*a^7*b^
13*c^3 - 3840*a^8*b^11*c^4 + 24960*a^9*b^9*c^5 - 97280*a^10*b^7*c^6 + 22732
8*a^11*b^5*c^7 - 294912*a^12*b^3*c^8))/(64*a^9*(4*a*c - b^2)^(15/2)*(a^6*b^
12 + 4096*a^12*c^6 - 24*a^7*b^10*c + 240*a^8*b^8*c^2 - 1280*a^9*b^6*c^3 + 3
840*a^10*b^4*c^4 - 6144*a^11*b^2*c^5)))*(3*A*b^8 + 160*A*a^4*c^4 - 39*A*a*b
^6*c + 18*B*a^4*b*c^3 + 180*A*a^2*b^4*c^2 - 325*A*a^3*b^2*c^3 - 6*B*a^3*b^3
*c^2))/(8*a^3*c^2*(4*a*c - b^2)^(13/2)*(6*A^2*b^10 - 6400*A^2*a^5*c^5 - 36*
B^2*a^6*c^4 + 960*A^2*a^2*b^6*c^2 - 3850*A^2*a^3*b^4*c^3 + 7775*A^2*a^4*b^2
*c^4 - 120*A^2*a*b^8*c + 6*A*B*a^3*b^5*c^2 - 60*A*B*a^4*b^3*c^3 + 180*A*B*a
^5*b*c^4)) + (((A^3*b^9*c^5 + 216*B^3*a^6*c^8 + 147*A^3*a^2*b^5*c^7 - 343*A
^3*a^3*b^3*c^8 - 21*A^3*a*b^7*c^6 - 756*A*B^2*a^5*b*c^8 + 108*A*B^2*a^4*b^3
*c^7 + 18*A^2*B*a^2*b^6*c^6 - 252*A^2*B*a^3*b^4*c^7 + 882*A^2*B*a^4*b^2*c^8
)/(a^6*b^12 + 4096*a^12*c^6 - 24*a^7*b^10*c + 240*a^8*b^8*c^2 - 1280*a^9*b^
6*c^3 + 3840*a^10*b^4*c^4 - 6144*a^11*b^2*c^5) - (((6*A^2*a^2*b^11*c^4 - 13
7*A^2*a^3*b^9*c^5 + 1217*A^2*a^4*b^7*c^6 - 5256*A^2*a^5*b^5*c^7 + 11024*A^2
*a^6*b^3*c^8 + 36*B^2*a^6*b^5*c^6 - 288*B^2*a^7*b^3*c^7 + 7680*A*B*a^8*c^9
- 8960*A^2*a^7*b*c^9 + 576*B^2*a^8*b*c^8 + 42*A*B*a^4*b^8*c^5 - 660*A*B*a^5
*b^6*c^6 + 3744*A*B*a^6*b^4*c^7 - 9024*A*B*a^7*b^2*c^8)/(a^6*b^12 + 4096*a^
12*c^6 - 24*a^7*b^10*c + 240*a^8*b^8*c^2 - 1280*a^9*b^6*c^3 + 3840*a^10*b^4
*c^4 - 6144*a^11*b^2*c^5) - (((30720*B*a^11*c^9 + 5120*A*a^10*b*c^9 + 2*A*a
^4*b^13*c^3 - 36*A*a^5*b^11*c^4 + 276*A*a^6*b^9*c^5 - 1216*A*a^7*b^7*c^6 +
3456*A*a^8*b^5*c^7 - 6144*A*a^9*b^3*c^8 - 48*B*a^6*b^10*c^4 + 888*B*a^7*b^8
*c^5 - 6528*B*a^8*b^6*c^6 + 23808*B*a^9*b^4*c^7 - 43008*B*a^10*b^2*c^8)/(a^
6*b^12 + 4096*a^12*c^6 - 24*a^7*b^10*c + 240*a^8*b^8*c^2 - 1280*a^9*b^6*c^3
+ 3840*a^10*b^4*c^4 - 6144*a^11*b^2*c^5) - ((2*A*b^10 - 2048*A*a^5*c^5 - 4
0*A*a*b^8*c + 320*A*a^2*b^6*c^2 - 1280*A*a^3*b^4*c^3 + 2560*A*a^4*b^2*c^4)*
(163840*a^13*b*c^9 - 12*a^6*b^15*c^2 + 328*a^7*b^13*c^3 - 3840*a^8*b^11*c^4
+ 24960*a^9*b^9*c^5 - 97280*a^10*b^7*c^6 + 227328*a^11*b^5*c^7 - 294912*a^
12*b^3*c^8))/(2*(4*a^3*b^10 - 4096*a^8*c^5 - 80*a^4*b^8*c + 640*a^5*b^6*c^2
- 2560*a^6*b^4*c^3 + 5120*a^7*b^2*c^4)*(a^6*b^12 + 4096*a^12*c^6 - 24*a^7*
b^10*c + 240*a^8*b^8*c^2 - 1280*a^9*b^6*c^3 + 3840*a^10*b^4*c^4 - 6144*a^11
*b^2*c^5)))*(2*A*b^10 - 2048*A*a^5*c^5 - 40*A*a*b^8*c + 320*A*a^2*b^6*c^2 -
1280*A*a^3*b^4*c^3 + 2560*A*a^4*b^2*c^4))/(2*(4*a^3*b^10 - 4096*a^8*c^5 -
80*a^4*b^8*c + 640*a^5*b^6*c^2 - 2560*a^6*b^4*c^3 + 5120*a^7*b^2*c^4)))*(2*
A*b^10 - 2048*A*a^5*c^5 - 40*A*a*b^8*c + 320*A*a^2*b^6*c^2 - 1280*A*a^3*b^4
*c^3 + 2560*A*a^4*b^2*c^4))/(2*(4*a^3*b^10 - 4096*a^8*c^5 - 80*a^4*b^8*c +
640*a^5*b^6*c^2 - 2560*a^6*b^4*c^3 + 5120*a^7*b^2*c^4)) - (((((30720*B*a^11
*c^9 + 5120*A*a^10*b*c^9 + 2*A*a^4*b^13*c^3 - 36*A*a^5*b^11*c^4 + 276*A*a^6

*b^9*c^5 - 1216*A*a^7*b^7*c^6 + 3456*A*a^8*b^5*c^7 - 6144*A*a^9*b^3*c^8 - 48*B*a^6*b^10*c^4 + 888*B*a^7*b^8*c^5 - 6528*B*a^8*b^6*c^6 + 23808*B*a^9*b^4*c^7 - 43008*B*a^10*b^2*c^8)/(a^6*b^12 + 4096*a^12*c^6 - 24*a^7*b^10*c + 240*a^8*b^8*c^2 - 1280*a^9*b^6*c^3 + 3840*a^10*b^4*c^4 - 6144*a^11*b^2*c^5) - ((2*A*b^10 - 2048*A*a^5*c^5 - 40*A*a*b^8*c + 320*A*a^2*b^6*c^2 - 1280*A*a^3*b^4*c^3 + 2560*A*a^4*b^2*c^4)*(163840*a^13*b*c^9 - 12*a^6*b^15*c^2 + 328*a^7*b^13*c^3 - 3840*a^8*b^11*c^4 + 24960*a^9*b^9*c^5 - 97280*a^10*b^7*c^6 + 227328*a^11*b^5*c^7 - 294912*a^12*b^3*c^8))/(2*(4*a^3*b^10 - 4096*a^8*c^5 - 80*a^4*b^8*c + 640*a^5*b^6*c^2 - 2560*a^6*b^4*c^3 + 5120*a^7*b^2*c^4)*(a^6*b^12 + 4096*a^12*c^6 - 24*a^7*b^10*c + 240*a^8*b^8*c^2 - 1280*a^9*b^6*c^3 + 3840*a^10*b^4*c^4 - 6144*a^11*b^2*c^5)))*(A*b^5 - 12*B*a^3*c^2 - 10*A*a*b^3*c + 30*A*a^2*b*c^2))/(4*a^3*(4*a*c - b^2)^(5/2)) - ((A*b^5 - 12*B*a^3*c^2 - 10*A*a*b^3*c + 30*A*a^2*b*c^2)*(2*A*b^10 - 2048*A*a^5*c^5 - 40*A*a*b^8*c + 320*A*a^2*b^6*c^2 - 1280*A*a^3*b^4*c^3 + 2560*A*a^4*b^2*c^4)*(163840*a^13*b*c^9 - 12*a^6*b^15*c^2 + 328*a^7*b^13*c^3 - 3840*a^8*b^11*c^4 + 24960*a^9*b^9*c^5 - 97280*a^10*b^7*c^6 + 227328*a^11*b^5*c^7 - 294912*a^12*b^3*c^8))/(8*a^3*(4*a*c - b^2)^(5/2)*(4*a^3*b^10 - 4096*a^8*c^5 - 80*a^4*b^8*c + 640*a^5*b^6*c^2 - 2560*a^6*b^4*c^3 + 5120*a^7*b^2*c^4)*(a^6*b^12 + 4096*a^12*c^6 - 24*a^7*b^10*c + 240*a^8*b^8*c^2 - 1280*a^9*b^6*c^3 + 3840*a^10*b^4*c^4 - 6144*a^11*b^2*c^5)))*(A*b^5 - 12*B*a^3*c^2 - 10*A*a*b^3*c + 30*A*a^2*b*c^2))/(4*a^3*(4*a*c - b^2)^(5/2)) + ((A*b^5 - 12*B*a^3*c^2 - 10*A*a*b^3*c + 30*A*a^2*b*c^2)^2*(2*A*b^10 - 2048*A*a^5*c^5 - 40*A*a*b^8*c + 320*A*a^2*b^6*c^2 - 1280*A*a^3*b^4*c^3 + 2560*A*a^4*b^2*c^4)*(163840*a^13*b*c^9 - 12*a^6*b^15*c^2 + 328*a^7*b^13*c^3 - 3840*a^8*b^11*c^4 + 24960*a^9*b^9*c^5 - 97280*a^10*b^7*c^6 + 227328*a^11*b^5*c^7 - 294912*a^12*b^3*c^8))/(32*a^6*(4*a*c - b^2)^5*(4*a^3*b^10 - 4096*a^8*c^5 - 80*a^4*b^8*c + 640*a^5*b^6*c^2 - 2560*a^6*b^4*c^3 + 5120*a^7*b^2*c^4)*(a^6*b^12 + 4096*a^12*c^6 - 24*a^7*b^10*c + 240*a^8*b^8*c^2 - 1280*a^9*b^6*c^3 + 3840*a^10*b^4*c^4 - 6144*a^11*b^2*c^5)))*(3*A*b^7 + 6*B*a^4*c^3 - 33*A*a*b^5*c - 135*A*a^3*b*c^3 + 120*A*a^2*b^3*c^2 - 6*B*a^3*b^2*c^2))/(8*a^3*c^2*(4*a*c - b^2)^6*(6*A^2*b^10 - 6400*A^2*a^5*c^5 - 36*B^2*a^6*c^4 + 960*A^2*a^2*b^6*c^2 - 3850*A^2*a^3*b^4*c^3 + 7775*A^2*a^4*b^2*c^4 - 120*A^2*a*b^8*c + 6*A*B*a^3*b^5*c^2 - 60*A*B*a^4*b^3*c^3 + 180*A*B*a^5*b*c^4)))*(16*a^9*b^12*(4*a*c - b^2)^(15/2) + 65536*a^15*c^6*(4*a*c - b^2)^(15/2) - 384*a^10*b^10*c*(4*a*c - b^2)^(15/2) + 3840*a^11*b^8*c^2*(4*a*c - b^2)^(15/2) - 20480*a^12*b^6*c^3*(4*a*c - b^2)^(15/2) + 61440*a^13*b^4*c^4*(4*a*c - b^2)^(15/2) - 98304*a^14*b^2*c^5*(4*a*c - b^2)^(15/2)))/(A^2*b^10*c^2 + 144*B^2*a^6*c^6 + 160*A^2*a^2*b^6*c^4 - 600*A^2*a^3*b^4*c^5 + 900*A^2*a^4*b^2*c^6 - 20*A^2*a*b^8*c^3 - 24*A*B*a^3*b^5*c^4 + 240*A*B*a^4*b^3*c^5 - 720*A*B*a^5*b*c^6) - (((((((384*B*a^9*b*c^6 - 4*A*a^4*b^10*c^2 + 68*A*a^5*b^8*c^3 - 444*A*a^6*b^6*c^4 + 1312*A*a^7*b^4*c^5 - 1472*A*a^8*b^2*c^6 + 24*B*a^7*b^5*c^4 - 192*B*a^8*b^3*c^5)/(a^6*b^8 + 256*a^10*c^4 - 16*a^7*b^6*c + 96*a^8*b^4*c^2 - 256*a^9*b^2*c^3) - ((4*a^7*b^10*c^2 - 64*a^8*b^8*c^3 + 384*a^9*b^6*c^4 - 1024*a^10*b^4*c^5 + 1024*a^11*b^2*c^6)*(2*A*b^10 - 2048*A*a^5*c^5 - 40*A*a*b^8*c + 320*A*a^2*b^6*c^2 - 1280*A*a^3*b^4*c^3 + 2560*A*a^4*b^2*c^4))/(2*(a^6*b^8 + 256*a^10*c^4 - 16*a^7*b^6*c +

96*a^8*b^4*c^2 - 256*a^9*b^2*c^3)*(4*a^3*b^10 - 4096*a^8*c^5 - 80*a^4*b^8*
c + 640*a^5*b^6*c^2 - 2560*a^6*b^4*c^3 + 5120*a^7*b^2*c^4)))*(A*b^5 - 12*B*
a^3*c^2 - 10*A*a*b^3*c + 30*A*a^2*b*c^2))/(4*a^3*(4*a*c - b^2)^(5/2)) - ((A
*b^5 - 12*B*a^3*c^2 - 10*A*a*b^3*c + 30*A*a^2*b*c^2)*(4*a^7*b^10*c^2 - 64*a
^8*b^8*c^3 + 384*a^9*b^6*c^4 - 1024*a^10*b^4*c^5 + 1024*a^11*b^2*c^6)*(2*A*
b^10 - 2048*A*a^5*c^5 - 40*A*a*b^8*c + 320*A*a^2*b^6*c^2 - 1280*A*a^3*b^4*c
^3 + 2560*A*a^4*b^2*c^4))/(8*a^3*(4*a*c - b^2)^(5/2)*(a^6*b^8 + 256*a^10*c^
4 - 16*a^7*b^6*c + 96*a^8*b^4*c^2 - 256*a^9*b^2*c^3)*(4*a^3*b^10 - 4096*a^8
*c^5 - 80*a^4*b^8*c + 640*a^5*b^6*c^2 - 2560*a^6*b^4*c^3 + 5120*a^7*b^2*c^4
)))*(2*A*b^10 - 2048*A*a^5*c^5 - 40*A*a*b^8*c + 320*A*a^2*b^6*c^2 - 1280*A*
a^3*b^4*c^3 + 2560*A*a^4*b^2*c^4))/(2*(4*a^3*b^10 - 4096*a^8*c^5 - 80*a^4*b
^8*c + 640*a^5*b^6*c^2 - 2560*a^6*b^4*c^3 + 5120*a^7*b^2*c^4)) - (((36*B^2*
a^7*c^6 - 4*A^2*a^2*b^8*c^3 + 61*A^2*a^3*b^6*c^4 - 302*A^2*a^4*b^4*c^5 + 49
7*A^2*a^5*b^2*c^6 - 24*A*B*a^4*b^5*c^4 + 204*A*B*a^5*b^3*c^5 - 468*A*B*a^6*
b*c^6)/(a^6*b^8 + 256*a^10*c^4 - 16*a^7*b^6*c + 96*a^8*b^4*c^2 - 256*a^9*b^
2*c^3) - (((384*B*a^9*b*c^6 - 4*A*a^4*b^10*c^2 + 68*A*a^5*b^8*c^3 - 444*A*a
^6*b^6*c^4 + 1312*A*a^7*b^4*c^5 - 1472*A*a^8*b^2*c^6 + 24*B*a^7*b^5*c^4 - 1
92*B*a^8*b^3*c^5)/(a^6*b^8 + 256*a^10*c^4 - 16*a^7*b^6*c + 96*a^8*b^4*c^2 -
256*a^9*b^2*c^3) - ((4*a^7*b^10*c^2 - 64*a^8*b^8*c^3 + 384*a^9*b^6*c^4 - 1
024*a^10*b^4*c^5 + 1024*a^11*b^2*c^6)*(2*A*b^10 - 2048*A*a^5*c^5 - 40*A*a*b
^8*c + 320*A*a^2*b^6*c^2 - 1280*A*a^3*b^4*c^3 + 2560*A*a^4*b^2*c^4))/(2*(a^
6*b^8 + 256*a^10*c^4 - 16*a^7*b^6*c + 96*a^8*b^4*c^2 - 256*a^9*b^2*c^3)*(4*
a^3*b^10 - 4096*a^8*c^5 - 80*a^4*b^8*c + 640*a^5*b^6*c^2 - 2560*a^6*b^4*c^3
+ 5120*a^7*b^2*c^4)))*(2*A*b^10 - 2048*A*a^5*c^5 - 40*A*a*b^8*c + 320*A*a^
2*b^6*c^2 - 1280*A*a^3*b^4*c^3 + 2560*A*a^4*b^2*c^4))/(2*(4*a^3*b^10 - 4096
*a^8*c^5 - 80*a^4*b^8*c + 640*a^5*b^6*c^2 - 2560*a^6*b^4*c^3 + 5120*a^7*b^2
c^4)))(A*b^5 - 12*B*a^3*c^2 - 10*A*a*b^3*c + 30*A*a^2*b*c^2))/(4*a^3*(4*a
*c - b^2)^(5/2)) + ((A*b^5 - 12*B*a^3*c^2 - 10*A*a*b^3*c + 30*A*a^2*b*c^2)^
3*(4*a^7*b^10*c^2 - 64*a^8*b^8*c^3 + 384*a^9*b^6*c^4 - 1024*a^10*b^4*c^5 +
1024*a^11*b^2*c^6))/(64*a^9*(4*a*c - b^2)^(15/2)*(a^6*b^8 + 256*a^10*c^4 -
16*a^7*b^6*c + 96*a^8*b^4*c^2 - 256*a^9*b^2*c^3)))*(16*a^9*b^12*(4*a*c - b^
2)^(15/2) + 65536*a^15*c^6*(4*a*c - b^2)^(15/2) - 384*a^10*b^10*c*(4*a*c -
b^2)^(15/2) + 3840*a^11*b^8*c^2*(4*a*c - b^2)^(15/2) - 20480*a^12*b^6*c^3*(
4*a*c - b^2)^(15/2) + 61440*a^13*b^4*c^4*(4*a*c - b^2)^(15/2) - 98304*a^14*
b^2*c^5*(4*a*c - b^2)^(15/2))*(3*A*b^8 + 160*A*a^4*c^4 - 39*A*a*b^6*c + 18*
B*a^4*b*c^3 + 180*A*a^2*b^4*c^2 - 325*A*a^3*b^2*c^3 - 6*B*a^3*b^3*c^2))/(8*
a^3*c^2*(4*a*c - b^2)^(13/2)*(A^2*b^10*c^2 + 144*B^2*a^6*c^6 + 160*A^2*a^2*
b^6*c^4 - 600*A^2*a^3*b^4*c^5 + 900*A^2*a^4*b^2*c^6 - 20*A^2*a*b^8*c^3 - 24
*A*B*a^3*b^5*c^4 + 240*A*B*a^4*b^3*c^5 - 720*A*B*a^5*b*c^6)*(6*A^2*b^10 - 6
400*A^2*a^5*c^5 - 36*B^2*a^6*c^4 + 960*A^2*a^2*b^6*c^2 - 3850*A^2*a^3*b^4*c
^3 + 7775*A^2*a^4*b^2*c^4 - 120*A^2*a*b^8*c + 6*A*B*a^3*b^5*c^2 - 60*A*B*a^
4*b^3*c^3 + 180*A*B*a^5*b*c^4)) + (((A^3*b^6*c^4 + 49*A^3*a^2*b^2*c^6 + 36*
A*B^2*a^4*c^6 - 14*A^3*a*b^4*c^5 - 84*A^2*B*a^3*b*c^6 + 12*A^2*B*a^2*b^3*c^
5)/(a^6*b^8 + 256*a^10*c^4 - 16*a^7*b^6*c + 96*a^8*b^4*c^2 - 256*a^9*b^2*c^
3) + (((36*B^2*a^7*c^6 - 4*A^2*a^2*b^8*c^3 + 61*A^2*a^3*b^6*c^4 - 302*A^2*a

^4*b^4*c^5 + 497*A^2*a^5*b^2*c^6 - 24*A*B*a^4*b^5*c^4 + 204*A*B*a^5*b^3*c^5 - 468*A*B*a^6*b*c^6)/(a^6*b^8 + 256*a^10*c^4 - 16*a^7*b^6*c + 96*a^8*b^4*c^2 - 256*a^9*b^2*c^3) - (((384*B*a^9*b*c^6 - 4*A*a^4*b^10*c^2 + 68*A*a^5*b^8*c^3 - 444*A*a^6*b^6*c^4 + 1312*A*a^7*b^4*c^5 - 1472*A*a^8*b^2*c^6 + 24*B*a^7*b^5*c^4 - 192*B*a^8*b^3*c^5)/(a^6*b^8 + 256*a^10*c^4 - 16*a^7*b^6*c + 96*a^8*b^4*c^2 - 256*a^9*b^2*c^3) - ((4*a^7*b^10*c^2 - 64*a^8*b^8*c^3 + 384*a^9*b^6*c^4 - 1024*a^10*b^4*c^5 + 1024*a^11*b^2*c^6)*(2*A*b^10 - 2048*A*a^5*c^5 - 40*A*a*b^8*c + 320*A*a^2*b^6*c^2 - 1280*A*a^3*b^4*c^3 + 2560*A*a^4*b^2*c^4))/(2*(a^6*b^8 + 256*a^10*c^4 - 16*a^7*b^6*c + 96*a^8*b^4*c^2 - 256*a^9*b^2*c^3)*(4*a^3*b^10 - 4096*a^8*c^5 - 80*a^4*b^8*c + 640*a^5*b^6*c^2 - 2560*a^6*b^4*c^3 + 5120*a^7*b^2*c^4)))*(2*A*b^10 - 2048*A*a^5*c^5 - 40*A*a*b^8*c + 320*A*a^2*b^6*c^2 - 1280*A*a^3*b^4*c^3 + 2560*A*a^4*b^2*c^4))/(2*(4*a^3*b^10 - 4096*a^8*c^5 - 80*a^4*b^8*c + 640*a^5*b^6*c^2 - 2560*a^6*b^4*c^3 + 5120*a^7*b^2*c^4)))*(2*A*b^10 - 2048*A*a^5*c^5 - 40*A*a*b^8*c + 320*A*a^2*b^6*c^2 - 1280*A*a^3*b^4*c^3 + 2560*A*a^4*b^2*c^4))/(2*(4*a^3*b^10 - 4096*a^8*c^5 - 80*a^4*b^8*c + 640*a^5*b^6*c^2 - 2560*a^6*b^4*c^3 + 5120*a^7*b^2*c^4)) + (((((384*B*a^9*b*c^6 - 4*A*a^4*b^10*c^2 + 68*A*a^5*b^8*c^3 - 444*A*a^6*b^6*c^4 + 1312*A*a^7*b^4*c^5 - 1472*A*a^8*b^2*c^6 + 24*B*a^7*b^5*c^4 - 192*B*a^8*b^3*c^5)/(a^6*b^8 + 256*a^10*c^4 - 16*a^7*b^6*c + 96*a^8*b^4*c^2 - 256*a^9*b^2*c^3) - ((4*a^7*b^10*c^2 - 64*a^8*b^8*c^3 + 384*a^9*b^6*c^4 - 1024*a^10*b^4*c^5 + 1024*a^11*b^2*c^6)*(2*A*b^10 - 2048*A*a^5*c^5 - 40*A*a*b^8*c + 320*A*a^2*b^6*c^2 - 1280*A*a^3*b^4*c^3 + 2560*A*a^4*b^2*c^4))/(2*(a^6*b^8 + 256*a^10*c^4 - 16*a^7*b^6*c + 96*a^8*b^4*c^2 - 256*a^9*b^2*c^3)*(4*a^3*b^10 - 4096*a^8*c^5 - 80*a^4*b^8*c + 640*a^5*b^6*c^2 - 2560*a^6*b^4*c^3 + 5120*a^7*b^2*c^4)))*(A*b^5 - 12*B*a^3*c^2 - 10*A*a*b^3*c + 30*A*a^2*b*c^2))/(4*a^3*(4*a*c - b^2)^(5/2)) - ((A*b^5 - 12*B*a^3*c^2 - 10*A*a*b^3*c + 30*A*a^2*b*c^2)*(4*a^7*b^10*c^2 - 64*a^8*b^8*c^3 + 384*a^9*b^6*c^4 - 1024*a^10*b^4*c^5 + 1024*a^11*b^2*c^6)*(2*A*b^10 - 2048*A*a^5*c^5 - 40*A*a*b^8*c + 320*A*a^2*b^6*c^2 - 1280*A*a^3*b^4*c^3 + 2560*A*a^4*b^2*c^4))/(8*a^3*(4*a*c - b^2)^(5/2)*(a^6*b^8 + 256*a^10*c^4 - 16*a^7*b^6*c + 96*a^8*b^4*c^2 - 256*a^9*b^2*c^3)*(4*a^3*b^10 - 4096*a^8*c^5 - 80*a^4*b^8*c + 640*a^5*b^6*c^2 - 2560*a^6*b^4*c^3 + 5120*a^7*b^2*c^4)))*(A*b^5 - 12*B*a^3*c^2 - 10*A*a*b^3*c + 30*A*a^2*b*c^2))/(4*a^3*(4*a*c - b^2)^(5/2)) - ((A*b^5 - 12*B*a^3*c^2 - 10*A*a*b^3*c + 30*A*a^2*b*c^2)^2*(4*a^7*b^10*c^2 - 64*a^8*b^8*c^3 + 384*a^9*b^6*c^4 - 1024*a^10*b^4*c^5 + 1024*a^11*b^2*c^6)*(2*A*b^10 - 2048*A*a^5*c^5 - 40*A*a*b^8*c + 320*A*a^2*b^6*c^2 - 1280*A*a^3*b^4*c^3 + 2560*A*a^4*b^2*c^4))/(32*a^6*(4*a*c - b^2)^5*(a^6*b^8 + 256*a^10*c^4 - 16*a^7*b^6*c + 96*a^8*b^4*c^2 - 256*a^9*b^2*c^3)*(4*a^3*b^10 - 4096*a^8*c^5 - 80*a^4*b^8*c + 640*a^5*b^6*c^2 - 2560*a^6*b^4*c^3 + 5120*a^7*b^2*c^4)))*(3*A*b^7 + 6*B*a^4*c^3 - 33*A*a*b^5*c - 135*A*a^3*b*c^3 + 120*A*a^2*b^3*c^2 - 6*B*a^3*b^2*c^2)*(16*a^9*b^12*(4*a*c - b^2)^(15/2) + 65536*a^15*c^6*(4*a*c - b^2)^(15/2) - 384*a^10*b^10*c*(4*a*c - b^2)^(15/2) + 3840*a^11*b^8*c^2*(4*a*c - b^2)^(15/2) - 20480*a^12*b^6*c^3*(4*a*c - b^2)^(15/2) + 61440*a^13*b^4*c^4*(4*a*c - b^2)^(15/2) - 98304*a^14*b^2*c^5*(4*a*c - b^2)^(15/2)))/(8*a^3*c^2*(4*a*c - b^2)^6*(A^2*b^10*c^2 + 144*B^2*a^6*c^6 + 160*A^2*a^2*b^6*c^4 - 600*A^2

```
*a^3*b^4*c^5 + 900*A^2*a^4*b^2*c^6 - 20*A^2*a*b^8*c^3 - 24*A*B*a^3*b^5*c^4
+ 240*A*B*a^4*b^3*c^5 - 720*A*B*a^5*b*c^6)*(6*A^2*b^10 - 6400*A^2*a^5*c^5 -
 36*B^2*a^6*c^4 + 960*A^2*a^2*b^6*c^2 - 3850*A^2*a^3*b^4*c^3 + 7775*A^2*a^4
*b^2*c^4 - 120*A^2*a*b^8*c + 6*A*B*a^3*b^5*c^2 - 60*A*B*a^4*b^3*c^3 + 180*A
*B*a^5*b*c^4)))*(A*b^5 - 12*B*a^3*c^2 - 10*A*a*b^3*c + 30*A*a^2*b*c^2))/(2*
a^3*(4*a*c - b^2)^(5/2))
```

3.131 $\int \frac{A+Bx^2}{x^3(a+bx^2+cx^4)^3}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 363

$$
\begin{aligned}
\int \frac{A+Bx^2}{x^3\left(a+bx^2+cx^4\right)^3}\,dx
&= \frac{abB(b^2-7ac)-3A(b^4-7ab^2c+10a^2c^2)}{2a^3\left(b^2-4ac\right)^2x^2} - \frac{abB-A(b^2-2ac)-(Ab-2aB)cx^2}{4a\left(b^2-4ac\right)x^2\left(a+bx^2+cx^4\right)^2} \\
&\quad - \frac{abB(b^2-10ac)-A(3b^4-20ab^2c+20a^2c^2)+c(aB(b^2-16ac)-3A(b^3-6abc))\,x^2}{4a^2\left(b^2-4ac\right)^2x^2\left(a+bx^2+cx^4\right)} \\
&\quad + \frac{(abB(b^4-10ab^2c+30a^2c^2)-3A(b^6-10ab^4c+30a^2b^2c^2-20a^3c^3))\operatorname{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2a^4\left(b^2-4ac\right)^{5/2}} \\
&\quad - \frac{(3Ab-aB)\log(x)}{a^4} + \frac{(3Ab-aB)\log\left(a+bx^2+cx^4\right)}{4a^4}
\end{aligned}
$$

```
[Out] 1/2*(a*b*B*(-7*a*c+b^2)-3*A*(10*a^2*c^2-7*a*b^2*c+b^4))/a^3/(-4*a*c+b^2)^2/
x^2+1/4*(-a*b*B+A*(-2*a*c+b^2)+(A*b-2*B*a)*c*x^2)/a/(-4*a*c+b^2)/x^2/(c*x^4
+b*x^2+a)^2+1/4*(-a*b*B*(-10*a*c+b^2)+A*(20*a^2*c^2-20*a*b^2*c+3*b^4)-c*(a*
B*(-16*a*c+b^2)-3*A*(-6*a*b*c+b^3))*x^2)/a^2/(-4*a*c+b^2)^2/x^2/(c*x^4+b*x^
2+a)+1/2*(a*b*B*(30*a^2*c^2-10*a*b^2*c+b^4)-3*A*(-20*a^3*c^3+30*a^2*b^2*c^2
-10*a*b^4*c+b^6))*arctanh((2*c*x^2+b)/(-4*a*c+b^2)^(1/2))/a^4/(-4*a*c+b^2)^
(5/2)-(3*A*b-B*a)*ln(x)/a^4+1/4*(3*A*b-B*a)*ln(c*x^4+b*x^2+a)/a^4
```

Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 363, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.280$, Rules used = {1265, 836, 814, 648, 632, 212, 642}

$$\int \frac{A+Bx^2}{x^3 (a+bx^2+cx^4)^3} \, dx = \frac{(3Ab-aB)\log(a+bx^2+cx^4)}{4a^4} - \frac{\log(x)(3Ab-aB)}{a^4} - \frac{-A(20a^2c^2-20ab^2c+3b^4)+cx^2(aB(b^2-16ac)-3A(b^3-6abc))+abB(b^2-10ac)}{4a^2x^2(b^2-4ac)^2(a+bx^2+cx^4)} + \frac{abB(b^2-7ac)-3A(10a^2c^2-7ab^2c+b^4)}{2a^3x^2(b^2-4ac)^2} + \frac{(abB(30a^2c^2-10ab^2c+b^4)-3A(-20a^3c^3+30a^2b^2c^2-10ab^4c+b^6))\operatorname{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2a^4(b^2-4ac)^{5/2}} - \frac{-A(b^2-2ac)-(cx^2(Ab-2aB))+abB}{4ax^2(b^2-4ac)(a+bx^2+cx^4)^2}$$

```
[In] Int[(A + B*x^2)/(x^3*(a + b*x^2 + c*x^4)^3),x]
```

```
[Out] (a*b*B*(b^2 - 7*a*c) - 3*A*(b^4 - 7*a*b^2*c + 10*a^2*c^2))/(2*a^3*(b^2 - 4*
a*c)^2*x^2) - (a*b*B - A*(b^2 - 2*a*c) - (A*b - 2*a*B)*c*x^2)/(4*a*(b^2 - 4
*a*c)*x^2*(a + b*x^2 + c*x^4)^2) - (a*b*B*(b^2 - 10*a*c) - A*(3*b^4 - 20*a*
b^2*c + 20*a^2*c^2) + c*(a*B*(b^2 - 16*a*c) - 3*A*(b^3 - 6*a*b*c))*x^2)/(4*
a^2*(b^2 - 4*a*c)^2*x^2*(a + b*x^2 + c*x^4)) + ((a*b*B*(b^4 - 10*a*b^2*c +
30*a^2*c^2) - 3*A*(b^6 - 10*a*b^4*c + 30*a^2*b^2*c^2 - 20*a^3*c^3))*ArcTanh
[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(2*a^4*(b^2 - 4*a*c)^(5/2)) - ((3*A*b -
a*B)*Log[x])/a^4 + ((3*A*b - a*B)*Log[a + b*x^2 + c*x^4])/(4*a^4)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
```

```
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 814

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)/(a +
 b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 836

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(f*(b*c*d - b^2*e + 2
*a*c*e) - a*g*(2*c*d - b*e) + c*(f*(2*c*d - b*e) - g*(b*d - 2*a*e))*x)*((a
+ b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2))), x]
 + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^m*
(a + b*x + c*x^2)^(p + 1)*Simp[f*(b*c*d*e*(2*p - m + 2) + b^2*e^2*(p + m +
2) - 2*c^2*d^2*(2*p + 3) - 2*a*c*e^2*(m + 2*p + 3)) - g*(a*e*(b*e - 2*c*d*m
 + b*e*m) - b*d*(3*c*d - b*e + 2*c*d*p - b*e*p)) + c*e*(g*(b*d - 2*a*e) - f
*(2*c*d - b*e))*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1
] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 1265

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rubi steps

$$\text{integral} = \frac{1}{2}\text{Subst}\left(\int \frac{A+Bx}{x^2\left(a+bx+cx^2\right)^3}\,dx, x, x^2\right)$$

$$= -\frac{abB-A(b^2-2ac)-(Ab-2aB)cx^2}{4a\left(b^2-4ac\right)x^2\left(a+bx^2+cx^4\right)^2} - \frac{\text{Subst}\left(\int \frac{-3Ab^2+abB+10aAc-4(Ab-2aB)cx}{x^2\left(a+bx+cx^2\right)^2}\,dx, x, x^2\right)}{4a\left(b^2-4ac\right)}$$

$$= -\frac{abB - A(b^2-2ac) - (Ab-2aB)cx^2}{4a\left(b^2-4ac\right)x^2\left(a+bx^2+cx^4\right)^2} - \frac{abB(b^2-10ac) - A(3b^4-20ab^2c+20a^2c^2) + c(aB(b^2-16ac)-3A(b^3-6abc))\,x^2}{4a^2\left(b^2-4ac\right)^2x^2\left(a+bx^2+cx^4\right)} + \frac{\text{Subst}\left(\int \frac{-2(abB(b^2-7ac)-3A(b^4-7ab^2c+10a^2c^2))-2c(aB(b^2-16ac)-3A(b^3-6abc))x}{x^2(a+bx+cx^2)}\,dx,x,x^2\right)}{4a^2\left(b^2-4ac\right)^2}$$

$$= -\frac{abB - A(b^2-2ac) - (Ab-2aB)cx^2}{4a\left(b^2-4ac\right)x^2\left(a+bx^2+cx^4\right)^2} - \frac{abB(b^2-10ac) - A(3b^4-20ab^2c+20a^2c^2) + c(aB(b^2-16ac)-3A(b^3-6abc))\,x^2}{4a^2\left(b^2-4ac\right)^2x^2\left(a+bx^2+cx^4\right)} + \frac{\text{Subst}\left(\int \left(\frac{2(-abB(b^2-7ac)+3A(b^4-7ab^2c+10a^2c^2))}{ax^2} + \frac{2(-3Ab+aB)(-b^2+4ac)^2}{a^2x} + \frac{2\left(-abB(b^4-9ab^2c+23a^2c^2)+3A(\right.}{}\right.\right.}{4a^2\left(b^2-4ac\right)^2}$$

$$= \frac{abB(b^2-7ac) - 3A(b^4-7ab^2c+10a^2c^2)}{2a^3\left(b^2-4ac\right)^2x^2} - \frac{abB - A(b^2-2ac) - (Ab-2aB)cx^2}{4a\left(b^2-4ac\right)x^2\left(a+bx^2+cx^4\right)^2} - \frac{abB(b^2-10ac) - A(3b^4-20ab^2c+20a^2c^2) + c(aB(b^2-16ac)-3A(b^3-6abc))\,x^2}{4a^2\left(b^2-4ac\right)^2x^2\left(a+bx^2+cx^4\right)} - \frac{(3Ab-aB)\log(x)}{a^4} + \frac{\text{Subst}\left(\int \frac{-abB(b^4-9ab^2c+23a^2c^2)+3A(b^6-9ab^4c+23a^2b^2c^2-10a^3c^3)+(3Ab-aB)c(b^2-4ac)^2x}{a+bx+cx^2}\,dx,x,x^2\right)}{2a^4\left(b^2-4ac\right)^2}$$

$$= \frac{abB(b^2-7ac) - 3A(b^4-7ab^2c+10a^2c^2)}{2a^3\left(b^2-4ac\right)^2x^2} - \frac{abB - A(b^2-2ac) - (Ab-2aB)cx^2}{4a\left(b^2-4ac\right)x^2\left(a+bx^2+cx^4\right)^2} - \frac{abB(b^2-10ac) - A(3b^4-20ab^2c+20a^2c^2) + c(aB(b^2-16ac)-3A(b^3-6abc))\,x^2}{4a^2\left(b^2-4ac\right)^2x^2\left(a+bx^2+cx^4\right)} - \frac{(3Ab-aB)\log(x)}{a^4} + \frac{(3Ab-aB)\text{Subst}\left(\int \frac{b+2cx}{a+bx+cx^2}\,dx,x,x^2\right)}{4a^4} - \frac{(abB(b^4-10ab^2c+30a^2c^2)-3A(b^6-10ab^4c+30a^2b^2c^2-20a^3c^3))\,\text{Subst}\left(\int \frac{1}{a+bx+cx^2}\,dx,x,x^2\right.}{4a^4\left(b^2-4ac\right)^2}$$

$$= \frac{abB(b^2-7ac) - 3A(b^4-7ab^2c+10a^2c^2)}{2a^3\left(b^2-4ac\right)^2x^2} - \frac{abB - A(b^2-2ac) - (Ab-2aB)cx^2}{4a\left(b^2-4ac\right)x^2\left(a+bx^2+cx^4\right)^2} - \frac{abB(b^2-10ac) - A(3b^4-20ab^2c+20a^2c^2) + c(aB(b^2-16ac)-3A(b^3-6abc))\,x^2}{4a^2\left(b^2-4ac\right)^2x^2\left(a+bx^2+cx^4\right)} - \frac{(3Ab-aB)\log(x)}{a^4} + \frac{(3Ab-aB)\log\left(a+bx^2+cx^4\right)}{4a^4} + \frac{(abB(b^4-10ab^2c+30a^2c^2)-3A(b^6-10ab^4c+30a^2b^2c^2-20a^3c^3))\,\text{Subst}\left(\int \frac{1}{b^2-4ac-x^2}\,dx,x,b\right.}{2a^4\left(b^2-4ac\right)^2}$$

$$
\begin{aligned}
&= \frac{abB(b^2-7ac)-3A(b^4-7ab^2c+10a^2c^2)}{2a^3\left(b^2-4ac\right)^2x^2} - \frac{abB-A(b^2-2ac)-(Ab-2aB)cx^2}{4a\left(b^2-4ac\right)x^2\left(a+bx^2+cx^4\right)^2} \\
&\quad - \frac{abB(b^2-10ac)-A(3b^4-20ab^2c+20a^2c^2)+c(aB(b^2-16ac)-3A(b^3-6abc))\,x^2}{4a^2\left(b^2-4ac\right)^2x^2\left(a+bx^2+cx^4\right)} \\
&\quad + \frac{(abB(b^4-10ab^2c+30a^2c^2)-3A(b^6-10ab^4c+30a^2b^2c^2-20a^3c^3))\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2a^4\left(b^2-4ac\right)^{5/2}} \\
&\quad - \frac{(3Ab-aB)\log(x)}{a^4} + \frac{(3Ab-aB)\log\left(a+bx^2+cx^4\right)}{4a^4}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.90 (sec) , antiderivative size = 642, normalized size of antiderivative = 1.77

$$
\int \frac{A+Bx^2}{x^3\left(a+bx^2+cx^4\right)^3}\,dx
$$

$$
= \frac{-\frac{2aA}{x^2} - \frac{a^2\left(aB\left(-b^2+2ac-bcx^2\right)+A\left(b^3-3abc+b^2cx^2-2ac^2x^2\right)\right)}{(b^2-4ac)(a+bx^2+cx^4)^2} + \frac{a\left(aB\left(2b^4-15ab^2c+16a^2c^2+2b^3cx^2-14abc^2x^2\right)-A\left(4b^5-29ab^3c+46a^2bc^2\right.\right.}{(b^2-4ac)^2(a+bx^2+cx^4)}}{}
$$

```
[In] Integrate[(A + B*x^2)/(x^3*(a + b*x^2 + c*x^4)^3),x]
```

```
[Out] ((-2*a*A)/x^2 - (a^2*(a*B*(-b^2 + 2*a*c - b*c*x^2) + A*(b^3 - 3*a*b*c + b^2
*c*x^2 - 2*a*c^2*x^2)))/((b^2 - 4*a*c)*(a + b*x^2 + c*x^4)^2) + (a*(a*B*(2*
b^4 - 15*a*b^2*c + 16*a^2*c^2 + 2*b^3*c*x^2 - 14*a*b*c^2*x^2) - A*(4*b^5 -
29*a*b^3*c + 46*a^2*b*c^2 + 4*b^4*c*x^2 - 26*a*b^2*c^2*x^2 + 28*a^2*c^3*x^2
)))/((b^2 - 4*a*c)^2*(a + b*x^2 + c*x^4)) + 4*(-3*A*b + a*B)*Log[x] + ((-(a
*B*(b^5 - 10*a*b^3*c + 30*a^2*b*c^2 + b^4*Sqrt[b^2 - 4*a*c] - 8*a*b^2*c*Sqr
t[b^2 - 4*a*c] + 16*a^2*c^2*Sqrt[b^2 - 4*a*c])) + 3*A*(b^6 - 10*a*b^4*c + 3
0*a^2*b^2*c^2 - 20*a^3*c^3 + b^5*Sqrt[b^2 - 4*a*c] - 8*a*b^3*c*Sqrt[b^2 - 4
*a*c] + 16*a^2*b*c^2*Sqrt[b^2 - 4*a*c]))*Log[b - Sqrt[b^2 - 4*a*c] + 2*c*x^
2])/(b^2 - 4*a*c)^(5/2) + ((a*B*(b^5 - 10*a*b^3*c + 30*a^2*b*c^2 - b^4*Sqrt
[b^2 - 4*a*c] + 8*a*b^2*c*Sqrt[b^2 - 4*a*c] - 16*a^2*c^2*Sqrt[b^2 - 4*a*c])
 + 3*A*(-b^6 + 10*a*b^4*c - 30*a^2*b^2*c^2 + 20*a^3*c^3 + b^5*Sqrt[b^2 - 4*
a*c] - 8*a*b^3*c*Sqrt[b^2 - 4*a*c] + 16*a^2*b*c^2*Sqrt[b^2 - 4*a*c]))*Log[b
 + Sqrt[b^2 - 4*a*c] + 2*c*x^2])/(b^2 - 4*a*c)^(5/2))/(4*a^4)
```

Maple [A] (verified)

Time = 0.28 (sec) , antiderivative size = 616, normalized size of antiderivative = 1.70

method	result
default	$-\frac{A}{2a^3x^2}+\frac{(-3Ab+Ba)\ln(x)}{a^4}-\frac{\frac{a\,c^2\left(14A\,a^2c^2-13Aa\,b^2c+2A\,b^4+7a^2bBc-Ba\,b^3\right)x^6}{16a^2c^2-8a\,b^2c+b^4}+\frac{ac\left(74A\,a^2b\,c^2-55Aa\,b^3c+8A\,b^5-16a^3B\,c^2+29B\,a\right.}{32a^2c^2-16a\,b^2c+2b^4}}{}$
risch	Expression too large to display

```
[In] int((B*x^2+A)/x^3/(c*x^4+b*x^2+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*A/a^3/x^2+(-3*A*b+B*a)/a^4*ln(x)-1/2/a^4*((a*c^2*(14*A*a^2*c^2-13*A*a*
b^2*c+2*A*b^4+7*B*a^2*b*c-B*a*b^3)/(16*a^2*c^2-8*a*b^2*c+b^4)*x^6+1/2*a*c*(
74*A*a^2*b*c^2-55*A*a*b^3*c+8*A*b^5-16*B*a^3*c^2+29*B*a^2*b^2*c-4*B*a*b^4)/
(16*a^2*c^2-8*a*b^2*c+b^4)*x^4+a*(18*A*a^3*c^3+7*A*a^2*b^2*c^2-12*A*a*b^4*c
+2*A*b^6+B*a^3*b*c^2+6*B*a^2*b^3*c-B*a*b^5)/(16*a^2*c^2-8*a*b^2*c+b^4)*x^2+
1/2*a^2*(58*A*a^2*b*c^2-36*A*a*b^3*c+5*A*b^5-24*B*a^3*c^2+21*B*a^2*b^2*c-3*
B*a*b^4)/(16*a^2*c^2-8*a*b^2*c+b^4))/(c*x^4+b*x^2+a)^2+1/(16*a^2*c^2-8*a*b^
2*c+b^4)*(1/2*(-48*A*a^2*b*c^3+24*A*a*b^3*c^2-3*A*b^5*c+16*B*a^3*c^3-8*B*a^
2*b^2*c^2+B*a*b^4*c)/c*ln(c*x^4+b*x^2+a)+2*(30*A*a^3*c^3-69*A*a^2*b^2*c^2+2
7*A*a*b^4*c-3*b^6*A+23*B*a^3*b*c^2-9*B*a^2*b^3*c+B*a*b^5-1/2*(-48*A*a^2*b*c
^3+24*A*a*b^3*c^2-3*A*b^5*c+16*B*a^3*c^3-8*B*a^2*b^2*c^2+B*a*b^4*c)*b/c)/(4
*a*c-b^2)^(1/2)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1966 vs. $2(346) = 692$.

Time = 3.98 (sec) , antiderivative size = 3956, normalized size of antiderivative = 10.90

$$\int \frac{A+Bx^2}{x^3\left(a+bx^2+cx^4\right)^3}\,dx = \text{Too large to display}$$

```
[In] integrate((B*x^2+A)/x^3/(c*x^4+b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] [-1/4*(2*A*a^3*b^6 - 24*A*a^4*b^4*c + 96*A*a^5*b^2*c^2 - 128*A*a^6*c^3 - 2*
(120*A*a^4*c^5 + 2*(14*B*a^4*b - 57*A*a^3*b^2)*c^4 - 11*(B*a^3*b^3 - 3*A*a^
2*b^4)*c^3 + (B*a^2*b^5 - 3*A*a*b^6)*c^2)*x^8 + (8*(8*B*a^5 - 69*A*a^4*b)*c
^4 - 6*(22*B*a^4*b^2 - 81*A*a^3*b^3)*c^3 + 45*(B*a^3*b^4 - 3*A*a^2*b^5)*c^2
 - 4*(B*a^2*b^6 - 3*A*a*b^7)*c)*x^6 - 2*(B*a^2*b^7 - 3*A*a*b^8 + 200*A*a^5*
c^4 + 2*(2*B*a^5*b - 11*A*a^4*b^2)*c^3 + (23*B*a^4*b^3 - 79*A*a^3*b^4)*c^2
- 10*(B*a^3*b^5 - 3*A*a^2*b^6)*c)*x^4 - (3*B*a^3*b^6 - 9*A*a^2*b^7 - 8*(12*
B*a^6 - 61*A*a^5*b)*c^3 + 2*(54*B*a^5*b^2 - 197*A*a^4*b^3)*c^2 - (33*B*a^4*
b^4 - 104*A*a^3*b^5)*c)*x^2 - ((60*A*a^3*c^5 + 30*(B*a^3*b - 3*A*a^2*b^2)*c
```

^4 - 10*(B*a^2*b^3 - 3*A*a*b^4)*c^3 + (B*a*b^5 - 3*A*b^6)*c^2)*x^10 + 2*(60
*A*a^3*b*c^4 + 30*(B*a^3*b^2 - 3*A*a^2*b^3)*c^3 - 10*(B*a^2*b^4 - 3*A*a*b^5
)*c^2 + (B*a*b^6 - 3*A*b^7)*c)*x^8 + (B*a*b^7 - 3*A*b^8 + 120*A*a^4*c^4 + 6
0*(B*a^4*b - 2*A*a^3*b^2)*c^3 + 10*(B*a^3*b^3 - 3*A*a^2*b^4)*c^2 - 8*(B*a^2
*b^5 - 3*A*a*b^6)*c)*x^6 + 2*(B*a^2*b^6 - 3*A*a*b^7 + 60*A*a^4*b*c^3 + 30*(
B*a^4*b^2 - 3*A*a^3*b^3)*c^2 - 10*(B*a^3*b^4 - 3*A*a^2*b^5)*c)*x^4 + (B*a^3
*b^5 - 3*A*a^2*b^6 + 60*A*a^5*c^3 + 30*(B*a^5*b - 3*A*a^4*b^2)*c^2 - 10*(B*
a^4*b^3 - 3*A*a^3*b^4)*c)*x^2)*sqrt(b^2 - 4*a*c)*log((2*c^2*x^4 + 2*b*c*x^2
 + b^2 - 2*a*c + (2*c*x^2 + b)*sqrt(b^2 - 4*a*c))/(c*x^4 + b*x^2 + a)) - ((
64*(B*a^4 - 3*A*a^3*b)*c^5 - 48*(B*a^3*b^2 - 3*A*a^2*b^3)*c^4 + 12*(B*a^2*b
^4 - 3*A*a*b^5)*c^3 - (B*a*b^6 - 3*A*b^7)*c^2)*x^10 + 2*(64*(B*a^4*b - 3*A*
a^3*b^2)*c^4 - 48*(B*a^3*b^3 - 3*A*a^2*b^4)*c^3 + 12*(B*a^2*b^5 - 3*A*a*b^6
)*c^2 - (B*a*b^7 - 3*A*b^8)*c)*x^8 - (B*a*b^8 - 3*A*b^9 - 128*(B*a^5 - 3*A*
a^4*b)*c^4 + 32*(B*a^4*b^2 - 3*A*a^3*b^3)*c^3 + 24*(B*a^3*b^4 - 3*A*a^2*b^5
)*c^2 - 10*(B*a^2*b^6 - 3*A*a*b^7)*c)*x^6 - 2*(B*a^2*b^7 - 3*A*a*b^8 - 64*(
B*a^5*b - 3*A*a^4*b^2)*c^3 + 48*(B*a^4*b^3 - 3*A*a^3*b^4)*c^2 - 12*(B*a^3*b
^5 - 3*A*a^2*b^6)*c)*x^4 - (B*a^3*b^6 - 3*A*a^2*b^7 - 64*(B*a^6 - 3*A*a^5*b
)*c^3 + 48*(B*a^5*b^2 - 3*A*a^4*b^3)*c^2 - 12*(B*a^4*b^4 - 3*A*a^3*b^5)*c)*
x^2)*log(c*x^4 + b*x^2 + a) + 4*((64*(B*a^4 - 3*A*a^3*b)*c^5 - 48*(B*a^3*b^
2 - 3*A*a^2*b^3)*c^4 + 12*(B*a^2*b^4 - 3*A*a*b^5)*c^3 - (B*a*b^6 - 3*A*b^7)
*c^2)*x^10 + 2*(64*(B*a^4*b - 3*A*a^3*b^2)*c^4 - 48*(B*a^3*b^3 - 3*A*a^2*b^
4)*c^3 + 12*(B*a^2*b^5 - 3*A*a*b^6)*c^2 - (B*a*b^7 - 3*A*b^8)*c)*x^8 - (B*a
*b^8 - 3*A*b^9 - 128*(B*a^5 - 3*A*a^4*b)*c^4 + 32*(B*a^4*b^2 - 3*A*a^3*b^3)
c^3 + 24(B*a^3*b^4 - 3*A*a^2*b^5)*c^2 - 10*(B*a^2*b^6 - 3*A*a*b^7)*c)*x^6
 - 2*(B*a^2*b^7 - 3*A*a*b^8 - 64*(B*a^5*b - 3*A*a^4*b^2)*c^3 + 48*(B*a^4*b^
3 - 3*A*a^3*b^4)*c^2 - 12*(B*a^3*b^5 - 3*A*a^2*b^6)*c)*x^4 - (B*a^3*b^6 - 3
*A*a^2*b^7 - 64*(B*a^6 - 3*A*a^5*b)*c^3 + 48*(B*a^5*b^2 - 3*A*a^4*b^3)*c^2
 - 12*(B*a^4*b^4 - 3*A*a^3*b^5)*c)*x^2)*log(x))/((a^4*b^6*c^2 - 12*a^5*b^4*c
^3 + 48*a^6*b^2*c^4 - 64*a^7*c^5)*x^10 + 2*(a^4*b^7*c - 12*a^5*b^5*c^2 + 48
*a^6*b^3*c^3 - 64*a^7*b*c^4)*x^8 + (a^4*b^8 - 10*a^5*b^6*c + 24*a^6*b^4*c^2
 + 32*a^7*b^2*c^3 - 128*a^8*c^4)*x^6 + 2*(a^5*b^7 - 12*a^6*b^5*c + 48*a^7*b
^3*c^2 - 64*a^8*b*c^3)*x^4 + (a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*
a^9*c^3)*x^2), -1/4*(2*A*a^3*b^6 - 24*A*a^4*b^4*c + 96*A*a^5*b^2*c^2 - 128*
A*a^6*c^3 - 2*(120*A*a^4*c^5 + 2*(14*B*a^4*b - 57*A*a^3*b^2)*c^4 - 11*(B*a^
3*b^3 - 3*A*a^2*b^4)*c^3 + (B*a^2*b^5 - 3*A*a*b^6)*c^2)*x^8 + (8*(8*B*a^5 -
 69*A*a^4*b)*c^4 - 6*(22*B*a^4*b^2 - 81*A*a^3*b^3)*c^3 + 45*(B*a^3*b^4 - 3*
A*a^2*b^5)*c^2 - 4*(B*a^2*b^6 - 3*A*a*b^7)*c)*x^6 - 2*(B*a^2*b^7 - 3*A*a*b^
8 + 200*A*a^5*c^4 + 2*(2*B*a^5*b - 11*A*a^4*b^2)*c^3 + (23*B*a^4*b^3 - 79*A
*a^3*b^4)*c^2 - 10*(B*a^3*b^5 - 3*A*a^2*b^6)*c)*x^4 - (3*B*a^3*b^6 - 9*A*a^
2*b^7 - 8*(12*B*a^6 - 61*A*a^5*b)*c^3 + 2*(54*B*a^5*b^2 - 197*A*a^4*b^3)*c^
2 - (33*B*a^4*b^4 - 104*A*a^3*b^5)*c)*x^2 - 2*((60*A*a^3*c^5 + 30*(B*a^3*b
 - 3*A*a^2*b^2)*c^4 - 10*(B*a^2*b^3 - 3*A*a*b^4)*c^3 + (B*a*b^5 - 3*A*b^6)*c
^2)*x^10 + 2*(60*A*a^3*b*c^4 + 30*(B*a^3*b^2 - 3*A*a^2*b^3)*c^3 - 10*(B*a^2
*b^4 - 3*A*a*b^5)*c^2 + (B*a*b^6 - 3*A*b^7)*c)*x^8 + (B*a*b^7 - 3*A*b^8 + 1
20*A*a^4*c^4 + 60*(B*a^4*b - 2*A*a^3*b^2)*c^3 + 10*(B*a^3*b^3 - 3*A*a^2*b^4

```
)*c^2 - 8*(B*a^2*b^5 - 3*A*a*b^6)*c)*x^6 + 2*(B*a^2*b^6 - 3*A*a*b^7 + 60*A*
a^4*b*c^3 + 30*(B*a^4*b^2 - 3*A*a^3*b^3)*c^2 - 10*(B*a^3*b^4 - 3*A*a^2*b^5)
*c)*x^4 + (B*a^3*b^5 - 3*A*a^2*b^6 + 60*A*a^5*c^3 + 30*(B*a^5*b - 3*A*a^4*b
^2)*c^2 - 10*(B*a^4*b^3 - 3*A*a^3*b^4)*c)*x^2)*sqrt(-b^2 + 4*a*c)*arctan(-(
2*c*x^2 + b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4*a*c)) - ((64*(B*a^4 - 3*A*a^3*b)*c
^5 - 48*(B*a^3*b^2 - 3*A*a^2*b^3)*c^4 + 12*(B*a^2*b^4 - 3*A*a*b^5)*c^3 - (B
*a*b^6 - 3*A*b^7)*c^2)*x^10 + 2*(64*(B*a^4*b - 3*A*a^3*b^2)*c^4 - 48*(B*a^3
*b^3 - 3*A*a^2*b^4)*c^3 + 12*(B*a^2*b^5 - 3*A*a*b^6)*c^2 - (B*a*b^7 - 3*A*b
^8)*c)*x^8 - (B*a*b^8 - 3*A*b^9 - 128*(B*a^5 - 3*A*a^4*b)*c^4 + 32*(B*a^4*b
^2 - 3*A*a^3*b^3)*c^3 + 24*(B*a^3*b^4 - 3*A*a^2*b^5)*c^2 - 10*(B*a^2*b^6 -
3*A*a*b^7)*c)*x^6 - 2*(B*a^2*b^7 - 3*A*a*b^8 - 64*(B*a^5*b - 3*A*a^4*b^2)*c
^3 + 48*(B*a^4*b^3 - 3*A*a^3*b^4)*c^2 - 12*(B*a^3*b^5 - 3*A*a^2*b^6)*c)*x^4
 - (B*a^3*b^6 - 3*A*a^2*b^7 - 64*(B*a^6 - 3*A*a^5*b)*c^3 + 48*(B*a^5*b^2 -
3*A*a^4*b^3)*c^2 - 12*(B*a^4*b^4 - 3*A*a^3*b^5)*c)*x^2)*log(c*x^4 + b*x^2 +
 a) + 4*((64*(B*a^4 - 3*A*a^3*b)*c^5 - 48*(B*a^3*b^2 - 3*A*a^2*b^3)*c^4 + 1
2*(B*a^2*b^4 - 3*A*a*b^5)*c^3 - (B*a*b^6 - 3*A*b^7)*c^2)*x^10 + 2*(64*(B*a^
4*b - 3*A*a^3*b^2)*c^4 - 48*(B*a^3*b^3 - 3*A*a^2*b^4)*c^3 + 12*(B*a^2*b^5 -
 3*A*a*b^6)*c^2 - (B*a*b^7 - 3*A*b^8)*c)*x^8 - (B*a*b^8 - 3*A*b^9 - 128*(B*
a^5 - 3*A*a^4*b)*c^4 + 32*(B*a^4*b^2 - 3*A*a^3*b^3)*c^3 + 24*(B*a^3*b^4 - 3
*A*a^2*b^5)*c^2 - 10*(B*a^2*b^6 - 3*A*a*b^7)*c)*x^6 - 2*(B*a^2*b^7 - 3*A*a*
b^8 - 64*(B*a^5*b - 3*A*a^4*b^2)*c^3 + 48*(B*a^4*b^3 - 3*A*a^3*b^4)*c^2 - 1
2*(B*a^3*b^5 - 3*A*a^2*b^6)*c)*x^4 - (B*a^3*b^6 - 3*A*a^2*b^7 - 64*(B*a^6 -
 3*A*a^5*b)*c^3 + 48*(B*a^5*b^2 - 3*A*a^4*b^3)*c^2 - 12*(B*a^4*b^4 - 3*A*a^
3*b^5)*c)*x^2)*log(x))/((a^4*b^6*c^2 - 12*a^5*b^4*c^3 + 48*a^6*b^2*c^4 - 64
*a^7*c^5)*x^10 + 2*(a^4*b^7*c - 12*a^5*b^5*c^2 + 48*a^6*b^3*c^3 - 64*a^7*b*
c^4)*x^8 + (a^4*b^8 - 10*a^5*b^6*c + 24*a^6*b^4*c^2 + 32*a^7*b^2*c^3 - 128*
a^8*c^4)*x^6 + 2*(a^5*b^7 - 12*a^6*b^5*c + 48*a^7*b^3*c^2 - 64*a^8*b*c^3)*x
^4 + (a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3)*x^2)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A+Bx^2}{x^3\left(a+bx^2+cx^4\right)^3}\,dx=\text{Timed out}$$

```
[In] integrate((B*x**2+A)/x**3/(c*x**4+b*x**2+a)**3,x)
[Out] Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{A + Bx^2}{x^3 (a + bx^2 + cx^4)^3} \, dx = \text{Exception raised: ValueError}$$

```
[In] integrate((B*x^2+A)/x^3/(c*x^4+b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

Giac [A] (verification not implemented)

none

Time = 1.43 (sec) , antiderivative size = 648, normalized size of antiderivative = 1.79

$$\int \frac{A + Bx^2}{x^3 (a + bx^2 + cx^4)^3} \, dx = -\frac{(Bab^5 - 3Ab^6 - 10Ba^2b^3c + 30Aab^4c + 30Ba^3bc^2 - 90Aa^2b^2c^2 + 60Aa^3c^3)\arctan\left(\frac{2cx^2+b}{\sqrt{-b^2+4ac}}\right)}{2(a^4b^4 - 8a^5b^2c + 16a^6c^2)\sqrt{-b^2+4ac}} + \frac{3Bab^4c^2x^8 - 9Ab^5c^2x^8 - 24Ba^2b^2c^3x^8 + 72Aab^3c^3x^8 + 48Ba^3c^4x^8 - 144Aa^2bc^4x^8 + 6Bab^5cx^6 - 18A}{} - \frac{(Ba - 3Ab)\log(cx^4 + bx^2 + a)}{4a^4} + \frac{(Ba - 3Ab)\log(x^2)}{2a^4} - \frac{Bax^2 - 3Abx^2 + Aa}{2a^4x^2}$$

```
[In] integrate((B*x^2+A)/x^3/(c*x^4+b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] -1/2*(B*a*b^5 - 3*A*b^6 - 10*B*a^2*b^3*c + 30*A*a*b^4*c + 30*B*a^3*b*c^2 -
90*A*a^2*b^2*c^2 + 60*A*a^3*c^3)*arctan((2*c*x^2 + b)/sqrt(-b^2 + 4*a*c))/(
(a^4*b^4 - 8*a^5*b^2*c + 16*a^6*c^2)*sqrt(-b^2 + 4*a*c)) + 1/8*(3*B*a*b^4*c
^2*x^8 - 9*A*b^5*c^2*x^8 - 24*B*a^2*b^2*c^3*x^8 + 72*A*a*b^3*c^3*x^8 + 48*B
*a^3*c^4*x^8 - 144*A*a^2*b*c^4*x^8 + 6*B*a*b^5*c*x^6 - 18*A*b^6*c*x^6 - 44*
B*a^2*b^3*c^2*x^6 + 136*A*a*b^4*c^2*x^6 + 68*B*a^3*b*c^3*x^6 - 236*A*a^2*b^
2*c^3*x^6 - 56*A*a^3*c^4*x^6 + 3*B*a*b^6*x^4 - 9*A*b^7*x^4 - 10*B*a^2*b^4*c
*x^4 + 38*A*a*b^5*c*x^4 - 58*B*a^3*b^2*c^2*x^4 + 110*A*a^2*b^3*c^2*x^4 + 12
8*B*a^4*c^3*x^4 - 436*A*a^3*b*c^3*x^4 + 10*B*a^2*b^5*x^2 - 26*A*a*b^6*x^2 -
 72*B*a^3*b^3*c*x^2 + 192*A*a^2*b^4*c*x^2 + 92*B*a^4*b*c^2*x^2 - 316*A*a^3*
b^2*c^2*x^2 - 72*A*a^4*c^3*x^2 + 9*B*a^3*b^4 - 19*A*a^2*b^5 - 66*B*a^4*b^2*
c + 144*A*a^3*b^3*c + 96*B*a^5*c^2 - 260*A*a^4*b*c^2)/((a^4*b^4 - 8*a^5*b^2
*c + 16*a^6*c^2)*(c*x^4 + b*x^2 + a)^2) - 1/4*(B*a - 3*A*b)*log(c*x^4 + b*x
^2 + a)/a^4 + 1/2*(B*a - 3*A*b)*log(x^2)/a^4 - 1/2*(B*a*x^2 - 3*A*b*x^2 + A
*a)/(a^4*x^2)
```

Mupad [B] (verification not implemented)

Time = 18.35 (sec) , antiderivative size = 16265, normalized size of antiderivative = 44.81

$$\int \frac{A+Bx^2}{x^3\left(a+bx^2+cx^4\right)^3}\,dx = \text{Too large to display}$$

```
[In] int((A + B*x^2)/(x^3*(a + b*x^2 + c*x^4)^3),x)
```

```
[Out] (log(((c^5*x^2*(3*A*b^4 + 30*A*a^2*c^2 - B*a*b^3 - 21*A*a*b^2*c + 7*B*a^2*b
*c)^3)/(a^9*(4*a*c - b^2)^6) - ((B*a - 3*A*b + a^4*(-(60*A*a^3*c^3 - 3*A*b^
6 + B*a*b^5 + 30*A*a*b^4*c - 10*B*a^2*b^3*c + 30*B*a^3*b*c^2 - 90*A*a^2*b^2
*c^2)^2/(a^8*(4*a*c - b^2)^5))^(1/2))*(((B*a - 3*A*b + a^4*(-(60*A*a^3*c^3
- 3*A*b^6 + B*a*b^5 + 30*A*a*b^4*c - 10*B*a^2*b^3*c + 30*B*a^3*b*c^2 - 90*A
*a^2*b^2*c^2)^2/(a^8*(4*a*c - b^2)^5))^(1/2))*((4*b*c^2*(30*A*a^3*c^3 - 3*A
*b^6 + B*a*b^5 + 27*A*a*b^4*c - 9*B*a^2*b^3*c + 23*B*a^3*b*c^2 - 69*A*a^2*b
^2*c^2))/(a^3*(4*a*c - b^2)^2) + (2*c^3*x^2*(B*a*b^5 - 300*A*a^3*c^3 - 3*A*
b^6 + 6*A*a*b^4*c - 2*B*a^2*b^3*c + 10*B*a^3*b*c^2 + 90*A*a^2*b^2*c^2))/(a^
3*(4*a*c - b^2)^2) + (b*c^2*(B*a - 3*A*b + a^4*(-(60*A*a^3*c^3 - 3*A*b^6 +
B*a*b^5 + 30*A*a*b^4*c - 10*B*a^2*b^3*c + 30*B*a^3*b*c^2 - 90*A*a^2*b^2*c^2
)^2/(a^8*(4*a*c - b^2)^5))^(1/2))*(a*b + 3*b^2*x^2 - 10*a*c*x^2))/a^4))/(4*
a^4) + (c^3*(900*A^2*a^5*c^5 - 36*A^2*b^10 - 4*B^2*a^2*b^8 + 24*A*B*a*b^9 -
 3078*A^2*a^2*b^6*c^2 + 7533*A^2*a^3*b^4*c^3 - 7020*A^2*a^4*b^2*c^4 - 302*B
^2*a^4*b^4*c^2 + 497*B^2*a^5*b^2*c^3 + 549*A^2*a*b^8*c + 61*B^2*a^3*b^6*c +
 1932*A*B*a^3*b^5*c^2 - 4002*A*B*a^4*b^3*c^3 - 366*A*B*a^2*b^7*c + 2340*A*B
*a^5*b*c^4))/(a^6*(4*a*c - b^2)^4) - (c^4*x^2*(54*A^2*b^9 + 6*B^2*a^2*b^7 -
 36*A*B*a*b^8 + 4311*A^2*a^2*b^5*c^2 - 9900*A^2*a^3*b^3*c^3 + 409*B^2*a^4*b
^3*c^2 - 2400*A*B*a^5*c^4 - 801*A^2*a*b^7*c + 8100*A^2*a^4*b*c^4 - 89*B^2*a
^3*b^5*c - 560*B^2*a^5*b*c^3 - 2664*A*B*a^3*b^4*c^2 + 4980*A*B*a^4*b^2*c^3
+ 534*A*B*a^2*b^6*c))/(a^6*(4*a*c - b^2)^4)))/(4*a^4) + (c^4*(3*A*b - B*a)*
(3*A*b^4 + 30*A*a^2*c^2 - B*a*b^3 - 21*A*a*b^2*c + 7*B*a^2*b*c)^2)/(a^9*(4*
a*c - b^2)^4))*((c^5*x^2*(3*A*b^4 + 30*A*a^2*c^2 - B*a*b^3 - 21*A*a*b^2*c +
 7*B*a^2*b*c)^3)/(a^9*(4*a*c - b^2)^6) - ((3*A*b - B*a + a^4*(-(60*A*a^3*c^
3 - 3*A*b^6 + B*a*b^5 + 30*A*a*b^4*c - 10*B*a^2*b^3*c + 30*B*a^3*b*c^2 - 90
*A*a^2*b^2*c^2)^2/(a^8*(4*a*c - b^2)^5))^(1/2))*(((3*A*b - B*a + a^4*(-(60*
A*a^3*c^3 - 3*A*b^6 + B*a*b^5 + 30*A*a*b^4*c - 10*B*a^2*b^3*c + 30*B*a^3*b*
c^2 - 90*A*a^2*b^2*c^2)^2/(a^8*(4*a*c - b^2)^5))^(1/2))*((4*b*c^2*(30*A*a^3
*c^3 - 3*A*b^6 + B*a*b^5 + 27*A*a*b^4*c - 9*B*a^2*b^3*c + 23*B*a^3*b*c^2 -
69*A*a^2*b^2*c^2))/(a^3*(4*a*c - b^2)^2) + (2*c^3*x^2*(B*a*b^5 - 300*A*a^3*
c^3 - 3*A*b^6 + 6*A*a*b^4*c - 2*B*a^2*b^3*c + 10*B*a^3*b*c^2 + 90*A*a^2*b^2
*c^2))/(a^3*(4*a*c - b^2)^2) - (b*c^2*(3*A*b - B*a + a^4*(-(60*A*a^3*c^3 -
3*A*b^6 + B*a*b^5 + 30*A*a*b^4*c - 10*B*a^2*b^3*c + 30*B*a^3*b*c^2 - 90*A*a
^2*b^2*c^2)^2/(a^8*(4*a*c - b^2)^5))^(1/2))*(a*b + 3*b^2*x^2 - 10*a*c*x^2))
/a^4))/(4*a^4) - (c^3*(900*A^2*a^5*c^5 - 36*A^2*b^10 - 4*B^2*a^2*b^8 + 24*A
*B*a*b^9 - 3078*A^2*a^2*b^6*c^2 + 7533*A^2*a^3*b^4*c^3 - 7020*A^2*a^4*b^2*c
```

^4 - 302*B^2*a^4*b^4*c^2 + 497*B^2*a^5*b^2*c^3 + 549*A^2*a*b^8*c + 61*B^2*a
^3*b^6*c + 1932*A*B*a^3*b^5*c^2 - 4002*A*B*a^4*b^3*c^3 - 366*A*B*a^2*b^7*c
+ 2340*A*B*a^5*b*c^4))/(a^6*(4*a*c - b^2)^4) + (c^4*x^2*(54*A^2*b^9 + 6*B^2
*a^2*b^7 - 36*A*B*a*b^8 + 4311*A^2*a^2*b^5*c^2 - 9900*A^2*a^3*b^3*c^3 + 409
*B^2*a^4*b^3*c^2 - 2400*A*B*a^5*c^4 - 801*A^2*a*b^7*c + 8100*A^2*a^4*b*c^4
- 89*B^2*a^3*b^5*c - 560*B^2*a^5*b*c^3 - 2664*A*B*a^3*b^4*c^2 + 4980*A*B*a^
4*b^2*c^3 + 534*A*B*a^2*b^6*c))/(a^6*(4*a*c - b^2)^4)))/(4*a^4) + (c^4*(3*A
*b - B*a)*(3*A*b^4 + 30*A*a^2*c^2 - B*a*b^3 - 21*A*a*b^2*c + 7*B*a^2*b*c)^2
)/(a^9*(4*a*c - b^2)^4)))*(6*A*b^11 + 2048*B*a^6*c^5 - 2*B*a*b^10 - 120*A*a
*b^9*c - 6144*A*a^5*b*c^5 + 40*B*a^2*b^8*c + 960*A*a^2*b^7*c^2 - 3840*A*a^3
*b^5*c^3 + 7680*A*a^4*b^3*c^4 - 320*B*a^3*b^6*c^2 + 1280*B*a^4*b^4*c^3 - 25
60*B*a^5*b^2*c^4))/(2*(4*a^4*b^10 - 4096*a^9*c^5 - 80*a^5*b^8*c + 640*a^6*b
^6*c^2 - 2560*a^7*b^4*c^3 + 5120*a^8*b^2*c^4)) - (log(x)*(3*A*b - B*a))/a^4
 - (A/(2*a) + (x^2*(9*A*b^5 - 24*B*a^3*c^2 - 3*B*a*b^4 - 68*A*a*b^3*c + 122
*A*a^2*b*c^2 + 21*B*a^2*b^2*c))/(4*a^2*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) - (x
^6*(16*B*a^3*c^3 - 12*A*b^5*c + 4*B*a*b^4*c + 87*A*a*b^3*c^2 - 138*A*a^2*b*
c^3 - 29*B*a^2*b^2*c^2))/(4*a^3*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (x^4*(3*A
*b^6 + 50*A*a^3*c^3 - B*a*b^5 - 18*A*a*b^4*c + 6*B*a^2*b^3*c + B*a^3*b*c^2
+ 7*A*a^2*b^2*c^2))/(2*a^3*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (c^2*x^8*(3*A*
b^4 + 30*A*a^2*c^2 - B*a*b^3 - 21*A*a*b^2*c + 7*B*a^2*b*c))/(2*a^3*(b^4 + 1
6*a^2*c^2 - 8*a*b^2*c)))/(x^6*(2*a*c + b^2) + a^2*x^2 + c^2*x^10 + 2*a*b*x^
4 + 2*b*c*x^8) - (atan((x^2*((((((((153600*A*a^13*c^10 - 5120*B*a^13*b*c^9
+ 6*A*a^6*b^14*c^3 - 108*A*a^7*b^12*c^4 + 588*A*a^8*b^10*c^5 + 792*A*a^9*b^
8*c^6 - 22272*A*a^10*b^6*c^7 + 100608*A*a^11*b^4*c^8 - 199680*A*a^12*b^2*c^
9 - 2*B*a^7*b^13*c^3 + 36*B*a^8*b^11*c^4 - 276*B*a^9*b^9*c^5 + 1216*B*a^10*
b^7*c^6 - 3456*B*a^11*b^5*c^7 + 6144*B*a^12*b^3*c^8)/(a^9*b^12 + 4096*a^15*
c^6 - 24*a^10*b^10*c + 240*a^11*b^8*c^2 - 1280*a^12*b^6*c^3 + 3840*a^13*b^4
*c^4 - 6144*a^14*b^2*c^5) - ((163840*a^16*b*c^9 - 12*a^9*b^15*c^2 + 328*a^1
0*b^13*c^3 - 3840*a^11*b^11*c^4 + 24960*a^12*b^9*c^5 - 97280*a^13*b^7*c^6 +
 227328*a^14*b^5*c^7 - 294912*a^15*b^3*c^8)*(6*A*b^11 + 2048*B*a^6*c^5 - 2*
B*a*b^10 - 120*A*a*b^9*c - 6144*A*a^5*b*c^5 + 40*B*a^2*b^8*c + 960*A*a^2*b^
7*c^2 - 3840*A*a^3*b^5*c^3 + 7680*A*a^4*b^3*c^4 - 320*B*a^3*b^6*c^2 + 1280*
B*a^4*b^4*c^3 - 2560*B*a^5*b^2*c^4))/(2*(4*a^4*b^10 - 4096*a^9*c^5 - 80*a^5
*b^8*c + 640*a^6*b^6*c^2 - 2560*a^7*b^4*c^3 + 5120*a^8*b^2*c^4)*(a^9*b^12 +
 4096*a^15*c^6 - 24*a^10*b^10*c + 240*a^11*b^8*c^2 - 1280*a^12*b^6*c^3 + 38
40*a^13*b^4*c^4 - 6144*a^14*b^2*c^5)))*(60*A*a^3*c^3 - 3*A*b^6 + B*a*b^5 +
30*A*a*b^4*c - 10*B*a^2*b^3*c + 30*B*a^3*b*c^2 - 90*A*a^2*b^2*c^2))/(4*a^4*
(4*a*c - b^2)^(5/2)) - ((60*A*a^3*c^3 - 3*A*b^6 + B*a*b^5 + 30*A*a*b^4*c -
10*B*a^2*b^3*c + 30*B*a^3*b*c^2 - 90*A*a^2*b^2*c^2)*(163840*a^16*b*c^9 - 12
*a^9*b^15*c^2 + 328*a^10*b^13*c^3 - 3840*a^11*b^11*c^4 + 24960*a^12*b^9*c^5
 - 97280*a^13*b^7*c^6 + 227328*a^14*b^5*c^7 - 294912*a^15*b^3*c^8)*(6*A*b^1
1 + 2048*B*a^6*c^5 - 2*B*a*b^10 - 120*A*a*b^9*c - 6144*A*a^5*b*c^5 + 40*B*a
^2*b^8*c + 960*A*a^2*b^7*c^2 - 3840*A*a^3*b^5*c^3 + 7680*A*a^4*b^3*c^4 - 32
0*B*a^3*b^6*c^2 + 1280*B*a^4*b^4*c^3 - 2560*B*a^5*b^2*c^4))/(8*a^4*(4*a*c -
 b^2)^(5/2)*(4*a^4*b^10 - 4096*a^9*c^5 - 80*a^5*b^8*c + 640*a^6*b^6*c^2 - 2

560*a^7*b^4*c^3 + 5120*a^8*b^2*c^4)*(a^9*b^12 + 4096*a^15*c^6 - 24*a^10*b^1
0*c + 240*a^11*b^8*c^2 - 1280*a^12*b^6*c^3 + 3840*a^13*b^4*c^4 - 6144*a^14*
b^2*c^5)))*(6*A*b^11 + 2048*B*a^6*c^5 - 2*B*a*b^10 - 120*A*a*b^9*c - 6144*A
*a^5*b*c^5 + 40*B*a^2*b^8*c + 960*A*a^2*b^7*c^2 - 3840*A*a^3*b^5*c^3 + 7680
*A*a^4*b^3*c^4 - 320*B*a^3*b^6*c^2 + 1280*B*a^4*b^4*c^3 - 2560*B*a^5*b^2*c^
4))/(2*(4*a^4*b^10 - 4096*a^9*c^5 - 80*a^5*b^8*c + 640*a^6*b^6*c^2 - 2560*a
^7*b^4*c^3 + 5120*a^8*b^2*c^4)) - (((54*A^2*a^3*b^13*c^4 - 1233*A^2*a^4*b^1
1*c^5 + 11583*A^2*a^5*b^9*c^6 - 57204*A^2*a^6*b^7*c^7 + 156276*A^2*a^7*b^5*
c^8 - 223200*A^2*a^8*b^3*c^9 + 6*B^2*a^5*b^11*c^4 - 137*B^2*a^6*b^9*c^5 + 1
217*B^2*a^7*b^7*c^6 - 5256*B^2*a^8*b^5*c^7 + 11024*B^2*a^9*b^3*c^8 - 38400*
A*B*a^10*c^10 + 129600*A^2*a^9*b*c^10 - 8960*B^2*a^10*b*c^9 - 36*A*B*a^4*b^
12*c^4 + 822*A*B*a^5*b^10*c^5 - 7512*A*B*a^6*b^8*c^6 + 34836*A*B*a^7*b^6*c^
7 - 84864*A*B*a^8*b^4*c^8 + 98880*A*B*a^9*b^2*c^9)/(a^9*b^12 + 4096*a^15*c^
6 - 24*a^10*b^10*c + 240*a^11*b^8*c^2 - 1280*a^12*b^6*c^3 + 3840*a^13*b^4*c
^4 - 6144*a^14*b^2*c^5) - (((153600*A*a^13*c^10 - 5120*B*a^13*b*c^9 + 6*A*a
^6*b^14*c^3 - 108*A*a^7*b^12*c^4 + 588*A*a^8*b^10*c^5 + 792*A*a^9*b^8*c^6 -
 22272*A*a^10*b^6*c^7 + 100608*A*a^11*b^4*c^8 - 199680*A*a^12*b^2*c^9 - 2*B
*a^7*b^13*c^3 + 36*B*a^8*b^11*c^4 - 276*B*a^9*b^9*c^5 + 1216*B*a^10*b^7*c^6
 - 3456*B*a^11*b^5*c^7 + 6144*B*a^12*b^3*c^8)/(a^9*b^12 + 4096*a^15*c^6 - 2
4*a^10*b^10*c + 240*a^11*b^8*c^2 - 1280*a^12*b^6*c^3 + 3840*a^13*b^4*c^4 -
6144*a^14*b^2*c^5) - ((163840*a^16*b*c^9 - 12*a^9*b^15*c^2 + 328*a^10*b^13*
c^3 - 3840*a^11*b^11*c^4 + 24960*a^12*b^9*c^5 - 97280*a^13*b^7*c^6 + 227328
*a^14*b^5*c^7 - 294912*a^15*b^3*c^8)*(6*A*b^11 + 2048*B*a^6*c^5 - 2*B*a*b^1
0 - 120*A*a*b^9*c - 6144*A*a^5*b*c^5 + 40*B*a^2*b^8*c + 960*A*a^2*b^7*c^2 -
 3840*A*a^3*b^5*c^3 + 7680*A*a^4*b^3*c^4 - 320*B*a^3*b^6*c^2 + 1280*B*a^4*b
^4*c^3 - 2560*B*a^5*b^2*c^4))/(2*(4*a^4*b^10 - 4096*a^9*c^5 - 80*a^5*b^8*c
+ 640*a^6*b^6*c^2 - 2560*a^7*b^4*c^3 + 5120*a^8*b^2*c^4)*(a^9*b^12 + 4096*a
^15*c^6 - 24*a^10*b^10*c + 240*a^11*b^8*c^2 - 1280*a^12*b^6*c^3 + 3840*a^13
*b^4*c^4 - 6144*a^14*b^2*c^5)))*(6*A*b^11 + 2048*B*a^6*c^5 - 2*B*a*b^10 - 1
20*A*a*b^9*c - 6144*A*a^5*b*c^5 + 40*B*a^2*b^8*c + 960*A*a^2*b^7*c^2 - 3840
*A*a^3*b^5*c^3 + 7680*A*a^4*b^3*c^4 - 320*B*a^3*b^6*c^2 + 1280*B*a^4*b^4*c^
3 - 2560*B*a^5*b^2*c^4))/(2*(4*a^4*b^10 - 4096*a^9*c^5 - 80*a^5*b^8*c + 640
*a^6*b^6*c^2 - 2560*a^7*b^4*c^3 + 5120*a^8*b^2*c^4)))*(60*A*a^3*c^3 - 3*A*b
^6 + B*a*b^5 + 30*A*a*b^4*c - 10*B*a^2*b^3*c + 30*B*a^3*b*c^2 - 90*A*a^2*b^
2*c^2))/(4*a^4*(4*a*c - b^2)^(5/2)) + ((60*A*a^3*c^3 - 3*A*b^6 + B*a*b^5 +
30*A*a*b^4*c - 10*B*a^2*b^3*c + 30*B*a^3*b*c^2 - 90*A*a^2*b^2*c^2)^3*(16384
0*a^16*b*c^9 - 12*a^9*b^15*c^2 + 328*a^10*b^13*c^3 - 3840*a^11*b^11*c^4 + 2
4960*a^12*b^9*c^5 - 97280*a^13*b^7*c^6 + 227328*a^14*b^5*c^7 - 294912*a^15*
b^3*c^8))/(64*a^12*(4*a*c - b^2)^(15/2)*(a^9*b^12 + 4096*a^15*c^6 - 24*a^10
*b^10*c + 240*a^11*b^8*c^2 - 1280*a^12*b^6*c^3 + 3840*a^13*b^4*c^4 - 6144*a
^14*b^2*c^5)))*(9*A*b^9 - 160*B*a^5*c^4 - 3*B*a*b^8 - 117*A*a*b^7*c + 570*A
*a^4*b*c^4 + 39*B*a^2*b^6*c + 540*A*a^2*b^5*c^2 - 1005*A*a^3*b^3*c^3 - 180*
B*a^3*b^4*c^2 + 325*B*a^4*b^2*c^3))/(8*(4*a*c - b^2)^(13/2)*(900*A^2*a^9*c^
8 + 6400*B^2*a^10*c^7 - 54*A^2*a^3*b^12*c^2 - 960*a^6*b^6*c^4*(B^2*a - 36*A
^2*c) + 120*a^5*b^8*c^3*(B^2*a - 72*A^2*c) - 6*a^4*b^10*c^2*(B^2*a - 180*A^

2*c) - 25*a^8*b^2*c^6*(311*B^2*a - 2196*A^2*c) + 25*a^7*b^4*c^5*(154*B^2*a
- 2763*A^2*c) + 36*A*B*a^4*b^11*c^2 - 720*A*B*a^5*b^9*c^3 + 5760*A*B*a^6*b^
7*c^4 - 23070*A*B*a^7*b^5*c^5 + 46350*A*B*a^8*b^3*c^6 - 37500*A*B*a^9*b*c^7
)) - (((27000*A^3*a^6*c^11 + 27*A^3*b^12*c^5 + 4779*A^3*a^2*b^8*c^7 - 20601
*A^3*a^3*b^6*c^8 + 47790*A^3*a^4*b^4*c^9 - 56700*A^3*a^5*b^2*c^10 - B^3*a^3
*b^9*c^5 + 21*B^3*a^4*b^7*c^6 - 147*B^3*a^5*b^5*c^7 + 343*B^3*a^6*b^3*c^8 -
567*A^3*a*b^10*c^6 - 27*A^2*B*a*b^11*c^5 + 18900*A^2*B*a^6*b*c^10 + 9*A*B^
2*a^2*b^10*c^5 - 189*A*B^2*a^3*b^8*c^6 + 1413*A*B^2*a^4*b^6*c^7 - 4347*A*B^
2*a^5*b^4*c^8 + 4410*A*B^2*a^6*b^2*c^9 + 567*A^2*B*a^2*b^9*c^6 - 4509*A^2*B
*a^3*b^7*c^7 + 16821*A^2*B*a^4*b^5*c^8 - 29160*A^2*B*a^5*b^3*c^9)/(a^9*b^12
+ 4096*a^15*c^6 - 24*a^10*b^10*c + 240*a^11*b^8*c^2 - 1280*a^12*b^6*c^3 +
3840*a^13*b^4*c^4 - 6144*a^14*b^2*c^5) - (((54*A^2*a^3*b^13*c^4 - 1233*A^2*
a^4*b^11*c^5 + 11583*A^2*a^5*b^9*c^6 - 57204*A^2*a^6*b^7*c^7 + 156276*A^2*a
^7*b^5*c^8 - 223200*A^2*a^8*b^3*c^9 + 6*B^2*a^5*b^11*c^4 - 137*B^2*a^6*b^9*
c^5 + 1217*B^2*a^7*b^7*c^6 - 5256*B^2*a^8*b^5*c^7 + 11024*B^2*a^9*b^3*c^8 -
38400*A*B*a^10*c^10 + 129600*A^2*a^9*b*c^10 - 8960*B^2*a^10*b*c^9 - 36*A*B
*a^4*b^12*c^4 + 822*A*B*a^5*b^10*c^5 - 7512*A*B*a^6*b^8*c^6 + 34836*A*B*a^7
*b^6*c^7 - 84864*A*B*a^8*b^4*c^8 + 98880*A*B*a^9*b^2*c^9)/(a^9*b^12 + 4096*
a^15*c^6 - 24*a^10*b^10*c + 240*a^11*b^8*c^2 - 1280*a^12*b^6*c^3 + 3840*a^1
3*b^4*c^4 - 6144*a^14*b^2*c^5) - (((153600*A*a^13*c^10 - 5120*B*a^13*b*c^9
+ 6*A*a^6*b^14*c^3 - 108*A*a^7*b^12*c^4 + 588*A*a^8*b^10*c^5 + 792*A*a^9*b^
8*c^6 - 22272*A*a^10*b^6*c^7 + 100608*A*a^11*b^4*c^8 - 199680*A*a^12*b^2*c^
9 - 2*B*a^7*b^13*c^3 + 36*B*a^8*b^11*c^4 - 276*B*a^9*b^9*c^5 + 1216*B*a^10*
b^7*c^6 - 3456*B*a^11*b^5*c^7 + 6144*B*a^12*b^3*c^8)/(a^9*b^12 + 4096*a^15*
c^6 - 24*a^10*b^10*c + 240*a^11*b^8*c^2 - 1280*a^12*b^6*c^3 + 3840*a^13*b^4
*c^4 - 6144*a^14*b^2*c^5) - ((163840*a^16*b*c^9 - 12*a^9*b^15*c^2 + 328*a^1
0*b^13*c^3 - 3840*a^11*b^11*c^4 + 24960*a^12*b^9*c^5 - 97280*a^13*b^7*c^6 +
227328*a^14*b^5*c^7 - 294912*a^15*b^3*c^8)*(6*A*b^11 + 2048*B*a^6*c^5 - 2*
B*a*b^10 - 120*A*a*b^9*c - 6144*A*a^5*b*c^5 + 40*B*a^2*b^8*c + 960*A*a^2*b^
7*c^2 - 3840*A*a^3*b^5*c^3 + 7680*A*a^4*b^3*c^4 - 320*B*a^3*b^6*c^2 + 1280*
B*a^4*b^4*c^3 - 2560*B*a^5*b^2*c^4))/(2*(4*a^4*b^10 - 4096*a^9*c^5 - 80*a^5
*b^8*c + 640*a^6*b^6*c^2 - 2560*a^7*b^4*c^3 + 5120*a^8*b^2*c^4)*(a^9*b^12 +
4096*a^15*c^6 - 24*a^10*b^10*c + 240*a^11*b^8*c^2 - 1280*a^12*b^6*c^3 + 38
40*a^13*b^4*c^4 - 6144*a^14*b^2*c^5)))*(6*A*b^11 + 2048*B*a^6*c^5 - 2*B*a*b
^10 - 120*A*a*b^9*c - 6144*A*a^5*b*c^5 + 40*B*a^2*b^8*c + 960*A*a^2*b^7*c^2
- 3840*A*a^3*b^5*c^3 + 7680*A*a^4*b^3*c^4 - 320*B*a^3*b^6*c^2 + 1280*B*a^4
*b^4*c^3 - 2560*B*a^5*b^2*c^4))/(2*(4*a^4*b^10 - 4096*a^9*c^5 - 80*a^5*b^8*
c + 640*a^6*b^6*c^2 - 2560*a^7*b^4*c^3 + 5120*a^8*b^2*c^4)))*(6*A*b^11 + 20
48*B*a^6*c^5 - 2*B*a*b^10 - 120*A*a*b^9*c - 6144*A*a^5*b*c^5 + 40*B*a^2*b^8
*c + 960*A*a^2*b^7*c^2 - 3840*A*a^3*b^5*c^3 + 7680*A*a^4*b^3*c^4 - 320*B*a^
3*b^6*c^2 + 1280*B*a^4*b^4*c^3 - 2560*B*a^5*b^2*c^4))/(2*(4*a^4*b^10 - 4096
*a^9*c^5 - 80*a^5*b^8*c + 640*a^6*b^6*c^2 - 2560*a^7*b^4*c^3 + 5120*a^8*b^2
*c^4)) - (((((153600*A*a^13*c^10 - 5120*B*a^13*b*c^9 + 6*A*a^6*b^14*c^3 - 1
08*A*a^7*b^12*c^4 + 588*A*a^8*b^10*c^5 + 792*A*a^9*b^8*c^6 - 22272*A*a^10*b
^6*c^7 + 100608*A*a^11*b^4*c^8 - 199680*A*a^12*b^2*c^9 - 2*B*a^7*b^13*c^3 +

36*B*a^8*b^11*c^4 - 276*B*a^9*b^9*c^5 + 1216*B*a^10*b^7*c^6 - 3456*B*a^11*
b^5*c^7 + 6144*B*a^12*b^3*c^8)/(a^9*b^12 + 4096*a^15*c^6 - 24*a^10*b^10*c +
240*a^11*b^8*c^2 - 1280*a^12*b^6*c^3 + 3840*a^13*b^4*c^4 - 6144*a^14*b^2*c
^5) - ((163840*a^16*b*c^9 - 12*a^9*b^15*c^2 + 328*a^10*b^13*c^3 - 3840*a^11
*b^11*c^4 + 24960*a^12*b^9*c^5 - 97280*a^13*b^7*c^6 + 227328*a^14*b^5*c^7 -
294912*a^15*b^3*c^8)*(6*A*b^11 + 2048*B*a^6*c^5 - 2*B*a*b^10 - 120*A*a*b^9
*c - 6144*A*a^5*b*c^5 + 40*B*a^2*b^8*c + 960*A*a^2*b^7*c^2 - 3840*A*a^3*b^5
*c^3 + 7680*A*a^4*b^3*c^4 - 320*B*a^3*b^6*c^2 + 1280*B*a^4*b^4*c^3 - 2560*B
*a^5*b^2*c^4))/(2*(4*a^4*b^10 - 4096*a^9*c^5 - 80*a^5*b^8*c + 640*a^6*b^6*c
^2 - 2560*a^7*b^4*c^3 + 5120*a^8*b^2*c^4)*(a^9*b^12 + 4096*a^15*c^6 - 24*a^
10*b^10*c + 240*a^11*b^8*c^2 - 1280*a^12*b^6*c^3 + 3840*a^13*b^4*c^4 - 6144
*a^14*b^2*c^5)))*(60*A*a^3*c^3 - 3*A*b^6 + B*a*b^5 + 30*A*a*b^4*c - 10*B*a^
2*b^3*c + 30*B*a^3*b*c^2 - 90*A*a^2*b^2*c^2))/(4*a^4*(4*a*c - b^2)^(5/2)) -
((60*A*a^3*c^3 - 3*A*b^6 + B*a*b^5 + 30*A*a*b^4*c - 10*B*a^2*b^3*c + 30*B*
a^3*b*c^2 - 90*A*a^2*b^2*c^2)*(163840*a^16*b*c^9 - 12*a^9*b^15*c^2 + 328*a^
10*b^13*c^3 - 3840*a^11*b^11*c^4 + 24960*a^12*b^9*c^5 - 97280*a^13*b^7*c^6
+ 227328*a^14*b^5*c^7 - 294912*a^15*b^3*c^8)*(6*A*b^11 + 2048*B*a^6*c^5 - 2
*B*a*b^10 - 120*A*a*b^9*c - 6144*A*a^5*b*c^5 + 40*B*a^2*b^8*c + 960*A*a^2*b
^7*c^2 - 3840*A*a^3*b^5*c^3 + 7680*A*a^4*b^3*c^4 - 320*B*a^3*b^6*c^2 + 1280
*B*a^4*b^4*c^3 - 2560*B*a^5*b^2*c^4))/(8*a^4*(4*a*c - b^2)^(5/2)*(4*a^4*b^1
0 - 4096*a^9*c^5 - 80*a^5*b^8*c + 640*a^6*b^6*c^2 - 2560*a^7*b^4*c^3 + 5120
*a^8*b^2*c^4)*(a^9*b^12 + 4096*a^15*c^6 - 24*a^10*b^10*c + 240*a^11*b^8*c^2
- 1280*a^12*b^6*c^3 + 3840*a^13*b^4*c^4 - 6144*a^14*b^2*c^5)))*(60*A*a^3*c
^3 - 3*A*b^6 + B*a*b^5 + 30*A*a*b^4*c - 10*B*a^2*b^3*c + 30*B*a^3*b*c^2 - 9
0*A*a^2*b^2*c^2))/(4*a^4*(4*a*c - b^2)^(5/2)) + ((60*A*a^3*c^3 - 3*A*b^6 +
B*a*b^5 + 30*A*a*b^4*c - 10*B*a^2*b^3*c + 30*B*a^3*b*c^2 - 90*A*a^2*b^2*c^2
)^2*(163840*a^16*b*c^9 - 12*a^9*b^15*c^2 + 328*a^10*b^13*c^3 - 3840*a^11*b^
11*c^4 + 24960*a^12*b^9*c^5 - 97280*a^13*b^7*c^6 + 227328*a^14*b^5*c^7 - 29
4912*a^15*b^3*c^8)*(6*A*b^11 + 2048*B*a^6*c^5 - 2*B*a*b^10 - 120*A*a*b^9*c
- 6144*A*a^5*b*c^5 + 40*B*a^2*b^8*c + 960*A*a^2*b^7*c^2 - 3840*A*a^3*b^5*c^
3 + 7680*A*a^4*b^3*c^4 - 320*B*a^3*b^6*c^2 + 1280*B*a^4*b^4*c^3 - 2560*B*a^
5*b^2*c^4))/(32*a^8*(4*a*c - b^2)^5*(4*a^4*b^10 - 4096*a^9*c^5 - 80*a^5*b^8
*c + 640*a^6*b^6*c^2 - 2560*a^7*b^4*c^3 + 5120*a^8*b^2*c^4)*(a^9*b^12 + 409
6*a^15*c^6 - 24*a^10*b^10*c + 240*a^11*b^8*c^2 - 1280*a^12*b^6*c^3 + 3840*a
^13*b^4*c^4 - 6144*a^14*b^2*c^5)))*(9*A*b^8 + 30*A*a^4*c^4 - 3*B*a*b^7 - 99
*A*a*b^6*c + 33*B*a^2*b^5*c + 135*B*a^4*b*c^3 + 360*A*a^2*b^4*c^2 - 435*A*a
^3*b^2*c^3 - 120*B*a^3*b^3*c^2))/(8*a^3*c^2*(4*a*c - b^2)^6*(900*A^2*a^6*c^
6 - 54*A^2*b^12 - 6*B^2*a^2*b^10 + 6400*B^2*a^7*c^5 + 36*A*B*a*b^11 - 8640*
A^2*a^2*b^8*c^2 + 34560*A^2*a^3*b^6*c^3 - 69075*A^2*a^4*b^4*c^4 + 54900*A^2
*a^5*b^2*c^5 - 960*B^2*a^4*b^6*c^2 + 3850*B^2*a^5*b^4*c^3 - 7775*B^2*a^6*b^
2*c^4 + 1080*A^2*a*b^10*c + 120*B^2*a^3*b^8*c + 5760*A*B*a^3*b^7*c^2 - 2307
0*A*B*a^4*b^5*c^3 + 46350*A*B*a^5*b^3*c^4 - 720*A*B*a^2*b^9*c - 37500*A*B*a
^6*b*c^5)))*(16*a^12*b^12*(4*a*c - b^2)^(15/2) + 65536*a^18*c^6*(4*a*c - b^
2)^(15/2) - 384*a^13*b^10*c*(4*a*c - b^2)^(15/2) + 3840*a^14*b^8*c^2*(4*a*c
- b^2)^(15/2) - 20480*a^15*b^6*c^3*(4*a*c - b^2)^(15/2) + 61440*a^16*b^4*c

^4*(4*a*c - b^2)^(15/2) - 98304*a^17*b^2*c^5*(4*a*c - b^2)^(15/2)))/(3600*A ^2*a^6*c^8 + 9*A^2*b^12*c^2 + 1440*A^2*a^2*b^8*c^4 - 5760*A^2*a^3*b^6*c^5 + 11700*A^2*a^4*b^4*c^6 - 10800*A^2*a^5*b^2*c^7 + B^2*a^2*b^10*c^2 - 20*B^2* a^3*b^8*c^3 + 160*B^2*a^4*b^6*c^4 - 600*B^2*a^5*b^4*c^5 + 900*B^2*a^6*b^2*c ^6 - 180*A^2*a*b^10*c^3 + 120*A*B*a^2*b^9*c^3 - 960*A*B*a^3*b^7*c^4 + 3720* A*B*a^4*b^5*c^5 - 6600*A*B*a^5*b^3*c^6 - 6*A*B*a*b^11*c^2 + 3600*A*B*a^6*b* c^7) - (((((((1920*A*a^11*b*c^7 - 12*A*a^6*b^11*c^2 + 204*A*a^7*b^9*c^3 - 1 332*A*a^8*b^7*c^4 + 4056*A*a^9*b^5*c^5 - 5376*A*a^10*b^3*c^6 + 4*B*a^7*b^10 *c^2 - 68*B*a^8*b^8*c^3 + 444*B*a^9*b^6*c^4 - 1312*B*a^10*b^4*c^5 + 1472*B* a^11*b^2*c^6)/(a^9*b^8 + 256*a^13*c^4 - 16*a^10*b^6*c + 96*a^11*b^4*c^2 - 2 56*a^12*b^2*c^3) - ((4*a^10*b^10*c^2 - 64*a^11*b^8*c^3 + 384*a^12*b^6*c^4 - 1024*a^13*b^4*c^5 + 1024*a^14*b^2*c^6)*(6*A*b^11 + 2048*B*a^6*c^5 - 2*B*a* b^10 - 120*A*a*b^9*c - 6144*A*a^5*b*c^5 + 40*B*a^2*b^8*c + 960*A*a^2*b^7*c^ 2 - 3840*A*a^3*b^5*c^3 + 7680*A*a^4*b^3*c^4 - 320*B*a^3*b^6*c^2 + 1280*B*a^ 4*b^4*c^3 - 2560*B*a^5*b^2*c^4))/(2*(a^9*b^8 + 256*a^13*c^4 - 16*a^10*b^6*c + 96*a^11*b^4*c^2 - 256*a^12*b^2*c^3)*(4*a^4*b^10 - 4096*a^9*c^5 - 80*a^5* b^8*c + 640*a^6*b^6*c^2 - 2560*a^7*b^4*c^3 + 5120*a^8*b^2*c^4)))*(60*A*a^3* c^3 - 3*A*b^6 + B*a*b^5 + 30*A*a*b^4*c - 10*B*a^2*b^3*c + 30*B*a^3*b*c^2 - 90*A*a^2*b^2*c^2))/(4*a^4*(4*a*c - b^2)^(5/2)) - ((4*a^10*b^10*c^2 - 64*a^1 1*b^8*c^3 + 384*a^12*b^6*c^4 - 1024*a^13*b^4*c^5 + 1024*a^14*b^2*c^6)*(60*A *a^3*c^3 - 3*A*b^6 + B*a*b^5 + 30*A*a*b^4*c - 10*B*a^2*b^3*c + 30*B*a^3*b*c ^2 - 90*A*a^2*b^2*c^2)*(6*A*b^11 + 2048*B*a^6*c^5 - 2*B*a*b^10 - 120*A*a*b^ 9*c - 6144*A*a^5*b*c^5 + 40*B*a^2*b^8*c + 960*A*a^2*b^7*c^2 - 3840*A*a^3*b^ 5*c^3 + 7680*A*a^4*b^3*c^4 - 320*B*a^3*b^6*c^2 + 1280*B*a^4*b^4*c^3 - 2560* B*a^5*b^2*c^4))/(8*a^4*(4*a*c - b^2)^(5/2)*(a^9*b^8 + 256*a^13*c^4 - 16*a^1 0*b^6*c + 96*a^11*b^4*c^2 - 256*a^12*b^2*c^3)*(4*a^4*b^10 - 4096*a^9*c^5 - 80*a^5*b^8*c + 640*a^6*b^6*c^2 - 2560*a^7*b^4*c^3 + 5120*a^8*b^2*c^4)))*(6* A*b^11 + 2048*B*a^6*c^5 - 2*B*a*b^10 - 120*A*a*b^9*c - 6144*A*a^5*b*c^5 + 4 0*B*a^2*b^8*c + 960*A*a^2*b^7*c^2 - 3840*A*a^3*b^5*c^3 + 7680*A*a^4*b^3*c^4 - 320*B*a^3*b^6*c^2 + 1280*B*a^4*b^4*c^3 - 2560*B*a^5*b^2*c^4))/(2*(4*a^4* b^10 - 4096*a^9*c^5 - 80*a^5*b^8*c + 640*a^6*b^6*c^2 - 2560*a^7*b^4*c^3 + 5 120*a^8*b^2*c^4)) - (((900*A^2*a^8*c^8 - 36*A^2*a^3*b^10*c^3 + 549*A^2*a^4* b^8*c^4 - 3078*A^2*a^5*b^6*c^5 + 7533*A^2*a^6*b^4*c^6 - 7020*A^2*a^7*b^2*c^ 7 - 4*B^2*a^5*b^8*c^3 + 61*B^2*a^6*b^6*c^4 - 302*B^2*a^7*b^4*c^5 + 497*B^2* a^8*b^2*c^6 + 24*A*B*a^4*b^9*c^3 - 366*A*B*a^5*b^7*c^4 + 1932*A*B*a^6*b^5*c ^5 - 4002*A*B*a^7*b^3*c^6 + 2340*A*B*a^8*b*c^7)/(a^9*b^8 + 256*a^13*c^4 - 1 6*a^10*b^6*c + 96*a^11*b^4*c^2 - 256*a^12*b^2*c^3) - (((1920*A*a^11*b*c^7 - 12*A*a^6*b^11*c^2 + 204*A*a^7*b^9*c^3 - 1332*A*a^8*b^7*c^4 + 4056*A*a^9*b^ 5*c^5 - 5376*A*a^10*b^3*c^6 + 4*B*a^7*b^10*c^2 - 68*B*a^8*b^8*c^3 + 444*B*a ^9*b^6*c^4 - 1312*B*a^10*b^4*c^5 + 1472*B*a^11*b^2*c^6)/(a^9*b^8 + 256*a^13 *c^4 - 16*a^10*b^6*c + 96*a^11*b^4*c^2 - 256*a^12*b^2*c^3) - ((4*a^10*b^10* c^2 - 64*a^11*b^8*c^3 + 384*a^12*b^6*c^4 - 1024*a^13*b^4*c^5 + 1024*a^14*b^ 2*c^6)*(6*A*b^11 + 2048*B*a^6*c^5 - 2*B*a*b^10 - 120*A*a*b^9*c - 6144*A*a^5 *b*c^5 + 40*B*a^2*b^8*c + 960*A*a^2*b^7*c^2 - 3840*A*a^3*b^5*c^3 + 7680*A*a ^4*b^3*c^4 - 320*B*a^3*b^6*c^2 + 1280*B*a^4*b^4*c^3 - 2560*B*a^5*b^2*c^4))/

(2*(a^9*b^8 + 256*a^13*c^4 - 16*a^10*b^6*c + 96*a^11*b^4*c^2 - 256*a^12*b^2
c^3)(4*a^4*b^10 - 4096*a^9*c^5 - 80*a^5*b^8*c + 640*a^6*b^6*c^2 - 2560*a^
7*b^4*c^3 + 5120*a^8*b^2*c^4)))*(6*A*b^11 + 2048*B*a^6*c^5 - 2*B*a*b^10 - 1
20*A*a*b^9*c - 6144*A*a^5*b*c^5 + 40*B*a^2*b^8*c + 960*A*a^2*b^7*c^2 - 3840
*A*a^3*b^5*c^3 + 7680*A*a^4*b^3*c^4 - 320*B*a^3*b^6*c^2 + 1280*B*a^4*b^4*c^
3 - 2560*B*a^5*b^2*c^4))/(2*(4*a^4*b^10 - 4096*a^9*c^5 - 80*a^5*b^8*c + 640
*a^6*b^6*c^2 - 2560*a^7*b^4*c^3 + 5120*a^8*b^2*c^4)))*(60*A*a^3*c^3 - 3*A*b
^6 + B*a*b^5 + 30*A*a*b^4*c - 10*B*a^2*b^3*c + 30*B*a^3*b*c^2 - 90*A*a^2*b^
2*c^2))/(4*a^4*(4*a*c - b^2)^(5/2)) + ((4*a^10*b^10*c^2 - 64*a^11*b^8*c^3 +
384*a^12*b^6*c^4 - 1024*a^13*b^4*c^5 + 1024*a^14*b^2*c^6)*(60*A*a^3*c^3 -
3*A*b^6 + B*a*b^5 + 30*A*a*b^4*c - 10*B*a^2*b^3*c + 30*B*a^3*b*c^2 - 90*A*a
^2*b^2*c^2)^3)/(64*a^12*(4*a*c - b^2)^(15/2)*(a^9*b^8 + 256*a^13*c^4 - 16*a
^10*b^6*c + 96*a^11*b^4*c^2 - 256*a^12*b^2*c^3)))*(16*a^12*b^12*(4*a*c - b^
2)^(15/2) + 65536*a^18*c^6*(4*a*c - b^2)^(15/2) - 384*a^13*b^10*c*(4*a*c -
b^2)^(15/2) + 3840*a^14*b^8*c^2*(4*a*c - b^2)^(15/2) - 20480*a^15*b^6*c^3*(
4*a*c - b^2)^(15/2) + 61440*a^16*b^4*c^4*(4*a*c - b^2)^(15/2) - 98304*a^17*
b^2*c^5*(4*a*c - b^2)^(15/2))*(9*A*b^9 - 160*B*a^5*c^4 - 3*B*a*b^8 - 117*A*
a*b^7*c + 570*A*a^4*b*c^4 + 39*B*a^2*b^6*c + 540*A*a^2*b^5*c^2 - 1005*A*a^3
*b^3*c^3 - 180*B*a^3*b^4*c^2 + 325*B*a^4*b^2*c^3))/(8*(4*a*c - b^2)^(13/2)*
(900*A^2*a^9*c^8 + 6400*B^2*a^10*c^7 - 54*A^2*a^3*b^12*c^2 - 960*a^6*b^6*c^
4*(B^2*a - 36*A^2*c) + 120*a^5*b^8*c^3*(B^2*a - 72*A^2*c) - 6*a^4*b^10*c^2*
(B^2*a - 180*A^2*c) - 25*a^8*b^2*c^6*(311*B^2*a - 2196*A^2*c) + 25*a^7*b^4*
c^5*(154*B^2*a - 2763*A^2*c) + 36*A*B*a^4*b^11*c^2 - 720*A*B*a^5*b^9*c^3 +
5760*A*B*a^6*b^7*c^4 - 23070*A*B*a^7*b^5*c^5 + 46350*A*B*a^8*b^3*c^6 - 3750
0*A*B*a^9*b*c^7)*(3600*A^2*a^6*c^8 + 9*A^2*b^12*c^2 + 1440*A^2*a^2*b^8*c^4
- 5760*A^2*a^3*b^6*c^5 + 11700*A^2*a^4*b^4*c^6 - 10800*A^2*a^5*b^2*c^7 + B^
2*a^2*b^10*c^2 - 20*B^2*a^3*b^8*c^3 + 160*B^2*a^4*b^6*c^4 - 600*B^2*a^5*b^4
*c^5 + 900*B^2*a^6*b^2*c^6 - 180*A^2*a*b^10*c^3 + 120*A*B*a^2*b^9*c^3 - 960
*A*B*a^3*b^7*c^4 + 3720*A*B*a^4*b^5*c^5 - 6600*A*B*a^5*b^3*c^6 - 6*A*B*a*b^
11*c^2 + 3600*A*B*a^6*b*c^7)) + (((3780*A^3*a^3*b^3*c^7 - 1863*A^3*a^2*b^5*
c^6 - 27*A^3*b^9*c^4 + B^3*a^3*b^6*c^4 - 14*B^3*a^4*b^4*c^5 + 49*B^3*a^5*b^
2*c^6 + 900*A^2*B*a^5*c^8 + 378*A^3*a*b^7*c^5 - 2700*A^3*a^4*b*c^8 + 420*A*
B^2*a^5*b*c^7 + 27*A^2*B*a*b^8*c^4 - 9*A*B^2*a^2*b^7*c^4 + 126*A*B^2*a^3*b^
5*c^5 - 501*A*B^2*a^4*b^3*c^6 - 378*A^2*B*a^2*b^6*c^5 + 1683*A^2*B*a^3*b^4*
c^6 - 2520*A^2*B*a^4*b^2*c^7)/(a^9*b^8 + 256*a^13*c^4 - 16*a^10*b^6*c + 96*
a^11*b^4*c^2 - 256*a^12*b^2*c^3) - (((900*A^2*a^8*c^8 - 36*A^2*a^3*b^10*c^3
+ 549*A^2*a^4*b^8*c^4 - 3078*A^2*a^5*b^6*c^5 + 7533*A^2*a^6*b^4*c^6 - 7020
*A^2*a^7*b^2*c^7 - 4*B^2*a^5*b^8*c^3 + 61*B^2*a^6*b^6*c^4 - 302*B^2*a^7*b^4
*c^5 + 497*B^2*a^8*b^2*c^6 + 24*A*B*a^4*b^9*c^3 - 366*A*B*a^5*b^7*c^4 + 193
2*A*B*a^6*b^5*c^5 - 4002*A*B*a^7*b^3*c^6 + 2340*A*B*a^8*b*c^7)/(a^9*b^8 + 2
56*a^13*c^4 - 16*a^10*b^6*c + 96*a^11*b^4*c^2 - 256*a^12*b^2*c^3) - (((1920
*A*a^11*b*c^7 - 12*A*a^6*b^11*c^2 + 204*A*a^7*b^9*c^3 - 1332*A*a^8*b^7*c^4
+ 4056*A*a^9*b^5*c^5 - 5376*A*a^10*b^3*c^6 + 4*B*a^7*b^10*c^2 - 68*B*a^8*b^
8*c^3 + 444*B*a^9*b^6*c^4 - 1312*B*a^10*b^4*c^5 + 1472*B*a^11*b^2*c^6)/(a^9
*b^8 + 256*a^13*c^4 - 16*a^10*b^6*c + 96*a^11*b^4*c^2 - 256*a^12*b^2*c^3) -

((4*a^10*b^10*c^2 - 64*a^11*b^8*c^3 + 384*a^12*b^6*c^4 - 1024*a^13*b^4*c^5
+ 1024*a^14*b^2*c^6)*(6*A*b^11 + 2048*B*a^6*c^5 - 2*B*a*b^10 - 120*A*a*b^9
*c - 6144*A*a^5*b*c^5 + 40*B*a^2*b^8*c + 960*A*a^2*b^7*c^2 - 3840*A*a^3*b^5
*c^3 + 7680*A*a^4*b^3*c^4 - 320*B*a^3*b^6*c^2 + 1280*B*a^4*b^4*c^3 - 2560*B
*a^5*b^2*c^4))/(2*(a^9*b^8 + 256*a^13*c^4 - 16*a^10*b^6*c + 96*a^11*b^4*c^2
- 256*a^12*b^2*c^3)*(4*a^4*b^10 - 4096*a^9*c^5 - 80*a^5*b^8*c + 640*a^6*b^
6*c^2 - 2560*a^7*b^4*c^3 + 5120*a^8*b^2*c^4)))*(6*A*b^11 + 2048*B*a^6*c^5 -
2*B*a*b^10 - 120*A*a*b^9*c - 6144*A*a^5*b*c^5 + 40*B*a^2*b^8*c + 960*A*a^2
*b^7*c^2 - 3840*A*a^3*b^5*c^3 + 7680*A*a^4*b^3*c^4 - 320*B*a^3*b^6*c^2 + 12
80*B*a^4*b^4*c^3 - 2560*B*a^5*b^2*c^4))/(2*(4*a^4*b^10 - 4096*a^9*c^5 - 80*
a^5*b^8*c + 640*a^6*b^6*c^2 - 2560*a^7*b^4*c^3 + 5120*a^8*b^2*c^4)))*(6*A*b
^11 + 2048*B*a^6*c^5 - 2*B*a*b^10 - 120*A*a*b^9*c - 6144*A*a^5*b*c^5 + 40*B
*a^2*b^8*c + 960*A*a^2*b^7*c^2 - 3840*A*a^3*b^5*c^3 + 7680*A*a^4*b^3*c^4 -
320*B*a^3*b^6*c^2 + 1280*B*a^4*b^4*c^3 - 2560*B*a^5*b^2*c^4))/(2*(4*a^4*b^1
0 - 4096*a^9*c^5 - 80*a^5*b^8*c + 640*a^6*b^6*c^2 - 2560*a^7*b^4*c^3 + 5120
*a^8*b^2*c^4)) - (((((1920*A*a^11*b*c^7 - 12*A*a^6*b^11*c^2 + 204*A*a^7*b^9
*c^3 - 1332*A*a^8*b^7*c^4 + 4056*A*a^9*b^5*c^5 - 5376*A*a^10*b^3*c^6 + 4*B*
a^7*b^10*c^2 - 68*B*a^8*b^8*c^3 + 444*B*a^9*b^6*c^4 - 1312*B*a^10*b^4*c^5 +
1472*B*a^11*b^2*c^6)/(a^9*b^8 + 256*a^13*c^4 - 16*a^10*b^6*c + 96*a^11*b^4
*c^2 - 256*a^12*b^2*c^3) - ((4*a^10*b^10*c^2 - 64*a^11*b^8*c^3 + 384*a^12*b
^6*c^4 - 1024*a^13*b^4*c^5 + 1024*a^14*b^2*c^6)*(6*A*b^11 + 2048*B*a^6*c^5
- 2*B*a*b^10 - 120*A*a*b^9*c - 6144*A*a^5*b*c^5 + 40*B*a^2*b^8*c + 960*A*a^
2*b^7*c^2 - 3840*A*a^3*b^5*c^3 + 7680*A*a^4*b^3*c^4 - 320*B*a^3*b^6*c^2 + 1
280*B*a^4*b^4*c^3 - 2560*B*a^5*b^2*c^4))/(2*(a^9*b^8 + 256*a^13*c^4 - 16*a^
10*b^6*c + 96*a^11*b^4*c^2 - 256*a^12*b^2*c^3)*(4*a^4*b^10 - 4096*a^9*c^5 -
80*a^5*b^8*c + 640*a^6*b^6*c^2 - 2560*a^7*b^4*c^3 + 5120*a^8*b^2*c^4)))*(6
0*A*a^3*c^3 - 3*A*b^6 + B*a*b^5 + 30*A*a*b^4*c - 10*B*a^2*b^3*c + 30*B*a^3*
b*c^2 - 90*A*a^2*b^2*c^2))/(4*a^4*(4*a*c - b^2)^(5/2)) - ((4*a^10*b^10*c^2
- 64*a^11*b^8*c^3 + 384*a^12*b^6*c^4 - 1024*a^13*b^4*c^5 + 1024*a^14*b^2*c^
6)*(60*A*a^3*c^3 - 3*A*b^6 + B*a*b^5 + 30*A*a*b^4*c - 10*B*a^2*b^3*c + 30*B
*a^3*b*c^2 - 90*A*a^2*b^2*c^2)*(6*A*b^11 + 2048*B*a^6*c^5 - 2*B*a*b^10 - 12
0*A*a*b^9*c - 6144*A*a^5*b*c^5 + 40*B*a^2*b^8*c + 960*A*a^2*b^7*c^2 - 3840*
A*a^3*b^5*c^3 + 7680*A*a^4*b^3*c^4 - 320*B*a^3*b^6*c^2 + 1280*B*a^4*b^4*c^3
- 2560*B*a^5*b^2*c^4))/(8*a^4*(4*a*c - b^2)^(5/2)*(a^9*b^8 + 256*a^13*c^4
- 16*a^10*b^6*c + 96*a^11*b^4*c^2 - 256*a^12*b^2*c^3)*(4*a^4*b^10 - 4096*a^
9*c^5 - 80*a^5*b^8*c + 640*a^6*b^6*c^2 - 2560*a^7*b^4*c^3 + 5120*a^8*b^2*c^
4)))*(60*A*a^3*c^3 - 3*A*b^6 + B*a*b^5 + 30*A*a*b^4*c - 10*B*a^2*b^3*c + 30
*B*a^3*b*c^2 - 90*A*a^2*b^2*c^2))/(4*a^4*(4*a*c - b^2)^(5/2)) + ((4*a^10*b^
10*c^2 - 64*a^11*b^8*c^3 + 384*a^12*b^6*c^4 - 1024*a^13*b^4*c^5 + 1024*a^14
*b^2*c^6)*(60*A*a^3*c^3 - 3*A*b^6 + B*a*b^5 + 30*A*a*b^4*c - 10*B*a^2*b^3*c
+ 30*B*a^3*b*c^2 - 90*A*a^2*b^2*c^2)^2*(6*A*b^11 + 2048*B*a^6*c^5 - 2*B*a*
b^10 - 120*A*a*b^9*c - 6144*A*a^5*b*c^5 + 40*B*a^2*b^8*c + 960*A*a^2*b^7*c^
2 - 3840*A*a^3*b^5*c^3 + 7680*A*a^4*b^3*c^4 - 320*B*a^3*b^6*c^2 + 1280*B*a^
4*b^4*c^3 - 2560*B*a^5*b^2*c^4))/(32*a^8*(4*a*c - b^2)^5*(a^9*b^8 + 256*a^1
3*c^4 - 16*a^10*b^6*c + 96*a^11*b^4*c^2 - 256*a^12*b^2*c^3)*(4*a^4*b^10 - 4

096*a^9*c^5 - 80*a^5*b^8*c + 640*a^6*b^6*c^2 - 2560*a^7*b^4*c^3 + 5120*a^8*
b^2*c^4)))*(16*a^12*b^12*(4*a*c - b^2)^(15/2) + 65536*a^18*c^6*(4*a*c - b^2
)^(15/2) - 384*a^13*b^10*c*(4*a*c - b^2)^(15/2) + 3840*a^14*b^8*c^2*(4*a*c
- b^2)^(15/2) - 20480*a^15*b^6*c^3*(4*a*c - b^2)^(15/2) + 61440*a^16*b^4*c^
4*(4*a*c - b^2)^(15/2) - 98304*a^17*b^2*c^5*(4*a*c - b^2)^(15/2))*(9*A*b^8
+ 30*A*a^4*c^4 - 3*B*a*b^7 - 99*A*a*b^6*c + 33*B*a^2*b^5*c + 135*B*a^4*b*c^
3 + 360*A*a^2*b^4*c^2 - 435*A*a^3*b^2*c^3 - 120*B*a^3*b^3*c^2))/(8*a^3*c^2*
(4*a*c - b^2)^6*(3600*A^2*a^6*c^8 + 9*A^2*b^12*c^2 + 1440*A^2*a^2*b^8*c^4 -
 5760*A^2*a^3*b^6*c^5 + 11700*A^2*a^4*b^4*c^6 - 10800*A^2*a^5*b^2*c^7 + B^2
*a^2*b^10*c^2 - 20*B^2*a^3*b^8*c^3 + 160*B^2*a^4*b^6*c^4 - 600*B^2*a^5*b^4*
c^5 + 900*B^2*a^6*b^2*c^6 - 180*A^2*a*b^10*c^3 + 120*A*B*a^2*b^9*c^3 - 960*
A*B*a^3*b^7*c^4 + 3720*A*B*a^4*b^5*c^5 - 6600*A*B*a^5*b^3*c^6 - 6*A*B*a*b^1
1*c^2 + 3600*A*B*a^6*b*c^7)*(900*A^2*a^6*c^6 - 54*A^2*b^12 - 6*B^2*a^2*b^10
 + 6400*B^2*a^7*c^5 + 36*A*B*a*b^11 - 8640*A^2*a^2*b^8*c^2 + 34560*A^2*a^3*
b^6*c^3 - 69075*A^2*a^4*b^4*c^4 + 54900*A^2*a^5*b^2*c^5 - 960*B^2*a^4*b^6*c
^2 + 3850*B^2*a^5*b^4*c^3 - 7775*B^2*a^6*b^2*c^4 + 1080*A^2*a*b^10*c + 120*
B^2*a^3*b^8*c + 5760*A*B*a^3*b^7*c^2 - 23070*A*B*a^4*b^5*c^3 + 46350*A*B*a^
5*b^3*c^4 - 720*A*B*a^2*b^9*c - 37500*A*B*a^6*b*c^5)))*(60*A*a^3*c^3 - 3*A*
b^6 + B*a*b^5 + 30*A*a*b^4*c - 10*B*a^2*b^3*c + 30*B*a^3*b*c^2 - 90*A*a^2*b
^2*c^2))/(2*a^4*(4*a*c - b^2)^(5/2))

3.132 $\int \frac{x^8(A+Bx^2)}{(a+bx^2+cx^4)^3}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 554

$$\int \frac{x^8(A+Bx^2)}{(a+bx^2+cx^4)^3}\,dx = -\frac{(3b^3B+Ab^2c-24abBc+20aAc^2)\,x}{8c^2\,(b^2-4ac)^2}$$
$$+\frac{(b^2B+12Abc-28aBc)\,x^3}{8c\,(b^2-4ac)^2}-\frac{x^7(Ab-2aB-(bB-2Ac)x^2)}{4\,(b^2-4ac)\,(a+bx^2+cx^4)^2}$$
$$-\frac{x^5(7Ab^2-12abB-4aAc+(b^2B+12Abc-28aBc)\,x^2)}{8\,(b^2-4ac)^2\,(a+bx^2+cx^4)}$$
$$+\frac{\left(3b^4B+Ab^3c-27ab^2Bc-16aAbc^2+84a^2Bc^2-\frac{3b^5B+Ab^4c-33ab^3Bc-18aAb^2c^2+132a^2bBc^2-40a^2Ac^3}{\sqrt{b^2-4ac}}\right)\arctan\Bigl(}{8\sqrt{2}c^{5/2}\,(b^2-4ac)^2\,\sqrt{b-\sqrt{b^2-4ac}}}$$
$$+\frac{\left(3b^4B+Ab^3c-27ab^2Bc-16aAbc^2+84a^2Bc^2+\frac{3b^5B+Ab^4c-33ab^3Bc-18aAb^2c^2+132a^2bBc^2-40a^2Ac^3}{\sqrt{b^2-4ac}}\right)\arctan\Bigl(}{8\sqrt{2}c^{5/2}\,(b^2-4ac)^2\,\sqrt{b+\sqrt{b^2-4ac}}}$$

```
[Out] -1/8*(20*A*a*c^2+A*b^2*c-24*B*a*b*c+3*B*b^3)*x/c^2/(-4*a*c+b^2)^2+1/8*(12*A
*b*c-28*B*a*c+B*b^2)*x^3/c/(-4*a*c+b^2)^2-1/4*x^7*(A*b-2*B*a-(-2*A*c+B*b)*x
^2)/(-4*a*c+b^2)/(c*x^4+b*x^2+a)^2-1/8*x^5*(7*A*b^2-12*a*b*B-4*A*a*c+(12*A*
b*c-28*B*a*c+B*b^2)*x^2)/(-4*a*c+b^2)^2/(c*x^4+b*x^2+a)+1/16*arctan(x*2^(1/
2)*c^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2))*(3*B*b^4+A*b^3*c-27*B*a*b^2*c-16*A
*a*b*c^2+84*B*a^2*c^2+(40*A*a^2*c^3+18*A*a*b^2*c^2-A*b^4*c-132*B*a^2*b*c^2+
33*B*a*b^3*c-3*B*b^5)/(-4*a*c+b^2)^(1/2))/c^(5/2)/(-4*a*c+b^2)^2*2^(1/2)/(b
-(-4*a*c+b^2)^(1/2))^(1/2)+1/16*arctan(x*2^(1/2)*c^(1/2)/(b+(-4*a*c+b^2)^(1
/2))^(1/2))*(3*B*b^4+A*b^3*c-27*B*a*b^2*c-16*A*a*b*c^2+84*B*a^2*c^2+(-40*A*
a^2*c^3-18*A*a*b^2*c^2+A*b^4*c+132*B*a^2*b*c^2-33*B*a*b^3*c+3*B*b^5)/(-4*a*
c+b^2)^(1/2))/c^(5/2)/(-4*a*c+b^2)^2*2^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2)
```

Rubi [A] (verified)

Time = 7.40 (sec) , antiderivative size = 554, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {1289, 1293, 1180, 211}

$$\int \frac{x^8(A+Bx^2)}{(a+bx^2+cx^4)^3}\,dx$$
$$= \frac{\left(-\frac{-40a^2Ac^3+132a^2bBc^2-18aAb^2c^2-33ab^3Bc+Ab^4c+3b^5B}{\sqrt{b^2-4ac}}+84a^2Bc^2-16aAbc^2-27ab^2Bc+Ab^3c+3b^4B\right)\arctan}{8\sqrt{2}c^{5/2}\left(b^2-4ac\right)^2\sqrt{b-\sqrt{b^2-4ac}}}$$
$$+\frac{\left(\frac{-40a^2Ac^3+132a^2bBc^2-18aAb^2c^2-33ab^3Bc+Ab^4c+3b^5B}{\sqrt{b^2-4ac}}+84a^2Bc^2-16aAbc^2-27ab^2Bc+Ab^3c+3b^4B\right)\arctan}{8\sqrt{2}c^{5/2}\left(b^2-4ac\right)^2\sqrt{\sqrt{b^2-4ac}+b}}$$
$$+\frac{x^3(-28aBc+12Abc+b^2B)}{8c\left(b^2-4ac\right)^2}-\frac{x^7(-2aB-(x^2(bB-2Ac))+Ab)}{4\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2}$$
$$-\frac{x^5(x^2(-28aBc+12Abc+b^2B)-4aAc-12abB+7Ab^2)}{8\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)}$$
$$-\frac{x(20aAc^2-24abBc+Ab^2c+3b^3B)}{8c^2\left(b^2-4ac\right)^2}$$

```
[In] Int[(x^8*(A + B*x^2))/(a + b*x^2 + c*x^4)^3,x]
```

```
[Out] -1/8*((3*b^3*B + A*b^2*c - 24*a*b*B*c + 20*a*A*c^2)*x)/(c^2*(b^2 - 4*a*c)^2
) + ((b^2*B + 12*A*b*c - 28*a*B*c)*x^3)/(8*c*(b^2 - 4*a*c)^2) - (x^7*(A*b -
 2*a*B - (b*B - 2*A*c)*x^2))/(4*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)^2) - (x^5
*(7*A*b^2 - 12*a*b*B - 4*a*A*c + (b^2*B + 12*A*b*c - 28*a*B*c)*x^2))/(8*(b^
2 - 4*a*c)^2*(a + b*x^2 + c*x^4)) + ((3*b^4*B + A*b^3*c - 27*a*b^2*B*c - 16
*a*A*b*c^2 + 84*a^2*B*c^2 - (3*b^5*B + A*b^4*c - 33*a*b^3*B*c - 18*a*A*b^2*
c^2 + 132*a^2*b*B*c^2 - 40*a^2*A*c^3)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sq
rt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(8*Sqrt[2]*c^(5/2)*(b^2 - 4*a*c)^2*S
qrt[b - Sqrt[b^2 - 4*a*c]]) + ((3*b^4*B + A*b^3*c - 27*a*b^2*B*c - 16*a*A*b
*c^2 + 84*a^2*B*c^2 + (3*b^5*B + A*b^4*c - 33*a*b^3*B*c - 18*a*A*b^2*c^2 +
132*a^2*b*B*c^2 - 40*a^2*A*c^3)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*
x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(8*Sqrt[2]*c^(5/2)*(b^2 - 4*a*c)^2*Sqrt[b
+ Sqrt[b^2 - 4*a*c]])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
```

```
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1289

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_.), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(a + b*x^2 + c*x^4)^(p + 1)
*((b*d - 2*a*e - (b*e - 2*c*d)*x^2)/(2*(p + 1)*(b^2 - 4*a*c))), x] - Dist[f
^2/(2*(p + 1)*(b^2 - 4*a*c)), Int[(f*x)^(m - 2)*(a + b*x^2 + c*x^4)^(p + 1)
*Simp[(m - 1)*(b*d - 2*a*e) - (4*p + 4 + m + 1)*(b*e - 2*c*d)*x^2, x], x],
x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] &&
GtQ[m, 1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1293

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[e*f*(f*x)^(m - 1)*((a + b*x^2 + c*x^4)^(p +
1)/(c*(m + 4*p + 3))), x] - Dist[f^2/(c*(m + 4*p + 3)), Int[(f*x)^(m - 2)*(
a + b*x^2 + c*x^4)^p*Simp[a*e*(m - 1) + (b*e*(m + 2*p + 1) - c*d*(m + 4*p +
 3))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c,
 0] && GtQ[m, 1] && NeQ[m + 4*p + 3, 0] && IntegerQ[2*p] && (IntegerQ[p] ||
 IntegerQ[m])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{x^7(Ab-2aB-(bB-2Ac)x^2)}{4\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2}+\frac{\int \frac{x^6\left(7(Ab-2aB)+(-bB+2Ac)x^2\right)}{\left(a+bx^2+cx^4\right)^2}\,dx}{4\left(b^2-4ac\right)} \\
&= -\frac{x^7(Ab-2aB-(bB-2Ac)x^2)}{4\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2} \\
&\quad -\frac{x^5(7Ab^2-12abB-4aAc+\left(b^2B+12Abc-28aBc\right)x^2)}{8\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)} \\
&\quad +\frac{\int \frac{x^4\left(5(7Ab^2-12abB-4aAc)+3\left(b^2B+12Abc-28aBc\right)x^2\right)}{a+bx^2+cx^4}\,dx}{8\left(b^2-4ac\right)^2} \\
&= \frac{\left(b^2B+12Abc-28aBc\right)x^3}{8c\left(b^2-4ac\right)^2}-\frac{x^7(Ab-2aB-(bB-2Ac)x^2)}{4\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2} \\
&\quad -\frac{x^5(7Ab^2-12abB-4aAc+\left(b^2B+12Abc-28aBc\right)x^2)}{8\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)} \\
&\quad -\frac{\int \frac{x^2\left(9a\left(b^2B+12Abc-28aBc\right)+3\left(3b^3B+Ab^2c-24abBc+20aAc^2\right)x^2\right)}{a+bx^2+cx^4}\,dx}{24c\left(b^2-4ac\right)^2}
\end{aligned}$$

$$= -\frac{(3b^3B + Ab^2c - 24abBc + 20aAc^2)\,x}{8c^2\,(b^2-4ac)^2} + \frac{(b^2B + 12Abc - 28aBc)\,x^3}{8c\,(b^2-4ac)^2} - \frac{x^7(Ab - 2aB - (bB - 2Ac)x^2)}{4\,(b^2-4ac)\,(a + bx^2 + cx^4)^2} - \frac{x^5(7Ab^2 - 12abB - 4aAc + (b^2B + 12Abc - 28aBc)\,x^2)}{8\,(b^2-4ac)^2\,(a + bx^2 + cx^4)} + \frac{\int \frac{3a(3b^3B+Ab^2c-24abBc+20aAc^2)+3(3b^4B+Ab^3c-27ab^2Bc-16aAbc^2+84a^2Bc^2)x^2}{a+bx^2+cx^4}\,dx}{24c^2\,(b^2-4ac)^2}$$

$$= -\frac{(3b^3B + Ab^2c - 24abBc + 20aAc^2)\,x}{8c^2\,(b^2-4ac)^2} + \frac{(b^2B + 12Abc - 28aBc)\,x^3}{8c\,(b^2-4ac)^2} - \frac{x^7(Ab - 2aB - (bB - 2Ac)x^2)}{4\,(b^2-4ac)\,(a + bx^2 + cx^4)^2} - \frac{x^5(7Ab^2 - 12abB - 4aAc + (b^2B + 12Abc - 28aBc)\,x^2)}{8\,(b^2-4ac)^2\,(a + bx^2 + cx^4)} + \frac{\left(3b^4B + Ab^3c - 27ab^2Bc - 16aAbc^2 + 84a^2Bc^2 - \frac{3b^5B+Ab^4c-33ab^3Bc-18aAb^2c^2+132a^2bBc^2-40a^2Ac^3}{\sqrt{b^2-4ac}}\right)}{16c^2\,(b^2-4ac)^2} + \frac{\left(3b^4B + Ab^3c - 27ab^2Bc - 16aAbc^2 + 84a^2Bc^2 + \frac{3b^5B+Ab^4c-33ab^3Bc-18aAb^2c^2+132a^2bBc^2-40a^2Ac^3}{\sqrt{b^2-4ac}}\right)}{16c^2\,(b^2-4ac)^2}$$

$$= -\frac{(3b^3B + Ab^2c - 24abBc + 20aAc^2)\,x}{8c^2\,(b^2-4ac)^2} + \frac{(b^2B + 12Abc - 28aBc)\,x^3}{8c\,(b^2-4ac)^2} - \frac{x^7(Ab - 2aB - (bB - 2Ac)x^2)}{4\,(b^2-4ac)\,(a + bx^2 + cx^4)^2} - \frac{x^5(7Ab^2 - 12abB - 4aAc + (b^2B + 12Abc - 28aBc)\,x^2)}{8\,(b^2-4ac)^2\,(a + bx^2 + cx^4)} + \frac{\left(3b^4B + Ab^3c - 27ab^2Bc - 16aAbc^2 + 84a^2Bc^2 - \frac{3b^5B+Ab^4c-33ab^3Bc-18aAb^2c^2+132a^2bBc^2-40a^2Ac^3}{\sqrt{b^2-4ac}}\right)}{8\sqrt{2}c^{5/2}\,(b^2-4ac)^2\,\sqrt{b-\sqrt{b^2-4ac}}} + \frac{\left(3b^4B + Ab^3c - 27ab^2Bc - 16aAbc^2 + 84a^2Bc^2 + \frac{3b^5B+Ab^4c-33ab^3Bc-18aAb^2c^2+132a^2bBc^2-40a^2Ac^3}{\sqrt{b^2-4ac}}\right)}{8\sqrt{2}c^{5/2}\,(b^2-4ac)^2\,\sqrt{b+\sqrt{b^2-4ac}}}$$

Mathematica [A] (verified)

Time = 1.52 (sec) , antiderivative size = 644, normalized size of antiderivative = 1.16

$$\int \frac{x^8(A+Bx^2)}{(a+bx^2+cx^4)^3}\,dx$$

$$= \frac{\frac{2x(2b^5B-b^4c(2A+5Bx^2)-4a^2c^3(9A+11Bx^2)+ab^2c^2(11A+37Bx^2)+16abc^2(3aB-Acx^2)+b^3c(-17aB+Acx^2))}{(b^2-4ac)^2(a+bx^2+cx^4)} - \frac{4x(b^3(bB-Ac)x^2+a^2c(-3$$

[In] Integrate[(x^8*(A + B*x^2))/(a + b*x^2 + c*x^4)^3,x]

[Out] ((2*x*(2*b^5*B - b^4*c*(2*A + 5*B*x^2) - 4*a^2*c^3*(9*A + 11*B*x^2) + a*b^2*c^2*(11*A + 37*B*x^2) + 16*a*b*c^2*(3*a*B - A*c*x^2) + b^3*c*(-17*a*B + A*c*x^2)))/((b^2 - 4*a*c)^2*(a + b*x^2 + c*x^4)) - (4*x*(b^3*(b*B - A*c)*x^2 + a^2*c*(-3*b*B + 2*c*(A + B*x^2)) + a*b*(b^2*B + 3*A*c^2*x^2 - b*c*(A + 4*B*x^2))))/((b^2 - 4*a*c)*(a + b*x^2 + c*x^4)^2) + (Sqrt[2]*Sqrt[c]*(-3*b^5*B + b^3*c*(33*a*B + A*Sqrt[b^2 - 4*a*c]) - 4*a*b*c^2*(33*a*B + 4*A*Sqrt[b^2 - 4*a*c]) + 9*a*b^2*c*(2*A*c - 3*B*Sqrt[b^2 - 4*a*c]) + b^4*(-(A*c) + 3*B*Sqrt[b^2 - 4*a*c]) + 4*a^2*c^2*(10*A*c + 21*B*Sqrt[b^2 - 4*a*c]))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/((b^2 - 4*a*c)^(5/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (Sqrt[2]*Sqrt[c]*(3*b^5*B + 4*a*b*c^2*(33*a*B - 4*A*Sqrt[b^2 - 4*a*c]) + b^4*(A*c + 3*B*Sqrt[b^2 - 4*a*c]) - 9*a*b^2*c*(2*A*c + 3*B*Sqrt[b^2 - 4*a*c]) + 4*a^2*c^2*(-10*A*c + 21*B*Sqrt[b^2 - 4*a*c]) + b^3*(-33*a*B*c + A*c*Sqrt[b^2 - 4*a*c]))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/((b^2 - 4*a*c)^(5/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]]))/(16*c^3)

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.17 (sec) , antiderivative size = 445, normalized size of antiderivative = 0.80

method	result
risch	$\frac{-\frac{\left(16Aab\,c^2-A\,b^3c+44B\,a^2c^2-37Ba\,b^2c+5B\,b^4\right)x^7}{8\left(16a^2c^2-8a\,b^2c+b^4\right)c}-\frac{\left(36A\,a^2c^3+5Aa\,b^2c^2+A\,b^4c-4B\,a^2b\,c^2-20Ba\,b^3c+3B\,b^5\right)x^5}{8c^2\left(16a^2c^2-8a\,b^2c+b^4\right)}-\frac{a\left(28Aab\,c^2+2A\,b^3c+28B\,a^2}{8c^2\left(16a^2c^2-8}}{(c\,x^4+b\,x^2+a)^2}$
default	$\frac{-\frac{\left(16Aab\,c^2-A\,b^3c+44B\,a^2c^2-37Ba\,b^2c+5B\,b^4\right)x^7}{8\left(16a^2c^2-8a\,b^2c+b^4\right)c}-\frac{\left(36A\,a^2c^3+5Aa\,b^2c^2+A\,b^4c-4B\,a^2b\,c^2-20Ba\,b^3c+3B\,b^5\right)x^5}{8c^2\left(16a^2c^2-8a\,b^2c+b^4\right)}-\frac{a\left(28Aab\,c^2+2A\,b^3c+28B\,a^2}{8c^2\left(16a^2c^2-8}}{(c\,x^4+b\,x^2+a)^2}$

```
[In] int(x^8*(B*x^2+A)/(c*x^4+b*x^2+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] (-1/8*(16*A*a*b*c^2-A*b^3*c+44*B*a^2*c^2-37*B*a*b^2*c+5*B*b^4)/(16*a^2*c^2-
8*a*b^2*c+b^4)/c*x^7-1/8*(36*A*a^2*c^3+5*A*a*b^2*c^2+A*b^4*c-4*B*a^2*b*c^2-
20*B*a*b^3*c+3*B*b^5)/c^2/(16*a^2*c^2-8*a*b^2*c+b^4)*x^5-1/8*a/c^2*(28*A*a*
b*c^2+2*A*b^3*c+28*B*a^2*c^2-49*B*a*b^2*c+6*B*b^4)/(16*a^2*c^2-8*a*b^2*c+b^
4)*x^3-1/8*a^2*(20*A*a*c^2+A*b^2*c-24*B*a*b*c+3*B*b^3)/c^2/(16*a^2*c^2-8*a*
b^2*c+b^4)*x)/(c*x^4+b*x^2+a)^2+1/16/c^2*sum((-(16*A*a*b*c^2-A*b^3*c-84*B*a
^2*c^2+27*B*a*b^2*c-3*B*b^4)/(16*a^2*c^2-8*a*b^2*c+b^4)*_R^2+a*(20*A*a*c^2+
A*b^2*c-24*B*a*b*c+3*B*b^3)/(16*a^2*c^2-8*a*b^2*c+b^4))/(2*_R^3*c+_R*b)*ln(
x-_R),_R=RootOf(_Z^4*c+_Z^2*b+a))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 9636 vs. $2(507) = 1014$.

Time = 18.80 (sec) , antiderivative size = 9636, normalized size of antiderivative = 17.39

$$\int \frac{x^8(A+Bx^2)}{(a+bx^2+cx^4)^3}\,dx = \text{Too large to display}$$

```
[In] integrate(x^8*(B*x^2+A)/(c*x^4+b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^8(A+Bx^2)}{(a+bx^2+cx^4)^3}\,dx = \text{Timed out}$$

```
[In] integrate(x**8*(B*x**2+A)/(c*x**4+b*x**2+a)**3,x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{x^8(A+Bx^2)}{(a+bx^2+cx^4)^3}\,dx = \int \frac{(Bx^2+A)x^8}{(cx^4+bx^2+a)^3}\,dx$$

```
[In] integrate(x^8*(B*x^2+A)/(c*x^4+b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] -1/8*((5*B*b^4*c + 4*(11*B*a^2 + 4*A*a*b)*c^3 - (37*B*a*b^2 + A*b^3)*c^2)*x
^7 + (3*B*b^5 + 36*A*a^2*c^3 - (4*B*a^2*b - 5*A*a*b^2)*c^2 - (20*B*a*b^3 -
A*b^4)*c)*x^5 + (6*B*a*b^4 + 28*(B*a^3 + A*a^2*b)*c^2 - (49*B*a^2*b^2 - 2*A
```

```
*a*b^3)*c)*x^3 + (3*B*a^2*b^3 + 20*A*a^3*c^2 - (24*B*a^3*b - A*a^2*b^2)*c)*
x)/((b^4*c^4 - 8*a*b^2*c^5 + 16*a^2*c^6)*x^8 + a^2*b^4*c^2 - 8*a^3*b^2*c^3
+ 16*a^4*c^4 + 2*(b^5*c^3 - 8*a*b^3*c^4 + 16*a^2*b*c^5)*x^6 + (b^6*c^2 - 6*
a*b^4*c^3 + 32*a^3*c^5)*x^4 + 2*(a*b^5*c^2 - 8*a^2*b^3*c^3 + 16*a^3*b*c^4)*
x^2) - 1/8*integrate(-(3*B*a*b^3 + 20*A*a^2*c^2 + (3*B*b^4 + 4*(21*B*a^2 -
4*A*a*b)*c^2 - (27*B*a*b^2 - A*b^3)*c)*x^2 - (24*B*a^2*b - A*a*b^2)*c)/(c*x
^4 + b*x^2 + a), x)/(b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3987 vs. $2(507) = 1014$.

Time = 2.02 (sec) , antiderivative size = 3987, normalized size of antiderivative = 7.20

$$\int \frac{x^8(A+Bx^2)}{(a+bx^2+cx^4)^3}\,dx = \text{Too large to display}$$

```
[In] integrate(x^8*(B*x^2+A)/(c*x^4+b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] 1/32*((sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^6*c + 12*sqrt(2)*sqrt(b*c
+ sqrt(b^2 - 4*a*c)*c)*a*b^4*c^2 - 2*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c
)*b^5*c^2 - 2*b^6*c^2 - 144*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^2
*c^3 - 32*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c^3 + sqrt(2)*sqrt(
b*c + sqrt(b^2 - 4*a*c)*c)*b^4*c^3 - 24*a*b^4*c^3 - 2*b^5*c^3 + 320*sqrt(2)
*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*c^4 + 160*sqrt(2)*sqrt(b*c + sqrt(b^2
- 4*a*c)*c)*a^2*b*c^4 + 16*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^
4 + 288*a^2*b^2*c^4 + 112*a*b^3*c^4 - 80*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*
c)*c)*a^2*c^5 - 640*a^3*c^5 - 416*a^2*b*c^5 + sqrt(2)*sqrt(b^2 - 4*a*c)*sqr
t(b*c + sqrt(b^2 - 4*a*c)*c)*b^5*c - 56*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c
+ sqrt(b^2 - 4*a*c)*c)*a*b^3*c^2 - 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + s
qrt(b^2 - 4*a*c)*c)*b^4*c^2 + 208*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt
(b^2 - 4*a*c)*c)*a^2*b*c^3 + 104*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(
b^2 - 4*a*c)*c)*a*b^2*c^3 + sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 -
 4*a*c)*c)*b^3*c^3 - 52*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a
*c)*c)*a*b*c^4 + 2*(b^2 - 4*a*c)*b^4*c^2 + 32*(b^2 - 4*a*c)*a*b^2*c^3 + 2*(
b^2 - 4*a*c)*b^3*c^3 - 160*(b^2 - 4*a*c)*a^2*c^4 - 104*(b^2 - 4*a*c)*a*b*c^
4)*A + 3*(sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^7 - 16*sqrt(2)*sqrt(b*c
 + sqrt(b^2 - 4*a*c)*c)*a*b^5*c - 2*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)
*b^6*c - 2*b^7*c + 80*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c^2 +
 24*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^4*c^2 + sqrt(2)*sqrt(b*c +
sqrt(b^2 - 4*a*c)*c)*b^5*c^2 + 32*a*b^5*c^2 - 2*b^6*c^2 - 128*sqrt(2)*sqrt(
b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b*c^3 - 64*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a
*c)*c)*a^2*b^2*c^3 - 12*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c^3 -
 160*a^2*b^3*c^3 + 28*a*b^4*c^3 + 32*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c
)*a^2*b*c^4 + 256*a^3*b*c^4 - 192*a^2*b^2*c^4 + 448*a^3*c^5 + sqrt(2)*sqrt(
```

b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^6 - 14*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^4*c - 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^5*c + 96*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^2 + 20*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c^2 + sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^4*c^2 - 224*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*c^3 - 112*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b*c^3 - 10*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^3 + 56*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*c^4 + 2*(b^2 - 4*a*c)*b^5*c - 24*(b^2 - 4*a*c)*a*b^3*c^2 + 2*(b^2 - 4*a*c)*b^4*c^2 + 64*(b^2 - 4*a*c)*a^2*b*c^3 - 20*(b^2 - 4*a*c)*a*b^2*c^3 + 112*(b^2 - 4*a*c)*a^2*c^4)*B)*arctan(2*sqrt(1/2)*x/sqrt((b^5*c^2 - 8*a*b^3*c^3 + 16*a^2*b*c^4 + sqrt((b^5*c^2 - 8*a*b^3*c^3 + 16*a^2*b*c^4)^2 - 4*(a*b^4*c^2 - 8*a^2*b^2*c^3 + 16*a^3*c^4)*(b^4*c^3 - 8*a*b^2*c^4 + 16*a^2*c^5)))/(b^4*c^3 - 8*a*b^2*c^4 + 16*a^2*c^5)))/((b^8*c^2 - 16*a*b^6*c^3 - 2*b^7*c^3 + 96*a^2*b^4*c^4 + 24*a*b^5*c^4 + b^6*c^4 - 256*a^3*b^2*c^5 - 96*a^2*b^3*c^5 - 12*a*b^4*c^5 + 256*a^4*c^6 + 128*a^3*b*c^6 + 48*a^2*b^2*c^6 - 64*a^3*c^7)*abs(c)) + 1/32*((sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^6*c + 12*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^4*c^2 - 2*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^5*c^2 + 2*b^6*c^2 - 144*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^3 - 32*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^3*c^3 + sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^4*c^3 + 24*a*b^4*c^3 + 2*b^5*c^3 + 320*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*c^4 + 160*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b*c^4 + 16*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^2*c^4 - 288*a^2*b^2*c^4 - 112*a*b^3*c^4 - 80*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*c^5 + 640*a^3*c^5 + 416*a^2*b*c^5 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^5*c + 56*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^3*c^2 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^4*c^2 - 208*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b*c^3 - 104*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^2*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^3*c^3 + 52*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b*c^4 - 2*(b^2 - 4*a*c)*b^4*c^2 - 32*(b^2 - 4*a*c)*a*b^2*c^3 - 2*(b^2 - 4*a*c)*b^3*c^3 + 160*(b^2 - 4*a*c)*a^2*c^4 + 104*(b^2 - 4*a*c)*a*b*c^4)*A + 3*(sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^7 - 16*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^5*c - 2*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^6*c + 2*b^7*c + 80*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c^2 + 24*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^4*c^2 + sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^5*c^2 - 32*a*b^5*c^2 + 2*b^6*c^2 - 128*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b*c^3 - 64*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^3 - 12*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^3*c^3 + 160*a^2*b^3*c^3 - 28*a*b^4*c^3 + 32*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b*c^4 - 256*a^3*b*c^4 + 192*a^2*b^2*c^4 - 448*a^3*c^5 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^6 + 14*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^4*c + 2*sqr

```
t(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^5*c - 96*sqrt(2)*s
qrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^2 - 20*sqrt(2)*s
qrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^3*c^2 - sqrt(2)*sqrt(b
^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^4*c^2 + 224*sqrt(2)*sqrt(b^2
- 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*c^3 + 112*sqrt(2)*sqrt(b^2 - 4
*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b*c^3 + 10*sqrt(2)*sqrt(b^2 - 4*a
*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^2*c^3 - 56*sqrt(2)*sqrt(b^2 - 4*a*c
)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*c^4 - 2*(b^2 - 4*a*c)*b^5*c + 24*(b^2
 - 4*a*c)*a*b^3*c^2 - 2*(b^2 - 4*a*c)*b^4*c^2 - 64*(b^2 - 4*a*c)*a^2*b*c^3
+ 20*(b^2 - 4*a*c)*a*b^2*c^3 - 112*(b^2 - 4*a*c)*a^2*c^4)*B)*arctan(2*sqrt(
1/2)*x/sqrt((b^5*c^2 - 8*a*b^3*c^3 + 16*a^2*b*c^4 - sqrt((b^5*c^2 - 8*a*b^3
*c^3 + 16*a^2*b*c^4)^2 - 4*(a*b^4*c^2 - 8*a^2*b^2*c^3 + 16*a^3*c^4)*(b^4*c^
3 - 8*a*b^2*c^4 + 16*a^2*c^5)))/(b^4*c^3 - 8*a*b^2*c^4 + 16*a^2*c^5)))/((b^
8*c^2 - 16*a*b^6*c^3 - 2*b^7*c^3 + 96*a^2*b^4*c^4 + 24*a*b^5*c^4 + b^6*c^4
- 256*a^3*b^2*c^5 - 96*a^2*b^3*c^5 - 12*a*b^4*c^5 + 256*a^4*c^6 + 128*a^3*b
*c^6 + 48*a^2*b^2*c^6 - 64*a^3*c^7)*abs(c)) - 1/8*(5*B*b^4*c*x^7 - 37*B*a*b
^2*c^2*x^7 - A*b^3*c^2*x^7 + 44*B*a^2*c^3*x^7 + 16*A*a*b*c^3*x^7 + 3*B*b^5*
x^5 - 20*B*a*b^3*c*x^5 + A*b^4*c*x^5 - 4*B*a^2*b*c^2*x^5 + 5*A*a*b^2*c^2*x^
5 + 36*A*a^2*c^3*x^5 + 6*B*a*b^4*x^3 - 49*B*a^2*b^2*c*x^3 + 2*A*a*b^3*c*x^3
 + 28*B*a^3*c^2*x^3 + 28*A*a^2*b*c^2*x^3 + 3*B*a^2*b^3*x - 24*B*a^3*b*c*x +
 A*a^2*b^2*c*x + 20*A*a^3*c^2*x)/((b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*(c*x
^4 + b*x^2 + a)^2)
```

Mupad [B] (verification not implemented)

Time = 10.32 (sec) , antiderivative size = 22911, normalized size of antiderivative = 41.36

$$\int \frac{x^8(A+Bx^2)}{(a+bx^2+cx^4)^3}\,dx = \text{Too large to display}$$

```
[In] int((x^8*(A + B*x^2))/(a + b*x^2 + c*x^4)^3,x)
```

```
[Out] - ((x^5*(3*B*b^5 + 36*A*a^2*c^3 + A*b^4*c - 20*B*a*b^3*c + 5*A*a*b^2*c^2 -
4*B*a^2*b*c^2))/(8*c^2*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (x^7*(5*B*b^4 + 44
*B*a^2*c^2 - A*b^3*c + 16*A*a*b*c^2 - 37*B*a*b^2*c))/(8*c*(b^4 + 16*a^2*c^2
 - 8*a*b^2*c)) + (x^3*(28*B*a^3*c^2 + 6*B*a*b^4 + 2*A*a*b^3*c + 28*A*a^2*b*
c^2 - 49*B*a^2*b^2*c))/(8*c^2*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (a^2*x*(3*B
*b^3 + 20*A*a*c^2 + A*b^2*c - 24*B*a*b*c))/(8*c^2*(b^4 + 16*a^2*c^2 - 8*a*b
^2*c)))/(x^4*(2*a*c + b^2) + a^2 + c^2*x^8 + 2*a*b*x^2 + 2*b*c*x^6) - atan(
((((256*A*a*b^12*c^4 - 5242880*A*a^7*c^10 + 768*B*a*b^13*c^3 + 6291456*B*a^
7*b*c^9 - 61440*A*a^3*b^8*c^6 + 655360*A*a^4*b^6*c^7 - 2949120*A*a^5*b^4*c^
8 + 6291456*A*a^6*b^2*c^9 - 21504*B*a^2*b^11*c^4 + 245760*B*a^3*b^9*c^5 - 1
474560*B*a^4*b^7*c^6 + 4915200*B*a^5*b^5*c^7 - 8650752*B*a^6*b^3*c^8)/(512*
(4096*a^6*c^9 + b^12*c^3 - 24*a*b^10*c^4 + 240*a^2*b^8*c^5 - 1280*a^3*b^6*c
^6 + 3840*a^4*b^4*c^7 - 6144*a^5*b^2*c^8)) - (x*(-(9*B^2*b^19 + A^2*b^17*c^
```

2 + 9*B^2*b^4*(-(4*a*c - b^2)^15)^(1/2) + 6*A*B*b^18*c + 1140*A^2*a^2*b^13*
c^4 - 10160*A^2*a^3*b^11*c^5 + 34880*A^2*a^4*b^9*c^6 + 43776*A^2*a^5*b^7*c^
7 - 680960*A^2*a^6*b^5*c^8 + 1863680*A^2*a^7*b^3*c^9 + 6921*B^2*a^2*b^15*c^
2 - 77580*B^2*a^3*b^13*c^3 + 570960*B^2*a^4*b^11*c^4 - 2851776*B^2*a^5*b^9*
c^5 + 9628416*B^2*a^6*b^7*c^6 - 21095424*B^2*a^7*b^5*c^7 + 27095040*B^2*a^8
*b^3*c^8 + A^2*b^2*c^2*(-(4*a*c - b^2)^15)^(1/2) + 441*B^2*a^2*c^2*(-(4*a*c
 - b^2)^15)^(1/2) + 6881280*A*B*a^9*c^10 - 369*B^2*a*b^17*c - 55*A^2*a*b^15
*c^3 - 1720320*A^2*a^8*b*c^10 - 25*A^2*a*c^3*(-(4*a*c - b^2)^15)^(1/2) - 15
482880*B^2*a^9*b*c^9 + 5580*A*B*a^2*b^14*c^3 - 59280*A*B*a^3*b^12*c^4 + 377
280*A*B*a^4*b^10*c^5 - 1430784*A*B*a^5*b^8*c^6 + 2860032*A*B*a^6*b^6*c^7 -
1290240*A*B*a^7*b^4*c^8 - 5160960*A*B*a^8*b^2*c^9 - 99*B^2*a*b^2*c*(-(4*a*c
 - b^2)^15)^(1/2) - 288*A*B*a*b^16*c^2 + 6*A*B*b^3*c*(-(4*a*c - b^2)^15)^(1
/2) - 108*A*B*a*b*c^2*(-(4*a*c - b^2)^15)^(1/2))/(512*(1048576*a^10*c^15 +
b^20*c^5 - 40*a*b^18*c^6 + 720*a^2*b^16*c^7 - 7680*a^3*b^14*c^8 + 53760*a^4
*b^12*c^9 - 258048*a^5*b^10*c^10 + 860160*a^6*b^8*c^11 - 1966080*a^7*b^6*c^
12 + 2949120*a^8*b^4*c^13 - 2621440*a^9*b^2*c^14)))^(1/2)*(256*b^11*c^5 - 5
120*a*b^9*c^6 - 262144*a^5*b*c^10 + 40960*a^2*b^7*c^7 - 163840*a^3*b^5*c^8
+ 327680*a^4*b^3*c^9))/(32*(256*a^4*c^7 + b^8*c^3 - 16*a*b^6*c^4 + 96*a^2*b
^4*c^5 - 256*a^3*b^2*c^6)))*(-(9*B^2*b^19 + A^2*b^17*c^2 + 9*B^2*b^4*(-(4*a
*c - b^2)^15)^(1/2) + 6*A*B*b^18*c + 1140*A^2*a^2*b^13*c^4 - 10160*A^2*a^3*
b^11*c^5 + 34880*A^2*a^4*b^9*c^6 + 43776*A^2*a^5*b^7*c^7 - 680960*A^2*a^6*b
^5*c^8 + 1863680*A^2*a^7*b^3*c^9 + 6921*B^2*a^2*b^15*c^2 - 77580*B^2*a^3*b^
13*c^3 + 570960*B^2*a^4*b^11*c^4 - 2851776*B^2*a^5*b^9*c^5 + 9628416*B^2*a^
6*b^7*c^6 - 21095424*B^2*a^7*b^5*c^7 + 27095040*B^2*a^8*b^3*c^8 + A^2*b^2*c
^2*(-(4*a*c - b^2)^15)^(1/2) + 441*B^2*a^2*c^2*(-(4*a*c - b^2)^15)^(1/2) +
6881280*A*B*a^9*c^10 - 369*B^2*a*b^17*c - 55*A^2*a*b^15*c^3 - 1720320*A^2*a
^8*b*c^10 - 25*A^2*a*c^3*(-(4*a*c - b^2)^15)^(1/2) - 15482880*B^2*a^9*b*c^9
 + 5580*A*B*a^2*b^14*c^3 - 59280*A*B*a^3*b^12*c^4 + 377280*A*B*a^4*b^10*c^5
 - 1430784*A*B*a^5*b^8*c^6 + 2860032*A*B*a^6*b^6*c^7 - 1290240*A*B*a^7*b^4*
c^8 - 5160960*A*B*a^8*b^2*c^9 - 99*B^2*a*b^2*c*(-(4*a*c - b^2)^15)^(1/2) -
288*A*B*a*b^16*c^2 + 6*A*B*b^3*c*(-(4*a*c - b^2)^15)^(1/2) - 108*A*B*a*b*c^
2*(-(4*a*c - b^2)^15)^(1/2))/(512*(1048576*a^10*c^15 + b^20*c^5 - 40*a*b^18
*c^6 + 720*a^2*b^16*c^7 - 7680*a^3*b^14*c^8 + 53760*a^4*b^12*c^9 - 258048*a
^5*b^10*c^10 + 860160*a^6*b^8*c^11 - 1966080*a^7*b^6*c^12 + 2949120*a^8*b^4
*c^13 - 2621440*a^9*b^2*c^14)))^(1/2) - (x*(9*B^2*b^10 + 800*A^2*a^4*c^6 +
A^2*b^8*c^2 - 14112*B^2*a^5*c^5 + 6*A*B*b^9*c + 314*A^2*a^2*b^4*c^4 + 208*A
^2*a^3*b^2*c^5 + 1881*B^2*a^2*b^6*c^2 - 9090*B^2*a^3*b^4*c^3 + 21312*B^2*a^
4*b^2*c^4 - 198*B^2*a*b^8*c - 36*A^2*a*b^6*c^3 + 1422*A*B*a^2*b^5*c^3 - 446
4*A*B*a^3*b^3*c^4 - 174*A*B*a*b^7*c^2 + 96*A*B*a^4*b*c^5))/(32*(256*a^4*c^7
 + b^8*c^3 - 16*a*b^6*c^4 + 96*a^2*b^4*c^5 - 256*a^3*b^2*c^6)))*(-(9*B^2*b^
19 + A^2*b^17*c^2 + 9*B^2*b^4*(-(4*a*c - b^2)^15)^(1/2) + 6*A*B*b^18*c + 11
40*A^2*a^2*b^13*c^4 - 10160*A^2*a^3*b^11*c^5 + 34880*A^2*a^4*b^9*c^6 + 4377
6*A^2*a^5*b^7*c^7 - 680960*A^2*a^6*b^5*c^8 + 1863680*A^2*a^7*b^3*c^9 + 6921
*B^2*a^2*b^15*c^2 - 77580*B^2*a^3*b^13*c^3 + 570960*B^2*a^4*b^11*c^4 - 2851
776*B^2*a^5*b^9*c^5 + 9628416*B^2*a^6*b^7*c^6 - 21095424*B^2*a^7*b^5*c^7 +

27095040*B^2*a^8*b^3*c^8 + A^2*b^2*c^2*(-(4*a*c - b^2)^15)^(1/2) + 441*B^2*a^2*c^2*(-(4*a*c - b^2)^15)^(1/2) + 6881280*A*B*a^9*c^10 - 369*B^2*a*b^17*c - 55*A^2*a*b^15*c^3 - 1720320*A^2*a^8*b*c^10 - 25*A^2*a*c^3*(-(4*a*c - b^2)^15)^(1/2) - 15482880*B^2*a^9*b*c^9 + 5580*A*B*a^2*b^14*c^3 - 59280*A*B*a^3*b^12*c^4 + 377280*A*B*a^4*b^10*c^5 - 1430784*A*B*a^5*b^8*c^6 + 2860032*A*B*a^6*b^6*c^7 - 1290240*A*B*a^7*b^4*c^8 - 5160960*A*B*a^8*b^2*c^9 - 99*B^2*a*b^2*c*(-(4*a*c - b^2)^15)^(1/2) - 288*A*B*a*b^16*c^2 + 6*A*B*b^3*c*(-(4*a*c - b^2)^15)^(1/2) - 108*A*B*a*b*c^2*(-(4*a*c - b^2)^15)^(1/2))/(512*(1048576*a^10*c^15 + b^20*c^5 - 40*a*b^18*c^6 + 720*a^2*b^16*c^7 - 7680*a^3*b^14*c^8 + 53760*a^4*b^12*c^9 - 258048*a^5*b^10*c^10 + 860160*a^6*b^8*c^11 - 1966080*a^7*b^6*c^12 + 2949120*a^8*b^4*c^13 - 2621440*a^9*b^2*c^14)))^(1/2)*1i - (((256*A*a*b^12*c^4 - 5242880*A*a^7*c^10 + 768*B*a*b^13*c^3 + 6291456*B*a^7*b*c^9 - 61440*A*a^3*b^8*c^6 + 655360*A*a^4*b^6*c^7 - 2949120*A*a^5*b^4*c^8 + 6291456*A*a^6*b^2*c^9 - 21504*B*a^2*b^11*c^4 + 245760*B*a^3*b^9*c^5 - 1474560*B*a^4*b^7*c^6 + 4915200*B*a^5*b^5*c^7 - 8650752*B*a^6*b^3*c^8)/(512*(4096*a^6*c^9 + b^12*c^3 - 24*a*b^10*c^4 + 240*a^2*b^8*c^5 - 1280*a^3*b^6*c^6 + 3840*a^4*b^4*c^7 - 6144*a^5*b^2*c^8)) + (x*(-(9*B^2*b^19 + A^2*b^17*c^2 + 9*B^2*b^4*(-(4*a*c - b^2)^15)^(1/2) + 6*A*B*b^18*c + 1140*A^2*a^2*b^13*c^4 - 10160*A^2*a^3*b^11*c^5 + 34880*A^2*a^4*b^9*c^6 + 43776*A^2*a^5*b^7*c^7 - 680960*A^2*a^6*b^5*c^8 + 1863680*A^2*a^7*b^3*c^9 + 6921*B^2*a^2*b^15*c^2 - 77580*B^2*a^3*b^13*c^3 + 570960*B^2*a^4*b^11*c^4 - 2851776*B^2*a^5*b^9*c^5 + 9628416*B^2*a^6*b^7*c^6 - 21095424*B^2*a^7*b^5*c^7 + 27095040*B^2*a^8*b^3*c^8 + A^2*b^2*c^2*(-(4*a*c - b^2)^15)^(1/2) + 441*B^2*a^2*c^2*(-(4*a*c - b^2)^15)^(1/2) + 6881280*A*B*a^9*c^10 - 369*B^2*a*b^17*c - 55*A^2*a*b^15*c^3 - 1720320*A^2*a^8*b*c^10 - 25*A^2*a*c^3*(-(4*a*c - b^2)^15)^(1/2) - 15482880*B^2*a^9*b*c^9 + 5580*A*B*a^2*b^14*c^3 - 59280*A*B*a^3*b^12*c^4 + 377280*A*B*a^4*b^10*c^5 - 1430784*A*B*a^5*b^8*c^6 + 2860032*A*B*a^6*b^6*c^7 - 1290240*A*B*a^7*b^4*c^8 - 5160960*A*B*a^8*b^2*c^9 - 99*B^2*a*b^2*c*(-(4*a*c - b^2)^15)^(1/2) - 288*A*B*a*b^16*c^2 + 6*A*B*b^3*c*(-(4*a*c - b^2)^15)^(1/2) - 108*A*B*a*b*c^2*(-(4*a*c - b^2)^15)^(1/2))/(512*(1048576*a^10*c^15 + b^20*c^5 - 40*a*b^18*c^6 + 720*a^2*b^16*c^7 - 7680*a^3*b^14*c^8 + 53760*a^4*b^12*c^9 - 258048*a^5*b^10*c^10 + 860160*a^6*b^8*c^11 - 1966080*a^7*b^6*c^12 + 2949120*a^8*b^4*c^13 - 2621440*a^9*b^2*c^14)))^(1/2)*(256*b^11*c^5 - 5120*a*b^9*c^6 - 262144*a^5*b*c^10 + 40960*a^2*b^7*c^7 - 163840*a^3*b^5*c^8 + 327680*a^4*b^3*c^9))/(32*(256*a^4*c^7 + b^8*c^3 - 16*a*b^6*c^4 + 96*a^2*b^4*c^5 - 256*a^3*b^2*c^6)))*(-(9*B^2*b^19 + A^2*b^17*c^2 + 9*B^2*b^4*(-(4*a*c - b^2)^15)^(1/2) + 6*A*B*b^18*c + 1140*A^2*a^2*b^13*c^4 - 10160*A^2*a^3*b^11*c^5 + 34880*A^2*a^4*b^9*c^6 + 43776*A^2*a^5*b^7*c^7 - 680960*A^2*a^6*b^5*c^8 + 1863680*A^2*a^7*b^3*c^9 + 6921*B^2*a^2*b^15*c^2 - 77580*B^2*a^3*b^13*c^3 + 570960*B^2*a^4*b^11*c^4 - 2851776*B^2*a^5*b^9*c^5 + 9628416*B^2*a^6*b^7*c^6 - 21095424*B^2*a^7*b^5*c^7 + 27095040*B^2*a^8*b^3*c^8 + A^2*b^2*c^2*(-(4*a*c - b^2)^15)^(1/2) + 441*B^2*a^2*c^2*(-(4*a*c - b^2)^15)^(1/2) + 6881280*A*B*a^9*c^10 - 369*B^2*a*b^17*c - 55*A^2*a*b^15*c^3 - 1720320*A^2*a^8*b*c^10 - 25*A^2*a*c^3*(-(4*a*c - b^2)^15)^(1/2) - 15482880*B^2*a^9*b*c^9 + 5580*A*B*a^2*b^14*c^3 - 59280*A*B*a^3*b^12*c^4 + 377280*A*B*a^4*b^10* c^5 - 1430784*A*B*a^5*b^8*c^6 + 2860032*A*B*a^6*b^6*c^7 - 1290240*A*B*a^7*b^4*c^8 - 5160960*A*B*a^8*b^2*c^9 - 99*B^2*a*b^2*c*(-(4*a*c - b^2)^15)^(1/2) - 288*A*B*a*b^16*c^2 + 6*A*B*b^3*c*(-(4*a*c - b^2)^15)^(1/2) - 108*A*B*a*b*c^2*(-(4*a*c - b^2)^15)^(1/2))/(512*(1048576*a^10*c^15 + b^20*c^5 - 40*a*b^18*c^6 + 720*a^2*b^16*c^7 - 7680*a^3*b^14*c^8 + 53760*a^4*b^12*c^9 - 258048*a^5*b^10*c^10 + 860160*a^6*b^8*c^11 - 1966080*a^7*b^6*c^12 + 2949120*a^8*b^4*c^13 - 2621440*a^9*b^2*c^14)))^(1/2) + (x*(9*B^2*b^10 + 800*A^2*a^4*c^6 + A^2*b^8*c^2 - 14112*B^2*a^5*c^5 + 6*A*B*b^9*c + 314*A^2*a^2*b^4*c^4 + 208*A^2*a^3*b^2*c^5 + 1881*B^2*a^2*b^6*c^2 - 9090*B^2*a^3*b^4*c^3 + 21312*B^2*a^4*b^2*c^4 - 198*B^2*a*b^8*c - 36*A^2*a*b^6*c^3 + 1422*A*B*a^2*b^5*c^3 - 4464*A*B*a^3*b^3*c^4 - 174*A*B*a*b^7*c^2 + 96*A*B*a^4*b*c^5))/(32*(256*a^4*c^7 + b^8*c^3 - 16*a*b^6*c^4 + 96*a^2*b^4*c^5 - 256*a^3*b^2*c^6)))*(-(9*B^2*b^19 + A^2*b^17*c^2 + 9*B^2*b^4*(-(4*a*c - b^2)^15)^(1/2) + 6*A*B*b^18*c + 1140*A^2*a^2*b^13*c^4 - 10160*A^2*a^3*b^11*c^5 + 34880*A^2*a^4*b^9*c^6 + 43776*A^2*a^5*b^7*c^7 - 680960*A^2*a^6*b^5*c^8 + 1863680*A^2*a^7*b^3*c^9 + 6921*B^2*a^2*b^15*c^2 - 77580*B^2*a^3*b^13*c^3 + 570960*B^2*a^4*b^11*c^4 - 2851776*B^2*a^5*b^9*c^5 + 9628416*B^2*a^6*b^7*c^6 - 21095424*B^2*a^7*b^5*c^7 + 27095040*B^2*a^8*b^3*c^8 + A^2*b^2*c^2*(-(4*a*c - b^2)^15)^(1/2) + 441*B^2*a^2*c^2*(-(4*a*c - b^2)^15)^(1/2) + 6881280*A*B*a^9*c^10 - 369*B^2*a*b^17*c - 55*A^2*a*b^15*c^3 - 1720320*A^2*a^8*b*c^10 - 25*A^2*a*c^3*(-(4*a*c - b^2)^15)^(1/2) - 15482880*B^2*a^9*b*c^9 + 5580*A*B*a^2*b^14*c^3 - 59280*A*B*a^3*b^12*c^4 + 377280*A*B*a^4*b^10*c^5 - 1430784*A*B*a^5*b^8*c^6 + 2860032*A*B*a^6*b^6*c^7 - 1290240*A*B*a^7*b^4*c^8 - 5160960*A*B*a^8*b^2*c^9 - 99*B^2*a*b^2*c*(-(4*a*c - b^2)^15)^(1/2) - 288*A*B*a*b^16*c^2 + 6*A*B*b^3*c*(-(4*a*c - b^2)^15)^(1/2) - 108*A*B*a*b*c^2*(-(4*a*c - b^2)^15)^(1/2))/(512*(1048576*a^10*c^15 + b^20*c^5 - 40*a*b^18*c^6 + 720*a^2*b^16*c^7 - 7680*a^3*b^14*c^8 + 53760*a^4*b^12*c^9 - 258048*a^5*b^10*c^10 + 860160*a^6*b^8*c^11 - 1966080*a^7*b^6*c^12 + 2949120*a^8*b^4*c^13 - 2621440*a^9*b^2*c^14)))^(1/2)*1i)/((((256*A*a*b^12*c^4 - 5242880*A*a^7*c^10 + 768*B*a*b^13*c^3 + 6291456*B*a^7*b*c^9 - 61440*A*a^3*b^8*c^6 + 655360*A*a^4*b^6*c^7 - 2949120*A*a^5*b^4*c^8 + 6291456*A*a^6*b^2*c^9 - 21504*B*a^2*b^11*c^4 + 245760*B*a^3*b^9*c^5 - 1474560*B*a^4*b^7*c^6 + 4915200*B*a^5*b^5*c^7 - 8650752*B*a^6*b^3*c^8)/(512*(4096*a^6*c^9 + b^12*c^3 - 24*a*b^10*c^4 + 240*a^2*b^8*c^5 - 1280*a^3*b^6*c^6 + 3840*a^4*b^4*c^7 - 6144*a^5*b^2*c^8)) - (x*(-(9*B^2*b^19 + A^2*b^17*c^2 + 9*B^2*b^4*(-(4*a*c - b^2)^15)^(1/2) + 6*A*B*b^18*c + 1140*A^2*a^2*b^13*c^4 - 10160*A^2*a^3*b^11*c^5 + 34880*A^2*a^4*b^9*c^6 + 43776*A^2*a^5*b^7*c^7 - 680960*A^2*a^6*b^5*c^8 + 1863680*A^2*a^7*b^3*c^9 + 6921*B^2*a^2*b^15*c^2 - 77580*B^2*a^3*b^13*c^3 + 570960*B^2*a^4*b^11*c^4 - 2851776*B^2*a^5*b^9*c^5 + 9628416*B^2*a^6*b^7*c^6 - 21095424*B^2*a^7*b^5*c^7 + 27095040*B^2*a^8*b^3*c^8 + A^2*b^2*c^2*(-(4*a*c - b^2)^15)^(1/2) + 441*B^2*a^2*c^2*(-(4*a*c - b^2)^15)^(1/2) + 6881280*A*B*a^9*c^10 - 369*B^2*a*b^17*c - 55*A^2*a*b^15*c^3 - 1720320*A^2*a^8*b*c^10 - 25*A^2*a*c^3*(-(4*a*c - b^2)^15)^(1/2) - 15482880*B^2*a^9*b*c^9 + 5580*A*B*a^2*b^14*c^3 - 59280*A*B*a^3*b^12*c^4 + 377280*A*B*a^4*b^10*c^5 - 1430784*A*B*a^5*b^8*c^6 + 2860032*A*B*a^6*b^6*c^7 - 1290240*A*B*a^7*b^4*c^8 - 5160960*A*B*a^8*b^2*c^9 - 99*B^2*a*b^2*c*(-

(4*a*c - b^2)^15)^(1/2) - 288*A*B*a*b^16*c^2 + 6*A*B*b^3*c*(-(4*a*c - b^2)^
15)^(1/2) - 108*A*B*a*b*c^2*(-(4*a*c - b^2)^15)^(1/2))/(512*(1048576*a^10*c
^15 + b^20*c^5 - 40*a*b^18*c^6 + 720*a^2*b^16*c^7 - 7680*a^3*b^14*c^8 + 537
60*a^4*b^12*c^9 - 258048*a^5*b^10*c^10 + 860160*a^6*b^8*c^11 - 1966080*a^7*
b^6*c^12 + 2949120*a^8*b^4*c^13 - 2621440*a^9*b^2*c^14)))^(1/2)*(256*b^11*c
^5 - 5120*a*b^9*c^6 - 262144*a^5*b*c^10 + 40960*a^2*b^7*c^7 - 163840*a^3*b^
5*c^8 + 327680*a^4*b^3*c^9))/(32*(256*a^4*c^7 + b^8*c^3 - 16*a*b^6*c^4 + 96
*a^2*b^4*c^5 - 256*a^3*b^2*c^6)))*(-(9*B^2*b^19 + A^2*b^17*c^2 + 9*B^2*b^4*
(-(4*a*c - b^2)^15)^(1/2) + 6*A*B*b^18*c + 1140*A^2*a^2*b^13*c^4 - 10160*A^
2*a^3*b^11*c^5 + 34880*A^2*a^4*b^9*c^6 + 43776*A^2*a^5*b^7*c^7 - 680960*A^2
*a^6*b^5*c^8 + 1863680*A^2*a^7*b^3*c^9 + 6921*B^2*a^2*b^15*c^2 - 77580*B^2*
a^3*b^13*c^3 + 570960*B^2*a^4*b^11*c^4 - 2851776*B^2*a^5*b^9*c^5 + 9628416*
B^2*a^6*b^7*c^6 - 21095424*B^2*a^7*b^5*c^7 + 27095040*B^2*a^8*b^3*c^8 + A^2
*b^2*c^2*(-(4*a*c - b^2)^15)^(1/2) + 441*B^2*a^2*c^2*(-(4*a*c - b^2)^15)^(1
/2) + 6881280*A*B*a^9*c^10 - 369*B^2*a*b^17*c - 55*A^2*a*b^15*c^3 - 1720320
*A^2*a^8*b*c^10 - 25*A^2*a*c^3*(-(4*a*c - b^2)^15)^(1/2) - 15482880*B^2*a^9
*b*c^9 + 5580*A*B*a^2*b^14*c^3 - 59280*A*B*a^3*b^12*c^4 + 377280*A*B*a^4*b^
10*c^5 - 1430784*A*B*a^5*b^8*c^6 + 2860032*A*B*a^6*b^6*c^7 - 1290240*A*B*a^
7*b^4*c^8 - 5160960*A*B*a^8*b^2*c^9 - 99*B^2*a*b^2*c*(-(4*a*c - b^2)^15)^(1
/2) - 288*A*B*a*b^16*c^2 + 6*A*B*b^3*c*(-(4*a*c - b^2)^15)^(1/2) - 108*A*B*
a*b*c^2*(-(4*a*c - b^2)^15)^(1/2))/(512*(1048576*a^10*c^15 + b^20*c^5 - 40*
a*b^18*c^6 + 720*a^2*b^16*c^7 - 7680*a^3*b^14*c^8 + 53760*a^4*b^12*c^9 - 25
8048*a^5*b^10*c^10 + 860160*a^6*b^8*c^11 - 1966080*a^7*b^6*c^12 + 2949120*a
^8*b^4*c^13 - 2621440*a^9*b^2*c^14)))^(1/2) - (x*(9*B^2*b^10 + 800*A^2*a^4*
c^6 + A^2*b^8*c^2 - 14112*B^2*a^5*c^5 + 6*A*B*b^9*c + 314*A^2*a^2*b^4*c^4 +
 208*A^2*a^3*b^2*c^5 + 1881*B^2*a^2*b^6*c^2 - 9090*B^2*a^3*b^4*c^3 + 21312*
B^2*a^4*b^2*c^4 - 198*B^2*a*b^8*c - 36*A^2*a*b^6*c^3 + 1422*A*B*a^2*b^5*c^3
 - 4464*A*B*a^3*b^3*c^4 - 174*A*B*a*b^7*c^2 + 96*A*B*a^4*b*c^5))/(32*(256*a
^4*c^7 + b^8*c^3 - 16*a*b^6*c^4 + 96*a^2*b^4*c^5 - 256*a^3*b^2*c^6)))*(-(9*
B^2*b^19 + A^2*b^17*c^2 + 9*B^2*b^4*(-(4*a*c - b^2)^15)^(1/2) + 6*A*B*b^18*
c + 1140*A^2*a^2*b^13*c^4 - 10160*A^2*a^3*b^11*c^5 + 34880*A^2*a^4*b^9*c^6
+ 43776*A^2*a^5*b^7*c^7 - 680960*A^2*a^6*b^5*c^8 + 1863680*A^2*a^7*b^3*c^9
+ 6921*B^2*a^2*b^15*c^2 - 77580*B^2*a^3*b^13*c^3 + 570960*B^2*a^4*b^11*c^4
- 2851776*B^2*a^5*b^9*c^5 + 9628416*B^2*a^6*b^7*c^6 - 21095424*B^2*a^7*b^5*
c^7 + 27095040*B^2*a^8*b^3*c^8 + A^2*b^2*c^2*(-(4*a*c - b^2)^15)^(1/2) + 44
1*B^2*a^2*c^2*(-(4*a*c - b^2)^15)^(1/2) + 6881280*A*B*a^9*c^10 - 369*B^2*a*
b^17*c - 55*A^2*a*b^15*c^3 - 1720320*A^2*a^8*b*c^10 - 25*A^2*a*c^3*(-(4*a*c
 - b^2)^15)^(1/2) - 15482880*B^2*a^9*b*c^9 + 5580*A*B*a^2*b^14*c^3 - 59280*
A*B*a^3*b^12*c^4 + 377280*A*B*a^4*b^10*c^5 - 1430784*A*B*a^5*b^8*c^6 + 2860
032*A*B*a^6*b^6*c^7 - 1290240*A*B*a^7*b^4*c^8 - 5160960*A*B*a^8*b^2*c^9 - 9
9*B^2*a*b^2*c*(-(4*a*c - b^2)^15)^(1/2) - 288*A*B*a*b^16*c^2 + 6*A*B*b^3*c*
(-(4*a*c - b^2)^15)^(1/2) - 108*A*B*a*b*c^2*(-(4*a*c - b^2)^15)^(1/2))/(512
*(1048576*a^10*c^15 + b^20*c^5 - 40*a*b^18*c^6 + 720*a^2*b^16*c^7 - 7680*a^
3*b^14*c^8 + 53760*a^4*b^12*c^9 - 258048*a^5*b^10*c^10 + 860160*a^6*b^8*c^1
1 - 1966080*a^7*b^6*c^12 + 2949120*a^8*b^4*c^13 - 2621440*a^9*b^2*c^14)))^(

1/2) + (((256*A*a*b^12*c^4 - 5242880*A*a^7*c^10 + 768*B*a*b^13*c^3 + 629145
6*B*a^7*b*c^9 - 61440*A*a^3*b^8*c^6 + 655360*A*a^4*b^6*c^7 - 2949120*A*a^5*
b^4*c^8 + 6291456*A*a^6*b^2*c^9 - 21504*B*a^2*b^11*c^4 + 245760*B*a^3*b^9*c
^5 - 1474560*B*a^4*b^7*c^6 + 4915200*B*a^5*b^5*c^7 - 8650752*B*a^6*b^3*c^8)
/(512*(4096*a^6*c^9 + b^12*c^3 - 24*a*b^10*c^4 + 240*a^2*b^8*c^5 - 1280*a^3
*b^6*c^6 + 3840*a^4*b^4*c^7 - 6144*a^5*b^2*c^8)) + (x*(-(9*B^2*b^19 + A^2*b
^17*c^2 + 9*B^2*b^4*(-(4*a*c - b^2)^15)^(1/2) + 6*A*B*b^18*c + 1140*A^2*a^2
*b^13*c^4 - 10160*A^2*a^3*b^11*c^5 + 34880*A^2*a^4*b^9*c^6 + 43776*A^2*a^5*
b^7*c^7 - 680960*A^2*a^6*b^5*c^8 + 1863680*A^2*a^7*b^3*c^9 + 6921*B^2*a^2*b
^15*c^2 - 77580*B^2*a^3*b^13*c^3 + 570960*B^2*a^4*b^11*c^4 - 2851776*B^2*a^
5*b^9*c^5 + 9628416*B^2*a^6*b^7*c^6 - 21095424*B^2*a^7*b^5*c^7 + 27095040*B
^2*a^8*b^3*c^8 + A^2*b^2*c^2*(-(4*a*c - b^2)^15)^(1/2) + 441*B^2*a^2*c^2*(-
(4*a*c - b^2)^15)^(1/2) + 6881280*A*B*a^9*c^10 - 369*B^2*a*b^17*c - 55*A^2*
a*b^15*c^3 - 1720320*A^2*a^8*b*c^10 - 25*A^2*a*c^3*(-(4*a*c - b^2)^15)^(1/2
) - 15482880*B^2*a^9*b*c^9 + 5580*A*B*a^2*b^14*c^3 - 59280*A*B*a^3*b^12*c^4
 + 377280*A*B*a^4*b^10*c^5 - 1430784*A*B*a^5*b^8*c^6 + 2860032*A*B*a^6*b^6*
c^7 - 1290240*A*B*a^7*b^4*c^8 - 5160960*A*B*a^8*b^2*c^9 - 99*B^2*a*b^2*c*(-
(4*a*c - b^2)^15)^(1/2) - 288*A*B*a*b^16*c^2 + 6*A*B*b^3*c*(-(4*a*c - b^2)^
15)^(1/2) - 108*A*B*a*b*c^2*(-(4*a*c - b^2)^15)^(1/2))/(512*(1048576*a^10*c
^15 + b^20*c^5 - 40*a*b^18*c^6 + 720*a^2*b^16*c^7 - 7680*a^3*b^14*c^8 + 537
60*a^4*b^12*c^9 - 258048*a^5*b^10*c^10 + 860160*a^6*b^8*c^11 - 1966080*a^7*
b^6*c^12 + 2949120*a^8*b^4*c^13 - 2621440*a^9*b^2*c^14)))^(1/2)*(256*b^11*c
^5 - 5120*a*b^9*c^6 - 262144*a^5*b*c^10 + 40960*a^2*b^7*c^7 - 163840*a^3*b^
5*c^8 + 327680*a^4*b^3*c^9))/(32*(256*a^4*c^7 + b^8*c^3 - 16*a*b^6*c^4 + 96
*a^2*b^4*c^5 - 256*a^3*b^2*c^6)))*(-(9*B^2*b^19 + A^2*b^17*c^2 + 9*B^2*b^4*
(-(4*a*c - b^2)^15)^(1/2) + 6*A*B*b^18*c + 1140*A^2*a^2*b^13*c^4 - 10160*A^
2*a^3*b^11*c^5 + 34880*A^2*a^4*b^9*c^6 + 43776*A^2*a^5*b^7*c^7 - 680960*A^2
*a^6*b^5*c^8 + 1863680*A^2*a^7*b^3*c^9 + 6921*B^2*a^2*b^15*c^2 - 77580*B^2*
a^3*b^13*c^3 + 570960*B^2*a^4*b^11*c^4 - 2851776*B^2*a^5*b^9*c^5 + 9628416*
B^2*a^6*b^7*c^6 - 21095424*B^2*a^7*b^5*c^7 + 27095040*B^2*a^8*b^3*c^8 + A^2
*b^2*c^2*(-(4*a*c - b^2)^15)^(1/2) + 441*B^2*a^2*c^2*(-(4*a*c - b^2)^15)^(1
/2) + 6881280*A*B*a^9*c^10 - 369*B^2*a*b^17*c - 55*A^2*a*b^15*c^3 - 1720320
*A^2*a^8*b*c^10 - 25*A^2*a*c^3*(-(4*a*c - b^2)^15)^(1/2) - 15482880*B^2*a^9
*b*c^9 + 5580*A*B*a^2*b^14*c^3 - 59280*A*B*a^3*b^12*c^4 + 377280*A*B*a^4*b^
10*c^5 - 1430784*A*B*a^5*b^8*c^6 + 2860032*A*B*a^6*b^6*c^7 - 1290240*A*B*a^
7*b^4*c^8 - 5160960*A*B*a^8*b^2*c^9 - 99*B^2*a*b^2*c*(-(4*a*c - b^2)^15)^(1
/2) - 288*A*B*a*b^16*c^2 + 6*A*B*b^3*c*(-(4*a*c - b^2)^15)^(1/2) - 108*A*B*
a*b*c^2*(-(4*a*c - b^2)^15)^(1/2))/(512*(1048576*a^10*c^15 + b^20*c^5 - 40*
a*b^18*c^6 + 720*a^2*b^16*c^7 - 7680*a^3*b^14*c^8 + 53760*a^4*b^12*c^9 - 25
8048*a^5*b^10*c^10 + 860160*a^6*b^8*c^11 - 1966080*a^7*b^6*c^12 + 2949120*a
^8*b^4*c^13 - 2621440*a^9*b^2*c^14)))^(1/2) + (x*(9*B^2*b^10 + 800*A^2*a^4*
c^6 + A^2*b^8*c^2 - 14112*B^2*a^5*c^5 + 6*A*B*b^9*c + 314*A^2*a^2*b^4*c^4 +
 208*A^2*a^3*b^2*c^5 + 1881*B^2*a^2*b^6*c^2 - 9090*B^2*a^3*b^4*c^3 + 21312*
B^2*a^4*b^2*c^4 - 198*B^2*a*b^8*c - 36*A^2*a*b^6*c^3 + 1422*A*B*a^2*b^5*c^3
 - 4464*A*B*a^3*b^3*c^4 - 174*A*B*a*b^7*c^2 + 96*A*B*a^4*b*c^5))/(32*(256*a

^4*c^7 + b^8*c^3 - 16*a*b^6*c^4 + 96*a^2*b^4*c^5 - 256*a^3*b^2*c^6)))*(-(9*
B^2*b^19 + A^2*b^17*c^2 + 9*B^2*b^4*(-(4*a*c - b^2)^15)^(1/2) + 6*A*B*b^18*
c + 1140*A^2*a^2*b^13*c^4 - 10160*A^2*a^3*b^11*c^5 + 34880*A^2*a^4*b^9*c^6
+ 43776*A^2*a^5*b^7*c^7 - 680960*A^2*a^6*b^5*c^8 + 1863680*A^2*a^7*b^3*c^9
+ 6921*B^2*a^2*b^15*c^2 - 77580*B^2*a^3*b^13*c^3 + 570960*B^2*a^4*b^11*c^4
- 2851776*B^2*a^5*b^9*c^5 + 9628416*B^2*a^6*b^7*c^6 - 21095424*B^2*a^7*b^5*
c^7 + 27095040*B^2*a^8*b^3*c^8 + A^2*b^2*c^2*(-(4*a*c - b^2)^15)^(1/2) + 44
1*B^2*a^2*c^2*(-(4*a*c - b^2)^15)^(1/2) + 6881280*A*B*a^9*c^10 - 369*B^2*a*
b^17*c - 55*A^2*a*b^15*c^3 - 1720320*A^2*a^8*b*c^10 - 25*A^2*a*c^3*(-(4*a*c
 - b^2)^15)^(1/2) - 15482880*B^2*a^9*b*c^9 + 5580*A*B*a^2*b^14*c^3 - 59280*
A*B*a^3*b^12*c^4 + 377280*A*B*a^4*b^10*c^5 - 1430784*A*B*a^5*b^8*c^6 + 2860
032*A*B*a^6*b^6*c^7 - 1290240*A*B*a^7*b^4*c^8 - 5160960*A*B*a^8*b^2*c^9 - 9
9*B^2*a*b^2*c*(-(4*a*c - b^2)^15)^(1/2) - 288*A*B*a*b^16*c^2 + 6*A*B*b^3*c*
(-(4*a*c - b^2)^15)^(1/2) - 108*A*B*a*b*c^2*(-(4*a*c - b^2)^15)^(1/2))/(512
*(1048576*a^10*c^15 + b^20*c^5 - 40*a*b^18*c^6 + 720*a^2*b^16*c^7 - 7680*a^
3*b^14*c^8 + 53760*a^4*b^12*c^9 - 258048*a^5*b^10*c^10 + 860160*a^6*b^8*c^1
1 - 1966080*a^7*b^6*c^12 + 2949120*a^8*b^4*c^13 - 2621440*a^9*b^2*c^14)))^(
1/2) + (35*A^3*a^2*b^7*c^2 - 592704*B^3*a^7*c^4 - 567*B^3*a^3*b^8 - 1176*A^
3*a^3*b^5*c^3 + 9456*A^3*a^4*b^3*c^4 - 89532*B^3*a^5*b^4*c^2 + 353808*B^3*a
^6*b^2*c^3 + 315*A*B^2*a^2*b^9 - 33600*A^2*B*a^6*c^5 + 6400*A^3*a^5*b*c^5 +
 10935*B^3*a^4*b^6*c - 6552*A*B^2*a^3*b^7*c + 560448*A*B^2*a^6*b*c^4 + 210*
A^2*B*a^2*b^8*c + 61524*A*B^2*a^4*b^5*c^2 - 280800*A*B^2*a^5*b^3*c^3 - 5649
*A^2*B*a^3*b^6*c^2 + 42516*A^2*B*a^4*b^4*c^3 - 126192*A^2*B*a^5*b^2*c^4)/(2
56*(4096*a^6*c^9 + b^12*c^3 - 24*a*b^10*c^4 + 240*a^2*b^8*c^5 - 1280*a^3*b^
6*c^6 + 3840*a^4*b^4*c^7 - 6144*a^5*b^2*c^8))))*(-(9*B^2*b^19 + A^2*b^17*c^
2 + 9*B^2*b^4*(-(4*a*c - b^2)^15)^(1/2) + 6*A*B*b^18*c + 1140*A^2*a^2*b^13*
c^4 - 10160*A^2*a^3*b^11*c^5 + 34880*A^2*a^4*b^9*c^6 + 43776*A^2*a^5*b^7*c^
7 - 680960*A^2*a^6*b^5*c^8 + 1863680*A^2*a^7*b^3*c^9 + 6921*B^2*a^2*b^15*c^
2 - 77580*B^2*a^3*b^13*c^3 + 570960*B^2*a^4*b^11*c^4 - 2851776*B^2*a^5*b^9*
c^5 + 9628416*B^2*a^6*b^7*c^6 - 21095424*B^2*a^7*b^5*c^7 + 27095040*B^2*a^8
*b^3*c^8 + A^2*b^2*c^2*(-(4*a*c - b^2)^15)^(1/2) + 441*B^2*a^2*c^2*(-(4*a*c
 - b^2)^15)^(1/2) + 6881280*A*B*a^9*c^10 - 369*B^2*a*b^17*c - 55*A^2*a*b^15
*c^3 - 1720320*A^2*a^8*b*c^10 - 25*A^2*a*c^3*(-(4*a*c - b^2)^15)^(1/2) - 15
482880*B^2*a^9*b*c^9 + 5580*A*B*a^2*b^14*c^3 - 59280*A*B*a^3*b^12*c^4 + 377
280*A*B*a^4*b^10*c^5 - 1430784*A*B*a^5*b^8*c^6 + 2860032*A*B*a^6*b^6*c^7 -
1290240*A*B*a^7*b^4*c^8 - 5160960*A*B*a^8*b^2*c^9 - 99*B^2*a*b^2*c*(-(4*a*c
 - b^2)^15)^(1/2) - 288*A*B*a*b^16*c^2 + 6*A*B*b^3*c*(-(4*a*c - b^2)^15)^(1
/2) - 108*A*B*a*b*c^2*(-(4*a*c - b^2)^15)^(1/2))/(512*(1048576*a^10*c^15 +
b^20*c^5 - 40*a*b^18*c^6 + 720*a^2*b^16*c^7 - 7680*a^3*b^14*c^8 + 53760*a^4
*b^12*c^9 - 258048*a^5*b^10*c^10 + 860160*a^6*b^8*c^11 - 1966080*a^7*b^6*c^
12 + 2949120*a^8*b^4*c^13 - 2621440*a^9*b^2*c^14)))^(1/2)*2i - atan(((((256
*A*a*b^12*c^4 - 5242880*A*a^7*c^10 + 768*B*a*b^13*c^3 + 6291456*B*a^7*b*c^9
 - 61440*A*a^3*b^8*c^6 + 655360*A*a^4*b^6*c^7 - 2949120*A*a^5*b^4*c^8 + 629
1456*A*a^6*b^2*c^9 - 21504*B*a^2*b^11*c^4 + 245760*B*a^3*b^9*c^5 - 1474560*
B*a^4*b^7*c^6 + 4915200*B*a^5*b^5*c^7 - 8650752*B*a^6*b^3*c^8)/(512*(4096*a

^6*c^9 + b^12*c^3 - 24*a*b^10*c^4 + 240*a^2*b^8*c^5 - 1280*a^3*b^6*c^6 + 38
40*a^4*b^4*c^7 - 6144*a^5*b^2*c^8)) - (x*(-(9*B^2*b^19 + A^2*b^17*c^2 - 9*B
^2*b^4*(-(4*a*c - b^2)^15)^(1/2) + 6*A*B*b^18*c + 1140*A^2*a^2*b^13*c^4 - 1
0160*A^2*a^3*b^11*c^5 + 34880*A^2*a^4*b^9*c^6 + 43776*A^2*a^5*b^7*c^7 - 680
960*A^2*a^6*b^5*c^8 + 1863680*A^2*a^7*b^3*c^9 + 6921*B^2*a^2*b^15*c^2 - 775
80*B^2*a^3*b^13*c^3 + 570960*B^2*a^4*b^11*c^4 - 2851776*B^2*a^5*b^9*c^5 + 9
628416*B^2*a^6*b^7*c^6 - 21095424*B^2*a^7*b^5*c^7 + 27095040*B^2*a^8*b^3*c^
8 - A^2*b^2*c^2*(-(4*a*c - b^2)^15)^(1/2) - 441*B^2*a^2*c^2*(-(4*a*c - b^2)
^15)^(1/2) + 6881280*A*B*a^9*c^10 - 369*B^2*a*b^17*c - 55*A^2*a*b^15*c^3 -
1720320*A^2*a^8*b*c^10 + 25*A^2*a*c^3*(-(4*a*c - b^2)^15)^(1/2) - 15482880*
B^2*a^9*b*c^9 + 5580*A*B*a^2*b^14*c^3 - 59280*A*B*a^3*b^12*c^4 + 377280*A*B
*a^4*b^10*c^5 - 1430784*A*B*a^5*b^8*c^6 + 2860032*A*B*a^6*b^6*c^7 - 1290240
*A*B*a^7*b^4*c^8 - 5160960*A*B*a^8*b^2*c^9 + 99*B^2*a*b^2*c*(-(4*a*c - b^2)
^15)^(1/2) - 288*A*B*a*b^16*c^2 - 6*A*B*b^3*c*(-(4*a*c - b^2)^15)^(1/2) + 1
08*A*B*a*b*c^2*(-(4*a*c - b^2)^15)^(1/2))/(512*(1048576*a^10*c^15 + b^20*c^
5 - 40*a*b^18*c^6 + 720*a^2*b^16*c^7 - 7680*a^3*b^14*c^8 + 53760*a^4*b^12*c
^9 - 258048*a^5*b^10*c^10 + 860160*a^6*b^8*c^11 - 1966080*a^7*b^6*c^12 + 29
49120*a^8*b^4*c^13 - 2621440*a^9*b^2*c^14)))^(1/2)*(256*b^11*c^5 - 5120*a*b
^9*c^6 - 262144*a^5*b*c^10 + 40960*a^2*b^7*c^7 - 163840*a^3*b^5*c^8 + 32768
0*a^4*b^3*c^9))/(32*(256*a^4*c^7 + b^8*c^3 - 16*a*b^6*c^4 + 96*a^2*b^4*c^5
- 256*a^3*b^2*c^6)))*(-(9*B^2*b^19 + A^2*b^17*c^2 - 9*B^2*b^4*(-(4*a*c - b^
2)^15)^(1/2) + 6*A*B*b^18*c + 1140*A^2*a^2*b^13*c^4 - 10160*A^2*a^3*b^11*c^
5 + 34880*A^2*a^4*b^9*c^6 + 43776*A^2*a^5*b^7*c^7 - 680960*A^2*a^6*b^5*c^8
+ 1863680*A^2*a^7*b^3*c^9 + 6921*B^2*a^2*b^15*c^2 - 77580*B^2*a^3*b^13*c^3
+ 570960*B^2*a^4*b^11*c^4 - 2851776*B^2*a^5*b^9*c^5 + 9628416*B^2*a^6*b^7*c
^6 - 21095424*B^2*a^7*b^5*c^7 + 27095040*B^2*a^8*b^3*c^8 - A^2*b^2*c^2*(-(4
*a*c - b^2)^15)^(1/2) - 441*B^2*a^2*c^2*(-(4*a*c - b^2)^15)^(1/2) + 6881280
*A*B*a^9*c^10 - 369*B^2*a*b^17*c - 55*A^2*a*b^15*c^3 - 1720320*A^2*a^8*b*c^
10 + 25*A^2*a*c^3*(-(4*a*c - b^2)^15)^(1/2) - 15482880*B^2*a^9*b*c^9 + 5580
*A*B*a^2*b^14*c^3 - 59280*A*B*a^3*b^12*c^4 + 377280*A*B*a^4*b^10*c^5 - 1430
784*A*B*a^5*b^8*c^6 + 2860032*A*B*a^6*b^6*c^7 - 1290240*A*B*a^7*b^4*c^8 - 5
160960*A*B*a^8*b^2*c^9 + 99*B^2*a*b^2*c*(-(4*a*c - b^2)^15)^(1/2) - 288*A*B
*a*b^16*c^2 - 6*A*B*b^3*c*(-(4*a*c - b^2)^15)^(1/2) + 108*A*B*a*b*c^2*(-(4*
a*c - b^2)^15)^(1/2))/(512*(1048576*a^10*c^15 + b^20*c^5 - 40*a*b^18*c^6 +
720*a^2*b^16*c^7 - 7680*a^3*b^14*c^8 + 53760*a^4*b^12*c^9 - 258048*a^5*b^10
*c^10 + 860160*a^6*b^8*c^11 - 1966080*a^7*b^6*c^12 + 2949120*a^8*b^4*c^13 -
2621440*a^9*b^2*c^14)))^(1/2) - (x*(9*B^2*b^10 + 800*A^2*a^4*c^6 + A^2*b^8
*c^2 - 14112*B^2*a^5*c^5 + 6*A*B*b^9*c + 314*A^2*a^2*b^4*c^4 + 208*A^2*a^3*
b^2*c^5 + 1881*B^2*a^2*b^6*c^2 - 9090*B^2*a^3*b^4*c^3 + 21312*B^2*a^4*b^2*c
^4 - 198*B^2*a*b^8*c - 36*A^2*a*b^6*c^3 + 1422*A*B*a^2*b^5*c^3 - 4464*A*B*a
^3*b^3*c^4 - 174*A*B*a*b^7*c^2 + 96*A*B*a^4*b*c^5))/(32*(256*a^4*c^7 + b^8*
c^3 - 16*a*b^6*c^4 + 96*a^2*b^4*c^5 - 256*a^3*b^2*c^6)))*(-(9*B^2*b^19 + A^
2*b^17*c^2 - 9*B^2*b^4*(-(4*a*c - b^2)^15)^(1/2) + 6*A*B*b^18*c + 1140*A^2*
a^2*b^13*c^4 - 10160*A^2*a^3*b^11*c^5 + 34880*A^2*a^4*b^9*c^6 + 43776*A^2*a
^5*b^7*c^7 - 680960*A^2*a^6*b^5*c^8 + 1863680*A^2*a^7*b^3*c^9 + 6921*B^2*a^

2*b^15*c^2 - 77580*B^2*a^3*b^13*c^3 + 570960*B^2*a^4*b^11*c^4 - 2851776*B^2
*a^5*b^9*c^5 + 9628416*B^2*a^6*b^7*c^6 - 21095424*B^2*a^7*b^5*c^7 + 2709504
0*B^2*a^8*b^3*c^8 - A^2*b^2*c^2*(-(4*a*c - b^2)^15)^(1/2) - 441*B^2*a^2*c^2
*(-(4*a*c - b^2)^15)^(1/2) + 6881280*A*B*a^9*c^10 - 369*B^2*a*b^17*c - 55*A
^2*a*b^15*c^3 - 1720320*A^2*a^8*b*c^10 + 25*A^2*a*c^3*(-(4*a*c - b^2)^15)^(
1/2) - 15482880*B^2*a^9*b*c^9 + 5580*A*B*a^2*b^14*c^3 - 59280*A*B*a^3*b^12*
c^4 + 377280*A*B*a^4*b^10*c^5 - 1430784*A*B*a^5*b^8*c^6 + 2860032*A*B*a^6*b
^6*c^7 - 1290240*A*B*a^7*b^4*c^8 - 5160960*A*B*a^8*b^2*c^9 + 99*B^2*a*b^2*c
*(-(4*a*c - b^2)^15)^(1/2) - 288*A*B*a*b^16*c^2 - 6*A*B*b^3*c*(-(4*a*c - b^
2)^15)^(1/2) + 108*A*B*a*b*c^2*(-(4*a*c - b^2)^15)^(1/2))/(512*(1048576*a^1
0*c^15 + b^20*c^5 - 40*a*b^18*c^6 + 720*a^2*b^16*c^7 - 7680*a^3*b^14*c^8 +
53760*a^4*b^12*c^9 - 258048*a^5*b^10*c^10 + 860160*a^6*b^8*c^11 - 1966080*a
^7*b^6*c^12 + 2949120*a^8*b^4*c^13 - 2621440*a^9*b^2*c^14)))^(1/2)*1i - (((
256*A*a*b^12*c^4 - 5242880*A*a^7*c^10 + 768*B*a*b^13*c^3 + 6291456*B*a^7*b*
c^9 - 61440*A*a^3*b^8*c^6 + 655360*A*a^4*b^6*c^7 - 2949120*A*a^5*b^4*c^8 +
6291456*A*a^6*b^2*c^9 - 21504*B*a^2*b^11*c^4 + 245760*B*a^3*b^9*c^5 - 14745
60*B*a^4*b^7*c^6 + 4915200*B*a^5*b^5*c^7 - 8650752*B*a^6*b^3*c^8)/(512*(409
6*a^6*c^9 + b^12*c^3 - 24*a*b^10*c^4 + 240*a^2*b^8*c^5 - 1280*a^3*b^6*c^6 +
3840*a^4*b^4*c^7 - 6144*a^5*b^2*c^8)) + (x*(-(9*B^2*b^19 + A^2*b^17*c^2 -
9*B^2*b^4*(-(4*a*c - b^2)^15)^(1/2) + 6*A*B*b^18*c + 1140*A^2*a^2*b^13*c^4
- 10160*A^2*a^3*b^11*c^5 + 34880*A^2*a^4*b^9*c^6 + 43776*A^2*a^5*b^7*c^7 -
680960*A^2*a^6*b^5*c^8 + 1863680*A^2*a^7*b^3*c^9 + 6921*B^2*a^2*b^15*c^2 -
77580*B^2*a^3*b^13*c^3 + 570960*B^2*a^4*b^11*c^4 - 2851776*B^2*a^5*b^9*c^5
+ 9628416*B^2*a^6*b^7*c^6 - 21095424*B^2*a^7*b^5*c^7 + 27095040*B^2*a^8*b^3
*c^8 - A^2*b^2*c^2*(-(4*a*c - b^2)^15)^(1/2) - 441*B^2*a^2*c^2*(-(4*a*c - b
^2)^15)^(1/2) + 6881280*A*B*a^9*c^10 - 369*B^2*a*b^17*c - 55*A^2*a*b^15*c^3
- 1720320*A^2*a^8*b*c^10 + 25*A^2*a*c^3*(-(4*a*c - b^2)^15)^(1/2) - 154828
80*B^2*a^9*b*c^9 + 5580*A*B*a^2*b^14*c^3 - 59280*A*B*a^3*b^12*c^4 + 377280*
A*B*a^4*b^10*c^5 - 1430784*A*B*a^5*b^8*c^6 + 2860032*A*B*a^6*b^6*c^7 - 1290
240*A*B*a^7*b^4*c^8 - 5160960*A*B*a^8*b^2*c^9 + 99*B^2*a*b^2*c*(-(4*a*c - b
^2)^15)^(1/2) - 288*A*B*a*b^16*c^2 - 6*A*B*b^3*c*(-(4*a*c - b^2)^15)^(1/2)
+ 108*A*B*a*b*c^2*(-(4*a*c - b^2)^15)^(1/2))/(512*(1048576*a^10*c^15 + b^20
*c^5 - 40*a*b^18*c^6 + 720*a^2*b^16*c^7 - 7680*a^3*b^14*c^8 + 53760*a^4*b^1
2*c^9 - 258048*a^5*b^10*c^10 + 860160*a^6*b^8*c^11 - 1966080*a^7*b^6*c^12 +
2949120*a^8*b^4*c^13 - 2621440*a^9*b^2*c^14)))^(1/2)*(256*b^11*c^5 - 5120*
a*b^9*c^6 - 262144*a^5*b*c^10 + 40960*a^2*b^7*c^7 - 163840*a^3*b^5*c^8 + 32
7680*a^4*b^3*c^9))/(32*(256*a^4*c^7 + b^8*c^3 - 16*a*b^6*c^4 + 96*a^2*b^4*c
^5 - 256*a^3*b^2*c^6)))*(-(9*B^2*b^19 + A^2*b^17*c^2 - 9*B^2*b^4*(-(4*a*c -
b^2)^15)^(1/2) + 6*A*B*b^18*c + 1140*A^2*a^2*b^13*c^4 - 10160*A^2*a^3*b^11
*c^5 + 34880*A^2*a^4*b^9*c^6 + 43776*A^2*a^5*b^7*c^7 - 680960*A^2*a^6*b^5*c
^8 + 1863680*A^2*a^7*b^3*c^9 + 6921*B^2*a^2*b^15*c^2 - 77580*B^2*a^3*b^13*c
^3 + 570960*B^2*a^4*b^11*c^4 - 2851776*B^2*a^5*b^9*c^5 + 9628416*B^2*a^6*b^
7*c^6 - 21095424*B^2*a^7*b^5*c^7 + 27095040*B^2*a^8*b^3*c^8 - A^2*b^2*c^2*(
-(4*a*c - b^2)^15)^(1/2) - 441*B^2*a^2*c^2*(-(4*a*c - b^2)^15)^(1/2) + 6881
280*A*B*a^9*c^10 - 369*B^2*a*b^17*c - 55*A^2*a*b^15*c^3 - 1720320*A^2*a^8*b

*c^10 + 25*A^2*a*c^3*(-(4*a*c - b^2)^15)^(1/2) - 15482880*B^2*a^9*b*c^9 + 5580*A*B*a^2*b^14*c^3 - 59280*A*B*a^3*b^12*c^4 + 377280*A*B*a^4*b^10*c^5 - 1430784*A*B*a^5*b^8*c^6 + 2860032*A*B*a^6*b^6*c^7 - 1290240*A*B*a^7*b^4*c^8 - 5160960*A*B*a^8*b^2*c^9 + 99*B^2*a*b^2*c*(-(4*a*c - b^2)^15)^(1/2) - 288*A*B*a*b^16*c^2 - 6*A*B*b^3*c*(-(4*a*c - b^2)^15)^(1/2) + 108*A*B*a*b*c^2*(-(4*a*c - b^2)^15)^(1/2))/(512*(1048576*a^10*c^15 + b^20*c^5 - 40*a*b^18*c^6 + 720*a^2*b^16*c^7 - 7680*a^3*b^14*c^8 + 53760*a^4*b^12*c^9 - 258048*a^5*b^10*c^10 + 860160*a^6*b^8*c^11 - 1966080*a^7*b^6*c^12 + 2949120*a^8*b^4*c^13 - 2621440*a^9*b^2*c^14)))^(1/2) + (x*(9*B^2*b^10 + 800*A^2*a^4*c^6 + A^2*b^8*c^2 - 14112*B^2*a^5*c^5 + 6*A*B*b^9*c + 314*A^2*a^2*b^4*c^4 + 208*A^2*a^3*b^2*c^5 + 1881*B^2*a^2*b^6*c^2 - 9090*B^2*a^3*b^4*c^3 + 21312*B^2*a^4*b^2*c^4 - 198*B^2*a*b^8*c - 36*A^2*a*b^6*c^3 + 1422*A*B*a^2*b^5*c^3 - 4464*A*B*a^3*b^3*c^4 - 174*A*B*a*b^7*c^2 + 96*A*B*a^4*b*c^5))/(32*(256*a^4*c^7 + b^8*c^3 - 16*a*b^6*c^4 + 96*a^2*b^4*c^5 - 256*a^3*b^2*c^6)))*(-(9*B^2*b^19 + A^2*b^17*c^2 - 9*B^2*b^4*(-(4*a*c - b^2)^15)^(1/2) + 6*A*B*b^18*c + 1140*A^2*a^2*b^13*c^4 - 10160*A^2*a^3*b^11*c^5 + 34880*A^2*a^4*b^9*c^6 + 43776*A^2*a^5*b^7*c^7 - 680960*A^2*a^6*b^5*c^8 + 1863680*A^2*a^7*b^3*c^9 + 6921*B^2*a^2*b^15*c^2 - 77580*B^2*a^3*b^13*c^3 + 570960*B^2*a^4*b^11*c^4 - 2851776*B^2*a^5*b^9*c^5 + 9628416*B^2*a^6*b^7*c^6 - 21095424*B^2*a^7*b^5*c^7 + 27095040*B^2*a^8*b^3*c^8 - A^2*b^2*c^2*(-(4*a*c - b^2)^15)^(1/2) - 441*B^2*a^2*c^2*(-(4*a*c - b^2)^15)^(1/2) + 6881280*A*B*a^9*c^10 - 369*B^2*a*b^17*c - 55*A^2*a*b^15*c^3 - 1720320*A^2*a^8*b*c^10 + 25*A^2*a*c^3*(-(4*a*c - b^2)^15)^(1/2) - 15482880*B^2*a^9*b*c^9 + 5580*A*B*a^2*b^14*c^3 - 59280*A*B*a^3*b^12*c^4 + 377280*A*B*a^4*b^10*c^5 - 1430784*A*B*a^5*b^8*c^6 + 2860032*A*B*a^6*b^6*c^7 - 1290240*A*B*a^7*b^4*c^8 - 5160960*A*B*a^8*b^2*c^9 + 99*B^2*a*b^2*c*(-(4*a*c - b^2)^15)^(1/2) - 288*A*B*a*b^16*c^2 - 6*A*B*b^3*c*(-(4*a*c - b^2)^15)^(1/2) + 108*A*B*a*b*c^2*(-(4*a*c - b^2)^15)^(1/2))/(512*(1048576*a^10*c^15 + b^20*c^5 - 40*a*b^18*c^6 + 720*a^2*b^16*c^7 - 7680*a^3*b^14*c^8 + 53760*a^4*b^12*c^9 - 258048*a^5*b^10*c^10 + 860160*a^6*b^8*c^11 - 1966080*a^7*b^6*c^12 + 2949120*a^8*b^4*c^13 - 2621440*a^9*b^2*c^14)))^(1/2)*1i)/((((256*A*a*b^12*c^4 - 5242880*A*a^7*c^10 + 768*B*a*b^13*c^3 + 6291456*B*a^7*b*c^9 - 61440*A*a^3*b^8*c^6 + 655360*A*a^4*b^6*c^7 - 2949120*A*a^5*b^4*c^8 + 6291456*A*a^6*b^2*c^9 - 21504*B*a^2*b^11*c^4 + 245760*B*a^3*b^9*c^5 - 1474560*B*a^4*b^7*c^6 + 4915200*B*a^5*b^5*c^7 - 8650752*B*a^6*b^3*c^8)/(512*(4096*a^6*c^9 + b^12*c^3 - 24*a*b^10*c^4 + 240*a^2*b^8*c^5 - 1280*a^3*b^6*c^6 + 3840*a^4*b^4*c^7 - 6144*a^5*b^2*c^8)) - (x*(-(9*B^2*b^19 + A^2*b^17*c^2 - 9*B^2*b^4*(-(4*a*c - b^2)^15)^(1/2) + 6*A*B*b^18*c + 1140*A^2*a^2*b^13*c^4 - 10160*A^2*a^3*b^11*c^5 + 34880*A^2*a^4*b^9*c^6 + 43776*A^2*a^5*b^7*c^7 - 680960*A^2*a^6*b^5*c^8 + 1863680*A^2*a^7*b^3*c^9 + 6921*B^2*a^2*b^15*c^2 - 77580*B^2*a^3*b^13*c^3 + 570960*B^2*a^4*b^11*c^4 - 2851776*B^2*a^5*b^9*c^5 + 9628416*B^2*a^6*b^7*c^6 - 21095424*B^2*a^7*b^5*c^7 + 27095040*B^2*a^8*b^3*c^8 - A^2*b^2*c^2*(-(4*a*c - b^2)^15)^(1/2) - 441*B^2*a^2*c^2*(-(4*a*c - b^2)^15)^(1/2) + 6881280*A*B*a^9*c^10 - 369*B^2*a*b^17*c - 55*A^2*a*b^15*c^3 - 1720320*A^2*a^8*b*c^10 + 25*A^2*a*c^3*(-(4*a*c - b^2)^15)^(1/2) - 15482880*B^2*a^9*b*c^9 + 5580*A*B*a^2*b^14*c^3 - 59280*A*B*a^3*b^12*c^4 + 3772

80*A*B*a^4*b^10*c^5 - 1430784*A*B*a^5*b^8*c^6 + 2860032*A*B*a^6*b^6*c^7 - 1
290240*A*B*a^7*b^4*c^8 - 5160960*A*B*a^8*b^2*c^9 + 99*B^2*a*b^2*c*(-(4*a*c
- b^2)^15)^(1/2) - 288*A*B*a*b^16*c^2 - 6*A*B*b^3*c*(-(4*a*c - b^2)^15)^(1/
2) + 108*A*B*a*b*c^2*(-(4*a*c - b^2)^15)^(1/2))/(512*(1048576*a^10*c^15 + b
^20*c^5 - 40*a*b^18*c^6 + 720*a^2*b^16*c^7 - 7680*a^3*b^14*c^8 + 53760*a^4*
b^12*c^9 - 258048*a^5*b^10*c^10 + 860160*a^6*b^8*c^11 - 1966080*a^7*b^6*c^1
2 + 2949120*a^8*b^4*c^13 - 2621440*a^9*b^2*c^14)))^(1/2)*(256*b^11*c^5 - 51
20*a*b^9*c^6 - 262144*a^5*b*c^10 + 40960*a^2*b^7*c^7 - 163840*a^3*b^5*c^8 +
327680*a^4*b^3*c^9))/(32*(256*a^4*c^7 + b^8*c^3 - 16*a*b^6*c^4 + 96*a^2*b^
4*c^5 - 256*a^3*b^2*c^6)))*(-(9*B^2*b^19 + A^2*b^17*c^2 - 9*B^2*b^4*(-(4*a*
c - b^2)^15)^(1/2) + 6*A*B*b^18*c + 1140*A^2*a^2*b^13*c^4 - 10160*A^2*a^3*b
^11*c^5 + 34880*A^2*a^4*b^9*c^6 + 43776*A^2*a^5*b^7*c^7 - 680960*A^2*a^6*b^
5*c^8 + 1863680*A^2*a^7*b^3*c^9 + 6921*B^2*a^2*b^15*c^2 - 77580*B^2*a^3*b^1
3*c^3 + 570960*B^2*a^4*b^11*c^4 - 2851776*B^2*a^5*b^9*c^5 + 9628416*B^2*a^6
*b^7*c^6 - 21095424*B^2*a^7*b^5*c^7 + 27095040*B^2*a^8*b^3*c^8 - A^2*b^2*c^
2*(-(4*a*c - b^2)^15)^(1/2) - 441*B^2*a^2*c^2*(-(4*a*c - b^2)^15)^(1/2) + 6
881280*A*B*a^9*c^10 - 369*B^2*a*b^17*c - 55*A^2*a*b^15*c^3 - 1720320*A^2*a^
8*b*c^10 + 25*A^2*a*c^3*(-(4*a*c - b^2)^15)^(1/2) - 15482880*B^2*a^9*b*c^9
+ 5580*A*B*a^2*b^14*c^3 - 59280*A*B*a^3*b^12*c^4 + 377280*A*B*a^4*b^10*c^5
- 1430784*A*B*a^5*b^8*c^6 + 2860032*A*B*a^6*b^6*c^7 - 1290240*A*B*a^7*b^4*c
^8 - 5160960*A*B*a^8*b^2*c^9 + 99*B^2*a*b^2*c*(-(4*a*c - b^2)^15)^(1/2) - 2
88*A*B*a*b^16*c^2 - 6*A*B*b^3*c*(-(4*a*c - b^2)^15)^(1/2) + 108*A*B*a*b*c^2
*(-(4*a*c - b^2)^15)^(1/2))/(512*(1048576*a^10*c^15 + b^20*c^5 - 40*a*b^18*
c^6 + 720*a^2*b^16*c^7 - 7680*a^3*b^14*c^8 + 53760*a^4*b^12*c^9 - 258048*a^
5*b^10*c^10 + 860160*a^6*b^8*c^11 - 1966080*a^7*b^6*c^12 + 2949120*a^8*b^4*
c^13 - 2621440*a^9*b^2*c^14)))^(1/2) - (x*(9*B^2*b^10 + 800*A^2*a^4*c^6 + A
^2*b^8*c^2 - 14112*B^2*a^5*c^5 + 6*A*B*b^9*c + 314*A^2*a^2*b^4*c^4 + 208*A^
2*a^3*b^2*c^5 + 1881*B^2*a^2*b^6*c^2 - 9090*B^2*a^3*b^4*c^3 + 21312*B^2*a^4
*b^2*c^4 - 198*B^2*a*b^8*c - 36*A^2*a*b^6*c^3 + 1422*A*B*a^2*b^5*c^3 - 4464
*A*B*a^3*b^3*c^4 - 174*A*B*a*b^7*c^2 + 96*A*B*a^4*b*c^5))/(32*(256*a^4*c^7
+ b^8*c^3 - 16*a*b^6*c^4 + 96*a^2*b^4*c^5 - 256*a^3*b^2*c^6)))*(-(9*B^2*b^1
9 + A^2*b^17*c^2 - 9*B^2*b^4*(-(4*a*c - b^2)^15)^(1/2) + 6*A*B*b^18*c + 114
0*A^2*a^2*b^13*c^4 - 10160*A^2*a^3*b^11*c^5 + 34880*A^2*a^4*b^9*c^6 + 43776
*A^2*a^5*b^7*c^7 - 680960*A^2*a^6*b^5*c^8 + 1863680*A^2*a^7*b^3*c^9 + 6921*
B^2*a^2*b^15*c^2 - 77580*B^2*a^3*b^13*c^3 + 570960*B^2*a^4*b^11*c^4 - 28517
76*B^2*a^5*b^9*c^5 + 9628416*B^2*a^6*b^7*c^6 - 21095424*B^2*a^7*b^5*c^7 + 2
7095040*B^2*a^8*b^3*c^8 - A^2*b^2*c^2*(-(4*a*c - b^2)^15)^(1/2) - 441*B^2*a
^2*c^2*(-(4*a*c - b^2)^15)^(1/2) + 6881280*A*B*a^9*c^10 - 369*B^2*a*b^17*c
- 55*A^2*a*b^15*c^3 - 1720320*A^2*a^8*b*c^10 + 25*A^2*a*c^3*(-(4*a*c - b^2)
^15)^(1/2) - 15482880*B^2*a^9*b*c^9 + 5580*A*B*a^2*b^14*c^3 - 59280*A*B*a^3
*b^12*c^4 + 377280*A*B*a^4*b^10*c^5 - 1430784*A*B*a^5*b^8*c^6 + 2860032*A*B
*a^6*b^6*c^7 - 1290240*A*B*a^7*b^4*c^8 - 5160960*A*B*a^8*b^2*c^9 + 99*B^2*a
*b^2*c*(-(4*a*c - b^2)^15)^(1/2) - 288*A*B*a*b^16*c^2 - 6*A*B*b^3*c*(-(4*a*
c - b^2)^15)^(1/2) + 108*A*B*a*b*c^2*(-(4*a*c - b^2)^15)^(1/2))/(512*(10485
76*a^10*c^15 + b^20*c^5 - 40*a*b^18*c^6 + 720*a^2*b^16*c^7 - 7680*a^3*b^14*

c^8 + 53760*a^4*b^12*c^9 - 258048*a^5*b^10*c^10 + 860160*a^6*b^8*c^11 - 196
6080*a^7*b^6*c^12 + 2949120*a^8*b^4*c^13 - 2621440*a^9*b^2*c^14)))^(1/2) +
(((256*A*a*b^12*c^4 - 5242880*A*a^7*c^10 + 768*B*a*b^13*c^3 + 6291456*B*a^7
*b*c^9 - 61440*A*a^3*b^8*c^6 + 655360*A*a^4*b^6*c^7 - 2949120*A*a^5*b^4*c^8
 + 6291456*A*a^6*b^2*c^9 - 21504*B*a^2*b^11*c^4 + 245760*B*a^3*b^9*c^5 - 14
74560*B*a^4*b^7*c^6 + 4915200*B*a^5*b^5*c^7 - 8650752*B*a^6*b^3*c^8)/(512*(
4096*a^6*c^9 + b^12*c^3 - 24*a*b^10*c^4 + 240*a^2*b^8*c^5 - 1280*a^3*b^6*c^
6 + 3840*a^4*b^4*c^7 - 6144*a^5*b^2*c^8)) + (x*(-(9*B^2*b^19 + A^2*b^17*c^2
 - 9*B^2*b^4*(-(4*a*c - b^2)^15)^(1/2) + 6*A*B*b^18*c + 1140*A^2*a^2*b^13*c
^4 - 10160*A^2*a^3*b^11*c^5 + 34880*A^2*a^4*b^9*c^6 + 43776*A^2*a^5*b^7*c^7
 - 680960*A^2*a^6*b^5*c^8 + 1863680*A^2*a^7*b^3*c^9 + 6921*B^2*a^2*b^15*c^2
 - 77580*B^2*a^3*b^13*c^3 + 570960*B^2*a^4*b^11*c^4 - 2851776*B^2*a^5*b^9*c
^5 + 9628416*B^2*a^6*b^7*c^6 - 21095424*B^2*a^7*b^5*c^7 + 27095040*B^2*a^8*
b^3*c^8 - A^2*b^2*c^2*(-(4*a*c - b^2)^15)^(1/2) - 441*B^2*a^2*c^2*(-(4*a*c
 - b^2)^15)^(1/2) + 6881280*A*B*a^9*c^10 - 369*B^2*a*b^17*c - 55*A^2*a*b^15*
c^3 - 1720320*A^2*a^8*b*c^10 + 25*A^2*a*c^3*(-(4*a*c - b^2)^15)^(1/2) - 154
82880*B^2*a^9*b*c^9 + 5580*A*B*a^2*b^14*c^3 - 59280*A*B*a^3*b^12*c^4 + 3772
80*A*B*a^4*b^10*c^5 - 1430784*A*B*a^5*b^8*c^6 + 2860032*A*B*a^6*b^6*c^7 - 1
290240*A*B*a^7*b^4*c^8 - 5160960*A*B*a^8*b^2*c^9 + 99*B^2*a*b^2*c*(-(4*a*c
 - b^2)^15)^(1/2) - 288*A*B*a*b^16*c^2 - 6*A*B*b^3*c*(-(4*a*c - b^2)^15)^(1/
2) + 108*A*B*a*b*c^2*(-(4*a*c - b^2)^15)^(1/2))/(512*(1048576*a^10*c^15 + b
^20*c^5 - 40*a*b^18*c^6 + 720*a^2*b^16*c^7 - 7680*a^3*b^14*c^8 + 53760*a^4*
b^12*c^9 - 258048*a^5*b^10*c^10 + 860160*a^6*b^8*c^11 - 1966080*a^7*b^6*c^1
2 + 2949120*a^8*b^4*c^13 - 2621440*a^9*b^2*c^14)))^(1/2)*(256*b^11*c^5 - 51
20*a*b^9*c^6 - 262144*a^5*b*c^10 + 40960*a^2*b^7*c^7 - 163840*a^3*b^5*c^8 +
 327680*a^4*b^3*c^9))/(32*(256*a^4*c^7 + b^8*c^3 - 16*a*b^6*c^4 + 96*a^2*b^
4*c^5 - 256*a^3*b^2*c^6)))*(-(9*B^2*b^19 + A^2*b^17*c^2 - 9*B^2*b^4*(-(4*a*
c - b^2)^15)^(1/2) + 6*A*B*b^18*c + 1140*A^2*a^2*b^13*c^4 - 10160*A^2*a^3*b
^11*c^5 + 34880*A^2*a^4*b^9*c^6 + 43776*A^2*a^5*b^7*c^7 - 680960*A^2*a^6*b^
5*c^8 + 1863680*A^2*a^7*b^3*c^9 + 6921*B^2*a^2*b^15*c^2 - 77580*B^2*a^3*b^1
3*c^3 + 570960*B^2*a^4*b^11*c^4 - 2851776*B^2*a^5*b^9*c^5 + 9628416*B^2*a^6
*b^7*c^6 - 21095424*B^2*a^7*b^5*c^7 + 27095040*B^2*a^8*b^3*c^8 - A^2*b^2*c^
2*(-(4*a*c - b^2)^15)^(1/2) - 441*B^2*a^2*c^2*(-(4*a*c - b^2)^15)^(1/2) + 6
881280*A*B*a^9*c^10 - 369*B^2*a*b^17*c - 55*A^2*a*b^15*c^3 - 1720320*A^2*a^
8*b*c^10 + 25*A^2*a*c^3*(-(4*a*c - b^2)^15)^(1/2) - 15482880*B^2*a^9*b*c^9
+ 5580*A*B*a^2*b^14*c^3 - 59280*A*B*a^3*b^12*c^4 + 377280*A*B*a^4*b^10*c^5
- 1430784*A*B*a^5*b^8*c^6 + 2860032*A*B*a^6*b^6*c^7 - 1290240*A*B*a^7*b^4*c
^8 - 5160960*A*B*a^8*b^2*c^9 + 99*B^2*a*b^2*c*(-(4*a*c - b^2)^15)^(1/2) - 2
88*A*B*a*b^16*c^2 - 6*A*B*b^3*c*(-(4*a*c - b^2)^15)^(1/2) + 108*A*B*a*b*c^2
*(-(4*a*c - b^2)^15)^(1/2))/(512*(1048576*a^10*c^15 + b^20*c^5 - 40*a*b^18*
c^6 + 720*a^2*b^16*c^7 - 7680*a^3*b^14*c^8 + 53760*a^4*b^12*c^9 - 258048*a^
5*b^10*c^10 + 860160*a^6*b^8*c^11 - 1966080*a^7*b^6*c^12 + 2949120*a^8*b^4*
c^13 - 2621440*a^9*b^2*c^14)))^(1/2) + (x*(9*B^2*b^10 + 800*A^2*a^4*c^6 + A
^2*b^8*c^2 - 14112*B^2*a^5*c^5 + 6*A*B*b^9*c + 314*A^2*a^2*b^4*c^4 + 208*A^
2*a^3*b^2*c^5 + 1881*B^2*a^2*b^6*c^2 - 9090*B^2*a^3*b^4*c^3 + 21312*B^2*a^4

*b^2*c^4 - 198*B^2*a*b^8*c - 36*A^2*a*b^6*c^3 + 1422*A*B*a^2*b^5*c^3 - 4464
*A*B*a^3*b^3*c^4 - 174*A*B*a*b^7*c^2 + 96*A*B*a^4*b*c^5))/(32*(256*a^4*c^7
+ b^8*c^3 - 16*a*b^6*c^4 + 96*a^2*b^4*c^5 - 256*a^3*b^2*c^6)))*(-(9*B^2*b^1
9 + A^2*b^17*c^2 - 9*B^2*b^4*(-(4*a*c - b^2)^15)^(1/2) + 6*A*B*b^18*c + 114
0*A^2*a^2*b^13*c^4 - 10160*A^2*a^3*b^11*c^5 + 34880*A^2*a^4*b^9*c^6 + 43776
*A^2*a^5*b^7*c^7 - 680960*A^2*a^6*b^5*c^8 + 1863680*A^2*a^7*b^3*c^9 + 6921*
B^2*a^2*b^15*c^2 - 77580*B^2*a^3*b^13*c^3 + 570960*B^2*a^4*b^11*c^4 - 28517
76*B^2*a^5*b^9*c^5 + 9628416*B^2*a^6*b^7*c^6 - 21095424*B^2*a^7*b^5*c^7 + 2
7095040*B^2*a^8*b^3*c^8 - A^2*b^2*c^2*(-(4*a*c - b^2)^15)^(1/2) - 441*B^2*a
^2*c^2*(-(4*a*c - b^2)^15)^(1/2) + 6881280*A*B*a^9*c^10 - 369*B^2*a*b^17*c
- 55*A^2*a*b^15*c^3 - 1720320*A^2*a^8*b*c^10 + 25*A^2*a*c^3*(-(4*a*c - b^2)
^15)^(1/2) - 15482880*B^2*a^9*b*c^9 + 5580*A*B*a^2*b^14*c^3 - 59280*A*B*a^3
*b^12*c^4 + 377280*A*B*a^4*b^10*c^5 - 1430784*A*B*a^5*b^8*c^6 + 2860032*A*B
*a^6*b^6*c^7 - 1290240*A*B*a^7*b^4*c^8 - 5160960*A*B*a^8*b^2*c^9 + 99*B^2*a
*b^2*c*(-(4*a*c - b^2)^15)^(1/2) - 288*A*B*a*b^16*c^2 - 6*A*B*b^3*c*(-(4*a*
c - b^2)^15)^(1/2) + 108*A*B*a*b*c^2*(-(4*a*c - b^2)^15)^(1/2))/(512*(10485
76*a^10*c^15 + b^20*c^5 - 40*a*b^18*c^6 + 720*a^2*b^16*c^7 - 7680*a^3*b^14*
c^8 + 53760*a^4*b^12*c^9 - 258048*a^5*b^10*c^10 + 860160*a^6*b^8*c^11 - 196
6080*a^7*b^6*c^12 + 2949120*a^8*b^4*c^13 - 2621440*a^9*b^2*c^14)))^(1/2) +
(35*A^3*a^2*b^7*c^2 - 592704*B^3*a^7*c^4 - 567*B^3*a^3*b^8 - 1176*A^3*a^3*b
^5*c^3 + 9456*A^3*a^4*b^3*c^4 - 89532*B^3*a^5*b^4*c^2 + 353808*B^3*a^6*b^2*
c^3 + 315*A*B^2*a^2*b^9 - 33600*A^2*B*a^6*c^5 + 6400*A^3*a^5*b*c^5 + 10935*
B^3*a^4*b^6*c - 6552*A*B^2*a^3*b^7*c + 560448*A*B^2*a^6*b*c^4 + 210*A^2*B*a
^2*b^8*c + 61524*A*B^2*a^4*b^5*c^2 - 280800*A*B^2*a^5*b^3*c^3 - 5649*A^2*B*
a^3*b^6*c^2 + 42516*A^2*B*a^4*b^4*c^3 - 126192*A^2*B*a^5*b^2*c^4)/(256*(409
6*a^6*c^9 + b^12*c^3 - 24*a*b^10*c^4 + 240*a^2*b^8*c^5 - 1280*a^3*b^6*c^6 +
3840*a^4*b^4*c^7 - 6144*a^5*b^2*c^8))))*(-(9*B^2*b^19 + A^2*b^17*c^2 - 9*B
^2*b^4*(-(4*a*c - b^2)^15)^(1/2) + 6*A*B*b^18*c + 1140*A^2*a^2*b^13*c^4 - 1
0160*A^2*a^3*b^11*c^5 + 34880*A^2*a^4*b^9*c^6 + 43776*A^2*a^5*b^7*c^7 - 680
960*A^2*a^6*b^5*c^8 + 1863680*A^2*a^7*b^3*c^9 + 6921*B^2*a^2*b^15*c^2 - 775
80*B^2*a^3*b^13*c^3 + 570960*B^2*a^4*b^11*c^4 - 2851776*B^2*a^5*b^9*c^5 + 9
628416*B^2*a^6*b^7*c^6 - 21095424*B^2*a^7*b^5*c^7 + 27095040*B^2*a^8*b^3*c^
8 - A^2*b^2*c^2*(-(4*a*c - b^2)^15)^(1/2) - 441*B^2*a^2*c^2*(-(4*a*c - b^2)
^15)^(1/2) + 6881280*A*B*a^9*c^10 - 369*B^2*a*b^17*c - 55*A^2*a*b^15*c^3 -
1720320*A^2*a^8*b*c^10 + 25*A^2*a*c^3*(-(4*a*c - b^2)^15)^(1/2) - 15482880*
B^2*a^9*b*c^9 + 5580*A*B*a^2*b^14*c^3 - 59280*A*B*a^3*b^12*c^4 + 377280*A*B
*a^4*b^10*c^5 - 1430784*A*B*a^5*b^8*c^6 + 2860032*A*B*a^6*b^6*c^7 - 1290240
*A*B*a^7*b^4*c^8 - 5160960*A*B*a^8*b^2*c^9 + 99*B^2*a*b^2*c*(-(4*a*c - b^2)
^15)^(1/2) - 288*A*B*a*b^16*c^2 - 6*A*B*b^3*c*(-(4*a*c - b^2)^15)^(1/2) + 1
08*A*B*a*b*c^2*(-(4*a*c - b^2)^15)^(1/2))/(512*(1048576*a^10*c^15 + b^20*c^
5 - 40*a*b^18*c^6 + 720*a^2*b^16*c^7 - 7680*a^3*b^14*c^8 + 53760*a^4*b^12*c
^9 - 258048*a^5*b^10*c^10 + 860160*a^6*b^8*c^11 - 1966080*a^7*b^6*c^12 + 29
49120*a^8*b^4*c^13 - 2621440*a^9*b^2*c^14)))^(1/2)*2i

3.133 $\int \frac{x^6(A+Bx^2)}{(a+bx^2+cx^4)^3}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 461

$$\int \frac{x^6(A+Bx^2)}{(a+bx^2+cx^4)^3}\,dx = -\frac{(b^2B-12Abc+20aBc)\,x}{8c\,(b^2-4ac)^2} - \frac{x^5(Ab-2aB-(bB-2Ac)x^2)}{4\,(b^2-4ac)\,(a+bx^2+cx^4)^2}$$
$$-\frac{x^3(5Ab^2-12abB+4aAc-(b^2B-12Abc+20aBc)\,x^2)}{8\,(b^2-4ac)^2\,(a+bx^2+cx^4)}$$
$$+\frac{\left(b^3B+3Ab^2c-16abBc+12aAc^2-\frac{b^4B+3Ab^3c-18ab^2Bc+36aAbc^2-40a^2Bc^2}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{8\sqrt{2}c^{3/2}\,(b^2-4ac)^2\,\sqrt{b-\sqrt{b^2-4ac}}}$$
$$+\frac{\left(b^3B+3Ab^2c-16abBc+12aAc^2+\frac{b^4B+3Ab^3c-18ab^2Bc+36aAbc^2-40a^2Bc^2}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{8\sqrt{2}c^{3/2}\,(b^2-4ac)^2\,\sqrt{b+\sqrt{b^2-4ac}}}$$

```
[Out] -1/8*(-12*A*b*c+20*B*a*c+B*b^2)*x/c/(-4*a*c+b^2)^2-1/4*x^5*(A*b-2*B*a-(-2*A
*c+B*b)*x^2)/(-4*a*c+b^2)/(c*x^4+b*x^2+a)^2-1/8*x^3*(5*A*b^2-12*a*b*B+4*A*a
*c-(-12*A*b*c+20*B*a*c+B*b^2)*x^2)/(-4*a*c+b^2)^2/(c*x^4+b*x^2+a)+1/16*arct
an(x*2^(1/2)*c^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2))*(B*b^3+3*A*b^2*c-16*B*a*
b*c+12*A*a*c^2+(-36*A*a*b*c^2-3*A*b^3*c+40*B*a^2*c^2+18*B*a*b^2*c-B*b^4)/(-
4*a*c+b^2)^(1/2))/c^(3/2)/(-4*a*c+b^2)^2*2^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/
2)+1/16*arctan(x*2^(1/2)*c^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2))*(B*b^3+3*A*b
^2*c-16*B*a*b*c+12*A*a*c^2+(36*A*a*b*c^2+3*A*b^3*c-40*B*a^2*c^2-18*B*a*b^2*
c+B*b^4)/(-4*a*c+b^2)^(1/2))/c^(3/2)/(-4*a*c+b^2)^2*2^(1/2)/(b+(-4*a*c+b^2)
^(1/2))^(1/2)
```

Rubi [A] (verified)

Time = 2.96 (sec) , antiderivative size = 461, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {1289, 1293, 1180, 211}

$$\int \frac{x^6(A+Bx^2)}{(a+bx^2+cx^4)^3}\,dx$$
$$= \frac{\left(-\frac{-40a^2Bc^2+36aAbc^2-18ab^2Bc+3Ab^3c+b^4B}{\sqrt{b^2-4ac}}+12aAc^2-16abBc+3Ab^2c+b^3B\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{8\sqrt{2}c^{3/2}(b^2-4ac)^2\sqrt{b-\sqrt{b^2-4ac}}}$$
$$+\frac{\left(\frac{-40a^2Bc^2+36aAbc^2-18ab^2Bc+3Ab^3c+b^4B}{\sqrt{b^2-4ac}}+12aAc^2-16abBc+3Ab^2c+b^3B\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{8\sqrt{2}c^{3/2}(b^2-4ac)^2\sqrt{\sqrt{b^2-4ac}+b}}$$
$$-\frac{x^5(-2aB-(x^2(bB-2Ac))+Ab)}{4(b^2-4ac)(a+bx^2+cx^4)^2}$$
$$-\frac{x^3(-x^2(20aBc-12Abc+b^2B)+4aAc-12abB+5Ab^2)}{8(b^2-4ac)^2(a+bx^2+cx^4)}-\frac{x(20aBc-12Abc+b^2B)}{8c(b^2-4ac)^2}$$

```
[In] Int[(x^6*(A + B*x^2))/(a + b*x^2 + c*x^4)^3,x]
```

```
[Out] -1/8*((b^2*B - 12*A*b*c + 20*a*B*c)*x)/(c*(b^2 - 4*a*c)^2) - (x^5*(A*b - 2*
a*B - (b*B - 2*A*c)*x^2))/(4*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)^2) - (x^3*(5
*A*b^2 - 12*a*b*B + 4*a*A*c - (b^2*B - 12*A*b*c + 20*a*B*c)*x^2))/(8*(b^2 -
 4*a*c)^2*(a + b*x^2 + c*x^4)) + ((b^3*B + 3*A*b^2*c - 16*a*b*B*c + 12*a*A*
c^2 - (b^4*B + 3*A*b^3*c - 18*a*b^2*B*c + 36*a*A*b*c^2 - 40*a^2*B*c^2)/Sqrt
[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(8*
Sqrt[2]*c^(3/2)*(b^2 - 4*a*c)^2*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + ((b^3*B + 3*
A*b^2*c - 16*a*b*B*c + 12*a*A*c^2 + (b^4*B + 3*A*b^3*c - 18*a*b^2*B*c + 36*
a*A*b*c^2 - 40*a^2*B*c^2)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqr
t[b + Sqrt[b^2 - 4*a*c]]])/(8*Sqrt[2]*c^(3/2)*(b^2 - 4*a*c)^2*Sqrt[b + Sqrt
[b^2 - 4*a*c]])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1289

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_.), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(a + b*x^2 + c*x^4)^(p + 1)
*((b*d - 2*a*e - (b*e - 2*c*d)*x^2)/(2*(p + 1)*(b^2 - 4*a*c))), x] - Dist[f
^2/(2*(p + 1)*(b^2 - 4*a*c)), Int[(f*x)^(m - 2)*(a + b*x^2 + c*x^4)^(p + 1)
*Simp[(m - 1)*(b*d - 2*a*e) - (4*p + 4 + m + 1)*(b*e - 2*c*d)*x^2, x], x],
x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] &&
GtQ[m, 1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1293

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[e*f*(f*x)^(m - 1)*((a + b*x^2 + c*x^4)^(p +
1)/(c*(m + 4*p + 3))), x] - Dist[f^2/(c*(m + 4*p + 3)), Int[(f*x)^(m - 2)*(
a + b*x^2 + c*x^4)^p*Simp[a*e*(m - 1) + (b*e*(m + 2*p + 1) - c*d*(m + 4*p +
 3))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c,
 0] && GtQ[m, 1] && NeQ[m + 4*p + 3, 0] && IntegerQ[2*p] && (IntegerQ[p] ||
 IntegerQ[m])
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{x^5(Ab-2aB-(bB-2Ac)x^2)}{4(b^2-4ac)(a+bx^2+cx^4)^2}+\frac{\int \frac{x^4(5(Ab-2aB)+(bB-2Ac)x^2)}{(a+bx^2+cx^4)^2}\,dx}{4(b^2-4ac)} \\
&= -\frac{x^5(Ab-2aB-(bB-2Ac)x^2)}{4(b^2-4ac)(a+bx^2+cx^4)^2} \\
&\quad -\frac{x^3(5Ab^2-12abB+4aAc-(b^2B-12Abc+20aBc)x^2)}{8(b^2-4ac)^2(a+bx^2+cx^4)} \\
&\quad +\frac{\int \frac{x^2(3(5Ab^2-12abB+4aAc)+(-b^2B+12Abc-20aBc)x^2)}{a+bx^2+cx^4}\,dx}{8(b^2-4ac)^2} \\
&= -\frac{(b^2B-12Abc+20aBc)x}{8c(b^2-4ac)^2}-\frac{x^5(Ab-2aB-(bB-2Ac)x^2)}{4(b^2-4ac)(a+bx^2+cx^4)^2} \\
&\quad -\frac{x^3(5Ab^2-12abB+4aAc-(b^2B-12Abc+20aBc)x^2)}{8(b^2-4ac)^2(a+bx^2+cx^4)} \\
&\quad -\frac{\int \frac{-a(b^2B-12Abc+20aBc)+(-b^3B-3Ab^2c+16abBc-12aAc^2)x^2}{a+bx^2+cx^4}\,dx}{8c(b^2-4ac)^2}
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{(b^2B-12Abc+20aBc)\,x}{8c\,(b^2-4ac)^2} - \frac{x^5(Ab-2aB-(bB-2Ac)x^2)}{4\,(b^2-4ac)\,(a+bx^2+cx^4)^2} \\
&\quad - \frac{x^3(5Ab^2-12abB+4aAc-(b^2B-12Abc+20aBc)\,x^2)}{8\,(b^2-4ac)^2\,(a+bx^2+cx^4)} \\
&\quad + \frac{\left(b^3B+3Ab^2c-16abBc+12aAc^2-\frac{b^4B+3Ab^3c-18ab^2Bc+36aAbc^2-40a^2Bc^2}{\sqrt{b^2-4ac}}\right)\int\frac{1}{\frac{b}{2}-\frac{1}{2}\sqrt{b^2-4ac}+cx^2}\,dx}{16c\,(b^2-4ac)^2} \\
&\quad + \frac{\left(b^3B+3Ab^2c-16abBc+12aAc^2+\frac{b^4B+3Ab^3c-18ab^2Bc+36aAbc^2-40a^2Bc^2}{\sqrt{b^2-4ac}}\right)\int\frac{1}{\frac{b}{2}+\frac{1}{2}\sqrt{b^2-4ac}+cx^2}\,dx}{16c\,(b^2-4ac)^2}
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{(b^2B-12Abc+20aBc)\,x}{8c\,(b^2-4ac)^2} - \frac{x^5(Ab-2aB-(bB-2Ac)x^2)}{4\,(b^2-4ac)\,(a+bx^2+cx^4)^2} \\
&\quad - \frac{x^3(5Ab^2-12abB+4aAc-(b^2B-12Abc+20aBc)\,x^2)}{8\,(b^2-4ac)^2\,(a+bx^2+cx^4)} \\
&\quad + \frac{\left(b^3B+3Ab^2c-16abBc+12aAc^2-\frac{b^4B+3Ab^3c-18ab^2Bc+36aAbc^2-40a^2Bc^2}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{8\sqrt{2}c^{3/2}\,(b^2-4ac)^2\,\sqrt{b-\sqrt{b^2-4ac}}} \\
&\quad + \frac{\left(b^3B+3Ab^2c-16abBc+12aAc^2+\frac{b^4B+3Ab^3c-18ab^2Bc+36aAbc^2-40a^2Bc^2}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{8\sqrt{2}c^{3/2}\,(b^2-4ac)^2\,\sqrt{b+\sqrt{b^2-4ac}}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 1.27 (sec) , antiderivative size = 543, normalized size of antiderivative = 1.18

$$\int \frac{x^6(A+Bx^2)}{(a+bx^2+cx^4)^3}\,dx$$

$$= \frac{\frac{2x(-2b^4B+4abc^2(A-4Bx^2)+b^3c(2A+Bx^2)+12ac^2(-3aB+Acx^2)+b^2c(11aB+3Acx^2))}{(b^2-4ac)^2(a+bx^2+cx^4)} - \frac{4x(2a^2Bc+b^2(-bB+Ac)x^2+a(-b^2B-2Ac^2x^2+bc}{(b^2-4ac)(a+bx^2+cx^4)^2}}{}$$

```
[In] Integrate[(x^6*(A + B*x^2))/(a + b*x^2 + c*x^4)^3,x]
```

```
[Out] ((2*x*(-2*b^4*B + 4*a*b*c^2*(A - 4*B*x^2) + b^3*c*(2*A + B*x^2) + 12*a*c^2*
(-3*a*B + A*c*x^2) + b^2*c*(11*a*B + 3*A*c*x^2)))/((b^2 - 4*a*c)^2*(a + b*x
^2 + c*x^4)) - (4*x*(2*a^2*B*c + b^2*(-(b*B) + A*c)*x^2 + a*(-(b^2*B) - 2*A
*c^2*x^2 + b*c*(A + 3*B*x^2))))/((b^2 - 4*a*c)*(a + b*x^2 + c*x^4)^2) + (Sq
rt[2]*Sqrt[c]*(-(b^4*B) + 3*b^2*c*(6*a*B + A*Sqrt[b^2 - 4*a*c]) + 4*a*c^2*(
10*a*B + 3*A*Sqrt[b^2 - 4*a*c]) + b^3*(-3*A*c + B*Sqrt[b^2 - 4*a*c]) - 4*a*
b*c*(9*A*c + 4*B*Sqrt[b^2 - 4*a*c]))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sq
rt[b^2 - 4*a*c]]])/((b^2 - 4*a*c)^(5/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (Sqr
t[2]*Sqrt[c]*(b^4*B + 3*b^2*c*(-6*a*B + A*Sqrt[b^2 - 4*a*c]) + 4*a*c^2*(-10
*a*B + 3*A*Sqrt[b^2 - 4*a*c]) + 4*a*b*c*(9*A*c - 4*B*Sqrt[b^2 - 4*a*c]) + b
^3*(3*A*c + B*Sqrt[b^2 - 4*a*c]))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[
b^2 - 4*a*c]]])/((b^2 - 4*a*c)^(5/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]]))/(16*c^2)
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.16 (sec) , antiderivative size = 375, normalized size of antiderivative = 0.81

method	result
risch	$\frac{\frac{\left(12Aa\,c^2+3A\,b^2c-16Babc+B\,b^3\right)x^7}{128a^2c^2-64a\,b^2c+8b^4}+\frac{\left(16Aab\,c^2+5A\,b^3c-36B\,a^2c^2-5Ba\,b^2c-B\,b^4\right)x^5}{8c\left(16a^2c^2-8a\,b^2c+b^4\right)}-\frac{a\left(4Aa\,c^2-19A\,b^2c+28Babc+2B\,b^3\right)x^3}{8c\left(16a^2c^2-8a\,b^2c+b^4\right)}+\frac{a^2\left(12Abc-2\right.}{8c\left(16a^2c^2-\right.}}{\left(c\,x^4+b\,x^2+a\right)^2}$
default	$\frac{\frac{\left(12Aa\,c^2+3A\,b^2c-16Babc+B\,b^3\right)x^7}{128a^2c^2-64a\,b^2c+8b^4}+\frac{\left(16Aab\,c^2+5A\,b^3c-36B\,a^2c^2-5Ba\,b^2c-B\,b^4\right)x^5}{8c\left(16a^2c^2-8a\,b^2c+b^4\right)}-\frac{a\left(4Aa\,c^2-19A\,b^2c+28Babc+2B\,b^3\right)x^3}{8c\left(16a^2c^2-8a\,b^2c+b^4\right)}+\frac{a^2\left(12Abc-2\right.}{8c\left(16a^2c^2-\right.}}{\left(c\,x^4+b\,x^2+a\right)^2}$

```
[In] int(x^6*(B*x^2+A)/(c*x^4+b*x^2+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] (1/8*(12*A*a*c^2+3*A*b^2*c-16*B*a*b*c+B*b^3)/(16*a^2*c^2-8*a*b^2*c+b^4)*x^7
+1/8*(16*A*a*b*c^2+5*A*b^3*c-36*B*a^2*c^2-5*B*a*b^2*c-B*b^4)/c/(16*a^2*c^2-
8*a*b^2*c+b^4)*x^5-1/8/c*a*(4*A*a*c^2-19*A*b^2*c+28*B*a*b*c+2*B*b^3)/(16*a^
2*c^2-8*a*b^2*c+b^4)*x^3+1/8*a^2*(12*A*b*c-20*B*a*c-B*b^2)/c/(16*a^2*c^2-8*
a*b^2*c+b^4)*x)/(c*x^4+b*x^2+a)^2+1/16/c*sum(((12*A*a*c^2+3*A*b^2*c-16*B*a*
b*c+B*b^3)/(16*a^2*c^2-8*a*b^2*c+b^4)*_R^2-a*(12*A*b*c-20*B*a*c-B*b^2)/(16*
a^2*c^2-8*a*b^2*c+b^4))/(2*_R^3*c+_R*b)*ln(x-_R),_R=RootOf(_Z^4*c+_Z^2*b+a)
)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 7060 vs. 2(414) = 828.

Time = 6.01 (sec) , antiderivative size = 7060, normalized size of antiderivative = 15.31

$$\int \frac{x^6(A+Bx^2)}{(a+bx^2+cx^4)^3}\,dx = \text{Too large to display}$$

```
[In] integrate(x^6*(B*x^2+A)/(c*x^4+b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^6(A+Bx^2)}{(a+bx^2+cx^4)^3}\,dx = \text{Timed out}$$

```
[In] integrate(x**6*(B*x**2+A)/(c*x**4+b*x**2+a)**3,x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{x^6(A+Bx^2)}{(a+bx^2+cx^4)^3}\,dx = \int \frac{(Bx^2+A)x^6}{(cx^4+bx^2+a)^3}\,dx$$

```
[In] integrate(x^6*(B*x^2+A)/(c*x^4+b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] 1/8*((B*b^3*c + 12*A*a*c^3 - (16*B*a*b - 3*A*b^2)*c^2)*x^7 - (B*b^4 + 4*(9*
B*a^2 - 4*A*a*b)*c^2 + 5*(B*a*b^2 - A*b^3)*c)*x^5 - (2*B*a*b^3 + 4*A*a^2*c^
2 + (28*B*a^2*b - 19*A*a*b^2)*c)*x^3 - (B*a^2*b^2 + 4*(5*B*a^3 - 3*A*a^2*b)
*c)*x)/((b^4*c^3 - 8*a*b^2*c^4 + 16*a^2*c^5)*x^8 + a^2*b^4*c - 8*a^3*b^2*c^
2 + 16*a^4*c^3 + 2*(b^5*c^2 - 8*a*b^3*c^3 + 16*a^2*b*c^4)*x^6 + (b^6*c - 6*
a*b^4*c^2 + 32*a^3*c^4)*x^4 + 2*(a*b^5*c - 8*a^2*b^3*c^2 + 16*a^3*b*c^3)*x^
2) + 1/8*integrate((B*a*b^2 + (B*b^3 + 12*A*a*c^2 - (16*B*a*b - 3*A*b^2)*c)
*x^2 + 4*(5*B*a^2 - 3*A*a*b)*c)/(c*x^4 + b*x^2 + a), x)/(b^4*c - 8*a*b^2*c^
2 + 16*a^2*c^3)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 7578 vs. $2(414) = 828$.

Time = 2.35 (sec) , antiderivative size = 7578, normalized size of antiderivative = 16.44

$$\int \frac{x^6(A+Bx^2)}{(a+bx^2+cx^4)^3}\,dx = \text{Too large to display}$$

```
[In] integrate(x^6*(B*x^2+A)/(c*x^4+b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] -1/64*(3*(2*b^4*c^3 - 32*a^2*c^5 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqr
t(b^2 - 4*a*c)*c)*b^4*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 -
 4*a*c)*c)*b^3*c^2 + 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a
*c)*c)*a^2*c^3 + 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c
)*a*b*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^2*c
^3 - 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*c^4 - 2*
```

(b^2 - 4*a*c)*b^2*c^3 - 8*(b^2 - 4*a*c)*a*c^4)*(b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)^2*A + (2*b^5*c^2 - 40*a*b^3*c^3 + 128*a^2*b*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^5 + 20*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^4*c - 64*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b*c^2 - 32*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^3*c^2 + 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c^3 - 2*(b^2 - 4*a*c)*b^3*c^2 + 32*(b^2 - 4*a*c)*a*b*c^3)*(b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)^2*B + 24*(sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^7*c^3 - 12*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^5*c^4 - 2*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^6*c^4 - 2*a*b^7*c^4 + 48*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^3*c^5 + 16*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^4*c^5 + sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^5*c^5 + 24*a^2*b^5*c^5 - 64*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b*c^6 - 32*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^2*c^6 - 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c^6 - 96*a^3*b^3*c^6 + 16*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b*c^7 + 128*a^4*b*c^7 + 2*(b^2 - 4*a*c)*a*b^5*c^4 - 16*(b^2 - 4*a*c)*a^2*b^3*c^5 + 32*(b^2 - 4*a*c)*a^3*b*c^6)*A*abs(b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3) - 2*(sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^8*c^2 + 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^6*c^3 - 2*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^7*c^3 - 2*a*b^8*c^3 - 192*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^4*c^4 - 24*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^5*c^4 + sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^6*c^4 - 16*a^2*b^6*c^4 + 896*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b^2*c^5 + 288*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^3*c^5 + 12*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^4*c^5 + 384*a^3*b^4*c^5 - 1280*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^5*c^6 - 640*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b*c^6 - 144*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^2*c^6 - 1792*a^4*b^2*c^6 + 320*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*c^7 + 2560*a^5*c^7 + 2*(b^2 - 4*a*c)*a*b^6*c^3 + 24*(b^2 - 4*a*c)*a^2*b^4*c^4 - 288*(b^2 - 4*a*c)*a^3*b^2*c^5 + 640*(b^2 - 4*a*c)*a^4*c^6)*B*abs(b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3) - 3*(2*b^12*c^5 - 8*a*b^10*c^6 - 192*a^2*b^8*c^7 + 1792*a^3*b^6*c^8 - 5632*a^4*b^4*c^9 + 6144*a^5*b^2*c^10 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^12*c^3 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^10*c^4 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^11*c^4 + 96*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^8*c^5 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^10*c^5 - 896*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^6*c^6 - 192*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^7*c^6 + 2816*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b^4*c^7 + 1024*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^5*c^7 + 96*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^6*c^7 - 3072*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^5*b^2*c^8 - 1536*sqrt(2)*sqrt(b^

2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b^3*c^8 - 512*sqrt(2)*sqrt(b
^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^4*c^8 + 768*sqrt(2)*sqrt(
b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b^2*c^9 - 2*(b^2 - 4*a*c)*
b^10*c^5 + 192*(b^2 - 4*a*c)*a^2*b^6*c^7 - 1024*(b^2 - 4*a*c)*a^3*b^4*c^8 +
1536*(b^2 - 4*a*c)*a^4*b^2*c^9)*A - (2*b^13*c^4 - 68*a*b^11*c^5 + 688*a^2*
b^9*c^6 - 2688*a^3*b^7*c^7 + 2048*a^4*b^5*c^8 + 11264*a^5*b^3*c^9 - 20480*a
^6*b*c^10 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^13*
c^2 + 34*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^11*c
^3 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^12*c^3 -
344*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^9*c^4
- 60*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^10*c^4 -
sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^11*c^4 + 1344*
sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^7*c^5 + 448
*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^8*c^5 + 30
*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^9*c^5 - 1024
*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b^5*c^6 - 89
6*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^6*c^6 - 2
24*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^7*c^6 -
5632*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^5*b^3*c^7
- 1536*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b^4*c^
7 + 448*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^5*c
^7 + 10240*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^6*b*
c^8 + 5120*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^5*b^
2*c^8 + 768*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b
^3*c^8 - 2560*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^5
*b*c^9 - 2*(b^2 - 4*a*c)*b^11*c^4 + 60*(b^2 - 4*a*c)*a*b^9*c^5 - 448*(b^2 -
4*a*c)*a^2*b^7*c^6 + 896*(b^2 - 4*a*c)*a^3*b^5*c^7 + 1536*(b^2 - 4*a*c)*a^
4*b^3*c^8 - 5120*(b^2 - 4*a*c)*a^5*b*c^9)*B)*arctan(2*sqrt(1/2)*x/sqrt((b^5
*c - 8*a*b^3*c^2 + 16*a^2*b*c^3 + sqrt((b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)
^2 - 4*(a*b^4*c - 8*a^2*b^2*c^2 + 16*a^3*c^3)*(b^4*c^2 - 8*a*b^2*c^3 + 16*a
^2*c^4)))/(b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)))/((a*b^10*c^3 - 20*a^2*b^8*
c^4 - 2*a*b^9*c^4 + 160*a^3*b^6*c^5 + 32*a^2*b^7*c^5 + a*b^8*c^5 - 640*a^4*
b^4*c^6 - 192*a^3*b^5*c^6 - 16*a^2*b^6*c^6 + 1280*a^5*b^2*c^7 + 512*a^4*b^3
*c^7 + 96*a^3*b^4*c^7 - 1024*a^6*c^8 - 512*a^5*b*c^8 - 256*a^4*b^2*c^8 + 25
6*a^5*c^9)*abs(b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*abs(c)) + 1/64*(3*(2*b^4*c
^3 - 32*a^2*c^5 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)
*b^4*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^3*c^
2 + 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*c^3 +
8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b*c^3 - sqrt(
2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^2*c^3 - 4*sqrt(2)*sq
rt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*c^4 - 2*(b^2 - 4*a*c)*b^2
c^3 - 8(b^2 - 4*a*c)*a*c^4)*(b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)^2*A + (2*b
^5*c^2 - 40*a*b^3*c^3 + 128*a^2*b*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c
- sqrt(b^2 - 4*a*c)*c)*b^5 + 20*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b
^2 - 4*a*c)*c)*a*b^3*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 -

4*a*c)*c)*b^4*c - 64*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)
*c)*a^2*b*c^2 - 32*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c
)*a*b^2*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^3
*c^2 + 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b*c^3
 - 2*(b^2 - 4*a*c)*b^3*c^2 + 32*(b^2 - 4*a*c)*a*b*c^3)*(b^4*c - 8*a*b^2*c^2
 + 16*a^2*c^3)^2*B - 24*(sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^7*c^3
- 12*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^5*c^4 - 2*sqrt(2)*sqrt(b
*c - sqrt(b^2 - 4*a*c)*c)*a*b^6*c^4 + 2*a*b^7*c^4 + 48*sqrt(2)*sqrt(b*c - s
qrt(b^2 - 4*a*c)*c)*a^3*b^3*c^5 + 16*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c
)*a^2*b^4*c^5 + sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^5*c^5 - 24*a^2*
b^5*c^5 - 64*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*b*c^6 - 32*sqrt(2)
*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^2*c^6 - 8*sqrt(2)*sqrt(b*c - sqrt(b^
2 - 4*a*c)*c)*a^2*b^3*c^6 + 96*a^3*b^3*c^6 + 16*sqrt(2)*sqrt(b*c - sqrt(b^2
 - 4*a*c)*c)*a^3*b*c^7 - 128*a^4*b*c^7 - 2*(b^2 - 4*a*c)*a*b^5*c^4 + 16*(b^
2 - 4*a*c)*a^2*b^3*c^5 - 32*(b^2 - 4*a*c)*a^3*b*c^6)*A*abs(b^4*c - 8*a*b^2*
c^2 + 16*a^2*c^3) + 2*(sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^8*c^2 +
8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^6*c^3 - 2*sqrt(2)*sqrt(b*c
- sqrt(b^2 - 4*a*c)*c)*a*b^7*c^3 + 2*a*b^8*c^3 - 192*sqrt(2)*sqrt(b*c - sqr
t(b^2 - 4*a*c)*c)*a^3*b^4*c^4 - 24*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*
a^2*b^5*c^4 + sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^6*c^4 + 16*a^2*b^
6*c^4 + 896*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*b^2*c^5 + 288*sqrt(
2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^3*c^5 + 12*sqrt(2)*sqrt(b*c - sqrt
(b^2 - 4*a*c)*c)*a^2*b^4*c^5 - 384*a^3*b^4*c^5 - 1280*sqrt(2)*sqrt(b*c - sq
rt(b^2 - 4*a*c)*c)*a^5*c^6 - 640*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^
4*b*c^6 - 144*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^2*c^6 + 1792*a^
4*b^2*c^6 + 320*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*c^7 - 2560*a^5*
c^7 - 2*(b^2 - 4*a*c)*a*b^6*c^3 - 24*(b^2 - 4*a*c)*a^2*b^4*c^4 + 288*(b^2 -
 4*a*c)*a^3*b^2*c^5 - 640*(b^2 - 4*a*c)*a^4*c^6)*B*abs(b^4*c - 8*a*b^2*c^2
+ 16*a^2*c^3) - 3*(2*b^12*c^5 - 8*a*b^10*c^6 - 192*a^2*b^8*c^7 + 1792*a^3*b
^6*c^8 - 5632*a^4*b^4*c^9 + 6144*a^5*b^2*c^10 - sqrt(2)*sqrt(b^2 - 4*a*c)*s
qrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^12*c^3 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(
b*c - sqrt(b^2 - 4*a*c)*c)*a*b^10*c^4 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*
c - sqrt(b^2 - 4*a*c)*c)*b^11*c^4 + 96*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c -
 sqrt(b^2 - 4*a*c)*c)*a^2*b^8*c^5 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sq
rt(b^2 - 4*a*c)*c)*b^10*c^5 - 896*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt
(b^2 - 4*a*c)*c)*a^3*b^6*c^6 - 192*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqr
t(b^2 - 4*a*c)*c)*a^2*b^7*c^6 + 2816*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - s
qrt(b^2 - 4*a*c)*c)*a^4*b^4*c^7 + 1024*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c -
 sqrt(b^2 - 4*a*c)*c)*a^3*b^5*c^7 + 96*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c -
 sqrt(b^2 - 4*a*c)*c)*a^2*b^6*c^7 - 3072*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c
 - sqrt(b^2 - 4*a*c)*c)*a^5*b^2*c^8 - 1536*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b
*c - sqrt(b^2 - 4*a*c)*c)*a^4*b^3*c^8 - 512*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(
b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^4*c^8 + 768*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt
(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*b^2*c^9 - 2*(b^2 - 4*a*c)*b^10*c^5 + 192*(b
^2 - 4*a*c)*a^2*b^6*c^7 - 1024*(b^2 - 4*a*c)*a^3*b^4*c^8 + 1536*(b^2 - 4*a*

```
c)*a^4*b^2*c^9)*A - (2*b^13*c^4 - 68*a*b^11*c^5 + 688*a^2*b^9*c^6 - 2688*a^
3*b^7*c^7 + 2048*a^4*b^5*c^8 + 11264*a^5*b^3*c^9 - 20480*a^6*b*c^10 - sqrt(
2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^13*c^2 + 34*sqrt(2)*
sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^11*c^3 + 2*sqrt(2)*sq
rt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^12*c^3 - 344*sqrt(2)*sqrt
(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^9*c^4 - 60*sqrt(2)*sqrt
(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^10*c^4 - sqrt(2)*sqrt(b^2
 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^11*c^4 + 1344*sqrt(2)*sqrt(b^2
- 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^7*c^5 + 448*sqrt(2)*sqrt(b^2
 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^8*c^5 + 30*sqrt(2)*sqrt(b^2
 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^9*c^5 - 1024*sqrt(2)*sqrt(b^2
 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*b^5*c^6 - 896*sqrt(2)*sqrt(b^
2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^6*c^6 - 224*sqrt(2)*sqrt(b
^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^7*c^6 - 5632*sqrt(2)*sqrt
(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^5*b^3*c^7 - 1536*sqrt(2)*sq
rt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*b^4*c^7 + 448*sqrt(2)*s
qrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^5*c^7 + 10240*sqrt(2
)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^6*b*c^8 + 5120*sqrt(2
)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^5*b^2*c^8 + 768*sqrt(
2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*b^3*c^8 - 2560*sqr
t(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^5*b*c^9 - 2*(b^2 -
 4*a*c)*b^11*c^4 + 60*(b^2 - 4*a*c)*a*b^9*c^5 - 448*(b^2 - 4*a*c)*a^2*b^7*c
^6 + 896*(b^2 - 4*a*c)*a^3*b^5*c^7 + 1536*(b^2 - 4*a*c)*a^4*b^3*c^8 - 5120*
(b^2 - 4*a*c)*a^5*b*c^9)*B)*arctan(2*sqrt(1/2)*x/sqrt((b^5*c - 8*a*b^3*c^2
+ 16*a^2*b*c^3 - sqrt((b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)^2 - 4*(a*b^4*c -
 8*a^2*b^2*c^2 + 16*a^3*c^3)*(b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)))/(b^4*c^
2 - 8*a*b^2*c^3 + 16*a^2*c^4)))/((a*b^10*c^3 - 20*a^2*b^8*c^4 - 2*a*b^9*c^4
 + 160*a^3*b^6*c^5 + 32*a^2*b^7*c^5 + a*b^8*c^5 - 640*a^4*b^4*c^6 - 192*a^3
*b^5*c^6 - 16*a^2*b^6*c^6 + 1280*a^5*b^2*c^7 + 512*a^4*b^3*c^7 + 96*a^3*b^4
*c^7 - 1024*a^6*c^8 - 512*a^5*b*c^8 - 256*a^4*b^2*c^8 + 256*a^5*c^9)*abs(b^
4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*abs(c)) + 1/8*(B*b^3*c*x^7 - 16*B*a*b*c^2*x
^7 + 3*A*b^2*c^2*x^7 + 12*A*a*c^3*x^7 - B*b^4*x^5 - 5*B*a*b^2*c*x^5 + 5*A*b
^3*c*x^5 - 36*B*a^2*c^2*x^5 + 16*A*a*b*c^2*x^5 - 2*B*a*b^3*x^3 - 28*B*a^2*b
*c*x^3 + 19*A*a*b^2*c*x^3 - 4*A*a^2*c^2*x^3 - B*a^2*b^2*x - 20*B*a^3*c*x +
12*A*a^2*b*c*x)/((b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*(c*x^4 + b*x^2 + a)^2)
```

Mupad [B] (verification not implemented)

Time = 9.74 (sec) , antiderivative size = 19041, normalized size of antiderivative = 41.30

$$\int \frac{x^6(A+Bx^2)}{(a+bx^2+cx^4)^3}\,dx = \text{Too large to display}$$

```
[In] int((x^6*(A + B*x^2))/(a + b*x^2 + c*x^4)^3,x)
```

```
[Out] atan(((((5242880*B*a^7*c^8 + 3072*A*a*b^11*c^3 - 3145728*A*a^6*b*c^8 - 256*
B*a*b^12*c^2 - 61440*A*a^2*b^9*c^4 + 491520*A*a^3*b^7*c^5 - 1966080*A*a^4*b
^5*c^6 + 3932160*A*a^5*b^3*c^7 + 61440*B*a^3*b^8*c^4 - 655360*B*a^4*b^6*c^5
 + 2949120*B*a^5*b^4*c^6 - 6291456*B*a^6*b^2*c^7)/(512*(b^12*c + 4096*a^6*c
^7 - 24*a*b^10*c^2 + 240*a^2*b^8*c^3 - 1280*a^3*b^6*c^4 + 3840*a^4*b^4*c^5
- 6144*a^5*b^2*c^6)) - (x*(-(B^2*b^17 + 9*A^2*b^15*c^2 + 9*A^2*c^2*(-(4*a*c
 - b^2)^15)^(1/2) + B^2*b^2*(-(4*a*c - b^2)^15)^(1/2) + 6*A*B*b^16*c - 5040
*A^2*a^2*b^11*c^4 + 37440*A^2*a^3*b^9*c^5 - 103680*A^2*a^4*b^7*c^6 - 9216*A
^2*a^5*b^5*c^7 + 552960*A^2*a^6*b^3*c^8 + 1140*B^2*a^2*b^13*c^2 - 10160*B^2
*a^3*b^11*c^3 + 34880*B^2*a^4*b^9*c^4 + 43776*B^2*a^5*b^7*c^5 - 680960*B^2*
a^6*b^5*c^6 + 1863680*B^2*a^7*b^3*c^7 + 983040*A*B*a^8*c^9 - 55*B^2*a*b^15*
c - 25*B^2*a*c*(-(4*a*c - b^2)^15)^(1/2) + 180*A^2*a*b^13*c^3 - 737280*A^2*
a^7*b*c^9 - 1720320*B^2*a^8*b*c^8 + 240*A*B*a^2*b^12*c^3 + 24000*A*B*a^3*b^
10*c^4 - 241920*A*B*a^4*b^8*c^5 + 992256*A*B*a^5*b^6*c^6 - 1781760*A*B*a^6*
b^4*c^7 + 737280*A*B*a^7*b^2*c^8 + 6*A*B*b*c*(-(4*a*c - b^2)^15)^(1/2) - 18
0*A*B*a*b^14*c^2)/(512*(1048576*a^10*c^13 + b^20*c^3 - 40*a*b^18*c^4 + 720*
a^2*b^16*c^5 - 7680*a^3*b^14*c^6 + 53760*a^4*b^12*c^7 - 258048*a^5*b^10*c^8
 + 860160*a^6*b^8*c^9 - 1966080*a^7*b^6*c^10 + 2949120*a^8*b^4*c^11 - 26214
40*a^9*b^2*c^12)))^(1/2)*(256*b^11*c^3 - 5120*a*b^9*c^4 - 262144*a^5*b*c^8
+ 40960*a^2*b^7*c^5 - 163840*a^3*b^5*c^6 + 327680*a^4*b^3*c^7))/(32*(b^8*c
+ 256*a^4*c^5 - 16*a*b^6*c^2 + 96*a^2*b^4*c^3 - 256*a^3*b^2*c^4)))*(-(B^2*b
^17 + 9*A^2*b^15*c^2 + 9*A^2*c^2*(-(4*a*c - b^2)^15)^(1/2) + B^2*b^2*(-(4*a
*c - b^2)^15)^(1/2) + 6*A*B*b^16*c - 5040*A^2*a^2*b^11*c^4 + 37440*A^2*a^3*
b^9*c^5 - 103680*A^2*a^4*b^7*c^6 - 9216*A^2*a^5*b^5*c^7 + 552960*A^2*a^6*b^
3*c^8 + 1140*B^2*a^2*b^13*c^2 - 10160*B^2*a^3*b^11*c^3 + 34880*B^2*a^4*b^9*
c^4 + 43776*B^2*a^5*b^7*c^5 - 680960*B^2*a^6*b^5*c^6 + 1863680*B^2*a^7*b^3*
c^7 + 983040*A*B*a^8*c^9 - 55*B^2*a*b^15*c - 25*B^2*a*c*(-(4*a*c - b^2)^15)
^(1/2) + 180*A^2*a*b^13*c^3 - 737280*A^2*a^7*b*c^9 - 1720320*B^2*a^8*b*c^8
+ 240*A*B*a^2*b^12*c^3 + 24000*A*B*a^3*b^10*c^4 - 241920*A*B*a^4*b^8*c^5 +
992256*A*B*a^5*b^6*c^6 - 1781760*A*B*a^6*b^4*c^7 + 737280*A*B*a^7*b^2*c^8 +
 6*A*B*b*c*(-(4*a*c - b^2)^15)^(1/2) - 180*A*B*a*b^14*c^2)/(512*(1048576*a^
10*c^13 + b^20*c^3 - 40*a*b^18*c^4 + 720*a^2*b^16*c^5 - 7680*a^3*b^14*c^6 +
 53760*a^4*b^12*c^7 - 258048*a^5*b^10*c^8 + 860160*a^6*b^8*c^9 - 1966080*a^
7*b^6*c^10 + 2949120*a^8*b^4*c^11 - 2621440*a^9*b^2*c^12)))^(1/2) - (x*(B^2
*b^8 - 288*A^2*a^3*c^5 + 9*A^2*b^6*c^2 + 800*B^2*a^4*c^4 + 6*A*B*b^7*c + 57
6*A^2*a^2*b^2*c^4 + 314*B^2*a^2*b^4*c^2 + 208*B^2*a^3*b^2*c^3 - 36*B^2*a*b^
6*c + 126*A^2*a*b^4*c^3 - 816*A*B*a^2*b^3*c^3 - 66*A*B*a*b^5*c^2 - 672*A*B*
a^3*b*c^4))/(32*(b^8*c + 256*a^4*c^5 - 16*a*b^6*c^2 + 96*a^2*b^4*c^3 - 256*
a^3*b^2*c^4)))*(-(B^2*b^17 + 9*A^2*b^15*c^2 + 9*A^2*c^2*(-(4*a*c - b^2)^15)
^(1/2) + B^2*b^2*(-(4*a*c - b^2)^15)^(1/2) + 6*A*B*b^16*c - 5040*A^2*a^2*b^
11*c^4 + 37440*A^2*a^3*b^9*c^5 - 103680*A^2*a^4*b^7*c^6 - 9216*A^2*a^5*b^5*
c^7 + 552960*A^2*a^6*b^3*c^8 + 1140*B^2*a^2*b^13*c^2 - 10160*B^2*a^3*b^11*c
^3 + 34880*B^2*a^4*b^9*c^4 + 43776*B^2*a^5*b^7*c^5 - 680960*B^2*a^6*b^5*c^6
 + 1863680*B^2*a^7*b^3*c^7 + 983040*A*B*a^8*c^9 - 55*B^2*a*b^15*c - 25*B^2*
a*c*(-(4*a*c - b^2)^15)^(1/2) + 180*A^2*a*b^13*c^3 - 737280*A^2*a^7*b*c^9 -
```

1720320*B^2*a^8*b*c^8 + 240*A*B*a^2*b^12*c^3 + 24000*A*B*a^3*b^10*c^4 - 24
1920*A*B*a^4*b^8*c^5 + 992256*A*B*a^5*b^6*c^6 - 1781760*A*B*a^6*b^4*c^7 + 7
37280*A*B*a^7*b^2*c^8 + 6*A*B*b*c*(-(4*a*c - b^2)^15)^(1/2) - 180*A*B*a*b^1
4*c^2)/(512*(1048576*a^10*c^13 + b^20*c^3 - 40*a*b^18*c^4 + 720*a^2*b^16*c^
5 - 7680*a^3*b^14*c^6 + 53760*a^4*b^12*c^7 - 258048*a^5*b^10*c^8 + 860160*a
^6*b^8*c^9 - 1966080*a^7*b^6*c^10 + 2949120*a^8*b^4*c^11 - 2621440*a^9*b^2*
c^12)))^(1/2)*1i - (((5242880*B*a^7*c^8 + 3072*A*a*b^11*c^3 - 3145728*A*a^6
*b*c^8 - 256*B*a*b^12*c^2 - 61440*A*a^2*b^9*c^4 + 491520*A*a^3*b^7*c^5 - 19
66080*A*a^4*b^5*c^6 + 3932160*A*a^5*b^3*c^7 + 61440*B*a^3*b^8*c^4 - 655360*
B*a^4*b^6*c^5 + 2949120*B*a^5*b^4*c^6 - 6291456*B*a^6*b^2*c^7)/(512*(b^12*c
 + 4096*a^6*c^7 - 24*a*b^10*c^2 + 240*a^2*b^8*c^3 - 1280*a^3*b^6*c^4 + 3840
*a^4*b^4*c^5 - 6144*a^5*b^2*c^6)) + (x*(-(B^2*b^17 + 9*A^2*b^15*c^2 + 9*A^2
c^2(-(4*a*c - b^2)^15)^(1/2) + B^2*b^2*(-(4*a*c - b^2)^15)^(1/2) + 6*A*B*
b^16*c - 5040*A^2*a^2*b^11*c^4 + 37440*A^2*a^3*b^9*c^5 - 103680*A^2*a^4*b^7
*c^6 - 9216*A^2*a^5*b^5*c^7 + 552960*A^2*a^6*b^3*c^8 + 1140*B^2*a^2*b^13*c^
2 - 10160*B^2*a^3*b^11*c^3 + 34880*B^2*a^4*b^9*c^4 + 43776*B^2*a^5*b^7*c^5
 - 680960*B^2*a^6*b^5*c^6 + 1863680*B^2*a^7*b^3*c^7 + 983040*A*B*a^8*c^9 - 5
5*B^2*a*b^15*c - 25*B^2*a*c*(-(4*a*c - b^2)^15)^(1/2) + 180*A^2*a*b^13*c^3
 - 737280*A^2*a^7*b*c^9 - 1720320*B^2*a^8*b*c^8 + 240*A*B*a^2*b^12*c^3 + 240
00*A*B*a^3*b^10*c^4 - 241920*A*B*a^4*b^8*c^5 + 992256*A*B*a^5*b^6*c^6 - 178
1760*A*B*a^6*b^4*c^7 + 737280*A*B*a^7*b^2*c^8 + 6*A*B*b*c*(-(4*a*c - b^2)^1
5)^(1/2) - 180*A*B*a*b^14*c^2)/(512*(1048576*a^10*c^13 + b^20*c^3 - 40*a*b^
18*c^4 + 720*a^2*b^16*c^5 - 7680*a^3*b^14*c^6 + 53760*a^4*b^12*c^7 - 258048
*a^5*b^10*c^8 + 860160*a^6*b^8*c^9 - 1966080*a^7*b^6*c^10 + 2949120*a^8*b^4
*c^11 - 2621440*a^9*b^2*c^12)))^(1/2)*(256*b^11*c^3 - 5120*a*b^9*c^4 - 2621
44*a^5*b*c^8 + 40960*a^2*b^7*c^5 - 163840*a^3*b^5*c^6 + 327680*a^4*b^3*c^7)
)/(32*(b^8*c + 256*a^4*c^5 - 16*a*b^6*c^2 + 96*a^2*b^4*c^3 - 256*a^3*b^2*c^
4)))*(-(B^2*b^17 + 9*A^2*b^15*c^2 + 9*A^2*c^2*(-(4*a*c - b^2)^15)^(1/2) + B
^2*b^2*(-(4*a*c - b^2)^15)^(1/2) + 6*A*B*b^16*c - 5040*A^2*a^2*b^11*c^4 + 3
7440*A^2*a^3*b^9*c^5 - 103680*A^2*a^4*b^7*c^6 - 9216*A^2*a^5*b^5*c^7 + 5529
60*A^2*a^6*b^3*c^8 + 1140*B^2*a^2*b^13*c^2 - 10160*B^2*a^3*b^11*c^3 + 34880
*B^2*a^4*b^9*c^4 + 43776*B^2*a^5*b^7*c^5 - 680960*B^2*a^6*b^5*c^6 + 1863680
*B^2*a^7*b^3*c^7 + 983040*A*B*a^8*c^9 - 55*B^2*a*b^15*c - 25*B^2*a*c*(-(4*a
*c - b^2)^15)^(1/2) + 180*A^2*a*b^13*c^3 - 737280*A^2*a^7*b*c^9 - 1720320*B
^2*a^8*b*c^8 + 240*A*B*a^2*b^12*c^3 + 24000*A*B*a^3*b^10*c^4 - 241920*A*B*a
^4*b^8*c^5 + 992256*A*B*a^5*b^6*c^6 - 1781760*A*B*a^6*b^4*c^7 + 737280*A*B*
a^7*b^2*c^8 + 6*A*B*b*c*(-(4*a*c - b^2)^15)^(1/2) - 180*A*B*a*b^14*c^2)/(51
2*(1048576*a^10*c^13 + b^20*c^3 - 40*a*b^18*c^4 + 720*a^2*b^16*c^5 - 7680*a
^3*b^14*c^6 + 53760*a^4*b^12*c^7 - 258048*a^5*b^10*c^8 + 860160*a^6*b^8*c^9
 - 1966080*a^7*b^6*c^10 + 2949120*a^8*b^4*c^11 - 2621440*a^9*b^2*c^12)))^(1
/2) + (x*(B^2*b^8 - 288*A^2*a^3*c^5 + 9*A^2*b^6*c^2 + 800*B^2*a^4*c^4 + 6*A
*B*b^7*c + 576*A^2*a^2*b^2*c^4 + 314*B^2*a^2*b^4*c^2 + 208*B^2*a^3*b^2*c^3
 - 36*B^2*a*b^6*c + 126*A^2*a*b^4*c^3 - 816*A*B*a^2*b^3*c^3 - 66*A*B*a*b^5*c
^2 - 672*A*B*a^3*b*c^4))/(32*(b^8*c + 256*a^4*c^5 - 16*a*b^6*c^2 + 96*a^2*b
^4*c^3 - 256*a^3*b^2*c^4)))*(-(B^2*b^17 + 9*A^2*b^15*c^2 + 9*A^2*c^2*(-(4*a

*c - b^2)^15)^(1/2) + B^2*b^2*(-(4*a*c - b^2)^15)^(1/2) + 6*A*B*b^16*c - 50
40*A^2*a^2*b^11*c^4 + 37440*A^2*a^3*b^9*c^5 - 103680*A^2*a^4*b^7*c^6 - 9216
*A^2*a^5*b^5*c^7 + 552960*A^2*a^6*b^3*c^8 + 1140*B^2*a^2*b^13*c^2 - 10160*B
^2*a^3*b^11*c^3 + 34880*B^2*a^4*b^9*c^4 + 43776*B^2*a^5*b^7*c^5 - 680960*B^
2*a^6*b^5*c^6 + 1863680*B^2*a^7*b^3*c^7 + 983040*A*B*a^8*c^9 - 55*B^2*a*b^1
5*c - 25*B^2*a*c*(-(4*a*c - b^2)^15)^(1/2) + 180*A^2*a*b^13*c^3 - 737280*A^
2*a^7*b*c^9 - 1720320*B^2*a^8*b*c^8 + 240*A*B*a^2*b^12*c^3 + 24000*A*B*a^3*
b^10*c^4 - 241920*A*B*a^4*b^8*c^5 + 992256*A*B*a^5*b^6*c^6 - 1781760*A*B*a^
6*b^4*c^7 + 737280*A*B*a^7*b^2*c^8 + 6*A*B*b*c*(-(4*a*c - b^2)^15)^(1/2) -
180*A*B*a*b^14*c^2)/(512*(1048576*a^10*c^13 + b^20*c^3 - 40*a*b^18*c^4 + 72
0*a^2*b^16*c^5 - 7680*a^3*b^14*c^6 + 53760*a^4*b^12*c^7 - 258048*a^5*b^10*c
^8 + 860160*a^6*b^8*c^9 - 1966080*a^7*b^6*c^10 + 2949120*a^8*b^4*c^11 - 262
1440*a^9*b^2*c^12)))^(1/2)*1i)/((((5242880*B*a^7*c^8 + 3072*A*a*b^11*c^3 -
3145728*A*a^6*b*c^8 - 256*B*a*b^12*c^2 - 61440*A*a^2*b^9*c^4 + 491520*A*a^3
*b^7*c^5 - 1966080*A*a^4*b^5*c^6 + 3932160*A*a^5*b^3*c^7 + 61440*B*a^3*b^8*
c^4 - 655360*B*a^4*b^6*c^5 + 2949120*B*a^5*b^4*c^6 - 6291456*B*a^6*b^2*c^7)
/(512*(b^12*c + 4096*a^6*c^7 - 24*a*b^10*c^2 + 240*a^2*b^8*c^3 - 1280*a^3*b
^6*c^4 + 3840*a^4*b^4*c^5 - 6144*a^5*b^2*c^6)) - (x*(-(B^2*b^17 + 9*A^2*b^1
5*c^2 + 9*A^2*c^2*(-(4*a*c - b^2)^15)^(1/2) + B^2*b^2*(-(4*a*c - b^2)^15)^(
1/2) + 6*A*B*b^16*c - 5040*A^2*a^2*b^11*c^4 + 37440*A^2*a^3*b^9*c^5 - 10368
0*A^2*a^4*b^7*c^6 - 9216*A^2*a^5*b^5*c^7 + 552960*A^2*a^6*b^3*c^8 + 1140*B^
2*a^2*b^13*c^2 - 10160*B^2*a^3*b^11*c^3 + 34880*B^2*a^4*b^9*c^4 + 43776*B^2
*a^5*b^7*c^5 - 680960*B^2*a^6*b^5*c^6 + 1863680*B^2*a^7*b^3*c^7 + 983040*A*
B*a^8*c^9 - 55*B^2*a*b^15*c - 25*B^2*a*c*(-(4*a*c - b^2)^15)^(1/2) + 180*A^
2*a*b^13*c^3 - 737280*A^2*a^7*b*c^9 - 1720320*B^2*a^8*b*c^8 + 240*A*B*a^2*b
^12*c^3 + 24000*A*B*a^3*b^10*c^4 - 241920*A*B*a^4*b^8*c^5 + 992256*A*B*a^5*
b^6*c^6 - 1781760*A*B*a^6*b^4*c^7 + 737280*A*B*a^7*b^2*c^8 + 6*A*B*b*c*(-(4
*a*c - b^2)^15)^(1/2) - 180*A*B*a*b^14*c^2)/(512*(1048576*a^10*c^13 + b^20*
c^3 - 40*a*b^18*c^4 + 720*a^2*b^16*c^5 - 7680*a^3*b^14*c^6 + 53760*a^4*b^12
*c^7 - 258048*a^5*b^10*c^8 + 860160*a^6*b^8*c^9 - 1966080*a^7*b^6*c^10 + 29
49120*a^8*b^4*c^11 - 2621440*a^9*b^2*c^12)))^(1/2)*(256*b^11*c^3 - 5120*a*b
^9*c^4 - 262144*a^5*b*c^8 + 40960*a^2*b^7*c^5 - 163840*a^3*b^5*c^6 + 327680
*a^4*b^3*c^7))/(32*(b^8*c + 256*a^4*c^5 - 16*a*b^6*c^2 + 96*a^2*b^4*c^3 - 2
56*a^3*b^2*c^4)))*(-(B^2*b^17 + 9*A^2*b^15*c^2 + 9*A^2*c^2*(-(4*a*c - b^2)^
15)^(1/2) + B^2*b^2*(-(4*a*c - b^2)^15)^(1/2) + 6*A*B*b^16*c - 5040*A^2*a^2
*b^11*c^4 + 37440*A^2*a^3*b^9*c^5 - 103680*A^2*a^4*b^7*c^6 - 9216*A^2*a^5*b
^5*c^7 + 552960*A^2*a^6*b^3*c^8 + 1140*B^2*a^2*b^13*c^2 - 10160*B^2*a^3*b^1
1*c^3 + 34880*B^2*a^4*b^9*c^4 + 43776*B^2*a^5*b^7*c^5 - 680960*B^2*a^6*b^5*
c^6 + 1863680*B^2*a^7*b^3*c^7 + 983040*A*B*a^8*c^9 - 55*B^2*a*b^15*c - 25*B
^2*a*c*(-(4*a*c - b^2)^15)^(1/2) + 180*A^2*a*b^13*c^3 - 737280*A^2*a^7*b*c^
9 - 1720320*B^2*a^8*b*c^8 + 240*A*B*a^2*b^12*c^3 + 24000*A*B*a^3*b^10*c^4 -
241920*A*B*a^4*b^8*c^5 + 992256*A*B*a^5*b^6*c^6 - 1781760*A*B*a^6*b^4*c^7
+ 737280*A*B*a^7*b^2*c^8 + 6*A*B*b*c*(-(4*a*c - b^2)^15)^(1/2) - 180*A*B*a*
b^14*c^2)/(512*(1048576*a^10*c^13 + b^20*c^3 - 40*a*b^18*c^4 + 720*a^2*b^16
*c^5 - 7680*a^3*b^14*c^6 + 53760*a^4*b^12*c^7 - 258048*a^5*b^10*c^8 + 86016

0*a^6*b^8*c^9 - 1966080*a^7*b^6*c^10 + 2949120*a^8*b^4*c^11 - 2621440*a^9*b
^2*c^12)))^(1/2) - (x*(B^2*b^8 - 288*A^2*a^3*c^5 + 9*A^2*b^6*c^2 + 800*B^2*
a^4*c^4 + 6*A*B*b^7*c + 576*A^2*a^2*b^2*c^4 + 314*B^2*a^2*b^4*c^2 + 208*B^2
*a^3*b^2*c^3 - 36*B^2*a*b^6*c + 126*A^2*a*b^4*c^3 - 816*A*B*a^2*b^3*c^3 - 6
6*A*B*a*b^5*c^2 - 672*A*B*a^3*b*c^4))/(32*(b^8*c + 256*a^4*c^5 - 16*a*b^6*c
^2 + 96*a^2*b^4*c^3 - 256*a^3*b^2*c^4)))*(-(B^2*b^17 + 9*A^2*b^15*c^2 + 9*A
^2*c^2*(-(4*a*c - b^2)^15)^(1/2) + B^2*b^2*(-(4*a*c - b^2)^15)^(1/2) + 6*A*
B*b^16*c - 5040*A^2*a^2*b^11*c^4 + 37440*A^2*a^3*b^9*c^5 - 103680*A^2*a^4*b
^7*c^6 - 9216*A^2*a^5*b^5*c^7 + 552960*A^2*a^6*b^3*c^8 + 1140*B^2*a^2*b^13*
c^2 - 10160*B^2*a^3*b^11*c^3 + 34880*B^2*a^4*b^9*c^4 + 43776*B^2*a^5*b^7*c^
5 - 680960*B^2*a^6*b^5*c^6 + 1863680*B^2*a^7*b^3*c^7 + 983040*A*B*a^8*c^9 -
 55*B^2*a*b^15*c - 25*B^2*a*c*(-(4*a*c - b^2)^15)^(1/2) + 180*A^2*a*b^13*c^
3 - 737280*A^2*a^7*b*c^9 - 1720320*B^2*a^8*b*c^8 + 240*A*B*a^2*b^12*c^3 + 2
4000*A*B*a^3*b^10*c^4 - 241920*A*B*a^4*b^8*c^5 + 992256*A*B*a^5*b^6*c^6 - 1
781760*A*B*a^6*b^4*c^7 + 737280*A*B*a^7*b^2*c^8 + 6*A*B*b*c*(-(4*a*c - b^2)
^15)^(1/2) - 180*A*B*a*b^14*c^2)/(512*(1048576*a^10*c^13 + b^20*c^3 - 40*a*
b^18*c^4 + 720*a^2*b^16*c^5 - 7680*a^3*b^14*c^6 + 53760*a^4*b^12*c^7 - 2580
48*a^5*b^10*c^8 + 860160*a^6*b^8*c^9 - 1966080*a^7*b^6*c^10 + 2949120*a^8*b
^4*c^11 - 2621440*a^9*b^2*c^12)))^(1/2) + (((5242880*B*a^7*c^8 + 3072*A*a*b
^11*c^3 - 3145728*A*a^6*b*c^8 - 256*B*a*b^12*c^2 - 61440*A*a^2*b^9*c^4 + 49
1520*A*a^3*b^7*c^5 - 1966080*A*a^4*b^5*c^6 + 3932160*A*a^5*b^3*c^7 + 61440*
B*a^3*b^8*c^4 - 655360*B*a^4*b^6*c^5 + 2949120*B*a^5*b^4*c^6 - 6291456*B*a^
6*b^2*c^7)/(512*(b^12*c + 4096*a^6*c^7 - 24*a*b^10*c^2 + 240*a^2*b^8*c^3 -
1280*a^3*b^6*c^4 + 3840*a^4*b^4*c^5 - 6144*a^5*b^2*c^6)) + (x*(-(B^2*b^17 +
 9*A^2*b^15*c^2 + 9*A^2*c^2*(-(4*a*c - b^2)^15)^(1/2) + B^2*b^2*(-(4*a*c -
b^2)^15)^(1/2) + 6*A*B*b^16*c - 5040*A^2*a^2*b^11*c^4 + 37440*A^2*a^3*b^9*c
^5 - 103680*A^2*a^4*b^7*c^6 - 9216*A^2*a^5*b^5*c^7 + 552960*A^2*a^6*b^3*c^8
 + 1140*B^2*a^2*b^13*c^2 - 10160*B^2*a^3*b^11*c^3 + 34880*B^2*a^4*b^9*c^4 +
 43776*B^2*a^5*b^7*c^5 - 680960*B^2*a^6*b^5*c^6 + 1863680*B^2*a^7*b^3*c^7 +
 983040*A*B*a^8*c^9 - 55*B^2*a*b^15*c - 25*B^2*a*c*(-(4*a*c - b^2)^15)^(1/2
) + 180*A^2*a*b^13*c^3 - 737280*A^2*a^7*b*c^9 - 1720320*B^2*a^8*b*c^8 + 240
*A*B*a^2*b^12*c^3 + 24000*A*B*a^3*b^10*c^4 - 241920*A*B*a^4*b^8*c^5 + 99225
6*A*B*a^5*b^6*c^6 - 1781760*A*B*a^6*b^4*c^7 + 737280*A*B*a^7*b^2*c^8 + 6*A*
B*b*c*(-(4*a*c - b^2)^15)^(1/2) - 180*A*B*a*b^14*c^2)/(512*(1048576*a^10*c^
13 + b^20*c^3 - 40*a*b^18*c^4 + 720*a^2*b^16*c^5 - 7680*a^3*b^14*c^6 + 5376
0*a^4*b^12*c^7 - 258048*a^5*b^10*c^8 + 860160*a^6*b^8*c^9 - 1966080*a^7*b^6
*c^10 + 2949120*a^8*b^4*c^11 - 2621440*a^9*b^2*c^12)))^(1/2)*(256*b^11*c^3
- 5120*a*b^9*c^4 - 262144*a^5*b*c^8 + 40960*a^2*b^7*c^5 - 163840*a^3*b^5*c^
6 + 327680*a^4*b^3*c^7))/(32*(b^8*c + 256*a^4*c^5 - 16*a*b^6*c^2 + 96*a^2*b
^4*c^3 - 256*a^3*b^2*c^4)))*(-(B^2*b^17 + 9*A^2*b^15*c^2 + 9*A^2*c^2*(-(4*a
*c - b^2)^15)^(1/2) + B^2*b^2*(-(4*a*c - b^2)^15)^(1/2) + 6*A*B*b^16*c - 50
40*A^2*a^2*b^11*c^4 + 37440*A^2*a^3*b^9*c^5 - 103680*A^2*a^4*b^7*c^6 - 9216
*A^2*a^5*b^5*c^7 + 552960*A^2*a^6*b^3*c^8 + 1140*B^2*a^2*b^13*c^2 - 10160*B
^2*a^3*b^11*c^3 + 34880*B^2*a^4*b^9*c^4 + 43776*B^2*a^5*b^7*c^5 - 680960*B^
2*a^6*b^5*c^6 + 1863680*B^2*a^7*b^3*c^7 + 983040*A*B*a^8*c^9 - 55*B^2*a*b^1

5*c - 25*B^2*a*c*(-(4*a*c - b^2)^15)^(1/2) + 180*A^2*a*b^13*c^3 - 737280*A^2*a^7*b*c^9 - 1720320*B^2*a^8*b*c^8 + 240*A*B*a^2*b^12*c^3 + 24000*A*B*a^3*b^10*c^4 - 241920*A*B*a^4*b^8*c^5 + 992256*A*B*a^5*b^6*c^6 - 1781760*A*B*a^6*b^4*c^7 + 737280*A*B*a^7*b^2*c^8 + 6*A*B*b*c*(-(4*a*c - b^2)^15)^(1/2) - 180*A*B*a*b^14*c^2)/(512*(1048576*a^10*c^13 + b^20*c^3 - 40*a*b^18*c^4 + 720*a^2*b^16*c^5 - 7680*a^3*b^14*c^6 + 53760*a^4*b^12*c^7 - 258048*a^5*b^10*c^8 + 860160*a^6*b^8*c^9 - 1966080*a^7*b^6*c^10 + 2949120*a^8*b^4*c^11 - 2621440*a^9*b^2*c^12)))^(1/2) + (x*(B^2*b^8 - 288*A^2*a^3*c^5 + 9*A^2*b^6*c^2 + 800*B^2*a^4*c^4 + 6*A*B*b^7*c + 576*A^2*a^2*b^2*c^4 + 314*B^2*a^2*b^4*c^2 + 208*B^2*a^3*b^2*c^3 - 36*B^2*a*b^6*c + 126*A^2*a*b^4*c^3 - 816*A*B*a^2*b^3*c^3 - 66*A*B*a*b^5*c^2 - 672*A*B*a^3*b*c^4))/(32*(b^8*c + 256*a^4*c^5 - 16*a*b^6*c^2 + 96*a^2*b^4*c^3 - 256*a^3*b^2*c^4)))*(-(B^2*b^17 + 9*A^2*b^15*c^2 + 9*A^2*c^2*(-(4*a*c - b^2)^15)^(1/2) + B^2*b^2*(-(4*a*c - b^2)^15)^(1/2) + 6*A*B*b^16*c - 5040*A^2*a^2*b^11*c^4 + 37440*A^2*a^3*b^9*c^5 - 103680*A^2*a^4*b^7*c^6 - 9216*A^2*a^5*b^5*c^7 + 552960*A^2*a^6*b^3*c^8 + 1140*B^2*a^2*b^13*c^2 - 10160*B^2*a^3*b^11*c^3 + 34880*B^2*a^4*b^9*c^4 + 43776*B^2*a^5*b^7*c^5 - 680960*B^2*a^6*b^5*c^6 + 1863680*B^2*a^7*b^3*c^7 + 983040*A*B*a^8*c^9 - 55*B^2*a*b^15*c - 25*B^2*a*c*(-(4*a*c - b^2)^15)^(1/2) + 180*A^2*a*b^13*c^3 - 737280*A^2*a^7*b*c^9 - 1720320*B^2*a^8*b*c^8 + 240*A*B*a^2*b^12*c^3 + 24000*A*B*a^3*b^10*c^4 - 241920*A*B*a^4*b^8*c^5 + 992256*A*B*a^5*b^6*c^6 - 1781760*A*B*a^6*b^4*c^7 + 737280*A*B*a^7*b^2*c^8 + 6*A*B*b*c*(-(4*a*c - b^2)^15)^(1/2) - 180*A*B*a*b^14*c^2)/(512*(1048576*a^10*c^13 + b^20*c^3 - 40*a*b^18*c^4 + 720*a^2*b^16*c^5 - 7680*a^3*b^14*c^6 + 53760*a^4*b^12*c^7 - 258048*a^5*b^10*c^8 + 860160*a^6*b^8*c^9 - 1966080*a^7*b^6*c^10 + 2949120*a^8*b^4*c^11 - 2621440*a^9*b^2*c^12)))^(1/2) + (1728*A^3*a^4*c^5 - 35*B^3*a^2*b^7 + 1620*A^3*a^2*b^4*c^3 + 4752*A^3*a^3*b^2*c^4 - 9456*B^3*a^4*b^3*c^2 + 15*A*B^2*a*b^8 + 4800*A*B^2*a^5*c^4 + 135*A^3*a*b^6*c^2 + 1176*B^3*a^3*b^5*c - 6400*B^3*a^5*b*c^3 - 705*A*B^2*a^2*b^6*c - 15552*A^2*B*a^4*b*c^4 + 6084*A*B^2*a^3*b^4*c^2 + 26256*A*B^2*a^4*b^2*c^3 - 1260*A^2*B*a^2*b^5*c^2 - 13248*A^2*B*a^3*b^3*c^3 + 90*A^2*B*a*b^7*c)/(256*(b^12*c + 4096*a^6*c^7 - 24*a*b^10*c^2 + 240*a^2*b^8*c^3 - 1280*a^3*b^6*c^4 + 3840*a^4*b^4*c^5 - 6144*a^5*b^2*c^6))))*(-(B^2*b^17 + 9*A^2*b^15*c^2 + 9*A^2*c^2*(-(4*a*c - b^2)^15)^(1/2) + B^2*b^2*(-(4*a*c - b^2)^15)^(1/2) + 6*A*B*b^16*c - 5040*A^2*a^2*b^11*c^4 + 37440*A^2*a^3*b^9*c^5 - 103680*A^2*a^4*b^7*c^6 - 9216*A^2*a^5*b^5*c^7 + 552960*A^2*a^6*b^3*c^8 + 1140*B^2*a^2*b^13*c^2 - 10160*B^2*a^3*b^11*c^3 + 34880*B^2*a^4*b^9*c^4 + 43776*B^2*a^5*b^7*c^5 - 680960*B^2*a^6*b^5*c^6 + 1863680*B^2*a^7*b^3*c^7 + 983040*A*B*a^8*c^9 - 55*B^2*a*b^15*c - 25*B^2*a*c*(-(4*a*c - b^2)^15)^(1/2) + 180*A^2*a*b^13*c^3 - 737280*A^2*a^7*b*c^9 - 1720320*B^2*a^8*b*c^8 + 240*A*B*a^2*b^12*c^3 + 24000*A*B*a^3*b^10*c^4 - 241920*A*B*a^4*b^8*c^5 + 992256*A*B*a^5*b^6*c^6 - 1781760*A*B*a^6*b^4*c^7 + 737280*A*B*a^7*b^2*c^8 + 6*A*B*b*c*(-(4*a*c - b^2)^15)^(1/2) - 180*A*B*a*b^14*c^2)/(512*(1048576*a^10*c^13 + b^20*c^3 - 40*a*b^18*c^4 + 720*a^2*b^16*c^5 - 7680*a^3*b^14*c^6 + 53760*a^4*b^12*c^7 - 258048*a^5*b^10*c^8 + 860160*a^6*b^8*c^9 - 1966080*a^7*b^6*c^10 + 2949120*a^8*b^4*c^11 - 2621440*a^9*b^2*c^12)))^(1/2)*2i - ((x^5*(B*b^4 + 36*B*a^2*c^2 - 5*A*b^3*c - 16*A*a* b*c^2 + 5*B*a*b^2*c))/(8*c*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) - (x^7*(B*b^3 + 12*A*a*c^2 + 3*A*b^2*c - 16*B*a*b*c))/(8*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (x^3*(4*A*a^2*c^2 + 2*B*a*b^3 - 19*A*a*b^2*c + 28*B*a^2*b*c))/(8*c*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (a^2*x*(B*b^2 - 12*A*b*c + 20*B*a*c))/(8*c*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)))/(x^4*(2*a*c + b^2) + a^2 + c^2*x^8 + 2*a*b*x^2 + 2*b*c*x^6) + atan(((((5242880*B*a^7*c^8 + 3072*A*a*b^11*c^3 - 3145728*A*a^6*b*c^8 - 256*B*a*b^12*c^2 - 61440*A*a^2*b^9*c^4 + 491520*A*a^3*b^7*c^5 - 1966080*A*a^4*b^5*c^6 + 3932160*A*a^5*b^3*c^7 + 61440*B*a^3*b^8*c^4 - 655360*B*a^4*b^6*c^5 + 2949120*B*a^5*b^4*c^6 - 6291456*B*a^6*b^2*c^7)/(512*(b^12*c + 4096*a^6*c^7 - 24*a*b^10*c^2 + 240*a^2*b^8*c^3 - 1280*a^3*b^6*c^4 + 3840*a^4*b^4*c^5 - 6144*a^5*b^2*c^6)) - (x*(-(B^2*b^17 + 9*A^2*b^15*c^2 - 9*A^2*c^2*(-(4*a*c - b^2)^15)^(1/2) - B^2*b^2*(-(4*a*c - b^2)^15)^(1/2) + 6*A*B*b^16*c - 5040*A^2*a^2*b^11*c^4 + 37440*A^2*a^3*b^9*c^5 - 103680*A^2*a^4*b^7*c^6 - 9216*A^2*a^5*b^5*c^7 + 552960*A^2*a^6*b^3*c^8 + 1140*B^2*a^2*b^13*c^2 - 10160*B^2*a^3*b^11*c^3 + 34880*B^2*a^4*b^9*c^4 + 43776*B^2*a^5*b^7*c^5 - 680960*B^2*a^6*b^5*c^6 + 1863680*B^2*a^7*b^3*c^7 + 983040*A*B*a^8*c^9 - 55*B^2*a*b^15*c + 25*B^2*a*c*(-(4*a*c - b^2)^15)^(1/2) + 180*A^2*a*b^13*c^3 - 737280*A^2*a^7*b*c^9 - 1720320*B^2*a^8*b*c^8 + 240*A*B*a^2*b^12*c^3 + 24000*A*B*a^3*b^10*c^4 - 241920*A*B*a^4*b^8*c^5 + 992256*A*B*a^5*b^6*c^6 - 1781760*A*B*a^6*b^4*c^7 + 737280*A*B*a^7*b^2*c^8 - 6*A*B*b*c*(-(4*a*c - b^2)^15)^(1/2) - 180*A*B*a*b^14*c^2)/(512*(1048576*a^10*c^13 + b^20*c^3 - 40*a*b^18*c^4 + 720*a^2*b^16*c^5 - 7680*a^3*b^14*c^6 + 53760*a^4*b^12*c^7 - 258048*a^5*b^10*c^8 + 860160*a^6*b^8*c^9 - 1966080*a^7*b^6*c^10 + 2949120*a^8*b^4*c^11 - 2621440*a^9*b^2*c^12)))^(1/2)*(256*b^11*c^3 - 5120*a*b^9*c^4 - 262144*a^5*b*c^8 + 40960*a^2*b^7*c^5 - 163840*a^3*b^5*c^6 + 327680*a^4*b^3*c^7))/(32*(b^8*c + 256*a^4*c^5 - 16*a*b^6*c^2 + 96*a^2*b^4*c^3 - 256*a^3*b^2*c^4)))*(-(B^2*b^17 + 9*A^2*b^15*c^2 - 9*A^2*c^2*(-(4*a*c - b^2)^15)^(1/2) - B^2*b^2*(-(4*a*c - b^2)^15)^(1/2) + 6*A*B*b^16*c - 5040*A^2*a^2*b^11*c^4 + 37440*A^2*a^3*b^9*c^5 - 103680*A^2*a^4*b^7*c^6 - 9216*A^2*a^5*b^5*c^7 + 552960*A^2*a^6*b^3*c^8 + 1140*B^2*a^2*b^13*c^2 - 10160*B^2*a^3*b^11*c^3 + 34880*B^2*a^4*b^9*c^4 + 43776*B^2*a^5*b^7*c^5 - 680960*B^2*a^6*b^5*c^6 + 1863680*B^2*a^7*b^3*c^7 + 983040*A*B*a^8*c^9 - 55*B^2*a*b^15*c + 25*B^2*a*c*(-(4*a*c - b^2)^15)^(1/2) + 180*A^2*a*b^13*c^3 - 737280*A^2*a^7*b*c^9 - 1720320*B^2*a^8*b*c^8 + 240*A*B*a^2*b^12*c^3 + 24000*A*B*a^3*b^10*c^4 - 241920*A*B*a^4*b^8*c^5 + 992256*A*B*a^5*b^6*c^6 - 1781760*A*B*a^6*b^4*c^7 + 737280*A*B*a^7*b^2*c^8 - 6*A*B*b*c*(-(4*a*c - b^2)^15)^(1/2) - 180*A*B*a*b^14*c^2)/(512*(1048576*a^10*c^13 + b^20*c^3 - 40*a*b^18*c^4 + 720*a^2*b^16*c^5 - 7680*a^3*b^14*c^6 + 53760*a^4*b^12*c^7 - 258048*a^5*b^10*c^8 + 860160*a^6*b^8*c^9 - 1966080*a^7*b^6*c^10 + 2949120*a^8*b^4*c^11 - 2621440*a^9*b^2*c^12)))^(1/2) - (x*(B^2*b^8 - 288*A^2*a^3*c^5 + 9*A^2*b^6*c^2 + 800*B^2*a^4*c^4 + 6*A*B*b^7*c + 576*A^2*a^2*b^2*c^4 + 314*B^2*a^2*b^4*c^2 + 208*B^2*a^3*b^2*c^3 - 36*B^2*a*b^6*c + 126*A^2*a*b^4*c^3 - 816*A*B*a^2*b^3*c^3 - 66*A*B*a*b^5*c^2 - 672*A*B*a^3*b*c^4))/(32*(b^8*c + 256*a^4*c^5 - 16*a*b^6*c^2 + 96*a^2*b^4*c^3 - 256*a^3*b^2*c^4)))*(-(B^2*b^17 + 9*A^2*b^15*c^2 - 9*A^2*c^2*(-(4*a*c - b^2)^15)^(1/2) - B^2*b^2*(-(4*a*c - b^2)^15)^(1/2) + 6*A*B*b^16*c - 5

040*A^2*a^2*b^11*c^4 + 37440*A^2*a^3*b^9*c^5 - 103680*A^2*a^4*b^7*c^6 - 921
6*A^2*a^5*b^5*c^7 + 552960*A^2*a^6*b^3*c^8 + 1140*B^2*a^2*b^13*c^2 - 10160*
B^2*a^3*b^11*c^3 + 34880*B^2*a^4*b^9*c^4 + 43776*B^2*a^5*b^7*c^5 - 680960*B
^2*a^6*b^5*c^6 + 1863680*B^2*a^7*b^3*c^7 + 983040*A*B*a^8*c^9 - 55*B^2*a*b^
15*c + 25*B^2*a*c*(-(4*a*c - b^2)^15)^(1/2) + 180*A^2*a*b^13*c^3 - 737280*A
^2*a^7*b*c^9 - 1720320*B^2*a^8*b*c^8 + 240*A*B*a^2*b^12*c^3 + 24000*A*B*a^3
*b^10*c^4 - 241920*A*B*a^4*b^8*c^5 + 992256*A*B*a^5*b^6*c^6 - 1781760*A*B*a
^6*b^4*c^7 + 737280*A*B*a^7*b^2*c^8 - 6*A*B*b*c*(-(4*a*c - b^2)^15)^(1/2) -
180*A*B*a*b^14*c^2)/(512*(1048576*a^10*c^13 + b^20*c^3 - 40*a*b^18*c^4 + 7
20*a^2*b^16*c^5 - 7680*a^3*b^14*c^6 + 53760*a^4*b^12*c^7 - 258048*a^5*b^10*
c^8 + 860160*a^6*b^8*c^9 - 1966080*a^7*b^6*c^10 + 2949120*a^8*b^4*c^11 - 26
21440*a^9*b^2*c^12)))^(1/2)*1i - (((5242880*B*a^7*c^8 + 3072*A*a*b^11*c^3 -
3145728*A*a^6*b*c^8 - 256*B*a*b^12*c^2 - 61440*A*a^2*b^9*c^4 + 491520*A*a^
3*b^7*c^5 - 1966080*A*a^4*b^5*c^6 + 3932160*A*a^5*b^3*c^7 + 61440*B*a^3*b^8
*c^4 - 655360*B*a^4*b^6*c^5 + 2949120*B*a^5*b^4*c^6 - 6291456*B*a^6*b^2*c^7
)/(512*(b^12*c + 4096*a^6*c^7 - 24*a*b^10*c^2 + 240*a^2*b^8*c^3 - 1280*a^3*
b^6*c^4 + 3840*a^4*b^4*c^5 - 6144*a^5*b^2*c^6)) + (x*(-(B^2*b^17 + 9*A^2*b^
15*c^2 - 9*A^2*c^2*(-(4*a*c - b^2)^15)^(1/2) - B^2*b^2*(-(4*a*c - b^2)^15)^
(1/2) + 6*A*B*b^16*c - 5040*A^2*a^2*b^11*c^4 + 37440*A^2*a^3*b^9*c^5 - 1036
80*A^2*a^4*b^7*c^6 - 9216*A^2*a^5*b^5*c^7 + 552960*A^2*a^6*b^3*c^8 + 1140*B
^2*a^2*b^13*c^2 - 10160*B^2*a^3*b^11*c^3 + 34880*B^2*a^4*b^9*c^4 + 43776*B^
2*a^5*b^7*c^5 - 680960*B^2*a^6*b^5*c^6 + 1863680*B^2*a^7*b^3*c^7 + 983040*A
*B*a^8*c^9 - 55*B^2*a*b^15*c + 25*B^2*a*c*(-(4*a*c - b^2)^15)^(1/2) + 180*A
^2*a*b^13*c^3 - 737280*A^2*a^7*b*c^9 - 1720320*B^2*a^8*b*c^8 + 240*A*B*a^2*
b^12*c^3 + 24000*A*B*a^3*b^10*c^4 - 241920*A*B*a^4*b^8*c^5 + 992256*A*B*a^5
*b^6*c^6 - 1781760*A*B*a^6*b^4*c^7 + 737280*A*B*a^7*b^2*c^8 - 6*A*B*b*c*(-(
4*a*c - b^2)^15)^(1/2) - 180*A*B*a*b^14*c^2)/(512*(1048576*a^10*c^13 + b^20
*c^3 - 40*a*b^18*c^4 + 720*a^2*b^16*c^5 - 7680*a^3*b^14*c^6 + 53760*a^4*b^1
2*c^7 - 258048*a^5*b^10*c^8 + 860160*a^6*b^8*c^9 - 1966080*a^7*b^6*c^10 + 2
949120*a^8*b^4*c^11 - 2621440*a^9*b^2*c^12)))^(1/2)*(256*b^11*c^3 - 5120*a*
b^9*c^4 - 262144*a^5*b*c^8 + 40960*a^2*b^7*c^5 - 163840*a^3*b^5*c^6 + 32768
0*a^4*b^3*c^7))/(32*(b^8*c + 256*a^4*c^5 - 16*a*b^6*c^2 + 96*a^2*b^4*c^3 -
256*a^3*b^2*c^4)))*(-(B^2*b^17 + 9*A^2*b^15*c^2 - 9*A^2*c^2*(-(4*a*c - b^2)
^15)^(1/2) - B^2*b^2*(-(4*a*c - b^2)^15)^(1/2) + 6*A*B*b^16*c - 5040*A^2*a^
2*b^11*c^4 + 37440*A^2*a^3*b^9*c^5 - 103680*A^2*a^4*b^7*c^6 - 9216*A^2*a^5*
b^5*c^7 + 552960*A^2*a^6*b^3*c^8 + 1140*B^2*a^2*b^13*c^2 - 10160*B^2*a^3*b^
11*c^3 + 34880*B^2*a^4*b^9*c^4 + 43776*B^2*a^5*b^7*c^5 - 680960*B^2*a^6*b^5
*c^6 + 1863680*B^2*a^7*b^3*c^7 + 983040*A*B*a^8*c^9 - 55*B^2*a*b^15*c + 25*
B^2*a*c*(-(4*a*c - b^2)^15)^(1/2) + 180*A^2*a*b^13*c^3 - 737280*A^2*a^7*b*c
^9 - 1720320*B^2*a^8*b*c^8 + 240*A*B*a^2*b^12*c^3 + 24000*A*B*a^3*b^10*c^4
- 241920*A*B*a^4*b^8*c^5 + 992256*A*B*a^5*b^6*c^6 - 1781760*A*B*a^6*b^4*c^7
+ 737280*A*B*a^7*b^2*c^8 - 6*A*B*b*c*(-(4*a*c - b^2)^15)^(1/2) - 180*A*B*a
*b^14*c^2)/(512*(1048576*a^10*c^13 + b^20*c^3 - 40*a*b^18*c^4 + 720*a^2*b^1
6*c^5 - 7680*a^3*b^14*c^6 + 53760*a^4*b^12*c^7 - 258048*a^5*b^10*c^8 + 8601
60*a^6*b^8*c^9 - 1966080*a^7*b^6*c^10 + 2949120*a^8*b^4*c^11 - 2621440*a^9*

b^2*c^12)))^(1/2) + (x*(B^2*b^8 - 288*A^2*a^3*c^5 + 9*A^2*b^6*c^2 + 800*B^2
*a^4*c^4 + 6*A*B*b^7*c + 576*A^2*a^2*b^2*c^4 + 314*B^2*a^2*b^4*c^2 + 208*B^
2*a^3*b^2*c^3 - 36*B^2*a*b^6*c + 126*A^2*a*b^4*c^3 - 816*A*B*a^2*b^3*c^3 -
66*A*B*a*b^5*c^2 - 672*A*B*a^3*b*c^4))/(32*(b^8*c + 256*a^4*c^5 - 16*a*b^6*
c^2 + 96*a^2*b^4*c^3 - 256*a^3*b^2*c^4)))*(-(B^2*b^17 + 9*A^2*b^15*c^2 - 9*
A^2*c^2*(-(4*a*c - b^2)^15)^(1/2) - B^2*b^2*(-(4*a*c - b^2)^15)^(1/2) + 6*A
*B*b^16*c - 5040*A^2*a^2*b^11*c^4 + 37440*A^2*a^3*b^9*c^5 - 103680*A^2*a^4*
b^7*c^6 - 9216*A^2*a^5*b^5*c^7 + 552960*A^2*a^6*b^3*c^8 + 1140*B^2*a^2*b^13
*c^2 - 10160*B^2*a^3*b^11*c^3 + 34880*B^2*a^4*b^9*c^4 + 43776*B^2*a^5*b^7*c
^5 - 680960*B^2*a^6*b^5*c^6 + 1863680*B^2*a^7*b^3*c^7 + 983040*A*B*a^8*c^9
- 55*B^2*a*b^15*c + 25*B^2*a*c*(-(4*a*c - b^2)^15)^(1/2) + 180*A^2*a*b^13*c
^3 - 737280*A^2*a^7*b*c^9 - 1720320*B^2*a^8*b*c^8 + 240*A*B*a^2*b^12*c^3 +
24000*A*B*a^3*b^10*c^4 - 241920*A*B*a^4*b^8*c^5 + 992256*A*B*a^5*b^6*c^6 -
1781760*A*B*a^6*b^4*c^7 + 737280*A*B*a^7*b^2*c^8 - 6*A*B*b*c*(-(4*a*c - b^2
)^15)^(1/2) - 180*A*B*a*b^14*c^2)/(512*(1048576*a^10*c^13 + b^20*c^3 - 40*a
*b^18*c^4 + 720*a^2*b^16*c^5 - 7680*a^3*b^14*c^6 + 53760*a^4*b^12*c^7 - 258
048*a^5*b^10*c^8 + 860160*a^6*b^8*c^9 - 1966080*a^7*b^6*c^10 + 2949120*a^8*
b^4*c^11 - 2621440*a^9*b^2*c^12)))^(1/2)*1i)/((((5242880*B*a^7*c^8 + 3072*A
*a*b^11*c^3 - 3145728*A*a^6*b*c^8 - 256*B*a*b^12*c^2 - 61440*A*a^2*b^9*c^4
+ 491520*A*a^3*b^7*c^5 - 1966080*A*a^4*b^5*c^6 + 3932160*A*a^5*b^3*c^7 + 61
440*B*a^3*b^8*c^4 - 655360*B*a^4*b^6*c^5 + 2949120*B*a^5*b^4*c^6 - 6291456*
B*a^6*b^2*c^7)/(512*(b^12*c + 4096*a^6*c^7 - 24*a*b^10*c^2 + 240*a^2*b^8*c^
3 - 1280*a^3*b^6*c^4 + 3840*a^4*b^4*c^5 - 6144*a^5*b^2*c^6)) - (x*(-(B^2*b^
17 + 9*A^2*b^15*c^2 - 9*A^2*c^2*(-(4*a*c - b^2)^15)^(1/2) - B^2*b^2*(-(4*a*
c - b^2)^15)^(1/2) + 6*A*B*b^16*c - 5040*A^2*a^2*b^11*c^4 + 37440*A^2*a^3*b
^9*c^5 - 103680*A^2*a^4*b^7*c^6 - 9216*A^2*a^5*b^5*c^7 + 552960*A^2*a^6*b^3
*c^8 + 1140*B^2*a^2*b^13*c^2 - 10160*B^2*a^3*b^11*c^3 + 34880*B^2*a^4*b^9*c
^4 + 43776*B^2*a^5*b^7*c^5 - 680960*B^2*a^6*b^5*c^6 + 1863680*B^2*a^7*b^3*c
^7 + 983040*A*B*a^8*c^9 - 55*B^2*a*b^15*c + 25*B^2*a*c*(-(4*a*c - b^2)^15)^
(1/2) + 180*A^2*a*b^13*c^3 - 737280*A^2*a^7*b*c^9 - 1720320*B^2*a^8*b*c^8 +
240*A*B*a^2*b^12*c^3 + 24000*A*B*a^3*b^10*c^4 - 241920*A*B*a^4*b^8*c^5 + 9
92256*A*B*a^5*b^6*c^6 - 1781760*A*B*a^6*b^4*c^7 + 737280*A*B*a^7*b^2*c^8 -
6*A*B*b*c*(-(4*a*c - b^2)^15)^(1/2) - 180*A*B*a*b^14*c^2)/(512*(1048576*a^1
0*c^13 + b^20*c^3 - 40*a*b^18*c^4 + 720*a^2*b^16*c^5 - 7680*a^3*b^14*c^6 +
53760*a^4*b^12*c^7 - 258048*a^5*b^10*c^8 + 860160*a^6*b^8*c^9 - 1966080*a^7
*b^6*c^10 + 2949120*a^8*b^4*c^11 - 2621440*a^9*b^2*c^12)))^(1/2)*(256*b^11*
c^3 - 5120*a*b^9*c^4 - 262144*a^5*b*c^8 + 40960*a^2*b^7*c^5 - 163840*a^3*b^
5*c^6 + 327680*a^4*b^3*c^7))/(32*(b^8*c + 256*a^4*c^5 - 16*a*b^6*c^2 + 96*a
^2*b^4*c^3 - 256*a^3*b^2*c^4)))*(-(B^2*b^17 + 9*A^2*b^15*c^2 - 9*A^2*c^2*(-
(4*a*c - b^2)^15)^(1/2) - B^2*b^2*(-(4*a*c - b^2)^15)^(1/2) + 6*A*B*b^16*c
- 5040*A^2*a^2*b^11*c^4 + 37440*A^2*a^3*b^9*c^5 - 103680*A^2*a^4*b^7*c^6 -
9216*A^2*a^5*b^5*c^7 + 552960*A^2*a^6*b^3*c^8 + 1140*B^2*a^2*b^13*c^2 - 101
60*B^2*a^3*b^11*c^3 + 34880*B^2*a^4*b^9*c^4 + 43776*B^2*a^5*b^7*c^5 - 68096
0*B^2*a^6*b^5*c^6 + 1863680*B^2*a^7*b^3*c^7 + 983040*A*B*a^8*c^9 - 55*B^2*a
*b^15*c + 25*B^2*a*c*(-(4*a*c - b^2)^15)^(1/2) + 180*A^2*a*b^13*c^3 - 73728

0*A^2*a^7*b*c^9 - 1720320*B^2*a^8*b*c^8 + 240*A*B*a^2*b^12*c^3 + 24000*A*B*a^3*b^10*c^4 - 241920*A*B*a^4*b^8*c^5 + 992256*A*B*a^5*b^6*c^6 - 1781760*A*B*a^6*b^4*c^7 + 737280*A*B*a^7*b^2*c^8 - 6*A*B*b*c*(-(4*a*c - b^2)^15)^(1/2) - 180*A*B*a*b^14*c^2)/(512*(1048576*a^10*c^13 + b^20*c^3 - 40*a*b^18*c^4 + 720*a^2*b^16*c^5 - 7680*a^3*b^14*c^6 + 53760*a^4*b^12*c^7 - 258048*a^5*b^10*c^8 + 860160*a^6*b^8*c^9 - 1966080*a^7*b^6*c^10 + 2949120*a^8*b^4*c^11 - 2621440*a^9*b^2*c^12)))^(1/2) - (x*(B^2*b^8 - 288*A^2*a^3*c^5 + 9*A^2*b^6*c^2 + 800*B^2*a^4*c^4 + 6*A*B*b^7*c + 576*A^2*a^2*b^2*c^4 + 314*B^2*a^2*b^4*c^2 + 208*B^2*a^3*b^2*c^3 - 36*B^2*a*b^6*c + 126*A^2*a*b^4*c^3 - 816*A*B*a^2*b^3*c^3 - 66*A*B*a*b^5*c^2 - 672*A*B*a^3*b*c^4))/(32*(b^8*c + 256*a^4*c^5 - 16*a*b^6*c^2 + 96*a^2*b^4*c^3 - 256*a^3*b^2*c^4)))*(-(B^2*b^17 + 9*A^2*b^15*c^2 - 9*A^2*c^2*(-(4*a*c - b^2)^15)^(1/2) - B^2*b^2*(-(4*a*c - b^2)^15)^(1/2) + 6*A*B*b^16*c - 5040*A^2*a^2*b^11*c^4 + 37440*A^2*a^3*b^9*c^5 - 103680*A^2*a^4*b^7*c^6 - 9216*A^2*a^5*b^5*c^7 + 552960*A^2*a^6*b^3*c^8 + 1140*B^2*a^2*b^13*c^2 - 10160*B^2*a^3*b^11*c^3 + 34880*B^2*a^4*b^9*c^4 + 43776*B^2*a^5*b^7*c^5 - 680960*B^2*a^6*b^5*c^6 + 1863680*B^2*a^7*b^3*c^7 + 983040*A*B*a^8*c^9 - 55*B^2*a*b^15*c + 25*B^2*a*c*(-(4*a*c - b^2)^15)^(1/2) + 180*A^2*a*b^13*c^3 - 737280*A^2*a^7*b*c^9 - 1720320*B^2*a^8*b*c^8 + 240*A*B*a^2*b^12*c^3 + 24000*A*B*a^3*b^10*c^4 - 241920*A*B*a^4*b^8*c^5 + 992256*A*B*a^5*b^6*c^6 - 1781760*A*B*a^6*b^4*c^7 + 737280*A*B*a^7*b^2*c^8 - 6*A*B*b*c*(-(4*a*c - b^2)^15)^(1/2) - 180*A*B*a*b^14*c^2)/(512*(1048576*a^10*c^13 + b^20*c^3 - 40*a*b^18*c^4 + 720*a^2*b^16*c^5 - 7680*a^3*b^14*c^6 + 53760*a^4*b^12*c^7 - 258048*a^5*b^10*c^8 + 860160*a^6*b^8*c^9 - 1966080*a^7*b^6*c^10 + 2949120*a^8*b^4*c^11 - 2621440*a^9*b^2*c^12)))^(1/2) + (((5242880*B*a^7*c^8 + 3072*A*a*b^11*c^3 - 3145728*A*a^6*b*c^8 - 256*B*a*b^12*c^2 - 61440*A*a^2*b^9*c^4 + 491520*A*a^3*b^7*c^5 - 1966080*A*a^4*b^5*c^6 + 3932160*A*a^5*b^3*c^7 + 61440*B*a^3*b^8*c^4 - 655360*B*a^4*b^6*c^5 + 2949120*B*a^5*b^4*c^6 - 6291456*B*a^6*b^2*c^7)/(512*(b^12*c + 4096*a^6*c^7 - 24*a*b^10*c^2 + 240*a^2*b^8*c^3 - 1280*a^3*b^6*c^4 + 3840*a^4*b^4*c^5 - 6144*a^5*b^2*c^6)) + (x*(-(B^2*b^17 + 9*A^2*b^15*c^2 - 9*A^2*c^2*(-(4*a*c - b^2)^15)^(1/2) - B^2*b^2*(-(4*a*c - b^2)^15)^(1/2) + 6*A*B*b^16*c - 5040*A^2*a^2*b^11*c^4 + 37440*A^2*a^3*b^9*c^5 - 103680*A^2*a^4*b^7*c^6 - 9216*A^2*a^5*b^5*c^7 + 552960*A^2*a^6*b^3*c^8 + 1140*B^2*a^2*b^13*c^2 - 10160*B^2*a^3*b^11*c^3 + 34880*B^2*a^4*b^9*c^4 + 43776*B^2*a^5*b^7*c^5 - 680960*B^2*a^6*b^5*c^6 + 1863680*B^2*a^7*b^3*c^7 + 983040*A*B*a^8*c^9 - 55*B^2*a*b^15*c + 25*B^2*a*c*(-(4*a*c - b^2)^15)^(1/2) + 180*A^2*a*b^13*c^3 - 737280*A^2*a^7*b*c^9 - 1720320*B^2*a^8*b*c^8 + 240*A*B*a^2*b^12*c^3 + 24000*A*B*a^3*b^10*c^4 - 241920*A*B*a^4*b^8*c^5 + 992256*A*B*a^5*b^6*c^6 - 1781760*A*B*a^6*b^4*c^7 + 737280*A*B*a^7*b^2*c^8 - 6*A*B*b*c*(-(4*a*c - b^2)^15)^(1/2) - 180*A*B*a*b^14*c^2)/(512*(1048576*a^10*c^13 + b^20*c^3 - 40*a*b^18*c^4 + 720*a^2*b^16*c^5 - 7680*a^3*b^14*c^6 + 53760*a^4*b^12*c^7 - 258048*a^5*b^10*c^8 + 860160*a^6*b^8*c^9 - 1966080*a^7*b^6*c^10 + 2949120*a^8*b^4*c^11 - 2621440*a^9*b^2*c^12)))^(1/2)*(256*b^11*c^3 - 5120*a*b^9*c^4 - 262144*a^5*b*c^8 + 40960*a^2*b^7*c^5 - 163840*a^3*b^5*c^6 + 327680*a^4*b^3*c^7))/(32*(b^8*c + 256*a^4*c^5 - 16*a*b^6*c^2 + 96*a^2*b^4*c^3 - 256*a^3*b^2*c^4)))*(-(B^2*b^17 + 9*A^2*b^15*c^2 - 9*

A^2*c^2*(-(4*a*c - b^2)^15)^(1/2) - B^2*b^2*(-(4*a*c - b^2)^15)^(1/2) + 6*A
*B*b^16*c - 5040*A^2*a^2*b^11*c^4 + 37440*A^2*a^3*b^9*c^5 - 103680*A^2*a^4*
b^7*c^6 - 9216*A^2*a^5*b^5*c^7 + 552960*A^2*a^6*b^3*c^8 + 1140*B^2*a^2*b^13
*c^2 - 10160*B^2*a^3*b^11*c^3 + 34880*B^2*a^4*b^9*c^4 + 43776*B^2*a^5*b^7*c
^5 - 680960*B^2*a^6*b^5*c^6 + 1863680*B^2*a^7*b^3*c^7 + 983040*A*B*a^8*c^9
- 55*B^2*a*b^15*c + 25*B^2*a*c*(-(4*a*c - b^2)^15)^(1/2) + 180*A^2*a*b^13*c
^3 - 737280*A^2*a^7*b*c^9 - 1720320*B^2*a^8*b*c^8 + 240*A*B*a^2*b^12*c^3 +
24000*A*B*a^3*b^10*c^4 - 241920*A*B*a^4*b^8*c^5 + 992256*A*B*a^5*b^6*c^6 -
1781760*A*B*a^6*b^4*c^7 + 737280*A*B*a^7*b^2*c^8 - 6*A*B*b*c*(-(4*a*c - b^2
)^15)^(1/2) - 180*A*B*a*b^14*c^2)/(512*(1048576*a^10*c^13 + b^20*c^3 - 40*a
*b^18*c^4 + 720*a^2*b^16*c^5 - 7680*a^3*b^14*c^6 + 53760*a^4*b^12*c^7 - 258
048*a^5*b^10*c^8 + 860160*a^6*b^8*c^9 - 1966080*a^7*b^6*c^10 + 2949120*a^8*
b^4*c^11 - 2621440*a^9*b^2*c^12)))^(1/2) + (x*(B^2*b^8 - 288*A^2*a^3*c^5 +
9*A^2*b^6*c^2 + 800*B^2*a^4*c^4 + 6*A*B*b^7*c + 576*A^2*a^2*b^2*c^4 + 314*B
^2*a^2*b^4*c^2 + 208*B^2*a^3*b^2*c^3 - 36*B^2*a*b^6*c + 126*A^2*a*b^4*c^3 -
816*A*B*a^2*b^3*c^3 - 66*A*B*a*b^5*c^2 - 672*A*B*a^3*b*c^4))/(32*(b^8*c +
256*a^4*c^5 - 16*a*b^6*c^2 + 96*a^2*b^4*c^3 - 256*a^3*b^2*c^4)))*(-(B^2*b^1
7 + 9*A^2*b^15*c^2 - 9*A^2*c^2*(-(4*a*c - b^2)^15)^(1/2) - B^2*b^2*(-(4*a*c
- b^2)^15)^(1/2) + 6*A*B*b^16*c - 5040*A^2*a^2*b^11*c^4 + 37440*A^2*a^3*b^
9*c^5 - 103680*A^2*a^4*b^7*c^6 - 9216*A^2*a^5*b^5*c^7 + 552960*A^2*a^6*b^3*
c^8 + 1140*B^2*a^2*b^13*c^2 - 10160*B^2*a^3*b^11*c^3 + 34880*B^2*a^4*b^9*c^
4 + 43776*B^2*a^5*b^7*c^5 - 680960*B^2*a^6*b^5*c^6 + 1863680*B^2*a^7*b^3*c^
7 + 983040*A*B*a^8*c^9 - 55*B^2*a*b^15*c + 25*B^2*a*c*(-(4*a*c - b^2)^15)^(
1/2) + 180*A^2*a*b^13*c^3 - 737280*A^2*a^7*b*c^9 - 1720320*B^2*a^8*b*c^8 +
240*A*B*a^2*b^12*c^3 + 24000*A*B*a^3*b^10*c^4 - 241920*A*B*a^4*b^8*c^5 + 99
2256*A*B*a^5*b^6*c^6 - 1781760*A*B*a^6*b^4*c^7 + 737280*A*B*a^7*b^2*c^8 - 6
*A*B*b*c*(-(4*a*c - b^2)^15)^(1/2) - 180*A*B*a*b^14*c^2)/(512*(1048576*a^10
*c^13 + b^20*c^3 - 40*a*b^18*c^4 + 720*a^2*b^16*c^5 - 7680*a^3*b^14*c^6 + 5
3760*a^4*b^12*c^7 - 258048*a^5*b^10*c^8 + 860160*a^6*b^8*c^9 - 1966080*a^7*
b^6*c^10 + 2949120*a^8*b^4*c^11 - 2621440*a^9*b^2*c^12)))^(1/2) + (1728*A^3
*a^4*c^5 - 35*B^3*a^2*b^7 + 1620*A^3*a^2*b^4*c^3 + 4752*A^3*a^3*b^2*c^4 - 9
456*B^3*a^4*b^3*c^2 + 15*A*B^2*a*b^8 + 4800*A*B^2*a^5*c^4 + 135*A^3*a*b^6*c
^2 + 1176*B^3*a^3*b^5*c - 6400*B^3*a^5*b*c^3 - 705*A*B^2*a^2*b^6*c - 15552*
A^2*B*a^4*b*c^4 + 6084*A*B^2*a^3*b^4*c^2 + 26256*A*B^2*a^4*b^2*c^3 - 1260*A
^2*B*a^2*b^5*c^2 - 13248*A^2*B*a^3*b^3*c^3 + 90*A^2*B*a*b^7*c)/(256*(b^12*c
+ 4096*a^6*c^7 - 24*a*b^10*c^2 + 240*a^2*b^8*c^3 - 1280*a^3*b^6*c^4 + 3840
*a^4*b^4*c^5 - 6144*a^5*b^2*c^6))))*(-(B^2*b^17 + 9*A^2*b^15*c^2 - 9*A^2*c^
2*(-(4*a*c - b^2)^15)^(1/2) - B^2*b^2*(-(4*a*c - b^2)^15)^(1/2) + 6*A*B*b^1
6*c - 5040*A^2*a^2*b^11*c^4 + 37440*A^2*a^3*b^9*c^5 - 103680*A^2*a^4*b^7*c^
6 - 9216*A^2*a^5*b^5*c^7 + 552960*A^2*a^6*b^3*c^8 + 1140*B^2*a^2*b^13*c^2 -
10160*B^2*a^3*b^11*c^3 + 34880*B^2*a^4*b^9*c^4 + 43776*B^2*a^5*b^7*c^5 - 6
80960*B^2*a^6*b^5*c^6 + 1863680*B^2*a^7*b^3*c^7 + 983040*A*B*a^8*c^9 - 55*B
^2*a*b^15*c + 25*B^2*a*c*(-(4*a*c - b^2)^15)^(1/2) + 180*A^2*a*b^13*c^3 - 7
37280*A^2*a^7*b*c^9 - 1720320*B^2*a^8*b*c^8 + 240*A*B*a^2*b^12*c^3 + 24000*
A*B*a^3*b^10*c^4 - 241920*A*B*a^4*b^8*c^5 + 992256*A*B*a^5*b^6*c^6 - 178176

0*A*B*a^6*b^4*c^7 + 737280*A*B*a^7*b^2*c^8 - 6*A*B*b*c*(-(4*a*c - b^2)^15)^
(1/2) - 180*A*B*a*b^14*c^2)/(512*(1048576*a^10*c^13 + b^20*c^3 - 40*a*b^18*
c^4 + 720*a^2*b^16*c^5 - 7680*a^3*b^14*c^6 + 53760*a^4*b^12*c^7 - 258048*a^
5*b^10*c^8 + 860160*a^6*b^8*c^9 - 1966080*a^7*b^6*c^10 + 2949120*a^8*b^4*c^
11 - 2621440*a^9*b^2*c^12)))^(1/2)*2i

3.134 $\int \frac{x^4(A+Bx^2)}{(a+bx^2+cx^4)^3}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 380

$$\begin{aligned}\int \frac{x^4(A+Bx^2)}{(a+bx^2+cx^4)^3}\,dx &= -\frac{x^3(Ab-2aB-(bB-2Ac)x^2)}{4(b^2-4ac)(a+bx^2+cx^4)^2}+\frac{3x(4abB-A(b^2+4ac)+(b^2B-4Abc+4aBc)x^2)}{8(b^2-4ac)^2(a+bx^2+cx^4)}\\ &+\frac{3\left(b^2B-4Abc+4aBc-\frac{b^3B-6Ab^2c+12abBc-8aAc^2}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{8\sqrt{2}\sqrt{c}(b^2-4ac)^2\sqrt{b-\sqrt{b^2-4ac}}}\\ &+\frac{3\left(b^2B-4Abc+4aBc+\frac{b^3B-6Ab^2c+12abBc-8aAc^2}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{8\sqrt{2}\sqrt{c}(b^2-4ac)^2\sqrt{b+\sqrt{b^2-4ac}}}\end{aligned}$$

```
[Out] -1/4*x^3*(A*b-2*B*a-(-2*A*c+B*b)*x^2)/(-4*a*c+b^2)/(c*x^4+b*x^2+a)^2+3/8*x*
(4*a*b*B-A*(4*a*c+b^2)+(-4*A*b*c+4*B*a*c+B*b^2)*x^2)/(-4*a*c+b^2)^2/(c*x^4+
b*x^2+a)+3/16*arctan(x*2^(1/2)*c^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2))*(B*b^2
-4*A*b*c+4*B*a*c+(8*A*a*c^2+6*A*b^2*c-12*B*a*b*c-B*b^3)/(-4*a*c+b^2)^(1/2))
/(-4*a*c+b^2)^2*2^(1/2)/c^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2)+3/16*arctan(x*
2^(1/2)*c^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2))*(B*b^2-4*A*b*c+4*B*a*c+(-8*A*
a*c^2-6*A*b^2*c+12*B*a*b*c+B*b^3)/(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^2*2^(1/2
)/c^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2)
```

Rubi [A] (verified)

Time = 0.99 (sec) , antiderivative size = 380, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {1289, 1180, 211}

$$\int \frac{x^4(A+Bx^2)}{(a+bx^2+cx^4)^3}\,dx$$
$$=\frac{3\left(-\frac{-8aAc^2+12abBc-6Ab^2c+b^3B}{\sqrt{b^2-4ac}}+4aBc-4Abc+b^2B\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{8\sqrt{2}\sqrt{c}\,(b^2-4ac)^2\sqrt{b-\sqrt{b^2-4ac}}}$$
$$+\frac{3\left(\frac{-8aAc^2+12abBc-6Ab^2c+b^3B}{\sqrt{b^2-4ac}}+4aBc-4Abc+b^2B\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{8\sqrt{2}\sqrt{c}\,(b^2-4ac)^2\sqrt{\sqrt{b^2-4ac}+b}}$$
$$+\frac{3x(x^2(4aBc-4Abc+b^2B)-A(4ac+b^2)+4abB)}{8\,(b^2-4ac)^2\,(a+bx^2+cx^4)}$$
$$-\frac{x^3(-2aB-(x^2(bB-2Ac))+Ab)}{4\,(b^2-4ac)\,(a+bx^2+cx^4)^2}$$

```
[In] Int[(x^4*(A + B*x^2))/(a + b*x^2 + c*x^4)^3,x]
```

```
[Out] -1/4*(x^3*(A*b - 2*a*B - (b*B - 2*A*c)*x^2))/((b^2 - 4*a*c)*(a + b*x^2 + c*
x^4)^2) + (3*x*(4*a*b*B - A*(b^2 + 4*a*c) + (b^2*B - 4*A*b*c + 4*a*B*c)*x^2
))/(8*(b^2 - 4*a*c)^2*(a + b*x^2 + c*x^4)) + (3*(b^2*B - 4*A*b*c + 4*a*B*c
- (b^3*B - 6*A*b^2*c + 12*a*b*B*c - 8*a*A*c^2)/Sqrt[b^2 - 4*a*c])*ArcTan[(S
qrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(8*Sqrt[2]*Sqrt[c]*(b^2 - 4
*a*c)^2*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (3*(b^2*B - 4*A*b*c + 4*a*B*c + (b^3
*B - 6*A*b^2*c + 12*a*b*B*c - 8*a*A*c^2)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]
*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(8*Sqrt[2]*Sqrt[c]*(b^2 - 4*a*c)^
2*Sqrt[b + Sqrt[b^2 - 4*a*c]])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1289

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_.), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(a + b*x^2 + c*x^4)^(p + 1)
*((b*d - 2*a*e - (b*e - 2*c*d)*x^2)/(2*(p + 1)*(b^2 - 4*a*c))), x] - Dist[f
^2/(2*(p + 1)*(b^2 - 4*a*c)), Int[(f*x)^(m - 2)*(a + b*x^2 + c*x^4)^(p + 1)
*Simp[(m - 1)*(b*d - 2*a*e) - (4*p + 4 + m + 1)*(b*e - 2*c*d)*x^2, x], x],
x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] &&
GtQ[m, 1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{x^3(Ab-2aB-(bB-2Ac)x^2)}{4(b^2-4ac)(a+bx^2+cx^4)^2}+\frac{\int \frac{x^2(3(Ab-2aB)+3(bB-2Ac)x^2)}{(a+bx^2+cx^4)^2}\,dx}{4(b^2-4ac)}\\
&= -\frac{x^3(Ab-2aB-(bB-2Ac)x^2)}{4(b^2-4ac)(a+bx^2+cx^4)^2}\\
&\quad+\frac{3x(4abB-A(b^2+4ac)+(b^2B-4Abc+4aBc)x^2)}{8(b^2-4ac)^2(a+bx^2+cx^4)}\\
&\quad+\frac{\int \frac{-3(4abB-A(b^2+4ac))+3(b^2B-4Abc+4aBc)x^2}{a+bx^2+cx^4}\,dx}{8(b^2-4ac)^2}\\
&= -\frac{x^3(Ab-2aB-(bB-2Ac)x^2)}{4(b^2-4ac)(a+bx^2+cx^4)^2}+\frac{3x(4abB-A(b^2+4ac)+(b^2B-4Abc+4aBc)x^2)}{8(b^2-4ac)^2(a+bx^2+cx^4)}\\
&\quad+\frac{\left(3\left(b^2B-4Abc+4aBc-\frac{b^3B-6Ab^2c+12abBc-8aAc^2}{\sqrt{b^2-4ac}}\right)\right)\int \frac{1}{\frac{b}{2}-\frac{1}{2}\sqrt{b^2-4ac}+cx^2}\,dx}{16(b^2-4ac)^2}\\
&\quad+\frac{\left(3\left(b^2B-4Abc+4aBc+\frac{b^3B-6Ab^2c+12abBc-8aAc^2}{\sqrt{b^2-4ac}}\right)\right)\int \frac{1}{\frac{b}{2}+\frac{1}{2}\sqrt{b^2-4ac}+cx^2}\,dx}{16(b^2-4ac)^2}\\
&= -\frac{x^3(Ab-2aB-(bB-2Ac)x^2)}{4(b^2-4ac)(a+bx^2+cx^4)^2}+\frac{3x(4abB-A(b^2+4ac)+(b^2B-4Abc+4aBc)x^2)}{8(b^2-4ac)^2(a+bx^2+cx^4)}\\
&\quad+\frac{3\left(b^2B-4Abc+4aBc-\frac{b^3B-6Ab^2c+12abBc-8aAc^2}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{8\sqrt{2}\sqrt{c}(b^2-4ac)^2\sqrt{b-\sqrt{b^2-4ac}}}\\
&\quad+\frac{3\left(b^2B-4Abc+4aBc+\frac{b^3B-6Ab^2c+12abBc-8aAc^2}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{8\sqrt{2}\sqrt{c}(b^2-4ac)^2\sqrt{b+\sqrt{b^2-4ac}}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 1.08 (sec) , antiderivative size = 447, normalized size of antiderivative = 1.18

$$\int \frac{x^4(A+Bx^2)}{(a+bx^2+cx^4)^3}\,dx$$
$$= \frac{\frac{-4abBx+4b(-bB+Ac)x^3+8acx(A+Bx^2)}{(b^2-4ac)(a+bx^2+cx^4)^2} + \frac{2x(2b^3B+4ac^2(A+3Bx^2)+4bc(aB-3Acx^2)+b^2(-7Ac+3Bcx^2))}{(b^2-4ac)^2(a+bx^2+cx^4)} + \frac{3\sqrt{2}\sqrt{c}\left(-b^3B-4bc\left(3aB\right.\right.}{}}{}$$

```
[In] Integrate[(x^4*(A + B*x^2))/(a + b*x^2 + c*x^4)^3,x]
```

```
[Out] ((-4*a*b*B*x + 4*b*(-(b*B) + A*c)*x^3 + 8*a*c*x*(A + B*x^2))/((b^2 - 4*a*c)
*(a + b*x^2 + c*x^4)^2) + (2*x*(2*b^3*B + 4*a*c^2*(A + 3*B*x^2) + 4*b*c*(a*
B - 3*A*c*x^2) + b^2*(-7*A*c + 3*B*c*x^2)))/((b^2 - 4*a*c)^2*(a + b*x^2 + c
*x^4)) + (3*Sqrt[2]*Sqrt[c]*(-(b^3*B) - 4*b*c*(3*a*B + A*Sqrt[b^2 - 4*a*c])
 + 4*a*c*(2*A*c + B*Sqrt[b^2 - 4*a*c]) + b^2*(6*A*c + B*Sqrt[b^2 - 4*a*c]))
*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/((b^2 - 4*a*c)^(5
/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (3*Sqrt[2]*Sqrt[c]*(b^3*B + 4*b*c*(3*a*B
 - A*Sqrt[b^2 - 4*a*c]) + b^2*(-6*A*c + B*Sqrt[b^2 - 4*a*c]) + 4*a*c*(-2*A*
c + B*Sqrt[b^2 - 4*a*c]))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*
a*c]]])/((b^2 - 4*a*c)^(5/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]]))/(16*c)
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.16 (sec) , antiderivative size = 328, normalized size of antiderivative = 0.86

method	result
risch	$\frac{-\frac{3c\left(4Abc-4Bac-B\,b^2\right)x^7}{8\left(16a^2c^2-8a\,b^2c+b^4\right)}+\frac{\left(4Aa\,c^2-19A\,b^2c+16Babc+5B\,b^3\right)x^5}{128a^2c^2-64a\,b^2c+8b^4}-\frac{\left(16Aabc+5A\,b^3+4a^2Bc-19Ba\,b^2\right)x^3}{8\left(16a^2c^2-8a\,b^2c+b^4\right)}-\frac{3a\left(4Aac+A\,b^2-4abB\right)x}{8\left(16a^2c^2-8a\,b^2c+b^4\right)}}{\left(c\,x^4+b\,x^2+a\right)^2}+\frac{3\left(\ldots\right.}{}$
default	$\frac{-\frac{3c\left(4Abc-4Bac-B\,b^2\right)x^7}{8\left(16a^2c^2-8a\,b^2c+b^4\right)}+\frac{\left(4Aa\,c^2-19A\,b^2c+16Babc+5B\,b^3\right)x^5}{128a^2c^2-64a\,b^2c+8b^4}-\frac{\left(16Aabc+5A\,b^3+4a^2Bc-19Ba\,b^2\right)x^3}{8\left(16a^2c^2-8a\,b^2c+b^4\right)}-\frac{3a\left(4Aac+A\,b^2-4abB\right)x}{8\left(16a^2c^2-8a\,b^2c+b^4\right)}}{\left(c\,x^4+b\,x^2+a\right)^2}+\frac{3c\left(\ldots\right.}{}$

```
[In] int(x^4*(B*x^2+A)/(c*x^4+b*x^2+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] (-3/8*c*(4*A*b*c-4*B*a*c-B*b^2)/(16*a^2*c^2-8*a*b^2*c+b^4)*x^7+1/8*(4*A*a*c
^2-19*A*b^2*c+16*B*a*b*c+5*B*b^3)/(16*a^2*c^2-8*a*b^2*c+b^4)*x^5-1/8*(16*A*
```

```
a*b*c+5*A*b^3+4*B*a^2*c-19*B*a*b^2)/(16*a^2*c^2-8*a*b^2*c+b^4)*x^3-3/8*a*(4
*A*a*c+A*b^2-4*B*a*b)/(16*a^2*c^2-8*a*b^2*c+b^4)*x)/(c*x^4+b*x^2+a)^2+3/16*
sum((-(4*A*b*c-4*B*a*c-B*b^2)/(16*a^2*c^2-8*a*b^2*c+b^4)*_R^2+(4*A*a*c+A*b^
2-4*B*a*b)/(16*a^2*c^2-8*a*b^2*c+b^4))/(2*_R^3*c+_R*b)*ln(x-_R),_R=RootOf(_
Z^4*c+_Z^2*b+a))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 5650 vs. $2(336) = 672$.

Time = 4.21 (sec) , antiderivative size = 5650, normalized size of antiderivative = 14.87

$$\int \frac{x^4(A+Bx^2)}{(a+bx^2+cx^4)^3}\,dx = \text{Too large to display}$$

```
[In] integrate(x^4*(B*x^2+A)/(c*x^4+b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^4(A+Bx^2)}{(a+bx^2+cx^4)^3}\,dx = \text{Timed out}$$

```
[In] integrate(x**4*(B*x**2+A)/(c*x**4+b*x**2+a)**3,x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{x^4(A+Bx^2)}{(a+bx^2+cx^4)^3}\,dx = \int \frac{(Bx^2+A)x^4}{(cx^4+bx^2+a)^3}\,dx$$

```
[In] integrate(x^4*(B*x^2+A)/(c*x^4+b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] 1/8*(3*(B*b^2*c + 4*(B*a - A*b)*c^2)*x^7 + (5*B*b^3 + 4*A*a*c^2 + (16*B*a*b
 - 19*A*b^2)*c)*x^5 + (19*B*a*b^2 - 5*A*b^3 - 4*(B*a^2 + 4*A*a*b)*c)*x^3 +
3*(4*B*a^2*b - A*a*b^2 - 4*A*a^2*c)*x)/((b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4
)*x^8 + 2*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*x^6 + a^2*b^4 - 8*a^3*b^2*c
+ 16*a^4*c^2 + (b^6 - 6*a*b^4*c + 32*a^3*c^3)*x^4 + 2*(a*b^5 - 8*a^2*b^3*c
+ 16*a^3*b*c^2)*x^2) - 3/8*integrate((4*B*a*b - A*b^2 - 4*A*a*c - (B*b^2 +
4*(B*a - A*b)*c)*x^2)/(c*x^4 + b*x^2 + a), x)/(b^4 - 8*a*b^2*c + 16*a^2*c^2
)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3164 vs. 2(336) = 672.

Time = 1.74 (sec) , antiderivative size = 3164, normalized size of antiderivative = 8.33

$$\int \frac{x^4(A+Bx^2)}{(a+bx^2+cx^4)^3}\,dx = \text{Too large to display}$$

```
[In] integrate(x^4*(B*x^2+A)/(c*x^4+b*x^2+a)^3,x, algorithm="giac")
[Out] 3/32*((sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^6 - 4*sqrt(2)*sqrt(b*c + s
qrt(b^2 - 4*a*c)*c)*a*b^4*c - 2*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^5
*c - 2*b^6*c - 16*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^2 + sqr
t(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^4*c^2 + 8*a*b^4*c^2 + 2*b^5*c^2 + 64
*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*c^3 + 32*sqrt(2)*sqrt(b*c + sq
rt(b^2 - 4*a*c)*c)*a^2*b*c^3 + 32*a^2*b^2*c^3 + 16*a*b^3*c^3 - 16*sqrt(2)*s
qrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*c^4 - 128*a^3*c^4 - 96*a^2*b*c^4 - sqrt(
2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^5 - 8*sqrt(2)*sqrt(b
^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c + 2*sqrt(2)*sqrt(b^2 -
4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^4*c + 48*sqrt(2)*sqrt(b^2 - 4*a*c)
*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b*c^2 + 24*sqrt(2)*sqrt(b^2 - 4*a*c)*s
qrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b
*c + sqrt(b^2 - 4*a*c)*c)*b^3*c^2 - 12*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c +
 sqrt(b^2 - 4*a*c)*c)*a*b*c^3 + 2*(b^2 - 4*a*c)*b^4*c - 2*(b^2 - 4*a*c)*b^3
*c^2 - 32*(b^2 - 4*a*c)*a^2*c^3 - 24*(b^2 - 4*a*c)*a*b*c^3)*A - 2*(2*sqrt(2
)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^5 - 16*sqrt(2)*sqrt(b*c + sqrt(b^2 -
4*a*c)*c)*a^2*b^3*c - 4*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^4*c - 4
*a*b^5*c + 32*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b*c^2 + 16*sqrt(2
)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^2 + 2*sqrt(2)*sqrt(b*c + sqrt(b
^2 - 4*a*c)*c)*a*b^3*c^2 + 32*a^2*b^3*c^2 + 6*a*b^4*c^2 - 8*sqrt(2)*sqrt(b*
c + sqrt(b^2 - 4*a*c)*c)*a^2*b*c^3 - 64*a^3*b*c^3 - 16*a^2*b^2*c^3 - 32*a^3
*c^4 - 3*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^4 +
8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c + 6*s
qrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c + 16*sqrt(
2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*c^2 + 8*sqrt(2)*sq
rt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b*c^2 - 3*sqrt(2)*sqrt(
b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^2 - 4*sqrt(2)*sqrt(b^2
 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*c^3 + 4*(b^2 - 4*a*c)*a*b^3*c
 - 16*(b^2 - 4*a*c)*a^2*b*c^2 - 6*(b^2 - 4*a*c)*a*b^2*c^2 - 8*(b^2 - 4*a*c)
*a^2*c^3)*B)*arctan(2*sqrt(1/2)*x/sqrt((b^5 - 8*a*b^3*c + 16*a^2*b*c^2 + sq
rt((b^5 - 8*a*b^3*c + 16*a^2*b*c^2)^2 - 4*(a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2
)*(b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)))/(b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)))
/((a*b^8 - 16*a^2*b^6*c - 2*a*b^7*c + 96*a^3*b^4*c^2 + 24*a^2*b^5*c^2 + a*b
^6*c^2 - 256*a^4*b^2*c^3 - 96*a^3*b^3*c^3 - 12*a^2*b^4*c^3 + 256*a^5*c^4 +
128*a^4*b*c^4 + 48*a^3*b^2*c^4 - 64*a^4*c^5)*abs(c)) + 3/32*((sqrt(2)*sqrt(
```

```
b*c - sqrt(b^2 - 4*a*c)*c)*b^6 - 4*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*
a*b^4*c - 2*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^5*c + 2*b^6*c - 16*sq
rt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^2 + sqrt(2)*sqrt(b*c - sqrt
(b^2 - 4*a*c)*c)*b^4*c^2 - 8*a*b^4*c^2 + 2*b^5*c^2 + 64*sqrt(2)*sqrt(b*c -
sqrt(b^2 - 4*a*c)*c)*a^3*c^3 + 32*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a
^2*b*c^3 - 32*a^2*b^2*c^3 + 16*a*b^3*c^3 - 16*sqrt(2)*sqrt(b*c - sqrt(b^2 -
 4*a*c)*c)*a^2*c^4 + 128*a^3*c^4 - 96*a^2*b*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)
*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^5 - 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c
 - sqrt(b^2 - 4*a*c)*c)*a*b^3*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sq
rt(b^2 - 4*a*c)*c)*b^4*c + 48*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2
 - 4*a*c)*c)*a^2*b*c^2 + 24*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 -
 4*a*c)*c)*a*b^2*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*
c)*c)*b^3*c^2 - 12*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c
)*a*b*c^3 - 2*(b^2 - 4*a*c)*b^4*c - 2*(b^2 - 4*a*c)*b^3*c^2 + 32*(b^2 - 4*a
*c)*a^2*c^3 - 24*(b^2 - 4*a*c)*a*b*c^3)*A - 2*(2*sqrt(2)*sqrt(b*c - sqrt(b^
2 - 4*a*c)*c)*a*b^5 - 16*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c
 - 4*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^4*c + 4*a*b^5*c + 32*sqrt(2
)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b*c^2 + 16*sqrt(2)*sqrt(b*c - sqrt(b^
2 - 4*a*c)*c)*a^2*b^2*c^2 + 2*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^3
*c^2 - 32*a^2*b^3*c^2 + 6*a*b^4*c^2 - 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c
)*c)*a^2*b*c^3 + 64*a^3*b*c^3 - 16*a^2*b^2*c^3 - 32*a^3*c^4 - 3*sqrt(2)*sqr
t(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^4 + 8*sqrt(2)*sqrt(b^2 -
 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c + 6*sqrt(2)*sqrt(b^2 - 4*
a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^3*c + 16*sqrt(2)*sqrt(b^2 - 4*a*c)
*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*c^2 + 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt
(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b*c^2 - 3*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*
c - sqrt(b^2 - 4*a*c)*c)*a*b^2*c^2 - 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c -
 sqrt(b^2 - 4*a*c)*c)*a^2*c^3 - 4*(b^2 - 4*a*c)*a*b^3*c + 16*(b^2 - 4*a*c)*
a^2*b*c^2 - 6*(b^2 - 4*a*c)*a*b^2*c^2 - 8*(b^2 - 4*a*c)*a^2*c^3)*B)*arctan(
2*sqrt(1/2)*x/sqrt((b^5 - 8*a*b^3*c + 16*a^2*b*c^2 - sqrt((b^5 - 8*a*b^3*c
+ 16*a^2*b*c^2)^2 - 4*(a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2)*(b^4*c - 8*a*b^2*c
^2 + 16*a^2*c^3)))/(b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)))/((a*b^8 - 16*a^2*b^
6*c - 2*a*b^7*c + 96*a^3*b^4*c^2 + 24*a^2*b^5*c^2 + a*b^6*c^2 - 256*a^4*b^2
*c^3 - 96*a^3*b^3*c^3 - 12*a^2*b^4*c^3 + 256*a^5*c^4 + 128*a^4*b*c^4 + 48*a
^3*b^2*c^4 - 64*a^4*c^5)*abs(c)) + 1/8*(3*B*b^2*c*x^7 + 12*B*a*c^2*x^7 - 12
*A*b*c^2*x^7 + 5*B*b^3*x^5 + 16*B*a*b*c*x^5 - 19*A*b^2*c*x^5 + 4*A*a*c^2*x^
5 + 19*B*a*b^2*x^3 - 5*A*b^3*x^3 - 4*B*a^2*c*x^3 - 16*A*a*b*c*x^3 + 12*B*a^
2*b*x - 3*A*a*b^2*x - 12*A*a^2*c*x)/((c*x^4 + b*x^2 + a)^2*(b^4 - 8*a*b^2*c
 + 16*a^2*c^2))
```

Mupad [B] (verification not implemented)

Time = 9.43 (sec) , antiderivative size = 16688, normalized size of antiderivative = 43.92

$$\int \frac{x^4(A+Bx^2)}{(a+bx^2+cx^4)^3}\,dx = \text{Too large to display}$$

[In] int((x^4*(A + B*x^2))/(a + b*x^2 + c*x^4)^3,x)

[Out] atan(((((3*(1048576*A*a^6*c^8 - 256*A*b^12*c^2 + 4096*A*a*b^10*c^3 + 1024*B
*a*b^11*c^2 - 1048576*B*a^6*b*c^7 - 20480*A*a^2*b^8*c^4 + 327680*A*a^4*b^4*
c^6 - 1048576*A*a^5*b^2*c^7 - 20480*B*a^2*b^9*c^3 + 163840*B*a^3*b^7*c^4 -
655360*B*a^4*b^5*c^5 + 1310720*B*a^5*b^3*c^6))/(512*(b^12 + 4096*a^6*c^6 +
240*a^2*b^8*c^2 - 1280*a^3*b^6*c^3 + 3840*a^4*b^4*c^4 - 6144*a^5*b^2*c^5 -
24*a*b^10*c)) - (x*(-(9*(B^2*a*b^15 + B^2*a*(-(4*a*c - b^2)^15)^(1/2) + A^2
*b^15*c - A^2*c*(-(4*a*c - b^2)^15)^(1/2) - 560*A^2*a^2*b^11*c^3 + 4160*A^2
*a^3*b^9*c^4 - 11520*A^2*a^4*b^7*c^5 - 1024*A^2*a^5*b^5*c^6 + 61440*A^2*a^6
*b^3*c^7 - 560*B^2*a^3*b^11*c^2 + 4160*B^2*a^4*b^9*c^3 - 11520*B^2*a^5*b^7*
c^4 - 1024*B^2*a^6*b^5*c^5 + 61440*B^2*a^7*b^3*c^6 + 65536*A*B*a^8*c^8 + 20
*A^2*a*b^13*c^2 - 81920*A^2*a^7*b*c^8 + 20*B^2*a^2*b^13*c - 81920*B^2*a^8*b
*c^7 + 240*A*B*a^2*b^12*c^2 - 64*A*B*a^3*b^10*c^3 - 11520*A*B*a^4*b^8*c^4 +
66560*A*B*a^5*b^6*c^5 - 143360*A*B*a^6*b^4*c^6 + 81920*A*B*a^7*b^2*c^7 - 2
0*A*B*a*b^14*c))/(512*(1048576*a^11*c^11 - 40*a^2*b^18*c^2 + 720*a^3*b^16*c
^3 - 7680*a^4*b^14*c^4 + 53760*a^5*b^12*c^5 - 258048*a^6*b^10*c^6 + 860160*
a^7*b^8*c^7 - 1966080*a^8*b^6*c^8 + 2949120*a^9*b^4*c^9 - 2621440*a^10*b^2*
c^10 + a*b^20*c)))^(1/2)*(256*b^11*c^2 - 5120*a*b^9*c^3 - 262144*a^5*b*c^7
+ 40960*a^2*b^7*c^4 - 163840*a^3*b^5*c^5 + 327680*a^4*b^3*c^6))/(32*(b^8 +
256*a^4*c^4 + 96*a^2*b^4*c^2 - 256*a^3*b^2*c^3 - 16*a*b^6*c)))*(-(9*(B^2*a*
b^15 + B^2*a*(-(4*a*c - b^2)^15)^(1/2) + A^2*b^15*c - A^2*c*(-(4*a*c - b^2)
^15)^(1/2) - 560*A^2*a^2*b^11*c^3 + 4160*A^2*a^3*b^9*c^4 - 11520*A^2*a^4*b^
7*c^5 - 1024*A^2*a^5*b^5*c^6 + 61440*A^2*a^6*b^3*c^7 - 560*B^2*a^3*b^11*c^2
+ 4160*B^2*a^4*b^9*c^3 - 11520*B^2*a^5*b^7*c^4 - 1024*B^2*a^6*b^5*c^5 + 61
440*B^2*a^7*b^3*c^6 + 65536*A*B*a^8*c^8 + 20*A^2*a*b^13*c^2 - 81920*A^2*a^7
*b*c^8 + 20*B^2*a^2*b^13*c - 81920*B^2*a^8*b*c^7 + 240*A*B*a^2*b^12*c^2 - 6
4*A*B*a^3*b^10*c^3 - 11520*A*B*a^4*b^8*c^4 + 66560*A*B*a^5*b^6*c^5 - 143360
*A*B*a^6*b^4*c^6 + 81920*A*B*a^7*b^2*c^7 - 20*A*B*a*b^14*c))/(512*(1048576*
a^11*c^11 - 40*a^2*b^18*c^2 + 720*a^3*b^16*c^3 - 7680*a^4*b^14*c^4 + 53760*
a^5*b^12*c^5 - 258048*a^6*b^10*c^6 + 860160*a^7*b^8*c^7 - 1966080*a^8*b^6*c
^8 + 2949120*a^9*b^4*c^9 - 2621440*a^10*b^2*c^10 + a*b^20*c)))^(1/2) - (x*(
9*B^2*b^6*c + 288*A^2*a^2*c^5 + 234*A^2*b^4*c^3 - 288*B^2*a^3*c^4 + 576*B^2
*a^2*b^2*c^3 - 90*A*B*b^5*c^2 + 144*A^2*a*b^2*c^4 + 126*B^2*a*b^4*c^2 - 720
*A*B*a*b^3*c^3 - 288*A*B*a^2*b*c^4))/(32*(b^8 + 256*a^4*c^4 + 96*a^2*b^4*c^
2 - 256*a^3*b^2*c^3 - 16*a*b^6*c)))*(-(9*(B^2*a*b^15 + B^2*a*(-(4*a*c - b^2
)^15)^(1/2) + A^2*b^15*c - A^2*c*(-(4*a*c - b^2)^15)^(1/2) - 560*A^2*a^2*b^
11*c^3 + 4160*A^2*a^3*b^9*c^4 - 11520*A^2*a^4*b^7*c^5 - 1024*A^2*a^5*b^5*c^

6 + 61440*A^2*a^6*b^3*c^7 - 560*B^2*a^3*b^11*c^2 + 4160*B^2*a^4*b^9*c^3 - 1
1520*B^2*a^5*b^7*c^4 - 1024*B^2*a^6*b^5*c^5 + 61440*B^2*a^7*b^3*c^6 + 65536
*A*B*a^8*c^8 + 20*A^2*a*b^13*c^2 - 81920*A^2*a^7*b*c^8 + 20*B^2*a^2*b^13*c
- 81920*B^2*a^8*b*c^7 + 240*A*B*a^2*b^12*c^2 - 64*A*B*a^3*b^10*c^3 - 11520*
A*B*a^4*b^8*c^4 + 66560*A*B*a^5*b^6*c^5 - 143360*A*B*a^6*b^4*c^6 + 81920*A*
B*a^7*b^2*c^7 - 20*A*B*a*b^14*c))/(512*(1048576*a^11*c^11 - 40*a^2*b^18*c^2
 + 720*a^3*b^16*c^3 - 7680*a^4*b^14*c^4 + 53760*a^5*b^12*c^5 - 258048*a^6*b
^10*c^6 + 860160*a^7*b^8*c^7 - 1966080*a^8*b^6*c^8 + 2949120*a^9*b^4*c^9 -
2621440*a^10*b^2*c^10 + a*b^20*c)))^(1/2)*1i - (((3*(1048576*A*a^6*c^8 - 25
6*A*b^12*c^2 + 4096*A*a*b^10*c^3 + 1024*B*a*b^11*c^2 - 1048576*B*a^6*b*c^7
- 20480*A*a^2*b^8*c^4 + 327680*A*a^4*b^4*c^6 - 1048576*A*a^5*b^2*c^7 - 2048
0*B*a^2*b^9*c^3 + 163840*B*a^3*b^7*c^4 - 655360*B*a^4*b^5*c^5 + 1310720*B*a
^5*b^3*c^6))/(512*(b^12 + 4096*a^6*c^6 + 240*a^2*b^8*c^2 - 1280*a^3*b^6*c^3
 + 3840*a^4*b^4*c^4 - 6144*a^5*b^2*c^5 - 24*a*b^10*c)) + (x*(-(9*(B^2*a*b^1
5 + B^2*a*(-(4*a*c - b^2)^15)^(1/2) + A^2*b^15*c - A^2*c*(-(4*a*c - b^2)^15
)^(1/2) - 560*A^2*a^2*b^11*c^3 + 4160*A^2*a^3*b^9*c^4 - 11520*A^2*a^4*b^7*c
^5 - 1024*A^2*a^5*b^5*c^6 + 61440*A^2*a^6*b^3*c^7 - 560*B^2*a^3*b^11*c^2 +
4160*B^2*a^4*b^9*c^3 - 11520*B^2*a^5*b^7*c^4 - 1024*B^2*a^6*b^5*c^5 + 61440
*B^2*a^7*b^3*c^6 + 65536*A*B*a^8*c^8 + 20*A^2*a*b^13*c^2 - 81920*A^2*a^7*b*
c^8 + 20*B^2*a^2*b^13*c - 81920*B^2*a^8*b*c^7 + 240*A*B*a^2*b^12*c^2 - 64*A
*B*a^3*b^10*c^3 - 11520*A*B*a^4*b^8*c^4 + 66560*A*B*a^5*b^6*c^5 - 143360*A*
B*a^6*b^4*c^6 + 81920*A*B*a^7*b^2*c^7 - 20*A*B*a*b^14*c))/(512*(1048576*a^1
1*c^11 - 40*a^2*b^18*c^2 + 720*a^3*b^16*c^3 - 7680*a^4*b^14*c^4 + 53760*a^5
*b^12*c^5 - 258048*a^6*b^10*c^6 + 860160*a^7*b^8*c^7 - 1966080*a^8*b^6*c^8
+ 2949120*a^9*b^4*c^9 - 2621440*a^10*b^2*c^10 + a*b^20*c)))^(1/2)*(256*b^11
*c^2 - 5120*a*b^9*c^3 - 262144*a^5*b*c^7 + 40960*a^2*b^7*c^4 - 163840*a^3*b
^5*c^5 + 327680*a^4*b^3*c^6))/(32*(b^8 + 256*a^4*c^4 + 96*a^2*b^4*c^2 - 256
*a^3*b^2*c^3 - 16*a*b^6*c)))*(-(9*(B^2*a*b^15 + B^2*a*(-(4*a*c - b^2)^15)^(
1/2) + A^2*b^15*c - A^2*c*(-(4*a*c - b^2)^15)^(1/2) - 560*A^2*a^2*b^11*c^3
+ 4160*A^2*a^3*b^9*c^4 - 11520*A^2*a^4*b^7*c^5 - 1024*A^2*a^5*b^5*c^6 + 614
40*A^2*a^6*b^3*c^7 - 560*B^2*a^3*b^11*c^2 + 4160*B^2*a^4*b^9*c^3 - 11520*B^
2*a^5*b^7*c^4 - 1024*B^2*a^6*b^5*c^5 + 61440*B^2*a^7*b^3*c^6 + 65536*A*B*a^
8*c^8 + 20*A^2*a*b^13*c^2 - 81920*A^2*a^7*b*c^8 + 20*B^2*a^2*b^13*c - 81920
*B^2*a^8*b*c^7 + 240*A*B*a^2*b^12*c^2 - 64*A*B*a^3*b^10*c^3 - 11520*A*B*a^4
*b^8*c^4 + 66560*A*B*a^5*b^6*c^5 - 143360*A*B*a^6*b^4*c^6 + 81920*A*B*a^7*b
^2*c^7 - 20*A*B*a*b^14*c))/(512*(1048576*a^11*c^11 - 40*a^2*b^18*c^2 + 720*
a^3*b^16*c^3 - 7680*a^4*b^14*c^4 + 53760*a^5*b^12*c^5 - 258048*a^6*b^10*c^6
 + 860160*a^7*b^8*c^7 - 1966080*a^8*b^6*c^8 + 2949120*a^9*b^4*c^9 - 2621440
*a^10*b^2*c^10 + a*b^20*c)))^(1/2) + (x*(9*B^2*b^6*c + 288*A^2*a^2*c^5 + 23
4*A^2*b^4*c^3 - 288*B^2*a^3*c^4 + 576*B^2*a^2*b^2*c^3 - 90*A*B*b^5*c^2 + 14
4*A^2*a*b^2*c^4 + 126*B^2*a*b^4*c^2 - 720*A*B*a*b^3*c^3 - 288*A*B*a^2*b*c^4
))/(32*(b^8 + 256*a^4*c^4 + 96*a^2*b^4*c^2 - 256*a^3*b^2*c^3 - 16*a*b^6*c))
)*(-(9*(B^2*a*b^15 + B^2*a*(-(4*a*c - b^2)^15)^(1/2) + A^2*b^15*c - A^2*c*(
-(4*a*c - b^2)^15)^(1/2) - 560*A^2*a^2*b^11*c^3 + 4160*A^2*a^3*b^9*c^4 - 11
520*A^2*a^4*b^7*c^5 - 1024*A^2*a^5*b^5*c^6 + 61440*A^2*a^6*b^3*c^7 - 560*B^

2*a^3*b^11*c^2 + 4160*B^2*a^4*b^9*c^3 - 11520*B^2*a^5*b^7*c^4 - 1024*B^2*a^
6*b^5*c^5 + 61440*B^2*a^7*b^3*c^6 + 65536*A*B*a^8*c^8 + 20*A^2*a*b^13*c^2 -
81920*A^2*a^7*b*c^8 + 20*B^2*a^2*b^13*c - 81920*B^2*a^8*b*c^7 + 240*A*B*a^
2*b^12*c^2 - 64*A*B*a^3*b^10*c^3 - 11520*A*B*a^4*b^8*c^4 + 66560*A*B*a^5*b^
6*c^5 - 143360*A*B*a^6*b^4*c^6 + 81920*A*B*a^7*b^2*c^7 - 20*A*B*a*b^14*c))/
(512*(1048576*a^11*c^11 - 40*a^2*b^18*c^2 + 720*a^3*b^16*c^3 - 7680*a^4*b^1
4*c^4 + 53760*a^5*b^12*c^5 - 258048*a^6*b^10*c^6 + 860160*a^7*b^8*c^7 - 196
6080*a^8*b^6*c^8 + 2949120*a^9*b^4*c^9 - 2621440*a^10*b^2*c^10 + a*b^20*c))
)^(1/2)*1i)/((((3*(1048576*A*a^6*c^8 - 256*A*b^12*c^2 + 4096*A*a*b^10*c^3 +
1024*B*a*b^11*c^2 - 1048576*B*a^6*b*c^7 - 20480*A*a^2*b^8*c^4 + 327680*A*a
^4*b^4*c^6 - 1048576*A*a^5*b^2*c^7 - 20480*B*a^2*b^9*c^3 + 163840*B*a^3*b^7
*c^4 - 655360*B*a^4*b^5*c^5 + 1310720*B*a^5*b^3*c^6))/(512*(b^12 + 4096*a^6
*c^6 + 240*a^2*b^8*c^2 - 1280*a^3*b^6*c^3 + 3840*a^4*b^4*c^4 - 6144*a^5*b^2
*c^5 - 24*a*b^10*c)) - (x*(-(9*(B^2*a*b^15 + B^2*a*(-(4*a*c - b^2)^15)^(1/2
) + A^2*b^15*c - A^2*c*(-(4*a*c - b^2)^15)^(1/2) - 560*A^2*a^2*b^11*c^3 + 4
160*A^2*a^3*b^9*c^4 - 11520*A^2*a^4*b^7*c^5 - 1024*A^2*a^5*b^5*c^6 + 61440*
A^2*a^6*b^3*c^7 - 560*B^2*a^3*b^11*c^2 + 4160*B^2*a^4*b^9*c^3 - 11520*B^2*a
^5*b^7*c^4 - 1024*B^2*a^6*b^5*c^5 + 61440*B^2*a^7*b^3*c^6 + 65536*A*B*a^8*c
^8 + 20*A^2*a*b^13*c^2 - 81920*A^2*a^7*b*c^8 + 20*B^2*a^2*b^13*c - 81920*B^
2*a^8*b*c^7 + 240*A*B*a^2*b^12*c^2 - 64*A*B*a^3*b^10*c^3 - 11520*A*B*a^4*b^
8*c^4 + 66560*A*B*a^5*b^6*c^5 - 143360*A*B*a^6*b^4*c^6 + 81920*A*B*a^7*b^2*
c^7 - 20*A*B*a*b^14*c))/(512*(1048576*a^11*c^11 - 40*a^2*b^18*c^2 + 720*a^3
*b^16*c^3 - 7680*a^4*b^14*c^4 + 53760*a^5*b^12*c^5 - 258048*a^6*b^10*c^6 +
860160*a^7*b^8*c^7 - 1966080*a^8*b^6*c^8 + 2949120*a^9*b^4*c^9 - 2621440*a^
10*b^2*c^10 + a*b^20*c)))^(1/2)*(256*b^11*c^2 - 5120*a*b^9*c^3 - 262144*a^5
*b*c^7 + 40960*a^2*b^7*c^4 - 163840*a^3*b^5*c^5 + 327680*a^4*b^3*c^6))/(32*
(b^8 + 256*a^4*c^4 + 96*a^2*b^4*c^2 - 256*a^3*b^2*c^3 - 16*a*b^6*c)))*(-(9*
(B^2*a*b^15 + B^2*a*(-(4*a*c - b^2)^15)^(1/2) + A^2*b^15*c - A^2*c*(-(4*a*c
- b^2)^15)^(1/2) - 560*A^2*a^2*b^11*c^3 + 4160*A^2*a^3*b^9*c^4 - 11520*A^2
*a^4*b^7*c^5 - 1024*A^2*a^5*b^5*c^6 + 61440*A^2*a^6*b^3*c^7 - 560*B^2*a^3*b
^11*c^2 + 4160*B^2*a^4*b^9*c^3 - 11520*B^2*a^5*b^7*c^4 - 1024*B^2*a^6*b^5*c
^5 + 61440*B^2*a^7*b^3*c^6 + 65536*A*B*a^8*c^8 + 20*A^2*a*b^13*c^2 - 81920*
A^2*a^7*b*c^8 + 20*B^2*a^2*b^13*c - 81920*B^2*a^8*b*c^7 + 240*A*B*a^2*b^12*
c^2 - 64*A*B*a^3*b^10*c^3 - 11520*A*B*a^4*b^8*c^4 + 66560*A*B*a^5*b^6*c^5 -
143360*A*B*a^6*b^4*c^6 + 81920*A*B*a^7*b^2*c^7 - 20*A*B*a*b^14*c))/(512*(1
048576*a^11*c^11 - 40*a^2*b^18*c^2 + 720*a^3*b^16*c^3 - 7680*a^4*b^14*c^4 +
53760*a^5*b^12*c^5 - 258048*a^6*b^10*c^6 + 860160*a^7*b^8*c^7 - 1966080*a^
8*b^6*c^8 + 2949120*a^9*b^4*c^9 - 2621440*a^10*b^2*c^10 + a*b^20*c)))^(1/2)
- (x*(9*B^2*b^6*c + 288*A^2*a^2*c^5 + 234*A^2*b^4*c^3 - 288*B^2*a^3*c^4 +
576*B^2*a^2*b^2*c^3 - 90*A*B*b^5*c^2 + 144*A^2*a*b^2*c^4 + 126*B^2*a*b^4*c^
2 - 720*A*B*a*b^3*c^3 - 288*A*B*a^2*b*c^4))/(32*(b^8 + 256*a^4*c^4 + 96*a^2
*b^4*c^2 - 256*a^3*b^2*c^3 - 16*a*b^6*c)))*(-(9*(B^2*a*b^15 + B^2*a*(-(4*a*
c - b^2)^15)^(1/2) + A^2*b^15*c - A^2*c*(-(4*a*c - b^2)^15)^(1/2) - 560*A^2
*a^2*b^11*c^3 + 4160*A^2*a^3*b^9*c^4 - 11520*A^2*a^4*b^7*c^5 - 1024*A^2*a^5
*b^5*c^6 + 61440*A^2*a^6*b^3*c^7 - 560*B^2*a^3*b^11*c^2 + 4160*B^2*a^4*b^9*

c^3 - 11520*B^2*a^5*b^7*c^4 - 1024*B^2*a^6*b^5*c^5 + 61440*B^2*a^7*b^3*c^6 + 65536*A*B*a^8*c^8 + 20*A^2*a*b^13*c^2 - 81920*A^2*a^7*b*c^8 + 20*B^2*a^2*b^13*c - 81920*B^2*a^8*b*c^7 + 240*A*B*a^2*b^12*c^2 - 64*A*B*a^3*b^10*c^3 - 11520*A*B*a^4*b^8*c^4 + 66560*A*B*a^5*b^6*c^5 - 143360*A*B*a^6*b^4*c^6 + 81920*A*B*a^7*b^2*c^7 - 20*A*B*a*b^14*c))/(512*(1048576*a^11*c^11 - 40*a^2*b^18*c^2 + 720*a^3*b^16*c^3 - 7680*a^4*b^14*c^4 + 53760*a^5*b^12*c^5 - 258048*a^6*b^10*c^6 + 860160*a^7*b^8*c^7 - 1966080*a^8*b^6*c^8 + 2949120*a^9*b^4*c^9 - 2621440*a^10*b^2*c^10 + a*b^20*c)))^(1/2) + (((3*(1048576*A*a^6*c^8 - 256*A*b^12*c^2 + 4096*A*a*b^10*c^3 + 1024*B*a*b^11*c^2 - 1048576*B*a^6*b*c^7 - 20480*A*a^2*b^8*c^4 + 327680*A*a^4*b^4*c^6 - 1048576*A*a^5*b^2*c^7 - 20480*B*a^2*b^9*c^3 + 163840*B*a^3*b^7*c^4 - 655360*B*a^4*b^5*c^5 + 1310720*B*a^5*b^3*c^6))/(512*(b^12 + 4096*a^6*c^6 + 240*a^2*b^8*c^2 - 1280*a^3*b^6*c^3 + 3840*a^4*b^4*c^4 - 6144*a^5*b^2*c^5 - 24*a*b^10*c)) + (x*(-(9*(B^2*a*b^15 + B^2*a*(-(4*a*c - b^2)^15)^(1/2) + A^2*b^15*c - A^2*c*(-(4*a*c - b^2)^15)^(1/2) - 560*A^2*a^2*b^11*c^3 + 4160*A^2*a^3*b^9*c^4 - 11520*A^2*a^4*b^7*c^5 - 1024*A^2*a^5*b^5*c^6 + 61440*A^2*a^6*b^3*c^7 - 560*B^2*a^3*b^11*c^2 + 4160*B^2*a^4*b^9*c^3 - 11520*B^2*a^5*b^7*c^4 - 1024*B^2*a^6*b^5*c^5 + 61440*B^2*a^7*b^3*c^6 + 65536*A*B*a^8*c^8 + 20*A^2*a*b^13*c^2 - 81920*A^2*a^7*b*c^8 + 20*B^2*a^2*b^13*c - 81920*B^2*a^8*b*c^7 + 240*A*B*a^2*b^12*c^2 - 64*A*B*a^3*b^10*c^3 - 11520*A*B*a^4*b^8*c^4 + 66560*A*B*a^5*b^6*c^5 - 143360*A*B*a^6*b^4*c^6 + 81920*A*B*a^7*b^2*c^7 - 20*A*B*a*b^14*c))/(512*(1048576*a^11*c^11 - 40*a^2*b^18*c^2 + 720*a^3*b^16*c^3 - 7680*a^4*b^14*c^4 + 53760*a^5*b^12*c^5 - 258048*a^6*b^10*c^6 + 860160*a^7*b^8*c^7 - 1966080*a^8*b^6*c^8 + 2949120*a^9*b^4*c^9 - 2621440*a^10*b^2*c^10 + a*b^20*c)))^(1/2)*(256*b^11*c^2 - 5120*a*b^9*c^3 - 262144*a^5*b*c^7 + 40960*a^2*b^7*c^4 - 163840*a^3*b^5*c^5 + 327680*a^4*b^3*c^6))/(32*(b^8 + 256*a^4*c^4 + 96*a^2*b^4*c^2 - 256*a^3*b^2*c^3 - 16*a*b^6*c)))*(-(9*(B^2*a*b^15 + B^2*a*(-(4*a*c - b^2)^15)^(1/2) + A^2*b^15*c - A^2*c*(-(4*a*c - b^2)^15)^(1/2) - 560*A^2*a^2*b^11*c^3 + 4160*A^2*a^3*b^9*c^4 - 11520*A^2*a^4*b^7*c^5 - 1024*A^2*a^5*b^5*c^6 + 61440*A^2*a^6*b^3*c^7 - 560*B^2*a^3*b^11*c^2 + 4160*B^2*a^4*b^9*c^3 - 11520*B^2*a^5*b^7*c^4 - 1024*B^2*a^6*b^5*c^5 + 61440*B^2*a^7*b^3*c^6 + 65536*A*B*a^8*c^8 + 20*A^2*a*b^13*c^2 - 81920*A^2*a^7*b*c^8 + 20*B^2*a^2*b^13*c - 81920*B^2*a^8*b*c^7 + 240*A*B*a^2*b^12*c^2 - 64*A*B*a^3*b^10*c^3 - 11520*A*B*a^4*b^8*c^4 + 66560*A*B*a^5*b^6*c^5 - 143360*A*B*a^6*b^4*c^6 + 81920*A*B*a^7*b^2*c^7 - 20*A*B*a*b^14*c))/(512*(1048576*a^11*c^11 - 40*a^2*b^18*c^2 + 720*a^3*b^16*c^3 - 7680*a^4*b^14*c^4 + 53760*a^5*b^12*c^5 - 258048*a^6*b^10*c^6 + 860160*a^7*b^8*c^7 - 1966080*a^8*b^6*c^8 + 2949120*a^9*b^4*c^9 - 2621440*a^10*b^2*c^10 + a*b^20*c)))^(1/2) + (x*(9*B^2*b^6*c + 288*A^2*a^2*c^5 + 234*A^2*b^4*c^3 - 288*B^2*a^3*c^4 + 576*B^2*a^2*b^2*c^3 - 90*A*B*b^5*c^2 + 144*A^2*a*b^2*c^4 + 126*B^2*a*b^4*c^2 - 720*A*B*a*b^3*c^3 - 288*A*B*a^2*b*c^4))/(32*(b^8 + 256*a^4*c^4 + 96*a^2*b^4*c^2 - 256*a^3*b^2*c^3 - 16*a*b^6*c)))*(-(9*(B^2*a*b^15 + B^2*a*(-(4*a*c - b^2)^15)^(1/2) + A^2*b^15*c - A^2*c*(-(4*a*c - b^2)^15)^(1/2) - 560*A^2*a^2*b^11*c^3 + 4160*A^2*a^3*b^9*c^4 - 11520*A^2*a^4*b^7*c^5 - 1024*A^2*a^5*b^5*c^6 + 61440*A^2*a^6*b^3*c^7 - 560*B^2*a^3*b^11*c^2 + 4160*B^2*a^4*b^9*c^3 - 11520*B^2*a^5*b^7*c^4 - 1024*B^

2*a^6*b^5*c^5 + 61440*B^2*a^7*b^3*c^6 + 65536*A*B*a^8*c^8 + 20*A^2*a*b^13*c
^2 - 81920*A^2*a^7*b*c^8 + 20*B^2*a^2*b^13*c - 81920*B^2*a^8*b*c^7 + 240*A*
B*a^2*b^12*c^2 - 64*A*B*a^3*b^10*c^3 - 11520*A*B*a^4*b^8*c^4 + 66560*A*B*a^
5*b^6*c^5 - 143360*A*B*a^6*b^4*c^6 + 81920*A*B*a^7*b^2*c^7 - 20*A*B*a*b^14*
c))/(512*(1048576*a^11*c^11 - 40*a^2*b^18*c^2 + 720*a^3*b^16*c^3 - 7680*a^4
*b^14*c^4 + 53760*a^5*b^12*c^5 - 258048*a^6*b^10*c^6 + 860160*a^7*b^8*c^7 -
 1966080*a^8*b^6*c^8 + 2949120*a^9*b^4*c^9 - 2621440*a^10*b^2*c^10 + a*b^20
c)))^(1/2) + (3(576*B^3*a^4*c^4 - 180*A^3*b^5*c^3 + 540*B^3*a^2*b^4*c^2 +
 1584*B^3*a^3*b^2*c^3 - 9*A*B^2*b^7*c + 45*B^3*a*b^6*c + 576*A^2*B*a^3*c^5
+ 81*A^2*B*b^6*c^2 - 1440*A^3*a*b^3*c^4 - 576*A^3*a^2*b*c^5 - 576*A*B^2*a*b
^5*c^2 - 3456*A*B^2*a^3*b*c^4 + 1980*A^2*B*a*b^4*c^3 - 3600*A*B^2*a^2*b^3*c
^3 + 4464*A^2*B*a^2*b^2*c^4))/(256*(b^12 + 4096*a^6*c^6 + 240*a^2*b^8*c^2 -
 1280*a^3*b^6*c^3 + 3840*a^4*b^4*c^4 - 6144*a^5*b^2*c^5 - 24*a*b^10*c))))*(
-(9*(B^2*a*b^15 + B^2*a*(-(4*a*c - b^2)^15)^(1/2) + A^2*b^15*c - A^2*c*(-(4
*a*c - b^2)^15)^(1/2) - 560*A^2*a^2*b^11*c^3 + 4160*A^2*a^3*b^9*c^4 - 11520
*A^2*a^4*b^7*c^5 - 1024*A^2*a^5*b^5*c^6 + 61440*A^2*a^6*b^3*c^7 - 560*B^2*a
^3*b^11*c^2 + 4160*B^2*a^4*b^9*c^3 - 11520*B^2*a^5*b^7*c^4 - 1024*B^2*a^6*b
^5*c^5 + 61440*B^2*a^7*b^3*c^6 + 65536*A*B*a^8*c^8 + 20*A^2*a*b^13*c^2 - 81
920*A^2*a^7*b*c^8 + 20*B^2*a^2*b^13*c - 81920*B^2*a^8*b*c^7 + 240*A*B*a^2*b
^12*c^2 - 64*A*B*a^3*b^10*c^3 - 11520*A*B*a^4*b^8*c^4 + 66560*A*B*a^5*b^6*c
^5 - 143360*A*B*a^6*b^4*c^6 + 81920*A*B*a^7*b^2*c^7 - 20*A*B*a*b^14*c))/(51
2*(1048576*a^11*c^11 - 40*a^2*b^18*c^2 + 720*a^3*b^16*c^3 - 7680*a^4*b^14*c
^4 + 53760*a^5*b^12*c^5 - 258048*a^6*b^10*c^6 + 860160*a^7*b^8*c^7 - 196608
0*a^8*b^6*c^8 + 2949120*a^9*b^4*c^9 - 2621440*a^10*b^2*c^10 + a*b^20*c)))^(
1/2)*2i - ((x^3*(5*A*b^3 - 19*B*a*b^2 + 4*B*a^2*c + 16*A*a*b*c))/(8*(b^4 +
16*a^2*c^2 - 8*a*b^2*c)) - (x^5*(5*B*b^3 + 4*A*a*c^2 - 19*A*b^2*c + 16*B*a*
b*c))/(8*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (3*a*x*(A*b^2 + 4*A*a*c - 4*B*a*
b))/(8*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) - (3*c*x^7*(B*b^2 - 4*A*b*c + 4*B*a*
c))/(8*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)))/(x^4*(2*a*c + b^2) + a^2 + c^2*x^8
+ 2*a*b*x^2 + 2*b*c*x^6) + atan(((((3*(1048576*A*a^6*c^8 - 256*A*b^12*c^2 +
 4096*A*a*b^10*c^3 + 1024*B*a*b^11*c^2 - 1048576*B*a^6*b*c^7 - 20480*A*a^2*
b^8*c^4 + 327680*A*a^4*b^4*c^6 - 1048576*A*a^5*b^2*c^7 - 20480*B*a^2*b^9*c^
3 + 163840*B*a^3*b^7*c^4 - 655360*B*a^4*b^5*c^5 + 1310720*B*a^5*b^3*c^6))/(
512*(b^12 + 4096*a^6*c^6 + 240*a^2*b^8*c^2 - 1280*a^3*b^6*c^3 + 3840*a^4*b^
4*c^4 - 6144*a^5*b^2*c^5 - 24*a*b^10*c)) - (x*(-(9*(B^2*a*b^15 - B^2*a*(-(4
*a*c - b^2)^15)^(1/2) + A^2*b^15*c + A^2*c*(-(4*a*c - b^2)^15)^(1/2) - 560*
A^2*a^2*b^11*c^3 + 4160*A^2*a^3*b^9*c^4 - 11520*A^2*a^4*b^7*c^5 - 1024*A^2*
a^5*b^5*c^6 + 61440*A^2*a^6*b^3*c^7 - 560*B^2*a^3*b^11*c^2 + 4160*B^2*a^4*b
^9*c^3 - 11520*B^2*a^5*b^7*c^4 - 1024*B^2*a^6*b^5*c^5 + 61440*B^2*a^7*b^3*c
^6 + 65536*A*B*a^8*c^8 + 20*A^2*a*b^13*c^2 - 81920*A^2*a^7*b*c^8 + 20*B^2*a
^2*b^13*c - 81920*B^2*a^8*b*c^7 + 240*A*B*a^2*b^12*c^2 - 64*A*B*a^3*b^10*c^
3 - 11520*A*B*a^4*b^8*c^4 + 66560*A*B*a^5*b^6*c^5 - 143360*A*B*a^6*b^4*c^6
+ 81920*A*B*a^7*b^2*c^7 - 20*A*B*a*b^14*c))/(512*(1048576*a^11*c^11 - 40*a^
2*b^18*c^2 + 720*a^3*b^16*c^3 - 7680*a^4*b^14*c^4 + 53760*a^5*b^12*c^5 - 25
8048*a^6*b^10*c^6 + 860160*a^7*b^8*c^7 - 1966080*a^8*b^6*c^8 + 2949120*a^9*

b^4*c^9 - 2621440*a^10*b^2*c^10 + a*b^20*c)))^(1/2)*(256*b^11*c^2 - 5120*a*
b^9*c^3 - 262144*a^5*b*c^7 + 40960*a^2*b^7*c^4 - 163840*a^3*b^5*c^5 + 32768
0*a^4*b^3*c^6))/(32*(b^8 + 256*a^4*c^4 + 96*a^2*b^4*c^2 - 256*a^3*b^2*c^3 -
16*a*b^6*c)))*(-(9*(B^2*a*b^15 - B^2*a*(-(4*a*c - b^2)^15)^(1/2) + A^2*b^1
5*c + A^2*c*(-(4*a*c - b^2)^15)^(1/2) - 560*A^2*a^2*b^11*c^3 + 4160*A^2*a^3
*b^9*c^4 - 11520*A^2*a^4*b^7*c^5 - 1024*A^2*a^5*b^5*c^6 + 61440*A^2*a^6*b^3
*c^7 - 560*B^2*a^3*b^11*c^2 + 4160*B^2*a^4*b^9*c^3 - 11520*B^2*a^5*b^7*c^4
- 1024*B^2*a^6*b^5*c^5 + 61440*B^2*a^7*b^3*c^6 + 65536*A*B*a^8*c^8 + 20*A^2
*a*b^13*c^2 - 81920*A^2*a^7*b*c^8 + 20*B^2*a^2*b^13*c - 81920*B^2*a^8*b*c^7
+ 240*A*B*a^2*b^12*c^2 - 64*A*B*a^3*b^10*c^3 - 11520*A*B*a^4*b^8*c^4 + 665
60*A*B*a^5*b^6*c^5 - 143360*A*B*a^6*b^4*c^6 + 81920*A*B*a^7*b^2*c^7 - 20*A*
B*a*b^14*c))/(512*(1048576*a^11*c^11 - 40*a^2*b^18*c^2 + 720*a^3*b^16*c^3 -
7680*a^4*b^14*c^4 + 53760*a^5*b^12*c^5 - 258048*a^6*b^10*c^6 + 860160*a^7*
b^8*c^7 - 1966080*a^8*b^6*c^8 + 2949120*a^9*b^4*c^9 - 2621440*a^10*b^2*c^10
+ a*b^20*c)))^(1/2) - (x*(9*B^2*b^6*c + 288*A^2*a^2*c^5 + 234*A^2*b^4*c^3
- 288*B^2*a^3*c^4 + 576*B^2*a^2*b^2*c^3 - 90*A*B*b^5*c^2 + 144*A^2*a*b^2*c^
4 + 126*B^2*a*b^4*c^2 - 720*A*B*a*b^3*c^3 - 288*A*B*a^2*b*c^4))/(32*(b^8 +
256*a^4*c^4 + 96*a^2*b^4*c^2 - 256*a^3*b^2*c^3 - 16*a*b^6*c)))*(-(9*(B^2*a*
b^15 - B^2*a*(-(4*a*c - b^2)^15)^(1/2) + A^2*b^15*c + A^2*c*(-(4*a*c - b^2)
^15)^(1/2) - 560*A^2*a^2*b^11*c^3 + 4160*A^2*a^3*b^9*c^4 - 11520*A^2*a^4*b^
7*c^5 - 1024*A^2*a^5*b^5*c^6 + 61440*A^2*a^6*b^3*c^7 - 560*B^2*a^3*b^11*c^2
+ 4160*B^2*a^4*b^9*c^3 - 11520*B^2*a^5*b^7*c^4 - 1024*B^2*a^6*b^5*c^5 + 61
440*B^2*a^7*b^3*c^6 + 65536*A*B*a^8*c^8 + 20*A^2*a*b^13*c^2 - 81920*A^2*a^7
*b*c^8 + 20*B^2*a^2*b^13*c - 81920*B^2*a^8*b*c^7 + 240*A*B*a^2*b^12*c^2 - 6
4*A*B*a^3*b^10*c^3 - 11520*A*B*a^4*b^8*c^4 + 66560*A*B*a^5*b^6*c^5 - 143360
*A*B*a^6*b^4*c^6 + 81920*A*B*a^7*b^2*c^7 - 20*A*B*a*b^14*c))/(512*(1048576*
a^11*c^11 - 40*a^2*b^18*c^2 + 720*a^3*b^16*c^3 - 7680*a^4*b^14*c^4 + 53760*
a^5*b^12*c^5 - 258048*a^6*b^10*c^6 + 860160*a^7*b^8*c^7 - 1966080*a^8*b^6*c
^8 + 2949120*a^9*b^4*c^9 - 2621440*a^10*b^2*c^10 + a*b^20*c)))^(1/2)*1i - (
((3*(1048576*A*a^6*c^8 - 256*A*b^12*c^2 + 4096*A*a*b^10*c^3 + 1024*B*a*b^11
*c^2 - 1048576*B*a^6*b*c^7 - 20480*A*a^2*b^8*c^4 + 327680*A*a^4*b^4*c^6 - 1
048576*A*a^5*b^2*c^7 - 20480*B*a^2*b^9*c^3 + 163840*B*a^3*b^7*c^4 - 655360*
B*a^4*b^5*c^5 + 1310720*B*a^5*b^3*c^6))/(512*(b^12 + 4096*a^6*c^6 + 240*a^2
*b^8*c^2 - 1280*a^3*b^6*c^3 + 3840*a^4*b^4*c^4 - 6144*a^5*b^2*c^5 - 24*a*b^
10*c)) + (x*(-(9*(B^2*a*b^15 - B^2*a*(-(4*a*c - b^2)^15)^(1/2) + A^2*b^15*c
+ A^2*c*(-(4*a*c - b^2)^15)^(1/2) - 560*A^2*a^2*b^11*c^3 + 4160*A^2*a^3*b^
9*c^4 - 11520*A^2*a^4*b^7*c^5 - 1024*A^2*a^5*b^5*c^6 + 61440*A^2*a^6*b^3*c^
7 - 560*B^2*a^3*b^11*c^2 + 4160*B^2*a^4*b^9*c^3 - 11520*B^2*a^5*b^7*c^4 - 1
024*B^2*a^6*b^5*c^5 + 61440*B^2*a^7*b^3*c^6 + 65536*A*B*a^8*c^8 + 20*A^2*a*
b^13*c^2 - 81920*A^2*a^7*b*c^8 + 20*B^2*a^2*b^13*c - 81920*B^2*a^8*b*c^7 +
240*A*B*a^2*b^12*c^2 - 64*A*B*a^3*b^10*c^3 - 11520*A*B*a^4*b^8*c^4 + 66560*
A*B*a^5*b^6*c^5 - 143360*A*B*a^6*b^4*c^6 + 81920*A*B*a^7*b^2*c^7 - 20*A*B*a
*b^14*c))/(512*(1048576*a^11*c^11 - 40*a^2*b^18*c^2 + 720*a^3*b^16*c^3 - 76
80*a^4*b^14*c^4 + 53760*a^5*b^12*c^5 - 258048*a^6*b^10*c^6 + 860160*a^7*b^8
*c^7 - 1966080*a^8*b^6*c^8 + 2949120*a^9*b^4*c^9 - 2621440*a^10*b^2*c^10 +

a*b^20*c)))^(1/2)*(256*b^11*c^2 - 5120*a*b^9*c^3 - 262144*a^5*b*c^7 + 40960
*a^2*b^7*c^4 - 163840*a^3*b^5*c^5 + 327680*a^4*b^3*c^6))/(32*(b^8 + 256*a^4
*c^4 + 96*a^2*b^4*c^2 - 256*a^3*b^2*c^3 - 16*a*b^6*c)))*(-(9*(B^2*a*b^15 -
B^2*a*(-(4*a*c - b^2)^15)^(1/2) + A^2*b^15*c + A^2*c*(-(4*a*c - b^2)^15)^(1
/2) - 560*A^2*a^2*b^11*c^3 + 4160*A^2*a^3*b^9*c^4 - 11520*A^2*a^4*b^7*c^5 -
1024*A^2*a^5*b^5*c^6 + 61440*A^2*a^6*b^3*c^7 - 560*B^2*a^3*b^11*c^2 + 4160
*B^2*a^4*b^9*c^3 - 11520*B^2*a^5*b^7*c^4 - 1024*B^2*a^6*b^5*c^5 + 61440*B^2
*a^7*b^3*c^6 + 65536*A*B*a^8*c^8 + 20*A^2*a*b^13*c^2 - 81920*A^2*a^7*b*c^8
+ 20*B^2*a^2*b^13*c - 81920*B^2*a^8*b*c^7 + 240*A*B*a^2*b^12*c^2 - 64*A*B*a
^3*b^10*c^3 - 11520*A*B*a^4*b^8*c^4 + 66560*A*B*a^5*b^6*c^5 - 143360*A*B*a^
6*b^4*c^6 + 81920*A*B*a^7*b^2*c^7 - 20*A*B*a*b^14*c))/(512*(1048576*a^11*c^
11 - 40*a^2*b^18*c^2 + 720*a^3*b^16*c^3 - 7680*a^4*b^14*c^4 + 53760*a^5*b^1
2*c^5 - 258048*a^6*b^10*c^6 + 860160*a^7*b^8*c^7 - 1966080*a^8*b^6*c^8 + 29
49120*a^9*b^4*c^9 - 2621440*a^10*b^2*c^10 + a*b^20*c)))^(1/2) + (x*(9*B^2*b
^6*c + 288*A^2*a^2*c^5 + 234*A^2*b^4*c^3 - 288*B^2*a^3*c^4 + 576*B^2*a^2*b^
2*c^3 - 90*A*B*b^5*c^2 + 144*A^2*a*b^2*c^4 + 126*B^2*a*b^4*c^2 - 720*A*B*a*
b^3*c^3 - 288*A*B*a^2*b*c^4))/(32*(b^8 + 256*a^4*c^4 + 96*a^2*b^4*c^2 - 256
*a^3*b^2*c^3 - 16*a*b^6*c)))*(-(9*(B^2*a*b^15 - B^2*a*(-(4*a*c - b^2)^15)^(
1/2) + A^2*b^15*c + A^2*c*(-(4*a*c - b^2)^15)^(1/2) - 560*A^2*a^2*b^11*c^3
+ 4160*A^2*a^3*b^9*c^4 - 11520*A^2*a^4*b^7*c^5 - 1024*A^2*a^5*b^5*c^6 + 614
40*A^2*a^6*b^3*c^7 - 560*B^2*a^3*b^11*c^2 + 4160*B^2*a^4*b^9*c^3 - 11520*B^
2*a^5*b^7*c^4 - 1024*B^2*a^6*b^5*c^5 + 61440*B^2*a^7*b^3*c^6 + 65536*A*B*a^
8*c^8 + 20*A^2*a*b^13*c^2 - 81920*A^2*a^7*b*c^8 + 20*B^2*a^2*b^13*c - 81920
*B^2*a^8*b*c^7 + 240*A*B*a^2*b^12*c^2 - 64*A*B*a^3*b^10*c^3 - 11520*A*B*a^4
*b^8*c^4 + 66560*A*B*a^5*b^6*c^5 - 143360*A*B*a^6*b^4*c^6 + 81920*A*B*a^7*b
^2*c^7 - 20*A*B*a*b^14*c))/(512*(1048576*a^11*c^11 - 40*a^2*b^18*c^2 + 720*
a^3*b^16*c^3 - 7680*a^4*b^14*c^4 + 53760*a^5*b^12*c^5 - 258048*a^6*b^10*c^6
+ 860160*a^7*b^8*c^7 - 1966080*a^8*b^6*c^8 + 2949120*a^9*b^4*c^9 - 2621440
*a^10*b^2*c^10 + a*b^20*c)))^(1/2)*1i)/((((3*(1048576*A*a^6*c^8 - 256*A*b^1
2*c^2 + 4096*A*a*b^10*c^3 + 1024*B*a*b^11*c^2 - 1048576*B*a^6*b*c^7 - 20480
*A*a^2*b^8*c^4 + 327680*A*a^4*b^4*c^6 - 1048576*A*a^5*b^2*c^7 - 20480*B*a^2
*b^9*c^3 + 163840*B*a^3*b^7*c^4 - 655360*B*a^4*b^5*c^5 + 1310720*B*a^5*b^3*
c^6))/(512*(b^12 + 4096*a^6*c^6 + 240*a^2*b^8*c^2 - 1280*a^3*b^6*c^3 + 3840
*a^4*b^4*c^4 - 6144*a^5*b^2*c^5 - 24*a*b^10*c)) - (x*(-(9*(B^2*a*b^15 - B^2
a(-(4*a*c - b^2)^15)^(1/2) + A^2*b^15*c + A^2*c*(-(4*a*c - b^2)^15)^(1/2)
- 560*A^2*a^2*b^11*c^3 + 4160*A^2*a^3*b^9*c^4 - 11520*A^2*a^4*b^7*c^5 - 10
24*A^2*a^5*b^5*c^6 + 61440*A^2*a^6*b^3*c^7 - 560*B^2*a^3*b^11*c^2 + 4160*B^
2*a^4*b^9*c^3 - 11520*B^2*a^5*b^7*c^4 - 1024*B^2*a^6*b^5*c^5 + 61440*B^2*a^
7*b^3*c^6 + 65536*A*B*a^8*c^8 + 20*A^2*a*b^13*c^2 - 81920*A^2*a^7*b*c^8 + 2
0*B^2*a^2*b^13*c - 81920*B^2*a^8*b*c^7 + 240*A*B*a^2*b^12*c^2 - 64*A*B*a^3*
b^10*c^3 - 11520*A*B*a^4*b^8*c^4 + 66560*A*B*a^5*b^6*c^5 - 143360*A*B*a^6*b
^4*c^6 + 81920*A*B*a^7*b^2*c^7 - 20*A*B*a*b^14*c))/(512*(1048576*a^11*c^11
- 40*a^2*b^18*c^2 + 720*a^3*b^16*c^3 - 7680*a^4*b^14*c^4 + 53760*a^5*b^12*c
^5 - 258048*a^6*b^10*c^6 + 860160*a^7*b^8*c^7 - 1966080*a^8*b^6*c^8 + 29491
20*a^9*b^4*c^9 - 2621440*a^10*b^2*c^10 + a*b^20*c)))^(1/2)*(256*b^11*c^2 -

5120*a*b^9*c^3 - 262144*a^5*b*c^7 + 40960*a^2*b^7*c^4 - 163840*a^3*b^5*c^5 + 327680*a^4*b^3*c^6))/(32*(b^8 + 256*a^4*c^4 + 96*a^2*b^4*c^2 - 256*a^3*b^2*c^3 - 16*a*b^6*c)))*(-(9*(B^2*a*b^15 - B^2*a*(-(4*a*c - b^2)^15)^(1/2) + A^2*b^15*c + A^2*c*(-(4*a*c - b^2)^15)^(1/2) - 560*A^2*a^2*b^11*c^3 + 4160*A^2*a^3*b^9*c^4 - 11520*A^2*a^4*b^7*c^5 - 1024*A^2*a^5*b^5*c^6 + 61440*A^2*a^6*b^3*c^7 - 560*B^2*a^3*b^11*c^2 + 4160*B^2*a^4*b^9*c^3 - 11520*B^2*a^5*b^7*c^4 - 1024*B^2*a^6*b^5*c^5 + 61440*B^2*a^7*b^3*c^6 + 65536*A*B*a^8*c^8 + 20*A^2*a*b^13*c^2 - 81920*A^2*a^7*b*c^8 + 20*B^2*a^2*b^13*c - 81920*B^2*a^8*b*c^7 + 240*A*B*a^2*b^12*c^2 - 64*A*B*a^3*b^10*c^3 - 11520*A*B*a^4*b^8*c^4 + 66560*A*B*a^5*b^6*c^5 - 143360*A*B*a^6*b^4*c^6 + 81920*A*B*a^7*b^2*c^7 - 20*A*B*a*b^14*c))/(512*(1048576*a^11*c^11 - 40*a^2*b^18*c^2 + 720*a^3*b^16*c^3 - 7680*a^4*b^14*c^4 + 53760*a^5*b^12*c^5 - 258048*a^6*b^10*c^6 + 860160*a^7*b^8*c^7 - 1966080*a^8*b^6*c^8 + 2949120*a^9*b^4*c^9 - 2621440*a^10*b^2*c^10 + a*b^20*c)))^(1/2) - (x*(9*B^2*b^6*c + 288*A^2*a^2*c^5 + 234*A^2*b^4*c^3 - 288*B^2*a^3*c^4 + 576*B^2*a^2*b^2*c^3 - 90*A*B*b^5*c^2 + 144*A^2*a*b^2*c^4 + 126*B^2*a*b^4*c^2 - 720*A*B*a*b^3*c^3 - 288*A*B*a^2*b*c^4))/(32*(b^8 + 256*a^4*c^4 + 96*a^2*b^4*c^2 - 256*a^3*b^2*c^3 - 16*a*b^6*c)))*(-(9*(B^2*a*b^15 - B^2*a*(-(4*a*c - b^2)^15)^(1/2) + A^2*b^15*c + A^2*c*(-(4*a*c - b^2)^15)^(1/2) - 560*A^2*a^2*b^11*c^3 + 4160*A^2*a^3*b^9*c^4 - 11520*A^2*a^4*b^7*c^5 - 1024*A^2*a^5*b^5*c^6 + 61440*A^2*a^6*b^3*c^7 - 560*B^2*a^3*b^11*c^2 + 4160*B^2*a^4*b^9*c^3 - 11520*B^2*a^5*b^7*c^4 - 1024*B^2*a^6*b^5*c^5 + 61440*B^2*a^7*b^3*c^6 + 65536*A*B*a^8*c^8 + 20*A^2*a*b^13*c^2 - 81920*A^2*a^7*b*c^8 + 20*B^2*a^2*b^13*c - 81920*B^2*a^8*b*c^7 + 240*A*B*a^2*b^12*c^2 - 64*A*B*a^3*b^10*c^3 - 11520*A*B*a^4*b^8*c^4 + 66560*A*B*a^5*b^6*c^5 - 143360*A*B*a^6*b^4*c^6 + 81920*A*B*a^7*b^2*c^7 - 20*A*B*a*b^14*c))/(512*(1048576*a^11*c^11 - 40*a^2*b^18*c^2 + 720*a^3*b^16*c^3 - 7680*a^4*b^14*c^4 + 53760*a^5*b^12*c^5 - 258048*a^6*b^10*c^6 + 860160*a^7*b^8*c^7 - 1966080*a^8*b^6*c^8 + 2949120*a^9*b^4*c^9 - 2621440*a^10*b^2*c^10 + a*b^20*c)))^(1/2) + (((3*(1048576*A*a^6*c^8 - 256*A*b^12*c^2 + 4096*A*a*b^10*c^3 + 1024*B*a*b^11*c^2 - 1048576*B*a^6*b*c^7 - 20480*A*a^2*b^8*c^4 + 327680*A*a^4*b^4*c^6 - 1048576*A*a^5*b^2*c^7 - 20480*B*a^2*b^9*c^3 + 163840*B*a^3*b^7*c^4 - 655360*B*a^4*b^5*c^5 + 1310720*B*a^5*b^3*c^6))/(512*(b^12 + 4096*a^6*c^6 + 240*a^2*b^8*c^2 - 1280*a^3*b^6*c^3 + 3840*a^4*b^4*c^4 - 6144*a^5*b^2*c^5 - 24*a*b^10*c)) + (x*(-(9*(B^2*a*b^15 - B^2*a*(-(4*a*c - b^2)^15)^(1/2) + A^2*b^15*c + A^2*c*(-(4*a*c - b^2)^15)^(1/2) - 560*A^2*a^2*b^11*c^3 + 4160*A^2*a^3*b^9*c^4 - 11520*A^2*a^4*b^7*c^5 - 1024*A^2*a^5*b^5*c^6 + 61440*A^2*a^6*b^3*c^7 - 560*B^2*a^3*b^11*c^2 + 4160*B^2*a^4*b^9*c^3 - 11520*B^2*a^5*b^7*c^4 - 1024*B^2*a^6*b^5*c^5 + 61440*B^2*a^7*b^3*c^6 + 65536*A*B*a^8*c^8 + 20*A^2*a*b^13*c^2 - 81920*A^2*a^7*b*c^8 + 20*B^2*a^2*b^13*c - 81920*B^2*a^8*b*c^7 + 240*A*B*a^2*b^12*c^2 - 64*A*B*a^3*b^10*c^3 - 11520*A*B*a^4*b^8*c^4 + 66560*A*B*a^5*b^6*c^5 - 143360*A*B*a^6*b^4*c^6 + 81920*A*B*a^7*b^2*c^7 - 20*A*B*a*b^14*c))/(512*(1048576*a^11*c^11 - 40*a^2*b^18*c^2 + 720*a^3*b^16*c^3 - 7680*a^4*b^14*c^4 + 53760*a^5*b^12*c^5 - 258048*a^6*b^10*c^6 + 860160*a^7*b^8*c^7 - 1966080*a^8*b^6*c^8 + 2949120*a^9*b^4*c^9 - 2621440*a^10*b^2*c^10 + a*b^20*c)))^(1/2)*(256*b^11*c^2 - 5120*a*b^9*c^3 - 262144*a^5*b*c^7 + 4

0960*a^2*b^7*c^4 - 163840*a^3*b^5*c^5 + 327680*a^4*b^3*c^6))/(32*(b^8 + 256
*a^4*c^4 + 96*a^2*b^4*c^2 - 256*a^3*b^2*c^3 - 16*a*b^6*c)))*(-(9*(B^2*a*b^1
5 - B^2*a*(-(4*a*c - b^2)^15)^(1/2) + A^2*b^15*c + A^2*c*(-(4*a*c - b^2)^15
)^(1/2) - 560*A^2*a^2*b^11*c^3 + 4160*A^2*a^3*b^9*c^4 - 11520*A^2*a^4*b^7*c
^5 - 1024*A^2*a^5*b^5*c^6 + 61440*A^2*a^6*b^3*c^7 - 560*B^2*a^3*b^11*c^2 +
4160*B^2*a^4*b^9*c^3 - 11520*B^2*a^5*b^7*c^4 - 1024*B^2*a^6*b^5*c^5 + 61440
*B^2*a^7*b^3*c^6 + 65536*A*B*a^8*c^8 + 20*A^2*a*b^13*c^2 - 81920*A^2*a^7*b*
c^8 + 20*B^2*a^2*b^13*c - 81920*B^2*a^8*b*c^7 + 240*A*B*a^2*b^12*c^2 - 64*A
*B*a^3*b^10*c^3 - 11520*A*B*a^4*b^8*c^4 + 66560*A*B*a^5*b^6*c^5 - 143360*A*
B*a^6*b^4*c^6 + 81920*A*B*a^7*b^2*c^7 - 20*A*B*a*b^14*c))/(512*(1048576*a^1
1*c^11 - 40*a^2*b^18*c^2 + 720*a^3*b^16*c^3 - 7680*a^4*b^14*c^4 + 53760*a^5
*b^12*c^5 - 258048*a^6*b^10*c^6 + 860160*a^7*b^8*c^7 - 1966080*a^8*b^6*c^8
+ 2949120*a^9*b^4*c^9 - 2621440*a^10*b^2*c^10 + a*b^20*c)))^(1/2) + (x*(9*B
^2*b^6*c + 288*A^2*a^2*c^5 + 234*A^2*b^4*c^3 - 288*B^2*a^3*c^4 + 576*B^2*a^
2*b^2*c^3 - 90*A*B*b^5*c^2 + 144*A^2*a*b^2*c^4 + 126*B^2*a*b^4*c^2 - 720*A*
B*a*b^3*c^3 - 288*A*B*a^2*b*c^4))/(32*(b^8 + 256*a^4*c^4 + 96*a^2*b^4*c^2 -
256*a^3*b^2*c^3 - 16*a*b^6*c)))*(-(9*(B^2*a*b^15 - B^2*a*(-(4*a*c - b^2)^1
5)^(1/2) + A^2*b^15*c + A^2*c*(-(4*a*c - b^2)^15)^(1/2) - 560*A^2*a^2*b^11*
c^3 + 4160*A^2*a^3*b^9*c^4 - 11520*A^2*a^4*b^7*c^5 - 1024*A^2*a^5*b^5*c^6 +
61440*A^2*a^6*b^3*c^7 - 560*B^2*a^3*b^11*c^2 + 4160*B^2*a^4*b^9*c^3 - 1152
0*B^2*a^5*b^7*c^4 - 1024*B^2*a^6*b^5*c^5 + 61440*B^2*a^7*b^3*c^6 + 65536*A*
B*a^8*c^8 + 20*A^2*a*b^13*c^2 - 81920*A^2*a^7*b*c^8 + 20*B^2*a^2*b^13*c - 8
1920*B^2*a^8*b*c^7 + 240*A*B*a^2*b^12*c^2 - 64*A*B*a^3*b^10*c^3 - 11520*A*B
*a^4*b^8*c^4 + 66560*A*B*a^5*b^6*c^5 - 143360*A*B*a^6*b^4*c^6 + 81920*A*B*a
^7*b^2*c^7 - 20*A*B*a*b^14*c))/(512*(1048576*a^11*c^11 - 40*a^2*b^18*c^2 +
720*a^3*b^16*c^3 - 7680*a^4*b^14*c^4 + 53760*a^5*b^12*c^5 - 258048*a^6*b^10
*c^6 + 860160*a^7*b^8*c^7 - 1966080*a^8*b^6*c^8 + 2949120*a^9*b^4*c^9 - 262
1440*a^10*b^2*c^10 + a*b^20*c)))^(1/2) + (3*(576*B^3*a^4*c^4 - 180*A^3*b^5*
c^3 + 540*B^3*a^2*b^4*c^2 + 1584*B^3*a^3*b^2*c^3 - 9*A*B^2*b^7*c + 45*B^3*a
*b^6*c + 576*A^2*B*a^3*c^5 + 81*A^2*B*b^6*c^2 - 1440*A^3*a*b^3*c^4 - 576*A^
3*a^2*b*c^5 - 576*A*B^2*a*b^5*c^2 - 3456*A*B^2*a^3*b*c^4 + 1980*A^2*B*a*b^4
*c^3 - 3600*A*B^2*a^2*b^3*c^3 + 4464*A^2*B*a^2*b^2*c^4))/(256*(b^12 + 4096*
a^6*c^6 + 240*a^2*b^8*c^2 - 1280*a^3*b^6*c^3 + 3840*a^4*b^4*c^4 - 6144*a^5*
b^2*c^5 - 24*a*b^10*c))))*(-(9*(B^2*a*b^15 - B^2*a*(-(4*a*c - b^2)^15)^(1/2
) + A^2*b^15*c + A^2*c*(-(4*a*c - b^2)^15)^(1/2) - 560*A^2*a^2*b^11*c^3 + 4
160*A^2*a^3*b^9*c^4 - 11520*A^2*a^4*b^7*c^5 - 1024*A^2*a^5*b^5*c^6 + 61440*
A^2*a^6*b^3*c^7 - 560*B^2*a^3*b^11*c^2 + 4160*B^2*a^4*b^9*c^3 - 11520*B^2*a
^5*b^7*c^4 - 1024*B^2*a^6*b^5*c^5 + 61440*B^2*a^7*b^3*c^6 + 65536*A*B*a^8*c
^8 + 20*A^2*a*b^13*c^2 - 81920*A^2*a^7*b*c^8 + 20*B^2*a^2*b^13*c - 81920*B^
2*a^8*b*c^7 + 240*A*B*a^2*b^12*c^2 - 64*A*B*a^3*b^10*c^3 - 11520*A*B*a^4*b^
8*c^4 + 66560*A*B*a^5*b^6*c^5 - 143360*A*B*a^6*b^4*c^6 + 81920*A*B*a^7*b^2*
c^7 - 20*A*B*a*b^14*c))/(512*(1048576*a^11*c^11 - 40*a^2*b^18*c^2 + 720*a^3
*b^16*c^3 - 7680*a^4*b^14*c^4 + 53760*a^5*b^12*c^5 - 258048*a^6*b^10*c^6 +
860160*a^7*b^8*c^7 - 1966080*a^8*b^6*c^8 + 2949120*a^9*b^4*c^9 - 2621440*a^
10*b^2*c^10 + a*b^20*c)))^(1/2)*2i

3.135 $\int \frac{x^2(A+Bx^2)}{(a+bx^2+cx^4)^3}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 438

$$\int \frac{x^2(A+Bx^2)}{(a+bx^2+cx^4)^3}\,dx = -\frac{x(Ab-2aB-(bB-2Ac)x^2)}{4(b^2-4ac)(a+bx^2+cx^4)^2}$$
$$-\frac{x(aB(7b^2-4ac)-A(b^3+8abc)+c(12abB-A(b^2+20ac))x^2)}{8a(b^2-4ac)^2(a+bx^2+cx^4)}$$
$$+\frac{\sqrt{c}\left(6aB\left(3b^2+4ac-2b\sqrt{b^2-4ac}\right)+A\left(b^3-52abc+b^2\sqrt{b^2-4ac}+20ac\sqrt{b^2-4ac}\right)\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{8\sqrt{2}a(b^2-4ac)^{5/2}\sqrt{b-\sqrt{b^2-4ac}}}$$
$$-\frac{\sqrt{c}\left(6aB\left(3b^2+4ac+2b\sqrt{b^2-4ac}\right)+A\left(b^3-52abc-b^2\sqrt{b^2-4ac}-20ac\sqrt{b^2-4ac}\right)\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{8\sqrt{2}a(b^2-4ac)^{5/2}\sqrt{b+\sqrt{b^2-4ac}}}$$

```
[Out] -1/4*x*(A*b-2*B*a-(-2*A*c+B*b)*x^2)/(-4*a*c+b^2)/(c*x^4+b*x^2+a)^2-1/8*x*(a
*B*(-4*a*c+7*b^2)-A*(8*a*b*c+b^3)+c*(12*a*b*B-A*(20*a*c+b^2))*x^2)/a/(-4*a*
c+b^2)^2/(c*x^4+b*x^2+a)+1/16*arctan(x*2^(1/2)*c^(1/2)/(b-(-4*a*c+b^2)^(1/2
))^(1/2))*c^(1/2)*(6*a*B*(3*b^2+4*a*c-2*b*(-4*a*c+b^2)^(1/2))+A*(b^3-52*a*b
*c+b^2*(-4*a*c+b^2)^(1/2)+20*a*c*(-4*a*c+b^2)^(1/2)))/a/(-4*a*c+b^2)^(5/2)*
2^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2)-1/16*arctan(x*2^(1/2)*c^(1/2)/(b+(-4*a
*c+b^2)^(1/2))^(1/2))*c^(1/2)*(6*a*B*(3*b^2+4*a*c+2*b*(-4*a*c+b^2)^(1/2))+A
*(b^3-52*a*b*c-b^2*(-4*a*c+b^2)^(1/2)-20*a*c*(-4*a*c+b^2)^(1/2)))/a/(-4*a*c
+b^2)^(5/2)*2^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2)
```

Rubi [A] (verified)

Time = 0.69 (sec) , antiderivative size = 438, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {1289, 1192, 1180, 211}

$$\int \frac{x^2(A+Bx^2)}{(a+bx^2+cx^4)^3}\,dx$$
$$= \frac{\sqrt{c}\left(A\left(b^2\sqrt{b^2-4ac}+20ac\sqrt{b^2-4ac}-52abc+b^3\right)+6aB\left(-2b\sqrt{b^2-4ac}+4ac+3b^2\right)\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}}{\sqrt{b-\sqrt{b^2}}}\right.}{8\sqrt{2}a\left(b^2-4ac\right)^{5/2}\sqrt{b-\sqrt{b^2-4ac}}}$$
$$-\frac{\sqrt{c}\left(A\left(-b^2\sqrt{b^2-4ac}-20ac\sqrt{b^2-4ac}-52abc+b^3\right)+6aB\left(2b\sqrt{b^2-4ac}+4ac+3b^2\right)\right)\arctan\left(\frac{\sqrt{2}}{\sqrt{\sqrt{b^2}}}\right.}{8\sqrt{2}a\left(b^2-4ac\right)^{5/2}\sqrt{\sqrt{b^2-4ac}+b}}$$
$$-\frac{x(-2aB-(x^2(bB-2Ac))+Ab)}{4\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2}$$
$$-\frac{x(-A(8abc+b^3)+cx^2(12abB-A(20ac+b^2))+aB(7b^2-4ac))}{8a\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)}$$

```
[In] Int[(x^2*(A + B*x^2))/(a + b*x^2 + c*x^4)^3,x]
```

```
[Out] -1/4*(x*(A*b - 2*a*B - (b*B - 2*A*c)*x^2))/((b^2 - 4*a*c)*(a + b*x^2 + c*x^
4)^2) - (x*(a*B*(7*b^2 - 4*a*c) - A*(b^3 + 8*a*b*c) + c*(12*a*b*B - A*(b^2
+ 20*a*c))*x^2))/(8*a*(b^2 - 4*a*c)^2*(a + b*x^2 + c*x^4)) + (Sqrt[c]*(6*a*
B*(3*b^2 + 4*a*c - 2*b*Sqrt[b^2 - 4*a*c]) + A*(b^3 - 52*a*b*c + b^2*Sqrt[b^
2 - 4*a*c] + 20*a*c*Sqrt[b^2 - 4*a*c]))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b -
 Sqrt[b^2 - 4*a*c]]])/(8*Sqrt[2]*a*(b^2 - 4*a*c)^(5/2)*Sqrt[b - Sqrt[b^2 -
4*a*c]]) - (Sqrt[c]*(6*a*B*(3*b^2 + 4*a*c + 2*b*Sqrt[b^2 - 4*a*c]) + A*(b^3
 - 52*a*b*c - b^2*Sqrt[b^2 - 4*a*c] - 20*a*c*Sqrt[b^2 - 4*a*c]))*ArcTan[(Sq
rt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(8*Sqrt[2]*a*(b^2 - 4*a*c)^(
5/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1192

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[x*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)*((a + b*x^2 +
 c*x^4)^(p + 1)/(2*a*(p + 1)*(b^2 - 4*a*c))), x] + Dist[1/(2*a*(p + 1)*(b^2
 - 4*a*c)), Int[Simp[(2*p + 3)*d*b^2 - a*b*e - 2*a*c*d*(4*p + 5) + (4*p + 7
)*(d*b - 2*a*e)*c*x^2, x]*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; FreeQ[{a,
b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
LtQ[p, -1] && IntegerQ[2*p]
```

Rule 1289

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_.), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(a + b*x^2 + c*x^4)^(p + 1)
*((b*d - 2*a*e - (b*e - 2*c*d)*x^2)/(2*(p + 1)*(b^2 - 4*a*c))), x] - Dist[f
^2/(2*(p + 1)*(b^2 - 4*a*c)), Int[(f*x)^(m - 2)*(a + b*x^2 + c*x^4)^(p + 1)
*Simp[(m - 1)*(b*d - 2*a*e) - (4*p + 4 + m + 1)*(b*e - 2*c*d)*x^2, x], x],
x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] &&
GtQ[m, 1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rubi steps

$$\text{integral} = -\frac{x(Ab-2aB-(bB-2Ac)x^2)}{4(b^2-4ac)(a+bx^2+cx^4)^2} + \frac{\int \frac{Ab-2aB+5(bB-2Ac)x^2}{(a+bx^2+cx^4)^2}\,dx}{4(b^2-4ac)}$$

$$= -\frac{x(Ab-2aB-(bB-2Ac)x^2)}{4(b^2-4ac)(a+bx^2+cx^4)^2} - \frac{x(aB(7b^2-4ac)-A(b^3+8abc)+c(12abB-A(b^2+20ac))x^2)}{8a(b^2-4ac)^2(a+bx^2+cx^4)} - \frac{\int \frac{-3aB(b^2+4ac)-A(b^3-16abc)+c(12abB-A(b^2+20ac))x^2}{a+bx^2+cx^4}\,dx}{8a(b^2-4ac)^2}$$

$$= -\frac{x(Ab-2aB-(bB-2Ac)x^2)}{4(b^2-4ac)(a+bx^2+cx^4)^2} - \frac{x(aB(7b^2-4ac)-A(b^3+8abc)+c(12abB-A(b^2+20ac))x^2)}{8a(b^2-4ac)^2(a+bx^2+cx^4)} - \frac{\left(c\left(6aB\left(3b^2+4ac+2b\sqrt{b^2-4ac}\right)+A\left(b^3-52abc-b^2\sqrt{b^2-4ac}-20ac\sqrt{b^2-4ac}\right)\right)\right)\int \frac{}{\frac{b}{2}+\frac{1}{2}\sqrt{b\cdots}}}{16a(b^2-4ac)^{5/2}} - \frac{\left(c\left(12abB-A(b^2+20ac)-\frac{6aB(3b^2+4ac)+A(b^3-52abc)}{\sqrt{b^2-4ac}}\right)\right)\int \frac{1}{\frac{b}{2}-\frac{1}{2}\sqrt{b^2-4ac}+cx^2}\,dx}{16a(b^2-4ac)^2}$$

$$= -\frac{x(Ab - 2aB - (bB - 2Ac)x^2)}{4(b^2 - 4ac)(a + bx^2 + cx^4)^2}$$
$$- \frac{x(aB(7b^2 - 4ac) - A(b^3 + 8abc) + c(12abB - A(b^2 + 20ac))x^2)}{8a(b^2 - 4ac)^2(a + bx^2 + cx^4)}$$
$$- \frac{\sqrt{c}\left(12abB - A(b^2 + 20ac) - \frac{6aB(3b^2+4ac)+A(b^3-52abc)}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{cx}}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{8\sqrt{2}a(b^2 - 4ac)^2\sqrt{b - \sqrt{b^2 - 4ac}}}$$
$$- \frac{\sqrt{c}\left(6aB\left(3b^2 + 4ac + 2b\sqrt{b^2 - 4ac}\right) + A\left(b^3 - 52abc - b^2\sqrt{b^2 - 4ac} - 20ac\sqrt{b^2 - 4ac}\right)\right)\tan^{-1}\left(\right.}{8\sqrt{2}a(b^2 - 4ac)^{5/2}\sqrt{b + \sqrt{b^2 - 4ac}}}$$

Mathematica [A] (verified)

Time = 1.01 (sec) , antiderivative size = 436, normalized size of antiderivative = 1.00

$$\int \frac{x^2(A + Bx^2)}{(a + bx^2 + cx^4)^3}\,dx = \frac{1}{16}\left(\frac{4x(B(2a + bx^2) - A(b + 2cx^2))}{(b^2 - 4ac)(a + bx^2 + cx^4)^2}\right.$$
$$+ \frac{2x(aB(-7b^2 + 4ac - 12bcx^2) + A(b^3 + 8abc + b^2cx^2 + 20ac^2x^2))}{a(b^2 - 4ac)^2(a + bx^2 + cx^4)}$$
$$+ \frac{\sqrt{2}\sqrt{c}\left(6aB\left(3b^2 + 4ac - 2b\sqrt{b^2 - 4ac}\right) + A\left(b^3 - 52abc + b^2\sqrt{b^2 - 4ac} + 20ac\sqrt{b^2 - 4ac}\right)\right)\arctan\left(\right.}{a(b^2 - 4ac)^{5/2}\sqrt{b - \sqrt{b^2 - 4ac}}}$$
$$+ \frac{\sqrt{2}\sqrt{c}\left(-6aB\left(3b^2 + 4ac + 2b\sqrt{b^2 - 4ac}\right) + A\left(-b^3 + 52abc + b^2\sqrt{b^2 - 4ac} + 20ac\sqrt{b^2 - 4ac}\right)\right)\arctan\left(\right.}{a(b^2 - 4ac)^{5/2}\sqrt{b + \sqrt{b^2 - 4ac}}}$$

```
[In] Integrate[(x^2*(A + B*x^2))/(a + b*x^2 + c*x^4)^3,x]
```

```
[Out] ((4*x*(B*(2*a + b*x^2) - A*(b + 2*c*x^2)))/((b^2 - 4*a*c)*(a + b*x^2 + c*x^
4)^2) + (2*x*(a*B*(-7*b^2 + 4*a*c - 12*b*c*x^2) + A*(b^3 + 8*a*b*c + b^2*c*
x^2 + 20*a*c^2*x^2)))/(a*(b^2 - 4*a*c)^2*(a + b*x^2 + c*x^4)) + (Sqrt[2]*Sq
rt[c]*(6*a*B*(3*b^2 + 4*a*c - 2*b*Sqrt[b^2 - 4*a*c]) + A*(b^3 - 52*a*b*c +
b^2*Sqrt[b^2 - 4*a*c] + 20*a*c*Sqrt[b^2 - 4*a*c]))*ArcTan[(Sqrt[2]*Sqrt[c]*
x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(a*(b^2 - 4*a*c)^(5/2)*Sqrt[b - Sqrt[b^2 -
 4*a*c]]) + (Sqrt[2]*Sqrt[c]*(-6*a*B*(3*b^2 + 4*a*c + 2*b*Sqrt[b^2 - 4*a*c]
) + A*(-b^3 + 52*a*b*c + b^2*Sqrt[b^2 - 4*a*c] + 20*a*c*Sqrt[b^2 - 4*a*c]))
*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(a*(b^2 - 4*a*c)^
(5/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]]))/16
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.15 (sec) , antiderivative size = 374, normalized size of antiderivative = 0.85

method	result
risch	$\frac{\frac{c^2(20Aac+Ab^2-12abB)x^7}{8a(16a^2c^2-8ab^2c+b^4)}+\frac{c(28Aabc+2Ab^3+4a^2Bc-19Bab^2)x^5}{8a(16a^2c^2-8ab^2c+b^4)}+\frac{(36Aa^2c^2+5Aab^2c+Ab^4-16a^2bBc-5Bab^3)x^3}{8a(16a^2c^2-8ab^2c+b^4)}+\frac{(16Aabc-Ab^3-12a^2Bc\cdots}{128a^2c^2-64ab^2c\cdots}}{(cx^4+bx^2+a)^2}$
default	$\frac{\frac{c^2(20Aac+Ab^2-12abB)x^7}{8a(16a^2c^2-8ab^2c+b^4)}+\frac{c(28Aabc+2Ab^3+4a^2Bc-19Bab^2)x^5}{8a(16a^2c^2-8ab^2c+b^4)}+\frac{(36Aa^2c^2+5Aab^2c+Ab^4-16a^2bBc-5Bab^3)x^3}{8a(16a^2c^2-8ab^2c+b^4)}+\frac{(16Aabc-Ab^3-12a^2Bc\cdots}{128a^2c^2-64ab^2c\cdots}}{(cx^4+bx^2+a)^2}$

```
[In] int(x^2*(B*x^2+A)/(c*x^4+b*x^2+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] (1/8*c^2*(20*A*a*c+A*b^2-12*B*a*b)/a/(16*a^2*c^2-8*a*b^2*c+b^4)*x^7+1/8/a*c
*(28*A*a*b*c+2*A*b^3+4*B*a^2*c-19*B*a*b^2)/(16*a^2*c^2-8*a*b^2*c+b^4)*x^5+1
/8*(36*A*a^2*c^2+5*A*a*b^2*c+A*b^4-16*B*a^2*b*c-5*B*a*b^3)/a/(16*a^2*c^2-8*
a*b^2*c+b^4)*x^3+1/8*(16*A*a*b*c-A*b^3-12*B*a^2*c-3*B*a*b^2)/(16*a^2*c^2-8*
a*b^2*c+b^4)*x)/(c*x^4+b*x^2+a)^2+1/16/a*sum((c*(20*A*a*c+A*b^2-12*B*a*b)/(
16*a^2*c^2-8*a*b^2*c+b^4)*_R^2-(16*A*a*b*c-A*b^3-12*B*a^2*c-3*B*a*b^2)/(16*
a^2*c^2-8*a*b^2*c+b^4))/(2*_R^3*c+_R*b)*ln(x-_R),_R=RootOf(_Z^4*c+_Z^2*b+a)
)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 7270 vs. 2(382) = 764.

Time = 8.33 (sec) , antiderivative size = 7270, normalized size of antiderivative = 16.60

$$\int \frac{x^2(A+Bx^2)}{(a+bx^2+cx^4)^3}\,dx = \text{Too large to display}$$

```
[In] integrate(x^2*(B*x^2+A)/(c*x^4+b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^2(A+Bx^2)}{(a+bx^2+cx^4)^3}\,dx = \text{Timed out}$$

```
[In] integrate(x**2*(B*x**2+A)/(c*x**4+b*x**2+a)**3,x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{x^2(A+Bx^2)}{(a+bx^2+cx^4)^3}\,dx = \int \frac{(Bx^2+A)x^2}{(cx^4+bx^2+a)^3}\,dx$$

```
[In] integrate(x^2*(B*x^2+A)/(c*x^4+b*x^2+a)^3,x, algorithm="maxima")
[Out] 1/8*((20*A*a*c^3 - (12*B*a*b - A*b^2)*c^2)*x^7 + (4*(B*a^2 + 7*A*a*b)*c^2 -
 (19*B*a*b^2 - 2*A*b^3)*c)*x^5 - (5*B*a*b^3 - A*b^4 - 36*A*a^2*c^2 + (16*B*
a^2*b - 5*A*a*b^2)*c)*x^3 - (3*B*a^2*b^2 + A*a*b^3 + 4*(3*B*a^3 - 4*A*a^2*b
)*c)*x)/((a*b^4*c^2 - 8*a^2*b^2*c^3 + 16*a^3*c^4)*x^8 + a^3*b^4 - 8*a^4*b^2
*c + 16*a^5*c^2 + 2*(a*b^5*c - 8*a^2*b^3*c^2 + 16*a^3*b*c^3)*x^6 + (a*b^6 -
 6*a^2*b^4*c + 32*a^4*c^3)*x^4 + 2*(a^2*b^5 - 8*a^3*b^3*c + 16*a^4*b*c^2)*x
^2) + 1/8*integrate((3*B*a*b^2 + A*b^3 + (20*A*a*c^2 - (12*B*a*b - A*b^2)*c
)*x^2 + 4*(3*B*a^2 - 4*A*a*b)*c)/(c*x^4 + b*x^2 + a), x)/(a*b^4 - 8*a^2*b^2
*c + 16*a^3*c^2)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 7267 vs. $2(382) = 764$.

Time = 2.18 (sec) , antiderivative size = 7267, normalized size of antiderivative = 16.59

$$\int \frac{x^2(A+Bx^2)}{(a+bx^2+cx^4)^3}\,dx = \text{Too large to display}$$

```
[In] integrate(x^2*(B*x^2+A)/(c*x^4+b*x^2+a)^3,x, algorithm="giac")
[Out] -1/64*((2*b^4*c^2 + 32*a*b^2*c^3 - 160*a^2*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*
sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^4 - 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c
 + sqrt(b^2 - 4*a*c)*c)*a*b^2*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sq
rt(b^2 - 4*a*c)*c)*b^3*c + 80*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2
 - 4*a*c)*c)*a^2*c^2 + 40*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4
*a*c)*c)*a*b*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c
```

)*b^2*c^2 - 20*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*
c^3 - 2*(b^2 - 4*a*c)*b^2*c^2 - 40*(b^2 - 4*a*c)*a*c^3)*(a*b^4 - 8*a^2*b^2*
c + 16*a^3*c^2)^2*A - 12*(2*a*b^3*c^2 - 8*a^2*b*c^3 - sqrt(2)*sqrt(b^2 - 4*
a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sq
rt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*
c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqr
t(b^2 - 4*a*c)*c)*a*b*c^2 - 2*(b^2 - 4*a*c)*a*b*c^2)*(a*b^4 - 8*a^2*b^2*c +
 16*a^3*c^2)^2*B - 2*(sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^9 - 28*sq
rt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^7*c - 2*sqrt(2)*sqrt(b*c + sqrt
(b^2 - 4*a*c)*c)*a*b^8*c - 2*a*b^9*c + 240*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*
a*c)*c)*a^3*b^5*c^2 + 48*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^6*c^
2 + sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^7*c^2 + 56*a^2*b^7*c^2 - 83
2*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b^3*c^3 - 288*sqrt(2)*sqrt(b*
c + sqrt(b^2 - 4*a*c)*c)*a^3*b^4*c^3 - 24*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a
*c)*c)*a^2*b^5*c^3 - 480*a^3*b^5*c^3 + 1024*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4
*a*c)*c)*a^5*b*c^4 + 512*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b^2*c^
4 + 144*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^3*c^4 + 1664*a^4*b^3*
c^4 - 256*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b*c^5 - 2048*a^5*b*c^
5 + 2*(b^2 - 4*a*c)*a*b^7*c - 48*(b^2 - 4*a*c)*a^2*b^5*c^2 + 288*(b^2 - 4*a
*c)*a^3*b^3*c^3 - 512*(b^2 - 4*a*c)*a^4*b*c^4)*A*abs(a*b^4 - 8*a^2*b^2*c +
16*a^3*c^2) - 6*(sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^8 - 8*sqrt(2
)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^6*c - 2*sqrt(2)*sqrt(b*c + sqrt(b^2
 - 4*a*c)*c)*a^2*b^7*c - 2*a^2*b^8*c + 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*
c)*c)*a^3*b^5*c^2 + sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^6*c^2 + 1
6*a^3*b^6*c^2 + 128*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^5*b^2*c^3 + 3
2*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b^3*c^3 - 4*sqrt(2)*sqrt(b*c
+ sqrt(b^2 - 4*a*c)*c)*a^3*b^4*c^3 - 256*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*
c)*c)*a^6*c^4 - 128*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^5*b*c^4 - 16*
sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b^2*c^4 - 256*a^5*b^2*c^4 + 64*
sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^5*c^5 + 512*a^6*c^5 + 2*(b^2 - 4*
a*c)*a^2*b^6*c - 8*(b^2 - 4*a*c)*a^3*b^4*c^2 - 32*(b^2 - 4*a*c)*a^4*b^2*c^3
 + 128*(b^2 - 4*a*c)*a^5*c^4)*B*abs(a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2) + (2*
a^2*b^12*c^2 - 136*a^3*b^10*c^3 + 1856*a^4*b^8*c^4 - 10496*a^5*b^6*c^5 + 27
136*a^6*b^4*c^6 - 26624*a^7*b^2*c^7 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c +
sqrt(b^2 - 4*a*c)*c)*a^2*b^12 + 68*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqr
t(b^2 - 4*a*c)*c)*a^3*b^10*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(
b^2 - 4*a*c)*c)*a^2*b^11*c - 928*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(
b^2 - 4*a*c)*c)*a^4*b^8*c^2 - 128*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt
(b^2 - 4*a*c)*c)*a^3*b^9*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^
2 - 4*a*c)*c)*a^2*b^10*c^2 + 5248*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt
(b^2 - 4*a*c)*c)*a^5*b^6*c^3 + 1344*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sq
rt(b^2 - 4*a*c)*c)*a^4*b^7*c^3 + 64*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sq
rt(b^2 - 4*a*c)*c)*a^3*b^8*c^3 - 13568*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c +
 sqrt(b^2 - 4*a*c)*c)*a^6*b^4*c^4 - 5120*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c
 + sqrt(b^2 - 4*a*c)*c)*a^5*b^5*c^4 - 672*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*

c + sqrt(b^2 - 4*a*c)*c)*a^4*b^6*c^4 + 13312*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt
(b*c + sqrt(b^2 - 4*a*c)*c)*a^7*b^2*c^5 + 6656*sqrt(2)*sqrt(b^2 - 4*a*c)*sq
rt(b*c + sqrt(b^2 - 4*a*c)*c)*a^6*b^3*c^5 + 2560*sqrt(2)*sqrt(b^2 - 4*a*c)*
sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^5*b^4*c^5 - 3328*sqrt(2)*sqrt(b^2 - 4*a*c
)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^6*b^2*c^6 - 2*(b^2 - 4*a*c)*a^2*b^10*c^
2 + 128*(b^2 - 4*a*c)*a^3*b^8*c^3 - 1344*(b^2 - 4*a*c)*a^4*b^6*c^4 + 5120*(
b^2 - 4*a*c)*a^5*b^4*c^5 - 6656*(b^2 - 4*a*c)*a^6*b^2*c^6)*A + 6*(6*a^3*b^1
1*c^2 - 88*a^4*b^9*c^3 + 448*a^5*b^7*c^4 - 768*a^6*b^5*c^5 - 512*a^7*b^3*c^
6 + 2048*a^8*b*c^7 - 3*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*
c)*c)*a^3*b^11 + 44*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*
c)*a^4*b^9*c + 6*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*
a^3*b^10*c - 224*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*
a^5*b^7*c^2 - 64*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*
a^4*b^8*c^2 - 3*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a
^3*b^9*c^2 + 384*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*
a^6*b^5*c^3 + 192*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)
*a^5*b^6*c^3 + 32*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)
*a^4*b^7*c^3 + 256*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c
)*a^7*b^3*c^4 - 96*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c
)*a^5*b^5*c^4 - 1024*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)
*c)*a^8*b*c^5 - 512*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*
c)*a^7*b^2*c^5 + 256*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)
*c)*a^7*b*c^6 - 6*(b^2 - 4*a*c)*a^3*b^9*c^2 + 64*(b^2 - 4*a*c)*a^4*b^7*c^3
- 192*(b^2 - 4*a*c)*a^5*b^5*c^4 + 512*(b^2 - 4*a*c)*a^7*b*c^6)*B)*arctan(2*
sqrt(1/2)*x/sqrt((a*b^5 - 8*a^2*b^3*c + 16*a^3*b*c^2 + sqrt((a*b^5 - 8*a^2*
b^3*c + 16*a^3*b*c^2)^2 - 4*(a^2*b^4 - 8*a^3*b^2*c + 16*a^4*c^2)*(a*b^4*c -
8*a^2*b^2*c^2 + 16*a^3*c^3)))/(a*b^4*c - 8*a^2*b^2*c^2 + 16*a^3*c^3)))/((a
^3*b^10 - 20*a^4*b^8*c - 2*a^3*b^9*c + 160*a^5*b^6*c^2 + 32*a^4*b^7*c^2 + a
^3*b^8*c^2 - 640*a^6*b^4*c^3 - 192*a^5*b^5*c^3 - 16*a^4*b^6*c^3 + 1280*a^7*
b^2*c^4 + 512*a^6*b^3*c^4 + 96*a^5*b^4*c^4 - 1024*a^8*c^5 - 512*a^7*b*c^5 -
256*a^6*b^2*c^5 + 256*a^7*c^6)*abs(a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2)*abs(c
)) - 1/64*((2*b^4*c^2 + 32*a*b^2*c^3 - 160*a^2*c^4 - sqrt(2)*sqrt(b^2 - 4*a
*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^4 - 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt
(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^2*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c
- sqrt(b^2 - 4*a*c)*c)*b^3*c + 80*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt
(b^2 - 4*a*c)*c)*a^2*c^2 + 40*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2
- 4*a*c)*c)*a*b*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*
c)*c)*b^2*c^2 - 20*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c
)*a*c^3 - 2*(b^2 - 4*a*c)*b^2*c^2 - 40*(b^2 - 4*a*c)*a*c^3)*(a*b^4 - 8*a^2*
b^2*c + 16*a^3*c^2)^2*A - 12*(2*a*b^3*c^2 - 8*a^2*b*c^3 - sqrt(2)*sqrt(b^2
- 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^3 + 4*sqrt(2)*sqrt(b^2 - 4*a*c
)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqr
t(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^2*c - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c -
sqrt(b^2 - 4*a*c)*c)*a*b*c^2 - 2*(b^2 - 4*a*c)*a*b*c^2)*(a*b^4 - 8*a^2*b^2
*c + 16*a^3*c^2)^2*B - 2*(sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^9 - 2

8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^7*c - 2*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^8*c + 2*a*b^9*c + 240*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^5*c^2 + 48*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^6*c^2 + sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^7*c^2 - 56*a^2*b^7*c^2 - 832*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*b^3*c^3 - 288*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^4*c^3 - 24*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^5*c^3 + 480*a^3*b^5*c^3 + 1024*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^5*b*c^4 + 512*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*b^2*c^4 + 144*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^3*c^4 - 1664*a^4*b^3*c^4 - 256*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*b*c^5 + 2048*a^5*b*c^5 - 2*(b^2 - 4*a*c)*a*b^7*c + 48*(b^2 - 4*a*c)*a^2*b^5*c^2 - 288*(b^2 - 4*a*c)*a^3*b^3*c^3 + 512*(b^2 - 4*a*c)*a^4*b*c^4)*A*abs(a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2) - 6*(sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^8 - 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^6*c - 2*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^7*c + 2*a^2*b^8*c + 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^5*c^2 + sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^6*c^2 - 16*a^3*b^6*c^2 + 128*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^5*b^2*c^3 + 32*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*b^3*c^3 - 4*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^4*c^3 - 256*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^6*c^4 - 128*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^5*b*c^4 - 16*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*b^2*c^4 + 256*a^5*b^2*c^4 + 64*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^5*c^5 - 512*a^6*c^5 - 2*(b^2 - 4*a*c)*a^2*b^6*c + 8*(b^2 - 4*a*c)*a^3*b^4*c^2 + 32*(b^2 - 4*a*c)*a^4*b^2*c^3 - 128*(b^2 - 4*a*c)*a^5*c^4)*B*abs(a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2) + (2*a^2*b^12*c^2 - 136*a^3*b^10*c^3 + 1856*a^4*b^8*c^4 - 10496*a^5*b^6*c^5 + 27136*a^6*b^4*c^6 - 26624*a^7*b^2*c^7 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^12 + 68*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^10*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^11*c - 928*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*b^8*c^2 - 128*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^9*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^10*c^2 + 5248*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^5*b^6*c^3 + 1344*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*b^7*c^3 + 64*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^8*c^3 - 13568*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^6*b^4*c^4 - 5120*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^5*b^5*c^4 - 672*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*b^6*c^4 + 13312*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^7*b^2*c^5 + 6656*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^6*b^3*c^5 + 2560*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^5*b^4*c^5 - 3328*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^6*b^2*c^6 - 2*(b^2 - 4*a*c)*a^2*b^10*c^2 + 128*(b^2 - 4*a*c)*a^3*b^8*c^3 - 1344*(b^2 - 4*a*c)*a^4*b^6*c^4 + 5120*(b^2 - 4*a*c)*a^5*b^4*c^5 - 6656*(b^2 - 4*a*c)*a^6*b^2*c^6)*A + 6*(6*a^3*b^11*c^2 - 88*a^4*b^9*c^3 + 448*a^5*b^7*c^4 - 768*a^6*b^5*c^5 - 512*a^7*b^

```
3*c^6 + 2048*a^8*b*c^7 - 3*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 -
4*a*c)*c)*a^3*b^11 + 44*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a
*c)*c)*a^4*b^9*c + 6*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)
*c)*a^3*b^10*c - 224*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)
*c)*a^5*b^7*c^2 - 64*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)
*c)*a^4*b^8*c^2 - 3*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*
c)*a^3*b^9*c^2 + 384*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)
*c)*a^6*b^5*c^3 + 192*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c
)*c)*a^5*b^6*c^3 + 32*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c
)*c)*a^4*b^7*c^3 + 256*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*
c)*c)*a^7*b^3*c^4 - 96*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*
c)*c)*a^5*b^5*c^4 - 1024*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*
a*c)*c)*a^8*b*c^5 - 512*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a
*c)*c)*a^7*b^2*c^5 + 256*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*
a*c)*c)*a^7*b*c^6 - 6*(b^2 - 4*a*c)*a^3*b^9*c^2 + 64*(b^2 - 4*a*c)*a^4*b^7*
c^3 - 192*(b^2 - 4*a*c)*a^5*b^5*c^4 + 512*(b^2 - 4*a*c)*a^7*b*c^6)*B)*arcta
n(2*sqrt(1/2)*x/sqrt((a*b^5 - 8*a^2*b^3*c + 16*a^3*b*c^2 - sqrt((a*b^5 - 8*
a^2*b^3*c + 16*a^3*b*c^2)^2 - 4*(a^2*b^4 - 8*a^3*b^2*c + 16*a^4*c^2)*(a*b^4
*c - 8*a^2*b^2*c^2 + 16*a^3*c^3)))/(a*b^4*c - 8*a^2*b^2*c^2 + 16*a^3*c^3)))
/((a^3*b^10 - 20*a^4*b^8*c - 2*a^3*b^9*c + 160*a^5*b^6*c^2 + 32*a^4*b^7*c^2
 + a^3*b^8*c^2 - 640*a^6*b^4*c^3 - 192*a^5*b^5*c^3 - 16*a^4*b^6*c^3 + 1280*
a^7*b^2*c^4 + 512*a^6*b^3*c^4 + 96*a^5*b^4*c^4 - 1024*a^8*c^5 - 512*a^7*b*c
^5 - 256*a^6*b^2*c^5 + 256*a^7*c^6)*abs(a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2)*a
bs(c)) - 1/8*(12*B*a*b*c^2*x^7 - A*b^2*c^2*x^7 - 20*A*a*c^3*x^7 + 19*B*a*b^
2*c*x^5 - 2*A*b^3*c*x^5 - 4*B*a^2*c^2*x^5 - 28*A*a*b*c^2*x^5 + 5*B*a*b^3*x^
3 - A*b^4*x^3 + 16*B*a^2*b*c*x^3 - 5*A*a*b^2*c*x^3 - 36*A*a^2*c^2*x^3 + 3*B
*a^2*b^2*x + A*a*b^3*x + 12*B*a^3*c*x - 16*A*a^2*b*c*x)/((a*b^4 - 8*a^2*b^2
*c + 16*a^3*c^2)*(c*x^4 + b*x^2 + a)^2)
```

Mupad [B] (verification not implemented)

Time = 9.69 (sec) , antiderivative size = 18992, normalized size of antiderivative = 43.36

$$\int \frac{x^2(A+Bx^2)}{(a+bx^2+cx^4)^3}\,dx = \text{Too large to display}$$

```
[In] int((x^2*(A + B*x^2))/(a + b*x^2 + c*x^4)^3,x)
```

```
[Out] ((x^3*(A*b^4 + 36*A*a^2*c^2 - 5*B*a*b^3 + 5*A*a*b^2*c - 16*B*a^2*b*c))/(8*a
*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) - (x*(A*b^3 + 3*B*a*b^2 + 12*B*a^2*c - 16*
A*a*b*c))/(8*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (x^5*(4*B*a^2*c^2 + 2*A*b^3*
c + 28*A*a*b*c^2 - 19*B*a*b^2*c))/(8*a*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (c
*x^7*(20*A*a*c^2 + A*b^2*c - 12*B*a*b*c))/(8*a*(b^4 + 16*a^2*c^2 - 8*a*b^2*
c)))/(x^4*(2*a*c + b^2) + a^2 + c^2*x^8 + 2*a*b*x^2 + 2*b*c*x^6) + atan((((
(256*A*a*b^13*c^2 - 3145728*B*a^8*c^8 + 4194304*A*a^7*b*c^8 - 9216*A*a^2*b^
```

11*c^3 + 122880*A*a^3*b^9*c^4 - 819200*A*a^4*b^7*c^5 + 2949120*A*a^5*b^5*c^6 - 5505024*A*a^6*b^3*c^7 + 768*B*a^2*b^12*c^2 - 12288*B*a^3*b^10*c^3 + 61440*B*a^4*b^8*c^4 - 983040*B*a^6*b^4*c^6 + 3145728*B*a^7*b^2*c^7)/(512*(a^2*b^12 + 4096*a^8*c^6 - 24*a^3*b^10*c + 240*a^4*b^8*c^2 - 1280*a^5*b^6*c^3 + 3840*a^6*b^4*c^4 - 6144*a^7*b^2*c^5)) - (x*(-(A^2*b^17 + 9*B^2*a^2*b^15 + A^2*b^2*(-(4*a*c - b^2)^15)^(1/2) + 9*B^2*a^2*(-(4*a*c - b^2)^15)^(1/2) + 6*A*B*a*b^16 + 1140*A^2*a^2*b^13*c^2 - 10160*A^2*a^3*b^11*c^3 + 34880*A^2*a^4*b^9*c^4 + 43776*A^2*a^5*b^7*c^5 - 680960*A^2*a^6*b^5*c^6 + 1863680*A^2*a^7*b^3*c^7 - 5040*B^2*a^4*b^11*c^2 + 37440*B^2*a^5*b^9*c^3 - 103680*B^2*a^6*b^7*c^4 - 9216*B^2*a^7*b^5*c^5 + 552960*B^2*a^8*b^3*c^6 + 983040*A*B*a^9*c^8 - 55*A^2*a*b^15*c - 25*A^2*a*c*(-(4*a*c - b^2)^15)^(1/2) - 1720320*A^2*a^8*b*c^8 + 180*B^2*a^3*b^13*c - 737280*B^2*a^9*b*c^7 + 240*A*B*a^3*b^12*c^2 + 24000*A*B*a^4*b^10*c^3 - 241920*A*B*a^5*b^8*c^4 + 992256*A*B*a^6*b^6*c^5 - 1781760*A*B*a^7*b^4*c^6 + 737280*A*B*a^8*b^2*c^7 + 6*A*B*a*b*(-(4*a*c - b^2)^15)^(1/2) - 180*A*B*a^2*b^14*c)/(512*(a^3*b^20 + 1048576*a^13*c^10 - 40*a^4*b^18*c + 720*a^5*b^16*c^2 - 7680*a^6*b^14*c^3 + 53760*a^7*b^12*c^4 - 258048*a^8*b^10*c^5 + 860160*a^9*b^8*c^6 - 1966080*a^10*b^6*c^7 + 2949120*a^11*b^4*c^8 - 2621440*a^12*b^2*c^9)))^(1/2)*(262144*a^7*b*c^7 - 256*a^2*b^11*c^2 + 5120*a^3*b^9*c^3 - 40960*a^4*b^7*c^4 + 163840*a^5*b^5*c^5 - 327680*a^6*b^3*c^6))/(32*(a^2*b^8 + 256*a^6*c^4 - 16*a^3*b^6*c + 96*a^4*b^4*c^2 - 256*a^5*b^2*c^3)))*(-(A^2*b^17 + 9*B^2*a^2*b^15 + A^2*b^2*(-(4*a*c - b^2)^15)^(1/2) + 9*B^2*a^2*(-(4*a*c - b^2)^15)^(1/2) + 6*A*B*a*b^16 + 1140*A^2*a^2*b^13*c^2 - 10160*A^2*a^3*b^11*c^3 + 34880*A^2*a^4*b^9*c^4 + 43776*A^2*a^5*b^7*c^5 - 680960*A^2*a^6*b^5*c^6 + 1863680*A^2*a^7*b^3*c^7 - 5040*B^2*a^4*b^11*c^2 + 37440*B^2*a^5*b^9*c^3 - 103680*B^2*a^6*b^7*c^4 - 9216*B^2*a^7*b^5*c^5 + 552960*B^2*a^8*b^3*c^6 + 983040*A*B*a^9*c^8 - 55*A^2*a*b^15*c - 25*A^2*a*c*(-(4*a*c - b^2)^15)^(1/2) - 1720320*A^2*a^8*b*c^8 + 180*B^2*a^3*b^13*c - 737280*B^2*a^9*b*c^7 + 240*A*B*a^3*b^12*c^2 + 24000*A*B*a^4*b^10*c^3 - 241920*A*B*a^5*b^8*c^4 + 992256*A*B*a^6*b^6*c^5 - 1781760*A*B*a^7*b^4*c^6 + 737280*A*B*a^8*b^2*c^7 + 6*A*B*a*b*(-(4*a*c - b^2)^15)^(1/2) - 180*A*B*a^2*b^14*c)/(512*(a^3*b^20 + 1048576*a^13*c^10 - 40*a^4*b^18*c + 720*a^5*b^16*c^2 - 7680*a^6*b^14*c^3 + 53760*a^7*b^12*c^4 - 258048*a^8*b^10*c^5 + 860160*a^9*b^8*c^6 - 1966080*a^10*b^6*c^7 + 2949120*a^11*b^4*c^8 - 2621440*a^12*b^2*c^9)))^(1/2) + (x*(A^2*b^6*c^3 - 800*A^2*a^3*c^6 + 288*B^2*a^4*c^5 + 1472*A^2*a^2*b^2*c^5 + 234*B^2*a^2*b^4*c^3 + 144*B^2*a^3*b^2*c^4 - 34*A^2*a*b^4*c^4 - 1104*A*B*a^2*b^3*c^4 + 6*A*B*a*b^5*c^3 - 288*A*B*a^3*b*c^5))/(32*(a^2*b^8 + 256*a^6*c^4 - 16*a^3*b^6*c + 96*a^4*b^4*c^2 - 256*a^5*b^2*c^3)))*(-(A^2*b^17 + 9*B^2*a^2*b^15 + A^2*b^2*(-(4*a*c - b^2)^15)^(1/2) + 9*B^2*a^2*(-(4*a*c - b^2)^15)^(1/2) + 6*A*B*a*b^16 + 1140*A^2*a^2*b^13*c^2 - 10160*A^2*a^3*b^11*c^3 + 34880*A^2*a^4*b^9*c^4 + 43776*A^2*a^5*b^7*c^5 - 680960*A^2*a^6*b^5*c^6 + 1863680*A^2*a^7*b^3*c^7 - 5040*B^2*a^4*b^11*c^2 + 37440*B^2*a^5*b^9*c^3 - 103680*B^2*a^6*b^7*c^4 - 9216*B^2*a^7*b^5*c^5 + 552960*B^2*a^8*b^3*c^6 + 983040*A*B*a^9*c^8 - 55*A^2*a*b^15*c - 25*A^2*a*c*(-(4*a*c - b^2)^15)^(1/2) - 1720320*A^2*a^8*b*c^8 + 180*B^2*a^3*b^13*c - 737280*B^2*a^9*b*c^7 + 240*A*B*a^3*b^12*c^2 + 24000*A*B*a^4*b^10*c^3 - 241920*A*B*a^5*b^8 *c^4 + 992256*A*B*a^6*b^6*c^5 - 1781760*A*B*a^7*b^4*c^6 + 737280*A*B*a^8*b^2*c^7 + 6*A*B*a*b*(-(4*a*c - b^2)^15)^(1/2) - 180*A*B*a^2*b^14*c)/(512*(a^3*b^20 + 1048576*a^13*c^10 - 40*a^4*b^18*c + 720*a^5*b^16*c^2 - 7680*a^6*b^14*c^3 + 53760*a^7*b^12*c^4 - 258048*a^8*b^10*c^5 + 860160*a^9*b^8*c^6 - 1966080*a^10*b^6*c^7 + 2949120*a^11*b^4*c^8 - 2621440*a^12*b^2*c^9)))^(1/2)*1i - (((256*A*a*b^13*c^2 - 3145728*B*a^8*c^8 + 4194304*A*a^7*b*c^8 - 9216*A*a^2*b^11*c^3 + 122880*A*a^3*b^9*c^4 - 819200*A*a^4*b^7*c^5 + 2949120*A*a^5*b^5*c^6 - 5505024*A*a^6*b^3*c^7 + 768*B*a^2*b^12*c^2 - 12288*B*a^3*b^10*c^3 + 61440*B*a^4*b^8*c^4 - 983040*B*a^6*b^4*c^6 + 3145728*B*a^7*b^2*c^7)/(512*(a^2*b^12 + 4096*a^8*c^6 - 24*a^3*b^10*c + 240*a^4*b^8*c^2 - 1280*a^5*b^6*c^3 + 3840*a^6*b^4*c^4 - 6144*a^7*b^2*c^5)) + (x*(-(A^2*b^17 + 9*B^2*a^2*b^15 + A^2*b^2*(-(4*a*c - b^2)^15)^(1/2) + 9*B^2*a^2*(-(4*a*c - b^2)^15)^(1/2) + 6*A*B*a*b^16 + 1140*A^2*a^2*b^13*c^2 - 10160*A^2*a^3*b^11*c^3 + 34880*A^2*a^4*b^9*c^4 + 43776*A^2*a^5*b^7*c^5 - 680960*A^2*a^6*b^5*c^6 + 1863680*A^2*a^7*b^3*c^7 - 5040*B^2*a^4*b^11*c^2 + 37440*B^2*a^5*b^9*c^3 - 103680*B^2*a^6*b^7*c^4 - 9216*B^2*a^7*b^5*c^5 + 552960*B^2*a^8*b^3*c^6 + 983040*A*B*a^9*c^8 - 55*A^2*a*b^15*c - 25*A^2*a*c*(-(4*a*c - b^2)^15)^(1/2) - 1720320*A^2*a^8*b*c^8 + 180*B^2*a^3*b^13*c - 737280*B^2*a^9*b*c^7 + 240*A*B*a^3*b^12*c^2 + 24000*A*B*a^4*b^10*c^3 - 241920*A*B*a^5*b^8*c^4 + 992256*A*B*a^6*b^6*c^5 - 1781760*A*B*a^7*b^4*c^6 + 737280*A*B*a^8*b^2*c^7 + 6*A*B*a*b*(-(4*a*c - b^2)^15)^(1/2) - 180*A*B*a^2*b^14*c)/(512*(a^3*b^20 + 1048576*a^13*c^10 - 40*a^4*b^18*c + 720*a^5*b^16*c^2 - 7680*a^6*b^14*c^3 + 53760*a^7*b^12*c^4 - 258048*a^8*b^10*c^5 + 860160*a^9*b^8*c^6 - 1966080*a^10*b^6*c^7 + 2949120*a^11*b^4*c^8 - 2621440*a^12*b^2*c^9)))^(1/2)*(262144*a^7*b*c^7 - 256*a^2*b^11*c^2 + 5120*a^3*b^9*c^3 - 40960*a^4*b^7*c^4 + 163840*a^5*b^5*c^5 - 327680*a^6*b^3*c^6))/(32*(a^2*b^8 + 256*a^6*c^4 - 16*a^3*b^6*c + 96*a^4*b^4*c^2 - 256*a^5*b^2*c^3)))*(-(A^2*b^17 + 9*B^2*a^2*b^15 + A^2*b^2*(-(4*a*c - b^2)^15)^(1/2) + 9*B^2*a^2*(-(4*a*c - b^2)^15)^(1/2) + 6*A*B*a*b^16 + 1140*A^2*a^2*b^13*c^2 - 10160*A^2*a^3*b^11*c^3 + 34880*A^2*a^4*b^9*c^4 + 43776*A^2*a^5*b^7*c^5 - 680960*A^2*a^6*b^5*c^6 + 1863680*A^2*a^7*b^3*c^7 - 5040*B^2*a^4*b^11*c^2 + 37440*B^2*a^5*b^9*c^3 - 103680*B^2*a^6*b^7*c^4 - 9216*B^2*a^7*b^5*c^5 + 552960*B^2*a^8*b^3*c^6 + 983040*A*B*a^9*c^8 - 55*A^2*a*b^15*c - 25*A^2*a*c*(-(4*a*c - b^2)^15)^(1/2) - 1720320*A^2*a^8*b*c^8 + 180*B^2*a^3*b^13*c - 737280*B^2*a^9*b*c^7 + 240*A*B*a^3*b^12*c^2 + 24000*A*B*a^4*b^10*c^3 - 241920*A*B*a^5*b^8*c^4 + 992256*A*B*a^6*b^6*c^5 - 1781760*A*B*a^7*b^4*c^6 + 737280*A*B*a^8*b^2*c^7 + 6*A*B*a*b*(-(4*a*c - b^2)^15)^(1/2) - 180*A*B*a^2*b^14*c)/(512*(a^3*b^20 + 1048576*a^13*c^10 - 40*a^4*b^18*c + 720*a^5*b^16*c^2 - 7680*a^6*b^14*c^3 + 53760*a^7*b^12*c^4 - 258048*a^8*b^10*c^5 + 860160*a^9*b^8*c^6 - 1966080*a^10*b^6*c^7 + 2949120*a^11*b^4*c^8 - 2621440*a^12*b^2*c^9)))^(1/2) - (x*(A^2*b^6*c^3 - 800*A^2*a^3*c^6 + 288*B^2*a^4*c^5 + 1472*A^2*a^2*b^2*c^5 + 234*B^2*a^2*b^4*c^3 + 144*B^2*a^3*b^2*c^4 - 34*A^2*a*b^4*c^4 - 1104*A*B*a^2*b^3*c^4 + 6*A*B*a*b^5*c^3 - 288*A*B*a^3*b*c^5))/(32*(a^2*b^8 + 256*a^6*c^4 - 16*a^3*b^6*c + 96*a^4*b^4*c^2 - 256*a^5*b^2*c^3)))*(-(A^2*b^17 + 9*B^2*a^2*b^15 + A^2*b^2*(-(4*a*c - b^2)^15)^(1/2) + 9*B^2*a^2*(-(4*a*c - b^2)^15)^(1/2) + 6*A*B*a*b^16 + 1140*A^2*a^2*b^13*c^2 - 10

160*A^2*a^3*b^11*c^3 + 34880*A^2*a^4*b^9*c^4 + 43776*A^2*a^5*b^7*c^5 - 6809
60*A^2*a^6*b^5*c^6 + 1863680*A^2*a^7*b^3*c^7 - 5040*B^2*a^4*b^11*c^2 + 3744
0*B^2*a^5*b^9*c^3 - 103680*B^2*a^6*b^7*c^4 - 9216*B^2*a^7*b^5*c^5 + 552960*
B^2*a^8*b^3*c^6 + 983040*A*B*a^9*c^8 - 55*A^2*a*b^15*c - 25*A^2*a*c*(-(4*a*
c - b^2)^15)^(1/2) - 1720320*A^2*a^8*b*c^8 + 180*B^2*a^3*b^13*c - 737280*B^
2*a^9*b*c^7 + 240*A*B*a^3*b^12*c^2 + 24000*A*B*a^4*b^10*c^3 - 241920*A*B*a^
5*b^8*c^4 + 992256*A*B*a^6*b^6*c^5 - 1781760*A*B*a^7*b^4*c^6 + 737280*A*B*a
^8*b^2*c^7 + 6*A*B*a*b*(-(4*a*c - b^2)^15)^(1/2) - 180*A*B*a^2*b^14*c)/(512
*(a^3*b^20 + 1048576*a^13*c^10 - 40*a^4*b^18*c + 720*a^5*b^16*c^2 - 7680*a^
6*b^14*c^3 + 53760*a^7*b^12*c^4 - 258048*a^8*b^10*c^5 + 860160*a^9*b^8*c^6
 - 1966080*a^10*b^6*c^7 + 2949120*a^11*b^4*c^8 - 2621440*a^12*b^2*c^9)))^(1/
2)*1i)/((((256*A*a*b^13*c^2 - 3145728*B*a^8*c^8 + 4194304*A*a^7*b*c^8 - 921
6*A*a^2*b^11*c^3 + 122880*A*a^3*b^9*c^4 - 819200*A*a^4*b^7*c^5 + 2949120*A*
a^5*b^5*c^6 - 5505024*A*a^6*b^3*c^7 + 768*B*a^2*b^12*c^2 - 12288*B*a^3*b^10
*c^3 + 61440*B*a^4*b^8*c^4 - 983040*B*a^6*b^4*c^6 + 3145728*B*a^7*b^2*c^7)/
(512*(a^2*b^12 + 4096*a^8*c^6 - 24*a^3*b^10*c + 240*a^4*b^8*c^2 - 1280*a^5*
b^6*c^3 + 3840*a^6*b^4*c^4 - 6144*a^7*b^2*c^5)) - (x*(-(A^2*b^17 + 9*B^2*a^
2*b^15 + A^2*b^2*(-(4*a*c - b^2)^15)^(1/2) + 9*B^2*a^2*(-(4*a*c - b^2)^15)^
(1/2) + 6*A*B*a*b^16 + 1140*A^2*a^2*b^13*c^2 - 10160*A^2*a^3*b^11*c^3 + 348
80*A^2*a^4*b^9*c^4 + 43776*A^2*a^5*b^7*c^5 - 680960*A^2*a^6*b^5*c^6 + 18636
80*A^2*a^7*b^3*c^7 - 5040*B^2*a^4*b^11*c^2 + 37440*B^2*a^5*b^9*c^3 - 103680
*B^2*a^6*b^7*c^4 - 9216*B^2*a^7*b^5*c^5 + 552960*B^2*a^8*b^3*c^6 + 983040*A
*B*a^9*c^8 - 55*A^2*a*b^15*c - 25*A^2*a*c*(-(4*a*c - b^2)^15)^(1/2) - 17203
20*A^2*a^8*b*c^8 + 180*B^2*a^3*b^13*c - 737280*B^2*a^9*b*c^7 + 240*A*B*a^3*
b^12*c^2 + 24000*A*B*a^4*b^10*c^3 - 241920*A*B*a^5*b^8*c^4 + 992256*A*B*a^6
*b^6*c^5 - 1781760*A*B*a^7*b^4*c^6 + 737280*A*B*a^8*b^2*c^7 + 6*A*B*a*b*(-(
4*a*c - b^2)^15)^(1/2) - 180*A*B*a^2*b^14*c)/(512*(a^3*b^20 + 1048576*a^13*
c^10 - 40*a^4*b^18*c + 720*a^5*b^16*c^2 - 7680*a^6*b^14*c^3 + 53760*a^7*b^1
2*c^4 - 258048*a^8*b^10*c^5 + 860160*a^9*b^8*c^6 - 1966080*a^10*b^6*c^7 + 2
949120*a^11*b^4*c^8 - 2621440*a^12*b^2*c^9)))^(1/2)*(262144*a^7*b*c^7 - 256
*a^2*b^11*c^2 + 5120*a^3*b^9*c^3 - 40960*a^4*b^7*c^4 + 163840*a^5*b^5*c^5 -
 327680*a^6*b^3*c^6))/(32*(a^2*b^8 + 256*a^6*c^4 - 16*a^3*b^6*c + 96*a^4*b^
4*c^2 - 256*a^5*b^2*c^3)))*(-(A^2*b^17 + 9*B^2*a^2*b^15 + A^2*b^2*(-(4*a*c
 - b^2)^15)^(1/2) + 9*B^2*a^2*(-(4*a*c - b^2)^15)^(1/2) + 6*A*B*a*b^16 + 114
0*A^2*a^2*b^13*c^2 - 10160*A^2*a^3*b^11*c^3 + 34880*A^2*a^4*b^9*c^4 + 43776
*A^2*a^5*b^7*c^5 - 680960*A^2*a^6*b^5*c^6 + 1863680*A^2*a^7*b^3*c^7 - 5040*
B^2*a^4*b^11*c^2 + 37440*B^2*a^5*b^9*c^3 - 103680*B^2*a^6*b^7*c^4 - 9216*B^
2*a^7*b^5*c^5 + 552960*B^2*a^8*b^3*c^6 + 983040*A*B*a^9*c^8 - 55*A^2*a*b^15
*c - 25*A^2*a*c*(-(4*a*c - b^2)^15)^(1/2) - 1720320*A^2*a^8*b*c^8 + 180*B^2
*a^3*b^13*c - 737280*B^2*a^9*b*c^7 + 240*A*B*a^3*b^12*c^2 + 24000*A*B*a^4*b
^10*c^3 - 241920*A*B*a^5*b^8*c^4 + 992256*A*B*a^6*b^6*c^5 - 1781760*A*B*a^7
*b^4*c^6 + 737280*A*B*a^8*b^2*c^7 + 6*A*B*a*b*(-(4*a*c - b^2)^15)^(1/2) - 1
80*A*B*a^2*b^14*c)/(512*(a^3*b^20 + 1048576*a^13*c^10 - 40*a^4*b^18*c + 720
*a^5*b^16*c^2 - 7680*a^6*b^14*c^3 + 53760*a^7*b^12*c^4 - 258048*a^8*b^10*c^
5 + 860160*a^9*b^8*c^6 - 1966080*a^10*b^6*c^7 + 2949120*a^11*b^4*c^8 - 2621

```
440*a^12*b^2*c^9)))^(1/2) + (x*(A^2*b^6*c^3 - 800*A^2*a^3*c^6 + 288*B^2*a^4
*c^5 + 1472*A^2*a^2*b^2*c^5 + 234*B^2*a^2*b^4*c^3 + 144*B^2*a^3*b^2*c^4 - 3
4*A^2*a*b^4*c^4 - 1104*A*B*a^2*b^3*c^4 + 6*A*B*a*b^5*c^3 - 288*A*B*a^3*b*c^
5))/(32*(a^2*b^8 + 256*a^6*c^4 - 16*a^3*b^6*c + 96*a^4*b^4*c^2 - 256*a^5*b^
2*c^3)))*(-(A^2*b^17 + 9*B^2*a^2*b^15 + A^2*b^2*(-(4*a*c - b^2)^15)^(1/2) +
 9*B^2*a^2*(-(4*a*c - b^2)^15)^(1/2) + 6*A*B*a*b^16 + 1140*A^2*a^2*b^13*c^2
 - 10160*A^2*a^3*b^11*c^3 + 34880*A^2*a^4*b^9*c^4 + 43776*A^2*a^5*b^7*c^5 -
 680960*A^2*a^6*b^5*c^6 + 1863680*A^2*a^7*b^3*c^7 - 5040*B^2*a^4*b^11*c^2 +
 37440*B^2*a^5*b^9*c^3 - 103680*B^2*a^6*b^7*c^4 - 9216*B^2*a^7*b^5*c^5 + 55
2960*B^2*a^8*b^3*c^6 + 983040*A*B*a^9*c^8 - 55*A^2*a*b^15*c - 25*A^2*a*c*(-
(4*a*c - b^2)^15)^(1/2) - 1720320*A^2*a^8*b*c^8 + 180*B^2*a^3*b^13*c - 7372
80*B^2*a^9*b*c^7 + 240*A*B*a^3*b^12*c^2 + 24000*A*B*a^4*b^10*c^3 - 241920*A
*B*a^5*b^8*c^4 + 992256*A*B*a^6*b^6*c^5 - 1781760*A*B*a^7*b^4*c^6 + 737280*
A*B*a^8*b^2*c^7 + 6*A*B*a*b*(-(4*a*c - b^2)^15)^(1/2) - 180*A*B*a^2*b^14*c)
/(512*(a^3*b^20 + 1048576*a^13*c^10 - 40*a^4*b^18*c + 720*a^5*b^16*c^2 - 76
80*a^6*b^14*c^3 + 53760*a^7*b^12*c^4 - 258048*a^8*b^10*c^5 + 860160*a^9*b^8
*c^6 - 1966080*a^10*b^6*c^7 + 2949120*a^11*b^4*c^8 - 2621440*a^12*b^2*c^9))
)^(1/2) + (((256*A*a*b^13*c^2 - 3145728*B*a^8*c^8 + 4194304*A*a^7*b*c^8 - 9
216*A*a^2*b^11*c^3 + 122880*A*a^3*b^9*c^4 - 819200*A*a^4*b^7*c^5 + 2949120*
A*a^5*b^5*c^6 - 5505024*A*a^6*b^3*c^7 + 768*B*a^2*b^12*c^2 - 12288*B*a^3*b^
10*c^3 + 61440*B*a^4*b^8*c^4 - 983040*B*a^6*b^4*c^6 + 3145728*B*a^7*b^2*c^7
)/(512*(a^2*b^12 + 4096*a^8*c^6 - 24*a^3*b^10*c + 240*a^4*b^8*c^2 - 1280*a^
5*b^6*c^3 + 3840*a^6*b^4*c^4 - 6144*a^7*b^2*c^5)) + (x*(-(A^2*b^17 + 9*B^2*
a^2*b^15 + A^2*b^2*(-(4*a*c - b^2)^15)^(1/2) + 9*B^2*a^2*(-(4*a*c - b^2)^15
)^(1/2) + 6*A*B*a*b^16 + 1140*A^2*a^2*b^13*c^2 - 10160*A^2*a^3*b^11*c^3 + 3
4880*A^2*a^4*b^9*c^4 + 43776*A^2*a^5*b^7*c^5 - 680960*A^2*a^6*b^5*c^6 + 186
3680*A^2*a^7*b^3*c^7 - 5040*B^2*a^4*b^11*c^2 + 37440*B^2*a^5*b^9*c^3 - 1036
80*B^2*a^6*b^7*c^4 - 9216*B^2*a^7*b^5*c^5 + 552960*B^2*a^8*b^3*c^6 + 983040
*A*B*a^9*c^8 - 55*A^2*a*b^15*c - 25*A^2*a*c*(-(4*a*c - b^2)^15)^(1/2) - 172
0320*A^2*a^8*b*c^8 + 180*B^2*a^3*b^13*c - 737280*B^2*a^9*b*c^7 + 240*A*B*a^
3*b^12*c^2 + 24000*A*B*a^4*b^10*c^3 - 241920*A*B*a^5*b^8*c^4 + 992256*A*B*a
^6*b^6*c^5 - 1781760*A*B*a^7*b^4*c^6 + 737280*A*B*a^8*b^2*c^7 + 6*A*B*a*b*(
-(4*a*c - b^2)^15)^(1/2) - 180*A*B*a^2*b^14*c)/(512*(a^3*b^20 + 1048576*a^1
3*c^10 - 40*a^4*b^18*c + 720*a^5*b^16*c^2 - 7680*a^6*b^14*c^3 + 53760*a^7*b
^12*c^4 - 258048*a^8*b^10*c^5 + 860160*a^9*b^8*c^6 - 1966080*a^10*b^6*c^7 +
 2949120*a^11*b^4*c^8 - 2621440*a^12*b^2*c^9)))^(1/2)*(262144*a^7*b*c^7 - 2
56*a^2*b^11*c^2 + 5120*a^3*b^9*c^3 - 40960*a^4*b^7*c^4 + 163840*a^5*b^5*c^5
 - 327680*a^6*b^3*c^6))/(32*(a^2*b^8 + 256*a^6*c^4 - 16*a^3*b^6*c + 96*a^4*
b^4*c^2 - 256*a^5*b^2*c^3)))*(-(A^2*b^17 + 9*B^2*a^2*b^15 + A^2*b^2*(-(4*a*
c - b^2)^15)^(1/2) + 9*B^2*a^2*(-(4*a*c - b^2)^15)^(1/2) + 6*A*B*a*b^16 + 1
140*A^2*a^2*b^13*c^2 - 10160*A^2*a^3*b^11*c^3 + 34880*A^2*a^4*b^9*c^4 + 437
76*A^2*a^5*b^7*c^5 - 680960*A^2*a^6*b^5*c^6 + 1863680*A^2*a^7*b^3*c^7 - 504
0*B^2*a^4*b^11*c^2 + 37440*B^2*a^5*b^9*c^3 - 103680*B^2*a^6*b^7*c^4 - 9216*
B^2*a^7*b^5*c^5 + 552960*B^2*a^8*b^3*c^6 + 983040*A*B*a^9*c^8 - 55*A^2*a*b^
15*c - 25*A^2*a*c*(-(4*a*c - b^2)^15)^(1/2) - 1720320*A^2*a^8*b*c^8 + 180*B
```

^2*a^3*b^13*c - 737280*B^2*a^9*b*c^7 + 240*A*B*a^3*b^12*c^2 + 24000*A*B*a^4*b^10*c^3 - 241920*A*B*a^5*b^8*c^4 + 992256*A*B*a^6*b^6*c^5 - 1781760*A*B*a^7*b^4*c^6 + 737280*A*B*a^8*b^2*c^7 + 6*A*B*a*b*(-(4*a*c - b^2)^15)^(1/2) - 180*A*B*a^2*b^14*c)/(512*(a^3*b^20 + 1048576*a^13*c^10 - 40*a^4*b^18*c + 720*a^5*b^16*c^2 - 7680*a^6*b^14*c^3 + 53760*a^7*b^12*c^4 - 258048*a^8*b^10*c^5 + 860160*a^9*b^8*c^6 - 1966080*a^10*b^6*c^7 + 2949120*a^11*b^4*c^8 - 2621440*a^12*b^2*c^9)))^(1/2) - (x*(A^2*b^6*c^3 - 800*A^2*a^3*c^6 + 288*B^2*a^4*c^5 + 1472*A^2*a^2*b^2*c^5 + 234*B^2*a^2*b^4*c^3 + 144*B^2*a^3*b^2*c^4 - 34*A^2*a*b^4*c^4 - 1104*A*B*a^2*b^3*c^4 + 6*A*B*a*b^5*c^3 - 288*A*B*a^3*b*c^5))/(32*(a^2*b^8 + 256*a^6*c^4 - 16*a^3*b^6*c + 96*a^4*b^4*c^2 - 256*a^5*b^2*c^3)))*(-(A^2*b^17 + 9*B^2*a^2*b^15 + A^2*b^2*(-(4*a*c - b^2)^15)^(1/2) + 9*B^2*a^2*(-(4*a*c - b^2)^15)^(1/2) + 6*A*B*a*b^16 + 1140*A^2*a^2*b^13*c^2 - 10160*A^2*a^3*b^11*c^3 + 34880*A^2*a^4*b^9*c^4 + 43776*A^2*a^5*b^7*c^5 - 680960*A^2*a^6*b^5*c^6 + 1863680*A^2*a^7*b^3*c^7 - 5040*B^2*a^4*b^11*c^2 + 37440*B^2*a^5*b^9*c^3 - 103680*B^2*a^6*b^7*c^4 - 9216*B^2*a^7*b^5*c^5 + 552960*B^2*a^8*b^3*c^6 + 983040*A*B*a^9*c^8 - 55*A^2*a*b^15*c - 25*A^2*a*c*(-(4*a*c - b^2)^15)^(1/2) - 1720320*A^2*a^8*b*c^8 + 180*B^2*a^3*b^13*c - 737280*B^2*a^9*b*c^7 + 240*A*B*a^3*b^12*c^2 + 24000*A*B*a^4*b^10*c^3 - 241920*A*B*a^5*b^8*c^4 + 992256*A*B*a^6*b^6*c^5 - 1781760*A*B*a^7*b^4*c^6 + 737280*A*B*a^8*b^2*c^7 + 6*A*B*a*b*(-(4*a*c - b^2)^15)^(1/2) - 180*A*B*a^2*b^14*c)/(512*(a^3*b^20 + 1048576*a^13*c^10 - 40*a^4*b^18*c + 720*a^5*b^16*c^2 - 7680*a^6*b^14*c^3 + 53760*a^7*b^12*c^4 - 258048*a^8*b^10*c^5 + 860160*a^9*b^8*c^6 - 1966080*a^10*b^6*c^7 + 2949120*a^11*b^4*c^8 - 2621440*a^12*b^2*c^9)))^(1/2) + (35*A^3*b^6*c^4 - 8000*A^3*a^3*c^7 - 12720*A^3*a^2*b^2*c^6 + 540*B^3*a^2*b^5*c^3 + 4320*B^3*a^3*b^3*c^4 - 2880*A*B^2*a^4*c^6 - 15*A^2*B*b^7*c^3 + 84*A^3*a*b^4*c^5 + 1728*B^3*a^4*b*c^5 + 135*A*B^2*a*b^6*c^3 - 360*A^2*B*a*b^5*c^4 + 26880*A^2*B*a^3*b*c^6 - 5580*A*B^2*a^2*b^4*c^4 - 20592*A*B^2*a^3*b^2*c^5 + 15696*A^2*B*a^2*b^3*c^5)/(256*(a^2*b^12 + 4096*a^8*c^6 - 24*a^3*b^10*c + 240*a^4*b^8*c^2 - 1280*a^5*b^6*c^3 + 3840*a^6*b^4*c^4 - 6144*a^7*b^2*c^5))))*(-(A^2*b^17 + 9*B^2*a^2*b^15 + A^2*b^2*(-(4*a*c - b^2)^15)^(1/2) + 9*B^2*a^2*(-(4*a*c - b^2)^15)^(1/2) + 6*A*B*a*b^16 + 1140*A^2*a^2*b^13*c^2 - 10160*A^2*a^3*b^11*c^3 + 34880*A^2*a^4*b^9*c^4 + 43776*A^2*a^5*b^7*c^5 - 680960*A^2*a^6*b^5*c^6 + 1863680*A^2*a^7*b^3*c^7 - 5040*B^2*a^4*b^11*c^2 + 37440*B^2*a^5*b^9*c^3 - 103680*B^2*a^6*b^7*c^4 - 9216*B^2*a^7*b^5*c^5 + 552960*B^2*a^8*b^3*c^6 + 983040*A*B*a^9*c^8 - 55*A^2*a*b^15*c - 25*A^2*a*c*(-(4*a*c - b^2)^15)^(1/2) - 1720320*A^2*a^8*b*c^8 + 180*B^2*a^3*b^13*c - 737280*B^2*a^9*b*c^7 + 240*A*B*a^3*b^12*c^2 + 24000*A*B*a^4*b^10*c^3 - 241920*A*B*a^5*b^8*c^4 + 992256*A*B*a^6*b^6*c^5 - 1781760*A*B*a^7*b^4*c^6 + 737280*A*B*a^8*b^2*c^7 + 6*A*B*a*b*(-(4*a*c - b^2)^15)^(1/2) - 180*A*B*a^2*b^14*c)/(512*(a^3*b^20 + 1048576*a^13*c^10 - 40*a^4*b^18*c + 720*a^5*b^16*c^2 - 7680*a^6*b^14*c^3 + 53760*a^7*b^12*c^4 - 258048*a^8*b^10*c^5 + 860160*a^9*b^8*c^6 - 1966080*a^10*b^6*c^7 + 2949120*a^11*b^4*c^8 - 2621440*a^12*b^2*c^9)))^(1/2)*2i + atan(((((256*A*a*b^13*c^2 - 3145728*B*a^8*c^8 + 4194304*A*a^7*b*c^8 - 9216*A*a^2*b^11*c^3 + 122880*A*a^3*b^9*c^4 - 819200*A*a^4*b^7*c^5 + 2949120*A*a^5*b^5*c^6 - 5505024*A*a^6*b^3*c^7 + 768*B*a^2*b^12*c^2

- 12288*B*a^3*b^10*c^3 + 61440*B*a^4*b^8*c^4 - 983040*B*a^6*b^4*c^6 + 3145
728*B*a^7*b^2*c^7)/(512*(a^2*b^12 + 4096*a^8*c^6 - 24*a^3*b^10*c + 240*a^4*
b^8*c^2 - 1280*a^5*b^6*c^3 + 3840*a^6*b^4*c^4 - 6144*a^7*b^2*c^5)) - (x*(-(
A^2*b^17 + 9*B^2*a^2*b^15 - A^2*b^2*(-(4*a*c - b^2)^15)^(1/2) - 9*B^2*a^2*(
-(4*a*c - b^2)^15)^(1/2) + 6*A*B*a*b^16 + 1140*A^2*a^2*b^13*c^2 - 10160*A^2
*a^3*b^11*c^3 + 34880*A^2*a^4*b^9*c^4 + 43776*A^2*a^5*b^7*c^5 - 680960*A^2*
a^6*b^5*c^6 + 1863680*A^2*a^7*b^3*c^7 - 5040*B^2*a^4*b^11*c^2 + 37440*B^2*a
^5*b^9*c^3 - 103680*B^2*a^6*b^7*c^4 - 9216*B^2*a^7*b^5*c^5 + 552960*B^2*a^8
*b^3*c^6 + 983040*A*B*a^9*c^8 - 55*A^2*a*b^15*c + 25*A^2*a*c*(-(4*a*c - b^2
)^15)^(1/2) - 1720320*A^2*a^8*b*c^8 + 180*B^2*a^3*b^13*c - 737280*B^2*a^9*b
*c^7 + 240*A*B*a^3*b^12*c^2 + 24000*A*B*a^4*b^10*c^3 - 241920*A*B*a^5*b^8*c
^4 + 992256*A*B*a^6*b^6*c^5 - 1781760*A*B*a^7*b^4*c^6 + 737280*A*B*a^8*b^2*
c^7 - 6*A*B*a*b*(-(4*a*c - b^2)^15)^(1/2) - 180*A*B*a^2*b^14*c)/(512*(a^3*b
^20 + 1048576*a^13*c^10 - 40*a^4*b^18*c + 720*a^5*b^16*c^2 - 7680*a^6*b^14*
c^3 + 53760*a^7*b^12*c^4 - 258048*a^8*b^10*c^5 + 860160*a^9*b^8*c^6 - 19660
80*a^10*b^6*c^7 + 2949120*a^11*b^4*c^8 - 2621440*a^12*b^2*c^9)))^(1/2)*(262
144*a^7*b*c^7 - 256*a^2*b^11*c^2 + 5120*a^3*b^9*c^3 - 40960*a^4*b^7*c^4 + 1
63840*a^5*b^5*c^5 - 327680*a^6*b^3*c^6))/(32*(a^2*b^8 + 256*a^6*c^4 - 16*a^
3*b^6*c + 96*a^4*b^4*c^2 - 256*a^5*b^2*c^3)))*(-(A^2*b^17 + 9*B^2*a^2*b^15
- A^2*b^2*(-(4*a*c - b^2)^15)^(1/2) - 9*B^2*a^2*(-(4*a*c - b^2)^15)^(1/2) +
6*A*B*a*b^16 + 1140*A^2*a^2*b^13*c^2 - 10160*A^2*a^3*b^11*c^3 + 34880*A^2*
a^4*b^9*c^4 + 43776*A^2*a^5*b^7*c^5 - 680960*A^2*a^6*b^5*c^6 + 1863680*A^2*
a^7*b^3*c^7 - 5040*B^2*a^4*b^11*c^2 + 37440*B^2*a^5*b^9*c^3 - 103680*B^2*a^
6*b^7*c^4 - 9216*B^2*a^7*b^5*c^5 + 552960*B^2*a^8*b^3*c^6 + 983040*A*B*a^9*
c^8 - 55*A^2*a*b^15*c + 25*A^2*a*c*(-(4*a*c - b^2)^15)^(1/2) - 1720320*A^2*
a^8*b*c^8 + 180*B^2*a^3*b^13*c - 737280*B^2*a^9*b*c^7 + 240*A*B*a^3*b^12*c^
2 + 24000*A*B*a^4*b^10*c^3 - 241920*A*B*a^5*b^8*c^4 + 992256*A*B*a^6*b^6*c^
5 - 1781760*A*B*a^7*b^4*c^6 + 737280*A*B*a^8*b^2*c^7 - 6*A*B*a*b*(-(4*a*c -
b^2)^15)^(1/2) - 180*A*B*a^2*b^14*c)/(512*(a^3*b^20 + 1048576*a^13*c^10 -
40*a^4*b^18*c + 720*a^5*b^16*c^2 - 7680*a^6*b^14*c^3 + 53760*a^7*b^12*c^4 -
258048*a^8*b^10*c^5 + 860160*a^9*b^8*c^6 - 1966080*a^10*b^6*c^7 + 2949120*
a^11*b^4*c^8 - 2621440*a^12*b^2*c^9)))^(1/2) + (x*(A^2*b^6*c^3 - 800*A^2*a^
3*c^6 + 288*B^2*a^4*c^5 + 1472*A^2*a^2*b^2*c^5 + 234*B^2*a^2*b^4*c^3 + 144*
B^2*a^3*b^2*c^4 - 34*A^2*a*b^4*c^4 - 1104*A*B*a^2*b^3*c^4 + 6*A*B*a*b^5*c^3
- 288*A*B*a^3*b*c^5))/(32*(a^2*b^8 + 256*a^6*c^4 - 16*a^3*b^6*c + 96*a^4*b
^4*c^2 - 256*a^5*b^2*c^3)))*(-(A^2*b^17 + 9*B^2*a^2*b^15 - A^2*b^2*(-(4*a*c
- b^2)^15)^(1/2) - 9*B^2*a^2*(-(4*a*c - b^2)^15)^(1/2) + 6*A*B*a*b^16 + 11
40*A^2*a^2*b^13*c^2 - 10160*A^2*a^3*b^11*c^3 + 34880*A^2*a^4*b^9*c^4 + 4377
6*A^2*a^5*b^7*c^5 - 680960*A^2*a^6*b^5*c^6 + 1863680*A^2*a^7*b^3*c^7 - 5040
*B^2*a^4*b^11*c^2 + 37440*B^2*a^5*b^9*c^3 - 103680*B^2*a^6*b^7*c^4 - 9216*B
^2*a^7*b^5*c^5 + 552960*B^2*a^8*b^3*c^6 + 983040*A*B*a^9*c^8 - 55*A^2*a*b^1
5*c + 25*A^2*a*c*(-(4*a*c - b^2)^15)^(1/2) - 1720320*A^2*a^8*b*c^8 + 180*B^
2*a^3*b^13*c - 737280*B^2*a^9*b*c^7 + 240*A*B*a^3*b^12*c^2 + 24000*A*B*a^4*
b^10*c^3 - 241920*A*B*a^5*b^8*c^4 + 992256*A*B*a^6*b^6*c^5 - 1781760*A*B*a^
7*b^4*c^6 + 737280*A*B*a^8*b^2*c^7 - 6*A*B*a*b*(-(4*a*c - b^2)^15)^(1/2) -

180*A*B*a^2*b^14*c)/(512*(a^3*b^20 + 1048576*a^13*c^10 - 40*a^4*b^18*c + 720*a^5*b^16*c^2 - 7680*a^6*b^14*c^3 + 53760*a^7*b^12*c^4 - 258048*a^8*b^10*c^5 + 860160*a^9*b^8*c^6 - 1966080*a^10*b^6*c^7 + 2949120*a^11*b^4*c^8 - 2621440*a^12*b^2*c^9)))^(1/2)*1i - (((256*A*a*b^13*c^2 - 3145728*B*a^8*c^8 + 4194304*A*a^7*b*c^8 - 9216*A*a^2*b^11*c^3 + 122880*A*a^3*b^9*c^4 - 819200*A*a^4*b^7*c^5 + 2949120*A*a^5*b^5*c^6 - 5505024*A*a^6*b^3*c^7 + 768*B*a^2*b^12*c^2 - 12288*B*a^3*b^10*c^3 + 61440*B*a^4*b^8*c^4 - 983040*B*a^6*b^4*c^6 + 3145728*B*a^7*b^2*c^7)/(512*(a^2*b^12 + 4096*a^8*c^6 - 24*a^3*b^10*c + 240*a^4*b^8*c^2 - 1280*a^5*b^6*c^3 + 3840*a^6*b^4*c^4 - 6144*a^7*b^2*c^5)) + (x*(-(A^2*b^17 + 9*B^2*a^2*b^15 - A^2*b^2*(-(4*a*c - b^2)^15)^(1/2) - 9*B^2*a^2*(-(4*a*c - b^2)^15)^(1/2) + 6*A*B*a*b^16 + 1140*A^2*a^2*b^13*c^2 - 10160*A^2*a^3*b^11*c^3 + 34880*A^2*a^4*b^9*c^4 + 43776*A^2*a^5*b^7*c^5 - 680960*A^2*a^6*b^5*c^6 + 1863680*A^2*a^7*b^3*c^7 - 5040*B^2*a^4*b^11*c^2 + 37440*B^2*a^5*b^9*c^3 - 103680*B^2*a^6*b^7*c^4 - 9216*B^2*a^7*b^5*c^5 + 552960*B^2*a^8*b^3*c^6 + 983040*A*B*a^9*c^8 - 55*A^2*a*b^15*c + 25*A^2*a*c*(-(4*a*c - b^2)^15)^(1/2) - 1720320*A^2*a^8*b*c^8 + 180*B^2*a^3*b^13*c - 737280*B^2*a^9*b*c^7 + 240*A*B*a^3*b^12*c^2 + 24000*A*B*a^4*b^10*c^3 - 241920*A*B*a^5*b^8*c^4 + 992256*A*B*a^6*b^6*c^5 - 1781760*A*B*a^7*b^4*c^6 + 737280*A*B*a^8*b^2*c^7 - 6*A*B*a*b*(-(4*a*c - b^2)^15)^(1/2) - 180*A*B*a^2*b^14*c)/(512*(a^3*b^20 + 1048576*a^13*c^10 - 40*a^4*b^18*c + 720*a^5*b^16*c^2 - 7680*a^6*b^14*c^3 + 53760*a^7*b^12*c^4 - 258048*a^8*b^10*c^5 + 860160*a^9*b^8*c^6 - 1966080*a^10*b^6*c^7 + 2949120*a^11*b^4*c^8 - 2621440*a^12*b^2*c^9)))^(1/2)*(262144*a^7*b*c^7 - 256*a^2*b^11*c^2 + 5120*a^3*b^9*c^3 - 40960*a^4*b^7*c^4 + 163840*a^5*b^5*c^5 - 327680*a^6*b^3*c^6))/(32*(a^2*b^8 + 256*a^6*c^4 - 16*a^3*b^6*c + 96*a^4*b^4*c^2 - 256*a^5*b^2*c^3)))*(-(A^2*b^17 + 9*B^2*a^2*b^15 - A^2*b^2*(-(4*a*c - b^2)^15)^(1/2) - 9*B^2*a^2*(-(4*a*c - b^2)^15)^(1/2) + 6*A*B*a*b^16 + 1140*A^2*a^2*b^13*c^2 - 10160*A^2*a^3*b^11*c^3 + 34880*A^2*a^4*b^9*c^4 + 43776*A^2*a^5*b^7*c^5 - 680960*A^2*a^6*b^5*c^6 + 1863680*A^2*a^7*b^3*c^7 - 5040*B^2*a^4*b^11*c^2 + 37440*B^2*a^5*b^9*c^3 - 103680*B^2*a^6*b^7*c^4 - 9216*B^2*a^7*b^5*c^5 + 552960*B^2*a^8*b^3*c^6 + 983040*A*B*a^9*c^8 - 55*A^2*a*b^15*c + 25*A^2*a*c*(-(4*a*c - b^2)^15)^(1/2) - 1720320*A^2*a^8*b*c^8 + 180*B^2*a^3*b^13*c - 737280*B^2*a^9*b*c^7 + 240*A*B*a^3*b^12*c^2 + 24000*A*B*a^4*b^10*c^3 - 241920*A*B*a^5*b^8*c^4 + 992256*A*B*a^6*b^6*c^5 - 1781760*A*B*a^7*b^4*c^6 + 737280*A*B*a^8*b^2*c^7 - 6*A*B*a*b*(-(4*a*c - b^2)^15)^(1/2) - 180*A*B*a^2*b^14*c)/(512*(a^3*b^20 + 1048576*a^13*c^10 - 40*a^4*b^18*c + 720*a^5*b^16*c^2 - 7680*a^6*b^14*c^3 + 53760*a^7*b^12*c^4 - 258048*a^8*b^10*c^5 + 860160*a^9*b^8*c^6 - 1966080*a^10*b^6*c^7 + 2949120*a^11*b^4*c^8 - 2621440*a^12*b^2*c^9)))^(1/2) - (x*(A^2*b^6*c^3 - 800*A^2*a^3*c^6 + 288*B^2*a^4*c^5 + 1472*A^2*a^2*b^2*c^5 + 234*B^2*a^2*b^4*c^3 + 144*B^2*a^3*b^2*c^4 - 34*A^2*a*b^4*c^4 - 1104*A*B*a^2*b^3*c^4 + 6*A*B*a*b^5*c^3 - 288*A*B*a^3*b*c^5))/(32*(a^2*b^8 + 256*a^6*c^4 - 16*a^3*b^6*c + 96*a^4*b^4*c^2 - 256*a^5*b^2*c^3)))*(-(A^2*b^17 + 9*B^2*a^2*b^15 - A^2*b^2*(-(4*a*c - b^2)^15)^(1/2) - 9*B^2*a^2*(-(4*a*c - b^2)^15)^(1/2) + 6*A*B*a*b^16 + 1140*A^2*a^2*b^13*c^2 - 10160*A^2*a^3*b^11*c^3 + 34880*A^2*a^4*b^9*c^4 + 43776*A^2*a^5*b^7*c^5 - 680960*A^2*a^6*b^5*c^6 + 1863680*A^2*a^7*b^3*c^7 -

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 5040*B^2*a^4*b^11*c^2 + 37440*B^2*a^5*b^9*c^3 - 103680*B^2*a^6*b^7*c^4 - 9
216*B^2*a^7*b^5*c^5 + 552960*B^2*a^8*b^3*c^6 + 983040*A*B*a^9*c^8 - 55*A^2*
a*b^15*c + 25*A^2*a*c*(-(4*a*c - b^2)^15)^(1/2) - 1720320*A^2*a^8*b*c^8 + 1
80*B^2*a^3*b^13*c - 737280*B^2*a^9*b*c^7 + 240*A*B*a^3*b^12*c^2 + 24000*A*B
*a^4*b^10*c^3 - 241920*A*B*a^5*b^8*c^4 + 992256*A*B*a^6*b^6*c^5 - 1781760*A
*B*a^7*b^4*c^6 + 737280*A*B*a^8*b^2*c^7 - 6*A*B*a*b*(-(4*a*c - b^2)^15)^(1/
2) - 180*A*B*a^2*b^14*c)/(512*(a^3*b^20 + 1048576*a^13*c^10 - 40*a^4*b^18*c
 + 720*a^5*b^16*c^2 - 7680*a^6*b^14*c^3 + 53760*a^7*b^12*c^4 - 258048*a^8*b
^10*c^5 + 860160*a^9*b^8*c^6 - 1966080*a^10*b^6*c^7 + 2949120*a^11*b^4*c^8
- 2621440*a^12*b^2*c^9)))^(1/2)*1i)/((((256*A*a*b^13*c^2 - 3145728*B*a^8*c^
8 + 4194304*A*a^7*b*c^8 - 9216*A*a^2*b^11*c^3 + 122880*A*a^3*b^9*c^4 - 8192
00*A*a^4*b^7*c^5 + 2949120*A*a^5*b^5*c^6 - 5505024*A*a^6*b^3*c^7 + 768*B*a^
2*b^12*c^2 - 12288*B*a^3*b^10*c^3 + 61440*B*a^4*b^8*c^4 - 983040*B*a^6*b^4*
c^6 + 3145728*B*a^7*b^2*c^7)/(512*(a^2*b^12 + 4096*a^8*c^6 - 24*a^3*b^10*c
+ 240*a^4*b^8*c^2 - 1280*a^5*b^6*c^3 + 3840*a^6*b^4*c^4 - 6144*a^7*b^2*c^5)
) - (x*(-(A^2*b^17 + 9*B^2*a^2*b^15 - A^2*b^2*(-(4*a*c - b^2)^15)^(1/2) - 9
*B^2*a^2*(-(4*a*c - b^2)^15)^(1/2) + 6*A*B*a*b^16 + 1140*A^2*a^2*b^13*c^2 -
 10160*A^2*a^3*b^11*c^3 + 34880*A^2*a^4*b^9*c^4 + 43776*A^2*a^5*b^7*c^5 - 6
80960*A^2*a^6*b^5*c^6 + 1863680*A^2*a^7*b^3*c^7 - 5040*B^2*a^4*b^11*c^2 + 3
7440*B^2*a^5*b^9*c^3 - 103680*B^2*a^6*b^7*c^4 - 9216*B^2*a^7*b^5*c^5 + 5529
60*B^2*a^8*b^3*c^6 + 983040*A*B*a^9*c^8 - 55*A^2*a*b^15*c + 25*A^2*a*c*(-(4
*a*c - b^2)^15)^(1/2) - 1720320*A^2*a^8*b*c^8 + 180*B^2*a^3*b^13*c - 737280
*B^2*a^9*b*c^7 + 240*A*B*a^3*b^12*c^2 + 24000*A*B*a^4*b^10*c^3 - 241920*A*B
*a^5*b^8*c^4 + 992256*A*B*a^6*b^6*c^5 - 1781760*A*B*a^7*b^4*c^6 + 737280*A*
B*a^8*b^2*c^7 - 6*A*B*a*b*(-(4*a*c - b^2)^15)^(1/2) - 180*A*B*a^2*b^14*c)/(
512*(a^3*b^20 + 1048576*a^13*c^10 - 40*a^4*b^18*c + 720*a^5*b^16*c^2 - 7680
*a^6*b^14*c^3 + 53760*a^7*b^12*c^4 - 258048*a^8*b^10*c^5 + 860160*a^9*b^8*c
^6 - 1966080*a^10*b^6*c^7 + 2949120*a^11*b^4*c^8 - 2621440*a^12*b^2*c^9)))^
(1/2)*(262144*a^7*b*c^7 - 256*a^2*b^11*c^2 + 5120*a^3*b^9*c^3 - 40960*a^4*b
^7*c^4 + 163840*a^5*b^5*c^5 - 327680*a^6*b^3*c^6))/(32*(a^2*b^8 + 256*a^6*c
^4 - 16*a^3*b^6*c + 96*a^4*b^4*c^2 - 256*a^5*b^2*c^3)))*(-(A^2*b^17 + 9*B^2
*a^2*b^15 - A^2*b^2*(-(4*a*c - b^2)^15)^(1/2) - 9*B^2*a^2*(-(4*a*c - b^2)^1
5)^(1/2) + 6*A*B*a*b^16 + 1140*A^2*a^2*b^13*c^2 - 10160*A^2*a^3*b^11*c^3 +
34880*A^2*a^4*b^9*c^4 + 43776*A^2*a^5*b^7*c^5 - 680960*A^2*a^6*b^5*c^6 + 18
63680*A^2*a^7*b^3*c^7 - 5040*B^2*a^4*b^11*c^2 + 37440*B^2*a^5*b^9*c^3 - 103
680*B^2*a^6*b^7*c^4 - 9216*B^2*a^7*b^5*c^5 + 552960*B^2*a^8*b^3*c^6 + 98304
0*A*B*a^9*c^8 - 55*A^2*a*b^15*c + 25*A^2*a*c*(-(4*a*c - b^2)^15)^(1/2) - 17
20320*A^2*a^8*b*c^8 + 180*B^2*a^3*b^13*c - 737280*B^2*a^9*b*c^7 + 240*A*B*a
^3*b^12*c^2 + 24000*A*B*a^4*b^10*c^3 - 241920*A*B*a^5*b^8*c^4 + 992256*A*B*
a^6*b^6*c^5 - 1781760*A*B*a^7*b^4*c^6 + 737280*A*B*a^8*b^2*c^7 - 6*A*B*a*b*
(-(4*a*c - b^2)^15)^(1/2) - 180*A*B*a^2*b^14*c)/(512*(a^3*b^20 + 1048576*a^
13*c^10 - 40*a^4*b^18*c + 720*a^5*b^16*c^2 - 7680*a^6*b^14*c^3 + 53760*a^7*
b^12*c^4 - 258048*a^8*b^10*c^5 + 860160*a^9*b^8*c^6 - 1966080*a^10*b^6*c^7
+ 2949120*a^11*b^4*c^8 - 2621440*a^12*b^2*c^9)))^(1/2) + (x*(A^2*b^6*c^3 -
800*A^2*a^3*c^6 + 288*B^2*a^4*c^5 + 1472*A^2*a^2*b^2*c^5 + 234*B^2*a^2*b^4*
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c^3 + 144*B^2*a^3*b^2*c^4 - 34*A^2*a*b^4*c^4 - 1104*A*B*a^2*b^3*c^4 + 6*A*B
*a*b^5*c^3 - 288*A*B*a^3*b*c^5))/(32*(a^2*b^8 + 256*a^6*c^4 - 16*a^3*b^6*c
+ 96*a^4*b^4*c^2 - 256*a^5*b^2*c^3)))*(-(A^2*b^17 + 9*B^2*a^2*b^15 - A^2*b^
2*(-(4*a*c - b^2)^15)^(1/2) - 9*B^2*a^2*(-(4*a*c - b^2)^15)^(1/2) + 6*A*B*a
*b^16 + 1140*A^2*a^2*b^13*c^2 - 10160*A^2*a^3*b^11*c^3 + 34880*A^2*a^4*b^9*
c^4 + 43776*A^2*a^5*b^7*c^5 - 680960*A^2*a^6*b^5*c^6 + 1863680*A^2*a^7*b^3*
c^7 - 5040*B^2*a^4*b^11*c^2 + 37440*B^2*a^5*b^9*c^3 - 103680*B^2*a^6*b^7*c^
4 - 9216*B^2*a^7*b^5*c^5 + 552960*B^2*a^8*b^3*c^6 + 983040*A*B*a^9*c^8 - 55
*A^2*a*b^15*c + 25*A^2*a*c*(-(4*a*c - b^2)^15)^(1/2) - 1720320*A^2*a^8*b*c^
8 + 180*B^2*a^3*b^13*c - 737280*B^2*a^9*b*c^7 + 240*A*B*a^3*b^12*c^2 + 2400
0*A*B*a^4*b^10*c^3 - 241920*A*B*a^5*b^8*c^4 + 992256*A*B*a^6*b^6*c^5 - 1781
760*A*B*a^7*b^4*c^6 + 737280*A*B*a^8*b^2*c^7 - 6*A*B*a*b*(-(4*a*c - b^2)^15
)^(1/2) - 180*A*B*a^2*b^14*c)/(512*(a^3*b^20 + 1048576*a^13*c^10 - 40*a^4*b
^18*c + 720*a^5*b^16*c^2 - 7680*a^6*b^14*c^3 + 53760*a^7*b^12*c^4 - 258048*
a^8*b^10*c^5 + 860160*a^9*b^8*c^6 - 1966080*a^10*b^6*c^7 + 2949120*a^11*b^4
*c^8 - 2621440*a^12*b^2*c^9)))^(1/2) + (((256*A*a*b^13*c^2 - 3145728*B*a^8*
c^8 + 4194304*A*a^7*b*c^8 - 9216*A*a^2*b^11*c^3 + 122880*A*a^3*b^9*c^4 - 81
9200*A*a^4*b^7*c^5 + 2949120*A*a^5*b^5*c^6 - 5505024*A*a^6*b^3*c^7 + 768*B*
a^2*b^12*c^2 - 12288*B*a^3*b^10*c^3 + 61440*B*a^4*b^8*c^4 - 983040*B*a^6*b^
4*c^6 + 3145728*B*a^7*b^2*c^7)/(512*(a^2*b^12 + 4096*a^8*c^6 - 24*a^3*b^10*
c + 240*a^4*b^8*c^2 - 1280*a^5*b^6*c^3 + 3840*a^6*b^4*c^4 - 6144*a^7*b^2*c^
5)) + (x*(-(A^2*b^17 + 9*B^2*a^2*b^15 - A^2*b^2*(-(4*a*c - b^2)^15)^(1/2) -
9*B^2*a^2*(-(4*a*c - b^2)^15)^(1/2) + 6*A*B*a*b^16 + 1140*A^2*a^2*b^13*c^2
- 10160*A^2*a^3*b^11*c^3 + 34880*A^2*a^4*b^9*c^4 + 43776*A^2*a^5*b^7*c^5 -
680960*A^2*a^6*b^5*c^6 + 1863680*A^2*a^7*b^3*c^7 - 5040*B^2*a^4*b^11*c^2 +
37440*B^2*a^5*b^9*c^3 - 103680*B^2*a^6*b^7*c^4 - 9216*B^2*a^7*b^5*c^5 + 55
2960*B^2*a^8*b^3*c^6 + 983040*A*B*a^9*c^8 - 55*A^2*a*b^15*c + 25*A^2*a*c*(-
(4*a*c - b^2)^15)^(1/2) - 1720320*A^2*a^8*b*c^8 + 180*B^2*a^3*b^13*c - 7372
80*B^2*a^9*b*c^7 + 240*A*B*a^3*b^12*c^2 + 24000*A*B*a^4*b^10*c^3 - 241920*A
*B*a^5*b^8*c^4 + 992256*A*B*a^6*b^6*c^5 - 1781760*A*B*a^7*b^4*c^6 + 737280*
A*B*a^8*b^2*c^7 - 6*A*B*a*b*(-(4*a*c - b^2)^15)^(1/2) - 180*A*B*a^2*b^14*c)
/(512*(a^3*b^20 + 1048576*a^13*c^10 - 40*a^4*b^18*c + 720*a^5*b^16*c^2 - 76
80*a^6*b^14*c^3 + 53760*a^7*b^12*c^4 - 258048*a^8*b^10*c^5 + 860160*a^9*b^8
*c^6 - 1966080*a^10*b^6*c^7 + 2949120*a^11*b^4*c^8 - 2621440*a^12*b^2*c^9))
)^(1/2)*(262144*a^7*b*c^7 - 256*a^2*b^11*c^2 + 5120*a^3*b^9*c^3 - 40960*a^4
*b^7*c^4 + 163840*a^5*b^5*c^5 - 327680*a^6*b^3*c^6))/(32*(a^2*b^8 + 256*a^6
*c^4 - 16*a^3*b^6*c + 96*a^4*b^4*c^2 - 256*a^5*b^2*c^3)))*(-(A^2*b^17 + 9*B
^2*a^2*b^15 - A^2*b^2*(-(4*a*c - b^2)^15)^(1/2) - 9*B^2*a^2*(-(4*a*c - b^2)
^15)^(1/2) + 6*A*B*a*b^16 + 1140*A^2*a^2*b^13*c^2 - 10160*A^2*a^3*b^11*c^3
+ 34880*A^2*a^4*b^9*c^4 + 43776*A^2*a^5*b^7*c^5 - 680960*A^2*a^6*b^5*c^6 +
1863680*A^2*a^7*b^3*c^7 - 5040*B^2*a^4*b^11*c^2 + 37440*B^2*a^5*b^9*c^3 - 1
03680*B^2*a^6*b^7*c^4 - 9216*B^2*a^7*b^5*c^5 + 552960*B^2*a^8*b^3*c^6 + 983
040*A*B*a^9*c^8 - 55*A^2*a*b^15*c + 25*A^2*a*c*(-(4*a*c - b^2)^15)^(1/2) -
1720320*A^2*a^8*b*c^8 + 180*B^2*a^3*b^13*c - 737280*B^2*a^9*b*c^7 + 240*A*B
*a^3*b^12*c^2 + 24000*A*B*a^4*b^10*c^3 - 241920*A*B*a^5*b^8*c^4 + 992256*A*

B*a^6*b^6*c^5 - 1781760*A*B*a^7*b^4*c^6 + 737280*A*B*a^8*b^2*c^7 - 6*A*B*a*b*(-(4*a*c - b^2)^15)^(1/2) - 180*A*B*a^2*b^14*c)/(512*(a^3*b^20 + 1048576*a^13*c^10 - 40*a^4*b^18*c + 720*a^5*b^16*c^2 - 7680*a^6*b^14*c^3 + 53760*a^7*b^12*c^4 - 258048*a^8*b^10*c^5 + 860160*a^9*b^8*c^6 - 1966080*a^10*b^6*c^7 + 2949120*a^11*b^4*c^8 - 2621440*a^12*b^2*c^9)))^(1/2) - (x*(A^2*b^6*c^3 - 800*A^2*a^3*c^6 + 288*B^2*a^4*c^5 + 1472*A^2*a^2*b^2*c^5 + 234*B^2*a^2*b^4*c^3 + 144*B^2*a^3*b^2*c^4 - 34*A^2*a*b^4*c^4 - 1104*A*B*a^2*b^3*c^4 + 6*A*B*a*b^5*c^3 - 288*A*B*a^3*b*c^5))/(32*(a^2*b^8 + 256*a^6*c^4 - 16*a^3*b^6*c + 96*a^4*b^4*c^2 - 256*a^5*b^2*c^3)))*(-(A^2*b^17 + 9*B^2*a^2*b^15 - A^2*b^2*(-(4*a*c - b^2)^15)^(1/2) - 9*B^2*a^2*(-(4*a*c - b^2)^15)^(1/2) + 6*A*B*a*b^16 + 1140*A^2*a^2*b^13*c^2 - 10160*A^2*a^3*b^11*c^3 + 34880*A^2*a^4*b^9*c^4 + 43776*A^2*a^5*b^7*c^5 - 680960*A^2*a^6*b^5*c^6 + 1863680*A^2*a^7*b^3*c^7 - 5040*B^2*a^4*b^11*c^2 + 37440*B^2*a^5*b^9*c^3 - 103680*B^2*a^6*b^7*c^4 - 9216*B^2*a^7*b^5*c^5 + 552960*B^2*a^8*b^3*c^6 + 983040*A*B*a^9*c^8 - 55*A^2*a*b^15*c + 25*A^2*a*c*(-(4*a*c - b^2)^15)^(1/2) - 1720320*A^2*a^8*b*c^8 + 180*B^2*a^3*b^13*c - 737280*B^2*a^9*b*c^7 + 240*A*B*a^3*b^12*c^2 + 24000*A*B*a^4*b^10*c^3 - 241920*A*B*a^5*b^8*c^4 + 992256*A*B*a^6*b^6*c^5 - 1781760*A*B*a^7*b^4*c^6 + 737280*A*B*a^8*b^2*c^7 - 6*A*B*a*b*(-(4*a*c - b^2)^15)^(1/2) - 180*A*B*a^2*b^14*c)/(512*(a^3*b^20 + 1048576*a^13*c^10 - 40*a^4*b^18*c + 720*a^5*b^16*c^2 - 7680*a^6*b^14*c^3 + 53760*a^7*b^12*c^4 - 258048*a^8*b^10*c^5 + 860160*a^9*b^8*c^6 - 1966080*a^10*b^6*c^7 + 2949120*a^11*b^4*c^8 - 2621440*a^12*b^2*c^9)))^(1/2) + (35*A^3*b^6*c^4 - 8000*A^3*a^3*c^7 - 12720*A^3*a^2*b^2*c^6 + 540*B^3*a^2*b^5*c^3 + 4320*B^3*a^3*b^3*c^4 - 2880*A*B^2*a^4*c^6 - 15*A^2*B*b^7*c^3 + 84*A^3*a*b^4*c^5 + 1728*B^3*a^4*b*c^5 + 135*A*B^2*a*b^6*c^3 - 360*A^2*B*a*b^5*c^4 + 26880*A^2*B*a^3*b*c^6 - 5580*A*B^2*a^2*b^4*c^4 - 20592*A*B^2*a^3*b^2*c^5 + 15696*A^2*B*a^2*b^3*c^5)/(256*(a^2*b^12 + 4096*a^8*c^6 - 24*a^3*b^10*c + 240*a^4*b^8*c^2 - 1280*a^5*b^6*c^3 + 3840*a^6*b^4*c^4 - 6144*a^7*b^2*c^5))))*(-(A^2*b^17 + 9*B^2*a^2*b^15 - A^2*b^2*(-(4*a*c - b^2)^15)^(1/2) - 9*B^2*a^2*(-(4*a*c - b^2)^15)^(1/2) + 6*A*B*a*b^16 + 1140*A^2*a^2*b^13*c^2 - 10160*A^2*a^3*b^11*c^3 + 34880*A^2*a^4*b^9*c^4 + 43776*A^2*a^5*b^7*c^5 - 680960*A^2*a^6*b^5*c^6 + 1863680*A^2*a^7*b^3*c^7 - 5040*B^2*a^4*b^11*c^2 + 37440*B^2*a^5*b^9*c^3 - 103680*B^2*a^6*b^7*c^4 - 9216*B^2*a^7*b^5*c^5 + 552960*B^2*a^8*b^3*c^6 + 983040*A*B*a^9*c^8 - 55*A^2*a*b^15*c + 25*A^2*a*c*(-(4*a*c - b^2)^15)^(1/2) - 1720320*A^2*a^8*b*c^8 + 180*B^2*a^3*b^13*c - 737280*B^2*a^9*b*c^7 + 240*A*B*a^3*b^12*c^2 + 24000*A*B*a^4*b^10*c^3 - 241920*A*B*a^5*b^8*c^4 + 992256*A*B*a^6*b^6*c^5 - 1781760*A*B*a^7*b^4*c^6 + 737280*A*B*a^8*b^2*c^7 - 6*A*B*a*b*(-(4*a*c - b^2)^15)^(1/2) - 180*A*B*a^2*b^14*c)/(512*(a^3*b^20 + 1048576*a^13*c^10 - 40*a^4*b^18*c + 720*a^5*b^16*c^2 - 7680*a^6*b^14*c^3 + 53760*a^7*b^12*c^4 - 258048*a^8*b^10*c^5 + 860160*a^9*b^8*c^6 - 1966080*a^10*b^6*c^7 + 2949120*a^11*b^4*c^8 - 2621440*a^12*b^2*c^9)))^(1/2)*2i

3.136 $\int \frac{A+Bx^2}{(a+bx^2+cx^4)^3} dx$

Optimal result

Integrand size = 22, antiderivative size = 460

$$\int \frac{A+Bx^2}{(a+bx^2+cx^4)^3} dx = \frac{x(Ab^2-abB-2aAc+(Ab-2aB)cx^2)}{4a(b^2-4ac)(a+bx^2+cx^4)^2} + \frac{x(abB(b^2+8ac)+A(3b^4-25ab^2c+28a^2c^2)+c(aB(b^2+20ac)+3A(b^3-8abc))x^2)}{8a^2(b^2-4ac)^2(a+bx^2+cx^4)} + \frac{\sqrt{c}\left(aB(b^2+20ac)+3A(b^3-8abc)+\frac{abB(b^2-52ac)+3A(b^4-10ab^2c+56a^2c^2)}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{8\sqrt{2}a^2(b^2-4ac)^2\sqrt{b-\sqrt{b^2-4ac}}} + \frac{\sqrt{c}\left(aB(b^2+20ac)+3A(b^3-8abc)-\frac{abB(b^2-52ac)+3A(b^4-10ab^2c+56a^2c^2)}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{8\sqrt{2}a^2(b^2-4ac)^2\sqrt{b+\sqrt{b^2-4ac}}}$$

```
[Out] 1/4*x*(A*b^2-a*b*B-2*A*a*c+(A*b-2*B*a)*c*x^2)/a/(-4*a*c+b^2)/(c*x^4+b*x^2+a
)^2+1/8*x*(a*b*B*(8*a*c+b^2)+A*(28*a^2*c^2-25*a*b^2*c+3*b^4)+c*(a*B*(20*a*c
+b^2)+3*A*(-8*a*b*c+b^3))*x^2)/a^2/(-4*a*c+b^2)^2/(c*x^4+b*x^2+a)+1/16*arct
an(x*2^(1/2)*c^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2))*c^(1/2)*(a*B*(20*a*c+b^2
)+3*A*(-8*a*b*c+b^3)+(a*b*B*(-52*a*c+b^2)+3*A*(56*a^2*c^2-10*a*b^2*c+b^4))/
(-4*a*c+b^2)^(1/2))/a^2/(-4*a*c+b^2)^2*2^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2)
+1/16*arctan(x*2^(1/2)*c^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2))*c^(1/2)*(a*B*(
20*a*c+b^2)+3*A*(-8*a*b*c+b^3)+(-a*b*B*(-52*a*c+b^2)-3*A*(56*a^2*c^2-10*a*b
^2*c+b^4))/(-4*a*c+b^2)^(1/2))/a^2/(-4*a*c+b^2)^2*2^(1/2)/(b+(-4*a*c+b^2)^(
1/2))^(1/2)
```

Rubi [A] (verified)

Time = 0.79 (sec) , antiderivative size = 460, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {1192, 1180, 211}

$$\int \frac{A+Bx^2}{(a+bx^2+cx^4)^3}\,dx$$
$$=\frac{\sqrt{c}\left(\frac{3A(56a^2c^2-10ab^2c+b^4)+abB(b^2-52ac)}{\sqrt{b^2-4ac}}+3A(b^3-8abc)+aB(20ac+b^2)\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{8\sqrt{2}a^2(b^2-4ac)^2\sqrt{b-\sqrt{b^2-4ac}}}$$
$$+\frac{\sqrt{c}\left(-\frac{3A(56a^2c^2-10ab^2c+b^4)+abB(b^2-52ac)}{\sqrt{b^2-4ac}}+3A(b^3-8abc)+aB(20ac+b^2)\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{8\sqrt{2}a^2(b^2-4ac)^2\sqrt{\sqrt{b^2-4ac}+b}}$$
$$+\frac{x(A(28a^2c^2-25ab^2c+3b^4)+cx^2(3A(b^3-8abc)+aB(20ac+b^2))+abB(8ac+b^2))}{8a^2(b^2-4ac)^2(a+bx^2+cx^4)}$$
$$+\frac{x(cx^2(Ab-2aB)-2aAc-abB+Ab^2)}{4a(b^2-4ac)(a+bx^2+cx^4)^2}$$

```
[In] Int[(A + B*x^2)/(a + b*x^2 + c*x^4)^3,x]
```

```
[Out] (x*(A*b^2 - a*b*B - 2*a*A*c + (A*b - 2*a*B)*c*x^2))/(4*a*(b^2 - 4*a*c)*(a +
 b*x^2 + c*x^4)^2) + (x*(a*b*B*(b^2 + 8*a*c) + A*(3*b^4 - 25*a*b^2*c + 28*a
^2*c^2) + c*(a*B*(b^2 + 20*a*c) + 3*A*(b^3 - 8*a*b*c))*x^2))/(8*a^2*(b^2 -
4*a*c)^2*(a + b*x^2 + c*x^4)) + (Sqrt[c]*(a*B*(b^2 + 20*a*c) + 3*A*(b^3 - 8
*a*b*c) + (a*b*B*(b^2 - 52*a*c) + 3*A*(b^4 - 10*a*b^2*c + 56*a^2*c^2))/Sqrt
[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(8*
Sqrt[2]*a^2*(b^2 - 4*a*c)^2*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (Sqrt[c]*(a*B*(b
^2 + 20*a*c) + 3*A*(b^3 - 8*a*b*c) - (a*b*B*(b^2 - 52*a*c) + 3*A*(b^4 - 10*
a*b^2*c + 56*a^2*c^2))/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b
 + Sqrt[b^2 - 4*a*c]]])/(8*Sqrt[2]*a^2*(b^2 - 4*a*c)^2*Sqrt[b + Sqrt[b^2 -
4*a*c]])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1192

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[x*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)*((a + b*x^2 +
 c*x^4)^(p + 1)/(2*a*(p + 1)*(b^2 - 4*a*c))), x] + Dist[1/(2*a*(p + 1)*(b^2
 - 4*a*c)), Int[Simp[(2*p + 3)*d*b^2 - a*b*e - 2*a*c*d*(4*p + 5) + (4*p + 7
)*(d*b - 2*a*e)*c*x^2, x]*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; FreeQ[{a,
b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
LtQ[p, -1] && IntegerQ[2*p]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{x(Ab^2-abB-2aAc+(Ab-2aB)cx^2)}{4a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2}-\frac{\int \frac{-3Ab^2-abB+14aAc-5(Ab-2aB)cx^2}{\left(a+bx^2+cx^4\right)^2}\,dx}{4a\left(b^2-4ac\right)}\\
&= \frac{x(Ab^2-abB-2aAc+(Ab-2aB)cx^2)}{4a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2}\\
&\quad+\frac{x(abB(b^2+8ac)+A(3b^4-25ab^2c+28a^2c^2)+c(aB(b^2+20ac)+3A(b^3-8abc))\,x^2)}{8a^2\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)}\\
&\quad+\frac{\int \frac{abB\left(b^2-16ac\right)+3A\left(b^4-9ab^2c+28a^2c^2\right)+c\left(aB\left(b^2+20ac\right)+3A\left(b^3-8abc\right)\right)x^2}{a+bx^2+cx^4}\,dx}{8a^2\left(b^2-4ac\right)^2}\\
&= \frac{x(Ab^2-abB-2aAc+(Ab-2aB)cx^2)}{4a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2}\\
&\quad+\frac{x(abB(b^2+8ac)+A(3b^4-25ab^2c+28a^2c^2)+c(aB(b^2+20ac)+3A(b^3-8abc))\,x^2)}{8a^2\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)}\\
&\quad+\frac{\left(c\left(aB(b^2+20ac)+3A(b^3-8abc)-\frac{abB\left(b^2-52ac\right)+3A\left(b^4-10ab^2c+56a^2c^2\right)}{\sqrt{b^2-4ac}}\right)\right)\int \frac{1}{\frac{b}{2}+\frac{1}{2}\sqrt{b^2-4ac}+cx^2}\,dx}{16a^2\left(b^2-4ac\right)^2}\\
&\quad+\frac{\left(c\left(aB(b^2+20ac)+3A(b^3-8abc)+\frac{abB\left(b^2-52ac\right)+3A\left(b^4-10ab^2c+56a^2c^2\right)}{\sqrt{b^2-4ac}}\right)\right)\int \frac{1}{\frac{b}{2}-\frac{1}{2}\sqrt{b^2-4ac}+cx^2}\,dx}{16a^2\left(b^2-4ac\right)^2}\\
&= \frac{x(Ab^2-abB-2aAc+(Ab-2aB)cx^2)}{4a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2}\\
&\quad+\frac{x(abB(b^2+8ac)+A(3b^4-25ab^2c+28a^2c^2)+c(aB(b^2+20ac)+3A(b^3-8abc))\,x^2)}{8a^2\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)}\\
&\quad+\frac{\sqrt{c}\left(aB(b^2+20ac)+3A(b^3-8abc)+\frac{abB\left(b^2-52ac\right)+3A\left(b^4-10ab^2c+56a^2c^2\right)}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{8\sqrt{2}a^2\left(b^2-4ac\right)^2\sqrt{b-\sqrt{b^2-4ac}}}\\
&\quad+\frac{\sqrt{c}\left(aB(b^2+20ac)+3A(b^3-8abc)-\frac{abB\left(b^2-52ac\right)+3A\left(b^4-10ab^2c+56a^2c^2\right)}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{8\sqrt{2}a^2\left(b^2-4ac\right)^2\sqrt{b+\sqrt{b^2-4ac}}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 1.30 (sec) , antiderivative size = 516, normalized size of antiderivative = 1.12

$$\int \frac{A+Bx^2}{(a+bx^2+cx^4)^3}\,dx$$

$$= \frac{-\frac{4ax(aB(b+2cx^2)-A(b^2-2ac+bcx^2))}{(b^2-4ac)(a+bx^2+cx^4)^2} + \frac{2x(aB(b^3+8abc+b^2cx^2+20ac^2x^2)+A(3b^4-25ab^2c+28a^2c^2+3b^3cx^2-24abc^2x^2))}{(b^2-4ac)^2(a+bx^2+cx^4)} + \frac{\sqrt{2}\sqrt{c}\left(aB\right(\cdots}{\cdots}}{\cdots}$$

```
[In] Integrate[(A + B*x^2)/(a + b*x^2 + c*x^4)^3,x]
```

```
[Out] ((-4*a*x*(a*B*(b + 2*c*x^2) - A*(b^2 - 2*a*c + b*c*x^2)))/((b^2 - 4*a*c)*(a
 + b*x^2 + c*x^4)^2) + (2*x*(a*B*(b^3 + 8*a*b*c + b^2*c*x^2 + 20*a*c^2*x^2)
 + A*(3*b^4 - 25*a*b^2*c + 28*a^2*c^2 + 3*b^3*c*x^2 - 24*a*b*c^2*x^2)))/((b
^2 - 4*a*c)^2*(a + b*x^2 + c*x^4)) + (Sqrt[2]*Sqrt[c]*(a*B*(b^3 - 52*a*b*c
+ b^2*Sqrt[b^2 - 4*a*c] + 20*a*c*Sqrt[b^2 - 4*a*c]) + 3*A*(b^4 - 10*a*b^2*c
 + 56*a^2*c^2 + b^3*Sqrt[b^2 - 4*a*c] - 8*a*b*c*Sqrt[b^2 - 4*a*c]))*ArcTan[
(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/((b^2 - 4*a*c)^(5/2)*Sqrt
[b - Sqrt[b^2 - 4*a*c]]) + (Sqrt[2]*Sqrt[c]*(a*B*(-b^3 + 52*a*b*c + b^2*Sqr
t[b^2 - 4*a*c] + 20*a*c*Sqrt[b^2 - 4*a*c]) + 3*A*(-b^4 + 10*a*b^2*c - 56*a^
2*c^2 + b^3*Sqrt[b^2 - 4*a*c] - 8*a*b*c*Sqrt[b^2 - 4*a*c]))*ArcTan[(Sqrt[2]
*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/((b^2 - 4*a*c)^(5/2)*Sqrt[b + Sqr
t[b^2 - 4*a*c]]))/(16*a^2)
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.39 (sec) , antiderivative size = 447, normalized size of antiderivative = 0.97

method	result
risch	$\frac{-\frac{c^2(24Aabc-3Ab^3-20a^2Bc-Bab^2)x^7}{8a^2(16a^2c^2-8ab^2c+b^4)}+\frac{c(28Aa^2c^2-49Aab^2c+6Ab^4+28a^2bBc+2Bab^3)x^5}{8a^2(16a^2c^2-8ab^2c+b^4)}-\frac{(4Aa^2bc^2+20Aab^3c-3Ab^5-36a^3Bc^2-5Ba^2\cdots}{8a^2(16a^2c^2-8ab^2c+b^4)}\cdots}{(cx^4+bx^2+a)^2}$
default	Expression too large to display

```
[In] int((B*x^2+A)/(c*x^4+b*x^2+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] (-1/8*c^2*(24*A*a*b*c-3*A*b^3-20*B*a^2*c-B*a*b^2)/a^2/(16*a^2*c^2-8*a*b^2*c
+b^4)*x^7+1/8/a^2*c*(28*A*a^2*c^2-49*A*a*b^2*c+6*A*b^4+28*B*a^2*b*c+2*B*a*b
^3)/(16*a^2*c^2-8*a*b^2*c+b^4)*x^5-1/8*(4*A*a^2*b*c^2+20*A*a*b^3*c-3*A*b^5-
36*B*a^3*c^2-5*B*a^2*b^2*c-B*a*b^4)/a^2/(16*a^2*c^2-8*a*b^2*c+b^4)*x^3+1/8*
(44*A*a*a^2*c^2-37*A*a*b^2*c+5*A*b^4+16*B*a^2*b*c-B*a*b^3)/(16*a^2*c^2-8*a*b^
```

```
2*c+b^4)/a*x)/(c*x^4+b*x^2+a)^2+1/16/a^2*sum((-c*(24*A*a*b*c-3*A*b^3-20*B*a
^2*c-B*a*b^2)/(16*a^2*c^2-8*a*b^2*c+b^4)*_R^2+(84*A*a^2*c^2-27*A*a*b^2*c+3*
A*b^4-16*B*a^2*b*c+B*a*b^3)/(16*a^2*c^2-8*a*b^2*c+b^4))/(2*_R^3*c+_R*b)*ln(
x-_R),_R=RootOf(_Z^4*c+_Z^2*b+a))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 9909 vs. $2(417) = 834$.

Time = 21.04 (sec) , antiderivative size = 9909, normalized size of antiderivative = 21.54

$$\int \frac{A+Bx^2}{(a+bx^2+cx^4)^3}\,dx = \text{Too large to display}$$

```
[In] integrate((B*x^2+A)/(c*x^4+b*x^2+a)^3,x, algorithm="fricas")
[Out] Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A+Bx^2}{(a+bx^2+cx^4)^3}\,dx = \text{Timed out}$$

```
[In] integrate((B*x**2+A)/(c*x**4+b*x**2+a)**3,x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{A+Bx^2}{(a+bx^2+cx^4)^3}\,dx = \int \frac{Bx^2+A}{(cx^4+bx^2+a)^3}\,dx$$

```
[In] integrate((B*x^2+A)/(c*x^4+b*x^2+a)^3,x, algorithm="maxima")
[Out] 1/8*((4*(5*B*a^2 - 6*A*a*b)*c^3 + (B*a*b^2 + 3*A*b^3)*c^2)*x^7 + (28*A*a^2*
c^3 + 7*(4*B*a^2*b - 7*A*a*b^2)*c^2 + 2*(B*a*b^3 + 3*A*b^4)*c)*x^5 + (B*a*b
^4 + 3*A*b^5 + 4*(9*B*a^3 - A*a^2*b)*c^2 + 5*(B*a^2*b^2 - 4*A*a*b^3)*c)*x^3
 - (B*a^2*b^3 - 5*A*a*b^4 - 44*A*a^3*c^2 - (16*B*a^3*b - 37*A*a^2*b^2)*c)*x
)/((a^2*b^4*c^2 - 8*a^3*b^2*c^3 + 16*a^4*c^4)*x^8 + a^4*b^4 - 8*a^5*b^2*c +
 16*a^6*c^2 + 2*(a^2*b^5*c - 8*a^3*b^3*c^2 + 16*a^4*b*c^3)*x^6 + (a^2*b^6 -
 6*a^3*b^4*c + 32*a^5*c^3)*x^4 + 2*(a^3*b^5 - 8*a^4*b^3*c + 16*a^5*b*c^2)*x
^2) - 1/8*integrate(-(B*a*b^3 + 3*A*b^4 + 84*A*a^2*c^2 + (4*(5*B*a^2 - 6*A*
a*b)*c^2 + (B*a*b^2 + 3*A*b^3)*c)*x^2 - (16*B*a^2*b + 27*A*a*b^2)*c)/(c*x^4
 + b*x^2 + a), x)/(a^2*b^4 - 8*a^3*b^2*c + 16*a^4*c^2)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 4605 vs. 2(417) = 834.

Time = 1.79 (sec) , antiderivative size = 4605, normalized size of antiderivative = 10.01

$$\int \frac{A+Bx^2}{(a+bx^2+cx^4)^3}\,dx = \text{Too large to display}$$

```
[In] integrate((B*x^2+A)/(c*x^4+b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] 1/32*(3*(sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^8 - 17*sqrt(2)*sqrt(b*c
+ sqrt(b^2 - 4*a*c)*c)*a*b^6*c - 2*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*
b^7*c - 2*b^8*c + 116*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^4*c^2 +
 26*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^5*c^2 + sqrt(2)*sqrt(b*c +
sqrt(b^2 - 4*a*c)*c)*b^6*c^2 + 34*a*b^6*c^2 - 2*b^7*c^2 - 368*sqrt(2)*sqrt(
b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^2*c^3 - 128*sqrt(2)*sqrt(b*c + sqrt(b^2 -
4*a*c)*c)*a^2*b^3*c^3 - 13*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^4*c^
3 - 232*a^2*b^4*c^3 + 30*a*b^5*c^3 + 448*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*
c)*c)*a^4*c^4 + 224*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b*c^4 + 64*
sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^4 + 736*a^3*b^2*c^4 - 176
*a^2*b^3*c^4 - 112*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*c^5 - 896*a^
4*c^5 + 352*a^3*b*c^5 + sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a
*c)*c)*b^7 - 15*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a
*b^5*c - 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^6*c
+ 88*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c^2
+ 22*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^4*c^2 +
sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^5*c^2 - 176*sqr
t(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b*c^3 - 88*sqrt(
2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^3 - 11*sqrt(
2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c^3 + 44*sqrt(2)
*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b*c^4 + 2*(b^2 - 4*a
*c)*b^6*c - 26*(b^2 - 4*a*c)*a*b^4*c^2 + 2*(b^2 - 4*a*c)*b^5*c^2 + 128*(b^2
 - 4*a*c)*a^2*b^2*c^3 - 22*(b^2 - 4*a*c)*a*b^3*c^3 - 224*(b^2 - 4*a*c)*a^3*
c^4 + 88*(b^2 - 4*a*c)*a^2*b*c^4)*A + (sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)
*c)*a*b^7 - 24*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^5*c - 2*sqrt(2
)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^6*c - 2*a*b^7*c + 144*sqrt(2)*sqrt(b*
c + sqrt(b^2 - 4*a*c)*c)*a^3*b^3*c^2 + 40*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a
*c)*c)*a^2*b^4*c^2 + sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^5*c^2 + 48
*a^2*b^5*c^2 - 2*a*b^6*c^2 - 256*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^
4*b*c^3 - 128*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^2*c^3 - 20*sqrt
(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c^3 - 288*a^3*b^3*c^3 + 44*a^2*
b^4*c^3 + 64*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b*c^4 + 512*a^4*b*
c^4 - 64*a^3*b^2*c^4 - 320*a^4*c^5 + sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + s
qrt(b^2 - 4*a*c)*c)*a*b^6 - 22*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^
2 - 4*a*c)*c)*a^2*b^4*c - 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 -
```

4*a*c)*c)*a*b^5*c + 32*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^2*c^2 + 36*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c^2 + sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^4*c^2 + 160*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*c^3 + 80*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b*c^3 - 18*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^3 - 40*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*c^4 + 2*(b^2 - 4*a*c)*a*b^5*c - 40*(b^2 - 4*a*c)*a^2*b^3*c^2 + 2*(b^2 - 4*a*c)*a*b^4*c^2 + 128*(b^2 - 4*a*c)*a^3*b*c^3 - 36*(b^2 - 4*a*c)*a^2*b^2*c^3 - 80*(b^2 - 4*a*c)*a^3*c^4)*B)*arctan(2*sqrt(1/2)*x/sqrt((a^2*b^5 - 8*a^3*b^3*c + 16*a^4*b*c^2 + sqrt((a^2*b^5 - 8*a^3*b^3*c + 16*a^4*b*c^2)^2 - 4*(a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2)*(a^2*b^4*c - 8*a^3*b^2*c^2 + 16*a^4*c^3)))/(a^2*b^4*c - 8*a^3*b^2*c^2 + 16*a^4*c^3)))/((a^3*b^8 - 16*a^4*b^6*c - 2*a^3*b^7*c + 96*a^5*b^4*c^2 + 24*a^4*b^5*c^2 + a^3*b^6*c^2 - 256*a^6*b^2*c^3 - 96*a^5*b^3*c^3 - 12*a^4*b^4*c^3 + 256*a^7*c^4 + 128*a^6*b*c^4 + 48*a^5*b^2*c^4 - 64*a^6*c^5)*abs(c)) + 1/32*(3*(sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^8 - 17*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^6*c - 2*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^7*c + 2*b^8*c + 116*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^4*c^2 + 26*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^5*c^2 + sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^6*c^2 - 34*a*b^6*c^2 - 2*b^7*c^2 - 368*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^2*c^3 - 128*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c^3 - 13*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^4*c^3 + 232*a^2*b^4*c^3 + 30*a*b^5*c^3 + 448*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*c^4 + 224*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b*c^4 + 64*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^4 - 736*a^3*b^2*c^4 - 176*a^2*b^3*c^4 - 112*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*c^5 + 896*a^4*c^5 + 352*a^3*b*c^5 + sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^7 - 15*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^5*c - 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^6*c + 88*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c^2 + 22*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^4*c^2 + sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^5*c^2 - 176*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b*c^3 - 88*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^3 - 11*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^3*c^3 + 44*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b*c^4 - 2*(b^2 - 4*a*c)*b^6*c + 26*(b^2 - 4*a*c)*a*b^4*c^2 + 2*(b^2 - 4*a*c)*b^5*c^2 - 128*(b^2 - 4*a*c)*a^2*b^2*c^3 - 22*(b^2 - 4*a*c)*a*b^3*c^3 + 224*(b^2 - 4*a*c)*a^3*c^4 + 88*(b^2 - 4*a*c)*a^2*b*c^4)*A + (sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^7 - 24*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^5*c - 2*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^6*c + 2*a*b^7*c + 144*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^3*c^2 + 40*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^4*c^2 + sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^5*c^2 - 48*a^2*b^5*c^2 - 2*a*b^6*c^2 - 256*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*b*c^3 - 128*sqrt(2)*sqrt(b*c - sqrt(b^2 -

```
4*a*c)*c)*a^3*b^2*c^3 - 20*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^3*
c^3 + 288*a^3*b^3*c^3 + 44*a^2*b^4*c^3 + 64*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4
*a*c)*c)*a^3*b*c^4 - 512*a^4*b*c^4 - 64*a^3*b^2*c^4 - 320*a^4*c^5 + sqrt(2)
*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^6 - 22*sqrt(2)*sqrt(
b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^4*c - 2*sqrt(2)*sqrt(b^2
 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^5*c + 32*sqrt(2)*sqrt(b^2 - 4
*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^2*c^2 + 36*sqrt(2)*sqrt(b^2 - 4
*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c^2 + sqrt(2)*sqrt(b^2 - 4*a*
c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^4*c^2 + 160*sqrt(2)*sqrt(b^2 - 4*a*c
)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*c^3 + 80*sqrt(2)*sqrt(b^2 - 4*a*c)*sq
rt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b*c^3 - 18*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt
(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^3 - 40*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt
(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*c^4 - 2*(b^2 - 4*a*c)*a*b^5*c + 40*(b^2 - 4
*a*c)*a^2*b^3*c^2 + 2*(b^2 - 4*a*c)*a*b^4*c^2 - 128*(b^2 - 4*a*c)*a^3*b*c^3
 - 36*(b^2 - 4*a*c)*a^2*b^2*c^3 - 80*(b^2 - 4*a*c)*a^3*c^4)*B)*arctan(2*sqr
t(1/2)*x/sqrt((a^2*b^5 - 8*a^3*b^3*c + 16*a^4*b*c^2 - sqrt((a^2*b^5 - 8*a^3
*b^3*c + 16*a^4*b*c^2)^2 - 4*(a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2)*(a^2*b^4*
c - 8*a^3*b^2*c^2 + 16*a^4*c^3)))/(a^2*b^4*c - 8*a^3*b^2*c^2 + 16*a^4*c^3))
)/((a^3*b^8 - 16*a^4*b^6*c - 2*a^3*b^7*c + 96*a^5*b^4*c^2 + 24*a^4*b^5*c^2
+ a^3*b^6*c^2 - 256*a^6*b^2*c^3 - 96*a^5*b^3*c^3 - 12*a^4*b^4*c^3 + 256*a^7
*c^4 + 128*a^6*b*c^4 + 48*a^5*b^2*c^4 - 64*a^6*c^5)*abs(c)) + 1/8*(B*a*b^2*
c^2*x^7 + 3*A*b^3*c^2*x^7 + 20*B*a^2*c^3*x^7 - 24*A*a*b*c^3*x^7 + 2*B*a*b^3
*c*x^5 + 6*A*b^4*c*x^5 + 28*B*a^2*b*c^2*x^5 - 49*A*a*b^2*c^2*x^5 + 28*A*a^2
*c^3*x^5 + B*a*b^4*x^3 + 3*A*b^5*x^3 + 5*B*a^2*b^2*c*x^3 - 20*A*a*b^3*c*x^3
 + 36*B*a^3*c^2*x^3 - 4*A*a^2*b*c^2*x^3 - B*a^2*b^3*x + 5*A*a*b^4*x + 16*B*
a^3*b*c*x - 37*A*a^2*b^2*c*x + 44*A*a^3*c^2*x)/((a^2*b^4 - 8*a^3*b^2*c + 16
*a^4*c^2)*(c*x^4 + b*x^2 + a)^2)
```

Mupad [B] (verification not implemented)

Time = 10.18 (sec) , antiderivative size = 22914, normalized size of antiderivative = 49.81

$$\int \frac{A+Bx^2}{(a+bx^2+cx^4)^3}\,dx = \text{Too large to display}$$

```
[In] int((A + B*x^2)/(a + b*x^2 + c*x^4)^3,x)
```

```
[Out] ((x^3*(3*A*b^5 + 36*B*a^3*c^2 + B*a*b^4 - 20*A*a*b^3*c - 4*A*a^2*b*c^2 + 5*
B*a^2*b^2*c))/(8*a^2*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (x^5*(28*A*a^2*c^3 +
 6*A*b^4*c + 2*B*a*b^3*c - 49*A*a*b^2*c^2 + 28*B*a^2*b*c^2))/(8*a^2*(b^4 +
16*a^2*c^2 - 8*a*b^2*c)) + (x*(5*A*b^4 + 44*A*a^2*c^2 - B*a*b^3 - 37*A*a*b^
2*c + 16*B*a^2*b*c))/(8*a*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (c*x^7*(20*B*a^
2*c^2 + 3*A*b^3*c - 24*A*a*b*c^2 + B*a*b^2*c))/(8*a^2*(b^4 + 16*a^2*c^2 - 8
*a*b^2*c)))/(x^4*(2*a*c + b^2) + a^2 + c^2*x^8 + 2*a*b*x^2 + 2*b*c*x^6) + a
tan(((((4194304*B*a^9*b*c^8 - 22020096*A*a^9*c^9 + 768*A*a^2*b^14*c^2 - 222
```

72*A*a^3*b^12*c^3 + 282624*A*a^4*b^10*c^4 - 2027520*A*a^5*b^8*c^5 + 8847360
*A*a^6*b^6*c^6 - 23396352*A*a^7*b^4*c^7 + 34603008*A*a^8*b^2*c^8 + 256*B*a^
3*b^13*c^2 - 9216*B*a^4*b^11*c^3 + 122880*B*a^5*b^9*c^4 - 819200*B*a^6*b^7*
c^5 + 2949120*B*a^7*b^5*c^6 - 5505024*B*a^8*b^3*c^7)/(512*(a^4*b^12 + 4096*
a^10*c^6 - 24*a^5*b^10*c + 240*a^6*b^8*c^2 - 1280*a^7*b^6*c^3 + 3840*a^8*b^
4*c^4 - 6144*a^9*b^2*c^5)) - (x*(-(9*A^2*b^19 + B^2*a^2*b^17 + 9*A^2*b^4*(-
(4*a*c - b^2)^15)^(1/2) + 6*A*B*a*b^18 + 6921*A^2*a^2*b^15*c^2 - 77580*A^2*
a^3*b^13*c^3 + 570960*A^2*a^4*b^11*c^4 - 2851776*A^2*a^5*b^9*c^5 + 9628416*
A^2*a^6*b^7*c^6 - 21095424*A^2*a^7*b^5*c^7 + 27095040*A^2*a^8*b^3*c^8 + 441
*A^2*a^2*c^2*(-(4*a*c - b^2)^15)^(1/2) + B^2*a^2*b^2*(-(4*a*c - b^2)^15)^(1
/2) + 1140*B^2*a^4*b^13*c^2 - 10160*B^2*a^5*b^11*c^3 + 34880*B^2*a^6*b^9*c^
4 + 43776*B^2*a^7*b^7*c^5 - 680960*B^2*a^8*b^5*c^6 + 1863680*B^2*a^9*b^3*c^
7 + 6881280*A*B*a^10*c^9 - 369*A^2*a*b^17*c - 15482880*A^2*a^9*b*c^9 - 55*B
^2*a^3*b^15*c - 1720320*B^2*a^10*b*c^8 - 25*B^2*a^3*c*(-(4*a*c - b^2)^15)^(
1/2) + 5580*A*B*a^3*b^14*c^2 - 59280*A*B*a^4*b^12*c^3 + 377280*A*B*a^5*b^10
*c^4 - 1430784*A*B*a^6*b^8*c^5 + 2860032*A*B*a^7*b^6*c^6 - 1290240*A*B*a^8*
b^4*c^7 - 5160960*A*B*a^9*b^2*c^8 - 99*A^2*a*b^2*c*(-(4*a*c - b^2)^15)^(1/2
) + 6*A*B*a*b^3*(-(4*a*c - b^2)^15)^(1/2) - 288*A*B*a^2*b^16*c - 108*A*B*a^
2*b*c*(-(4*a*c - b^2)^15)^(1/2))/(512*(a^5*b^20 + 1048576*a^15*c^10 - 40*a^
6*b^18*c + 720*a^7*b^16*c^2 - 7680*a^8*b^14*c^3 + 53760*a^9*b^12*c^4 - 2580
48*a^10*b^10*c^5 + 860160*a^11*b^8*c^6 - 1966080*a^12*b^6*c^7 + 2949120*a^1
3*b^4*c^8 - 2621440*a^14*b^2*c^9)))^(1/2)*(262144*a^9*b*c^7 - 256*a^4*b^11*
c^2 + 5120*a^5*b^9*c^3 - 40960*a^6*b^7*c^4 + 163840*a^7*b^5*c^5 - 327680*a^
8*b^3*c^6))/(32*(a^4*b^8 + 256*a^8*c^4 - 16*a^5*b^6*c + 96*a^6*b^4*c^2 - 25
6*a^7*b^2*c^3)))*(-(9*A^2*b^19 + B^2*a^2*b^17 + 9*A^2*b^4*(-(4*a*c - b^2)^1
5)^(1/2) + 6*A*B*a*b^18 + 6921*A^2*a^2*b^15*c^2 - 77580*A^2*a^3*b^13*c^3 +
570960*A^2*a^4*b^11*c^4 - 2851776*A^2*a^5*b^9*c^5 + 9628416*A^2*a^6*b^7*c^6
 - 21095424*A^2*a^7*b^5*c^7 + 27095040*A^2*a^8*b^3*c^8 + 441*A^2*a^2*c^2*(-
(4*a*c - b^2)^15)^(1/2) + B^2*a^2*b^2*(-(4*a*c - b^2)^15)^(1/2) + 1140*B^2*
a^4*b^13*c^2 - 10160*B^2*a^5*b^11*c^3 + 34880*B^2*a^6*b^9*c^4 + 43776*B^2*a
^7*b^7*c^5 - 680960*B^2*a^8*b^5*c^6 + 1863680*B^2*a^9*b^3*c^7 + 6881280*A*B
*a^10*c^9 - 369*A^2*a*b^17*c - 15482880*A^2*a^9*b*c^9 - 55*B^2*a^3*b^15*c -
 1720320*B^2*a^10*b*c^8 - 25*B^2*a^3*c*(-(4*a*c - b^2)^15)^(1/2) + 5580*A*B
*a^3*b^14*c^2 - 59280*A*B*a^4*b^12*c^3 + 377280*A*B*a^5*b^10*c^4 - 1430784*
A*B*a^6*b^8*c^5 + 2860032*A*B*a^7*b^6*c^6 - 1290240*A*B*a^8*b^4*c^7 - 51609
60*A*B*a^9*b^2*c^8 - 99*A^2*a*b^2*c*(-(4*a*c - b^2)^15)^(1/2) + 6*A*B*a*b^3
*(-(4*a*c - b^2)^15)^(1/2) - 288*A*B*a^2*b^16*c - 108*A*B*a^2*b*c*(-(4*a*c
- b^2)^15)^(1/2))/(512*(a^5*b^20 + 1048576*a^15*c^10 - 40*a^6*b^18*c + 720*
a^7*b^16*c^2 - 7680*a^8*b^14*c^3 + 53760*a^9*b^12*c^4 - 258048*a^10*b^10*c^
5 + 860160*a^11*b^8*c^6 - 1966080*a^12*b^6*c^7 + 2949120*a^13*b^4*c^8 - 262
1440*a^14*b^2*c^9)))^(1/2) + (x*(14112*A^2*a^4*c^7 + 9*A^2*b^8*c^3 - 800*B^
2*a^5*c^6 + 1530*A^2*a^2*b^4*c^5 - 6192*A^2*a^3*b^2*c^6 + B^2*a^2*b^6*c^3 -
 34*B^2*a^3*b^4*c^4 + 1472*B^2*a^4*b^2*c^5 - 180*A^2*a*b^6*c^4 - 162*A*B*a^
2*b^5*c^4 + 1104*A*B*a^3*b^3*c^5 + 6*A*B*a*b^7*c^3 - 6816*A*B*a^4*b*c^6))/(
32*(a^4*b^8 + 256*a^8*c^4 - 16*a^5*b^6*c + 96*a^6*b^4*c^2 - 256*a^7*b^2*c^3

)))*(-(9*A^2*b^19 + B^2*a^2*b^17 + 9*A^2*b^4*(-(4*a*c - b^2)^15)^(1/2) + 6*A*B*a*b^18 + 6921*A^2*a^2*b^15*c^2 - 77580*A^2*a^3*b^13*c^3 + 570960*A^2*a^4*b^11*c^4 - 2851776*A^2*a^5*b^9*c^5 + 9628416*A^2*a^6*b^7*c^6 - 21095424*A^2*a^7*b^5*c^7 + 27095040*A^2*a^8*b^3*c^8 + 441*A^2*a^2*c^2*(-(4*a*c - b^2)^15)^(1/2) + B^2*a^2*b^2*(-(4*a*c - b^2)^15)^(1/2) + 1140*B^2*a^4*b^13*c^2 - 10160*B^2*a^5*b^11*c^3 + 34880*B^2*a^6*b^9*c^4 + 43776*B^2*a^7*b^7*c^5 - 680960*B^2*a^8*b^5*c^6 + 1863680*B^2*a^9*b^3*c^7 + 6881280*A*B*a^10*c^9 - 369*A^2*a*b^17*c - 15482880*A^2*a^9*b*c^9 - 55*B^2*a^3*b^15*c - 1720320*B^2*a^10*b*c^8 - 25*B^2*a^3*c*(-(4*a*c - b^2)^15)^(1/2) + 5580*A*B*a^3*b^14*c^2 - 59280*A*B*a^4*b^12*c^3 + 377280*A*B*a^5*b^10*c^4 - 1430784*A*B*a^6*b^8*c^5 + 2860032*A*B*a^7*b^6*c^6 - 1290240*A*B*a^8*b^4*c^7 - 5160960*A*B*a^9*b^2*c^8 - 99*A^2*a*b^2*c*(-(4*a*c - b^2)^15)^(1/2) + 6*A*B*a*b^3*(-(4*a*c - b^2)^15)^(1/2) - 288*A*B*a^2*b^16*c - 108*A*B*a^2*b*c*(-(4*a*c - b^2)^15)^(1/2))/(512*(a^5*b^20 + 1048576*a^15*c^10 - 40*a^6*b^18*c + 720*a^7*b^16*c^2 - 7680*a^8*b^14*c^3 + 53760*a^9*b^12*c^4 - 258048*a^10*b^10*c^5 + 860160*a^11*b^8*c^6 - 1966080*a^12*b^6*c^7 + 2949120*a^13*b^4*c^8 - 2621440*a^14*b^2*c^9)))^(1/2)*1i - (((4194304*B*a^9*b*c^8 - 22020096*A*a^9*c^9 + 768*A*a^2*b^14*c^2 - 22272*A*a^3*b^12*c^3 + 282624*A*a^4*b^10*c^4 - 2027520*A*a^5*b^8*c^5 + 8847360*A*a^6*b^6*c^6 - 23396352*A*a^7*b^4*c^7 + 34603008*A*a^8*b^2*c^8 + 256*B*a^3*b^13*c^2 - 9216*B*a^4*b^11*c^3 + 122880*B*a^5*b^9*c^4 - 819200*B*a^6*b^7*c^5 + 2949120*B*a^7*b^5*c^6 - 5505024*B*a^8*b^3*c^7)/(512*(a^4*b^12 + 4096*a^10*c^6 - 24*a^5*b^10*c + 240*a^6*b^8*c^2 - 1280*a^7*b^6*c^3 + 3840*a^8*b^4*c^4 - 6144*a^9*b^2*c^5)) + (x*(-(9*A^2*b^19 + B^2*a^2*b^17 + 9*A^2*b^4*(-(4*a*c - b^2)^15)^(1/2) + 6*A*B*a*b^18 + 6921*A^2*a^2*b^15*c^2 - 77580*A^2*a^3*b^13*c^3 + 570960*A^2*a^4*b^11*c^4 - 2851776*A^2*a^5*b^9*c^5 + 9628416*A^2*a^6*b^7*c^6 - 21095424*A^2*a^7*b^5*c^7 + 27095040*A^2*a^8*b^3*c^8 + 441*A^2*a^2*c^2*(-(4*a*c - b^2)^15)^(1/2) + B^2*a^2*b^2*(-(4*a*c - b^2)^15)^(1/2) + 1140*B^2*a^4*b^13*c^2 - 10160*B^2*a^5*b^11*c^3 + 34880*B^2*a^6*b^9*c^4 + 43776*B^2*a^7*b^7*c^5 - 680960*B^2*a^8*b^5*c^6 + 1863680*B^2*a^9*b^3*c^7 + 6881280*A*B*a^10*c^9 - 369*A^2*a*b^17*c - 15482880*A^2*a^9*b*c^9 - 55*B^2*a^3*b^15*c - 1720320*B^2*a^10*b*c^8 - 25*B^2*a^3*c*(-(4*a*c - b^2)^15)^(1/2) + 5580*A*B*a^3*b^14*c^2 - 59280*A*B*a^4*b^12*c^3 + 377280*A*B*a^5*b^10*c^4 - 1430784*A*B*a^6*b^8*c^5 + 2860032*A*B*a^7*b^6*c^6 - 1290240*A*B*a^8*b^4*c^7 - 5160960*A*B*a^9*b^2*c^8 - 99*A^2*a*b^2*c*(-(4*a*c - b^2)^15)^(1/2) + 6*A*B*a*b^3*(-(4*a*c - b^2)^15)^(1/2) - 288*A*B*a^2*b^16*c - 108*A*B*a^2*b*c*(-(4*a*c - b^2)^15)^(1/2))/(512*(a^5*b^20 + 1048576*a^15*c^10 - 40*a^6*b^18*c + 720*a^7*b^16*c^2 - 7680*a^8*b^14*c^3 + 53760*a^9*b^12*c^4 - 258048*a^10*b^10*c^5 + 860160*a^11*b^8*c^6 - 1966080*a^12*b^6*c^7 + 2949120*a^13*b^4*c^8 - 2621440*a^14*b^2*c^9)))^(1/2)*(262144*a^9*b*c^7 - 256*a^4*b^11*c^2 + 5120*a^5*b^9*c^3 - 40960*a^6*b^7*c^4 + 163840*a^7*b^5*c^5 - 327680*a^8*b^3*c^6))/(32*(a^4*b^8 + 256*a^8*c^4 - 16*a^5*b^6*c + 96*a^6*b^4*c^2 - 256*a^7*b^2*c^3)))*(-(9*A^2*b^19 + B^2*a^2*b^17 + 9*A^2*b^4*(-(4*a*c - b^2)^15)^(1/2) + 6*A*B*a*b^18 + 6921*A^2*a^2*b^15*c^2 - 77580*A^2*a^3*b^13*c^3 + 570960*A^2*a^4*b^11*c^4 - 2851776*A^2*a^5*b^9*c^5 + 9628416*A^2*a^6*b^7*c^6 - 21095424*A^2*a^7*b^5*c^7 + 27095040*A^2*a^8*b^3*c^8 + 441*

A^2*a^2*c^2*(-(4*a*c - b^2)^15)^(1/2) + B^2*a^2*b^2*(-(4*a*c - b^2)^15)^(1/
2) + 1140*B^2*a^4*b^13*c^2 - 10160*B^2*a^5*b^11*c^3 + 34880*B^2*a^6*b^9*c^4
 + 43776*B^2*a^7*b^7*c^5 - 680960*B^2*a^8*b^5*c^6 + 1863680*B^2*a^9*b^3*c^7
 + 6881280*A*B*a^10*c^9 - 369*A^2*a*b^17*c - 15482880*A^2*a^9*b*c^9 - 55*B^
2*a^3*b^15*c - 1720320*B^2*a^10*b*c^8 - 25*B^2*a^3*c*(-(4*a*c - b^2)^15)^(1
/2) + 5580*A*B*a^3*b^14*c^2 - 59280*A*B*a^4*b^12*c^3 + 377280*A*B*a^5*b^10*
c^4 - 1430784*A*B*a^6*b^8*c^5 + 2860032*A*B*a^7*b^6*c^6 - 1290240*A*B*a^8*b
^4*c^7 - 5160960*A*B*a^9*b^2*c^8 - 99*A^2*a*b^2*c*(-(4*a*c - b^2)^15)^(1/2)
 + 6*A*B*a*b^3*(-(4*a*c - b^2)^15)^(1/2) - 288*A*B*a^2*b^16*c - 108*A*B*a^2
*b*c*(-(4*a*c - b^2)^15)^(1/2))/(512*(a^5*b^20 + 1048576*a^15*c^10 - 40*a^6
*b^18*c + 720*a^7*b^16*c^2 - 7680*a^8*b^14*c^3 + 53760*a^9*b^12*c^4 - 25804
8*a^10*b^10*c^5 + 860160*a^11*b^8*c^6 - 1966080*a^12*b^6*c^7 + 2949120*a^13
*b^4*c^8 - 2621440*a^14*b^2*c^9)))^(1/2) - (x*(14112*A^2*a^4*c^7 + 9*A^2*b^
8*c^3 - 800*B^2*a^5*c^6 + 1530*A^2*a^2*b^4*c^5 - 6192*A^2*a^3*b^2*c^6 + B^2
*a^2*b^6*c^3 - 34*B^2*a^3*b^4*c^4 + 1472*B^2*a^4*b^2*c^5 - 180*A^2*a*b^6*c^
4 - 162*A*B*a^2*b^5*c^4 + 1104*A*B*a^3*b^3*c^5 + 6*A*B*a*b^7*c^3 - 6816*A*B
*a^4*b*c^6))/(32*(a^4*b^8 + 256*a^8*c^4 - 16*a^5*b^6*c + 96*a^6*b^4*c^2 - 2
56*a^7*b^2*c^3)))*(-(9*A^2*b^19 + B^2*a^2*b^17 + 9*A^2*b^4*(-(4*a*c - b^2)^
15)^(1/2) + 6*A*B*a*b^18 + 6921*A^2*a^2*b^15*c^2 - 77580*A^2*a^3*b^13*c^3 +
 570960*A^2*a^4*b^11*c^4 - 2851776*A^2*a^5*b^9*c^5 + 9628416*A^2*a^6*b^7*c^
6 - 21095424*A^2*a^7*b^5*c^7 + 27095040*A^2*a^8*b^3*c^8 + 441*A^2*a^2*c^2*(
-(4*a*c - b^2)^15)^(1/2) + B^2*a^2*b^2*(-(4*a*c - b^2)^15)^(1/2) + 1140*B^2
*a^4*b^13*c^2 - 10160*B^2*a^5*b^11*c^3 + 34880*B^2*a^6*b^9*c^4 + 43776*B^2*
a^7*b^7*c^5 - 680960*B^2*a^8*b^5*c^6 + 1863680*B^2*a^9*b^3*c^7 + 6881280*A*
B*a^10*c^9 - 369*A^2*a*b^17*c - 15482880*A^2*a^9*b*c^9 - 55*B^2*a^3*b^15*c
 - 1720320*B^2*a^10*b*c^8 - 25*B^2*a^3*c*(-(4*a*c - b^2)^15)^(1/2) + 5580*A*
B*a^3*b^14*c^2 - 59280*A*B*a^4*b^12*c^3 + 377280*A*B*a^5*b^10*c^4 - 1430784
*A*B*a^6*b^8*c^5 + 2860032*A*B*a^7*b^6*c^6 - 1290240*A*B*a^8*b^4*c^7 - 5160
960*A*B*a^9*b^2*c^8 - 99*A^2*a*b^2*c*(-(4*a*c - b^2)^15)^(1/2) + 6*A*B*a*b^
3*(-(4*a*c - b^2)^15)^(1/2) - 288*A*B*a^2*b^16*c - 108*A*B*a^2*b*c*(-(4*a*c
 - b^2)^15)^(1/2))/(512*(a^5*b^20 + 1048576*a^15*c^10 - 40*a^6*b^18*c + 720
*a^7*b^16*c^2 - 7680*a^8*b^14*c^3 + 53760*a^9*b^12*c^4 - 258048*a^10*b^10*c
^5 + 860160*a^11*b^8*c^6 - 1966080*a^12*b^6*c^7 + 2949120*a^13*b^4*c^8 - 26
21440*a^14*b^2*c^9)))^(1/2)*1i)/((((4194304*B*a^9*b*c^8 - 22020096*A*a^9*c^
9 + 768*A*a^2*b^14*c^2 - 22272*A*a^3*b^12*c^3 + 282624*A*a^4*b^10*c^4 - 202
7520*A*a^5*b^8*c^5 + 8847360*A*a^6*b^6*c^6 - 23396352*A*a^7*b^4*c^7 + 34603
008*A*a^8*b^2*c^8 + 256*B*a^3*b^13*c^2 - 9216*B*a^4*b^11*c^3 + 122880*B*a^5
*b^9*c^4 - 819200*B*a^6*b^7*c^5 + 2949120*B*a^7*b^5*c^6 - 5505024*B*a^8*b^3
c^7)/(512(a^4*b^12 + 4096*a^10*c^6 - 24*a^5*b^10*c + 240*a^6*b^8*c^2 - 12
80*a^7*b^6*c^3 + 3840*a^8*b^4*c^4 - 6144*a^9*b^2*c^5)) - (x*(-(9*A^2*b^19 +
 B^2*a^2*b^17 + 9*A^2*b^4*(-(4*a*c - b^2)^15)^(1/2) + 6*A*B*a*b^18 + 6921*A
^2*a^2*b^15*c^2 - 77580*A^2*a^3*b^13*c^3 + 570960*A^2*a^4*b^11*c^4 - 285177
6*A^2*a^5*b^9*c^5 + 9628416*A^2*a^6*b^7*c^6 - 21095424*A^2*a^7*b^5*c^7 + 27
095040*A^2*a^8*b^3*c^8 + 441*A^2*a^2*c^2*(-(4*a*c - b^2)^15)^(1/2) + B^2*a^
2*b^2*(-(4*a*c - b^2)^15)^(1/2) + 1140*B^2*a^4*b^13*c^2 - 10160*B^2*a^5*b^1

1*c^3 + 34880*B^2*a^6*b^9*c^4 + 43776*B^2*a^7*b^7*c^5 - 680960*B^2*a^8*b^5*
c^6 + 1863680*B^2*a^9*b^3*c^7 + 6881280*A*B*a^10*c^9 - 369*A^2*a*b^17*c - 1
5482880*A^2*a^9*b*c^9 - 55*B^2*a^3*b^15*c - 1720320*B^2*a^10*b*c^8 - 25*B^2
*a^3*c*(-(4*a*c - b^2)^15)^(1/2) + 5580*A*B*a^3*b^14*c^2 - 59280*A*B*a^4*b^
12*c^3 + 377280*A*B*a^5*b^10*c^4 - 1430784*A*B*a^6*b^8*c^5 + 2860032*A*B*a^
7*b^6*c^6 - 1290240*A*B*a^8*b^4*c^7 - 5160960*A*B*a^9*b^2*c^8 - 99*A^2*a*b^
2*c*(-(4*a*c - b^2)^15)^(1/2) + 6*A*B*a*b^3*(-(4*a*c - b^2)^15)^(1/2) - 288
*A*B*a^2*b^16*c - 108*A*B*a^2*b*c*(-(4*a*c - b^2)^15)^(1/2))/(512*(a^5*b^20
 + 1048576*a^15*c^10 - 40*a^6*b^18*c + 720*a^7*b^16*c^2 - 7680*a^8*b^14*c^3
 + 53760*a^9*b^12*c^4 - 258048*a^10*b^10*c^5 + 860160*a^11*b^8*c^6 - 196608
0*a^12*b^6*c^7 + 2949120*a^13*b^4*c^8 - 2621440*a^14*b^2*c^9)))^(1/2)*(2621
44*a^9*b*c^7 - 256*a^4*b^11*c^2 + 5120*a^5*b^9*c^3 - 40960*a^6*b^7*c^4 + 16
3840*a^7*b^5*c^5 - 327680*a^8*b^3*c^6))/(32*(a^4*b^8 + 256*a^8*c^4 - 16*a^5
*b^6*c + 96*a^6*b^4*c^2 - 256*a^7*b^2*c^3)))*(-(9*A^2*b^19 + B^2*a^2*b^17 +
 9*A^2*b^4*(-(4*a*c - b^2)^15)^(1/2) + 6*A*B*a*b^18 + 6921*A^2*a^2*b^15*c^2
 - 77580*A^2*a^3*b^13*c^3 + 570960*A^2*a^4*b^11*c^4 - 2851776*A^2*a^5*b^9*c
^5 + 9628416*A^2*a^6*b^7*c^6 - 21095424*A^2*a^7*b^5*c^7 + 27095040*A^2*a^8*
b^3*c^8 + 441*A^2*a^2*c^2*(-(4*a*c - b^2)^15)^(1/2) + B^2*a^2*b^2*(-(4*a*c
 - b^2)^15)^(1/2) + 1140*B^2*a^4*b^13*c^2 - 10160*B^2*a^5*b^11*c^3 + 34880*B
^2*a^6*b^9*c^4 + 43776*B^2*a^7*b^7*c^5 - 680960*B^2*a^8*b^5*c^6 + 1863680*B
^2*a^9*b^3*c^7 + 6881280*A*B*a^10*c^9 - 369*A^2*a*b^17*c - 15482880*A^2*a^9
*b*c^9 - 55*B^2*a^3*b^15*c - 1720320*B^2*a^10*b*c^8 - 25*B^2*a^3*c*(-(4*a*c
 - b^2)^15)^(1/2) + 5580*A*B*a^3*b^14*c^2 - 59280*A*B*a^4*b^12*c^3 + 377280
*A*B*a^5*b^10*c^4 - 1430784*A*B*a^6*b^8*c^5 + 2860032*A*B*a^7*b^6*c^6 - 129
0240*A*B*a^8*b^4*c^7 - 5160960*A*B*a^9*b^2*c^8 - 99*A^2*a*b^2*c*(-(4*a*c -
b^2)^15)^(1/2) + 6*A*B*a*b^3*(-(4*a*c - b^2)^15)^(1/2) - 288*A*B*a^2*b^16*c
 - 108*A*B*a^2*b*c*(-(4*a*c - b^2)^15)^(1/2))/(512*(a^5*b^20 + 1048576*a^15
*c^10 - 40*a^6*b^18*c + 720*a^7*b^16*c^2 - 7680*a^8*b^14*c^3 + 53760*a^9*b^
12*c^4 - 258048*a^10*b^10*c^5 + 860160*a^11*b^8*c^6 - 1966080*a^12*b^6*c^7
+ 2949120*a^13*b^4*c^8 - 2621440*a^14*b^2*c^9)))^(1/2) + (x*(14112*A^2*a^4*
c^7 + 9*A^2*b^8*c^3 - 800*B^2*a^5*c^6 + 1530*A^2*a^2*b^4*c^5 - 6192*A^2*a^3
*b^2*c^6 + B^2*a^2*b^6*c^3 - 34*B^2*a^3*b^4*c^4 + 1472*B^2*a^4*b^2*c^5 - 18
0*A^2*a*b^6*c^4 - 162*A*B*a^2*b^5*c^4 + 1104*A*B*a^3*b^3*c^5 + 6*A*B*a*b^7*
c^3 - 6816*A*B*a^4*b*c^6))/(32*(a^4*b^8 + 256*a^8*c^4 - 16*a^5*b^6*c + 96*a
^6*b^4*c^2 - 256*a^7*b^2*c^3)))*(-(9*A^2*b^19 + B^2*a^2*b^17 + 9*A^2*b^4*(-
(4*a*c - b^2)^15)^(1/2) + 6*A*B*a*b^18 + 6921*A^2*a^2*b^15*c^2 - 77580*A^2*
a^3*b^13*c^3 + 570960*A^2*a^4*b^11*c^4 - 2851776*A^2*a^5*b^9*c^5 + 9628416*
A^2*a^6*b^7*c^6 - 21095424*A^2*a^7*b^5*c^7 + 27095040*A^2*a^8*b^3*c^8 + 441
*A^2*a^2*c^2*(-(4*a*c - b^2)^15)^(1/2) + B^2*a^2*b^2*(-(4*a*c - b^2)^15)^(1
/2) + 1140*B^2*a^4*b^13*c^2 - 10160*B^2*a^5*b^11*c^3 + 34880*B^2*a^6*b^9*c^
4 + 43776*B^2*a^7*b^7*c^5 - 680960*B^2*a^8*b^5*c^6 + 1863680*B^2*a^9*b^3*c^
7 + 6881280*A*B*a^10*c^9 - 369*A^2*a*b^17*c - 15482880*A^2*a^9*b*c^9 - 55*B
^2*a^3*b^15*c - 1720320*B^2*a^10*b*c^8 - 25*B^2*a^3*c*(-(4*a*c - b^2)^15)^(
1/2) + 5580*A*B*a^3*b^14*c^2 - 59280*A*B*a^4*b^12*c^3 + 377280*A*B*a^5*b^10
*c^4 - 1430784*A*B*a^6*b^8*c^5 + 2860032*A*B*a^7*b^6*c^6 - 1290240*A*B*a^8*

b^4*c^7 - 5160960*A*B*a^9*b^2*c^8 - 99*A^2*a*b^2*c*(-(4*a*c - b^2)^15)^(1/2
) + 6*A*B*a*b^3*(-(4*a*c - b^2)^15)^(1/2) - 288*A*B*a^2*b^16*c - 108*A*B*a^
2*b*c*(-(4*a*c - b^2)^15)^(1/2))/(512*(a^5*b^20 + 1048576*a^15*c^10 - 40*a^
6*b^18*c + 720*a^7*b^16*c^2 - 7680*a^8*b^14*c^3 + 53760*a^9*b^12*c^4 - 2580
48*a^10*b^10*c^5 + 860160*a^11*b^8*c^6 - 1966080*a^12*b^6*c^7 + 2949120*a^1
3*b^4*c^8 - 2621440*a^14*b^2*c^9)))^(1/2) - (567*A^3*b^7*c^5 + 8000*B^3*a^5
*c^7 + 67824*A^3*a^2*b^3*c^7 - 35*B^3*a^2*b^6*c^4 - 84*B^3*a^3*b^4*c^5 + 12
720*B^3*a^4*b^2*c^6 + 141120*A^2*B*a^4*c^8 - 315*A^2*B*b^8*c^4 - 10368*A^3*
a*b^5*c^6 - 169344*A^3*a^3*b*c^8 - 210*A*B^2*a*b^7*c^4 - 116160*A*B^2*a^4*b
*c^7 + 6237*A^2*B*a*b^6*c^5 + 1764*A*B^2*a^2*b^5*c^5 + 4608*A*B^2*a^3*b^3*c
^6 - 42372*A^2*B*a^2*b^4*c^6 + 96048*A^2*B*a^3*b^2*c^7)/(256*(a^4*b^12 + 40
96*a^10*c^6 - 24*a^5*b^10*c + 240*a^6*b^8*c^2 - 1280*a^7*b^6*c^3 + 3840*a^8
*b^4*c^4 - 6144*a^9*b^2*c^5)) + (((4194304*B*a^9*b*c^8 - 22020096*A*a^9*c^9
 + 768*A*a^2*b^14*c^2 - 22272*A*a^3*b^12*c^3 + 282624*A*a^4*b^10*c^4 - 2027
520*A*a^5*b^8*c^5 + 8847360*A*a^6*b^6*c^6 - 23396352*A*a^7*b^4*c^7 + 346030
08*A*a^8*b^2*c^8 + 256*B*a^3*b^13*c^2 - 9216*B*a^4*b^11*c^3 + 122880*B*a^5*
b^9*c^4 - 819200*B*a^6*b^7*c^5 + 2949120*B*a^7*b^5*c^6 - 5505024*B*a^8*b^3*
c^7)/(512*(a^4*b^12 + 4096*a^10*c^6 - 24*a^5*b^10*c + 240*a^6*b^8*c^2 - 128
0*a^7*b^6*c^3 + 3840*a^8*b^4*c^4 - 6144*a^9*b^2*c^5)) + (x*(-(9*A^2*b^19 +
B^2*a^2*b^17 + 9*A^2*b^4*(-(4*a*c - b^2)^15)^(1/2) + 6*A*B*a*b^18 + 6921*A^
2*a^2*b^15*c^2 - 77580*A^2*a^3*b^13*c^3 + 570960*A^2*a^4*b^11*c^4 - 2851776
*A^2*a^5*b^9*c^5 + 9628416*A^2*a^6*b^7*c^6 - 21095424*A^2*a^7*b^5*c^7 + 270
95040*A^2*a^8*b^3*c^8 + 441*A^2*a^2*c^2*(-(4*a*c - b^2)^15)^(1/2) + B^2*a^2
b^2(-(4*a*c - b^2)^15)^(1/2) + 1140*B^2*a^4*b^13*c^2 - 10160*B^2*a^5*b^11
*c^3 + 34880*B^2*a^6*b^9*c^4 + 43776*B^2*a^7*b^7*c^5 - 680960*B^2*a^8*b^5*c
^6 + 1863680*B^2*a^9*b^3*c^7 + 6881280*A*B*a^10*c^9 - 369*A^2*a*b^17*c - 15
482880*A^2*a^9*b*c^9 - 55*B^2*a^3*b^15*c - 1720320*B^2*a^10*b*c^8 - 25*B^2*
a^3*c*(-(4*a*c - b^2)^15)^(1/2) + 5580*A*B*a^3*b^14*c^2 - 59280*A*B*a^4*b^1
2*c^3 + 377280*A*B*a^5*b^10*c^4 - 1430784*A*B*a^6*b^8*c^5 + 2860032*A*B*a^7
*b^6*c^6 - 1290240*A*B*a^8*b^4*c^7 - 5160960*A*B*a^9*b^2*c^8 - 99*A^2*a*b^2
c(-(4*a*c - b^2)^15)^(1/2) + 6*A*B*a*b^3*(-(4*a*c - b^2)^15)^(1/2) - 288*
A*B*a^2*b^16*c - 108*A*B*a^2*b*c*(-(4*a*c - b^2)^15)^(1/2))/(512*(a^5*b^20
+ 1048576*a^15*c^10 - 40*a^6*b^18*c + 720*a^7*b^16*c^2 - 7680*a^8*b^14*c^3
+ 53760*a^9*b^12*c^4 - 258048*a^10*b^10*c^5 + 860160*a^11*b^8*c^6 - 1966080
*a^12*b^6*c^7 + 2949120*a^13*b^4*c^8 - 2621440*a^14*b^2*c^9)))^(1/2)*(26214
4*a^9*b*c^7 - 256*a^4*b^11*c^2 + 5120*a^5*b^9*c^3 - 40960*a^6*b^7*c^4 + 163
840*a^7*b^5*c^5 - 327680*a^8*b^3*c^6))/(32*(a^4*b^8 + 256*a^8*c^4 - 16*a^5*
b^6*c + 96*a^6*b^4*c^2 - 256*a^7*b^2*c^3)))*(-(9*A^2*b^19 + B^2*a^2*b^17 +
9*A^2*b^4*(-(4*a*c - b^2)^15)^(1/2) + 6*A*B*a*b^18 + 6921*A^2*a^2*b^15*c^2
- 77580*A^2*a^3*b^13*c^3 + 570960*A^2*a^4*b^11*c^4 - 2851776*A^2*a^5*b^9*c^
5 + 9628416*A^2*a^6*b^7*c^6 - 21095424*A^2*a^7*b^5*c^7 + 27095040*A^2*a^8*b
^3*c^8 + 441*A^2*a^2*c^2*(-(4*a*c - b^2)^15)^(1/2) + B^2*a^2*b^2*(-(4*a*c -
 b^2)^15)^(1/2) + 1140*B^2*a^4*b^13*c^2 - 10160*B^2*a^5*b^11*c^3 + 34880*B^
2*a^6*b^9*c^4 + 43776*B^2*a^7*b^7*c^5 - 680960*B^2*a^8*b^5*c^6 + 1863680*B^
2*a^9*b^3*c^7 + 6881280*A*B*a^10*c^9 - 369*A^2*a*b^17*c - 15482880*A^2*a^9*

b*c^9 - 55*B^2*a^3*b^15*c - 1720320*B^2*a^10*b*c^8 - 25*B^2*a^3*c*(-(4*a*c
- b^2)^15)^(1/2) + 5580*A*B*a^3*b^14*c^2 - 59280*A*B*a^4*b^12*c^3 + 377280*
A*B*a^5*b^10*c^4 - 1430784*A*B*a^6*b^8*c^5 + 2860032*A*B*a^7*b^6*c^6 - 1290
240*A*B*a^8*b^4*c^7 - 5160960*A*B*a^9*b^2*c^8 - 99*A^2*a*b^2*c*(-(4*a*c - b
^2)^15)^(1/2) + 6*A*B*a*b^3*(-(4*a*c - b^2)^15)^(1/2) - 288*A*B*a^2*b^16*c
- 108*A*B*a^2*b*c*(-(4*a*c - b^2)^15)^(1/2))/(512*(a^5*b^20 + 1048576*a^15*
c^10 - 40*a^6*b^18*c + 720*a^7*b^16*c^2 - 7680*a^8*b^14*c^3 + 53760*a^9*b^1
2*c^4 - 258048*a^10*b^10*c^5 + 860160*a^11*b^8*c^6 - 1966080*a^12*b^6*c^7 +
2949120*a^13*b^4*c^8 - 2621440*a^14*b^2*c^9)))^(1/2) - (x*(14112*A^2*a^4*c
^7 + 9*A^2*b^8*c^3 - 800*B^2*a^5*c^6 + 1530*A^2*a^2*b^4*c^5 - 6192*A^2*a^3*
b^2*c^6 + B^2*a^2*b^6*c^3 - 34*B^2*a^3*b^4*c^4 + 1472*B^2*a^4*b^2*c^5 - 180
*A^2*a*b^6*c^4 - 162*A*B*a^2*b^5*c^4 + 1104*A*B*a^3*b^3*c^5 + 6*A*B*a*b^7*c
^3 - 6816*A*B*a^4*b*c^6))/(32*(a^4*b^8 + 256*a^8*c^4 - 16*a^5*b^6*c + 96*a^
6*b^4*c^2 - 256*a^7*b^2*c^3)))*(-(9*A^2*b^19 + B^2*a^2*b^17 + 9*A^2*b^4*(-(
4*a*c - b^2)^15)^(1/2) + 6*A*B*a*b^18 + 6921*A^2*a^2*b^15*c^2 - 77580*A^2*a
^3*b^13*c^3 + 570960*A^2*a^4*b^11*c^4 - 2851776*A^2*a^5*b^9*c^5 + 9628416*A
^2*a^6*b^7*c^6 - 21095424*A^2*a^7*b^5*c^7 + 27095040*A^2*a^8*b^3*c^8 + 441*
A^2*a^2*c^2*(-(4*a*c - b^2)^15)^(1/2) + B^2*a^2*b^2*(-(4*a*c - b^2)^15)^(1/
2) + 1140*B^2*a^4*b^13*c^2 - 10160*B^2*a^5*b^11*c^3 + 34880*B^2*a^6*b^9*c^4
+ 43776*B^2*a^7*b^7*c^5 - 680960*B^2*a^8*b^5*c^6 + 1863680*B^2*a^9*b^3*c^7
+ 6881280*A*B*a^10*c^9 - 369*A^2*a*b^17*c - 15482880*A^2*a^9*b*c^9 - 55*B^
2*a^3*b^15*c - 1720320*B^2*a^10*b*c^8 - 25*B^2*a^3*c*(-(4*a*c - b^2)^15)^(1
/2) + 5580*A*B*a^3*b^14*c^2 - 59280*A*B*a^4*b^12*c^3 + 377280*A*B*a^5*b^10*
c^4 - 1430784*A*B*a^6*b^8*c^5 + 2860032*A*B*a^7*b^6*c^6 - 1290240*A*B*a^8*b
^4*c^7 - 5160960*A*B*a^9*b^2*c^8 - 99*A^2*a*b^2*c*(-(4*a*c - b^2)^15)^(1/2)
+ 6*A*B*a*b^3*(-(4*a*c - b^2)^15)^(1/2) - 288*A*B*a^2*b^16*c - 108*A*B*a^2
*b*c*(-(4*a*c - b^2)^15)^(1/2))/(512*(a^5*b^20 + 1048576*a^15*c^10 - 40*a^6
*b^18*c + 720*a^7*b^16*c^2 - 7680*a^8*b^14*c^3 + 53760*a^9*b^12*c^4 - 25804
8*a^10*b^10*c^5 + 860160*a^11*b^8*c^6 - 1966080*a^12*b^6*c^7 + 2949120*a^13
*b^4*c^8 - 2621440*a^14*b^2*c^9)))^(1/2)))*(-(9*A^2*b^19 + B^2*a^2*b^17 + 9
*A^2*b^4*(-(4*a*c - b^2)^15)^(1/2) + 6*A*B*a*b^18 + 6921*A^2*a^2*b^15*c^2 -
77580*A^2*a^3*b^13*c^3 + 570960*A^2*a^4*b^11*c^4 - 2851776*A^2*a^5*b^9*c^5
+ 9628416*A^2*a^6*b^7*c^6 - 21095424*A^2*a^7*b^5*c^7 + 27095040*A^2*a^8*b^
3*c^8 + 441*A^2*a^2*c^2*(-(4*a*c - b^2)^15)^(1/2) + B^2*a^2*b^2*(-(4*a*c -
b^2)^15)^(1/2) + 1140*B^2*a^4*b^13*c^2 - 10160*B^2*a^5*b^11*c^3 + 34880*B^2
*a^6*b^9*c^4 + 43776*B^2*a^7*b^7*c^5 - 680960*B^2*a^8*b^5*c^6 + 1863680*B^2
*a^9*b^3*c^7 + 6881280*A*B*a^10*c^9 - 369*A^2*a*b^17*c - 15482880*A^2*a^9*b
*c^9 - 55*B^2*a^3*b^15*c - 1720320*B^2*a^10*b*c^8 - 25*B^2*a^3*c*(-(4*a*c -
b^2)^15)^(1/2) + 5580*A*B*a^3*b^14*c^2 - 59280*A*B*a^4*b^12*c^3 + 377280*A
*B*a^5*b^10*c^4 - 1430784*A*B*a^6*b^8*c^5 + 2860032*A*B*a^7*b^6*c^6 - 12902
40*A*B*a^8*b^4*c^7 - 5160960*A*B*a^9*b^2*c^8 - 99*A^2*a*b^2*c*(-(4*a*c - b^
2)^15)^(1/2) + 6*A*B*a*b^3*(-(4*a*c - b^2)^15)^(1/2) - 288*A*B*a^2*b^16*c -
108*A*B*a^2*b*c*(-(4*a*c - b^2)^15)^(1/2))/(512*(a^5*b^20 + 1048576*a^15*c
^10 - 40*a^6*b^18*c + 720*a^7*b^16*c^2 - 7680*a^8*b^14*c^3 + 53760*a^9*b^12
*c^4 - 258048*a^10*b^10*c^5 + 860160*a^11*b^8*c^6 - 1966080*a^12*b^6*c^7 +

2949120*a^13*b^4*c^8 - 2621440*a^14*b^2*c^9)))^(1/2)*2i + atan(((((4194304*
B*a^9*b*c^8 - 22020096*A*a^9*c^9 + 768*A*a^2*b^14*c^2 - 22272*A*a^3*b^12*c^
3 + 282624*A*a^4*b^10*c^4 - 2027520*A*a^5*b^8*c^5 + 8847360*A*a^6*b^6*c^6 -
23396352*A*a^7*b^4*c^7 + 34603008*A*a^8*b^2*c^8 + 256*B*a^3*b^13*c^2 - 921
6*B*a^4*b^11*c^3 + 122880*B*a^5*b^9*c^4 - 819200*B*a^6*b^7*c^5 + 2949120*B*
a^7*b^5*c^6 - 5505024*B*a^8*b^3*c^7)/(512*(a^4*b^12 + 4096*a^10*c^6 - 24*a^
5*b^10*c + 240*a^6*b^8*c^2 - 1280*a^7*b^6*c^3 + 3840*a^8*b^4*c^4 - 6144*a^9
*b^2*c^5)) - (x*(-(9*A^2*b^19 + B^2*a^2*b^17 - 9*A^2*b^4*(-(4*a*c - b^2)^15
)^(1/2) + 6*A*B*a*b^18 + 6921*A^2*a^2*b^15*c^2 - 77580*A^2*a^3*b^13*c^3 + 5
70960*A^2*a^4*b^11*c^4 - 2851776*A^2*a^5*b^9*c^5 + 9628416*A^2*a^6*b^7*c^6
- 21095424*A^2*a^7*b^5*c^7 + 27095040*A^2*a^8*b^3*c^8 - 441*A^2*a^2*c^2*(-(
4*a*c - b^2)^15)^(1/2) - B^2*a^2*b^2*(-(4*a*c - b^2)^15)^(1/2) + 1140*B^2*a
^4*b^13*c^2 - 10160*B^2*a^5*b^11*c^3 + 34880*B^2*a^6*b^9*c^4 + 43776*B^2*a^
7*b^7*c^5 - 680960*B^2*a^8*b^5*c^6 + 1863680*B^2*a^9*b^3*c^7 + 6881280*A*B*
a^10*c^9 - 369*A^2*a*b^17*c - 15482880*A^2*a^9*b*c^9 - 55*B^2*a^3*b^15*c -
1720320*B^2*a^10*b*c^8 + 25*B^2*a^3*c*(-(4*a*c - b^2)^15)^(1/2) + 5580*A*B*
a^3*b^14*c^2 - 59280*A*B*a^4*b^12*c^3 + 377280*A*B*a^5*b^10*c^4 - 1430784*A
*B*a^6*b^8*c^5 + 2860032*A*B*a^7*b^6*c^6 - 1290240*A*B*a^8*b^4*c^7 - 516096
0*A*B*a^9*b^2*c^8 + 99*A^2*a*b^2*c*(-(4*a*c - b^2)^15)^(1/2) - 6*A*B*a*b^3*
(-(4*a*c - b^2)^15)^(1/2) - 288*A*B*a^2*b^16*c + 108*A*B*a^2*b*c*(-(4*a*c -
b^2)^15)^(1/2))/(512*(a^5*b^20 + 1048576*a^15*c^10 - 40*a^6*b^18*c + 720*a
^7*b^16*c^2 - 7680*a^8*b^14*c^3 + 53760*a^9*b^12*c^4 - 258048*a^10*b^10*c^5
+ 860160*a^11*b^8*c^6 - 1966080*a^12*b^6*c^7 + 2949120*a^13*b^4*c^8 - 2621
440*a^14*b^2*c^9)))^(1/2)*(262144*a^9*b*c^7 - 256*a^4*b^11*c^2 + 5120*a^5*b
^9*c^3 - 40960*a^6*b^7*c^4 + 163840*a^7*b^5*c^5 - 327680*a^8*b^3*c^6))/(32*
(a^4*b^8 + 256*a^8*c^4 - 16*a^5*b^6*c + 96*a^6*b^4*c^2 - 256*a^7*b^2*c^3)))
*(-(9*A^2*b^19 + B^2*a^2*b^17 - 9*A^2*b^4*(-(4*a*c - b^2)^15)^(1/2) + 6*A*B
*a*b^18 + 6921*A^2*a^2*b^15*c^2 - 77580*A^2*a^3*b^13*c^3 + 570960*A^2*a^4*b
^11*c^4 - 2851776*A^2*a^5*b^9*c^5 + 9628416*A^2*a^6*b^7*c^6 - 21095424*A^2*
a^7*b^5*c^7 + 27095040*A^2*a^8*b^3*c^8 - 441*A^2*a^2*c^2*(-(4*a*c - b^2)^15
)^(1/2) - B^2*a^2*b^2*(-(4*a*c - b^2)^15)^(1/2) + 1140*B^2*a^4*b^13*c^2 - 1
0160*B^2*a^5*b^11*c^3 + 34880*B^2*a^6*b^9*c^4 + 43776*B^2*a^7*b^7*c^5 - 680
960*B^2*a^8*b^5*c^6 + 1863680*B^2*a^9*b^3*c^7 + 6881280*A*B*a^10*c^9 - 369*
A^2*a*b^17*c - 15482880*A^2*a^9*b*c^9 - 55*B^2*a^3*b^15*c - 1720320*B^2*a^1
0*b*c^8 + 25*B^2*a^3*c*(-(4*a*c - b^2)^15)^(1/2) + 5580*A*B*a^3*b^14*c^2 -
59280*A*B*a^4*b^12*c^3 + 377280*A*B*a^5*b^10*c^4 - 1430784*A*B*a^6*b^8*c^5
+ 2860032*A*B*a^7*b^6*c^6 - 1290240*A*B*a^8*b^4*c^7 - 5160960*A*B*a^9*b^2*c
^8 + 99*A^2*a*b^2*c*(-(4*a*c - b^2)^15)^(1/2) - 6*A*B*a*b^3*(-(4*a*c - b^2)
^15)^(1/2) - 288*A*B*a^2*b^16*c + 108*A*B*a^2*b*c*(-(4*a*c - b^2)^15)^(1/2)
)/(512*(a^5*b^20 + 1048576*a^15*c^10 - 40*a^6*b^18*c + 720*a^7*b^16*c^2 - 7
680*a^8*b^14*c^3 + 53760*a^9*b^12*c^4 - 258048*a^10*b^10*c^5 + 860160*a^11*
b^8*c^6 - 1966080*a^12*b^6*c^7 + 2949120*a^13*b^4*c^8 - 2621440*a^14*b^2*c^
9)))^(1/2) + (x*(14112*A^2*a^4*c^7 + 9*A^2*b^8*c^3 - 800*B^2*a^5*c^6 + 1530
*A^2*a^2*b^4*c^5 - 6192*A^2*a^3*b^2*c^6 + B^2*a^2*b^6*c^3 - 34*B^2*a^3*b^4*
c^4 + 1472*B^2*a^4*b^2*c^5 - 180*A^2*a*b^6*c^4 - 162*A*B*a^2*b^5*c^4 + 1104

*A*B*a^3*b^3*c^5 + 6*A*B*a*b^7*c^3 - 6816*A*B*a^4*b*c^6))/(32*(a^4*b^8 + 256*a^8*c^4 - 16*a^5*b^6*c + 96*a^6*b^4*c^2 - 256*a^7*b^2*c^3)))*(-(9*A^2*b^19 + B^2*a^2*b^17 - 9*A^2*b^4*(-(4*a*c - b^2)^15)^(1/2) + 6*A*B*a*b^18 + 6921*A^2*a^2*b^15*c^2 - 77580*A^2*a^3*b^13*c^3 + 570960*A^2*a^4*b^11*c^4 - 2851776*A^2*a^5*b^9*c^5 + 9628416*A^2*a^6*b^7*c^6 - 21095424*A^2*a^7*b^5*c^7 + 27095040*A^2*a^8*b^3*c^8 - 441*A^2*a^2*c^2*(-(4*a*c - b^2)^15)^(1/2) - B^2*a^2*b^2*(-(4*a*c - b^2)^15)^(1/2) + 1140*B^2*a^4*b^13*c^2 - 10160*B^2*a^5*b^11*c^3 + 34880*B^2*a^6*b^9*c^4 + 43776*B^2*a^7*b^7*c^5 - 680960*B^2*a^8*b^5*c^6 + 1863680*B^2*a^9*b^3*c^7 + 6881280*A*B*a^10*c^9 - 369*A^2*a*b^17*c - 15482880*A^2*a^9*b*c^9 - 55*B^2*a^3*b^15*c - 1720320*B^2*a^10*b*c^8 + 25*B^2*a^3*c*(-(4*a*c - b^2)^15)^(1/2) + 5580*A*B*a^3*b^14*c^2 - 59280*A*B*a^4*b^12*c^3 + 377280*A*B*a^5*b^10*c^4 - 1430784*A*B*a^6*b^8*c^5 + 2860032*A*B*a^7*b^6*c^6 - 1290240*A*B*a^8*b^4*c^7 - 5160960*A*B*a^9*b^2*c^8 + 99*A^2*a*b^2*c*(-(4*a*c - b^2)^15)^(1/2) - 6*A*B*a*b^3*(-(4*a*c - b^2)^15)^(1/2) - 288*A*B*a^2*b^16*c + 108*A*B*a^2*b*c*(-(4*a*c - b^2)^15)^(1/2))/(512*(a^5*b^20 + 1048576*a^15*c^10 - 40*a^6*b^18*c + 720*a^7*b^16*c^2 - 7680*a^8*b^14*c^3 + 53760*a^9*b^12*c^4 - 258048*a^10*b^10*c^5 + 860160*a^11*b^8*c^6 - 1966080*a^12*b^6*c^7 + 2949120*a^13*b^4*c^8 - 2621440*a^14*b^2*c^9)))^(1/2)*1i - (((4194304*B*a^9*b*c^8 - 22020096*A*a^9*c^9 + 768*A*a^2*b^14*c^2 - 22272*A*a^3*b^12*c^3 + 282624*A*a^4*b^10*c^4 - 2027520*A*a^5*b^8*c^5 + 8847360*A*a^6*b^6*c^6 - 23396352*A*a^7*b^4*c^7 + 34603008*A*a^8*b^2*c^8 + 256*B*a^3*b^13*c^2 - 9216*B*a^4*b^11*c^3 + 122880*B*a^5*b^9*c^4 - 819200*B*a^6*b^7*c^5 + 2949120*B*a^7*b^5*c^6 - 5505024*B*a^8*b^3*c^7)/(512*(a^4*b^12 + 4096*a^10*c^6 - 24*a^5*b^10*c + 240*a^6*b^8*c^2 - 1280*a^7*b^6*c^3 + 3840*a^8*b^4*c^4 - 6144*a^9*b^2*c^5)) + (x*(-(9*A^2*b^19 + B^2*a^2*b^17 - 9*A^2*b^4*(-(4*a*c - b^2)^15)^(1/2) + 6*A*B*a*b^18 + 6921*A^2*a^2*b^15*c^2 - 77580*A^2*a^3*b^13*c^3 + 570960*A^2*a^4*b^11*c^4 - 2851776*A^2*a^5*b^9*c^5 + 9628416*A^2*a^6*b^7*c^6 - 21095424*A^2*a^7*b^5*c^7 + 27095040*A^2*a^8*b^3*c^8 - 441*A^2*a^2*c^2*(-(4*a*c - b^2)^15)^(1/2) - B^2*a^2*b^2*(-(4*a*c - b^2)^15)^(1/2) + 1140*B^2*a^4*b^13*c^2 - 10160*B^2*a^5*b^11*c^3 + 34880*B^2*a^6*b^9*c^4 + 43776*B^2*a^7*b^7*c^5 - 680960*B^2*a^8*b^5*c^6 + 1863680*B^2*a^9*b^3*c^7 + 6881280*A*B*a^10*c^9 - 369*A^2*a*b^17*c - 15482880*A^2*a^9*b*c^9 - 55*B^2*a^3*b^15*c - 1720320*B^2*a^10*b*c^8 + 25*B^2*a^3*c*(-(4*a*c - b^2)^15)^(1/2) + 5580*A*B*a^3*b^14*c^2 - 59280*A*B*a^4*b^12*c^3 + 377280*A*B*a^5*b^10*c^4 - 1430784*A*B*a^6*b^8*c^5 + 2860032*A*B*a^7*b^6*c^6 - 1290240*A*B*a^8*b^4*c^7 - 5160960*A*B*a^9*b^2*c^8 + 99*A^2*a*b^2*c*(-(4*a*c - b^2)^15)^(1/2) - 6*A*B*a*b^3*(-(4*a*c - b^2)^15)^(1/2) - 288*A*B*a^2*b^16*c + 108*A*B*a^2*b*c*(-(4*a*c - b^2)^15)^(1/2))/(512*(a^5*b^20 + 1048576*a^15*c^10 - 40*a^6*b^18*c + 720*a^7*b^16*c^2 - 7680*a^8*b^14*c^3 + 53760*a^9*b^12*c^4 - 258048*a^10*b^10*c^5 + 860160*a^11*b^8*c^6 - 1966080*a^12*b^6*c^7 + 2949120*a^13*b^4*c^8 - 2621440*a^14*b^2*c^9)))^(1/2)*(262144*a^9*b*c^7 - 256*a^4*b^11*c^2 + 5120*a^5*b^9*c^3 - 40960*a^6*b^7*c^4 + 163840*a^7*b^5*c^5 - 327680*a^8*b^3*c^6))/(32*(a^4*b^8 + 256*a^8*c^4 - 16*a^5*b^6*c + 96*a^6*b^4*c^2 - 256*a^7*b^2*c^3)))*(-(9*A^2*b^19 + B^2*a^2*b^17 - 9*A^2*b^4*(-(4*a*c - b^2)^15)^(1/2) + 6*A*B*a*b^18 + 6921*A^2*a^2*b^15*c^2 - 77580*A^2*a^3*b^13*c^3 + 57

0960*A^2*a^4*b^11*c^4 - 2851776*A^2*a^5*b^9*c^5 + 9628416*A^2*a^6*b^7*c^6 - 21095424*A^2*a^7*b^5*c^7 + 27095040*A^2*a^8*b^3*c^8 - 441*A^2*a^2*c^2*(-(4*a*c - b^2)^15)^(1/2) - B^2*a^2*b^2*(-(4*a*c - b^2)^15)^(1/2) + 1140*B^2*a^4*b^13*c^2 - 10160*B^2*a^5*b^11*c^3 + 34880*B^2*a^6*b^9*c^4 + 43776*B^2*a^7*b^7*c^5 - 680960*B^2*a^8*b^5*c^6 + 1863680*B^2*a^9*b^3*c^7 + 6881280*A*B*a^10*c^9 - 369*A^2*a*b^17*c - 15482880*A^2*a^9*b*c^9 - 55*B^2*a^3*b^15*c - 1720320*B^2*a^10*b*c^8 + 25*B^2*a^3*c*(-(4*a*c - b^2)^15)^(1/2) + 5580*A*B*a^3*b^14*c^2 - 59280*A*B*a^4*b^12*c^3 + 377280*A*B*a^5*b^10*c^4 - 1430784*A*B*a^6*b^8*c^5 + 2860032*A*B*a^7*b^6*c^6 - 1290240*A*B*a^8*b^4*c^7 - 5160960*A*B*a^9*b^2*c^8 + 99*A^2*a*b^2*c*(-(4*a*c - b^2)^15)^(1/2) - 6*A*B*a*b^3*(-(4*a*c - b^2)^15)^(1/2) - 288*A*B*a^2*b^16*c + 108*A*B*a^2*b*c*(-(4*a*c - b^2)^15)^(1/2))/(512*(a^5*b^20 + 1048576*a^15*c^10 - 40*a^6*b^18*c + 720*a^7*b^16*c^2 - 7680*a^8*b^14*c^3 + 53760*a^9*b^12*c^4 - 258048*a^10*b^10*c^5 + 860160*a^11*b^8*c^6 - 1966080*a^12*b^6*c^7 + 2949120*a^13*b^4*c^8 - 2621440*a^14*b^2*c^9)))^(1/2) - (x*(14112*A^2*a^4*c^7 + 9*A^2*b^8*c^3 - 800*B^2*a^5*c^6 + 1530*A^2*a^2*b^4*c^5 - 6192*A^2*a^3*b^2*c^6 + B^2*a^2*b^6*c^3 - 34*B^2*a^3*b^4*c^4 + 1472*B^2*a^4*b^2*c^5 - 180*A^2*a*b^6*c^4 - 162*A*B*a^2*b^5*c^4 + 1104*A*B*a^3*b^3*c^5 + 6*A*B*a*b^7*c^3 - 6816*A*B*a^4*b*c^6))/(32*(a^4*b^8 + 256*a^8*c^4 - 16*a^5*b^6*c + 96*a^6*b^4*c^2 - 256*a^7*b^2*c^3)))*(-(9*A^2*b^19 + B^2*a^2*b^17 - 9*A^2*b^4*(-(4*a*c - b^2)^15)^(1/2) + 6*A*B*a*b^18 + 6921*A^2*a^2*b^15*c^2 - 77580*A^2*a^3*b^13*c^3 + 570960*A^2*a^4*b^11*c^4 - 2851776*A^2*a^5*b^9*c^5 + 9628416*A^2*a^6*b^7*c^6 - 21095424*A^2*a^7*b^5*c^7 + 27095040*A^2*a^8*b^3*c^8 - 441*A^2*a^2*c^2*(-(4*a*c - b^2)^15)^(1/2) - B^2*a^2*b^2*(-(4*a*c - b^2)^15)^(1/2) + 1140*B^2*a^4*b^13*c^2 - 10160*B^2*a^5*b^11*c^3 + 34880*B^2*a^6*b^9*c^4 + 43776*B^2*a^7*b^7*c^5 - 680960*B^2*a^8*b^5*c^6 + 1863680*B^2*a^9*b^3*c^7 + 6881280*A*B*a^10*c^9 - 369*A^2*a*b^17*c - 15482880*A^2*a^9*b*c^9 - 55*B^2*a^3*b^15*c - 1720320*B^2*a^10*b*c^8 + 25*B^2*a^3*c*(-(4*a*c - b^2)^15)^(1/2) + 5580*A*B*a^3*b^14*c^2 - 59280*A*B*a^4*b^12*c^3 + 377280*A*B*a^5*b^10*c^4 - 1430784*A*B*a^6*b^8*c^5 + 2860032*A*B*a^7*b^6*c^6 - 1290240*A*B*a^8*b^4*c^7 - 5160960*A*B*a^9*b^2*c^8 + 99*A^2*a*b^2*c*(-(4*a*c - b^2)^15)^(1/2) - 6*A*B*a*b^3*(-(4*a*c - b^2)^15)^(1/2) - 288*A*B*a^2*b^16*c + 108*A*B*a^2*b*c*(-(4*a*c - b^2)^15)^(1/2))/(512*(a^5*b^20 + 1048576*a^15*c^10 - 40*a^6*b^18*c + 720*a^7*b^16*c^2 - 7680*a^8*b^14*c^3 + 53760*a^9*b^12*c^4 - 258048*a^10*b^10*c^5 + 860160*a^11*b^8*c^6 - 1966080*a^12*b^6*c^7 + 2949120*a^13*b^4*c^8 - 2621440*a^14*b^2*c^9)))^(1/2)*1i)/((((4194304*B*a^9*b*c^8 - 22020096*A*a^9*c^9 + 768*A*a^2*b^14*c^2 - 22272*A*a^3*b^12*c^3 + 282624*A*a^4*b^10*c^4 - 2027520*A*a^5*b^8*c^5 + 8847360*A*a^6*b^6*c^6 - 23396352*A*a^7*b^4*c^7 + 34603008*A*a^8*b^2*c^8 + 256*B*a^3*b^13*c^2 - 9216*B*a^4*b^11*c^3 + 122880*B*a^5*b^9*c^4 - 819200*B*a^6*b^7*c^5 + 2949120*B*a^7*b^5*c^6 - 5505024*B*a^8*b^3*c^7)/(512*(a^4*b^12 + 4096*a^10*c^6 - 24*a^5*b^10*c + 240*a^6*b^8*c^2 - 1280*a^7*b^6*c^3 + 3840*a^8*b^4*c^4 - 6144*a^9*b^2*c^5)) - (x*(-(9*A^2*b^19 + B^2*a^2*b^17 - 9*A^2*b^4*(-(4*a*c - b^2)^15)^(1/2) + 6*A*B*a*b^18 + 6921*A^2*a^2*b^15*c^2 - 77580*A^2*a^3*b^13*c^3 + 570960*A^2*a^4*b^11*c^4 - 2851776*A^2*a^5*b^9*c^5 + 9628416*A^2*a^6*b^7*c^6 - 21095424*A^2*a^7*b^5*c^7 + 27095040*A^2*a^8*b

^3*c^8 - 441*A^2*a^2*c^2*(-(4*a*c - b^2)^15)^(1/2) - B^2*a^2*b^2*(-(4*a*c - b^2)^15)^(1/2) + 1140*B^2*a^4*b^13*c^2 - 10160*B^2*a^5*b^11*c^3 + 34880*B^2*a^6*b^9*c^4 + 43776*B^2*a^7*b^7*c^5 - 680960*B^2*a^8*b^5*c^6 + 1863680*B^2*a^9*b^3*c^7 + 6881280*A*B*a^10*c^9 - 369*A^2*a*b^17*c - 15482880*A^2*a^9*b*c^9 - 55*B^2*a^3*b^15*c - 1720320*B^2*a^10*b*c^8 + 25*B^2*a^3*c*(-(4*a*c - b^2)^15)^(1/2) + 5580*A*B*a^3*b^14*c^2 - 59280*A*B*a^4*b^12*c^3 + 377280*A*B*a^5*b^10*c^4 - 1430784*A*B*a^6*b^8*c^5 + 2860032*A*B*a^7*b^6*c^6 - 1290240*A*B*a^8*b^4*c^7 - 5160960*A*B*a^9*b^2*c^8 + 99*A^2*a*b^2*c*(-(4*a*c - b^2)^15)^(1/2) - 6*A*B*a*b^3*(-(4*a*c - b^2)^15)^(1/2) - 288*A*B*a^2*b^16*c + 108*A*B*a^2*b*c*(-(4*a*c - b^2)^15)^(1/2))/(512*(a^5*b^20 + 1048576*a^15*c^10 - 40*a^6*b^18*c + 720*a^7*b^16*c^2 - 7680*a^8*b^14*c^3 + 53760*a^9*b^12*c^4 - 258048*a^10*b^10*c^5 + 860160*a^11*b^8*c^6 - 1966080*a^12*b^6*c^7 + 2949120*a^13*b^4*c^8 - 2621440*a^14*b^2*c^9)))^(1/2)*(262144*a^9*b*c^7 - 256*a^4*b^11*c^2 + 5120*a^5*b^9*c^3 - 40960*a^6*b^7*c^4 + 163840*a^7*b^5*c^5 - 327680*a^8*b^3*c^6))/(32*(a^4*b^8 + 256*a^8*c^4 - 16*a^5*b^6*c + 96*a^6*b^4*c^2 - 256*a^7*b^2*c^3)))*(-(9*A^2*b^19 + B^2*a^2*b^17 - 9*A^2*b^4*(-(4*a*c - b^2)^15)^(1/2) + 6*A*B*a*b^18 + 6921*A^2*a^2*b^15*c^2 - 77580*A^2*a^3*b^13*c^3 + 570960*A^2*a^4*b^11*c^4 - 2851776*A^2*a^5*b^9*c^5 + 9628416*A^2*a^6*b^7*c^6 - 21095424*A^2*a^7*b^5*c^7 + 27095040*A^2*a^8*b^3*c^8 - 441*A^2*a^2*c^2*(-(4*a*c - b^2)^15)^(1/2) - B^2*a^2*b^2*(-(4*a*c - b^2)^15)^(1/2) + 1140*B^2*a^4*b^13*c^2 - 10160*B^2*a^5*b^11*c^3 + 34880*B^2*a^6*b^9*c^4 + 43776*B^2*a^7*b^7*c^5 - 680960*B^2*a^8*b^5*c^6 + 1863680*B^2*a^9*b^3*c^7 + 6881280*A*B*a^10*c^9 - 369*A^2*a*b^17*c - 15482880*A^2*a^9*b*c^9 - 55*B^2*a^3*b^15*c - 1720320*B^2*a^10*b*c^8 + 25*B^2*a^3*c*(-(4*a*c - b^2)^15)^(1/2) + 5580*A*B*a^3*b^14*c^2 - 59280*A*B*a^4*b^12*c^3 + 377280*A*B*a^5*b^10*c^4 - 1430784*A*B*a^6*b^8*c^5 + 2860032*A*B*a^7*b^6*c^6 - 1290240*A*B*a^8*b^4*c^7 - 5160960*A*B*a^9*b^2*c^8 + 99*A^2*a*b^2*c*(-(4*a*c - b^2)^15)^(1/2) - 6*A*B*a*b^3*(-(4*a*c - b^2)^15)^(1/2) - 288*A*B*a^2*b^16*c + 108*A*B*a^2*b*c*(-(4*a*c - b^2)^15)^(1/2))/(512*(a^5*b^20 + 1048576*a^15*c^10 - 40*a^6*b^18*c + 720*a^7*b^16*c^2 - 7680*a^8*b^14*c^3 + 53760*a^9*b^12*c^4 - 258048*a^10*b^10*c^5 + 860160*a^11*b^8*c^6 - 1966080*a^12*b^6*c^7 + 2949120*a^13*b^4*c^8 - 2621440*a^14*b^2*c^9)))^(1/2) + (x*(14112*A^2*a^4*c^7 + 9*A^2*b^8*c^3 - 800*B^2*a^5*c^6 + 1530*A^2*a^2*b^4*c^5 - 6192*A^2*a^3*b^2*c^6 + B^2*a^2*b^6*c^3 - 34*B^2*a^3*b^4*c^4 + 1472*B^2*a^4*b^2*c^5 - 180*A^2*a*b^6*c^4 - 162*A*B*a^2*b^5*c^4 + 1104*A*B*a^3*b^3*c^5 + 6*A*B*a*b^7*c^3 - 6816*A*B*a^4*b*c^6))/(32*(a^4*b^8 + 256*a^8*c^4 - 16*a^5*b^6*c + 96*a^6*b^4*c^2 - 256*a^7*b^2*c^3)))*(-(9*A^2*b^19 + B^2*a^2*b^17 - 9*A^2*b^4*(-(4*a*c - b^2)^15)^(1/2) + 6*A*B*a*b^18 + 6921*A^2*a^2*b^15*c^2 - 77580*A^2*a^3*b^13*c^3 + 570960*A^2*a^4*b^11*c^4 - 2851776*A^2*a^5*b^9*c^5 + 9628416*A^2*a^6*b^7*c^6 - 21095424*A^2*a^7*b^5*c^7 + 27095040*A^2*a^8*b^3*c^8 - 441*A^2*a^2*c^2*(-(4*a*c - b^2)^15)^(1/2) - B^2*a^2*b^2*(-(4*a*c - b^2)^15)^(1/2) + 1140*B^2*a^4*b^13*c^2 - 10160*B^2*a^5*b^11*c^3 + 34880*B^2*a^6*b^9*c^4 + 43776*B^2*a^7*b^7*c^5 - 680960*B^2*a^8*b^5*c^6 + 1863680*B^2*a^9*b^3*c^7 + 6881280*A*B*a^10*c^9 - 369*A^2*a*b^17*c - 15482880*A^2*a^9*b*c^9 - 55*B^2*a^3*b^15*c - 1720320*B^2*a^10*b*c^8 + 25*B^2*a^3*c*(-(4*a*c - b^2)^15)^(1/2) + 5580*A*B*

a^3*b^14*c^2 - 59280*A*B*a^4*b^12*c^3 + 377280*A*B*a^5*b^10*c^4 - 1430784*A
*B*a^6*b^8*c^5 + 2860032*A*B*a^7*b^6*c^6 - 1290240*A*B*a^8*b^4*c^7 - 516096
0*A*B*a^9*b^2*c^8 + 99*A^2*a*b^2*c*(-(4*a*c - b^2)^15)^(1/2) - 6*A*B*a*b^3*
(-(4*a*c - b^2)^15)^(1/2) - 288*A*B*a^2*b^16*c + 108*A*B*a^2*b*c*(-(4*a*c -
b^2)^15)^(1/2))/(512*(a^5*b^20 + 1048576*a^15*c^10 - 40*a^6*b^18*c + 720*a
^7*b^16*c^2 - 7680*a^8*b^14*c^3 + 53760*a^9*b^12*c^4 - 258048*a^10*b^10*c^5
+ 860160*a^11*b^8*c^6 - 1966080*a^12*b^6*c^7 + 2949120*a^13*b^4*c^8 - 2621
440*a^14*b^2*c^9)))^(1/2) - (567*A^3*b^7*c^5 + 8000*B^3*a^5*c^7 + 67824*A^3
*a^2*b^3*c^7 - 35*B^3*a^2*b^6*c^4 - 84*B^3*a^3*b^4*c^5 + 12720*B^3*a^4*b^2*
c^6 + 141120*A^2*B*a^4*c^8 - 315*A^2*B*b^8*c^4 - 10368*A^3*a*b^5*c^6 - 1693
44*A^3*a^3*b*c^8 - 210*A*B^2*a*b^7*c^4 - 116160*A*B^2*a^4*b*c^7 + 6237*A^2*
B*a*b^6*c^5 + 1764*A*B^2*a^2*b^5*c^5 + 4608*A*B^2*a^3*b^3*c^6 - 42372*A^2*B
*a^2*b^4*c^6 + 96048*A^2*B*a^3*b^2*c^7)/(256*(a^4*b^12 + 4096*a^10*c^6 - 24
*a^5*b^10*c + 240*a^6*b^8*c^2 - 1280*a^7*b^6*c^3 + 3840*a^8*b^4*c^4 - 6144*
a^9*b^2*c^5)) + (((4194304*B*a^9*b*c^8 - 22020096*A*a^9*c^9 + 768*A*a^2*b^1
4*c^2 - 22272*A*a^3*b^12*c^3 + 282624*A*a^4*b^10*c^4 - 2027520*A*a^5*b^8*c^
5 + 8847360*A*a^6*b^6*c^6 - 23396352*A*a^7*b^4*c^7 + 34603008*A*a^8*b^2*c^8
+ 256*B*a^3*b^13*c^2 - 9216*B*a^4*b^11*c^3 + 122880*B*a^5*b^9*c^4 - 819200
*B*a^6*b^7*c^5 + 2949120*B*a^7*b^5*c^6 - 5505024*B*a^8*b^3*c^7)/(512*(a^4*b
^12 + 4096*a^10*c^6 - 24*a^5*b^10*c + 240*a^6*b^8*c^2 - 1280*a^7*b^6*c^3 +
3840*a^8*b^4*c^4 - 6144*a^9*b^2*c^5)) + (x*(-(9*A^2*b^19 + B^2*a^2*b^17 - 9
*A^2*b^4*(-(4*a*c - b^2)^15)^(1/2) + 6*A*B*a*b^18 + 6921*A^2*a^2*b^15*c^2 -
77580*A^2*a^3*b^13*c^3 + 570960*A^2*a^4*b^11*c^4 - 2851776*A^2*a^5*b^9*c^5
+ 9628416*A^2*a^6*b^7*c^6 - 21095424*A^2*a^7*b^5*c^7 + 27095040*A^2*a^8*b^
3*c^8 - 441*A^2*a^2*c^2*(-(4*a*c - b^2)^15)^(1/2) - B^2*a^2*b^2*(-(4*a*c -
b^2)^15)^(1/2) + 1140*B^2*a^4*b^13*c^2 - 10160*B^2*a^5*b^11*c^3 + 34880*B^2
*a^6*b^9*c^4 + 43776*B^2*a^7*b^7*c^5 - 680960*B^2*a^8*b^5*c^6 + 1863680*B^2
*a^9*b^3*c^7 + 6881280*A*B*a^10*c^9 - 369*A^2*a*b^17*c - 15482880*A^2*a^9*b
*c^9 - 55*B^2*a^3*b^15*c - 1720320*B^2*a^10*b*c^8 + 25*B^2*a^3*c*(-(4*a*c -
b^2)^15)^(1/2) + 5580*A*B*a^3*b^14*c^2 - 59280*A*B*a^4*b^12*c^3 + 377280*A
*B*a^5*b^10*c^4 - 1430784*A*B*a^6*b^8*c^5 + 2860032*A*B*a^7*b^6*c^6 - 12902
40*A*B*a^8*b^4*c^7 - 5160960*A*B*a^9*b^2*c^8 + 99*A^2*a*b^2*c*(-(4*a*c - b^
2)^15)^(1/2) - 6*A*B*a*b^3*(-(4*a*c - b^2)^15)^(1/2) - 288*A*B*a^2*b^16*c +
108*A*B*a^2*b*c*(-(4*a*c - b^2)^15)^(1/2))/(512*(a^5*b^20 + 1048576*a^15*c
^10 - 40*a^6*b^18*c + 720*a^7*b^16*c^2 - 7680*a^8*b^14*c^3 + 53760*a^9*b^12
*c^4 - 258048*a^10*b^10*c^5 + 860160*a^11*b^8*c^6 - 1966080*a^12*b^6*c^7 +
2949120*a^13*b^4*c^8 - 2621440*a^14*b^2*c^9)))^(1/2)*(262144*a^9*b*c^7 - 25
6*a^4*b^11*c^2 + 5120*a^5*b^9*c^3 - 40960*a^6*b^7*c^4 + 163840*a^7*b^5*c^5
- 327680*a^8*b^3*c^6))/(32*(a^4*b^8 + 256*a^8*c^4 - 16*a^5*b^6*c + 96*a^6*b
^4*c^2 - 256*a^7*b^2*c^3)))*(-(9*A^2*b^19 + B^2*a^2*b^17 - 9*A^2*b^4*(-(4*a
*c - b^2)^15)^(1/2) + 6*A*B*a*b^18 + 6921*A^2*a^2*b^15*c^2 - 77580*A^2*a^3*
b^13*c^3 + 570960*A^2*a^4*b^11*c^4 - 2851776*A^2*a^5*b^9*c^5 + 9628416*A^2*
a^6*b^7*c^6 - 21095424*A^2*a^7*b^5*c^7 + 27095040*A^2*a^8*b^3*c^8 - 441*A^2
*a^2*c^2*(-(4*a*c - b^2)^15)^(1/2) - B^2*a^2*b^2*(-(4*a*c - b^2)^15)^(1/2)
+ 1140*B^2*a^4*b^13*c^2 - 10160*B^2*a^5*b^11*c^3 + 34880*B^2*a^6*b^9*c^4 +

43776*B^2*a^7*b^7*c^5 - 680960*B^2*a^8*b^5*c^6 + 1863680*B^2*a^9*b^3*c^7 + 6881280*A*B*a^10*c^9 - 369*A^2*a*b^17*c - 15482880*A^2*a^9*b*c^9 - 55*B^2*a^3*b^15*c - 1720320*B^2*a^10*b*c^8 + 25*B^2*a^3*c*(-(4*a*c - b^2)^15)^(1/2) + 5580*A*B*a^3*b^14*c^2 - 59280*A*B*a^4*b^12*c^3 + 377280*A*B*a^5*b^10*c^4 - 1430784*A*B*a^6*b^8*c^5 + 2860032*A*B*a^7*b^6*c^6 - 1290240*A*B*a^8*b^4*c^7 - 5160960*A*B*a^9*b^2*c^8 + 99*A^2*a*b^2*c*(-(4*a*c - b^2)^15)^(1/2) - 6*A*B*a*b^3*(-(4*a*c - b^2)^15)^(1/2) - 288*A*B*a^2*b^16*c + 108*A*B*a^2*b*c*(-(4*a*c - b^2)^15)^(1/2))/(512*(a^5*b^20 + 1048576*a^15*c^10 - 40*a^6*b^18*c + 720*a^7*b^16*c^2 - 7680*a^8*b^14*c^3 + 53760*a^9*b^12*c^4 - 258048*a^10*b^10*c^5 + 860160*a^11*b^8*c^6 - 1966080*a^12*b^6*c^7 + 2949120*a^13*b^4*c^8 - 2621440*a^14*b^2*c^9)))^(1/2) - (x*(14112*A^2*a^4*c^7 + 9*A^2*b^8*c^3 - 800*B^2*a^5*c^6 + 1530*A^2*a^2*b^4*c^5 - 6192*A^2*a^3*b^2*c^6 + B^2*a^2*b^6*c^3 - 34*B^2*a^3*b^4*c^4 + 1472*B^2*a^4*b^2*c^5 - 180*A^2*a*b^6*c^4 - 162*A*B*a^2*b^5*c^4 + 1104*A*B*a^3*b^3*c^5 + 6*A*B*a*b^7*c^3 - 6816*A*B*a^4*b*c^6))/(32*(a^4*b^8 + 256*a^8*c^4 - 16*a^5*b^6*c + 96*a^6*b^4*c^2 - 256*a^7*b^2*c^3)))*(-(9*A^2*b^19 + B^2*a^2*b^17 - 9*A^2*b^4*(-(4*a*c - b^2)^15)^(1/2) + 6*A*B*a*b^18 + 6921*A^2*a^2*b^15*c^2 - 77580*A^2*a^3*b^13*c^3 + 570960*A^2*a^4*b^11*c^4 - 2851776*A^2*a^5*b^9*c^5 + 9628416*A^2*a^6*b^7*c^6 - 21095424*A^2*a^7*b^5*c^7 + 27095040*A^2*a^8*b^3*c^8 - 441*A^2*a^2*c^2*(-(4*a*c - b^2)^15)^(1/2) - B^2*a^2*b^2*(-(4*a*c - b^2)^15)^(1/2) + 1140*B^2*a^4*b^13*c^2 - 10160*B^2*a^5*b^11*c^3 + 34880*B^2*a^6*b^9*c^4 + 43776*B^2*a^7*b^7*c^5 - 680960*B^2*a^8*b^5*c^6 + 1863680*B^2*a^9*b^3*c^7 + 6881280*A*B*a^10*c^9 - 369*A^2*a*b^17*c - 15482880*A^2*a^9*b*c^9 - 55*B^2*a^3*b^15*c - 1720320*B^2*a^10*b*c^8 + 25*B^2*a^3*c*(-(4*a*c - b^2)^15)^(1/2) + 5580*A*B*a^3*b^14*c^2 - 59280*A*B*a^4*b^12*c^3 + 377280*A*B*a^5*b^10*c^4 - 1430784*A*B*a^6*b^8*c^5 + 2860032*A*B*a^7*b^6*c^6 - 1290240*A*B*a^8*b^4*c^7 - 5160960*A*B*a^9*b^2*c^8 + 99*A^2*a*b^2*c*(-(4*a*c - b^2)^15)^(1/2) - 6*A*B*a*b^3*(-(4*a*c - b^2)^15)^(1/2) - 288*A*B*a^2*b^16*c + 108*A*B*a^2*b*c*(-(4*a*c - b^2)^15)^(1/2))/(512*(a^5*b^20 + 1048576*a^15*c^10 - 40*a^6*b^18*c + 720*a^7*b^16*c^2 - 7680*a^8*b^14*c^3 + 53760*a^9*b^12*c^4 - 258048*a^10*b^10*c^5 + 860160*a^11*b^8*c^6 - 1966080*a^12*b^6*c^7 + 2949120*a^13*b^4*c^8 - 2621440*a^14*b^2*c^9)))^(1/2)))*(-(9*A^2*b^19 + B^2*a^2*b^17 - 9*A^2*b^4*(-(4*a*c - b^2)^15)^(1/2) + 6*A*B*a*b^18 + 6921*A^2*a^2*b^15*c^2 - 77580*A^2*a^3*b^13*c^3 + 570960*A^2*a^4*b^11*c^4 - 2851776*A^2*a^5*b^9*c^5 + 9628416*A^2*a^6*b^7*c^6 - 21095424*A^2*a^7*b^5*c^7 + 27095040*A^2*a^8*b^3*c^8 - 441*A^2*a^2*c^2*(-(4*a*c - b^2)^15)^(1/2) - B^2*a^2*b^2*(-(4*a*c - b^2)^15)^(1/2) + 1140*B^2*a^4*b^13*c^2 - 10160*B^2*a^5*b^11*c^3 + 34880*B^2*a^6*b^9*c^4 + 43776*B^2*a^7*b^7*c^5 - 680960*B^2*a^8*b^5*c^6 + 1863680*B^2*a^9*b^3*c^7 + 6881280*A*B*a^10*c^9 - 369*A^2*a*b^17*c - 15482880*A^2*a^9*b*c^9 - 55*B^2*a^3*b^15*c - 1720320*B^2*a^10*b*c^8 + 25*B^2*a^3*c*(-(4*a*c - b^2)^15)^(1/2) + 5580*A*B*a^3*b^14*c^2 - 59280*A*B*a^4*b^12*c^3 + 377280*A*B*a^5*b^10*c^4 - 1430784*A*B*a^6*b^8*c^5 + 2860032*A*B*a^7*b^6*c^6 - 1290240*A*B*a^8*b^4*c^7 - 5160960*A*B*a^9*b^2*c^8 + 99*A^2*a*b^2*c*(-(4*a*c - b^2)^15)^(1/2) - 6*A*B*a*b^3*(-(4*a*c - b^2)^15)^(1/2) - 288*A*B*a^2*b^16*c + 108*A*B*a^2*b*c*(-(4*a*c - b^2)^15)^(1/2))/(512*(a^5*b^20 + 1048576*a^15*c^10 - 40*a^6*b^1

```
8*c + 720*a^7*b^16*c^2 - 7680*a^8*b^14*c^3 + 53760*a^9*b^12*c^4 - 258048*a^
10*b^10*c^5 + 860160*a^11*b^8*c^6 - 1966080*a^12*b^6*c^7 + 2949120*a^13*b^4
*c^8 - 2621440*a^14*b^2*c^9)))^(1/2)*2i
```

3.137 $\int \frac{x(-7+4x^2)}{4-5x^2+x^4}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 25

$$\int \frac{x(-7+4x^2)}{4-5x^2+x^4}\,dx = \frac{1}{2}\log\left(1-x^2\right)+\frac{3}{2}\log\left(4-x^2\right)$$

```
[Out] 1/2*ln(-x^2+1)+3/2*ln(-x^2+4)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {1261, 646, 31}

$$\int \frac{x(-7+4x^2)}{4-5x^2+x^4}\,dx = \frac{1}{2}\log\left(1-x^2\right)+\frac{3}{2}\log\left(4-x^2\right)$$

```
[In] Int[(x*(-7 + 4*x^2))/(4 - 5*x^2 + x^4),x]
[Out] Log[1 - x^2]/2 + (3*Log[4 - x^2])/2
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 646

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> W
ith[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(c*d - e*(b/2 - q/2))/q, Int[1/(b/2 - q/
2 + c*x), x], x] - Dist[(c*d - e*(b/2 + q/2))/q, Int[1/(b/2 + q/2 + c*x), x
], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a
```

```
*c, 0] && NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1261

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(
p_.), x_Symbol] :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x],
 x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{1}{2}\text{Subst}\left(\int \frac{-7+4x}{4-5x+x^2}\,dx, x, x^2\right)\\ &= \frac{1}{2}\text{Subst}\left(\int \frac{1}{-1+x}\,dx, x, x^2\right)+\frac{3}{2}\text{Subst}\left(\int \frac{1}{-4+x}\,dx, x, x^2\right)\\ &= \frac{1}{2}\log\left(1-x^2\right)+\frac{3}{2}\log\left(4-x^2\right)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \frac{x(-7+4x^2)}{4-5x^2+x^4}\,dx = \frac{1}{2}\log\left(1-x^2\right)+\frac{3}{2}\log\left(4-x^2\right)$$

```
[In] Integrate[(x*(-7 + 4*x^2))/(4 - 5*x^2 + x^4),x]
[Out] Log[1 - x^2]/2 + (3*Log[4 - x^2])/2
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.72

method	result	size
default	$\frac{\ln(x^2-1)}{2}+\frac{3\ln(x^2-4)}{2}$	18
risch	$\frac{\ln(x^2-1)}{2}+\frac{3\ln(x^2-4)}{2}$	18
norman	$\frac{3\ln(x-2)}{2}+\frac{\ln(x-1)}{2}+\frac{\ln(x+1)}{2}+\frac{3\ln(x+2)}{2}$	26
parallelrisch	$\frac{3\ln(x-2)}{2}+\frac{\ln(x-1)}{2}+\frac{\ln(x+1)}{2}+\frac{3\ln(x+2)}{2}$	26

```
[In] int(x*(4*x^2-7)/(x^4-5*x^2+4),x,method=_RETURNVERBOSE)
[Out] 1/2*ln(x^2-1)+3/2*ln(x^2-4)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.68

$$\int \frac{x(-7+4x^2)}{4-5x^2+x^4}\,dx = \frac{1}{2}\,\log\left(x^2-1\right)+\frac{3}{2}\,\log\left(x^2-4\right)$$

```
[In] integrate(x*(4*x^2-7)/(x^4-5*x^2+4),x, algorithm="fricas")
[Out] 1/2*log(x^2 - 1) + 3/2*log(x^2 - 4)
```

Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.68

$$\int \frac{x(-7+4x^2)}{4-5x^2+x^4}\,dx = \frac{3\log\left(x^2-4\right)}{2}+\frac{\log\left(x^2-1\right)}{2}$$

```
[In] integrate(x*(4*x**2-7)/(x**4-5*x**2+4),x)
[Out] 3*log(x**2 - 4)/2 + log(x**2 - 1)/2
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.68

$$\int \frac{x(-7+4x^2)}{4-5x^2+x^4}\,dx = \frac{1}{2}\,\log\left(x^2-1\right)+\frac{3}{2}\,\log\left(x^2-4\right)$$

```
[In] integrate(x*(4*x^2-7)/(x^4-5*x^2+4),x, algorithm="maxima")
[Out] 1/2*log(x^2 - 1) + 3/2*log(x^2 - 4)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.76

$$\int \frac{x(-7+4x^2)}{4-5x^2+x^4}\,dx = \frac{1}{2}\,\log\left(\left|x^2-1\right|\right)+\frac{3}{2}\,\log\left(\left|x^2-4\right|\right)$$

```
[In] integrate(x*(4*x^2-7)/(x^4-5*x^2+4),x, algorithm="giac")
[Out] 1/2*log(abs(x^2 - 1)) + 3/2*log(abs(x^2 - 4))
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.68

$$\int \frac{x(-7+4x^2)}{4-5x^2+x^4}\,dx = \frac{\ln\left(x^2-1\right)}{2}+\frac{3\,\ln\left(x^2-4\right)}{2}$$

```
[In] int((x*(4*x^2 - 7))/(x^4 - 5*x^2 + 4),x)
[Out] log(x^2 - 1)/2 + (3*log(x^2 - 4))/2
```

3.138 $\int \frac{-7x+4x^3}{4-5x^2+x^4} dx$

Optimal result

Integrand size = 22, antiderivative size = 25

$$\int \frac{-7x+4x^3}{4-5x^2+x^4} \, dx = \frac{1}{2}\log\left(1-x^2\right)+\frac{3}{2}\log\left(4-x^2\right)$$

```
[Out] 1/2*ln(-x^2+1)+3/2*ln(-x^2+4)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {1607, 1261, 646, 31}

$$\int \frac{-7x+4x^3}{4-5x^2+x^4} \, dx = \frac{1}{2}\log\left(1-x^2\right)+\frac{3}{2}\log\left(4-x^2\right)$$

```
[In] Int[(-7*x + 4*x^3)/(4 - 5*x^2 + x^4),x]
```

```
[Out] Log[1 - x^2]/2 + (3*Log[4 - x^2])/2
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 646

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> W
ith[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(c*d - e*(b/2 - q/2))/q, Int[1/(b/2 - q/
2 + c*x), x], x] - Dist[(c*d - e*(b/2 + q/2))/q, Int[1/(b/2 + q/2 + c*x), x
], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a
```

```
*c, 0] && NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1261

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(
p_.), x_Symbol] :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x],
 x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]
```

Rule 1607

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \frac{x(-7+4x^2)}{4-5x^2+x^4}\,dx \\
&= \frac{1}{2}\text{Subst}\left(\int \frac{-7+4x}{4-5x+x^2}\,dx, x, x^2\right) \\
&= \frac{1}{2}\text{Subst}\left(\int \frac{1}{-1+x}\,dx, x, x^2\right) + \frac{3}{2}\text{Subst}\left(\int \frac{1}{-4+x}\,dx, x, x^2\right) \\
&= \frac{1}{2}\log\left(1-x^2\right) + \frac{3}{2}\log\left(4-x^2\right)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \frac{-7x+4x^3}{4-5x^2+x^4}\,dx = \frac{1}{2}\log\left(1-x^2\right) + \frac{3}{2}\log\left(4-x^2\right)$$

```
[In] Integrate[(-7*x + 4*x^3)/(4 - 5*x^2 + x^4),x]
[Out] Log[1 - x^2]/2 + (3*Log[4 - x^2])/2
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.72

method	result	size
default	$\frac{\ln(x^2-1)}{2}+\frac{3\ln(x^2-4)}{2}$	18
risch	$\frac{\ln(x^2-1)}{2}+\frac{3\ln(x^2-4)}{2}$	18
norman	$\frac{3\ln(x-2)}{2}+\frac{\ln(x-1)}{2}+\frac{\ln(x+1)}{2}+\frac{3\ln(x+2)}{2}$	26
parallelrisch	$\frac{3\ln(x-2)}{2}+\frac{\ln(x-1)}{2}+\frac{\ln(x+1)}{2}+\frac{3\ln(x+2)}{2}$	26

```
[In] int((4*x^3-7*x)/(x^4-5*x^2+4),x,method=_RETURNVERBOSE)
[Out] 1/2*ln(x^2-1)+3/2*ln(x^2-4)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.68

$$\int \frac{-7x+4x^3}{4-5x^2+x^4}\,dx=\frac{1}{2}\,\log\left(x^2-1\right)+\frac{3}{2}\,\log\left(x^2-4\right)$$

```
[In] integrate((4*x^3-7*x)/(x^4-5*x^2+4),x, algorithm="fricas")
[Out] 1/2*log(x^2 - 1) + 3/2*log(x^2 - 4)
```

Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.68

$$\int \frac{-7x+4x^3}{4-5x^2+x^4}\,dx=\frac{3\log\left(x^2-4\right)}{2}+\frac{\log\left(x^2-1\right)}{2}$$

```
[In] integrate((4*x**3-7*x)/(x**4-5*x**2+4),x)
[Out] 3*log(x**2 - 4)/2 + log(x**2 - 1)/2
```

Maxima [A] (verification not implemented)

none

Time = 0.18 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \frac{-7x+4x^3}{4-5x^2+x^4}\,dx = \frac{3}{2}\log(x+2)+\frac{1}{2}\log(x+1)+\frac{1}{2}\log(x-1)+\frac{3}{2}\log(x-2)$$

```
[In] integrate((4*x^3-7*x)/(x^4-5*x^2+4),x, algorithm="maxima")
[Out] 3/2*log(x + 2) + 1/2*log(x + 1) + 1/2*log(x - 1) + 3/2*log(x - 2)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.76

$$\int \frac{-7x+4x^3}{4-5x^2+x^4}\,dx = \frac{1}{2}\log\left(\left|x^2-1\right|\right)+\frac{3}{2}\log\left(\left|x^2-4\right|\right)$$

```
[In] integrate((4*x^3-7*x)/(x^4-5*x^2+4),x, algorithm="giac")
[Out] 1/2*log(abs(x^2 - 1)) + 3/2*log(abs(x^2 - 4))
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.68

$$\int \frac{-7x+4x^3}{4-5x^2+x^4}\,dx = \frac{\ln(x^2-1)}{2}+\frac{3\ln(x^2-4)}{2}$$

```
[In] int(-(7*x - 4*x^3)/(x^4 - 5*x^2 + 4),x)
[Out] log(x^2 - 1)/2 + (3*log(x^2 - 4))/2
```

3.139 $\int \frac{x(2+x^2)}{1+x^2+x^4} \, dx$

Optimal result

Integrand size = 17, antiderivative size = 37

$$\int \frac{x(2+x^2)}{1+x^2+x^4} \, dx = \frac{1}{2}\sqrt{3}\arctan\left(\frac{1+2x^2}{\sqrt{3}}\right) + \frac{1}{4}\log\left(1+x^2+x^4\right)$$

```
[Out] 1/4*ln(x^4+x^2+1)+1/2*arctan(1/3*(2*x^2+1)*3^(1/2))*3^(1/2)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.294, Rules used = {1261, 648, 632, 210, 642}

$$\int \frac{x(2+x^2)}{1+x^2+x^4} \, dx = \frac{1}{2}\sqrt{3}\arctan\left(\frac{2x^2+1}{\sqrt{3}}\right) + \frac{1}{4}\log\left(x^4+x^2+1\right)$$

```
[In] Int[(x*(2 + x^2))/(1 + x^2 + x^4),x]
```

```
[Out] (Sqrt[3]*ArcTan[(1 + 2*x^2)/Sqrt[3]])/2 + Log[1 + x^2 + x^4]/4
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
```

```
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1261

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(
p_.), x_Symbol] :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x],
 x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{2}\text{Subst}\left(\int \frac{2+x}{1+x+x^2}\,dx, x, x^2\right)\\
&= \frac{1}{4}\text{Subst}\left(\int \frac{1+2x}{1+x+x^2}\,dx, x, x^2\right) + \frac{3}{4}\text{Subst}\left(\int \frac{1}{1+x+x^2}\,dx, x, x^2\right)\\
&= \frac{1}{4}\log\left(1+x^2+x^4\right) - \frac{3}{2}\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1+2x^2\right)\\
&= \frac{1}{2}\sqrt{3}\tan^{-1}\left(\frac{1+2x^2}{\sqrt{3}}\right) + \frac{1}{4}\log\left(1+x^2+x^4\right)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.00

$$\int \frac{x(2+x^2)}{1+x^2+x^4}\,dx = \frac{1}{2}\sqrt{3}\arctan\left(\frac{1+2x^2}{\sqrt{3}}\right) + \frac{1}{4}\log\left(1+x^2+x^4\right)$$

```
[In] Integrate[(x*(2 + x^2))/(1 + x^2 + x^4),x]
[Out] (Sqrt[3]*ArcTan[(1 + 2*x^2)/Sqrt[3]])/2 + Log[1 + x^2 + x^4]/4
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.84

method	result	size
default	$\frac{\ln\left(x^4+x^2+1\right)}{4}+\frac{\arctan\left(\frac{\left(2x^2+1\right)\sqrt{3}}{3}\right)\sqrt{3}}{2}$	31
risch	$\frac{\ln\left(4x^4+4x^2+4\right)}{4}+\frac{\arctan\left(\frac{\left(2x^2+1\right)\sqrt{3}}{3}\right)\sqrt{3}}{2}$	35

```
[In] int(x*(x^2+2)/(x^4+x^2+1),x,method=_RETURNVERBOSE)
[Out] 1/4*ln(x^4+x^2+1)+1/2*arctan(1/3*(2*x^2+1)*3^(1/2))*3^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.81

$$\int \frac{x(2+x^2)}{1+x^2+x^4}\,dx = \frac{1}{2}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x^2+1)\right)+\frac{1}{4}\log\left(x^4+x^2+1\right)$$

```
[In] integrate(x*(x^2+2)/(x^4+x^2+1),x, algorithm="fricas")
[Out] 1/2*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^2 + 1)) + 1/4*log(x^4 + x^2 + 1)
```

Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.00

$$\int \frac{x(2+x^2)}{1+x^2+x^4}\,dx = \frac{\log\left(x^4+x^2+1\right)}{4}+\frac{\sqrt{3}\,\mathrm{atan}\left(\frac{2\sqrt{3}x^2}{3}+\frac{\sqrt{3}}{3}\right)}{2}$$

```
[In] integrate(x*(x**2+2)/(x**4+x**2+1),x)
[Out] log(x**4 + x**2 + 1)/4 + sqrt(3)*atan(2*sqrt(3)*x**2/3 + sqrt(3)/3)/2
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.81

$$\int \frac{x(2+x^2)}{1+x^2+x^4}\,dx = \frac{1}{2}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x^2+1)\right) + \frac{1}{4}\log\left(x^4+x^2+1\right)$$

```
[In] integrate(x*(x^2+2)/(x^4+x^2+1),x, algorithm="maxima")
[Out] 1/2*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^2 + 1)) + 1/4*log(x^4 + x^2 + 1)
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.81

$$\int \frac{x(2+x^2)}{1+x^2+x^4}\,dx = \frac{1}{2}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x^2+1)\right) + \frac{1}{4}\log\left(x^4+x^2+1\right)$$

```
[In] integrate(x*(x^2+2)/(x^4+x^2+1),x, algorithm="giac")
[Out] 1/2*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^2 + 1)) + 1/4*log(x^4 + x^2 + 1)
```

Mupad [B] (verification not implemented)

Time = 7.56 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.86

$$\int \frac{x(2+x^2)}{1+x^2+x^4}\,dx = \frac{\ln\left(x^4+x^2+1\right)}{4} + \frac{\sqrt{3}\,\mathrm{atan}\left(\frac{2\sqrt{3}\,x^2}{3}+\frac{\sqrt{3}}{3}\right)}{2}$$

```
[In] int((x*(x^2 + 2))/(x^2 + x^4 + 1),x)
[Out] log(x^2 + x^4 + 1)/4 + (3^(1/2)*atan(3^(1/2)/3 + (2*3^(1/2)*x^2)/3))/2
```

3.140 $\int \frac{2x+x^3}{1+x^2+x^4} \, dx$

Optimal result

Integrand size = 18, antiderivative size = 37

$$\int \frac{2x+x^3}{1+x^2+x^4} \, dx = \frac{1}{2}\sqrt{3}\arctan\left(\frac{1+2x^2}{\sqrt{3}}\right)+\frac{1}{4}\log\left(1+x^2+x^4\right)$$

```
[Out] 1/4*ln(x^4+x^2+1)+1/2*arctan(1/3*(2*x^2+1)*3^(1/2))*3^(1/2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {1607, 1261, 648, 632, 210, 642}

$$\int \frac{2x+x^3}{1+x^2+x^4} \, dx = \frac{1}{2}\sqrt{3}\arctan\left(\frac{2x^2+1}{\sqrt{3}}\right)+\frac{1}{4}\log\left(x^4+x^2+1\right)$$

```
[In] Int[(2*x + x^3)/(1 + x^2 + x^4),x]
```

```
[Out] (Sqrt[3]*ArcTan[(1 + 2*x^2)/Sqrt[3]])/2 + Log[1 + x^2 + x^4]/4
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
```

```
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1261

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(
p_.), x_Symbol] :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x],
 x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]
```

Rule 1607

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \frac{x(2+x^2)}{1+x^2+x^4}\,dx \\
&= \frac{1}{2}\text{Subst}\left(\int \frac{2+x}{1+x+x^2}\,dx, x, x^2\right) \\
&= \frac{1}{4}\text{Subst}\left(\int \frac{1+2x}{1+x+x^2}\,dx, x, x^2\right) + \frac{3}{4}\text{Subst}\left(\int \frac{1}{1+x+x^2}\,dx, x, x^2\right) \\
&= \frac{1}{4}\log\left(1+x^2+x^4\right) - \frac{3}{2}\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1+2x^2\right) \\
&= \frac{1}{2}\sqrt{3}\tan^{-1}\left(\frac{1+2x^2}{\sqrt{3}}\right) + \frac{1}{4}\log\left(1+x^2+x^4\right)
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.00

$$\int \frac{2x + x^3}{1 + x^2 + x^4} \, dx = \frac{1}{2}\sqrt{3} \arctan\left(\frac{1 + 2x^2}{\sqrt{3}}\right) + \frac{1}{4}\log\left(1 + x^2 + x^4\right)$$

```
[In] Integrate[(2*x + x^3)/(1 + x^2 + x^4),x]
[Out] (Sqrt[3]*ArcTan[(1 + 2*x^2)/Sqrt[3]])/2 + Log[1 + x^2 + x^4]/4
```

Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.84

method	result	size
default	$\frac{\ln\left(x^4+x^2+1\right)}{4}+\frac{\arctan\left(\frac{\left(2x^2+1\right)\sqrt{3}}{3}\right)\sqrt{3}}{2}$	31
risch	$\frac{\ln\left(4x^4+4x^2+4\right)}{4}+\frac{\arctan\left(\frac{\left(2x^2+1\right)\sqrt{3}}{3}\right)\sqrt{3}}{2}$	35

```
[In] int((x^3+2*x)/(x^4+x^2+1),x,method=_RETURNVERBOSE)
[Out] 1/4*ln(x^4+x^2+1)+1/2*arctan(1/3*(2*x^2+1)*3^(1/2))*3^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.81

$$\int \frac{2x + x^3}{1 + x^2 + x^4} \, dx = \frac{1}{2}\sqrt{3} \arctan\left(\frac{1}{3}\sqrt{3}(2x^2 + 1)\right) + \frac{1}{4}\log\left(x^4 + x^2 + 1\right)$$

```
[In] integrate((x^3+2*x)/(x^4+x^2+1),x, algorithm="fricas")
[Out] 1/2*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^2 + 1)) + 1/4*log(x^4 + x^2 + 1)
```

Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.00

$$\int \frac{2x+x^3}{1+x^2+x^4}\,dx = \frac{\log\left(x^4+x^2+1\right)}{4} + \frac{\sqrt{3}\,\mathrm{atan}\left(\frac{2\sqrt{3}x^2}{3}+\frac{\sqrt{3}}{3}\right)}{2}$$

```
[In] integrate((x**3+2*x)/(x**4+x**2+1),x)
[Out] log(x**4 + x**2 + 1)/4 + sqrt(3)*atan(2*sqrt(3)*x**2/3 + sqrt(3)/3)/2
```

Maxima [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.43

$$\int \frac{2x+x^3}{1+x^2+x^4}\,dx = -\frac{1}{2}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right) + \frac{1}{2}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right) + \frac{1}{4}\log\left(x^2+x+1\right) + \frac{1}{4}\log\left(x^2-x+1\right)$$

```
[In] integrate((x^3+2*x)/(x^4+x^2+1),x, algorithm="maxima")
[Out] -1/2*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) + 1/2*sqrt(3)*arctan(1/3*sqrt(3)
*(2*x - 1)) + 1/4*log(x^2 + x + 1) + 1/4*log(x^2 - x + 1)
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.81

$$\int \frac{2x+x^3}{1+x^2+x^4}\,dx = \frac{1}{2}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x^2+1)\right) + \frac{1}{4}\log\left(x^4+x^2+1\right)$$

```
[In] integrate((x^3+2*x)/(x^4+x^2+1),x, algorithm="giac")
[Out] 1/2*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^2 + 1)) + 1/4*log(x^4 + x^2 + 1)
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.86

$$\int \frac{2x+x^3}{1+x^2+x^4}\,dx = \frac{\ln\left(x^4+x^2+1\right)}{4}+\frac{\sqrt{3}\,\mathrm{atan}\left(\frac{2\sqrt{3}\,x^2}{3}+\frac{\sqrt{3}}{3}\right)}{2}$$

```
[In] int((2*x + x^3)/(x^2 + x^4 + 1),x)
[Out] log(x^2 + x^4 + 1)/4 + (3^(1/2)*atan(3^(1/2)/3 + (2*3^(1/2)*x^2)/3))/2
```

3.141 $\int \frac{11x+2x^3}{(3+2x^2+x^4)^2}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 45

$$\int \frac{11x+2x^3}{(3+2x^2+x^4)^2}\,dx = \frac{5+9x^2}{8(3+2x^2+x^4)}+\frac{9\arctan\left(\frac{1+x^2}{\sqrt{2}}\right)}{8\sqrt{2}}$$

```
[Out] 1/8*(9*x^2+5)/(x^4+2*x^2+3)+9/16*arctan(1/2*(x^2+1)*2^(1/2))*2^(1/2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {1607, 1261, 652, 632, 210}

$$\int \frac{11x+2x^3}{(3+2x^2+x^4)^2}\,dx = \frac{9\arctan\left(\frac{x^2+1}{\sqrt{2}}\right)}{8\sqrt{2}}+\frac{9x^2+5}{8(x^4+2x^2+3)}$$

```
[In] Int[(11*x + 2*x^3)/(3 + 2*x^2 + x^4)^2,x]
```

```
[Out] (5 + 9*x^2)/(8*(3 + 2*x^2 + x^4)) + (9*ArcTan[(1 + x^2)/Sqrt[2]])/(8*Sqrt[2
])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 652

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[((b*d - 2*a*e + (2*c*d - b*e)*x)/((p + 1)*(b^2 - 4*a*c)))*(a + b*
x + c*x^2)^(p + 1), x] - Dist[(2*p + 3)*((2*c*d - b*e)/((p + 1)*(b^2 - 4*a*
c))), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] &&
 NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2]
```

Rule 1261

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(
p_.), x_Symbol] :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x],
 x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]
```

Rule 1607

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \frac{x(11+2x^2)}{(3+2x^2+x^4)^2}\,dx \\
&= \frac{1}{2}\text{Subst}\left(\int \frac{11+2x}{(3+2x+x^2)^2}\,dx, x, x^2\right) \\
&= \frac{5+9x^2}{8\left(3+2x^2+x^4\right)} + \frac{9}{8}\text{Subst}\left(\int \frac{1}{3+2x+x^2}\,dx, x, x^2\right) \\
&= \frac{5+9x^2}{8\left(3+2x^2+x^4\right)} - \frac{9}{4}\text{Subst}\left(\int \frac{1}{-8-x^2}\,dx, x, 2\left(1+x^2\right)\right) \\
&= \frac{5+9x^2}{8\left(3+2x^2+x^4\right)} + \frac{9\tan^{-1}\left(\frac{1+x^2}{\sqrt{2}}\right)}{8\sqrt{2}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.00

$$\int \frac{11x+2x^3}{(3+2x^2+x^4)^2}\,dx=\frac{5+9x^2}{8(3+2x^2+x^4)}+\frac{9\arctan\left(\frac{1+x^2}{\sqrt{2}}\right)}{8\sqrt{2}}$$

```
[In] Integrate[(11*x + 2*x^3)/(3 + 2*x^2 + x^4)^2,x]
[Out] (5 + 9*x^2)/(8*(3 + 2*x^2 + x^4)) + (9*ArcTan[(1 + x^2)/Sqrt[2]])/(8*Sqrt[2
])
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.84

method	result	size
risch	$\frac{\frac{9x^2}{8}+\frac{5}{8}}{x^4+2x^2+3}+\frac{9\arctan\left(\frac{(x^2+1)\sqrt{2}}{2}\right)\sqrt{2}}{16}$	38
default	$\frac{18x^2+10}{16x^4+32x^2+48}+\frac{9\sqrt{2}\,\arctan\left(\frac{(2x^2+2)\sqrt{2}}{4}\right)}{16}$	41

```
[In] int((2*x^3+11*x)/(x^4+2*x^2+3)^2,x,method=_RETURNVERBOSE)
[Out] (9/8*x^2+5/8)/(x^4+2*x^2+3)+9/16*arctan(1/2*(x^2+1)*2^(1/2))*2^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.04

$$\int \frac{11x+2x^3}{(3+2x^2+x^4)^2}\,dx=\frac{9\sqrt{2}(x^4+2x^2+3)\arctan\left(\frac{1}{2}\sqrt{2}(x^2+1)\right)+18x^2+10}{16(x^4+2x^2+3)}$$

```
[In] integrate((2*x^3+11*x)/(x^4+2*x^2+3)^2,x, algorithm="fricas")
[Out] 1/16*(9*sqrt(2)*(x^4 + 2*x^2 + 3)*arctan(1/2*sqrt(2)*(x^2 + 1)) + 18*x^2 +
10)/(x^4 + 2*x^2 + 3)
```

Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.98

$$\int \frac{11x+2x^3}{(3+2x^2+x^4)^2}\,dx = \frac{9x^2+5}{8x^4+16x^2+24} + \frac{9\sqrt{2}\,\text{atan}\left(\frac{\sqrt{2}x^2}{2}+\frac{\sqrt{2}}{2}\right)}{16}$$

```
[In] integrate((2*x**3+11*x)/(x**4+2*x**2+3)**2,x)
[Out] (9*x**2 + 5)/(8*x**4 + 16*x**2 + 24) + 9*sqrt(2)*atan(sqrt(2)*x**2/2 + sqrt
(2)/2)/16
```

Maxima [F]

$$\int \frac{11x+2x^3}{(3+2x^2+x^4)^2}\,dx = \int \frac{2\,x^3+11\,x}{(x^4+2\,x^2+3)^2}\,dx$$

```
[In] integrate((2*x^3+11*x)/(x^4+2*x^2+3)^2,x, algorithm="maxima")
[Out] 1/8*(9*x^2 + 5)/(x^4 + 2*x^2 + 3) + 9/4*integrate(x/(x^4 + 2*x^2 + 3), x)
```

Giac [A] (verification not implemented)

none

Time = 0.41 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.84

$$\int \frac{11x+2x^3}{(3+2x^2+x^4)^2}\,dx = \frac{9}{16}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}(x^2+1)\right) + \frac{9\,x^2+5}{8\,(x^4+2\,x^2+3)}$$

```
[In] integrate((2*x^3+11*x)/(x^4+2*x^2+3)^2,x, algorithm="giac")
[Out] 9/16*sqrt(2)*arctan(1/2*sqrt(2)*(x^2 + 1)) + 1/8*(9*x^2 + 5)/(x^4 + 2*x^2 +
 3)
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.91

$$\int \frac{11x+2x^3}{(3+2x^2+x^4)^2}\,dx = \frac{\frac{9\,x^2}{8}+\frac{5}{8}}{x^4+2\,x^2+3} + \frac{9\,\sqrt{2}\,\text{atan}\left(\frac{\sqrt{2}\,x^2}{2}+\frac{\sqrt{2}}{2}\right)}{16}$$

```
[In] int((11*x + 2*x^3)/(2*x^2 + x^4 + 3)^2,x)
[Out] ((9*x^2)/8 + 5/8)/(2*x^2 + x^4 + 3) + (9*2^(1/2)*atan(2^(1/2)/2 + (2^(1/2)*
x^2)/2))/16
```

3.142 $\int x^5(2+3x^2)\sqrt{3+5x^2+x^4}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 102

$$\int x^5(2+3x^2)\sqrt{3+5x^2+x^4}\,dx = -\frac{1633}{256}(5+2x^2)\sqrt{3+5x^2+x^4}+\frac{3}{10}x^4(3+5x^2+x^4)^{3/2} + \frac{1}{480}(1837-510x^2)(3+5x^2+x^4)^{3/2} + \frac{21229}{512}\operatorname{arctanh}\left(\frac{5+2x^2}{2\sqrt{3+5x^2+x^4}}\right)$$

```
[Out] 3/10*x^4*(x^4+5*x^2+3)^(3/2)+1/480*(-510*x^2+1837)*(x^4+5*x^2+3)^(3/2)+2122
9/512*arctanh(1/2*(2*x^2+5)/(x^4+5*x^2+3)^(1/2))-1633/256*(2*x^2+5)*(x^4+5*
x^2+3)^(1/2)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {1265, 846, 793, 626, 635, 212}

$$\int x^5(2+3x^2)\sqrt{3+5x^2+x^4}\,dx = \frac{21229}{512}\operatorname{arctanh}\left(\frac{2x^2+5}{2\sqrt{x^4+5x^2+3}}\right)+\frac{3}{10}(x^4+5x^2+3)^{3/2}x^4 + \frac{1}{480}(1837-510x^2)(x^4+5x^2+3)^{3/2} - \frac{1633}{256}(2x^2+5)\sqrt{x^4+5x^2+3}$$

```
[In] Int[x^5*(2 + 3*x^2)*Sqrt[3 + 5*x^2 + x^4],x]
```

```
[Out] (-1633*(5 + 2*x^2)*Sqrt[3 + 5*x^2 + x^4])/256 + (3*x^4*(3 + 5*x^2 + x^4)^(3
/2))/10 + ((1837 - 510*x^2)*(3 + 5*x^2 + x^4)^(3/2))/480 + (21229*ArcTanh[(
5 + 2*x^2)/(2*Sqrt[3 + 5*x^2 + x^4])])/512
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 626

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Dist[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1))), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 635

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 793

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> Simp[(-(b*e*g*(p + 2) - c*(e*f + d*g)*(2*p + 3) -
 2*c*e*g*(p + 1)*x))*((a + b*x + c*x^2)^(p + 1)/(2*c^2*(p + 1)*(2*p + 3))),
 x] + Dist[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(2*c*d*f - b*(e*f + d*g))*(2*p
+ 3))/(2*c^2*(2*p + 3)), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c,
 d, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rule 846

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g*(d + e*x)^m*((a + b*x + c*x^2)^(p +
1)/(c*(m + 2*p + 2))), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)
*(a + b*x + c*x^2)^p*Simp[m*(c*d*f - a*e*g) + d*(2*c*f - b*g)*(p + 1) + (m*
(c*e*f + c*d*g - b*e*g) + e*(p + 1)*(2*c*f - b*g))*x, x], x], x] /; FreeQ[{
a, b, c, d, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a
*e^2, 0] && GtQ[m, 0] && NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p]
 || IntegersQ[2*m, 2*p]) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

Rule 1265

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{1}{2}\text{Subst}\left(\int x^2(2+3x)\sqrt{3+5x+x^2}\,dx,x,x^2\right)\\
&= \frac{3}{10}x^4\left(3+5x^2+x^4\right)^{3/2}+\frac{1}{10}\text{Subst}\left(\int\left(-18-\frac{85x}{2}\right)x\sqrt{3+5x+x^2}\,dx,x,x^2\right)\\
&= \frac{3}{10}x^4\left(3+5x^2+x^4\right)^{3/2}\\
&\quad+\frac{1}{480}\left(1837-510x^2\right)\left(3+5x^2+x^4\right)^{3/2}-\frac{1633}{64}\text{Subst}\left(\int\sqrt{3+5x+x^2}\,dx,x,x^2\right)\\
&= -\frac{1633}{256}\left(5+2x^2\right)\sqrt{3+5x^2+x^4}+\frac{3}{10}x^4\left(3+5x^2+x^4\right)^{3/2}\\
&\quad+\frac{1}{480}\left(1837-510x^2\right)\left(3+5x^2+x^4\right)^{3/2}+\frac{21229}{512}\text{Subst}\left(\int\frac{1}{\sqrt{3+5x+x^2}}\,dx,x,x^2\right)\\
&= -\frac{1633}{256}\left(5+2x^2\right)\sqrt{3+5x^2+x^4}+\frac{3}{10}x^4\left(3+5x^2+x^4\right)^{3/2}\\
&\quad+\frac{1}{480}\left(1837-510x^2\right)\left(3+5x^2+x^4\right)^{3/2}\\
&\quad+\frac{21229}{256}\text{Subst}\left(\int\frac{1}{4-x^2}\,dx,x,\frac{5+2x^2}{\sqrt{3+5x^2+x^4}}\right)\\
&= -\frac{1633}{256}\left(5+2x^2\right)\sqrt{3+5x^2+x^4}+\frac{3}{10}x^4\left(3+5x^2+x^4\right)^{3/2}\\
&\quad+\frac{1}{480}\left(1837-510x^2\right)\left(3+5x^2+x^4\right)^{3/2}+\frac{21229}{512}\tanh^{-1}\left(\frac{5+2x^2}{2\sqrt{3+5x^2+x^4}}\right)
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.68

$$
\begin{aligned}
&\int x^5\left(2+3x^2\right)\sqrt{3+5x^2+x^4}\,dx\\
&=\frac{\sqrt{3+5x^2+x^4}(-78387+12250x^2-2248x^4+1680x^6+1152x^8)}{3840}\\
&\quad-\frac{21229}{512}\log\left(-5-2x^2+2\sqrt{3+5x^2+x^4}\right)
\end{aligned}
$$

```
[In] Integrate[x^5*(2 + 3*x^2)*Sqrt[3 + 5*x^2 + x^4],x]
```

```
[Out] (Sqrt[3 + 5*x^2 + x^4]*(-78387 + 12250*x^2 - 2248*x^4 + 1680*x^6 + 1152*x^8
))/3840 - (21229*Log[-5 - 2*x^2 + 2*Sqrt[3 + 5*x^2 + x^4]])/512
```

Maple [A] (verified)

Time = 0.33 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.57

method	result
risch	$\frac{\left(1152x^{8}+1680x^{6}-2248x^{4}+12250x^{2}-78387\right)\sqrt{x^{4}+5x^{2}+3}}{3840}+\frac{21229\ln\left(\frac{5}{2}+x^{2}+\sqrt{x^{4}+5x^{2}+3}\right)}{512}$
trager	$\left(\frac{3}{10}x^{8}+\frac{7}{16}x^{6}-\frac{281}{480}x^{4}+\frac{1225}{384}x^{2}-\frac{26129}{1280}\right)\sqrt{x^{4}+5x^{2}+3}-\frac{21229\ln\left(-2x^{2}+2\sqrt{x^{4}+5x^{2}+3}-5\right)}{512}$
pseudoelliptic	$\frac{21229\ln\left(2x^{2}+5+2\sqrt{x^{4}+5x^{2}+3}\right)}{512}+\frac{\left(1152x^{8}+1680x^{6}-2248x^{4}+12250x^{2}-78387\right)\sqrt{x^{4}+5x^{2}+3}}{3840}$
default	$\frac{3x^{4}\left(x^{4}+5x^{2}+3\right)^{\frac{3}{2}}}{10}-\frac{17x^{2}\left(x^{4}+5x^{2}+3\right)^{\frac{3}{2}}}{16}+\frac{1837\left(x^{4}+5x^{2}+3\right)^{\frac{3}{2}}}{480}-\frac{1633\left(2x^{2}+5\right)\sqrt{x^{4}+5x^{2}+3}}{256}+\frac{21229\ln\left(\frac{5}{2}+x^{2}+\sqrt{\right.}}{512}$
elliptic	$\frac{3x^{8}\sqrt{x^{4}+5x^{2}+3}}{10}+\frac{7x^{6}\sqrt{x^{4}+5x^{2}+3}}{16}-\frac{281x^{4}\sqrt{x^{4}+5x^{2}+3}}{480}+\frac{1225x^{2}\sqrt{x^{4}+5x^{2}+3}}{384}-\frac{26129\sqrt{x^{4}+5x^{2}+3}}{1280}+\frac{21229\ln}{}$

```
[In] int(x^5*(3*x^2+2)*(x^4+5*x^2+3)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3840*(1152*x^8+1680*x^6-2248*x^4+12250*x^2-78387)*(x^4+5*x^2+3)^(1/2)+212
29/512*ln(5/2+x^2+(x^4+5*x^2+3)^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.60

$$\int x^5(2+3x^2)\sqrt{3+5x^2+x^4}\,dx = \frac{1}{3840}\left(1152\,x^8+1680\,x^6-2248\,x^4+12250\,x^2-78387\right)\sqrt{x^4+5\,x^2+3} - \frac{21229}{512}\log\left(-2\,x^2+2\,\sqrt{x^4+5\,x^2+3}-5\right)$$

```
[In] integrate(x^5*(3*x^2+2)*(x^4+5*x^2+3)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/3840*(1152*x^8 + 1680*x^6 - 2248*x^4 + 12250*x^2 - 78387)*sqrt(x^4 + 5*x^
2 + 3) - 21229/512*log(-2*x^2 + 2*sqrt(x^4 + 5*x^2 + 3) - 5)
```

Sympy [A] (verification not implemented)

Time = 0.78 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.03

$$\int x^5(2+3x^2)\sqrt{3+5x^2+x^4}\,dx = \sqrt{x^4+5x^2+3}\left(\frac{x^6}{4}+\frac{5x^4}{24}-\frac{89x^2}{96}+\frac{365}{64}\right) + \frac{3\sqrt{x^4+5x^2+3}\left(\frac{x^8}{5}+\frac{x^6}{8}-\frac{127x^4}{240}+\frac{527x^2}{192}-\frac{11143}{640}\right)}{2} + \frac{21229\log\left(2x^2+2\sqrt{x^4+5x^2+3}+5\right)}{512}$$

```
[In] integrate(x**5*(3*x**2+2)*(x**4+5*x**2+3)**(1/2),x)
```

```
[Out] sqrt(x**4 + 5*x**2 + 3)*(x**6/4 + 5*x**4/24 - 89*x**2/96 + 365/64) + 3*sqrt
(x**4 + 5*x**2 + 3)*(x**8/5 + x**6/8 - 127*x**4/240 + 527*x**2/192 - 11143/
640)/2 + 21229*log(2*x**2 + 2*sqrt(x**4 + 5*x**2 + 3) + 5)/512
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.02

$$\int x^5(2+3x^2)\sqrt{3+5x^2+x^4}\,dx = \frac{3}{10}\left(x^4+5x^2+3\right)^{\frac{3}{2}}x^4 - \frac{17}{16}\left(x^4+5x^2+3\right)^{\frac{3}{2}}x^2 - \frac{1633}{128}\sqrt{x^4+5x^2+3}x^2 + \frac{1837}{480}\left(x^4+5x^2+3\right)^{\frac{3}{2}} - \frac{8165}{256}\sqrt{x^4+5x^2+3} + \frac{21229}{512}\log\left(2x^2+2\sqrt{x^4+5x^2+3}+5\right)$$

```
[In] integrate(x^5*(3*x^2+2)*(x^4+5*x^2+3)^(1/2),x, algorithm="maxima")
```

```
[Out] 3/10*(x^4 + 5*x^2 + 3)^(3/2)*x^4 - 17/16*(x^4 + 5*x^2 + 3)^(3/2)*x^2 - 1633
/128*sqrt(x^4 + 5*x^2 + 3)*x^2 + 1837/480*(x^4 + 5*x^2 + 3)^(3/2) - 8165/25
6*sqrt(x^4 + 5*x^2 + 3) + 21229/512*log(2*x^2 + 2*sqrt(x^4 + 5*x^2 + 3) + 5
)
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.00

$$\begin{aligned}\int x^5(2+3x^2)\sqrt{3+5x^2+x^4}\,dx &= \frac{1}{1280}\sqrt{x^4+5x^2+3}\left(2\left(4\left(6\left(8x^2+5\right)x^2-127\right)x^2+2635\right)x^2-33429\right)\\ &\quad+\frac{1}{192}\sqrt{x^4+5x^2+3}\left(2\left(4\left(6x^2+5\right)x^2-89\right)x^2+1095\right)\\ &\quad-\frac{21229}{512}\log\left(2x^2-2\sqrt{x^4+5x^2+3}+5\right)\end{aligned}$$

```
[In] integrate(x^5*(3*x^2+2)*(x^4+5*x^2+3)^(1/2),x, algorithm="giac")
```

```
[Out] 1/1280*sqrt(x^4 + 5*x^2 + 3)*(2*(4*(6*(8*x^2 + 5)*x^2 - 127)*x^2 + 2635)*x^
2 - 33429) + 1/192*sqrt(x^4 + 5*x^2 + 3)*(2*(4*(6*x^2 + 5)*x^2 - 89)*x^2 +
1095) - 21229/512*log(2*x^2 - 2*sqrt(x^4 + 5*x^2 + 3) + 5)
```

Mupad [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.00

$$\begin{aligned}\int x^5(2+3x^2)\sqrt{3+5x^2+x^4}\,dx &= \frac{21229\ln\left(\sqrt{x^4+5x^2+3}+x^2+\frac{5}{2}\right)}{512}\\ &\quad-\frac{17x^2\left(x^4+5x^2+3\right)^{3/2}}{16}+\frac{3x^4\left(x^4+5x^2+3\right)^{3/2}}{10}\\ &\quad+\frac{51\left(\frac{x^2}{2}+\frac{5}{4}\right)\sqrt{x^4+5x^2+3}}{16}\\ &\quad+\frac{1837\sqrt{x^4+5x^2+3}\left(8x^4+10x^2-51\right)}{3840}\end{aligned}$$

```
[In] int(x^5*(3*x^2 + 2)*(5*x^2 + x^4 + 3)^(1/2),x)
```

```
[Out] (21229*log((5*x^2 + x^4 + 3)^(1/2) + x^2 + 5/2))/512 - (17*x^2*(5*x^2 + x^4
 + 3)^(3/2))/16 + (3*x^4*(5*x^2 + x^4 + 3)^(3/2))/10 + (51*(x^2/2 + 5/4)*(5
*x^2 + x^4 + 3)^(1/2))/16 + (1837*(5*x^2 + x^4 + 3)^(1/2)*(10*x^2 + 8*x^4 -
 51))/3840
```

3.143 $\int x^3(2+3x^2)\sqrt{3+5x^2+x^4}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 81

$$\int x^3(2+3x^2)\sqrt{3+5x^2+x^4}\,dx=\frac{259}{128}(5+2x^2)\sqrt{3+5x^2+x^4}-\frac{1}{48}(59-18x^2)\left(3+5x^2+x^4\right)^{3/2}-\frac{3367}{256}\text{arctanh}\left(\frac{5+2x^2}{2\sqrt{3+5x^2+x^4}}\right)$$

```
[Out] -1/48*(-18*x^2+59)*(x^4+5*x^2+3)^(3/2)-3367/256*arctanh(1/2*(2*x^2+5)/(x^4+
5*x^2+3)^(1/2))+259/128*(2*x^2+5)*(x^4+5*x^2+3)^(1/2)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {1265, 793, 626, 635, 212}

$$\int x^3(2+3x^2)\sqrt{3+5x^2+x^4}\,dx=-\frac{3367}{256}\text{arctanh}\left(\frac{2x^2+5}{2\sqrt{x^4+5x^2+3}}\right)-\frac{1}{48}(59-18x^2)\left(x^4+5x^2+3\right)^{3/2}+\frac{259}{128}(2x^2+5)\sqrt{x^4+5x^2+3}$$

```
[In] Int[x^3*(2 + 3*x^2)*Sqrt[3 + 5*x^2 + x^4],x]
```

```
[Out] (259*(5 + 2*x^2)*Sqrt[3 + 5*x^2 + x^4])/128 - ((59 - 18*x^2)*(3 + 5*x^2 + x
^4)^(3/2))/48 - (3367*ArcTanh[(5 + 2*x^2)/(2*Sqrt[3 + 5*x^2 + x^4])])/256
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 626

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Dist[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1))), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 635

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 793

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> Simp[(-(b*e*g*(p + 2) - c*(e*f + d*g)*(2*p + 3) -
 2*c*e*g*(p + 1)*x))*((a + b*x + c*x^2)^(p + 1)/(2*c^2*(p + 1)*(2*p + 3))),
 x] + Dist[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(2*c*d*f - b*(e*f + d*g))*(2*p
+ 3))/(2*c^2*(2*p + 3)), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c,
 d, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rule 1265

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{2}\text{Subst}\left(\int x(2+3x)\sqrt{3+5x+x^2}\,dx,x,x^2\right)\\
&= -\frac{1}{48}\left(59-18x^2\right)\left(3+5x^2+x^4\right)^{3/2}+\frac{259}{32}\text{Subst}\left(\int \sqrt{3+5x+x^2}\,dx,x,x^2\right)\\
&= \frac{259}{128}\left(5+2x^2\right)\sqrt{3+5x^2+x^4}-\frac{1}{48}\left(59-18x^2\right)\left(3+5x^2+x^4\right)^{3/2}\\
&\quad -\frac{3367}{256}\text{Subst}\left(\int \frac{1}{\sqrt{3+5x+x^2}}\,dx,x,x^2\right)
\end{aligned}$$

$$= \frac{259}{128}(5+2x^2)\sqrt{3+5x^2+x^4} - \frac{1}{48}(59-18x^2)\left(3+5x^2+x^4\right)^{3/2} - \frac{3367}{128}\text{Subst}\left(\int \frac{1}{4-x^2}\,dx, x, \frac{5+2x^2}{\sqrt{3+5x^2+x^4}}\right)$$

$$= \frac{259}{128}(5+2x^2)\sqrt{3+5x^2+x^4} - \frac{1}{48}(59-18x^2)\left(3+5x^2+x^4\right)^{3/2} - \frac{3367}{256}\tanh^{-1}\left(\frac{5+2x^2}{2\sqrt{3+5x^2+x^4}}\right)$$

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.79

$$\int x^3(2+3x^2)\sqrt{3+5x^2+x^4}\,dx = \frac{1}{384}\sqrt{3+5x^2+x^4}(2469-374x^2+248x^4+144x^6) + \frac{3367}{256}\log\left(-5-2x^2+2\sqrt{3+5x^2+x^4}\right)$$

```
[In] Integrate[x^3*(2 + 3*x^2)*Sqrt[3 + 5*x^2 + x^4],x]
```

```
[Out] (Sqrt[3 + 5*x^2 + x^4]*(2469 - 374*x^2 + 248*x^4 + 144*x^6))/384 + (3367*Lo
g[-5 - 2*x^2 + 2*Sqrt[3 + 5*x^2 + x^4]])/256
```

Maple [A] (verified)

Time = 0.20 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.65

method	result	si
risch	$\frac{(144x^6+248x^4-374x^2+2469)\sqrt{x^4+5x^2+3}}{384} - \frac{3367\ln\left(\frac{5}{2}+x^2+\sqrt{x^4+5x^2+3}\right)}{256}$	53
trager	$\left(\frac{3}{8}x^6+\frac{31}{48}x^4-\frac{187}{192}x^2+\frac{823}{128}\right)\sqrt{x^4+5x^2+3} - \frac{3367\ln\left(2x^2+5+2\sqrt{x^4+5x^2+3}\right)}{256}$	56
pseudoelliptic	$-\frac{3367\ln\left(2x^2+5+2\sqrt{x^4+5x^2+3}\right)}{256} + \frac{(288x^6+496x^4-748x^2+4938)\sqrt{x^4+5x^2+3}}{768}$	57
default	$\frac{3x^2(x^4+5x^2+3)^{\frac{3}{2}}}{8} - \frac{59(x^4+5x^2+3)^{\frac{3}{2}}}{48} + \frac{259(2x^2+5)\sqrt{x^4+5x^2+3}}{128} - \frac{3367\ln\left(\frac{5}{2}+x^2+\sqrt{x^4+5x^2+3}\right)}{256}$	74
elliptic	$\frac{823\sqrt{x^4+5x^2+3}}{128} - \frac{3367\ln\left(\frac{5}{2}+x^2+\sqrt{x^4+5x^2+3}\right)}{256} + \frac{3x^6\sqrt{x^4+5x^2+3}}{8} + \frac{31x^4\sqrt{x^4+5x^2+3}}{48} - \frac{187x^2\sqrt{x^4+5x^2+3}}{192}$	87

```
[In] int(x^3*(3*x^2+2)*(x^4+5*x^2+3)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/384*(144*x^6+248*x^4-374*x^2+2469)*(x^4+5*x^2+3)^(1/2)-3367/256*ln(5/2+x^
2+(x^4+5*x^2+3)^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.69

$$\int x^3(2+3x^2)\sqrt{3+5x^2+x^4}\,dx = \frac{1}{384}\left(144\,x^6+248\,x^4-374\,x^2+2469\right)\sqrt{x^4+5\,x^2+3} + \frac{3367}{256}\log\left(-2\,x^2+2\,\sqrt{x^4+5\,x^2+3}-5\right)$$

```
[In] integrate(x^3*(3*x^2+2)*(x^4+5*x^2+3)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/384*(144*x^6 + 248*x^4 - 374*x^2 + 2469)*sqrt(x^4 + 5*x^2 + 3) + 3367/256
*log(-2*x^2 + 2*sqrt(x^4 + 5*x^2 + 3) - 5)
```

Sympy [A] (verification not implemented)

Time = 0.73 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.16

$$\int x^3(2+3x^2)\sqrt{3+5x^2+x^4}\,dx = \left(\frac{x^4}{3}+\frac{5x^2}{12}-\frac{17}{8}\right)\sqrt{x^4+5x^2+3} + \frac{3\sqrt{x^4+5x^2+3}\left(\frac{x^6}{4}+\frac{5x^4}{24}-\frac{89x^2}{96}+\frac{365}{64}\right)}{2} - \frac{3367\log\left(2x^2+2\sqrt{x^4+5x^2+3}+5\right)}{256}$$

```
[In] integrate(x**3*(3*x**2+2)*(x**4+5*x**2+3)**(1/2),x)
```

```
[Out] (x**4/3 + 5*x**2/12 - 17/8)*sqrt(x**4 + 5*x**2 + 3) + 3*sqrt(x**4 + 5*x**2
+ 3)*(x**6/4 + 5*x**4/24 - 89*x**2/96 + 365/64)/2 - 3367*log(2*x**2 + 2*sqr
t(x**4 + 5*x**2 + 3) + 5)/256
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.07

$$\int x^3(2+3x^2)\sqrt{3+5x^2+x^4}\,dx = \frac{3}{8}\left(x^4+5\,x^2+3\right)^{\frac{3}{2}}x^2 + \frac{259}{64}\sqrt{x^4+5\,x^2+3}x^2 - \frac{59}{48}\left(x^4+5\,x^2+3\right)^{\frac{3}{2}} + \frac{1295}{128}\sqrt{x^4+5\,x^2+3} - \frac{3367}{256}\log\left(2\,x^2+2\,\sqrt{x^4+5\,x^2+3}+5\right)$$

```
[In] integrate(x^3*(3*x^2+2)*(x^4+5*x^2+3)^(1/2),x, algorithm="maxima")
```

```
[Out] 3/8*(x^4 + 5*x^2 + 3)^(3/2)*x^2 + 259/64*sqrt(x^4 + 5*x^2 + 3)*x^2 - 59/48*
(x^4 + 5*x^2 + 3)^(3/2) + 1295/128*sqrt(x^4 + 5*x^2 + 3) - 3367/256*log(2*x
^2 + 2*sqrt(x^4 + 5*x^2 + 3) + 5)
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.09

$$\int x^3(2+3x^2)\sqrt{3+5x^2+x^4}\,dx = \frac{1}{128}\sqrt{x^4+5x^2+3}\left(2\left(4\left(6x^2+5\right)x^2-89\right)x^2+1095\right) + \frac{1}{24}\sqrt{x^4+5x^2+3}\left(2\left(4x^2+5\right)x^2-51\right) + \frac{3367}{256}\log\left(2x^2-2\sqrt{x^4+5x^2+3}+5\right)$$

```
[In] integrate(x^3*(3*x^2+2)*(x^4+5*x^2+3)^(1/2),x, algorithm="giac")
```

```
[Out] 1/128*sqrt(x^4 + 5*x^2 + 3)*(2*(4*(6*x^2 + 5)*x^2 - 89)*x^2 + 1095) + 1/24*
sqrt(x^4 + 5*x^2 + 3)*(2*(4*x^2 + 5)*x^2 - 51) + 3367/256*log(2*x^2 - 2*sqr
t(x^4 + 5*x^2 + 3) + 5)
```

Mupad [B] (verification not implemented)

Time = 7.68 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.05

$$\int x^3(2+3x^2)\sqrt{3+5x^2+x^4}\,dx = \frac{3x^2\left(x^4+5x^2+3\right)^{3/2}}{8} - \frac{3367\ln\left(\sqrt{x^4+5x^2+3}+x^2+\frac{5}{2}\right)}{256} - \frac{9\left(\frac{x^2}{2}+\frac{5}{4}\right)\sqrt{x^4+5x^2+3}}{8} - \frac{59\sqrt{x^4+5x^2+3}\left(8x^4+10x^2-51\right)}{384}$$

```
[In] int(x^3*(3*x^2 + 2)*(5*x^2 + x^4 + 3)^(1/2),x)
```

```
[Out] (3*x^2*(5*x^2 + x^4 + 3)^(3/2))/8 - (3367*log((5*x^2 + x^4 + 3)^(1/2) + x^2
 + 5/2))/256 - (9*(x^2/2 + 5/4)*(5*x^2 + x^4 + 3)^(1/2))/8 - (59*(5*x^2 + x
^4 + 3)^(1/2)*(10*x^2 + 8*x^4 - 51))/384
```

3.144 $\int x(2+3x^2)\sqrt{3+5x^2+x^4}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 74

$$\int x(2+3x^2)\sqrt{3+5x^2+x^4}\,dx = -\frac{11}{16}(5+2x^2)\sqrt{3+5x^2+x^4}+\frac{1}{2}(3+5x^2+x^4)^{3/2} + \frac{143}{32}\operatorname{arctanh}\left(\frac{5+2x^2}{2\sqrt{3+5x^2+x^4}}\right)$$

```
[Out] 1/2*(x^4+5*x^2+3)^(3/2)+143/32*arctanh(1/2*(2*x^2+5)/(x^4+5*x^2+3)^(1/2))-1
1/16*(2*x^2+5)*(x^4+5*x^2+3)^(1/2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {1261, 654, 626, 635, 212}

$$\int x(2+3x^2)\sqrt{3+5x^2+x^4}\,dx = \frac{143}{32}\operatorname{arctanh}\left(\frac{2x^2+5}{2\sqrt{x^4+5x^2+3}}\right) + \frac{1}{2}(x^4+5x^2+3)^{3/2}-\frac{11}{16}(2x^2+5)\sqrt{x^4+5x^2+3}$$

```
[In] Int[x*(2 + 3*x^2)*Sqrt[3 + 5*x^2 + x^4],x]
```

```
[Out] (-11*(5 + 2*x^2)*Sqrt[3 + 5*x^2 + x^4])/16 + (3 + 5*x^2 + x^4)^(3/2)/2 + (1
43*ArcTanh[(5 + 2*x^2)/(2*Sqrt[3 + 5*x^2 + x^4])])/32
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rule 626

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Dist[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1))), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 635

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 654

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 1261

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(
p_.), x_Symbol] :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x],
 x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{1}{2}\text{Subst}\left(\int (2+3x)\sqrt{3+5x+x^2}\,dx, x, x^2\right) \\
&= \frac{1}{2}\left(3+5x^2+x^4\right)^{3/2} - \frac{11}{4}\text{Subst}\left(\int \sqrt{3+5x+x^2}\,dx, x, x^2\right) \\
&= -\frac{11}{16}\left(5+2x^2\right)\sqrt{3+5x^2+x^4} + \frac{1}{2}\left(3+5x^2+x^4\right)^{3/2} + \frac{143}{32}\text{Subst}\left(\int \frac{1}{\sqrt{3+5x+x^2}}\,dx, x, x^2\right) \\
&= -\frac{11}{16}\left(5+2x^2\right)\sqrt{3+5x^2+x^4} + \frac{1}{2}\left(3+5x^2+x^4\right)^{3/2} \\
&\quad + \frac{143}{16}\text{Subst}\left(\int \frac{1}{4-x^2}\,dx, x, \frac{5+2x^2}{\sqrt{3+5x^2+x^4}}\right) \\
&= -\frac{11}{16}\left(5+2x^2\right)\sqrt{3+5x^2+x^4} + \frac{1}{2}\left(3+5x^2+x^4\right)^{3/2} + \frac{143}{32}\tanh^{-1}\left(\frac{5+2x^2}{2\sqrt{3+5x^2+x^4}}\right)
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.80

$$\int x(2+3x^2)\sqrt{3+5x^2+x^4}\,dx = \frac{1}{16}\sqrt{3+5x^2+x^4}(-31+18x^2+8x^4) - \frac{143}{32}\log\left(-5-2x^2+2\sqrt{3+5x^2+x^4}\right)$$

```
[In] Integrate[x*(2 + 3*x^2)*Sqrt[3 + 5*x^2 + x^4],x]
[Out] (Sqrt[3 + 5*x^2 + x^4]*(-31 + 18*x^2 + 8*x^4))/16 - (143*Log[-5 - 2*x^2 + 2
*Sqrt[3 + 5*x^2 + x^4]])/32
```

Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.65

method	result	size
risch	$\frac{(8x^4+18x^2-31)\sqrt{x^4+5x^2+3}}{16}+\frac{143\ln\left(\frac{5}{2}+x^2+\sqrt{x^4+5x^2+3}\right)}{32}$	48
trager	$\left(\frac{1}{2}x^4+\frac{9}{8}x^2-\frac{31}{16}\right)\sqrt{x^4+5x^2+3}+\frac{143\ln\left(2x^2+5+2\sqrt{x^4+5x^2+3}\right)}{32}$	51
pseudoelliptic	$\frac{143\ln\left(2x^2+5+2\sqrt{x^4+5x^2+3}\right)}{32}+\frac{(8x^4+18x^2-31)\sqrt{x^4+5x^2+3}}{16}$	52
default	$-\frac{11(2x^2+5)\sqrt{x^4+5x^2+3}}{16}+\frac{143\ln\left(\frac{5}{2}+x^2+\sqrt{x^4+5x^2+3}\right)}{32}+\frac{(x^4+5x^2+3)^{\frac{3}{2}}}{2}$	57
elliptic	$\frac{143\ln\left(\frac{5}{2}+x^2+\sqrt{x^4+5x^2+3}\right)}{32}-\frac{31\sqrt{x^4+5x^2+3}}{16}+\frac{x^4\sqrt{x^4+5x^2+3}}{2}+\frac{9x^2\sqrt{x^4+5x^2+3}}{8}$	70

```
[In] int(x*(3*x^2+2)*(x^4+5*x^2+3)^(1/2),x,method=_RETURNVERBOSE)
[Out] 1/16*(8*x^4+18*x^2-31)*(x^4+5*x^2+3)^(1/2)+143/32*ln(5/2+x^2+(x^4+5*x^2+3)^
(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.69

$$\int x(2+3x^2)\sqrt{3+5x^2+x^4}\,dx = \frac{1}{16}\left(8x^4+18x^2-31\right)\sqrt{x^4+5x^2+3} - \frac{143}{32}\log\left(-2x^2+2\sqrt{x^4+5x^2+3}-5\right)$$

```
[In] integrate(x*(3*x^2+2)*(x^4+5*x^2+3)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/16*(8*x^4 + 18*x^2 - 31)*sqrt(x^4 + 5*x^2 + 3) - 143/32*log(-2*x^2 + 2*sq
rt(x^4 + 5*x^2 + 3) - 5)
```

Sympy [A] (verification not implemented)

Time = 0.70 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.08

$$\int x(2+3x^2)\sqrt{3+5x^2+x^4}\,dx = \left(\frac{x^2}{2}+\frac{5}{4}\right)\sqrt{x^4+5x^2+3} + \frac{3\left(\frac{x^4}{3}+\frac{5x^2}{12}-\frac{17}{8}\right)\sqrt{x^4+5x^2+3}}{2} + \frac{143\log\left(2x^2+2\sqrt{x^4+5x^2+3}+5\right)}{32}$$

```
[In] integrate(x*(3*x**2+2)*(x**4+5*x**2+3)**(1/2),x)
```

```
[Out] (x**2/2 + 5/4)*sqrt(x**4 + 5*x**2 + 3) + 3*(x**4/3 + 5*x**2/12 - 17/8)*sqrt
(x**4 + 5*x**2 + 3)/2 + 143*log(2*x**2 + 2*sqrt(x**4 + 5*x**2 + 3) + 5)/32
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.95

$$\int x(2+3x^2)\sqrt{3+5x^2+x^4}\,dx = -\frac{11}{8}\sqrt{x^4+5x^2+3}x^2 + \frac{1}{2}\left(x^4+5x^2+3\right)^{\frac{3}{2}} - \frac{55}{16}\sqrt{x^4+5x^2+3} + \frac{143}{32}\log\left(2x^2+2\sqrt{x^4+5x^2+3}+5\right)$$

```
[In] integrate(x*(3*x^2+2)*(x^4+5*x^2+3)^(1/2),x, algorithm="maxima")
```

```
[Out] -11/8*sqrt(x^4 + 5*x^2 + 3)*x^2 + 1/2*(x^4 + 5*x^2 + 3)^(3/2) - 55/16*sqrt(
x^4 + 5*x^2 + 3) + 143/32*log(2*x^2 + 2*sqrt(x^4 + 5*x^2 + 3) + 5)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.00

$$\int x(2+3x^2)\sqrt{3+5x^2+x^4}\,dx = \frac{1}{16}\sqrt{x^4+5x^2+3}\big(2\left(4x^2+5\right)x^2-51\big) + \frac{1}{4}\sqrt{x^4+5x^2+3}\big(2x^2+5\big) - \frac{143}{32}\log\left(2x^2-2\sqrt{x^4+5x^2+3}+5\right)$$

```
[In] integrate(x*(3*x^2+2)*(x^4+5*x^2+3)^(1/2),x, algorithm="giac")
```

```
[Out] 1/16*sqrt(x^4 + 5*x^2 + 3)*(2*(4*x^2 + 5)*x^2 - 51) + 1/4*sqrt(x^4 + 5*x^2
+ 3)*(2*x^2 + 5) - 143/32*log(2*x^2 - 2*sqrt(x^4 + 5*x^2 + 3) + 5)
```

Mupad [B] (verification not implemented)

Time = 7.67 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.91

$$\int x(2+3x^2)\sqrt{3+5x^2+x^4}\,dx = \frac{143\,\ln\left(\sqrt{x^4+5x^2+3}+x^2+\frac{5}{2}\right)}{32} + \left(\frac{x^2}{2}+\frac{5}{4}\right)\sqrt{x^4+5x^2+3} + \frac{\sqrt{x^4+5x^2+3}\,(8x^4+10x^2-51)}{16}$$

```
[In] int(x*(3*x^2 + 2)*(5*x^2 + x^4 + 3)^(1/2),x)
```

```
[Out] (143*log((5*x^2 + x^4 + 3)^(1/2) + x^2 + 5/2))/32 + (x^2/2 + 5/4)*(5*x^2 +
x^4 + 3)^(1/2) + ((5*x^2 + x^4 + 3)^(1/2)*(10*x^2 + 8*x^4 - 51))/16
```

3.145 $\int \frac{(2+3x^2)\sqrt{3+5x^2+x^4}}{x}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 94

$$\int \frac{(2+3x^2)\sqrt{3+5x^2+x^4}}{x}\,dx = \frac{1}{8}(23+6x^2)\sqrt{3+5x^2+x^4} + \frac{1}{16}\text{arctanh}\left(\frac{5+2x^2}{2\sqrt{3+5x^2+x^4}}\right) - \sqrt{3}\text{arctanh}\left(\frac{6+5x^2}{2\sqrt{3}\sqrt{3+5x^2+x^4}}\right)$$

```
[Out] 1/16*arctanh(1/2*(2*x^2+5)/(x^4+5*x^2+3)^(1/2))-arctanh(1/6*(5*x^2+6)*3^(1/
2)/(x^4+5*x^2+3)^(1/2))*3^(1/2)+1/8*(6*x^2+23)*(x^4+5*x^2+3)^(1/2)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {1265, 828, 857, 635, 212, 738}

$$\int \frac{(2+3x^2)\sqrt{3+5x^2+x^4}}{x}\,dx = \frac{1}{16}\text{arctanh}\left(\frac{2x^2+5}{2\sqrt{x^4+5x^2+3}}\right) - \sqrt{3}\text{arctanh}\left(\frac{5x^2+6}{2\sqrt{3}\sqrt{x^4+5x^2+3}}\right) + \frac{1}{8}\sqrt{x^4+5x^2+3}(6x^2+23)$$

```
[In] Int[((2 + 3*x^2)*Sqrt[3 + 5*x^2 + x^4])/x,x]
```

[Out] ((23 + 6*x^2)*Sqrt[3 + 5*x^2 + x^4])/8 + ArcTanh[(5 + 2*x^2)/(2*Sqrt[3 + 5*x^2 + x^4])]/16 - Sqrt[3]*ArcTanh[(6 + 5*x^2)/(2*Sqrt[3]*Sqrt[3 + 5*x^2 + x^4])]

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 635

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 828

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2)
 - g*(c*d + 2*c*d*p - b*e*p) + g*c*e*(m + 2*p + 1)*x)*((a + b*x + c*x^2)^p/
(c*e^2*(m + 2*p + 1)*(m + 2*p + 2))), x] - Dist[p/(c*e^2*(m + 2*p + 1)*(m +
 2*p + 2)), Int[(d + e*x)^m*(a + b*x + c*x^2)^(p - 1)*Simp[c*e*f*(b*d - 2*a
*e)*(m + 2*p + 2) + g*(a*e*(b*e - 2*c*d*m + b*e*m) + b*d*(b*e*p - c*d - 2*c
*d*p)) + (c*e*f*(2*c*d - b*e)*(m + 2*p + 2) + g*(b^2*e^2*(p + m + 1) - 2*c^
2*d^2*(1 + 2*p) - c*e*(b*d*(m - 2*p) + 2*a*e*(m + 2*p + 1))))*x, x], x], x]
 /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
- b*d*e + a*e^2, 0] && GtQ[p, 0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[
m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p]
|| IntegersQ[2*m, 2*p])
```

Rule 857

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1265

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{2}\text{Subst}\left(\int \frac{(2+3x)\sqrt{3+5x+x^2}}{x}\,dx,x,x^2\right)\\
&= \frac{1}{8}(23+6x^2)\sqrt{3+5x^2+x^4}-\frac{1}{8}\text{Subst}\left(\int \frac{-24-\frac{x}{2}}{x\sqrt{3+5x+x^2}}\,dx,x,x^2\right)\\
&= \frac{1}{8}(23+6x^2)\sqrt{3+5x^2+x^4}+\frac{1}{16}\text{Subst}\left(\int \frac{1}{\sqrt{3+5x+x^2}}\,dx,x,x^2\right)\\
&\quad+3\text{Subst}\left(\int \frac{1}{x\sqrt{3+5x+x^2}}\,dx,x,x^2\right)\\
&= \frac{1}{8}(23+6x^2)\sqrt{3+5x^2+x^4}+\frac{1}{8}\text{Subst}\left(\int \frac{1}{4-x^2}\,dx,x,\frac{5+2x^2}{\sqrt{3+5x^2+x^4}}\right)\\
&\quad-6\text{Subst}\left(\int \frac{1}{12-x^2}\,dx,x,\frac{6+5x^2}{\sqrt{3+5x^2+x^4}}\right)\\
&= \frac{1}{8}(23+6x^2)\sqrt{3+5x^2+x^4}+\frac{1}{16}\tanh^{-1}\left(\frac{5+2x^2}{2\sqrt{3+5x^2+x^4}}\right)\\
&\quad-\sqrt{3}\tanh^{-1}\left(\frac{6+5x^2}{2\sqrt{3}\sqrt{3+5x^2+x^4}}\right)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.94

$$\begin{aligned}
\int \frac{(2+3x^2)\sqrt{3+5x^2+x^4}}{x}\,dx &= \frac{1}{8}(23+6x^2)\sqrt{3+5x^2+x^4}\\
&\quad+2\sqrt{3}\text{arctanh}\left(\frac{x^2-\sqrt{3+5x^2+x^4}}{\sqrt{3}}\right)\\
&\quad-\frac{1}{16}\log\left(-5-2x^2+2\sqrt{3+5x^2+x^4}\right)
\end{aligned}$$

```
[In] Integrate[((2 + 3*x^2)*Sqrt[3 + 5*x^2 + x^4])/x,x]
```

```
[Out] ((23 + 6*x^2)*Sqrt[3 + 5*x^2 + x^4])/8 + 2*Sqrt[3]*ArcTanh[(x^2 - Sqrt[3 +
5*x^2 + x^4])/Sqrt[3]] - Log[-5 - 2*x^2 + 2*Sqrt[3 + 5*x^2 + x^4]]/16
```

Maple [A] (verified)

Time = 0.52 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.88

method	result
elliptic	$\frac{\ln\left(\frac{5}{2}+x^2+\sqrt{x^4+5x^2+3}\right)}{16}+\frac{23\sqrt{x^4+5x^2+3}}{8}+\frac{3x^2\sqrt{x^4+5x^2+3}}{4}-\operatorname{arctanh}\left(\frac{(5x^2+6)\sqrt{3}}{6\sqrt{x^4+5x^2+3}}\right)\sqrt{3}$
default	$\frac{3(2x^2+5)\sqrt{x^4+5x^2+3}}{8}+\frac{\ln\left(\frac{5}{2}+x^2+\sqrt{x^4+5x^2+3}\right)}{16}+\sqrt{x^4+5x^2+3}-\operatorname{arctanh}\left(\frac{(5x^2+6)\sqrt{3}}{6\sqrt{x^4+5x^2+3}}\right)\sqrt{3}$
pseudoelliptic	$\frac{3x^2\sqrt{x^4+5x^2+3}}{4}+\frac{23\sqrt{x^4+5x^2+3}}{8}+\frac{\ln\left(2x^2+5+2\sqrt{x^4+5x^2+3}\right)}{16}-\operatorname{arctanh}\left(\frac{(5x^2+6)\sqrt{3}}{6\sqrt{x^4+5x^2+3}}\right)\sqrt{3}$
trager	$\left(\frac{3x^2}{4}+\frac{23}{8}\right)\sqrt{x^4+5x^2+3}+\operatorname{RootOf}\left(\_Z^2-3\right)\ln\left(-\frac{-5\operatorname{RootOf}\left(\_Z^2-3\right)x^2+6\sqrt{x^4+5x^2+3}-6\operatorname{Roo}}{x^2}\right.$

```
[In] int((3*x^2+2)*(x^4+5*x^2+3)^(1/2)/x,x,method=_RETURNVERBOSE)
```

```
[Out] 1/16*ln(5/2+x^2+(x^4+5*x^2+3)^(1/2))+23/8*(x^4+5*x^2+3)^(1/2)+3/4*x^2*(x^4+
5*x^2+3)^(1/2)-arctanh(1/6*(5*x^2+6)*3^(1/2)/(x^4+5*x^2+3)^(1/2))*3^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.01

$$\begin{aligned}\int \frac{(2+3x^2)\sqrt{3+5x^2+x^4}}{x}\,dx &= \frac{1}{8}\sqrt{x^4+5x^2+3}(6x^2+23) \\ &\quad+\sqrt{3}\log\left(\frac{25x^2-2\sqrt{3}(5x^2+6)-2\sqrt{x^4+5x^2+3}(5\sqrt{3}-6)+30}{x^2}\right) \\ &\quad-\frac{1}{16}\log\left(-2x^2+2\sqrt{x^4+5x^2+3}-5\right)\end{aligned}$$

```
[In] integrate((3*x^2+2)*(x^4+5*x^2+3)^(1/2)/x,x, algorithm="fricas")
```

```
[Out] 1/8*sqrt(x^4 + 5*x^2 + 3)*(6*x^2 + 23) + sqrt(3)*log((25*x^2 - 2*sqrt(3)*(5
*x^2 + 6) - 2*sqrt(x^4 + 5*x^2 + 3)*(5*sqrt(3) - 6) + 30)/x^2) - 1/16*log(-
2*x^2 + 2*sqrt(x^4 + 5*x^2 + 3) - 5)
```

Sympy [F]

$$\int \frac{(2+3x^2)\sqrt{3+5x^2+x^4}}{x}\,dx = \int \frac{(3x^2+2)\sqrt{x^4+5x^2+3}}{x}\,dx$$

```
[In] integrate((3*x**2+2)*(x**4+5*x**2+3)**(1/2)/x,x)
[Out] Integral((3*x**2 + 2)*sqrt(x**4 + 5*x**2 + 3)/x, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.95

$$\int \frac{(2+3x^2)\sqrt{3+5x^2+x^4}}{x}\,dx = \frac{3}{4}\sqrt{x^4+5\,x^2+3}x^2 - \sqrt{3}\log\left(\frac{2\sqrt{3}\sqrt{x^4+5\,x^2+3}}{x^2}+\frac{6}{x^2}+5\right) + \frac{23}{8}\sqrt{x^4+5\,x^2+3}+\frac{1}{16}\log\left(2\,x^2+2\sqrt{x^4+5\,x^2+3}+5\right)$$

```
[In] integrate((3*x^2+2)*(x^4+5*x^2+3)^(1/2)/x,x, algorithm="maxima")
[Out] 3/4*sqrt(x^4 + 5*x^2 + 3)*x^2 - sqrt(3)*log(2*sqrt(3)*sqrt(x^4 + 5*x^2 + 3)
/x^2 + 6/x^2 + 5) + 23/8*sqrt(x^4 + 5*x^2 + 3) + 1/16*log(2*x^2 + 2*sqrt(x^
4 + 5*x^2 + 3) + 5)
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.04

$$\int \frac{(2+3x^2)\sqrt{3+5x^2+x^4}}{x}\,dx = \frac{1}{8}\sqrt{x^4+5\,x^2+3}(6\,x^2+23) + \sqrt{3}\log\left(\frac{x^2+\sqrt{3}-\sqrt{x^4+5\,x^2+3}}{x^2-\sqrt{3}-\sqrt{x^4+5\,x^2+3}}\right) - \frac{1}{16}\log\left(2\,x^2-2\sqrt{x^4+5\,x^2+3}+5\right)$$

```
[In] integrate((3*x^2+2)*(x^4+5*x^2+3)^(1/2)/x,x, algorithm="giac")
[Out] 1/8*sqrt(x^4 + 5*x^2 + 3)*(6*x^2 + 23) + sqrt(3)*log((x^2 + sqrt(3) - sqrt(
x^4 + 5*x^2 + 3))/(x^2 - sqrt(3) - sqrt(x^4 + 5*x^2 + 3))) - 1/16*log(2*x^2
 - 2*sqrt(x^4 + 5*x^2 + 3) + 5)
```

Mupad [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.91

$$\int \frac{(2+3x^2)\sqrt{3+5x^2+x^4}}{x}\,dx = \frac{\ln\left(\sqrt{x^4+5\,x^2+3}+x^2+\frac{5}{2}\right)}{16} - \sqrt{3}\,\ln\left(\frac{3}{x^2}+\frac{\sqrt{3}\,\sqrt{x^4+5\,x^2+3}}{x^2}+\frac{5}{2}\right) + \frac{3\left(\frac{x^2}{2}+\frac{5}{4}\right)\,\sqrt{x^4+5\,x^2+3}}{2}+\sqrt{x^4+5\,x^2+3}$$

```
[In] int(((3*x^2 + 2)*(5*x^2 + x^4 + 3)^(1/2))/x,x)
```

```
[Out] log((5*x^2 + x^4 + 3)^(1/2) + x^2 + 5/2)/16 - 3^(1/2)*log(3/x^2 + (3^(1/2)*
(5*x^2 + x^4 + 3)^(1/2))/x^2 + 5/2) + (3*(x^2/2 + 5/4)*(5*x^2 + x^4 + 3)^(1
/2))/2 + (5*x^2 + x^4 + 3)^(1/2)
```

3.146 $\int \frac{(2+3x^2)\sqrt{3+5x^2+x^4}}{x^3}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 97

$$\int \frac{(2+3x^2)\sqrt{3+5x^2+x^4}}{x^3}\,dx = -\frac{(2-3x^2)\sqrt{3+5x^2+x^4}}{2x^2} + \frac{19}{4}\operatorname{arctanh}\left(\frac{5+2x^2}{2\sqrt{3+5x^2+x^4}}\right) - \frac{7\operatorname{arctanh}\left(\frac{6+5x^2}{2\sqrt{3}\sqrt{3+5x^2+x^4}}\right)}{\sqrt{3}}$$

```
[Out] 19/4*arctanh(1/2*(2*x^2+5)/(x^4+5*x^2+3)^(1/2))-7/3*arctanh(1/6*(5*x^2+6)*3
^(1/2)/(x^4+5*x^2+3)^(1/2))*3^(1/2)-1/2*(-3*x^2+2)*(x^4+5*x^2+3)^(1/2)/x^2
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {1265, 826, 857, 635, 212, 738}

$$\int \frac{(2+3x^2)\sqrt{3+5x^2+x^4}}{x^3}\,dx = \frac{19}{4}\operatorname{arctanh}\left(\frac{2x^2+5}{2\sqrt{x^4+5x^2+3}}\right) - \frac{7\operatorname{arctanh}\left(\frac{5x^2+6}{2\sqrt{3}\sqrt{x^4+5x^2+3}}\right)}{\sqrt{3}} - \frac{\sqrt{x^4+5x^2+3}(2-3x^2)}{2x^2}$$

```
[In] Int[((2 + 3*x^2)*Sqrt[3 + 5*x^2 + x^4])/x^3,x]
```

```
[Out] -1/2*((2 - 3*x^2)*Sqrt[3 + 5*x^2 + x^4])/x^2 + (19*ArcTanh[(5 + 2*x^2)/(2*S
qrt[3 + 5*x^2 + x^4])])/4 - (7*ArcTanh[(6 + 5*x^2)/(2*Sqrt[3]*Sqrt[3 + 5*x^
2 + x^4])])/Sqrt[3]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 635

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 826

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(e*f*(m + 2*p + 2) -
 d*g*(2*p + 1) + e*g*(m + 1)*x)*((a + b*x + c*x^2)^p/(e^2*(m + 1)*(m + 2*p
+ 2))), x] + Dist[p/(e^2*(m + 1)*(m + 2*p + 2)), Int[(d + e*x)^(m + 1)*(a +
 b*x + c*x^2)^(p - 1)*Simp[g*(b*d + 2*a*e + 2*a*e*m + 2*b*d*p) - f*b*e*(m +
 2*p + 2) + (g*(2*c*d + b*e + b*e*m + 4*c*d*p) - 2*c*e*f*(m + 2*p + 2))*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] && RationalQ[p] && p > 0 && (LtQ[m, -1] || Eq
Q[p, 1] || (IntegerQ[p] &&  !RationalQ[m])) && NeQ[m, -1] &&  !ILtQ[m + 2*p
 + 1, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 857

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1265

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rubi steps

$$\text{integral} = \frac{1}{2}\text{Subst}\left(\int \frac{(2+3x)\sqrt{3+5x+x^2}}{x^2}\,dx,x,x^2\right)$$
$$= -\frac{(2-3x^2)\sqrt{3+5x^2+x^4}}{2x^2} - \frac{1}{4}\text{Subst}\left(\int \frac{-28-19x}{x\sqrt{3+5x+x^2}}\,dx,x,x^2\right)$$
$$= -\frac{(2-3x^2)\sqrt{3+5x^2+x^4}}{2x^2} + \frac{19}{4}\text{Subst}\left(\int \frac{1}{\sqrt{3+5x+x^2}}\,dx,x,x^2\right) + 7\text{Subst}\left(\int \frac{1}{x\sqrt{3+5x+x^2}}\,dx,x,x^2\right)$$
$$= -\frac{(2-3x^2)\sqrt{3+5x^2+x^4}}{2x^2} + \frac{19}{2}\text{Subst}\left(\int \frac{1}{4-x^2}\,dx,x,\frac{5+2x^2}{\sqrt{3+5x^2+x^4}}\right) - 14\text{Subst}\left(\int \frac{1}{12-x^2}\,dx,x,\frac{6+5x^2}{\sqrt{3+5x^2+x^4}}\right)$$
$$= -\frac{(2-3x^2)\sqrt{3+5x^2+x^4}}{2x^2} + \frac{19}{4}\tanh^{-1}\left(\frac{5+2x^2}{2\sqrt{3+5x^2+x^4}}\right) - \frac{7\tanh^{-1}\left(\frac{6+5x^2}{2\sqrt{3}\sqrt{3+5x^2+x^4}}\right)}{\sqrt{3}}$$

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.94

$$\int \frac{(2+3x^2)\sqrt{3+5x^2+x^4}}{x^3}\,dx = \frac{(-2+3x^2)\sqrt{3+5x^2+x^4}}{2x^2} + \frac{14\text{arctanh}\left(\frac{x^2-\sqrt{3+5x^2+x^4}}{\sqrt{3}}\right)}{\sqrt{3}} - \frac{19}{4}\log\left(-5-2x^2+2\sqrt{3+5x^2+x^4}\right)$$

```
[In] Integrate[((2 + 3*x^2)*Sqrt[3 + 5*x^2 + x^4])/x^3,x]
```

```
[Out] ((-2 + 3*x^2)*Sqrt[3 + 5*x^2 + x^4])/(2*x^2) + (14*ArcTanh[(x^2 - Sqrt[3 +
5*x^2 + x^4])/Sqrt[3]])/Sqrt[3] - (19*Log[-5 - 2*x^2 + 2*Sqrt[3 + 5*x^2 + x
^4]])/4
```

Maple [A] (verified)

Time = 0.50 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.86

method	result
risch	$-\frac{\sqrt{x^4+5x^2+3}}{x^2}+\frac{19\ln\left(\frac{5}{2}+x^2+\sqrt{x^4+5x^2+3}\right)}{4}+\frac{3\sqrt{x^4+5x^2+3}}{2}-\frac{7\,\mathrm{arctanh}\left(\frac{(5x^2+6)\sqrt{3}}{6\sqrt{x^4+5x^2+3}}\right)\sqrt{3}}{3}$
elliptic	$-\frac{\sqrt{x^4+5x^2+3}}{x^2}+\frac{19\ln\left(\frac{5}{2}+x^2+\sqrt{x^4+5x^2+3}\right)}{4}+\frac{3\sqrt{x^4+5x^2+3}}{2}-\frac{7\,\mathrm{arctanh}\left(\frac{(5x^2+6)\sqrt{3}}{6\sqrt{x^4+5x^2+3}}\right)\sqrt{3}}{3}$
trager	$\frac{(3x^2-2)\sqrt{x^4+5x^2+3}}{2x^2}+\frac{7\,\mathrm{RootOf}\left(\_Z^2-3\right)\ln\left(-\frac{-5\,\mathrm{RootOf}\left(\_Z^2-3\right)x^2+6\sqrt{x^4+5x^2+3}-6\,\mathrm{RootOf}\left(\_Z^2-3\right)}{x^2}\right)}{3}+\frac{19\ln}{}$
pseudoelliptic	$\frac{-28\,\mathrm{arctanh}\left(\frac{(5x^2+6)\sqrt{3}}{6\sqrt{x^4+5x^2+3}}\right)\sqrt{3}\,x^2+18x^2\sqrt{x^4+5x^2+3}+57\ln\left(2x^2+5+2\sqrt{x^4+5x^2+3}\right)x^2-12\sqrt{x^4+5x^2+3}}{12x^2}$
default	$\frac{7\sqrt{x^4+5x^2+3}}{3}+\frac{19\ln\left(\frac{5}{2}+x^2+\sqrt{x^4+5x^2+3}\right)}{4}-\frac{7\,\mathrm{arctanh}\left(\frac{(5x^2+6)\sqrt{3}}{6\sqrt{x^4+5x^2+3}}\right)\sqrt{3}}{3}-\frac{(x^4+5x^2+3)^{\frac{3}{2}}}{3x^2}+\frac{(2x^2+5)\sqrt{x^4+5x}}{6}$

```
[In] int((3*x^2+2)*(x^4+5*x^2+3)^(1/2)/x^3,x,method=_RETURNVERBOSE)
```

```
[Out] -(x^4+5*x^2+3)^(1/2)/x^2+19/4*ln(5/2+x^2+(x^4+5*x^2+3)^(1/2))+3/2*(x^4+5*x^
2+3)^(1/2)-7/3*arctanh(1/6*(5*x^2+6)*3^(1/2)/(x^4+5*x^2+3)^(1/2))*3^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.15

$$\int \frac{(2+3x^2)\sqrt{3+5x^2+x^4}}{x^3}\,dx$$
$$=\frac{56\sqrt{3}x^2\log\left(\frac{25\,x^2-2\,\sqrt{3}(5\,x^2+6)-2\,\sqrt{x^4+5\,x^2+3}\left(5\,\sqrt{3}-6\right)+30}{x^2}\right)-114\,x^2\log\left(-2\,x^2+2\,\sqrt{x^4+5\,x^2+3}-5\right)+21\,}{24\,x^2}$$

```
[In] integrate((3*x^2+2)*(x^4+5*x^2+3)^(1/2)/x^3,x, algorithm="fricas")
```

```
[Out] 1/24*(56*sqrt(3)*x^2*log((25*x^2 - 2*sqrt(3)*(5*x^2 + 6) - 2*sqrt(x^4 + 5*x
^2 + 3)*(5*sqrt(3) - 6) + 30)/x^2) - 114*x^2*log(-2*x^2 + 2*sqrt(x^4 + 5*x^
2 + 3) - 5) + 21*x^2 + 12*sqrt(x^4 + 5*x^2 + 3)*(3*x^2 - 2))/x^2
```

Sympy [F]

$$\int \frac{(2+3x^2)\sqrt{3+5x^2+x^4}}{x^3}\,dx = \int \frac{(3x^2+2)\sqrt{x^4+5x^2+3}}{x^3}\,dx$$

```
[In] integrate((3*x**2+2)*(x**4+5*x**2+3)**(1/2)/x**3,x)
[Out] Integral((3*x**2 + 2)*sqrt(x**4 + 5*x**2 + 3)/x**3, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.92

$$\int \frac{(2+3x^2)\sqrt{3+5x^2+x^4}}{x^3}\,dx = -\frac{7}{3}\sqrt{3}\log\left(\frac{2\sqrt{3}\sqrt{x^4+5x^2+3}}{x^2}+\frac{6}{x^2}+5\right) + \frac{3}{2}\sqrt{x^4+5x^2+3} - \frac{\sqrt{x^4+5x^2+3}}{x^2} + \frac{19}{4}\log\left(2x^2+2\sqrt{x^4+5x^2+3}+5\right)$$

```
[In] integrate((3*x^2+2)*(x^4+5*x^2+3)^(1/2)/x^3,x, algorithm="maxima")
[Out] -7/3*sqrt(3)*log(2*sqrt(3)*sqrt(x^4 + 5*x^2 + 3)/x^2 + 6/x^2 + 5) + 3/2*sqr
t(x^4 + 5*x^2 + 3) - sqrt(x^4 + 5*x^2 + 3)/x^2 + 19/4*log(2*x^2 + 2*sqrt(x^
4 + 5*x^2 + 3) + 5)
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.42

$$\int \frac{(2+3x^2)\sqrt{3+5x^2+x^4}}{x^3}\,dx = \frac{7}{3}\sqrt{3}\log\left(\frac{x^2+\sqrt{3}-\sqrt{x^4+5x^2+3}}{x^2-\sqrt{3}-\sqrt{x^4+5x^2+3}}\right) + \frac{3}{2}\sqrt{x^4+5x^2+3} + \frac{5x^2-5\sqrt{x^4+5x^2+3}+6}{\left(x^2-\sqrt{x^4+5x^2+3}\right)^2-3} - \frac{19}{4}\log\left(2x^2-2\sqrt{x^4+5x^2+3}+5\right)$$

```
[In] integrate((3*x^2+2)*(x^4+5*x^2+3)^(1/2)/x^3,x, algorithm="giac")
[Out] 7/3*sqrt(3)*log((x^2 + sqrt(3) - sqrt(x^4 + 5*x^2 + 3))/(x^2 - sqrt(3) - sq
rt(x^4 + 5*x^2 + 3))) + 3/2*sqrt(x^4 + 5*x^2 + 3) + (5*x^2 - 5*sqrt(x^4 + 5
*x^2 + 3) + 6)/((x^2 - sqrt(x^4 + 5*x^2 + 3))^2 - 3) - 19/4*log(2*x^2 - 2*s
qrt(x^4 + 5*x^2 + 3) + 5)
```

Mupad [B] (verification not implemented)

Time = 8.01 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.87

$$\int \frac{(2+3x^2)\sqrt{3+5x^2+x^4}}{x^3}\,dx = \frac{19\ln\left(\sqrt{x^4+5\,x^2+3}+x^2+\frac{5}{2}\right)}{4}-\frac{\sqrt{x^4+5\,x^2+3}}{x^2}-\frac{7\sqrt{3}\ln\left(\frac{3}{x^2}+\frac{\sqrt{3}\sqrt{x^4+5\,x^2+3}}{x^2}+\frac{5}{2}\right)}{3}+\frac{3\sqrt{x^4+5\,x^2+3}}{2}$$

```
[In] int(((3*x^2 + 2)*(5*x^2 + x^4 + 3)^(1/2))/x^3,x)
```

```
[Out] (19*log((5*x^2 + x^4 + 3)^(1/2) + x^2 + 5/2))/4 - (5*x^2 + x^4 + 3)^(1/2)/x
^2 - (7*3^(1/2)*log(3/x^2 + (3^(1/2)*(5*x^2 + x^4 + 3)^(1/2))/x^2 + 5/2))/3
 + (3*(5*x^2 + x^4 + 3)^(1/2))/2
```

3.147 $\int \frac{(2+3x^2)\sqrt{3+5x^2+x^4}}{x^5} \, dx$

Optimal result

Integrand size = 25, antiderivative size = 99

$$\int \frac{(2+3x^2)\sqrt{3+5x^2+x^4}}{x^5} \, dx = -\frac{(6+23x^2)\sqrt{3+5x^2+x^4}}{12x^4} + \frac{3}{2}\text{arctanh}\left(\frac{5+2x^2}{2\sqrt{3+5x^2+x^4}}\right) - \frac{77\text{arctanh}\left(\frac{6+5x^2}{2\sqrt{3}\sqrt{3+5x^2+x^4}}\right)}{24\sqrt{3}}$$

```
[Out] 3/2*arctanh(1/2*(2*x^2+5)/(x^4+5*x^2+3)^(1/2))-77/72*arctanh(1/6*(5*x^2+6)*
3^(1/2)/(x^4+5*x^2+3)^(1/2))*3^(1/2)-1/12*(23*x^2+6)*(x^4+5*x^2+3)^(1/2)/x^
4
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {1265, 824, 857, 635, 212, 738}

$$\int \frac{(2+3x^2)\sqrt{3+5x^2+x^4}}{x^5} \, dx = \frac{3}{2}\text{arctanh}\left(\frac{2x^2+5}{2\sqrt{x^4+5x^2+3}}\right) - \frac{77\text{arctanh}\left(\frac{5x^2+6}{2\sqrt{3}\sqrt{x^4+5x^2+3}}\right)}{24\sqrt{3}} - \frac{\sqrt{x^4+5x^2+3}(23x^2+6)}{12x^4}$$

```
[In] Int[((2 + 3*x^2)*Sqrt[3 + 5*x^2 + x^4])/x^5,x]
```

```
[Out] -1/12*((6 + 23*x^2)*Sqrt[3 + 5*x^2 + x^4])/x^4 + (3*ArcTanh[(5 + 2*x^2)/(2*
Sqrt[3 + 5*x^2 + x^4])])/2 - (77*ArcTanh[(6 + 5*x^2)/(2*Sqrt[3]*Sqrt[3 + 5*
x^2 + x^4])])/(24*Sqrt[3])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 635

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 824

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(-(d + e*x)^(m + 1))*((a + b*x + c*x^2
)^p/(e^2*(m + 1)*(m + 2)*(c*d^2 - b*d*e + a*e^2)))*((d*g - e*f*(m + 2))*(c*
d^2 - b*d*e + a*e^2) - d*p*(2*c*d - b*e)*(e*f - d*g) - e*(g*(m + 1)*(c*d^2
- b*d*e + a*e^2) + p*(2*c*d - b*e)*(e*f - d*g))*x), x] - Dist[p/(e^2*(m + 1
)*(m + 2)*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m + 2)*(a + b*x + c*x^2)
^(p - 1)*Simp[2*a*c*e*(e*f - d*g)*(m + 2) + b^2*e*(d*g*(p + 1) - e*f*(m + p
 + 2)) + b*(a*e^2*g*(m + 1) - c*d*(d*g*(2*p + 1) - e*f*(m + 2*p + 2))) - c*
(2*c*d*(d*g*(2*p + 1) - e*f*(m + 2*p + 2)) - e*(2*a*e*g*(m + 1) - b*(d*g*(m
 - 2*p) + e*f*(m + 2*p + 2))))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g}
, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && GtQ[p, 0] &
& LtQ[m, -2] && LtQ[m + 2*p, 0] &&  !ILtQ[m + 2*p + 3, 0]
```

Rule 857

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1265

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{2}\text{Subst}\left(\int \frac{(2+3x)\sqrt{3+5x+x^2}}{x^3}\,dx, x, x^2\right)\\
&= -\frac{(6+23x^2)\sqrt{3+5x^2+x^4}}{12x^4} - \frac{1}{24}\text{Subst}\left(\int \frac{-77-36x}{x\sqrt{3+5x+x^2}}\,dx, x, x^2\right)\\
&= -\frac{(6+23x^2)\sqrt{3+5x^2+x^4}}{12x^4} + \frac{3}{2}\text{Subst}\left(\int \frac{1}{\sqrt{3+5x+x^2}}\,dx, x, x^2\right)\\
&\quad + \frac{77}{24}\text{Subst}\left(\int \frac{1}{x\sqrt{3+5x+x^2}}\,dx, x, x^2\right)\\
&= -\frac{(6+23x^2)\sqrt{3+5x^2+x^4}}{12x^4} + 3\text{Subst}\left(\int \frac{1}{4-x^2}\,dx, x, \frac{5+2x^2}{\sqrt{3+5x^2+x^4}}\right)\\
&\quad - \frac{77}{12}\text{Subst}\left(\int \frac{1}{12-x^2}\,dx, x, \frac{6+5x^2}{\sqrt{3+5x^2+x^4}}\right)\\
&= -\frac{(6+23x^2)\sqrt{3+5x^2+x^4}}{12x^4} + \frac{3}{2}\tanh^{-1}\left(\frac{5+2x^2}{2\sqrt{3+5x^2+x^4}}\right) - \frac{77\tanh^{-1}\left(\frac{6+5x^2}{2\sqrt{3}\sqrt{3+5x^2+x^4}}\right)}{24\sqrt{3}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.92

$$\int \frac{(2+3x^2)\sqrt{3+5x^2+x^4}}{x^5}\,dx = \frac{1}{36}\left(-\frac{3(6+23x^2)\sqrt{3+5x^2+x^4}}{x^4} + 77\sqrt{3}\text{arctanh}\left(\frac{x^2-\sqrt{3+5x^2+x^4}}{\sqrt{3}}\right) - 54\log\left(-5-2x^2+2\sqrt{3+5x^2+x^4}\right)\right)$$

```
[In] Integrate[((2 + 3*x^2)*Sqrt[3 + 5*x^2 + x^4])/x^5,x]
```

```
[Out] ((-3*(6 + 23*x^2)*Sqrt[3 + 5*x^2 + x^4])/x^4 + 77*Sqrt[3]*ArcTanh[(x^2 - Sq
rt[3 + 5*x^2 + x^4])/Sqrt[3]] - 54*Log[-5 - 2*x^2 + 2*Sqrt[3 + 5*x^2 + x^4]
])/36
```

Maple [A] (verified)

Time = 0.48 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.87

method	result
risch	$-\frac{23x^6+121x^4+99x^2+18}{12x^4\sqrt{x^4+5x^2+3}}+\frac{3\ln\left(\frac{5}{2}+x^2+\sqrt{x^4+5x^2+3}\right)}{2}-\frac{77\,\operatorname{arctanh}\left(\frac{(5x^2+6)\sqrt{3}}{6\sqrt{x^4+5x^2+3}}\right)\sqrt{3}}{72}$
elliptic	$\frac{3\ln\left(\frac{5}{2}+x^2+\sqrt{x^4+5x^2+3}\right)}{2}-\frac{\sqrt{x^4+5x^2+3}}{2x^4}-\frac{23\sqrt{x^4+5x^2+3}}{12x^2}-\frac{77\,\operatorname{arctanh}\left(\frac{(5x^2+6)\sqrt{3}}{6\sqrt{x^4+5x^2+3}}\right)\sqrt{3}}{72}$
trager	$-\frac{(23x^2+6)\sqrt{x^4+5x^2+3}}{12x^4}+\frac{3\ln\left(-2x^2-2\sqrt{x^4+5x^2+3}-5\right)}{2}+\frac{77\,\mathrm{RootOf}\left(\_Z^2-3\right)\ln\left(-\frac{-5\,\mathrm{RootOf}\left(\_Z^2-3\right)x^2+6\sqrt{x}}{x}\right.}{72}$
pseudoelliptic	$\frac{-77\,\operatorname{arctanh}\left(\frac{(5x^2+6)\sqrt{3}}{6\sqrt{x^4+5x^2+3}}\right)\sqrt{3}\,x^4+108\ln\left(2x^2+5+2\sqrt{x^4+5x^2+3}\right)x^4-138x^2\sqrt{x^4+5x^2+3}-36\sqrt{x^4+5x^2+3}}{72x^4}$
default	$-\frac{13(x^4+5x^2+3)^{\frac{3}{2}}}{36x^2}+\frac{77\sqrt{x^4+5x^2+3}}{72}+\frac{3\ln\left(\frac{5}{2}+x^2+\sqrt{x^4+5x^2+3}\right)}{2}-\frac{77\,\operatorname{arctanh}\left(\frac{(5x^2+6)\sqrt{3}}{6\sqrt{x^4+5x^2+3}}\right)\sqrt{3}}{72}+\frac{13(2x^2+5)}{}$

```
[In] int((3*x^2+2)*(x^4+5*x^2+3)^(1/2)/x^5,x,method=_RETURNVERBOSE)
```

```
[Out] -1/12*(23*x^6+121*x^4+99*x^2+18)/x^4/(x^4+5*x^2+3)^(1/2)+3/2*ln(5/2+x^2+(x^
4+5*x^2+3)^(1/2))-77/72*arctanh(1/6*(5*x^2+6)*3^(1/2)/(x^4+5*x^2+3)^(1/2))*
3^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.13

$$\int \frac{(2+3x^2)\sqrt{3+5x^2+x^4}}{x^5}\,dx$$
$$=\frac{77\sqrt{3}x^4\log\left(\frac{25\,x^2-2\,\sqrt{3}(5\,x^2+6)-2\,\sqrt{x^4+5\,x^2+3}\left(5\,\sqrt{3}-6\right)+30}{x^2}\right)-108\,x^4\log\left(-2\,x^2+2\,\sqrt{x^4+5\,x^2+3}-5\right)-138}{72\,x^4}$$

```
[In] integrate((3*x^2+2)*(x^4+5*x^2+3)^(1/2)/x^5,x, algorithm="fricas")
```

```
[Out] 1/72*(77*sqrt(3)*x^4*log((25*x^2 - 2*sqrt(3)*(5*x^2 + 6) - 2*sqrt(x^4 + 5*x
^2 + 3)*(5*sqrt(3) - 6) + 30)/x^2) - 108*x^4*log(-2*x^2 + 2*sqrt(x^4 + 5*x^
2 + 3) - 5) - 138*x^4 - 6*sqrt(x^4 + 5*x^2 + 3)*(23*x^2 + 6))/x^4
```

Sympy [F]

$$\int \frac{(2+3x^2)\sqrt{3+5x^2+x^4}}{x^5}\,dx = \int \frac{(3x^2+2)\sqrt{x^4+5x^2+3}}{x^5}\,dx$$

```
[In] integrate((3*x**2+2)*(x**4+5*x**2+3)**(1/2)/x**5,x)
[Out] Integral((3*x**2 + 2)*sqrt(x**4 + 5*x**2 + 3)/x**5, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.07

$$\int \frac{(2+3x^2)\sqrt{3+5x^2+x^4}}{x^5}\,dx = -\frac{77}{72}\sqrt{3}\log\left(\frac{2\sqrt{3}\sqrt{x^4+5x^2+3}}{x^2}+\frac{6}{x^2}+5\right) + \frac{1}{6}\sqrt{x^4+5x^2+3} - \frac{13\sqrt{x^4+5x^2+3}}{12x^2} - \frac{(x^4+5x^2+3)^{\frac{3}{2}}}{6x^4} + \frac{3}{2}\log\left(2x^2+2\sqrt{x^4+5x^2+3}+5\right)$$

```
[In] integrate((3*x^2+2)*(x^4+5*x^2+3)^(1/2)/x^5,x, algorithm="maxima")
[Out] -77/72*sqrt(3)*log(2*sqrt(3)*sqrt(x^4 + 5*x^2 + 3)/x^2 + 6/x^2 + 5) + 1/6*s
qrt(x^4 + 5*x^2 + 3) - 13/12*sqrt(x^4 + 5*x^2 + 3)/x^2 - 1/6*(x^4 + 5*x^2 +
 3)^(3/2)/x^4 + 3/2*log(2*x^2 + 2*sqrt(x^4 + 5*x^2 + 3) + 5)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 169 vs. 2(79) = 158.

Time = 0.32 (sec) , antiderivative size = 169, normalized size of antiderivative = 1.71

$$\int \frac{(2+3x^2)\sqrt{3+5x^2+x^4}}{x^5}\,dx = \frac{77}{72}\sqrt{3}\log\left(\frac{x^2+\sqrt{3}-\sqrt{x^4+5x^2+3}}{x^2-\sqrt{3}-\sqrt{x^4+5x^2+3}}\right) + \frac{127\left(x^2-\sqrt{x^4+5x^2+3}\right)^3+228\left(x^2-\sqrt{x^4+5x^2+3}\right)^2-159x^2+159\sqrt{x^4+5x^2+3}-324}{12\left(\left(x^2-\sqrt{x^4+5x^2+3}\right)^2-3\right)^2} - \frac{3}{2}\log\left(2x^2-2\sqrt{x^4+5x^2+3}+5\right)$$

```
[In] integrate((3*x^2+2)*(x^4+5*x^2+3)^(1/2)/x^5,x, algorithm="giac")
```

```
[Out] 77/72*sqrt(3)*log((x^2 + sqrt(3) - sqrt(x^4 + 5*x^2 + 3))/(x^2 - sqrt(3) -
sqrt(x^4 + 5*x^2 + 3))) + 1/12*(127*(x^2 - sqrt(x^4 + 5*x^2 + 3))^3 + 228*(
x^2 - sqrt(x^4 + 5*x^2 + 3))^2 - 159*x^2 + 159*sqrt(x^4 + 5*x^2 + 3) - 324)
/((x^2 - sqrt(x^4 + 5*x^2 + 3))^2 - 3)^2 - 3/2*log(2*x^2 - 2*sqrt(x^4 + 5*x
^2 + 3) + 5)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(2+3x^2)\sqrt{3+5x^2+x^4}}{x^5}\,dx=\int \frac{\left(3\,x^2+2\right)\,\sqrt{x^4+5\,x^2+3}}{x^5}\,dx$$

```
[In] int(((3*x^2 + 2)*(5*x^2 + x^4 + 3)^(1/2))/x^5,x)
```

```
[Out] int(((3*x^2 + 2)*(5*x^2 + x^4 + 3)^(1/2))/x^5, x)
```

3.148 $\int \frac{(2+3x^2)\sqrt{3+5x^2+x^4}}{x^7}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 90

$$\int \frac{(2+3x^2)\sqrt{3+5x^2+x^4}}{x^7}\,dx = -\frac{(6+5x^2)\sqrt{3+5x^2+x^4}}{18x^4} - \frac{(3+5x^2+x^4)^{3/2}}{9x^6} + \frac{13\text{arctanh}\left(\frac{6+5x^2}{2\sqrt{3}\sqrt{3+5x^2+x^4}}\right)}{36\sqrt{3}}$$

```
[Out] -1/9*(x^4+5*x^2+3)^(3/2)/x^6+13/108*arctanh(1/6*(5*x^2+6)*3^(1/2)/(x^4+5*x^
2+3)^(1/2))*3^(1/2)-1/18*(5*x^2+6)*(x^4+5*x^2+3)^(1/2)/x^4
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {1265, 820, 734, 738, 212}

$$\int \frac{(2+3x^2)\sqrt{3+5x^2+x^4}}{x^7}\,dx = \frac{13\text{arctanh}\left(\frac{5x^2+6}{2\sqrt{3}\sqrt{x^4+5x^2+3}}\right)}{36\sqrt{3}} - \frac{(5x^2+6)\sqrt{x^4+5x^2+3}}{18x^4} - \frac{(x^4+5x^2+3)^{3/2}}{9x^6}$$

```
[In] Int[((2 + 3*x^2)*Sqrt[3 + 5*x^2 + x^4])/x^7,x]
```

```
[Out] -1/18*((6 + 5*x^2)*Sqrt[3 + 5*x^2 + x^4])/x^4 - (3 + 5*x^2 + x^4)^(3/2)/(9*
x^6) + (13*ArcTanh[(6 + 5*x^2)/(2*Sqrt[3]*Sqrt[3 + 5*x^2 + x^4])])/(36*Sqrt
[3])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 734

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(-(d + e*x)^(m + 1))*(d*b - 2*a*e + (2*c*d - b*e)*x)*((a + b
*x + c*x^2)^p/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Dist[p*((b^2 - 4*a
*c)/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2))), Int[(d + e*x)^(m + 2)*(a + b*x +
c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0]
&& NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && EqQ[m + 2*p + 2,
 0] && GtQ[p, 0]
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 820

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a +
 b*x + c*x^2)^(p + 1)/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2))), x] - Dist[(b*(e
*f + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(
m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[Simplify[m
+ 2*p + 3], 0]
```

Rule 1265

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{1}{2}\text{Subst}\left(\int \frac{(2+3x)\sqrt{3+5x+x^2}}{x^4}\,dx, x, x^2\right)\\ &= -\frac{(3+5x^2+x^4)^{3/2}}{9x^6}+\frac{2}{3}\text{Subst}\left(\int \frac{\sqrt{3+5x+x^2}}{x^3}\,dx, x, x^2\right)\end{aligned}$$

$$= -\frac{(6+5x^2)\sqrt{3+5x^2+x^4}}{18x^4} - \frac{(3+5x^2+x^4)^{3/2}}{9x^6} - \frac{13}{36}\text{Subst}\left(\int \frac{1}{x\sqrt{3+5x+x^2}}\,dx, x, x^2\right)$$

$$= -\frac{(6+5x^2)\sqrt{3+5x^2+x^4}}{18x^4} - \frac{(3+5x^2+x^4)^{3/2}}{9x^6} + \frac{13}{18}\text{Subst}\left(\int \frac{1}{12-x^2}\,dx, x, \frac{6+5x^2}{\sqrt{3+5x^2+x^4}}\right)$$

$$= -\frac{(6+5x^2)\sqrt{3+5x^2+x^4}}{18x^4} - \frac{(3+5x^2+x^4)^{3/2}}{9x^6} + \frac{13\tanh^{-1}\left(\frac{6+5x^2}{2\sqrt{3}\sqrt{3+5x^2+x^4}}\right)}{36\sqrt{3}}$$

Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.78

$$\int \frac{(2+3x^2)\sqrt{3+5x^2+x^4}}{x^7}\,dx = \frac{1}{54}\left(-\frac{3\sqrt{3+5x^2+x^4}(6+16x^2+7x^4)}{x^6} - 13\sqrt{3}\text{arctanh}\left(\frac{x^2-\sqrt{3+5x^2+x^4}}{\sqrt{3}}\right)\right)$$

```
[In] Integrate[((2 + 3*x^2)*Sqrt[3 + 5*x^2 + x^4])/x^7,x]
[Out] ((-3*Sqrt[3 + 5*x^2 + x^4]*(6 + 16*x^2 + 7*x^4))/x^6 - 13*Sqrt[3]*ArcTanh[(
x^2 - Sqrt[3 + 5*x^2 + x^4])/Sqrt[3]])/54
```

Maple [A] (verified)

Time = 0.41 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.73

method	result
pseudoelliptic	$\frac{13 \operatorname{arctanh}\left(\frac{\left(5x^2+6\right)\sqrt{3}}{6\sqrt{x^4+5x^2+3}}\right)\sqrt{3}\,x^6-6\sqrt{x^4+5x^2+3}\,(7x^4+16x^2+6)}{108x^6}$
risch	$-\frac{7x^8+51x^6+107x^4+78x^2+18}{18x^6\sqrt{x^4+5x^2+3}}+\frac{13 \operatorname{arctanh}\left(\frac{\left(5x^2+6\right)\sqrt{3}}{6\sqrt{x^4+5x^2+3}}\right)\sqrt{3}}{108}$
trager	$-\frac{(7x^4+16x^2+6)\sqrt{x^4+5x^2+3}}{18x^6}+\frac{13\,\mathrm{RootOf}\left(\_Z^2-3\right)\ln\left(-\frac{5\,\mathrm{RootOf}\left(\_Z^2-3\right)x^2+6\sqrt{x^4+5x^2+3}+6\,\mathrm{RootOf}\left(\_Z^2-3\right)}{x^2}\right)}{108}$
elliptic	$\frac{13 \operatorname{arctanh}\left(\frac{\left(5x^2+6\right)\sqrt{3}}{6\sqrt{x^4+5x^2+3}}\right)\sqrt{3}}{108}-\frac{\sqrt{x^4+5x^2+3}}{3x^6}-\frac{8\sqrt{x^4+5x^2+3}}{9x^4}-\frac{7\sqrt{x^4+5x^2+3}}{18x^2}$
default	$-\frac{(x^4+5x^2+3)^{\frac{3}{2}}}{9x^6}-\frac{(x^4+5x^2+3)^{\frac{3}{2}}}{9x^4}+\frac{5(x^4+5x^2+3)^{\frac{3}{2}}}{54x^2}-\frac{13\sqrt{x^4+5x^2+3}}{108}+\frac{13 \operatorname{arctanh}\left(\frac{\left(5x^2+6\right)\sqrt{3}}{6\sqrt{x^4+5x^2+3}}\right)\sqrt{3}}{108}-\frac{5(2x$

```
[In] int((3*x^2+2)*(x^4+5*x^2+3)^(1/2)/x^7,x,method=_RETURNVERBOSE)
```

```
[Out] 1/108*(13*arctanh(1/6*(5*x^2+6)*3^(1/2)/(x^4+5*x^2+3)^(1/2))*3^(1/2)*x^6-6*
(x^4+5*x^2+3)^(1/2)*(7*x^4+16*x^2+6))/x^6
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.00

$$\int \frac{(2+3x^2)\sqrt{3+5x^2+x^4}}{x^7}\,dx$$
$$=\frac{13\sqrt{3}x^6\log\left(\frac{25\,x^2+2\sqrt{3}(5\,x^2+6)+2\sqrt{x^4+5\,x^2+3}\left(5\sqrt{3}+6\right)+30}{x^2}\right)-42\,x^6-6\,(7\,x^4+16\,x^2+6)\sqrt{x^4+5\,x^2+3}}{108\,x^6}$$

```
[In] integrate((3*x^2+2)*(x^4+5*x^2+3)^(1/2)/x^7,x, algorithm="fricas")
```

```
[Out] 1/108*(13*sqrt(3)*x^6*log((25*x^2 + 2*sqrt(3)*(5*x^2 + 6) + 2*sqrt(x^4 + 5*
x^2 + 3)*(5*sqrt(3) + 6) + 30)/x^2) - 42*x^6 - 6*(7*x^4 + 16*x^2 + 6)*sqrt(
x^4 + 5*x^2 + 3))/x^6
```

Sympy [F]

$$\int \frac{(2+3x^2)\sqrt{3+5x^2+x^4}}{x^7}\,dx = \int \frac{(3x^2+2)\sqrt{x^4+5x^2+3}}{x^7}\,dx$$

```
[In] integrate((3*x**2+2)*(x**4+5*x**2+3)**(1/2)/x**7,x)
[Out] Integral((3*x**2 + 2)*sqrt(x**4 + 5*x**2 + 3)/x**7, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.10

$$\int \frac{(2+3x^2)\sqrt{3+5x^2+x^4}}{x^7}\,dx = \frac{13}{108}\sqrt{3}\log\left(\frac{2\sqrt{3}\sqrt{x^4+5x^2+3}}{x^2}+\frac{6}{x^2}+5\right) + \frac{1}{9}\sqrt{x^4+5x^2+3} + \frac{5\sqrt{x^4+5x^2+3}}{18x^2} - \frac{(x^4+5x^2+3)^{\frac{3}{2}}}{9x^4} - \frac{(x^4+5x^2+3)^{\frac{3}{2}}}{9x^6}$$

```
[In] integrate((3*x^2+2)*(x^4+5*x^2+3)^(1/2)/x^7,x, algorithm="maxima")
[Out] 13/108*sqrt(3)*log(2*sqrt(3)*sqrt(x^4 + 5*x^2 + 3)/x^2 + 6/x^2 + 5) + 1/9*s
qrt(x^4 + 5*x^2 + 3) + 5/18*sqrt(x^4 + 5*x^2 + 3)/x^2 - 1/9*(x^4 + 5*x^2 +
3)^(3/2)/x^4 - 1/9*(x^4 + 5*x^2 + 3)^(3/2)/x^6
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 189 vs. 2(72) = 144.

Time = 0.31 (sec) , antiderivative size = 189, normalized size of antiderivative = 2.10

$$\int \frac{(2+3x^2)\sqrt{3+5x^2+x^4}}{x^7}\,dx = -\frac{13}{108}\sqrt{3}\log\left(\frac{x^2+\sqrt{3}-\sqrt{x^4+5x^2+3}}{x^2-\sqrt{3}-\sqrt{x^4+5x^2+3}}\right) + \frac{67\left(x^2-\sqrt{x^4+5x^2+3}\right)^5+306\left(x^2-\sqrt{x^4+5x^2+3}\right)^4+430\left(x^2-\sqrt{x^4+5x^2+3}\right)^3+90\left(x^2-\sqrt{x^4\cdots}\right.}{18\left(\left(x^2-\sqrt{x^4+5x^2+3}\right)^2-3\right)^3}$$

```
[In] integrate((3*x^2+2)*(x^4+5*x^2+3)^(1/2)/x^7,x, algorithm="giac")
[Out] -13/108*sqrt(3)*log((x^2 + sqrt(3) - sqrt(x^4 + 5*x^2 + 3))/(x^2 - sqrt(3)
- sqrt(x^4 + 5*x^2 + 3))) + 1/18*(67*(x^2 - sqrt(x^4 + 5*x^2 + 3))^5 + 306*
(x^2 - sqrt(x^4 + 5*x^2 + 3))^4 + 430*(x^2 - sqrt(x^4 + 5*x^2 + 3))^3 + 90*
(x^2 - sqrt(x^4 + 5*x^2 + 3))^2 - 63*x^2 + 63*sqrt(x^4 + 5*x^2 + 3) + 108)/
((x^2 - sqrt(x^4 + 5*x^2 + 3))^2 - 3)^3
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(2+3x^2)\sqrt{3+5x^2+x^4}}{x^7}\,dx = \int \frac{(3\,x^2+2)\,\sqrt{x^4+5\,x^2+3}}{x^7}\,dx$$

```
[In] int(((3*x^2 + 2)*(5*x^2 + x^4 + 3)^(1/2))/x^7,x)
[Out] int(((3*x^2 + 2)*(5*x^2 + x^4 + 3)^(1/2))/x^7, x)
```

3.149 $\int \frac{(2+3x^2)\sqrt{3+5x^2+x^4}}{x^9}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 111

$$\int \frac{(2+3x^2)\sqrt{3+5x^2+x^4}}{x^9}\,dx = \frac{67(6+5x^2)\sqrt{3+5x^2+x^4}}{1728x^4} - \frac{(3+5x^2+x^4)^{3/2}}{12x^8} - \frac{11(3+5x^2+x^4)^{3/2}}{216x^6} - \frac{871\operatorname{arctanh}\left(\frac{6+5x^2}{2\sqrt{3}\sqrt{3+5x^2+x^4}}\right)}{3456\sqrt{3}}$$

```
[Out] -1/12*(x^4+5*x^2+3)^(3/2)/x^8-11/216*(x^4+5*x^2+3)^(3/2)/x^6-871/10368*arct
anh(1/6*(5*x^2+6)*3^(1/2)/(x^4+5*x^2+3)^(1/2))*3^(1/2)+67/1728*(5*x^2+6)*(x
^4+5*x^2+3)^(1/2)/x^4
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {1265, 848, 820, 734, 738, 212}

$$\int \frac{(2+3x^2)\sqrt{3+5x^2+x^4}}{x^9}\,dx = -\frac{871\operatorname{arctanh}\left(\frac{5x^2+6}{2\sqrt{3}\sqrt{x^4+5x^2+3}}\right)}{3456\sqrt{3}} + \frac{67(5x^2+6)\sqrt{x^4+5x^2+3}}{1728x^4} - \frac{(x^4+5x^2+3)^{3/2}}{12x^8} - \frac{11(x^4+5x^2+3)^{3/2}}{216x^6}$$

```
[In] Int[((2 + 3*x^2)*Sqrt[3 + 5*x^2 + x^4])/x^9,x]
```

```
[Out] (67*(6 + 5*x^2)*Sqrt[3 + 5*x^2 + x^4])/(1728*x^4) - (3 + 5*x^2 + x^4)^(3/2)
/(12*x^8) - (11*(3 + 5*x^2 + x^4)^(3/2))/(216*x^6) - (871*ArcTanh[(6 + 5*x^
2)/(2*Sqrt[3]*Sqrt[3 + 5*x^2 + x^4])])/(3456*Sqrt[3])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 734

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(-(d + e*x)^(m + 1))*(d*b - 2*a*e + (2*c*d - b*e)*x)*((a + b
*x + c*x^2)^p/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Dist[p*((b^2 - 4*a
*c)/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2))), Int[(d + e*x)^(m + 2)*(a + b*x +
c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0]
&& NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && EqQ[m + 2*p + 2,
 0] && GtQ[p, 0]
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 820

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a +
 b*x + c*x^2)^(p + 1)/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2))), x] - Dist[(b*(e
*f + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(
m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[Simplify[m
+ 2*p + 3], 0]
```

Rule 848

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*f - d*g)*(d + e*x)^(m + 1)*((a + b*
x + c*x^2)^(p + 1)/((m + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Dist[1/((m + 1)
*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*Simp[(
c*d*f - f*b*e + a*e*g)*(m + 1) + b*(d*g - e*f)*(p + 1) - c*(e*f - d*g)*(m +
 2*p + 3)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] ||
 IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 1265

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rubi steps

$$\text{integral} = \frac{1}{2}\text{Subst}\left(\int \frac{(2+3x)\sqrt{3+5x+x^2}}{x^5}\,dx, x, x^2\right)$$

$$= -\frac{(3+5x^2+x^4)^{3/2}}{12x^8} - \frac{1}{24}\text{Subst}\left(\int \frac{(-11+2x)\sqrt{3+5x+x^2}}{x^4}\,dx, x, x^2\right)$$

$$= -\frac{(3+5x^2+x^4)^{3/2}}{12x^8} - \frac{11(3+5x^2+x^4)^{3/2}}{216x^6} - \frac{67}{144}\text{Subst}\left(\int \frac{\sqrt{3+5x+x^2}}{x^3}\,dx, x, x^2\right)$$

$$= \frac{67(6+5x^2)\sqrt{3+5x^2+x^4}}{1728x^4} - \frac{(3+5x^2+x^4)^{3/2}}{12x^8} - \frac{11(3+5x^2+x^4)^{3/2}}{216x^6} + \frac{871\text{Subst}\left(\int \frac{1}{x\sqrt{3+5x+x^2}}\,dx, x, x^2\right)}{3456}$$

$$= \frac{67(6+5x^2)\sqrt{3+5x^2+x^4}}{1728x^4} - \frac{(3+5x^2+x^4)^{3/2}}{12x^8} - \frac{11(3+5x^2+x^4)^{3/2}}{216x^6} - \frac{871\text{Subst}\left(\int \frac{1}{12-x^2}\,dx, x, \frac{6+5x^2}{\sqrt{3+5x^2+x^4}}\right)}{1728}$$

$$= \frac{67(6+5x^2)\sqrt{3+5x^2+x^4}}{1728x^4} - \frac{(3+5x^2+x^4)^{3/2}}{12x^8} - \frac{11(3+5x^2+x^4)^{3/2}}{216x^6} - \frac{871\tanh^{-1}\left(\frac{6+5x^2}{2\sqrt{3}\sqrt{3+5x^2+x^4}}\right)}{3456\sqrt{3}}$$

Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.72

$$\int \frac{(2+3x^2)\sqrt{3+5x^2+x^4}}{x^9}\,dx = \frac{\sqrt{3+5x^2+x^4}(-432-984x^2-182x^4+247x^6)}{1728x^8} + \frac{871\text{arctanh}\left(\frac{x^2}{\sqrt{3}}-\frac{\sqrt{3+5x^2+x^4}}{\sqrt{3}}\right)}{1728\sqrt{3}}$$

```
[In] Integrate[((2 + 3*x^2)*Sqrt[3 + 5*x^2 + x^4])/x^9,x]
[Out] (Sqrt[3 + 5*x^2 + x^4]*(-432 - 984*x^2 - 182*x^4 + 247*x^6))/(1728*x^8) + (
871*ArcTanh[x^2/Sqrt[3] - Sqrt[3 + 5*x^2 + x^4]/Sqrt[3]])/(1728*Sqrt[3])
```

Maple [A] (verified)

Time = 0.40 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.64

method	result
pseudoelliptic	$\frac{-871\,\operatorname{arctanh}\left(\frac{(5x^2+6)\sqrt{3}}{6\sqrt{x^4+5x^2+3}}\right)\sqrt{3}\,x^8+6\sqrt{x^4+5x^2+3}\,(247x^6-182x^4-984x^2-432)}{10368x^8}$
risch	$\frac{247x^{10}+1053x^8-1153x^6-5898x^4-5112x^2-1296}{1728x^8\sqrt{x^4+5x^2+3}}-\frac{871\,\operatorname{arctanh}\left(\frac{(5x^2+6)\sqrt{3}}{6\sqrt{x^4+5x^2+3}}\right)\sqrt{3}}{10368}$
trager	$\frac{(247x^6-182x^4-984x^2-432)\sqrt{x^4+5x^2+3}}{1728x^8}-\frac{871\,\mathrm{RootOf}\left(\_Z^2-3\right)\ln\left(-\frac{5\,\mathrm{RootOf}\left(\_Z^2-3\right)x^2+6\sqrt{x^4+5x^2+3}+6\,\mathrm{RootOf}\left(\right.}{x^2}\right.}{10368}$
elliptic	$\frac{247\sqrt{x^4+5x^2+3}}{1728x^2}-\frac{871\,\operatorname{arctanh}\left(\frac{(5x^2+6)\sqrt{3}}{6\sqrt{x^4+5x^2+3}}\right)\sqrt{3}}{10368}-\frac{\sqrt{x^4+5x^2+3}}{4x^8}-\frac{41\sqrt{x^4+5x^2+3}}{72x^6}-\frac{91\sqrt{x^4+5x^2+3}}{864x^4}$
default	$-\frac{(x^4+5x^2+3)^{\frac{3}{2}}}{12x^8}-\frac{11(x^4+5x^2+3)^{\frac{3}{2}}}{216x^6}+\frac{67(x^4+5x^2+3)^{\frac{3}{2}}}{864x^4}-\frac{335(x^4+5x^2+3)^{\frac{3}{2}}}{5184x^2}+\frac{871\sqrt{x^4+5x^2+3}}{10368}-\frac{871\,\operatorname{arctanh}}{}$

```
[In] int((3*x^2+2)*(x^4+5*x^2+3)^(1/2)/x^9,x,method=_RETURNVERBOSE)
```

```
[Out] 1/10368*(-871*arctanh(1/6*(5*x^2+6)*3^(1/2)/(x^4+5*x^2+3)^(1/2))*3^(1/2)*x^
8+6*(x^4+5*x^2+3)^(1/2)*(247*x^6-182*x^4-984*x^2-432))/x^8
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.86

$$\int \frac{(2+3x^2)\sqrt{3+5x^2+x^4}}{x^9}\,dx$$
$$=\frac{871\sqrt{3}x^8\log\left(\frac{25\,x^2-2\sqrt{3}(5\,x^2+6)-2\sqrt{x^4+5\,x^2+3}\left(5\sqrt{3}-6\right)+30}{x^2}\right)+1482\,x^8+6\,(247\,x^6-182\,x^4-984\,x^2-432)\sqrt{}}{10368\,x^8}$$

```
[In] integrate((3*x^2+2)*(x^4+5*x^2+3)^(1/2)/x^9,x, algorithm="fricas")
```

```
[Out] 1/10368*(871*sqrt(3)*x^8*log((25*x^2 - 2*sqrt(3)*(5*x^2 + 6) - 2*sqrt(x^4 +
 5*x^2 + 3)*(5*sqrt(3) - 6) + 30)/x^2) + 1482*x^8 + 6*(247*x^6 - 182*x^4 -
984*x^2 - 432)*sqrt(x^4 + 5*x^2 + 3))/x^8
```

Sympy [F]

$$\int \frac{(2+3x^2)\sqrt{3+5x^2+x^4}}{x^9}\,dx=\int \frac{(3x^2+2)\sqrt{x^4+5x^2+3}}{x^9}\,dx$$

```
[In] integrate((3*x**2+2)*(x**4+5*x**2+3)**(1/2)/x**9,x)
[Out] Integral((3*x**2 + 2)*sqrt(x**4 + 5*x**2 + 3)/x**9, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.05

$$\int \frac{(2+3x^2)\sqrt{3+5x^2+x^4}}{x^9}\,dx=-\frac{871}{10368}\sqrt{3}\log\left(\frac{2\sqrt{3}\sqrt{x^4+5\,x^2+3}}{x^2}+\frac{6}{x^2}+5\right)-\frac{67}{864}\sqrt{x^4+5\,x^2+3}-\frac{335\sqrt{x^4+5\,x^2+3}}{1728\,x^2}+\frac{67\,(x^4+5\,x^2+3)^{\frac{3}{2}}}{864\,x^4}-\frac{11\,(x^4+5\,x^2+3)^{\frac{3}{2}}}{216\,x^6}-\frac{(x^4+5\,x^2+3)^{\frac{3}{2}}}{12\,x^8}$$

```
[In] integrate((3*x^2+2)*(x^4+5*x^2+3)^(1/2)/x^9,x, algorithm="maxima")
[Out] -871/10368*sqrt(3)*log(2*sqrt(3)*sqrt(x^4 + 5*x^2 + 3)/x^2 + 6/x^2 + 5) - 6
7/864*sqrt(x^4 + 5*x^2 + 3) - 335/1728*sqrt(x^4 + 5*x^2 + 3)/x^2 + 67/864*(
x^4 + 5*x^2 + 3)^(3/2)/x^4 - 11/216*(x^4 + 5*x^2 + 3)^(3/2)/x^6 - 1/12*(x^4
 + 5*x^2 + 3)^(3/2)/x^8
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 233 vs. 2(89) = 178.

Time = 0.29 (sec) , antiderivative size = 233, normalized size of antiderivative = 2.10

$$\int \frac{(2+3x^2)\sqrt{3+5x^2+x^4}}{x^9}\,dx=\frac{871}{10368}\sqrt{3}\log\left(\frac{x^2+\sqrt{3}-\sqrt{x^4+5\,x^2+3}}{x^2-\sqrt{3}-\sqrt{x^4+5\,x^2+3}}\right)-\frac{871\left(x^2-\sqrt{x^4+5\,x^2+3}\right)^7-5184\left(x^2-\sqrt{x^4+5\,x^2+3}\right)^6-57389\left(x^2-\sqrt{x^4+5\,x^2+3}\right)^5-165888\,(x}{1728}$$

```
[In] integrate((3*x^2+2)*(x^4+5*x^2+3)^(1/2)/x^9,x, algorithm="giac")
```

```
[Out] 871/10368*sqrt(3)*log((x^2 + sqrt(3) - sqrt(x^4 + 5*x^2 + 3))/(x^2 - sqrt(3
) - sqrt(x^4 + 5*x^2 + 3))) - 1/1728*(871*(x^2 - sqrt(x^4 + 5*x^2 + 3))^7 -
 5184*(x^2 - sqrt(x^4 + 5*x^2 + 3))^6 - 57389*(x^2 - sqrt(x^4 + 5*x^2 + 3))
^5 - 165888*(x^2 - sqrt(x^4 + 5*x^2 + 3))^4 - 204807*(x^2 - sqrt(x^4 + 5*x^
2 + 3))^3 - 93312*(x^2 - sqrt(x^4 + 5*x^2 + 3))^2 - 2403*x^2 + 2403*sqrt(x^
4 + 5*x^2 + 3) - 5184)/((x^2 - sqrt(x^4 + 5*x^2 + 3))^2 - 3)^4
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(2+3x^2)\sqrt{3+5x^2+x^4}}{x^9}\,dx = \int \frac{(3\,x^2+2)\,\sqrt{x^4+5\,x^2+3}}{x^9}\,dx$$

```
[In] int(((3*x^2 + 2)*(5*x^2 + x^4 + 3)^(1/2))/x^9,x)
[Out] int(((3*x^2 + 2)*(5*x^2 + x^4 + 3)^(1/2))/x^9, x)
```

3.150 $\int \frac{(2+3x^2)\sqrt{3+5x^2+x^4}}{x^{11}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 132

$$\int \frac{(2+3x^2)\sqrt{3+5x^2+x^4}}{x^{11}}\,dx = -\frac{161(6+5x^2)\sqrt{3+5x^2+x^4}}{5184x^4} - \frac{(3+5x^2+x^4)^{3/2}}{15x^{10}} - \frac{(3+5x^2+x^4)^{3/2}}{36x^8} + \frac{173(3+5x^2+x^4)^{3/2}}{3240x^6} + \frac{2093\,\text{arctanh}\left(\frac{6+5x^2}{2\sqrt{3}\sqrt{3+5x^2+x^4}}\right)}{10368\sqrt{3}}$$

```
[Out] -1/15*(x^4+5*x^2+3)^(3/2)/x^10-1/36*(x^4+5*x^2+3)^(3/2)/x^8+173/3240*(x^4+5
*x^2+3)^(3/2)/x^6+2093/31104*arctanh(1/6*(5*x^2+6)*3^(1/2)/(x^4+5*x^2+3)^(1
/2))*3^(1/2)-161/5184*(5*x^2+6)*(x^4+5*x^2+3)^(1/2)/x^4
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {1265, 848, 820, 734, 738, 212}

$$\int \frac{(2+3x^2)\sqrt{3+5x^2+x^4}}{x^{11}}\,dx = \frac{2093\,\text{arctanh}\left(\frac{5x^2+6}{2\sqrt{3}\sqrt{x^4+5x^2+3}}\right)}{10368\sqrt{3}} - \frac{161(5x^2+6)\sqrt{x^4+5x^2+3}}{5184x^4} - \frac{(x^4+5x^2+3)^{3/2}}{15x^{10}} - \frac{(x^4+5x^2+3)^{3/2}}{36x^8} + \frac{173(x^4+5x^2+3)^{3/2}}{3240x^6}$$

```
[In] Int[((2 + 3*x^2)*Sqrt[3 + 5*x^2 + x^4])/x^11,x]
```

```
[Out] (-161*(6 + 5*x^2)*Sqrt[3 + 5*x^2 + x^4])/(5184*x^4) - (3 + 5*x^2 + x^4)^(3/
2)/(15*x^10) - (3 + 5*x^2 + x^4)^(3/2)/(36*x^8) + (173*(3 + 5*x^2 + x^4)^(3
/2))/(3240*x^6) + (2093*ArcTanh[(6 + 5*x^2)/(2*Sqrt[3]*Sqrt[3 + 5*x^2 + x^4
])])/(10368*Sqrt[3])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 734

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(-(d + e*x)^(m + 1))*(d*b - 2*a*e + (2*c*d - b*e)*x)*((a + b
*x + c*x^2)^p/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Dist[p*((b^2 - 4*a
*c)/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2))), Int[(d + e*x)^(m + 2)*(a + b*x +
c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0]
&& NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && EqQ[m + 2*p + 2,
 0] && GtQ[p, 0]
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 820

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a +
 b*x + c*x^2)^(p + 1)/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2))), x] - Dist[(b*(e
*f + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(
m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[Simplify[m
+ 2*p + 3], 0]
```

Rule 848

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*f - d*g)*(d + e*x)^(m + 1)*((a + b*
x + c*x^2)^(p + 1)/((m + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Dist[1/((m + 1)
*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*Simp[(
c*d*f - f*b*e + a*e*g)*(m + 1) + b*(d*g - e*f)*(p + 1) - c*(e*f - d*g)*(m +
 2*p + 3)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] ||
```

```
IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 1265

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{2}\text{Subst}\left(\int \frac{(2+3x)\sqrt{3+5x+x^2}}{x^6}\,dx, x, x^2\right) \\
&= -\frac{(3+5x^2+x^4)^{3/2}}{15x^{10}} - \frac{1}{30}\text{Subst}\left(\int \frac{(-10+4x)\sqrt{3+5x+x^2}}{x^5}\,dx, x, x^2\right) \\
&= -\frac{(3+5x^2+x^4)^{3/2}}{15x^{10}} - \frac{(3+5x^2+x^4)^{3/2}}{36x^8} \\
&\quad + \frac{1}{360}\text{Subst}\left(\int \frac{(-173-10x)\sqrt{3+5x+x^2}}{x^4}\,dx, x, x^2\right) \\
&= -\frac{(3+5x^2+x^4)^{3/2}}{15x^{10}} - \frac{(3+5x^2+x^4)^{3/2}}{36x^8} + \frac{173(3+5x^2+x^4)^{3/2}}{3240x^6} \\
&\quad + \frac{161}{432}\text{Subst}\left(\int \frac{\sqrt{3+5x+x^2}}{x^3}\,dx, x, x^2\right) \\
&= -\frac{161(6+5x^2)\sqrt{3+5x^2+x^4}}{5184x^4} - \frac{(3+5x^2+x^4)^{3/2}}{15x^{10}} - \frac{(3+5x^2+x^4)^{3/2}}{36x^8} \\
&\quad + \frac{173(3+5x^2+x^4)^{3/2}}{3240x^6} - \frac{2093\text{Subst}\left(\int \frac{1}{x\sqrt{3+5x+x^2}}\,dx, x, x^2\right)}{10368} \\
&= -\frac{161(6+5x^2)\sqrt{3+5x^2+x^4}}{5184x^4} - \frac{(3+5x^2+x^4)^{3/2}}{15x^{10}} - \frac{(3+5x^2+x^4)^{3/2}}{36x^8} \\
&\quad + \frac{173(3+5x^2+x^4)^{3/2}}{3240x^6} + \frac{2093\text{Subst}\left(\int \frac{1}{12-x^2}\,dx, x, \frac{6+5x^2}{\sqrt{3+5x^2+x^4}}\right)}{5184} \\
&= -\frac{161(6+5x^2)\sqrt{3+5x^2+x^4}}{5184x^4} - \frac{(3+5x^2+x^4)^{3/2}}{15x^{10}} - \frac{(3+5x^2+x^4)^{3/2}}{36x^8} \\
&\quad + \frac{173(3+5x^2+x^4)^{3/2}}{3240x^6} + \frac{2093\tanh^{-1}\left(\frac{6+5x^2}{2\sqrt{3}\sqrt{3+5x^2+x^4}}\right)}{10368\sqrt{3}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.31 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.61

$$\int \frac{(2+3x^2)\sqrt{3+5x^2+x^4}}{x^{11}}\,dx = \frac{-\frac{3\sqrt{3+5x^2+x^4}(5184+10800x^2+1176x^4-1370x^6+2641x^8)}{x^{10}}-10465\sqrt{3}\text{arctanh}\left(\frac{x^2-\sqrt{3+5x^2+x^4}}{\sqrt{3}}\right)}{77760}$$

```
[In] Integrate[((2 + 3*x^2)*Sqrt[3 + 5*x^2 + x^4])/x^11,x]
```

```
[Out] ((-3*Sqrt[3 + 5*x^2 + x^4]*(5184 + 10800*x^2 + 1176*x^4 - 1370*x^6 + 2641*x
^8))/x^10 - 10465*Sqrt[3]*ArcTanh[(x^2 - Sqrt[3 + 5*x^2 + x^4])/Sqrt[3]])/7
7760
```

Maple [A] (verified)

Time = 0.44 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.58

method	result
pseudoelliptic	$\frac{10465\,\text{arctanh}\left(\frac{(5x^2+6)\sqrt{3}}{6\sqrt{x^4+5x^2+3}}\right)\sqrt{3}\,x^{10}-6\sqrt{x^4+5x^2+3}\,(2641x^8-1370x^6+1176x^4+10800x^2+5184)}{155520x^{10}}$
risch	$-\frac{2641x^{12}+11835x^{10}+2249x^8+12570x^6+62712x^4+58320x^2+15552}{25920x^{10}\sqrt{x^4+5x^2+3}}+\frac{2093\,\text{arctanh}\left(\frac{(5x^2+6)\sqrt{3}}{6\sqrt{x^4+5x^2+3}}\right)\sqrt{3}}{31104}$
trager	$-\frac{(2641x^8-1370x^6+1176x^4+10800x^2+5184)\sqrt{x^4+5x^2+3}}{25920x^{10}}-\frac{2093\,\text{RootOf}\left(\_Z^2-3\right)\ln\left(-\frac{-5\,\text{RootOf}\left(\_Z^2-3\right)x^2+6\sqrt{}}{}\right.}{31104}$
elliptic	$-\frac{\sqrt{x^4+5x^2+3}}{5x^{10}}-\frac{5\sqrt{x^4+5x^2+3}}{12x^8}-\frac{49\sqrt{x^4+5x^2+3}}{1080x^6}+\frac{137\sqrt{x^4+5x^2+3}}{2592x^4}-\frac{2641\sqrt{x^4+5x^2+3}}{25920x^2}+\frac{2093\,\text{arctanh}\left(\frac{(5x^2}{6\sqrt{x^4}}\right.}{31104}$
default	$-\frac{(x^4+5x^2+3)^{\frac{3}{2}}}{36x^8}+\frac{173(x^4+5x^2+3)^{\frac{3}{2}}}{3240x^6}-\frac{161(x^4+5x^2+3)^{\frac{3}{2}}}{2592x^4}+\frac{805(x^4+5x^2+3)^{\frac{3}{2}}}{15552x^2}-\frac{2093\sqrt{x^4+5x^2+3}}{31104}+\frac{2093\,\text{arct}}{}$

```
[In] int((3*x^2+2)*(x^4+5*x^2+3)^(1/2)/x^11,x,method=_RETURNVERBOSE)
```

```
[Out] 1/155520*(10465*arctanh(1/6*(5*x^2+6)*3^(1/2)/(x^4+5*x^2+3)^(1/2))*3^(1/2)*
x^10-6*(x^4+5*x^2+3)^(1/2)*(2641*x^8-1370*x^6+1176*x^4+10800*x^2+5184))/x^1
0
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.76

$$\int \frac{(2+3x^2)\sqrt{3+5x^2+x^4}}{x^{11}}\,dx = \frac{10465\,\sqrt{3}x^{10}\log\left(\frac{25\,x^2+2\,\sqrt{3}(5\,x^2+6)+2\,\sqrt{x^4+5\,x^2+3}\left(5\,\sqrt{3}+6\right)+30}{x^2}\right) - 15846\,x^{10} - 6\left(2641\,x^8 - 1370\,x^6 + 1176\,x^4 + \cdots\right.}{155520\,x^{10}}$$

```
[In] integrate((3*x^2+2)*(x^4+5*x^2+3)^(1/2)/x^11,x, algorithm="fricas")
[Out] 1/155520*(10465*sqrt(3)*x^10*log((25*x^2 + 2*sqrt(3)*(5*x^2 + 6) + 2*sqrt(x
^4 + 5*x^2 + 3)*(5*sqrt(3) + 6) + 30)/x^2) - 15846*x^10 - 6*(2641*x^8 - 137
0*x^6 + 1176*x^4 + 10800*x^2 + 5184)*sqrt(x^4 + 5*x^2 + 3))/x^10
```

Sympy [F]

$$\int \frac{(2+3x^2)\sqrt{3+5x^2+x^4}}{x^{11}}\,dx = \int \frac{(3x^2+2)\sqrt{x^4+5x^2+3}}{x^{11}}\,dx$$

```
[In] integrate((3*x**2+2)*(x**4+5*x**2+3)**(1/2)/x**11,x)
[Out] Integral((3*x**2 + 2)*sqrt(x**4 + 5*x**2 + 3)/x**11, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.01

$$\begin{aligned}\int \frac{(2+3x^2)\sqrt{3+5x^2+x^4}}{x^{11}}\,dx &= \frac{2093}{31104}\,\sqrt{3}\log\left(\frac{2\,\sqrt{3}\sqrt{x^4+5\,x^2+3}}{x^2}+\frac{6}{x^2}+5\right)\\ &\quad+\frac{161}{2592}\,\sqrt{x^4+5\,x^2+3}+\frac{805\,\sqrt{x^4+5\,x^2+3}}{5184\,x^2}\\ &\quad-\frac{161\,(x^4+5\,x^2+3)^{\frac{3}{2}}}{2592\,x^4}+\frac{173\,(x^4+5\,x^2+3)^{\frac{3}{2}}}{3240\,x^6}\\ &\quad-\frac{(x^4+5\,x^2+3)^{\frac{3}{2}}}{36\,x^8}-\frac{(x^4+5\,x^2+3)^{\frac{3}{2}}}{15\,x^{10}}\end{aligned}$$

```
[In] integrate((3*x^2+2)*(x^4+5*x^2+3)^(1/2)/x^11,x, algorithm="maxima")
[Out] 2093/31104*sqrt(3)*log(2*sqrt(3)*sqrt(x^4 + 5*x^2 + 3)/x^2 + 6/x^2 + 5) + 1
61/2592*sqrt(x^4 + 5*x^2 + 3) + 805/5184*sqrt(x^4 + 5*x^2 + 3)/x^2 - 161/25
92*(x^4 + 5*x^2 + 3)^(3/2)/x^4 + 173/3240*(x^4 + 5*x^2 + 3)^(3/2)/x^6 - 1/3
6*(x^4 + 5*x^2 + 3)^(3/2)/x^8 - 1/15*(x^4 + 5*x^2 + 3)^(3/2)/x^10
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 255 vs. $2(106) = 212$.

Time = 0.31 (sec) , antiderivative size = 255, normalized size of antiderivative = 1.93

$$\int \frac{(2+3x^2)\sqrt{3+5x^2+x^4}}{x^{11}}\,dx = -\frac{2093}{31104}\sqrt{3}\log\left(\frac{x^2+\sqrt{3}-\sqrt{x^4+5\,x^2+3}}{x^2-\sqrt{3}-\sqrt{x^4+5\,x^2+3}}\right) + \frac{10465\,\left(x^2-\sqrt{x^4+5\,x^2+3}\right)^9 - 42830\,\left(x^2-\sqrt{x^4+5\,x^2+3}\right)^7 + 1270080\,\left(x^2-\sqrt{x^4+5\,x^2+3}\right)^6 + 70\cdots}{\cdots}$$

```
[In] integrate((3*x^2+2)*(x^4+5*x^2+3)^(1/2)/x^11,x, algorithm="giac")
```

```
[Out] -2093/31104*sqrt(3)*log((x^2 + sqrt(3) - sqrt(x^4 + 5*x^2 + 3))/(x^2 - sqrt
(3) - sqrt(x^4 + 5*x^2 + 3))) + 1/25920*(10465*(x^2 - sqrt(x^4 + 5*x^2 + 3)
)^9 - 42830*(x^2 - sqrt(x^4 + 5*x^2 + 3))^7 + 1270080*(x^2 - sqrt(x^4 + 5*x
^2 + 3))^6 + 7060800*(x^2 - sqrt(x^4 + 5*x^2 + 3))^5 + 15310080*(x^2 - sqrt
(x^4 + 5*x^2 + 3))^4 + 16095870*(x^2 - sqrt(x^4 + 5*x^2 + 3))^3 + 7568640*(
x^2 - sqrt(x^4 + 5*x^2 + 3))^2 + 1096335*x^2 - 1096335*sqrt(x^4 + 5*x^2 + 3
) + 202176)/((x^2 - sqrt(x^4 + 5*x^2 + 3))^2 - 3)^5
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(2+3x^2)\sqrt{3+5x^2+x^4}}{x^{11}}\,dx = \int \frac{(3\,x^2+2)\,\sqrt{x^4+5\,x^2+3}}{x^{11}}\,dx$$

```
[In] int(((3*x^2 + 2)*(5*x^2 + x^4 + 3)^(1/2))/x^11,x)
```

```
[Out] int(((3*x^2 + 2)*(5*x^2 + x^4 + 3)^(1/2))/x^11, x)
```

3.151 $\int x^4(2+3x^2)\sqrt{3+5x^2+x^4}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 322

$$\int x^4(2+3x^2)\sqrt{3+5x^2+x^4}\,dx = -\frac{1924x\left(5+\sqrt{13}+2x^2\right)}{105\sqrt{3+5x^2+x^4}} + \frac{13}{3}x\sqrt{3+5x^2+x^4} - \frac{26}{35}x^3\sqrt{3+5x^2+x^4} + \frac{1}{21}x^5\left(11+7x^2\right)\sqrt{3+5x^2+x^4} + \frac{962\sqrt{\frac{2}{3}\left(5+\sqrt{13}\right)}\sqrt{\frac{6+\left(5-\sqrt{13}\right)x^2}{6+\left(5+\sqrt{13}\right)x^2}}\left(6+\left(5+\sqrt{13}\right)x^2\right)E\left(\arctan\left(\sqrt{\frac{1}{6}\left(5+\sqrt{13}\right)}x\right)|\frac{1}{6}\left(-13+5\sqrt{13}\right)\right)}{105\sqrt{3+5x^2+x^4}} - \frac{13\sqrt{\frac{6+\left(5-\sqrt{13}\right)x^2}{6+\left(5+\sqrt{13}\right)x^2}}\left(6+\left(5+\sqrt{13}\right)x^2\right)\operatorname{EllipticF}\left(\arctan\left(\sqrt{\frac{1}{6}\left(5+\sqrt{13}\right)}x\right),\frac{1}{6}\left(-13+5\sqrt{13}\right)\right)}{\sqrt{6\left(5+\sqrt{13}\right)}\sqrt{3+5x^2+x^4}}$$

```
[Out] -1924/105*x*(5+2*x^2+13^(1/2))/(x^4+5*x^2+3)^(1/2)+13/3*x*(x^4+5*x^2+3)^(1/
2)-26/35*x^3*(x^4+5*x^2+3)^(1/2)+1/21*x^5*(7*x^2+11)*(x^4+5*x^2+3)^(1/2)+96
2/315*(1/(36+x^2*(30+6*13^(1/2))))^(1/2)*(36+x^2*(30+6*13^(1/2)))^(1/2)*Ell
ipticE(x*(30+6*13^(1/2))^(1/2)/(36+x^2*(30+6*13^(1/2)))^(1/2),1/6*(-78+30*1
3^(1/2))^(1/2))*(6+x^2*(5+13^(1/2)))*(30+6*13^(1/2))^(1/2)*((6+x^2*(5-13^(1
/2)))/(6+x^2*(5+13^(1/2))))^(1/2)/(x^4+5*x^2+3)^(1/2)-13*(1/(36+x^2*(30+6*1
3^(1/2))))^(1/2)*(36+x^2*(30+6*13^(1/2)))^(1/2)*EllipticF(x*(30+6*13^(1/2))
^(1/2)/(36+x^2*(30+6*13^(1/2)))^(1/2),1/6*(-78+30*13^(1/2))^(1/2))*(6+x^2*(
5+13^(1/2)))*((6+x^2*(5-13^(1/2)))/(6+x^2*(5+13^(1/2))))^(1/2)/(x^4+5*x^2+3
)^(1/2)/(30+6*13^(1/2))^(1/2)
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 322, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {1287, 1293, 1203, 1113, 1149}

$$\int x^4(2+3x^2)\sqrt{3+5x^2+x^4}\,dx = -\frac{13\sqrt{\frac{(5-\sqrt{13})x^2+6}{(5+\sqrt{13})x^2+6}}\left((5+\sqrt{13})x^2+6\right)\text{EllipticF}\left(\arctan\left(\sqrt{\frac{1}{6}(5+\sqrt{13})}x\right),\frac{1}{6}(-13+5\sqrt{13})\right)}{\sqrt{6(5+\sqrt{13})}\sqrt{x^4+5x^2+3}} + \frac{962\sqrt{\frac{2}{3}(5+\sqrt{13})}\sqrt{\frac{(5-\sqrt{13})x^2+6}{(5+\sqrt{13})x^2+6}}\left((5+\sqrt{13})x^2+6\right)E\left(\arctan\left(\sqrt{\frac{1}{6}(5+\sqrt{13})}x\right)|\frac{1}{6}(-13+5\sqrt{13})\right)}{105\sqrt{x^4+5x^2+3}} + \frac{13}{3}\sqrt{x^4+5x^2+3}x - \frac{1924(2x^2+\sqrt{13}+5)x}{105\sqrt{x^4+5x^2+3}} + \frac{1}{21}(7x^2+11)\sqrt{x^4+5x^2+3}x^5 - \frac{26}{35}\sqrt{x^4+5x^2+3}x^3$$

```
[In] Int[x^4*(2 + 3*x^2)*Sqrt[3 + 5*x^2 + x^4],x]
```

```
[Out] (-1924*x*(5 + Sqrt[13] + 2*x^2))/(105*Sqrt[3 + 5*x^2 + x^4]) + (13*x*Sqrt[3
 + 5*x^2 + x^4])/3 - (26*x^3*Sqrt[3 + 5*x^2 + x^4])/35 + (x^5*(11 + 7*x^2)*
Sqrt[3 + 5*x^2 + x^4])/21 + (962*Sqrt[(2*(5 + Sqrt[13]))/3]*Sqrt[(6 + (5 -
Sqrt[13])*x^2)/(6 + (5 + Sqrt[13])*x^2)]*(6 + (5 + Sqrt[13])*x^2)*EllipticE
[ArcTan[Sqrt[(5 + Sqrt[13])/6]*x], (-13 + 5*Sqrt[13])/6])/(105*Sqrt[3 + 5*x
^2 + x^4]) - (13*Sqrt[(6 + (5 - Sqrt[13])*x^2)/(6 + (5 + Sqrt[13])*x^2)]*(6
 + (5 + Sqrt[13])*x^2)*EllipticF[ArcTan[Sqrt[(5 + Sqrt[13])/6]*x], (-13 + 5
*Sqrt[13])/6])/(Sqrt[6*(5 + Sqrt[13])]*Sqrt[3 + 5*x^2 + x^4])
```

Rule 1113

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[(2*a + (b + q)*x^2)*(Sqrt[(2*a + (b - q)*x^2)/(2*a +
(b + q)*x^2)]/(2*a*Rt[(b + q)/(2*a), 2]*Sqrt[a + b*x^2 + c*x^4]))*EllipticF
[ArcTan[Rt[(b + q)/(2*a), 2]*x], 2*(q/(b + q))], x] /; PosQ[(b + q)/a] &&
!(PosQ[(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[
{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1149

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[b^2 - 4*a*c, 2]}, Simp[x*((b + q + 2*c*x^2)/(2*c*Sqrt[a + b*x^2 + c*x^4
])), x] - Simp[Rt[(b + q)/(2*a), 2]*(2*a + (b + q)*x^2)*(Sqrt[(2*a + (b - q
```

```
)*x^2)/(2*a + (b + q)*x^2)]/(2*c*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[ArcTan
[Rt[(b + q)/(2*a), 2]*x], 2*(q/(b + q))], x] /; PosQ[(b + q)/a] &&  !(PosQ[
(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[{a, b,
c}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1203

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[d, Int[1/Sqrt[a + b*x^2 + c*x^4],
 x], x] + Dist[e, Int[x^2/Sqrt[a + b*x^2 + c*x^4], x], x] /; PosQ[(b + q)/a
] || PosQ[(b - q)/a]] /; FreeQ[{a, b, c, d, e}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1287

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(a + b*x^2 + c*x^4)^p*((b*e*2
*p + c*d*(m + 4*p + 3) + c*e*(4*p + m + 1)*x^2)/(c*f*(4*p + m + 1)*(m + 4*p
 + 3))), x] + Dist[2*(p/(c*(4*p + m + 1)*(m + 4*p + 3))), Int[(f*x)^m*(a +
b*x^2 + c*x^4)^(p - 1)*Simp[2*a*c*d*(m + 4*p + 3) - a*b*e*(m + 1) + (2*a*c*
e*(4*p + m + 1) + b*c*d*(m + 4*p + 3) - b^2*e*(m + 2*p + 1))*x^2, x], x], x
] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b^2 - 4*a*c, 0] && GtQ[p, 0] &&
 NeQ[4*p + m + 1, 0] && NeQ[m + 4*p + 3, 0] && IntegerQ[2*p] && (IntegerQ[p
] || IntegerQ[m])
```

Rule 1293

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[e*f*(f*x)^(m - 1)*((a + b*x^2 + c*x^4)^(p +
1)/(c*(m + 4*p + 3))), x] - Dist[f^2/(c*(m + 4*p + 3)), Int[(f*x)^(m - 2)*(
a + b*x^2 + c*x^4)^p*Simp[a*e*(m - 1) + (b*e*(m + 2*p + 1) - c*d*(m + 4*p +
 3))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c,
 0] && GtQ[m, 1] && NeQ[m + 4*p + 3, 0] && IntegerQ[2*p] && (IntegerQ[p] ||
 IntegerQ[m])
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{1}{21}x^5(11+7x^2)\sqrt{3+5x^2+x^4}+\frac{1}{63}\int\frac{x^4(-117-234x^2)}{\sqrt{3+5x^2+x^4}}\,dx\\
&= -\frac{26}{35}x^3\sqrt{3+5x^2+x^4}+\frac{1}{21}x^5(11+7x^2)\sqrt{3+5x^2+x^4}-\frac{1}{315}\int\frac{x^2(-2106-4095x^2)}{\sqrt{3+5x^2+x^4}}\,dx\\
&= \frac{13}{3}x\sqrt{3+5x^2+x^4}-\frac{26}{35}x^3\sqrt{3+5x^2+x^4}\\
&\quad+\frac{1}{21}x^5(11+7x^2)\sqrt{3+5x^2+x^4}+\frac{1}{945}\int\frac{-12285-34632x^2}{\sqrt{3+5x^2+x^4}}\,dx
\end{aligned}
$$

$$= \frac{13}{3}x\sqrt{3+5x^2+x^4} - \frac{26}{35}x^3\sqrt{3+5x^2+x^4} + \frac{1}{21}x^5\left(11+7x^2\right)\sqrt{3+5x^2+x^4} - 13\int \frac{1}{\sqrt{3+5x^2+x^4}}\,dx - \frac{3848}{105}\int \frac{x^2}{\sqrt{3+5x^2+x^4}}\,dx$$

$$= -\frac{1924x\left(5+\sqrt{13}+2x^2\right)}{105\sqrt{3+5x^2+x^4}} + \frac{13}{3}x\sqrt{3+5x^2+x^4} - \frac{26}{35}x^3\sqrt{3+5x^2+x^4} + \frac{1}{21}x^5\left(11+7x^2\right)\sqrt{3+5x^2+x^4} + \frac{962\sqrt{\frac{2}{3}\left(5+\sqrt{13}\right)}\sqrt{\frac{6+\left(5-\sqrt{13}\right)x^2}{6+\left(5+\sqrt{13}\right)x^2}}\left(6+\left(5+\sqrt{13}\right)x^2\right)E\left(\tan^{-1}\left(\sqrt{\frac{1}{6}\left(5+\sqrt{13}\right)}x\right)|\frac{1}{6}\left(-13+5\sqrt{\ }\right.\right.}{105\sqrt{3+5x^2+x^4}} - \frac{13\sqrt{\frac{6+\left(5-\sqrt{13}\right)x^2}{6+\left(5+\sqrt{13}\right)x^2}}\left(6+\left(5+\sqrt{13}\right)x^2\right)F\left(\tan^{-1}\left(\sqrt{\frac{1}{6}\left(5+\sqrt{13}\right)}x\right)|\frac{1}{6}\left(-13+5\sqrt{13}\right)\right)}{\sqrt{6\left(5+\sqrt{13}\right)}\sqrt{3+5x^2+x^4}}$$

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 6.06 (sec) , antiderivative size = 237, normalized size of antiderivative = 0.74

$$\int x^4\left(2+3x^2\right)\sqrt{3+5x^2+x^4}\,dx = \frac{2730x+4082x^3+460x^5+604x^7+460x^9+70x^{11}-1924i\sqrt{2}\left(-5+\sqrt{13}\right)\sqrt{\frac{-5+\sqrt{13}-2x^2}{-5+\sqrt{13}}}\sqrt{5+\sqrt{13}+2x}}{}$$

```
[In] Integrate[x^4*(2 + 3*x^2)*Sqrt[3 + 5*x^2 + x^4],x]
```

```
[Out] (2730*x + 4082*x^3 + 460*x^5 + 604*x^7 + 460*x^9 + 70*x^11 - (1924*I)*Sqrt[
2]*(-5 + Sqrt[13])*Sqrt[(-5 + Sqrt[13] - 2*x^2)/(-5 + Sqrt[13])]*Sqrt[5 + S
qrt[13] + 2*x^2]*EllipticE[I*ArcSinh[Sqrt[2/(5 + Sqrt[13])]*x], 19/6 + (5*S
qrt[13])/6] + (13*I)*Sqrt[2]*(-635 + 148*Sqrt[13])*Sqrt[(-5 + Sqrt[13] - 2*
x^2)/(-5 + Sqrt[13])]*Sqrt[5 + Sqrt[13] + 2*x^2]*EllipticF[I*ArcSinh[Sqrt[2
/(5 + Sqrt[13])]*x], 19/6 + (5*Sqrt[13])/6])/(210*Sqrt[3 + 5*x^2 + x^4])
```

Maple [A] (verified)

Time = 5.58 (sec) , antiderivative size = 226, normalized size of antiderivative = 0.70

method	result
risch	$\frac{x(35x^6+55x^4-78x^2+455)\sqrt{x^4+5x^2+3}}{105} - \frac{78\sqrt{1-\left(-\frac{5}{6}+\frac{\sqrt{13}}{6}\right)x^2}\,\sqrt{1-\left(-\frac{5}{6}-\frac{\sqrt{13}}{6}\right)x^2}\,F\left(\frac{x\sqrt{-30+6\sqrt{13}}}{6}, \frac{5\sqrt{3}}{6}+\frac{\sqrt{39}}{6}\right)}{\sqrt{-30+6\sqrt{13}}\,\sqrt{x^4+5x^2+3}} + \frac{46176\sqrt{1-}}{}$
default	$\frac{x^7\sqrt{x^4+5x^2+3}}{3} + \frac{11x^5\sqrt{x^4+5x^2+3}}{21} - \frac{26x^3\sqrt{x^4+5x^2+3}}{35} + \frac{13x\sqrt{x^4+5x^2+3}}{3} - \frac{78\sqrt{1-\left(-\frac{5}{6}+\frac{\sqrt{13}}{6}\right)x^2}\,\sqrt{1-\left(-\frac{5}{6}-\frac{\sqrt{13}}{6}\right)x^2}\,F\left(\right.}{\sqrt{-30+6\sqrt{13}}\,\sqrt{x^4+5x^2}}$
elliptic	$\frac{x^7\sqrt{x^4+5x^2+3}}{3} + \frac{11x^5\sqrt{x^4+5x^2+3}}{21} - \frac{26x^3\sqrt{x^4+5x^2+3}}{35} + \frac{13x\sqrt{x^4+5x^2+3}}{3} - \frac{78\sqrt{1-\left(-\frac{5}{6}+\frac{\sqrt{13}}{6}\right)x^2}\,\sqrt{1-\left(-\frac{5}{6}-\frac{\sqrt{13}}{6}\right)x^2}\,F\left(\right.}{\sqrt{-30+6\sqrt{13}}\,\sqrt{x^4+5x^2}}$

```
[In] int(x^4*(3*x^2+2)*(x^4+5*x^2+3)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/105*x*(35*x^6+55*x^4-78*x^2+455)*(x^4+5*x^2+3)^(1/2)-78/(-30+6*13^(1/2))^
(1/2)*(1-(-5/6+1/6*13^(1/2))*x^2)^(1/2)*(1-(-5/6-1/6*13^(1/2))*x^2)^(1/2)/(
x^4+5*x^2+3)^(1/2)*EllipticF(1/6*x*(-30+6*13^(1/2))^(1/2),5/6*3^(1/2)+1/6*3
9^(1/2))+46176/35/(-30+6*13^(1/2))^(1/2)*(1-(-5/6+1/6*13^(1/2))*x^2)^(1/2)*
(1-(-5/6-1/6*13^(1/2))*x^2)^(1/2)/(x^4+5*x^2+3)^(1/2)/(5+13^(1/2))*(Ellipti
cF(1/6*x*(-30+6*13^(1/2))^(1/2),5/6*3^(1/2)+1/6*39^(1/2))-EllipticE(1/6*x*(
-30+6*13^(1/2))^(1/2),5/6*3^(1/2)+1/6*39^(1/2)))
```

Fricas [A] (verification not implemented)

none

Time = 0.09 (sec) , antiderivative size = 139, normalized size of antiderivative = 0.43

$$\int x^4(2+3x^2)\sqrt{3+5x^2+x^4}\,dx = -\frac{3848\left(\sqrt{13}\sqrt{2}x-5\sqrt{2}x\right)\sqrt{\sqrt{13}-5}E(\arcsin\left(\frac{\sqrt{2}\sqrt{\sqrt{13}-5}}{2x}\right) \mid \frac{5}{6}\sqrt{13}+\frac{19}{6}) - 13\left(261\sqrt{13}\sqrt{2}x - 1655\sqrt{2}x\right.}{4}$$

```
[In] integrate(x^4*(3*x^2+2)*(x^4+5*x^2+3)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/420*(3848*(sqrt(13)*sqrt(2)*x - 5*sqrt(2)*x)*sqrt(sqrt(13) - 5)*elliptic
_e(arcsin(1/2*sqrt(2)*sqrt(sqrt(13) - 5)/x), 5/6*sqrt(13) + 19/6) - 13*(261
*sqrt(13)*sqrt(2)*x - 1655*sqrt(2)*x)*sqrt(sqrt(13) - 5)*elliptic_f(arcsin(
1/2*sqrt(2)*sqrt(sqrt(13) - 5)/x), 5/6*sqrt(13) + 19/6) - 4*(35*x^8 + 55*x^
6 - 78*x^4 + 455*x^2 - 3848)*sqrt(x^4 + 5*x^2 + 3))/x
```

Sympy [F]

$$\int x^4(2+3x^2)\sqrt{3+5x^2+x^4}\,dx = \int x^4\cdot(3x^2+2)\sqrt{x^4+5x^2+3}\,dx$$

```
[In] integrate(x**4*(3*x**2+2)*(x**4+5*x**2+3)**(1/2),x)
[Out] Integral(x**4*(3*x**2 + 2)*sqrt(x**4 + 5*x**2 + 3), x)
```

Maxima [F]

$$\int x^4(2+3x^2)\sqrt{3+5x^2+x^4}\,dx = \int \sqrt{x^4+5\,x^2+3}(3\,x^2+2)x^4\,dx$$

```
[In] integrate(x^4*(3*x^2+2)*(x^4+5*x^2+3)^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(x^4 + 5*x^2 + 3)*(3*x^2 + 2)*x^4, x)
```

Giac [F]

$$\int x^4(2+3x^2)\sqrt{3+5x^2+x^4}\,dx = \int \sqrt{x^4+5\,x^2+3}(3\,x^2+2)x^4\,dx$$

```
[In] integrate(x^4*(3*x^2+2)*(x^4+5*x^2+3)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(x^4 + 5*x^2 + 3)*(3*x^2 + 2)*x^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^4(2+3x^2)\sqrt{3+5x^2+x^4}\,dx = \int x^4\,\left(3\,x^2+2\right)\,\sqrt{x^4+5\,x^2+3}\,dx$$

```
[In] int(x^4*(3*x^2 + 2)*(5*x^2 + x^4 + 3)^(1/2),x)
[Out] int(x^4*(3*x^2 + 2)*(5*x^2 + x^4 + 3)^(1/2), x)
```

3.152 $\int x^2(2+3x^2)\sqrt{3+5x^2+x^4}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 305

$$\int x^2(2+3x^2)\sqrt{3+5x^2+x^4}\,dx$$
$$= \frac{1247x(5+\sqrt{13}+2x^2)}{210\sqrt{3+5x^2+x^4}} - \frac{4}{3}x\sqrt{3+5x^2+x^4} + \frac{1}{35}x^3(29+15x^2)\sqrt{3+5x^2+x^4}$$
$$- \frac{1247\sqrt{\frac{1}{6}(5+\sqrt{13})}\sqrt{\frac{6+(5-\sqrt{13})x^2}{6+(5+\sqrt{13})x^2}}(6+(5+\sqrt{13})x^2)\,E\left(\arctan\left(\sqrt{\frac{1}{6}(5+\sqrt{13})}x\right)|\frac{1}{6}(-13+5\sqrt{13})\right)}{210\sqrt{3+5x^2+x^4}}$$
$$+ \frac{2\sqrt{\frac{2}{3(5+\sqrt{13})}}\sqrt{\frac{6+(5-\sqrt{13})x^2}{6+(5+\sqrt{13})x^2}}(6+(5+\sqrt{13})x^2)\,\text{EllipticF}\left(\arctan\left(\sqrt{\frac{1}{6}(5+\sqrt{13})}x\right),\frac{1}{6}(-13+5\sqrt{13})\right)}{\sqrt{3+5x^2+x^4}}$$

```
[Out] 1247/210*x*(5+2*x^2+13^(1/2))/(x^4+5*x^2+3)^(1/2)-4/3*x*(x^4+5*x^2+3)^(1/2)
+1/35*x^3*(15*x^2+29)*(x^4+5*x^2+3)^(1/2)+2/3*(1/(36+x^2*(30+6*13^(1/2))))^
(1/2)*(36+x^2*(30+6*13^(1/2)))^(1/2)*EllipticF(x*(30+6*13^(1/2))^(1/2)/(36+
x^2*(30+6*13^(1/2)))^(1/2),1/6*(-78+30*13^(1/2))^(1/2))*(6+x^2*(5+13^(1/2))
)*6^(1/2)/(5+13^(1/2))^(1/2)*((6+x^2*(5-13^(1/2)))/(6+x^2*(5+13^(1/2))))^(1
/2)/(x^4+5*x^2+3)^(1/2)-1247/1260*(1/(36+x^2*(30+6*13^(1/2))))^(1/2)*(36+x^
2*(30+6*13^(1/2)))^(1/2)*EllipticE(x*(30+6*13^(1/2))^(1/2)/(36+x^2*(30+6*13
^(1/2)))^(1/2),1/6*(-78+30*13^(1/2))^(1/2))*(6+x^2*(5+13^(1/2)))*(30+6*13^(
1/2))^(1/2)*((6+x^2*(5-13^(1/2)))/(6+x^2*(5+13^(1/2))))^(1/2)/(x^4+5*x^2+3)
^(1/2)
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 305, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {1287, 1293, 1203, 1113, 1149}

$$\int x^2(2+3x^2)\sqrt{3+5x^2+x^4}\,dx$$
$$= \frac{2\sqrt{\frac{2}{3(5+\sqrt{13})}}\sqrt{\frac{(5-\sqrt{13})x^2+6}{(5+\sqrt{13})x^2+6}}((5+\sqrt{13})\,x^2+6)\,\text{EllipticF}\left(\arctan\left(\sqrt{\frac{1}{6}\,(5+\sqrt{13})}x\right),\frac{1}{6}(-13+5\sqrt{13})\right)}{\sqrt{x^4+5x^2+3}}$$
$$-\frac{1247\sqrt{\frac{1}{6}\,(5+\sqrt{13})}\sqrt{\frac{(5-\sqrt{13})x^2+6}{(5+\sqrt{13})x^2+6}}((5+\sqrt{13})\,x^2+6)\,E\left(\arctan\left(\sqrt{\frac{1}{6}\,(5+\sqrt{13})}x\right)|\frac{1}{6}(-13+5\sqrt{13})\right)}{210\sqrt{x^4+5x^2+3}}$$
$$-\frac{4}{3}\sqrt{x^4+5x^2+3}x+\frac{1247(2x^2+\sqrt{13}+5)\,x}{210\sqrt{x^4+5x^2+3}}+\frac{1}{35}(15x^2+29)\sqrt{x^4+5x^2+3}x^3$$

```
[In] Int[x^2*(2 + 3*x^2)*Sqrt[3 + 5*x^2 + x^4],x]
```

```
[Out] (1247*x*(5 + Sqrt[13] + 2*x^2))/(210*Sqrt[3 + 5*x^2 + x^4]) - (4*x*Sqrt[3 +
 5*x^2 + x^4])/3 + (x^3*(29 + 15*x^2)*Sqrt[3 + 5*x^2 + x^4])/35 - (1247*Sqr
t[(5 + Sqrt[13])/6]*Sqrt[(6 + (5 - Sqrt[13])*x^2)/(6 + (5 + Sqrt[13])*x^2)]
*(6 + (5 + Sqrt[13])*x^2)*EllipticE[ArcTan[Sqrt[(5 + Sqrt[13])/6]*x], (-13
+ 5*Sqrt[13])/6])/(210*Sqrt[3 + 5*x^2 + x^4]) + (2*Sqrt[2/(3*(5 + Sqrt[13])
)]*Sqrt[(6 + (5 - Sqrt[13])*x^2)/(6 + (5 + Sqrt[13])*x^2)]*(6 + (5 + Sqrt[1
3])*x^2)*EllipticF[ArcTan[Sqrt[(5 + Sqrt[13])/6]*x], (-13 + 5*Sqrt[13])/6])
/Sqrt[3 + 5*x^2 + x^4]
```

Rule 1113

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[(2*a + (b + q)*x^2)*(Sqrt[(2*a + (b - q)*x^2)/(2*a +
(b + q)*x^2)]/(2*a*Rt[(b + q)/(2*a), 2]*Sqrt[a + b*x^2 + c*x^4]))*EllipticF
[ArcTan[Rt[(b + q)/(2*a), 2]*x], 2*(q/(b + q))], x] /; PosQ[(b + q)/a] &&
!(PosQ[(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[
{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1149

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[b^2 - 4*a*c, 2]}, Simp[x*((b + q + 2*c*x^2)/(2*c*Sqrt[a + b*x^2 + c*x^4
])), x] - Simp[Rt[(b + q)/(2*a), 2]*(2*a + (b + q)*x^2)*(Sqrt[(2*a + (b - q
)*x^2)/(2*a + (b + q)*x^2)]/(2*c*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[ArcTan
[Rt[(b + q)/(2*a), 2]*x], 2*(q/(b + q))], x] /; PosQ[(b + q)/a] &&  !(PosQ[
(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[{a, b,
```

```
c}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1203

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[d, Int[1/Sqrt[a + b*x^2 + c*x^4],
 x], x] + Dist[e, Int[x^2/Sqrt[a + b*x^2 + c*x^4], x], x] /; PosQ[(b + q)/a
] || PosQ[(b - q)/a]] /; FreeQ[{a, b, c, d, e}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1287

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(a + b*x^2 + c*x^4)^p*((b*e*2
*p + c*d*(m + 4*p + 3) + c*e*(4*p + m + 1)*x^2)/(c*f*(4*p + m + 1)*(m + 4*p
 + 3))), x] + Dist[2*(p/(c*(4*p + m + 1)*(m + 4*p + 3))), Int[(f*x)^m*(a +
b*x^2 + c*x^4)^(p - 1)*Simp[2*a*c*d*(m + 4*p + 3) - a*b*e*(m + 1) + (2*a*c*
e*(4*p + m + 1) + b*c*d*(m + 4*p + 3) - b^2*e*(m + 2*p + 1))*x^2, x], x], x
] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b^2 - 4*a*c, 0] && GtQ[p, 0] &&
 NeQ[4*p + m + 1, 0] && NeQ[m + 4*p + 3, 0] && IntegerQ[2*p] && (IntegerQ[p
] || IntegerQ[m])
```

Rule 1293

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[e*f*(f*x)^(m - 1)*((a + b*x^2 + c*x^4)^(p +
1)/(c*(m + 4*p + 3))), x] - Dist[f^2/(c*(m + 4*p + 3)), Int[(f*x)^(m - 2)*(
a + b*x^2 + c*x^4)^p*Simp[a*e*(m - 1) + (b*e*(m + 2*p + 1) - c*d*(m + 4*p +
 3))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c,
 0] && GtQ[m, 1] && NeQ[m + 4*p + 3, 0] && IntegerQ[2*p] && (IntegerQ[p] ||
 IntegerQ[m])
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{1}{35}x^3\left(29+15x^2\right)\sqrt{3+5x^2+x^4}+\frac{1}{35}\int\frac{x^2\left(-51-140x^2\right)}{\sqrt{3+5x^2+x^4}}\,dx\\
&= -\frac{4}{3}x\sqrt{3+5x^2+x^4}+\frac{1}{35}x^3\left(29+15x^2\right)\sqrt{3+5x^2+x^4}-\frac{1}{105}\int\frac{-420-1247x^2}{\sqrt{3+5x^2+x^4}}\,dx\\
&= -\frac{4}{3}x\sqrt{3+5x^2+x^4}+\frac{1}{35}x^3\left(29+15x^2\right)\sqrt{3+5x^2+x^4}\\
&\quad+4\int\frac{1}{\sqrt{3+5x^2+x^4}}\,dx+\frac{1247}{105}\int\frac{x^2}{\sqrt{3+5x^2+x^4}}\,dx
\end{aligned}
$$

$$= \frac{1247x(5+\sqrt{13}+2x^2)}{210\sqrt{3+5x^2+x^4}} - \frac{4}{3}x\sqrt{3+5x^2+x^4} + \frac{1}{35}x^3(29+15x^2)\sqrt{3+5x^2+x^4}$$

$$- \frac{1247\sqrt{\frac{1}{6}(5+\sqrt{13})}\sqrt{\frac{6+(5-\sqrt{13})x^2}{6+(5+\sqrt{13})x^2}}(6+(5+\sqrt{13})x^2)E\left(\tan^{-1}\left(\sqrt{\frac{1}{6}(5+\sqrt{13})}x\right)|\frac{1}{6}(-13+5\right.}{210\sqrt{3+5x^2+x^4}}$$

$$+ \frac{2\sqrt{\frac{2}{3(5+\sqrt{13})}}\sqrt{\frac{6+(5-\sqrt{13})x^2}{6+(5+\sqrt{13})x^2}}(6+(5+\sqrt{13})x^2)F\left(\tan^{-1}\left(\sqrt{\frac{1}{6}(5+\sqrt{13})}x\right)|\frac{1}{6}(-13+5\sqrt{13})\right)}{\sqrt{3+5x^2+x^4}}$$

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 5.09 (sec) , antiderivative size = 234, normalized size of antiderivative = 0.77

$$\int x^2(2+3x^2)\sqrt{3+5x^2+x^4}\,dx$$

$$= \frac{4x(-420-439x^2+430x^4+312x^6+45x^8)+1247i\sqrt{2}(-5+\sqrt{13})\sqrt{\frac{-5+\sqrt{13}-2x^2}{-5+\sqrt{13}}}\sqrt{5+\sqrt{13}+2x^2}E(ia}{}$$

```
[In] Integrate[x^2*(2 + 3*x^2)*Sqrt[3 + 5*x^2 + x^4],x]
[Out] (4*x*(-420 - 439*x^2 + 430*x^4 + 312*x^6 + 45*x^8) + (1247*I)*Sqrt[2]*(-5 +
 Sqrt[13])*Sqrt[(-5 + Sqrt[13] - 2*x^2)/(-5 + Sqrt[13])]*Sqrt[5 + Sqrt[13]
+ 2*x^2]*EllipticE[I*ArcSinh[Sqrt[2/(5 + Sqrt[13])]*x], 19/6 + (5*Sqrt[13])
/6] - I*Sqrt[2]*(-5395 + 1247*Sqrt[13])*Sqrt[(-5 + Sqrt[13] - 2*x^2)/(-5 +
Sqrt[13])]*Sqrt[5 + Sqrt[13] + 2*x^2]*EllipticF[I*ArcSinh[Sqrt[2/(5 + Sqrt[
13])]*x], 19/6 + (5*Sqrt[13])/6])/(420*Sqrt[3 + 5*x^2 + x^4])
```

Maple [A] (verified)

Time = 2.00 (sec) , antiderivative size = 221, normalized size of antiderivative = 0.72

method	result
risch	$\frac{x(45x^4+87x^2-140)\sqrt{x^4+5x^2+3}}{105} + \frac{24\sqrt{1-\left(-\frac{5}{6}+\frac{\sqrt{13}}{6}\right)x^2}\sqrt{1-\left(-\frac{5}{6}-\frac{\sqrt{13}}{6}\right)x^2}F\left(\frac{x\sqrt{-30+6\sqrt{13}}}{6},\frac{5\sqrt{3}}{6}+\frac{\sqrt{39}}{6}\right)}{\sqrt{-30+6\sqrt{13}}\sqrt{x^4+5x^2+3}} - \frac{14964\sqrt{1-\left(-\frac{5}{6}\right.}}{}$
default	$\frac{3x^5\sqrt{x^4+5x^2+3}}{7} + \frac{29x^3\sqrt{x^4+5x^2+3}}{35} - \frac{4x\sqrt{x^4+5x^2+3}}{3} + \frac{24\sqrt{1-\left(-\frac{5}{6}+\frac{\sqrt{13}}{6}\right)x^2}\sqrt{1-\left(-\frac{5}{6}-\frac{\sqrt{13}}{6}\right)x^2}F\left(\frac{x\sqrt{-30+6\sqrt{13}}}{6},\frac{5\sqrt{3}}{6}+\right.}{\sqrt{-30+6\sqrt{13}}\sqrt{x^4+5x^2+3}}$
elliptic	$\frac{3x^5\sqrt{x^4+5x^2+3}}{7} + \frac{29x^3\sqrt{x^4+5x^2+3}}{35} - \frac{4x\sqrt{x^4+5x^2+3}}{3} + \frac{24\sqrt{1-\left(-\frac{5}{6}+\frac{\sqrt{13}}{6}\right)x^2}\sqrt{1-\left(-\frac{5}{6}-\frac{\sqrt{13}}{6}\right)x^2}F\left(\frac{x\sqrt{-30+6\sqrt{13}}}{6},\frac{5\sqrt{3}}{6}+\right.}{\sqrt{-30+6\sqrt{13}}\sqrt{x^4+5x^2+3}}$

```
[In] int(x^2*(3*x^2+2)*(x^4+5*x^2+3)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/105*x*(45*x^4+87*x^2-140)*(x^4+5*x^2+3)^(1/2)+24/(-30+6*13^(1/2))^(1/2)*(
1-(-5/6+1/6*13^(1/2))*x^2)^(1/2)*(1-(-5/6-1/6*13^(1/2))*x^2)^(1/2)/(x^4+5*x
^2+3)^(1/2)*EllipticF(1/6*x*(-30+6*13^(1/2))^(1/2),5/6*3^(1/2)+1/6*39^(1/2)
)-14964/35/(-30+6*13^(1/2))^(1/2)*(1-(-5/6+1/6*13^(1/2))*x^2)^(1/2)*(1-(-5/
6-1/6*13^(1/2))*x^2)^(1/2)/(x^4+5*x^2+3)^(1/2)/(5+13^(1/2))*(EllipticF(1/6*
x*(-30+6*13^(1/2))^(1/2),5/6*3^(1/2)+1/6*39^(1/2))-EllipticE(1/6*x*(-30+6*1
3^(1/2))^(1/2),5/6*3^(1/2)+1/6*39^(1/2)))
```

Fricas [A] (verification not implemented)

none

Time = 0.10 (sec) , antiderivative size = 134, normalized size of antiderivative = 0.44

$$\int x^2(2+3x^2)\sqrt{3+5x^2+x^4}\,dx = \frac{1247\left(\sqrt{13}\sqrt{2}x-5\sqrt{2}x\right)\sqrt{\sqrt{13}-5}E(\arcsin\left(\frac{\sqrt{2}\sqrt{\sqrt{13}-5}}{2x}\right) \mid \frac{5}{6}\sqrt{13}+\frac{19}{6}) - \left(1107\sqrt{13}\sqrt{2}x-6935\sqrt{2}x\right)\sqrt{}}{420\,x}$$

```
[In] integrate(x^2*(3*x^2+2)*(x^4+5*x^2+3)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/420*(1247*(sqrt(13)*sqrt(2)*x - 5*sqrt(2)*x)*sqrt(sqrt(13) - 5)*elliptic_
e(arcsin(1/2*sqrt(2)*sqrt(sqrt(13) - 5)/x), 5/6*sqrt(13) + 19/6) - (1107*sq
rt(13)*sqrt(2)*x - 6935*sqrt(2)*x)*sqrt(sqrt(13) - 5)*elliptic_f(arcsin(1/2
*sqrt(2)*sqrt(sqrt(13) - 5)/x), 5/6*sqrt(13) + 19/6) + 4*(45*x^6 + 87*x^4 -
 140*x^2 + 1247)*sqrt(x^4 + 5*x^2 + 3))/x
```

Sympy [F]

$$\int x^2(2+3x^2)\sqrt{3+5x^2+x^4}\,dx = \int x^2\cdot\left(3x^2+2\right)\sqrt{x^4+5x^2+3}\,dx$$

```
[In] integrate(x**2*(3*x**2+2)*(x**4+5*x**2+3)**(1/2),x)
```

```
[Out] Integral(x**2*(3*x**2 + 2)*sqrt(x**4 + 5*x**2 + 3), x)
```

Maxima [F]

$$\int x^2(2+3x^2)\sqrt{3+5x^2+x^4}\,dx = \int \sqrt{x^4+5\,x^2+3}(3\,x^2+2)x^2\,dx$$

```
[In] integrate(x^2*(3*x^2+2)*(x^4+5*x^2+3)^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(x^4 + 5*x^2 + 3)*(3*x^2 + 2)*x^2, x)
```

Giac [F]

$$\int x^2(2+3x^2)\sqrt{3+5x^2+x^4}\,dx = \int \sqrt{x^4+5\,x^2+3}(3\,x^2+2)x^2\,dx$$

```
[In] integrate(x^2*(3*x^2+2)*(x^4+5*x^2+3)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(x^4 + 5*x^2 + 3)*(3*x^2 + 2)*x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^2(2+3x^2)\sqrt{3+5x^2+x^4}\,dx = \int x^2\,\left(3\,x^2+2\right)\,\sqrt{x^4+5\,x^2+3}\,dx$$

```
[In] int(x^2*(3*x^2 + 2)*(5*x^2 + x^4 + 3)^(1/2),x)
[Out] int(x^2*(3*x^2 + 2)*(5*x^2 + x^4 + 3)^(1/2), x)
```

3.153 $\int (2+3x^2)\sqrt{3+5x^2+x^4}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 279

$$\int (2+3x^2)\sqrt{3+5x^2+x^4}\,dx = -\frac{23x\left(5+\sqrt{13}+2x^2\right)}{15\sqrt{3+5x^2+x^4}} + \frac{1}{15}x(25+9x^2)\sqrt{3+5x^2+x^4}$$
$$+ \frac{23\sqrt{\frac{1}{6}\left(5+\sqrt{13}\right)}\sqrt{\frac{6+\left(5-\sqrt{13}\right)x^2}{6+\left(5+\sqrt{13}\right)x^2}}\left(6+\left(5+\sqrt{13}\right)x^2\right)E\left(\arctan\left(\sqrt{\frac{1}{6}\left(5+\sqrt{13}\right)}x\right)|\frac{1}{6}\left(-13+5\sqrt{13}\right)\right)}{15\sqrt{3+5x^2+x^4}}$$
$$+ \frac{\sqrt{\frac{6+\left(5-\sqrt{13}\right)x^2}{6+\left(5+\sqrt{13}\right)x^2}}\left(6+\left(5+\sqrt{13}\right)x^2\right)\operatorname{EllipticF}\left(\arctan\left(\sqrt{\frac{1}{6}\left(5+\sqrt{13}\right)}x\right),\frac{1}{6}\left(-13+5\sqrt{13}\right)\right)}{\sqrt{6\left(5+\sqrt{13}\right)}\sqrt{3+5x^2+x^4}}$$

```
[Out] -23/15*x*(5+2*x^2+13^(1/2))/(x^4+5*x^2+3)^(1/2)+1/15*x*(9*x^2+25)*(x^4+5*x^
2+3)^(1/2)+23/90*(1/(36+x^2*(30+6*13^(1/2))))^(1/2)*(36+x^2*(30+6*13^(1/2))
)^(1/2)*EllipticE(x*(30+6*13^(1/2))^(1/2)/(36+x^2*(30+6*13^(1/2)))^(1/2),1/
6*(-78+30*13^(1/2))^(1/2))*(6+x^2*(5+13^(1/2)))*(30+6*13^(1/2))^(1/2)*((6+x
^2*(5-13^(1/2)))/(6+x^2*(5+13^(1/2))))^(1/2)/(x^4+5*x^2+3)^(1/2)+(1/(36+x^2
*(30+6*13^(1/2))))^(1/2)*(36+x^2*(30+6*13^(1/2)))^(1/2)*EllipticF(x*(30+6*1
3^(1/2))^(1/2)/(36+x^2*(30+6*13^(1/2)))^(1/2),1/6*(-78+30*13^(1/2))^(1/2))*
(6+x^2*(5+13^(1/2)))*((6+x^2*(5-13^(1/2)))/(6+x^2*(5+13^(1/2))))^(1/2)/(x^4
+5*x^2+3)^(1/2)/(30+6*13^(1/2))^(1/2)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 279, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {1190, 1203, 1113, 1149}

$$\int \left(2+3x^2\right)\sqrt{3+5x^2+x^4}\,dx$$
$$=\frac{\sqrt{\frac{\left(5-\sqrt{13}\right)x^2+6}{\left(5+\sqrt{13}\right)x^2+6}}\left(\left(5+\sqrt{13}\right)x^2+6\right)\text{EllipticF}\left(\arctan\left(\sqrt{\frac{1}{6}\left(5+\sqrt{13}\right)}x\right),\frac{1}{6}\left(-13+5\sqrt{13}\right)\right)}{\sqrt{6\left(5+\sqrt{13}\right)}\sqrt{x^4+5x^2+3}}$$
$$+\frac{23\sqrt{\frac{1}{6}\left(5+\sqrt{13}\right)}\sqrt{\frac{\left(5-\sqrt{13}\right)x^2+6}{\left(5+\sqrt{13}\right)x^2+6}}\left(\left(5+\sqrt{13}\right)x^2+6\right)E\left(\arctan\left(\sqrt{\frac{1}{6}\left(5+\sqrt{13}\right)}x\right)|\frac{1}{6}\left(-13+5\sqrt{13}\right)\right)}{15\sqrt{x^4+5x^2+3}}$$
$$-\frac{23x\left(2x^2+\sqrt{13}+5\right)}{15\sqrt{x^4+5x^2+3}}+\frac{1}{15}x\left(9x^2+25\right)\sqrt{x^4+5x^2+3}$$

```
[In] Int[(2 + 3*x^2)*Sqrt[3 + 5*x^2 + x^4],x]
```

```
[Out] (-23*x*(5 + Sqrt[13] + 2*x^2))/(15*Sqrt[3 + 5*x^2 + x^4]) + (x*(25 + 9*x^2)
*Sqrt[3 + 5*x^2 + x^4])/15 + (23*Sqrt[(5 + Sqrt[13])/6]*Sqrt[(6 + (5 - Sqrt
[13])*x^2)/(6 + (5 + Sqrt[13])*x^2)]*(6 + (5 + Sqrt[13])*x^2)*EllipticE[Arc
Tan[Sqrt[(5 + Sqrt[13])/6]*x], (-13 + 5*Sqrt[13])/6])/(15*Sqrt[3 + 5*x^2 +
x^4]) + (Sqrt[(6 + (5 - Sqrt[13])*x^2)/(6 + (5 + Sqrt[13])*x^2)]*(6 + (5 +
Sqrt[13])*x^2)*EllipticF[ArcTan[Sqrt[(5 + Sqrt[13])/6]*x], (-13 + 5*Sqrt[13
])/6])/(Sqrt[6*(5 + Sqrt[13])]*Sqrt[3 + 5*x^2 + x^4])
```

Rule 1113

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[(2*a + (b + q)*x^2)*(Sqrt[(2*a + (b - q)*x^2)/(2*a +
(b + q)*x^2)]/(2*a*Rt[(b + q)/(2*a), 2]*Sqrt[a + b*x^2 + c*x^4]))*EllipticF
[ArcTan[Rt[(b + q)/(2*a), 2]*x], 2*(q/(b + q))], x] /; PosQ[(b + q)/a] &&
!(PosQ[(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[
{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1149

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[b^2 - 4*a*c, 2]}, Simp[x*((b + q + 2*c*x^2)/(2*c*Sqrt[a + b*x^2 + c*x^4
])), x] - Simp[Rt[(b + q)/(2*a), 2]*(2*a + (b + q)*x^2)*(Sqrt[(2*a + (b - q
)*x^2)/(2*a + (b + q)*x^2)]/(2*c*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[ArcTan
[Rt[(b + q)/(2*a), 2]*x], 2*(q/(b + q))], x] /; PosQ[(b + q)/a] &&  !(PosQ[
(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[{a, b,
```

```
c}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1190

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[x*(2*b*e*p + c*d*(4*p + 3) + c*e*(4*p + 1)*x^2)*((a + b*x^2 + c
*x^4)^p/(c*(4*p + 1)*(4*p + 3))), x] + Dist[2*(p/(c*(4*p + 1)*(4*p + 3))),
Int[Simp[2*a*c*d*(4*p + 3) - a*b*e + (2*a*c*e*(4*p + 1) + b*c*d*(4*p + 3) -
 b^2*e*(2*p + 1))*x^2, x]*(a + b*x^2 + c*x^4)^(p - 1), x], x] /; FreeQ[{a,
b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
GtQ[p, 0] && FractionQ[p] && IntegerQ[2*p]
```

Rule 1203

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[d, Int[1/Sqrt[a + b*x^2 + c*x^4],
 x], x] + Dist[e, Int[x^2/Sqrt[a + b*x^2 + c*x^4], x], x] /; PosQ[(b + q)/a
] || PosQ[(b - q)/a]] /; FreeQ[{a, b, c, d, e}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{1}{15}x\left(25+9x^2\right)\sqrt{3+5x^2+x^4}+\frac{1}{15}\int\frac{15-46x^2}{\sqrt{3+5x^2+x^4}}\,dx \\
&= \frac{1}{15}x\left(25+9x^2\right)\sqrt{3+5x^2+x^4}-\frac{46}{15}\int\frac{x^2}{\sqrt{3+5x^2+x^4}}\,dx+\int\frac{1}{\sqrt{3+5x^2+x^4}}\,dx \\
&= -\frac{23x\left(5+\sqrt{13}+2x^2\right)}{15\sqrt{3+5x^2+x^4}}+\frac{1}{15}x\left(25+9x^2\right)\sqrt{3+5x^2+x^4} \\
&\quad+\frac{23\sqrt{\frac{1}{6}\left(5+\sqrt{13}\right)}\sqrt{\frac{6+\left(5-\sqrt{13}\right)x^2}{6+\left(5+\sqrt{13}\right)x^2}}\left(6+\left(5+\sqrt{13}\right)x^2\right)E\left(\tan^{-1}\left(\sqrt{\frac{1}{6}\left(5+\sqrt{13}\right)}x\right)|\frac{1}{6}\left(-13+5\sqrt{13}\right)\right)}{15\sqrt{3+5x^2+x^4}} \\
&\quad+\frac{\sqrt{\frac{6+\left(5-\sqrt{13}\right)x^2}{6+\left(5+\sqrt{13}\right)x^2}}\left(6+\left(5+\sqrt{13}\right)x^2\right)F\left(\tan^{-1}\left(\sqrt{\frac{1}{6}\left(5+\sqrt{13}\right)}x\right)|\frac{1}{6}\left(-13+5\sqrt{13}\right)\right)}{\sqrt{6\left(5+\sqrt{13}\right)}\sqrt{3+5x^2+x^4}}
\end{aligned}
$$

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 5.06 (sec) , antiderivative size = 229, normalized size of antiderivative = 0.82

$$\int (2+3x^2)\sqrt{3+5x^2+x^4}\,dx$$
$$= \frac{2x(75+152x^2+70x^4+9x^6)-23i\sqrt{2}(-5+\sqrt{13})\sqrt{\frac{-5+\sqrt{13}-2x^2}{-5+\sqrt{13}}}\sqrt{5+\sqrt{13}+2x^2}E\left(i\operatorname{arcsinh}\left(\sqrt{\frac{2}{5+\sqrt{13}}}x\right.\right.}{30\sqrt{}}$$

```
[In] Integrate[(2 + 3*x^2)*Sqrt[3 + 5*x^2 + x^4],x]
```

```
[Out] (2*x*(75 + 152*x^2 + 70*x^4 + 9*x^6) - (23*I)*Sqrt[2]*(-5 + Sqrt[13])*Sqrt[
(-5 + Sqrt[13] - 2*x^2)/(-5 + Sqrt[13])]*Sqrt[5 + Sqrt[13] + 2*x^2]*Ellipti
cE[I*ArcSinh[Sqrt[2/(5 + Sqrt[13])]*x], 19/6 + (5*Sqrt[13])/6] + I*Sqrt[2]*
(-130 + 23*Sqrt[13])*Sqrt[(-5 + Sqrt[13] - 2*x^2)/(-5 + Sqrt[13])]*Sqrt[5 +
 Sqrt[13] + 2*x^2]*EllipticF[I*ArcSinh[Sqrt[2/(5 + Sqrt[13])]*x], 19/6 + (5
*Sqrt[13])/6])/(30*Sqrt[3 + 5*x^2 + x^4])
```

Maple [A] (verified)

Time = 1.74 (sec) , antiderivative size = 216, normalized size of antiderivative = 0.77

method	result
risch	$\frac{x(9x^2+25)\sqrt{x^4+5x^2+3}}{15}+\frac{6\sqrt{1-\left(-\frac{5}{6}+\frac{\sqrt{13}}{6}\right)x^2}\sqrt{1-\left(-\frac{5}{6}-\frac{\sqrt{13}}{6}\right)x^2}F\left(\frac{x\sqrt{-30+6\sqrt{13}}}{6},\frac{5\sqrt{3}}{6}+\frac{\sqrt{39}}{6}\right)}{\sqrt{-30+6\sqrt{13}}\sqrt{x^4+5x^2+3}}+\frac{552\sqrt{1-\left(-\frac{5}{6}+\frac{\sqrt{13}}{6}\right)x^2}}{}$
default	$\frac{5x\sqrt{x^4+5x^2+3}}{3}+\frac{6\sqrt{1-\left(-\frac{5}{6}+\frac{\sqrt{13}}{6}\right)x^2}\sqrt{1-\left(-\frac{5}{6}-\frac{\sqrt{13}}{6}\right)x^2}F\left(\frac{x\sqrt{-30+6\sqrt{13}}}{6},\frac{5\sqrt{3}}{6}+\frac{\sqrt{39}}{6}\right)}{\sqrt{-30+6\sqrt{13}}\sqrt{x^4+5x^2+3}}+\frac{552\sqrt{1-\left(-\frac{5}{6}+\frac{\sqrt{13}}{6}\right)x^2}\sqrt{1-\left(-\frac{5}{6}\right.}}{}$
elliptic	$\frac{5x\sqrt{x^4+5x^2+3}}{3}+\frac{6\sqrt{1-\left(-\frac{5}{6}+\frac{\sqrt{13}}{6}\right)x^2}\sqrt{1-\left(-\frac{5}{6}-\frac{\sqrt{13}}{6}\right)x^2}F\left(\frac{x\sqrt{-30+6\sqrt{13}}}{6},\frac{5\sqrt{3}}{6}+\frac{\sqrt{39}}{6}\right)}{\sqrt{-30+6\sqrt{13}}\sqrt{x^4+5x^2+3}}+\frac{552\sqrt{1-\left(-\frac{5}{6}+\frac{\sqrt{13}}{6}\right)x^2}\sqrt{1-\left(-\frac{5}{6}\right.}}{}$

```
[In] int((3*x^2+2)*(x^4+5*x^2+3)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/15*x*(9*x^2+25)*(x^4+5*x^2+3)^(1/2)+6/(-30+6*13^(1/2))^(1/2)*(1-(-5/6+1/6
*13^(1/2))*x^2)^(1/2)*(1-(-5/6-1/6*13^(1/2))*x^2)^(1/2)/(x^4+5*x^2+3)^(1/2)
*EllipticF(1/6*x*(-30+6*13^(1/2))^(1/2),5/6*3^(1/2)+1/6*39^(1/2))+552/5/(-3
0+6*13^(1/2))^(1/2)*(1-(-5/6+1/6*13^(1/2))*x^2)^(1/2)*(1-(-5/6-1/6*13^(1/2)
)*x^2)^(1/2)/(x^4+5*x^2+3)^(1/2)/(5+13^(1/2))*(EllipticF(1/6*x*(-30+6*13^(1
/2))^(1/2),5/6*3^(1/2)+1/6*39^(1/2))-EllipticE(1/6*x*(-30+6*13^(1/2))^(1/2)
,5/6*3^(1/2)+1/6*39^(1/2)))
```

Fricas [A] (verification not implemented)

none

Time = 0.09 (sec) , antiderivative size = 129, normalized size of antiderivative = 0.46

$$\int \left(2+3x^2\right)\sqrt{3+5x^2+x^4}\,dx = -\frac{46\left(\sqrt{13}\sqrt{2}x-5\sqrt{2}x\right)\sqrt{\sqrt{13}-5}E\left(\arcsin\left(\frac{\sqrt{2}\sqrt{\sqrt{13}-5}}{2x}\right)\mid\frac{5}{6}\sqrt{13}+\frac{19}{6}\right)-\left(51\sqrt{13}\sqrt{2}x-205\sqrt{2}x\right)\sqrt{\sqrt{13}}}{60x}$$

```
[In] integrate((3*x^2+2)*(x^4+5*x^2+3)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/60*(46*(sqrt(13)*sqrt(2)*x - 5*sqrt(2)*x)*sqrt(sqrt(13) - 5)*elliptic_e(
arcsin(1/2*sqrt(2)*sqrt(sqrt(13) - 5)/x), 5/6*sqrt(13) + 19/6) - (51*sqrt(1
3)*sqrt(2)*x - 205*sqrt(2)*x)*sqrt(sqrt(13) - 5)*elliptic_f(arcsin(1/2*sqrt
(2)*sqrt(sqrt(13) - 5)/x), 5/6*sqrt(13) + 19/6) - 4*(9*x^4 + 25*x^2 - 46)*s
qrt(x^4 + 5*x^2 + 3))/x
```

Sympy [F]

$$\int \left(2+3x^2\right)\sqrt{3+5x^2+x^4}\,dx = \int \left(3x^2+2\right)\sqrt{x^4+5x^2+3}\,dx$$

```
[In] integrate((3*x**2+2)*(x**4+5*x**2+3)**(1/2),x)
```

```
[Out] Integral((3*x**2 + 2)*sqrt(x**4 + 5*x**2 + 3), x)
```

Maxima [F]

$$\int \left(2+3x^2\right)\sqrt{3+5x^2+x^4}\,dx = \int \sqrt{x^4+5\,x^2+3}\left(3\,x^2+2\right)dx$$

```
[In] integrate((3*x^2+2)*(x^4+5*x^2+3)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(x^4 + 5*x^2 + 3)*(3*x^2 + 2), x)
```

Giac [F]

$$\int (2+3x^2)\sqrt{3+5x^2+x^4}\,dx = \int \sqrt{x^4+5\,x^2+3}(3\,x^2+2)\,dx$$

```
[In] integrate((3*x^2+2)*(x^4+5*x^2+3)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(x^4 + 5*x^2 + 3)*(3*x^2 + 2), x)
```

Mupad [F(-1)]

Timed out.

$$\int (2+3x^2)\sqrt{3+5x^2+x^4}\,dx = \int \left(3\,x^2+2\right)\,\sqrt{x^4+5\,x^2+3}\,dx$$

```
[In] int((3*x^2 + 2)*(5*x^2 + x^4 + 3)^(1/2),x)
[Out] int((3*x^2 + 2)*(5*x^2 + x^4 + 3)^(1/2), x)
```

3.154 $\int \frac{(2+3x^2)\sqrt{3+5x^2+x^4}}{x^2}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 284

$$\int \frac{(2+3x^2)\sqrt{3+5x^2+x^4}}{x^2}\,dx = \frac{9x\left(5+\sqrt{13}+2x^2\right)}{2\sqrt{3+5x^2+x^4}} - \frac{(2-x^2)\sqrt{3+5x^2+x^4}}{x} - \frac{3\sqrt{\frac{3}{2}\left(5+\sqrt{13}\right)}\sqrt{\frac{6+\left(5-\sqrt{13}\right)x^2}{6+\left(5+\sqrt{13}\right)x^2}}\left(6+\left(5+\sqrt{13}\right)x^2\right)E\left(\arctan\left(\sqrt{\frac{1}{6}\left(5+\sqrt{13}\right)}x\right)\middle|\frac{1}{6}\left(-13+5\sqrt{13}\right)\right)}{2\sqrt{3+5x^2+x^4}} + \frac{8\sqrt{\frac{2}{3\left(5+\sqrt{13}\right)}}\sqrt{\frac{6+\left(5-\sqrt{13}\right)x^2}{6+\left(5+\sqrt{13}\right)x^2}}\left(6+\left(5+\sqrt{13}\right)x^2\right)\text{EllipticF}\left(\arctan\left(\sqrt{\frac{1}{6}\left(5+\sqrt{13}\right)}x\right),\frac{1}{6}\left(-13+5\sqrt{13}\right)\right)}{\sqrt{3+5x^2+x^4}}$$

```
[Out] 9/2*x*(5+2*x^2+13^(1/2))/(x^4+5*x^2+3)^(1/2)-(-x^2+2)*(x^4+5*x^2+3)^(1/2)/x
+8/3*(1/(36+x^2*(30+6*13^(1/2))))^(1/2)*(36+x^2*(30+6*13^(1/2)))^(1/2)*Elli
pticF(x*(30+6*13^(1/2))^(1/2)/(36+x^2*(30+6*13^(1/2)))^(1/2),1/6*(-78+30*13
^(1/2))^(1/2))*(6+x^2*(5+13^(1/2)))*6^(1/2)/(5+13^(1/2))^(1/2)*((6+x^2*(5-1
3^(1/2)))/(6+x^2*(5+13^(1/2))))^(1/2)/(x^4+5*x^2+3)^(1/2)-3/4*(1/(36+x^2*(3
0+6*13^(1/2))))^(1/2)*(36+x^2*(30+6*13^(1/2)))^(1/2)*EllipticE(x*(30+6*13^(
1/2))^(1/2)/(36+x^2*(30+6*13^(1/2)))^(1/2),1/6*(-78+30*13^(1/2))^(1/2))*(6+
x^2*(5+13^(1/2)))*(30+6*13^(1/2))^(1/2)*((6+x^2*(5-13^(1/2)))/(6+x^2*(5+13^
(1/2))))^(1/2)/(x^4+5*x^2+3)^(1/2)
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 284, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {1285, 1203, 1113, 1149}

$$\int \frac{(2+3x^2)\sqrt{3+5x^2+x^4}}{x^2}\,dx$$
$$= \frac{8\sqrt{\frac{2}{3\left(5+\sqrt{13}\right)}}\sqrt{\frac{\left(5-\sqrt{13}\right)x^2+6}{\left(5+\sqrt{13}\right)x^2+6}}\left(\left(5+\sqrt{13}\right)x^2+6\right)\text{EllipticF}\left(\arctan\left(\sqrt{\frac{1}{6}\left(5+\sqrt{13}\right)}x\right),\frac{1}{6}\left(-13+5\sqrt{13}\right)\right)}{\sqrt{x^4+5x^2+3}}$$
$$-\frac{3\sqrt{\frac{3}{2}\left(5+\sqrt{13}\right)}\sqrt{\frac{\left(5-\sqrt{13}\right)x^2+6}{\left(5+\sqrt{13}\right)x^2+6}}\left(\left(5+\sqrt{13}\right)x^2+6\right)E\left(\arctan\left(\sqrt{\frac{1}{6}\left(5+\sqrt{13}\right)}x\right)|\frac{1}{6}\left(-13+5\sqrt{13}\right)\right)}{2\sqrt{x^4+5x^2+3}}$$
$$-\frac{\sqrt{x^4+5x^2+3}(2-x^2)}{x}+\frac{9x\left(2x^2+\sqrt{13}+5\right)}{2\sqrt{x^4+5x^2+3}}$$

```
[In] Int[((2 + 3*x^2)*Sqrt[3 + 5*x^2 + x^4])/x^2,x]
```

```
[Out] (9*x*(5 + Sqrt[13] + 2*x^2))/(2*Sqrt[3 + 5*x^2 + x^4]) - ((2 - x^2)*Sqrt[3
+ 5*x^2 + x^4])/x - (3*Sqrt[(3*(5 + Sqrt[13]))/2]*Sqrt[(6 + (5 - Sqrt[13])*
x^2)/(6 + (5 + Sqrt[13])*x^2)]*(6 + (5 + Sqrt[13])*x^2)*EllipticE[ArcTan[Sq
rt[(5 + Sqrt[13])/6]*x], (-13 + 5*Sqrt[13])/6])/(2*Sqrt[3 + 5*x^2 + x^4]) +
 (8*Sqrt[2/(3*(5 + Sqrt[13]))]*Sqrt[(6 + (5 - Sqrt[13])*x^2)/(6 + (5 + Sqrt
[13])*x^2)]*(6 + (5 + Sqrt[13])*x^2)*EllipticF[ArcTan[Sqrt[(5 + Sqrt[13])/6
]*x], (-13 + 5*Sqrt[13])/6])/Sqrt[3 + 5*x^2 + x^4]
```

Rule 1113

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[(2*a + (b + q)*x^2)*(Sqrt[(2*a + (b - q)*x^2)/(2*a +
(b + q)*x^2)]/(2*a*Rt[(b + q)/(2*a), 2]*Sqrt[a + b*x^2 + c*x^4]))*EllipticF
[ArcTan[Rt[(b + q)/(2*a), 2]*x], 2*(q/(b + q))], x] /; PosQ[(b + q)/a] &&
!(PosQ[(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[
{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1149

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[b^2 - 4*a*c, 2]}, Simp[x*((b + q + 2*c*x^2)/(2*c*Sqrt[a + b*x^2 + c*x^4
])), x] - Simp[Rt[(b + q)/(2*a), 2]*(2*a + (b + q)*x^2)*(Sqrt[(2*a + (b - q
)*x^2)/(2*a + (b + q)*x^2)]/(2*c*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[ArcTan
[Rt[(b + q)/(2*a), 2]*x], 2*(q/(b + q))], x] /; PosQ[(b + q)/a] &&  !(PosQ[
(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[{a, b,
```

```
c}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1203

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[d, Int[1/Sqrt[a + b*x^2 + c*x^4],
 x], x] + Dist[e, Int[x^2/Sqrt[a + b*x^2 + c*x^4], x], x] /; PosQ[(b + q)/a
] || PosQ[(b - q)/a]] /; FreeQ[{a, b, c, d, e}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1285

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(a + b*x^2 + c*x^4)^p*((d*(m
+ 4*p + 3) + e*(m + 1)*x^2)/(f*(m + 1)*(m + 4*p + 3))), x] + Dist[2*(p/(f^2
*(m + 1)*(m + 4*p + 3))), Int[(f*x)^(m + 2)*(a + b*x^2 + c*x^4)^(p - 1)*Sim
p[2*a*e*(m + 1) - b*d*(m + 4*p + 3) + (b*e*(m + 1) - 2*c*d*(m + 4*p + 3))*x
^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b^2 - 4*a*c, 0] && Gt
Q[p, 0] && LtQ[m, -1] && m + 4*p + 3 != 0 && IntegerQ[2*p] && (IntegerQ[p]
|| IntegerQ[m])
```

Rubi steps

$$\text{integral} = -\frac{(2-x^2)\sqrt{3+5x^2+x^4}}{x} - \frac{1}{3}\int \frac{-48-27x^2}{\sqrt{3+5x^2+x^4}}\,dx$$

$$= -\frac{(2-x^2)\sqrt{3+5x^2+x^4}}{x} + 9\int \frac{x^2}{\sqrt{3+5x^2+x^4}}\,dx + 16\int \frac{1}{\sqrt{3+5x^2+x^4}}\,dx$$

$$= \frac{9x\left(5+\sqrt{13}+2x^2\right)}{2\sqrt{3+5x^2+x^4}} - \frac{(2-x^2)\sqrt{3+5x^2+x^4}}{x} - \frac{3\sqrt{\frac{3}{2}\left(5+\sqrt{13}\right)}\sqrt{\frac{6+\left(5-\sqrt{13}\right)x^2}{6+\left(5+\sqrt{13}\right)x^2}}\left(6+\left(5+\sqrt{13}\right)x^2\right)E\left(\tan^{-1}\left(\sqrt{\frac{1}{6}\left(5+\sqrt{13}\right)}x\right)|\frac{1}{6}\left(-13+5\sqrt{13}\right)\right)}{2\sqrt{3+5x^2+x^4}} + \frac{8\sqrt{\frac{2}{3\left(5+\sqrt{13}\right)}}\sqrt{\frac{6+\left(5-\sqrt{13}\right)x^2}{6+\left(5+\sqrt{13}\right)x^2}}\left(6+\left(5+\sqrt{13}\right)x^2\right)F\left(\tan^{-1}\left(\sqrt{\frac{1}{6}\left(5+\sqrt{13}\right)}x\right)|\frac{1}{6}\left(-13+5\sqrt{13}\right)\right)}{\sqrt{3+5x^2+x^4}}$$

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 4.81 (sec) , antiderivative size = 231, normalized size of antiderivative = 0.81

$$\int \frac{(2+3x^2)\sqrt{3+5x^2+x^4}}{x^2}\,dx = \frac{4(-6-7x^2+3x^4+x^6)+9i\sqrt{2}(-5+\sqrt{13})\,x\sqrt{\frac{-5+\sqrt{13}-2x^2}{-5+\sqrt{13}}}\sqrt{5+\sqrt{13}+2x^2}E\left(i\text{arcsinh}\left(\sqrt{\frac{2}{5+\sqrt{13}}}x\right)|\frac{19}{6}\right.}{4x\sqrt{3+}}$$

```
[In] Integrate[((2 + 3*x^2)*Sqrt[3 + 5*x^2 + x^4])/x^2,x]
```

```
[Out] (4*(-6 - 7*x^2 + 3*x^4 + x^6) + (9*I)*Sqrt[2]*(-5 + Sqrt[13])*x*Sqrt[(-5 +
Sqrt[13] - 2*x^2)/(-5 + Sqrt[13])]*Sqrt[5 + Sqrt[13] + 2*x^2]*EllipticE[I*A
rcSinh[Sqrt[2/(5 + Sqrt[13])]*x], 19/6 + (5*Sqrt[13])/6] - I*Sqrt[2]*(-13 +
 9*Sqrt[13])*x*Sqrt[(-5 + Sqrt[13] - 2*x^2)/(-5 + Sqrt[13])]*Sqrt[5 + Sqrt[
13] + 2*x^2]*EllipticF[I*ArcSinh[Sqrt[2/(5 + Sqrt[13])]*x], 19/6 + (5*Sqrt[
13])/6])/(4*x*Sqrt[3 + 5*x^2 + x^4])
```

Maple [A] (verified)

Time = 1.41 (sec) , antiderivative size = 225, normalized size of antiderivative = 0.79

method	result
default	$x\sqrt{x^4+5x^2+3}+\frac{96\sqrt{1-\left(-\frac{5}{6}+\frac{\sqrt{13}}{6}\right)x^2}\,\sqrt{1-\left(-\frac{5}{6}-\frac{\sqrt{13}}{6}\right)x^2}\,F\left(\frac{x\sqrt{-30+6\sqrt{13}}}{6},\frac{5\sqrt{3}}{6}+\frac{\sqrt{39}}{6}\right)}{\sqrt{-30+6\sqrt{13}}\,\sqrt{x^4+5x^2+3}}-\frac{324\sqrt{1-\left(-\frac{5}{6}+\frac{\sqrt{13}}{6}\right)x^2}\,\sqrt{}}{}$
risch	$\frac{x^6+3x^4-7x^2-6}{x\sqrt{x^4+5x^2+3}}+\frac{96\sqrt{1-\left(-\frac{5}{6}+\frac{\sqrt{13}}{6}\right)x^2}\,\sqrt{1-\left(-\frac{5}{6}-\frac{\sqrt{13}}{6}\right)x^2}\,F\left(\frac{x\sqrt{-30+6\sqrt{13}}}{6},\frac{5\sqrt{3}}{6}+\frac{\sqrt{39}}{6}\right)}{\sqrt{-30+6\sqrt{13}}\,\sqrt{x^4+5x^2+3}}-\frac{324\sqrt{1-\left(-\frac{5}{6}+\frac{\sqrt{13}}{6}\right)x^2}\,\sqrt{1-(}}{}$
elliptic	$x\sqrt{x^4+5x^2+3}+\frac{96\sqrt{1-\left(-\frac{5}{6}+\frac{\sqrt{13}}{6}\right)x^2}\,\sqrt{1-\left(-\frac{5}{6}-\frac{\sqrt{13}}{6}\right)x^2}\,F\left(\frac{x\sqrt{-30+6\sqrt{13}}}{6},\frac{5\sqrt{3}}{6}+\frac{\sqrt{39}}{6}\right)}{\sqrt{-30+6\sqrt{13}}\,\sqrt{x^4+5x^2+3}}-\frac{324\sqrt{1-\left(-\frac{5}{6}+\frac{\sqrt{13}}{6}\right)x^2}\,\sqrt{}}{}$

```
[In] int((3*x^2+2)*(x^4+5*x^2+3)^(1/2)/x^2,x,method=_RETURNVERBOSE)
```

```
[Out] x*(x^4+5*x^2+3)^(1/2)+96/(-30+6*13^(1/2))^(1/2)*(1-(-5/6+1/6*13^(1/2))*x^2)
^(1/2)*(1-(-5/6-1/6*13^(1/2))*x^2)^(1/2)/(x^4+5*x^2+3)^(1/2)*EllipticF(1/6*
x*(-30+6*13^(1/2))^(1/2),5/6*3^(1/2)+1/6*39^(1/2))-324/(-30+6*13^(1/2))^(1/
2)*(1-(-5/6+1/6*13^(1/2))*x^2)^(1/2)*(1-(-5/6-1/6*13^(1/2))*x^2)^(1/2)/(x^4
+5*x^2+3)^(1/2)/(5+13^(1/2))*(EllipticF(1/6*x*(-30+6*13^(1/2))^(1/2),5/6*3^
(1/2)+1/6*39^(1/2))-EllipticE(1/6*x*(-30+6*13^(1/2))^(1/2),5/6*3^(1/2)+1/6*
39^(1/2)))-2*(x^4+5*x^2+3)^(1/2)/x
```

Fricas [F]

$$\int \frac{(2+3x^2)\sqrt{3+5x^2+x^4}}{x^2}\,dx=\int \frac{\sqrt{x^4+5\,x^2+3}(3\,x^2+2)}{x^2}\,dx$$

```
[In] integrate((3*x^2+2)*(x^4+5*x^2+3)^(1/2)/x^2,x, algorithm="fricas")
[Out] integral(sqrt(x^4 + 5*x^2 + 3)*(3*x^2 + 2)/x^2, x)
```

Sympy [F]

$$\int \frac{(2+3x^2)\sqrt{3+5x^2+x^4}}{x^2}\,dx=\int \frac{(3x^2+2)\sqrt{x^4+5x^2+3}}{x^2}\,dx$$

```
[In] integrate((3*x**2+2)*(x**4+5*x**2+3)**(1/2)/x**2,x)
[Out] Integral((3*x**2 + 2)*sqrt(x**4 + 5*x**2 + 3)/x**2, x)
```

Maxima [F]

$$\int \frac{(2+3x^2)\sqrt{3+5x^2+x^4}}{x^2}\,dx=\int \frac{\sqrt{x^4+5\,x^2+3}(3\,x^2+2)}{x^2}\,dx$$

```
[In] integrate((3*x^2+2)*(x^4+5*x^2+3)^(1/2)/x^2,x, algorithm="maxima")
[Out] integrate(sqrt(x^4 + 5*x^2 + 3)*(3*x^2 + 2)/x^2, x)
```

Giac [F]

$$\int \frac{(2+3x^2)\sqrt{3+5x^2+x^4}}{x^2}\,dx=\int \frac{\sqrt{x^4+5\,x^2+3}(3\,x^2+2)}{x^2}\,dx$$

```
[In] integrate((3*x^2+2)*(x^4+5*x^2+3)^(1/2)/x^2,x, algorithm="giac")
[Out] integrate(sqrt(x^4 + 5*x^2 + 3)*(3*x^2 + 2)/x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(2+3x^2)\sqrt{3+5x^2+x^4}}{x^2}\,dx = \int \frac{(3\,x^2+2)\,\sqrt{x^4+5\,x^2+3}}{x^2}\,dx$$

```
[In] int(((3*x^2 + 2)*(5*x^2 + x^4 + 3)^(1/2))/x^2,x)
[Out] int(((3*x^2 + 2)*(5*x^2 + x^4 + 3)^(1/2))/x^2, x)
```

3.155 $\int \frac{(2+3x^2)\sqrt{3+5x^2+x^4}}{x^4}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 305

$$\int \frac{(2+3x^2)\sqrt{3+5x^2+x^4}}{x^4}\,dx = \frac{32x\left(5+\sqrt{13}+2x^2\right)}{9\sqrt{3+5x^2+x^4}} - \frac{64\sqrt{3+5x^2+x^4}}{9x} - \frac{(2-9x^2)\sqrt{3+5x^2+x^4}}{3x^3} - \frac{16\sqrt{\frac{2}{3}\left(5+\sqrt{13}\right)}\sqrt{\frac{6+\left(5-\sqrt{13}\right)x^2}{6+\left(5+\sqrt{13}\right)x^2}}\left(6+\left(5+\sqrt{13}\right)x^2\right)E\left(\arctan\left(\sqrt{\frac{1}{6}\left(5+\sqrt{13}\right)}x\right)|\frac{1}{6}\left(-13+5\sqrt{13}\right)\right)}{9\sqrt{3+5x^2+x^4}} + \frac{49\sqrt{\frac{6+\left(5-\sqrt{13}\right)x^2}{6+\left(5+\sqrt{13}\right)x^2}}\left(6+\left(5+\sqrt{13}\right)x^2\right)\text{EllipticF}\left(\arctan\left(\sqrt{\frac{1}{6}\left(5+\sqrt{13}\right)}x\right),\frac{1}{6}\left(-13+5\sqrt{13}\right)\right)}{3\sqrt{6\left(5+\sqrt{13}\right)}\sqrt{3+5x^2+x^4}}$$

```
[Out] 32/9*x*(5+2*x^2+13^(1/2))/(x^4+5*x^2+3)^(1/2)-64/9*(x^4+5*x^2+3)^(1/2)/x-1/
3*(-9*x^2+2)*(x^4+5*x^2+3)^(1/2)/x^3-16/27*(1/(36+x^2*(30+6*13^(1/2))))^(1/
2)*(36+x^2*(30+6*13^(1/2)))^(1/2)*EllipticE(x*(30+6*13^(1/2))^(1/2)/(36+x^2
*(30+6*13^(1/2)))^(1/2),1/6*(-78+30*13^(1/2))^(1/2))*(6+x^2*(5+13^(1/2)))*(
30+6*13^(1/2))^(1/2)*((6+x^2*(5-13^(1/2)))/(6+x^2*(5+13^(1/2))))^(1/2)/(x^4
+5*x^2+3)^(1/2)+49/3*(1/(36+x^2*(30+6*13^(1/2))))^(1/2)*(36+x^2*(30+6*13^(1
/2)))^(1/2)*EllipticF(x*(30+6*13^(1/2))^(1/2)/(36+x^2*(30+6*13^(1/2)))^(1/2
),1/6*(-78+30*13^(1/2))^(1/2))*(6+x^2*(5+13^(1/2)))*((6+x^2*(5-13^(1/2)))/(
6+x^2*(5+13^(1/2))))^(1/2)/(x^4+5*x^2+3)^(1/2)/(30+6*13^(1/2))^(1/2)
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 305, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {1285, 1295, 1203, 1113, 1149}

$$\int \frac{(2+3x^2)\sqrt{3+5x^2+x^4}}{x^4}\,dx$$
$$= \frac{49\sqrt{\frac{\left(5-\sqrt{13}\right)x^2+6}{\left(5+\sqrt{13}\right)x^2+6}}\left(\left(5+\sqrt{13}\right)x^2+6\right)\text{EllipticF}\left(\arctan\left(\sqrt{\frac{1}{6}\left(5+\sqrt{13}\right)}x\right),\frac{1}{6}\left(-13+5\sqrt{13}\right)\right)}{3\sqrt{6\left(5+\sqrt{13}\right)}\sqrt{x^4+5x^2+3}}$$
$$-\frac{16\sqrt{\frac{2}{3}\left(5+\sqrt{13}\right)}\sqrt{\frac{\left(5-\sqrt{13}\right)x^2+6}{\left(5+\sqrt{13}\right)x^2+6}}\left(\left(5+\sqrt{13}\right)x^2+6\right)E\left(\arctan\left(\sqrt{\frac{1}{6}\left(5+\sqrt{13}\right)}x\right)|\frac{1}{6}\left(-13+5\sqrt{13}\right)\right)}{9\sqrt{x^4+5x^2+3}}$$
$$-\frac{64\sqrt{x^4+5x^2+3}}{9x}+\frac{32x\left(2x^2+\sqrt{13}+5\right)}{9\sqrt{x^4+5x^2+3}}-\frac{\sqrt{x^4+5x^2+3}(2-9x^2)}{3x^3}$$

```
[In] Int[((2 + 3*x^2)*Sqrt[3 + 5*x^2 + x^4])/x^4,x]
```

```
[Out] (32*x*(5 + Sqrt[13] + 2*x^2))/(9*Sqrt[3 + 5*x^2 + x^4]) - (64*Sqrt[3 + 5*x^
2 + x^4])/(9*x) - ((2 - 9*x^2)*Sqrt[3 + 5*x^2 + x^4])/(3*x^3) - (16*Sqrt[(2
*(5 + Sqrt[13]))/3]*Sqrt[(6 + (5 - Sqrt[13])*x^2)/(6 + (5 + Sqrt[13])*x^2)]
*(6 + (5 + Sqrt[13])*x^2)*EllipticE[ArcTan[Sqrt[(5 + Sqrt[13])/6]*x], (-13
+ 5*Sqrt[13])/6])/(9*Sqrt[3 + 5*x^2 + x^4]) + (49*Sqrt[(6 + (5 - Sqrt[13])*
x^2)/(6 + (5 + Sqrt[13])*x^2)]*(6 + (5 + Sqrt[13])*x^2)*EllipticF[ArcTan[Sq
rt[(5 + Sqrt[13])/6]*x], (-13 + 5*Sqrt[13])/6])/(3*Sqrt[6*(5 + Sqrt[13])]*S
qrt[3 + 5*x^2 + x^4])
```

Rule 1113

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[(2*a + (b + q)*x^2)*(Sqrt[(2*a + (b - q)*x^2)/(2*a +
(b + q)*x^2)]/(2*a*Rt[(b + q)/(2*a), 2]*Sqrt[a + b*x^2 + c*x^4]))*EllipticF
[ArcTan[Rt[(b + q)/(2*a), 2]*x], 2*(q/(b + q))], x] /; PosQ[(b + q)/a] &&
!(PosQ[(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[
{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1149

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[b^2 - 4*a*c, 2]}, Simp[x*((b + q + 2*c*x^2)/(2*c*Sqrt[a + b*x^2 + c*x^4
])), x] - Simp[Rt[(b + q)/(2*a), 2]*(2*a + (b + q)*x^2)*(Sqrt[(2*a + (b - q
)*x^2)/(2*a + (b + q)*x^2)]/(2*c*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[ArcTan
[Rt[(b + q)/(2*a), 2]*x], 2*(q/(b + q))], x] /; PosQ[(b + q)/a] &&  !(PosQ[
```

```
(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[{a, b,
c}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1203

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[d, Int[1/Sqrt[a + b*x^2 + c*x^4],
 x], x] + Dist[e, Int[x^2/Sqrt[a + b*x^2 + c*x^4], x], x] /; PosQ[(b + q)/a
] || PosQ[(b - q)/a]] /; FreeQ[{a, b, c, d, e}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1285

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(a + b*x^2 + c*x^4)^p*((d*(m
+ 4*p + 3) + e*(m + 1)*x^2)/(f*(m + 1)*(m + 4*p + 3))), x] + Dist[2*(p/(f^2
*(m + 1)*(m + 4*p + 3))), Int[(f*x)^(m + 2)*(a + b*x^2 + c*x^4)^(p - 1)*Sim
p[2*a*e*(m + 1) - b*d*(m + 4*p + 3) + (b*e*(m + 1) - 2*c*d*(m + 4*p + 3))*x
^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b^2 - 4*a*c, 0] && Gt
Q[p, 0] && LtQ[m, -1] && m + 4*p + 3 != 0 && IntegerQ[2*p] && (IntegerQ[p]
|| IntegerQ[m])
```

Rule 1295

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[d*(f*x)^(m + 1)*((a + b*x^2 + c*x^4)^(p + 1)
/(a*f*(m + 1))), x] + Dist[1/(a*f^2*(m + 1)), Int[(f*x)^(m + 2)*(a + b*x^2
+ c*x^4)^p*Simp[a*e*(m + 1) - b*d*(m + 2*p + 3) - c*d*(m + 4*p + 5)*x^2, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[m
, -1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{(2-9x^2)\sqrt{3+5x^2+x^4}}{3x^3}-\frac{1}{3}\int\frac{-64-49x^2}{x^2\sqrt{3+5x^2+x^4}}\,dx\\
&= -\frac{64\sqrt{3+5x^2+x^4}}{9x}-\frac{(2-9x^2)\sqrt{3+5x^2+x^4}}{3x^3}+\frac{1}{9}\int\frac{147+64x^2}{\sqrt{3+5x^2+x^4}}\,dx\\
&= -\frac{64\sqrt{3+5x^2+x^4}}{9x}-\frac{(2-9x^2)\sqrt{3+5x^2+x^4}}{3x^3}\\
&\quad+\frac{64}{9}\int\frac{x^2}{\sqrt{3+5x^2+x^4}}\,dx+\frac{49}{3}\int\frac{1}{\sqrt{3+5x^2+x^4}}\,dx
\end{aligned}$$

$$= \frac{32x\left(5+\sqrt{13}+2x^2\right)}{9\sqrt{3+5x^2+x^4}} - \frac{64\sqrt{3+5x^2+x^4}}{9x} - \frac{(2-9x^2)\sqrt{3+5x^2+x^4}}{3x^3}$$

$$- \frac{16\sqrt{\frac{2}{3}\left(5+\sqrt{13}\right)}\sqrt{\frac{6+\left(5-\sqrt{13}\right)x^2}{6+\left(5+\sqrt{13}\right)x^2}}\left(6+\left(5+\sqrt{13}\right)x^2\right)E\left(\tan^{-1}\left(\sqrt{\frac{1}{6}\left(5+\sqrt{13}\right)}x\right)|\frac{1}{6}(-13+5\sqrt{}\right.}{9\sqrt{3+5x^2+x^4}}$$

$$+ \frac{49\sqrt{\frac{6+\left(5-\sqrt{13}\right)x^2}{6+\left(5+\sqrt{13}\right)x^2}}\left(6+\left(5+\sqrt{13}\right)x^2\right)F\left(\tan^{-1}\left(\sqrt{\frac{1}{6}\left(5+\sqrt{13}\right)}x\right)|\frac{1}{6}\left(-13+5\sqrt{13}\right)\right)}{3\sqrt{6\left(5+\sqrt{13}\right)}\sqrt{3+5x^2+x^4}}$$

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 9.68 (sec) , antiderivative size = 237, normalized size of antiderivative = 0.78

$$\int \frac{(2+3x^2)\sqrt{3+5x^2+x^4}}{x^4}\,dx$$

$$= \frac{-2(18+141x^2+191x^4+37x^6)+32i\sqrt{2}\left(-5+\sqrt{13}\right)x^3\sqrt{\frac{-5+\sqrt{13}-2x^2}{-5+\sqrt{13}}}\sqrt{5+\sqrt{13}+2x^2}E\left(i\text{arcsinh}\left(\sqrt{\frac{}{5+}}\right.\right.}{18}$$

```
[In] Integrate[((2 + 3*x^2)*Sqrt[3 + 5*x^2 + x^4])/x^4,x]
```

```
[Out] (-2*(18 + 141*x^2 + 191*x^4 + 37*x^6) + (32*I)*Sqrt[2]*(-5 + Sqrt[13])*x^3*
Sqrt[(-5 + Sqrt[13] - 2*x^2)/(-5 + Sqrt[13])]*Sqrt[5 + Sqrt[13] + 2*x^2]*El
lipticE[I*ArcSinh[Sqrt[2/(5 + Sqrt[13])]*x], 19/6 + (5*Sqrt[13])/6] - I*Sqr
t[2]*(-13 + 32*Sqrt[13])*x^3*Sqrt[(-5 + Sqrt[13] - 2*x^2)/(-5 + Sqrt[13])]*
Sqrt[5 + Sqrt[13] + 2*x^2]*EllipticF[I*ArcSinh[Sqrt[2/(5 + Sqrt[13])]*x], 1
9/6 + (5*Sqrt[13])/6])/(18*x^3*Sqrt[3 + 5*x^2 + x^4])
```

Maple [A] (verified)

Time = 2.15 (sec) , antiderivative size = 228, normalized size of antiderivative = 0.75

method	result
default	$-\frac{37\sqrt{x^4+5x^2+3}}{9x}+\frac{98\sqrt{1-\left(-\frac{5}{6}+\frac{\sqrt{13}}{6}\right)x^2}\,\sqrt{1-\left(-\frac{5}{6}-\frac{\sqrt{13}}{6}\right)x^2}\,F\left(\frac{x\sqrt{-30+6\sqrt{13}}}{6},\frac{5\sqrt{3}}{6}+\frac{\sqrt{39}}{6}\right)}{\sqrt{-30+6\sqrt{13}}\,\sqrt{x^4+5x^2+3}}-\frac{256\sqrt{1-\left(-\frac{5}{6}+\frac{\sqrt{13}}{6}\right)x^2}\,\sqrt{1-\left(-\right.}}{}$
risch	$-\frac{37x^6+191x^4+141x^2+18}{9x^3\sqrt{x^4+5x^2+3}}+\frac{98\sqrt{1-\left(-\frac{5}{6}+\frac{\sqrt{13}}{6}\right)x^2}\,\sqrt{1-\left(-\frac{5}{6}-\frac{\sqrt{13}}{6}\right)x^2}\,F\left(\frac{x\sqrt{-30+6\sqrt{13}}}{6},\frac{5\sqrt{3}}{6}+\frac{\sqrt{39}}{6}\right)}{\sqrt{-30+6\sqrt{13}}\,\sqrt{x^4+5x^2+3}}-\frac{256\sqrt{1-\left(-\frac{5}{6}+\frac{\sqrt{13}}{6}\right)x^2}}{}$
elliptic	$-\frac{37\sqrt{x^4+5x^2+3}}{9x}+\frac{98\sqrt{1-\left(-\frac{5}{6}+\frac{\sqrt{13}}{6}\right)x^2}\,\sqrt{1-\left(-\frac{5}{6}-\frac{\sqrt{13}}{6}\right)x^2}\,F\left(\frac{x\sqrt{-30+6\sqrt{13}}}{6},\frac{5\sqrt{3}}{6}+\frac{\sqrt{39}}{6}\right)}{\sqrt{-30+6\sqrt{13}}\,\sqrt{x^4+5x^2+3}}-\frac{256\sqrt{1-\left(-\frac{5}{6}+\frac{\sqrt{13}}{6}\right)x^2}\,\sqrt{1-\left(-\right.}}{}$

```
[In] int((3*x^2+2)*(x^4+5*x^2+3)^(1/2)/x^4,x,method=_RETURNVERBOSE)
```

```
[Out] -37/9*(x^4+5*x^2+3)^(1/2)/x+98/(-30+6*13^(1/2))^(1/2)*(1-(-5/6+1/6*13^(1/2)
)*x^2)^(1/2)*(1-(-5/6-1/6*13^(1/2))*x^2)^(1/2)/(x^4+5*x^2+3)^(1/2)*Elliptic
F(1/6*x*(-30+6*13^(1/2))^(1/2),5/6*3^(1/2)+1/6*39^(1/2))-256/(-30+6*13^(1/2
))^(1/2)*(1-(-5/6+1/6*13^(1/2))*x^2)^(1/2)*(1-(-5/6-1/6*13^(1/2))*x^2)^(1/2
)/(x^4+5*x^2+3)^(1/2)/(5+13^(1/2))*(EllipticF(1/6*x*(-30+6*13^(1/2))^(1/2),
5/6*3^(1/2)+1/6*39^(1/2))-EllipticE(1/6*x*(-30+6*13^(1/2))^(1/2),5/6*3^(1/2
)+1/6*39^(1/2)))-2/3*(x^4+5*x^2+3)^(1/2)/x^3
```

Fricas [F]

$$\int \frac{(2+3x^2)\sqrt{3+5x^2+x^4}}{x^4}\,dx=\int \frac{\sqrt{x^4+5\,x^2+3}(3\,x^2+2)}{x^4}\,dx$$

```
[In] integrate((3*x^2+2)*(x^4+5*x^2+3)^(1/2)/x^4,x, algorithm="fricas")
```

```
[Out] integral(sqrt(x^4 + 5*x^2 + 3)*(3*x^2 + 2)/x^4, x)
```

Sympy [F]

$$\int \frac{(2+3x^2)\sqrt{3+5x^2+x^4}}{x^4}\,dx=\int \frac{(3x^2+2)\sqrt{x^4+5x^2+3}}{x^4}\,dx$$

```
[In] integrate((3*x**2+2)*(x**4+5*x**2+3)**(1/2)/x**4,x)
```

```
[Out] Integral((3*x**2 + 2)*sqrt(x**4 + 5*x**2 + 3)/x**4, x)
```

Maxima [F]

$$\int \frac{(2+3x^2)\sqrt{3+5x^2+x^4}}{x^4}\,dx = \int \frac{\sqrt{x^4+5\,x^2+3}(3\,x^2+2)}{x^4}\,dx$$

```
[In] integrate((3*x^2+2)*(x^4+5*x^2+3)^(1/2)/x^4,x, algorithm="maxima")
[Out] integrate(sqrt(x^4 + 5*x^2 + 3)*(3*x^2 + 2)/x^4, x)
```

Giac [F]

$$\int \frac{(2+3x^2)\sqrt{3+5x^2+x^4}}{x^4}\,dx = \int \frac{\sqrt{x^4+5\,x^2+3}(3\,x^2+2)}{x^4}\,dx$$

```
[In] integrate((3*x^2+2)*(x^4+5*x^2+3)^(1/2)/x^4,x, algorithm="giac")
[Out] integrate(sqrt(x^4 + 5*x^2 + 3)*(3*x^2 + 2)/x^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(2+3x^2)\sqrt{3+5x^2+x^4}}{x^4}\,dx = \int \frac{(3\,x^2+2)\,\sqrt{x^4+5\,x^2+3}}{x^4}\,dx$$

```
[In] int(((3*x^2 + 2)*(5*x^2 + x^4 + 3)^(1/2))/x^4,x)
[Out] int(((3*x^2 + 2)*(5*x^2 + x^4 + 3)^(1/2))/x^4, x)
```

3.156 $\int x^5(2+3x^2)\left(3+5x^2+x^4\right)^{3/2}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 127

$$\int x^5\left(2+3x^2\right)\left(3+5x^2+x^4\right)^{3/2}\,dx=\frac{28379(5+2x^2)\sqrt{3+5x^2+x^4}}{2048}-\frac{2183}{768}\left(5+2x^2\right)\left(3+5x^2+x^4\right)^{3/2}+\frac{3}{14}x^4\left(3+5x^2+x^4\right)^{5/2}+\frac{(3313-1070x^2)\left(3+5x^2+x^4\right)^{5/2}}{1680}-\frac{368927\text{arctanh}\left(\frac{5+2x^2}{2\sqrt{3+5x^2+x^4}}\right)}{4096}$$

```
[Out] -2183/768*(2*x^2+5)*(x^4+5*x^2+3)^(3/2)+3/14*x^4*(x^4+5*x^2+3)^(5/2)+1/1680
*(-1070*x^2+3313)*(x^4+5*x^2+3)^(5/2)-368927/4096*arctanh(1/2*(2*x^2+5)/(x^
4+5*x^2+3)^(1/2))+28379/2048*(2*x^2+5)*(x^4+5*x^2+3)^(1/2)
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {1265, 846, 793, 626, 635, 212}

$$\int x^5\left(2+3x^2\right)\left(3+5x^2+x^4\right)^{3/2}\,dx=-\frac{368927\text{arctanh}\left(\frac{2x^2+5}{2\sqrt{x^4+5x^2+3}}\right)}{4096}+\frac{3}{14}\left(x^4+5x^2+3\right)^{5/2}x^4+\frac{(3313-1070x^2)\left(x^4+5x^2+3\right)^{5/2}}{1680}-\frac{2183}{768}\left(2x^2+5\right)\left(x^4+5x^2+3\right)^{3/2}+\frac{28379(2x^2+5)\sqrt{x^4+5x^2+3}}{2048}$$

```
[In] Int[x^5*(2 + 3*x^2)*(3 + 5*x^2 + x^4)^(3/2),x]
```

```
[Out] (28379*(5 + 2*x^2)*Sqrt[3 + 5*x^2 + x^4])/2048 - (2183*(5 + 2*x^2)*(3 + 5*x
^2 + x^4)^(3/2))/768 + (3*x^4*(3 + 5*x^2 + x^4)^(5/2))/14 + ((3313 - 1070*x
^2)*(3 + 5*x^2 + x^4)^(5/2))/1680 - (368927*ArcTanh[(5 + 2*x^2)/(2*Sqrt[3 +
 5*x^2 + x^4])])/4096
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 626

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Dist[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1))), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 635

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 793

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> Simp[(-(b*e*g*(p + 2) - c*(e*f + d*g)*(2*p + 3) -
 2*c*e*g*(p + 1)*x))*((a + b*x + c*x^2)^(p + 1)/(2*c^2*(p + 1)*(2*p + 3))),
 x] + Dist[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(2*c*d*f - b*(e*f + d*g))*(2*p
+ 3))/(2*c^2*(2*p + 3)), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c,
 d, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rule 846

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g*(d + e*x)^m*((a + b*x + c*x^2)^(p +
1)/(c*(m + 2*p + 2))), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)
*(a + b*x + c*x^2)^p*Simp[m*(c*d*f - a*e*g) + d*(2*c*f - b*g)*(p + 1) + (m*
(c*e*f + c*d*g - b*e*g) + e*(p + 1)*(2*c*f - b*g))*x, x], x], x] /; FreeQ[{
a, b, c, d, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a
*e^2, 0] && GtQ[m, 0] && NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p]
 || IntegersQ[2*m, 2*p]) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

Rule 1265

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
```

```
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rubi steps

$$\text{integral} = \frac{1}{2}\text{Subst}\left(\int x^2(2+3x)\left(3+5x+x^2\right)^{3/2}\,dx,x,x^2\right)$$

$$= \frac{3}{14}x^4\left(3+5x^2+x^4\right)^{5/2}+\frac{1}{14}\text{Subst}\left(\int\left(-18-\frac{107x}{2}\right)x\left(3+5x+x^2\right)^{3/2}\,dx,x,x^2\right)$$

$$= \frac{3}{14}x^4\left(3+5x^2+x^4\right)^{5/2}+\frac{(3313-1070x^2)\left(3+5x^2+x^4\right)^{5/2}}{1680} - \frac{2183}{96}\text{Subst}\left(\int\left(3+5x+x^2\right)^{3/2}\,dx,x,x^2\right)$$

$$= -\frac{2183}{768}\left(5+2x^2\right)\left(3+5x^2+x^4\right)^{3/2}+\frac{3}{14}x^4\left(3+5x^2+x^4\right)^{5/2} + \frac{(3313-1070x^2)\left(3+5x^2+x^4\right)^{5/2}}{1680}+\frac{28379}{512}\text{Subst}\left(\int\sqrt{3+5x+x^2}\,dx,x,x^2\right)$$

$$= \frac{28379(5+2x^2)\sqrt{3+5x^2+x^4}}{2048}-\frac{2183}{768}\left(5+2x^2\right)\left(3+5x^2+x^4\right)^{3/2}+\frac{3}{14}x^4\left(3+5x^2+x^4\right)^{5/2} + \frac{(3313-1070x^2)\left(3+5x^2+x^4\right)^{5/2}}{1680}-\frac{368927\text{Subst}\left(\int\frac{1}{\sqrt{3+5x+x^2}}\,dx,x,x^2\right)}{4096}$$

$$= \frac{28379(5+2x^2)\sqrt{3+5x^2+x^4}}{2048}-\frac{2183}{768}\left(5+2x^2\right)\left(3+5x^2+x^4\right)^{3/2}+\frac{3}{14}x^4\left(3+5x^2+x^4\right)^{5/2} + \frac{(3313-1070x^2)\left(3+5x^2+x^4\right)^{5/2}}{1680}-\frac{368927\text{Subst}\left(\int\frac{1}{4-x^2}\,dx,x,\frac{5+2x^2}{\sqrt{3+5x^2+x^4}}\right)}{2048}$$

$$= \frac{28379(5+2x^2)\sqrt{3+5x^2+x^4}}{2048}-\frac{2183}{768}\left(5+2x^2\right)\left(3+5x^2+x^4\right)^{3/2}+\frac{3}{14}x^4\left(3+5x^2+x^4\right)^{5/2} + \frac{(3313-1070x^2)\left(3+5x^2+x^4\right)^{5/2}}{1680}-\frac{368927\tanh^{-1}\left(\frac{5+2x^2}{2\sqrt{3+5x^2+x^4}}\right)}{4096}$$

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.62

$$\int x^5(2+3x^2)(3+5x^2+x^4)^{3/2}\,dx = \frac{\sqrt{3+5x^2+x^4}(9546951-1499570x^2+283304x^4+154800x^6+482944x^8+323840x^{10}+460}{215040} + \frac{368927\log\left(-5-2x^2+2\sqrt{3+5x^2+x^4}\right)}{4096}$$

```
[In] Integrate[x^5*(2 + 3*x^2)*(3 + 5*x^2 + x^4)^(3/2),x]
```

```
[Out] (Sqrt[3 + 5*x^2 + x^4]*(9546951 - 1499570*x^2 + 283304*x^4 + 154800*x^6 + 4
82944*x^8 + 323840*x^10 + 46080*x^12))/215040 + (368927*Log[-5 - 2*x^2 + 2*
Sqrt[3 + 5*x^2 + x^4]])/4096
```

Maple [A] (verified)

Time = 0.24 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.54

method	result
risch	$\frac{(46080x^{12}+323840x^{10}+482944x^8+154800x^6+283304x^4-1499570x^2+9546951)\sqrt{x^4+5x^2+3}}{215040} - \frac{368927\ln\left(\frac{5}{2}+x^2+\sqrt{x^4+\right.}{4096}$
trager	$\left(\frac{3}{14}x^{12}+\frac{253}{168}x^{10}+\frac{539}{240}x^8+\frac{645}{896}x^6+\frac{5059}{3840}x^4-\frac{149957}{21504}x^2+\frac{3182317}{71680}\right)\sqrt{x^4+5x^2+3} - \frac{368927\ln\left(2\right.}{}$
pseudoelliptic	$-\frac{368927\ln\left(2x^2+5+2\sqrt{x^4+5x^2+3}\right)}{4096} + \frac{(46080x^{12}+323840x^{10}+482944x^8+154800x^6+283304x^4-1499570x^2+9546951)}{215040}$
default	$\frac{3x^{12}\sqrt{x^4+5x^2+3}}{14} + \frac{253x^{10}\sqrt{x^4+5x^2+3}}{168} + \frac{539x^8\sqrt{x^4+5x^2+3}}{240} + \frac{645x^6\sqrt{x^4+5x^2+3}}{896} + \frac{5059x^4\sqrt{x^4+5x^2+3}}{3840} - 1499$
elliptic	$\frac{3x^{12}\sqrt{x^4+5x^2+3}}{14} + \frac{253x^{10}\sqrt{x^4+5x^2+3}}{168} + \frac{539x^8\sqrt{x^4+5x^2+3}}{240} + \frac{645x^6\sqrt{x^4+5x^2+3}}{896} + \frac{5059x^4\sqrt{x^4+5x^2+3}}{3840} - 1499$

```
[In] int(x^5*(3*x^2+2)*(x^4+5*x^2+3)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/215040*(46080*x^12+323840*x^10+482944*x^8+154800*x^6+283304*x^4-1499570*x
^2+9546951)*(x^4+5*x^2+3)^(1/2)-368927/4096*ln(5/2+x^2+(x^4+5*x^2+3)^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.56

$$\int x^5 (2+3x^2) (3+5x^2+x^4)^{3/2} \, dx = \frac{1}{215040} (46080 \, x^{12} + 323840 \, x^{10} + 482944 \, x^8 + 154800 \, x^6 + 283304 \, x^4 - 1499570 \, x^2 + 9546951 + \frac{368927}{4096} \log \left(-2 \, x^2 + 2 \, \sqrt{x^4 + 5 \, x^2 + 3} - 5\right)$$

```
[In] integrate(x^5*(3*x^2+2)*(x^4+5*x^2+3)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/215040*(46080*x^12 + 323840*x^10 + 482944*x^8 + 154800*x^6 + 283304*x^4 -
 1499570*x^2 + 9546951)*sqrt(x^4 + 5*x^2 + 3) + 368927/4096*log(-2*x^2 + 2*
sqrt(x^4 + 5*x^2 + 3) - 5)
```

Sympy [A] (verification not implemented)

Time = 1.21 (sec) , antiderivative size = 218, normalized size of antiderivative = 1.72

$$\begin{aligned}\int x^5 (2+3x^2) (3+5x^2+x^4)^{3/2} \, dx &= 3\sqrt{x^4+5x^2+3}\left(\frac{x^6}{4}+\frac{5x^4}{24}-\frac{89x^2}{96}+\frac{365}{64}\right) \\ &+ \frac{19\sqrt{x^4+5x^2+3}\left(\frac{x^8}{5}+\frac{x^6}{8}-\frac{127x^4}{240}+\frac{527x^2}{192}-\frac{11143}{640}\right)}{2} \\ &+ \frac{17\sqrt{x^4+5x^2+3}\left(\frac{x^{10}}{6}+\frac{x^8}{12}-\frac{11x^6}{32}+\frac{107x^4}{64}-\frac{2279x^2}{256}+\frac{29049}{512}\right)}{2} \\ &+ \frac{3\sqrt{x^4+5x^2+3}\left(\frac{x^{12}}{7}+\frac{5x^{10}}{84}-\frac{29x^8}{120}+\frac{509x^6}{448}-\frac{3623x^4}{640}+\frac{108481x^2}{3584}-\frac{6918747}{35840}\right)}{2} \\ &- \frac{368927\log\left(2x^2+2\sqrt{x^4+5x^2+3}+5\right)}{4096}\end{aligned}$$

```
[In] integrate(x**5*(3*x**2+2)*(x**4+5*x**2+3)**(3/2),x)
```

```
[Out] 3*sqrt(x**4 + 5*x**2 + 3)*(x**6/4 + 5*x**4/24 - 89*x**2/96 + 365/64) + 19*s
qrt(x**4 + 5*x**2 + 3)*(x**8/5 + x**6/8 - 127*x**4/240 + 527*x**2/192 - 111
43/640)/2 + 17*sqrt(x**4 + 5*x**2 + 3)*(x**10/6 + x**8/12 - 11*x**6/32 + 10
7*x**4/64 - 2279*x**2/256 + 29049/512)/2 + 3*sqrt(x**4 + 5*x**2 + 3)*(x**12
/7 + 5*x**10/84 - 29*x**8/120 + 509*x**6/448 - 3623*x**4/640 + 108481*x**2/
3584 - 6918747/35840)/2 - 368927*log(2*x**2 + 2*sqrt(x**4 + 5*x**2 + 3) + 5
)/4096
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.06

$$\int x^5(2+3x^2)\left(3+5x^2+x^4\right)^{3/2}\,dx = \frac{3}{14}\left(x^4+5\,x^2+3\right)^{\frac{5}{2}}x^4 - \frac{107}{168}\left(x^4+5\,x^2+3\right)^{\frac{5}{2}}x^2 - \frac{2183}{384}\left(x^4+5\,x^2+3\right)^{\frac{3}{2}}x^2 + \frac{3313}{1680}\left(x^4+5\,x^2+3\right)^{\frac{5}{2}} + \frac{28379}{1024}\sqrt{x^4+5\,x^2+3}x^2 - \frac{10915}{768}\left(x^4+5\,x^2+3\right)^{\frac{3}{2}} + \frac{141895}{2048}\sqrt{x^4+5\,x^2+3} - \frac{368927}{4096}\log\left(2\,x^2+2\,\sqrt{x^4+5\,x^2+3}+5\right)$$

```
[In] integrate(x^5*(3*x^2+2)*(x^4+5*x^2+3)^(3/2),x, algorithm="maxima")
```

```
[Out] 3/14*(x^4 + 5*x^2 + 3)^(5/2)*x^4 - 107/168*(x^4 + 5*x^2 + 3)^(5/2)*x^2 - 21
83/384*(x^4 + 5*x^2 + 3)^(3/2)*x^2 + 3313/1680*(x^4 + 5*x^2 + 3)^(5/2) + 28
379/1024*sqrt(x^4 + 5*x^2 + 3)*x^2 - 10915/768*(x^4 + 5*x^2 + 3)^(3/2) + 14
1895/2048*sqrt(x^4 + 5*x^2 + 3) - 368927/4096*log(2*x^2 + 2*sqrt(x^4 + 5*x^
2 + 3) + 5)
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 207, normalized size of antiderivative = 1.63

$$\int x^5(2+3x^2)\left(3+5x^2+x^4\right)^{3/2}\,dx = \frac{1}{71680}\sqrt{x^4+5\,x^2+3}\left(2\left(4\left(2\left(8\left(10\left(12\,x^2+5\right)x^2-203\right)x^2+7635\right)x^2-76083\right)x^2+1627215\right. + \frac{17}{3072}\sqrt{x^4+5\,x^2+3}\left(2\left(4\left(2\left(8\left(2\,x^2+1\right)x^2-33\right)x^2+321\right)x^2-6837\right)x^2+87147\right) + \frac{19}{3840}\sqrt{x^4+5\,x^2+3}\left(2\left(4\left(6\left(8\,x^2+5\right)x^2-127\right)x^2+2635\right)x^2-33429\right) + \frac{1}{64}\sqrt{x^4+5\,x^2+3}\left(2\left(4\left(6\,x^2+5\right)x^2-89\right)x^2+1095\right) + \frac{368927}{4096}\log\left(2\,x^2-2\,\sqrt{x^4+5\,x^2+3}+5\right)$$

```
[In] integrate(x^5*(3*x^2+2)*(x^4+5*x^2+3)^(3/2),x, algorithm="giac")
```

```
[Out] 1/71680*sqrt(x^4 + 5*x^2 + 3)*(2*(4*(2*(8*(10*(12*x^2 + 5)*x^2 - 203)*x^2 +
 7635)*x^2 - 76083)*x^2 + 1627215)*x^2 - 20756241) + 17/3072*sqrt(x^4 + 5*x
^2 + 3)*(2*(4*(2*(8*(2*x^2 + 1)*x^2 - 33)*x^2 + 321)*x^2 - 6837)*x^2 + 8714
7) + 19/3840*sqrt(x^4 + 5*x^2 + 3)*(2*(4*(6*(8*x^2 + 5)*x^2 - 127)*x^2 + 26
35)*x^2 - 33429) + 1/64*sqrt(x^4 + 5*x^2 + 3)*(2*(4*(6*x^2 + 5)*x^2 - 89)*x
^2 + 1095) + 368927/4096*log(2*x^2 - 2*sqrt(x^4 + 5*x^2 + 3) + 5)
```

Mupad [F(-1)]

Timed out.

$$\int x^5(2+3x^2)\left(3+5x^2+x^4\right)^{3/2}\,dx=\int x^5\,\left(3\,x^2+2\right)\,\left(x^4+5\,x^2+3\right)^{3/2}\,dx$$

```
[In] int(x^5*(3*x^2 + 2)*(5*x^2 + x^4 + 3)^(3/2),x)
[Out] int(x^5*(3*x^2 + 2)*(5*x^2 + x^4 + 3)^(3/2), x)
```

3.157 $\int x^3(2+3x^2)\left(3+5x^2+x^4\right)^{3/2}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 106

$$\int x^3(2+3x^2)\left(3+5x^2+x^4\right)^{3/2}\,dx = -\frac{4797(5+2x^2)\sqrt{3+5x^2+x^4}}{1024}+\frac{123}{128}(5+2x^2)\left(3+5x^2+x^4\right)^{3/2} -\frac{1}{40}(27-10x^2)\left(3+5x^2+x^4\right)^{5/2}+\frac{62361\text{arctanh}\left(\frac{5+2x^2}{2\sqrt{3+5x^2+x^4}}\right)}{2048}$$

```
[Out] 123/128*(2*x^2+5)*(x^4+5*x^2+3)^(3/2)-1/40*(-10*x^2+27)*(x^4+5*x^2+3)^(5/2)
+62361/2048*arctanh(1/2*(2*x^2+5)/(x^4+5*x^2+3)^(1/2))-4797/1024*(2*x^2+5)*
(x^4+5*x^2+3)^(1/2)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {1265, 793, 626, 635, 212}

$$\int x^3(2+3x^2)\left(3+5x^2+x^4\right)^{3/2}\,dx = \frac{62361\text{arctanh}\left(\frac{2x^2+5}{2\sqrt{x^4+5x^2+3}}\right)}{2048} -\frac{1}{40}(27-10x^2)\left(x^4+5x^2+3\right)^{5/2} +\frac{123}{128}(2x^2+5)\left(x^4+5x^2+3\right)^{3/2}-\frac{4797(2x^2+5)\sqrt{x^4+5x^2+3}}{1024}$$

```
[In] Int[x^3*(2 + 3*x^2)*(3 + 5*x^2 + x^4)^(3/2),x]
```

```
[Out] (-4797*(5 + 2*x^2)*Sqrt[3 + 5*x^2 + x^4])/1024 + (123*(5 + 2*x^2)*(3 + 5*x^
2 + x^4)^(3/2))/128 - ((27 - 10*x^2)*(3 + 5*x^2 + x^4)^(5/2))/40 + (62361*A
rcTanh[(5 + 2*x^2)/(2*Sqrt[3 + 5*x^2 + x^4])])/2048
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 626

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Dist[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1))), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 635

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 793

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> Simp[(-(b*e*g*(p + 2) - c*(e*f + d*g)*(2*p + 3) -
 2*c*e*g*(p + 1)*x))*((a + b*x + c*x^2)^(p + 1)/(2*c^2*(p + 1)*(2*p + 3))),
 x] + Dist[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(2*c*d*f - b*(e*f + d*g))*(2*p
+ 3))/(2*c^2*(2*p + 3)), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c,
 d, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rule 1265

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{1}{2}\text{Subst}\left(\int x(2+3x)\left(3+5x+x^2\right)^{3/2}\,dx,x,x^2\right)\\ &= -\frac{1}{40}\left(27-10x^2\right)\left(3+5x^2+x^4\right)^{5/2}+\frac{123}{16}\text{Subst}\left(\int\left(3+5x+x^2\right)^{3/2}\,dx,x,x^2\right)\end{aligned}$$

$$= \frac{123}{128}(5+2x^2)\left(3+5x^2+x^4\right)^{3/2} - \frac{1}{40}(27-10x^2)\left(3+5x^2+x^4\right)^{5/2} - \frac{4797}{256}\text{Subst}\left(\int \sqrt{3+5x+x^2}\,dx, x, x^2\right)$$

$$= -\frac{4797(5+2x^2)\sqrt{3+5x^2+x^4}}{1024} + \frac{123}{128}(5+2x^2)\left(3+5x^2+x^4\right)^{3/2} - \frac{1}{40}(27-10x^2)\left(3+5x^2+x^4\right)^{5/2} + \frac{62361\text{Subst}\left(\int \frac{1}{\sqrt{3+5x+x^2}}\,dx, x, x^2\right)}{2048}$$

$$= -\frac{4797(5+2x^2)\sqrt{3+5x^2+x^4}}{1024} + \frac{123}{128}(5+2x^2)\left(3+5x^2+x^4\right)^{3/2} - \frac{1}{40}(27-10x^2)\left(3+5x^2+x^4\right)^{5/2} + \frac{62361\text{Subst}\left(\int \frac{1}{4-x^2}\,dx, x, \frac{5+2x^2}{\sqrt{3+5x^2+x^4}}\right)}{1024}$$

$$= -\frac{4797(5+2x^2)\sqrt{3+5x^2+x^4}}{1024} + \frac{123}{128}(5+2x^2)\left(3+5x^2+x^4\right)^{3/2} - \frac{1}{40}(27-10x^2)\left(3+5x^2+x^4\right)^{5/2} + \frac{62361\tanh^{-1}\left(\frac{5+2x^2}{2\sqrt{3+5x^2+x^4}}\right)}{2048}$$

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.70

$$\int x^3(2+3x^2)\left(3+5x^2+x^4\right)^{3/2}\,dx = \frac{\sqrt{3+5x^2+x^4}(-77229+12390x^2+5064x^4+14960x^6+9344x^8+1280x^{10})}{5120} - \frac{62361\log\left(-5-2x^2+2\sqrt{3+5x^2+x^4}\right)}{2048}$$

```
[In] Integrate[x^3*(2 + 3*x^2)*(3 + 5*x^2 + x^4)^(3/2),x]
```

```
[Out] (Sqrt[3 + 5*x^2 + x^4]*(-77229 + 12390*x^2 + 5064*x^4 + 14960*x^6 + 9344*x^
8 + 1280*x^10))/5120 - (62361*Log[-5 - 2*x^2 + 2*Sqrt[3 + 5*x^2 + x^4]])/20
48
```

Maple [A] (verified)

Time = 0.21 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.59

method	result
risch	$\frac{\left(1280x^{10}+9344x^{8}+14960x^{6}+5064x^{4}+12390x^{2}-77229\right)\sqrt{x^{4}+5x^{2}+3}}{5120}+\frac{62361\ln\left(\frac{5}{2}+x^{2}+\sqrt{x^{4}+5x^{2}+3}\right)}{2048}$
trager	$\left(\frac{1}{4}x^{10}+\frac{73}{40}x^{8}+\frac{187}{64}x^{6}+\frac{633}{640}x^{4}+\frac{1239}{512}x^{2}-\frac{77229}{5120}\right)\sqrt{x^{4}+5x^{2}+3}+\frac{62361\ln\left(2x^{2}+5+2\sqrt{x^{4}+5x^{2}+3}\right)}{2048}$
pseudoelliptic	$\frac{62361\ln\left(2x^{2}+5+2\sqrt{x^{4}+5x^{2}+3}\right)}{2048}+\frac{\left(1280x^{10}+9344x^{8}+14960x^{6}+5064x^{4}+12390x^{2}-77229\right)\sqrt{x^{4}+5x^{2}+3}}{5120}$
default	$\frac{x^{10}\sqrt{x^{4}+5x^{2}+3}}{4}+\frac{73x^{8}\sqrt{x^{4}+5x^{2}+3}}{40}+\frac{187x^{6}\sqrt{x^{4}+5x^{2}+3}}{64}+\frac{633x^{4}\sqrt{x^{4}+5x^{2}+3}}{640}+\frac{1239x^{2}\sqrt{x^{4}+5x^{2}+3}}{512}-\frac{77229\sqrt{x}}{51}$
elliptic	$\frac{x^{10}\sqrt{x^{4}+5x^{2}+3}}{4}+\frac{73x^{8}\sqrt{x^{4}+5x^{2}+3}}{40}+\frac{187x^{6}\sqrt{x^{4}+5x^{2}+3}}{64}+\frac{633x^{4}\sqrt{x^{4}+5x^{2}+3}}{640}+\frac{1239x^{2}\sqrt{x^{4}+5x^{2}+3}}{512}-\frac{77229\sqrt{x}}{51}$

```
[In] int(x^3*(3*x^2+2)*(x^4+5*x^2+3)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/5120*(1280*x^10+9344*x^8+14960*x^6+5064*x^4+12390*x^2-77229)*(x^4+5*x^2+3
)^(1/2)+62361/2048*ln(5/2+x^2+(x^4+5*x^2+3)^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.62

$$\int x^3(2+3x^2)\left(3+5x^2+x^4\right)^{3/2}dx=\frac{1}{5120}\left(1280x^{10}+9344x^{8}+14960x^{6}+5064x^{4}+12390x^{2}-77229\right)\sqrt{x^{4}+5x^{2}+3}-\frac{62361}{2048}\log\left(-2x^{2}+2\sqrt{x^{4}+5x^{2}+3}-5\right)$$

```
[In] integrate(x^3*(3*x^2+2)*(x^4+5*x^2+3)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/5120*(1280*x^10 + 9344*x^8 + 14960*x^6 + 5064*x^4 + 12390*x^2 - 77229)*sq
rt(x^4 + 5*x^2 + 3) - 62361/2048*log(-2*x^2 + 2*sqrt(x^4 + 5*x^2 + 3) - 5)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 190 vs. $2(94) = 188$.

Time = 1.16 (sec) , antiderivative size = 190, normalized size of antiderivative = 1.79

$$\int x^3(2+3x^2)\left(3+5x^2+x^4\right)^{3/2}\,dx = 3\left(\frac{x^4}{3}+\frac{5x^2}{12}-\frac{17}{8}\right)\sqrt{x^4+5x^2+3} + \frac{19\sqrt{x^4+5x^2+3}\left(\frac{x^6}{4}+\frac{5x^4}{24}-\frac{89x^2}{96}+\frac{365}{64}\right)}{2} + \frac{17\sqrt{x^4+5x^2+3}\left(\frac{x^8}{5}+\frac{x^6}{8}-\frac{127x^4}{240}+\frac{527x^2}{192}-\frac{11143}{640}\right)}{2} + \frac{3\sqrt{x^4+5x^2+3}\left(\frac{x^{10}}{6}+\frac{x^8}{12}-\frac{11x^6}{32}+\frac{107x^4}{64}-\frac{2279x^2}{256}+\frac{29049}{512}\right)}{2} + \frac{62361\log\left(2x^2+2\sqrt{x^4+5x^2+3}+5\right)}{2048}$$

```
[In] integrate(x**3*(3*x**2+2)*(x**4+5*x**2+3)**(3/2),x)
```

```
[Out] 3*(x**4/3 + 5*x**2/12 - 17/8)*sqrt(x**4 + 5*x**2 + 3) + 19*sqrt(x**4 + 5*x*
*2 + 3)*(x**6/4 + 5*x**4/24 - 89*x**2/96 + 365/64)/2 + 17*sqrt(x**4 + 5*x**
2 + 3)*(x**8/5 + x**6/8 - 127*x**4/240 + 527*x**2/192 - 11143/640)/2 + 3*sq
rt(x**4 + 5*x**2 + 3)*(x**10/6 + x**8/12 - 11*x**6/32 + 107*x**4/64 - 2279*
x**2/256 + 29049/512)/2 + 62361*log(2*x**2 + 2*sqrt(x**4 + 5*x**2 + 3) + 5)
/2048
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.11

$$\int x^3(2+3x^2)\left(3+5x^2+x^4\right)^{3/2}\,dx = \frac{1}{4}\left(x^4+5\,x^2+3\right)^{\frac{5}{2}}x^2 + \frac{123}{64}\left(x^4+5\,x^2+3\right)^{\frac{3}{2}}x^2 - \frac{27}{40}\left(x^4+5\,x^2+3\right)^{\frac{5}{2}} - \frac{4797}{512}\sqrt{x^4+5\,x^2+3}x^2 + \frac{615}{128}\left(x^4+5\,x^2+3\right)^{\frac{3}{2}} - \frac{23985}{1024}\sqrt{x^4+5\,x^2+3} + \frac{62361}{2048}\log\left(2\,x^2+2\sqrt{x^4+5\,x^2+3}+5\right)$$

```
[In] integrate(x^3*(3*x^2+2)*(x^4+5*x^2+3)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/4*(x^4 + 5*x^2 + 3)^(5/2)*x^2 + 123/64*(x^4 + 5*x^2 + 3)^(3/2)*x^2 - 27/4
0*(x^4 + 5*x^2 + 3)^(5/2) - 4797/512*sqrt(x^4 + 5*x^2 + 3)*x^2 + 615/128*(x
^4 + 5*x^2 + 3)^(3/2) - 23985/1024*sqrt(x^4 + 5*x^2 + 3) + 62361/2048*log(2
*x^2 + 2*sqrt(x^4 + 5*x^2 + 3) + 5)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 179 vs. 2(88) = 176.

Time = 0.32 (sec) , antiderivative size = 179, normalized size of antiderivative = 1.69

$$\int x^3(2+3x^2)\left(3+5x^2+x^4\right)^{3/2}dx = \frac{1}{1024}\sqrt{x^4+5x^2+3}(2(4(2(8(2x^2+1)x^2-33)x^2+321)x^2-6837)x^2+87147) + \frac{17}{3840}\sqrt{x^4+5x^2+3}(2(4(6(8x^2+5)x^2-127)x^2+2635)x^2-33429) + \frac{19}{384}\sqrt{x^4+5x^2+3}(2(4(6x^2+5)x^2-89)x^2+1095) + \frac{1}{8}\sqrt{x^4+5x^2+3}(2(4x^2+5)x^2-51) - \frac{62361}{2048}\log\left(2x^2-2\sqrt{x^4+5x^2+3}+5\right)$$

```
[In] integrate(x^3*(3*x^2+2)*(x^4+5*x^2+3)^(3/2),x, algorithm="giac")
```

```
[Out] 1/1024*sqrt(x^4 + 5*x^2 + 3)*(2*(4*(2*(8*(2*x^2 + 1)*x^2 - 33)*x^2 + 321)*x
^2 - 6837)*x^2 + 87147) + 17/3840*sqrt(x^4 + 5*x^2 + 3)*(2*(4*(6*(8*x^2 + 5
)*x^2 - 127)*x^2 + 2635)*x^2 - 33429) + 19/384*sqrt(x^4 + 5*x^2 + 3)*(2*(4*
(6*x^2 + 5)*x^2 - 89)*x^2 + 1095) + 1/8*sqrt(x^4 + 5*x^2 + 3)*(2*(4*x^2 + 5
)*x^2 - 51) - 62361/2048*log(2*x^2 - 2*sqrt(x^4 + 5*x^2 + 3) + 5)
```

Mupad [F(-1)]

Timed out.

$$\int x^3(2+3x^2)\left(3+5x^2+x^4\right)^{3/2}dx = \int x^3\left(3x^2+2\right)\left(x^4+5x^2+3\right)^{3/2}dx$$

```
[In] int(x^3*(3*x^2 + 2)*(5*x^2 + x^4 + 3)^(3/2),x)
```

```
[Out] int(x^3*(3*x^2 + 2)*(5*x^2 + x^4 + 3)^(3/2), x)
```

3.158 $\int x(2+3x^2)\left(3+5x^2+x^4\right)^{3/2}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 99

$$\int x(2+3x^2)\left(3+5x^2+x^4\right)^{3/2}\,dx=\frac{429}{256}(5+2x^2)\sqrt{3+5x^2+x^4}-\frac{11}{32}(5+2x^2)\left(3+5x^2+x^4\right)^{3/2}+\frac{3}{10}\left(3+5x^2+x^4\right)^{5/2}-\frac{5577}{512}\operatorname{arctanh}\left(\frac{5+2x^2}{2\sqrt{3+5x^2+x^4}}\right)$$

```
[Out] -11/32*(2*x^2+5)*(x^4+5*x^2+3)^(3/2)+3/10*(x^4+5*x^2+3)^(5/2)-5577/512*arct
anh(1/2*(2*x^2+5)/(x^4+5*x^2+3)^(1/2))+429/256*(2*x^2+5)*(x^4+5*x^2+3)^(1/2
)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {1261, 654, 626, 635, 212}

$$\int x(2+3x^2)\left(3+5x^2+x^4\right)^{3/2}\,dx=-\frac{5577}{512}\operatorname{arctanh}\left(\frac{2x^2+5}{2\sqrt{x^4+5x^2+3}}\right)+\frac{3}{10}\left(x^4+5x^2+3\right)^{5/2}-\frac{11}{32}(2x^2+5)\left(x^4+5x^2+3\right)^{3/2}+\frac{429}{256}(2x^2+5)\sqrt{x^4+5x^2+3}$$

```
[In] Int[x*(2 + 3*x^2)*(3 + 5*x^2 + x^4)^(3/2),x]
```

```
[Out] (429*(5 + 2*x^2)*Sqrt[3 + 5*x^2 + x^4])/256 - (11*(5 + 2*x^2)*(3 + 5*x^2 +
x^4)^(3/2))/32 + (3*(3 + 5*x^2 + x^4)^(5/2))/10 - (5577*ArcTanh[(5 + 2*x^2)
/(2*Sqrt[3 + 5*x^2 + x^4])])/512
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 626

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Dist[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1))), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 635

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 654

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 1261

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(
p_.), x_Symbol] :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x],
 x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{1}{2}\text{Subst}\left(\int (2+3x)\left(3+5x+x^2\right)^{3/2}\,dx,x,x^2\right)\\
&= \frac{3}{10}\left(3+5x^2+x^4\right)^{5/2}-\frac{11}{4}\text{Subst}\left(\int \left(3+5x+x^2\right)^{3/2}\,dx,x,x^2\right)\\
&= -\frac{11}{32}\left(5+2x^2\right)\left(3+5x^2+x^4\right)^{3/2}\\
&\quad+\frac{3}{10}\left(3+5x^2+x^4\right)^{5/2}+\frac{429}{64}\text{Subst}\left(\int \sqrt{3+5x+x^2}\,dx,x,x^2\right)\\
&= \frac{429}{256}\left(5+2x^2\right)\sqrt{3+5x^2+x^4}-\frac{11}{32}\left(5+2x^2\right)\left(3+5x^2+x^4\right)^{3/2}\\
&\quad+\frac{3}{10}\left(3+5x^2+x^4\right)^{5/2}-\frac{5577}{512}\text{Subst}\left(\int \frac{1}{\sqrt{3+5x+x^2}}\,dx,x,x^2\right)
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{429}{256}\left(5+2x^2\right)\sqrt{3+5x^2+x^4} - \frac{11}{32}\left(5+2x^2\right)\left(3+5x^2+x^4\right)^{3/2} \\
&\quad + \frac{3}{10}\left(3+5x^2+x^4\right)^{5/2} - \frac{5577}{256}\text{Subst}\left(\int \frac{1}{4-x^2}\,dx, x, \frac{5+2x^2}{\sqrt{3+5x^2+x^4}}\right) \\
&= \frac{429}{256}\left(5+2x^2\right)\sqrt{3+5x^2+x^4} - \frac{11}{32}\left(5+2x^2\right)\left(3+5x^2+x^4\right)^{3/2} \\
&\quad + \frac{3}{10}\left(3+5x^2+x^4\right)^{5/2} - \frac{5577}{512}\tanh^{-1}\left(\frac{5+2x^2}{2\sqrt{3+5x^2+x^4}}\right)
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.70

$$
\begin{aligned}
&\int x\left(2+3x^2\right)\left(3+5x^2 + x^4\right)^{3/2}\,dx = \frac{\sqrt{3+5x^2+x^4}\,(7581+2170x^2+5304x^4+2960x^6+384x^8)}{1280} \\
&\quad + \frac{5577}{512}\log\left(-5-2x^2+2\sqrt{3+5x^2+x^4}\right)
\end{aligned}
$$

```
[In] Integrate[x*(2 + 3*x^2)*(3 + 5*x^2 + x^4)^(3/2),x]
```

```
[Out] (Sqrt[3 + 5*x^2 + x^4]*(7581 + 2170*x^2 + 5304*x^4 + 2960*x^6 + 384*x^8))/1
280 + (5577*Log[-5 - 2*x^2 + 2*Sqrt[3 + 5*x^2 + x^4]])/512
```

Maple [A] (verified)

Time = 0.19 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.59

method	result
risch	$\frac{(384x^8+2960x^6+5304x^4+2170x^2+7581)\sqrt{x^4+5x^2+3}}{1280} - \frac{5577\ln\left(\frac{5}{2}+x^2+\sqrt{x^4+5x^2+3}\right)}{512}$
trager	$\left(\frac{3}{10}x^8+\frac{37}{16}x^6+\frac{663}{160}x^4+\frac{217}{128}x^2+\frac{7581}{1280}\right)\sqrt{x^4+5x^2+3} - \frac{5577\ln\left(2x^2+5+2\sqrt{x^4+5x^2+3}\right)}{512}$
pseudoelliptic	$-\frac{5577\ln\left(2x^2+5+2\sqrt{x^4+5x^2+3}\right)}{512} + \frac{(384x^8+2960x^6+5304x^4+2170x^2+7581)\sqrt{x^4+5x^2+3}}{1280}$
default	$-\frac{5577\ln\left(\frac{5}{2}+x^2+\sqrt{x^4+5x^2+3}\right)}{512} + \frac{37x^6\sqrt{x^4+5x^2+3}}{16} + \frac{663x^4\sqrt{x^4+5x^2+3}}{160} + \frac{217x^2\sqrt{x^4+5x^2+3}}{128} + \frac{7581\sqrt{x^4+5x^2+}}{1280}$
elliptic	$-\frac{5577\ln\left(\frac{5}{2}+x^2+\sqrt{x^4+5x^2+3}\right)}{512} + \frac{37x^6\sqrt{x^4+5x^2+3}}{16} + \frac{663x^4\sqrt{x^4+5x^2+3}}{160} + \frac{217x^2\sqrt{x^4+5x^2+3}}{128} + \frac{7581\sqrt{x^4+5x^2+}}{1280}$

```
[In] int(x*(3*x^2+2)*(x^4+5*x^2+3)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/1280*(384*x^8+2960*x^6+5304*x^4+2170*x^2+7581)*(x^4+5*x^2+3)^(1/2)-5577/5
12*ln(5/2+x^2+(x^4+5*x^2+3)^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.62

$$\int x(2+3x^2)\left(3+5x^2+x^4\right)^{3/2}\,dx = \frac{1}{1280}\left(384\,x^8+2960\,x^6+5304\,x^4+2170\,x^2+7581\right)\sqrt{x^4+5\,x^2+3} + \frac{5577}{512}\,\log\left(-2\,x^2+2\,\sqrt{x^4+5\,x^2+3}-5\right)$$

```
[In] integrate(x*(3*x^2+2)*(x^4+5*x^2+3)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/1280*(384*x^8 + 2960*x^6 + 5304*x^4 + 2170*x^2 + 7581)*sqrt(x^4 + 5*x^2 +
 3) + 5577/512*log(-2*x^2 + 2*sqrt(x^4 + 5*x^2 + 3) - 5)
```

Sympy [A] (verification not implemented)

Time = 1.10 (sec) , antiderivative size = 165, normalized size of antiderivative = 1.67

$$\int x(2+3x^2)\left(3+5x^2+x^4\right)^{3/2}\,dx = 3\left(\frac{x^2}{2}+\frac{5}{4}\right)\sqrt{x^4+5x^2+3} + \frac{19\left(\frac{x^4}{3}+\frac{5x^2}{12}-\frac{17}{8}\right)\sqrt{x^4+5x^2+3}}{2} + \frac{17\sqrt{x^4+5x^2+3}\left(\frac{x^6}{4}+\frac{5x^4}{24}-\frac{89x^2}{96}+\frac{365}{64}\right)}{2} + \frac{3\sqrt{x^4+5x^2+3}\left(\frac{x^8}{5}+\frac{x^6}{8}-\frac{127x^4}{240}+\frac{527x^2}{192}-\frac{11143}{640}\right)}{2} - \frac{5577\log\left(2x^2+2\sqrt{x^4+5x^2+3}+5\right)}{512}$$

```
[In] integrate(x*(3*x**2+2)*(x**4+5*x**2+3)**(3/2),x)
```

```
[Out] 3*(x**2/2 + 5/4)*sqrt(x**4 + 5*x**2 + 3) + 19*(x**4/3 + 5*x**2/12 - 17/8)*s
qrt(x**4 + 5*x**2 + 3)/2 + 17*sqrt(x**4 + 5*x**2 + 3)*(x**6/4 + 5*x**4/24 -
 89*x**2/96 + 365/64)/2 + 3*sqrt(x**4 + 5*x**2 + 3)*(x**8/5 + x**6/8 - 127*
x**4/240 + 527*x**2/192 - 11143/640)/2 - 5577*log(2*x**2 + 2*sqrt(x**4 + 5*
x**2 + 3) + 5)/512
```

Maxima [A] (verification not implemented)

none

Time = 0.18 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.02

$$\int x(2+3x^2)\left(3+5x^2+x^4\right)^{3/2}\,dx = -\frac{11}{16}\left(x^4+5\,x^2+3\right)^{\frac{3}{2}}x^2 + \frac{3}{10}\left(x^4+5\,x^2+3\right)^{\frac{5}{2}} + \frac{429}{128}\sqrt{x^4+5\,x^2+3}x^2 - \frac{55}{32}\left(x^4+5\,x^2+3\right)^{\frac{3}{2}} + \frac{2145}{256}\sqrt{x^4+5\,x^2+3} - \frac{5577}{512}\log\left(2\,x^2+2\sqrt{x^4+5\,x^2+3}+5\right)$$

```
[In] integrate(x*(3*x^2+2)*(x^4+5*x^2+3)^(3/2),x, algorithm="maxima")
```

```
[Out] -11/16*(x^4 + 5*x^2 + 3)^(3/2)*x^2 + 3/10*(x^4 + 5*x^2 + 3)^(5/2) + 429/128
*sqrt(x^4 + 5*x^2 + 3)*x^2 - 55/32*(x^4 + 5*x^2 + 3)^(3/2) + 2145/256*sqrt(
x^4 + 5*x^2 + 3) - 5577/512*log(2*x^2 + 2*sqrt(x^4 + 5*x^2 + 3) + 5)
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.53

$$\int x(2+3x^2)\left(3+5x^2+x^4\right)^{3/2}\,dx = \frac{1}{1280}\sqrt{x^4+5\,x^2+3}\left(2\left(4\left(6\left(8\,x^2+5\right)x^2-127\right)x^2+2635\right)x^2-33429\right) + \frac{17}{384}\sqrt{x^4+5\,x^2+3}\left(2\left(4\left(6\,x^2+5\right)x^2-89\right)x^2+1095\right) + \frac{19}{48}\sqrt{x^4+5\,x^2+3}\left(2\left(4\,x^2+5\right)x^2-51\right) + \frac{3}{4}\sqrt{x^4+5\,x^2+3}\left(2\,x^2+5\right) + \frac{5577}{512}\log\left(2\,x^2-2\sqrt{x^4+5\,x^2+3}+5\right)$$

```
[In] integrate(x*(3*x^2+2)*(x^4+5*x^2+3)^(3/2),x, algorithm="giac")
```

```
[Out] 1/1280*sqrt(x^4 + 5*x^2 + 3)*(2*(4*(6*(8*x^2 + 5)*x^2 - 127)*x^2 + 2635)*x^
2 - 33429) + 17/384*sqrt(x^4 + 5*x^2 + 3)*(2*(4*(6*x^2 + 5)*x^2 - 89)*x^2 +
 1095) + 19/48*sqrt(x^4 + 5*x^2 + 3)*(2*(4*x^2 + 5)*x^2 - 51) + 3/4*sqrt(x^
4 + 5*x^2 + 3)*(2*x^2 + 5) + 5577/512*log(2*x^2 - 2*sqrt(x^4 + 5*x^2 + 3) +
 5)
```

Mupad [B] (verification not implemented)

Time = 7.79 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.28

$$\int x(2+3x^2)\,(3+5x^2+x^4)^{3/2}\,dx=\frac{\left(x^2+\frac{5}{2}\right)\,(x^4+5\,x^2+3)^{3/2}}{4}-\frac{15\,x^2\,(x^4+5\,x^2+3)^{3/2}}{16}-\frac{5577\,\ln\left(\sqrt{x^4+5\,x^2+3}+x^2+\frac{5}{2}\right)}{512}+\frac{585\,(2\,x^2+5)\,\sqrt{x^4+5\,x^2+3}}{256}-\frac{39\left(\frac{x^2}{2}+\frac{5}{4}\right)\,\sqrt{x^4+5\,x^2+3}}{16}-\frac{75\,(x^4+5\,x^2+3)^{3/2}}{32}+\frac{3\,(x^4+5\,x^2+3)^{5/2}}{10}$$

```
[In] int(x*(3*x^2 + 2)*(5*x^2 + x^4 + 3)^(3/2),x)
```

```
[Out] ((x^2 + 5/2)*(5*x^2 + x^4 + 3)^(3/2))/4 - (15*x^2*(5*x^2 + x^4 + 3)^(3/2))/
16 - (5577*log((5*x^2 + x^4 + 3)^(1/2) + x^2 + 5/2))/512 + (585*(2*x^2 + 5)
*(5*x^2 + x^4 + 3)^(1/2))/256 - (39*(x^2/2 + 5/4)*(5*x^2 + x^4 + 3)^(1/2))/
16 - (75*(5*x^2 + x^4 + 3)^(3/2))/32 + (3*(5*x^2 + x^4 + 3)^(5/2))/10
```

3.159 $\int \frac{(2+3x^2)(3+5x^2+x^4)^{3/2}}{x} dx$

Optimal result

Integrand size = 25, antiderivative size = 119

$$\int \frac{(2+3x^2)(3+5x^2+x^4)^{3/2}}{x} dx = \frac{1}{128}(199-74x^2)\sqrt{3+5x^2+x^4} + \frac{1}{48}(61+18x^2)(3+5x^2+x^4)^{3/2} + \frac{2401}{256}\text{arctanh}\left(\frac{5+2x^2}{2\sqrt{3+5x^2+x^4}}\right) - 3\sqrt{3}\text{arctanh}\left(\frac{6+5x^2}{2\sqrt{3}\sqrt{3+5x^2+x^4}}\right)$$

```
[Out] 1/48*(18*x^2+61)*(x^4+5*x^2+3)^(3/2)+2401/256*arctanh(1/2*(2*x^2+5)/(x^4+5*
x^2+3)^(1/2))-3*arctanh(1/6*(5*x^2+6)*3^(1/2)/(x^4+5*x^2+3)^(1/2))*3^(1/2)+
1/128*(-74*x^2+199)*(x^4+5*x^2+3)^(1/2)
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {1265, 828, 857, 635, 212, 738}

$$\int \frac{(2+3x^2)(3+5x^2+x^4)^{3/2}}{x} dx = \frac{2401}{256}\text{arctanh}\left(\frac{2x^2+5}{2\sqrt{x^4+5x^2+3}}\right) - 3\sqrt{3}\text{arctanh}\left(\frac{5x^2+6}{2\sqrt{3}\sqrt{x^4+5x^2+3}}\right) + \frac{1}{48}(18x^2+61)(x^4+5x^2+3)^{3/2} + \frac{1}{128}(199-74x^2)\sqrt{x^4+5x^2+3}$$

```
[In] Int[((2 + 3*x^2)*(3 + 5*x^2 + x^4)^(3/2))/x,x]
```

```
[Out] ((199 - 74*x^2)*Sqrt[3 + 5*x^2 + x^4])/128 + ((61 + 18*x^2)*(3 + 5*x^2 + x^
4)^(3/2))/48 + (2401*ArcTanh[(5 + 2*x^2)/(2*Sqrt[3 + 5*x^2 + x^4])])/256 -
3*Sqrt[3]*ArcTanh[(6 + 5*x^2)/(2*Sqrt[3]*Sqrt[3 + 5*x^2 + x^4])]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 635

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 828

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2)
 - g*(c*d + 2*c*d*p - b*e*p) + g*c*e*(m + 2*p + 1)*x)*((a + b*x + c*x^2)^p/
(c*e^2*(m + 2*p + 1)*(m + 2*p + 2))), x] - Dist[p/(c*e^2*(m + 2*p + 1)*(m +
 2*p + 2)), Int[(d + e*x)^m*(a + b*x + c*x^2)^(p - 1)*Simp[c*e*f*(b*d - 2*a
*e)*(m + 2*p + 2) + g*(a*e*(b*e - 2*c*d*m + b*e*m) + b*d*(b*e*p - c*d - 2*c
*d*p)) + (c*e*f*(2*c*d - b*e)*(m + 2*p + 2) + g*(b^2*e^2*(p + m + 1) - 2*c^
2*d^2*(1 + 2*p) - c*e*(b*d*(m - 2*p) + 2*a*e*(m + 2*p + 1))))*x, x], x], x]
 /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
- b*d*e + a*e^2, 0] && GtQ[p, 0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[
m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p]
|| IntegersQ[2*m, 2*p])
```

Rule 857

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1265

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{2}\text{Subst}\left(\int \frac{(2+3x)\left(3+5x+x^2\right)^{3/2}}{x}\,dx, x, x^2\right)\\
&= \frac{1}{48}\left(61+18x^2\right)\left(3+5x^2+x^4\right)^{3/2}-\frac{1}{16}\text{Subst}\left(\int \frac{\left(-48+\frac{37x}{2}\right)\sqrt{3+5x+x^2}}{x}\,dx, x, x^2\right)\\
&= \frac{1}{128}\left(199-74x^2\right)\sqrt{3+5x^2+x^4}+\frac{1}{48}\left(61+18x^2\right)\left(3+5x^2+x^4\right)^{3/2}\\
&\quad+\frac{1}{64}\text{Subst}\left(\int \frac{576+\frac{2401x}{4}}{x\sqrt{3+5x+x^2}}\,dx, x, x^2\right)\\
&= \frac{1}{128}\left(199-74x^2\right)\sqrt{3+5x^2+x^4}+\frac{1}{48}\left(61+18x^2\right)\left(3+5x^2+x^4\right)^{3/2}\\
&\quad+9\text{Subst}\left(\int \frac{1}{x\sqrt{3+5x+x^2}}\,dx, x, x^2\right)+\frac{2401}{256}\text{Subst}\left(\int \frac{1}{\sqrt{3+5x+x^2}}\,dx, x, x^2\right)\\
&= \frac{1}{128}\left(199-74x^2\right)\sqrt{3+5x^2+x^4}+\frac{1}{48}\left(61+18x^2\right)\left(3+5x^2+x^4\right)^{3/2}\\
&\quad-18\text{Subst}\left(\int \frac{1}{12-x^2}\,dx, x, \frac{6+5x^2}{\sqrt{3+5x^2+x^4}}\right)\\
&\quad+\frac{2401}{128}\text{Subst}\left(\int \frac{1}{4-x^2}\,dx, x, \frac{5+2x^2}{\sqrt{3+5x^2+x^4}}\right)\\
&= \frac{1}{128}\left(199-74x^2\right)\sqrt{3+5x^2+x^4}+\frac{1}{48}\left(61+18x^2\right)\left(3+5x^2+x^4\right)^{3/2}\\
&\quad+\frac{2401}{256}\tanh^{-1}\left(\frac{5+2x^2}{2\sqrt{3+5x^2+x^4}}\right)-3\sqrt{3}\tanh^{-1}\left(\frac{6+5x^2}{2\sqrt{3}\sqrt{3+5x^2+x^4}}\right)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.83

$$\begin{aligned}
&\int \frac{(2+3x^2)\left(3+5x^2+x^4\right)^{3/2}}{x}\,dx = 6\sqrt{3}\text{arctanh}\left(\frac{x^2-\sqrt{3+5x^2+x^4}}{\sqrt{3}}\right)\\
&+\frac{1}{768}\left(2\sqrt{3+5x^2+x^4}\left(2061+2650x^2+1208x^4+144x^6\right)\right.\\
&\left.-7203\log\left(-5-2x^2+2\sqrt{3+5x^2+x^4}\right)\right)
\end{aligned}$$

[In] Integrate[((2 + 3*x^2)*(3 + 5*x^2 + x^4)^(3/2))/x,x]

[Out] 6*Sqrt[3]*ArcTanh[(x^2 - Sqrt[3 + 5*x^2 + x^4])/Sqrt[3]] + (2*Sqrt[3 + 5*x^2 + x^4]*(2061 + 2650*x^2 + 1208*x^4 + 144*x^6) - 7203*Log[-5 - 2*x^2 + 2*Sqrt[3 + 5*x^2 + x^4]])/768

Maple [A] (verified)

Time = 0.48 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.73

method	result
pseudoelliptic	$-3\,\operatorname{arctanh}\left(\frac{(5x^2+6)\sqrt{3}}{6\sqrt{x^4+5x^2+3}}\right)\sqrt{3}+\frac{2401\ln\left(2x^2+5+2\sqrt{x^4+5x^2+3}\right)}{256}+\frac{(288x^6+2416x^4+5300x^2+4122)\sqrt{x^4+5x^2+}}{768}$
trager	$\left(\frac{3}{8}x^6+\frac{151}{48}x^4+\frac{1325}{192}x^2+\frac{687}{128}\right)\sqrt{x^4+5x^2+3}+3\,\mathrm{RootOf}\left(\_Z^2-3\right)\ln\left(-\frac{-5\,\mathrm{RootOf}\left(\_Z^2-3\right)}{}\right.$
default	$\frac{2401\ln\left(\frac{5}{2}+x^2+\sqrt{x^4+5x^2+3}\right)}{256}+\frac{3x^6\sqrt{x^4+5x^2+3}}{8}+\frac{151x^4\sqrt{x^4+5x^2+3}}{48}+\frac{1325x^2\sqrt{x^4+5x^2+3}}{192}+\frac{687\sqrt{x^4+5x^2+3}}{128}-$
elliptic	$\frac{2401\ln\left(\frac{5}{2}+x^2+\sqrt{x^4+5x^2+3}\right)}{256}+\frac{3x^6\sqrt{x^4+5x^2+3}}{8}+\frac{151x^4\sqrt{x^4+5x^2+3}}{48}+\frac{1325x^2\sqrt{x^4+5x^2+3}}{192}+\frac{687\sqrt{x^4+5x^2+3}}{128}-$

[In] int((3*x^2+2)*(x^4+5*x^2+3)^(3/2)/x,x,method=_RETURNVERBOSE)

[Out] -3*arctanh(1/6*(5*x^2+6)*3^(1/2)/(x^4+5*x^2+3)^(1/2))*3^(1/2)+2401/256*ln(2*x^2+5+2*(x^4+5*x^2+3)^(1/2))+1/768*(288*x^6+2416*x^4+5300*x^2+4122)*(x^4+5*x^2+3)^(1/2)

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.89

$$\int \frac{(2+3x^2)\left(3+5x^2+x^4\right)^{3/2}}{x}\,dx = \frac{1}{384}\left(144\,x^6+1208\,x^4+2650\,x^2+2061\right)\sqrt{x^4+5\,x^2+3}$$
$$+3\sqrt{3}\log\left(\frac{25\,x^2-2\sqrt{3}(5\,x^2+6)-2\sqrt{x^4+5\,x^2+3}(5\sqrt{3}-6)+30}{x^2}\right)$$
$$-\frac{2401}{256}\log\left(-2\,x^2+2\sqrt{x^4+5\,x^2+3}-5\right)$$

[In] integrate((3*x^2+2)*(x^4+5*x^2+3)^(3/2)/x,x, algorithm="fricas")

[Out] 1/384*(144*x^6 + 1208*x^4 + 2650*x^2 + 2061)*sqrt(x^4 + 5*x^2 + 3) + 3*sqrt(3)*log((25*x^2 - 2*sqrt(3)*(5*x^2 + 6) - 2*sqrt(x^4 + 5*x^2 + 3)*(5*sqrt(3) - 6) + 30)/x^2) - 2401/256*log(-2*x^2 + 2*sqrt(x^4 + 5*x^2 + 3) - 5)

Sympy [F]

$$\int \frac{(2+3x^2)\left(3+5x^2+x^4\right)^{3/2}}{x}\,dx = \int \frac{(3x^2+2)\left(x^4+5x^2+3\right)^{\frac{3}{2}}}{x}\,dx$$

[In] integrate((3*x**2+2)*(x**4+5*x**2+3)**(3/2)/x,x)

[Out] Integral((3*x**2 + 2)*(x**4 + 5*x**2 + 3)**(3/2)/x, x)

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.01

$$\int \frac{(2+3x^2)\left(3+5x^2+x^4\right)^{3/2}}{x}\,dx = \frac{3}{8}\left(x^4+5\,x^2+3\right)^{\frac{3}{2}}x^2 - \frac{37}{64}\sqrt{x^4+5\,x^2+3}x^2 + \frac{61}{48}\left(x^4+5\,x^2+3\right)^{\frac{3}{2}} - 3\sqrt{3}\log\left(\frac{2\sqrt{3}\sqrt{x^4+5\,x^2+3}}{x^2}+\frac{6}{x^2}+5\right) + \frac{199}{128}\sqrt{x^4+5\,x^2+3} + \frac{2401}{256}\log\left(2\,x^2+2\sqrt{x^4+5\,x^2+3}+5\right)$$

[In] integrate((3*x^2+2)*(x^4+5*x^2+3)^(3/2)/x,x, algorithm="maxima")

[Out] 3/8*(x^4 + 5*x^2 + 3)^(3/2)*x^2 - 37/64*sqrt(x^4 + 5*x^2 + 3)*x^2 + 61/48*(x^4 + 5*x^2 + 3)^(3/2) - 3*sqrt(3)*log(2*sqrt(3)*sqrt(x^4 + 5*x^2 + 3)/x^2 + 6/x^2 + 5) + 199/128*sqrt(x^4 + 5*x^2 + 3) + 2401/256*log(2*x^2 + 2*sqrt(x^4 + 5*x^2 + 3) + 5)

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.95

$$\int \frac{(2+3x^2)\left(3+5x^2+x^4\right)^{3/2}}{x}\,dx = \frac{1}{384}\sqrt{x^4+5\,x^2+3}\left(2\left(4\left(18\,x^2+151\right)x^2+1325\right)x^2+2061\right) + 3\sqrt{3}\log\left(\frac{x^2+\sqrt{3}-\sqrt{x^4+5\,x^2+3}}{x^2-\sqrt{3}-\sqrt{x^4+5\,x^2+3}}\right) - \frac{2401}{256}\log\left(2\,x^2-2\sqrt{x^4+5\,x^2+3}+5\right)$$

[In] integrate((3*x^2+2)*(x^4+5*x^2+3)^(3/2)/x,x, algorithm="giac")

[Out] 1/384*sqrt(x^4 + 5*x^2 + 3)*(2*(4*(18*x^2 + 151)*x^2 + 1325)*x^2 + 2061) + 3*sqrt(3)*log((x^2 + sqrt(3) - sqrt(x^4 + 5*x^2 + 3))/(x^2 - sqrt(3) - sqrt(x^4 + 5*x^2 + 3))) - 2401/256*log(2*x^2 - 2*sqrt(x^4 + 5*x^2 + 3) + 5)

Mupad [F(-1)]

Timed out.

$$\int \frac{(2+3x^2)\,(3+5x^2+x^4)^{3/2}}{x}\,dx = \int \frac{(3\,x^2+2)\,\left(x^4+5\,x^2+3\right)^{3/2}}{x}\,dx$$

```
[In] int(((3*x^2 + 2)*(5*x^2 + x^4 + 3)^(3/2))/x,x)
[Out] int(((3*x^2 + 2)*(5*x^2 + x^4 + 3)^(3/2))/x, x)
```

3.160 $\int \frac{(2+3x^2)(3+5x^2+x^4)^{3/2}}{x^3}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 122

$$\int \frac{(2+3x^2)(3+5x^2+x^4)^{3/2}}{x^3}\,dx = \frac{3}{16}(109+18x^2)\sqrt{3+5x^2+x^4} - \frac{(2-x^2)(3+5x^2+x^4)^{3/2}}{2x^2} + \frac{609}{32}\operatorname{arctanh}\left(\frac{5+2x^2}{2\sqrt{3+5x^2+x^4}}\right) - 12\sqrt{3}\operatorname{arctanh}\left(\frac{6+5x^2}{2\sqrt{3}\sqrt{3+5x^2+x^4}}\right)$$

```
[Out] -1/2*(-x^2+2)*(x^4+5*x^2+3)^(3/2)/x^2+609/32*arctanh(1/2*(2*x^2+5)/(x^4+5*x
^2+3)^(1/2))-12*arctanh(1/6*(5*x^2+6)*3^(1/2)/(x^4+5*x^2+3)^(1/2))*3^(1/2)+
3/16*(18*x^2+109)*(x^4+5*x^2+3)^(1/2)
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.280, Rules used = {1265, 826, 828, 857, 635, 212, 738}

$$\int \frac{(2+3x^2)(3+5x^2+x^4)^{3/2}}{x^3}\,dx = \frac{609}{32}\operatorname{arctanh}\left(\frac{2x^2+5}{2\sqrt{x^4+5x^2+3}}\right) - 12\sqrt{3}\operatorname{arctanh}\left(\frac{5x^2+6}{2\sqrt{3}\sqrt{x^4+5x^2+3}}\right) - \frac{(2-x^2)(x^4+5x^2+3)^{3/2}}{2x^2} + \frac{3}{16}(18x^2+109)\sqrt{x^4+5x^2+3}$$

```
[In] Int[((2 + 3*x^2)*(3 + 5*x^2 + x^4)^(3/2))/x^3,x]
```

```
[Out] (3*(109 + 18*x^2)*Sqrt[3 + 5*x^2 + x^4])/16 - ((2 - x^2)*(3 + 5*x^2 + x^4)^
(3/2))/(2*x^2) + (609*ArcTanh[(5 + 2*x^2)/(2*Sqrt[3 + 5*x^2 + x^4])])/32 -
12*Sqrt[3]*ArcTanh[(6 + 5*x^2)/(2*Sqrt[3]*Sqrt[3 + 5*x^2 + x^4])]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 635

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 826

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(e*f*(m + 2*p + 2) -
 d*g*(2*p + 1) + e*g*(m + 1)*x)*((a + b*x + c*x^2)^p/(e^2*(m + 1)*(m + 2*p
+ 2))), x] + Dist[p/(e^2*(m + 1)*(m + 2*p + 2)), Int[(d + e*x)^(m + 1)*(a +
 b*x + c*x^2)^(p - 1)*Simp[g*(b*d + 2*a*e + 2*a*e*m + 2*b*d*p) - f*b*e*(m +
 2*p + 2) + (g*(2*c*d + b*e + b*e*m + 4*c*d*p) - 2*c*e*f*(m + 2*p + 2))*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] && RationalQ[p] && p > 0 && (LtQ[m, -1] || Eq
Q[p, 1] || (IntegerQ[p] &&  !RationalQ[m])) && NeQ[m, -1] &&  !ILtQ[m + 2*p
 + 1, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 828

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2)
 - g*(c*d + 2*c*d*p - b*e*p) + g*c*e*(m + 2*p + 1)*x)*((a + b*x + c*x^2)^p/
(c*e^2*(m + 2*p + 1)*(m + 2*p + 2))), x] - Dist[p/(c*e^2*(m + 2*p + 1)*(m +
 2*p + 2)), Int[(d + e*x)^m*(a + b*x + c*x^2)^(p - 1)*Simp[c*e*f*(b*d - 2*a
*e)*(m + 2*p + 2) + g*(a*e*(b*e - 2*c*d*m + b*e*m) + b*d*(b*e*p - c*d - 2*c
*d*p)) + (c*e*f*(2*c*d - b*e)*(m + 2*p + 2) + g*(b^2*e^2*(p + m + 1) - 2*c^
2*d^2*(1 + 2*p) - c*e*(b*d*(m - 2*p) + 2*a*e*(m + 2*p + 1))))*x, x], x], x]
 /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
```

```
- b*d*e + a*e^2, 0] && GtQ[p, 0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[
m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p]
|| IntegersQ[2*m, 2*p])
```

Rule 857

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1265

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{1}{2}\text{Subst}\left(\int \frac{(2+3x)\left(3+5x+x^2\right)^{3/2}}{x^2}\,dx, x, x^2\right)\\
&= -\frac{\left(2-x^2\right)\left(3+5x^2+x^4\right)^{3/2}}{2x^2}-\frac{1}{4}\text{Subst}\left(\int \frac{(-48-27x)\sqrt{3+5x+x^2}}{x}\,dx, x, x^2\right)\\
&= \frac{3}{16}\left(109+18x^2\right)\sqrt{3+5x^2+x^4}-\frac{\left(2-x^2\right)\left(3+5x^2+x^4\right)^{3/2}}{2x^2}\\
&\quad+\frac{1}{16}\text{Subst}\left(\int \frac{576+\frac{609x}{2}}{x\sqrt{3+5x+x^2}}\,dx, x, x^2\right)\\
&= \frac{3}{16}\left(109+18x^2\right)\sqrt{3+5x^2+x^4}-\frac{\left(2-x^2\right)\left(3+5x^2+x^4\right)^{3/2}}{2x^2}\\
&\quad+\frac{609}{32}\text{Subst}\left(\int \frac{1}{\sqrt{3+5x+x^2}}\,dx, x, x^2\right)+36\text{Subst}\left(\int \frac{1}{x\sqrt{3+5x+x^2}}\,dx, x, x^2\right)\\
&= \frac{3}{16}\left(109+18x^2\right)\sqrt{3+5x^2+x^4}-\frac{\left(2-x^2\right)\left(3+5x^2+x^4\right)^{3/2}}{2x^2}\\
&\quad+\frac{609}{16}\text{Subst}\left(\int \frac{1}{4-x^2}\,dx, x, \frac{5+2x^2}{\sqrt{3+5x^2+x^4}}\right)\\
&\quad-72\text{Subst}\left(\int \frac{1}{12-x^2}\,dx, x, \frac{6+5x^2}{\sqrt{3+5x^2+x^4}}\right)
\end{aligned}
$$

$$= \frac{3}{16}(109+18x^2)\sqrt{3+5x^2+x^4} - \frac{(2-x^2)(3+5x^2+x^4)^{3/2}}{2x^2} + \frac{609}{32}\tanh^{-1}\left(\frac{5+2x^2}{2\sqrt{3+5x^2+x^4}}\right) - 12\sqrt{3}\tanh^{-1}\left(\frac{6+5x^2}{2\sqrt{3}\sqrt{3+5x^2+x^4}}\right)$$

Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.84

$$\int \frac{(2+3x^2)(3+5x^2+x^4)^{3/2}}{x^3}\,dx = 24\sqrt{3}\text{arctanh}\left(\frac{x^2-\sqrt{3+5x^2+x^4}}{\sqrt{3}}\right) + \frac{1}{32}\left(\frac{2\sqrt{3+5x^2+x^4}(-48+271x^2+78x^4+8x^6)}{x^2} - 609\log\left(-5-2x^2+2\sqrt{3+5x^2+x^4}\right)\right)$$

```
[In] Integrate[((2 + 3*x^2)*(3 + 5*x^2 + x^4)^(3/2))/x^3,x]
```

```
[Out] 24*Sqrt[3]*ArcTanh[(x^2 - Sqrt[3 + 5*x^2 + x^4])/Sqrt[3]] + ((2*Sqrt[3 + 5*
x^2 + x^4]*(-48 + 271*x^2 + 78*x^4 + 8*x^6))/x^2 - 609*Log[-5 - 2*x^2 + 2*S
qrt[3 + 5*x^2 + x^4]])/32
```

Maple [A] (verified)

Time = 0.53 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.79

method	result
pseudoelliptic	$\frac{-384\,\text{arctanh}\left(\frac{(5x^2+6)\sqrt{3}}{6\sqrt{x^4+5x^2+3}}\right)\sqrt{3}\,x^2+609\ln\left(2x^2+5+2\sqrt{x^4+5x^2+3}\right)x^2+16\sqrt{x^4+5x^2+3}\,\left(x^6+\frac{39}{4}x^4+\frac{271}{8}x^2-6\right)}{32x^2}$
trager	$\frac{(8x^6+78x^4+271x^2-48)\sqrt{x^4+5x^2+3}}{16x^2} - 12\,\text{RootOf}\left(\_Z^2-3\right)\ln\left(-\frac{5\,\text{RootOf}\left(\_Z^2-3\right)x^2+6\sqrt{x^4+5x^2+3}+6\,\text{R}}{x^2}\right.$
default	$\frac{609\ln\left(\frac{5}{2}+x^2+\sqrt{x^4+5x^2+3}\right)}{32}+\frac{x^4\sqrt{x^4+5x^2+3}}{2}+\frac{39x^2\sqrt{x^4+5x^2+3}}{8}+\frac{271\sqrt{x^4+5x^2+3}}{16}-12\,\text{arctanh}\left(\frac{(5x^2+6)}{6\sqrt{x^4+5x}}\right.$
risch	$\frac{609\ln\left(\frac{5}{2}+x^2+\sqrt{x^4+5x^2+3}\right)}{32}+\frac{x^4\sqrt{x^4+5x^2+3}}{2}+\frac{39x^2\sqrt{x^4+5x^2+3}}{8}+\frac{271\sqrt{x^4+5x^2+3}}{16}-12\,\text{arctanh}\left(\frac{(5x^2+6)}{6\sqrt{x^4+5x}}\right.$
elliptic	$\frac{609\ln\left(\frac{5}{2}+x^2+\sqrt{x^4+5x^2+3}\right)}{32}+\frac{x^4\sqrt{x^4+5x^2+3}}{2}+\frac{39x^2\sqrt{x^4+5x^2+3}}{8}+\frac{271\sqrt{x^4+5x^2+3}}{16}-12\,\text{arctanh}\left(\frac{(5x^2+6)}{6\sqrt{x^4+5x}}\right.$

```
[In] int((3*x^2+2)*(x^4+5*x^2+3)^(3/2)/x^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/32*(-384*arctanh(1/6*(5*x^2+6)*3^(1/2)/(x^4+5*x^2+3)^(1/2))*3^(1/2)*x^2+6
09*ln(2*x^2+5+2*(x^4+5*x^2+3)^(1/2))*x^2+16*(x^4+5*x^2+3)^(1/2)*(x^6+39/4*x
^4+271/8*x^2-6))/x^2
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.00

$$\int \frac{(2+3x^2)(3+5x^2+x^4)^{3/2}}{x^3}\,dx = \frac{1536\sqrt{3}x^2\log\left(\frac{25\,x^2-2\,\sqrt{3}(5\,x^2+6)-2\,\sqrt{x^4+5\,x^2+3}\left(5\,\sqrt{3}-6\right)+30}{x^2}\right)-2436\,x^2\log\left(\right.}{}$$

```
[In] integrate((3*x^2+2)*(x^4+5*x^2+3)^(3/2)/x^3,x, algorithm="fricas")
```

```
[Out] 1/128*(1536*sqrt(3)*x^2*log((25*x^2 - 2*sqrt(3)*(5*x^2 + 6) - 2*sqrt(x^4 +
5*x^2 + 3)*(5*sqrt(3) - 6) + 30)/x^2) - 2436*x^2*log(-2*x^2 + 2*sqrt(x^4 +
5*x^2 + 3) - 5) + 1541*x^2 + 8*(8*x^6 + 78*x^4 + 271*x^2 - 48)*sqrt(x^4 + 5
*x^2 + 3))/x^2
```

Sympy [F]

$$\int \frac{(2+3x^2)(3+5x^2+x^4)^{3/2}}{x^3}\,dx = \int \frac{(3x^2+2)(x^4+5x^2+3)^{\frac{3}{2}}}{x^3}\,dx$$

```
[In] integrate((3*x**2+2)*(x**4+5*x**2+3)**(3/2)/x**3,x)
```

```
[Out] Integral((3*x**2 + 2)*(x**4 + 5*x**2 + 3)**(3/2)/x**3, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 120, normalized size of antiderivative = 0.98

$$\begin{aligned}\int \frac{(2+3x^2)(3+5x^2+x^4)^{3/2}}{x^3}\,dx &= \frac{27}{8}\,\sqrt{x^4+5\,x^2+3}x^2+\frac{1}{2}\,(x^4+5\,x^2+3)^{\frac{3}{2}} \\ &\quad -12\,\sqrt{3}\log\left(\frac{2\,\sqrt{3}\sqrt{x^4+5\,x^2+3}}{x^2}+\frac{6}{x^2}+5\right)+\frac{327}{16}\,\sqrt{x^4+5\,x^2+3} \\ &\quad -\frac{(x^4+5\,x^2+3)^{\frac{3}{2}}}{x^2}+\frac{609}{32}\,\log\left(2\,x^2+2\,\sqrt{x^4+5\,x^2+3}+5\right)\end{aligned}$$

```
[In] integrate((3*x^2+2)*(x^4+5*x^2+3)^(3/2)/x^3,x, algorithm="maxima")
```

```
[Out] 27/8*sqrt(x^4 + 5*x^2 + 3)*x^2 + 1/2*(x^4 + 5*x^2 + 3)^(3/2) - 12*sqrt(3)*l
og(2*sqrt(3)*sqrt(x^4 + 5*x^2 + 3)/x^2 + 6/x^2 + 5) + 327/16*sqrt(x^4 + 5*x
^2 + 3) - (x^4 + 5*x^2 + 3)^(3/2)/x^2 + 609/32*log(2*x^2 + 2*sqrt(x^4 + 5*x
^2 + 3) + 5)
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.25

$$\int \frac{(2+3x^2)\,(3+5x^2+x^4)^{3/2}}{x^3}\,dx = \frac{1}{16}\sqrt{x^4+5\,x^2+3}\left(2\left(4\,x^2+39\right)x^2+271\right) + 12\,\sqrt{3}\log\left(\frac{x^2+\sqrt{3}-\sqrt{x^4+5\,x^2+3}}{x^2-\sqrt{3}-\sqrt{x^4+5\,x^2+3}}\right) + \frac{3\left(5\,x^2-5\,\sqrt{x^4+5\,x^2+3}+6\right)}{\left(x^2-\sqrt{x^4+5\,x^2+3}\right)^2-3} - \frac{609}{32}\log\left(2\,x^2-2\,\sqrt{x^4+5\,x^2+3}+5\right)$$

```
[In] integrate((3*x^2+2)*(x^4+5*x^2+3)^(3/2)/x^3,x, algorithm="giac")
```

```
[Out] 1/16*sqrt(x^4 + 5*x^2 + 3)*(2*(4*x^2 + 39)*x^2 + 271) + 12*sqrt(3)*log((x^2
 + sqrt(3) - sqrt(x^4 + 5*x^2 + 3))/(x^2 - sqrt(3) - sqrt(x^4 + 5*x^2 + 3))
) + 3*(5*x^2 - 5*sqrt(x^4 + 5*x^2 + 3) + 6)/((x^2 - sqrt(x^4 + 5*x^2 + 3))^
2 - 3) - 609/32*log(2*x^2 - 2*sqrt(x^4 + 5*x^2 + 3) + 5)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(2+3x^2)\,(3+5x^2+x^4)^{3/2}}{x^3}\,dx = \int \frac{\left(3\,x^2+2\right)\,\left(x^4+5\,x^2+3\right)^{3/2}}{x^3}\,dx$$

```
[In] int(((3*x^2 + 2)*(5*x^2 + x^4 + 3)^(3/2))/x^3,x)
```

```
[Out] int(((3*x^2 + 2)*(5*x^2 + x^4 + 3)^(3/2))/x^3, x)
```

3.161 $\int \frac{(2+3x^2)(3+5x^2+x^4)^{3/2}}{x^5} dx$

Optimal result

Integrand size = 25, antiderivative size = 127

$$\int \frac{(2+3x^2)(3+5x^2+x^4)^{3/2}}{x^5} dx = -\frac{3(28-19x^2)\sqrt{3+5x^2+x^4}}{8x^2} - \frac{(2-3x^2)(3+5x^2+x^4)^{3/2}}{4x^4} + \frac{453}{16}\text{arctanh}\left(\frac{5+2x^2}{2\sqrt{3+5x^2+x^4}}\right) - \frac{127}{8}\sqrt{3}\text{arctanh}\left(\frac{6+5x^2}{2\sqrt{3}\sqrt{3+5x^2+x^4}}\right)$$

```
[Out] -1/4*(-3*x^2+2)*(x^4+5*x^2+3)^(3/2)/x^4+453/16*arctanh(1/2*(2*x^2+5)/(x^4+5
*x^2+3)^(1/2))-127/8*arctanh(1/6*(5*x^2+6)*3^(1/2)/(x^4+5*x^2+3)^(1/2))*3^(
1/2)-3/8*(-19*x^2+28)*(x^4+5*x^2+3)^(1/2)/x^2
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {1265, 826, 857, 635, 212, 738}

$$\int \frac{(2+3x^2)(3+5x^2+x^4)^{3/2}}{x^5} dx = \frac{453}{16}\text{arctanh}\left(\frac{2x^2+5}{2\sqrt{x^4+5x^2+3}}\right) - \frac{127}{8}\sqrt{3}\text{arctanh}\left(\frac{5x^2+6}{2\sqrt{3}\sqrt{x^4+5x^2+3}}\right) - \frac{(2-3x^2)(x^4+5x^2+3)^{3/2}}{4x^4} - \frac{3(28-19x^2)\sqrt{x^4+5x^2+3}}{8x^2}$$

```
[In] Int[((2 + 3*x^2)*(3 + 5*x^2 + x^4)^(3/2))/x^5,x]
```

```
[Out] (-3*(28 - 19*x^2)*Sqrt[3 + 5*x^2 + x^4])/(8*x^2) - ((2 - 3*x^2)*(3 + 5*x^2
+ x^4)^(3/2))/(4*x^4) + (453*ArcTanh[(5 + 2*x^2)/(2*Sqrt[3 + 5*x^2 + x^4])]
)/16 - (127*Sqrt[3]*ArcTanh[(6 + 5*x^2)/(2*Sqrt[3]*Sqrt[3 + 5*x^2 + x^4])])
/8
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 635

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 826

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(e*f*(m + 2*p + 2) -
 d*g*(2*p + 1) + e*g*(m + 1)*x)*((a + b*x + c*x^2)^p/(e^2*(m + 1)*(m + 2*p
+ 2))), x] + Dist[p/(e^2*(m + 1)*(m + 2*p + 2)), Int[(d + e*x)^(m + 1)*(a +
 b*x + c*x^2)^(p - 1)*Simp[g*(b*d + 2*a*e + 2*a*e*m + 2*b*d*p) - f*b*e*(m +
 2*p + 2) + (g*(2*c*d + b*e + b*e*m + 4*c*d*p) - 2*c*e*f*(m + 2*p + 2))*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] && RationalQ[p] && p > 0 && (LtQ[m, -1] || Eq
Q[p, 1] || (IntegerQ[p] &&  !RationalQ[m])) && NeQ[m, -1] &&  !ILtQ[m + 2*p
 + 1, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 857

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1265

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{2}\text{Subst}\left(\int \frac{(2+3x)\left(3+5x+x^2\right)^{3/2}}{x^3}\,dx,x,x^2\right)\\
&= -\frac{\left(2-3x^2\right)\left(3+5x^2+x^4\right)^{3/2}}{4x^4}-\frac{3}{16}\text{Subst}\left(\int \frac{(-56-38x)\sqrt{3+5x+x^2}}{x^2}\,dx,x,x^2\right)\\
&= -\frac{3(28-19x^2)\sqrt{3+5x^2+x^4}}{8x^2}-\frac{\left(2-3x^2\right)\left(3+5x^2+x^4\right)^{3/2}}{4x^4}\\
&\quad+\frac{3}{32}\text{Subst}\left(\int \frac{508+302x}{x\sqrt{3+5x+x^2}}\,dx,x,x^2\right)\\
&= -\frac{3(28-19x^2)\sqrt{3+5x^2+x^4}}{8x^2}-\frac{\left(2-3x^2\right)\left(3+5x^2+x^4\right)^{3/2}}{4x^4}\\
&\quad+\frac{453}{16}\text{Subst}\left(\int \frac{1}{\sqrt{3+5x+x^2}}\,dx,x,x^2\right)\\
&\quad+\frac{381}{8}\text{Subst}\left(\int \frac{1}{x\sqrt{3+5x+x^2}}\,dx,x,x^2\right)\\
&= -\frac{3(28-19x^2)\sqrt{3+5x^2+x^4}}{8x^2}-\frac{\left(2-3x^2\right)\left(3+5x^2+x^4\right)^{3/2}}{4x^4}\\
&\quad+\frac{453}{8}\text{Subst}\left(\int \frac{1}{4-x^2}\,dx,x,\frac{5+2x^2}{\sqrt{3+5x^2+x^4}}\right)\\
&\quad-\frac{381}{4}\text{Subst}\left(\int \frac{1}{12-x^2}\,dx,x,\frac{6+5x^2}{\sqrt{3+5x^2+x^4}}\right)\\
&= -\frac{3(28-19x^2)\sqrt{3+5x^2+x^4}}{8x^2}-\frac{\left(2-3x^2\right)\left(3+5x^2+x^4\right)^{3/2}}{4x^4}\\
&\quad+\frac{453}{16}\tanh^{-1}\left(\frac{5+2x^2}{2\sqrt{3+5x^2+x^4}}\right)-\frac{127}{8}\sqrt{3}\tanh^{-1}\left(\frac{6+5x^2}{2\sqrt{3}\sqrt{3+5x^2+x^4}}\right)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.25 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.80

$$\int \frac{(2+3x^2)(3+5x^2+x^4)^{3/2}}{x^5}\,dx = \frac{1}{16}\left(\frac{2\sqrt{3+5x^2+x^4}(-12-86x^2+83x^4+6x^6)}{x^4} + 508\sqrt{3}\text{arctanh}\left(\frac{x^2-\sqrt{3+5x^2+x^4}}{\sqrt{3}}\right) - 453\log\left(-5-2x^2+2\sqrt{3+5x^2+x^4}\right)\right)$$

```
[In] Integrate[((2 + 3*x^2)*(3 + 5*x^2 + x^4)^(3/2))/x^5,x]
```

```
[Out] ((2*Sqrt[3 + 5*x^2 + x^4]*(-12 - 86*x^2 + 83*x^4 + 6*x^6))/x^4 + 508*Sqrt[3
]*ArcTanh[(x^2 - Sqrt[3 + 5*x^2 + x^4])/Sqrt[3]] - 453*Log[-5 - 2*x^2 + 2*S
qrt[3 + 5*x^2 + x^4]])/16
```

Maple [A] (verified)

Time = 0.53 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.76

method	result
pseudoelliptic	$\frac{-254\ \text{arctanh}\left(\frac{(5x^2+6)\sqrt{3}}{6\sqrt{x^4+5x^2+3}}\right)\sqrt{3}\,x^4+453\ln\left(2x^2+5+2\sqrt{x^4+5x^2+3}\right)x^4+12\sqrt{x^4+5x^2+3}\,\left(x^6+\frac{83}{6}x^4-\frac{43}{3}x^2-2\right)}{16x^4}$
trager	$\frac{(6x^6+83x^4-86x^2-12)\sqrt{x^4+5x^2+3}}{8x^4}+\frac{453\ln\left(-2x^2-2\sqrt{x^4+5x^2+3}-5\right)}{16}+\frac{127\,\text{RootOf}\left(\_Z^2-3\right)\ln\left(-\frac{-5\,\text{RootOf}\left(\_Z^2 \right.}{}\right.}{8}$
default	$\frac{453\ln\left(\frac{5}{2}+x^2+\sqrt{x^4+5x^2+3}\right)}{16}+\frac{3x^2\sqrt{x^4+5x^2+3}}{4}+\frac{83\sqrt{x^4+5x^2+3}}{8}-\frac{43\sqrt{x^4+5x^2+3}}{4x^2}-\frac{127\ \text{arctanh}\left(\frac{(5x^2+6)\sqrt{3}}{6\sqrt{x^4+5x^2+3}}\right)}{8}$
risch	$-\frac{43x^6+221x^4+159x^2+18}{4x^4\sqrt{x^4+5x^2+3}}+\frac{453\ln\left(\frac{5}{2}+x^2+\sqrt{x^4+5x^2+3}\right)}{16}+\frac{3x^2\sqrt{x^4+5x^2+3}}{4}+\frac{83\sqrt{x^4+5x^2+3}}{8}-\frac{127\ \text{arctanh}\left(\right.}{8}$
elliptic	$\frac{453\ln\left(\frac{5}{2}+x^2+\sqrt{x^4+5x^2+3}\right)}{16}+\frac{3x^2\sqrt{x^4+5x^2+3}}{4}+\frac{83\sqrt{x^4+5x^2+3}}{8}-\frac{43\sqrt{x^4+5x^2+3}}{4x^2}-\frac{127\ \text{arctanh}\left(\frac{(5x^2+6)\sqrt{3}}{6\sqrt{x^4+5x^2+3}}\right)}{8}$

```
[In] int((3*x^2+2)*(x^4+5*x^2+3)^(3/2)/x^5,x,method=_RETURNVERBOSE)
```

```
[Out] 1/16*(-254*arctanh(1/6*(5*x^2+6)*3^(1/2)/(x^4+5*x^2+3)^(1/2))*3^(1/2)*x^4+4
53*ln(2*x^2+5+2*(x^4+5*x^2+3)^(1/2))*x^4+12*(x^4+5*x^2+3)^(1/2)*(x^6+83/6*x
^4-43/3*x^2-2))/x^4
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.96

$$\int \frac{(2+3x^2)(3+5x^2+x^4)^{3/2}}{x^5}\,dx = \frac{1016\sqrt{3}x^4\log\left(\frac{25\,x^2-2\,\sqrt{3}(5\,x^2+6)-2\,\sqrt{x^4+5\,x^2+3}\left(5\,\sqrt{3}-6\right)+30}{x^2}\right)-1812\,x^4\log\left(\right.}{}$$

```
[In] integrate((3*x^2+2)*(x^4+5*x^2+3)^(3/2)/x^5,x, algorithm="fricas")
```

```
[Out] 1/64*(1016*sqrt(3)*x^4*log((25*x^2 - 2*sqrt(3)*(5*x^2 + 6) - 2*sqrt(x^4 + 5
*x^2 + 3)*(5*sqrt(3) - 6) + 30)/x^2) - 1812*x^4*log(-2*x^2 + 2*sqrt(x^4 + 5
*x^2 + 3) - 5) + 67*x^4 + 8*(6*x^6 + 83*x^4 - 86*x^2 - 12)*sqrt(x^4 + 5*x^2
 + 3))/x^4
```

Sympy [F]

$$\int \frac{(2+3x^2)(3+5x^2+x^4)^{3/2}}{x^5}\,dx = \int \frac{(3x^2+2)(x^4+5x^2+3)^{\frac{3}{2}}}{x^5}\,dx$$

```
[In] integrate((3*x**2+2)*(x**4+5*x**2+3)**(3/2)/x**5,x)
```

```
[Out] Integral((3*x**2 + 2)*(x**4 + 5*x**2 + 3)**(3/2)/x**5, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.08

$$\begin{aligned}\int \frac{(2+3x^2)(3+5x^2+x^4)^{3/2}}{x^5}\,dx &= \frac{7}{2}\sqrt{x^4+5\,x^2+3}x^2+\frac{1}{6}\left(x^4+5\,x^2+3\right)^{\frac{3}{2}}\\&\quad-\frac{127}{8}\sqrt{3}\log\left(\frac{2\sqrt{3}\sqrt{x^4+5\,x^2+3}}{x^2}+\frac{6}{x^2}+5\right)+\frac{197}{8}\sqrt{x^4+5\,x^2+3}\\&\quad-\frac{23\left(x^4+5\,x^2+3\right)^{\frac{3}{2}}}{12\,x^2}-\frac{\left(x^4+5\,x^2+3\right)^{\frac{5}{2}}}{6\,x^4}+\frac{453}{16}\log\left(2\,x^2+2\sqrt{x^4+5\,x^2+3}+5\right)\end{aligned}$$

```
[In] integrate((3*x^2+2)*(x^4+5*x^2+3)^(3/2)/x^5,x, algorithm="maxima")
```

```
[Out] 7/2*sqrt(x^4 + 5*x^2 + 3)*x^2 + 1/6*(x^4 + 5*x^2 + 3)^(3/2) - 127/8*sqrt(3)
*log(2*sqrt(3)*sqrt(x^4 + 5*x^2 + 3)/x^2 + 6/x^2 + 5) + 197/8*sqrt(x^4 + 5*
x^2 + 3) - 23/12*(x^4 + 5*x^2 + 3)^(3/2)/x^2 - 1/6*(x^4 + 5*x^2 + 3)^(5/2)/
x^4 + 453/16*log(2*x^2 + 2*sqrt(x^4 + 5*x^2 + 3) + 5)
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 190, normalized size of antiderivative = 1.50

$$\int \frac{(2+3x^2)(3+5x^2+x^4)^{3/2}}{x^5}\,dx = \frac{1}{8}\sqrt{x^4+5x^2+3}(6x^2+83) + \frac{127}{8}\sqrt{3}\log\left(\frac{x^2+\sqrt{3}-\sqrt{x^4+5x^2+3}}{x^2-\sqrt{3}-\sqrt{x^4+5x^2+3}}\right) + \frac{227\left(x^2-\sqrt{x^4+5x^2+3}\right)^3+348\left(x^2-\sqrt{x^4+5x^2+3}\right)^2-459x^2+459\sqrt{x^4+5x^2+3}-684}{4\left(\left(x^2-\sqrt{x^4+5x^2+3}\right)^2-3\right)^2} - \frac{453}{16}\log\left(2x^2-2\sqrt{x^4+5x^2+3}+5\right)$$

```
[In] integrate((3*x^2+2)*(x^4+5*x^2+3)^(3/2)/x^5,x, algorithm="giac")
```

```
[Out] 1/8*sqrt(x^4 + 5*x^2 + 3)*(6*x^2 + 83) + 127/8*sqrt(3)*log((x^2 + sqrt(3) -
 sqrt(x^4 + 5*x^2 + 3))/(x^2 - sqrt(3) - sqrt(x^4 + 5*x^2 + 3))) + 1/4*(227
*(x^2 - sqrt(x^4 + 5*x^2 + 3))^3 + 348*(x^2 - sqrt(x^4 + 5*x^2 + 3))^2 - 45
9*x^2 + 459*sqrt(x^4 + 5*x^2 + 3) - 684)/((x^2 - sqrt(x^4 + 5*x^2 + 3))^2 -
 3)^2 - 453/16*log(2*x^2 - 2*sqrt(x^4 + 5*x^2 + 3) + 5)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(2+3x^2)(3+5x^2+x^4)^{3/2}}{x^5}\,dx = \int \frac{(3x^2+2)\,(x^4+5x^2+3)^{3/2}}{x^5}\,dx$$

```
[In] int(((3*x^2 + 2)*(5*x^2 + x^4 + 3)^(3/2))/x^5,x)
```

```
[Out] int(((3*x^2 + 2)*(5*x^2 + x^4 + 3)^(3/2))/x^5, x)
```

3.162 $\int \frac{(2+3x^2)(3+5x^2+x^4)^{3/2}}{x^7} dx$

Optimal result

Integrand size = 25, antiderivative size = 127

$$\int \frac{(2+3x^2)(3+5x^2+x^4)^{3/2}}{x^7} dx = -\frac{(67-32x^2)\sqrt{3+5x^2+x^4}}{12x^2} - \frac{(2+7x^2)(3+5x^2+x^4)^{3/2}}{6x^6} + \frac{49}{4}\text{arctanh}\left(\frac{5+2x^2}{2\sqrt{3+5x^2+x^4}}\right) - \frac{527\text{arctanh}\left(\frac{6+5x^2}{2\sqrt{3}\sqrt{3+5x^2+x^4}}\right)}{24\sqrt{3}}$$

```
[Out] -1/6*(7*x^2+2)*(x^4+5*x^2+3)^(3/2)/x^6+49/4*arctanh(1/2*(2*x^2+5)/(x^4+5*x^
2+3)^(1/2))-527/72*arctanh(1/6*(5*x^2+6)*3^(1/2)/(x^4+5*x^2+3)^(1/2))*3^(1/
2)-1/12*(-32*x^2+67)*(x^4+5*x^2+3)^(1/2)/x^2
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.280, Rules used = {1265, 824, 826, 857, 635, 212, 738}

$$\int \frac{(2+3x^2)(3+5x^2+x^4)^{3/2}}{x^7} dx = \frac{49}{4}\text{arctanh}\left(\frac{2x^2+5}{2\sqrt{x^4+5x^2+3}}\right) - \frac{527\text{arctanh}\left(\frac{5x^2+6}{2\sqrt{3}\sqrt{x^4+5x^2+3}}\right)}{24\sqrt{3}} - \frac{(67-32x^2)\sqrt{x^4+5x^2+3}}{12x^2} - \frac{(7x^2+2)(x^4+5x^2+3)^{3/2}}{6x^6}$$

```
[In] Int[((2 + 3*x^2)*(3 + 5*x^2 + x^4)^(3/2))/x^7,x]
```

```
[Out] -1/12*((67 - 32*x^2)*Sqrt[3 + 5*x^2 + x^4])/x^2 - ((2 + 7*x^2)*(3 + 5*x^2 +
 x^4)^(3/2))/(6*x^6) + (49*ArcTanh[(5 + 2*x^2)/(2*Sqrt[3 + 5*x^2 + x^4])])/
4 - (527*ArcTanh[(6 + 5*x^2)/(2*Sqrt[3]*Sqrt[3 + 5*x^2 + x^4])])/(24*Sqrt[3
])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 635

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 824

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(-(d + e*x)^(m + 1))*((a + b*x + c*x^2
)^p/(e^2*(m + 1)*(m + 2)*(c*d^2 - b*d*e + a*e^2)))*((d*g - e*f*(m + 2))*(c*
d^2 - b*d*e + a*e^2) - d*p*(2*c*d - b*e)*(e*f - d*g) - e*(g*(m + 1)*(c*d^2
- b*d*e + a*e^2) + p*(2*c*d - b*e)*(e*f - d*g))*x), x] - Dist[p/(e^2*(m + 1
)*(m + 2)*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m + 2)*(a + b*x + c*x^2)
^(p - 1)*Simp[2*a*c*e*(e*f - d*g)*(m + 2) + b^2*e*(d*g*(p + 1) - e*f*(m + p
 + 2)) + b*(a*e^2*g*(m + 1) - c*d*(d*g*(2*p + 1) - e*f*(m + 2*p + 2))) - c*
(2*c*d*(d*g*(2*p + 1) - e*f*(m + 2*p + 2)) - e*(2*a*e*g*(m + 1) - b*(d*g*(m
 - 2*p) + e*f*(m + 2*p + 2))))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g}
, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && GtQ[p, 0] &
& LtQ[m, -2] && LtQ[m + 2*p, 0] &&  !ILtQ[m + 2*p + 3, 0]
```

Rule 826

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(e*f*(m + 2*p + 2) -
 d*g*(2*p + 1) + e*g*(m + 1)*x)*((a + b*x + c*x^2)^p/(e^2*(m + 1)*(m + 2*p
+ 2))), x] + Dist[p/(e^2*(m + 1)*(m + 2*p + 2)), Int[(d + e*x)^(m + 1)*(a +
```

```
 b*x + c*x^2)^(p - 1)*Simp[g*(b*d + 2*a*e + 2*a*e*m + 2*b*d*p) - f*b*e*(m +
 2*p + 2) + (g*(2*c*d + b*e + b*e*m + 4*c*d*p) - 2*c*e*f*(m + 2*p + 2))*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] && RationalQ[p] && p > 0 && (LtQ[m, -1] || Eq
Q[p, 1] || (IntegerQ[p] &&  !RationalQ[m])) && NeQ[m, -1] &&  !ILtQ[m + 2*p
 + 1, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 857

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1265

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{2}\text{Subst}\left(\int \frac{(2+3x)\left(3+5x+x^2\right)^{3/2}}{x^4}\,dx,x,x^2\right)\\
&= -\frac{\left(2+7x^2\right)\left(3+5x^2+x^4\right)^{3/2}}{6x^6}-\frac{1}{24}\text{Subst}\left(\int \frac{(-134-64x)\sqrt{3+5x+x^2}}{x^2}\,dx,x,x^2\right)\\
&= -\frac{\left(67-32x^2\right)\sqrt{3+5x^2+x^4}}{12x^2}-\frac{\left(2+7x^2\right)\left(3+5x^2+x^4\right)^{3/2}}{6x^6}\\
&\quad+\frac{1}{48}\text{Subst}\left(\int \frac{1054+588x}{x\sqrt{3+5x+x^2}}\,dx,x,x^2\right)\\
&= -\frac{\left(67-32x^2\right)\sqrt{3+5x^2+x^4}}{12x^2}-\frac{\left(2+7x^2\right)\left(3+5x^2+x^4\right)^{3/2}}{6x^6}\\
&\quad+\frac{49}{4}\text{Subst}\left(\int \frac{1}{\sqrt{3+5x+x^2}}\,dx,x,x^2\right)+\frac{527}{24}\text{Subst}\left(\int \frac{1}{x\sqrt{3+5x+x^2}}\,dx,x,x^2\right)\\
&= -\frac{\left(67-32x^2\right)\sqrt{3+5x^2+x^4}}{12x^2}-\frac{\left(2+7x^2\right)\left(3+5x^2+x^4\right)^{3/2}}{6x^6}\\
&\quad+\frac{49}{2}\text{Subst}\left(\int \frac{1}{4-x^2}\,dx,x,\frac{5+2x^2}{\sqrt{3+5x^2+x^4}}\right)\\
&\quad-\frac{527}{12}\text{Subst}\left(\int \frac{1}{12-x^2}\,dx,x,\frac{6+5x^2}{\sqrt{3+5x^2+x^4}}\right)
\end{aligned}$$

$$= -\frac{(67-32x^2)\sqrt{3+5x^2+x^4}}{12x^2} - \frac{(2+7x^2)(3+5x^2+x^4)^{3/2}}{6x^6} + \frac{49}{4}\tanh^{-1}\left(\frac{5+2x^2}{2\sqrt{3+5x^2+x^4}}\right) - \frac{527\tanh^{-1}\left(\frac{6+5x^2}{2\sqrt{3}\sqrt{3+5x^2+x^4}}\right)}{24\sqrt{3}}$$

Mathematica [A] (verified)

Time = 0.34 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.80

$$\int \frac{(2+3x^2)(3+5x^2+x^4)^{3/2}}{x^7}\,dx = \frac{1}{36}\left(\frac{3\sqrt{3+5x^2+x^4}(-12-62x^2-141x^4+18x^6)}{x^6} + 527\sqrt{3}\text{arctanh}\left(\frac{x^2-\sqrt{3+5x^2+x^4}}{\sqrt{3}}\right) - 441\log\left(-5-2x^2+2\sqrt{3+5x^2+x^4}\right)\right)$$

```
[In] Integrate[((2 + 3*x^2)*(3 + 5*x^2 + x^4)^(3/2))/x^7,x]
```

```
[Out] ((3*Sqrt[3 + 5*x^2 + x^4]*(-12 - 62*x^2 - 141*x^4 + 18*x^6))/x^6 + 527*Sqrt
[3]*ArcTanh[(x^2 - Sqrt[3 + 5*x^2 + x^4])/Sqrt[3]] - 441*Log[-5 - 2*x^2 + 2
*Sqrt[3 + 5*x^2 + x^4]])/36
```

Maple [A] (verified)

Time = 0.55 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.76

method	result
pseudoelliptic	$\frac{-527\,\text{arctanh}\left(\frac{(5x^2+6)\sqrt{3}}{6\sqrt{x^4+5x^2+3}}\right)\sqrt{3}\,x^6+882\ln\left(2x^2+5+2\sqrt{x^4+5x^2+3}\right)x^6+108\left(x^6-\frac{47}{6}x^4-\frac{31}{9}x^2-\frac{2}{3}\right)\sqrt{x^4+5x^2+3}}{72x^6}$
risch	$-\frac{141x^8+767x^6+745x^4+246x^2+36}{12x^6\sqrt{x^4+5x^2+3}}+\frac{49\ln\left(\frac{5}{2}+x^2+\sqrt{x^4+5x^2+3}\right)}{4}+\frac{3\sqrt{x^4+5x^2+3}}{2}-\frac{527\,\text{arctanh}\left(\frac{(5x^2+6)\sqrt{3}}{6\sqrt{x^4+5x^2+3}}\right)\sqrt{3}}{72}$
trager	$\frac{(18x^6-141x^4-62x^2-12)\sqrt{x^4+5x^2+3}}{12x^6}-\frac{49\ln\left(2x^2-2\sqrt{x^4+5x^2+3}+5\right)}{4}+\frac{527\,\text{RootOf}\left(\_Z^2-3\right)\ln\left(-\frac{-5\,\text{RootOf}\left(\_Z^2\right.}{}\right.}{72}$
default	$\frac{49\ln\left(\frac{5}{2}+x^2+\sqrt{x^4+5x^2+3}\right)}{4}-\frac{\sqrt{x^4+5x^2+3}}{x^6}-\frac{31\sqrt{x^4+5x^2+3}}{6x^4}-\frac{47\sqrt{x^4+5x^2+3}}{4x^2}-\frac{527\,\text{arctanh}\left(\frac{(5x^2+6)\sqrt{3}}{6\sqrt{x^4+5x^2+3}}\right)\sqrt{3}}{72}-$
elliptic	$\frac{49\ln\left(\frac{5}{2}+x^2+\sqrt{x^4+5x^2+3}\right)}{4}-\frac{\sqrt{x^4+5x^2+3}}{x^6}-\frac{31\sqrt{x^4+5x^2+3}}{6x^4}-\frac{47\sqrt{x^4+5x^2+3}}{4x^2}-\frac{527\,\text{arctanh}\left(\frac{(5x^2+6)\sqrt{3}}{6\sqrt{x^4+5x^2+3}}\right)\sqrt{3}}{72}-$

```
[In] int((3*x^2+2)*(x^4+5*x^2+3)^(3/2)/x^7,x,method=_RETURNVERBOSE)
```

```
[Out] 1/72*(-527*arctanh(1/6*(5*x^2+6)*3^(1/2)/(x^4+5*x^2+3)^(1/2))*3^(1/2)*x^6+8
82*ln(2*x^2+5+2*(x^4+5*x^2+3)^(1/2))*x^6+108*(x^6-47/6*x^4-31/9*x^2-2/3)*(x
^4+5*x^2+3)^(1/2))/x^6
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.96

$$\int \frac{(2+3x^2)(3+5x^2+x^4)^{3/2}}{x^7}\,dx = \frac{527\sqrt{3}x^6\log\left(\frac{25\,x^2-2\,\sqrt{3}(5\,x^2+6)-2\,\sqrt{x^4+5\,x^2+3}\left(5\,\sqrt{3}-6\right)+30}{x^2}\right)-882\,x^6\log\left(-\right.}{}$$

```
[In] integrate((3*x^2+2)*(x^4+5*x^2+3)^(3/2)/x^7,x, algorithm="fricas")
```

```
[Out] 1/72*(527*sqrt(3)*x^6*log((25*x^2 - 2*sqrt(3)*(5*x^2 + 6) - 2*sqrt(x^4 + 5*
x^2 + 3)*(5*sqrt(3) - 6) + 30)/x^2) - 882*x^6*log(-2*x^2 + 2*sqrt(x^4 + 5*x
^2 + 3) - 5) - 711*x^6 + 6*(18*x^6 - 141*x^4 - 62*x^2 - 12)*sqrt(x^4 + 5*x^
2 + 3))/x^6
```

Sympy [F]

$$\int \frac{(2+3x^2)(3+5x^2+x^4)^{3/2}}{x^7}\,dx = \int \frac{(3x^2+2)(x^4+5x^2+3)^{\frac{3}{2}}}{x^7}\,dx$$

```
[In] integrate((3*x**2+2)*(x**4+5*x**2+3)**(3/2)/x**7,x)
```

```
[Out] Integral((3*x**2 + 2)*(x**4 + 5*x**2 + 3)**(3/2)/x**7, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.21

$$\begin{aligned}\int \frac{(2+3x^2)(3+5x^2+x^4)^{3/2}}{x^7}\,dx &= \frac{67}{36}\,\sqrt{x^4+5\,x^2+3}x^2 \\ &+ \frac{11}{54}\,(x^4+5\,x^2+3)^{\frac{3}{2}} - \frac{527}{72}\,\sqrt{3}\log\left(\frac{2\,\sqrt{3}\sqrt{x^4+5\,x^2+3}}{x^2}+\frac{6}{x^2}+5\right) \\ &+ \frac{431}{36}\,\sqrt{x^4+5\,x^2+3} - \frac{79\,(x^4+5\,x^2+3)^{\frac{3}{2}}}{108\,x^2} - \frac{11\,(x^4+5\,x^2+3)^{\frac{5}{2}}}{54\,x^4} \\ &- \frac{(x^4+5\,x^2+3)^{\frac{5}{2}}}{9\,x^6} + \frac{49}{4}\,\log\left(2\,x^2+2\,\sqrt{x^4+5\,x^2+3}+5\right)\end{aligned}$$

```
[In] integrate((3*x^2+2)*(x^4+5*x^2+3)^(3/2)/x^7,x, algorithm="maxima")
```

```
[Out] 67/36*sqrt(x^4 + 5*x^2 + 3)*x^2 + 11/54*(x^4 + 5*x^2 + 3)^(3/2) - 527/72*sq
rt(3)*log(2*sqrt(3)*sqrt(x^4 + 5*x^2 + 3)/x^2 + 6/x^2 + 5) + 431/36*sqrt(x^
4 + 5*x^2 + 3) - 79/108*(x^4 + 5*x^2 + 3)^(3/2)/x^2 - 11/54*(x^4 + 5*x^2 +
3)^(5/2)/x^4 - 1/9*(x^4 + 5*x^2 + 3)^(5/2)/x^6 + 49/4*log(2*x^2 + 2*sqrt(x^
4 + 5*x^2 + 3) + 5)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 227 vs. $2(103) = 206$.

Time = 0.32 (sec) , antiderivative size = 227, normalized size of antiderivative = 1.79

$$\int \frac{(2+3x^2)(3+5x^2+x^4)^{3/2}}{x^7}\,dx = \frac{527}{72}\sqrt{3}\log\left(\frac{x^2+\sqrt{3}-\sqrt{x^4+5\,x^2+3}}{x^2-\sqrt{3}-\sqrt{x^4+5\,x^2+3}}\right) + \frac{3}{2}\sqrt{x^4+5\,x^2+3} + \frac{829\,\left(x^2-\sqrt{x^4+5\,x^2+3}\right)^5+1824\,\left(x^2-\sqrt{x^4+5\,x^2+3}\right)^4-2200\,\left(x^2-\sqrt{x^4+5\,x^2+3}\right)^3-5292\,\left(x^2-\sqrt{\right.}}{12\,\left(\left(x^2-\sqrt{x^4+5\,x^2+3}\right)^2-3\right)^3} - \frac{49}{4}\log\left(2\,x^2-2\,\sqrt{x^4+5\,x^2+3}+5\right)$$

```
[In] integrate((3*x^2+2)*(x^4+5*x^2+3)^(3/2)/x^7,x, algorithm="giac")
```

```
[Out] 527/72*sqrt(3)*log((x^2 + sqrt(3) - sqrt(x^4 + 5*x^2 + 3))/(x^2 - sqrt(3) -
 sqrt(x^4 + 5*x^2 + 3))) + 3/2*sqrt(x^4 + 5*x^2 + 3) + 1/12*(829*(x^2 - sqr
t(x^4 + 5*x^2 + 3))^5 + 1824*(x^2 - sqrt(x^4 + 5*x^2 + 3))^4 - 2200*(x^2 -
sqrt(x^4 + 5*x^2 + 3))^3 - 5292*(x^2 - sqrt(x^4 + 5*x^2 + 3))^2 + 2799*x^2
- 2799*sqrt(x^4 + 5*x^2 + 3) + 5724)/((x^2 - sqrt(x^4 + 5*x^2 + 3))^2 - 3)^
3 - 49/4*log(2*x^2 - 2*sqrt(x^4 + 5*x^2 + 3) + 5)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(2+3x^2)(3+5x^2+x^4)^{3/2}}{x^7}\,dx = \int \frac{\left(3\,x^2+2\right)\,\left(x^4+5\,x^2+3\right)^{3/2}}{x^7}\,dx$$

```
[In] int(((3*x^2 + 2)*(5*x^2 + x^4 + 3)^(3/2))/x^7,x)
```

```
[Out] int(((3*x^2 + 2)*(5*x^2 + x^4 + 3)^(3/2))/x^7, x)
```

3.163 $\int x^4(2+3x^2)\left(3+5x^2+x^4\right)^{3/2}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 356

$$\int x^4\left(2+3x^2\right)\left(3+5x^2+x^4\right)^{3/2}\,dx=\frac{176723x\left(5+\sqrt{13}+2x^2\right)}{4290\sqrt{3+5x^2+x^4}}-\frac{4210}{429}x\sqrt{3+5x^2+x^4}+\frac{1251}{715}x^3\sqrt{3+5x^2+x^4}-\frac{1}{429}x^5\left(283+272x^2\right)\sqrt{3+5x^2+x^4}+\frac{1}{143}x^5\left(71+33x^2\right)\left(3+5x^2+x^4\right)^{3/2}-\frac{176723\sqrt{\frac{1}{6}\left(5+\sqrt{13}\right)}\sqrt{\frac{6+\left(5-\sqrt{13}\right)x^2}{6+\left(5+\sqrt{13}\right)x^2}}\left(6+\left(5+\sqrt{13}\right)x^2\right)E\left(\arctan\left(\sqrt{\frac{1}{6}\left(5+\sqrt{13}\right)}x\right)\middle|\frac{1}{6}\left(-13+5\sqrt{13}\right)\right)}{4290\sqrt{3+5x^2+x^4}}+\frac{2105\sqrt{\frac{2}{3\left(5+\sqrt{13}\right)}}\sqrt{\frac{6+\left(5-\sqrt{13}\right)x^2}{6+\left(5+\sqrt{13}\right)x^2}}\left(6+\left(5+\sqrt{13}\right)x^2\right)\operatorname{EllipticF}\left(\arctan\left(\sqrt{\frac{1}{6}\left(5+\sqrt{13}\right)}x\right),\frac{1}{6}\left(-13+5\sqrt{13}\right)\right)}{143\sqrt{3+5x^2+x^4}}$$

```
[Out] 1/143*x^5*(33*x^2+71)*(x^4+5*x^2+3)^(3/2)+176723/4290*x*(5+2*x^2+13^(1/2))/
(x^4+5*x^2+3)^(1/2)-4210/429*x*(x^4+5*x^2+3)^(1/2)+1251/715*x^3*(x^4+5*x^2+
3)^(1/2)-1/429*x^5*(272*x^2+283)*(x^4+5*x^2+3)^(1/2)+2105/429*(1/(36+x^2*(3
0+6*13^(1/2))))^(1/2)*(36+x^2*(30+6*13^(1/2)))^(1/2)*EllipticF(x*(30+6*13^(
1/2))^(1/2)/(36+x^2*(30+6*13^(1/2)))^(1/2),1/6*(-78+30*13^(1/2))^(1/2))*(6+
x^2*(5+13^(1/2)))*6^(1/2)/(5+13^(1/2))^(1/2)*((6+x^2*(5-13^(1/2)))/(6+x^2*(
5+13^(1/2))))^(1/2)/(x^4+5*x^2+3)^(1/2)-176723/25740*(1/(36+x^2*(30+6*13^(1
/2))))^(1/2)*(36+x^2*(30+6*13^(1/2)))^(1/2)*EllipticE(x*(30+6*13^(1/2))^(1/
2)/(36+x^2*(30+6*13^(1/2)))^(1/2),1/6*(-78+30*13^(1/2))^(1/2))*(6+x^2*(5+13
^(1/2)))*(30+6*13^(1/2))^(1/2)*((6+x^2*(5-13^(1/2)))/(6+x^2*(5+13^(1/2))))^
(1/2)/(x^4+5*x^2+3)^(1/2)
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 356, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {1287, 1293, 1203, 1113, 1149}

$$\int x^4(2+3x^2)(3+5x^2+x^4)^{3/2}\,dx = \frac{2105\sqrt{\frac{2}{3(5+\sqrt{13})}}\sqrt{\frac{(5-\sqrt{13})x^2+6}{(5+\sqrt{13})x^2+6}}\left((5+\sqrt{13})x^2+6\right)\text{EllipticF}\left(\arctan\left(\sqrt{\frac{1}{6}(5+\sqrt{13})}x\right),\frac{1}{6}(-13+5\sqrt{13})\right)}{143\sqrt{x^4+5x^2+3}} - \frac{176723\sqrt{\frac{1}{6}(5+\sqrt{13})}\sqrt{\frac{(5-\sqrt{13})x^2+6}{(5+\sqrt{13})x^2+6}}\left((5+\sqrt{13})x^2+6\right)E\left(\arctan\left(\sqrt{\frac{1}{6}(5+\sqrt{13})}x\right)|\frac{1}{6}(-13+5\sqrt{13})\right)}{4290\sqrt{x^4+5x^2+3}} - \frac{4210}{429}\sqrt{x^4+5x^2+3}x + \frac{176723(2x^2+\sqrt{13}+5)x}{4290\sqrt{x^4+5x^2+3}} + \frac{1}{143}(33x^2+71)(x^4+5x^2+3)^{3/2}x^5 - \frac{1}{429}(272x^2+283)\sqrt{x^4+5x^2+3}x^5 + \frac{1251}{715}\sqrt{x^4+5x^2+3}x^3$$

```
[In] Int[x^4*(2 + 3*x^2)*(3 + 5*x^2 + x^4)^(3/2),x]
```

```
[Out] (176723*x*(5 + Sqrt[13] + 2*x^2))/(4290*Sqrt[3 + 5*x^2 + x^4]) - (4210*x*Sq
rt[3 + 5*x^2 + x^4])/429 + (1251*x^3*Sqrt[3 + 5*x^2 + x^4])/715 - (x^5*(283
 + 272*x^2)*Sqrt[3 + 5*x^2 + x^4])/429 + (x^5*(71 + 33*x^2)*(3 + 5*x^2 + x^
4)^(3/2))/143 - (176723*Sqrt[(5 + Sqrt[13])/6]*Sqrt[(6 + (5 - Sqrt[13])*x^2
)/(6 + (5 + Sqrt[13])*x^2)]*(6 + (5 + Sqrt[13])*x^2)*EllipticE[ArcTan[Sqrt[
(5 + Sqrt[13])/6]*x], (-13 + 5*Sqrt[13])/6])/(4290*Sqrt[3 + 5*x^2 + x^4]) +
 (2105*Sqrt[2/(3*(5 + Sqrt[13]))]*Sqrt[(6 + (5 - Sqrt[13])*x^2)/(6 + (5 + S
qrt[13])*x^2)]*(6 + (5 + Sqrt[13])*x^2)*EllipticF[ArcTan[Sqrt[(5 + Sqrt[13]
)/6]*x], (-13 + 5*Sqrt[13])/6])/(143*Sqrt[3 + 5*x^2 + x^4])
```

Rule 1113

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[(2*a + (b + q)*x^2)*(Sqrt[(2*a + (b - q)*x^2)/(2*a +
(b + q)*x^2)]/(2*a*Rt[(b + q)/(2*a), 2]*Sqrt[a + b*x^2 + c*x^4]))*EllipticF
[ArcTan[Rt[(b + q)/(2*a), 2]*x], 2*(q/(b + q))], x] /; PosQ[(b + q)/a] &&
!(PosQ[(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[
{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1149

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[b^2 - 4*a*c, 2]}, Simp[x*((b + q + 2*c*x^2)/(2*c*Sqrt[a + b*x^2 + c*x^4
])), x] - Simp[Rt[(b + q)/(2*a), 2]*(2*a + (b + q)*x^2)*(Sqrt[(2*a + (b - q
```

```
)*x^2)/(2*a + (b + q)*x^2)]/(2*c*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[ArcTan
[Rt[(b + q)/(2*a), 2]*x], 2*(q/(b + q))], x] /; PosQ[(b + q)/a] &&  !(PosQ[
(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[{a, b,
c}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1203

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[d, Int[1/Sqrt[a + b*x^2 + c*x^4],
 x], x] + Dist[e, Int[x^2/Sqrt[a + b*x^2 + c*x^4], x], x] /; PosQ[(b + q)/a
] || PosQ[(b - q)/a]] /; FreeQ[{a, b, c, d, e}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1287

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(a + b*x^2 + c*x^4)^p*((b*e*2
*p + c*d*(m + 4*p + 3) + c*e*(4*p + m + 1)*x^2)/(c*f*(4*p + m + 1)*(m + 4*p
 + 3))), x] + Dist[2*(p/(c*(4*p + m + 1)*(m + 4*p + 3))), Int[(f*x)^m*(a +
b*x^2 + c*x^4)^(p - 1)*Simp[2*a*c*d*(m + 4*p + 3) - a*b*e*(m + 1) + (2*a*c*
e*(4*p + m + 1) + b*c*d*(m + 4*p + 3) - b^2*e*(m + 2*p + 1))*x^2, x], x], x
] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b^2 - 4*a*c, 0] && GtQ[p, 0] &&
 NeQ[4*p + m + 1, 0] && NeQ[m + 4*p + 3, 0] && IntegerQ[2*p] && (IntegerQ[p
] || IntegerQ[m])
```

Rule 1293

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[e*f*(f*x)^(m - 1)*((a + b*x^2 + c*x^4)^(p +
1)/(c*(m + 4*p + 3))), x] - Dist[f^2/(c*(m + 4*p + 3)), Int[(f*x)^(m - 2)*(
a + b*x^2 + c*x^4)^p*Simp[a*e*(m - 1) + (b*e*(m + 2*p + 1) - c*d*(m + 4*p +
 3))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c,
 0] && GtQ[m, 1] && NeQ[m + 4*p + 3, 0] && IntegerQ[2*p] && (IntegerQ[p] ||
 IntegerQ[m])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{143}x^5(71+33x^2)\left(3+5x^2+x^4\right)^{3/2}+\frac{3}{143}\int x^4(-69-272x^2)\sqrt{3+5x^2+x^4}\,dx \\
&= -\frac{1}{429}x^5(283+272x^2)\sqrt{3+5x^2+x^4} \\
&\quad +\frac{1}{143}x^5(71+33x^2)\left(3+5x^2+x^4\right)^{3/2}+\frac{\int\frac{x^4(16674+26271x^2)}{\sqrt{3+5x^2+x^4}}\,dx}{3003}
\end{aligned}$$

$$= \frac{1251}{715}x^3\sqrt{3+5x^2+x^4} - \frac{1}{429}x^5\left(283+272x^2\right)\sqrt{3+5x^2+x^4} + \frac{1}{143}x^5\left(71+33x^2\right)\left(3+5x^2+x^4\right)^{3/2} - \frac{\int \frac{x^2(236439+442050x^2)}{\sqrt{3+5x^2+x^4}}\,dx}{15015}$$

$$= -\frac{4210}{429}x\sqrt{3+5x^2+x^4} + \frac{1251}{715}x^3\sqrt{3+5x^2+x^4} - \frac{1}{429}x^5\left(283+272x^2\right)\sqrt{3+5x^2+x^4} + \frac{1}{143}x^5\left(71+33x^2\right)\left(3+5x^2+x^4\right)^{3/2} + \frac{\int \frac{1326150+3711183x^2}{\sqrt{3+5x^2+x^4}}\,dx}{45045}$$

$$= -\frac{4210}{429}x\sqrt{3+5x^2+x^4} + \frac{1251}{715}x^3\sqrt{3+5x^2+x^4} - \frac{1}{429}x^5\left(283+272x^2\right)\sqrt{3+5x^2+x^4} + \frac{1}{143}x^5\left(71+33x^2\right)\left(3+5x^2+x^4\right)^{3/2} + \frac{4210}{143}\int \frac{1}{\sqrt{3+5x^2+x^4}}\,dx + \frac{176723\int \frac{x^2}{\sqrt{3+5x^2+x^4}}\,dx}{2145}$$

$$= \frac{176723x\left(5+\sqrt{13}+2x^2\right)}{4290\sqrt{3+5x^2+x^4}} - \frac{4210}{429}x\sqrt{3+5x^2+x^4} + \frac{1251}{715}x^3\sqrt{3+5x^2+x^4} - \frac{1}{429}x^5\left(283+272x^2\right)\sqrt{3+5x^2+x^4} + \frac{1}{143}x^5\left(71+33x^2\right)\left(3+5x^2+x^4\right)^{3/2} - \frac{176723\sqrt{\frac{1}{6}\left(5+\sqrt{13}\right)}\sqrt{\frac{6+\left(5-\sqrt{13}\right)x^2}{6+\left(5+\sqrt{13}\right)x^2}}\left(6+\left(5+\sqrt{13}\right)x^2\right)E\left(\tan^{-1}\left(\sqrt{\frac{1}{6}\left(5+\sqrt{13}\right)}x\right)|\frac{1}{6}(-13+\right.}{4290\sqrt{3+5x^2+x^4}} + \frac{2105\sqrt{\frac{2}{3\left(5+\sqrt{13}\right)}}\sqrt{\frac{6+\left(5-\sqrt{13}\right)x^2}{6+\left(5+\sqrt{13}\right)x^2}}\left(6+\left(5+\sqrt{13}\right)x^2\right)F\left(\tan^{-1}\left(\sqrt{\frac{1}{6}\left(5+\sqrt{13}\right)}x\right)|\frac{1}{6}(-13+5\sqrt{13}\right.}{143\sqrt{3+5x^2+x^4}}$$

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 9.17 (sec) , antiderivative size = 249, normalized size of antiderivative = 0.70

$$\int x^4\left(2+3x^2\right)\left(3+5x^2+x^4\right)^{3/2}\,dx = \frac{4x(-63150-93991x^2+3055x^4+29003x^6+39650x^8+24635x^{10}+6015x^{12}+495x^{14})+1767}{}$$

```
[In] Integrate[x^4*(2 + 3*x^2)*(3 + 5*x^2 + x^4)^(3/2),x]
```

```
[Out] (4*x*(-63150 - 93991*x^2 + 3055*x^4 + 29003*x^6 + 39650*x^8 + 24635*x^10 +
6015*x^12 + 495*x^14) + (176723*I)*Sqrt[2]*(-5 + Sqrt[13])*Sqrt[(-5 + Sqrt[
13] - 2*x^2)/(-5 + Sqrt[13])]*Sqrt[5 + Sqrt[13] + 2*x^2]*EllipticE[I*ArcSin
```

```
h[Sqrt[2/(5 + Sqrt[13])]*x], 19/6 + (5*Sqrt[13])/6] - I*Sqrt[2]*(-757315 +
176723*Sqrt[13])*Sqrt[(-5 + Sqrt[13] - 2*x^2)/(-5 + Sqrt[13])]*Sqrt[5 + Sqr
t[13] + 2*x^2]*EllipticF[I*ArcSinh[Sqrt[2/(5 + Sqrt[13])]*x], 19/6 + (5*Sqr
t[13])/6])/(8580*Sqrt[3 + 5*x^2 + x^4])
```

Maple [A] (verified)

Time = 4.21 (sec) , antiderivative size = 236, normalized size of antiderivative = 0.66

method	result
risch	$\frac{x(495x^{10}+3540x^8+5450x^6+1780x^4+3753x^2-21050)\sqrt{x^4+5x^2+3}}{2145}+\frac{25260\sqrt{1-\left(-\frac{5}{6}+\frac{\sqrt{13}}{6}\right)x^2}\sqrt{1-\left(-\frac{5}{6}-\frac{\sqrt{13}}{6}\right)x^2}F\left(\frac{x\sqrt{-30+6}}{6}\right.}{143\sqrt{-30+6\sqrt{13}}\sqrt{x^4+5x^2+3}}$
default	$\frac{3x^{11}\sqrt{x^4+5x^2+3}}{13}+\frac{236x^9\sqrt{x^4+5x^2+3}}{143}+\frac{1090x^7\sqrt{x^4+5x^2+3}}{429}+\frac{356x^5\sqrt{x^4+5x^2+3}}{429}+\frac{1251x^3\sqrt{x^4+5x^2+3}}{715}-\frac{4210x\sqrt{x^4+}}{429}$
elliptic	$\frac{3x^{11}\sqrt{x^4+5x^2+3}}{13}+\frac{236x^9\sqrt{x^4+5x^2+3}}{143}+\frac{1090x^7\sqrt{x^4+5x^2+3}}{429}+\frac{356x^5\sqrt{x^4+5x^2+3}}{429}+\frac{1251x^3\sqrt{x^4+5x^2+3}}{715}-\frac{4210x\sqrt{x^4+}}{429}$

```
[In] int(x^4*(3*x^2+2)*(x^4+5*x^2+3)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2145*x*(495*x^10+3540*x^8+5450*x^6+1780*x^4+3753*x^2-21050)*(x^4+5*x^2+3)
^(1/2)+25260/143/(-30+6*13^(1/2))^(1/2)*(1-(-5/6+1/6*13^(1/2))*x^2)^(1/2)*(
1-(-5/6-1/6*13^(1/2))*x^2)^(1/2)/(x^4+5*x^2+3)^(1/2)*EllipticF(1/6*x*(-30+6
*13^(1/2))^(1/2),5/6*3^(1/2)+1/6*39^(1/2))-2120676/715/(-30+6*13^(1/2))^(1/
2)*(1-(-5/6+1/6*13^(1/2))*x^2)^(1/2)*(1-(-5/6-1/6*13^(1/2))*x^2)^(1/2)/(x^4
+5*x^2+3)^(1/2)/(5+13^(1/2))*(EllipticF(1/6*x*(-30+6*13^(1/2))^(1/2),5/6*3^
(1/2)+1/6*39^(1/2))-EllipticE(1/6*x*(-30+6*13^(1/2))^(1/2),5/6*3^(1/2)+1/6*
39^(1/2)))
```

Fricas [A] (verification not implemented)

none

Time = 0.08 (sec) , antiderivative size = 149, normalized size of antiderivative = 0.42

$$\int x^4(2+3x^2)\left(3+5x^2+x^4\right)^{3/2}\,dx = \frac{176723\left(\sqrt{13}\sqrt{2}x-5\sqrt{2}x\right)\sqrt{\sqrt{13}-5}E(\arcsin\left(\frac{\sqrt{2}\sqrt{\sqrt{13}-5}}{2x}\right)\mid\frac{5}{6}\sqrt{13}+\frac{19}{6})-\left(155673\sqrt{13}\sqrt{2}\right.}{}$$

```
[In] integrate(x^4*(3*x^2+2)*(x^4+5*x^2+3)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/8580*(176723*(sqrt(13)*sqrt(2)*x - 5*sqrt(2)*x)*sqrt(sqrt(13) - 5)*ellipt
ic_e(arcsin(1/2*sqrt(2)*sqrt(sqrt(13) - 5)/x), 5/6*sqrt(13) + 19/6) - (1556
```

```
73*sqrt(13)*sqrt(2)*x - 988865*sqrt(2)*x)*sqrt(sqrt(13) - 5)*elliptic_f(arc
sin(1/2*sqrt(2)*sqrt(sqrt(13) - 5)/x), 5/6*sqrt(13) + 19/6) + 4*(495*x^12 +
 3540*x^10 + 5450*x^8 + 1780*x^6 + 3753*x^4 - 21050*x^2 + 176723)*sqrt(x^4
+ 5*x^2 + 3))/x
```

Sympy [F]

$$\int x^4 (2+3x^2) (3+5x^2+x^4)^{3/2} \, dx = \int x^4 \cdot (3x^2+2) (x^4+5x^2+3)^{\frac{3}{2}} \, dx$$

```
[In] integrate(x**4*(3*x**2+2)*(x**4+5*x**2+3)**(3/2),x)
[Out] Integral(x**4*(3*x**2 + 2)*(x**4 + 5*x**2 + 3)**(3/2), x)
```

Maxima [F]

$$\int x^4 (2+3x^2) (3+5x^2+x^4)^{3/2} \, dx = \int (x^4+5x^2+3)^{\frac{3}{2}} (3x^2+2) x^4 \, dx$$

```
[In] integrate(x^4*(3*x^2+2)*(x^4+5*x^2+3)^(3/2),x, algorithm="maxima")
[Out] integrate((x^4 + 5*x^2 + 3)^(3/2)*(3*x^2 + 2)*x^4, x)
```

Giac [F]

$$\int x^4 (2+3x^2) (3+5x^2+x^4)^{3/2} \, dx = \int (x^4+5x^2+3)^{\frac{3}{2}} (3x^2+2) x^4 \, dx$$

```
[In] integrate(x^4*(3*x^2+2)*(x^4+5*x^2+3)^(3/2),x, algorithm="giac")
[Out] integrate((x^4 + 5*x^2 + 3)^(3/2)*(3*x^2 + 2)*x^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^4 (2+3x^2) (3+5x^2+x^4)^{3/2} \, dx = \int x^4 (3x^2+2) (x^4+5x^2+3)^{3/2} \, dx$$

```
[In] int(x^4*(3*x^2 + 2)*(5*x^2 + x^4 + 3)^(3/2),x)
[Out] int(x^4*(3*x^2 + 2)*(5*x^2 + x^4 + 3)^(3/2), x)
```

3.164 $\int x^2(2+3x^2)\left(3+5x^2+x^4\right)^{3/2}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 331

$$\int x^2(2+3x^2)\left(3+5x^2+x^4\right)^{3/2}\,dx=-\frac{49949x\left(5+\sqrt{13}+2x^2\right)}{3465\sqrt{3+5x^2+x^4}}+\frac{353}{99}x\sqrt{3+5x^2+x^4}-\frac{x^3(911+890x^2)\sqrt{3+5x^2+x^4}}{1155}+\frac{1}{99}x^3\left(67+27x^2\right)\left(3+5x^2+x^4\right)^{3/2}+\frac{49949\sqrt{\frac{1}{6}\left(5+\sqrt{13}\right)}\sqrt{\frac{6+\left(5-\sqrt{13}\right)x^2}{6+\left(5+\sqrt{13}\right)x^2}}\left(6+\left(5+\sqrt{13}\right)x^2\right)E\left(\arctan\left(\sqrt{\frac{1}{6}\left(5+\sqrt{13}\right)}x\right)|\frac{1}{6}\left(-13+5\sqrt{13}\right)\right)}{3465\sqrt{3+5x^2+x^4}}-\frac{353\sqrt{\frac{6+\left(5-\sqrt{13}\right)x^2}{6+\left(5+\sqrt{13}\right)x^2}}\left(6+\left(5+\sqrt{13}\right)x^2\right)\text{EllipticF}\left(\arctan\left(\sqrt{\frac{1}{6}\left(5+\sqrt{13}\right)}x\right),\frac{1}{6}\left(-13+5\sqrt{13}\right)\right)}{33\sqrt{6\left(5+\sqrt{13}\right)}\sqrt{3+5x^2+x^4}}$$

```
[Out] 1/99*x^3*(27*x^2+67)*(x^4+5*x^2+3)^(3/2)-49949/3465*x*(5+2*x^2+13^(1/2))/(x
^4+5*x^2+3)^(1/2)+353/99*x*(x^4+5*x^2+3)^(1/2)-1/1155*x^3*(890*x^2+911)*(x^
4+5*x^2+3)^(1/2)+49949/20790*(1/(36+x^2*(30+6*13^(1/2))))^(1/2)*(36+x^2*(30
+6*13^(1/2)))^(1/2)*EllipticE(x*(30+6*13^(1/2))^(1/2)/(36+x^2*(30+6*13^(1/2
)))^(1/2),1/6*(-78+30*13^(1/2))^(1/2))*(6+x^2*(5+13^(1/2)))*(30+6*13^(1/2))
^(1/2)*((6+x^2*(5-13^(1/2)))/(6+x^2*(5+13^(1/2))))^(1/2)/(x^4+5*x^2+3)^(1/2
)-353/33*(1/(36+x^2*(30+6*13^(1/2))))^(1/2)*(36+x^2*(30+6*13^(1/2)))^(1/2)*
EllipticF(x*(30+6*13^(1/2))^(1/2)/(36+x^2*(30+6*13^(1/2)))^(1/2),1/6*(-78+3
0*13^(1/2))^(1/2))*(6+x^2*(5+13^(1/2)))*((6+x^2*(5-13^(1/2)))/(6+x^2*(5+13^
(1/2))))^(1/2)/(x^4+5*x^2+3)^(1/2)/(30+6*13^(1/2))^(1/2)
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 331, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {1287, 1293, 1203, 1113, 1149}

$$\int x^2(2+3x^2)\left(3+5x^2+x^4\right)^{3/2}dx = -\frac{353\sqrt{\frac{(5-\sqrt{13})x^2+6}{(5+\sqrt{13})x^2+6}}((5+\sqrt{13})x^2+6)\,\text{EllipticF}\left(\arctan\left(\sqrt{\frac{1}{6}(5+\sqrt{13})}x\right),\frac{1}{6}(-13+5\sqrt{13})\right)}{33\sqrt{6(5+\sqrt{13})}\sqrt{x^4+5x^2+3}}$$
$$+\frac{49949\sqrt{\frac{1}{6}(5+\sqrt{13})}\sqrt{\frac{(5-\sqrt{13})x^2+6}{(5+\sqrt{13})x^2+6}}((5+\sqrt{13})x^2+6)\,E\left(\arctan\left(\sqrt{\frac{1}{6}(5+\sqrt{13})}x\right)|\frac{1}{6}(-13+5\sqrt{13})\right)}{3465\sqrt{x^4+5x^2+3}}$$
$$+\frac{353}{99}\sqrt{x^4+5x^2+3}x-\frac{49949(2x^2+\sqrt{13}+5)x}{3465\sqrt{x^4+5x^2+3}}$$
$$+\frac{1}{99}(27x^2+67)\left(x^4+5x^2+3\right)^{3/2}x^3-\frac{(890x^2+911)\sqrt{x^4+5x^2+3}x^3}{1155}$$

```
[In] Int[x^2*(2 + 3*x^2)*(3 + 5*x^2 + x^4)^(3/2),x]
```

```
[Out] (-49949*x*(5 + Sqrt[13] + 2*x^2))/(3465*Sqrt[3 + 5*x^2 + x^4]) + (353*x*Sqr
t[3 + 5*x^2 + x^4])/99 - (x^3*(911 + 890*x^2)*Sqrt[3 + 5*x^2 + x^4])/1155 +
 (x^3*(67 + 27*x^2)*(3 + 5*x^2 + x^4)^(3/2))/99 + (49949*Sqrt[(5 + Sqrt[13]
)/6]*Sqrt[(6 + (5 - Sqrt[13])*x^2)/(6 + (5 + Sqrt[13])*x^2)]*(6 + (5 + Sqrt
[13])*x^2)*EllipticE[ArcTan[Sqrt[(5 + Sqrt[13])/6]*x], (-13 + 5*Sqrt[13])/6
])/(3465*Sqrt[3 + 5*x^2 + x^4]) - (353*Sqrt[(6 + (5 - Sqrt[13])*x^2)/(6 + (
5 + Sqrt[13])*x^2)]*(6 + (5 + Sqrt[13])*x^2)*EllipticF[ArcTan[Sqrt[(5 + Sqr
t[13])/6]*x], (-13 + 5*Sqrt[13])/6])/(33*Sqrt[6*(5 + Sqrt[13])]*Sqrt[3 + 5*
x^2 + x^4])
```

Rule 1113

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[(2*a + (b + q)*x^2)*(Sqrt[(2*a + (b - q)*x^2)/(2*a +
(b + q)*x^2)]/(2*a*Rt[(b + q)/(2*a), 2]*Sqrt[a + b*x^2 + c*x^4]))*EllipticF
[ArcTan[Rt[(b + q)/(2*a), 2]*x], 2*(q/(b + q))], x] /; PosQ[(b + q)/a] &&
!(PosQ[(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[
{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1149

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[b^2 - 4*a*c, 2]}, Simp[x*((b + q + 2*c*x^2)/(2*c*Sqrt[a + b*x^2 + c*x^4
```

```
])), x] - Simp[Rt[(b + q)/(2*a), 2]*(2*a + (b + q)*x^2)*(Sqrt[(2*a + (b - q
)*x^2)/(2*a + (b + q)*x^2)]/(2*c*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[ArcTan
[Rt[(b + q)/(2*a), 2]*x], 2*(q/(b + q))], x] /; PosQ[(b + q)/a] &&  !(PosQ[
(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[{a, b,
c}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1203

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[d, Int[1/Sqrt[a + b*x^2 + c*x^4],
 x], x] + Dist[e, Int[x^2/Sqrt[a + b*x^2 + c*x^4], x], x] /; PosQ[(b + q)/a
] || PosQ[(b - q)/a]] /; FreeQ[{a, b, c, d, e}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1287

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(a + b*x^2 + c*x^4)^p*((b*e*2
*p + c*d*(m + 4*p + 3) + c*e*(4*p + m + 1)*x^2)/(c*f*(4*p + m + 1)*(m + 4*p
 + 3))), x] + Dist[2*(p/(c*(4*p + m + 1)*(m + 4*p + 3))), Int[(f*x)^m*(a +
b*x^2 + c*x^4)^(p - 1)*Simp[2*a*c*d*(m + 4*p + 3) - a*b*e*(m + 1) + (2*a*c*
e*(4*p + m + 1) + b*c*d*(m + 4*p + 3) - b^2*e*(m + 2*p + 1))*x^2, x], x], x
] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b^2 - 4*a*c, 0] && GtQ[p, 0] &&
 NeQ[4*p + m + 1, 0] && NeQ[m + 4*p + 3, 0] && IntegerQ[2*p] && (IntegerQ[p
] || IntegerQ[m])
```

Rule 1293

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[e*f*(f*x)^(m - 1)*((a + b*x^2 + c*x^4)^(p +
1)/(c*(m + 4*p + 3))), x] - Dist[f^2/(c*(m + 4*p + 3)), Int[(f*x)^(m - 2)*(
a + b*x^2 + c*x^4)^p*Simp[a*e*(m - 1) + (b*e*(m + 2*p + 1) - c*d*(m + 4*p +
 3))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c,
 0] && GtQ[m, 1] && NeQ[m + 4*p + 3, 0] && IntegerQ[2*p] && (IntegerQ[p] ||
 IntegerQ[m])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{99}x^3(67+27x^2)\left(3+5x^2+x^4\right)^{3/2}+\frac{1}{33}\int x^2(-3-178x^2)\sqrt{3+5x^2+x^4}\,dx \\
&= -\frac{x^3(911+890x^2)\sqrt{3+5x^2+x^4}}{1155}+\frac{1}{99}x^3(67+27x^2)\left(3+5x^2+x^4\right)^{3/2}+\frac{\int\frac{x^2(7884+12355x^2)}{\sqrt{3+5x^2+x^4}}\,dx}{1155} \\
&= \frac{353}{99}x\sqrt{3+5x^2+x^4}-\frac{x^3(911+890x^2)\sqrt{3+5x^2+x^4}}{1155} \\
&\quad +\frac{1}{99}x^3(67+27x^2)\left(3+5x^2+x^4\right)^{3/2}-\frac{\int\frac{37065+99898x^2}{\sqrt{3+5x^2+x^4}}\,dx}{3465}
\end{aligned}$$

$$= \frac{353}{99}x\sqrt{3+5x^2+x^4} - \frac{x^3(911+890x^2)\sqrt{3+5x^2+x^4}}{1155} + \frac{1}{99}x^3(67+27x^2)\left(3+5x^2+x^4\right)^{3/2} - \frac{353}{33}\int \frac{1}{\sqrt{3+5x^2+x^4}}\,dx - \frac{99898\int \frac{x^2}{\sqrt{3+5x^2+x^4}}\,dx}{3465}$$

$$= -\frac{49949x(5+\sqrt{13}+2x^2)}{3465\sqrt{3+5x^2+x^4}} + \frac{353}{99}x\sqrt{3+5x^2+x^4} - \frac{x^3(911+890x^2)\sqrt{3+5x^2+x^4}}{1155} + \frac{1}{99}x^3(67+27x^2)\left(3+5x^2+x^4\right)^{3/2} + \frac{49949\sqrt{\frac{1}{6}\left(5+\sqrt{13}\right)}\sqrt{\frac{6+\left(5-\sqrt{13}\right)x^2}{6+\left(5+\sqrt{13}\right)x^2}}\left(6+\left(5+\sqrt{13}\right)x^2\right)E\left(\tan^{-1}\left(\sqrt{\frac{1}{6}\left(5+\sqrt{13}\right)}x\right)|\frac{1}{6}\left(-13+5\right.\right.}{3465\sqrt{3+5x^2+x^4}} - \frac{353\sqrt{\frac{6+\left(5-\sqrt{13}\right)x^2}{6+\left(5+\sqrt{13}\right)x^2}}\left(6+\left(5+\sqrt{13}\right)x^2\right)F\left(\tan^{-1}\left(\sqrt{\frac{1}{6}\left(5+\sqrt{13}\right)}x\right)|\frac{1}{6}\left(-13+5\sqrt{13}\right)\right)}{33\sqrt{6\left(5+\sqrt{13}\right)}\sqrt{3+5x^2+x^4}}$$

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 7.62 (sec) , antiderivative size = 244, normalized size of antiderivative = 0.74

$$\int x^2(2+3x^2)\left(3+5x^2+x^4\right)^{3/2}\,dx = \frac{2x(37065+74681x^2+69535x^4+84962x^6+50075x^8+11795x^{10}+945x^{12})-49949i\sqrt{2}(-5+}{}$$

```
[In] Integrate[x^2*(2 + 3*x^2)*(3 + 5*x^2 + x^4)^(3/2),x]
```

```
[Out] (2*x*(37065 + 74681*x^2 + 69535*x^4 + 84962*x^6 + 50075*x^8 + 11795*x^10 +
945*x^12) - (49949*I)*Sqrt[2]*(-5 + Sqrt[13])*Sqrt[(-5 + Sqrt[13] - 2*x^2)/
(-5 + Sqrt[13])]*Sqrt[5 + Sqrt[13] + 2*x^2]*EllipticE[I*ArcSinh[Sqrt[2/(5 +
 Sqrt[13])]*x], 19/6 + (5*Sqrt[13])/6] + I*Sqrt[2]*(-212680 + 49949*Sqrt[13
])*Sqrt[(-5 + Sqrt[13] - 2*x^2)/(-5 + Sqrt[13])]*Sqrt[5 + Sqrt[13] + 2*x^2]
*EllipticF[I*ArcSinh[Sqrt[2/(5 + Sqrt[13])]*x], 19/6 + (5*Sqrt[13])/6])/(69
30*Sqrt[3 + 5*x^2 + x^4])
```

Maple [A] (verified)

Time = 2.78 (sec) , antiderivative size = 231, normalized size of antiderivative = 0.70

method	result
risch	$\frac{x(945x^8+7070x^6+11890x^4+4302x^2+12355)\sqrt{x^4+5x^2+3}}{3465}-\frac{706\sqrt{1-\left(-\frac{5}{6}+\frac{\sqrt{13}}{6}\right)x^2}\sqrt{1-\left(-\frac{5}{6}-\frac{\sqrt{13}}{6}\right)x^2}\,F\left(\frac{x\sqrt{-30+6\sqrt{13}}}{6},\frac{5\sqrt{3}}{6}+\right.}{11\sqrt{-30+6\sqrt{13}}\sqrt{x^4+5x^2+3}}$
default	$\frac{3x^9\sqrt{x^4+5x^2+3}}{11}+\frac{202x^7\sqrt{x^4+5x^2+3}}{99}+\frac{2378x^5\sqrt{x^4+5x^2+3}}{693}+\frac{478x^3\sqrt{x^4+5x^2+3}}{385}+\frac{353x\sqrt{x^4+5x^2+3}}{99}-\frac{706\sqrt{1-\left(-\frac{5}{6}+\right.}}{}$
elliptic	$\frac{3x^9\sqrt{x^4+5x^2+3}}{11}+\frac{202x^7\sqrt{x^4+5x^2+3}}{99}+\frac{2378x^5\sqrt{x^4+5x^2+3}}{693}+\frac{478x^3\sqrt{x^4+5x^2+3}}{385}+\frac{353x\sqrt{x^4+5x^2+3}}{99}-\frac{706\sqrt{1-\left(-\frac{5}{6}+\right.}}{}$

```
[In] int(x^2*(3*x^2+2)*(x^4+5*x^2+3)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3465*x*(945*x^8+7070*x^6+11890*x^4+4302*x^2+12355)*(x^4+5*x^2+3)^(1/2)-70
6/11/(-30+6*13^(1/2))^(1/2)*(1-(-5/6+1/6*13^(1/2))*x^2)^(1/2)*(1-(-5/6-1/6*
13^(1/2))*x^2)^(1/2)/(x^4+5*x^2+3)^(1/2)*EllipticF(1/6*x*(-30+6*13^(1/2))^(
1/2),5/6*3^(1/2)+1/6*39^(1/2))+399592/385/(-30+6*13^(1/2))^(1/2)*(1-(-5/6+1
/6*13^(1/2))*x^2)^(1/2)*(1-(-5/6-1/6*13^(1/2))*x^2)^(1/2)/(x^4+5*x^2+3)^(1/
2)/(5+13^(1/2))*(EllipticF(1/6*x*(-30+6*13^(1/2))^(1/2),5/6*3^(1/2)+1/6*39^
(1/2))-EllipticE(1/6*x*(-30+6*13^(1/2))^(1/2),5/6*3^(1/2)+1/6*39^(1/2)))
```

Fricas [A] (verification not implemented)

none

Time = 0.08 (sec) , antiderivative size = 144, normalized size of antiderivative = 0.44

$$\int x^2(2+3x^2)\left(3+5x^2+x^4\right)^{3/2}\,dx =$$

$$-\frac{99898\left(\sqrt{13}\sqrt{2}x-5\sqrt{2}x\right)\sqrt{\sqrt{13}-5}E(\arcsin\left(\frac{\sqrt{2}\sqrt{\sqrt{13}-5}}{2\,x}\right)\mid\frac{5}{6}\sqrt{13}+\frac{19}{6})-\left(87543\sqrt{13}\sqrt{2}x-561265\sqrt{2}\right.}{}$$

```
[In] integrate(x^2*(3*x^2+2)*(x^4+5*x^2+3)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/13860*(99898*(sqrt(13)*sqrt(2)*x - 5*sqrt(2)*x)*sqrt(sqrt(13) - 5)*ellip
tic_e(arcsin(1/2*sqrt(2)*sqrt(sqrt(13) - 5)/x), 5/6*sqrt(13) + 19/6) - (875
43*sqrt(13)*sqrt(2)*x - 561265*sqrt(2)*x)*sqrt(sqrt(13) - 5)*elliptic_f(arc
sin(1/2*sqrt(2)*sqrt(sqrt(13) - 5)/x), 5/6*sqrt(13) + 19/6) - 4*(945*x^10 +
 7070*x^8 + 11890*x^6 + 4302*x^4 + 12355*x^2 - 99898)*sqrt(x^4 + 5*x^2 + 3)
)/x
```

Sympy [F]

$$\int x^2 \left(2+3 x^2\right) \left(3+5 x^2+x^4\right)^{3/2} \, dx=\int x^{2} \cdot \left(3 x^{2} + 2\right) \left(x^{4} + 5 x^{2} + 3\right)^{\frac{3}{2}}\, dx$$

```
[In] integrate(x**2*(3*x**2+2)*(x**4+5*x**2+3)**(3/2),x)
[Out] Integral(x**2*(3*x**2 + 2)*(x**4 + 5*x**2 + 3)**(3/2), x)
```

Maxima [F]

$$\int x^2 \left(2+3 x^2\right) \left(3+5 x^2+x^4\right)^{3/2} \, dx=\int { {\left(x^{4} + 5 \, x^{2} + 3\right)}^{\frac{3}{2}} {\left(3 \, x^{2} + 2\right)} x^{2} \,d x }$$

```
[In] integrate(x^2*(3*x^2+2)*(x^4+5*x^2+3)^(3/2),x, algorithm="maxima")
[Out] integrate((x^4 + 5*x^2 + 3)^(3/2)*(3*x^2 + 2)*x^2, x)
```

Giac [F]

$$\int x^2 \left(2+3 x^2\right) \left(3+5 x^2+x^4\right)^{3/2} \, dx=\int { {\left(x^{4} + 5 \, x^{2} + 3\right)}^{\frac{3}{2}} {\left(3 \, x^{2} + 2\right)} x^{2} \,d x }$$

```
[In] integrate(x^2*(3*x^2+2)*(x^4+5*x^2+3)^(3/2),x, algorithm="giac")
[Out] integrate((x^4 + 5*x^2 + 3)^(3/2)*(3*x^2 + 2)*x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^2 \left(2+3 x^2\right) \left(3+5 x^2+x^4\right)^{3/2} \, dx=\int x^2\,\left(3\,x^2+2\right)\,{\left(x^4+5\,x^2+3\right)}^{3/2} \,d x$$

```
[In] int(x^2*(3*x^2 + 2)*(5*x^2 + x^4 + 3)^(3/2),x)
[Out] int(x^2*(3*x^2 + 2)*(5*x^2 + x^4 + 3)^(3/2), x)
```

3.165 $\int (2+3x^2)(3+5x^2+x^4)^{3/2}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 308

$$
\begin{aligned}
\int (2+3x^2)(3+5x^2+x^4)^{3/2}\,dx &= \frac{203x(5+\sqrt{13}+2x^2)}{30\sqrt{3+5x^2+x^4}} \\
&- \frac{1}{15}x(5+12x^2)\sqrt{3+5x^2+x^4} + \frac{1}{3}x(3+x^2)(3+5x^2+x^4)^{3/2} \\
&- \frac{203\sqrt{\frac{1}{6}(5+\sqrt{13})}\sqrt{\frac{6+(5-\sqrt{13})x^2}{6+(5+\sqrt{13})x^2}}(6+(5+\sqrt{13})x^2)\,E\left(\arctan\left(\sqrt{\frac{1}{6}(5+\sqrt{13})}x\right)|\frac{1}{6}(-13+5\sqrt{13})\right)}{30\sqrt{3+5x^2+x^4}} \\
&+ \frac{5\sqrt{\frac{2}{3(5+\sqrt{13})}}\sqrt{\frac{6+(5-\sqrt{13})x^2}{6+(5+\sqrt{13})x^2}}(6+(5+\sqrt{13})x^2)\,\text{EllipticF}\left(\arctan\left(\sqrt{\frac{1}{6}(5+\sqrt{13})}x\right),\frac{1}{6}(-13+5\sqrt{13})\right)}{\sqrt{3+5x^2+x^4}}
\end{aligned}
$$

```
[Out] 1/3*x*(x^2+3)*(x^4+5*x^2+3)^(3/2)+203/30*x*(5+2*x^2+13^(1/2))/(x^4+5*x^2+3)
^(1/2)-1/15*x*(12*x^2+5)*(x^4+5*x^2+3)^(1/2)+5/3*(1/(36+x^2*(30+6*13^(1/2))
))^(1/2)*(36+x^2*(30+6*13^(1/2)))^(1/2)*EllipticF(x*(30+6*13^(1/2))^(1/2)/(
36+x^2*(30+6*13^(1/2)))^(1/2),1/6*(-78+30*13^(1/2))^(1/2))*(6+x^2*(5+13^(1/
2)))*6^(1/2)/(5+13^(1/2))^(1/2)*((6+x^2*(5-13^(1/2)))/(6+x^2*(5+13^(1/2))))
^(1/2)/(x^4+5*x^2+3)^(1/2)-203/180*(1/(36+x^2*(30+6*13^(1/2))))^(1/2)*(36+x
^2*(30+6*13^(1/2)))^(1/2)*EllipticE(x*(30+6*13^(1/2))^(1/2)/(36+x^2*(30+6*1
3^(1/2)))^(1/2),1/6*(-78+30*13^(1/2))^(1/2))*(6+x^2*(5+13^(1/2)))*(30+6*13^
(1/2))^(1/2)*((6+x^2*(5-13^(1/2)))/(6+x^2*(5+13^(1/2))))^(1/2)/(x^4+5*x^2+3
)^(1/2)
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 308, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {1190, 1203, 1113, 1149}

$$\int (2+3x^2)\left(3+5x^2+x^4\right)^{3/2}\,dx = \frac{5\sqrt{\frac{2}{3\left(5+\sqrt{13}\right)}}\sqrt{\frac{\left(5-\sqrt{13}\right)x^2+6}{\left(5+\sqrt{13}\right)x^2+6}}\left(\left(5+\sqrt{13}\right)x^2+6\right)\text{EllipticF}\left(\arctan\left(\sqrt{\frac{1}{6}\left(5+\sqrt{13}\right)}x\right),\frac{1}{6}\left(-13+\right.\right.}{\sqrt{x^4+5x^2+3}}$$
$$-\frac{203\sqrt{\frac{1}{6}\left(5+\sqrt{13}\right)}\sqrt{\frac{\left(5-\sqrt{13}\right)x^2+6}{\left(5+\sqrt{13}\right)x^2+6}}\left(\left(5+\sqrt{13}\right)x^2+6\right)E\left(\arctan\left(\sqrt{\frac{1}{6}\left(5+\sqrt{13}\right)}x\right)|\frac{1}{6}\left(-13+5\sqrt{13}\right)\right)}{30\sqrt{x^4+5x^2+3}}$$
$$+\frac{1}{3}x\left(x^2+3\right)\left(x^4+5x^2+3\right)^{3/2}-\frac{1}{15}x\left(12x^2+5\right)\sqrt{x^4+5x^2+3}+\frac{203x\left(2x^2+\sqrt{13}+5\right)}{30\sqrt{x^4+5x^2+3}}$$

```
[In] Int[(2 + 3*x^2)*(3 + 5*x^2 + x^4)^(3/2),x]
```

```
[Out] (203*x*(5 + Sqrt[13] + 2*x^2))/(30*Sqrt[3 + 5*x^2 + x^4]) - (x*(5 + 12*x^2)
*Sqrt[3 + 5*x^2 + x^4])/15 + (x*(3 + x^2)*(3 + 5*x^2 + x^4)^(3/2))/3 - (203
*Sqrt[(5 + Sqrt[13])/6]*Sqrt[(6 + (5 - Sqrt[13])*x^2)/(6 + (5 + Sqrt[13])*x
^2)]*(6 + (5 + Sqrt[13])*x^2)*EllipticE[ArcTan[Sqrt[(5 + Sqrt[13])/6]*x], (
-13 + 5*Sqrt[13])/6])/(30*Sqrt[3 + 5*x^2 + x^4]) + (5*Sqrt[2/(3*(5 + Sqrt[1
3]))]*Sqrt[(6 + (5 - Sqrt[13])*x^2)/(6 + (5 + Sqrt[13])*x^2)]*(6 + (5 + Sqr
t[13])*x^2)*EllipticF[ArcTan[Sqrt[(5 + Sqrt[13])/6]*x], (-13 + 5*Sqrt[13])/
6])/Sqrt[3 + 5*x^2 + x^4]
```

Rule 1113

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[(2*a + (b + q)*x^2)*(Sqrt[(2*a + (b - q)*x^2)/(2*a +
(b + q)*x^2)]/(2*a*Rt[(b + q)/(2*a), 2]*Sqrt[a + b*x^2 + c*x^4]))*EllipticF
[ArcTan[Rt[(b + q)/(2*a), 2]*x], 2*(q/(b + q))], x] /; PosQ[(b + q)/a] &&
!(PosQ[(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[
{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1149

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[b^2 - 4*a*c, 2]}, Simp[x*((b + q + 2*c*x^2)/(2*c*Sqrt[a + b*x^2 + c*x^4
])), x] - Simp[Rt[(b + q)/(2*a), 2]*(2*a + (b + q)*x^2)*(Sqrt[(2*a + (b - q
)*x^2)/(2*a + (b + q)*x^2)]/(2*c*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[ArcTan
[Rt[(b + q)/(2*a), 2]*x], 2*(q/(b + q))], x] /; PosQ[(b + q)/a] &&  !(PosQ[
(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[{a, b,
```

```
c}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1190

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[x*(2*b*e*p + c*d*(4*p + 3) + c*e*(4*p + 1)*x^2)*((a + b*x^2 + c
*x^4)^p/(c*(4*p + 1)*(4*p + 3))), x] + Dist[2*(p/(c*(4*p + 1)*(4*p + 3))),
Int[Simp[2*a*c*d*(4*p + 3) - a*b*e + (2*a*c*e*(4*p + 1) + b*c*d*(4*p + 3) -
 b^2*e*(2*p + 1))*x^2, x]*(a + b*x^2 + c*x^4)^(p - 1), x], x] /; FreeQ[{a,
b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
GtQ[p, 0] && FractionQ[p] && IntegerQ[2*p]
```

Rule 1203

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[d, Int[1/Sqrt[a + b*x^2 + c*x^4],
 x], x] + Dist[e, Int[x^2/Sqrt[a + b*x^2 + c*x^4], x], x] /; PosQ[(b + q)/a
] || PosQ[(b - q)/a]] /; FreeQ[{a, b, c, d, e}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{1}{3}x\left(3+x^2\right)\left(3+5x^2+x^4\right)^{3/2}+\frac{1}{21}\int\left(63-84x^2\right)\sqrt{3+5x^2+x^4}\,dx\\
&=-\frac{1}{15}x\left(5+12x^2\right)\sqrt{3+5x^2+x^4}+\frac{1}{3}x\left(3+x^2\right)\left(3+5x^2+x^4\right)^{3/2}+\frac{1}{315}\int\frac{3150+4263x^2}{\sqrt{3+5x^2+x^4}}\,dx\\
&=-\frac{1}{15}x\left(5+12x^2\right)\sqrt{3+5x^2+x^4}+\frac{1}{3}x\left(3+x^2\right)\left(3+5x^2+x^4\right)^{3/2}\\
&\quad+10\int\frac{1}{\sqrt{3+5x^2+x^4}}\,dx+\frac{203}{15}\int\frac{x^2}{\sqrt{3+5x^2+x^4}}\,dx\\
&=\frac{203x\left(5+\sqrt{13}+2x^2\right)}{30\sqrt{3+5x^2+x^4}}-\frac{1}{15}x\left(5+12x^2\right)\sqrt{3+5x^2+x^4}+\frac{1}{3}x\left(3+x^2\right)\left(3+5x^2+x^4\right)^{3/2}\\
&\quad-\frac{203\sqrt{\frac{1}{6}\left(5+\sqrt{13}\right)}\sqrt{\frac{6+\left(5-\sqrt{13}\right)x^2}{6+\left(5+\sqrt{13}\right)x^2}}\left(6+\left(5+\sqrt{13}\right)x^2\right)E\left(\tan^{-1}\left(\sqrt{\frac{1}{6}\left(5+\sqrt{13}\right)}x\right)|\frac{1}{6}\left(-13+5\right.\right.}{30\sqrt{3+5x^2+x^4}}\\
&\quad+\frac{5\sqrt{\frac{2}{3\left(5+\sqrt{13}\right)}}\sqrt{\frac{6+\left(5-\sqrt{13}\right)x^2}{6+\left(5+\sqrt{13}\right)x^2}}\left(6+\left(5+\sqrt{13}\right)x^2\right)F\left(\tan^{-1}\left(\sqrt{\frac{1}{6}\left(5+\sqrt{13}\right)}x\right)|\frac{1}{6}\left(-13+5\sqrt{13}\right)\right)}{\sqrt{3+5x^2+x^4}}
\end{aligned}
$$

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 6.66 (sec) , antiderivative size = 239, normalized size of antiderivative = 0.78

$$\int (2+3x^2)(3+5x^2+x^4)^{3/2}\,dx = \frac{4x(120+434x^2+550x^4+293x^6+65x^8+5x^{10})+203i\sqrt{2}(-5+\sqrt{13})\sqrt{\frac{-5+\sqrt{13}-2x^2}{-5+\sqrt{13}}}\sqrt{5+\sqrt{1}}$$

```
[In] Integrate[(2 + 3*x^2)*(3 + 5*x^2 + x^4)^(3/2),x]
```

```
[Out] (4*x*(120 + 434*x^2 + 550*x^4 + 293*x^6 + 65*x^8 + 5*x^10) + (203*I)*Sqrt[2
]*(-5 + Sqrt[13])*Sqrt[(-5 + Sqrt[13] - 2*x^2)/(-5 + Sqrt[13])]*Sqrt[5 + Sq
rt[13] + 2*x^2]*EllipticE[I*ArcSinh[Sqrt[2/(5 + Sqrt[13])]*x], 19/6 + (5*Sq
rt[13])/6] - I*Sqrt[2]*(-715 + 203*Sqrt[13])*Sqrt[(-5 + Sqrt[13] - 2*x^2)/(
-5 + Sqrt[13])]*Sqrt[5 + Sqrt[13] + 2*x^2]*EllipticF[I*ArcSinh[Sqrt[2/(5 +
Sqrt[13])]*x], 19/6 + (5*Sqrt[13])/6])/(60*Sqrt[3 + 5*x^2 + x^4])
```

Maple [A] (verified)

Time = 1.40 (sec) , antiderivative size = 226, normalized size of antiderivative = 0.73

method	result
risch	$\frac{x(5x^6+40x^4+78x^2+40)\sqrt{x^4+5x^2+3}}{15}+\frac{60\sqrt{1-\left(-\frac{5}{6}+\frac{\sqrt{13}}{6}\right)x^2}\sqrt{1-\left(-\frac{5}{6}-\frac{\sqrt{13}}{6}\right)x^2}\,F\left(\frac{x\sqrt{-30+6\sqrt{13}}}{6},\frac{5\sqrt{3}}{6}+\frac{\sqrt{39}}{6}\right)}{\sqrt{-30+6\sqrt{13}}\sqrt{x^4+5x^2+3}}-\frac{2436\sqrt{1-(-}{}$
default	$\frac{8x^5\sqrt{x^4+5x^2+3}}{3}+\frac{26x^3\sqrt{x^4+5x^2+3}}{5}+\frac{8x\sqrt{x^4+5x^2+3}}{3}+\frac{60\sqrt{1-\left(-\frac{5}{6}+\frac{\sqrt{13}}{6}\right)x^2}\sqrt{1-\left(-\frac{5}{6}-\frac{\sqrt{13}}{6}\right)x^2}\,F\left(\frac{x\sqrt{-30+6\sqrt{13}}}{6},\frac{5\sqrt{3}}{6}+\frac{\sqrt{}}{}\right.}{\sqrt{-30+6\sqrt{13}}\sqrt{x^4+5x^2+3}}$
elliptic	$\frac{8x^5\sqrt{x^4+5x^2+3}}{3}+\frac{26x^3\sqrt{x^4+5x^2+3}}{5}+\frac{8x\sqrt{x^4+5x^2+3}}{3}+\frac{60\sqrt{1-\left(-\frac{5}{6}+\frac{\sqrt{13}}{6}\right)x^2}\sqrt{1-\left(-\frac{5}{6}-\frac{\sqrt{13}}{6}\right)x^2}\,F\left(\frac{x\sqrt{-30+6\sqrt{13}}}{6},\frac{5\sqrt{3}}{6}+\frac{\sqrt{}}{}\right.}{\sqrt{-30+6\sqrt{13}}\sqrt{x^4+5x^2+3}}$

```
[In] int((3*x^2+2)*(x^4+5*x^2+3)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/15*x*(5*x^6+40*x^4+78*x^2+40)*(x^4+5*x^2+3)^(1/2)+60/(-30+6*13^(1/2))^(1/
2)*(1-(-5/6+1/6*13^(1/2))*x^2)^(1/2)*(1-(-5/6-1/6*13^(1/2))*x^2)^(1/2)/(x^4
+5*x^2+3)^(1/2)*EllipticF(1/6*x*(-30+6*13^(1/2))^(1/2),5/6*3^(1/2)+1/6*39^(
1/2))-2436/5/(-30+6*13^(1/2))^(1/2)*(1-(-5/6+1/6*13^(1/2))*x^2)^(1/2)*(1-(-
5/6-1/6*13^(1/2))*x^2)^(1/2)/(x^4+5*x^2+3)^(1/2)/(5+13^(1/2))*(EllipticF(1/
6*x*(-30+6*13^(1/2))^(1/2),5/6*3^(1/2)+1/6*39^(1/2))-EllipticE(1/6*x*(-30+6
*13^(1/2))^(1/2),5/6*3^(1/2)+1/6*39^(1/2)))
```

Fricas [A] (verification not implemented)

none

Time = 0.10 (sec) , antiderivative size = 139, normalized size of antiderivative = 0.45

$$\int \left(2+3x^2\right)\left(3+5x^2+x^4\right)^{3/2}\,dx = \frac{203\left(\sqrt{13}\sqrt{2}x-5\sqrt{2}x\right)\sqrt{\sqrt{13}-5}E\left(\arcsin\left(\frac{\sqrt{2}\sqrt{\sqrt{13}-5}}{2x}\right)\mid\frac{5}{6}\sqrt{13}+\frac{19}{6}\right)-\left(153\sqrt{13}\sqrt{2}x-126\right.}{}$$

```
[In] integrate((3*x^2+2)*(x^4+5*x^2+3)^(3/2),x, algorithm="fricas")
[Out] 1/60*(203*(sqrt(13)*sqrt(2)*x - 5*sqrt(2)*x)*sqrt(sqrt(13) - 5)*elliptic_e(
arcsin(1/2*sqrt(2)*sqrt(sqrt(13) - 5)/x), 5/6*sqrt(13) + 19/6) - (153*sqrt(
13)*sqrt(2)*x - 1265*sqrt(2)*x)*sqrt(sqrt(13) - 5)*elliptic_f(arcsin(1/2*sq
rt(2)*sqrt(sqrt(13) - 5)/x), 5/6*sqrt(13) + 19/6) + 4*(5*x^8 + 40*x^6 + 78*
x^4 + 40*x^2 + 203)*sqrt(x^4 + 5*x^2 + 3))/x
```

Sympy [F]

$$\int \left(2+3x^2\right)\left(3+5x^2+x^4\right)^{3/2}\,dx = \int \left(3x^2+2\right)\left(x^4+5x^2+3\right)^{\frac{3}{2}}\,dx$$

```
[In] integrate((3*x**2+2)*(x**4+5*x**2+3)**(3/2),x)
[Out] Integral((3*x**2 + 2)*(x**4 + 5*x**2 + 3)**(3/2), x)
```

Maxima [F]

$$\int \left(2+3x^2\right)\left(3+5x^2+x^4\right)^{3/2}\,dx = \int \left(x^4+5\,x^2+3\right)^{\frac{3}{2}}\left(3\,x^2+2\right)dx$$

```
[In] integrate((3*x^2+2)*(x^4+5*x^2+3)^(3/2),x, algorithm="maxima")
[Out] integrate((x^4 + 5*x^2 + 3)^(3/2)*(3*x^2 + 2), x)
```

Giac [F]

$$\int (2+3x^2) (3+5x^2+x^4)^{3/2} \, dx = \int (x^4+5\,x^2+3)^{\frac{3}{2}}(3\,x^2+2)\, dx$$

```
[In] integrate((3*x^2+2)*(x^4+5*x^2+3)^(3/2),x, algorithm="giac")
[Out] integrate((x^4 + 5*x^2 + 3)^(3/2)*(3*x^2 + 2), x)
```

Mupad [F(-1)]

Timed out.

$$\int (2+3x^2) (3+5x^2+x^4)^{3/2} \, dx = \int (3\,x^2+2)\, (x^4+5\,x^2+3)^{3/2}\, dx$$

```
[In] int((3*x^2 + 2)*(5*x^2 + x^4 + 3)^(3/2),x)
[Out] int((3*x^2 + 2)*(5*x^2 + x^4 + 3)^(3/2), x)
```

3.166 $\int \frac{(2+3x^2)(3+5x^2+x^4)^{3/2}}{x^2} \, dx$

Optimal result

Integrand size = 25, antiderivative size = 312

$$\int \frac{(2+3x^2)(3+5x^2+x^4)^{3/2}}{x^2} \, dx = \frac{412x(5+\sqrt{13}+2x^2)}{35\sqrt{3+5x^2+x^4}} + \frac{1}{35}x(655+129x^2)\sqrt{3+5x^2+x^4} - \frac{(14-3x^2)(3+5x^2+x^4)^{3/2}}{7x} - \frac{206\sqrt{\frac{2}{3}(5+\sqrt{13})}\sqrt{\frac{6+(5-\sqrt{13})x^2}{6+(5+\sqrt{13})x^2}}(6+(5+\sqrt{13})x^2)E\left(\arctan\left(\sqrt{\frac{1}{6}(5+\sqrt{13})}x\right)|\frac{1}{6}(-13+5\sqrt{13})\right)}{35\sqrt{3+5x^2+x^4}} + \frac{19\sqrt{\frac{3}{2(5+\sqrt{13})}}\sqrt{\frac{6+(5-\sqrt{13})x^2}{6+(5+\sqrt{13})x^2}}(6+(5+\sqrt{13})x^2)\operatorname{EllipticF}\left(\arctan\left(\sqrt{\frac{1}{6}(5+\sqrt{13})}x\right),\frac{1}{6}(-13+5\sqrt{13})\right)}{\sqrt{3+5x^2+x^4}}$$

```
[Out] -1/7*(-3*x^2+14)*(x^4+5*x^2+3)^(3/2)/x+412/35*x*(5+2*x^2+13^(1/2))/(x^4+5*x
^2+3)^(1/2)+1/35*x*(129*x^2+655)*(x^4+5*x^2+3)^(1/2)+19/2*(1/(36+x^2*(30+6*
13^(1/2))))^(1/2)*(36+x^2*(30+6*13^(1/2)))^(1/2)*EllipticF(x*(30+6*13^(1/2)
)^(1/2)/(36+x^2*(30+6*13^(1/2)))^(1/2),1/6*(-78+30*13^(1/2))^(1/2))*(6+x^2*
(5+13^(1/2)))*6^(1/2)/(5+13^(1/2))^(1/2)*((6+x^2*(5-13^(1/2)))/(6+x^2*(5+13
^(1/2))))^(1/2)/(x^4+5*x^2+3)^(1/2)-206/105*(1/(36+x^2*(30+6*13^(1/2))))^(1
/2)*(36+x^2*(30+6*13^(1/2)))^(1/2)*EllipticE(x*(30+6*13^(1/2))^(1/2)/(36+x^
2*(30+6*13^(1/2)))^(1/2),1/6*(-78+30*13^(1/2))^(1/2))*(6+x^2*(5+13^(1/2)))*
(30+6*13^(1/2))^(1/2)*((6+x^2*(5-13^(1/2)))/(6+x^2*(5+13^(1/2))))^(1/2)/(x^
4+5*x^2+3)^(1/2)
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 312, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {1285, 1190, 1203, 1113, 1149}

$$\int \frac{(2+3x^2)(3+5x^2+x^4)^{3/2}}{x^2}\,dx = \frac{19\sqrt{\frac{3}{2\left(5+\sqrt{13}\right)}}\sqrt{\frac{\left(5-\sqrt{13}\right)x^2+6}{\left(5+\sqrt{13}\right)x^2+6}}\left(\left(5+\sqrt{13}\right)x^2+6\right)\text{EllipticF}\left(\arctan\left(\sqrt{\frac{1}{6}\left(\right.}\right.\right.}{\sqrt{x^4+5x^2+3}}$$

$$-\frac{206\sqrt{\frac{2}{3}\left(5+\sqrt{13}\right)}\sqrt{\frac{\left(5-\sqrt{13}\right)x^2+6}{\left(5+\sqrt{13}\right)x^2+6}}\left(\left(5+\sqrt{13}\right)x^2+6\right)E\left(\arctan\left(\sqrt{\frac{1}{6}\left(5+\sqrt{13}\right)}x\right)|\frac{1}{6}\left(-13+5\sqrt{13}\right)\right)}{35\sqrt{x^4+5x^2+3}}$$

$$-\frac{(14-3x^2)(x^4+5x^2+3)^{3/2}}{7x}+\frac{1}{35}x\left(129x^2+655\right)\sqrt{x^4+5x^2+3}+\frac{412x\left(2x^2+\sqrt{13}+5\right)}{35\sqrt{x^4+5x^2+3}}$$

```
[In] Int[((2 + 3*x^2)*(3 + 5*x^2 + x^4)^(3/2))/x^2,x]
```

```
[Out] (412*x*(5 + Sqrt[13] + 2*x^2))/(35*Sqrt[3 + 5*x^2 + x^4]) + (x*(655 + 129*x
^2)*Sqrt[3 + 5*x^2 + x^4])/35 - ((14 - 3*x^2)*(3 + 5*x^2 + x^4)^(3/2))/(7*x
) - (206*Sqrt[(2*(5 + Sqrt[13]))/3]*Sqrt[(6 + (5 - Sqrt[13])*x^2)/(6 + (5 +
 Sqrt[13])*x^2)]*(6 + (5 + Sqrt[13])*x^2)*EllipticE[ArcTan[Sqrt[(5 + Sqrt[1
3])/6]*x], (-13 + 5*Sqrt[13])/6])/(35*Sqrt[3 + 5*x^2 + x^4]) + (19*Sqrt[3/(
2*(5 + Sqrt[13]))]*Sqrt[(6 + (5 - Sqrt[13])*x^2)/(6 + (5 + Sqrt[13])*x^2)]*
(6 + (5 + Sqrt[13])*x^2)*EllipticF[ArcTan[Sqrt[(5 + Sqrt[13])/6]*x], (-13 +
 5*Sqrt[13])/6])/Sqrt[3 + 5*x^2 + x^4]
```

Rule 1113

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[(2*a + (b + q)*x^2)*(Sqrt[(2*a + (b - q)*x^2)/(2*a +
(b + q)*x^2)]/(2*a*Rt[(b + q)/(2*a), 2]*Sqrt[a + b*x^2 + c*x^4]))*EllipticF
[ArcTan[Rt[(b + q)/(2*a), 2]*x], 2*(q/(b + q))], x] /; PosQ[(b + q)/a] &&
!(PosQ[(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[
{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1149

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[b^2 - 4*a*c, 2]}, Simp[x*((b + q + 2*c*x^2)/(2*c*Sqrt[a + b*x^2 + c*x^4
])), x] - Simp[Rt[(b + q)/(2*a), 2]*(2*a + (b + q)*x^2)*(Sqrt[(2*a + (b - q
)*x^2)/(2*a + (b + q)*x^2)]/(2*c*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[ArcTan
[Rt[(b + q)/(2*a), 2]*x], 2*(q/(b + q))], x] /; PosQ[(b + q)/a] &&  !(PosQ[
(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[{a, b,
c}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1190

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[x*(2*b*e*p + c*d*(4*p + 3) + c*e*(4*p + 1)*x^2)*((a + b*x^2 + c
*x^4)^p/(c*(4*p + 1)*(4*p + 3))), x] + Dist[2*(p/(c*(4*p + 1)*(4*p + 3))),
Int[Simp[2*a*c*d*(4*p + 3) - a*b*e + (2*a*c*e*(4*p + 1) + b*c*d*(4*p + 3) -
 b^2*e*(2*p + 1))*x^2, x]*(a + b*x^2 + c*x^4)^(p - 1), x], x] /; FreeQ[{a,
b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
GtQ[p, 0] && FractionQ[p] && IntegerQ[2*p]
```

Rule 1203

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[d, Int[1/Sqrt[a + b*x^2 + c*x^4],
 x], x] + Dist[e, Int[x^2/Sqrt[a + b*x^2 + c*x^4], x], x] /; PosQ[(b + q)/a
] || PosQ[(b - q)/a]] /; FreeQ[{a, b, c, d, e}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1285

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(a + b*x^2 + c*x^4)^p*((d*(m
+ 4*p + 3) + e*(m + 1)*x^2)/(f*(m + 1)*(m + 4*p + 3))), x] + Dist[2*(p/(f^2
*(m + 1)*(m + 4*p + 3))), Int[(f*x)^(m + 2)*(a + b*x^2 + c*x^4)^(p - 1)*Sim
p[2*a*e*(m + 1) - b*d*(m + 4*p + 3) + (b*e*(m + 1) - 2*c*d*(m + 4*p + 3))*x
^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b^2 - 4*a*c, 0] && Gt
Q[p, 0] && LtQ[m, -1] && m + 4*p + 3 != 0 && IntegerQ[2*p] && (IntegerQ[p]
|| IntegerQ[m])
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{(14-3x^2)\,(3+5x^2+x^4)^{3/2}}{7x} - \frac{3}{7}\int \left(-88-43x^2\right)\sqrt{3+5x^2+x^4}\,dx \\
&= \frac{1}{35}x\left(655+129x^2\right)\sqrt{3+5x^2+x^4} - \frac{(14-3x^2)\,(3+5x^2+x^4)^{3/2}}{7x} - \frac{1}{35}\int \frac{-1995-824x^2}{\sqrt{3+5x^2+x^4}}\,dx \\
&= \frac{1}{35}x\left(655+129x^2\right)\sqrt{3+5x^2+x^4} - \frac{(14-3x^2)\,(3+5x^2+x^4)^{3/2}}{7x} \\
&\quad + \frac{824}{35}\int \frac{x^2}{\sqrt{3+5x^2+x^4}}\,dx + 57\int \frac{1}{\sqrt{3+5x^2+x^4}}\,dx
\end{aligned}
$$

$$= \frac{412x\left(5+\sqrt{13}+2x^2\right)}{35\sqrt{3+5x^2+x^4}}+\frac{1}{35}x\left(655+129x^2\right)\sqrt{3+5x^2+x^4}-\frac{\left(14-3x^2\right)\left(3+5x^2+x^4\right)^{3/2}}{7x}$$
$$-\frac{206\sqrt{\frac{2}{3}\left(5+\sqrt{13}\right)}\sqrt{\frac{6+\left(5-\sqrt{13}\right)x^2}{6+\left(5+\sqrt{13}\right)x^2}}\left(6+\left(5+\sqrt{13}\right)x^2\right)E\left(\tan^{-1}\left(\sqrt{\frac{1}{6}\left(5+\sqrt{13}\right)}x\right)|\frac{1}{6}\left(-13+5\sqrt{\right.\right.}}{35\sqrt{3+5x^2+x^4}}$$
$$+\frac{19\sqrt{\frac{3}{2\left(5+\sqrt{13}\right)}}\sqrt{\frac{6+\left(5-\sqrt{13}\right)x^2}{6+\left(5+\sqrt{13}\right)x^2}}\left(6+\left(5+\sqrt{13}\right)x^2\right)F\left(\tan^{-1}\left(\sqrt{\frac{1}{6}\left(5+\sqrt{13}\right)}x\right)|\frac{1}{6}\left(-13+5\sqrt{13}\right)\right)}{\sqrt{3+5x^2+x^4}}$$

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 6.51 (sec) , antiderivative size = 235, normalized size of antiderivative = 0.75

$$\int \frac{\left(2+3x^2\right)\left(3+5x^2+x^4\right)^{3/2}}{x^2}\,dx = \frac{-1260+3884x^4+2130x^6+418x^8+30x^{10}+412i\sqrt{2}\left(-5+\sqrt{13}\right)x\sqrt{\frac{-5}{}}}{}$$

```
[In] Integrate[((2 + 3*x^2)*(3 + 5*x^2 + x^4)^(3/2))/x^2,x]
```

```
[Out] (-1260 + 3884*x^4 + 2130*x^6 + 418*x^8 + 30*x^10 + (412*I)*Sqrt[2]*(-5 + Sq
rt[13])*x*Sqrt[(-5 + Sqrt[13] - 2*x^2)/(-5 + Sqrt[13])]*Sqrt[5 + Sqrt[13] +
 2*x^2]*EllipticE[I*ArcSinh[Sqrt[2/(5 + Sqrt[13])]*x], 19/6 + (5*Sqrt[13])/
6] - I*Sqrt[2]*(-65 + 412*Sqrt[13])*x*Sqrt[(-5 + Sqrt[13] - 2*x^2)/(-5 + Sq
rt[13])]*Sqrt[5 + Sqrt[13] + 2*x^2]*EllipticF[I*ArcSinh[Sqrt[2/(5 + Sqrt[13
])]*x], 19/6 + (5*Sqrt[13])/6])/(70*x*Sqrt[3 + 5*x^2 + x^4])
```

Maple [A] (verified)

Time = 1.46 (sec) , antiderivative size = 233, normalized size of antiderivative = 0.75

method	result
risch	$\frac{15x^{10}+209x^8+1065x^6+1942x^4-630}{35x\sqrt{x^4+5x^2+3}}+\frac{342\sqrt{1-\left(-\frac{5}{6}+\frac{\sqrt{13}}{6}\right)x^2}\sqrt{1-\left(-\frac{5}{6}-\frac{\sqrt{13}}{6}\right)x^2}\,F\left(\frac{x\sqrt{-30+6\sqrt{13}}}{6},\frac{5\sqrt{3}}{6}+\frac{\sqrt{39}}{6}\right)}{\sqrt{-30+6\sqrt{13}}\sqrt{x^4+5x^2+3}}-\frac{29664\sqrt{1-\left(\right.}}{}$
default	$\frac{3x^5\sqrt{x^4+5x^2+3}}{7}+\frac{134x^3\sqrt{x^4+5x^2+3}}{35}+10x\sqrt{x^4+5x^2+3}+\frac{342\sqrt{1-\left(-\frac{5}{6}+\frac{\sqrt{13}}{6}\right)x^2}\sqrt{1-\left(-\frac{5}{6}-\frac{\sqrt{13}}{6}\right)x^2}\,F\left(\frac{x\sqrt{-30+6\sqrt{}}}{6}\right.}{\sqrt{-30+6\sqrt{13}}\sqrt{x^4+5x^2+3}}$
elliptic	$\frac{3x^5\sqrt{x^4+5x^2+3}}{7}+\frac{134x^3\sqrt{x^4+5x^2+3}}{35}+10x\sqrt{x^4+5x^2+3}+\frac{342\sqrt{1-\left(-\frac{5}{6}+\frac{\sqrt{13}}{6}\right)x^2}\sqrt{1-\left(-\frac{5}{6}-\frac{\sqrt{13}}{6}\right)x^2}\,F\left(\frac{x\sqrt{-30+6\sqrt{}}}{6}\right.}{\sqrt{-30+6\sqrt{13}}\sqrt{x^4+5x^2+3}}$

```
[In] int((3*x^2+2)*(x^4+5*x^2+3)^(3/2)/x^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/35*(15*x^10+209*x^8+1065*x^6+1942*x^4-630)/x/(x^4+5*x^2+3)^(1/2)+342/(-30
+6*13^(1/2))^(1/2)*(1-(-5/6+1/6*13^(1/2))*x^2)^(1/2)*(1-(-5/6-1/6*13^(1/2))
*x^2)^(1/2)/(x^4+5*x^2+3)^(1/2)*EllipticF(1/6*x*(-30+6*13^(1/2))^(1/2),5/6*
3^(1/2)+1/6*39^(1/2))-29664/35/(-30+6*13^(1/2))^(1/2)*(1-(-5/6+1/6*13^(1/2)
)*x^2)^(1/2)*(1-(-5/6-1/6*13^(1/2))*x^2)^(1/2)/(x^4+5*x^2+3)^(1/2)/(5+13^(1
/2))*(EllipticF(1/6*x*(-30+6*13^(1/2))^(1/2),5/6*3^(1/2)+1/6*39^(1/2))-Elli
pticE(1/6*x*(-30+6*13^(1/2))^(1/2),5/6*3^(1/2)+1/6*39^(1/2)))
```

Fricas [F]

$$\int \frac{(2+3x^2)(3+5x^2+x^4)^{3/2}}{x^2}\,dx = \int \frac{(x^4+5\,x^2+3)^{\frac{3}{2}}(3\,x^2+2)}{x^2}\,dx$$

```
[In] integrate((3*x^2+2)*(x^4+5*x^2+3)^(3/2)/x^2,x, algorithm="fricas")
[Out] integral((3*x^6 + 17*x^4 + 19*x^2 + 6)*sqrt(x^4 + 5*x^2 + 3)/x^2, x)
```

Sympy [F]

$$\int \frac{(2+3x^2)(3+5x^2+x^4)^{3/2}}{x^2}\,dx = \int \frac{(3x^2+2)(x^4+5x^2+3)^{\frac{3}{2}}}{x^2}\,dx$$

```
[In] integrate((3*x**2+2)*(x**4+5*x**2+3)**(3/2)/x**2,x)
[Out] Integral((3*x**2 + 2)*(x**4 + 5*x**2 + 3)**(3/2)/x**2, x)
```

Maxima [F]

$$\int \frac{(2+3x^2)(3+5x^2+x^4)^{3/2}}{x^2}\,dx = \int \frac{(x^4+5\,x^2+3)^{\frac{3}{2}}(3\,x^2+2)}{x^2}\,dx$$

```
[In] integrate((3*x^2+2)*(x^4+5*x^2+3)^(3/2)/x^2,x, algorithm="maxima")
[Out] integrate((x^4 + 5*x^2 + 3)^(3/2)*(3*x^2 + 2)/x^2, x)
```

Giac [F]

$$\int \frac{(2+3x^2)(3+5x^2+x^4)^{3/2}}{x^2}\,dx = \int \frac{(x^4+5\,x^2+3)^{\frac{3}{2}}(3\,x^2+2)}{x^2}\,dx$$

```
[In] integrate((3*x^2+2)*(x^4+5*x^2+3)^(3/2)/x^2,x, algorithm="giac")
[Out] integrate((x^4 + 5*x^2 + 3)^(3/2)*(3*x^2 + 2)/x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(2+3x^2)(3+5x^2+x^4)^{3/2}}{x^2}\,dx = \int \frac{(3\,x^2+2)\,(x^4+5\,x^2+3)^{3/2}}{x^2}\,dx$$

```
[In] int(((3*x^2 + 2)*(5*x^2 + x^4 + 3)^(3/2))/x^2,x)
[Out] int(((3*x^2 + 2)*(5*x^2 + x^4 + 3)^(3/2))/x^2, x)
```

3.167 $\int \frac{(2+3x^2)(3+5x^2+x^4)^{3/2}}{x^4} dx$

Optimal result

Integrand size = 25, antiderivative size = 314

$$\int \frac{(2+3x^2)(3+5x^2+x^4)^{3/2}}{x^4} dx = \frac{949x(5+\sqrt{13}+2x^2)}{30\sqrt{3+5x^2+x^4}} - \frac{13(24-5x^2)\sqrt{3+5x^2+x^4}}{15x} - \frac{(10-9x^2)(3+5x^2+x^4)^{3/2}}{15x^3} - \frac{949\sqrt{\frac{1}{6}(5+\sqrt{13})}\sqrt{\frac{6+(5-\sqrt{13})x^2}{6+(5+\sqrt{13})x^2}}(6+(5+\sqrt{13})x^2)E\left(\arctan\left(\sqrt{\frac{1}{6}(5+\sqrt{13})}x\right)|\frac{1}{6}(-13+5\sqrt{13})\right)}{30\sqrt{3+5x^2+x^4}} + \frac{65\sqrt{\frac{2}{3(5+\sqrt{13})}}\sqrt{\frac{6+(5-\sqrt{13})x^2}{6+(5+\sqrt{13})x^2}}(6+(5+\sqrt{13})x^2)\operatorname{EllipticF}\left(\arctan\left(\sqrt{\frac{1}{6}(5+\sqrt{13})}x\right),\frac{1}{6}(-13+5\sqrt{13})\right)}{\sqrt{3+5x^2+x^4}}$$

```
[Out] -1/15*(-9*x^2+10)*(x^4+5*x^2+3)^(3/2)/x^3+949/30*x*(5+2*x^2+13^(1/2))/(x^4+
5*x^2+3)^(1/2)-13/15*(-5*x^2+24)*(x^4+5*x^2+3)^(1/2)/x+65/3*(1/(36+x^2*(30+
6*13^(1/2))))^(1/2)*(36+x^2*(30+6*13^(1/2)))^(1/2)*EllipticF(x*(30+6*13^(1/
2))^(1/2)/(36+x^2*(30+6*13^(1/2)))^(1/2),1/6*(-78+30*13^(1/2))^(1/2))*(6+x^
2*(5+13^(1/2)))*6^(1/2)/(5+13^(1/2))^(1/2)*((6+x^2*(5-13^(1/2)))/(6+x^2*(5+
13^(1/2))))^(1/2)/(x^4+5*x^2+3)^(1/2)-949/180*(1/(36+x^2*(30+6*13^(1/2))))^
(1/2)*(36+x^2*(30+6*13^(1/2)))^(1/2)*EllipticE(x*(30+6*13^(1/2))^(1/2)/(36+
x^2*(30+6*13^(1/2)))^(1/2),1/6*(-78+30*13^(1/2))^(1/2))*(6+x^2*(5+13^(1/2))
)*(30+6*13^(1/2))^(1/2)*((6+x^2*(5-13^(1/2)))/(6+x^2*(5+13^(1/2))))^(1/2)/(
x^4+5*x^2+3)^(1/2)
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 314, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {1285, 1203, 1113, 1149}

$$\int \frac{(2+3x^2)(3+5x^2+x^4)^{3/2}}{x^4}\,dx = \frac{65\sqrt{\frac{2}{3\left(5+\sqrt{13}\right)}}\sqrt{\frac{\left(5-\sqrt{13}\right)x^2+6}{\left(5+\sqrt{13}\right)x^2+6}}\left(\left(5+\sqrt{13}\right)x^2+6\right)\text{EllipticF}\left(\arctan\left(\sqrt{\frac{1}{6}\left(\right.}\right.\right.}{\sqrt{x^4+5x^2+3}}$$
$$-\frac{949\sqrt{\frac{1}{6}\left(5+\sqrt{13}\right)}\sqrt{\frac{\left(5-\sqrt{13}\right)x^2+6}{\left(5+\sqrt{13}\right)x^2+6}}\left(\left(5+\sqrt{13}\right)x^2+6\right)E\left(\arctan\left(\sqrt{\frac{1}{6}\left(5+\sqrt{13}\right)}x\right)|\frac{1}{6}\left(-13+5\sqrt{13}\right)\right)}{30\sqrt{x^4+5x^2+3}}$$
$$-\frac{13(24-5x^2)\sqrt{x^4+5x^2+3}}{15x}+\frac{949x\left(2x^2+\sqrt{13}+5\right)}{30\sqrt{x^4+5x^2+3}}-\frac{(10-9x^2)(x^4+5x^2+3)^{3/2}}{15x^3}$$

```
[In] Int[((2 + 3*x^2)*(3 + 5*x^2 + x^4)^(3/2))/x^4,x]
```

```
[Out] (949*x*(5 + Sqrt[13] + 2*x^2))/(30*Sqrt[3 + 5*x^2 + x^4]) - (13*(24 - 5*x^2
)*Sqrt[3 + 5*x^2 + x^4])/(15*x) - ((10 - 9*x^2)*(3 + 5*x^2 + x^4)^(3/2))/(1
5*x^3) - (949*Sqrt[(5 + Sqrt[13])/6]*Sqrt[(6 + (5 - Sqrt[13])*x^2)/(6 + (5
+ Sqrt[13])*x^2)]*(6 + (5 + Sqrt[13])*x^2)*EllipticE[ArcTan[Sqrt[(5 + Sqrt[
13])/6]*x], (-13 + 5*Sqrt[13])/6])/(30*Sqrt[3 + 5*x^2 + x^4]) + (65*Sqrt[2/
(3*(5 + Sqrt[13]))]*Sqrt[(6 + (5 - Sqrt[13])*x^2)/(6 + (5 + Sqrt[13])*x^2)]
*(6 + (5 + Sqrt[13])*x^2)*EllipticF[ArcTan[Sqrt[(5 + Sqrt[13])/6]*x], (-13
+ 5*Sqrt[13])/6])/Sqrt[3 + 5*x^2 + x^4]
```

Rule 1113

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[(2*a + (b + q)*x^2)*(Sqrt[(2*a + (b - q)*x^2)/(2*a +
(b + q)*x^2)]/(2*a*Rt[(b + q)/(2*a), 2]*Sqrt[a + b*x^2 + c*x^4]))*EllipticF
[ArcTan[Rt[(b + q)/(2*a), 2]*x], 2*(q/(b + q))], x] /; PosQ[(b + q)/a] &&
!(PosQ[(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[
{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1149

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[b^2 - 4*a*c, 2]}, Simp[x*((b + q + 2*c*x^2)/(2*c*Sqrt[a + b*x^2 + c*x^4
])), x] - Simp[Rt[(b + q)/(2*a), 2]*(2*a + (b + q)*x^2)*(Sqrt[(2*a + (b - q
)*x^2)/(2*a + (b + q)*x^2)]/(2*c*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[ArcTan
[Rt[(b + q)/(2*a), 2]*x], 2*(q/(b + q))], x] /; PosQ[(b + q)/a] &&  !(PosQ[
(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[{a, b,
c}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1203

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[d, Int[1/Sqrt[a + b*x^2 + c*x^4],
 x], x] + Dist[e, Int[x^2/Sqrt[a + b*x^2 + c*x^4], x], x] /; PosQ[(b + q)/a
] || PosQ[(b - q)/a]] /; FreeQ[{a, b, c, d, e}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1285

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(a + b*x^2 + c*x^4)^p*((d*(m
+ 4*p + 3) + e*(m + 1)*x^2)/(f*(m + 1)*(m + 4*p + 3))), x] + Dist[2*(p/(f^2
*(m + 1)*(m + 4*p + 3))), Int[(f*x)^(m + 2)*(a + b*x^2 + c*x^4)^(p - 1)*Sim
p[2*a*e*(m + 1) - b*d*(m + 4*p + 3) + (b*e*(m + 1) - 2*c*d*(m + 4*p + 3))*x
^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b^2 - 4*a*c, 0] && Gt
Q[p, 0] && LtQ[m, -1] && m + 4*p + 3 != 0 && IntegerQ[2*p] && (IntegerQ[p]
|| IntegerQ[m])
```

Rubi steps

$$\text{integral} = -\frac{(10-9x^2)\,(3+5x^2+x^4)^{3/2}}{15x^3} - \frac{1}{5}\int \frac{(-104-65x^2)\sqrt{3+5x^2+x^4}}{x^2}\,dx$$

$$= -\frac{13(24-5x^2)\sqrt{3+5x^2+x^4}}{15x} - \frac{(10-9x^2)\,(3+5x^2+x^4)^{3/2}}{15x^3} + \frac{1}{15}\int \frac{1950+949x^2}{\sqrt{3+5x^2+x^4}}\,dx$$

$$= -\frac{13(24-5x^2)\sqrt{3+5x^2+x^4}}{15x} - \frac{(10-9x^2)\,(3+5x^2+x^4)^{3/2}}{15x^3} + \frac{949}{15}\int \frac{x^2}{\sqrt{3+5x^2+x^4}}\,dx + 130\int \frac{1}{\sqrt{3+5x^2+x^4}}\,dx$$

$$= \frac{949x\left(5+\sqrt{13}+2x^2\right)}{30\sqrt{3+5x^2+x^4}} - \frac{13(24-5x^2)\sqrt{3+5x^2+x^4}}{15x} - \frac{(10-9x^2)\,(3+5x^2+x^4)^{3/2}}{15x^3} - \frac{949\sqrt{\frac{1}{6}\left(5+\sqrt{13}\right)}\sqrt{\frac{6+\left(5-\sqrt{13}\right)x^2}{6+\left(5+\sqrt{13}\right)x^2}}\left(6+\left(5+\sqrt{13}\right)x^2\right)E\left(\tan^{-1}\left(\sqrt{\frac{1}{6}\left(5+\sqrt{13}\right)}x\right)\middle|\frac{1}{6}\left(-13+5\sqrt{\ldots}\right.\right.}{30\sqrt{3+5x^2+x^4}} + \frac{65\sqrt{\frac{2}{3\left(5+\sqrt{13}\right)}}\sqrt{\frac{6+\left(5-\sqrt{13}\right)x^2}{6+\left(5+\sqrt{13}\right)x^2}}\left(6+\left(5+\sqrt{13}\right)x^2\right)F\left(\tan^{-1}\left(\sqrt{\frac{1}{6}\left(5+\sqrt{13}\right)}x\right)\middle|\frac{1}{6}\left(-13+5\sqrt{13}\right)\ldots\right.}{\sqrt{3+5x^2+x^4}}$$

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 10.25 (sec) , antiderivative size = 247, normalized size of antiderivative = 0.79

$$\int \frac{(2+3x^2)(3+5x^2+x^4)^{3/2}}{x^4}\,dx = \frac{4(-90-1155x^2-1405x^4+192x^6+145x^8+9x^{10})+949i\sqrt{2}(-5+\sqrt{1}}{}$$

```
[In] Integrate[((2 + 3*x^2)*(3 + 5*x^2 + x^4)^(3/2))/x^4,x]
```

```
[Out] (4*(-90 - 1155*x^2 - 1405*x^4 + 192*x^6 + 145*x^8 + 9*x^10) + (949*I)*Sqrt[
2]*(-5 + Sqrt[13])*x^3*Sqrt[(-5 + Sqrt[13] - 2*x^2)/(-5 + Sqrt[13])]*Sqrt[5
 + Sqrt[13] + 2*x^2]*EllipticE[I*ArcSinh[Sqrt[2/(5 + Sqrt[13])]*x], 19/6 +
(5*Sqrt[13])/6] - (13*I)*Sqrt[2]*(-65 + 73*Sqrt[13])*x^3*Sqrt[(-5 + Sqrt[13
] - 2*x^2)/(-5 + Sqrt[13])]*Sqrt[5 + Sqrt[13] + 2*x^2]*EllipticF[I*ArcSinh[
Sqrt[2/(5 + Sqrt[13])]*x], 19/6 + (5*Sqrt[13])/6])/(60*x^3*Sqrt[3 + 5*x^2 +
 x^4])
```

Maple [A] (verified)

Time = 2.13 (sec) , antiderivative size = 238, normalized size of antiderivative = 0.76

method	result
risch	$\frac{9x^{10}+145x^8+192x^6-1405x^4-1155x^2-90}{15x^3\sqrt{x^4+5x^2+3}}+\frac{780\sqrt{1-\left(-\frac{5}{6}+\frac{\sqrt{13}}{6}\right)x^2}\sqrt{1-\left(-\frac{5}{6}-\frac{\sqrt{13}}{6}\right)x^2}\,F\left(\frac{x\sqrt{-30+6\sqrt{13}}}{6},\frac{5\sqrt{3}}{6}+\frac{\sqrt{39}}{6}\right)}{\sqrt{-30+6\sqrt{13}}\sqrt{x^4+5x^2+3}}-\frac{11388}{}$
default	$-\frac{67\sqrt{x^4+5x^2+3}}{3x}+\frac{3x^3\sqrt{x^4+5x^2+3}}{5}+\frac{20x\sqrt{x^4+5x^2+3}}{3}+\frac{780\sqrt{1-\left(-\frac{5}{6}+\frac{\sqrt{13}}{6}\right)x^2}\sqrt{1-\left(-\frac{5}{6}-\frac{\sqrt{13}}{6}\right)x^2}\,F\left(\frac{x\sqrt{-30+6\sqrt{13}}}{6},\frac{5\sqrt{3}}{6}+\right.}{\sqrt{-30+6\sqrt{13}}\sqrt{x^4+5x^2+3}}$
elliptic	$-\frac{67\sqrt{x^4+5x^2+3}}{3x}+\frac{3x^3\sqrt{x^4+5x^2+3}}{5}+\frac{20x\sqrt{x^4+5x^2+3}}{3}+\frac{780\sqrt{1-\left(-\frac{5}{6}+\frac{\sqrt{13}}{6}\right)x^2}\sqrt{1-\left(-\frac{5}{6}-\frac{\sqrt{13}}{6}\right)x^2}\,F\left(\frac{x\sqrt{-30+6\sqrt{13}}}{6},\frac{5\sqrt{3}}{6}+\right.}{\sqrt{-30+6\sqrt{13}}\sqrt{x^4+5x^2+3}}$

```
[In] int((3*x^2+2)*(x^4+5*x^2+3)^(3/2)/x^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/15*(9*x^10+145*x^8+192*x^6-1405*x^4-1155*x^2-90)/x^3/(x^4+5*x^2+3)^(1/2)+
780/(-30+6*13^(1/2))^(1/2)*(1-(-5/6+1/6*13^(1/2))*x^2)^(1/2)*(1-(-5/6-1/6*1
3^(1/2))*x^2)^(1/2)/(x^4+5*x^2+3)^(1/2)*EllipticF(1/6*x*(-30+6*13^(1/2))^(1
/2),5/6*3^(1/2)+1/6*39^(1/2))-11388/5/(-30+6*13^(1/2))^(1/2)*(1-(-5/6+1/6*1
3^(1/2))*x^2)^(1/2)*(1-(-5/6-1/6*13^(1/2))*x^2)^(1/2)/(x^4+5*x^2+3)^(1/2)/(
5+13^(1/2))*(EllipticF(1/6*x*(-30+6*13^(1/2))^(1/2),5/6*3^(1/2)+1/6*39^(1/2
))-EllipticE(1/6*x*(-30+6*13^(1/2))^(1/2),5/6*3^(1/2)+1/6*39^(1/2)))
```

Fricas [F]

$$\int \frac{(2+3x^2)(3+5x^2+x^4)^{3/2}}{x^4}\,dx = \int \frac{(x^4+5\,x^2+3)^{\frac{3}{2}}(3\,x^2+2)}{x^4}\,dx$$

```
[In] integrate((3*x^2+2)*(x^4+5*x^2+3)^(3/2)/x^4,x, algorithm="fricas")
[Out] integral((3*x^6 + 17*x^4 + 19*x^2 + 6)*sqrt(x^4 + 5*x^2 + 3)/x^4, x)
```

Sympy [F]

$$\int \frac{(2+3x^2)(3+5x^2+x^4)^{3/2}}{x^4}\,dx = \int \frac{(3x^2+2)(x^4+5x^2+3)^{\frac{3}{2}}}{x^4}\,dx$$

```
[In] integrate((3*x**2+2)*(x**4+5*x**2+3)**(3/2)/x**4,x)
[Out] Integral((3*x**2 + 2)*(x**4 + 5*x**2 + 3)**(3/2)/x**4, x)
```

Maxima [F]

$$\int \frac{(2+3x^2)(3+5x^2+x^4)^{3/2}}{x^4}\,dx = \int \frac{(x^4+5\,x^2+3)^{\frac{3}{2}}(3\,x^2+2)}{x^4}\,dx$$

```
[In] integrate((3*x^2+2)*(x^4+5*x^2+3)^(3/2)/x^4,x, algorithm="maxima")
[Out] integrate((x^4 + 5*x^2 + 3)^(3/2)*(3*x^2 + 2)/x^4, x)
```

Giac [F]

$$\int \frac{(2+3x^2)(3+5x^2+x^4)^{3/2}}{x^4}\,dx = \int \frac{(x^4+5\,x^2+3)^{\frac{3}{2}}(3\,x^2+2)}{x^4}\,dx$$

```
[In] integrate((3*x^2+2)*(x^4+5*x^2+3)^(3/2)/x^4,x, algorithm="giac")
[Out] integrate((x^4 + 5*x^2 + 3)^(3/2)*(3*x^2 + 2)/x^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(2+3x^2)\,(3+5x^2+x^4)^{3/2}}{x^4}\,dx = \int \frac{(3\,x^2+2)\,\left(x^4+5\,x^2+3\right)^{3/2}}{x^4}\,dx$$

```
[In] int(((3*x^2 + 2)*(5*x^2 + x^4 + 3)^(3/2))/x^4,x)
[Out] int(((3*x^2 + 2)*(5*x^2 + x^4 + 3)^(3/2))/x^4, x)
```

3.168 $\int \frac{(2+3x^2)(3+5x^2+x^4)^{3/2}}{x^6}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 331

$$\int \frac{(2+3x^2)(3+5x^2+x^4)^{3/2}}{x^6}\,dx = \frac{361x(5+\sqrt{13}+2x^2)}{15\sqrt{3+5x^2+x^4}} - \frac{722\sqrt{3+5x^2+x^4}}{15x} - \frac{(40-87x^2)\sqrt{3+5x^2+x^4}}{5x^3} - \frac{(2-5x^2)(3+5x^2+x^4)^{3/2}}{5x^5} - \frac{361\sqrt{\frac{1}{6}(5+\sqrt{13})}\sqrt{\frac{6+(5-\sqrt{13})x^2}{6+(5+\sqrt{13})x^2}}(6+(5+\sqrt{13})x^2)\,E\left(\arctan\left(\sqrt{\frac{1}{6}(5+\sqrt{13})}x\right)|\frac{1}{6}(-13+5\sqrt{13})\right)}{15\sqrt{3+5x^2+x^4}} + \frac{103\sqrt{\frac{6+(5-\sqrt{13})x^2}{6+(5+\sqrt{13})x^2}}(6+(5+\sqrt{13})x^2)\,\text{EllipticF}\left(\arctan\left(\sqrt{\frac{1}{6}(5+\sqrt{13})}x\right),\frac{1}{6}(-13+5\sqrt{13})\right)}{\sqrt{6(5+\sqrt{13})}\sqrt{3+5x^2+x^4}}$$

```
[Out] -1/5*(-5*x^2+2)*(x^4+5*x^2+3)^(3/2)/x^5+361/15*x*(5+2*x^2+13^(1/2))/(x^4+5*
x^2+3)^(1/2)-722/15*(x^4+5*x^2+3)^(1/2)/x-1/5*(-87*x^2+40)*(x^4+5*x^2+3)^(1
/2)/x^3-361/90*(1/(36+x^2*(30+6*13^(1/2))))^(1/2)*(36+x^2*(30+6*13^(1/2)))^
(1/2)*EllipticE(x*(30+6*13^(1/2))^(1/2)/(36+x^2*(30+6*13^(1/2)))^(1/2),1/6*
(-78+30*13^(1/2))^(1/2))*(6+x^2*(5+13^(1/2)))*(30+6*13^(1/2))^(1/2)*((6+x^2
*(5-13^(1/2)))/(6+x^2*(5+13^(1/2))))^(1/2)/(x^4+5*x^2+3)^(1/2)+103*(1/(36+x
^2*(30+6*13^(1/2))))^(1/2)*(36+x^2*(30+6*13^(1/2)))^(1/2)*EllipticF(x*(30+6
*13^(1/2))^(1/2)/(36+x^2*(30+6*13^(1/2)))^(1/2),1/6*(-78+30*13^(1/2))^(1/2)
)*(6+x^2*(5+13^(1/2)))*((6+x^2*(5-13^(1/2)))/(6+x^2*(5+13^(1/2))))^(1/2)/(x
^4+5*x^2+3)^(1/2)/(30+6*13^(1/2))^(1/2)
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 331, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {1285, 1295, 1203, 1113, 1149}

$$\int \frac{(2+3x^2)(3+5x^2+x^4)^{3/2}}{x^6}\,dx = \frac{103\sqrt{\frac{(5-\sqrt{13})x^2+6}{(5+\sqrt{13})x^2+6}}\left((5+\sqrt{13})x^2+6\right)\text{EllipticF}\left(\arctan\left(\sqrt{\frac{1}{6}(5+\sqrt{13})}\right.\right.}{\sqrt{6(5+\sqrt{13})}\sqrt{x^4+5x^2+3}}$$
$$-\frac{361\sqrt{\frac{1}{6}(5+\sqrt{13})}\sqrt{\frac{(5-\sqrt{13})x^2+6}{(5+\sqrt{13})x^2+6}}\left((5+\sqrt{13})x^2+6\right)E\left(\arctan\left(\sqrt{\frac{1}{6}(5+\sqrt{13})}x\right)|\frac{1}{6}(-13+5\sqrt{13})\right)}{15\sqrt{x^4+5x^2+3}}$$
$$-\frac{722\sqrt{x^4+5x^2+3}}{15x}+\frac{361x(2x^2+\sqrt{13}+5)}{15\sqrt{x^4+5x^2+3}}$$
$$-\frac{(2-5x^2)(x^4+5x^2+3)^{3/2}}{5x^5}-\frac{(40-87x^2)\sqrt{x^4+5x^2+3}}{5x^3}$$

```
[In] Int[((2 + 3*x^2)*(3 + 5*x^2 + x^4)^(3/2))/x^6,x]
```

```
[Out] (361*x*(5 + Sqrt[13] + 2*x^2))/(15*Sqrt[3 + 5*x^2 + x^4]) - (722*Sqrt[3 + 5
*x^2 + x^4])/(15*x) - ((40 - 87*x^2)*Sqrt[3 + 5*x^2 + x^4])/(5*x^3) - ((2 -
 5*x^2)*(3 + 5*x^2 + x^4)^(3/2))/(5*x^5) - (361*Sqrt[(5 + Sqrt[13])/6]*Sqrt
[(6 + (5 - Sqrt[13])*x^2)/(6 + (5 + Sqrt[13])*x^2)]*(6 + (5 + Sqrt[13])*x^2
)*EllipticE[ArcTan[Sqrt[(5 + Sqrt[13])/6]*x], (-13 + 5*Sqrt[13])/6])/(15*Sq
rt[3 + 5*x^2 + x^4]) + (103*Sqrt[(6 + (5 - Sqrt[13])*x^2)/(6 + (5 + Sqrt[13
])*x^2)]*(6 + (5 + Sqrt[13])*x^2)*EllipticF[ArcTan[Sqrt[(5 + Sqrt[13])/6]*x
], (-13 + 5*Sqrt[13])/6])/(Sqrt[6*(5 + Sqrt[13])]*Sqrt[3 + 5*x^2 + x^4])
```

Rule 1113

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[(2*a + (b + q)*x^2)*(Sqrt[(2*a + (b - q)*x^2)/(2*a +
(b + q)*x^2)]/(2*a*Rt[(b + q)/(2*a), 2]*Sqrt[a + b*x^2 + c*x^4]))*EllipticF
[ArcTan[Rt[(b + q)/(2*a), 2]*x], 2*(q/(b + q))], x] /; PosQ[(b + q)/a] &&
!(PosQ[(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[
{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1149

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[b^2 - 4*a*c, 2]}, Simp[x*((b + q + 2*c*x^2)/(2*c*Sqrt[a + b*x^2 + c*x^4
])), x] - Simp[Rt[(b + q)/(2*a), 2]*(2*a + (b + q)*x^2)*(Sqrt[(2*a + (b - q
)*x^2)/(2*a + (b + q)*x^2)]/(2*c*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[ArcTan
[Rt[(b + q)/(2*a), 2]*x], 2*(q/(b + q))], x] /; PosQ[(b + q)/a] &&  !(PosQ[
```

```
(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[{a, b,
c}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1203

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[d, Int[1/Sqrt[a + b*x^2 + c*x^4],
 x], x] + Dist[e, Int[x^2/Sqrt[a + b*x^2 + c*x^4], x], x] /; PosQ[(b + q)/a
] || PosQ[(b - q)/a]] /; FreeQ[{a, b, c, d, e}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1285

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(a + b*x^2 + c*x^4)^p*((d*(m
+ 4*p + 3) + e*(m + 1)*x^2)/(f*(m + 1)*(m + 4*p + 3))), x] + Dist[2*(p/(f^2
*(m + 1)*(m + 4*p + 3))), Int[(f*x)^(m + 2)*(a + b*x^2 + c*x^4)^(p - 1)*Sim
p[2*a*e*(m + 1) - b*d*(m + 4*p + 3) + (b*e*(m + 1) - 2*c*d*(m + 4*p + 3))*x
^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b^2 - 4*a*c, 0] && Gt
Q[p, 0] && LtQ[m, -1] && m + 4*p + 3 != 0 && IntegerQ[2*p] && (IntegerQ[p]
|| IntegerQ[m])
```

Rule 1295

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[d*(f*x)^(m + 1)*((a + b*x^2 + c*x^4)^(p + 1)
/(a*f*(m + 1))), x] + Dist[1/(a*f^2*(m + 1)), Int[(f*x)^(m + 2)*(a + b*x^2
+ c*x^4)^p*Simp[a*e*(m + 1) - b*d*(m + 2*p + 3) - c*d*(m + 4*p + 5)*x^2, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[m
, -1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{(2-5x^2)(3+5x^2+x^4)^{3/2}}{5x^5} - \frac{1}{5}\int \frac{(-120-87x^2)\sqrt{3+5x^2+x^4}}{x^4}\,dx \\
&= -\frac{(40-87x^2)\sqrt{3+5x^2+x^4}}{5x^3} - \frac{(2-5x^2)(3+5x^2+x^4)^{3/2}}{5x^5} + \frac{1}{15}\int \frac{2166+1545x^2}{x^2\sqrt{3+5x^2+x^4}}\,dx \\
&= -\frac{722\sqrt{3+5x^2+x^4}}{15x} - \frac{(40-87x^2)\sqrt{3+5x^2+x^4}}{5x^3} \\
&\quad - \frac{(2-5x^2)(3+5x^2+x^4)^{3/2}}{5x^5} - \frac{1}{45}\int \frac{-4635-2166x^2}{\sqrt{3+5x^2+x^4}}\,dx \\
&= -\frac{722\sqrt{3+5x^2+x^4}}{15x} - \frac{(40-87x^2)\sqrt{3+5x^2+x^4}}{5x^3} - \frac{(2-5x^2)(3+5x^2+x^4)^{3/2}}{5x^5} \\
&\quad + \frac{722}{15}\int \frac{x^2}{\sqrt{3+5x^2+x^4}}\,dx + 103\int \frac{1}{\sqrt{3+5x^2+x^4}}\,dx
\end{aligned}$$

$$= \frac{361x\left(5+\sqrt{13}+2x^2\right)}{15\sqrt{3+5x^2+x^4}} - \frac{722\sqrt{3+5x^2+x^4}}{15x} - \frac{(40-87x^2)\sqrt{3+5x^2+x^4}}{5x^3} - \frac{(2-5x^2)(3+5x^2+x^4)^{3/2}}{5x^5} - \frac{361\sqrt{\frac{1}{6}\left(5+\sqrt{13}\right)}\sqrt{\frac{6+\left(5-\sqrt{13}\right)x^2}{6+\left(5+\sqrt{13}\right)x^2}}\left(6+\left(5+\sqrt{13}\right)x^2\right)E\left(\tan^{-1}\left(\sqrt{\frac{1}{6}\left(5+\sqrt{13}\right)}x\right)|\frac{1}{6}\left(-13+5\sqrt{1\right.\right.}{15\sqrt{3+5x^2+x^4}} + \frac{103\sqrt{\frac{6+\left(5-\sqrt{13}\right)x^2}{6+\left(5+\sqrt{13}\right)x^2}}\left(6+\left(5+\sqrt{13}\right)x^2\right)F\left(\tan^{-1}\left(\sqrt{\frac{1}{6}\left(5+\sqrt{13}\right)}x\right)|\frac{1}{6}\left(-13+5\sqrt{13}\right)\right)}{\sqrt{6\left(5+\sqrt{13}\right)}\sqrt{3+5x^2+x^4}}$$

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 10.27 (sec) , antiderivative size = 244, normalized size of antiderivative = 0.74

$$\int \frac{(2+3x^2)(3+5x^2+x^4)^{3/2}}{x^6}\,dx = \frac{-108-810x^2-3438x^4-4040x^6-634x^8+30x^{10}+361i\sqrt{2}\left(-5+\sqrt{13}\right.}{}$$

```
[In] Integrate[((2 + 3*x^2)*(3 + 5*x^2 + x^4)^(3/2))/x^6,x]
```

```
[Out] (-108 - 810*x^2 - 3438*x^4 - 4040*x^6 - 634*x^8 + 30*x^10 + (361*I)*Sqrt[2]
*(-5 + Sqrt[13])*x^5*Sqrt[(-5 + Sqrt[13] - 2*x^2)/(-5 + Sqrt[13])]*Sqrt[5 +
 Sqrt[13] + 2*x^2]*EllipticE[I*ArcSinh[Sqrt[2/(5 + Sqrt[13])]*x], 19/6 + (5
*Sqrt[13])/6] - I*Sqrt[2]*(-260 + 361*Sqrt[13])*x^5*Sqrt[(-5 + Sqrt[13] - 2
*x^2)/(-5 + Sqrt[13])]*Sqrt[5 + Sqrt[13] + 2*x^2]*EllipticF[I*ArcSinh[Sqrt[
2/(5 + Sqrt[13])]*x], 19/6 + (5*Sqrt[13])/6])/(30*x^5*Sqrt[3 + 5*x^2 + x^4]
)
```

Maple [A] (verified)

Time = 4.09 (sec) , antiderivative size = 238, normalized size of antiderivative = 0.72

method	result
risch	$\frac{15x^{10}-317x^8-2020x^6-1719x^4-405x^2-54}{15x^5\sqrt{x^4+5x^2+3}}+\frac{618\sqrt{1-\left(-\frac{5}{6}+\frac{\sqrt{13}}{6}\right)x^2}\,\sqrt{1-\left(-\frac{5}{6}-\frac{\sqrt{13}}{6}\right)x^2}\,F\left(\frac{x\sqrt{-30+6\sqrt{13}}}{6},\frac{5\sqrt{3}}{6}+\frac{\sqrt{39}}{6}\right)}{\sqrt{-30+6\sqrt{13}}\,\sqrt{x^4+5x^2+3}}-\frac{866}{}$
default	$-\frac{6\sqrt{x^4+5x^2+3}}{5x^5}-\frac{7\sqrt{x^4+5x^2+3}}{x^3}-\frac{392\sqrt{x^4+5x^2+3}}{15x}+\frac{618\sqrt{1-\left(-\frac{5}{6}+\frac{\sqrt{13}}{6}\right)x^2}\,\sqrt{1-\left(-\frac{5}{6}-\frac{\sqrt{13}}{6}\right)x^2}\,F\left(\frac{x\sqrt{-30+6\sqrt{13}}}{6},\frac{5\sqrt{3}}{6}+\sqrt{}\right.}{\sqrt{-30+6\sqrt{13}}\,\sqrt{x^4+5x^2+3}}$
elliptic	$-\frac{6\sqrt{x^4+5x^2+3}}{5x^5}-\frac{7\sqrt{x^4+5x^2+3}}{x^3}-\frac{392\sqrt{x^4+5x^2+3}}{15x}+\frac{618\sqrt{1-\left(-\frac{5}{6}+\frac{\sqrt{13}}{6}\right)x^2}\,\sqrt{1-\left(-\frac{5}{6}-\frac{\sqrt{13}}{6}\right)x^2}\,F\left(\frac{x\sqrt{-30+6\sqrt{13}}}{6},\frac{5\sqrt{3}}{6}+\sqrt{}\right.}{\sqrt{-30+6\sqrt{13}}\,\sqrt{x^4+5x^2+3}}$

```
[In] int((3*x^2+2)*(x^4+5*x^2+3)^(3/2)/x^6,x,method=_RETURNVERBOSE)
```

```
[Out] 1/15*(15*x^10-317*x^8-2020*x^6-1719*x^4-405*x^2-54)/x^5/(x^4+5*x^2+3)^(1/2)
+618/(-30+6*13^(1/2))^(1/2)*(1-(-5/6+1/6*13^(1/2))*x^2)^(1/2)*(1-(-5/6-1/6*
13^(1/2))*x^2)^(1/2)/(x^4+5*x^2+3)^(1/2)*EllipticF(1/6*x*(-30+6*13^(1/2))^(
1/2),5/6*3^(1/2)+1/6*39^(1/2))-8664/5/(-30+6*13^(1/2))^(1/2)*(1-(-5/6+1/6*1
3^(1/2))*x^2)^(1/2)*(1-(-5/6-1/6*13^(1/2))*x^2)^(1/2)/(x^4+5*x^2+3)^(1/2)/(
5+13^(1/2))*(EllipticF(1/6*x*(-30+6*13^(1/2))^(1/2),5/6*3^(1/2)+1/6*39^(1/2
))-EllipticE(1/6*x*(-30+6*13^(1/2))^(1/2),5/6*3^(1/2)+1/6*39^(1/2)))
```

Fricas [F]

$$\int \frac{(2+3x^2)\left(3+5x^2+x^4\right)^{3/2}}{x^6}\,dx=\int \frac{\left(x^4+5\,x^2+3\right)^{\frac{3}{2}}\left(3\,x^2+2\right)}{x^6}\,dx$$

```
[In] integrate((3*x^2+2)*(x^4+5*x^2+3)^(3/2)/x^6,x, algorithm="fricas")
```

```
[Out] integral((3*x^6 + 17*x^4 + 19*x^2 + 6)*sqrt(x^4 + 5*x^2 + 3)/x^6, x)
```

Sympy [F]

$$\int \frac{(2+3x^2)\left(3+5x^2+x^4\right)^{3/2}}{x^6}\,dx=\int \frac{(3x^2+2)\left(x^4+5x^2+3\right)^{\frac{3}{2}}}{x^6}\,dx$$

```
[In] integrate((3*x**2+2)*(x**4+5*x**2+3)**(3/2)/x**6,x)
```

```
[Out] Integral((3*x**2 + 2)*(x**4 + 5*x**2 + 3)**(3/2)/x**6, x)
```

Maxima [F]

$$\int \frac{(2+3x^2)(3+5x^2+x^4)^{3/2}}{x^6}\,dx = \int \frac{(x^4+5\,x^2+3)^{\frac{3}{2}}(3\,x^2+2)}{x^6}\,dx$$

```
[In] integrate((3*x^2+2)*(x^4+5*x^2+3)^(3/2)/x^6,x, algorithm="maxima")
[Out] integrate((x^4 + 5*x^2 + 3)^(3/2)*(3*x^2 + 2)/x^6, x)
```

Giac [F]

$$\int \frac{(2+3x^2)(3+5x^2+x^4)^{3/2}}{x^6}\,dx = \int \frac{(x^4+5\,x^2+3)^{\frac{3}{2}}(3\,x^2+2)}{x^6}\,dx$$

```
[In] integrate((3*x^2+2)*(x^4+5*x^2+3)^(3/2)/x^6,x, algorithm="giac")
[Out] integrate((x^4 + 5*x^2 + 3)^(3/2)*(3*x^2 + 2)/x^6, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(2+3x^2)(3+5x^2+x^4)^{3/2}}{x^6}\,dx = \int \frac{(3\,x^2+2)\,(x^4+5\,x^2+3)^{3/2}}{x^6}\,dx$$

```
[In] int(((3*x^2 + 2)*(5*x^2 + x^4 + 3)^(3/2))/x^6,x)
[Out] int(((3*x^2 + 2)*(5*x^2 + x^4 + 3)^(3/2))/x^6, x)
```

3.169 $\int \frac{x^5(A+Bx^2)}{\sqrt{a+bx^2+cx^4}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 153

$$\int \frac{x^5(A+Bx^2)}{\sqrt{a+bx^2+cx^4}}\,dx=\frac{Bx^4\sqrt{a+bx^2+cx^4}}{6c}+\frac{(15b^2B-18Abc-16aBc-2c(5bB-6Ac)x^2)\sqrt{a+bx^2+cx^4}}{48c^3}-\frac{(5b^3B-6Ab^2c-12abBc+8aAc^2)\operatorname{arctanh}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{32c^{7/2}}$$

```
[Out] -1/32*(8*A*a*c^2-6*A*b^2*c-12*B*a*b*c+5*B*b^3)*arctanh(1/2*(2*c*x^2+b)/c^(1
/2)/(c*x^4+b*x^2+a)^(1/2))/c^(7/2)+1/6*B*x^4*(c*x^4+b*x^2+a)^(1/2)/c+1/48*(
15*B*b^2-18*A*b*c-16*B*a*c-2*c*(-6*A*c+5*B*b)*x^2)*(c*x^4+b*x^2+a)^(1/2)/c^
3
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {1265, 846, 793, 635, 212}

$$\int \frac{x^5(A+Bx^2)}{\sqrt{a+bx^2+cx^4}}\,dx=-\frac{(8aAc^2-12abBc-6Ab^2c+5b^3B)\operatorname{arctanh}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{32c^{7/2}}+\frac{\sqrt{a+bx^2+cx^4}(-16aBc-2cx^2(5bB-6Ac)-18Abc+15b^2B)}{48c^3}+\frac{Bx^4\sqrt{a+bx^2+cx^4}}{6c}$$

```
[In] Int[(x^5*(A + B*x^2))/Sqrt[a + b*x^2 + c*x^4],x]
```

```
[Out] (B*x^4*Sqrt[a + b*x^2 + c*x^4])/(6*c) + ((15*b^2*B - 18*A*b*c - 16*a*B*c -
2*c*(5*b*B - 6*A*c)*x^2)*Sqrt[a + b*x^2 + c*x^4])/(48*c^3) - ((5*b^3*B - 6*
A*b^2*c - 12*a*b*B*c + 8*a*A*c^2)*ArcTanh[(b + 2*c*x^2)/(2*Sqrt[c]*Sqrt[a +
 b*x^2 + c*x^4])])/(32*c^(7/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 635

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 793

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> Simp[(-(b*e*g*(p + 2) - c*(e*f + d*g)*(2*p + 3) -
 2*c*e*g*(p + 1)*x))*((a + b*x + c*x^2)^(p + 1)/(2*c^2*(p + 1)*(2*p + 3))),
 x] + Dist[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(2*c*d*f - b*(e*f + d*g))*(2*p
+ 3))/(2*c^2*(2*p + 3)), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c,
 d, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rule 846

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g*(d + e*x)^m*((a + b*x + c*x^2)^(p +
1)/(c*(m + 2*p + 2))), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)
*(a + b*x + c*x^2)^p*Simp[m*(c*d*f - a*e*g) + d*(2*c*f - b*g)*(p + 1) + (m*
(c*e*f + c*d*g - b*e*g) + e*(p + 1)*(2*c*f - b*g))*x, x], x], x] /; FreeQ[{
a, b, c, d, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a
*e^2, 0] && GtQ[m, 0] && NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p]
 || IntegersQ[2*m, 2*p]) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

Rule 1265

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rubi steps

$$\text{integral} = \frac{1}{2}\text{Subst}\left(\int \frac{x^2(A+Bx)}{\sqrt{a+bx+cx^2}}\,dx, x, x^2\right)$$
$$= \frac{Bx^4\sqrt{a+bx^2+cx^4}}{6c} + \frac{\text{Subst}\left(\int \frac{x\left(-2aB-\frac{1}{2}(5bB-6Ac)x\right)}{\sqrt{a+bx+cx^2}}\,dx, x, x^2\right)}{6c}$$
$$= \frac{Bx^4\sqrt{a+bx^2+cx^4}}{6c} + \frac{(15b^2B-18Abc-16aBc-2c(5bB-6Ac)x^2)\sqrt{a+bx^2+cx^4}}{48c^3} - \frac{(5b^3B-6Ab^2c-12abBc+8aAc^2)\,\text{Subst}\left(\int \frac{1}{\sqrt{a+bx+cx^2}}\,dx, x, x^2\right)}{32c^3}$$
$$= \frac{Bx^4\sqrt{a+bx^2+cx^4}}{6c} + \frac{(15b^2B-18Abc-16aBc-2c(5bB-6Ac)x^2)\sqrt{a+bx^2+cx^4}}{48c^3} - \frac{(5b^3B-6Ab^2c-12abBc+8aAc^2)\,\text{Subst}\left(\int \frac{1}{4c-x^2}\,dx, x, \frac{b+2cx^2}{\sqrt{a+bx^2+cx^4}}\right)}{16c^3}$$
$$= \frac{Bx^4\sqrt{a+bx^2+cx^4}}{6c} + \frac{(15b^2B-18Abc-16aBc-2c(5bB-6Ac)x^2)\sqrt{a+bx^2+cx^4}}{48c^3} - \frac{(5b^3B-6Ab^2c-12abBc+8aAc^2)\tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{32c^{7/2}}$$

Mathematica [A] (verified)

Time = 0.52 (sec) , antiderivative size = 135, normalized size of antiderivative = 0.88

$$\int \frac{x^5(A+Bx^2)}{\sqrt{a+bx^2+cx^4}}\,dx = \frac{\sqrt{a+bx^2+cx^4}(15b^2B-18Abc-16aBc-10bBcx^2+12Ac^2x^2+8Bc^2x^4)}{48c^3} + \frac{(5b^3B-6Ab^2c-12abBc+8aAc^2)\log\left(b+2cx^2-2\sqrt{c}\sqrt{a+bx^2+cx^4}\right)}{32c^{7/2}}$$

```
[In] Integrate[(x^5*(A + B*x^2))/Sqrt[a + b*x^2 + c*x^4],x]
```

```
[Out] (Sqrt[a + b*x^2 + c*x^4]*(15*b^2*B - 18*A*b*c - 16*a*B*c - 10*b*B*c*x^2 + 1
2*A*c^2*x^2 + 8*B*c^2*x^4))/(48*c^3) + ((5*b^3*B - 6*A*b^2*c - 12*a*b*B*c +
 8*a*A*c^2)*Log[b + 2*c*x^2 - 2*Sqrt[c]*Sqrt[a + b*x^2 + c*x^4]])/(32*c^(7/
2))
```

Maple [A] (verified)

Time = 0.19 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.82

method	result
risch	$-\frac{\left(-8B\,c^{2}x^{4}-12A\,c^{2}x^{2}+10Bbc\,x^{2}+18Abc+16Bac-15B\,b^{2}\right)\sqrt{c\,x^{4}+b\,x^{2}+a}}{48c^{3}}-\frac{\left(8Aa\,c^{2}-6A\,b^{2}c-12Babc+5B\,b^{3}\right)\ln\left(\frac{\frac{b}{2}+c\,x}{\sqrt{c}}\right.}{32c^{\frac{7}{2}}}$
pseudoelliptic	$-\frac{3\left(\sqrt{c\,x^{4}+b\,x^{2}+a}\left(\left(\left(\frac{5B\,x^{2}}{9}+A\right)b+\frac{8Ba}{9}\right)c^{\frac{3}{2}}-\frac{2\sqrt{c\,x^{4}+b\,x^{2}+a}\,\left(\frac{2B\,x^{2}}{3}+A\right)x^{2}c^{\frac{5}{2}}}{3}-\frac{5B\,b^{2}\sqrt{c\,x^{4}+b\,x^{2}+a}\,\sqrt{c}}{6}+\frac{2\left(Aa\,c^{2}-\frac{3}{4}A\,b^{2}c-\frac{3}{2}\right.}{}\right.\right.}{8c^{\frac{7}{2}}}$
default	$B\left(\frac{x^{4}\sqrt{c\,x^{4}+b\,x^{2}+a}}{6c}-\frac{5b\,x^{2}\sqrt{c\,x^{4}+b\,x^{2}+a}}{24c^{2}}+\frac{5b^{2}\sqrt{c\,x^{4}+b\,x^{2}+a}}{16c^{3}}-\frac{5b^{3}\ln\left(\frac{\frac{b}{2}+c\,x^{2}}{\sqrt{c}}+\sqrt{c\,x^{4}+b\,x^{2}+a}\right)}{32c^{\frac{7}{2}}}+\frac{3ba\ln\left(\frac{\frac{b}{2}+c\,x}{\sqrt{c}}\right.}{}\right.$
elliptic	$\frac{B\,x^{4}\sqrt{c\,x^{4}+b\,x^{2}+a}}{6c}-\frac{5Bb\,x^{2}\sqrt{c\,x^{4}+b\,x^{2}+a}}{24c^{2}}+\frac{5B\,b^{2}\sqrt{c\,x^{4}+b\,x^{2}+a}}{16c^{3}}-\frac{5B\,b^{3}\ln\left(\frac{\frac{b}{2}+c\,x^{2}}{\sqrt{c}}+\sqrt{c\,x^{4}+b\,x^{2}+a}\right)}{32c^{\frac{7}{2}}}+\frac{3Bba\ln\left(\right.}{}$

```
[In] int(x^5*(B*x^2+A)/(c*x^4+b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/48*(-8*B*c^2*x^4-12*A*c^2*x^2+10*B*b*c*x^2+18*A*b*c+16*B*a*c-15*B*b^2)*(
c*x^4+b*x^2+a)^(1/2)/c^3-1/32*(8*A*a*c^2-6*A*b^2*c-12*B*a*b*c+5*B*b^3)/c^(7
/2)*ln((1/2*b+c*x^2)/c^(1/2)+(c*x^4+b*x^2+a)^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 315, normalized size of antiderivative = 2.06

$$\int \frac{x^5(A+Bx^2)}{\sqrt{a+bx^2+cx^4}}\,dx$$
$$=\left[\frac{3\,(5\,Bb^3+8\,Aac^2-6\,(2\,Bab+Ab^2)c)\sqrt{c}\log\left(-8\,c^2x^4-8\,bcx^2-b^2+4\,\sqrt{cx^4+bx^2+a}(2\,cx^2+b)\sqrt{c}-4\right.}{192\,c^4}\right.$$

```
[In] integrate(x^5*(B*x^2+A)/(c*x^4+b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/192*(3*(5*B*b^3 + 8*A*a*c^2 - 6*(2*B*a*b + A*b^2)*c)*sqrt(c)*log(-8*c^2*
x^4 - 8*b*c*x^2 - b^2 + 4*sqrt(c*x^4 + b*x^2 + a)*(2*c*x^2 + b)*sqrt(c) - 4
*a*c) + 4*(8*B*c^3*x^4 + 15*B*b^2*c - 2*(8*B*a + 9*A*b)*c^2 - 2*(5*B*b*c^2
- 6*A*c^3)*x^2)*sqrt(c*x^4 + b*x^2 + a))/c^4, 1/96*(3*(5*B*b^3 + 8*A*a*c^2
- 6*(2*B*a*b + A*b^2)*c)*sqrt(-c)*arctan(1/2*sqrt(c*x^4 + b*x^2 + a)*(2*c*x
^2 + b)*sqrt(-c)/(c^2*x^4 + b*c*x^2 + a*c)) + 2*(8*B*c^3*x^4 + 15*B*b^2*c -
 2*(8*B*a + 9*A*b)*c^2 - 2*(5*B*b*c^2 - 6*A*c^3)*x^2)*sqrt(c*x^4 + b*x^2 +
a))/c^4]
```

Sympy [A] (verification not implemented)

Time = 1.05 (sec) , antiderivative size = 325, normalized size of antiderivative = 2.12

$$\int \frac{x^5(A+Bx^2)}{\sqrt{a+bx^2+cx^4}}\,dx = \frac{\begin{cases} \left(-\frac{a\left(A-\frac{5Bb}{6c}\right)}{2c}-\frac{b\left(-\frac{2Ba}{3c}-\frac{3b\left(A-\frac{5Bb}{6c}\right)}{4c}\right)}{2c}\right)\left(\begin{cases}\frac{\log\left(b+2\sqrt{c}\sqrt{a+bx^2+cx^4}+2cx^2\right)}{\sqrt{c}} & \text{for } a-\frac{b^2}{4c}\neq 0\\ \frac{\left(\frac{b}{2c}+x^2\right)\log\left(\frac{b}{2c}+x^2\right)}{\sqrt{c\left(\frac{b}{2c}+x^2\right)^2}} & \text{otherwise}\end{cases}\right)+\sqrt{a+bx^2+cx^4}\cdots \\ \frac{\frac{2A\left(a^2\sqrt{a+bx^2}-\frac{2a(a+bx^2)^{\frac{3}{2}}}{3}+\frac{(a+bx^2)^{\frac{5}{2}}}{5}\right)}{b^2}+\frac{2B\left(-a^3\sqrt{a+bx^2}+a^2(a+bx^2)^{\frac{3}{2}}-\frac{3a(a+bx^2)^{\frac{5}{2}}}{5}+\frac{(a+bx^2)^{\frac{7}{2}}}{7}\right)}{b^3}}{b} \\ \frac{\frac{Ax^6}{3}+\frac{Bx^8}{4}}{\sqrt{a}} \end{cases}}{2}$$

```
[In] integrate(x**5*(B*x**2+A)/(c*x**4+b*x**2+a)**(1/2),x)
```

```
[Out] Piecewise(((-a*(A - 5*B*b/(6*c))/(2*c) - b*(-2*B*a/(3*c) - 3*b*(A - 5*B*b/(
6*c))/(4*c))/(2*c))*Piecewise((log(b + 2*sqrt(c)*sqrt(a + b*x**2 + c*x**4)
+ 2*c*x**2)/sqrt(c), Ne(a - b**2/(4*c), 0)), ((b/(2*c) + x**2)*log(b/(2*c)
+ x**2)/sqrt(c*(b/(2*c) + x**2)**2), True)) + sqrt(a + b*x**2 + c*x**4)*(B*
x**4/(3*c) + x**2*(A - 5*B*b/(6*c))/(2*c) + (-2*B*a/(3*c) - 3*b*(A - 5*B*b/
(6*c))/(4*c))/c), Ne(c, 0)), ((2*A*(a**2*sqrt(a + b*x**2) - 2*a*(a + b*x**2
)**(3/2)/3 + (a + b*x**2)**(5/2)/5)/b**2 + 2*B*(-a**3*sqrt(a + b*x**2) + a*
*2*(a + b*x**2)**(3/2) - 3*a*(a + b*x**2)**(5/2)/5 + (a + b*x**2)**(7/2)/7)
/b**3)/b, Ne(b, 0)), ((A*x**6/3 + B*x**8/4)/sqrt(a), True))/2
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^5(A+Bx^2)}{\sqrt{a+bx^2+cx^4}}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(x^5*(B*x^2+A)/(c*x^4+b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

Giac [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 136, normalized size of antiderivative = 0.89

$$\int \frac{x^5(A+Bx^2)}{\sqrt{a+bx^2+cx^4}}\,dx$$
$$= \frac{1}{48}\sqrt{cx^4+bx^2+a}\left(2\left(\frac{4\,Bx^2}{c}-\frac{5\,Bbc-6\,Ac^2}{c^3}\right)x^2+\frac{15\,Bb^2-16\,Bac-18\,Abc}{c^3}\right)$$
$$+\frac{(5\,Bb^3-12\,Babc-6\,Ab^2c+8\,Aac^2)\log\left(\left|2\left(\sqrt{c}x^2-\sqrt{cx^4+bx^2+a}\right)\sqrt{c}+b\right|\right)}{32\,c^{\frac{7}{2}}}$$

```
[In] integrate(x^5*(B*x^2+A)/(c*x^4+b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] 1/48*sqrt(c*x^4 + b*x^2 + a)*(2*(4*B*x^2/c - (5*B*b*c - 6*A*c^2)/c^3)*x^2 +
 (15*B*b^2 - 16*B*a*c - 18*A*b*c)/c^3) + 1/32*(5*B*b^3 - 12*B*a*b*c - 6*A*b
^2*c + 8*A*a*c^2)*log(abs(2*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))*sqrt(c)
 + b))/c^(7/2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^5(A+Bx^2)}{\sqrt{a+bx^2+cx^4}}\,dx=\int \frac{x^5\,(B\,x^2+A)}{\sqrt{c\,x^4+b\,x^2+a}}\,dx$$

```
[In] int((x^5*(A + B*x^2))/(a + b*x^2 + c*x^4)^(1/2),x)
```

```
[Out] int((x^5*(A + B*x^2))/(a + b*x^2 + c*x^4)^(1/2), x)
```

3.170 $\int \frac{x^3(A+Bx^2)}{\sqrt{a+bx^2+cx^4}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 100

$$\int \frac{x^3(A+Bx^2)}{\sqrt{a+bx^2+cx^4}}\,dx = -\frac{(3bB-4Ac-2Bcx^2)\sqrt{a+bx^2+cx^4}}{8c^2} + \frac{(3b^2B-4Abc-4aBc)\,\text{arctanh}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{16c^{5/2}}$$

```
[Out] 1/16*(-4*A*b*c-4*B*a*c+3*B*b^2)*arctanh(1/2*(2*c*x^2+b)/c^(1/2)/(c*x^4+b*x^
2+a)^(1/2))/c^(5/2)-1/8*(-2*B*c*x^2-4*A*c+3*B*b)*(c*x^4+b*x^2+a)^(1/2)/c^2
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {1265, 793, 635, 212}

$$\int \frac{x^3(A+Bx^2)}{\sqrt{a+bx^2+cx^4}}\,dx = \frac{(-4aBc-4Abc+3b^2B)\,\text{arctanh}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{16c^{5/2}} - \frac{\sqrt{a+bx^2+cx^4}(-4Ac+3bB-2Bcx^2)}{8c^2}$$

```
[In] Int[(x^3*(A + B*x^2))/Sqrt[a + b*x^2 + c*x^4],x]
```

```
[Out] -1/8*((3*b*B - 4*A*c - 2*B*c*x^2)*Sqrt[a + b*x^2 + c*x^4])/c^2 + ((3*b^2*B
- 4*A*b*c - 4*a*B*c)*ArcTanh[(b + 2*c*x^2)/(2*Sqrt[c]*Sqrt[a + b*x^2 + c*x^
4])])/(16*c^(5/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 635

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 793

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> Simp[(-(b*e*g*(p + 2) - c*(e*f + d*g)*(2*p + 3) -
 2*c*e*g*(p + 1)*x))*((a + b*x + c*x^2)^(p + 1)/(2*c^2*(p + 1)*(2*p + 3))),
 x] + Dist[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(2*c*d*f - b*(e*f + d*g))*(2*p
+ 3))/(2*c^2*(2*p + 3)), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c,
 d, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rule 1265

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{2}\text{Subst}\left(\int \frac{x(A+Bx)}{\sqrt{a+bx+cx^2}}\,dx, x, x^2\right)\\
&= -\frac{(3bB-4Ac-2Bcx^2)\sqrt{a+bx^2+cx^4}}{8c^2}\\
&\quad+\frac{(3b^2B-4Abc-4aBc)\,\text{Subst}\left(\int \frac{1}{\sqrt{a+bx+cx^2}}\,dx, x, x^2\right)}{16c^2}\\
&= -\frac{(3bB-4Ac-2Bcx^2)\sqrt{a+bx^2+cx^4}}{8c^2}\\
&\quad+\frac{(3b^2B-4Abc-4aBc)\,\text{Subst}\left(\int \frac{1}{4c-x^2}\,dx, x, \frac{b+2cx^2}{\sqrt{a+bx^2+cx^4}}\right)}{8c^2}\\
&= -\frac{(3bB-4Ac-2Bcx^2)\sqrt{a+bx^2+cx^4}}{8c^2}+\frac{(3b^2B-4Abc-4aBc)\tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{16c^{5/2}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.33 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.04

$$\int \frac{x^3(A+Bx^2)}{\sqrt{a+bx^2+cx^4}}\,dx = \frac{(-3bB+4Ac+2Bcx^2)\sqrt{a+bx^2+cx^4}}{8c^2} + \frac{(-3b^2B+4Abc+4aBc)\log\left(bc^2+2c^3x^2-2c^{5/2}\sqrt{a+bx^2+cx^4}\right)}{16c^{5/2}}$$

```
[In] Integrate[(x^3*(A + B*x^2))/Sqrt[a + b*x^2 + c*x^4],x]
```

```
[Out] ((-3*b*B + 4*A*c + 2*B*c*x^2)*Sqrt[a + b*x^2 + c*x^4])/(8*c^2) + ((-3*b^2*B
 + 4*A*b*c + 4*a*B*c)*Log[b*c^2 + 2*c^3*x^2 - 2*c^(5/2)*Sqrt[a + b*x^2 + c*
x^4]])/(16*c^(5/2))
```

Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.88

method	result
risch	$\frac{(2B\,x^2c+4Ac-3Bb)\sqrt{c\,x^4+b\,x^2+a}}{8c^2}-\frac{(4Abc+4Bac-3B\,b^2)\ln\left(\frac{\frac{b}{2}+c\,x^2}{\sqrt{c}}+\sqrt{c\,x^4+b\,x^2+a}\right)}{16c^{\frac{5}{2}}}$
default	$B\left(\frac{x^2\sqrt{c\,x^4+b\,x^2+a}}{4c}-\frac{3b\sqrt{c\,x^4+b\,x^2+a}}{8c^2}+\frac{3b^2\ln\left(\frac{\frac{b}{2}+c\,x^2}{\sqrt{c}}+\sqrt{c\,x^4+b\,x^2+a}\right)}{16c^{\frac{5}{2}}}-\frac{a\ln\left(\frac{\frac{b}{2}+c\,x^2}{\sqrt{c}}+\sqrt{c\,x^4+b\,x^2+a}\right)}{4c^{\frac{3}{2}}}\right)+$
elliptic	$\frac{B\,x^2\sqrt{c\,x^4+b\,x^2+a}}{4c}-\frac{3Bb\sqrt{c\,x^4+b\,x^2+a}}{8c^2}+\frac{3B\,b^2\ln\left(\frac{\frac{b}{2}+c\,x^2}{\sqrt{c}}+\sqrt{c\,x^4+b\,x^2+a}\right)}{16c^{\frac{5}{2}}}-\frac{Ba\ln\left(\frac{\frac{b}{2}+c\,x^2}{\sqrt{c}}+\sqrt{c\,x^4+b\,x^2+a}\right)}{4c^{\frac{3}{2}}}+$
pseudoelliptic	$\frac{4B\,c^{\frac{3}{2}}x^2\sqrt{c\,x^4+b\,x^2+a}+8A\,c^{\frac{3}{2}}\sqrt{c\,x^4+b\,x^2+a}-4A\ln\left(\frac{2c\,x^2+2\sqrt{c\,x^4+b\,x^2+a}\,\sqrt{c}+b}{\sqrt{c}}\right)bc+4A\ln(2)bc-6Bb\sqrt{c\,x^4+b\,x^2+a}\,\sqrt{c}-}{16c^{\frac{5}{2}}}$

```
[In] int(x^3*(B*x^2+A)/(c*x^4+b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/8*(2*B*c*x^2+4*A*c-3*B*b)*(c*x^4+b*x^2+a)^(1/2)/c^2-1/16*(4*A*b*c+4*B*a*c
-3*B*b^2)/c^(5/2)*ln((1/2*b+c*x^2)/c^(1/2)+(c*x^4+b*x^2+a)^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 233, normalized size of antiderivative = 2.33

$$\int \frac{x^3(A+Bx^2)}{\sqrt{a+bx^2+cx^4}}\,dx$$
$$=\left[-\frac{(3\,Bb^2-4\,(Ba+Ab)c)\sqrt{c}\log\left(-8\,c^2x^4-8\,bcx^2-b^2+4\,\sqrt{cx^4+bx^2+a}(2\,cx^2+b)\sqrt{c}-4\,ac\right)-4\,(2\,B\right.$$
$$-\frac{(3\,Bb^2-4\,(Ba+Ab)c)\sqrt{-c}\arctan\left(\frac{\sqrt{cx^4+bx^2+a}(2\,cx^2+b)\sqrt{-c}}{2\,(c^2x^4+bcx^2+ac)}\right)-2\,(2\,Bc^2x^2-3\,Bbc+4\,Ac^2)\sqrt{cx^4+bx^2+}}{16\,c^3}$$

```
[In] integrate(x^3*(B*x^2+A)/(c*x^4+b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/32*((3*B*b^2 - 4*(B*a + A*b)*c)*sqrt(c)*log(-8*c^2*x^4 - 8*b*c*x^2 - b^
2 + 4*sqrt(c*x^4 + b*x^2 + a)*(2*c*x^2 + b)*sqrt(c) - 4*a*c) - 4*(2*B*c^2*x
^2 - 3*B*b*c + 4*A*c^2)*sqrt(c*x^4 + b*x^2 + a))/c^3, -1/16*((3*B*b^2 - 4*(
B*a + A*b)*c)*sqrt(-c)*arctan(1/2*sqrt(c*x^4 + b*x^2 + a)*(2*c*x^2 + b)*sqr
t(-c)/(c^2*x^4 + b*c*x^2 + a*c)) - 2*(2*B*c^2*x^2 - 3*B*b*c + 4*A*c^2)*sqrt
(c*x^4 + b*x^2 + a))/c^3]
```

Sympy [A] (verification not implemented)

Time = 0.92 (sec) , antiderivative size = 233, normalized size of antiderivative = 2.33

$$\int \frac{x^3(A+Bx^2)}{\sqrt{a+bx^2+cx^4}}\,dx$$
$$=\frac{\begin{cases}\left(-\frac{Ba}{2c}-\frac{b\left(A-\frac{3Bb}{4c}\right)}{2c}\right)\left(\begin{cases}\frac{\log\left(b+2\sqrt{c}\sqrt{a+bx^2+cx^4}+2cx^2\right)}{\sqrt{c}} & \text{for } a-\frac{b^2}{4c}\neq 0\\ \frac{\left(\frac{b}{2c}+x^2\right)\log\left(\frac{b}{2c}+x^2\right)}{\sqrt{c\left(\frac{b}{2c}+x^2\right)^2}} & \text{otherwise}\end{cases}\right)+\left(\frac{Bx^2}{2c}+\frac{A-\frac{3Bb}{4c}}{c}\right)\sqrt{a+bx^2+cx^4} \\ \frac{\frac{2A\left(-a\sqrt{a+bx^2}+\frac{(a+bx^2)^{\frac{3}{2}}}{3}\right)}{b}+\frac{2B\left(a^2\sqrt{a+bx^2}-\frac{2a(a+bx^2)^{\frac{3}{2}}}{3}+\frac{(a+bx^2)^{\frac{5}{2}}}{5}\right)}{b^2}}{b} \\ \frac{\frac{Ax^4}{2}+\frac{Bx^6}{3}}{\sqrt{a}}\end{cases}}{2}$$

```
[In] integrate(x**3*(B*x**2+A)/(c*x**4+b*x**2+a)**(1/2),x)
```

```
[Out] Piecewise(((-B*a/(2*c) - b*(A - 3*B*b/(4*c))/(2*c))*Piecewise((log(b + 2*sq
rt(c)*sqrt(a + b*x**2 + c*x**4) + 2*c*x**2)/sqrt(c), Ne(a - b**2/(4*c), 0))
, ((b/(2*c) + x**2)*log(b/(2*c) + x**2)/sqrt(c*(b/(2*c) + x**2)**2), True))
 + (B*x**2/(2*c) + (A - 3*B*b/(4*c))/c)*sqrt(a + b*x**2 + c*x**4), Ne(c, 0)
), ((2*A*(-a*sqrt(a + b*x**2) + (a + b*x**2)**(3/2)/3)/b + 2*B*(a**2*sqrt(a
 + b*x**2) - 2*a*(a + b*x**2)**(3/2)/3 + (a + b*x**2)**(5/2)/5)/b**2)/b, Ne
(b, 0)), ((A*x**4/2 + B*x**6/3)/sqrt(a), True))/2
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^3(A+Bx^2)}{\sqrt{a+bx^2+cx^4}}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(x^3*(B*x^2+A)/(c*x^4+b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.96

$$\begin{aligned}\int \frac{x^3(A+Bx^2)}{\sqrt{a+bx^2+cx^4}}\,dx &= \frac{1}{8}\sqrt{cx^4+bx^2+a}\left(\frac{2\,Bx^2}{c}-\frac{3\,Bb-4\,Ac}{c^2}\right)\\&\quad-\frac{(3\,Bb^2-4\,Bac-4\,Abc)\log\left(\left|2\left(\sqrt{c}x^2-\sqrt{cx^4+bx^2+a}\right)\sqrt{c}+b\right|\right)}{16\,c^{\frac{5}{2}}}\end{aligned}$$

```
[In] integrate(x^3*(B*x^2+A)/(c*x^4+b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] 1/8*sqrt(c*x^4 + b*x^2 + a)*(2*B*x^2/c - (3*B*b - 4*A*c)/c^2) - 1/16*(3*B*b
^2 - 4*B*a*c - 4*A*b*c)*log(abs(2*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))*s
qrt(c) + b))/c^(5/2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3(A+Bx^2)}{\sqrt{a+bx^2+cx^4}}\,dx = \int \frac{x^3\,(B\,x^2+A)}{\sqrt{c\,x^4+b\,x^2+a}}\,dx$$

```
[In] int((x^3*(A + B*x^2))/(a + b*x^2 + c*x^4)^(1/2),x)
[Out] int((x^3*(A + B*x^2))/(a + b*x^2 + c*x^4)^(1/2), x)
```

3.171 $\int \frac{x(A+Bx^2)}{\sqrt{a+bx^2+cx^4}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 76

$$\int \frac{x(A+Bx^2)}{\sqrt{a+bx^2+cx^4}}\,dx = \frac{B\sqrt{a+bx^2+cx^4}}{2c} - \frac{(bB-2Ac)\,\text{arctanh}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{4c^{3/2}}$$

```
[Out] -1/4*(-2*A*c+B*b)*arctanh(1/2*(2*c*x^2+b)/c^(1/2)/(c*x^4+b*x^2+a)^(1/2))/c^
(3/2)+1/2*B*(c*x^4+b*x^2+a)^(1/2)/c
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {1261, 654, 635, 212}

$$\int \frac{x(A+Bx^2)}{\sqrt{a+bx^2+cx^4}}\,dx = \frac{B\sqrt{a+bx^2+cx^4}}{2c} - \frac{(bB-2Ac)\,\text{arctanh}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{4c^{3/2}}$$

```
[In] Int[(x*(A + B*x^2))/Sqrt[a + b*x^2 + c*x^4],x]
```

```
[Out] (B*Sqrt[a + b*x^2 + c*x^4])/(2*c) - ((b*B - 2*A*c)*ArcTanh[(b + 2*c*x^2)/(2
*Sqrt[c]*Sqrt[a + b*x^2 + c*x^4])])/(4*c^(3/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 635

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 654

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 1261

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(
p_.), x_Symbol] :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x],
 x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{2}\text{Subst}\left(\int \frac{A+Bx}{\sqrt{a+bx+cx^2}}\,dx, x, x^2\right)\\
&= \frac{B\sqrt{a+bx^2+cx^4}}{2c} + \frac{(-bB+2Ac)\text{Subst}\left(\int \frac{1}{\sqrt{a+bx+cx^2}}\,dx, x, x^2\right)}{4c}\\
&= \frac{B\sqrt{a+bx^2+cx^4}}{2c} + \frac{(-bB+2Ac)\text{Subst}\left(\int \frac{1}{4c-x^2}\,dx, x, \frac{b+2cx^2}{\sqrt{a+bx^2+cx^4}}\right)}{2c}\\
&= \frac{B\sqrt{a+bx^2+cx^4}}{2c} - \frac{(bB-2Ac)\tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{4c^{3/2}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.29 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.01

$$\int \frac{x(A+Bx^2)}{\sqrt{a+bx^2+cx^4}}\,dx = \frac{B\sqrt{a+bx^2+cx^4}}{2c} + \frac{(-bB+2Ac)\text{arctanh}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{4c^{3/2}}$$

```
[In] Integrate[(x*(A + B*x^2))/Sqrt[a + b*x^2 + c*x^4],x]
[Out] (B*Sqrt[a + b*x^2 + c*x^4])/(2*c) + ((-(b*B) + 2*A*c)*ArcTanh[(b + 2*c*x^2)
/(2*Sqrt[c]*Sqrt[a + b*x^2 + c*x^4])])/(4*c^(3/2))
```

Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.86

method	result	si
risch	$\frac{B\sqrt{c\,x^{4}+b\,x^{2}+a}}{2c}+\frac{(2Ac-Bb)\ln\left(\frac{\frac{b}{2}+c\,x^{2}}{\sqrt{c}}+\sqrt{c\,x^{4}+b\,x^{2}+a}\right)}{4c^{\frac{3}{2}}}$	6
elliptic	$\frac{A\ln\left(\frac{\frac{b}{2}+c\,x^{2}}{\sqrt{c}}+\sqrt{c\,x^{4}+b\,x^{2}+a}\right)}{2\sqrt{c}}+\frac{B\sqrt{c\,x^{4}+b\,x^{2}+a}}{2c}-\frac{Bb\ln\left(\frac{\frac{b}{2}+c\,x^{2}}{\sqrt{c}}+\sqrt{c\,x^{4}+b\,x^{2}+a}\right)}{4c^{\frac{3}{2}}}$	9
default	$B\left(\frac{\sqrt{c\,x^{4}+b\,x^{2}+a}}{2c}-\frac{b\ln\left(\frac{\frac{b}{2}+c\,x^{2}}{\sqrt{c}}+\sqrt{c\,x^{4}+b\,x^{2}+a}\right)}{4c^{\frac{3}{2}}}\right)+\frac{A\ln\left(\frac{\frac{b}{2}+c\,x^{2}}{\sqrt{c}}+\sqrt{c\,x^{4}+b\,x^{2}+a}\right)}{2\sqrt{c}}$	9
pseudoelliptic	$\frac{2A\ln\left(\frac{2c\,x^{2}+2\sqrt{c\,x^{4}+b\,x^{2}+a}\,\sqrt{c}+b}{\sqrt{c}}\right)c-2A\ln(2)c-B\ln\left(\frac{2c\,x^{2}+2\sqrt{c\,x^{4}+b\,x^{2}+a}\,\sqrt{c}+b}{\sqrt{c}}\right)b+B\ln(2)b+2B\sqrt{c\,x^{4}+b\,x^{2}+a}\,\sqrt{c}}{4c^{\frac{3}{2}}}$	1

```
[In] int(x*(B*x^2+A)/(c*x^4+b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*B*(c*x^4+b*x^2+a)^(1/2)/c+1/4*(2*A*c-B*b)/c^(3/2)*ln((1/2*b+c*x^2)/c^(1
/2)+(c*x^4+b*x^2+a)^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 178, normalized size of antiderivative = 2.34

$$\int \frac{x(A+Bx^2)}{\sqrt{a+bx^2+cx^4}}\,dx$$
$$=\left[\frac{4\sqrt{cx^4+bx^2+a}Bc-(Bb-2\,Ac)\sqrt{c}\log\left(-8\,c^2x^4-8\,bcx^2-b^2-4\sqrt{cx^4+bx^2+a}(2\,cx^2+b)\sqrt{c}-4\,\right.}{8\,c^2}\right.$$

```
[In] integrate(x*(B*x^2+A)/(c*x^4+b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/8*(4*sqrt(c*x^4 + b*x^2 + a)*B*c - (B*b - 2*A*c)*sqrt(c)*log(-8*c^2*x^4
- 8*b*c*x^2 - b^2 - 4*sqrt(c*x^4 + b*x^2 + a)*(2*c*x^2 + b)*sqrt(c) - 4*a*c
))/c^2, 1/4*(2*sqrt(c*x^4 + b*x^2 + a)*B*c + (B*b - 2*A*c)*sqrt(-c)*arctan(
1/2*sqrt(c*x^4 + b*x^2 + a)*(2*c*x^2 + b)*sqrt(-c)/(c^2*x^4 + b*c*x^2 + a*c
)))/c^2]
```

Sympy [A] (verification not implemented)

Time = 0.83 (sec) , antiderivative size = 165, normalized size of antiderivative = 2.17

$$\int \frac{x(A+Bx^2)}{\sqrt{a+bx^2+cx^4}}\,dx = \frac{\begin{cases} \frac{B\sqrt{a+bx^2+cx^4}}{c} + \left(A-\frac{Bb}{2c}\right)\left(\begin{cases} \frac{\log\left(b+2\sqrt{c}\sqrt{a+bx^2+cx^4}+2cx^2\right)}{\sqrt{c}} & \text{for } a-\frac{b^2}{4c}\neq 0 \\ \frac{\left(\frac{b}{2c}+x^2\right)\log\left(\frac{b}{2c}+x^2\right)}{\sqrt{c\left(\frac{b}{2c}+x^2\right)^2}} & \text{otherwise} \end{cases}\right) & \text{for } c\neq 0 \\ \frac{2A\sqrt{a+bx^2}+\frac{2B\left(-a\sqrt{a+bx^2}+\frac{(a+bx^2)^{\frac{3}{2}}}{3}\right)}{b}}{b} & \text{for } b\neq 0 \\ \frac{Ax^2+\frac{Bx^4}{2}}{\sqrt{a}} & \text{otherwise} \end{cases}}{2}$$

```
[In] integrate(x*(B*x**2+A)/(c*x**4+b*x**2+a)**(1/2),x)
```

```
[Out] Piecewise((B*sqrt(a + b*x**2 + c*x**4)/c + (A - B*b/(2*c))*Piecewise((log(b
 + 2*sqrt(c)*sqrt(a + b*x**2 + c*x**4) + 2*c*x**2)/sqrt(c), Ne(a - b**2/(4*
c), 0)), ((b/(2*c) + x**2)*log(b/(2*c) + x**2)/sqrt(c*(b/(2*c) + x**2)**2),
 True)), Ne(c, 0)), ((2*A*sqrt(a + b*x**2) + 2*B*(-a*sqrt(a + b*x**2) + (a
+ b*x**2)**(3/2)/3)/b)/b, Ne(b, 0)), ((A*x**2 + B*x**4/2)/sqrt(a), True))/2
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x(A+Bx^2)}{\sqrt{a+bx^2+cx^4}}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(x*(B*x^2+A)/(c*x^4+b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.88

$$\int \frac{x(A+Bx^2)}{\sqrt{a+bx^2+cx^4}}\,dx = \frac{\sqrt{cx^4+bx^2+a}B}{2\,c} + \frac{(Bb-2\,Ac)\log\left(\left|2\left(\sqrt{c}x^2-\sqrt{cx^4+bx^2+a}\right)\sqrt{c}+b\right|\right)}{4\,c^{\frac{3}{2}}}$$

```
[In] integrate(x*(B*x^2+A)/(c*x^4+b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(c*x^4 + b*x^2 + a)*B/c + 1/4*(B*b - 2*A*c)*log(abs(2*(sqrt(c)*x^2
- sqrt(c*x^4 + b*x^2 + a))*sqrt(c) + b))/c^(3/2)
```

Mupad [B] (verification not implemented)

Time = 8.17 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.21

$$\int \frac{x(A+Bx^2)}{\sqrt{a+bx^2+cx^4}}\,dx = \frac{A\,\ln\left(\sqrt{c\,x^4+b\,x^2+a}+\frac{c\,x^2+\frac{b}{2}}{\sqrt{c}}\right)}{2\,\sqrt{c}} + \frac{B\,\sqrt{c\,x^4+b\,x^2+a}}{2\,c} - \frac{B\,b\,\ln\left(\sqrt{c\,x^4+b\,x^2+a}+\frac{c\,x^2+\frac{b}{2}}{\sqrt{c}}\right)}{4\,c^{3/2}}$$

```
[In] int((x*(A + B*x^2))/(a + b*x^2 + c*x^4)^(1/2),x)
```

```
[Out] (A*log((a + b*x^2 + c*x^4)^(1/2) + (b/2 + c*x^2)/c^(1/2)))/(2*c^(1/2)) + (B
*(a + b*x^2 + c*x^4)^(1/2))/(2*c) - (B*b*log((a + b*x^2 + c*x^4)^(1/2) + (b
/2 + c*x^2)/c^(1/2)))/(4*c^(3/2))
```

3.172 $\int \frac{A+Bx^2}{x\sqrt{a+bx^2+cx^4}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 90

$$\int \frac{A+Bx^2}{x\sqrt{a+bx^2+cx^4}}\,dx=-\frac{A\text{arctanh}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{2\sqrt{a}}+\frac{B\text{arctanh}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{2\sqrt{c}}$$

```
[Out] -1/2*A*arctanh(1/2*(b*x^2+2*a)/a^(1/2)/(c*x^4+b*x^2+a)^(1/2))/a^(1/2)+1/2*B
*arctanh(1/2*(2*c*x^2+b)/c^(1/2)/(c*x^4+b*x^2+a)^(1/2))/c^(1/2)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {1265, 857, 635, 212, 738}

$$\int \frac{A+Bx^2}{x\sqrt{a+bx^2+cx^4}}\,dx=\frac{B\text{arctanh}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{2\sqrt{c}}-\frac{A\text{arctanh}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{2\sqrt{a}}$$

```
[In] Int[(A + B*x^2)/(x*Sqrt[a + b*x^2 + c*x^4]),x]
```

```
[Out] -1/2*(A*ArcTanh[(2*a + b*x^2)/(2*Sqrt[a]*Sqrt[a + b*x^2 + c*x^4])])/Sqrt[a]
 + (B*ArcTanh[(b + 2*c*x^2)/(2*Sqrt[c]*Sqrt[a + b*x^2 + c*x^4])])/(2*Sqrt[c
])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 635

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 857

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1265

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{1}{2}\text{Subst}\left(\int \frac{A+Bx}{x\sqrt{a+bx+cx^2}}\,dx, x, x^2\right) \\
&= \frac{1}{2}A\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx+cx^2}}\,dx, x, x^2\right) + \frac{1}{2}B\text{Subst}\left(\int \frac{1}{\sqrt{a+bx+cx^2}}\,dx, x, x^2\right) \\
&= -\left(A\text{Subst}\left(\int \frac{1}{4a-x^2}\,dx, x, \frac{2a+bx^2}{\sqrt{a+bx^2+cx^4}}\right)\right) \\
&\quad + B\text{Subst}\left(\int \frac{1}{4c-x^2}\,dx, x, \frac{b+2cx^2}{\sqrt{a+bx^2+cx^4}}\right) \\
&= -\frac{A\tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{2\sqrt{a}} + \frac{B\tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{2\sqrt{c}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.94

$$\int \frac{A+Bx^2}{x\sqrt{a+bx^2+cx^4}}\,dx = \frac{A\text{arctanh}\left(\frac{\sqrt{c}x^2-\sqrt{a+bx^2+cx^4}}{\sqrt{a}}\right)}{\sqrt{a}} - \frac{B\log\left(b+2cx^2-2\sqrt{c}\sqrt{a+bx^2+cx^4}\right)}{2\sqrt{c}}$$

[In] Integrate[(A + B*x^2)/(x*Sqrt[a + b*x^2 + c*x^4]),x]

[Out] (A*ArcTanh[(Sqrt[c]*x^2 - Sqrt[a + b*x^2 + c*x^4])/Sqrt[a]])/Sqrt[a] - (B*Log[b + 2*c*x^2 - 2*Sqrt[c]*Sqrt[a + b*x^2 + c*x^4]])/(2*Sqrt[c])

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.84

method	result	size
default	$\frac{B\ln\left(\frac{\frac{b}{2}+c\,x^2}{\sqrt{c}}+\sqrt{c\,x^4+b\,x^2+a}\right)}{2\sqrt{c}}-\frac{A\ln\left(\frac{2a+b\,x^2+2\sqrt{a}\,\sqrt{c\,x^4+b\,x^2+a}}{x^2}\right)}{2\sqrt{a}}$	76
elliptic	$\frac{B\ln\left(\frac{\frac{b}{2}+c\,x^2}{\sqrt{c}}+\sqrt{c\,x^4+b\,x^2+a}\right)}{2\sqrt{c}}-\frac{A\ln\left(\frac{2a+b\,x^2+2\sqrt{a}\,\sqrt{c\,x^4+b\,x^2+a}}{x^2}\right)}{2\sqrt{a}}$	76
pseudoelliptic	$-\frac{A\ln\left(\frac{2a+b\,x^2+2\sqrt{a}\,\sqrt{c\,x^4+b\,x^2+a}}{x^2}\right)\sqrt{c}-B\sqrt{a}\left(-\ln(2)+\ln\left(\frac{2c\,x^2+2\sqrt{c\,x^4+b\,x^2+a}\,\sqrt{c}+b}{\sqrt{c}}\right)\right)}{2\sqrt{a}\,\sqrt{c}}$	91

[In] int((B*x^2+A)/x/(c*x^4+b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)

[Out] 1/2*B*ln((1/2*b+c*x^2)/c^(1/2)+(c*x^4+b*x^2+a)^(1/2))/c^(1/2)-1/2*A/a^(1/2)*ln((2*a+b*x^2+2*a^(1/2)*(c*x^4+b*x^2+a)^(1/2))/x^2)

Fricas [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 517, normalized size of antiderivative = 5.74

$$\int \frac{A + Bx^2}{x\sqrt{a + bx^2 + cx^4}} \, dx$$
$$= \left[\frac{Ba\sqrt{c} \log \left(-8\, c^2 x^4 - 8\, bcx^2 - b^2 - 4\, \sqrt{cx^4 + bx^2 + a}(2\, cx^2 + b)\sqrt{c} - 4\, ac\right) + A\sqrt{a}c \log \left(-\frac{(b^2+4\,ac)x^4+8\,a\right.}{4\,ac} \right.$$
$$-\frac{2\, Ba\sqrt{-c} \arctan \left(\frac{\sqrt{cx^4+bx^2+a}(2\,cx^2+b)\sqrt{-c}}{2\,(c^2x^4+bcx^2+ac)}\right) - A\sqrt{a}c \log \left(-\frac{(b^2+4\,ac)x^4+8\,abx^2-4\,\sqrt{cx^4+bx^2+a}(bx^2+2\,a)\sqrt{a}+8\,a^2}{x^4}\right)}{4\,ac},$$

[In] integrate((B*x^2+A)/x/(c*x^4+b*x^2+a)^(1/2),x, algorithm="fricas")

[Out] [1/4*(B*a*sqrt(c)*log(-8*c^2*x^4 - 8*b*c*x^2 - b^2 - 4*sqrt(c*x^4 + b*x^2 + a)*(2*c*x^2 + b)*sqrt(c) - 4*a*c) + A*sqrt(a)*c*log(-((b^2 + 4*a*c)*x^4 + 8*a*b*x^2 - 4*sqrt(c*x^4 + b*x^2 + a)*(b*x^2 + 2*a)*sqrt(a) + 8*a^2)/x^4))/(a*c), -1/4*(2*B*a*sqrt(-c)*arctan(1/2*sqrt(c*x^4 + b*x^2 + a)*(2*c*x^2 + b)*sqrt(-c)/(c^2*x^4 + b*c*x^2 + a*c)) - A*sqrt(a)*c*log(-((b^2 + 4*a*c)*x^4 + 8*a*b*x^2 - 4*sqrt(c*x^4 + b*x^2 + a)*(b*x^2 + 2*a)*sqrt(a) + 8*a^2)/x^4))/(a*c), 1/4*(2*A*sqrt(-a)*c*arctan(1/2*sqrt(c*x^4 + b*x^2 + a)*(b*x^2 + 2*a)*sqrt(-a)/(a*c*x^4 + a*b*x^2 + a^2)) + B*a*sqrt(c)*log(-8*c^2*x^4 - 8*b*c*x^2 - b^2 - 4*sqrt(c*x^4 + b*x^2 + a)*(2*c*x^2 + b)*sqrt(c) - 4*a*c))/(a*c), 1/2*(A*sqrt(-a)*c*arctan(1/2*sqrt(c*x^4 + b*x^2 + a)*(b*x^2 + 2*a)*sqrt(-a)/(a*c*x^4 + a*b*x^2 + a^2)) - B*a*sqrt(-c)*arctan(1/2*sqrt(c*x^4 + b*x^2 + a)*(2*c*x^2 + b)*sqrt(-c)/(c^2*x^4 + b*c*x^2 + a*c)))/(a*c)]

Sympy [F]

$$\int \frac{A + Bx^2}{x\sqrt{a + bx^2 + cx^4}} \, dx = \int \frac{A + Bx^2}{x\sqrt{a + bx^2 + cx^4}} \, dx$$

[In] integrate((B*x**2+A)/x/(c*x**4+b*x**2+a)**(1/2),x)

[Out] Integral((A + B*x**2)/(x*sqrt(a + b*x**2 + c*x**4)), x)

Maxima [F(-2)]

Exception generated.

$$\int \frac{A+Bx^2}{x\sqrt{a+bx^2+cx^4}}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((B*x^2+A)/x/(c*x^4+b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

Giac [F(-2)]

Exception generated.

$$\int \frac{A+Bx^2}{x\sqrt{a+bx^2+cx^4}}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate((B*x^2+A)/x/(c*x^4+b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument Type
```

Mupad [B] (verification not implemented)

Time = 8.00 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.90

$$\int \frac{A+Bx^2}{x\sqrt{a+bx^2+cx^4}}\,dx = \frac{B\,\ln\left(\sqrt{c\,x^4+b\,x^2+a}+\frac{c\,x^2+\frac{b}{2}}{\sqrt{c}}\right)}{2\,\sqrt{c}} - \frac{A\,\ln\left(\frac{1}{x^2}\right)}{2\,\sqrt{a}} - \frac{A\,\ln\left(2\,a+2\,\sqrt{a}\,\sqrt{c\,x^4+b\,x^2+a}+b\,x^2\right)}{2\,\sqrt{a}}$$

```
[In] int((A + B*x^2)/(x*(a + b*x^2 + c*x^4)^(1/2)),x)
```

```
[Out] (B*log((a + b*x^2 + c*x^4)^(1/2) + (b/2 + c*x^2)/c^(1/2)))/(2*c^(1/2)) - (A
*log(1/x^2))/(2*a^(1/2)) - (A*log(2*a + 2*a^(1/2)*(a + b*x^2 + c*x^4)^(1/2)
 + b*x^2))/(2*a^(1/2))
```

3.173 $\int \frac{A+Bx^2}{x^3\sqrt{a+bx^2+cx^4}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 80

$$\int \frac{A+Bx^2}{x^3\sqrt{a+bx^2+cx^4}}\,dx = -\frac{A\sqrt{a+bx^2+cx^4}}{2ax^2} + \frac{(Ab-2aB)\text{arctanh}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{4a^{3/2}}$$

```
[Out] 1/4*(A*b-2*B*a)*arctanh(1/2*(b*x^2+2*a)/a^(1/2)/(c*x^4+b*x^2+a)^(1/2))/a^(3
/2)-1/2*A*(c*x^4+b*x^2+a)^(1/2)/a/x^2
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {1265, 820, 738, 212}

$$\int \frac{A+Bx^2}{x^3\sqrt{a+bx^2+cx^4}}\,dx = \frac{(Ab-2aB)\text{arctanh}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{4a^{3/2}} - \frac{A\sqrt{a+bx^2+cx^4}}{2ax^2}$$

```
[In] Int[(A + B*x^2)/(x^3*Sqrt[a + b*x^2 + c*x^4]),x]
```

```
[Out] -1/2*(A*Sqrt[a + b*x^2 + c*x^4])/(a*x^2) + ((A*b - 2*a*B)*ArcTanh[(2*a + b*
x^2)/(2*Sqrt[a]*Sqrt[a + b*x^2 + c*x^4])])/(4*a^(3/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 820

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a +
 b*x + c*x^2)^(p + 1)/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2))), x] - Dist[(b*(e
*f + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(
m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[Simplify[m
+ 2*p + 3], 0]
```

Rule 1265

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{2}\text{Subst}\left(\int \frac{A+Bx}{x^2\sqrt{a+bx+cx^2}}\,dx, x, x^2\right)\\
&= -\frac{A\sqrt{a+bx^2+cx^4}}{2ax^2} - \frac{(Ab-2aB)\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx+cx^2}}\,dx, x, x^2\right)}{4a}\\
&= -\frac{A\sqrt{a+bx^2+cx^4}}{2ax^2} + \frac{(Ab-2aB)\text{Subst}\left(\int \frac{1}{4a-x^2}\,dx, x, \frac{2a+bx^2}{\sqrt{a+bx^2+cx^4}}\right)}{2a}\\
&= -\frac{A\sqrt{a+bx^2+cx^4}}{2ax^2} + \frac{(Ab-2aB)\tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{4a^{3/2}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.33 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.01

$$\int \frac{A+Bx^2}{x^3\sqrt{a+bx^2+cx^4}}\,dx = -\frac{A\sqrt{a+bx^2+cx^4}}{2ax^2}+\frac{(-Ab+2aB)\text{arctanh}\left(\frac{\sqrt{c}x^2-\sqrt{a+bx^2+cx^4}}{\sqrt{a}}\right)}{2a^{3/2}}$$

```
[In] Integrate[(A + B*x^2)/(x^3*Sqrt[a + b*x^2 + c*x^4]),x]
[Out] -1/2*(A*Sqrt[a + b*x^2 + c*x^4])/(a*x^2) + ((-(A*b) + 2*a*B)*ArcTanh[(Sqrt[
c]*x^2 - Sqrt[a + b*x^2 + c*x^4])/Sqrt[a]])/(2*a^(3/2))
```

Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.89

method	result	size
risch	$-\frac{A\sqrt{c\,x^4+b\,x^2+a}}{2a\,x^2}+\frac{(Ab-2Ba)\ln\left(\frac{2a+b\,x^2+2\sqrt{a}\,\sqrt{c\,x^4+b\,x^2+a}}{x^2}\right)}{4a^{\frac{3}{2}}}$	71
pseudoelliptic	$\frac{x^2(Ab-2Ba)\ln\left(\frac{2a+b\,x^2+2\sqrt{a}\,\sqrt{c\,x^4+b\,x^2+a}}{x^2}\right)-2A\sqrt{a}\,\sqrt{c\,x^4+b\,x^2+a}}{4a^{\frac{3}{2}}\,x^2}$	75
elliptic	$-\frac{A\sqrt{c\,x^4+b\,x^2+a}}{2a\,x^2}+\frac{Ab\ln\left(\frac{2a+b\,x^2+2\sqrt{a}\,\sqrt{c\,x^4+b\,x^2+a}}{x^2}\right)}{4a^{\frac{3}{2}}}-\frac{B\ln\left(\frac{2a+b\,x^2+2\sqrt{a}\,\sqrt{c\,x^4+b\,x^2+a}}{x^2}\right)}{2\sqrt{a}}$	104
default	$-\frac{B\ln\left(\frac{2a+b\,x^2+2\sqrt{a}\,\sqrt{c\,x^4+b\,x^2+a}}{x^2}\right)}{2\sqrt{a}}+A\left(-\frac{\sqrt{c\,x^4+b\,x^2+a}}{2a\,x^2}+\frac{b\ln\left(\frac{2a+b\,x^2+2\sqrt{a}\,\sqrt{c\,x^4+b\,x^2+a}}{x^2}\right)}{4a^{\frac{3}{2}}}\right)$	105

```
[In] int((B*x^2+A)/x^3/(c*x^4+b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
[Out] -1/2*A*(c*x^4+b*x^2+a)^(1/2)/a/x^2+1/4*(A*b-2*B*a)/a^(3/2)*ln((2*a+b*x^2+2*
a^(1/2)*(c*x^4+b*x^2+a)^(1/2))/x^2)
```

Fricas [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 197, normalized size of antiderivative = 2.46

$$\int \frac{A+Bx^2}{x^3\sqrt{a+bx^2+cx^4}}\,dx = \left[-\frac{(2\,Ba-Ab)\sqrt{a}x^2\log\left(-\frac{(b^2+4\,ac)x^4+8\,abx^2+4\,\sqrt{cx^4+bx^2+a}(bx^2+2\,a)\sqrt{a}+8\,a^2}{x^4}\right)+4\,\sqrt{cx^4+bx^2+a}Aa}{8\,a^2x^2}, \frac{(2\,Ba-\right.$$

```
[In] integrate((B*x^2+A)/x^3/(c*x^4+b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/8*((2*B*a - A*b)*sqrt(a)*x^2*log(-((b^2 + 4*a*c)*x^4 + 8*a*b*x^2 + 4*sq
rt(c*x^4 + b*x^2 + a)*(b*x^2 + 2*a)*sqrt(a) + 8*a^2)/x^4) + 4*sqrt(c*x^4 +
b*x^2 + a)*A*a)/(a^2*x^2), 1/4*((2*B*a - A*b)*sqrt(-a)*x^2*arctan(1/2*sqrt(
c*x^4 + b*x^2 + a)*(b*x^2 + 2*a)*sqrt(-a)/(a*c*x^4 + a*b*x^2 + a^2)) - 2*sq
rt(c*x^4 + b*x^2 + a)*A*a)/(a^2*x^2)]
```

Sympy [F]

$$\int \frac{A+Bx^2}{x^3\sqrt{a+bx^2+cx^4}}\,dx = \int \frac{A+Bx^2}{x^3\sqrt{a+bx^2+cx^4}}\,dx$$

```
[In] integrate((B*x**2+A)/x**3/(c*x**4+b*x**2+a)**(1/2),x)
```

```
[Out] Integral((A + B*x**2)/(x**3*sqrt(a + b*x**2 + c*x**4)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{A+Bx^2}{x^3\sqrt{a+bx^2+cx^4}}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((B*x^2+A)/x^3/(c*x^4+b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.55

$$\int \frac{A+Bx^2}{x^3\sqrt{a+bx^2+cx^4}}\,dx = \frac{(2\,Ba-Ab)\arctan\left(-\frac{\sqrt{c}x^2-\sqrt{cx^4+bx^2+a}}{\sqrt{-a}}\right)}{2\,\sqrt{-a}a} + \frac{\left(\sqrt{c}x^2-\sqrt{cx^4+bx^2+a}\right)Ab+2\,Aa\sqrt{c}}{2\left(\left(\sqrt{c}x^2-\sqrt{cx^4+bx^2+a}\right)^2-a\right)a}$$

```
[In] integrate((B*x^2+A)/x^3/(c*x^4+b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*(2*B*a - A*b)*arctan(-(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))/sqrt(-a))
/(sqrt(-a)*a) + 1/2*((sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))*A*b + 2*A*a*sq
rt(c))/(((sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))^2 - a)*a)
```

Mupad [B] (verification not implemented)

Time = 8.15 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.29

$$\int \frac{A+Bx^2}{x^3\sqrt{a+bx^2+cx^4}}\,dx = \frac{A\,b\,\mathrm{atanh}\left(\frac{\frac{b\,x^2}{2}+a}{\sqrt{a}\,\sqrt{c\,x^4+b\,x^2+a}}\right)}{4\,a^{3/2}} - \frac{B\,\ln\left(2\,a+2\,\sqrt{a}\,\sqrt{c\,x^4+b\,x^2+a}+b\,x^2\right)}{2\,\sqrt{a}} - \frac{A\,\sqrt{c\,x^4+b\,x^2+a}}{2\,a\,x^2} - \frac{B\,\ln\left(\frac{1}{x^2}\right)}{2\,\sqrt{a}}$$

```
[In] int((A + B*x^2)/(x^3*(a + b*x^2 + c*x^4)^(1/2)),x)
```

```
[Out] (A*b*atanh((a + (b*x^2)/2)/(a^(1/2)*(a + b*x^2 + c*x^4)^(1/2))))/(4*a^(3/2)
) - (B*log(2*a + 2*a^(1/2)*(a + b*x^2 + c*x^4)^(1/2) + b*x^2))/(2*a^(1/2))
- (A*(a + b*x^2 + c*x^4)^(1/2))/(2*a*x^2) - (B*log(1/x^2))/(2*a^(1/2))
```

3.174 $\int \frac{A+Bx^2}{x^5\sqrt{a+bx^2+cx^4}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 124

$$\int \frac{A+Bx^2}{x^5\sqrt{a+bx^2+cx^4}}\,dx = -\frac{A\sqrt{a+bx^2+cx^4}}{4ax^4}+\frac{(3Ab-4aB)\sqrt{a+bx^2+cx^4}}{8a^2x^2} - \frac{(3Ab^2-4abB-4aAc)\operatorname{arctanh}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{16a^{5/2}}$$

```
[Out] -1/16*(-4*A*a*c+3*A*b^2-4*B*a*b)*arctanh(1/2*(b*x^2+2*a)/a^(1/2)/(c*x^4+b*x
^2+a)^(1/2))/a^(5/2)-1/4*A*(c*x^4+b*x^2+a)^(1/2)/a/x^4+1/8*(3*A*b-4*B*a)*(c
*x^4+b*x^2+a)^(1/2)/a^2/x^2
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {1265, 848, 820, 738, 212}

$$\int \frac{A+Bx^2}{x^5\sqrt{a+bx^2+cx^4}}\,dx = -\frac{(-4aAc-4abB+3Ab^2)\operatorname{arctanh}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{16a^{5/2}} + \frac{(3Ab-4aB)\sqrt{a+bx^2+cx^4}}{8a^2x^2}-\frac{A\sqrt{a+bx^2+cx^4}}{4ax^4}$$

```
[In] Int[(A + B*x^2)/(x^5*Sqrt[a + b*x^2 + c*x^4]),x]
```

```
[Out] -1/4*(A*Sqrt[a + b*x^2 + c*x^4])/(a*x^4) + ((3*A*b - 4*a*B)*Sqrt[a + b*x^2
+ c*x^4])/(8*a^2*x^2) - ((3*A*b^2 - 4*a*b*B - 4*a*A*c)*ArcTanh[(2*a + b*x^2
)/(2*Sqrt[a]*Sqrt[a + b*x^2 + c*x^4])])/(16*a^(5/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 820

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a +
 b*x + c*x^2)^(p + 1)/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2))), x] - Dist[(b*(e
*f + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(
m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[Simplify[m
+ 2*p + 3], 0]
```

Rule 848

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*f - d*g)*(d + e*x)^(m + 1)*((a + b*
x + c*x^2)^(p + 1)/((m + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Dist[1/((m + 1)
*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*Simp[(
c*d*f - f*b*e + a*e*g)*(m + 1) + b*(d*g - e*f)*(p + 1) - c*(e*f - d*g)*(m +
 2*p + 3)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] ||
 IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 1265

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rubi steps

$$\text{integral} = \frac{1}{2}\text{Subst}\left(\int \frac{A+Bx}{x^3\sqrt{a+bx+cx^2}}\,dx, x, x^2\right)$$

$$= -\frac{A\sqrt{a+bx^2+cx^4}}{4ax^4} - \frac{\text{Subst}\left(\int \frac{\frac{1}{2}(3Ab-4aB)+Acx}{x^2\sqrt{a+bx+cx^2}}\,dx, x, x^2\right)}{4a}$$

$$= -\frac{A\sqrt{a+bx^2+cx^4}}{4ax^4} + \frac{(3Ab-4aB)\sqrt{a+bx^2+cx^4}}{8a^2x^2} + \frac{(3Ab^2-4abB-4aAc)\,\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx+cx^2}}\,dx, x, x^2\right)}{16a^2}$$

$$= -\frac{A\sqrt{a+bx^2+cx^4}}{4ax^4} + \frac{(3Ab-4aB)\sqrt{a+bx^2+cx^4}}{8a^2x^2} - \frac{(3Ab^2-4abB-4aAc)\,\text{Subst}\left(\int \frac{1}{4a-x^2}\,dx, x, \frac{2a+bx^2}{\sqrt{a+bx^2+cx^4}}\right)}{8a^2}$$

$$= -\frac{A\sqrt{a+bx^2+cx^4}}{4ax^4} + \frac{(3Ab-4aB)\sqrt{a+bx^2+cx^4}}{8a^2x^2} - \frac{(3Ab^2-4abB-4aAc)\tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{16a^{5/2}}$$

Mathematica [A] (verified)

Time = 0.57 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.17

$$\int \frac{A+Bx^2}{x^5\sqrt{a+bx^2+cx^4}}\,dx$$
$$= \frac{\sqrt{a}\sqrt{a+bx^2+cx^4}(3Abx^2-2a(A+2Bx^2)) + 3Ab^2x^4\text{arctanh}\left(\frac{\sqrt{c}x^2-\sqrt{a+bx^2+cx^4}}{\sqrt{a}}\right) + 4a(bB+Ac)x^4\text{arctanh}}{8a^{5/2}x^4}$$

```
[In] Integrate[(A + B*x^2)/(x^5*Sqrt[a + b*x^2 + c*x^4]),x]
```

```
[Out] (Sqrt[a]*Sqrt[a + b*x^2 + c*x^4]*(3*A*b*x^2 - 2*a*(A + 2*B*x^2)) + 3*A*b^2*
x^4*ArcTanh[(Sqrt[c]*x^2 - Sqrt[a + b*x^2 + c*x^4])/Sqrt[a]] + 4*a*(b*B + A
*c)*x^4*ArcTanh[(-(Sqrt[c]*x^2) + Sqrt[a + b*x^2 + c*x^4])/Sqrt[a]])/(8*a^(
5/2)*x^4)
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.79

method	result
risch	$-\frac{\sqrt{c\,x^{4}+b\,x^{2}+a}\,\left(-3Ab\,x^{2}+4Ba\,x^{2}+2Aa\right)}{8a^{2}x^{4}}+\frac{\left(4Aac-3A\,b^{2}+4abB\right)\ln\left(\frac{2a+b\,x^{2}+2\sqrt{a}\,\sqrt{c\,x^{4}+b\,x^{2}+a}}{x^{2}}\right)}{16a^{\frac{5}{2}}}$
pseudoelliptic	$\frac{\frac{\left(\left(ac-\frac{3b^{2}}{4}\right)A+abB\right)x^{4}\ln\left(\frac{2a+b\,x^{2}+2\sqrt{a}\,\sqrt{c\,x^{4}+b\,x^{2}+a}}{x^{2}}\right)}{4}+\frac{3\left(\frac{2\left(-2B\,x^{2}-A\right)a^{\frac{3}{2}}}{3}+A\sqrt{a}\,b\,x^{2}\right)\sqrt{c\,x^{4}+b\,x^{2}+a}}{8}}{a^{\frac{5}{2}}x^{4}}$
default	$B\left(-\frac{\sqrt{c\,x^{4}+b\,x^{2}+a}}{2a\,x^{2}}+\frac{b\ln\left(\frac{2a+b\,x^{2}+2\sqrt{a}\,\sqrt{c\,x^{4}+b\,x^{2}+a}}{x^{2}}\right)}{4a^{\frac{3}{2}}}\right)+A\left(-\frac{\sqrt{c\,x^{4}+b\,x^{2}+a}}{4a\,x^{4}}+\frac{3b\sqrt{c\,x^{4}+b\,x^{2}+a}}{8a^{2}x^{2}}-\frac{3b^{2}\ln(}{}\right.$
elliptic	$-\frac{B\sqrt{c\,x^{4}+b\,x^{2}+a}}{2a\,x^{2}}+\frac{Bb\ln\left(\frac{2a+b\,x^{2}+2\sqrt{a}\,\sqrt{c\,x^{4}+b\,x^{2}+a}}{x^{2}}\right)}{4a^{\frac{3}{2}}}-\frac{A\sqrt{c\,x^{4}+b\,x^{2}+a}}{4a\,x^{4}}+\frac{3Ab\sqrt{c\,x^{4}+b\,x^{2}+a}}{8a^{2}x^{2}}-\frac{3A\,b^{2}\ln\left(\frac{2a+b}{}\right.}{}$

```
[In] int((B*x^2+A)/x^5/(c*x^4+b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/8*(c*x^4+b*x^2+a)^(1/2)*(-3*A*b*x^2+4*B*a*x^2+2*A*a)/a^2/x^4+1/16*(4*A*a
*c-3*A*b^2+4*B*a*b)/a^(5/2)*ln((2*a+b*x^2+2*a^(1/2)*(c*x^4+b*x^2+a)^(1/2))/
x^2)
```

Fricas [A] (verification not implemented)

none

Time = 0.35 (sec) , antiderivative size = 255, normalized size of antiderivative = 2.06

$$\int \frac{A+Bx^2}{x^5\sqrt{a+bx^2+cx^4}}\,dx$$
$$=\left[\frac{(4\,Bab-3\,Ab^2+4\,Aac)\sqrt{a}x^4\log\left(-\frac{(b^2+4\,ac)x^4+8\,abx^2+4\,\sqrt{cx^4+bx^2+a}(bx^2+2\,a)\sqrt{a}+8\,a^2}{x^4}\right)-4\,\sqrt{cx^4+bx^2+a}(2\,\cdots}{32\,a^3x^4}\right.$$
$$-\frac{(4\,Bab-3\,Ab^2+4\,Aac)\sqrt{-a}x^4\arctan\left(\frac{\sqrt{cx^4+bx^2+a}(bx^2+2\,a)\sqrt{-a}}{2\,(acx^4+abx^2+a^2)}\right)+2\,\sqrt{cx^4+bx^2+a}(2\,Aa^2+(4\,Ba^2-\cdots}{16\,a^3x^4}$$

```
[In] integrate((B*x^2+A)/x^5/(c*x^4+b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/32*((4*B*a*b - 3*A*b^2 + 4*A*a*c)*sqrt(a)*x^4*log(-((b^2 + 4*a*c)*x^4 +
8*a*b*x^2 + 4*sqrt(c*x^4 + b*x^2 + a)*(b*x^2 + 2*a)*sqrt(a) + 8*a^2)/x^4) -
 4*sqrt(c*x^4 + b*x^2 + a)*(2*A*a^2 + (4*B*a^2 - 3*A*a*b)*x^2))/(a^3*x^4),
-1/16*((4*B*a*b - 3*A*b^2 + 4*A*a*c)*sqrt(-a)*x^4*arctan(1/2*sqrt(c*x^4 + b
*x^2 + a)*(b*x^2 + 2*a)*sqrt(-a)/(a*c*x^4 + a*b*x^2 + a^2)) + 2*sqrt(c*x^4
+ b*x^2 + a)*(2*A*a^2 + (4*B*a^2 - 3*A*a*b)*x^2))/(a^3*x^4)]
```

Sympy [F]

$$\int \frac{A+Bx^2}{x^5\sqrt{a+bx^2+cx^4}}\,dx = \int \frac{A+Bx^2}{x^5\sqrt{a+bx^2+cx^4}}\,dx$$

```
[In] integrate((B*x**2+A)/x**5/(c*x**4+b*x**2+a)**(1/2),x)
[Out] Integral((A + B*x**2)/(x**5*sqrt(a + b*x**2 + c*x**4)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{A+Bx^2}{x^5\sqrt{a+bx^2+cx^4}}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((B*x^2+A)/x^5/(c*x^4+b*x^2+a)^(1/2),x, algorithm="maxima")
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 339 vs. $2(106) = 212$.

Time = 0.31 (sec) , antiderivative size = 339, normalized size of antiderivative = 2.73

$$\int \frac{A+Bx^2}{x^5\sqrt{a+bx^2+cx^4}}\,dx = -\frac{(4\,Bab-3\,Ab^2+4\,Aac)\arctan\left(-\frac{\sqrt{c}x^2-\sqrt{cx^4+bx^2+a}}{\sqrt{-a}}\right)}{8\,\sqrt{-a}a^2} + \frac{4\left(\sqrt{c}x^2-\sqrt{cx^4+bx^2+a}\right)^3Bab-3\left(\sqrt{c}x^2-\sqrt{cx^4+bx^2+a}\right)^3Ab^2+4\left(\sqrt{c}x^2-\sqrt{cx^4+bx^2+a}\right)^3Aac}{}$$

```
[In] integrate((B*x^2+A)/x^5/(c*x^4+b*x^2+a)^(1/2),x, algorithm="giac")
[Out] -1/8*(4*B*a*b - 3*A*b^2 + 4*A*a*c)*arctan(-(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^
2 + a))/sqrt(-a))/(sqrt(-a)*a^2) + 1/8*(4*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2
 + a))^3*B*a*b - 3*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))^3*A*b^2 + 4*(sqr
t(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))^3*A*a*c + 8*(sqrt(c)*x^2 - sqrt(c*x^4 +
 b*x^2 + a))^2*B*a^2*sqrt(c) - 4*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))*B*
a^2*b + 5*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))*A*a*b^2 + 4*(sqrt(c)*x^2
- sqrt(c*x^4 + b*x^2 + a))*A*a^2*c - 8*B*a^3*sqrt(c) + 8*A*a^2*b*sqrt(c))/(
((sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))^2 - a)^2*a^2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx^2}{x^5\sqrt{a+bx^2+cx^4}}\,dx=\int \frac{B\,x^2+A}{x^5\,\sqrt{c\,x^4+b\,x^2+a}}\,dx$$

```
[In] int((A + B*x^2)/(x^5*(a + b*x^2 + c*x^4)^(1/2)),x)
[Out] int((A + B*x^2)/(x^5*(a + b*x^2 + c*x^4)^(1/2)), x)
```

3.175 $\int \frac{A+Bx^2}{x^7\sqrt{a+bx^2+cx^4}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 177

$$\int \frac{A+Bx^2}{x^7\sqrt{a+bx^2+cx^4}}\,dx = -\frac{A\sqrt{a+bx^2+cx^4}}{6ax^6}+\frac{(5Ab-6aB)\sqrt{a+bx^2+cx^4}}{24a^2x^4} - \frac{(15Ab^2-18abB-16aAc)\sqrt{a+bx^2+cx^4}}{48a^3x^2} + \frac{(5Ab^3-6ab^2B-12aAbc+8a^2Bc)\operatorname{arctanh}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{32a^{7/2}}$$

```
[Out] 1/32*(-12*A*a*b*c+5*A*b^3+8*B*a^2*c-6*B*a*b^2)*arctanh(1/2*(b*x^2+2*a)/a^(1
/2)/(c*x^4+b*x^2+a)^(1/2))/a^(7/2)-1/6*A*(c*x^4+b*x^2+a)^(1/2)/a/x^6+1/24*(
5*A*b-6*B*a)*(c*x^4+b*x^2+a)^(1/2)/a^2/x^4-1/48*(-16*A*a*c+15*A*b^2-18*B*a*
b)*(c*x^4+b*x^2+a)^(1/2)/a^3/x^2
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 177, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {1265, 848, 820, 738, 212}

$$\int \frac{A+Bx^2}{x^7\sqrt{a+bx^2+cx^4}}\,dx = -\frac{\sqrt{a+bx^2+cx^4}(-16aAc-18abB+15Ab^2)}{48a^3x^2} + \frac{(5Ab-6aB)\sqrt{a+bx^2+cx^4}}{24a^2x^4} + \frac{(8a^2Bc-12aAbc-6ab^2B+5Ab^3)\operatorname{arctanh}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{32a^{7/2}} - \frac{A\sqrt{a+bx^2+cx^4}}{6ax^6}$$

```
[In] Int[(A + B*x^2)/(x^7*Sqrt[a + b*x^2 + c*x^4]),x]
```

```
[Out] -1/6*(A*Sqrt[a + b*x^2 + c*x^4])/(a*x^6) + ((5*A*b - 6*a*B)*Sqrt[a + b*x^2
+ c*x^4])/(24*a^2*x^4) - ((15*A*b^2 - 18*a*b*B - 16*a*A*c)*Sqrt[a + b*x^2 +
 c*x^4])/(48*a^3*x^2) + ((5*A*b^3 - 6*a*b^2*B - 12*a*A*b*c + 8*a^2*B*c)*Arc
Tanh[(2*a + b*x^2)/(2*Sqrt[a]*Sqrt[a + b*x^2 + c*x^4])])/(32*a^(7/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 820

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a +
 b*x + c*x^2)^(p + 1)/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2))), x] - Dist[(b*(e
*f + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(
m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[Simplify[m
+ 2*p + 3], 0]
```

Rule 848

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*f - d*g)*(d + e*x)^(m + 1)*((a + b*
x + c*x^2)^(p + 1)/((m + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Dist[1/((m + 1)
*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*Simp[(
c*d*f - f*b*e + a*e*g)*(m + 1) + b*(d*g - e*f)*(p + 1) - c*(e*f - d*g)*(m +
 2*p + 3)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] ||
 IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 1265

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rubi steps

$$\text{integral} = \frac{1}{2}\text{Subst}\left(\int \frac{A+Bx}{x^4\sqrt{a+bx+cx^2}}\,dx, x, x^2\right)$$

$$= -\frac{A\sqrt{a+bx^2+cx^4}}{6ax^6} - \frac{\text{Subst}\left(\int \frac{\frac{1}{2}(5Ab-6aB)+2Acx}{x^3\sqrt{a+bx+cx^2}}\,dx, x, x^2\right)}{6a}$$

$$= -\frac{A\sqrt{a+bx^2+cx^4}}{6ax^6} + \frac{(5Ab-6aB)\sqrt{a+bx^2+cx^4}}{24a^2x^4} + \frac{\text{Subst}\left(\int \frac{\frac{1}{4}(15Ab^2-18abB-16aAc)+\frac{1}{2}(5Ab-6aB)cx}{x^2\sqrt{a+bx+cx^2}}\,dx, x, x^2\right)}{12a^2}$$

$$= -\frac{A\sqrt{a+bx^2+cx^4}}{6ax^6} + \frac{(5Ab-6aB)\sqrt{a+bx^2+cx^4}}{24a^2x^4} - \frac{(15Ab^2-18abB-16aAc)\sqrt{a+bx^2+cx^4}}{48a^3x^2} - \frac{(5Ab^3-6ab^2B-12aAbc+8a^2Bc)\,\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx+cx^2}}\,dx, x, x^2\right)}{32a^3}$$

$$= -\frac{A\sqrt{a+bx^2+cx^4}}{6ax^6} + \frac{(5Ab-6aB)\sqrt{a+bx^2+cx^4}}{24a^2x^4} - \frac{(15Ab^2-18abB-16aAc)\sqrt{a+bx^2+cx^4}}{48a^3x^2} + \frac{(5Ab^3-6ab^2B-12aAbc+8a^2Bc)\,\text{Subst}\left(\int \frac{1}{4a-x^2}\,dx, x, \frac{2a+bx^2}{\sqrt{a+bx^2+cx^4}}\right)}{16a^3}$$

$$= -\frac{A\sqrt{a+bx^2+cx^4}}{6ax^6} + \frac{(5Ab-6aB)\sqrt{a+bx^2+cx^4}}{24a^2x^4} - \frac{(15Ab^2-18abB-16aAc)\sqrt{a+bx^2+cx^4}}{48a^3x^2} + \frac{(5Ab^3-6ab^2B-12aAbc+8a^2Bc)\tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{32a^{7/2}}$$

Mathematica [A] (verified)

Time = 0.81 (sec) , antiderivative size = 186, normalized size of antiderivative = 1.05

$$\int \frac{A+Bx^2}{x^7\sqrt{a+bx^2+cx^4}}\,dx = \frac{\sqrt{a+bx^2+cx^4}(-8a^2A+10aAbx^2-12a^2Bx^2-15Ab^2x^4+18abBx^4+16aAcx^4)}{48a^3x^6} + \frac{(-5Ab^3-8a^2Bc)\operatorname{arctanh}\left(\frac{\sqrt{c}x^2-\sqrt{a+bx^2+cx^4}}{\sqrt{a}}\right)}{16a^{7/2}} - \frac{3b(bB+2Ac)\operatorname{arctanh}\left(\frac{-\sqrt{c}x^2+\sqrt{a+bx^2+cx^4}}{\sqrt{a}}\right)}{8a^{5/2}}$$

```
[In] Integrate[(A + B*x^2)/(x^7*Sqrt[a + b*x^2 + c*x^4]),x]
```

```
[Out] (Sqrt[a + b*x^2 + c*x^4]*(-8*a^2*A + 10*a*A*b*x^2 - 12*a^2*B*x^2 - 15*A*b^2
*x^4 + 18*a*b*B*x^4 + 16*a*A*c*x^4))/(48*a^3*x^6) + ((-5*A*b^3 - 8*a^2*B*c)
*ArcTanh[(Sqrt[c]*x^2 - Sqrt[a + b*x^2 + c*x^4])/Sqrt[a]])/(16*a^(7/2)) - (
3*b*(b*B + 2*A*c)*ArcTanh[(-(Sqrt[c]*x^2) + Sqrt[a + b*x^2 + c*x^4])/Sqrt[a
]])/(8*a^(5/2))
```

Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.78

method	result
risch	$-\frac{\sqrt{c\,x^4+b\,x^2+a}\,(-16Aac\,x^4+15A\,b^2x^4-18Bab\,x^4-10Aab\,x^2+12B\,a^2x^2+8A\,a^2)}{48a^3x^6} - \frac{(12Aabc-5A\,b^3-8a^2Bc+6Ba\,b^2)\ln \ldots}{32a^{\frac{7}{2}} \ldots}$
pseudoelliptic	$-\frac{\frac{9\left(\left(abc-\frac{5}{12}b^3\right)A-\frac{2\left(ac-\frac{3b^2}{4}\right)aB}{3}\right)x^6\ln\left(\frac{2a+b\,x^2+2\sqrt{a}\,\sqrt{c\,x^4+b\,x^2+a}}{x^2}\right)}{4}+(((-2Ac-\frac{9Bb}{4}) x^4-\frac{5Ab\,x^2}{4}) a^{\frac{3}{2}}+(\frac{3B\,x^2}{2}+A) a^{\frac{5}{2}} \ldots}{6a^{\frac{7}{2}}x^6}$
default	$A(-\frac{\sqrt{c\,x^4+b\,x^2+a}}{6a\,x^6}+\frac{5b\sqrt{c\,x^4+b\,x^2+a}}{24a^2x^4}-\frac{5b^2\sqrt{c\,x^4+b\,x^2+a}}{16a^3x^2}+\frac{5b^3\ln\left(\frac{2a+b\,x^2+2\sqrt{a}\,\sqrt{c\,x^4+b\,x^2+a}}{x^2}\right)}{32a^{\frac{7}{2}}}-\frac{3bc\ln(\frac{2a+ \ldots}{\ldots}}{\ldots}$
elliptic	$-\frac{B\sqrt{c\,x^4+b\,x^2+a}}{4a\,x^4}+\frac{3Bb\sqrt{c\,x^4+b\,x^2+a}}{8a^2x^2}-\frac{3B\,b^2\ln\left(\frac{2a+b\,x^2+2\sqrt{a}\,\sqrt{c\,x^4+b\,x^2+a}}{x^2}\right)}{16a^{\frac{5}{2}}}+\frac{Bc\ln(\frac{2a+b\,x^2+2\sqrt{a}\,\sqrt{c\,x^4+b\,x^2+ \ldots}}{x^2} \ldots}{4a^{\frac{3}{2}}}$

```
[In] int((B*x^2+A)/x^7/(c*x^4+b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/48*(c*x^4+b*x^2+a)^(1/2)*(-16*A*a*c*x^4+15*A*b^2*x^4-18*B*a*b*x^4-10*A*a
*b*x^2+12*B*a^2*x^2+8*A*a^2)/a^3/x^6-1/32/a^(7/2)*(12*A*a*b*c-5*A*b^3-8*B*a
^2*c+6*B*a*b^2)*ln((2*a+b*x^2+2*a^(1/2)*(c*x^4+b*x^2+a)^(1/2))/x^2)
```

Fricas [A] (verification not implemented)

none

Time = 0.41 (sec) , antiderivative size = 339, normalized size of antiderivative = 1.92

$$\int \frac{A+Bx^2}{x^7\sqrt{a+bx^2+cx^4}}\,dx = \left[\frac{3\,(6\,Bab^2-5\,Ab^3-4\,(2\,Ba^2-3\,Aab)c)\sqrt{a}x^6\log\left(-\frac{(b^2+4\,ac)x^4+8\,abx^2-4\,\sqrt{cx^4+bx^2+a}(bx^2+2\,a)\sqrt{a}+8\,a^2}{x^4}\right)+4\,((18}{192\,a^4x^6}\right.$$

[In] integrate((B*x^2+A)/x^7/(c*x^4+b*x^2+a)^(1/2),x, algorithm="fricas")

[Out] [1/192*(3*(6*B*a*b^2 - 5*A*b^3 - 4*(2*B*a^2 - 3*A*a*b)*c)*sqrt(a)*x^6*log(-((b^2 + 4*a*c)*x^4 + 8*a*b*x^2 - 4*sqrt(c*x^4 + b*x^2 + a)*(b*x^2 + 2*a)*sqrt(a) + 8*a^2)/x^4) + 4*((18*B*a^2*b - 15*A*a*b^2 + 16*A*a^2*c)*x^4 - 8*A*a^3 - 2*(6*B*a^3 - 5*A*a^2*b)*x^2)*sqrt(c*x^4 + b*x^2 + a))/(a^4*x^6), 1/96*(3*(6*B*a*b^2 - 5*A*b^3 - 4*(2*B*a^2 - 3*A*a*b)*c)*sqrt(-a)*x^6*arctan(1/2*sqrt(c*x^4 + b*x^2 + a)*(b*x^2 + 2*a)*sqrt(-a)/(a*c*x^4 + a*b*x^2 + a^2)) + 2*((18*B*a^2*b - 15*A*a*b^2 + 16*A*a^2*c)*x^4 - 8*A*a^3 - 2*(6*B*a^3 - 5*A*a^2*b)*x^2)*sqrt(c*x^4 + b*x^2 + a))/(a^4*x^6)]

Sympy [F]

$$\int \frac{A+Bx^2}{x^7\sqrt{a+bx^2+cx^4}}\,dx = \int \frac{A+Bx^2}{x^7\sqrt{a+bx^2+cx^4}}\,dx$$

[In] integrate((B*x**2+A)/x**7/(c*x**4+b*x**2+a)**(1/2),x)

[Out] Integral((A + B*x**2)/(x**7*sqrt(a + b*x**2 + c*x**4)), x)

Maxima [F(-2)]

Exception generated.

$$\int \frac{A+Bx^2}{x^7\sqrt{a+bx^2+cx^4}}\,dx = \text{Exception raised: ValueError}$$

[In] integrate((B*x^2+A)/x^7/(c*x^4+b*x^2+a)^(1/2),x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for more deta

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 571 vs. $2(155) = 310$.

Time = 0.31 (sec) , antiderivative size = 571, normalized size of antiderivative = 3.23

$$\int \frac{A + Bx^2}{x^7\sqrt{a + bx^2 + cx^4}}\, dx = \frac{(6\, Bab^2 - 5\, Ab^3 - 8\, Ba^2c + 12\, Aabc) \arctan\left(-\frac{\sqrt{c}x^2 - \sqrt{cx^4 + bx^2 + a}}{\sqrt{-a}}\right)}{16\,\sqrt{-a}a^3}$$
$$- \frac{18\left(\sqrt{c}x^2 - \sqrt{cx^4 + bx^2 + a}\right)^5 Bab^2 - 15\left(\sqrt{c}x^2 - \sqrt{cx^4 + bx^2 + a}\right)^5 Ab^3 - 24\left(\sqrt{c}x^2 - \sqrt{cx^4 + bx^2 + a}\right.}{}$$

```
[In] integrate((B*x^2+A)/x^7/(c*x^4+b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] 1/16*(6*B*a*b^2 - 5*A*b^3 - 8*B*a^2*c + 12*A*a*b*c)*arctan(-(sqrt(c)*x^2 -
sqrt(c*x^4 + b*x^2 + a))/sqrt(-a))/(sqrt(-a)*a^3) - 1/48*(18*(sqrt(c)*x^2 -
 sqrt(c*x^4 + b*x^2 + a))^5*B*a*b^2 - 15*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2
+ a))^5*A*b^3 - 24*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))^5*B*a^2*c + 36*(
sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))^5*A*a*b*c - 48*(sqrt(c)*x^2 - sqrt(c
*x^4 + b*x^2 + a))^3*B*a^2*b^2 + 40*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))
^3*A*a*b^3 - 96*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))^3*A*a^2*b*c - 48*(s
qrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))^2*B*a^3*b*sqrt(c) - 96*(sqrt(c)*x^2 -
 sqrt(c*x^4 + b*x^2 + a))^2*A*a^3*c^(3/2) + 30*(sqrt(c)*x^2 - sqrt(c*x^4 +
b*x^2 + a))*B*a^3*b^2 - 33*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))*A*a^2*b^
3 + 24*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))*B*a^4*c - 36*(sqrt(c)*x^2 -
sqrt(c*x^4 + b*x^2 + a))*A*a^3*b*c + 48*B*a^4*b*sqrt(c) - 48*A*a^3*b^2*sqrt
(c) + 32*A*a^4*c^(3/2))/(((sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))^2 - a)^3*
a^3)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A + Bx^2}{x^7\sqrt{a + bx^2 + cx^4}}\, dx = \int \frac{B\, x^2 + A}{x^7\,\sqrt{c\, x^4 + b\, x^2 + a}}\, dx$$

```
[In] int((A + B*x^2)/(x^7*(a + b*x^2 + c*x^4)^(1/2)),x)
```

```
[Out] int((A + B*x^2)/(x^7*(a + b*x^2 + c*x^4)^(1/2)), x)
```

3.176 $\int \frac{x^4(A+Bx^2)}{\sqrt{a+bx^2+cx^4}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 403

$$\begin{aligned}\int \frac{x^4(A+Bx^2)}{\sqrt{a+bx^2+cx^4}}\,dx &= -\frac{(4bB-5Ac)x\sqrt{a+bx^2+cx^4}}{15c^2} \\ &+ \frac{Bx^3\sqrt{a+bx^2+cx^4}}{5c} + \frac{(8b^2B-10Abc-9aBc)\,x\sqrt{a+bx^2+cx^4}}{15c^{5/2}\left(\sqrt{a}+\sqrt{c}x^2\right)} \\ &- \frac{\sqrt[4]{a}(8b^2B-10Abc-9aBc)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{15c^{11/4}\sqrt{a+bx^2+cx^4}} \\ &+ \frac{\sqrt[4]{a}\left(8b^2B-10Abc-9aBc+\sqrt{a}\sqrt{c}(4bB-5Ac)\right)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right.\right.}{30c^{11/4}\sqrt{a+bx^2+cx^4}}\end{aligned}$$

```
[Out] -1/15*(-5*A*c+4*B*b)*x*(c*x^4+b*x^2+a)^(1/2)/c^2+1/5*B*x^3*(c*x^4+b*x^2+a)^
(1/2)/c+1/15*(-10*A*b*c-9*B*a*c+8*B*b^2)*x*(c*x^4+b*x^2+a)^(1/2)/c^(5/2)/(a
^(1/2)+x^2*c^(1/2))-1/15*a^(1/4)*(-10*A*b*c-9*B*a*c+8*B*b^2)*(cos(2*arctan(
c^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*x/a^(1/4)))*EllipticE(sin
(2*arctan(c^(1/4)*x/a^(1/4))),1/2*(2-b/a^(1/2)/c^(1/2))^(1/2))*(a^(1/2)+x^2
*c^(1/2))*((c*x^4+b*x^2+a)/(a^(1/2)+x^2*c^(1/2))^2)^(1/2)/c^(11/4)/(c*x^4+b
*x^2+a)^(1/2)+1/30*a^(1/4)*(cos(2*arctan(c^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2
*arctan(c^(1/4)*x/a^(1/4)))*EllipticF(sin(2*arctan(c^(1/4)*x/a^(1/4))),1/2*
(2-b/a^(1/2)/c^(1/2))^(1/2))*(a^(1/2)+x^2*c^(1/2))*(8*B*b^2-10*A*b*c-9*B*a*
c+(-5*A*c+4*B*b)*a^(1/2)*c^(1/2))*((c*x^4+b*x^2+a)/(a^(1/2)+x^2*c^(1/2))^2)
^(1/2)/c^(11/4)/(c*x^4+b*x^2+a)^(1/2)
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 403, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {1293, 1211, 1117, 1209}

$$\int \frac{x^4(A+Bx^2)}{\sqrt{a+bx^2+cx^4}}\,dx$$
$$= \frac{\sqrt[4]{a}(\sqrt{a}+\sqrt{c}x^2)\sqrt{\frac{a+bx^2+cx^4}{(\sqrt{a}+\sqrt{c}x^2)^2}}(\sqrt{a}\sqrt{c}(4bB-5Ac)-9aBc-10Abc+8b^2B)\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\right.}{30c^{11/4}\sqrt{a+bx^2+cx^4}}$$
$$-\frac{\sqrt[4]{a}(\sqrt{a}+\sqrt{c}x^2)\sqrt{\frac{a+bx^2+cx^4}{(\sqrt{a}+\sqrt{c}x^2)^2}}(-9aBc-10Abc+8b^2B)\,E\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{15c^{11/4}\sqrt{a+bx^2+cx^4}}$$
$$+\frac{x\sqrt{a+bx^2+cx^4}(-9aBc-10Abc+8b^2B)}{15c^{5/2}(\sqrt{a}+\sqrt{c}x^2)}$$
$$-\frac{x\sqrt{a+bx^2+cx^4}(4bB-5Ac)}{15c^2}+\frac{Bx^3\sqrt{a+bx^2+cx^4}}{5c}$$

```
[In] Int[(x^4*(A + B*x^2))/Sqrt[a + b*x^2 + c*x^4],x]
```

```
[Out] -1/15*((4*b*B - 5*A*c)*x*Sqrt[a + b*x^2 + c*x^4])/c^2 + (B*x^3*Sqrt[a + b*x
^2 + c*x^4])/(5*c) + ((8*b^2*B - 10*A*b*c - 9*a*B*c)*x*Sqrt[a + b*x^2 + c*x
^4])/(15*c^(5/2)*(Sqrt[a] + Sqrt[c]*x^2)) - (a^(1/4)*(8*b^2*B - 10*A*b*c -
9*a*B*c)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c
]*x^2)^2]*EllipticE[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c])
)/4])/(15*c^(11/4)*Sqrt[a + b*x^2 + c*x^4]) + (a^(1/4)*(8*b^2*B - 10*A*b*c
- 9*a*B*c + Sqrt[a]*Sqrt[c]*(4*b*B - 5*A*c))*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(
a + b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)*x
)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(30*c^(11/4)*Sqrt[a + b*x^2 + c*x
^4])
```

Rule 1117

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1209

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
```

```
- 4*a*c, 0] && PosQ[c/a]
```

Rule 1211

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + b*x^2 + c*x^4
], x], x] - Dist[e/q, Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /; Ne
Q[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[
c/a]
```

Rule 1293

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[e*f*(f*x)^(m - 1)*((a + b*x^2 + c*x^4)^(p +
1)/(c*(m + 4*p + 3))), x] - Dist[f^2/(c*(m + 4*p + 3)), Int[(f*x)^(m - 2)*(
a + b*x^2 + c*x^4)^p*Simp[a*e*(m - 1) + (b*e*(m + 2*p + 1) - c*d*(m + 4*p +
 3))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c,
 0] && GtQ[m, 1] && NeQ[m + 4*p + 3, 0] && IntegerQ[2*p] && (IntegerQ[p] ||
 IntegerQ[m])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{Bx^3\sqrt{a+bx^2+cx^4}}{5c} - \frac{\int \frac{x^2(3aB+(4bB-5Ac)x^2)}{\sqrt{a+bx^2+cx^4}}\,dx}{5c} \\
&= -\frac{(4bB-5Ac)x\sqrt{a+bx^2+cx^4}}{15c^2} + \frac{Bx^3\sqrt{a+bx^2+cx^4}}{5c} + \frac{\int \frac{a(4bB-5Ac)+(8b^2B-10Abc-9aBc)x^2}{\sqrt{a+bx^2+cx^4}}\,dx}{15c^2} \\
&= -\frac{(4bB-5Ac)x\sqrt{a+bx^2+cx^4}}{15c^2} + \frac{Bx^3\sqrt{a+bx^2+cx^4}}{5c} \\
&\quad - \frac{\left(\sqrt{a}(8b^2B-10Abc-9aBc)\right)\int \frac{1-\frac{\sqrt{c}x^2}{\sqrt{a}}}{\sqrt{a+bx^2+cx^4}}\,dx}{15c^{5/2}} \\
&\quad + \frac{\left(\sqrt{a}\left(8b^2B-10Abc-9aBc+\sqrt{a}\sqrt{c}(4bB-5Ac)\right)\right)\int \frac{1}{\sqrt{a+bx^2+cx^4}}\,dx}{15c^{5/2}} \\
&= -\frac{(4bB-5Ac)x\sqrt{a+bx^2+cx^4}}{15c^2} + \frac{Bx^3\sqrt{a+bx^2+cx^4}}{5c} \\
&\quad + \frac{(8b^2B-10Abc-9aBc)\,x\sqrt{a+bx^2+cx^4}}{15c^{5/2}\left(\sqrt{a}+\sqrt{c}x^2\right)} \\
&\quad - \frac{\sqrt[4]{a}(8b^2B-10Abc-9aBc)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{15c^{11/4}\sqrt{a+bx^2+cx^4}} \\
&\quad + \frac{\sqrt[4]{a}\left(8b^2B-10Abc-9aBc+\sqrt{a}\sqrt{c}(4bB-5Ac)\right)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right.\right.}{30c^{11/4}\sqrt{a+bx^2+cx^4}}
\end{aligned}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 11.51 (sec) , antiderivative size = 532, normalized size of antiderivative = 1.32

$$\int \frac{x^4(A+Bx^2)}{\sqrt{a+bx^2+cx^4}}\,dx$$
$$= \frac{4c\sqrt{\frac{c}{b+\sqrt{b^2-4ac}}}x(-4bB+5Ac+3Bcx^2)\left(a+bx^2+cx^4\right)+i(8b^2B-10Abc-9aBc)\left(-b+\sqrt{b^2-4ac}\right)\sqrt{}}{}$$

```
[In] Integrate[(x^4*(A + B*x^2))/Sqrt[a + b*x^2 + c*x^4],x]
```

```
[Out] (4*c*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x*(-4*b*B + 5*A*c + 3*B*c*x^2)*(a + b*
x^2 + c*x^4) + I*(8*b^2*B - 10*A*b*c - 9*a*B*c)*(-b + Sqrt[b^2 - 4*a*c])*Sq
rt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*Sqrt[(2*b - 2
*Sqrt[b^2 - 4*a*c] + 4*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*EllipticE[I*ArcSinh[
Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x], (b + Sqrt[b^2 - 4*a*c])/(b - Sq
rt[b^2 - 4*a*c])] - I*(-8*b^3*B + b*c*(17*a*B - 10*A*Sqrt[b^2 - 4*a*c]) + 2
*b^2*(5*A*c + 4*B*Sqrt[b^2 - 4*a*c]) - a*c*(10*A*c + 9*B*Sqrt[b^2 - 4*a*c])
)*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*Sqrt[(2*b
 - 2*Sqrt[b^2 - 4*a*c] + 4*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*EllipticF[I*ArcS
inh[Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x], (b + Sqrt[b^2 - 4*a*c])/(b
- Sqrt[b^2 - 4*a*c])])/(60*c^3*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*Sqrt[a + b*x
^2 + c*x^4])
```

Maple [A] (verified)

Time = 5.78 (sec) , antiderivative size = 448, normalized size of antiderivative = 1.11

method	result
elliptic	$\frac{B\,x^{3} \sqrt{c\,x^{4}+b\,x^{2}+a}}{5c}+\frac{\left(A-\frac{4Bb}{5c}\right) x \sqrt{c\,x^{4}+b\,x^{2}+a}}{3c}-\frac{a\left(A-\frac{4Bb}{5c}\right) \sqrt{2}\, \sqrt{4-\frac{2\left(-b+\sqrt{-4ac+b^{2}}\right) x^{2}}{a}}\, \sqrt{4+\frac{2\left(b+\sqrt{-4ac+b^{2}}\right) x^{2}}{a}}\, F\left(\frac{x\sqrt{2}\,\sqrt{-}}{}\right.}{12c\sqrt{\frac{-b+\sqrt{-4ac+b^{2}}}{a}}\, \sqrt{c\,x^{4}+b\,x^{2}+a}}$
risch	$\frac{x\left(3B\,x^{2}c+5Ac-4Bb\right) \sqrt{c\,x^{4}+b\,x^{2}+a}}{15c^{2}}-\frac{\frac{5Aac\sqrt{2}\, \sqrt{4-\frac{2\left(-b+\sqrt{-4ac+b^{2}}\right) x^{2}}{a}}\, \sqrt{4+\frac{2\left(b+\sqrt{-4ac+b^{2}}\right) x^{2}}{a}}\, F\left(\frac{x\sqrt{2}\, \sqrt{\frac{-b+\sqrt{-4ac+b^{2}}}{a}}}{2}, \frac{\sqrt{-4+\frac{2b(}{}}}{}\right.}{4\sqrt{\frac{-b+\sqrt{-4ac+b^{2}}}{a}}\, \sqrt{c\,x^{4}+b\,x^{2}+a}}}{}$
default	$B\left(\frac{x^{3} \sqrt{c\,x^{4}+b\,x^{2}+a}}{5c}-\frac{4bx\sqrt{c\,x^{4}+b\,x^{2}+a}}{15c^{2}}+\frac{ab\sqrt{2}\, \sqrt{4-\frac{2\left(-b+\sqrt{-4ac+b^{2}}\right) x^{2}}{a}}\, \sqrt{4+\frac{2\left(b+\sqrt{-4ac+b^{2}}\right) x^{2}}{a}}\, F\left(\frac{x\sqrt{2}\, \sqrt{\frac{-b+\sqrt{-4ac+b^{2}}}{a}}}{2}, \right.}{15c^{2} \sqrt{\frac{-b+\sqrt{-4ac+b^{2}}}{a}}\, \sqrt{c\,x^{4}+b\,x^{2}+a}}\right.$

```
[In] int(x^4*(B*x^2+A)/(c*x^4+b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/5*B*x^3*(c*x^4+b*x^2+a)^(1/2)/c+1/3*(A-4/5*B*b/c)/c*x*(c*x^4+b*x^2+a)^(1/
2)-1/12*a/c*(A-4/5*B*b/c)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*(4-2*(-
b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)*(4+2*(b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)
/(c*x^4+b*x^2+a)^(1/2)*EllipticF(1/2*x*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^
(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2))-1/2*(-3/5*a/c*B-2/3*b/
c*(A-4/5*B*b/c))*a*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*(4-2*(-b+(-4*a
*c+b^2)^(1/2))/a*x^2)^(1/2)*(4+2*(b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)/(c*x^4
+b*x^2+a)^(1/2)/(b+(-4*a*c+b^2)^(1/2))*(EllipticF(1/2*x*2^(1/2)*((-b+(-4*a*
c+b^2)^(1/2))/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2))-Ellip
ticE(1/2*x*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c
+b^2)^(1/2))/a/c)^(1/2)))
```

Fricas [A] (verification not implemented)

none

Time = 0.09 (sec) , antiderivative size = 431, normalized size of antiderivative = 1.07

$$\int \frac{x^4(A+Bx^2)}{\sqrt{a+bx^2+cx^4}}\,dx$$

$$= \frac{\sqrt{\frac{1}{2}}\left(\left(8\,Bb^2c-(9\,Ba+10\,Ab)c^2\right)x\sqrt{\frac{b^2-4\,ac}{c^2}}-\left(8\,Bb^3-(9\,Bab+10\,Ab^2)c\right)x\right)\sqrt{c}\sqrt{\frac{c\sqrt{\frac{b^2-4\,ac}{c^2}}-b}{c}}E\left(\arcsin\left(\right.\right.}{}$$

```
[In] integrate(x^4*(B*x^2+A)/(c*x^4+b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/30*(sqrt(1/2)*((8*B*b^2*c - (9*B*a + 10*A*b)*c^2)*x*sqrt((b^2 - 4*a*c)/c^
2) - (8*B*b^3 - (9*B*a*b + 10*A*b^2)*c)*x)*sqrt(c)*sqrt((c*sqrt((b^2 - 4*a*
c)/c^2) - b)/c)*elliptic_e(arcsin(sqrt(1/2)*sqrt((c*sqrt((b^2 - 4*a*c)/c^2)
 - b)/c)/x), 1/2*(b*c*sqrt((b^2 - 4*a*c)/c^2) + b^2 - 2*a*c)/(a*c)) - sqrt(
1/2)*((8*B*b^2*c + 5*A*c^3 - (9*B*a + 2*(5*A + 2*B)*b)*c^2)*x*sqrt((b^2 - 4
*a*c)/c^2) - (8*B*b^3 - 5*A*b*c^2 - (9*B*a*b + 2*(5*A - 2*B)*b^2)*c)*x)*sqr
t(c)*sqrt((c*sqrt((b^2 - 4*a*c)/c^2) - b)/c)*elliptic_f(arcsin(sqrt(1/2)*sq
rt((c*sqrt((b^2 - 4*a*c)/c^2) - b)/c)/x), 1/2*(b*c*sqrt((b^2 - 4*a*c)/c^2)
+ b^2 - 2*a*c)/(a*c)) + 2*(3*B*c^3*x^4 + 8*B*b^2*c - (9*B*a + 10*A*b)*c^2 -
 (4*B*b*c^2 - 5*A*c^3)*x^2)*sqrt(c*x^4 + b*x^2 + a))/(c^4*x)
```

Sympy [F]

$$\int \frac{x^4(A+Bx^2)}{\sqrt{a+bx^2+cx^4}}\,dx = \int \frac{x^4(A+Bx^2)}{\sqrt{a+bx^2+cx^4}}\,dx$$

```
[In] integrate(x**4*(B*x**2+A)/(c*x**4+b*x**2+a)**(1/2),x)
[Out] Integral(x**4*(A + B*x**2)/sqrt(a + b*x**2 + c*x**4), x)
```

Maxima [F]

$$\int \frac{x^4(A+Bx^2)}{\sqrt{a+bx^2+cx^4}}\,dx = \int \frac{(Bx^2+A)x^4}{\sqrt{cx^4+bx^2+a}}\,dx$$

```
[In] integrate(x^4*(B*x^2+A)/(c*x^4+b*x^2+a)^(1/2),x, algorithm="maxima")
[Out] integrate((B*x^2 + A)*x^4/sqrt(c*x^4 + b*x^2 + a), x)
```

Giac [F]

$$\int \frac{x^4(A+Bx^2)}{\sqrt{a+bx^2+cx^4}}\,dx = \int \frac{(Bx^2+A)x^4}{\sqrt{cx^4+bx^2+a}}\,dx$$

```
[In] integrate(x^4*(B*x^2+A)/(c*x^4+b*x^2+a)^(1/2),x, algorithm="giac")
[Out] integrate((B*x^2 + A)*x^4/sqrt(c*x^4 + b*x^2 + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^4(A+Bx^2)}{\sqrt{a+bx^2+cx^4}}\,dx=\int \frac{x^4\,\left(B\,x^2+A\right)}{\sqrt{c\,x^4+b\,x^2+a}}\,dx$$

```
[In] int((x^4*(A + B*x^2))/(a + b*x^2 + c*x^4)^(1/2),x)
[Out] int((x^4*(A + B*x^2))/(a + b*x^2 + c*x^4)^(1/2), x)
```

3.177 $\int \frac{x^2(A+Bx^2)}{\sqrt{a+bx^2+cx^4}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 336

$$\int \frac{x^2(A+Bx^2)}{\sqrt{a+bx^2+cx^4}}\,dx = \frac{Bx\sqrt{a+bx^2+cx^4}}{3c} - \frac{(2bB-3Ac)x\sqrt{a+bx^2+cx^4}}{3c^{3/2}\left(\sqrt{a}+\sqrt{c}x^2\right)} + \frac{\sqrt[4]{a}(2bB-3Ac)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{3c^{7/4}\sqrt{a+bx^2+cx^4}} - \frac{\sqrt[4]{a}\left(2bB+\sqrt{a}B\sqrt{c}-3Ac\right)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{6c^{7/4}\sqrt{a+bx^2+cx^4}}$$

```
[Out] 1/3*B*x*(c*x^4+b*x^2+a)^(1/2)/c-1/3*(-3*A*c+2*B*b)*x*(c*x^4+b*x^2+a)^(1/2)/
c^(3/2)/(a^(1/2)+x^2*c^(1/2))+1/3*a^(1/4)*(-3*A*c+2*B*b)*(cos(2*arctan(c^(1
/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*x/a^(1/4)))*EllipticE(sin(2*a
rctan(c^(1/4)*x/a^(1/4))),1/2*(2-b/a^(1/2)/c^(1/2))^(1/2))*(a^(1/2)+x^2*c^(
1/2))*((c*x^4+b*x^2+a)/(a^(1/2)+x^2*c^(1/2))^2)^(1/2)/c^(7/4)/(c*x^4+b*x^2+
a)^(1/2)-1/6*a^(1/4)*(cos(2*arctan(c^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arcta
n(c^(1/4)*x/a^(1/4)))*EllipticF(sin(2*arctan(c^(1/4)*x/a^(1/4))),1/2*(2-b/a
^(1/2)/c^(1/2))^(1/2))*(a^(1/2)+x^2*c^(1/2))*(2*B*b-3*A*c+B*a^(1/2)*c^(1/2)
)*((c*x^4+b*x^2+a)/(a^(1/2)+x^2*c^(1/2))^2)^(1/2)/c^(7/4)/(c*x^4+b*x^2+a)^(
1/2)
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 336, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {1293, 1211, 1117, 1209}

$$\int \frac{x^2(A+Bx^2)}{\sqrt{a+bx^2+cx^4}}\,dx = -\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\left(\sqrt{a}B\sqrt{c}-3Ac+2bB\right)\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{6c^{7/4}\sqrt{a+bx^2+cx^4}} + \frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}(2bB-3Ac)E\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{3c^{7/4}\sqrt{a+bx^2+cx^4}} - \frac{x\sqrt{a+bx^2+cx^4}(2bB-3Ac)}{3c^{3/2}\left(\sqrt{a}+\sqrt{c}x^2\right)} + \frac{Bx\sqrt{a+bx^2+cx^4}}{3c}$$

```
[In] Int[(x^2*(A + B*x^2))/Sqrt[a + b*x^2 + c*x^4],x]
```

```
[Out] (B*x*Sqrt[a + b*x^2 + c*x^4])/(3*c) - ((2*b*B - 3*A*c)*x*Sqrt[a + b*x^2 + c
*x^4])/(3*c^(3/2)*(Sqrt[a] + Sqrt[c]*x^2)) + (a^(1/4)*(2*b*B - 3*A*c)*(Sqrt
[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*Elli
pticE[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(3*c^(7/
4)*Sqrt[a + b*x^2 + c*x^4]) - (a^(1/4)*(2*b*B + Sqrt[a]*B*Sqrt[c] - 3*A*c)*
(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]
*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(6*
c^(7/4)*Sqrt[a + b*x^2 + c*x^4])
```

Rule 1117

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1209

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

Rule 1211

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + b*x^2 + c*x^4
], x], x] - Dist[e/q, Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /; Ne
Q[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[
c/a]
```

Rule 1293

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[e*f*(f*x)^(m - 1)*((a + b*x^2 + c*x^4)^(p +
1)/(c*(m + 4*p + 3))), x] - Dist[f^2/(c*(m + 4*p + 3)), Int[(f*x)^(m - 2)*(
a + b*x^2 + c*x^4)^p*Simp[a*e*(m - 1) + (b*e*(m + 2*p + 1) - c*d*(m + 4*p +
 3))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c,
 0] && GtQ[m, 1] && NeQ[m + 4*p + 3, 0] && IntegerQ[2*p] && (IntegerQ[p] ||
 IntegerQ[m])
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{Bx\sqrt{a+bx^2+cx^4}}{3c} - \frac{\int \frac{aB+(2bB-3Ac)x^2}{\sqrt{a+bx^2+cx^4}}\,dx}{3c} \\
&= \frac{Bx\sqrt{a+bx^2+cx^4}}{3c} + \frac{\left(\sqrt{a}(2bB-3Ac)\right)\int \frac{1-\frac{\sqrt{c}x^2}{\sqrt{a}}}{\sqrt{a+bx^2+cx^4}}\,dx}{3c^{3/2}} \\
&\quad - \frac{\left(\sqrt{a}\left(2bB+\sqrt{a}B\sqrt{c}-3Ac\right)\right)\int \frac{1}{\sqrt{a+bx^2+cx^4}}\,dx}{3c^{3/2}} \\
&= \frac{Bx\sqrt{a+bx^2+cx^4}}{3c} - \frac{(2bB-3Ac)x\sqrt{a+bx^2+cx^4}}{3c^{3/2}\left(\sqrt{a}+\sqrt{c}x^2\right)} \\
&\quad + \frac{\sqrt[4]{a}(2bB-3Ac)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{3c^{7/4}\sqrt{a+bx^2+cx^4}} \\
&\quad - \frac{\sqrt[4]{a}\left(2bB+\sqrt{a}B\sqrt{c}-3Ac\right)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{6c^{7/4}\sqrt{a+bx^2+cx^4}}
\end{aligned}
$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 11.01 (sec) , antiderivative size = 479, normalized size of antiderivative = 1.43

$$\int \frac{x^2(A+Bx^2)}{\sqrt{a+bx^2+cx^4}}\,dx$$
$$= \frac{4Bc\sqrt{\frac{c}{b+\sqrt{b^2-4ac}}}x(a+bx^2+cx^4)-i(2bB-3Ac)\left(-b+\sqrt{b^2-4ac}\right)\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx^2}{b+\sqrt{b^2-4ac}}}\sqrt{\frac{2b-2\sqrt{b^2-4ac}+4cx^2}{b-\sqrt{b^2-4ac}}}E\Big(}{}$$

```
[In] Integrate[(x^2*(A + B*x^2))/Sqrt[a + b*x^2 + c*x^4],x]
```

```
[Out] (4*B*c*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x*(a + b*x^2 + c*x^4) - I*(2*b*B - 3
*A*c)*(-b + Sqrt[b^2 - 4*a*c])*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b +
Sqrt[b^2 - 4*a*c])]*Sqrt[(2*b - 2*Sqrt[b^2 - 4*a*c] + 4*c*x^2)/(b - Sqrt[b^
2 - 4*a*c])]*EllipticE[I*ArcSinh[Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x]
, (b + Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c])] + I*(-2*b^2*B + 3*A*b*c
+ 2*a*B*c + 2*b*B*Sqrt[b^2 - 4*a*c] - 3*A*c*Sqrt[b^2 - 4*a*c])*Sqrt[(b + Sq
rt[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*Sqrt[(2*b - 2*Sqrt[b^2
- 4*a*c] + 4*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*EllipticF[I*ArcSinh[Sqrt[2]*Sq
rt[c/(b + Sqrt[b^2 - 4*a*c])]*x], (b + Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4
*a*c])])/(12*c^2*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*Sqrt[a + b*x^2 + c*x^4])
```

Maple [A] (verified)

Time = 2.40 (sec) , antiderivative size = 395, normalized size of antiderivative = 1.18

method	result
elliptic	$\frac{Bx\sqrt{c\,x^4+b\,x^2+a}}{3c}-\frac{aB\sqrt{2}\,\sqrt{4-\frac{2\left(-b+\sqrt{-4ac+b^2}\right)x^2}{a}}\,\sqrt{4+\frac{2\left(b+\sqrt{-4ac+b^2}\right)x^2}{a}}\,F\left(\frac{x\sqrt{2}\,\sqrt{\frac{-b+\sqrt{-4ac+b^2}}{a}}}{2},\frac{\sqrt{-4+\frac{2b\left(b+\sqrt{-4ac+b^2}\right)}{ac}}}{2}\right)}{12c\sqrt{\frac{-b+\sqrt{-4ac+b^2}}{a}}\,\sqrt{c\,x^4+b\,x^2+a}}$
risch	$\frac{Bx\sqrt{c\,x^4+b\,x^2+a}}{3c}+\frac{-\frac{Ba\sqrt{2}\,\sqrt{4-\frac{2\left(-b+\sqrt{-4ac+b^2}\right)x^2}{a}}\,\sqrt{4+\frac{2\left(b+\sqrt{-4ac+b^2}\right)x^2}{a}}\,F\left(\frac{x\sqrt{2}\,\sqrt{\frac{-b+\sqrt{-4ac+b^2}}{a}}}{2},\frac{\sqrt{-4+\frac{2b\left(b+\sqrt{-4ac+b^2}\right)}{ac}}}{2}\right)}{4\sqrt{\frac{-b+\sqrt{-4ac+b^2}}{a}}\,\sqrt{c\,x^4+b\,x^2+a}}}{}$
default	$B\left(\frac{x\sqrt{c\,x^4+b\,x^2+a}}{3c}-\frac{a\sqrt{2}\,\sqrt{4-\frac{2\left(-b+\sqrt{-4ac+b^2}\right)x^2}{a}}\,\sqrt{4+\frac{2\left(b+\sqrt{-4ac+b^2}\right)x^2}{a}}\,F\left(\frac{x\sqrt{2}\,\sqrt{\frac{-b+\sqrt{-4ac+b^2}}{a}}}{2},\frac{\sqrt{-4+\frac{2b\left(b+\sqrt{-4ac+b^2}\right)}{ac}}}{2}\right.}{12c\sqrt{\frac{-b+\sqrt{-4ac+b^2}}{a}}\,\sqrt{c\,x^4+b\,x^2+a}}\right.$

```
[In] int(x^2*(B*x^2+A)/(c*x^4+b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*B*x*(c*x^4+b*x^2+a)^(1/2)/c-1/12*a/c*B*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))
/a)^(1/2)*(4-2*(-b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)*(4+2*(b+(-4*a*c+b^2)^(1
/2))/a*x^2)^(1/2)/(c*x^4+b*x^2+a)^(1/2)*EllipticF(1/2*x*2^(1/2)*((-b+(-4*a*
c+b^2)^(1/2))/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2))-1/2*(
A-2/3*B*b/c)*a*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*(4-2*(-b+(-4*a*c+b
^2)^(1/2))/a*x^2)^(1/2)*(4+2*(b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)/(c*x^4+b*x
^2+a)^(1/2)/(b+(-4*a*c+b^2)^(1/2))*(EllipticF(1/2*x*2^(1/2)*((-b+(-4*a*c+b^
2)^(1/2))/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2))-EllipticE
(1/2*x*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2
)^(1/2))/a/c)^(1/2)))
```

Fricas [A] (verification not implemented)

none

Time = 0.10 (sec) , antiderivative size = 346, normalized size of antiderivative = 1.03

$$\int \frac{x^2(A+Bx^2)}{\sqrt{a+bx^2+cx^4}}\,dx =$$

$$-\frac{\sqrt{\frac{1}{2}}\left(\left(2\,Bbc-3\,Ac^2\right)x\sqrt{\frac{b^2-4\,ac}{c^2}}-\left(2\,Bb^2-3\,Abc\right)x\right)\sqrt{c}\sqrt{\frac{c\sqrt{\frac{b^2-4\,ac}{c^2}}-b}{c}}E(\arcsin\left(\frac{\sqrt{\frac{1}{2}}\sqrt{\frac{c\sqrt{\frac{b^2-4\,ac}{c^2}}-b}{c}}}{x}\right)\mid\frac{bc\sqrt{}}{}}{}$$

```
[In] integrate(x^2*(B*x^2+A)/(c*x^4+b*x^2+a)^(1/2),x, algorithm="fricas")
```

[Out] -1/6*(sqrt(1/2)*((2*B*b*c - 3*A*c^2)*x*sqrt((b^2 - 4*a*c)/c^2) - (2*B*b^2 - 3*A*b*c)*x)*sqrt(c)*sqrt((c*sqrt((b^2 - 4*a*c)/c^2) - b)/c)*elliptic_e(arcsin(sqrt(1/2)*sqrt((c*sqrt((b^2 - 4*a*c)/c^2) - b)/c)/x), 1/2*(b*c*sqrt((b^2 - 4*a*c)/c^2) + b^2 - 2*a*c)/(a*c)) - sqrt(1/2)*((2*B*b*c - (3*A + B)*c^2)*x*sqrt((b^2 - 4*a*c)/c^2) - (2*B*b^2 - (3*A - B)*b*c)*x)*sqrt(c)*sqrt((c*sqrt((b^2 - 4*a*c)/c^2) - b)/c)*elliptic_f(arcsin(sqrt(1/2)*sqrt((c*sqrt((b^2 - 4*a*c)/c^2) - b)/c)/x), 1/2*(b*c*sqrt((b^2 - 4*a*c)/c^2) + b^2 - 2*a*c)/(a*c)) - 2*(B*c^2*x^2 - 2*B*b*c + 3*A*c^2)*sqrt(c*x^4 + b*x^2 + a))/(c^3*x)

Sympy [F]

$$\int \frac{x^2(A+Bx^2)}{\sqrt{a+bx^2+cx^4}}\,dx = \int \frac{x^2(A+Bx^2)}{\sqrt{a+bx^2+cx^4}}\,dx$$

[In] integrate(x**2*(B*x**2+A)/(c*x**4+b*x**2+a)**(1/2),x)

[Out] Integral(x**2*(A + B*x**2)/sqrt(a + b*x**2 + c*x**4), x)

Maxima [F]

$$\int \frac{x^2(A+Bx^2)}{\sqrt{a+bx^2+cx^4}}\,dx = \int \frac{(Bx^2+A)x^2}{\sqrt{cx^4+bx^2+a}}\,dx$$

[In] integrate(x^2*(B*x^2+A)/(c*x^4+b*x^2+a)^(1/2),x, algorithm="maxima")

[Out] integrate((B*x^2 + A)*x^2/sqrt(c*x^4 + b*x^2 + a), x)

Giac [F]

$$\int \frac{x^2(A+Bx^2)}{\sqrt{a+bx^2+cx^4}}\,dx = \int \frac{(Bx^2+A)x^2}{\sqrt{cx^4+bx^2+a}}\,dx$$

[In] integrate(x^2*(B*x^2+A)/(c*x^4+b*x^2+a)^(1/2),x, algorithm="giac")

[Out] integrate((B*x^2 + A)*x^2/sqrt(c*x^4 + b*x^2 + a), x)

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2(A+Bx^2)}{\sqrt{a+bx^2+cx^4}}\,dx=\int \frac{x^2\,\left(B\,x^2+A\right)}{\sqrt{c\,x^4+b\,x^2+a}}\,dx$$

```
[In] int((x^2*(A + B*x^2))/(a + b*x^2 + c*x^4)^(1/2),x)
[Out] int((x^2*(A + B*x^2))/(a + b*x^2 + c*x^4)^(1/2), x)
```

3.178 $\int \frac{A+Bx^2}{\sqrt{a+bx^2+cx^4}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 283

$$
\begin{aligned}
\int \frac{A+Bx^2}{\sqrt{a+bx^2+cx^4}}\,dx &= \frac{Bx\sqrt{a+bx^2+cx^4}}{\sqrt{c}\left(\sqrt{a}+\sqrt{c}x^2\right)} \\
&\quad - \frac{\sqrt[4]{a}B\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{c^{3/4}\sqrt{a+bx^2+cx^4}} \\
&\quad + \frac{\sqrt[4]{a}\left(B+\frac{A\sqrt{c}}{\sqrt{a}}\right)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{2c^{3/4}\sqrt{a+bx^2+cx^4}}
\end{aligned}
$$

```
[Out] B*x*(c*x^4+b*x^2+a)^(1/2)/c^(1/2)/(a^(1/2)+x^2*c^(1/2))-a^(1/4)*B*(cos(2*ar
ctan(c^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*x/a^(1/4)))*Elliptic
E(sin(2*arctan(c^(1/4)*x/a^(1/4))),1/2*(2-b/a^(1/2)/c^(1/2))^(1/2))*(a^(1/2
)+x^2*c^(1/2))*((c*x^4+b*x^2+a)/(a^(1/2)+x^2*c^(1/2))^2)^(1/2)/c^(3/4)/(c*x
^4+b*x^2+a)^(1/2)+1/2*a^(1/4)*(cos(2*arctan(c^(1/4)*x/a^(1/4)))^2)^(1/2)/co
s(2*arctan(c^(1/4)*x/a^(1/4)))*EllipticF(sin(2*arctan(c^(1/4)*x/a^(1/4))),1
/2*(2-b/a^(1/2)/c^(1/2))^(1/2))*(a^(1/2)+x^2*c^(1/2))*(B+A*c^(1/2)/a^(1/2))
*((c*x^4+b*x^2+a)/(a^(1/2)+x^2*c^(1/2))^2)^(1/2)/c^(3/4)/(c*x^4+b*x^2+a)^(1
/2)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 283, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1211, 1117, 1209}

$$\int \frac{A+Bx^2}{\sqrt{a+bx^2+cx^4}}\,dx$$
$$=\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{c}x^2\right)\left(\frac{A\sqrt{c}}{\sqrt{a}}+B\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{2c^{3/4}\sqrt{a+bx^2+cx^4}}$$
$$-\frac{\sqrt[4]{a}B\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{c^{3/4}\sqrt{a+bx^2+cx^4}}$$
$$+\frac{Bx\sqrt{a+bx^2+cx^4}}{\sqrt{c}\left(\sqrt{a}+\sqrt{c}x^2\right)}$$

```
[In] Int[(A + B*x^2)/Sqrt[a + b*x^2 + c*x^4],x]
```

```
[Out] (B*x*Sqrt[a + b*x^2 + c*x^4])/(Sqrt[c]*(Sqrt[a] + Sqrt[c]*x^2)) - (a^(1/4)*
B*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^
2]*EllipticE[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(
c^(3/4)*Sqrt[a + b*x^2 + c*x^4]) + (a^(1/4)*(B + (A*Sqrt[c])/Sqrt[a])*(Sqrt
[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*Elli
pticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(2*c^(3/
4)*Sqrt[a + b*x^2 + c*x^4])
```

Rule 1117

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1209

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1211

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + b*x^2 + c*x^4
```

```
], x], x] - Dist[e/q, Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /; Ne
Q[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[
c/a]
```

Rubi steps

$$\text{integral} = \left(A+\frac{\sqrt{a}B}{\sqrt{c}}\right)\int \frac{1}{\sqrt{a+bx^2+cx^4}}\,dx - \frac{\left(\sqrt{a}B\right)\int \frac{1-\frac{\sqrt{c}x^2}{\sqrt{a}}}{\sqrt{a+bx^2+cx^4}}\,dx}{\sqrt{c}}$$

$$= \frac{Bx\sqrt{a+bx^2+cx^4}}{\sqrt{c}\left(\sqrt{a}+\sqrt{c}x^2\right)} - \frac{\sqrt[4]{a}B\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\middle|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{c^{3/4}\sqrt{a+bx^2+cx^4}}$$
$$+ \frac{\left(\sqrt{a}B+A\sqrt{c}\right)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\middle|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{2\sqrt[4]{a}c^{3/4}\sqrt{a+bx^2+cx^4}}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.18 (sec) , antiderivative size = 302, normalized size of antiderivative = 1.07

$$\int \frac{A+Bx^2}{\sqrt{a+bx^2+cx^4}}\,dx$$
$$= \frac{i\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx^2}{b+\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\left(B\left(-b+\sqrt{b^2-4ac}\right)E\left(i\text{arcsinh}\left(\sqrt{2}\sqrt{\frac{c}{b+\sqrt{b^2-4ac}}}x\right)\middle|\frac{b+\sqrt{b^2-4ac}}{b-\sqrt{b^2-4ac}}\right)+(bB\right.}{2\sqrt{2}c\sqrt{\frac{c}{b+\sqrt{b^2-4ac}}}\sqrt{a+bx^2+cx^4}}$$

```
[In] Integrate[(A + B*x^2)/Sqrt[a + b*x^2 + c*x^4],x]
```

```
[Out] ((I/2)*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*Sqrt
[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*(B*(-b + Sqrt[b^2 - 4*a*c])*Ellipti
cE[I*ArcSinh[Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x], (b + Sqrt[b^2 - 4*
a*c])/(b - Sqrt[b^2 - 4*a*c])] + (b*B - 2*A*c - B*Sqrt[b^2 - 4*a*c])*Ellipt
icF[I*ArcSinh[Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x], (b + Sqrt[b^2 - 4
*a*c])/(b - Sqrt[b^2 - 4*a*c])]))/(Sqrt[2]*c*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])
]*Sqrt[a + b*x^2 + c*x^4])
```

Maple [A] (verified)

Time = 0.99 (sec) , antiderivative size = 362, normalized size of antiderivative = 1.28

method	result
default	$\frac{A\sqrt{2}\,\sqrt{4-\frac{2\left(-b+\sqrt{-4ac+b^{2}}\right)x^{2}}{a}}\,\sqrt{4+\frac{2\left(b+\sqrt{-4ac+b^{2}}\right)x^{2}}{a}}\,F\left(\frac{x\sqrt{2}\,\sqrt{\frac{-b+\sqrt{-4ac+b^{2}}}{a}}}{2},\frac{\sqrt{-4+\frac{2b\left(b+\sqrt{-4ac+b^{2}}\right)}{ac}}}{2}\right)}{4\sqrt{\frac{-b+\sqrt{-4ac+b^{2}}}{a}}\,\sqrt{c\,x^{4}+b\,x^{2}+a}}-\frac{Ba\sqrt{2}\,\sqrt{4-\frac{2(-b+}{}}}{}$
elliptic	$\frac{A\sqrt{2}\,\sqrt{4-\frac{2\left(-b+\sqrt{-4ac+b^{2}}\right)x^{2}}{a}}\,\sqrt{4+\frac{2\left(b+\sqrt{-4ac+b^{2}}\right)x^{2}}{a}}\,F\left(\frac{x\sqrt{2}\,\sqrt{\frac{-b+\sqrt{-4ac+b^{2}}}{a}}}{2},\frac{\sqrt{-4+\frac{2b\left(b+\sqrt{-4ac+b^{2}}\right)}{ac}}}{2}\right)}{4\sqrt{\frac{-b+\sqrt{-4ac+b^{2}}}{a}}\,\sqrt{c\,x^{4}+b\,x^{2}+a}}-\frac{Ba\sqrt{2}\,\sqrt{4-\frac{2(-b+}{}}}{}$

```
[In] int((B*x^2+A)/(c*x^4+b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*A*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*(4-2*(-b+(-4*a*c+b^2)^(1/2)
)/a*x^2)^(1/2)*(4+2*(b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)/(c*x^4+b*x^2+a)^(1/
2)*EllipticF(1/2*x*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4+2*b*(b
+(-4*a*c+b^2)^(1/2))/a/c)^(1/2))-1/2*B*a*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))/a
)^(1/2)*(4-2*(-b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)*(4+2*(b+(-4*a*c+b^2)^(1/2
))/a*x^2)^(1/2)/(c*x^4+b*x^2+a)^(1/2)/(b+(-4*a*c+b^2)^(1/2))*(EllipticF(1/2
*x*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1
/2))/a/c)^(1/2))-EllipticE(1/2*x*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),
1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2)))
```

Fricas [A] (verification not implemented)

none

Time = 0.10 (sec) , antiderivative size = 300, normalized size of antiderivative = 1.06

$$\int \frac{A+Bx^2}{\sqrt{a+bx^2+cx^4}}\,dx = \frac{2\sqrt{cx^4+bx^2+a}Bac+\sqrt{\frac{1}{2}}\left(Bacx\sqrt{\frac{b^2-4ac}{c^2}}-Babx\right)\sqrt{c}\sqrt{\frac{c\sqrt{\frac{b^2-4ac}{c^2}}-b}{c}}E(\arcsin\left(\frac{\sqrt{\frac{1}{2}}\sqrt{\frac{c\sqrt{\frac{b^2-4ac}{c^2}}-b}{c}}}{x}\right)\mid\frac{bc\sqrt{b}}{}}{}$$

```
[In] integrate((B*x^2+A)/(c*x^4+b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2*(2*sqrt(c*x^4 + b*x^2 + a)*B*a*c + sqrt(1/2)*(B*a*c*x*sqrt((b^2 - 4*a*c
)/c^2) - B*a*b*x)*sqrt(c)*sqrt((c*sqrt((b^2 - 4*a*c)/c^2) - b)/c)*elliptic_
e(arcsin(sqrt(1/2)*sqrt((c*sqrt((b^2 - 4*a*c)/c^2) - b)/c)/x), 1/2*(b*c*sqr
t((b^2 - 4*a*c)/c^2) + b^2 - 2*a*c)/(a*c)) - sqrt(1/2)*((B*a*c - A*c^2)*x*s
```

```
qrt((b^2 - 4*a*c)/c^2) - (B*a*b + A*b*c)*x)*sqrt(c)*sqrt((c*sqrt((b^2 - 4*a
*c)/c^2) - b)/c)*elliptic_f(arcsin(sqrt(1/2)*sqrt((c*sqrt((b^2 - 4*a*c)/c^2
) - b)/c)/x), 1/2*(b*c*sqrt((b^2 - 4*a*c)/c^2) + b^2 - 2*a*c)/(a*c)))/(a*c^
2*x)
```

Sympy [F]

$$\int \frac{A+Bx^2}{\sqrt{a+bx^2+cx^4}}\,dx=\int \frac{A+Bx^2}{\sqrt{a+bx^2+cx^4}}\,dx$$

```
[In] integrate((B*x**2+A)/(c*x**4+b*x**2+a)**(1/2),x)
[Out] Integral((A + B*x**2)/sqrt(a + b*x**2 + c*x**4), x)
```

Maxima [F]

$$\int \frac{A+Bx^2}{\sqrt{a+bx^2+cx^4}}\,dx=\int \frac{Bx^2+A}{\sqrt{cx^4+bx^2+a}}\,dx$$

```
[In] integrate((B*x^2+A)/(c*x^4+b*x^2+a)^(1/2),x, algorithm="maxima")
[Out] integrate((B*x^2 + A)/sqrt(c*x^4 + b*x^2 + a), x)
```

Giac [F]

$$\int \frac{A+Bx^2}{\sqrt{a+bx^2+cx^4}}\,dx=\int \frac{Bx^2+A}{\sqrt{cx^4+bx^2+a}}\,dx$$

```
[In] integrate((B*x^2+A)/(c*x^4+b*x^2+a)^(1/2),x, algorithm="giac")
[Out] integrate((B*x^2 + A)/sqrt(c*x^4 + b*x^2 + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx^2}{\sqrt{a+bx^2+cx^4}}\,dx=\int \frac{B\,x^2+A}{\sqrt{c\,x^4+b\,x^2+a}}\,dx$$

```
[In] int((A + B*x^2)/(a + b*x^2 + c*x^4)^(1/2),x)
[Out] int((A + B*x^2)/(a + b*x^2 + c*x^4)^(1/2), x)
```

3.179 $\int \frac{A+Bx^2}{x^2\sqrt{a+bx^2+cx^4}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 312

$$\begin{aligned}\int \frac{A+Bx^2}{x^2\sqrt{a+bx^2+cx^4}}\,dx &= -\frac{A\sqrt{a+bx^2+cx^4}}{ax}+\frac{A\sqrt{c}x\sqrt{a+bx^2+cx^4}}{a\left(\sqrt{a}+\sqrt{c}x^2\right)}\\
&\quad-\frac{A\sqrt[4]{c}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{a^{3/4}\sqrt{a+bx^2+cx^4}}\\
&\quad+\frac{\left(\sqrt{a}B+A\sqrt{c}\right)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{2a^{3/4}\sqrt[4]{c}\sqrt{a+bx^2+cx^4}}\end{aligned}$$

```
[Out] -A*(c*x^4+b*x^2+a)^(1/2)/a/x+A*x*c^(1/2)*(c*x^4+b*x^2+a)^(1/2)/a/(a^(1/2)+x
^2*c^(1/2))-A*c^(1/4)*(cos(2*arctan(c^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arct
an(c^(1/4)*x/a^(1/4)))*EllipticE(sin(2*arctan(c^(1/4)*x/a^(1/4))),1/2*(2-b/
a^(1/2)/c^(1/2))^(1/2))*(a^(1/2)+x^2*c^(1/2))*((c*x^4+b*x^2+a)/(a^(1/2)+x^2
*c^(1/2))^2)^(1/2)/a^(3/4)/(c*x^4+b*x^2+a)^(1/2)+1/2*(cos(2*arctan(c^(1/4)*
x/a^(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*x/a^(1/4)))*EllipticF(sin(2*arcta
n(c^(1/4)*x/a^(1/4))),1/2*(2-b/a^(1/2)/c^(1/2))^(1/2))*(B*a^(1/2)+A*c^(1/2)
)*(a^(1/2)+x^2*c^(1/2))*((c*x^4+b*x^2+a)/(a^(1/2)+x^2*c^(1/2))^2)^(1/2)/a^(
3/4)/c^(1/4)/(c*x^4+b*x^2+a)^(1/2)
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 312, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.148$, Rules used = {1295, 1211, 1117, 1209}

$$\int \frac{A+Bx^2}{x^2\sqrt{a+bx^2+cx^4}}\,dx$$
$$=\frac{\left(\sqrt{a}+\sqrt{c}x^2\right)\left(\sqrt{a}B+A\sqrt{c}\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{2a^{3/4}\sqrt[4]{c}\sqrt{a+bx^2+cx^4}}$$
$$-\frac{A\sqrt[4]{c}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{a^{3/4}\sqrt{a+bx^2+cx^4}}$$
$$-\frac{A\sqrt{a+bx^2+cx^4}}{ax}+\frac{A\sqrt{c}x\sqrt{a+bx^2+cx^4}}{a\left(\sqrt{a}+\sqrt{c}x^2\right)}$$

```
[In] Int[(A + B*x^2)/(x^2*Sqrt[a + b*x^2 + c*x^4]),x]
```

```
[Out] -((A*Sqrt[a + b*x^2 + c*x^4])/(a*x)) + (A*Sqrt[c]*x*Sqrt[a + b*x^2 + c*x^4]
)/(a*(Sqrt[a] + Sqrt[c]*x^2)) - (A*c^(1/4)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a
+ b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticE[2*ArcTan[(c^(1/4)*x)/
a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(a^(3/4)*Sqrt[a + b*x^2 + c*x^4]) +
 ((Sqrt[a]*B + A*Sqrt[c])*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/
(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 - b/
(Sqrt[a]*Sqrt[c]))/4])/(2*a^(3/4)*c^(1/4)*Sqrt[a + b*x^2 + c*x^4])
```

Rule 1117

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1209

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

Rule 1211

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + b*x^2 + c*x^4
```

```
], x], x] - Dist[e/q, Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /; Ne
Q[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[
c/a]
```

Rule 1295

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[d*(f*x)^(m + 1)*((a + b*x^2 + c*x^4)^(p + 1)
/(a*f*(m + 1))), x] + Dist[1/(a*f^2*(m + 1)), Int[(f*x)^(m + 2)*(a + b*x^2
+ c*x^4)^p*Simp[a*e*(m + 1) - b*d*(m + 2*p + 3) - c*d*(m + 4*p + 5)*x^2, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[m
, -1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{A\sqrt{a+bx^2+cx^4}}{ax} - \frac{\int \frac{-aB-Acx^2}{\sqrt{a+bx^2+cx^4}}\,dx}{a} \\
&= -\frac{A\sqrt{a+bx^2+cx^4}}{ax} + \left(B+\frac{A\sqrt{c}}{\sqrt{a}}\right)\int \frac{1}{\sqrt{a+bx^2+cx^4}}\,dx - \frac{\left(A\sqrt{c}\right)\int \frac{1-\frac{\sqrt{c}x^2}{\sqrt{a}}}{\sqrt{a+bx^2+cx^4}}\,dx}{\sqrt{a}} \\
&= -\frac{A\sqrt{a+bx^2+cx^4}}{ax} + \frac{A\sqrt{c}x\sqrt{a+bx^2+cx^4}}{a\left(\sqrt{a}+\sqrt{c}x^2\right)} \\
&\quad - \frac{A\sqrt[4]{c}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{a^{3/4}\sqrt{a+bx^2+cx^4}} \\
&\quad + \frac{\left(\sqrt{a}B+A\sqrt{c}\right)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{2a^{3/4}\sqrt[4]{c}\sqrt{a+bx^2+cx^4}}
\end{aligned}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.71 (sec) , antiderivative size = 448, normalized size of antiderivative = 1.44

$$\int \frac{A+Bx^2}{x^2\sqrt{a+bx^2+cx^4}}\,dx = \frac{-4A\sqrt{\frac{c}{b+\sqrt{b^2-4ac}}}(a+bx^2+cx^4)+iA\left(-b+\sqrt{b^2-4ac}\right)x\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx^2}{b+\sqrt{b^2-4ac}}}\sqrt{\frac{2b-2\sqrt{b^2-4ac}+4cx^2}{b-\sqrt{b^2-4ac}}}E\left(i\text{arcsinh}\right.}{}$$

```
[In] Integrate[(A + B*x^2)/(x^2*Sqrt[a + b*x^2 + c*x^4]),x]
```

```
[Out] (-4*A*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*(a + b*x^2 + c*x^4) + I*A*(-b + Sqrt[
b^2 - 4*a*c])*x*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*
c])]*Sqrt[(2*b - 2*Sqrt[b^2 - 4*a*c] + 4*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*El
lipticE[I*ArcSinh[Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x], (b + Sqrt[b^2
 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c])] - I*(2*a*B + A*(-b + Sqrt[b^2 - 4*a*c])
)*x*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*Sqrt[(2
*b - 2*Sqrt[b^2 - 4*a*c] + 4*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*EllipticF[I*Ar
cSinh[Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x], (b + Sqrt[b^2 - 4*a*c])/(
b - Sqrt[b^2 - 4*a*c])])/(4*a*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x*Sqrt[a + b*
x^2 + c*x^4])
```

Maple [A] (verified)

Time = 1.94 (sec) , antiderivative size = 385, normalized size of antiderivative = 1.23

method	result
elliptic	$-\frac{A\sqrt{c\,x^4+b\,x^2+a}}{a x}+\frac{B\sqrt{2}\,\sqrt{4-\frac{2\left(-b+\sqrt{-4ac+b^2}\right)x^2}{a}}\,\sqrt{4+\frac{2\left(b+\sqrt{-4ac+b^2}\right)x^2}{a}}\,F\left(\frac{x\sqrt{2}\,\sqrt{\frac{-b+\sqrt{-4ac+b^2}}{a}}}{2},\frac{\sqrt{-4+\frac{2b\left(b+\sqrt{-4ac+b^2}\right)}{ac}}}{2}\right)}{4\sqrt{\frac{-b+\sqrt{-4ac+b^2}}{a}}\,\sqrt{c\,x^4+b\,x^2+a}}$
default	$\frac{B\sqrt{2}\,\sqrt{4-\frac{2\left(-b+\sqrt{-4ac+b^2}\right)x^2}{a}}\,\sqrt{4+\frac{2\left(b+\sqrt{-4ac+b^2}\right)x^2}{a}}\,F\left(\frac{x\sqrt{2}\,\sqrt{\frac{-b+\sqrt{-4ac+b^2}}{a}}}{2},\frac{\sqrt{-4+\frac{2b\left(b+\sqrt{-4ac+b^2}\right)}{ac}}}{2}\right)}{4\sqrt{\frac{-b+\sqrt{-4ac+b^2}}{a}}\,\sqrt{c\,x^4+b\,x^2+a}}+A\left(-\frac{\sqrt{c\,x^4+b\,x^2+}}{a x}\right.$
risch	$-\frac{A\sqrt{c\,x^4+b\,x^2+a}}{a x}+\frac{\frac{B a\sqrt{2}\,\sqrt{4-\frac{2\left(-b+\sqrt{-4ac+b^2}\right)x^2}{a}}\,\sqrt{4+\frac{2\left(b+\sqrt{-4ac+b^2}\right)x^2}{a}}\,F\left(\frac{x\sqrt{2}\,\sqrt{\frac{-b+\sqrt{-4ac+b^2}}{a}}}{2},\frac{\sqrt{-4+\frac{2b\left(b+\sqrt{-4ac+b^2}\right)}{ac}}}{2}\right)}{4\sqrt{\frac{-b+\sqrt{-4ac+b^2}}{a}}\,\sqrt{c\,x^4+b\,x^2+a}}-}{}$

```
[In] int((B*x^2+A)/x^2/(c*x^4+b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -A*(c*x^4+b*x^2+a)^(1/2)/a/x+1/4*B*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2
)*(4-2*(-b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)*(4+2*(b+(-4*a*c+b^2)^(1/2))/a*x
^2)^(1/2)/(c*x^4+b*x^2+a)^(1/2)*EllipticF(1/2*x*2^(1/2)*((-b+(-4*a*c+b^2)^(
1/2))/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2))-1/2*c*A*2^(1/
2)/((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*(4-2*(-b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1
/2)*(4+2*(b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)/(c*x^4+b*x^2+a)^(1/2)/(b+(-4*a
*c+b^2)^(1/2))*(EllipticF(1/2*x*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1
/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2))-EllipticE(1/2*x*2^(1/2)*((-b+
(-4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2))
)
```

Fricas [A] (verification not implemented)

none

Time = 0.09 (sec) , antiderivative size = 294, normalized size of antiderivative = 0.94

$$\int \frac{A+Bx^2}{x^2\sqrt{a+bx^2+cx^4}}\,dx =$$

$$-\frac{2\sqrt{cx^4+bx^2+a}Aac+\sqrt{\frac{1}{2}}\left(Aacx\sqrt{\frac{b^2-4\,ac}{a^2}}-Abcx\right)\sqrt{a}\sqrt{\frac{a\sqrt{\frac{b^2-4\,ac}{a^2}}-b}{a}}E\left(\arcsin\left(\sqrt{\frac{1}{2}}x\sqrt{\frac{a\sqrt{\frac{b^2-4\,ac}{a^2}}-b}{a}}\right)\right.}{}$$

```
[In] integrate((B*x^2+A)/x^2/(c*x^4+b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/2*(2*sqrt(c*x^4 + b*x^2 + a)*A*a*c + sqrt(1/2)*(A*a*c*x*sqrt((b^2 - 4*a*
c)/a^2) - A*b*c*x)*sqrt(a)*sqrt((a*sqrt((b^2 - 4*a*c)/a^2) - b)/a)*elliptic
_e(arcsin(sqrt(1/2)*x*sqrt((a*sqrt((b^2 - 4*a*c)/a^2) - b)/a)), 1/2*(a*b*sq
rt((b^2 - 4*a*c)/a^2) + b^2 - 2*a*c)/(a*c)) + sqrt(1/2)*((B*a^2 - A*a*c)*x*
sqrt((b^2 - 4*a*c)/a^2) + (B*a*b + A*b*c)*x)*sqrt(a)*sqrt((a*sqrt((b^2 - 4*
a*c)/a^2) - b)/a)*elliptic_f(arcsin(sqrt(1/2)*x*sqrt((a*sqrt((b^2 - 4*a*c)/
a^2) - b)/a)), 1/2*(a*b*sqrt((b^2 - 4*a*c)/a^2) + b^2 - 2*a*c)/(a*c)))/(a^2
*c*x)
```

Sympy [F]

$$\int \frac{A+Bx^2}{x^2\sqrt{a+bx^2+cx^4}}\,dx = \int \frac{A+Bx^2}{x^2\sqrt{a+bx^2+cx^4}}\,dx$$

```
[In] integrate((B*x**2+A)/x**2/(c*x**4+b*x**2+a)**(1/2),x)
```

```
[Out] Integral((A + B*x**2)/(x**2*sqrt(a + b*x**2 + c*x**4)), x)
```

Maxima [F]

$$\int \frac{A+Bx^2}{x^2\sqrt{a+bx^2+cx^4}}\,dx = \int \frac{Bx^2+A}{\sqrt{cx^4+bx^2+a}x^2}\,dx$$

```
[In] integrate((B*x^2+A)/x^2/(c*x^4+b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((B*x^2 + A)/(sqrt(c*x^4 + b*x^2 + a)*x^2), x)
```

Giac [F]

$$\int \frac{A + Bx^2}{x^2\sqrt{a + bx^2 + cx^4}}\,dx = \int \frac{Bx^2 + A}{\sqrt{cx^4 + bx^2 + a}x^2}\,dx$$

```
[In] integrate((B*x^2+A)/x^2/(c*x^4+b*x^2+a)^(1/2),x, algorithm="giac")
[Out] integrate((B*x^2 + A)/(sqrt(c*x^4 + b*x^2 + a)*x^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A + Bx^2}{x^2\sqrt{a + bx^2 + cx^4}}\,dx = \int \frac{B\,x^2 + A}{x^2\,\sqrt{c\,x^4 + b\,x^2 + a}}\,dx$$

```
[In] int((A + B*x^2)/(x^2*(a + b*x^2 + c*x^4)^(1/2)),x)
[Out] int((A + B*x^2)/(x^2*(a + b*x^2 + c*x^4)^(1/2)), x)
```

3.180 $\int \frac{A+Bx^2}{x^4\sqrt{a+bx^2+cx^4}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 376

$$\int \frac{A+Bx^2}{x^4\sqrt{a+bx^2+cx^4}}\,dx = -\frac{A\sqrt{a+bx^2+cx^4}}{3ax^3}+\frac{(2Ab-3aB)\sqrt{a+bx^2+cx^4}}{3a^2x}-\frac{(2Ab-3aB)\sqrt{c}x\sqrt{a+bx^2+cx^4}}{3a^2\left(\sqrt{a}+\sqrt{c}x^2\right)}+\frac{(2Ab-3aB)\sqrt[4]{c}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{3a^{7/4}\sqrt{a+bx^2+cx^4}}-\frac{\left(2Ab-3aB+\sqrt{a}A\sqrt{c}\right)\sqrt[4]{c}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{6a^{7/4}\sqrt{a+bx^2+cx^4}}$$

```
[Out] -1/3*A*(c*x^4+b*x^2+a)^(1/2)/a/x^3+1/3*(2*A*b-3*B*a)*(c*x^4+b*x^2+a)^(1/2)/
a^2/x-1/3*(2*A*b-3*B*a)*x*c^(1/2)*(c*x^4+b*x^2+a)^(1/2)/a^2/(a^(1/2)+x^2*c^
(1/2))+1/3*(2*A*b-3*B*a)*c^(1/4)*(cos(2*arctan(c^(1/4)*x/a^(1/4)))^2)^(1/2)
/cos(2*arctan(c^(1/4)*x/a^(1/4)))*EllipticE(sin(2*arctan(c^(1/4)*x/a^(1/4))
),1/2*(2-b/a^(1/2)/c^(1/2))^(1/2))*(a^(1/2)+x^2*c^(1/2))*((c*x^4+b*x^2+a)/(
a^(1/2)+x^2*c^(1/2))^2)^(1/2)/a^(7/4)/(c*x^4+b*x^2+a)^(1/2)-1/6*c^(1/4)*(co
s(2*arctan(c^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*x/a^(1/4)))*El
lipticF(sin(2*arctan(c^(1/4)*x/a^(1/4))),1/2*(2-b/a^(1/2)/c^(1/2))^(1/2))*(
a^(1/2)+x^2*c^(1/2))*(2*A*b-3*B*a+A*a^(1/2)*c^(1/2))*((c*x^4+b*x^2+a)/(a^(1
/2)+x^2*c^(1/2))^2)^(1/2)/a^(7/4)/(c*x^4+b*x^2+a)^(1/2)
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 376, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {1295, 1211, 1117, 1209}

$$\int \frac{A+Bx^2}{x^4\sqrt{a+bx^2+cx^4}}\,dx = -\frac{\sqrt[4]{c}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\left(\sqrt{a}A\sqrt{c}-3aB+2Ab\right)\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{6a^{7/4}\sqrt{a+bx^2+cx^4}} + \frac{\sqrt[4]{c}\left(\sqrt{a}+\sqrt{c}x^2\right)(2Ab-3aB)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{3a^{7/4}\sqrt{a+bx^2+cx^4}} + \frac{(2Ab-3aB)\sqrt{a+bx^2+cx^4}}{3a^2x} - \frac{\sqrt{c}x(2Ab-3aB)\sqrt{a+bx^2+cx^4}}{3a^2\left(\sqrt{a}+\sqrt{c}x^2\right)} - \frac{A\sqrt{a+bx^2+cx^4}}{3ax^3}$$

```
[In] Int[(A + B*x^2)/(x^4*Sqrt[a + b*x^2 + c*x^4]),x]
```

```
[Out] -1/3*(A*Sqrt[a + b*x^2 + c*x^4])/(a*x^3) + ((2*A*b - 3*a*B)*Sqrt[a + b*x^2
+ c*x^4])/(3*a^2*x) - ((2*A*b - 3*a*B)*Sqrt[c]*x*Sqrt[a + b*x^2 + c*x^4])/(
3*a^2*(Sqrt[a] + Sqrt[c]*x^2)) + ((2*A*b - 3*a*B)*c^(1/4)*(Sqrt[a] + Sqrt[c
]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticE[2*ArcT
an[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(3*a^(7/4)*Sqrt[a +
b*x^2 + c*x^4]) - ((2*A*b - 3*a*B + Sqrt[a]*A*Sqrt[c])*c^(1/4)*(Sqrt[a] + S
qrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticF[2
*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(6*a^(7/4)*Sqrt
[a + b*x^2 + c*x^4])
```

Rule 1117

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1209

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

Rule 1211

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + b*x^2 + c*x^4
], x], x] - Dist[e/q, Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /; Ne
Q[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[
c/a]
```

Rule 1295

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[d*(f*x)^(m + 1)*((a + b*x^2 + c*x^4)^(p + 1)
/(a*f*(m + 1))), x] + Dist[1/(a*f^2*(m + 1)), Int[(f*x)^(m + 2)*(a + b*x^2
+ c*x^4)^p*Simp[a*e*(m + 1) - b*d*(m + 2*p + 3) - c*d*(m + 4*p + 5)*x^2, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[m
, -1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{A\sqrt{a+bx^2+cx^4}}{3ax^3} - \frac{\int \frac{2Ab-3aB+Acx^2}{x^2\sqrt{a+bx^2+cx^4}}\,dx}{3a} \\
&= -\frac{A\sqrt{a+bx^2+cx^4}}{3ax^3} + \frac{(2Ab-3aB)\sqrt{a+bx^2+cx^4}}{3a^2x} + \frac{\int \frac{-aAc-(2Ab-3aB)cx^2}{\sqrt{a+bx^2+cx^4}}\,dx}{3a^2} \\
&= -\frac{A\sqrt{a+bx^2+cx^4}}{3ax^3} + \frac{(2Ab-3aB)\sqrt{a+bx^2+cx^4}}{3a^2x} \\
&\quad + \frac{\left((2Ab-3aB)\sqrt{c}\right)\int \frac{1-\frac{\sqrt{c}x^2}{\sqrt{a}}}{\sqrt{a+bx^2+cx^4}}\,dx}{3a^{3/2}} \\
&\quad - \frac{\left(\left(2Ab-3aB+\sqrt{a}A\sqrt{c}\right)\sqrt{c}\right)\int \frac{1}{\sqrt{a+bx^2+cx^4}}\,dx}{3a^{3/2}} \\
&= -\frac{A\sqrt{a+bx^2+cx^4}}{3ax^3} + \frac{(2Ab-3aB)\sqrt{a+bx^2+cx^4}}{3a^2x} - \frac{(2Ab-3aB)\sqrt{c}x\sqrt{a+bx^2+cx^4}}{3a^2\left(\sqrt{a}+\sqrt{c}x^2\right)} \\
&\quad + \frac{(2Ab-3aB)\sqrt[4]{c}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{3a^{7/4}\sqrt{a+bx^2+cx^4}} \\
&\quad - \frac{\left(2Ab-3aB+\sqrt{a}A\sqrt{c}\right)\sqrt[4]{c}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{6a^{7/4}\sqrt{a+bx^2+cx^4}}
\end{aligned}
$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.47 (sec) , antiderivative size = 373, normalized size of antiderivative = 0.99

$$\int \frac{A+Bx^2}{x^4\sqrt{a+bx^2+cx^4}}\,dx$$
$$= \frac{-\frac{4(a+bx^2+cx^4)(-2Abx^2+a(A+3Bx^2))}{x^3} + \frac{i\sqrt{2}\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx^2}{b+\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\left(-\left((2Ab-3aB)\left(-b+\sqrt{b^2-4ac}\right)E\left(i\text{arcsinh}\left(\sqrt{2}\sqrt{\frac{\cdots}{b+\cdots}}\right.\right.\right.\right.}{\cdots}}{12a^2\sqrt{a+bx\cdots}}$$

```
[In] Integrate[(A + B*x^2)/(x^4*Sqrt[a + b*x^2 + c*x^4]),x]
```

```
[Out] ((-4*(a + b*x^2 + c*x^4)*(-2*A*b*x^2 + a*(A + 3*B*x^2)))/x^3 + (I*Sqrt[2]*S
qrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*
c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*(-((2*A*b - 3*a*B)*(-b + Sqrt[b^2 - 4*a*c])
*EllipticE[I*ArcSinh[Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x], (b + Sqrt[
b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c])]) + (3*a*B*(b - Sqrt[b^2 - 4*a*c]) +
2*A*(-b^2 + a*c + b*Sqrt[b^2 - 4*a*c]))*EllipticF[I*ArcSinh[Sqrt[2]*Sqrt[c/
(b + Sqrt[b^2 - 4*a*c])]*x], (b + Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c]
)]))/Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]])/(12*a^2*Sqrt[a + b*x^2 + c*x^4])
```

Maple [A] (verified)

Time = 3.38 (sec) , antiderivative size = 418, normalized size of antiderivative = 1.11

method	result
risch	$-\frac{\sqrt{c\,x^4+b\,x^2+a}\,(-2Ab\,x^2+3Ba\,x^2+Aa)}{3a^2x^3} - \frac{c\left(\frac{Aa\sqrt{2}\sqrt{4-\frac{2\left(-b+\sqrt{-4ac+b^2}\right)x^2}{a}}\sqrt{4+\frac{2\left(b+\sqrt{-4ac+b^2}\right)x^2}{a}}\,F\left(\frac{x\sqrt{2}\sqrt{\frac{-b+\sqrt{-4ac+b^2}}{a}}}{2},\sqrt{\cdots}\right.}{4\sqrt{\frac{-b+\sqrt{-4ac+b^2}}{a}}\sqrt{c\,x^4+b\,x^2+a}}\cdots\right.}{\cdots}$
elliptic	$-\frac{A\sqrt{c\,x^4+b\,x^2+a}}{3a\,x^3} + \frac{(2Ab-3Ba)\sqrt{c\,x^4+b\,x^2+a}}{3a^2x} - \frac{cA\sqrt{2}\sqrt{4-\frac{2\left(-b+\sqrt{-4ac+b^2}\right)x^2}{a}}\sqrt{4+\frac{2\left(b+\sqrt{-4ac+b^2}\right)x^2}{a}}\,F\left(\frac{x\sqrt{2}\sqrt{\frac{-b+\sqrt{-4ac\cdots}}{a}}}{2}\cdots\right.}{12a\sqrt{\frac{-b+\sqrt{-4ac+b^2}}{a}}\sqrt{c\,x^4+b\,x^2+a}\cdots}$
default	$B\left(-\frac{\sqrt{c\,x^4+b\,x^2+a}}{ax} - \frac{c\sqrt{2}\sqrt{4-\frac{2\left(-b+\sqrt{-4ac+b^2}\right)x^2}{a}}\sqrt{4+\frac{2\left(b+\sqrt{-4ac+b^2}\right)x^2}{a}}\left(F\left(\frac{x\sqrt{2}\sqrt{\frac{-b+\sqrt{-4ac+b^2}}{a}}}{2},\frac{\sqrt{-4+\frac{2b\left(b+\sqrt{-4ac+b^2}\right)}{ac}}}{2}\cdots\right.\right.}{2\sqrt{\frac{-b+\sqrt{-4ac+b^2}}{a}}\sqrt{c\,x^4+b\,x^2+a}\left(b+\sqrt{-4ac+b^2}\cdots\right.}\cdots\right.$

```
[In] int((B*x^2+A)/x^4/(c*x^4+b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/3*(c*x^4+b*x^2+a)^(1/2)*(-2*A*b*x^2+3*B*a*x^2+A*a)/a^2/x^3-1/3*c/a^2*(1/
4*A*a*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*(4-2*(-b+(-4*a*c+b^2)^(1/2)
)/a*x^2)^(1/2)*(4+2*(b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)/(c*x^4+b*x^2+a)^(1/
2)*EllipticF(1/2*x*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4+2*b*(b
+(-4*a*c+b^2)^(1/2))/a/c)^(1/2))-1/2*(2*A*b-3*B*a)*a*2^(1/2)/((-b+(-4*a*c+b
^2)^(1/2))/a)^(1/2)*(4-2*(-b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)*(4+2*(b+(-4*a
*c+b^2)^(1/2))/a*x^2)^(1/2)/(c*x^4+b*x^2+a)^(1/2)/(b+(-4*a*c+b^2)^(1/2))*(E
llipticF(1/2*x*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4+2*b*(b+(-4
*a*c+b^2)^(1/2))/a/c)^(1/2))-EllipticE(1/2*x*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2
))/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2))))
```

Fricas [A] (verification not implemented)

none

Time = 0.09 (sec) , antiderivative size = 347, normalized size of antiderivative = 0.92

$$\int \frac{A+Bx^2}{x^4\sqrt{a+bx^2+cx^4}}\,dx = -\frac{\sqrt{\frac{1}{2}}\left((3\,Ba^2-2\,Aab)x^3\sqrt{\frac{b^2-4\,ac}{a^2}}-(3\,Bab-2\,Ab^2)x^3\right)\sqrt{a}\sqrt{\frac{a\sqrt{\frac{b^2-4\,ac}{a^2}}-b}{a}}E\left(\arcsin\left(\sqrt{\frac{1}{2}}x\sqrt{\frac{a\sqrt{\frac{b^2-4\,ac}{a^2}}-b}{a}}\right)\right.}{}$$

```
[In] integrate((B*x^2+A)/x^4/(c*x^4+b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/6*(sqrt(1/2)*((3*B*a^2 - 2*A*a*b)*x^3*sqrt((b^2 - 4*a*c)/a^2) - (3*B*a*b
 - 2*A*b^2)*x^3)*sqrt(a)*sqrt((a*sqrt((b^2 - 4*a*c)/a^2) - b)/a)*elliptic_e
(arcsin(sqrt(1/2)*x*sqrt((a*sqrt((b^2 - 4*a*c)/a^2) - b)/a)), 1/2*(a*b*sqrt
((b^2 - 4*a*c)/a^2) + b^2 - 2*a*c)/(a*c)) - sqrt(1/2)*(((A + 3*B)*a^2 - 2*A
*a*b)*x^3*sqrt((b^2 - 4*a*c)/a^2) + ((A - 3*B)*a*b + 2*A*b^2)*x^3)*sqrt(a)*
sqrt((a*sqrt((b^2 - 4*a*c)/a^2) - b)/a)*elliptic_f(arcsin(sqrt(1/2)*x*sqrt(
(a*sqrt((b^2 - 4*a*c)/a^2) - b)/a)), 1/2*(a*b*sqrt((b^2 - 4*a*c)/a^2) + b^2
 - 2*a*c)/(a*c)) + 2*sqrt(c*x^4 + b*x^2 + a)*(A*a^2 + (3*B*a^2 - 2*A*a*b)*x
^2))/(a^3*x^3)
```

Sympy [F]

$$\int \frac{A+Bx^2}{x^4\sqrt{a+bx^2+cx^4}}\,dx=\int \frac{A+Bx^2}{x^4\sqrt{a+bx^2+cx^4}}\,dx$$

```
[In] integrate((B*x**2+A)/x**4/(c*x**4+b*x**2+a)**(1/2),x)
[Out] Integral((A + B*x**2)/(x**4*sqrt(a + b*x**2 + c*x**4)), x)
```

Maxima [F]

$$\int \frac{A+Bx^2}{x^4\sqrt{a+bx^2+cx^4}}\,dx=\int \frac{Bx^2+A}{\sqrt{cx^4+bx^2+a}x^4}\,dx$$

```
[In] integrate((B*x^2+A)/x^4/(c*x^4+b*x^2+a)^(1/2),x, algorithm="maxima")
[Out] integrate((B*x^2 + A)/(sqrt(c*x^4 + b*x^2 + a)*x^4), x)
```

Giac [F]

$$\int \frac{A+Bx^2}{x^4\sqrt{a+bx^2+cx^4}}\,dx=\int \frac{Bx^2+A}{\sqrt{cx^4+bx^2+a}x^4}\,dx$$

```
[In] integrate((B*x^2+A)/x^4/(c*x^4+b*x^2+a)^(1/2),x, algorithm="giac")
[Out] integrate((B*x^2 + A)/(sqrt(c*x^4 + b*x^2 + a)*x^4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx^2}{x^4\sqrt{a+bx^2+cx^4}}\,dx=\int \frac{B\,x^2+A}{x^4\,\sqrt{c\,x^4+b\,x^2+a}}\,dx$$

```
[In] int((A + B*x^2)/(x^4*(a + b*x^2 + c*x^4)^(1/2)),x)
[Out] int((A + B*x^2)/(x^4*(a + b*x^2 + c*x^4)^(1/2)), x)
```

3.181 $\int \frac{x^7(2+3x^2)}{\sqrt{3+5x^2+x^4}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 98

$$\int \frac{x^7(2+3x^2)}{\sqrt{3+5x^2+x^4}}\,dx = -\frac{89}{48}x^4\sqrt{3+5x^2+x^4}+\frac{3}{8}x^6\sqrt{3+5x^2+x^4} -\frac{1}{384}\left(24243-3802x^2\right)\sqrt{3+5x^2+x^4} +\frac{32801}{256}\text{arctanh}\left(\frac{5+2x^2}{2\sqrt{3+5x^2+x^4}}\right)$$

```
[Out] 32801/256*arctanh(1/2*(2*x^2+5)/(x^4+5*x^2+3)^(1/2))-89/48*x^4*(x^4+5*x^2+3
)^(1/2)+3/8*x^6*(x^4+5*x^2+3)^(1/2)-1/384*(-3802*x^2+24243)*(x^4+5*x^2+3)^(
1/2)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {1265, 846, 793, 635, 212}

$$\int \frac{x^7(2+3x^2)}{\sqrt{3+5x^2+x^4}}\,dx = \frac{32801}{256}\text{arctanh}\left(\frac{2x^2+5}{2\sqrt{x^4+5x^2+3}}\right)-\frac{89}{48}\sqrt{x^4+5x^2+3}x^4 -\frac{1}{384}\left(24243-3802x^2\right)\sqrt{x^4+5x^2+3}+\frac{3}{8}\sqrt{x^4+5x^2+3}x^6$$

```
[In] Int[(x^7*(2 + 3*x^2))/Sqrt[3 + 5*x^2 + x^4],x]
```

```
[Out] (-89*x^4*Sqrt[3 + 5*x^2 + x^4])/48 + (3*x^6*Sqrt[3 + 5*x^2 + x^4])/8 - ((24
243 - 3802*x^2)*Sqrt[3 + 5*x^2 + x^4])/384 + (32801*ArcTanh[(5 + 2*x^2)/(2*
Sqrt[3 + 5*x^2 + x^4])])/256
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 635

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 793

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> Simp[(-(b*e*g*(p + 2) - c*(e*f + d*g)*(2*p + 3) -
 2*c*e*g*(p + 1)*x))*((a + b*x + c*x^2)^(p + 1)/(2*c^2*(p + 1)*(2*p + 3))),
 x] + Dist[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(2*c*d*f - b*(e*f + d*g))*(2*p
+ 3))/(2*c^2*(2*p + 3)), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c,
 d, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rule 846

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g*(d + e*x)^m*((a + b*x + c*x^2)^(p +
1)/(c*(m + 2*p + 2))), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)
*(a + b*x + c*x^2)^p*Simp[m*(c*d*f - a*e*g) + d*(2*c*f - b*g)*(p + 1) + (m*
(c*e*f + c*d*g - b*e*g) + e*(p + 1)*(2*c*f - b*g))*x, x], x], x] /; FreeQ[{
a, b, c, d, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a
*e^2, 0] && GtQ[m, 0] && NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p]
 || IntegersQ[2*m, 2*p]) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

Rule 1265

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{1}{2}\text{Subst}\left(\int \frac{x^3(2+3x)}{\sqrt{3+5x+x^2}}\,dx, x, x^2\right)\\ &= \frac{3}{8}x^6\sqrt{3+5x^2+x^4}+\frac{1}{8}\text{Subst}\left(\int \frac{\left(-27-\frac{89x}{2}\right)x^2}{\sqrt{3+5x+x^2}}\,dx, x, x^2\right)\end{aligned}$$

$$= -\frac{89}{48}x^4\sqrt{3+5x^2+x^4}+\frac{3}{8}x^6\sqrt{3+5x^2+x^4}+\frac{1}{24}\text{Subst}\left(\int \frac{x\left(267+\frac{1901x}{4}\right)}{\sqrt{3+5x+x^2}}\,dx,x,x^2\right)$$

$$= -\frac{89}{48}x^4\sqrt{3+5x^2+x^4}+\frac{3}{8}x^6\sqrt{3+5x^2+x^4}-\frac{1}{384}\left(24243-3802x^2\right)\sqrt{3+5x^2+x^4} + \frac{32801}{256}\text{Subst}\left(\int \frac{1}{\sqrt{3+5x+x^2}}\,dx,x,x^2\right)$$

$$= -\frac{89}{48}x^4\sqrt{3+5x^2+x^4}+\frac{3}{8}x^6\sqrt{3+5x^2+x^4}-\frac{1}{384}\left(24243-3802x^2\right)\sqrt{3+5x^2+x^4} + \frac{32801}{128}\text{Subst}\left(\int \frac{1}{4-x^2}\,dx,x,\frac{5+2x^2}{\sqrt{3+5x^2+x^4}}\right)$$

$$= -\frac{89}{48}x^4\sqrt{3+5x^2+x^4}+\frac{3}{8}x^6\sqrt{3+5x^2+x^4} - \frac{1}{384}\left(24243-3802x^2\right)\sqrt{3+5x^2+x^4}+\frac{32801}{256}\tanh^{-1}\left(\frac{5+2x^2}{2\sqrt{3+5x^2+x^4}}\right)$$

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.65

$$\int \frac{x^7(2+3x^2)}{\sqrt{3+5x^2+x^4}}\,dx = \frac{1}{384}\sqrt{3+5x^2+x^4}\left(-24243+3802x^2-712x^4+144x^6\right) - \frac{32801}{256}\log\left(-5-2x^2+2\sqrt{3+5x^2+x^4}\right)$$

```
[In] Integrate[(x^7*(2 + 3*x^2))/Sqrt[3 + 5*x^2 + x^4],x]
```

```
[Out] (Sqrt[3 + 5*x^2 + x^4]*(-24243 + 3802*x^2 - 712*x^4 + 144*x^6))/384 - (3280
1*Log[-5 - 2*x^2 + 2*Sqrt[3 + 5*x^2 + x^4]])/256
```

Maple [A] (verified)

Time = 0.21 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.54

method	result
risch	$\frac{(144x^6-712x^4+3802x^2-24243)\sqrt{x^4+5x^2+3}}{384}+\frac{32801\ln\left(\frac{5}{2}+x^2+\sqrt{x^4+5x^2+3}\right)}{256}$
trager	$\left(\frac{3}{8}x^6-\frac{89}{48}x^4+\frac{1901}{192}x^2-\frac{8081}{128}\right)\sqrt{x^4+5x^2+3}-\frac{32801\ln\left(-2x^2+2\sqrt{x^4+5x^2+3}-5\right)}{256}$
pseudoelliptic	$\frac{32801\ln\left(2x^2+5+2\sqrt{x^4+5x^2+3}\right)}{256}+\frac{(144x^6-712x^4+3802x^2-24243)\sqrt{x^4+5x^2+3}}{384}$
default	$\frac{3x^6\sqrt{x^4+5x^2+3}}{8}-\frac{89x^4\sqrt{x^4+5x^2+3}}{48}+\frac{1901x^2\sqrt{x^4+5x^2+3}}{192}-\frac{8081\sqrt{x^4+5x^2+3}}{128}+\frac{32801\ln\left(\frac{5}{2}+x^2+\sqrt{x^4+5x^2+3}\right)}{256}$
elliptic	$\frac{3x^6\sqrt{x^4+5x^2+3}}{8}-\frac{89x^4\sqrt{x^4+5x^2+3}}{48}+\frac{1901x^2\sqrt{x^4+5x^2+3}}{192}-\frac{8081\sqrt{x^4+5x^2+3}}{128}+\frac{32801\ln\left(\frac{5}{2}+x^2+\sqrt{x^4+5x^2+3}\right)}{256}$

```
[In] int(x^7*(3*x^2+2)/(x^4+5*x^2+3)^(1/2),x,method=_RETURNVERBOSE)
[Out] 1/384*(144*x^6-712*x^4+3802*x^2-24243)*(x^4+5*x^2+3)^(1/2)+32801/256*ln(5/2
+x^2+(x^4+5*x^2+3)^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.57

$$\int \frac{x^7(2+3x^2)}{\sqrt{3+5x^2+x^4}}\,dx = \frac{1}{384}\left(144\,x^6-712\,x^4+3802\,x^2-24243\right)\sqrt{x^4+5\,x^2+3} - \frac{32801}{256}\log\left(-2\,x^2+2\,\sqrt{x^4+5\,x^2+3}-5\right)$$

```
[In] integrate(x^7*(3*x^2+2)/(x^4+5*x^2+3)^(1/2),x, algorithm="fricas")
[Out] 1/384*(144*x^6 - 712*x^4 + 3802*x^2 - 24243)*sqrt(x^4 + 5*x^2 + 3) - 32801/
256*log(-2*x^2 + 2*sqrt(x^4 + 5*x^2 + 3) - 5)
```

Sympy [A] (verification not implemented)

Time = 0.91 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.66

$$\int \frac{x^7(2+3x^2)}{\sqrt{3+5x^2+x^4}}\,dx = \frac{\sqrt{x^4+5x^2+3}\cdot\left(\frac{3x^6}{4}-\frac{89x^4}{24}+\frac{1901x^2}{96}-\frac{8081}{64}\right)}{2} + \frac{32801\log\left(2x^2+2\sqrt{x^4+5x^2+3}+5\right)}{256}$$

```
[In] integrate(x**7*(3*x**2+2)/(x**4+5*x**2+3)**(1/2),x)
[Out] sqrt(x**4 + 5*x**2 + 3)*(3*x**6/4 - 89*x**4/24 + 1901*x**2/96 - 8081/64)/2
+ 32801*log(2*x**2 + 2*sqrt(x**4 + 5*x**2 + 3) + 5)/256
```

Maxima [A] (verification not implemented)

none

Time = 0.18 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.92

$$\int \frac{x^7(2+3x^2)}{\sqrt{3+5x^2+x^4}}\,dx = \frac{3}{8}\sqrt{x^4+5\,x^2+3}x^6 - \frac{89}{48}\sqrt{x^4+5\,x^2+3}x^4 + \frac{1901}{192}\sqrt{x^4+5\,x^2+3}x^2 - \frac{8081}{128}\sqrt{x^4+5\,x^2+3} + \frac{32801}{256}\log\left(2\,x^2+2\sqrt{x^4+5\,x^2+3}+5\right)$$

```
[In] integrate(x^7*(3*x^2+2)/(x^4+5*x^2+3)^(1/2),x, algorithm="maxima")
```

```
[Out] 3/8*sqrt(x^4 + 5*x^2 + 3)*x^6 - 89/48*sqrt(x^4 + 5*x^2 + 3)*x^4 + 1901/192*
sqrt(x^4 + 5*x^2 + 3)*x^2 - 8081/128*sqrt(x^4 + 5*x^2 + 3) + 32801/256*log(
2*x^2 + 2*sqrt(x^4 + 5*x^2 + 3) + 5)
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.61

$$\int \frac{x^7(2+3x^2)}{\sqrt{3+5x^2+x^4}}\,dx = \frac{1}{384}\sqrt{x^4+5\,x^2+3}\left(2\left(4\left(18\,x^2-89\right)x^2+1901\right)x^2-24243\right) - \frac{32801}{256}\log\left(2\,x^2-2\sqrt{x^4+5\,x^2+3}+5\right)$$

```
[In] integrate(x^7*(3*x^2+2)/(x^4+5*x^2+3)^(1/2),x, algorithm="giac")
```

```
[Out] 1/384*sqrt(x^4 + 5*x^2 + 3)*(2*(4*(18*x^2 - 89)*x^2 + 1901)*x^2 - 24243) -
32801/256*log(2*x^2 - 2*sqrt(x^4 + 5*x^2 + 3) + 5)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^7(2+3x^2)}{\sqrt{3+5x^2+x^4}}\,dx = \int \frac{x^7\,\left(3\,x^2+2\right)}{\sqrt{x^4+5\,x^2+3}}\,dx$$

```
[In] int((x^7*(3*x^2 + 2))/(5*x^2 + x^4 + 3)^(1/2),x)
```

```
[Out] int((x^7*(3*x^2 + 2))/(5*x^2 + x^4 + 3)^(1/2), x)
```

3.182 $\int \frac{x^5(2+3x^2)}{\sqrt{3+5x^2+x^4}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 77

$$\int \frac{x^5(2+3x^2)}{\sqrt{3+5x^2+x^4}}\,dx = \frac{1}{2}x^4\sqrt{3+5x^2+x^4}+\frac{3}{16}(89-14x^2)\sqrt{3+5x^2+x^4} - \frac{1083}{32}\text{arctanh}\left(\frac{5+2x^2}{2\sqrt{3+5x^2+x^4}}\right)$$

```
[Out] -1083/32*arctanh(1/2*(2*x^2+5)/(x^4+5*x^2+3)^(1/2))+1/2*x^4*(x^4+5*x^2+3)^(
1/2)+3/16*(-14*x^2+89)*(x^4+5*x^2+3)^(1/2)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {1265, 846, 793, 635, 212}

$$\int \frac{x^5(2+3x^2)}{\sqrt{3+5x^2+x^4}}\,dx = -\frac{1083}{32}\text{arctanh}\left(\frac{2x^2+5}{2\sqrt{x^4+5x^2+3}}\right) + \frac{1}{2}\sqrt{x^4+5x^2+3}x^4+\frac{3}{16}(89-14x^2)\sqrt{x^4+5x^2+3}$$

```
[In] Int[(x^5*(2 + 3*x^2))/Sqrt[3 + 5*x^2 + x^4],x]
```

```
[Out] (x^4*Sqrt[3 + 5*x^2 + x^4])/2 + (3*(89 - 14*x^2)*Sqrt[3 + 5*x^2 + x^4])/16
- (1083*ArcTanh[(5 + 2*x^2)/(2*Sqrt[3 + 5*x^2 + x^4])])/32
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rule 635

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 793

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> Simp[(-(b*e*g*(p + 2) - c*(e*f + d*g)*(2*p + 3) -
 2*c*e*g*(p + 1)*x))*((a + b*x + c*x^2)^(p + 1)/(2*c^2*(p + 1)*(2*p + 3))),
 x] + Dist[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(2*c*d*f - b*(e*f + d*g))*(2*p
+ 3))/(2*c^2*(2*p + 3)), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c,
 d, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rule 846

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g*(d + e*x)^m*((a + b*x + c*x^2)^(p +
1)/(c*(m + 2*p + 2))), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)
*(a + b*x + c*x^2)^p*Simp[m*(c*d*f - a*e*g) + d*(2*c*f - b*g)*(p + 1) + (m*
(c*e*f + c*d*g - b*e*g) + e*(p + 1)*(2*c*f - b*g))*x, x], x], x] /; FreeQ[{
a, b, c, d, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a
*e^2, 0] && GtQ[m, 0] && NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p]
 || IntegersQ[2*m, 2*p]) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

Rule 1265

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{2}\text{Subst}\left(\int \frac{x^2(2+3x)}{\sqrt{3+5x+x^2}}\,dx, x, x^2\right)\\
&= \frac{1}{2}x^4\sqrt{3+5x^2+x^4}+\frac{1}{6}\text{Subst}\left(\int \frac{\left(-18-\frac{63x}{2}\right)x}{\sqrt{3+5x+x^2}}\,dx, x, x^2\right)\\
&= \frac{1}{2}x^4\sqrt{3+5x^2+x^4}+\frac{3}{16}\left(89-14x^2\right)\sqrt{3+5x^2+x^4}\\
&\quad -\frac{1083}{32}\text{Subst}\left(\int \frac{1}{\sqrt{3+5x+x^2}}\,dx, x, x^2\right)
\end{aligned}$$

$$= \frac{1}{2}x^4\sqrt{3+5x^2+x^4} + \frac{3}{16}(89-14x^2)\sqrt{3+5x^2+x^4} - \frac{1083}{16}\text{Subst}\left(\int \frac{1}{4-x^2}\,dx, x, \frac{5+2x^2}{\sqrt{3+5x^2+x^4}}\right)$$

$$= \frac{1}{2}x^4\sqrt{3+5x^2+x^4} + \frac{3}{16}(89-14x^2)\sqrt{3+5x^2+x^4} - \frac{1083}{32}\tanh^{-1}\left(\frac{5+2x^2}{2\sqrt{3+5x^2+x^4}}\right)$$

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.77

$$\int \frac{x^5(2+3x^2)}{\sqrt{3+5x^2+x^4}}\,dx = \frac{1}{16}\sqrt{3+5x^2+x^4}(267-42x^2+8x^4) + \frac{1083}{32}\log\left(-5-2x^2+2\sqrt{3+5x^2+x^4}\right)$$

```
[In] Integrate[(x^5*(2 + 3*x^2))/Sqrt[3 + 5*x^2 + x^4],x]
[Out] (Sqrt[3 + 5*x^2 + x^4]*(267 - 42*x^2 + 8*x^4))/16 + (1083*Log[-5 - 2*x^2 +
2*Sqrt[3 + 5*x^2 + x^4]])/32
```

Maple [A] (verified)

Time = 0.20 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.62

method	result	size
risch	$\frac{(8x^4-42x^2+267)\sqrt{x^4+5x^2+3}}{16}-\frac{1083\ln\left(\frac{5}{2}+x^2+\sqrt{x^4+5x^2+3}\right)}{32}$	48
trager	$\left(\frac{1}{2}x^4-\frac{21}{8}x^2+\frac{267}{16}\right)\sqrt{x^4+5x^2+3}-\frac{1083\ln\left(2x^2+5+2\sqrt{x^4+5x^2+3}\right)}{32}$	51
pseudoelliptic	$-\frac{1083\ln\left(2x^2+5+2\sqrt{x^4+5x^2+3}\right)}{32}+\frac{(8x^4-42x^2+267)\sqrt{x^4+5x^2+3}}{16}$	52
default	$\frac{x^4\sqrt{x^4+5x^2+3}}{2}-\frac{21x^2\sqrt{x^4+5x^2+3}}{8}+\frac{267\sqrt{x^4+5x^2+3}}{16}-\frac{1083\ln\left(\frac{5}{2}+x^2+\sqrt{x^4+5x^2+3}\right)}{32}$	70
elliptic	$\frac{x^4\sqrt{x^4+5x^2+3}}{2}-\frac{21x^2\sqrt{x^4+5x^2+3}}{8}+\frac{267\sqrt{x^4+5x^2+3}}{16}-\frac{1083\ln\left(\frac{5}{2}+x^2+\sqrt{x^4+5x^2+3}\right)}{32}$	70

```
[In] int(x^5*(3*x^2+2)/(x^4+5*x^2+3)^(1/2),x,method=_RETURNVERBOSE)
[Out] 1/16*(8*x^4-42*x^2+267)*(x^4+5*x^2+3)^(1/2)-1083/32*ln(5/2+x^2+(x^4+5*x^2+3
)^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.66

$$\int \frac{x^5(2+3x^2)}{\sqrt{3+5x^2+x^4}}\,dx = \frac{1}{16}\left(8\,x^4-42\,x^2+267\right)\sqrt{x^4+5\,x^2+3} + \frac{1083}{32}\,\log\left(-2\,x^2+2\,\sqrt{x^4+5\,x^2+3}-5\right)$$

```
[In] integrate(x^5*(3*x^2+2)/(x^4+5*x^2+3)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/16*(8*x^4 - 42*x^2 + 267)*sqrt(x^4 + 5*x^2 + 3) + 1083/32*log(-2*x^2 + 2*
sqrt(x^4 + 5*x^2 + 3) - 5)
```

Sympy [A] (verification not implemented)

Time = 0.87 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.70

$$\int \frac{x^5(2+3x^2)}{\sqrt{3+5x^2+x^4}}\,dx = \frac{\left(x^4-\frac{21x^2}{4}+\frac{267}{8}\right)\sqrt{x^4+5x^2+3}}{2} - \frac{1083\log\left(2x^2+2\sqrt{x^4+5x^2+3}+5\right)}{32}$$

```
[In] integrate(x**5*(3*x**2+2)/(x**4+5*x**2+3)**(1/2),x)
```

```
[Out] (x**4 - 21*x**2/4 + 267/8)*sqrt(x**4 + 5*x**2 + 3)/2 - 1083*log(2*x**2 + 2*
sqrt(x**4 + 5*x**2 + 3) + 5)/32
```

Maxima [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.95

$$\int \frac{x^5(2+3x^2)}{\sqrt{3+5x^2+x^4}}\,dx = \frac{1}{2}\,\sqrt{x^4+5\,x^2+3}x^4 - \frac{21}{8}\,\sqrt{x^4+5\,x^2+3}x^2 + \frac{267}{16}\,\sqrt{x^4+5\,x^2+3} - \frac{1083}{32}\,\log\left(2\,x^2+2\,\sqrt{x^4+5\,x^2+3}+5\right)$$

```
[In] integrate(x^5*(3*x^2+2)/(x^4+5*x^2+3)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/2*sqrt(x^4 + 5*x^2 + 3)*x^4 - 21/8*sqrt(x^4 + 5*x^2 + 3)*x^2 + 267/16*sqr
t(x^4 + 5*x^2 + 3) - 1083/32*log(2*x^2 + 2*sqrt(x^4 + 5*x^2 + 3) + 5)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.69

$$\int \frac{x^5(2+3x^2)}{\sqrt{3+5x^2+x^4}}\,dx = \frac{1}{16}\sqrt{x^4+5\,x^2+3}\big(2\left(4\,x^2-21\right)x^2+267\big) + \frac{1083}{32}\log\left(2\,x^2-2\,\sqrt{x^4+5\,x^2+3}+5\right)$$

```
[In] integrate(x^5*(3*x^2+2)/(x^4+5*x^2+3)^(1/2),x, algorithm="giac")
[Out] 1/16*sqrt(x^4 + 5*x^2 + 3)*(2*(4*x^2 - 21)*x^2 + 267) + 1083/32*log(2*x^2 -
 2*sqrt(x^4 + 5*x^2 + 3) + 5)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^5(2+3x^2)}{\sqrt{3+5x^2+x^4}}\,dx = \int \frac{x^5\left(3\,x^2+2\right)}{\sqrt{x^4+5\,x^2+3}}\,dx$$

```
[In] int((x^5*(3*x^2 + 2))/(5*x^2 + x^4 + 3)^(1/2),x)
[Out] int((x^5*(3*x^2 + 2))/(5*x^2 + x^4 + 3)^(1/2), x)
```

3.183 $\int \frac{x^3(2+3x^2)}{\sqrt{3+5x^2+x^4}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 56

$$\int \frac{x^3(2+3x^2)}{\sqrt{3+5x^2+x^4}}\,dx=-\frac{1}{8}(37-6x^2)\sqrt{3+5x^2+x^4}+\frac{149}{16}\operatorname{arctanh}\left(\frac{5+2x^2}{2\sqrt{3+5x^2+x^4}}\right)$$

```
[Out] 149/16*arctanh(1/2*(2*x^2+5)/(x^4+5*x^2+3)^(1/2))-1/8*(-6*x^2+37)*(x^4+5*x^
2+3)^(1/2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {1265, 793, 635, 212}

$$\int \frac{x^3(2+3x^2)}{\sqrt{3+5x^2+x^4}}\,dx=\frac{149}{16}\operatorname{arctanh}\left(\frac{2x^2+5}{2\sqrt{x^4+5x^2+3}}\right)-\frac{1}{8}(37-6x^2)\sqrt{x^4+5x^2+3}$$

```
[In] Int[(x^3*(2 + 3*x^2))/Sqrt[3 + 5*x^2 + x^4],x]
```

```
[Out] -1/8*((37 - 6*x^2)*Sqrt[3 + 5*x^2 + x^4]) + (149*ArcTanh[(5 + 2*x^2)/(2*Sqr
t[3 + 5*x^2 + x^4])])/16
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 635

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 793

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> Simp[(-(b*e*g*(p + 2) - c*(e*f + d*g)*(2*p + 3) -
 2*c*e*g*(p + 1)*x))*((a + b*x + c*x^2)^(p + 1)/(2*c^2*(p + 1)*(2*p + 3))),
 x] + Dist[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(2*c*d*f - b*(e*f + d*g))*(2*p
+ 3))/(2*c^2*(2*p + 3)), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c,
 d, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rule 1265

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{2}\text{Subst}\left(\int \frac{x(2+3x)}{\sqrt{3+5x+x^2}}\,dx, x, x^2\right)\\
&= -\frac{1}{8}\left(37-6x^2\right)\sqrt{3+5x^2+x^4}+\frac{149}{16}\text{Subst}\left(\int \frac{1}{\sqrt{3+5x+x^2}}\,dx, x, x^2\right)\\
&= -\frac{1}{8}\left(37-6x^2\right)\sqrt{3+5x^2+x^4}+\frac{149}{8}\text{Subst}\left(\int \frac{1}{4-x^2}\,dx, x, \frac{5+2x^2}{\sqrt{3+5x^2+x^4}}\right)\\
&= -\frac{1}{8}\left(37-6x^2\right)\sqrt{3+5x^2+x^4}+\frac{149}{16}\tanh^{-1}\left(\frac{5+2x^2}{2\sqrt{3+5x^2+x^4}}\right)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.96

$$\int \frac{x^3(2+3x^2)}{\sqrt{3+5x^2+x^4}}\,dx = \frac{1}{8}\left(-37+6x^2\right)\sqrt{3+5x^2+x^4}-\frac{149}{16}\log\left(-5-2x^2+2\sqrt{3+5x^2+x^4}\right)$$

```
[In] Integrate[(x^3*(2 + 3*x^2))/Sqrt[3 + 5*x^2 + x^4],x]
[Out] ((-37 + 6*x^2)*Sqrt[3 + 5*x^2 + x^4])/8 - (149*Log[-5 - 2*x^2 + 2*Sqrt[3 +
5*x^2 + x^4]])/16
```

Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.77

method	result	size
risch	$\frac{(6x^2-37)\sqrt{x^4+5x^2+3}}{8}+\frac{149\ln\left(\frac{5}{2}+x^2+\sqrt{x^4+5x^2+3}\right)}{16}$	43
trager	$\left(\frac{3x^2}{4}-\frac{37}{8}\right)\sqrt{x^4+5x^2+3}+\frac{149\ln\left(2x^2+5+2\sqrt{x^4+5x^2+3}\right)}{16}$	46
default	$\frac{3x^2\sqrt{x^4+5x^2+3}}{4}-\frac{37\sqrt{x^4+5x^2+3}}{8}+\frac{149\ln\left(\frac{5}{2}+x^2+\sqrt{x^4+5x^2+3}\right)}{16}$	53
elliptic	$\frac{3x^2\sqrt{x^4+5x^2+3}}{4}-\frac{37\sqrt{x^4+5x^2+3}}{8}+\frac{149\ln\left(\frac{5}{2}+x^2+\sqrt{x^4+5x^2+3}\right)}{16}$	53
pseudoelliptic	$\frac{149\ln\left(2x^2+5+2\sqrt{x^4+5x^2+3}\right)}{16}-\frac{37\sqrt{x^4+5x^2+3}}{8}+\frac{3x^2\sqrt{x^4+5x^2+3}}{4}$	57

```
[In] int(x^3*(3*x^2+2)/(x^4+5*x^2+3)^(1/2),x,method=_RETURNVERBOSE)
[Out] 1/8*(6*x^2-37)*(x^4+5*x^2+3)^(1/2)+149/16*ln(5/2+x^2+(x^4+5*x^2+3)^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.82

$$\int \frac{x^3(2+3x^2)}{\sqrt{3+5x^2+x^4}}\,dx = \frac{1}{8}\sqrt{x^4+5x^2+3}\left(6x^2-37\right)-\frac{149}{16}\log\left(-2x^2+2\sqrt{x^4+5x^2+3}-5\right)$$

```
[In] integrate(x^3*(3*x^2+2)/(x^4+5*x^2+3)^(1/2),x, algorithm="fricas")
[Out] 1/8*sqrt(x^4 + 5*x^2 + 3)*(6*x^2 - 37) - 149/16*log(-2*x^2 + 2*sqrt(x^4 + 5
*x^2 + 3) - 5)
```

Sympy [A] (verification not implemented)

Time = 0.82 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.91

$$\int \frac{x^3(2+3x^2)}{\sqrt{3+5x^2+x^4}}\,dx = \frac{\left(\frac{3x^2}{2}-\frac{37}{4}\right)\sqrt{x^4+5x^2+3}}{2}+\frac{149\log\left(2x^2+2\sqrt{x^4+5x^2+3}+5\right)}{16}$$

```
[In] integrate(x**3*(3*x**2+2)/(x**4+5*x**2+3)**(1/2),x)
[Out] (3*x**2/2 - 37/4)*sqrt(x**4 + 5*x**2 + 3)/2 + 149*log(2*x**2 + 2*sqrt(x**4
+ 5*x**2 + 3) + 5)/16
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.00

$$\int \frac{x^3(2+3x^2)}{\sqrt{3+5x^2+x^4}}\,dx = \frac{3}{4}\sqrt{x^4+5\,x^2+3}x^2 - \frac{37}{8}\sqrt{x^4+5\,x^2+3} + \frac{149}{16}\log\left(2\,x^2+2\,\sqrt{x^4+5\,x^2+3}+5\right)$$

```
[In] integrate(x^3*(3*x^2+2)/(x^4+5*x^2+3)^(1/2),x, algorithm="maxima")
[Out] 3/4*sqrt(x^4 + 5*x^2 + 3)*x^2 - 37/8*sqrt(x^4 + 5*x^2 + 3) + 149/16*log(2*x
^2 + 2*sqrt(x^4 + 5*x^2 + 3) + 5)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.82

$$\int \frac{x^3(2+3x^2)}{\sqrt{3+5x^2+x^4}}\,dx = \frac{1}{8}\sqrt{x^4+5\,x^2+3}\left(6\,x^2-37\right) - \frac{149}{16}\log\left(2\,x^2-2\,\sqrt{x^4+5\,x^2+3}+5\right)$$

```
[In] integrate(x^3*(3*x^2+2)/(x^4+5*x^2+3)^(1/2),x, algorithm="giac")
[Out] 1/8*sqrt(x^4 + 5*x^2 + 3)*(6*x^2 - 37) - 149/16*log(2*x^2 - 2*sqrt(x^4 + 5*
x^2 + 3) + 5)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3(2+3x^2)}{\sqrt{3+5x^2+x^4}}\,dx = \int \frac{x^3\left(3\,x^2+2\right)}{\sqrt{x^4+5\,x^2+3}}\,dx$$

```
[In] int((x^3*(3*x^2 + 2))/(5*x^2 + x^4 + 3)^(1/2),x)
[Out] int((x^3*(3*x^2 + 2))/(5*x^2 + x^4 + 3)^(1/2), x)
```

3.184 $\int \frac{x(2+3x^2)}{\sqrt{3+5x^2+x^4}} \, dx$

Optimal result

Integrand size = 23, antiderivative size = 49

$$\int \frac{x(2+3x^2)}{\sqrt{3+5x^2+x^4}} \, dx = \frac{3}{2}\sqrt{3+5x^2+x^4} - \frac{11}{4}\text{arctanh}\left(\frac{5+2x^2}{2\sqrt{3+5x^2+x^4}}\right)$$

```
[Out] -11/4*arctanh(1/2*(2*x^2+5)/(x^4+5*x^2+3)^(1/2))+3/2*(x^4+5*x^2+3)^(1/2)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {1261, 654, 635, 212}

$$\int \frac{x(2+3x^2)}{\sqrt{3+5x^2+x^4}} \, dx = \frac{3}{2}\sqrt{x^4+5x^2+3} - \frac{11}{4}\text{arctanh}\left(\frac{2x^2+5}{2\sqrt{x^4+5x^2+3}}\right)$$

```
[In] Int[(x*(2 + 3*x^2))/Sqrt[3 + 5*x^2 + x^4],x]
```

```
[Out] (3*Sqrt[3 + 5*x^2 + x^4])/2 - (11*ArcTanh[(5 + 2*x^2)/(2*Sqrt[3 + 5*x^2 + x
^4])])/4
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 635

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
```

```
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 654

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 1261

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(
p_.), x_Symbol] :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x],
 x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{1}{2}\text{Subst}\left(\int \frac{2+3x}{\sqrt{3+5x+x^2}}\,dx, x, x^2\right)\\
&= \frac{3}{2}\sqrt{3+5x^2+x^4} - \frac{11}{4}\text{Subst}\left(\int \frac{1}{\sqrt{3+5x+x^2}}\,dx, x, x^2\right)\\
&= \frac{3}{2}\sqrt{3+5x^2+x^4} - \frac{11}{2}\text{Subst}\left(\int \frac{1}{4-x^2}\,dx, x, \frac{5+2x^2}{\sqrt{3+5x^2+x^4}}\right)\\
&= \frac{3}{2}\sqrt{3+5x^2+x^4} - \frac{11}{4}\tanh^{-1}\left(\frac{5+2x^2}{2\sqrt{3+5x^2+x^4}}\right)
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.96

$$
\int \frac{x(2+3x^2)}{\sqrt{3+5x^2+x^4}}\,dx = \frac{3}{2}\sqrt{3+5x^2+x^4} + \frac{11}{4}\log\left(-5-2x^2+2\sqrt{3+5x^2+x^4}\right)
$$

```
[In] Integrate[(x*(2 + 3*x^2))/Sqrt[3 + 5*x^2 + x^4],x]
[Out] (3*Sqrt[3 + 5*x^2 + x^4])/2 + (11*Log[-5 - 2*x^2 + 2*Sqrt[3 + 5*x^2 + x^4]]
)/4
```

Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.73

method	result	size
default	$-\frac{11 \ln\left(\frac{5}{2}+x^2+\sqrt{x^4+5x^2+3}\right)}{4}+\frac{3\sqrt{x^4+5x^2+3}}{2}$	36
risch	$-\frac{11 \ln\left(\frac{5}{2}+x^2+\sqrt{x^4+5x^2+3}\right)}{4}+\frac{3\sqrt{x^4+5x^2+3}}{2}$	36
elliptic	$-\frac{11 \ln\left(\frac{5}{2}+x^2+\sqrt{x^4+5x^2+3}\right)}{4}+\frac{3\sqrt{x^4+5x^2+3}}{2}$	36
trager	$\frac{3\sqrt{x^4+5x^2+3}}{2}+\frac{11 \ln\left(-2x^2+2\sqrt{x^4+5x^2+3}-5\right)}{4}$	40
pseudoelliptic	$-\frac{11 \ln\left(2x^2+5+2\sqrt{x^4+5x^2+3}\right)}{4}+\frac{3\sqrt{x^4+5x^2+3}}{2}$	40

```
[In] int(x*(3*x^2+2)/(x^4+5*x^2+3)^(1/2),x,method=_RETURNVERBOSE)
[Out] -11/4*ln(5/2+x^2+(x^4+5*x^2+3)^(1/2))+3/2*(x^4+5*x^2+3)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.80

$$\int \frac{x(2+3x^2)}{\sqrt{3+5x^2+x^4}}\,dx = \frac{3}{2}\,\sqrt{x^4+5\,x^2+3}+\frac{11}{4}\,\log\left(-2\,x^2+2\,\sqrt{x^4+5\,x^2+3}-5\right)$$

```
[In] integrate(x*(3*x^2+2)/(x^4+5*x^2+3)^(1/2),x, algorithm="fricas")
[Out] 3/2*sqrt(x^4 + 5*x^2 + 3) + 11/4*log(-2*x^2 + 2*sqrt(x^4 + 5*x^2 + 3) - 5)
```

Sympy [A] (verification not implemented)

Time = 0.80 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.86

$$\int \frac{x(2+3x^2)}{\sqrt{3+5x^2+x^4}}\,dx = \frac{3\sqrt{x^4+5x^2+3}}{2}-\frac{11\log\left(2x^2+2\sqrt{x^4+5x^2+3}+5\right)}{4}$$

```
[In] integrate(x*(3*x**2+2)/(x**4+5*x**2+3)**(1/2),x)
[Out] 3*sqrt(x**4 + 5*x**2 + 3)/2 - 11*log(2*x**2 + 2*sqrt(x**4 + 5*x**2 + 3) + 5
)/4
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.80

$$\int \frac{x(2+3x^2)}{\sqrt{3+5x^2+x^4}}\,dx = \frac{3}{2}\,\sqrt{x^4+5\,x^2+3}-\frac{11}{4}\,\log\left(2\,x^2+2\,\sqrt{x^4+5\,x^2+3}+5\right)$$

```
[In] integrate(x*(3*x^2+2)/(x^4+5*x^2+3)^(1/2),x, algorithm="maxima")
[Out] 3/2*sqrt(x^4 + 5*x^2 + 3) - 11/4*log(2*x^2 + 2*sqrt(x^4 + 5*x^2 + 3) + 5)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.80

$$\int \frac{x(2+3x^2)}{\sqrt{3+5x^2+x^4}}\,dx = \frac{3}{2}\,\sqrt{x^4+5\,x^2+3}+\frac{11}{4}\,\log\left(2\,x^2-2\,\sqrt{x^4+5\,x^2+3}+5\right)$$

```
[In] integrate(x*(3*x^2+2)/(x^4+5*x^2+3)^(1/2),x, algorithm="giac")
[Out] 3/2*sqrt(x^4 + 5*x^2 + 3) + 11/4*log(2*x^2 - 2*sqrt(x^4 + 5*x^2 + 3) + 5)
```

Mupad [B] (verification not implemented)

Time = 8.10 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.71

$$\int \frac{x(2+3x^2)}{\sqrt{3+5x^2+x^4}}\,dx = \frac{3\,\sqrt{x^4+5\,x^2+3}}{2}-\frac{11\,\ln\left(\sqrt{x^4+5\,x^2+3}+x^2+\frac{5}{2}\right)}{4}$$

```
[In] int((x*(3*x^2 + 2))/(5*x^2 + x^4 + 3)^(1/2),x)
[Out] (3*(5*x^2 + x^4 + 3)^(1/2))/2 - (11*log((5*x^2 + x^4 + 3)^(1/2) + x^2 + 5/2
))/4
```

3.185 $\int \frac{2+3x^2}{x\sqrt{3+5x^2+x^4}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 69

$$\int \frac{2+3x^2}{x\sqrt{3+5x^2+x^4}}\,dx = \frac{3}{2}\text{arctanh}\left(\frac{5+2x^2}{2\sqrt{3+5x^2+x^4}}\right) - \frac{\text{arctanh}\left(\frac{6+5x^2}{2\sqrt{3}\sqrt{3+5x^2+x^4}}\right)}{\sqrt{3}}$$

```
[Out] 3/2*arctanh(1/2*(2*x^2+5)/(x^4+5*x^2+3)^(1/2))-1/3*arctanh(1/6*(5*x^2+6)*3^
(1/2)/(x^4+5*x^2+3)^(1/2))*3^(1/2)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {1265, 857, 635, 212, 738}

$$\int \frac{2+3x^2}{x\sqrt{3+5x^2+x^4}}\,dx = \frac{3}{2}\text{arctanh}\left(\frac{2x^2+5}{2\sqrt{x^4+5x^2+3}}\right) - \frac{\text{arctanh}\left(\frac{5x^2+6}{2\sqrt{3}\sqrt{x^4+5x^2+3}}\right)}{\sqrt{3}}$$

```
[In] Int[(2 + 3*x^2)/(x*Sqrt[3 + 5*x^2 + x^4]),x]
```

```
[Out] (3*ArcTanh[(5 + 2*x^2)/(2*Sqrt[3 + 5*x^2 + x^4])])/2 - ArcTanh[(6 + 5*x^2)/
(2*Sqrt[3]*Sqrt[3 + 5*x^2 + x^4])]/Sqrt[3]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 635

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 857

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1265

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{2}\text{Subst}\left(\int \frac{2+3x}{x\sqrt{3+5x+x^2}}\,dx, x, x^2\right)\\
&= \frac{3}{2}\text{Subst}\left(\int \frac{1}{\sqrt{3+5x+x^2}}\,dx, x, x^2\right) + \text{Subst}\left(\int \frac{1}{x\sqrt{3+5x+x^2}}\,dx, x, x^2\right)\\
&= -\left(2\text{Subst}\left(\int \frac{1}{12-x^2}\,dx, x, \frac{6+5x^2}{\sqrt{3+5x^2+x^4}}\right)\right) + 3\text{Subst}\left(\int \frac{1}{4-x^2}\,dx, x, \frac{5+2x^2}{\sqrt{3+5x^2+x^4}}\right)\\
&= \frac{3}{2}\tanh^{-1}\left(\frac{5+2x^2}{2\sqrt{3+5x^2+x^4}}\right) - \frac{\tanh^{-1}\left(\frac{6+5x^2}{2\sqrt{3}\sqrt{3+5x^2+x^4}}\right)}{\sqrt{3}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.91

$$\int \frac{2+3x^2}{x\sqrt{3+5x^2+x^4}}\,dx = \frac{2\text{arctanh}\left(\frac{x^2-\sqrt{3+5x^2+x^4}}{\sqrt{3}}\right)}{\sqrt{3}} - \frac{3}{2}\log\left(-5-2x^2+2\sqrt{3+5x^2+x^4}\right)$$

```
[In] Integrate[(2 + 3*x^2)/(x*Sqrt[3 + 5*x^2 + x^4]),x]
[Out] (2*ArcTanh[(x^2 - Sqrt[3 + 5*x^2 + x^4])/Sqrt[3]])/Sqrt[3] - (3*Log[-5 - 2*
x^2 + 2*Sqrt[3 + 5*x^2 + x^4]])/2
```

Maple [A] (verified)

Time = 0.45 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.75

method	result
default	$\frac{3\ln\left(\frac{5}{2}+x^2+\sqrt{x^4+5x^2+3}\right)}{2}-\frac{\text{arctanh}\left(\frac{(5x^2+6)\sqrt{3}}{6\sqrt{x^4+5x^2+3}}\right)\sqrt{3}}{3}$
elliptic	$\frac{3\ln\left(\frac{5}{2}+x^2+\sqrt{x^4+5x^2+3}\right)}{2}-\frac{\text{arctanh}\left(\frac{(5x^2+6)\sqrt{3}}{6\sqrt{x^4+5x^2+3}}\right)\sqrt{3}}{3}$
pseudoelliptic	$\frac{3\ln\left(2x^2+5+2\sqrt{x^4+5x^2+3}\right)}{2}-\frac{\text{arctanh}\left(\frac{(5x^2+6)\sqrt{3}}{6\sqrt{x^4+5x^2+3}}\right)\sqrt{3}}{3}$
trager	$-\frac{\text{RootOf}\left(\_Z^2-3\right)\ln\left(-\frac{5\,\text{RootOf}\left(\_Z^2-3\right)x^2+6\sqrt{x^4+5x^2+3}+6\,\text{RootOf}\left(\_Z^2-3\right)}{x^2}\right)}{3}+\frac{3\ln\left(-2x^2-2\sqrt{x^4+5x^2+3}-5\right)}{2}$

```
[In] int((3*x^2+2)/x/(x^4+5*x^2+3)^(1/2),x,method=_RETURNVERBOSE)
[Out] 3/2*ln(5/2+x^2+(x^4+5*x^2+3)^(1/2))-1/3*arctanh(1/6*(5*x^2+6)*3^(1/2)/(x^4+
5*x^2+3)^(1/2))*3^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.09

$$\begin{aligned}\int \frac{2+3x^2}{x\sqrt{3+5x^2+x^4}}\,dx &= \frac{1}{3}\sqrt{3}\log\left(\frac{25\,x^2-2\sqrt{3}(5\,x^2+6)-2\sqrt{x^4+5\,x^2+3}(5\sqrt{3}-6)+30}{x^2}\right)\\ &\quad -\frac{3}{2}\log\left(-2\,x^2+2\sqrt{x^4+5\,x^2+3}-5\right)\end{aligned}$$

```
[In] integrate((3*x^2+2)/x/(x^4+5*x^2+3)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/3*sqrt(3)*log((25*x^2 - 2*sqrt(3)*(5*x^2 + 6) - 2*sqrt(x^4 + 5*x^2 + 3)*(
5*sqrt(3) - 6) + 30)/x^2) - 3/2*log(-2*x^2 + 2*sqrt(x^4 + 5*x^2 + 3) - 5)
```

Sympy [F]

$$\int \frac{2+3x^2}{x\sqrt{3+5x^2+x^4}}\,dx = \int \frac{3x^2+2}{x\sqrt{x^4+5x^2+3}}\,dx$$

```
[In] integrate((3*x**2+2)/x/(x**4+5*x**2+3)**(1/2),x)
```

```
[Out] Integral((3*x**2 + 2)/(x*sqrt(x**4 + 5*x**2 + 3)), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.84

$$\int \frac{2+3x^2}{x\sqrt{3+5x^2+x^4}}\,dx = -\frac{1}{3}\sqrt{3}\log\left(\frac{2\sqrt{3}\sqrt{x^4+5x^2+3}}{x^2}+\frac{6}{x^2}+5\right) + \frac{3}{2}\log\left(2x^2+2\sqrt{x^4+5x^2+3}+5\right)$$

```
[In] integrate((3*x^2+2)/x/(x^4+5*x^2+3)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/3*sqrt(3)*log(2*sqrt(3)*sqrt(x^4 + 5*x^2 + 3)/x^2 + 6/x^2 + 5) + 3/2*log
(2*x^2 + 2*sqrt(x^4 + 5*x^2 + 3) + 5)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.13

$$\int \frac{2+3x^2}{x\sqrt{3+5x^2+x^4}}\,dx = \frac{1}{3}\sqrt{3}\log\left(\frac{x^2+\sqrt{3}-\sqrt{x^4+5x^2+3}}{x^2-\sqrt{3}-\sqrt{x^4+5x^2+3}}\right) - \frac{3}{2}\log\left(2x^2-2\sqrt{x^4+5x^2+3}+5\right)$$

```
[In] integrate((3*x^2+2)/x/(x^4+5*x^2+3)^(1/2),x, algorithm="giac")
```

```
[Out] 1/3*sqrt(3)*log((x^2 + sqrt(3) - sqrt(x^4 + 5*x^2 + 3))/(x^2 - sqrt(3) - sq
rt(x^4 + 5*x^2 + 3))) - 3/2*log(2*x^2 - 2*sqrt(x^4 + 5*x^2 + 3) + 5)
```

Mupad [B] (verification not implemented)

Time = 8.20 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.81

$$\int \frac{2+3x^2}{x\sqrt{3+5x^2+x^4}}\,dx = \frac{3\,\ln\left(\sqrt{x^4+5\,x^2+3}+x^2+\frac{5}{2}\right)}{2} - \frac{\sqrt{3}\,\left(\ln\left(\frac{1}{x^2}\right)+\ln\left(2\,\sqrt{3}\,\sqrt{x^4+5\,x^2+3}+5\,x^2+6\right)\right)}{3}$$

```
[In] int((3*x^2 + 2)/(x*(5*x^2 + x^4 + 3)^(1/2)),x)
```

```
[Out] (3*log((5*x^2 + x^4 + 3)^(1/2) + x^2 + 5/2))/2 - (3^(1/2)*(log(1/x^2) + log
(2*3^(1/2)*(5*x^2 + x^4 + 3)^(1/2) + 5*x^2 + 6)))/3
```

3.186 $\int \frac{2+3x^2}{x^3\sqrt{3+5x^2+x^4}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 62

$$\int \frac{2+3x^2}{x^3\sqrt{3+5x^2+x^4}}\,dx = -\frac{\sqrt{3+5x^2+x^4}}{3x^2} - \frac{2\text{arctanh}\left(\frac{6+5x^2}{2\sqrt{3}\sqrt{3+5x^2+x^4}}\right)}{3\sqrt{3}}$$

```
[Out] -2/9*arctanh(1/6*(5*x^2+6)*3^(1/2)/(x^4+5*x^2+3)^(1/2))*3^(1/2)-1/3*(x^4+5*
x^2+3)^(1/2)/x^2
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {1265, 820, 738, 212}

$$\int \frac{2+3x^2}{x^3\sqrt{3+5x^2+x^4}}\,dx = -\frac{2\text{arctanh}\left(\frac{5x^2+6}{2\sqrt{3}\sqrt{x^4+5x^2+3}}\right)}{3\sqrt{3}} - \frac{\sqrt{x^4+5x^2+3}}{3x^2}$$

```
[In] Int[(2 + 3*x^2)/(x^3*Sqrt[3 + 5*x^2 + x^4]),x]
```

```
[Out] -1/3*Sqrt[3 + 5*x^2 + x^4]/x^2 - (2*ArcTanh[(6 + 5*x^2)/(2*Sqrt[3]*Sqrt[3 +
 5*x^2 + x^4])])/(3*Sqrt[3])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 820

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a +
 b*x + c*x^2)^(p + 1)/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2))), x] - Dist[(b*(e
*f + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(
m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[Simplify[m
+ 2*p + 3], 0]
```

Rule 1265

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{2}\text{Subst}\left(\int \frac{2+3x}{x^2\sqrt{3+5x+x^2}}\,dx, x, x^2\right)\\
&= -\frac{\sqrt{3+5x^2+x^4}}{3x^2}+\frac{2}{3}\text{Subst}\left(\int \frac{1}{x\sqrt{3+5x+x^2}}\,dx, x, x^2\right)\\
&= -\frac{\sqrt{3+5x^2+x^4}}{3x^2}-\frac{4}{3}\text{Subst}\left(\int \frac{1}{12-x^2}\,dx, x, \frac{6+5x^2}{\sqrt{3+5x^2+x^4}}\right)\\
&= -\frac{\sqrt{3+5x^2+x^4}}{3x^2}-\frac{2\tanh^{-1}\left(\frac{6+5x^2}{2\sqrt{3}\sqrt{3+5x^2+x^4}}\right)}{3\sqrt{3}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.94

$$\int \frac{2+3x^2}{x^3\sqrt{3+5x^2+x^4}}\,dx = \frac{1}{9}\left(-\frac{3\sqrt{3+5x^2+x^4}}{x^2}+4\sqrt{3}\text{arctanh}\left(\frac{x^2-\sqrt{3+5x^2+x^4}}{\sqrt{3}}\right)\right)$$

```
[In] Integrate[(2 + 3*x^2)/(x^3*Sqrt[3 + 5*x^2 + x^4]),x]
[Out] ((-3*Sqrt[3 + 5*x^2 + x^4])/x^2 + 4*Sqrt[3]*ArcTanh[(x^2 - Sqrt[3 + 5*x^2 +
 x^4])/Sqrt[3]])/9
```

Maple [A] (verified)

Time = 0.33 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.79

method	result	size
default	$-\frac{2\,\mathrm{arctanh}\left(\frac{\left(5x^2+6\right)\sqrt{3}}{6\sqrt{x^4+5x^2+3}}\right)\sqrt{3}}{9}-\frac{\sqrt{x^4+5x^2+3}}{3x^2}$	49
risch	$-\frac{2\,\mathrm{arctanh}\left(\frac{\left(5x^2+6\right)\sqrt{3}}{6\sqrt{x^4+5x^2+3}}\right)\sqrt{3}}{9}-\frac{\sqrt{x^4+5x^2+3}}{3x^2}$	49
elliptic	$-\frac{2\,\mathrm{arctanh}\left(\frac{\left(5x^2+6\right)\sqrt{3}}{6\sqrt{x^4+5x^2+3}}\right)\sqrt{3}}{9}-\frac{\sqrt{x^4+5x^2+3}}{3x^2}$	49
pseudoelliptic	$\frac{-2\,\mathrm{arctanh}\left(\frac{\left(5x^2+6\right)\sqrt{3}}{6\sqrt{x^4+5x^2+3}}\right)\sqrt{3}\,x^2-3\sqrt{x^4+5x^2+3}}{9x^2}$	54
trager	$-\frac{\sqrt{x^4+5x^2+3}}{3x^2}-\frac{2\,\mathrm{RootOf}\left(\_Z^2-3\right)\ln\left(-\frac{5\,\mathrm{RootOf}\left(\_Z^2-3\right)x^2+6\sqrt{x^4+5x^2+3}+6\,\mathrm{RootOf}\left(\_Z^2-3\right)}{x^2}\right)}{9}$	67

```
[In] int((3*x^2+2)/x^3/(x^4+5*x^2+3)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/9*arctanh(1/6*(5*x^2+6)*3^(1/2)/(x^4+5*x^2+3)^(1/2))*3^(1/2)-1/3*(x^4+5*
x^2+3)^(1/2)/x^2
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.26

$$\int \frac{2+3x^2}{x^3\sqrt{3+5x^2+x^4}}\,dx = \frac{2\sqrt{3}x^2\log\left(\frac{25\,x^2-2\,\sqrt{3}(5\,x^2+6)-2\,\sqrt{x^4+5\,x^2+3}\left(5\,\sqrt{3}-6\right)+30}{x^2}\right)-3\,x^2-3\,\sqrt{x^4+5\,x^2+3}}{9\,x^2}$$

```
[In] integrate((3*x^2+2)/x^3/(x^4+5*x^2+3)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/9*(2*sqrt(3)*x^2*log((25*x^2 - 2*sqrt(3)*(5*x^2 + 6) - 2*sqrt(x^4 + 5*x^2
 + 3)*(5*sqrt(3) - 6) + 30)/x^2) - 3*x^2 - 3*sqrt(x^4 + 5*x^2 + 3))/x^2
```

Sympy [F]

$$\int \frac{2+3x^2}{x^3\sqrt{3+5x^2+x^4}}\,dx = \int \frac{3x^2+2}{x^3\sqrt{x^4+5x^2+3}}\,dx$$

```
[In] integrate((3*x**2+2)/x**3/(x**4+5*x**2+3)**(1/2),x)
[Out] Integral((3*x**2 + 2)/(x**3*sqrt(x**4 + 5*x**2 + 3)), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.82

$$\int \frac{2+3x^2}{x^3\sqrt{3+5x^2+x^4}}\,dx = -\frac{2}{9}\sqrt{3}\log\left(\frac{2\sqrt{3}\sqrt{x^4+5x^2+3}}{x^2}+\frac{6}{x^2}+5\right)-\frac{\sqrt{x^4+5x^2+3}}{3x^2}$$

```
[In] integrate((3*x^2+2)/x^3/(x^4+5*x^2+3)^(1/2),x, algorithm="maxima")
[Out] -2/9*sqrt(3)*log(2*sqrt(3)*sqrt(x^4 + 5*x^2 + 3)/x^2 + 6/x^2 + 5) - 1/3*sqr
t(x^4 + 5*x^2 + 3)/x^2
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 101 vs. $2(48) = 96$.

Time = 0.29 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.63

$$\int \frac{2+3x^2}{x^3\sqrt{3+5x^2+x^4}}\,dx = \frac{2}{9}\sqrt{3}\log\left(\frac{x^2+\sqrt{3}-\sqrt{x^4+5x^2+3}}{x^2-\sqrt{3}-\sqrt{x^4+5x^2+3}}\right) + \frac{5x^2-5\sqrt{x^4+5x^2+3}+6}{3\left(\left(x^2-\sqrt{x^4+5x^2+3}\right)^2-3\right)}$$

```
[In] integrate((3*x^2+2)/x^3/(x^4+5*x^2+3)^(1/2),x, algorithm="giac")
[Out] 2/9*sqrt(3)*log((x^2 + sqrt(3) - sqrt(x^4 + 5*x^2 + 3))/(x^2 - sqrt(3) - sq
rt(x^4 + 5*x^2 + 3))) + 1/3*(5*x^2 - 5*sqrt(x^4 + 5*x^2 + 3) + 6)/((x^2 - s
qrt(x^4 + 5*x^2 + 3))^2 - 3)
```

Mupad [B] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.34

$$\int \frac{2+3x^2}{x^3\sqrt{3+5x^2+x^4}}\,dx = \frac{5\sqrt{3}\,\mathrm{atanh}\left(\frac{\sqrt{3}\,(5\,x^2+6)}{6\,\sqrt{x^4+5\,x^2+3}}\right)}{18} - \frac{\sqrt{x^4+5\,x^2+3}}{3\,x^2} - \frac{\sqrt{3}\,\left(\ln\left(\frac{1}{x^2}\right)+\ln\left(2\,\sqrt{3}\,\sqrt{x^4+5\,x^2+3}+5\,x^2+6\right)\right)}{2}$$

```
[In] int((3*x^2 + 2)/(x^3*(5*x^2 + x^4 + 3)^(1/2)),x)
```

```
[Out] (5*3^(1/2)*atanh((3^(1/2)*(5*x^2 + 6))/(6*(5*x^2 + x^4 + 3)^(1/2))))/18 - (
5*x^2 + x^4 + 3)^(1/2)/(3*x^2) - (3^(1/2)*(log(1/x^2) + log(2*3^(1/2)*(5*x^
2 + x^4 + 3)^(1/2) + 5*x^2 + 6)))/2
```

3.187 $\int \frac{2+3x^2}{x^5\sqrt{3+5x^2+x^4}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 83

$$\int \frac{2+3x^2}{x^5\sqrt{3+5x^2+x^4}}\,dx=-\frac{\sqrt{3+5x^2+x^4}}{6x^4}-\frac{\sqrt{3+5x^2+x^4}}{12x^2}+\frac{1}{8}\sqrt{3}\text{arctanh}\left(\frac{6+5x^2}{2\sqrt{3}\sqrt{3+5x^2+x^4}}\right)$$

```
[Out] 1/8*arctanh(1/6*(5*x^2+6)*3^(1/2)/(x^4+5*x^2+3)^(1/2))*3^(1/2)-1/6*(x^4+5*x
^2+3)^(1/2)/x^4-1/12*(x^4+5*x^2+3)^(1/2)/x^2
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {1265, 848, 820, 738, 212}

$$\int \frac{2+3x^2}{x^5\sqrt{3+5x^2+x^4}}\,dx=\frac{1}{8}\sqrt{3}\text{arctanh}\left(\frac{5x^2+6}{2\sqrt{3}\sqrt{x^4+5x^2+3}}\right)-\frac{\sqrt{x^4+5x^2+3}}{12x^2}-\frac{\sqrt{x^4+5x^2+3}}{6x^4}$$

```
[In] Int[(2 + 3*x^2)/(x^5*Sqrt[3 + 5*x^2 + x^4]),x]
```

```
[Out] -1/6*Sqrt[3 + 5*x^2 + x^4]/x^4 - Sqrt[3 + 5*x^2 + x^4]/(12*x^2) + (Sqrt[3]*
ArcTanh[(6 + 5*x^2)/(2*Sqrt[3]*Sqrt[3 + 5*x^2 + x^4])])/8
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 820

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a +
 b*x + c*x^2)^(p + 1)/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2))), x] - Dist[(b*(e
*f + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(
m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[Simplify[m
+ 2*p + 3], 0]
```

Rule 848

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*f - d*g)*(d + e*x)^(m + 1)*((a + b*
x + c*x^2)^(p + 1)/((m + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Dist[1/((m + 1)
*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*Simp[(
c*d*f - f*b*e + a*e*g)*(m + 1) + b*(d*g - e*f)*(p + 1) - c*(e*f - d*g)*(m +
 2*p + 3)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] ||
 IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 1265

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{2}\text{Subst}\left(\int \frac{2+3x}{x^3\sqrt{3+5x+x^2}}\,dx,x,x^2\right)\\
&= -\frac{\sqrt{3+5x^2+x^4}}{6x^4}-\frac{1}{12}\text{Subst}\left(\int \frac{-3+2x}{x^2\sqrt{3+5x+x^2}}\,dx,x,x^2\right)\\
&= -\frac{\sqrt{3+5x^2+x^4}}{6x^4}-\frac{\sqrt{3+5x^2+x^4}}{12x^2}-\frac{3}{8}\text{Subst}\left(\int \frac{1}{x\sqrt{3+5x+x^2}}\,dx,x,x^2\right)
\end{aligned}$$

$$= -\frac{\sqrt{3+5x^2+x^4}}{6x^4} - \frac{\sqrt{3+5x^2+x^4}}{12x^2} + \frac{3}{4}\text{Subst}\left(\int \frac{1}{12-x^2}\,dx, x, \frac{6+5x^2}{\sqrt{3+5x^2+x^4}}\right)$$

$$= -\frac{\sqrt{3+5x^2+x^4}}{6x^4} - \frac{\sqrt{3+5x^2+x^4}}{12x^2} + \frac{1}{8}\sqrt{3}\tanh^{-1}\left(\frac{6+5x^2}{2\sqrt{3}\sqrt{3+5x^2+x^4}}\right)$$

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.76

$$\int \frac{2+3x^2}{x^5\sqrt{3+5x^2+x^4}}\,dx = -\frac{(2+x^2)\sqrt{3+5x^2+x^4}}{12x^4} - \frac{1}{4}\sqrt{3}\text{arctanh}\left(\frac{x^2-\sqrt{3+5x^2+x^4}}{\sqrt{3}}\right)$$

```
[In] Integrate[(2 + 3*x^2)/(x^5*Sqrt[3 + 5*x^2 + x^4]),x]
[Out] -1/12*((2 + x^2)*Sqrt[3 + 5*x^2 + x^4])/x^4 - (Sqrt[3]*ArcTanh[(x^2 - Sqrt[
3 + 5*x^2 + x^4])/Sqrt[3]])/4
```

Maple [A] (verified)

Time = 0.36 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.77

method	result	size
risch	$-\frac{x^6+7x^4+13x^2+6}{12x^4\sqrt{x^4+5x^2+3}}+\frac{\text{arctanh}\left(\frac{(5x^2+6)\sqrt{3}}{6\sqrt{x^4+5x^2+3}}\right)\sqrt{3}}{8}$	64
default	$\frac{\text{arctanh}\left(\frac{(5x^2+6)\sqrt{3}}{6\sqrt{x^4+5x^2+3}}\right)\sqrt{3}}{8}-\frac{\sqrt{x^4+5x^2+3}}{6x^4}-\frac{\sqrt{x^4+5x^2+3}}{12x^2}$	66
elliptic	$\frac{\text{arctanh}\left(\frac{(5x^2+6)\sqrt{3}}{6\sqrt{x^4+5x^2+3}}\right)\sqrt{3}}{8}-\frac{\sqrt{x^4+5x^2+3}}{6x^4}-\frac{\sqrt{x^4+5x^2+3}}{12x^2}$	66
pseudoelliptic	$\frac{3\,\text{arctanh}\left(\frac{(5x^2+6)\sqrt{3}}{6\sqrt{x^4+5x^2+3}}\right)\sqrt{3}\,x^4-2x^2\sqrt{x^4+5x^2+3}-4\sqrt{x^4+5x^2+3}}{24x^4}$	71
trager	$-\frac{(x^2+2)\sqrt{x^4+5x^2+3}}{12x^4}-\frac{\text{RootOf}\left(\_Z^2-3\right)\ln\left(-\frac{-5\,\text{RootOf}\left(\_Z^2-3\right)x^2+6\sqrt{x^4+5x^2+3}-6\,\text{RootOf}\left(\_Z^2-3\right)}{x^2}\right)}{8}$	72

```
[In] int((3*x^2+2)/x^5/(x^4+5*x^2+3)^(1/2),x,method=_RETURNVERBOSE)
[Out] -1/12*(x^6+7*x^4+13*x^2+6)/x^4/(x^4+5*x^2+3)^(1/2)+1/8*arctanh(1/6*(5*x^2+6
)*3^(1/2)/(x^4+5*x^2+3)^(1/2))*3^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.00

$$\int \frac{2+3x^2}{x^5\sqrt{3+5x^2+x^4}}\,dx = \frac{3\sqrt{3}x^4\log\left(\frac{25\,x^2+2\,\sqrt{3}(5\,x^2+6)+2\,\sqrt{x^4+5\,x^2+3}\left(5\,\sqrt{3}+6\right)+30}{x^2}\right)-2\,x^4-2\,\sqrt{x^4+5\,x^2+3}(x^2+2)}{24\,x^4}$$

```
[In] integrate((3*x^2+2)/x^5/(x^4+5*x^2+3)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/24*(3*sqrt(3)*x^4*log((25*x^2 + 2*sqrt(3)*(5*x^2 + 6) + 2*sqrt(x^4 + 5*x^
2 + 3)*(5*sqrt(3) + 6) + 30)/x^2) - 2*x^4 - 2*sqrt(x^4 + 5*x^2 + 3)*(x^2 +
2))/x^4
```

Sympy [F]

$$\int \frac{2+3x^2}{x^5\sqrt{3+5x^2+x^4}}\,dx = \int \frac{3x^2+2}{x^5\sqrt{x^4+5x^2+3}}\,dx$$

```
[In] integrate((3*x**2+2)/x**5/(x**4+5*x**2+3)**(1/2),x)
```

```
[Out] Integral((3*x**2 + 2)/(x**5*sqrt(x**4 + 5*x**2 + 3)), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.82

$$\int \frac{2+3x^2}{x^5\sqrt{3+5x^2+x^4}}\,dx = \frac{1}{8}\sqrt{3}\log\left(\frac{2\,\sqrt{3}\sqrt{x^4+5\,x^2+3}}{x^2}+\frac{6}{x^2}+5\right) - \frac{\sqrt{x^4+5\,x^2+3}}{12\,x^2} - \frac{\sqrt{x^4+5\,x^2+3}}{6\,x^4}$$

```
[In] integrate((3*x^2+2)/x^5/(x^4+5*x^2+3)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/8*sqrt(3)*log(2*sqrt(3)*sqrt(x^4 + 5*x^2 + 3)/x^2 + 6/x^2 + 5) - 1/12*sqr
t(x^4 + 5*x^2 + 3)/x^2 - 1/6*sqrt(x^4 + 5*x^2 + 3)/x^4
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 145 vs. $2(65) = 130$.

Time = 0.29 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.75

$$\int \frac{2+3x^2}{x^5\sqrt{3+5x^2+x^4}}\,dx = -\frac{1}{8}\sqrt{3}\log\left(\frac{x^2+\sqrt{3}-\sqrt{x^4+5x^2+3}}{x^2-\sqrt{3}-\sqrt{x^4+5x^2+3}}\right) + \frac{9\left(x^2-\sqrt{x^4+5x^2+3}\right)^3+36\left(x^2-\sqrt{x^4+5x^2+3}\right)^2+47x^2-47\sqrt{x^4+5x^2+3}+12}{12\left(\left(x^2-\sqrt{x^4+5x^2+3}\right)^2-3\right)^2}$$

```
[In] integrate((3*x^2+2)/x^5/(x^4+5*x^2+3)^(1/2),x, algorithm="giac")
```

```
[Out] -1/8*sqrt(3)*log((x^2 + sqrt(3) - sqrt(x^4 + 5*x^2 + 3))/(x^2 - sqrt(3) - s
qrt(x^4 + 5*x^2 + 3))) + 1/12*(9*(x^2 - sqrt(x^4 + 5*x^2 + 3))^3 + 36*(x^2
- sqrt(x^4 + 5*x^2 + 3))^2 + 47*x^2 - 47*sqrt(x^4 + 5*x^2 + 3) + 12)/((x^2
- sqrt(x^4 + 5*x^2 + 3))^2 - 3)^2
```

Mupad [F(-1)]

Timed out.

$$\int \frac{2+3x^2}{x^5\sqrt{3+5x^2+x^4}}\,dx = \int \frac{3x^2+2}{x^5\sqrt{x^4+5x^2+3}}\,dx$$

```
[In] int((3*x^2 + 2)/(x^5*(5*x^2 + x^4 + 3)^(1/2)),x)
```

```
[Out] int((3*x^2 + 2)/(x^5*(5*x^2 + x^4 + 3)^(1/2)), x)
```

3.188 $\int \frac{2+3x^2}{x^7\sqrt{3+5x^2+x^4}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 104

$$\int \frac{2+3x^2}{x^7\sqrt{3+5x^2+x^4}}\,dx = -\frac{\sqrt{3+5x^2+x^4}}{9x^6}-\frac{\sqrt{3+5x^2+x^4}}{54x^4}+\frac{13\sqrt{3+5x^2+x^4}}{108x^2}-\frac{61\text{arctanh}\left(\frac{6+5x^2}{2\sqrt{3}\sqrt{3+5x^2+x^4}}\right)}{216\sqrt{3}}$$

```
[Out] -61/648*arctanh(1/6*(5*x^2+6)*3^(1/2)/(x^4+5*x^2+3)^(1/2))*3^(1/2)-1/9*(x^4
+5*x^2+3)^(1/2)/x^6-1/54*(x^4+5*x^2+3)^(1/2)/x^4+13/108*(x^4+5*x^2+3)^(1/2)
/x^2
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {1265, 848, 820, 738, 212}

$$\int \frac{2+3x^2}{x^7\sqrt{3+5x^2+x^4}}\,dx = -\frac{61\text{arctanh}\left(\frac{5x^2+6}{2\sqrt{3}\sqrt{x^4+5x^2+3}}\right)}{216\sqrt{3}}+\frac{13\sqrt{x^4+5x^2+3}}{108x^2}-\frac{\sqrt{x^4+5x^2+3}}{54x^4}-\frac{\sqrt{x^4+5x^2+3}}{9x^6}$$

```
[In] Int[(2 + 3*x^2)/(x^7*Sqrt[3 + 5*x^2 + x^4]),x]
```

```
[Out] -1/9*Sqrt[3 + 5*x^2 + x^4]/x^6 - Sqrt[3 + 5*x^2 + x^4]/(54*x^4) + (13*Sqrt[
3 + 5*x^2 + x^4])/(108*x^2) - (61*ArcTanh[(6 + 5*x^2)/(2*Sqrt[3]*Sqrt[3 + 5
*x^2 + x^4])])/(216*Sqrt[3])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 820

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a +
 b*x + c*x^2)^(p + 1)/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2))), x] - Dist[(b*(e
*f + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(
m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[Simplify[m
+ 2*p + 3], 0]
```

Rule 848

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*f - d*g)*(d + e*x)^(m + 1)*((a + b*
x + c*x^2)^(p + 1)/((m + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Dist[1/((m + 1)
*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*Simp[(
c*d*f - f*b*e + a*e*g)*(m + 1) + b*(d*g - e*f)*(p + 1) - c*(e*f - d*g)*(m +
 2*p + 3)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] ||
 IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 1265

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rubi steps

$$\text{integral} = \frac{1}{2}\text{Subst}\left(\int \frac{2+3x}{x^4\sqrt{3+5x+x^2}}\,dx, x, x^2\right)$$

$$= -\frac{\sqrt{3+5x^2+x^4}}{9x^6} - \frac{1}{18}\text{Subst}\left(\int \frac{-2+4x}{x^3\sqrt{3+5x+x^2}}\,dx, x, x^2\right)$$

$$= -\frac{\sqrt{3+5x^2+x^4}}{9x^6} - \frac{\sqrt{3+5x^2+x^4}}{54x^4} + \frac{1}{108}\text{Subst}\left(\int \frac{-39-2x}{x^2\sqrt{3+5x+x^2}}\,dx, x, x^2\right)$$

$$= -\frac{\sqrt{3+5x^2+x^4}}{9x^6} - \frac{\sqrt{3+5x^2+x^4}}{54x^4} + \frac{13\sqrt{3+5x^2+x^4}}{108x^2} + \frac{61}{216}\text{Subst}\left(\int \frac{1}{x\sqrt{3+5x+x^2}}\,dx, x, x^2\right)$$

$$= -\frac{\sqrt{3+5x^2+x^4}}{9x^6} - \frac{\sqrt{3+5x^2+x^4}}{54x^4} + \frac{13\sqrt{3+5x^2+x^4}}{108x^2} - \frac{61}{108}\text{Subst}\left(\int \frac{1}{12-x^2}\,dx, x, \frac{6+5x^2}{\sqrt{3+5x^2+x^4}}\right)$$

$$= -\frac{\sqrt{3+5x^2+x^4}}{9x^6} - \frac{\sqrt{3+5x^2+x^4}}{54x^4} + \frac{13\sqrt{3+5x^2+x^4}}{108x^2} - \frac{61\tanh^{-1}\left(\frac{6+5x^2}{2\sqrt{3}\sqrt{3+5x^2+x^4}}\right)}{216\sqrt{3}}$$

Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.72

$$\int \frac{2+3x^2}{x^7\sqrt{3+5x^2+x^4}}\,dx = \frac{\sqrt{3+5x^2+x^4}(-12-2x^2+13x^4)}{108x^6} + \frac{61\text{arctanh}\left(\frac{x^2}{\sqrt{3}} - \frac{\sqrt{3+5x^2+x^4}}{\sqrt{3}}\right)}{108\sqrt{3}}$$

```
[In] Integrate[(2 + 3*x^2)/(x^7*Sqrt[3 + 5*x^2 + x^4]),x]
[Out] (Sqrt[3 + 5*x^2 + x^4]*(-12 - 2*x^2 + 13*x^4))/(108*x^6) + (61*ArcTanh[x^2/
Sqrt[3] - Sqrt[3 + 5*x^2 + x^4]/Sqrt[3]])/(108*Sqrt[3])
```

Maple [A] (verified)

Time = 0.38 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.63

method	result
pseudoelliptic	$\frac{-61\,\mathrm{arctanh}\left(\frac{\left(5x^2+6\right)\sqrt{3}}{6\sqrt{x^4+5x^2+3}}\right)\sqrt{3}\,x^6+6\sqrt{x^4+5x^2+3}\,\left(13x^4-2x^2-12\right)}{648x^6}$
risch	$\frac{13x^8+63x^6+17x^4-66x^2-36}{108x^6\sqrt{x^4+5x^2+3}}-\frac{61\,\mathrm{arctanh}\left(\frac{\left(5x^2+6\right)\sqrt{3}}{6\sqrt{x^4+5x^2+3}}\right)\sqrt{3}}{648}$
trager	$\frac{\left(13x^4-2x^2-12\right)\sqrt{x^4+5x^2+3}}{108x^6}+\frac{61\,\mathrm{RootOf}\left(\_Z^2-3\right)\ln\left(-\frac{-5\,\mathrm{RootOf}\left(\_Z^2-3\right)x^2+6\sqrt{x^4+5x^2+3}-6\,\mathrm{RootOf}\left(\_Z^2-3\right)}{x^2}\right)}{648}$
default	$-\frac{61\,\mathrm{arctanh}\left(\frac{\left(5x^2+6\right)\sqrt{3}}{6\sqrt{x^4+5x^2+3}}\right)\sqrt{3}}{648}-\frac{\sqrt{x^4+5x^2+3}}{9x^6}-\frac{\sqrt{x^4+5x^2+3}}{54x^4}+\frac{13\sqrt{x^4+5x^2+3}}{108x^2}$
elliptic	$-\frac{61\,\mathrm{arctanh}\left(\frac{\left(5x^2+6\right)\sqrt{3}}{6\sqrt{x^4+5x^2+3}}\right)\sqrt{3}}{648}-\frac{\sqrt{x^4+5x^2+3}}{9x^6}-\frac{\sqrt{x^4+5x^2+3}}{54x^4}+\frac{13\sqrt{x^4+5x^2+3}}{108x^2}$

```
[In] int((3*x^2+2)/x^7/(x^4+5*x^2+3)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/648*(-61*arctanh(1/6*(5*x^2+6)*3^(1/2)/(x^4+5*x^2+3)^(1/2))*3^(1/2)*x^6+6
*(x^4+5*x^2+3)^(1/2)*(13*x^4-2*x^2-12))/x^6
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.87

$$\int \frac{2+3x^2}{x^7\sqrt{3+5x^2+x^4}}\,dx$$
$$=\frac{61\,\sqrt{3}x^6\log\left(\frac{25\,x^2-2\,\sqrt{3}(5\,x^2+6)-2\,\sqrt{x^4+5\,x^2+3}\left(5\,\sqrt{3}-6\right)+30}{x^2}\right)+78\,x^6+6\,(13\,x^4-2\,x^2-12)\sqrt{x^4+5\,x^2+3}}{648\,x^6}$$

```
[In] integrate((3*x^2+2)/x^7/(x^4+5*x^2+3)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/648*(61*sqrt(3)*x^6*log((25*x^2 - 2*sqrt(3)*(5*x^2 + 6) - 2*sqrt(x^4 + 5*
x^2 + 3)*(5*sqrt(3) - 6) + 30)/x^2) + 78*x^6 + 6*(13*x^4 - 2*x^2 - 12)*sqrt
(x^4 + 5*x^2 + 3))/x^6
```

Sympy [F]

$$\int \frac{2+3x^2}{x^7\sqrt{3+5x^2+x^4}}\,dx = \int \frac{3x^2+2}{x^7\sqrt{x^4+5x^2+3}}\,dx$$

```
[In] integrate((3*x**2+2)/x**7/(x**4+5*x**2+3)**(1/2),x)
[Out] Integral((3*x**2 + 2)/(x**7*sqrt(x**4 + 5*x**2 + 3)), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.82

$$\int \frac{2+3x^2}{x^7\sqrt{3+5x^2+x^4}}\,dx = -\frac{61}{648}\sqrt{3}\log\left(\frac{2\sqrt{3}\sqrt{x^4+5x^2+3}}{x^2}+\frac{6}{x^2}+5\right) + \frac{13\sqrt{x^4+5x^2+3}}{108x^2} - \frac{\sqrt{x^4+5x^2+3}}{54x^4} - \frac{\sqrt{x^4+5x^2+3}}{9x^6}$$

```
[In] integrate((3*x^2+2)/x^7/(x^4+5*x^2+3)^(1/2),x, algorithm="maxima")
[Out] -61/648*sqrt(3)*log(2*sqrt(3)*sqrt(x^4 + 5*x^2 + 3)/x^2 + 6/x^2 + 5) + 13/1
08*sqrt(x^4 + 5*x^2 + 3)/x^2 - 1/54*sqrt(x^4 + 5*x^2 + 3)/x^4 - 1/9*sqrt(x^
4 + 5*x^2 + 3)/x^6
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 167 vs. 2(82) = 164.

Time = 0.30 (sec) , antiderivative size = 167, normalized size of antiderivative = 1.61

$$\int \frac{2+3x^2}{x^7\sqrt{3+5x^2+x^4}}\,dx = \frac{61}{648}\sqrt{3}\log\left(\frac{x^2+\sqrt{3}-\sqrt{x^4+5x^2+3}}{x^2-\sqrt{3}-\sqrt{x^4+5x^2+3}}\right) - \frac{61\left(x^2-\sqrt{x^4+5x^2+3}\right)^5 - 920\left(x^2-\sqrt{x^4+5x^2+3}\right)^3 - 2052\left(x^2-\sqrt{x^4+5x^2+3}\right)^2 - 1449x^2 + 14}{108\left(\left(x^2-\sqrt{x^4+5x^2+3}\right)^2-3\right)^3}$$

```
[In] integrate((3*x^2+2)/x^7/(x^4+5*x^2+3)^(1/2),x, algorithm="giac")
[Out] 61/648*sqrt(3)*log((x^2 + sqrt(3) - sqrt(x^4 + 5*x^2 + 3))/(x^2 - sqrt(3) -
 sqrt(x^4 + 5*x^2 + 3))) - 1/108*(61*(x^2 - sqrt(x^4 + 5*x^2 + 3))^5 - 920*
(x^2 - sqrt(x^4 + 5*x^2 + 3))^3 - 2052*(x^2 - sqrt(x^4 + 5*x^2 + 3))^2 - 14
49*x^2 + 1449*sqrt(x^4 + 5*x^2 + 3) - 108)/((x^2 - sqrt(x^4 + 5*x^2 + 3))^2
 - 3)^3
```

Mupad [F(-1)]

Timed out.

$$\int \frac{2+3x^2}{x^7\sqrt{3+5x^2+x^4}}\,dx = \int \frac{3\,x^2+2}{x^7\,\sqrt{x^4+5\,x^2+3}}\,dx$$

```
[In] int((3*x^2 + 2)/(x^7*(5*x^2 + x^4 + 3)^(1/2)),x)
[Out] int((3*x^2 + 2)/(x^7*(5*x^2 + x^4 + 3)^(1/2)), x)
```

3.189 $\int \frac{x^4(2+3x^2)}{\sqrt{3+5x^2+x^4}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 298

$$\int \frac{x^4(2+3x^2)}{\sqrt{3+5x^2+x^4}}\,dx = \frac{419x\left(5+\sqrt{13}+2x^2\right)}{30\sqrt{3+5x^2+x^4}} - \frac{10}{3}x\sqrt{3+5x^2+x^4} + \frac{3}{5}x^3\sqrt{3+5x^2+x^4}$$
$$- \frac{419\sqrt{\frac{1}{6}\left(5+\sqrt{13}\right)}\sqrt{\frac{6+\left(5-\sqrt{13}\right)x^2}{6+\left(5+\sqrt{13}\right)x^2}}\left(6+\left(5+\sqrt{13}\right)x^2\right)E\left(\arctan\left(\sqrt{\frac{1}{6}\left(5+\sqrt{13}\right)}x\right)|\frac{1}{6}\left(-13+5\sqrt{13}\right)\right)}{30\sqrt{3+5x^2+x^4}}$$
$$+ \frac{5\sqrt{\frac{2}{3\left(5+\sqrt{13}\right)}}\sqrt{\frac{6+\left(5-\sqrt{13}\right)x^2}{6+\left(5+\sqrt{13}\right)x^2}}\left(6+\left(5+\sqrt{13}\right)x^2\right)\text{EllipticF}\left(\arctan\left(\sqrt{\frac{1}{6}\left(5+\sqrt{13}\right)}x\right),\frac{1}{6}\left(-13+5\sqrt{13}\right)\right)}{\sqrt{3+5x^2+x^4}}$$

```
[Out] 419/30*x*(5+2*x^2+13^(1/2))/(x^4+5*x^2+3)^(1/2)-10/3*x*(x^4+5*x^2+3)^(1/2)+
3/5*x^3*(x^4+5*x^2+3)^(1/2)+5/3*(1/(36+x^2*(30+6*13^(1/2))))^(1/2)*(36+x^2*
(30+6*13^(1/2)))^(1/2)*EllipticF(x*(30+6*13^(1/2))^(1/2)/(36+x^2*(30+6*13^(
1/2)))^(1/2),1/6*(-78+30*13^(1/2))^(1/2))*(6+x^2*(5+13^(1/2)))*6^(1/2)/(5+1
3^(1/2))^(1/2)*((6+x^2*(5-13^(1/2)))/(6+x^2*(5+13^(1/2))))^(1/2)/(x^4+5*x^2
+3)^(1/2)-419/180*(1/(36+x^2*(30+6*13^(1/2))))^(1/2)*(36+x^2*(30+6*13^(1/2)
))^(1/2)*EllipticE(x*(30+6*13^(1/2))^(1/2)/(36+x^2*(30+6*13^(1/2)))^(1/2),1
/6*(-78+30*13^(1/2))^(1/2))*(6+x^2*(5+13^(1/2)))*(30+6*13^(1/2))^(1/2)*((6+
x^2*(5-13^(1/2)))/(6+x^2*(5+13^(1/2))))^(1/2)/(x^4+5*x^2+3)^(1/2)
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 298, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {1293, 1203, 1113, 1149}

$$\int \frac{x^4(2+3x^2)}{\sqrt{3+5x^2+x^4}}\,dx$$
$$= \frac{5\sqrt{\frac{2}{3\left(5+\sqrt{13}\right)}}\sqrt{\frac{\left(5-\sqrt{13}\right)x^2+6}{\left(5+\sqrt{13}\right)x^2+6}}\left(\left(5+\sqrt{13}\right)x^2+6\right)\text{EllipticF}\left(\arctan\left(\sqrt{\frac{1}{6}\left(5+\sqrt{13}\right)}x\right),\frac{1}{6}\left(-13+5\sqrt{13}\right)\right)}{\sqrt{x^4+5x^2+3}}$$
$$-\frac{419\sqrt{\frac{1}{6}\left(5+\sqrt{13}\right)}\sqrt{\frac{\left(5-\sqrt{13}\right)x^2+6}{\left(5+\sqrt{13}\right)x^2+6}}\left(\left(5+\sqrt{13}\right)x^2+6\right)E\left(\arctan\left(\sqrt{\frac{1}{6}\left(5+\sqrt{13}\right)}x\right)|\frac{1}{6}\left(-13+5\sqrt{13}\right)\right)}{30\sqrt{x^4+5x^2+3}}$$
$$-\frac{10}{3}\sqrt{x^4+5x^2+3}x+\frac{419\left(2x^2+\sqrt{13}+5\right)x}{30\sqrt{x^4+5x^2+3}}+\frac{3}{5}\sqrt{x^4+5x^2+3}x^3$$

```
[In] Int[(x^4*(2 + 3*x^2))/Sqrt[3 + 5*x^2 + x^4],x]
```

```
[Out] (419*x*(5 + Sqrt[13] + 2*x^2))/(30*Sqrt[3 + 5*x^2 + x^4]) - (10*x*Sqrt[3 +
5*x^2 + x^4])/3 + (3*x^3*Sqrt[3 + 5*x^2 + x^4])/5 - (419*Sqrt[(5 + Sqrt[13]
)/6]*Sqrt[(6 + (5 - Sqrt[13])*x^2)/(6 + (5 + Sqrt[13])*x^2)]*(6 + (5 + Sqrt
[13])*x^2)*EllipticE[ArcTan[Sqrt[(5 + Sqrt[13])/6]*x], (-13 + 5*Sqrt[13])/6
])/(30*Sqrt[3 + 5*x^2 + x^4]) + (5*Sqrt[2/(3*(5 + Sqrt[13]))]*Sqrt[(6 + (5
- Sqrt[13])*x^2)/(6 + (5 + Sqrt[13])*x^2)]*(6 + (5 + Sqrt[13])*x^2)*Ellipti
cF[ArcTan[Sqrt[(5 + Sqrt[13])/6]*x], (-13 + 5*Sqrt[13])/6])/Sqrt[3 + 5*x^2
+ x^4]
```

Rule 1113

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[(2*a + (b + q)*x^2)*(Sqrt[(2*a + (b - q)*x^2)/(2*a +
(b + q)*x^2)]/(2*a*Rt[(b + q)/(2*a), 2]*Sqrt[a + b*x^2 + c*x^4]))*EllipticF
[ArcTan[Rt[(b + q)/(2*a), 2]*x], 2*(q/(b + q))], x] /; PosQ[(b + q)/a] &&
!(PosQ[(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[
{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1149

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[b^2 - 4*a*c, 2]}, Simp[x*((b + q + 2*c*x^2)/(2*c*Sqrt[a + b*x^2 + c*x^4
])), x] - Simp[Rt[(b + q)/(2*a), 2]*(2*a + (b + q)*x^2)*(Sqrt[(2*a + (b - q
)*x^2)/(2*a + (b + q)*x^2)]/(2*c*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[ArcTan
[Rt[(b + q)/(2*a), 2]*x], 2*(q/(b + q))], x] /; PosQ[(b + q)/a] &&  !(PosQ[
(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[{a, b,
```

```
c}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1203

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[d, Int[1/Sqrt[a + b*x^2 + c*x^4],
 x], x] + Dist[e, Int[x^2/Sqrt[a + b*x^2 + c*x^4], x], x] /; PosQ[(b + q)/a
] || PosQ[(b - q)/a]] /; FreeQ[{a, b, c, d, e}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1293

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[e*f*(f*x)^(m - 1)*((a + b*x^2 + c*x^4)^(p +
1)/(c*(m + 4*p + 3))), x] - Dist[f^2/(c*(m + 4*p + 3)), Int[(f*x)^(m - 2)*(
a + b*x^2 + c*x^4)^p*Simp[a*e*(m - 1) + (b*e*(m + 2*p + 1) - c*d*(m + 4*p +
 3))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c,
 0] && GtQ[m, 1] && NeQ[m + 4*p + 3, 0] && IntegerQ[2*p] && (IntegerQ[p] ||
 IntegerQ[m])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{3}{5}x^3\sqrt{3+5x^2+x^4}-\frac{1}{5}\int \frac{x^2(27+50x^2)}{\sqrt{3+5x^2+x^4}}\,dx \\
&= -\frac{10}{3}x\sqrt{3+5x^2+x^4}+\frac{3}{5}x^3\sqrt{3+5x^2+x^4}+\frac{1}{15}\int \frac{150+419x^2}{\sqrt{3+5x^2+x^4}}\,dx \\
&= -\frac{10}{3}x\sqrt{3+5x^2+x^4}+\frac{3}{5}x^3\sqrt{3+5x^2+x^4} \\
&\quad +10\int \frac{1}{\sqrt{3+5x^2+x^4}}\,dx+\frac{419}{15}\int \frac{x^2}{\sqrt{3+5x^2+x^4}}\,dx \\
&= \frac{419x\left(5+\sqrt{13}+2x^2\right)}{30\sqrt{3+5x^2+x^4}}-\frac{10}{3}x\sqrt{3+5x^2+x^4}+\frac{3}{5}x^3\sqrt{3+5x^2+x^4} \\
&\quad -\frac{419\sqrt{\frac{1}{6}\left(5+\sqrt{13}\right)}\sqrt{\frac{6+\left(5-\sqrt{13}\right)x^2}{6+\left(5+\sqrt{13}\right)x^2}}\left(6+\left(5+\sqrt{13}\right)x^2\right)E\left(\tan^{-1}\left(\sqrt{\frac{1}{6}\left(5+\sqrt{13}\right)}x\right)|\frac{1}{6}\left(-13+5\sqrt{\right.\right.}{30\sqrt{3+5x^2+x^4}} \\
&\quad +\frac{5\sqrt{\frac{2}{3\left(5+\sqrt{13}\right)}}\sqrt{\frac{6+\left(5-\sqrt{13}\right)x^2}{6+\left(5+\sqrt{13}\right)x^2}}\left(6+\left(5+\sqrt{13}\right)x^2\right)F\left(\tan^{-1}\left(\sqrt{\frac{1}{6}\left(5+\sqrt{13}\right)}x\right)|\frac{1}{6}\left(-13+5\sqrt{13}\right)\right)}{\sqrt{3+5x^2+x^4}}
\end{aligned}$$

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 10.23 (sec) , antiderivative size = 229, normalized size of antiderivative = 0.77

$$\int \frac{x^4(2+3x^2)}{\sqrt{3+5x^2+x^4}}\,dx$$
$$= \frac{4x(-150-223x^2-5x^4+9x^6)+419i\sqrt{2}(-5+\sqrt{13})\sqrt{\frac{-5+\sqrt{13}-2x^2}{-5+\sqrt{13}}}\sqrt{5+\sqrt{13}+2x^2}E\Big(i\text{arcsinh}\Big(\sqrt{\frac{2}{5+\sqrt{1}}}}{60}$$

[In] Integrate[(x^4*(2 + 3*x^2))/Sqrt[3 + 5*x^2 + x^4],x]

[Out] (4*x*(-150 - 223*x^2 - 5*x^4 + 9*x^6) + (419*I)*Sqrt[2]*(-5 + Sqrt[13])*Sqrt[(-5 + Sqrt[13] - 2*x^2)/(-5 + Sqrt[13])]*Sqrt[5 + Sqrt[13] + 2*x^2]*EllipticE[I*ArcSinh[Sqrt[2/(5 + Sqrt[13])]*x], 19/6 + (5*Sqrt[13])/6] - I*Sqrt[2]*(-1795 + 419*Sqrt[13])*Sqrt[(-5 + Sqrt[13] - 2*x^2)/(-5 + Sqrt[13])]*Sqrt[5 + Sqrt[13] + 2*x^2]*EllipticF[I*ArcSinh[Sqrt[2/(5 + Sqrt[13])]*x], 19/6 + (5*Sqrt[13])/6])/(60*Sqrt[3 + 5*x^2 + x^4])

Maple [A] (verified)

Time = 3.89 (sec) , antiderivative size = 216, normalized size of antiderivative = 0.72

method	result
risch	$\frac{x(9x^2-50)\sqrt{x^4+5x^2+3}}{15}+\frac{60\sqrt{1-\left(-\frac{5}{6}+\frac{\sqrt{13}}{6}\right)x^2}\sqrt{1-\left(-\frac{5}{6}-\frac{\sqrt{13}}{6}\right)x^2}\,F\left(\frac{x\sqrt{-30+6\sqrt{13}}}{6},\frac{5\sqrt{3}}{6}+\frac{\sqrt{39}}{6}\right)}{\sqrt{-30+6\sqrt{13}}\sqrt{x^4+5x^2+3}}-\frac{5028\sqrt{1-\left(-\frac{5}{6}+\frac{\sqrt{13}}{6}\right)x^2}}{}$
default	$\frac{3x^3\sqrt{x^4+5x^2+3}}{5}-\frac{10x\sqrt{x^4+5x^2+3}}{3}+\frac{60\sqrt{1-\left(-\frac{5}{6}+\frac{\sqrt{13}}{6}\right)x^2}\sqrt{1-\left(-\frac{5}{6}-\frac{\sqrt{13}}{6}\right)x^2}\,F\left(\frac{x\sqrt{-30+6\sqrt{13}}}{6},\frac{5\sqrt{3}}{6}+\frac{\sqrt{39}}{6}\right)}{\sqrt{-30+6\sqrt{13}}\sqrt{x^4+5x^2+3}}-\frac{5028\sqrt{1-}}{}$
elliptic	$\frac{3x^3\sqrt{x^4+5x^2+3}}{5}-\frac{10x\sqrt{x^4+5x^2+3}}{3}+\frac{60\sqrt{1-\left(-\frac{5}{6}+\frac{\sqrt{13}}{6}\right)x^2}\sqrt{1-\left(-\frac{5}{6}-\frac{\sqrt{13}}{6}\right)x^2}\,F\left(\frac{x\sqrt{-30+6\sqrt{13}}}{6},\frac{5\sqrt{3}}{6}+\frac{\sqrt{39}}{6}\right)}{\sqrt{-30+6\sqrt{13}}\sqrt{x^4+5x^2+3}}-\frac{5028\sqrt{1-}}{}$

[In] int(x^4*(3*x^2+2)/(x^4+5*x^2+3)^(1/2),x,method=_RETURNVERBOSE)

[Out] 1/15*x*(9*x^2-50)*(x^4+5*x^2+3)^(1/2)+60/(-30+6*13^(1/2))^(1/2)*(1-(-5/6+1/6*13^(1/2))*x^2)^(1/2)*(1-(-5/6-1/6*13^(1/2))*x^2)^(1/2)/(x^4+5*x^2+3)^(1/2)*EllipticF(1/6*x*(-30+6*13^(1/2))^(1/2),5/6*3^(1/2)+1/6*39^(1/2))-5028/5/(-30+6*13^(1/2))^(1/2)*(1-(-5/6+1/6*13^(1/2))*x^2)^(1/2)*(1-(-5/6-1/6*13^(1/2))*x^2)^(1/2)/(x^4+5*x^2+3)^(1/2)/(5+13^(1/2))*(EllipticF(1/6*x*(-30+6*13^(1/2))^(1/2),5/6*3^(1/2)+1/6*39^(1/2))-EllipticE(1/6*x*(-30+6*13^(1/2))^(1/2),5/6*3^(1/2)+1/6*39^(1/2)))

Fricas [A] (verification not implemented)

none

Time = 0.10 (sec) , antiderivative size = 129, normalized size of antiderivative = 0.43

$$\int \frac{x^4(2+3x^2)}{\sqrt{3+5x^2+x^4}}\,dx = \frac{419\left(\sqrt{13}\sqrt{2}x-5\sqrt{2}x\right)\sqrt{\sqrt{13}-5}E(\arcsin\left(\frac{\sqrt{2}\sqrt{\sqrt{13}-5}}{2x}\right) \mid \frac{5}{6}\sqrt{13}+\frac{19}{6}) - \left(369\sqrt{13}\sqrt{2}x-2345\sqrt{2}x\right)\sqrt{\sqrt{1}}}{60\,x}$$

```
[In] integrate(x^4*(3*x^2+2)/(x^4+5*x^2+3)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/60*(419*(sqrt(13)*sqrt(2)*x - 5*sqrt(2)*x)*sqrt(sqrt(13) - 5)*elliptic_e(
arcsin(1/2*sqrt(2)*sqrt(sqrt(13) - 5)/x), 5/6*sqrt(13) + 19/6) - (369*sqrt(
13)*sqrt(2)*x - 2345*sqrt(2)*x)*sqrt(sqrt(13) - 5)*elliptic_f(arcsin(1/2*sq
rt(2)*sqrt(sqrt(13) - 5)/x), 5/6*sqrt(13) + 19/6) + 4*(9*x^4 - 50*x^2 + 419
)*sqrt(x^4 + 5*x^2 + 3))/x
```

Sympy [F]

$$\int \frac{x^4(2+3x^2)}{\sqrt{3+5x^2+x^4}}\,dx = \int \frac{x^4\cdot(3x^2+2)}{\sqrt{x^4+5x^2+3}}\,dx$$

```
[In] integrate(x**4*(3*x**2+2)/(x**4+5*x**2+3)**(1/2),x)
```

```
[Out] Integral(x**4*(3*x**2 + 2)/sqrt(x**4 + 5*x**2 + 3), x)
```

Maxima [F]

$$\int \frac{x^4(2+3x^2)}{\sqrt{3+5x^2+x^4}}\,dx = \int \frac{(3\,x^2+2)x^4}{\sqrt{x^4+5\,x^2+3}}\,dx$$

```
[In] integrate(x^4*(3*x^2+2)/(x^4+5*x^2+3)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((3*x^2 + 2)*x^4/sqrt(x^4 + 5*x^2 + 3), x)
```

Giac [F]

$$\int \frac{x^4(2+3x^2)}{\sqrt{3+5x^2+x^4}}\,dx = \int \frac{(3\,x^2+2)x^4}{\sqrt{x^4+5\,x^2+3}}\,dx$$

```
[In] integrate(x^4*(3*x^2+2)/(x^4+5*x^2+3)^(1/2),x, algorithm="giac")
[Out] integrate((3*x^2 + 2)*x^4/sqrt(x^4 + 5*x^2 + 3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^4(2+3x^2)}{\sqrt{3+5x^2+x^4}}\,dx = \int \frac{x^4\,(3\,x^2+2)}{\sqrt{x^4+5\,x^2+3}}\,dx$$

```
[In] int((x^4*(3*x^2 + 2))/(5*x^2 + x^4 + 3)^(1/2),x)
[Out] int((x^4*(3*x^2 + 2))/(5*x^2 + x^4 + 3)^(1/2), x)
```

3.190 $\int \frac{x^2(2+3x^2)}{\sqrt{3+5x^2+x^4}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 270

$$\int \frac{x^2(2+3x^2)}{\sqrt{3+5x^2+x^4}}\,dx = -\frac{4x\left(5+\sqrt{13}+2x^2\right)}{\sqrt{3+5x^2+x^4}}+x\sqrt{3+5x^2+x^4}$$
$$+\frac{2\sqrt{\frac{2}{3}\left(5+\sqrt{13}\right)}\sqrt{\frac{6+\left(5-\sqrt{13}\right)x^2}{6+\left(5+\sqrt{13}\right)x^2}}\left(6+\left(5+\sqrt{13}\right)x^2\right)E\left(\arctan\left(\sqrt{\frac{1}{6}\left(5+\sqrt{13}\right)}x\right)|\frac{1}{6}\left(-13+5\sqrt{13}\right)\right)}{\sqrt{3+5x^2+x^4}}$$
$$-\frac{\sqrt{\frac{3}{2\left(5+\sqrt{13}\right)}}\sqrt{\frac{6+\left(5-\sqrt{13}\right)x^2}{6+\left(5+\sqrt{13}\right)x^2}}\left(6+\left(5+\sqrt{13}\right)x^2\right)\text{EllipticF}\left(\arctan\left(\sqrt{\frac{1}{6}\left(5+\sqrt{13}\right)}x\right),\frac{1}{6}\left(-13+5\sqrt{13}\right)\right)}{\sqrt{3+5x^2+x^4}}$$

```
[Out] -4*x*(5+2*x^2+13^(1/2))/(x^4+5*x^2+3)^(1/2)+x*(x^4+5*x^2+3)^(1/2)-1/2*(1/(3
6+x^2*(30+6*13^(1/2))))^(1/2)*(36+x^2*(30+6*13^(1/2)))^(1/2)*EllipticF(x*(3
0+6*13^(1/2))^(1/2)/(36+x^2*(30+6*13^(1/2)))^(1/2),1/6*(-78+30*13^(1/2))^(1
/2))*(6+x^2*(5+13^(1/2)))*6^(1/2)/(5+13^(1/2))^(1/2)*((6+x^2*(5-13^(1/2)))/
(6+x^2*(5+13^(1/2))))^(1/2)/(x^4+5*x^2+3)^(1/2)+2/3*(1/(36+x^2*(30+6*13^(1/
2))))^(1/2)*(36+x^2*(30+6*13^(1/2)))^(1/2)*EllipticE(x*(30+6*13^(1/2))^(1/2
)/(36+x^2*(30+6*13^(1/2)))^(1/2),1/6*(-78+30*13^(1/2))^(1/2))*(6+x^2*(5+13^
(1/2)))*(30+6*13^(1/2))^(1/2)*((6+x^2*(5-13^(1/2)))/(6+x^2*(5+13^(1/2))))^(
1/2)/(x^4+5*x^2+3)^(1/2)
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 270, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {1293, 1203, 1113, 1149}

$$\int \frac{x^2(2+3x^2)}{\sqrt{3+5x^2+x^4}}\,dx = -\frac{\sqrt{\frac{3}{2\left(5+\sqrt{13}\right)}}\sqrt{\frac{\left(5-\sqrt{13}\right)x^2+6}{\left(5+\sqrt{13}\right)x^2+6}}\left(\left(5+\sqrt{13}\right)x^2+6\right)\text{EllipticF}\left(\arctan\left(\sqrt{\frac{1}{6}\left(5+\sqrt{13}\right)}x\right),\frac{1}{6}\left(-13+5\sqrt{13}\right)\right)}{\sqrt{x^4+5x^2+3}}$$
$$+\frac{2\sqrt{\frac{2}{3}\left(5+\sqrt{13}\right)}\sqrt{\frac{\left(5-\sqrt{13}\right)x^2+6}{\left(5+\sqrt{13}\right)x^2+6}}\left(\left(5+\sqrt{13}\right)x^2+6\right)E\left(\arctan\left(\sqrt{\frac{1}{6}\left(5+\sqrt{13}\right)}x\right)|\frac{1}{6}\left(-13+5\sqrt{13}\right)\right)}{\sqrt{x^4+5x^2+3}}$$
$$+\sqrt{x^4+5x^2+3}x-\frac{4\left(2x^2+\sqrt{13}+5\right)x}{\sqrt{x^4+5x^2+3}}$$

```
[In] Int[(x^2*(2 + 3*x^2))/Sqrt[3 + 5*x^2 + x^4],x]
```

```
[Out] (-4*x*(5 + Sqrt[13] + 2*x^2))/Sqrt[3 + 5*x^2 + x^4] + x*Sqrt[3 + 5*x^2 + x^
4] + (2*Sqrt[(2*(5 + Sqrt[13]))/3]*Sqrt[(6 + (5 - Sqrt[13])*x^2)/(6 + (5 +
Sqrt[13])*x^2)]*(6 + (5 + Sqrt[13])*x^2)*EllipticE[ArcTan[Sqrt[(5 + Sqrt[13
])/6]*x], (-13 + 5*Sqrt[13])/6])/Sqrt[3 + 5*x^2 + x^4] - (Sqrt[3/(2*(5 + Sq
rt[13]))]*Sqrt[(6 + (5 - Sqrt[13])*x^2)/(6 + (5 + Sqrt[13])*x^2)]*(6 + (5 +
 Sqrt[13])*x^2)*EllipticF[ArcTan[Sqrt[(5 + Sqrt[13])/6]*x], (-13 + 5*Sqrt[1
3])/6])/Sqrt[3 + 5*x^2 + x^4]
```

Rule 1113

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[(2*a + (b + q)*x^2)*(Sqrt[(2*a + (b - q)*x^2)/(2*a +
(b + q)*x^2)]/(2*a*Rt[(b + q)/(2*a), 2]*Sqrt[a + b*x^2 + c*x^4]))*EllipticF
[ArcTan[Rt[(b + q)/(2*a), 2]*x], 2*(q/(b + q))], x] /; PosQ[(b + q)/a] &&
!(PosQ[(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[
{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1149

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[b^2 - 4*a*c, 2]}, Simp[x*((b + q + 2*c*x^2)/(2*c*Sqrt[a + b*x^2 + c*x^4
])), x] - Simp[Rt[(b + q)/(2*a), 2]*(2*a + (b + q)*x^2)*(Sqrt[(2*a + (b - q
)*x^2)/(2*a + (b + q)*x^2)]/(2*c*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[ArcTan
[Rt[(b + q)/(2*a), 2]*x], 2*(q/(b + q))], x] /; PosQ[(b + q)/a] &&  !(PosQ[
(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[{a, b,
```

```
c}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1203

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[d, Int[1/Sqrt[a + b*x^2 + c*x^4],
 x], x] + Dist[e, Int[x^2/Sqrt[a + b*x^2 + c*x^4], x], x] /; PosQ[(b + q)/a
] || PosQ[(b - q)/a]] /; FreeQ[{a, b, c, d, e}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1293

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[e*f*(f*x)^(m - 1)*((a + b*x^2 + c*x^4)^(p +
1)/(c*(m + 4*p + 3))), x] - Dist[f^2/(c*(m + 4*p + 3)), Int[(f*x)^(m - 2)*(
a + b*x^2 + c*x^4)^p*Simp[a*e*(m - 1) + (b*e*(m + 2*p + 1) - c*d*(m + 4*p +
 3))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c,
 0] && GtQ[m, 1] && NeQ[m + 4*p + 3, 0] && IntegerQ[2*p] && (IntegerQ[p] ||
 IntegerQ[m])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= x\sqrt{3+5x^2+x^4}-\frac{1}{3}\int\frac{9+24x^2}{\sqrt{3+5x^2+x^4}}\,dx\\
&= x\sqrt{3+5x^2+x^4}-3\int\frac{1}{\sqrt{3+5x^2+x^4}}\,dx-8\int\frac{x^2}{\sqrt{3+5x^2+x^4}}\,dx\\
&= -\frac{4x\left(5+\sqrt{13}+2x^2\right)}{\sqrt{3+5x^2+x^4}}+x\sqrt{3+5x^2+x^4}\\
&\quad+\frac{2\sqrt{\frac{2}{3}\left(5+\sqrt{13}\right)}\sqrt{\frac{6+\left(5-\sqrt{13}\right)x^2}{6+\left(5+\sqrt{13}\right)x^2}}\left(6+\left(5+\sqrt{13}\right)x^2\right)E\left(\tan^{-1}\left(\sqrt{\frac{1}{6}\left(5+\sqrt{13}\right)}x\right)\left|\frac{1}{6}\left(-13+5\sqrt{13}\right)\right.\right)}{\sqrt{3+5x^2+x^4}}\\
&\quad-\frac{\sqrt{\frac{3}{2\left(5+\sqrt{13}\right)}}\sqrt{\frac{6+\left(5-\sqrt{13}\right)x^2}{6+\left(5+\sqrt{13}\right)x^2}}\left(6+\left(5+\sqrt{13}\right)x^2\right)F\left(\tan^{-1}\left(\sqrt{\frac{1}{6}\left(5+\sqrt{13}\right)}x\right)\left|\frac{1}{6}\left(-13+5\sqrt{13}\right)\right.\right)}{\sqrt{3+5x^2+x^4}}
\end{aligned}$$

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 10.23 (sec) , antiderivative size = 222, normalized size of antiderivative = 0.82

$$\int \frac{x^2(2+3x^2)}{\sqrt{3+5x^2+x^4}}\,dx$$
$$= \frac{2x(3+5x^2+x^4) - 4i\sqrt{2}(-5+\sqrt{13})\sqrt{\frac{-5+\sqrt{13}-2x^2}{-5+\sqrt{13}}}\sqrt{5+\sqrt{13}+2x^2}E\left(i\operatorname{arcsinh}\left(\sqrt{\frac{2}{5+\sqrt{13}}}x\right)|\frac{19}{6}+\frac{5\sqrt{13}}{6}\right)}{2\sqrt{3+5x^2}}$$

```
[In] Integrate[(x^2*(2 + 3*x^2))/Sqrt[3 + 5*x^2 + x^4],x]
```

```
[Out] (2*x*(3 + 5*x^2 + x^4) - (4*I)*Sqrt[2]*(-5 + Sqrt[13])*Sqrt[(-5 + Sqrt[13]
 - 2*x^2)/(-5 + Sqrt[13])]*Sqrt[5 + Sqrt[13] + 2*x^2]*EllipticE[I*ArcSinh[Sq
rt[2/(5 + Sqrt[13])]*x], 19/6 + (5*Sqrt[13])/6] + I*Sqrt[2]*(-17 + 4*Sqrt[1
3])*Sqrt[(-5 + Sqrt[13] - 2*x^2)/(-5 + Sqrt[13])]*Sqrt[5 + Sqrt[13] + 2*x^2
]*EllipticF[I*ArcSinh[Sqrt[2/(5 + Sqrt[13])]*x], 19/6 + (5*Sqrt[13])/6])/(2
*Sqrt[3 + 5*x^2 + x^4])
```

Maple [A] (verified)

Time = 2.60 (sec) , antiderivative size = 208, normalized size of antiderivative = 0.77

method	result
default	$x\sqrt{x^4+5x^2+3} - \frac{18\sqrt{1-\left(-\frac{5}{6}+\frac{\sqrt{13}}{6}\right)x^2}\sqrt{1-\left(-\frac{5}{6}-\frac{\sqrt{13}}{6}\right)x^2}\,F\left(\frac{x\sqrt{-30+6\sqrt{13}}}{6},\frac{5\sqrt{3}}{6}+\frac{\sqrt{39}}{6}\right)}{\sqrt{-30+6\sqrt{13}}\sqrt{x^4+5x^2+3}} + \frac{288\sqrt{1-\left(-\frac{5}{6}+\frac{\sqrt{13}}{6}\right)x^2}\sqrt{}}{}$
risch	$x\sqrt{x^4+5x^2+3} - \frac{18\sqrt{1-\left(-\frac{5}{6}+\frac{\sqrt{13}}{6}\right)x^2}\sqrt{1-\left(-\frac{5}{6}-\frac{\sqrt{13}}{6}\right)x^2}\,F\left(\frac{x\sqrt{-30+6\sqrt{13}}}{6},\frac{5\sqrt{3}}{6}+\frac{\sqrt{39}}{6}\right)}{\sqrt{-30+6\sqrt{13}}\sqrt{x^4+5x^2+3}} + \frac{288\sqrt{1-\left(-\frac{5}{6}+\frac{\sqrt{13}}{6}\right)x^2}\sqrt{}}{}$
elliptic	$x\sqrt{x^4+5x^2+3} - \frac{18\sqrt{1-\left(-\frac{5}{6}+\frac{\sqrt{13}}{6}\right)x^2}\sqrt{1-\left(-\frac{5}{6}-\frac{\sqrt{13}}{6}\right)x^2}\,F\left(\frac{x\sqrt{-30+6\sqrt{13}}}{6},\frac{5\sqrt{3}}{6}+\frac{\sqrt{39}}{6}\right)}{\sqrt{-30+6\sqrt{13}}\sqrt{x^4+5x^2+3}} + \frac{288\sqrt{1-\left(-\frac{5}{6}+\frac{\sqrt{13}}{6}\right)x^2}\sqrt{}}{}$

```
[In] int(x^2*(3*x^2+2)/(x^4+5*x^2+3)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] x*(x^4+5*x^2+3)^(1/2)-18/(-30+6*13^(1/2))^(1/2)*(1-(-5/6+1/6*13^(1/2))*x^2)
^(1/2)*(1-(-5/6-1/6*13^(1/2))*x^2)^(1/2)/(x^4+5*x^2+3)^(1/2)*EllipticF(1/6*
x*(-30+6*13^(1/2))^(1/2),5/6*3^(1/2)+1/6*39^(1/2))+288/(-30+6*13^(1/2))^(1/
2)*(1-(-5/6+1/6*13^(1/2))*x^2)^(1/2)*(1-(-5/6-1/6*13^(1/2))*x^2)^(1/2)/(x^4
+5*x^2+3)^(1/2)/(5+13^(1/2))*(EllipticF(1/6*x*(-30+6*13^(1/2))^(1/2),5/6*3^
(1/2)+1/6*39^(1/2))-EllipticE(1/6*x*(-30+6*13^(1/2))^(1/2),5/6*3^(1/2)+1/6*
39^(1/2)))
```

Fricas [A] (verification not implemented)

none

Time = 0.09 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.45

$$\int \frac{x^2(2+3x^2)}{\sqrt{3+5x^2+x^4}}\,dx = -\frac{8\left(\sqrt{13}\sqrt{2}x-5\sqrt{2}x\right)\sqrt{\sqrt{13}-5}E(\arcsin\left(\frac{\sqrt{2}\sqrt{\sqrt{13}-5}}{2x}\right)\mid\frac{5}{6}\sqrt{13}+\frac{19}{6})-\left(7\sqrt{13}\sqrt{2}x-45\sqrt{2}x\right)\sqrt{\sqrt{13}-}}{4x}$$

```
[In] integrate(x^2*(3*x^2+2)/(x^4+5*x^2+3)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/4*(8*(sqrt(13)*sqrt(2)*x - 5*sqrt(2)*x)*sqrt(sqrt(13) - 5)*elliptic_e(ar
csin(1/2*sqrt(2)*sqrt(sqrt(13) - 5)/x), 5/6*sqrt(13) + 19/6) - (7*sqrt(13)*
sqrt(2)*x - 45*sqrt(2)*x)*sqrt(sqrt(13) - 5)*elliptic_f(arcsin(1/2*sqrt(2)*
sqrt(sqrt(13) - 5)/x), 5/6*sqrt(13) + 19/6) - 4*sqrt(x^4 + 5*x^2 + 3)*(x^2
- 8))/x
```

Sympy [F]

$$\int \frac{x^2(2+3x^2)}{\sqrt{3+5x^2+x^4}}\,dx = \int \frac{x^2\cdot(3x^2+2)}{\sqrt{x^4+5x^2+3}}\,dx$$

```
[In] integrate(x**2*(3*x**2+2)/(x**4+5*x**2+3)**(1/2),x)
```

```
[Out] Integral(x**2*(3*x**2 + 2)/sqrt(x**4 + 5*x**2 + 3), x)
```

Maxima [F]

$$\int \frac{x^2(2+3x^2)}{\sqrt{3+5x^2+x^4}}\,dx = \int \frac{(3\,x^2+2)x^2}{\sqrt{x^4+5\,x^2+3}}\,dx$$

```
[In] integrate(x^2*(3*x^2+2)/(x^4+5*x^2+3)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((3*x^2 + 2)*x^2/sqrt(x^4 + 5*x^2 + 3), x)
```

Giac [F]

$$\int \frac{x^2(2+3x^2)}{\sqrt{3+5x^2+x^4}}\,dx = \int \frac{(3\,x^2+2)x^2}{\sqrt{x^4+5\,x^2+3}}\,dx$$

[In] integrate(x^2*(3*x^2+2)/(x^4+5*x^2+3)^(1/2),x, algorithm="giac")

[Out] integrate((3*x^2 + 2)*x^2/sqrt(x^4 + 5*x^2 + 3), x)

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2(2+3x^2)}{\sqrt{3+5x^2+x^4}}\,dx = \int \frac{x^2\,(3\,x^2+2)}{\sqrt{x^4+5\,x^2+3}}\,dx$$

[In] int((x^2*(3*x^2 + 2))/(5*x^2 + x^4 + 3)^(1/2),x)

[Out] int((x^2*(3*x^2 + 2))/(5*x^2 + x^4 + 3)^(1/2), x)

3.191 $\int \frac{2+3x^2}{\sqrt{3+5x^2+x^4}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 257

$$\int \frac{2+3x^2}{\sqrt{3+5x^2+x^4}}\,dx = \frac{3x\left(5+\sqrt{13}+2x^2\right)}{2\sqrt{3+5x^2+x^4}} - \frac{\sqrt{\frac{3}{2}\left(5+\sqrt{13}\right)}\sqrt{\frac{6+\left(5-\sqrt{13}\right)x^2}{6+\left(5+\sqrt{13}\right)x^2}}\left(6+\left(5+\sqrt{13}\right)x^2\right)E\left(\arctan\left(\sqrt{\frac{1}{6}\left(5+\sqrt{13}\right)}x\right)\middle|\frac{1}{6}\left(-13+5\sqrt{13}\right)\right)}{2\sqrt{3+5x^2+x^4}} + \frac{\sqrt{\frac{2}{3\left(5+\sqrt{13}\right)}}\sqrt{\frac{6+\left(5-\sqrt{13}\right)x^2}{6+\left(5+\sqrt{13}\right)x^2}}\left(6+\left(5+\sqrt{13}\right)x^2\right)\operatorname{EllipticF}\left(\arctan\left(\sqrt{\frac{1}{6}\left(5+\sqrt{13}\right)}x\right),\frac{1}{6}\left(-13+5\sqrt{13}\right)\right)}{\sqrt{3+5x^2+x^4}}$$

```
[Out] 3/2*x*(5+2*x^2+13^(1/2))/(x^4+5*x^2+3)^(1/2)+1/3*(1/(36+x^2*(30+6*13^(1/2))
))^(1/2)*(36+x^2*(30+6*13^(1/2)))^(1/2)*EllipticF(x*(30+6*13^(1/2))^(1/2)/(
36+x^2*(30+6*13^(1/2)))^(1/2),1/6*(-78+30*13^(1/2))^(1/2))*(6+x^2*(5+13^(1/
2)))*6^(1/2)/(5+13^(1/2))^(1/2)*((6+x^2*(5-13^(1/2)))/(6+x^2*(5+13^(1/2))))
^(1/2)/(x^4+5*x^2+3)^(1/2)-1/4*(1/(36+x^2*(30+6*13^(1/2))))^(1/2)*(36+x^2*(
30+6*13^(1/2)))^(1/2)*EllipticE(x*(30+6*13^(1/2))^(1/2)/(36+x^2*(30+6*13^(1
/2)))^(1/2),1/6*(-78+30*13^(1/2))^(1/2))*(6+x^2*(5+13^(1/2)))*(30+6*13^(1/2
))^(1/2)*((6+x^2*(5-13^(1/2)))/(6+x^2*(5+13^(1/2))))^(1/2)/(x^4+5*x^2+3)^(1
/2)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 257, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {1203, 1113, 1149}

$$\int \frac{2+3x^2}{\sqrt{3+5x^2+x^4}}\,dx$$
$$=\frac{\sqrt{\frac{2}{3\left(5+\sqrt{13}\right)}}\sqrt{\frac{\left(5-\sqrt{13}\right)x^2+6}{\left(5+\sqrt{13}\right)x^2+6}}\left(\left(5+\sqrt{13}\right)x^2+6\right)\text{EllipticF}\left(\arctan\left(\sqrt{\frac{1}{6}\left(5+\sqrt{13}\right)}x\right),\frac{1}{6}\left(-13+5\sqrt{13}\right)\right)}{\sqrt{x^4+5x^2+3}}$$
$$-\frac{\sqrt{\frac{3}{2}\left(5+\sqrt{13}\right)}\sqrt{\frac{\left(5-\sqrt{13}\right)x^2+6}{\left(5+\sqrt{13}\right)x^2+6}}\left(\left(5+\sqrt{13}\right)x^2+6\right)E\left(\arctan\left(\sqrt{\frac{1}{6}\left(5+\sqrt{13}\right)}x\right)|\frac{1}{6}\left(-13+5\sqrt{13}\right)\right)}{2\sqrt{x^4+5x^2+3}}$$
$$+\frac{3x\left(2x^2+\sqrt{13}+5\right)}{2\sqrt{x^4+5x^2+3}}$$

```
[In] Int[(2 + 3*x^2)/Sqrt[3 + 5*x^2 + x^4],x]
```

```
[Out] (3*x*(5 + Sqrt[13] + 2*x^2))/(2*Sqrt[3 + 5*x^2 + x^4]) - (Sqrt[(3*(5 + Sqrt
[13]))/2]*Sqrt[(6 + (5 - Sqrt[13])*x^2)/(6 + (5 + Sqrt[13])*x^2)]*(6 + (5 +
 Sqrt[13])*x^2)*EllipticE[ArcTan[Sqrt[(5 + Sqrt[13])/6]*x], (-13 + 5*Sqrt[1
3])/6]])/(2*Sqrt[3 + 5*x^2 + x^4]) + (Sqrt[2/(3*(5 + Sqrt[13]))]*Sqrt[(6 + (
5 - Sqrt[13])*x^2)/(6 + (5 + Sqrt[13])*x^2)]*(6 + (5 + Sqrt[13])*x^2)*Ellip
ticF[ArcTan[Sqrt[(5 + Sqrt[13])/6]*x], (-13 + 5*Sqrt[13])/6])/Sqrt[3 + 5*x^
2 + x^4]
```

Rule 1113

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[(2*a + (b + q)*x^2)*(Sqrt[(2*a + (b - q)*x^2)/(2*a +
(b + q)*x^2)]/(2*a*Rt[(b + q)/(2*a), 2]*Sqrt[a + b*x^2 + c*x^4]))*EllipticF
[ArcTan[Rt[(b + q)/(2*a), 2]*x], 2*(q/(b + q))], x] /; PosQ[(b + q)/a] &&
!(PosQ[(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[
{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1149

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[b^2 - 4*a*c, 2]}, Simp[x*((b + q + 2*c*x^2)/(2*c*Sqrt[a + b*x^2 + c*x^4
])), x] - Simp[Rt[(b + q)/(2*a), 2]*(2*a + (b + q)*x^2)*(Sqrt[(2*a + (b - q
)*x^2)/(2*a + (b + q)*x^2)]/(2*c*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[ArcTan
[Rt[(b + q)/(2*a), 2]*x], 2*(q/(b + q))], x] /; PosQ[(b + q)/a] &&  !(PosQ[
(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[{a, b,
```

```
c}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1203

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[d, Int[1/Sqrt[a + b*x^2 + c*x^4],
 x], x] + Dist[e, Int[x^2/Sqrt[a + b*x^2 + c*x^4], x], x] /; PosQ[(b + q)/a
] || PosQ[(b - q)/a]] /; FreeQ[{a, b, c, d, e}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= 2\int \frac{1}{\sqrt{3+5x^2+x^4}}\,dx + 3\int \frac{x^2}{\sqrt{3+5x^2+x^4}}\,dx \\ &= \frac{3x\left(5+\sqrt{13}+2x^2\right)}{2\sqrt{3+5x^2+x^4}} \\ &\quad - \frac{\sqrt{\frac{3}{2}\left(5+\sqrt{13}\right)}\sqrt{\frac{6+\left(5-\sqrt{13}\right)x^2}{6+\left(5+\sqrt{13}\right)x^2}}\left(6+\left(5+\sqrt{13}\right)x^2\right)E\left(\tan^{-1}\left(\sqrt{\frac{1}{6}\left(5+\sqrt{13}\right)}x\right)|\frac{1}{6}\left(-13+5\sqrt{13}\right)\right.}{2\sqrt{3+5x^2+x^4}} \\ &\quad + \frac{\sqrt{\frac{2}{3\left(5+\sqrt{13}\right)}}\sqrt{\frac{6+\left(5-\sqrt{13}\right)x^2}{6+\left(5+\sqrt{13}\right)x^2}}\left(6+\left(5+\sqrt{13}\right)x^2\right)F\left(\tan^{-1}\left(\sqrt{\frac{1}{6}\left(5+\sqrt{13}\right)}x\right)|\frac{1}{6}\left(-13+5\sqrt{13}\right)\right)}{\sqrt{3+5x^2+x^4}}\end{aligned}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.12 (sec) , antiderivative size = 159, normalized size of antiderivative = 0.62

$$\int \frac{2+3x^2}{\sqrt{3+5x^2+x^4}}\,dx = \frac{i\sqrt{\frac{-5+\sqrt{13}-2x^2}{-5+\sqrt{13}}}\sqrt{5+\sqrt{13}+2x^2}\left(3\left(-5+\sqrt{13}\right)E\left(i\text{arcsinh}\left(\sqrt{\frac{2}{5+\sqrt{13}}}x\right)|\frac{19}{6}+\frac{5\sqrt{13}}{6}\right)+\left(11-3\sqrt{13}\right)\text{Elliptic}\right.}{2\sqrt{2}\sqrt{3+5x^2+x^4}}$$

```
[In] Integrate[(2 + 3*x^2)/Sqrt[3 + 5*x^2 + x^4],x]
```

```
[Out] ((I/2)*Sqrt[(-5 + Sqrt[13] - 2*x^2)/(-5 + Sqrt[13])]*Sqrt[5 + Sqrt[13] + 2*
x^2]*(3*(-5 + Sqrt[13])*EllipticE[I*ArcSinh[Sqrt[2/(5 + Sqrt[13])]*x], 19/6
 + (5*Sqrt[13])/6] + (11 - 3*Sqrt[13])*EllipticF[I*ArcSinh[Sqrt[2/(5 + Sqrt
[13])]*x], 19/6 + (5*Sqrt[13])/6]))/(Sqrt[2]*Sqrt[3 + 5*x^2 + x^4])
```

Maple [A] (verified)

Time = 0.84 (sec) , antiderivative size = 194, normalized size of antiderivative = 0.75

method	result
default	$\frac{12\sqrt{1-\left(-\frac{5}{6}+\frac{\sqrt{13}}{6}\right)x^2}\sqrt{1-\left(-\frac{5}{6}-\frac{\sqrt{13}}{6}\right)x^2}\,F\left(\frac{x\sqrt{-30+6\sqrt{13}}}{6},\frac{5\sqrt{3}}{6}+\frac{\sqrt{39}}{6}\right)}{\sqrt{-30+6\sqrt{13}}\sqrt{x^4+5x^2+3}}-\frac{108\sqrt{1-\left(-\frac{5}{6}+\frac{\sqrt{13}}{6}\right)x^2}\sqrt{1-\left(-\frac{5}{6}-\frac{\sqrt{13}}{6}\right)x^2}\left(F\left(\frac{x\sqrt{}}{}\right.\right.}{\sqrt{-30+6\sqrt{13}}\sqrt{x}}$
elliptic	$\frac{12\sqrt{1-\left(-\frac{5}{6}+\frac{\sqrt{13}}{6}\right)x^2}\sqrt{1-\left(-\frac{5}{6}-\frac{\sqrt{13}}{6}\right)x^2}\,F\left(\frac{x\sqrt{-30+6\sqrt{13}}}{6},\frac{5\sqrt{3}}{6}+\frac{\sqrt{39}}{6}\right)}{\sqrt{-30+6\sqrt{13}}\sqrt{x^4+5x^2+3}}-\frac{108\sqrt{1-\left(-\frac{5}{6}+\frac{\sqrt{13}}{6}\right)x^2}\sqrt{1-\left(-\frac{5}{6}-\frac{\sqrt{13}}{6}\right)x^2}\left(F\left(\frac{x\sqrt{}}{}\right.\right.}{\sqrt{-30+6\sqrt{13}}\sqrt{x}}$

```
[In] int((3*x^2+2)/(x^4+5*x^2+3)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 12/(-30+6*13^(1/2))^(1/2)*(1-(-5/6+1/6*13^(1/2))*x^2)^(1/2)*(1-(-5/6-1/6*13
^(1/2))*x^2)^(1/2)/(x^4+5*x^2+3)^(1/2)*EllipticF(1/6*x*(-30+6*13^(1/2))^(1/
2),5/6*3^(1/2)+1/6*39^(1/2))-108/(-30+6*13^(1/2))^(1/2)*(1-(-5/6+1/6*13^(1/
2))*x^2)^(1/2)*(1-(-5/6-1/6*13^(1/2))*x^2)^(1/2)/(x^4+5*x^2+3)^(1/2)/(5+13^
(1/2))*(EllipticF(1/6*x*(-30+6*13^(1/2))^(1/2),5/6*3^(1/2)+1/6*39^(1/2))-El
lipticE(1/6*x*(-30+6*13^(1/2))^(1/2),5/6*3^(1/2)+1/6*39^(1/2)))
```

Fricas [A] (verification not implemented)

none

Time = 0.09 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.46

$$\int \frac{2+3x^2}{\sqrt{3+5x^2+x^4}}\,dx$$
$$=\frac{9\left(\sqrt{13}\sqrt{2}x-5\sqrt{2}x\right)\sqrt{\sqrt{13}-5}E(\arcsin\left(\frac{\sqrt{2}\sqrt{\sqrt{13}-5}}{2x}\right)\mid\frac{5}{6}\sqrt{13}+\frac{19}{6})-\left(7\sqrt{13}\sqrt{2}x-55\sqrt{2}x\right)\sqrt{\sqrt{13}-5}}{12x}$$

```
[In] integrate((3*x^2+2)/(x^4+5*x^2+3)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/12*(9*(sqrt(13)*sqrt(2)*x - 5*sqrt(2)*x)*sqrt(sqrt(13) - 5)*elliptic_e(ar
csin(1/2*sqrt(2)*sqrt(sqrt(13) - 5)/x), 5/6*sqrt(13) + 19/6) - (7*sqrt(13)*
sqrt(2)*x - 55*sqrt(2)*x)*sqrt(sqrt(13) - 5)*elliptic_f(arcsin(1/2*sqrt(2)*
sqrt(sqrt(13) - 5)/x), 5/6*sqrt(13) + 19/6) + 36*sqrt(x^4 + 5*x^2 + 3))/x
```

Sympy [F]

$$\int \frac{2+3x^2}{\sqrt{3+5x^2+x^4}}\,dx=\int \frac{3x^2+2}{\sqrt{x^4+5x^2+3}}\,dx$$

```
[In] integrate((3*x**2+2)/(x**4+5*x**2+3)**(1/2),x)
[Out] Integral((3*x**2 + 2)/sqrt(x**4 + 5*x**2 + 3), x)
```

Maxima [F]

$$\int \frac{2+3x^2}{\sqrt{3+5x^2+x^4}}\,dx=\int \frac{3\,x^2+2}{\sqrt{x^4+5\,x^2+3}}\,dx$$

```
[In] integrate((3*x^2+2)/(x^4+5*x^2+3)^(1/2),x, algorithm="maxima")
[Out] integrate((3*x^2 + 2)/sqrt(x^4 + 5*x^2 + 3), x)
```

Giac [F]

$$\int \frac{2+3x^2}{\sqrt{3+5x^2+x^4}}\,dx=\int \frac{3\,x^2+2}{\sqrt{x^4+5\,x^2+3}}\,dx$$

```
[In] integrate((3*x^2+2)/(x^4+5*x^2+3)^(1/2),x, algorithm="giac")
[Out] integrate((3*x^2 + 2)/sqrt(x^4 + 5*x^2 + 3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{2+3x^2}{\sqrt{3+5x^2+x^4}}\,dx=\int \frac{3\,x^2+2}{\sqrt{x^4+5\,x^2+3}}\,dx$$

```
[In] int((3*x^2 + 2)/(5*x^2 + x^4 + 3)^(1/2),x)
[Out] int((3*x^2 + 2)/(5*x^2 + x^4 + 3)^(1/2), x)
```

3.192 $\int \frac{2+3x^2}{x^2\sqrt{3+5x^2+x^4}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 278

$$\int \frac{2+3x^2}{x^2\sqrt{3+5x^2+x^4}}\,dx = \frac{x\left(5+\sqrt{13}+2x^2\right)}{3\sqrt{3+5x^2+x^4}} - \frac{2\sqrt{3+5x^2+x^4}}{3x} - \frac{\sqrt{\frac{1}{6}\left(5+\sqrt{13}\right)}\sqrt{\frac{6+\left(5-\sqrt{13}\right)x^2}{6+\left(5+\sqrt{13}\right)x^2}}\left(6+\left(5+\sqrt{13}\right)x^2\right)E\left(\arctan\left(\sqrt{\frac{1}{6}\left(5+\sqrt{13}\right)}x\right)\middle|\frac{1}{6}\left(-13+5\sqrt{13}\right)\right)}{3\sqrt{3+5x^2+x^4}} + \frac{\sqrt{\frac{3}{2\left(5+\sqrt{13}\right)}}\sqrt{\frac{6+\left(5-\sqrt{13}\right)x^2}{6+\left(5+\sqrt{13}\right)x^2}}\left(6+\left(5+\sqrt{13}\right)x^2\right)\text{EllipticF}\left(\arctan\left(\sqrt{\frac{1}{6}\left(5+\sqrt{13}\right)}x\right),\frac{1}{6}\left(-13+5\sqrt{13}\right)\right)}{\sqrt{3+5x^2+x^4}}$$

```
[Out] 1/3*x*(5+2*x^2+13^(1/2))/(x^4+5*x^2+3)^(1/2)-2/3*(x^4+5*x^2+3)^(1/2)/x+1/2*
(1/(36+x^2*(30+6*13^(1/2))))^(1/2)*(36+x^2*(30+6*13^(1/2)))^(1/2)*EllipticF
(x*(30+6*13^(1/2))^(1/2)/(36+x^2*(30+6*13^(1/2)))^(1/2),1/6*(-78+30*13^(1/2
))^(1/2))*(6+x^2*(5+13^(1/2)))*6^(1/2)/(5+13^(1/2))^(1/2)*((6+x^2*(5-13^(1/
2)))/(6+x^2*(5+13^(1/2))))^(1/2)/(x^4+5*x^2+3)^(1/2)-1/18*(1/(36+x^2*(30+6*
13^(1/2))))^(1/2)*(36+x^2*(30+6*13^(1/2)))^(1/2)*EllipticE(x*(30+6*13^(1/2)
)^(1/2)/(36+x^2*(30+6*13^(1/2)))^(1/2),1/6*(-78+30*13^(1/2))^(1/2))*(6+x^2*
(5+13^(1/2)))*(30+6*13^(1/2))^(1/2)*((6+x^2*(5-13^(1/2)))/(6+x^2*(5+13^(1/2
))))^(1/2)/(x^4+5*x^2+3)^(1/2)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 278, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {1295, 1203, 1113, 1149}

$$\int \frac{2+3x^2}{x^2\sqrt{3+5x^2+x^4}}\,dx$$
$$=\frac{\sqrt{\frac{3}{2\left(5+\sqrt{13}\right)}}\sqrt{\frac{\left(5-\sqrt{13}\right)x^2+6}{\left(5+\sqrt{13}\right)x^2+6}}\left(\left(5+\sqrt{13}\right)x^2+6\right)\text{EllipticF}\left(\arctan\left(\sqrt{\frac{1}{6}\left(5+\sqrt{13}\right)}x\right),\frac{1}{6}\left(-13+5\sqrt{13}\right)\right)}{\sqrt{x^4+5x^2+3}}$$
$$-\frac{\sqrt{\frac{1}{6}\left(5+\sqrt{13}\right)}\sqrt{\frac{\left(5-\sqrt{13}\right)x^2+6}{\left(5+\sqrt{13}\right)x^2+6}}\left(\left(5+\sqrt{13}\right)x^2+6\right)E\left(\arctan\left(\sqrt{\frac{1}{6}\left(5+\sqrt{13}\right)}x\right)|\frac{1}{6}\left(-13+5\sqrt{13}\right)\right)}{3\sqrt{x^4+5x^2+3}}$$
$$+\frac{x\left(2x^2+\sqrt{13}+5\right)}{3\sqrt{x^4+5x^2+3}}-\frac{2\sqrt{x^4+5x^2+3}}{3x}$$

```
[In] Int[(2 + 3*x^2)/(x^2*Sqrt[3 + 5*x^2 + x^4]),x]
```

```
[Out] (x*(5 + Sqrt[13] + 2*x^2))/(3*Sqrt[3 + 5*x^2 + x^4]) - (2*Sqrt[3 + 5*x^2 +
x^4])/(3*x) - (Sqrt[(5 + Sqrt[13])/6]*Sqrt[(6 + (5 - Sqrt[13])*x^2)/(6 + (5
 + Sqrt[13])*x^2)]*(6 + (5 + Sqrt[13])*x^2)*EllipticE[ArcTan[Sqrt[(5 + Sqrt
[13])/6]*x], (-13 + 5*Sqrt[13])/6])/(3*Sqrt[3 + 5*x^2 + x^4]) + (Sqrt[3/(2*
(5 + Sqrt[13]))]*Sqrt[(6 + (5 - Sqrt[13])*x^2)/(6 + (5 + Sqrt[13])*x^2)]*(6
 + (5 + Sqrt[13])*x^2)*EllipticF[ArcTan[Sqrt[(5 + Sqrt[13])/6]*x], (-13 + 5
*Sqrt[13])/6])/Sqrt[3 + 5*x^2 + x^4]
```

Rule 1113

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[(2*a + (b + q)*x^2)*(Sqrt[(2*a + (b - q)*x^2)/(2*a +
(b + q)*x^2)]/(2*a*Rt[(b + q)/(2*a), 2]*Sqrt[a + b*x^2 + c*x^4]))*EllipticF
[ArcTan[Rt[(b + q)/(2*a), 2]*x], 2*(q/(b + q))], x] /; PosQ[(b + q)/a] &&
!(PosQ[(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[
{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1149

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[b^2 - 4*a*c, 2]}, Simp[x*((b + q + 2*c*x^2)/(2*c*Sqrt[a + b*x^2 + c*x^4
])), x] - Simp[Rt[(b + q)/(2*a), 2]*(2*a + (b + q)*x^2)*(Sqrt[(2*a + (b - q
)*x^2)/(2*a + (b + q)*x^2)]/(2*c*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[ArcTan
[Rt[(b + q)/(2*a), 2]*x], 2*(q/(b + q))], x] /; PosQ[(b + q)/a] &&  !(PosQ[
(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[{a, b,
```

```
c}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1203

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[d, Int[1/Sqrt[a + b*x^2 + c*x^4],
 x], x] + Dist[e, Int[x^2/Sqrt[a + b*x^2 + c*x^4], x], x] /; PosQ[(b + q)/a
] || PosQ[(b - q)/a]] /; FreeQ[{a, b, c, d, e}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1295

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[d*(f*x)^(m + 1)*((a + b*x^2 + c*x^4)^(p + 1)
/(a*f*(m + 1))), x] + Dist[1/(a*f^2*(m + 1)), Int[(f*x)^(m + 2)*(a + b*x^2
+ c*x^4)^p*Simp[a*e*(m + 1) - b*d*(m + 2*p + 3) - c*d*(m + 4*p + 5)*x^2, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[m
, -1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{2\sqrt{3+5x^2+x^4}}{3x} - \frac{1}{3}\int \frac{-9-2x^2}{\sqrt{3+5x^2+x^4}}\,dx \\
&= -\frac{2\sqrt{3+5x^2+x^4}}{3x} + \frac{2}{3}\int \frac{x^2}{\sqrt{3+5x^2+x^4}}\,dx + 3\int \frac{1}{\sqrt{3+5x^2+x^4}}\,dx \\
&= \frac{x\left(5+\sqrt{13}+2x^2\right)}{3\sqrt{3+5x^2+x^4}} - \frac{2\sqrt{3+5x^2+x^4}}{3x} \\
&\quad - \frac{\sqrt{\frac{1}{6}\left(5+\sqrt{13}\right)}\sqrt{\frac{6+\left(5-\sqrt{13}\right)x^2}{6+\left(5+\sqrt{13}\right)x^2}}\left(6+\left(5+\sqrt{13}\right)x^2\right)E\left(\tan^{-1}\left(\sqrt{\frac{1}{6}\left(5+\sqrt{13}\right)}x\right)|\frac{1}{6}\left(-13+5\sqrt{13}\right)\right)}{3\sqrt{3+5x^2+x^4}} \\
&\quad + \frac{\sqrt{\frac{3}{2\left(5+\sqrt{13}\right)}}\sqrt{\frac{6+\left(5-\sqrt{13}\right)x^2}{6+\left(5+\sqrt{13}\right)x^2}}\left(6+\left(5+\sqrt{13}\right)x^2\right)F\left(\tan^{-1}\left(\sqrt{\frac{1}{6}\left(5+\sqrt{13}\right)}x\right)|\frac{1}{6}\left(-13+5\sqrt{13}\right)\right)}{\sqrt{3+5x^2+x^4}}
\end{aligned}$$

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 10.23 (sec) , antiderivative size = 224, normalized size of antiderivative = 0.81

$$\int \frac{2+3x^2}{x^2\sqrt{3+5x^2+x^4}}\,dx$$
$$= \frac{-4(3+5x^2+x^4)+i\sqrt{2}(-5+\sqrt{13})\,x\sqrt{\frac{-5+\sqrt{13}-2x^2}{-5+\sqrt{13}}}\sqrt{5+\sqrt{13}+2x^2}E\left(i\text{arcsinh}\left(\sqrt{\frac{2}{5+\sqrt{13}}}x\right)|\frac{19}{6}+\frac{5\sqrt{13}}{6}\right)}{6x\sqrt{3+5x^2+}}$$

```
[In] Integrate[(2 + 3*x^2)/(x^2*Sqrt[3 + 5*x^2 + x^4]),x]
```

```
[Out] (-4*(3 + 5*x^2 + x^4) + I*Sqrt[2]*(-5 + Sqrt[13])*x*Sqrt[(-5 + Sqrt[13] - 2
*x^2)/(-5 + Sqrt[13])]*Sqrt[5 + Sqrt[13] + 2*x^2]*EllipticE[I*ArcSinh[Sqrt[
2/(5 + Sqrt[13])]*x], 19/6 + (5*Sqrt[13])/6] - I*Sqrt[2]*(4 + Sqrt[13])*x*S
qrt[(-5 + Sqrt[13] - 2*x^2)/(-5 + Sqrt[13])]*Sqrt[5 + Sqrt[13] + 2*x^2]*Ell
ipticF[I*ArcSinh[Sqrt[2/(5 + Sqrt[13])]*x], 19/6 + (5*Sqrt[13])/6])/(6*x*Sq
rt[3 + 5*x^2 + x^4])
```

Maple [A] (verified)

Time = 1.40 (sec) , antiderivative size = 211, normalized size of antiderivative = 0.76

method	result
default	$\frac{18\sqrt{1-\left(-\frac{5}{6}+\frac{\sqrt{13}}{6}\right)x^2}\sqrt{1-\left(-\frac{5}{6}-\frac{\sqrt{13}}{6}\right)x^2}\,F\left(\frac{x\sqrt{-30+6\sqrt{13}}}{6},\frac{5\sqrt{3}}{6}+\frac{\sqrt{39}}{6}\right)}{\sqrt{-30+6\sqrt{13}}\sqrt{x^4+5x^2+3}}-\frac{2\sqrt{x^4+5x^2+3}}{3x}-\frac{24\sqrt{1-\left(-\frac{5}{6}+\frac{\sqrt{13}}{6}\right)x^2}\sqrt{1-\left(-\frac{5}{6}-\right.}}{}$
risch	$\frac{18\sqrt{1-\left(-\frac{5}{6}+\frac{\sqrt{13}}{6}\right)x^2}\sqrt{1-\left(-\frac{5}{6}-\frac{\sqrt{13}}{6}\right)x^2}\,F\left(\frac{x\sqrt{-30+6\sqrt{13}}}{6},\frac{5\sqrt{3}}{6}+\frac{\sqrt{39}}{6}\right)}{\sqrt{-30+6\sqrt{13}}\sqrt{x^4+5x^2+3}}-\frac{2\sqrt{x^4+5x^2+3}}{3x}-\frac{24\sqrt{1-\left(-\frac{5}{6}+\frac{\sqrt{13}}{6}\right)x^2}\sqrt{1-\left(-\frac{5}{6}-\right.}}{}$
elliptic	$\frac{18\sqrt{1-\left(-\frac{5}{6}+\frac{\sqrt{13}}{6}\right)x^2}\sqrt{1-\left(-\frac{5}{6}-\frac{\sqrt{13}}{6}\right)x^2}\,F\left(\frac{x\sqrt{-30+6\sqrt{13}}}{6},\frac{5\sqrt{3}}{6}+\frac{\sqrt{39}}{6}\right)}{\sqrt{-30+6\sqrt{13}}\sqrt{x^4+5x^2+3}}-\frac{2\sqrt{x^4+5x^2+3}}{3x}-\frac{24\sqrt{1-\left(-\frac{5}{6}+\frac{\sqrt{13}}{6}\right)x^2}\sqrt{1-\left(-\frac{5}{6}-\right.}}{}$

```
[In] int((3*x^2+2)/x^2/(x^4+5*x^2+3)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 18/(-30+6*13^(1/2))^(1/2)*(1-(-5/6+1/6*13^(1/2))*x^2)^(1/2)*(1-(-5/6-1/6*13
^(1/2))*x^2)^(1/2)/(x^4+5*x^2+3)^(1/2)*EllipticF(1/6*x*(-30+6*13^(1/2))^(1/
2),5/6*3^(1/2)+1/6*39^(1/2))-2/3*(x^4+5*x^2+3)^(1/2)/x-24/(-30+6*13^(1/2))^
(1/2)*(1-(-5/6+1/6*13^(1/2))*x^2)^(1/2)*(1-(-5/6-1/6*13^(1/2))*x^2)^(1/2)/(
x^4+5*x^2+3)^(1/2)/(5+13^(1/2))*(EllipticF(1/6*x*(-30+6*13^(1/2))^(1/2),5/6
*3^(1/2)+1/6*39^(1/2))-EllipticE(1/6*x*(-30+6*13^(1/2))^(1/2),5/6*3^(1/2)+1
/6*39^(1/2)))
```

Fricas [A] (verification not implemented)

none

Time = 0.08 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.45

$$\int \frac{2+3x^2}{x^2\sqrt{3+5x^2+x^4}}\,dx = -\frac{2\left(\sqrt{13}\sqrt{6}\sqrt{3}x-5\sqrt{6}\sqrt{3}x\right)\sqrt{\sqrt{13}-5}E(\arcsin\left(\frac{1}{6}\sqrt{6}x\sqrt{\sqrt{13}-5}\right)\mid\frac{5}{6}\sqrt{13}+\frac{19}{6})+\left(7\sqrt{13}\sqrt{6}\sqrt{3}x+\right.}{108\,x}$$

```
[In] integrate((3*x^2+2)/x^2/(x^4+5*x^2+3)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/108*(2*(sqrt(13)*sqrt(6)*sqrt(3)*x - 5*sqrt(6)*sqrt(3)*x)*sqrt(sqrt(13)
- 5)*elliptic_e(arcsin(1/6*sqrt(6)*x*sqrt(sqrt(13) - 5)), 5/6*sqrt(13) + 19
/6) + (7*sqrt(13)*sqrt(6)*sqrt(3)*x + 55*sqrt(6)*sqrt(3)*x)*sqrt(sqrt(13) -
 5)*elliptic_f(arcsin(1/6*sqrt(6)*x*sqrt(sqrt(13) - 5)), 5/6*sqrt(13) + 19/
6) + 72*sqrt(x^4 + 5*x^2 + 3))/x
```

Sympy [F]

$$\int \frac{2+3x^2}{x^2\sqrt{3+5x^2+x^4}}\,dx = \int \frac{3x^2+2}{x^2\sqrt{x^4+5x^2+3}}\,dx$$

```
[In] integrate((3*x**2+2)/x**2/(x**4+5*x**2+3)**(1/2),x)
```

```
[Out] Integral((3*x**2 + 2)/(x**2*sqrt(x**4 + 5*x**2 + 3)), x)
```

Maxima [F]

$$\int \frac{2+3x^2}{x^2\sqrt{3+5x^2+x^4}}\,dx = \int \frac{3\,x^2+2}{\sqrt{x^4+5\,x^2+3}x^2}\,dx$$

```
[In] integrate((3*x^2+2)/x^2/(x^4+5*x^2+3)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((3*x^2 + 2)/(sqrt(x^4 + 5*x^2 + 3)*x^2), x)
```

Giac [F]

$$\int \frac{2+3x^2}{x^2\sqrt{3+5x^2+x^4}}\,dx = \int \frac{3\,x^2+2}{\sqrt{x^4+5\,x^2+3}x^2}\,dx$$

```
[In] integrate((3*x^2+2)/x^2/(x^4+5*x^2+3)^(1/2),x, algorithm="giac")
[Out] integrate((3*x^2 + 2)/(sqrt(x^4 + 5*x^2 + 3)*x^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{2+3x^2}{x^2\sqrt{3+5x^2+x^4}}\,dx = \int \frac{3\,x^2+2}{x^2\,\sqrt{x^4+5\,x^2+3}}\,dx$$

```
[In] int((3*x^2 + 2)/(x^2*(5*x^2 + x^4 + 3)^(1/2)),x)
[Out] int((3*x^2 + 2)/(x^2*(5*x^2 + x^4 + 3)^(1/2)), x)
```

3.193 $\int \frac{2+3x^2}{x^4\sqrt{3+5x^2+x^4}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 302

$$\int \frac{2+3x^2}{x^4\sqrt{3+5x^2+x^4}}\,dx = \frac{7x\left(5+\sqrt{13}+2x^2\right)}{54\sqrt{3+5x^2+x^4}} - \frac{2\sqrt{3+5x^2+x^4}}{9x^3} - \frac{7\sqrt{3+5x^2+x^4}}{27x}$$
$$- \frac{7\sqrt{\frac{1}{6}\left(5+\sqrt{13}\right)}\sqrt{\frac{6+\left(5-\sqrt{13}\right)x^2}{6+\left(5+\sqrt{13}\right)x^2}}\left(6+\left(5+\sqrt{13}\right)x^2\right)E\left(\arctan\left(\sqrt{\frac{1}{6}\left(5+\sqrt{13}\right)}x\right)|\frac{1}{6}\left(-13+5\sqrt{13}\right)\right)}{54\sqrt{3+5x^2+x^4}}$$
$$- \frac{\sqrt{\frac{2}{3\left(5+\sqrt{13}\right)}}\sqrt{\frac{6+\left(5-\sqrt{13}\right)x^2}{6+\left(5+\sqrt{13}\right)x^2}}\left(6+\left(5+\sqrt{13}\right)x^2\right)\text{EllipticF}\left(\arctan\left(\sqrt{\frac{1}{6}\left(5+\sqrt{13}\right)}x\right),\frac{1}{6}\left(-13+5\sqrt{13}\right)\right)}{9\sqrt{3+5x^2+x^4}}$$

```
[Out] 7/54*x*(5+2*x^2+13^(1/2))/(x^4+5*x^2+3)^(1/2)-2/9*(x^4+5*x^2+3)^(1/2)/x^3-7
/27*(x^4+5*x^2+3)^(1/2)/x-1/27*(1/(36+x^2*(30+6*13^(1/2))))^(1/2)*(36+x^2*(
30+6*13^(1/2)))^(1/2)*EllipticF(x*(30+6*13^(1/2))^(1/2)/(36+x^2*(30+6*13^(1
/2)))^(1/2),1/6*(-78+30*13^(1/2))^(1/2))*(6+x^2*(5+13^(1/2)))*6^(1/2)/(5+13
^(1/2))^(1/2)*((6+x^2*(5-13^(1/2)))/(6+x^2*(5+13^(1/2))))^(1/2)/(x^4+5*x^2+
3)^(1/2)-7/324*(1/(36+x^2*(30+6*13^(1/2))))^(1/2)*(36+x^2*(30+6*13^(1/2)))^
(1/2)*EllipticE(x*(30+6*13^(1/2))^(1/2)/(36+x^2*(30+6*13^(1/2)))^(1/2),1/6*
(-78+30*13^(1/2))^(1/2))*(6+x^2*(5+13^(1/2)))*(30+6*13^(1/2))^(1/2)*((6+x^2
*(5-13^(1/2)))/(6+x^2*(5+13^(1/2))))^(1/2)/(x^4+5*x^2+3)^(1/2)
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 302, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {1295, 1203, 1113, 1149}

$$\int \frac{2+3x^2}{x^4\sqrt{3+5x^2+x^4}}\,dx = -\frac{\sqrt{\frac{2}{3\left(5+\sqrt{13}\right)}}\sqrt{\frac{\left(5-\sqrt{13}\right)x^2+6}{\left(5+\sqrt{13}\right)x^2+6}}\left(\left(5+\sqrt{13}\right)x^2+6\right)\text{EllipticF}\left(\arctan\left(\sqrt{\frac{1}{6}\left(5+\sqrt{13}\right)}x\right),\frac{1}{6}\left(-13+5\sqrt{13}\right)\right)}{9\sqrt{x^4+5x^2+3}} - \frac{7\sqrt{\frac{1}{6}\left(5+\sqrt{13}\right)}\sqrt{\frac{\left(5-\sqrt{13}\right)x^2+6}{\left(5+\sqrt{13}\right)x^2+6}}\left(\left(5+\sqrt{13}\right)x^2+6\right)E\left(\arctan\left(\sqrt{\frac{1}{6}\left(5+\sqrt{13}\right)}x\right)|\frac{1}{6}\left(-13+5\sqrt{13}\right)\right)}{54\sqrt{x^4+5x^2+3}} + \frac{7x\left(2x^2+\sqrt{13}+5\right)}{54\sqrt{x^4+5x^2+3}} - \frac{7\sqrt{x^4+5x^2+3}}{27x} - \frac{2\sqrt{x^4+5x^2+3}}{9x^3}$$

```
[In] Int[(2 + 3*x^2)/(x^4*Sqrt[3 + 5*x^2 + x^4]),x]
```

```
[Out] (7*x*(5 + Sqrt[13] + 2*x^2))/(54*Sqrt[3 + 5*x^2 + x^4]) - (2*Sqrt[3 + 5*x^2
 + x^4])/(9*x^3) - (7*Sqrt[3 + 5*x^2 + x^4])/(27*x) - (7*Sqrt[(5 + Sqrt[13]
)/6]*Sqrt[(6 + (5 - Sqrt[13])*x^2)/(6 + (5 + Sqrt[13])*x^2)]*(6 + (5 + Sqrt
[13])*x^2)*EllipticE[ArcTan[Sqrt[(5 + Sqrt[13])/6]*x], (-13 + 5*Sqrt[13])/6
])/(54*Sqrt[3 + 5*x^2 + x^4]) - (Sqrt[2/(3*(5 + Sqrt[13]))]*Sqrt[(6 + (5 -
Sqrt[13])*x^2)/(6 + (5 + Sqrt[13])*x^2)]*(6 + (5 + Sqrt[13])*x^2)*EllipticF
[ArcTan[Sqrt[(5 + Sqrt[13])/6]*x], (-13 + 5*Sqrt[13])/6])/(9*Sqrt[3 + 5*x^2
 + x^4])
```

Rule 1113

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[(2*a + (b + q)*x^2)*(Sqrt[(2*a + (b - q)*x^2)/(2*a +
(b + q)*x^2)]/(2*a*Rt[(b + q)/(2*a), 2]*Sqrt[a + b*x^2 + c*x^4]))*EllipticF
[ArcTan[Rt[(b + q)/(2*a), 2]*x], 2*(q/(b + q))], x] /; PosQ[(b + q)/a] &&
!(PosQ[(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[
{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1149

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[b^2 - 4*a*c, 2]}, Simp[x*((b + q + 2*c*x^2)/(2*c*Sqrt[a + b*x^2 + c*x^4
])), x] - Simp[Rt[(b + q)/(2*a), 2]*(2*a + (b + q)*x^2)*(Sqrt[(2*a + (b - q
)*x^2)/(2*a + (b + q)*x^2)]/(2*c*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[ArcTan
[Rt[(b + q)/(2*a), 2]*x], 2*(q/(b + q))], x] /; PosQ[(b + q)/a] &&  !(PosQ[
(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[{a, b,
```

```
c}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1203

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[d, Int[1/Sqrt[a + b*x^2 + c*x^4],
 x], x] + Dist[e, Int[x^2/Sqrt[a + b*x^2 + c*x^4], x], x] /; PosQ[(b + q)/a
] || PosQ[(b - q)/a]] /; FreeQ[{a, b, c, d, e}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1295

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[d*(f*x)^(m + 1)*((a + b*x^2 + c*x^4)^(p + 1)
/(a*f*(m + 1))), x] + Dist[1/(a*f^2*(m + 1)), Int[(f*x)^(m + 2)*(a + b*x^2
+ c*x^4)^p*Simp[a*e*(m + 1) - b*d*(m + 2*p + 3) - c*d*(m + 4*p + 5)*x^2, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[m
, -1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{2\sqrt{3+5x^2+x^4}}{9x^3} - \frac{1}{9}\int \frac{-7+2x^2}{x^2\sqrt{3+5x^2+x^4}}\,dx \\
&= -\frac{2\sqrt{3+5x^2+x^4}}{9x^3} - \frac{7\sqrt{3+5x^2+x^4}}{27x} + \frac{1}{27}\int \frac{-6+7x^2}{\sqrt{3+5x^2+x^4}}\,dx \\
&= -\frac{2\sqrt{3+5x^2+x^4}}{9x^3} - \frac{7\sqrt{3+5x^2+x^4}}{27x} - \frac{2}{9}\int \frac{1}{\sqrt{3+5x^2+x^4}}\,dx + \frac{7}{27}\int \frac{x^2}{\sqrt{3+5x^2+x^4}}\,dx \\
&= \frac{7x\left(5+\sqrt{13}+2x^2\right)}{54\sqrt{3+5x^2+x^4}} - \frac{2\sqrt{3+5x^2+x^4}}{9x^3} - \frac{7\sqrt{3+5x^2+x^4}}{27x} \\
&\quad - \frac{7\sqrt{\frac{1}{6}\left(5+\sqrt{13}\right)}\sqrt{\frac{6+\left(5-\sqrt{13}\right)x^2}{6+\left(5+\sqrt{13}\right)x^2}}\left(6+\left(5+\sqrt{13}\right)x^2\right)E\left(\tan^{-1}\left(\sqrt{\frac{1}{6}\left(5+\sqrt{13}\right)}x\right)|\frac{1}{6}\left(-13+5\sqrt{1\right.\right.}}{54\sqrt{3+5x^2+x^4}} \\
&\quad - \frac{\sqrt{\frac{2}{3\left(5+\sqrt{13}\right)}}\sqrt{\frac{6+\left(5-\sqrt{13}\right)x^2}{6+\left(5+\sqrt{13}\right)x^2}}\left(6+\left(5+\sqrt{13}\right)x^2\right)F\left(\tan^{-1}\left(\sqrt{\frac{1}{6}\left(5+\sqrt{13}\right)}x\right)|\frac{1}{6}\left(-13+5\sqrt{13}\right)\right)}{9\sqrt{3+5x^2+x^4}}
\end{aligned}$$

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 10.24 (sec) , antiderivative size = 237, normalized size of antiderivative = 0.78

$$\int \frac{2+3x^2}{x^4\sqrt{3+5x^2+x^4}}\,dx$$
$$= \frac{-4(18+51x^2+41x^4+7x^6)+7i\sqrt{2}(-5+\sqrt{13})\,x^3\sqrt{\frac{-5+\sqrt{13}-2x^2}{-5+\sqrt{13}}}\sqrt{5+\sqrt{13}+2x^2}E\left(i\text{arcsinh}\left(\sqrt{\frac{2}{5+\sqrt{13}}}x\right)\right.}{108x^3\sqrt{}}$$

```
[In] Integrate[(2 + 3*x^2)/(x^4*Sqrt[3 + 5*x^2 + x^4]),x]
```

```
[Out] (-4*(18 + 51*x^2 + 41*x^4 + 7*x^6) + (7*I)*Sqrt[2]*(-5 + Sqrt[13])*x^3*Sqrt
[(-5 + Sqrt[13] - 2*x^2)/(-5 + Sqrt[13])]*Sqrt[5 + Sqrt[13] + 2*x^2]*Ellipt
icE[I*ArcSinh[Sqrt[2/(5 + Sqrt[13])]*x], 19/6 + (5*Sqrt[13])/6] - I*Sqrt[2]
*(-47 + 7*Sqrt[13])*x^3*Sqrt[(-5 + Sqrt[13] - 2*x^2)/(-5 + Sqrt[13])]*Sqrt[
5 + Sqrt[13] + 2*x^2]*EllipticF[I*ArcSinh[Sqrt[2/(5 + Sqrt[13])]*x], 19/6 +
 (5*Sqrt[13])/6])/(108*x^3*Sqrt[3 + 5*x^2 + x^4])
```

Maple [A] (verified)

Time = 2.11 (sec) , antiderivative size = 228, normalized size of antiderivative = 0.75

method	result
default	$-\frac{7\sqrt{x^4+5x^2+3}}{27x}-\frac{28\sqrt{1-\left(-\frac{5}{6}+\frac{\sqrt{13}}{6}\right)x^2}\sqrt{1-\left(-\frac{5}{6}-\frac{\sqrt{13}}{6}\right)x^2}\left(F\left(\frac{x\sqrt{-30+6\sqrt{13}}}{6},\frac{5\sqrt{3}}{6}+\frac{\sqrt{39}}{6}\right)-E\left(\frac{x\sqrt{-30+6\sqrt{13}}}{6},\frac{5\sqrt{3}}{6}+\frac{\sqrt{39}}{6}\right)\right)}{3\sqrt{-30+6\sqrt{13}}\sqrt{x^4+5x^2+3}\left(5+\sqrt{13}\right)}-$
risch	$-\frac{7x^6+41x^4+51x^2+18}{27x^3\sqrt{x^4+5x^2+3}}-\frac{4\sqrt{1-\left(-\frac{5}{6}+\frac{\sqrt{13}}{6}\right)x^2}\sqrt{1-\left(-\frac{5}{6}-\frac{\sqrt{13}}{6}\right)x^2}F\left(\frac{x\sqrt{-30+6\sqrt{13}}}{6},\frac{5\sqrt{3}}{6}+\frac{\sqrt{39}}{6}\right)}{3\sqrt{-30+6\sqrt{13}}\sqrt{x^4+5x^2+3}}-\frac{28\sqrt{1-\left(-\frac{5}{6}+\frac{\sqrt{13}}{6}\right)x^2}\sqrt{1-}}{}$
elliptic	$-\frac{7\sqrt{x^4+5x^2+3}}{27x}-\frac{28\sqrt{1-\left(-\frac{5}{6}+\frac{\sqrt{13}}{6}\right)x^2}\sqrt{1-\left(-\frac{5}{6}-\frac{\sqrt{13}}{6}\right)x^2}\left(F\left(\frac{x\sqrt{-30+6\sqrt{13}}}{6},\frac{5\sqrt{3}}{6}+\frac{\sqrt{39}}{6}\right)-E\left(\frac{x\sqrt{-30+6\sqrt{13}}}{6},\frac{5\sqrt{3}}{6}+\frac{\sqrt{39}}{6}\right)\right)}{3\sqrt{-30+6\sqrt{13}}\sqrt{x^4+5x^2+3}\left(5+\sqrt{13}\right)}-$

```
[In] int((3*x^2+2)/x^4/(x^4+5*x^2+3)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -7/27*(x^4+5*x^2+3)^(1/2)/x-28/3/(-30+6*13^(1/2))^(1/2)*(1-(-5/6+1/6*13^(1/
2))*x^2)^(1/2)*(1-(-5/6-1/6*13^(1/2))*x^2)^(1/2)/(x^4+5*x^2+3)^(1/2)/(5+13^
(1/2))*(EllipticF(1/6*x*(-30+6*13^(1/2))^(1/2),5/6*3^(1/2)+1/6*39^(1/2))-El
lipticE(1/6*x*(-30+6*13^(1/2))^(1/2),5/6*3^(1/2)+1/6*39^(1/2)))-2/9*(x^4+5*
x^2+3)^(1/2)/x^3-4/3/(-30+6*13^(1/2))^(1/2)*(1-(-5/6+1/6*13^(1/2))*x^2)^(1/
2)*(1-(-5/6-1/6*13^(1/2))*x^2)^(1/2)/(x^4+5*x^2+3)^(1/2)*EllipticF(1/6*x*(-
30+6*13^(1/2))^(1/2),5/6*3^(1/2)+1/6*39^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.08 (sec) , antiderivative size = 140, normalized size of antiderivative = 0.46

$$\int \frac{2+3x^2}{x^4\sqrt{3+5x^2+x^4}}\,dx = -\frac{7\left(\sqrt{13}\sqrt{6}\sqrt{3}x^3-5\sqrt{6}\sqrt{3}x^3\right)\sqrt{\sqrt{13}-5}E(\arcsin\left(\frac{1}{6}\sqrt{6}x\sqrt{\sqrt{13}-5}\right)\mid\frac{5}{6}\sqrt{13}+\frac{19}{6})-(13\sqrt{13}\sqrt{6}\sqrt{3}x}{972}$$

```
[In] integrate((3*x^2+2)/x^4/(x^4+5*x^2+3)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/972*(7*(sqrt(13)*sqrt(6)*sqrt(3)*x^3 - 5*sqrt(6)*sqrt(3)*x^3)*sqrt(sqrt(
13) - 5)*elliptic_e(arcsin(1/6*sqrt(6)*x*sqrt(sqrt(13) - 5)), 5/6*sqrt(13)
+ 19/6) - (13*sqrt(13)*sqrt(6)*sqrt(3)*x^3 - 5*sqrt(6)*sqrt(3)*x^3)*sqrt(sq
rt(13) - 5)*elliptic_f(arcsin(1/6*sqrt(6)*x*sqrt(sqrt(13) - 5)), 5/6*sqrt(1
3) + 19/6) + 36*sqrt(x^4 + 5*x^2 + 3)*(7*x^2 + 6))/x^3
```

Sympy [F]

$$\int \frac{2+3x^2}{x^4\sqrt{3+5x^2+x^4}}\,dx = \int \frac{3x^2+2}{x^4\sqrt{x^4+5x^2+3}}\,dx$$

```
[In] integrate((3*x**2+2)/x**4/(x**4+5*x**2+3)**(1/2),x)
```

```
[Out] Integral((3*x**2 + 2)/(x**4*sqrt(x**4 + 5*x**2 + 3)), x)
```

Maxima [F]

$$\int \frac{2+3x^2}{x^4\sqrt{3+5x^2+x^4}}\,dx = \int \frac{3\,x^2+2}{\sqrt{x^4+5\,x^2+3}x^4}\,dx$$

```
[In] integrate((3*x^2+2)/x^4/(x^4+5*x^2+3)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((3*x^2 + 2)/(sqrt(x^4 + 5*x^2 + 3)*x^4), x)
```

Giac [F]

$$\int \frac{2+3x^2}{x^4\sqrt{3+5x^2+x^4}}\,dx = \int \frac{3\,x^2+2}{\sqrt{x^4+5\,x^2+3}x^4}\,dx$$

```
[In] integrate((3*x^2+2)/x^4/(x^4+5*x^2+3)^(1/2),x, algorithm="giac")
[Out] integrate((3*x^2 + 2)/(sqrt(x^4 + 5*x^2 + 3)*x^4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{2+3x^2}{x^4\sqrt{3+5x^2+x^4}}\,dx = \int \frac{3\,x^2+2}{x^4\,\sqrt{x^4+5\,x^2+3}}\,dx$$

```
[In] int((3*x^2 + 2)/(x^4*(5*x^2 + x^4 + 3)^(1/2)),x)
[Out] int((3*x^2 + 2)/(x^4*(5*x^2 + x^4 + 3)^(1/2)), x)
```

3.194 $\int \frac{x^5(2+3x^2)}{(3+5x^2+x^4)^{3/2}} dx$

Optimal result

Integrand size = 25, antiderivative size = 77

$$\int \frac{x^5(2+3x^2)}{(3+5x^2+x^4)^{3/2}} dx = -\frac{x^2(33+47x^2)}{13\sqrt{3+5x^2+x^4}} + \frac{133}{26}\sqrt{3+5x^2+x^4} - \frac{41}{4}\text{arctanh}\left(\frac{5+2x^2}{2\sqrt{3+5x^2+x^4}}\right)$$

```
[Out] -41/4*arctanh(1/2*(2*x^2+5)/(x^4+5*x^2+3)^(1/2))-1/13*x^2*(47*x^2+33)/(x^4+
5*x^2+3)^(1/2)+133/26*(x^4+5*x^2+3)^(1/2)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {1265, 832, 654, 635, 212}

$$\int \frac{x^5(2+3x^2)}{(3+5x^2+x^4)^{3/2}} dx = -\frac{41}{4}\text{arctanh}\left(\frac{2x^2+5}{2\sqrt{x^4+5x^2+3}}\right) - \frac{(47x^2+33)x^2}{13\sqrt{x^4+5x^2+3}} + \frac{133}{26}\sqrt{x^4+5x^2+3}$$

```
[In] Int[(x^5*(2 + 3*x^2))/(3 + 5*x^2 + x^4)^(3/2),x]
```

```
[Out] -1/13*(x^2*(33 + 47*x^2))/Sqrt[3 + 5*x^2 + x^4] + (133*Sqrt[3 + 5*x^2 + x^4
])/26 - (41*ArcTanh[(5 + 2*x^2)/(2*Sqrt[3 + 5*x^2 + x^4])])/4
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 635

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 654

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 832

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(-(d + e*x)^(m - 1))*(a + b*x + c*x^2)
^(p + 1)*((2*a*c*(e*f + d*g) - b*(c*d*f + a*e*g) - (2*c^2*d*f + b^2*e*g - c
*(b*e*f + b*d*g + 2*a*e*g))*x)/(c*(p + 1)*(b^2 - 4*a*c))), x] - Dist[1/(c*(
p + 1)*(b^2 - 4*a*c)), Int[(d + e*x)^(m - 2)*(a + b*x + c*x^2)^(p + 1)*Simp
[2*c^2*d^2*f*(2*p + 3) + b*e*g*(a*e*(m - 1) + b*d*(p + 2)) - c*(2*a*e*(e*f*
(m - 1) + d*g*m) + b*d*(d*g*(2*p + 3) - e*f*(m - 2*p - 4))) + e*(b^2*e*g*(m
 + p + 1) + 2*c^2*d*f*(m + 2*p + 2) - c*(2*a*e*g*m + b*(e*f + d*g)*(m + 2*p
 + 2)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*c
, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 1] && ((EqQ[m
, 2] && EqQ[p, -3] && RationalQ[a, b, c, d, e, f, g]) ||  !ILtQ[m + 2*p + 3
, 0])
```

Rule 1265

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rubi steps

$$\text{integral} = \frac{1}{2}\text{Subst}\left(\int \frac{x^2(2+3x)}{(3+5x+x^2)^{3/2}}\,dx, x, x^2\right)$$

$$= -\frac{x^2(33+47x^2)}{13\sqrt{3+5x^2+x^4}} + \frac{1}{13}\text{Subst}\left(\int \frac{33+\frac{133x}{2}}{\sqrt{3+5x+x^2}}\,dx, x, x^2\right)$$

$$= -\frac{x^2(33+47x^2)}{13\sqrt{3+5x^2+x^4}} + \frac{133}{26}\sqrt{3+5x^2+x^4} - \frac{41}{4}\text{Subst}\left(\int \frac{1}{\sqrt{3+5x+x^2}}\,dx, x, x^2\right)$$

$$= -\frac{x^2(33+47x^2)}{13\sqrt{3+5x^2+x^4}} + \frac{133}{26}\sqrt{3+5x^2+x^4} - \frac{41}{2}\text{Subst}\left(\int \frac{1}{4-x^2}\,dx, x, \frac{5+2x^2}{\sqrt{3+5x^2+x^4}}\right)$$

$$= -\frac{x^2(33+47x^2)}{13\sqrt{3+5x^2+x^4}} + \frac{133}{26}\sqrt{3+5x^2+x^4} - \frac{41}{4}\tanh^{-1}\left(\frac{5+2x^2}{2\sqrt{3+5x^2+x^4}}\right)$$

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.77

$$\int \frac{x^5(2+3x^2)}{(3+5x^2+x^4)^{3/2}}\,dx = \frac{399+599x^2+39x^4}{26\sqrt{3+5x^2+x^4}} + \frac{41}{4}\log\left(-5-2x^2+2\sqrt{3+5x^2+x^4}\right)$$

```
[In] Integrate[(x^5*(2 + 3*x^2))/(3 + 5*x^2 + x^4)^(3/2),x]
```

```
[Out] (399 + 599*x^2 + 39*x^4)/(26*Sqrt[3 + 5*x^2 + x^4]) + (41*Log[-5 - 2*x^2 + 2*Sqrt[3 + 5*x^2 + x^4]])/4
```

Maple [A] (verified)

Time = 0.22 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.62

method	result	size
risch	$\frac{39x^4+599x^2+399}{26\sqrt{x^4+5x^2+3}} - \frac{41\ln\left(\frac{5}{2}+x^2+\sqrt{x^4+5x^2+3}\right)}{4}$	48
trager	$\frac{39x^4+599x^2+399}{26\sqrt{x^4+5x^2+3}} + \frac{41\ln\left(-2x^2+2\sqrt{x^4+5x^2+3}-5\right)}{4}$	52
pseudoelliptic	$\frac{78x^4-533\ln\left(2x^2+5+2\sqrt{x^4+5x^2+3}\right)\sqrt{x^4+5x^2+3}+1198x^2+798}{52\sqrt{x^4+5x^2+3}}$	63
default	$\frac{3x^4}{2\sqrt{x^4+5x^2+3}} + \frac{41x^2}{4\sqrt{x^4+5x^2+3}} - \frac{133}{8\sqrt{x^4+5x^2+3}} + \frac{\frac{665x^2}{52}+\frac{3325}{104}}{\sqrt{x^4+5x^2+3}} - \frac{41\ln\left(\frac{5}{2}+x^2+\sqrt{x^4+5x^2+3}\right)}{4}$	91
elliptic	$\frac{3x^4}{2\sqrt{x^4+5x^2+3}} + \frac{41x^2}{4\sqrt{x^4+5x^2+3}} - \frac{133}{8\sqrt{x^4+5x^2+3}} + \frac{\frac{665x^2}{52}+\frac{3325}{104}}{\sqrt{x^4+5x^2+3}} - \frac{41\ln\left(\frac{5}{2}+x^2+\sqrt{x^4+5x^2+3}\right)}{4}$	91

```
[In] int(x^5*(3*x^2+2)/(x^4+5*x^2+3)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/26*(39*x^4+599*x^2+399)/(x^4+5*x^2+3)^(1/2)-41/4*ln(5/2+x^2+(x^4+5*x^2+3)^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.12

$$\int \frac{x^5(2+3x^2)}{(3+5x^2+x^4)^{3/2}}\,dx = \frac{1811\,x^4+9055\,x^2+1066\,(x^4+5\,x^2+3)\log\left(-2\,x^2+2\,\sqrt{x^4+5\,x^2+3}-5\right)+4\,(3}{104\,(x^4+5\,x^2+3)}$$

```
[In] integrate(x^5*(3*x^2+2)/(x^4+5*x^2+3)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/104*(1811*x^4 + 9055*x^2 + 1066*(x^4 + 5*x^2 + 3)*log(-2*x^2 + 2*sqrt(x^4
 + 5*x^2 + 3) - 5) + 4*(39*x^4 + 599*x^2 + 399)*sqrt(x^4 + 5*x^2 + 3) + 543
3)/(x^4 + 5*x^2 + 3)
```

Sympy [F]

$$\int \frac{x^5(2+3x^2)}{(3+5x^2+x^4)^{3/2}}\,dx = \int \frac{x^5\cdot(3x^2+2)}{(x^4+5x^2+3)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(x**5*(3*x**2+2)/(x**4+5*x**2+3)**(3/2),x)
```

```
[Out] Integral(x**5*(3*x**2 + 2)/(x**4 + 5*x**2 + 3)**(3/2), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.95

$$\int \frac{x^5(2+3x^2)}{(3+5x^2+x^4)^{3/2}}\,dx = \frac{3\,x^4}{2\,\sqrt{x^4+5\,x^2+3}} + \frac{599\,x^2}{26\,\sqrt{x^4+5\,x^2+3}} + \frac{399}{26\,\sqrt{x^4+5\,x^2+3}} - \frac{41}{4}\,\log\left(2\,x^2+2\,\sqrt{x^4+5\,x^2+3}+5\right)$$

```
[In] integrate(x^5*(3*x^2+2)/(x^4+5*x^2+3)^(3/2),x, algorithm="maxima")
```

```
[Out] 3/2*x^4/sqrt(x^4 + 5*x^2 + 3) + 599/26*x^2/sqrt(x^4 + 5*x^2 + 3) + 399/26/s
qrt(x^4 + 5*x^2 + 3) - 41/4*log(2*x^2 + 2*sqrt(x^4 + 5*x^2 + 3) + 5)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.68

$$\int \frac{x^5(2+3x^2)}{(3+5x^2+x^4)^{3/2}}\,dx = \frac{(39\,x^2+599)x^2+399}{26\,\sqrt{x^4+5\,x^2+3}} + \frac{41}{4}\,\log\left(2\,x^2-2\,\sqrt{x^4+5\,x^2+3}+5\right)$$

```
[In] integrate(x^5*(3*x^2+2)/(x^4+5*x^2+3)^(3/2),x, algorithm="giac")
[Out] 1/26*((39*x^2 + 599)*x^2 + 399)/sqrt(x^4 + 5*x^2 + 3) + 41/4*log(2*x^2 - 2*
sqrt(x^4 + 5*x^2 + 3) + 5)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^5(2+3x^2)}{(3+5x^2+x^4)^{3/2}}\,dx = \int \frac{x^5\,(3\,x^2+2)}{(x^4+5\,x^2+3)^{3/2}}\,dx$$

```
[In] int((x^5*(3*x^2 + 2))/(5*x^2 + x^4 + 3)^(3/2),x)
[Out] int((x^5*(3*x^2 + 2))/(5*x^2 + x^4 + 3)^(3/2), x)
```

3.195 $\int \frac{x^3(2+3x^2)}{(3+5x^2+x^4)^{3/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 56

$$\int \frac{x^3(2+3x^2)}{(3+5x^2+x^4)^{3/2}}\,dx = \frac{-33-47x^2}{13\sqrt{3+5x^2+x^4}} + \frac{3}{2}\text{arctanh}\left(\frac{5+2x^2}{2\sqrt{3+5x^2+x^4}}\right)$$

```
[Out] 3/2*arctanh(1/2*(2*x^2+5)/(x^4+5*x^2+3)^(1/2))+1/13*(-47*x^2-33)/(x^4+5*x^2
+3)^(1/2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {1265, 791, 635, 212}

$$\int \frac{x^3(2+3x^2)}{(3+5x^2+x^4)^{3/2}}\,dx = \frac{3}{2}\text{arctanh}\left(\frac{2x^2+5}{2\sqrt{x^4+5x^2+3}}\right) - \frac{47x^2+33}{13\sqrt{x^4+5x^2+3}}$$

```
[In] Int[(x^3*(2 + 3*x^2))/(3 + 5*x^2 + x^4)^(3/2),x]
```

```
[Out] -1/13*(33 + 47*x^2)/Sqrt[3 + 5*x^2 + x^4] + (3*ArcTanh[(5 + 2*x^2)/(2*Sqrt[
3 + 5*x^2 + x^4])])/2
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 635

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 791

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> Simp[(-(2*a*c*(e*f + d*g) - b*(c*d*f + a*e*g) - (
b^2*e*g - b*c*(e*f + d*g) + 2*c*(c*d*f - a*e*g))*x))*((a + b*x + c*x^2)^(p
+ 1)/(c*(p + 1)*(b^2 - 4*a*c))), x] - Dist[(b^2*e*g*(p + 2) - 2*a*c*e*g + c
*(2*c*d*f - b*(e*f + d*g))*(2*p + 3))/(c*(p + 1)*(b^2 - 4*a*c)), Int[(a + b
*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 -
 4*a*c, 0] && LtQ[p, -1]
```

Rule 1265

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{2}\text{Subst}\left(\int \frac{x(2+3x)}{(3+5x+x^2)^{3/2}}\,dx, x, x^2\right)\\
&= -\frac{33+47x^2}{13\sqrt{3+5x^2+x^4}}+\frac{3}{2}\text{Subst}\left(\int \frac{1}{\sqrt{3+5x+x^2}}\,dx, x, x^2\right)\\
&= -\frac{33+47x^2}{13\sqrt{3+5x^2+x^4}}+3\text{Subst}\left(\int \frac{1}{4-x^2}\,dx, x, \frac{5+2x^2}{\sqrt{3+5x^2+x^4}}\right)\\
&= -\frac{33+47x^2}{13\sqrt{3+5x^2+x^4}}+\frac{3}{2}\tanh^{-1}\left(\frac{5+2x^2}{2\sqrt{3+5x^2+x^4}}\right)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.96

$$\int \frac{x^3(2+3x^2)}{(3+5x^2+x^4)^{3/2}}\,dx = \frac{-33-47x^2}{13\sqrt{3+5x^2+x^4}}-\frac{3}{2}\log\left(-5-2x^2+2\sqrt{3+5x^2+x^4}\right)$$

```
[In] Integrate[(x^3*(2 + 3*x^2))/(3 + 5*x^2 + x^4)^(3/2),x]
[Out] (-33 - 47*x^2)/(13*Sqrt[3 + 5*x^2 + x^4]) - (3*Log[-5 - 2*x^2 + 2*Sqrt[3 +
5*x^2 + x^4]])/2
```

Maple [A] (verified)

Time = 0.20 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.77

method	result	size
risch	$-\frac{47x^2+33}{13\sqrt{x^4+5x^2+3}}+\frac{3\ln\left(\frac{5}{2}+x^2+\sqrt{x^4+5x^2+3}\right)}{2}$	43
trager	$-\frac{47x^2+33}{13\sqrt{x^4+5x^2+3}}+\frac{3\ln\left(2x^2+5+2\sqrt{x^4+5x^2+3}\right)}{2}$	47
pseudoelliptic	$\frac{39\ln\left(2x^2+5+2\sqrt{x^4+5x^2+3}\right)\sqrt{x^4+5x^2+3}-94x^2-66}{26\sqrt{x^4+5x^2+3}}$	58
elliptic	$\frac{11}{4\sqrt{x^4+5x^2+3}}-\frac{55(2x^2+5)}{52\sqrt{x^4+5x^2+3}}-\frac{3x^2}{2\sqrt{x^4+5x^2+3}}+\frac{3\ln\left(\frac{5}{2}+x^2+\sqrt{x^4+5x^2+3}\right)}{2}$	74
default	$-\frac{3x^2}{2\sqrt{x^4+5x^2+3}}+\frac{15}{4\sqrt{x^4+5x^2+3}}-\frac{75(2x^2+5)}{52\sqrt{x^4+5x^2+3}}+\frac{3\ln\left(\frac{5}{2}+x^2+\sqrt{x^4+5x^2+3}\right)}{2}+\frac{\frac{10x^2}{13}+\frac{12}{13}}{\sqrt{x^4+5x^2+3}}$	95

```
[In] int(x^3*(3*x^2+2)/(x^4+5*x^2+3)^(3/2),x,method=_RETURNVERBOSE)
[Out] -1/13*(47*x^2+33)/(x^4+5*x^2+3)^(1/2)+3/2*ln(5/2+x^2+(x^4+5*x^2+3)^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.45

$$\int \frac{x^3(2+3x^2)}{(3+5x^2+x^4)^{3/2}}\,dx =$$
$$-\frac{94\,x^4+470\,x^2+39\,(x^4+5\,x^2+3)\log\left(-2\,x^2+2\,\sqrt{x^4+5\,x^2+3}-5\right)+2\,\sqrt{x^4+5\,x^2+3}(47\,x^2+33)+28}{26\,(x^4+5\,x^2+3)}$$

```
[In] integrate(x^3*(3*x^2+2)/(x^4+5*x^2+3)^(3/2),x, algorithm="fricas")
[Out] -1/26*(94*x^4 + 470*x^2 + 39*(x^4 + 5*x^2 + 3)*log(-2*x^2 + 2*sqrt(x^4 + 5*
x^2 + 3) - 5) + 2*sqrt(x^4 + 5*x^2 + 3)*(47*x^2 + 33) + 282)/(x^4 + 5*x^2 +
 3)
```

Sympy [F]

$$\int \frac{x^3(2+3x^2)}{(3+5x^2+x^4)^{3/2}}\,dx = \int \frac{x^3\cdot(3x^2+2)}{(x^4+5x^2+3)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(x**3*(3*x**2+2)/(x**4+5*x**2+3)**(3/2),x)
```

```
[Out] Integral(x**3*(3*x**2 + 2)/(x**4 + 5*x**2 + 3)**(3/2), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.00

$$\int \frac{x^3(2+3x^2)}{(3+5x^2+x^4)^{3/2}}\,dx = -\frac{47\,x^2}{13\,\sqrt{x^4+5\,x^2+3}} - \frac{33}{13\,\sqrt{x^4+5\,x^2+3}} + \frac{3}{2}\,\log\left(2\,x^2+2\,\sqrt{x^4+5\,x^2+3}+5\right)$$

```
[In] integrate(x^3*(3*x^2+2)/(x^4+5*x^2+3)^(3/2),x, algorithm="maxima")
```

```
[Out] -47/13*x^2/sqrt(x^4 + 5*x^2 + 3) - 33/13/sqrt(x^4 + 5*x^2 + 3) + 3/2*log(2*
x^2 + 2*sqrt(x^4 + 5*x^2 + 3) + 5)
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.82

$$\int \frac{x^3(2+3x^2)}{(3+5x^2+x^4)^{3/2}}\,dx = -\frac{47\,x^2+33}{13\,\sqrt{x^4+5\,x^2+3}} - \frac{3}{2}\,\log\left(2\,x^2-2\,\sqrt{x^4+5\,x^2+3}+5\right)$$

```
[In] integrate(x^3*(3*x^2+2)/(x^4+5*x^2+3)^(3/2),x, algorithm="giac")
```

```
[Out] -1/13*(47*x^2 + 33)/sqrt(x^4 + 5*x^2 + 3) - 3/2*log(2*x^2 - 2*sqrt(x^4 + 5*
x^2 + 3) + 5)
```

Mupad [B] (verification not implemented)

Time = 7.85 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.93

$$\int \frac{x^3(2+3x^2)}{(3+5x^2+x^4)^{3/2}}\,dx = \frac{3\,\ln\left(\sqrt{x^4+5\,x^2+3}+x^2+\frac{5}{2}\right)}{2} - \frac{47\,x^2}{13\,\sqrt{x^4+5\,x^2+3}} - \frac{33}{13\,\sqrt{x^4+5\,x^2+3}}$$

```
[In] int((x^3*(3*x^2 + 2))/(5*x^2 + x^4 + 3)^(3/2),x)
[Out] (3*log((5*x^2 + x^4 + 3)^(1/2) + x^2 + 5/2))/2 - (47*x^2)/(13*(5*x^2 + x^4
+ 3)^(1/2)) - 33/(13*(5*x^2 + x^4 + 3)^(1/2))
```

3.196 $\int \frac{x(2+3x^2)}{(3+5x^2+x^4)^{3/2}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 25

$$\int \frac{x(2+3x^2)}{(3+5x^2+x^4)^{3/2}}\,dx = \frac{8+11x^2}{13\sqrt{3+5x^2+x^4}}$$

```
[Out] 1/13*(11*x^2+8)/(x^4+5*x^2+3)^(1/2)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {1261, 650}

$$\int \frac{x(2+3x^2)}{(3+5x^2+x^4)^{3/2}}\,dx = \frac{11x^2+8}{13\sqrt{x^4+5x^2+3}}$$

```
[In] Int[(x*(2 + 3*x^2))/(3 + 5*x^2 + x^4)^(3/2),x]
```

```
[Out] (8 + 11*x^2)/(13*Sqrt[3 + 5*x^2 + x^4])
```

Rule 650

```
Int[((d_.) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(3/2), x_Symbo
l] :> Simp[-2*((b*d - 2*a*e + (2*c*d - b*e)*x)/((b^2 - 4*a*c)*Sqrt[a + b*x
+ c*x^2])), x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b
^2 - 4*a*c, 0]
```

Rule 1261

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(
p_.), x_Symbol] :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x],
```

```
x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]
```

Rubi steps

$$\text{integral} = \frac{1}{2}\text{Subst}\left(\int \frac{2+3x}{(3+5x+x^2)^{3/2}}\,dx, x, x^2\right)$$
$$= \frac{8+11x^2}{13\sqrt{3+5x^2+x^4}}$$

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \frac{x(2+3x^2)}{(3+5x^2+x^4)^{3/2}}\,dx = \frac{8+11x^2}{13\sqrt{3+5x^2+x^4}}$$

```
[In] Integrate[(x*(2 + 3*x^2))/(3 + 5*x^2 + x^4)^(3/2),x]
[Out] (8 + 11*x^2)/(13*Sqrt[3 + 5*x^2 + x^4])
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.88

method	result	size
gosper	$\frac{11x^2+8}{13\sqrt{x^4+5x^2+3}}$	22
trager	$\frac{11x^2+8}{13\sqrt{x^4+5x^2+3}}$	22
risch	$\frac{11x^2+8}{13\sqrt{x^4+5x^2+3}}$	22
elliptic	$\frac{11x^2+8}{13\sqrt{x^4+5x^2+3}}$	22
pseudoelliptic	$\frac{11x^2+8}{13\sqrt{x^4+5x^2+3}}$	22
default	$-\frac{2(2x^2+5)}{13\sqrt{x^4+5x^2+3}}+\frac{\frac{15x^2}{13}+\frac{18}{13}}{\sqrt{x^4+5x^2+3}}$	44

```
[In] int(x*(3*x^2+2)/(x^4+5*x^2+3)^(3/2),x,method=_RETURNVERBOSE)
[Out] 1/13*(11*x^2+8)/(x^4+5*x^2+3)^(1/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 46 vs. $2(21) = 42$.

Time = 0.26 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.84

$$\int \frac{x(2+3x^2)}{(3+5x^2+x^4)^{3/2}}\,dx = \frac{11\,x^4+55\,x^2+\sqrt{x^4+5\,x^2+3}(11\,x^2+8)+33}{13\,(x^4+5\,x^2+3)}$$

```
[In] integrate(x*(3*x^2+2)/(x^4+5*x^2+3)^(3/2),x, algorithm="fricas")
[Out] 1/13*(11*x^4 + 55*x^2 + sqrt(x^4 + 5*x^2 + 3)*(11*x^2 + 8) + 33)/(x^4 + 5*x
^2 + 3)
```

Sympy [F]

$$\int \frac{x(2+3x^2)}{(3+5x^2+x^4)^{3/2}}\,dx = \int \frac{x(3x^2+2)}{(x^4+5x^2+3)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(x*(3*x**2+2)/(x**4+5*x**2+3)**(3/2),x)
[Out] Integral(x*(3*x**2 + 2)/(x**4 + 5*x**2 + 3)**(3/2), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.28

$$\int \frac{x(2+3x^2)}{(3+5x^2+x^4)^{3/2}}\,dx = \frac{11\,x^2}{13\,\sqrt{x^4+5\,x^2+3}} + \frac{8}{13\,\sqrt{x^4+5\,x^2+3}}$$

```
[In] integrate(x*(3*x^2+2)/(x^4+5*x^2+3)^(3/2),x, algorithm="maxima")
[Out] 11/13*x^2/sqrt(x^4 + 5*x^2 + 3) + 8/13/sqrt(x^4 + 5*x^2 + 3)
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.84

$$\int \frac{x(2+3x^2)}{(3+5x^2+x^4)^{3/2}}\,dx = \frac{11\,x^2+8}{13\,\sqrt{x^4+5\,x^2+3}}$$

```
[In] integrate(x*(3*x^2+2)/(x^4+5*x^2+3)^(3/2),x, algorithm="giac")
[Out] 1/13*(11*x^2 + 8)/sqrt(x^4 + 5*x^2 + 3)
```

Mupad [B] (verification not implemented)

Time = 7.58 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.84

$$\int \frac{x(2+3x^2)}{(3+5x^2+x^4)^{3/2}}\,dx = \frac{11\,x^2+8}{13\,\sqrt{x^4+5\,x^2+3}}$$

```
[In] int((x*(3*x^2 + 2))/(5*x^2 + x^4 + 3)^(3/2),x)
[Out] (11*x^2 + 8)/(13*(5*x^2 + x^4 + 3)^(1/2))
```

3.197 $\int \frac{2+3x^2}{x\left(3+5x^2+x^4\right)^{3/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 66

$$\int \frac{2+3x^2}{x\left(3+5x^2+x^4\right)^{3/2}}\,dx = \frac{-7-8x^2}{39\sqrt{3+5x^2+x^4}} - \frac{\operatorname{arctanh}\left(\frac{6+5x^2}{2\sqrt{3}\sqrt{3+5x^2+x^4}}\right)}{3\sqrt{3}}$$

```
[Out] -1/9*arctanh(1/6*(5*x^2+6)*3^(1/2)/(x^4+5*x^2+3)^(1/2))*3^(1/2)+1/39*(-8*x^
2-7)/(x^4+5*x^2+3)^(1/2)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {1265, 836, 12, 738, 212}

$$\int \frac{2+3x^2}{x\left(3+5x^2+x^4\right)^{3/2}}\,dx = -\frac{\operatorname{arctanh}\left(\frac{5x^2+6}{2\sqrt{3}\sqrt{x^4+5x^2+3}}\right)}{3\sqrt{3}} - \frac{8x^2+7}{39\sqrt{x^4+5x^2+3}}$$

```
[In] Int[(2 + 3*x^2)/(x*(3 + 5*x^2 + x^4)^(3/2)),x]
```

```
[Out] -1/39*(7 + 8*x^2)/Sqrt[3 + 5*x^2 + x^4] - ArcTanh[(6 + 5*x^2)/(2*Sqrt[3]*Sq
rt[3 + 5*x^2 + x^4])]/(3*Sqrt[3])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 836

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(f*(b*c*d - b^2*e + 2
*a*c*e) - a*g*(2*c*d - b*e) + c*(f*(2*c*d - b*e) - g*(b*d - 2*a*e))*x)*((a
+ b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2))), x]
 + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^m*
(a + b*x + c*x^2)^(p + 1)*Simp[f*(b*c*d*e*(2*p - m + 2) + b^2*e^2*(p + m +
2) - 2*c^2*d^2*(2*p + 3) - 2*a*c*e^2*(m + 2*p + 3)) - g*(a*e*(b*e - 2*c*d*m
 + b*e*m) - b*d*(3*c*d - b*e + 2*c*d*p - b*e*p)) + c*e*(g*(b*d - 2*a*e) - f
*(2*c*d - b*e))*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1
] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 1265

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{2}\text{Subst}\left(\int \frac{2+3x}{x\left(3+5x+x^2\right)^{3/2}}\,dx, x, x^2\right)\\
&= -\frac{7+8x^2}{39\sqrt{3+5x^2+x^4}} - \frac{1}{39}\text{Subst}\left(\int -\frac{13}{x\sqrt{3+5x+x^2}}\,dx, x, x^2\right)\\
&= -\frac{7+8x^2}{39\sqrt{3+5x^2+x^4}} + \frac{1}{3}\text{Subst}\left(\int \frac{1}{x\sqrt{3+5x+x^2}}\,dx, x, x^2\right)\\
&= -\frac{7+8x^2}{39\sqrt{3+5x^2+x^4}} - \frac{2}{3}\text{Subst}\left(\int \frac{1}{12-x^2}\,dx, x, \frac{6+5x^2}{\sqrt{3+5x^2+x^4}}\right)
\end{aligned}$$

$$= -\frac{7+8x^2}{39\sqrt{3+5x^2+x^4}} - \frac{\tanh^{-1}\left(\frac{6+5x^2}{2\sqrt{3}\sqrt{3+5x^2+x^4}}\right)}{3\sqrt{3}}$$

Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.94

$$\int \frac{2+3x^2}{x\left(3+5x^2+x^4\right)^{3/2}}\,dx = \frac{-7-8x^2}{39\sqrt{3+5x^2+x^4}} + \frac{2\text{arctanh}\left(\frac{x^2-\sqrt{3+5x^2+x^4}}{\sqrt{3}}\right)}{3\sqrt{3}}$$

```
[In] Integrate[(2 + 3*x^2)/(x*(3 + 5*x^2 + x^4)^(3/2)),x]
[Out] (-7 - 8*x^2)/(39*Sqrt[3 + 5*x^2 + x^4]) + (2*ArcTanh[(x^2 - Sqrt[3 + 5*x^2
+ x^4])/Sqrt[3]])/(3*Sqrt[3])
```

Maple [A] (verified)

Time = 0.34 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.80

method	result	size
risch	$-\frac{8x^2+7}{39\sqrt{x^4+5x^2+3}}-\frac{\operatorname{arctanh}\left(\frac{\left(5x^2+6\right)\sqrt{3}}{6\sqrt{x^4+5x^2+3}}\right)\sqrt{3}}{9}$	53
pseudoelliptic	$\frac{-13\operatorname{arctanh}\left(\frac{\left(5x^2+6\right)\sqrt{3}}{6\sqrt{x^4+5x^2+3}}\right)\sqrt{3}\,\sqrt{x^4+5x^2+3}-24x^2-21}{117\sqrt{x^4+5x^2+3}}$	64
default	$-\frac{4\left(2x^2+5\right)}{39\sqrt{x^4+5x^2+3}}+\frac{1}{3\sqrt{x^4+5x^2+3}}-\frac{\operatorname{arctanh}\left(\frac{\left(5x^2+6\right)\sqrt{3}}{6\sqrt{x^4+5x^2+3}}\right)\sqrt{3}}{9}$	67
elliptic	$-\frac{4\left(2x^2+5\right)}{39\sqrt{x^4+5x^2+3}}+\frac{1}{3\sqrt{x^4+5x^2+3}}-\frac{\operatorname{arctanh}\left(\frac{\left(5x^2+6\right)\sqrt{3}}{6\sqrt{x^4+5x^2+3}}\right)\sqrt{3}}{9}$	67
trager	$-\frac{8x^2+7}{39\sqrt{x^4+5x^2+3}}-\frac{\operatorname{RootOf}\left(\_Z^2-3\right)\ln\left(-\frac{5\operatorname{RootOf}\left(\_Z^2-3\right)x^2+6\sqrt{x^4+5x^2+3}+6\operatorname{RootOf}\left(\_Z^2-3\right)}{x^2}\right)}{9}$	71

```
[In] int((3*x^2+2)/x/(x^4+5*x^2+3)^(3/2),x,method=_RETURNVERBOSE)
[Out] -1/39*(8*x^2+7)/(x^4+5*x^2+3)^(1/2)-1/9*arctanh(1/6*(5*x^2+6)*3^(1/2)/(x^4+
5*x^2+3)^(1/2))*3^(1/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 107 vs. $2(52) = 104$.

Time = 0.27 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.62

$$\int \frac{2+3x^2}{x\left(3+5x^2+x^4\right)^{3/2}}\,dx = -\frac{24\,x^4-13\,\sqrt{3}(x^4+5\,x^2+3)\log\left(\frac{25\,x^2-2\,\sqrt{3}(5\,x^2+6)-2\,\sqrt{x^4+5\,x^2+3}\left(5\,\sqrt{3}-6\right)+30}{x^2}\right)+120\,x^2+3\,\sqrt{x^4+5\,x^2+3}(8\,x}{117\,(x^4+5\,x^2+3)}$$

```
[In] integrate((3*x^2+2)/x/(x^4+5*x^2+3)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/117*(24*x^4 - 13*sqrt(3)*(x^4 + 5*x^2 + 3)*log((25*x^2 - 2*sqrt(3)*(5*x^
2 + 6) - 2*sqrt(x^4 + 5*x^2 + 3)*(5*sqrt(3) - 6) + 30)/x^2) + 120*x^2 + 3*s
qrt(x^4 + 5*x^2 + 3)*(8*x^2 + 7) + 72)/(x^4 + 5*x^2 + 3)
```

Sympy [F]

$$\int \frac{2+3x^2}{x\left(3+5x^2+x^4\right)^{3/2}}\,dx = \int \frac{3x^2+2}{x\left(x^4+5x^2+3\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((3*x**2+2)/x/(x**4+5*x**2+3)**(3/2),x)
```

```
[Out] Integral((3*x**2 + 2)/(x*(x**4 + 5*x**2 + 3)**(3/2)), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.98

$$\int \frac{2+3x^2}{x\left(3+5x^2+x^4\right)^{3/2}}\,dx = -\frac{8\,x^2}{39\,\sqrt{x^4+5\,x^2+3}} - \frac{1}{9}\,\sqrt{3}\log\left(\frac{2\,\sqrt{3}\sqrt{x^4+5\,x^2+3}}{x^2}+\frac{6}{x^2}+5\right) - \frac{7}{39\,\sqrt{x^4+5\,x^2+3}}$$

```
[In] integrate((3*x^2+2)/x/(x^4+5*x^2+3)^(3/2),x, algorithm="maxima")
```

```
[Out] -8/39*x^2/sqrt(x^4 + 5*x^2 + 3) - 1/9*sqrt(3)*log(2*sqrt(3)*sqrt(x^4 + 5*x^
2 + 3)/x^2 + 6/x^2 + 5) - 7/39/sqrt(x^4 + 5*x^2 + 3)
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.18

$$\int \frac{2+3x^2}{x\,(3+5x^2+x^4)^{3/2}}\,dx = -\frac{1}{9}\,\sqrt{3}\log\left(-x^2+\sqrt{3}+\sqrt{x^4+5\,x^2+3}\right) + \frac{1}{9}\,\sqrt{3}\log\left(-x^2-\sqrt{3}+\sqrt{x^4+5\,x^2+3}\right) - \frac{8\,x^2+7}{39\,\sqrt{x^4+5\,x^2+3}}$$

```
[In] integrate((3*x^2+2)/x/(x^4+5*x^2+3)^(3/2),x, algorithm="giac")
[Out] -1/9*sqrt(3)*log(-x^2 + sqrt(3) + sqrt(x^4 + 5*x^2 + 3)) + 1/9*sqrt(3)*log(
-x^2 - sqrt(3) + sqrt(x^4 + 5*x^2 + 3)) - 1/39*(8*x^2 + 7)/sqrt(x^4 + 5*x^2
 + 3)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{2+3x^2}{x\,(3+5x^2+x^4)^{3/2}}\,dx = \int \frac{3\,x^2+2}{x\,(x^4+5\,x^2+3)^{3/2}}\,dx$$

```
[In] int((3*x^2 + 2)/(x*(5*x^2 + x^4 + 3)^(3/2)),x)
[Out] int((3*x^2 + 2)/(x*(5*x^2 + x^4 + 3)^(3/2)), x)
```

3.198 $\int \frac{2+3x^2}{x^3(3+5x^2+x^4)^{3/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 90

$$\int \frac{2+3x^2}{x^3\left(3+5x^2+x^4\right)^{3/2}}\,dx=\frac{-7-8x^2}{39x^2\sqrt{3+5x^2+x^4}}-\frac{2\sqrt{3+5x^2+x^4}}{39x^2}+\frac{\operatorname{arctanh}\left(\frac{6+5x^2}{2\sqrt{3}\sqrt{3+5x^2+x^4}}\right)}{3\sqrt{3}}$$

```
[Out] 1/9*arctanh(1/6*(5*x^2+6)*3^(1/2)/(x^4+5*x^2+3)^(1/2))*3^(1/2)+1/39*(-8*x^2
-7)/x^2/(x^4+5*x^2+3)^(1/2)-2/39*(x^4+5*x^2+3)^(1/2)/x^2
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {1265, 836, 820, 738, 212}

$$\int \frac{2+3x^2}{x^3\left(3+5x^2+x^4\right)^{3/2}}\,dx=\frac{\operatorname{arctanh}\left(\frac{5x^2+6}{2\sqrt{3}\sqrt{x^4+5x^2+3}}\right)}{3\sqrt{3}}-\frac{8x^2+7}{39x^2\sqrt{x^4+5x^2+3}}-\frac{2\sqrt{x^4+5x^2+3}}{39x^2}$$

```
[In] Int[(2 + 3*x^2)/(x^3*(3 + 5*x^2 + x^4)^(3/2)),x]
```

```
[Out] -1/39*(7 + 8*x^2)/(x^2*Sqrt[3 + 5*x^2 + x^4]) - (2*Sqrt[3 + 5*x^2 + x^4])/(
39*x^2) + ArcTanh[(6 + 5*x^2)/(2*Sqrt[3]*Sqrt[3 + 5*x^2 + x^4])]/(3*Sqrt[3]
)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 820

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a +
 b*x + c*x^2)^(p + 1)/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2))), x] - Dist[(b*(e
*f + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(
m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[Simplify[m
+ 2*p + 3], 0]
```

Rule 836

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(f*(b*c*d - b^2*e + 2
*a*c*e) - a*g*(2*c*d - b*e) + c*(f*(2*c*d - b*e) - g*(b*d - 2*a*e))*x)*((a
+ b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2))), x]
 + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^m*
(a + b*x + c*x^2)^(p + 1)*Simp[f*(b*c*d*e*(2*p - m + 2) + b^2*e^2*(p + m +
2) - 2*c^2*d^2*(2*p + 3) - 2*a*c*e^2*(m + 2*p + 3)) - g*(a*e*(b*e - 2*c*d*m
 + b*e*m) - b*d*(3*c*d - b*e + 2*c*d*p - b*e*p)) + c*e*(g*(b*d - 2*a*e) - f
*(2*c*d - b*e))*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1
] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 1265

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rubi steps

$$\text{integral} = \frac{1}{2}\text{Subst}\left(\int \frac{2+3x}{x^2\,(3+5x+x^2)^{3/2}}\,dx, x, x^2\right)$$

$$= -\frac{7+8x^2}{39x^2\sqrt{3+5x^2+x^4}} - \frac{1}{39}\text{Subst}\left(\int \frac{-6+8x}{x^2\sqrt{3+5x+x^2}}\,dx, x, x^2\right)$$

$$= -\frac{7+8x^2}{39x^2\sqrt{3+5x^2+x^4}} - \frac{2\sqrt{3+5x^2+x^4}}{39x^2} - \frac{1}{3}\text{Subst}\left(\int \frac{1}{x\sqrt{3+5x+x^2}}\,dx, x, x^2\right)$$

$$= -\frac{7+8x^2}{39x^2\sqrt{3+5x^2+x^4}} - \frac{2\sqrt{3+5x^2+x^4}}{39x^2} + \frac{2}{3}\text{Subst}\left(\int \frac{1}{12-x^2}\,dx, x, \frac{6+5x^2}{\sqrt{3+5x^2+x^4}}\right)$$

$$= -\frac{7+8x^2}{39x^2\sqrt{3+5x^2+x^4}} - \frac{2\sqrt{3+5x^2+x^4}}{39x^2} + \frac{\tanh^{-1}\left(\frac{6+5x^2}{2\sqrt{3}\sqrt{3+5x^2+x^4}}\right)}{3\sqrt{3}}$$

Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.78

$$\int \frac{2+3x^2}{x^3(3+5x^2+x^4)^{3/2}}\,dx = \frac{-13-18x^2-2x^4}{39x^2\sqrt{3+5x^2+x^4}} - \frac{2\text{arctanh}\left(\frac{x^2-\sqrt{3+5x^2+x^4}}{\sqrt{3}}\right)}{3\sqrt{3}}$$

```
[In] Integrate[(2 + 3*x^2)/(x^3*(3 + 5*x^2 + x^4)^(3/2)),x]
[Out] (-13 - 18*x^2 - 2*x^4)/(39*x^2*Sqrt[3 + 5*x^2 + x^4]) - (2*ArcTanh[(x^2 - S
qrt[3 + 5*x^2 + x^4])/Sqrt[3]])/(3*Sqrt[3])
```

Maple [A] (verified)

Time = 0.37 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.68

method	result	size
risch	$-\frac{2x^4+18x^2+13}{39x^2\sqrt{x^4+5x^2+3}}+\frac{\text{arctanh}\left(\frac{(5x^2+6)\sqrt{3}}{6\sqrt{x^4+5x^2+3}}\right)\sqrt{3}}{9}$	61
pseudoelliptic	$\frac{13\ \text{arctanh}\left(\frac{(5x^2+6)\sqrt{3}}{6\sqrt{x^4+5x^2+3}}\right)\sqrt{3}\,x^2\sqrt{x^4+5x^2+3}-6x^4-54x^2-39}{117x^2\sqrt{x^4+5x^2+3}}$	75
trager	$-\frac{2x^4+18x^2+13}{39x^2\sqrt{x^4+5x^2+3}}-\frac{\text{RootOf}\left(\_Z^2-3\right)\ln\left(-\frac{-5\,\text{RootOf}\left(\_Z^2-3\right)x^2+6\sqrt{x^4+5x^2+3}-6\,\text{RootOf}\left(\_Z^2-3\right)}{x^2}\right)}{9}$	79
default	$-\frac{1}{3\sqrt{x^4+5x^2+3}}-\frac{2x^2+5}{39\sqrt{x^4+5x^2+3}}+\frac{\text{arctanh}\left(\frac{(5x^2+6)\sqrt{3}}{6\sqrt{x^4+5x^2+3}}\right)\sqrt{3}}{9}-\frac{1}{3x^2\sqrt{x^4+5x^2+3}}$	84
elliptic	$-\frac{1}{3\sqrt{x^4+5x^2+3}}-\frac{2x^2+5}{39\sqrt{x^4+5x^2+3}}+\frac{\text{arctanh}\left(\frac{(5x^2+6)\sqrt{3}}{6\sqrt{x^4+5x^2+3}}\right)\sqrt{3}}{9}-\frac{1}{3x^2\sqrt{x^4+5x^2+3}}$	84

```
[In] int((3*x^2+2)/x^3/(x^4+5*x^2+3)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/39*(2*x^4+18*x^2+13)/x^2/(x^4+5*x^2+3)^(1/2)+1/9*arctanh(1/6*(5*x^2+6)*3
^(1/2)/(x^4+5*x^2+3)^(1/2))*3^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.38

$$\int \frac{2+3x^2}{x^3\left(3+5x^2+x^4\right)^{3/2}}\,dx =$$

$$-\frac{6\,x^6+30\,x^4-13\,\sqrt{3}(x^6+5\,x^4+3\,x^2)\log\left(\frac{25\,x^2+2\,\sqrt{3}(5\,x^2+6)+2\,\sqrt{x^4+5\,x^2+3}\left(5\,\sqrt{3}+6\right)+30}{x^2}\right)+18\,x^2+3\,(2\,x^4+1}{117\,(x^6+5\,x^4+3\,x^2)}$$

```
[In] integrate((3*x^2+2)/x^3/(x^4+5*x^2+3)^(3/2),x, algorithm="fricas")
[Out] -1/117*(6*x^6 + 30*x^4 - 13*sqrt(3)*(x^6 + 5*x^4 + 3*x^2)*log((25*x^2 + 2*s
qrt(3)*(5*x^2 + 6) + 2*sqrt(x^4 + 5*x^2 + 3)*(5*sqrt(3) + 6) + 30)/x^2) + 1
8*x^2 + 3*(2*x^4 + 18*x^2 + 13)*sqrt(x^4 + 5*x^2 + 3))/(x^6 + 5*x^4 + 3*x^2
)
```

Sympy [F]

$$\int \frac{2+3x^2}{x^3\left(3+5x^2+x^4\right)^{3/2}}\,dx = \int \frac{3x^2+2}{x^3\left(x^4+5x^2+3\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((3*x**2+2)/x**3/(x**4+5*x**2+3)**(3/2),x)
[Out] Integral((3*x**2 + 2)/(x**3*(x**4 + 5*x**2 + 3)**(3/2)), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.91

$$\int \frac{2+3x^2}{x^3\left(3+5x^2+x^4\right)^{3/2}}\,dx = -\frac{2\,x^2}{39\,\sqrt{x^4+5\,x^2+3}} + \frac{1}{9}\,\sqrt{3}\log\left(\frac{2\,\sqrt{3}\sqrt{x^4+5\,x^2+3}}{x^2}+\frac{6}{x^2}+5\right) - \frac{6}{13\,\sqrt{x^4+5\,x^2+3}} - \frac{1}{3\,\sqrt{x^4+5\,x^2+3}x^2}$$

```
[In] integrate((3*x^2+2)/x^3/(x^4+5*x^2+3)^(3/2),x, algorithm="maxima")
[Out] -2/39*x^2/sqrt(x^4 + 5*x^2 + 3) + 1/9*sqrt(3)*log(2*sqrt(3)*sqrt(x^4 + 5*x^
2 + 3)/x^2 + 6/x^2 + 5) - 6/13/sqrt(x^4 + 5*x^2 + 3) - 1/3/(sqrt(x^4 + 5*x^
2 + 3)*x^2)
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.36

$$\int \frac{2+3x^2}{x^3 (3+5x^2+x^4)^{3/2}} \, dx = -\frac{1}{9} \sqrt{3} \log \left(\frac{x^2+\sqrt{3}-\sqrt{x^4+5\,x^2+3}}{x^2-\sqrt{3}-\sqrt{x^4+5\,x^2+3}}\right) + \frac{7\,x^2+11}{117\,\sqrt{x^4+5\,x^2+3}} + \frac{5\,x^2-5\,\sqrt{x^4+5\,x^2+3}+6}{9\left(\left(x^2-\sqrt{x^4+5\,x^2+3}\right)^2-3\right)}$$

```
[In] integrate((3*x^2+2)/x^3/(x^4+5*x^2+3)^(3/2),x, algorithm="giac")
```

```
[Out] -1/9*sqrt(3)*log((x^2 + sqrt(3) - sqrt(x^4 + 5*x^2 + 3))/(x^2 - sqrt(3) - s
qrt(x^4 + 5*x^2 + 3))) + 1/117*(7*x^2 + 11)/sqrt(x^4 + 5*x^2 + 3) + 1/9*(5*
x^2 - 5*sqrt(x^4 + 5*x^2 + 3) + 6)/((x^2 - sqrt(x^4 + 5*x^2 + 3))^2 - 3)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{2+3x^2}{x^3 (3+5x^2+x^4)^{3/2}} \, dx = \int \frac{3\,x^2+2}{x^3\,\left(x^4+5\,x^2+3\right)^{3/2}} \, dx$$

```
[In] int((3*x^2 + 2)/(x^3*(5*x^2 + x^4 + 3)^(3/2)),x)
```

```
[Out] int((3*x^2 + 2)/(x^3*(5*x^2 + x^4 + 3)^(3/2)), x)
```

3.199 $\int \frac{x^4(2+3x^2)}{(3+5x^2+x^4)^{3/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 307

$$\int \frac{x^4(2+3x^2)}{(3+5x^2+x^4)^{3/2}}\,dx = \frac{43x(5+\sqrt{13}+2x^2)}{13\sqrt{3+5x^2+x^4}} + \frac{x^3(8+11x^2)}{13\sqrt{3+5x^2+x^4}} - \frac{11}{13}x\sqrt{3+5x^2+x^4} - \frac{43\sqrt{\frac{1}{6}(5+\sqrt{13})}\sqrt{\frac{6+(5-\sqrt{13})x^2}{6+(5+\sqrt{13})x^2}}(6+(5+\sqrt{13})x^2)\,E\left(\arctan\left(\sqrt{\frac{1}{6}(5+\sqrt{13})}x\right)|\frac{1}{6}(-13+5\sqrt{13})\right)}{13\sqrt{3+5x^2+x^4}} + \frac{11\sqrt{\frac{3}{2(5+\sqrt{13})}}\sqrt{\frac{6+(5-\sqrt{13})x^2}{6+(5+\sqrt{13})x^2}}(6+(5+\sqrt{13})x^2)\,\text{EllipticF}\left(\arctan\left(\sqrt{\frac{1}{6}(5+\sqrt{13})}x\right),\frac{1}{6}(-13+5\sqrt{13})\right)}{13\sqrt{3+5x^2+x^4}}$$

```
[Out] 1/13*x^3*(11*x^2+8)/(x^4+5*x^2+3)^(1/2)+43/13*x*(5+2*x^2+13^(1/2))/(x^4+5*x
^2+3)^(1/2)-11/13*x*(x^4+5*x^2+3)^(1/2)+11/26*(1/(36+x^2*(30+6*13^(1/2))))^
(1/2)*(36+x^2*(30+6*13^(1/2)))^(1/2)*EllipticF(x*(30+6*13^(1/2))^(1/2)/(36+
x^2*(30+6*13^(1/2)))^(1/2),1/6*(-78+30*13^(1/2))^(1/2))*(6+x^2*(5+13^(1/2))
)*6^(1/2)/(5+13^(1/2))^(1/2)*((6+x^2*(5-13^(1/2)))/(6+x^2*(5+13^(1/2))))^(1
/2)/(x^4+5*x^2+3)^(1/2)-43/78*(1/(36+x^2*(30+6*13^(1/2))))^(1/2)*(36+x^2*(3
0+6*13^(1/2)))^(1/2)*EllipticE(x*(30+6*13^(1/2))^(1/2)/(36+x^2*(30+6*13^(1/
2)))^(1/2),1/6*(-78+30*13^(1/2))^(1/2))*(6+x^2*(5+13^(1/2)))*(30+6*13^(1/2)
)^(1/2)*((6+x^2*(5-13^(1/2)))/(6+x^2*(5+13^(1/2))))^(1/2)/(x^4+5*x^2+3)^(1/
2)
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 307, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {1289, 1293, 1203, 1113, 1149}

$$\int \frac{x^4(2+3x^2)}{(3+5x^2+x^4)^{3/2}}\,dx = \frac{11\sqrt{\frac{3}{2\left(5+\sqrt{13}\right)}}\sqrt{\frac{\left(5-\sqrt{13}\right)x^2+6}{\left(5+\sqrt{13}\right)x^2+6}}\left(\left(5+\sqrt{13}\right)x^2+6\right)\operatorname{EllipticF}\left(\arctan\left(\sqrt{\frac{1}{6}\left(5+\sqrt{13}\right)}\right.\right.}{13\sqrt{x^4+5x^2+3}}$$
$$-\frac{43\sqrt{\frac{1}{6}\left(5+\sqrt{13}\right)}\sqrt{\frac{\left(5-\sqrt{13}\right)x^2+6}{\left(5+\sqrt{13}\right)x^2+6}}\left(\left(5+\sqrt{13}\right)x^2+6\right)E\left(\arctan\left(\sqrt{\frac{1}{6}\left(5+\sqrt{13}\right)}x\right)|\frac{1}{6}\left(-13+5\sqrt{13}\right)\right)}{13\sqrt{x^4+5x^2+3}}$$
$$-\frac{11}{13}\sqrt{x^4+5x^2+3}x+\frac{43\left(2x^2+\sqrt{13}+5\right)x}{13\sqrt{x^4+5x^2+3}}+\frac{\left(11x^2+8\right)x^3}{13\sqrt{x^4+5x^2+3}}$$

```
[In] Int[(x^4*(2 + 3*x^2))/(3 + 5*x^2 + x^4)^(3/2),x]
```

```
[Out] (43*x*(5 + Sqrt[13] + 2*x^2))/(13*Sqrt[3 + 5*x^2 + x^4]) + (x^3*(8 + 11*x^2
))/(13*Sqrt[3 + 5*x^2 + x^4]) - (11*x*Sqrt[3 + 5*x^2 + x^4])/13 - (43*Sqrt[
(5 + Sqrt[13])/6]*Sqrt[(6 + (5 - Sqrt[13])*x^2)/(6 + (5 + Sqrt[13])*x^2)]*(
6 + (5 + Sqrt[13])*x^2)*EllipticE[ArcTan[Sqrt[(5 + Sqrt[13])/6]*x], (-13 +
5*Sqrt[13])/6])/(13*Sqrt[3 + 5*x^2 + x^4]) + (11*Sqrt[3/(2*(5 + Sqrt[13]))]
*Sqrt[(6 + (5 - Sqrt[13])*x^2)/(6 + (5 + Sqrt[13])*x^2)]*(6 + (5 + Sqrt[13]
)*x^2)*EllipticF[ArcTan[Sqrt[(5 + Sqrt[13])/6]*x], (-13 + 5*Sqrt[13])/6])/(
13*Sqrt[3 + 5*x^2 + x^4])
```

Rule 1113

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[(2*a + (b + q)*x^2)*(Sqrt[(2*a + (b - q)*x^2)/(2*a +
(b + q)*x^2)]/(2*a*Rt[(b + q)/(2*a), 2]*Sqrt[a + b*x^2 + c*x^4]))*EllipticF
[ArcTan[Rt[(b + q)/(2*a), 2]*x], 2*(q/(b + q))], x] /; PosQ[(b + q)/a] &&
!(PosQ[(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[
{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1149

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[b^2 - 4*a*c, 2]}, Simp[x*((b + q + 2*c*x^2)/(2*c*Sqrt[a + b*x^2 + c*x^4
])), x] - Simp[Rt[(b + q)/(2*a), 2]*(2*a + (b + q)*x^2)*(Sqrt[(2*a + (b - q
)*x^2)/(2*a + (b + q)*x^2)]/(2*c*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[ArcTan
[Rt[(b + q)/(2*a), 2]*x], 2*(q/(b + q))], x] /; PosQ[(b + q)/a] &&  !(PosQ[
(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[{a, b,
c}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1203

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[d, Int[1/Sqrt[a + b*x^2 + c*x^4],
 x], x] + Dist[e, Int[x^2/Sqrt[a + b*x^2 + c*x^4], x], x] /; PosQ[(b + q)/a
] || PosQ[(b - q)/a]] /; FreeQ[{a, b, c, d, e}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1289

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_.), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(a + b*x^2 + c*x^4)^(p + 1)
*((b*d - 2*a*e - (b*e - 2*c*d)*x^2)/(2*(p + 1)*(b^2 - 4*a*c))), x] - Dist[f
^2/(2*(p + 1)*(b^2 - 4*a*c)), Int[(f*x)^(m - 2)*(a + b*x^2 + c*x^4)^(p + 1)
*Simp[(m - 1)*(b*d - 2*a*e) - (4*p + 4 + m + 1)*(b*e - 2*c*d)*x^2, x], x],
x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] &&
GtQ[m, 1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1293

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[e*f*(f*x)^(m - 1)*((a + b*x^2 + c*x^4)^(p +
1)/(c*(m + 4*p + 3))), x] - Dist[f^2/(c*(m + 4*p + 3)), Int[(f*x)^(m - 2)*(
a + b*x^2 + c*x^4)^p*Simp[a*e*(m - 1) + (b*e*(m + 2*p + 1) - c*d*(m + 4*p +
 3))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c,
 0] && GtQ[m, 1] && NeQ[m + 4*p + 3, 0] && IntegerQ[2*p] && (IntegerQ[p] ||
 IntegerQ[m])
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{x^3(8+11x^2)}{13\sqrt{3+5x^2+x^4}}+\frac{1}{13}\int\frac{x^2(-24-33x^2)}{\sqrt{3+5x^2+x^4}}\,dx\\
&= \frac{x^3(8+11x^2)}{13\sqrt{3+5x^2+x^4}}-\frac{11}{13}x\sqrt{3+5x^2+x^4}-\frac{1}{39}\int\frac{-99-258x^2}{\sqrt{3+5x^2+x^4}}\,dx\\
&= \frac{x^3(8+11x^2)}{13\sqrt{3+5x^2+x^4}}-\frac{11}{13}x\sqrt{3+5x^2+x^4}+\frac{33}{13}\int\frac{1}{\sqrt{3+5x^2+x^4}}\,dx+\frac{86}{13}\int\frac{x^2}{\sqrt{3+5x^2+x^4}}\,dx\\
&= \frac{43x\left(5+\sqrt{13}+2x^2\right)}{13\sqrt{3+5x^2+x^4}}+\frac{x^3(8+11x^2)}{13\sqrt{3+5x^2+x^4}}-\frac{11}{13}x\sqrt{3+5x^2+x^4}\\
&\quad-\frac{43\sqrt{\frac{1}{6}\left(5+\sqrt{13}\right)}\sqrt{\frac{6+\left(5-\sqrt{13}\right)x^2}{6+\left(5+\sqrt{13}\right)x^2}}\left(6+\left(5+\sqrt{13}\right)x^2\right)E\left(\tan^{-1}\left(\sqrt{\frac{1}{6}\left(5+\sqrt{13}\right)}x\right)|\frac{1}{6}\left(-13+5\sqrt{\right.}\right.}{13\sqrt{3+5x^2+x^4}}\\
&\quad+\frac{11\sqrt{\frac{3}{2\left(5+\sqrt{13}\right)}}\sqrt{\frac{6+\left(5-\sqrt{13}\right)x^2}{6+\left(5+\sqrt{13}\right)x^2}}\left(6+\left(5+\sqrt{13}\right)x^2\right)F\left(\tan^{-1}\left(\sqrt{\frac{1}{6}\left(5+\sqrt{13}\right)}x\right)|\frac{1}{6}\left(-13+5\sqrt{13}\right)\right.}{13\sqrt{3+5x^2+x^4}}
\end{aligned}
$$

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 10.23 (sec) , antiderivative size = 219, normalized size of antiderivative = 0.71

$$\int \frac{x^4(2+3x^2)}{(3+5x^2+x^4)^{3/2}}\,dx = \frac{-2x(33+47x^2)+43i\sqrt{2}(-5+\sqrt{13})\sqrt{\frac{-5+\sqrt{13}-2x^2}{-5+\sqrt{13}}}\sqrt{5+\sqrt{13}+2x^2}\,E\Big(i\operatorname{arcsinh}\Big(\ldots}{\ldots}$$

```
[In] Integrate[(x^4*(2 + 3*x^2))/(3 + 5*x^2 + x^4)^(3/2),x]
```

```
[Out] (-2*x*(33 + 47*x^2) + (43*I)*Sqrt[2]*(-5 + Sqrt[13])*Sqrt[(-5 + Sqrt[13] -
2*x^2)/(-5 + Sqrt[13])]*Sqrt[5 + Sqrt[13] + 2*x^2]*EllipticE[I*ArcSinh[Sqrt
[2/(5 + Sqrt[13])]*x], 19/6 + (5*Sqrt[13])/6] - I*Sqrt[2]*(-182 + 43*Sqrt[1
3])*Sqrt[(-5 + Sqrt[13] - 2*x^2)/(-5 + Sqrt[13])]*Sqrt[5 + Sqrt[13] + 2*x^2
]*EllipticF[I*ArcSinh[Sqrt[2/(5 + Sqrt[13])]*x], 19/6 + (5*Sqrt[13])/6])/(2
6*Sqrt[3 + 5*x^2 + x^4])
```

Maple [A] (verified)

Time = 4.03 (sec) , antiderivative size = 216, normalized size of antiderivative = 0.70

method	result
risch	$-\frac{x(47x^2+33)}{13\sqrt{x^4+5x^2+3}}+\frac{198\sqrt{1-\left(-\frac{5}{6}+\frac{\sqrt{13}}{6}\right)x^2}\,\sqrt{1-\left(-\frac{5}{6}-\frac{\sqrt{13}}{6}\right)x^2}\,F\left(\frac{x\sqrt{-30+6\sqrt{13}}}{6},\frac{5\sqrt{3}}{6}+\frac{\sqrt{39}}{6}\right)}{13\sqrt{-30+6\sqrt{13}}\,\sqrt{x^4+5x^2+3}}-\frac{3096\sqrt{1-\left(-\frac{5}{6}+\frac{\sqrt{13}}{6}\right)x^2}\,\sqrt{1-(\ldots}{\ldots}$
elliptic	$-\frac{2\left(\frac{47}{26}x^3+\frac{33}{26}x\right)}{\sqrt{x^4+5x^2+3}}+\frac{198\sqrt{1-\left(-\frac{5}{6}+\frac{\sqrt{13}}{6}\right)x^2}\,\sqrt{1-\left(-\frac{5}{6}-\frac{\sqrt{13}}{6}\right)x^2}\,F\left(\frac{x\sqrt{-30+6\sqrt{13}}}{6},\frac{5\sqrt{3}}{6}+\frac{\sqrt{39}}{6}\right)}{13\sqrt{-30+6\sqrt{13}}\,\sqrt{x^4+5x^2+3}}-\frac{3096\sqrt{1-\left(-\frac{5}{6}+\frac{\sqrt{13}}{6}\right)x^2}\,\sqrt{1-(\ldots}{\ldots}$
default	$-\frac{6\left(\frac{19}{26}x^3+\frac{15}{26}x\right)}{\sqrt{x^4+5x^2+3}}+\frac{198\sqrt{1-\left(-\frac{5}{6}+\frac{\sqrt{13}}{6}\right)x^2}\,\sqrt{1-\left(-\frac{5}{6}-\frac{\sqrt{13}}{6}\right)x^2}\,F\left(\frac{x\sqrt{-30+6\sqrt{13}}}{6},\frac{5\sqrt{3}}{6}+\frac{\sqrt{39}}{6}\right)}{13\sqrt{-30+6\sqrt{13}}\,\sqrt{x^4+5x^2+3}}-\frac{3096\sqrt{1-\left(-\frac{5}{6}+\frac{\sqrt{13}}{6}\right)x^2}\,\sqrt{1-(\ldots}{\ldots}$

```
[In] int(x^4*(3*x^2+2)/(x^4+5*x^2+3)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/13*x*(47*x^2+33)/(x^4+5*x^2+3)^(1/2)+198/13/(-30+6*13^(1/2))^(1/2)*(1-(-
5/6+1/6*13^(1/2))*x^2)^(1/2)*(1-(-5/6-1/6*13^(1/2))*x^2)^(1/2)/(x^4+5*x^2+3
)^(1/2)*EllipticF(1/6*x*(-30+6*13^(1/2))^(1/2),5/6*3^(1/2)+1/6*39^(1/2))-30
96/13/(-30+6*13^(1/2))^(1/2)*(1-(-5/6+1/6*13^(1/2))*x^2)^(1/2)*(1-(-5/6-1/6
*13^(1/2))*x^2)^(1/2)/(x^4+5*x^2+3)^(1/2)/(5+13^(1/2))*(EllipticF(1/6*x*(-3
0+6*13^(1/2))^(1/2),5/6*3^(1/2)+1/6*39^(1/2))-EllipticE(1/6*x*(-30+6*13^(1/
2))^(1/2),5/6*3^(1/2)+1/6*39^(1/2)))
```

Fricas [A] (verification not implemented)

none

Time = 0.08 (sec) , antiderivative size = 184, normalized size of antiderivative = 0.60

$$\int \frac{x^4(2+3x^2)}{(3+5x^2+x^4)^{3/2}}\,dx = \frac{86\left(\sqrt{13}\sqrt{2}(x^5+5\,x^3+3\,x)-5\,\sqrt{2}(x^5+5\,x^3+3\,x)\right)\sqrt{\sqrt{13}-5}E(\arcsin\left(\frac{\sqrt{2}\sqrt{\sqrt{}}}{2\,x}\right.$$

```
[In] integrate(x^4*(3*x^2+2)/(x^4+5*x^2+3)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/52*(86*(sqrt(13)*sqrt(2)*(x^5 + 5*x^3 + 3*x) - 5*sqrt(2)*(x^5 + 5*x^3 + 3
*x))*sqrt(sqrt(13) - 5)*elliptic_e(arcsin(1/2*sqrt(2)*sqrt(sqrt(13) - 5)/x)
, 5/6*sqrt(13) + 19/6) - 5*(15*sqrt(13)*sqrt(2)*(x^5 + 5*x^3 + 3*x) - 97*sq
rt(2)*(x^5 + 5*x^3 + 3*x))*sqrt(sqrt(13) - 5)*elliptic_f(arcsin(1/2*sqrt(2)
*sqrt(sqrt(13) - 5)/x), 5/6*sqrt(13) + 19/6) + 4*(39*x^4 + 397*x^2 + 258)*s
qrt(x^4 + 5*x^2 + 3))/(x^5 + 5*x^3 + 3*x)
```

Sympy [F]

$$\int \frac{x^4(2+3x^2)}{(3+5x^2+x^4)^{3/2}}\,dx = \int \frac{x^4\cdot(3x^2+2)}{(x^4+5x^2+3)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(x**4*(3*x**2+2)/(x**4+5*x**2+3)**(3/2),x)
```

```
[Out] Integral(x**4*(3*x**2 + 2)/(x**4 + 5*x**2 + 3)**(3/2), x)
```

Maxima [F]

$$\int \frac{x^4(2+3x^2)}{(3+5x^2+x^4)^{3/2}}\,dx = \int \frac{(3\,x^2+2)x^4}{(x^4+5\,x^2+3)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(x^4*(3*x^2+2)/(x^4+5*x^2+3)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((3*x^2 + 2)*x^4/(x^4 + 5*x^2 + 3)^(3/2), x)
```

Giac [F]

$$\int \frac{x^4(2+3x^2)}{(3+5x^2+x^4)^{3/2}}\,dx = \int \frac{(3\,x^2+2)x^4}{(x^4+5\,x^2+3)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(x^4*(3*x^2+2)/(x^4+5*x^2+3)^(3/2),x, algorithm="giac")
[Out] integrate((3*x^2 + 2)*x^4/(x^4 + 5*x^2 + 3)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^4(2+3x^2)}{(3+5x^2+x^4)^{3/2}}\,dx = \int \frac{x^4\,(3\,x^2+2)}{(x^4+5\,x^2+3)^{3/2}}\,dx$$

```
[In] int((x^4*(3*x^2 + 2))/(5*x^2 + x^4 + 3)^(3/2),x)
[Out] int((x^4*(3*x^2 + 2))/(5*x^2 + x^4 + 3)^(3/2), x)
```

3.200 $\int \frac{x^2(2+3x^2)}{(3+5x^2+x^4)^{3/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 286

$$\int \frac{x^2(2+3x^2)}{(3+5x^2+x^4)^{3/2}}\,dx = -\frac{11x(5+\sqrt{13}+2x^2)}{26\sqrt{3+5x^2+x^4}}+\frac{x(8+11x^2)}{13\sqrt{3+5x^2+x^4}}$$
$$+\frac{11\sqrt{\frac{1}{6}\left(5+\sqrt{13}\right)}\sqrt{\frac{6+\left(5-\sqrt{13}\right)x^2}{6+\left(5+\sqrt{13}\right)x^2}}\left(6+\left(5+\sqrt{13}\right)x^2\right)E\left(\arctan\left(\sqrt{\frac{1}{6}\left(5+\sqrt{13}\right)}x\right)|\frac{1}{6}\left(-13+5\sqrt{13}\right)\right)}{26\sqrt{3+5x^2+x^4}}$$
$$-\frac{4\sqrt{\frac{2}{3\left(5+\sqrt{13}\right)}}\sqrt{\frac{6+\left(5-\sqrt{13}\right)x^2}{6+\left(5+\sqrt{13}\right)x^2}}\left(6+\left(5+\sqrt{13}\right)x^2\right)\operatorname{EllipticF}\left(\arctan\left(\sqrt{\frac{1}{6}\left(5+\sqrt{13}\right)}x\right),\frac{1}{6}\left(-13+5\sqrt{13}\right)\right)}{13\sqrt{3+5x^2+x^4}}$$

```
[Out] 1/13*x*(11*x^2+8)/(x^4+5*x^2+3)^(1/2)-11/26*x*(5+2*x^2+13^(1/2))/(x^4+5*x^2
+3)^(1/2)-4/39*(1/(36+x^2*(30+6*13^(1/2))))^(1/2)*(36+x^2*(30+6*13^(1/2)))^
(1/2)*EllipticF(x*(30+6*13^(1/2))^(1/2)/(36+x^2*(30+6*13^(1/2)))^(1/2),1/6*
(-78+30*13^(1/2))^(1/2))*(6+x^2*(5+13^(1/2)))*6^(1/2)/(5+13^(1/2))^(1/2)*((
6+x^2*(5-13^(1/2)))/(6+x^2*(5+13^(1/2))))^(1/2)/(x^4+5*x^2+3)^(1/2)+11/156*
(1/(36+x^2*(30+6*13^(1/2))))^(1/2)*(36+x^2*(30+6*13^(1/2)))^(1/2)*EllipticE
(x*(30+6*13^(1/2))^(1/2)/(36+x^2*(30+6*13^(1/2)))^(1/2),1/6*(-78+30*13^(1/2
))^(1/2))*(6+x^2*(5+13^(1/2)))*(30+6*13^(1/2))^(1/2)*((6+x^2*(5-13^(1/2)))/
(6+x^2*(5+13^(1/2))))^(1/2)/(x^4+5*x^2+3)^(1/2)
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 286, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {1289, 1203, 1113, 1149}

$$\int \frac{x^2(2+3x^2)}{(3+5x^2+x^4)^{3/2}}\,dx = -\frac{4\sqrt{\frac{2}{3(5+\sqrt{13})}}\sqrt{\frac{(5-\sqrt{13})x^2+6}{(5+\sqrt{13})x^2+6}}\left((5+\sqrt{13})x^2+6\right)\text{EllipticF}\left(\arctan\left(\sqrt{\frac{1}{6}(5+\sqrt{13})}x\right),\frac{1}{6}(-13+5\sqrt{13})\right)}{13\sqrt{x^4+5x^2+3}}$$
$$+\frac{11\sqrt{\frac{1}{6}(5+\sqrt{13})}\sqrt{\frac{(5-\sqrt{13})x^2+6}{(5+\sqrt{13})x^2+6}}\left((5+\sqrt{13})x^2+6\right)E\left(\arctan\left(\sqrt{\frac{1}{6}(5+\sqrt{13})}x\right)|\frac{1}{6}(-13+5\sqrt{13})\right)}{26\sqrt{x^4+5x^2+3}}$$
$$-\frac{11x(2x^2+\sqrt{13}+5)}{26\sqrt{x^4+5x^2+3}}+\frac{x(11x^2+8)}{13\sqrt{x^4+5x^2+3}}$$

```
[In] Int[(x^2*(2 + 3*x^2))/(3 + 5*x^2 + x^4)^(3/2),x]
```

```
[Out] (-11*x*(5 + Sqrt[13] + 2*x^2))/(26*Sqrt[3 + 5*x^2 + x^4]) + (x*(8 + 11*x^2)
)/(13*Sqrt[3 + 5*x^2 + x^4]) + (11*Sqrt[(5 + Sqrt[13])/6]*Sqrt[(6 + (5 - Sq
rt[13])*x^2)/(6 + (5 + Sqrt[13])*x^2)]*(6 + (5 + Sqrt[13])*x^2)*EllipticE[A
rcTan[Sqrt[(5 + Sqrt[13])/6]*x], (-13 + 5*Sqrt[13])/6])/(26*Sqrt[3 + 5*x^2
+ x^4]) - (4*Sqrt[2/(3*(5 + Sqrt[13]))]*Sqrt[(6 + (5 - Sqrt[13])*x^2)/(6 +
(5 + Sqrt[13])*x^2)]*(6 + (5 + Sqrt[13])*x^2)*EllipticF[ArcTan[Sqrt[(5 + Sq
rt[13])/6]*x], (-13 + 5*Sqrt[13])/6])/(13*Sqrt[3 + 5*x^2 + x^4])
```

Rule 1113

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[(2*a + (b + q)*x^2)*(Sqrt[(2*a + (b - q)*x^2)/(2*a +
(b + q)*x^2)]/(2*a*Rt[(b + q)/(2*a), 2]*Sqrt[a + b*x^2 + c*x^4]))*EllipticF
[ArcTan[Rt[(b + q)/(2*a), 2]*x], 2*(q/(b + q))], x] /; PosQ[(b + q)/a] &&
!(PosQ[(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[
{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1149

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[b^2 - 4*a*c, 2]}, Simp[x*((b + q + 2*c*x^2)/(2*c*Sqrt[a + b*x^2 + c*x^4
])), x] - Simp[Rt[(b + q)/(2*a), 2]*(2*a + (b + q)*x^2)*(Sqrt[(2*a + (b - q
)*x^2)/(2*a + (b + q)*x^2)]/(2*c*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[ArcTan
[Rt[(b + q)/(2*a), 2]*x], 2*(q/(b + q))], x] /; PosQ[(b + q)/a] &&  !(PosQ[
(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[{a, b,
```

```
c}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1203

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[d, Int[1/Sqrt[a + b*x^2 + c*x^4],
 x], x] + Dist[e, Int[x^2/Sqrt[a + b*x^2 + c*x^4], x], x] /; PosQ[(b + q)/a
] || PosQ[(b - q)/a]] /; FreeQ[{a, b, c, d, e}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1289

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_.), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(a + b*x^2 + c*x^4)^(p + 1)
*((b*d - 2*a*e - (b*e - 2*c*d)*x^2)/(2*(p + 1)*(b^2 - 4*a*c))), x] - Dist[f
^2/(2*(p + 1)*(b^2 - 4*a*c)), Int[(f*x)^(m - 2)*(a + b*x^2 + c*x^4)^(p + 1)
*Simp[(m - 1)*(b*d - 2*a*e) - (4*p + 4 + m + 1)*(b*e - 2*c*d)*x^2, x], x],
x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] &&
GtQ[m, 1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{x(8+11x^2)}{13\sqrt{3+5x^2+x^4}}+\frac{1}{13}\int\frac{-8-11x^2}{\sqrt{3+5x^2+x^4}}\,dx\\
&= \frac{x(8+11x^2)}{13\sqrt{3+5x^2+x^4}}-\frac{8}{13}\int\frac{1}{\sqrt{3+5x^2+x^4}}\,dx-\frac{11}{13}\int\frac{x^2}{\sqrt{3+5x^2+x^4}}\,dx\\
&= -\frac{11x\left(5+\sqrt{13}+2x^2\right)}{26\sqrt{3+5x^2+x^4}}+\frac{x(8+11x^2)}{13\sqrt{3+5x^2+x^4}}\\
&\quad+\frac{11\sqrt{\frac{1}{6}\left(5+\sqrt{13}\right)}\sqrt{\frac{6+\left(5-\sqrt{13}\right)x^2}{6+\left(5+\sqrt{13}\right)x^2}}\left(6+\left(5+\sqrt{13}\right)x^2\right)E\left(\tan^{-1}\left(\sqrt{\frac{1}{6}\left(5+\sqrt{13}\right)}x\right)|\frac{1}{6}\left(-13+5\sqrt{\right.\right.}}{26\sqrt{3+5x^2+x^4}}\\
&\quad-\frac{4\sqrt{\frac{2}{3\left(5+\sqrt{13}\right)}}\sqrt{\frac{6+\left(5-\sqrt{13}\right)x^2}{6+\left(5+\sqrt{13}\right)x^2}}\left(6+\left(5+\sqrt{13}\right)x^2\right)F\left(\tan^{-1}\left(\sqrt{\frac{1}{6}\left(5+\sqrt{13}\right)}x\right)|\frac{1}{6}\left(-13+5\sqrt{13}\right)\right)}{13\sqrt{3+5x^2+x^4}}
\end{aligned}$$

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 10.25 (sec) , antiderivative size = 219, normalized size of antiderivative = 0.77

$$\int \frac{x^2(2+3x^2)}{(3+5x^2+x^4)^{3/2}}\,dx = \frac{4x(8+11x^2)-11i\sqrt{2}(-5+\sqrt{13})\sqrt{\frac{-5+\sqrt{13}-2x^2}{-5+\sqrt{13}}}\sqrt{5+\sqrt{13}+2x^2}E\Big(i\text{arcsinh}\Big(\sqrt{\ldots}}{\ldots}$$

[In] Integrate[(x^2*(2 + 3*x^2))/(3 + 5*x^2 + x^4)^(3/2),x]

[Out] (4*x*(8 + 11*x^2) - (11*I)*Sqrt[2]*(-5 + Sqrt[13])*Sqrt[(-5 + Sqrt[13] - 2*x^2)/(-5 + Sqrt[13])]*Sqrt[5 + Sqrt[13] + 2*x^2]*EllipticE[I*ArcSinh[Sqrt[2/(5 + Sqrt[13])]*x], 19/6 + (5*Sqrt[13])/6] + I*Sqrt[2]*(-39 + 11*Sqrt[13])*Sqrt[(-5 + Sqrt[13] - 2*x^2)/(-5 + Sqrt[13])]*Sqrt[5 + Sqrt[13] + 2*x^2]*EllipticF[I*ArcSinh[Sqrt[2/(5 + Sqrt[13])]*x], 19/6 + (5*Sqrt[13])/6])/(52*Sqrt[3 + 5*x^2 + x^4])

Maple [A] (verified)

Time = 2.62 (sec) , antiderivative size = 216, normalized size of antiderivative = 0.76

method	result
risch	$\frac{x(11x^2+8)}{13\sqrt{x^4+5x^2+3}} - \frac{48\sqrt{1-\left(-\frac{5}{6}+\frac{\sqrt{13}}{6}\right)x^2}\sqrt{1-\left(-\frac{5}{6}-\frac{\sqrt{13}}{6}\right)x^2}\,F\left(\frac{x\sqrt{-30+6\sqrt{13}}}{6},\frac{5\sqrt{3}}{6}+\frac{\sqrt{39}}{6}\right)}{13\sqrt{-30+6\sqrt{13}}\sqrt{x^4+5x^2+3}} + \frac{396\sqrt{1-\left(-\frac{5}{6}+\frac{\sqrt{13}}{6}\right)x^2}\sqrt{1-\left(-\frac{5}{6}\ldots}{\ldots}$
elliptic	$-\frac{2\left(-\frac{11}{26}x^3-\frac{4}{13}x\right)}{\sqrt{x^4+5x^2+3}} - \frac{48\sqrt{1-\left(-\frac{5}{6}+\frac{\sqrt{13}}{6}\right)x^2}\sqrt{1-\left(-\frac{5}{6}-\frac{\sqrt{13}}{6}\right)x^2}\,F\left(\frac{x\sqrt{-30+6\sqrt{13}}}{6},\frac{5\sqrt{3}}{6}+\frac{\sqrt{39}}{6}\right)}{13\sqrt{-30+6\sqrt{13}}\sqrt{x^4+5x^2+3}} + \frac{396\sqrt{1-\left(-\frac{5}{6}+\frac{\sqrt{13}}{6}\right)x^2}\sqrt{1-\left(\ldots}{\ldots}$
default	$-\frac{6\left(-\frac{5}{26}x^3-\frac{3}{13}x\right)}{\sqrt{x^4+5x^2+3}} - \frac{48\sqrt{1-\left(-\frac{5}{6}+\frac{\sqrt{13}}{6}\right)x^2}\sqrt{1-\left(-\frac{5}{6}-\frac{\sqrt{13}}{6}\right)x^2}\,F\left(\frac{x\sqrt{-30+6\sqrt{13}}}{6},\frac{5\sqrt{3}}{6}+\frac{\sqrt{39}}{6}\right)}{13\sqrt{-30+6\sqrt{13}}\sqrt{x^4+5x^2+3}} + \frac{396\sqrt{1-\left(-\frac{5}{6}+\frac{\sqrt{13}}{6}\right)x^2}\sqrt{1-\left(\ldots}{\ldots}$

[In] int(x^2*(3*x^2+2)/(x^4+5*x^2+3)^(3/2),x,method=_RETURNVERBOSE)

[Out] 1/13*x*(11*x^2+8)/(x^4+5*x^2+3)^(1/2)-48/13/(-30+6*13^(1/2))^(1/2)*(1-(-5/6+1/6*13^(1/2))*x^2)^(1/2)*(1-(-5/6-1/6*13^(1/2))*x^2)^(1/2)/(x^4+5*x^2+3)^(1/2)*EllipticF(1/6*x*(-30+6*13^(1/2))^(1/2),5/6*3^(1/2)+1/6*39^(1/2))+396/13/(-30+6*13^(1/2))^(1/2)*(1-(-5/6+1/6*13^(1/2))*x^2)^(1/2)*(1-(-5/6-1/6*13^(1/2))*x^2)^(1/2)/(x^4+5*x^2+3)^(1/2)/(5+13^(1/2))*(EllipticF(1/6*x*(-30+6*13^(1/2))^(1/2),5/6*3^(1/2)+1/6*39^(1/2))-EllipticE(1/6*x*(-30+6*13^(1/2))^(1/2),5/6*3^(1/2)+1/6*39^(1/2)))

Fricas [A] (verification not implemented)

none

Time = 0.08 (sec) , antiderivative size = 179, normalized size of antiderivative = 0.63

$$\int \frac{x^2(2+3x^2)}{(3+5x^2+x^4)^{3/2}}\,dx = \frac{11\left(\sqrt{13}\sqrt{6}\sqrt{3}(x^4+5\,x^2+3)-5\,\sqrt{6}\sqrt{3}(x^4+5\,x^2+3)\right)\sqrt{\sqrt{13}-5}E\left(\arcsin\left(\frac{1}{6}\sqrt{\cdots}\right.\right.}{\cdots}$$

```
[In] integrate(x^2*(3*x^2+2)/(x^4+5*x^2+3)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/468*(11*(sqrt(13)*sqrt(6)*sqrt(3)*(x^4 + 5*x^2 + 3) - 5*sqrt(6)*sqrt(3)*(
x^4 + 5*x^2 + 3))*sqrt(sqrt(13) - 5)*elliptic_e(arcsin(1/6*sqrt(6)*x*sqrt(s
qrt(13) - 5)), 5/6*sqrt(13) + 19/6) - (3*sqrt(13)*sqrt(6)*sqrt(3)*(x^4 + 5*
x^2 + 3) - 95*sqrt(6)*sqrt(3)*(x^4 + 5*x^2 + 3))*sqrt(sqrt(13) - 5)*ellipti
c_f(arcsin(1/6*sqrt(6)*x*sqrt(sqrt(13) - 5)), 5/6*sqrt(13) + 19/6) + 36*sqr
t(x^4 + 5*x^2 + 3)*(11*x^3 + 8*x))/(x^4 + 5*x^2 + 3)
```

Sympy [F]

$$\int \frac{x^2(2+3x^2)}{(3+5x^2+x^4)^{3/2}}\,dx = \int \frac{x^2\cdot(3x^2+2)}{(x^4+5x^2+3)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(x**2*(3*x**2+2)/(x**4+5*x**2+3)**(3/2),x)
```

```
[Out] Integral(x**2*(3*x**2 + 2)/(x**4 + 5*x**2 + 3)**(3/2), x)
```

Maxima [F]

$$\int \frac{x^2(2+3x^2)}{(3+5x^2+x^4)^{3/2}}\,dx = \int \frac{(3\,x^2+2)x^2}{(x^4+5\,x^2+3)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(x^2*(3*x^2+2)/(x^4+5*x^2+3)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((3*x^2 + 2)*x^2/(x^4 + 5*x^2 + 3)^(3/2), x)
```

Giac [F]

$$\int \frac{x^2(2+3x^2)}{(3+5x^2+x^4)^{3/2}}\,dx = \int \frac{(3\,x^2+2)x^2}{(x^4+5\,x^2+3)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(x^2*(3*x^2+2)/(x^4+5*x^2+3)^(3/2),x, algorithm="giac")
[Out] integrate((3*x^2 + 2)*x^2/(x^4 + 5*x^2 + 3)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2(2+3x^2)}{(3+5x^2+x^4)^{3/2}}\,dx = \int \frac{x^2\,(3\,x^2+2)}{(x^4+5\,x^2+3)^{3/2}}\,dx$$

```
[In] int((x^2*(3*x^2 + 2))/(5*x^2 + x^4 + 3)^(3/2),x)
[Out] int((x^2*(3*x^2 + 2))/(5*x^2 + x^4 + 3)^(3/2), x)
```

3.201 $\int \frac{2+3x^2}{(3+5x^2+x^4)^{3/2}} \, dx$

Optimal result

Integrand size = 22, antiderivative size = 282

$$\int \frac{2+3x^2}{(3+5x^2+x^4)^{3/2}} \, dx = \frac{4x\left(5+\sqrt{13}+2x^2\right)}{39\sqrt{3+5x^2+x^4}} - \frac{x(7+8x^2)}{39\sqrt{3+5x^2+x^4}}$$
$$- \frac{2\sqrt{\frac{2}{3}\left(5+\sqrt{13}\right)}\sqrt{\frac{6+\left(5-\sqrt{13}\right)x^2}{6+\left(5+\sqrt{13}\right)x^2}}\left(6+\left(5+\sqrt{13}\right)x^2\right)E\left(\arctan\left(\sqrt{\frac{1}{6}\left(5+\sqrt{13}\right)}x\right)|\frac{1}{6}\left(-13+5\sqrt{13}\right)\right)}{39\sqrt{3+5x^2+x^4}}$$
$$+ \frac{11\sqrt{\frac{6+\left(5-\sqrt{13}\right)x^2}{6+\left(5+\sqrt{13}\right)x^2}}\left(6+\left(5+\sqrt{13}\right)x^2\right)\text{EllipticF}\left(\arctan\left(\sqrt{\frac{1}{6}\left(5+\sqrt{13}\right)}x\right),\frac{1}{6}\left(-13+5\sqrt{13}\right)\right)}{13\sqrt{6\left(5+\sqrt{13}\right)}\sqrt{3+5x^2+x^4}}$$

```
[Out] -1/39*x*(8*x^2+7)/(x^4+5*x^2+3)^(1/2)+4/39*x*(5+2*x^2+13^(1/2))/(x^4+5*x^2+
3)^(1/2)-2/117*(1/(36+x^2*(30+6*13^(1/2))))^(1/2)*(36+x^2*(30+6*13^(1/2)))^
(1/2)*EllipticE(x*(30+6*13^(1/2))^(1/2)/(36+x^2*(30+6*13^(1/2)))^(1/2),1/6*
(-78+30*13^(1/2))^(1/2))*(6+x^2*(5+13^(1/2)))*(30+6*13^(1/2))^(1/2)*((6+x^2
*(5-13^(1/2)))/(6+x^2*(5+13^(1/2))))^(1/2)/(x^4+5*x^2+3)^(1/2)+11/13*(1/(36
+x^2*(30+6*13^(1/2))))^(1/2)*(36+x^2*(30+6*13^(1/2)))^(1/2)*EllipticF(x*(30
+6*13^(1/2))^(1/2)/(36+x^2*(30+6*13^(1/2)))^(1/2),1/6*(-78+30*13^(1/2))^(1/
2))*(6+x^2*(5+13^(1/2)))*((6+x^2*(5-13^(1/2)))/(6+x^2*(5+13^(1/2))))^(1/2)/
(x^4+5*x^2+3)^(1/2)/(30+6*13^(1/2))^(1/2)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 282, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {1192, 1203, 1113, 1149}

$$\int \frac{2+3x^2}{(3+5x^2+x^4)^{3/2}}\,dx = \frac{11\sqrt{\frac{(5-\sqrt{13})x^2+6}{(5+\sqrt{13})x^2+6}}\left((5+\sqrt{13})\,x^2+6\right)\text{EllipticF}\left(\arctan\left(\sqrt{\frac{1}{6}\,(5+\sqrt{13})}x\right),\frac{1}{6}(-13$$
$$\overline{13\sqrt{6\,(5+\sqrt{13})}\sqrt{x^4+5x^2+3}}$$
$$-\frac{2\sqrt{\frac{2}{3}\,(5+\sqrt{13})}\sqrt{\frac{(5-\sqrt{13})x^2+6}{(5+\sqrt{13})x^2+6}}\left((5+\sqrt{13})\,x^2+6\right)E\left(\arctan\left(\sqrt{\frac{1}{6}\,(5+\sqrt{13})}x\right)|\frac{1}{6}(-13+5\sqrt{13})\right)}{39\sqrt{x^4+5x^2+3}}$$
$$+\frac{4x\left(2x^2+\sqrt{13}+5\right)}{39\sqrt{x^4+5x^2+3}}-\frac{x(8x^2+7)}{39\sqrt{x^4+5x^2+3}}$$

```
[In] Int[(2 + 3*x^2)/(3 + 5*x^2 + x^4)^(3/2),x]
```

```
[Out] (4*x*(5 + Sqrt[13] + 2*x^2))/(39*Sqrt[3 + 5*x^2 + x^4]) - (x*(7 + 8*x^2))/(
39*Sqrt[3 + 5*x^2 + x^4]) - (2*Sqrt[(2*(5 + Sqrt[13]))/3]*Sqrt[(6 + (5 - Sq
rt[13])*x^2)/(6 + (5 + Sqrt[13])*x^2)]*(6 + (5 + Sqrt[13])*x^2)*EllipticE[A
rcTan[Sqrt[(5 + Sqrt[13])/6]*x], (-13 + 5*Sqrt[13])/6])/(39*Sqrt[3 + 5*x^2
+ x^4]) + (11*Sqrt[(6 + (5 - Sqrt[13])*x^2)/(6 + (5 + Sqrt[13])*x^2)]*(6 +
(5 + Sqrt[13])*x^2)*EllipticF[ArcTan[Sqrt[(5 + Sqrt[13])/6]*x], (-13 + 5*Sq
rt[13])/6])/(13*Sqrt[6*(5 + Sqrt[13])]*Sqrt[3 + 5*x^2 + x^4])
```

Rule 1113

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[(2*a + (b + q)*x^2)*(Sqrt[(2*a + (b - q)*x^2)/(2*a +
(b + q)*x^2)]/(2*a*Rt[(b + q)/(2*a), 2]*Sqrt[a + b*x^2 + c*x^4]))*EllipticF
[ArcTan[Rt[(b + q)/(2*a), 2]*x], 2*(q/(b + q))], x] /; PosQ[(b + q)/a] &&
!(PosQ[(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[
{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1149

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[b^2 - 4*a*c, 2]}, Simp[x*((b + q + 2*c*x^2)/(2*c*Sqrt[a + b*x^2 + c*x^4
])), x] - Simp[Rt[(b + q)/(2*a), 2]*(2*a + (b + q)*x^2)*(Sqrt[(2*a + (b - q
)*x^2)/(2*a + (b + q)*x^2)]/(2*c*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[ArcTan
[Rt[(b + q)/(2*a), 2]*x], 2*(q/(b + q))], x] /; PosQ[(b + q)/a] &&  !(PosQ[
(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[{a, b,
c}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1192

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[x*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)*((a + b*x^2 +
 c*x^4)^(p + 1)/(2*a*(p + 1)*(b^2 - 4*a*c))), x] + Dist[1/(2*a*(p + 1)*(b^2
 - 4*a*c)), Int[Simp[(2*p + 3)*d*b^2 - a*b*e - 2*a*c*d*(4*p + 5) + (4*p + 7
)*(d*b - 2*a*e)*c*x^2, x]*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; FreeQ[{a,
b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
LtQ[p, -1] && IntegerQ[2*p]
```

Rule 1203

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[d, Int[1/Sqrt[a + b*x^2 + c*x^4],
 x], x] + Dist[e, Int[x^2/Sqrt[a + b*x^2 + c*x^4], x], x] /; PosQ[(b + q)/a
] || PosQ[(b - q)/a]] /; FreeQ[{a, b, c, d, e}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{x(7+8x^2)}{39\sqrt{3+5x^2+x^4}} - \frac{1}{39}\int \frac{-33-8x^2}{\sqrt{3+5x^2+x^4}}\,dx \\
&= -\frac{x(7+8x^2)}{39\sqrt{3+5x^2+x^4}} + \frac{8}{39}\int \frac{x^2}{\sqrt{3+5x^2+x^4}}\,dx + \frac{11}{13}\int \frac{1}{\sqrt{3+5x^2+x^4}}\,dx \\
&= \frac{4x\left(5+\sqrt{13}+2x^2\right)}{39\sqrt{3+5x^2+x^4}} - \frac{x(7+8x^2)}{39\sqrt{3+5x^2+x^4}} \\
&\quad - \frac{2\sqrt{\frac{2}{3}\left(5+\sqrt{13}\right)}\sqrt{\frac{6+\left(5-\sqrt{13}\right)x^2}{6+\left(5+\sqrt{13}\right)x^2}}\left(6+\left(5+\sqrt{13}\right)x^2\right)E\left(\tan^{-1}\left(\sqrt{\frac{1}{6}\left(5+\sqrt{13}\right)}x\right)|\frac{1}{6}\left(-13+5\sqrt{1\ldots}\right.\right.}{39\sqrt{3+5x^2+x^4}} \\
&\quad + \frac{11\sqrt{\frac{6+\left(5-\sqrt{13}\right)x^2}{6+\left(5+\sqrt{13}\right)x^2}}\left(6+\left(5+\sqrt{13}\right)x^2\right)F\left(\tan^{-1}\left(\sqrt{\frac{1}{6}\left(5+\sqrt{13}\right)}x\right)|\frac{1}{6}\left(-13+5\sqrt{13}\right)\right)}{13\sqrt{6\left(5+\sqrt{13}\right)}\sqrt{3+5x^2+x^4}}
\end{aligned}
$$

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 10.25 (sec) , antiderivative size = 219, normalized size of antiderivative = 0.78

$$
\int \frac{2+3x^2}{\left(3+5x^2+x^4\right)^{3/2}}\,dx = \frac{-2x(7+8x^2)+4i\sqrt{2}\left(-5+\sqrt{13}\right)\sqrt{\frac{-5+\sqrt{13}-2x^2}{-5+\sqrt{13}}}\sqrt{5+\sqrt{13}+2x^2}\,E\left(i\operatorname{arcsinh}\left(\sqrt{\ldots}\right.\right.}{\ldots}
$$

```
[In] Integrate[(2 + 3*x^2)/(3 + 5*x^2 + x^4)^(3/2),x]
```

```
[Out] (-2*x*(7 + 8*x^2) + (4*I)*Sqrt[2]*(-5 + Sqrt[13])*Sqrt[(-5 + Sqrt[13] - 2*x
^2)/(-5 + Sqrt[13])]*Sqrt[5 + Sqrt[13] + 2*x^2]*EllipticE[I*ArcSinh[Sqrt[2/
(5 + Sqrt[13])]*x], 19/6 + (5*Sqrt[13])/6] - I*Sqrt[2]*(13 + 4*Sqrt[13])*Sq
rt[(-5 + Sqrt[13] - 2*x^2)/(-5 + Sqrt[13])]*Sqrt[5 + Sqrt[13] + 2*x^2]*Elli
pticF[I*ArcSinh[Sqrt[2/(5 + Sqrt[13])]*x], 19/6 + (5*Sqrt[13])/6])/(78*Sqrt
[3 + 5*x^2 + x^4])
```

Maple [A] (verified)

Time = 1.31 (sec) , antiderivative size = 216, normalized size of antiderivative = 0.77

method	result
risch	$-\frac{x(8x^2+7)}{39\sqrt{x^4+5x^2+3}}+\frac{66\sqrt{1-\left(-\frac{5}{6}+\frac{\sqrt{13}}{6}\right)x^2}\,\sqrt{1-\left(-\frac{5}{6}-\frac{\sqrt{13}}{6}\right)x^2}\,F\left(\frac{x\sqrt{-30+6\sqrt{13}}}{6},\frac{5\sqrt{3}}{6}+\frac{\sqrt{39}}{6}\right)}{13\sqrt{-30+6\sqrt{13}}\,\sqrt{x^4+5x^2+3}}-\frac{96\sqrt{1-\left(-\frac{5}{6}+\frac{\sqrt{13}}{6}\right)x^2}\,\sqrt{1-\left(-\right.}}{}$
elliptic	$-\frac{2\left(\frac{4}{39}x^3+\frac{7}{78}x\right)}{\sqrt{x^4+5x^2+3}}+\frac{66\sqrt{1-\left(-\frac{5}{6}+\frac{\sqrt{13}}{6}\right)x^2}\,\sqrt{1-\left(-\frac{5}{6}-\frac{\sqrt{13}}{6}\right)x^2}\,F\left(\frac{x\sqrt{-30+6\sqrt{13}}}{6},\frac{5\sqrt{3}}{6}+\frac{\sqrt{39}}{6}\right)}{13\sqrt{-30+6\sqrt{13}}\,\sqrt{x^4+5x^2+3}}-\frac{96\sqrt{1-\left(-\frac{5}{6}+\frac{\sqrt{13}}{6}\right)x^2}\,\sqrt{1-\left(-\frac{5}{6}\right.}}{}$
default	$-\frac{4\left(-\frac{19}{78}x-\frac{5}{78}x^3\right)}{\sqrt{x^4+5x^2+3}}+\frac{66\sqrt{1-\left(-\frac{5}{6}+\frac{\sqrt{13}}{6}\right)x^2}\,\sqrt{1-\left(-\frac{5}{6}-\frac{\sqrt{13}}{6}\right)x^2}\,F\left(\frac{x\sqrt{-30+6\sqrt{13}}}{6},\frac{5\sqrt{3}}{6}+\frac{\sqrt{39}}{6}\right)}{13\sqrt{-30+6\sqrt{13}}\,\sqrt{x^4+5x^2+3}}-\frac{96\sqrt{1-\left(-\frac{5}{6}+\frac{\sqrt{13}}{6}\right)x^2}\,\sqrt{1-\left(-\right.}}{}$

```
[In] int((3*x^2+2)/(x^4+5*x^2+3)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/39*x*(8*x^2+7)/(x^4+5*x^2+3)^(1/2)+66/13/(-30+6*13^(1/2))^(1/2)*(1-(-5/6
+1/6*13^(1/2))*x^2)^(1/2)*(1-(-5/6-1/6*13^(1/2))*x^2)^(1/2)/(x^4+5*x^2+3)^(
1/2)*EllipticF(1/6*x*(-30+6*13^(1/2))^(1/2),5/6*3^(1/2)+1/6*39^(1/2))-96/13
/(-30+6*13^(1/2))^(1/2)*(1-(-5/6+1/6*13^(1/2))*x^2)^(1/2)*(1-(-5/6-1/6*13^(
1/2))*x^2)^(1/2)/(x^4+5*x^2+3)^(1/2)/(5+13^(1/2))*(EllipticF(1/6*x*(-30+6*1
3^(1/2))^(1/2),5/6*3^(1/2)+1/6*39^(1/2))-EllipticE(1/6*x*(-30+6*13^(1/2))^(
1/2),5/6*3^(1/2)+1/6*39^(1/2)))
```

Fricas [A] (verification not implemented)

none

Time = 0.08 (sec) , antiderivative size = 179, normalized size of antiderivative = 0.63

$$\int \frac{2+3x^2}{\left(3+5x^2+x^4\right)^{3/2}}\,dx =$$

$$-\frac{8\left(\sqrt{13}\sqrt{6}\sqrt{3}(x^4+5\,x^2+3)-5\,\sqrt{6}\sqrt{3}(x^4+5\,x^2+3)\right)\sqrt{\sqrt{13}-5}\,E\left(\arcsin\left(\frac{1}{6}\,\sqrt{6}x\sqrt{\sqrt{13}-5}\right)\,|\,\frac{5}{6}\,\sqrt{13}+\right.}{}$$

```
[In] integrate((3*x^2+2)/(x^4+5*x^2+3)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/1404*(8*(sqrt(13)*sqrt(6)*sqrt(3)*(x^4 + 5*x^2 + 3) - 5*sqrt(6)*sqrt(3)*
(x^4 + 5*x^2 + 3))*sqrt(sqrt(13) - 5)*elliptic_e(arcsin(1/6*sqrt(6)*x*sqrt(
sqrt(13) - 5)), 5/6*sqrt(13) + 19/6) + 5*(5*sqrt(13)*sqrt(6)*sqrt(3)*(x^4 +
 5*x^2 + 3) + 41*sqrt(6)*sqrt(3)*(x^4 + 5*x^2 + 3))*sqrt(sqrt(13) - 5)*elli
ptic_f(arcsin(1/6*sqrt(6)*x*sqrt(sqrt(13) - 5)), 5/6*sqrt(13) + 19/6) + 36*
sqrt(x^4 + 5*x^2 + 3)*(8*x^3 + 7*x))/(x^4 + 5*x^2 + 3)
```

Sympy [F]

$$\int \frac{2+3x^2}{(3+5x^2+x^4)^{3/2}}\,dx = \int \frac{3x^2+2}{(x^4+5x^2+3)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((3*x**2+2)/(x**4+5*x**2+3)**(3/2),x)
[Out] Integral((3*x**2 + 2)/(x**4 + 5*x**2 + 3)**(3/2), x)
```

Maxima [F]

$$\int \frac{2+3x^2}{(3+5x^2+x^4)^{3/2}}\,dx = \int \frac{3\,x^2+2}{(x^4+5\,x^2+3)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((3*x^2+2)/(x^4+5*x^2+3)^(3/2),x, algorithm="maxima")
[Out] integrate((3*x^2 + 2)/(x^4 + 5*x^2 + 3)^(3/2), x)
```

Giac [F]

$$\int \frac{2+3x^2}{(3+5x^2+x^4)^{3/2}}\,dx = \int \frac{3\,x^2+2}{(x^4+5\,x^2+3)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((3*x^2+2)/(x^4+5*x^2+3)^(3/2),x, algorithm="giac")
[Out] integrate((3*x^2 + 2)/(x^4 + 5*x^2 + 3)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{2+3x^2}{(3+5x^2+x^4)^{3/2}}\,dx = \int \frac{3\,x^2+2}{(x^4+5\,x^2+3)^{3/2}}\,dx$$

```
[In] int((3*x^2 + 2)/(5*x^2 + x^4 + 3)^(3/2),x)
[Out] int((3*x^2 + 2)/(5*x^2 + x^4 + 3)^(3/2), x)
```

3.202 $\int \frac{2+3x^2}{x^2(3+5x^2+x^4)^{3/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 309

$$\int \frac{2+3x^2}{x^2(3+5x^2+x^4)^{3/2}}\,dx = \frac{19x(5+\sqrt{13}+2x^2)}{234\sqrt{3+5x^2+x^4}} - \frac{7+8x^2}{39x\sqrt{3+5x^2+x^4}} - \frac{19\sqrt{3+5x^2+x^4}}{117x}$$
$$- \frac{19\sqrt{\frac{1}{6}(5+\sqrt{13})}\sqrt{\frac{6+(5-\sqrt{13})x^2}{6+(5+\sqrt{13})x^2}}\,(6+(5+\sqrt{13})\,x^2)\,E\left(\arctan\left(\sqrt{\frac{1}{6}(5+\sqrt{13})}x\right)|\frac{1}{6}(-13+5\sqrt{13})\right)}{234\sqrt{3+5x^2+x^4}}$$
$$- \frac{4\sqrt{\frac{2}{3(5+\sqrt{13})}}\sqrt{\frac{6+(5-\sqrt{13})x^2}{6+(5+\sqrt{13})x^2}}\,(6+(5+\sqrt{13})\,x^2)\,\text{EllipticF}\left(\arctan\left(\sqrt{\frac{1}{6}(5+\sqrt{13})}x\right),\frac{1}{6}(-13+5\sqrt{13})\right)}{39\sqrt{3+5x^2+x^4}}$$

```
[Out] 1/39*(-8*x^2-7)/x/(x^4+5*x^2+3)^(1/2)+19/234*x*(5+2*x^2+13^(1/2))/(x^4+5*x^
2+3)^(1/2)-19/117*(x^4+5*x^2+3)^(1/2)/x-4/117*(1/(36+x^2*(30+6*13^(1/2))))^
(1/2)*(36+x^2*(30+6*13^(1/2)))^(1/2)*EllipticF(x*(30+6*13^(1/2))^(1/2)/(36+
x^2*(30+6*13^(1/2)))^(1/2),1/6*(-78+30*13^(1/2))^(1/2))*(6+x^2*(5+13^(1/2))
)*6^(1/2)/(5+13^(1/2))^(1/2)*((6+x^2*(5-13^(1/2)))/(6+x^2*(5+13^(1/2))))^(1
/2)/(x^4+5*x^2+3)^(1/2)-19/1404*(1/(36+x^2*(30+6*13^(1/2))))^(1/2)*(36+x^2*
(30+6*13^(1/2)))^(1/2)*EllipticE(x*(30+6*13^(1/2))^(1/2)/(36+x^2*(30+6*13^(
1/2)))^(1/2),1/6*(-78+30*13^(1/2))^(1/2))*(6+x^2*(5+13^(1/2)))*(30+6*13^(1/
2))^(1/2)*((6+x^2*(5-13^(1/2)))/(6+x^2*(5+13^(1/2))))^(1/2)/(x^4+5*x^2+3)^(
1/2)
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 309, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {1291, 1295, 1203, 1113, 1149}

$$\int \frac{2+3x^2}{x^2(3+5x^2+x^4)^{3/2}}\,dx =$$

$$-\frac{4\sqrt{\frac{2}{3(5+\sqrt{13})}}\sqrt{\frac{(5-\sqrt{13})x^2+6}{(5+\sqrt{13})x^2+6}}\left((5+\sqrt{13})x^2+6\right)\text{EllipticF}\left(\arctan\left(\sqrt{\frac{1}{6}(5+\sqrt{13})}x\right),\frac{1}{6}(-13+5\sqrt{13})\right)}{39\sqrt{x^4+5x^2+3}}$$

$$-\frac{19\sqrt{\frac{1}{6}(5+\sqrt{13})}\sqrt{\frac{(5-\sqrt{13})x^2+6}{(5+\sqrt{13})x^2+6}}\left((5+\sqrt{13})x^2+6\right)E\left(\arctan\left(\sqrt{\frac{1}{6}(5+\sqrt{13})}x\right)|\frac{1}{6}(-13+5\sqrt{13})\right)}{234\sqrt{x^4+5x^2+3}}$$

$$+\frac{19x(2x^2+\sqrt{13}+5)}{234\sqrt{x^4+5x^2+3}}-\frac{19\sqrt{x^4+5x^2+3}}{117x}-\frac{8x^2+7}{39x\sqrt{x^4+5x^2+3}}$$

```
[In] Int[(2 + 3*x^2)/(x^2*(3 + 5*x^2 + x^4)^(3/2)),x]
```

```
[Out] (19*x*(5 + Sqrt[13] + 2*x^2))/(234*Sqrt[3 + 5*x^2 + x^4]) - (7 + 8*x^2)/(39
*x*Sqrt[3 + 5*x^2 + x^4]) - (19*Sqrt[3 + 5*x^2 + x^4])/(117*x) - (19*Sqrt[(
5 + Sqrt[13])/6]*Sqrt[(6 + (5 - Sqrt[13])*x^2)/(6 + (5 + Sqrt[13])*x^2)]*(6
 + (5 + Sqrt[13])*x^2)*EllipticE[ArcTan[Sqrt[(5 + Sqrt[13])/6]*x], (-13 + 5
*Sqrt[13])/6])/(234*Sqrt[3 + 5*x^2 + x^4]) - (4*Sqrt[2/(3*(5 + Sqrt[13]))]*
Sqrt[(6 + (5 - Sqrt[13])*x^2)/(6 + (5 + Sqrt[13])*x^2)]*(6 + (5 + Sqrt[13])
*x^2)*EllipticF[ArcTan[Sqrt[(5 + Sqrt[13])/6]*x], (-13 + 5*Sqrt[13])/6])/(3
9*Sqrt[3 + 5*x^2 + x^4])
```

Rule 1113

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[(2*a + (b + q)*x^2)*(Sqrt[(2*a + (b - q)*x^2)/(2*a +
(b + q)*x^2)]/(2*a*Rt[(b + q)/(2*a), 2]*Sqrt[a + b*x^2 + c*x^4]))*EllipticF
[ArcTan[Rt[(b + q)/(2*a), 2]*x], 2*(q/(b + q))], x] /; PosQ[(b + q)/a] &&
!(PosQ[(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[
{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1149

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[b^2 - 4*a*c, 2]}, Simp[x*((b + q + 2*c*x^2)/(2*c*Sqrt[a + b*x^2 + c*x^4
])), x] - Simp[Rt[(b + q)/(2*a), 2]*(2*a + (b + q)*x^2)*(Sqrt[(2*a + (b - q
)*x^2)/(2*a + (b + q)*x^2)]/(2*c*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[ArcTan
[Rt[(b + q)/(2*a), 2]*x], 2*(q/(b + q))], x] /; PosQ[(b + q)/a] &&  !(PosQ[
```

```
(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[{a, b,
c}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1203

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[d, Int[1/Sqrt[a + b*x^2 + c*x^4],
 x], x] + Dist[e, Int[x^2/Sqrt[a + b*x^2 + c*x^4], x], x] /; PosQ[(b + q)/a
] || PosQ[(b - q)/a]] /; FreeQ[{a, b, c, d, e}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1291

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[(-(f*x)^(m + 1))*(a + b*x^2 + c*x^4)^(p + 1)
*((d*(b^2 - 2*a*c) - a*b*e + (b*d - 2*a*e)*c*x^2)/(2*a*f*(p + 1)*(b^2 - 4*a
*c))), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), Int[(f*x)^m*(a + b*x^2 + c*
x^4)^(p + 1)*Simp[d*(b^2*(m + 2*(p + 1) + 1) - 2*a*c*(m + 4*(p + 1) + 1)) -
 a*b*e*(m + 1) + c*(m + 2*(2*p + 3) + 1)*(b*d - 2*a*e)*x^2, x], x], x] /; F
reeQ[{a, b, c, d, e, f, m}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && Inte
gerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1295

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[d*(f*x)^(m + 1)*((a + b*x^2 + c*x^4)^(p + 1)
/(a*f*(m + 1))), x] + Dist[1/(a*f^2*(m + 1)), Int[(f*x)^(m + 2)*(a + b*x^2
+ c*x^4)^p*Simp[a*e*(m + 1) - b*d*(m + 2*p + 3) - c*d*(m + 4*p + 5)*x^2, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[m
, -1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{7+8x^2}{39x\sqrt{3+5x^2+x^4}} - \frac{1}{39}\int \frac{-19+8x^2}{x^2\sqrt{3+5x^2+x^4}}\,dx \\
&= -\frac{7+8x^2}{39x\sqrt{3+5x^2+x^4}} - \frac{19\sqrt{3+5x^2+x^4}}{117x} + \frac{1}{117}\int \frac{-24+19x^2}{\sqrt{3+5x^2+x^4}}\,dx \\
&= -\frac{7+8x^2}{39x\sqrt{3+5x^2+x^4}} - \frac{19\sqrt{3+5x^2+x^4}}{117x} \\
&\quad + \frac{19}{117}\int \frac{x^2}{\sqrt{3+5x^2+x^4}}\,dx - \frac{8}{39}\int \frac{1}{\sqrt{3+5x^2+x^4}}\,dx
\end{aligned}$$

$$= \frac{19x(5+\sqrt{13}+2x^2)}{234\sqrt{3+5x^2+x^4}} - \frac{7+8x^2}{39x\sqrt{3+5x^2+x^4}} - \frac{19\sqrt{3+5x^2+x^4}}{117x}$$
$$- \frac{19\sqrt{\frac{1}{6}\left(5+\sqrt{13}\right)}\sqrt{\frac{6+\left(5-\sqrt{13}\right)x^2}{6+\left(5+\sqrt{13}\right)x^2}}\left(6+\left(5+\sqrt{13}\right)x^2\right)E\left(\tan^{-1}\left(\sqrt{\frac{1}{6}\left(5+\sqrt{13}\right)}x\right)|\frac{1}{6}(-13+5\sqrt{13}\right.}{234\sqrt{3+5x^2+x^4}}$$
$$- \frac{4\sqrt{\frac{2}{3\left(5+\sqrt{13}\right)}}\sqrt{\frac{6+\left(5-\sqrt{13}\right)x^2}{6+\left(5+\sqrt{13}\right)x^2}}\left(6+\left(5+\sqrt{13}\right)x^2\right)F\left(\tan^{-1}\left(\sqrt{\frac{1}{6}\left(5+\sqrt{13}\right)}x\right)|\frac{1}{6}\left(-13+5\sqrt{13}\right)\right)}{39\sqrt{3+5x^2+x^4}}$$

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 10.26 (sec) , antiderivative size = 228, normalized size of antiderivative = 0.74

$$\int \frac{2+3x^2}{x^2\left(3+5x^2+x^4\right)^{3/2}}\,dx = \frac{-4(78+119x^2+19x^4)+19i\sqrt{2}\left(-5+\sqrt{13}\right)x\sqrt{\frac{-5+\sqrt{13}-2x^2}{-5+\sqrt{13}}}\sqrt{5+\sqrt{13}+2x^2}E}{}$$

```
[In] Integrate[(2 + 3*x^2)/(x^2*(3 + 5*x^2 + x^4)^(3/2)),x]
```

```
[Out] (-4*(78 + 119*x^2 + 19*x^4) + (19*I)*Sqrt[2]*(-5 + Sqrt[13])*x*Sqrt[(-5 + S
qrt[13] - 2*x^2)/(-5 + Sqrt[13])]*Sqrt[5 + Sqrt[13] + 2*x^2]*EllipticE[I*Ar
cSinh[Sqrt[2/(5 + Sqrt[13])]*x], 19/6 + (5*Sqrt[13])/6] - I*Sqrt[2]*(-143 +
 19*Sqrt[13])*x*Sqrt[(-5 + Sqrt[13] - 2*x^2)/(-5 + Sqrt[13])]*Sqrt[5 + Sqrt
[13] + 2*x^2]*EllipticF[I*ArcSinh[Sqrt[2/(5 + Sqrt[13])]*x], 19/6 + (5*Sqrt
[13])/6])/(468*x*Sqrt[3 + 5*x^2 + x^4])
```

Maple [A] (verified)

Time = 1.51 (sec) , antiderivative size = 223, normalized size of antiderivative = 0.72

method	result
risch	$-\frac{19x^4+119x^2+78}{117x\sqrt{x^4+5x^2+3}} - \frac{16\sqrt{1-\left(-\frac{5}{6}+\frac{\sqrt{13}}{6}\right)x^2}\sqrt{1-\left(-\frac{5}{6}-\frac{\sqrt{13}}{6}\right)x^2}F\left(\frac{x\sqrt{-30+6\sqrt{13}}}{6},\frac{5\sqrt{3}}{6}+\frac{\sqrt{39}}{6}\right)}{13\sqrt{-30+6\sqrt{13}}\sqrt{x^4+5x^2+3}} - \frac{76\sqrt{1-\left(-\frac{5}{6}+\frac{\sqrt{13}}{6}\right)x^2}\sqrt{1-\left(\right.}}{}$
elliptic	$-\frac{2\left(-\frac{7}{234}x^3-\frac{11}{234}x\right)}{\sqrt{x^4+5x^2+3}} - \frac{2\sqrt{x^4+5x^2+3}}{9x} - \frac{16\sqrt{1-\left(-\frac{5}{6}+\frac{\sqrt{13}}{6}\right)x^2}\sqrt{1-\left(-\frac{5}{6}-\frac{\sqrt{13}}{6}\right)x^2}F\left(\frac{x\sqrt{-30+6\sqrt{13}}}{6},\frac{5\sqrt{3}}{6}+\frac{\sqrt{39}}{6}\right)}{13\sqrt{-30+6\sqrt{13}}\sqrt{x^4+5x^2+3}} - \frac{76\sqrt{1-\left(-\right.}}{}$
default	$-\frac{6\left(-\frac{19}{78}x-\frac{5}{78}x^3\right)}{\sqrt{x^4+5x^2+3}} - \frac{16\sqrt{1-\left(-\frac{5}{6}+\frac{\sqrt{13}}{6}\right)x^2}\sqrt{1-\left(-\frac{5}{6}-\frac{\sqrt{13}}{6}\right)x^2}F\left(\frac{x\sqrt{-30+6\sqrt{13}}}{6},\frac{5\sqrt{3}}{6}+\frac{\sqrt{39}}{6}\right)}{13\sqrt{-30+6\sqrt{13}}\sqrt{x^4+5x^2+3}} - \frac{76\sqrt{1-\left(-\frac{5}{6}+\frac{\sqrt{13}}{6}\right)x^2}\sqrt{1-\left(-\right.}}{}$

```
[In] int((3*x^2+2)/x^2/(x^4+5*x^2+3)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/117*(19*x^4+119*x^2+78)/x/(x^4+5*x^2+3)^(1/2)-16/13/(-30+6*13^(1/2))^(1/
2)*(1-(-5/6+1/6*13^(1/2))*x^2)^(1/2)*(1-(-5/6-1/6*13^(1/2))*x^2)^(1/2)/(x^4
+5*x^2+3)^(1/2)*EllipticF(1/6*x*(-30+6*13^(1/2))^(1/2),5/6*3^(1/2)+1/6*39^(
1/2))-76/13/(-30+6*13^(1/2))^(1/2)*(1-(-5/6+1/6*13^(1/2))*x^2)^(1/2)*(1-(-5
/6-1/6*13^(1/2))*x^2)^(1/2)/(x^4+5*x^2+3)^(1/2)/(5+13^(1/2))*(EllipticF(1/6
*x*(-30+6*13^(1/2))^(1/2),5/6*3^(1/2)+1/6*39^(1/2))-EllipticE(1/6*x*(-30+6*
13^(1/2))^(1/2),5/6*3^(1/2)+1/6*39^(1/2)))
```

Fricas [A] (verification not implemented)

none

Time = 0.08 (sec) , antiderivative size = 192, normalized size of antiderivative = 0.62

$$\int \frac{2+3x^2}{x^2\left(3+5x^2+x^4\right)^{3/2}}\,dx =$$

$$-\frac{19\left(\sqrt{13}\sqrt{6}\sqrt{3}(x^5+5\,x^3+3\,x)-5\,\sqrt{6}\sqrt{3}(x^5+5\,x^3+3\,x)\right)\sqrt{\sqrt{13}-5}E(\arcsin\left(\frac{1}{6}\,\sqrt{6}x\sqrt{\sqrt{13}-5}\right)\mid\frac{5}{6}\,\sqrt{}}{}$$

```
[In] integrate((3*x^2+2)/x^2/(x^4+5*x^2+3)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/4212*(19*(sqrt(13)*sqrt(6)*sqrt(3)*(x^5 + 5*x^3 + 3*x) - 5*sqrt(6)*sqrt(
3)*(x^5 + 5*x^3 + 3*x))*sqrt(sqrt(13) - 5)*elliptic_e(arcsin(1/6*sqrt(6)*x*
sqrt(sqrt(13) - 5)), 5/6*sqrt(13) + 19/6) - (43*sqrt(13)*sqrt(6)*sqrt(3)*(x
^5 + 5*x^3 + 3*x) + 25*sqrt(6)*sqrt(3)*(x^5 + 5*x^3 + 3*x))*sqrt(sqrt(13) -
 5)*elliptic_f(arcsin(1/6*sqrt(6)*x*sqrt(sqrt(13) - 5)), 5/6*sqrt(13) + 19/
6) + 36*(19*x^4 + 119*x^2 + 78)*sqrt(x^4 + 5*x^2 + 3))/(x^5 + 5*x^3 + 3*x)
```

Sympy [F]

$$\int \frac{2+3x^2}{x^2\left(3+5x^2+x^4\right)^{3/2}}\,dx = \int \frac{3x^2+2}{x^2\left(x^4+5x^2+3\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((3*x**2+2)/x**2/(x**4+5*x**2+3)**(3/2),x)
```

```
[Out] Integral((3*x**2 + 2)/(x**2*(x**4 + 5*x**2 + 3)**(3/2)), x)
```

Maxima [F]

$$\int \frac{2+3x^2}{x^2\left(3+5x^2+x^4\right)^{3/2}}\,dx=\int \frac{3\,x^2+2}{\left(x^4+5\,x^2+3\right)^{\frac{3}{2}}x^2}\,dx$$

```
[In] integrate((3*x^2+2)/x^2/(x^4+5*x^2+3)^(3/2),x, algorithm="maxima")
[Out] integrate((3*x^2 + 2)/((x^4 + 5*x^2 + 3)^(3/2)*x^2), x)
```

Giac [F]

$$\int \frac{2+3x^2}{x^2\left(3+5x^2+x^4\right)^{3/2}}\,dx=\int \frac{3\,x^2+2}{\left(x^4+5\,x^2+3\right)^{\frac{3}{2}}x^2}\,dx$$

```
[In] integrate((3*x^2+2)/x^2/(x^4+5*x^2+3)^(3/2),x, algorithm="giac")
[Out] integrate((3*x^2 + 2)/((x^4 + 5*x^2 + 3)^(3/2)*x^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{2+3x^2}{x^2\left(3+5x^2+x^4\right)^{3/2}}\,dx=\int \frac{3\,x^2+2}{x^2\,\left(x^4+5\,x^2+3\right)^{3/2}}\,dx$$

```
[In] int((3*x^2 + 2)/(x^2*(5*x^2 + x^4 + 3)^(3/2)),x)
[Out] int((3*x^2 + 2)/(x^2*(5*x^2 + x^4 + 3)^(3/2)), x)
```

3.203 $\int \frac{2+3x^2}{x^4(3+5x^2+x^4)^{3/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 326

$$\int \frac{2+3x^2}{x^4(3+5x^2+x^4)^{3/2}}\,dx = -\frac{133x(5+\sqrt{13}+2x^2)}{1053\sqrt{3+5x^2+x^4}} - \frac{7+8x^2}{39x^3\sqrt{3+5x^2+x^4}} - \frac{5\sqrt{3+5x^2+x^4}}{351x^3} + \frac{266\sqrt{3+5x^2+x^4}}{1053x} + \frac{133\sqrt{\frac{1}{6}(5+\sqrt{13})}\sqrt{\frac{6+(5-\sqrt{13})x^2}{6+(5+\sqrt{13})x^2}}(6+(5+\sqrt{13})x^2)E\left(\arctan\left(\sqrt{\frac{1}{6}(5+\sqrt{13})}x\right)|\frac{1}{6}(-13+5\sqrt{13})\right)}{1053\sqrt{3+5x^2+x^4}} - \frac{5\sqrt{\frac{6+(5-\sqrt{13})x^2}{6+(5+\sqrt{13})x^2}}(6+(5+\sqrt{13})x^2)\,\text{EllipticF}\left(\arctan\left(\sqrt{\frac{1}{6}(5+\sqrt{13})}x\right),\frac{1}{6}(-13+5\sqrt{13})\right)}{351\sqrt{6(5+\sqrt{13})}\sqrt{3+5x^2+x^4}}$$

```
[Out] 1/39*(-8*x^2-7)/x^3/(x^4+5*x^2+3)^(1/2)-133/1053*x*(5+2*x^2+13^(1/2))/(x^4+
5*x^2+3)^(1/2)-5/351*(x^4+5*x^2+3)^(1/2)/x^3+266/1053*(x^4+5*x^2+3)^(1/2)/x
+133/6318*(1/(36+x^2*(30+6*13^(1/2))))^(1/2)*(36+x^2*(30+6*13^(1/2)))^(1/2)
*EllipticE(x*(30+6*13^(1/2))^(1/2)/(36+x^2*(30+6*13^(1/2)))^(1/2),1/6*(-78+
30*13^(1/2))^(1/2))*(6+x^2*(5+13^(1/2)))*(30+6*13^(1/2))^(1/2)*((6+x^2*(5-1
3^(1/2)))/(6+x^2*(5+13^(1/2))))^(1/2)/(x^4+5*x^2+3)^(1/2)-5/351*(1/(36+x^2*
(30+6*13^(1/2))))^(1/2)*(36+x^2*(30+6*13^(1/2)))^(1/2)*EllipticF(x*(30+6*13
^(1/2))^(1/2)/(36+x^2*(30+6*13^(1/2)))^(1/2),1/6*(-78+30*13^(1/2))^(1/2))*(
6+x^2*(5+13^(1/2)))*((6+x^2*(5-13^(1/2)))/(6+x^2*(5+13^(1/2))))^(1/2)/(x^4+
5*x^2+3)^(1/2)/(30+6*13^(1/2))^(1/2)
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 326, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {1291, 1295, 1203, 1113, 1149}

$$\int \frac{2+3x^2}{x^4\left(3+5x^2+x^4\right)^{3/2}}\,dx = -\frac{5\sqrt{\frac{\left(5-\sqrt{13}\right)x^2+6}{\left(5+\sqrt{13}\right)x^2+6}}\left(\left(5+\sqrt{13}\right)x^2+6\right)\text{EllipticF}\left(\arctan\left(\sqrt{\frac{1}{6}\left(5+\sqrt{13}\right)}x\right),\frac{1}{6}\left(-13+5\sqrt{13}\right)\right)}{351\sqrt{6\left(5+\sqrt{13}\right)}\sqrt{x^4+5x^2+3}} + \frac{133\sqrt{\frac{1}{6}\left(5+\sqrt{13}\right)}\sqrt{\frac{\left(5-\sqrt{13}\right)x^2+6}{\left(5+\sqrt{13}\right)x^2+6}}\left(\left(5+\sqrt{13}\right)x^2+6\right)E\left(\arctan\left(\sqrt{\frac{1}{6}\left(5+\sqrt{13}\right)}x\right)|\frac{1}{6}\left(-13+5\sqrt{13}\right)\right)}{1053\sqrt{x^4+5x^2+3}} - \frac{133x\left(2x^2+\sqrt{13}+5\right)}{1053\sqrt{x^4+5x^2+3}} + \frac{266\sqrt{x^4+5x^2+3}}{1053x} - \frac{5\sqrt{x^4+5x^2+3}}{351x^3} - \frac{8x^2+7}{39x^3\sqrt{x^4+5x^2+3}}$$

```
[In] Int[(2 + 3*x^2)/(x^4*(3 + 5*x^2 + x^4)^(3/2)),x]
```

```
[Out] (-133*x*(5 + Sqrt[13] + 2*x^2))/(1053*Sqrt[3 + 5*x^2 + x^4]) - (7 + 8*x^2)/
(39*x^3*Sqrt[3 + 5*x^2 + x^4]) - (5*Sqrt[3 + 5*x^2 + x^4])/(351*x^3) + (266
*Sqrt[3 + 5*x^2 + x^4])/(1053*x) + (133*Sqrt[(5 + Sqrt[13])/6]*Sqrt[(6 + (5
 - Sqrt[13])*x^2)/(6 + (5 + Sqrt[13])*x^2)]*(6 + (5 + Sqrt[13])*x^2)*Ellipt
icE[ArcTan[Sqrt[(5 + Sqrt[13])/6]*x], (-13 + 5*Sqrt[13])/6])/(1053*Sqrt[3 +
 5*x^2 + x^4]) - (5*Sqrt[(6 + (5 - Sqrt[13])*x^2)/(6 + (5 + Sqrt[13])*x^2)]
*(6 + (5 + Sqrt[13])*x^2)*EllipticF[ArcTan[Sqrt[(5 + Sqrt[13])/6]*x], (-13
+ 5*Sqrt[13])/6])/(351*Sqrt[6*(5 + Sqrt[13])]*Sqrt[3 + 5*x^2 + x^4])
```

Rule 1113

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[(2*a + (b + q)*x^2)*(Sqrt[(2*a + (b - q)*x^2)/(2*a +
(b + q)*x^2)]/(2*a*Rt[(b + q)/(2*a), 2]*Sqrt[a + b*x^2 + c*x^4]))*EllipticF
[ArcTan[Rt[(b + q)/(2*a), 2]*x], 2*(q/(b + q))], x] /; PosQ[(b + q)/a] &&
!(PosQ[(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[
{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1149

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[b^2 - 4*a*c, 2]}, Simp[x*((b + q + 2*c*x^2)/(2*c*Sqrt[a + b*x^2 + c*x^4
])), x] - Simp[Rt[(b + q)/(2*a), 2]*(2*a + (b + q)*x^2)*(Sqrt[(2*a + (b - q
)*x^2)/(2*a + (b + q)*x^2)]/(2*c*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[ArcTan
[Rt[(b + q)/(2*a), 2]*x], 2*(q/(b + q))], x] /; PosQ[(b + q)/a] &&  !(PosQ[
```

```
(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[{a, b,
c}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1203

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[d, Int[1/Sqrt[a + b*x^2 + c*x^4],
 x], x] + Dist[e, Int[x^2/Sqrt[a + b*x^2 + c*x^4], x], x] /; PosQ[(b + q)/a
] || PosQ[(b - q)/a]] /; FreeQ[{a, b, c, d, e}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1291

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[(-(f*x)^(m + 1))*(a + b*x^2 + c*x^4)^(p + 1)
*((d*(b^2 - 2*a*c) - a*b*e + (b*d - 2*a*e)*c*x^2)/(2*a*f*(p + 1)*(b^2 - 4*a
*c))), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), Int[(f*x)^m*(a + b*x^2 + c*
x^4)^(p + 1)*Simp[d*(b^2*(m + 2*(p + 1) + 1) - 2*a*c*(m + 4*(p + 1) + 1)) -
 a*b*e*(m + 1) + c*(m + 2*(2*p + 3) + 1)*(b*d - 2*a*e)*x^2, x], x], x] /; F
reeQ[{a, b, c, d, e, f, m}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && Inte
gerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1295

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[d*(f*x)^(m + 1)*((a + b*x^2 + c*x^4)^(p + 1)
/(a*f*(m + 1))), x] + Dist[1/(a*f^2*(m + 1)), Int[(f*x)^(m + 2)*(a + b*x^2
+ c*x^4)^p*Simp[a*e*(m + 1) - b*d*(m + 2*p + 3) - c*d*(m + 4*p + 5)*x^2, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[m
, -1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{7+8x^2}{39x^3\sqrt{3+5x^2+x^4}}-\frac{1}{39}\int\frac{-5+24x^2}{x^4\sqrt{3+5x^2+x^4}}\,dx\\
&= -\frac{7+8x^2}{39x^3\sqrt{3+5x^2+x^4}}-\frac{5\sqrt{3+5x^2+x^4}}{351x^3}+\frac{1}{351}\int\frac{-266-5x^2}{x^2\sqrt{3+5x^2+x^4}}\,dx\\
&= -\frac{7+8x^2}{39x^3\sqrt{3+5x^2+x^4}}-\frac{5\sqrt{3+5x^2+x^4}}{351x^3}+\frac{266\sqrt{3+5x^2+x^4}}{1053x}-\frac{\int\frac{15+266x^2}{\sqrt{3+5x^2+x^4}}\,dx}{1053}\\
&= -\frac{7+8x^2}{39x^3\sqrt{3+5x^2+x^4}}-\frac{5\sqrt{3+5x^2+x^4}}{351x^3}+\frac{266\sqrt{3+5x^2+x^4}}{1053x}\\
&\quad-\frac{5}{351}\int\frac{1}{\sqrt{3+5x^2+x^4}}\,dx-\frac{266\int\frac{x^2}{\sqrt{3+5x^2+x^4}}\,dx}{1053}
\end{aligned}
$$

$$=-\frac{133x(5+\sqrt{13}+2x^2)}{1053\sqrt{3+5x^2+x^4}}-\frac{7+8x^2}{39x^3\sqrt{3+5x^2+x^4}}-\frac{5\sqrt{3+5x^2+x^4}}{351x^3}+\frac{266\sqrt{3+5x^2+x^4}}{1053x}$$
$$+\frac{133\sqrt{\frac{1}{6}\left(5+\sqrt{13}\right)}\sqrt{\frac{6+\left(5-\sqrt{13}\right)x^2}{6+\left(5+\sqrt{13}\right)x^2}}\left(6+\left(5+\sqrt{13}\right)x^2\right)E\left(\tan^{-1}\left(\sqrt{\frac{1}{6}\left(5+\sqrt{13}\right)}x\right)|\frac{1}{6}\left(-13+5\sqrt{13}\right)\right)}{1053\sqrt{3+5x^2+x^4}}$$
$$-\frac{5\sqrt{\frac{6+\left(5-\sqrt{13}\right)x^2}{6+\left(5+\sqrt{13}\right)x^2}}\left(6+\left(5+\sqrt{13}\right)x^2\right)F\left(\tan^{-1}\left(\sqrt{\frac{1}{6}\left(5+\sqrt{13}\right)}x\right)|\frac{1}{6}\left(-13+5\sqrt{13}\right)\right)}{351\sqrt{6\left(5+\sqrt{13}\right)}\sqrt{3+5x^2+x^4}}$$

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 10.26 (sec) , antiderivative size = 234, normalized size of antiderivative = 0.72

$$\int\frac{2+3x^2}{x^4\left(3+5x^2+x^4\right)^{3/2}}\,dx=\frac{-468+1014x^2+2630x^4+532x^6-133i\sqrt{2}\left(-5+\sqrt{13}\right)x^3\sqrt{\frac{-5+\sqrt{13}-2x^2}{-5+\sqrt{13}}}\sqrt{5+\cdots}}{\cdots}$$

```
[In] Integrate[(2 + 3*x^2)/(x^4*(3 + 5*x^2 + x^4)^(3/2)),x]
```

```
[Out] (-468 + 1014*x^2 + 2630*x^4 + 532*x^6 - (133*I)*Sqrt[2]*(-5 + Sqrt[13])*x^3
*Sqrt[(-5 + Sqrt[13] - 2*x^2)/(-5 + Sqrt[13])]*Sqrt[5 + Sqrt[13] + 2*x^2]*E
llipticE[I*ArcSinh[Sqrt[2/(5 + Sqrt[13])]*x], 19/6 + (5*Sqrt[13])/6] + I*Sq
rt[2]*(-650 + 133*Sqrt[13])*x^3*Sqrt[(-5 + Sqrt[13] - 2*x^2)/(-5 + Sqrt[13]
)]*Sqrt[5 + Sqrt[13] + 2*x^2]*EllipticF[I*ArcSinh[Sqrt[2/(5 + Sqrt[13])]*x]
, 19/6 + (5*Sqrt[13])/6])/(2106*x^3*Sqrt[3 + 5*x^2 + x^4])
```

Maple [A] (verified)

Time = 2.98 (sec) , antiderivative size = 228, normalized size of antiderivative = 0.70

method	result
risch	$\frac{266x^6+1315x^4+507x^2-234}{1053x^3\sqrt{x^4+5x^2+3}}-\frac{10\sqrt{1-\left(-\frac{5}{6}+\frac{\sqrt{13}}{6}\right)x^2}\sqrt{1-\left(-\frac{5}{6}-\frac{\sqrt{13}}{6}\right)x^2}F\left(\frac{x\sqrt{-30+6\sqrt{13}}}{6},\frac{5\sqrt{3}}{6}+\frac{\sqrt{39}}{6}\right)}{117\sqrt{-30+6\sqrt{13}}\sqrt{x^4+5x^2+3}}+\frac{1064\sqrt{1-\left(-\frac{5}{6}+\frac{\sqrt{13}}{6}\right)\cdots}}{\cdots}$
elliptic	$-\frac{2\left(\frac{11}{702}x^3+\frac{17}{351}x\right)}{\sqrt{x^4+5x^2+3}}-\frac{2\sqrt{x^4+5x^2+3}}{27x^3}+\frac{23\sqrt{x^4+5x^2+3}}{81x}-\frac{10\sqrt{1-\left(-\frac{5}{6}+\frac{\sqrt{13}}{6}\right)x^2}\sqrt{1-\left(-\frac{5}{6}-\frac{\sqrt{13}}{6}\right)x^2}F\left(\frac{x\sqrt{-30+6\sqrt{13}}}{6},\frac{5\sqrt{3}}{6}+\frac{\sqrt{39}}{6}\cdots\right)}{117\sqrt{-30+6\sqrt{13}}\sqrt{x^4+5x^2+3}}$
default	$-\frac{6\left(\frac{19}{234}x^3+\frac{40}{117}x\right)}{\sqrt{x^4+5x^2+3}}+\frac{23\sqrt{x^4+5x^2+3}}{81x}-\frac{10\sqrt{1-\left(-\frac{5}{6}+\frac{\sqrt{13}}{6}\right)x^2}\sqrt{1-\left(-\frac{5}{6}-\frac{\sqrt{13}}{6}\right)x^2}F\left(\frac{x\sqrt{-30+6\sqrt{13}}}{6},\frac{5\sqrt{3}}{6}+\frac{\sqrt{39}}{6}\right)}{117\sqrt{-30+6\sqrt{13}}\sqrt{x^4+5x^2+3}}+\frac{1064\sqrt{1-\left(\cdots\right.}}{\cdots}$

```
[In] int((3*x^2+2)/x^4/(x^4+5*x^2+3)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/1053*(266*x^6+1315*x^4+507*x^2-234)/x^3/(x^4+5*x^2+3)^(1/2)-10/117/(-30+6
*13^(1/2))^(1/2)*(1-(-5/6+1/6*13^(1/2))*x^2)^(1/2)*(1-(-5/6-1/6*13^(1/2))*x
^2)^(1/2)/(x^4+5*x^2+3)^(1/2)*EllipticF(1/6*x*(-30+6*13^(1/2))^(1/2),5/6*3^
(1/2)+1/6*39^(1/2))+1064/117/(-30+6*13^(1/2))^(1/2)*(1-(-5/6+1/6*13^(1/2))*
x^2)^(1/2)*(1-(-5/6-1/6*13^(1/2))*x^2)^(1/2)/(x^4+5*x^2+3)^(1/2)/(5+13^(1/2
))*(EllipticF(1/6*x*(-30+6*13^(1/2))^(1/2),5/6*3^(1/2)+1/6*39^(1/2))-Ellipt
icE(1/6*x*(-30+6*13^(1/2))^(1/2),5/6*3^(1/2)+1/6*39^(1/2)))
```

Fricas [A] (verification not implemented)

none

Time = 0.08 (sec) , antiderivative size = 207, normalized size of antiderivative = 0.63

$$\int \frac{2+3x^2}{x^4\left(3+5x^2+x^4\right)^{3/2}}\,dx = \frac{266\left(\sqrt{13}\sqrt{6}\sqrt{3}(x^7+5\,x^5+3\,x^3)-5\,\sqrt{6}\sqrt{3}(x^7+5\,x^5+3\,x^3)\right)\sqrt{\sqrt{13}-5}E(\mathrm{a}}{}$$

```
[In] integrate((3*x^2+2)/x^4/(x^4+5*x^2+3)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/37908*(266*(sqrt(13)*sqrt(6)*sqrt(3)*(x^7 + 5*x^5 + 3*x^3) - 5*sqrt(6)*sq
rt(3)*(x^7 + 5*x^5 + 3*x^3))*sqrt(sqrt(13) - 5)*elliptic_e(arcsin(1/6*sqrt(
6)*x*sqrt(sqrt(13) - 5)), 5/6*sqrt(13) + 19/6) - (251*sqrt(13)*sqrt(6)*sqrt
(3)*(x^7 + 5*x^5 + 3*x^3) - 1405*sqrt(6)*sqrt(3)*(x^7 + 5*x^5 + 3*x^3))*sqr
t(sqrt(13) - 5)*elliptic_f(arcsin(1/6*sqrt(6)*x*sqrt(sqrt(13) - 5)), 5/6*sq
rt(13) + 19/6) + 36*(266*x^6 + 1315*x^4 + 507*x^2 - 234)*sqrt(x^4 + 5*x^2 +
 3))/(x^7 + 5*x^5 + 3*x^3)
```

Sympy [F]

$$\int \frac{2+3x^2}{x^4\left(3+5x^2+x^4\right)^{3/2}}\,dx = \int \frac{3x^2+2}{x^4\left(x^4+5x^2+3\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((3*x**2+2)/x**4/(x**4+5*x**2+3)**(3/2),x)
```

```
[Out] Integral((3*x**2 + 2)/(x**4*(x**4 + 5*x**2 + 3)**(3/2)), x)
```

Maxima [F]

$$\int \frac{2+3x^2}{x^4 (3+5x^2+x^4)^{3/2}}\,dx = \int \frac{3\,x^2+2}{(x^4+5\,x^2+3)^{\frac{3}{2}} x^4}\,dx$$

```
[In] integrate((3*x^2+2)/x^4/(x^4+5*x^2+3)^(3/2),x, algorithm="maxima")
[Out] integrate((3*x^2 + 2)/((x^4 + 5*x^2 + 3)^(3/2)*x^4), x)
```

Giac [F]

$$\int \frac{2+3x^2}{x^4 (3+5x^2+x^4)^{3/2}}\,dx = \int \frac{3\,x^2+2}{(x^4+5\,x^2+3)^{\frac{3}{2}} x^4}\,dx$$

```
[In] integrate((3*x^2+2)/x^4/(x^4+5*x^2+3)^(3/2),x, algorithm="giac")
[Out] integrate((3*x^2 + 2)/((x^4 + 5*x^2 + 3)^(3/2)*x^4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{2+3x^2}{x^4 (3+5x^2+x^4)^{3/2}}\,dx = \int \frac{3\,x^2+2}{x^4\,(x^4+5\,x^2+3)^{3/2}}\,dx$$

```
[In] int((3*x^2 + 2)/(x^4*(5*x^2 + x^4 + 3)^(3/2)),x)
[Out] int((3*x^2 + 2)/(x^4*(5*x^2 + x^4 + 3)^(3/2)), x)
```

3.204 $\int (fx)^{3/2} (d+ex^2) \sqrt{a+bx^2+cx^4} \, dx$

Optimal result

Integrand size = 31, antiderivative size = 297

$$\int (fx)^{3/2} (d+ex^2) \sqrt{a+bx^2+cx^4} \, dx = \frac{2d(fx)^{5/2}\sqrt{a+bx^2+cx^4}\,\text{AppellF1}\left(\frac{5}{4},-\frac{1}{2},-\frac{1}{2},\frac{9}{4},-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{5f\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}} + \frac{2e(fx)^{9/2}\sqrt{a+bx^2+cx^4}\,\text{AppellF1}\left(\frac{9}{4},-\frac{1}{2},-\frac{1}{2},\frac{13}{4},-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{9f^3\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}}$$

```
[Out] 2/5*d*(f*x)^(5/2)*AppellF1(5/4,-1/2,-1/2,9/4,-2*c*x^2/(b-(-4*a*c+b^2)^(1/2)
),-2*c*x^2/(b+(-4*a*c+b^2)^(1/2)))*(c*x^4+b*x^2+a)^(1/2)/f/(1+2*c*x^2/(b-(-
4*a*c+b^2)^(1/2)))^(1/2)/(1+2*c*x^2/(b+(-4*a*c+b^2)^(1/2)))^(1/2)+2/9*e*(f*
x)^(9/2)*AppellF1(9/4,-1/2,-1/2,13/4,-2*c*x^2/(b-(-4*a*c+b^2)^(1/2)),-2*c*x
^2/(b+(-4*a*c+b^2)^(1/2)))*(c*x^4+b*x^2+a)^(1/2)/f^3/(1+2*c*x^2/(b-(-4*a*c+
b^2)^(1/2)))^(1/2)/(1+2*c*x^2/(b+(-4*a*c+b^2)^(1/2)))^(1/2)
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 297, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.097, Rules used

$= \{1349, 1155, 524\}$

$$\int (fx)^{3/2} \left(d + ex^2\right) \sqrt{a+bx^2+cx^4}\,dx = \frac{2d(fx)^{5/2}\sqrt{a+bx^2+cx^4}\,\text{AppellF1}\left(\frac{5}{4},-\frac{1}{2},-\frac{1}{2},\frac{9}{4},-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{5f\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}} + \frac{2e(fx)^{9/2}\sqrt{a+bx^2+cx^4}\,\text{AppellF1}\left(\frac{9}{4},-\frac{1}{2},-\frac{1}{2},\frac{13}{4},-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{9f^3\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}}$$

```
[In] Int[(f*x)^(3/2)*(d + e*x^2)*Sqrt[a + b*x^2 + c*x^4],x]
```

```
[Out] (2*d*(f*x)^(5/2)*Sqrt[a + b*x^2 + c*x^4]*AppellF1[5/4, -1/2, -1/2, 9/4, (-2
*c*x^2)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])])/(5*f*
Sqrt[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^2)/(b + Sqrt[b^
2 - 4*a*c])]) + (2*e*(f*x)^(9/2)*Sqrt[a + b*x^2 + c*x^4]*AppellF1[9/4, -1/2
, -1/2, 13/4, (-2*c*x^2)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^2)/(b + Sqrt[b^2
- 4*a*c])])/(9*f^3*Sqrt[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*
c*x^2)/(b + Sqrt[b^2 - 4*a*c])])
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 1155

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[a^IntPart[p]*((a + b*x^2 + c*x^4)^FracPart[p]/((1 + 2*c*(x^2/(b +
Rt[b^2 - 4*a*c, 2])))^FracPart[p]*(1 + 2*c*(x^2/(b - Rt[b^2 - 4*a*c, 2])))^
FracPart[p])), Int[(d*x)^m*(1 + 2*c*(x^2/(b + Sqrt[b^2 - 4*a*c])))^p*(1 + 2
*c*(x^2/(b - Sqrt[b^2 - 4*a*c])))^p, x], x] /; FreeQ[{a, b, c, d, m, p}, x]
```

Rule 1349

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*
(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, p, q}, x] && N
eQ[b^2 - 4*a*c, 0] && (IGtQ[p, 0] || IGtQ[q, 0] || IntegersQ[m, q])
```

Rubi steps

$$\text{integral} = \int \left(d(fx)^{3/2}\sqrt{a+bx^2+cx^4} + \frac{e(fx)^{7/2}\sqrt{a+bx^2+cx^4}}{f^2}\right) dx$$

$$= d\int (fx)^{3/2}\sqrt{a+bx^2+cx^4}\,dx + \frac{e\int (fx)^{7/2}\sqrt{a+bx^2+cx^4}\,dx}{f^2}$$

$$= \frac{\left(d\sqrt{a+bx^2+cx^4}\right)\int (fx)^{3/2}\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}\,dx}{\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}} + \frac{\left(e\sqrt{a+bx^2+cx^4}\right)\int (fx)^{7/2}\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}\,dx}{f^2\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}}$$

$$= \frac{2d(fx)^{5/2}\sqrt{a+bx^2+cx^4}\,F_1\left(\frac{5}{4};-\frac{1}{2},-\frac{1}{2};\frac{9}{4};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{5f\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}} + \frac{2e(fx)^{9/2}\sqrt{a+bx^2+cx^4}\,F_1\left(\frac{9}{4};-\frac{1}{2},-\frac{1}{2};\frac{13}{4};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{9f^3\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}}$$

Mathematica [A] (verified)

Time = 11.76 (sec) , antiderivative size = 430, normalized size of antiderivative = 1.45

$$\int (fx)^{3/2}\left(d+ex^2\right)\sqrt{a+bx^2+cx^4}\,dx = \frac{2f\sqrt{fx}\left(5\left(a+bx^2+cx^4\right)\left(-14b^2e+2bc(13d+5ex^2)+c(36ae+65cdx^2+45ce\right.\right.}{}$$

```
[In] Integrate[(f*x)^(3/2)*(d + e*x^2)*Sqrt[a + b*x^2 + c*x^4],x]
```

```
[Out] (2*f*Sqrt[f*x]*(5*(a + b*x^2 + c*x^4)*(-14*b^2*e + 2*b*c*(13*d + 5*e*x^2) +
 c*(36*a*e + 65*c*d*x^2 + 45*c*e*x^4)) + 10*a*(-13*b*c*d + 7*b^2*e - 18*a*c
*e)*Sqrt[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[(b
 + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[1/4, 1/2,
 1/2, 5/4, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(-b + Sqrt[b^2 - 4
*a*c])] + 2*(-39*b^2*c*d + 130*a*c^2*d + 21*b^3*e - 79*a*b*c*e)*x^2*Sqrt[(b
 - Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[(b + Sqrt[b^2
 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[5/4, 1/2, 1/2, 9/4,
(-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])]))/(
2925*c^2*Sqrt[a + b*x^2 + c*x^4])
```

Maple [F]

$$\int (fx)^{\frac{3}{2}} \left(e\,x^{2}+d\right) \sqrt{c\,x^{4}+b\,x^{2}+a}dx$$

```
[In] int((f*x)^(3/2)*(e*x^2+d)*(c*x^4+b*x^2+a)^(1/2),x)
```

```
[Out] int((f*x)^(3/2)*(e*x^2+d)*(c*x^4+b*x^2+a)^(1/2),x)
```

Fricas [F]

$$\int (fx)^{3/2} \left(d+ex^2\right) \sqrt{a+bx^2+cx^4}\,dx = \int \sqrt{cx^4+bx^2+a}\left(ex^2+d\right)(fx)^{\frac{3}{2}}\,dx$$

```
[In] integrate((f*x)^(3/2)*(e*x^2+d)*(c*x^4+b*x^2+a)^(1/2),x, algorithm="fricas"
)
```

```
[Out] integral((e*f*x^3 + d*f*x)*sqrt(c*x^4 + b*x^2 + a)*sqrt(f*x), x)
```

Sympy [F]

$$\int (fx)^{3/2} \left(d+ex^2\right) \sqrt{a+bx^2+cx^4}\,dx = \int (fx)^{\frac{3}{2}} \left(d+ex^2\right) \sqrt{a+bx^2+cx^4}\,dx$$

```
[In] integrate((f*x)**(3/2)*(e*x**2+d)*(c*x**4+b*x**2+a)**(1/2),x)
```

```
[Out] Integral((f*x)**(3/2)*(d + e*x**2)*sqrt(a + b*x**2 + c*x**4), x)
```

Maxima [F]

$$\int (fx)^{3/2} \left(d+ex^2\right) \sqrt{a+bx^2+cx^4}\,dx = \int \sqrt{cx^4+bx^2+a}\left(ex^2+d\right)(fx)^{\frac{3}{2}}\,dx$$

```
[In] integrate((f*x)^(3/2)*(e*x^2+d)*(c*x^4+b*x^2+a)^(1/2),x, algorithm="maxima"
)
```

```
[Out] integrate(sqrt(c*x^4 + b*x^2 + a)*(e*x^2 + d)*(f*x)^(3/2), x)
```

Giac [F]

$$\int (fx)^{3/2}\left(d+ex^2\right)\sqrt{a+bx^2+cx^4}\,dx = \int \sqrt{cx^4+bx^2+a}\left(ex^2+d\right)(fx)^{\frac{3}{2}}\,dx$$

```
[In] integrate((f*x)^(3/2)*(e*x^2+d)*(c*x^4+b*x^2+a)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(c*x^4 + b*x^2 + a)*(e*x^2 + d)*(f*x)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int (fx)^{3/2}\left(d+ex^2\right)\sqrt{a+bx^2+cx^4}\,dx = \int (f\,x)^{3/2}\,\left(e\,x^2+d\right)\,\sqrt{c\,x^4+b\,x^2+a}\,dx$$

```
[In] int((f*x)^(3/2)*(d + e*x^2)*(a + b*x^2 + c*x^4)^(1/2),x)
[Out] int((f*x)^(3/2)*(d + e*x^2)*(a + b*x^2 + c*x^4)^(1/2), x)
```

3.205 $\int \sqrt{fx}(d+ex^2)\sqrt{a+bx^2+cx^4}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 297

$$\int \sqrt{fx}(d+ex^2)\sqrt{a+bx^2+cx^4}\,dx = \frac{2d(fx)^{3/2}\sqrt{a+bx^2+cx^4}\,\text{AppellF1}\left(\frac{3}{4},-\frac{1}{2},-\frac{1}{2},\frac{7}{4},-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{3f\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}} + \frac{2e(fx)^{7/2}\sqrt{a+bx^2+cx^4}\,\text{AppellF1}\left(\frac{7}{4},-\frac{1}{2},-\frac{1}{2},\frac{11}{4},-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{7f^3\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}}$$

```
[Out] 2/3*d*(f*x)^(3/2)*AppellF1(3/4,-1/2,-1/2,7/4,-2*c*x^2/(b-(-4*a*c+b^2)^(1/2)
),-2*c*x^2/(b+(-4*a*c+b^2)^(1/2)))*(c*x^4+b*x^2+a)^(1/2)/f/(1+2*c*x^2/(b-(-
4*a*c+b^2)^(1/2)))^(1/2)/(1+2*c*x^2/(b+(-4*a*c+b^2)^(1/2)))^(1/2)+2/7*e*(f*
x)^(7/2)*AppellF1(7/4,-1/2,-1/2,11/4,-2*c*x^2/(b-(-4*a*c+b^2)^(1/2)),-2*c*x
^2/(b+(-4*a*c+b^2)^(1/2)))*(c*x^4+b*x^2+a)^(1/2)/f^3/(1+2*c*x^2/(b-(-4*a*c+
b^2)^(1/2)))^(1/2)/(1+2*c*x^2/(b+(-4*a*c+b^2)^(1/2)))^(1/2)
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 297, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.097$, Rules used

= {1349, 1155, 524}

$$\int \sqrt{fx}(d+ex^2)\sqrt{a+bx^2+cx^4}\,dx = \frac{2d(fx)^{3/2}\sqrt{a+bx^2+cx^4}\,\text{AppellF1}\left(\frac{3}{4},-\frac{1}{2},-\frac{1}{2},\frac{7}{4},-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{3f\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}} + \frac{2e(fx)^{7/2}\sqrt{a+bx^2+cx^4}\,\text{AppellF1}\left(\frac{7}{4},-\frac{1}{2},-\frac{1}{2},\frac{11}{4},-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{7f^3\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}}$$

```
[In] Int[Sqrt[f*x]*(d + e*x^2)*Sqrt[a + b*x^2 + c*x^4],x]
```

```
[Out] (2*d*(f*x)^(3/2)*Sqrt[a + b*x^2 + c*x^4]*AppellF1[3/4, -1/2, -1/2, 7/4, (-2
*c*x^2)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])])/(3*f*
Sqrt[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^2)/(b + Sqrt[b^
2 - 4*a*c])]) + (2*e*(f*x)^(7/2)*Sqrt[a + b*x^2 + c*x^4]*AppellF1[7/4, -1/2
, -1/2, 11/4, (-2*c*x^2)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^2)/(b + Sqrt[b^2
- 4*a*c])])/(7*f^3*Sqrt[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*
c*x^2)/(b + Sqrt[b^2 - 4*a*c])])
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 1155

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[a^IntPart[p]*((a + b*x^2 + c*x^4)^FracPart[p]/((1 + 2*c*(x^2/(b +
Rt[b^2 - 4*a*c, 2])))^FracPart[p]*(1 + 2*c*(x^2/(b - Rt[b^2 - 4*a*c, 2])))^
FracPart[p])), Int[(d*x)^m*(1 + 2*c*(x^2/(b + Sqrt[b^2 - 4*a*c])))^p*(1 + 2
*c*(x^2/(b - Sqrt[b^2 - 4*a*c])))^p, x], x] /; FreeQ[{a, b, c, d, m, p}, x]
```

Rule 1349

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*
(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, p, q}, x] && N
eQ[b^2 - 4*a*c, 0] && (IGtQ[p, 0] || IGtQ[q, 0] || IntegersQ[m, q])
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \left(d\sqrt{fx}\sqrt{a+bx^2+cx^4}+\frac{e(fx)^{5/2}\sqrt{a+bx^2+cx^4}}{f^2}\right)dx\\
&= d\int \sqrt{fx}\sqrt{a+bx^2+cx^4}\,dx+\frac{e\int (fx)^{5/2}\sqrt{a+bx^2+cx^4}\,dx}{f^2}\\
&= \frac{\left(d\sqrt{a+bx^2+cx^4}\right)\int \sqrt{fx}\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}\,dx}{\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}}\\
&\quad+\frac{\left(e\sqrt{a+bx^2+cx^4}\right)\int (fx)^{5/2}\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}\,dx}{f^2\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}}\\
&= \frac{2d(fx)^{3/2}\sqrt{a+bx^2+cx^4}F_1\left(\frac{3}{4};-\frac{1}{2},-\frac{1}{2};\frac{7}{4};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{3f\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}}\\
&\quad+\frac{2e(fx)^{7/2}\sqrt{a+bx^2+cx^4}F_1\left(\frac{7}{4};-\frac{1}{2},-\frac{1}{2};\frac{11}{4};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{7f^3\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 11.61 (sec) , antiderivative size = 386, normalized size of antiderivative = 1.30

$$\int \sqrt{fx}\left(d+ex^2\right)\sqrt{a+bx^2+cx^4}\,dx$$

$$= \frac{2x\sqrt{fx}\Big(21(11cd+2be+7cex^2)\left(a+bx^2+cx^4\right)+14a(22cd-3be)\sqrt{\frac{b-\sqrt{b^2-4ac}+2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx^2}{b+\sqrt{b^2-4ac}}}\,\text{Appel}}{}$$

```
[In] Integrate[Sqrt[f*x]*(d + e*x^2)*Sqrt[a + b*x^2 + c*x^4],x]
```

```
[Out] (2*x*Sqrt[f*x]*(21*(11*c*d + 2*b*e + 7*c*e*x^2)*(a + b*x^2 + c*x^4) + 14*a*
(22*c*d - 3*b*e)*Sqrt[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b - Sqrt[b^2 - 4*a
*c])]*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*Appel
lF1[3/4, 1/2, 1/2, 7/4, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(-b +
 Sqrt[b^2 - 4*a*c])] + 6*(11*b*c*d - 5*b^2*e + 14*a*c*e)*x^2*Sqrt[(b - Sqrt
[b^2 - 4*a*c] + 2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[(b + Sqrt[b^2 - 4*a*
c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[7/4, 1/2, 1/2, 11/4, (-2*c*
x^2)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])]))/(1617*c
*Sqrt[a + b*x^2 + c*x^4])
```

Maple [F]

$$\int \sqrt{fx}\,(e\,x^2+d)\,\sqrt{c\,x^4+b\,x^2+a}dx$$

[In] int((f*x)^(1/2)*(e*x^2+d)*(c*x^4+b*x^2+a)^(1/2),x)

[Out] int((f*x)^(1/2)*(e*x^2+d)*(c*x^4+b*x^2+a)^(1/2),x)

Fricas [F]

$$\int \sqrt{fx}(d+ex^2)\,\sqrt{a+bx^2+cx^4}\,dx = \int \sqrt{cx^4+bx^2+a}(ex^2+d)\sqrt{fx}\,dx$$

[In] integrate((f*x)^(1/2)*(e*x^2+d)*(c*x^4+b*x^2+a)^(1/2),x, algorithm="fricas"
)

[Out] integral(sqrt(c*x^4 + b*x^2 + a)*(e*x^2 + d)*sqrt(f*x), x)

Sympy [F]

$$\int \sqrt{fx}(d+ex^2)\,\sqrt{a+bx^2+cx^4}\,dx = \int \sqrt{fx}(d+ex^2)\,\sqrt{a+bx^2+cx^4}\,dx$$

[In] integrate((f*x)**(1/2)*(e*x**2+d)*(c*x**4+b*x**2+a)**(1/2),x)

[Out] Integral(sqrt(f*x)*(d + e*x**2)*sqrt(a + b*x**2 + c*x**4), x)

Maxima [F]

$$\int \sqrt{fx}(d+ex^2)\,\sqrt{a+bx^2+cx^4}\,dx = \int \sqrt{cx^4+bx^2+a}(ex^2+d)\sqrt{fx}\,dx$$

[In] integrate((f*x)^(1/2)*(e*x^2+d)*(c*x^4+b*x^2+a)^(1/2),x, algorithm="maxima"
)

[Out] integrate(sqrt(c*x^4 + b*x^2 + a)*(e*x^2 + d)*sqrt(f*x), x)

Giac [F]

$$\int \sqrt{fx}(d+ex^2)\sqrt{a+bx^2+cx^4}\,dx = \int \sqrt{cx^4+bx^2+a}(ex^2+d)\sqrt{fx}\,dx$$

```
[In] integrate((f*x)^(1/2)*(e*x^2+d)*(c*x^4+b*x^2+a)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(c*x^4 + b*x^2 + a)*(e*x^2 + d)*sqrt(f*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{fx}(d+ex^2)\sqrt{a+bx^2+cx^4}\,dx = \int \sqrt{f\,x}\,\left(e\,x^2+d\right)\,\sqrt{c\,x^4+b\,x^2+a}\,dx$$

```
[In] int((f*x)^(1/2)*(d + e*x^2)*(a + b*x^2 + c*x^4)^(1/2),x)
[Out] int((f*x)^(1/2)*(d + e*x^2)*(a + b*x^2 + c*x^4)^(1/2), x)
```

3.206 $\int \frac{(d+ex^2)\sqrt{a+bx^2+cx^4}}{\sqrt{fx}}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 295

$$\int \frac{(d+ex^2)\sqrt{a+bx^2+cx^4}}{\sqrt{fx}}\,dx = \frac{2d\sqrt{fx}\sqrt{a+bx^2+cx^4}\,\mathrm{AppellF1}\left(\frac{1}{4},-\frac{1}{2},-\frac{1}{2},\frac{5}{4},-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{f\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}} + \frac{2e(fx)^{5/2}\sqrt{a+bx^2+cx^4}\,\mathrm{AppellF1}\left(\frac{5}{4},-\frac{1}{2},-\frac{1}{2},\frac{9}{4},-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{5f^3\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}}$$

```
[Out] 2/5*e*(f*x)^(5/2)*AppellF1(5/4,-1/2,-1/2,9/4,-2*c*x^2/(b-(-4*a*c+b^2)^(1/2)
),-2*c*x^2/(b+(-4*a*c+b^2)^(1/2)))*(c*x^4+b*x^2+a)^(1/2)/f^3/(1+2*c*x^2/(b-
(-4*a*c+b^2)^(1/2)))^(1/2)/(1+2*c*x^2/(b+(-4*a*c+b^2)^(1/2)))^(1/2)+2*d*App
ellF1(1/4,-1/2,-1/2,5/4,-2*c*x^2/(b-(-4*a*c+b^2)^(1/2)),-2*c*x^2/(b+(-4*a*c
+b^2)^(1/2)))*(f*x)^(1/2)*(c*x^4+b*x^2+a)^(1/2)/f/(1+2*c*x^2/(b-(-4*a*c+b^2
)^(1/2)))^(1/2)/(1+2*c*x^2/(b+(-4*a*c+b^2)^(1/2)))^(1/2)
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 295, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.097$, Rules used

= {1349, 1155, 524}

$$\int \frac{(d+ex^2)\sqrt{a+bx^2+cx^4}}{\sqrt{fx}}\,dx$$
$$= \frac{2d\sqrt{fx}\sqrt{a+bx^2+cx^4}\,\text{AppellF1}\left(\frac{1}{4},-\frac{1}{2},-\frac{1}{2},\frac{5}{4},-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{f\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}}$$
$$+\frac{2e(fx)^{5/2}\sqrt{a+bx^2+cx^4}\,\text{AppellF1}\left(\frac{5}{4},-\frac{1}{2},-\frac{1}{2},\frac{9}{4},-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{5f^3\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}}$$

```
[In] Int[((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4])/Sqrt[f*x],x]
```

```
[Out] (2*d*Sqrt[f*x]*Sqrt[a + b*x^2 + c*x^4]*AppellF1[1/4, -1/2, -1/2, 5/4, (-2*c
*x^2)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])])/(f*Sqrt
[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^2)/(b + Sqrt[b^2 -
4*a*c])]) + (2*e*(f*x)^(5/2)*Sqrt[a + b*x^2 + c*x^4]*AppellF1[5/4, -1/2, -1
/2, 9/4, (-2*c*x^2)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^2)/(b + Sqrt[b^2 - 4*a
*c])])/(5*f^3*Sqrt[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^2
)/(b + Sqrt[b^2 - 4*a*c])])
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 1155

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[a^IntPart[p]*((a + b*x^2 + c*x^4)^FracPart[p]/((1 + 2*c*(x^2/(b +
Rt[b^2 - 4*a*c, 2])))^FracPart[p]*(1 + 2*c*(x^2/(b - Rt[b^2 - 4*a*c, 2])))^
FracPart[p])), Int[(d*x)^m*(1 + 2*c*(x^2/(b + Sqrt[b^2 - 4*a*c])))^p*(1 + 2
*c*(x^2/(b - Sqrt[b^2 - 4*a*c])))^p, x], x] /; FreeQ[{a, b, c, d, m, p}, x]
```

Rule 1349

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*
(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, p, q}, x] && N
eQ[b^2 - 4*a*c, 0] && (IGtQ[p, 0] || IGtQ[q, 0] || IntegersQ[m, q])
```

Rubi steps

$$\text{integral} = \int \left(\frac{d\sqrt{a+bx^2+cx^4}}{\sqrt{fx}} + \frac{e(fx)^{3/2}\sqrt{a+bx^2+cx^4}}{f^2}\right) dx$$

$$= d\int \frac{\sqrt{a+bx^2+cx^4}}{\sqrt{fx}}\,dx + \frac{e\int (fx)^{3/2}\sqrt{a+bx^2+cx^4}\,dx}{f^2}$$

$$= \frac{\left(d\sqrt{a+bx^2+cx^4}\right)\int \frac{\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}}{\sqrt{fx}}\,dx}{\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}} + \frac{\left(e\sqrt{a+bx^2+cx^4}\right)\int (fx)^{3/2}\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}\,dx}{f^2\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}}$$

$$= \frac{2d\sqrt{fx}\sqrt{a+bx^2+cx^4}F_1\left(\frac{1}{4};-\frac{1}{2},-\frac{1}{2};\frac{5}{4};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{f\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}} + \frac{2e(fx)^{5/2}\sqrt{a+bx^2+cx^4}F_1\left(\frac{5}{4};-\frac{1}{2},-\frac{1}{2};\frac{9}{4};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{5f^3\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}}$$

Mathematica [A] (verified)

Time = 11.56 (sec) , antiderivative size = 386, normalized size of antiderivative = 1.31

$$\int \frac{(d+ex^2)\sqrt{a+bx^2+cx^4}}{\sqrt{fx}}\,dx = \frac{2x\left(5(9cd+2be+5cex^2)(a+bx^2+cx^4)+10a(18cd-be)\sqrt{\frac{b-\sqrt{b^2-4ac}+2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx^2}{b+\sqrt{b^2-4ac}}}\,\text{AppellF1}\left(\frac{1}{4}\right.\right.}{}$$

```
[In] Integrate[((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4])/Sqrt[f*x],x]
```

```
[Out] (2*x*(5*(9*c*d + 2*b*e + 5*c*e*x^2)*(a + b*x^2 + c*x^4) + 10*a*(18*c*d - b*
e)*Sqrt[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[(b
+ Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[1/4, 1/2,
1/2, 5/4, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(-b + Sqrt[b^2 - 4*
a*c])] + 2*(9*b*c*d - 3*b^2*e + 10*a*c*e)*x^2*Sqrt[(b - Sqrt[b^2 - 4*a*c] +
 2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(
b + Sqrt[b^2 - 4*a*c])]*AppellF1[5/4, 1/2, 1/2, 9/4, (-2*c*x^2)/(b + Sqrt[b
^2 - 4*a*c]), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])]))/(225*c*Sqrt[f*x]*Sqrt[a
 + b*x^2 + c*x^4])
```

Maple [F]

$$\int \frac{(e\,x^2+d)\,\sqrt{c\,x^4+b\,x^2+a}}{\sqrt{fx}}dx$$

```
[In] int((e*x^2+d)*(c*x^4+b*x^2+a)^(1/2)/(f*x)^(1/2),x)
[Out] int((e*x^2+d)*(c*x^4+b*x^2+a)^(1/2)/(f*x)^(1/2),x)
```

Fricas [F]

$$\int \frac{(d+ex^2)\,\sqrt{a+bx^2+cx^4}}{\sqrt{fx}}\,dx = \int \frac{\sqrt{cx^4+bx^2+a}(ex^2+d)}{\sqrt{fx}}\,dx$$

```
[In] integrate((e*x^2+d)*(c*x^4+b*x^2+a)^(1/2)/(f*x)^(1/2),x, algorithm="fricas"
)
[Out] integral(sqrt(c*x^4 + b*x^2 + a)*(e*x^2 + d)*sqrt(f*x)/(f*x), x)
```

Sympy [F]

$$\int \frac{(d+ex^2)\,\sqrt{a+bx^2+cx^4}}{\sqrt{fx}}\,dx = \int \frac{(d+ex^2)\,\sqrt{a+bx^2+cx^4}}{\sqrt{fx}}\,dx$$

```
[In] integrate((e*x**2+d)*(c*x**4+b*x**2+a)**(1/2)/(f*x)**(1/2),x)
[Out] Integral((d + e*x**2)*sqrt(a + b*x**2 + c*x**4)/sqrt(f*x), x)
```

Maxima [F]

$$\int \frac{(d+ex^2)\,\sqrt{a+bx^2+cx^4}}{\sqrt{fx}}\,dx = \int \frac{\sqrt{cx^4+bx^2+a}(ex^2+d)}{\sqrt{fx}}\,dx$$

```
[In] integrate((e*x^2+d)*(c*x^4+b*x^2+a)^(1/2)/(f*x)^(1/2),x, algorithm="maxima"
)
[Out] integrate(sqrt(c*x^4 + b*x^2 + a)*(e*x^2 + d)/sqrt(f*x), x)
```

Giac [F]

$$\int \frac{(d+ex^2)\sqrt{a+bx^2+cx^4}}{\sqrt{fx}}\,dx = \int \frac{\sqrt{cx^4+bx^2+a}(ex^2+d)}{\sqrt{fx}}\,dx$$

```
[In] integrate((e*x^2+d)*(c*x^4+b*x^2+a)^(1/2)/(f*x)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(c*x^4 + b*x^2 + a)*(e*x^2 + d)/sqrt(f*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex^2)\sqrt{a+bx^2+cx^4}}{\sqrt{fx}}\,dx = \int \frac{(e\,x^2+d)\,\sqrt{c\,x^4+b\,x^2+a}}{\sqrt{f\,x}}\,dx$$

```
[In] int(((d + e*x^2)*(a + b*x^2 + c*x^4)^(1/2))/(f*x)^(1/2),x)
[Out] int(((d + e*x^2)*(a + b*x^2 + c*x^4)^(1/2))/(f*x)^(1/2), x)
```

3.207 $\int \frac{(d+ex^2)\sqrt{a+bx^2+cx^4}}{(fx)^{3/2}} dx$

Optimal result

Integrand size = 31, antiderivative size = 295

$$\int \frac{(d+ex^2)\sqrt{a+bx^2+cx^4}}{(fx)^{3/2}} dx = -\frac{2d\sqrt{a+bx^2+cx^4}\operatorname{AppellF1}\left(-\frac{1}{4},-\frac{1}{2},-\frac{1}{2},\frac{3}{4},-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{f\sqrt{fx}\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}} + \frac{2e(fx)^{3/2}\sqrt{a+bx^2+cx^4}\operatorname{AppellF1}\left(\frac{3}{4},-\frac{1}{2},-\frac{1}{2},\frac{7}{4},-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{3f^3\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}}$$

```
[Out] 2/3*e*(f*x)^(3/2)*AppellF1(3/4,-1/2,-1/2,7/4,-2*c*x^2/(b-(-4*a*c+b^2)^(1/2)
),-2*c*x^2/(b+(-4*a*c+b^2)^(1/2)))*(c*x^4+b*x^2+a)^(1/2)/f^3/(1+2*c*x^2/(b-
(-4*a*c+b^2)^(1/2)))^(1/2)/(1+2*c*x^2/(b+(-4*a*c+b^2)^(1/2)))^(1/2)-2*d*App
ellF1(-1/4,-1/2,-1/2,3/4,-2*c*x^2/(b-(-4*a*c+b^2)^(1/2)),-2*c*x^2/(b+(-4*a*
c+b^2)^(1/2)))*(c*x^4+b*x^2+a)^(1/2)/f/(f*x)^(1/2)/(1+2*c*x^2/(b-(-4*a*c+b^
2)^(1/2)))^(1/2)/(1+2*c*x^2/(b+(-4*a*c+b^2)^(1/2)))^(1/2)
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 295, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.097, Rules used

= {1349, 1155, 524}

$$\int \frac{(d+ex^2)\sqrt{a+bx^2+cx^4}}{(fx)^{3/2}}\,dx = \frac{2e(fx)^{3/2}\sqrt{a+bx^2+cx^4}\,\text{AppellF1}\left(\frac{3}{4},-\frac{1}{2},-\frac{1}{2},\frac{7}{4},-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4a}}\right.}{3f^3\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}}$$
$$-\frac{2d\sqrt{a+bx^2+cx^4}\,\text{AppellF1}\left(-\frac{1}{4},-\frac{1}{2},-\frac{1}{2},\frac{3}{4},-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{f\sqrt{fx}\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}}$$

```
[In] Int[((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4])/(f*x)^(3/2),x]
```

```
[Out] (-2*d*Sqrt[a + b*x^2 + c*x^4]*AppellF1[-1/4, -1/2, -1/2, 3/4, (-2*c*x^2)/(b
 - Sqrt[b^2 - 4*a*c]), (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])])/(f*Sqrt[f*x]*Sq
rt[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^2)/(b + Sqrt[b^2
- 4*a*c])]) + (2*e*(f*x)^(3/2)*Sqrt[a + b*x^2 + c*x^4]*AppellF1[3/4, -1/2,
-1/2, 7/4, (-2*c*x^2)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^2)/(b + Sqrt[b^2 - 4
*a*c])])/(3*f^3*Sqrt[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x
^2)/(b + Sqrt[b^2 - 4*a*c])])
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 1155

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[a^IntPart[p]*((a + b*x^2 + c*x^4)^FracPart[p]/((1 + 2*c*(x^2/(b +
Rt[b^2 - 4*a*c, 2])))^FracPart[p]*(1 + 2*c*(x^2/(b - Rt[b^2 - 4*a*c, 2])))^
FracPart[p])), Int[(d*x)^m*(1 + 2*c*(x^2/(b + Sqrt[b^2 - 4*a*c])))^p*(1 + 2
*c*(x^2/(b - Sqrt[b^2 - 4*a*c])))^p, x], x] /; FreeQ[{a, b, c, d, m, p}, x]
```

Rule 1349

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*
(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, p, q}, x] && N
eQ[b^2 - 4*a*c, 0] && (IGtQ[p, 0] || IGtQ[q, 0] || IntegersQ[m, q])
```

Rubi steps

$$\text{integral} = \int \left(\frac{d\sqrt{a+bx^2+cx^4}}{(fx)^{3/2}} + \frac{e\sqrt{fx}\sqrt{a+bx^2+cx^4}}{f^2}\right) dx$$

$$= d\int \frac{\sqrt{a+bx^2+cx^4}}{(fx)^{3/2}}\,dx + \frac{e\int \sqrt{fx}\sqrt{a+bx^2+cx^4}\,dx}{f^2}$$

$$= \frac{\left(d\sqrt{a+bx^2+cx^4}\right)\int \frac{\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}}{(fx)^{3/2}}\,dx}{\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}} + \frac{\left(e\sqrt{a+bx^2+cx^4}\right)\int \sqrt{fx}\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}\,dx}{f^2\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}}$$

$$= -\frac{2d\sqrt{a+bx^2+cx^4}F_1\left(-\frac{1}{4};-\frac{1}{2},-\frac{1}{2};\frac{3}{4};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{f\sqrt{fx}\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}} + \frac{2e(fx)^{3/2}\sqrt{a+bx^2+cx^4}F_1\left(\frac{3}{4};-\frac{1}{2},-\frac{1}{2};\frac{7}{4};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{3f^3\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}}$$

Mathematica [A] (verified)

Time = 11.65 (sec) , antiderivative size = 370, normalized size of antiderivative = 1.25

$$\int \frac{(d+ex^2)\sqrt{a+bx^2+cx^4}}{(fx)^{3/2}}\,dx = \frac{x\left(-42(7d-ex^2)(a+bx^2+cx^4)+28(7bd+2ae)x^2\sqrt{\frac{b-\sqrt{b^2-4ac}+2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{\frac{b+\cdots}{\cdots}}\right.}{\cdots}$$

```
[In] Integrate[((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4])/(f*x)^(3/2),x]
```

```
[Out] (x*(-42*(7*d - e*x^2)*(a + b*x^2 + c*x^4) + 28*(7*b*d + 2*a*e)*x^2*Sqrt[(b
- Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[(b + Sqrt[b^2
- 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[3/4, 1/2, 1/2, 7/4, (
-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])] + 12
*(14*c*d + b*e)*x^4*Sqrt[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b - Sqrt[b^2 -
4*a*c])]*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*Ap
pellF1[7/4, 1/2, 1/2, 11/4, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(
-b + Sqrt[b^2 - 4*a*c])]))/(147*(f*x)^(3/2)*Sqrt[a + b*x^2 + c*x^4])
```

Maple [F]

$$\int \frac{(e\,x^2+d)\,\sqrt{c\,x^4+b\,x^2+a}}{(fx)^{\frac{3}{2}}}dx$$

```
[In] int((e*x^2+d)*(c*x^4+b*x^2+a)^(1/2)/(f*x)^(3/2),x)
[Out] int((e*x^2+d)*(c*x^4+b*x^2+a)^(1/2)/(f*x)^(3/2),x)
```

Fricas [F]

$$\int \frac{(d+ex^2)\,\sqrt{a+bx^2+cx^4}}{(fx)^{3/2}}\,dx = \int \frac{\sqrt{cx^4+bx^2+a}(ex^2+d)}{(fx)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((e*x^2+d)*(c*x^4+b*x^2+a)^(1/2)/(f*x)^(3/2),x, algorithm="fricas"
)
[Out] integral(sqrt(c*x^4 + b*x^2 + a)*(e*x^2 + d)*sqrt(f*x)/(f^2*x^2), x)
```

Sympy [F]

$$\int \frac{(d+ex^2)\,\sqrt{a+bx^2+cx^4}}{(fx)^{3/2}}\,dx = \int \frac{(d+ex^2)\,\sqrt{a+bx^2+cx^4}}{(fx)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((e*x**2+d)*(c*x**4+b*x**2+a)**(1/2)/(f*x)**(3/2),x)
[Out] Integral((d + e*x**2)*sqrt(a + b*x**2 + c*x**4)/(f*x)**(3/2), x)
```

Maxima [F]

$$\int \frac{(d+ex^2)\,\sqrt{a+bx^2+cx^4}}{(fx)^{3/2}}\,dx = \int \frac{\sqrt{cx^4+bx^2+a}(ex^2+d)}{(fx)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((e*x^2+d)*(c*x^4+b*x^2+a)^(1/2)/(f*x)^(3/2),x, algorithm="maxima"
)
[Out] integrate(sqrt(c*x^4 + b*x^2 + a)*(e*x^2 + d)/(f*x)^(3/2), x)
```

Giac [F]

$$\int \frac{(d+ex^2)\sqrt{a+bx^2+cx^4}}{(fx)^{3/2}}\,dx = \int \frac{\sqrt{cx^4+bx^2+a}(ex^2+d)}{(fx)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((e*x^2+d)*(c*x^4+b*x^2+a)^(1/2)/(f*x)^(3/2),x, algorithm="giac")
[Out] integrate(sqrt(c*x^4 + b*x^2 + a)*(e*x^2 + d)/(f*x)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex^2)\sqrt{a+bx^2+cx^4}}{(fx)^{3/2}}\,dx = \int \frac{(e\,x^2+d)\,\sqrt{c\,x^4+b\,x^2+a}}{(f\,x)^{3/2}}\,dx$$

```
[In] int(((d + e*x^2)*(a + b*x^2 + c*x^4)^(1/2))/(f*x)^(3/2),x)
[Out] int(((d + e*x^2)*(a + b*x^2 + c*x^4)^(1/2))/(f*x)^(3/2), x)
```

3.208 $\int (fx)^{3/2} (d+ex^2) (a+bx^2+cx^4)^{3/2} \, dx$

Optimal result

Integrand size = 31, antiderivative size = 299

$$\int (fx)^{3/2} (d+ex^2) (a+bx^2+cx^4)^{3/2} \, dx = \frac{2ad(fx)^{5/2}\sqrt{a+bx^2+cx^4}\,\text{AppellF1}\left(\frac{5}{4},-\frac{3}{2},-\frac{3}{2},\frac{9}{4},-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{5f\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}} + \frac{2ae(fx)^{9/2}\sqrt{a+bx^2+cx^4}\,\text{AppellF1}\left(\frac{9}{4},-\frac{3}{2},-\frac{3}{2},\frac{13}{4},-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{9f^3\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}}$$

```
[Out] 2/5*a*d*(f*x)^(5/2)*AppellF1(5/4,-3/2,-3/2,9/4,-2*c*x^2/(b-(-4*a*c+b^2)^(1/
2)),-2*c*x^2/(b+(-4*a*c+b^2)^(1/2)))*(c*x^4+b*x^2+a)^(1/2)/f/(1+2*c*x^2/(b-
(-4*a*c+b^2)^(1/2)))^(1/2)/(1+2*c*x^2/(b+(-4*a*c+b^2)^(1/2)))^(1/2)+2/9*a*e
*(f*x)^(9/2)*AppellF1(9/4,-3/2,-3/2,13/4,-2*c*x^2/(b-(-4*a*c+b^2)^(1/2)),-2
*c*x^2/(b+(-4*a*c+b^2)^(1/2)))*(c*x^4+b*x^2+a)^(1/2)/f^3/(1+2*c*x^2/(b-(-4*
a*c+b^2)^(1/2)))^(1/2)/(1+2*c*x^2/(b+(-4*a*c+b^2)^(1/2)))^(1/2)
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 299, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.097, Rules used

$= \{1349, 1155, 524\}$

$$\int (fx)^{3/2} \left(d+ex^2\right) \left(a+bx^2+cx^4\right)^{3/2} dx = \frac{2ad(fx)^{5/2}\sqrt{a+bx^2+cx^4}\,\text{AppellF1}\left(\frac{5}{4},-\frac{3}{2},-\frac{3}{2},\frac{9}{4},-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{5f\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}} + \frac{2ae(fx)^{9/2}\sqrt{a+bx^2+cx^4}\,\text{AppellF1}\left(\frac{9}{4},-\frac{3}{2},-\frac{3}{2},\frac{13}{4},-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{9f^3\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}}$$

```
[In] Int[(f*x)^(3/2)*(d + e*x^2)*(a + b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] (2*a*d*(f*x)^(5/2)*Sqrt[a + b*x^2 + c*x^4]*AppellF1[5/4, -3/2, -3/2, 9/4, (
-2*c*x^2)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])])/(5*
f*Sqrt[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^2)/(b + Sqrt[
b^2 - 4*a*c])]) + (2*a*e*(f*x)^(9/2)*Sqrt[a + b*x^2 + c*x^4]*AppellF1[9/4,
-3/2, -3/2, 13/4, (-2*c*x^2)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^2)/(b + Sqrt[
b^2 - 4*a*c])])/(9*f^3*Sqrt[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 +
 (2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])])
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 1155

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[a^IntPart[p]*((a + b*x^2 + c*x^4)^FracPart[p]/((1 + 2*c*(x^2/(b +
Rt[b^2 - 4*a*c, 2])))^FracPart[p]*(1 + 2*c*(x^2/(b - Rt[b^2 - 4*a*c, 2])))^
FracPart[p])), Int[(d*x)^m*(1 + 2*c*(x^2/(b + Sqrt[b^2 - 4*a*c])))^p*(1 + 2
*c*(x^2/(b - Sqrt[b^2 - 4*a*c])))^p, x], x] /; FreeQ[{a, b, c, d, m, p}, x]
```

Rule 1349

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*
(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, p, q}, x] && N
eQ[b^2 - 4*a*c, 0] && (IGtQ[p, 0] || IGtQ[q, 0] || IntegersQ[m, q])
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \left(d(fx)^{3/2}\left(a+bx^2+cx^4\right)^{3/2}+\frac{e(fx)^{7/2}\left(a+bx^2+cx^4\right)^{3/2}}{f^2}\right)dx\\
&= d\int (fx)^{3/2}\left(a+bx^2+cx^4\right)^{3/2}\,dx+\frac{e\int (fx)^{7/2}\left(a+bx^2+cx^4\right)^{3/2}\,dx}{f^2}\\
&= \frac{\left(ad\sqrt{a+bx^2+cx^4}\right)\int (fx)^{3/2}\left(1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}\right)^{3/2}\left(1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)^{3/2}dx}{\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}}\\
&\quad+\frac{\left(ae\sqrt{a+bx^2+cx^4}\right)\int (fx)^{7/2}\left(1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}\right)^{3/2}\left(1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)^{3/2}dx}{f^2\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}}\\
&= \frac{2ad(fx)^{5/2}\sqrt{a+bx^2+cx^4}F_1\left(\frac{5}{4};-\frac{3}{2},-\frac{3}{2};\frac{9}{4};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{5f\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}}\\
&\quad+\frac{2ae(fx)^{9/2}\sqrt{a+bx^2+cx^4}F_1\left(\frac{9}{4};-\frac{3}{2},-\frac{3}{2};\frac{13}{4};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{9f^3\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}}\end{aligned}$$

Mathematica [A] (verified)

Time = 12.00 (sec) , antiderivative size = 567, normalized size of antiderivative = 1.90

$$\int (fx)^{3/2}\left(d+ex^2\right)\left(a+bx^2+cx^4\right)^{3/2}\,dx=\frac{2f\sqrt{fx}\Big(5(a+bx^2+cx^4)\left(308b^4e-4b^3c(147d+55ex^2)+12b^2c(-167ae+5cx^2(7d+3ex^2))\right.}{}$$

```
[In] Integrate[(f*x)^(3/2)*(d + e*x^2)*(a + b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] (2*f*Sqrt[f*x]*(5*(a + b*x^2 + c*x^4)*(308*b^4*e - 4*b^3*c*(147*d + 55*e*x^
2) + 12*b^2*c*(-167*a*e + 5*c*x^2*(7*d + 3*e*x^2)) + 3*b*c^2*(16*a*(77*d +
25*e*x^2) + 5*c*x^4*(399*d + 299*e*x^2)) + 3*c^2*(816*a^2*e + 65*c^2*x^6*(2
1*d + 17*e*x^2) + 5*a*c*x^2*(637*d + 425*e*x^2))) - 20*a*(-147*b^3*c*d + 92
4*a*b*c^2*d + 77*b^4*e - 501*a*b^2*c*e + 612*a^2*c^2*e)*Sqrt[(b - Sqrt[b^2
- 4*a*c] + 2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[(b + Sqrt[b^2 - 4*a*c] +
2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[1/4, 1/2, 1/2, 5/4, (-2*c*x^2)/(
b + Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])] - 4*(-441*b^4*c
*d + 3297*a*b^2*c^2*d - 5460*a^2*c^3*d + 231*b^5*e - 1778*a*b^3*c*e + 3336*
a^2*b*c^2*e)*x^2*Sqrt[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b - Sqrt[b^2 - 4*a
```

```
*c])]*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*Appel
lF1[5/4, 1/2, 1/2, 9/4, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(-b +
 Sqrt[b^2 - 4*a*c])]))/(348075*c^3*Sqrt[a + b*x^2 + c*x^4])
```

Maple [F]

$$\int (fx)^{\frac{3}{2}} \left(e\,x^{2}+d\right) \left(c\,x^{4}+b\,x^{2}+a\right)^{\frac{3}{2}} dx$$

```
[In] int((f*x)^(3/2)*(e*x^2+d)*(c*x^4+b*x^2+a)^(3/2),x)
[Out] int((f*x)^(3/2)*(e*x^2+d)*(c*x^4+b*x^2+a)^(3/2),x)
```

Fricas [F]

$$\int (fx)^{3/2} \left(d+ex^2\right) \left(a+bx^2+cx^4\right)^{3/2} dx = \int \left(cx^4+bx^2+a\right)^{\frac{3}{2}} \left(ex^2+d\right) (fx)^{\frac{3}{2}} dx$$

```
[In] integrate((f*x)^(3/2)*(e*x^2+d)*(c*x^4+b*x^2+a)^(3/2),x, algorithm="fricas"
)
[Out] integral((c*e*f*x^7 + (c*d + b*e)*f*x^5 + (b*d + a*e)*f*x^3 + a*d*f*x)*sqrt
(c*x^4 + b*x^2 + a)*sqrt(f*x), x)
```

Sympy [F]

$$\int (fx)^{3/2} \left(d+ex^2\right) \left(a+bx^2+cx^4\right)^{3/2} dx = \int (fx)^{\frac{3}{2}} \left(d+ex^2\right) \left(a+bx^2+cx^4\right)^{\frac{3}{2}} dx$$

```
[In] integrate((f*x)**(3/2)*(e*x**2+d)*(c*x**4+b*x**2+a)**(3/2),x)
[Out] Integral((f*x)**(3/2)*(d + e*x**2)*(a + b*x**2 + c*x**4)**(3/2), x)
```

Maxima [F]

$$\int (fx)^{3/2} \left(d+ex^2\right) \left(a+bx^2+cx^4\right)^{3/2} dx = \int \left(cx^4+bx^2+a\right)^{\frac{3}{2}} \left(ex^2+d\right) (fx)^{\frac{3}{2}} dx$$

```
[In] integrate((f*x)^(3/2)*(e*x^2+d)*(c*x^4+b*x^2+a)^(3/2),x, algorithm="maxima"
)
[Out] integrate((c*x^4 + b*x^2 + a)^(3/2)*(e*x^2 + d)*(f*x)^(3/2), x)
```

Giac [F]

$$\int (fx)^{3/2} (d+ex^2) (a+bx^2+cx^4)^{3/2} \, dx = \int (cx^4+bx^2+a)^{\frac{3}{2}} (ex^2+d)(fx)^{\frac{3}{2}} \, dx$$

```
[In] integrate((f*x)^(3/2)*(e*x^2+d)*(c*x^4+b*x^2+a)^(3/2),x, algorithm="giac")
[Out] integrate((c*x^4 + b*x^2 + a)^(3/2)*(e*x^2 + d)*(f*x)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int (fx)^{3/2} (d+ex^2) (a+bx^2+cx^4)^{3/2} \, dx = \int (f\,x)^{3/2} \left(e\,x^2+d\right) \left(c\,x^4+b\,x^2+a\right)^{3/2} dx$$

```
[In] int((f*x)^(3/2)*(d + e*x^2)*(a + b*x^2 + c*x^4)^(3/2),x)
[Out] int((f*x)^(3/2)*(d + e*x^2)*(a + b*x^2 + c*x^4)^(3/2), x)
```

3.209 $\int \sqrt{fx}(d+ex^2)\left(a+bx^2+cx^4\right)^{3/2}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 299

$$\int \sqrt{fx}(d+ex^2)\left(a+bx^2+cx^4\right)^{3/2}\,dx = \frac{2ad(fx)^{3/2}\sqrt{a+bx^2+cx^4}\,\text{AppellF1}\left(\frac{3}{4},-\frac{3}{2},-\frac{3}{2},\frac{7}{4},-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{3f\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}} + \frac{2ae(fx)^{7/2}\sqrt{a+bx^2+cx^4}\,\text{AppellF1}\left(\frac{7}{4},-\frac{3}{2},-\frac{3}{2},\frac{11}{4},-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{7f^3\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}}$$

```
[Out] 2/3*a*d*(f*x)^(3/2)*AppellF1(3/4,-3/2,-3/2,7/4,-2*c*x^2/(b-(-4*a*c+b^2)^(1/
2)),-2*c*x^2/(b+(-4*a*c+b^2)^(1/2)))*(c*x^4+b*x^2+a)^(1/2)/f/(1+2*c*x^2/(b-
(-4*a*c+b^2)^(1/2)))^(1/2)/(1+2*c*x^2/(b+(-4*a*c+b^2)^(1/2)))^(1/2)+2/7*a*e
*(f*x)^(7/2)*AppellF1(7/4,-3/2,-3/2,11/4,-2*c*x^2/(b-(-4*a*c+b^2)^(1/2)),-2
*c*x^2/(b+(-4*a*c+b^2)^(1/2)))*(c*x^4+b*x^2+a)^(1/2)/f^3/(1+2*c*x^2/(b-(-4*
a*c+b^2)^(1/2)))^(1/2)/(1+2*c*x^2/(b+(-4*a*c+b^2)^(1/2)))^(1/2)
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 299, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.097, Rules used

$= \{1349, 1155, 524\}$

$$\int \sqrt{fx}(d+ex^2)\left(a+bx^2+cx^4\right)^{3/2}\,dx = \frac{2ad(fx)^{3/2}\sqrt{a+bx^2+cx^4}\,\text{AppellF1}\left(\frac{3}{4},-\frac{3}{2},-\frac{3}{2},\frac{7}{4},-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{3f\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}} + \frac{2ae(fx)^{7/2}\sqrt{a+bx^2+cx^4}\,\text{AppellF1}\left(\frac{7}{4},-\frac{3}{2},-\frac{3}{2},\frac{11}{4},-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{7f^3\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}}$$

```
[In] Int[Sqrt[f*x]*(d + e*x^2)*(a + b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] (2*a*d*(f*x)^(3/2)*Sqrt[a + b*x^2 + c*x^4]*AppellF1[3/4, -3/2, -3/2, 7/4, (
-2*c*x^2)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])])/(3*
f*Sqrt[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^2)/(b + Sqrt[
b^2 - 4*a*c])]) + (2*a*e*(f*x)^(7/2)*Sqrt[a + b*x^2 + c*x^4]*AppellF1[7/4,
-3/2, -3/2, 11/4, (-2*c*x^2)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^2)/(b + Sqrt[
b^2 - 4*a*c])])/(7*f^3*Sqrt[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 +
 (2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])])
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 1155

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[a^IntPart[p]*((a + b*x^2 + c*x^4)^FracPart[p]/((1 + 2*c*(x^2/(b +
Rt[b^2 - 4*a*c, 2])))^FracPart[p]*(1 + 2*c*(x^2/(b - Rt[b^2 - 4*a*c, 2])))^
FracPart[p])), Int[(d*x)^m*(1 + 2*c*(x^2/(b + Sqrt[b^2 - 4*a*c])))^p*(1 + 2
*c*(x^2/(b - Sqrt[b^2 - 4*a*c])))^p, x], x] /; FreeQ[{a, b, c, d, m, p}, x]
```

Rule 1349

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*
(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, p, q}, x] && N
eQ[b^2 - 4*a*c, 0] && (IGtQ[p, 0] || IGtQ[q, 0] || IntegersQ[m, q])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(d\sqrt{fx}\left(a+bx^2+cx^4\right)^{3/2}+\frac{e(fx)^{5/2}\left(a+bx^2+cx^4\right)^{3/2}}{f^2}\right)dx\\
&= d\int \sqrt{fx}\left(a+bx^2+cx^4\right)^{3/2}\,dx+\frac{e\int (fx)^{5/2}\left(a+bx^2+cx^4\right)^{3/2}\,dx}{f^2}\\
&= \frac{\left(ad\sqrt{a+bx^2+cx^4}\right)\int \sqrt{fx}\left(1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}\right)^{3/2}\left(1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)^{3/2}dx}{\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}}\\
&\quad+\frac{\left(ae\sqrt{a+bx^2+cx^4}\right)\int (fx)^{5/2}\left(1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}\right)^{3/2}\left(1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)^{3/2}dx}{f^2\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}}\\
&= \frac{2ad(fx)^{3/2}\sqrt{a+bx^2+cx^4}F_1\left(\frac{3}{4};-\frac{3}{2},-\frac{3}{2};\frac{7}{4};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{3f\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}}\\
&\quad+\frac{2ae(fx)^{7/2}\sqrt{a+bx^2+cx^4}F_1\left(\frac{7}{4};-\frac{3}{2},-\frac{3}{2};\frac{11}{4};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{7f^3\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 11.94 (sec) , antiderivative size = 490, normalized size of antiderivative = 1.64

$$\int \sqrt{fx}\left(d+ex^2\right)\left(a+bx^2+cx^4\right)^{3/2}\,dx=\frac{2x\sqrt{fx}\Big(7\left(a+bx^2+cx^4\right)\left(-108b^3e+12b^2c(19d+7ex^2)+bc(624ae+7cx^2(323d+231ex^2))\right.}{}$$

```
[In] Integrate[Sqrt[f*x]*(d + e*x^2)*(a + b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] (2*x*Sqrt[f*x]*(7*(a + b*x^2 + c*x^4)*(-108*b^3*e + 12*b^2*c*(19*d + 7*e*x^
2) + b*c*(624*a*e + 7*c*x^2*(323*d + 231*e*x^2)) + c^2*(77*c*x^4*(19*d + 15
*e*x^2) + a*(3971*d + 2415*e*x^2))) + 28*a*(-57*b^2*c*d + 836*a*c^2*d + 27*
b^3*e - 156*a*b*c*e)*Sqrt[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b - Sqrt[b^2 -
 4*a*c])]*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*A
ppellF1[3/4, 1/2, 1/2, 7/4, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(
-b + Sqrt[b^2 - 4*a*c])] + 12*(-95*b^3*c*d + 684*a*b*c^2*d + 45*b^4*e - 309
*a*b^2*c*e + 420*a^2*c^2*e)*x^2*Sqrt[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b -
 Sqrt[b^2 - 4*a*c])]*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 -
 4*a*c])]*AppellF1[7/4, 1/2, 1/2, 11/4, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]),
 (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])]))/(153615*c^2*Sqrt[a + b*x^2 + c*x^4])
```

Maple [F]

$$\int \sqrt{fx}\,(e\,x^2+d)\,(c\,x^4+b\,x^2+a)^{\frac{3}{2}}\,dx$$

```
[In] int((f*x)^(1/2)*(e*x^2+d)*(c*x^4+b*x^2+a)^(3/2),x)
[Out] int((f*x)^(1/2)*(e*x^2+d)*(c*x^4+b*x^2+a)^(3/2),x)
```

Fricas [F]

$$\int \sqrt{fx}(d+ex^2)\left(a+bx^2+cx^4\right)^{3/2}\,dx = \int \left(cx^4+bx^2+a\right)^{\frac{3}{2}}\left(ex^2+d\right)\sqrt{fx}\,dx$$

```
[In] integrate((f*x)^(1/2)*(e*x^2+d)*(c*x^4+b*x^2+a)^(3/2),x, algorithm="fricas"
)
[Out] integral((c*e*x^6 + (c*d + b*e)*x^4 + (b*d + a*e)*x^2 + a*d)*sqrt(c*x^4 + b
*x^2 + a)*sqrt(f*x), x)
```

Sympy [F]

$$\int \sqrt{fx}(d+ex^2)\left(a+bx^2+cx^4\right)^{3/2}\,dx = \int \sqrt{fx}\left(d+ex^2\right)\left(a+bx^2+cx^4\right)^{\frac{3}{2}}\,dx$$

```
[In] integrate((f*x)**(1/2)*(e*x**2+d)*(c*x**4+b*x**2+a)**(3/2),x)
[Out] Integral(sqrt(f*x)*(d + e*x**2)*(a + b*x**2 + c*x**4)**(3/2), x)
```

Maxima [F]

$$\int \sqrt{fx}(d+ex^2)\left(a+bx^2+cx^4\right)^{3/2}\,dx = \int \left(cx^4+bx^2+a\right)^{\frac{3}{2}}\left(ex^2+d\right)\sqrt{fx}\,dx$$

```
[In] integrate((f*x)^(1/2)*(e*x^2+d)*(c*x^4+b*x^2+a)^(3/2),x, algorithm="maxima"
)
[Out] integrate((c*x^4 + b*x^2 + a)^(3/2)*(e*x^2 + d)*sqrt(f*x), x)
```

Giac [F]

$$\int \sqrt{fx}(d+ex^2)\left(a+bx^2+cx^4\right)^{3/2}\,dx = \int \left(cx^4+bx^2+a\right)^{\frac{3}{2}}\left(ex^2+d\right)\sqrt{fx}\,dx$$

```
[In] integrate((f*x)^(1/2)*(e*x^2+d)*(c*x^4+b*x^2+a)^(3/2),x, algorithm="giac")
[Out] integrate((c*x^4 + b*x^2 + a)^(3/2)*(e*x^2 + d)*sqrt(f*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{fx}(d+ex^2)\left(a+bx^2+cx^4\right)^{3/2}\,dx = \int \sqrt{f\,x}\,\left(e\,x^2+d\right)\,\left(c\,x^4+b\,x^2+a\right)^{3/2}\,dx$$

```
[In] int((f*x)^(1/2)*(d + e*x^2)*(a + b*x^2 + c*x^4)^(3/2),x)
[Out] int((f*x)^(1/2)*(d + e*x^2)*(a + b*x^2 + c*x^4)^(3/2), x)
```

3.210 $\int \frac{(d+ex^2)(a+bx^2+cx^4)^{3/2}}{\sqrt{fx}}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 297

$$\int \frac{(d+ex^2)(a+bx^2+cx^4)^{3/2}}{\sqrt{fx}}\,dx = \frac{2ad\sqrt{fx}\sqrt{a+bx^2+cx^4}\,\text{AppellF1}\left(\frac{1}{4},-\frac{3}{2},-\frac{3}{2},\frac{5}{4},-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4}}\right.}{f\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}} + \frac{2ae(fx)^{5/2}\sqrt{a+bx^2+cx^4}\,\text{AppellF1}\left(\frac{5}{4},-\frac{3}{2},-\frac{3}{2},\frac{9}{4},-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{5f^3\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}}$$

```
[Out] 2/5*a*e*(f*x)^(5/2)*AppellF1(5/4,-3/2,-3/2,9/4,-2*c*x^2/(b-(-4*a*c+b^2)^(1/
2)),-2*c*x^2/(b+(-4*a*c+b^2)^(1/2)))*(c*x^4+b*x^2+a)^(1/2)/f^3/(1+2*c*x^2/(
b-(-4*a*c+b^2)^(1/2)))^(1/2)/(1+2*c*x^2/(b+(-4*a*c+b^2)^(1/2)))^(1/2)+2*a*d
*AppellF1(1/4,-3/2,-3/2,5/4,-2*c*x^2/(b-(-4*a*c+b^2)^(1/2)),-2*c*x^2/(b+(-4
*a*c+b^2)^(1/2)))*(f*x)^(1/2)*(c*x^4+b*x^2+a)^(1/2)/f/(1+2*c*x^2/(b-(-4*a*c
+b^2)^(1/2)))^(1/2)/(1+2*c*x^2/(b+(-4*a*c+b^2)^(1/2)))^(1/2)
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 297, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.097, Rules used = {1349, 1155, 524}

$$\int \frac{(d+ex^2)(a+bx^2+cx^4)^{3/2}}{\sqrt{fx}}\,dx = \frac{2ad\sqrt{fx}\sqrt{a+bx^2+cx^4}\,\text{AppellF1}\left(\frac{1}{4},-\frac{3}{2},-\frac{3}{2},\frac{5}{4},-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4}}\right.}{f\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}} + \frac{2ae(fx)^{5/2}\sqrt{a+bx^2+cx^4}\,\text{AppellF1}\left(\frac{5}{4},-\frac{3}{2},-\frac{3}{2},\frac{9}{4},-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{5f^3\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}}$$

```
[In] Int[((d + e*x^2)*(a + b*x^2 + c*x^4)^(3/2))/Sqrt[f*x],x]
```

```
[Out] (2*a*d*Sqrt[f*x]*Sqrt[a + b*x^2 + c*x^4]*AppellF1[1/4, -3/2, -3/2, 5/4, (-2
*c*x^2)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])])/(f*Sq
rt[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^2)/(b + Sqrt[b^2
- 4*a*c])]) + (2*a*e*(f*x)^(5/2)*Sqrt[a + b*x^2 + c*x^4]*AppellF1[5/4, -3/2
, -3/2, 9/4, (-2*c*x^2)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^2)/(b + Sqrt[b^2 -
 4*a*c])])/(5*f^3*Sqrt[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c
*x^2)/(b + Sqrt[b^2 - 4*a*c])])
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 1155

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[a^IntPart[p]*((a + b*x^2 + c*x^4)^FracPart[p]/((1 + 2*c*(x^2/(b +
Rt[b^2 - 4*a*c, 2])))^FracPart[p]*(1 + 2*c*(x^2/(b - Rt[b^2 - 4*a*c, 2])))^
FracPart[p])), Int[(d*x)^m*(1 + 2*c*(x^2/(b + Sqrt[b^2 - 4*a*c])))^p*(1 + 2
*c*(x^2/(b - Sqrt[b^2 - 4*a*c])))^p, x], x] /; FreeQ[{a, b, c, d, m, p}, x]
```

Rule 1349

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*
(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, p, q}, x] && N
eQ[b^2 - 4*a*c, 0] && (IGtQ[p, 0] || IGtQ[q, 0] || IntegersQ[m, q])
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \left(\frac{d\left(a+bx^2+cx^4\right)^{3/2}}{\sqrt{fx}}+\frac{e(fx)^{3/2}\left(a+bx^2+cx^4\right)^{3/2}}{f^2}\right) dx\\
&= d\int \frac{\left(a+bx^2+cx^4\right)^{3/2}}{\sqrt{fx}}\,dx+\frac{e\int (fx)^{3/2}\left(a+bx^2+cx^4\right)^{3/2}\,dx}{f^2}\\
&= \frac{\left(ad\sqrt{a+bx^2+cx^4}\right)\int \frac{\left(1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}\right)^{3/2}\left(1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)^{3/2}}{\sqrt{fx}}\,dx}{\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}}\\
&\quad+\frac{\left(ae\sqrt{a+bx^2+cx^4}\right)\int (fx)^{3/2}\left(1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}\right)^{3/2}\left(1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)^{3/2}\,dx}{f^2\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}}
\end{aligned}
$$

$$= \frac{2ad\sqrt{fx}\sqrt{a+bx^2+cx^4}F_1\left(\frac{1}{4};-\frac{3}{2},-\frac{3}{2};\frac{5}{4};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{f\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}} + \frac{2ae(fx)^{5/2}\sqrt{a+bx^2+cx^4}F_1\left(\frac{5}{4};-\frac{3}{2},-\frac{3}{2};\frac{9}{4};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{5f^3\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}}$$

Mathematica [A] (verified)

Time = 11.91 (sec) , antiderivative size = 487, normalized size of antiderivative = 1.64

$$\int \frac{(d+ex^2)(a+bx^2+cx^4)^{3/2}}{\sqrt{fx}}\,dx = \frac{2x\Big(5(a+bx^2+cx^4)(-28b^3e+4b^2c(17d+5ex^2)+c^2(867ad+455aex}{}$$

```
[In] Integrate[((d + e*x^2)*(a + b*x^2 + c*x^4)^(3/2))/Sqrt[f*x],x]
```

```
[Out] (2*x*(5*(a + b*x^2 + c*x^4)*(-28*b^3*e + 4*b^2*c*(17*d + 5*e*x^2) + c^2*(86
7*a*d + 455*a*e*x^2 + 255*c*d*x^4 + 195*c*e*x^6) + b*c*(176*a*e + 5*c*x^2*(
85*d + 57*e*x^2))) + 20*a*(-17*b^2*c*d + 612*a*c^2*d + 7*b^3*e - 44*a*b*c*e
)*Sqrt[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[(b +
 Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[1/4, 1/2, 1
/2, 5/4, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a
*c])] + 4*(-51*b^3*c*d + 476*a*b*c^2*d + 21*b^4*e - 157*a*b^2*c*e + 260*a^2
*c^2*e)*x^2*Sqrt[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]
*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[5
/4, 1/2, 1/2, 9/4, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(-b + Sqrt
[b^2 - 4*a*c])]))/(16575*c^2*Sqrt[f*x]*Sqrt[a + b*x^2 + c*x^4])
```

Maple [F]

$$\int \frac{(e\,x^2+d)\,(c\,x^4+b\,x^2+a)^{\frac{3}{2}}}{\sqrt{fx}}dx$$

```
[In] int((e*x^2+d)*(c*x^4+b*x^2+a)^(3/2)/(f*x)^(1/2),x)
```

```
[Out] int((e*x^2+d)*(c*x^4+b*x^2+a)^(3/2)/(f*x)^(1/2),x)
```

Fricas [F]

$$\int \frac{(d+ex^2)(a+bx^2+cx^4)^{3/2}}{\sqrt{fx}}\,dx = \int \frac{(cx^4+bx^2+a)^{\frac{3}{2}}(ex^2+d)}{\sqrt{fx}}\,dx$$

```
[In] integrate((e*x^2+d)*(c*x^4+b*x^2+a)^(3/2)/(f*x)^(1/2),x, algorithm="fricas"
)
```

```
[Out] integral((c*e*x^6 + (c*d + b*e)*x^4 + (b*d + a*e)*x^2 + a*d)*sqrt(c*x^4 + b
*x^2 + a)*sqrt(f*x)/(f*x), x)
```

Sympy [F]

$$\int \frac{(d+ex^2)(a+bx^2+cx^4)^{3/2}}{\sqrt{fx}}\,dx = \int \frac{(d+ex^2)(a+bx^2+cx^4)^{\frac{3}{2}}}{\sqrt{fx}}\,dx$$

```
[In] integrate((e*x**2+d)*(c*x**4+b*x**2+a)**(3/2)/(f*x)**(1/2),x)
```

```
[Out] Integral((d + e*x**2)*(a + b*x**2 + c*x**4)**(3/2)/sqrt(f*x), x)
```

Maxima [F]

$$\int \frac{(d+ex^2)(a+bx^2+cx^4)^{3/2}}{\sqrt{fx}}\,dx = \int \frac{(cx^4+bx^2+a)^{\frac{3}{2}}(ex^2+d)}{\sqrt{fx}}\,dx$$

```
[In] integrate((e*x^2+d)*(c*x^4+b*x^2+a)^(3/2)/(f*x)^(1/2),x, algorithm="maxima"
)
```

```
[Out] integrate((c*x^4 + b*x^2 + a)^(3/2)*(e*x^2 + d)/sqrt(f*x), x)
```

Giac [F]

$$\int \frac{(d+ex^2)(a+bx^2+cx^4)^{3/2}}{\sqrt{fx}}\,dx = \int \frac{(cx^4+bx^2+a)^{\frac{3}{2}}(ex^2+d)}{\sqrt{fx}}\,dx$$

```
[In] integrate((e*x^2+d)*(c*x^4+b*x^2+a)^(3/2)/(f*x)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((c*x^4 + b*x^2 + a)^(3/2)*(e*x^2 + d)/sqrt(f*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex^2)\left(a+bx^2+cx^4\right)^{3/2}}{\sqrt{fx}}\,dx=\int \frac{\left(e\,x^2+d\right)\,\left(c\,x^4+b\,x^2+a\right)^{3/2}}{\sqrt{f\,x}}\,dx$$

```
[In] int(((d + e*x^2)*(a + b*x^2 + c*x^4)^(3/2))/(f*x)^(1/2),x)
[Out] int(((d + e*x^2)*(a + b*x^2 + c*x^4)^(3/2))/(f*x)^(1/2), x)
```

3.211 $\int \frac{(d+ex^2)(a+bx^2+cx^4)^{3/2}}{(fx)^{3/2}}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 297

$$\int \frac{(d+ex^2)(a+bx^2+cx^4)^{3/2}}{(fx)^{3/2}}\,dx = -\frac{2ad\sqrt{a+bx^2+cx^4}\,\text{AppellF1}\left(-\frac{1}{4},-\frac{3}{2},-\frac{3}{2},\frac{3}{4},-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{f\sqrt{fx}\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}} + \frac{2ae(fx)^{3/2}\sqrt{a+bx^2+cx^4}\,\text{AppellF1}\left(\frac{3}{4},-\frac{3}{2},-\frac{3}{2},\frac{7}{4},-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{3f^3\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}}$$

```
[Out] 2/3*a*e*(f*x)^(3/2)*AppellF1(3/4,-3/2,-3/2,7/4,-2*c*x^2/(b-(-4*a*c+b^2)^(1/
2)),-2*c*x^2/(b+(-4*a*c+b^2)^(1/2)))*(c*x^4+b*x^2+a)^(1/2)/f^3/(1+2*c*x^2/(
b-(-4*a*c+b^2)^(1/2)))^(1/2)/(1+2*c*x^2/(b+(-4*a*c+b^2)^(1/2)))^(1/2)-2*a*d
*AppellF1(-1/4,-3/2,-3/2,3/4,-2*c*x^2/(b-(-4*a*c+b^2)^(1/2)),-2*c*x^2/(b+(-
4*a*c+b^2)^(1/2)))*(c*x^4+b*x^2+a)^(1/2)/f/(f*x)^(1/2)/(1+2*c*x^2/(b-(-4*a*
c+b^2)^(1/2)))^(1/2)/(1+2*c*x^2/(b+(-4*a*c+b^2)^(1/2)))^(1/2)
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 297, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.097, Rules used

= {1349, 1155, 524}

$$\int \frac{(d+ex^2)(a+bx^2+cx^4)^{3/2}}{(fx)^{3/2}}\,dx = \frac{2ae(fx)^{3/2}\sqrt{a+bx^2+cx^4}\,\text{AppellF1}\left(\frac{3}{4},-\frac{3}{2},-\frac{3}{2},\frac{7}{4},-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2c}{b+\sqrt{b^2}}\right.}{3f^3\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}}$$

$$-\frac{2ad\sqrt{a+bx^2+cx^4}\,\text{AppellF1}\left(-\frac{1}{4},-\frac{3}{2},-\frac{3}{2},\frac{3}{4},-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{f\sqrt{fx}\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}}$$

```
[In] Int[((d + e*x^2)*(a + b*x^2 + c*x^4)^(3/2))/(f*x)^(3/2),x]
```

```
[Out] (-2*a*d*Sqrt[a + b*x^2 + c*x^4]*AppellF1[-1/4, -3/2, -3/2, 3/4, (-2*c*x^2)/
(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])])/(f*Sqrt[f*x]*
Sqrt[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^2)/(b + Sqrt[b^
2 - 4*a*c])]) + (2*a*e*(f*x)^(3/2)*Sqrt[a + b*x^2 + c*x^4]*AppellF1[3/4, -3
/2, -3/2, 7/4, (-2*c*x^2)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^2)/(b + Sqrt[b^2
 - 4*a*c])])/(3*f^3*Sqrt[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2
*c*x^2)/(b + Sqrt[b^2 - 4*a*c])])
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 1155

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[a^IntPart[p]*((a + b*x^2 + c*x^4)^FracPart[p]/((1 + 2*c*(x^2/(b +
Rt[b^2 - 4*a*c, 2])))^FracPart[p]*(1 + 2*c*(x^2/(b - Rt[b^2 - 4*a*c, 2])))^
FracPart[p])), Int[(d*x)^m*(1 + 2*c*(x^2/(b + Sqrt[b^2 - 4*a*c])))^p*(1 + 2
*c*(x^2/(b - Sqrt[b^2 - 4*a*c])))^p, x], x] /; FreeQ[{a, b, c, d, m, p}, x]
```

Rule 1349

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*
(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, p, q}, x] && N
eQ[b^2 - 4*a*c, 0] && (IGtQ[p, 0] || IGtQ[q, 0] || IntegersQ[m, q])
```

Rubi steps

$$\text{integral} = \int \left(\frac{d(a+bx^2+cx^4)^{3/2}}{(fx)^{3/2}} + \frac{e\sqrt{fx}(a+bx^2+cx^4)^{3/2}}{f^2}\right) dx$$

$$= d\int \frac{(a+bx^2+cx^4)^{3/2}}{(fx)^{3/2}}\,dx + \frac{e\int \sqrt{fx}(a+bx^2+cx^4)^{3/2}\,dx}{f^2}$$

$$= \frac{\left(ad\sqrt{a+bx^2+cx^4}\right)\int \frac{\left(1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}\right)^{3/2}\left(1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)^{3/2}}{(fx)^{3/2}}\,dx}{\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}} + \frac{\left(ae\sqrt{a+bx^2+cx^4}\right)\int \sqrt{fx}\left(1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}\right)^{3/2}\left(1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)^{3/2}\,dx}{f^2\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}}$$

$$= -\frac{2ad\sqrt{a+bx^2+cx^4}\operatorname{F}_1\left(-\frac{1}{4};-\frac{3}{2},-\frac{3}{2};\frac{3}{4};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{f\sqrt{fx}\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}} + \frac{2ae(fx)^{3/2}\sqrt{a+bx^2+cx^4}\operatorname{F}_1\left(\frac{3}{4};-\frac{3}{2},-\frac{3}{2};\frac{7}{4};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{3f^3\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}}$$

Mathematica [A] (verified)

Time = 11.75 (sec) , antiderivative size = 447, normalized size of antiderivative = 1.51

$$\int \frac{(d+ex^2)(a+bx^2+cx^4)^{3/2}}{(fx)^{3/2}}\,dx = \frac{x\Big(14(a+bx^2+cx^4)\left(ac(-1155d+209ex^2)+x^2(12b^2e+7c^2x^2(15d+1\right.}{}$$

```
[In] Integrate[((d + e*x^2)*(a + b*x^2 + c*x^4)^(3/2))/(f*x)^(3/2),x]
```

```
[Out] (x*(14*(a + b*x^2 + c*x^4)*(a*c*(-1155*d + 209*e*x^2) + x^2*(12*b^2*e + 7*c
^2*x^2*(15*d + 11*e*x^2) + b*c*(195*d + 119*e*x^2))) - 56*a*(-240*b*c*d + 3
*b^2*e - 44*a*c*e)*x^2*Sqrt[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b - Sqrt[b^2
 - 4*a*c])]*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]
*AppellF1[3/4, 1/2, 1/2, 7/4, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^2)
/(-b + Sqrt[b^2 - 4*a*c])] + 24*(15*b^2*c*d + 420*a*c^2*d - 5*b^3*e + 36*a*
b*c*e)*x^4*Sqrt[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*
Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[7/
4, 1/2, 1/2, 11/4, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(-b + Sqrt
[b^2 - 4*a*c])]))/(8085*c*(f*x)^(3/2)*Sqrt[a + b*x^2 + c*x^4])
```

Maple [F]

$$\int \frac{(e\,x^2+d)\,(c\,x^4+b\,x^2+a)^{\frac{3}{2}}}{(fx)^{\frac{3}{2}}}dx$$

```
[In] int((e*x^2+d)*(c*x^4+b*x^2+a)^(3/2)/(f*x)^(3/2),x)
[Out] int((e*x^2+d)*(c*x^4+b*x^2+a)^(3/2)/(f*x)^(3/2),x)
```

Fricas [F]

$$\int \frac{(d+ex^2)\,(a+bx^2+cx^4)^{3/2}}{(fx)^{3/2}}\,dx = \int \frac{(cx^4+bx^2+a)^{\frac{3}{2}}(ex^2+d)}{(fx)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((e*x^2+d)*(c*x^4+b*x^2+a)^(3/2)/(f*x)^(3/2),x, algorithm="fricas"
)
[Out] integral((c*e*x^6 + (c*d + b*e)*x^4 + (b*d + a*e)*x^2 + a*d)*sqrt(c*x^4 + b
*x^2 + a)*sqrt(f*x)/(f^2*x^2), x)
```

Sympy [F]

$$\int \frac{(d+ex^2)\,(a+bx^2+cx^4)^{3/2}}{(fx)^{3/2}}\,dx = \int \frac{(d+ex^2)\,(a+bx^2+cx^4)^{\frac{3}{2}}}{(fx)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((e*x**2+d)*(c*x**4+b*x**2+a)**(3/2)/(f*x)**(3/2),x)
[Out] Integral((d + e*x**2)*(a + b*x**2 + c*x**4)**(3/2)/(f*x)**(3/2), x)
```

Maxima [F]

$$\int \frac{(d+ex^2)\,(a+bx^2+cx^4)^{3/2}}{(fx)^{3/2}}\,dx = \int \frac{(cx^4+bx^2+a)^{\frac{3}{2}}(ex^2+d)}{(fx)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((e*x^2+d)*(c*x^4+b*x^2+a)^(3/2)/(f*x)^(3/2),x, algorithm="maxima"
)
[Out] integrate((c*x^4 + b*x^2 + a)^(3/2)*(e*x^2 + d)/(f*x)^(3/2), x)
```

Giac [F]

$$\int \frac{(d+ex^2)\,(a+bx^2+cx^4)^{3/2}}{(fx)^{3/2}}\,dx = \int \frac{(cx^4+bx^2+a)^{\frac{3}{2}}(ex^2+d)}{(fx)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((e*x^2+d)*(c*x^4+b*x^2+a)^(3/2)/(f*x)^(3/2),x, algorithm="giac")
[Out] integrate((c*x^4 + b*x^2 + a)^(3/2)*(e*x^2 + d)/(f*x)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex^2)\,(a+bx^2+cx^4)^{3/2}}{(fx)^{3/2}}\,dx = \int \frac{(e\,x^2+d)\,\left(c\,x^4+b\,x^2+a\right)^{3/2}}{(f\,x)^{3/2}}\,dx$$

```
[In] int(((d + e*x^2)*(a + b*x^2 + c*x^4)^(3/2))/(f*x)^(3/2),x)
[Out] int(((d + e*x^2)*(a + b*x^2 + c*x^4)^(3/2))/(f*x)^(3/2), x)
```

3.212 $\int \frac{(fx)^{3/2}(d+ex^2)}{\sqrt{a+bx^2+cx^4}}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 297

$$\int \frac{(fx)^{3/2}(d+ex^2)}{\sqrt{a+bx^2+cx^4}}\,dx = \frac{2d(fx)^{5/2}\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}\,\text{AppellF1}\left(\frac{5}{4},\frac{1}{2},\frac{1}{2},\frac{9}{4},-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{5f\sqrt{a+bx^2+cx^4}} + \frac{2e(fx)^{9/2}\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}\,\text{AppellF1}\left(\frac{9}{4},\frac{1}{2},\frac{1}{2},\frac{13}{4},-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{9f^3\sqrt{a+bx^2+cx^4}}$$

```
[Out] 2/5*d*(f*x)^(5/2)*AppellF1(5/4,1/2,1/2,9/4,-2*c*x^2/(b-(-4*a*c+b^2)^(1/2)),
-2*c*x^2/(b+(-4*a*c+b^2)^(1/2)))*(1+2*c*x^2/(b-(-4*a*c+b^2)^(1/2)))^(1/2)*(
1+2*c*x^2/(b+(-4*a*c+b^2)^(1/2)))^(1/2)/f/(c*x^4+b*x^2+a)^(1/2)+2/9*e*(f*x)
^(9/2)*AppellF1(9/4,1/2,1/2,13/4,-2*c*x^2/(b-(-4*a*c+b^2)^(1/2)),-2*c*x^2/(
b+(-4*a*c+b^2)^(1/2)))*(1+2*c*x^2/(b-(-4*a*c+b^2)^(1/2)))^(1/2)*(1+2*c*x^2/
(b+(-4*a*c+b^2)^(1/2)))^(1/2)/f^3/(c*x^4+b*x^2+a)^(1/2)
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 297, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.097, Rules used = {1349, 1155, 524}

$$\int \frac{(fx)^{3/2}(d+ex^2)}{\sqrt{a+bx^2+cx^4}}\,dx = \frac{2d(fx)^{5/2}\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}\,\text{AppellF1}\left(\frac{5}{4},\frac{1}{2},\frac{1}{2},\frac{9}{4},-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{5f\sqrt{a+bx^2+cx^4}} + \frac{2e(fx)^{9/2}\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}\,\text{AppellF1}\left(\frac{9}{4},\frac{1}{2},\frac{1}{2},\frac{13}{4},-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{9f^3\sqrt{a+bx^2+cx^4}}$$

```
[In] Int[((f*x)^(3/2)*(d + e*x^2))/Sqrt[a + b*x^2 + c*x^4],x]
```

```
[Out] (2*d*(f*x)^(5/2)*Sqrt[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*
x^2)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[5/4, 1/2, 1/2, 9/4, (-2*c*x^2)/(b -
Sqrt[b^2 - 4*a*c]), (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])])/(5*f*Sqrt[a + b*x^
2 + c*x^4]) + (2*e*(f*x)^(9/2)*Sqrt[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*
Sqrt[1 + (2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[9/4, 1/2, 1/2, 13/4, (
-2*c*x^2)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])])/(9*
f^3*Sqrt[a + b*x^2 + c*x^4])
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 1155

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[a^IntPart[p]*((a + b*x^2 + c*x^4)^FracPart[p]/((1 + 2*c*(x^2/(b +
Rt[b^2 - 4*a*c, 2])))^FracPart[p]*(1 + 2*c*(x^2/(b - Rt[b^2 - 4*a*c, 2])))^
FracPart[p])), Int[(d*x)^m*(1 + 2*c*(x^2/(b + Sqrt[b^2 - 4*a*c])))^p*(1 + 2
*c*(x^2/(b - Sqrt[b^2 - 4*a*c])))^p, x], x] /; FreeQ[{a, b, c, d, m, p}, x]
```

Rule 1349

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*
(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, p, q}, x] && N
eQ[b^2 - 4*a*c, 0] && (IGtQ[p, 0] || IGtQ[q, 0] || IntegersQ[m, q])
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \left(\frac{d(fx)^{3/2}}{\sqrt{a+bx^2+cx^4}}+\frac{e(fx)^{7/2}}{f^2\sqrt{a+bx^2+cx^4}}\right)dx \\
&= d\int \frac{(fx)^{3/2}}{\sqrt{a+bx^2+cx^4}}\,dx+\frac{e\int \frac{(fx)^{7/2}}{\sqrt{a+bx^2+cx^4}}\,dx}{f^2} \\
&= \frac{\left(d\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}\right)\int \frac{(fx)^{3/2}}{\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}}\,dx}{\sqrt{a+bx^2+cx^4}} \\
&\quad+\frac{\left(e\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}\right)\int \frac{(fx)^{7/2}}{\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}}\,dx}{f^2\sqrt{a+bx^2+cx^4}}
\end{aligned}
$$

$$= \frac{2d(fx)^{5/2}\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}F_1\left(\frac{5}{4};\frac{1}{2},\frac{1}{2};\frac{9}{4};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{5f\sqrt{a+bx^2+cx^4}} + \frac{2e(fx)^{9/2}\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}F_1\left(\frac{9}{4};\frac{1}{2},\frac{1}{2};\frac{13}{4};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{9f^3\sqrt{a+bx^2+cx^4}}$$

Mathematica [A] (verified)

Time = 11.50 (sec) , antiderivative size = 354, normalized size of antiderivative = 1.19

$$\int \frac{(fx)^{3/2}(d+ex^2)}{\sqrt{a+bx^2+cx^4}}\,dx = \frac{2f\sqrt{fx}\left(5e(a+bx^2+cx^4)-5ae\sqrt{\frac{b-\sqrt{b^2-4ac}+2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx^2}{b+\sqrt{b^2-4ac}}}\operatorname{AppellF1}\left(\frac{1}{4},\frac{1}{2}\right.\right.}{}$$

```
[In] Integrate[((f*x)^(3/2)*(d + e*x^2))/Sqrt[a + b*x^2 + c*x^4],x]
```

```
[Out] (2*f*Sqrt[f*x]*(5*e*(a + b*x^2 + c*x^4) - 5*a*e*Sqrt[(b - Sqrt[b^2 - 4*a*c]
 + 2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)
/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[1/4, 1/2, 1/2, 5/4, (-2*c*x^2)/(b + Sqrt
[b^2 - 4*a*c]), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])] + (5*c*d - 3*b*e)*x^2*S
qrt[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[(b + Sq
rt[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[5/4, 1/2, 1/2,
 9/4, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c]
)]))/(25*c*Sqrt[a + b*x^2 + c*x^4])
```

Maple [F]

$$\int \frac{(fx)^{\frac{3}{2}}\left(e\,x^2+d\right)}{\sqrt{c\,x^4+b\,x^2+a}}dx$$

```
[In] int((f*x)^(3/2)*(e*x^2+d)/(c*x^4+b*x^2+a)^(1/2),x)
```

```
[Out] int((f*x)^(3/2)*(e*x^2+d)/(c*x^4+b*x^2+a)^(1/2),x)
```

Fricas [F]

$$\int \frac{(fx)^{3/2}(d+ex^2)}{\sqrt{a+bx^2+cx^4}}\,dx = \int \frac{(ex^2+d)(fx)^{\frac{3}{2}}}{\sqrt{cx^4+bx^2+a}}\,dx$$

```
[In] integrate((f*x)^(3/2)*(e*x^2+d)/(c*x^4+b*x^2+a)^(1/2),x, algorithm="fricas"
)
```

```
[Out] integral((e*f*x^3 + d*f*x)*sqrt(f*x)/sqrt(c*x^4 + b*x^2 + a), x)
```

Sympy [F]

$$\int \frac{(fx)^{3/2}(d+ex^2)}{\sqrt{a+bx^2+cx^4}}\,dx = \int \frac{(fx)^{\frac{3}{2}}(d+ex^2)}{\sqrt{a+bx^2+cx^4}}\,dx$$

```
[In] integrate((f*x)**(3/2)*(e*x**2+d)/(c*x**4+b*x**2+a)**(1/2),x)
[Out] Integral((f*x)**(3/2)*(d + e*x**2)/sqrt(a + b*x**2 + c*x**4), x)
```

Maxima [F]

$$\int \frac{(fx)^{3/2}(d+ex^2)}{\sqrt{a+bx^2+cx^4}}\,dx = \int \frac{(ex^2+d)(fx)^{\frac{3}{2}}}{\sqrt{cx^4+bx^2+a}}\,dx$$

```
[In] integrate((f*x)^(3/2)*(e*x^2+d)/(c*x^4+b*x^2+a)^(1/2),x, algorithm="maxima"
)
[Out] integrate((e*x^2 + d)*(f*x)^(3/2)/sqrt(c*x^4 + b*x^2 + a), x)
```

Giac [F]

$$\int \frac{(fx)^{3/2}(d+ex^2)}{\sqrt{a+bx^2+cx^4}}\,dx = \int \frac{(ex^2+d)(fx)^{\frac{3}{2}}}{\sqrt{cx^4+bx^2+a}}\,dx$$

```
[In] integrate((f*x)^(3/2)*(e*x^2+d)/(c*x^4+b*x^2+a)^(1/2),x, algorithm="giac")
[Out] integrate((e*x^2 + d)*(f*x)^(3/2)/sqrt(c*x^4 + b*x^2 + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(fx)^{3/2}(d+ex^2)}{\sqrt{a+bx^2+cx^4}}\,dx = \int \frac{(f\,x)^{3/2}\,(e\,x^2+d)}{\sqrt{c\,x^4+b\,x^2+a}}\,dx$$

```
[In] int(((f*x)^(3/2)*(d + e*x^2))/(a + b*x^2 + c*x^4)^(1/2),x)
[Out] int(((f*x)^(3/2)*(d + e*x^2))/(a + b*x^2 + c*x^4)^(1/2), x)
```

3.213 $\int \frac{\sqrt{fx}(d+ex^2)}{\sqrt{a+bx^2+cx^4}}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 297

$$\int \frac{\sqrt{fx}(d+ex^2)}{\sqrt{a+bx^2+cx^4}}\,dx = \frac{2d(fx)^{3/2}\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}\operatorname{AppellF1}\left(\frac{3}{4},\frac{1}{2},\frac{1}{2},\frac{7}{4},-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{3f\sqrt{a+bx^2+cx^4}} + \frac{2e(fx)^{7/2}\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}\operatorname{AppellF1}\left(\frac{7}{4},\frac{1}{2},\frac{1}{2},\frac{11}{4},-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{7f^3\sqrt{a+bx^2+cx^4}}$$

```
[Out] 2/3*d*(f*x)^(3/2)*AppellF1(3/4,1/2,1/2,7/4,-2*c*x^2/(b-(-4*a*c+b^2)^(1/2)),
-2*c*x^2/(b+(-4*a*c+b^2)^(1/2)))*(1+2*c*x^2/(b-(-4*a*c+b^2)^(1/2)))^(1/2)*(
1+2*c*x^2/(b+(-4*a*c+b^2)^(1/2)))^(1/2)/f/(c*x^4+b*x^2+a)^(1/2)+2/7*e*(f*x)
^(7/2)*AppellF1(7/4,1/2,1/2,11/4,-2*c*x^2/(b-(-4*a*c+b^2)^(1/2)),-2*c*x^2/(
b+(-4*a*c+b^2)^(1/2)))*(1+2*c*x^2/(b-(-4*a*c+b^2)^(1/2)))^(1/2)*(1+2*c*x^2/
(b+(-4*a*c+b^2)^(1/2)))^(1/2)/f^3/(c*x^4+b*x^2+a)^(1/2)
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 297, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.097, Rules used

$= \{1349, 1155, 524\}$

$$\int \frac{\sqrt{fx}(d+ex^2)}{\sqrt{a+bx^2+cx^4}}\,dx$$
$$= \frac{2d(fx)^{3/2}\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}\,\text{AppellF1}\left(\frac{3}{4},\frac{1}{2},\frac{1}{2},\frac{7}{4},-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{3f\sqrt{a+bx^2+cx^4}}$$
$$+\frac{2e(fx)^{7/2}\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}\,\text{AppellF1}\left(\frac{7}{4},\frac{1}{2},\frac{1}{2},\frac{11}{4},-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{7f^3\sqrt{a+bx^2+cx^4}}$$

```
[In] Int[(Sqrt[f*x]*(d + e*x^2))/Sqrt[a + b*x^2 + c*x^4],x]
```

```
[Out] (2*d*(f*x)^(3/2)*Sqrt[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*
x^2)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[3/4, 1/2, 1/2, 7/4, (-2*c*x^2)/(b -
Sqrt[b^2 - 4*a*c]), (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])])/(3*f*Sqrt[a + b*x^
2 + c*x^4]) + (2*e*(f*x)^(7/2)*Sqrt[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*
Sqrt[1 + (2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[7/4, 1/2, 1/2, 11/4, (
-2*c*x^2)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])])/(7*
f^3*Sqrt[a + b*x^2 + c*x^4])
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 1155

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[a^IntPart[p]*((a + b*x^2 + c*x^4)^FracPart[p]/((1 + 2*c*(x^2/(b +
Rt[b^2 - 4*a*c, 2])))^FracPart[p]*(1 + 2*c*(x^2/(b - Rt[b^2 - 4*a*c, 2])))^
FracPart[p])), Int[(d*x)^m*(1 + 2*c*(x^2/(b + Sqrt[b^2 - 4*a*c])))^p*(1 + 2
*c*(x^2/(b - Sqrt[b^2 - 4*a*c])))^p, x], x] /; FreeQ[{a, b, c, d, m, p}, x]
```

Rule 1349

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*
(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, p, q}, x] && N
eQ[b^2 - 4*a*c, 0] && (IGtQ[p, 0] || IGtQ[q, 0] || IntegersQ[m, q])
```

Rubi steps

$$\text{integral} = \int \left(\frac{d\sqrt{fx}}{\sqrt{a+bx^2+cx^4}} + \frac{e(fx)^{5/2}}{f^2\sqrt{a+bx^2+cx^4}}\right) dx$$

$$= d\int \frac{\sqrt{fx}}{\sqrt{a+bx^2+cx^4}}\,dx + \frac{e\int \frac{(fx)^{5/2}}{\sqrt{a+bx^2+cx^4}}\,dx}{f^2}$$

$$= \frac{\left(d\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}\right)\int \frac{\sqrt{fx}}{\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}}\,dx}{\sqrt{a+bx^2+cx^4}} + \frac{\left(e\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}\right)\int \frac{(fx)^{5/2}}{\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}}\,dx}{f^2\sqrt{a+bx^2+cx^4}}$$

$$= \frac{2d(fx)^{3/2}\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}F_1\left(\frac{3}{4};\frac{1}{2},\frac{1}{2};\frac{7}{4};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{3f\sqrt{a+bx^2+cx^4}} + \frac{2e(fx)^{7/2}\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}F_1\left(\frac{7}{4};\frac{1}{2},\frac{1}{2};\frac{11}{4};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{7f^3\sqrt{a+bx^2+cx^4}}$$

Mathematica [A] (verified)

Time = 11.23 (sec) , antiderivative size = 242, normalized size of antiderivative = 0.81

$$\int \frac{\sqrt{fx}(d+ex^2)}{\sqrt{a+bx^2+cx^4}}\,dx = \frac{2\sqrt{fx}\sqrt{\frac{b-\sqrt{b^2-4ac}+2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx^2}{b+\sqrt{b^2-4ac}}}\left(7dx\,\text{AppellF1}\left(\frac{3}{4},\frac{1}{2},\frac{1}{2},\frac{7}{4},-\frac{2cx^2}{b+\sqrt{b^2-4ac}},\frac{2cx^2}{-b+\sqrt{b^2-4ac}}\right)+3ex^3\,\text{AppellF1}\right.}{21\sqrt{a+bx^2+cx^4}}$$

```
[In] Integrate[(Sqrt[f*x]*(d + e*x^2))/Sqrt[a + b*x^2 + c*x^4],x]
```

```
[Out] (2*Sqrt[f*x]*Sqrt[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])
]*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*(7*d*x*Ap
pellF1[3/4, 1/2, 1/2, 7/4, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(-
b + Sqrt[b^2 - 4*a*c])] + 3*e*x^3*AppellF1[7/4, 1/2, 1/2, 11/4, (-2*c*x^2)/
(b + Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])]))/(21*Sqrt[a +
 b*x^2 + c*x^4])
```

Maple [F]

$$\int \frac{\sqrt{f x}\,(e\,x^2+d)}{\sqrt{c\,x^4+b\,x^2+a}}dx$$

```
[In] int((f*x)^(1/2)*(e*x^2+d)/(c*x^4+b*x^2+a)^(1/2),x)
[Out] int((f*x)^(1/2)*(e*x^2+d)/(c*x^4+b*x^2+a)^(1/2),x)
```

Fricas [F]

$$\int \frac{\sqrt{fx}(d+ex^2)}{\sqrt{a+bx^2+cx^4}}\,dx = \int \frac{(ex^2+d)\sqrt{fx}}{\sqrt{cx^4+bx^2+a}}\,dx$$

```
[In] integrate((f*x)^(1/2)*(e*x^2+d)/(c*x^4+b*x^2+a)^(1/2),x, algorithm="fricas"
)
[Out] integral((e*x^2 + d)*sqrt(f*x)/sqrt(c*x^4 + b*x^2 + a), x)
```

Sympy [F]

$$\int \frac{\sqrt{fx}(d+ex^2)}{\sqrt{a+bx^2+cx^4}}\,dx = \int \frac{\sqrt{fx}\,(d+ex^2)}{\sqrt{a+bx^2+cx^4}}\,dx$$

```
[In] integrate((f*x)**(1/2)*(e*x**2+d)/(c*x**4+b*x**2+a)**(1/2),x)
[Out] Integral(sqrt(f*x)*(d + e*x**2)/sqrt(a + b*x**2 + c*x**4), x)
```

Maxima [F]

$$\int \frac{\sqrt{fx}(d+ex^2)}{\sqrt{a+bx^2+cx^4}}\,dx = \int \frac{(ex^2+d)\sqrt{fx}}{\sqrt{cx^4+bx^2+a}}\,dx$$

```
[In] integrate((f*x)^(1/2)*(e*x^2+d)/(c*x^4+b*x^2+a)^(1/2),x, algorithm="maxima"
)
[Out] integrate((e*x^2 + d)*sqrt(f*x)/sqrt(c*x^4 + b*x^2 + a), x)
```

Giac [F]

$$\int \frac{\sqrt{fx}(d+ex^2)}{\sqrt{a+bx^2+cx^4}}\,dx = \int \frac{(ex^2+d)\sqrt{fx}}{\sqrt{cx^4+bx^2+a}}\,dx$$

```
[In] integrate((f*x)^(1/2)*(e*x^2+d)/(c*x^4+b*x^2+a)^(1/2),x, algorithm="giac")
[Out] integrate((e*x^2 + d)*sqrt(f*x)/sqrt(c*x^4 + b*x^2 + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{fx}(d+ex^2)}{\sqrt{a+bx^2+cx^4}}\,dx = \int \frac{\sqrt{f\,x}\,(e\,x^2+d)}{\sqrt{c\,x^4+b\,x^2+a}}\,dx$$

```
[In] int(((f*x)^(1/2)*(d + e*x^2))/(a + b*x^2 + c*x^4)^(1/2),x)
[Out] int(((f*x)^(1/2)*(d + e*x^2))/(a + b*x^2 + c*x^4)^(1/2), x)
```

3.214 $\int \frac{d+ex^2}{\sqrt{fx}\sqrt{a+bx^2+cx^4}}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 295

$$\int \frac{d+ex^2}{\sqrt{fx}\sqrt{a+bx^2+cx^4}}\,dx = \frac{2d\sqrt{fx}\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}\,\text{AppellF1}\left(\frac{1}{4},\frac{1}{2},\frac{1}{2},\frac{5}{4},-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{f\sqrt{a+bx^2+cx^4}} + \frac{2e(fx)^{5/2}\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}\,\text{AppellF1}\left(\frac{5}{4},\frac{1}{2},\frac{1}{2},\frac{9}{4},-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{5f^3\sqrt{a+bx^2+cx^4}}$$

```
[Out] 2/5*e*(f*x)^(5/2)*AppellF1(5/4,1/2,1/2,9/4,-2*c*x^2/(b-(-4*a*c+b^2)^(1/2)),
-2*c*x^2/(b+(-4*a*c+b^2)^(1/2)))*(1+2*c*x^2/(b-(-4*a*c+b^2)^(1/2)))^(1/2)*(
1+2*c*x^2/(b+(-4*a*c+b^2)^(1/2)))^(1/2)/f^3/(c*x^4+b*x^2+a)^(1/2)+2*d*Appel
lF1(1/4,1/2,1/2,5/4,-2*c*x^2/(b-(-4*a*c+b^2)^(1/2)),-2*c*x^2/(b+(-4*a*c+b^2
)^(1/2)))*(f*x)^(1/2)*(1+2*c*x^2/(b-(-4*a*c+b^2)^(1/2)))^(1/2)*(1+2*c*x^2/(
b+(-4*a*c+b^2)^(1/2)))^(1/2)/f/(c*x^4+b*x^2+a)^(1/2)
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 295, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.097$, Rules used

$= \{1349, 1155, 524\}$

$$\int \frac{d+ex^2}{\sqrt{fx}\sqrt{a+bx^2+cx^4}}\,dx$$
$$= \frac{2d\sqrt{fx}\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}\,\text{AppellF1}\left(\frac{1}{4},\frac{1}{2},\frac{1}{2},\frac{5}{4},-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{f\sqrt{a+bx^2+cx^4}} + \frac{2e(fx)^{5/2}\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}\,\text{AppellF1}\left(\frac{5}{4},\frac{1}{2},\frac{1}{2},\frac{9}{4},-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{5f^3\sqrt{a+bx^2+cx^4}}$$

```
[In] Int[(d + e*x^2)/(Sqrt[f*x]*Sqrt[a + b*x^2 + c*x^4]),x]
```

```
[Out] (2*d*Sqrt[f*x]*Sqrt[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^
2)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[1/4, 1/2, 1/2, 5/4, (-2*c*x^2)/(b - Sq
rt[b^2 - 4*a*c]), (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])])/(f*Sqrt[a + b*x^2 +
c*x^4]) + (2*e*(f*x)^(5/2)*Sqrt[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt
[1 + (2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[5/4, 1/2, 1/2, 9/4, (-2*c*
x^2)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])])/(5*f^3*S
qrt[a + b*x^2 + c*x^4])
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 1155

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[a^IntPart[p]*((a + b*x^2 + c*x^4)^FracPart[p]/((1 + 2*c*(x^2/(b +
Rt[b^2 - 4*a*c, 2])))^FracPart[p]*(1 + 2*c*(x^2/(b - Rt[b^2 - 4*a*c, 2])))^
FracPart[p])), Int[(d*x)^m*(1 + 2*c*(x^2/(b + Sqrt[b^2 - 4*a*c])))^p*(1 + 2
*c*(x^2/(b - Sqrt[b^2 - 4*a*c])))^p, x], x] /; FreeQ[{a, b, c, d, m, p}, x]
```

Rule 1349

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*
(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, p, q}, x] && N
eQ[b^2 - 4*a*c, 0] && (IGtQ[p, 0] || IGtQ[q, 0] || IntegersQ[m, q])
```

Rubi steps

$$\text{integral} = \int \left(\frac{d}{\sqrt{fx}\sqrt{a+bx^2+cx^4}} + \frac{e(fx)^{3/2}}{f^2\sqrt{a+bx^2+cx^4}}\right) dx$$

$$= d\int \frac{1}{\sqrt{fx}\sqrt{a+bx^2+cx^4}}\, dx + \frac{e\int \frac{(fx)^{3/2}}{\sqrt{a+bx^2+cx^4}}\, dx}{f^2}$$

$$= \frac{\left(d\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}\right)\int \frac{1}{\sqrt{fx}\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}}\, dx}{\sqrt{a+bx^2+cx^4}} + \frac{\left(e\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}\right)\int \frac{(fx)^{3/2}}{\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}}\, dx}{f^2\sqrt{a+bx^2+cx^4}}$$

$$= \frac{2d\sqrt{fx}\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}F_1\left(\frac{1}{4};\frac{1}{2},\frac{1}{2};\frac{5}{4};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{f\sqrt{a+bx^2+cx^4}} + \frac{2e(fx)^{5/2}\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}F_1\left(\frac{5}{4};\frac{1}{2},\frac{1}{2};\frac{9}{4};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{5f^3\sqrt{a+bx^2+cx^4}}$$

Mathematica [A] (verified)

Time = 11.22 (sec) , antiderivative size = 241, normalized size of antiderivative = 0.82

$$\int \frac{d+ex^2}{\sqrt{fx}\sqrt{a+bx^2+cx^4}}\, dx = \frac{2\sqrt{\frac{b-\sqrt{b^2-4ac}+2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx^2}{b+\sqrt{b^2-4ac}}}\left(5dx\,\text{AppellF1}\left(\frac{1}{4},\frac{1}{2},\frac{1}{2},\frac{5}{4},-\frac{2cx^2}{b+\sqrt{b^2-4ac}},\frac{2cx^2}{-b+\sqrt{b^2-4ac}}\right)+ex^3\,\text{AppellF1}\left(\frac{5}{4},\frac{1}{2}\right.\right.}{5\sqrt{fx}\sqrt{a+bx^2+cx^4}}$$

```
[In] Integrate[(d + e*x^2)/(Sqrt[f*x]*Sqrt[a + b*x^2 + c*x^4]),x]
```

```
[Out] (2*Sqrt[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[(b
+ Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*(5*d*x*AppellF1[1/4
, 1/2, 1/2, 5/4, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(-b + Sqrt[b
^2 - 4*a*c])] + e*x^3*AppellF1[5/4, 1/2, 1/2, 9/4, (-2*c*x^2)/(b + Sqrt[b^2
 - 4*a*c]), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])]))/(5*Sqrt[f*x]*Sqrt[a + b*x
^2 + c*x^4])
```

Maple [F]

$$\int \frac{e\,x^2+d}{\sqrt{fx}\,\sqrt{c\,x^4+b\,x^2+a}}dx$$

```
[In] int((e*x^2+d)/(f*x)^(1/2)/(c*x^4+b*x^2+a)^(1/2),x)
[Out] int((e*x^2+d)/(f*x)^(1/2)/(c*x^4+b*x^2+a)^(1/2),x)
```

Fricas [F]

$$\int \frac{d+ex^2}{\sqrt{fx}\sqrt{a+bx^2+cx^4}}\,dx = \int \frac{ex^2+d}{\sqrt{cx^4+bx^2+a}\sqrt{fx}}\,dx$$

```
[In] integrate((e*x^2+d)/(f*x)^(1/2)/(c*x^4+b*x^2+a)^(1/2),x, algorithm="fricas"
)
[Out] integral(sqrt(c*x^4 + b*x^2 + a)*(e*x^2 + d)*sqrt(f*x)/(c*f*x^5 + b*f*x^3 +
 a*f*x), x)
```

Sympy [F]

$$\int \frac{d+ex^2}{\sqrt{fx}\sqrt{a+bx^2+cx^4}}\,dx = \int \frac{d+ex^2}{\sqrt{fx}\sqrt{a+bx^2+cx^4}}\,dx$$

```
[In] integrate((e*x**2+d)/(f*x)**(1/2)/(c*x**4+b*x**2+a)**(1/2),x)
[Out] Integral((d + e*x**2)/(sqrt(f*x)*sqrt(a + b*x**2 + c*x**4)), x)
```

Maxima [F]

$$\int \frac{d+ex^2}{\sqrt{fx}\sqrt{a+bx^2+cx^4}}\,dx = \int \frac{ex^2+d}{\sqrt{cx^4+bx^2+a}\sqrt{fx}}\,dx$$

```
[In] integrate((e*x^2+d)/(f*x)^(1/2)/(c*x^4+b*x^2+a)^(1/2),x, algorithm="maxima"
)
[Out] integrate((e*x^2 + d)/(sqrt(c*x^4 + b*x^2 + a)*sqrt(f*x)), x)
```

Giac [F]

$$\int \frac{d+ex^2}{\sqrt{fx}\sqrt{a+bx^2+cx^4}}\,dx=\int \frac{ex^2+d}{\sqrt{cx^4+bx^2+a}\sqrt{fx}}\,dx$$

```
[In] integrate((e*x^2+d)/(f*x)^(1/2)/(c*x^4+b*x^2+a)^(1/2),x, algorithm="giac")
[Out] integrate((e*x^2 + d)/(sqrt(c*x^4 + b*x^2 + a)*sqrt(f*x)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{d+ex^2}{\sqrt{fx}\sqrt{a+bx^2+cx^4}}\,dx=\int \frac{e\,x^2+d}{\sqrt{f\,x}\,\sqrt{c\,x^4+b\,x^2+a}}\,dx$$

```
[In] int((d + e*x^2)/((f*x)^(1/2)*(a + b*x^2 + c*x^4)^(1/2)),x)
[Out] int((d + e*x^2)/((f*x)^(1/2)*(a + b*x^2 + c*x^4)^(1/2)), x)
```

3.215 $\int \frac{d+ex^2}{(fx)^{3/2}\sqrt{a+bx^2+cx^4}}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 295

$$\int \frac{d+ex^2}{(fx)^{3/2}\sqrt{a+bx^2+cx^4}}\,dx = -\frac{2d\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}\,\text{AppellF1}\left(-\frac{1}{4},\frac{1}{2},\frac{1}{2},\frac{3}{4},-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{f\sqrt{fx}\sqrt{a+bx^2+cx^4}} + \frac{2e(fx)^{3/2}\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}\,\text{AppellF1}\left(\frac{3}{4},\frac{1}{2},\frac{1}{2},\frac{7}{4},-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{3f^3\sqrt{a+bx^2+cx^4}}$$

```
[Out] 2/3*e*(f*x)^(3/2)*AppellF1(3/4,1/2,1/2,7/4,-2*c*x^2/(b-(-4*a*c+b^2)^(1/2)),
-2*c*x^2/(b+(-4*a*c+b^2)^(1/2)))*(1+2*c*x^2/(b-(-4*a*c+b^2)^(1/2)))^(1/2)*(
1+2*c*x^2/(b+(-4*a*c+b^2)^(1/2)))^(1/2)/f^3/(c*x^4+b*x^2+a)^(1/2)-2*d*Appel
lF1(-1/4,1/2,1/2,3/4,-2*c*x^2/(b-(-4*a*c+b^2)^(1/2)),-2*c*x^2/(b+(-4*a*c+b^
2)^(1/2)))*(1+2*c*x^2/(b-(-4*a*c+b^2)^(1/2)))^(1/2)*(1+2*c*x^2/(b+(-4*a*c+b
^2)^(1/2)))^(1/2)/f/(f*x)^(1/2)/(c*x^4+b*x^2+a)^(1/2)
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 295, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.097, Rules used = {1349, 1155, 524}

$$\int \frac{d+ex^2}{(fx)^{3/2}\sqrt{a+bx^2+cx^4}}\,dx = \frac{2e(fx)^{3/2}\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}\,\text{AppellF1}\left(\frac{3}{4},\frac{1}{2},\frac{1}{2},\frac{7}{4},-\frac{2cx^2}{b-\sqrt{b^2-4ac}},\right.}{3f^3\sqrt{a+bx^2+cx^4}} - \frac{2d\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}\,\text{AppellF1}\left(-\frac{1}{4},\frac{1}{2},\frac{1}{2},\frac{3}{4},-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{f\sqrt{fx}\sqrt{a+bx^2+cx^4}}$$

```
[In] Int[(d + e*x^2)/((f*x)^(3/2)*Sqrt[a + b*x^2 + c*x^4]),x]
```

```
[Out] (-2*d*Sqrt[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^2)/(b + S
qrt[b^2 - 4*a*c])]*AppellF1[-1/4, 1/2, 1/2, 3/4, (-2*c*x^2)/(b - Sqrt[b^2 -
 4*a*c]), (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])])/(f*Sqrt[f*x]*Sqrt[a + b*x^2
+ c*x^4]) + (2*e*(f*x)^(3/2)*Sqrt[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Sq
rt[1 + (2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[3/4, 1/2, 1/2, 7/4, (-2*
c*x^2)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])])/(3*f^3
*Sqrt[a + b*x^2 + c*x^4])
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 1155

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[a^IntPart[p]*((a + b*x^2 + c*x^4)^FracPart[p]/((1 + 2*c*(x^2/(b +
Rt[b^2 - 4*a*c, 2])))^FracPart[p]*(1 + 2*c*(x^2/(b - Rt[b^2 - 4*a*c, 2])))^
FracPart[p])), Int[(d*x)^m*(1 + 2*c*(x^2/(b + Sqrt[b^2 - 4*a*c])))^p*(1 + 2
*c*(x^2/(b - Sqrt[b^2 - 4*a*c])))^p, x], x] /; FreeQ[{a, b, c, d, m, p}, x]
```

Rule 1349

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*
(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, p, q}, x] && N
eQ[b^2 - 4*a*c, 0] && (IGtQ[p, 0] || IGtQ[q, 0] || IntegersQ[m, q])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(\frac{d}{(fx)^{3/2}\sqrt{a+bx^2+cx^4}}+\frac{e\sqrt{fx}}{f^2\sqrt{a+bx^2+cx^4}}\right) dx \\
&= d\int \frac{1}{(fx)^{3/2}\sqrt{a+bx^2+cx^4}}\,dx+\frac{e\int \frac{\sqrt{fx}}{\sqrt{a+bx^2+cx^4}}\,dx}{f^2} \\
&= \frac{\left(d\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}\right)\int \frac{1}{(fx)^{3/2}\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}}\,dx}{\sqrt{a+bx^2+cx^4}} \\
&\quad+\frac{\left(e\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}\right)\int \frac{\sqrt{fx}}{\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}}\,dx}{f^2\sqrt{a+bx^2+cx^4}}
\end{aligned}$$

$$= -\frac{2d\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}F_1\left(-\frac{1}{4};\frac{1}{2},\frac{1}{2};\frac{3}{4};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{f\sqrt{fx}\sqrt{a+bx^2+cx^4}} + \frac{2e(fx)^{3/2}\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}F_1\left(\frac{3}{4};\frac{1}{2},\frac{1}{2};\frac{7}{4};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{3f^3\sqrt{a+bx^2+cx^4}}$$

Mathematica [A] (verified)

Time = 11.54 (sec) , antiderivative size = 356, normalized size of antiderivative = 1.21

$$\int \frac{d+ex^2}{(fx)^{3/2}\sqrt{a+bx^2+cx^4}}\,dx = \frac{2x\left(-21d(a+bx^2+cx^4)+7(bd+ae)x^2\sqrt{\frac{b-\sqrt{b^2-4ac}+2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx^2}{b+\sqrt{b^2-4ac}}}\,\mathrm{A}$$

```
[In] Integrate[(d + e*x^2)/((f*x)^(3/2)*Sqrt[a + b*x^2 + c*x^4]),x]
```

```
[Out] (2*x*(-21*d*(a + b*x^2 + c*x^4) + 7*(b*d + a*e)*x^2*Sqrt[(b - Sqrt[b^2 - 4*
a*c] + 2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*
x^2)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[3/4, 1/2, 1/2, 7/4, (-2*c*x^2)/(b +
Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])] + 9*c*d*x^4*Sqrt[(b
 - Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[(b + Sqrt[b^2
 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[7/4, 1/2, 1/2, 11/4,
 (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])]))/
(21*a*(f*x)^(3/2)*Sqrt[a + b*x^2 + c*x^4])
```

Maple [F]

$$\int \frac{e\,x^2+d}{(fx)^{\frac{3}{2}}\sqrt{c\,x^4+b\,x^2+a}}dx$$

```
[In] int((e*x^2+d)/(f*x)^(3/2)/(c*x^4+b*x^2+a)^(1/2),x)
```

```
[Out] int((e*x^2+d)/(f*x)^(3/2)/(c*x^4+b*x^2+a)^(1/2),x)
```

Fricas [F]

$$\int \frac{d+ex^2}{(fx)^{3/2}\sqrt{a+bx^2+cx^4}}\,dx = \int \frac{ex^2+d}{\sqrt{cx^4+bx^2+a}\,(fx)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((e*x^2+d)/(f*x)^(3/2)/(c*x^4+b*x^2+a)^(1/2),x, algorithm="fricas"
)
```

```
[Out] integral(sqrt(c*x^4 + b*x^2 + a)*(e*x^2 + d)*sqrt(f*x)/(c*f^2*x^6 + b*f^2*x
^4 + a*f^2*x^2), x)
```

Sympy [F]

$$\int \frac{d+ex^2}{(fx)^{3/2}\sqrt{a+bx^2+cx^4}}\,dx = \int \frac{d+ex^2}{(fx)^{\frac{3}{2}}\sqrt{a+bx^2+cx^4}}\,dx$$

```
[In] integrate((e*x**2+d)/(f*x)**(3/2)/(c*x**4+b*x**2+a)**(1/2),x)
[Out] Integral((d + e*x**2)/((f*x)**(3/2)*sqrt(a + b*x**2 + c*x**4)), x)
```

Maxima [F]

$$\int \frac{d+ex^2}{(fx)^{3/2}\sqrt{a+bx^2+cx^4}}\,dx = \int \frac{ex^2+d}{\sqrt{cx^4+bx^2+a}\,(fx)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((e*x^2+d)/(f*x)^(3/2)/(c*x^4+b*x^2+a)^(1/2),x, algorithm="maxima"
)
[Out] integrate((e*x^2 + d)/(sqrt(c*x^4 + b*x^2 + a)*(f*x)^(3/2)), x)
```

Giac [F]

$$\int \frac{d+ex^2}{(fx)^{3/2}\sqrt{a+bx^2+cx^4}}\,dx = \int \frac{ex^2+d}{\sqrt{cx^4+bx^2+a}\,(fx)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((e*x^2+d)/(f*x)^(3/2)/(c*x^4+b*x^2+a)^(1/2),x, algorithm="giac")
[Out] integrate((e*x^2 + d)/(sqrt(c*x^4 + b*x^2 + a)*(f*x)^(3/2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{d+ex^2}{(fx)^{3/2}\sqrt{a+bx^2+cx^4}}\,dx = \int \frac{e\,x^2+d}{(f\,x)^{3/2}\,\sqrt{c\,x^4+b\,x^2+a}}\,dx$$

```
[In] int((d + e*x^2)/((f*x)^(3/2)*(a + b*x^2 + c*x^4)^(1/2)),x)
[Out] int((d + e*x^2)/((f*x)^(3/2)*(a + b*x^2 + c*x^4)^(1/2)), x)
```

3.216 $\int \frac{(fx)^{3/2}(d+ex^2)}{(a+bx^2+cx^4)^{3/2}}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 303

$$\int \frac{(fx)^{3/2}\,(d+ex^2)}{(a+bx^2+cx^4)^{3/2}}\,dx = \frac{2d(fx)^{5/2}\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}\,\text{AppellF1}\left(\frac{5}{4},\frac{3}{2},\frac{3}{2},\frac{9}{4},-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{5af\sqrt{a+bx^2+cx^4}} + \frac{2e(fx)^{9/2}\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}\,\text{AppellF1}\left(\frac{9}{4},\frac{3}{2},\frac{3}{2},\frac{13}{4},-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{9af^3\sqrt{a+bx^2+cx^4}}$$

```
[Out] 2/5*d*(f*x)^(5/2)*AppellF1(5/4,3/2,3/2,9/4,-2*c*x^2/(b-(-4*a*c+b^2)^(1/2)),
-2*c*x^2/(b+(-4*a*c+b^2)^(1/2)))*(1+2*c*x^2/(b-(-4*a*c+b^2)^(1/2)))^(1/2)*(
1+2*c*x^2/(b+(-4*a*c+b^2)^(1/2)))^(1/2)/a/f/(c*x^4+b*x^2+a)^(1/2)+2/9*e*(f*
x)^(9/2)*AppellF1(9/4,3/2,3/2,13/4,-2*c*x^2/(b-(-4*a*c+b^2)^(1/2)),-2*c*x^2
/(b+(-4*a*c+b^2)^(1/2)))*(1+2*c*x^2/(b-(-4*a*c+b^2)^(1/2)))^(1/2)*(1+2*c*x^
2/(b+(-4*a*c+b^2)^(1/2)))^(1/2)/a/f^3/(c*x^4+b*x^2+a)^(1/2)
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 303, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.097, Rules used = {1349, 1155, 524}

$$\int \frac{(fx)^{3/2}\,(d+ex^2)}{(a+bx^2+cx^4)^{3/2}}\,dx = \frac{2d(fx)^{5/2}\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}\,\text{AppellF1}\left(\frac{5}{4},\frac{3}{2},\frac{3}{2},\frac{9}{4},-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{5af\sqrt{a+bx^2+cx^4}} + \frac{2e(fx)^{9/2}\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}\,\text{AppellF1}\left(\frac{9}{4},\frac{3}{2},\frac{3}{2},\frac{13}{4},-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{9af^3\sqrt{a+bx^2+cx^4}}$$

```
[In] Int[((f*x)^(3/2)*(d + e*x^2))/(a + b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] (2*d*(f*x)^(5/2)*Sqrt[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*
x^2)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[5/4, 3/2, 3/2, 9/4, (-2*c*x^2)/(b -
Sqrt[b^2 - 4*a*c]), (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])])/(5*a*f*Sqrt[a + b*
x^2 + c*x^4]) + (2*e*(f*x)^(9/2)*Sqrt[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])
]*Sqrt[1 + (2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[9/4, 3/2, 3/2, 13/4,
 (-2*c*x^2)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])])/(
9*a*f^3*Sqrt[a + b*x^2 + c*x^4])
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 1155

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[a^IntPart[p]*((a + b*x^2 + c*x^4)^FracPart[p]/((1 + 2*c*(x^2/(b +
Rt[b^2 - 4*a*c, 2])))^FracPart[p]*(1 + 2*c*(x^2/(b - Rt[b^2 - 4*a*c, 2])))^
FracPart[p])), Int[(d*x)^m*(1 + 2*c*(x^2/(b + Sqrt[b^2 - 4*a*c])))^p*(1 + 2
*c*(x^2/(b - Sqrt[b^2 - 4*a*c])))^p, x], x] /; FreeQ[{a, b, c, d, m, p}, x]
```

Rule 1349

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*
(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, p, q}, x] && N
eQ[b^2 - 4*a*c, 0] && (IGtQ[p, 0] || IGtQ[q, 0] || IntegersQ[m, q])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(\frac{d(fx)^{3/2}}{(a+bx^2+cx^4)^{3/2}}+\frac{e(fx)^{7/2}}{f^2\,(a+bx^2+cx^4)^{3/2}}\right)dx\\
&= d\int \frac{(fx)^{3/2}}{(a+bx^2+cx^4)^{3/2}}\,dx+\frac{e\int \frac{(fx)^{7/2}}{(a+bx^2+cx^4)^{3/2}}\,dx}{f^2}\\
&= \frac{\left(d\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}\right)\int \frac{(fx)^{3/2}}{\left(1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}\right)^{3/2}\left(1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)^{3/2}}\,dx}{a\sqrt{a+bx^2+cx^4}}\\
&\quad+\frac{\left(e\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}\right)\int \frac{(fx)^{7/2}}{\left(1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}\right)^{3/2}\left(1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)^{3/2}}\,dx}{af^2\sqrt{a+bx^2+cx^4}}
\end{aligned}$$

$$= \frac{2d(fx)^{5/2}\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}F_1\left(\frac{5}{4};\frac{3}{2},\frac{3}{2};\frac{9}{4};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{5af\sqrt{a+bx^2+cx^4}} + \frac{2e(fx)^{9/2}\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}F_1\left(\frac{9}{4};\frac{3}{2},\frac{3}{2};\frac{13}{4};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{9af^3\sqrt{a+bx^2+cx^4}}$$

Mathematica [A] (verified)

Time = 11.60 (sec) , antiderivative size = 375, normalized size of antiderivative = 1.24

$$\int \frac{(fx)^{3/2}(d+ex^2)}{(a+bx^2+cx^4)^{3/2}}\,dx =$$

$$-\frac{f\sqrt{fx}\left(5(bd-2ae+2cdx^2-bex^2)-5(bd-2ae)\sqrt{\frac{b-\sqrt{b^2-4ac}+2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx^2}{b+\sqrt{b^2-4ac}}}\,\text{AppellF1}\left(\frac{1}{4},\frac{1}{2},\frac{1}{2},\frac{5}{4},-\right.\right.}{5\,(b^2-}$$

```
[In] Integrate[((f*x)^(3/2)*(d + e*x^2))/(a + b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] -1/5*(f*Sqrt[f*x]*(5*(b*d - 2*a*e + 2*c*d*x^2 - b*e*x^2) - 5*(b*d - 2*a*e)*
Sqrt[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[(b + S
qrt[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[1/4, 1/2, 1/2
, 5/4, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c
])] + (-2*c*d + b*e)*x^2*Sqrt[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b - Sqrt[b
^2 - 4*a*c])]*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]
)]*AppellF1[5/4, 1/2, 1/2, 9/4, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^
2)/(-b + Sqrt[b^2 - 4*a*c])]))/((b^2 - 4*a*c)*Sqrt[a + b*x^2 + c*x^4])
```

Maple [F]

$$\int \frac{(fx)^{\frac{3}{2}}\left(e\,x^2+d\right)}{\left(c\,x^4+b\,x^2+a\right)^{\frac{3}{2}}}dx$$

```
[In] int((f*x)^(3/2)*(e*x^2+d)/(c*x^4+b*x^2+a)^(3/2),x)
```

```
[Out] int((f*x)^(3/2)*(e*x^2+d)/(c*x^4+b*x^2+a)^(3/2),x)
```

Fricas [F]

$$\int \frac{(fx)^{3/2}(d+ex^2)}{(a+bx^2+cx^4)^{3/2}}\,dx=\int \frac{(ex^2+d)(fx)^{\frac{3}{2}}}{(cx^4+bx^2+a)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((f*x)^(3/2)*(e*x^2+d)/(c*x^4+b*x^2+a)^(3/2),x, algorithm="fricas"
)
[Out] integral((e*f*x^3 + d*f*x)*sqrt(c*x^4 + b*x^2 + a)*sqrt(f*x)/(c^2*x^8 + 2*b
*c*x^6 + (b^2 + 2*a*c)*x^4 + 2*a*b*x^2 + a^2), x)
```

Sympy [F]

$$\int \frac{(fx)^{3/2}(d+ex^2)}{(a+bx^2+cx^4)^{3/2}}\,dx=\int \frac{(fx)^{\frac{3}{2}}(d+ex^2)}{(a+bx^2+cx^4)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((f*x)**(3/2)*(e*x**2+d)/(c*x**4+b*x**2+a)**(3/2),x)
[Out] Integral((f*x)**(3/2)*(d + e*x**2)/(a + b*x**2 + c*x**4)**(3/2), x)
```

Maxima [F]

$$\int \frac{(fx)^{3/2}(d+ex^2)}{(a+bx^2+cx^4)^{3/2}}\,dx=\int \frac{(ex^2+d)(fx)^{\frac{3}{2}}}{(cx^4+bx^2+a)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((f*x)^(3/2)*(e*x^2+d)/(c*x^4+b*x^2+a)^(3/2),x, algorithm="maxima"
)
[Out] integrate((e*x^2 + d)*(f*x)^(3/2)/(c*x^4 + b*x^2 + a)^(3/2), x)
```

Giac [F]

$$\int \frac{(fx)^{3/2}(d+ex^2)}{(a+bx^2+cx^4)^{3/2}}\,dx=\int \frac{(ex^2+d)(fx)^{\frac{3}{2}}}{(cx^4+bx^2+a)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((f*x)^(3/2)*(e*x^2+d)/(c*x^4+b*x^2+a)^(3/2),x, algorithm="giac")
[Out] integrate((e*x^2 + d)*(f*x)^(3/2)/(c*x^4 + b*x^2 + a)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(fx)^{3/2}\,(d+ex^2)}{(a+bx^2+cx^4)^{3/2}}\,dx=\int \frac{{\left(f\,x\right)}^{3/2}\,\left(e\,x^2+d\right)}{{\left(c\,x^4+b\,x^2+a\right)}^{3/2}}\,dx$$

```
[In] int(((f*x)^(3/2)*(d + e*x^2))/(a + b*x^2 + c*x^4)^(3/2),x)
[Out] int(((f*x)^(3/2)*(d + e*x^2))/(a + b*x^2 + c*x^4)^(3/2), x)
```

3.217 $\int \frac{\sqrt{fx}\left(d+ex^2\right)}{\left(a+bx^2+cx^4\right)^{3/2}}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 303

$$\int \frac{\sqrt{fx}(d+ex^2)}{(a+bx^2+cx^4)^{3/2}}\,dx = \frac{2d(fx)^{3/2}\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}\,\text{AppellF1}\left(\frac{3}{4},\frac{3}{2},\frac{3}{2},\frac{7}{4},-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{3af\sqrt{a+bx^2+cx^4}} + \frac{2e(fx)^{7/2}\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}\,\text{AppellF1}\left(\frac{7}{4},\frac{3}{2},\frac{3}{2},\frac{11}{4},-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{7af^3\sqrt{a+bx^2+cx^4}}$$

```
[Out] 2/3*d*(f*x)^(3/2)*AppellF1(3/4,3/2,3/2,7/4,-2*c*x^2/(b-(-4*a*c+b^2)^(1/2)),
-2*c*x^2/(b+(-4*a*c+b^2)^(1/2)))*(1+2*c*x^2/(b-(-4*a*c+b^2)^(1/2)))^(1/2)*(
1+2*c*x^2/(b+(-4*a*c+b^2)^(1/2)))^(1/2)/a/f/(c*x^4+b*x^2+a)^(1/2)+2/7*e*(f*
x)^(7/2)*AppellF1(7/4,3/2,3/2,11/4,-2*c*x^2/(b-(-4*a*c+b^2)^(1/2)),-2*c*x^2
/(b+(-4*a*c+b^2)^(1/2)))*(1+2*c*x^2/(b-(-4*a*c+b^2)^(1/2)))^(1/2)*(1+2*c*x^
2/(b+(-4*a*c+b^2)^(1/2)))^(1/2)/a/f^3/(c*x^4+b*x^2+a)^(1/2)
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 303, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.097, Rules used = {1349, 1155, 524}

$$\int \frac{\sqrt{fx}(d+ex^2)}{(a+bx^2+cx^4)^{3/2}}\,dx = \frac{2d(fx)^{3/2}\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}\,\text{AppellF1}\left(\frac{3}{4},\frac{3}{2},\frac{3}{2},\frac{7}{4},-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{3af\sqrt{a+bx^2+cx^4}} + \frac{2e(fx)^{7/2}\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}\,\text{AppellF1}\left(\frac{7}{4},\frac{3}{2},\frac{3}{2},\frac{11}{4},-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{7af^3\sqrt{a+bx^2+cx^4}}$$

```
[In] Int[(Sqrt[f*x]*(d + e*x^2))/(a + b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] (2*d*(f*x)^(3/2)*Sqrt[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*
x^2)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[3/4, 3/2, 3/2, 7/4, (-2*c*x^2)/(b -
Sqrt[b^2 - 4*a*c]), (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])])/(3*a*f*Sqrt[a + b*
x^2 + c*x^4]) + (2*e*(f*x)^(7/2)*Sqrt[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])
]*Sqrt[1 + (2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[7/4, 3/2, 3/2, 11/4,
 (-2*c*x^2)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])])/(
7*a*f^3*Sqrt[a + b*x^2 + c*x^4])
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 1155

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[a^IntPart[p]*((a + b*x^2 + c*x^4)^FracPart[p]/((1 + 2*c*(x^2/(b +
Rt[b^2 - 4*a*c, 2])))^FracPart[p]*(1 + 2*c*(x^2/(b - Rt[b^2 - 4*a*c, 2])))^
FracPart[p])), Int[(d*x)^m*(1 + 2*c*(x^2/(b + Sqrt[b^2 - 4*a*c])))^p*(1 + 2
*c*(x^2/(b - Sqrt[b^2 - 4*a*c])))^p, x], x] /; FreeQ[{a, b, c, d, m, p}, x]
```

Rule 1349

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*
(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, p, q}, x] && N
eQ[b^2 - 4*a*c, 0] && (IGtQ[p, 0] || IGtQ[q, 0] || IntegersQ[m, q])
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \left(\frac{d\sqrt{fx}}{(a+bx^2+cx^4)^{3/2}}+\frac{e(fx)^{5/2}}{f^2(a+bx^2+cx^4)^{3/2}}\right)dx\\
&= d\int \frac{\sqrt{fx}}{(a+bx^2+cx^4)^{3/2}}\,dx+\frac{e\int \frac{(fx)^{5/2}}{(a+bx^2+cx^4)^{3/2}}\,dx}{f^2}\\
&= \frac{\left(d\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}\right)\int \frac{\sqrt{fx}}{\left(1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}\right)^{3/2}\left(1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)^{3/2}}\,dx}{a\sqrt{a+bx^2+cx^4}}\\
&\quad+\frac{\left(e\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}\right)\int \frac{(fx)^{5/2}}{\left(1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}\right)^{3/2}\left(1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)^{3/2}}\,dx}{af^2\sqrt{a+bx^2+cx^4}}
\end{aligned}
$$

$$= \frac{2d(fx)^{3/2}\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}F_1\left(\frac{3}{4};\frac{3}{2},\frac{3}{2};\frac{7}{4};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{3af\sqrt{a+bx^2+cx^4}} + \frac{2e(fx)^{7/2}\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}F_1\left(\frac{7}{4};\frac{3}{2},\frac{3}{2};\frac{11}{4};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{7af^3\sqrt{a+bx^2+cx^4}}$$

Mathematica [A] (verified)

Time = 11.60 (sec) , antiderivative size = 397, normalized size of antiderivative = 1.31

$$\int \frac{\sqrt{fx}(d+ex^2)}{(a+bx^2+cx^4)^{3/2}}\,dx = \frac{x\sqrt{fx}\left(-21b^2d+21b(ae-cdx^2)+42ac(d+ex^2)+7(b^2d+2acd-3abe)\sqrt{\frac{b-\sqrt{b^2}}{b-}}\right.}{}$$

```
[In] Integrate[(Sqrt[f*x]*(d + e*x^2))/(a + b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] (x*Sqrt[f*x]*(-21*b^2*d + 21*b*(a*e - c*d*x^2) + 42*a*c*(d + e*x^2) + 7*(b^
2*d + 2*a*c*d - 3*a*b*e)*Sqrt[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b - Sqrt[b
^2 - 4*a*c])]*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]
)]*AppellF1[3/4, 1/2, 1/2, 7/4, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^
2)/(-b + Sqrt[b^2 - 4*a*c])] + 9*c*(b*d - 2*a*e)*x^2*Sqrt[(b - Sqrt[b^2 - 4
*a*c] + 2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c
*x^2)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[7/4, 1/2, 1/2, 11/4, (-2*c*x^2)/(b
+ Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])]))/(21*a*(-b^2 + 4
*a*c)*Sqrt[a + b*x^2 + c*x^4])
```

Maple [F]

$$\int \frac{\sqrt{fx}\,(e\,x^2+d)}{(c\,x^4+b\,x^2+a)^{\frac{3}{2}}}dx$$

```
[In] int((f*x)^(1/2)*(e*x^2+d)/(c*x^4+b*x^2+a)^(3/2),x)
```

```
[Out] int((f*x)^(1/2)*(e*x^2+d)/(c*x^4+b*x^2+a)^(3/2),x)
```

Fricas [F]

$$\int \frac{\sqrt{fx}(d+ex^2)}{(a+bx^2+cx^4)^{3/2}}\,dx = \int \frac{(ex^2+d)\sqrt{fx}}{(cx^4+bx^2+a)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((f*x)^(1/2)*(e*x^2+d)/(c*x^4+b*x^2+a)^(3/2),x, algorithm="fricas"
)
```

```
[Out] integral(sqrt(c*x^4 + b*x^2 + a)*(e*x^2 + d)*sqrt(f*x)/(c^2*x^8 + 2*b*c*x^6
 + (b^2 + 2*a*c)*x^4 + 2*a*b*x^2 + a^2), x)
```

Sympy [F]

$$\int \frac{\sqrt{fx}(d+ex^2)}{(a+bx^2+cx^4)^{3/2}}\,dx = \int \frac{\sqrt{fx}(d+ex^2)}{(a+bx^2+cx^4)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((f*x)**(1/2)*(e*x**2+d)/(c*x**4+b*x**2+a)**(3/2),x)
[Out] Integral(sqrt(f*x)*(d + e*x**2)/(a + b*x**2 + c*x**4)**(3/2), x)
```

Maxima [F]

$$\int \frac{\sqrt{fx}(d+ex^2)}{(a+bx^2+cx^4)^{3/2}}\,dx = \int \frac{(ex^2+d)\sqrt{fx}}{(cx^4+bx^2+a)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((f*x)^(1/2)*(e*x^2+d)/(c*x^4+b*x^2+a)^(3/2),x, algorithm="maxima"
)
[Out] integrate((e*x^2 + d)*sqrt(f*x)/(c*x^4 + b*x^2 + a)^(3/2), x)
```

Giac [F]

$$\int \frac{\sqrt{fx}(d+ex^2)}{(a+bx^2+cx^4)^{3/2}}\,dx = \int \frac{(ex^2+d)\sqrt{fx}}{(cx^4+bx^2+a)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((f*x)^(1/2)*(e*x^2+d)/(c*x^4+b*x^2+a)^(3/2),x, algorithm="giac")
[Out] integrate((e*x^2 + d)*sqrt(f*x)/(c*x^4 + b*x^2 + a)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{fx}(d+ex^2)}{(a+bx^2+cx^4)^{3/2}}\,dx = \int \frac{\sqrt{f\,x}\,(e\,x^2+d)}{(c\,x^4+b\,x^2+a)^{3/2}}\,dx$$

```
[In] int(((f*x)^(1/2)*(d + e*x^2))/(a + b*x^2 + c*x^4)^(3/2),x)
[Out] int(((f*x)^(1/2)*(d + e*x^2))/(a + b*x^2 + c*x^4)^(3/2), x)
```

3.218 $\int \frac{d+ex^2}{\sqrt{fx}\left(a+bx^2+cx^4\right)^{3/2}}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 301

$$\int \frac{d+ex^2}{\sqrt{fx}\left(a+bx^2+cx^4\right)^{3/2}}\,dx = \frac{2d\sqrt{fx}\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}\operatorname{AppellF1}\left(\frac{1}{4},\frac{3}{2},\frac{3}{2},\frac{5}{4},-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{}{b+}\right.}{af\sqrt{a+bx^2+cx^4}}$$
$$+\frac{2e(fx)^{5/2}\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}\operatorname{AppellF1}\left(\frac{5}{4},\frac{3}{2},\frac{3}{2},\frac{9}{4},-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{5af^3\sqrt{a+bx^2+cx^4}}$$

```
[Out] 2/5*e*(f*x)^(5/2)*AppellF1(5/4,3/2,3/2,9/4,-2*c*x^2/(b-(-4*a*c+b^2)^(1/2)),
-2*c*x^2/(b+(-4*a*c+b^2)^(1/2)))*(1+2*c*x^2/(b-(-4*a*c+b^2)^(1/2)))^(1/2)*(
1+2*c*x^2/(b+(-4*a*c+b^2)^(1/2)))^(1/2)/a/f^3/(c*x^4+b*x^2+a)^(1/2)+2*d*App
ellF1(1/4,3/2,3/2,5/4,-2*c*x^2/(b-(-4*a*c+b^2)^(1/2)),-2*c*x^2/(b+(-4*a*c+b
^2)^(1/2)))*(f*x)^(1/2)*(1+2*c*x^2/(b-(-4*a*c+b^2)^(1/2)))^(1/2)*(1+2*c*x^2
/(b+(-4*a*c+b^2)^(1/2)))^(1/2)/a/f/(c*x^4+b*x^2+a)^(1/2)
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 301, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.097, Rules used = {1349, 1155, 524}

$$\int \frac{d+ex^2}{\sqrt{fx}\left(a+bx^2+cx^4\right)^{3/2}}\,dx = \frac{2d\sqrt{fx}\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}\operatorname{AppellF1}\left(\frac{1}{4},\frac{3}{2},\frac{3}{2},\frac{5}{4},-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{}{b+}\right.}{af\sqrt{a+bx^2+cx^4}}$$
$$+\frac{2e(fx)^{5/2}\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}\operatorname{AppellF1}\left(\frac{5}{4},\frac{3}{2},\frac{3}{2},\frac{9}{4},-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{5af^3\sqrt{a+bx^2+cx^4}}$$

```
[In] Int[(d + e*x^2)/(Sqrt[f*x]*(a + b*x^2 + c*x^4)^(3/2)),x]
```

```
[Out] (2*d*Sqrt[f*x]*Sqrt[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^
2)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[1/4, 3/2, 3/2, 5/4, (-2*c*x^2)/(b - Sq
rt[b^2 - 4*a*c]), (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])])/(a*f*Sqrt[a + b*x^2
+ c*x^4]) + (2*e*(f*x)^(5/2)*Sqrt[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Sq
rt[1 + (2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[5/4, 3/2, 3/2, 9/4, (-2*
c*x^2)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])])/(5*a*f
^3*Sqrt[a + b*x^2 + c*x^4])
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 1155

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[a^IntPart[p]*((a + b*x^2 + c*x^4)^FracPart[p]/((1 + 2*c*(x^2/(b +
Rt[b^2 - 4*a*c, 2])))^FracPart[p]*(1 + 2*c*(x^2/(b - Rt[b^2 - 4*a*c, 2])))^
FracPart[p])), Int[(d*x)^m*(1 + 2*c*(x^2/(b + Sqrt[b^2 - 4*a*c])))^p*(1 + 2
*c*(x^2/(b - Sqrt[b^2 - 4*a*c])))^p, x], x] /; FreeQ[{a, b, c, d, m, p}, x]
```

Rule 1349

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*
(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, p, q}, x] && N
eQ[b^2 - 4*a*c, 0] && (IGtQ[p, 0] || IGtQ[q, 0] || IntegersQ[m, q])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(\frac{d}{\sqrt{fx}\,(a+bx^2+cx^4)^{3/2}}+\frac{e(fx)^{3/2}}{f^2\,(a+bx^2+cx^4)^{3/2}}\right)dx\\
&= d\int \frac{1}{\sqrt{fx}\,(a+bx^2+cx^4)^{3/2}}\,dx+\frac{e\int \frac{(fx)^{3/2}}{(a+bx^2+cx^4)^{3/2}}\,dx}{f^2}\\
&= \frac{\left(d\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}\right)\int \frac{1}{\sqrt{fx}\left(1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}\right)^{3/2}\left(1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)^{3/2}}\,dx}{a\sqrt{a+bx^2+cx^4}}\\
&\quad+\frac{\left(e\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}\right)\int \frac{(fx)^{3/2}}{\left(1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}\right)^{3/2}\left(1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)^{3/2}}\,dx}{af^2\sqrt{a+bx^2+cx^4}}
\end{aligned}$$

$$= \frac{2d\sqrt{fx}\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}F_1\left(\frac{1}{4};\frac{3}{2},\frac{3}{2};\frac{5}{4};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{af\sqrt{a+bx^2+cx^4}} + \frac{2e(fx)^{5/2}\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}F_1\left(\frac{5}{4};\frac{3}{2},\frac{3}{2};\frac{9}{4};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{5af^3\sqrt{a+bx^2+cx^4}}$$

Mathematica [A] (verified)

Time = 11.57 (sec) , antiderivative size = 395, normalized size of antiderivative = 1.31

$$\int \frac{d+ex^2}{\sqrt{fx}\,(a+bx^2+cx^4)^{3/2}}\,dx = \frac{x\left(-5b^2d+5b(ae-cdx^2)+10ac(d+ex^2)-5(b^2d-6acd+abe)\sqrt{\frac{b-\sqrt{b^2-4ac}}{b-\sqrt{b^2}}}\right.}{}$$

```
[In] Integrate[(d + e*x^2)/(Sqrt[f*x]*(a + b*x^2 + c*x^4)^(3/2)),x]
```

```
[Out] (x*(-5*b^2*d + 5*b*(a*e - c*d*x^2) + 10*a*c*(d + e*x^2) - 5*(b^2*d - 6*a*c*
d + a*b*e)*Sqrt[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*
Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[1/
4, 1/2, 1/2, 5/4, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(-b + Sqrt[
b^2 - 4*a*c])] + c*(b*d - 2*a*e)*x^2*Sqrt[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^2)
/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[
b^2 - 4*a*c])]*AppellF1[5/4, 1/2, 1/2, 9/4, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*
c]), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])]))/(5*a*(-b^2 + 4*a*c)*Sqrt[f*x]*Sq
rt[a + b*x^2 + c*x^4])
```

Maple [F]

$$\int \frac{e\,x^2+d}{\sqrt{fx}\,\left(c\,x^4+b\,x^2+a\right)^{\frac{3}{2}}}dx$$

```
[In] int((e*x^2+d)/(f*x)^(1/2)/(c*x^4+b*x^2+a)^(3/2),x)
```

```
[Out] int((e*x^2+d)/(f*x)^(1/2)/(c*x^4+b*x^2+a)^(3/2),x)
```

Fricas [F]

$$\int \frac{d+ex^2}{\sqrt{fx}\,(a+bx^2+cx^4)^{3/2}}\,dx = \int \frac{ex^2+d}{(cx^4+bx^2+a)^{\frac{3}{2}}\sqrt{fx}}\,dx$$

```
[In] integrate((e*x^2+d)/(f*x)^(1/2)/(c*x^4+b*x^2+a)^(3/2),x, algorithm="fricas"
)
```

```
[Out] integral(sqrt(c*x^4 + b*x^2 + a)*(e*x^2 + d)*sqrt(f*x)/(c^2*f*x^9 + 2*b*c*f
*x^7 + (b^2 + 2*a*c)*f*x^5 + 2*a*b*f*x^3 + a^2*f*x), x)
```

Sympy [F]

$$\int \frac{d+ex^2}{\sqrt{fx}\,(a+bx^2+cx^4)^{3/2}}\,dx = \int \frac{d+ex^2}{\sqrt{fx}\,(a+bx^2+cx^4)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((e*x**2+d)/(f*x)**(1/2)/(c*x**4+b*x**2+a)**(3/2),x)
[Out] Integral((d + e*x**2)/(sqrt(f*x)*(a + b*x**2 + c*x**4)**(3/2)), x)
```

Maxima [F]

$$\int \frac{d+ex^2}{\sqrt{fx}\,(a+bx^2+cx^4)^{3/2}}\,dx = \int \frac{ex^2+d}{(cx^4+bx^2+a)^{\frac{3}{2}}\sqrt{fx}}\,dx$$

```
[In] integrate((e*x^2+d)/(f*x)^(1/2)/(c*x^4+b*x^2+a)^(3/2),x, algorithm="maxima"
)
[Out] integrate((e*x^2 + d)/((c*x^4 + b*x^2 + a)^(3/2)*sqrt(f*x)), x)
```

Giac [F]

$$\int \frac{d+ex^2}{\sqrt{fx}\,(a+bx^2+cx^4)^{3/2}}\,dx = \int \frac{ex^2+d}{(cx^4+bx^2+a)^{\frac{3}{2}}\sqrt{fx}}\,dx$$

```
[In] integrate((e*x^2+d)/(f*x)^(1/2)/(c*x^4+b*x^2+a)^(3/2),x, algorithm="giac")
[Out] integrate((e*x^2 + d)/((c*x^4 + b*x^2 + a)^(3/2)*sqrt(f*x)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{d+ex^2}{\sqrt{fx}\,(a+bx^2+cx^4)^{3/2}}\,dx = \int \frac{e\,x^2+d}{\sqrt{f\,x}\,(c\,x^4+b\,x^2+a)^{3/2}}\,dx$$

```
[In] int((d + e*x^2)/((f*x)^(1/2)*(a + b*x^2 + c*x^4)^(3/2)),x)
[Out] int((d + e*x^2)/((f*x)^(1/2)*(a + b*x^2 + c*x^4)^(3/2)), x)
```

3.219 $\int \frac{d+ex^2}{(fx)^{3/2}(a+bx^2+cx^4)^{3/2}}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 301

$$\int \frac{d+ex^2}{(fx)^{3/2}\left(a+bx^2+cx^4\right)^{3/2}}\,dx = -\frac{2d\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}\,\text{AppellF1}\left(-\frac{1}{4},\frac{3}{2},\frac{3}{2},\frac{3}{4},-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{af\sqrt{fx}\sqrt{a+bx^2+cx^4}} + \frac{2e(fx)^{3/2}\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}\,\text{AppellF1}\left(\frac{3}{4},\frac{3}{2},\frac{3}{2},\frac{7}{4},-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{3af^3\sqrt{a+bx^2+cx^4}}$$

```
[Out] 2/3*e*(f*x)^(3/2)*AppellF1(3/4,3/2,3/2,7/4,-2*c*x^2/(b-(-4*a*c+b^2)^(1/2)),
-2*c*x^2/(b+(-4*a*c+b^2)^(1/2)))*(1+2*c*x^2/(b-(-4*a*c+b^2)^(1/2)))^(1/2)*(
1+2*c*x^2/(b+(-4*a*c+b^2)^(1/2)))^(1/2)/a/f^3/(c*x^4+b*x^2+a)^(1/2)-2*d*App
ellF1(-1/4,3/2,3/2,3/4,-2*c*x^2/(b-(-4*a*c+b^2)^(1/2)),-2*c*x^2/(b+(-4*a*c+
b^2)^(1/2)))*(1+2*c*x^2/(b-(-4*a*c+b^2)^(1/2)))^(1/2)*(1+2*c*x^2/(b+(-4*a*c
+b^2)^(1/2)))^(1/2)/a/f/(f*x)^(1/2)/(c*x^4+b*x^2+a)^(1/2)
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 301, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.097, Rules used = {1349, 1155, 524}

$$\int \frac{d+ex^2}{(fx)^{3/2}\left(a+bx^2+cx^4\right)^{3/2}}\,dx = \frac{2e(fx)^{3/2}\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}\,\text{AppellF1}\left(\frac{3}{4},\frac{3}{2},\frac{3}{2},\frac{7}{4},-\frac{2cx^2}{b-\sqrt{b^2-4ac}}\right.}{3af^3\sqrt{a+bx^2+cx^4}} - \frac{2d\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}\,\text{AppellF1}\left(-\frac{1}{4},\frac{3}{2},\frac{3}{2},\frac{3}{4},-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{af\sqrt{fx}\sqrt{a+bx^2+cx^4}}$$

```
[In] Int[(d + e*x^2)/((f*x)^(3/2)*(a + b*x^2 + c*x^4)^(3/2)),x]
```

```
[Out] (-2*d*Sqrt[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^2)/(b + S
qrt[b^2 - 4*a*c])]*AppellF1[-1/4, 3/2, 3/2, 3/4, (-2*c*x^2)/(b - Sqrt[b^2 -
 4*a*c]), (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])])/(a*f*Sqrt[f*x]*Sqrt[a + b*x^
2 + c*x^4]) + (2*e*(f*x)^(3/2)*Sqrt[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*
Sqrt[1 + (2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[3/4, 3/2, 3/2, 7/4, (-
2*c*x^2)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])])/(3*a
*f^3*Sqrt[a + b*x^2 + c*x^4])
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 1155

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[a^IntPart[p]*((a + b*x^2 + c*x^4)^FracPart[p]/((1 + 2*c*(x^2/(b +
Rt[b^2 - 4*a*c, 2])))^FracPart[p]*(1 + 2*c*(x^2/(b - Rt[b^2 - 4*a*c, 2])))^
FracPart[p])), Int[(d*x)^m*(1 + 2*c*(x^2/(b + Sqrt[b^2 - 4*a*c])))^p*(1 + 2
*c*(x^2/(b - Sqrt[b^2 - 4*a*c])))^p, x], x] /; FreeQ[{a, b, c, d, m, p}, x]
```

Rule 1349

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*
(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, p, q}, x] && N
eQ[b^2 - 4*a*c, 0] && (IGtQ[p, 0] || IGtQ[q, 0] || IntegersQ[m, q])
```

Rubi steps

$$\text{integral} = \int \left(\frac{d}{(fx)^{3/2} \left(a+bx^2+cx^4\right)^{3/2}} + \frac{e\sqrt{fx}}{f^2 \left(a+bx^2+cx^4\right)^{3/2}} \right) dx$$

$$= d \int \frac{1}{(fx)^{3/2} \left(a+bx^2+cx^4\right)^{3/2}} \, dx + \frac{e \int \frac{\sqrt{fx}}{\left(a+bx^2+cx^4\right)^{3/2}} \, dx}{f^2}$$

$$= \frac{\left(d\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}\right) \int \frac{1}{(fx)^{3/2}\left(1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}\right)^{3/2}\left(1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)^{3/2}} \, dx}{a\sqrt{a+bx^2+cx^4}}$$

$$+ \frac{\left(e\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}\right) \int \frac{\sqrt{fx}}{\left(1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}\right)^{3/2}\left(1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)^{3/2}} \, dx}{af^2\sqrt{a+bx^2+cx^4}}$$

$$= -\frac{2d\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}F_1\left(-\frac{1}{4};\frac{3}{2},\frac{3}{2};\frac{3}{4};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{af\sqrt{fx}\sqrt{a+bx^2+cx^4}} + \frac{2e(fx)^{3/2}\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}F_1\left(\frac{3}{4};\frac{3}{2},\frac{3}{2};\frac{7}{4};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{3af^3\sqrt{a+bx^2+cx^4}}$$

Mathematica [A] (verified)

Time = 11.74 (sec) , antiderivative size = 460, normalized size of antiderivative = 1.53

$$\int \frac{d+ex^2}{(fx)^{3/2}(a+bx^2+cx^4)^{3/2}}\,dx = -\frac{x\Big(-21(-3b^2dx^2(b+cx^2)+a^2c(8d-2ex^2)+a(10c^2dx^4+b^2(-2d+ex^2)+bcx^2(11d+ex^2)))+7(-3b^3d\cdots}{}$$

```
[In] Integrate[(d + e*x^2)/((f*x)^(3/2)*(a + b*x^2 + c*x^4)^(3/2)),x]
```

```
[Out] -1/21*(x*(-21*(-3*b^2*d*x^2*(b + c*x^2) + a^2*c*(8*d - 2*e*x^2) + a*(10*c^2
*d*x^4 + b^2*(-2*d + e*x^2) + b*c*x^2*(11*d + e*x^2))) + 7*(-3*b^3*d + 9*a*
b*c*d + a*b^2*e + 2*a^2*c*e)*x^2*Sqrt[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b
- Sqrt[b^2 - 4*a*c])]*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2
- 4*a*c])]*AppellF1[3/4, 1/2, 1/2, 7/4, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]),
 (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])] - 9*c*(3*b^2*d - 10*a*c*d - a*b*e)*x^4
*Sqrt[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[(b +
Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[7/4, 1/2, 1/
2, 11/4, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a
*c])]))/(a^2*(b^2 - 4*a*c)*(f*x)^(3/2)*Sqrt[a + b*x^2 + c*x^4])
```

Maple [F]

$$\int \frac{e\,x^2+d}{(fx)^{\frac{3}{2}}\,(c\,x^4+b\,x^2+a)^{\frac{3}{2}}}dx$$

```
[In] int((e*x^2+d)/(f*x)^(3/2)/(c*x^4+b*x^2+a)^(3/2),x)
```

```
[Out] int((e*x^2+d)/(f*x)^(3/2)/(c*x^4+b*x^2+a)^(3/2),x)
```

Fricas [F]

$$\int \frac{d+ex^2}{(fx)^{3/2}(a+bx^2+cx^4)^{3/2}}\,dx = \int \frac{ex^2+d}{(cx^4+bx^2+a)^{\frac{3}{2}}(fx)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((e*x^2+d)/(f*x)^(3/2)/(c*x^4+b*x^2+a)^(3/2),x, algorithm="fricas"
)
[Out] integral(sqrt(c*x^4 + b*x^2 + a)*(e*x^2 + d)*sqrt(f*x)/(c^2*f^2*x^10 + 2*b*
c*f^2*x^8 + (b^2 + 2*a*c)*f^2*x^6 + 2*a*b*f^2*x^4 + a^2*f^2*x^2), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{d+ex^2}{(fx)^{3/2}(a+bx^2+cx^4)^{3/2}}\,dx = \text{Timed out}$$

```
[In] integrate((e*x**2+d)/(f*x)**(3/2)/(c*x**4+b*x**2+a)**(3/2),x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{d+ex^2}{(fx)^{3/2}(a+bx^2+cx^4)^{3/2}}\,dx = \int \frac{ex^2+d}{(cx^4+bx^2+a)^{\frac{3}{2}}(fx)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((e*x^2+d)/(f*x)^(3/2)/(c*x^4+b*x^2+a)^(3/2),x, algorithm="maxima"
)
[Out] integrate((e*x^2 + d)/((c*x^4 + b*x^2 + a)^(3/2)*(f*x)^(3/2)), x)
```

Giac [F]

$$\int \frac{d+ex^2}{(fx)^{3/2}(a+bx^2+cx^4)^{3/2}}\,dx = \int \frac{ex^2+d}{(cx^4+bx^2+a)^{\frac{3}{2}}(fx)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((e*x^2+d)/(f*x)^(3/2)/(c*x^4+b*x^2+a)^(3/2),x, algorithm="giac")
[Out] integrate((e*x^2 + d)/((c*x^4 + b*x^2 + a)^(3/2)*(f*x)^(3/2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{d+ex^2}{(fx)^{3/2}\left(a+bx^2+cx^4\right)^{3/2}}\,dx=\int \frac{e\,x^2+d}{\left(f\,x\right)^{3/2}\left(c\,x^4+b\,x^2+a\right)^{3/2}}\,dx$$

```
[In] int((d + e*x^2)/((f*x)^(3/2)*(a + b*x^2 + c*x^4)^(3/2)),x)
[Out] int((d + e*x^2)/((f*x)^(3/2)*(a + b*x^2 + c*x^4)^(3/2)), x)
```

3.220 $\int (fx)^m (d+ex^2) (a+bx^2+cx^4)^3 \, dx$

Optimal result

Integrand size = 27, antiderivative size = 243

$$\int (fx)^m (d+ex^2) (a+bx^2+cx^4)^3 \, dx = \frac{a^3d(fx)^{1+m}}{f(1+m)} + \frac{a^2(3bd+ae)(fx)^{3+m}}{f^3(3+m)} + \frac{3a(b^2d+acd+abe)(fx)^{5+m}}{f^5(5+m)} + \frac{(b^3d+6abcd+3ab^2e+3a^2ce)(fx)^{7+m}}{f^7(7+m)} + \frac{(3b^2cd+3ac^2d+b^3e+6abce)(fx)^{9+m}}{f^9(9+m)} + \frac{3c(bcd+b^2e+ace)(fx)^{11+m}}{f^{11}(11+m)} + \frac{c^2(cd+3be)(fx)^{13+m}}{f^{13}(13+m)} + \frac{c^3e(fx)^{15+m}}{f^{15}(15+m)}$$

```
[Out] a^3*d*(f*x)^(1+m)/f/(1+m)+a^2*(a*e+3*b*d)*(f*x)^(3+m)/f^3/(3+m)+3*a*(a*b*e+
a*c*d+b^2*d)*(f*x)^(5+m)/f^5/(5+m)+(3*a^2*c*e+3*a*b^2*e+6*a*b*c*d+b^3*d)*(f
*x)^(7+m)/f^7/(7+m)+(6*a*b*c*e+3*a*c^2*d+b^3*e+3*b^2*c*d)*(f*x)^(9+m)/f^9/(
9+m)+3*c*(a*c*e+b^2*e+b*c*d)*(f*x)^(11+m)/f^11/(11+m)+c^2*(3*b*e+c*d)*(f*x)
^(13+m)/f^13/(13+m)+c^3*e*(f*x)^(15+m)/f^15/(15+m)
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 243, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.037, Rules used = {1275}

$$\int (fx)^m\left(d+ex^2\right)\left(a+bx^2+cx^4\right)^3\,dx=\frac{a^3d(fx)^{m+1}}{f(m+1)}+\frac{(fx)^{m+7}\left(3a^2ce+3ab^2e+6abcd+b^3d\right)}{f^7(m+7)}+\frac{a^2(fx)^{m+3}(ae+3bd)}{f^3(m+3)}+\frac{3c(fx)^{m+11}\left(ace+b^2e+bcd\right)}{f^{11}(m+11)}+\frac{3a(fx)^{m+5}\left(abe+acd+b^2d\right)}{f^5(m+5)}+\frac{(fx)^{m+9}\left(6abce+3ac^2d+b^3e+3b^2cd\right)}{f^9(m+9)}+\frac{c^2(fx)^{m+13}(3be+cd)}{f^{13}(m+13)}+\frac{c^3e(fx)^{m+15}}{f^{15}(m+15)}$$

```
[In] Int[(f*x)^m*(d + e*x^2)*(a + b*x^2 + c*x^4)^3,x]
```

```
[Out] (a^3*d*(f*x)^(1 + m))/(f*(1 + m)) + (a^2*(3*b*d + a*e)*(f*x)^(3 + m))/(f^3*
(3 + m)) + (3*a*(b^2*d + a*c*d + a*b*e)*(f*x)^(5 + m))/(f^5*(5 + m)) + ((b^
3*d + 6*a*b*c*d + 3*a*b^2*e + 3*a^2*c*e)*(f*x)^(7 + m))/(f^7*(7 + m)) + ((3
*b^2*c*d + 3*a*c^2*d + b^3*e + 6*a*b*c*e)*(f*x)^(9 + m))/(f^9*(9 + m)) + (3
*c*(b*c*d + b^2*e + a*c*e)*(f*x)^(11 + m))/(f^11*(11 + m)) + (c^2*(c*d + 3*
b*e)*(f*x)^(13 + m))/(f^13*(13 + m)) + (c^3*e*(f*x)^(15 + m))/(f^15*(15 + m
))
```

Rule 1275

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*
(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && NeQ[
b^2 - 4*a*c, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rubi steps

$$\text{integral} = \int \left(a^3d(fx)^m + \frac{a^2(3bd+ae)(fx)^{2+m}}{f^2} + \frac{3a(b^2d+acd+abe)(fx)^{4+m}}{f^4} + \frac{(b^3d+6abcd+3ab^2e+3a^2ce)(fx)^{6+m}}{f^6} + \frac{(3b^2cd+3ac^2d+b^3e+6abce)(fx)^{8+m}}{f^8} + \frac{3c(bcd+b^2e+ace)(fx)^{10+m}}{f^{10}} + \frac{c^2(cd+3be)(fx)^{12+m}}{f^{12}} + \frac{c^3e(fx)^{14+m}}{f^{14}}\right) dx$$

$$= \frac{a^3d(fx)^{1+m}}{f(1+m)} + \frac{a^2(3bd+ae)(fx)^{3+m}}{f^3(3+m)} + \frac{3a(b^2d+acd+abe)(fx)^{5+m}}{f^5(5+m)} + \frac{(b^3d+6abcd+3ab^2e+3a^2ce)(fx)^{7+m}}{f^7(7+m)} + \frac{(3b^2cd+3ac^2d+b^3e+6abce)(fx)^{9+m}}{f^9(9+m)} + \frac{3c(bcd+b^2e+ace)(fx)^{11+m}}{f^{11}(11+m)} + \frac{c^2(cd+3be)(fx)^{13+m}}{f^{13}(13+m)} + \frac{c^3e(fx)^{15+m}}{f^{15}(15+m)}$$

Mathematica [A] (verified)

Time = 0.81 (sec) , antiderivative size = 191, normalized size of antiderivative = 0.79

$$\int (fx)^m (d+ex^2) (a+bx^2+cx^4)^3 \, dx = x(fx)^m \left(\frac{a^3d}{1+m} + \frac{a^2(3bd+ae)x^2}{3+m} + \frac{3a(b^2d+acd+abe)x^4}{5+m} + \frac{(b^3d+6abcd+3ab^2e+3a^2ce)x^6}{7+m} + \frac{(3b^2cd+3ac^2d+b^3e+6abce)x^8}{9+m} + \frac{3c(bcd+b^2e+ace)x^{10}}{11+m} + \frac{c^2(cd+3be)x^{12}}{13+m} + \frac{c^3ex^{14}}{15+m}\right)$$

```
[In] Integrate[(f*x)^m*(d + e*x^2)*(a + b*x^2 + c*x^4)^3,x]
```

```
[Out] x*(f*x)^m*((a^3*d)/(1 + m) + (a^2*(3*b*d + a*e)*x^2)/(3 + m) + (3*a*(b^2*d
+ a*c*d + a*b*e)*x^4)/(5 + m) + ((b^3*d + 6*a*b*c*d + 3*a*b^2*e + 3*a^2*c*e
)*x^6)/(7 + m) + ((3*b^2*c*d + 3*a*c^2*d + b^3*e + 6*a*b*c*e)*x^8)/(9 + m)
+ (3*c*(b*c*d + b^2*e + a*c*e)*x^10)/(11 + m) + (c^2*(c*d + 3*b*e)*x^12)/(1
3 + m) + (c^3*e*x^14)/(15 + m))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1934 vs. $2(243) = 486$.

Time = 0.35 (sec) , antiderivative size = 1935, normalized size of antiderivative = 7.96

method	result	size
gosper	Expression too large to display	1935
risch	Expression too large to display	1935
parallelrisch	Expression too large to display	2737

```
[In] int((f*x)^m*(e*x^2+d)*(c*x^4+b*x^2+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] x*(c^3*e*m^7*x^14+49*c^3*e*m^6*x^14+3*b*c^2*e*m^7*x^12+c^3*d*m^7*x^12+973*c
^3*e*m^5*x^14+153*b*c^2*e*m^6*x^12+51*c^3*d*m^6*x^12+10045*c^3*e*m^4*x^14+3
*a*c^2*e*m^7*x^10+3*b^2*c*e*m^7*x^10+3*b*c^2*d*m^7*x^10+3135*b*c^2*e*m^5*x^
12+1045*c^3*d*m^5*x^12+57379*c^3*e*m^3*x^14+159*a*c^2*e*m^6*x^10+159*b^2*c*
e*m^6*x^10+159*b*c^2*d*m^6*x^10+33165*b*c^2*e*m^4*x^12+11055*c^3*d*m^4*x^12
+177331*c^3*e*m^2*x^14+6*a*b*c*e*m^7*x^8+3*a*c^2*d*m^7*x^8+3375*a*c^2*e*m^5
*x^10+b^3*e*m^7*x^8+3*b^2*c*d*m^7*x^8+3375*b^2*c*e*m^5*x^10+3375*b*c^2*d*m^
5*x^10+193017*b*c^2*e*m^3*x^12+64339*c^3*d*m^3*x^12+264207*c^3*e*m*x^14+330
*a*b*c*e*m^6*x^8+165*a*c^2*d*m^6*x^8+36795*a*c^2*e*m^4*x^10+55*b^3*e*m^6*x^
8+165*b^2*c*d*m^6*x^8+36795*b^2*c*e*m^4*x^10+36795*b*c^2*d*m^4*x^10+604827*
b*c^2*e*m^2*x^12+201609*c^3*d*m^2*x^12+135135*c^3*e*x^14+3*a^2*c*e*m^7*x^6+
3*a*b^2*e*m^7*x^6+6*a*b*c*d*m^7*x^6+7278*a*b*c*e*m^5*x^8+3639*a*c^2*d*m^5*x
^8+219417*a*c^2*e*m^3*x^10+b^3*d*m^7*x^6+1213*b^3*e*m^5*x^8+3639*b^2*c*d*m^
5*x^8+219417*b^2*c*e*m^3*x^10+219417*b*c^2*d*m^3*x^10+909765*b*c^2*e*m*x^12
+303255*c^3*d*m*x^12+171*a^2*c*e*m^6*x^6+171*a*b^2*e*m^6*x^6+342*a*b*c*d*m^
6*x^6+82338*a*b*c*e*m^4*x^8+41169*a*c^2*d*m^4*x^8+700461*a*c^2*e*m^2*x^10+5
7*b^3*d*m^6*x^6+13723*b^3*e*m^4*x^8+41169*b^2*c*d*m^4*x^8+700461*b^2*c*e*m^
2*x^10+700461*b*c^2*d*m^2*x^10+467775*b*c^2*e*x^12+155925*c^3*d*x^12+3*a^2*
b*e*m^7*x^4+3*a^2*c*d*m^7*x^4+3927*a^2*c*e*m^5*x^6+3*a*b^2*d*m^7*x^4+3927*a
*b^2*e*m^5*x^6+7854*a*b*c*d*m^5*x^6+507282*a*b*c*e*m^3*x^8+253641*a*c^2*d*m
^3*x^8+1067445*a*c^2*e*m*x^10+1309*b^3*d*m^5*x^6+84547*b^3*e*m^3*x^8+253641
*b^2*c*d*m^3*x^8+1067445*b^2*c*e*m*x^10+1067445*b*c^2*d*m*x^10+177*a^2*b*e*
m^6*x^4+177*a^2*c*d*m^6*x^4+46431*a^2*c*e*m^4*x^6+177*a*b^2*d*m^6*x^4+46431
*a*b^2*e*m^4*x^6+92862*a*b*c*d*m^4*x^6+1662558*a*b*c*e*m^2*x^8+831279*a*c^2
*d*m^2*x^8+552825*a*c^2*e*x^10+15477*b^3*d*m^4*x^6+277093*b^3*e*m^2*x^8+831
279*b^2*c*d*m^2*x^8+552825*b^2*c*e*x^10+552825*b*c^2*d*x^10+a^3*e*m^7*x^2+3
*a^2*b*d*m^7*x^2+4239*a^2*b*e*m^5*x^4+4239*a^2*c*d*m^5*x^4+299145*a^2*c*e*m
^3*x^6+4239*a*b^2*d*m^5*x^4+299145*a*b^2*e*m^3*x^6+598290*a*b*c*d*m^3*x^6+2
582010*a*b*c*e*m*x^8+1291005*a*c^2*d*m*x^8+99715*b^3*d*m^3*x^6+430335*b^3*e
*m*x^8+1291005*b^2*c*d*m*x^8+61*a^3*e*m^6*x^2+183*a^2*b*d*m^6*x^2+52725*a^2
*b*e*m^4*x^4+52725*a^2*c*d*m^4*x^4+1020033*a^2*c*e*m^2*x^6+52725*a*b^2*d*m^
4*x^4+1020033*a*b^2*e*m^2*x^6+2040066*a*b*c*d*m^2*x^6+1351350*a*b*c*e*x^8+6
75675*a*c^2*d*x^8+340011*b^3*d*m^2*x^6+225225*b^3*e*x^8+675675*b^2*c*d*x^8+
```

```
a^3*d*m^7+1525*a^3*e*m^5*x^2+4575*a^2*b*d*m^5*x^2+360537*a^2*b*e*m^3*x^4+36
0537*a^2*c*d*m^3*x^4+1632285*a^2*c*e*m*x^6+360537*a*b^2*d*m^3*x^4+1632285*a
*b^2*e*m*x^6+3264570*a*b*c*d*m*x^6+544095*b^3*d*m*x^6+63*a^3*d*m^6+20065*a^
3*e*m^4*x^2+60195*a^2*b*d*m^4*x^2+1311363*a^2*b*e*m^2*x^4+1311363*a^2*c*d*m
^2*x^4+868725*a^2*c*e*x^6+1311363*a*b^2*d*m^2*x^4+868725*a*b^2*e*x^6+173745
0*a*b*c*d*x^6+289575*b^3*d*x^6+1645*a^3*d*m^5+147859*a^3*e*m^3*x^2+443577*a
^2*b*d*m^3*x^2+2215701*a^2*b*e*m*x^4+2215701*a^2*c*d*m*x^4+2215701*a*b^2*d*
m*x^4+22995*a^3*d*m^4+594439*a^3*e*m^2*x^2+1783317*a^2*b*d*m^2*x^2+1216215*
a^2*b*e*x^4+1216215*a^2*c*d*x^4+1216215*a*b^2*d*x^4+185059*a^3*d*m^3+114085
5*a^3*e*m*x^2+3422565*a^2*b*d*m*x^2+852957*a^3*d*m^2+675675*a^3*e*x^2+20270
25*a^2*b*d*x^2+2071215*a^3*d*m+2027025*a^3*d)*(f*x)^m/(1+m)/(3+m)/(5+m)/(7+
m)/(9+m)/(11+m)/(13+m)/(15+m)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1357 vs. 2(243) = 486.

Time = 0.26 (sec) , antiderivative size = 1357, normalized size of antiderivative = 5.58

$$\int (fx)^m \left(d+ex^2\right) \left(a+bx^2+cx^4\right)^3 \, dx = \text{Too large to display}$$

```
[In] integrate((f*x)^m*(e*x^2+d)*(c*x^4+b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] ((c^3*e*m^7 + 49*c^3*e*m^6 + 973*c^3*e*m^5 + 10045*c^3*e*m^4 + 57379*c^3*e*
m^3 + 177331*c^3*e*m^2 + 264207*c^3*e*m + 135135*c^3*e)*x^15 + ((c^3*d + 3*
b*c^2*e)*m^7 + 51*(c^3*d + 3*b*c^2*e)*m^6 + 1045*(c^3*d + 3*b*c^2*e)*m^5 +
11055*(c^3*d + 3*b*c^2*e)*m^4 + 155925*c^3*d + 467775*b*c^2*e + 64339*(c^3*
d + 3*b*c^2*e)*m^3 + 201609*(c^3*d + 3*b*c^2*e)*m^2 + 303255*(c^3*d + 3*b*c
^2*e)*m)*x^13 + 3*((b*c^2*d + (b^2*c + a*c^2)*e)*m^7 + 53*(b*c^2*d + (b^2*c
 + a*c^2)*e)*m^6 + 1125*(b*c^2*d + (b^2*c + a*c^2)*e)*m^5 + 12265*(b*c^2*d
+ (b^2*c + a*c^2)*e)*m^4 + 184275*b*c^2*d + 73139*(b*c^2*d + (b^2*c + a*c^2
)*e)*m^3 + 233487*(b*c^2*d + (b^2*c + a*c^2)*e)*m^2 + 184275*(b^2*c + a*c^2
)*e + 355815*(b*c^2*d + (b^2*c + a*c^2)*e)*m)*x^11 + ((3*(b^2*c + a*c^2)*d
+ (b^3 + 6*a*b*c)*e)*m^7 + 55*(3*(b^2*c + a*c^2)*d + (b^3 + 6*a*b*c)*e)*m^6
 + 1213*(3*(b^2*c + a*c^2)*d + (b^3 + 6*a*b*c)*e)*m^5 + 13723*(3*(b^2*c + a
*c^2)*d + (b^3 + 6*a*b*c)*e)*m^4 + 84547*(3*(b^2*c + a*c^2)*d + (b^3 + 6*a*
b*c)*e)*m^3 + 277093*(3*(b^2*c + a*c^2)*d + (b^3 + 6*a*b*c)*e)*m^2 + 675675
*(b^2*c + a*c^2)*d + 225225*(b^3 + 6*a*b*c)*e + 430335*(3*(b^2*c + a*c^2)*d
 + (b^3 + 6*a*b*c)*e)*m)*x^9 + (((b^3 + 6*a*b*c)*d + 3*(a*b^2 + a^2*c)*e)*m
^7 + 57*((b^3 + 6*a*b*c)*d + 3*(a*b^2 + a^2*c)*e)*m^6 + 1309*((b^3 + 6*a*b*
c)*d + 3*(a*b^2 + a^2*c)*e)*m^5 + 15477*((b^3 + 6*a*b*c)*d + 3*(a*b^2 + a^2
*c)*e)*m^4 + 99715*((b^3 + 6*a*b*c)*d + 3*(a*b^2 + a^2*c)*e)*m^3 + 340011*(
(b^3 + 6*a*b*c)*d + 3*(a*b^2 + a^2*c)*e)*m^2 + 289575*(b^3 + 6*a*b*c)*d + 8
68725*(a*b^2 + a^2*c)*e + 544095*((b^3 + 6*a*b*c)*d + 3*(a*b^2 + a^2*c)*e)*
m)*x^7 + 3*((a^2*b*e + (a*b^2 + a^2*c)*d)*m^7 + 59*(a^2*b*e + (a*b^2 + a^2*
```

```
c)*d)*m^6 + 1413*(a^2*b*e + (a*b^2 + a^2*c)*d)*m^5 + 17575*(a^2*b*e + (a*b^
2 + a^2*c)*d)*m^4 + 405405*a^2*b*e + 120179*(a^2*b*e + (a*b^2 + a^2*c)*d)*m
^3 + 437121*(a^2*b*e + (a*b^2 + a^2*c)*d)*m^2 + 405405*(a*b^2 + a^2*c)*d +
738567*(a^2*b*e + (a*b^2 + a^2*c)*d)*m)*x^5 + ((3*a^2*b*d + a^3*e)*m^7 + 61
*(3*a^2*b*d + a^3*e)*m^6 + 1525*(3*a^2*b*d + a^3*e)*m^5 + 20065*(3*a^2*b*d
+ a^3*e)*m^4 + 2027025*a^2*b*d + 675675*a^3*e + 147859*(3*a^2*b*d + a^3*e)*
m^3 + 594439*(3*a^2*b*d + a^3*e)*m^2 + 1140855*(3*a^2*b*d + a^3*e)*m)*x^3 +
 (a^3*d*m^7 + 63*a^3*d*m^6 + 1645*a^3*d*m^5 + 22995*a^3*d*m^4 + 185059*a^3*
d*m^3 + 852957*a^3*d*m^2 + 2071215*a^3*d*m + 2027025*a^3*d)*x)*(f*x)^m/(m^8
 + 64*m^7 + 1708*m^6 + 24640*m^5 + 208054*m^4 + 1038016*m^3 + 2924172*m^2 +
 4098240*m + 2027025)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 11266 vs. 2(238) = 476.

Time = 1.53 (sec) , antiderivative size = 11266, normalized size of antiderivative = 46.36

$$\int (fx)^m \left(d+ex^2\right) \left(a+bx^2+cx^4\right)^3 \, dx = \text{Too large to display}$$

```
[In] integrate((f*x)**m*(e*x**2+d)*(c*x**4+b*x**2+a)**3,x)
```

```
[Out] Piecewise(((-a**3*d/(14*x**14) - a**3*e/(12*x**12) - a**2*b*d/(4*x**12) - 3
*a**2*b*e/(10*x**10) - 3*a**2*c*d/(10*x**10) - 3*a**2*c*e/(8*x**8) - 3*a*b*
*2*d/(10*x**10) - 3*a*b**2*e/(8*x**8) - 3*a*b*c*d/(4*x**8) - a*b*c*e/x**6 -
 a*c**2*d/(2*x**6) - 3*a*c**2*e/(4*x**4) - b**3*d/(8*x**8) - b**3*e/(6*x**6
) - b**2*c*d/(2*x**6) - 3*b**2*c*e/(4*x**4) - 3*b*c**2*d/(4*x**4) - 3*b*c**
2*e/(2*x**2) - c**3*d/(2*x**2) + c**3*e*log(x))/f**15, Eq(m, -15)), ((-a**3
*d/(12*x**12) - a**3*e/(10*x**10) - 3*a**2*b*d/(10*x**10) - 3*a**2*b*e/(8*x
**8) - 3*a**2*c*d/(8*x**8) - a**2*c*e/(2*x**6) - 3*a*b**2*d/(8*x**8) - a*b*
*2*e/(2*x**6) - a*b*c*d/x**6 - 3*a*b*c*e/(2*x**4) - 3*a*c**2*d/(4*x**4) - 3
*a*c**2*e/(2*x**2) - b**3*d/(6*x**6) - b**3*e/(4*x**4) - 3*b**2*c*d/(4*x**4
) - 3*b**2*c*e/(2*x**2) - 3*b*c**2*d/(2*x**2) + 3*b*c**2*e*log(x) + c**3*d*
log(x) + c**3*e*x**2/2)/f**13, Eq(m, -13)), ((-a**3*d/(10*x**10) - a**3*e/(
8*x**8) - 3*a**2*b*d/(8*x**8) - a**2*b*e/(2*x**6) - a**2*c*d/(2*x**6) - 3*a
**2*c*e/(4*x**4) - a*b**2*d/(2*x**6) - 3*a*b**2*e/(4*x**4) - 3*a*b*c*d/(2*x
**4) - 3*a*b*c*e/x**2 - 3*a*c**2*d/(2*x**2) + 3*a*c**2*e*log(x) - b**3*d/(4
*x**4) - b**3*e/(2*x**2) - 3*b**2*c*d/(2*x**2) + 3*b**2*c*e*log(x) + 3*b*c*
*2*d*log(x) + 3*b*c**2*e*x**2/2 + c**3*d*x**2/2 + c**3*e*x**4/4)/f**11, Eq(
m, -11)), ((-a**3*d/(8*x**8) - a**3*e/(6*x**6) - a**2*b*d/(2*x**6) - 3*a**2
*b*e/(4*x**4) - 3*a**2*c*d/(4*x**4) - 3*a**2*c*e/(2*x**2) - 3*a*b**2*d/(4*x
**4) - 3*a*b**2*e/(2*x**2) - 3*a*b*c*d/x**2 + 6*a*b*c*e*log(x) + 3*a*c**2*d
*log(x) + 3*a*c**2*e*x**2/2 - b**3*d/(2*x**2) + b**3*e*log(x) + 3*b**2*c*d*
log(x) + 3*b**2*c*e*x**2/2 + 3*b*c**2*d*x**2/2 + 3*b*c**2*e*x**4/4 + c**3*d
*x**4/4 + c**3*e*x**6/6)/f**9, Eq(m, -9)), ((-a**3*d/(6*x**6) - a**3*e/(4*x
```

4) - 3*a2*b*d/(4*x**4) - 3*a**2*b*e/(2*x**2) - 3*a**2*c*d/(2*x**2) + 3*
a**2*c*e*log(x) - 3*a*b**2*d/(2*x**2) + 3*a*b**2*e*log(x) + 6*a*b*c*d*log(x
) + 3*a*b*c*e*x**2 + 3*a*c**2*d*x**2/2 + 3*a*c**2*e*x**4/4 + b**3*d*log(x)
+ b**3*e*x**2/2 + 3*b**2*c*d*x**2/2 + 3*b**2*c*e*x**4/4 + 3*b*c**2*d*x**4/4
 + b*c**2*e*x**6/2 + c**3*d*x**6/6 + c**3*e*x**8/8)/f**7, Eq(m, -7)), ((-a*
*3*d/(4*x**4) - a**3*e/(2*x**2) - 3*a**2*b*d/(2*x**2) + 3*a**2*b*e*log(x) +
 3*a**2*c*d*log(x) + 3*a**2*c*e*x**2/2 + 3*a*b**2*d*log(x) + 3*a*b**2*e*x**
2/2 + 3*a*b*c*d*x**2 + 3*a*b*c*e*x**4/2 + 3*a*c**2*d*x**4/4 + a*c**2*e*x**6
/2 + b**3*d*x**2/2 + b**3*e*x**4/4 + 3*b**2*c*d*x**4/4 + b**2*c*e*x**6/2 +
b*c**2*d*x**6/2 + 3*b*c**2*e*x**8/8 + c**3*d*x**8/8 + c**3*e*x**10/10)/f**5
, Eq(m, -5)), ((-a**3*d/(2*x**2) + a**3*e*log(x) + 3*a**2*b*d*log(x) + 3*a*
*2*b*e*x**2/2 + 3*a**2*c*d*x**2/2 + 3*a**2*c*e*x**4/4 + 3*a*b**2*d*x**2/2 +
 3*a*b**2*e*x**4/4 + 3*a*b*c*d*x**4/2 + a*b*c*e*x**6 + a*c**2*d*x**6/2 + 3*
a*c**2*e*x**8/8 + b**3*d*x**4/4 + b**3*e*x**6/6 + b**2*c*d*x**6/2 + 3*b**2*
c*e*x**8/8 + 3*b*c**2*d*x**8/8 + 3*b*c**2*e*x**10/10 + c**3*d*x**10/10 + c*
*3*e*x**12/12)/f**3, Eq(m, -3)), ((a**3*d*log(x) + a**3*e*x**2/2 + 3*a**2*b
*d*x**2/2 + 3*a**2*b*e*x**4/4 + 3*a**2*c*d*x**4/4 + a**2*c*e*x**6/2 + 3*a*b
2*d*x4/4 + a*b**2*e*x**6/2 + a*b*c*d*x**6 + 3*a*b*c*e*x**8/4 + 3*a*c**2
*d*x**8/8 + 3*a*c**2*e*x**10/10 + b**3*d*x**6/6 + b**3*e*x**8/8 + 3*b**2*c*
d*x**8/8 + 3*b**2*c*e*x**10/10 + 3*b*c**2*d*x**10/10 + b*c**2*e*x**12/4 + c
3*d*x12/12 + c**3*e*x**14/14)/f, Eq(m, -1)), (a**3*d*m**7*x*(f*x)**m/(m
8 + 64*m7 + 1708*m**6 + 24640*m**5 + 208054*m**4 + 1038016*m**3 + 29241
72*m**2 + 4098240*m + 2027025) + 63*a**3*d*m**6*x*(f*x)**m/(m**8 + 64*m**7
+ 1708*m**6 + 24640*m**5 + 208054*m**4 + 1038016*m**3 + 2924172*m**2 + 4098
240*m + 2027025) + 1645*a**3*d*m**5*x*(f*x)**m/(m**8 + 64*m**7 + 1708*m**6
+ 24640*m**5 + 208054*m**4 + 1038016*m**3 + 2924172*m**2 + 4098240*m + 2027
025) + 22995*a**3*d*m**4*x*(f*x)**m/(m**8 + 64*m**7 + 1708*m**6 + 24640*m**
5 + 208054*m**4 + 1038016*m**3 + 2924172*m**2 + 4098240*m + 2027025) + 1850
59*a**3*d*m**3*x*(f*x)**m/(m**8 + 64*m**7 + 1708*m**6 + 24640*m**5 + 208054
*m**4 + 1038016*m**3 + 2924172*m**2 + 4098240*m + 2027025) + 852957*a**3*d*
m**2*x*(f*x)**m/(m**8 + 64*m**7 + 1708*m**6 + 24640*m**5 + 208054*m**4 + 10
38016*m**3 + 2924172*m**2 + 4098240*m + 2027025) + 2071215*a**3*d*m*x*(f*x)
m/(m8 + 64*m**7 + 1708*m**6 + 24640*m**5 + 208054*m**4 + 1038016*m**3 +
 2924172*m**2 + 4098240*m + 2027025) + 2027025*a**3*d*x*(f*x)**m/(m**8 + 64
*m**7 + 1708*m**6 + 24640*m**5 + 208054*m**4 + 1038016*m**3 + 2924172*m**2
+ 4098240*m + 2027025) + a**3*e*m**7*x**3*(f*x)**m/(m**8 + 64*m**7 + 1708*m
6 + 24640*m5 + 208054*m**4 + 1038016*m**3 + 2924172*m**2 + 4098240*m +
2027025) + 61*a**3*e*m**6*x**3*(f*x)**m/(m**8 + 64*m**7 + 1708*m**6 + 24640
*m**5 + 208054*m**4 + 1038016*m**3 + 2924172*m**2 + 4098240*m + 2027025) +
1525*a**3*e*m**5*x**3*(f*x)**m/(m**8 + 64*m**7 + 1708*m**6 + 24640*m**5 + 2
08054*m**4 + 1038016*m**3 + 2924172*m**2 + 4098240*m + 2027025) + 20065*a**
3*e*m**4*x**3*(f*x)**m/(m**8 + 64*m**7 + 1708*m**6 + 24640*m**5 + 208054*m*
*4 + 1038016*m**3 + 2924172*m**2 + 4098240*m + 2027025) + 147859*a**3*e*m**
3*x**3*(f*x)**m/(m**8 + 64*m**7 + 1708*m**6 + 24640*m**5 + 208054*m**4 + 10
38016*m**3 + 2924172*m**2 + 4098240*m + 2027025) + 594439*a**3*e*m**2*x**3*

(f*x)**m/(m**8 + 64*m**7 + 1708*m**6 + 24640*m**5 + 208054*m**4 + 1038016*m**3 + 2924172*m**2 + 4098240*m + 2027025) + 1140855*a**3*e*m*x**3*(f*x)**m/(m**8 + 64*m**7 + 1708*m**6 + 24640*m**5 + 208054*m**4 + 1038016*m**3 + 2924172*m**2 + 4098240*m + 2027025) + 675675*a**3*e*x**3*(f*x)**m/(m**8 + 64*m**7 + 1708*m**6 + 24640*m**5 + 208054*m**4 + 1038016*m**3 + 2924172*m**2 + 4098240*m + 2027025) + 3*a**2*b*d*m**7*x**3*(f*x)**m/(m**8 + 64*m**7 + 1708*m**6 + 24640*m**5 + 208054*m**4 + 1038016*m**3 + 2924172*m**2 + 4098240*m + 2027025) + 183*a**2*b*d*m**6*x**3*(f*x)**m/(m**8 + 64*m**7 + 1708*m**6 + 24640*m**5 + 208054*m**4 + 1038016*m**3 + 2924172*m**2 + 4098240*m + 2027025) + 4575*a**2*b*d*m**5*x**3*(f*x)**m/(m**8 + 64*m**7 + 1708*m**6 + 24640*m**5 + 208054*m**4 + 1038016*m**3 + 2924172*m**2 + 4098240*m + 2027025) + 60195*a**2*b*d*m**4*x**3*(f*x)**m/(m**8 + 64*m**7 + 1708*m**6 + 24640*m**5 + 208054*m**4 + 1038016*m**3 + 2924172*m**2 + 4098240*m + 2027025) + 443577*a**2*b*d*m**3*x**3*(f*x)**m/(m**8 + 64*m**7 + 1708*m**6 + 24640*m**5 + 208054*m**4 + 1038016*m**3 + 2924172*m**2 + 4098240*m + 2027025) + 1783317*a**2*b*d*m**2*x**3*(f*x)**m/(m**8 + 64*m**7 + 1708*m**6 + 24640*m**5 + 208054*m**4 + 1038016*m**3 + 2924172*m**2 + 4098240*m + 2027025) + 3422565*a**2*b*d*m*x**3*(f*x)**m/(m**8 + 64*m**7 + 1708*m**6 + 24640*m**5 + 208054*m**4 + 1038016*m**3 + 2924172*m**2 + 4098240*m + 2027025) + 2027025*a**2*b*d*x**3*(f*x)**m/(m**8 + 64*m**7 + 1708*m**6 + 24640*m**5 + 208054*m**4 + 1038016*m**3 + 2924172*m**2 + 4098240*m + 2027025) + 3*a**2*b*e*m**7*x**5*(f*x)**m/(m**8 + 64*m**7 + 1708*m**6 + 24640*m**5 + 208054*m**4 + 1038016*m**3 + 2924172*m**2 + 4098240*m + 2027025) + 177*a**2*b*e*m**6*x**5*(f*x)**m/(m**8 + 64*m**7 + 1708*m**6 + 24640*m**5 + 208054*m**4 + 1038016*m**3 + 2924172*m**2 + 4098240*m + 2027025) + 4239*a**2*b*e*m**5*x**5*(f*x)**m/(m**8 + 64*m**7 + 1708*m**6 + 24640*m**5 + 208054*m**4 + 1038016*m**3 + 2924172*m**2 + 4098240*m + 2027025) + 52725*a**2*b*e*m**4*x**5*(f*x)**m/(m**8 + 64*m**7 + 1708*m**6 + 24640*m**5 + 208054*m**4 + 1038016*m**3 + 2924172*m**2 + 4098240*m + 2027025) + 360537*a**2*b*e*m**3*x**5*(f*x)**m/(m**8 + 64*m**7 + 1708*m**6 + 24640*m**5 + 208054*m**4 + 1038016*m**3 + 2924172*m**2 + 4098240*m + 2027025) + 1311363*a**2*b*e*m**2*x**5*(f*x)**m/(m**8 + 64*m**7 + 1708*m**6 + 24640*m**5 + 208054*m**4 + 1038016*m**3 + 2924172*m**2 + 4098240*m + 2027025) + 2215701*a**2*b*e*m*x**5*(f*x)**m/(m**8 + 64*m**7 + 1708*m**6 + 24640*m**5 + 208054*m**4 + 1038016*m**3 + 2924172*m**2 + 4098240*m + 2027025) + 1216215*a**2*b*e*x**5*(f*x)**m/(m**8 + 64*m**7 + 1708*m**6 + 24640*m**5 + 208054*m**4 + 1038016*m**3 + 2924172*m**2 + 4098240*m + 2027025) + 3*a**2*c*d*m**7*x**5*(f*x)**m/(m**8 + 64*m**7 + 1708*m**6 + 24640*m**5 + 208054*m**4 + 1038016*m**3 + 2924172*m**2 + 4098240*m + 2027025) + 177*a**2*c*d*m**6*x**5*(f*x)**m/(m**8 + 64*m**7 + 1708*m**6 + 24640*m**5 + 208054*m**4 + 1038016*m**3 + 2924172*m**2 + 4098240*m + 2027025) + 4239*a**2*c*d*m**5*x**5*(f*x)**m/(m**8 + 64*m**7 + 1708*m**6 + 24640*m**5 + 208054*m**4 + 1038016*m**3 + 2924172*m**2 + 4098240*m + 2027025) + 52725*a**2*c*d*m**4*x**5*(f*x)**m/(m**8 + 64*m**7 + 1708*m**6 + 24640*m**5 + 208054*m**4 + 1038016*m**3 + 2924172*m**2 + 4098240*m + 2027025) + 360537*a**2*c*d*m**3*x**5*(f*x)**m/(m**8 + 64*m**7 + 1708*m**6 + 24640*m**5 + 208054*m**4 + 1038016*m**3 + 2924172*m**2

+ 4098240*m + 2027025) + 1311363*a**2*c*d*m**2*x**5*(f*x)**m/(m**8 + 64*m**
7 + 1708*m**6 + 24640*m**5 + 208054*m**4 + 1038016*m**3 + 2924172*m**2 + 40
98240*m + 2027025) + 2215701*a**2*c*d*m*x**5*(f*x)**m/(m**8 + 64*m**7 + 170
8*m**6 + 24640*m**5 + 208054*m**4 + 1038016*m**3 + 2924172*m**2 + 4098240*m
 + 2027025) + 1216215*a**2*c*d*x**5*(f*x)**m/(m**8 + 64*m**7 + 1708*m**6 +
24640*m**5 + 208054*m**4 + 1038016*m**3 + 2924172*m**2 + 4098240*m + 202702
5) + 3*a**2*c*e*m**7*x**7*(f*x)**m/(m**8 + 64*m**7 + 1708*m**6 + 24640*m**5
 + 208054*m**4 + 1038016*m**3 + 2924172*m**2 + 4098240*m + 2027025) + 171*a
2*c*e*m6*x**7*(f*x)**m/(m**8 + 64*m**7 + 1708*m**6 + 24640*m**5 + 20805
4*m**4 + 1038016*m**3 + 2924172*m**2 + 4098240*m + 2027025) + 3927*a**2*c*e
*m**5*x**7*(f*x)**m/(m**8 + 64*m**7 + 1708*m**6 + 24640*m**5 + 208054*m**4
+ 1038016*m**3 + 2924172*m**2 + 4098240*m + 2027025) + 46431*a**2*c*e*m**4*
x**7*(f*x)**m/(m**8 + 64*m**7 + 1708*m**6 + 24640*m**5 + 208054*m**4 + 1038
016*m**3 + 2924172*m**2 + 4098240*m + 2027025) + 299145*a**2*c*e*m**3*x**7*
(f*x)**m/(m**8 + 64*m**7 + 1708*m**6 + 24640*m**5 + 208054*m**4 + 1038016*m
3 + 2924172*m2 + 4098240*m + 2027025) + 1020033*a**2*c*e*m**2*x**7*(f*x
)**m/(m**8 + 64*m**7 + 1708*m**6 + 24640*m**5 + 208054*m**4 + 1038016*m**3
+ 2924172*m**2 + 4098240*m + 2027025) + 1632285*a**2*c*e*m*x**7*(f*x)**m/(m
8 + 64*m7 + 1708*m**6 + 24640*m**5 + 208054*m**4 + 1038016*m**3 + 29241
72*m**2 + 4098240*m + 2027025) + 868725*a**2*c*e*x**7*(f*x)**m/(m**8 + 64*m
7 + 1708*m6 + 24640*m**5 + 208054*m**4 + 1038016*m**3 + 2924172*m**2 +
4098240*m + 2027025) + 3*a*b**2*d*m**7*x**5*(f*x)**m/(m**8 + 64*m**7 + 1708
*m**6 + 24640*m**5 + 208054*m**4 + 1038016*m**3 + 2924172*m**2 + 4098240*m
+ 2027025) + 177*a*b**2*d*m**6*x**5*(f*x)**m/(m**8 + 64*m**7 + 1708*m**6 +
24640*m**5 + 208054*m**4 + 1038016*m**3 + 2924172*m**2 + 4098240*m + 202702
5) + 4239*a*b**2*d*m**5*x**5*(f*x)**m/(m**8 + 64*m**7 + 1708*m**6 + 24640*m
5 + 208054*m4 + 1038016*m**3 + 2924172*m**2 + 4098240*m + 2027025) + 52
725*a*b**2*d*m**4*x**5*(f*x)**m/(m**8 + 64*m**7 + 1708*m**6 + 24640*m**5 +
208054*m**4 + 1038016*m**3 + 2924172*m**2 + 4098240*m + 2027025) + 360537*a
*b**2*d*m**3*x**5*(f*x)**m/(m**8 + 64*m**7 + 1708*m**6 + 24640*m**5 + 20805
4*m**4 + 1038016*m**3 + 2924172*m**2 + 4098240*m + 2027025) + 1311363*a*b**
2*d*m**2*x**5*(f*x)**m/(m**8 + 64*m**7 + 1708*m**6 + 24640*m**5 + 208054*m*
*4 + 1038016*m**3 + 2924172*m**2 + 4098240*m + 2027025) + 2215701*a*b**2*d*
m*x**5*(f*x)**m/(m**8 + 64*m**7 + 1708*m**6 + 24640*m**5 + 208054*m**4 + 10
38016*m**3 + 2924172*m**2 + 4098240*m + 2027025) + 1216215*a*b**2*d*x**5*(f
*x)**m/(m**8 + 64*m**7 + 1708*m**6 + 24640*m**5 + 208054*m**4 + 1038016*m**
3 + 2924172*m**2 + 4098240*m + 2027025) + 3*a*b**2*e*m**7*x**7*(f*x)**m/(m*
*8 + 64*m**7 + 1708*m**6 + 24640*m**5 + 208054*m**4 + 1038016*m**3 + 292417
2*m**2 + 4098240*m + 2027025) + 171*a*b**2*e*m**6*x**7*(f*x)**m/(m**8 + 64*
m**7 + 1708*m**6 + 24640*m**5 + 208054*m**4 + 1038016*m**3 + 2924172*m**2 +
 4098240*m + 2027025) + 3927*a*b**2*e*m**5*x**7*(f*x)**m/(m**8 + 64*m**7 +
1708*m**6 + 24640*m**5 + 208054*m**4 + 1038016*m**3 + 2924172*m**2 + 409824
0*m + 2027025) + 46431*a*b**2*e*m**4*x**7*(f*x)**m/(m**8 + 64*m**7 + 1708*m
6 + 24640*m5 + 208054*m**4 + 1038016*m**3 + 2924172*m**2 + 4098240*m +
2027025) + 299145*a*b**2*e*m**3*x**7*(f*x)**m/(m**8 + 64*m**7 + 1708*m**6 +

24640*m**5 + 208054*m**4 + 1038016*m**3 + 2924172*m**2 + 4098240*m + 20270
25) + 1020033*a*b**2*e*m**2*x**7*(f*x)**m/(m**8 + 64*m**7 + 1708*m**6 + 246
40*m**5 + 208054*m**4 + 1038016*m**3 + 2924172*m**2 + 4098240*m + 2027025)
+ 1632285*a*b**2*e*m*x**7*(f*x)**m/(m**8 + 64*m**7 + 1708*m**6 + 24640*m**5
+ 208054*m**4 + 1038016*m**3 + 2924172*m**2 + 4098240*m + 2027025) + 86872
5*a*b**2*e*x**7*(f*x)**m/(m**8 + 64*m**7 + 1708*m**6 + 24640*m**5 + 208054*
m**4 + 1038016*m**3 + 2924172*m**2 + 4098240*m + 2027025) + 6*a*b*c*d*m**7*
x**7*(f*x)**m/(m**8 + 64*m**7 + 1708*m**6 + 24640*m**5 + 208054*m**4 + 1038
016*m**3 + 2924172*m**2 + 4098240*m + 2027025) + 342*a*b*c*d*m**6*x**7*(f*x
)**m/(m**8 + 64*m**7 + 1708*m**6 + 24640*m**5 + 208054*m**4 + 1038016*m**3
+ 2924172*m**2 + 4098240*m + 2027025) + 7854*a*b*c*d*m**5*x**7*(f*x)**m/(m*
*8 + 64*m**7 + 1708*m**6 + 24640*m**5 + 208054*m**4 + 1038016*m**3 + 292417
2*m**2 + 4098240*m + 2027025) + 92862*a*b*c*d*m**4*x**7*(f*x)**m/(m**8 + 64
*m**7 + 1708*m**6 + 24640*m**5 + 208054*m**4 + 1038016*m**3 + 2924172*m**2
+ 4098240*m + 2027025) + 598290*a*b*c*d*m**3*x**7*(f*x)**m/(m**8 + 64*m**7
+ 1708*m**6 + 24640*m**5 + 208054*m**4 + 1038016*m**3 + 2924172*m**2 + 4098
240*m + 2027025) + 2040066*a*b*c*d*m**2*x**7*(f*x)**m/(m**8 + 64*m**7 + 170
8*m**6 + 24640*m**5 + 208054*m**4 + 1038016*m**3 + 2924172*m**2 + 4098240*m
+ 2027025) + 3264570*a*b*c*d*m*x**7*(f*x)**m/(m**8 + 64*m**7 + 1708*m**6 +
24640*m**5 + 208054*m**4 + 1038016*m**3 + 2924172*m**2 + 4098240*m + 20270
25) + 1737450*a*b*c*d*x**7*(f*x)**m/(m**8 + 64*m**7 + 1708*m**6 + 24640*m**
5 + 208054*m**4 + 1038016*m**3 + 2924172*m**2 + 4098240*m + 2027025) + 6*a*
b*c*e*m**7*x**9*(f*x)**m/(m**8 + 64*m**7 + 1708*m**6 + 24640*m**5 + 208054*
m**4 + 1038016*m**3 + 2924172*m**2 + 4098240*m + 2027025) + 330*a*b*c*e*m**
6*x**9*(f*x)**m/(m**8 + 64*m**7 + 1708*m**6 + 24640*m**5 + 208054*m**4 + 10
38016*m**3 + 2924172*m**2 + 4098240*m + 2027025) + 7278*a*b*c*e*m**5*x**9*(
f*x)**m/(m**8 + 64*m**7 + 1708*m**6 + 24640*m**5 + 208054*m**4 + 1038016*m*
*3 + 2924172*m**2 + 4098240*m + 2027025) + 82338*a*b*c*e*m**4*x**9*(f*x)**m
/(m**8 + 64*m**7 + 1708*m**6 + 24640*m**5 + 208054*m**4 + 1038016*m**3 + 29
24172*m**2 + 4098240*m + 2027025) + 507282*a*b*c*e*m**3*x**9*(f*x)**m/(m**8
+ 64*m**7 + 1708*m**6 + 24640*m**5 + 208054*m**4 + 1038016*m**3 + 2924172*
m**2 + 4098240*m + 2027025) + 1662558*a*b*c*e*m**2*x**9*(f*x)**m/(m**8 + 64
*m**7 + 1708*m**6 + 24640*m**5 + 208054*m**4 + 1038016*m**3 + 2924172*m**2
+ 4098240*m + 2027025) + 2582010*a*b*c*e*m*x**9*(f*x)**m/(m**8 + 64*m**7 +
1708*m**6 + 24640*m**5 + 208054*m**4 + 1038016*m**3 + 2924172*m**2 + 409824
0*m + 2027025) + 1351350*a*b*c*e*x**9*(f*x)**m/(m**8 + 64*m**7 + 1708*m**6
+ 24640*m**5 + 208054*m**4 + 1038016*m**3 + 2924172*m**2 + 4098240*m + 2027
025) + 3*a*c**2*d*m**7*x**9*(f*x)**m/(m**8 + 64*m**7 + 1708*m**6 + 24640*m*
*5 + 208054*m**4 + 1038016*m**3 + 2924172*m**2 + 4098240*m + 2027025) + 165
*a*c**2*d*m**6*x**9*(f*x)**m/(m**8 + 64*m**7 + 1708*m**6 + 24640*m**5 + 208
054*m**4 + 1038016*m**3 + 2924172*m**2 + 4098240*m + 2027025) + 3639*a*c**2
*d*m**5*x**9*(f*x)**m/(m**8 + 64*m**7 + 1708*m**6 + 24640*m**5 + 208054*m**
4 + 1038016*m**3 + 2924172*m**2 + 4098240*m + 2027025) + 41169*a*c**2*d*m**
4*x**9*(f*x)**m/(m**8 + 64*m**7 + 1708*m**6 + 24640*m**5 + 208054*m**4 + 10
38016*m**3 + 2924172*m**2 + 4098240*m + 2027025) + 253641*a*c**2*d*m**3*x**

9*(f*x)**m/(m**8 + 64*m**7 + 1708*m**6 + 24640*m**5 + 208054*m**4 + 1038016
*m**3 + 2924172*m**2 + 4098240*m + 2027025) + 831279*a*c**2*d*m**2*x**9*(f*
x)**m/(m**8 + 64*m**7 + 1708*m**6 + 24640*m**5 + 208054*m**4 + 1038016*m**3
 + 2924172*m**2 + 4098240*m + 2027025) + 1291005*a*c**2*d*m*x**9*(f*x)**m/(
m**8 + 64*m**7 + 1708*m**6 + 24640*m**5 + 208054*m**4 + 1038016*m**3 + 2924
172*m**2 + 4098240*m + 2027025) + 675675*a*c**2*d*x**9*(f*x)**m/(m**8 + 64*
m**7 + 1708*m**6 + 24640*m**5 + 208054*m**4 + 1038016*m**3 + 2924172*m**2 +
 4098240*m + 2027025) + 3*a*c**2*e*m**7*x**11*(f*x)**m/(m**8 + 64*m**7 + 17
08*m**6 + 24640*m**5 + 208054*m**4 + 1038016*m**3 + 2924172*m**2 + 4098240*
m + 2027025) + 159*a*c**2*e*m**6*x**11*(f*x)**m/(m**8 + 64*m**7 + 1708*m**6
 + 24640*m**5 + 208054*m**4 + 1038016*m**3 + 2924172*m**2 + 4098240*m + 202
7025) + 3375*a*c**2*e*m**5*x**11*(f*x)**m/(m**8 + 64*m**7 + 1708*m**6 + 246
40*m**5 + 208054*m**4 + 1038016*m**3 + 2924172*m**2 + 4098240*m + 2027025)
+ 36795*a*c**2*e*m**4*x**11*(f*x)**m/(m**8 + 64*m**7 + 1708*m**6 + 24640*m*
*5 + 208054*m**4 + 1038016*m**3 + 2924172*m**2 + 4098240*m + 2027025) + 219
417*a*c**2*e*m**3*x**11*(f*x)**m/(m**8 + 64*m**7 + 1708*m**6 + 24640*m**5 +
 208054*m**4 + 1038016*m**3 + 2924172*m**2 + 4098240*m + 2027025) + 700461*
a*c**2*e*m**2*x**11*(f*x)**m/(m**8 + 64*m**7 + 1708*m**6 + 24640*m**5 + 208
054*m**4 + 1038016*m**3 + 2924172*m**2 + 4098240*m + 2027025) + 1067445*a*c
2*e*m*x11*(f*x)**m/(m**8 + 64*m**7 + 1708*m**6 + 24640*m**5 + 208054*m*
*4 + 1038016*m**3 + 2924172*m**2 + 4098240*m + 2027025) + 552825*a*c**2*e*x
11*(f*x)m/(m**8 + 64*m**7 + 1708*m**6 + 24640*m**5 + 208054*m**4 + 1038
016*m**3 + 2924172*m**2 + 4098240*m + 2027025) + b**3*d*m**7*x**7*(f*x)**m/
(m**8 + 64*m**7 + 1708*m**6 + 24640*m**5 + 208054*m**4 + 1038016*m**3 + 292
4172*m**2 + 4098240*m + 2027025) + 57*b**3*d*m**6*x**7*(f*x)**m/(m**8 + 64*
m**7 + 1708*m**6 + 24640*m**5 + 208054*m**4 + 1038016*m**3 + 2924172*m**2 +
 4098240*m + 2027025) + 1309*b**3*d*m**5*x**7*(f*x)**m/(m**8 + 64*m**7 + 17
08*m**6 + 24640*m**5 + 208054*m**4 + 1038016*m**3 + 2924172*m**2 + 4098240*
m + 2027025) + 15477*b**3*d*m**4*x**7*(f*x)**m/(m**8 + 64*m**7 + 1708*m**6
+ 24640*m**5 + 208054*m**4 + 1038016*m**3 + 2924172*m**2 + 4098240*m + 2027
025) + 99715*b**3*d*m**3*x**7*(f*x)**m/(m**8 + 64*m**7 + 1708*m**6 + 24640*
m**5 + 208054*m**4 + 1038016*m**3 + 2924172*m**2 + 4098240*m + 2027025) + 3
40011*b**3*d*m**2*x**7*(f*x)**m/(m**8 + 64*m**7 + 1708*m**6 + 24640*m**5 +
208054*m**4 + 1038016*m**3 + 2924172*m**2 + 4098240*m + 2027025) + 544095*b
3*d*m*x7*(f*x)**m/(m**8 + 64*m**7 + 1708*m**6 + 24640*m**5 + 208054*m**
4 + 1038016*m**3 + 2924172*m**2 + 4098240*m + 2027025) + 289575*b**3*d*x**7
*(f*x)**m/(m**8 + 64*m**7 + 1708*m**6 + 24640*m**5 + 208054*m**4 + 1038016*
m**3 + 2924172*m**2 + 4098240*m + 2027025) + b**3*e*m**7*x**9*(f*x)**m/(m**
8 + 64*m**7 + 1708*m**6 + 24640*m**5 + 208054*m**4 + 1038016*m**3 + 2924172
*m**2 + 4098240*m + 2027025) + 55*b**3*e*m**6*x**9*(f*x)**m/(m**8 + 64*m**7
 + 1708*m**6 + 24640*m**5 + 208054*m**4 + 1038016*m**3 + 2924172*m**2 + 409
8240*m + 2027025) + 1213*b**3*e*m**5*x**9*(f*x)**m/(m**8 + 64*m**7 + 1708*m
6 + 24640*m5 + 208054*m**4 + 1038016*m**3 + 2924172*m**2 + 4098240*m +
2027025) + 13723*b**3*e*m**4*x**9*(f*x)**m/(m**8 + 64*m**7 + 1708*m**6 + 24
640*m**5 + 208054*m**4 + 1038016*m**3 + 2924172*m**2 + 4098240*m + 2027025)

+ 84547*b**3*e*m**3*x**9*(f*x)**m/(m**8 + 64*m**7 + 1708*m**6 + 24640*m**5
+ 208054*m**4 + 1038016*m**3 + 2924172*m**2 + 4098240*m + 2027025) + 27709
3*b**3*e*m**2*x**9*(f*x)**m/(m**8 + 64*m**7 + 1708*m**6 + 24640*m**5 + 2080
54*m**4 + 1038016*m**3 + 2924172*m**2 + 4098240*m + 2027025) + 430335*b**3*
e*m*x**9*(f*x)**m/(m**8 + 64*m**7 + 1708*m**6 + 24640*m**5 + 208054*m**4 +
1038016*m**3 + 2924172*m**2 + 4098240*m + 2027025) + 225225*b**3*e*x**9*(f*
x)**m/(m**8 + 64*m**7 + 1708*m**6 + 24640*m**5 + 208054*m**4 + 1038016*m**3
+ 2924172*m**2 + 4098240*m + 2027025) + 3*b**2*c*d*m**7*x**9*(f*x)**m/(m**
8 + 64*m**7 + 1708*m**6 + 24640*m**5 + 208054*m**4 + 1038016*m**3 + 2924172
*m**2 + 4098240*m + 2027025) + 165*b**2*c*d*m**6*x**9*(f*x)**m/(m**8 + 64*m
7 + 1708*m6 + 24640*m**5 + 208054*m**4 + 1038016*m**3 + 2924172*m**2 +
4098240*m + 2027025) + 3639*b**2*c*d*m**5*x**9*(f*x)**m/(m**8 + 64*m**7 + 1
708*m**6 + 24640*m**5 + 208054*m**4 + 1038016*m**3 + 2924172*m**2 + 4098240
*m + 2027025) + 41169*b**2*c*d*m**4*x**9*(f*x)**m/(m**8 + 64*m**7 + 1708*m*
*6 + 24640*m**5 + 208054*m**4 + 1038016*m**3 + 2924172*m**2 + 4098240*m + 2
027025) + 253641*b**2*c*d*m**3*x**9*(f*x)**m/(m**8 + 64*m**7 + 1708*m**6 +
24640*m**5 + 208054*m**4 + 1038016*m**3 + 2924172*m**2 + 4098240*m + 202702
5) + 831279*b**2*c*d*m**2*x**9*(f*x)**m/(m**8 + 64*m**7 + 1708*m**6 + 24640
*m**5 + 208054*m**4 + 1038016*m**3 + 2924172*m**2 + 4098240*m + 2027025) +
1291005*b**2*c*d*m*x**9*(f*x)**m/(m**8 + 64*m**7 + 1708*m**6 + 24640*m**5 +
208054*m**4 + 1038016*m**3 + 2924172*m**2 + 4098240*m + 2027025) + 675675*
b**2*c*d*x**9*(f*x)**m/(m**8 + 64*m**7 + 1708*m**6 + 24640*m**5 + 208054*m*
*4 + 1038016*m**3 + 2924172*m**2 + 4098240*m + 2027025) + 3*b**2*c*e*m**7*x
11*(f*x)m/(m**8 + 64*m**7 + 1708*m**6 + 24640*m**5 + 208054*m**4 + 1038
016*m**3 + 2924172*m**2 + 4098240*m + 2027025) + 159*b**2*c*e*m**6*x**11*(f
*x)**m/(m**8 + 64*m**7 + 1708*m**6 + 24640*m**5 + 208054*m**4 + 1038016*m**
3 + 2924172*m**2 + 4098240*m + 2027025) + 3375*b**2*c*e*m**5*x**11*(f*x)**m
/(m**8 + 64*m**7 + 1708*m**6 + 24640*m**5 + 208054*m**4 + 1038016*m**3 + 29
24172*m**2 + 4098240*m + 2027025) + 36795*b**2*c*e*m**4*x**11*(f*x)**m/(m**
8 + 64*m**7 + 1708*m**6 + 24640*m**5 + 208054*m**4 + 1038016*m**3 + 2924172
*m**2 + 4098240*m + 2027025) + 219417*b**2*c*e*m**3*x**11*(f*x)**m/(m**8 +
64*m**7 + 1708*m**6 + 24640*m**5 + 208054*m**4 + 1038016*m**3 + 2924172*m**
2 + 4098240*m + 2027025) + 700461*b**2*c*e*m**2*x**11*(f*x)**m/(m**8 + 64*m
7 + 1708*m6 + 24640*m**5 + 208054*m**4 + 1038016*m**3 + 2924172*m**2 +
4098240*m + 2027025) + 1067445*b**2*c*e*m*x**11*(f*x)**m/(m**8 + 64*m**7 +
1708*m**6 + 24640*m**5 + 208054*m**4 + 1038016*m**3 + 2924172*m**2 + 409824
0*m + 2027025) + 552825*b**2*c*e*x**11*(f*x)**m/(m**8 + 64*m**7 + 1708*m**6
+ 24640*m**5 + 208054*m**4 + 1038016*m**3 + 2924172*m**2 + 4098240*m + 202
7025) + 3*b*c**2*d*m**7*x**11*(f*x)**m/(m**8 + 64*m**7 + 1708*m**6 + 24640*
m**5 + 208054*m**4 + 1038016*m**3 + 2924172*m**2 + 4098240*m + 2027025) + 1
59*b*c**2*d*m**6*x**11*(f*x)**m/(m**8 + 64*m**7 + 1708*m**6 + 24640*m**5 +
208054*m**4 + 1038016*m**3 + 2924172*m**2 + 4098240*m + 2027025) + 3375*b*c
2*d*m5*x**11*(f*x)**m/(m**8 + 64*m**7 + 1708*m**6 + 24640*m**5 + 208054
*m**4 + 1038016*m**3 + 2924172*m**2 + 4098240*m + 2027025) + 36795*b*c**2*d
*m**4*x**11*(f*x)**m/(m**8 + 64*m**7 + 1708*m**6 + 24640*m**5 + 208054*m**4

+ 1038016*m**3 + 2924172*m**2 + 4098240*m + 2027025) + 219417*b*c**2*d*m**
3*x**11*(f*x)**m/(m**8 + 64*m**7 + 1708*m**6 + 24640*m**5 + 208054*m**4 + 1
038016*m**3 + 2924172*m**2 + 4098240*m + 2027025) + 700461*b*c**2*d*m**2*x*
11(f*x)**m/(m**8 + 64*m**7 + 1708*m**6 + 24640*m**5 + 208054*m**4 + 10380
16*m**3 + 2924172*m**2 + 4098240*m + 2027025) + 1067445*b*c**2*d*m*x**11*(f
*x)**m/(m**8 + 64*m**7 + 1708*m**6 + 24640*m**5 + 208054*m**4 + 1038016*m**
3 + 2924172*m**2 + 4098240*m + 2027025) + 552825*b*c**2*d*x**11*(f*x)**m/(m
8 + 64*m7 + 1708*m**6 + 24640*m**5 + 208054*m**4 + 1038016*m**3 + 29241
72*m**2 + 4098240*m + 2027025) + 3*b*c**2*e*m**7*x**13*(f*x)**m/(m**8 + 64*
m**7 + 1708*m**6 + 24640*m**5 + 208054*m**4 + 1038016*m**3 + 2924172*m**2 +
4098240*m + 2027025) + 153*b*c**2*e*m**6*x**13*(f*x)**m/(m**8 + 64*m**7 +
1708*m**6 + 24640*m**5 + 208054*m**4 + 1038016*m**3 + 2924172*m**2 + 409824
0*m + 2027025) + 3135*b*c**2*e*m**5*x**13*(f*x)**m/(m**8 + 64*m**7 + 1708*m
6 + 24640*m5 + 208054*m**4 + 1038016*m**3 + 2924172*m**2 + 4098240*m +
2027025) + 33165*b*c**2*e*m**4*x**13*(f*x)**m/(m**8 + 64*m**7 + 1708*m**6 +
24640*m**5 + 208054*m**4 + 1038016*m**3 + 2924172*m**2 + 4098240*m + 20270
25) + 193017*b*c**2*e*m**3*x**13*(f*x)**m/(m**8 + 64*m**7 + 1708*m**6 + 246
40*m**5 + 208054*m**4 + 1038016*m**3 + 2924172*m**2 + 4098240*m + 2027025)
+ 604827*b*c**2*e*m**2*x**13*(f*x)**m/(m**8 + 64*m**7 + 1708*m**6 + 24640*m
5 + 208054*m4 + 1038016*m**3 + 2924172*m**2 + 4098240*m + 2027025) + 90
9765*b*c**2*e*m*x**13*(f*x)**m/(m**8 + 64*m**7 + 1708*m**6 + 24640*m**5 + 2
08054*m**4 + 1038016*m**3 + 2924172*m**2 + 4098240*m + 2027025) + 467775*b*
c**2*e*x**13*(f*x)**m/(m**8 + 64*m**7 + 1708*m**6 + 24640*m**5 + 208054*m**
4 + 1038016*m**3 + 2924172*m**2 + 4098240*m + 2027025) + c**3*d*m**7*x**13*
(f*x)**m/(m**8 + 64*m**7 + 1708*m**6 + 24640*m**5 + 208054*m**4 + 1038016*m
3 + 2924172*m2 + 4098240*m + 2027025) + 51*c**3*d*m**6*x**13*(f*x)**m/(
m**8 + 64*m**7 + 1708*m**6 + 24640*m**5 + 208054*m**4 + 1038016*m**3 + 2924
172*m**2 + 4098240*m + 2027025) + 1045*c**3*d*m**5*x**13*(f*x)**m/(m**8 + 6
4*m**7 + 1708*m**6 + 24640*m**5 + 208054*m**4 + 1038016*m**3 + 2924172*m**2
+ 4098240*m + 2027025) + 11055*c**3*d*m**4*x**13*(f*x)**m/(m**8 + 64*m**7
+ 1708*m**6 + 24640*m**5 + 208054*m**4 + 1038016*m**3 + 2924172*m**2 + 4098
240*m + 2027025) + 64339*c**3*d*m**3*x**13*(f*x)**m/(m**8 + 64*m**7 + 1708*
m**6 + 24640*m**5 + 208054*m**4 + 1038016*m**3 + 2924172*m**2 + 4098240*m +
2027025) + 201609*c**3*d*m**2*x**13*(f*x)**m/(m**8 + 64*m**7 + 1708*m**6 +
24640*m**5 + 208054*m**4 + 1038016*m**3 + 2924172*m**2 + 4098240*m + 20270
25) + 303255*c**3*d*m*x**13*(f*x)**m/(m**8 + 64*m**7 + 1708*m**6 + 24640*m*
*5 + 208054*m**4 + 1038016*m**3 + 2924172*m**2 + 4098240*m + 2027025) + 155
925*c**3*d*x**13*(f*x)**m/(m**8 + 64*m**7 + 1708*m**6 + 24640*m**5 + 208054
*m**4 + 1038016*m**3 + 2924172*m**2 + 4098240*m + 2027025) + c**3*e*m**7*x*
15(f*x)**m/(m**8 + 64*m**7 + 1708*m**6 + 24640*m**5 + 208054*m**4 + 10380
16*m**3 + 2924172*m**2 + 4098240*m + 2027025) + 49*c**3*e*m**6*x**15*(f*x)*
*m/(m**8 + 64*m**7 + 1708*m**6 + 24640*m**5 + 208054*m**4 + 1038016*m**3 +
2924172*m**2 + 4098240*m + 2027025) + 973*c**3*e*m**5*x**15*(f*x)**m/(m**8
+ 64*m**7 + 1708*m**6 + 24640*m**5 + 208054*m**4 + 1038016*m**3 + 2924172*m
2 + 4098240*m + 2027025) + 10045*c3*e*m**4*x**15*(f*x)**m/(m**8 + 64*m*

```
*7 + 1708*m**6 + 24640*m**5 + 208054*m**4 + 1038016*m**3 + 2924172*m**2 + 4
098240*m + 2027025) + 57379*c**3*e*m**3*x**15*(f*x)**m/(m**8 + 64*m**7 + 17
08*m**6 + 24640*m**5 + 208054*m**4 + 1038016*m**3 + 2924172*m**2 + 4098240*
m + 2027025) + 177331*c**3*e*m**2*x**15*(f*x)**m/(m**8 + 64*m**7 + 1708*m**
6 + 24640*m**5 + 208054*m**4 + 1038016*m**3 + 2924172*m**2 + 4098240*m + 20
27025) + 264207*c**3*e*m*x**15*(f*x)**m/(m**8 + 64*m**7 + 1708*m**6 + 24640
*m**5 + 208054*m**4 + 1038016*m**3 + 2924172*m**2 + 4098240*m + 2027025) +
135135*c**3*e*x**15*(f*x)**m/(m**8 + 64*m**7 + 1708*m**6 + 24640*m**5 + 208
054*m**4 + 1038016*m**3 + 2924172*m**2 + 4098240*m + 2027025), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 408, normalized size of antiderivative = 1.68

$$\int (fx)^m (d+ex^2) (a+bx^2+cx^4)^3 \, dx = \frac{c^3ef^mx^{15}x^m}{m+15} + \frac{c^3df^mx^{13}x^m}{m+13} + \frac{3\,bc^2ef^mx^{13}x^m}{m+13} + \frac{3\,bc^2df^mx^{11}x^m}{m+11} + \frac{3\,b^2cef^mx^{11}x^m}{m+11} + \frac{3\,ac^2ef^mx^{11}x^m}{m+11} + \frac{3\,b^2cdf^mx^9x^m}{m+9} + \frac{3\,ac^2df^mx^9x^m}{m+9} + \frac{b^3ef^mx^9x^m}{m+9} + \frac{6\,abcef^mx^9x^m}{m+9} + \frac{b^3df^mx^7x^m}{m+7} + \frac{6\,abcdf^mx^7x^m}{m+7} + \frac{3\,ab^2ef^mx^7x^m}{m+7} + \frac{3\,a^2cef^mx^7x^m}{m+7} + \frac{3\,ab^2df^mx^5x^m}{m+5} + \frac{3\,a^2cdf^mx^5x^m}{m+5} + \frac{3\,a^2bef^mx^5x^m}{m+5} + \frac{3\,a^2bdf^mx^3x^m}{m+3} + \frac{a^3ef^mx^3x^m}{m+3} + \frac{(fx)^{m+1}\,a^3d}{f(m+1)}$$

```
[In] integrate((f*x)^m*(e*x^2+d)*(c*x^4+b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] c^3*e*f^m*x^15*x^m/(m + 15) + c^3*d*f^m*x^13*x^m/(m + 13) + 3*b*c^2*e*f^m*x
^13*x^m/(m + 13) + 3*b*c^2*d*f^m*x^11*x^m/(m + 11) + 3*b^2*c*e*f^m*x^11*x^m
/(m + 11) + 3*a*c^2*e*f^m*x^11*x^m/(m + 11) + 3*b^2*c*d*f^m*x^9*x^m/(m + 9)
 + 3*a*c^2*d*f^m*x^9*x^m/(m + 9) + b^3*e*f^m*x^9*x^m/(m + 9) + 6*a*b*c*e*f^
m*x^9*x^m/(m + 9) + b^3*d*f^m*x^7*x^m/(m + 7) + 6*a*b*c*d*f^m*x^7*x^m/(m +
7) + 3*a*b^2*e*f^m*x^7*x^m/(m + 7) + 3*a^2*c*e*f^m*x^7*x^m/(m + 7) + 3*a*b^
2*d*f^m*x^5*x^m/(m + 5) + 3*a^2*c*d*f^m*x^5*x^m/(m + 5) + 3*a^2*b*e*f^m*x^5
*x^m/(m + 5) + 3*a^2*b*d*f^m*x^3*x^m/(m + 3) + a^3*e*f^m*x^3*x^m/(m + 3) +
(f*x)^(m + 1)*a^3*d/(f*(m + 1))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2736 vs. 2(243) = 486.

Time = 0.36 (sec) , antiderivative size = 2736, normalized size of antiderivative = 11.26

$$\int (fx)^m (d+ex^2) (a+bx^2+cx^4)^3 \, dx = \text{Too large to display}$$

```
[In] integrate((f*x)^m*(e*x^2+d)*(c*x^4+b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] ((f*x)^m*c^3*e*m^7*x^15 + 49*(f*x)^m*c^3*e*m^6*x^15 + (f*x)^m*c^3*d*m^7*x^1
3 + 3*(f*x)^m*b*c^2*e*m^7*x^13 + 973*(f*x)^m*c^3*e*m^5*x^15 + 51*(f*x)^m*c^
3*d*m^6*x^13 + 153*(f*x)^m*b*c^2*e*m^6*x^13 + 10045*(f*x)^m*c^3*e*m^4*x^15
+ 3*(f*x)^m*b*c^2*d*m^7*x^11 + 3*(f*x)^m*b^2*c*e*m^7*x^11 + 3*(f*x)^m*a*c^2
*e*m^7*x^11 + 1045*(f*x)^m*c^3*d*m^5*x^13 + 3135*(f*x)^m*b*c^2*e*m^5*x^13 +
 57379*(f*x)^m*c^3*e*m^3*x^15 + 159*(f*x)^m*b*c^2*d*m^6*x^11 + 159*(f*x)^m*
b^2*c*e*m^6*x^11 + 159*(f*x)^m*a*c^2*e*m^6*x^11 + 11055*(f*x)^m*c^3*d*m^4*x
^13 + 33165*(f*x)^m*b*c^2*e*m^4*x^13 + 177331*(f*x)^m*c^3*e*m^2*x^15 + 3*(f
*x)^m*b^2*c*d*m^7*x^9 + 3*(f*x)^m*a*c^2*d*m^7*x^9 + (f*x)^m*b^3*e*m^7*x^9 +
 6*(f*x)^m*a*b*c*e*m^7*x^9 + 3375*(f*x)^m*b*c^2*d*m^5*x^11 + 3375*(f*x)^m*b
^2*c*e*m^5*x^11 + 3375*(f*x)^m*a*c^2*e*m^5*x^11 + 64339*(f*x)^m*c^3*d*m^3*x
^13 + 193017*(f*x)^m*b*c^2*e*m^3*x^13 + 264207*(f*x)^m*c^3*e*m*x^15 + 165*(
f*x)^m*b^2*c*d*m^6*x^9 + 165*(f*x)^m*a*c^2*d*m^6*x^9 + 55*(f*x)^m*b^3*e*m^6
*x^9 + 330*(f*x)^m*a*b*c*e*m^6*x^9 + 36795*(f*x)^m*b*c^2*d*m^4*x^11 + 36795
*(f*x)^m*b^2*c*e*m^4*x^11 + 36795*(f*x)^m*a*c^2*e*m^4*x^11 + 201609*(f*x)^m
*c^3*d*m^2*x^13 + 604827*(f*x)^m*b*c^2*e*m^2*x^13 + 135135*(f*x)^m*c^3*e*x^
15 + (f*x)^m*b^3*d*m^7*x^7 + 6*(f*x)^m*a*b*c*d*m^7*x^7 + 3*(f*x)^m*a*b^2*e*
m^7*x^7 + 3*(f*x)^m*a^2*c*e*m^7*x^7 + 3639*(f*x)^m*b^2*c*d*m^5*x^9 + 3639*(
f*x)^m*a*c^2*d*m^5*x^9 + 1213*(f*x)^m*b^3*e*m^5*x^9 + 7278*(f*x)^m*a*b*c*e*
m^5*x^9 + 219417*(f*x)^m*b*c^2*d*m^3*x^11 + 219417*(f*x)^m*b^2*c*e*m^3*x^11
 + 219417*(f*x)^m*a*c^2*e*m^3*x^11 + 303255*(f*x)^m*c^3*d*m*x^13 + 909765*(
f*x)^m*b*c^2*e*m*x^13 + 57*(f*x)^m*b^3*d*m^6*x^7 + 342*(f*x)^m*a*b*c*d*m^6*
x^7 + 171*(f*x)^m*a*b^2*e*m^6*x^7 + 171*(f*x)^m*a^2*c*e*m^6*x^7 + 41169*(f*
x)^m*b^2*c*d*m^4*x^9 + 41169*(f*x)^m*a*c^2*d*m^4*x^9 + 13723*(f*x)^m*b^3*e*
m^4*x^9 + 82338*(f*x)^m*a*b*c*e*m^4*x^9 + 700461*(f*x)^m*b*c^2*d*m^2*x^11 +
 700461*(f*x)^m*b^2*c*e*m^2*x^11 + 700461*(f*x)^m*a*c^2*e*m^2*x^11 + 155925
*(f*x)^m*c^3*d*x^13 + 467775*(f*x)^m*b*c^2*e*x^13 + 3*(f*x)^m*a*b^2*d*m^7*x
^5 + 3*(f*x)^m*a^2*c*d*m^7*x^5 + 3*(f*x)^m*a^2*b*e*m^7*x^5 + 1309*(f*x)^m*b
^3*d*m^5*x^7 + 7854*(f*x)^m*a*b*c*d*m^5*x^7 + 3927*(f*x)^m*a*b^2*e*m^5*x^7
+ 3927*(f*x)^m*a^2*c*e*m^5*x^7 + 253641*(f*x)^m*b^2*c*d*m^3*x^9 + 253641*(f
*x)^m*a*c^2*d*m^3*x^9 + 84547*(f*x)^m*b^3*e*m^3*x^9 + 507282*(f*x)^m*a*b*c*
e*m^3*x^9 + 1067445*(f*x)^m*b*c^2*d*m*x^11 + 1067445*(f*x)^m*b^2*c*e*m*x^11
 + 1067445*(f*x)^m*a*c^2*e*m*x^11 + 177*(f*x)^m*a*b^2*d*m^6*x^5 + 177*(f*x)
^m*a^2*c*d*m^6*x^5 + 177*(f*x)^m*a^2*b*e*m^6*x^5 + 15477*(f*x)^m*b^3*d*m^4*
x^7 + 92862*(f*x)^m*a*b*c*d*m^4*x^7 + 46431*(f*x)^m*a*b^2*e*m^4*x^7 + 46431
```

*(f*x)^m*a^2*c*e*m^4*x^7 + 831279*(f*x)^m*b^2*c*d*m^2*x^9 + 831279*(f*x)^m*a*c^2*d*m^2*x^9 + 277093*(f*x)^m*b^3*e*m^2*x^9 + 1662558*(f*x)^m*a*b*c*e*m^2*x^9 + 552825*(f*x)^m*b*c^2*d*x^11 + 552825*(f*x)^m*b^2*c*e*x^11 + 552825*(f*x)^m*a*c^2*e*x^11 + 3*(f*x)^m*a^2*b*d*m^7*x^3 + (f*x)^m*a^3*e*m^7*x^3 + 4239*(f*x)^m*a*b^2*d*m^5*x^5 + 4239*(f*x)^m*a^2*c*d*m^5*x^5 + 4239*(f*x)^m*a^2*b*e*m^5*x^5 + 99715*(f*x)^m*b^3*d*m^3*x^7 + 598290*(f*x)^m*a*b*c*d*m^3*x^7 + 299145*(f*x)^m*a*b^2*e*m^3*x^7 + 299145*(f*x)^m*a^2*c*e*m^3*x^7 + 1291005*(f*x)^m*b^2*c*d*m*x^9 + 1291005*(f*x)^m*a*c^2*d*m*x^9 + 430335*(f*x)^m*b^3*e*m*x^9 + 2582010*(f*x)^m*a*b*c*e*m*x^9 + 183*(f*x)^m*a^2*b*d*m^6*x^3 + 61*(f*x)^m*a^3*e*m^6*x^3 + 52725*(f*x)^m*a*b^2*d*m^4*x^5 + 52725*(f*x)^m*a^2*c*d*m^4*x^5 + 52725*(f*x)^m*a^2*b*e*m^4*x^5 + 340011*(f*x)^m*b^3*d*m^2*x^7 + 2040066*(f*x)^m*a*b*c*d*m^2*x^7 + 1020033*(f*x)^m*a*b^2*e*m^2*x^7 + 1020033*(f*x)^m*a^2*c*e*m^2*x^7 + 675675*(f*x)^m*b^2*c*d*x^9 + 675675*(f*x)^m*a*c^2*d*x^9 + 225225*(f*x)^m*b^3*e*x^9 + 1351350*(f*x)^m*a*b*c*e*x^9 + (f*x)^m*a^3*d*m^7*x + 4575*(f*x)^m*a^2*b*d*m^5*x^3 + 1525*(f*x)^m*a^3*e*m^5*x^3 + 360537*(f*x)^m*a*b^2*d*m^3*x^5 + 360537*(f*x)^m*a^2*c*d*m^3*x^5 + 360537*(f*x)^m*a^2*b*e*m^3*x^5 + 544095*(f*x)^m*b^3*d*m*x^7 + 3264570*(f*x)^m*a*b*c*d*m*x^7 + 1632285*(f*x)^m*a*b^2*e*m*x^7 + 1632285*(f*x)^m*a^2*c*e*m*x^7 + 63*(f*x)^m*a^3*d*m^6*x + 60195*(f*x)^m*a^2*b*d*m^4*x^3 + 20065*(f*x)^m*a^3*e*m^4*x^3 + 1311363*(f*x)^m*a*b^2*d*m^2*x^5 + 1311363*(f*x)^m*a^2*c*d*m^2*x^5 + 1311363*(f*x)^m*a^2*b*e*m^2*x^5 + 289575*(f*x)^m*b^3*d*x^7 + 1737450*(f*x)^m*a*b*c*d*x^7 + 868725*(f*x)^m*a*b^2*e*x^7 + 868725*(f*x)^m*a^2*c*e*x^7 + 1645*(f*x)^m*a^3*d*m^5*x + 443577*(f*x)^m*a^2*b*d*m^3*x^3 + 147859*(f*x)^m*a^3*e*m^3*x^3 + 2215701*(f*x)^m*a*b^2*d*m*x^5 + 2215701*(f*x)^m*a^2*c*d*m*x^5 + 2215701*(f*x)^m*a^2*b*e*m*x^5 + 22995*(f*x)^m*a^3*d*m^4*x + 1783317*(f*x)^m*a^2*b*d*m^2*x^3 + 594439*(f*x)^m*a^3*e*m^2*x^3 + 1216215*(f*x)^m*a*b^2*d*x^5 + 1216215*(f*x)^m*a^2*c*d*x^5 + 1216215*(f*x)^m*a^2*b*e*x^5 + 185059*(f*x)^m*a^3*d*m^3*x + 3422565*(f*x)^m*a^2*b*d*m*x^3 + 1140855*(f*x)^m*a^3*e*m*x^3 + 852957*(f*x)^m*a^3*d*m^2*x + 2027025*(f*x)^m*a^2*b*d*x^3 + 675675*(f*x)^m*a^3*e*x^3 + 2071215*(f*x)^m*a^3*d*m*x + 2027025*(f*x)^m*a^3*d*x)/(m^8 + 64*m^7 + 1708*m^6 + 24640*m^5 + 208054*m^4 + 1038016*m^3 + 2924172*m^2 + 4098240*m + 2027025)

Mupad [B] (verification not implemented)

Time = 8.07 (sec) , antiderivative size = 769, normalized size of antiderivative = 3.16

$$\int (fx)^m \left(d+ex^2\right) \left(a+bx^2+cx^4\right)^3 \, dx$$
$$= \frac{x^7\,(f\,x)^m\,(3\,c\,e\,a^2+3\,e\,a\,b^2+6\,c\,d\,a\,b+d\,b^3)\,\left(m^7+57\,m^6+1309\,m^5+15477\,m^4+99715\,m^3+340011\,m\right.}{m^8+64\,m^7+1708\,m^6+24640\,m^5+208054\,m^4+1038016\,m^3+2924172\,m^2+4098240\,m}$$
$$+\frac{x^9\,(f\,x)^m\,(e\,b^3+3\,d\,b^2\,c+6\,a\,e\,b\,c+3\,a\,d\,c^2)\,\left(m^7+55\,m^6+1213\,m^5+13723\,m^4+84547\,m^3+27709\right.}{m^8+64\,m^7+1708\,m^6+24640\,m^5+208054\,m^4+1038016\,m^3+2924172\,m^2+4098240}$$
$$+\frac{a^3\,d\,x\,(f\,x)^m\,(m^7+63\,m^6+1645\,m^5+22995\,m^4+185059\,m^3+852957\,m^2+2071215\,m+2027025)}{m^8+64\,m^7+1708\,m^6+24640\,m^5+208054\,m^4+1038016\,m^3+2924172\,m^2+4098240\,m+2027025}$$
$$+\frac{c^3\,e\,x^{15}\,(f\,x)^m\,(m^7+49\,m^6+973\,m^5+10045\,m^4+57379\,m^3+177331\,m^2+264207\,m+135135)}{m^8+64\,m^7+1708\,m^6+24640\,m^5+208054\,m^4+1038016\,m^3+2924172\,m^2+4098240\,m+2027025}$$
$$+\frac{3\,a\,x^5\,(f\,x)^m\,(d\,b^2+a\,e\,b+a\,c\,d)\,\left(m^7+59\,m^6+1413\,m^5+17575\,m^4+120179\,m^3+437121\,m^2+738\right.}{m^8+64\,m^7+1708\,m^6+24640\,m^5+208054\,m^4+1038016\,m^3+2924172\,m^2+4098240\,m+}$$
$$+\frac{3\,c\,x^{11}\,(f\,x)^m\,(e\,b^2+c\,d\,b+a\,c\,e)\,\left(m^7+53\,m^6+1125\,m^5+12265\,m^4+73139\,m^3+233487\,m^2+3558\right.}{m^8+64\,m^7+1708\,m^6+24640\,m^5+208054\,m^4+1038016\,m^3+2924172\,m^2+4098240\,m+2}$$
$$+\frac{a^2\,x^3\,(f\,x)^m\,(a\,e+3\,b\,d)\,\left(m^7+61\,m^6+1525\,m^5+20065\,m^4+147859\,m^3+594439\,m^2+1140855\,m+\right.}{m^8+64\,m^7+1708\,m^6+24640\,m^5+208054\,m^4+1038016\,m^3+2924172\,m^2+4098240\,m+202}$$
$$+\frac{c^2\,x^{13}\,(f\,x)^m\,(3\,b\,e+c\,d)\,\left(m^7+51\,m^6+1045\,m^5+11055\,m^4+64339\,m^3+201609\,m^2+303255\,m+1\right.}{m^8+64\,m^7+1708\,m^6+24640\,m^5+208054\,m^4+1038016\,m^3+2924172\,m^2+4098240\,m+2027}$$

```
[In] int((f*x)^m*(d + e*x^2)*(a + b*x^2 + c*x^4)^3,x)
```

```
[Out] (x^7*(f*x)^m*(b^3*d + 3*a*b^2*e + 3*a^2*c*e + 6*a*b*c*d)*(544095*m + 340011
*m^2 + 99715*m^3 + 15477*m^4 + 1309*m^5 + 57*m^6 + m^7 + 289575))/(4098240*
m + 2924172*m^2 + 1038016*m^3 + 208054*m^4 + 24640*m^5 + 1708*m^6 + 64*m^7
+ m^8 + 2027025) + (x^9*(f*x)^m*(b^3*e + 3*a*c^2*d + 3*b^2*c*d + 6*a*b*c*e)
*(430335*m + 277093*m^2 + 84547*m^3 + 13723*m^4 + 1213*m^5 + 55*m^6 + m^7 +
 225225))/(4098240*m + 2924172*m^2 + 1038016*m^3 + 208054*m^4 + 24640*m^5 +
 1708*m^6 + 64*m^7 + m^8 + 2027025) + (a^3*d*x*(f*x)^m*(2071215*m + 852957*
m^2 + 185059*m^3 + 22995*m^4 + 1645*m^5 + 63*m^6 + m^7 + 2027025))/(4098240
*m + 2924172*m^2 + 1038016*m^3 + 208054*m^4 + 24640*m^5 + 1708*m^6 + 64*m^7
 + m^8 + 2027025) + (c^3*e*x^15*(f*x)^m*(264207*m + 177331*m^2 + 57379*m^3
+ 10045*m^4 + 973*m^5 + 49*m^6 + m^7 + 135135))/(4098240*m + 2924172*m^2 +
1038016*m^3 + 208054*m^4 + 24640*m^5 + 1708*m^6 + 64*m^7 + m^8 + 2027025) +
 (3*a*x^5*(f*x)^m*(b^2*d + a*b*e + a*c*d)*(738567*m + 437121*m^2 + 120179*m
^3 + 17575*m^4 + 1413*m^5 + 59*m^6 + m^7 + 405405))/(4098240*m + 2924172*m^
2 + 1038016*m^3 + 208054*m^4 + 24640*m^5 + 1708*m^6 + 64*m^7 + m^8 + 202702
5) + (3*c*x^11*(f*x)^m*(b^2*e + a*c*e + b*c*d)*(355815*m + 233487*m^2 + 731
39*m^3 + 12265*m^4 + 1125*m^5 + 53*m^6 + m^7 + 184275))/(4098240*m + 292417
2*m^2 + 1038016*m^3 + 208054*m^4 + 24640*m^5 + 1708*m^6 + 64*m^7 + m^8 + 20
27025) + (a^2*x^3*(f*x)^m*(a*e + 3*b*d)*(1140855*m + 594439*m^2 + 147859*m^
3 + 20065*m^4 + 1525*m^5 + 61*m^6 + m^7 + 675675))/(4098240*m + 2924172*m^2
```

```
 + 1038016*m^3 + 208054*m^4 + 24640*m^5 + 1708*m^6 + 64*m^7 + m^8 + 2027025
) + (c^2*x^13*(f*x)^m*(3*b*e + c*d)*(303255*m + 201609*m^2 + 64339*m^3 + 11
055*m^4 + 1045*m^5 + 51*m^6 + m^7 + 155925))/(4098240*m + 2924172*m^2 + 103
8016*m^3 + 208054*m^4 + 24640*m^5 + 1708*m^6 + 64*m^7 + m^8 + 2027025)
```

3.221 $\int (fx)^m (d+ex^2) (a+bx^2+cx^4)^2 \, dx$

Optimal result

Integrand size = 27, antiderivative size = 155

$$\int (fx)^m (d+ex^2) (a+bx^2+cx^4)^2 \, dx = \frac{a^2 d (fx)^{1+m}}{f(1+m)} + \frac{a(2bd+ae)(fx)^{3+m}}{f^3(3+m)} + \frac{(b^2d+2acd+2abe)(fx)^{5+m}}{f^5(5+m)} + \frac{(2bcd+b^2e+2ace)(fx)^{7+m}}{f^7(7+m)} + \frac{c(cd+2be)(fx)^{9+m}}{f^9(9+m)} + \frac{c^2e(fx)^{11+m}}{f^{11}(11+m)}$$

```
[Out] a^2*d*(f*x)^(1+m)/f/(1+m)+a*(a*e+2*b*d)*(f*x)^(3+m)/f^3/(3+m)+(2*a*b*e+2*a*
c*d+b^2*d)*(f*x)^(5+m)/f^5/(5+m)+(2*a*c*e+b^2*e+2*b*c*d)*(f*x)^(7+m)/f^7/(7
+m)+c*(2*b*e+c*d)*(f*x)^(9+m)/f^9/(9+m)+c^2*e*(f*x)^(11+m)/f^11/(11+m)
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.037, Rules used = {1275}

$$\int (fx)^m (d+ex^2) (a+bx^2+cx^4)^2 \, dx = \frac{a^2 d (fx)^{m+1}}{f(m+1)} + \frac{(fx)^{m+7} (2ace+b^2e+2bcd)}{f^7(m+7)} + \frac{(fx)^{m+5} (2abe+2acd+b^2d)}{f^5(m+5)} + \frac{a(fx)^{m+3}(ae+2bd)}{f^3(m+3)} + \frac{c(fx)^{m+9}(2be+cd)}{f^9(m+9)} + \frac{c^2e(fx)^{m+11}}{f^{11}(m+11)}$$

```
[In] Int[(f*x)^m*(d + e*x^2)*(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] (a^2*d*(f*x)^(1 + m))/(f*(1 + m)) + (a*(2*b*d + a*e)*(f*x)^(3 + m))/(f^3*(3
 + m)) + ((b^2*d + 2*a*c*d + 2*a*b*e)*(f*x)^(5 + m))/(f^5*(5 + m)) + ((2*b*
c*d + b^2*e + 2*a*c*e)*(f*x)^(7 + m))/(f^7*(7 + m)) + (c*(c*d + 2*b*e)*(f*x
)^(9 + m))/(f^9*(9 + m)) + (c^2*e*(f*x)^(11 + m))/(f^11*(11 + m))
```

Rule 1275

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*
(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && NeQ[
b^2 - 4*a*c, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \left(a^2d(fx)^m + \frac{a(2bd+ae)(fx)^{2+m}}{f^2} + \frac{(b^2d+2acd+2abe)(fx)^{4+m}}{f^4} + \frac{(2bcd+b^2e+2ace)(fx)^{6+m}}{f^6} + \frac{c(cd+2be)(fx)^{8+m}}{f^8} + \frac{c^2e(fx)^{10+m}}{f^{10}}\right) dx \\ &= \frac{a^2d(fx)^{1+m}}{f(1+m)} + \frac{a(2bd+ae)(fx)^{3+m}}{f^3(3+m)} + \frac{(b^2d+2acd+2abe)(fx)^{5+m}}{f^5(5+m)} + \frac{(2bcd+b^2e+2ace)(fx)^{7+m}}{f^7(7+m)} + \frac{c(cd+2be)(fx)^{9+m}}{f^9(9+m)} + \frac{c^2e(fx)^{11+m}}{f^{11}(11+m)}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.45 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.75

$$\int (fx)^m (d+ex^2)(a+bx^2+cx^4)^2 \, dx = x(fx)^m \left(\frac{a^2d}{1+m} + \frac{a(2bd+ae)x^2}{3+m} + \frac{(b^2d+2acd+2abe)x^4}{5+m} + \frac{(2bcd+b^2e+2ace)x^6}{7+m} + \frac{c(cd+2be)x^8}{9+m} + \frac{c^2ex^{10}}{11+m}\right)$$

```
[In] Integrate[(f*x)^m*(d + e*x^2)*(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] x*(f*x)^m*((a^2*d)/(1 + m) + (a*(2*b*d + a*e)*x^2)/(3 + m) + ((b^2*d + 2*a*
c*d + 2*a*b*e)*x^4)/(5 + m) + ((2*b*c*d + b^2*e + 2*a*c*e)*x^6)/(7 + m) + (
c*(c*d + 2*b*e)*x^8)/(9 + m) + (c^2*e*x^10)/(11 + m))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 782 vs. $2(155) = 310$.

Time = 0.17 (sec) , antiderivative size = 783, normalized size of antiderivative = 5.05

method	result
gosper	$\frac{x(c^2 e m^5 x^{10}+25c^2 e m^4 x^{10}+2bce m^5 x^8+c^2 d m^5 x^8+230c^2 e m^3 x^{10}+54bce m^4 x^8+27c^2 d m^4 x^8+950c^2 e m^2 x^{10}+2ace m^5 x^6+b^2}{}$
risch	$\frac{x(c^2 e m^5 x^{10}+25c^2 e m^4 x^{10}+2bce m^5 x^8+c^2 d m^5 x^8+230c^2 e m^3 x^{10}+54bce m^4 x^8+27c^2 d m^4 x^8+950c^2 e m^2 x^{10}+2ace m^5 x^6+b^2}{}$
parallelrisch	Expression too large to display

```
[In] int((f*x)^m*(e*x^2+d)*(c*x^4+b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] x*(c^2*e*m^5*x^10+25*c^2*e*m^4*x^10+2*b*c*e*m^5*x^8+c^2*d*m^5*x^8+230*c^2*e
*m^3*x^10+54*b*c*e*m^4*x^8+27*c^2*d*m^4*x^8+950*c^2*e*m^2*x^10+2*a*c*e*m^5*
x^6+b^2*e*m^5*x^6+2*b*c*d*m^5*x^6+524*b*c*e*m^3*x^8+262*c^2*d*m^3*x^8+1689*
c^2*e*m*x^10+58*a*c*e*m^4*x^6+29*b^2*e*m^4*x^6+58*b*c*d*m^4*x^6+2244*b*c*e*
m^2*x^8+1122*c^2*d*m^2*x^8+945*c^2*e*x^10+2*a*b*e*m^5*x^4+2*a*c*d*m^5*x^4+6
04*a*c*e*m^3*x^6+b^2*d*m^5*x^4+302*b^2*e*m^3*x^6+604*b*c*d*m^3*x^6+4082*b*c
*e*m*x^8+2041*c^2*d*m*x^8+62*a*b*e*m^4*x^4+62*a*c*d*m^4*x^4+2732*a*c*e*m^2*
x^6+31*b^2*d*m^4*x^4+1366*b^2*e*m^2*x^6+2732*b*c*d*m^2*x^6+2310*b*c*e*x^8+1
155*c^2*d*x^8+a^2*e*m^5*x^2+2*a*b*d*m^5*x^2+700*a*b*e*m^3*x^4+700*a*c*d*m^3
*x^4+5154*a*c*e*m*x^6+350*b^2*d*m^3*x^4+2577*b^2*e*m*x^6+5154*b*c*d*m*x^6+3
3*a^2*e*m^4*x^2+66*a*b*d*m^4*x^2+3460*a*b*e*m^2*x^4+3460*a*c*d*m^2*x^4+2970
*a*c*e*x^6+1730*b^2*d*m^2*x^4+1485*b^2*e*x^6+2970*b*c*d*x^6+a^2*d*m^5+406*a
^2*e*m^3*x^2+812*a*b*d*m^3*x^2+6978*a*b*e*m*x^4+6978*a*c*d*m*x^4+3489*b^2*d
*m*x^4+35*a^2*d*m^4+2262*a^2*e*m^2*x^2+4524*a*b*d*m^2*x^2+4158*a*b*e*x^4+41
58*a*c*d*x^4+2079*b^2*d*x^4+470*a^2*d*m^3+5353*a^2*e*m*x^2+10706*a*b*d*m*x^
2+3010*a^2*d*m^2+3465*a^2*e*x^2+6930*a*b*d*x^2+9129*a^2*d*m+10395*a^2*d)*(f
*x)^m/(11+m)/(9+m)/(7+m)/(5+m)/(3+m)/(1+m)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 573 vs. $2(155) = 310$.

Time = 0.25 (sec) , antiderivative size = 573, normalized size of antiderivative = 3.70

$$\int (fx)^m \left(d+ex^2\right) \left(a+bx^2+cx^4\right)^2 \, dx$$
$$= \frac{((c^2em^5 + 25\,c^2em^4 + 230\,c^2em^3 + 950\,c^2em^2 + 1689\,c^2em + 945\,c^2e)x^{11} + ((c^2d + 2\,bce)m^5 + 27\,(c^2d +}{}$$

```
[In] integrate((f*x)^m*(e*x^2+d)*(c*x^4+b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] ((c^2*e*m^5 + 25*c^2*e*m^4 + 230*c^2*e*m^3 + 950*c^2*e*m^2 + 1689*c^2*e*m +
 945*c^2*e)*x^11 + ((c^2*d + 2*b*c*e)*m^5 + 27*(c^2*d + 2*b*c*e)*m^4 + 262*
```

```
(c^2*d + 2*b*c*e)*m^3 + 1155*c^2*d + 2310*b*c*e + 1122*(c^2*d + 2*b*c*e)*m^
2 + 2041*(c^2*d + 2*b*c*e)*m)*x^9 + ((2*b*c*d + (b^2 + 2*a*c)*e)*m^5 + 29*(
2*b*c*d + (b^2 + 2*a*c)*e)*m^4 + 302*(2*b*c*d + (b^2 + 2*a*c)*e)*m^3 + 2970
*b*c*d + 1366*(2*b*c*d + (b^2 + 2*a*c)*e)*m^2 + 1485*(b^2 + 2*a*c)*e + 2577
*(2*b*c*d + (b^2 + 2*a*c)*e)*m)*x^7 + ((2*a*b*e + (b^2 + 2*a*c)*d)*m^5 + 31
*(2*a*b*e + (b^2 + 2*a*c)*d)*m^4 + 350*(2*a*b*e + (b^2 + 2*a*c)*d)*m^3 + 41
58*a*b*e + 1730*(2*a*b*e + (b^2 + 2*a*c)*d)*m^2 + 2079*(b^2 + 2*a*c)*d + 34
89*(2*a*b*e + (b^2 + 2*a*c)*d)*m)*x^5 + ((2*a*b*d + a^2*e)*m^5 + 33*(2*a*b*
d + a^2*e)*m^4 + 406*(2*a*b*d + a^2*e)*m^3 + 6930*a*b*d + 3465*a^2*e + 2262
*(2*a*b*d + a^2*e)*m^2 + 5353*(2*a*b*d + a^2*e)*m)*x^3 + (a^2*d*m^5 + 35*a^
2*d*m^4 + 470*a^2*d*m^3 + 3010*a^2*d*m^2 + 9129*a^2*d*m + 10395*a^2*d)*x)*(
f*x)^m/(m^6 + 36*m^5 + 505*m^4 + 3480*m^3 + 12139*m^2 + 19524*m + 10395)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 4068 vs. $2(146) = 292$.

Time = 0.88 (sec) , antiderivative size = 4068, normalized size of antiderivative = 26.25

$$\int (fx)^m \left(d + ex^2\right) \left(a + bx^2 + cx^4\right)^2 \, dx = \text{Too large to display}$$

```
[In] integrate((f*x)**m*(e*x**2+d)*(c*x**4+b*x**2+a)**2,x)
```

```
[Out] Piecewise(((-a**2*d/(10*x**10) - a**2*e/(8*x**8) - a*b*d/(4*x**8) - a*b*e/(
3*x**6) - a*c*d/(3*x**6) - a*c*e/(2*x**4) - b**2*d/(6*x**6) - b**2*e/(4*x**
4) - b*c*d/(2*x**4) - b*c*e/x**2 - c**2*d/(2*x**2) + c**2*e*log(x))/f**11,
Eq(m, -11)), ((-a**2*d/(8*x**8) - a**2*e/(6*x**6) - a*b*d/(3*x**6) - a*b*e/
(2*x**4) - a*c*d/(2*x**4) - a*c*e/x**2 - b**2*d/(4*x**4) - b**2*e/(2*x**2)
- b*c*d/x**2 + 2*b*c*e*log(x) + c**2*d*log(x) + c**2*e*x**2/2)/f**9, Eq(m,
-9)), ((-a**2*d/(6*x**6) - a**2*e/(4*x**4) - a*b*d/(2*x**4) - a*b*e/x**2 -
a*c*d/x**2 + 2*a*c*e*log(x) - b**2*d/(2*x**2) + b**2*e*log(x) + 2*b*c*d*log
(x) + b*c*e*x**2 + c**2*d*x**2/2 + c**2*e*x**4/4)/f**7, Eq(m, -7)), ((-a**2
*d/(4*x**4) - a**2*e/(2*x**2) - a*b*d/x**2 + 2*a*b*e*log(x) + 2*a*c*d*log(x
) + a*c*e*x**2 + b**2*d*log(x) + b**2*e*x**2/2 + b*c*d*x**2 + b*c*e*x**4/2
+ c**2*d*x**4/4 + c**2*e*x**6/6)/f**5, Eq(m, -5)), ((-a**2*d/(2*x**2) + a**
2*e*log(x) + 2*a*b*d*log(x) + a*b*e*x**2 + a*c*d*x**2 + a*c*e*x**4/2 + b**2
*d*x**2/2 + b**2*e*x**4/4 + b*c*d*x**4/2 + b*c*e*x**6/3 + c**2*d*x**6/6 + c
**2*e*x**8/8)/f**3, Eq(m, -3)), ((a**2*d*log(x) + a**2*e*x**2/2 + a*b*d*x**
2 + a*b*e*x**4/2 + a*c*d*x**4/2 + a*c*e*x**6/3 + b**2*d*x**4/4 + b**2*e*x**
6/6 + b*c*d*x**6/3 + b*c*e*x**8/4 + c**2*d*x**8/8 + c**2*e*x**10/10)/f, Eq(
m, -1)), (a**2*d*m**5*x*(f*x)**m/(m**6 + 36*m**5 + 505*m**4 + 3480*m**3 + 1
2139*m**2 + 19524*m + 10395) + 35*a**2*d*m**4*x*(f*x)**m/(m**6 + 36*m**5 +
505*m**4 + 3480*m**3 + 12139*m**2 + 19524*m + 10395) + 470*a**2*d*m**3*x*(f
*x)**m/(m**6 + 36*m**5 + 505*m**4 + 3480*m**3 + 12139*m**2 + 19524*m + 1039
5) + 3010*a**2*d*m**2*x*(f*x)**m/(m**6 + 36*m**5 + 505*m**4 + 3480*m**3 + 1
```

2139*m**2 + 19524*m + 10395) + 9129*a**2*d*m*x*(f*x)**m/(m**6 + 36*m**5 + 5
05*m**4 + 3480*m**3 + 12139*m**2 + 19524*m + 10395) + 10395*a**2*d*x*(f*x)*
*m/(m**6 + 36*m**5 + 505*m**4 + 3480*m**3 + 12139*m**2 + 19524*m + 10395) +
 a**2*e*m**5*x**3*(f*x)**m/(m**6 + 36*m**5 + 505*m**4 + 3480*m**3 + 12139*m
2 + 19524*m + 10395) + 33*a2*e*m**4*x**3*(f*x)**m/(m**6 + 36*m**5 + 505
*m**4 + 3480*m**3 + 12139*m**2 + 19524*m + 10395) + 406*a**2*e*m**3*x**3*(f
*x)**m/(m**6 + 36*m**5 + 505*m**4 + 3480*m**3 + 12139*m**2 + 19524*m + 1039
5) + 2262*a**2*e*m**2*x**3*(f*x)**m/(m**6 + 36*m**5 + 505*m**4 + 3480*m**3
 + 12139*m**2 + 19524*m + 10395) + 5353*a**2*e*m*x**3*(f*x)**m/(m**6 + 36*m*
*5 + 505*m**4 + 3480*m**3 + 12139*m**2 + 19524*m + 10395) + 3465*a**2*e*x**
3*(f*x)**m/(m**6 + 36*m**5 + 505*m**4 + 3480*m**3 + 12139*m**2 + 19524*m +
10395) + 2*a*b*d*m**5*x**3*(f*x)**m/(m**6 + 36*m**5 + 505*m**4 + 3480*m**3
 + 12139*m**2 + 19524*m + 10395) + 66*a*b*d*m**4*x**3*(f*x)**m/(m**6 + 36*m*
*5 + 505*m**4 + 3480*m**3 + 12139*m**2 + 19524*m + 10395) + 812*a*b*d*m**3*
x**3*(f*x)**m/(m**6 + 36*m**5 + 505*m**4 + 3480*m**3 + 12139*m**2 + 19524*m
 + 10395) + 4524*a*b*d*m**2*x**3*(f*x)**m/(m**6 + 36*m**5 + 505*m**4 + 3480
*m**3 + 12139*m**2 + 19524*m + 10395) + 10706*a*b*d*m*x**3*(f*x)**m/(m**6 +
 36*m**5 + 505*m**4 + 3480*m**3 + 12139*m**2 + 19524*m + 10395) + 6930*a*b*
d*x**3*(f*x)**m/(m**6 + 36*m**5 + 505*m**4 + 3480*m**3 + 12139*m**2 + 19524
*m + 10395) + 2*a*b*e*m**5*x**5*(f*x)**m/(m**6 + 36*m**5 + 505*m**4 + 3480*
m**3 + 12139*m**2 + 19524*m + 10395) + 62*a*b*e*m**4*x**5*(f*x)**m/(m**6 +
36*m**5 + 505*m**4 + 3480*m**3 + 12139*m**2 + 19524*m + 10395) + 700*a*b*e*
m**3*x**5*(f*x)**m/(m**6 + 36*m**5 + 505*m**4 + 3480*m**3 + 12139*m**2 + 19
524*m + 10395) + 3460*a*b*e*m**2*x**5*(f*x)**m/(m**6 + 36*m**5 + 505*m**4 +
 3480*m**3 + 12139*m**2 + 19524*m + 10395) + 6978*a*b*e*m*x**5*(f*x)**m/(m*
*6 + 36*m**5 + 505*m**4 + 3480*m**3 + 12139*m**2 + 19524*m + 10395) + 4158*
a*b*e*x**5*(f*x)**m/(m**6 + 36*m**5 + 505*m**4 + 3480*m**3 + 12139*m**2 + 1
9524*m + 10395) + 2*a*c*d*m**5*x**5*(f*x)**m/(m**6 + 36*m**5 + 505*m**4 + 3
480*m**3 + 12139*m**2 + 19524*m + 10395) + 62*a*c*d*m**4*x**5*(f*x)**m/(m**
6 + 36*m**5 + 505*m**4 + 3480*m**3 + 12139*m**2 + 19524*m + 10395) + 700*a*
c*d*m**3*x**5*(f*x)**m/(m**6 + 36*m**5 + 505*m**4 + 3480*m**3 + 12139*m**2
 + 19524*m + 10395) + 3460*a*c*d*m**2*x**5*(f*x)**m/(m**6 + 36*m**5 + 505*m*
*4 + 3480*m**3 + 12139*m**2 + 19524*m + 10395) + 6978*a*c*d*m*x**5*(f*x)**m
/(m**6 + 36*m**5 + 505*m**4 + 3480*m**3 + 12139*m**2 + 19524*m + 10395) + 4
158*a*c*d*x**5*(f*x)**m/(m**6 + 36*m**5 + 505*m**4 + 3480*m**3 + 12139*m**2
 + 19524*m + 10395) + 2*a*c*e*m**5*x**7*(f*x)**m/(m**6 + 36*m**5 + 505*m**4
 + 3480*m**3 + 12139*m**2 + 19524*m + 10395) + 58*a*c*e*m**4*x**7*(f*x)**m/
(m**6 + 36*m**5 + 505*m**4 + 3480*m**3 + 12139*m**2 + 19524*m + 10395) + 60
4*a*c*e*m**3*x**7*(f*x)**m/(m**6 + 36*m**5 + 505*m**4 + 3480*m**3 + 12139*m
2 + 19524*m + 10395) + 2732*a*c*e*m2*x**7*(f*x)**m/(m**6 + 36*m**5 + 50
5*m**4 + 3480*m**3 + 12139*m**2 + 19524*m + 10395) + 5154*a*c*e*m*x**7*(f*x
)**m/(m**6 + 36*m**5 + 505*m**4 + 3480*m**3 + 12139*m**2 + 19524*m + 10395)
 + 2970*a*c*e*x**7*(f*x)**m/(m**6 + 36*m**5 + 505*m**4 + 3480*m**3 + 12139*
m**2 + 19524*m + 10395) + b**2*d*m**5*x**5*(f*x)**m/(m**6 + 36*m**5 + 505*m
4 + 3480*m3 + 12139*m**2 + 19524*m + 10395) + 31*b**2*d*m**4*x**5*(f*x)

m/(m6 + 36*m**5 + 505*m**4 + 3480*m**3 + 12139*m**2 + 19524*m + 10395)
+ 350*b**2*d*m**3*x**5*(f*x)**m/(m**6 + 36*m**5 + 505*m**4 + 3480*m**3 + 12
139*m**2 + 19524*m + 10395) + 1730*b**2*d*m**2*x**5*(f*x)**m/(m**6 + 36*m**
5 + 505*m**4 + 3480*m**3 + 12139*m**2 + 19524*m + 10395) + 3489*b**2*d*m*x*
5(f*x)**m/(m**6 + 36*m**5 + 505*m**4 + 3480*m**3 + 12139*m**2 + 19524*m +
 10395) + 2079*b**2*d*x**5*(f*x)**m/(m**6 + 36*m**5 + 505*m**4 + 3480*m**3
+ 12139*m**2 + 19524*m + 10395) + b**2*e*m**5*x**7*(f*x)**m/(m**6 + 36*m**5
 + 505*m**4 + 3480*m**3 + 12139*m**2 + 19524*m + 10395) + 29*b**2*e*m**4*x*
7(f*x)**m/(m**6 + 36*m**5 + 505*m**4 + 3480*m**3 + 12139*m**2 + 19524*m +
 10395) + 302*b**2*e*m**3*x**7*(f*x)**m/(m**6 + 36*m**5 + 505*m**4 + 3480*m
3 + 12139*m2 + 19524*m + 10395) + 1366*b**2*e*m**2*x**7*(f*x)**m/(m**6
+ 36*m**5 + 505*m**4 + 3480*m**3 + 12139*m**2 + 19524*m + 10395) + 2577*b**
2*e*m*x**7*(f*x)**m/(m**6 + 36*m**5 + 505*m**4 + 3480*m**3 + 12139*m**2 + 1
9524*m + 10395) + 1485*b**2*e*x**7*(f*x)**m/(m**6 + 36*m**5 + 505*m**4 + 34
80*m**3 + 12139*m**2 + 19524*m + 10395) + 2*b*c*d*m**5*x**7*(f*x)**m/(m**6
+ 36*m**5 + 505*m**4 + 3480*m**3 + 12139*m**2 + 19524*m + 10395) + 58*b*c*d
*m**4*x**7*(f*x)**m/(m**6 + 36*m**5 + 505*m**4 + 3480*m**3 + 12139*m**2 + 1
9524*m + 10395) + 604*b*c*d*m**3*x**7*(f*x)**m/(m**6 + 36*m**5 + 505*m**4 +
 3480*m**3 + 12139*m**2 + 19524*m + 10395) + 2732*b*c*d*m**2*x**7*(f*x)**m/
(m**6 + 36*m**5 + 505*m**4 + 3480*m**3 + 12139*m**2 + 19524*m + 10395) + 51
54*b*c*d*m*x**7*(f*x)**m/(m**6 + 36*m**5 + 505*m**4 + 3480*m**3 + 12139*m**
2 + 19524*m + 10395) + 2970*b*c*d*x**7*(f*x)**m/(m**6 + 36*m**5 + 505*m**4
+ 3480*m**3 + 12139*m**2 + 19524*m + 10395) + 2*b*c*e*m**5*x**9*(f*x)**m/(m
6 + 36*m5 + 505*m**4 + 3480*m**3 + 12139*m**2 + 19524*m + 10395) + 54*b
*c*e*m**4*x**9*(f*x)**m/(m**6 + 36*m**5 + 505*m**4 + 3480*m**3 + 12139*m**2
 + 19524*m + 10395) + 524*b*c*e*m**3*x**9*(f*x)**m/(m**6 + 36*m**5 + 505*m*
*4 + 3480*m**3 + 12139*m**2 + 19524*m + 10395) + 2244*b*c*e*m**2*x**9*(f*x)
m/(m6 + 36*m**5 + 505*m**4 + 3480*m**3 + 12139*m**2 + 19524*m + 10395)
+ 4082*b*c*e*m*x**9*(f*x)**m/(m**6 + 36*m**5 + 505*m**4 + 3480*m**3 + 12139
*m**2 + 19524*m + 10395) + 2310*b*c*e*x**9*(f*x)**m/(m**6 + 36*m**5 + 505*m
4 + 3480*m3 + 12139*m**2 + 19524*m + 10395) + c**2*d*m**5*x**9*(f*x)**m
/(m**6 + 36*m**5 + 505*m**4 + 3480*m**3 + 12139*m**2 + 19524*m + 10395) + 2
7*c**2*d*m**4*x**9*(f*x)**m/(m**6 + 36*m**5 + 505*m**4 + 3480*m**3 + 12139*
m**2 + 19524*m + 10395) + 262*c**2*d*m**3*x**9*(f*x)**m/(m**6 + 36*m**5 + 5
05*m**4 + 3480*m**3 + 12139*m**2 + 19524*m + 10395) + 1122*c**2*d*m**2*x**9
*(f*x)**m/(m**6 + 36*m**5 + 505*m**4 + 3480*m**3 + 12139*m**2 + 19524*m + 1
0395) + 2041*c**2*d*m*x**9*(f*x)**m/(m**6 + 36*m**5 + 505*m**4 + 3480*m**3
+ 12139*m**2 + 19524*m + 10395) + 1155*c**2*d*x**9*(f*x)**m/(m**6 + 36*m**5
 + 505*m**4 + 3480*m**3 + 12139*m**2 + 19524*m + 10395) + c**2*e*m**5*x**11
*(f*x)**m/(m**6 + 36*m**5 + 505*m**4 + 3480*m**3 + 12139*m**2 + 19524*m + 1
0395) + 25*c**2*e*m**4*x**11*(f*x)**m/(m**6 + 36*m**5 + 505*m**4 + 3480*m**
3 + 12139*m**2 + 19524*m + 10395) + 230*c**2*e*m**3*x**11*(f*x)**m/(m**6 +
36*m**5 + 505*m**4 + 3480*m**3 + 12139*m**2 + 19524*m + 10395) + 950*c**2*e
*m**2*x**11*(f*x)**m/(m**6 + 36*m**5 + 505*m**4 + 3480*m**3 + 12139*m**2 +
19524*m + 10395) + 1689*c**2*e*m*x**11*(f*x)**m/(m**6 + 36*m**5 + 505*m**4

```
+ 3480*m**3 + 12139*m**2 + 19524*m + 10395) + 945*c**2*e*x**11*(f*x)**m/(m*
*6 + 36*m**5 + 505*m**4 + 3480*m**3 + 12139*m**2 + 19524*m + 10395), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 230, normalized size of antiderivative = 1.48

$$\int (fx)^m\left(d+ex^2\right)\left(a+bx^2+cx^4\right)^2\,dx=\frac{c^2ef^mx^{11}x^m}{m+11}+\frac{c^2df^mx^9x^m}{m+9}+\frac{2\,bcef^mx^9x^m}{m+9}+\frac{2\,bcdf^mx^7x^m}{m+7}+\frac{b^2ef^mx^7x^m}{m+7}+\frac{2\,acef^mx^7x^m}{m+7}+\frac{b^2df^mx^5x^m}{m+5}+\frac{2\,acdf^mx^5x^m}{m+5}+\frac{2\,abef^mx^5x^m}{m+5}+\frac{2\,abdf^mx^3x^m}{m+3}+\frac{a^2ef^mx^3x^m}{m+3}+\frac{(fx)^{m+1}\,a^2d}{f(m+1)}$$

```
[In] integrate((f*x)^m*(e*x^2+d)*(c*x^4+b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] c^2*e*f^m*x^11*x^m/(m + 11) + c^2*d*f^m*x^9*x^m/(m + 9) + 2*b*c*e*f^m*x^9*x
^m/(m + 9) + 2*b*c*d*f^m*x^7*x^m/(m + 7) + b^2*e*f^m*x^7*x^m/(m + 7) + 2*a*
c*e*f^m*x^7*x^m/(m + 7) + b^2*d*f^m*x^5*x^m/(m + 5) + 2*a*c*d*f^m*x^5*x^m/(
m + 5) + 2*a*b*e*f^m*x^5*x^m/(m + 5) + 2*a*b*d*f^m*x^3*x^m/(m + 3) + a^2*e*
f^m*x^3*x^m/(m + 3) + (f*x)^(m + 1)*a^2*d/(f*(m + 1))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1142 vs. 2(155) = 310.

Time = 0.32 (sec) , antiderivative size = 1142, normalized size of antiderivative = 7.37

$$\int (fx)^m\left(d+ex^2\right)\left(a+bx^2+cx^4\right)^2\,dx=\text{Too large to display}$$

```
[In] integrate((f*x)^m*(e*x^2+d)*(c*x^4+b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] ((f*x)^m*c^2*e*m^5*x^11 + 25*(f*x)^m*c^2*e*m^4*x^11 + (f*x)^m*c^2*d*m^5*x^9
 + 2*(f*x)^m*b*c*e*m^5*x^9 + 230*(f*x)^m*c^2*e*m^3*x^11 + 27*(f*x)^m*c^2*d*
m^4*x^9 + 54*(f*x)^m*b*c*e*m^4*x^9 + 950*(f*x)^m*c^2*e*m^2*x^11 + 2*(f*x)^m
*b*c*d*m^5*x^7 + (f*x)^m*b^2*e*m^5*x^7 + 2*(f*x)^m*a*c*e*m^5*x^7 + 262*(f*x
)^m*c^2*d*m^3*x^9 + 524*(f*x)^m*b*c*e*m^3*x^9 + 1689*(f*x)^m*c^2*e*m*x^11 +
 58*(f*x)^m*b*c*d*m^4*x^7 + 29*(f*x)^m*b^2*e*m^4*x^7 + 58*(f*x)^m*a*c*e*m^4
*x^7 + 1122*(f*x)^m*c^2*d*m^2*x^9 + 2244*(f*x)^m*b*c*e*m^2*x^9 + 945*(f*x)^
m*c^2*e*x^11 + (f*x)^m*b^2*d*m^5*x^5 + 2*(f*x)^m*a*c*d*m^5*x^5 + 2*(f*x)^m*
a*b*e*m^5*x^5 + 604*(f*x)^m*b*c*d*m^3*x^7 + 302*(f*x)^m*b^2*e*m^3*x^7 + 604
```

```
*(f*x)^m*a*c*e*m^3*x^7 + 2041*(f*x)^m*c^2*d*m*x^9 + 4082*(f*x)^m*b*c*e*m*x^
9 + 31*(f*x)^m*b^2*d*m^4*x^5 + 62*(f*x)^m*a*c*d*m^4*x^5 + 62*(f*x)^m*a*b*e*
m^4*x^5 + 2732*(f*x)^m*b*c*d*m^2*x^7 + 1366*(f*x)^m*b^2*e*m^2*x^7 + 2732*(f
*x)^m*a*c*e*m^2*x^7 + 1155*(f*x)^m*c^2*d*x^9 + 2310*(f*x)^m*b*c*e*x^9 + 2*(
f*x)^m*a*b*d*m^5*x^3 + (f*x)^m*a^2*e*m^5*x^3 + 350*(f*x)^m*b^2*d*m^3*x^5 +
700*(f*x)^m*a*c*d*m^3*x^5 + 700*(f*x)^m*a*b*e*m^3*x^5 + 5154*(f*x)^m*b*c*d*
m*x^7 + 2577*(f*x)^m*b^2*e*m*x^7 + 5154*(f*x)^m*a*c*e*m*x^7 + 66*(f*x)^m*a*
b*d*m^4*x^3 + 33*(f*x)^m*a^2*e*m^4*x^3 + 1730*(f*x)^m*b^2*d*m^2*x^5 + 3460*
(f*x)^m*a*c*d*m^2*x^5 + 3460*(f*x)^m*a*b*e*m^2*x^5 + 2970*(f*x)^m*b*c*d*x^7
 + 1485*(f*x)^m*b^2*e*x^7 + 2970*(f*x)^m*a*c*e*x^7 + (f*x)^m*a^2*d*m^5*x +
812*(f*x)^m*a*b*d*m^3*x^3 + 406*(f*x)^m*a^2*e*m^3*x^3 + 3489*(f*x)^m*b^2*d*
m*x^5 + 6978*(f*x)^m*a*c*d*m*x^5 + 6978*(f*x)^m*a*b*e*m*x^5 + 35*(f*x)^m*a^
2*d*m^4*x + 4524*(f*x)^m*a*b*d*m^2*x^3 + 2262*(f*x)^m*a^2*e*m^2*x^3 + 2079*
(f*x)^m*b^2*d*x^5 + 4158*(f*x)^m*a*c*d*x^5 + 4158*(f*x)^m*a*b*e*x^5 + 470*(
f*x)^m*a^2*d*m^3*x + 10706*(f*x)^m*a*b*d*m*x^3 + 5353*(f*x)^m*a^2*e*m*x^3 +
 3010*(f*x)^m*a^2*d*m^2*x + 6930*(f*x)^m*a*b*d*x^3 + 3465*(f*x)^m*a^2*e*x^3
 + 9129*(f*x)^m*a^2*d*m*x + 10395*(f*x)^m*a^2*d*x)/(m^6 + 36*m^5 + 505*m^4
+ 3480*m^3 + 12139*m^2 + 19524*m + 10395)
```

Mupad [B] (verification not implemented)

Time = 7.87 (sec) , antiderivative size = 429, normalized size of antiderivative = 2.77

$$\int (fx)^m \left(d+ex^2\right) \left(a+bx^2+cx^4\right)^2 dx = \frac{x^5\,(f\,x)^m\,(d\,b^2+2\,a\,e\,b+2\,a\,c\,d)\,\left(m^5+31\,m^4+350\,m^3+1730\,m^2+3489\,m+2079\right)}{m^6+36\,m^5+505\,m^4+3480\,m^3+12139\,m^2+19524\,m+10395} + \frac{x^7\,(f\,x)^m\,(e\,b^2+2\,c\,d\,b+2\,a\,c\,e)\,\left(m^5+29\,m^4+302\,m^3+1366\,m^2+2577\,m+1485\right)}{m^6+36\,m^5+505\,m^4+3480\,m^3+12139\,m^2+19524\,m+10395} + \frac{a^2\,d\,x\,(f\,x)^m\,\left(m^5+35\,m^4+470\,m^3+3010\,m^2+9129\,m+10395\right)}{m^6+36\,m^5+505\,m^4+3480\,m^3+12139\,m^2+19524\,m+10395} + \frac{a\,x^3\,(f\,x)^m\,(a\,e+2\,b\,d)\,\left(m^5+33\,m^4+406\,m^3+2262\,m^2+5353\,m+3465\right)}{m^6+36\,m^5+505\,m^4+3480\,m^3+12139\,m^2+19524\,m+10395} + \frac{c\,x^9\,(f\,x)^m\,(2\,b\,e+c\,d)\,\left(m^5+27\,m^4+262\,m^3+1122\,m^2+2041\,m+1155\right)}{m^6+36\,m^5+505\,m^4+3480\,m^3+12139\,m^2+19524\,m+10395} + \frac{c^2\,e\,x^{11}\,(f\,x)^m\,\left(m^5+25\,m^4+230\,m^3+950\,m^2+1689\,m+945\right)}{m^6+36\,m^5+505\,m^4+3480\,m^3+12139\,m^2+19524\,m+10395}$$

```
[In] int((f*x)^m*(d + e*x^2)*(a + b*x^2 + c*x^4)^2,x)
```

```
[Out] (x^5*(f*x)^m*(b^2*d + 2*a*b*e + 2*a*c*d)*(3489*m + 1730*m^2 + 350*m^3 + 31*
m^4 + m^5 + 2079))/(19524*m + 12139*m^2 + 3480*m^3 + 505*m^4 + 36*m^5 + m^6
 + 10395) + (x^7*(f*x)^m*(b^2*e + 2*a*c*e + 2*b*c*d)*(2577*m + 1366*m^2 + 3
02*m^3 + 29*m^4 + m^5 + 1485))/(19524*m + 12139*m^2 + 3480*m^3 + 505*m^4 +
36*m^5 + m^6 + 10395) + (a^2*d*x*(f*x)^m*(9129*m + 3010*m^2 + 470*m^3 + 35*
```

```
m^4 + m^5 + 10395))/(19524*m + 12139*m^2 + 3480*m^3 + 505*m^4 + 36*m^5 + m^
6 + 10395) + (a*x^3*(f*x)^m*(a*e + 2*b*d)*(5353*m + 2262*m^2 + 406*m^3 + 33
*m^4 + m^5 + 3465))/(19524*m + 12139*m^2 + 3480*m^3 + 505*m^4 + 36*m^5 + m^
6 + 10395) + (c*x^9*(f*x)^m*(2*b*e + c*d)*(2041*m + 1122*m^2 + 262*m^3 + 27
*m^4 + m^5 + 1155))/(19524*m + 12139*m^2 + 3480*m^3 + 505*m^4 + 36*m^5 + m^
6 + 10395) + (c^2*e*x^11*(f*x)^m*(1689*m + 950*m^2 + 230*m^3 + 25*m^4 + m^5
 + 945))/(19524*m + 12139*m^2 + 3480*m^3 + 505*m^4 + 36*m^5 + m^6 + 10395)
```

3.222 $\int (fx)^m (d+ex^2) (a+bx^2+cx^4) \, dx$

Optimal result

Integrand size = 25, antiderivative size = 83

$$\int (fx)^m (d+ex^2) (a+bx^2+cx^4) \, dx = \frac{ad(fx)^{1+m}}{f(1+m)} + \frac{(bd+ae)(fx)^{3+m}}{f^3(3+m)} + \frac{(cd+be)(fx)^{5+m}}{f^5(5+m)} + \frac{ce(fx)^{7+m}}{f^7(7+m)}$$

```
[Out] a*d*(f*x)^(1+m)/f/(1+m)+(a*e+b*d)*(f*x)^(3+m)/f^3/(3+m)+(b*e+c*d)*(f*x)^(5+
m)/f^5/(5+m)+c*e*(f*x)^(7+m)/f^7/(7+m)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.040, Rules used = {1275}

$$\int (fx)^m (d+ex^2) (a+bx^2+cx^4) \, dx = \frac{(fx)^{m+3}(ae+bd)}{f^3(m+3)} + \frac{ad(fx)^{m+1}}{f(m+1)} + \frac{(fx)^{m+5}(be+cd)}{f^5(m+5)} + \frac{ce(fx)^{m+7}}{f^7(m+7)}$$

```
[In] Int[(f*x)^m*(d + e*x^2)*(a + b*x^2 + c*x^4),x]
```

```
[Out] (a*d*(f*x)^(1 + m))/(f*(1 + m)) + ((b*d + a*e)*(f*x)^(3 + m))/(f^3*(3 + m))
 + ((c*d + b*e)*(f*x)^(5 + m))/(f^5*(5 + m)) + (c*e*(f*x)^(7 + m))/(f^7*(7
+ m))
```

Rule 1275

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*
```

```
(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && NeQ[
b^2 - 4*a*c, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \left(ad(fx)^m + \frac{(bd+ae)(fx)^{2+m}}{f^2} + \frac{(cd+be)(fx)^{4+m}}{f^4} + \frac{ce(fx)^{6+m}}{f^6}\right) dx \\ &= \frac{ad(fx)^{1+m}}{f(1+m)} + \frac{(bd+ae)(fx)^{3+m}}{f^3(3+m)} + \frac{(cd+be)(fx)^{5+m}}{f^5(5+m)} + \frac{ce(fx)^{7+m}}{f^7(7+m)}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.71

$$\int (fx)^m \left(d+ex^2\right) \left(a+bx^2+cx^4\right) dx = x(fx)^m \left(\frac{ad}{1+m} + \frac{(bd+ae)x^2}{3+m} + \frac{(cd+be)x^4}{5+m} + \frac{cex^6}{7+m}\right)$$

```
[In] Integrate[(f*x)^m*(d + e*x^2)*(a + b*x^2 + c*x^4),x]
[Out] x*(f*x)^m*((a*d)/(1 + m) + ((b*d + a*e)*x^2)/(3 + m) + ((c*d + b*e)*x^4)/(5
 + m) + (c*e*x^6)/(7 + m))
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.99

method	result
norman	$\frac{(ae+bd)x^3 e^{m\ln(fx)}}{3+m}+\frac{(be+cd)x^5 e^{m\ln(fx)}}{5+m}+\frac{dax\, e^{m\ln(fx)}}{1+m}+\frac{ec\, x^7 e^{m\ln(fx)}}{7+m}$
gosper	$\frac{x\left(ce\, m^3x^6+9ce\, m^2x^6+be\, m^3x^4+cd\, m^3x^4+23cem\, x^6+11be\, m^2x^4+11cd\, m^2x^4+15ce\, x^6+ae\, m^3x^2+bd\, m^3x^2+31bem\, x^4+31cd\right.}{(7+m)(5+}$
risch	$\frac{x\left(ce\, m^3x^6+9ce\, m^2x^6+be\, m^3x^4+cd\, m^3x^4+23cem\, x^6+11be\, m^2x^4+11cd\, m^2x^4+15ce\, x^6+ae\, m^3x^2+bd\, m^3x^2+31bem\, x^4+31cd\right.}{(7+m)(5+}$
parallelrisch	$\frac{15x^7(fx)^m ce+21x^5(fx)^m be+21x^5(fx)^m cd+35x^3(fx)^m ae+35x^3(fx)^m bd+105x(fx)^m ad+71x(fx)^m adm+47x^3(fx)^m aem-}{}$

```
[In] int((f*x)^m*(e*x^2+d)*(c*x^4+b*x^2+a),x,method=_RETURNVERBOSE)
[Out] (a*e+b*d)/(3+m)*x^3*exp(m*ln(f*x))+(b*e+c*d)/(5+m)*x^5*exp(m*ln(f*x))+d*a/(
1+m)*x*exp(m*ln(f*x))+e*c/(7+m)*x^7*exp(m*ln(f*x))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 171 vs. $2(83) = 166$.

Time = 0.25 (sec) , antiderivative size = 171, normalized size of antiderivative = 2.06

$$\int (fx)^m (d+ex^2) (a+bx^2+cx^4) \, dx$$
$$= \frac{((cem^3+9\,cem^2+23\,cem+15\,ce)x^7+((cd+be)m^3+11\,(cd+be)m^2+21\,cd+21\,be+31\,(cd+be)m)x^5}{m^4+16\,m^3}$$

```
[In] integrate((f*x)^m*(e*x^2+d)*(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] ((c*e*m^3 + 9*c*e*m^2 + 23*c*e*m + 15*c*e)*x^7 + ((c*d + b*e)*m^3 + 11*(c*d
 + b*e)*m^2 + 21*c*d + 21*b*e + 31*(c*d + b*e)*m)*x^5 + ((b*d + a*e)*m^3 +
13*(b*d + a*e)*m^2 + 35*b*d + 35*a*e + 47*(b*d + a*e)*m)*x^3 + (a*d*m^3 + 1
5*a*d*m^2 + 71*a*d*m + 105*a*d)*x)*(f*x)^m/(m^4 + 16*m^3 + 86*m^2 + 176*m +
 105)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1015 vs. $2(71) = 142$.

Time = 0.47 (sec) , antiderivative size = 1015, normalized size of antiderivative = 12.23

$$\int (fx)^m (d+ex^2) (a+bx^2+cx^4) \, dx$$
$$= \begin{cases} \frac{-\frac{ad}{6x^6}-\frac{ae}{4x^4}-\frac{bd}{4x^4}-\frac{be}{2x^2}-\frac{cd}{2x^2}+ce\log(x)}{f^7} \\ \frac{-\frac{ad}{4x^4}-\frac{ae}{2x^2}-\frac{bd}{2x^2}+be\log(x)+cd\log(x)+\frac{cex^2}{2}}{f^5} \\ \frac{-\frac{ad}{2x^2}+ae\log(x)+bd\log(x)+\frac{bex^2}{2}+\frac{cdx^2}{2}+\frac{cex^4}{4}}{f^3} \\ \frac{ad\log(x)+\frac{aex^2}{2}+\frac{bdx^2}{2}+\frac{bex^4}{4}+\frac{cdx^4}{4}+\frac{cex^6}{6}}{f} \\ \frac{adm^3x(fx)^m}{m^4+16m^3+86m^2+176m+105}+\frac{15adm^2x(fx)^m}{m^4+16m^3+86m^2+176m+105}+\frac{71admx(fx)^m}{m^4+16m^3+86m^2+176m+105}+\frac{105adx(fx)^m}{m^4+16m^3+86m^2+176m+105}+ \end{cases}$$

```
[In] integrate((f*x)**m*(e*x**2+d)*(c*x**4+b*x**2+a),x)
```

```
[Out] Piecewise(((-a*d/(6*x**6) - a*e/(4*x**4) - b*d/(4*x**4) - b*e/(2*x**2) - c*
d/(2*x**2) + c*e*log(x))/f**7, Eq(m, -7)), ((-a*d/(4*x**4) - a*e/(2*x**2) -
 b*d/(2*x**2) + b*e*log(x) + c*d*log(x) + c*e*x**2/2)/f**5, Eq(m, -5)), ((-
a*d/(2*x**2) + a*e*log(x) + b*d*log(x) + b*e*x**2/2 + c*d*x**2/2 + c*e*x**4
/4)/f**3, Eq(m, -3)), ((a*d*log(x) + a*e*x**2/2 + b*d*x**2/2 + b*e*x**4/4 +
 c*d*x**4/4 + c*e*x**6/6)/f, Eq(m, -1)), (a*d*m**3*x*(f*x)**m/(m**4 + 16*m*
*3 + 86*m**2 + 176*m + 105) + 15*a*d*m**2*x*(f*x)**m/(m**4 + 16*m**3 + 86*m
```

```
**2 + 176*m + 105) + 71*a*d*m*x*(f*x)**m/(m**4 + 16*m**3 + 86*m**2 + 176*m
+ 105) + 105*a*d*x*(f*x)**m/(m**4 + 16*m**3 + 86*m**2 + 176*m + 105) + a*e*
m**3*x**3*(f*x)**m/(m**4 + 16*m**3 + 86*m**2 + 176*m + 105) + 13*a*e*m**2*x
**3*(f*x)**m/(m**4 + 16*m**3 + 86*m**2 + 176*m + 105) + 47*a*e*m*x**3*(f*x)
**m/(m**4 + 16*m**3 + 86*m**2 + 176*m + 105) + 35*a*e*x**3*(f*x)**m/(m**4 +
 16*m**3 + 86*m**2 + 176*m + 105) + b*d*m**3*x**3*(f*x)**m/(m**4 + 16*m**3
+ 86*m**2 + 176*m + 105) + 13*b*d*m**2*x**3*(f*x)**m/(m**4 + 16*m**3 + 86*m
**2 + 176*m + 105) + 47*b*d*m*x**3*(f*x)**m/(m**4 + 16*m**3 + 86*m**2 + 176
*m + 105) + 35*b*d*x**3*(f*x)**m/(m**4 + 16*m**3 + 86*m**2 + 176*m + 105) +
 b*e*m**3*x**5*(f*x)**m/(m**4 + 16*m**3 + 86*m**2 + 176*m + 105) + 11*b*e*m
**2*x**5*(f*x)**m/(m**4 + 16*m**3 + 86*m**2 + 176*m + 105) + 31*b*e*m*x**5*
(f*x)**m/(m**4 + 16*m**3 + 86*m**2 + 176*m + 105) + 21*b*e*x**5*(f*x)**m/(m
**4 + 16*m**3 + 86*m**2 + 176*m + 105) + c*d*m**3*x**5*(f*x)**m/(m**4 + 16*
m**3 + 86*m**2 + 176*m + 105) + 11*c*d*m**2*x**5*(f*x)**m/(m**4 + 16*m**3 +
 86*m**2 + 176*m + 105) + 31*c*d*m*x**5*(f*x)**m/(m**4 + 16*m**3 + 86*m**2
+ 176*m + 105) + 21*c*d*x**5*(f*x)**m/(m**4 + 16*m**3 + 86*m**2 + 176*m + 1
05) + c*e*m**3*x**7*(f*x)**m/(m**4 + 16*m**3 + 86*m**2 + 176*m + 105) + 9*c
*e*m**2*x**7*(f*x)**m/(m**4 + 16*m**3 + 86*m**2 + 176*m + 105) + 23*c*e*m*x
**7*(f*x)**m/(m**4 + 16*m**3 + 86*m**2 + 176*m + 105) + 15*c*e*x**7*(f*x)**
m/(m**4 + 16*m**3 + 86*m**2 + 176*m + 105), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.25

$$\int (fx)^m \left(d+ex^2\right) \left(a+bx^2+cx^4\right) \, dx = \frac{cef^m x^7 x^m}{m+7} + \frac{cdf^m x^5 x^m}{m+5} + \frac{bef^m x^5 x^m}{m+5} + \frac{bdf^m x^3 x^m}{m+3} + \frac{aef^m x^3 x^m}{m+3} + \frac{(fx)^{m+1}\, ad}{f(m+1)}$$

```
[In] integrate((f*x)^m*(e*x^2+d)*(c*x^4+b*x^2+a),x, algorithm="maxima")
```

```
[Out] c*e*f^m*x^7*x^m/(m + 7) + c*d*f^m*x^5*x^m/(m + 5) + b*e*f^m*x^5*x^m/(m + 5)
 + b*d*f^m*x^3*x^m/(m + 3) + a*e*f^m*x^3*x^m/(m + 3) + (f*x)^(m + 1)*a*d/(f
*(m + 1))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 338 vs. 2(83) = 166.

Time = 0.30 (sec) , antiderivative size = 338, normalized size of antiderivative = 4.07

$$\int (fx)^m (d+ex^2) (a+bx^2+cx^4) \, dx = \frac{(fx)^m cem^3x^7 + 9(fx)^m cem^2x^7 + (fx)^m cdm^3x^5 + (fx)^m bem^3x^5 + 23(fx)^m cemx^7 + 11(fx)^m cdm^2x^5 -}{}$$

[In] integrate((f*x)^m*(e*x^2+d)*(c*x^4+b*x^2+a),x, algorithm="giac")

[Out] ((f*x)^m*c*e*m^3*x^7 + 9*(f*x)^m*c*e*m^2*x^7 + (f*x)^m*c*d*m^3*x^5 + (f*x)^m*b*e*m^3*x^5 + 23*(f*x)^m*c*e*m*x^7 + 11*(f*x)^m*c*d*m^2*x^5 + 11*(f*x)^m*b*e*m^2*x^5 + 15*(f*x)^m*c*e*x^7 + (f*x)^m*b*d*m^3*x^3 + (f*x)^m*a*e*m^3*x^3 + 31*(f*x)^m*c*d*m*x^5 + 31*(f*x)^m*b*e*m*x^5 + 13*(f*x)^m*b*d*m^2*x^3 + 13*(f*x)^m*a*e*m^2*x^3 + 21*(f*x)^m*c*d*x^5 + 21*(f*x)^m*b*e*x^5 + (f*x)^m*a*d*m^3*x + 47*(f*x)^m*b*d*m*x^3 + 47*(f*x)^m*a*e*m*x^3 + 15*(f*x)^m*a*d*m^2*x + 35*(f*x)^m*b*d*x^3 + 35*(f*x)^m*a*e*x^3 + 71*(f*x)^m*a*d*m*x + 105*(f*x)^m*a*d*x)/(m^4 + 16*m^3 + 86*m^2 + 176*m + 105)

Mupad [B] (verification not implemented)

Time = 7.68 (sec) , antiderivative size = 171, normalized size of antiderivative = 2.06

$$\int (fx)^m (d+ex^2) (a+bx^2+cx^4) \, dx = (f\,x)^m \left(\frac{x^3\,(a\,e+b\,d)\,(m^3+13\,m^2+47\,m+35)}{m^4+16\,m^3+86\,m^2+176\,m+105} + \frac{x^5\,(b\,e+c\,d)\,(m^3+11\,m^2+31\,m+21)}{m^4+16\,m^3+86\,m^2+176\,m+105} + \frac{a\,d\,x\,(m^3+15\,m^2+71\,m+105)}{m^4+16\,m^3+86\,m^2+176\,m+105} + \frac{c\,e\,x^7\,(m^3+9\,m^2+23\,m+15)}{m^4+16\,m^3+86\,m^2+176\,m+105} \right)$$

[In] int((f*x)^m*(d + e*x^2)*(a + b*x^2 + c*x^4),x)

[Out] (f*x)^m*((x^3*(a*e + b*d)*(47*m + 13*m^2 + m^3 + 35))/(176*m + 86*m^2 + 16*m^3 + m^4 + 105) + (x^5*(b*e + c*d)*(31*m + 11*m^2 + m^3 + 21))/(176*m + 86*m^2 + 16*m^3 + m^4 + 105) + (a*d*x*(71*m + 15*m^2 + m^3 + 105))/(176*m + 86*m^2 + 16*m^3 + m^4 + 105) + (c*e*x^7*(23*m + 9*m^2 + m^3 + 15))/(176*m + 86*m^2 + 16*m^3 + m^4 + 105))

3.223 $\int \frac{(fx)^m(d+ex^2)}{a+bx^2+cx^4}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 194

$$\int \frac{(fx)^m\left(d+ex^2\right)}{a+bx^2+cx^4}\,dx = \frac{\left(e+\frac{2cd-be}{\sqrt{b^2-4ac}}\right)(fx)^{1+m}\,\text{Hypergeometric2F1}\left(1,\frac{1+m}{2},\frac{3+m}{2},-\frac{2cx^2}{b-\sqrt{b^2-4ac}}\right)}{\left(b-\sqrt{b^2-4ac}\right)f(1+m)} + \frac{\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)(fx)^{1+m}\,\text{Hypergeometric2F1}\left(1,\frac{1+m}{2},\frac{3+m}{2},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{\left(b+\sqrt{b^2-4ac}\right)f(1+m)}$$

```
[Out] (f*x)^(1+m)*hypergeom([1, 1/2+1/2*m],[3/2+1/2*m],-2*c*x^2/(b-(-4*a*c+b^2)^(
1/2)))*(e+(-b*e+2*c*d)/(-4*a*c+b^2)^(1/2))/f/(1+m)/(b-(-4*a*c+b^2)^(1/2))+(
f*x)^(1+m)*hypergeom([1, 1/2+1/2*m],[3/2+1/2*m],-2*c*x^2/(b+(-4*a*c+b^2)^(1
/2)))*(e+(b*e-2*c*d)/(-4*a*c+b^2)^(1/2))/f/(1+m)/(b+(-4*a*c+b^2)^(1/2))
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 194, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.074$, Rules used = {1299, 371}

$$\int \frac{(fx)^m\left(d+ex^2\right)}{a+bx^2+cx^4}\,dx = \frac{(fx)^{m+1}\left(\frac{2cd-be}{\sqrt{b^2-4ac}}+e\right)\text{Hypergeometric2F1}\left(1,\frac{m+1}{2},\frac{m+3}{2},-\frac{2cx^2}{b-\sqrt{b^2-4ac}}\right)}{f(m+1)\left(b-\sqrt{b^2-4ac}\right)} + \frac{(fx)^{m+1}\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\text{Hypergeometric2F1}\left(1,\frac{m+1}{2},\frac{m+3}{2},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{f(m+1)\left(\sqrt{b^2-4ac}+b\right)}$$

```
[In] Int[((f*x)^m*(d + e*x^2))/(a + b*x^2 + c*x^4),x]
```

```
[Out] ((e + (2*c*d - b*e)/Sqrt[b^2 - 4*a*c])*(f*x)^(1 + m)*Hypergeometric2F1[1, (
1 + m)/2, (3 + m)/2, (-2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])])/((b - Sqrt[b^2 -
4*a*c])*f*(1 + m)) + ((e - (2*c*d - b*e)/Sqrt[b^2 - 4*a*c])*(f*x)^(1 + m)*H
ypergeometric2F1[1, (1 + m)/2, (3 + m)/2, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]
)])/((b + Sqrt[b^2 - 4*a*c])*f*(1 + m))
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 1299

```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2))/((a_) + (b_.)*(x_)^2 + (c_.)
*(x_)^4), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b
*e)/(2*q), Int[(f*x)^m/(b/2 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*
e)/(2*q), Int[(f*x)^m/(b/2 + q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e,
 f, m}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{2}\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\int\frac{(fx)^m}{\frac{b}{2}+\frac{1}{2}\sqrt{b^2-4ac}+cx^2}\,dx \\
&\quad+\frac{1}{2}\left(e+\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\int\frac{(fx)^m}{\frac{b}{2}-\frac{1}{2}\sqrt{b^2-4ac}+cx^2}\,dx \\
&= \frac{\left(e+\frac{2cd-be}{\sqrt{b^2-4ac}}\right)(fx)^{1+m}\,{}_2F_1\left(1,\frac{1+m}{2};\frac{3+m}{2};-\frac{2cx^2}{b-\sqrt{b^2-4ac}}\right)}{\left(b-\sqrt{b^2-4ac}\right)f(1+m)} \\
&\quad+\frac{\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)(fx)^{1+m}\,{}_2F_1\left(1,\frac{1+m}{2};\frac{3+m}{2};-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{\left(b+\sqrt{b^2-4ac}\right)f(1+m)}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.35 (sec) , antiderivative size = 156, normalized size of antiderivative = 0.80

$$\int\frac{(fx)^m\,(d+ex^2)}{a+bx^2+cx^4}\,dx = \frac{x(fx)^m\left(\left(bd+\sqrt{b^2-4ac}\,d-2ae\right)\text{Hypergeometric2F1}\left(1,\frac{1+m}{2},\frac{3+m}{2},\frac{2cx^2}{-b+\sqrt{b^2-4ac}}\right)+\left(-bd+\sqrt{b^2-4ac}\,d+\right.\right.}{2a\sqrt{b^2-4ac}(1+m)}$$

```
[In] Integrate[((f*x)^m*(d + e*x^2))/(a + b*x^2 + c*x^4),x]
```

```
[Out] (x*(f*x)^m*((b*d + Sqrt[b^2 - 4*a*c]*d - 2*a*e)*Hypergeometric2F1[1, (1 + m
)/2, (3 + m)/2, (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])] + (-(b*d) + Sqrt[b^2 -
4*a*c]*d + 2*a*e)*Hypergeometric2F1[1, (1 + m)/2, (3 + m)/2, (-2*c*x^2)/(b
+ Sqrt[b^2 - 4*a*c])]))/(2*a*Sqrt[b^2 - 4*a*c]*(1 + m))
```

Maple [F]

$$\int \frac{\left(f x\right)^{m} \left(e\,x^{2}+d\right)}{c\,x^{4}+b\,x^{2}+a}dx$$

```
[In] int((f*x)^m*(e*x^2+d)/(c*x^4+b*x^2+a),x)
```

```
[Out] int((f*x)^m*(e*x^2+d)/(c*x^4+b*x^2+a),x)
```

Fricas [F]

$$\int \frac{(fx)^m (d+ex^2)}{a+bx^2+cx^4} \, dx = \int \frac{(ex^2+d)(fx)^m}{cx^4+bx^2+a} \, dx$$

```
[In] integrate((f*x)^m*(e*x^2+d)/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] integral((e*x^2 + d)*(f*x)^m/(c*x^4 + b*x^2 + a), x)
```

Sympy [F]

$$\int \frac{(fx)^m (d+ex^2)}{a+bx^2+cx^4} \, dx = \int \frac{(fx)^m (d+ex^2)}{a+bx^2+cx^4} \, dx$$

```
[In] integrate((f*x)**m*(e*x**2+d)/(c*x**4+b*x**2+a),x)
```

```
[Out] Integral((f*x)**m*(d + e*x**2)/(a + b*x**2 + c*x**4), x)
```

Maxima [F]

$$\int \frac{(fx)^m (d+ex^2)}{a+bx^2+cx^4} \, dx = \int \frac{(ex^2+d)(fx)^m}{cx^4+bx^2+a} \, dx$$

```
[In] integrate((f*x)^m*(e*x^2+d)/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

```
[Out] integrate((e*x^2 + d)*(f*x)^m/(c*x^4 + b*x^2 + a), x)
```

Giac [F]

$$\int \frac{(fx)^m (d+ex^2)}{a+bx^2+cx^4}\,dx = \int \frac{(ex^2+d)(fx)^m}{cx^4+bx^2+a}\,dx$$

```
[In] integrate((f*x)^m*(e*x^2+d)/(c*x^4+b*x^2+a),x, algorithm="giac")
[Out] integrate((e*x^2 + d)*(f*x)^m/(c*x^4 + b*x^2 + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(fx)^m (d+ex^2)}{a+bx^2+cx^4}\,dx = \int \frac{(f\,x)^m\,(e\,x^2+d)}{c\,x^4+b\,x^2+a}\,dx$$

```
[In] int(((f*x)^m*(d + e*x^2))/(a + b*x^2 + c*x^4),x)
[Out] int(((f*x)^m*(d + e*x^2))/(a + b*x^2 + c*x^4), x)
```

3.224 $\int \frac{(fx)^m(d+ex^2)}{(a+bx^2+cx^4)^2}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 392

$$\int \frac{(fx)^m\,(d+ex^2)}{(a+bx^2+cx^4)^2}\,dx = \frac{(fx)^{1+m}\,(b^2d-2acd-abe+c(bd-2ae)x^2)}{2a\,(b^2-4ac)\,f\,(a+bx^2+cx^4)}$$
$$+\frac{c\Big(b\big(4ae+\sqrt{b^2-4ac}d(1-m)\big)-2a\big(\sqrt{b^2-4ac}e(1-m)+2cd(3-m)\big)+b^2(d-dm)\Big)\,(fx)^{1+m}\,\text{Hype}}{2a\,(b^2-4ac)^{3/2}\,\big(b-\sqrt{b^2-4ac}\big)\,f(1+m)}$$
$$-\frac{c\Big(b\big(4ae-\sqrt{b^2-4ac}d(1-m)\big)+2a\big(\sqrt{b^2-4ac}e(1-m)-2cd(3-m)\big)+b^2d(1-m)\Big)\,(fx)^{1+m}\,\text{Hype}}{2a\,(b^2-4ac)^{3/2}\,\big(b+\sqrt{b^2-4ac}\big)\,f(1+m)}$$

```
[Out] 1/2*(f*x)^(1+m)*(b^2*d-2*a*c*d-a*b*e+c*(-2*a*e+b*d)*x^2)/a/(-4*a*c+b^2)/f/(
c*x^4+b*x^2+a)-1/2*c*(f*x)^(1+m)*hypergeom([1, 1/2+1/2*m],[3/2+1/2*m],-2*c*
x^2/(b+(-4*a*c+b^2)^(1/2)))*(b^2*d*(1-m)+b*(4*a*e-d*(1-m)*(-4*a*c+b^2)^(1/2
))+2*a*(-2*c*d*(3-m)+e*(1-m)*(-4*a*c+b^2)^(1/2)))/a/(-4*a*c+b^2)^(3/2)/f/(1
+m)/(b+(-4*a*c+b^2)^(1/2))+1/2*c*(f*x)^(1+m)*hypergeom([1, 1/2+1/2*m],[3/2+
1/2*m],-2*c*x^2/(b-(-4*a*c+b^2)^(1/2)))*(b^2*(-d*m+d)+b*(4*a*e+d*(1-m)*(-4*
a*c+b^2)^(1/2))-2*a*(2*c*d*(3-m)+e*(1-m)*(-4*a*c+b^2)^(1/2)))/a/(-4*a*c+b^2
)^(3/2)/f/(1+m)/(b-(-4*a*c+b^2)^(1/2))
```

Rubi [A] (verified)

Time = 1.75 (sec) , antiderivative size = 358, normalized size of antiderivative = 0.91, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {1291, 1299, 371}

$$\int \frac{(fx)^m (d+ex^2)}{(a+bx^2+cx^4)^2} dx$$
$$= \frac{c(fx)^{m+1} \left((1-m)\sqrt{b^2-4ac}(bd-2ae)+4abe-4acd(3-m)+b^2(d-dm)\right) \text{Hypergeometric2F1}\left(1, \frac{m+1}{2}\right.}{2af(m+1)\left(b^2-4ac\right)^{3/2}\left(b-\sqrt{b^2-4ac}\right)}$$
$$- \frac{c(fx)^{m+1} \left(-(1-m)\sqrt{b^2-4ac}(bd-2ae)+4abe-4acd(3-m)+b^2(d-dm)\right) \text{Hypergeometric2F1}\left(1,\right.}{2af(m+1)\left(b^2-4ac\right)^{3/2}\left(\sqrt{b^2-4ac}+b\right)}$$
$$+ \frac{(fx)^{m+1}\left(cx^2(bd-2ae)-abe-2acd+b^2d\right)}{2af\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}$$

```
[In] Int[((f*x)^m*(d + e*x^2))/(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] ((f*x)^(1 + m)*(b^2*d - 2*a*c*d - a*b*e + c*(b*d - 2*a*e)*x^2))/(2*a*(b^2 -
 4*a*c)*f*(a + b*x^2 + c*x^4)) + (c*(4*a*b*e + Sqrt[b^2 - 4*a*c]*(b*d - 2*a
*e)*(1 - m) - 4*a*c*d*(3 - m) + b^2*(d - d*m))*(f*x)^(1 + m)*Hypergeometric
2F1[1, (1 + m)/2, (3 + m)/2, (-2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])])/(2*a*(b^2
 - 4*a*c)^(3/2)*(b - Sqrt[b^2 - 4*a*c])*f*(1 + m)) - (c*(4*a*b*e - Sqrt[b^2
 - 4*a*c]*(b*d - 2*a*e)*(1 - m) - 4*a*c*d*(3 - m) + b^2*(d - d*m))*(f*x)^(1
 + m)*Hypergeometric2F1[1, (1 + m)/2, (3 + m)/2, (-2*c*x^2)/(b + Sqrt[b^2 -
 4*a*c])])/(2*a*(b^2 - 4*a*c)^(3/2)*(b + Sqrt[b^2 - 4*a*c])*f*(1 + m))
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 1291

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[(-(f*x)^(m + 1))*(a + b*x^2 + c*x^4)^(p + 1)
*((d*(b^2 - 2*a*c) - a*b*e + (b*d - 2*a*e)*c*x^2)/(2*a*f*(p + 1)*(b^2 - 4*a
*c))), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), Int[(f*x)^m*(a + b*x^2 + c*
x^4)^(p + 1)*Simp[d*(b^2*(m + 2*(p + 1) + 1) - 2*a*c*(m + 4*(p + 1) + 1)) -
 a*b*e*(m + 1) + c*(m + 2*(2*p + 3) + 1)*(b*d - 2*a*e)*x^2, x], x], x] /; F
reeQ[{a, b, c, d, e, f, m}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && Inte
gerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1299

```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2))/((a_) + (b_.)*(x_)^2 + (c_.)
*(x_)^4), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b
*e)/(2*q), Int[(f*x)^m/(b/2 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*
e)/(2*q), Int[(f*x)^m/(b/2 + q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e,
 f, m}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\text{integral} = \frac{(fx)^{1+m}\left(b^2d-2acd-abe+c(bd-2ae)x^2\right)}{2a\left(b^2-4ac\right)f\left(a+bx^2+cx^4\right)} - \frac{\int \frac{(fx)^m\left(-b^2d(1-m)+2acd(3-m)-abe(1+m)-c(bd-2ae)(1-m)x^2\right)}{a+bx^2+cx^4}\,dx}{2a\left(b^2-4ac\right)}$$

$$= \frac{(fx)^{1+m}\left(b^2d-2acd-abe+c(bd-2ae)x^2\right)}{2a\left(b^2-4ac\right)f\left(a+bx^2+cx^4\right)} + \frac{\left(c\left(4abe+b^2d(1-m)+\sqrt{b^2-4ac}(bd-2ae)(1-m)-4acd(3-m)\right)\right)\int \frac{(fx)^m}{\frac{b}{2}-\frac{1}{2}\sqrt{b^2-4ac}+cx^2}\,dx}{4a\left(b^2-4ac\right)^{3/2}} - \frac{\left(c\left(4abe-\sqrt{b^2-4ac}(bd-2ae)(1-m)-4acd(3-m)+b^2(d-dm)\right)\right)\int \frac{(fx)^m}{\frac{b}{2}+\frac{1}{2}\sqrt{b^2-4ac}+cx^2}\,dx}{4a\left(b^2-4ac\right)^{3/2}}$$

$$= \frac{(fx)^{1+m}\left(b^2d-2acd-abe+c(bd-2ae)x^2\right)}{2a\left(b^2-4ac\right)f\left(a+bx^2+cx^4\right)} + \frac{c\left(4abe+b^2d(1-m)+\sqrt{b^2-4ac}(bd-2ae)(1-m)-4acd(3-m)\right)(fx)^{1+m}\,{}_2F_1\left(1,\frac{1+m}{2};\frac{3+m}{2}\right.}{2a\left(b^2-4ac\right)^{3/2}\left(b-\sqrt{b^2-4ac}\right)f(1+m)} - \frac{c\left(4abe-\sqrt{b^2-4ac}(bd-2ae)(1-m)-4acd(3-m)+b^2(d-dm)\right)(fx)^{1+m}\,{}_2F_1\left(1,\frac{1+m}{2};\frac{3+m}{2}\right.}{2a\left(b^2-4ac\right)^{3/2}\left(b+\sqrt{b^2-4ac}\right)f(1+m)}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 6 vs. order 5 in optimal.

Time = 1.53 (sec) , antiderivative size = 160, normalized size of antiderivative = 0.41

$$\int \frac{(fx)^m\left(d+ex^2\right)}{\left(a+bx^2+cx^4\right)^2}\,dx$$

$$= \frac{x(fx)^m\left(d(3+m)\,\text{AppellF1}\left(\frac{1+m}{2},2,2,\frac{3+m}{2},-\frac{2cx^2}{b+\sqrt{b^2-4ac}},\frac{2cx^2}{-b+\sqrt{b^2-4ac}}\right)+e(1+m)x^2\,\text{AppellF1}\left(\frac{3+m}{2},2,2,\right.\right.}{a^2(1+m)(3+m)}$$

```
[In] Integrate[((f*x)^m*(d + e*x^2))/(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] (x*(f*x)^m*(d*(3 + m)*AppellF1[(1 + m)/2, 2, 2, (3 + m)/2, (-2*c*x^2)/(b +
Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])] + e*(1 + m)*x^2*App
ellF1[(3 + m)/2, 2, 2, (5 + m)/2, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]), (2*c*
x^2)/(-b + Sqrt[b^2 - 4*a*c])]))/(a^2*(1 + m)*(3 + m))
```

Maple [F]

$$\int \frac{(fx)^m (e\,x^2+d)}{(c\,x^4+b\,x^2+a)^2}dx$$

```
[In] int((f*x)^m*(e*x^2+d)/(c*x^4+b*x^2+a)^2,x)
```

```
[Out] int((f*x)^m*(e*x^2+d)/(c*x^4+b*x^2+a)^2,x)
```

Fricas [F]

$$\int \frac{(fx)^m (d+ex^2)}{(a+bx^2+cx^4)^2}\,dx = \int \frac{(ex^2+d)(fx)^m}{(cx^4+bx^2+a)^2}\,dx$$

```
[In] integrate((f*x)^m*(e*x^2+d)/(c*x^4+b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] integral((e*x^2 + d)*(f*x)^m/(c^2*x^8 + 2*b*c*x^6 + (b^2 + 2*a*c)*x^4 + 2*a
*b*x^2 + a^2), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(fx)^m (d+ex^2)}{(a+bx^2+cx^4)^2}\,dx = \text{Timed out}$$

```
[In] integrate((f*x)**m*(e*x**2+d)/(c*x**4+b*x**2+a)**2,x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{(fx)^m (d+ex^2)}{(a+bx^2+cx^4)^2}\,dx = \int \frac{(ex^2+d)(fx)^m}{(cx^4+bx^2+a)^2}\,dx$$

```
[In] integrate((f*x)^m*(e*x^2+d)/(c*x^4+b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] integrate((e*x^2 + d)*(f*x)^m/(c*x^4 + b*x^2 + a)^2, x)
```

Giac [F]

$$\int \frac{(fx)^m (d+ex^2)}{(a+bx^2+cx^4)^2} \, dx = \int \frac{(ex^2+d)(fx)^m}{(cx^4+bx^2+a)^2} \, dx$$

```
[In] integrate((f*x)^m*(e*x^2+d)/(c*x^4+b*x^2+a)^2,x, algorithm="giac")
[Out] integrate((e*x^2 + d)*(f*x)^m/(c*x^4 + b*x^2 + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(fx)^m (d+ex^2)}{(a+bx^2+cx^4)^2} \, dx = \int \frac{(f\,x)^m\,(e\,x^2+d)}{(c\,x^4+b\,x^2+a)^2} \, dx$$

```
[In] int(((f*x)^m*(d + e*x^2))/(a + b*x^2 + c*x^4)^2,x)
[Out] int(((f*x)^m*(d + e*x^2))/(a + b*x^2 + c*x^4)^2, x)
```

3.225 $\int (fx)^m (d+ex^2) (a+bx^2+cx^4)^{3/2} dx$

Optimal result

Integrand size = 29, antiderivative size = 319

$$\int (fx)^m (d+ex^2) (a+bx^2+cx^4)^{3/2} dx = \frac{ad(fx)^{1+m}\sqrt{a+bx^2+cx^4}\,\text{AppellF1}\left(\frac{1+m}{2},-\frac{3}{2},-\frac{3}{2},\frac{3+m}{2},-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{f(1+m)\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}} + \frac{ae(fx)^{3+m}\sqrt{a+bx^2+cx^4}\,\text{AppellF1}\left(\frac{3+m}{2},-\frac{3}{2},-\frac{3}{2},\frac{5+m}{2},-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{f^3(3+m)\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}}$$

```
[Out] a*d*(f*x)^(1+m)*AppellF1(1/2+1/2*m,-3/2,-3/2,3/2+1/2*m,-2*c*x^2/(b-(-4*a*c+
b^2)^(1/2)),-2*c*x^2/(b+(-4*a*c+b^2)^(1/2)))*(c*x^4+b*x^2+a)^(1/2)/f/(1+m)/
(1+2*c*x^2/(b-(-4*a*c+b^2)^(1/2)))^(1/2)/(1+2*c*x^2/(b+(-4*a*c+b^2)^(1/2)))
^(1/2)+a*e*(f*x)^(3+m)*AppellF1(3/2+1/2*m,-3/2,-3/2,5/2+1/2*m,-2*c*x^2/(b-(
-4*a*c+b^2)^(1/2)),-2*c*x^2/(b+(-4*a*c+b^2)^(1/2)))*(c*x^4+b*x^2+a)^(1/2)/f
^3/(3+m)/(1+2*c*x^2/(b-(-4*a*c+b^2)^(1/2)))^(1/2)/(1+2*c*x^2/(b+(-4*a*c+b^2
)^(1/2)))^(1/2)
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 319, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used

$= \{1349, 1155, 524\}$

$$\int (fx)^m (d+ex^2)(a+bx^2+cx^4)^{3/2}\,dx = \frac{ad(fx)^{m+1}\sqrt{a+bx^2+cx^4}\,\text{AppellF1}\left(\frac{m+1}{2},-\frac{3}{2},-\frac{3}{2},\frac{m+3}{2},-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{f(m+1)\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}} + \frac{ae(fx)^{m+3}\sqrt{a+bx^2+cx^4}\,\text{AppellF1}\left(\frac{m+3}{2},-\frac{3}{2},-\frac{3}{2},\frac{m+5}{2},-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{f^3(m+3)\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}}$$

```
[In] Int[(f*x)^m*(d + e*x^2)*(a + b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] (a*d*(f*x)^(1 + m)*Sqrt[a + b*x^2 + c*x^4]*AppellF1[(1 + m)/2, -3/2, -3/2,
(3 + m)/2, (-2*c*x^2)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^2)/(b + Sqrt[b^2 - 4
*a*c])])/(f*(1 + m)*Sqrt[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2
*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]) + (a*e*(f*x)^(3 + m)*Sqrt[a + b*x^2 + c*x
^4]*AppellF1[(3 + m)/2, -3/2, -3/2, (5 + m)/2, (-2*c*x^2)/(b - Sqrt[b^2 - 4
*a*c]), (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])])/(f^3*(3 + m)*Sqrt[1 + (2*c*x^2
)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])])
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 1155

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[a^IntPart[p]*((a + b*x^2 + c*x^4)^FracPart[p]/((1 + 2*c*(x^2/(b +
Rt[b^2 - 4*a*c, 2])))^FracPart[p]*(1 + 2*c*(x^2/(b - Rt[b^2 - 4*a*c, 2])))^
FracPart[p])), Int[(d*x)^m*(1 + 2*c*(x^2/(b + Sqrt[b^2 - 4*a*c])))^p*(1 + 2
*c*(x^2/(b - Sqrt[b^2 - 4*a*c])))^p, x], x] /; FreeQ[{a, b, c, d, m, p}, x]
```

Rule 1349

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*
(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, p, q}, x] && N
eQ[b^2 - 4*a*c, 0] && (IGtQ[p, 0] || IGtQ[q, 0] || IntegersQ[m, q])
```

Rubi steps

$$\text{integral} = \int \left(d(fx)^m \left(a+bx^2+cx^4\right)^{3/2}+\frac{e(fx)^{2+m} \left(a+bx^2+cx^4\right)^{3/2}}{f^2}\right) dx$$

$$= d\int (fx)^m \left(a+bx^2+cx^4\right)^{3/2} dx+\frac{e\int (fx)^{2+m} \left(a+bx^2+cx^4\right)^{3/2} dx}{f^2}$$

$$= \frac{\left(ad\sqrt{a+bx^2+cx^4}\right)\int (fx)^m \left(1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}\right)^{3/2} \left(1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)^{3/2} dx}{\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}} + \frac{\left(ae\sqrt{a+bx^2+cx^4}\right)\int (fx)^{2+m} \left(1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}\right)^{3/2} \left(1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)^{3/2} dx}{f^2\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}}$$

$$= \frac{ad(fx)^{1+m}\sqrt{a+bx^2+cx^4}F_1\left(\frac{1+m}{2};-\frac{3}{2},-\frac{3}{2};\frac{3+m}{2};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{f(1+m)\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}} + \frac{ae(fx)^{3+m}\sqrt{a+bx^2+cx^4}F_1\left(\frac{3+m}{2};-\frac{3}{2},-\frac{3}{2};\frac{5+m}{2};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{f^3(3+m)\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}}$$

Mathematica [A] (warning: unable to verify)

Time = 2.90 (sec) , antiderivative size = 466, normalized size of antiderivative = 1.46

$$\int (fx)^m \left(d+ex^2\right) \left(a+bx^2+cx^4\right)^{3/2} dx = \frac{x(fx)^m\sqrt{a+bx^2+cx^4}\left(ad(105+71m+15m^2+m^3)\,\text{AppellF1}\left(\frac{1+m}{2},-\frac{1}{2},-\frac{1}{2},\frac{3+m}{2},-\frac{2cx^2}{b+\sqrt{b^2-4}}\right.\right.}{}$$

```
[In] Integrate[(f*x)^m*(d + e*x^2)*(a + b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] (x*(f*x)^m*Sqrt[a + b*x^2 + c*x^4]*(a*d*(105 + 71*m + 15*m^2 + m^3)*AppellF
1[(1 + m)/2, -1/2, -1/2, (3 + m)/2, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]), (2*
c*x^2)/(-b + Sqrt[b^2 - 4*a*c])] + (1 + m)*x^2*((b*d + a*e)*(35 + 12*m + m^
2)*AppellF1[(3 + m)/2, -1/2, -1/2, (5 + m)/2, (-2*c*x^2)/(b + Sqrt[b^2 - 4*
a*c]), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])] + (3 + m)*x^2*((c*d + b*e)*(7 +
m)*AppellF1[(5 + m)/2, -1/2, -1/2, (7 + m)/2, (-2*c*x^2)/(b + Sqrt[b^2 - 4*
a*c]), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])] + c*e*(5 + m)*x^2*AppellF1[(7 +
m)/2, -1/2, -1/2, (9 + m)/2, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^2)/
(-b + Sqrt[b^2 - 4*a*c])]))))/((1 + m)*(3 + m)*(5 + m)*(7 + m)*Sqrt[(b - Sq
rt[b^2 - 4*a*c] + 2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[(b + Sqrt[b^2 - 4*
a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])])
```

Maple [F]

$$\int (fx)^m \left(e\,x^2+d\right) \left(c\,x^4+b\,x^2+a\right)^{\frac{3}{2}} dx$$

```
[In] int((f*x)^m*(e*x^2+d)*(c*x^4+b*x^2+a)^(3/2),x)
[Out] int((f*x)^m*(e*x^2+d)*(c*x^4+b*x^2+a)^(3/2),x)
```

Fricas [F]

$$\int (fx)^m \left(d+ex^2\right) \left(a+bx^2+cx^4\right)^{3/2} dx = \int \left(cx^4+bx^2+a\right)^{\frac{3}{2}} \left(ex^2+d\right)(fx)^m \, dx$$

```
[In] integrate((f*x)^m*(e*x^2+d)*(c*x^4+b*x^2+a)^(3/2),x, algorithm="fricas")
[Out] integral((c*e*x^6 + (c*d + b*e)*x^4 + (b*d + a*e)*x^2 + a*d)*sqrt(c*x^4 + b
*x^2 + a)*(f*x)^m, x)
```

Sympy [F]

$$\int (fx)^m \left(d+ex^2\right) \left(a+bx^2+cx^4\right)^{3/2} dx = \int (fx)^m \left(d+ex^2\right) \left(a+bx^2+cx^4\right)^{\frac{3}{2}} dx$$

```
[In] integrate((f*x)**m*(e*x**2+d)*(c*x**4+b*x**2+a)**(3/2),x)
[Out] Integral((f*x)**m*(d + e*x**2)*(a + b*x**2 + c*x**4)**(3/2), x)
```

Maxima [F]

$$\int (fx)^m \left(d+ex^2\right) \left(a+bx^2+cx^4\right)^{3/2} dx = \int \left(cx^4+bx^2+a\right)^{\frac{3}{2}} \left(ex^2+d\right)(fx)^m \, dx$$

```
[In] integrate((f*x)^m*(e*x^2+d)*(c*x^4+b*x^2+a)^(3/2),x, algorithm="maxima")
[Out] integrate((c*x^4 + b*x^2 + a)^(3/2)*(e*x^2 + d)*(f*x)^m, x)
```

Giac [F]

$$\int (fx)^m (d+ex^2) (a+bx^2+cx^4)^{3/2} \, dx = \int (cx^4+bx^2+a)^{\frac{3}{2}} (ex^2+d)(fx)^m \, dx$$

```
[In] integrate((f*x)^m*(e*x^2+d)*(c*x^4+b*x^2+a)^(3/2),x, algorithm="giac")
[Out] integrate((c*x^4 + b*x^2 + a)^(3/2)*(e*x^2 + d)*(f*x)^m, x)
```

Mupad [F(-1)]

Timed out.

$$\int (fx)^m (d+ex^2) (a+bx^2+cx^4)^{3/2} \, dx = \int (f\,x)^m \left(e\,x^2+d\right) \left(c\,x^4+b\,x^2+a\right)^{3/2} dx$$

```
[In] int((f*x)^m*(d + e*x^2)*(a + b*x^2 + c*x^4)^(3/2),x)
[Out] int((f*x)^m*(d + e*x^2)*(a + b*x^2 + c*x^4)^(3/2), x)
```

3.226 $\int (fx)^m (d+ex^2) \sqrt{a+bx^2+cx^4} \, dx$

Optimal result

Integrand size = 29, antiderivative size = 317

$$\int (fx)^m (d+ex^2) \sqrt{a+bx^2+cx^4} \, dx = \frac{d(fx)^{1+m}\sqrt{a+bx^2+cx^4}\,\text{AppellF1}\left(\frac{1+m}{2},-\frac{1}{2},-\frac{1}{2},\frac{3+m}{2},-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{f(1+m)\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}} + \frac{e(fx)^{3+m}\sqrt{a+bx^2+cx^4}\,\text{AppellF1}\left(\frac{3+m}{2},-\frac{1}{2},-\frac{1}{2},\frac{5+m}{2},-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{f^3(3+m)\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}}$$

```
[Out] d*(f*x)^(1+m)*AppellF1(1/2+1/2*m,-1/2,-1/2,3/2+1/2*m,-2*c*x^2/(b-(-4*a*c+b^
2)^(1/2)),-2*c*x^2/(b+(-4*a*c+b^2)^(1/2)))*(c*x^4+b*x^2+a)^(1/2)/f/(1+m)/(1
+2*c*x^2/(b-(-4*a*c+b^2)^(1/2)))^(1/2)/(1+2*c*x^2/(b+(-4*a*c+b^2)^(1/2)))^(
1/2)+e*(f*x)^(3+m)*AppellF1(3/2+1/2*m,-1/2,-1/2,5/2+1/2*m,-2*c*x^2/(b-(-4*a
*c+b^2)^(1/2)),-2*c*x^2/(b+(-4*a*c+b^2)^(1/2)))*(c*x^4+b*x^2+a)^(1/2)/f^3/(
3+m)/(1+2*c*x^2/(b-(-4*a*c+b^2)^(1/2)))^(1/2)/(1+2*c*x^2/(b+(-4*a*c+b^2)^(1
/2)))^(1/2)
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 317, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.103$, Rules used

$= \{1349, 1155, 524\}$

$$\int (fx)^m \left(d+ex^2\right) \sqrt{a+bx^2+cx^4}\,dx = \frac{d(fx)^{m+1}\sqrt{a+bx^2+cx^4}\operatorname{AppellF1}\left(\frac{m+1}{2},-\frac{1}{2},-\frac{1}{2},\frac{m+3}{2},-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{f(m+1)\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}} + \frac{e(fx)^{m+3}\sqrt{a+bx^2+cx^4}\operatorname{AppellF1}\left(\frac{m+3}{2},-\frac{1}{2},-\frac{1}{2},\frac{m+5}{2},-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{f^3(m+3)\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}}$$

```
[In] Int[(f*x)^m*(d + e*x^2)*Sqrt[a + b*x^2 + c*x^4],x]
```

```
[Out] (d*(f*x)^(1 + m)*Sqrt[a + b*x^2 + c*x^4]*AppellF1[(1 + m)/2, -1/2, -1/2, (3
 + m)/2, (-2*c*x^2)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^2)/(b + Sqrt[b^2 - 4*a
*c])])/(f*(1 + m)*Sqrt[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c
*x^2)/(b + Sqrt[b^2 - 4*a*c])]) + (e*(f*x)^(3 + m)*Sqrt[a + b*x^2 + c*x^4]*
AppellF1[(3 + m)/2, -1/2, -1/2, (5 + m)/2, (-2*c*x^2)/(b - Sqrt[b^2 - 4*a*c
]), (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])])/(f^3*(3 + m)*Sqrt[1 + (2*c*x^2)/(b
 - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])])
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 1155

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[a^IntPart[p]*((a + b*x^2 + c*x^4)^FracPart[p]/((1 + 2*c*(x^2/(b +
Rt[b^2 - 4*a*c, 2])))^FracPart[p]*(1 + 2*c*(x^2/(b - Rt[b^2 - 4*a*c, 2])))^
FracPart[p])), Int[(d*x)^m*(1 + 2*c*(x^2/(b + Sqrt[b^2 - 4*a*c])))^p*(1 + 2
*c*(x^2/(b - Sqrt[b^2 - 4*a*c])))^p, x], x] /; FreeQ[{a, b, c, d, m, p}, x]
```

Rule 1349

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*
(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, p, q}, x] && N
eQ[b^2 - 4*a*c, 0] && (IGtQ[p, 0] || IGtQ[q, 0] || IntegersQ[m, q])
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \left(d(fx)^m\sqrt{a+bx^2+cx^4}+\frac{e(fx)^{2+m}\sqrt{a+bx^2+cx^4}}{f^2}\right)dx\\
&= d\int (fx)^m\sqrt{a+bx^2+cx^4}\,dx+\frac{e\int (fx)^{2+m}\sqrt{a+bx^2+cx^4}\,dx}{f^2}\\
&= \frac{\left(d\sqrt{a+bx^2+cx^4}\right)\int (fx)^m\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}\,dx}{\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}}\\
&\quad+\frac{\left(e\sqrt{a+bx^2+cx^4}\right)\int (fx)^{2+m}\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}\,dx}{f^2\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}}\\
&= \frac{d(fx)^{1+m}\sqrt{a+bx^2+cx^4}F_1\left(\frac{1+m}{2};-\frac{1}{2},-\frac{1}{2};\frac{3+m}{2};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{f(1+m)\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}}\\
&\quad+\frac{e(fx)^{3+m}\sqrt{a+bx^2+cx^4}F_1\left(\frac{3+m}{2};-\frac{1}{2},-\frac{1}{2};\frac{5+m}{2};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{f^3(3+m)\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 1.98 (sec) , antiderivative size = 267, normalized size of antiderivative = 0.84

$$
\int (fx)^m\left(d+ex^2\right)\sqrt{a+bx^2+cx^4}\,dx
$$

$$
=\frac{x(fx)^m\sqrt{a+bx^2+cx^4}\Big(d(3+m)\,\text{AppellF1}\left(\frac{1+m}{2},-\frac{1}{2},-\frac{1}{2},\frac{3+m}{2},-\frac{2cx^2}{b+\sqrt{b^2-4ac}},\frac{2cx^2}{-b+\sqrt{b^2-4ac}}\right)+e(1+m)x^2\,\text{A}}{(1+m)(3+m)\sqrt{\frac{b-\sqrt{b^2-4ac}+2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx^2}{b+\sqrt{b^2-4ac}}}}
$$

```
[In] Integrate[(f*x)^m*(d + e*x^2)*Sqrt[a + b*x^2 + c*x^4],x]
```

```
[Out] (x*(f*x)^m*Sqrt[a + b*x^2 + c*x^4]*(d*(3 + m)*AppellF1[(1 + m)/2, -1/2, -1/
2, (3 + m)/2, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(-b + Sqrt[b^2
- 4*a*c])] + e*(1 + m)*x^2*AppellF1[(3 + m)/2, -1/2, -1/2, (5 + m)/2, (-2*c
*x^2)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])]))/((1 +
m)*(3 + m)*Sqrt[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*
Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])])
```

Maple [F]

$$\int (fx)^m \left(e\,x^2+d\right)\sqrt{c\,x^4+b\,x^2+a}dx$$

```
[In] int((f*x)^m*(e*x^2+d)*(c*x^4+b*x^2+a)^(1/2),x)
```

```
[Out] int((f*x)^m*(e*x^2+d)*(c*x^4+b*x^2+a)^(1/2),x)
```

Fricas [F]

$$\int (fx)^m \left(d+ex^2\right)\sqrt{a+bx^2+cx^4}\,dx = \int \sqrt{cx^4+bx^2+a}\left(ex^2+d\right)(fx)^m\,dx$$

```
[In] integrate((f*x)^m*(e*x^2+d)*(c*x^4+b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c*x^4 + b*x^2 + a)*(e*x^2 + d)*(f*x)^m, x)
```

Sympy [F]

$$\int (fx)^m \left(d+ex^2\right)\sqrt{a+bx^2+cx^4}\,dx = \int (fx)^m \left(d+ex^2\right)\sqrt{a+bx^2+cx^4}\,dx$$

```
[In] integrate((f*x)**m*(e*x**2+d)*(c*x**4+b*x**2+a)**(1/2),x)
```

```
[Out] Integral((f*x)**m*(d + e*x**2)*sqrt(a + b*x**2 + c*x**4), x)
```

Maxima [F]

$$\int (fx)^m \left(d+ex^2\right)\sqrt{a+bx^2+cx^4}\,dx = \int \sqrt{cx^4+bx^2+a}\left(ex^2+d\right)(fx)^m\,dx$$

```
[In] integrate((f*x)^m*(e*x^2+d)*(c*x^4+b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(c*x^4 + b*x^2 + a)*(e*x^2 + d)*(f*x)^m, x)
```

Giac [F]

$$\int (fx)^m \left(d+ex^2\right) \sqrt{a+bx^2+cx^4}\,dx = \int \sqrt{cx^4+bx^2+a}\left(ex^2+d\right)(fx)^m\,dx$$

```
[In] integrate((f*x)^m*(e*x^2+d)*(c*x^4+b*x^2+a)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(c*x^4 + b*x^2 + a)*(e*x^2 + d)*(f*x)^m, x)
```

Mupad [F(-1)]

Timed out.

$$\int (fx)^m \left(d+ex^2\right) \sqrt{a+bx^2+cx^4}\,dx = \int (f\,x)^m\,\left(e\,x^2+d\right)\,\sqrt{c\,x^4+b\,x^2+a}\,dx$$

```
[In] int((f*x)^m*(d + e*x^2)*(a + b*x^2 + c*x^4)^(1/2),x)
[Out] int((f*x)^m*(d + e*x^2)*(a + b*x^2 + c*x^4)^(1/2), x)
```

3.227 $\int \frac{(fx)^m(d+ex^2)}{\sqrt{a+bx^2+cx^4}}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 317

$$\int \frac{(fx)^m\,(d+ex^2)}{\sqrt{a+bx^2+cx^4}}\,dx = \frac{d(fx)^{1+m}\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}\,\text{AppellF1}\left(\frac{1+m}{2},\frac{1}{2},\frac{1}{2},\frac{3+m}{2},-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{f(1+m)\sqrt{a+bx^2+cx^4}} + \frac{e(fx)^{3+m}\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}\,\text{AppellF1}\left(\frac{3+m}{2},\frac{1}{2},\frac{1}{2},\frac{5+m}{2},-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{f^3(3+m)\sqrt{a+bx^2+cx^4}}$$

```
[Out] d*(f*x)^(1+m)*AppellF1(1/2+1/2*m,1/2,1/2,3/2+1/2*m,-2*c*x^2/(b-(-4*a*c+b^2)
^(1/2)),-2*c*x^2/(b+(-4*a*c+b^2)^(1/2)))*(1+2*c*x^2/(b-(-4*a*c+b^2)^(1/2)))
^(1/2)*(1+2*c*x^2/(b+(-4*a*c+b^2)^(1/2)))^(1/2)/f/(1+m)/(c*x^4+b*x^2+a)^(1/
2)+e*(f*x)^(3+m)*AppellF1(3/2+1/2*m,1/2,1/2,5/2+1/2*m,-2*c*x^2/(b-(-4*a*c+b
^2)^(1/2)),-2*c*x^2/(b+(-4*a*c+b^2)^(1/2)))*(1+2*c*x^2/(b-(-4*a*c+b^2)^(1/2
)))^(1/2)*(1+2*c*x^2/(b+(-4*a*c+b^2)^(1/2)))^(1/2)/f^3/(3+m)/(c*x^4+b*x^2+a
)^(1/2)
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 317, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used

$= \{1349, 1155, 524\}$

$$\int \frac{(fx)^m (d+ex^2)}{\sqrt{a+bx^2+cx^4}}\,dx$$
$$= \frac{d(fx)^{m+1}\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}\,\text{AppellF1}\left(\frac{m+1}{2},\frac{1}{2},\frac{1}{2},\frac{m+3}{2},-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{f(m+1)\sqrt{a+bx^2+cx^4}}$$
$$+\frac{e(fx)^{m+3}\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}\,\text{AppellF1}\left(\frac{m+3}{2},\frac{1}{2},\frac{1}{2},\frac{m+5}{2},-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{f^3(m+3)\sqrt{a+bx^2+cx^4}}$$

```
[In] Int[((f*x)^m*(d + e*x^2))/Sqrt[a + b*x^2 + c*x^4],x]
```

```
[Out] (d*(f*x)^(1 + m)*Sqrt[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*
x^2)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[(1 + m)/2, 1/2, 1/2, (3 + m)/2, (-2*
c*x^2)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])])/(f*(1
+ m)*Sqrt[a + b*x^2 + c*x^4]) + (e*(f*x)^(3 + m)*Sqrt[1 + (2*c*x^2)/(b - Sq
rt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[(3 +
 m)/2, 1/2, 1/2, (5 + m)/2, (-2*c*x^2)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^2)/
(b + Sqrt[b^2 - 4*a*c])])/(f^3*(3 + m)*Sqrt[a + b*x^2 + c*x^4])
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 1155

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[a^IntPart[p]*((a + b*x^2 + c*x^4)^FracPart[p]/((1 + 2*c*(x^2/(b +
Rt[b^2 - 4*a*c, 2])))^FracPart[p]*(1 + 2*c*(x^2/(b - Rt[b^2 - 4*a*c, 2])))^
FracPart[p])), Int[(d*x)^m*(1 + 2*c*(x^2/(b + Sqrt[b^2 - 4*a*c])))^p*(1 + 2
*c*(x^2/(b - Sqrt[b^2 - 4*a*c])))^p, x], x] /; FreeQ[{a, b, c, d, m, p}, x]
```

Rule 1349

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*
(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, p, q}, x] && N
eQ[b^2 - 4*a*c, 0] && (IGtQ[p, 0] || IGtQ[q, 0] || IntegersQ[m, q])
```

Rubi steps

$$\text{integral} = \int \left(\frac{d(fx)^m}{\sqrt{a+bx^2+cx^4}}+\frac{e(fx)^{2+m}}{f^2\sqrt{a+bx^2+cx^4}}\right)\,dx$$

$$= d\int \frac{(fx)^m}{\sqrt{a+bx^2+cx^4}}\,dx+\frac{e\int \frac{(fx)^{2+m}}{\sqrt{a+bx^2+cx^4}}\,dx}{f^2}$$

$$= \frac{\left(d\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}\right)\int \frac{(fx)^m}{\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}}\,dx}{\sqrt{a+bx^2+cx^4}}$$
$$+\frac{\left(e\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}\right)\int \frac{(fx)^{2+m}}{\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}}\,dx}{f^2\sqrt{a+bx^2+cx^4}}$$

$$= \frac{d(fx)^{1+m}\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}F_1\left(\frac{1+m}{2};\frac{1}{2},\frac{1}{2};\frac{3+m}{2};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{f(1+m)\sqrt{a+bx^2+cx^4}}$$
$$+\frac{e(fx)^{3+m}\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}F_1\left(\frac{3+m}{2};\frac{1}{2},\frac{1}{2};\frac{5+m}{2};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{f^3(3+m)\sqrt{a+bx^2+cx^4}}$$

Mathematica [A] (verified)

Time = 2.83 (sec) , antiderivative size = 267, normalized size of antiderivative = 0.84

$$\int \frac{(fx)^m\,(d+ex^2)}{\sqrt{a+bx^2+cx^4}}\,dx$$
$$= \frac{x(fx)^m\sqrt{\frac{b-\sqrt{b^2-4ac}+2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx^2}{b+\sqrt{b^2-4ac}}}\left(d(3+m)\,\text{AppellF1}\left(\frac{1+m}{2},\frac{1}{2},\frac{1}{2},\frac{3+m}{2},-\frac{2cx^2}{b+\sqrt{b^2-4ac}},\frac{2cx^2}{-b+\sqrt{b^2-4ac}}\right)+e\right.}{(1+m)(3+m)\sqrt{a+bx^2+cx^4}}$$

```
[In] Integrate[((f*x)^m*(d + e*x^2))/Sqrt[a + b*x^2 + c*x^4],x]
```

```
[Out] (x*(f*x)^m*Sqrt[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*
Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*(d*(3 + m)*
AppellF1[(1 + m)/2, 1/2, 1/2, (3 + m)/2, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])
, (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])] + e*(1 + m)*x^2*AppellF1[(3 + m)/2, 1
/2, 1/2, (5 + m)/2, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(-b + Sqr
t[b^2 - 4*a*c])]))/((1 + m)*(3 + m)*Sqrt[a + b*x^2 + c*x^4])
```

Maple [F]

$$\int \frac{(fx)^m (e\,x^2+d)}{\sqrt{c\,x^4+b\,x^2+a}}dx$$

```
[In] int((f*x)^m*(e*x^2+d)/(c*x^4+b*x^2+a)^(1/2),x)
[Out] int((f*x)^m*(e*x^2+d)/(c*x^4+b*x^2+a)^(1/2),x)
```

Fricas [F]

$$\int \frac{(fx)^m (d+ex^2)}{\sqrt{a+bx^2+cx^4}}\,dx = \int \frac{(ex^2+d)(fx)^m}{\sqrt{cx^4+bx^2+a}}\,dx$$

```
[In] integrate((f*x)^m*(e*x^2+d)/(c*x^4+b*x^2+a)^(1/2),x, algorithm="fricas")
[Out] integral((e*x^2 + d)*(f*x)^m/sqrt(c*x^4 + b*x^2 + a), x)
```

Sympy [F]

$$\int \frac{(fx)^m (d+ex^2)}{\sqrt{a+bx^2+cx^4}}\,dx = \int \frac{(fx)^m (d+ex^2)}{\sqrt{a+bx^2+cx^4}}\,dx$$

```
[In] integrate((f*x)**m*(e*x**2+d)/(c*x**4+b*x**2+a)**(1/2),x)
[Out] Integral((f*x)**m*(d + e*x**2)/sqrt(a + b*x**2 + c*x**4), x)
```

Maxima [F]

$$\int \frac{(fx)^m (d+ex^2)}{\sqrt{a+bx^2+cx^4}}\,dx = \int \frac{(ex^2+d)(fx)^m}{\sqrt{cx^4+bx^2+a}}\,dx$$

```
[In] integrate((f*x)^m*(e*x^2+d)/(c*x^4+b*x^2+a)^(1/2),x, algorithm="maxima")
[Out] integrate((e*x^2 + d)*(f*x)^m/sqrt(c*x^4 + b*x^2 + a), x)
```

Giac [F]

$$\int \frac{(fx)^m (d+ex^2)}{\sqrt{a+bx^2+cx^4}}\,dx = \int \frac{(ex^2+d)(fx)^m}{\sqrt{cx^4+bx^2+a}}\,dx$$

```
[In] integrate((f*x)^m*(e*x^2+d)/(c*x^4+b*x^2+a)^(1/2),x, algorithm="giac")
[Out] integrate((e*x^2 + d)*(f*x)^m/sqrt(c*x^4 + b*x^2 + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(fx)^m (d+ex^2)}{\sqrt{a+bx^2+cx^4}}\,dx = \int \frac{(f\,x)^m\,(e\,x^2+d)}{\sqrt{c\,x^4+b\,x^2+a}}\,dx$$

```
[In] int(((f*x)^m*(d + e*x^2))/(a + b*x^2 + c*x^4)^(1/2),x)
[Out] int(((f*x)^m*(d + e*x^2))/(a + b*x^2 + c*x^4)^(1/2), x)
```

3.228 $\int \frac{(fx)^m(d+ex^2)}{(a+bx^2+cx^4)^{3/2}}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 323

$$\int \frac{(fx)^m\,(d+ex^2)}{(a+bx^2+cx^4)^{3/2}}\,dx = \frac{d(fx)^{1+m}\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}\,\text{AppellF1}\left(\frac{1+m}{2},\frac{3}{2},\frac{3}{2},\frac{3+m}{2},-\frac{2cx^2}{b-\sqrt{b^2-4ac}},\right.}{af(1+m)\sqrt{a+bx^2+cx^4}}$$
$$+\frac{e(fx)^{3+m}\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}\,\text{AppellF1}\left(\frac{3+m}{2},\frac{3}{2},\frac{3}{2},\frac{5+m}{2},-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{af^3(3+m)\sqrt{a+bx^2+cx^4}}$$

```
[Out] d*(f*x)^(1+m)*AppellF1(1/2+1/2*m,3/2,3/2,3/2+1/2*m,-2*c*x^2/(b-(-4*a*c+b^2)
^(1/2)),-2*c*x^2/(b+(-4*a*c+b^2)^(1/2)))*(1+2*c*x^2/(b-(-4*a*c+b^2)^(1/2)))
^(1/2)*(1+2*c*x^2/(b+(-4*a*c+b^2)^(1/2)))^(1/2)/a/f/(1+m)/(c*x^4+b*x^2+a)^(
1/2)+e*(f*x)^(3+m)*AppellF1(3/2+1/2*m,3/2,3/2,5/2+1/2*m,-2*c*x^2/(b-(-4*a*c
+b^2)^(1/2)),-2*c*x^2/(b+(-4*a*c+b^2)^(1/2)))*(1+2*c*x^2/(b-(-4*a*c+b^2)^(1
/2)))^(1/2)*(1+2*c*x^2/(b+(-4*a*c+b^2)^(1/2)))^(1/2)/a/f^3/(3+m)/(c*x^4+b*x
^2+a)^(1/2)
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 323, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {1349, 1155, 524}

$$\int \frac{(fx)^m\,(d+ex^2)}{(a+bx^2+cx^4)^{3/2}}\,dx = \frac{d(fx)^{m+1}\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}\,\text{AppellF1}\left(\frac{m+1}{2},\frac{3}{2},\frac{3}{2},\frac{m+3}{2},-\frac{2cx^2}{b-\sqrt{b^2-4ac}},\right.}{af(m+1)\sqrt{a+bx^2+cx^4}}$$
$$+\frac{e(fx)^{m+3}\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}\,\text{AppellF1}\left(\frac{m+3}{2},\frac{3}{2},\frac{3}{2},\frac{m+5}{2},-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{af^3(m+3)\sqrt{a+bx^2+cx^4}}$$

```
[In] Int[((f*x)^m*(d + e*x^2))/(a + b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] (d*(f*x)^(1 + m)*Sqrt[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*
x^2)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[(1 + m)/2, 3/2, 3/2, (3 + m)/2, (-2*
c*x^2)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])])/(a*f*(
1 + m)*Sqrt[a + b*x^2 + c*x^4]) + (e*(f*x)^(3 + m)*Sqrt[1 + (2*c*x^2)/(b -
Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[(3
 + m)/2, 3/2, 3/2, (5 + m)/2, (-2*c*x^2)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^2
)/(b + Sqrt[b^2 - 4*a*c])])/(a*f^3*(3 + m)*Sqrt[a + b*x^2 + c*x^4])
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 1155

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[a^IntPart[p]*((a + b*x^2 + c*x^4)^FracPart[p]/((1 + 2*c*(x^2/(b +
Rt[b^2 - 4*a*c, 2])))^FracPart[p]*(1 + 2*c*(x^2/(b - Rt[b^2 - 4*a*c, 2])))^
FracPart[p])), Int[(d*x)^m*(1 + 2*c*(x^2/(b + Sqrt[b^2 - 4*a*c])))^p*(1 + 2
*c*(x^2/(b - Sqrt[b^2 - 4*a*c])))^p, x], x] /; FreeQ[{a, b, c, d, m, p}, x]
```

Rule 1349

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*
(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, p, q}, x] && N
eQ[b^2 - 4*a*c, 0] && (IGtQ[p, 0] || IGtQ[q, 0] || IntegersQ[m, q])
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \left(\frac{d(fx)^m}{(a+bx^2+cx^4)^{3/2}}+\frac{e(fx)^{2+m}}{f^2\,(a+bx^2+cx^4)^{3/2}}\right)dx\\
&= d\int \frac{(fx)^m}{(a+bx^2+cx^4)^{3/2}}\,dx+\frac{e\int \frac{(fx)^{2+m}}{(a+bx^2+cx^4)^{3/2}}\,dx}{f^2}\\
&= \frac{\left(d\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}\right)\int \frac{(fx)^m}{\left(1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}\right)^{3/2}\left(1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)^{3/2}}\,dx}{a\sqrt{a+bx^2+cx^4}}\\
&\quad+\frac{\left(e\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}\right)\int \frac{(fx)^{2+m}}{\left(1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}\right)^{3/2}\left(1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)^{3/2}}\,dx}{af^2\sqrt{a+bx^2+cx^4}}
\end{aligned}
$$

$$= \frac{d(fx)^{1+m}\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}F_1\left(\frac{1+m}{2};\frac{3}{2},\frac{3}{2};\frac{3+m}{2};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{af(1+m)\sqrt{a+bx^2+cx^4}} + \frac{e(fx)^{3+m}\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}F_1\left(\frac{3+m}{2};\frac{3}{2},\frac{3}{2};\frac{5+m}{2};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{af^3(3+m)\sqrt{a+bx^2+cx^4}}$$

Mathematica [A] (verified)

Time = 11.34 (sec) , antiderivative size = 307, normalized size of antiderivative = 0.95

$$\int \frac{(fx)^m(d+ex^2)}{(a+bx^2+cx^4)^{3/2}}\,dx = \frac{x(fx)^m\left(-b+\sqrt{b^2-4ac}-2cx^2\right)\sqrt{\frac{b-\sqrt{b^2-4ac}+2cx^2}{b-\sqrt{b^2-4ac}}}\left(\frac{b+\sqrt{b^2-4ac}+2cx^2}{b+\sqrt{b^2-4ac}}\right)^{3/2}\left(d(3+m\right.}{\left(-b+\right.}$$

```
[In] Integrate[((f*x)^m*(d + e*x^2))/(a + b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] (x*(f*x)^m*(-b + Sqrt[b^2 - 4*a*c] - 2*c*x^2)*Sqrt[(b - Sqrt[b^2 - 4*a*c] +
 2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*((b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b +
Sqrt[b^2 - 4*a*c]))^(3/2)*(d*(3 + m)*AppellF1[(1 + m)/2, 3/2, 3/2, (3 + m)/
2, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])]
+ e*(1 + m)*x^2*AppellF1[(3 + m)/2, 3/2, 3/2, (5 + m)/2, (-2*c*x^2)/(b + Sq
rt[b^2 - 4*a*c]), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])]))/((-b + Sqrt[b^2 - 4
*a*c])*(1 + m)*(3 + m)*(a + b*x^2 + c*x^4)^(3/2))
```

Maple [F]

$$\int \frac{(fx)^m\left(e\,x^2+d\right)}{\left(c\,x^4+b\,x^2+a\right)^{\frac{3}{2}}}dx$$

```
[In] int((f*x)^m*(e*x^2+d)/(c*x^4+b*x^2+a)^(3/2),x)
```

```
[Out] int((f*x)^m*(e*x^2+d)/(c*x^4+b*x^2+a)^(3/2),x)
```

Fricas [F]

$$\int \frac{(fx)^m(d+ex^2)}{(a+bx^2+cx^4)^{3/2}}\,dx = \int \frac{(ex^2+d)(fx)^m}{(cx^4+bx^2+a)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((f*x)^m*(e*x^2+d)/(c*x^4+b*x^2+a)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c*x^4 + b*x^2 + a)*(e*x^2 + d)*(f*x)^m/(c^2*x^8 + 2*b*c*x^6 +
 (b^2 + 2*a*c)*x^4 + 2*a*b*x^2 + a^2), x)
```

Sympy [F]

$$\int \frac{(fx)^m (d+ex^2)}{(a+bx^2+cx^4)^{3/2}}\,dx = \int \frac{(fx)^m (d+ex^2)}{(a+bx^2+cx^4)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((f*x)**m*(e*x**2+d)/(c*x**4+b*x**2+a)**(3/2),x)
[Out] Integral((f*x)**m*(d + e*x**2)/(a + b*x**2 + c*x**4)**(3/2), x)
```

Maxima [F]

$$\int \frac{(fx)^m (d+ex^2)}{(a+bx^2+cx^4)^{3/2}}\,dx = \int \frac{(ex^2+d)(fx)^m}{(cx^4+bx^2+a)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((f*x)^m*(e*x^2+d)/(c*x^4+b*x^2+a)^(3/2),x, algorithm="maxima")
[Out] integrate((e*x^2 + d)*(f*x)^m/(c*x^4 + b*x^2 + a)^(3/2), x)
```

Giac [F]

$$\int \frac{(fx)^m (d+ex^2)}{(a+bx^2+cx^4)^{3/2}}\,dx = \int \frac{(ex^2+d)(fx)^m}{(cx^4+bx^2+a)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((f*x)^m*(e*x^2+d)/(c*x^4+b*x^2+a)^(3/2),x, algorithm="giac")
[Out] integrate((e*x^2 + d)*(f*x)^m/(c*x^4 + b*x^2 + a)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(fx)^m (d+ex^2)}{(a+bx^2+cx^4)^{3/2}}\,dx = \int \frac{(f\,x)^m\,(e\,x^2+d)}{(c\,x^4+b\,x^2+a)^{3/2}}\,dx$$

```
[In] int(((f*x)^m*(d + e*x^2))/(a + b*x^2 + c*x^4)^(3/2),x)
[Out] int(((f*x)^m*(d + e*x^2))/(a + b*x^2 + c*x^4)^(3/2), x)
```

3.229 $\int \frac{x^9}{(d+ex^2)(a+cx^4)}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 134

$$\int \frac{x^9}{(d+ex^2)(a+cx^4)}\,dx = -\frac{dx^2}{2ce^2}+\frac{x^4}{4ce}+\frac{a^{3/2}d\arctan\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{2c^{3/2}(cd^2+ae^2)} + \frac{d^4\log(d+ex^2)}{2e^3(cd^2+ae^2)}-\frac{a^2e\log(a+cx^4)}{4c^2(cd^2+ae^2)}$$

```
[Out] -1/2*d*x^2/c/e^2+1/4*x^4/c/e+1/2*a^(3/2)*d*arctan(x^2*c^(1/2)/a^(1/2))/c^(3
/2)/(a*e^2+c*d^2)+1/2*d^4*ln(e*x^2+d)/e^3/(a*e^2+c*d^2)-1/4*a^2*e*ln(c*x^4+
a)/c^2/(a*e^2+c*d^2)
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {1266, 1643, 649, 211, 266}

$$\int \frac{x^9}{(d+ex^2)(a+cx^4)}\,dx = \frac{a^{3/2}d\arctan\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{2c^{3/2}(ae^2+cd^2)}-\frac{a^2e\log(a+cx^4)}{4c^2(ae^2+cd^2)} + \frac{d^4\log(d+ex^2)}{2e^3(ae^2+cd^2)}-\frac{dx^2}{2ce^2}+\frac{x^4}{4ce}$$

```
[In] Int[x^9/((d + e*x^2)*(a + c*x^4)),x]
```

```
[Out] -1/2*(d*x^2)/(c*e^2) + x^4/(4*c*e) + (a^(3/2)*d*ArcTan[(Sqrt[c]*x^2)/Sqrt[a
]])/(2*c^(3/2)*(c*d^2 + a*e^2)) + (d^4*Log[d + e*x^2])/(2*e^3*(c*d^2 + a*e^
2)) - (a^2*e*Log[a + c*x^4])/(4*c^2*(c*d^2 + a*e^2))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[(-a)*c]
```

Rule 1266

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_S
ymbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + c*x^2)^p, x],
x, x^2], x] /; FreeQ[{a, c, d, e, p, q}, x] && IntegerQ[(m + 1)/2]
```

Rule 1643

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol]
:> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + c*x^2)^p, x], x] /; FreeQ[{a, c,
 d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{1}{2}\text{Subst}\left(\int \frac{x^4}{(d+ex)\left(a+cx^2\right)}\,dx, x, x^2\right)\\
&= \frac{1}{2}\text{Subst}\left(\int \left(-\frac{d}{ce^2}+\frac{x}{ce}+\frac{d^4}{e^2\left(cd^2+ae^2\right)(d+ex)}+\frac{a^2(d-ex)}{c\left(cd^2+ae^2\right)\left(a+cx^2\right)}\right)\,dx, x, x^2\right)\\
&= -\frac{dx^2}{2ce^2}+\frac{x^4}{4ce}+\frac{d^4\log\left(d+ex^2\right)}{2e^3\left(cd^2+ae^2\right)}+\frac{a^2\text{Subst}\left(\int \frac{d-ex}{a+cx^2}\,dx, x, x^2\right)}{2c\left(cd^2+ae^2\right)}\\
&= -\frac{dx^2}{2ce^2}+\frac{x^4}{4ce}+\frac{d^4\log\left(d+ex^2\right)}{2e^3\left(cd^2+ae^2\right)}+\frac{\left(a^2d\right)\text{Subst}\left(\int \frac{1}{a+cx^2}\,dx, x, x^2\right)}{2c\left(cd^2+ae^2\right)}\\
&\quad -\frac{\left(a^2e\right)\text{Subst}\left(\int \frac{x}{a+cx^2}\,dx, x, x^2\right)}{2c\left(cd^2+ae^2\right)}\\
&= -\frac{dx^2}{2ce^2}+\frac{x^4}{4ce}+\frac{a^{3/2}d\tan^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{2c^{3/2}\left(cd^2+ae^2\right)}+\frac{d^4\log\left(d+ex^2\right)}{2e^3\left(cd^2+ae^2\right)}-\frac{a^2e\log\left(a+cx^4\right)}{4c^2\left(cd^2+ae^2\right)}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.00

$$\int \frac{x^9}{(d+ex^2)(a+cx^4)}\,dx = -\frac{dx^2}{2ce^2}+\frac{x^4}{4ce}+\frac{a^{3/2}d\arctan\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{2c^{3/2}(cd^2+ae^2)}+\frac{d^4\log(d+ex^2)}{2e^3(cd^2+ae^2)}-\frac{a^2e\log(a+cx^4)}{4c^2(cd^2+ae^2)}$$

```
[In] Integrate[x^9/((d + e*x^2)*(a + c*x^4)),x]
```

```
[Out] -1/2*(d*x^2)/(c*e^2) + x^4/(4*c*e) + (a^(3/2)*d*ArcTan[(Sqrt[c]*x^2)/Sqrt[a
]])/(2*c^(3/2)*(c*d^2 + a*e^2)) + (d^4*Log[d + e*x^2])/(2*e^3*(c*d^2 + a*e^
2)) - (a^2*e*Log[a + c*x^4])/(4*c^2*(c*d^2 + a*e^2))
```

Maple [A] (verified)

Time = 0.41 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.77

method	result
default	$\frac{(-e\,x^2+d)^2}{4c\,e^3}+\frac{a^2\left(-\frac{e\ln(c\,x^4+a)}{2c}+\frac{d\arctan\left(\frac{c\,x^2}{\sqrt{ac}}\right)}{\sqrt{ac}}\right)}{2(a\,e^2+c\,d^2)c}+\frac{d^4\ln(e\,x^2+d)}{2e^3(a\,e^2+c\,d^2)}$
risch	$\frac{x^4}{4ce}-\frac{d\,x^2}{2c\,e^2}+\frac{d^2}{4c\,e^3}+\frac{d^4\ln(e\,x^2+d)}{2e^3(a\,e^2+c\,d^2)}+\frac{\sum_{\_R=\text{RootOf}\left((a\,c^2e^2+c^3d^2)\_Z^2+2a^2c\,e^3\_Z+e^4a^3\right)}\_R\ln\left(\left((2a\,c^2e^4-2c^3d^2e^2)\_R^2\right.\right.}{}$

```
[In] int(x^9/(e*x^2+d)/(c*x^4+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*(-e*x^2+d)^2/c/e^3+1/2*a^2/(a*e^2+c*d^2)/c*(-1/2*e/c*ln(c*x^4+a)+d/(a*c
)^(1/2)*arctan(c*x^2/(a*c)^(1/2)))+1/2*d^4*ln(e*x^2+d)/e^3/(a*e^2+c*d^2)
```

Fricas [A] (verification not implemented)

none

Time = 2.89 (sec) , antiderivative size = 277, normalized size of antiderivative = 2.07

$$\int \frac{x^9}{(d+ex^2)(a+cx^4)}\,dx = \left[\frac{acde^3\sqrt{-\frac{a}{c}}\log\left(\frac{cx^4+2\,cx^2\sqrt{-\frac{a}{c}}-a}{cx^4+a}\right)-a^2e^4\log(cx^4+a)+2\,c^2d^4\log(ex^2+d)+(c^2d^2e^2+ace^4)x^4-2\,(c^2}{4\,(c^3d^2e^3+ac^2e^5)}\right.$$

```
[In] integrate(x^9/(e*x^2+d)/(c*x^4+a),x, algorithm="fricas")
```

```
[Out] [1/4*(a*c*d*e^3*sqrt(-a/c)*log((c*x^4 + 2*c*x^2*sqrt(-a/c) - a)/(c*x^4 + a)
) - a^2*e^4*log(c*x^4 + a) + 2*c^2*d^4*log(e*x^2 + d) + (c^2*d^2*e^2 + a*c*
e^4)*x^4 - 2*(c^2*d^3*e + a*c*d*e^3)*x^2)/(c^3*d^2*e^3 + a*c^2*e^5), 1/4*(2
*a*c*d*e^3*sqrt(a/c)*arctan(c*x^2*sqrt(a/c)/a) - a^2*e^4*log(c*x^4 + a) + 2
*c^2*d^4*log(e*x^2 + d) + (c^2*d^2*e^2 + a*c*e^4)*x^4 - 2*(c^2*d^3*e + a*c*
d*e^3)*x^2)/(c^3*d^2*e^3 + a*c^2*e^5)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^9}{(d+ex^2)(a+cx^4)}\,dx = \text{Timed out}$$

```
[In] integrate(x**9/(e*x**2+d)/(c*x**4+a),x)
```

```
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 120, normalized size of antiderivative = 0.90

$$\int \frac{x^9}{(d+ex^2)(a+cx^4)}\,dx = \frac{d^4\log(ex^2+d)}{2(cd^2e^3+ae^5)} - \frac{a^2e\log(cx^4+a)}{4(c^3d^2+ac^2e^2)} + \frac{a^2d\arctan\left(\frac{cx^2}{\sqrt{ac}}\right)}{2(c^2d^2+ace^2)\sqrt{ac}} + \frac{ex^4-2\,dx^2}{4\,ce^2}$$

```
[In] integrate(x^9/(e*x^2+d)/(c*x^4+a),x, algorithm="maxima")
```

```
[Out] 1/2*d^4*log(e*x^2 + d)/(c*d^2*e^3 + a*e^5) - 1/4*a^2*e*log(c*x^4 + a)/(c^3*
d^2 + a*c^2*e^2) + 1/2*a^2*d*arctan(c*x^2/sqrt(a*c))/((c^2*d^2 + a*c*e^2)*s
qrt(a*c)) + 1/4*(e*x^4 - 2*d*x^2)/(c*e^2)
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.92

$$\int \frac{x^9}{(d+ex^2)(a+cx^4)}\,dx = \frac{d^4\log(|ex^2+d|)}{2(cd^2e^3+ae^5)} - \frac{a^2e\log(cx^4+a)}{4(c^3d^2+ac^2e^2)} + \frac{a^2d\arctan\left(\frac{cx^2}{\sqrt{ac}}\right)}{2(c^2d^2+ace^2)\sqrt{ac}} + \frac{cex^4-2\,cdx^2}{4\,c^2e^2}$$

```
[In] integrate(x^9/(e*x^2+d)/(c*x^4+a),x, algorithm="giac")
```

```
[Out] 1/2*d^4*log(abs(e*x^2 + d))/(c*d^2*e^3 + a*e^5) - 1/4*a^2*e*log(c*x^4 + a)/
(c^3*d^2 + a*c^2*e^2) + 1/2*a^2*d*arctan(c*x^2/sqrt(a*c))/((c^2*d^2 + a*c*e
^2)*sqrt(a*c)) + 1/4*(c*e*x^4 - 2*c*d*x^2)/(c^2*e^2)
```

Mupad [B] (verification not implemented)

Time = 8.01 (sec) , antiderivative size = 181, normalized size of antiderivative = 1.35

$$\int \frac{x^9}{(d+ex^2)(a+cx^4)}\,dx = \frac{\ln\left(\sqrt{-a^3\,c^5}+a\,c^3\,x^2\right)\,\left(d\,\sqrt{-a^3\,c^5}-a^2\,c^2\,e\right)}{4\,c^5\,d^2+4\,a\,c^4\,e^2} - \frac{\ln\left(\sqrt{-a^3\,c^5}-a\,c^3\,x^2\right)\,\left(d\,\sqrt{-a^3\,c^5}+a^2\,c^2\,e\right)}{4\,\left(c^5\,d^2+a\,c^4\,e^2\right)} + \frac{d^4\,\ln\left(e\,x^2+d\right)}{2\,c\,d^2\,e^3+2\,a\,e^5} + \frac{x^4}{4\,c\,e} - \frac{d\,x^2}{2\,c\,e^2}$$

```
[In] int(x^9/((a + c*x^4)*(d + e*x^2)),x)
```

```
[Out] (log((-a^3*c^5)^(1/2) + a*c^3*x^2)*(d*(-a^3*c^5)^(1/2) - a^2*c^2*e))/(4*c^5
*d^2 + 4*a*c^4*e^2) - (log((-a^3*c^5)^(1/2) - a*c^3*x^2)*(d*(-a^3*c^5)^(1/2
) + a^2*c^2*e))/(4*(c^5*d^2 + a*c^4*e^2)) + (d^4*log(d + e*x^2))/(2*a*e^5 +
 2*c*d^2*e^3) + x^4/(4*c*e) - (d*x^2)/(2*c*e^2)
```

3.230 $\int \frac{x^7}{(d+ex^2)(a+cx^4)}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 118

$$\int \frac{x^7}{(d+ex^2)(a+cx^4)}\,dx=\frac{x^2}{2ce}-\frac{a^{3/2}e\arctan\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{2c^{3/2}(cd^2+ae^2)}-\frac{d^3\log(d+ex^2)}{2e^2(cd^2+ae^2)}-\frac{ad\log(a+cx^4)}{4c(cd^2+ae^2)}$$

```
[Out] 1/2*x^2/c/e-1/2*a^(3/2)*e*arctan(x^2*c^(1/2)/a^(1/2))/c^(3/2)/(a*e^2+c*d^2)
-1/2*d^3*ln(e*x^2+d)/e^2/(a*e^2+c*d^2)-1/4*a*d*ln(c*x^4+a)/c/(a*e^2+c*d^2)
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {1266, 1643, 649, 211, 266}

$$\int \frac{x^7}{(d+ex^2)(a+cx^4)}\,dx=-\frac{a^{3/2}e\arctan\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{2c^{3/2}(ae^2+cd^2)}-\frac{ad\log(a+cx^4)}{4c(ae^2+cd^2)}-\frac{d^3\log(d+ex^2)}{2e^2(ae^2+cd^2)}+\frac{x^2}{2ce}$$

```
[In] Int[x^7/((d + e*x^2)*(a + c*x^4)),x]
```

```
[Out] x^2/(2*c*e) - (a^(3/2)*e*ArcTan[(Sqrt[c]*x^2)/Sqrt[a]])/(2*c^(3/2)*(c*d^2 +
 a*e^2)) - (d^3*Log[d + e*x^2])/(2*e^2*(c*d^2 + a*e^2)) - (a*d*Log[a + c*x^
4])/(4*c*(c*d^2 + a*e^2))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[(-a)*c]
```

Rule 1266

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_S
ymbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + c*x^2)^p, x],
x, x^2], x] /; FreeQ[{a, c, d, e, p, q}, x] && IntegerQ[(m + 1)/2]
```

Rule 1643

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol]
:> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + c*x^2)^p, x], x] /; FreeQ[{a, c,
 d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{1}{2}\text{Subst}\left(\int \frac{x^3}{(d+ex)\left(a+cx^2\right)}\,dx, x, x^2\right)\\
&= \frac{1}{2}\text{Subst}\left(\int \left(\frac{1}{ce}-\frac{d^3}{e\left(cd^2+ae^2\right)(d+ex)}-\frac{a(ae+cdx)}{c\left(cd^2+ae^2\right)\left(a+cx^2\right)}\right)\,dx, x, x^2\right)\\
&= \frac{x^2}{2ce}-\frac{d^3\log\left(d+ex^2\right)}{2e^2\left(cd^2+ae^2\right)}-\frac{a\text{Subst}\left(\int \frac{ae+cdx}{a+cx^2}\,dx, x, x^2\right)}{2c\left(cd^2+ae^2\right)}\\
&= \frac{x^2}{2ce}-\frac{d^3\log\left(d+ex^2\right)}{2e^2\left(cd^2+ae^2\right)}-\frac{(ad)\text{Subst}\left(\int \frac{x}{a+cx^2}\,dx, x, x^2\right)}{2\left(cd^2+ae^2\right)}-\frac{\left(a^2e\right)\text{Subst}\left(\int \frac{1}{a+cx^2}\,dx, x, x^2\right)}{2c\left(cd^2+ae^2\right)}\\
&= \frac{x^2}{2ce}-\frac{a^{3/2}e\tan^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{2c^{3/2}\left(cd^2+ae^2\right)}-\frac{d^3\log\left(d+ex^2\right)}{2e^2\left(cd^2+ae^2\right)}-\frac{ad\log\left(a+cx^4\right)}{4c\left(cd^2+ae^2\right)}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.84

$$\int \frac{x^7}{(d+ex^2)(a+cx^4)}\,dx = \frac{-\frac{2a^{3/2}e^3\arctan\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{c^{3/2}} - 2d^3\log(d+ex^2) + \frac{e(2(cd^2+ae^2)x^2-ade\log(a+cx^4))}{c}}{4e^2(cd^2+ae^2)}$$

```
[In] Integrate[x^7/((d + e*x^2)*(a + c*x^4)),x]
```

```
[Out] ((-2*a^(3/2)*e^3*ArcTan[(Sqrt[c]*x^2)/Sqrt[a]])/c^(3/2) - 2*d^3*Log[d + e*x
^2] + (e*(2*(c*d^2 + a*e^2)*x^2 - a*d*e*Log[a + c*x^4]))/c)/(4*e^2*(c*d^2 +
 a*e^2))
```

Maple [A] (verified)

Time = 0.41 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.78

method	result
default	$\frac{x^2}{2ce} - \frac{a\left(\frac{d\ln(c\,x^4+a)}{2}+\frac{ae\arctan\left(\frac{c\,x^2}{\sqrt{ac}}\right)}{\sqrt{ac}}\right)}{2(a\,e^2+c\,d^2)c} - \frac{d^3\ln(e\,x^2+d)}{2e^2(a\,e^2+c\,d^2)}$
risch	$\frac{x^2}{2ce} + \frac{a\ln((-\sqrt{-ac}\,a^2e^5+3\sqrt{-ac}\,ac\,d^2e^3-4\sqrt{-ac}\,c^2d^4e+3a^2cd\,e^4-3a\,c^2d^3e^2+2c^3d^5)x^2-3\sqrt{-ac}\,a^2d\,e^4+3\sqrt{-ac}\,ac\,d^3e^2-2\sqrt{-ac}\,c}{4c^2(a\,e^2+c\,d^2)}$

```
[In] int(x^7/(e*x^2+d)/(c*x^4+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*x^2/c/e-1/2*a/(a*e^2+c*d^2)/c*(1/2*d*ln(c*x^4+a)+a*e/(a*c)^(1/2)*arctan
(c*x^2/(a*c)^(1/2)))-1/2*d^3*ln(e*x^2+d)/e^2/(a*e^2+c*d^2)
```

Fricas [A] (verification not implemented)

none

Time = 1.18 (sec) , antiderivative size = 212, normalized size of antiderivative = 1.80

$$\int \frac{x^7}{(d+ex^2)(a+cx^4)}\,dx = \left[\frac{ae^3\sqrt{-\frac{a}{c}}\log\left(\frac{cx^4-2\,cx^2\sqrt{-\frac{a}{c}}-a}{cx^4+a}\right)-ade^2\log\left(cx^4+a\right)-2\,cd^3\log\left(ex^2+d\right)+2\,(cd^2e+ae^3)x^2}{4\,(c^2d^2e^2+ace^4)}, -\frac{2\,ae^3\sqrt{\frac{a}{c}}\arctan\left(\frac{cx^2\sqrt{\frac{a}{c}}}{a}\right)+ade^2\log\left(cx^4+a\right)+2\,cd^3\log\left(ex^2+d\right)-2\,(cd^2e+ae^3)x^2}{4\,(c^2d^2e^2+ace^4)}\right]$$

```
[In] integrate(x^7/(e*x^2+d)/(c*x^4+a),x, algorithm="fricas")
```

```
[Out] [1/4*(a*e^3*sqrt(-a/c)*log((c*x^4 - 2*c*x^2*sqrt(-a/c) - a)/(c*x^4 + a)) -
a*d*e^2*log(c*x^4 + a) - 2*c*d^3*log(e*x^2 + d) + 2*(c*d^2*e + a*e^3)*x^2)/
(c^2*d^2*e^2 + a*c*e^4), -1/4*(2*a*e^3*sqrt(a/c)*arctan(c*x^2*sqrt(a/c)/a)
+ a*d*e^2*log(c*x^4 + a) + 2*c*d^3*log(e*x^2 + d) - 2*(c*d^2*e + a*e^3)*x^2
)/(c^2*d^2*e^2 + a*c*e^4)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^7}{(d+ex^2)(a+cx^4)}\,dx = \text{Timed out}$$

```
[In] integrate(x**7/(e*x**2+d)/(c*x**4+a),x)
```

```
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.91

$$\int \frac{x^7}{(d+ex^2)(a+cx^4)}\,dx = -\frac{d^3\log\left(ex^2+d\right)}{2\,(cd^2e^2+ae^4)}-\frac{a^2e\arctan\left(\frac{cx^2}{\sqrt{ac}}\right)}{2\,(c^2d^2+ace^2)\sqrt{ac}}-\frac{ad\log\left(cx^4+a\right)}{4\,(c^2d^2+ace^2)}+\frac{x^2}{2\,ce}$$

```
[In] integrate(x^7/(e*x^2+d)/(c*x^4+a),x, algorithm="maxima")
```

```
[Out] -1/2*d^3*log(e*x^2 + d)/(c*d^2*e^2 + a*e^4) - 1/2*a^2*e*arctan(c*x^2/sqrt(a
*c))/((c^2*d^2 + a*c*e^2)*sqrt(a*c)) - 1/4*a*d*log(c*x^4 + a)/(c^2*d^2 + a*
c*e^2) + 1/2*x^2/(c*e)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.92

$$\int \frac{x^7}{(d+ex^2)(a+cx^4)}\,dx = -\frac{d^3\log(|ex^2+d|)}{2(cd^2e^2+ae^4)} - \frac{a^2e\arctan\left(\frac{cx^2}{\sqrt{ac}}\right)}{2(c^2d^2+ace^2)\sqrt{ac}} - \frac{ad\log(cx^4+a)}{4(c^2d^2+ace^2)} + \frac{x^2}{2ce}$$

```
[In] integrate(x^7/(e*x^2+d)/(c*x^4+a),x, algorithm="giac")
```

```
[Out] -1/2*d^3*log(abs(e*x^2 + d))/(c*d^2*e^2 + a*e^4) - 1/2*a^2*e*arctan(c*x^2/s
qrt(a*c))/((c^2*d^2 + a*c*e^2)*sqrt(a*c)) - 1/4*a*d*log(c*x^4 + a)/(c^2*d^2
 + a*c*e^2) + 1/2*x^2/(c*e)
```

Mupad [B] (verification not implemented)

Time = 7.88 (sec) , antiderivative size = 166, normalized size of antiderivative = 1.41

$$\begin{aligned}\int \frac{x^7}{(d+ex^2)(a+cx^4)}\,dx &= \frac{x^2}{2\,c\,e} - \frac{d^3\,\ln(e\,x^2+d)}{2\,c\,d^2\,e^2+2\,a\,e^4} \\ &\quad - \frac{\ln\left(\sqrt{-a^3\,c^3}+a\,c^2\,x^2\right)\,\left(e\,\sqrt{-a^3\,c^3}+a\,c^2\,d\right)}{4\,\left(c^4\,d^2+a\,c^3\,e^2\right)} \\ &\quad + \frac{\ln\left(\sqrt{-a^3\,c^3}-a\,c^2\,x^2\right)\,\left(e\,\sqrt{-a^3\,c^3}-a\,c^2\,d\right)}{4\,c^4\,d^2+4\,a\,c^3\,e^2}\end{aligned}$$

```
[In] int(x^7/((a + c*x^4)*(d + e*x^2)),x)
```

```
[Out] x^2/(2*c*e) - (d^3*log(d + e*x^2))/(2*a*e^4 + 2*c*d^2*e^2) - (log((-a^3*c^3
)^(1/2) + a*c^2*x^2)*(e*(-a^3*c^3)^(1/2) + a*c^2*d))/(4*(c^4*d^2 + a*c^3*e^
2)) + (log((-a^3*c^3)^(1/2) - a*c^2*x^2)*(e*(-a^3*c^3)^(1/2) - a*c^2*d))/(4
*c^4*d^2 + 4*a*c^3*e^2)
```

3.231 $\int \frac{x^5}{(d+ex^2)(a+cx^4)}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 105

$$\int \frac{x^5}{(d+ex^2)(a+cx^4)}\,dx = -\frac{\sqrt{a}\,d\arctan\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{2\sqrt{c}\,(cd^2+ae^2)} + \frac{d^2\log(d+ex^2)}{2e\,(cd^2+ae^2)} + \frac{ae\log(a+cx^4)}{4c\,(cd^2+ae^2)}$$

```
[Out] 1/2*d^2*ln(e*x^2+d)/e/(a*e^2+c*d^2)+1/4*a*e*ln(c*x^4+a)/c/(a*e^2+c*d^2)-1/2
*d*arctan(x^2*c^(1/2)/a^(1/2))*a^(1/2)/(a*e^2+c*d^2)/c^(1/2)
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {1266, 1643, 649, 211, 266}

$$\int \frac{x^5}{(d+ex^2)(a+cx^4)}\,dx = -\frac{\sqrt{a}\,d\arctan\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{2\sqrt{c}\,(ae^2+cd^2)} + \frac{ae\log(a+cx^4)}{4c\,(ae^2+cd^2)} + \frac{d^2\log(d+ex^2)}{2e\,(ae^2+cd^2)}$$

```
[In] Int[x^5/((d + e*x^2)*(a + c*x^4)),x]
```

```
[Out] -1/2*(Sqrt[a]*d*ArcTan[(Sqrt[c]*x^2)/Sqrt[a]])/(Sqrt[c]*(c*d^2 + a*e^2)) +
(d^2*Log[d + e*x^2])/(2*e*(c*d^2 + a*e^2)) + (a*e*Log[a + c*x^4])/(4*c*(c*d
^2 + a*e^2))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[(-a)*c]
```

Rule 1266

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_S
ymbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + c*x^2)^p, x],
x, x^2], x] /; FreeQ[{a, c, d, e, p, q}, x] && IntegerQ[(m + 1)/2]
```

Rule 1643

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol]
:> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + c*x^2)^p, x], x] /; FreeQ[{a, c,
 d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{2}\text{Subst}\left(\int \frac{x^2}{(d+ex)\left(a+cx^2\right)}\,dx,x,x^2\right)\\
&= \frac{1}{2}\text{Subst}\left(\int \left(\frac{d^2}{\left(cd^2+ae^2\right)(d+ex)}-\frac{a(d-ex)}{\left(cd^2+ae^2\right)\left(a+cx^2\right)}\right)\,dx,x,x^2\right)\\
&= \frac{d^2\log\left(d+ex^2\right)}{2e\left(cd^2+ae^2\right)}-\frac{a\text{Subst}\left(\int \frac{d-ex}{a+cx^2}\,dx,x,x^2\right)}{2\left(cd^2+ae^2\right)}\\
&= \frac{d^2\log\left(d+ex^2\right)}{2e\left(cd^2+ae^2\right)}-\frac{(ad)\text{Subst}\left(\int \frac{1}{a+cx^2}\,dx,x,x^2\right)}{2\left(cd^2+ae^2\right)}+\frac{(ae)\text{Subst}\left(\int \frac{x}{a+cx^2}\,dx,x,x^2\right)}{2\left(cd^2+ae^2\right)}\\
&= -\frac{\sqrt{a}d\tan^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{2\sqrt{c}\left(cd^2+ae^2\right)}+\frac{d^2\log\left(d+ex^2\right)}{2e\left(cd^2+ae^2\right)}+\frac{ae\log\left(a+cx^4\right)}{4c\left(cd^2+ae^2\right)}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.73

$$\int \frac{x^5}{(d+ex^2)(a+cx^4)}\,dx = \frac{-2\sqrt{a}\sqrt{c}de\arctan\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)+2cd^2\log(d+ex^2)+ae^2\log(a+cx^4)}{4c^2d^2e+4ace^3}$$

```
[In] Integrate[x^5/((d + e*x^2)*(a + c*x^4)),x]
```

```
[Out] (-2*Sqrt[a]*Sqrt[c]*d*e*ArcTan[(Sqrt[c]*x^2)/Sqrt[a]] + 2*c*d^2*Log[d + e*x
^2] + a*e^2*Log[a + c*x^4])/(4*c^2*d^2*e + 4*a*c*e^3)
```

Maple [A] (verified)

Time = 0.40 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.76

method	result
default	$-\frac{a\left(-\frac{e\ln\left(c\,x^4+a\right)}{2c}+\frac{d\arctan\left(\frac{c\,x^2}{\sqrt{ac}}\right)}{\sqrt{ac}}\right)}{2(a\,e^2+c\,d^2)}+\frac{d^2\ln\left(e\,x^2+d\right)}{2e(a\,e^2+c\,d^2)}$
risch	$\frac{\ln\left(\left(-a^2c\,e^4+5a\,c^2d^2e^2-3\sqrt{-ac}\,acd\,e^3+5\sqrt{-ac}\,c^2d^3e-2c^3d^4\right)x^2-3a^2cd\,e^3+5a\,c^2d^3e+\sqrt{-ac}\,a^2e^4-5\sqrt{-ac}\,ac\,d^2e^2+2\sqrt{-ac}\,c^2d^4\right)d}{4(a\,e^2+c\,d^2)c}$

```
[In] int(x^5/(e*x^2+d)/(c*x^4+a),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*a/(a*e^2+c*d^2)*(-1/2*e/c*ln(c*x^4+a)+d/(a*c)^(1/2)*arctan(c*x^2/(a*c)
^(1/2)))+1/2*d^2*ln(e*x^2+d)/e/(a*e^2+c*d^2)
```

Fricas [A] (verification not implemented)

none

Time = 0.73 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.62

$$\int \frac{x^5}{(d+ex^2)(a+cx^4)}\,dx = \left[\frac{cde\sqrt{-\frac{a}{c}}\log\left(\frac{cx^4-2\,cx^2\sqrt{-\frac{a}{c}}-a}{cx^4+a}\right)+ae^2\log\left(cx^4+a\right)+2\,cd^2\log\left(ex^2+d\right)}{4\,(c^2d^2e+ace^3)}, -\frac{2\,cde\sqrt{\frac{a}{c}}\arctan\left(\frac{cx^2\sqrt{\frac{a}{c}}}{a}\right)-ae^2\log\left(cx^4+a\right)-2\,cd^2\log\left(ex^2+d\right)}{4\,(c^2d^2e+ace^3)}\right]$$

```
[In] integrate(x^5/(e*x^2+d)/(c*x^4+a),x, algorithm="fricas")
```

```
[Out] [1/4*(c*d*e*sqrt(-a/c)*log((c*x^4 - 2*c*x^2*sqrt(-a/c) - a)/(c*x^4 + a)) +
a*e^2*log(c*x^4 + a) + 2*c*d^2*log(e*x^2 + d))/(c^2*d^2*e + a*c*e^3), -1/4*
(2*c*d*e*sqrt(a/c)*arctan(c*x^2*sqrt(a/c)/a) - a*e^2*log(c*x^4 + a) - 2*c*d
^2*log(e*x^2 + d))/(c^2*d^2*e + a*c*e^3)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^5}{(d+ex^2)(a+cx^4)}\,dx = \text{Timed out}$$

```
[In] integrate(x**5/(e*x**2+d)/(c*x**4+a),x)
```

```
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.85

$$\int \frac{x^5}{(d+ex^2)(a+cx^4)}\,dx = \frac{ae\log\left(cx^4+a\right)}{4\,(c^2d^2+ace^2)}+\frac{d^2\log\left(ex^2+d\right)}{2\,(cd^2e+ae^3)}-\frac{ad\arctan\left(\frac{cx^2}{\sqrt{ac}}\right)}{2\,(cd^2+ae^2)\sqrt{ac}}$$

```
[In] integrate(x^5/(e*x^2+d)/(c*x^4+a),x, algorithm="maxima")
```

```
[Out] 1/4*a*e*log(c*x^4 + a)/(c^2*d^2 + a*c*e^2) + 1/2*d^2*log(e*x^2 + d)/(c*d^2*
e + a*e^3) - 1/2*a*d*arctan(c*x^2/sqrt(a*c))/((c*d^2 + a*e^2)*sqrt(a*c))
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.86

$$\int \frac{x^5}{(d+ex^2)(a+cx^4)}\,dx = \frac{ae\log(cx^4+a)}{4(c^2d^2+ace^2)} + \frac{d^2\log(|ex^2+d|)}{2(cd^2e+ae^3)} - \frac{ad\arctan\left(\frac{cx^2}{\sqrt{ac}}\right)}{2(cd^2+ae^2)\sqrt{ac}}$$

```
[In] integrate(x^5/(e*x^2+d)/(c*x^4+a),x, algorithm="giac")
```

```
[Out] 1/4*a*e*log(c*x^4 + a)/(c^2*d^2 + a*c*e^2) + 1/2*d^2*log(abs(e*x^2 + d))/(c
*d^2*e + a*e^3) - 1/2*a*d*arctan(c*x^2/sqrt(a*c))/((c*d^2 + a*e^2)*sqrt(a*c
))
```

Mupad [B] (verification not implemented)

Time = 8.06 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.31

$$\int \frac{x^5}{(d+ex^2)(a+cx^4)}\,dx = \frac{d^2\ln(e\,x^2+d)}{2\,c\,d^2\,e+2\,a\,e^3} - \frac{\ln\left(\sqrt{-a\,c^3}+c^2\,x^2\right)\left(d\sqrt{-a\,c^3}-a\,c\,e\right)}{4\left(c^3\,d^2+a\,c^2\,e^2\right)} + \frac{\ln\left(\sqrt{-a\,c^3}-c^2\,x^2\right)\left(d\sqrt{-a\,c^3}+a\,c\,e\right)}{4\,c^3\,d^2+4\,a\,c^2\,e^2}$$

```
[In] int(x^5/((a + c*x^4)*(d + e*x^2)),x)
```

```
[Out] (d^2*log(d + e*x^2))/(2*a*e^3 + 2*c*d^2*e) - (log((-a*c^3)^(1/2) + c^2*x^2)
*(d*(-a*c^3)^(1/2) - a*c*e))/(4*(c^3*d^2 + a*c^2*e^2)) + (log((-a*c^3)^(1/2
) - c^2*x^2)*(d*(-a*c^3)^(1/2) + a*c*e))/(4*c^3*d^2 + 4*a*c^2*e^2)
```

3.232 $\int \frac{x^3}{(d+ex^2)(a+cx^4)} \, dx$

Optimal result

Integrand size = 22, antiderivative size = 96

$$\int \frac{x^3}{(d+ex^2)(a+cx^4)} \, dx = \frac{\sqrt{a}e \arctan\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{2\sqrt{c}\,(cd^2+ae^2)} - \frac{d\log(d+ex^2)}{2\,(cd^2+ae^2)} + \frac{d\log(a+cx^4)}{4\,(cd^2+ae^2)}$$

```
[Out] -1/2*d*ln(e*x^2+d)/(a*e^2+c*d^2)+1/4*d*ln(c*x^4+a)/(a*e^2+c*d^2)+1/2*e*arct
an(x^2*c^(1/2)/a^(1/2))*a^(1/2)/(a*e^2+c*d^2)/c^(1/2)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used = {1266, 815, 649, 211, 266}

$$\int \frac{x^3}{(d+ex^2)(a+cx^4)} \, dx = \frac{\sqrt{a}e \arctan\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{2\sqrt{c}\,(ae^2+cd^2)} + \frac{d\log(a+cx^4)}{4\,(ae^2+cd^2)} - \frac{d\log(d+ex^2)}{2\,(ae^2+cd^2)}$$

```
[In] Int[x^3/((d + e*x^2)*(a + c*x^4)),x]
```

```
[Out] (Sqrt[a]*e*ArcTan[(Sqrt[c]*x^2)/Sqrt[a]])/(2*Sqrt[c]*(c*d^2 + a*e^2)) - (d*
Log[d + e*x^2])/(2*(c*d^2 + a*e^2)) + (d*Log[a + c*x^4])/(4*(c*d^2 + a*e^2)
)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[(-a)*c]
```

Rule 815

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)/(a + c*x^2)), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rule 1266

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_S
ymbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + c*x^2)^p, x],
x, x^2], x] /; FreeQ[{a, c, d, e, p, q}, x] && IntegerQ[(m + 1)/2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{2}\text{Subst}\left(\int \frac{x}{(d+ex)\,(a+cx^2)}\,dx, x, x^2\right)\\
&= \frac{1}{2}\text{Subst}\left(\int \left(-\frac{de}{(cd^2+ae^2)\,(d+ex)}+\frac{ae+cdx}{(cd^2+ae^2)\,(a+cx^2)}\right)\,dx, x, x^2\right)\\
&= -\frac{d\log\left(d+ex^2\right)}{2\,(cd^2+ae^2)}+\frac{\text{Subst}\left(\int \frac{ae+cdx}{a+cx^2}\,dx, x, x^2\right)}{2\,(cd^2+ae^2)}\\
&= -\frac{d\log\left(d+ex^2\right)}{2\,(cd^2+ae^2)}+\frac{(cd)\text{Subst}\left(\int \frac{x}{a+cx^2}\,dx, x, x^2\right)}{2\,(cd^2+ae^2)}+\frac{(ae)\text{Subst}\left(\int \frac{1}{a+cx^2}\,dx, x, x^2\right)}{2\,(cd^2+ae^2)}\\
&= \frac{\sqrt{a}e\tan^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{2\sqrt{c}\,(cd^2+ae^2)}-\frac{d\log\left(d+ex^2\right)}{2\,(cd^2+ae^2)}+\frac{d\log\left(a+cx^4\right)}{4\,(cd^2+ae^2)}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.69

$$\int \frac{x^3}{(d+ex^2)(a+cx^4)}\,dx=\frac{\frac{2\sqrt{a}e\arctan\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{\sqrt{c}}-2d\log(d+ex^2)+d\log(a+cx^4)}{4cd^2+4ae^2}$$

```
[In] Integrate[x^3/((d + e*x^2)*(a + c*x^4)),x]
```

```
[Out] ((2*Sqrt[a]*e*ArcTan[(Sqrt[c]*x^2)/Sqrt[a]])/Sqrt[c] - 2*d*Log[d + e*x^2] +
 d*Log[a + c*x^4])/(4*c*d^2 + 4*a*e^2)
```

Maple [A] (verified)

Time = 0.39 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.75

method	result
default	$\frac{\frac{d\ln(cx^4+a)}{2}+\frac{ae\arctan\left(\frac{cx^2}{\sqrt{ac}}\right)}{\sqrt{ac}}}{2ae^2+2cd^2}-\frac{d\ln(ex^2+d)}{2(ae^2+cd^2)}$
risch	$\frac{\ln((\sqrt{-ac}\,ae^3-7\sqrt{-ac}\,cd^2e+5acde^2-3c^2d^3)x^2+5\sqrt{-ac}\,ade^2-3\sqrt{-ac}\,cd^3-e^3a^2+7acd^2e)e\sqrt{-ac}}{4c(ae^2+cd^2)}+\frac{\ln((\sqrt{-ac}\,ae^3-7\sqrt{-ac}\,cd^2e$

```
[In] int(x^3/(e*x^2+d)/(c*x^4+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2/(a*e^2+c*d^2)*(1/2*d*ln(c*x^4+a)+a*e/(a*c)^(1/2)*arctan(c*x^2/(a*c)^(1/
2)))-1/2*d*ln(e*x^2+d)/(a*e^2+c*d^2)
```

Fricas [A] (verification not implemented)

none

Time = 0.37 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.51

$$\int \frac{x^3}{(d+ex^2)(a+cx^4)}\,dx$$
$$=\left[\frac{e\sqrt{-\frac{a}{c}}\log\left(\frac{cx^4+2\,cx^2\sqrt{-\frac{a}{c}}-a}{cx^4+a}\right)+d\log(cx^4+a)-2\,d\log(ex^2+d)}{4\,(cd^2+ae^2)},\frac{2\,e\sqrt{\frac{a}{c}}\arctan\left(\frac{cx^2\sqrt{\frac{a}{c}}}{a}\right)+d\log(cx^4+}{4\,(cd^2+ae^2)}\right.$$

```
[In] integrate(x^3/(e*x^2+d)/(c*x^4+a),x, algorithm="fricas")
```

```
[Out] [1/4*(e*sqrt(-a/c)*log((c*x^4 + 2*c*x^2*sqrt(-a/c) - a)/(c*x^4 + a)) + d*lo
g(c*x^4 + a) - 2*d*log(e*x^2 + d))/(c*d^2 + a*e^2), 1/4*(2*e*sqrt(a/c)*arct
```

```
an(c*x^2*sqrt(a/c)/a) + d*log(c*x^4 + a) - 2*d*log(e*x^2 + d))/(c*d^2 + a*e
^2)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^3}{(d + ex^2)(a + cx^4)} \, dx = \text{Timed out}$$

```
[In] integrate(x**3/(e*x**2+d)/(c*x**4+a),x)
```

```
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.85

$$\int \frac{x^3}{(d + ex^2)(a + cx^4)} \, dx = \frac{ae \arctan\left(\frac{cx^2}{\sqrt{ac}}\right)}{2(cd^2 + ae^2)\sqrt{ac}} + \frac{d \log(cx^4 + a)}{4(cd^2 + ae^2)} - \frac{d \log(ex^2 + d)}{2(cd^2 + ae^2)}$$

```
[In] integrate(x^3/(e*x^2+d)/(c*x^4+a),x, algorithm="maxima")
```

```
[Out] 1/2*a*e*arctan(c*x^2/sqrt(a*c))/((c*d^2 + a*e^2)*sqrt(a*c)) + 1/4*d*log(c*x
^4 + a)/(c*d^2 + a*e^2) - 1/2*d*log(e*x^2 + d)/(c*d^2 + a*e^2)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.89

$$\int \frac{x^3}{(d + ex^2)(a + cx^4)} \, dx = -\frac{de \log(|ex^2 + d|)}{2(cd^2e + ae^3)} + \frac{ae \arctan\left(\frac{cx^2}{\sqrt{ac}}\right)}{2(cd^2 + ae^2)\sqrt{ac}} + \frac{d \log(cx^4 + a)}{4(cd^2 + ae^2)}$$

```
[In] integrate(x^3/(e*x^2+d)/(c*x^4+a),x, algorithm="giac")
```

```
[Out] -1/2*d*e*log(abs(e*x^2 + d))/(c*d^2*e + a*e^3) + 1/2*a*e*arctan(c*x^2/sqrt(
a*c))/((c*d^2 + a*e^2)*sqrt(a*c)) + 1/4*d*log(c*x^4 + a)/(c*d^2 + a*e^2)
```

Mupad [B] (verification not implemented)

Time = 8.74 (sec) , antiderivative size = 944, normalized size of antiderivative = 9.83

$$\int \frac{x^3}{(d+ex^2)(a+cx^4)}\,dx$$
$$= \frac{c\,d\,\ln\Big(a^4\,e^6-9\,a\,c^3\,d^6-39\,a^3\,c\,d^2\,e^4+a^3\,e^6\,x^2\,\sqrt{-a\,c}-9\,c^3\,d^6\,x^2\,\sqrt{-a\,c}+79\,a^2\,c^2\,d^4\,e^2-42\,c\,d^5\,e\,(-a\,c)^3}{}$$
$$-\frac{d\,\ln\left(e\,x^2+d\right)}{2\,\left(c\,d^2+a\,e^2\right)}$$
$$+\frac{c\,d\,\ln\left(9\,a\,c^3\,d^6-a^4\,e^6+39\,a^3\,c\,d^2\,e^4+a^3\,e^6\,x^2\,\sqrt{-a\,c}-9\,c^3\,d^6\,x^2\,\sqrt{-a\,c}-79\,a^2\,c^2\,d^4\,e^2+10\,a^3\,d\,e^5\,\sqrt{-}\right.}{}$$
$$-\frac{e\,\ln\Big(a^4\,e^6-9\,a\,c^3\,d^6-39\,a^3\,c\,d^2\,e^4+a^3\,e^6\,x^2\,\sqrt{-a\,c}-9\,c^3\,d^6\,x^2\,\sqrt{-a\,c}+79\,a^2\,c^2\,d^4\,e^2-42\,c\,d^5\,e\,(-a\,c)}{}$$
$$+\frac{e\,\ln\left(9\,a\,c^3\,d^6-a^4\,e^6+39\,a^3\,c\,d^2\,e^4+a^3\,e^6\,x^2\,\sqrt{-a\,c}-9\,c^3\,d^6\,x^2\,\sqrt{-a\,c}-79\,a^2\,c^2\,d^4\,e^2+10\,a^3\,d\,e^5\,\sqrt{-a}\right.}{}$$

```
[In] int(x^3/((a + c*x^4)*(d + e*x^2)),x)
```

```
[Out] (c*d*log(a^4*e^6 - 9*a*c^3*d^6 - 39*a^3*c*d^2*e^4 + a^3*e^6*x^2*(-a*c)^(1/2
) - 9*c^3*d^6*x^2*(-a*c)^(1/2) + 79*a^2*c^2*d^4*e^2 - 42*c*d^5*e*(-a*c)^(3/
2) + 76*a*d^3*e^3*(-a*c)^(3/2) + 10*a^3*d*e^5*(-a*c)^(1/2) + 76*a^2*c^2*d^3
*e^3*x^2 - 42*a*c^3*d^5*e*x^2 - 10*a^3*c*d*e^5*x^2 + 39*a*d^2*e^4*x^2*(-a*c
)^(3/2) - 79*c*d^4*e^2*x^2*(-a*c)^(3/2)))/(4*c^2*d^2 + 4*a*c*e^2) - (d*log(
d + e*x^2))/(2*(a*e^2 + c*d^2)) + (c*d*log(9*a*c^3*d^6 - a^4*e^6 + 39*a^3*c
*d^2*e^4 + a^3*e^6*x^2*(-a*c)^(1/2) - 9*c^3*d^6*x^2*(-a*c)^(1/2) - 79*a^2*c
^2*d^4*e^2 + 10*a^3*d*e^5*(-a*c)^(1/2) + 42*a*c^2*d^5*e*(-a*c)^(1/2) - 76*a
^2*c^2*d^3*e^3*x^2 + 42*a*c^3*d^5*e*x^2 + 10*a^3*c*d*e^5*x^2 - 76*a^2*c*d^3
*e^3*(-a*c)^(1/2) + 79*a*c^2*d^4*e^2*x^2*(-a*c)^(1/2) - 39*a^2*c*d^2*e^4*x^
2*(-a*c)^(1/2)))/(4*c^2*d^2 + 4*a*c*e^2) - (e*log(a^4*e^6 - 9*a*c^3*d^6 - 3
9*a^3*c*d^2*e^4 + a^3*e^6*x^2*(-a*c)^(1/2) - 9*c^3*d^6*x^2*(-a*c)^(1/2) + 7
9*a^2*c^2*d^4*e^2 - 42*c*d^5*e*(-a*c)^(3/2) + 76*a*d^3*e^3*(-a*c)^(3/2) + 1
0*a^3*d*e^5*(-a*c)^(1/2) + 76*a^2*c^2*d^3*e^3*x^2 - 42*a*c^3*d^5*e*x^2 - 10
*a^3*c*d*e^5*x^2 + 39*a*d^2*e^4*x^2*(-a*c)^(3/2) - 79*c*d^4*e^2*x^2*(-a*c)^
(3/2))*(-a*c)^(1/2))/(4*c^2*d^2 + 4*a*c*e^2) + (e*log(9*a*c^3*d^6 - a^4*e^6
 + 39*a^3*c*d^2*e^4 + a^3*e^6*x^2*(-a*c)^(1/2) - 9*c^3*d^6*x^2*(-a*c)^(1/2)
 - 79*a^2*c^2*d^4*e^2 + 10*a^3*d*e^5*(-a*c)^(1/2) + 42*a*c^2*d^5*e*(-a*c)^(
1/2) - 76*a^2*c^2*d^3*e^3*x^2 + 42*a*c^3*d^5*e*x^2 + 10*a^3*c*d*e^5*x^2 - 7
6*a^2*c*d^3*e^3*(-a*c)^(1/2) + 79*a*c^2*d^4*e^2*x^2*(-a*c)^(1/2) - 39*a^2*c
*d^2*e^4*x^2*(-a*c)^(1/2))*(-a*c)^(1/2))/(4*c^2*d^2 + 4*a*c*e^2)
```

3.233 $\int \frac{x}{(d+ex^2)(a+cx^4)} dx$

Optimal result

Integrand size = 20, antiderivative size = 96

$$\int \frac{x}{(d+ex^2)(a+cx^4)} dx = \frac{\sqrt{c}d\arctan\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{2\sqrt{a}(cd^2+ae^2)} + \frac{e\log(d+ex^2)}{2(cd^2+ae^2)} - \frac{e\log(a+cx^4)}{4(cd^2+ae^2)}$$

```
[Out] 1/2*e*ln(e*x^2+d)/(a*e^2+c*d^2)-1/4*e*ln(c*x^4+a)/(a*e^2+c*d^2)+1/2*d*arcta
n(x^2*c^(1/2)/a^(1/2))*c^(1/2)/(a*e^2+c*d^2)/a^(1/2)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {1262, 720, 31, 649, 211, 266}

$$\int \frac{x}{(d+ex^2)(a+cx^4)} dx = \frac{\sqrt{c}d\arctan\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{2\sqrt{a}(ae^2+cd^2)} - \frac{e\log(a+cx^4)}{4(ae^2+cd^2)} + \frac{e\log(d+ex^2)}{2(ae^2+cd^2)}$$

```
[In] Int[x/((d + e*x^2)*(a + c*x^4)),x]
```

```
[Out] (Sqrt[c]*d*ArcTan[(Sqrt[c]*x^2)/Sqrt[a]])/(2*Sqrt[a]*(c*d^2 + a*e^2)) + (e*
Log[d + e*x^2])/(2*(c*d^2 + a*e^2)) - (e*Log[a + c*x^4])/(4*(c*d^2 + a*e^2)
)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[(-a)*c]
```

Rule 720

```
Int[1/(((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)), x_Symbol] :> Dist[e^2/(c
*d^2 + a*e^2), Int[1/(d + e*x), x], x] + Dist[1/(c*d^2 + a*e^2), Int[(c*d -
 c*e*x)/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2,
 0]
```

Rule 1262

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol]
 :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + c*x^2)^p, x], x, x^2], x] /; FreeQ
[{a, c, d, e, p, q}, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{2}\text{Subst}\left(\int \frac{1}{(d+ex)\,(a+cx^2)}\,dx, x, x^2\right)\\
&= \frac{\text{Subst}\left(\int \frac{cd-cex}{a+cx^2}\,dx, x, x^2\right)}{2\,(cd^2+ae^2)} + \frac{e^2\text{Subst}\left(\int \frac{1}{d+ex}\,dx, x, x^2\right)}{2\,(cd^2+ae^2)}\\
&= \frac{e\log\left(d+ex^2\right)}{2\,(cd^2+ae^2)} + \frac{(cd)\text{Subst}\left(\int \frac{1}{a+cx^2}\,dx, x, x^2\right)}{2\,(cd^2+ae^2)} - \frac{(ce)\text{Subst}\left(\int \frac{x}{a+cx^2}\,dx, x, x^2\right)}{2\,(cd^2+ae^2)}\\
&= \frac{\sqrt{c}d\tan^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{2\sqrt{a}\,(cd^2+ae^2)} + \frac{e\log\left(d+ex^2\right)}{2\,(cd^2+ae^2)} - \frac{e\log\left(a+cx^4\right)}{4\,(cd^2+ae^2)}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.70

$$\int \frac{x}{(d+ex^2)(a+cx^4)}\,dx = \frac{\frac{2\sqrt{c}d\arctan\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{\sqrt{a}} + 2e\log(d+ex^2) - e\log(a+cx^4)}{4cd^2+4ae^2}$$

```
[In] Integrate[x/((d + e*x^2)*(a + c*x^4)),x]
```

```
[Out] ((2*Sqrt[c]*d*ArcTan[(Sqrt[c]*x^2)/Sqrt[a]])/Sqrt[a] + 2*e*Log[d + e*x^2] -
 e*Log[a + c*x^4])/(4*c*d^2 + 4*a*e^2)
```

Maple [A] (verified)

Time = 0.38 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.78

method	result
default	$\frac{c\left(-\frac{e\ln\left(c\,x^4+a\right)}{2c}+\frac{d\arctan\left(\frac{c\,x^2}{\sqrt{ac}}\right)}{\sqrt{ac}}\right)}{2a\,e^2+2c\,d^2}+\frac{e\ln\left(e\,x^2+d\right)}{2a\,e^2+2c\,d^2}$
risch	$\frac{\ln((-3a^2c\,e^3+5a\,c^2d^2e+7\sqrt{-ac}\,acd\,e^2-\sqrt{-ac}\,c^2d^3)x^2-7a^2cd\,e^2+a\,c^2d^3-3\sqrt{-ac}\,a^2e^3+5\sqrt{-ac}\,ac\,d^2e)d\sqrt{-ac}}{4a(a\,e^2+c\,d^2)} - \frac{\ln((-3a^2c\,e^3+5}{}$

```
[In] int(x/(e*x^2+d)/(c*x^4+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*c/(a*e^2+c*d^2)*(-1/2*e/c*ln(c*x^4+a)+d/(a*c)^(1/2)*arctan(c*x^2/(a*c)^
(1/2)))+1/2*e*ln(e*x^2+d)/(a*e^2+c*d^2)
```

Fricas [A] (verification not implemented)

none

Time = 0.43 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.52

$$\int \frac{x}{(d+ex^2)(a+cx^4)}\,dx$$
$$= \left[\frac{d\sqrt{-\frac{c}{a}}\log\left(\frac{cx^4+2\,ax^2\sqrt{-\frac{c}{a}}-a}{cx^4+a}\right) - e\log(cx^4+a) + 2\,e\log(ex^2+d)}{4\,(cd^2+ae^2)},\right.$$
$$\left. -\frac{2\,d\sqrt{\frac{c}{a}}\arctan\left(\frac{a\sqrt{\frac{c}{a}}}{cx^2}\right) + e\log(cx^4+a) - 2\,e\log(ex^2+d)}{4\,(cd^2+ae^2)}\right]$$

```
[In] integrate(x/(e*x^2+d)/(c*x^4+a),x, algorithm="fricas")
```

```
[Out] [1/4*(d*sqrt(-c/a)*log((c*x^4 + 2*a*x^2*sqrt(-c/a) - a)/(c*x^4 + a)) - e*lo
g(c*x^4 + a) + 2*e*log(e*x^2 + d))/(c*d^2 + a*e^2), -1/4*(2*d*sqrt(c/a)*arc
tan(a*sqrt(c/a)/(c*x^2)) + e*log(c*x^4 + a) - 2*e*log(e*x^2 + d))/(c*d^2 +
a*e^2)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x}{(d+ex^2)(a+cx^4)}\,dx = \text{Timed out}$$

```
[In] integrate(x/(e*x**2+d)/(c*x**4+a),x)
```

```
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.85

$$\int \frac{x}{(d+ex^2)(a+cx^4)}\,dx = \frac{cd\arctan\left(\frac{cx^2}{\sqrt{ac}}\right)}{2(cd^2+ae^2)\sqrt{ac}} - \frac{e\log(cx^4+a)}{4(cd^2+ae^2)} + \frac{e\log(ex^2+d)}{2(cd^2+ae^2)}$$

```
[In] integrate(x/(e*x^2+d)/(c*x^4+a),x, algorithm="maxima")
```

```
[Out] 1/2*c*d*arctan(c*x^2/sqrt(a*c))/((c*d^2 + a*e^2)*sqrt(a*c)) - 1/4*e*log(c*x
^4 + a)/(c*d^2 + a*e^2) + 1/2*e*log(e*x^2 + d)/(c*d^2 + a*e^2)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.90

$$\int \frac{x}{(d+ex^2)(a+cx^4)}\,dx = \frac{e^2\log(|ex^2+d|)}{2(cd^2e+ae^3)} + \frac{cd\arctan\left(\frac{cx^2}{\sqrt{ac}}\right)}{2(cd^2+ae^2)\sqrt{ac}} - \frac{e\log(cx^4+a)}{4(cd^2+ae^2)}$$

```
[In] integrate(x/(e*x^2+d)/(c*x^4+a),x, algorithm="giac")
```

```
[Out] 1/2*e^2*log(abs(e*x^2 + d))/(c*d^2*e + a*e^3) + 1/2*c*d*arctan(c*x^2/sqrt(a
*c))/((c*d^2 + a*e^2)*sqrt(a*c)) - 1/4*e*log(c*x^4 + a)/(c*d^2 + a*e^2)
```

Mupad [B] (verification not implemented)

Time = 8.31 (sec) , antiderivative size = 328, normalized size of antiderivative = 3.42

$$\int \frac{x}{(d+ex^2)(a+cx^4)}\,dx = \frac{e\,\ln\left(e\,x^2+d\right)}{2\,c\,d^2+2\,a\,e^2}$$

$$-\frac{\ln\Big(a\,c^5\,d^6\,x^2-c^3\,d^6\,(-a\,c)^{3/2}-9\,a^3\,e^6\,(-a\,c)^{3/2}+9\,a^4\,c^2\,e^6\,x^2+19\,a\,d^2\,e^4\,(-a\,c)^{5/2}+11\,c\,d^4\,e^2\,(-a\,c)^{5}}{4\,\left(a^2\,e^2+c\,a\,d^2\right)}$$

$$-\frac{\ln\Big(9\,a^3\,e^6\,(-a\,c)^{3/2}+c^3\,d^6\,(-a\,c)^{3/2}+a\,c^5\,d^6\,x^2+9\,a^4\,c^2\,e^6\,x^2-19\,a\,d^2\,e^4\,(-a\,c)^{5/2}-11\,c\,d^4\,e^2\,(-a\,c)^{5}}{4\,\left(a^2\,e^2+c\,a\,d^2\right)}$$

```
[In] int(x/((a + c*x^4)*(d + e*x^2)),x)
```

```
[Out] (e*log(d + e*x^2))/(2*a*e^2 + 2*c*d^2) - (log(a*c^5*d^6*x^2 - c^3*d^6*(-a*c
)^(3/2) - 9*a^3*e^6*(-a*c)^(3/2) + 9*a^4*c^2*e^6*x^2 + 19*a*d^2*e^4*(-a*c)^
(5/2) + 11*c*d^4*e^2*(-a*c)^(5/2) + 11*a^2*c^4*d^4*e^2*x^2 + 19*a^3*c^3*d^2
*e^4*x^2)*(a*e - d*(-a*c)^(1/2)))/(4*(a^2*e^2 + a*c*d^2)) - (log(9*a^3*e^6*
(-a*c)^(3/2) + c^3*d^6*(-a*c)^(3/2) + a*c^5*d^6*x^2 + 9*a^4*c^2*e^6*x^2 - 1
9*a*d^2*e^4*(-a*c)^(5/2) - 11*c*d^4*e^2*(-a*c)^(5/2) + 11*a^2*c^4*d^4*e^2*x
^2 + 19*a^3*c^3*d^2*e^4*x^2)*(a*e + d*(-a*c)^(1/2)))/(4*(a^2*e^2 + a*c*d^2)
)
```

3.234 $\int \frac{1}{x(d+ex^2)(a+cx^4)}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 114

$$\int \frac{1}{x(d+ex^2)(a+cx^4)}\,dx = -\frac{\sqrt{c}e\arctan\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{2\sqrt{a}(cd^2+ae^2)} + \frac{\log(x)}{ad} - \frac{e^2\log(d+ex^2)}{2d(cd^2+ae^2)} - \frac{cd\log(a+cx^4)}{4a(cd^2+ae^2)}$$

```
[Out] ln(x)/a/d-1/2*e^2*ln(e*x^2+d)/d/(a*e^2+c*d^2)-1/4*c*d*ln(c*x^4+a)/a/(a*e^2+
c*d^2)-1/2*e*arctan(x^2*c^(1/2)/a^(1/2))*c^(1/2)/(a*e^2+c*d^2)/a^(1/2)
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {1266, 908, 649, 211, 266}

$$\int \frac{1}{x(d+ex^2)(a+cx^4)}\,dx = -\frac{\sqrt{c}e\arctan\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{2\sqrt{a}(ae^2+cd^2)} - \frac{cd\log(a+cx^4)}{4a(ae^2+cd^2)} - \frac{e^2\log(d+ex^2)}{2d(ae^2+cd^2)} + \frac{\log(x)}{ad}$$

```
[In] Int[1/(x*(d + e*x^2)*(a + c*x^4)),x]
```

```
[Out] -1/2*(Sqrt[c]*e*ArcTan[(Sqrt[c]*x^2)/Sqrt[a]])/(Sqrt[a]*(c*d^2 + a*e^2)) +
Log[x]/(a*d) - (e^2*Log[d + e*x^2])/(2*d*(c*d^2 + a*e^2)) - (c*d*Log[a + c*
x^4])/(4*a*(c*d^2 + a*e^2))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[(-a)*c]
```

Rule 908

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && IntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ
[m, 0] && ILtQ[n, 0]))
```

Rule 1266

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_S
ymbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + c*x^2)^p, x],
x, x^2], x] /; FreeQ[{a, c, d, e, p, q}, x] && IntegerQ[(m + 1)/2]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{1}{2}\text{Subst}\left(\int \frac{1}{x(d+ex)\left(a+cx^2\right)}\,dx, x, x^2\right)\\
&= \frac{1}{2}\text{Subst}\left(\int \left(\frac{1}{adx}-\frac{e^3}{d\left(cd^2+ae^2\right)(d+ex)}-\frac{c(ae+cdx)}{a\left(cd^2+ae^2\right)\left(a+cx^2\right)}\right)\,dx, x, x^2\right)\\
&= \frac{\log(x)}{ad}-\frac{e^2\log\left(d+ex^2\right)}{2d\left(cd^2+ae^2\right)}-\frac{c\text{Subst}\left(\int \frac{ae+cdx}{a+cx^2}\,dx, x, x^2\right)}{2a\left(cd^2+ae^2\right)}\\
&= \frac{\log(x)}{ad}-\frac{e^2\log\left(d+ex^2\right)}{2d\left(cd^2+ae^2\right)}-\frac{\left(c^2d\right)\text{Subst}\left(\int \frac{x}{a+cx^2}\,dx, x, x^2\right)}{2a\left(cd^2+ae^2\right)}-\frac{(ce)\text{Subst}\left(\int \frac{1}{a+cx^2}\,dx, x, x^2\right)}{2\left(cd^2+ae^2\right)}\\
&= -\frac{\sqrt{c}e\tan^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{2\sqrt{a}\left(cd^2+ae^2\right)}+\frac{\log(x)}{ad}-\frac{e^2\log\left(d+ex^2\right)}{2d\left(cd^2+ae^2\right)}-\frac{cd\log\left(a+cx^4\right)}{4a\left(cd^2+ae^2\right)}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.18

$$\int \frac{1}{x(d+ex^2)(a+cx^4)}\,dx = \frac{2\sqrt{a}\sqrt{c}de\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)+2\sqrt{a}\sqrt{c}de\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)+4cd^2\log(x)+4ae^2\log(x)-2ae^2\log(}{4acd^3+4a^2de^2}$$

```
[In] Integrate[1/(x*(d + e*x^2)*(a + c*x^4)),x]
[Out] (2*Sqrt[a]*Sqrt[c]*d*e*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)] + 2*Sqrt[a]*
Sqrt[c]*d*e*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)] + 4*c*d^2*Log[x] + 4*a*
e^2*Log[x] - 2*a*e^2*Log[d + e*x^2] - c*d^2*Log[a + c*x^4])/(4*a*c*d^3 + 4*
a^2*d*e^2)
```

Maple [A] (verified)

Time = 0.40 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.79

method	result
default	$\frac{\ln(x)}{ad}-\frac{c\left(\frac{d\ln(cx^4+a)}{2}+\frac{ae\arctan\left(\frac{cx^2}{\sqrt{ac}}\right)}{\sqrt{ac}}\right)}{2(ae^2+cd^2)a}-\frac{e^2\ln(ex^2+d)}{2d(ae^2+cd^2)}$
risch	$\frac{\ln(x)}{ad}+\frac{\left(\sum_{\_R=\text{RootOf}\left(\left(e^2a^3+ca^2d^2\right)\_Z^2+2ad\_Zc+c\right)}\_R\ln\left(\left((-6e^4a^3-7a^2cd^2e^2-5ac^2d^4)\_R^2+(-22acde^2-5c^2d^3)\_R-14c\right.\right.\right.}{4}$

```
[In] int(1/x/(e*x^2+d)/(c*x^4+a),x,method=_RETURNVERBOSE)
[Out] ln(x)/a/d-1/2*c/(a*e^2+c*d^2)/a*(1/2*d*ln(c*x^4+a)+a*e/(a*c)^(1/2)*arctan(c
*x^2/(a*c)^(1/2)))-1/2*e^2*ln(e*x^2+d)/d/(a*e^2+c*d^2)
```

Fricas [A] (verification not implemented)

none

Time = 4.43 (sec) , antiderivative size = 201, normalized size of antiderivative = 1.76

$$\int \frac{1}{x(d+ex^2)(a+cx^4)}\,dx = \left[\frac{ade\sqrt{-\frac{c}{a}}\log\left(\frac{cx^4-2\,ax^2\sqrt{-\frac{c}{a}}-a}{cx^4+a}\right)-cd^2\log\left(cx^4+a\right)-2\,ae^2\log\left(ex^2+d\right)+4\left(cd^2+ae^2\right)\log\left(x\right)}{4\left(acd^3+a^2de^2\right)}, \frac{2\,ade\sqrt{\frac{c}{a}}}{}\right.$$

```
[In] integrate(1/x/(e*x^2+d)/(c*x^4+a),x, algorithm="fricas")
```

```
[Out] [1/4*(a*d*e*sqrt(-c/a)*log((c*x^4 - 2*a*x^2*sqrt(-c/a) - a)/(c*x^4 + a)) -
c*d^2*log(c*x^4 + a) - 2*a*e^2*log(e*x^2 + d) + 4*(c*d^2 + a*e^2)*log(x))/(
a*c*d^3 + a^2*d*e^2), 1/4*(2*a*d*e*sqrt(c/a)*arctan(a*sqrt(c/a)/(c*x^2)) -
c*d^2*log(c*x^4 + a) - 2*a*e^2*log(e*x^2 + d) + 4*(c*d^2 + a*e^2)*log(x))/(
a*c*d^3 + a^2*d*e^2)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{x\,(d+ex^2)\,(a+cx^4)}\,dx = \text{Timed out}$$

```
[In] integrate(1/x/(e*x**2+d)/(c*x**4+a),x)
```

```
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.89

$$\int \frac{1}{x\,(d+ex^2)\,(a+cx^4)}\,dx = -\frac{cd\log\left(cx^4+a\right)}{4\,(acd^2+a^2e^2)} - \frac{e^2\log\left(ex^2+d\right)}{2\,(cd^3+ade^2)} - \frac{ce\arctan\left(\frac{cx^2}{\sqrt{ac}}\right)}{2\,(cd^2+ae^2)\sqrt{ac}} + \frac{\log\left(x^2\right)}{2\,ad}$$

```
[In] integrate(1/x/(e*x^2+d)/(c*x^4+a),x, algorithm="maxima")
```

```
[Out] -1/4*c*d*log(c*x^4 + a)/(a*c*d^2 + a^2*e^2) - 1/2*e^2*log(e*x^2 + d)/(c*d^3
 + a*d*e^2) - 1/2*c*e*arctan(c*x^2/sqrt(a*c))/((c*d^2 + a*e^2)*sqrt(a*c)) +
 1/2*log(x^2)/(a*d)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.90

$$\int \frac{1}{x\,(d+ex^2)\,(a+cx^4)}\,dx = -\frac{e^3\log\left(|ex^2+d|\right)}{2\,(cd^3e+ade^3)} - \frac{cd\log\left(cx^4+a\right)}{4\,(acd^2+a^2e^2)} - \frac{ce\arctan\left(\frac{cx^2}{\sqrt{ac}}\right)}{2\,(cd^2+ae^2)\sqrt{ac}} + \frac{\log\left(x^2\right)}{2\,ad}$$

```
[In] integrate(1/x/(e*x^2+d)/(c*x^4+a),x, algorithm="giac")
```

```
[Out] -1/2*e^3*log(abs(e*x^2 + d))/(c*d^3*e + a*d*e^3) - 1/4*c*d*log(c*x^4 + a)/(
a*c*d^2 + a^2*e^2) - 1/2*c*e*arctan(c*x^2/sqrt(a*c))/((c*d^2 + a*e^2)*sqrt(
a*c)) + 1/2*log(x^2)/(a*d)
```

Mupad [B] (verification not implemented)

Time = 8.04 (sec) , antiderivative size = 527, normalized size of antiderivative = 4.62

$$\int \frac{1}{x(d+ex^2)(a+cx^4)}\,dx$$
$$= \frac{\ln\left(64\,a^7\,c\,e^{10}\,x^2-64\,a^6\,e^{10}\,\sqrt{-a^3\,c}-25\,a\,c^5\,d^{10}\,\sqrt{-a^3\,c}+25\,a^2\,c^6\,d^{10}\,x^2+180\,a^2\,d^2\,e^8\,{\left(-a^3\,c\right)}^{3/2}-41\,c^2\,d^6\right.}{}$$
$$-\frac{\ln\left(64\,a^6\,e^{10}\,\sqrt{-a^3\,c}+64\,a^7\,c\,e^{10}\,x^2+25\,a\,c^5\,d^{10}\,\sqrt{-a^3\,c}+25\,a^2\,c^6\,d^{10}\,x^2-180\,a^2\,d^2\,e^8\,{\left(-a^3\,c\right)}^{3/2}+41\,c^2\right.}{}$$
$$-\frac{e^2\,\ln\left(e\,x^2+d\right)}{2\,c\,d^3+2\,a\,d\,e^2}+\frac{\ln\left(x\right)}{a\,d}$$

```
[In] int(1/(x*(a + c*x^4)*(d + e*x^2)),x)
```

```
[Out] (log(64*a^7*c*e^10*x^2 - 64*a^6*e^10*(-a^3*c)^(1/2) - 25*a*c^5*d^10*(-a^3*c
)^(1/2) + 25*a^2*c^6*d^10*x^2 + 180*a^2*d^2*e^8*(-a^3*c)^(3/2) - 41*c^2*d^6
*e^4*(-a^3*c)^(3/2) - 9*a^3*c^5*d^8*e^2*x^2 - 41*a^4*c^4*d^6*e^4*x^2 + 109*
a^5*c^3*d^4*e^6*x^2 + 180*a^6*c^2*d^2*e^8*x^2 + 9*a^2*c^4*d^8*e^2*(-a^3*c)^
(1/2) + 109*a*c*d^4*e^6*(-a^3*c)^(3/2))*(e*(-a^3*c)^(1/2) - a*c*d))/(4*a^3*
e^2 + 4*a^2*c*d^2) - (log(64*a^6*e^10*(-a^3*c)^(1/2) + 64*a^7*c*e^10*x^2 +
25*a*c^5*d^10*(-a^3*c)^(1/2) + 25*a^2*c^6*d^10*x^2 - 180*a^2*d^2*e^8*(-a^3*
c)^(3/2) + 41*c^2*d^6*e^4*(-a^3*c)^(3/2) - 9*a^3*c^5*d^8*e^2*x^2 - 41*a^4*c
^4*d^6*e^4*x^2 + 109*a^5*c^3*d^4*e^6*x^2 + 180*a^6*c^2*d^2*e^8*x^2 - 9*a^2*
c^4*d^8*e^2*(-a^3*c)^(1/2) - 109*a*c*d^4*e^6*(-a^3*c)^(3/2))*(e*(-a^3*c)^(1
/2) + a*c*d))/(4*(a^3*e^2 + a^2*c*d^2)) - (e^2*log(d + e*x^2))/(2*c*d^3 + 2
*a*d*e^2) + log(x)/(a*d)
```

3.235 $\int \frac{1}{x^3(d+ex^2)(a+cx^4)}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 129

$$\int \frac{1}{x^3\left(d+ex^2\right)\left(a+cx^4\right)}\,dx = -\frac{1}{2adx^2}-\frac{c^{3/2}d\arctan\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{2a^{3/2}\left(cd^2+ae^2\right)}-\frac{e\log(x)}{ad^2}+\frac{e^3\log\left(d+ex^2\right)}{2d^2\left(cd^2+ae^2\right)}+\frac{ce\log\left(a+cx^4\right)}{4a\left(cd^2+ae^2\right)}$$

```
[Out] -1/2/a/d/x^2-1/2*c^(3/2)*d*arctan(x^2*c^(1/2)/a^(1/2))/a^(3/2)/(a*e^2+c*d^2
)-e*ln(x)/a/d^2+1/2*e^3*ln(e*x^2+d)/d^2/(a*e^2+c*d^2)+1/4*c*e*ln(c*x^4+a)/a
/(a*e^2+c*d^2)
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {1266, 908, 649, 211, 266}

$$\int \frac{1}{x^3\left(d+ex^2\right)\left(a+cx^4\right)}\,dx = -\frac{c^{3/2}d\arctan\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{2a^{3/2}\left(ae^2+cd^2\right)}+\frac{ce\log\left(a+cx^4\right)}{4a\left(ae^2+cd^2\right)}+\frac{e^3\log\left(d+ex^2\right)}{2d^2\left(ae^2+cd^2\right)}-\frac{e\log(x)}{ad^2}-\frac{1}{2adx^2}$$

```
[In] Int[1/(x^3*(d + e*x^2)*(a + c*x^4)),x]
```

```
[Out] -1/2*1/(a*d*x^2) - (c^(3/2)*d*ArcTan[(Sqrt[c]*x^2)/Sqrt[a]])/(2*a^(3/2)*(c*
d^2 + a*e^2)) - (e*Log[x])/(a*d^2) + (e^3*Log[d + e*x^2])/(2*d^2*(c*d^2 + a
*e^2)) + (c*e*Log[a + c*x^4])/(4*a*(c*d^2 + a*e^2))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[(-a)*c]
```

Rule 908

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && IntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ
[m, 0] && ILtQ[n, 0]))
```

Rule 1266

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_S
ymbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + c*x^2)^p, x],
x, x^2], x] /; FreeQ[{a, c, d, e, p, q}, x] && IntegerQ[(m + 1)/2]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{1}{2}\text{Subst}\left(\int \frac{1}{x^2(d+ex)\,(a+cx^2)}\,dx, x, x^2\right)\\
&= \frac{1}{2}\text{Subst}\left(\int \left(\frac{1}{adx^2}-\frac{e}{ad^2x}+\frac{e^4}{d^2\,(cd^2+ae^2)\,(d+ex)}-\frac{c^2(d-ex)}{a\,(cd^2+ae^2)\,(a+cx^2)}\right)\,dx, x, x^2\right)\\
&= -\frac{1}{2adx^2}-\frac{e\log(x)}{ad^2}+\frac{e^3\log\left(d+ex^2\right)}{2d^2\,(cd^2+ae^2)}-\frac{c^2\text{Subst}\left(\int \frac{d-ex}{a+cx^2}\,dx, x, x^2\right)}{2a\,(cd^2+ae^2)}\\
&= -\frac{1}{2adx^2}-\frac{e\log(x)}{ad^2}+\frac{e^3\log\left(d+ex^2\right)}{2d^2\,(cd^2+ae^2)}\\
&\quad -\frac{(c^2d)\,\text{Subst}\left(\int \frac{1}{a+cx^2}\,dx, x, x^2\right)}{2a\,(cd^2+ae^2)}+\frac{(c^2e)\,\text{Subst}\left(\int \frac{x}{a+cx^2}\,dx, x, x^2\right)}{2a\,(cd^2+ae^2)}\\
&= -\frac{1}{2adx^2}-\frac{c^{3/2}d\tan^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{2a^{3/2}\,(cd^2+ae^2)}-\frac{e\log(x)}{ad^2}+\frac{e^3\log\left(d+ex^2\right)}{2d^2\,(cd^2+ae^2)}+\frac{ce\log\left(a+cx^4\right)}{4a\,(cd^2+ae^2)}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 169, normalized size of antiderivative = 1.31

$$\int \frac{1}{x^3 (d + ex^2) (a + cx^4)} dx$$
$$= \frac{2c^{3/2}d^3x^2 \arctan\left(1 - \frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right) + 2c^{3/2}d^3x^2 \arctan\left(1 + \frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right) + \sqrt{a}(-2cd^3 - 2ade^2 - 4e(cd^2 + ae^2)\,x}{4a^{3/2}d^2(cd^2 + ae^2)\,x^2}$$

```
[In] Integrate[1/(x^3*(d + e*x^2)*(a + c*x^4)),x]
```

```
[Out] (2*c^(3/2)*d^3*x^2*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)] + 2*c^(3/2)*d^3*
x^2*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)] + Sqrt[a]*(-2*c*d^3 - 2*a*d*e^2
 - 4*e*(c*d^2 + a*e^2)*x^2*Log[x] + 2*a*e^3*x^2*Log[d + e*x^2] + c*d^2*e*x^
2*Log[a + c*x^4]))/(4*a^(3/2)*d^2*(c*d^2 + a*e^2)*x^2)
```

Maple [A] (verified)

Time = 0.39 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.83

method	result
default	$-\frac{1}{2ad\,x^2} - \frac{e\ln(x)}{a\,d^2} - \frac{c^2\left(-\frac{e\ln\left(c\,x^4+a\right)}{2c} + \frac{d\arctan\left(\frac{c\,x^2}{\sqrt{ac}}\right)}{\sqrt{ac}}\right)}{2(a\,e^2+c\,d^2)a} + \frac{e^3\ln(e\,x^2+d)}{2d^2(a\,e^2+c\,d^2)}$
risch	$-\frac{1}{2ad\,x^2} - \frac{e\ln(x)}{a\,d^2} + \frac{e^3\ln(-e\,x^2-d)}{2d^2(a\,e^2+c\,d^2)} + \frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\left(e^2a^4+a^3c\,d^2\right)\_Z^2-2a^2ce\_Z+c^2\right)} \_R\ln\left(\left(\left(-6a^5d^2e^4-7a^4c\,d^4e^2-5a^3\right.\right.\right.}{}$

```
[In] int(1/x^3/(e*x^2+d)/(c*x^4+a),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2/a/d/x^2-e*ln(x)/a/d^2-1/2*c^2/(a*e^2+c*d^2)/a*(-1/2*e/c*ln(c*x^4+a)+d/
(a*c)^(1/2)*arctan(c*x^2/(a*c)^(1/2)))+1/2*e^3*ln(e*x^2+d)/d^2/(a*e^2+c*d^2
)
```

Fricas [A] (verification not implemented)

none

Time = 22.70 (sec) , antiderivative size = 265, normalized size of antiderivative = 2.05

$$\int \frac{1}{x^3(d+ex^2)(a+cx^4)}\,dx = \left[\frac{cd^3x^2\sqrt{-\frac{c}{a}}\log\left(\frac{cx^4-2\,ax^2\sqrt{-\frac{c}{a}}-a}{cx^4+a}\right)+cd^2ex^2\log(cx^4+a)+2\,ae^3x^2\log(ex^2+d)-2\,cd^3-2\,ade^2-4\,(cd^2\ldots}{4\,(acd^4+a^2d^2e^2)x^2}\right.$$

```
[In] integrate(1/x^3/(e*x^2+d)/(c*x^4+a),x, algorithm="fricas")
```

```
[Out] [1/4*(c*d^3*x^2*sqrt(-c/a)*log((c*x^4 - 2*a*x^2*sqrt(-c/a) - a)/(c*x^4 + a)
) + c*d^2*e*x^2*log(c*x^4 + a) + 2*a*e^3*x^2*log(e*x^2 + d) - 2*c*d^3 - 2*a
*d*e^2 - 4*(c*d^2*e + a*e^3)*x^2*log(x))/((a*c*d^4 + a^2*d^2*e^2)*x^2), 1/4
*(2*c*d^3*x^2*sqrt(c/a)*arctan(a*sqrt(c/a)/(c*x^2)) + c*d^2*e*x^2*log(c*x^4
 + a) + 2*a*e^3*x^2*log(e*x^2 + d) - 2*c*d^3 - 2*a*d*e^2 - 4*(c*d^2*e + a*e
^3)*x^2*log(x))/((a*c*d^4 + a^2*d^2*e^2)*x^2)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{x^3(d+ex^2)(a+cx^4)}\,dx = \text{Timed out}$$

```
[In] integrate(1/x**3/(e*x**2+d)/(c*x**4+a),x)
```

```
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 120, normalized size of antiderivative = 0.93

$$\int \frac{1}{x^3(d+ex^2)(a+cx^4)}\,dx = \frac{e^3\log(ex^2+d)}{2\,(cd^4+ad^2e^2)} - \frac{c^2d\arctan\left(\frac{cx^2}{\sqrt{ac}}\right)}{2\,(acd^2+a^2e^2)\sqrt{ac}} + \frac{ce\log(cx^4+a)}{4\,(acd^2+a^2e^2)} - \frac{e\log(x^2)}{2\,ad^2} - \frac{1}{2\,adx^2}$$

```
[In] integrate(1/x^3/(e*x^2+d)/(c*x^4+a),x, algorithm="maxima")
```

```
[Out] 1/2*e^3*log(e*x^2 + d)/(c*d^4 + a*d^2*e^2) - 1/2*c^2*d*arctan(c*x^2/sqrt(a*
c))/((a*c*d^2 + a^2*e^2)*sqrt(a*c)) + 1/4*c*e*log(c*x^4 + a)/(a*c*d^2 + a^2
*e^2) - 1/2*e*log(x^2)/(a*d^2) - 1/2/(a*d*x^2)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.02

$$\int \frac{1}{x^3(d+ex^2)(a+cx^4)}\,dx = \frac{e^4\log(|ex^2+d|)}{2(cd^4e+ad^2e^3)} - \frac{c^2d\arctan\left(\frac{cx^2}{\sqrt{ac}}\right)}{2(acd^2+a^2e^2)\sqrt{ac}} + \frac{ce\log(cx^4+a)}{4(acd^2+a^2e^2)} - \frac{e\log(x^2)}{2ad^2} + \frac{ex^2-d}{2ad^2x^2}$$

```
[In] integrate(1/x^3/(e*x^2+d)/(c*x^4+a),x, algorithm="giac")
```

```
[Out] 1/2*e^4*log(abs(e*x^2 + d))/(c*d^4*e + a*d^2*e^3) - 1/2*c^2*d*arctan(c*x^2/
sqrt(a*c))/((a*c*d^2 + a^2*e^2)*sqrt(a*c)) + 1/4*c*e*log(c*x^4 + a)/(a*c*d^
2 + a^2*e^2) - 1/2*e*log(x^2)/(a*d^2) + 1/2*(e*x^2 - d)/(a*d^2*x^2)
```

Mupad [B] (verification not implemented)

Time = 8.29 (sec) , antiderivative size = 820, normalized size of antiderivative = 6.36

$$\int \frac{1}{x^3(d+ex^2)(a+cx^4)}\,dx = \frac{\ln\left(a^6\,c^{12}\,d^{16}\,x^2+64\,a^{14}\,c^4\,e^{16}\,x^2+a^2\,c^7\,d^{16}\,{\left(-a^3\,c^3\right)}^{3/2}-64\,a^{13}\,c^2\,e^{16}\,\sqrt{-a^3\,c^3}+63\,a^3\,d^8\,e^8\,{\left(-a^3\,c^3\right)}^{5/2}+\right.}{} - \frac{\ln\left(a^6\,c^{12}\,d^{16}\,x^2+64\,a^{14}\,c^4\,e^{16}\,x^2-a^2\,c^7\,d^{16}\,{\left(-a^3\,c^3\right)}^{3/2}+64\,a^{13}\,c^2\,e^{16}\,\sqrt{-a^3\,c^3}-63\,a^3\,d^8\,e^8\,{\left(-a^3\,c^3\right)}^{5/2}\right.}{} + \frac{e^3\,\ln\left(e\,x^2+d\right)}{2\,c\,d^4+2\,a\,d^2\,e^2} - \frac{1}{2\,a\,d\,x^2} - \frac{e\,\ln\left(x\right)}{a\,d^2}$$

```
[In] int(1/(x^3*(a + c*x^4)*(d + e*x^2)),x)
```

```
[Out] (log(a^6*c^12*d^16*x^2 + 64*a^14*c^4*e^16*x^2 + a^2*c^7*d^16*(-a^3*c^3)^(3/
2) - 64*a^13*c^2*e^16*(-a^3*c^3)^(1/2) + 63*a^3*d^8*e^8*(-a^3*c^3)^(5/2) +
224*a^9*d^2*e^14*(-a^3*c^3)^(3/2) - 28*c^3*d^14*e^2*(-a^3*c^3)^(5/2) + 28*a
^7*c^11*d^14*e^2*x^2 + 114*a^8*c^10*d^12*e^4*x^2 + 108*a^9*c^9*d^10*e^6*x^2
 - 63*a^10*c^8*d^8*e^8*x^2 - 32*a^11*c^7*d^6*e^10*x^2 + 212*a^12*c^6*d^4*e^
12*x^2 + 224*a^13*c^5*d^2*e^14*x^2 - 114*a*c^2*d^12*e^4*(-a^3*c^3)^(5/2) -
108*a^2*c*d^10*e^6*(-a^3*c^3)^(5/2) + 212*a^8*c*d^4*e^12*(-a^3*c^3)^(3/2) -
 32*a^7*c^2*d^6*e^10*(-a^3*c^3)^(3/2))*(d*(-a^3*c^3)^(1/2) + a^2*c*e))/(4*a
^4*e^2 + 4*a^3*c*d^2) - (log(a^6*c^12*d^16*x^2 + 64*a^14*c^4*e^16*x^2 - a^2
*c^7*d^16*(-a^3*c^3)^(3/2) + 64*a^13*c^2*e^16*(-a^3*c^3)^(1/2) - 63*a^3*d^8
*e^8*(-a^3*c^3)^(5/2) - 224*a^9*d^2*e^14*(-a^3*c^3)^(3/2) + 28*c^3*d^14*e^2
```

```
*(-a^3*c^3)^(5/2) + 28*a^7*c^11*d^14*e^2*x^2 + 114*a^8*c^10*d^12*e^4*x^2 +
108*a^9*c^9*d^10*e^6*x^2 - 63*a^10*c^8*d^8*e^8*x^2 - 32*a^11*c^7*d^6*e^10*x
^2 + 212*a^12*c^6*d^4*e^12*x^2 + 224*a^13*c^5*d^2*e^14*x^2 + 114*a*c^2*d^12
*e^4*(-a^3*c^3)^(5/2) + 108*a^2*c*d^10*e^6*(-a^3*c^3)^(5/2) - 212*a^8*c*d^4
*e^12*(-a^3*c^3)^(3/2) + 32*a^7*c^2*d^6*e^10*(-a^3*c^3)^(3/2))*(d*(-a^3*c^3
)^(1/2) - a^2*c*e))/(4*(a^4*e^2 + a^3*c*d^2)) + (e^3*log(d + e*x^2))/(2*c*d
^4 + 2*a*d^2*e^2) - 1/(2*a*d*x^2) - (e*log(x))/(a*d^2)
```

3.236 $\int \frac{1}{x^5(d+ex^2)(a+cx^4)}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 156

$$\int \frac{1}{x^5\left(d+ex^2\right)\left(a+cx^4\right)}\,dx = -\frac{1}{4adx^4}+\frac{e}{2ad^2x^2}+\frac{c^{3/2}e\arctan\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{2a^{3/2}\left(cd^2+ae^2\right)} - \frac{\left(cd^2-ae^2\right)\log(x)}{a^2d^3}-\frac{e^4\log\left(d+ex^2\right)}{2d^3\left(cd^2+ae^2\right)}+\frac{c^2d\log\left(a+cx^4\right)}{4a^2\left(cd^2+ae^2\right)}$$

```
[Out] -1/4/a/d/x^4+1/2*e/a/d^2/x^2+1/2*c^(3/2)*e*arctan(x^2*c^(1/2)/a^(1/2))/a^(3
/2)/(a*e^2+c*d^2)-(-a*e^2+c*d^2)*ln(x)/a^2/d^3-1/2*e^4*ln(e*x^2+d)/d^3/(a*e
^2+c*d^2)+1/4*c^2*d*ln(c*x^4+a)/a^2/(a*e^2+c*d^2)
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {1266, 908, 649, 211, 266}

$$\int \frac{1}{x^5\left(d+ex^2\right)\left(a+cx^4\right)}\,dx = \frac{c^{3/2}e\arctan\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{2a^{3/2}\left(ae^2+cd^2\right)}+\frac{c^2d\log\left(a+cx^4\right)}{4a^2\left(ae^2+cd^2\right)} - \frac{\log(x)\left(cd^2-ae^2\right)}{a^2d^3}-\frac{e^4\log\left(d+ex^2\right)}{2d^3\left(ae^2+cd^2\right)}+\frac{e}{2ad^2x^2}-\frac{1}{4adx^4}$$

```
[In] Int[1/(x^5*(d + e*x^2)*(a + c*x^4)),x]
```

```
[Out] -1/4*1/(a*d*x^4) + e/(2*a*d^2*x^2) + (c^(3/2)*e*ArcTan[(Sqrt[c]*x^2)/Sqrt[a
]])/(2*a^(3/2)*(c*d^2 + a*e^2)) - ((c*d^2 - a*e^2)*Log[x])/(a^2*d^3) - (e^4
*Log[d + e*x^2])/(2*d^3*(c*d^2 + a*e^2)) + (c^2*d*Log[a + c*x^4])/(4*a^2*(c
*d^2 + a*e^2))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[(-a)*c]
```

Rule 908

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && IntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ
[m, 0] && ILtQ[n, 0]))
```

Rule 1266

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_S
ymbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + c*x^2)^p, x],
x, x^2], x] /; FreeQ[{a, c, d, e, p, q}, x] && IntegerQ[(m + 1)/2]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{1}{2}\text{Subst}\left(\int \frac{1}{x^3(d+ex)\,(a+cx^2)}\,dx, x, x^2\right)\\
&= \frac{1}{2}\text{Subst}\left(\int \left(\frac{1}{adx^3}-\frac{e}{ad^2x^2}+\frac{-cd^2+ae^2}{a^2d^3x}-\frac{e^5}{d^3\,(cd^2+ae^2)\,(d+ex)}\right.\right.\\
&\qquad\left.\left.+\frac{c^2(ae+cdx)}{a^2\,(cd^2+ae^2)\,(a+cx^2)}\right)\,dx, x, x^2\right)\\
&= -\frac{1}{4adx^4}+\frac{e}{2ad^2x^2}-\frac{(cd^2-ae^2)\log(x)}{a^2d^3}-\frac{e^4\log\left(d+ex^2\right)}{2d^3\,(cd^2+ae^2)}+\frac{c^2\text{Subst}\left(\int\frac{ae+cdx}{a+cx^2}\,dx, x, x^2\right)}{2a^2\,(cd^2+ae^2)}\\
&= -\frac{1}{4adx^4}+\frac{e}{2ad^2x^2}-\frac{(cd^2-ae^2)\log(x)}{a^2d^3}-\frac{e^4\log\left(d+ex^2\right)}{2d^3\,(cd^2+ae^2)}\\
&\quad+\frac{(c^3d)\,\text{Subst}\left(\int\frac{x}{a+cx^2}\,dx, x, x^2\right)}{2a^2\,(cd^2+ae^2)}+\frac{(c^2e)\,\text{Subst}\left(\int\frac{1}{a+cx^2}\,dx, x, x^2\right)}{2a\,(cd^2+ae^2)}
\end{aligned}
$$

$$= -\frac{1}{4adx^4} + \frac{e}{2ad^2x^2} + \frac{c^{3/2}e\tan^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{2a^{3/2}(cd^2+ae^2)} - \frac{(cd^2-ae^2)\log(x)}{a^2d^3} - \frac{e^4\log(d+ex^2)}{2d^3(cd^2+ae^2)} + \frac{c^2d\log(a+cx^4)}{4a^2(cd^2+ae^2)}$$

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 209, normalized size of antiderivative = 1.34

$$\int \frac{1}{x^5(d+ex^2)(a+cx^4)}\,dx = -\frac{acd^4 + a^2d^2e^2 - 2acd^3ex^2 - 2a^2de^3x^2 + 2\sqrt{a}c^{3/2}d^3ex^4\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right) + 2\sqrt{a}c^{3/2}d^3ex^4\arctan\left(1\right.}{4a^2d^3(cd^2+ae}$$

```
[In] Integrate[1/(x^5*(d + e*x^2)*(a + c*x^4)),x]
```

```
[Out] -1/4*(a*c*d^4 + a^2*d^2*e^2 - 2*a*c*d^3*e*x^2 - 2*a^2*d*e^3*x^2 + 2*Sqrt[a]
*c^(3/2)*d^3*e*x^4*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)] + 2*Sqrt[a]*c^(3
/2)*d^3*e*x^4*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)] + 4*c^2*d^4*x^4*Log[x
] - 4*a^2*e^4*x^4*Log[x] + 2*a^2*e^4*x^4*Log[d + e*x^2] - c^2*d^4*x^4*Log[a
 + c*x^4])/(a^2*d^3*(c*d^2 + a*e^2)*x^4)
```

Maple [A] (verified)

Time = 0.42 (sec) , antiderivative size = 127, normalized size of antiderivative = 0.81

method	result
default	$-\frac{1}{4adx^4} + \frac{(ae^2-cd^2)\ln(x)}{d^3a^2} + \frac{e}{2ad^2x^2} + \frac{c^2\left(\frac{d\ln(cx^4+a)}{2} + \frac{ae\arctan\left(\frac{cx^2}{\sqrt{ac}}\right)}{\sqrt{ac}}\right)}{2(ae^2+cd^2)a^2} - \frac{e^4\ln(ex^2+d)}{2d^3(ae^2+cd^2)}$
risch	$\frac{\frac{ex^2}{2d^2a}-\frac{1}{4da}}{x^4} + \frac{\ln(x)e^2}{d^3a} - \frac{\ln(x)c}{da^2} - \frac{e^4\ln(ex^2+d)}{2d^3(ae^2+cd^2)} + \frac{\left(\sum_{\_R=\mathrm{RootOf}\left((a^5e^2+d^2a^4c)\_Z^2-2a^2c^2d\_Z+c^3\right)}\_R\ln\left(\left(\left(-6a^6d^4e^4-7\right.\right.\right.\right.}{}$

```
[In] int(1/x^5/(e*x^2+d)/(c*x^4+a),x,method=_RETURNVERBOSE)
```

```
[Out] -1/4/a/d/x^4+(a*e^2-c*d^2)/d^3/a^2*ln(x)+1/2*e/a/d^2/x^2+1/2*c^2/(a*e^2+c*d
^2)/a^2*(1/2*d*ln(c*x^4+a)+a*e/(a*c)^(1/2)*arctan(c*x^2/(a*c)^(1/2)))-1/2*e
^4*ln(e*x^2+d)/d^3/(a*e^2+c*d^2)
```

Fricas [A] (verification not implemented)

none

Time = 49.46 (sec) , antiderivative size = 338, normalized size of antiderivative = 2.17

$$\int \frac{1}{x^5 (d+ex^2) (a+cx^4)} \, dx$$

$$= \left[\frac{acd^3ex^4\sqrt{-\frac{c}{a}}\log\left(\frac{cx^4+2\,ax^2\sqrt{-\frac{c}{a}}-a}{cx^4+a}\right) + c^2d^4x^4\log\left(cx^4+a\right) - 2\,a^2e^4x^4\log\left(ex^2+d\right) - acd^4 - a^2d^2e^2 - 4\,(}{4\,(a^2cd^5+a^3d^3e^2)x^4} \right.$$

$$- \frac{2\,acd^3ex^4\sqrt{\frac{c}{a}}\arctan\left(\frac{a\sqrt{\frac{c}{a}}}{cx^2}\right) - c^2d^4x^4\log\left(cx^4+a\right) + 2\,a^2e^4x^4\log\left(ex^2+d\right) + acd^4 + a^2d^2e^2 + 4\,(c^2d^4 -}{4\,(a^2cd^5+a^3d^3e^2)x^4}$$

```
[In] integrate(1/x^5/(e*x^2+d)/(c*x^4+a),x, algorithm="fricas")
```

```
[Out] [1/4*(a*c*d^3*e*x^4*sqrt(-c/a)*log((c*x^4 + 2*a*x^2*sqrt(-c/a) - a)/(c*x^4
+ a)) + c^2*d^4*x^4*log(c*x^4 + a) - 2*a^2*e^4*x^4*log(e*x^2 + d) - a*c*d^4
 - a^2*d^2*e^2 - 4*(c^2*d^4 - a^2*e^4)*x^4*log(x) + 2*(a*c*d^3*e + a^2*d*e^
3)*x^2)/((a^2*c*d^5 + a^3*d^3*e^2)*x^4), -1/4*(2*a*c*d^3*e*x^4*sqrt(c/a)*ar
ctan(a*sqrt(c/a)/(c*x^2)) - c^2*d^4*x^4*log(c*x^4 + a) + 2*a^2*e^4*x^4*log(
e*x^2 + d) + a*c*d^4 + a^2*d^2*e^2 + 4*(c^2*d^4 - a^2*e^4)*x^4*log(x) - 2*(
a*c*d^3*e + a^2*d*e^3)*x^2)/((a^2*c*d^5 + a^3*d^3*e^2)*x^4)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{x^5 (d+ex^2) (a+cx^4)} \, dx = \text{Timed out}$$

```
[In] integrate(1/x**5/(e*x**2+d)/(c*x**4+a),x)
```

```
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 145, normalized size of antiderivative = 0.93

$$\int \frac{1}{x^5 (d+ex^2) (a+cx^4)} \, dx = -\frac{e^4 \log (ex^2+d)}{2 (cd^5+ad^3e^2)} + \frac{c^2 d \log (cx^4+a)}{4 (a^2cd^2+a^3e^2)} + \frac{c^2 e \arctan \left(\frac{cx^2}{\sqrt{ac}}\right)}{2 (acd^2+a^2e^2)\sqrt{ac}} - \frac{(cd^2-ae^2) \log (x^2)}{2 a^2d^3} + \frac{2 ex^2-d}{4 ad^2x^4}$$

```
[In] integrate(1/x^5/(e*x^2+d)/(c*x^4+a),x, algorithm="maxima")
```

```
[Out] -1/2*e^4*log(e*x^2 + d)/(c*d^5 + a*d^3*e^2) + 1/4*c^2*d*log(c*x^4 + a)/(a^2
*c*d^2 + a^3*e^2) + 1/2*c^2*e*arctan(c*x^2/sqrt(a*c))/((a*c*d^2 + a^2*e^2)*
sqrt(a*c)) - 1/2*(c*d^2 - a*e^2)*log(x^2)/(a^2*d^3) + 1/4*(2*e*x^2 - d)/(a*
d^2*x^4)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.09

$$\int \frac{1}{x^5 (d+ex^2) (a+cx^4)} \, dx = -\frac{e^5 \log (|ex^2+d|)}{2 (cd^5e+ad^3e^3)} + \frac{c^2 d \log (cx^4+a)}{4 (a^2cd^2+a^3e^2)} + \frac{c^2 e \arctan \left(\frac{cx^2}{\sqrt{ac}}\right)}{2 (acd^2+a^2e^2)\sqrt{ac}} - \frac{(cd^2-ae^2) \log (x^2)}{2 a^2d^3} + \frac{3 cd^2x^4-3 ae^2x^4+2 adex^2-ad^2}{4 a^2d^3x^4}$$

```
[In] integrate(1/x^5/(e*x^2+d)/(c*x^4+a),x, algorithm="giac")
```

```
[Out] -1/2*e^5*log(abs(e*x^2 + d))/(c*d^5*e + a*d^3*e^3) + 1/4*c^2*d*log(c*x^4 +
a)/(a^2*c*d^2 + a^3*e^2) + 1/2*c^2*e*arctan(c*x^2/sqrt(a*c))/((a*c*d^2 + a^
2*e^2)*sqrt(a*c)) - 1/2*(c*d^2 - a*e^2)*log(x^2)/(a^2*d^3) + 1/4*(3*c*d^2*x
^4 - 3*a*e^2*x^4 + 2*a*d*e*x^2 - a*d^2)/(a^2*d^3*x^4)
```

Mupad [B] (verification not implemented)

Time = 8.60 (sec) , antiderivative size = 1017, normalized size of antiderivative = 6.52

$$\int \frac{1}{x^5\,(d+ex^2)\,(a+cx^4)}\,dx$$
$$= \frac{\ln\Big(25\,a^2\,c^9\,d^{20}\,(-a^5\,c^3)^{3/2}-64\,a^{19}\,c^4\,e^{20}\,x^2-25\,a^9\,c^{14}\,d^{20}\,x^2-64\,a^{17}\,c^2\,e^{20}\,\sqrt{-a^5\,c^3}+100\,a^3\,d^8\,e^{12}\,(-a^5\,c^3$$
$$-\frac{e^4\,\ln(e\,x^2+d)}{2\,(c\,d^5+a\,d^3\,e^2)}$$
$$-\frac{\ln\Big(25\,a^9\,c^{14}\,d^{20}\,x^2+64\,a^{19}\,c^4\,e^{20}\,x^2+25\,a^2\,c^9\,d^{20}\,(-a^5\,c^3)^{3/2}-64\,a^{17}\,c^2\,e^{20}\,\sqrt{-a^5\,c^3}+100\,a^3\,d^8\,e^{12}\,(-a^5$$
$$-\frac{\frac{1}{4\,a\,d}-\frac{e\,x^2}{2\,a\,d^2}}{x^4}+\frac{\ln(x)\,(a\,e^2-c\,d^2)}{a^2\,d^3}$$

```
[In] int(1/(x^5*(a + c*x^4)*(d + e*x^2)),x)
```

```
[Out] (log(25*a^2*c^9*d^20*(-a^5*c^3)^(3/2) - 64*a^19*c^4*e^20*x^2 - 25*a^9*c^14*
d^20*x^2 - 64*a^17*c^2*e^20*(-a^5*c^3)^(1/2) + 100*a^3*d^8*e^12*(-a^5*c^3)^
(5/2) + 128*a^11*d^2*e^18*(-a^5*c^3)^(3/2) - 112*c^3*d^14*e^6*(-a^5*c^3)^(5
/2) - 76*a^10*c^13*d^18*e^2*x^2 - 138*a^11*c^12*d^16*e^4*x^2 - 112*a^12*c^1
1*d^14*e^6*x^2 + 55*a^13*c^10*d^12*e^8*x^2 + 104*a^14*c^9*d^10*e^10*x^2 + 1
00*a^15*c^8*d^8*e^12*x^2 + 172*a^16*c^7*d^6*e^14*x^2 + 32*a^17*c^6*d^4*e^16
*x^2 - 128*a^18*c^5*d^2*e^18*x^2 + 55*a*c^2*d^12*e^8*(-a^5*c^3)^(5/2) + 104
*a^2*c*d^10*e^10*(-a^5*c^3)^(5/2) - 32*a^10*c*d^4*e^16*(-a^5*c^3)^(3/2) + 7
6*a^3*c^8*d^18*e^2*(-a^5*c^3)^(3/2) + 138*a^4*c^7*d^16*e^4*(-a^5*c^3)^(3/2)
 - 172*a^9*c^2*d^6*e^14*(-a^5*c^3)^(3/2))*(e*(-a^5*c^3)^(1/2) + a^2*c^2*d))
/(4*a^5*e^2 + 4*a^4*c*d^2) - (e^4*log(d + e*x^2))/(2*(c*d^5 + a*d^3*e^2)) -
 (log(25*a^9*c^14*d^20*x^2 + 64*a^19*c^4*e^20*x^2 + 25*a^2*c^9*d^20*(-a^5*c
^3)^(3/2) - 64*a^17*c^2*e^20*(-a^5*c^3)^(1/2) + 100*a^3*d^8*e^12*(-a^5*c^3)
^(5/2) + 128*a^11*d^2*e^18*(-a^5*c^3)^(3/2) - 112*c^3*d^14*e^6*(-a^5*c^3)^(
5/2) + 76*a^10*c^13*d^18*e^2*x^2 + 138*a^11*c^12*d^16*e^4*x^2 + 112*a^12*c^
11*d^14*e^6*x^2 - 55*a^13*c^10*d^12*e^8*x^2 - 104*a^14*c^9*d^10*e^10*x^2 -
100*a^15*c^8*d^8*e^12*x^2 - 172*a^16*c^7*d^6*e^14*x^2 - 32*a^17*c^6*d^4*e^1
6*x^2 + 128*a^18*c^5*d^2*e^18*x^2 + 55*a*c^2*d^12*e^8*(-a^5*c^3)^(5/2) + 10
4*a^2*c*d^10*e^10*(-a^5*c^3)^(5/2) - 32*a^10*c*d^4*e^16*(-a^5*c^3)^(3/2) +
76*a^3*c^8*d^18*e^2*(-a^5*c^3)^(3/2) + 138*a^4*c^7*d^16*e^4*(-a^5*c^3)^(3/2
) - 172*a^9*c^2*d^6*e^14*(-a^5*c^3)^(3/2))*(e*(-a^5*c^3)^(1/2) - a^2*c^2*d)
)/(4*(a^5*e^2 + a^4*c*d^2)) - (1/(4*a*d) - (e*x^2)/(2*a*d^2))/x^4 + (log(x)
*(a*e^2 - c*d^2))/(a^2*d^3)
```

3.237 $\int \frac{x^8}{(d+ex^2)(a+cx^4)}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 359

$$\begin{aligned}\int \frac{x^8}{(d+ex^2)(a+cx^4)}\,dx = & -\frac{dx}{ce^2}+\frac{x^3}{3ce}+\frac{d^{7/2}\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{e^{5/2}(cd^2+ae^2)} \\ & -\frac{a^{5/4}(\sqrt{c}d-\sqrt{a}e)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}c^{7/4}(cd^2+ae^2)} \\ & +\frac{a^{5/4}(\sqrt{c}d-\sqrt{a}e)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}c^{7/4}(cd^2+ae^2)} \\ & -\frac{a^{5/4}(\sqrt{c}d+\sqrt{a}e)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)}{4\sqrt{2}c^{7/4}(cd^2+ae^2)} \\ & +\frac{a^{5/4}(\sqrt{c}d+\sqrt{a}e)\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)}{4\sqrt{2}c^{7/4}(cd^2+ae^2)}\end{aligned}$$

```
[Out] -d*x/c/e^2+1/3*x^3/c/e+d^(7/2)*arctan(x*e^(1/2)/d^(1/2))/e^(5/2)/(a*e^2+c*d
^2)+1/4*a^(5/4)*arctan(-1+c^(1/4)*x*2^(1/2)/a^(1/4))*(-e*a^(1/2)+d*c^(1/2))
/c^(7/4)/(a*e^2+c*d^2)*2^(1/2)+1/4*a^(5/4)*arctan(1+c^(1/4)*x*2^(1/2)/a^(1/
4))*(-e*a^(1/2)+d*c^(1/2))/c^(7/4)/(a*e^2+c*d^2)*2^(1/2)-1/8*a^(5/4)*ln(-a^
(1/4)*c^(1/4)*x*2^(1/2)+a^(1/2)+x^2*c^(1/2))*(e*a^(1/2)+d*c^(1/2))/c^(7/4)/
(a*e^2+c*d^2)*2^(1/2)+1/8*a^(5/4)*ln(a^(1/4)*c^(1/4)*x*2^(1/2)+a^(1/2)+x^2*
c^(1/2))*(e*a^(1/2)+d*c^(1/2))/c^(7/4)/(a*e^2+c*d^2)*2^(1/2)
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 359, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$, Rules used = {1302, 211, 1182, 1176, 631, 210, 1179, 642}

$$\int \frac{x^8}{(d+ex^2)(a+cx^4)}\,dx = -\frac{a^{5/4}\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\left(\sqrt{c}d-\sqrt{a}e\right)}{2\sqrt{2}c^{7/4}\left(ae^2+cd^2\right)} + \frac{a^{5/4}\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)\left(\sqrt{c}d-\sqrt{a}e\right)}{2\sqrt{2}c^{7/4}\left(ae^2+cd^2\right)} - \frac{a^{5/4}\left(\sqrt{a}e+\sqrt{c}d\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{4\sqrt{2}c^{7/4}\left(ae^2+cd^2\right)} + \frac{a^{5/4}\left(\sqrt{a}e+\sqrt{c}d\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{4\sqrt{2}c^{7/4}\left(ae^2+cd^2\right)} + \frac{d^{7/2}\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{e^{5/2}\left(ae^2+cd^2\right)} - \frac{dx}{ce^2} + \frac{x^3}{3ce}$$

```
[In] Int[x^8/((d + e*x^2)*(a + c*x^4)),x]
```

```
[Out] -((d*x)/(c*e^2)) + x^3/(3*c*e) + (d^(7/2)*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(e^(
5/2)*(c*d^2 + a*e^2)) - (a^(5/4)*(Sqrt[c]*d - Sqrt[a]*e)*ArcTan[1 - (Sqrt[2
]*c^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*c^(7/4)*(c*d^2 + a*e^2)) + (a^(5/4)*(Sqrt
[c]*d - Sqrt[a]*e)*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*c^(7
/4)*(c*d^2 + a*e^2)) - (a^(5/4)*(Sqrt[c]*d + Sqrt[a]*e)*Log[Sqrt[a] - Sqrt[
2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(4*Sqrt[2]*c^(7/4)*(c*d^2 + a*e^2)) +
(a^(5/4)*(Sqrt[c]*d + Sqrt[a]*e)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x +
Sqrt[c]*x^2])/(4*Sqrt[2]*c^(7/4)*(c*d^2 + a*e^2))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
```

```
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1182

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[(-a)
*c]
```

Rule 1302

```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.))/((a_) + (c_.)*(x_)^4),
 x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*((d + e*x^2)^q/(a + c*x^4)), x],
x] /; FreeQ[{a, c, d, e, f, m}, x] && IntegerQ[q] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(-\frac{d}{ce^2}+\frac{x^2}{ce}+\frac{d^4}{e^2\left(cd^2+ae^2\right)\left(d+ex^2\right)}+\frac{a^2(d-ex^2)}{c\left(cd^2+ae^2\right)\left(a+cx^4\right)}\right)dx \\
&= -\frac{dx}{ce^2}+\frac{x^3}{3ce}+\frac{a^2\int\frac{d-ex^2}{a+cx^4}\,dx}{c\left(cd^2+ae^2\right)}+\frac{d^4\int\frac{1}{d+ex^2}\,dx}{e^2\left(cd^2+ae^2\right)}
\end{aligned}$$

$$= -\frac{dx}{ce^2} + \frac{x^3}{3ce} + \frac{d^{7/2}\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{e^{5/2}(cd^2+ae^2)} + \frac{\left(a^2\left(\frac{\sqrt{c}d}{\sqrt{a}}-e\right)\right)\int \frac{\sqrt{a}\sqrt{c}+cx^2}{a+cx^4}\,dx}{2c^2(cd^2+ae^2)} + \frac{\left(a^2\left(\frac{\sqrt{c}d}{\sqrt{a}}+e\right)\right)\int \frac{\sqrt{a}\sqrt{c}-cx^2}{a+cx^4}\,dx}{2c^2(cd^2+ae^2)}$$

$$= -\frac{dx}{ce^2} + \frac{x^3}{3ce} + \frac{d^{7/2}\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{e^{5/2}(cd^2+ae^2)} + \frac{\left(a^2\left(\frac{\sqrt{c}d}{\sqrt{a}}-e\right)\right)\int \frac{1}{\frac{\sqrt{a}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}+x^2}\,dx}{4c^2(cd^2+ae^2)} + \frac{\left(a^2\left(\frac{\sqrt{c}d}{\sqrt{a}}-e\right)\right)\int \frac{1}{\frac{\sqrt{a}}{\sqrt{c}}+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}+x^2}\,dx}{4c^2(cd^2+ae^2)} - \frac{\left(a^{5/4}\left(\sqrt{c}d+\sqrt{a}e\right)\right)\int \frac{\frac{\sqrt{2}\sqrt[4]{a}}{\sqrt[4]{c}}+2x}{-\frac{\sqrt{a}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}-x^2}\,dx}{4\sqrt{2}c^{7/4}(cd^2+ae^2)} - \frac{\left(a^{5/4}\left(\sqrt{c}d+\sqrt{a}e\right)\right)\int \frac{\frac{\sqrt{2}\sqrt[4]{a}}{\sqrt[4]{c}}-2x}{-\frac{\sqrt{a}}{\sqrt{c}}+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}-x^2}\,dx}{4\sqrt{2}c^{7/4}(cd^2+ae^2)}$$

$$= -\frac{dx}{ce^2} + \frac{x^3}{3ce} + \frac{d^{7/2}\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{e^{5/2}(cd^2+ae^2)} - \frac{a^{5/4}\left(\sqrt{c}d+\sqrt{a}e\right)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)}{4\sqrt{2}c^{7/4}(cd^2+ae^2)} + \frac{a^{5/4}\left(\sqrt{c}d+\sqrt{a}e\right)\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)}{4\sqrt{2}c^{7/4}(cd^2+ae^2)} + \frac{\left(a^{7/4}\left(\frac{\sqrt{c}d}{\sqrt{a}}-e\right)\right)\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}c^{7/4}(cd^2+ae^2)} - \frac{\left(a^{7/4}\left(\frac{\sqrt{c}d}{\sqrt{a}}-e\right)\right)\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1+\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}c^{7/4}(cd^2+ae^2)}$$

$$= -\frac{dx}{ce^2} + \frac{x^3}{3ce} + \frac{d^{7/2}\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{e^{5/2}(cd^2+ae^2)} - \frac{a^{7/4}\left(\frac{\sqrt{c}d}{\sqrt{a}}-e\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}c^{7/4}(cd^2+ae^2)} + \frac{a^{7/4}\left(\frac{\sqrt{c}d}{\sqrt{a}}-e\right)\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}c^{7/4}(cd^2+ae^2)} - \frac{a^{5/4}\left(\sqrt{c}d+\sqrt{a}e\right)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)}{4\sqrt{2}c^{7/4}(cd^2+ae^2)} + \frac{a^{5/4}\left(\sqrt{c}d+\sqrt{a}e\right)\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)}{4\sqrt{2}c^{7/4}(cd^2+ae^2)}$$

Mathematica [A] (verified)

Time = 0.26 (sec) , antiderivative size = 344, normalized size of antiderivative = 0.96

$$\int \frac{x^8}{(d+ex^2)(a+cx^4)}\,dx$$
$$= \frac{-24c^{3/4}d\sqrt{e}(cd^2+ae^2)\,x+8c^{3/4}e^{3/2}(cd^2+ae^2)\,x^3+24c^{7/4}d^{7/2}\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)+6\sqrt{2}a^{5/4}e^{5/2}\left(-\sqrt{c}d+\sqrt{c}\right.}{}$$

[In] Integrate[x^8/((d + e*x^2)*(a + c*x^4)),x]

[Out] (-24*c^(3/4)*d*Sqrt[e]*(c*d^2 + a*e^2)*x + 8*c^(3/4)*e^(3/2)*(c*d^2 + a*e^2
)*x^3 + 24*c^(7/4)*d^(7/2)*ArcTan[(Sqrt[e]*x)/Sqrt[d]] + 6*Sqrt[2]*a^(5/4)*
e^(5/2)*(-(Sqrt[c]*d) + Sqrt[a]*e)*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)]
- 6*Sqrt[2]*a^(5/4)*e^(5/2)*(-(Sqrt[c]*d) + Sqrt[a]*e)*ArcTan[1 + (Sqrt[2]*
c^(1/4)*x)/a^(1/4)] - 3*Sqrt[2]*a*e^(5/2)*(a^(1/4)*Sqrt[c]*d + a^(3/4)*e)*L
og[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2] + 3*Sqrt[2]*a*e^(5/2)
*(a^(1/4)*Sqrt[c]*d + a^(3/4)*e)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x +
Sqrt[c]*x^2])/(24*c^(7/4)*e^(5/2)*(c*d^2 + a*e^2))

Maple [A] (verified)

Time = 0.41 (sec) , antiderivative size = 279, normalized size of antiderivative = 0.78

method	result
default	$-\frac{-\frac{1}{3}e\,x^3+dx}{c\,e^2}+\frac{a^2\left(\frac{d\left(\frac{a}{c}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x^2+\left(\frac{a}{c}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{c}}}{x^2-\left(\frac{a}{c}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{c}}}\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{c}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{c}\right)^{\frac{1}{4}}}-1\right)\right)}{8a}-\frac{e\sqrt{2}\left(\ln\left(\frac{x^2-\left(\frac{a}{c}\right)^{\frac{1}{4}}x\sqrt{2}+\cdots}{x^2+\left(\frac{a}{c}\right)^{\frac{1}{4}}x\sqrt{2}+\cdots}\right.\right.}{}\right.}{(a\,e^2+c\,d^2)c}$
risch	$\frac{x^3}{3ce}-\frac{dx}{c\,e^2}+\frac{\sqrt{-ed}\,d^3\ln\left(\left(-16(-ed)^{\frac{5}{2}}a\,c^7d^{12}e^2+16(-ed)^{\frac{5}{2}}c^8d^{14}+14a^4c^4d^7e^9(-ed)^{\frac{3}{2}}-4a^3c^5d^9e^7(-ed)^{\frac{3}{2}}-2a^2c^6d^{11}e^5(-ed)^{\frac{3}{2}}\cdots\right.\right.}{}$

[In] int(x^8/(e*x^2+d)/(c*x^4+a),x,method=_RETURNVERBOSE)

[Out] -1/c/e^2*(-1/3*e*x^3+d*x)+a^2/(a*e^2+c*d^2)/c*(1/8*d*(a/c)^(1/4)/a*2^(1/2)*
(ln((x^2+(a/c)^(1/4)*x*2^(1/2)+(a/c)^(1/2))/(x^2-(a/c)^(1/4)*x*2^(1/2)+(a/c
)^(1/2)))+2*arctan(2^(1/2)/(a/c)^(1/4)*x+1)+2*arctan(2^(1/2)/(a/c)^(1/4)*x-
1))-1/8*e/c/(a/c)^(1/4)*2^(1/2)*(ln((x^2-(a/c)^(1/4)*x*2^(1/2)+(a/c)^(1/2))
/(x^2+(a/c)^(1/4)*x*2^(1/2)+(a/c)^(1/2)))+2*arctan(2^(1/2)/(a/c)^(1/4)*x+1)
+2*arctan(2^(1/2)/(a/c)^(1/4)*x-1)))+1/e^2*d^4/(a*e^2+c*d^2)/(e*d)^(1/2)*ar
ctan(e*x/(e*d)^(1/2))

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2197 vs. $2(268) = 536$.

Time = 5.53 (sec) , antiderivative size = 4414, normalized size of antiderivative = 12.30

$$\int \frac{x^8}{(d+ex^2)(a+cx^4)}\,dx = \text{Too large to display}$$

```
[In] integrate(x^8/(e*x^2+d)/(c*x^4+a),x, algorithm="fricas")
```

```
[Out] [1/12*(6*c*d^3*sqrt(-d/e)*log((e*x^2 + 2*e*x*sqrt(-d/e) - d)/(e*x^2 + d)) +
 4*(c*d^2*e + a*e^3)*x^3 - 3*(c^2*d^2*e^2 + a*c*e^4)*sqrt((2*a^3*d*e + (c^5
*d^4 + 2*a*c^4*d^2*e^2 + a^2*c^3*e^4)*sqrt(-(a^5*c^2*d^4 - 2*a^6*c*d^2*e^2
+ a^7*e^4)/(c^11*d^8 + 4*a*c^10*d^6*e^2 + 6*a^2*c^9*d^4*e^4 + 4*a^3*c^8*d^2
*e^6 + a^4*c^7*e^8)))/(c^5*d^4 + 2*a*c^4*d^2*e^2 + a^2*c^3*e^4))*log(-(a^3*
c*d^2 - a^4*e^2)*x + (a^2*c^3*d^3 - a^3*c^2*d*e^2 + (c^7*d^4*e + 2*a*c^6*d^
2*e^3 + a^2*c^5*e^5)*sqrt(-(a^5*c^2*d^4 - 2*a^6*c*d^2*e^2 + a^7*e^4)/(c^11*
d^8 + 4*a*c^10*d^6*e^2 + 6*a^2*c^9*d^4*e^4 + 4*a^3*c^8*d^2*e^6 + a^4*c^7*e^
8)))*sqrt((2*a^3*d*e + (c^5*d^4 + 2*a*c^4*d^2*e^2 + a^2*c^3*e^4)*sqrt(-(a^5
*c^2*d^4 - 2*a^6*c*d^2*e^2 + a^7*e^4)/(c^11*d^8 + 4*a*c^10*d^6*e^2 + 6*a^2*
c^9*d^4*e^4 + 4*a^3*c^8*d^2*e^6 + a^4*c^7*e^8)))/(c^5*d^4 + 2*a*c^4*d^2*e^2
 + a^2*c^3*e^4))) + 3*(c^2*d^2*e^2 + a*c*e^4)*sqrt((2*a^3*d*e + (c^5*d^4 +
2*a*c^4*d^2*e^2 + a^2*c^3*e^4)*sqrt(-(a^5*c^2*d^4 - 2*a^6*c*d^2*e^2 + a^7*e
^4)/(c^11*d^8 + 4*a*c^10*d^6*e^2 + 6*a^2*c^9*d^4*e^4 + 4*a^3*c^8*d^2*e^6 +
a^4*c^7*e^8)))/(c^5*d^4 + 2*a*c^4*d^2*e^2 + a^2*c^3*e^4))*log(-(a^3*c*d^2 -
 a^4*e^2)*x - (a^2*c^3*d^3 - a^3*c^2*d*e^2 + (c^7*d^4*e + 2*a*c^6*d^2*e^3 +
 a^2*c^5*e^5)*sqrt(-(a^5*c^2*d^4 - 2*a^6*c*d^2*e^2 + a^7*e^4)/(c^11*d^8 + 4
*a*c^10*d^6*e^2 + 6*a^2*c^9*d^4*e^4 + 4*a^3*c^8*d^2*e^6 + a^4*c^7*e^8)))*sq
rt((2*a^3*d*e + (c^5*d^4 + 2*a*c^4*d^2*e^2 + a^2*c^3*e^4)*sqrt(-(a^5*c^2*d^
4 - 2*a^6*c*d^2*e^2 + a^7*e^4)/(c^11*d^8 + 4*a*c^10*d^6*e^2 + 6*a^2*c^9*d^4
*e^4 + 4*a^3*c^8*d^2*e^6 + a^4*c^7*e^8)))/(c^5*d^4 + 2*a*c^4*d^2*e^2 + a^2*
c^3*e^4))) - 3*(c^2*d^2*e^2 + a*c*e^4)*sqrt((2*a^3*d*e - (c^5*d^4 + 2*a*c^4
*d^2*e^2 + a^2*c^3*e^4)*sqrt(-(a^5*c^2*d^4 - 2*a^6*c*d^2*e^2 + a^7*e^4)/(c^
11*d^8 + 4*a*c^10*d^6*e^2 + 6*a^2*c^9*d^4*e^4 + 4*a^3*c^8*d^2*e^6 + a^4*c^7
*e^8)))/(c^5*d^4 + 2*a*c^4*d^2*e^2 + a^2*c^3*e^4))*log(-(a^3*c*d^2 - a^4*e^
2)*x + (a^2*c^3*d^3 - a^3*c^2*d*e^2 - (c^7*d^4*e + 2*a*c^6*d^2*e^3 + a^2*c^
5*e^5)*sqrt(-(a^5*c^2*d^4 - 2*a^6*c*d^2*e^2 + a^7*e^4)/(c^11*d^8 + 4*a*c^10
*d^6*e^2 + 6*a^2*c^9*d^4*e^4 + 4*a^3*c^8*d^2*e^6 + a^4*c^7*e^8)))*sqrt((2*a
^3*d*e - (c^5*d^4 + 2*a*c^4*d^2*e^2 + a^2*c^3*e^4)*sqrt(-(a^5*c^2*d^4 - 2*a
^6*c*d^2*e^2 + a^7*e^4)/(c^11*d^8 + 4*a*c^10*d^6*e^2 + 6*a^2*c^9*d^4*e^4 +
4*a^3*c^8*d^2*e^6 + a^4*c^7*e^8)))/(c^5*d^4 + 2*a*c^4*d^2*e^2 + a^2*c^3*e^4
))) + 3*(c^2*d^2*e^2 + a*c*e^4)*sqrt((2*a^3*d*e - (c^5*d^4 + 2*a*c^4*d^2*e^
2 + a^2*c^3*e^4)*sqrt(-(a^5*c^2*d^4 - 2*a^6*c*d^2*e^2 + a^7*e^4)/(c^11*d^8
+ 4*a*c^10*d^6*e^2 + 6*a^2*c^9*d^4*e^4 + 4*a^3*c^8*d^2*e^6 + a^4*c^7*e^8)))
/(c^5*d^4 + 2*a*c^4*d^2*e^2 + a^2*c^3*e^4))*log(-(a^3*c*d^2 - a^4*e^2)*x -
```

(a^2*c^3*d^3 - a^3*c^2*d*e^2 - (c^7*d^4*e + 2*a*c^6*d^2*e^3 + a^2*c^5*e^5)*
sqrt(-(a^5*c^2*d^4 - 2*a^6*c*d^2*e^2 + a^7*e^4)/(c^11*d^8 + 4*a*c^10*d^6*e^
2 + 6*a^2*c^9*d^4*e^4 + 4*a^3*c^8*d^2*e^6 + a^4*c^7*e^8)))*sqrt((2*a^3*d*e
- (c^5*d^4 + 2*a*c^4*d^2*e^2 + a^2*c^3*e^4)*sqrt(-(a^5*c^2*d^4 - 2*a^6*c*d^
2*e^2 + a^7*e^4)/(c^11*d^8 + 4*a*c^10*d^6*e^2 + 6*a^2*c^9*d^4*e^4 + 4*a^3*c
^8*d^2*e^6 + a^4*c^7*e^8)))/(c^5*d^4 + 2*a*c^4*d^2*e^2 + a^2*c^3*e^4))) - 1
2*(c*d^3 + a*d*e^2)*x)/(c^2*d^2*e^2 + a*c*e^4), 1/12*(12*c*d^3*sqrt(d/e)*ar
ctan(e*x*sqrt(d/e)/d) + 4*(c*d^2*e + a*e^3)*x^3 - 3*(c^2*d^2*e^2 + a*c*e^4)
*sqrt((2*a^3*d*e + (c^5*d^4 + 2*a*c^4*d^2*e^2 + a^2*c^3*e^4)*sqrt(-(a^5*c^2
*d^4 - 2*a^6*c*d^2*e^2 + a^7*e^4)/(c^11*d^8 + 4*a*c^10*d^6*e^2 + 6*a^2*c^9*
d^4*e^4 + 4*a^3*c^8*d^2*e^6 + a^4*c^7*e^8)))/(c^5*d^4 + 2*a*c^4*d^2*e^2 + a
^2*c^3*e^4))*log(-(a^3*c*d^2 - a^4*e^2)*x + (a^2*c^3*d^3 - a^3*c^2*d*e^2 +
(c^7*d^4*e + 2*a*c^6*d^2*e^3 + a^2*c^5*e^5)*sqrt(-(a^5*c^2*d^4 - 2*a^6*c*d^
2*e^2 + a^7*e^4)/(c^11*d^8 + 4*a*c^10*d^6*e^2 + 6*a^2*c^9*d^4*e^4 + 4*a^3*c
^8*d^2*e^6 + a^4*c^7*e^8)))*sqrt((2*a^3*d*e + (c^5*d^4 + 2*a*c^4*d^2*e^2 +
a^2*c^3*e^4)*sqrt(-(a^5*c^2*d^4 - 2*a^6*c*d^2*e^2 + a^7*e^4)/(c^11*d^8 + 4*
a*c^10*d^6*e^2 + 6*a^2*c^9*d^4*e^4 + 4*a^3*c^8*d^2*e^6 + a^4*c^7*e^8)))/(c^
5*d^4 + 2*a*c^4*d^2*e^2 + a^2*c^3*e^4))) + 3*(c^2*d^2*e^2 + a*c*e^4)*sqrt((
2*a^3*d*e + (c^5*d^4 + 2*a*c^4*d^2*e^2 + a^2*c^3*e^4)*sqrt(-(a^5*c^2*d^4 -
2*a^6*c*d^2*e^2 + a^7*e^4)/(c^11*d^8 + 4*a*c^10*d^6*e^2 + 6*a^2*c^9*d^4*e^4
 + 4*a^3*c^8*d^2*e^6 + a^4*c^7*e^8)))/(c^5*d^4 + 2*a*c^4*d^2*e^2 + a^2*c^3*
e^4))*log(-(a^3*c*d^2 - a^4*e^2)*x - (a^2*c^3*d^3 - a^3*c^2*d*e^2 + (c^7*d^
4*e + 2*a*c^6*d^2*e^3 + a^2*c^5*e^5)*sqrt(-(a^5*c^2*d^4 - 2*a^6*c*d^2*e^2 +
 a^7*e^4)/(c^11*d^8 + 4*a*c^10*d^6*e^2 + 6*a^2*c^9*d^4*e^4 + 4*a^3*c^8*d^2*
e^6 + a^4*c^7*e^8)))*sqrt((2*a^3*d*e + (c^5*d^4 + 2*a*c^4*d^2*e^2 + a^2*c^3
*e^4)*sqrt(-(a^5*c^2*d^4 - 2*a^6*c*d^2*e^2 + a^7*e^4)/(c^11*d^8 + 4*a*c^10*
d^6*e^2 + 6*a^2*c^9*d^4*e^4 + 4*a^3*c^8*d^2*e^6 + a^4*c^7*e^8)))/(c^5*d^4 +
 2*a*c^4*d^2*e^2 + a^2*c^3*e^4))) - 3*(c^2*d^2*e^2 + a*c*e^4)*sqrt((2*a^3*d
*e - (c^5*d^4 + 2*a*c^4*d^2*e^2 + a^2*c^3*e^4)*sqrt(-(a^5*c^2*d^4 - 2*a^6*c
*d^2*e^2 + a^7*e^4)/(c^11*d^8 + 4*a*c^10*d^6*e^2 + 6*a^2*c^9*d^4*e^4 + 4*a^
3*c^8*d^2*e^6 + a^4*c^7*e^8)))/(c^5*d^4 + 2*a*c^4*d^2*e^2 + a^2*c^3*e^4))*l
og(-(a^3*c*d^2 - a^4*e^2)*x + (a^2*c^3*d^3 - a^3*c^2*d*e^2 - (c^7*d^4*e + 2
*a*c^6*d^2*e^3 + a^2*c^5*e^5)*sqrt(-(a^5*c^2*d^4 - 2*a^6*c*d^2*e^2 + a^7*e^
4)/(c^11*d^8 + 4*a*c^10*d^6*e^2 + 6*a^2*c^9*d^4*e^4 + 4*a^3*c^8*d^2*e^6 + a
^4*c^7*e^8)))*sqrt((2*a^3*d*e - (c^5*d^4 + 2*a*c^4*d^2*e^2 + a^2*c^3*e^4)*s
qrt(-(a^5*c^2*d^4 - 2*a^6*c*d^2*e^2 + a^7*e^4)/(c^11*d^8 + 4*a*c^10*d^6*e^2
 + 6*a^2*c^9*d^4*e^4 + 4*a^3*c^8*d^2*e^6 + a^4*c^7*e^8)))/(c^5*d^4 + 2*a*c^
4*d^2*e^2 + a^2*c^3*e^4))) + 3*(c^2*d^2*e^2 + a*c*e^4)*sqrt((2*a^3*d*e - (c
^5*d^4 + 2*a*c^4*d^2*e^2 + a^2*c^3*e^4)*sqrt(-(a^5*c^2*d^4 - 2*a^6*c*d^2*e^
2 + a^7*e^4)/(c^11*d^8 + 4*a*c^10*d^6*e^2 + 6*a^2*c^9*d^4*e^4 + 4*a^3*c^8*d
^2*e^6 + a^4*c^7*e^8)))/(c^5*d^4 + 2*a*c^4*d^2*e^2 + a^2*c^3*e^4))*log(-(a^
3*c*d^2 - a^4*e^2)*x - (a^2*c^3*d^3 - a^3*c^2*d*e^2 - (c^7*d^4*e + 2*a*c^6*
d^2*e^3 + a^2*c^5*e^5)*sqrt(-(a^5*c^2*d^4 - 2*a^6*c*d^2*e^2 + a^7*e^4)/(c^1
1*d^8 + 4*a*c^10*d^6*e^2 + 6*a^2*c^9*d^4*e^4 + 4*a^3*c^8*d^2*e^6 + a^4*c^7*
e^8)))*sqrt((2*a^3*d*e - (c^5*d^4 + 2*a*c^4*d^2*e^2 + a^2*c^3*e^4)*sqrt(-(a

```
^5*c^2*d^4 - 2*a^6*c*d^2*e^2 + a^7*e^4)/(c^11*d^8 + 4*a*c^10*d^6*e^2 + 6*a^
2*c^9*d^4*e^4 + 4*a^3*c^8*d^2*e^6 + a^4*c^7*e^8)))/(c^5*d^4 + 2*a*c^4*d^2*e
^2 + a^2*c^3*e^4))) - 12*(c*d^3 + a*d*e^2)*x)/(c^2*d^2*e^2 + a*c*e^4)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^8}{(d+ex^2)(a+cx^4)}\,dx = \text{Timed out}$$

```
[In] integrate(x**8/(e*x**2+d)/(c*x**4+a),x)
```

```
[Out] Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^8}{(d+ex^2)(a+cx^4)}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(x^8/(e*x^2+d)/(c*x^4+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 370, normalized size of antiderivative = 1.03

$$\int \frac{x^8}{(d+ex^2)(a+cx^4)}\,dx = \frac{d^4\arctan\left(\frac{ex}{\sqrt{de}}\right)}{(cd^2e^2+ae^4)\sqrt{de}} + \frac{\left((ac^3)^{\frac{1}{4}}\,ac^2d-(ac^3)^{\frac{3}{4}}\,ae\right)\arctan\left(\frac{\sqrt{2}\left(2\,x+\sqrt{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{2\left(\sqrt{2}c^5d^2+\sqrt{2}ac^4e^2\right)} + \frac{\left((ac^3)^{\frac{1}{4}}\,ac^2d-(ac^3)^{\frac{3}{4}}\,ae\right)\arctan\left(\frac{\sqrt{2}\left(2\,x-\sqrt{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{2\left(\sqrt{2}c^5d^2+\sqrt{2}ac^4e^2\right)} + \frac{\left((ac^3)^{\frac{1}{4}}\,ac^2d+(ac^3)^{\frac{3}{4}}\,ae\right)\log\left(x^2+\sqrt{2}x\left(\frac{a}{c}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{c}}\right)}{4\left(\sqrt{2}c^5d^2+\sqrt{2}ac^4e^2\right)} - \frac{\left((ac^3)^{\frac{1}{4}}\,ac^2d+(ac^3)^{\frac{3}{4}}\,ae\right)\log\left(x^2-\sqrt{2}x\left(\frac{a}{c}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{c}}\right)}{4\left(\sqrt{2}c^5d^2+\sqrt{2}ac^4e^2\right)} + \frac{c^2e^2x^3-3\,c^2dex}{3\,c^3e^3}$$

```
[In] integrate(x^8/(e*x^2+d)/(c*x^4+a),x, algorithm="giac")
```

```
[Out] d^4*arctan(e*x/sqrt(d*e))/((c*d^2*e^2 + a*e^4)*sqrt(d*e)) + 1/2*((a*c^3)^(1
/4)*a*c^2*d - (a*c^3)^(3/4)*a*e)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(a/c)^(1
/4))/(a/c)^(1/4))/(sqrt(2)*c^5*d^2 + sqrt(2)*a*c^4*e^2) + 1/2*((a*c^3)^(1/4
)*a*c^2*d - (a*c^3)^(3/4)*a*e)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(a/c)^(1/4
))/(a/c)^(1/4))/(sqrt(2)*c^5*d^2 + sqrt(2)*a*c^4*e^2) + 1/4*((a*c^3)^(1/4)*
a*c^2*d + (a*c^3)^(3/4)*a*e)*log(x^2 + sqrt(2)*x*(a/c)^(1/4) + sqrt(a/c))/(
sqrt(2)*c^5*d^2 + sqrt(2)*a*c^4*e^2) - 1/4*((a*c^3)^(1/4)*a*c^2*d + (a*c^3)
^(3/4)*a*e)*log(x^2 - sqrt(2)*x*(a/c)^(1/4) + sqrt(a/c))/(sqrt(2)*c^5*d^2 +
 sqrt(2)*a*c^4*e^2) + 1/3*(c^2*e^2*x^3 - 3*c^2*d*e*x)/(c^3*e^3)
```

Mupad [B] (verification not implemented)

Time = 8.48 (sec) , antiderivative size = 6097, normalized size of antiderivative = 16.98

$$\int \frac{x^8}{(d+ex^2)(a+cx^4)}\,dx = \text{Too large to display}$$

```
[In] int(x^8/((a + c*x^4)*(d + e*x^2)),x)
```

```
[Out] (log(a^7*d^4*e^26 + 16*c^7*d^18*e^12 - 16*c^7*x*(-d^7*e^5)^(5/2) + 2*a^6*c*
d^6*e^24 + 16*a^3*c^4*d^12*e^18 + a^5*c^2*d^8*e^22 - a^7*e^24*x*(-d^7*e^5)^
(1/2) - a^5*c^2*d^4*e^20*x*(-d^7*e^5)^(1/2) + 16*a^3*c^4*d*e^11*x*(-d^7*e^5
)^(3/2) - 2*a^6*c*d^2*e^22*x*(-d^7*e^5)^(1/2))*(-d^7*e^5)^(1/2))/(2*a*e^7 +
 2*c*d^2*e^5) - atan((((((((192*a^3*c^8*d^6*e^5 + 384*a^4*c^7*d^4*e^7 + 192*
a^5*c^6*d^2*e^9)/(c^3*e^3) - (2*x*((c*d^2*(-a^5*c^7)^(1/2) - a*e^2*(-a^5*c^
7)^(1/2) + 2*a^3*c^4*d*e)/(16*(c^9*d^4 + a^2*c^7*e^4 + 2*a*c^8*d^2*e^2)))^(
1/2)*(256*a^5*c^7*e^12 - 256*a^2*c^10*d^6*e^6 - 256*a^3*c^9*d^4*e^8 + 256*a
^4*c^8*d^2*e^10))/(c^3*e^3))*((c*d^2*(-a^5*c^7)^(1/2) - a*e^2*(-a^5*c^7)^(1
/2) + 2*a^3*c^4*d*e)/(16*(c^9*d^4 + a^2*c^7*e^4 + 2*a*c^8*d^2*e^2)))^(1/2)
+ (2*x*(64*a^2*c^8*d^9*e + 56*a^6*c^4*d*e^9 - 8*a^4*c^6*d^5*e^5 - 16*a^5*c^
5*d^3*e^7))/(c^3*e^3))*((c*d^2*(-a^5*c^7)^(1/2) - a*e^2*(-a^5*c^7)^(1/2) +
2*a^3*c^4*d*e)/(16*(c^9*d^4 + a^2*c^7*e^4 + 2*a*c^8*d^2*e^2)))^(1/2) - (16*
a^3*c^6*d^9 + 4*a^7*c^2*d*e^8 - 64*a^4*c^5*d^7*e^2 + 64*a^5*c^4*d^5*e^4 + 4
*a^6*c^3*d^3*e^6)/(c^3*e^3))*((c*d^2*(-a^5*c^7)^(1/2) - a*e^2*(-a^5*c^7)^(1
/2) + 2*a^3*c^4*d*e)/(16*(c^9*d^4 + a^2*c^7*e^4 + 2*a*c^8*d^2*e^2)))^(1/2)
- (2*x*(a^8*e^8 + 2*a^4*c^4*d^8))/(c^3*e^3))*((c*d^2*(-a^5*c^7)^(1/2) - a*e
^2*(-a^5*c^7)^(1/2) + 2*a^3*c^4*d*e)/(16*(c^9*d^4 + a^2*c^7*e^4 + 2*a*c^8*d
^2*e^2)))^(1/2)*1i - (((((192*a^3*c^8*d^6*e^5 + 384*a^4*c^7*d^4*e^7 + 192*a
^5*c^6*d^2*e^9)/(c^3*e^3) + (2*x*((c*d^2*(-a^5*c^7)^(1/2) - a*e^2*(-a^5*c^7
)^(1/2) + 2*a^3*c^4*d*e)/(16*(c^9*d^4 + a^2*c^7*e^4 + 2*a*c^8*d^2*e^2)))^(1
/2)*(256*a^5*c^7*e^12 - 256*a^2*c^10*d^6*e^6 - 256*a^3*c^9*d^4*e^8 + 256*a^
4*c^8*d^2*e^10))/(c^3*e^3))*((c*d^2*(-a^5*c^7)^(1/2) - a*e^2*(-a^5*c^7)^(1/
2) + 2*a^3*c^4*d*e)/(16*(c^9*d^4 + a^2*c^7*e^4 + 2*a*c^8*d^2*e^2)))^(1/2) -
 (2*x*(64*a^2*c^8*d^9*e + 56*a^6*c^4*d*e^9 - 8*a^4*c^6*d^5*e^5 - 16*a^5*c^5
*d^3*e^7))/(c^3*e^3))*((c*d^2*(-a^5*c^7)^(1/2) - a*e^2*(-a^5*c^7)^(1/2) + 2
*a^3*c^4*d*e)/(16*(c^9*d^4 + a^2*c^7*e^4 + 2*a*c^8*d^2*e^2)))^(1/2) - (16*a
^3*c^6*d^9 + 4*a^7*c^2*d*e^8 - 64*a^4*c^5*d^7*e^2 + 64*a^5*c^4*d^5*e^4 + 4*
a^6*c^3*d^3*e^6)/(c^3*e^3))*((c*d^2*(-a^5*c^7)^(1/2) - a*e^2*(-a^5*c^7)^(1/
2) + 2*a^3*c^4*d*e)/(16*(c^9*d^4 + a^2*c^7*e^4 + 2*a*c^8*d^2*e^2)))^(1/2) +
 (2*x*(a^8*e^8 + 2*a^4*c^4*d^8))/(c^3*e^3))*((c*d^2*(-a^5*c^7)^(1/2) - a*e^
2*(-a^5*c^7)^(1/2) + 2*a^3*c^4*d*e)/(16*(c^9*d^4 + a^2*c^7*e^4 + 2*a*c^8*d^
2*e^2)))^(1/2)*1i)/((((((192*a^3*c^8*d^6*e^5 + 384*a^4*c^7*d^4*e^7 + 192*a^
5*c^6*d^2*e^9)/(c^3*e^3) - (2*x*((c*d^2*(-a^5*c^7)^(1/2) - a*e^2*(-a^5*c^7)
^(1/2) + 2*a^3*c^4*d*e)/(16*(c^9*d^4 + a^2*c^7*e^4 + 2*a*c^8*d^2*e^2)))^(1/
2)*(256*a^5*c^7*e^12 - 256*a^2*c^10*d^6*e^6 - 256*a^3*c^9*d^4*e^8 + 256*a^4
```

*c^8*d^2*e^10))/(c^3*e^3))*((c*d^2*(-a^5*c^7)^(1/2) - a*e^2*(-a^5*c^7)^(1/2
) + 2*a^3*c^4*d*e)/(16*(c^9*d^4 + a^2*c^7*e^4 + 2*a*c^8*d^2*e^2)))^(1/2) +
(2*x*(64*a^2*c^8*d^9*e + 56*a^6*c^4*d*e^9 - 8*a^4*c^6*d^5*e^5 - 16*a^5*c^5*
d^3*e^7))/(c^3*e^3))*((c*d^2*(-a^5*c^7)^(1/2) - a*e^2*(-a^5*c^7)^(1/2) + 2*
a^3*c^4*d*e)/(16*(c^9*d^4 + a^2*c^7*e^4 + 2*a*c^8*d^2*e^2)))^(1/2) - (16*a^
3*c^6*d^9 + 4*a^7*c^2*d*e^8 - 64*a^4*c^5*d^7*e^2 + 64*a^5*c^4*d^5*e^4 + 4*a
^6*c^3*d^3*e^6)/(c^3*e^3))*((c*d^2*(-a^5*c^7)^(1/2) - a*e^2*(-a^5*c^7)^(1/2
) + 2*a^3*c^4*d*e)/(16*(c^9*d^4 + a^2*c^7*e^4 + 2*a*c^8*d^2*e^2)))^(1/2) -
(2*x*(a^8*e^8 + 2*a^4*c^4*d^8))/(c^3*e^3))*((c*d^2*(-a^5*c^7)^(1/2) - a*e^2
*(-a^5*c^7)^(1/2) + 2*a^3*c^4*d*e)/(16*(c^9*d^4 + a^2*c^7*e^4 + 2*a*c^8*d^2
*e^2)))^(1/2) + (((((192*a^3*c^8*d^6*e^5 + 384*a^4*c^7*d^4*e^7 + 192*a^5*c^
6*d^2*e^9)/(c^3*e^3) + (2*x*((c*d^2*(-a^5*c^7)^(1/2) - a*e^2*(-a^5*c^7)^(1/
2) + 2*a^3*c^4*d*e)/(16*(c^9*d^4 + a^2*c^7*e^4 + 2*a*c^8*d^2*e^2)))^(1/2)*(
256*a^5*c^7*e^12 - 256*a^2*c^10*d^6*e^6 - 256*a^3*c^9*d^4*e^8 + 256*a^4*c^8
*d^2*e^10))/(c^3*e^3))*((c*d^2*(-a^5*c^7)^(1/2) - a*e^2*(-a^5*c^7)^(1/2) +
2*a^3*c^4*d*e)/(16*(c^9*d^4 + a^2*c^7*e^4 + 2*a*c^8*d^2*e^2)))^(1/2) - (2*x
*(64*a^2*c^8*d^9*e + 56*a^6*c^4*d*e^9 - 8*a^4*c^6*d^5*e^5 - 16*a^5*c^5*d^3*
e^7))/(c^3*e^3))*((c*d^2*(-a^5*c^7)^(1/2) - a*e^2*(-a^5*c^7)^(1/2) + 2*a^3*
c^4*d*e)/(16*(c^9*d^4 + a^2*c^7*e^4 + 2*a*c^8*d^2*e^2)))^(1/2) - (16*a^3*c^
6*d^9 + 4*a^7*c^2*d*e^8 - 64*a^4*c^5*d^7*e^2 + 64*a^5*c^4*d^5*e^4 + 4*a^6*c
^3*d^3*e^6)/(c^3*e^3))*((c*d^2*(-a^5*c^7)^(1/2) - a*e^2*(-a^5*c^7)^(1/2) +
2*a^3*c^4*d*e)/(16*(c^9*d^4 + a^2*c^7*e^4 + 2*a*c^8*d^2*e^2)))^(1/2) + (2*x
*(a^8*e^8 + 2*a^4*c^4*d^8))/(c^3*e^3))*((c*d^2*(-a^5*c^7)^(1/2) - a*e^2*(-a
^5*c^7)^(1/2) + 2*a^3*c^4*d*e)/(16*(c^9*d^4 + a^2*c^7*e^4 + 2*a*c^8*d^2*e^2
)))^(1/2) + (2*(a^7*d^4*e^3 - a^6*c*d^6*e))/(c^3*e^3)))*((c*d^2*(-a^5*c^7)^
(1/2) - a*e^2*(-a^5*c^7)^(1/2) + 2*a^3*c^4*d*e)/(16*(c^9*d^4 + a^2*c^7*e^4
+ 2*a*c^8*d^2*e^2)))^(1/2)*2i - (log(a^7*d^4*e^26 + 16*c^7*d^18*e^12 + 16*c
^7*x*(-d^7*e^5)^(5/2) + 2*a^6*c*d^6*e^24 + 16*a^3*c^4*d^12*e^18 + a^5*c^2*d
^8*e^22 + a^7*e^24*x*(-d^7*e^5)^(1/2) + a^5*c^2*d^4*e^20*x*(-d^7*e^5)^(1/2)
 - 16*a^3*c^4*d*e^11*x*(-d^7*e^5)^(3/2) + 2*a^6*c*d^2*e^22*x*(-d^7*e^5)^(1/
2))*(-d^7*e^5)^(1/2))/(2*(a*e^7 + c*d^2*e^5)) - atan((((((192*a^3*c^8*d^6*
e^5 + 384*a^4*c^7*d^4*e^7 + 192*a^5*c^6*d^2*e^9)/(c^3*e^3) - (2*x*((a*e^2*(
-a^5*c^7)^(1/2) - c*d^2*(-a^5*c^7)^(1/2) + 2*a^3*c^4*d*e)/(16*(c^9*d^4 + a^
2*c^7*e^4 + 2*a*c^8*d^2*e^2)))^(1/2)*(256*a^5*c^7*e^12 - 256*a^2*c^10*d^6*e
^6 - 256*a^3*c^9*d^4*e^8 + 256*a^4*c^8*d^2*e^10))/(c^3*e^3))*((a*e^2*(-a^5*
c^7)^(1/2) - c*d^2*(-a^5*c^7)^(1/2) + 2*a^3*c^4*d*e)/(16*(c^9*d^4 + a^2*c^7
*e^4 + 2*a*c^8*d^2*e^2)))^(1/2) + (2*x*(64*a^2*c^8*d^9*e + 56*a^6*c^4*d*e^9
 - 8*a^4*c^6*d^5*e^5 - 16*a^5*c^5*d^3*e^7))/(c^3*e^3))*((a*e^2*(-a^5*c^7)^(
1/2) - c*d^2*(-a^5*c^7)^(1/2) + 2*a^3*c^4*d*e)/(16*(c^9*d^4 + a^2*c^7*e^4 +
 2*a*c^8*d^2*e^2)))^(1/2) - (16*a^3*c^6*d^9 + 4*a^7*c^2*d*e^8 - 64*a^4*c^5*
d^7*e^2 + 64*a^5*c^4*d^5*e^4 + 4*a^6*c^3*d^3*e^6)/(c^3*e^3))*((a*e^2*(-a^5*
c^7)^(1/2) - c*d^2*(-a^5*c^7)^(1/2) + 2*a^3*c^4*d*e)/(16*(c^9*d^4 + a^2*c^7
*e^4 + 2*a*c^8*d^2*e^2)))^(1/2) - (2*x*(a^8*e^8 + 2*a^4*c^4*d^8))/(c^3*e^3)
)*((a*e^2*(-a^5*c^7)^(1/2) - c*d^2*(-a^5*c^7)^(1/2) + 2*a^3*c^4*d*e)/(16*(c
^9*d^4 + a^2*c^7*e^4 + 2*a*c^8*d^2*e^2)))^(1/2)*1i - (((((192*a^3*c^8*d^6*e

```
^5 + 384*a^4*c^7*d^4*e^7 + 192*a^5*c^6*d^2*e^9)/(c^3*e^3) + (2*x*((a*e^2*(-
a^5*c^7)^(1/2) - c*d^2*(-a^5*c^7)^(1/2) + 2*a^3*c^4*d*e)/(16*(c^9*d^4 + a^2
*c^7*e^4 + 2*a*c^8*d^2*e^2)))^(1/2)*(256*a^5*c^7*e^12 - 256*a^2*c^10*d^6*e^
6 - 256*a^3*c^9*d^4*e^8 + 256*a^4*c^8*d^2*e^10))/(c^3*e^3))*((a*e^2*(-a^5*c
^7)^(1/2) - c*d^2*(-a^5*c^7)^(1/2) + 2*a^3*c^4*d*e)/(16*(c^9*d^4 + a^2*c^7*
e^4 + 2*a*c^8*d^2*e^2)))^(1/2) - (2*x*(64*a^2*c^8*d^9*e + 56*a^6*c^4*d*e^9
- 8*a^4*c^6*d^5*e^5 - 16*a^5*c^5*d^3*e^7))/(c^3*e^3))*((a*e^2*(-a^5*c^7)^(1
/2) - c*d^2*(-a^5*c^7)^(1/2) + 2*a^3*c^4*d*e)/(16*(c^9*d^4 + a^2*c^7*e^4 +
2*a*c^8*d^2*e^2)))^(1/2) - (16*a^3*c^6*d^9 + 4*a^7*c^2*d*e^8 - 64*a^4*c^5*d
^7*e^2 + 64*a^5*c^4*d^5*e^4 + 4*a^6*c^3*d^3*e^6)/(c^3*e^3))*((a*e^2*(-a^5*c
^7)^(1/2) - c*d^2*(-a^5*c^7)^(1/2) + 2*a^3*c^4*d*e)/(16*(c^9*d^4 + a^2*c^7*
e^4 + 2*a*c^8*d^2*e^2)))^(1/2) + (2*x*(a^8*e^8 + 2*a^4*c^4*d^8))/(c^3*e^3))
*((a*e^2*(-a^5*c^7)^(1/2) - c*d^2*(-a^5*c^7)^(1/2) + 2*a^3*c^4*d*e)/(16*(c^
9*d^4 + a^2*c^7*e^4 + 2*a*c^8*d^2*e^2)))^(1/2)*1i)/((((((192*a^3*c^8*d^6*e^
5 + 384*a^4*c^7*d^4*e^7 + 192*a^5*c^6*d^2*e^9)/(c^3*e^3) - (2*x*((a*e^2*(-a
^5*c^7)^(1/2) - c*d^2*(-a^5*c^7)^(1/2) + 2*a^3*c^4*d*e)/(16*(c^9*d^4 + a^2*
c^7*e^4 + 2*a*c^8*d^2*e^2)))^(1/2)*(256*a^5*c^7*e^12 - 256*a^2*c^10*d^6*e^6
 - 256*a^3*c^9*d^4*e^8 + 256*a^4*c^8*d^2*e^10))/(c^3*e^3))*((a*e^2*(-a^5*c^
7)^(1/2) - c*d^2*(-a^5*c^7)^(1/2) + 2*a^3*c^4*d*e)/(16*(c^9*d^4 + a^2*c^7*e
^4 + 2*a*c^8*d^2*e^2)))^(1/2) + (2*x*(64*a^2*c^8*d^9*e + 56*a^6*c^4*d*e^9 -
 8*a^4*c^6*d^5*e^5 - 16*a^5*c^5*d^3*e^7))/(c^3*e^3))*((a*e^2*(-a^5*c^7)^(1/
2) - c*d^2*(-a^5*c^7)^(1/2) + 2*a^3*c^4*d*e)/(16*(c^9*d^4 + a^2*c^7*e^4 + 2
*a*c^8*d^2*e^2)))^(1/2) - (16*a^3*c^6*d^9 + 4*a^7*c^2*d*e^8 - 64*a^4*c^5*d^
7*e^2 + 64*a^5*c^4*d^5*e^4 + 4*a^6*c^3*d^3*e^6)/(c^3*e^3))*((a*e^2*(-a^5*c^
7)^(1/2) - c*d^2*(-a^5*c^7)^(1/2) + 2*a^3*c^4*d*e)/(16*(c^9*d^4 + a^2*c^7*e
^4 + 2*a*c^8*d^2*e^2)))^(1/2) - (2*x*(a^8*e^8 + 2*a^4*c^4*d^8))/(c^3*e^3))*
((a*e^2*(-a^5*c^7)^(1/2) - c*d^2*(-a^5*c^7)^(1/2) + 2*a^3*c^4*d*e)/(16*(c^9
*d^4 + a^2*c^7*e^4 + 2*a*c^8*d^2*e^2)))^(1/2) + (((((192*a^3*c^8*d^6*e^5 +
384*a^4*c^7*d^4*e^7 + 192*a^5*c^6*d^2*e^9)/(c^3*e^3) + (2*x*((a*e^2*(-a^5*c
^7)^(1/2) - c*d^2*(-a^5*c^7)^(1/2) + 2*a^3*c^4*d*e)/(16*(c^9*d^4 + a^2*c^7*
e^4 + 2*a*c^8*d^2*e^2)))^(1/2)*(256*a^5*c^7*e^12 - 256*a^2*c^10*d^6*e^6 - 2
56*a^3*c^9*d^4*e^8 + 256*a^4*c^8*d^2*e^10))/(c^3*e^3))*((a*e^2*(-a^5*c^7)^(
1/2) - c*d^2*(-a^5*c^7)^(1/2) + 2*a^3*c^4*d*e)/(16*(c^9*d^4 + a^2*c^7*e^4 +
 2*a*c^8*d^2*e^2)))^(1/2) - (2*x*(64*a^2*c^8*d^9*e + 56*a^6*c^4*d*e^9 - 8*a
^4*c^6*d^5*e^5 - 16*a^5*c^5*d^3*e^7))/(c^3*e^3))*((a*e^2*(-a^5*c^7)^(1/2) -
 c*d^2*(-a^5*c^7)^(1/2) + 2*a^3*c^4*d*e)/(16*(c^9*d^4 + a^2*c^7*e^4 + 2*a*c
^8*d^2*e^2)))^(1/2) - (16*a^3*c^6*d^9 + 4*a^7*c^2*d*e^8 - 64*a^4*c^5*d^7*e^
2 + 64*a^5*c^4*d^5*e^4 + 4*a^6*c^3*d^3*e^6)/(c^3*e^3))*((a*e^2*(-a^5*c^7)^(
1/2) - c*d^2*(-a^5*c^7)^(1/2) + 2*a^3*c^4*d*e)/(16*(c^9*d^4 + a^2*c^7*e^4 +
 2*a*c^8*d^2*e^2)))^(1/2) + (2*x*(a^8*e^8 + 2*a^4*c^4*d^8))/(c^3*e^3))*((a*
e^2*(-a^5*c^7)^(1/2) - c*d^2*(-a^5*c^7)^(1/2) + 2*a^3*c^4*d*e)/(16*(c^9*d^4
 + a^2*c^7*e^4 + 2*a*c^8*d^2*e^2)))^(1/2) + (2*(a^7*d^4*e^3 - a^6*c*d^6*e))
/(c^3*e^3)))*((a*e^2*(-a^5*c^7)^(1/2) - c*d^2*(-a^5*c^7)^(1/2) + 2*a^3*c^4*
d*e)/(16*(c^9*d^4 + a^2*c^7*e^4 + 2*a*c^8*d^2*e^2)))^(1/2)*2i + x^3/(3*c*e)
 - (d*x)/(c*e^2)
```

3.238 $\int \frac{x^6}{(d+ex^2)(a+cx^4)}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 345

$$\int \frac{x^6}{(d+ex^2)(a+cx^4)}\,dx = \frac{x}{ce} - \frac{d^{5/2}\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{e^{3/2}(cd^2+ae^2)} + \frac{a^{3/4}\left(\sqrt{c}d+\sqrt{a}e\right)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}c^{5/4}(cd^2+ae^2)} - \frac{a^{3/4}\left(\sqrt{c}d+\sqrt{a}e\right)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}c^{5/4}(cd^2+ae^2)} - \frac{a^{3/4}\left(\sqrt{c}d-\sqrt{a}e\right)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)}{4\sqrt{2}c^{5/4}(cd^2+ae^2)} + \frac{a^{3/4}\left(\sqrt{c}d-\sqrt{a}e\right)\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)}{4\sqrt{2}c^{5/4}(cd^2+ae^2)}$$

```
[Out] x/c/e-d^(5/2)*arctan(x*e^(1/2)/d^(1/2))/e^(3/2)/(a*e^2+c*d^2)-1/8*a^(3/4)*l
n(-a^(1/4)*c^(1/4)*x*2^(1/2)+a^(1/2)+x^2*c^(1/2))*(-e*a^(1/2)+d*c^(1/2))/c^
(5/4)/(a*e^2+c*d^2)*2^(1/2)+1/8*a^(3/4)*ln(a^(1/4)*c^(1/4)*x*2^(1/2)+a^(1/2
)+x^2*c^(1/2))*(-e*a^(1/2)+d*c^(1/2))/c^(5/4)/(a*e^2+c*d^2)*2^(1/2)-1/4*a^(
3/4)*arctan(-1+c^(1/4)*x*2^(1/2)/a^(1/4))*(e*a^(1/2)+d*c^(1/2))/c^(5/4)/(a*
e^2+c*d^2)*2^(1/2)-1/4*a^(3/4)*arctan(1+c^(1/4)*x*2^(1/2)/a^(1/4))*(e*a^(1/
2)+d*c^(1/2))/c^(5/4)/(a*e^2+c*d^2)*2^(1/2)
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 345, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {1302, 211, 1182, 1176, 631, 210, 1179, 642}

$$\int \frac{x^6}{(d+ex^2)(a+cx^4)}\,dx = \frac{a^{3/4}\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)(\sqrt{a}e+\sqrt{c}d)}{2\sqrt{2}c^{5/4}(ae^2+cd^2)} - \frac{a^{3/4}\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)(\sqrt{a}e+\sqrt{c}d)}{2\sqrt{2}c^{5/4}(ae^2+cd^2)} - \frac{a^{3/4}(\sqrt{c}d-\sqrt{a}e)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{4\sqrt{2}c^{5/4}(ae^2+cd^2)} + \frac{a^{3/4}(\sqrt{c}d-\sqrt{a}e)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{4\sqrt{2}c^{5/4}(ae^2+cd^2)} - \frac{d^{5/2}\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{e^{3/2}(ae^2+cd^2)} + \frac{x}{ce}$$

```
[In] Int[x^6/((d + e*x^2)*(a + c*x^4)),x]
```

```
[Out] x/(c*e) - (d^(5/2)*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(e^(3/2)*(c*d^2 + a*e^2)) +
 (a^(3/4)*(Sqrt[c]*d + Sqrt[a]*e)*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/
(2*Sqrt[2]*c^(5/4)*(c*d^2 + a*e^2)) - (a^(3/4)*(Sqrt[c]*d + Sqrt[a]*e)*ArcT
an[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*c^(5/4)*(c*d^2 + a*e^2)) -
(a^(3/4)*(Sqrt[c]*d - Sqrt[a]*e)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x +
Sqrt[c]*x^2])/(4*Sqrt[2]*c^(5/4)*(c*d^2 + a*e^2)) + (a^(3/4)*(Sqrt[c]*d - S
qrt[a]*e)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(4*Sqrt[2
]*c^(5/4)*(c*d^2 + a*e^2))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
```

```
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1182

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[(-a)
*c]
```

Rule 1302

```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.))/((a_) + (c_.)*(x_)^4),
 x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*((d + e*x^2)^q/(a + c*x^4)), x],
x] /; FreeQ[{a, c, d, e, f, m}, x] && IntegerQ[q] && IntegerQ[m]
```

Rubi steps

$$\text{integral} = \int \left(\frac{1}{ce} - \frac{d^3}{e\left(cd^2+ae^2\right)\left(d+ex^2\right)} - \frac{a\left(ae+cdx^2\right)}{c\left(cd^2+ae^2\right)\left(a+cx^4\right)}\right) dx$$

$$= \frac{x}{ce} - \frac{a\int \frac{ae+cdx^2}{a+cx^4}\,dx}{c\left(cd^2+ae^2\right)} - \frac{d^3\int \frac{1}{d+ex^2}\,dx}{e\left(cd^2+ae^2\right)}$$

$$=\frac{x}{ce}-\frac{d^{5/2}\tan^{-1}\left(\frac{\sqrt{ex}}{\sqrt{d}}\right)}{e^{3/2}\left(cd^2+ae^2\right)}+\frac{\left(a\left(d-\frac{\sqrt{a}e}{\sqrt{c}}\right)\right)\int\frac{\sqrt{a}\sqrt{c}-cx^2}{a+cx^4}\,dx}{2c\left(cd^2+ae^2\right)}-\frac{\left(a\left(d+\frac{\sqrt{a}e}{\sqrt{c}}\right)\right)\int\frac{\sqrt{a}\sqrt{c}+cx^2}{a+cx^4}\,dx}{2c\left(cd^2+ae^2\right)}$$

$$=\frac{x}{ce}-\frac{d^{5/2}\tan^{-1}\left(\frac{\sqrt{ex}}{\sqrt{d}}\right)}{e^{3/2}\left(cd^2+ae^2\right)}-\frac{\left(a^{3/4}\left(d-\frac{\sqrt{a}e}{\sqrt{c}}\right)\right)\int\frac{\frac{\sqrt{2}\sqrt[4]{a}}{\sqrt[4]{c}}+2x}{-\frac{\sqrt{a}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}-x^2}\,dx}{4\sqrt{2}c^{3/4}\left(cd^2+ae^2\right)}$$

$$-\frac{\left(a^{3/4}\left(d-\frac{\sqrt{a}e}{\sqrt{c}}\right)\right)\int\frac{\frac{\sqrt{2}\sqrt[4]{a}}{\sqrt[4]{c}}-2x}{-\frac{\sqrt{a}}{\sqrt{c}}+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}-x^2}\,dx}{4\sqrt{2}c^{3/4}\left(cd^2+ae^2\right)}$$

$$-\frac{\left(a\left(d+\frac{\sqrt{a}e}{\sqrt{c}}\right)\right)\int\frac{1}{\frac{\sqrt{a}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}+x^2}\,dx}{4c\left(cd^2+ae^2\right)}-\frac{\left(a\left(d+\frac{\sqrt{a}e}{\sqrt{c}}\right)\right)\int\frac{1}{\frac{\sqrt{a}}{\sqrt{c}}+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}+x^2}\,dx}{4c\left(cd^2+ae^2\right)}$$

$$=\frac{x}{ce}-\frac{d^{5/2}\tan^{-1}\left(\frac{\sqrt{ex}}{\sqrt{d}}\right)}{e^{3/2}\left(cd^2+ae^2\right)}-\frac{a^{3/4}\left(d-\frac{\sqrt{a}e}{\sqrt{c}}\right)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)}{4\sqrt{2}c^{3/4}\left(cd^2+ae^2\right)}$$

$$+\frac{a^{3/4}\left(d-\frac{\sqrt{a}e}{\sqrt{c}}\right)\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)}{4\sqrt{2}c^{3/4}\left(cd^2+ae^2\right)}$$

$$-\frac{\left(a^{3/4}\left(\sqrt{c}d+\sqrt{a}e\right)\right)\text{Subst}\left(\int\frac{1}{-1-x^2}\,dx,x,1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}c^{5/4}\left(cd^2+ae^2\right)}$$

$$+\frac{\left(a^{3/4}\left(\sqrt{c}d+\sqrt{a}e\right)\right)\text{Subst}\left(\int\frac{1}{-1-x^2}\,dx,x,1+\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}c^{5/4}\left(cd^2+ae^2\right)}$$

$$=\frac{x}{ce}-\frac{d^{5/2}\tan^{-1}\left(\frac{\sqrt{ex}}{\sqrt{d}}\right)}{e^{3/2}\left(cd^2+ae^2\right)}+\frac{a^{3/4}\left(\sqrt{c}d+\sqrt{a}e\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}c^{5/4}\left(cd^2+ae^2\right)}$$

$$-\frac{a^{3/4}\left(\sqrt{c}d+\sqrt{a}e\right)\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}c^{5/4}\left(cd^2+ae^2\right)}$$

$$-\frac{a^{3/4}\left(d-\frac{\sqrt{a}e}{\sqrt{c}}\right)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)}{4\sqrt{2}c^{3/4}\left(cd^2+ae^2\right)}$$

$$+\frac{a^{3/4}\left(d-\frac{\sqrt{a}e}{\sqrt{c}}\right)\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)}{4\sqrt{2}c^{3/4}\left(cd^2+ae^2\right)}$$

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 373, normalized size of antiderivative = 1.08

$$\int \frac{x^6}{(d+ex^2)(a+cx^4)}\,dx = \frac{x}{ce} - \frac{d^{5/2}\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{e^{3/2}(cd^2+ae^2)} - \frac{\left(a^{3/4}cd+a^{5/4}\sqrt{c}e\right)\arctan\left(\frac{-\sqrt{2}\sqrt[4]{a}+2\sqrt[4]{c}x}{\sqrt{2}\sqrt[4]{a}}\right)}{2\sqrt{2}c^{7/4}(cd^2+ae^2)} - \frac{\left(a^{3/4}cd+a^{5/4}\sqrt{c}e\right)\arctan\left(\frac{\sqrt{2}\sqrt[4]{a}+2\sqrt[4]{c}x}{\sqrt{2}\sqrt[4]{a}}\right)}{2\sqrt{2}c^{7/4}(cd^2+ae^2)} - \frac{\left(a^{3/4}cd-a^{5/4}\sqrt{c}e\right)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)}{4\sqrt{2}c^{7/4}(cd^2+ae^2)} + \frac{\left(a^{3/4}cd-a^{5/4}\sqrt{c}e\right)\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)}{4\sqrt{2}c^{7/4}(cd^2+ae^2)}$$

```
[In] Integrate[x^6/((d + e*x^2)*(a + c*x^4)),x]
```

```
[Out] x/(c*e) - (d^(5/2)*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(e^(3/2)*(c*d^2 + a*e^2)) -
 ((a^(3/4)*c*d + a^(5/4)*Sqrt[c]*e)*ArcTan[(-(Sqrt[2]*a^(1/4)) + 2*c^(1/4)*
x)/(Sqrt[2]*a^(1/4))])/(2*Sqrt[2]*c^(7/4)*(c*d^2 + a*e^2)) - ((a^(3/4)*c*d
+ a^(5/4)*Sqrt[c]*e)*ArcTan[(Sqrt[2]*a^(1/4) + 2*c^(1/4)*x)/(Sqrt[2]*a^(1/4
))])/(2*Sqrt[2]*c^(7/4)*(c*d^2 + a*e^2)) - ((a^(3/4)*c*d - a^(5/4)*Sqrt[c]*
e)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(4*Sqrt[2]*c^(7/
4)*(c*d^2 + a*e^2)) + ((a^(3/4)*c*d - a^(5/4)*Sqrt[c]*e)*Log[Sqrt[a] + Sqrt
[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(4*Sqrt[2]*c^(7/4)*(c*d^2 + a*e^2))
```

Maple [A] (verified)

Time = 0.41 (sec) , antiderivative size = 263, normalized size of antiderivative = 0.76

method	result
default	$\frac{x}{ce} - \frac{a\left(\frac{e\left(\frac{a}{c}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x^2+\left(\frac{a}{c}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{c}}}{x^2-\left(\frac{a}{c}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{c}}}\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{c}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{c}\right)^{\frac{1}{4}}}-1\right)\right)}{8}+\frac{d\sqrt{2}\left(\ln\left(\frac{x^2-\left(\frac{a}{c}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{c}}}{x^2+\left(\frac{a}{c}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{c}}}\right)+2\arctan}{8\left(\frac{a}{c}\right)}\right.}{(a e^2+c d^2)c}$
risch	$\frac{x}{ce} + \frac{\sqrt{-ed}\,d^2\ln\left(\left(-16(-ed)^{\frac{5}{2}}a c^5 d^8 e^2+16(-ed)^{\frac{5}{2}}c^6 d^{10}-14a^3c^3d^5e^7(-ed)^{\frac{3}{2}}+4a^2c^4d^7e^5(-ed)^{\frac{3}{2}}+2a c^5 d^9(-ed)^{\frac{3}{2}}e^3+16c^6d^{11}(-\right.\right.}{}$

```
[In] int(x^6/(e*x^2+d)/(c*x^4+a),x,method=_RETURNVERBOSE)
```

```
[Out] x/c/e-a/(a*e^2+c*d^2)/c*(1/8*e*(a/c)^(1/4)*2^(1/2)*(ln((x^2+(a/c)^(1/4)*x*2
^(1/2)+(a/c)^(1/2))/(x^2-(a/c)^(1/4)*x*2^(1/2)+(a/c)^(1/2)))+2*arctan(2^(1/
2)/(a/c)^(1/4)*x+1)+2*arctan(2^(1/2)/(a/c)^(1/4)*x-1))+1/8*d/(a/c)^(1/4)*2^
(1/2)*(ln((x^2-(a/c)^(1/4)*x*2^(1/2)+(a/c)^(1/2))/(x^2+(a/c)^(1/4)*x*2^(1/2
)+(a/c)^(1/2)))+2*arctan(2^(1/2)/(a/c)^(1/4)*x+1)+2*arctan(2^(1/2)/(a/c)^(1
/4)*x-1)))-1/e*d^3/(a*e^2+c*d^2)/(e*d)^(1/2)*arctan(e*x/(e*d)^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2167 vs. $2(256) = 512$.

Time = 1.08 (sec) , antiderivative size = 4354, normalized size of antiderivative = 12.62

$$\int \frac{x^6}{(d+ex^2)(a+cx^4)}\,dx = \text{Too large to display}$$

```
[In] integrate(x^6/(e*x^2+d)/(c*x^4+a),x, algorithm="fricas")
```

```
[Out] [1/4*(2*c*d^2*sqrt(-d/e)*log((e*x^2 - 2*e*x*sqrt(-d/e) - d)/(e*x^2 + d)) +
(c^2*d^2*e + a*c*e^3)*sqrt(-(2*a^2*d*e + (c^4*d^4 + 2*a*c^3*d^2*e^2 + a^2*c
^2*e^4)*sqrt(-(a^3*c^2*d^4 - 2*a^4*c*d^2*e^2 + a^5*e^4)/(c^9*d^8 + 4*a*c^8*
d^6*e^2 + 6*a^2*c^7*d^4*e^4 + 4*a^3*c^6*d^2*e^6 + a^4*c^5*e^8)))/(c^4*d^4 +
 2*a*c^3*d^2*e^2 + a^2*c^2*e^4))*log(-(a^2*c*d^2 - a^3*e^2)*x + (a^2*c^2*d^
2*e - a^3*c*e^3 - (c^6*d^5 + 2*a*c^5*d^3*e^2 + a^2*c^4*d*e^4)*sqrt(-(a^3*c^
2*d^4 - 2*a^4*c*d^2*e^2 + a^5*e^4)/(c^9*d^8 + 4*a*c^8*d^6*e^2 + 6*a^2*c^7*d
^4*e^4 + 4*a^3*c^6*d^2*e^6 + a^4*c^5*e^8)))*sqrt(-(2*a^2*d*e + (c^4*d^4 + 2
*a*c^3*d^2*e^2 + a^2*c^2*e^4)*sqrt(-(a^3*c^2*d^4 - 2*a^4*c*d^2*e^2 + a^5*e^
4)/(c^9*d^8 + 4*a*c^8*d^6*e^2 + 6*a^2*c^7*d^4*e^4 + 4*a^3*c^6*d^2*e^6 + a^4
*c^5*e^8)))/(c^4*d^4 + 2*a*c^3*d^2*e^2 + a^2*c^2*e^4))) - (c^2*d^2*e + a*c*
e^3)*sqrt(-(2*a^2*d*e + (c^4*d^4 + 2*a*c^3*d^2*e^2 + a^2*c^2*e^4)*sqrt(-(a^
3*c^2*d^4 - 2*a^4*c*d^2*e^2 + a^5*e^4)/(c^9*d^8 + 4*a*c^8*d^6*e^2 + 6*a^2*c
^7*d^4*e^4 + 4*a^3*c^6*d^2*e^6 + a^4*c^5*e^8)))/(c^4*d^4 + 2*a*c^3*d^2*e^2
+ a^2*c^2*e^4))*log(-(a^2*c*d^2 - a^3*e^2)*x - (a^2*c^2*d^2*e - a^3*c*e^3 -
 (c^6*d^5 + 2*a*c^5*d^3*e^2 + a^2*c^4*d*e^4)*sqrt(-(a^3*c^2*d^4 - 2*a^4*c*d
^2*e^2 + a^5*e^4)/(c^9*d^8 + 4*a*c^8*d^6*e^2 + 6*a^2*c^7*d^4*e^4 + 4*a^3*c^
6*d^2*e^6 + a^4*c^5*e^8)))*sqrt(-(2*a^2*d*e + (c^4*d^4 + 2*a*c^3*d^2*e^2 +
a^2*c^2*e^4)*sqrt(-(a^3*c^2*d^4 - 2*a^4*c*d^2*e^2 + a^5*e^4)/(c^9*d^8 + 4*a
*c^8*d^6*e^2 + 6*a^2*c^7*d^4*e^4 + 4*a^3*c^6*d^2*e^6 + a^4*c^5*e^8)))/(c^4*
d^4 + 2*a*c^3*d^2*e^2 + a^2*c^2*e^4))) + (c^2*d^2*e + a*c*e^3)*sqrt(-(2*a^2
*d*e - (c^4*d^4 + 2*a*c^3*d^2*e^2 + a^2*c^2*e^4)*sqrt(-(a^3*c^2*d^4 - 2*a^4
*c*d^2*e^2 + a^5*e^4)/(c^9*d^8 + 4*a*c^8*d^6*e^2 + 6*a^2*c^7*d^4*e^4 + 4*a^
3*c^6*d^2*e^6 + a^4*c^5*e^8)))/(c^4*d^4 + 2*a*c^3*d^2*e^2 + a^2*c^2*e^4))*l
og(-(a^2*c*d^2 - a^3*e^2)*x + (a^2*c^2*d^2*e - a^3*c*e^3 + (c^6*d^5 + 2*a*c
^5*d^3*e^2 + a^2*c^4*d*e^4)*sqrt(-(a^3*c^2*d^4 - 2*a^4*c*d^2*e^2 + a^5*e^4)
```

/(c^9*d^8 + 4*a*c^8*d^6*e^2 + 6*a^2*c^7*d^4*e^4 + 4*a^3*c^6*d^2*e^6 + a^4*c
^5*e^8)))*sqrt(-(2*a^2*d*e - (c^4*d^4 + 2*a*c^3*d^2*e^2 + a^2*c^2*e^4)*sqrt
(-(a^3*c^2*d^4 - 2*a^4*c*d^2*e^2 + a^5*e^4)/(c^9*d^8 + 4*a*c^8*d^6*e^2 + 6*
a^2*c^7*d^4*e^4 + 4*a^3*c^6*d^2*e^6 + a^4*c^5*e^8)))/(c^4*d^4 + 2*a*c^3*d^2
*e^2 + a^2*c^2*e^4))) - (c^2*d^2*e + a*c*e^3)*sqrt(-(2*a^2*d*e - (c^4*d^4 +
 2*a*c^3*d^2*e^2 + a^2*c^2*e^4)*sqrt(-(a^3*c^2*d^4 - 2*a^4*c*d^2*e^2 + a^5*
e^4)/(c^9*d^8 + 4*a*c^8*d^6*e^2 + 6*a^2*c^7*d^4*e^4 + 4*a^3*c^6*d^2*e^6 + a
^4*c^5*e^8)))/(c^4*d^4 + 2*a*c^3*d^2*e^2 + a^2*c^2*e^4))*log(-(a^2*c*d^2 -
a^3*e^2)*x - (a^2*c^2*d^2*e - a^3*c*e^3 + (c^6*d^5 + 2*a*c^5*d^3*e^2 + a^2*
c^4*d*e^4)*sqrt(-(a^3*c^2*d^4 - 2*a^4*c*d^2*e^2 + a^5*e^4)/(c^9*d^8 + 4*a*c
^8*d^6*e^2 + 6*a^2*c^7*d^4*e^4 + 4*a^3*c^6*d^2*e^6 + a^4*c^5*e^8)))*sqrt(-(
2*a^2*d*e - (c^4*d^4 + 2*a*c^3*d^2*e^2 + a^2*c^2*e^4)*sqrt(-(a^3*c^2*d^4 -
2*a^4*c*d^2*e^2 + a^5*e^4)/(c^9*d^8 + 4*a*c^8*d^6*e^2 + 6*a^2*c^7*d^4*e^4 +
 4*a^3*c^6*d^2*e^6 + a^4*c^5*e^8)))/(c^4*d^4 + 2*a*c^3*d^2*e^2 + a^2*c^2*e^
4))) + 4*(c*d^2 + a*e^2)*x)/(c^2*d^2*e + a*c*e^3), -1/4*(4*c*d^2*sqrt(d/e)*
arctan(e*x*sqrt(d/e)/d) - (c^2*d^2*e + a*c*e^3)*sqrt(-(2*a^2*d*e + (c^4*d^4
 + 2*a*c^3*d^2*e^2 + a^2*c^2*e^4)*sqrt(-(a^3*c^2*d^4 - 2*a^4*c*d^2*e^2 + a^
5*e^4)/(c^9*d^8 + 4*a*c^8*d^6*e^2 + 6*a^2*c^7*d^4*e^4 + 4*a^3*c^6*d^2*e^6 +
 a^4*c^5*e^8)))/(c^4*d^4 + 2*a*c^3*d^2*e^2 + a^2*c^2*e^4))*log(-(a^2*c*d^2
- a^3*e^2)*x + (a^2*c^2*d^2*e - a^3*c*e^3 - (c^6*d^5 + 2*a*c^5*d^3*e^2 + a^
2*c^4*d*e^4)*sqrt(-(a^3*c^2*d^4 - 2*a^4*c*d^2*e^2 + a^5*e^4)/(c^9*d^8 + 4*a
*c^8*d^6*e^2 + 6*a^2*c^7*d^4*e^4 + 4*a^3*c^6*d^2*e^6 + a^4*c^5*e^8)))*sqrt(
-(2*a^2*d*e + (c^4*d^4 + 2*a*c^3*d^2*e^2 + a^2*c^2*e^4)*sqrt(-(a^3*c^2*d^4
- 2*a^4*c*d^2*e^2 + a^5*e^4)/(c^9*d^8 + 4*a*c^8*d^6*e^2 + 6*a^2*c^7*d^4*e^4
 + 4*a^3*c^6*d^2*e^6 + a^4*c^5*e^8)))/(c^4*d^4 + 2*a*c^3*d^2*e^2 + a^2*c^2*
e^4))) + (c^2*d^2*e + a*c*e^3)*sqrt(-(2*a^2*d*e + (c^4*d^4 + 2*a*c^3*d^2*e^
2 + a^2*c^2*e^4)*sqrt(-(a^3*c^2*d^4 - 2*a^4*c*d^2*e^2 + a^5*e^4)/(c^9*d^8 +
 4*a*c^8*d^6*e^2 + 6*a^2*c^7*d^4*e^4 + 4*a^3*c^6*d^2*e^6 + a^4*c^5*e^8)))/(
c^4*d^4 + 2*a*c^3*d^2*e^2 + a^2*c^2*e^4))*log(-(a^2*c*d^2 - a^3*e^2)*x - (a
^2*c^2*d^2*e - a^3*c*e^3 - (c^6*d^5 + 2*a*c^5*d^3*e^2 + a^2*c^4*d*e^4)*sqrt
(-(a^3*c^2*d^4 - 2*a^4*c*d^2*e^2 + a^5*e^4)/(c^9*d^8 + 4*a*c^8*d^6*e^2 + 6*
a^2*c^7*d^4*e^4 + 4*a^3*c^6*d^2*e^6 + a^4*c^5*e^8)))*sqrt(-(2*a^2*d*e + (c^
4*d^4 + 2*a*c^3*d^2*e^2 + a^2*c^2*e^4)*sqrt(-(a^3*c^2*d^4 - 2*a^4*c*d^2*e^2
 + a^5*e^4)/(c^9*d^8 + 4*a*c^8*d^6*e^2 + 6*a^2*c^7*d^4*e^4 + 4*a^3*c^6*d^2*
e^6 + a^4*c^5*e^8)))/(c^4*d^4 + 2*a*c^3*d^2*e^2 + a^2*c^2*e^4))) - (c^2*d^2
*e + a*c*e^3)*sqrt(-(2*a^2*d*e - (c^4*d^4 + 2*a*c^3*d^2*e^2 + a^2*c^2*e^4)*
sqrt(-(a^3*c^2*d^4 - 2*a^4*c*d^2*e^2 + a^5*e^4)/(c^9*d^8 + 4*a*c^8*d^6*e^2
+ 6*a^2*c^7*d^4*e^4 + 4*a^3*c^6*d^2*e^6 + a^4*c^5*e^8)))/(c^4*d^4 + 2*a*c^3
*d^2*e^2 + a^2*c^2*e^4))*log(-(a^2*c*d^2 - a^3*e^2)*x + (a^2*c^2*d^2*e - a^
3*c*e^3 + (c^6*d^5 + 2*a*c^5*d^3*e^2 + a^2*c^4*d*e^4)*sqrt(-(a^3*c^2*d^4 -
2*a^4*c*d^2*e^2 + a^5*e^4)/(c^9*d^8 + 4*a*c^8*d^6*e^2 + 6*a^2*c^7*d^4*e^4 +
 4*a^3*c^6*d^2*e^6 + a^4*c^5*e^8)))*sqrt(-(2*a^2*d*e - (c^4*d^4 + 2*a*c^3*d
^2*e^2 + a^2*c^2*e^4)*sqrt(-(a^3*c^2*d^4 - 2*a^4*c*d^2*e^2 + a^5*e^4)/(c^9*
d^8 + 4*a*c^8*d^6*e^2 + 6*a^2*c^7*d^4*e^4 + 4*a^3*c^6*d^2*e^6 + a^4*c^5*e^8
)))/(c^4*d^4 + 2*a*c^3*d^2*e^2 + a^2*c^2*e^4))) + (c^2*d^2*e + a*c*e^3)*sqr

```
t(-(2*a^2*d*e - (c^4*d^4 + 2*a*c^3*d^2*e^2 + a^2*c^2*e^4)*sqrt(-(a^3*c^2*d^
4 - 2*a^4*c*d^2*e^2 + a^5*e^4)/(c^9*d^8 + 4*a*c^8*d^6*e^2 + 6*a^2*c^7*d^4*e
^4 + 4*a^3*c^6*d^2*e^6 + a^4*c^5*e^8)))/(c^4*d^4 + 2*a*c^3*d^2*e^2 + a^2*c^
2*e^4))*log(-(a^2*c*d^2 - a^3*e^2)*x - (a^2*c^2*d^2*e - a^3*c*e^3 + (c^6*d^
5 + 2*a*c^5*d^3*e^2 + a^2*c^4*d*e^4)*sqrt(-(a^3*c^2*d^4 - 2*a^4*c*d^2*e^2 +
 a^5*e^4)/(c^9*d^8 + 4*a*c^8*d^6*e^2 + 6*a^2*c^7*d^4*e^4 + 4*a^3*c^6*d^2*e^
6 + a^4*c^5*e^8)))*sqrt(-(2*a^2*d*e - (c^4*d^4 + 2*a*c^3*d^2*e^2 + a^2*c^2*
e^4)*sqrt(-(a^3*c^2*d^4 - 2*a^4*c*d^2*e^2 + a^5*e^4)/(c^9*d^8 + 4*a*c^8*d^6
*e^2 + 6*a^2*c^7*d^4*e^4 + 4*a^3*c^6*d^2*e^6 + a^4*c^5*e^8)))/(c^4*d^4 + 2*
a*c^3*d^2*e^2 + a^2*c^2*e^4))) - 4*(c*d^2 + a*e^2)*x)/(c^2*d^2*e + a*c*e^3)
]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^6}{(d+ex^2)(a+cx^4)}\,dx = \text{Timed out}$$

```
[In] integrate(x**6/(e*x**2+d)/(c*x**4+a),x)
```

```
[Out] Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^6}{(d+ex^2)(a+cx^4)}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(x^6/(e*x^2+d)/(c*x^4+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 338, normalized size of antiderivative = 0.98

$$\int \frac{x^6}{(d+ex^2)(a+cx^4)}\,dx = -\frac{d^3\arctan\left(\frac{ex}{\sqrt{de}}\right)}{(cd^2e+ae^3)\sqrt{de}} - \frac{\left((ac^3)^{\frac{1}{4}}ace+(ac^3)^{\frac{3}{4}}d\right)\arctan\left(\frac{\sqrt{2}\left(2x+\sqrt{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{2\left(\sqrt{2}c^4d^2+\sqrt{2}ac^3e^2\right)} - \frac{\left((ac^3)^{\frac{1}{4}}ace+(ac^3)^{\frac{3}{4}}d\right)\arctan\left(\frac{\sqrt{2}\left(2x-\sqrt{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{2\left(\sqrt{2}c^4d^2+\sqrt{2}ac^3e^2\right)} - \frac{\left((ac^3)^{\frac{1}{4}}ace-(ac^3)^{\frac{3}{4}}d\right)\log\left(x^2+\sqrt{2}x\left(\frac{a}{c}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{c}}\right)}{4\left(\sqrt{2}c^4d^2+\sqrt{2}ac^3e^2\right)} + \frac{\left((ac^3)^{\frac{1}{4}}ace-(ac^3)^{\frac{3}{4}}d\right)\log\left(x^2-\sqrt{2}x\left(\frac{a}{c}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{c}}\right)}{4\left(\sqrt{2}c^4d^2+\sqrt{2}ac^3e^2\right)} + \frac{x}{ce}$$

```
[In] integrate(x^6/(e*x^2+d)/(c*x^4+a),x, algorithm="giac")
```

```
[Out] -d^3*arctan(e*x/sqrt(d*e))/((c*d^2*e + a*e^3)*sqrt(d*e)) - 1/2*((a*c^3)^(1/
4)*a*c*e + (a*c^3)^(3/4)*d)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(a/c)^(1/4))/
(a/c)^(1/4))/(sqrt(2)*c^4*d^2 + sqrt(2)*a*c^3*e^2) - 1/2*((a*c^3)^(1/4)*a*c
*e + (a*c^3)^(3/4)*d)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(a/c)^(1/4))/(a/c)^
(1/4))/(sqrt(2)*c^4*d^2 + sqrt(2)*a*c^3*e^2) - 1/4*((a*c^3)^(1/4)*a*c*e - (
a*c^3)^(3/4)*d)*log(x^2 + sqrt(2)*x*(a/c)^(1/4) + sqrt(a/c))/(sqrt(2)*c^4*d
^2 + sqrt(2)*a*c^3*e^2) + 1/4*((a*c^3)^(1/4)*a*c*e - (a*c^3)^(3/4)*d)*log(x
^2 - sqrt(2)*x*(a/c)^(1/4) + sqrt(a/c))/(sqrt(2)*c^4*d^2 + sqrt(2)*a*c^3*e^
2) + x/(c*e)
```

Mupad [B] (verification not implemented)

Time = 8.44 (sec) , antiderivative size = 5908, normalized size of antiderivative = 17.12

$$\int \frac{x^6}{(d+ex^2)(a+cx^4)}\,dx = \text{Too large to display}$$

```
[In] int(x^6/((a + c*x^4)*(d + e*x^2)),x)
```

```
[Out] atan((((((((64*a^5*c^4*d*e^8 + 64*a^3*c^6*d^5*e^4 + 128*a^4*c^5*d^3*e^6)/(c*
e) - (2*x*(-(a*e^2*(-a^3*c^5)^(1/2) - c*d^2*(-a^3*c^5)^(1/2) + 2*a^2*c^3*d*
```

e)/(16*(c^7*d^4 + a^2*c^5*e^4 + 2*a*c^6*d^2*e^2)))^(1/2)*(256*a^5*c^5*e^10
- 256*a^2*c^8*d^6*e^4 - 256*a^3*c^7*d^4*e^6 + 256*a^4*c^6*d^2*e^8))/(c*e))*
(-(a*e^2*(-a^3*c^5)^(1/2) - c*d^2*(-a^3*c^5)^(1/2) + 2*a^2*c^3*d*e)/(16*(c^
7*d^4 + a^2*c^5*e^4 + 2*a*c^6*d^2*e^2)))^(1/2) + (2*x*(64*a^2*c^6*d^7*e - 5
6*a^5*c^3*d*e^7 + 8*a^3*c^5*d^5*e^3 + 16*a^4*c^4*d^3*e^5))/(c*e))*(-(a*e^2*
(-a^3*c^5)^(1/2) - c*d^2*(-a^3*c^5)^(1/2) + 2*a^2*c^3*d*e)/(16*(c^7*d^4 + a
^2*c^5*e^4 + 2*a*c^6*d^2*e^2)))^(1/2) - (48*a^3*c^4*d^6*e - 60*a^4*c^3*d^4*
e^3 + 4*a^5*c^2*d^2*e^5)/(c*e))*(-(a*e^2*(-a^3*c^5)^(1/2) - c*d^2*(-a^3*c^5
)^(1/2) + 2*a^2*c^3*d*e)/(16*(c^7*d^4 + a^2*c^5*e^4 + 2*a*c^6*d^2*e^2)))^(1
/2) - (2*x*(a^6*e^6 - 2*a^3*c^3*d^6))/(c*e))*(-(a*e^2*(-a^3*c^5)^(1/2) - c*
d^2*(-a^3*c^5)^(1/2) + 2*a^2*c^3*d*e)/(16*(c^7*d^4 + a^2*c^5*e^4 + 2*a*c^6*
d^2*e^2)))^(1/2)*1i - (((((64*a^5*c^4*d*e^8 + 64*a^3*c^6*d^5*e^4 + 128*a^4*
c^5*d^3*e^6)/(c*e) + (2*x*(-(a*e^2*(-a^3*c^5)^(1/2) - c*d^2*(-a^3*c^5)^(1/2
) + 2*a^2*c^3*d*e)/(16*(c^7*d^4 + a^2*c^5*e^4 + 2*a*c^6*d^2*e^2)))^(1/2)*(2
56*a^5*c^5*e^10 - 256*a^2*c^8*d^6*e^4 - 256*a^3*c^7*d^4*e^6 + 256*a^4*c^6*d
^2*e^8))/(c*e))*(-(a*e^2*(-a^3*c^5)^(1/2) - c*d^2*(-a^3*c^5)^(1/2) + 2*a^2*
c^3*d*e)/(16*(c^7*d^4 + a^2*c^5*e^4 + 2*a*c^6*d^2*e^2)))^(1/2) - (2*x*(64*a
^2*c^6*d^7*e - 56*a^5*c^3*d*e^7 + 8*a^3*c^5*d^5*e^3 + 16*a^4*c^4*d^3*e^5))/
(c*e))*(-(a*e^2*(-a^3*c^5)^(1/2) - c*d^2*(-a^3*c^5)^(1/2) + 2*a^2*c^3*d*e)/
(16*(c^7*d^4 + a^2*c^5*e^4 + 2*a*c^6*d^2*e^2)))^(1/2) - (48*a^3*c^4*d^6*e -
 60*a^4*c^3*d^4*e^3 + 4*a^5*c^2*d^2*e^5)/(c*e))*(-(a*e^2*(-a^3*c^5)^(1/2) -
 c*d^2*(-a^3*c^5)^(1/2) + 2*a^2*c^3*d*e)/(16*(c^7*d^4 + a^2*c^5*e^4 + 2*a*c
^6*d^2*e^2)))^(1/2) + (2*x*(a^6*e^6 - 2*a^3*c^3*d^6))/(c*e))*(-(a*e^2*(-a^3
*c^5)^(1/2) - c*d^2*(-a^3*c^5)^(1/2) + 2*a^2*c^3*d*e)/(16*(c^7*d^4 + a^2*c^
5*e^4 + 2*a*c^6*d^2*e^2)))^(1/2)*1i)/((((((64*a^5*c^4*d*e^8 + 64*a^3*c^6*d^
5*e^4 + 128*a^4*c^5*d^3*e^6)/(c*e) - (2*x*(-(a*e^2*(-a^3*c^5)^(1/2) - c*d^2
*(-a^3*c^5)^(1/2) + 2*a^2*c^3*d*e)/(16*(c^7*d^4 + a^2*c^5*e^4 + 2*a*c^6*d^2
e^2)))^(1/2)(256*a^5*c^5*e^10 - 256*a^2*c^8*d^6*e^4 - 256*a^3*c^7*d^4*e^6
 + 256*a^4*c^6*d^2*e^8))/(c*e))*(-(a*e^2*(-a^3*c^5)^(1/2) - c*d^2*(-a^3*c^5
)^(1/2) + 2*a^2*c^3*d*e)/(16*(c^7*d^4 + a^2*c^5*e^4 + 2*a*c^6*d^2*e^2)))^(1
/2) + (2*x*(64*a^2*c^6*d^7*e - 56*a^5*c^3*d*e^7 + 8*a^3*c^5*d^5*e^3 + 16*a^
4*c^4*d^3*e^5))/(c*e))*(-(a*e^2*(-a^3*c^5)^(1/2) - c*d^2*(-a^3*c^5)^(1/2) +
 2*a^2*c^3*d*e)/(16*(c^7*d^4 + a^2*c^5*e^4 + 2*a*c^6*d^2*e^2)))^(1/2) - (48
*a^3*c^4*d^6*e - 60*a^4*c^3*d^4*e^3 + 4*a^5*c^2*d^2*e^5)/(c*e))*(-(a*e^2*(-
a^3*c^5)^(1/2) - c*d^2*(-a^3*c^5)^(1/2) + 2*a^2*c^3*d*e)/(16*(c^7*d^4 + a^2
*c^5*e^4 + 2*a*c^6*d^2*e^2)))^(1/2) - (2*x*(a^6*e^6 - 2*a^3*c^3*d^6))/(c*e)
)*(-(a*e^2*(-a^3*c^5)^(1/2) - c*d^2*(-a^3*c^5)^(1/2) + 2*a^2*c^3*d*e)/(16*(
c^7*d^4 + a^2*c^5*e^4 + 2*a*c^6*d^2*e^2)))^(1/2) + (((((64*a^5*c^4*d*e^8 +
64*a^3*c^6*d^5*e^4 + 128*a^4*c^5*d^3*e^6)/(c*e) + (2*x*(-(a*e^2*(-a^3*c^5)^
(1/2) - c*d^2*(-a^3*c^5)^(1/2) + 2*a^2*c^3*d*e)/(16*(c^7*d^4 + a^2*c^5*e^4
+ 2*a*c^6*d^2*e^2)))^(1/2)*(256*a^5*c^5*e^10 - 256*a^2*c^8*d^6*e^4 - 256*a^
3*c^7*d^4*e^6 + 256*a^4*c^6*d^2*e^8))/(c*e))*(-(a*e^2*(-a^3*c^5)^(1/2) - c*
d^2*(-a^3*c^5)^(1/2) + 2*a^2*c^3*d*e)/(16*(c^7*d^4 + a^2*c^5*e^4 + 2*a*c^6*
d^2*e^2)))^(1/2) - (2*x*(64*a^2*c^6*d^7*e - 56*a^5*c^3*d*e^7 + 8*a^3*c^5*d^
5*e^3 + 16*a^4*c^4*d^3*e^5))/(c*e))*(-(a*e^2*(-a^3*c^5)^(1/2) - c*d^2*(-a^3

*c^5)^(1/2) + 2*a^2*c^3*d*e)/(16*(c^7*d^4 + a^2*c^5*e^4 + 2*a*c^6*d^2*e^2))
)^(1/2) - (48*a^3*c^4*d^6*e - 60*a^4*c^3*d^4*e^3 + 4*a^5*c^2*d^2*e^5)/(c*e)
)*(-(a*e^2*(-a^3*c^5)^(1/2) - c*d^2*(-a^3*c^5)^(1/2) + 2*a^2*c^3*d*e)/(16*(
c^7*d^4 + a^2*c^5*e^4 + 2*a*c^6*d^2*e^2)))^(1/2) + (2*x*(a^6*e^6 - 2*a^3*c^
3*d^6))/(c*e))*(-(a*e^2*(-a^3*c^5)^(1/2) - c*d^2*(-a^3*c^5)^(1/2) + 2*a^2*c
^3*d*e)/(16*(c^7*d^4 + a^2*c^5*e^4 + 2*a*c^6*d^2*e^2)))^(1/2) - (2*(a^4*c*d
^5 - a^5*d^3*e^2))/(c*e)))*(-(a*e^2*(-a^3*c^5)^(1/2) - c*d^2*(-a^3*c^5)^(1/
2) + 2*a^2*c^3*d*e)/(16*(c^7*d^4 + a^2*c^5*e^4 + 2*a*c^6*d^2*e^2)))^(1/2)*2
i + atan((((((64*a^5*c^4*d*e^8 + 64*a^3*c^6*d^5*e^4 + 128*a^4*c^5*d^3*e^6)
/(c*e) - (2*x*(-(c*d^2*(-a^3*c^5)^(1/2) - a*e^2*(-a^3*c^5)^(1/2) + 2*a^2*c^
3*d*e)/(16*(c^7*d^4 + a^2*c^5*e^4 + 2*a*c^6*d^2*e^2)))^(1/2)*(256*a^5*c^5*e
^10 - 256*a^2*c^8*d^6*e^4 - 256*a^3*c^7*d^4*e^6 + 256*a^4*c^6*d^2*e^8))/(c*
e))*(-(c*d^2*(-a^3*c^5)^(1/2) - a*e^2*(-a^3*c^5)^(1/2) + 2*a^2*c^3*d*e)/(16
*(c^7*d^4 + a^2*c^5*e^4 + 2*a*c^6*d^2*e^2)))^(1/2) + (2*x*(64*a^2*c^6*d^7*e
 - 56*a^5*c^3*d*e^7 + 8*a^3*c^5*d^5*e^3 + 16*a^4*c^4*d^3*e^5))/(c*e))*(-(c*
d^2*(-a^3*c^5)^(1/2) - a*e^2*(-a^3*c^5)^(1/2) + 2*a^2*c^3*d*e)/(16*(c^7*d^4
 + a^2*c^5*e^4 + 2*a*c^6*d^2*e^2)))^(1/2) - (48*a^3*c^4*d^6*e - 60*a^4*c^3*
d^4*e^3 + 4*a^5*c^2*d^2*e^5)/(c*e))*(-(c*d^2*(-a^3*c^5)^(1/2) - a*e^2*(-a^3
*c^5)^(1/2) + 2*a^2*c^3*d*e)/(16*(c^7*d^4 + a^2*c^5*e^4 + 2*a*c^6*d^2*e^2))
)^(1/2) - (2*x*(a^6*e^6 - 2*a^3*c^3*d^6))/(c*e))*(-(c*d^2*(-a^3*c^5)^(1/2)
- a*e^2*(-a^3*c^5)^(1/2) + 2*a^2*c^3*d*e)/(16*(c^7*d^4 + a^2*c^5*e^4 + 2*a*
c^6*d^2*e^2)))^(1/2)*1i - (((((64*a^5*c^4*d*e^8 + 64*a^3*c^6*d^5*e^4 + 128*
a^4*c^5*d^3*e^6)/(c*e) + (2*x*(-(c*d^2*(-a^3*c^5)^(1/2) - a*e^2*(-a^3*c^5)^
(1/2) + 2*a^2*c^3*d*e)/(16*(c^7*d^4 + a^2*c^5*e^4 + 2*a*c^6*d^2*e^2)))^(1/2
)*(256*a^5*c^5*e^10 - 256*a^2*c^8*d^6*e^4 - 256*a^3*c^7*d^4*e^6 + 256*a^4*c
^6*d^2*e^8))/(c*e))*(-(c*d^2*(-a^3*c^5)^(1/2) - a*e^2*(-a^3*c^5)^(1/2) + 2*
a^2*c^3*d*e)/(16*(c^7*d^4 + a^2*c^5*e^4 + 2*a*c^6*d^2*e^2)))^(1/2) - (2*x*(
64*a^2*c^6*d^7*e - 56*a^5*c^3*d*e^7 + 8*a^3*c^5*d^5*e^3 + 16*a^4*c^4*d^3*e^
5))/(c*e))*(-(c*d^2*(-a^3*c^5)^(1/2) - a*e^2*(-a^3*c^5)^(1/2) + 2*a^2*c^3*d
e)/(16(c^7*d^4 + a^2*c^5*e^4 + 2*a*c^6*d^2*e^2)))^(1/2) - (48*a^3*c^4*d^6
*e - 60*a^4*c^3*d^4*e^3 + 4*a^5*c^2*d^2*e^5)/(c*e))*(-(c*d^2*(-a^3*c^5)^(1/
2) - a*e^2*(-a^3*c^5)^(1/2) + 2*a^2*c^3*d*e)/(16*(c^7*d^4 + a^2*c^5*e^4 + 2
*a*c^6*d^2*e^2)))^(1/2) + (2*x*(a^6*e^6 - 2*a^3*c^3*d^6))/(c*e))*(-(c*d^2*(
-a^3*c^5)^(1/2) - a*e^2*(-a^3*c^5)^(1/2) + 2*a^2*c^3*d*e)/(16*(c^7*d^4 + a^
2*c^5*e^4 + 2*a*c^6*d^2*e^2)))^(1/2)*1i)/((((((64*a^5*c^4*d*e^8 + 64*a^3*c^
6*d^5*e^4 + 128*a^4*c^5*d^3*e^6)/(c*e) - (2*x*(-(c*d^2*(-a^3*c^5)^(1/2) - a
e^2(-a^3*c^5)^(1/2) + 2*a^2*c^3*d*e)/(16*(c^7*d^4 + a^2*c^5*e^4 + 2*a*c^6
*d^2*e^2)))^(1/2)*(256*a^5*c^5*e^10 - 256*a^2*c^8*d^6*e^4 - 256*a^3*c^7*d^4
*e^6 + 256*a^4*c^6*d^2*e^8))/(c*e))*(-(c*d^2*(-a^3*c^5)^(1/2) - a*e^2*(-a^3
*c^5)^(1/2) + 2*a^2*c^3*d*e)/(16*(c^7*d^4 + a^2*c^5*e^4 + 2*a*c^6*d^2*e^2))
)^(1/2) + (2*x*(64*a^2*c^6*d^7*e - 56*a^5*c^3*d*e^7 + 8*a^3*c^5*d^5*e^3 + 1
6*a^4*c^4*d^3*e^5))/(c*e))*(-(c*d^2*(-a^3*c^5)^(1/2) - a*e^2*(-a^3*c^5)^(1/
2) + 2*a^2*c^3*d*e)/(16*(c^7*d^4 + a^2*c^5*e^4 + 2*a*c^6*d^2*e^2)))^(1/2) -
 (48*a^3*c^4*d^6*e - 60*a^4*c^3*d^4*e^3 + 4*a^5*c^2*d^2*e^5)/(c*e))*(-(c*d^
2*(-a^3*c^5)^(1/2) - a*e^2*(-a^3*c^5)^(1/2) + 2*a^2*c^3*d*e)/(16*(c^7*d^4 +

```
 a^2*c^5*e^4 + 2*a*c^6*d^2*e^2)))^(1/2) - (2*x*(a^6*e^6 - 2*a^3*c^3*d^6))/(
c*e))*(-(c*d^2*(-a^3*c^5)^(1/2) - a*e^2*(-a^3*c^5)^(1/2) + 2*a^2*c^3*d*e)/(
16*(c^7*d^4 + a^2*c^5*e^4 + 2*a*c^6*d^2*e^2)))^(1/2) + (((((64*a^5*c^4*d*e^
8 + 64*a^3*c^6*d^5*e^4 + 128*a^4*c^5*d^3*e^6)/(c*e) + (2*x*(-(c*d^2*(-a^3*c
^5)^(1/2) - a*e^2*(-a^3*c^5)^(1/2) + 2*a^2*c^3*d*e)/(16*(c^7*d^4 + a^2*c^5*
e^4 + 2*a*c^6*d^2*e^2)))^(1/2)*(256*a^5*c^5*e^10 - 256*a^2*c^8*d^6*e^4 - 25
6*a^3*c^7*d^4*e^6 + 256*a^4*c^6*d^2*e^8))/(c*e))*(-(c*d^2*(-a^3*c^5)^(1/2)
- a*e^2*(-a^3*c^5)^(1/2) + 2*a^2*c^3*d*e)/(16*(c^7*d^4 + a^2*c^5*e^4 + 2*a*
c^6*d^2*e^2)))^(1/2) - (2*x*(64*a^2*c^6*d^7*e - 56*a^5*c^3*d*e^7 + 8*a^3*c^
5*d^5*e^3 + 16*a^4*c^4*d^3*e^5))/(c*e))*(-(c*d^2*(-a^3*c^5)^(1/2) - a*e^2*(
-a^3*c^5)^(1/2) + 2*a^2*c^3*d*e)/(16*(c^7*d^4 + a^2*c^5*e^4 + 2*a*c^6*d^2*e
^2)))^(1/2) - (48*a^3*c^4*d^6*e - 60*a^4*c^3*d^4*e^3 + 4*a^5*c^2*d^2*e^5)/(
c*e))*(-(c*d^2*(-a^3*c^5)^(1/2) - a*e^2*(-a^3*c^5)^(1/2) + 2*a^2*c^3*d*e)/(
16*(c^7*d^4 + a^2*c^5*e^4 + 2*a*c^6*d^2*e^2)))^(1/2) + (2*x*(a^6*e^6 - 2*a^
3*c^3*d^6))/(c*e))*(-(c*d^2*(-a^3*c^5)^(1/2) - a*e^2*(-a^3*c^5)^(1/2) + 2*a
^2*c^3*d*e)/(16*(c^7*d^4 + a^2*c^5*e^4 + 2*a*c^6*d^2*e^2)))^(1/2) - (2*(a^4
*c*d^5 - a^5*d^3*e^2))/(c*e)))*(-(c*d^2*(-a^3*c^5)^(1/2) - a*e^2*(-a^3*c^5)
^(1/2) + 2*a^2*c^3*d*e)/(16*(c^7*d^4 + a^2*c^5*e^4 + 2*a*c^6*d^2*e^2)))^(1/
2)*2i + x/(c*e) - (log(16*c^5*x*(-d^5*e^3)^(5/2) - 16*c^5*d^13*e^7 - a^5*d^
3*e^17 - 2*a^4*c*d^5*e^15 + 16*a^2*c^3*d^9*e^11 - a^3*c^2*d^7*e^13 + a^5*e^
16*x*(-d^5*e^3)^(1/2) + a^3*c^2*d^4*e^12*x*(-d^5*e^3)^(1/2) + 16*a^2*c^3*d*
e^7*x*(-d^5*e^3)^(3/2) + 2*a^4*c*d^2*e^14*x*(-d^5*e^3)^(1/2))*(-d^5*e^3)^(1
/2))/(2*(a*e^5 + c*d^2*e^3)) + (log(a^5*d^3*e^17 + 16*c^5*d^13*e^7 + 16*c^5
*x*(-d^5*e^3)^(5/2) + 2*a^4*c*d^5*e^15 - 16*a^2*c^3*d^9*e^11 + a^3*c^2*d^7*
e^13 + a^5*e^16*x*(-d^5*e^3)^(1/2) + a^3*c^2*d^4*e^12*x*(-d^5*e^3)^(1/2) +
16*a^2*c^3*d*e^7*x*(-d^5*e^3)^(3/2) + 2*a^4*c*d^2*e^14*x*(-d^5*e^3)^(1/2))*
(-d^5*e^3)^(1/2))/(2*a*e^5 + 2*c*d^2*e^3)
```

3.239 $\int \frac{x^4}{(d+ex^2)(a+cx^4)}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 336

$$\int \frac{x^4}{(d+ex^2)(a+cx^4)}\,dx = \frac{d^{3/2}\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{e}(cd^2+ae^2)} + \frac{\sqrt[4]{a}(\sqrt{c}d-\sqrt{a}e)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}c^{3/4}(cd^2+ae^2)} - \frac{\sqrt[4]{a}(\sqrt{c}d-\sqrt{a}e)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}c^{3/4}(cd^2+ae^2)} + \frac{\sqrt[4]{a}(\sqrt{c}d+\sqrt{a}e)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)}{4\sqrt{2}c^{3/4}(cd^2+ae^2)} - \frac{\sqrt[4]{a}(\sqrt{c}d+\sqrt{a}e)\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)}{4\sqrt{2}c^{3/4}(cd^2+ae^2)}$$

```
[Out] -1/4*a^(1/4)*arctan(-1+c^(1/4)*x*2^(1/2)/a^(1/4))*(-e*a^(1/2)+d*c^(1/2))/c^
(3/4)/(a*e^2+c*d^2)*2^(1/2)-1/4*a^(1/4)*arctan(1+c^(1/4)*x*2^(1/2)/a^(1/4))
*(-e*a^(1/2)+d*c^(1/2))/c^(3/4)/(a*e^2+c*d^2)*2^(1/2)+1/8*a^(1/4)*ln(-a^(1/
4)*c^(1/4)*x*2^(1/2)+a^(1/2)+x^2*c^(1/2))*(e*a^(1/2)+d*c^(1/2))/c^(3/4)/(a*
e^2+c*d^2)*2^(1/2)-1/8*a^(1/4)*ln(a^(1/4)*c^(1/4)*x*2^(1/2)+a^(1/2)+x^2*c^(
1/2))*(e*a^(1/2)+d*c^(1/2))/c^(3/4)/(a*e^2+c*d^2)*2^(1/2)+d^(3/2)*arctan(x*
e^(1/2)/d^(1/2))/(a*e^2+c*d^2)/e^(1/2)
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 336, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {1302, 211, 1182, 1176, 631, 210, 1179, 642}

$$\int \frac{x^4}{(d+ex^2)(a+cx^4)}\,dx = \frac{\sqrt[4]{a}\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\left(\sqrt{c}d-\sqrt{a}e\right)}{2\sqrt{2}c^{3/4}(ae^2+cd^2)} - \frac{\sqrt[4]{a}\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)\left(\sqrt{c}d-\sqrt{a}e\right)}{2\sqrt{2}c^{3/4}(ae^2+cd^2)} + \frac{d^{3/2}\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{e}(ae^2+cd^2)} + \frac{\sqrt[4]{a}\left(\sqrt{a}e+\sqrt{c}d\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{4\sqrt{2}c^{3/4}(ae^2+cd^2)} - \frac{\sqrt[4]{a}\left(\sqrt{a}e+\sqrt{c}d\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{4\sqrt{2}c^{3/4}(ae^2+cd^2)}$$

```
[In] Int[x^4/((d + e*x^2)*(a + c*x^4)),x]
```

```
[Out] (d^(3/2)*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(Sqrt[e]*(c*d^2 + a*e^2)) + (a^(1/4)*
(Sqrt[c]*d - Sqrt[a]*e)*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]
*c^(3/4)*(c*d^2 + a*e^2)) - (a^(1/4)*(Sqrt[c]*d - Sqrt[a]*e)*ArcTan[1 + (Sq
rt[2]*c^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*c^(3/4)*(c*d^2 + a*e^2)) + (a^(1/4)*(
Sqrt[c]*d + Sqrt[a]*e)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^
2])/(4*Sqrt[2]*c^(3/4)*(c*d^2 + a*e^2)) - (a^(1/4)*(Sqrt[c]*d + Sqrt[a]*e)*
Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(4*Sqrt[2]*c^(3/4)*
(c*d^2 + a*e^2))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1182

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[(-a)
*c]
```

Rule 1302

```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.))/((a_) + (c_.)*(x_)^4),
 x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*((d + e*x^2)^q/(a + c*x^4)), x],
x] /; FreeQ[{a, c, d, e, f, m}, x] && IntegerQ[q] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(\frac{d^2}{(cd^2+ae^2)(d+ex^2)}-\frac{a(d-ex^2)}{(cd^2+ae^2)(a+cx^4)}\right)dx\\
&= -\frac{a\int \frac{d-ex^2}{a+cx^4}\,dx}{cd^2+ae^2}+\frac{d^2\int \frac{1}{d+ex^2}\,dx}{cd^2+ae^2}\\
&= \frac{d^{3/2}\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{e}\,(cd^2+ae^2)}-\frac{\left(a\left(\frac{\sqrt{c}d}{\sqrt{a}}-e\right)\right)\int \frac{\sqrt{a}\sqrt{c}+cx^2}{a+cx^4}\,dx}{2c\,(cd^2+ae^2)}-\frac{\left(a\left(\frac{\sqrt{c}d}{\sqrt{a}}+e\right)\right)\int \frac{\sqrt{a}\sqrt{c}-cx^2}{a+cx^4}\,dx}{2c\,(cd^2+ae^2)}
\end{aligned}$$

$$=\frac{d^{3/2}\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{e}\left(cd^2+ae^2\right)}-\frac{\left(a\left(\frac{\sqrt{c}d}{\sqrt{a}}-e\right)\right)\int\frac{1}{\frac{\sqrt{a}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}+x^2}dx}{4c\left(cd^2+ae^2\right)}-\frac{\left(a\left(\frac{\sqrt{c}d}{\sqrt{a}}-e\right)\right)\int\frac{1}{\frac{\sqrt{a}}{\sqrt{c}}+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}+x^2}dx}{4c\left(cd^2+ae^2\right)}$$
$$+\frac{\left(\sqrt[4]{a}\left(\sqrt{c}d+\sqrt{a}e\right)\right)\int\frac{\frac{\sqrt{2}\sqrt[4]{a}}{\sqrt[4]{c}}+2x}{-\frac{\sqrt{a}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}-x^2}dx}{4\sqrt{2}c^{3/4}\left(cd^2+ae^2\right)}+\frac{\left(\sqrt[4]{a}\left(\sqrt{c}d+\sqrt{a}e\right)\right)\int\frac{\frac{\sqrt{2}\sqrt[4]{a}}{\sqrt[4]{c}}-2x}{-\frac{\sqrt{a}}{\sqrt{c}}+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}-x^2}dx}{4\sqrt{2}c^{3/4}\left(cd^2+ae^2\right)}$$

$$=\frac{d^{3/2}\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{e}\left(cd^2+ae^2\right)}+\frac{\sqrt[4]{a}\left(\sqrt{c}d+\sqrt{a}e\right)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)}{4\sqrt{2}c^{3/4}\left(cd^2+ae^2\right)}$$
$$-\frac{\sqrt[4]{a}\left(\sqrt{c}d+\sqrt{a}e\right)\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)}{4\sqrt{2}c^{3/4}\left(cd^2+ae^2\right)}$$
$$-\frac{\left(a^{3/4}\left(\frac{\sqrt{c}d}{\sqrt{a}}-e\right)\right)\text{Subst}\left(\int\frac{1}{-1-x^2}dx,x,1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}c^{3/4}\left(cd^2+ae^2\right)}$$
$$+\frac{\left(a^{3/4}\left(\frac{\sqrt{c}d}{\sqrt{a}}-e\right)\right)\text{Subst}\left(\int\frac{1}{-1-x^2}dx,x,1+\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}c^{3/4}\left(cd^2+ae^2\right)}$$

$$=\frac{d^{3/2}\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{e}\left(cd^2+ae^2\right)}+\frac{a^{3/4}\left(\frac{\sqrt{c}d}{\sqrt{a}}-e\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}c^{3/4}\left(cd^2+ae^2\right)}$$
$$-\frac{a^{3/4}\left(\frac{\sqrt{c}d}{\sqrt{a}}-e\right)\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}c^{3/4}\left(cd^2+ae^2\right)}$$
$$+\frac{\sqrt[4]{a}\left(\sqrt{c}d+\sqrt{a}e\right)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)}{4\sqrt{2}c^{3/4}\left(cd^2+ae^2\right)}$$
$$-\frac{\sqrt[4]{a}\left(\sqrt{c}d+\sqrt{a}e\right)\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)}{4\sqrt{2}c^{3/4}\left(cd^2+ae^2\right)}$$

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 233, normalized size of antiderivative = 0.69

$$\int\frac{x^4}{(d+ex^2)(a+cx^4)}dx$$
$$=\frac{8c^{3/4}d^{3/2}\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)+\sqrt{2}\sqrt[4]{a}\sqrt{e}\left(2\left(\sqrt{c}d-\sqrt{a}e\right)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)+\left(-2\sqrt{c}d+2\sqrt{a}e\right)\arctan\left(1-\right.\right.}{8c^{3/4}\sqrt{e}\left(cd^2+\right.}$$

```
[In] Integrate[x^4/((d + e*x^2)*(a + c*x^4)),x]
```

```
[Out] (8*c^(3/4)*d^(3/2)*ArcTan[(Sqrt[e]*x)/Sqrt[d]] + Sqrt[2]*a^(1/4)*Sqrt[e]*(2
*(Sqrt[c]*d - Sqrt[a]*e)*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)] + (-2*Sqrt
[c]*d + 2*Sqrt[a]*e)*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)] + (Sqrt[c]*d +
 Sqrt[a]*e)*(Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2] - Log[S
qrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])))/(8*c^(3/4)*Sqrt[e]*(c*
d^2 + a*e^2))
```

Maple [A] (verified)

Time = 0.41 (sec) , antiderivative size = 254, normalized size of antiderivative = 0.76

method	result
default	$-\frac{a\Bigl(\frac{d\left(\frac{a}{c}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x^{2}+\left(\frac{a}{c}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{c}}}{x^{2}-\left(\frac{a}{c}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{c}}}\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{c}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{c}\right)^{\frac{1}{4}}}-1\right)\right)}{8a}-\frac{e\sqrt{2}\Bigl(\ln\left(\frac{x^{2}-\left(\frac{a}{c}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{c}}}{x^{2}+\left(\frac{a}{c}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{c}}}\right)+2\arctan\Bigl(\frac{\sqrt{}}{(}}{8c\left(\frac{a}{c}\right)^{\frac{1}{4}}}}{a\,e^{2}+c\,d^{2}}$
risch	$\frac{\Bigl(\sum_{\_R=\mathrm{RootOf}\left(\left(a^{2}c^{3}e^{4}+2a\,c^{4}d^{2}e^{2}+c^{5}d^{4}\right)\_Z^{4}-4a\,c^{2}de\,\_Z^{2}+a\right)}\_R\ln\Bigl(\Bigl(\bigl(-2a^{3}c^{3}e^{8}-2a^{2}c^{4}d^{2}e^{6}+2a\,c^{5}d^{4}e^{4}+2c^{6}d^{6}e^{2}\bigr)\_R^{5}+(7a^{2}c^{2}}{}$

```
[In] int(x^4/(e*x^2+d)/(c*x^4+a),x,method=_RETURNVERBOSE)
```

```
[Out] -a/(a*e^2+c*d^2)*(1/8*d*(a/c)^(1/4)/a*2^(1/2)*(ln((x^2+(a/c)^(1/4)*x*2^(1/2
)+(a/c)^(1/2))/(x^2-(a/c)^(1/4)*x*2^(1/2)+(a/c)^(1/2)))+2*arctan(2^(1/2)/(a
/c)^(1/4)*x+1)+2*arctan(2^(1/2)/(a/c)^(1/4)*x-1))-1/8*e/c/(a/c)^(1/4)*2^(1/
2)*(ln((x^2-(a/c)^(1/4)*x*2^(1/2)+(a/c)^(1/2))/(x^2+(a/c)^(1/4)*x*2^(1/2)+(
a/c)^(1/2)))+2*arctan(2^(1/2)/(a/c)^(1/4)*x+1)+2*arctan(2^(1/2)/(a/c)^(1/4)
*x-1)))+d^2/(a*e^2+c*d^2)/(e*d)^(1/2)*arctan(e*x/(e*d)^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2010 vs. $2(247) = 494$.

Time = 0.47 (sec) , antiderivative size = 4040, normalized size of antiderivative = 12.02

$$\int \frac{x^4}{(d+ex^2)(a+cx^4)}\,dx = \text{Too large to display}$$

```
[In] integrate(x^4/(e*x^2+d)/(c*x^4+a),x, algorithm="fricas")
```

```
[Out] [1/4*((c*d^2 + a*e^2)*sqrt((2*a*d*e + (c^3*d^4 + 2*a*c^2*d^2*e^2 + a^2*c*e^
4)*sqrt(-(a*c^2*d^4 - 2*a^2*c*d^2*e^2 + a^3*e^4)/(c^7*d^8 + 4*a*c^6*d^6*e^2
 + 6*a^2*c^5*d^4*e^4 + 4*a^3*c^4*d^2*e^6 + a^4*c^3*e^8)))/(c^3*d^4 + 2*a*c^
```

```
2*d^2*e^2 + a^2*c*e^4))*log(-(c*d^2 - a*e^2)*x + (c^2*d^3 - a*c*d*e^2 + (c^
4*d^4*e + 2*a*c^3*d^2*e^3 + a^2*c^2*e^5)*sqrt(-(a*c^2*d^4 - 2*a^2*c*d^2*e^2
 + a^3*e^4)/(c^7*d^8 + 4*a*c^6*d^6*e^2 + 6*a^2*c^5*d^4*e^4 + 4*a^3*c^4*d^2*
e^6 + a^4*c^3*e^8)))*sqrt((2*a*d*e + (c^3*d^4 + 2*a*c^2*d^2*e^2 + a^2*c*e^4
)*sqrt(-(a*c^2*d^4 - 2*a^2*c*d^2*e^2 + a^3*e^4)/(c^7*d^8 + 4*a*c^6*d^6*e^2
+ 6*a^2*c^5*d^4*e^4 + 4*a^3*c^4*d^2*e^6 + a^4*c^3*e^8)))/(c^3*d^4 + 2*a*c^2
*d^2*e^2 + a^2*c*e^4))) - (c*d^2 + a*e^2)*sqrt((2*a*d*e + (c^3*d^4 + 2*a*c^
2*d^2*e^2 + a^2*c*e^4)*sqrt(-(a*c^2*d^4 - 2*a^2*c*d^2*e^2 + a^3*e^4)/(c^7*d
^8 + 4*a*c^6*d^6*e^2 + 6*a^2*c^5*d^4*e^4 + 4*a^3*c^4*d^2*e^6 + a^4*c^3*e^8)
))/(c^3*d^4 + 2*a*c^2*d^2*e^2 + a^2*c*e^4))*log(-(c*d^2 - a*e^2)*x - (c^2*d
^3 - a*c*d*e^2 + (c^4*d^4*e + 2*a*c^3*d^2*e^3 + a^2*c^2*e^5)*sqrt(-(a*c^2*d
^4 - 2*a^2*c*d^2*e^2 + a^3*e^4)/(c^7*d^8 + 4*a*c^6*d^6*e^2 + 6*a^2*c^5*d^4*
e^4 + 4*a^3*c^4*d^2*e^6 + a^4*c^3*e^8)))*sqrt((2*a*d*e + (c^3*d^4 + 2*a*c^2
*d^2*e^2 + a^2*c*e^4)*sqrt(-(a*c^2*d^4 - 2*a^2*c*d^2*e^2 + a^3*e^4)/(c^7*d^
8 + 4*a*c^6*d^6*e^2 + 6*a^2*c^5*d^4*e^4 + 4*a^3*c^4*d^2*e^6 + a^4*c^3*e^8))
)/(c^3*d^4 + 2*a*c^2*d^2*e^2 + a^2*c*e^4))) + (c*d^2 + a*e^2)*sqrt((2*a*d*e
 - (c^3*d^4 + 2*a*c^2*d^2*e^2 + a^2*c*e^4)*sqrt(-(a*c^2*d^4 - 2*a^2*c*d^2*e
^2 + a^3*e^4)/(c^7*d^8 + 4*a*c^6*d^6*e^2 + 6*a^2*c^5*d^4*e^4 + 4*a^3*c^4*d^
2*e^6 + a^4*c^3*e^8)))/(c^3*d^4 + 2*a*c^2*d^2*e^2 + a^2*c*e^4))*log(-(c*d^2
 - a*e^2)*x + (c^2*d^3 - a*c*d*e^2 - (c^4*d^4*e + 2*a*c^3*d^2*e^3 + a^2*c^2
*e^5)*sqrt(-(a*c^2*d^4 - 2*a^2*c*d^2*e^2 + a^3*e^4)/(c^7*d^8 + 4*a*c^6*d^6*
e^2 + 6*a^2*c^5*d^4*e^4 + 4*a^3*c^4*d^2*e^6 + a^4*c^3*e^8)))*sqrt((2*a*d*e
- (c^3*d^4 + 2*a*c^2*d^2*e^2 + a^2*c*e^4)*sqrt(-(a*c^2*d^4 - 2*a^2*c*d^2*e^
2 + a^3*e^4)/(c^7*d^8 + 4*a*c^6*d^6*e^2 + 6*a^2*c^5*d^4*e^4 + 4*a^3*c^4*d^2
*e^6 + a^4*c^3*e^8)))/(c^3*d^4 + 2*a*c^2*d^2*e^2 + a^2*c*e^4))) - (c*d^2 +
a*e^2)*sqrt((2*a*d*e - (c^3*d^4 + 2*a*c^2*d^2*e^2 + a^2*c*e^4)*sqrt(-(a*c^2
*d^4 - 2*a^2*c*d^2*e^2 + a^3*e^4)/(c^7*d^8 + 4*a*c^6*d^6*e^2 + 6*a^2*c^5*d^
4*e^4 + 4*a^3*c^4*d^2*e^6 + a^4*c^3*e^8)))/(c^3*d^4 + 2*a*c^2*d^2*e^2 + a^2
*c*e^4))*log(-(c*d^2 - a*e^2)*x - (c^2*d^3 - a*c*d*e^2 - (c^4*d^4*e + 2*a*c
^3*d^2*e^3 + a^2*c^2*e^5)*sqrt(-(a*c^2*d^4 - 2*a^2*c*d^2*e^2 + a^3*e^4)/(c^
7*d^8 + 4*a*c^6*d^6*e^2 + 6*a^2*c^5*d^4*e^4 + 4*a^3*c^4*d^2*e^6 + a^4*c^3*e
^8)))*sqrt((2*a*d*e - (c^3*d^4 + 2*a*c^2*d^2*e^2 + a^2*c*e^4)*sqrt(-(a*c^2*
d^4 - 2*a^2*c*d^2*e^2 + a^3*e^4)/(c^7*d^8 + 4*a*c^6*d^6*e^2 + 6*a^2*c^5*d^4
*e^4 + 4*a^3*c^4*d^2*e^6 + a^4*c^3*e^8)))/(c^3*d^4 + 2*a*c^2*d^2*e^2 + a^2*
c*e^4))) + 2*d*sqrt(-d/e)*log((e*x^2 + 2*e*x*sqrt(-d/e) - d)/(e*x^2 + d)))/
(c*d^2 + a*e^2), 1/4*(4*d*sqrt(d/e)*arctan(e*x*sqrt(d/e)/d) + (c*d^2 + a*e^
2)*sqrt((2*a*d*e + (c^3*d^4 + 2*a*c^2*d^2*e^2 + a^2*c*e^4)*sqrt(-(a*c^2*d^4
 - 2*a^2*c*d^2*e^2 + a^3*e^4)/(c^7*d^8 + 4*a*c^6*d^6*e^2 + 6*a^2*c^5*d^4*e^
4 + 4*a^3*c^4*d^2*e^6 + a^4*c^3*e^8)))/(c^3*d^4 + 2*a*c^2*d^2*e^2 + a^2*c*e
^4))*log(-(c*d^2 - a*e^2)*x + (c^2*d^3 - a*c*d*e^2 + (c^4*d^4*e + 2*a*c^3*d
^2*e^3 + a^2*c^2*e^5)*sqrt(-(a*c^2*d^4 - 2*a^2*c*d^2*e^2 + a^3*e^4)/(c^7*d^
8 + 4*a*c^6*d^6*e^2 + 6*a^2*c^5*d^4*e^4 + 4*a^3*c^4*d^2*e^6 + a^4*c^3*e^8))
)*sqrt((2*a*d*e + (c^3*d^4 + 2*a*c^2*d^2*e^2 + a^2*c*e^4)*sqrt(-(a*c^2*d^4
- 2*a^2*c*d^2*e^2 + a^3*e^4)/(c^7*d^8 + 4*a*c^6*d^6*e^2 + 6*a^2*c^5*d^4*e^4
 + 4*a^3*c^4*d^2*e^6 + a^4*c^3*e^8)))/(c^3*d^4 + 2*a*c^2*d^2*e^2 + a^2*c*e^
```

```
4))) - (c*d^2 + a*e^2)*sqrt((2*a*d*e + (c^3*d^4 + 2*a*c^2*d^2*e^2 + a^2*c*e
^4)*sqrt(-(a*c^2*d^4 - 2*a^2*c*d^2*e^2 + a^3*e^4)/(c^7*d^8 + 4*a*c^6*d^6*e^
2 + 6*a^2*c^5*d^4*e^4 + 4*a^3*c^4*d^2*e^6 + a^4*c^3*e^8)))/(c^3*d^4 + 2*a*c
^2*d^2*e^2 + a^2*c*e^4))*log(-(c*d^2 - a*e^2)*x - (c^2*d^3 - a*c*d*e^2 + (c
^4*d^4*e + 2*a*c^3*d^2*e^3 + a^2*c^2*e^5)*sqrt(-(a*c^2*d^4 - 2*a^2*c*d^2*e^
2 + a^3*e^4)/(c^7*d^8 + 4*a*c^6*d^6*e^2 + 6*a^2*c^5*d^4*e^4 + 4*a^3*c^4*d^2
*e^6 + a^4*c^3*e^8)))*sqrt((2*a*d*e + (c^3*d^4 + 2*a*c^2*d^2*e^2 + a^2*c*e^
4)*sqrt(-(a*c^2*d^4 - 2*a^2*c*d^2*e^2 + a^3*e^4)/(c^7*d^8 + 4*a*c^6*d^6*e^2
 + 6*a^2*c^5*d^4*e^4 + 4*a^3*c^4*d^2*e^6 + a^4*c^3*e^8)))/(c^3*d^4 + 2*a*c^
2*d^2*e^2 + a^2*c*e^4))) + (c*d^2 + a*e^2)*sqrt((2*a*d*e - (c^3*d^4 + 2*a*c
^2*d^2*e^2 + a^2*c*e^4)*sqrt(-(a*c^2*d^4 - 2*a^2*c*d^2*e^2 + a^3*e^4)/(c^7*
d^8 + 4*a*c^6*d^6*e^2 + 6*a^2*c^5*d^4*e^4 + 4*a^3*c^4*d^2*e^6 + a^4*c^3*e^8
)))/(c^3*d^4 + 2*a*c^2*d^2*e^2 + a^2*c*e^4))*log(-(c*d^2 - a*e^2)*x + (c^2*
d^3 - a*c*d*e^2 - (c^4*d^4*e + 2*a*c^3*d^2*e^3 + a^2*c^2*e^5)*sqrt(-(a*c^2*
d^4 - 2*a^2*c*d^2*e^2 + a^3*e^4)/(c^7*d^8 + 4*a*c^6*d^6*e^2 + 6*a^2*c^5*d^4
*e^4 + 4*a^3*c^4*d^2*e^6 + a^4*c^3*e^8)))*sqrt((2*a*d*e - (c^3*d^4 + 2*a*c^
2*d^2*e^2 + a^2*c*e^4)*sqrt(-(a*c^2*d^4 - 2*a^2*c*d^2*e^2 + a^3*e^4)/(c^7*d
^8 + 4*a*c^6*d^6*e^2 + 6*a^2*c^5*d^4*e^4 + 4*a^3*c^4*d^2*e^6 + a^4*c^3*e^8)
))/(c^3*d^4 + 2*a*c^2*d^2*e^2 + a^2*c*e^4))) - (c*d^2 + a*e^2)*sqrt((2*a*d*
e - (c^3*d^4 + 2*a*c^2*d^2*e^2 + a^2*c*e^4)*sqrt(-(a*c^2*d^4 - 2*a^2*c*d^2*
e^2 + a^3*e^4)/(c^7*d^8 + 4*a*c^6*d^6*e^2 + 6*a^2*c^5*d^4*e^4 + 4*a^3*c^4*d
^2*e^6 + a^4*c^3*e^8)))/(c^3*d^4 + 2*a*c^2*d^2*e^2 + a^2*c*e^4))*log(-(c*d^
2 - a*e^2)*x - (c^2*d^3 - a*c*d*e^2 - (c^4*d^4*e + 2*a*c^3*d^2*e^3 + a^2*c^
2*e^5)*sqrt(-(a*c^2*d^4 - 2*a^2*c*d^2*e^2 + a^3*e^4)/(c^7*d^8 + 4*a*c^6*d^6
*e^2 + 6*a^2*c^5*d^4*e^4 + 4*a^3*c^4*d^2*e^6 + a^4*c^3*e^8)))*sqrt((2*a*d*e
 - (c^3*d^4 + 2*a*c^2*d^2*e^2 + a^2*c*e^4)*sqrt(-(a*c^2*d^4 - 2*a^2*c*d^2*e
^2 + a^3*e^4)/(c^7*d^8 + 4*a*c^6*d^6*e^2 + 6*a^2*c^5*d^4*e^4 + 4*a^3*c^4*d^
2*e^6 + a^4*c^3*e^8)))/(c^3*d^4 + 2*a*c^2*d^2*e^2 + a^2*c*e^4))))/(c*d^2 +
a*e^2)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^4}{(d+ex^2)(a+cx^4)}\,dx = \text{Timed out}$$

```
[In] integrate(x**4/(e*x**2+d)/(c*x**4+a),x)
```

```
[Out] Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^4}{(d+ex^2)(a+cx^4)}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(x^4/(e*x^2+d)/(c*x^4+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 332, normalized size of antiderivative = 0.99

$$\int \frac{x^4}{(d+ex^2)(a+cx^4)}\,dx = \frac{d^2\arctan\left(\frac{ex}{\sqrt{de}}\right)}{(cd^2+ae^2)\sqrt{de}} - \frac{\left((ac^3)^{\frac{1}{4}}c^2d-(ac^3)^{\frac{3}{4}}e\right)\arctan\left(\frac{\sqrt{2}\left(2x+\sqrt{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{2\left(\sqrt{2}c^4d^2+\sqrt{2}ac^3e^2\right)} - \frac{\left((ac^3)^{\frac{1}{4}}c^2d-(ac^3)^{\frac{3}{4}}e\right)\arctan\left(\frac{\sqrt{2}\left(2x-\sqrt{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{2\left(\sqrt{2}c^4d^2+\sqrt{2}ac^3e^2\right)} - \frac{\left((ac^3)^{\frac{1}{4}}c^2d+(ac^3)^{\frac{3}{4}}e\right)\log\left(x^2+\sqrt{2}x\left(\frac{a}{c}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{c}}\right)}{4\left(\sqrt{2}c^4d^2+\sqrt{2}ac^3e^2\right)} + \frac{\left((ac^3)^{\frac{1}{4}}c^2d+(ac^3)^{\frac{3}{4}}e\right)\log\left(x^2-\sqrt{2}x\left(\frac{a}{c}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{c}}\right)}{4\left(\sqrt{2}c^4d^2+\sqrt{2}ac^3e^2\right)}$$

```
[In] integrate(x^4/(e*x^2+d)/(c*x^4+a),x, algorithm="giac")
```

```
[Out] d^2*arctan(e*x/sqrt(d*e))/((c*d^2 + a*e^2)*sqrt(d*e)) - 1/2*((a*c^3)^(1/4)*
c^2*d - (a*c^3)^(3/4)*e)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(a/c)^(1/4))/(a/
c)^(1/4))/(sqrt(2)*c^4*d^2 + sqrt(2)*a*c^3*e^2) - 1/2*((a*c^3)^(1/4)*c^2*d
- (a*c^3)^(3/4)*e)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(a/c)^(1/4))/(a/c)^(1/
4))/(sqrt(2)*c^4*d^2 + sqrt(2)*a*c^3*e^2) - 1/4*((a*c^3)^(1/4)*c^2*d + (a*c
^3)^(3/4)*e)*log(x^2 + sqrt(2)*x*(a/c)^(1/4) + sqrt(a/c))/(sqrt(2)*c^4*d^2
+ sqrt(2)*a*c^3*e^2) + 1/4*((a*c^3)^(1/4)*c^2*d + (a*c^3)^(3/4)*e)*log(x^2
- sqrt(2)*x*(a/c)^(1/4) + sqrt(a/c))/(sqrt(2)*c^4*d^2 + sqrt(2)*a*c^3*e^2)
```

Mupad [B] (verification not implemented)

Time = 8.65 (sec) , antiderivative size = 5111, normalized size of antiderivative = 15.21

$$\int \frac{x^4}{(d+ex^2)(a+cx^4)}\,dx = \text{Too large to display}$$

```
[In] int(x^4/((a + c*x^4)*(d + e*x^2)),x)
```

```
[Out] atan(-((((a*e^2*(-a*c^3)^(1/2) - c*d^2*(-a*c^3)^(1/2) + 2*a*c^2*d*e)/(16*(c
^5*d^4 + a^2*c^3*e^4 + 2*a*c^4*d^2*e^2)))^(1/2)*((x*(112*a^4*c^3*d*e^6 + 11
2*a^2*c^5*d^5*e^2 - 32*a^3*c^4*d^3*e^4) + ((a*e^2*(-a*c^3)^(1/2) - c*d^2*(-
a*c^3)^(1/2) + 2*a*c^2*d*e)/(16*(c^5*d^4 + a^2*c^3*e^4 + 2*a*c^4*d^2*e^2)))
^(1/2)*(64*a^2*c^6*d^6*e^2 - x*((a*e^2*(-a*c^3)^(1/2) - c*d^2*(-a*c^3)^(1/2
) + 2*a*c^2*d*e)/(16*(c^5*d^4 + a^2*c^3*e^4 + 2*a*c^4*d^2*e^2)))^(1/2)*(512
*a^5*c^4*e^9 - 512*a^2*c^7*d^6*e^3 - 512*a^3*c^6*d^4*e^5 + 512*a^4*c^5*d^2*
e^7) + 128*a^3*c^5*d^4*e^4 + 64*a^4*c^4*d^2*e^6))*((a*e^2*(-a*c^3)^(1/2) -
c*d^2*(-a*c^3)^(1/2) + 2*a*c^2*d*e)/(16*(c^5*d^4 + a^2*c^3*e^4 + 2*a*c^4*d^
2*e^2)))^(1/2) + 16*a^2*c^4*d^5*e + 4*a^4*c^2*d*e^5 - 60*a^3*c^3*d^3*e^3) -
 x*(2*a^4*c*e^5 + 4*a^2*c^3*d^4*e))*((a*e^2*(-a*c^3)^(1/2) - c*d^2*(-a*c^3)
^(1/2) + 2*a*c^2*d*e)/(16*(c^5*d^4 + a^2*c^3*e^4 + 2*a*c^4*d^2*e^2)))^(1/2)
*1i + (((a*e^2*(-a*c^3)^(1/2) - c*d^2*(-a*c^3)^(1/2) + 2*a*c^2*d*e)/(16*(c^
5*d^4 + a^2*c^3*e^4 + 2*a*c^4*d^2*e^2)))^(1/2)*((x*(112*a^4*c^3*d*e^6 + 112
*a^2*c^5*d^5*e^2 - 32*a^3*c^4*d^3*e^4) - ((a*e^2*(-a*c^3)^(1/2) - c*d^2*(-a
*c^3)^(1/2) + 2*a*c^2*d*e)/(16*(c^5*d^4 + a^2*c^3*e^4 + 2*a*c^4*d^2*e^2)))^
(1/2)*(x*((a*e^2*(-a*c^3)^(1/2) - c*d^2*(-a*c^3)^(1/2) + 2*a*c^2*d*e)/(16*(
c^5*d^4 + a^2*c^3*e^4 + 2*a*c^4*d^2*e^2)))^(1/2)*(512*a^5*c^4*e^9 - 512*a^2
*c^7*d^6*e^3 - 512*a^3*c^6*d^4*e^5 + 512*a^4*c^5*d^2*e^7) + 64*a^2*c^6*d^6*
e^2 + 128*a^3*c^5*d^4*e^4 + 64*a^4*c^4*d^2*e^6))*((a*e^2*(-a*c^3)^(1/2) - c
*d^2*(-a*c^3)^(1/2) + 2*a*c^2*d*e)/(16*(c^5*d^4 + a^2*c^3*e^4 + 2*a*c^4*d^2
*e^2)))^(1/2) - 16*a^2*c^4*d^5*e - 4*a^4*c^2*d*e^5 + 60*a^3*c^3*d^3*e^3) -
x*(2*a^4*c*e^5 + 4*a^2*c^3*d^4*e))*((a*e^2*(-a*c^3)^(1/2) - c*d^2*(-a*c^3)^
(1/2) + 2*a*c^2*d*e)/(16*(c^5*d^4 + a^2*c^3*e^4 + 2*a*c^4*d^2*e^2)))^(1/2)*
1i)/((((a*e^2*(-a*c^3)^(1/2) - c*d^2*(-a*c^3)^(1/2) + 2*a*c^2*d*e)/(16*(c^5
*d^4 + a^2*c^3*e^4 + 2*a*c^4*d^2*e^2)))^(1/2)*((x*(112*a^4*c^3*d*e^6 + 112*
a^2*c^5*d^5*e^2 - 32*a^3*c^4*d^3*e^4) + ((a*e^2*(-a*c^3)^(1/2) - c*d^2*(-a*
c^3)^(1/2) + 2*a*c^2*d*e)/(16*(c^5*d^4 + a^2*c^3*e^4 + 2*a*c^4*d^2*e^2)))^(
1/2)*(64*a^2*c^6*d^6*e^2 - x*((a*e^2*(-a*c^3)^(1/2) - c*d^2*(-a*c^3)^(1/2)
+ 2*a*c^2*d*e)/(16*(c^5*d^4 + a^2*c^3*e^4 + 2*a*c^4*d^2*e^2)))^(1/2)*(512*a
^5*c^4*e^9 - 512*a^2*c^7*d^6*e^3 - 512*a^3*c^6*d^4*e^5 + 512*a^4*c^5*d^2*e^
7) + 128*a^3*c^5*d^4*e^4 + 64*a^4*c^4*d^2*e^6))*((a*e^2*(-a*c^3)^(1/2) - c*
d^2*(-a*c^3)^(1/2) + 2*a*c^2*d*e)/(16*(c^5*d^4 + a^2*c^3*e^4 + 2*a*c^4*d^2*
e^2)))^(1/2) + 16*a^2*c^4*d^5*e + 4*a^4*c^2*d*e^5 - 60*a^3*c^3*d^3*e^3) - x
*(2*a^4*c*e^5 + 4*a^2*c^3*d^4*e))*((a*e^2*(-a*c^3)^(1/2) - c*d^2*(-a*c^3)^(
1/2) + 2*a*c^2*d*e)/(16*(c^5*d^4 + a^2*c^3*e^4 + 2*a*c^4*d^2*e^2)))^(1/2) -
```

```
(((a*e^2*(-a*c^3)^(1/2) - c*d^2*(-a*c^3)^(1/2) + 2*a*c^2*d*e)/(16*(c^5*d^4
 + a^2*c^3*e^4 + 2*a*c^4*d^2*e^2)))^(1/2)*((x*(112*a^4*c^3*d*e^6 + 112*a^2*
c^5*d^5*e^2 - 32*a^3*c^4*d^3*e^4) - ((a*e^2*(-a*c^3)^(1/2) - c*d^2*(-a*c^3)
^(1/2) + 2*a*c^2*d*e)/(16*(c^5*d^4 + a^2*c^3*e^4 + 2*a*c^4*d^2*e^2)))^(1/2)
*(x*((a*e^2*(-a*c^3)^(1/2) - c*d^2*(-a*c^3)^(1/2) + 2*a*c^2*d*e)/(16*(c^5*d
^4 + a^2*c^3*e^4 + 2*a*c^4*d^2*e^2)))^(1/2)*(512*a^5*c^4*e^9 - 512*a^2*c^7*
d^6*e^3 - 512*a^3*c^6*d^4*e^5 + 512*a^4*c^5*d^2*e^7) + 64*a^2*c^6*d^6*e^2 +
 128*a^3*c^5*d^4*e^4 + 64*a^4*c^4*d^2*e^6))*((a*e^2*(-a*c^3)^(1/2) - c*d^2*
(-a*c^3)^(1/2) + 2*a*c^2*d*e)/(16*(c^5*d^4 + a^2*c^3*e^4 + 2*a*c^4*d^2*e^2)
))^(1/2) - 16*a^2*c^4*d^5*e - 4*a^4*c^2*d*e^5 + 60*a^3*c^3*d^3*e^3) - x*(2*
a^4*c*e^5 + 4*a^2*c^3*d^4*e))*((a*e^2*(-a*c^3)^(1/2) - c*d^2*(-a*c^3)^(1/2)
 + 2*a*c^2*d*e)/(16*(c^5*d^4 + a^2*c^3*e^4 + 2*a*c^4*d^2*e^2)))^(1/2) + 2*a
^3*c*d^2*e^2))*((a*e^2*(-a*c^3)^(1/2) - c*d^2*(-a*c^3)^(1/2) + 2*a*c^2*d*e)
/(16*(c^5*d^4 + a^2*c^3*e^4 + 2*a*c^4*d^2*e^2)))^(1/2)*2i + atan(-((((c*d^2
*(-a*c^3)^(1/2) - a*e^2*(-a*c^3)^(1/2) + 2*a*c^2*d*e)/(16*(c^5*d^4 + a^2*c^
3*e^4 + 2*a*c^4*d^2*e^2)))^(1/2)*((x*(112*a^4*c^3*d*e^6 + 112*a^2*c^5*d^5*e
^2 - 32*a^3*c^4*d^3*e^4) + ((c*d^2*(-a*c^3)^(1/2) - a*e^2*(-a*c^3)^(1/2) +
2*a*c^2*d*e)/(16*(c^5*d^4 + a^2*c^3*e^4 + 2*a*c^4*d^2*e^2)))^(1/2)*(64*a^2*
c^6*d^6*e^2 - x*((c*d^2*(-a*c^3)^(1/2) - a*e^2*(-a*c^3)^(1/2) + 2*a*c^2*d*e
)/(16*(c^5*d^4 + a^2*c^3*e^4 + 2*a*c^4*d^2*e^2)))^(1/2)*(512*a^5*c^4*e^9 -
512*a^2*c^7*d^6*e^3 - 512*a^3*c^6*d^4*e^5 + 512*a^4*c^5*d^2*e^7) + 128*a^3*
c^5*d^4*e^4 + 64*a^4*c^4*d^2*e^6))*((c*d^2*(-a*c^3)^(1/2) - a*e^2*(-a*c^3)^
(1/2) + 2*a*c^2*d*e)/(16*(c^5*d^4 + a^2*c^3*e^4 + 2*a*c^4*d^2*e^2)))^(1/2)
+ 16*a^2*c^4*d^5*e + 4*a^4*c^2*d*e^5 - 60*a^3*c^3*d^3*e^3) - x*(2*a^4*c*e^5
 + 4*a^2*c^3*d^4*e))*((c*d^2*(-a*c^3)^(1/2) - a*e^2*(-a*c^3)^(1/2) + 2*a*c^
2*d*e)/(16*(c^5*d^4 + a^2*c^3*e^4 + 2*a*c^4*d^2*e^2)))^(1/2)*1i + (((c*d^2*
(-a*c^3)^(1/2) - a*e^2*(-a*c^3)^(1/2) + 2*a*c^2*d*e)/(16*(c^5*d^4 + a^2*c^3
*e^4 + 2*a*c^4*d^2*e^2)))^(1/2)*((x*(112*a^4*c^3*d*e^6 + 112*a^2*c^5*d^5*e^
2 - 32*a^3*c^4*d^3*e^4) - ((c*d^2*(-a*c^3)^(1/2) - a*e^2*(-a*c^3)^(1/2) + 2
*a*c^2*d*e)/(16*(c^5*d^4 + a^2*c^3*e^4 + 2*a*c^4*d^2*e^2)))^(1/2)*(x*((c*d^
2*(-a*c^3)^(1/2) - a*e^2*(-a*c^3)^(1/2) + 2*a*c^2*d*e)/(16*(c^5*d^4 + a^2*c
^3*e^4 + 2*a*c^4*d^2*e^2)))^(1/2)*(512*a^5*c^4*e^9 - 512*a^2*c^7*d^6*e^3 -
512*a^3*c^6*d^4*e^5 + 512*a^4*c^5*d^2*e^7) + 64*a^2*c^6*d^6*e^2 + 128*a^3*c
^5*d^4*e^4 + 64*a^4*c^4*d^2*e^6))*((c*d^2*(-a*c^3)^(1/2) - a*e^2*(-a*c^3)^(
1/2) + 2*a*c^2*d*e)/(16*(c^5*d^4 + a^2*c^3*e^4 + 2*a*c^4*d^2*e^2)))^(1/2) -
 16*a^2*c^4*d^5*e - 4*a^4*c^2*d*e^5 + 60*a^3*c^3*d^3*e^3) - x*(2*a^4*c*e^5
+ 4*a^2*c^3*d^4*e))*((c*d^2*(-a*c^3)^(1/2) - a*e^2*(-a*c^3)^(1/2) + 2*a*c^2
*d*e)/(16*(c^5*d^4 + a^2*c^3*e^4 + 2*a*c^4*d^2*e^2)))^(1/2)*1i)/((((c*d^2*(
-a*c^3)^(1/2) - a*e^2*(-a*c^3)^(1/2) + 2*a*c^2*d*e)/(16*(c^5*d^4 + a^2*c^3*
e^4 + 2*a*c^4*d^2*e^2)))^(1/2)*((x*(112*a^4*c^3*d*e^6 + 112*a^2*c^5*d^5*e^2
 - 32*a^3*c^4*d^3*e^4) + ((c*d^2*(-a*c^3)^(1/2) - a*e^2*(-a*c^3)^(1/2) + 2*
a*c^2*d*e)/(16*(c^5*d^4 + a^2*c^3*e^4 + 2*a*c^4*d^2*e^2)))^(1/2)*(64*a^2*c^
6*d^6*e^2 - x*((c*d^2*(-a*c^3)^(1/2) - a*e^2*(-a*c^3)^(1/2) + 2*a*c^2*d*e)/
(16*(c^5*d^4 + a^2*c^3*e^4 + 2*a*c^4*d^2*e^2)))^(1/2)*(512*a^5*c^4*e^9 - 51
2*a^2*c^7*d^6*e^3 - 512*a^3*c^6*d^4*e^5 + 512*a^4*c^5*d^2*e^7) + 128*a^3*c^
```

5*d^4*e^4 + 64*a^4*c^4*d^2*e^6))*((c*d^2*(-a*c^3)^(1/2) - a*e^2*(-a*c^3)^(1/2) + 2*a*c^2*d*e)/(16*(c^5*d^4 + a^2*c^3*e^4 + 2*a*c^4*d^2*e^2)))^(1/2) + 16*a^2*c^4*d^5*e + 4*a^4*c^2*d*e^5 - 60*a^3*c^3*d^3*e^3) - x*(2*a^4*c*e^5 + 4*a^2*c^3*d^4*e))*((c*d^2*(-a*c^3)^(1/2) - a*e^2*(-a*c^3)^(1/2) + 2*a*c^2*d*e)/(16*(c^5*d^4 + a^2*c^3*e^4 + 2*a*c^4*d^2*e^2)))^(1/2) - (((c*d^2*(-a*c^3)^(1/2) - a*e^2*(-a*c^3)^(1/2) + 2*a*c^2*d*e)/(16*(c^5*d^4 + a^2*c^3*e^4 + 2*a*c^4*d^2*e^2)))^(1/2)*((x*(112*a^4*c^3*d*e^6 + 112*a^2*c^5*d^5*e^2 - 32*a^3*c^4*d^3*e^4) - ((c*d^2*(-a*c^3)^(1/2) - a*e^2*(-a*c^3)^(1/2) + 2*a*c^2*d*e)/(16*(c^5*d^4 + a^2*c^3*e^4 + 2*a*c^4*d^2*e^2)))^(1/2)*(x*((c*d^2*(-a*c^3)^(1/2) - a*e^2*(-a*c^3)^(1/2) + 2*a*c^2*d*e)/(16*(c^5*d^4 + a^2*c^3*e^4 + 2*a*c^4*d^2*e^2)))^(1/2)*(512*a^5*c^4*e^9 - 512*a^2*c^7*d^6*e^3 - 512*a^3*c^6*d^4*e^5 + 512*a^4*c^5*d^2*e^7) + 64*a^2*c^6*d^6*e^2 + 128*a^3*c^5*d^4*e^4 + 64*a^4*c^4*d^2*e^6))*((c*d^2*(-a*c^3)^(1/2) - a*e^2*(-a*c^3)^(1/2) + 2*a*c^2*d*e)/(16*(c^5*d^4 + a^2*c^3*e^4 + 2*a*c^4*d^2*e^2)))^(1/2) - 16*a^2*c^4*d^5*e - 4*a^4*c^2*d*e^5 + 60*a^3*c^3*d^3*e^3) - x*(2*a^4*c*e^5 + 4*a^2*c^3*d^4*e))*((c*d^2*(-a*c^3)^(1/2) - a*e^2*(-a*c^3)^(1/2) + 2*a*c^2*d*e)/(16*(c^5*d^4 + a^2*c^3*e^4 + 2*a*c^4*d^2*e^2)))^(1/2) + 2*a^3*c*d^2*e^2))*((c*d^2*(-a*c^3)^(1/2) - a*e^2*(-a*c^3)^(1/2) + 2*a*c^2*d*e)/(16*(c^5*d^4 + a^2*c^3*e^4 + 2*a*c^4*d^2*e^2)))^(1/2)*2i - (log(16*c^3*x*(-d^3*e)^(5/2) + a^3*d^2*e^8 + 16*c^3*d^8*e^2 + 17*a*c^2*d^6*e^4 + 2*a^2*c*d^4*e^6 + a^3*e^8*x*(-d^3*e)^(1/2) - 17*a*c^2*d*e^3*x*(-d^3*e)^(3/2) + 2*a^2*c*d^2*e^6*x*(-d^3*e)^(1/2))*(-d^3*e)^(1/2))/(2*(a*e^3 + c*d^2*e)) + (log(a^3*d^2*e^8 - 16*c^3*x*(-d^3*e)^(5/2) + 16*c^3*d^8*e^2 + 17*a*c^2*d^6*e^4 + 2*a^2*c*d^4*e^6 - a^3*e^8*x*(-d^3*e)^(1/2) + 17*a*c^2*d*e^3*x*(-d^3*e)^(3/2) - 2*a^2*c*d^2*e^6*x*(-d^3*e)^(1/2))*(-d^3*e)^(1/2))/(2*a*e^3 + 2*c*d^2*e)

3.240 $\int \frac{x^2}{(d+ex^2)(a+cx^4)} dx$

Optimal result

Integrand size = 22, antiderivative size = 337

$$\begin{aligned}\int \frac{x^2}{(d+ex^2)(a+cx^4)} dx = & -\frac{\sqrt{d}\sqrt{e}\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{cd^2+ae^2} - \frac{\left(\sqrt{c}d+\sqrt{a}e\right)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}\,(cd^2+ae^2)} \\ & + \frac{\left(\sqrt{c}d+\sqrt{a}e\right)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}\,(cd^2+ae^2)} \\ & + \frac{\left(\sqrt{c}d-\sqrt{a}e\right)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)}{4\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}\,(cd^2+ae^2)} \\ & - \frac{\left(\sqrt{c}d-\sqrt{a}e\right)\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)}{4\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}\,(cd^2+ae^2)}\end{aligned}$$

```
[Out] 1/8*ln(-a^(1/4)*c^(1/4)*x*2^(1/2)+a^(1/2)+x^2*c^(1/2))*(-e*a^(1/2)+d*c^(1/2
))/a^(1/4)/c^(1/4)/(a*e^2+c*d^2)*2^(1/2)-1/8*ln(a^(1/4)*c^(1/4)*x*2^(1/2)+a
^(1/2)+x^2*c^(1/2))*(-e*a^(1/2)+d*c^(1/2))/a^(1/4)/c^(1/4)/(a*e^2+c*d^2)*2^
(1/2)+1/4*arctan(-1+c^(1/4)*x*2^(1/2)/a^(1/4))*(e*a^(1/2)+d*c^(1/2))/a^(1/4
)/c^(1/4)/(a*e^2+c*d^2)*2^(1/2)+1/4*arctan(1+c^(1/4)*x*2^(1/2)/a^(1/4))*(e*
a^(1/2)+d*c^(1/2))/a^(1/4)/c^(1/4)/(a*e^2+c*d^2)*2^(1/2)-arctan(x*e^(1/2)/d
^(1/2))*d^(1/2)*e^(1/2)/(a*e^2+c*d^2)
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 337, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {1302, 211, 1182, 1176, 631, 210, 1179, 642}

$$\int \frac{x^2}{(d+ex^2)(a+cx^4)}\,dx = -\frac{\sqrt{d}\sqrt{e}\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{ae^2+cd^2} - \frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)(\sqrt{a}e+\sqrt{c}d)}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}(ae^2+cd^2)} + \frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)(\sqrt{a}e+\sqrt{c}d)}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}(ae^2+cd^2)} + \frac{(\sqrt{c}d-\sqrt{a}e)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{4\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}(ae^2+cd^2)} - \frac{(\sqrt{c}d-\sqrt{a}e)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{4\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}(ae^2+cd^2)}$$

```
[In] Int[x^2/((d + e*x^2)*(a + c*x^4)),x]
```

```
[Out] -((Sqrt[d]*Sqrt[e]*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(c*d^2 + a*e^2)) - ((Sqrt[c
]*d + Sqrt[a]*e)*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(1/4
)*c^(1/4)*(c*d^2 + a*e^2)) + ((Sqrt[c]*d + Sqrt[a]*e)*ArcTan[1 + (Sqrt[2]*c
^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(1/4)*c^(1/4)*(c*d^2 + a*e^2)) + ((Sqrt[c]
*d - Sqrt[a]*e)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(4*
Sqrt[2]*a^(1/4)*c^(1/4)*(c*d^2 + a*e^2)) - ((Sqrt[c]*d - Sqrt[a]*e)*Log[Sqr
t[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(4*Sqrt[2]*a^(1/4)*c^(1/4)
*(c*d^2 + a*e^2))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1182

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[(-a)
*c]
```

Rule 1302

```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.))/((a_) + (c_.)*(x_)^4),
 x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*((d + e*x^2)^q/(a + c*x^4)), x],
x] /; FreeQ[{a, c, d, e, f, m}, x] && IntegerQ[q] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(-\frac{de}{(cd^2+ae^2)\,(d+ex^2)}+\frac{ae+cdx^2}{(cd^2+ae^2)\,(a+cx^4)}\right)dx \\
&= \frac{\int \frac{ae+cdx^2}{a+cx^4}\,dx}{cd^2+ae^2}-\frac{(de)\int \frac{1}{d+ex^2}\,dx}{cd^2+ae^2} \\
&= -\frac{\sqrt{d}\sqrt{e}\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{cd^2+ae^2}-\frac{\left(d-\frac{\sqrt{a}e}{\sqrt{c}}\right)\int \frac{\sqrt{a}\sqrt{c}-cx^2}{a+cx^4}\,dx}{2\,(cd^2+ae^2)}+\frac{\left(d+\frac{\sqrt{a}e}{\sqrt{c}}\right)\int \frac{\sqrt{a}\sqrt{c}+cx^2}{a+cx^4}\,dx}{2\,(cd^2+ae^2)}
\end{aligned}$$

$$= -\frac{\sqrt{d}\sqrt{e}\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{cd^2+ae^2} + \frac{\left(\sqrt[4]{c}\left(d-\frac{\sqrt{a}e}{\sqrt{c}}\right)\right)\int \frac{\frac{\sqrt{2}\sqrt[4]{a}}{\sqrt[4]{c}}+2x}{-\frac{\sqrt{a}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}-x^2}\,dx}{4\sqrt{2}\sqrt[4]{a}\,(cd^2+ae^2)}$$

$$+ \frac{\left(\sqrt[4]{c}\left(d-\frac{\sqrt{a}e}{\sqrt{c}}\right)\right)\int \frac{\frac{\sqrt{2}\sqrt[4]{a}}{\sqrt[4]{c}}-2x}{-\frac{\sqrt{a}}{\sqrt{c}}+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}-x^2}\,dx}{4\sqrt{2}\sqrt[4]{a}\,(cd^2+ae^2)}$$

$$+ \frac{\left(d+\frac{\sqrt{a}e}{\sqrt{c}}\right)\int \frac{1}{\frac{\sqrt{a}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}+x^2}\,dx}{4\,(cd^2+ae^2)} + \frac{\left(d+\frac{\sqrt{a}e}{\sqrt{c}}\right)\int \frac{1}{\frac{\sqrt{a}}{\sqrt{c}}+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}+x^2}\,dx}{4\,(cd^2+ae^2)}$$

$$= -\frac{\sqrt{d}\sqrt{e}\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{cd^2+ae^2} + \frac{\sqrt[4]{c}\left(d-\frac{\sqrt{a}e}{\sqrt{c}}\right)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)}{4\sqrt{2}\sqrt[4]{a}\,(cd^2+ae^2)}$$

$$- \frac{\sqrt[4]{c}\left(d-\frac{\sqrt{a}e}{\sqrt{c}}\right)\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)}{4\sqrt{2}\sqrt[4]{a}\,(cd^2+ae^2)}$$

$$+ \frac{\left(\sqrt[4]{c}\left(d+\frac{\sqrt{a}e}{\sqrt{c}}\right)\right)\mathrm{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}\sqrt[4]{a}\,(cd^2+ae^2)}$$

$$- \frac{\left(\sqrt[4]{c}\left(d+\frac{\sqrt{a}e}{\sqrt{c}}\right)\right)\mathrm{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1+\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}\sqrt[4]{a}\,(cd^2+ae^2)}$$

$$= -\frac{\sqrt{d}\sqrt{e}\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{cd^2+ae^2} - \frac{\sqrt[4]{c}\left(d+\frac{\sqrt{a}e}{\sqrt{c}}\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}\sqrt[4]{a}\,(cd^2+ae^2)}$$

$$+ \frac{\sqrt[4]{c}\left(d+\frac{\sqrt{a}e}{\sqrt{c}}\right)\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}\sqrt[4]{a}\,(cd^2+ae^2)}$$

$$+ \frac{\sqrt[4]{c}\left(d-\frac{\sqrt{a}e}{\sqrt{c}}\right)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)}{4\sqrt{2}\sqrt[4]{a}\,(cd^2+ae^2)}$$

$$- \frac{\sqrt[4]{c}\left(d-\frac{\sqrt{a}e}{\sqrt{c}}\right)\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)}{4\sqrt{2}\sqrt[4]{a}\,(cd^2+ae^2)}$$

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 232, normalized size of antiderivative = 0.69

$$\int \frac{x^2}{(d+ex^2)(a+cx^4)}\,dx = \frac{-8\sqrt[4]{a}\sqrt[4]{c}\sqrt{d}\sqrt{e}\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)+\sqrt{2}\left(-2\left(\sqrt{c}d+\sqrt{a}e\right)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)+2\left(\sqrt{c}d+\sqrt{a}e\right)\arctan\left(1+\right.\right.}{8\sqrt[4]{a}\sqrt[4]{c}\,(cd^2+}$$

[In] Integrate[x^2/((d + e*x^2)*(a + c*x^4)),x]

[Out] (-8*a^(1/4)*c^(1/4)*Sqrt[d]*Sqrt[e]*ArcTan[(Sqrt[e]*x)/Sqrt[d]] + Sqrt[2]*(-2*(Sqrt[c]*d + Sqrt[a]*e)*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)] + 2*(Sqrt[c]*d + Sqrt[a]*e)*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)] + (Sqrt[c]*d - Sqrt[a]*e)*(Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2] - Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])))/(8*a^(1/4)*c^(1/4)*(c*d^2 + a*e^2))

Maple [A] (verified)

Time = 0.39 (sec) , antiderivative size = 246, normalized size of antiderivative = 0.73

method	result
default	$\frac{\frac{e\left(\frac{a}{c}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x^2+\left(\frac{a}{c}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{c}}}{x^2-\left(\frac{a}{c}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{c}}}\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{c}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{c}\right)^{\frac{1}{4}}}-1\right)\right)}{8}+\frac{d\sqrt{2}\left(\ln\left(\frac{x^2-\left(\frac{a}{c}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{c}}}{x^2+\left(\frac{a}{c}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{c}}}\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{c}\right)^{\frac{1}{4}}}+1\right)\right.}{8\left(\frac{a}{c}\right)^{\frac{1}{4}}}}{a\,e^2+c\,d^2}$
risch	$\frac{\sqrt{-ed}\,\ln\left(\left(-16(-ed)^{\frac{5}{2}}a^2c\,e^3+16(-ed)^{\frac{5}{2}}a\,c^2d^2e-14a^2cd\,e^4(-ed)^{\frac{3}{2}}+20a\,c^2d^3e^2(-ed)^{\frac{3}{2}}+2d^5(-ed)^{\frac{3}{2}}c^3-\sqrt{-ed}\,a^3e^7+3\sqrt{-ed}\,a\,c^2d^4e\right.\right.}{2a\,e^2+2c\,d^2}$

[In] int(x^2/(e*x^2+d)/(c*x^4+a),x,method=_RETURNVERBOSE)

[Out] 1/(a*e^2+c*d^2)*(1/8*e*(a/c)^(1/4)*2^(1/2)*(ln((x^2+(a/c)^(1/4)*x*2^(1/2)+(a/c)^(1/2))/(x^2-(a/c)^(1/4)*x*2^(1/2)+(a/c)^(1/2)))+2*arctan(2^(1/2)/(a/c)^(1/4)*x+1)+2*arctan(2^(1/2)/(a/c)^(1/4)*x-1))+1/8*d/(a/c)^(1/4)*2^(1/2)*(ln((x^2-(a/c)^(1/4)*x*2^(1/2)+(a/c)^(1/2))/(x^2+(a/c)^(1/4)*x*2^(1/2)+(a/c)^(1/2)))+2*arctan(2^(1/2)/(a/c)^(1/4)*x+1)+2*arctan(2^(1/2)/(a/c)^(1/4)*x-1)))-d*e/(a*e^2+c*d^2)/(e*d)^(1/2)*arctan(e*x/(e*d)^(1/2))

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1936 vs. 2(248) = 496.

Time = 0.36 (sec) , antiderivative size = 3892, normalized size of antiderivative = 11.55

$$\int \frac{x^2}{(d+ex^2)(a+cx^4)} dx = \text{Too large to display}$$

```
[In] integrate(x^2/(e*x^2+d)/(c*x^4+a),x, algorithm="fricas")
[Out] [-1/4*((c*d^2 + a*e^2)*sqrt(-(2*d*e + (c^2*d^4 + 2*a*c*d^2*e^2 + a^2*e^4)*s
qrt(-(c^2*d^4 - 2*a*c*d^2*e^2 + a^2*e^4)/(a*c^5*d^8 + 4*a^2*c^4*d^6*e^2 + 6
*a^3*c^3*d^4*e^4 + 4*a^4*c^2*d^2*e^6 + a^5*c*e^8)))/(c^2*d^4 + 2*a*c*d^2*e^
2 + a^2*e^4))*log(-(c*d^2 - a*e^2)*x + (a*c*d^2*e - a^2*e^3 - (a*c^3*d^5 +
2*a^2*c^2*d^3*e^2 + a^3*c*d*e^4)*sqrt(-(c^2*d^4 - 2*a*c*d^2*e^2 + a^2*e^4)/
(a*c^5*d^8 + 4*a^2*c^4*d^6*e^2 + 6*a^3*c^3*d^4*e^4 + 4*a^4*c^2*d^2*e^6 + a^
5*c*e^8)))*sqrt(-(2*d*e + (c^2*d^4 + 2*a*c*d^2*e^2 + a^2*e^4)*sqrt(-(c^2*d^
4 - 2*a*c*d^2*e^2 + a^2*e^4)/(a*c^5*d^8 + 4*a^2*c^4*d^6*e^2 + 6*a^3*c^3*d^4
*e^4 + 4*a^4*c^2*d^2*e^6 + a^5*c*e^8)))/(c^2*d^4 + 2*a*c*d^2*e^2 + a^2*e^4)
)) - (c*d^2 + a*e^2)*sqrt(-(2*d*e + (c^2*d^4 + 2*a*c*d^2*e^2 + a^2*e^4)*sqr
t(-(c^2*d^4 - 2*a*c*d^2*e^2 + a^2*e^4)/(a*c^5*d^8 + 4*a^2*c^4*d^6*e^2 + 6*a
^3*c^3*d^4*e^4 + 4*a^4*c^2*d^2*e^6 + a^5*c*e^8)))/(c^2*d^4 + 2*a*c*d^2*e^2
+ a^2*e^4))*log(-(c*d^2 - a*e^2)*x - (a*c*d^2*e - a^2*e^3 - (a*c^3*d^5 + 2*
a^2*c^2*d^3*e^2 + a^3*c*d*e^4)*sqrt(-(c^2*d^4 - 2*a*c*d^2*e^2 + a^2*e^4)/(a
*c^5*d^8 + 4*a^2*c^4*d^6*e^2 + 6*a^3*c^3*d^4*e^4 + 4*a^4*c^2*d^2*e^6 + a^5*
c*e^8)))*sqrt(-(2*d*e + (c^2*d^4 + 2*a*c*d^2*e^2 + a^2*e^4)*sqrt(-(c^2*d^4
- 2*a*c*d^2*e^2 + a^2*e^4)/(a*c^5*d^8 + 4*a^2*c^4*d^6*e^2 + 6*a^3*c^3*d^4*e
^4 + 4*a^4*c^2*d^2*e^6 + a^5*c*e^8)))/(c^2*d^4 + 2*a*c*d^2*e^2 + a^2*e^4)))
 + (c*d^2 + a*e^2)*sqrt(-(2*d*e - (c^2*d^4 + 2*a*c*d^2*e^2 + a^2*e^4)*sqrt(
-(c^2*d^4 - 2*a*c*d^2*e^2 + a^2*e^4)/(a*c^5*d^8 + 4*a^2*c^4*d^6*e^2 + 6*a^3
*c^3*d^4*e^4 + 4*a^4*c^2*d^2*e^6 + a^5*c*e^8)))/(c^2*d^4 + 2*a*c*d^2*e^2 +
a^2*e^4))*log(-(c*d^2 - a*e^2)*x + (a*c*d^2*e - a^2*e^3 + (a*c^3*d^5 + 2*a^
2*c^2*d^3*e^2 + a^3*c*d*e^4)*sqrt(-(c^2*d^4 - 2*a*c*d^2*e^2 + a^2*e^4)/(a*c
^5*d^8 + 4*a^2*c^4*d^6*e^2 + 6*a^3*c^3*d^4*e^4 + 4*a^4*c^2*d^2*e^6 + a^5*c*
e^8)))*sqrt(-(2*d*e - (c^2*d^4 + 2*a*c*d^2*e^2 + a^2*e^4)*sqrt(-(c^2*d^4 -
2*a*c*d^2*e^2 + a^2*e^4)/(a*c^5*d^8 + 4*a^2*c^4*d^6*e^2 + 6*a^3*c^3*d^4*e^4
 + 4*a^4*c^2*d^2*e^6 + a^5*c*e^8)))/(c^2*d^4 + 2*a*c*d^2*e^2 + a^2*e^4))) -
 (c*d^2 + a*e^2)*sqrt(-(2*d*e - (c^2*d^4 + 2*a*c*d^2*e^2 + a^2*e^4)*sqrt(-(
c^2*d^4 - 2*a*c*d^2*e^2 + a^2*e^4)/(a*c^5*d^8 + 4*a^2*c^4*d^6*e^2 + 6*a^3*c
^3*d^4*e^4 + 4*a^4*c^2*d^2*e^6 + a^5*c*e^8)))/(c^2*d^4 + 2*a*c*d^2*e^2 + a^
2*e^4))*log(-(c*d^2 - a*e^2)*x - (a*c*d^2*e - a^2*e^3 + (a*c^3*d^5 + 2*a^2*
c^2*d^3*e^2 + a^3*c*d*e^4)*sqrt(-(c^2*d^4 - 2*a*c*d^2*e^2 + a^2*e^4)/(a*c^5
*d^8 + 4*a^2*c^4*d^6*e^2 + 6*a^3*c^3*d^4*e^4 + 4*a^4*c^2*d^2*e^6 + a^5*c*e^
8)))*sqrt(-(2*d*e - (c^2*d^4 + 2*a*c*d^2*e^2 + a^2*e^4)*sqrt(-(c^2*d^4 - 2*
a*c*d^2*e^2 + a^2*e^4)/(a*c^5*d^8 + 4*a^2*c^4*d^6*e^2 + 6*a^3*c^3*d^4*e^4 +
```

```
 4*a^4*c^2*d^2*e^6 + a^5*c*e^8)))/(c^2*d^4 + 2*a*c*d^2*e^2 + a^2*e^4))) - 2
*sqrt(-d*e)*log((e*x^2 - 2*sqrt(-d*e)*x - d)/(e*x^2 + d)))/(c*d^2 + a*e^2),
 -1/4*((c*d^2 + a*e^2)*sqrt(-(2*d*e + (c^2*d^4 + 2*a*c*d^2*e^2 + a^2*e^4)*s
qrt(-(c^2*d^4 - 2*a*c*d^2*e^2 + a^2*e^4)/(a*c^5*d^8 + 4*a^2*c^4*d^6*e^2 + 6
*a^3*c^3*d^4*e^4 + 4*a^4*c^2*d^2*e^6 + a^5*c*e^8)))/(c^2*d^4 + 2*a*c*d^2*e^
2 + a^2*e^4))*log(-(c*d^2 - a*e^2)*x + (a*c*d^2*e - a^2*e^3 - (a*c^3*d^5 +
2*a^2*c^2*d^3*e^2 + a^3*c*d*e^4)*sqrt(-(c^2*d^4 - 2*a*c*d^2*e^2 + a^2*e^4)/
(a*c^5*d^8 + 4*a^2*c^4*d^6*e^2 + 6*a^3*c^3*d^4*e^4 + 4*a^4*c^2*d^2*e^6 + a^
5*c*e^8)))*sqrt(-(2*d*e + (c^2*d^4 + 2*a*c*d^2*e^2 + a^2*e^4)*sqrt(-(c^2*d^
4 - 2*a*c*d^2*e^2 + a^2*e^4)/(a*c^5*d^8 + 4*a^2*c^4*d^6*e^2 + 6*a^3*c^3*d^4
*e^4 + 4*a^4*c^2*d^2*e^6 + a^5*c*e^8)))/(c^2*d^4 + 2*a*c*d^2*e^2 + a^2*e^4)
)) - (c*d^2 + a*e^2)*sqrt(-(2*d*e + (c^2*d^4 + 2*a*c*d^2*e^2 + a^2*e^4)*sqr
t(-(c^2*d^4 - 2*a*c*d^2*e^2 + a^2*e^4)/(a*c^5*d^8 + 4*a^2*c^4*d^6*e^2 + 6*a
^3*c^3*d^4*e^4 + 4*a^4*c^2*d^2*e^6 + a^5*c*e^8)))/(c^2*d^4 + 2*a*c*d^2*e^2
+ a^2*e^4))*log(-(c*d^2 - a*e^2)*x - (a*c*d^2*e - a^2*e^3 - (a*c^3*d^5 + 2*
a^2*c^2*d^3*e^2 + a^3*c*d*e^4)*sqrt(-(c^2*d^4 - 2*a*c*d^2*e^2 + a^2*e^4)/(a
*c^5*d^8 + 4*a^2*c^4*d^6*e^2 + 6*a^3*c^3*d^4*e^4 + 4*a^4*c^2*d^2*e^6 + a^5*
c*e^8)))*sqrt(-(2*d*e + (c^2*d^4 + 2*a*c*d^2*e^2 + a^2*e^4)*sqrt(-(c^2*d^4
- 2*a*c*d^2*e^2 + a^2*e^4)/(a*c^5*d^8 + 4*a^2*c^4*d^6*e^2 + 6*a^3*c^3*d^4*e
^4 + 4*a^4*c^2*d^2*e^6 + a^5*c*e^8)))/(c^2*d^4 + 2*a*c*d^2*e^2 + a^2*e^4)))
 + (c*d^2 + a*e^2)*sqrt(-(2*d*e - (c^2*d^4 + 2*a*c*d^2*e^2 + a^2*e^4)*sqrt(
-(c^2*d^4 - 2*a*c*d^2*e^2 + a^2*e^4)/(a*c^5*d^8 + 4*a^2*c^4*d^6*e^2 + 6*a^3
*c^3*d^4*e^4 + 4*a^4*c^2*d^2*e^6 + a^5*c*e^8)))/(c^2*d^4 + 2*a*c*d^2*e^2 +
a^2*e^4))*log(-(c*d^2 - a*e^2)*x + (a*c*d^2*e - a^2*e^3 + (a*c^3*d^5 + 2*a^
2*c^2*d^3*e^2 + a^3*c*d*e^4)*sqrt(-(c^2*d^4 - 2*a*c*d^2*e^2 + a^2*e^4)/(a*c
^5*d^8 + 4*a^2*c^4*d^6*e^2 + 6*a^3*c^3*d^4*e^4 + 4*a^4*c^2*d^2*e^6 + a^5*c*
e^8)))*sqrt(-(2*d*e - (c^2*d^4 + 2*a*c*d^2*e^2 + a^2*e^4)*sqrt(-(c^2*d^4 -
2*a*c*d^2*e^2 + a^2*e^4)/(a*c^5*d^8 + 4*a^2*c^4*d^6*e^2 + 6*a^3*c^3*d^4*e^4
 + 4*a^4*c^2*d^2*e^6 + a^5*c*e^8)))/(c^2*d^4 + 2*a*c*d^2*e^2 + a^2*e^4))) -
 (c*d^2 + a*e^2)*sqrt(-(2*d*e - (c^2*d^4 + 2*a*c*d^2*e^2 + a^2*e^4)*sqrt(-(
c^2*d^4 - 2*a*c*d^2*e^2 + a^2*e^4)/(a*c^5*d^8 + 4*a^2*c^4*d^6*e^2 + 6*a^3*c
^3*d^4*e^4 + 4*a^4*c^2*d^2*e^6 + a^5*c*e^8)))/(c^2*d^4 + 2*a*c*d^2*e^2 + a^
2*e^4))*log(-(c*d^2 - a*e^2)*x - (a*c*d^2*e - a^2*e^3 + (a*c^3*d^5 + 2*a^2*
c^2*d^3*e^2 + a^3*c*d*e^4)*sqrt(-(c^2*d^4 - 2*a*c*d^2*e^2 + a^2*e^4)/(a*c^5
*d^8 + 4*a^2*c^4*d^6*e^2 + 6*a^3*c^3*d^4*e^4 + 4*a^4*c^2*d^2*e^6 + a^5*c*e^
8)))*sqrt(-(2*d*e - (c^2*d^4 + 2*a*c*d^2*e^2 + a^2*e^4)*sqrt(-(c^2*d^4 - 2*
a*c*d^2*e^2 + a^2*e^4)/(a*c^5*d^8 + 4*a^2*c^4*d^6*e^2 + 6*a^3*c^3*d^4*e^4 +
 4*a^4*c^2*d^2*e^6 + a^5*c*e^8)))/(c^2*d^4 + 2*a*c*d^2*e^2 + a^2*e^4))) + 4
*sqrt(d*e)*arctan(sqrt(d*e)*x/d))/(c*d^2 + a*e^2)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^2}{(d+ex^2)(a+cx^4)}\,dx = \text{Timed out}$$

```
[In] integrate(x**2/(e*x**2+d)/(c*x**4+a),x)
[Out] Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^2}{(d+ex^2)(a+cx^4)}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(x^2/(e*x^2+d)/(c*x^4+a),x, algorithm="maxima")
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 340, normalized size of antiderivative = 1.01

$$\begin{aligned}\int \frac{x^2}{(d+ex^2)(a+cx^4)}\,dx = &-\frac{de\arctan\left(\frac{ex}{\sqrt{de}}\right)}{(cd^2+ae^2)\sqrt{de}} \\ &+\frac{\left((ac^3)^{\frac{1}{4}}ace+(ac^3)^{\frac{3}{4}}d\right)\arctan\left(\frac{\sqrt{2}\left(2x+\sqrt{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{2\left(\sqrt{2}ac^3d^2+\sqrt{2}a^2c^2e^2\right)} \\ &+\frac{\left((ac^3)^{\frac{1}{4}}ace+(ac^3)^{\frac{3}{4}}d\right)\arctan\left(\frac{\sqrt{2}\left(2x-\sqrt{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{2\left(\sqrt{2}ac^3d^2+\sqrt{2}a^2c^2e^2\right)} \\ &+\frac{\left((ac^3)^{\frac{1}{4}}ace-(ac^3)^{\frac{3}{4}}d\right)\log\left(x^2+\sqrt{2}x\left(\frac{a}{c}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{c}}\right)}{4\left(\sqrt{2}ac^3d^2+\sqrt{2}a^2c^2e^2\right)} \\ &-\frac{\left((ac^3)^{\frac{1}{4}}ace-(ac^3)^{\frac{3}{4}}d\right)\log\left(x^2-\sqrt{2}x\left(\frac{a}{c}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{c}}\right)}{4\left(\sqrt{2}ac^3d^2+\sqrt{2}a^2c^2e^2\right)}\end{aligned}$$

```
[In] integrate(x^2/(e*x^2+d)/(c*x^4+a),x, algorithm="giac")
```

```
[Out] -d*e*arctan(e*x/sqrt(d*e))/((c*d^2 + a*e^2)*sqrt(d*e)) + 1/2*((a*c^3)^(1/4)
*a*c*e + (a*c^3)^(3/4)*d)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(a/c)^(1/4))/(a
/c)^(1/4))/(sqrt(2)*a*c^3*d^2 + sqrt(2)*a^2*c^2*e^2) + 1/2*((a*c^3)^(1/4)*a
*c*e + (a*c^3)^(3/4)*d)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(a/c)^(1/4))/(a/c
)^(1/4))/(sqrt(2)*a*c^3*d^2 + sqrt(2)*a^2*c^2*e^2) + 1/4*((a*c^3)^(1/4)*a*c
*e - (a*c^3)^(3/4)*d)*log(x^2 + sqrt(2)*x*(a/c)^(1/4) + sqrt(a/c))/(sqrt(2)
*a*c^3*d^2 + sqrt(2)*a^2*c^2*e^2) - 1/4*((a*c^3)^(1/4)*a*c*e - (a*c^3)^(3/4
)*d)*log(x^2 - sqrt(2)*x*(a/c)^(1/4) + sqrt(a/c))/(sqrt(2)*a*c^3*d^2 + sqrt
(2)*a^2*c^2*e^2)
```

Mupad [B] (verification not implemented)

Time = 8.46 (sec) , antiderivative size = 4720, normalized size of antiderivative = 14.01

$$\int \frac{x^2}{(d+ex^2)(a+cx^4)}\,dx = \text{Too large to display}$$

```
[In] int(x^2/((a + c*x^4)*(d + e*x^2)),x)
```

```
[Out] (log(a^2*d*e^7 + c^2*d^5*e^3 - c^2*d*x*(-d*e)^(7/2) + 2*a*c*d^3*e^5 + a^2*e
^7*x*(-d*e)^(1/2) + 2*a*c*e^3*x*(-d*e)^(5/2))*(-d*e)^(1/2))/(2*a*e^2 + 2*c*
d^2) - atan((((-(c*d^2*(-a*c)^(1/2) - a*e^2*(-a*c)^(1/2) + 2*a*c*d*e)/(16*(
a*c^3*d^4 + a^3*c*e^4 + 2*a^2*c^2*d^2*e^2)))^(1/2)*(((-(c*d^2*(-a*c)^(1/2)
- a*e^2*(-a*c)^(1/2) + 2*a*c*d*e)/(16*(a*c^3*d^4 + a^3*c*e^4 + 2*a^2*c^2*d^
2*e^2)))^(1/2)*(x*(-(c*d^2*(-a*c)^(1/2) - a*e^2*(-a*c)^(1/2) + 2*a*c*d*e)/(
16*(a*c^3*d^4 + a^3*c*e^4 + 2*a^2*c^2*d^2*e^2)))^(1/2)*(512*a^5*c^4*e^9 - 5
12*a^2*c^7*d^6*e^3 - 512*a^3*c^6*d^4*e^5 + 512*a^4*c^5*d^2*e^7) + 192*a^4*c
^4*d*e^7 + 192*a^2*c^6*d^5*e^3 + 384*a^3*c^5*d^3*e^5) - x*(16*a*c^6*d^5*e^2
 - 112*a^3*c^4*d*e^6 + 160*a^2*c^5*d^3*e^4))*(-(c*d^2*(-a*c)^(1/2) - a*e^2*
(-a*c)^(1/2) + 2*a*c*d*e)/(16*(a*c^3*d^4 + a^3*c*e^4 + 2*a^2*c^2*d^2*e^2)))
^(1/2) + 4*a*c^5*d^4*e^2 + 52*a^2*c^4*d^2*e^4) + x*(2*a^2*c^3*e^5 - 4*a*c^4
*d^2*e^3))*(-(c*d^2*(-a*c)^(1/2) - a*e^2*(-a*c)^(1/2) + 2*a*c*d*e)/(16*(a*c
^3*d^4 + a^3*c*e^4 + 2*a^2*c^2*d^2*e^2)))^(1/2)*1i - ((-(c*d^2*(-a*c)^(1/2)
 - a*e^2*(-a*c)^(1/2) + 2*a*c*d*e)/(16*(a*c^3*d^4 + a^3*c*e^4 + 2*a^2*c^2*d
^2*e^2)))^(1/2)*(((-(c*d^2*(-a*c)^(1/2) - a*e^2*(-a*c)^(1/2) + 2*a*c*d*e)/(
16*(a*c^3*d^4 + a^3*c*e^4 + 2*a^2*c^2*d^2*e^2)))^(1/2)*(192*a^4*c^4*d*e^7 -
 x*(-(c*d^2*(-a*c)^(1/2) - a*e^2*(-a*c)^(1/2) + 2*a*c*d*e)/(16*(a*c^3*d^4 +
 a^3*c*e^4 + 2*a^2*c^2*d^2*e^2)))^(1/2)*(512*a^5*c^4*e^9 - 512*a^2*c^7*d^6*
e^3 - 512*a^3*c^6*d^4*e^5 + 512*a^4*c^5*d^2*e^7) + 192*a^2*c^6*d^5*e^3 + 38
4*a^3*c^5*d^3*e^5) + x*(16*a*c^6*d^5*e^2 - 112*a^3*c^4*d*e^6 + 160*a^2*c^5*
d^3*e^4))*(-(c*d^2*(-a*c)^(1/2) - a*e^2*(-a*c)^(1/2) + 2*a*c*d*e)/(16*(a*c^
3*d^4 + a^3*c*e^4 + 2*a^2*c^2*d^2*e^2)))^(1/2) + 4*a*c^5*d^4*e^2 + 52*a^2*c
^4*d^2*e^4) - x*(2*a^2*c^3*e^5 - 4*a*c^4*d^2*e^3))*(-(c*d^2*(-a*c)^(1/2) -
a*e^2*(-a*c)^(1/2) + 2*a*c*d*e)/(16*(a*c^3*d^4 + a^3*c*e^4 + 2*a^2*c^2*d^2*
```

```
e^2)))^(1/2)*1i)/(((-(c*d^2*(-a*c)^(1/2) - a*e^2*(-a*c)^(1/2) + 2*a*c*d*e)/
(16*(a*c^3*d^4 + a^3*c*e^4 + 2*a^2*c^2*d^2*e^2)))^(1/2)*(((-(c*d^2*(-a*c)^(
1/2) - a*e^2*(-a*c)^(1/2) + 2*a*c*d*e)/(16*(a*c^3*d^4 + a^3*c*e^4 + 2*a^2*c
^2*d^2*e^2)))^(1/2)*(x*(-(c*d^2*(-a*c)^(1/2) - a*e^2*(-a*c)^(1/2) + 2*a*c*d
*e)/(16*(a*c^3*d^4 + a^3*c*e^4 + 2*a^2*c^2*d^2*e^2)))^(1/2)*(512*a^5*c^4*e^
9 - 512*a^2*c^7*d^6*e^3 - 512*a^3*c^6*d^4*e^5 + 512*a^4*c^5*d^2*e^7) + 192*
a^4*c^4*d*e^7 + 192*a^2*c^6*d^5*e^3 + 384*a^3*c^5*d^3*e^5) - x*(16*a*c^6*d^
5*e^2 - 112*a^3*c^4*d*e^6 + 160*a^2*c^5*d^3*e^4))*(-(c*d^2*(-a*c)^(1/2) - a
*e^2*(-a*c)^(1/2) + 2*a*c*d*e)/(16*(a*c^3*d^4 + a^3*c*e^4 + 2*a^2*c^2*d^2*e
^2)))^(1/2) + 4*a*c^5*d^4*e^2 + 52*a^2*c^4*d^2*e^4) + x*(2*a^2*c^3*e^5 - 4*
a*c^4*d^2*e^3))*(-(c*d^2*(-a*c)^(1/2) - a*e^2*(-a*c)^(1/2) + 2*a*c*d*e)/(16
*(a*c^3*d^4 + a^3*c*e^4 + 2*a^2*c^2*d^2*e^2)))^(1/2) + ((-(c*d^2*(-a*c)^(1/
2) - a*e^2*(-a*c)^(1/2) + 2*a*c*d*e)/(16*(a*c^3*d^4 + a^3*c*e^4 + 2*a^2*c^2
*d^2*e^2)))^(1/2)*(((-(c*d^2*(-a*c)^(1/2) - a*e^2*(-a*c)^(1/2) + 2*a*c*d*e)
/(16*(a*c^3*d^4 + a^3*c*e^4 + 2*a^2*c^2*d^2*e^2)))^(1/2)*(192*a^4*c^4*d*e^7
 - x*(-(c*d^2*(-a*c)^(1/2) - a*e^2*(-a*c)^(1/2) + 2*a*c*d*e)/(16*(a*c^3*d^4
 + a^3*c*e^4 + 2*a^2*c^2*d^2*e^2)))^(1/2)*(512*a^5*c^4*e^9 - 512*a^2*c^7*d^
6*e^3 - 512*a^3*c^6*d^4*e^5 + 512*a^4*c^5*d^2*e^7) + 192*a^2*c^6*d^5*e^3 +
384*a^3*c^5*d^3*e^5) + x*(16*a*c^6*d^5*e^2 - 112*a^3*c^4*d*e^6 + 160*a^2*c^
5*d^3*e^4))*(-(c*d^2*(-a*c)^(1/2) - a*e^2*(-a*c)^(1/2) + 2*a*c*d*e)/(16*(a*
c^3*d^4 + a^3*c*e^4 + 2*a^2*c^2*d^2*e^2)))^(1/2) + 4*a*c^5*d^4*e^2 + 52*a^2
*c^4*d^2*e^4) - x*(2*a^2*c^3*e^5 - 4*a*c^4*d^2*e^3))*(-(c*d^2*(-a*c)^(1/2)
- a*e^2*(-a*c)^(1/2) + 2*a*c*d*e)/(16*(a*c^3*d^4 + a^3*c*e^4 + 2*a^2*c^2*d^
2*e^2)))^(1/2) + 2*a*c^3*d*e^3))*(-(c*d^2*(-a*c)^(1/2) - a*e^2*(-a*c)^(1/2)
 + 2*a*c*d*e)/(16*(a*c^3*d^4 + a^3*c*e^4 + 2*a^2*c^2*d^2*e^2)))^(1/2)*2i -
(log(a^2*d*e^7 + c^2*d^5*e^3 + c^2*d*x*(-d*e)^(7/2) + 2*a*c*d^3*e^5 - a^2*e
^7*x*(-d*e)^(1/2) - 2*a*c*e^3*x*(-d*e)^(5/2))*(-d*e)^(1/2))/(2*(a*e^2 + c*d
^2)) - atan((((-(a*e^2*(-a*c)^(1/2) - c*d^2*(-a*c)^(1/2) + 2*a*c*d*e)/(16*(
a*c^3*d^4 + a^3*c*e^4 + 2*a^2*c^2*d^2*e^2)))^(1/2)*(((-(a*e^2*(-a*c)^(1/2)
- c*d^2*(-a*c)^(1/2) + 2*a*c*d*e)/(16*(a*c^3*d^4 + a^3*c*e^4 + 2*a^2*c^2*d^
2*e^2)))^(1/2)*(x*(-(a*e^2*(-a*c)^(1/2) - c*d^2*(-a*c)^(1/2) + 2*a*c*d*e)/(
16*(a*c^3*d^4 + a^3*c*e^4 + 2*a^2*c^2*d^2*e^2)))^(1/2)*(512*a^5*c^4*e^9 - 5
12*a^2*c^7*d^6*e^3 - 512*a^3*c^6*d^4*e^5 + 512*a^4*c^5*d^2*e^7) + 192*a^4*c
^4*d*e^7 + 192*a^2*c^6*d^5*e^3 + 384*a^3*c^5*d^3*e^5) - x*(16*a*c^6*d^5*e^2
 - 112*a^3*c^4*d*e^6 + 160*a^2*c^5*d^3*e^4))*(-(a*e^2*(-a*c)^(1/2) - c*d^2*
(-a*c)^(1/2) + 2*a*c*d*e)/(16*(a*c^3*d^4 + a^3*c*e^4 + 2*a^2*c^2*d^2*e^2)))
^(1/2) + 4*a*c^5*d^4*e^2 + 52*a^2*c^4*d^2*e^4) + x*(2*a^2*c^3*e^5 - 4*a*c^4
*d^2*e^3))*(-(a*e^2*(-a*c)^(1/2) - c*d^2*(-a*c)^(1/2) + 2*a*c*d*e)/(16*(a*c
^3*d^4 + a^3*c*e^4 + 2*a^2*c^2*d^2*e^2)))^(1/2)*1i - ((-(a*e^2*(-a*c)^(1/2)
 - c*d^2*(-a*c)^(1/2) + 2*a*c*d*e)/(16*(a*c^3*d^4 + a^3*c*e^4 + 2*a^2*c^2*d
^2*e^2)))^(1/2)*(((-(a*e^2*(-a*c)^(1/2) - c*d^2*(-a*c)^(1/2) + 2*a*c*d*e)/(
16*(a*c^3*d^4 + a^3*c*e^4 + 2*a^2*c^2*d^2*e^2)))^(1/2)*(192*a^4*c^4*d*e^7 -
 x*(-(a*e^2*(-a*c)^(1/2) - c*d^2*(-a*c)^(1/2) + 2*a*c*d*e)/(16*(a*c^3*d^4 +
 a^3*c*e^4 + 2*a^2*c^2*d^2*e^2)))^(1/2)*(512*a^5*c^4*e^9 - 512*a^2*c^7*d^6*
e^3 - 512*a^3*c^6*d^4*e^5 + 512*a^4*c^5*d^2*e^7) + 192*a^2*c^6*d^5*e^3 + 38
```

4*a^3*c^5*d^3*e^5) + x*(16*a*c^6*d^5*e^2 - 112*a^3*c^4*d*e^6 + 160*a^2*c^5*d^3*e^4))*(-(a*e^2*(-a*c)^(1/2) - c*d^2*(-a*c)^(1/2) + 2*a*c*d*e)/(16*(a*c^3*d^4 + a^3*c*e^4 + 2*a^2*c^2*d^2*e^2)))^(1/2) + 4*a*c^5*d^4*e^2 + 52*a^2*c^4*d^2*e^4) - x*(2*a^2*c^3*e^5 - 4*a*c^4*d^2*e^3))*(-(a*e^2*(-a*c)^(1/2) - c*d^2*(-a*c)^(1/2) + 2*a*c*d*e)/(16*(a*c^3*d^4 + a^3*c*e^4 + 2*a^2*c^2*d^2*e^2)))^(1/2)*1i)/(((-(a*e^2*(-a*c)^(1/2) - c*d^2*(-a*c)^(1/2) + 2*a*c*d*e)/(16*(a*c^3*d^4 + a^3*c*e^4 + 2*a^2*c^2*d^2*e^2)))^(1/2)*(((-(a*e^2*(-a*c)^(1/2) - c*d^2*(-a*c)^(1/2) + 2*a*c*d*e)/(16*(a*c^3*d^4 + a^3*c*e^4 + 2*a^2*c^2*d^2*e^2)))^(1/2)*(x*(-(a*e^2*(-a*c)^(1/2) - c*d^2*(-a*c)^(1/2) + 2*a*c*d*e)/(16*(a*c^3*d^4 + a^3*c*e^4 + 2*a^2*c^2*d^2*e^2)))^(1/2)*(512*a^5*c^4*e^9 - 512*a^2*c^7*d^6*e^3 - 512*a^3*c^6*d^4*e^5 + 512*a^4*c^5*d^2*e^7) + 192*a^4*c^4*d*e^7 + 192*a^2*c^6*d^5*e^3 + 384*a^3*c^5*d^3*e^5) - x*(16*a*c^6*d^5*e^2 - 112*a^3*c^4*d*e^6 + 160*a^2*c^5*d^3*e^4))*(-(a*e^2*(-a*c)^(1/2) - c*d^2*(-a*c)^(1/2) + 2*a*c*d*e)/(16*(a*c^3*d^4 + a^3*c*e^4 + 2*a^2*c^2*d^2*e^2)))^(1/2) + 4*a*c^5*d^4*e^2 + 52*a^2*c^4*d^2*e^4) + x*(2*a^2*c^3*e^5 - 4*a*c^4*d^2*e^3))*(-(a*e^2*(-a*c)^(1/2) - c*d^2*(-a*c)^(1/2) + 2*a*c*d*e)/(16*(a*c^3*d^4 + a^3*c*e^4 + 2*a^2*c^2*d^2*e^2)))^(1/2) + ((-(a*e^2*(-a*c)^(1/2) - c*d^2*(-a*c)^(1/2) + 2*a*c*d*e)/(16*(a*c^3*d^4 + a^3*c*e^4 + 2*a^2*c^2*d^2*e^2)))^(1/2)*(((-(a*e^2*(-a*c)^(1/2) - c*d^2*(-a*c)^(1/2) + 2*a*c*d*e)/(16*(a*c^3*d^4 + a^3*c*e^4 + 2*a^2*c^2*d^2*e^2)))^(1/2)*(192*a^4*c^4*d*e^7 - x*(-(a*e^2*(-a*c)^(1/2) - c*d^2*(-a*c)^(1/2) + 2*a*c*d*e)/(16*(a*c^3*d^4 + a^3*c*e^4 + 2*a^2*c^2*d^2*e^2)))^(1/2)*(512*a^5*c^4*e^9 - 512*a^2*c^7*d^6*e^3 - 512*a^3*c^6*d^4*e^5 + 512*a^4*c^5*d^2*e^7) + 192*a^2*c^6*d^5*e^3 + 384*a^3*c^5*d^3*e^5) + x*(16*a*c^6*d^5*e^2 - 112*a^3*c^4*d*e^6 + 160*a^2*c^5*d^3*e^4))*(-(a*e^2*(-a*c)^(1/2) - c*d^2*(-a*c)^(1/2) + 2*a*c*d*e)/(16*(a*c^3*d^4 + a^3*c*e^4 + 2*a^2*c^2*d^2*e^2)))^(1/2) + 4*a*c^5*d^4*e^2 + 52*a^2*c^4*d^2*e^4) - x*(2*a^2*c^3*e^5 - 4*a*c^4*d^2*e^3))*(-(a*e^2*(-a*c)^(1/2) - c*d^2*(-a*c)^(1/2) + 2*a*c*d*e)/(16*(a*c^3*d^4 + a^3*c*e^4 + 2*a^2*c^2*d^2*e^2)))^(1/2) + 2*a*c^3*d*e^3))*(-(a*e^2*(-a*c)^(1/2) - c*d^2*(-a*c)^(1/2) + 2*a*c*d*e)/(16*(a*c^3*d^4 + a^3*c*e^4 + 2*a^2*c^2*d^2*e^2)))^(1/2)*2i

3.241 $\int \frac{1}{(d+ex^2)(a+cx^4)}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 336

$$\int \frac{1}{(d+ex^2)(a+cx^4)}\,dx = \frac{e^{3/2}\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{d}\,(cd^2+ae^2)} - \frac{\sqrt[4]{c}(\sqrt{c}d-\sqrt{a}e)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}\,(cd^2+ae^2)} + \frac{\sqrt[4]{c}(\sqrt{c}d-\sqrt{a}e)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}\,(cd^2+ae^2)} - \frac{\sqrt[4]{c}(\sqrt{c}d+\sqrt{a}e)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)}{4\sqrt{2}a^{3/4}\,(cd^2+ae^2)} + \frac{\sqrt[4]{c}(\sqrt{c}d+\sqrt{a}e)\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)}{4\sqrt{2}a^{3/4}\,(cd^2+ae^2)}$$

```
[Out] 1/4*c^(1/4)*arctan(-1+c^(1/4)*x*2^(1/2)/a^(1/4))*(-e*a^(1/2)+d*c^(1/2))/a^(
3/4)/(a*e^2+c*d^2)*2^(1/2)+1/4*c^(1/4)*arctan(1+c^(1/4)*x*2^(1/2)/a^(1/4))*
(-e*a^(1/2)+d*c^(1/2))/a^(3/4)/(a*e^2+c*d^2)*2^(1/2)-1/8*c^(1/4)*ln(-a^(1/4
)*c^(1/4)*x*2^(1/2)+a^(1/2)+x^2*c^(1/2))*(e*a^(1/2)+d*c^(1/2))/a^(3/4)/(a*e
^2+c*d^2)*2^(1/2)+1/8*c^(1/4)*ln(a^(1/4)*c^(1/4)*x*2^(1/2)+a^(1/2)+x^2*c^(1
/2))*(e*a^(1/2)+d*c^(1/2))/a^(3/4)/(a*e^2+c*d^2)*2^(1/2)+e^(3/2)*arctan(x*e
^(1/2)/d^(1/2))/(a*e^2+c*d^2)/d^(1/2)
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 336, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.421$, Rules used = {1185, 211, 1182, 1176, 631, 210, 1179, 642}

$$\int \frac{1}{(d+ex^2)(a+cx^4)}\,dx = -\frac{\sqrt[4]{c}\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\left(\sqrt{c}d-\sqrt{a}e\right)}{2\sqrt{2}a^{3/4}(ae^2+cd^2)} + \frac{\sqrt[4]{c}\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)\left(\sqrt{c}d-\sqrt{a}e\right)}{2\sqrt{2}a^{3/4}(ae^2+cd^2)} - \frac{\sqrt[4]{c}\left(\sqrt{a}e+\sqrt{c}d\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{4\sqrt{2}a^{3/4}(ae^2+cd^2)} + \frac{\sqrt[4]{c}\left(\sqrt{a}e+\sqrt{c}d\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{4\sqrt{2}a^{3/4}(ae^2+cd^2)} + \frac{e^{3/2}\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{d}(ae^2+cd^2)}$$

```
[In] Int[1/((d + e*x^2)*(a + c*x^4)),x]
```

```
[Out] (e^(3/2)*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(Sqrt[d]*(c*d^2 + a*e^2)) - (c^(1/4)*
(Sqrt[c]*d - Sqrt[a]*e)*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]
*a^(3/4)*(c*d^2 + a*e^2)) + (c^(1/4)*(Sqrt[c]*d - Sqrt[a]*e)*ArcTan[1 + (Sq
rt[2]*c^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*(c*d^2 + a*e^2)) - (c^(1/4)*(
Sqrt[c]*d + Sqrt[a]*e)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^
2])/(4*Sqrt[2]*a^(3/4)*(c*d^2 + a*e^2)) + (c^(1/4)*(Sqrt[c]*d + Sqrt[a]*e)*
Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(4*Sqrt[2]*a^(3/4)*
(c*d^2 + a*e^2))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
```

```
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1182

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[(-a)
*c]
```

Rule 1185

```
Int[((d_) + (e_.)*(x_)^2)^(q_)/((a_) + (c_.)*(x_)^4), x_Symbol] :> Int[Expa
ndIntegrand[(d + e*x^2)^q/(a + c*x^4), x], x] /; FreeQ[{a, c, d, e}, x] &&
NeQ[c*d^2 + a*e^2, 0] && IntegerQ[q]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(\frac{e^2}{(cd^2+ae^2)\,(d+ex^2)}+\frac{c(d-ex^2)}{(cd^2+ae^2)\,(a+cx^4)}\right) dx \\
&= \frac{c\int \frac{d-ex^2}{a+cx^4}\,dx}{cd^2+ae^2}+\frac{e^2\int \frac{1}{d+ex^2}\,dx}{cd^2+ae^2} \\
&= \frac{e^{3/2}\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{d}\,(cd^2+ae^2)}+\frac{\left(\frac{\sqrt{c}d}{\sqrt{a}}-e\right)\int \frac{\sqrt{a}\sqrt{c}+cx^2}{a+cx^4}\,dx}{2\,(cd^2+ae^2)}+\frac{\left(\frac{\sqrt{c}d}{\sqrt{a}}+e\right)\int \frac{\sqrt{a}\sqrt{c}-cx^2}{a+cx^4}\,dx}{2\,(cd^2+ae^2)}
\end{aligned}$$

$$= \frac{e^{3/2}\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{d}\,(cd^2+ae^2)} + \frac{\left(\frac{\sqrt{c}d}{\sqrt{a}}-e\right)\int\frac{1}{\frac{\sqrt{a}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}+x^2}dx}{4\,(cd^2+ae^2)} + \frac{\left(\frac{\sqrt{c}d}{\sqrt{a}}-e\right)\int\frac{1}{\frac{\sqrt{a}}{\sqrt{c}}+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}+x^2}dx}{4\,(cd^2+ae^2)}$$
$$- \frac{\left(\sqrt[4]{c}\left(\sqrt{c}d+\sqrt{a}e\right)\right)\int\frac{\frac{\sqrt{2}\sqrt[4]{a}}{\sqrt[4]{c}}+2x}{-\frac{\sqrt{a}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}-x^2}dx}{4\sqrt{2}a^{3/4}\,(cd^2+ae^2)} - \frac{\left(\sqrt[4]{c}\left(\sqrt{c}d+\sqrt{a}e\right)\right)\int\frac{\frac{\sqrt{2}\sqrt[4]{a}}{\sqrt[4]{c}}-2x}{-\frac{\sqrt{a}}{\sqrt{c}}+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}-x^2}dx}{4\sqrt{2}a^{3/4}\,(cd^2+ae^2)}$$

$$= \frac{e^{3/2}\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{d}\,(cd^2+ae^2)} - \frac{\sqrt[4]{c}\left(\sqrt{c}d+\sqrt{a}e\right)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)}{4\sqrt{2}a^{3/4}\,(cd^2+ae^2)}$$
$$+ \frac{\sqrt[4]{c}\left(\sqrt{c}d+\sqrt{a}e\right)\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)}{4\sqrt{2}a^{3/4}\,(cd^2+ae^2)}$$
$$+ \frac{\left(\sqrt[4]{c}\left(\sqrt{c}d-\sqrt{a}e\right)\right)\text{Subst}\left(\int\frac{1}{-1-x^2}dx, x, 1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}\,(cd^2+ae^2)}$$
$$- \frac{\left(\sqrt[4]{c}\left(\sqrt{c}d-\sqrt{a}e\right)\right)\text{Subst}\left(\int\frac{1}{-1-x^2}dx, x, 1+\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}\,(cd^2+ae^2)}$$

$$= \frac{e^{3/2}\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{d}\,(cd^2+ae^2)} - \frac{\sqrt[4]{c}\left(\sqrt{c}d-\sqrt{a}e\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}\,(cd^2+ae^2)}$$
$$+ \frac{\sqrt[4]{c}\left(\sqrt{c}d-\sqrt{a}e\right)\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}\,(cd^2+ae^2)}$$
$$- \frac{\sqrt[4]{c}\left(\sqrt{c}d+\sqrt{a}e\right)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)}{4\sqrt{2}a^{3/4}\,(cd^2+ae^2)}$$
$$+ \frac{\sqrt[4]{c}\left(\sqrt{c}d+\sqrt{a}e\right)\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)}{4\sqrt{2}a^{3/4}\,(cd^2+ae^2)}$$

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 234, normalized size of antiderivative = 0.70

$$\int\frac{1}{(d+ex^2)\,(a+cx^4)}\,dx$$
$$= \frac{8a^{3/4}e^{3/2}\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)+\sqrt{2}\sqrt[4]{c}\sqrt{d}\left(\left(-2\sqrt{c}d+2\sqrt{a}e\right)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)+2\left(\sqrt{c}d-\sqrt{a}e\right)\arctan\left(1-\right.\right.}{8a^{3/4}\sqrt{d}\,(cd^2+\right.}$$

```
[In] Integrate[1/((d + e*x^2)*(a + c*x^4)),x]
```

```
[Out] (8*a^(3/4)*e^(3/2)*ArcTan[(Sqrt[e]*x)/Sqrt[d]] + Sqrt[2]*c^(1/4)*Sqrt[d]*((
-2*Sqrt[c]*d + 2*Sqrt[a]*e)*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)] + 2*(Sq
rt[c]*d - Sqrt[a]*e)*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)] - (Sqrt[c]*d +
 Sqrt[a]*e)*(Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2] - Log[S
qrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])))/(8*a^(3/4)*Sqrt[d]*(c*
d^2 + a*e^2))
```

Maple [A] (verified)

Time = 0.33 (sec) , antiderivative size = 253, normalized size of antiderivative = 0.75

method	result
default	$\frac{c\left(\frac{d\left(\frac{a}{c}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x^2+\left(\frac{a}{c}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{c}}}{x^2-\left(\frac{a}{c}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{c}}}\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{c}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{c}\right)^{\frac{1}{4}}}-1\right)\right)}{8a}-\frac{e\sqrt{2}\left(\ln\left(\frac{x^2-\left(\frac{a}{c}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{c}}}{x^2+\left(\frac{a}{c}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{c}}}\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right.\right.}{8c\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right.}{a\,e^2+c\,d^2}$
risch	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\left(a^5e^4+2a^4c\,d^2e^2+a^3c^2d^4\right)\_Z^4-4a^2cde\_Z^2+c\right)}\_R\ln\left(\left(\left(-2a^5e^7-2a^4c\,d^2e^5+2a^3c^2d^4e^3+2a^2c^3d^6e\right)\_R^4+(15a^2cd\right.\right.\right.}{4}$

```
[In] int(1/(e*x^2+d)/(c*x^4+a),x,method=_RETURNVERBOSE)
```

```
[Out] c/(a*e^2+c*d^2)*(1/8*d*(a/c)^(1/4)/a*2^(1/2)*(ln((x^2+(a/c)^(1/4)*x*2^(1/2)
+(a/c)^(1/2))/(x^2-(a/c)^(1/4)*x*2^(1/2)+(a/c)^(1/2)))+2*arctan(2^(1/2)/(a/
c)^(1/4)*x+1)+2*arctan(2^(1/2)/(a/c)^(1/4)*x-1))-1/8*e/c/(a/c)^(1/4)*2^(1/2
)*(ln((x^2-(a/c)^(1/4)*x*2^(1/2)+(a/c)^(1/2))/(x^2+(a/c)^(1/4)*x*2^(1/2)+(a
/c)^(1/2)))+2*arctan(2^(1/2)/(a/c)^(1/4)*x+1)+2*arctan(2^(1/2)/(a/c)^(1/4)*
x-1)))+e^2/(a*e^2+c*d^2)/(e*d)^(1/2)*arctan(e*x/(e*d)^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2030 vs. 2(247) = 494.

Time = 0.68 (sec) , antiderivative size = 4084, normalized size of antiderivative = 12.15

$$\int \frac{1}{(d+ex^2)\,(a+cx^4)}\,dx = \text{Too large to display}$$

```
[In] integrate(1/(e*x^2+d)/(c*x^4+a),x, algorithm="fricas")
```

```
[Out] [-1/4*((c*d^2 + a*e^2)*sqrt((2*c*d*e + (a*c^2*d^4 + 2*a^2*c*d^2*e^2 + a^3*e
^4)*sqrt(-(c^3*d^4 - 2*a*c^2*d^2*e^2 + a^2*c*e^4)/(a^3*c^4*d^8 + 4*a^4*c^3*
d^6*e^2 + 6*a^5*c^2*d^4*e^4 + 4*a^6*c*d^2*e^6 + a^7*e^8)))/(a*c^2*d^4 + 2*a
```

^2*c*d^2*e^2 + a^3*e^4))*log(-(c^2*d^2 - a*c*e^2)*x + (a*c^2*d^3 - a^2*c*d*e^2 + (a^3*c^2*d^4*e + 2*a^4*c*d^2*e^3 + a^5*e^5)*sqrt(-(c^3*d^4 - 2*a*c^2*d^2*e^2 + a^2*c*e^4)/(a^3*c^4*d^8 + 4*a^4*c^3*d^6*e^2 + 6*a^5*c^2*d^4*e^4 + 4*a^6*c*d^2*e^6 + a^7*e^8)))*sqrt((2*c*d*e + (a*c^2*d^4 + 2*a^2*c*d^2*e^2 + a^3*e^4)*sqrt(-(c^3*d^4 - 2*a*c^2*d^2*e^2 + a^2*c*e^4)/(a^3*c^4*d^8 + 4*a^4*c^3*d^6*e^2 + 6*a^5*c^2*d^4*e^4 + 4*a^6*c*d^2*e^6 + a^7*e^8)))/(a*c^2*d^4 + 2*a^2*c*d^2*e^2 + a^3*e^4))) - (c*d^2 + a*e^2)*sqrt((2*c*d*e + (a*c^2*d^4 + 2*a^2*c*d^2*e^2 + a^3*e^4)*sqrt(-(c^3*d^4 - 2*a*c^2*d^2*e^2 + a^2*c*e^4)/(a^3*c^4*d^8 + 4*a^4*c^3*d^6*e^2 + 6*a^5*c^2*d^4*e^4 + 4*a^6*c*d^2*e^6 + a^7*e^8)))/(a*c^2*d^4 + 2*a^2*c*d^2*e^2 + a^3*e^4))*log(-(c^2*d^2 - a*c*e^2)*x - (a*c^2*d^3 - a^2*c*d*e^2 + (a^3*c^2*d^4*e + 2*a^4*c*d^2*e^3 + a^5*e^5)*sqrt(-(c^3*d^4 - 2*a*c^2*d^2*e^2 + a^2*c*e^4)/(a^3*c^4*d^8 + 4*a^4*c^3*d^6*e^2 + 6*a^5*c^2*d^4*e^4 + 4*a^6*c*d^2*e^6 + a^7*e^8)))*sqrt((2*c*d*e + (a*c^2*d^4 + 2*a^2*c*d^2*e^2 + a^3*e^4)*sqrt(-(c^3*d^4 - 2*a*c^2*d^2*e^2 + a^2*c*e^4)/(a^3*c^4*d^8 + 4*a^4*c^3*d^6*e^2 + 6*a^5*c^2*d^4*e^4 + 4*a^6*c*d^2*e^6 + a^7*e^8)))/(a*c^2*d^4 + 2*a^2*c*d^2*e^2 + a^3*e^4))) + (c*d^2 + a*e^2)*sqrt((2*c*d*e - (a*c^2*d^4 + 2*a^2*c*d^2*e^2 + a^3*e^4)*sqrt(-(c^3*d^4 - 2*a*c^2*d^2*e^2 + a^2*c*e^4)/(a^3*c^4*d^8 + 4*a^4*c^3*d^6*e^2 + 6*a^5*c^2*d^4*e^4 + 4*a^6*c*d^2*e^6 + a^7*e^8)))/(a*c^2*d^4 + 2*a^2*c*d^2*e^2 + a^3*e^4))*log(-(c^2*d^2 - a*c*e^2)*x + (a*c^2*d^3 - a^2*c*d*e^2 - (a^3*c^2*d^4*e + 2*a^4*c*d^2*e^3 + a^5*e^5)*sqrt(-(c^3*d^4 - 2*a*c^2*d^2*e^2 + a^2*c*e^4)/(a^3*c^4*d^8 + 4*a^4*c^3*d^6*e^2 + 6*a^5*c^2*d^4*e^4 + 4*a^6*c*d^2*e^6 + a^7*e^8)))*sqrt((2*c*d*e - (a*c^2*d^4 + 2*a^2*c*d^2*e^2 + a^3*e^4)*sqrt(-(c^3*d^4 - 2*a*c^2*d^2*e^2 + a^2*c*e^4)/(a^3*c^4*d^8 + 4*a^4*c^3*d^6*e^2 + 6*a^5*c^2*d^4*e^4 + 4*a^6*c*d^2*e^6 + a^7*e^8)))/(a*c^2*d^4 + 2*a^2*c*d^2*e^2 + a^3*e^4))) - (c*d^2 + a*e^2)*sqrt((2*c*d*e - (a*c^2*d^4 + 2*a^2*c*d^2*e^2 + a^3*e^4)*sqrt(-(c^3*d^4 - 2*a*c^2*d^2*e^2 + a^2*c*e^4)/(a^3*c^4*d^8 + 4*a^4*c^3*d^6*e^2 + 6*a^5*c^2*d^4*e^4 + 4*a^6*c*d^2*e^6 + a^7*e^8)))/(a*c^2*d^4 + 2*a^2*c*d^2*e^2 + a^3*e^4))*log(-(c^2*d^2 - a*c*e^2)*x - (a*c^2*d^3 - a^2*c*d*e^2 - (a^3*c^2*d^4*e + 2*a^4*c*d^2*e^3 + a^5*e^5)*sqrt(-(c^3*d^4 - 2*a*c^2*d^2*e^2 + a^2*c*e^4)/(a^3*c^4*d^8 + 4*a^4*c^3*d^6*e^2 + 6*a^5*c^2*d^4*e^4 + 4*a^6*c*d^2*e^6 + a^7*e^8)))*sqrt((2*c*d*e - (a*c^2*d^4 + 2*a^2*c*d^2*e^2 + a^3*e^4)*sqrt(-(c^3*d^4 - 2*a*c^2*d^2*e^2 + a^2*c*e^4)/(a^3*c^4*d^8 + 4*a^4*c^3*d^6*e^2 + 6*a^5*c^2*d^4*e^4 + 4*a^6*c*d^2*e^6 + a^7*e^8)))/(a*c^2*d^4 + 2*a^2*c*d^2*e^2 + a^3*e^4))) - 2*e*sqrt(-e/d)*log((e*x^2 + 2*d*x*sqrt(-e/d) - d)/(e*x^2 + d)))/(c*d^2 + a*e^2), 1/4*(4*e*sqrt(e/d)*arctan(x*sqrt(e/d)) - (c*d^2 + a*e^2)*sqrt((2*c*d*e + (a*c^2*d^4 + 2*a^2*c*d^2*e^2 + a^3*e^4)*sqrt(-(c^3*d^4 - 2*a*c^2*d^2*e^2 + a^2*c*e^4)/(a^3*c^4*d^8 + 4*a^4*c^3*d^6*e^2 + 6*a^5*c^2*d^4*e^4 + 4*a^6*c*d^2*e^6 + a^7*e^8)))/(a*c^2*d^4 + 2*a^2*c*d^2*e^2 + a^3*e^4))*log(-(c^2*d^2 - a*c*e^2)*x + (a*c^2*d^3 - a^2*c*d*e^2 + (a^3*c^2*d^4*e + 2*a^4*c*d^2*e^3 + a^5*e^5)*sqrt(-(c^3*d^4 - 2*a*c^2*d^2*e^2 + a^2*c*e^4)/(a^3*c^4*d^8 + 4*a^4*c^3*d^6*e^2 + 6*a^5*c^2*d^4*e^4 + 4*a^6*c*d^2*e^6 + a^7*e^8)))*sqrt((2*c*d*e + (a*c^2*d^4 + 2*a^2*c*d^2*e^2 + a^3*e^4)*sqrt(-(c^3*d^4 - 2*a*c^2*d^2*e^2 + a^2*c*e^4)/(a^3*c^4*d^8 + 4*a^4*c^3*d^6*e^2 + 6*a^5*c^2*d^4*e^4 + 4*a^6*c*d^2*e^6 + a^7*e^8)))/

```
(a*c^2*d^4 + 2*a^2*c*d^2*e^2 + a^3*e^4))) + (c*d^2 + a*e^2)*sqrt((2*c*d*e +
 (a*c^2*d^4 + 2*a^2*c*d^2*e^2 + a^3*e^4)*sqrt(-(c^3*d^4 - 2*a*c^2*d^2*e^2 +
 a^2*c*e^4)/(a^3*c^4*d^8 + 4*a^4*c^3*d^6*e^2 + 6*a^5*c^2*d^4*e^4 + 4*a^6*c*
d^2*e^6 + a^7*e^8)))/(a*c^2*d^4 + 2*a^2*c*d^2*e^2 + a^3*e^4))*log(-(c^2*d^2
 - a*c*e^2)*x - (a*c^2*d^3 - a^2*c*d*e^2 + (a^3*c^2*d^4*e + 2*a^4*c*d^2*e^3
 + a^5*e^5)*sqrt(-(c^3*d^4 - 2*a*c^2*d^2*e^2 + a^2*c*e^4)/(a^3*c^4*d^8 + 4*
a^4*c^3*d^6*e^2 + 6*a^5*c^2*d^4*e^4 + 4*a^6*c*d^2*e^6 + a^7*e^8)))*sqrt((2*
c*d*e + (a*c^2*d^4 + 2*a^2*c*d^2*e^2 + a^3*e^4)*sqrt(-(c^3*d^4 - 2*a*c^2*d^
2*e^2 + a^2*c*e^4)/(a^3*c^4*d^8 + 4*a^4*c^3*d^6*e^2 + 6*a^5*c^2*d^4*e^4 + 4
*a^6*c*d^2*e^6 + a^7*e^8)))/(a*c^2*d^4 + 2*a^2*c*d^2*e^2 + a^3*e^4))) - (c*
d^2 + a*e^2)*sqrt((2*c*d*e - (a*c^2*d^4 + 2*a^2*c*d^2*e^2 + a^3*e^4)*sqrt(-
(c^3*d^4 - 2*a*c^2*d^2*e^2 + a^2*c*e^4)/(a^3*c^4*d^8 + 4*a^4*c^3*d^6*e^2 +
6*a^5*c^2*d^4*e^4 + 4*a^6*c*d^2*e^6 + a^7*e^8)))/(a*c^2*d^4 + 2*a^2*c*d^2*e
^2 + a^3*e^4))*log(-(c^2*d^2 - a*c*e^2)*x + (a*c^2*d^3 - a^2*c*d*e^2 - (a^3
*c^2*d^4*e + 2*a^4*c*d^2*e^3 + a^5*e^5)*sqrt(-(c^3*d^4 - 2*a*c^2*d^2*e^2 +
a^2*c*e^4)/(a^3*c^4*d^8 + 4*a^4*c^3*d^6*e^2 + 6*a^5*c^2*d^4*e^4 + 4*a^6*c*d
^2*e^6 + a^7*e^8)))*sqrt((2*c*d*e - (a*c^2*d^4 + 2*a^2*c*d^2*e^2 + a^3*e^4)
*sqrt(-(c^3*d^4 - 2*a*c^2*d^2*e^2 + a^2*c*e^4)/(a^3*c^4*d^8 + 4*a^4*c^3*d^6
*e^2 + 6*a^5*c^2*d^4*e^4 + 4*a^6*c*d^2*e^6 + a^7*e^8)))/(a*c^2*d^4 + 2*a^2*
c*d^2*e^2 + a^3*e^4))) + (c*d^2 + a*e^2)*sqrt((2*c*d*e - (a*c^2*d^4 + 2*a^2
*c*d^2*e^2 + a^3*e^4)*sqrt(-(c^3*d^4 - 2*a*c^2*d^2*e^2 + a^2*c*e^4)/(a^3*c^
4*d^8 + 4*a^4*c^3*d^6*e^2 + 6*a^5*c^2*d^4*e^4 + 4*a^6*c*d^2*e^6 + a^7*e^8))
)/(a*c^2*d^4 + 2*a^2*c*d^2*e^2 + a^3*e^4))*log(-(c^2*d^2 - a*c*e^2)*x - (a*
c^2*d^3 - a^2*c*d*e^2 - (a^3*c^2*d^4*e + 2*a^4*c*d^2*e^3 + a^5*e^5)*sqrt(-(
c^3*d^4 - 2*a*c^2*d^2*e^2 + a^2*c*e^4)/(a^3*c^4*d^8 + 4*a^4*c^3*d^6*e^2 + 6
*a^5*c^2*d^4*e^4 + 4*a^6*c*d^2*e^6 + a^7*e^8)))*sqrt((2*c*d*e - (a*c^2*d^4
+ 2*a^2*c*d^2*e^2 + a^3*e^4)*sqrt(-(c^3*d^4 - 2*a*c^2*d^2*e^2 + a^2*c*e^4)/
(a^3*c^4*d^8 + 4*a^4*c^3*d^6*e^2 + 6*a^5*c^2*d^4*e^4 + 4*a^6*c*d^2*e^6 + a^
7*e^8)))/(a*c^2*d^4 + 2*a^2*c*d^2*e^2 + a^3*e^4))))/(c*d^2 + a*e^2)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(d+ex^2)(a+cx^4)}\,dx = \text{Timed out}$$

```
[In] integrate(1/(e*x**2+d)/(c*x**4+a),x)
```

```
[Out] Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{(d+ex^2)(a+cx^4)}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(1/(e*x^2+d)/(c*x^4+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 344, normalized size of antiderivative = 1.02

$$\int \frac{1}{(d+ex^2)(a+cx^4)}\,dx = \frac{e^2\arctan\left(\frac{ex}{\sqrt{de}}\right)}{(cd^2+ae^2)\sqrt{de}} + \frac{\left((ac^3)^{\frac14}c^2d-(ac^3)^{\frac34}e\right)\arctan\left(\frac{\sqrt{2}\left(2x+\sqrt{2}\left(\frac{a}{c}\right)^{\frac14}\right)}{2\left(\frac{a}{c}\right)^{\frac14}}\right)}{2\left(\sqrt{2}ac^3d^2+\sqrt{2}a^2c^2e^2\right)} + \frac{\left((ac^3)^{\frac14}c^2d-(ac^3)^{\frac34}e\right)\arctan\left(\frac{\sqrt{2}\left(2x-\sqrt{2}\left(\frac{a}{c}\right)^{\frac14}\right)}{2\left(\frac{a}{c}\right)^{\frac14}}\right)}{2\left(\sqrt{2}ac^3d^2+\sqrt{2}a^2c^2e^2\right)} + \frac{\left((ac^3)^{\frac14}c^2d+(ac^3)^{\frac34}e\right)\log\left(x^2+\sqrt{2}x\left(\frac{a}{c}\right)^{\frac14}+\sqrt{\frac{a}{c}}\right)}{4\left(\sqrt{2}ac^3d^2+\sqrt{2}a^2c^2e^2\right)} - \frac{\left((ac^3)^{\frac14}c^2d+(ac^3)^{\frac34}e\right)\log\left(x^2-\sqrt{2}x\left(\frac{a}{c}\right)^{\frac14}+\sqrt{\frac{a}{c}}\right)}{4\left(\sqrt{2}ac^3d^2+\sqrt{2}a^2c^2e^2\right)}$$

```
[In] integrate(1/(e*x^2+d)/(c*x^4+a),x, algorithm="giac")
```

```
[Out] e^2*arctan(e*x/sqrt(d*e))/((c*d^2 + a*e^2)*sqrt(d*e)) + 1/2*((a*c^3)^(1/4)*
c^2*d - (a*c^3)^(3/4)*e)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(a/c)^(1/4))/(a/
c)^(1/4))/(sqrt(2)*a*c^3*d^2 + sqrt(2)*a^2*c^2*e^2) + 1/2*((a*c^3)^(1/4)*c^
2*d - (a*c^3)^(3/4)*e)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(a/c)^(1/4))/(a/c)
^(1/4))/(sqrt(2)*a*c^3*d^2 + sqrt(2)*a^2*c^2*e^2) + 1/4*((a*c^3)^(1/4)*c^2*
d + (a*c^3)^(3/4)*e)*log(x^2 + sqrt(2)*x*(a/c)^(1/4) + sqrt(a/c))/(sqrt(2)*
```

```
a*c^3*d^2 + sqrt(2)*a^2*c^2*e^2) - 1/4*((a*c^3)^(1/4)*c^2*d + (a*c^3)^(3/4)
*e)*log(x^2 - sqrt(2)*x*(a/c)^(1/4) + sqrt(a/c))/(sqrt(2)*a*c^3*d^2 + sqrt(
2)*a^2*c^2*e^2)
```

Mupad [B] (verification not implemented)

Time = 8.52 (sec) , antiderivative size = 4802, normalized size of antiderivative = 14.29

$$\int \frac{1}{(d+ex^2)(a+cx^4)}\,dx = \text{Too large to display}$$

```
[In] int(1/((a + c*x^4)*(d + e*x^2)),x)
```

```
[Out] atan(((((a*e^2*(-a^3*c)^(1/2) - c*d^2*(-a^3*c)^(1/2) + 2*a^2*c*d*e)/(16*(a^
5*e^4 + a^3*c^2*d^4 + 2*a^4*c*d^2*e^2)))^(1/2)*(4*c^6*d^3*e^3 - (((a*e^2*(-
a^3*c)^(1/2) - c*d^2*(-a^3*c)^(1/2) + 2*a^2*c*d*e)/(16*(a^5*e^4 + a^3*c^2*d
^4 + 2*a^4*c*d^2*e^2)))^(1/2)*(256*a^4*c^4*e^8 + x*((a*e^2*(-a^3*c)^(1/2) -
 c*d^2*(-a^3*c)^(1/2) + 2*a^2*c*d*e)/(16*(a^5*e^4 + a^3*c^2*d^4 + 2*a^4*c*d
^2*e^2)))^(1/2)*(512*a^5*c^4*e^9 - 512*a^2*c^7*d^6*e^3 - 512*a^3*c^6*d^4*e^
5 + 512*a^4*c^5*d^2*e^7) - 64*a*c^7*d^6*e^2 + 128*a^2*c^6*d^4*e^4 + 448*a^3
*c^5*d^2*e^6) + x*(16*c^7*d^5*e^2 + 32*a*c^6*d^3*e^4 - 240*a^2*c^5*d*e^6))*
((a*e^2*(-a^3*c)^(1/2) - c*d^2*(-a^3*c)^(1/2) + 2*a^2*c*d*e)/(16*(a^5*e^4 +
 a^3*c^2*d^4 + 2*a^4*c*d^2*e^2)))^(1/2) + 20*a*c^5*d*e^5) - 6*c^5*e^5*x)*((
a*e^2*(-a^3*c)^(1/2) - c*d^2*(-a^3*c)^(1/2) + 2*a^2*c*d*e)/(16*(a^5*e^4 + a
^3*c^2*d^4 + 2*a^4*c*d^2*e^2)))^(1/2)*1i - (((a*e^2*(-a^3*c)^(1/2) - c*d^2*
(-a^3*c)^(1/2) + 2*a^2*c*d*e)/(16*(a^5*e^4 + a^3*c^2*d^4 + 2*a^4*c*d^2*e^2)
))^(1/2)*(4*c^6*d^3*e^3 - (((a*e^2*(-a^3*c)^(1/2) - c*d^2*(-a^3*c)^(1/2) +
2*a^2*c*d*e)/(16*(a^5*e^4 + a^3*c^2*d^4 + 2*a^4*c*d^2*e^2)))^(1/2)*(256*a^4
*c^4*e^8 - x*((a*e^2*(-a^3*c)^(1/2) - c*d^2*(-a^3*c)^(1/2) + 2*a^2*c*d*e)/(
16*(a^5*e^4 + a^3*c^2*d^4 + 2*a^4*c*d^2*e^2)))^(1/2)*(512*a^5*c^4*e^9 - 512
*a^2*c^7*d^6*e^3 - 512*a^3*c^6*d^4*e^5 + 512*a^4*c^5*d^2*e^7) - 64*a*c^7*d^
6*e^2 + 128*a^2*c^6*d^4*e^4 + 448*a^3*c^5*d^2*e^6) - x*(16*c^7*d^5*e^2 + 32
*a*c^6*d^3*e^4 - 240*a^2*c^5*d*e^6))*((a*e^2*(-a^3*c)^(1/2) - c*d^2*(-a^3*c
)^(1/2) + 2*a^2*c*d*e)/(16*(a^5*e^4 + a^3*c^2*d^4 + 2*a^4*c*d^2*e^2)))^(1/2
) + 20*a*c^5*d*e^5) + 6*c^5*e^5*x)*((a*e^2*(-a^3*c)^(1/2) - c*d^2*(-a^3*c)^
(1/2) + 2*a^2*c*d*e)/(16*(a^5*e^4 + a^3*c^2*d^4 + 2*a^4*c*d^2*e^2)))^(1/2)*
1i)/((((a*e^2*(-a^3*c)^(1/2) - c*d^2*(-a^3*c)^(1/2) + 2*a^2*c*d*e)/(16*(a^5
*e^4 + a^3*c^2*d^4 + 2*a^4*c*d^2*e^2)))^(1/2)*(4*c^6*d^3*e^3 - (((a*e^2*(-a
^3*c)^(1/2) - c*d^2*(-a^3*c)^(1/2) + 2*a^2*c*d*e)/(16*(a^5*e^4 + a^3*c^2*d^
4 + 2*a^4*c*d^2*e^2)))^(1/2)*(256*a^4*c^4*e^8 + x*((a*e^2*(-a^3*c)^(1/2) -
c*d^2*(-a^3*c)^(1/2) + 2*a^2*c*d*e)/(16*(a^5*e^4 + a^3*c^2*d^4 + 2*a^4*c*d^
2*e^2)))^(1/2)*(512*a^5*c^4*e^9 - 512*a^2*c^7*d^6*e^3 - 512*a^3*c^6*d^4*e^5
 + 512*a^4*c^5*d^2*e^7) - 64*a*c^7*d^6*e^2 + 128*a^2*c^6*d^4*e^4 + 448*a^3*
c^5*d^2*e^6) + x*(16*c^7*d^5*e^2 + 32*a*c^6*d^3*e^4 - 240*a^2*c^5*d*e^6))*(
(a*e^2*(-a^3*c)^(1/2) - c*d^2*(-a^3*c)^(1/2) + 2*a^2*c*d*e)/(16*(a^5*e^4 +
```

a^3*c^2*d^4 + 2*a^4*c*d^2*e^2)))^(1/2) + 20*a*c^5*d*e^5) - 6*c^5*e^5*x)*((a
e^2(-a^3*c)^(1/2) - c*d^2*(-a^3*c)^(1/2) + 2*a^2*c*d*e)/(16*(a^5*e^4 + a^
3*c^2*d^4 + 2*a^4*c*d^2*e^2)))^(1/2) + (((a*e^2*(-a^3*c)^(1/2) - c*d^2*(-a^
3*c)^(1/2) + 2*a^2*c*d*e)/(16*(a^5*e^4 + a^3*c^2*d^4 + 2*a^4*c*d^2*e^2)))^(
1/2)*(4*c^6*d^3*e^3 - (((a*e^2*(-a^3*c)^(1/2) - c*d^2*(-a^3*c)^(1/2) + 2*a^
2*c*d*e)/(16*(a^5*e^4 + a^3*c^2*d^4 + 2*a^4*c*d^2*e^2)))^(1/2)*(256*a^4*c^4
e^8 - x((a*e^2*(-a^3*c)^(1/2) - c*d^2*(-a^3*c)^(1/2) + 2*a^2*c*d*e)/(16*(
a^5*e^4 + a^3*c^2*d^4 + 2*a^4*c*d^2*e^2)))^(1/2)*(512*a^5*c^4*e^9 - 512*a^2
*c^7*d^6*e^3 - 512*a^3*c^6*d^4*e^5 + 512*a^4*c^5*d^2*e^7) - 64*a*c^7*d^6*e^
2 + 128*a^2*c^6*d^4*e^4 + 448*a^3*c^5*d^2*e^6) - x*(16*c^7*d^5*e^2 + 32*a*c
^6*d^3*e^4 - 240*a^2*c^5*d*e^6))*((a*e^2*(-a^3*c)^(1/2) - c*d^2*(-a^3*c)^(1
/2) + 2*a^2*c*d*e)/(16*(a^5*e^4 + a^3*c^2*d^4 + 2*a^4*c*d^2*e^2)))^(1/2) +
20*a*c^5*d*e^5) + 6*c^5*e^5*x)*((a*e^2*(-a^3*c)^(1/2) - c*d^2*(-a^3*c)^(1/2
) + 2*a^2*c*d*e)/(16*(a^5*e^4 + a^3*c^2*d^4 + 2*a^4*c*d^2*e^2)))^(1/2)))*((
a*e^2*(-a^3*c)^(1/2) - c*d^2*(-a^3*c)^(1/2) + 2*a^2*c*d*e)/(16*(a^5*e^4 + a
^3*c^2*d^4 + 2*a^4*c*d^2*e^2)))^(1/2)*2i + atan(((((c*d^2*(-a^3*c)^(1/2) -
a*e^2*(-a^3*c)^(1/2) + 2*a^2*c*d*e)/(16*(a^5*e^4 + a^3*c^2*d^4 + 2*a^4*c*d^
2*e^2)))^(1/2)*(4*c^6*d^3*e^3 - (((c*d^2*(-a^3*c)^(1/2) - a*e^2*(-a^3*c)^(1
/2) + 2*a^2*c*d*e)/(16*(a^5*e^4 + a^3*c^2*d^4 + 2*a^4*c*d^2*e^2)))^(1/2)*(2
56*a^4*c^4*e^8 + x*((c*d^2*(-a^3*c)^(1/2) - a*e^2*(-a^3*c)^(1/2) + 2*a^2*c*
d*e)/(16*(a^5*e^4 + a^3*c^2*d^4 + 2*a^4*c*d^2*e^2)))^(1/2)*(512*a^5*c^4*e^9
 - 512*a^2*c^7*d^6*e^3 - 512*a^3*c^6*d^4*e^5 + 512*a^4*c^5*d^2*e^7) - 64*a*
c^7*d^6*e^2 + 128*a^2*c^6*d^4*e^4 + 448*a^3*c^5*d^2*e^6) + x*(16*c^7*d^5*e^
2 + 32*a*c^6*d^3*e^4 - 240*a^2*c^5*d*e^6))*((c*d^2*(-a^3*c)^(1/2) - a*e^2*(
-a^3*c)^(1/2) + 2*a^2*c*d*e)/(16*(a^5*e^4 + a^3*c^2*d^4 + 2*a^4*c*d^2*e^2))
)^(1/2) + 20*a*c^5*d*e^5) - 6*c^5*e^5*x)*((c*d^2*(-a^3*c)^(1/2) - a*e^2*(-a
^3*c)^(1/2) + 2*a^2*c*d*e)/(16*(a^5*e^4 + a^3*c^2*d^4 + 2*a^4*c*d^2*e^2)))^
(1/2)*1i - (((c*d^2*(-a^3*c)^(1/2) - a*e^2*(-a^3*c)^(1/2) + 2*a^2*c*d*e)/(1
6*(a^5*e^4 + a^3*c^2*d^4 + 2*a^4*c*d^2*e^2)))^(1/2)*(4*c^6*d^3*e^3 - (((c*d
^2*(-a^3*c)^(1/2) - a*e^2*(-a^3*c)^(1/2) + 2*a^2*c*d*e)/(16*(a^5*e^4 + a^3*
c^2*d^4 + 2*a^4*c*d^2*e^2)))^(1/2)*(256*a^4*c^4*e^8 - x*((c*d^2*(-a^3*c)^(1
/2) - a*e^2*(-a^3*c)^(1/2) + 2*a^2*c*d*e)/(16*(a^5*e^4 + a^3*c^2*d^4 + 2*a^
4*c*d^2*e^2)))^(1/2)*(512*a^5*c^4*e^9 - 512*a^2*c^7*d^6*e^3 - 512*a^3*c^6*d
^4*e^5 + 512*a^4*c^5*d^2*e^7) - 64*a*c^7*d^6*e^2 + 128*a^2*c^6*d^4*e^4 + 44
8*a^3*c^5*d^2*e^6) - x*(16*c^7*d^5*e^2 + 32*a*c^6*d^3*e^4 - 240*a^2*c^5*d*e
^6))*((c*d^2*(-a^3*c)^(1/2) - a*e^2*(-a^3*c)^(1/2) + 2*a^2*c*d*e)/(16*(a^5*
e^4 + a^3*c^2*d^4 + 2*a^4*c*d^2*e^2)))^(1/2) + 20*a*c^5*d*e^5) + 6*c^5*e^5*
x)*((c*d^2*(-a^3*c)^(1/2) - a*e^2*(-a^3*c)^(1/2) + 2*a^2*c*d*e)/(16*(a^5*e^
4 + a^3*c^2*d^4 + 2*a^4*c*d^2*e^2)))^(1/2)*1i)/((((c*d^2*(-a^3*c)^(1/2) - a
e^2(-a^3*c)^(1/2) + 2*a^2*c*d*e)/(16*(a^5*e^4 + a^3*c^2*d^4 + 2*a^4*c*d^2
e^2)))^(1/2)(4*c^6*d^3*e^3 - (((c*d^2*(-a^3*c)^(1/2) - a*e^2*(-a^3*c)^(1/
2) + 2*a^2*c*d*e)/(16*(a^5*e^4 + a^3*c^2*d^4 + 2*a^4*c*d^2*e^2)))^(1/2)*(25
6*a^4*c^4*e^8 + x*((c*d^2*(-a^3*c)^(1/2) - a*e^2*(-a^3*c)^(1/2) + 2*a^2*c*d
e)/(16(a^5*e^4 + a^3*c^2*d^4 + 2*a^4*c*d^2*e^2)))^(1/2)*(512*a^5*c^4*e^9
- 512*a^2*c^7*d^6*e^3 - 512*a^3*c^6*d^4*e^5 + 512*a^4*c^5*d^2*e^7) - 64*a*c

^7*d^6*e^2 + 128*a^2*c^6*d^4*e^4 + 448*a^3*c^5*d^2*e^6) + x*(16*c^7*d^5*e^2 + 32*a*c^6*d^3*e^4 - 240*a^2*c^5*d*e^6))*((c*d^2*(-a^3*c)^(1/2) - a*e^2*(-a^3*c)^(1/2) + 2*a^2*c*d*e)/(16*(a^5*e^4 + a^3*c^2*d^4 + 2*a^4*c*d^2*e^2)))^(1/2) + 20*a*c^5*d*e^5) - 6*c^5*e^5*x)*((c*d^2*(-a^3*c)^(1/2) - a*e^2*(-a^3*c)^(1/2) + 2*a^2*c*d*e)/(16*(a^5*e^4 + a^3*c^2*d^4 + 2*a^4*c*d^2*e^2)))^(1/2) + (((c*d^2*(-a^3*c)^(1/2) - a*e^2*(-a^3*c)^(1/2) + 2*a^2*c*d*e)/(16*(a^5*e^4 + a^3*c^2*d^4 + 2*a^4*c*d^2*e^2)))^(1/2)*(4*c^6*d^3*e^3 - (((c*d^2*(-a^3*c)^(1/2) - a*e^2*(-a^3*c)^(1/2) + 2*a^2*c*d*e)/(16*(a^5*e^4 + a^3*c^2*d^4 + 2*a^4*c*d^2*e^2)))^(1/2)*(256*a^4*c^4*e^8 - x*((c*d^2*(-a^3*c)^(1/2) - a*e^2*(-a^3*c)^(1/2) + 2*a^2*c*d*e)/(16*(a^5*e^4 + a^3*c^2*d^4 + 2*a^4*c*d^2*e^2)))^(1/2)*(512*a^5*c^4*e^9 - 512*a^2*c^7*d^6*e^3 - 512*a^3*c^6*d^4*e^5 + 512*a^4*c^5*d^2*e^7) - 64*a*c^7*d^6*e^2 + 128*a^2*c^6*d^4*e^4 + 448*a^3*c^5*d^2*e^6) - x*(16*c^7*d^5*e^2 + 32*a*c^6*d^3*e^4 - 240*a^2*c^5*d*e^6))*((c*d^2*(-a^3*c)^(1/2) - a*e^2*(-a^3*c)^(1/2) + 2*a^2*c*d*e)/(16*(a^5*e^4 + a^3*c^2*d^4 + 2*a^4*c*d^2*e^2)))^(1/2) + 20*a*c^5*d*e^5) + 6*c^5*e^5*x)*((c*d^2*(-a^3*c)^(1/2) - a*e^2*(-a^3*c)^(1/2) + 2*a^2*c*d*e)/(16*(a^5*e^4 + a^3*c^2*d^4 + 2*a^4*c*d^2*e^2)))^(1/2)))*((c*d^2*(-a^3*c)^(1/2) - a*e^2*(-a^3*c)^(1/2) + 2*a^2*c*d*e)/(16*(a^5*e^4 + a^3*c^2*d^4 + 2*a^4*c*d^2*e^2)))^(1/2)*2i - (log(16*a^2*e^2*(-d*e^3)^(3/2) + c^2*d^5*e^3*x - c^2*d^5*e*(-d*e^3)^(1/2) + 16*a^2*d*e^7*x + a*c*d^2*(-d*e^3)^(3/2) + a*c*d^3*e^5*x)*(-d*e^3)^(1/2))/(2*(c*d^3 + a*d*e^2)) + (log(c^2*d^5*e^3*x - 16*a^2*e^2*(-d*e^3)^(3/2) + c^2*d^5*e*(-d*e^3)^(1/2) + 16*a^2*d*e^7*x + 4*a*c*d^2*(-d*e^3)^(3/2) + a*c*d^3*e^5*x + 5*a*c*d^3*e^3*(-d*e^3)^(1/2))*(-d*e^3)^(1/2))/(2*c*d^3 + 2*a*d*e^2)

3.242 $\int \frac{1}{x^2(d+ex^2)(a+cx^4)}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 348

$$\begin{aligned}\int \frac{1}{x^2(d+ex^2)(a+cx^4)}\,dx = &-\frac{1}{adx}-\frac{e^{5/2}\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{d^{3/2}(cd^2+ae^2)}\\&+\frac{c^{3/4}\left(\sqrt{c}d+\sqrt{a}e\right)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{5/4}(cd^2+ae^2)}\\&-\frac{c^{3/4}\left(\sqrt{c}d+\sqrt{a}e\right)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{5/4}(cd^2+ae^2)}\\&-\frac{c^{3/4}\left(\sqrt{c}d-\sqrt{a}e\right)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)}{4\sqrt{2}a^{5/4}(cd^2+ae^2)}\\&+\frac{c^{3/4}\left(\sqrt{c}d-\sqrt{a}e\right)\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)}{4\sqrt{2}a^{5/4}(cd^2+ae^2)}\end{aligned}$$

```
[Out] -1/a/d/x-e^(5/2)*arctan(x*e^(1/2)/d^(1/2))/d^(3/2)/(a*e^2+c*d^2)-1/8*c^(3/4
)*ln(-a^(1/4)*c^(1/4)*x*2^(1/2)+a^(1/2)+x^2*c^(1/2))*(-e*a^(1/2)+d*c^(1/2))
/a^(5/4)/(a*e^2+c*d^2)*2^(1/2)+1/8*c^(3/4)*ln(a^(1/4)*c^(1/4)*x*2^(1/2)+a^(
1/2)+x^2*c^(1/2))*(-e*a^(1/2)+d*c^(1/2))/a^(5/4)/(a*e^2+c*d^2)*2^(1/2)-1/4*
c^(3/4)*arctan(-1+c^(1/4)*x*2^(1/2)/a^(1/4))*(e*a^(1/2)+d*c^(1/2))/a^(5/4)/
(a*e^2+c*d^2)*2^(1/2)-1/4*c^(3/4)*arctan(1+c^(1/4)*x*2^(1/2)/a^(1/4))*(e*a^
(1/2)+d*c^(1/2))/a^(5/4)/(a*e^2+c*d^2)*2^(1/2)
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 348, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {1302, 211, 1182, 1176, 631, 210, 1179, 642}

$$\int \frac{1}{x^2(d+ex^2)(a+cx^4)}\,dx = \frac{c^{3/4}\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\left(\sqrt{a}e+\sqrt{c}d\right)}{2\sqrt{2}a^{5/4}(ae^2+cd^2)} - \frac{c^{3/4}\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)\left(\sqrt{a}e+\sqrt{c}d\right)}{2\sqrt{2}a^{5/4}(ae^2+cd^2)} - \frac{c^{3/4}\left(\sqrt{c}d-\sqrt{a}e\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{4\sqrt{2}a^{5/4}(ae^2+cd^2)} + \frac{c^{3/4}\left(\sqrt{c}d-\sqrt{a}e\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{4\sqrt{2}a^{5/4}(ae^2+cd^2)} - \frac{e^{5/2}\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{d^{3/2}(ae^2+cd^2)} - \frac{1}{adx}$$

```
[In] Int[1/(x^2*(d + e*x^2)*(a + c*x^4)),x]
```

```
[Out] -(1/(a*d*x)) - (e^(5/2)*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(d^(3/2)*(c*d^2 + a*e^
2)) + (c^(3/4)*(Sqrt[c]*d + Sqrt[a]*e)*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/
4)])/(2*Sqrt[2]*a^(5/4)*(c*d^2 + a*e^2)) - (c^(3/4)*(Sqrt[c]*d + Sqrt[a]*e)
*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(5/4)*(c*d^2 + a*e^2
)) - (c^(3/4)*(Sqrt[c]*d - Sqrt[a]*e)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)
*x + Sqrt[c]*x^2])/(4*Sqrt[2]*a^(5/4)*(c*d^2 + a*e^2)) + (c^(3/4)*(Sqrt[c]*
d - Sqrt[a]*e)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(4*S
qrt[2]*a^(5/4)*(c*d^2 + a*e^2))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
```

```
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1182

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[(-a)
*c]
```

Rule 1302

```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.))/((a_) + (c_.)*(x_)^4),
 x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*((d + e*x^2)^q/(a + c*x^4)), x],
x] /; FreeQ[{a, c, d, e, f, m}, x] && IntegerQ[q] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \left(\frac{1}{adx^2} - \frac{e^3}{d\,(cd^2+ae^2)\,(d+ex^2)} - \frac{c(ae+cdx^2)}{a\,(cd^2+ae^2)\,(a+cx^4)}\right) dx \\ &= -\frac{1}{adx} - \frac{c\int \frac{ae+cdx^2}{a+cx^4}\,dx}{a\,(cd^2+ae^2)} - \frac{e^3\int \frac{1}{d+ex^2}\,dx}{d\,(cd^2+ae^2)}\end{aligned}$$

$$=-\frac{1}{adx}-\frac{e^{5/2}\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{d^{3/2}\left(cd^2+ae^2\right)}+\frac{\left(c\left(d-\frac{\sqrt{a}e}{\sqrt{c}}\right)\right)\int\frac{\sqrt{a}\sqrt{c}-cx^2}{a+cx^4}\,dx}{2a\left(cd^2+ae^2\right)}-\frac{\left(c\left(d+\frac{\sqrt{a}e}{\sqrt{c}}\right)\right)\int\frac{\sqrt{a}\sqrt{c}+cx^2}{a+cx^4}\,dx}{2a\left(cd^2+ae^2\right)}$$

$$=-\frac{1}{adx}-\frac{e^{5/2}\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{d^{3/2}\left(cd^2+ae^2\right)}-\frac{\left(c^{5/4}\left(d-\frac{\sqrt{a}e}{\sqrt{c}}\right)\right)\int\frac{\frac{\sqrt{2}\sqrt[4]{a}}{\sqrt[4]{c}}+2x}{-\frac{\sqrt{a}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}-x^2}\,dx}{4\sqrt{2}a^{5/4}\left(cd^2+ae^2\right)}-\frac{\left(c^{5/4}\left(d-\frac{\sqrt{a}e}{\sqrt{c}}\right)\right)\int\frac{\frac{\sqrt{2}\sqrt[4]{a}}{\sqrt[4]{c}}-2x}{-\frac{\sqrt{a}}{\sqrt{c}}+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}-x^2}\,dx}{4\sqrt{2}a^{5/4}\left(cd^2+ae^2\right)}-\frac{\left(c\left(d+\frac{\sqrt{a}e}{\sqrt{c}}\right)\right)\int\frac{1}{\frac{\sqrt{a}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}+x^2}\,dx}{4a\left(cd^2+ae^2\right)}-\frac{\left(c\left(d+\frac{\sqrt{a}e}{\sqrt{c}}\right)\right)\int\frac{1}{\frac{\sqrt{a}}{\sqrt{c}}+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}+x^2}\,dx}{4a\left(cd^2+ae^2\right)}$$

$$=-\frac{1}{adx}-\frac{e^{5/2}\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{d^{3/2}\left(cd^2+ae^2\right)}-\frac{c^{5/4}\left(d-\frac{\sqrt{a}e}{\sqrt{c}}\right)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)}{4\sqrt{2}a^{5/4}\left(cd^2+ae^2\right)}+\frac{c^{5/4}\left(d-\frac{\sqrt{a}e}{\sqrt{c}}\right)\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)}{4\sqrt{2}a^{5/4}\left(cd^2+ae^2\right)}-\frac{\left(c^{3/4}\left(\sqrt{c}d+\sqrt{a}e\right)\right)\text{Subst}\left(\int\frac{1}{-1-x^2}\,dx,x,1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{5/4}\left(cd^2+ae^2\right)}+\frac{\left(c^{3/4}\left(\sqrt{c}d+\sqrt{a}e\right)\right)\text{Subst}\left(\int\frac{1}{-1-x^2}\,dx,x,1+\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{5/4}\left(cd^2+ae^2\right)}$$

$$=-\frac{1}{adx}-\frac{e^{5/2}\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{d^{3/2}\left(cd^2+ae^2\right)}+\frac{c^{3/4}\left(\sqrt{c}d+\sqrt{a}e\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{5/4}\left(cd^2+ae^2\right)}-\frac{c^{3/4}\left(\sqrt{c}d+\sqrt{a}e\right)\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{5/4}\left(cd^2+ae^2\right)}-\frac{c^{5/4}\left(d-\frac{\sqrt{a}e}{\sqrt{c}}\right)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)}{4\sqrt{2}a^{5/4}\left(cd^2+ae^2\right)}+\frac{c^{5/4}\left(d-\frac{\sqrt{a}e}{\sqrt{c}}\right)\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)}{4\sqrt{2}a^{5/4}\left(cd^2+ae^2\right)}$$

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 389, normalized size of antiderivative = 1.12

$$\int \frac{1}{x^2 (d+ex^2) (a+cx^4)} \, dx$$
$$= \frac{-8a^{5/4}e^{5/2}x \arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right) - \sqrt{d}\left(8\sqrt[4]{a}cd^2 + 8a^{5/4}e^2 - 2\sqrt{2}c^{3/4}d(\sqrt{c}d + \sqrt{a}e)\, x \arctan\left(1 - \frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right) + 2\sqrt{}\right.}{}$$

[In] Integrate[1/(x^2*(d + e*x^2)*(a + c*x^4)),x]

[Out] (-8*a^(5/4)*e^(5/2)*x*ArcTan[(Sqrt[e]*x)/Sqrt[d]] - Sqrt[d]*(8*a^(1/4)*c*d^2 + 8*a^(5/4)*e^2 - 2*Sqrt[2]*c^(3/4)*d*(Sqrt[c]*d + Sqrt[a]*e)*x*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)] + 2*Sqrt[2]*c^(3/4)*d*(Sqrt[c]*d + Sqrt[a]*e)*x*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)] + Sqrt[2]*c^(5/4)*d^2*x*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2] - Sqrt[2]*Sqrt[a]*c^(3/4)*d*e*x*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2] - Sqrt[2]*c^(5/4)*d^2*x*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2] + Sqrt[2]*Sqrt[a]*c^(3/4)*d*e*x*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2]))/(8*a^(5/4)*d^(3/2)*(c*d^2 + a*e^2)*x)

Maple [A] (verified)

Time = 0.40 (sec) , antiderivative size = 266, normalized size of antiderivative = 0.76

method	result
default	$-\frac{c\left(\frac{e\left(\frac{a}{c}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x^2+\left(\frac{a}{c}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{c}}}{x^2-\left(\frac{a}{c}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{c}}}\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{c}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{c}\right)^{\frac{1}{4}}}-1\right)\right)}{8}+\frac{d\sqrt{2}\left(\ln\left(\frac{x^2-\left(\frac{a}{c}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{c}}}{x^2+\left(\frac{a}{c}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{c}}}\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{c}\right)}\right.\right.}{8\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right.}{(a\,e^2+c\,d^2)a}$
risch	$-\frac{1}{adx}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\left(a^7e^4+2a^6c\,d^2e^2+a^5c^2d^4\right)\_Z^4+4a^3c^2de\,\_Z^2+c^3\right)}\_R\ln\left(\left(\left(6e^8d^3a^9+19e^6d^5c\,a^8+25e^4d^7c^2a^7+17e^2d^9c^3a^6-\right.\right.\right.\right.}{}$

[In] int(1/x^2/(e*x^2+d)/(c*x^4+a),x,method=_RETURNVERBOSE)

[Out] -c/(a*e^2+c*d^2)/a*(1/8*e*(a/c)^(1/4)*2^(1/2)*(ln((x^2+(a/c)^(1/4)*x*2^(1/2)+(a/c)^(1/2))/(x^2-(a/c)^(1/4)*x*2^(1/2)+(a/c)^(1/2)))+2*arctan(2^(1/2)/(a/c)^(1/4)*x+1)+2*arctan(2^(1/2)/(a/c)^(1/4)*x-1))+1/8*d/(a/c)^(1/4)*2^(1/2)*(ln((x^2-(a/c)^(1/4)*x*2^(1/2)+(a/c)^(1/2))/(x^2+(a/c)^(1/4)*x*2^(1/2)+(a/c)^(1/2)))+2*arctan(2^(1/2)/(a/c)^(1/4)*x+1)+2*arctan(2^(1/2)/(a/c)^(1/4)*x-1)))-1/d*e^3/(a*e^2+c*d^2)/(e*d)^(1/2)*arctan(e*x/(e*d)^(1/2))-1/a/d/x

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2169 vs. 2(259) = 518.

Time = 1.81 (sec) , antiderivative size = 4362, normalized size of antiderivative = 12.53

$$\int \frac{1}{x^2 (d + ex^2) (a + cx^4)} \, dx = \text{Too large to display}$$

```
[In] integrate(1/x^2/(e*x^2+d)/(c*x^4+a),x, algorithm="fricas")
```

```
[Out] [1/4*(2*a*e^2*x*sqrt(-e/d)*log((e*x^2 - 2*d*x*sqrt(-e/d) - d)/(e*x^2 + d))
+ (a*c*d^3 + a^2*d*e^2)*x*sqrt(-(2*c^2*d*e + (a^2*c^2*d^4 + 2*a^3*c*d^2*e^2
 + a^4*e^4)*sqrt(-(c^5*d^4 - 2*a*c^4*d^2*e^2 + a^2*c^3*e^4)/(a^5*c^4*d^8 +
4*a^6*c^3*d^6*e^2 + 6*a^7*c^2*d^4*e^4 + 4*a^8*c*d^2*e^6 + a^9*e^8)))/(a^2*c
^2*d^4 + 2*a^3*c*d^2*e^2 + a^4*e^4))*log(-(c^3*d^2 - a*c^2*e^2)*x + (a^2*c^
2*d^2*e - a^3*c*e^3 - (a^4*c^2*d^5 + 2*a^5*c*d^3*e^2 + a^6*d*e^4)*sqrt(-(c^
5*d^4 - 2*a*c^4*d^2*e^2 + a^2*c^3*e^4)/(a^5*c^4*d^8 + 4*a^6*c^3*d^6*e^2 + 6
*a^7*c^2*d^4*e^4 + 4*a^8*c*d^2*e^6 + a^9*e^8)))*sqrt(-(2*c^2*d*e + (a^2*c^2
*d^4 + 2*a^3*c*d^2*e^2 + a^4*e^4)*sqrt(-(c^5*d^4 - 2*a*c^4*d^2*e^2 + a^2*c^
3*e^4)/(a^5*c^4*d^8 + 4*a^6*c^3*d^6*e^2 + 6*a^7*c^2*d^4*e^4 + 4*a^8*c*d^2*e
^6 + a^9*e^8)))/(a^2*c^2*d^4 + 2*a^3*c*d^2*e^2 + a^4*e^4))) - (a*c*d^3 + a^
2*d*e^2)*x*sqrt(-(2*c^2*d*e + (a^2*c^2*d^4 + 2*a^3*c*d^2*e^2 + a^4*e^4)*sqr
t(-(c^5*d^4 - 2*a*c^4*d^2*e^2 + a^2*c^3*e^4)/(a^5*c^4*d^8 + 4*a^6*c^3*d^6*e
^2 + 6*a^7*c^2*d^4*e^4 + 4*a^8*c*d^2*e^6 + a^9*e^8)))/(a^2*c^2*d^4 + 2*a^3*
c*d^2*e^2 + a^4*e^4))*log(-(c^3*d^2 - a*c^2*e^2)*x - (a^2*c^2*d^2*e - a^3*c
*e^3 - (a^4*c^2*d^5 + 2*a^5*c*d^3*e^2 + a^6*d*e^4)*sqrt(-(c^5*d^4 - 2*a*c^4
*d^2*e^2 + a^2*c^3*e^4)/(a^5*c^4*d^8 + 4*a^6*c^3*d^6*e^2 + 6*a^7*c^2*d^4*e^
4 + 4*a^8*c*d^2*e^6 + a^9*e^8)))*sqrt(-(2*c^2*d*e + (a^2*c^2*d^4 + 2*a^3*c*
d^2*e^2 + a^4*e^4)*sqrt(-(c^5*d^4 - 2*a*c^4*d^2*e^2 + a^2*c^3*e^4)/(a^5*c^4
*d^8 + 4*a^6*c^3*d^6*e^2 + 6*a^7*c^2*d^4*e^4 + 4*a^8*c*d^2*e^6 + a^9*e^8)))
/(a^2*c^2*d^4 + 2*a^3*c*d^2*e^2 + a^4*e^4))) + (a*c*d^3 + a^2*d*e^2)*x*sqrt
(-(2*c^2*d*e - (a^2*c^2*d^4 + 2*a^3*c*d^2*e^2 + a^4*e^4)*sqrt(-(c^5*d^4 - 2
*a*c^4*d^2*e^2 + a^2*c^3*e^4)/(a^5*c^4*d^8 + 4*a^6*c^3*d^6*e^2 + 6*a^7*c^2*
d^4*e^4 + 4*a^8*c*d^2*e^6 + a^9*e^8)))/(a^2*c^2*d^4 + 2*a^3*c*d^2*e^2 + a^4
*e^4))*log(-(c^3*d^2 - a*c^2*e^2)*x + (a^2*c^2*d^2*e - a^3*c*e^3 + (a^4*c^2
*d^5 + 2*a^5*c*d^3*e^2 + a^6*d*e^4)*sqrt(-(c^5*d^4 - 2*a*c^4*d^2*e^2 + a^2*
c^3*e^4)/(a^5*c^4*d^8 + 4*a^6*c^3*d^6*e^2 + 6*a^7*c^2*d^4*e^4 + 4*a^8*c*d^2
*e^6 + a^9*e^8)))*sqrt(-(2*c^2*d*e - (a^2*c^2*d^4 + 2*a^3*c*d^2*e^2 + a^4*e
^4)*sqrt(-(c^5*d^4 - 2*a*c^4*d^2*e^2 + a^2*c^3*e^4)/(a^5*c^4*d^8 + 4*a^6*c^
3*d^6*e^2 + 6*a^7*c^2*d^4*e^4 + 4*a^8*c*d^2*e^6 + a^9*e^8)))/(a^2*c^2*d^4 +
 2*a^3*c*d^2*e^2 + a^4*e^4))) - (a*c*d^3 + a^2*d*e^2)*x*sqrt(-(2*c^2*d*e -
(a^2*c^2*d^4 + 2*a^3*c*d^2*e^2 + a^4*e^4)*sqrt(-(c^5*d^4 - 2*a*c^4*d^2*e^2
+ a^2*c^3*e^4)/(a^5*c^4*d^8 + 4*a^6*c^3*d^6*e^2 + 6*a^7*c^2*d^4*e^4 + 4*a^8
*c*d^2*e^6 + a^9*e^8)))/(a^2*c^2*d^4 + 2*a^3*c*d^2*e^2 + a^4*e^4))*log(-(c^
3*d^2 - a*c^2*e^2)*x - (a^2*c^2*d^2*e - a^3*c*e^3 + (a^4*c^2*d^5 + 2*a^5*c*
```

```
d^3*e^2 + a^6*d*e^4)*sqrt(-(c^5*d^4 - 2*a*c^4*d^2*e^2 + a^2*c^3*e^4)/(a^5*c
^4*d^8 + 4*a^6*c^3*d^6*e^2 + 6*a^7*c^2*d^4*e^4 + 4*a^8*c*d^2*e^6 + a^9*e^8)
))*sqrt(-(2*c^2*d*e - (a^2*c^2*d^4 + 2*a^3*c*d^2*e^2 + a^4*e^4)*sqrt(-(c^5*
d^4 - 2*a*c^4*d^2*e^2 + a^2*c^3*e^4)/(a^5*c^4*d^8 + 4*a^6*c^3*d^6*e^2 + 6*a
^7*c^2*d^4*e^4 + 4*a^8*c*d^2*e^6 + a^9*e^8)))/(a^2*c^2*d^4 + 2*a^3*c*d^2*e^
2 + a^4*e^4))) - 4*c*d^2 - 4*a*e^2)/((a*c*d^3 + a^2*d*e^2)*x), -1/4*(4*a*e^
2*x*sqrt(e/d)*arctan(x*sqrt(e/d)) - (a*c*d^3 + a^2*d*e^2)*x*sqrt(-(2*c^2*d*
e + (a^2*c^2*d^4 + 2*a^3*c*d^2*e^2 + a^4*e^4)*sqrt(-(c^5*d^4 - 2*a*c^4*d^2*
e^2 + a^2*c^3*e^4)/(a^5*c^4*d^8 + 4*a^6*c^3*d^6*e^2 + 6*a^7*c^2*d^4*e^4 + 4
*a^8*c*d^2*e^6 + a^9*e^8)))/(a^2*c^2*d^4 + 2*a^3*c*d^2*e^2 + a^4*e^4))*log(
-(c^3*d^2 - a*c^2*e^2)*x + (a^2*c^2*d^2*e - a^3*c*e^3 - (a^4*c^2*d^5 + 2*a^
5*c*d^3*e^2 + a^6*d*e^4)*sqrt(-(c^5*d^4 - 2*a*c^4*d^2*e^2 + a^2*c^3*e^4)/(a
^5*c^4*d^8 + 4*a^6*c^3*d^6*e^2 + 6*a^7*c^2*d^4*e^4 + 4*a^8*c*d^2*e^6 + a^9*
e^8)))*sqrt(-(2*c^2*d*e + (a^2*c^2*d^4 + 2*a^3*c*d^2*e^2 + a^4*e^4)*sqrt(-(
c^5*d^4 - 2*a*c^4*d^2*e^2 + a^2*c^3*e^4)/(a^5*c^4*d^8 + 4*a^6*c^3*d^6*e^2 +
 6*a^7*c^2*d^4*e^4 + 4*a^8*c*d^2*e^6 + a^9*e^8)))/(a^2*c^2*d^4 + 2*a^3*c*d^
2*e^2 + a^4*e^4))) + (a*c*d^3 + a^2*d*e^2)*x*sqrt(-(2*c^2*d*e + (a^2*c^2*d^
4 + 2*a^3*c*d^2*e^2 + a^4*e^4)*sqrt(-(c^5*d^4 - 2*a*c^4*d^2*e^2 + a^2*c^3*e
^4)/(a^5*c^4*d^8 + 4*a^6*c^3*d^6*e^2 + 6*a^7*c^2*d^4*e^4 + 4*a^8*c*d^2*e^6
+ a^9*e^8)))/(a^2*c^2*d^4 + 2*a^3*c*d^2*e^2 + a^4*e^4))*log(-(c^3*d^2 - a*c
^2*e^2)*x - (a^2*c^2*d^2*e - a^3*c*e^3 - (a^4*c^2*d^5 + 2*a^5*c*d^3*e^2 + a
^6*d*e^4)*sqrt(-(c^5*d^4 - 2*a*c^4*d^2*e^2 + a^2*c^3*e^4)/(a^5*c^4*d^8 + 4*
a^6*c^3*d^6*e^2 + 6*a^7*c^2*d^4*e^4 + 4*a^8*c*d^2*e^6 + a^9*e^8)))*sqrt(-(2
*c^2*d*e + (a^2*c^2*d^4 + 2*a^3*c*d^2*e^2 + a^4*e^4)*sqrt(-(c^5*d^4 - 2*a*c
^4*d^2*e^2 + a^2*c^3*e^4)/(a^5*c^4*d^8 + 4*a^6*c^3*d^6*e^2 + 6*a^7*c^2*d^4*
e^4 + 4*a^8*c*d^2*e^6 + a^9*e^8)))/(a^2*c^2*d^4 + 2*a^3*c*d^2*e^2 + a^4*e^4
))) - (a*c*d^3 + a^2*d*e^2)*x*sqrt(-(2*c^2*d*e - (a^2*c^2*d^4 + 2*a^3*c*d^2
*e^2 + a^4*e^4)*sqrt(-(c^5*d^4 - 2*a*c^4*d^2*e^2 + a^2*c^3*e^4)/(a^5*c^4*d^
8 + 4*a^6*c^3*d^6*e^2 + 6*a^7*c^2*d^4*e^4 + 4*a^8*c*d^2*e^6 + a^9*e^8)))/(a
^2*c^2*d^4 + 2*a^3*c*d^2*e^2 + a^4*e^4))*log(-(c^3*d^2 - a*c^2*e^2)*x + (a^
2*c^2*d^2*e - a^3*c*e^3 + (a^4*c^2*d^5 + 2*a^5*c*d^3*e^2 + a^6*d*e^4)*sqrt(
-(c^5*d^4 - 2*a*c^4*d^2*e^2 + a^2*c^3*e^4)/(a^5*c^4*d^8 + 4*a^6*c^3*d^6*e^2
 + 6*a^7*c^2*d^4*e^4 + 4*a^8*c*d^2*e^6 + a^9*e^8)))*sqrt(-(2*c^2*d*e - (a^2
*c^2*d^4 + 2*a^3*c*d^2*e^2 + a^4*e^4)*sqrt(-(c^5*d^4 - 2*a*c^4*d^2*e^2 + a^
2*c^3*e^4)/(a^5*c^4*d^8 + 4*a^6*c^3*d^6*e^2 + 6*a^7*c^2*d^4*e^4 + 4*a^8*c*d
^2*e^6 + a^9*e^8)))/(a^2*c^2*d^4 + 2*a^3*c*d^2*e^2 + a^4*e^4))) + (a*c*d^3
+ a^2*d*e^2)*x*sqrt(-(2*c^2*d*e - (a^2*c^2*d^4 + 2*a^3*c*d^2*e^2 + a^4*e^4)
*sqrt(-(c^5*d^4 - 2*a*c^4*d^2*e^2 + a^2*c^3*e^4)/(a^5*c^4*d^8 + 4*a^6*c^3*d
^6*e^2 + 6*a^7*c^2*d^4*e^4 + 4*a^8*c*d^2*e^6 + a^9*e^8)))/(a^2*c^2*d^4 + 2*
a^3*c*d^2*e^2 + a^4*e^4))*log(-(c^3*d^2 - a*c^2*e^2)*x - (a^2*c^2*d^2*e - a
^3*c*e^3 + (a^4*c^2*d^5 + 2*a^5*c*d^3*e^2 + a^6*d*e^4)*sqrt(-(c^5*d^4 - 2*a
*c^4*d^2*e^2 + a^2*c^3*e^4)/(a^5*c^4*d^8 + 4*a^6*c^3*d^6*e^2 + 6*a^7*c^2*d^
4*e^4 + 4*a^8*c*d^2*e^6 + a^9*e^8)))*sqrt(-(2*c^2*d*e - (a^2*c^2*d^4 + 2*a^
3*c*d^2*e^2 + a^4*e^4)*sqrt(-(c^5*d^4 - 2*a*c^4*d^2*e^2 + a^2*c^3*e^4)/(a^5
*c^4*d^8 + 4*a^6*c^3*d^6*e^2 + 6*a^7*c^2*d^4*e^4 + 4*a^8*c*d^2*e^6 + a^9*e^
```

```
8)))/(a^2*c^2*d^4 + 2*a^3*c*d^2*e^2 + a^4*e^4))) + 4*c*d^2 + 4*a*e^2)/((a*c
*d^3 + a^2*d*e^2)*x)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{x^2 (d+ex^2) (a+cx^4)} dx = \text{Timed out}$$

```
[In] integrate(1/x**2/(e*x**2+d)/(c*x**4+a),x)
[Out] Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{x^2 (d+ex^2) (a+cx^4)} dx = \text{Exception raised: ValueError}$$

```
[In] integrate(1/x^2/(e*x^2+d)/(c*x^4+a),x, algorithm="maxima")
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 353, normalized size of antiderivative = 1.01

$$\int \frac{1}{x^2(d+ex^2)(a+cx^4)}\,dx = -\frac{e^3\arctan\left(\frac{ex}{\sqrt{de}}\right)}{(cd^3+ade^2)\sqrt{de}} - \frac{\left((ac^3)^{\frac{1}{4}}ace + (ac^3)^{\frac{3}{4}}d\right)\arctan\left(\frac{\sqrt{2}\left(2x+\sqrt{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{2\left(\sqrt{2}a^2c^2d^2+\sqrt{2}a^3ce^2\right)} - \frac{\left((ac^3)^{\frac{1}{4}}ace + (ac^3)^{\frac{3}{4}}d\right)\arctan\left(\frac{\sqrt{2}\left(2x-\sqrt{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{2\left(\sqrt{2}a^2c^2d^2+\sqrt{2}a^3ce^2\right)} - \frac{\left((ac^3)^{\frac{1}{4}}ace - (ac^3)^{\frac{3}{4}}d\right)\log\left(x^2+\sqrt{2}x\left(\frac{a}{c}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{c}}\right)}{4\left(\sqrt{2}a^2c^2d^2+\sqrt{2}a^3ce^2\right)} + \frac{\left((ac^3)^{\frac{1}{4}}ace - (ac^3)^{\frac{3}{4}}d\right)\log\left(x^2-\sqrt{2}x\left(\frac{a}{c}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{c}}\right)}{4\left(\sqrt{2}a^2c^2d^2+\sqrt{2}a^3ce^2\right)} - \frac{1}{adx}$$

```
[In] integrate(1/x^2/(e*x^2+d)/(c*x^4+a),x, algorithm="giac")
```

```
[Out] -e^3*arctan(e*x/sqrt(d*e))/((c*d^3 + a*d*e^2)*sqrt(d*e)) - 1/2*((a*c^3)^(1/
4)*a*c*e + (a*c^3)^(3/4)*d)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(a/c)^(1/4))/
(a/c)^(1/4))/(sqrt(2)*a^2*c^2*d^2 + sqrt(2)*a^3*c*e^2) - 1/2*((a*c^3)^(1/4)
*a*c*e + (a*c^3)^(3/4)*d)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(a/c)^(1/4))/(a
/c)^(1/4))/(sqrt(2)*a^2*c^2*d^2 + sqrt(2)*a^3*c*e^2) - 1/4*((a*c^3)^(1/4)*a
*c*e - (a*c^3)^(3/4)*d)*log(x^2 + sqrt(2)*x*(a/c)^(1/4) + sqrt(a/c))/(sqrt(
2)*a^2*c^2*d^2 + sqrt(2)*a^3*c*e^2) + 1/4*((a*c^3)^(1/4)*a*c*e - (a*c^3)^(3
/4)*d)*log(x^2 - sqrt(2)*x*(a/c)^(1/4) + sqrt(a/c))/(sqrt(2)*a^2*c^2*d^2 +
sqrt(2)*a^3*c*e^2) - 1/(a*d*x)
```

Mupad [B] (verification not implemented)

Time = 8.68 (sec) , antiderivative size = 5761, normalized size of antiderivative = 16.55

$$\int \frac{1}{x^2(d+ex^2)(a+cx^4)}\,dx = \text{Too large to display}$$

```
[In] int(1/(x^2*(a + c*x^4)*(d + e*x^2)),x)
```

```
[Out] atan(((x*(2*a^7*c^7*d^9*e^5 - 4*a^8*c^6*d^7*e^7) - (-(a*e^2*(-a^5*c^3)^(1/2
) - c*d^2*(-a^5*c^3)^(1/2) + 2*a^3*c^2*d*e)/(16*(a^7*e^4 + a^5*c^2*d^4 + 2*
```

a^6*c*d^2*e^2)))^(1/2)*(((-(a*e^2*(-a^5*c^3)^(1/2) - c*d^2*(-a^5*c^3)^(1/2)
 + 2*a^3*c^2*d*e)/(16*(a^7*e^4 + a^5*c^2*d^4 + 2*a^6*c*d^2*e^2)))^(1/2)*(x*
(-(a*e^2*(-a^5*c^3)^(1/2) - c*d^2*(-a^5*c^3)^(1/2) + 2*a^3*c^2*d*e)/(16*(a^
7*e^4 + a^5*c^2*d^4 + 2*a^6*c*d^2*e^2)))^(1/2)*(512*a^11*c^7*d^15*e^3 + 512
*a^12*c^6*d^13*e^5 - 512*a^13*c^5*d^11*e^7 - 512*a^14*c^4*d^9*e^9) - 192*a^
10*c^7*d^14*e^3 - 128*a^11*c^6*d^12*e^5 + 320*a^12*c^5*d^10*e^7 + 256*a^13*
c^4*d^8*e^9) + x*(16*a^8*c^8*d^14*e^2 + 32*a^9*c^7*d^12*e^4 - 112*a^10*c^6*
d^10*e^6 + 128*a^11*c^5*d^8*e^8))*(-(a*e^2*(-a^5*c^3)^(1/2) - c*d^2*(-a^5*c
^3)^(1/2) + 2*a^3*c^2*d*e)/(16*(a^7*e^4 + a^5*c^2*d^4 + 2*a^6*c*d^2*e^2)))^
(1/2) - 4*a^7*c^8*d^13*e^2 - 4*a^8*c^7*d^11*e^4 + 16*a^10*c^5*d^7*e^8))*(-(
a*e^2*(-a^5*c^3)^(1/2) - c*d^2*(-a^5*c^3)^(1/2) + 2*a^3*c^2*d*e)/(16*(a^7*e
^4 + a^5*c^2*d^4 + 2*a^6*c*d^2*e^2)))^(1/2)*1i + (x*(2*a^7*c^7*d^9*e^5 - 4*
a^8*c^6*d^7*e^7) - (-(a*e^2*(-a^5*c^3)^(1/2) - c*d^2*(-a^5*c^3)^(1/2) + 2*a
^3*c^2*d*e)/(16*(a^7*e^4 + a^5*c^2*d^4 + 2*a^6*c*d^2*e^2)))^(1/2)*(((-(a*e^
2*(-a^5*c^3)^(1/2) - c*d^2*(-a^5*c^3)^(1/2) + 2*a^3*c^2*d*e)/(16*(a^7*e^4 +
 a^5*c^2*d^4 + 2*a^6*c*d^2*e^2)))^(1/2)*(x*(-(a*e^2*(-a^5*c^3)^(1/2) - c*d^
2*(-a^5*c^3)^(1/2) + 2*a^3*c^2*d*e)/(16*(a^7*e^4 + a^5*c^2*d^4 + 2*a^6*c*d^
2*e^2)))^(1/2)*(512*a^11*c^7*d^15*e^3 + 512*a^12*c^6*d^13*e^5 - 512*a^13*c^
5*d^11*e^7 - 512*a^14*c^4*d^9*e^9) + 192*a^10*c^7*d^14*e^3 + 128*a^11*c^6*d
^12*e^5 - 320*a^12*c^5*d^10*e^7 - 256*a^13*c^4*d^8*e^9) + x*(16*a^8*c^8*d^1
4*e^2 + 32*a^9*c^7*d^12*e^4 - 112*a^10*c^6*d^10*e^6 + 128*a^11*c^5*d^8*e^8)
)*(-(a*e^2*(-a^5*c^3)^(1/2) - c*d^2*(-a^5*c^3)^(1/2) + 2*a^3*c^2*d*e)/(16*(
a^7*e^4 + a^5*c^2*d^4 + 2*a^6*c*d^2*e^2)))^(1/2) + 4*a^7*c^8*d^13*e^2 + 4*a
^8*c^7*d^11*e^4 - 16*a^10*c^5*d^7*e^8))*(-(a*e^2*(-a^5*c^3)^(1/2) - c*d^2*(
-a^5*c^3)^(1/2) + 2*a^3*c^2*d*e)/(16*(a^7*e^4 + a^5*c^2*d^4 + 2*a^6*c*d^2*e
^2)))^(1/2)*1i)/((x*(2*a^7*c^7*d^9*e^5 - 4*a^8*c^6*d^7*e^7) - (-(a*e^2*(-a^
5*c^3)^(1/2) - c*d^2*(-a^5*c^3)^(1/2) + 2*a^3*c^2*d*e)/(16*(a^7*e^4 + a^5*c
^2*d^4 + 2*a^6*c*d^2*e^2)))^(1/2)*(((-(a*e^2*(-a^5*c^3)^(1/2) - c*d^2*(-a^5
*c^3)^(1/2) + 2*a^3*c^2*d*e)/(16*(a^7*e^4 + a^5*c^2*d^4 + 2*a^6*c*d^2*e^2))
)^(1/2)*(x*(-(a*e^2*(-a^5*c^3)^(1/2) - c*d^2*(-a^5*c^3)^(1/2) + 2*a^3*c^2*d
e)/(16(a^7*e^4 + a^5*c^2*d^4 + 2*a^6*c*d^2*e^2)))^(1/2)*(512*a^11*c^7*d^1
5*e^3 + 512*a^12*c^6*d^13*e^5 - 512*a^13*c^5*d^11*e^7 - 512*a^14*c^4*d^9*e^
9) - 192*a^10*c^7*d^14*e^3 - 128*a^11*c^6*d^12*e^5 + 320*a^12*c^5*d^10*e^7
 + 256*a^13*c^4*d^8*e^9) + x*(16*a^8*c^8*d^14*e^2 + 32*a^9*c^7*d^12*e^4 - 11
2*a^10*c^6*d^10*e^6 + 128*a^11*c^5*d^8*e^8))*(-(a*e^2*(-a^5*c^3)^(1/2) - c*
d^2*(-a^5*c^3)^(1/2) + 2*a^3*c^2*d*e)/(16*(a^7*e^4 + a^5*c^2*d^4 + 2*a^6*c*
d^2*e^2)))^(1/2) - 4*a^7*c^8*d^13*e^2 - 4*a^8*c^7*d^11*e^4 + 16*a^10*c^5*d^
7*e^8))*(-(a*e^2*(-a^5*c^3)^(1/2) - c*d^2*(-a^5*c^3)^(1/2) + 2*a^3*c^2*d*e)
/(16*(a^7*e^4 + a^5*c^2*d^4 + 2*a^6*c*d^2*e^2)))^(1/2) - (x*(2*a^7*c^7*d^9*
e^5 - 4*a^8*c^6*d^7*e^7) - (-(a*e^2*(-a^5*c^3)^(1/2) - c*d^2*(-a^5*c^3)^(1/
2) + 2*a^3*c^2*d*e)/(16*(a^7*e^4 + a^5*c^2*d^4 + 2*a^6*c*d^2*e^2)))^(1/2)*(
((-(a*e^2*(-a^5*c^3)^(1/2) - c*d^2*(-a^5*c^3)^(1/2) + 2*a^3*c^2*d*e)/(16*(a
^7*e^4 + a^5*c^2*d^4 + 2*a^6*c*d^2*e^2)))^(1/2)*(x*(-(a*e^2*(-a^5*c^3)^(1/2
) - c*d^2*(-a^5*c^3)^(1/2) + 2*a^3*c^2*d*e)/(16*(a^7*e^4 + a^5*c^2*d^4 + 2*
a^6*c*d^2*e^2)))^(1/2)*(512*a^11*c^7*d^15*e^3 + 512*a^12*c^6*d^13*e^5 - 512

*a^13*c^5*d^11*e^7 - 512*a^14*c^4*d^9*e^9) + 192*a^10*c^7*d^14*e^3 + 128*a^11*c^6*d^12*e^5 - 320*a^12*c^5*d^10*e^7 - 256*a^13*c^4*d^8*e^9) + x*(16*a^8*c^8*d^14*e^2 + 32*a^9*c^7*d^12*e^4 - 112*a^10*c^6*d^10*e^6 + 128*a^11*c^5*d^8*e^8))*(-(a*e^2*(-a^5*c^3)^(1/2) - c*d^2*(-a^5*c^3)^(1/2) + 2*a^3*c^2*d*e)/(16*(a^7*e^4 + a^5*c^2*d^4 + 2*a^6*c*d^2*e^2)))^(1/2) + 4*a^7*c^8*d^13*e^2 + 4*a^8*c^7*d^11*e^4 - 16*a^10*c^5*d^7*e^8))*(-(a*e^2*(-a^5*c^3)^(1/2) - c*d^2*(-a^5*c^3)^(1/2) + 2*a^3*c^2*d*e)/(16*(a^7*e^4 + a^5*c^2*d^4 + 2*a^6*c*d^2*e^2)))^(1/2)))*(-(a*e^2*(-a^5*c^3)^(1/2) - c*d^2*(-a^5*c^3)^(1/2) + 2*a^3*c^2*d*e)/(16*(a^7*e^4 + a^5*c^2*d^4 + 2*a^6*c*d^2*e^2)))^(1/2)*2i + atan(((x*(2*a^7*c^7*d^9*e^5 - 4*a^8*c^6*d^7*e^7) - (-(c*d^2*(-a^5*c^3)^(1/2) - a*e^2*(-a^5*c^3)^(1/2) + 2*a^3*c^2*d*e)/(16*(a^7*e^4 + a^5*c^2*d^4 + 2*a^6*c*d^2*e^2)))^(1/2)*(((-(c*d^2*(-a^5*c^3)^(1/2) - a*e^2*(-a^5*c^3)^(1/2) + 2*a^3*c^2*d*e)/(16*(a^7*e^4 + a^5*c^2*d^4 + 2*a^6*c*d^2*e^2)))^(1/2)*(x*(-(c*d^2*(-a^5*c^3)^(1/2) - a*e^2*(-a^5*c^3)^(1/2) + 2*a^3*c^2*d*e)/(16*(a^7*e^4 + a^5*c^2*d^4 + 2*a^6*c*d^2*e^2)))^(1/2)*(512*a^11*c^7*d^15*e^3 + 512*a^12*c^6*d^13*e^5 - 512*a^13*c^5*d^11*e^7 - 512*a^14*c^4*d^9*e^9) - 192*a^10*c^7*d^14*e^3 - 128*a^11*c^6*d^12*e^5 + 320*a^12*c^5*d^10*e^7 + 256*a^13*c^4*d^8*e^9) + x*(16*a^8*c^8*d^14*e^2 + 32*a^9*c^7*d^12*e^4 - 112*a^10*c^6*d^10*e^6 + 128*a^11*c^5*d^8*e^8))*(-(c*d^2*(-a^5*c^3)^(1/2) - a*e^2*(-a^5*c^3)^(1/2) + 2*a^3*c^2*d*e)/(16*(a^7*e^4 + a^5*c^2*d^4 + 2*a^6*c*d^2*e^2)))^(1/2) - 4*a^7*c^8*d^13*e^2 - 4*a^8*c^7*d^11*e^4 + 16*a^10*c^5*d^7*e^8))*(-(c*d^2*(-a^5*c^3)^(1/2) - a*e^2*(-a^5*c^3)^(1/2) + 2*a^3*c^2*d*e)/(16*(a^7*e^4 + a^5*c^2*d^4 + 2*a^6*c*d^2*e^2)))^(1/2)*1i + (x*(2*a^7*c^7*d^9*e^5 - 4*a^8*c^6*d^7*e^7) - (-(c*d^2*(-a^5*c^3)^(1/2) - a*e^2*(-a^5*c^3)^(1/2) + 2*a^3*c^2*d*e)/(16*(a^7*e^4 + a^5*c^2*d^4 + 2*a^6*c*d^2*e^2)))^(1/2)*(((-(c*d^2*(-a^5*c^3)^(1/2) - a*e^2*(-a^5*c^3)^(1/2) + 2*a^3*c^2*d*e)/(16*(a^7*e^4 + a^5*c^2*d^4 + 2*a^6*c*d^2*e^2)))^(1/2)*(x*(-(c*d^2*(-a^5*c^3)^(1/2) - a*e^2*(-a^5*c^3)^(1/2) + 2*a^3*c^2*d*e)/(16*(a^7*e^4 + a^5*c^2*d^4 + 2*a^6*c*d^2*e^2)))^(1/2)*(512*a^11*c^7*d^15*e^3 + 512*a^12*c^6*d^13*e^5 - 512*a^13*c^5*d^11*e^7 - 512*a^14*c^4*d^9*e^9) + 192*a^10*c^7*d^14*e^3 + 128*a^11*c^6*d^12*e^5 - 320*a^12*c^5*d^10*e^7 - 256*a^13*c^4*d^8*e^9) + x*(16*a^8*c^8*d^14*e^2 + 32*a^9*c^7*d^12*e^4 - 112*a^10*c^6*d^10*e^6 + 128*a^11*c^5*d^8*e^8))*(-(c*d^2*(-a^5*c^3)^(1/2) - a*e^2*(-a^5*c^3)^(1/2) + 2*a^3*c^2*d*e)/(16*(a^7*e^4 + a^5*c^2*d^4 + 2*a^6*c*d^2*e^2)))^(1/2) + 4*a^7*c^8*d^13*e^2 + 4*a^8*c^7*d^11*e^4 - 16*a^10*c^5*d^7*e^8))*(-(c*d^2*(-a^5*c^3)^(1/2) - a*e^2*(-a^5*c^3)^(1/2) + 2*a^3*c^2*d*e)/(16*(a^7*e^4 + a^5*c^2*d^4 + 2*a^6*c*d^2*e^2)))^(1/2)*1i)/((x*(2*a^7*c^7*d^9*e^5 - 4*a^8*c^6*d^7*e^7) - (-(c*d^2*(-a^5*c^3)^(1/2) - a*e^2*(-a^5*c^3)^(1/2) + 2*a^3*c^2*d*e)/(16*(a^7*e^4 + a^5*c^2*d^4 + 2*a^6*c*d^2*e^2)))^(1/2)*(((-(c*d^2*(-a^5*c^3)^(1/2) - a*e^2*(-a^5*c^3)^(1/2) + 2*a^3*c^2*d*e)/(16*(a^7*e^4 + a^5*c^2*d^4 + 2*a^6*c*d^2*e^2)))^(1/2)*(x*(-(c*d^2*(-a^5*c^3)^(1/2) - a*e^2*(-a^5*c^3)^(1/2) + 2*a^3*c^2*d*e)/(16*(a^7*e^4 + a^5*c^2*d^4 + 2*a^6*c*d^2*e^2)))^(1/2)*(512*a^11*c^7*d^15*e^3 + 512*a^12*c^6*d^13*e^5 - 512*a^13*c^5*d^11*e^7 - 512*a^14*c^4*d^9*e^9) - 192*a^10*c^7*d^14*e^3 - 128*a^11*c^6*d^12*e^5 + 320*a^12*c^5*d^10*e^7 + 256*a^13*c^4*d^8*e^9) + x*(16*a^8*c^8*d^14*e^2 + 32*a^9*c^7*d^12*e^4 - 112

```
*a^10*c^6*d^10*e^6 + 128*a^11*c^5*d^8*e^8))*(-(c*d^2*(-a^5*c^3)^(1/2) - a*e
^2*(-a^5*c^3)^(1/2) + 2*a^3*c^2*d*e)/(16*(a^7*e^4 + a^5*c^2*d^4 + 2*a^6*c*d
^2*e^2)))^(1/2) - 4*a^7*c^8*d^13*e^2 - 4*a^8*c^7*d^11*e^4 + 16*a^10*c^5*d^7
*e^8))*(-(c*d^2*(-a^5*c^3)^(1/2) - a*e^2*(-a^5*c^3)^(1/2) + 2*a^3*c^2*d*e)/
(16*(a^7*e^4 + a^5*c^2*d^4 + 2*a^6*c*d^2*e^2)))^(1/2) - (x*(2*a^7*c^7*d^9*e
^5 - 4*a^8*c^6*d^7*e^7) - (-(c*d^2*(-a^5*c^3)^(1/2) - a*e^2*(-a^5*c^3)^(1/2
) + 2*a^3*c^2*d*e)/(16*(a^7*e^4 + a^5*c^2*d^4 + 2*a^6*c*d^2*e^2)))^(1/2)*((
(-(c*d^2*(-a^5*c^3)^(1/2) - a*e^2*(-a^5*c^3)^(1/2) + 2*a^3*c^2*d*e)/(16*(a^
7*e^4 + a^5*c^2*d^4 + 2*a^6*c*d^2*e^2)))^(1/2)*(x*(-(c*d^2*(-a^5*c^3)^(1/2)
 - a*e^2*(-a^5*c^3)^(1/2) + 2*a^3*c^2*d*e)/(16*(a^7*e^4 + a^5*c^2*d^4 + 2*a
^6*c*d^2*e^2)))^(1/2)*(512*a^11*c^7*d^15*e^3 + 512*a^12*c^6*d^13*e^5 - 512*
a^13*c^5*d^11*e^7 - 512*a^14*c^4*d^9*e^9) + 192*a^10*c^7*d^14*e^3 + 128*a^1
1*c^6*d^12*e^5 - 320*a^12*c^5*d^10*e^7 - 256*a^13*c^4*d^8*e^9) + x*(16*a^8*
c^8*d^14*e^2 + 32*a^9*c^7*d^12*e^4 - 112*a^10*c^6*d^10*e^6 + 128*a^11*c^5*d
^8*e^8))*(-(c*d^2*(-a^5*c^3)^(1/2) - a*e^2*(-a^5*c^3)^(1/2) + 2*a^3*c^2*d*e
)/(16*(a^7*e^4 + a^5*c^2*d^4 + 2*a^6*c*d^2*e^2)))^(1/2) + 4*a^7*c^8*d^13*e^
2 + 4*a^8*c^7*d^11*e^4 - 16*a^10*c^5*d^7*e^8))*(-(c*d^2*(-a^5*c^3)^(1/2) -
a*e^2*(-a^5*c^3)^(1/2) + 2*a^3*c^2*d*e)/(16*(a^7*e^4 + a^5*c^2*d^4 + 2*a^6*
c*d^2*e^2)))^(1/2)))*(-(c*d^2*(-a^5*c^3)^(1/2) - a*e^2*(-a^5*c^3)^(1/2) + 2
*a^3*c^2*d*e)/(16*(a^7*e^4 + a^5*c^2*d^4 + 2*a^6*c*d^2*e^2)))^(1/2)*2i - 1/
(a*d*x) - (log(c^4*d^11*(-d^3*e^5)^(1/2) - 16*a^4*e^3*(-d^3*e^5)^(3/2) + 16
*a^4*d^4*e^11*x + c^4*d^12*e^3*x + a*c^3*d^9*e^2*(-d^3*e^5)^(1/2) + a*c^3*d
^10*e^5*x - 16*a^3*c*d^6*e^9*x + 16*a^3*c*d^2*e*(-d^3*e^5)^(3/2))*(-d^3*e^5
)^(1/2))/(2*(c*d^5 + a*d^3*e^2)) + (log(16*a^4*e^3*(-d^3*e^5)^(3/2) - c^4*d
^11*(-d^3*e^5)^(1/2) + 16*a^4*d^4*e^11*x + c^4*d^12*e^3*x - a*c^3*d^9*e^2*(
-d^3*e^5)^(1/2) + a*c^3*d^10*e^5*x - 16*a^3*c*d^6*e^9*x - 16*a^3*c*d^2*e*(-
d^3*e^5)^(3/2))*(-d^3*e^5)^(1/2))/(2*c*d^5 + 2*a*d^3*e^2)
```

3.243 $\int \frac{1}{x^4(d+ex^2)(a+cx^4)}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 360

$$\begin{aligned}\int \frac{1}{x^4(d+ex^2)(a+cx^4)}\,dx =& -\frac{1}{3adx^3}+\frac{e}{ad^2x}+\frac{e^{7/2}\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{d^{5/2}(cd^2+ae^2)}\\ &+\frac{c^{5/4}\left(\sqrt{c}d-\sqrt{a}e\right)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{7/4}(cd^2+ae^2)}\\ &-\frac{c^{5/4}\left(\sqrt{c}d-\sqrt{a}e\right)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{7/4}(cd^2+ae^2)}\\ &+\frac{c^{5/4}\left(\sqrt{c}d+\sqrt{a}e\right)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)}{4\sqrt{2}a^{7/4}(cd^2+ae^2)}\\ &-\frac{c^{5/4}\left(\sqrt{c}d+\sqrt{a}e\right)\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)}{4\sqrt{2}a^{7/4}(cd^2+ae^2)}\end{aligned}$$

```
[Out] -1/3/a/d/x^3+e/a/d^2/x+e^(7/2)*arctan(x*e^(1/2)/d^(1/2))/d^(5/2)/(a*e^2+c*d
^2)-1/4*c^(5/4)*arctan(-1+c^(1/4)*x*2^(1/2)/a^(1/4))*(-e*a^(1/2)+d*c^(1/2))
/a^(7/4)/(a*e^2+c*d^2)*2^(1/2)-1/4*c^(5/4)*arctan(1+c^(1/4)*x*2^(1/2)/a^(1/
4))*(-e*a^(1/2)+d*c^(1/2))/a^(7/4)/(a*e^2+c*d^2)*2^(1/2)+1/8*c^(5/4)*ln(-a^
(1/4)*c^(1/4)*x*2^(1/2)+a^(1/2)+x^2*c^(1/2))*(e*a^(1/2)+d*c^(1/2))/a^(7/4)/
(a*e^2+c*d^2)*2^(1/2)-1/8*c^(5/4)*ln(a^(1/4)*c^(1/4)*x*2^(1/2)+a^(1/2)+x^2*
c^(1/2))*(e*a^(1/2)+d*c^(1/2))/a^(7/4)/(a*e^2+c*d^2)*2^(1/2)
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 360, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {1302, 211, 1182, 1176, 631, 210, 1179, 642}

$$\int \frac{1}{x^4(d+ex^2)(a+cx^4)}\,dx = \frac{c^{5/4}\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)(\sqrt{c}d-\sqrt{a}e)}{2\sqrt{2}a^{7/4}(ae^2+cd^2)} - \frac{c^{5/4}\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)(\sqrt{c}d-\sqrt{a}e)}{2\sqrt{2}a^{7/4}(ae^2+cd^2)} + \frac{c^{5/4}(\sqrt{a}e+\sqrt{c}d)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{4\sqrt{2}a^{7/4}(ae^2+cd^2)} - \frac{c^{5/4}(\sqrt{a}e+\sqrt{c}d)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{4\sqrt{2}a^{7/4}(ae^2+cd^2)} + \frac{e^{7/2}\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{d^{5/2}(ae^2+cd^2)} + \frac{e}{ad^2x} - \frac{1}{3adx^3}$$

```
[In] Int[1/(x^4*(d + e*x^2)*(a + c*x^4)),x]
```

```
[Out] -1/3*1/(a*d*x^3) + e/(a*d^2*x) + (e^(7/2)*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(d^(
5/2)*(c*d^2 + a*e^2)) + (c^(5/4)*(Sqrt[c]*d - Sqrt[a]*e)*ArcTan[1 - (Sqrt[2
]*c^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(7/4)*(c*d^2 + a*e^2)) - (c^(5/4)*(Sqrt
[c]*d - Sqrt[a]*e)*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(7
/4)*(c*d^2 + a*e^2)) + (c^(5/4)*(Sqrt[c]*d + Sqrt[a]*e)*Log[Sqrt[a] - Sqrt[
2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(4*Sqrt[2]*a^(7/4)*(c*d^2 + a*e^2)) -
(c^(5/4)*(Sqrt[c]*d + Sqrt[a]*e)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x +
Sqrt[c]*x^2])/(4*Sqrt[2]*a^(7/4)*(c*d^2 + a*e^2))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
```

```
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1182

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[(-a)
*c]
```

Rule 1302

```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.))/((a_) + (c_.)*(x_)^4),
 x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*((d + e*x^2)^q/(a + c*x^4)), x],
x] /; FreeQ[{a, c, d, e, f, m}, x] && IntegerQ[q] && IntegerQ[m]
```

Rubi steps

$$\text{integral} = \int \left(\frac{1}{adx^4} - \frac{e}{ad^2x^2} + \frac{e^4}{d^2(cd^2+ae^2)(d+ex^2)} - \frac{c^2(d-ex^2)}{a(cd^2+ae^2)(a+cx^4)}\right) dx$$

$$= -\frac{1}{3adx^3} + \frac{e}{ad^2x} - \frac{c^2 \int \frac{d-ex^2}{a+cx^4} dx}{a(cd^2+ae^2)} + \frac{e^4 \int \frac{1}{d+ex^2} dx}{d^2(cd^2+ae^2)}$$

$$= -\frac{1}{3adx^3} + \frac{e}{ad^2x} + \frac{e^{7/2}\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{d^{5/2}(cd^2+ae^2)} - \frac{\left(c\left(\frac{\sqrt{c}d}{\sqrt{a}}-e\right)\right)\int\frac{\sqrt{a}\sqrt{c}+cx^2}{a+cx^4}\,dx}{2a(cd^2+ae^2)} - \frac{\left(c\left(\frac{\sqrt{c}d}{\sqrt{a}}+e\right)\right)\int\frac{\sqrt{a}\sqrt{c}-cx^2}{a+cx^4}\,dx}{2a(cd^2+ae^2)}$$

$$= -\frac{1}{3adx^3} + \frac{e}{ad^2x} + \frac{e^{7/2}\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{d^{5/2}(cd^2+ae^2)} - \frac{\left(c\left(\frac{\sqrt{c}d}{\sqrt{a}}-e\right)\right)\int\frac{1}{\frac{\sqrt{a}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}+x^2}\,dx}{4a(cd^2+ae^2)} - \frac{\left(c\left(\frac{\sqrt{c}d}{\sqrt{a}}-e\right)\right)\int\frac{1}{\frac{\sqrt{a}}{\sqrt{c}}+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}+x^2}\,dx}{4a(cd^2+ae^2)} + \frac{\left(c^{5/4}\left(\sqrt{c}d+\sqrt{a}e\right)\right)\int\frac{\frac{\sqrt{2}\sqrt[4]{a}}{\sqrt[4]{c}}+2x}{-\frac{\sqrt{a}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}-x^2}\,dx}{4\sqrt{2}a^{7/4}(cd^2+ae^2)} + \frac{\left(c^{5/4}\left(\sqrt{c}d+\sqrt{a}e\right)\right)\int\frac{\frac{\sqrt{2}\sqrt[4]{a}}{\sqrt[4]{c}}-2x}{-\frac{\sqrt{a}}{\sqrt{c}}+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}-x^2}\,dx}{4\sqrt{2}a^{7/4}(cd^2+ae^2)}$$

$$= -\frac{1}{3adx^3} + \frac{e}{ad^2x} + \frac{e^{7/2}\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{d^{5/2}(cd^2+ae^2)} + \frac{c^{5/4}\left(\sqrt{c}d+\sqrt{a}e\right)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)}{4\sqrt{2}a^{7/4}(cd^2+ae^2)} - \frac{c^{5/4}\left(\sqrt{c}d+\sqrt{a}e\right)\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)}{4\sqrt{2}a^{7/4}(cd^2+ae^2)} - \frac{\left(c^{5/4}\left(\sqrt{c}d-\sqrt{a}e\right)\right)\text{Subst}\left(\int\frac{1}{-1-x^2}\,dx, x, 1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{7/4}(cd^2+ae^2)} + \frac{\left(c^{5/4}\left(\sqrt{c}d-\sqrt{a}e\right)\right)\text{Subst}\left(\int\frac{1}{-1-x^2}\,dx, x, 1+\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{7/4}(cd^2+ae^2)}$$

$$= -\frac{1}{3adx^3} + \frac{e}{ad^2x} + \frac{e^{7/2}\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{d^{5/2}(cd^2+ae^2)} + \frac{c^{5/4}(\sqrt{c}d-\sqrt{a}e)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{7/4}(cd^2+ae^2)}$$
$$- \frac{c^{5/4}(\sqrt{c}d-\sqrt{a}e)\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{7/4}(cd^2+ae^2)}$$
$$+ \frac{c^{5/4}(\sqrt{c}d+\sqrt{a}e)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)}{4\sqrt{2}a^{7/4}(cd^2+ae^2)}$$
$$- \frac{c^{5/4}(\sqrt{c}d+\sqrt{a}e)\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)}{4\sqrt{2}a^{7/4}(cd^2+ae^2)}$$

Mathematica [A] (verified)

Time = 0.30 (sec) , antiderivative size = 367, normalized size of antiderivative = 1.02

$$\int \frac{1}{x^4(d+ex^2)(a+cx^4)}\,dx$$
$$= \frac{-8ad^{3/2}(cd^2+ae^2)+24a\sqrt{d}e(cd^2+ae^2)x^2+24a^2e^{7/2}x^3\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)+6\sqrt{2}\sqrt[4]{a}c^{5/4}d^{5/2}(\sqrt{c}d-\sqrt{a}e)x^3}{}$$

```
[In] Integrate[1/(x^4*(d + e*x^2)*(a + c*x^4)),x]
```

```
[Out] (-8*a*d^(3/2)*(c*d^2 + a*e^2) + 24*a*Sqrt[d]*e*(c*d^2 + a*e^2)*x^2 + 24*a^2
*e^(7/2)*x^3*ArcTan[(Sqrt[e]*x)/Sqrt[d]] + 6*Sqrt[2]*a^(1/4)*c^(5/4)*d^(5/2
)*(Sqrt[c]*d - Sqrt[a]*e)*x^3*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)] + 6*S
qrt[2]*a^(1/4)*c^(5/4)*d^(5/2)*(-(Sqrt[c]*d) + Sqrt[a]*e)*x^3*ArcTan[1 + (S
qrt[2]*c^(1/4)*x)/a^(1/4)] + 3*Sqrt[2]*c^(5/4)*d^(5/2)*(a^(1/4)*Sqrt[c]*d +
 a^(3/4)*e)*x^3*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2] - 3*
Sqrt[2]*c^(5/4)*d^(5/2)*(a^(1/4)*Sqrt[c]*d + a^(3/4)*e)*x^3*Log[Sqrt[a] + S
qrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(24*a^2*d^(5/2)*(c*d^2 + a*e^2)*x^
3)
```

Maple [A] (verified)

Time = 0.40 (sec) , antiderivative size = 284, normalized size of antiderivative = 0.79

method	result
default	$-\frac{1}{3adx^3}+\frac{e}{ad^2x}-\frac{c^2\left(\frac{d\left(\frac{a}{c}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x^2+\left(\frac{a}{c}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{c}}}{x^2-\left(\frac{a}{c}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{c}}}\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{c}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{c}\right)^{\frac{1}{4}}}-1\right)\right)}{8a}-\frac{e\sqrt{2}\left(\ln\left(\frac{x^2-\left(\frac{a}{c}\right)^{\frac{1}{4}}x\cdots}{x^2+\left(\frac{a}{c}\right)^{\frac{1}{4}}x\cdots}\right.\right.\cdots}{\cdots}\right.\cdots}{(a\,e^2+c\,d^2)a}\cdots$
risch	$\frac{\frac{e\,x^2}{d^2a}-\frac{1}{3da}}{x^3}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\left(a^2d^5e^4+2c\,e^2a\,d^7+c^2d^9\right)\_Z^2+e^7\right)}\_R\ln\left(\left(\left(48a^{11}d^5e^8+152a^{10}c\,d^7e^6+200a^9c^2d^9e^4+136a^8c^3d^{11}e^2+\cdots\right.\right.\right.\right.}{\cdots}$

[In] int(1/x^4/(e*x^2+d)/(c*x^4+a),x,method=_RETURNVERBOSE)

[Out] -1/3/a/d/x^3+e/a/d^2/x-c^2/(a*e^2+c*d^2)/a*(1/8*d*(a/c)^(1/4)/a*2^(1/2)*(ln((x^2+(a/c)^(1/4)*x*2^(1/2)+(a/c)^(1/2))/(x^2-(a/c)^(1/4)*x*2^(1/2)+(a/c)^(1/2)))+2*arctan(2^(1/2)/(a/c)^(1/4)*x+1)+2*arctan(2^(1/2)/(a/c)^(1/4)*x-1))-1/8*e/c/(a/c)^(1/4)*2^(1/2)*(ln((x^2-(a/c)^(1/4)*x*2^(1/2)+(a/c)^(1/2))/(x^2+(a/c)^(1/4)*x*2^(1/2)+(a/c)^(1/2)))+2*arctan(2^(1/2)/(a/c)^(1/4)*x+1)+2*arctan(2^(1/2)/(a/c)^(1/4)*x-1)))+1/d^2*e^4/(a*e^2+c*d^2)/(e*d)^(1/2)*arctan(e*x/(e*d)^(1/2))

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2209 vs. $2(269) = 538$.

Time = 5.98 (sec) , antiderivative size = 4442, normalized size of antiderivative = 12.34

$$\int \frac{1}{x^4(d+ex^2)(a+cx^4)}\,dx = \text{Too large to display}$$

[In] integrate(1/x^4/(e*x^2+d)/(c*x^4+a),x, algorithm="fricas")

[Out] [1/12*(6*a*e^3*x^3*sqrt(-e/d)*log((e*x^2 + 2*d*x*sqrt(-e/d) - d)/(e*x^2 + d)) + 3*(a*c*d^4 + a^2*d^2*e^2)*x^3*sqrt((2*c^3*d*e + (a^3*c^2*d^4 + 2*a^4*c*d^2*e^2 + a^5*e^4)*sqrt(-(c^7*d^4 - 2*a*c^6*d^2*e^2 + a^2*c^5*e^4)/(a^7*c^4*d^8 + 4*a^8*c^3*d^6*e^2 + 6*a^9*c^2*d^4*e^4 + 4*a^10*c*d^2*e^6 + a^11*e^8)))/(a^3*c^2*d^4 + 2*a^4*c*d^2*e^2 + a^5*e^4))*log(-(c^5*d^2 - a*c^4*e^2)*x + (a^2*c^4*d^3 - a^3*c^3*d*e^2 + (a^6*c^2*d^4*e + 2*a^7*c*d^2*e^3 + a^8*e^5)*sqrt(-(c^7*d^4 - 2*a*c^6*d^2*e^2 + a^2*c^5*e^4)/(a^7*c^4*d^8 + 4*a^8*c^3*d^6*e^2 + 6*a^9*c^2*d^4*e^4 + 4*a^10*c*d^2*e^6 + a^11*e^8)))*sqrt((2*c^3*d*e + (a^3*c^2*d^4 + 2*a^4*c*d^2*e^2 + a^5*e^4)*sqrt(-(c^7*d^4 - 2*a*c^6*d^2*e^2 + a^2*c^5*e^4)/(a^7*c^4*d^8 + 4*a^8*c^3*d^6*e^2 + 6*a^9*c^2*d^4*e^4 + 4*a^10*c*d^2*e^6 + a^11*e^8)))/(a^3*c^2*d^4 + 2*a^4*c*d^2*e^2 + a^5*e^4))) - 3*(a*c*d^4 + a^2*d^2*e^2)*x^3*sqrt((2*c^3*d*e + (a^3*c^2*d^4 + 2*a^4*c*d^2*e^2 + a^5*e^4)*sqrt(-(c^7*d^4 - 2*a*c^6*d^2*e^2 + a^2*c^5*e^4)/(a^7*c^4*d^8 + 4*a^8*c^3*d^6*e^2 + 6*a^9*c^2*d^4*e^4 + 4*a^10*c*d^2*e^6 + a^11*e^8)))

/(a^3*c^2*d^4 + 2*a^4*c*d^2*e^2 + a^5*e^4))*log(-(c^5*d^2 - a*c^4*e^2)*x - (a^2*c^4*d^3 - a^3*c^3*d*e^2 + (a^6*c^2*d^4*e + 2*a^7*c*d^2*e^3 + a^8*e^5)*sqrt(-(c^7*d^4 - 2*a*c^6*d^2*e^2 + a^2*c^5*e^4)/(a^7*c^4*d^8 + 4*a^8*c^3*d^6*e^2 + 6*a^9*c^2*d^4*e^4 + 4*a^10*c*d^2*e^6 + a^11*e^8)))*sqrt((2*c^3*d*e + (a^3*c^2*d^4 + 2*a^4*c*d^2*e^2 + a^5*e^4)*sqrt(-(c^7*d^4 - 2*a*c^6*d^2*e^2 + a^2*c^5*e^4)/(a^7*c^4*d^8 + 4*a^8*c^3*d^6*e^2 + 6*a^9*c^2*d^4*e^4 + 4*a^10*c*d^2*e^6 + a^11*e^8)))/(a^3*c^2*d^4 + 2*a^4*c*d^2*e^2 + a^5*e^4))) + 3*(a*c*d^4 + a^2*d^2*e^2)*x^3*sqrt((2*c^3*d*e - (a^3*c^2*d^4 + 2*a^4*c*d^2*e^2 + a^5*e^4)*sqrt(-(c^7*d^4 - 2*a*c^6*d^2*e^2 + a^2*c^5*e^4)/(a^7*c^4*d^8 + 4*a^8*c^3*d^6*e^2 + 6*a^9*c^2*d^4*e^4 + 4*a^10*c*d^2*e^6 + a^11*e^8)))/(a^3*c^2*d^4 + 2*a^4*c*d^2*e^2 + a^5*e^4))*log(-(c^5*d^2 - a*c^4*e^2)*x + (a^2*c^4*d^3 - a^3*c^3*d*e^2 - (a^6*c^2*d^4*e + 2*a^7*c*d^2*e^3 + a^8*e^5)*sqrt(-(c^7*d^4 - 2*a*c^6*d^2*e^2 + a^2*c^5*e^4)/(a^7*c^4*d^8 + 4*a^8*c^3*d^6*e^2 + 6*a^9*c^2*d^4*e^4 + 4*a^10*c*d^2*e^6 + a^11*e^8)))*sqrt((2*c^3*d*e - (a^3*c^2*d^4 + 2*a^4*c*d^2*e^2 + a^5*e^4)*sqrt(-(c^7*d^4 - 2*a*c^6*d^2*e^2 + a^2*c^5*e^4)/(a^7*c^4*d^8 + 4*a^8*c^3*d^6*e^2 + 6*a^9*c^2*d^4*e^4 + 4*a^10*c*d^2*e^6 + a^11*e^8)))/(a^3*c^2*d^4 + 2*a^4*c*d^2*e^2 + a^5*e^4))) - 3*(a*c*d^4 + a^2*d^2*e^2)*x^3*sqrt((2*c^3*d*e - (a^3*c^2*d^4 + 2*a^4*c*d^2*e^2 + a^5*e^4)*sqrt(-(c^7*d^4 - 2*a*c^6*d^2*e^2 + a^2*c^5*e^4)/(a^7*c^4*d^8 + 4*a^8*c^3*d^6*e^2 + 6*a^9*c^2*d^4*e^4 + 4*a^10*c*d^2*e^6 + a^11*e^8)))/(a^3*c^2*d^4 + 2*a^4*c*d^2*e^2 + a^5*e^4))*log(-(c^5*d^2 - a*c^4*e^2)*x - (a^2*c^4*d^3 - a^3*c^3*d*e^2 - (a^6*c^2*d^4*e + 2*a^7*c*d^2*e^3 + a^8*e^5)*sqrt(-(c^7*d^4 - 2*a*c^6*d^2*e^2 + a^2*c^5*e^4)/(a^7*c^4*d^8 + 4*a^8*c^3*d^6*e^2 + 6*a^9*c^2*d^4*e^4 + 4*a^10*c*d^2*e^6 + a^11*e^8)))*sqrt((2*c^3*d*e - (a^3*c^2*d^4 + 2*a^4*c*d^2*e^2 + a^5*e^4)*sqrt(-(c^7*d^4 - 2*a*c^6*d^2*e^2 + a^2*c^5*e^4)/(a^7*c^4*d^8 + 4*a^8*c^3*d^6*e^2 + 6*a^9*c^2*d^4*e^4 + 4*a^10*c*d^2*e^6 + a^11*e^8)))/(a^3*c^2*d^4 + 2*a^4*c*d^2*e^2 + a^5*e^4))) - 4*c*d^3 - 4*a*d*e^2 + 12*(c*d^2*e + a*e^3)*x^2)/((a*c*d^4 + a^2*d^2*e^2)*x^3), 1/12*(12*a*e^3*x^3*sqrt(e/d)*arctan(x*sqrt(e/d)) + 3*(a*c*d^4 + a^2*d^2*e^2)*x^3*sqrt((2*c^3*d*e + (a^3*c^2*d^4 + 2*a^4*c*d^2*e^2 + a^5*e^4)*sqrt(-(c^7*d^4 - 2*a*c^6*d^2*e^2 + a^2*c^5*e^4)/(a^7*c^4*d^8 + 4*a^8*c^3*d^6*e^2 + 6*a^9*c^2*d^4*e^4 + 4*a^10*c*d^2*e^6 + a^11*e^8)))/(a^3*c^2*d^4 + 2*a^4*c*d^2*e^2 + a^5*e^4))*log(-(c^5*d^2 - a*c^4*e^2)*x + (a^2*c^4*d^3 - a^3*c^3*d*e^2 + (a^6*c^2*d^4*e + 2*a^7*c*d^2*e^3 + a^8*e^5)*sqrt(-(c^7*d^4 - 2*a*c^6*d^2*e^2 + a^2*c^5*e^4)/(a^7*c^4*d^8 + 4*a^8*c^3*d^6*e^2 + 6*a^9*c^2*d^4*e^4 + 4*a^10*c*d^2*e^6 + a^11*e^8)))*sqrt((2*c^3*d*e + (a^3*c^2*d^4 + 2*a^4*c*d^2*e^2 + a^5*e^4)*sqrt(-(c^7*d^4 - 2*a*c^6*d^2*e^2 + a^2*c^5*e^4)/(a^7*c^4*d^8 + 4*a^8*c^3*d^6*e^2 + 6*a^9*c^2*d^4*e^4 + 4*a^10*c*d^2*e^6 + a^11*e^8)))/(a^3*c^2*d^4 + 2*a^4*c*d^2*e^2 + a^5*e^4))) - 3*(a*c*d^4 + a^2*d^2*e^2)*x^3*sqrt((2*c^3*d*e + (a^3*c^2*d^4 + 2*a^4*c*d^2*e^2 + a^5*e^4)*sqrt(-(c^7*d^4 - 2*a*c^6*d^2*e^2 + a^2*c^5*e^4)/(a^7*c^4*d^8 + 4*a^8*c^3*d^6*e^2 + 6*a^9*c^2*d^4*e^4 + 4*a^10*c*d^2*e^6 + a^11*e^8)))/(a^3*c^2*d^4 + 2*a^4*c*d^2*e^2 + a^5*e^4))*log(-(c^5*d^2 - a*c^4*e^2)*x - (a^2*c^4*d^3 - a^3*c^3*d*e^2 + (a^6*c^2*d^4*e + 2*a^7*c*d^2*e^3 + a^8*e^5)*sqrt(-(c^7*d^4 - 2*a*c^6*d^2*e^2 + a^2*c^5*e^4)/(a^7*c^4*d^8 + 4*a^8*c^3*d^6*e^2 + 6*a^9*c^2*d^4*e^4 + 4*a^

```
10*c*d^2*e^6 + a^11*e^8)))*sqrt((2*c^3*d*e + (a^3*c^2*d^4 + 2*a^4*c*d^2*e^2
 + a^5*e^4)*sqrt(-(c^7*d^4 - 2*a*c^6*d^2*e^2 + a^2*c^5*e^4)/(a^7*c^4*d^8 +
4*a^8*c^3*d^6*e^2 + 6*a^9*c^2*d^4*e^4 + 4*a^10*c*d^2*e^6 + a^11*e^8)))/(a^3
*c^2*d^4 + 2*a^4*c*d^2*e^2 + a^5*e^4))) + 3*(a*c*d^4 + a^2*d^2*e^2)*x^3*sqr
t((2*c^3*d*e - (a^3*c^2*d^4 + 2*a^4*c*d^2*e^2 + a^5*e^4)*sqrt(-(c^7*d^4 - 2
*a*c^6*d^2*e^2 + a^2*c^5*e^4)/(a^7*c^4*d^8 + 4*a^8*c^3*d^6*e^2 + 6*a^9*c^2*
d^4*e^4 + 4*a^10*c*d^2*e^6 + a^11*e^8)))/(a^3*c^2*d^4 + 2*a^4*c*d^2*e^2 + a
^5*e^4))*log(-(c^5*d^2 - a*c^4*e^2)*x + (a^2*c^4*d^3 - a^3*c^3*d*e^2 - (a^6
*c^2*d^4*e + 2*a^7*c*d^2*e^3 + a^8*e^5)*sqrt(-(c^7*d^4 - 2*a*c^6*d^2*e^2 +
a^2*c^5*e^4)/(a^7*c^4*d^8 + 4*a^8*c^3*d^6*e^2 + 6*a^9*c^2*d^4*e^4 + 4*a^10*
c*d^2*e^6 + a^11*e^8)))*sqrt((2*c^3*d*e - (a^3*c^2*d^4 + 2*a^4*c*d^2*e^2 +
a^5*e^4)*sqrt(-(c^7*d^4 - 2*a*c^6*d^2*e^2 + a^2*c^5*e^4)/(a^7*c^4*d^8 + 4*a
^8*c^3*d^6*e^2 + 6*a^9*c^2*d^4*e^4 + 4*a^10*c*d^2*e^6 + a^11*e^8)))/(a^3*c^
2*d^4 + 2*a^4*c*d^2*e^2 + a^5*e^4))) - 3*(a*c*d^4 + a^2*d^2*e^2)*x^3*sqrt((
2*c^3*d*e - (a^3*c^2*d^4 + 2*a^4*c*d^2*e^2 + a^5*e^4)*sqrt(-(c^7*d^4 - 2*a*
c^6*d^2*e^2 + a^2*c^5*e^4)/(a^7*c^4*d^8 + 4*a^8*c^3*d^6*e^2 + 6*a^9*c^2*d^4
*e^4 + 4*a^10*c*d^2*e^6 + a^11*e^8)))/(a^3*c^2*d^4 + 2*a^4*c*d^2*e^2 + a^5*
e^4))*log(-(c^5*d^2 - a*c^4*e^2)*x - (a^2*c^4*d^3 - a^3*c^3*d*e^2 - (a^6*c^
2*d^4*e + 2*a^7*c*d^2*e^3 + a^8*e^5)*sqrt(-(c^7*d^4 - 2*a*c^6*d^2*e^2 + a^2
*c^5*e^4)/(a^7*c^4*d^8 + 4*a^8*c^3*d^6*e^2 + 6*a^9*c^2*d^4*e^4 + 4*a^10*c*d
^2*e^6 + a^11*e^8)))*sqrt((2*c^3*d*e - (a^3*c^2*d^4 + 2*a^4*c*d^2*e^2 + a^5
*e^4)*sqrt(-(c^7*d^4 - 2*a*c^6*d^2*e^2 + a^2*c^5*e^4)/(a^7*c^4*d^8 + 4*a^8*
c^3*d^6*e^2 + 6*a^9*c^2*d^4*e^4 + 4*a^10*c*d^2*e^6 + a^11*e^8)))/(a^3*c^2*d
^4 + 2*a^4*c*d^2*e^2 + a^5*e^4))) - 4*c*d^3 - 4*a*d*e^2 + 12*(c*d^2*e + a*e
^3)*x^2)/((a*c*d^4 + a^2*d^2*e^2)*x^3)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{x^4 (d + ex^2) (a + cx^4)} \, dx = \text{Timed out}$$

```
[In] integrate(1/x**4/(e*x**2+d)/(c*x**4+a),x)
```

```
[Out] Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{x^4 (d+ex^2)(a+cx^4)}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(1/x^4/(e*x^2+d)/(c*x^4+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 368, normalized size of antiderivative = 1.02

$$\int \frac{1}{x^4 (d+ex^2)(a+cx^4)}\,dx = \frac{e^4 \arctan\left(\frac{ex}{\sqrt{de}}\right)}{(cd^4+ad^2e^2)\sqrt{de}} - \frac{\left((ac^3)^{\frac{1}{4}}c^2d-(ac^3)^{\frac{3}{4}}e\right)\arctan\left(\frac{\sqrt{2}\left(2x+\sqrt{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{2\left(\sqrt{2}a^2c^2d^2+\sqrt{2}a^3ce^2\right)} - \frac{\left((ac^3)^{\frac{1}{4}}c^2d-(ac^3)^{\frac{3}{4}}e\right)\arctan\left(\frac{\sqrt{2}\left(2x-\sqrt{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{2\left(\sqrt{2}a^2c^2d^2+\sqrt{2}a^3ce^2\right)} - \frac{\left((ac^3)^{\frac{1}{4}}c^2d+(ac^3)^{\frac{3}{4}}e\right)\log\left(x^2+\sqrt{2}x\left(\frac{a}{c}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{c}}\right)}{4\left(\sqrt{2}a^2c^2d^2+\sqrt{2}a^3ce^2\right)} + \frac{\left((ac^3)^{\frac{1}{4}}c^2d+(ac^3)^{\frac{3}{4}}e\right)\log\left(x^2-\sqrt{2}x\left(\frac{a}{c}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{c}}\right)}{4\left(\sqrt{2}a^2c^2d^2+\sqrt{2}a^3ce^2\right)} + \frac{3\,ex^2-d}{3\,ad^2x^3}$$

```
[In] integrate(1/x^4/(e*x^2+d)/(c*x^4+a),x, algorithm="giac")
```

```
[Out] e^4*arctan(e*x/sqrt(d*e))/((c*d^4 + a*d^2*e^2)*sqrt(d*e)) - 1/2*((a*c^3)^(1
/4)*c^2*d - (a*c^3)^(3/4)*e)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(a/c)^(1/4))
/(a/c)^(1/4))/(sqrt(2)*a^2*c^2*d^2 + sqrt(2)*a^3*c*e^2) - 1/2*((a*c^3)^(1/4
)*c^2*d - (a*c^3)^(3/4)*e)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(a/c)^(1/4))/(
```

```
a/c)^(1/4))/(sqrt(2)*a^2*c^2*d^2 + sqrt(2)*a^3*c*e^2) - 1/4*((a*c^3)^(1/4)*
c^2*d + (a*c^3)^(3/4)*e)*log(x^2 + sqrt(2)*x*(a/c)^(1/4) + sqrt(a/c))/(sqrt
(2)*a^2*c^2*d^2 + sqrt(2)*a^3*c*e^2) + 1/4*((a*c^3)^(1/4)*c^2*d + (a*c^3)^(
3/4)*e)*log(x^2 - sqrt(2)*x*(a/c)^(1/4) + sqrt(a/c))/(sqrt(2)*a^2*c^2*d^2 +
 sqrt(2)*a^3*c*e^2) + 1/3*(3*e*x^2 - d)/(a*d^2*x^3)
```

Mupad [B] (verification not implemented)

Time = 8.71 (sec) , antiderivative size = 5972, normalized size of antiderivative = 16.59

$$\int \frac{1}{x^4 (d + ex^2) (a + cx^4)} \, dx = \text{Too large to display}$$

```
[In] int(1/(x^4*(a + c*x^4)*(d + e*x^2)),x)
```

```
[Out] atan(((x*(2*a^5*c^9*d^18*e^5 + 4*a^7*c^7*d^14*e^9) - ((a*e^2*(-a^7*c^5)^(1/
2) - c*d^2*(-a^7*c^5)^(1/2) + 2*a^4*c^3*d*e)/(16*(a^9*e^4 + a^7*c^2*d^4 + 2
*a^8*c*d^2*e^2)))^(1/2)*((((a*e^2*(-a^7*c^5)^(1/2) - c*d^2*(-a^7*c^5)^(1/2)
 + 2*a^4*c^3*d*e)/(16*(a^9*e^4 + a^7*c^2*d^4 + 2*a^8*c*d^2*e^2)))^(1/2)*(x*
((a*e^2*(-a^7*c^5)^(1/2) - c*d^2*(-a^7*c^5)^(1/2) + 2*a^4*c^3*d*e)/(16*(a^9
*e^4 + a^7*c^2*d^4 + 2*a^8*c*d^2*e^2)))^(1/2)*(512*a^11*c^7*d^24*e^3 + 512*
a^12*c^6*d^22*e^5 - 512*a^13*c^5*d^20*e^7 - 512*a^14*c^4*d^18*e^9) - 64*a^9
*c^8*d^24*e^2 + 128*a^10*c^7*d^22*e^4 + 192*a^11*c^6*d^20*e^6 - 256*a^12*c^
5*d^18*e^8 - 256*a^13*c^4*d^16*e^10) - x*(16*a^7*c^9*d^23*e^2 + 32*a^8*c^8*
d^21*e^4 - 112*a^9*c^7*d^19*e^6 - 128*a^11*c^5*d^15*e^10))*((a*e^2*(-a^7*c^
5)^(1/2) - c*d^2*(-a^7*c^5)^(1/2) + 2*a^4*c^3*d*e)/(16*(a^9*e^4 + a^7*c^2*d
^4 + 2*a^8*c*d^2*e^2)))^(1/2) - 4*a^6*c^9*d^21*e^3 - 4*a^7*c^8*d^19*e^5 + 4
8*a^9*c^6*d^15*e^9))*((a*e^2*(-a^7*c^5)^(1/2) - c*d^2*(-a^7*c^5)^(1/2) + 2*
a^4*c^3*d*e)/(16*(a^9*e^4 + a^7*c^2*d^4 + 2*a^8*c*d^2*e^2)))^(1/2)*1i + (x*
(2*a^5*c^9*d^18*e^5 + 4*a^7*c^7*d^14*e^9) - ((a*e^2*(-a^7*c^5)^(1/2) - c*d^
2*(-a^7*c^5)^(1/2) + 2*a^4*c^3*d*e)/(16*(a^9*e^4 + a^7*c^2*d^4 + 2*a^8*c*d^
2*e^2)))^(1/2)*((((a*e^2*(-a^7*c^5)^(1/2) - c*d^2*(-a^7*c^5)^(1/2) + 2*a^4*
c^3*d*e)/(16*(a^9*e^4 + a^7*c^2*d^4 + 2*a^8*c*d^2*e^2)))^(1/2)*(x*((a*e^2*(
-a^7*c^5)^(1/2) - c*d^2*(-a^7*c^5)^(1/2) + 2*a^4*c^3*d*e)/(16*(a^9*e^4 + a^
7*c^2*d^4 + 2*a^8*c*d^2*e^2)))^(1/2)*(512*a^11*c^7*d^24*e^3 + 512*a^12*c^6*
d^22*e^5 - 512*a^13*c^5*d^20*e^7 - 512*a^14*c^4*d^18*e^9) + 64*a^9*c^8*d^24
*e^2 - 128*a^10*c^7*d^22*e^4 - 192*a^11*c^6*d^20*e^6 + 256*a^12*c^5*d^18*e^
8 + 256*a^13*c^4*d^16*e^10) - x*(16*a^7*c^9*d^23*e^2 + 32*a^8*c^8*d^21*e^4
- 112*a^9*c^7*d^19*e^6 - 128*a^11*c^5*d^15*e^10))*((a*e^2*(-a^7*c^5)^(1/2)
- c*d^2*(-a^7*c^5)^(1/2) + 2*a^4*c^3*d*e)/(16*(a^9*e^4 + a^7*c^2*d^4 + 2*a^
8*c*d^2*e^2)))^(1/2) + 4*a^6*c^9*d^21*e^3 + 4*a^7*c^8*d^19*e^5 - 48*a^9*c^6
*d^15*e^9))*((a*e^2*(-a^7*c^5)^(1/2) - c*d^2*(-a^7*c^5)^(1/2) + 2*a^4*c^3*d
*e)/(16*(a^9*e^4 + a^7*c^2*d^4 + 2*a^8*c*d^2*e^2)))^(1/2)*1i)/((x*(2*a^5*c^
9*d^18*e^5 + 4*a^7*c^7*d^14*e^9) - ((a*e^2*(-a^7*c^5)^(1/2) - c*d^2*(-a^7*c
^5)^(1/2) + 2*a^4*c^3*d*e)/(16*(a^9*e^4 + a^7*c^2*d^4 + 2*a^8*c*d^2*e^2)))^
```

(1/2)*((((a*e^2*(-a^7*c^5)^(1/2) - c*d^2*(-a^7*c^5)^(1/2) + 2*a^4*c^3*d*e)/(16*(a^9*e^4 + a^7*c^2*d^4 + 2*a^8*c*d^2*e^2)))^(1/2)*(x*((a*e^2*(-a^7*c^5)^(1/2) - c*d^2*(-a^7*c^5)^(1/2) + 2*a^4*c^3*d*e)/(16*(a^9*e^4 + a^7*c^2*d^4 + 2*a^8*c*d^2*e^2)))^(1/2)*(512*a^11*c^7*d^24*e^3 + 512*a^12*c^6*d^22*e^5 - 512*a^13*c^5*d^20*e^7 - 512*a^14*c^4*d^18*e^9) - 64*a^9*c^8*d^24*e^2 + 128*a^10*c^7*d^22*e^4 + 192*a^11*c^6*d^20*e^6 - 256*a^12*c^5*d^18*e^8 - 256*a^13*c^4*d^16*e^10) - x*(16*a^7*c^9*d^23*e^2 + 32*a^8*c^8*d^21*e^4 - 112*a^9*c^7*d^19*e^6 - 128*a^11*c^5*d^15*e^10))*((a*e^2*(-a^7*c^5)^(1/2) - c*d^2*(-a^7*c^5)^(1/2) + 2*a^4*c^3*d*e)/(16*(a^9*e^4 + a^7*c^2*d^4 + 2*a^8*c*d^2*e^2)))^(1/2) - 4*a^6*c^9*d^21*e^3 - 4*a^7*c^8*d^19*e^5 + 48*a^9*c^6*d^15*e^9))*((a*e^2*(-a^7*c^5)^(1/2) - c*d^2*(-a^7*c^5)^(1/2) + 2*a^4*c^3*d*e)/(16*(a^9*e^4 + a^7*c^2*d^4 + 2*a^8*c*d^2*e^2)))^(1/2) - (x*(2*a^5*c^9*d^18*e^5 + 4*a^7*c^7*d^14*e^9) - ((a*e^2*(-a^7*c^5)^(1/2) - c*d^2*(-a^7*c^5)^(1/2) + 2*a^4*c^3*d*e)/(16*(a^9*e^4 + a^7*c^2*d^4 + 2*a^8*c*d^2*e^2)))^(1/2)*((((a*e^2*(-a^7*c^5)^(1/2) - c*d^2*(-a^7*c^5)^(1/2) + 2*a^4*c^3*d*e)/(16*(a^9*e^4 + a^7*c^2*d^4 + 2*a^8*c*d^2*e^2)))^(1/2)*(x*((a*e^2*(-a^7*c^5)^(1/2) - c*d^2*(-a^7*c^5)^(1/2) + 2*a^4*c^3*d*e)/(16*(a^9*e^4 + a^7*c^2*d^4 + 2*a^8*c*d^2*e^2)))^(1/2)*(512*a^11*c^7*d^24*e^3 + 512*a^12*c^6*d^22*e^5 - 512*a^13*c^5*d^20*e^7 - 512*a^14*c^4*d^18*e^9) + 64*a^9*c^8*d^24*e^2 - 128*a^10*c^7*d^22*e^4 - 192*a^11*c^6*d^20*e^6 + 256*a^12*c^5*d^18*e^8 + 256*a^13*c^4*d^16*e^10) - x*(16*a^7*c^9*d^23*e^2 + 32*a^8*c^8*d^21*e^4 - 112*a^9*c^7*d^19*e^6 - 128*a^11*c^5*d^15*e^10))*((a*e^2*(-a^7*c^5)^(1/2) - c*d^2*(-a^7*c^5)^(1/2) + 2*a^4*c^3*d*e)/(16*(a^9*e^4 + a^7*c^2*d^4 + 2*a^8*c*d^2*e^2)))^(1/2) + 4*a^6*c^9*d^21*e^3 + 4*a^7*c^8*d^19*e^5 - 48*a^9*c^6*d^15*e^9))*((a*e^2*(-a^7*c^5)^(1/2) - c*d^2*(-a^7*c^5)^(1/2) + 2*a^4*c^3*d*e)/(16*(a^9*e^4 + a^7*c^2*d^4 + 2*a^8*c*d^2*e^2)))^(1/2) + 2*a^5*c^8*d^14*e^8))*((a*e^2*(-a^7*c^5)^(1/2) - c*d^2*(-a^7*c^5)^(1/2) + 2*a^4*c^3*d*e)/(16*(a^9*e^4 + a^7*c^2*d^4 + 2*a^8*c*d^2*e^2)))^(1/2)*2i + atan(((x*(2*a^5*c^9*d^18*e^5 + 4*a^7*c^7*d^14*e^9) - ((c*d^2*(-a^7*c^5)^(1/2) - a*e^2*(-a^7*c^5)^(1/2) + 2*a^4*c^3*d*e)/(16*(a^9*e^4 + a^7*c^2*d^4 + 2*a^8*c*d^2*e^2)))^(1/2)*((((c*d^2*(-a^7*c^5)^(1/2) - a*e^2*(-a^7*c^5)^(1/2) + 2*a^4*c^3*d*e)/(16*(a^9*e^4 + a^7*c^2*d^4 + 2*a^8*c*d^2*e^2)))^(1/2)*(x*((c*d^2*(-a^7*c^5)^(1/2) - a*e^2*(-a^7*c^5)^(1/2) + 2*a^4*c^3*d*e)/(16*(a^9*e^4 + a^7*c^2*d^4 + 2*a^8*c*d^2*e^2)))^(1/2)*(512*a^11*c^7*d^24*e^3 + 512*a^12*c^6*d^22*e^5 - 512*a^13*c^5*d^20*e^7 - 512*a^14*c^4*d^18*e^9) - 64*a^9*c^8*d^24*e^2 + 128*a^10*c^7*d^22*e^4 + 192*a^11*c^6*d^20*e^6 - 256*a^12*c^5*d^18*e^8 - 256*a^13*c^4*d^16*e^10) - x*(16*a^7*c^9*d^23*e^2 + 32*a^8*c^8*d^21*e^4 - 112*a^9*c^7*d^19*e^6 - 128*a^11*c^5*d^15*e^10))*((c*d^2*(-a^7*c^5)^(1/2) - a*e^2*(-a^7*c^5)^(1/2) + 2*a^4*c^3*d*e)/(16*(a^9*e^4 + a^7*c^2*d^4 + 2*a^8*c*d^2*e^2)))^(1/2) - 4*a^6*c^9*d^21*e^3 - 4*a^7*c^8*d^19*e^5 + 48*a^9*c^6*d^15*e^9))*((c*d^2*(-a^7*c^5)^(1/2) - a*e^2*(-a^7*c^5)^(1/2) + 2*a^4*c^3*d*e)/(16*(a^9*e^4 + a^7*c^2*d^4 + 2*a^8*c*d^2*e^2)))^(1/2)*1i + (x*(2*a^5*c^9*d^18*e^5 + 4*a^7*c^7*d^14*e^9) - ((c*d^2*(-a^7*c^5)^(1/2) - a*e^2*(-a^7*c^5)^(1/2) + 2*a^4*c^3*d*e)/(16*(a^9*e^4 + a^7*c^2*d^4 + 2*a^8*c*d^2*e^2)))^(1/2)*((((c*d^2*(-a^7*c^5)^(1/2) - a*e^2*(-a^7*c^5)^(1/2) + 2*a^4*c^3*d*e)/(16*(a^9*e^4 + a^7*c^2*d^4 + 2

*a^8*c*d^2*e^2)))^(1/2)*(x*((c*d^2*(-a^7*c^5)^(1/2) - a*e^2*(-a^7*c^5)^(1/2
) + 2*a^4*c^3*d*e)/(16*(a^9*e^4 + a^7*c^2*d^4 + 2*a^8*c*d^2*e^2)))^(1/2)*(5
12*a^11*c^7*d^24*e^3 + 512*a^12*c^6*d^22*e^5 - 512*a^13*c^5*d^20*e^7 - 512*
a^14*c^4*d^18*e^9) + 64*a^9*c^8*d^24*e^2 - 128*a^10*c^7*d^22*e^4 - 192*a^11
*c^6*d^20*e^6 + 256*a^12*c^5*d^18*e^8 + 256*a^13*c^4*d^16*e^10) - x*(16*a^7
*c^9*d^23*e^2 + 32*a^8*c^8*d^21*e^4 - 112*a^9*c^7*d^19*e^6 - 128*a^11*c^5*d
^15*e^10))*((c*d^2*(-a^7*c^5)^(1/2) - a*e^2*(-a^7*c^5)^(1/2) + 2*a^4*c^3*d*
e)/(16*(a^9*e^4 + a^7*c^2*d^4 + 2*a^8*c*d^2*e^2)))^(1/2) + 4*a^6*c^9*d^21*e
^3 + 4*a^7*c^8*d^19*e^5 - 48*a^9*c^6*d^15*e^9))*((c*d^2*(-a^7*c^5)^(1/2) -
a*e^2*(-a^7*c^5)^(1/2) + 2*a^4*c^3*d*e)/(16*(a^9*e^4 + a^7*c^2*d^4 + 2*a^8*
c*d^2*e^2)))^(1/2)*1i)/((x*(2*a^5*c^9*d^18*e^5 + 4*a^7*c^7*d^14*e^9) - ((c*
d^2*(-a^7*c^5)^(1/2) - a*e^2*(-a^7*c^5)^(1/2) + 2*a^4*c^3*d*e)/(16*(a^9*e^4
 + a^7*c^2*d^4 + 2*a^8*c*d^2*e^2)))^(1/2)*((((c*d^2*(-a^7*c^5)^(1/2) - a*e^
2*(-a^7*c^5)^(1/2) + 2*a^4*c^3*d*e)/(16*(a^9*e^4 + a^7*c^2*d^4 + 2*a^8*c*d^
2*e^2)))^(1/2)*(x*((c*d^2*(-a^7*c^5)^(1/2) - a*e^2*(-a^7*c^5)^(1/2) + 2*a^4
*c^3*d*e)/(16*(a^9*e^4 + a^7*c^2*d^4 + 2*a^8*c*d^2*e^2)))^(1/2)*(512*a^11*c
^7*d^24*e^3 + 512*a^12*c^6*d^22*e^5 - 512*a^13*c^5*d^20*e^7 - 512*a^14*c^4*
d^18*e^9) - 64*a^9*c^8*d^24*e^2 + 128*a^10*c^7*d^22*e^4 + 192*a^11*c^6*d^20
*e^6 - 256*a^12*c^5*d^18*e^8 - 256*a^13*c^4*d^16*e^10) - x*(16*a^7*c^9*d^23
*e^2 + 32*a^8*c^8*d^21*e^4 - 112*a^9*c^7*d^19*e^6 - 128*a^11*c^5*d^15*e^10)
)*((c*d^2*(-a^7*c^5)^(1/2) - a*e^2*(-a^7*c^5)^(1/2) + 2*a^4*c^3*d*e)/(16*(a
^9*e^4 + a^7*c^2*d^4 + 2*a^8*c*d^2*e^2)))^(1/2) - 4*a^6*c^9*d^21*e^3 - 4*a^
7*c^8*d^19*e^5 + 48*a^9*c^6*d^15*e^9))*((c*d^2*(-a^7*c^5)^(1/2) - a*e^2*(-a
^7*c^5)^(1/2) + 2*a^4*c^3*d*e)/(16*(a^9*e^4 + a^7*c^2*d^4 + 2*a^8*c*d^2*e^2
)))^(1/2) - (x*(2*a^5*c^9*d^18*e^5 + 4*a^7*c^7*d^14*e^9) - ((c*d^2*(-a^7*c^
5)^(1/2) - a*e^2*(-a^7*c^5)^(1/2) + 2*a^4*c^3*d*e)/(16*(a^9*e^4 + a^7*c^2*d
^4 + 2*a^8*c*d^2*e^2)))^(1/2)*((((c*d^2*(-a^7*c^5)^(1/2) - a*e^2*(-a^7*c^5)
^(1/2) + 2*a^4*c^3*d*e)/(16*(a^9*e^4 + a^7*c^2*d^4 + 2*a^8*c*d^2*e^2)))^(1/
2)*(x*((c*d^2*(-a^7*c^5)^(1/2) - a*e^2*(-a^7*c^5)^(1/2) + 2*a^4*c^3*d*e)/(1
6*(a^9*e^4 + a^7*c^2*d^4 + 2*a^8*c*d^2*e^2)))^(1/2)*(512*a^11*c^7*d^24*e^3
+ 512*a^12*c^6*d^22*e^5 - 512*a^13*c^5*d^20*e^7 - 512*a^14*c^4*d^18*e^9) +
64*a^9*c^8*d^24*e^2 - 128*a^10*c^7*d^22*e^4 - 192*a^11*c^6*d^20*e^6 + 256*a
^12*c^5*d^18*e^8 + 256*a^13*c^4*d^16*e^10) - x*(16*a^7*c^9*d^23*e^2 + 32*a^
8*c^8*d^21*e^4 - 112*a^9*c^7*d^19*e^6 - 128*a^11*c^5*d^15*e^10))*((c*d^2*(-
a^7*c^5)^(1/2) - a*e^2*(-a^7*c^5)^(1/2) + 2*a^4*c^3*d*e)/(16*(a^9*e^4 + a^7
*c^2*d^4 + 2*a^8*c*d^2*e^2)))^(1/2) + 4*a^6*c^9*d^21*e^3 + 4*a^7*c^8*d^19*e
^5 - 48*a^9*c^6*d^15*e^9))*((c*d^2*(-a^7*c^5)^(1/2) - a*e^2*(-a^7*c^5)^(1/2
) + 2*a^4*c^3*d*e)/(16*(a^9*e^4 + a^7*c^2*d^4 + 2*a^8*c*d^2*e^2)))^(1/2) +
2*a^5*c^8*d^14*e^8))*((c*d^2*(-a^7*c^5)^(1/2) - a*e^2*(-a^7*c^5)^(1/2) + 2*
a^4*c^3*d*e)/(16*(a^9*e^4 + a^7*c^2*d^4 + 2*a^8*c*d^2*e^2)))^(1/2)*2i - (1/
(3*a*d) - (e*x^2)/(a*d^2))/x^3 - (log(16*a^7*d^13*e^20 + c^7*d^27*e^6 + 2*a
*c^6*d^25*e^8 + a^2*c^5*d^23*e^10 + 16*a^4*c^3*d^19*e^14 + 16*a^7*e^3*x*(-d
^5*e^7)^(5/2) - a^2*c^5*d^15*x*(-d^5*e^7)^(3/2) + c^7*d^24*e^3*x*(-d^5*e^7)
^(1/2) - 16*a^4*c^3*d^11*e^4*x*(-d^5*e^7)^(3/2) + 2*a*c^6*d^22*e^5*x*(-d^5*
e^7)^(1/2))*(-d^5*e^7)^(1/2))/(2*(c*d^7 + a*d^5*e^2)) + (log(16*a^7*d^13*e^

```
20 + c^7*d^27*e^6 + 2*a*c^6*d^25*e^8 + a^2*c^5*d^23*e^10 + 16*a^4*c^3*d^19*
e^14 - 16*a^7*e^3*x*(-d^5*e^7)^(5/2) + a^2*c^5*d^15*x*(-d^5*e^7)^(3/2) - c^
7*d^24*e^3*x*(-d^5*e^7)^(1/2) + 16*a^4*c^3*d^11*e^4*x*(-d^5*e^7)^(3/2) - 2*
a*c^6*d^22*e^5*x*(-d^5*e^7)^(1/2))*(-d^5*e^7)^(1/2))/(2*c*d^7 + 2*a*d^5*e^2
)
```

3.244 $\int \frac{x^9}{(d+ex^2)(a+cx^4)^2}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 169

$$\int \frac{x^9}{(d+ex^2)(a+cx^4)^2}\,dx = \frac{a(ae+cdx^2)}{4c^2(cd^2+ae^2)(a+cx^4)} - \frac{\sqrt{a}d(3cd^2+ae^2)\arctan\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{4c^{3/2}(cd^2+ae^2)^2} + \frac{d^4\log(d+ex^2)}{2e(cd^2+ae^2)^2} + \frac{ae(2cd^2+ae^2)\log(a+cx^4)}{4c^2(cd^2+ae^2)^2}$$

```
[Out] 1/4*a*(c*d*x^2+a*e)/c^2/(a*e^2+c*d^2)/(c*x^4+a)+1/2*d^4*ln(e*x^2+d)/e/(a*e^
2+c*d^2)^2+1/4*a*e*(a*e^2+2*c*d^2)*ln(c*x^4+a)/c^2/(a*e^2+c*d^2)^2-1/4*d*(a
*e^2+3*c*d^2)*arctan(x^2*c^(1/2)/a^(1/2))*a^(1/2)/c^(3/2)/(a*e^2+c*d^2)^2
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 169, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {1266, 1661, 1643, 649, 211, 266}

$$\int \frac{x^9}{(d+ex^2)(a+cx^4)^2}\,dx = -\frac{\sqrt{a}d\arctan\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)(ae^2+3cd^2)}{4c^{3/2}(ae^2+cd^2)^2} + \frac{ae(ae^2+2cd^2)\log(a+cx^4)}{4c^2(ae^2+cd^2)^2} + \frac{a(ae+cdx^2)}{4c^2(a+cx^4)(ae^2+cd^2)} + \frac{d^4\log(d+ex^2)}{2e(ae^2+cd^2)^2}$$

```
[In] Int[x^9/((d + e*x^2)*(a + c*x^4)^2),x]
```

```
[Out] (a*(a*e + c*d*x^2))/(4*c^2*(c*d^2 + a*e^2)*(a + c*x^4)) - (Sqrt[a]*d*(3*c*d
^2 + a*e^2)*ArcTan[(Sqrt[c]*x^2)/Sqrt[a]])/(4*c^(3/2)*(c*d^2 + a*e^2)^2) +
(d^4*Log[d + e*x^2])/(2*e*(c*d^2 + a*e^2)^2) + (a*e*(2*c*d^2 + a*e^2)*Log[a
 + c*x^4])/(4*c^2*(c*d^2 + a*e^2)^2)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[(-a)*c]
```

Rule 1266

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_S
ymbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + c*x^2)^p, x],
x, x^2], x] /; FreeQ[{a, c, d, e, p, q}, x] && IntegerQ[(m + 1)/2]
```

Rule 1643

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol]
:> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + c*x^2)^p, x], x] /; FreeQ[{a, c,
 d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 1661

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[(d + e*x)^m*Pq, a + c*x^2, x], f = Coeff[Pol
ynomialRemainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 0], g = Coeff[Polynomial
Remainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 1]}, Simp[(a*g - c*f*x)*((a + c
*x^2)^(p + 1)/(2*a*c*(p + 1))), x] + Dist[1/(2*a*c*(p + 1)), Int[(d + e*x)^
m*(a + c*x^2)^(p + 1)*ExpandToSum[(2*a*c*(p + 1)*Q)/(d + e*x)^m + (c*f*(2*p
 + 3))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, c, d, e}, x] && PolyQ[Pq, x] &
& NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rubi steps

$$\text{integral} = \frac{1}{2}\text{Subst}\left(\int \frac{x^4}{(d+ex)\left(a+cx^2\right)^2}\,dx, x, x^2\right)$$

$$= \frac{a(ae+cdx^2)}{4c^2\left(cd^2+ae^2\right)\left(a+cx^4\right)} - \frac{\text{Subst}\left(\int \frac{\frac{a^2d^2}{cd^2+ae^2} - \frac{a^2dex}{cd^2+ae^2} - 2ax^2}{(d+ex)(a+cx^2)}\,dx, x, x^2\right)}{4ac}$$

$$= \frac{a(ae+cdx^2)}{4c^2\left(cd^2+ae^2\right)\left(a+cx^4\right)} - \frac{\text{Subst}\left(\int \left(-\frac{2acd^4}{(cd^2+ae^2)^2(d+ex)} + \frac{a^2\left(d(3cd^2+ae^2)-2e(2cd^2+ae^2)x\right)}{(cd^2+ae^2)^2(a+cx^2)}\right)dx, x, x^2\right)}{4ac}$$

$$= \frac{a(ae+cdx^2)}{4c^2\left(cd^2+ae^2\right)\left(a+cx^4\right)} + \frac{d^4\log\left(d+ex^2\right)}{2e\left(cd^2+ae^2\right)^2} - \frac{a\text{Subst}\left(\int \frac{d(3cd^2+ae^2)-2e(2cd^2+ae^2)x}{a+cx^2}\,dx, x, x^2\right)}{4c\left(cd^2+ae^2\right)^2}$$

$$= \frac{a(ae+cdx^2)}{4c^2\left(cd^2+ae^2\right)\left(a+cx^4\right)} + \frac{d^4\log\left(d+ex^2\right)}{2e\left(cd^2+ae^2\right)^2} + \frac{(ae(2cd^2+ae^2))\,\text{Subst}\left(\int \frac{x}{a+cx^2}\,dx, x, x^2\right)}{2c\left(cd^2+ae^2\right)^2} - \frac{(ad(3cd^2+ae^2))\,\text{Subst}\left(\int \frac{1}{a+cx^2}\,dx, x, x^2\right)}{4c\left(cd^2+ae^2\right)^2}$$

$$= \frac{a(ae+cdx^2)}{4c^2\left(cd^2+ae^2\right)\left(a+cx^4\right)} - \frac{\sqrt{a}d(3cd^2+ae^2)\tan^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{4c^{3/2}\left(cd^2+ae^2\right)^2} + \frac{d^4\log\left(d+ex^2\right)}{2e\left(cd^2+ae^2\right)^2} + \frac{ae(2cd^2+ae^2)\log\left(a+cx^4\right)}{4c^2\left(cd^2+ae^2\right)^2}$$

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 159, normalized size of antiderivative = 0.94

$$\int \frac{x^9}{(d+ex^2)\left(a+cx^4\right)^2}\,dx$$
$$= \frac{-\sqrt{a}\sqrt{c}de(3cd^2+ae^2)\left(a+cx^4\right)\arctan\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right) + 2c^2d^4(a+cx^4)\log\left(d+ex^2\right) + ae((cd^2+ae^2)\,(ae+cdx}{4c^2e\left(cd^2+ae^2\right)^2\left(a+cx^4\right)}$$

```
[In] Integrate[x^9/((d + e*x^2)*(a + c*x^4)^2),x]
[Out] (-(Sqrt[a]*Sqrt[c]*d*e*(3*c*d^2 + a*e^2)*(a + c*x^4)*ArcTan[(Sqrt[c]*x^2)/S
qrt[a]]) + 2*c^2*d^4*(a + c*x^4)*Log[d + e*x^2] + a*e*((c*d^2 + a*e^2)*(a*e
 + c*d*x^2) + e*(2*c*d^2 + a*e^2)*(a + c*x^4)*Log[a + c*x^4]))/(4*c^2*e*(c*
d^2 + a*e^2)^2*(a + c*x^4))
```

Maple [A] (verified)

Time = 0.43 (sec) , antiderivative size = 160, normalized size of antiderivative = 0.95

method	result
default	$-\frac{a\left(\frac{-\frac{d\left(a\,e^{2}+c\,d^{2}\right)x^{2}}{2c}-\frac{ae\left(a\,e^{2}+c\,d^{2}\right)}{2c^{2}}}{c\,x^{4}+a}+\frac{\frac{\left(-2a\,e^{3}-4c\,d^{2}e\right)\ln\left(c\,x^{4}+a\right)}{2c}+\frac{\left(d\,e^{2}a+3d^{3}c\right)\arctan\left(\frac{c\,x^{2}}{\sqrt{ac}}\right)}{\sqrt{ac}}}{2c}\right)}{2\left(a\,e^{2}+c\,d^{2}\right)^{2}}+\frac{d^{4}\ln\left(e\,x^{2}+d\right)}{2e\left(a\,e^{2}+c\,d^{2}\right)^{2}}$
risch	$\frac{\frac{da\,x^{2}}{4c\left(a\,e^{2}+c\,d^{2}\right)}+\frac{a^{2}e}{4c^{2}\left(a\,e^{2}+c\,d^{2}\right)}}{c\,x^{4}+a}+\frac{d^{4}\ln\left(e\,x^{2}+d\right)}{2e\left(a^{2}e^{4}+2ac\,d^{2}e^{2}+c^{2}d^{4}\right)}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\left(c^{4}e^{4}a^{2}+2a\,c^{5}d^{2}e^{2}+c^{6}d^{4}\right)\_Z^{2}+\left(-4e^{3}c^{2}a^{2}-8a\,c^{3}d^{2}e\right)\_\right.}\right.}{}$

[In] int(x^9/(e*x^2+d)/(c*x^4+a)^2,x,method=_RETURNVERBOSE)

[Out] -1/2*a/(a*e^2+c*d^2)^2*((-1/2*d*(a*e^2+c*d^2)/c*x^2-1/2*a*e*(a*e^2+c*d^2)/c^2)/(c*x^4+a)+1/2/c*(1/2*(-2*a*e^3-4*c*d^2*e)/c*ln(c*x^4+a)+(a*d*e^2+3*c*d^3)/(a*c)^(1/2)*arctan(c*x^2/(a*c)^(1/2))))+1/2*d^4*ln(e*x^2+d)/e/(a*e^2+c*d^2)^2

Fricas [A] (verification not implemented)

none

Time = 8.77 (sec) , antiderivative size = 555, normalized size of antiderivative = 3.28

$$\int \frac{x^9}{(d+ex^2)(a+cx^4)^2}\,dx = \left[\frac{2\,a^2cd^2e^2+2\,a^3e^4+2\,(ac^2d^3e+a^2cde^3)x^2+(3\,ac^2d^3e+a^2cde^3+(3\,c^3d^3e+ac^2de^3)x^4)\sqrt{-\frac{a}{c}}\log\left(\frac{cx^4-2\,c}{c}\right.}{8\,(ac^4d^4e+2\,a^2c^3d^2e^3+a^3c^2e^5+}\right.$$

[In] integrate(x^9/(e*x^2+d)/(c*x^4+a)^2,x, algorithm="fricas")

[Out] [1/8*(2*a^2*c*d^2*e^2 + 2*a^3*e^4 + 2*(a*c^2*d^3*e + a^2*c*d*e^3)*x^2 + (3*a*c^2*d^3*e + a^2*c*d*e^3 + (3*c^3*d^3*e + a*c^2*d*e^3)*x^4)*sqrt(-a/c)*log((c*x^4 - 2*c*x^2*sqrt(-a/c) - a)/(c*x^4 + a)) + 2*(2*a^2*c*d^2*e^2 + a^3*e^4 + (2*a*c^2*d^2*e^2 + a^2*c*e^4)*x^4)*log(c*x^4 + a) + 4*(c^3*d^4*x^4 + a*c^2*d^4)*log(e*x^2 + d))/(a*c^4*d^4*e + 2*a^2*c^3*d^2*e^3 + a^3*c^2*e^5 + (c^5*d^4*e + 2*a*c^4*d^2*e^3 + a^2*c^3*e^5)*x^4), 1/4*(a^2*c*d^2*e^2 + a^3*e^4 + (a*c^2*d^3*e + a^2*c*d*e^3)*x^2 - (3*a*c^2*d^3*e + a^2*c*d*e^3 + (3*c^3*d^3*e + a*c^2*d*e^3)*x^4)*sqrt(a/c)*arctan(c*x^2*sqrt(a/c)/a) + (2*a^2*c*d^2*e^2 + a^3*e^4 + (2*a*c^2*d^2*e^2 + a^2*c*e^4)*x^4)*log(c*x^4 + a) + 2*(c^3*d^4*x^4 + a*c^2*d^4)*log(e*x^2 + d))/(a*c^4*d^4*e + 2*a^2*c^3*d^2*e^3 + a^3*c^2*e^5 + (c^5*d^4*e + 2*a*c^4*d^2*e^3 + a^2*c^3*e^5)*x^4)]

Sympy [F(-1)]

Timed out.

$$\int \frac{x^9}{(d+ex^2)\left(a+cx^4\right)^2}\,dx = \text{Timed out}$$

```
[In] integrate(x**9/(e*x**2+d)/(c*x**4+a)**2,x)
```

```
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 220, normalized size of antiderivative = 1.30

$$\int \frac{x^9}{(d+ex^2)\left(a+cx^4\right)^2}\,dx = \frac{d^4\log\left(ex^2+d\right)}{2\left(c^2d^4e+2\,acd^2e^3+a^2e^5\right)} + \frac{\left(2\,acd^2e+a^2e^3\right)\log\left(cx^4+a\right)}{4\left(c^4d^4+2\,ac^3d^2e^2+a^2c^2e^4\right)} - \frac{\left(3\,acd^3+a^2de^2\right)\arctan\left(\frac{cx^2}{\sqrt{ac}}\right)}{4\left(c^3d^4+2\,ac^2d^2e^2+a^2ce^4\right)\sqrt{ac}} + \frac{acdx^2+a^2e}{4\left(ac^3d^2+a^2c^2e^2+\left(c^4d^2+ac^3e^2\right)x^4\right)}$$

```
[In] integrate(x^9/(e*x^2+d)/(c*x^4+a)^2,x, algorithm="maxima")
```

```
[Out] 1/2*d^4*log(e*x^2 + d)/(c^2*d^4*e + 2*a*c*d^2*e^3 + a^2*e^5) + 1/4*(2*a*c*d
^2*e + a^2*e^3)*log(c*x^4 + a)/(c^4*d^4 + 2*a*c^3*d^2*e^2 + a^2*c^2*e^4) -
1/4*(3*a*c*d^3 + a^2*d*e^2)*arctan(c*x^2/sqrt(a*c))/((c^3*d^4 + 2*a*c^2*d^2
*e^2 + a^2*c*e^4)*sqrt(a*c)) + 1/4*(a*c*d*x^2 + a^2*e)/(a*c^3*d^2 + a^2*c^2
*e^2 + (c^4*d^2 + a*c^3*e^2)*x^4)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 258, normalized size of antiderivative = 1.53

$$\int \frac{x^9}{(d+ex^2)\left(a+cx^4\right)^2}\,dx = \frac{d^4\log\left(|ex^2+d|\right)}{2\left(c^2d^4e+2\,acd^2e^3+a^2e^5\right)} + \frac{\left(2\,acd^2e+a^2e^3\right)\log\left(cx^4+a\right)}{4\left(c^4d^4+2\,ac^3d^2e^2+a^2c^2e^4\right)} - \frac{\left(3\,acd^3+a^2de^2\right)\arctan\left(\frac{cx^2}{\sqrt{ac}}\right)}{4\left(c^3d^4+2\,ac^2d^2e^2+a^2ce^4\right)\sqrt{ac}} - \frac{2\,acd^2ex^4+a^2e^3x^4-acd^3x^2-a^2de^2x^2+a^2d^2e}{4\left(c^3d^4+2\,ac^2d^2e^2+a^2ce^4\right)\left(cx^4+a\right)}$$

```
[In] integrate(x^9/(e*x^2+d)/(c*x^4+a)^2,x, algorithm="giac")
```

```
[Out] 1/2*d^4*log(abs(e*x^2 + d))/(c^2*d^4*e + 2*a*c*d^2*e^3 + a^2*e^5) + 1/4*(2*
a*c*d^2*e + a^2*e^3)*log(c*x^4 + a)/(c^4*d^4 + 2*a*c^3*d^2*e^2 + a^2*c^2*e^
4) - 1/4*(3*a*c*d^3 + a^2*d*e^2)*arctan(c*x^2/sqrt(a*c))/((c^3*d^4 + 2*a*c^
2*d^2*e^2 + a^2*c*e^4)*sqrt(a*c)) - 1/4*(2*a*c*d^2*e*x^4 + a^2*e^3*x^4 - a*
c*d^3*x^2 - a^2*d*e^2*x^2 + a^2*d^2*e)/((c^3*d^4 + 2*a*c^2*d^2*e^2 + a^2*c*
e^4)*(c*x^4 + a))
```

Mupad [B] (verification not implemented)

Time = 8.25 (sec) , antiderivative size = 305, normalized size of antiderivative = 1.80

$$\int \frac{x^9}{(d+ex^2)\,(a+cx^4)^2}\,dx$$
$$= \frac{\frac{a^2\,e}{4\,c^2\,(c\,d^2+a\,e^2)}+\frac{a\,d\,x^2}{4\,c\,(c\,d^2+a\,e^2)}}{c\,x^4+a} - \frac{\ln\left(\sqrt{-a\,c^5}+c^3\,x^2\right)\,\left(3\,c\,d^3\,\sqrt{-a\,c^5}-2\,a^2\,c^2\,e^3-4\,a\,c^3\,d^2\,e+a\,d\,e^2\,\sqrt{-a\,c^5}\right)}{8\,\left(a^2\,c^4\,e^4+2\,a\,c^5\,d^2\,e^2+c^6\,d^4\right)} + \frac{\ln\left(\sqrt{-a\,c^5}-c^3\,x^2\right)\,\left(3\,c\,d^3\,\sqrt{-a\,c^5}+2\,a^2\,c^2\,e^3+4\,a\,c^3\,d^2\,e+a\,d\,e^2\,\sqrt{-a\,c^5}\right)}{8\,\left(a^2\,c^4\,e^4+2\,a\,c^5\,d^2\,e^2+c^6\,d^4\right)} + \frac{d^4\,\ln\left(e\,x^2+d\right)}{2\,a^2\,e^5+4\,a\,c\,d^2\,e^3+2\,c^2\,d^4\,e}$$

```
[In] int(x^9/((a + c*x^4)^2*(d + e*x^2)),x)
```

```
[Out] ((a^2*e)/(4*c^2*(a*e^2 + c*d^2)) + (a*d*x^2)/(4*c*(a*e^2 + c*d^2)))/(a + c*
x^4) - (log((-a*c^5)^(1/2) + c^3*x^2)*(3*c*d^3*(-a*c^5)^(1/2) - 2*a^2*c^2*e
^3 - 4*a*c^3*d^2*e + a*d*e^2*(-a*c^5)^(1/2)))/(8*(c^6*d^4 + a^2*c^4*e^4 + 2
*a*c^5*d^2*e^2)) + (log((-a*c^5)^(1/2) - c^3*x^2)*(3*c*d^3*(-a*c^5)^(1/2) +
 2*a^2*c^2*e^3 + 4*a*c^3*d^2*e + a*d*e^2*(-a*c^5)^(1/2)))/(8*(c^6*d^4 + a^2
*c^4*e^4 + 2*a*c^5*d^2*e^2)) + (d^4*log(d + e*x^2))/(2*a^2*e^5 + 2*c^2*d^4*
e + 4*a*c*d^2*e^3)
```

3.245 $\int \frac{x^7}{(d+ex^2)(a+cx^4)^2}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 150

$$\int \frac{x^7}{(d+ex^2)(a+cx^4)^2}\,dx = \frac{a(d-ex^2)}{4c(cd^2+ae^2)(a+cx^4)} + \frac{\sqrt{a}e(3cd^2+ae^2)\arctan\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{4c^{3/2}(cd^2+ae^2)^2} - \frac{d^3\log(d+ex^2)}{2(cd^2+ae^2)^2} + \frac{d^3\log(a+cx^4)}{4(cd^2+ae^2)^2}$$

```
[Out] 1/4*a*(-e*x^2+d)/c/(a*e^2+c*d^2)/(c*x^4+a)-1/2*d^3*ln(e*x^2+d)/(a*e^2+c*d^2
)^2+1/4*d^3*ln(c*x^4+a)/(a*e^2+c*d^2)^2+1/4*e*(a*e^2+3*c*d^2)*arctan(x^2*c^
(1/2)/a^(1/2))*a^(1/2)/c^(3/2)/(a*e^2+c*d^2)^2
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {1266, 1661, 815, 649, 211, 266}

$$\int \frac{x^7}{(d+ex^2)(a+cx^4)^2}\,dx = \frac{\sqrt{a}e\arctan\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)(ae^2+3cd^2)}{4c^{3/2}(ae^2+cd^2)^2} + \frac{a(d-ex^2)}{4c(a+cx^4)(ae^2+cd^2)} + \frac{d^3\log(a+cx^4)}{4(ae^2+cd^2)^2} - \frac{d^3\log(d+ex^2)}{2(ae^2+cd^2)^2}$$

```
[In] Int[x^7/((d + e*x^2)*(a + c*x^4)^2),x]
```

```
[Out] (a*(d - e*x^2))/(4*c*(c*d^2 + a*e^2)*(a + c*x^4)) + (Sqrt[a]*e*(3*c*d^2 + a
*e^2)*ArcTan[(Sqrt[c]*x^2)/Sqrt[a]])/(4*c^(3/2)*(c*d^2 + a*e^2)^2) - (d^3*L
```

```
og[d + e*x^2])/(2*(c*d^2 + a*e^2)^2) + (d^3*Log[a + c*x^4])/(4*(c*d^2 + a*e
^2)^2)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[(-a)*c]
```

Rule 815

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)/(a + c*x^2)), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rule 1266

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_S
ymbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + c*x^2)^p, x],
x, x^2], x] /; FreeQ[{a, c, d, e, p, q}, x] && IntegerQ[(m + 1)/2]
```

Rule 1661

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[(d + e*x)^m*Pq, a + c*x^2, x], f = Coeff[Pol
ynomialRemainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 0], g = Coeff[Polynomial
Remainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 1]}, Simp[(a*g - c*f*x)*((a + c
*x^2)^(p + 1)/(2*a*c*(p + 1))), x] + Dist[1/(2*a*c*(p + 1)), Int[(d + e*x)^
m*(a + c*x^2)^(p + 1)*ExpandToSum[(2*a*c*(p + 1)*Q)/(d + e*x)^m + (c*f*(2*p
 + 3))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, c, d, e}, x] && PolyQ[Pq, x] &
& NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rubi steps

$$\text{integral} = \frac{1}{2}\text{Subst}\left(\int \frac{x^3}{(d+ex)\,(a+cx^2)^2}\,dx, x, x^2\right)$$

$$= \frac{a(d-ex^2)}{4c(cd^2+ae^2)(a+cx^4)} - \frac{\text{Subst}\left(\int \frac{-\frac{a^2de}{cd^2+ae^2}-\frac{a(2cd^2+ae^2)x}{cd^2+ae^2}}{(d+ex)(a+cx^2)}\,dx, x, x^2\right)}{4ac}$$

$$= \frac{a(d-ex^2)}{4c(cd^2+ae^2)(a+cx^4)} - \frac{\text{Subst}\left(\int \left(\frac{2acd^3e}{(cd^2+ae^2)^2(d+ex)} - \frac{a(3acd^2e+a^2e^3+2c^2d^3x)}{(cd^2+ae^2)^2(a+cx^2)}\right)\,dx, x, x^2\right)}{4ac}$$

$$= \frac{a(d-ex^2)}{4c(cd^2+ae^2)(a+cx^4)} - \frac{d^3\log(d+ex^2)}{2(cd^2+ae^2)^2} + \frac{\text{Subst}\left(\int \frac{3acd^2e+a^2e^3+2c^2d^3x}{a+cx^2}\,dx, x, x^2\right)}{4c(cd^2+ae^2)^2}$$

$$= \frac{a(d-ex^2)}{4c(cd^2+ae^2)(a+cx^4)} - \frac{d^3\log(d+ex^2)}{2(cd^2+ae^2)^2} + \frac{(cd^3)\,\text{Subst}\left(\int \frac{x}{a+cx^2}\,dx, x, x^2\right)}{2(cd^2+ae^2)^2} + \frac{(ae(3cd^2+ae^2))\,\text{Subst}\left(\int \frac{1}{a+cx^2}\,dx, x, x^2\right)}{4c(cd^2+ae^2)^2}$$

$$= \frac{a(d-ex^2)}{4c(cd^2+ae^2)(a+cx^4)} + \frac{\sqrt{a}e(3cd^2+ae^2)\tan^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{4c^{3/2}(cd^2+ae^2)^2} - \frac{d^3\log(d+ex^2)}{2(cd^2+ae^2)^2} + \frac{d^3\log(a+cx^4)}{4(cd^2+ae^2)^2}$$

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 142, normalized size of antiderivative = 0.95

$$\int \frac{x^7}{(d+ex^2)(a+cx^4)^2}\,dx$$
$$= \frac{\sqrt{a}e(3cd^2+ae^2)(a+cx^4)\arctan\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right) + \sqrt{c}(a(cd^2+ae^2)(d-ex^2) - 2cd^3(a+cx^4)\log(d+ex^2) + cd}{4c^{3/2}(cd^2+ae^2)^2(a+cx^4)}$$

```
[In] Integrate[x^7/((d + e*x^2)*(a + c*x^4)^2),x]
```

```
[Out] (Sqrt[a]*e*(3*c*d^2 + a*e^2)*(a + c*x^4)*ArcTan[(Sqrt[c]*x^2)/Sqrt[a]] + Sq
rt[c]*(a*(c*d^2 + a*e^2)*(d - e*x^2) - 2*c*d^3*(a + c*x^4)*Log[d + e*x^2] +
 c*d^3*(a + c*x^4)*Log[a + c*x^4]))/(4*c^(3/2)*(c*d^2 + a*e^2)^2*(a + c*x^4
))
```

Maple [A] (verified)

Time = 0.50 (sec) , antiderivative size = 146, normalized size of antiderivative = 0.97

method	result	size
default	$\frac{\frac{-\frac{ae\left(a\,e^{2}+c\,d^{2}\right)x^{2}}{2c}+\frac{da\left(a\,e^{2}+c\,d^{2}\right)}{2c}}{c\,x^{4}+a}+\frac{c\,d^{3}\ln\left(c\,x^{4}+a\right)+\frac{\left(e^{3}a^{2}+3ac\,d^{2}e\right)\arctan\left(\frac{c\,x^{2}}{\sqrt{ac}}\right)}{\sqrt{ac}}}{2c}}{2\left(a\,e^{2}+c\,d^{2}\right)^{2}}-\frac{d^{3}\ln\left(e\,x^{2}+d\right)}{2\left(a\,e^{2}+c\,d^{2}\right)^{2}}$	146
risch	Expression too large to display	1428

```
[In] int(x^7/(e*x^2+d)/(c*x^4+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2/(a*e^2+c*d^2)^2*((-1/2*a*e*(a*e^2+c*d^2)/c*x^2+1/2*d*a*(a*e^2+c*d^2)/c)
/(c*x^4+a)+1/2/c*(c*d^3*ln(c*x^4+a)+(a^2*e^3+3*a*c*d^2*e)/(a*c)^(1/2)*arcta
n(c*x^2/(a*c)^(1/2))))-1/2*d^3*ln(e*x^2+d)/(a*e^2+c*d^2)^2
```

Fricas [A] (verification not implemented)

none

Time = 4.23 (sec) , antiderivative size = 457, normalized size of antiderivative = 3.05

$$\int \frac{x^7}{(d+ex^2)\left(a+cx^4\right)^2}\,dx$$
$$= \left[\frac{2\,acd^3+2\,a^2de^2-2\,(acd^2e+a^2e^3)x^2+(3\,acd^2e+a^2e^3+(3\,c^2d^2e+ace^3)x^4)\sqrt{-\frac{a}{c}}\log\left(\frac{cx^4+2\,cx^2\sqrt{-\frac{a}{c}}-a}{cx^4+a}\right)}{8\,(ac^3d^4+2\,a^2c^2d^2e^2+a^3ce^4+(c^4d^4+2\,ac^3d^2e^2\cdots}\right.$$

```
[In] integrate(x^7/(e*x^2+d)/(c*x^4+a)^2,x, algorithm="fricas")
```

```
[Out] [1/8*(2*a*c*d^3 + 2*a^2*d*e^2 - 2*(a*c*d^2*e + a^2*e^3)*x^2 + (3*a*c*d^2*e
+ a^2*e^3 + (3*c^2*d^2*e + a*c*e^3)*x^4)*sqrt(-a/c)*log((c*x^4 + 2*c*x^2*sq
rt(-a/c) - a)/(c*x^4 + a)) + 2*(c^2*d^3*x^4 + a*c*d^3)*log(c*x^4 + a) - 4*(
c^2*d^3*x^4 + a*c*d^3)*log(e*x^2 + d))/(a*c^3*d^4 + 2*a^2*c^2*d^2*e^2 + a^3
*c*e^4 + (c^4*d^4 + 2*a*c^3*d^2*e^2 + a^2*c^2*e^4)*x^4), 1/4*(a*c*d^3 + a^2
*d*e^2 - (a*c*d^2*e + a^2*e^3)*x^2 + (3*a*c*d^2*e + a^2*e^3 + (3*c^2*d^2*e
+ a*c*e^3)*x^4)*sqrt(a/c)*arctan(c*x^2*sqrt(a/c)/a) + (c^2*d^3*x^4 + a*c*d^
3)*log(c*x^4 + a) - 2*(c^2*d^3*x^4 + a*c*d^3)*log(e*x^2 + d))/(a*c^3*d^4 +
2*a^2*c^2*d^2*e^2 + a^3*c*e^4 + (c^4*d^4 + 2*a*c^3*d^2*e^2 + a^2*c^2*e^4)*x
^4)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^7}{(d+ex^2)(a+cx^4)^2}\,dx = \text{Timed out}$$

```
[In] integrate(x**7/(e*x**2+d)/(c*x**4+a)**2,x)
```

```
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 197, normalized size of antiderivative = 1.31

$$\int \frac{x^7}{(d+ex^2)(a+cx^4)^2}\,dx = \frac{d^3\log(cx^4+a)}{4(c^2d^4+2\,acd^2e^2+a^2e^4)} - \frac{d^3\log(ex^2+d)}{2(c^2d^4+2\,acd^2e^2+a^2e^4)} + \frac{(3\,acd^2e+a^2e^3)\arctan\left(\frac{cx^2}{\sqrt{ac}}\right)}{4(c^3d^4+2\,ac^2d^2e^2+a^2ce^4)\sqrt{ac}} - \frac{aex^2-ad}{4(ac^2d^2+a^2ce^2+(c^3d^2+ac^2e^2)x^4)}$$

```
[In] integrate(x^7/(e*x^2+d)/(c*x^4+a)^2,x, algorithm="maxima")
```

```
[Out] 1/4*d^3*log(c*x^4 + a)/(c^2*d^4 + 2*a*c*d^2*e^2 + a^2*e^4) - 1/2*d^3*log(e*
x^2 + d)/(c^2*d^4 + 2*a*c*d^2*e^2 + a^2*e^4) + 1/4*(3*a*c*d^2*e + a^2*e^3)*
arctan(c*x^2/sqrt(a*c))/((c^3*d^4 + 2*a*c^2*d^2*e^2 + a^2*c*e^4)*sqrt(a*c))
 - 1/4*(a*e*x^2 - a*d)/(a*c^2*d^2 + a^2*c*e^2 + (c^3*d^2 + a*c^2*e^2)*x^4)
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 229, normalized size of antiderivative = 1.53

$$\int \frac{x^7}{(d+ex^2)(a+cx^4)^2}\,dx = -\frac{d^3e\log(|ex^2+d|)}{2(c^2d^4e+2\,acd^2e^3+a^2e^5)} + \frac{d^3\log(cx^4+a)}{4(c^2d^4+2\,acd^2e^2+a^2e^4)} + \frac{(3\,acd^2e+a^2e^3)\arctan\left(\frac{cx^2}{\sqrt{ac}}\right)}{4(c^3d^4+2\,ac^2d^2e^2+a^2ce^4)\sqrt{ac}} - \frac{c^2d^3x^4+acd^2ex^2+a^2e^3x^2-a^2de^2}{4(c^3d^4+2\,ac^2d^2e^2+a^2ce^4)(cx^4+a)}$$

```
[In] integrate(x^7/(e*x^2+d)/(c*x^4+a)^2,x, algorithm="giac")
```

```
[Out] -1/2*d^3*e*log(abs(e*x^2 + d))/(c^2*d^4*e + 2*a*c*d^2*e^3 + a^2*e^5) + 1/4*
d^3*log(c*x^4 + a)/(c^2*d^4 + 2*a*c*d^2*e^2 + a^2*e^4) + 1/4*(3*a*c*d^2*e +
 a^2*e^3)*arctan(c*x^2/sqrt(a*c))/((c^3*d^4 + 2*a*c^2*d^2*e^2 + a^2*c*e^4)*
sqrt(a*c)) - 1/4*(c^2*d^3*x^4 + a*c*d^2*e*x^2 + a^2*e^3*x^2 - a^2*d*e^2)/((
c^3*d^4 + 2*a*c^2*d^2*e^2 + a^2*c*e^4)*(c*x^4 + a))
```

Mupad [B] (verification not implemented)

Time = 8.26 (sec) , antiderivative size = 647, normalized size of antiderivative = 4.31

$$\int \frac{x^7}{(d+ex^2)(a+cx^4)^2}\,dx = \frac{\frac{a\,d}{4\,c\,(c\,d^2+a\,e^2)}-\frac{a\,e\,x^2}{4\,c\,(c\,d^2+a\,e^2)}}{c\,x^4+a} - \frac{d^3\,\ln\left(e\,x^2+d\right)}{2\,\left(a^2\,e^4+2\,a\,c\,d^2\,e^2+c^2\,d^4\right)}$$
$$+\frac{\ln\Big(36\,c^8\,d^{10}\,x^2+36\,c^6\,d^{10}\,\sqrt{-a\,c^3}+a^5\,c\,e^{10}\,\sqrt{-a\,c^3}+a^5\,c^3\,e^{10}\,x^2-22\,a^2\,d^4\,e^6\,\left(-a\,c^3\right)^{3/2}-81\,c^2\,d^8\,e^2\,(-}{}$$
$$-\frac{\ln\Big(36\,c^8\,d^{10}\,x^2-36\,c^6\,d^{10}\,\sqrt{-a\,c^3}-a^5\,c\,e^{10}\,\sqrt{-a\,c^3}+a^5\,c^3\,e^{10}\,x^2+22\,a^2\,d^4\,e^6\,\left(-a\,c^3\right)^{3/2}+81\,c^2\,d^8\,e^2\,(-}{}$$

```
[In] int(x^7/((a + c*x^4)^2*(d + e*x^2)),x)
```

```
[Out] ((a*d)/(4*c*(a*e^2 + c*d^2)) - (a*e*x^2)/(4*c*(a*e^2 + c*d^2)))/(a + c*x^4)
 - (d^3*log(d + e*x^2))/(2*(a^2*e^4 + c^2*d^4 + 2*a*c*d^2*e^2)) + (log(36*c
^8*d^10*x^2 + 36*c^6*d^10*(-a*c^3)^(1/2) + a^5*c*e^10*(-a*c^3)^(1/2) + a^5*
c^3*e^10*x^2 - 22*a^2*d^4*e^6*(-a*c^3)^(3/2) - 81*c^2*d^8*e^2*(-a*c^3)^(3/2
) + 60*a^2*c^6*d^6*e^4*x^2 + 22*a^3*c^5*d^4*e^6*x^2 + 8*a^4*c^4*d^2*e^8*x^2
 + 8*a^4*c^2*d^2*e^8*(-a*c^3)^(1/2) - 60*a*c*d^6*e^4*(-a*c^3)^(3/2) + 81*a*
c^7*d^8*e^2*x^2)*(2*c^3*d^3 + a*e^3*(-a*c^3)^(1/2) + 3*c*d^2*e*(-a*c^3)^(1/
2)))/(8*(c^5*d^4 + a^2*c^3*e^4 + 2*a*c^4*d^2*e^2)) - (log(36*c^8*d^10*x^2 -
 36*c^6*d^10*(-a*c^3)^(1/2) - a^5*c*e^10*(-a*c^3)^(1/2) + a^5*c^3*e^10*x^2
+ 22*a^2*d^4*e^6*(-a*c^3)^(3/2) + 81*c^2*d^8*e^2*(-a*c^3)^(3/2) + 60*a^2*c^
6*d^6*e^4*x^2 + 22*a^3*c^5*d^4*e^6*x^2 + 8*a^4*c^4*d^2*e^8*x^2 - 8*a^4*c^2*
d^2*e^8*(-a*c^3)^(1/2) + 60*a*c*d^6*e^4*(-a*c^3)^(3/2) + 81*a*c^7*d^8*e^2*x
^2)*(a*e^3*(-a*c^3)^(1/2) - 2*c^3*d^3 + 3*c*d^2*e*(-a*c^3)^(1/2)))/(8*(c^5*
d^4 + a^2*c^3*e^4 + 2*a*c^4*d^2*e^2))
```

3.246 $\int \frac{x^5}{(d+ex^2)(a+cx^4)^2}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 155

$$\int \frac{x^5}{(d+ex^2)(a+cx^4)^2}\,dx=\frac{-ae-cdx^2}{4c(cd^2+ae^2)(a+cx^4)}+\frac{d(cd^2-ae^2)\arctan\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{4\sqrt{a}\sqrt{c}(cd^2+ae^2)^2}+\frac{d^2e\log(d+ex^2)}{2(cd^2+ae^2)^2}-\frac{d^2e\log(a+cx^4)}{4(cd^2+ae^2)^2}$$

```
[Out] 1/4*(-c*d*x^2-a*e)/c/(a*e^2+c*d^2)/(c*x^4+a)+1/2*d^2*e*ln(e*x^2+d)/(a*e^2+c
*d^2)^2-1/4*d^2*e*ln(c*x^4+a)/(a*e^2+c*d^2)^2+1/4*d*(-a*e^2+c*d^2)*arctan(x
^2*c^(1/2)/a^(1/2))/(a*e^2+c*d^2)^2/a^(1/2)/c^(1/2)
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 153, normalized size of antiderivative = 0.99, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {1266, 1661, 815, 649, 211, 266}

$$\int \frac{x^5}{(d+ex^2)(a+cx^4)^2}\,dx=\frac{d\arctan\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)(cd^2-ae^2)}{4\sqrt{a}\sqrt{c}(ae^2+cd^2)^2}-\frac{d^2e\log(a+cx^4)}{4(ae^2+cd^2)^2}+\frac{d^2e\log(d+ex^2)}{2(ae^2+cd^2)^2}-\frac{ae+cdx^2}{4c(a+cx^4)(ae^2+cd^2)}$$

```
[In] Int[x^5/((d + e*x^2)*(a + c*x^4)^2),x]
```

```
[Out] -1/4*(a*e + c*d*x^2)/(c*(c*d^2 + a*e^2)*(a + c*x^4)) + (d*(c*d^2 - a*e^2)*A
rcTan[(Sqrt[c]*x^2)/Sqrt[a]])/(4*Sqrt[a]*Sqrt[c]*(c*d^2 + a*e^2)^2) + (d^2*
```

```
e*Log[d + e*x^2])/(2*(c*d^2 + a*e^2)^2) - (d^2*e*Log[a + c*x^4])/(4*(c*d^2
+ a*e^2)^2)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[(-a)*c]
```

Rule 815

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)/(a + c*x^2)), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rule 1266

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_S
ymbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + c*x^2)^p, x],
x, x^2], x] /; FreeQ[{a, c, d, e, p, q}, x] && IntegerQ[(m + 1)/2]
```

Rule 1661

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[(d + e*x)^m*Pq, a + c*x^2, x], f = Coeff[Pol
ynomialRemainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 0], g = Coeff[Polynomial
Remainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 1]}, Simp[(a*g - c*f*x)*((a + c
*x^2)^(p + 1)/(2*a*c*(p + 1))), x] + Dist[1/(2*a*c*(p + 1)), Int[(d + e*x)^
m*(a + c*x^2)^(p + 1)*ExpandToSum[(2*a*c*(p + 1)*Q)/(d + e*x)^m + (c*f*(2*p
 + 3))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, c, d, e}, x] && PolyQ[Pq, x] &
& NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rubi steps

$$\text{integral} = \frac{1}{2}\text{Subst}\left(\int \frac{x^2}{(d+ex)\left(a+cx^2\right)^2}\,dx, x, x^2\right)$$

$$= -\frac{ae+cdx^2}{4c(cd^2+ae^2)(a+cx^4)} - \frac{\text{Subst}\left(\int \frac{-\frac{acd^2}{cd^2+ae^2}+\frac{acdex}{cd^2+ae^2}}{(d+ex)(a+cx^2)}\,dx, x, x^2\right)}{4ac}$$

$$= -\frac{ae+cdx^2}{4c(cd^2+ae^2)(a+cx^4)} - \frac{\text{Subst}\left(\int \left(-\frac{2acd^2e^2}{(cd^2+ae^2)^2(d+ex)} + \frac{acd(-cd^2+ae^2+2cdex)}{(cd^2+ae^2)^2(a+cx^2)}\right)\,dx, x, x^2\right)}{4ac}$$

$$= -\frac{ae+cdx^2}{4c(cd^2+ae^2)(a+cx^4)} + \frac{d^2e\log(d+ex^2)}{2(cd^2+ae^2)^2} - \frac{d\text{Subst}\left(\int \frac{-cd^2+ae^2+2cdex}{a+cx^2}\,dx, x, x^2\right)}{4(cd^2+ae^2)^2}$$

$$= -\frac{ae+cdx^2}{4c(cd^2+ae^2)(a+cx^4)} + \frac{d^2e\log(d+ex^2)}{2(cd^2+ae^2)^2} - \frac{(cd^2e)\,\text{Subst}\left(\int \frac{x}{a+cx^2}\,dx, x, x^2\right)}{2(cd^2+ae^2)^2} + \frac{(d(cd^2-ae^2))\,\text{Subst}\left(\int \frac{1}{a+cx^2}\,dx, x, x^2\right)}{4(cd^2+ae^2)^2}$$

$$= -\frac{ae+cdx^2}{4c(cd^2+ae^2)(a+cx^4)} + \frac{d(cd^2-ae^2)\tan^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{4\sqrt{a}\sqrt{c}(cd^2+ae^2)^2} + \frac{d^2e\log(d+ex^2)}{2(cd^2+ae^2)^2} - \frac{d^2e\log(a+cx^4)}{4(cd^2+ae^2)^2}$$

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 149, normalized size of antiderivative = 0.96

$$\int \frac{x^5}{(d+ex^2)(a+cx^4)^2}\,dx$$
$$= \frac{\sqrt{c}d(cd^2-ae^2)(a+cx^4)\arctan\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right) - \sqrt{a}((cd^2+ae^2)(ae+cdx^2) - 2cd^2e(a+cx^4)\log(d+ex^2) + c}{4\sqrt{a}c(cd^2+ae^2)^2(a+cx^4)}$$

```
[In] Integrate[x^5/((d + e*x^2)*(a + c*x^4)^2),x]
```

```
[Out] (Sqrt[c]*d*(c*d^2 - a*e^2)*(a + c*x^4)*ArcTan[(Sqrt[c]*x^2)/Sqrt[a]] - Sqrt
[a]*((c*d^2 + a*e^2)*(a*e + c*d*x^2) - 2*c*d^2*e*(a + c*x^4)*Log[d + e*x^2]
 + c*d^2*e*(a + c*x^4)*Log[a + c*x^4]))/(4*Sqrt[a]*c*(c*d^2 + a*e^2)^2*(a +
 c*x^4))
```

Maple [A] (verified)

Time = 0.42 (sec) , antiderivative size = 136, normalized size of antiderivative = 0.88

method	result
default	$-\frac{\frac{\left(\frac{1}{2} d\,e^{2} a+\frac{1}{2} d^{3} c\right) x^{2}+\frac{a e \left(a\,e^{2}+c\,d^{2}\right)}{2 c}}{c\,x^{4}+a}+\frac{d \left(d e \ln \left(c\,x^{4}+a\right)+\frac{\left(a\,e^{2}-c\,d^{2}\right) \arctan \left(\frac{c\,x^{2}}{\sqrt{a c}}\right)}{\sqrt{a c}}\right)}{2}}{2 (a\,e^{2}+c\,d^{2})^{2}}+\frac{d^{2} e \ln \left(e\,x^{2}+d\right)}{2 (a\,e^{2}+c\,d^{2})^{2}}$
risch	$\frac{-\frac{d\,x^{2}}{4 \left(a\,e^{2}+c\,d^{2}\right)}-\frac{a e}{4 c \left(a\,e^{2}+c\,d^{2}\right)}}{c\,x^{4}+a}+\frac{d^{2} e \ln \left(e\,x^{2}+d\right)}{2 a^{2} e^{4}+4 a c\,d^{2} e^{2}+2 c^{2} d^{4}}+\frac{\left(\sum_{\_R =\operatorname{RootOf}\left(\left(a^{3} c\,e^{4}+2 a^{2} c^{2} d^{2} e^{2}+a\,c^{3} d^{4}\right) \_Z^{2}+4 a c\,d^{2} e \_Z +d^{2}\right)}\_R \ln \left(\right.\right.}{}$

[In] int(x^5/(e*x^2+d)/(c*x^4+a)^2,x,method=_RETURNVERBOSE)

[Out] -1/2/(a*e^2+c*d^2)^2*(((1/2*d*e^2*a+1/2*d^3*c)*x^2+1/2*a*e*(a*e^2+c*d^2)/c)/(c*x^4+a)+1/2*d*(d*e*ln(c*x^4+a)+(a*e^2-c*d^2)/(a*c)^(1/2)*arctan(c*x^2/(a*c)^(1/2))))+1/2*d^2*e*ln(e*x^2+d)/(a*e^2+c*d^2)^2

Fricas [A] (verification not implemented)

none

Time = 1.79 (sec) , antiderivative size = 487, normalized size of antiderivative = 3.14

$$\int \frac{x^5}{(d+ex^2)(a+cx^4)^2}\,dx$$
$$= \left[-\frac{2\,a^2cd^2e + 2\,a^3e^3 + 2\,(ac^2d^3 + a^2cde^2)x^2 - (acd^3 - a^2de^2 + (c^2d^3 - acde^2)x^4)\sqrt{-ac}\log\left(\frac{cx^4+2\,\sqrt{-ac}x^2-a}{cx^4+a}\right.}{8\,(a^2c^3d^4 + 2\,a^3c^2d^2e^2 + a^4ce^4 + (ac^4d^4 + 2\,a^2}\right.$$
$$-\frac{a^2cd^2e + a^3e^3 + (ac^2d^3 + a^2cde^2)x^2 + (acd^3 - a^2de^2 + (c^2d^3 - acde^2)x^4)\sqrt{ac}\arctan\left(\frac{\sqrt{ac}}{cx^2}\right) + (ac^2d^2ex^4}{4\,(a^2c^3d^4 + 2\,a^3c^2d^2e^2 + a^4ce^4 + (ac^4d^4 + 2\,a^2c^3d^2e^2 +}$$

[In] integrate(x^5/(e*x^2+d)/(c*x^4+a)^2,x, algorithm="fricas")

[Out] [-1/8*(2*a^2*c*d^2*e + 2*a^3*e^3 + 2*(a*c^2*d^3 + a^2*c*d*e^2)*x^2 - (a*c*d^3 - a^2*d*e^2 + (c^2*d^3 - a*c*d*e^2)*x^4)*sqrt(-a*c)*log((c*x^4 + 2*sqrt(-a*c)*x^2 - a)/(c*x^4 + a)) + 2*(a*c^2*d^2*e*x^4 + a^2*c*d^2*e)*log(c*x^4 + a) - 4*(a*c^2*d^2*e*x^4 + a^2*c*d^2*e)*log(e*x^2 + d))/(a^2*c^3*d^4 + 2*a^3*c^2*d^2*e^2 + a^4*c*e^4 + (a*c^4*d^4 + 2*a^2*c^3*d^2*e^2 + a^3*c^2*e^4)*x^4), -1/4*(a^2*c*d^2*e + a^3*e^3 + (a*c^2*d^3 + a^2*c*d*e^2)*x^2 + (a*c*d^3 - a^2*d*e^2 + (c^2*d^3 - a*c*d*e^2)*x^4)*sqrt(a*c)*arctan(sqrt(a*c)/(c*x^2)) + (a*c^2*d^2*e*x^4 + a^2*c*d^2*e)*log(c*x^4 + a) - 2*(a*c^2*d^2*e*x^4 + a^2*c*d^2*e)*log(e*x^2 + d))/(a^2*c^3*d^4 + 2*a^3*c^2*d^2*e^2 + a^4*c*e^4 + (a*c^4*d^4 + 2*a^2*c^3*d^2*e^2 + a^3*c^2*e^4)*x^4)]

Sympy [F(-1)]

Timed out.

$$\int \frac{x^5}{(d+ex^2)(a+cx^4)^2}\,dx = \text{Timed out}$$

```
[In] integrate(x**5/(e*x**2+d)/(c*x**4+a)**2,x)
```

```
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 192, normalized size of antiderivative = 1.24

$$\begin{aligned}\int \frac{x^5}{(d+ex^2)(a+cx^4)^2}\,dx = &-\frac{d^2e\log(cx^4+a)}{4(c^2d^4+2\,acd^2e^2+a^2e^4)}+\frac{d^2e\log(ex^2+d)}{2(c^2d^4+2\,acd^2e^2+a^2e^4)}\\&+\frac{(cd^3-ade^2)\arctan\left(\frac{cx^2}{\sqrt{ac}}\right)}{4(c^2d^4+2\,acd^2e^2+a^2e^4)\sqrt{ac}}\\&-\frac{cdx^2+ae}{4(ac^2d^2+a^2ce^2+(c^3d^2+ac^2e^2)x^4)}\end{aligned}$$

```
[In] integrate(x^5/(e*x^2+d)/(c*x^4+a)^2,x, algorithm="maxima")
```

```
[Out] -1/4*d^2*e*log(c*x^4 + a)/(c^2*d^4 + 2*a*c*d^2*e^2 + a^2*e^4) + 1/2*d^2*e*l
og(e*x^2 + d)/(c^2*d^4 + 2*a*c*d^2*e^2 + a^2*e^4) + 1/4*(c*d^3 - a*d*e^2)*a
rctan(c*x^2/sqrt(a*c))/((c^2*d^4 + 2*a*c*d^2*e^2 + a^2*e^4)*sqrt(a*c)) - 1/
4*(c*d*x^2 + a*e)/(a*c^2*d^2 + a^2*c*e^2 + (c^3*d^2 + a*c^2*e^2)*x^4)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 228, normalized size of antiderivative = 1.47

$$\begin{aligned}\int \frac{x^5}{(d+ex^2)(a+cx^4)^2}\,dx = &\frac{d^2e^2\log(|ex^2+d|)}{2(c^2d^4e+2\,acd^2e^3+a^2e^5)}-\frac{d^2e\log(cx^4+a)}{4(c^2d^4+2\,acd^2e^2+a^2e^4)}\\&+\frac{(cd^3-ade^2)\arctan\left(\frac{cx^2}{\sqrt{ac}}\right)}{4(c^2d^4+2\,acd^2e^2+a^2e^4)\sqrt{ac}}\\&+\frac{c^2d^2ex^4-c^2d^3x^2-acde^2x^2-a^2e^3}{4(c^3d^4+2\,ac^2d^2e^2+a^2ce^4)(cx^4+a)}\end{aligned}$$

```
[In] integrate(x^5/(e*x^2+d)/(c*x^4+a)^2,x, algorithm="giac")
```

```
[Out] 1/2*d^2*e^2*log(abs(e*x^2 + d))/(c^2*d^4*e + 2*a*c*d^2*e^3 + a^2*e^5) - 1/4
*d^2*e*log(c*x^4 + a)/(c^2*d^4 + 2*a*c*d^2*e^2 + a^2*e^4) + 1/4*(c*d^3 - a*
d*e^2)*arctan(c*x^2/sqrt(a*c))/((c^2*d^4 + 2*a*c*d^2*e^2 + a^2*e^4)*sqrt(a*
c)) + 1/4*(c^2*d^2*e*x^4 - c^2*d^3*x^2 - a*c*d*e^2*x^2 - a^2*e^3)/((c^3*d^4
 + 2*a*c^2*d^2*e^2 + a^2*c*e^4)*(c*x^4 + a))
```

Mupad [B] (verification not implemented)

Time = 8.28 (sec) , antiderivative size = 528, normalized size of antiderivative = 3.41

$$\int \frac{x^5}{(d+ex^2)(a+cx^4)^2}\,dx$$

$$= \frac{\ln\left(a^4\,e^8\,\sqrt{-a\,c}+c^4\,d^8\,\sqrt{-a\,c}+70\,d^4\,e^4\,(-a\,c)^{5/2}+c^5\,d^8\,x^2+a^4\,c\,e^8\,x^2-36\,a^2\,d^2\,e^6\,(-a\,c)^{3/2}-36\,c^2\,d^6\,e^2\right.}{a^3\,c\,e^4+2\,a^2\,c^2\,d^2}$$

$$-\frac{\frac{d\,x^2}{4\,(c\,d^2+a\,e^2)}+\frac{a\,e}{4\,c\,(c\,d^2+a\,e^2)}}{c\,x^4+a}$$

$$-\frac{\ln\left(c^5\,d^8\,x^2-c^4\,d^8\,\sqrt{-a\,c}-70\,d^4\,e^4\,(-a\,c)^{5/2}-a^4\,e^8\,\sqrt{-a\,c}+a^4\,c\,e^8\,x^2+36\,a^2\,d^2\,e^6\,(-a\,c)^{3/2}+36\,c^2\,d^6\right.}{a^3\,c\,e^4+2\,a^2\,c^2}$$

$$+\frac{d^2\,e\,\ln\left(e\,x^2+d\right)}{2\,\left(a^2\,e^4+2\,a\,c\,d^2\,e^2+c^2\,d^4\right)}$$

```
[In] int(x^5/((a + c*x^4)^2*(d + e*x^2)),x)
```

```
[Out] (log(a^4*e^8*(-a*c)^(1/2) + c^4*d^8*(-a*c)^(1/2) + 70*d^4*e^4*(-a*c)^(5/2)
+ c^5*d^8*x^2 + a^4*c*e^8*x^2 - 36*a^2*d^2*e^6*(-a*c)^(3/2) - 36*c^2*d^6*e^
2*(-a*c)^(3/2) + 70*a^2*c^3*d^4*e^4*x^2 + 36*a^3*c^2*d^2*e^6*x^2 + 36*a*c^4
*d^6*e^2*x^2)*(c*((d^3*(-a*c)^(1/2))/8 - (a*d^2*e)/4) - (a*d*e^2*(-a*c)^(1/
2))/8))/(a*c^3*d^4 + a^3*c*e^4 + 2*a^2*c^2*d^2*e^2) - ((d*x^2)/(4*(a*e^2 +
c*d^2)) + (a*e)/(4*c*(a*e^2 + c*d^2)))/(a + c*x^4) - (log(c^5*d^8*x^2 - c^4
*d^8*(-a*c)^(1/2) - 70*d^4*e^4*(-a*c)^(5/2) - a^4*e^8*(-a*c)^(1/2) + a^4*c*
e^8*x^2 + 36*a^2*d^2*e^6*(-a*c)^(3/2) + 36*c^2*d^6*e^2*(-a*c)^(3/2) + 70*a^
2*c^3*d^4*e^4*x^2 + 36*a^3*c^2*d^2*e^6*x^2 + 36*a*c^4*d^6*e^2*x^2)*(c*((d^3
*(-a*c)^(1/2))/8 + (a*d^2*e)/4) - (a*d*e^2*(-a*c)^(1/2))/8))/(a*c^3*d^4 + a
^3*c*e^4 + 2*a^2*c^2*d^2*e^2) + (d^2*e*log(d + e*x^2))/(2*(a^2*e^4 + c^2*d^
4 + 2*a*c*d^2*e^2))
```

3.247 $\int \frac{x^3}{(d+ex^2)(a+cx^4)^2} dx$

Optimal result

Integrand size = 22, antiderivative size = 149

$$\int \frac{x^3}{(d+ex^2)(a+cx^4)^2} dx = \frac{-d+ex^2}{4(cd^2+ae^2)(a+cx^4)} - \frac{e(cd^2-ae^2)\arctan\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{4\sqrt{a}\sqrt{c}(cd^2+ae^2)^2} - \frac{de^2\log(d+ex^2)}{2(cd^2+ae^2)^2} + \frac{de^2\log(a+cx^4)}{4(cd^2+ae^2)^2}$$

```
[Out] 1/4*(e*x^2-d)/(a*e^2+c*d^2)/(c*x^4+a)-1/2*d*e^2*ln(e*x^2+d)/(a*e^2+c*d^2)^2
+1/4*d*e^2*ln(c*x^4+a)/(a*e^2+c*d^2)^2-1/4*e*(-a*e^2+c*d^2)*arctan(x^2*c^(1
/2)/a^(1/2))/(a*e^2+c*d^2)^2/a^(1/2)/c^(1/2)
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 148, normalized size of antiderivative = 0.99, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {1266, 837, 815, 649, 211, 266}

$$\int \frac{x^3}{(d+ex^2)(a+cx^4)^2} dx = -\frac{e\arctan\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)(cd^2-ae^2)}{4\sqrt{a}\sqrt{c}(ae^2+cd^2)^2} + \frac{de^2\log(a+cx^4)}{4(ae^2+cd^2)^2} - \frac{de^2\log(d+ex^2)}{2(ae^2+cd^2)^2} - \frac{d-ex^2}{4(a+cx^4)(ae^2+cd^2)}$$

```
[In] Int[x^3/((d + e*x^2)*(a + c*x^4)^2),x]
```

```
[Out] -1/4*(d - e*x^2)/((c*d^2 + a*e^2)*(a + c*x^4)) - (e*(c*d^2 - a*e^2)*ArcTan[
(Sqrt[c]*x^2)/Sqrt[a]])/(4*Sqrt[a]*Sqrt[c]*(c*d^2 + a*e^2)^2) - (d*e^2*Log[
```

```
d + e*x^2])/(2*(c*d^2 + a*e^2)^2) + (d*e^2*Log[a + c*x^4])/(4*(c*d^2 + a*e^
2)^2)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[(-a)*c]
```

Rule 815

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)/(a + c*x^2)), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rule 837

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> Simp[(-(d + e*x)^(m + 1))*(f*a*c*e - a*g*c*d + c*(c*d*f +
a*e*g)*x)*((a + c*x^2)^(p + 1)/(2*a*c*(p + 1)*(c*d^2 + a*e^2))), x] + Dist[
1/(2*a*c*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*(a + c*x^2)^(p + 1)*Simp
[f*(c^2*d^2*(2*p + 3) + a*c*e^2*(m + 2*p + 3)) - a*c*d*e*g*m + c*e*(c*d*f +
 a*e*g)*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[
c*d^2 + a*e^2, 0] && LtQ[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ
[2*m, 2*p])
```

Rule 1266

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_S
ymbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + c*x^2)^p, x],
x, x^2], x] /; FreeQ[{a, c, d, e, p, q}, x] && IntegerQ[(m + 1)/2]
```

Rubi steps

$$\text{integral} = \frac{1}{2}\text{Subst}\left(\int \frac{x}{(d+ex)\,(a+cx^2)^2}\,dx, x, x^2\right)$$

$$= -\frac{d-ex^2}{4(cd^2+ae^2)(a+cx^4)} - \frac{\text{Subst}\left(\int \frac{acde-ace^2x}{(d+ex)(a+cx^2)}\,dx, x, x^2\right)}{4ac(cd^2+ae^2)}$$

$$= -\frac{d-ex^2}{4(cd^2+ae^2)(a+cx^4)} - \frac{\text{Subst}\left(\int \left(\frac{2acde^3}{(cd^2+ae^2)(d+ex)} - \frac{ace(-cd^2+ae^2+2cdex)}{(cd^2+ae^2)(a+cx^2)}\right)\,dx, x, x^2\right)}{4ac(cd^2+ae^2)}$$

$$= -\frac{d-ex^2}{4(cd^2+ae^2)(a+cx^4)} - \frac{de^2\log(d+ex^2)}{2(cd^2+ae^2)^2} + \frac{e\text{Subst}\left(\int \frac{-cd^2+ae^2+2cdex}{a+cx^2}\,dx, x, x^2\right)}{4(cd^2+ae^2)^2}$$

$$= -\frac{d-ex^2}{4(cd^2+ae^2)(a+cx^4)} - \frac{de^2\log(d+ex^2)}{2(cd^2+ae^2)^2} + \frac{(cde^2)\,\text{Subst}\left(\int \frac{x}{a+cx^2}\,dx, x, x^2\right)}{2(cd^2+ae^2)^2} - \frac{(e(cd^2-ae^2))\,\text{Subst}\left(\int \frac{1}{a+cx^2}\,dx, x, x^2\right)}{4(cd^2+ae^2)^2}$$

$$= -\frac{d-ex^2}{4(cd^2+ae^2)(a+cx^4)} - \frac{e(cd^2-ae^2)\tan^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{4\sqrt{a}\sqrt{c}(cd^2+ae^2)^2} - \frac{de^2\log(d+ex^2)}{2(cd^2+ae^2)^2} + \frac{de^2\log(a+cx^4)}{4(cd^2+ae^2)^2}$$

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.77

$$\int \frac{x^3}{(d+ex^2)(a+cx^4)^2}\,dx = \frac{\frac{(cd^2+ae^2)(-d+ex^2)}{a+cx^4} + \frac{e(-cd^2+ae^2)\arctan\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{c}} - 2de^2\log(d+ex^2) + de^2\log(a+cx^4)}{4(cd^2+ae^2)^2}$$

```
[In] Integrate[x^3/((d + e*x^2)*(a + c*x^4)^2),x]
[Out] (((c*d^2 + a*e^2)*(-d + e*x^2))/(a + c*x^4) + (e*(-(c*d^2) + a*e^2)*ArcTan[
(Sqrt[c]*x^2)/Sqrt[a]])/(Sqrt[a]*Sqrt[c]) - 2*d*e^2*Log[d + e*x^2] + d*e^2*
Log[a + c*x^4])/(4*(c*d^2 + a*e^2)^2)
```

Maple [A] (verified)

Time = 0.42 (sec) , antiderivative size = 132, normalized size of antiderivative = 0.89

method	result
default	$\frac{\frac{\left(\frac{1}{2} a\, e^{3}+\frac{1}{2} c\, d^{2} e\right) x^{2}-\frac{d\left(a\, e^{2}+c\, d^{2}\right)}{2}}{c\, x^{4}+a}+\frac{e\left(d e \ln\left(c\, x^{4}+a\right)+\frac{\left(a\, e^{2}-c\, d^{2}\right) \arctan\left(\frac{c\, x^{2}}{\sqrt{a c}}\right)}{\sqrt{a c}}\right)}{2}}{2\left(a\, e^{2}+c\, d^{2}\right)^{2}}-\frac{d\, e^{2} \ln\left(e\, x^{2}+d\right)}{2\left(a\, e^{2}+c\, d^{2}\right)^{2}}$
risch	$\frac{\frac{e\, x^{2}}{4 a\, e^{2}+4 c\, d^{2}}-\frac{d}{4\left(a\, e^{2}+c\, d^{2}\right)}}{c\, x^{4}+a}-\frac{d\, e^{2} \ln\left(e\, x^{2}+d\right)}{2\left(a^{2} e^{4}+2 a c\, d^{2} e^{2}+c^{2} d^{4}\right)}+\frac{\left(\sum_{\_R=\operatorname{RootOf}\left(\left(a^{3} c\, e^{4}+2 a^{2} c^{2} d^{2} e^{2}+a\, c^{3} d^{4}\right)\_Z^{2}-4 a c d\, e^{2}\_Z+e^{2}\right)}\_R \ln\left(\left(\left(2\right.\right.\right.}{}$

[In] int(x^3/(e*x^2+d)/(c*x^4+a)^2,x,method=_RETURNVERBOSE)

[Out] 1/2/(a*e^2+c*d^2)^2*(((1/2*a*e^3+1/2*c*d^2*e)*x^2-1/2*d*(a*e^2+c*d^2))/(c*x^4+a)+1/2*e*(d*e*ln(c*x^4+a)+(a*e^2-c*d^2)/(a*c)^(1/2)*arctan(c*x^2/(a*c)^(1/2))))-1/2*d*e^2*ln(e*x^2+d)/(a*e^2+c*d^2)^2

Fricas [A] (verification not implemented)

none

Time = 1.75 (sec) , antiderivative size = 492, normalized size of antiderivative = 3.30

$$\int \frac{x^3}{(d+ex^2)(a+cx^4)^2}\,dx$$
$$= \left[-\frac{2\,ac^2d^3+2\,a^2cde^2-2\,(ac^2d^2e+a^2ce^3)x^2-(acd^2e-a^2e^3+(c^2d^2e-ace^3)x^4)\sqrt{-ac}\log\left(\frac{cx^4-2\,\sqrt{-ac}x^2-a}{cx^4+a}\right) \cdots}{8\,(a^2c^3d^4+2\,a^3c^2d^2e^2+a^4ce^4+(ac^4d^4+2\,a^2 \cdots}\right.$$
$$\left.-\frac{ac^2d^3+a^2cde^2-(ac^2d^2e+a^2ce^3)x^2-(acd^2e-a^2e^3+(c^2d^2e-ace^3)x^4)\sqrt{ac}\arctan\left(\frac{\sqrt{ac}}{cx^2}\right)-(ac^2de^2x^4 \cdots}{4\,(a^2c^3d^4+2\,a^3c^2d^2e^2+a^4ce^4+(ac^4d^4+2\,a^2c^3d^2e^2+\cdots}\right]$$

[In] integrate(x^3/(e*x^2+d)/(c*x^4+a)^2,x, algorithm="fricas")

[Out] [-1/8*(2*a*c^2*d^3 + 2*a^2*c*d*e^2 - 2*(a*c^2*d^2*e + a^2*c*e^3)*x^2 - (a*c*d^2*e - a^2*e^3 + (c^2*d^2*e - a*c*e^3)*x^4)*sqrt(-a*c)*log((c*x^4 - 2*sqrt(-a*c)*x^2 - a)/(c*x^4 + a)) - 2*(a*c^2*d*e^2*x^4 + a^2*c*d*e^2)*log(c*x^4 + a) + 4*(a*c^2*d*e^2*x^4 + a^2*c*d*e^2)*log(e*x^2 + d))/(a^2*c^3*d^4 + 2*a^3*c^2*d^2*e^2 + a^4*c*e^4 + (a*c^4*d^4 + 2*a^2*c^3*d^2*e^2 + a^3*c^2*e^4)*x^4), -1/4*(a*c^2*d^3 + a^2*c*d*e^2 - (a*c^2*d^2*e + a^2*c*e^3)*x^2 - (a*c*d^2*e - a^2*e^3 + (c^2*d^2*e - a*c*e^3)*x^4)*sqrt(a*c)*arctan(sqrt(a*c)/(c*x^2)) - (a*c^2*d*e^2*x^4 + a^2*c*d*e^2)*log(c*x^4 + a) + 2*(a*c^2*d*e^2*x^4 + a^2*c*d*e^2)*log(e*x^2 + d))/(a^2*c^3*d^4 + 2*a^3*c^2*d^2*e^2 + a^4*c*e^4 + (a*c^4*d^4 + 2*a^2*c^3*d^2*e^2 + a^3*c^2*e^4)*x^4)]

Sympy [F(-1)]

Timed out.

$$\int \frac{x^3}{(d+ex^2)(a+cx^4)^2}\,dx = \text{Timed out}$$

```
[In] integrate(x**3/(e*x**2+d)/(c*x**4+a)**2,x)
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 186, normalized size of antiderivative = 1.25

$$\int \frac{x^3}{(d+ex^2)(a+cx^4)^2}\,dx = \frac{de^2\log(cx^4+a)}{4(c^2d^4+2\,acd^2e^2+a^2e^4)} - \frac{de^2\log(ex^2+d)}{2(c^2d^4+2\,acd^2e^2+a^2e^4)} - \frac{(cd^2e-ae^3)\arctan\left(\frac{cx^2}{\sqrt{ac}}\right)}{4(c^2d^4+2\,acd^2e^2+a^2e^4)\sqrt{ac}} + \frac{ex^2-d}{4((c^2d^2+ace^2)x^4+acd^2+a^2e^2)}$$

```
[In] integrate(x^3/(e*x^2+d)/(c*x^4+a)^2,x, algorithm="maxima")
[Out] 1/4*d*e^2*log(c*x^4 + a)/(c^2*d^4 + 2*a*c*d^2*e^2 + a^2*e^4) - 1/2*d*e^2*lo
g(e*x^2 + d)/(c^2*d^4 + 2*a*c*d^2*e^2 + a^2*e^4) - 1/4*(c*d^2*e - a*e^3)*ar
ctan(c*x^2/sqrt(a*c))/((c^2*d^4 + 2*a*c*d^2*e^2 + a^2*e^4)*sqrt(a*c)) + 1/4
*(e*x^2 - d)/((c^2*d^2 + a*c*e^2)*x^4 + a*c*d^2 + a^2*e^2)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 196, normalized size of antiderivative = 1.32

$$\int \frac{x^3}{(d+ex^2)(a+cx^4)^2}\,dx = -\frac{de^3\log(|ex^2+d|)}{2(c^2d^4e+2\,acd^2e^3+a^2e^5)} + \frac{de^2\log(cx^4+a)}{4(c^2d^4+2\,acd^2e^2+a^2e^4)} - \frac{(cd^2e-ae^3)\arctan\left(\frac{cx^2}{\sqrt{ac}}\right)}{4(c^2d^4+2\,acd^2e^2+a^2e^4)\sqrt{ac}} - \frac{cd^3+ade^2-(cd^2e+ae^3)x^2}{4(cx^4+a)(cd^2+ae^2)^2}$$

```
[In] integrate(x^3/(e*x^2+d)/(c*x^4+a)^2,x, algorithm="giac")
[Out] -1/2*d*e^3*log(abs(e*x^2 + d))/(c^2*d^4*e + 2*a*c*d^2*e^3 + a^2*e^5) + 1/4*
d*e^2*log(c*x^4 + a)/(c^2*d^4 + 2*a*c*d^2*e^2 + a^2*e^4) - 1/4*(c*d^2*e - a
*e^3)*arctan(c*x^2/sqrt(a*c))/((c^2*d^4 + 2*a*c*d^2*e^2 + a^2*e^4)*sqrt(a*c
)) - 1/4*(c*d^3 + a*d*e^2 - (c*d^2*e + a*e^3)*x^2)/((c*x^4 + a)*(c*d^2 + a*
e^2)^2)
```

Mupad [B] (verification not implemented)

Time = 8.24 (sec) , antiderivative size = 527, normalized size of antiderivative = 3.54

$$\int \frac{x^3}{(d+ex^2)(a+cx^4)^2}\,dx$$
$$= \frac{\ln\left(a^4 e^8 \sqrt{-a\,c} + c^4 d^8 \sqrt{-a\,c} + 70\,d^4 e^4 (-a\,c)^{5/2} + c^5 d^8 x^2 + a^4 c\,e^8 x^2 - 36\,a^2 d^2 e^6 (-a\,c)^{3/2} - 36\,c^2 d^6 e^2 \cdots}{a^3 c\,e^4 + 2\,a^2 c^2 d^2 e \cdots}$$
$$- \frac{\frac{d}{4\,(c\,d^2+a\,e^2)} - \frac{e\,x^2}{4\,(c\,d^2+a\,e^2)}}{c\,x^4+a}$$
$$- \frac{\ln\left(c^5 d^8 x^2 - c^4 d^8 \sqrt{-a\,c} - 70\,d^4 e^4 (-a\,c)^{5/2} - a^4 e^8 \sqrt{-a\,c} + a^4 c\,e^8 x^2 + 36\,a^2 d^2 e^6 (-a\,c)^{3/2} + 36\,c^2 d^6 \cdots}{a^3 c\,e^4 + 2\,a^2 c^2 \cdots}$$
$$- \frac{d\,e^2 \ln\left(e\,x^2+d\right)}{2\,\left(a^2 e^4 + 2\,a\,c\,d^2 e^2 + c^2 d^4\right)}$$

```
[In] int(x^3/((a + c*x^4)^2*(d + e*x^2)),x)
```

```
[Out] (log(a^4*e^8*(-a*c)^(1/2) + c^4*d^8*(-a*c)^(1/2) + 70*d^4*e^4*(-a*c)^(5/2)
+ c^5*d^8*x^2 + a^4*c*e^8*x^2 - 36*a^2*d^2*e^6*(-a*c)^(3/2) - 36*c^2*d^6*e^
2*(-a*c)^(3/2) + 70*a^2*c^3*d^4*e^4*x^2 + 36*a^3*c^2*d^2*e^6*x^2 + 36*a*c^4
*d^6*e^2*x^2)*(a*((e^3*(-a*c)^(1/2))/8 + (c*d*e^2)/4) - (c*d^2*e*(-a*c)^(1/
2))/8))/(a*c^3*d^4 + a^3*c*e^4 + 2*a^2*c^2*d^2*e^2) - (d/(4*(a*e^2 + c*d^2)
) - (e*x^2)/(4*(a*e^2 + c*d^2)))/(a + c*x^4) - (log(c^5*d^8*x^2 - c^4*d^8*(
-a*c)^(1/2) - 70*d^4*e^4*(-a*c)^(5/2) - a^4*e^8*(-a*c)^(1/2) + a^4*c*e^8*x^
2 + 36*a^2*d^2*e^6*(-a*c)^(3/2) + 36*c^2*d^6*e^2*(-a*c)^(3/2) + 70*a^2*c^3*
d^4*e^4*x^2 + 36*a^3*c^2*d^2*e^6*x^2 + 36*a*c^4*d^6*e^2*x^2)*(a*((e^3*(-a*c
)^(1/2))/8 - (c*d*e^2)/4) - (c*d^2*e*(-a*c)^(1/2))/8))/(a*c^3*d^4 + a^3*c*e
^4 + 2*a^2*c^2*d^2*e^2) - (d*e^2*log(d + e*x^2))/(2*(a^2*e^4 + c^2*d^4 + 2*
a*c*d^2*e^2))
```

3.248 $\int \frac{x}{(d+ex^2)(a+cx^4)^2}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 151

$$\int \frac{x}{(d+ex^2)(a+cx^4)^2}\,dx = \frac{ae+cdx^2}{4a(cd^2+ae^2)(a+cx^4)} + \frac{\sqrt{c}d(cd^2+3ae^2)\arctan\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{4a^{3/2}(cd^2+ae^2)^2} + \frac{e^3\log(d+ex^2)}{2(cd^2+ae^2)^2} - \frac{e^3\log(a+cx^4)}{4(cd^2+ae^2)^2}$$

```
[Out] 1/4*(c*d*x^2+a*e)/a/(a*e^2+c*d^2)/(c*x^4+a)+1/2*e^3*ln(e*x^2+d)/(a*e^2+c*d^
2)^2-1/4*e^3*ln(c*x^4+a)/(a*e^2+c*d^2)^2+1/4*d*(3*a*e^2+c*d^2)*arctan(x^2*c
^(1/2)/a^(1/2))*c^(1/2)/a^(3/2)/(a*e^2+c*d^2)^2
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {1262, 755, 815, 649, 211, 266}

$$\int \frac{x}{(d+ex^2)(a+cx^4)^2}\,dx = \frac{\sqrt{c}d\arctan\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)(3ae^2+cd^2)}{4a^{3/2}(ae^2+cd^2)^2} + \frac{ae+cdx^2}{4a(a+cx^4)(ae^2+cd^2)} - \frac{e^3\log(a+cx^4)}{4(ae^2+cd^2)^2} + \frac{e^3\log(d+ex^2)}{2(ae^2+cd^2)^2}$$

```
[In] Int[x/((d + e*x^2)*(a + c*x^4)^2),x]
```

```
[Out] (a*e + c*d*x^2)/(4*a*(c*d^2 + a*e^2)*(a + c*x^4)) + (Sqrt[c]*d*(c*d^2 + 3*a
*e^2)*ArcTan[(Sqrt[c]*x^2)/Sqrt[a]])/(4*a^(3/2)*(c*d^2 + a*e^2)^2) + (e^3*L
```

```
og[d + e*x^2])/(2*(c*d^2 + a*e^2)^2) - (e^3*Log[a + c*x^4])/(4*(c*d^2 + a*e
^2)^2)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[(-a)*c]
```

Rule 755

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(-(d + e*x)^(m + 1))*(a*e + c*d*x)*((a + c*x^2)^(p + 1)/(2*a*(p + 1)*(c*d^2
 + a*e^2))), x] + Dist[1/(2*a*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*Sim
p[c*d^2*(2*p + 3) + a*e^2*(m + 2*p + 3) + c*e*d*(m + 2*p + 4)*x, x]*(a + c*
x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, m}, x] && NeQ[c*d^2 + a*e^2, 0]
&& LtQ[p, -1] && IntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 815

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)/(a + c*x^2)), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rule 1262

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol]
 :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + c*x^2)^p, x], x, x^2], x] /; FreeQ
[{a, c, d, e, p, q}, x]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{1}{2}\text{Subst}\left(\int \frac{1}{(d+ex)\,(a+cx^2)^2}\,dx, x, x^2\right)\\ &= \frac{ae+cdx^2}{4a\,(cd^2+ae^2)\,(a+cx^4)} - \frac{\text{Subst}\left(\int \frac{-cd^2-2ae^2-cdex}{(d+ex)(a+cx^2)}\,dx, x, x^2\right)}{4a\,(cd^2+ae^2)}\end{aligned}$$

$$= \frac{ae+cdx^2}{4a(cd^2+ae^2)(a+cx^4)} - \frac{\text{Subst}\left(\int\left(-\frac{2ae^4}{(cd^2+ae^2)(d+ex)} - \frac{c(cd^3+3ade^2-2ae^3x)}{(cd^2+ae^2)(a+cx^2)}\right)dx, x, x^2\right)}{4a(cd^2+ae^2)}$$

$$= \frac{ae+cdx^2}{4a(cd^2+ae^2)(a+cx^4)} + \frac{e^3\log(d+ex^2)}{2(cd^2+ae^2)^2} + \frac{c\text{Subst}\left(\int\frac{cd^3+3ade^2-2ae^3x}{a+cx^2}dx, x, x^2\right)}{4a(cd^2+ae^2)^2}$$

$$= \frac{ae+cdx^2}{4a(cd^2+ae^2)(a+cx^4)} + \frac{e^3\log(d+ex^2)}{2(cd^2+ae^2)^2} - \frac{(ce^3)\text{Subst}\left(\int\frac{x}{a+cx^2}dx, x, x^2\right)}{2(cd^2+ae^2)^2} + \frac{(cd(cd^2+3ae^2))\text{Subst}\left(\int\frac{1}{a+cx^2}dx, x, x^2\right)}{4a(cd^2+ae^2)^2}$$

$$= \frac{ae+cdx^2}{4a(cd^2+ae^2)(a+cx^4)} + \frac{\sqrt{c}d(cd^2+3ae^2)\tan^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{4a^{3/2}(cd^2+ae^2)^2} + \frac{e^3\log(d+ex^2)}{2(cd^2+ae^2)^2} - \frac{e^3\log(a+cx^4)}{4(cd^2+ae^2)^2}$$

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 144, normalized size of antiderivative = 0.95

$$\int\frac{x}{(d+ex^2)(a+cx^4)^2}dx$$
$$= \frac{\sqrt{c}d(cd^2+3ae^2)(a+cx^4)\arctan\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right) + \sqrt{a}((cd^2+ae^2)(ae+cdx^2) + 2ae^3(a+cx^4)\log(d+ex^2) - a}{4a^{3/2}(cd^2+ae^2)^2(a+cx^4)}$$

```
[In] Integrate[x/((d + e*x^2)*(a + c*x^4)^2),x]
```

```
[Out] (Sqrt[c]*d*(c*d^2 + 3*a*e^2)*(a + c*x^4)*ArcTan[(Sqrt[c]*x^2)/Sqrt[a]] + Sq
rt[a]*((c*d^2 + a*e^2)*(a*e + c*d*x^2) + 2*a*e^3*(a + c*x^4)*Log[d + e*x^2]
 - a*e^3*(a + c*x^4)*Log[a + c*x^4]))/(4*a^(3/2)*(c*d^2 + a*e^2)^2*(a + c*x
^4))
```

Maple [A] (verified)

Time = 0.42 (sec) , antiderivative size = 146, normalized size of antiderivative = 0.97

method	result
default	$\frac{c\left(\frac{\frac{d\left(a\,e^{2}+c\,d^{2}\right)x^{2}}{2a}+\frac{e\left(a\,e^{2}+c\,d^{2}\right)}{2c}}{c\,x^{4}+a}+\frac{-\frac{a\,e^{3}\ln\left(c\,x^{4}+a\right)}{c}+\frac{\left(3d\,e^{2}a+d^{3}c\right)\arctan\left(\frac{c\,x^{2}}{\sqrt{ac}}\right)}{\sqrt{ac}}}{2a}\right)}{2(a\,e^{2}+c\,d^{2})^{2}}+\frac{e^{3}\ln(e\,x^{2}+d)}{2(a\,e^{2}+c\,d^{2})^{2}}$
risch	$\frac{\frac{cd\,x^{2}}{4a\left(a\,e^{2}+c\,d^{2}\right)}+\frac{e}{4a\,e^{2}+4c\,d^{2}}}{c\,x^{4}+a}+\frac{e^{3}\ln(e\,x^{2}+d)}{2a^{2}e^{4}+4ac\,d^{2}e^{2}+2c^{2}d^{4}}+\left(\sum_{\_R=\mathrm{RootOf}\left(\left(a^{5}e^{4}+2a^{4}c\,d^{2}e^{2}+a^{3}c^{2}d^{4}\right)\_Z^{2}+4a^{3}e^{3}\_Z+4a\,e^{2}+c\,d^{2}\right)}\_R\ln$

[In] int(x/(e*x^2+d)/(c*x^4+a)^2,x,method=_RETURNVERBOSE)

[Out] 1/2*c/(a*e^2+c*d^2)^2*((1/2*d*(a*e^2+c*d^2)/a*x^2+1/2*e*(a*e^2+c*d^2)/c)/(c*x^4+a)+1/2/a*(-a*e^3/c*ln(c*x^4+a)+(3*a*d*e^2+c*d^3)/(a*c)^(1/2)*arctan(c*x^2/(a*c)^(1/2))))+1/2*e^3*ln(e*x^2+d)/(a*e^2+c*d^2)^2

Fricas [A] (verification not implemented)

none

Time = 4.22 (sec) , antiderivative size = 458, normalized size of antiderivative = 3.03

$$\int \frac{x}{(d+ex^2)(a+cx^4)^2}\,dx = \left[\frac{2\,acd^2e+2\,a^2e^3+2\,(c^2d^3+acde^2)x^2+(acd^3+3\,a^2de^2+(c^2d^3+3\,acde^2)x^4)\sqrt{-\frac{c}{a}}\log\left(\frac{cx^4+2\,ax^2\sqrt{-\frac{c}{a}}-a}{cx^4+a}\right)}{8\,(a^2c^2d^4+2\,a^3cd^2e^2+a^4e^4+(ac^3d^4+2\,a^2c^2d^2e^2}\right.$$

[In] integrate(x/(e*x^2+d)/(c*x^4+a)^2,x, algorithm="fricas")

[Out] [1/8*(2*a*c*d^2*e + 2*a^2*e^3 + 2*(c^2*d^3 + a*c*d*e^2)*x^2 + (a*c*d^3 + 3*a^2*d*e^2 + (c^2*d^3 + 3*a*c*d*e^2)*x^4)*sqrt(-c/a)*log((c*x^4 + 2*a*x^2*sqrt(-c/a) - a)/(c*x^4 + a)) - 2*(a*c*e^3*x^4 + a^2*e^3)*log(c*x^4 + a) + 4*(a*c*e^3*x^4 + a^2*e^3)*log(e*x^2 + d))/(a^2*c^2*d^4 + 2*a^3*c*d^2*e^2 + a^4*e^4 + (a*c^3*d^4 + 2*a^2*c^2*d^2*e^2 + a^3*c*e^4)*x^4), 1/4*(a*c*d^2*e + a^2*e^3 + (c^2*d^3 + a*c*d*e^2)*x^2 - (a*c*d^3 + 3*a^2*d*e^2 + (c^2*d^3 + 3*a*c*d*e^2)*x^4)*sqrt(c/a)*arctan(a*sqrt(c/a)/(c*x^2)) - (a*c*e^3*x^4 + a^2*e^3)*log(c*x^4 + a) + 2*(a*c*e^3*x^4 + a^2*e^3)*log(e*x^2 + d))/(a^2*c^2*d^4 + 2*a^3*c*d^2*e^2 + a^4*e^4 + (a*c^3*d^4 + 2*a^2*c^2*d^2*e^2 + a^3*c*e^4)*x^4)]

Sympy [F(-1)]

Timed out.

$$\int \frac{x}{(d+ex^2)(a+cx^4)^2}\,dx = \text{Timed out}$$

```
[In] integrate(x/(e*x**2+d)/(c*x**4+a)**2,x)
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 196, normalized size of antiderivative = 1.30

$$\int \frac{x}{(d+ex^2)(a+cx^4)^2}\,dx = -\frac{e^3\log(cx^4+a)}{4(c^2d^4+2acd^2e^2+a^2e^4)} + \frac{e^3\log(ex^2+d)}{2(c^2d^4+2acd^2e^2+a^2e^4)} + \frac{(c^2d^3+3acde^2)\arctan\left(\frac{cx^2}{\sqrt{ac}}\right)}{4(ac^2d^4+2a^2cd^2e^2+a^3e^4)\sqrt{ac}} + \frac{cdx^2+ae}{4(a^2cd^2+a^3e^2+(ac^2d^2+a^2ce^2)x^4)}$$

```
[In] integrate(x/(e*x^2+d)/(c*x^4+a)^2,x, algorithm="maxima")
[Out] -1/4*e^3*log(c*x^4 + a)/(c^2*d^4 + 2*a*c*d^2*e^2 + a^2*e^4) + 1/2*e^3*log(e
*x^2 + d)/(c^2*d^4 + 2*a*c*d^2*e^2 + a^2*e^4) + 1/4*(c^2*d^3 + 3*a*c*d*e^2)
*arctan(c*x^2/sqrt(a*c))/((a*c^2*d^4 + 2*a^2*c*d^2*e^2 + a^3*e^4)*sqrt(a*c)
) + 1/4*(c*d*x^2 + a*e)/(a^2*c*d^2 + a^3*e^2 + (a*c^2*d^2 + a^2*c*e^2)*x^4)
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 208, normalized size of antiderivative = 1.38

$$\int \frac{x}{(d+ex^2)(a+cx^4)^2}\,dx = \frac{e^4\log(|ex^2+d|)}{2(c^2d^4e+2acd^2e^3+a^2e^5)} - \frac{e^3\log(cx^4+a)}{4(c^2d^4+2acd^2e^2+a^2e^4)} + \frac{(c^2d^3+3acde^2)\arctan\left(\frac{cx^2}{\sqrt{ac}}\right)}{4(ac^2d^4+2a^2cd^2e^2+a^3e^4)\sqrt{ac}} + \frac{acd^2e+a^2e^3+(c^2d^3+acde^2)x^2}{4(cx^4+a)(cd^2+ae^2)^2a}$$

```
[In] integrate(x/(e*x^2+d)/(c*x^4+a)^2,x, algorithm="giac")
```

```
[Out] 1/2*e^4*log(abs(e*x^2 + d))/(c^2*d^4*e + 2*a*c*d^2*e^3 + a^2*e^5) - 1/4*e^3
*log(c*x^4 + a)/(c^2*d^4 + 2*a*c*d^2*e^2 + a^2*e^4) + 1/4*(c^2*d^3 + 3*a*c*
d*e^2)*arctan(c*x^2/sqrt(a*c))/((a*c^2*d^4 + 2*a^2*c*d^2*e^2 + a^3*e^4)*sqr
t(a*c)) + 1/4*(a*c*d^2*e + a^2*e^3 + (c^2*d^3 + a*c*d*e^2)*x^2)/((c*x^4 + a
)*(c*d^2 + a*e^2)^2*a)
```

Mupad [B] (verification not implemented)

Time = 8.41 (sec) , antiderivative size = 649, normalized size of antiderivative = 4.30

$$\int \frac{x}{(d+ex^2)(a+cx^4)^2}\,dx = \frac{\frac{e}{4\,(c\,d^2+a\,e^2)}+\frac{c\,d\,x^2}{4\,a\,(c\,d^2+a\,e^2)}}{c\,x^4+a}+\frac{e^3\,\ln\left(e\,x^2+d\right)}{2\,\left(a^2\,e^4+2\,a\,c\,d^2\,e^2+c^2\,d^4\right)}$$
$$+\frac{\ln\Big(36\,a^6\,e^{10}\,\sqrt{-a^3\,c}+36\,a^7\,c\,e^{10}\,x^2+a\,c^5\,d^{10}\,\sqrt{-a^3\,c}+a^2\,c^6\,d^{10}\,x^2-81\,a^2\,d^2\,e^8\,\left(-a^3\,c\right)^{3/2}-22\,c^2\,d^6\,e^4\,(}{}$$
$$-\frac{\ln\Big(36\,a^7\,c\,e^{10}\,x^2-36\,a^6\,e^{10}\,\sqrt{-a^3\,c}-a\,c^5\,d^{10}\,\sqrt{-a^3\,c}+a^2\,c^6\,d^{10}\,x^2+81\,a^2\,d^2\,e^8\,\left(-a^3\,c\right)^{3/2}+22\,c^2\,d^6\,e^4\,(}{}$$

```
[In] int(x/((a + c*x^4)^2*(d + e*x^2)),x)
```

```
[Out] (e/(4*(a*e^2 + c*d^2)) + (c*d*x^2)/(4*a*(a*e^2 + c*d^2)))/(a + c*x^4) + (e^
3*log(d + e*x^2))/(2*(a^2*e^4 + c^2*d^4 + 2*a*c*d^2*e^2)) + (log(36*a^6*e^1
0*(-a^3*c)^(1/2) + 36*a^7*c*e^10*x^2 + a*c^5*d^10*(-a^3*c)^(1/2) + a^2*c^6*
d^10*x^2 - 81*a^2*d^2*e^8*(-a^3*c)^(3/2) - 22*c^2*d^6*e^4*(-a^3*c)^(3/2) +
8*a^3*c^5*d^8*e^2*x^2 + 22*a^4*c^4*d^6*e^4*x^2 + 60*a^5*c^3*d^4*e^6*x^2 + 8
1*a^6*c^2*d^2*e^8*x^2 + 8*a^2*c^4*d^8*e^2*(-a^3*c)^(1/2) - 60*a*c*d^4*e^6*(
-a^3*c)^(3/2))*(c*d^3*(-a^3*c)^(1/2) - 2*a^3*e^3 + 3*a*d*e^2*(-a^3*c)^(1/2)
))/(8*(a^5*e^4 + a^3*c^2*d^4 + 2*a^4*c*d^2*e^2)) - (log(36*a^7*c*e^10*x^2 -
 36*a^6*e^10*(-a^3*c)^(1/2) - a*c^5*d^10*(-a^3*c)^(1/2) + a^2*c^6*d^10*x^2
+ 81*a^2*d^2*e^8*(-a^3*c)^(3/2) + 22*c^2*d^6*e^4*(-a^3*c)^(3/2) + 8*a^3*c^5
*d^8*e^2*x^2 + 22*a^4*c^4*d^6*e^4*x^2 + 60*a^5*c^3*d^4*e^6*x^2 + 81*a^6*c^2
*d^2*e^8*x^2 - 8*a^2*c^4*d^8*e^2*(-a^3*c)^(1/2) + 60*a*c*d^4*e^6*(-a^3*c)^(
3/2))*(2*a^3*e^3 + c*d^3*(-a^3*c)^(1/2) + 3*a*d*e^2*(-a^3*c)^(1/2)))/(8*(a^
5*e^4 + a^3*c^2*d^4 + 2*a^4*c*d^2*e^2))
```

3.249 $\int \frac{1}{x(d+ex^2)(a+cx^4)^2} dx$

Optimal result

Integrand size = 22, antiderivative size = 209

$$\int \frac{1}{x(d+ex^2)(a+cx^4)^2} dx = \frac{c(d-ex^2)}{4a(cd^2+ae^2)(a+cx^4)} - \frac{\sqrt{c}e^3\arctan\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{2\sqrt{a}(cd^2+ae^2)^2} - \frac{\sqrt{c}e\arctan\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{4a^{3/2}(cd^2+ae^2)} + \frac{\log(x)}{a^2d} - \frac{e^4\log(d+ex^2)}{2d(cd^2+ae^2)^2} - \frac{cd(cd^2+2ae^2)\log(a+cx^4)}{4a^2(cd^2+ae^2)^2}$$

```
[Out] 1/4*c*(-e*x^2+d)/a/(a*e^2+c*d^2)/(c*x^4+a)+ln(x)/a^2/d-1/2*e^4*ln(e*x^2+d)/
d/(a*e^2+c*d^2)^2-1/4*c*d*(2*a*e^2+c*d^2)*ln(c*x^4+a)/a^2/(a*e^2+c*d^2)^2-1
/4*e*arctan(x^2*c^(1/2)/a^(1/2))*c^(1/2)/a^(3/2)/(a*e^2+c*d^2)-1/2*e^3*arct
an(x^2*c^(1/2)/a^(1/2))*c^(1/2)/(a*e^2+c*d^2)^2/a^(1/2)
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 209, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {1266, 908, 653, 211, 649, 266}

$$\int \frac{1}{x(d+ex^2)(a+cx^4)^2} dx = -\frac{\sqrt{c}e\arctan\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{4a^{3/2}(ae^2+cd^2)} - \frac{cd(2ae^2+cd^2)\log(a+cx^4)}{4a^2(ae^2+cd^2)^2} + \frac{\log(x)}{a^2d} - \frac{\sqrt{c}e^3\arctan\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{2\sqrt{a}(ae^2+cd^2)^2} + \frac{c(d-ex^2)}{4a(a+cx^4)(ae^2+cd^2)} - \frac{e^4\log(d+ex^2)}{2d(ae^2+cd^2)^2}$$

```
[In] Int[1/(x*(d + e*x^2)*(a + c*x^4)^2),x]
```

```
[Out] (c*(d - e*x^2))/(4*a*(c*d^2 + a*e^2)*(a + c*x^4)) - (Sqrt[c]*e^3*ArcTan[(Sq
rt[c]*x^2)/Sqrt[a]])/(2*Sqrt[a]*(c*d^2 + a*e^2)^2) - (Sqrt[c]*e*ArcTan[(Sqr
t[c]*x^2)/Sqrt[a]])/(4*a^(3/2)*(c*d^2 + a*e^2)) + Log[x]/(a^2*d) - (e^4*Log
[d + e*x^2])/(2*d*(c*d^2 + a*e^2)^2) - (c*d*(c*d^2 + 2*a*e^2)*Log[a + c*x^4
])/(4*a^2*(c*d^2 + a*e^2)^2)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[(-a)*c]
```

Rule 653

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((a*e
 - c*d*x)/(2*a*c*(p + 1)))*(a + c*x^2)^(p + 1), x] + Dist[d*((2*p + 3)/(2*a
*(p + 1))), Int[(a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e}, x] && Lt
Q[p, -1] && NeQ[p, -3/2]
```

Rule 908

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && IntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ
[m, 0] && ILtQ[n, 0]))
```

Rule 1266

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_S
ymbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + c*x^2)^p, x],
x, x^2], x] /; FreeQ[{a, c, d, e, p, q}, x] && IntegerQ[(m + 1)/2]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{1}{2}\text{Subst}\left(\int \frac{1}{x(d+ex)\left(a+cx^2\right)^2}\,dx, x, x^2\right)\\
&= \frac{1}{2}\text{Subst}\left(\int \left(\frac{1}{a^2dx} - \frac{e^5}{d\left(cd^2+ae^2\right)^2(d+ex)} - \frac{c(ae+cdx)}{a\left(cd^2+ae^2\right)\left(a+cx^2\right)^2} + \frac{c(-a^2e^3-cd(cd^2+2ae^2)\,x)}{a^2\left(cd^2+ae^2\right)^2\left(a+cx^2\right)}\right) dx, x, x^2\right)\\
&= \frac{\log(x)}{a^2d} - \frac{e^4\log\left(d+ex^2\right)}{2d\left(cd^2+ae^2\right)^2} + \frac{c\text{Subst}\left(\int \frac{-a^2e^3-cd(cd^2+2ae^2)x}{a+cx^2}\,dx, x, x^2\right)}{2a^2\left(cd^2+ae^2\right)^2} - \frac{c\text{Subst}\left(\int \frac{ae+cdx}{(a+cx^2)^2}\,dx, x, x^2\right)}{2a\left(cd^2+ae^2\right)}\\
&= \frac{c(d-ex^2)}{4a\left(cd^2+ae^2\right)\left(a+cx^4\right)} + \frac{\log(x)}{a^2d} - \frac{e^4\log\left(d+ex^2\right)}{2d\left(cd^2+ae^2\right)^2} - \frac{(ce^3)\,\text{Subst}\left(\int \frac{1}{a+cx^2}\,dx, x, x^2\right)}{2\left(cd^2+ae^2\right)^2}\\
&\quad - \frac{(ce)\text{Subst}\left(\int \frac{1}{a+cx^2}\,dx, x, x^2\right)}{4a\left(cd^2+ae^2\right)} - \frac{(c^2d(cd^2+2ae^2))\,\text{Subst}\left(\int \frac{x}{a+cx^2}\,dx, x, x^2\right)}{2a^2\left(cd^2+ae^2\right)^2}\\
&= \frac{c(d-ex^2)}{4a\left(cd^2+ae^2\right)\left(a+cx^4\right)} - \frac{\sqrt{c}e^3\tan^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{2\sqrt{a}\left(cd^2+ae^2\right)^2} - \frac{\sqrt{c}e\tan^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{4a^{3/2}\left(cd^2+ae^2\right)}\\
&\quad + \frac{\log(x)}{a^2d} - \frac{e^4\log\left(d+ex^2\right)}{2d\left(cd^2+ae^2\right)^2} - \frac{cd(cd^2+2ae^2)\log\left(a+cx^4\right)}{4a^2\left(cd^2+ae^2\right)^2}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 241, normalized size of antiderivative = 1.15

$$\int \frac{1}{x\left(d+ex^2\right)\left(a+cx^4\right)^2}\,dx$$

$$= \frac{acd(cd^2+ae^2)\left(d-ex^2\right) + \sqrt{a}\sqrt{c}de(cd^2+3ae^2)\left(a+cx^4\right)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right) + \sqrt{a}\sqrt{c}de(cd^2+3ae^2)\left(\right.}{}$$

```
[In] Integrate[1/(x*(d + e*x^2)*(a + c*x^4)^2),x]
```

```
[Out] (a*c*d*(c*d^2 + a*e^2)*(d - e*x^2) + Sqrt[a]*Sqrt[c]*d*e*(c*d^2 + 3*a*e^2)*
(a + c*x^4)*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)] + Sqrt[a]*Sqrt[c]*d*e*(
c*d^2 + 3*a*e^2)*(a + c*x^4)*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)] + 4*(c
*d^2 + a*e^2)^2*(a + c*x^4)*Log[x] - 2*a^2*e^4*(a + c*x^4)*Log[d + e*x^2] -
 c*d^2*(c*d^2 + 2*a*e^2)*(a + c*x^4)*Log[a + c*x^4])/(4*a^2*d*(c*d^2 + a*e^
2)^2*(a + c*x^4))
```

Maple [A] (verified)

Time = 0.46 (sec) , antiderivative size = 171, normalized size of antiderivative = 0.82

method	result
default	$\frac{\ln(x)}{a^2 d}-\frac{c\left(\frac{\left(\frac{1}{2}e^3 a^2+\frac{1}{2}ac\,d^2 e\right)x^2-\frac{ad\left(a\,e^2+c\,d^2\right)}{2}}{c\,x^4+a}+\frac{\left(4acd\,e^2+2c^2 d^3\right)\ln\left(c\,x^4+a\right)}{4c}+\frac{\left(3e^3 a^2+ac\,d^2 e\right)\arctan\left(\frac{c\,x^2}{\sqrt{ac}}\right)}{2\sqrt{ac}}\right)}{2(a\,e^2+c\,d^2)^2 a^2}-\frac{e^4\ln\left(e\,x^2+d\right)}{2d(a\,e^2+c\,d^2)^2}$
risch	$\frac{-\frac{ec\,x^2}{4a\left(a\,e^2+c\,d^2\right)}+\frac{cd}{4a\left(a\,e^2+c\,d^2\right)}}{c\,x^4+a}+\frac{\ln(x)}{a^2 d}-\frac{e^4\ln\left(e\,x^2+d\right)}{2d(a^2e^4+2ac\,d^2e^2+c^2d^4)}+\frac{\left(\sum_{\_R=\text{RootOf}\left(\left(a^6e^4+2d^2a^5c\,e^2+a^4d^4c^2\right)\_Z^2+\left(8a^3cd\,e^2+4\right.\right.}\right.}{}$

[In] int(1/x/(e*x^2+d)/(c*x^4+a)^2,x,method=_RETURNVERBOSE)

[Out] ln(x)/a^2/d-1/2*c/(a*e^2+c*d^2)^2/a^2*(((1/2*e^3*a^2+1/2*a*c*d^2*e)*x^2-1/2*a*d*(a*e^2+c*d^2))/(c*x^4+a)+1/4*(4*a*c*d*e^2+2*c^2*d^3)/c*ln(c*x^4+a)+1/2*(3*a^2*e^3+a*c*d^2*e)/(a*c)^(1/2)*arctan(c*x^2/(a*c)^(1/2)))-1/2*e^4*ln(e*x^2+d)/d/(a*e^2+c*d^2)^2

Fricas [A] (verification not implemented)

none

Time = 74.98 (sec) , antiderivative size = 686, normalized size of antiderivative = 3.28

$$\int \frac{1}{x\left(d+ex^2\right)\left(a+cx^4\right)^2}\,dx$$

$$=\left[\frac{2\,ac^2d^4+2\,a^2cd^2e^2-2\left(ac^2d^3e+a^2cde^3\right)x^2+\left(a^2cd^3e+3\,a^3de^3+\left(ac^2d^3e+3\,a^2cde^3\right)x^4\right)\sqrt{-\frac{c}{a}}\log\left(\frac{cx^4-2}{\cdots}\right.}{\cdots}\right.$$

[In] integrate(1/x/(e*x^2+d)/(c*x^4+a)^2,x, algorithm="fricas")

[Out] [1/8*(2*a*c^2*d^4 + 2*a^2*c*d^2*e^2 - 2*(a*c^2*d^3*e + a^2*c*d*e^3)*x^2 + (a^2*c*d^3*e + 3*a^3*d*e^3 + (a*c^2*d^3*e + 3*a^2*c*d*e^3)*x^4)*sqrt(-c/a)*log((c*x^4 - 2*a*x^2*sqrt(-c/a) - a)/(c*x^4 + a)) - 2*(a*c^2*d^4 + 2*a^2*c*d^2*e^2 + (c^3*d^4 + 2*a*c^2*d^2*e^2)*x^4)*log(c*x^4 + a) - 4*(a^2*c*e^4*x^4 + a^3*e^4)*log(e*x^2 + d) + 8*(a*c^2*d^4 + 2*a^2*c*d^2*e^2 + a^3*e^4 + (c^3*d^4 + 2*a*c^2*d^2*e^2 + a^2*c*e^4)*x^4)*log(x))/(a^3*c^2*d^5 + 2*a^4*c*d^3*e^2 + a^5*d*e^4 + (a^2*c^3*d^5 + 2*a^3*c^2*d^3*e^2 + a^4*c*d*e^4)*x^4), 1/4*(a*c^2*d^4 + a^2*c*d^2*e^2 - (a*c^2*d^3*e + a^2*c*d*e^3)*x^2 + (a^2*c*d^3*e + 3*a^3*d*e^3 + (a*c^2*d^3*e + 3*a^2*c*d*e^3)*x^4)*sqrt(c/a)*arctan(a*sqrt(c/a)/(c*x^2)) - (a*c^2*d^4 + 2*a^2*c*d^2*e^2 + (c^3*d^4 + 2*a*c^2*d^2*e^2)*x^4)*log(c*x^4 + a) - 2*(a^2*c*e^4*x^4 + a^3*e^4)*log(e*x^2 + d) + 4*(a

```
*c^2*d^4 + 2*a^2*c*d^2*e^2 + a^3*e^4 + (c^3*d^4 + 2*a*c^2*d^2*e^2 + a^2*c*e
^4)*x^4)*log(x))/(a^3*c^2*d^5 + 2*a^4*c*d^3*e^2 + a^5*d*e^4 + (a^2*c^3*d^5
+ 2*a^3*c^2*d^3*e^2 + a^4*c*d*e^4)*x^4)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{x\left(d+ex^2\right)\left(a+cx^4\right)^2}\,dx = \text{Timed out}$$

```
[In] integrate(1/x/(e*x**2+d)/(c*x**4+a)**2,x)
```

```
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 228, normalized size of antiderivative = 1.09

$$\int \frac{1}{x\left(d+ex^2\right)\left(a+cx^4\right)^2}\,dx = -\frac{e^4\log\left(ex^2+d\right)}{2\left(c^2d^5+2\,acd^3e^2+a^2de^4\right)} - \frac{\left(c^2d^3+2\,acde^2\right)\log\left(cx^4+a\right)}{4\left(a^2c^2d^4+2\,a^3cd^2e^2+a^4e^4\right)} - \frac{\left(c^2d^2e+3\,ace^3\right)\arctan\left(\frac{cx^2}{\sqrt{ac}}\right)}{4\left(ac^2d^4+2\,a^2cd^2e^2+a^3e^4\right)\sqrt{ac}} - \frac{cex^2-cd}{4\left(a^2cd^2+a^3e^2+\left(ac^2d^2+a^2ce^2\right)x^4\right)} + \frac{\log\left(x^2\right)}{2\,a^2d}$$

```
[In] integrate(1/x/(e*x^2+d)/(c*x^4+a)^2,x, algorithm="maxima")
```

```
[Out] -1/2*e^4*log(e*x^2 + d)/(c^2*d^5 + 2*a*c*d^3*e^2 + a^2*d*e^4) - 1/4*(c^2*d^
3 + 2*a*c*d*e^2)*log(c*x^4 + a)/(a^2*c^2*d^4 + 2*a^3*c*d^2*e^2 + a^4*e^4) -
 1/4*(c^2*d^2*e + 3*a*c*e^3)*arctan(c*x^2/sqrt(a*c))/((a*c^2*d^4 + 2*a^2*c*
d^2*e^2 + a^3*e^4)*sqrt(a*c)) - 1/4*(c*e*x^2 - c*d)/(a^2*c*d^2 + a^3*e^2 +
(a*c^2*d^2 + a^2*c*e^2)*x^4) + 1/2*log(x^2)/(a^2*d)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 289, normalized size of antiderivative = 1.38

$$\int \frac{1}{x\left(d+ex^2\right)\left(a+cx^4\right)^2}\,dx = -\frac{e^5\log\left(|ex^2+d|\right)}{2\left(c^2d^5e+2\,acd^3e^3+a^2de^5\right)} - \frac{\left(c^2d^3+2\,acde^2\right)\log\left(cx^4+a\right)}{4\left(a^2c^2d^4+2\,a^3cd^2e^2+a^4e^4\right)} - \frac{\left(c^2d^2e+3\,ace^3\right)\arctan\left(\frac{cx^2}{\sqrt{ac}}\right)}{4\left(ac^2d^4+2\,a^2cd^2e^2+a^3e^4\right)\sqrt{ac}} + \frac{c^3d^3x^4+2\,ac^2de^2x^4-ac^2d^2ex^2-a^2ce^3x^2+2\,ac^2d^3+3\,a^2cde^2}{4\left(a^2c^2d^4+2\,a^3cd^2e^2+a^4e^4\right)\left(cx^4+a\right)} + \frac{\log\left(x^2\right)}{2\,a^2d}$$

```
[In] integrate(1/x/(e*x^2+d)/(c*x^4+a)^2,x, algorithm="giac")
```

```
[Out] -1/2*e^5*log(abs(e*x^2 + d))/(c^2*d^5*e + 2*a*c*d^3*e^3 + a^2*d*e^5) - 1/4*
(c^2*d^3 + 2*a*c*d*e^2)*log(c*x^4 + a)/(a^2*c^2*d^4 + 2*a^3*c*d^2*e^2 + a^4
*e^4) - 1/4*(c^2*d^2*e + 3*a*c*e^3)*arctan(c*x^2/sqrt(a*c))/((a*c^2*d^4 + 2
*a^2*c*d^2*e^2 + a^3*e^4)*sqrt(a*c)) + 1/4*(c^3*d^3*x^4 + 2*a*c^2*d*e^2*x^4
 - a*c^2*d^2*e*x^2 - a^2*c*e^3*x^2 + 2*a*c^2*d^3 + 3*a^2*c*d*e^2)/((a^2*c^2
*d^4 + 2*a^3*c*d^2*e^2 + a^4*e^4)*(c*x^4 + a)) + 1/2*log(x^2)/(a^2*d)
```

Mupad [B] (verification not implemented)

Time = 9.05 (sec) , antiderivative size = 1082, normalized size of antiderivative = 5.18

$$\int \frac{1}{x\left(d+ex^2\right)\left(a+cx^4\right)^2}\,dx = \frac{\frac{c\,d}{4\,a\,\left(c\,d^2+a\,e^2\right)}-\frac{c\,e\,x^2}{4\,a\,\left(c\,d^2+a\,e^2\right)}}{c\,x^4+a} - \frac{\ln\left(400\,a^9\,c^{12}\,d^{20}\,x^2-10481\,d^4\,e^{16}\,{\left(-a^5\,c\right)}^{7/2}-1024\,a^{12}\,e^{20}\,{\left(-a^5\,c\right)}^{3/2}+1024\,a^{19}\,c^2\,e^{20}\,x^2-400\,a^2\,c^{10}\,d^{20}\cdots\right.}{\cdots} + \frac{\ln\left(1024\,a^{12}\,e^{20}\,{\left(-a^5\,c\right)}^{3/2}+10481\,d^4\,e^{16}\,{\left(-a^5\,c\right)}^{7/2}+400\,a^9\,c^{12}\,d^{20}\,x^2+1024\,a^{19}\,c^2\,e^{20}\,x^2+400\,a^2\,c^{10}\,d^{20}\cdots\right.}{\cdots} - \frac{e^4\,\ln\left(e\,x^2+d\right)}{2\,a^2\,d\,e^4+4\,a\,c\,d^3\,e^2+2\,c^2\,d^5} + \frac{\ln\left(x\right)}{a^2\,d}$$

```
[In] int(1/(x*(a + c*x^4)^2*(d + e*x^2)),x)
```

```
[Out] ((c*d)/(4*a*(a*e^2 + c*d^2)) - (c*e*x^2)/(4*a*(a*e^2 + c*d^2)))/(a + c*x^4)
 - (log(400*a^9*c^12*d^20*x^2 - 10481*d^4*e^16*(-a^5*c)^(7/2) - 1024*a^12*e
^20*(-a^5*c)^(3/2) + 1024*a^19*c^2*e^20*x^2 - 400*a^2*c^10*d^20*(-a^5*c)^(3
```

/2) + 5840*a^6*d^2*e^18*(-a^5*c)^(5/2) + 33710*c^6*d^14*e^6*(-a^5*c)^(5/2) + 4104*a^10*c^11*d^18*e^2*x^2 + 16689*a^11*c^10*d^16*e^4*x^2 + 33710*a^12*c^9*d^14*e^6*x^2 + 33391*a^13*c^8*d^12*e^8*x^2 + 10748*a^14*c^7*d^10*e^10*x^2 - 3585*a^15*c^6*d^8*e^12*x^2 + 3998*a^16*c^5*d^6*e^14*x^2 + 10481*a^17*c^4*d^4*e^16*x^2 + 5840*a^18*c^3*d^2*e^18*x^2 + 10748*a^2*c^4*d^10*e^10*(-a^5*c)^(5/2) - 3585*a^3*c^3*d^8*e^12*(-a^5*c)^(5/2) + 3998*a^4*c^2*d^6*e^14*(-a^5*c)^(5/2) - 4104*a^3*c^9*d^18*e^2*(-a^5*c)^(3/2) - 16689*a^4*c^8*d^16*e^4*(-a^5*c)^(3/2) + 33391*a*c^5*d^12*e^8*(-a^5*c)^(5/2))*(3*a*e^3*(-a^5*c)^(1/2) + 2*a^2*c^2*d^3 + 4*a^3*c*d*e^2 + c*d^2*e*(-a^5*c)^(1/2)))/(8*(a^6*e^4 + a^4*c^2*d^4 + 2*a^5*c*d^2*e^2)) + (log(1024*a^12*e^20*(-a^5*c)^(3/2) + 10481*d^4*e^16*(-a^5*c)^(7/2) + 400*a^9*c^12*d^20*x^2 + 1024*a^19*c^2*e^20*x^2 + 400*a^2*c^10*d^20*(-a^5*c)^(3/2) - 5840*a^6*d^2*e^18*(-a^5*c)^(5/2) - 33710*c^6*d^14*e^6*(-a^5*c)^(5/2) + 4104*a^10*c^11*d^18*e^2*x^2 + 16689*a^11*c^10*d^16*e^4*x^2 + 33710*a^12*c^9*d^14*e^6*x^2 + 33391*a^13*c^8*d^12*e^8*x^2 + 10748*a^14*c^7*d^10*e^10*x^2 - 3585*a^15*c^6*d^8*e^12*x^2 + 3998*a^16*c^5*d^6*e^14*x^2 + 10481*a^17*c^4*d^4*e^16*x^2 + 5840*a^18*c^3*d^2*e^18*x^2 - 10748*a^2*c^4*d^10*e^10*(-a^5*c)^(5/2) + 3585*a^3*c^3*d^8*e^12*(-a^5*c)^(5/2) - 3998*a^4*c^2*d^6*e^14*(-a^5*c)^(5/2) + 4104*a^3*c^9*d^18*e^2*(-a^5*c)^(3/2) + 16689*a^4*c^8*d^16*e^4*(-a^5*c)^(3/2) - 33391*a*c^5*d^12*e^8*(-a^5*c)^(5/2))*(3*a*e^3*(-a^5*c)^(1/2) - 2*a^2*c^2*d^3 - 4*a^3*c*d*e^2 + c*d^2*e*(-a^5*c)^(1/2)))/(8*(a^6*e^4 + a^4*c^2*d^4 + 2*a^5*c*d^2*e^2)) - (e^4*log(d + e*x^2))/(2*c^2*d^5 + 2*a^2*d*e^4 + 4*a*c*d^3*e^2) + log(x)/(a^2*d)

3.250 $\int \frac{1}{x^3(d+ex^2)(a+cx^4)^2}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 236

$$\int \frac{1}{x^3(d+ex^2)(a+cx^4)^2}\,dx = -\frac{1}{2a^2dx^2} - \frac{c(ae+cdx^2)}{4a^2(cd^2+ae^2)(a+cx^4)} - \frac{c^{3/2}d\arctan\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{4a^{5/2}(cd^2+ae^2)} - \frac{c^{3/2}d(cd^2+2ae^2)\arctan\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{2a^{5/2}(cd^2+ae^2)^2} - \frac{e\log(x)}{a^2d^2} + \frac{e^5\log(d+ex^2)}{2d^2(cd^2+ae^2)^2} + \frac{ce(cd^2+2ae^2)\log(a+cx^4)}{4a^2(cd^2+ae^2)^2}$$

```
[Out] -1/2/a^2/d/x^2-1/4*c*(c*d*x^2+a*e)/a^2/(a*e^2+c*d^2)/(c*x^4+a)-1/4*c^(3/2)*
d*arctan(x^2*c^(1/2)/a^(1/2))/a^(5/2)/(a*e^2+c*d^2)-1/2*c^(3/2)*d*(2*a*e^2+
c*d^2)*arctan(x^2*c^(1/2)/a^(1/2))/a^(5/2)/(a*e^2+c*d^2)^2-e*ln(x)/a^2/d^2+
1/2*e^5*ln(e*x^2+d)/d^2/(a*e^2+c*d^2)^2+1/4*c*e*(2*a*e^2+c*d^2)*ln(c*x^4+a)
/a^2/(a*e^2+c*d^2)^2
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 236, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {1266, 908, 653, 211, 649, 266}

$$\int \frac{1}{x^3(d+ex^2)(a+cx^4)^2}\,dx = -\frac{c^{3/2}d\arctan\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)(2ae^2+cd^2)}{2a^{5/2}(ae^2+cd^2)^2} - \frac{c^{3/2}d\arctan\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{4a^{5/2}(ae^2+cd^2)} + \frac{ce(2ae^2+cd^2)\log(a+cx^4)}{4a^2(ae^2+cd^2)^2} - \frac{c(ae+cdx^2)}{4a^2(a+cx^4)(ae^2+cd^2)} - \frac{e\log(x)}{a^2d^2} - \frac{1}{2a^2dx^2} + \frac{e^5\log(d+ex^2)}{2d^2(ae^2+cd^2)^2}$$

```
[In] Int[1/(x^3*(d + e*x^2)*(a + c*x^4)^2),x]
```

```
[Out] -1/2*1/(a^2*d*x^2) - (c*(a*e + c*d*x^2))/(4*a^2*(c*d^2 + a*e^2)*(a + c*x^4)
) - (c^(3/2)*d*ArcTan[(Sqrt[c]*x^2)/Sqrt[a]])/(4*a^(5/2)*(c*d^2 + a*e^2)) -
 (c^(3/2)*d*(c*d^2 + 2*a*e^2)*ArcTan[(Sqrt[c]*x^2)/Sqrt[a]])/(2*a^(5/2)*(c*
d^2 + a*e^2)^2) - (e*Log[x])/(a^2*d^2) + (e^5*Log[d + e*x^2])/(2*d^2*(c*d^2
 + a*e^2)^2) + (c*e*(c*d^2 + 2*a*e^2)*Log[a + c*x^4])/(4*a^2*(c*d^2 + a*e^2
)^2)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[(-a)*c]
```

Rule 653

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((a*e
 - c*d*x)/(2*a*c*(p + 1)))*(a + c*x^2)^(p + 1), x] + Dist[d*((2*p + 3)/(2*a
*(p + 1))), Int[(a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e}, x] && Lt
Q[p, -1] && NeQ[p, -3/2]
```

Rule 908

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && IntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ
[m, 0] && ILtQ[n, 0]))
```

Rule 1266

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_S
ymbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + c*x^2)^p, x],
x, x^2], x] /; FreeQ[{a, c, d, e, p, q}, x] && IntegerQ[(m + 1)/2]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{1}{2}\text{Subst}\left(\int \frac{1}{x^2(d+ex)\left(a+cx^2\right)^2}\,dx,x,x^2\right)\\
&= \frac{1}{2}\text{Subst}\left(\int \left(\frac{1}{a^2dx^2}-\frac{e}{a^2d^2x}+\frac{e^6}{d^2\left(cd^2+ae^2\right)^2(d+ex)}-\frac{c^2(d-ex)}{a\left(cd^2+ae^2\right)\left(a+cx^2\right)^2}\right.\right.\\
&\qquad\left.\left.-\frac{c^2\left(cd^2+2ae^2\right)(d-ex)}{a^2\left(cd^2+ae^2\right)^2\left(a+cx^2\right)}\right)dx,x,x^2\right)\\
&= -\frac{1}{2a^2dx^2}-\frac{e\log(x)}{a^2d^2}+\frac{e^5\log\left(d+ex^2\right)}{2d^2\left(cd^2+ae^2\right)^2}-\frac{c^2\text{Subst}\left(\int \frac{d-ex}{\left(a+cx^2\right)^2}\,dx,x,x^2\right)}{2a\left(cd^2+ae^2\right)}\\
&\quad-\frac{\left(c^2\left(cd^2+2ae^2\right)\right)\text{Subst}\left(\int \frac{d-ex}{a+cx^2}\,dx,x,x^2\right)}{2a^2\left(cd^2+ae^2\right)^2}\\
&= -\frac{1}{2a^2dx^2}-\frac{c\left(ae+cdx^2\right)}{4a^2\left(cd^2+ae^2\right)\left(a+cx^4\right)}-\frac{e\log(x)}{a^2d^2}+\frac{e^5\log\left(d+ex^2\right)}{2d^2\left(cd^2+ae^2\right)^2}\\
&\quad-\frac{\left(c^2d\right)\text{Subst}\left(\int \frac{1}{a+cx^2}\,dx,x,x^2\right)}{4a^2\left(cd^2+ae^2\right)}-\frac{\left(c^2d\left(cd^2+2ae^2\right)\right)\text{Subst}\left(\int \frac{1}{a+cx^2}\,dx,x,x^2\right)}{2a^2\left(cd^2+ae^2\right)^2}\\
&\quad+\frac{\left(c^2e\left(cd^2+2ae^2\right)\right)\text{Subst}\left(\int \frac{x}{a+cx^2}\,dx,x,x^2\right)}{2a^2\left(cd^2+ae^2\right)^2}\\
&= -\frac{1}{2a^2dx^2}-\frac{c\left(ae+cdx^2\right)}{4a^2\left(cd^2+ae^2\right)\left(a+cx^4\right)}-\frac{c^{3/2}d\tan^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{4a^{5/2}\left(cd^2+ae^2\right)}\\
&\quad-\frac{c^{3/2}d\left(cd^2+2ae^2\right)\tan^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{2a^{5/2}\left(cd^2+ae^2\right)^2}-\frac{e\log(x)}{a^2d^2}\\
&\quad+\frac{e^5\log\left(d+ex^2\right)}{2d^2\left(cd^2+ae^2\right)^2}+\frac{ce\left(cd^2+2ae^2\right)\log\left(a+cx^4\right)}{4a^2\left(cd^2+ae^2\right)^2}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.31 (sec) , antiderivative size = 248, normalized size of antiderivative = 1.05

$$\int \frac{1}{x^3 (d+ex^2) (a+cx^4)^2} \, dx = \frac{1}{4} \left(-\frac{2}{a^2dx^2} - \frac{c(ae+cdx^2)}{a^2 (cd^2+ae^2) (a+cx^4)} + \frac{c^{3/2}d(3cd^2+5ae^2) \arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{a^{5/2} (cd^2+ae^2)^2} + \frac{c^{3/2}d(3cd^2+5ae^2) \arctan\left(1+\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{a^{5/2} (cd^2+ae^2)^2} - \frac{4e\log(x)}{a^2d^2} + \frac{2e^5 \log(d+ex^2)}{(cd^3+ade^2)^2} + \frac{c(cd^2e+2ae^3) \log(a+cx^4)}{a^2 (cd^2+ae^2)^2} \right)$$

```
[In] Integrate[1/(x^3*(d + e*x^2)*(a + c*x^4)^2),x]
```

```
[Out] (-2/(a^2*d*x^2) - (c*(a*e + c*d*x^2))/(a^2*(c*d^2 + a*e^2)*(a + c*x^4)) + (
c^(3/2)*d*(3*c*d^2 + 5*a*e^2)*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(a^(
5/2)*(c*d^2 + a*e^2)^2) + (c^(3/2)*d*(3*c*d^2 + 5*a*e^2)*ArcTan[1 + (Sqrt[2
]*c^(1/4)*x)/a^(1/4)])/(a^(5/2)*(c*d^2 + a*e^2)^2) - (4*e*Log[x])/(a^2*d^2)
 + (2*e^5*Log[d + e*x^2])/(c*d^3 + a*d*e^2)^2 + (c*(c*d^2*e + 2*a*e^3)*Log[
a + c*x^4])/(a^2*(c*d^2 + a*e^2)^2))/4
```

Maple [A] (verified)

Time = 0.49 (sec) , antiderivative size = 181, normalized size of antiderivative = 0.77

method	result
default	$-\frac{1}{2a^2 d\,x^2}-\frac{e\ln(x)}{a^2 d^2}-\frac{c^2\left(\frac{\left(\frac{1}{2}d\,e^2 a+\frac{1}{2}d^3 c\right)x^2+\frac{ae\left(a\,e^2+c\,d^2\right)}{2c}}{c\,x^4+a}+\frac{\left(-4a\,e^3-2c\,d^2 e\right)\ln\left(c\,x^4+a\right)}{4c}+\frac{\left(5d\,e^2 a+3d^3 c\right)\arctan\left(\frac{c\,x^2}{\sqrt{ac}}\right)}{2\sqrt{ac}}\right)}{2(a\,e^2+c\,d^2)^2 a^2}+\frac{e^5 \cdots}{2d^2 \cdots}$
risch	$\frac{-\frac{c\left(2a\,e^2+3c\,d^2\right)x^4}{4d\,a^2\left(a\,e^2+c\,d^2\right)}-\frac{ec\,x^2}{4a\left(a\,e^2+c\,d^2\right)}-\frac{1}{2da}}{x^2(c\,x^4+a)}-\frac{e\ln(x)}{a^2 d^2}+\frac{e^5\ln\left(-e\,x^2-d\right)}{2d^2(a^2e^4+2ac\,d^2e^2+c^2d^4)}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\left(a^7 e^4+2a^6 c\,d^2 e^2+a^5 c^2 d^4\right)\_Z^2 \cdots\right)}\cdots\right)}{\cdots}$

```
[In] int(1/x^3/(e*x^2+d)/(c*x^4+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/2/a^2/d/x^2-e*ln(x)/a^2/d^2-1/2*c^2/(a*e^2+c*d^2)^2/a^2*(((1/2*d*e^2*a+1
/2*d^3*c)*x^2+1/2*a*e*(a*e^2+c*d^2)/c)/(c*x^4+a)+1/4*(-4*a*e^3-2*c*d^2*e)/c
```

```
*ln(c*x^4+a)+1/2*(5*a*d*e^2+3*c*d^3)/(a*c)^(1/2)*arctan(c*x^2/(a*c)^(1/2)))
+1/2*e^5*ln(e*x^2+d)/d^2/(a*e^2+c*d^2)^2
```

Fricas [A] (verification not implemented)

none

Time = 224.20 (sec) , antiderivative size = 852, normalized size of antiderivative = 3.61

$$\int \frac{1}{x^3 (d+ex^2) (a+cx^4)^2} \, dx$$

$$= \left[-\frac{4\,ac^2d^5 + 8\,a^2cd^3e^2 + 4\,a^3de^4 + 2\,(3\,c^3d^5 + 5\,ac^2d^3e^2 + 2\,a^2cde^4)x^4 + 2\,(ac^2d^4e + a^2cd^2e^3)x^2 - ((3\,c^3d^5 \cdots}{\cdots} \right.$$

$$-\frac{2\,ac^2d^5 + 4\,a^2cd^3e^2 + 2\,a^3de^4 + (3\,c^3d^5 + 5\,ac^2d^3e^2 + 2\,a^2cde^4)x^4 + (ac^2d^4e + a^2cd^2e^3)x^2 - ((3\,c^3d^5 + 5\,\cdots}{\cdots}$$

```
[In] integrate(1/x^3/(e*x^2+d)/(c*x^4+a)^2,x, algorithm="fricas")
```

```
[Out] [-1/8*(4*a*c^2*d^5 + 8*a^2*c*d^3*e^2 + 4*a^3*d*e^4 + 2*(3*c^3*d^5 + 5*a*c^2
*d^3*e^2 + 2*a^2*c*d*e^4)*x^4 + 2*(a*c^2*d^4*e + a^2*c*d^2*e^3)*x^2 - ((3*c
^3*d^5 + 5*a*c^2*d^3*e^2)*x^6 + (3*a*c^2*d^5 + 5*a^2*c*d^3*e^2)*x^2)*sqrt(-
c/a)*log((c*x^4 - 2*a*x^2*sqrt(-c/a) - a)/(c*x^4 + a)) - 2*((c^3*d^4*e + 2*
a*c^2*d^2*e^3)*x^6 + (a*c^2*d^4*e + 2*a^2*c*d^2*e^3)*x^2)*log(c*x^4 + a) -
4*(a^2*c*e^5*x^6 + a^3*e^5*x^2)*log(e*x^2 + d) + 8*((c^3*d^4*e + 2*a*c^2*d^
2*e^3 + a^2*c*e^5)*x^6 + (a*c^2*d^4*e + 2*a^2*c*d^2*e^3 + a^3*e^5)*x^2)*log
(x))/((a^2*c^3*d^6 + 2*a^3*c^2*d^4*e^2 + a^4*c*d^2*e^4)*x^6 + (a^3*c^2*d^6
+ 2*a^4*c*d^4*e^2 + a^5*d^2*e^4)*x^2), -1/4*(2*a*c^2*d^5 + 4*a^2*c*d^3*e^2
+ 2*a^3*d*e^4 + (3*c^3*d^5 + 5*a*c^2*d^3*e^2 + 2*a^2*c*d*e^4)*x^4 + (a*c^2*
d^4*e + a^2*c*d^2*e^3)*x^2 - ((3*c^3*d^5 + 5*a*c^2*d^3*e^2)*x^6 + (3*a*c^2*
d^5 + 5*a^2*c*d^3*e^2)*x^2)*sqrt(c/a)*arctan(a*sqrt(c/a)/(c*x^2)) - ((c^3*d
^4*e + 2*a*c^2*d^2*e^3)*x^6 + (a*c^2*d^4*e + 2*a^2*c*d^2*e^3)*x^2)*log(c*x^
4 + a) - 2*(a^2*c*e^5*x^6 + a^3*e^5*x^2)*log(e*x^2 + d) + 4*((c^3*d^4*e + 2
*a*c^2*d^2*e^3 + a^2*c*e^5)*x^6 + (a*c^2*d^4*e + 2*a^2*c*d^2*e^3 + a^3*e^5)
*x^2)*log(x))/((a^2*c^3*d^6 + 2*a^3*c^2*d^4*e^2 + a^4*c*d^2*e^4)*x^6 + (a^3
*c^2*d^6 + 2*a^4*c*d^4*e^2 + a^5*d^2*e^4)*x^2)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{x^3 (d + ex^2) (a + cx^4)^2} \, dx = \text{Timed out}$$

```
[In] integrate(1/x**3/(e*x**2+d)/(c*x**4+a)**2,x)
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 278, normalized size of antiderivative = 1.18

$$\begin{aligned}\int \frac{1}{x^3 (d + ex^2) (a + cx^4)^2} \, dx = & \frac{e^5 \log(ex^2 + d)}{2 (c^2 d^6 + 2 acd^4 e^2 + a^2 d^2 e^4)} + \frac{(c^2 d^2 e + 2 ace^3) \log(cx^4 + a)}{4 (a^2 c^2 d^4 + 2 a^3 cd^2 e^2 + a^4 e^4)} \\ & - \frac{(3 c^3 d^3 + 5 ac^2 de^2) \arctan\left(\frac{cx^2}{\sqrt{ac}}\right)}{4 (a^2 c^2 d^4 + 2 a^3 cd^2 e^2 + a^4 e^4) \sqrt{ac}} \\ & - \frac{acdex^2 + (3 c^2 d^2 + 2 ace^2)x^4 + 2 acd^2 + 2 a^2 e^2}{4 ((a^2 c^2 d^3 + a^3 cde^2)x^6 + (a^3 cd^3 + a^4 de^2)x^2)} - \frac{e \log(x^2)}{2 a^2 d^2}\end{aligned}$$

```
[In] integrate(1/x^3/(e*x^2+d)/(c*x^4+a)^2,x, algorithm="maxima")
[Out] 1/2*e^5*log(e*x^2 + d)/(c^2*d^6 + 2*a*c*d^4*e^2 + a^2*d^2*e^4) + 1/4*(c^2*d
^2*e + 2*a*c*e^3)*log(c*x^4 + a)/(a^2*c^2*d^4 + 2*a^3*c*d^2*e^2 + a^4*e^4)
- 1/4*(3*c^3*d^3 + 5*a*c^2*d*e^2)*arctan(c*x^2/sqrt(a*c))/((a^2*c^2*d^4 + 2
*a^3*c*d^2*e^2 + a^4*e^4)*sqrt(a*c)) - 1/4*(a*c*d*e*x^2 + (3*c^2*d^2 + 2*a*
c*e^2)*x^4 + 2*a*c*d^2 + 2*a^2*e^2)/((a^2*c^2*d^3 + a^3*c*d*e^2)*x^6 + (a^3
*c*d^3 + a^4*d*e^2)*x^2) - 1/2*e*log(x^2)/(a^2*d^2)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 357, normalized size of antiderivative = 1.51

$$\begin{aligned}\int \frac{1}{x^3 (d + ex^2) (a + cx^4)^2} \, dx = & \frac{e^6 \log(|ex^2 + d|)}{2 (c^2 d^6 e + 2 acd^4 e^3 + a^2 d^2 e^5)} \\ & + \frac{(c^2 d^2 e + 2 ace^3) \log(cx^4 + a)}{4 (a^2 c^2 d^4 + 2 a^3 cd^2 e^2 + a^4 e^4)} - \frac{(3 c^3 d^3 + 5 ac^2 de^2) \arctan\left(\frac{cx^2}{\sqrt{ac}}\right)}{4 (a^2 c^2 d^4 + 2 a^3 cd^2 e^2 + a^4 e^4) \sqrt{ac}} \\ & + \frac{2 a^2 ce^5 x^6 - 9 c^3 d^5 x^4 - 15 ac^2 d^3 e^2 x^4 - 6 a^2 cde^4 x^4 - 3 ac^2 d^4 ex^2 - 3 a^2 cd^2 e^3 x^2 + 2 a^3 e^5 x^2 - 6 ac^2 d^5 - 12}{12 (a^2 c^2 d^6 + 2 a^3 cd^4 e^2 + a^4 d^2 e^4)(cx^6 + ax^2)} \\ & - \frac{e \log(x^2)}{2 a^2 d^2}\end{aligned}$$

```
[In] integrate(1/x^3/(e*x^2+d)/(c*x^4+a)^2,x, algorithm="giac")
```

```
[Out] 1/2*e^6*log(abs(e*x^2 + d))/(c^2*d^6*e + 2*a*c*d^4*e^3 + a^2*d^2*e^5) + 1/4
*(c^2*d^2*e + 2*a*c*e^3)*log(c*x^4 + a)/(a^2*c^2*d^4 + 2*a^3*c*d^2*e^2 + a^
4*e^4) - 1/4*(3*c^3*d^3 + 5*a*c^2*d*e^2)*arctan(c*x^2/sqrt(a*c))/((a^2*c^2*
d^4 + 2*a^3*c*d^2*e^2 + a^4*e^4)*sqrt(a*c)) + 1/12*(2*a^2*c*e^5*x^6 - 9*c^3
*d^5*x^4 - 15*a*c^2*d^3*e^2*x^4 - 6*a^2*c*d*e^4*x^4 - 3*a*c^2*d^4*e*x^2 - 3
*a^2*c*d^2*e^3*x^2 + 2*a^3*e^5*x^2 - 6*a*c^2*d^5 - 12*a^2*c*d^3*e^2 - 6*a^3
*d*e^4)/((a^2*c^2*d^6 + 2*a^3*c*d^4*e^2 + a^4*d^2*e^4)*(c*x^6 + a*x^2)) - 1
/2*e*log(x^2)/(a^2*d^2)
```

Mupad [B] (verification not implemented)

Time = 9.14 (sec) , antiderivative size = 1337, normalized size of antiderivative = 5.67

$$\int \frac{1}{x^3 (d+ex^2) (a+cx^4)^2} \, dx = \text{Too large to display}$$

```
[In] int(1/(x^3*(a + c*x^4)^2*(d + e*x^2)),x)
```

```
[Out] (log(81*a^10*c^16*d^24*x^2 + 1024*a^22*c^4*e^24*x^2 - 81*a^3*c^11*d^24*(-a^
5*c^3)^(3/2) + 1024*a^20*c^2*e^24*(-a^5*c^3)^(1/2) - 14496*a^6*d^8*e^16*(-a
^5*c^3)^(5/2) - 5120*a^14*d^2*e^22*(-a^5*c^3)^(3/2) + 11647*c^6*d^20*e^4*(-
a^5*c^3)^(5/2) + 1638*a^11*c^15*d^22*e^2*x^2 + 11647*a^12*c^14*d^20*e^4*x^2
 + 43524*a^13*c^13*d^18*e^6*x^2 + 97311*a^14*c^12*d^16*e^8*x^2 + 133334*a^1
5*c^11*d^14*e^10*x^2 + 103633*a^16*c^10*d^12*e^12*x^2 + 29456*a^17*c^9*d^10
*e^14*x^2 - 14496*a^18*c^8*d^8*e^16*x^2 - 7984*a^19*c^7*d^6*e^18*x^2 + 5888
*a^20*c^6*d^4*e^20*x^2 + 5120*a^21*c^5*d^2*e^22*x^2 + 43524*a*c^5*d^18*e^6*
(-a^5*c^3)^(5/2) + 29456*a^5*c*d^10*e^14*(-a^5*c^3)^(5/2) - 5888*a^13*c*d^4
*e^20*(-a^5*c^3)^(3/2) + 97311*a^2*c^4*d^16*e^8*(-a^5*c^3)^(5/2) + 133334*a
^3*c^3*d^14*e^10*(-a^5*c^3)^(5/2) + 103633*a^4*c^2*d^12*e^12*(-a^5*c^3)^(5/
2) - 1638*a^4*c^10*d^22*e^2*(-a^5*c^3)^(3/2) + 7984*a^12*c^2*d^6*e^18*(-a^5
*c^3)^(3/2))*(4*a^4*c*e^3 - 3*c*d^3*(-a^5*c^3)^(1/2) + 2*a^3*c^2*d^2*e - 5*
a*d*e^2*(-a^5*c^3)^(1/2)))/(8*(a^7*e^4 + a^5*c^2*d^4 + 2*a^6*c*d^2*e^2)) -
(1/(2*a*d) + (c*e*x^2)/(4*a*(a*e^2 + c*d^2)) + (c*x^4*(2*a*e^2 + 3*c*d^2))/
(4*a^2*d*(a*e^2 + c*d^2)))/(a*x^2 + c*x^6) + (log(81*a^10*c^16*d^24*x^2 + 1
024*a^22*c^4*e^24*x^2 + 81*a^3*c^11*d^24*(-a^5*c^3)^(3/2) - 1024*a^20*c^2*e
^24*(-a^5*c^3)^(1/2) + 14496*a^6*d^8*e^16*(-a^5*c^3)^(5/2) + 5120*a^14*d^2*
e^22*(-a^5*c^3)^(3/2) - 11647*c^6*d^20*e^4*(-a^5*c^3)^(5/2) + 1638*a^11*c^1
5*d^22*e^2*x^2 + 11647*a^12*c^14*d^20*e^4*x^2 + 43524*a^13*c^13*d^18*e^6*x^
2 + 97311*a^14*c^12*d^16*e^8*x^2 + 133334*a^15*c^11*d^14*e^10*x^2 + 103633*
a^16*c^10*d^12*e^12*x^2 + 29456*a^17*c^9*d^10*e^14*x^2 - 14496*a^18*c^8*d^8
*e^16*x^2 - 7984*a^19*c^7*d^6*e^18*x^2 + 5888*a^20*c^6*d^4*e^20*x^2 + 5120*
a^21*c^5*d^2*e^22*x^2 - 43524*a*c^5*d^18*e^6*(-a^5*c^3)^(5/2) - 29456*a^5*c
*d^10*e^14*(-a^5*c^3)^(5/2) + 5888*a^13*c*d^4*e^20*(-a^5*c^3)^(3/2) - 97311
```

*a^2*c^4*d^16*e^8*(-a^5*c^3)^(5/2) - 133334*a^3*c^3*d^14*e^10*(-a^5*c^3)^(5/2) - 103633*a^4*c^2*d^12*e^12*(-a^5*c^3)^(5/2) + 1638*a^4*c^10*d^22*e^2*(-a^5*c^3)^(3/2) - 7984*a^12*c^2*d^6*e^18*(-a^5*c^3)^(3/2))*(4*a^4*c*e^3 + 3*c*d^3*(-a^5*c^3)^(1/2) + 2*a^3*c^2*d^2*e + 5*a*d*e^2*(-a^5*c^3)^(1/2)))/(8*(a^7*e^4 + a^5*c^2*d^4 + 2*a^6*c*d^2*e^2)) + (e^5*log(d + e*x^2))/(2*c^2*d^6 + 2*a^2*d^2*e^4 + 4*a*c*d^4*e^2) - (e*log(x))/(a^2*d^2)

3.251 $\int \frac{1}{x^5(d+ex^2)(a+cx^4)^2}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 265

$$\begin{aligned}\int \frac{1}{x^5\left(d+ex^2\right)\left(a+cx^4\right)^2}\,dx =& -\frac{1}{4a^2dx^4}+\frac{e}{2a^2d^2x^2}-\frac{c^2(d-ex^2)}{4a^2\left(cd^2+ae^2\right)\left(a+cx^4\right)} \\ &+\frac{c^{3/2}e\arctan\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{4a^{5/2}\left(cd^2+ae^2\right)}+\frac{c^{3/2}e(cd^2+2ae^2)\arctan\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{2a^{5/2}\left(cd^2+ae^2\right)^2} \\ &-\frac{(2cd^2-ae^2)\log(x)}{a^3d^3}-\frac{e^6\log\left(d+ex^2\right)}{2d^3\left(cd^2+ae^2\right)^2} \\ &+\frac{c^2d(2cd^2+3ae^2)\log\left(a+cx^4\right)}{4a^3\left(cd^2+ae^2\right)^2}\end{aligned}$$

```
[Out] -1/4/a^2/d/x^4+1/2*e/a^2/d^2/x^2-1/4*c^2*(-e*x^2+d)/a^2/(a*e^2+c*d^2)/(c*x^
4+a)+1/4*c^(3/2)*e*arctan(x^2*c^(1/2)/a^(1/2))/a^(5/2)/(a*e^2+c*d^2)+1/2*c^
(3/2)*e*(2*a*e^2+c*d^2)*arctan(x^2*c^(1/2)/a^(1/2))/a^(5/2)/(a*e^2+c*d^2)^2
-(-a*e^2+2*c*d^2)*ln(x)/a^3/d^3-1/2*e^6*ln(e*x^2+d)/d^3/(a*e^2+c*d^2)^2+1/4
*c^2*d*(3*a*e^2+2*c*d^2)*ln(c*x^4+a)/a^3/(a*e^2+c*d^2)^2
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 265, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used

= {1266, 908, 653, 211, 649, 266}

$$\int \frac{1}{x^5 (d+ex^2) (a+cx^4)^2} dx = \frac{c^{3/2} e \arctan\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right) (2ae^2+cd^2)}{2a^{5/2} (ae^2+cd^2)^2} + \frac{c^{3/2} e \arctan\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{4a^{5/2} (ae^2+cd^2)} + \frac{c^2 d(3ae^2+2cd^2) \log(a+cx^4)}{4a^3 (ae^2+cd^2)^2} - \frac{\log(x) (2cd^2-ae^2)}{a^3d^3} - \frac{c^2(d-ex^2)}{4a^2 (a+cx^4) (ae^2+cd^2)} + \frac{e}{2a^2d^2x^2} - \frac{1}{4a^2dx^4} - \frac{e^6 \log(d+ex^2)}{2d^3 (ae^2+cd^2)^2}$$

```
[In] Int[1/(x^5*(d + e*x^2)*(a + c*x^4)^2),x]
```

```
[Out] -1/4*1/(a^2*d*x^4) + e/(2*a^2*d^2*x^2) - (c^2*(d - e*x^2))/(4*a^2*(c*d^2 +
a*e^2)*(a + c*x^4)) + (c^(3/2)*e*ArcTan[(Sqrt[c]*x^2)/Sqrt[a]])/(4*a^(5/2)*
(c*d^2 + a*e^2)) + (c^(3/2)*e*(c*d^2 + 2*a*e^2)*ArcTan[(Sqrt[c]*x^2)/Sqrt[a
]])/(2*a^(5/2)*(c*d^2 + a*e^2)^2) - ((2*c*d^2 - a*e^2)*Log[x])/(a^3*d^3) -
(e^6*Log[d + e*x^2])/(2*d^3*(c*d^2 + a*e^2)^2) + (c^2*d*(2*c*d^2 + 3*a*e^2)
*Log[a + c*x^4])/(4*a^3*(c*d^2 + a*e^2)^2)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[(-a)*c]
```

Rule 653

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((a*e
 - c*d*x)/(2*a*c*(p + 1)))*(a + c*x^2)^(p + 1), x] + Dist[d*((2*p + 3)/(2*a
*(p + 1))), Int[(a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e}, x] && Lt
Q[p, -1] && NeQ[p, -3/2]
```

Rule 908

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && IntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ
[m, 0] && ILtQ[n, 0]))
```

Rule 1266

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_S
ymbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + c*x^2)^p, x],
x, x^2], x] /; FreeQ[{a, c, d, e, p, q}, x] && IntegerQ[(m + 1)/2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{2}\text{Subst}\left(\int \frac{1}{x^3(d+ex)\left(a+cx^2\right)^2}\,dx, x, x^2\right) \\
&= \frac{1}{2}\text{Subst}\left(\int \left(\frac{1}{a^2dx^3} - \frac{e}{a^2d^2x^2} + \frac{-2cd^2+ae^2}{a^3d^3x} - \frac{e^7}{d^3\left(cd^2+ae^2\right)^2(d+ex)} \right.\right. \\
&\quad \left.\left. + \frac{c^2(ae+cdx)}{a^2\left(cd^2+ae^2\right)\left(a+cx^2\right)^2} + \frac{c^2\left(ae\left(cd^2+2ae^2\right)+cd\left(2cd^2+3ae^2\right)x\right)}{a^3\left(cd^2+ae^2\right)^2\left(a+cx^2\right)}\right)dx, x, x^2\right) \\
&= -\frac{1}{4a^2dx^4} + \frac{e}{2a^2d^2x^2} - \frac{\left(2cd^2-ae^2\right)\log(x)}{a^3d^3} - \frac{e^6\log\left(d+ex^2\right)}{2d^3\left(cd^2+ae^2\right)^2} \\
&\quad + \frac{c^2\text{Subst}\left(\int \frac{ae\left(cd^2+2ae^2\right)+cd\left(2cd^2+3ae^2\right)x}{a+cx^2}\,dx, x, x^2\right)}{2a^3\left(cd^2+ae^2\right)^2} + \frac{c^2\text{Subst}\left(\int \frac{ae+cdx}{\left(a+cx^2\right)^2}\,dx, x, x^2\right)}{2a^2\left(cd^2+ae^2\right)} \\
&= -\frac{1}{4a^2dx^4} + \frac{e}{2a^2d^2x^2} - \frac{c^2\left(d-ex^2\right)}{4a^2\left(cd^2+ae^2\right)\left(a+cx^4\right)} - \frac{\left(2cd^2-ae^2\right)\log(x)}{a^3d^3} \\
&\quad - \frac{e^6\log\left(d+ex^2\right)}{2d^3\left(cd^2+ae^2\right)^2} + \frac{\left(c^2e\right)\text{Subst}\left(\int \frac{1}{a+cx^2}\,dx, x, x^2\right)}{4a^2\left(cd^2+ae^2\right)} \\
&\quad + \frac{\left(c^2e\left(cd^2+2ae^2\right)\right)\text{Subst}\left(\int \frac{1}{a+cx^2}\,dx, x, x^2\right)}{2a^2\left(cd^2+ae^2\right)^2} \\
&\quad + \frac{\left(c^3d\left(2cd^2+3ae^2\right)\right)\text{Subst}\left(\int \frac{x}{a+cx^2}\,dx, x, x^2\right)}{2a^3\left(cd^2+ae^2\right)^2} \\
&= -\frac{1}{4a^2dx^4} + \frac{e}{2a^2d^2x^2} - \frac{c^2\left(d-ex^2\right)}{4a^2\left(cd^2+ae^2\right)\left(a+cx^4\right)} + \frac{c^{3/2}e\tan^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{4a^{5/2}\left(cd^2+ae^2\right)} \\
&\quad + \frac{c^{3/2}e\left(cd^2+2ae^2\right)\tan^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{2a^{5/2}\left(cd^2+ae^2\right)^2} - \frac{\left(2cd^2-ae^2\right)\log(x)}{a^3d^3} \\
&\quad - \frac{e^6\log\left(d+ex^2\right)}{2d^3\left(cd^2+ae^2\right)^2} + \frac{c^2d\left(2cd^2+3ae^2\right)\log\left(a+cx^4\right)}{4a^3\left(cd^2+ae^2\right)^2}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.30 (sec) , antiderivative size = 278, normalized size of antiderivative = 1.05

$$\int \frac{1}{x^5 (d+ex^2) (a+cx^4)^2} \, dx = \frac{1}{4} \left(-\frac{1}{a^2dx^4} + \frac{2e}{a^2d^2x^2} + \frac{c^2(-d+ex^2)}{a^2 (cd^2+ae^2) (a+cx^4)} - \frac{c^{3/2}e(3cd^2+5ae^2) \arctan \left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{a^{5/2} (cd^2+ae^2)^2} - \frac{c^{3/2}e(3cd^2+5ae^2) \arctan \left(1+\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{a^{5/2} (cd^2+ae^2)^2} + \frac{4(-2cd^2+ae^2)\log(x)}{a^3d^3} - \frac{2e^6 \log (d+ex^2)}{d^3 (cd^2+ae^2)^2} + \frac{c^2(2cd^3+3ade^2) \log (a+cx^4)}{a^3 (cd^2+ae^2)^2} \right)$$

```
[In] Integrate[1/(x^5*(d + e*x^2)*(a + c*x^4)^2),x]
```

```
[Out] (-(1/(a^2*d*x^4)) + (2*e)/(a^2*d^2*x^2) + (c^2*(-d + e*x^2))/(a^2*(c*d^2 +
a*e^2)*(a + c*x^4)) - (c^(3/2)*e*(3*c*d^2 + 5*a*e^2)*ArcTan[1 - (Sqrt[2]*c^
(1/4)*x)/a^(1/4)])/(a^(5/2)*(c*d^2 + a*e^2)^2) - (c^(3/2)*e*(3*c*d^2 + 5*a*
e^2)*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(a^(5/2)*(c*d^2 + a*e^2)^2) +
 (4*(-2*c*d^2 + a*e^2)*Log[x])/(a^3*d^3) - (2*e^6*Log[d + e*x^2])/(d^3*(c*d
^2 + a*e^2)^2) + (c^2*(2*c*d^3 + 3*a*d*e^2)*Log[a + c*x^4])/(a^3*(c*d^2 + a
*e^2)^2))/4
```

Maple [A] (verified)

Time = 0.48 (sec) , antiderivative size = 209, normalized size of antiderivative = 0.79

method	result
default	$-\frac{1}{4a^2d\,x^4}+\frac{\left(a\,e^2-2c\,d^2\right)\ln(x)}{a^3d^3}+\frac{e}{2a^2d^2x^2}+\frac{c^2\left(\frac{\left(\frac{1}{2}e^3a^2+\frac{1}{2}ac\,d^2e\right)x^2-\frac{ad\left(a\,e^2+c\,d^2\right)}{2}}{c\,x^4+a}+\frac{\left(6acd\,e^2+4c^2d^3\right)\ln\left(c\,x^4+a\right)}{4c}+\frac{\left(5e^3a^2+3a\right.}{}\right.}{2(a\,e^2+c\,d^2)^2a^3}$
risch	$\frac{\frac{ec\left(2a\,e^2+3c\,d^2\right)x^6}{4\left(a\,e^2+c\,d^2\right)a^2d^2}-\frac{c\left(a\,e^2+2c\,d^2\right)x^4}{4d\,a^2\left(a\,e^2+c\,d^2\right)}+\frac{e\,x^2}{2d^2a}-\frac{1}{4da}}{x^4(c\,x^4+a)}+\frac{\ln(x)e^2}{a^2d^3}-\frac{2\ln(x)c}{a^3d}-\frac{e^6\ln\left(e\,x^2+d\right)}{2d^3(a^2e^4+2ac\,d^2e^2+c^2d^4)}+\frac{\left(\_R=\text{RootOf}\left(\left(a^8e^4+2\right.\right.\right.}{}$

```
[In] int(1/x^5/(e*x^2+d)/(c*x^4+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/4/a^2/d/x^4+(a*e^2-2*c*d^2)/a^3/d^3*ln(x)+1/2*e/a^2/d^2/x^2+1/2*c^2/(a*e
^2+c*d^2)^2/a^3*(((1/2*e^3*a^2+1/2*a*c*d^2*e)*x^2-1/2*a*d*(a*e^2+c*d^2))/(c
*x^4+a)+1/4*(6*a*c*d*e^2+4*c^2*d^3)/c*ln(c*x^4+a)+1/2*(5*a^2*e^3+3*a*c*d^2*
e)/(a*c)^(1/2)*arctan(c*x^2/(a*c)^(1/2)))-1/2*e^6*ln(e*x^2+d)/d^3/(a*e^2+c*
d^2)^2
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{x^5\left(d+ex^2\right)\left(a+cx^4\right)^2}\,dx=\text{Timed out}$$

```
[In] integrate(1/x^5/(e*x^2+d)/(c*x^4+a)^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{x^5\left(d+ex^2\right)\left(a+cx^4\right)^2}\,dx=\text{Timed out}$$

```
[In] integrate(1/x**5/(e*x**2+d)/(c*x**4+a)**2,x)
```

```
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 332, normalized size of antiderivative = 1.25

$$\begin{aligned}\int \frac{1}{x^5\left(d+ex^2\right)\left(a+cx^4\right)^2}\,dx &= -\frac{e^6\log\left(ex^2+d\right)}{2\left(c^2d^7+2\,acd^5e^2+a^2d^3e^4\right)}+\frac{\left(2\,c^3d^3+3\,ac^2de^2\right)\log\left(cx^4+a\right)}{4\left(a^3c^2d^4+2\,a^4cd^2e^2+a^5e^4\right)}\\&\quad+\frac{\left(3\,c^3d^2e+5\,ac^2e^3\right)\arctan\left(\frac{cx^2}{\sqrt{ac}}\right)}{4\left(a^2c^2d^4+2\,a^3cd^2e^2+a^4e^4\right)\sqrt{ac}}\\&\quad+\frac{\left(3\,c^2d^2e+2\,ace^3\right)x^6-acd^3-a^2de^2-\left(2\,c^2d^3+acde^2\right)x^4+2\left(acd^2e+a^2e^3\right)x^2}{4\left(\left(a^2c^2d^4+a^3cd^2e^2\right)x^8+\left(a^3cd^4+a^4d^2e^2\right)x^4\right)}\\&\quad-\frac{\left(2\,cd^2-ae^2\right)\log\left(x^2\right)}{2\,a^3d^3}\end{aligned}$$

```
[In] integrate(1/x^5/(e*x^2+d)/(c*x^4+a)^2,x, algorithm="maxima")
```

```
[Out] -1/2*e^6*log(e*x^2 + d)/(c^2*d^7 + 2*a*c*d^5*e^2 + a^2*d^3*e^4) + 1/4*(2*c^
3*d^3 + 3*a*c^2*d*e^2)*log(c*x^4 + a)/(a^3*c^2*d^4 + 2*a^4*c*d^2*e^2 + a^5*
e^4) + 1/4*(3*c^3*d^2*e + 5*a*c^2*e^3)*arctan(c*x^2/sqrt(a*c))/((a^2*c^2*d^
4 + 2*a^3*c*d^2*e^2 + a^4*e^4)*sqrt(a*c)) + 1/4*((3*c^2*d^2*e + 2*a*c*e^3)*
x^6 - a*c*d^3 - a^2*d*e^2 - (2*c^2*d^3 + a*c*d*e^2)*x^4 + 2*(a*c*d^2*e + a^
2*e^3)*x^2)/((a^2*c^2*d^4 + a^3*c*d^2*e^2)*x^8 + (a^3*c*d^4 + a^4*d^2*e^2)*
x^4) - 1/2*(2*c*d^2 - a*e^2)*log(x^2)/(a^3*d^3)
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 361, normalized size of antiderivative = 1.36

$$\begin{aligned}
&\int \frac{1}{x^5\left(d+ex^2\right)\left(a+cx^4\right)^2}\,dx \\
&= -\frac{e^7\log\left(\left|ex^2+d\right|\right)}{2\left(c^2d^7e+2\,acd^5e^3+a^2d^3e^5\right)} + \frac{\left(2\,c^3d^3+3\,ac^2de^2\right)\log\left(cx^4+a\right)}{4\left(a^3c^2d^4+2\,a^4cd^2e^2+a^5e^4\right)} \\
&\quad + \frac{\left(3\,c^3d^2e+5\,ac^2e^3\right)\arctan\left(\frac{cx^2}{\sqrt{ac}}\right)}{4\left(a^2c^2d^4+2\,a^3cd^2e^2+a^4e^4\right)\sqrt{ac}} \\
&\quad - \frac{2\,c^4d^3x^4+3\,ac^3de^2x^4-ac^3d^2ex^2-a^2c^2e^3x^2+3\,ac^3d^3+4\,a^2c^2de^2}{4\left(a^3c^2d^4+2\,a^4cd^2e^2+a^5e^4\right)\left(cx^4+a\right)} \\
&\quad - \frac{\left(2\,cd^2-ae^2\right)\log\left(x^2\right)}{2\,a^3d^3} + \frac{6\,cd^2x^4-3\,ae^2x^4+2\,adex^2-ad^2}{4\,a^3d^3x^4}
\end{aligned}$$

```
[In] integrate(1/x^5/(e*x^2+d)/(c*x^4+a)^2,x, algorithm="giac")
```

```
[Out] -1/2*e^7*log(abs(e*x^2 + d))/(c^2*d^7*e + 2*a*c*d^5*e^3 + a^2*d^3*e^5) + 1/
4*(2*c^3*d^3 + 3*a*c^2*d*e^2)*log(c*x^4 + a)/(a^3*c^2*d^4 + 2*a^4*c*d^2*e^2
 + a^5*e^4) + 1/4*(3*c^3*d^2*e + 5*a*c^2*e^3)*arctan(c*x^2/sqrt(a*c))/((a^2
*c^2*d^4 + 2*a^3*c*d^2*e^2 + a^4*e^4)*sqrt(a*c)) - 1/4*(2*c^4*d^3*x^4 + 3*a
*c^3*d*e^2*x^4 - a*c^3*d^2*e*x^2 - a^2*c^2*e^3*x^2 + 3*a*c^3*d^3 + 4*a^2*c^
2*d*e^2)/((a^3*c^2*d^4 + 2*a^4*c*d^2*e^2 + a^5*e^4)*(c*x^4 + a)) - 1/2*(2*c
*d^2 - a*e^2)*log(x^2)/(a^3*d^3) + 1/4*(6*c*d^2*x^4 - 3*a*e^2*x^4 + 2*a*d*e
*x^2 - a*d^2)/(a^3*d^3*x^4)
```

Mupad [B] (verification not implemented)

Time = 9.49 (sec) , antiderivative size = 1545, normalized size of antiderivative = 5.83

$$\int \frac{1}{x^5 (d+ex^2) (a+cx^4)^2} dx = \text{Too large to display}$$

```
[In] int(1/(x^5*(a + c*x^4)^2*(d + e*x^2)),x)
```

```
[Out] (log(6400*a^13*c^18*d^28*x^2 + 1024*a^27*c^4*e^28*x^2 - 6400*a^3*c^13*d^28*
(-a^7*c^3)^(3/2) + 1024*a^24*c^2*e^28*(-a^7*c^3)^(1/2) - 10688*a^6*d^8*e^20
*(-a^7*c^3)^(5/2) - 2048*a^16*d^2*e^26*(-a^7*c^3)^(3/2) + 536959*c^6*d^20*e
^8*(-a^7*c^3)^(5/2) + 54944*a^14*c^17*d^26*e^2*x^2 + 200881*a^15*c^16*d^24*
e^4*x^2 + 413414*a^16*c^15*d^22*e^6*x^2 + 536959*a^17*c^14*d^20*e^8*x^2 + 4
65092*a^18*c^13*d^18*e^10*x^2 + 256991*a^19*c^12*d^16*e^12*x^2 + 52822*a^20
*c^11*d^14*e^14*x^2 - 37423*a^21*c^10*d^12*e^16*x^2 - 27472*a^22*c^9*d^10*e
^18*x^2 - 10688*a^23*c^8*d^8*e^20*x^2 - 10288*a^24*c^7*d^6*e^22*x^2 - 3584*
a^25*c^6*d^4*e^24*x^2 + 2048*a^26*c^5*d^2*e^26*x^2 + 465092*a*c^5*d^18*e^10
*(-a^7*c^3)^(5/2) - 27472*a^5*c*d^10*e^18*(-a^7*c^3)^(5/2) + 3584*a^15*c*d^
4*e^24*(-a^7*c^3)^(3/2) + 256991*a^2*c^4*d^16*e^12*(-a^7*c^3)^(5/2) + 52822
*a^3*c^3*d^14*e^14*(-a^7*c^3)^(5/2) - 37423*a^4*c^2*d^12*e^16*(-a^7*c^3)^(5
/2) - 54944*a^4*c^12*d^26*e^2*(-a^7*c^3)^(3/2) - 200881*a^5*c^11*d^24*e^4*(
-a^7*c^3)^(3/2) - 413414*a^6*c^10*d^22*e^6*(-a^7*c^3)^(3/2) + 10288*a^14*c^
2*d^6*e^22*(-a^7*c^3)^(3/2))*(4*a^3*c^3*d^3 + 5*a*e^3*(-a^7*c^3)^(1/2) + 6*
a^4*c^2*d*e^2 + 3*c*d^2*e*(-a^7*c^3)^(1/2)))/(8*(a^8*e^4 + a^6*c^2*d^4 + 2*
a^7*c*d^2*e^2)) - (e^6*log(d + e*x^2))/(2*(c^2*d^7 + a^2*d^3*e^4 + 2*a*c*d^
5*e^2)) - (1/(4*a*d) - (e*x^2)/(2*a*d^2) + (x^4*(2*c^2*d^2 + a*c*e^2))/(4*a
^2*d*(a*e^2 + c*d^2)) - (c*e*x^6*(2*a*e^2 + 3*c*d^2))/(4*a^2*d^2*(a*e^2 + c
*d^2)))/(a*x^4 + c*x^8) + (log(6400*a^13*c^18*d^28*x^2 + 1024*a^27*c^4*e^28
*x^2 + 6400*a^3*c^13*d^28*(-a^7*c^3)^(3/2) - 1024*a^24*c^2*e^28*(-a^7*c^3)^
(1/2) + 10688*a^6*d^8*e^20*(-a^7*c^3)^(5/2) + 2048*a^16*d^2*e^26*(-a^7*c^3)
^(3/2) - 536959*c^6*d^20*e^8*(-a^7*c^3)^(5/2) + 54944*a^14*c^17*d^26*e^2*x^
2 + 200881*a^15*c^16*d^24*e^4*x^2 + 413414*a^16*c^15*d^22*e^6*x^2 + 536959*
a^17*c^14*d^20*e^8*x^2 + 465092*a^18*c^13*d^18*e^10*x^2 + 256991*a^19*c^12*
d^16*e^12*x^2 + 52822*a^20*c^11*d^14*e^14*x^2 - 37423*a^21*c^10*d^12*e^16*x
^2 - 27472*a^22*c^9*d^10*e^18*x^2 - 10688*a^23*c^8*d^8*e^20*x^2 - 10288*a^2
4*c^7*d^6*e^22*x^2 - 3584*a^25*c^6*d^4*e^24*x^2 + 2048*a^26*c^5*d^2*e^26*x^
2 - 465092*a*c^5*d^18*e^10*(-a^7*c^3)^(5/2) + 27472*a^5*c*d^10*e^18*(-a^7*c
^3)^(5/2) - 3584*a^15*c*d^4*e^24*(-a^7*c^3)^(3/2) - 256991*a^2*c^4*d^16*e^1
2*(-a^7*c^3)^(5/2) - 52822*a^3*c^3*d^14*e^14*(-a^7*c^3)^(5/2) + 37423*a^4*c
^2*d^12*e^16*(-a^7*c^3)^(5/2) + 54944*a^4*c^12*d^26*e^2*(-a^7*c^3)^(3/2) +
200881*a^5*c^11*d^24*e^4*(-a^7*c^3)^(3/2) + 413414*a^6*c^10*d^22*e^6*(-a^7*
c^3)^(3/2) - 10288*a^14*c^2*d^6*e^22*(-a^7*c^3)^(3/2))*(4*a^3*c^3*d^3 - 5*a
*e^3*(-a^7*c^3)^(1/2) + 6*a^4*c^2*d*e^2 - 3*c*d^2*e*(-a^7*c^3)^(1/2)))/(8*(
a^8*e^4 + a^6*c^2*d^4 + 2*a^7*c*d^2*e^2)) + (log(x)*(a*e^2 - 2*c*d^2))/(a^3
*d^3)
```

3.252 $\int \frac{x^8}{(d+ex^2)(a+cx^4)^2} dx$

Optimal result

Integrand size = 22, antiderivative size = 712

$$\begin{aligned}\int \frac{x^8}{(d+ex^2)(a+cx^4)^2} dx = {} & \frac{dx}{4c(cd^2+ae^2)} - \frac{x^3(ae+cdx^2)}{4c(cd^2+ae^2)(a+cx^4)} + \frac{d^{7/2}\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{e}(cd^2+ae^2)^2} \\
& + \frac{\sqrt[4]{a}d^2(\sqrt{c}d-\sqrt{a}e)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}c^{3/4}(cd^2+ae^2)^2} \\
& + \frac{\sqrt[4]{a}(\sqrt{c}d-3\sqrt{a}e)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{8\sqrt{2}c^{7/4}(cd^2+ae^2)} \\
& - \frac{\sqrt[4]{a}d^2(\sqrt{c}d-\sqrt{a}e)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}c^{3/4}(cd^2+ae^2)^2} \\
& - \frac{\sqrt[4]{a}(\sqrt{c}d-3\sqrt{a}e)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{8\sqrt{2}c^{7/4}(cd^2+ae^2)} \\
& + \frac{\sqrt[4]{a}d^2(\sqrt{c}d+\sqrt{a}e)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)}{4\sqrt{2}c^{3/4}(cd^2+ae^2)^2} \\
& + \frac{\sqrt[4]{a}(\sqrt{c}d+3\sqrt{a}e)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)}{16\sqrt{2}c^{7/4}(cd^2+ae^2)} \\
& - \frac{\sqrt[4]{a}d^2(\sqrt{c}d+\sqrt{a}e)\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)}{4\sqrt{2}c^{3/4}(cd^2+ae^2)^2} \\
& - \frac{\sqrt[4]{a}(\sqrt{c}d+3\sqrt{a}e)\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)}{16\sqrt{2}c^{7/4}(cd^2+ae^2)}\end{aligned}$$

```
[Out] 1/4*d*x/c/(a*e^2+c*d^2)-1/4*x^3*(c*d*x^2+a*e)/c/(a*e^2+c*d^2)/(c*x^4+a)-1/1
6*a^(1/4)*arctan(-1+c^(1/4)*x*2^(1/2)/a^(1/4))*(-3*e*a^(1/2)+d*c^(1/2))/c^(
7/4)/(a*e^2+c*d^2)*2^(1/2)-1/16*a^(1/4)*arctan(1+c^(1/4)*x*2^(1/2)/a^(1/4))
*(-3*e*a^(1/2)+d*c^(1/2))/c^(7/4)/(a*e^2+c*d^2)*2^(1/2)-1/4*a^(1/4)*d^2*arc
tan(-1+c^(1/4)*x*2^(1/2)/a^(1/4))*(-e*a^(1/2)+d*c^(1/2))/c^(3/4)/(a*e^2+c*d
^2)^2*2^(1/2)-1/4*a^(1/4)*d^2*arctan(1+c^(1/4)*x*2^(1/2)/a^(1/4))*(-e*a^(1/
2)+d*c^(1/2))/c^(3/4)/(a*e^2+c*d^2)^2*2^(1/2)+1/8*a^(1/4)*d^2*ln(-a^(1/4)*c
^(1/4)*x*2^(1/2)+a^(1/2)+x^2*c^(1/2))*(e*a^(1/2)+d*c^(1/2))/c^(3/4)/(a*e^2+
c*d^2)^2*2^(1/2)-1/8*a^(1/4)*d^2*ln(a^(1/4)*c^(1/4)*x*2^(1/2)+a^(1/2)+x^2*c
^(1/2))*(e*a^(1/2)+d*c^(1/2))/c^(3/4)/(a*e^2+c*d^2)^2*2^(1/2)+1/32*a^(1/4)*
ln(-a^(1/4)*c^(1/4)*x*2^(1/2)+a^(1/2)+x^2*c^(1/2))*(3*e*a^(1/2)+d*c^(1/2))/
c^(7/4)/(a*e^2+c*d^2)*2^(1/2)-1/32*a^(1/4)*ln(a^(1/4)*c^(1/4)*x*2^(1/2)+a^(
1/2)+x^2*c^(1/2))*(3*e*a^(1/2)+d*c^(1/2))/c^(7/4)/(a*e^2+c*d^2)*2^(1/2)+d^(
7/2)*arctan(x*e^(1/2)/d^(1/2))/(a*e^2+c*d^2)^2/e^(1/2)
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 712, normalized size of antiderivative = 1.00, number of steps used = 24, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules

used = {1328, 1290, 1294, 1182, 1176, 631, 210, 1179, 642, 1302, 211}

$$\int \frac{x^8}{(d+ex^2)(a+cx^4)^2}\,dx = \frac{\sqrt[4]{a}d^2 \arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)(\sqrt{c}d-\sqrt{a}e)}{2\sqrt{2}c^{3/4}(ae^2+cd^2)^2} - \frac{\sqrt[4]{a}d^2 \arctan\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)(\sqrt{c}d-\sqrt{a}e)}{2\sqrt{2}c^{3/4}(ae^2+cd^2)^2} + \frac{\sqrt[4]{a}\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)(\sqrt{c}d-3\sqrt{a}e)}{8\sqrt{2}c^{7/4}(ae^2+cd^2)} - \frac{\sqrt[4]{a}\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)(\sqrt{c}d-3\sqrt{a}e)}{8\sqrt{2}c^{7/4}(ae^2+cd^2)} + \frac{d^{7/2}\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{e}(ae^2+cd^2)^2} + \frac{\sqrt[4]{a}d^2(\sqrt{a}e+\sqrt{c}d)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{4\sqrt{2}c^{3/4}(ae^2+cd^2)^2} - \frac{\sqrt[4]{a}d^2(\sqrt{a}e+\sqrt{c}d)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{4\sqrt{2}c^{3/4}(ae^2+cd^2)^2} + \frac{\sqrt[4]{a}(3\sqrt{a}e+\sqrt{c}d)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{16\sqrt{2}c^{7/4}(ae^2+cd^2)} - \frac{\sqrt[4]{a}(3\sqrt{a}e+\sqrt{c}d)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{16\sqrt{2}c^{7/4}(ae^2+cd^2)} - \frac{x^3(ae+cdx^2)}{4c(a+cx^4)(ae^2+cd^2)} + \frac{dx}{4c(ae^2+cd^2)}$$

```
[In] Int[x^8/((d + e*x^2)*(a + c*x^4)^2),x]
```

```
[Out] (d*x)/(4*c*(c*d^2 + a*e^2)) - (x^3*(a*e + c*d*x^2))/(4*c*(c*d^2 + a*e^2)*(a
 + c*x^4)) + (d^(7/2)*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(Sqrt[e]*(c*d^2 + a*e^2)
^2) + (a^(1/4)*d^2*(Sqrt[c]*d - Sqrt[a]*e)*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a
^(1/4)])/(2*Sqrt[2]*c^(3/4)*(c*d^2 + a*e^2)^2) + (a^(1/4)*(Sqrt[c]*d - 3*Sq
rt[a]*e)*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(8*Sqrt[2]*c^(7/4)*(c*d^2
 + a*e^2)) - (a^(1/4)*d^2*(Sqrt[c]*d - Sqrt[a]*e)*ArcTan[1 + (Sqrt[2]*c^(1/
4)*x)/a^(1/4)])/(2*Sqrt[2]*c^(3/4)*(c*d^2 + a*e^2)^2) - (a^(1/4)*(Sqrt[c]*d
 - 3*Sqrt[a]*e)*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(8*Sqrt[2]*c^(7/4)
*(c*d^2 + a*e^2)) + (a^(1/4)*d^2*(Sqrt[c]*d + Sqrt[a]*e)*Log[Sqrt[a] - Sqrt
[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(4*Sqrt[2]*c^(3/4)*(c*d^2 + a*e^2)^2)
 + (a^(1/4)*(Sqrt[c]*d + 3*Sqrt[a]*e)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)
*x + Sqrt[c]*x^2])/(16*Sqrt[2]*c^(7/4)*(c*d^2 + a*e^2)) - (a^(1/4)*d^2*(Sqr
```

```
t[c]*d + Sqrt[a]*e)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])
/(4*Sqrt[2]*c^(3/4)*(c*d^2 + a*e^2)^2) - (a^(1/4)*(Sqrt[c]*d + 3*Sqrt[a]*e)
*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(16*Sqrt[2]*c^(7/4
)*(c*d^2 + a*e^2))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1182

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
```

```
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[(-a)
*c]
```

Rule 1290

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (c_.)*(x_)^4)^(p_.), x
_Symbol] :> Simp[f*(f*x)^(m - 1)*(a + c*x^4)^(p + 1)*((a*e - c*d*x^2)/(4*a*
c*(p + 1))), x] - Dist[f^2/(4*a*c*(p + 1)), Int[(f*x)^(m - 2)*(a + c*x^4)^(
p + 1)*(a*e*(m - 1) - c*d*(4*p + 4 + m + 1)*x^2), x], x] /; FreeQ[{a, c, d,
 e, f}, x] && LtQ[p, -1] && GtQ[m, 1] && IntegerQ[2*p] && (IntegerQ[p] || I
ntegerQ[m])
```

Rule 1294

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (c_.)*(x_)^4)^(p_), x_
Symbol] :> Simp[e*f*(f*x)^(m - 1)*((a + c*x^4)^(p + 1)/(c*(m + 4*p + 3))),
x] - Dist[f^2/(c*(m + 4*p + 3)), Int[(f*x)^(m - 2)*(a + c*x^4)^p*(a*e*(m -
1) - c*d*(m + 4*p + 3)*x^2), x], x] /; FreeQ[{a, c, d, e, f, p}, x] && GtQ[
m, 1] && NeQ[m + 4*p + 3, 0] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m
])
```

Rule 1302

```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.))/((a_) + (c_.)*(x_)^4),
 x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*((d + e*x^2)^q/(a + c*x^4)), x],
x] /; FreeQ[{a, c, d, e, f, m}, x] && IntegerQ[q] && IntegerQ[m]
```

Rule 1328

```
Int[(((f_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^4)^(p_))/((d_.) + (e_.)*(x_)^2),
 x_Symbol] :> Dist[(-a)*(f^4/(c*d^2 + a*e^2)), Int[(f*x)^(m - 4)*(d - e*x^2
)*(a + c*x^4)^p, x], x] + Dist[d^2*(f^4/(c*d^2 + a*e^2)), Int[(f*x)^(m - 4)
*((a + c*x^4)^(p + 1)/(d + e*x^2)), x], x] /; FreeQ[{a, c, d, e, f}, x] &&
LtQ[p, -1] && GtQ[m, 2]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{a\int \frac{x^4(d-ex^2)}{(a+cx^4)^2}\,dx}{cd^2+ae^2}+\frac{d^2\int \frac{x^4}{(d+ex^2)(a+cx^4)}\,dx}{cd^2+ae^2}\\ &= -\frac{x^3(ae+cdx^2)}{4c\,(cd^2+ae^2)\,(a+cx^4)}-\frac{\int \frac{x^2(-3ae-cdx^2)}{a+cx^4}\,dx}{4c\,(cd^2+ae^2)}+\frac{d^2\int \left(\frac{d^2}{(cd^2+ae^2)(d+ex^2)}-\frac{a(d-ex^2)}{(cd^2+ae^2)(a+cx^4)}\right)dx}{cd^2+ae^2}\end{aligned}$$

$$= \frac{dx}{4c\left(cd^2+ae^2\right)} - \frac{x^3(ae+cdx^2)}{4c\left(cd^2+ae^2\right)\left(a+cx^4\right)} - \frac{\left(ad^2\right)\int\frac{d-ex^2}{a+cx^4}\,dx}{\left(cd^2+ae^2\right)^2} + \frac{d^4\int\frac{1}{d+ex^2}\,dx}{\left(cd^2+ae^2\right)^2} + \frac{\int\frac{-acd+3acex^2}{a+cx^4}\,dx}{4c^2\left(cd^2+ae^2\right)}$$

$$= \frac{dx}{4c\left(cd^2+ae^2\right)} - \frac{x^3(ae+cdx^2)}{4c\left(cd^2+ae^2\right)\left(a+cx^4\right)} + \frac{d^{7/2}\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{e}\left(cd^2+ae^2\right)^2} - \frac{\left(ad^2\left(\frac{\sqrt{c}d}{\sqrt{a}}-e\right)\right)\int\frac{\sqrt{a}\sqrt{c}+cx^2}{a+cx^4}\,dx}{2c\left(cd^2+ae^2\right)^2} - \frac{\left(ad^2\left(\frac{\sqrt{c}d}{\sqrt{a}}+e\right)\right)\int\frac{\sqrt{a}\sqrt{c}-cx^2}{a+cx^4}\,dx}{2c\left(cd^2+ae^2\right)^2} - \frac{\left(\sqrt{a}\left(\sqrt{c}d-3\sqrt{a}e\right)\right)\int\frac{\sqrt{a}\sqrt{c}+cx^2}{a+cx^4}\,dx}{8c^2\left(cd^2+ae^2\right)} - \frac{\left(\sqrt{a}\left(\sqrt{c}d+3\sqrt{a}e\right)\right)\int\frac{\sqrt{a}\sqrt{c}-cx^2}{a+cx^4}\,dx}{8c^2\left(cd^2+ae^2\right)}$$

$$= \frac{dx}{4c\left(cd^2+ae^2\right)} - \frac{x^3(ae+cdx^2)}{4c\left(cd^2+ae^2\right)\left(a+cx^4\right)} + \frac{d^{7/2}\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{e}\left(cd^2+ae^2\right)^2}$$

$$- \frac{\left(ad^2\left(\frac{\sqrt{c}d}{\sqrt{a}}-e\right)\right)\int\frac{1}{\frac{\sqrt{a}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}+x^2}\,dx}{4c\left(cd^2+ae^2\right)^2} - \frac{\left(ad^2\left(\frac{\sqrt{c}d}{\sqrt{a}}-e\right)\right)\int\frac{1}{\frac{\sqrt{a}}{\sqrt{c}}+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}+x^2}\,dx}{4c\left(cd^2+ae^2\right)^2}$$

$$+ \frac{\left(\sqrt[4]{a}d^2\left(\sqrt{c}d+\sqrt{a}e\right)\right)\int\frac{\frac{\sqrt{2}\sqrt[4]{a}}{\sqrt[4]{c}}+2x}{-\frac{\sqrt{a}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}-x^2}\,dx}{4\sqrt{2}c^{3/4}\left(cd^2+ae^2\right)^2}$$

$$+ \frac{\left(\sqrt[4]{a}d^2\left(\sqrt{c}d+\sqrt{a}e\right)\right)\int\frac{\frac{\sqrt{2}\sqrt[4]{a}}{\sqrt[4]{c}}-2x}{-\frac{\sqrt{a}}{\sqrt{c}}+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}-x^2}\,dx}{4\sqrt{2}c^{3/4}\left(cd^2+ae^2\right)^2}$$

$$- \frac{\left(\sqrt{a}\left(\sqrt{c}d-3\sqrt{a}e\right)\right)\int\frac{1}{\frac{\sqrt{a}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}+x^2}\,dx}{16c^2\left(cd^2+ae^2\right)}$$

$$- \frac{\left(\sqrt{a}\left(\sqrt{c}d-3\sqrt{a}e\right)\right)\int\frac{1}{\frac{\sqrt{a}}{\sqrt{c}}+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}+x^2}\,dx}{16c^2\left(cd^2+ae^2\right)}$$

$$+ \frac{\left(\sqrt[4]{a}\left(\sqrt{c}d+3\sqrt{a}e\right)\right)\int\frac{\frac{\sqrt{2}\sqrt[4]{a}}{\sqrt[4]{c}}+2x}{-\frac{\sqrt{a}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}-x^2}\,dx}{16\sqrt{2}c^{7/4}\left(cd^2+ae^2\right)}$$

$$+ \frac{\left(\sqrt[4]{a}\left(\sqrt{c}d+3\sqrt{a}e\right)\right)\int\frac{\frac{\sqrt{2}\sqrt[4]{a}}{\sqrt[4]{c}}-2x}{-\frac{\sqrt{a}}{\sqrt{c}}+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}-x^2}\,dx}{16\sqrt{2}c^{7/4}\left(cd^2+ae^2\right)}$$

$$= \frac{dx}{4c\left(cd^2+ae^2\right)} - \frac{x^3(ae+cdx^2)}{4c\left(cd^2+ae^2\right)\left(a+cx^4\right)} + \frac{d^{7/2}\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{e}\left(cd^2+ae^2\right)^2}$$
$$+ \frac{\sqrt[4]{a}d^2\left(\sqrt{c}d+\sqrt{a}e\right)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)}{4\sqrt{2}c^{3/4}\left(cd^2+ae^2\right)^2}$$
$$+ \frac{\sqrt[4]{a}\left(\sqrt{c}d+3\sqrt{a}e\right)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)}{16\sqrt{2}c^{7/4}\left(cd^2+ae^2\right)}$$
$$- \frac{\sqrt[4]{a}d^2\left(\sqrt{c}d+\sqrt{a}e\right)\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)}{4\sqrt{2}c^{3/4}\left(cd^2+ae^2\right)^2}$$
$$- \frac{\sqrt[4]{a}\left(\sqrt{c}d+3\sqrt{a}e\right)\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)}{16\sqrt{2}c^{7/4}\left(cd^2+ae^2\right)}$$
$$- \frac{\left(a^{3/4}d^2\left(\frac{\sqrt{c}d}{\sqrt{a}}-e\right)\right)\text{Subst}\left(\int\frac{1}{-1-x^2}\,dx,x,1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}c^{3/4}\left(cd^2+ae^2\right)^2}$$
$$+ \frac{\left(a^{3/4}d^2\left(\frac{\sqrt{c}d}{\sqrt{a}}-e\right)\right)\text{Subst}\left(\int\frac{1}{-1-x^2}\,dx,x,1+\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}c^{3/4}\left(cd^2+ae^2\right)^2}$$
$$- \frac{\left(\sqrt[4]{a}\left(\sqrt{c}d-3\sqrt{a}e\right)\right)\text{Subst}\left(\int\frac{1}{-1-x^2}\,dx,x,1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{8\sqrt{2}c^{7/4}\left(cd^2+ae^2\right)}$$
$$+ \frac{\left(\sqrt[4]{a}\left(\sqrt{c}d-3\sqrt{a}e\right)\right)\text{Subst}\left(\int\frac{1}{-1-x^2}\,dx,x,1+\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{8\sqrt{2}c^{7/4}\left(cd^2+ae^2\right)}$$

$$
\begin{aligned}
&= \frac{dx}{4c\left(cd^2+ae^2\right)} - \frac{x^3\left(ae+cdx^2\right)}{4c\left(cd^2+ae^2\right)\left(a+cx^4\right)} + \frac{d^{7/2}\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{e}\left(cd^2+ae^2\right)^2} \\
&\quad + \frac{a^{3/4}d^2\left(\frac{\sqrt{c}d}{\sqrt{a}}-e\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}c^{3/4}\left(cd^2+ae^2\right)^2} + \frac{\sqrt[4]{a}\left(\sqrt{c}d-3\sqrt{a}e\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{8\sqrt{2}c^{7/4}\left(cd^2+ae^2\right)} \\
&\quad - \frac{a^{3/4}d^2\left(\frac{\sqrt{c}d}{\sqrt{a}}-e\right)\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}c^{3/4}\left(cd^2+ae^2\right)^2} - \frac{\sqrt[4]{a}\left(\sqrt{c}d-3\sqrt{a}e\right)\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{8\sqrt{2}c^{7/4}\left(cd^2+ae^2\right)} \\
&\quad + \frac{\sqrt[4]{a}d^2\left(\sqrt{c}d+\sqrt{a}e\right)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)}{4\sqrt{2}c^{3/4}\left(cd^2+ae^2\right)^2} \\
&\quad + \frac{\sqrt[4]{a}\left(\sqrt{c}d+3\sqrt{a}e\right)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)}{16\sqrt{2}c^{7/4}\left(cd^2+ae^2\right)} \\
&\quad - \frac{\sqrt[4]{a}d^2\left(\sqrt{c}d+\sqrt{a}e\right)\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)}{4\sqrt{2}c^{3/4}\left(cd^2+ae^2\right)^2} \\
&\quad - \frac{\sqrt[4]{a}\left(\sqrt{c}d+3\sqrt{a}e\right)\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)}{16\sqrt{2}c^{7/4}\left(cd^2+ae^2\right)}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 431, normalized size of antiderivative = 0.61

$$
\int \frac{x^8}{\left(d+ex^2\right)\left(a+cx^4\right)^2}\,dx
$$

$$
= \frac{\frac{8a\left(cd^2+ae^2\right)x\left(d-ex^2\right)}{c\left(a+cx^4\right)} + \frac{32d^{7/2}\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{e}} - \frac{2\sqrt{2}\sqrt[4]{a}\left(-5c^{3/2}d^3+7\sqrt{a}cd^2e-a\sqrt{c}de^2+3a^{3/2}e^3\right)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{c^{7/4}} + \frac{2\sqrt{2}\sqrt[4]{a}(-\cdots}{\cdots}}{\cdots}
$$

```
[In] Integrate[x^8/((d + e*x^2)*(a + c*x^4)^2),x]
```

```
[Out] ((8*a*(c*d^2 + a*e^2)*x*(d - e*x^2))/(c*(a + c*x^4)) + (32*d^(7/2)*ArcTan[(
Sqrt[e]*x)/Sqrt[d]])/Sqrt[e] - (2*Sqrt[2]*a^(1/4)*(-5*c^(3/2)*d^3 + 7*Sqrt[
a]*c*d^2*e - a*Sqrt[c]*d*e^2 + 3*a^(3/2)*e^3)*ArcTan[1 - (Sqrt[2]*c^(1/4)*x
)/a^(1/4)])/c^(7/4) + (2*Sqrt[2]*a^(1/4)*(-5*c^(3/2)*d^3 + 7*Sqrt[a]*c*d^2*
e - a*Sqrt[c]*d*e^2 + 3*a^(3/2)*e^3)*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)
])/c^(7/4) + (Sqrt[2]*a^(1/4)*(5*c^(3/2)*d^3 + 7*Sqrt[a]*c*d^2*e + a*Sqrt[c
]*d*e^2 + 3*a^(3/2)*e^3)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*
x^2])/c^(7/4) - (Sqrt[2]*a^(1/4)*(5*c^(3/2)*d^3 + 7*Sqrt[a]*c*d^2*e + a*Sqr
t[c]*d*e^2 + 3*a^(3/2)*e^3)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[
c]*x^2])/c^(7/4))/(32*(c*d^2 + a*e^2)^2)
```

Maple [A] (verified)

Time = 0.46 (sec) , antiderivative size = 334, normalized size of antiderivative = 0.47

method	result
default	$-\frac{a\left(\frac{\frac{e\left(a\,e^{2}+c\,d^{2}\right)x^{3}}{4c}-\frac{d\left(a\,e^{2}+c\,d^{2}\right)x}{4c}}{c\,x^{4}+a}+\frac{\frac{\left(d\,e^{2}a+5d^{3}c\right)\left(\frac{a}{c}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x^{2}+\left(\frac{a}{c}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{c}}}{x^{2}-\left(\frac{a}{c}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{c}}}\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{c}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{c}\right)^{\frac{1}{4}}}-1\right)\right)}{8a}+}{4c}\right.}{\left(a\,e^{2}+c\,d^{2}\right)^{2}}$
risch	Expression too large to display

```
[In] int(x^8/(e*x^2+d)/(c*x^4+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -a/(a*e^2+c*d^2)^2*((1/4*e*(a*e^2+c*d^2)/c*x^3-1/4*d*(a*e^2+c*d^2)/c*x)/(c*
x^4+a)+1/4/c*(1/8*(a*d*e^2+5*c*d^3)*(a/c)^(1/4)/a*2^(1/2)*(ln((x^2+(a/c)^(1
/4)*x*2^(1/2)+(a/c)^(1/2))/(x^2-(a/c)^(1/4)*x*2^(1/2)+(a/c)^(1/2)))+2*arcta
n(2^(1/2)/(a/c)^(1/4)*x+1)+2*arctan(2^(1/2)/(a/c)^(1/4)*x-1))+1/8*(-3*a*e^3
-7*c*d^2*e)/c/(a/c)^(1/4)*2^(1/2)*(ln((x^2-(a/c)^(1/4)*x*2^(1/2)+(a/c)^(1/2
))/(x^2+(a/c)^(1/4)*x*2^(1/2)+(a/c)^(1/2)))+2*arctan(2^(1/2)/(a/c)^(1/4)*x+
1)+2*arctan(2^(1/2)/(a/c)^(1/4)*x-1))))+1/(a*e^2+c*d^2)^2*d^4/(e*d)^(1/2)*a
rctan(e*x/(e*d)^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 4918 vs. 2(538) = 1076.

Time = 9.62 (sec) , antiderivative size = 9856, normalized size of antiderivative = 13.84

$$\int \frac{x^8}{(d+ex^2)\,(a+cx^4)^2}\,dx = \text{Too large to display}$$

```
[In] integrate(x^8/(e*x^2+d)/(c*x^4+a)^2,x, algorithm="fricas")
```

```
[Out] Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^8}{(d+ex^2)(a+cx^4)^2}\,dx = \text{Timed out}$$

```
[In] integrate(x**8/(e*x**2+d)/(c*x**4+a)**2,x)
```

```
[Out] Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^8}{(d+ex^2)(a+cx^4)^2}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(x^8/(e*x^2+d)/(c*x^4+a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 599, normalized size of antiderivative = 0.84

$$\int \frac{x^8}{(d+ex^2)(a+cx^4)^2}\,dx = \frac{d^4\arctan\left(\frac{ex}{\sqrt{de}}\right)}{(c^2d^4+2\,acd^2e^2+a^2e^4)\sqrt{de}} - \frac{\left(5\,(ac^3)^{\frac{1}{4}}\,c^3d^3+(ac^3)^{\frac{1}{4}}\,ac^2de^2-7\,(ac^3)^{\frac{3}{4}}\,cd^2e-3\,(ac^3)^{\frac{3}{4}}\,ae^3\right)\arctan\left(\frac{\sqrt{2}\left(2\,x+\sqrt{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{8\left(\sqrt{2}c^6d^4+2\sqrt{2}ac^5d^2e^2+\sqrt{2}a^2c^4e^4\right)} - \frac{\left(5\,(ac^3)^{\frac{1}{4}}\,c^3d^3+(ac^3)^{\frac{1}{4}}\,ac^2de^2-7\,(ac^3)^{\frac{3}{4}}\,cd^2e-3\,(ac^3)^{\frac{3}{4}}\,ae^3\right)\arctan\left(\frac{\sqrt{2}\left(2\,x-\sqrt{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{8\left(\sqrt{2}c^6d^4+2\sqrt{2}ac^5d^2e^2+\sqrt{2}a^2c^4e^4\right)} - \frac{\left(5\,(ac^3)^{\frac{1}{4}}\,c^3d^3+(ac^3)^{\frac{1}{4}}\,ac^2de^2+7\,(ac^3)^{\frac{3}{4}}\,cd^2e+3\,(ac^3)^{\frac{3}{4}}\,ae^3\right)\log\left(x^2+\sqrt{2}x\left(\frac{a}{c}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{c}}\right)}{16\left(\sqrt{2}c^6d^4+2\sqrt{2}ac^5d^2e^2+\sqrt{2}a^2c^4e^4\right)} + \frac{\left(5\,(ac^3)^{\frac{1}{4}}\,c^3d^3+(ac^3)^{\frac{1}{4}}\,ac^2de^2+7\,(ac^3)^{\frac{3}{4}}\,cd^2e+3\,(ac^3)^{\frac{3}{4}}\,ae^3\right)\log\left(x^2-\sqrt{2}x\left(\frac{a}{c}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{c}}\right)}{16\left(\sqrt{2}c^6d^4+2\sqrt{2}ac^5d^2e^2+\sqrt{2}a^2c^4e^4\right)} - \frac{aex^3-adx}{4\,(cx^4+a)(c^2d^2+ace^2)}$$

```
[In] integrate(x^8/(e*x^2+d)/(c*x^4+a)^2,x, algorithm="giac")
```

```
[Out] d^4*arctan(e*x/sqrt(d*e))/((c^2*d^4 + 2*a*c*d^2*e^2 + a^2*e^4)*sqrt(d*e)) -
 1/8*(5*(a*c^3)^(1/4)*c^3*d^3 + (a*c^3)^(1/4)*a*c^2*d*e^2 - 7*(a*c^3)^(3/4)
*c*d^2*e - 3*(a*c^3)^(3/4)*a*e^3)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(a/c)^(
1/4))/(a/c)^(1/4))/(sqrt(2)*c^6*d^4 + 2*sqrt(2)*a*c^5*d^2*e^2 + sqrt(2)*a^2
*c^4*e^4) - 1/8*(5*(a*c^3)^(1/4)*c^3*d^3 + (a*c^3)^(1/4)*a*c^2*d*e^2 - 7*(a
*c^3)^(3/4)*c*d^2*e - 3*(a*c^3)^(3/4)*a*e^3)*arctan(1/2*sqrt(2)*(2*x - sqrt
(2)*(a/c)^(1/4))/(a/c)^(1/4))/(sqrt(2)*c^6*d^4 + 2*sqrt(2)*a*c^5*d^2*e^2 +
sqrt(2)*a^2*c^4*e^4) - 1/16*(5*(a*c^3)^(1/4)*c^3*d^3 + (a*c^3)^(1/4)*a*c^2*
d*e^2 + 7*(a*c^3)^(3/4)*c*d^2*e + 3*(a*c^3)^(3/4)*a*e^3)*log(x^2 + sqrt(2)*
x*(a/c)^(1/4) + sqrt(a/c))/(sqrt(2)*c^6*d^4 + 2*sqrt(2)*a*c^5*d^2*e^2 + sqr
t(2)*a^2*c^4*e^4) + 1/16*(5*(a*c^3)^(1/4)*c^3*d^3 + (a*c^3)^(1/4)*a*c^2*d*e
^2 + 7*(a*c^3)^(3/4)*c*d^2*e + 3*(a*c^3)^(3/4)*a*e^3)*log(x^2 - sqrt(2)*x*(
a/c)^(1/4) + sqrt(a/c))/(sqrt(2)*c^6*d^4 + 2*sqrt(2)*a*c^5*d^2*e^2 + sqrt(2
)*a^2*c^4*e^4) - 1/4*(a*e*x^3 - a*d*x)/((c*x^4 + a)*(c^2*d^2 + a*c*e^2))
```

Mupad [B] (verification not implemented)

Time = 9.06 (sec) , antiderivative size = 18343, normalized size of antiderivative = 25.76

$$\int \frac{x^8}{(d+ex^2)(a+cx^4)^2}\,dx = \text{Too large to display}$$

[In] int(x^8/((a + c*x^4)^2*(d + e*x^2)),x)

[Out] ((a*d*x)/(4*c*(a*e^2 + c*d^2)) - (a*e*x^3)/(4*c*(a*e^2 + c*d^2)))/(a + c*x^4) + atan(((((5120*a^2*c^8*d^13*e + 432*a^8*c^2*d*e^13 - 17232*a^3*c^7*d^11*e^3 - 37776*a^4*c^6*d^9*e^5 - 13600*a^5*c^5*d^7*e^7 + 4320*a^6*c^4*d^5*e^9 + 2928*a^7*c^3*d^3*e^11)/(256*(c^7*d^8 + a^4*c^3*e^8 + 4*a*c^6*d^6*e^2 + 6*a^2*c^5*d^4*e^4 + 4*a^3*c^4*d^2*e^6)) - (((81920*a^5*c^9*d^8*e^8 - 73728*a^3*c^11*d^12*e^4 - 61440*a^4*c^10*d^10*e^6 - 20480*a^2*c^12*d^14*e^2 + 184320*a^6*c^8*d^6*e^10 + 122880*a^7*c^7*d^4*e^12 + 28672*a^8*c^6*d^2*e^14)/(256*(c^7*d^8 + a^4*c^3*e^8 + 4*a*c^6*d^6*e^2 + 6*a^2*c^5*d^4*e^4 + 4*a^3*c^4*d^2*e^6)) - (x*((25*c^3*d^6*(-a*c^7)^(1/2) - 9*a^3*e^6*(-a*c^7)^(1/2) + 6*a^3*c^4*d*e^5 + 44*a^2*c^5*d^3*e^3 + 70*a*c^6*d^5*e - 39*a*c^2*d^4*e^2*(-a*c^7)^(1/2) - 41*a^2*c*d^2*e^4*(-a*c^7)^(1/2))/(256*(c^11*d^8 + a^4*c^7*e^8 + 4*a*c^10*d^6*e^2 + 6*a^2*c^9*d^4*e^4 + 4*a^3*c^8*d^2*e^6)))^(1/2)*(65536*a^9*c^7*e^17 - 65536*a^2*c^14*d^14*e^3 - 327680*a^3*c^13*d^12*e^5 - 589824*a^4*c^12*d^10*e^7 - 327680*a^5*c^11*d^8*e^9 + 327680*a^6*c^10*d^6*e^11 + 589824*a^7*c^9*d^4*e^13 + 327680*a^8*c^8*d^2*e^15))/(128*(c^7*d^8 + a^4*c^3*e^8 + 4*a*c^6*d^6*e^2 + 6*a^2*c^5*d^4*e^4 + 4*a^3*c^4*d^2*e^6)))*((25*c^3*d^6*(-a*c^7)^(1/2) - 9*a^3*e^6*(-a*c^7)^(1/2) + 6*a^3*c^4*d*e^5 + 44*a^2*c^5*d^3*e^3 + 70*a*c^6*d^5*e - 39*a*c^2*d^4*e^2*(-a*c^7)^(1/2) - 41*a^2*c*d^2*e^4*(-a*c^7)^(1/2))/(256*(c^11*d^8 + a^4*c^7*e^8 + 4*a*c^10*d^6*e^2 + 6*a^2*c^9*d^4*e^4 + 4*a^3*c^8*d^2*e^6)))^(1/2) + (x*(1920*a^8*c^4*d*e^14 + 13184*a^2*c^10*d^13*e^2 + 16640*a^3*c^9*d^11*e^4 + 18560*a^4*c^8*d^9*e^6 + 56832*a^5*c^7*d^7*e^8 + 60544*a^6*c^6*d^5*e^10 + 20736*a^7*c^5*d^3*e^12))/(128*(c^7*d^8 + a^4*c^3*e^8 + 4*a*c^6*d^6*e^2 + 6*a^2*c^5*d^4*e^4 + 4*a^3*c^4*d^2*e^6)))*((25*c^3*d^6*(-a*c^7)^(1/2) - 9*a^3*e^6*(-a*c^7)^(1/2) + 6*a^3*c^4*d*e^5 + 44*a^2*c^5*d^3*e^3 + 70*a*c^6*d^5*e - 39*a*c^2*d^4*e^2*(-a*c^7)^(1/2) - 41*a^2*c*d^2*e^4*(-a*c^7)^(1/2))/(256*(c^11*d^8 + a^4*c^7*e^8 + 4*a*c^10*d^6*e^2 + 6*a^2*c^9*d^4*e^4 + 4*a^3*c^8*d^2*e^6)))^(1/2))*((25*c^3*d^6*(-a*c^7)^(1/2) - 9*a^3*e^6*(-a*c^7)^(1/2) + 6*a^3*c^4*d*e^5 + 44*a^2*c^5*d^3*e^3 + 70*a*c^6*d^5*e - 39*a*c^2*d^4*e^2*(-a*c^7)^(1/2) - 41*a^2*c*d^2*e^4*(-a*c^7)^(1/2))/(256*(c^11*d^8 + a^4*c^7*e^8 + 4*a*c^10*d^6*e^2 + 6*a^2*c^9*d^4*e^4 + 4*a^3*c^8*d^2*e^6)))^(1/2) + (x*(81*a^8*e^13 + 800*a^2*c^6*d^12*e + 612*a^7*c*d^2*e^11 + 832*a^3*c^5*d^10*e^3 + 913*a^4*c^4*d^8*e^5 + 1700*a^5*c^3*d^6*e^7 + 1606*a^6*c^2*d^4*e^9))/(128*(c^7*d^8 + a^4*c^3*e^8 + 4*a*c^6*d^6*e^2 + 6*a^2*c^5*d^4*e^4 + 4*a^3*c^4*d^2*e^6)))*((25*c^3*d^6*(-a*c^7)^(1/2) - 9*a^3*e^6*(-a*c^7)^(1/2) + 6*a^3*c^4*d*e^5 + 44*a^2*c^5*d^3*e^3 + 70*a*c^6*d^5*e - 39*a*c^2*d^4*e^2*(-a*c^7)^(1/2) - 41*a^2*c*d^2*e^4*(-a*c^7)

^(1/2))/(256*(c^11*d^8 + a^4*c^7*e^8 + 4*a*c^10*d^6*e^2 + 6*a^2*c^9*d^4*e^4 + 4*a^3*c^8*d^2*e^6)))^(1/2)*1i - (((5120*a^2*c^8*d^13*e + 432*a^8*c^2*d*e^13 - 17232*a^3*c^7*d^11*e^3 - 37776*a^4*c^6*d^9*e^5 - 13600*a^5*c^5*d^7*e^7 + 4320*a^6*c^4*d^5*e^9 + 2928*a^7*c^3*d^3*e^11)/(256*(c^7*d^8 + a^4*c^3*e^8 + 4*a*c^6*d^6*e^2 + 6*a^2*c^5*d^4*e^4 + 4*a^3*c^4*d^2*e^6)) - (((81920*a^5*c^9*d^8*e^8 - 73728*a^3*c^11*d^12*e^4 - 61440*a^4*c^10*d^10*e^6 - 20480*a^2*c^12*d^14*e^2 + 184320*a^6*c^8*d^6*e^10 + 122880*a^7*c^7*d^4*e^12 + 28672*a^8*c^6*d^2*e^14)/(256*(c^7*d^8 + a^4*c^3*e^8 + 4*a*c^6*d^6*e^2 + 6*a^2*c^5*d^4*e^4 + 4*a^3*c^4*d^2*e^6)) + (x*((25*c^3*d^6*(-a*c^7)^(1/2) - 9*a^3*e^6*(-a*c^7)^(1/2) + 6*a^3*c^4*d*e^5 + 44*a^2*c^5*d^3*e^3 + 70*a*c^6*d^5*e - 39*a*c^2*d^4*e^2*(-a*c^7)^(1/2) - 41*a^2*c*d^2*e^4*(-a*c^7)^(1/2))/(256*(c^11*d^8 + a^4*c^7*e^8 + 4*a*c^10*d^6*e^2 + 6*a^2*c^9*d^4*e^4 + 4*a^3*c^8*d^2*e^6)))^(1/2)*(65536*a^9*c^7*e^17 - 65536*a^2*c^14*d^14*e^3 - 327680*a^3*c^13*d^12*e^5 - 589824*a^4*c^12*d^10*e^7 - 327680*a^5*c^11*d^8*e^9 + 327680*a^6*c^10*d^6*e^11 + 589824*a^7*c^9*d^4*e^13 + 327680*a^8*c^8*d^2*e^15))/(128*(c^7*d^8 + a^4*c^3*e^8 + 4*a*c^6*d^6*e^2 + 6*a^2*c^5*d^4*e^4 + 4*a^3*c^4*d^2*e^6)))*((25*c^3*d^6*(-a*c^7)^(1/2) - 9*a^3*e^6*(-a*c^7)^(1/2) + 6*a^3*c^4*d*e^5 + 44*a^2*c^5*d^3*e^3 + 70*a*c^6*d^5*e - 39*a*c^2*d^4*e^2*(-a*c^7)^(1/2) - 41*a^2*c*d^2*e^4*(-a*c^7)^(1/2))/(256*(c^11*d^8 + a^4*c^7*e^8 + 4*a*c^10*d^6*e^2 + 6*a^2*c^9*d^4*e^4 + 4*a^3*c^8*d^2*e^6)))^(1/2) - (x*(1920*a^8*c^4*d*e^14 + 13184*a^2*c^10*d^13*e^2 + 16640*a^3*c^9*d^11*e^4 + 18560*a^4*c^8*d^9*e^6 + 56832*a^5*c^7*d^7*e^8 + 60544*a^6*c^6*d^5*e^10 + 20736*a^7*c^5*d^3*e^12))/(128*(c^7*d^8 + a^4*c^3*e^8 + 4*a*c^6*d^6*e^2 + 6*a^2*c^5*d^4*e^4 + 4*a^3*c^4*d^2*e^6)))*((25*c^3*d^6*(-a*c^7)^(1/2) - 9*a^3*e^6*(-a*c^7)^(1/2) + 6*a^3*c^4*d*e^5 + 44*a^2*c^5*d^3*e^3 + 70*a*c^6*d^5*e - 39*a*c^2*d^4*e^2*(-a*c^7)^(1/2) - 41*a^2*c*d^2*e^4*(-a*c^7)^(1/2))/(256*(c^11*d^8 + a^4*c^7*e^8 + 4*a*c^10*d^6*e^2 + 6*a^2*c^9*d^4*e^4 + 4*a^3*c^8*d^2*e^6)))^(1/2))*((25*c^3*d^6*(-a*c^7)^(1/2) - 9*a^3*e^6*(-a*c^7)^(1/2) + 6*a^3*c^4*d*e^5 + 44*a^2*c^5*d^3*e^3 + 70*a*c^6*d^5*e - 39*a*c^2*d^4*e^2*(-a*c^7)^(1/2) - 41*a^2*c*d^2*e^4*(-a*c^7)^(1/2))/(256*(c^11*d^8 + a^4*c^7*e^8 + 4*a*c^10*d^6*e^2 + 6*a^2*c^9*d^4*e^4 + 4*a^3*c^8*d^2*e^6)))^(1/2) - (x*(81*a^8*e^13 + 800*a^2*c^6*d^12*e + 612*a^7*c*d^2*e^11 + 832*a^3*c^5*d^10*e^3 + 913*a^4*c^4*d^8*e^5 + 1700*a^5*c^3*d^6*e^7 + 1606*a^6*c^2*d^4*e^9))/(128*(c^7*d^8 + a^4*c^3*e^8 + 4*a*c^6*d^6*e^2 + 6*a^2*c^5*d^4*e^4 + 4*a^3*c^4*d^2*e^6)))*((25*c^3*d^6*(-a*c^7)^(1/2) - 9*a^3*e^6*(-a*c^7)^(1/2) + 6*a^3*c^4*d*e^5 + 44*a^2*c^5*d^3*e^3 + 70*a*c^6*d^5*e - 39*a*c^2*d^4*e^2*(-a*c^7)^(1/2) - 41*a^2*c*d^2*e^4*(-a*c^7)^(1/2))/(256*(c^11*d^8 + a^4*c^7*e^8 + 4*a*c^10*d^6*e^2 + 6*a^2*c^9*d^4*e^4 + 4*a^3*c^8*d^2*e^6)))^(1/2)*1i)/((81*a^6*d^4*e^8 + 450*a^5*c*d^6*e^6 + 300*a^3*c^3*d^10*e^2 + 733*a^4*c^2*d^8*e^4)/(128*(c^7*d^8 + a^4*c^3*e^8 + 4*a*c^6*d^6*e^2 + 6*a^2*c^5*d^4*e^4 + 4*a^3*c^4*d^2*e^6)) + (((5120*a^2*c^8*d^13*e + 432*a^8*c^2*d*e^13 - 17232*a^3*c^7*d^11*e^3 - 37776*a^4*c^6*d^9*e^5 - 13600*a^5*c^5*d^7*e^7 + 4320*a^6*c^4*d^5*e^9 + 2928*a^7*c^3*d^3*e^11)/(256*(c^7*d^8 + a^4*c^3*e^8 + 4*a*c^6*d^6*e^2 + 6*a^2*c^5*d^4*e^4 + 4*a^3*c^4*d^2*e^6)) - (((81920*a^5*c^9*d^8*e^8 - 73728*a^3*c^11*d^12*e^4 - 61440*a^4*c^10*d^10*e^6 - 20480*a^2*c^12*d^14*e^2 + 184320*a^

6*c^8*d^6*e^10 + 122880*a^7*c^7*d^4*e^12 + 28672*a^8*c^6*d^2*e^14)/(256*(c^
7*d^8 + a^4*c^3*e^8 + 4*a*c^6*d^6*e^2 + 6*a^2*c^5*d^4*e^4 + 4*a^3*c^4*d^2*e
^6)) - (x*((25*c^3*d^6*(-a*c^7)^(1/2) - 9*a^3*e^6*(-a*c^7)^(1/2) + 6*a^3*c^
4*d*e^5 + 44*a^2*c^5*d^3*e^3 + 70*a*c^6*d^5*e - 39*a*c^2*d^4*e^2*(-a*c^7)^(
1/2) - 41*a^2*c*d^2*e^4*(-a*c^7)^(1/2))/(256*(c^11*d^8 + a^4*c^7*e^8 + 4*a*
c^10*d^6*e^2 + 6*a^2*c^9*d^4*e^4 + 4*a^3*c^8*d^2*e^6)))^(1/2)*(65536*a^9*c^
7*e^17 - 65536*a^2*c^14*d^14*e^3 - 327680*a^3*c^13*d^12*e^5 - 589824*a^4*c^
12*d^10*e^7 - 327680*a^5*c^11*d^8*e^9 + 327680*a^6*c^10*d^6*e^11 + 589824*a
^7*c^9*d^4*e^13 + 327680*a^8*c^8*d^2*e^15))/(128*(c^7*d^8 + a^4*c^3*e^8 + 4
*a*c^6*d^6*e^2 + 6*a^2*c^5*d^4*e^4 + 4*a^3*c^4*d^2*e^6)))*((25*c^3*d^6*(-a*
c^7)^(1/2) - 9*a^3*e^6*(-a*c^7)^(1/2) + 6*a^3*c^4*d*e^5 + 44*a^2*c^5*d^3*e^
3 + 70*a*c^6*d^5*e - 39*a*c^2*d^4*e^2*(-a*c^7)^(1/2) - 41*a^2*c*d^2*e^4*(-a
c^7)^(1/2))/(256(c^11*d^8 + a^4*c^7*e^8 + 4*a*c^10*d^6*e^2 + 6*a^2*c^9*d^
4*e^4 + 4*a^3*c^8*d^2*e^6)))^(1/2) + (x*(1920*a^8*c^4*d*e^14 + 13184*a^2*c^
10*d^13*e^2 + 16640*a^3*c^9*d^11*e^4 + 18560*a^4*c^8*d^9*e^6 + 56832*a^5*c^
7*d^7*e^8 + 60544*a^6*c^6*d^5*e^10 + 20736*a^7*c^5*d^3*e^12))/(128*(c^7*d^8
 + a^4*c^3*e^8 + 4*a*c^6*d^6*e^2 + 6*a^2*c^5*d^4*e^4 + 4*a^3*c^4*d^2*e^6)))
*((25*c^3*d^6*(-a*c^7)^(1/2) - 9*a^3*e^6*(-a*c^7)^(1/2) + 6*a^3*c^4*d*e^5 +
 44*a^2*c^5*d^3*e^3 + 70*a*c^6*d^5*e - 39*a*c^2*d^4*e^2*(-a*c^7)^(1/2) - 41
*a^2*c*d^2*e^4*(-a*c^7)^(1/2))/(256*(c^11*d^8 + a^4*c^7*e^8 + 4*a*c^10*d^6*
e^2 + 6*a^2*c^9*d^4*e^4 + 4*a^3*c^8*d^2*e^6)))^(1/2))*((25*c^3*d^6*(-a*c^7)
^(1/2) - 9*a^3*e^6*(-a*c^7)^(1/2) + 6*a^3*c^4*d*e^5 + 44*a^2*c^5*d^3*e^3 +
70*a*c^6*d^5*e - 39*a*c^2*d^4*e^2*(-a*c^7)^(1/2) - 41*a^2*c*d^2*e^4*(-a*c^7
)^(1/2))/(256*(c^11*d^8 + a^4*c^7*e^8 + 4*a*c^10*d^6*e^2 + 6*a^2*c^9*d^4*e^
4 + 4*a^3*c^8*d^2*e^6)))^(1/2) + (x*(81*a^8*e^13 + 800*a^2*c^6*d^12*e + 612
*a^7*c*d^2*e^11 + 832*a^3*c^5*d^10*e^3 + 913*a^4*c^4*d^8*e^5 + 1700*a^5*c^3
*d^6*e^7 + 1606*a^6*c^2*d^4*e^9))/(128*(c^7*d^8 + a^4*c^3*e^8 + 4*a*c^6*d^6
*e^2 + 6*a^2*c^5*d^4*e^4 + 4*a^3*c^4*d^2*e^6)))*((25*c^3*d^6*(-a*c^7)^(1/2)
 - 9*a^3*e^6*(-a*c^7)^(1/2) + 6*a^3*c^4*d*e^5 + 44*a^2*c^5*d^3*e^3 + 70*a*c
^6*d^5*e - 39*a*c^2*d^4*e^2*(-a*c^7)^(1/2) - 41*a^2*c*d^2*e^4*(-a*c^7)^(1/2
))/(256*(c^11*d^8 + a^4*c^7*e^8 + 4*a*c^10*d^6*e^2 + 6*a^2*c^9*d^4*e^4 + 4*
a^3*c^8*d^2*e^6)))^(1/2) + (((5120*a^2*c^8*d^13*e + 432*a^8*c^2*d*e^13 - 17
232*a^3*c^7*d^11*e^3 - 37776*a^4*c^6*d^9*e^5 - 13600*a^5*c^5*d^7*e^7 + 4320
*a^6*c^4*d^5*e^9 + 2928*a^7*c^3*d^3*e^11)/(256*(c^7*d^8 + a^4*c^3*e^8 + 4*a
*c^6*d^6*e^2 + 6*a^2*c^5*d^4*e^4 + 4*a^3*c^4*d^2*e^6)) - (((81920*a^5*c^9*d
^8*e^8 - 73728*a^3*c^11*d^12*e^4 - 61440*a^4*c^10*d^10*e^6 - 20480*a^2*c^12
*d^14*e^2 + 184320*a^6*c^8*d^6*e^10 + 122880*a^7*c^7*d^4*e^12 + 28672*a^8*c
^6*d^2*e^14)/(256*(c^7*d^8 + a^4*c^3*e^8 + 4*a*c^6*d^6*e^2 + 6*a^2*c^5*d^4*
e^4 + 4*a^3*c^4*d^2*e^6)) + (x*((25*c^3*d^6*(-a*c^7)^(1/2) - 9*a^3*e^6*(-a*
c^7)^(1/2) + 6*a^3*c^4*d*e^5 + 44*a^2*c^5*d^3*e^3 + 70*a*c^6*d^5*e - 39*a*c
^2*d^4*e^2*(-a*c^7)^(1/2) - 41*a^2*c*d^2*e^4*(-a*c^7)^(1/2))/(256*(c^11*d^8
 + a^4*c^7*e^8 + 4*a*c^10*d^6*e^2 + 6*a^2*c^9*d^4*e^4 + 4*a^3*c^8*d^2*e^6))
)^(1/2)*(65536*a^9*c^7*e^17 - 65536*a^2*c^14*d^14*e^3 - 327680*a^3*c^13*d^1
2*e^5 - 589824*a^4*c^12*d^10*e^7 - 327680*a^5*c^11*d^8*e^9 + 327680*a^6*c^1
0*d^6*e^11 + 589824*a^7*c^9*d^4*e^13 + 327680*a^8*c^8*d^2*e^15))/(128*(c^7*

d^8 + a^4*c^3*e^8 + 4*a*c^6*d^6*e^2 + 6*a^2*c^5*d^4*e^4 + 4*a^3*c^4*d^2*e^6
)))*((25*c^3*d^6*(-a*c^7)^(1/2) - 9*a^3*e^6*(-a*c^7)^(1/2) + 6*a^3*c^4*d*e^
5 + 44*a^2*c^5*d^3*e^3 + 70*a*c^6*d^5*e - 39*a*c^2*d^4*e^2*(-a*c^7)^(1/2) -
41*a^2*c*d^2*e^4*(-a*c^7)^(1/2))/(256*(c^11*d^8 + a^4*c^7*e^8 + 4*a*c^10*d
^6*e^2 + 6*a^2*c^9*d^4*e^4 + 4*a^3*c^8*d^2*e^6)))^(1/2) - (x*(1920*a^8*c^4*
d*e^14 + 13184*a^2*c^10*d^13*e^2 + 16640*a^3*c^9*d^11*e^4 + 18560*a^4*c^8*d
^9*e^6 + 56832*a^5*c^7*d^7*e^8 + 60544*a^6*c^6*d^5*e^10 + 20736*a^7*c^5*d^3
e^12))/(128(c^7*d^8 + a^4*c^3*e^8 + 4*a*c^6*d^6*e^2 + 6*a^2*c^5*d^4*e^4 +
4*a^3*c^4*d^2*e^6)))*((25*c^3*d^6*(-a*c^7)^(1/2) - 9*a^3*e^6*(-a*c^7)^(1/2
) + 6*a^3*c^4*d*e^5 + 44*a^2*c^5*d^3*e^3 + 70*a*c^6*d^5*e - 39*a*c^2*d^4*e^
2*(-a*c^7)^(1/2) - 41*a^2*c*d^2*e^4*(-a*c^7)^(1/2))/(256*(c^11*d^8 + a^4*c^
7*e^8 + 4*a*c^10*d^6*e^2 + 6*a^2*c^9*d^4*e^4 + 4*a^3*c^8*d^2*e^6)))^(1/2))*
((25*c^3*d^6*(-a*c^7)^(1/2) - 9*a^3*e^6*(-a*c^7)^(1/2) + 6*a^3*c^4*d*e^5 +
44*a^2*c^5*d^3*e^3 + 70*a*c^6*d^5*e - 39*a*c^2*d^4*e^2*(-a*c^7)^(1/2) - 41*
a^2*c*d^2*e^4*(-a*c^7)^(1/2))/(256*(c^11*d^8 + a^4*c^7*e^8 + 4*a*c^10*d^6*e
^2 + 6*a^2*c^9*d^4*e^4 + 4*a^3*c^8*d^2*e^6)))^(1/2) - (x*(81*a^8*e^13 + 800
*a^2*c^6*d^12*e + 612*a^7*c*d^2*e^11 + 832*a^3*c^5*d^10*e^3 + 913*a^4*c^4*d
^8*e^5 + 1700*a^5*c^3*d^6*e^7 + 1606*a^6*c^2*d^4*e^9))/(128*(c^7*d^8 + a^4*
c^3*e^8 + 4*a*c^6*d^6*e^2 + 6*a^2*c^5*d^4*e^4 + 4*a^3*c^4*d^2*e^6)))*((25*c
^3*d^6*(-a*c^7)^(1/2) - 9*a^3*e^6*(-a*c^7)^(1/2) + 6*a^3*c^4*d*e^5 + 44*a^2
*c^5*d^3*e^3 + 70*a*c^6*d^5*e - 39*a*c^2*d^4*e^2*(-a*c^7)^(1/2) - 41*a^2*c*
d^2*e^4*(-a*c^7)^(1/2))/(256*(c^11*d^8 + a^4*c^7*e^8 + 4*a*c^10*d^6*e^2 + 6
*a^2*c^9*d^4*e^4 + 4*a^3*c^8*d^2*e^6)))^(1/2)))*((25*c^3*d^6*(-a*c^7)^(1/2)
- 9*a^3*e^6*(-a*c^7)^(1/2) + 6*a^3*c^4*d*e^5 + 44*a^2*c^5*d^3*e^3 + 70*a*c
^6*d^5*e - 39*a*c^2*d^4*e^2*(-a*c^7)^(1/2) - 41*a^2*c*d^2*e^4*(-a*c^7)^(1/2
))/(256*(c^11*d^8 + a^4*c^7*e^8 + 4*a*c^10*d^6*e^2 + 6*a^2*c^9*d^4*e^4 + 4*
a^3*c^8*d^2*e^6)))^(1/2)*2i + atan(((((5120*a^2*c^8*d^13*e + 432*a^8*c^2*d*
e^13 - 17232*a^3*c^7*d^11*e^3 - 37776*a^4*c^6*d^9*e^5 - 13600*a^5*c^5*d^7*e
^7 + 4320*a^6*c^4*d^5*e^9 + 2928*a^7*c^3*d^3*e^11)/(256*(c^7*d^8 + a^4*c^3*
e^8 + 4*a*c^6*d^6*e^2 + 6*a^2*c^5*d^4*e^4 + 4*a^3*c^4*d^2*e^6)) - (((81920*
a^5*c^9*d^8*e^8 - 73728*a^3*c^11*d^12*e^4 - 61440*a^4*c^10*d^10*e^6 - 20480
*a^2*c^12*d^14*e^2 + 184320*a^6*c^8*d^6*e^10 + 122880*a^7*c^7*d^4*e^12 + 28
672*a^8*c^6*d^2*e^14)/(256*(c^7*d^8 + a^4*c^3*e^8 + 4*a*c^6*d^6*e^2 + 6*a^2
*c^5*d^4*e^4 + 4*a^3*c^4*d^2*e^6)) - (x*((9*a^3*e^6*(-a*c^7)^(1/2) - 25*c^3
d^6(-a*c^7)^(1/2) + 6*a^3*c^4*d*e^5 + 44*a^2*c^5*d^3*e^3 + 70*a*c^6*d^5*e
+ 39*a*c^2*d^4*e^2*(-a*c^7)^(1/2) + 41*a^2*c*d^2*e^4*(-a*c^7)^(1/2))/(256*
(c^11*d^8 + a^4*c^7*e^8 + 4*a*c^10*d^6*e^2 + 6*a^2*c^9*d^4*e^4 + 4*a^3*c^8*
d^2*e^6)))^(1/2)*(65536*a^9*c^7*e^17 - 65536*a^2*c^14*d^14*e^3 - 327680*a^3
*c^13*d^12*e^5 - 589824*a^4*c^12*d^10*e^7 - 327680*a^5*c^11*d^8*e^9 + 32768
0*a^6*c^10*d^6*e^11 + 589824*a^7*c^9*d^4*e^13 + 327680*a^8*c^8*d^2*e^15))/(
128*(c^7*d^8 + a^4*c^3*e^8 + 4*a*c^6*d^6*e^2 + 6*a^2*c^5*d^4*e^4 + 4*a^3*c^
4*d^2*e^6)))*((9*a^3*e^6*(-a*c^7)^(1/2) - 25*c^3*d^6*(-a*c^7)^(1/2) + 6*a^3
*c^4*d*e^5 + 44*a^2*c^5*d^3*e^3 + 70*a*c^6*d^5*e + 39*a*c^2*d^4*e^2*(-a*c^7
)^(1/2) + 41*a^2*c*d^2*e^4*(-a*c^7)^(1/2))/(256*(c^11*d^8 + a^4*c^7*e^8 + 4
*a*c^10*d^6*e^2 + 6*a^2*c^9*d^4*e^4 + 4*a^3*c^8*d^2*e^6)))^(1/2) + (x*(1920

*a^8*c^4*d*e^14 + 13184*a^2*c^10*d^13*e^2 + 16640*a^3*c^9*d^11*e^4 + 18560*
a^4*c^8*d^9*e^6 + 56832*a^5*c^7*d^7*e^8 + 60544*a^6*c^6*d^5*e^10 + 20736*a^
7*c^5*d^3*e^12))/(128*(c^7*d^8 + a^4*c^3*e^8 + 4*a*c^6*d^6*e^2 + 6*a^2*c^5*
d^4*e^4 + 4*a^3*c^4*d^2*e^6)))*((9*a^3*e^6*(-a*c^7)^(1/2) - 25*c^3*d^6*(-a*
c^7)^(1/2) + 6*a^3*c^4*d*e^5 + 44*a^2*c^5*d^3*e^3 + 70*a*c^6*d^5*e + 39*a*c
^2*d^4*e^2*(-a*c^7)^(1/2) + 41*a^2*c*d^2*e^4*(-a*c^7)^(1/2))/(256*(c^11*d^8
 + a^4*c^7*e^8 + 4*a*c^10*d^6*e^2 + 6*a^2*c^9*d^4*e^4 + 4*a^3*c^8*d^2*e^6))
)^(1/2))*((9*a^3*e^6*(-a*c^7)^(1/2) - 25*c^3*d^6*(-a*c^7)^(1/2) + 6*a^3*c^4
*d*e^5 + 44*a^2*c^5*d^3*e^3 + 70*a*c^6*d^5*e + 39*a*c^2*d^4*e^2*(-a*c^7)^(1
/2) + 41*a^2*c*d^2*e^4*(-a*c^7)^(1/2))/(256*(c^11*d^8 + a^4*c^7*e^8 + 4*a*c
^10*d^6*e^2 + 6*a^2*c^9*d^4*e^4 + 4*a^3*c^8*d^2*e^6)))^(1/2) + (x*(81*a^8*e
^13 + 800*a^2*c^6*d^12*e + 612*a^7*c*d^2*e^11 + 832*a^3*c^5*d^10*e^3 + 913*
a^4*c^4*d^8*e^5 + 1700*a^5*c^3*d^6*e^7 + 1606*a^6*c^2*d^4*e^9))/(128*(c^7*d
^8 + a^4*c^3*e^8 + 4*a*c^6*d^6*e^2 + 6*a^2*c^5*d^4*e^4 + 4*a^3*c^4*d^2*e^6)
))*((9*a^3*e^6*(-a*c^7)^(1/2) - 25*c^3*d^6*(-a*c^7)^(1/2) + 6*a^3*c^4*d*e^5
 + 44*a^2*c^5*d^3*e^3 + 70*a*c^6*d^5*e + 39*a*c^2*d^4*e^2*(-a*c^7)^(1/2) +
41*a^2*c*d^2*e^4*(-a*c^7)^(1/2))/(256*(c^11*d^8 + a^4*c^7*e^8 + 4*a*c^10*d^
6*e^2 + 6*a^2*c^9*d^4*e^4 + 4*a^3*c^8*d^2*e^6)))^(1/2)*1i - (((5120*a^2*c^8
*d^13*e + 432*a^8*c^2*d*e^13 - 17232*a^3*c^7*d^11*e^3 - 37776*a^4*c^6*d^9*e
^5 - 13600*a^5*c^5*d^7*e^7 + 4320*a^6*c^4*d^5*e^9 + 2928*a^7*c^3*d^3*e^11)/
(256*(c^7*d^8 + a^4*c^3*e^8 + 4*a*c^6*d^6*e^2 + 6*a^2*c^5*d^4*e^4 + 4*a^3*c
^4*d^2*e^6)) - (((81920*a^5*c^9*d^8*e^8 - 73728*a^3*c^11*d^12*e^4 - 61440*a
^4*c^10*d^10*e^6 - 20480*a^2*c^12*d^14*e^2 + 184320*a^6*c^8*d^6*e^10 + 1228
80*a^7*c^7*d^4*e^12 + 28672*a^8*c^6*d^2*e^14)/(256*(c^7*d^8 + a^4*c^3*e^8 +
 4*a*c^6*d^6*e^2 + 6*a^2*c^5*d^4*e^4 + 4*a^3*c^4*d^2*e^6)) + (x*((9*a^3*e^6
*(-a*c^7)^(1/2) - 25*c^3*d^6*(-a*c^7)^(1/2) + 6*a^3*c^4*d*e^5 + 44*a^2*c^5*
d^3*e^3 + 70*a*c^6*d^5*e + 39*a*c^2*d^4*e^2*(-a*c^7)^(1/2) + 41*a^2*c*d^2*e
^4*(-a*c^7)^(1/2))/(256*(c^11*d^8 + a^4*c^7*e^8 + 4*a*c^10*d^6*e^2 + 6*a^2*
c^9*d^4*e^4 + 4*a^3*c^8*d^2*e^6)))^(1/2)*(65536*a^9*c^7*e^17 - 65536*a^2*c^
14*d^14*e^3 - 327680*a^3*c^13*d^12*e^5 - 589824*a^4*c^12*d^10*e^7 - 327680*
a^5*c^11*d^8*e^9 + 327680*a^6*c^10*d^6*e^11 + 589824*a^7*c^9*d^4*e^13 + 327
680*a^8*c^8*d^2*e^15))/(128*(c^7*d^8 + a^4*c^3*e^8 + 4*a*c^6*d^6*e^2 + 6*a^
2*c^5*d^4*e^4 + 4*a^3*c^4*d^2*e^6)))*((9*a^3*e^6*(-a*c^7)^(1/2) - 25*c^3*d^
6*(-a*c^7)^(1/2) + 6*a^3*c^4*d*e^5 + 44*a^2*c^5*d^3*e^3 + 70*a*c^6*d^5*e +
39*a*c^2*d^4*e^2*(-a*c^7)^(1/2) + 41*a^2*c*d^2*e^4*(-a*c^7)^(1/2))/(256*(c^
11*d^8 + a^4*c^7*e^8 + 4*a*c^10*d^6*e^2 + 6*a^2*c^9*d^4*e^4 + 4*a^3*c^8*d^2
e^6)))^(1/2) - (x(1920*a^8*c^4*d*e^14 + 13184*a^2*c^10*d^13*e^2 + 16640*a
^3*c^9*d^11*e^4 + 18560*a^4*c^8*d^9*e^6 + 56832*a^5*c^7*d^7*e^8 + 60544*a^6
*c^6*d^5*e^10 + 20736*a^7*c^5*d^3*e^12))/(128*(c^7*d^8 + a^4*c^3*e^8 + 4*a*
c^6*d^6*e^2 + 6*a^2*c^5*d^4*e^4 + 4*a^3*c^4*d^2*e^6)))*((9*a^3*e^6*(-a*c^7)
^(1/2) - 25*c^3*d^6*(-a*c^7)^(1/2) + 6*a^3*c^4*d*e^5 + 44*a^2*c^5*d^3*e^3 +
 70*a*c^6*d^5*e + 39*a*c^2*d^4*e^2*(-a*c^7)^(1/2) + 41*a^2*c*d^2*e^4*(-a*c^
7)^(1/2))/(256*(c^11*d^8 + a^4*c^7*e^8 + 4*a*c^10*d^6*e^2 + 6*a^2*c^9*d^4*e
^4 + 4*a^3*c^8*d^2*e^6)))^(1/2))*((9*a^3*e^6*(-a*c^7)^(1/2) - 25*c^3*d^6*(-
a*c^7)^(1/2) + 6*a^3*c^4*d*e^5 + 44*a^2*c^5*d^3*e^3 + 70*a*c^6*d^5*e + 39*a

*c^2*d^4*e^2*(-a*c^7)^(1/2) + 41*a^2*c*d^2*e^4*(-a*c^7)^(1/2))/(256*(c^11*d
^8 + a^4*c^7*e^8 + 4*a*c^10*d^6*e^2 + 6*a^2*c^9*d^4*e^4 + 4*a^3*c^8*d^2*e^6
)))^(1/2) - (x*(81*a^8*e^13 + 800*a^2*c^6*d^12*e + 612*a^7*c*d^2*e^11 + 832
*a^3*c^5*d^10*e^3 + 913*a^4*c^4*d^8*e^5 + 1700*a^5*c^3*d^6*e^7 + 1606*a^6*c
^2*d^4*e^9))/(128*(c^7*d^8 + a^4*c^3*e^8 + 4*a*c^6*d^6*e^2 + 6*a^2*c^5*d^4*
e^4 + 4*a^3*c^4*d^2*e^6)))*((9*a^3*e^6*(-a*c^7)^(1/2) - 25*c^3*d^6*(-a*c^7)
^(1/2) + 6*a^3*c^4*d*e^5 + 44*a^2*c^5*d^3*e^3 + 70*a*c^6*d^5*e + 39*a*c^2*d
^4*e^2*(-a*c^7)^(1/2) + 41*a^2*c*d^2*e^4*(-a*c^7)^(1/2))/(256*(c^11*d^8 + a
^4*c^7*e^8 + 4*a*c^10*d^6*e^2 + 6*a^2*c^9*d^4*e^4 + 4*a^3*c^8*d^2*e^6)))^(1
/2)*1i)/((81*a^6*d^4*e^8 + 450*a^5*c*d^6*e^6 + 300*a^3*c^3*d^10*e^2 + 733*a
^4*c^2*d^8*e^4)/(128*(c^7*d^8 + a^4*c^3*e^8 + 4*a*c^6*d^6*e^2 + 6*a^2*c^5*d
^4*e^4 + 4*a^3*c^4*d^2*e^6)) + (((5120*a^2*c^8*d^13*e + 432*a^8*c^2*d*e^13
- 17232*a^3*c^7*d^11*e^3 - 37776*a^4*c^6*d^9*e^5 - 13600*a^5*c^5*d^7*e^7 +
4320*a^6*c^4*d^5*e^9 + 2928*a^7*c^3*d^3*e^11)/(256*(c^7*d^8 + a^4*c^3*e^8 +
4*a*c^6*d^6*e^2 + 6*a^2*c^5*d^4*e^4 + 4*a^3*c^4*d^2*e^6)) - (((81920*a^5*c
^9*d^8*e^8 - 73728*a^3*c^11*d^12*e^4 - 61440*a^4*c^10*d^10*e^6 - 20480*a^2*
c^12*d^14*e^2 + 184320*a^6*c^8*d^6*e^10 + 122880*a^7*c^7*d^4*e^12 + 28672*a
^8*c^6*d^2*e^14)/(256*(c^7*d^8 + a^4*c^3*e^8 + 4*a*c^6*d^6*e^2 + 6*a^2*c^5*
d^4*e^4 + 4*a^3*c^4*d^2*e^6)) - (x*((9*a^3*e^6*(-a*c^7)^(1/2) - 25*c^3*d^6*
(-a*c^7)^(1/2) + 6*a^3*c^4*d*e^5 + 44*a^2*c^5*d^3*e^3 + 70*a*c^6*d^5*e + 39
*a*c^2*d^4*e^2*(-a*c^7)^(1/2) + 41*a^2*c*d^2*e^4*(-a*c^7)^(1/2))/(256*(c^11
*d^8 + a^4*c^7*e^8 + 4*a*c^10*d^6*e^2 + 6*a^2*c^9*d^4*e^4 + 4*a^3*c^8*d^2*e
^6)))^(1/2)*(65536*a^9*c^7*e^17 - 65536*a^2*c^14*d^14*e^3 - 327680*a^3*c^13
*d^12*e^5 - 589824*a^4*c^12*d^10*e^7 - 327680*a^5*c^11*d^8*e^9 + 327680*a^6
*c^10*d^6*e^11 + 589824*a^7*c^9*d^4*e^13 + 327680*a^8*c^8*d^2*e^15))/(128*(
c^7*d^8 + a^4*c^3*e^8 + 4*a*c^6*d^6*e^2 + 6*a^2*c^5*d^4*e^4 + 4*a^3*c^4*d^2
e^6)))((9*a^3*e^6*(-a*c^7)^(1/2) - 25*c^3*d^6*(-a*c^7)^(1/2) + 6*a^3*c^4*
d*e^5 + 44*a^2*c^5*d^3*e^3 + 70*a*c^6*d^5*e + 39*a*c^2*d^4*e^2*(-a*c^7)^(1/
2) + 41*a^2*c*d^2*e^4*(-a*c^7)^(1/2))/(256*(c^11*d^8 + a^4*c^7*e^8 + 4*a*c^
10*d^6*e^2 + 6*a^2*c^9*d^4*e^4 + 4*a^3*c^8*d^2*e^6)))^(1/2) + (x*(1920*a^8*
c^4*d*e^14 + 13184*a^2*c^10*d^13*e^2 + 16640*a^3*c^9*d^11*e^4 + 18560*a^4*c
^8*d^9*e^6 + 56832*a^5*c^7*d^7*e^8 + 60544*a^6*c^6*d^5*e^10 + 20736*a^7*c^5
*d^3*e^12))/(128*(c^7*d^8 + a^4*c^3*e^8 + 4*a*c^6*d^6*e^2 + 6*a^2*c^5*d^4*e
^4 + 4*a^3*c^4*d^2*e^6)))*((9*a^3*e^6*(-a*c^7)^(1/2) - 25*c^3*d^6*(-a*c^7)^
(1/2) + 6*a^3*c^4*d*e^5 + 44*a^2*c^5*d^3*e^3 + 70*a*c^6*d^5*e + 39*a*c^2*d^
4*e^2*(-a*c^7)^(1/2) + 41*a^2*c*d^2*e^4*(-a*c^7)^(1/2))/(256*(c^11*d^8 + a^
4*c^7*e^8 + 4*a*c^10*d^6*e^2 + 6*a^2*c^9*d^4*e^4 + 4*a^3*c^8*d^2*e^6)))^(1/
2))*((9*a^3*e^6*(-a*c^7)^(1/2) - 25*c^3*d^6*(-a*c^7)^(1/2) + 6*a^3*c^4*d*e^
5 + 44*a^2*c^5*d^3*e^3 + 70*a*c^6*d^5*e + 39*a*c^2*d^4*e^2*(-a*c^7)^(1/2) +
41*a^2*c*d^2*e^4*(-a*c^7)^(1/2))/(256*(c^11*d^8 + a^4*c^7*e^8 + 4*a*c^10*d
^6*e^2 + 6*a^2*c^9*d^4*e^4 + 4*a^3*c^8*d^2*e^6)))^(1/2) + (x*(81*a^8*e^13 +
800*a^2*c^6*d^12*e + 612*a^7*c*d^2*e^11 + 832*a^3*c^5*d^10*e^3 + 913*a^4*c
^4*d^8*e^5 + 1700*a^5*c^3*d^6*e^7 + 1606*a^6*c^2*d^4*e^9))/(128*(c^7*d^8 +
a^4*c^3*e^8 + 4*a*c^6*d^6*e^2 + 6*a^2*c^5*d^4*e^4 + 4*a^3*c^4*d^2*e^6)))*((
9*a^3*e^6*(-a*c^7)^(1/2) - 25*c^3*d^6*(-a*c^7)^(1/2) + 6*a^3*c^4*d*e^5 + 44

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*a^2*c^5*d^3*e^3 + 70*a*c^6*d^5*e + 39*a*c^2*d^4*e^2*(-a*c^7)^(1/2) + 41*a^
2*c*d^2*e^4*(-a*c^7)^(1/2))/(256*(c^11*d^8 + a^4*c^7*e^8 + 4*a*c^10*d^6*e^2
 + 6*a^2*c^9*d^4*e^4 + 4*a^3*c^8*d^2*e^6)))^(1/2) + (((5120*a^2*c^8*d^13*e
+ 432*a^8*c^2*d*e^13 - 17232*a^3*c^7*d^11*e^3 - 37776*a^4*c^6*d^9*e^5 - 136
00*a^5*c^5*d^7*e^7 + 4320*a^6*c^4*d^5*e^9 + 2928*a^7*c^3*d^3*e^11)/(256*(c^
7*d^8 + a^4*c^3*e^8 + 4*a*c^6*d^6*e^2 + 6*a^2*c^5*d^4*e^4 + 4*a^3*c^4*d^2*e
^6)) - (((81920*a^5*c^9*d^8*e^8 - 73728*a^3*c^11*d^12*e^4 - 61440*a^4*c^10*
d^10*e^6 - 20480*a^2*c^12*d^14*e^2 + 184320*a^6*c^8*d^6*e^10 + 122880*a^7*c
^7*d^4*e^12 + 28672*a^8*c^6*d^2*e^14)/(256*(c^7*d^8 + a^4*c^3*e^8 + 4*a*c^6
*d^6*e^2 + 6*a^2*c^5*d^4*e^4 + 4*a^3*c^4*d^2*e^6)) + (x*((9*a^3*e^6*(-a*c^7
)^(1/2) - 25*c^3*d^6*(-a*c^7)^(1/2) + 6*a^3*c^4*d*e^5 + 44*a^2*c^5*d^3*e^3
+ 70*a*c^6*d^5*e + 39*a*c^2*d^4*e^2*(-a*c^7)^(1/2) + 41*a^2*c*d^2*e^4*(-a*c
^7)^(1/2))/(256*(c^11*d^8 + a^4*c^7*e^8 + 4*a*c^10*d^6*e^2 + 6*a^2*c^9*d^4*
e^4 + 4*a^3*c^8*d^2*e^6)))^(1/2)*(65536*a^9*c^7*e^17 - 65536*a^2*c^14*d^14*
e^3 - 327680*a^3*c^13*d^12*e^5 - 589824*a^4*c^12*d^10*e^7 - 327680*a^5*c^11
*d^8*e^9 + 327680*a^6*c^10*d^6*e^11 + 589824*a^7*c^9*d^4*e^13 + 327680*a^8*
c^8*d^2*e^15))/(128*(c^7*d^8 + a^4*c^3*e^8 + 4*a*c^6*d^6*e^2 + 6*a^2*c^5*d^
4*e^4 + 4*a^3*c^4*d^2*e^6)))*((9*a^3*e^6*(-a*c^7)^(1/2) - 25*c^3*d^6*(-a*c^
7)^(1/2) + 6*a^3*c^4*d*e^5 + 44*a^2*c^5*d^3*e^3 + 70*a*c^6*d^5*e + 39*a*c^2
*d^4*e^2*(-a*c^7)^(1/2) + 41*a^2*c*d^2*e^4*(-a*c^7)^(1/2))/(256*(c^11*d^8 +
 a^4*c^7*e^8 + 4*a*c^10*d^6*e^2 + 6*a^2*c^9*d^4*e^4 + 4*a^3*c^8*d^2*e^6)))^
(1/2) - (x*(1920*a^8*c^4*d*e^14 + 13184*a^2*c^10*d^13*e^2 + 16640*a^3*c^9*d
^11*e^4 + 18560*a^4*c^8*d^9*e^6 + 56832*a^5*c^7*d^7*e^8 + 60544*a^6*c^6*d^5
*e^10 + 20736*a^7*c^5*d^3*e^12))/(128*(c^7*d^8 + a^4*c^3*e^8 + 4*a*c^6*d^6*
e^2 + 6*a^2*c^5*d^4*e^4 + 4*a^3*c^4*d^2*e^6)))*((9*a^3*e^6*(-a*c^7)^(1/2) -
 25*c^3*d^6*(-a*c^7)^(1/2) + 6*a^3*c^4*d*e^5 + 44*a^2*c^5*d^3*e^3 + 70*a*c^
6*d^5*e + 39*a*c^2*d^4*e^2*(-a*c^7)^(1/2) + 41*a^2*c*d^2*e^4*(-a*c^7)^(1/2)
)/(256*(c^11*d^8 + a^4*c^7*e^8 + 4*a*c^10*d^6*e^2 + 6*a^2*c^9*d^4*e^4 + 4*a
^3*c^8*d^2*e^6)))^(1/2))*((9*a^3*e^6*(-a*c^7)^(1/2) - 25*c^3*d^6*(-a*c^7)^(
1/2) + 6*a^3*c^4*d*e^5 + 44*a^2*c^5*d^3*e^3 + 70*a*c^6*d^5*e + 39*a*c^2*d^4
*e^2*(-a*c^7)^(1/2) + 41*a^2*c*d^2*e^4*(-a*c^7)^(1/2))/(256*(c^11*d^8 + a^4
*c^7*e^8 + 4*a*c^10*d^6*e^2 + 6*a^2*c^9*d^4*e^4 + 4*a^3*c^8*d^2*e^6)))^(1/2
) - (x*(81*a^8*e^13 + 800*a^2*c^6*d^12*e + 612*a^7*c*d^2*e^11 + 832*a^3*c^5
*d^10*e^3 + 913*a^4*c^4*d^8*e^5 + 1700*a^5*c^3*d^6*e^7 + 1606*a^6*c^2*d^4*e
^9))/(128*(c^7*d^8 + a^4*c^3*e^8 + 4*a*c^6*d^6*e^2 + 6*a^2*c^5*d^4*e^4 + 4*
a^3*c^4*d^2*e^6)))*((9*a^3*e^6*(-a*c^7)^(1/2) - 25*c^3*d^6*(-a*c^7)^(1/2) +
 6*a^3*c^4*d*e^5 + 44*a^2*c^5*d^3*e^3 + 70*a*c^6*d^5*e + 39*a*c^2*d^4*e^2*(
-a*c^7)^(1/2) + 41*a^2*c*d^2*e^4*(-a*c^7)^(1/2))/(256*(c^11*d^8 + a^4*c^7*e
^8 + 4*a*c^10*d^6*e^2 + 6*a^2*c^9*d^4*e^4 + 4*a^3*c^8*d^2*e^6)))^(1/2)))*((
9*a^3*e^6*(-a*c^7)^(1/2) - 25*c^3*d^6*(-a*c^7)^(1/2) + 6*a^3*c^4*d*e^5 + 44
*a^2*c^5*d^3*e^3 + 70*a*c^6*d^5*e + 39*a*c^2*d^4*e^2*(-a*c^7)^(1/2) + 41*a^
2*c*d^2*e^4*(-a*c^7)^(1/2))/(256*(c^11*d^8 + a^4*c^7*e^8 + 4*a*c^10*d^6*e^2
 + 6*a^2*c^9*d^4*e^4 + 4*a^3*c^8*d^2*e^6)))^(1/2)*2i + (atan(((((x*(81*a^8*
e^13 + 800*a^2*c^6*d^12*e + 612*a^7*c*d^2*e^11 + 832*a^3*c^5*d^10*e^3 + 913
*a^4*c^4*d^8*e^5 + 1700*a^5*c^3*d^6*e^7 + 1606*a^6*c^2*d^4*e^9))/(256*(c^7*
```

d^8 + a^4*c^3*e^8 + 4*a*c^6*d^6*e^2 + 6*a^2*c^5*d^4*e^4 + 4*a^3*c^4*d^2*e^6
)) + (((20*a^2*c^8*d^13*e + (27*a^8*c^2*d*e^13)/16 - (1077*a^3*c^7*d^11*e^3
)/16 - (2361*a^4*c^6*d^9*e^5)/16 - (425*a^5*c^5*d^7*e^7)/8 + (135*a^6*c^4*d
^5*e^9)/8 + (183*a^7*c^3*d^3*e^11)/16)/(2*(c^7*d^8 + a^4*c^3*e^8 + 4*a*c^6*
d^6*e^2 + 6*a^2*c^5*d^4*e^4 + 4*a^3*c^4*d^2*e^6)) - ((-d^7*e)^(1/2)*(((-d^7
e)^(1/2)((320*a^5*c^9*d^8*e^8 - 288*a^3*c^11*d^12*e^4 - 240*a^4*c^10*d^10
*e^6 - 80*a^2*c^12*d^14*e^2 + 720*a^6*c^8*d^6*e^10 + 480*a^7*c^7*d^4*e^12 +
112*a^8*c^6*d^2*e^14)/(2*(c^7*d^8 + a^4*c^3*e^8 + 4*a*c^6*d^6*e^2 + 6*a^2*
c^5*d^4*e^4 + 4*a^3*c^4*d^2*e^6)) - (x*(-d^7*e)^(1/2)*(65536*a^9*c^7*e^17 -
65536*a^2*c^14*d^14*e^3 - 327680*a^3*c^13*d^12*e^5 - 589824*a^4*c^12*d^10*
e^7 - 327680*a^5*c^11*d^8*e^9 + 327680*a^6*c^10*d^6*e^11 + 589824*a^7*c^9*d
^4*e^13 + 327680*a^8*c^8*d^2*e^15))/(512*(a^2*e^5 + c^2*d^4*e + 2*a*c*d^2*e
^3)*(c^7*d^8 + a^4*c^3*e^8 + 4*a*c^6*d^6*e^2 + 6*a^2*c^5*d^4*e^4 + 4*a^3*c^
4*d^2*e^6))))/(2*(a^2*e^5 + c^2*d^4*e + 2*a*c*d^2*e^3)) + (x*(1920*a^8*c^4*
d*e^14 + 13184*a^2*c^10*d^13*e^2 + 16640*a^3*c^9*d^11*e^4 + 18560*a^4*c^8*d
^9*e^6 + 56832*a^5*c^7*d^7*e^8 + 60544*a^6*c^6*d^5*e^10 + 20736*a^7*c^5*d^3
e^12))/(256(c^7*d^8 + a^4*c^3*e^8 + 4*a*c^6*d^6*e^2 + 6*a^2*c^5*d^4*e^4 +
4*a^3*c^4*d^2*e^6))))/(2*(a^2*e^5 + c^2*d^4*e + 2*a*c*d^2*e^3)))*(-d^7*e)^
(1/2))/(2*(a^2*e^5 + c^2*d^4*e + 2*a*c*d^2*e^3)))*(-d^7*e)^(1/2)*1i)/(a^2*e
^5 + c^2*d^4*e + 2*a*c*d^2*e^3) + (((x*(81*a^8*e^13 + 800*a^2*c^6*d^12*e +
612*a^7*c*d^2*e^11 + 832*a^3*c^5*d^10*e^3 + 913*a^4*c^4*d^8*e^5 + 1700*a^5*
c^3*d^6*e^7 + 1606*a^6*c^2*d^4*e^9))/(256*(c^7*d^8 + a^4*c^3*e^8 + 4*a*c^6*
d^6*e^2 + 6*a^2*c^5*d^4*e^4 + 4*a^3*c^4*d^2*e^6)) - (((20*a^2*c^8*d^13*e +
(27*a^8*c^2*d*e^13)/16 - (1077*a^3*c^7*d^11*e^3)/16 - (2361*a^4*c^6*d^9*e^5
)/16 - (425*a^5*c^5*d^7*e^7)/8 + (135*a^6*c^4*d^5*e^9)/8 + (183*a^7*c^3*d^3
e^11)/16)/(2(c^7*d^8 + a^4*c^3*e^8 + 4*a*c^6*d^6*e^2 + 6*a^2*c^5*d^4*e^4
+ 4*a^3*c^4*d^2*e^6)) - ((-d^7*e)^(1/2)*(((-d^7*e)^(1/2)*((320*a^5*c^9*d^8*
e^8 - 288*a^3*c^11*d^12*e^4 - 240*a^4*c^10*d^10*e^6 - 80*a^2*c^12*d^14*e^2
+ 720*a^6*c^8*d^6*e^10 + 480*a^7*c^7*d^4*e^12 + 112*a^8*c^6*d^2*e^14)/(2*(c
^7*d^8 + a^4*c^3*e^8 + 4*a*c^6*d^6*e^2 + 6*a^2*c^5*d^4*e^4 + 4*a^3*c^4*d^2*
e^6)) + (x*(-d^7*e)^(1/2)*(65536*a^9*c^7*e^17 - 65536*a^2*c^14*d^14*e^3 - 3
27680*a^3*c^13*d^12*e^5 - 589824*a^4*c^12*d^10*e^7 - 327680*a^5*c^11*d^8*e^
9 + 327680*a^6*c^10*d^6*e^11 + 589824*a^7*c^9*d^4*e^13 + 327680*a^8*c^8*d^2
e^15))/(512(a^2*e^5 + c^2*d^4*e + 2*a*c*d^2*e^3)*(c^7*d^8 + a^4*c^3*e^8 +
4*a*c^6*d^6*e^2 + 6*a^2*c^5*d^4*e^4 + 4*a^3*c^4*d^2*e^6))))/(2*(a^2*e^5 +
c^2*d^4*e + 2*a*c*d^2*e^3)) - (x*(1920*a^8*c^4*d*e^14 + 13184*a^2*c^10*d^13
*e^2 + 16640*a^3*c^9*d^11*e^4 + 18560*a^4*c^8*d^9*e^6 + 56832*a^5*c^7*d^7*e
^8 + 60544*a^6*c^6*d^5*e^10 + 20736*a^7*c^5*d^3*e^12))/(256*(c^7*d^8 + a^4*
c^3*e^8 + 4*a*c^6*d^6*e^2 + 6*a^2*c^5*d^4*e^4 + 4*a^3*c^4*d^2*e^6))))/(2*(a
^2*e^5 + c^2*d^4*e + 2*a*c*d^2*e^3)))*(-d^7*e)^(1/2))/(2*(a^2*e^5 + c^2*d^4
*e + 2*a*c*d^2*e^3)))*(-d^7*e)^(1/2)*1i)/(a^2*e^5 + c^2*d^4*e + 2*a*c*d^2*e
^3))/(((81*a^6*d^4*e^8)/128 + (225*a^5*c*d^6*e^6)/64 + (75*a^3*c^3*d^10*e^2
)/32 + (733*a^4*c^2*d^8*e^4)/128)/(c^7*d^8 + a^4*c^3*e^8 + 4*a*c^6*d^6*e^2
+ 6*a^2*c^5*d^4*e^4 + 4*a^3*c^4*d^2*e^6) + (((x*(81*a^8*e^13 + 800*a^2*c^6*
d^12*e + 612*a^7*c*d^2*e^11 + 832*a^3*c^5*d^10*e^3 + 913*a^4*c^4*d^8*e^5 +

1700*a^5*c^3*d^6*e^7 + 1606*a^6*c^2*d^4*e^9))/(256*(c^7*d^8 + a^4*c^3*e^8 + 4*a*c^6*d^6*e^2 + 6*a^2*c^5*d^4*e^4 + 4*a^3*c^4*d^2*e^6)) + (((20*a^2*c^8*d^13*e + (27*a^8*c^2*d*e^13)/16 - (1077*a^3*c^7*d^11*e^3)/16 - (2361*a^4*c^6*d^9*e^5)/16 - (425*a^5*c^5*d^7*e^7)/8 + (135*a^6*c^4*d^5*e^9)/8 + (183*a^7*c^3*d^3*e^11)/16)/(2*(c^7*d^8 + a^4*c^3*e^8 + 4*a*c^6*d^6*e^2 + 6*a^2*c^5*d^4*e^4 + 4*a^3*c^4*d^2*e^6)) - ((-d^7*e)^(1/2)*(((-d^7*e)^(1/2)*((320*a^5*c^9*d^8*e^8 - 288*a^3*c^11*d^12*e^4 - 240*a^4*c^10*d^10*e^6 - 80*a^2*c^12*d^14*e^2 + 720*a^6*c^8*d^6*e^10 + 480*a^7*c^7*d^4*e^12 + 112*a^8*c^6*d^2*e^14)/(2*(c^7*d^8 + a^4*c^3*e^8 + 4*a*c^6*d^6*e^2 + 6*a^2*c^5*d^4*e^4 + 4*a^3*c^4*d^2*e^6)) - (x*(-d^7*e)^(1/2)*(65536*a^9*c^7*e^17 - 65536*a^2*c^14*d^14*e^3 - 327680*a^3*c^13*d^12*e^5 - 589824*a^4*c^12*d^10*e^7 - 327680*a^5*c^11*d^8*e^9 + 327680*a^6*c^10*d^6*e^11 + 589824*a^7*c^9*d^4*e^13 + 327680*a^8*c^8*d^2*e^15))/(512*(a^2*e^5 + c^2*d^4*e + 2*a*c*d^2*e^3)*(c^7*d^8 + a^4*c^3*e^8 + 4*a*c^6*d^6*e^2 + 6*a^2*c^5*d^4*e^4 + 4*a^3*c^4*d^2*e^6))))/(2*(a^2*e^5 + c^2*d^4*e + 2*a*c*d^2*e^3)) + (x*(1920*a^8*c^4*d*e^14 + 13184*a^2*c^10*d^13*e^2 + 16640*a^3*c^9*d^11*e^4 + 18560*a^4*c^8*d^9*e^6 + 56832*a^5*c^7*d^7*e^8 + 60544*a^6*c^6*d^5*e^10 + 20736*a^7*c^5*d^3*e^12))/(256*(c^7*d^8 + a^4*c^3*e^8 + 4*a*c^6*d^6*e^2 + 6*a^2*c^5*d^4*e^4 + 4*a^3*c^4*d^2*e^6))))/(2*(a^2*e^5 + c^2*d^4*e + 2*a*c*d^2*e^3)))*(-d^7*e)^(1/2))/(2*(a^2*e^5 + c^2*d^4*e + 2*a*c*d^2*e^3)))*(-d^7*e)^(1/2))/(a^2*e^5 + c^2*d^4*e + 2*a*c*d^2*e^3) - (((x*(81*a^8*e^13 + 800*a^2*c^6*d^12*e + 612*a^7*c*d^2*e^11 + 832*a^3*c^5*d^10*e^3 + 913*a^4*c^4*d^8*e^5 + 1700*a^5*c^3*d^6*e^7 + 1606*a^6*c^2*d^4*e^9))/(256*(c^7*d^8 + a^4*c^3*e^8 + 4*a*c^6*d^6*e^2 + 6*a^2*c^5*d^4*e^4 + 4*a^3*c^4*d^2*e^6)) - (((20*a^2*c^8*d^13*e + (27*a^8*c^2*d*e^13)/16 - (1077*a^3*c^7*d^11*e^3)/16 - (2361*a^4*c^6*d^9*e^5)/16 - (425*a^5*c^5*d^7*e^7)/8 + (135*a^6*c^4*d^5*e^9)/8 + (183*a^7*c^3*d^3*e^11)/16)/(2*(c^7*d^8 + a^4*c^3*e^8 + 4*a*c^6*d^6*e^2 + 6*a^2*c^5*d^4*e^4 + 4*a^3*c^4*d^2*e^6)) - ((-d^7*e)^(1/2)*(((-d^7*e)^(1/2)*((320*a^5*c^9*d^8*e^8 - 288*a^3*c^11*d^12*e^4 - 240*a^4*c^10*d^10*e^6 - 80*a^2*c^12*d^14*e^2 + 720*a^6*c^8*d^6*e^10 + 480*a^7*c^7*d^4*e^12 + 112*a^8*c^6*d^2*e^14)/(2*(c^7*d^8 + a^4*c^3*e^8 + 4*a*c^6*d^6*e^2 + 6*a^2*c^5*d^4*e^4 + 4*a^3*c^4*d^2*e^6)) + (x*(-d^7*e)^(1/2)*(65536*a^9*c^7*e^17 - 65536*a^2*c^14*d^14*e^3 - 327680*a^3*c^13*d^12*e^5 - 589824*a^4*c^12*d^10*e^7 - 327680*a^5*c^11*d^8*e^9 + 327680*a^6*c^10*d^6*e^11 + 589824*a^7*c^9*d^4*e^13 + 327680*a^8*c^8*d^2*e^15))/(512*(a^2*e^5 + c^2*d^4*e + 2*a*c*d^2*e^3)*(c^7*d^8 + a^4*c^3*e^8 + 4*a*c^6*d^6*e^2 + 6*a^2*c^5*d^4*e^4 + 4*a^3*c^4*d^2*e^6))))/(2*(a^2*e^5 + c^2*d^4*e + 2*a*c*d^2*e^3)) - (x*(1920*a^8*c^4*d*e^14 + 13184*a^2*c^10*d^13*e^2 + 16640*a^3*c^9*d^11*e^4 + 18560*a^4*c^8*d^9*e^6 + 56832*a^5*c^7*d^7*e^8 + 60544*a^6*c^6*d^5*e^10 + 20736*a^7*c^5*d^3*e^12))/(256*(c^7*d^8 + a^4*c^3*e^8 + 4*a*c^6*d^6*e^2 + 6*a^2*c^5*d^4*e^4 + 4*a^3*c^4*d^2*e^6))))/(2*(a^2*e^5 + c^2*d^4*e + 2*a*c*d^2*e^3)))*(-d^7*e)^(1/2))/(2*(a^2*e^5 + c^2*d^4*e + 2*a*c*d^2*e^3)))*(-d^7*e)^(1/2))/(a^2*e^5 + c^2*d^4*e + 2*a*c*d^2*e^3)))*(-d^7*e)^(1/2)*1i)/(a^2*e^5 + c^2*d^4*e + 2*a*c*d^2*e^3)

3.253 $\int \frac{x^6}{(d+ex^2)(a+cx^4)^2}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 687

$$
\begin{aligned}
\int \frac{x^6}{(d+ex^2)(a+cx^4)^2}\,dx = & -\frac{x(ae+cdx^2)}{4c(cd^2+ae^2)(a+cx^4)} - \frac{d^{5/2}\sqrt{e}\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{(cd^2+ae^2)^2} \\
& - \frac{d^2(\sqrt{c}d+\sqrt{a}e)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}(cd^2+ae^2)^2} \\
& + \frac{(\sqrt{c}d-\sqrt{a}e)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{8\sqrt{2}\sqrt[4]{a}c^{5/4}(cd^2+ae^2)} \\
& + \frac{d^2(\sqrt{c}d+\sqrt{a}e)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}(cd^2+ae^2)^2} \\
& - \frac{(\sqrt{c}d-\sqrt{a}e)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{8\sqrt{2}\sqrt[4]{a}c^{5/4}(cd^2+ae^2)} \\
& + \frac{d^2(\sqrt{c}d-\sqrt{a}e)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)}{4\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}(cd^2+ae^2)^2} \\
& - \frac{(\sqrt{c}d+\sqrt{a}e)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)}{16\sqrt{2}\sqrt[4]{a}c^{5/4}(cd^2+ae^2)} \\
& - \frac{d^2(\sqrt{c}d-\sqrt{a}e)\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)}{4\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}(cd^2+ae^2)^2} \\
& + \frac{(\sqrt{c}d+\sqrt{a}e)\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)}{16\sqrt{2}\sqrt[4]{a}c^{5/4}(cd^2+ae^2)}
\end{aligned}
$$

```
[Out] -1/4*x*(c*d*x^2+a*e)/c/(a*e^2+c*d^2)/(c*x^4+a)-1/16*arctan(-1+c^(1/4)*x*2^(
1/2)/a^(1/4))*(-e*a^(1/2)+d*c^(1/2))/a^(1/4)/c^(5/4)/(a*e^2+c*d^2)*2^(1/2)-
1/16*arctan(1+c^(1/4)*x*2^(1/2)/a^(1/4))*(-e*a^(1/2)+d*c^(1/2))/a^(1/4)/c^(
5/4)/(a*e^2+c*d^2)*2^(1/2)+1/8*d^2*ln(-a^(1/4)*c^(1/4)*x*2^(1/2)+a^(1/2)+x^
2*c^(1/2))*(-e*a^(1/2)+d*c^(1/2))/a^(1/4)/c^(1/4)/(a*e^2+c*d^2)^2*2^(1/2)-1
/8*d^2*ln(a^(1/4)*c^(1/4)*x*2^(1/2)+a^(1/2)+x^2*c^(1/2))*(-e*a^(1/2)+d*c^(1
/2))/a^(1/4)/c^(1/4)/(a*e^2+c*d^2)^2*2^(1/2)+1/4*d^2*arctan(-1+c^(1/4)*x*2^
(1/2)/a^(1/4))*(e*a^(1/2)+d*c^(1/2))/a^(1/4)/c^(1/4)/(a*e^2+c*d^2)^2*2^(1/2
)+1/4*d^2*arctan(1+c^(1/4)*x*2^(1/2)/a^(1/4))*(e*a^(1/2)+d*c^(1/2))/a^(1/4)
/c^(1/4)/(a*e^2+c*d^2)^2*2^(1/2)-1/32*ln(-a^(1/4)*c^(1/4)*x*2^(1/2)+a^(1/2)
+x^2*c^(1/2))*(e*a^(1/2)+d*c^(1/2))/a^(1/4)/c^(5/4)/(a*e^2+c*d^2)*2^(1/2)+1
/32*ln(a^(1/4)*c^(1/4)*x*2^(1/2)+a^(1/2)+x^2*c^(1/2))*(e*a^(1/2)+d*c^(1/2))
/a^(1/4)/c^(5/4)/(a*e^2+c*d^2)*2^(1/2)-d^(5/2)*arctan(x*e^(1/2)/d^(1/2))*e^
(1/2)/(a*e^2+c*d^2)^2
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 687, normalized size of antiderivative = 1.00, number of steps used = 23, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.455$, Rules

used = {1328, 1290, 1182, 1176, 631, 210, 1179, 642, 1302, 211}

$$\int \frac{x^6}{(d+ex^2)\,(a+cx^4)^2}\,dx = \frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)(\sqrt{c}d-\sqrt{a}e)}{8\sqrt{2}\sqrt[4]{a}c^{5/4}\,(ae^2+cd^2)} - \frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)(\sqrt{c}d-\sqrt{a}e)}{8\sqrt{2}\sqrt[4]{a}c^{5/4}\,(ae^2+cd^2)} - \frac{d^2\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)(\sqrt{a}e+\sqrt{c}d)}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}\,(ae^2+cd^2)^2} + \frac{d^2\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)(\sqrt{a}e+\sqrt{c}d)}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}\,(ae^2+cd^2)^2} - \frac{d^{5/2}\sqrt{e}\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{(ae^2+cd^2)^2} - \frac{(\sqrt{a}e+\sqrt{c}d)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{16\sqrt{2}\sqrt[4]{a}c^{5/4}\,(ae^2+cd^2)} + \frac{(\sqrt{a}e+\sqrt{c}d)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{16\sqrt{2}\sqrt[4]{a}c^{5/4}\,(ae^2+cd^2)} + \frac{d^2(\sqrt{c}d-\sqrt{a}e)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{4\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}\,(ae^2+cd^2)^2} - \frac{d^2(\sqrt{c}d-\sqrt{a}e)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{4\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}\,(ae^2+cd^2)^2} - \frac{x(ae+cdx^2)}{4c\,(a+cx^4)\,(ae^2+cd^2)}$$

```
[In] Int[x^6/((d + e*x^2)*(a + c*x^4)^2),x]
```

```
[Out] -1/4*(x*(a*e + c*d*x^2))/(c*(c*d^2 + a*e^2)*(a + c*x^4)) - (d^(5/2)*Sqrt[e]
*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(c*d^2 + a*e^2)^2 - (d^2*(Sqrt[c]*d + Sqrt[a]
*e)*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(1/4)*c^(1/4)*(c*
d^2 + a*e^2)^2) + ((Sqrt[c]*d - Sqrt[a]*e)*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a
^(1/4)])/(8*Sqrt[2]*a^(1/4)*c^(5/4)*(c*d^2 + a*e^2)) + (d^2*(Sqrt[c]*d + Sq
rt[a]*e)*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(1/4)*c^(1/4
)*(c*d^2 + a*e^2)^2) - ((Sqrt[c]*d - Sqrt[a]*e)*ArcTan[1 + (Sqrt[2]*c^(1/4)
*x)/a^(1/4)])/(8*Sqrt[2]*a^(1/4)*c^(5/4)*(c*d^2 + a*e^2)) + (d^2*(Sqrt[c]*d
 - Sqrt[a]*e)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(4*Sq
rt[2]*a^(1/4)*c^(1/4)*(c*d^2 + a*e^2)^2) - ((Sqrt[c]*d + Sqrt[a]*e)*Log[Sqr
t[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(16*Sqrt[2]*a^(1/4)*c^(5/4
)*(c*d^2 + a*e^2)) - (d^2*(Sqrt[c]*d - Sqrt[a]*e)*Log[Sqrt[a] + Sqrt[2]*a^(
```

```
1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(4*Sqrt[2]*a^(1/4)*c^(1/4)*(c*d^2 + a*e^2)^2
) + ((Sqrt[c]*d + Sqrt[a]*e)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt
[c]*x^2])/(16*Sqrt[2]*a^(1/4)*c^(5/4)*(c*d^2 + a*e^2))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1182

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
```

```
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[(-a)
*c]
```

Rule 1290

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (c_.)*(x_)^4)^(p_.), x
_Symbol] :> Simp[f*(f*x)^(m - 1)*(a + c*x^4)^(p + 1)*((a*e - c*d*x^2)/(4*a*
c*(p + 1))), x] - Dist[f^2/(4*a*c*(p + 1)), Int[(f*x)^(m - 2)*(a + c*x^4)^(
p + 1)*(a*e*(m - 1) - c*d*(4*p + 4 + m + 1)*x^2), x], x] /; FreeQ[{a, c, d,
 e, f}, x] && LtQ[p, -1] && GtQ[m, 1] && IntegerQ[2*p] && (IntegerQ[p] || I
ntegerQ[m])
```

Rule 1302

```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.))/((a_) + (c_.)*(x_)^4),
 x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*((d + e*x^2)^q/(a + c*x^4)), x],
x] /; FreeQ[{a, c, d, e, f, m}, x] && IntegerQ[q] && IntegerQ[m]
```

Rule 1328

```
Int[(((f_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^4)^(p_))/((d_.) + (e_.)*(x_)^2),
 x_Symbol] :> Dist[(-a)*(f^4/(c*d^2 + a*e^2)), Int[(f*x)^(m - 4)*(d - e*x^2
)*(a + c*x^4)^p, x], x] + Dist[d^2*(f^4/(c*d^2 + a*e^2)), Int[(f*x)^(m - 4)
*((a + c*x^4)^(p + 1)/(d + e*x^2)), x], x] /; FreeQ[{a, c, d, e, f}, x] &&
LtQ[p, -1] && GtQ[m, 2]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{a\int \frac{x^2(d-ex^2)}{(a+cx^4)^2}\,dx}{cd^2+ae^2}+\frac{d^2\int \frac{x^2}{(d+ex^2)(a+cx^4)}\,dx}{cd^2+ae^2}\\
&= -\frac{x(ae+cdx^2)}{4c\,(cd^2+ae^2)\,(a+cx^4)}-\frac{\int \frac{-ae+cdx^2}{a+cx^4}\,dx}{4c\,(cd^2+ae^2)}+\frac{d^2\int \left(-\frac{de}{(cd^2+ae^2)(d+ex^2)}+\frac{ae+cdx^2}{(cd^2+ae^2)(a+cx^4)}\right)dx}{cd^2+ae^2}\\
&= -\frac{x(ae+cdx^2)}{4c\,(cd^2+ae^2)\,(a+cx^4)}+\frac{d^2\int \frac{ae+cdx^2}{a+cx^4}\,dx}{(cd^2+ae^2)^2}-\frac{(d^3e)\int \frac{1}{d+ex^2}\,dx}{(cd^2+ae^2)^2}\\
&\quad -\frac{\left(d-\frac{\sqrt{a}e}{\sqrt{c}}\right)\int \frac{\sqrt{a}\sqrt{c}+cx^2}{a+cx^4}\,dx}{8c\,(cd^2+ae^2)}+\frac{\left(d+\frac{\sqrt{a}e}{\sqrt{c}}\right)\int \frac{\sqrt{a}\sqrt{c}-cx^2}{a+cx^4}\,dx}{8c\,(cd^2+ae^2)}
\end{aligned}
$$

$$= -\frac{x(ae+cdx^2)}{4c(cd^2+ae^2)(a+cx^4)} - \frac{d^{5/2}\sqrt{e}\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{(cd^2+ae^2)^2} - \frac{\left(d^2\left(d-\frac{\sqrt{a}e}{\sqrt{c}}\right)\right)\int\frac{\sqrt{a}\sqrt{c}-cx^2}{a+cx^4}\,dx}{2(cd^2+ae^2)^2} + \frac{\left(d^2\left(d+\frac{\sqrt{a}e}{\sqrt{c}}\right)\right)\int\frac{\sqrt{a}\sqrt{c}+cx^2}{a+cx^4}\,dx}{2(cd^2+ae^2)^2} - \frac{\left(\sqrt{c}d+\sqrt{a}e\right)\int\frac{\frac{\sqrt{2}\sqrt[4]{a}}{\sqrt[4]{c}}+2x}{-\frac{\sqrt{a}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}-x^2}\,dx}{16\sqrt{2}\sqrt[4]{a}c^{5/4}(cd^2+ae^2)} - \frac{\left(\sqrt{c}d+\sqrt{a}e\right)\int\frac{\frac{\sqrt{2}\sqrt[4]{a}}{\sqrt[4]{c}}-2x}{-\frac{\sqrt{a}}{\sqrt{c}}+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}-x^2}\,dx}{16\sqrt{2}\sqrt[4]{a}c^{5/4}(cd^2+ae^2)} - \frac{\left(d-\frac{\sqrt{a}e}{\sqrt{c}}\right)\int\frac{1}{\frac{\sqrt{a}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}+x^2}\,dx}{16c(cd^2+ae^2)} - \frac{\left(d-\frac{\sqrt{a}e}{\sqrt{c}}\right)\int\frac{1}{\frac{\sqrt{a}}{\sqrt{c}}+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}+x^2}\,dx}{16c(cd^2+ae^2)}$$

$$= -\frac{x(ae+cdx^2)}{4c(cd^2+ae^2)(a+cx^4)} - \frac{d^{5/2}\sqrt{e}\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{(cd^2+ae^2)^2} - \frac{\left(\sqrt{c}d+\sqrt{a}e\right)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)}{16\sqrt{2}\sqrt[4]{a}c^{5/4}(cd^2+ae^2)} + \frac{\left(\sqrt{c}d+\sqrt{a}e\right)\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)}{16\sqrt{2}\sqrt[4]{a}c^{5/4}(cd^2+ae^2)} + \frac{\left(\sqrt[4]{c}d^2\left(d-\frac{\sqrt{a}e}{\sqrt{c}}\right)\right)\int\frac{\frac{\sqrt{2}\sqrt[4]{a}}{\sqrt[4]{c}}+2x}{-\frac{\sqrt{a}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}-x^2}\,dx}{4\sqrt{2}\sqrt[4]{a}(cd^2+ae^2)^2} + \frac{\left(\sqrt[4]{c}d^2\left(d-\frac{\sqrt{a}e}{\sqrt{c}}\right)\right)\int\frac{\frac{\sqrt{2}\sqrt[4]{a}}{\sqrt[4]{c}}-2x}{-\frac{\sqrt{a}}{\sqrt{c}}+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}-x^2}\,dx}{4\sqrt{2}\sqrt[4]{a}(cd^2+ae^2)^2} + \frac{\left(d^2\left(d+\frac{\sqrt{a}e}{\sqrt{c}}\right)\right)\int\frac{1}{\frac{\sqrt{a}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}+x^2}\,dx}{4(cd^2+ae^2)^2} + \frac{\left(d^2\left(d+\frac{\sqrt{a}e}{\sqrt{c}}\right)\right)\int\frac{1}{\frac{\sqrt{a}}{\sqrt{c}}+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}+x^2}\,dx}{4(cd^2+ae^2)^2} - \frac{\left(d-\frac{\sqrt{a}e}{\sqrt{c}}\right)\text{Subst}\left(\int\frac{1}{-1-x^2}\,dx,x,1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{8\sqrt{2}\sqrt[4]{a}c^{3/4}(cd^2+ae^2)} + \frac{\left(d-\frac{\sqrt{a}e}{\sqrt{c}}\right)\text{Subst}\left(\int\frac{1}{-1-x^2}\,dx,x,1+\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{8\sqrt{2}\sqrt[4]{a}c^{3/4}(cd^2+ae^2)}$$

$$= -\frac{x(ae+cdx^2)}{4c(cd^2+ae^2)(a+cx^4)} - \frac{d^{5/2}\sqrt{e}\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{(cd^2+ae^2)^2} + \frac{\left(d-\frac{\sqrt{a}e}{\sqrt{c}}\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{8\sqrt{2}\sqrt[4]{a}c^{3/4}(cd^2+ae^2)} - \frac{\left(d-\frac{\sqrt{a}e}{\sqrt{c}}\right)\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{8\sqrt{2}\sqrt[4]{a}c^{3/4}(cd^2+ae^2)} + \frac{\sqrt[4]{c}d^2\left(d-\frac{\sqrt{a}e}{\sqrt{c}}\right)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)}{4\sqrt{2}\sqrt[4]{a}(cd^2+ae^2)^2} - \frac{\left(\sqrt{c}d+\sqrt{a}e\right)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)}{16\sqrt{2}\sqrt[4]{a}c^{5/4}(cd^2+ae^2)} - \frac{\sqrt[4]{c}d^2\left(d-\frac{\sqrt{a}e}{\sqrt{c}}\right)\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)}{4\sqrt{2}\sqrt[4]{a}(cd^2+ae^2)^2} + \frac{\left(\sqrt{c}d+\sqrt{a}e\right)\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)}{16\sqrt{2}\sqrt[4]{a}c^{5/4}(cd^2+ae^2)} + \frac{\left(\sqrt[4]{c}d^2\left(d+\frac{\sqrt{a}e}{\sqrt{c}}\right)\right)\text{Subst}\left(\int\frac{1}{-1-x^2}\,dx,x,1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}\sqrt[4]{a}(cd^2+ae^2)^2} - \frac{\left(\sqrt[4]{c}d^2\left(d+\frac{\sqrt{a}e}{\sqrt{c}}\right)\right)\text{Subst}\left(\int\frac{1}{-1-x^2}\,dx,x,1+\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}\sqrt[4]{a}(cd^2+ae^2)^2}$$

$$= -\frac{x(ae+cdx^2)}{4c(cd^2+ae^2)(a+cx^4)} - \frac{d^{5/2}\sqrt{e}\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{(cd^2+ae^2)^2} - \frac{\sqrt[4]{c}d^2\left(d+\frac{\sqrt{a}e}{\sqrt{c}}\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}\sqrt[4]{a}(cd^2+ae^2)^2} + \frac{\left(d-\frac{\sqrt{a}e}{\sqrt{c}}\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{8\sqrt{2}\sqrt[4]{a}c^{3/4}(cd^2+ae^2)} + \frac{\sqrt[4]{c}d^2\left(d+\frac{\sqrt{a}e}{\sqrt{c}}\right)\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}\sqrt[4]{a}(cd^2+ae^2)^2} - \frac{\left(d-\frac{\sqrt{a}e}{\sqrt{c}}\right)\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{8\sqrt{2}\sqrt[4]{a}c^{3/4}(cd^2+ae^2)} + \frac{\sqrt[4]{c}d^2\left(d-\frac{\sqrt{a}e}{\sqrt{c}}\right)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)}{4\sqrt{2}\sqrt[4]{a}(cd^2+ae^2)^2} - \frac{\left(\sqrt{c}d+\sqrt{a}e\right)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)}{16\sqrt{2}\sqrt[4]{a}c^{5/4}(cd^2+ae^2)} - \frac{\sqrt[4]{c}d^2\left(d-\frac{\sqrt{a}e}{\sqrt{c}}\right)\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)}{4\sqrt{2}\sqrt[4]{a}(cd^2+ae^2)^2} + \frac{\left(\sqrt{c}d+\sqrt{a}e\right)\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)}{16\sqrt{2}\sqrt[4]{a}c^{5/4}(cd^2+ae^2)}$$

Mathematica [A] (verified)

Time = 0.26 (sec) , antiderivative size = 428, normalized size of antiderivative = 0.62

$$\int \frac{x^6}{(d+ex^2)(a+cx^4)^2}\,dx =$$

$$-\frac{\frac{8(cd^2+ae^2)(aex+cdx^3)}{c(a+cx^4)}+32d^{5/2}\sqrt{e}\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)+\frac{2\sqrt{2}(3c^{3/2}d^3+5\sqrt{a}cd^2e-a\sqrt{c}de^2+a^{3/2}e^3)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{\sqrt[4]{a}c^{5/4}}-\frac{2\sqrt{2}(}{}}{}$$

```
[In] Integrate[x^6/((d + e*x^2)*(a + c*x^4)^2),x]
```

```
[Out] -1/32*((8*(c*d^2 + a*e^2)*(a*e*x + c*d*x^3))/(c*(a + c*x^4)) + 32*d^(5/2)*S
qrt[e]*ArcTan[(Sqrt[e]*x)/Sqrt[d]] + (2*Sqrt[2]*(3*c^(3/2)*d^3 + 5*Sqrt[a]*
c*d^2*e - a*Sqrt[c]*d*e^2 + a^(3/2)*e^3)*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(
1/4)])/(a^(1/4)*c^(5/4)) - (2*Sqrt[2]*(3*c^(3/2)*d^3 + 5*Sqrt[a]*c*d^2*e -
a*Sqrt[c]*d*e^2 + a^(3/2)*e^3)*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(a^
(1/4)*c^(5/4)) + (Sqrt[2]*(-3*c^(3/2)*d^3 + 5*Sqrt[a]*c*d^2*e + a*Sqrt[c]*d
*e^2 + a^(3/2)*e^3)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])
/(a^(1/4)*c^(5/4)) - (Sqrt[2]*(-3*c^(3/2)*d^3 + 5*Sqrt[a]*c*d^2*e + a*Sqrt[
c]*d*e^2 + a^(3/2)*e^3)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x
^2])/(a^(1/4)*c^(5/4)))/(c*d^2 + a*e^2)^2
```

Maple [A] (verified)

Time = 0.46 (sec) , antiderivative size = 339, normalized size of antiderivative = 0.49

method	result
default	$\frac{\frac{\left(-\frac{1}{4}de^2a-\frac{1}{4}d^3c\right)x^3-\frac{ae\left(ae^2+cd^2\right)x}{4c}}{cx^4+a}+\frac{\frac{\left(e^3a^2+5acd^2e\right)\left(\frac{a}{c}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x^2+\left(\frac{a}{c}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{c}}}{x^2-\left(\frac{a}{c}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{c}}}\right)+2\arctan\left(\frac{\sqrt{2}x}{\left(\frac{a}{c}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}x}{\left(\frac{a}{c}\right)^{\frac{1}{4}}}-1\right)\right)}{8a}}{4c}}{\left(ae^2+cd^2\right)^2}$
risch	Expression too large to display

```
[In] int(x^6/(e*x^2+d)/(c*x^4+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/(a*e^2+c*d^2)^2*(((-1/4*d*e^2*a-1/4*d^3*c)*x^3-1/4*a*e*(a*e^2+c*d^2)/c*x)
/(c*x^4+a)+1/4/c*(1/8*(a^2*e^3+5*a*c*d^2*e)*(a/c)^(1/4)/a*2^(1/2)*(ln((x^2+
(a/c)^(1/4)*x*2^(1/2)+(a/c)^(1/2))/(x^2-(a/c)^(1/4)*x*2^(1/2)+(a/c)^(1/2)))
+2*arctan(2^(1/2)/(a/c)^(1/4)*x+1)+2*arctan(2^(1/2)/(a/c)^(1/4)*x-1))+1/8*(
-a*c*d*e^2+3*c^2*d^3)/c/(a/c)^(1/4)*2^(1/2)*(ln((x^2-(a/c)^(1/4)*x*2^(1/2)+
(a/c)^(1/2))/(x^2+(a/c)^(1/4)*x*2^(1/2)+(a/c)^(1/2)))+2*arctan(2^(1/2)/(a/c
```

```
)^(1/4)*x+1)+2*arctan(2^(1/2)/(a/c)^(1/4)*x-1))))-e/(a*e^2+c*d^2)^2*d^3/(e*
d)^(1/2)*arctan(e*x/(e*d)^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 4901 vs. 2(515) = 1030.

Time = 6.61 (sec) , antiderivative size = 9822, normalized size of antiderivative = 14.30

$$\int \frac{x^6}{(d+ex^2)(a+cx^4)^2}\,dx = \text{Too large to display}$$

```
[In] integrate(x^6/(e*x^2+d)/(c*x^4+a)^2,x, algorithm="fricas")
```

```
[Out] Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^6}{(d+ex^2)(a+cx^4)^2}\,dx = \text{Timed out}$$

```
[In] integrate(x**6/(e*x**2+d)/(c*x**4+a)**2,x)
```

```
[Out] Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^6}{(d+ex^2)(a+cx^4)^2}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(x^6/(e*x^2+d)/(c*x^4+a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 614, normalized size of antiderivative = 0.89

$$\int \frac{x^6}{(d+ex^2)(a+cx^4)^2}\,dx = -\frac{d^3e\arctan\left(\frac{ex}{\sqrt{de}}\right)}{(c^2d^4+2\,acd^2e^2+a^2e^4)\sqrt{de}} + \frac{\left(5\,(ac^3)^{\frac{1}{4}}\,ac^2d^2e+(ac^3)^{\frac{1}{4}}\,a^2ce^3+3\,(ac^3)^{\frac{3}{4}}\,cd^3-(ac^3)^{\frac{3}{4}}\,ade^2\right)\arctan\left(\frac{\sqrt{2}\left(2\,x+\sqrt{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{8\left(\sqrt{2}ac^5d^4+2\,\sqrt{2}a^2c^4d^2e^2+\sqrt{2}a^3c^3e^4\right)} + \frac{\left(5\,(ac^3)^{\frac{1}{4}}\,ac^2d^2e+(ac^3)^{\frac{1}{4}}\,a^2ce^3+3\,(ac^3)^{\frac{3}{4}}\,cd^3-(ac^3)^{\frac{3}{4}}\,ade^2\right)\arctan\left(\frac{\sqrt{2}\left(2\,x-\sqrt{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{8\left(\sqrt{2}ac^5d^4+2\,\sqrt{2}a^2c^4d^2e^2+\sqrt{2}a^3c^3e^4\right)} + \frac{\left(5\,(ac^3)^{\frac{1}{4}}\,ac^2d^2e+(ac^3)^{\frac{1}{4}}\,a^2ce^3-3\,(ac^3)^{\frac{3}{4}}\,cd^3+(ac^3)^{\frac{3}{4}}\,ade^2\right)\log\left(x^2+\sqrt{2}x\left(\frac{a}{c}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{c}}\right)}{16\left(\sqrt{2}ac^5d^4+2\,\sqrt{2}a^2c^4d^2e^2+\sqrt{2}a^3c^3e^4\right)} - \frac{\left(5\,(ac^3)^{\frac{1}{4}}\,ac^2d^2e+(ac^3)^{\frac{1}{4}}\,a^2ce^3-3\,(ac^3)^{\frac{3}{4}}\,cd^3+(ac^3)^{\frac{3}{4}}\,ade^2\right)\log\left(x^2-\sqrt{2}x\left(\frac{a}{c}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{c}}\right)}{16\left(\sqrt{2}ac^5d^4+2\,\sqrt{2}a^2c^4d^2e^2+\sqrt{2}a^3c^3e^4\right)} - \frac{cdx^3+aex}{4\,(cx^4+a)(c^2d^2+ace^2)}$$

```
[In] integrate(x^6/(e*x^2+d)/(c*x^4+a)^2,x, algorithm="giac")
```

```
[Out] -d^3*e*arctan(e*x/sqrt(d*e))/((c^2*d^4 + 2*a*c*d^2*e^2 + a^2*e^4)*sqrt(d*e)
) + 1/8*(5*(a*c^3)^(1/4)*a*c^2*d^2*e + (a*c^3)^(1/4)*a^2*c*e^3 + 3*(a*c^3)^
(3/4)*c*d^3 - (a*c^3)^(3/4)*a*d*e^2)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(a/c
)^(1/4))/(a/c)^(1/4))/(sqrt(2)*a*c^5*d^4 + 2*sqrt(2)*a^2*c^4*d^2*e^2 + sqrt
(2)*a^3*c^3*e^4) + 1/8*(5*(a*c^3)^(1/4)*a*c^2*d^2*e + (a*c^3)^(1/4)*a^2*c*e
^3 + 3*(a*c^3)^(3/4)*c*d^3 - (a*c^3)^(3/4)*a*d*e^2)*arctan(1/2*sqrt(2)*(2*x
 - sqrt(2)*(a/c)^(1/4))/(a/c)^(1/4))/(sqrt(2)*a*c^5*d^4 + 2*sqrt(2)*a^2*c^4
*d^2*e^2 + sqrt(2)*a^3*c^3*e^4) + 1/16*(5*(a*c^3)^(1/4)*a*c^2*d^2*e + (a*c^
3)^(1/4)*a^2*c*e^3 - 3*(a*c^3)^(3/4)*c*d^3 + (a*c^3)^(3/4)*a*d*e^2)*log(x^2
 + sqrt(2)*x*(a/c)^(1/4) + sqrt(a/c))/(sqrt(2)*a*c^5*d^4 + 2*sqrt(2)*a^2*c^
4*d^2*e^2 + sqrt(2)*a^3*c^3*e^4) - 1/16*(5*(a*c^3)^(1/4)*a*c^2*d^2*e + (a*c
^3)^(1/4)*a^2*c*e^3 - 3*(a*c^3)^(3/4)*c*d^3 + (a*c^3)^(3/4)*a*d*e^2)*log(x^
2 - sqrt(2)*x*(a/c)^(1/4) + sqrt(a/c))/(sqrt(2)*a*c^5*d^4 + 2*sqrt(2)*a^2*c
^4*d^2*e^2 + sqrt(2)*a^3*c^3*e^4) - 1/4*(c*d*x^3 + a*e*x)/((c*x^4 + a)*(c^2
*d^2 + a*c*e^2))
```

Mupad [B] (verification not implemented)

Time = 9.02 (sec) , antiderivative size = 17909, normalized size of antiderivative = 26.07

$$\int \frac{x^6}{(d+ex^2)(a+cx^4)^2}\,dx = \text{Too large to display}$$

```
[In] int(x^6/((a + c*x^4)^2*(d + e*x^2)),x)
```

```
[Out] atan(((((432*a*c^7*d^12*e^2 + 13040*a^2*c^6*d^10*e^4 + 12000*a^3*c^5*d^8*e^
6 - 1056*a^4*c^4*d^6*e^8 - 400*a^5*c^3*d^4*e^10 + 48*a^6*c^2*d^2*e^12)/(256
*(c^5*d^8 + a^4*c*e^8 + 4*a*c^4*d^6*e^2 + 6*a^2*c^3*d^4*e^4 + 4*a^3*c^2*d^2
*e^6)) + (((45056*a^2*c^10*d^13*e^3 - 4096*a^8*c^4*d*e^15 + 221184*a^3*c^9*
d^11*e^5 + 430080*a^4*c^8*d^9*e^7 + 409600*a^5*c^7*d^7*e^9 + 184320*a^6*c^6
*d^5*e^11 + 24576*a^7*c^5*d^3*e^13)/(256*(c^5*d^8 + a^4*c*e^8 + 4*a*c^4*d^6
*e^2 + 6*a^2*c^3*d^4*e^4 + 4*a^3*c^2*d^2*e^6)) - (x*(-(a^3*e^6*(-a*c^5)^(1/
2) - 9*c^3*d^6*(-a*c^5)^(1/2) - 2*a^3*c^3*d*e^5 - 4*a^2*c^4*d^3*e^3 + 30*a*
c^5*d^5*e + 31*a*c^2*d^4*e^2*(-a*c^5)^(1/2) + 9*a^2*c*d^2*e^4*(-a*c^5)^(1/2
))/(256*(a*c^9*d^8 + a^5*c^5*e^8 + 4*a^2*c^8*d^6*e^2 + 6*a^3*c^7*d^4*e^4 +
4*a^4*c^6*d^2*e^6)))^(1/2)*(65536*a^9*c^5*e^17 - 65536*a^2*c^12*d^14*e^3 -
327680*a^3*c^11*d^12*e^5 - 589824*a^4*c^10*d^10*e^7 - 327680*a^5*c^9*d^8*e^
9 + 327680*a^6*c^8*d^6*e^11 + 589824*a^7*c^7*d^4*e^13 + 327680*a^8*c^6*d^2*
e^15))/(128*(c^5*d^8 + a^4*c*e^8 + 4*a*c^4*d^6*e^2 + 6*a^2*c^3*d^4*e^4 + 4*
a^3*c^2*d^2*e^6)))*(-(a^3*e^6*(-a*c^5)^(1/2) - 9*c^3*d^6*(-a*c^5)^(1/2) - 2
*a^3*c^3*d*e^5 - 4*a^2*c^4*d^3*e^3 + 30*a*c^5*d^5*e + 31*a*c^2*d^4*e^2*(-a*
c^5)^(1/2) + 9*a^2*c*d^2*e^4*(-a*c^5)^(1/2))/(256*(a*c^9*d^8 + a^5*c^5*e^8
+ 4*a^2*c^8*d^6*e^2 + 6*a^3*c^7*d^4*e^4 + 4*a^4*c^6*d^2*e^6)))^(1/2) + (x*(
1152*a*c^9*d^13*e^2 + 1152*a^7*c^3*d*e^14 + 21248*a^2*c^8*d^11*e^4 + 25472*
a^3*c^7*d^9*e^6 - 5632*a^4*c^6*d^7*e^8 - 7296*a^5*c^5*d^5*e^10 + 4864*a^6*c
^4*d^3*e^12))/(128*(c^5*d^8 + a^4*c*e^8 + 4*a*c^4*d^6*e^2 + 6*a^2*c^3*d^4*e
^4 + 4*a^3*c^2*d^2*e^6)))*(-(a^3*e^6*(-a*c^5)^(1/2) - 9*c^3*d^6*(-a*c^5)^(1
/2) - 2*a^3*c^3*d*e^5 - 4*a^2*c^4*d^3*e^3 + 30*a*c^5*d^5*e + 31*a*c^2*d^4*e
^2*(-a*c^5)^(1/2) + 9*a^2*c*d^2*e^4*(-a*c^5)^(1/2))/(256*(a*c^9*d^8 + a^5*c
^5*e^8 + 4*a^2*c^8*d^6*e^2 + 6*a^3*c^7*d^4*e^4 + 4*a^4*c^6*d^2*e^6)))^(1/2)
)*(-(a^3*e^6*(-a*c^5)^(1/2) - 9*c^3*d^6*(-a*c^5)^(1/2) - 2*a^3*c^3*d*e^5 -
4*a^2*c^4*d^3*e^3 + 30*a*c^5*d^5*e + 31*a*c^2*d^4*e^2*(-a*c^5)^(1/2) + 9*a^
2*c*d^2*e^4*(-a*c^5)^(1/2))/(256*(a*c^9*d^8 + a^5*c^5*e^8 + 4*a^2*c^8*d^6*e
^2 + 6*a^3*c^7*d^4*e^4 + 4*a^4*c^6*d^2*e^6)))^(1/2) - (x*(a^6*e^13 - 288*a*
c^5*d^10*e^3 + 20*a^5*c*d^2*e^11 + 17*a^2*c^4*d^8*e^5 + 148*a^3*c^3*d^6*e^7
 + 118*a^4*c^2*d^4*e^9))/(128*(c^5*d^8 + a^4*c*e^8 + 4*a*c^4*d^6*e^2 + 6*a^
2*c^3*d^4*e^4 + 4*a^3*c^2*d^2*e^6)))*(-(a^3*e^6*(-a*c^5)^(1/2) - 9*c^3*d^6*
(-a*c^5)^(1/2) - 2*a^3*c^3*d*e^5 - 4*a^2*c^4*d^3*e^3 + 30*a*c^5*d^5*e + 31*
a*c^2*d^4*e^2*(-a*c^5)^(1/2) + 9*a^2*c*d^2*e^4*(-a*c^5)^(1/2))/(256*(a*c^9*
d^8 + a^5*c^5*e^8 + 4*a^2*c^8*d^6*e^2 + 6*a^3*c^7*d^4*e^4 + 4*a^4*c^6*d^2*e
^6)))^(1/2)*1i - (((432*a*c^7*d^12*e^2 + 13040*a^2*c^6*d^10*e^4 + 12000*a^3
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*c^5*d^8*e^6 - 1056*a^4*c^4*d^6*e^8 - 400*a^5*c^3*d^4*e^10 + 48*a^6*c^2*d^2
e^12)/(256(c^5*d^8 + a^4*c*e^8 + 4*a*c^4*d^6*e^2 + 6*a^2*c^3*d^4*e^4 + 4*
a^3*c^2*d^2*e^6)) + (((45056*a^2*c^10*d^13*e^3 - 4096*a^8*c^4*d*e^15 + 2211
84*a^3*c^9*d^11*e^5 + 430080*a^4*c^8*d^9*e^7 + 409600*a^5*c^7*d^7*e^9 + 184
320*a^6*c^6*d^5*e^11 + 24576*a^7*c^5*d^3*e^13)/(256*(c^5*d^8 + a^4*c*e^8 +
4*a*c^4*d^6*e^2 + 6*a^2*c^3*d^4*e^4 + 4*a^3*c^2*d^2*e^6)) + (x*(-(a^3*e^6*(
-a*c^5)^(1/2) - 9*c^3*d^6*(-a*c^5)^(1/2) - 2*a^3*c^3*d*e^5 - 4*a^2*c^4*d^3*
e^3 + 30*a*c^5*d^5*e + 31*a*c^2*d^4*e^2*(-a*c^5)^(1/2) + 9*a^2*c*d^2*e^4*(-
a*c^5)^(1/2))/(256*(a*c^9*d^8 + a^5*c^5*e^8 + 4*a^2*c^8*d^6*e^2 + 6*a^3*c^7
*d^4*e^4 + 4*a^4*c^6*d^2*e^6)))^(1/2)*(65536*a^9*c^5*e^17 - 65536*a^2*c^12*
d^14*e^3 - 327680*a^3*c^11*d^12*e^5 - 589824*a^4*c^10*d^10*e^7 - 327680*a^5
*c^9*d^8*e^9 + 327680*a^6*c^8*d^6*e^11 + 589824*a^7*c^7*d^4*e^13 + 327680*a
^8*c^6*d^2*e^15))/(128*(c^5*d^8 + a^4*c*e^8 + 4*a*c^4*d^6*e^2 + 6*a^2*c^3*d
^4*e^4 + 4*a^3*c^2*d^2*e^6)))*(-(a^3*e^6*(-a*c^5)^(1/2) - 9*c^3*d^6*(-a*c^5
)^(1/2) - 2*a^3*c^3*d*e^5 - 4*a^2*c^4*d^3*e^3 + 30*a*c^5*d^5*e + 31*a*c^2*d
^4*e^2*(-a*c^5)^(1/2) + 9*a^2*c*d^2*e^4*(-a*c^5)^(1/2))/(256*(a*c^9*d^8 + a
^5*c^5*e^8 + 4*a^2*c^8*d^6*e^2 + 6*a^3*c^7*d^4*e^4 + 4*a^4*c^6*d^2*e^6)))^(
1/2) - (x*(1152*a*c^9*d^13*e^2 + 1152*a^7*c^3*d*e^14 + 21248*a^2*c^8*d^11*e
^4 + 25472*a^3*c^7*d^9*e^6 - 5632*a^4*c^6*d^7*e^8 - 7296*a^5*c^5*d^5*e^10 +
 4864*a^6*c^4*d^3*e^12))/(128*(c^5*d^8 + a^4*c*e^8 + 4*a*c^4*d^6*e^2 + 6*a^
2*c^3*d^4*e^4 + 4*a^3*c^2*d^2*e^6)))*(-(a^3*e^6*(-a*c^5)^(1/2) - 9*c^3*d^6*
(-a*c^5)^(1/2) - 2*a^3*c^3*d*e^5 - 4*a^2*c^4*d^3*e^3 + 30*a*c^5*d^5*e + 31*
a*c^2*d^4*e^2*(-a*c^5)^(1/2) + 9*a^2*c*d^2*e^4*(-a*c^5)^(1/2))/(256*(a*c^9*
d^8 + a^5*c^5*e^8 + 4*a^2*c^8*d^6*e^2 + 6*a^3*c^7*d^4*e^4 + 4*a^4*c^6*d^2*e
^6)))^(1/2))*(-(a^3*e^6*(-a*c^5)^(1/2) - 9*c^3*d^6*(-a*c^5)^(1/2) - 2*a^3*c
^3*d*e^5 - 4*a^2*c^4*d^3*e^3 + 30*a*c^5*d^5*e + 31*a*c^2*d^4*e^2*(-a*c^5)^(
1/2) + 9*a^2*c*d^2*e^4*(-a*c^5)^(1/2))/(256*(a*c^9*d^8 + a^5*c^5*e^8 + 4*a^
2*c^8*d^6*e^2 + 6*a^3*c^7*d^4*e^4 + 4*a^4*c^6*d^2*e^6)))^(1/2) + (x*(a^6*e^
13 - 288*a*c^5*d^10*e^3 + 20*a^5*c*d^2*e^11 + 17*a^2*c^4*d^8*e^5 + 148*a^3*
c^3*d^6*e^7 + 118*a^4*c^2*d^4*e^9))/(128*(c^5*d^8 + a^4*c*e^8 + 4*a*c^4*d^6
*e^2 + 6*a^2*c^3*d^4*e^4 + 4*a^3*c^2*d^2*e^6)))*(-(a^3*e^6*(-a*c^5)^(1/2) -
 9*c^3*d^6*(-a*c^5)^(1/2) - 2*a^3*c^3*d*e^5 - 4*a^2*c^4*d^3*e^3 + 30*a*c^5*
d^5*e + 31*a*c^2*d^4*e^2*(-a*c^5)^(1/2) + 9*a^2*c*d^2*e^4*(-a*c^5)^(1/2))/(
256*(a*c^9*d^8 + a^5*c^5*e^8 + 4*a^2*c^8*d^6*e^2 + 6*a^3*c^7*d^4*e^4 + 4*a^
4*c^6*d^2*e^6)))^(1/2)*1i)/((((432*a*c^7*d^12*e^2 + 13040*a^2*c^6*d^10*e^4
+ 12000*a^3*c^5*d^8*e^6 - 1056*a^4*c^4*d^6*e^8 - 400*a^5*c^3*d^4*e^10 + 48*
a^6*c^2*d^2*e^12)/(256*(c^5*d^8 + a^4*c*e^8 + 4*a*c^4*d^6*e^2 + 6*a^2*c^3*d
^4*e^4 + 4*a^3*c^2*d^2*e^6)) + (((45056*a^2*c^10*d^13*e^3 - 4096*a^8*c^4*d*
e^15 + 221184*a^3*c^9*d^11*e^5 + 430080*a^4*c^8*d^9*e^7 + 409600*a^5*c^7*d^
7*e^9 + 184320*a^6*c^6*d^5*e^11 + 24576*a^7*c^5*d^3*e^13)/(256*(c^5*d^8 + a
^4*c*e^8 + 4*a*c^4*d^6*e^2 + 6*a^2*c^3*d^4*e^4 + 4*a^3*c^2*d^2*e^6)) - (x*(
-(a^3*e^6*(-a*c^5)^(1/2) - 9*c^3*d^6*(-a*c^5)^(1/2) - 2*a^3*c^3*d*e^5 - 4*a
^2*c^4*d^3*e^3 + 30*a*c^5*d^5*e + 31*a*c^2*d^4*e^2*(-a*c^5)^(1/2) + 9*a^2*c
*d^2*e^4*(-a*c^5)^(1/2))/(256*(a*c^9*d^8 + a^5*c^5*e^8 + 4*a^2*c^8*d^6*e^2
+ 6*a^3*c^7*d^4*e^4 + 4*a^4*c^6*d^2*e^6)))^(1/2)*(65536*a^9*c^5*e^17 - 6553

6*a^2*c^12*d^14*e^3 - 327680*a^3*c^11*d^12*e^5 - 589824*a^4*c^10*d^10*e^7 - 327680*a^5*c^9*d^8*e^9 + 327680*a^6*c^8*d^6*e^11 + 589824*a^7*c^7*d^4*e^13 + 327680*a^8*c^6*d^2*e^15))/(128*(c^5*d^8 + a^4*c*e^8 + 4*a*c^4*d^6*e^2 + 6*a^2*c^3*d^4*e^4 + 4*a^3*c^2*d^2*e^6)))*(-(a^3*e^6*(-a*c^5)^(1/2) - 9*c^3*d^6*(-a*c^5)^(1/2) - 2*a^3*c^3*d*e^5 - 4*a^2*c^4*d^3*e^3 + 30*a*c^5*d^5*e + 31*a*c^2*d^4*e^2*(-a*c^5)^(1/2) + 9*a^2*c*d^2*e^4*(-a*c^5)^(1/2))/(256*(a*c^9*d^8 + a^5*c^5*e^8 + 4*a^2*c^8*d^6*e^2 + 6*a^3*c^7*d^4*e^4 + 4*a^4*c^6*d^2*e^6)))^(1/2) + (x*(1152*a*c^9*d^13*e^2 + 1152*a^7*c^3*d*e^14 + 21248*a^2*c^8*d^11*e^4 + 25472*a^3*c^7*d^9*e^6 - 5632*a^4*c^6*d^7*e^8 - 7296*a^5*c^5*d^5*e^10 + 4864*a^6*c^4*d^3*e^12))/(128*(c^5*d^8 + a^4*c*e^8 + 4*a*c^4*d^6*e^2 + 6*a^2*c^3*d^4*e^4 + 4*a^3*c^2*d^2*e^6)))*(-(a^3*e^6*(-a*c^5)^(1/2) - 9*c^3*d^6*(-a*c^5)^(1/2) - 2*a^3*c^3*d*e^5 - 4*a^2*c^4*d^3*e^3 + 30*a*c^5*d^5*e + 31*a*c^2*d^4*e^2*(-a*c^5)^(1/2) + 9*a^2*c*d^2*e^4*(-a*c^5)^(1/2))/(256*(a*c^9*d^8 + a^5*c^5*e^8 + 4*a^2*c^8*d^6*e^2 + 6*a^3*c^7*d^4*e^4 + 4*a^4*c^6*d^2*e^6)))^(1/2))*(-(a^3*e^6*(-a*c^5)^(1/2) - 9*c^3*d^6*(-a*c^5)^(1/2) - 2*a^3*c^3*d*e^5 - 4*a^2*c^4*d^3*e^3 + 30*a*c^5*d^5*e + 31*a*c^2*d^4*e^2*(-a*c^5)^(1/2) + 9*a^2*c*d^2*e^4*(-a*c^5)^(1/2))/(256*(a*c^9*d^8 + a^5*c^5*e^8 + 4*a^2*c^8*d^6*e^2 + 6*a^3*c^7*d^4*e^4 + 4*a^4*c^6*d^2*e^6)))^(1/2) - (x*(a^6*e^13 - 288*a*c^5*d^10*e^3 + 20*a^5*c*d^2*e^11 + 17*a^2*c^4*d^8*e^5 + 148*a^3*c^3*d^6*e^7 + 118*a^4*c^2*d^4*e^9))/(128*(c^5*d^8 + a^4*c*e^8 + 4*a*c^4*d^6*e^2 + 6*a^2*c^3*d^4*e^4 + 4*a^3*c^2*d^2*e^6)))*(-(a^3*e^6*(-a*c^5)^(1/2) - 9*c^3*d^6*(-a*c^5)^(1/2) - 2*a^3*c^3*d*e^5 - 4*a^2*c^4*d^3*e^3 + 30*a*c^5*d^5*e + 31*a*c^2*d^4*e^2*(-a*c^5)^(1/2) + 9*a^2*c*d^2*e^4*(-a*c^5)^(1/2))/(256*(a*c^9*d^8 + a^5*c^5*e^8 + 4*a^2*c^8*d^6*e^2 + 6*a^3*c^7*d^4*e^4 + 4*a^4*c^6*d^2*e^6)))^(1/2) + (((432*a*c^7*d^12*e^2 + 13040*a^2*c^6*d^10*e^4 + 12000*a^3*c^5*d^8*e^6 - 1056*a^4*c^4*d^6*e^8 - 400*a^5*c^3*d^4*e^10 + 48*a^6*c^2*d^2*e^12)/(256*(c^5*d^8 + a^4*c*e^8 + 4*a*c^4*d^6*e^2 + 6*a^2*c^3*d^4*e^4 + 4*a^3*c^2*d^2*e^6)) + (((45056*a^2*c^10*d^13*e^3 - 4096*a^8*c^4*d*e^15 + 221184*a^3*c^9*d^11*e^5 + 430080*a^4*c^8*d^9*e^7 + 409600*a^5*c^7*d^7*e^9 + 184320*a^6*c^6*d^5*e^11 + 24576*a^7*c^5*d^3*e^13)/(256*(c^5*d^8 + a^4*c*e^8 + 4*a*c^4*d^6*e^2 + 6*a^2*c^3*d^4*e^4 + 4*a^3*c^2*d^2*e^6)) + (x*(-(a^3*e^6*(-a*c^5)^(1/2) - 9*c^3*d^6*(-a*c^5)^(1/2) - 2*a^3*c^3*d*e^5 - 4*a^2*c^4*d^3*e^3 + 30*a*c^5*d^5*e + 31*a*c^2*d^4*e^2*(-a*c^5)^(1/2) + 9*a^2*c*d^2*e^4*(-a*c^5)^(1/2))/(256*(a*c^9*d^8 + a^5*c^5*e^8 + 4*a^2*c^8*d^6*e^2 + 6*a^3*c^7*d^4*e^4 + 4*a^4*c^6*d^2*e^6)))^(1/2)*(65536*a^9*c^5*e^17 - 65536*a^2*c^12*d^14*e^3 - 327680*a^3*c^11*d^12*e^5 - 589824*a^4*c^10*d^10*e^7 - 327680*a^5*c^9*d^8*e^9 + 327680*a^6*c^8*d^6*e^11 + 589824*a^7*c^7*d^4*e^13 + 327680*a^8*c^6*d^2*e^15))/(128*(c^5*d^8 + a^4*c*e^8 + 4*a*c^4*d^6*e^2 + 6*a^2*c^3*d^4*e^4 + 4*a^3*c^2*d^2*e^6)))*(-(a^3*e^6*(-a*c^5)^(1/2) - 9*c^3*d^6*(-a*c^5)^(1/2) - 2*a^3*c^3*d*e^5 - 4*a^2*c^4*d^3*e^3 + 30*a*c^5*d^5*e + 31*a*c^2*d^4*e^2*(-a*c^5)^(1/2) + 9*a^2*c*d^2*e^4*(-a*c^5)^(1/2))/(256*(a*c^9*d^8 + a^5*c^5*e^8 + 4*a^2*c^8*d^6*e^2 + 6*a^3*c^7*d^4*e^4 + 4*a^4*c^6*d^2*e^6)))^(1/2) - (x*(1152*a*c^9*d^13*e^2 + 1152*a^7*c^3*d*e^14 + 21248*a^2*c^8*d^11*e^4 + 25472*a^3*c^7*d^9*e^6 - 5632*a^4*c^6*d^7*e^8 - 7296*a^5*c^5*d^5*e^10 + 4864*a^6*c^4*d^3*e^12))/(128*(c^5*d^8 + a^4*c*e^8 + 4*a

*c^4*d^6*e^2 + 6*a^2*c^3*d^4*e^4 + 4*a^3*c^2*d^2*e^6)))*(-(a^3*e^6*(-a*c^5)
^(1/2) - 9*c^3*d^6*(-a*c^5)^(1/2) - 2*a^3*c^3*d*e^5 - 4*a^2*c^4*d^3*e^3 + 3
0*a*c^5*d^5*e + 31*a*c^2*d^4*e^2*(-a*c^5)^(1/2) + 9*a^2*c*d^2*e^4*(-a*c^5)^
(1/2))/(256*(a*c^9*d^8 + a^5*c^5*e^8 + 4*a^2*c^8*d^6*e^2 + 6*a^3*c^7*d^4*e^
4 + 4*a^4*c^6*d^2*e^6)))^(1/2))*(-(a^3*e^6*(-a*c^5)^(1/2) - 9*c^3*d^6*(-a*c
^5)^(1/2) - 2*a^3*c^3*d*e^5 - 4*a^2*c^4*d^3*e^3 + 30*a*c^5*d^5*e + 31*a*c^2
*d^4*e^2*(-a*c^5)^(1/2) + 9*a^2*c*d^2*e^4*(-a*c^5)^(1/2))/(256*(a*c^9*d^8 +
 a^5*c^5*e^8 + 4*a^2*c^8*d^6*e^2 + 6*a^3*c^7*d^4*e^4 + 4*a^4*c^6*d^2*e^6)))
^(1/2) + (x*(a^6*e^13 - 288*a*c^5*d^10*e^3 + 20*a^5*c*d^2*e^11 + 17*a^2*c^4
*d^8*e^5 + 148*a^3*c^3*d^6*e^7 + 118*a^4*c^2*d^4*e^9))/(128*(c^5*d^8 + a^4*
c*e^8 + 4*a*c^4*d^6*e^2 + 6*a^2*c^3*d^4*e^4 + 4*a^3*c^2*d^2*e^6)))*(-(a^3*e
^6*(-a*c^5)^(1/2) - 9*c^3*d^6*(-a*c^5)^(1/2) - 2*a^3*c^3*d*e^5 - 4*a^2*c^4*
d^3*e^3 + 30*a*c^5*d^5*e + 31*a*c^2*d^4*e^2*(-a*c^5)^(1/2) + 9*a^2*c*d^2*e^
4*(-a*c^5)^(1/2))/(256*(a*c^9*d^8 + a^5*c^5*e^8 + 4*a^2*c^8*d^6*e^2 + 6*a^3
*c^7*d^4*e^4 + 4*a^4*c^6*d^2*e^6)))^(1/2) + (a^4*d^3*e^9 + 108*a*c^3*d^9*e^
3 + 18*a^3*c*d^5*e^7 + 93*a^2*c^2*d^7*e^5)/(128*(c^5*d^8 + a^4*c*e^8 + 4*a*
c^4*d^6*e^2 + 6*a^2*c^3*d^4*e^4 + 4*a^3*c^2*d^2*e^6))))*(-(a^3*e^6*(-a*c^5)
^(1/2) - 9*c^3*d^6*(-a*c^5)^(1/2) - 2*a^3*c^3*d*e^5 - 4*a^2*c^4*d^3*e^3 + 3
0*a*c^5*d^5*e + 31*a*c^2*d^4*e^2*(-a*c^5)^(1/2) + 9*a^2*c*d^2*e^4*(-a*c^5)^
(1/2))/(256*(a*c^9*d^8 + a^5*c^5*e^8 + 4*a^2*c^8*d^6*e^2 + 6*a^3*c^7*d^4*e^
4 + 4*a^4*c^6*d^2*e^6)))^(1/2)*2i - ((d*x^3)/(4*(a*e^2 + c*d^2)) + (a*e*x)/
(4*c*(a*e^2 + c*d^2)))/(a + c*x^4) + atan(((((432*a*c^7*d^12*e^2 + 13040*a^
2*c^6*d^10*e^4 + 12000*a^3*c^5*d^8*e^6 - 1056*a^4*c^4*d^6*e^8 - 400*a^5*c^3
*d^4*e^10 + 48*a^6*c^2*d^2*e^12)/(256*(c^5*d^8 + a^4*c*e^8 + 4*a*c^4*d^6*e^
2 + 6*a^2*c^3*d^4*e^4 + 4*a^3*c^2*d^2*e^6)) + (((45056*a^2*c^10*d^13*e^3 -
4096*a^8*c^4*d*e^15 + 221184*a^3*c^9*d^11*e^5 + 430080*a^4*c^8*d^9*e^7 + 40
9600*a^5*c^7*d^7*e^9 + 184320*a^6*c^6*d^5*e^11 + 24576*a^7*c^5*d^3*e^13)/(2
56*(c^5*d^8 + a^4*c*e^8 + 4*a*c^4*d^6*e^2 + 6*a^2*c^3*d^4*e^4 + 4*a^3*c^2*d
^2*e^6)) - (x*((a^3*e^6*(-a*c^5)^(1/2) - 9*c^3*d^6*(-a*c^5)^(1/2) + 2*a^3*c
^3*d*e^5 + 4*a^2*c^4*d^3*e^3 - 30*a*c^5*d^5*e + 31*a*c^2*d^4*e^2*(-a*c^5)^(
1/2) + 9*a^2*c*d^2*e^4*(-a*c^5)^(1/2))/(256*(a*c^9*d^8 + a^5*c^5*e^8 + 4*a^
2*c^8*d^6*e^2 + 6*a^3*c^7*d^4*e^4 + 4*a^4*c^6*d^2*e^6)))^(1/2)*(65536*a^9*c
^5*e^17 - 65536*a^2*c^12*d^14*e^3 - 327680*a^3*c^11*d^12*e^5 - 589824*a^4*c
^10*d^10*e^7 - 327680*a^5*c^9*d^8*e^9 + 327680*a^6*c^8*d^6*e^11 + 589824*a^
7*c^7*d^4*e^13 + 327680*a^8*c^6*d^2*e^15))/(128*(c^5*d^8 + a^4*c*e^8 + 4*a*
c^4*d^6*e^2 + 6*a^2*c^3*d^4*e^4 + 4*a^3*c^2*d^2*e^6)))*((a^3*e^6*(-a*c^5)^(
1/2) - 9*c^3*d^6*(-a*c^5)^(1/2) + 2*a^3*c^3*d*e^5 + 4*a^2*c^4*d^3*e^3 - 30*
a*c^5*d^5*e + 31*a*c^2*d^4*e^2*(-a*c^5)^(1/2) + 9*a^2*c*d^2*e^4*(-a*c^5)^(1
/2))/(256*(a*c^9*d^8 + a^5*c^5*e^8 + 4*a^2*c^8*d^6*e^2 + 6*a^3*c^7*d^4*e^4
+ 4*a^4*c^6*d^2*e^6)))^(1/2) + (x*(1152*a*c^9*d^13*e^2 + 1152*a^7*c^3*d*e^1
4 + 21248*a^2*c^8*d^11*e^4 + 25472*a^3*c^7*d^9*e^6 - 5632*a^4*c^6*d^7*e^8 -
 7296*a^5*c^5*d^5*e^10 + 4864*a^6*c^4*d^3*e^12))/(128*(c^5*d^8 + a^4*c*e^8
+ 4*a*c^4*d^6*e^2 + 6*a^2*c^3*d^4*e^4 + 4*a^3*c^2*d^2*e^6)))*((a^3*e^6*(-a*
c^5)^(1/2) - 9*c^3*d^6*(-a*c^5)^(1/2) + 2*a^3*c^3*d*e^5 + 4*a^2*c^4*d^3*e^3
 - 30*a*c^5*d^5*e + 31*a*c^2*d^4*e^2*(-a*c^5)^(1/2) + 9*a^2*c*d^2*e^4*(-a*c

^5)^(1/2))/(256*(a*c^9*d^8 + a^5*c^5*e^8 + 4*a^2*c^8*d^6*e^2 + 6*a^3*c^7*d^4*e^4 + 4*a^4*c^6*d^2*e^6)))^(1/2))*((a^3*e^6*(-a*c^5)^(1/2) - 9*c^3*d^6*(-a*c^5)^(1/2) + 2*a^3*c^3*d*e^5 + 4*a^2*c^4*d^3*e^3 - 30*a*c^5*d^5*e + 31*a*c^2*d^4*e^2*(-a*c^5)^(1/2) + 9*a^2*c*d^2*e^4*(-a*c^5)^(1/2))/(256*(a*c^9*d^8 + a^5*c^5*e^8 + 4*a^2*c^8*d^6*e^2 + 6*a^3*c^7*d^4*e^4 + 4*a^4*c^6*d^2*e^6)))^(1/2) - (x*(a^6*e^13 - 288*a*c^5*d^10*e^3 + 20*a^5*c*d^2*e^11 + 17*a^2*c^4*d^8*e^5 + 148*a^3*c^3*d^6*e^7 + 118*a^4*c^2*d^4*e^9))/(128*(c^5*d^8 + a^4*c*e^8 + 4*a*c^4*d^6*e^2 + 6*a^2*c^3*d^4*e^4 + 4*a^3*c^2*d^2*e^6)))*((a^3*e^6*(-a*c^5)^(1/2) - 9*c^3*d^6*(-a*c^5)^(1/2) + 2*a^3*c^3*d*e^5 + 4*a^2*c^4*d^3*e^3 - 30*a*c^5*d^5*e + 31*a*c^2*d^4*e^2*(-a*c^5)^(1/2) + 9*a^2*c*d^2*e^4*(-a*c^5)^(1/2))/(256*(a*c^9*d^8 + a^5*c^5*e^8 + 4*a^2*c^8*d^6*e^2 + 6*a^3*c^7*d^4*e^4 + 4*a^4*c^6*d^2*e^6)))^(1/2)*1i - (((432*a*c^7*d^12*e^2 + 13040*a^2*c^6*d^10*e^4 + 12000*a^3*c^5*d^8*e^6 - 1056*a^4*c^4*d^6*e^8 - 400*a^5*c^3*d^4*e^10 + 48*a^6*c^2*d^2*e^12)/(256*(c^5*d^8 + a^4*c*e^8 + 4*a*c^4*d^6*e^2 + 6*a^2*c^3*d^4*e^4 + 4*a^3*c^2*d^2*e^6)) + (((45056*a^2*c^10*d^13*e^3 - 4096*a^8*c^4*d*e^15 + 221184*a^3*c^9*d^11*e^5 + 430080*a^4*c^8*d^9*e^7 + 409600*a^5*c^7*d^7*e^9 + 184320*a^6*c^6*d^5*e^11 + 24576*a^7*c^5*d^3*e^13)/(256*(c^5*d^8 + a^4*c*e^8 + 4*a*c^4*d^6*e^2 + 6*a^2*c^3*d^4*e^4 + 4*a^3*c^2*d^2*e^6)) + (x*((a^3*e^6*(-a*c^5)^(1/2) - 9*c^3*d^6*(-a*c^5)^(1/2) + 2*a^3*c^3*d*e^5 + 4*a^2*c^4*d^3*e^3 - 30*a*c^5*d^5*e + 31*a*c^2*d^4*e^2*(-a*c^5)^(1/2) + 9*a^2*c*d^2*e^4*(-a*c^5)^(1/2))/(256*(a*c^9*d^8 + a^5*c^5*e^8 + 4*a^2*c^8*d^6*e^2 + 6*a^3*c^7*d^4*e^4 + 4*a^4*c^6*d^2*e^6)))^(1/2)*(65536*a^9*c^5*e^17 - 65536*a^2*c^12*d^14*e^3 - 327680*a^3*c^11*d^12*e^5 - 589824*a^4*c^10*d^10*e^7 - 327680*a^5*c^9*d^8*e^9 + 327680*a^6*c^8*d^6*e^11 + 589824*a^7*c^7*d^4*e^13 + 327680*a^8*c^6*d^2*e^15))/(128*(c^5*d^8 + a^4*c*e^8 + 4*a*c^4*d^6*e^2 + 6*a^2*c^3*d^4*e^4 + 4*a^3*c^2*d^2*e^6)))*((a^3*e^6*(-a*c^5)^(1/2) - 9*c^3*d^6*(-a*c^5)^(1/2) + 2*a^3*c^3*d*e^5 + 4*a^2*c^4*d^3*e^3 - 30*a*c^5*d^5*e + 31*a*c^2*d^4*e^2*(-a*c^5)^(1/2) + 9*a^2*c*d^2*e^4*(-a*c^5)^(1/2))/(256*(a*c^9*d^8 + a^5*c^5*e^8 + 4*a^2*c^8*d^6*e^2 + 6*a^3*c^7*d^4*e^4 + 4*a^4*c^6*d^2*e^6)))^(1/2) - (x*(1152*a*c^9*d^13*e^2 + 1152*a^7*c^3*d*e^14 + 21248*a^2*c^8*d^11*e^4 + 25472*a^3*c^7*d^9*e^6 - 5632*a^4*c^6*d^7*e^8 - 7296*a^5*c^5*d^5*e^10 + 4864*a^6*c^4*d^3*e^12))/(128*(c^5*d^8 + a^4*c*e^8 + 4*a*c^4*d^6*e^2 + 6*a^2*c^3*d^4*e^4 + 4*a^3*c^2*d^2*e^6)))*((a^3*e^6*(-a*c^5)^(1/2) - 9*c^3*d^6*(-a*c^5)^(1/2) + 2*a^3*c^3*d*e^5 + 4*a^2*c^4*d^3*e^3 - 30*a*c^5*d^5*e + 31*a*c^2*d^4*e^2*(-a*c^5)^(1/2) + 9*a^2*c*d^2*e^4*(-a*c^5)^(1/2))/(256*(a*c^9*d^8 + a^5*c^5*e^8 + 4*a^2*c^8*d^6*e^2 + 6*a^3*c^7*d^4*e^4 + 4*a^4*c^6*d^2*e^6)))^(1/2))*((a^3*e^6*(-a*c^5)^(1/2) - 9*c^3*d^6*(-a*c^5)^(1/2) + 2*a^3*c^3*d*e^5 + 4*a^2*c^4*d^3*e^3 - 30*a*c^5*d^5*e + 31*a*c^2*d^4*e^2*(-a*c^5)^(1/2) + 9*a^2*c*d^2*e^4*(-a*c^5)^(1/2))/(256*(a*c^9*d^8 + a^5*c^5*e^8 + 4*a^2*c^8*d^6*e^2 + 6*a^3*c^7*d^4*e^4 + 4*a^4*c^6*d^2*e^6)))^(1/2) + (x*(a^6*e^13 - 288*a*c^5*d^10*e^3 + 20*a^5*c*d^2*e^11 + 17*a^2*c^4*d^8*e^5 + 148*a^3*c^3*d^6*e^7 + 118*a^4*c^2*d^4*e^9))/(128*(c^5*d^8 + a^4*c*e^8 + 4*a*c^4*d^6*e^2 + 6*a^2*c^3*d^4*e^4 + 4*a^3*c^2*d^2*e^6)))*((a^3*e^6*(-a*c^5)^(1/2) - 9*c^3*d^6*(-a*c^5)^(1/2) + 2*a^3*c^3*d*e^5 + 4*a^2*c^4*d^3*e^3 - 30*a*c^5*d^5*e + 31*a*c^2*d^4*e^2*(-a*c^5)^(1/2) + 9*a^2* c*d^2*e^4*(-a*c^5)^(1/2))/(256*(a*c^9*d^8 + a^5*c^5*e^8 + 4*a^2*c^8*d^6*e^2 + 6*a^3*c^7*d^4*e^4 + 4*a^4*c^6*d^2*e^6)))^(1/2)*1i)/((((432*a*c^7*d^12*e^2 + 13040*a^2*c^6*d^10*e^4 + 12000*a^3*c^5*d^8*e^6 - 1056*a^4*c^4*d^6*e^8 - 400*a^5*c^3*d^4*e^10 + 48*a^6*c^2*d^2*e^12)/(256*(c^5*d^8 + a^4*c*e^8 + 4*a*c^4*d^6*e^2 + 6*a^2*c^3*d^4*e^4 + 4*a^3*c^2*d^2*e^6)) + (((45056*a^2*c^10*d^13*e^3 - 4096*a^8*c^4*d*e^15 + 221184*a^3*c^9*d^11*e^5 + 430080*a^4*c^8*d^9*e^7 + 409600*a^5*c^7*d^7*e^9 + 184320*a^6*c^6*d^5*e^11 + 24576*a^7*c^5*d^3*e^13)/(256*(c^5*d^8 + a^4*c*e^8 + 4*a*c^4*d^6*e^2 + 6*a^2*c^3*d^4*e^4 + 4*a^3*c^2*d^2*e^6)) - (x*((a^3*e^6*(-a*c^5)^(1/2) - 9*c^3*d^6*(-a*c^5)^(1/2) + 2*a^3*c^3*d*e^5 + 4*a^2*c^4*d^3*e^3 - 30*a*c^5*d^5*e + 31*a*c^2*d^4*e^2*(-a*c^5)^(1/2) + 9*a^2*c*d^2*e^4*(-a*c^5)^(1/2))/(256*(a*c^9*d^8 + a^5*c^5*e^8 + 4*a^2*c^8*d^6*e^2 + 6*a^3*c^7*d^4*e^4 + 4*a^4*c^6*d^2*e^6)))^(1/2)*(65536*a^9*c^5*e^17 - 65536*a^2*c^12*d^14*e^3 - 327680*a^3*c^11*d^12*e^5 - 589824*a^4*c^10*d^10*e^7 - 327680*a^5*c^9*d^8*e^9 + 327680*a^6*c^8*d^6*e^11 + 589824*a^7*c^7*d^4*e^13 + 327680*a^8*c^6*d^2*e^15))/(128*(c^5*d^8 + a^4*c*e^8 + 4*a*c^4*d^6*e^2 + 6*a^2*c^3*d^4*e^4 + 4*a^3*c^2*d^2*e^6)))*((a^3*e^6*(-a*c^5)^(1/2) - 9*c^3*d^6*(-a*c^5)^(1/2) + 2*a^3*c^3*d*e^5 + 4*a^2*c^4*d^3*e^3 - 30*a*c^5*d^5*e + 31*a*c^2*d^4*e^2*(-a*c^5)^(1/2) + 9*a^2*c*d^2*e^4*(-a*c^5)^(1/2))/(256*(a*c^9*d^8 + a^5*c^5*e^8 + 4*a^2*c^8*d^6*e^2 + 6*a^3*c^7*d^4*e^4 + 4*a^4*c^6*d^2*e^6)))^(1/2) + (x*(1152*a*c^9*d^13*e^2 + 1152*a^7*c^3*d*e^14 + 21248*a^2*c^8*d^11*e^4 + 25472*a^3*c^7*d^9*e^6 - 5632*a^4*c^6*d^7*e^8 - 7296*a^5*c^5*d^5*e^10 + 4864*a^6*c^4*d^3*e^12))/(128*(c^5*d^8 + a^4*c*e^8 + 4*a*c^4*d^6*e^2 + 6*a^2*c^3*d^4*e^4 + 4*a^3*c^2*d^2*e^6)))*((a^3*e^6*(-a*c^5)^(1/2) - 9*c^3*d^6*(-a*c^5)^(1/2) + 2*a^3*c^3*d*e^5 + 4*a^2*c^4*d^3*e^3 - 30*a*c^5*d^5*e + 31*a*c^2*d^4*e^2*(-a*c^5)^(1/2) + 9*a^2*c*d^2*e^4*(-a*c^5)^(1/2))/(256*(a*c^9*d^8 + a^5*c^5*e^8 + 4*a^2*c^8*d^6*e^2 + 6*a^3*c^7*d^4*e^4 + 4*a^4*c^6*d^2*e^6)))^(1/2))*((a^3*e^6*(-a*c^5)^(1/2) - 9*c^3*d^6*(-a*c^5)^(1/2) + 2*a^3*c^3*d*e^5 + 4*a^2*c^4*d^3*e^3 - 30*a*c^5*d^5*e + 31*a*c^2*d^4*e^2*(-a*c^5)^(1/2) + 9*a^2*c*d^2*e^4*(-a*c^5)^(1/2))/(256*(a*c^9*d^8 + a^5*c^5*e^8 + 4*a^2*c^8*d^6*e^2 + 6*a^3*c^7*d^4*e^4 + 4*a^4*c^6*d^2*e^6)))^(1/2) - (x*(a^6*e^13 - 288*a*c^5*d^10*e^3 + 20*a^5*c*d^2*e^11 + 17*a^2*c^4*d^8*e^5 + 148*a^3*c^3*d^6*e^7 + 118*a^4*c^2*d^4*e^9))/(128*(c^5*d^8 + a^4*c*e^8 + 4*a*c^4*d^6*e^2 + 6*a^2*c^3*d^4*e^4 + 4*a^3*c^2*d^2*e^6)))*((a^3*e^6*(-a*c^5)^(1/2) - 9*c^3*d^6*(-a*c^5)^(1/2) + 2*a^3*c^3*d*e^5 + 4*a^2*c^4*d^3*e^3 - 30*a*c^5*d^5*e + 31*a*c^2*d^4*e^2*(-a*c^5)^(1/2) + 9*a^2*c*d^2*e^4*(-a*c^5)^(1/2))/(256*(a*c^9*d^8 + a^5*c^5*e^8 + 4*a^2*c^8*d^6*e^2 + 6*a^3*c^7*d^4*e^4 + 4*a^4*c^6*d^2*e^6)))^(1/2) + (((432*a*c^7*d^12*e^2 + 13040*a^2*c^6*d^10*e^4 + 12000*a^3*c^5*d^8*e^6 - 1056*a^4*c^4*d^6*e^8 - 400*a^5*c^3*d^4*e^10 + 48*a^6*c^2*d^2*e^12)/(256*(c^5*d^8 + a^4*c*e^8 + 4*a*c^4*d^6*e^2 + 6*a^2*c^3*d^4*e^4 + 4*a^3*c^2*d^2*e^6)) + (((45056*a^2*c^10*d^13*e^3 - 4096*a^8*c^4*d*e^15 + 221184*a^3*c^9*d^11*e^5 + 430080*a^4*c^8*d^9*e^7 + 409600*a^5*c^7*d^7*e^9 + 184320*a^6*c^6*d^5*e^11 + 24576*a^7*c^5*d^3*e^13)/(256*(c^5*d^8 + a^4*c*e^8 + 4*a*c^4*d^6*e^2 + 6*a^2*c^3*d^4*e^4 + 4*a^3*c^2*d^2*e^6)) + (x*((a^3*e^6*(-a*c^5)^(1/2) - 9*c^3*d^6*(-a*c^5)^(1/2) + 2*a^3*c^3*d*e^5 + 4*a^2*c^4*d^3*e^3 - 30*a*c^5*d^5*e + 31*a*c^2*d^4

e^2(-a*c^5)^(1/2) + 9*a^2*c*d^2*e^4*(-a*c^5)^(1/2))/(256*(a*c^9*d^8 + a^5
*c^5*e^8 + 4*a^2*c^8*d^6*e^2 + 6*a^3*c^7*d^4*e^4 + 4*a^4*c^6*d^2*e^6)))^(1/
2)*(65536*a^9*c^5*e^17 - 65536*a^2*c^12*d^14*e^3 - 327680*a^3*c^11*d^12*e^5
 - 589824*a^4*c^10*d^10*e^7 - 327680*a^5*c^9*d^8*e^9 + 327680*a^6*c^8*d^6*e
^11 + 589824*a^7*c^7*d^4*e^13 + 327680*a^8*c^6*d^2*e^15))/(128*(c^5*d^8 + a
^4*c*e^8 + 4*a*c^4*d^6*e^2 + 6*a^2*c^3*d^4*e^4 + 4*a^3*c^2*d^2*e^6)))*((a^3
e^6(-a*c^5)^(1/2) - 9*c^3*d^6*(-a*c^5)^(1/2) + 2*a^3*c^3*d*e^5 + 4*a^2*c^
4*d^3*e^3 - 30*a*c^5*d^5*e + 31*a*c^2*d^4*e^2*(-a*c^5)^(1/2) + 9*a^2*c*d^2*
e^4*(-a*c^5)^(1/2))/(256*(a*c^9*d^8 + a^5*c^5*e^8 + 4*a^2*c^8*d^6*e^2 + 6*a
^3*c^7*d^4*e^4 + 4*a^4*c^6*d^2*e^6)))^(1/2) - (x*(1152*a*c^9*d^13*e^2 + 115
2*a^7*c^3*d*e^14 + 21248*a^2*c^8*d^11*e^4 + 25472*a^3*c^7*d^9*e^6 - 5632*a^
4*c^6*d^7*e^8 - 7296*a^5*c^5*d^5*e^10 + 4864*a^6*c^4*d^3*e^12))/(128*(c^5*d
^8 + a^4*c*e^8 + 4*a*c^4*d^6*e^2 + 6*a^2*c^3*d^4*e^4 + 4*a^3*c^2*d^2*e^6)))
*((a^3*e^6*(-a*c^5)^(1/2) - 9*c^3*d^6*(-a*c^5)^(1/2) + 2*a^3*c^3*d*e^5 + 4*
a^2*c^4*d^3*e^3 - 30*a*c^5*d^5*e + 31*a*c^2*d^4*e^2*(-a*c^5)^(1/2) + 9*a^2*
c*d^2*e^4*(-a*c^5)^(1/2))/(256*(a*c^9*d^8 + a^5*c^5*e^8 + 4*a^2*c^8*d^6*e^2
 + 6*a^3*c^7*d^4*e^4 + 4*a^4*c^6*d^2*e^6)))^(1/2))*((a^3*e^6*(-a*c^5)^(1/2)
 - 9*c^3*d^6*(-a*c^5)^(1/2) + 2*a^3*c^3*d*e^5 + 4*a^2*c^4*d^3*e^3 - 30*a*c^
5*d^5*e + 31*a*c^2*d^4*e^2*(-a*c^5)^(1/2) + 9*a^2*c*d^2*e^4*(-a*c^5)^(1/2))
/(256*(a*c^9*d^8 + a^5*c^5*e^8 + 4*a^2*c^8*d^6*e^2 + 6*a^3*c^7*d^4*e^4 + 4*
a^4*c^6*d^2*e^6)))^(1/2) + (x*(a^6*e^13 - 288*a*c^5*d^10*e^3 + 20*a^5*c*d^2
*e^11 + 17*a^2*c^4*d^8*e^5 + 148*a^3*c^3*d^6*e^7 + 118*a^4*c^2*d^4*e^9))/(1
28*(c^5*d^8 + a^4*c*e^8 + 4*a*c^4*d^6*e^2 + 6*a^2*c^3*d^4*e^4 + 4*a^3*c^2*d
^2*e^6)))*((a^3*e^6*(-a*c^5)^(1/2) - 9*c^3*d^6*(-a*c^5)^(1/2) + 2*a^3*c^3*d
*e^5 + 4*a^2*c^4*d^3*e^3 - 30*a*c^5*d^5*e + 31*a*c^2*d^4*e^2*(-a*c^5)^(1/2)
 + 9*a^2*c*d^2*e^4*(-a*c^5)^(1/2))/(256*(a*c^9*d^8 + a^5*c^5*e^8 + 4*a^2*c^
8*d^6*e^2 + 6*a^3*c^7*d^4*e^4 + 4*a^4*c^6*d^2*e^6)))^(1/2) + (a^4*d^3*e^9 +
 108*a*c^3*d^9*e^3 + 18*a^3*c*d^5*e^7 + 93*a^2*c^2*d^7*e^5)/(128*(c^5*d^8 +
 a^4*c*e^8 + 4*a*c^4*d^6*e^2 + 6*a^2*c^3*d^4*e^4 + 4*a^3*c^2*d^2*e^6))))*((
a^3*e^6*(-a*c^5)^(1/2) - 9*c^3*d^6*(-a*c^5)^(1/2) + 2*a^3*c^3*d*e^5 + 4*a^2
*c^4*d^3*e^3 - 30*a*c^5*d^5*e + 31*a*c^2*d^4*e^2*(-a*c^5)^(1/2) + 9*a^2*c*d
^2*e^4*(-a*c^5)^(1/2))/(256*(a*c^9*d^8 + a^5*c^5*e^8 + 4*a^2*c^8*d^6*e^2 +
6*a^3*c^7*d^4*e^4 + 4*a^4*c^6*d^2*e^6)))^(1/2)*2i - (atan(-(((-d^5*e)^(1/2)
*(((-d^5*e)^(1/2)*(((27*a*c^7*d^12*e^2)/16 + (815*a^2*c^6*d^10*e^4)/16 + (3
75*a^3*c^5*d^8*e^6)/8 - (33*a^4*c^4*d^6*e^8)/8 - (25*a^5*c^3*d^4*e^10)/16 +
 (3*a^6*c^2*d^2*e^12)/16)/(2*(c^5*d^8 + a^4*c*e^8 + 4*a*c^4*d^6*e^2 + 6*a^2
*c^3*d^4*e^4 + 4*a^3*c^2*d^2*e^6)) + (((x*(1152*a*c^9*d^13*e^2 + 1152*a^7*c
^3*d*e^14 + 21248*a^2*c^8*d^11*e^4 + 25472*a^3*c^7*d^9*e^6 - 5632*a^4*c^6*d
^7*e^8 - 7296*a^5*c^5*d^5*e^10 + 4864*a^6*c^4*d^3*e^12))/(256*(c^5*d^8 + a^
4*c*e^8 + 4*a*c^4*d^6*e^2 + 6*a^2*c^3*d^4*e^4 + 4*a^3*c^2*d^2*e^6)) + (((17
6*a^2*c^10*d^13*e^3 - 16*a^8*c^4*d*e^15 + 864*a^3*c^9*d^11*e^5 + 1680*a^4*c
^8*d^9*e^7 + 1600*a^5*c^7*d^7*e^9 + 720*a^6*c^6*d^5*e^11 + 96*a^7*c^5*d^3*e
^13)/(2*(c^5*d^8 + a^4*c*e^8 + 4*a*c^4*d^6*e^2 + 6*a^2*c^3*d^4*e^4 + 4*a^3*
c^2*d^2*e^6)) - (x*(-d^5*e)^(1/2)*(65536*a^9*c^5*e^17 - 65536*a^2*c^12*d^14
*e^3 - 327680*a^3*c^11*d^12*e^5 - 589824*a^4*c^10*d^10*e^7 - 327680*a^5*c^9

```
*d^8*e^9 + 327680*a^6*c^8*d^6*e^11 + 589824*a^7*c^7*d^4*e^13 + 327680*a^8*c
^6*d^2*e^15))/(512*(a^2*e^4 + c^2*d^4 + 2*a*c*d^2*e^2)*(c^5*d^8 + a^4*c*e^8
 + 4*a*c^4*d^6*e^2 + 6*a^2*c^3*d^4*e^4 + 4*a^3*c^2*d^2*e^6)))*(-d^5*e)^(1/2
))/(2*(a^2*e^4 + c^2*d^4 + 2*a*c*d^2*e^2)))*(-d^5*e)^(1/2))/(2*(a^2*e^4 + c
^2*d^4 + 2*a*c*d^2*e^2))))/(2*(a^2*e^4 + c^2*d^4 + 2*a*c*d^2*e^2)) - (x*(a^
6*e^13 - 288*a*c^5*d^10*e^3 + 20*a^5*c*d^2*e^11 + 17*a^2*c^4*d^8*e^5 + 148*
a^3*c^3*d^6*e^7 + 118*a^4*c^2*d^4*e^9))/(256*(c^5*d^8 + a^4*c*e^8 + 4*a*c^4
*d^6*e^2 + 6*a^2*c^3*d^4*e^4 + 4*a^3*c^2*d^2*e^6)))*1i)/(a^2*e^4 + c^2*d^4
+ 2*a*c*d^2*e^2) - ((-d^5*e)^(1/2)*(((-d^5*e)^(1/2)*(((27*a*c^7*d^12*e^2)/1
6 + (815*a^2*c^6*d^10*e^4)/16 + (375*a^3*c^5*d^8*e^6)/8 - (33*a^4*c^4*d^6*e
^8)/8 - (25*a^5*c^3*d^4*e^10)/16 + (3*a^6*c^2*d^2*e^12)/16)/(2*(c^5*d^8 + a
^4*c*e^8 + 4*a*c^4*d^6*e^2 + 6*a^2*c^3*d^4*e^4 + 4*a^3*c^2*d^2*e^6)) - (((x
*(1152*a*c^9*d^13*e^2 + 1152*a^7*c^3*d*e^14 + 21248*a^2*c^8*d^11*e^4 + 2547
2*a^3*c^7*d^9*e^6 - 5632*a^4*c^6*d^7*e^8 - 7296*a^5*c^5*d^5*e^10 + 4864*a^6
*c^4*d^3*e^12))/(256*(c^5*d^8 + a^4*c*e^8 + 4*a*c^4*d^6*e^2 + 6*a^2*c^3*d^4
*e^4 + 4*a^3*c^2*d^2*e^6)) - (((176*a^2*c^10*d^13*e^3 - 16*a^8*c^4*d*e^15 +
 864*a^3*c^9*d^11*e^5 + 1680*a^4*c^8*d^9*e^7 + 1600*a^5*c^7*d^7*e^9 + 720*a
^6*c^6*d^5*e^11 + 96*a^7*c^5*d^3*e^13)/(2*(c^5*d^8 + a^4*c*e^8 + 4*a*c^4*d^
6*e^2 + 6*a^2*c^3*d^4*e^4 + 4*a^3*c^2*d^2*e^6)) + (x*(-d^5*e)^(1/2)*(65536*
a^9*c^5*e^17 - 65536*a^2*c^12*d^14*e^3 - 327680*a^3*c^11*d^12*e^5 - 589824*
a^4*c^10*d^10*e^7 - 327680*a^5*c^9*d^8*e^9 + 327680*a^6*c^8*d^6*e^11 + 5898
24*a^7*c^7*d^4*e^13 + 327680*a^8*c^6*d^2*e^15))/(512*(a^2*e^4 + c^2*d^4 + 2
*a*c*d^2*e^2)*(c^5*d^8 + a^4*c*e^8 + 4*a*c^4*d^6*e^2 + 6*a^2*c^3*d^4*e^4 +
4*a^3*c^2*d^2*e^6)))*(-d^5*e)^(1/2))/(2*(a^2*e^4 + c^2*d^4 + 2*a*c*d^2*e^2)
))*(-d^5*e)^(1/2))/(2*(a^2*e^4 + c^2*d^4 + 2*a*c*d^2*e^2))))/(2*(a^2*e^4 +
c^2*d^4 + 2*a*c*d^2*e^2)) + (x*(a^6*e^13 - 288*a*c^5*d^10*e^3 + 20*a^5*c*d^
2*e^11 + 17*a^2*c^4*d^8*e^5 + 148*a^3*c^3*d^6*e^7 + 118*a^4*c^2*d^4*e^9))/(
256*(c^5*d^8 + a^4*c*e^8 + 4*a*c^4*d^6*e^2 + 6*a^2*c^3*d^4*e^4 + 4*a^3*c^2*
d^2*e^6)))*1i)/(a^2*e^4 + c^2*d^4 + 2*a*c*d^2*e^2))/(((a^4*d^3*e^9)/128 + (
27*a*c^3*d^9*e^3)/32 + (9*a^3*c*d^5*e^7)/64 + (93*a^2*c^2*d^7*e^5)/128)/(c^
5*d^8 + a^4*c*e^8 + 4*a*c^4*d^6*e^2 + 6*a^2*c^3*d^4*e^4 + 4*a^3*c^2*d^2*e^6
) + ((-d^5*e)^(1/2)*(((-d^5*e)^(1/2)*(((27*a*c^7*d^12*e^2)/16 + (815*a^2*c^
6*d^10*e^4)/16 + (375*a^3*c^5*d^8*e^6)/8 - (33*a^4*c^4*d^6*e^8)/8 - (25*a^5
*c^3*d^4*e^10)/16 + (3*a^6*c^2*d^2*e^12)/16)/(2*(c^5*d^8 + a^4*c*e^8 + 4*a*
c^4*d^6*e^2 + 6*a^2*c^3*d^4*e^4 + 4*a^3*c^2*d^2*e^6)) + (((x*(1152*a*c^9*d^
13*e^2 + 1152*a^7*c^3*d*e^14 + 21248*a^2*c^8*d^11*e^4 + 25472*a^3*c^7*d^9*e
^6 - 5632*a^4*c^6*d^7*e^8 - 7296*a^5*c^5*d^5*e^10 + 4864*a^6*c^4*d^3*e^12))
/(256*(c^5*d^8 + a^4*c*e^8 + 4*a*c^4*d^6*e^2 + 6*a^2*c^3*d^4*e^4 + 4*a^3*c^
2*d^2*e^6)) + (((176*a^2*c^10*d^13*e^3 - 16*a^8*c^4*d*e^15 + 864*a^3*c^9*d^
11*e^5 + 1680*a^4*c^8*d^9*e^7 + 1600*a^5*c^7*d^7*e^9 + 720*a^6*c^6*d^5*e^11
 + 96*a^7*c^5*d^3*e^13)/(2*(c^5*d^8 + a^4*c*e^8 + 4*a*c^4*d^6*e^2 + 6*a^2*c
^3*d^4*e^4 + 4*a^3*c^2*d^2*e^6)) - (x*(-d^5*e)^(1/2)*(65536*a^9*c^5*e^17 -
65536*a^2*c^12*d^14*e^3 - 327680*a^3*c^11*d^12*e^5 - 589824*a^4*c^10*d^10*e
^7 - 327680*a^5*c^9*d^8*e^9 + 327680*a^6*c^8*d^6*e^11 + 589824*a^7*c^7*d^4*
e^13 + 327680*a^8*c^6*d^2*e^15))/(512*(a^2*e^4 + c^2*d^4 + 2*a*c*d^2*e^2)*(
```

```
c^5*d^8 + a^4*c*e^8 + 4*a*c^4*d^6*e^2 + 6*a^2*c^3*d^4*e^4 + 4*a^3*c^2*d^2*e
^6)))*(-d^5*e)^(1/2))/(2*(a^2*e^4 + c^2*d^4 + 2*a*c*d^2*e^2)))*(-d^5*e)^(1/
2))/(2*(a^2*e^4 + c^2*d^4 + 2*a*c*d^2*e^2))))/(2*(a^2*e^4 + c^2*d^4 + 2*a*c
*d^2*e^2)) - (x*(a^6*e^13 - 288*a*c^5*d^10*e^3 + 20*a^5*c*d^2*e^11 + 17*a^2
*c^4*d^8*e^5 + 148*a^3*c^3*d^6*e^7 + 118*a^4*c^2*d^4*e^9))/(256*(c^5*d^8 +
a^4*c*e^8 + 4*a*c^4*d^6*e^2 + 6*a^2*c^3*d^4*e^4 + 4*a^3*c^2*d^2*e^6))))/(a^
2*e^4 + c^2*d^4 + 2*a*c*d^2*e^2) + ((-d^5*e)^(1/2)*(((-d^5*e)^(1/2)*(((27*a
*c^7*d^12*e^2)/16 + (815*a^2*c^6*d^10*e^4)/16 + (375*a^3*c^5*d^8*e^6)/8 - (
33*a^4*c^4*d^6*e^8)/8 - (25*a^5*c^3*d^4*e^10)/16 + (3*a^6*c^2*d^2*e^12)/16)
/(2*(c^5*d^8 + a^4*c*e^8 + 4*a*c^4*d^6*e^2 + 6*a^2*c^3*d^4*e^4 + 4*a^3*c^2*
d^2*e^6)) - (((x*(1152*a*c^9*d^13*e^2 + 1152*a^7*c^3*d*e^14 + 21248*a^2*c^8
*d^11*e^4 + 25472*a^3*c^7*d^9*e^6 - 5632*a^4*c^6*d^7*e^8 - 7296*a^5*c^5*d^5
*e^10 + 4864*a^6*c^4*d^3*e^12))/(256*(c^5*d^8 + a^4*c*e^8 + 4*a*c^4*d^6*e^2
 + 6*a^2*c^3*d^4*e^4 + 4*a^3*c^2*d^2*e^6)) - (((176*a^2*c^10*d^13*e^3 - 16*
a^8*c^4*d*e^15 + 864*a^3*c^9*d^11*e^5 + 1680*a^4*c^8*d^9*e^7 + 1600*a^5*c^7
*d^7*e^9 + 720*a^6*c^6*d^5*e^11 + 96*a^7*c^5*d^3*e^13)/(2*(c^5*d^8 + a^4*c*
e^8 + 4*a*c^4*d^6*e^2 + 6*a^2*c^3*d^4*e^4 + 4*a^3*c^2*d^2*e^6)) + (x*(-d^5*
e)^(1/2)*(65536*a^9*c^5*e^17 - 65536*a^2*c^12*d^14*e^3 - 327680*a^3*c^11*d^
12*e^5 - 589824*a^4*c^10*d^10*e^7 - 327680*a^5*c^9*d^8*e^9 + 327680*a^6*c^8
*d^6*e^11 + 589824*a^7*c^7*d^4*e^13 + 327680*a^8*c^6*d^2*e^15))/(512*(a^2*e
^4 + c^2*d^4 + 2*a*c*d^2*e^2)*(c^5*d^8 + a^4*c*e^8 + 4*a*c^4*d^6*e^2 + 6*a^
2*c^3*d^4*e^4 + 4*a^3*c^2*d^2*e^6)))*(-d^5*e)^(1/2))/(2*(a^2*e^4 + c^2*d^4
+ 2*a*c*d^2*e^2)))*(-d^5*e)^(1/2))/(2*(a^2*e^4 + c^2*d^4 + 2*a*c*d^2*e^2)))
)/(2*(a^2*e^4 + c^2*d^4 + 2*a*c*d^2*e^2)) + (x*(a^6*e^13 - 288*a*c^5*d^10*e
^3 + 20*a^5*c*d^2*e^11 + 17*a^2*c^4*d^8*e^5 + 148*a^3*c^3*d^6*e^7 + 118*a^4
*c^2*d^4*e^9))/(256*(c^5*d^8 + a^4*c*e^8 + 4*a*c^4*d^6*e^2 + 6*a^2*c^3*d^4*
e^4 + 4*a^3*c^2*d^2*e^6))))/(a^2*e^4 + c^2*d^4 + 2*a*c*d^2*e^2)))*(-d^5*e)^
(1/2)*1i)/(a^2*e^4 + c^2*d^4 + 2*a*c*d^2*e^2)
```

3.254 $\int \frac{x^4}{(d+ex^2)(a+cx^4)^2} dx$

Optimal result

Integrand size = 22, antiderivative size = 685

$$\begin{aligned}
\int \frac{x^4}{(d+ex^2)(a+cx^4)^2}\,dx &= -\frac{x(d-ex^2)}{4(cd^2+ae^2)(a+cx^4)} + \frac{d^{3/2}e^{3/2}\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{(cd^2+ae^2)^2} \\
&\quad - \frac{\sqrt[4]{c}d^2(\sqrt{c}d-\sqrt{a}e)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}(cd^2+ae^2)^2} \\
&\quad + \frac{(3\sqrt{c}d-\sqrt{a}e)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{8\sqrt{2}a^{3/4}c^{3/4}(cd^2+ae^2)} \\
&\quad + \frac{\sqrt[4]{c}d^2(\sqrt{c}d-\sqrt{a}e)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}(cd^2+ae^2)^2} \\
&\quad - \frac{(3\sqrt{c}d-\sqrt{a}e)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{8\sqrt{2}a^{3/4}c^{3/4}(cd^2+ae^2)} \\
&\quad - \frac{\sqrt[4]{c}d^2(\sqrt{c}d+\sqrt{a}e)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)}{4\sqrt{2}a^{3/4}(cd^2+ae^2)^2} \\
&\quad + \frac{(3\sqrt{c}d+\sqrt{a}e)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)}{16\sqrt{2}a^{3/4}c^{3/4}(cd^2+ae^2)} \\
&\quad + \frac{\sqrt[4]{c}d^2(\sqrt{c}d+\sqrt{a}e)\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)}{4\sqrt{2}a^{3/4}(cd^2+ae^2)^2} \\
&\quad - \frac{(3\sqrt{c}d+\sqrt{a}e)\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)}{16\sqrt{2}a^{3/4}c^{3/4}(cd^2+ae^2)}
\end{aligned}$$

```
[Out] -1/4*x*(-e*x^2+d)/(a*e^2+c*d^2)/(c*x^4+a)+d^(3/2)*e^(3/2)*arctan(x*e^(1/2)/
d^(1/2))/(a*e^2+c*d^2)^2+1/4*c^(1/4)*d^2*arctan(-1+c^(1/4)*x*2^(1/2)/a^(1/4
))*(-e*a^(1/2)+d*c^(1/2))/a^(3/4)/(a*e^2+c*d^2)^2*2^(1/2)+1/4*c^(1/4)*d^2*a
rctan(1+c^(1/4)*x*2^(1/2)/a^(1/4))*(-e*a^(1/2)+d*c^(1/2))/a^(3/4)/(a*e^2+c*
d^2)^2*2^(1/2)-1/8*c^(1/4)*d^2*ln(-a^(1/4)*c^(1/4)*x*2^(1/2)+a^(1/2)+x^2*c^
(1/2))*(e*a^(1/2)+d*c^(1/2))/a^(3/4)/(a*e^2+c*d^2)^2*2^(1/2)+1/8*c^(1/4)*d^
2*ln(a^(1/4)*c^(1/4)*x*2^(1/2)+a^(1/2)+x^2*c^(1/2))*(e*a^(1/2)+d*c^(1/2))/a
^(3/4)/(a*e^2+c*d^2)^2*2^(1/2)-1/16*arctan(-1+c^(1/4)*x*2^(1/2)/a^(1/4))*(-
e*a^(1/2)+3*d*c^(1/2))/a^(3/4)/c^(3/4)/(a*e^2+c*d^2)*2^(1/2)-1/16*arctan(1+
c^(1/4)*x*2^(1/2)/a^(1/4))*(-e*a^(1/2)+3*d*c^(1/2))/a^(3/4)/c^(3/4)/(a*e^2+
c*d^2)*2^(1/2)+1/32*ln(-a^(1/4)*c^(1/4)*x*2^(1/2)+a^(1/2)+x^2*c^(1/2))*(e*a
^(1/2)+3*d*c^(1/2))/a^(3/4)/c^(3/4)/(a*e^2+c*d^2)*2^(1/2)-1/32*ln(a^(1/4)*c
^(1/4)*x*2^(1/2)+a^(1/2)+x^2*c^(1/2))*(e*a^(1/2)+3*d*c^(1/2))/a^(3/4)/c^(3/
4)/(a*e^2+c*d^2)*2^(1/2)
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 685, normalized size of antiderivative = 1.00, number of steps used = 23, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.455$, Rules

used = {1328, 1193, 1182, 1176, 631, 210, 1179, 642, 1185, 211}

$$\int \frac{x^4}{(d+ex^2)(a+cx^4)^2}\,dx = \frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\left(3\sqrt{c}d-\sqrt{a}e\right)}{8\sqrt{2}a^{3/4}c^{3/4}\left(ae^2+cd^2\right)} - \frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)\left(3\sqrt{c}d-\sqrt{a}e\right)}{8\sqrt{2}a^{3/4}c^{3/4}\left(ae^2+cd^2\right)} - \frac{\sqrt[4]{c}d^2\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\left(\sqrt{c}d-\sqrt{a}e\right)}{2\sqrt{2}a^{3/4}\left(ae^2+cd^2\right)^2} + \frac{\sqrt[4]{c}d^2\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)\left(\sqrt{c}d-\sqrt{a}e\right)}{2\sqrt{2}a^{3/4}\left(ae^2+cd^2\right)^2} + \frac{\left(\sqrt{a}e+3\sqrt{c}d\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{16\sqrt{2}a^{3/4}c^{3/4}\left(ae^2+cd^2\right)} - \frac{\left(\sqrt{a}e+3\sqrt{c}d\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{16\sqrt{2}a^{3/4}c^{3/4}\left(ae^2+cd^2\right)} - \frac{\sqrt[4]{c}d^2\left(\sqrt{a}e+\sqrt{c}d\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{4\sqrt{2}a^{3/4}\left(ae^2+cd^2\right)^2} + \frac{\sqrt[4]{c}d^2\left(\sqrt{a}e+\sqrt{c}d\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{4\sqrt{2}a^{3/4}\left(ae^2+cd^2\right)^2} + \frac{d^{3/2}e^{3/2}\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\left(ae^2+cd^2\right)^2} - \frac{x(d-ex^2)}{4\left(a+cx^4\right)\left(ae^2+cd^2\right)}$$

```
[In] Int[x^4/((d + e*x^2)*(a + c*x^4)^2),x]
```

```
[Out] -1/4*(x*(d - e*x^2))/((c*d^2 + a*e^2)*(a + c*x^4)) + (d^(3/2)*e^(3/2)*ArcTa
n[(Sqrt[e]*x)/Sqrt[d]])/(c*d^2 + a*e^2)^2 - (c^(1/4)*d^2*(Sqrt[c]*d - Sqrt[
a]*e)*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*(c*d^2 +
a*e^2)^2) + ((3*Sqrt[c]*d - Sqrt[a]*e)*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/
4)])/(8*Sqrt[2]*a^(3/4)*c^(3/4)*(c*d^2 + a*e^2)) + (c^(1/4)*d^2*(Sqrt[c]*d
- Sqrt[a]*e)*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*(c
*d^2 + a*e^2)^2) - ((3*Sqrt[c]*d - Sqrt[a]*e)*ArcTan[1 + (Sqrt[2]*c^(1/4)*x
)/a^(1/4)])/(8*Sqrt[2]*a^(3/4)*c^(3/4)*(c*d^2 + a*e^2)) - (c^(1/4)*d^2*(Sqr
t[c]*d + Sqrt[a]*e)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])
/(4*Sqrt[2]*a^(3/4)*(c*d^2 + a*e^2)^2) + ((3*Sqrt[c]*d + Sqrt[a]*e)*Log[Sqr
t[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(16*Sqrt[2]*a^(3/4)*c^(3/4
)*(c*d^2 + a*e^2)) + (c^(1/4)*d^2*(Sqrt[c]*d + Sqrt[a]*e)*Log[Sqrt[a] + Sqr
t[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(4*Sqrt[2]*a^(3/4)*(c*d^2 + a*e^2)^2
) - ((3*Sqrt[c]*d + Sqrt[a]*e)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sq
rt[c]*x^2])/(16*Sqrt[2]*a^(3/4)*c^(3/4)*(c*d^2 + a*e^2))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1182

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[(-a)
*c]
```

Rule 1185

```
Int[((d_) + (e_.)*(x_)^2)^(q_)/((a_) + (c_.)*(x_)^4), x_Symbol] :> Int[Expa
ndIntegrand[(d + e*x^2)^q/(a + c*x^4), x], x] /; FreeQ[{a, c, d, e}, x] &&
NeQ[c*d^2 + a*e^2, 0] && IntegerQ[q]
```

Rule 1193

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[(-x
)*(d + e*x^2)*((a + c*x^4)^(p + 1)/(4*a*(p + 1))), x] + Dist[1/(4*a*(p + 1)
), Int[Simp[d*(4*p + 5) + e*(4*p + 7)*x^2, x]*(a + c*x^4)^(p + 1), x], x] /
; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && IntegerQ
[2*p]
```

Rule 1328

```
Int[(((f_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^4)^(p_))/((d_.) + (e_.)*(x_)^2),
 x_Symbol] :> Dist[(-a)*(f^4/(c*d^2 + a*e^2)), Int[(f*x)^(m - 4)*(d - e*x^2
)*(a + c*x^4)^p, x], x] + Dist[d^2*(f^4/(c*d^2 + a*e^2)), Int[(f*x)^(m - 4)
*((a + c*x^4)^(p + 1)/(d + e*x^2)), x], x] /; FreeQ[{a, c, d, e, f}, x] &&
LtQ[p, -1] && GtQ[m, 2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{a\int \frac{d-ex^2}{(a+cx^4)^2}\,dx}{cd^2+ae^2}+\frac{d^2\int \frac{1}{(d+ex^2)(a+cx^4)}\,dx}{cd^2+ae^2}\\
&= -\frac{x(d-ex^2)}{4\left(cd^2+ae^2\right)\left(a+cx^4\right)}+\frac{\int \frac{-3d+ex^2}{a+cx^4}\,dx}{4\left(cd^2+ae^2\right)}+\frac{d^2\int \left(\frac{e^2}{(cd^2+ae^2)(d+ex^2)}+\frac{c(d-ex^2)}{(cd^2+ae^2)(a+cx^4)}\right)dx}{cd^2+ae^2}\\
&= -\frac{x(d-ex^2)}{4\left(cd^2+ae^2\right)\left(a+cx^4\right)}+\frac{\left(cd^2\right)\int \frac{d-ex^2}{a+cx^4}\,dx}{\left(cd^2+ae^2\right)^2}+\frac{\left(d^2e^2\right)\int \frac{1}{d+ex^2}\,dx}{\left(cd^2+ae^2\right)^2}\\
&\quad -\frac{\left(\frac{3\sqrt{c}d}{\sqrt{a}}-e\right)\int \frac{\sqrt{a}\sqrt{c}+cx^2}{a+cx^4}\,dx}{8c\left(cd^2+ae^2\right)}-\frac{\left(\frac{3\sqrt{c}d}{\sqrt{a}}+e\right)\int \frac{\sqrt{a}\sqrt{c}-cx^2}{a+cx^4}\,dx}{8c\left(cd^2+ae^2\right)}
\end{aligned}$$

$$=-\frac{x(d-ex^2)}{4(cd^2+ae^2)(a+cx^4)}+\frac{d^{3/2}e^{3/2}\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{(cd^2+ae^2)^2}+\frac{\left(d^2\left(\frac{\sqrt{c}d}{\sqrt{a}}-e\right)\right)\int\frac{\sqrt{a}\sqrt{c}+cx^2}{a+cx^4}\,dx}{2(cd^2+ae^2)^2}+\frac{\left(d^2\left(\frac{\sqrt{c}d}{\sqrt{a}}+e\right)\right)\int\frac{\sqrt{a}\sqrt{c}-cx^2}{a+cx^4}\,dx}{2(cd^2+ae^2)^2}-\frac{\left(\frac{3\sqrt{c}d}{\sqrt{a}}-e\right)\int\frac{1}{\frac{\sqrt{a}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}+x^2}\,dx}{16c(cd^2+ae^2)}-\frac{\left(\frac{3\sqrt{c}d}{\sqrt{a}}-e\right)\int\frac{1}{\frac{\sqrt{a}}{\sqrt{c}}+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}+x^2}\,dx}{16c(cd^2+ae^2)}+\frac{(3\sqrt{c}d+\sqrt{a}e)\int\frac{\frac{\sqrt{2}\sqrt[4]{a}}{\sqrt[4]{c}}+2x}{-\frac{\sqrt{a}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}-x^2}\,dx}{16\sqrt{2}a^{3/4}c^{3/4}(cd^2+ae^2)}+\frac{(3\sqrt{c}d+\sqrt{a}e)\int\frac{\frac{\sqrt{2}\sqrt[4]{a}}{\sqrt[4]{c}}-2x}{-\frac{\sqrt{a}}{\sqrt{c}}+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}-x^2}\,dx}{16\sqrt{2}a^{3/4}c^{3/4}(cd^2+ae^2)}$$

$$=-\frac{x(d-ex^2)}{4(cd^2+ae^2)(a+cx^4)}+\frac{d^{3/2}e^{3/2}\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{(cd^2+ae^2)^2}+\frac{(3\sqrt{c}d+\sqrt{a}e)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)}{16\sqrt{2}a^{3/4}c^{3/4}(cd^2+ae^2)}-\frac{(3\sqrt{c}d+\sqrt{a}e)\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)}{16\sqrt{2}a^{3/4}c^{3/4}(cd^2+ae^2)}+\frac{\left(d^2\left(\frac{\sqrt{c}d}{\sqrt{a}}-e\right)\right)\int\frac{1}{\frac{\sqrt{a}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}+x^2}\,dx}{4(cd^2+ae^2)^2}+\frac{\left(d^2\left(\frac{\sqrt{c}d}{\sqrt{a}}-e\right)\right)\int\frac{1}{\frac{\sqrt{a}}{\sqrt{c}}+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}+x^2}\,dx}{4(cd^2+ae^2)^2}-\frac{\left(\sqrt[4]{c}d^2(\sqrt{c}d+\sqrt{a}e)\right)\int\frac{\frac{\sqrt{2}\sqrt[4]{a}}{\sqrt[4]{c}}+2x}{-\frac{\sqrt{a}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}-x^2}\,dx}{4\sqrt{2}a^{3/4}(cd^2+ae^2)^2}-\frac{\left(\sqrt[4]{c}d^2(\sqrt{c}d+\sqrt{a}e)\right)\int\frac{\frac{\sqrt{2}\sqrt[4]{a}}{\sqrt[4]{c}}-2x}{-\frac{\sqrt{a}}{\sqrt{c}}+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}-x^2}\,dx}{4\sqrt{2}a^{3/4}(cd^2+ae^2)^2}-\frac{(3\sqrt{c}d-\sqrt{a}e)\,\text{Subst}\left(\int\frac{1}{-1-x^2}\,dx,x,1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{8\sqrt{2}a^{3/4}c^{3/4}(cd^2+ae^2)}+\frac{(3\sqrt{c}d-\sqrt{a}e)\,\text{Subst}\left(\int\frac{1}{-1-x^2}\,dx,x,1+\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{8\sqrt{2}a^{3/4}c^{3/4}(cd^2+ae^2)}$$

$$=-\frac{x(d-ex^2)}{4(cd^2+ae^2)(a+cx^4)}+\frac{d^{3/2}e^{3/2}\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{(cd^2+ae^2)^2}$$
$$+\frac{(3\sqrt{c}d-\sqrt{a}e)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{8\sqrt{2}a^{3/4}c^{3/4}(cd^2+ae^2)}-\frac{(3\sqrt{c}d-\sqrt{a}e)\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{8\sqrt{2}a^{3/4}c^{3/4}(cd^2+ae^2)}$$
$$-\frac{\sqrt[4]{c}d^2(\sqrt{c}d+\sqrt{a}e)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)}{4\sqrt{2}a^{3/4}(cd^2+ae^2)^2}$$
$$+\frac{(3\sqrt{c}d+\sqrt{a}e)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)}{16\sqrt{2}a^{3/4}c^{3/4}(cd^2+ae^2)}$$
$$+\frac{\sqrt[4]{c}d^2(\sqrt{c}d+\sqrt{a}e)\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)}{4\sqrt{2}a^{3/4}(cd^2+ae^2)^2}$$
$$-\frac{(3\sqrt{c}d+\sqrt{a}e)\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)}{16\sqrt{2}a^{3/4}c^{3/4}(cd^2+ae^2)}$$
$$+\frac{\left(\sqrt[4]{c}d^2(\sqrt{c}d-\sqrt{a}e)\right)\text{Subst}\left(\int\frac{1}{-1-x^2}\,dx,x,1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}(cd^2+ae^2)^2}$$
$$-\frac{\left(\sqrt[4]{c}d^2(\sqrt{c}d-\sqrt{a}e)\right)\text{Subst}\left(\int\frac{1}{-1-x^2}\,dx,x,1+\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}(cd^2+ae^2)^2}$$

$$=-\frac{x(d-ex^2)}{4(cd^2+ae^2)(a+cx^4)}+\frac{d^{3/2}e^{3/2}\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{(cd^2+ae^2)^2}$$
$$-\frac{\sqrt[4]{c}d^2(\sqrt{c}d-\sqrt{a}e)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}(cd^2+ae^2)^2}+\frac{(3\sqrt{c}d-\sqrt{a}e)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{8\sqrt{2}a^{3/4}c^{3/4}(cd^2+ae^2)}$$
$$+\frac{\sqrt[4]{c}d^2(\sqrt{c}d-\sqrt{a}e)\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}(cd^2+ae^2)^2}-\frac{(3\sqrt{c}d-\sqrt{a}e)\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{8\sqrt{2}a^{3/4}c^{3/4}(cd^2+ae^2)}$$
$$-\frac{\sqrt[4]{c}d^2(\sqrt{c}d+\sqrt{a}e)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)}{4\sqrt{2}a^{3/4}(cd^2+ae^2)^2}$$
$$+\frac{(3\sqrt{c}d+\sqrt{a}e)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)}{16\sqrt{2}a^{3/4}c^{3/4}(cd^2+ae^2)}$$
$$+\frac{\sqrt[4]{c}d^2(\sqrt{c}d+\sqrt{a}e)\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)}{4\sqrt{2}a^{3/4}(cd^2+ae^2)^2}$$
$$-\frac{(3\sqrt{c}d+\sqrt{a}e)\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)}{16\sqrt{2}a^{3/4}c^{3/4}(cd^2+ae^2)}$$

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 423, normalized size of antiderivative = 0.62

$$\int \frac{x^4}{(d+ex^2)\,(a+cx^4)^2}\,dx$$

$$= \frac{\frac{8(cd^2+ae^2)(-dx+ex^3)}{a+cx^4} + 32d^{3/2}e^{3/2}\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right) - \frac{2\sqrt{2}(c^{3/2}d^3-3\sqrt{a}cd^2e-3a\sqrt{c}de^2+a^{3/2}e^3)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{a^{3/4}c^{3/4}} + \frac{2\sqrt{2}(c^{3/2}c}{}}{}$$

```
[In] Integrate[x^4/((d + e*x^2)*(a + c*x^4)^2),x]
```

```
[Out] ((8*(c*d^2 + a*e^2)*(-(d*x) + e*x^3))/(a + c*x^4) + 32*d^(3/2)*e^(3/2)*ArcT
an[(Sqrt[e]*x)/Sqrt[d]] - (2*Sqrt[2]*(c^(3/2)*d^3 - 3*Sqrt[a]*c*d^2*e - 3*a
*Sqrt[c]*d*e^2 + a^(3/2)*e^3)*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(a^(
3/4)*c^(3/4)) + (2*Sqrt[2]*(c^(3/2)*d^3 - 3*Sqrt[a]*c*d^2*e - 3*a*Sqrt[c]*d
*e^2 + a^(3/2)*e^3)*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(a^(3/4)*c^(3/
4)) + (Sqrt[2]*(-(c^(3/2)*d^3) - 3*Sqrt[a]*c*d^2*e + 3*a*Sqrt[c]*d*e^2 + a^
(3/2)*e^3)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(a^(3/4)
*c^(3/4)) + (Sqrt[2]*(c^(3/2)*d^3 + 3*Sqrt[a]*c*d^2*e - 3*a*Sqrt[c]*d*e^2 -
 a^(3/2)*e^3)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(a^(3
/4)*c^(3/4)))/(32*(c*d^2 + a*e^2)^2)
```

Maple [A] (verified)

Time = 0.48 (sec) , antiderivative size = 327, normalized size of antiderivative = 0.48

method	result
default	$-\frac{\frac{\left(-\frac{1}{4}a\,e^3-\frac{1}{4}c\,d^2e\right)x^3+\left(\frac{1}{4}d\,e^2a+\frac{1}{4}d^3c\right)x}{c\,x^4+a}+\frac{\left(3d\,e^2a-d^3c\right)\left(\frac{a}{c}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x^2+\left(\frac{a}{c}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{c}}}{x^2-\left(\frac{a}{c}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{c}}}\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{c}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{c}\right)^{\frac{1}{4}}}-1\right)\right)}{32a}}{\left(a\,e^2+c\,d^2\right)^2}$
risch	Expression too large to display

```
[In] int(x^4/(e*x^2+d)/(c*x^4+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/(a*e^2+c*d^2)^2*(((-1/4*a*e^3-1/4*c*d^2*e)*x^3+(1/4*d*e^2*a+1/4*d^3*c)*x
)/(c*x^4+a)+1/32*(3*a*d*e^2-c*d^3)*(a/c)^(1/4)/a*2^(1/2)*(ln((x^2+(a/c)^(1/
4)*x*2^(1/2)+(a/c)^(1/2))/(x^2-(a/c)^(1/4)*x*2^(1/2)+(a/c)^(1/2)))+2*arctan
(2^(1/2)/(a/c)^(1/4)*x+1)+2*arctan(2^(1/2)/(a/c)^(1/4)*x-1))+1/32*(-a*e^3+3
*c*d^2*e)/c/(a/c)^(1/4)*2^(1/2)*(ln((x^2-(a/c)^(1/4)*x*2^(1/2)+(a/c)^(1/2))
/(x^2+(a/c)^(1/4)*x*2^(1/2)+(a/c)^(1/2)))+2*arctan(2^(1/2)/(a/c)^(1/4)*x+1)
+2*arctan(2^(1/2)/(a/c)^(1/4)*x-1)))+d^2*e^2/(a*e^2+c*d^2)^2/(e*d)^(1/2)*ar
ctan(e*x/(e*d)^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 4829 vs. 2(514) = 1028.

Time = 5.78 (sec) , antiderivative size = 9678, normalized size of antiderivative = 14.13

$$\int \frac{x^4}{(d+ex^2)(a+cx^4)^2}\,dx = \text{Too large to display}$$

```
[In] integrate(x^4/(e*x^2+d)/(c*x^4+a)^2,x, algorithm="fricas")
```

```
[Out] Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^4}{(d+ex^2)(a+cx^4)^2}\,dx = \text{Timed out}$$

```
[In] integrate(x**4/(e*x**2+d)/(c*x**4+a)**2,x)
```

```
[Out] Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^4}{(d+ex^2)(a+cx^4)^2}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(x^4/(e*x^2+d)/(c*x^4+a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 607, normalized size of antiderivative = 0.89

$$\int \frac{x^4}{(d+ex^2)(a+cx^4)^2}\,dx = \frac{d^2e^2\arctan\left(\frac{ex}{\sqrt{de}}\right)}{(c^2d^4+2\,acd^2e^2+a^2e^4)\sqrt{de}} + \frac{\left((ac^3)^{\frac{1}{4}}\,c^3d^3-3\,(ac^3)^{\frac{1}{4}}\,ac^2de^2-3\,(ac^3)^{\frac{3}{4}}\,cd^2e+(ac^3)^{\frac{3}{4}}\,ae^3\right)\arctan\left(\frac{\sqrt{2}\left(2\,x+\sqrt{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{8\left(\sqrt{2}ac^5d^4+2\,\sqrt{2}a^2c^4d^2e^2+\sqrt{2}a^3c^3e^4\right)} + \frac{\left((ac^3)^{\frac{1}{4}}\,c^3d^3-3\,(ac^3)^{\frac{1}{4}}\,ac^2de^2-3\,(ac^3)^{\frac{3}{4}}\,cd^2e+(ac^3)^{\frac{3}{4}}\,ae^3\right)\arctan\left(\frac{\sqrt{2}\left(2\,x-\sqrt{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{8\left(\sqrt{2}ac^5d^4+2\,\sqrt{2}a^2c^4d^2e^2+\sqrt{2}a^3c^3e^4\right)} + \frac{\left((ac^3)^{\frac{1}{4}}\,c^3d^3-3\,(ac^3)^{\frac{1}{4}}\,ac^2de^2+3\,(ac^3)^{\frac{3}{4}}\,cd^2e-(ac^3)^{\frac{3}{4}}\,ae^3\right)\log\left(x^2+\sqrt{2}x\left(\frac{a}{c}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{c}}\right)}{16\left(\sqrt{2}ac^5d^4+2\,\sqrt{2}a^2c^4d^2e^2+\sqrt{2}a^3c^3e^4\right)} - \frac{\left((ac^3)^{\frac{1}{4}}\,c^3d^3-3\,(ac^3)^{\frac{1}{4}}\,ac^2de^2+3\,(ac^3)^{\frac{3}{4}}\,cd^2e-(ac^3)^{\frac{3}{4}}\,ae^3\right)\log\left(x^2-\sqrt{2}x\left(\frac{a}{c}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{c}}\right)}{16\left(\sqrt{2}ac^5d^4+2\,\sqrt{2}a^2c^4d^2e^2+\sqrt{2}a^3c^3e^4\right)} + \frac{ex^3-dx}{4\,(cx^4+a)(cd^2+ae^2)}$$

```
[In] integrate(x^4/(e*x^2+d)/(c*x^4+a)^2,x, algorithm="giac")
```

```
[Out] d^2*e^2*arctan(e*x/sqrt(d*e))/((c^2*d^4 + 2*a*c*d^2*e^2 + a^2*e^4)*sqrt(d*e
)) + 1/8*((a*c^3)^(1/4)*c^3*d^3 - 3*(a*c^3)^(1/4)*a*c^2*d*e^2 - 3*(a*c^3)^(
3/4)*c*d^2*e + (a*c^3)^(3/4)*a*e^3)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(a/c)
^(1/4))/(a/c)^(1/4))/(sqrt(2)*a*c^5*d^4 + 2*sqrt(2)*a^2*c^4*d^2*e^2 + sqrt(
2)*a^3*c^3*e^4) + 1/8*((a*c^3)^(1/4)*c^3*d^3 - 3*(a*c^3)^(1/4)*a*c^2*d*e^2
- 3*(a*c^3)^(3/4)*c*d^2*e + (a*c^3)^(3/4)*a*e^3)*arctan(1/2*sqrt(2)*(2*x -
sqrt(2)*(a/c)^(1/4))/(a/c)^(1/4))/(sqrt(2)*a*c^5*d^4 + 2*sqrt(2)*a^2*c^4*d^
2*e^2 + sqrt(2)*a^3*c^3*e^4) + 1/16*((a*c^3)^(1/4)*c^3*d^3 - 3*(a*c^3)^(1/4
)*a*c^2*d*e^2 + 3*(a*c^3)^(3/4)*c*d^2*e - (a*c^3)^(3/4)*a*e^3)*log(x^2 + sq
rt(2)*x*(a/c)^(1/4) + sqrt(a/c))/(sqrt(2)*a*c^5*d^4 + 2*sqrt(2)*a^2*c^4*d^2
*e^2 + sqrt(2)*a^3*c^3*e^4) - 1/16*((a*c^3)^(1/4)*c^3*d^3 - 3*(a*c^3)^(1/4)
*a*c^2*d*e^2 + 3*(a*c^3)^(3/4)*c*d^2*e - (a*c^3)^(3/4)*a*e^3)*log(x^2 - sqr
t(2)*x*(a/c)^(1/4) + sqrt(a/c))/(sqrt(2)*a*c^5*d^4 + 2*sqrt(2)*a^2*c^4*d^2*
e^2 + sqrt(2)*a^3*c^3*e^4) + 1/4*(e*x^3 - d*x)/((c*x^4 + a)*(c*d^2 + a*e^2)
)
```

Mupad [B] (verification not implemented)

Time = 10.46 (sec) , antiderivative size = 17180, normalized size of antiderivative = 25.08

$$\int \frac{x^4}{(d+ex^2)(a+cx^4)^2} dx = \text{Too large to display}$$

```
[In] int(x^4/((a + c*x^4)^2*(d + e*x^2)),x)
```

```
[Out] - atan((((((28672*a^2*c^8*d^10*e^4 - 4096*a*c^9*d^12*e^2 + 155648*a^3*c^7*
d^8*e^6 + 253952*a^4*c^6*d^6*e^8 + 176128*a^5*c^5*d^4*e^10 + 45056*a^6*c^4*
d^2*e^12)/(256*(a^3*e^6 + c^3*d^6 + 3*a*c^2*d^4*e^2 + 3*a^2*c*d^2*e^4)) - (
x*((a^3*e^6*(-a^3*c^3)^(1/2) - c^3*d^6*(-a^3*c^3)^(1/2) + 6*a^2*c^4*d^5*e +
 6*a^4*c^2*d*e^5 - 20*a^3*c^3*d^3*e^3 + 15*a*c^2*d^4*e^2*(-a^3*c^3)^(1/2) -
 15*a^2*c*d^2*e^4*(-a^3*c^3)^(1/2))/(256*(a^3*c^7*d^8 + a^7*c^3*e^8 + 4*a^4
*c^6*d^6*e^2 + 6*a^5*c^5*d^4*e^4 + 4*a^6*c^4*d^2*e^6)))^(1/2)*(65536*a^9*c^
4*e^17 - 65536*a^2*c^11*d^14*e^3 - 327680*a^3*c^10*d^12*e^5 - 589824*a^4*c^
9*d^10*e^7 - 327680*a^5*c^8*d^8*e^9 + 327680*a^6*c^7*d^6*e^11 + 589824*a^7*
c^6*d^4*e^13 + 327680*a^8*c^5*d^2*e^15))/(128*(a^4*e^8 + c^4*d^8 + 4*a*c^3*
d^6*e^2 + 4*a^3*c*d^2*e^6 + 6*a^2*c^2*d^4*e^4)))*((a^3*e^6*(-a^3*c^3)^(1/2)
 - c^3*d^6*(-a^3*c^3)^(1/2) + 6*a^2*c^4*d^5*e + 6*a^4*c^2*d*e^5 - 20*a^3*c^
3*d^3*e^3 + 15*a*c^2*d^4*e^2*(-a^3*c^3)^(1/2) - 15*a^2*c*d^2*e^4*(-a^3*c^3)
^(1/2))/(256*(a^3*c^7*d^8 + a^7*c^3*e^8 + 4*a^4*c^6*d^6*e^2 + 6*a^5*c^5*d^4
*e^4 + 4*a^6*c^4*d^2*e^6)))^(1/2) + (x*(256*a*c^8*d^11*e^4 - 128*c^9*d^13*e
^2 + 2944*a^6*c^3*d*e^14 + 21632*a^2*c^7*d^9*e^6 + 32256*a^3*c^6*d^7*e^8 +
4224*a^4*c^5*d^5*e^10 - 3840*a^5*c^4*d^3*e^12))/(128*(a^4*e^8 + c^4*d^8 + 4
*a*c^3*d^6*e^2 + 4*a^3*c*d^2*e^6 + 6*a^2*c^2*d^4*e^4)))*((a^3*e^6*(-a^3*c^3
)^(1/2) - c^3*d^6*(-a^3*c^3)^(1/2) + 6*a^2*c^4*d^5*e + 6*a^4*c^2*d*e^5 - 20
*a^3*c^3*d^3*e^3 + 15*a*c^2*d^4*e^2*(-a^3*c^3)^(1/2) - 15*a^2*c*d^2*e^4*(-a
^3*c^3)^(1/2))/(256*(a^3*c^7*d^8 + a^7*c^3*e^8 + 4*a^4*c^6*d^6*e^2 + 6*a^5*
c^5*d^4*e^4 + 4*a^6*c^4*d^2*e^6)))^(1/2) + (16*c^6*d^9*e^3 - 960*a*c^5*d^7*
e^5 + 16*a^4*c^2*d*e^11 + 8288*a^2*c^4*d^5*e^7 - 3008*a^3*c^3*d^3*e^9)/(256
*(a^3*e^6 + c^3*d^6 + 3*a*c^2*d^4*e^2 + 3*a^2*c*d^2*e^4)))*((a^3*e^6*(-a^3*
c^3)^(1/2) - c^3*d^6*(-a^3*c^3)^(1/2) + 6*a^2*c^4*d^5*e + 6*a^4*c^2*d*e^5 -
 20*a^3*c^3*d^3*e^3 + 15*a*c^2*d^4*e^2*(-a^3*c^3)^(1/2) - 15*a^2*c*d^2*e^4*
(-a^3*c^3)^(1/2))/(256*(a^3*c^7*d^8 + a^7*c^3*e^8 + 4*a^4*c^6*d^6*e^2 + 6*a
^5*c^5*d^4*e^4 + 4*a^6*c^4*d^2*e^6)))^(1/2) - (x*(a^4*c*e^13 + 33*c^5*d^8*e
^5 - 188*a*c^4*d^6*e^7 + 38*a^2*c^3*d^4*e^9 + 4*a^3*c^2*d^2*e^11))/(128*(a^
4*e^8 + c^4*d^8 + 4*a*c^3*d^6*e^2 + 4*a^3*c*d^2*e^6 + 6*a^2*c^2*d^4*e^4)))*
((a^3*e^6*(-a^3*c^3)^(1/2) - c^3*d^6*(-a^3*c^3)^(1/2) + 6*a^2*c^4*d^5*e + 6
*a^4*c^2*d*e^5 - 20*a^3*c^3*d^3*e^3 + 15*a*c^2*d^4*e^2*(-a^3*c^3)^(1/2) - 1
5*a^2*c*d^2*e^4*(-a^3*c^3)^(1/2))/(256*(a^3*c^7*d^8 + a^7*c^3*e^8 + 4*a^4*c
^6*d^6*e^2 + 6*a^5*c^5*d^4*e^4 + 4*a^6*c^4*d^2*e^6)))^(1/2)*1i - (((((28672
*a^2*c^8*d^10*e^4 - 4096*a*c^9*d^12*e^2 + 155648*a^3*c^7*d^8*e^6 + 253952*a
^4*c^6*d^6*e^8 + 176128*a^5*c^5*d^4*e^10 + 45056*a^6*c^4*d^2*e^12)/(256*(a^
```

3*e^6 + c^3*d^6 + 3*a*c^2*d^4*e^2 + 3*a^2*c*d^2*e^4)) + (x*((a^3*e^6*(-a^3*c^3)^(1/2) - c^3*d^6*(-a^3*c^3)^(1/2) + 6*a^2*c^4*d^5*e + 6*a^4*c^2*d*e^5 - 20*a^3*c^3*d^3*e^3 + 15*a*c^2*d^4*e^2*(-a^3*c^3)^(1/2) - 15*a^2*c*d^2*e^4*(-a^3*c^3)^(1/2))/(256*(a^3*c^7*d^8 + a^7*c^3*e^8 + 4*a^4*c^6*d^6*e^2 + 6*a^5*c^5*d^4*e^4 + 4*a^6*c^4*d^2*e^6)))^(1/2)*(65536*a^9*c^4*e^17 - 65536*a^2*c^11*d^14*e^3 - 327680*a^3*c^10*d^12*e^5 - 589824*a^4*c^9*d^10*e^7 - 327680*a^5*c^8*d^8*e^9 + 327680*a^6*c^7*d^6*e^11 + 589824*a^7*c^6*d^4*e^13 + 327680*a^8*c^5*d^2*e^15))/(128*(a^4*e^8 + c^4*d^8 + 4*a*c^3*d^6*e^2 + 4*a^3*c*d^2*e^6 + 6*a^2*c^2*d^4*e^4)))*((a^3*e^6*(-a^3*c^3)^(1/2) - c^3*d^6*(-a^3*c^3)^(1/2) + 6*a^2*c^4*d^5*e + 6*a^4*c^2*d*e^5 - 20*a^3*c^3*d^3*e^3 + 15*a*c^2*d^4*e^2*(-a^3*c^3)^(1/2) - 15*a^2*c*d^2*e^4*(-a^3*c^3)^(1/2))/(256*(a^3*c^7*d^8 + a^7*c^3*e^8 + 4*a^4*c^6*d^6*e^2 + 6*a^5*c^5*d^4*e^4 + 4*a^6*c^4*d^2*e^6)))^(1/2) - (x*(256*a*c^8*d^11*e^4 - 128*c^9*d^13*e^2 + 2944*a^6*c^3*d*e^14 + 21632*a^2*c^7*d^9*e^6 + 32256*a^3*c^6*d^7*e^8 + 4224*a^4*c^5*d^5*e^10 - 3840*a^5*c^4*d^3*e^12))/(128*(a^4*e^8 + c^4*d^8 + 4*a*c^3*d^6*e^2 + 4*a^3*c*d^2*e^6 + 6*a^2*c^2*d^4*e^4)))*((a^3*e^6*(-a^3*c^3)^(1/2) - c^3*d^6*(-a^3*c^3)^(1/2) + 6*a^2*c^4*d^5*e + 6*a^4*c^2*d*e^5 - 20*a^3*c^3*d^3*e^3 + 15*a*c^2*d^4*e^2*(-a^3*c^3)^(1/2) - 15*a^2*c*d^2*e^4*(-a^3*c^3)^(1/2))/(256*(a^3*c^7*d^8 + a^7*c^3*e^8 + 4*a^4*c^6*d^6*e^2 + 6*a^5*c^5*d^4*e^4 + 4*a^6*c^4*d^2*e^6)))^(1/2) + (16*c^6*d^9*e^3 - 960*a*c^5*d^7*e^5 + 16*a^4*c^2*d*e^11 + 8288*a^2*c^4*d^5*e^7 - 3008*a^3*c^3*d^3*e^9)/(256*(a^3*e^6 + c^3*d^6 + 3*a*c^2*d^4*e^2 + 3*a^2*c*d^2*e^4)))*((a^3*e^6*(-a^3*c^3)^(1/2) - c^3*d^6*(-a^3*c^3)^(1/2) + 6*a^2*c^4*d^5*e + 6*a^4*c^2*d*e^5 - 20*a^3*c^3*d^3*e^3 + 15*a*c^2*d^4*e^2*(-a^3*c^3)^(1/2) - 15*a^2*c*d^2*e^4*(-a^3*c^3)^(1/2))/(256*(a^3*c^7*d^8 + a^7*c^3*e^8 + 4*a^4*c^6*d^6*e^2 + 6*a^5*c^5*d^4*e^4 + 4*a^6*c^4*d^2*e^6)))^(1/2) + (x*(a^4*c*e^13 + 33*c^5*d^8*e^5 - 188*a*c^4*d^6*e^7 + 38*a^2*c^3*d^4*e^9 + 4*a^3*c^2*d^2*e^11))/(128*(a^4*e^8 + c^4*d^8 + 4*a*c^3*d^6*e^2 + 4*a^3*c*d^2*e^6 + 6*a^2*c^2*d^4*e^4)))*((a^3*e^6*(-a^3*c^3)^(1/2) - c^3*d^6*(-a^3*c^3)^(1/2) + 6*a^2*c^4*d^5*e + 6*a^4*c^2*d*e^5 - 20*a^3*c^3*d^3*e^3 + 15*a*c^2*d^4*e^2*(-a^3*c^3)^(1/2) - 15*a^2*c*d^2*e^4*(-a^3*c^3)^(1/2))/(256*(a^3*c^7*d^8 + a^7*c^3*e^8 + 4*a^4*c^6*d^6*e^2 + 6*a^5*c^5*d^4*e^4 + 4*a^6*c^4*d^2*e^6)))^(1/2)*1i)/((5*c^2*d^4*e^6 + a*c*d^2*e^8)/(128*(a^3*e^6 + c^3*d^6 + 3*a*c^2*d^4*e^2 + 3*a^2*c*d^2*e^4)) + (((((28672*a^2*c^8*d^10*e^4 - 4096*a*c^9*d^12*e^2 + 155648*a^3*c^7*d^8*e^6 + 253952*a^4*c^6*d^6*e^8 + 176128*a^5*c^5*d^4*e^10 + 45056*a^6*c^4*d^2*e^12)/(256*(a^3*e^6 + c^3*d^6 + 3*a*c^2*d^4*e^2 + 3*a^2*c*d^2*e^4)) - (x*((a^3*e^6*(-a^3*c^3)^(1/2) - c^3*d^6*(-a^3*c^3)^(1/2) + 6*a^2*c^4*d^5*e + 6*a^4*c^2*d*e^5 - 20*a^3*c^3*d^3*e^3 + 15*a*c^2*d^4*e^2*(-a^3*c^3)^(1/2) - 15*a^2*c*d^2*e^4*(-a^3*c^3)^(1/2))/(256*(a^3*c^7*d^8 + a^7*c^3*e^8 + 4*a^4*c^6*d^6*e^2 + 6*a^5*c^5*d^4*e^4 + 4*a^6*c^4*d^2*e^6)))^(1/2)*(65536*a^9*c^4*e^17 - 65536*a^2*c^11*d^14*e^3 - 327680*a^3*c^10*d^12*e^5 - 589824*a^4*c^9*d^10*e^7 - 327680*a^5*c^8*d^8*e^9 + 327680*a^6*c^7*d^6*e^11 + 589824*a^7*c^6*d^4*e^13 + 327680*a^8*c^5*d^2*e^15))/(128*(a^4*e^8 + c^4*d^8 + 4*a*c^3*d^6*e^2 + 4*a^3*c*d^2*e^6 + 6*a^2*c^2*d^4*e^4)))*((a^3*e^6*(-a^3*c^3)^(1/2) - c^3*d^6*(-a^3*c^3)^(1/2) + 6*a^2*c^4*d^5*e + 6*a^4*c^2*d*e^5 - 20*a^3*c^3*d^3*e^3 + 15*a*

c^2*d^4*e^2*(-a^3*c^3)^(1/2) - 15*a^2*c*d^2*e^4*(-a^3*c^3)^(1/2))/(256*(a^3
*c^7*d^8 + a^7*c^3*e^8 + 4*a^4*c^6*d^6*e^2 + 6*a^5*c^5*d^4*e^4 + 4*a^6*c^4*
d^2*e^6)))^(1/2) + (x*(256*a*c^8*d^11*e^4 - 128*c^9*d^13*e^2 + 2944*a^6*c^3
*d*e^14 + 21632*a^2*c^7*d^9*e^6 + 32256*a^3*c^6*d^7*e^8 + 4224*a^4*c^5*d^5*
e^10 - 3840*a^5*c^4*d^3*e^12))/(128*(a^4*e^8 + c^4*d^8 + 4*a*c^3*d^6*e^2 +
4*a^3*c*d^2*e^6 + 6*a^2*c^2*d^4*e^4)))*((a^3*e^6*(-a^3*c^3)^(1/2) - c^3*d^6
*(-a^3*c^3)^(1/2) + 6*a^2*c^4*d^5*e + 6*a^4*c^2*d*e^5 - 20*a^3*c^3*d^3*e^3
+ 15*a*c^2*d^4*e^2*(-a^3*c^3)^(1/2) - 15*a^2*c*d^2*e^4*(-a^3*c^3)^(1/2))/(2
56*(a^3*c^7*d^8 + a^7*c^3*e^8 + 4*a^4*c^6*d^6*e^2 + 6*a^5*c^5*d^4*e^4 + 4*a
^6*c^4*d^2*e^6)))^(1/2) + (16*c^6*d^9*e^3 - 960*a*c^5*d^7*e^5 + 16*a^4*c^2*
d*e^11 + 8288*a^2*c^4*d^5*e^7 - 3008*a^3*c^3*d^3*e^9)/(256*(a^3*e^6 + c^3*d
^6 + 3*a*c^2*d^4*e^2 + 3*a^2*c*d^2*e^4)))*((a^3*e^6*(-a^3*c^3)^(1/2) - c^3*
d^6*(-a^3*c^3)^(1/2) + 6*a^2*c^4*d^5*e + 6*a^4*c^2*d*e^5 - 20*a^3*c^3*d^3*e
^3 + 15*a*c^2*d^4*e^2*(-a^3*c^3)^(1/2) - 15*a^2*c*d^2*e^4*(-a^3*c^3)^(1/2))
/(256*(a^3*c^7*d^8 + a^7*c^3*e^8 + 4*a^4*c^6*d^6*e^2 + 6*a^5*c^5*d^4*e^4 +
4*a^6*c^4*d^2*e^6)))^(1/2) - (x*(a^4*c*e^13 + 33*c^5*d^8*e^5 - 188*a*c^4*d^
6*e^7 + 38*a^2*c^3*d^4*e^9 + 4*a^3*c^2*d^2*e^11))/(128*(a^4*e^8 + c^4*d^8 +
4*a*c^3*d^6*e^2 + 4*a^3*c*d^2*e^6 + 6*a^2*c^2*d^4*e^4)))*((a^3*e^6*(-a^3*c
^3)^(1/2) - c^3*d^6*(-a^3*c^3)^(1/2) + 6*a^2*c^4*d^5*e + 6*a^4*c^2*d*e^5 -
20*a^3*c^3*d^3*e^3 + 15*a*c^2*d^4*e^2*(-a^3*c^3)^(1/2) - 15*a^2*c*d^2*e^4*(
-a^3*c^3)^(1/2))/(256*(a^3*c^7*d^8 + a^7*c^3*e^8 + 4*a^4*c^6*d^6*e^2 + 6*a^
5*c^5*d^4*e^4 + 4*a^6*c^4*d^2*e^6)))^(1/2) + (((((28672*a^2*c^8*d^10*e^4 -
4096*a*c^9*d^12*e^2 + 155648*a^3*c^7*d^8*e^6 + 253952*a^4*c^6*d^6*e^8 + 176
128*a^5*c^5*d^4*e^10 + 45056*a^6*c^4*d^2*e^12)/(256*(a^3*e^6 + c^3*d^6 + 3*
a*c^2*d^4*e^2 + 3*a^2*c*d^2*e^4)) + (x*((a^3*e^6*(-a^3*c^3)^(1/2) - c^3*d^6
*(-a^3*c^3)^(1/2) + 6*a^2*c^4*d^5*e + 6*a^4*c^2*d*e^5 - 20*a^3*c^3*d^3*e^3
+ 15*a*c^2*d^4*e^2*(-a^3*c^3)^(1/2) - 15*a^2*c*d^2*e^4*(-a^3*c^3)^(1/2))/(2
56*(a^3*c^7*d^8 + a^7*c^3*e^8 + 4*a^4*c^6*d^6*e^2 + 6*a^5*c^5*d^4*e^4 + 4*a
^6*c^4*d^2*e^6)))^(1/2)*(65536*a^9*c^4*e^17 - 65536*a^2*c^11*d^14*e^3 - 327
680*a^3*c^10*d^12*e^5 - 589824*a^4*c^9*d^10*e^7 - 327680*a^5*c^8*d^8*e^9 +
327680*a^6*c^7*d^6*e^11 + 589824*a^7*c^6*d^4*e^13 + 327680*a^8*c^5*d^2*e^15
))/(128*(a^4*e^8 + c^4*d^8 + 4*a*c^3*d^6*e^2 + 4*a^3*c*d^2*e^6 + 6*a^2*c^2*
d^4*e^4)))*((a^3*e^6*(-a^3*c^3)^(1/2) - c^3*d^6*(-a^3*c^3)^(1/2) + 6*a^2*c^
4*d^5*e + 6*a^4*c^2*d*e^5 - 20*a^3*c^3*d^3*e^3 + 15*a*c^2*d^4*e^2*(-a^3*c^3
)^(1/2) - 15*a^2*c*d^2*e^4*(-a^3*c^3)^(1/2))/(256*(a^3*c^7*d^8 + a^7*c^3*e^
8 + 4*a^4*c^6*d^6*e^2 + 6*a^5*c^5*d^4*e^4 + 4*a^6*c^4*d^2*e^6)))^(1/2) - (x
*(256*a*c^8*d^11*e^4 - 128*c^9*d^13*e^2 + 2944*a^6*c^3*d*e^14 + 21632*a^2*c
^7*d^9*e^6 + 32256*a^3*c^6*d^7*e^8 + 4224*a^4*c^5*d^5*e^10 - 3840*a^5*c^4*d
^3*e^12))/(128*(a^4*e^8 + c^4*d^8 + 4*a*c^3*d^6*e^2 + 4*a^3*c*d^2*e^6 + 6*a
^2*c^2*d^4*e^4)))*((a^3*e^6*(-a^3*c^3)^(1/2) - c^3*d^6*(-a^3*c^3)^(1/2) + 6
*a^2*c^4*d^5*e + 6*a^4*c^2*d*e^5 - 20*a^3*c^3*d^3*e^3 + 15*a*c^2*d^4*e^2*(-
a^3*c^3)^(1/2) - 15*a^2*c*d^2*e^4*(-a^3*c^3)^(1/2))/(256*(a^3*c^7*d^8 + a^7
*c^3*e^8 + 4*a^4*c^6*d^6*e^2 + 6*a^5*c^5*d^4*e^4 + 4*a^6*c^4*d^2*e^6)))^(1/
2) + (16*c^6*d^9*e^3 - 960*a*c^5*d^7*e^5 + 16*a^4*c^2*d*e^11 + 8288*a^2*c^4
*d^5*e^7 - 3008*a^3*c^3*d^3*e^9)/(256*(a^3*e^6 + c^3*d^6 + 3*a*c^2*d^4*e^2

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+ 3*a^2*c*d^2*e^4)))*((a^3*e^6*(-a^3*c^3)^(1/2) - c^3*d^6*(-a^3*c^3)^(1/2)
+ 6*a^2*c^4*d^5*e + 6*a^4*c^2*d*e^5 - 20*a^3*c^3*d^3*e^3 + 15*a*c^2*d^4*e^2
*(-a^3*c^3)^(1/2) - 15*a^2*c*d^2*e^4*(-a^3*c^3)^(1/2))/(256*(a^3*c^7*d^8 +
a^7*c^3*e^8 + 4*a^4*c^6*d^6*e^2 + 6*a^5*c^5*d^4*e^4 + 4*a^6*c^4*d^2*e^6)))^
(1/2) + (x*(a^4*c*e^13 + 33*c^5*d^8*e^5 - 188*a*c^4*d^6*e^7 + 38*a^2*c^3*d^
4*e^9 + 4*a^3*c^2*d^2*e^11))/(128*(a^4*e^8 + c^4*d^8 + 4*a*c^3*d^6*e^2 + 4*
a^3*c*d^2*e^6 + 6*a^2*c^2*d^4*e^4)))*((a^3*e^6*(-a^3*c^3)^(1/2) - c^3*d^6*(
-a^3*c^3)^(1/2) + 6*a^2*c^4*d^5*e + 6*a^4*c^2*d*e^5 - 20*a^3*c^3*d^3*e^3 +
15*a*c^2*d^4*e^2*(-a^3*c^3)^(1/2) - 15*a^2*c*d^2*e^4*(-a^3*c^3)^(1/2))/(256
*(a^3*c^7*d^8 + a^7*c^3*e^8 + 4*a^4*c^6*d^6*e^2 + 6*a^5*c^5*d^4*e^4 + 4*a^6
*c^4*d^2*e^6)))^(1/2)))*((a^3*e^6*(-a^3*c^3)^(1/2) - c^3*d^6*(-a^3*c^3)^(1/
2) + 6*a^2*c^4*d^5*e + 6*a^4*c^2*d*e^5 - 20*a^3*c^3*d^3*e^3 + 15*a*c^2*d^4*
e^2*(-a^3*c^3)^(1/2) - 15*a^2*c*d^2*e^4*(-a^3*c^3)^(1/2))/(256*(a^3*c^7*d^8
 + a^7*c^3*e^8 + 4*a^4*c^6*d^6*e^2 + 6*a^5*c^5*d^4*e^4 + 4*a^6*c^4*d^2*e^6)
))^(1/2)*2i - atan((((((28672*a^2*c^8*d^10*e^4 - 4096*a*c^9*d^12*e^2 + 155
648*a^3*c^7*d^8*e^6 + 253952*a^4*c^6*d^6*e^8 + 176128*a^5*c^5*d^4*e^10 + 45
056*a^6*c^4*d^2*e^12)/(256*(a^3*e^6 + c^3*d^6 + 3*a*c^2*d^4*e^2 + 3*a^2*c*d
^2*e^4)) - (x*((c^3*d^6*(-a^3*c^3)^(1/2) - a^3*e^6*(-a^3*c^3)^(1/2) + 6*a^2
*c^4*d^5*e + 6*a^4*c^2*d*e^5 - 20*a^3*c^3*d^3*e^3 - 15*a*c^2*d^4*e^2*(-a^3*
c^3)^(1/2) + 15*a^2*c*d^2*e^4*(-a^3*c^3)^(1/2))/(256*(a^3*c^7*d^8 + a^7*c^3
*e^8 + 4*a^4*c^6*d^6*e^2 + 6*a^5*c^5*d^4*e^4 + 4*a^6*c^4*d^2*e^6)))^(1/2)*(
65536*a^9*c^4*e^17 - 65536*a^2*c^11*d^14*e^3 - 327680*a^3*c^10*d^12*e^5 - 5
89824*a^4*c^9*d^10*e^7 - 327680*a^5*c^8*d^8*e^9 + 327680*a^6*c^7*d^6*e^11 +
 589824*a^7*c^6*d^4*e^13 + 327680*a^8*c^5*d^2*e^15))/(128*(a^4*e^8 + c^4*d^
8 + 4*a*c^3*d^6*e^2 + 4*a^3*c*d^2*e^6 + 6*a^2*c^2*d^4*e^4)))*((c^3*d^6*(-a^
3*c^3)^(1/2) - a^3*e^6*(-a^3*c^3)^(1/2) + 6*a^2*c^4*d^5*e + 6*a^4*c^2*d*e^5
 - 20*a^3*c^3*d^3*e^3 - 15*a*c^2*d^4*e^2*(-a^3*c^3)^(1/2) + 15*a^2*c*d^2*e^
4*(-a^3*c^3)^(1/2))/(256*(a^3*c^7*d^8 + a^7*c^3*e^8 + 4*a^4*c^6*d^6*e^2 + 6
*a^5*c^5*d^4*e^4 + 4*a^6*c^4*d^2*e^6)))^(1/2) + (x*(256*a*c^8*d^11*e^4 - 12
8*c^9*d^13*e^2 + 2944*a^6*c^3*d*e^14 + 21632*a^2*c^7*d^9*e^6 + 32256*a^3*c^
6*d^7*e^8 + 4224*a^4*c^5*d^5*e^10 - 3840*a^5*c^4*d^3*e^12))/(128*(a^4*e^8 +
 c^4*d^8 + 4*a*c^3*d^6*e^2 + 4*a^3*c*d^2*e^6 + 6*a^2*c^2*d^4*e^4)))*((c^3*d
^6*(-a^3*c^3)^(1/2) - a^3*e^6*(-a^3*c^3)^(1/2) + 6*a^2*c^4*d^5*e + 6*a^4*c^
2*d*e^5 - 20*a^3*c^3*d^3*e^3 - 15*a*c^2*d^4*e^2*(-a^3*c^3)^(1/2) + 15*a^2*c
*d^2*e^4*(-a^3*c^3)^(1/2))/(256*(a^3*c^7*d^8 + a^7*c^3*e^8 + 4*a^4*c^6*d^6*
e^2 + 6*a^5*c^5*d^4*e^4 + 4*a^6*c^4*d^2*e^6)))^(1/2) + (16*c^6*d^9*e^3 - 96
0*a*c^5*d^7*e^5 + 16*a^4*c^2*d*e^11 + 8288*a^2*c^4*d^5*e^7 - 3008*a^3*c^3*d
^3*e^9)/(256*(a^3*e^6 + c^3*d^6 + 3*a*c^2*d^4*e^2 + 3*a^2*c*d^2*e^4)))*((c^
3*d^6*(-a^3*c^3)^(1/2) - a^3*e^6*(-a^3*c^3)^(1/2) + 6*a^2*c^4*d^5*e + 6*a^4
*c^2*d*e^5 - 20*a^3*c^3*d^3*e^3 - 15*a*c^2*d^4*e^2*(-a^3*c^3)^(1/2) + 15*a^
2*c*d^2*e^4*(-a^3*c^3)^(1/2))/(256*(a^3*c^7*d^8 + a^7*c^3*e^8 + 4*a^4*c^6*d
^6*e^2 + 6*a^5*c^5*d^4*e^4 + 4*a^6*c^4*d^2*e^6)))^(1/2) - (x*(a^4*c*e^13 +
33*c^5*d^8*e^5 - 188*a*c^4*d^6*e^7 + 38*a^2*c^3*d^4*e^9 + 4*a^3*c^2*d^2*e^1
1))/(128*(a^4*e^8 + c^4*d^8 + 4*a*c^3*d^6*e^2 + 4*a^3*c*d^2*e^6 + 6*a^2*c^2
*d^4*e^4)))*((c^3*d^6*(-a^3*c^3)^(1/2) - a^3*e^6*(-a^3*c^3)^(1/2) + 6*a^2*c
```

^4*d^5*e + 6*a^4*c^2*d*e^5 - 20*a^3*c^3*d^3*e^3 - 15*a*c^2*d^4*e^2*(-a^3*c^3)^(1/2) + 15*a^2*c*d^2*e^4*(-a^3*c^3)^(1/2))/(256*(a^3*c^7*d^8 + a^7*c^3*e^8 + 4*a^4*c^6*d^6*e^2 + 6*a^5*c^5*d^4*e^4 + 4*a^6*c^4*d^2*e^6)))^(1/2)*1i - (((((28672*a^2*c^8*d^10*e^4 - 4096*a*c^9*d^12*e^2 + 155648*a^3*c^7*d^8*e^6 + 253952*a^4*c^6*d^6*e^8 + 176128*a^5*c^5*d^4*e^10 + 45056*a^6*c^4*d^2*e^12)/(256*(a^3*e^6 + c^3*d^6 + 3*a*c^2*d^4*e^2 + 3*a^2*c*d^2*e^4)) + (x*((c^3*d^6*(-a^3*c^3)^(1/2) - a^3*e^6*(-a^3*c^3)^(1/2) + 6*a^2*c^4*d^5*e + 6*a^4*c^2*d*e^5 - 20*a^3*c^3*d^3*e^3 - 15*a*c^2*d^4*e^2*(-a^3*c^3)^(1/2) + 15*a^2*c*d^2*e^4*(-a^3*c^3)^(1/2))/(256*(a^3*c^7*d^8 + a^7*c^3*e^8 + 4*a^4*c^6*d^6*e^2 + 6*a^5*c^5*d^4*e^4 + 4*a^6*c^4*d^2*e^6)))^(1/2)*(65536*a^9*c^4*e^17 - 65536*a^2*c^11*d^14*e^3 - 327680*a^3*c^10*d^12*e^5 - 589824*a^4*c^9*d^10*e^7 - 327680*a^5*c^8*d^8*e^9 + 327680*a^6*c^7*d^6*e^11 + 589824*a^7*c^6*d^4*e^13 + 327680*a^8*c^5*d^2*e^15))/(128*(a^4*e^8 + c^4*d^8 + 4*a*c^3*d^6*e^2 + 4*a^3*c*d^2*e^6 + 6*a^2*c^2*d^4*e^4)))*((c^3*d^6*(-a^3*c^3)^(1/2) - a^3*e^6*(-a^3*c^3)^(1/2) + 6*a^2*c^4*d^5*e + 6*a^4*c^2*d*e^5 - 20*a^3*c^3*d^3*e^3 - 15*a*c^2*d^4*e^2*(-a^3*c^3)^(1/2) + 15*a^2*c*d^2*e^4*(-a^3*c^3)^(1/2))/(256*(a^3*c^7*d^8 + a^7*c^3*e^8 + 4*a^4*c^6*d^6*e^2 + 6*a^5*c^5*d^4*e^4 + 4*a^6*c^4*d^2*e^6)))^(1/2) - (x*(256*a*c^8*d^11*e^4 - 128*c^9*d^13*e^2 + 2944*a^6*c^3*d*e^14 + 21632*a^2*c^7*d^9*e^6 + 32256*a^3*c^6*d^7*e^8 + 4224*a^4*c^5*d^5*e^10 - 3840*a^5*c^4*d^3*e^12))/(128*(a^4*e^8 + c^4*d^8 + 4*a*c^3*d^6*e^2 + 4*a^3*c*d^2*e^6 + 6*a^2*c^2*d^4*e^4)))*((c^3*d^6*(-a^3*c^3)^(1/2) - a^3*e^6*(-a^3*c^3)^(1/2) + 6*a^2*c^4*d^5*e + 6*a^4*c^2*d*e^5 - 20*a^3*c^3*d^3*e^3 - 15*a*c^2*d^4*e^2*(-a^3*c^3)^(1/2) + 15*a^2*c*d^2*e^4*(-a^3*c^3)^(1/2))/(256*(a^3*c^7*d^8 + a^7*c^3*e^8 + 4*a^4*c^6*d^6*e^2 + 6*a^5*c^5*d^4*e^4 + 4*a^6*c^4*d^2*e^6)))^(1/2) + (16*c^6*d^9*e^3 - 960*a*c^5*d^7*e^5 + 16*a^4*c^2*d*e^11 + 8288*a^2*c^4*d^5*e^7 - 3008*a^3*c^3*d^3*e^9)/(256*(a^3*e^6 + c^3*d^6 + 3*a*c^2*d^4*e^2 + 3*a^2*c*d^2*e^4)))*((c^3*d^6*(-a^3*c^3)^(1/2) - a^3*e^6*(-a^3*c^3)^(1/2) + 6*a^2*c^4*d^5*e + 6*a^4*c^2*d*e^5 - 20*a^3*c^3*d^3*e^3 - 15*a*c^2*d^4*e^2*(-a^3*c^3)^(1/2) + 15*a^2*c*d^2*e^4*(-a^3*c^3)^(1/2))/(256*(a^3*c^7*d^8 + a^7*c^3*e^8 + 4*a^4*c^6*d^6*e^2 + 6*a^5*c^5*d^4*e^4 + 4*a^6*c^4*d^2*e^6)))^(1/2) + (x*(a^4*c*e^13 + 33*c^5*d^8*e^5 - 188*a*c^4*d^6*e^7 + 38*a^2*c^3*d^4*e^9 + 4*a^3*c^2*d^2*e^11))/(128*(a^4*e^8 + c^4*d^8 + 4*a*c^3*d^6*e^2 + 4*a^3*c*d^2*e^6 + 6*a^2*c^2*d^4*e^4)))*((c^3*d^6*(-a^3*c^3)^(1/2) - a^3*e^6*(-a^3*c^3)^(1/2) + 6*a^2*c^4*d^5*e + 6*a^4*c^2*d*e^5 - 20*a^3*c^3*d^3*e^3 - 15*a*c^2*d^4*e^2*(-a^3*c^3)^(1/2) + 15*a^2*c*d^2*e^4*(-a^3*c^3)^(1/2))/(256*(a^3*c^7*d^8 + a^7*c^3*e^8 + 4*a^4*c^6*d^6*e^2 + 6*a^5*c^5*d^4*e^4 + 4*a^6*c^4*d^2*e^6)))^(1/2)*1i)/((5*c^2*d^4*e^6 + a*c*d^2*e^8)/(128*(a^3*e^6 + c^3*d^6 + 3*a*c^2*d^4*e^2 + 3*a^2*c*d^2*e^4)) + (((((28672*a^2*c^8*d^10*e^4 - 4096*a*c^9*d^12*e^2 + 155648*a^3*c^7*d^8*e^6 + 253952*a^4*c^6*d^6*e^8 + 176128*a^5*c^5*d^4*e^10 + 45056*a^6*c^4*d^2*e^12)/(256*(a^3*e^6 + c^3*d^6 + 3*a*c^2*d^4*e^2 + 3*a^2*c*d^2*e^4)) - (x*((c^3*d^6*(-a^3*c^3)^(1/2) - a^3*e^6*(-a^3*c^3)^(1/2) + 6*a^2*c^4*d^5*e + 6*a^4*c^2*d*e^5 - 20*a^3*c^3*d^3*e^3 - 15*a*c^2*d^4*e^2*(-a^3*c^3)^(1/2) + 15*a^2*c*d^2*e^4*(-a^3*c^3)^(1/2))/(256*(a^3*c^7*d^8 + a^7*c^3*e^8 + 4*a^4*c^6*d^6*e^2 + 6*a^5*c^5*d^4*e^4 + 4*a^6*c^4*d^2*e^6)))^(1/2)*(65536*a^9*c^4*e^1

7 - 65536*a^2*c^11*d^14*e^3 - 327680*a^3*c^10*d^12*e^5 - 589824*a^4*c^9*d^1
0*e^7 - 327680*a^5*c^8*d^8*e^9 + 327680*a^6*c^7*d^6*e^11 + 589824*a^7*c^6*d
^4*e^13 + 327680*a^8*c^5*d^2*e^15))/(128*(a^4*e^8 + c^4*d^8 + 4*a*c^3*d^6*e
^2 + 4*a^3*c*d^2*e^6 + 6*a^2*c^2*d^4*e^4)))*((c^3*d^6*(-a^3*c^3)^(1/2) - a^
3*e^6*(-a^3*c^3)^(1/2) + 6*a^2*c^4*d^5*e + 6*a^4*c^2*d*e^5 - 20*a^3*c^3*d^3
*e^3 - 15*a*c^2*d^4*e^2*(-a^3*c^3)^(1/2) + 15*a^2*c*d^2*e^4*(-a^3*c^3)^(1/2
))/(256*(a^3*c^7*d^8 + a^7*c^3*e^8 + 4*a^4*c^6*d^6*e^2 + 6*a^5*c^5*d^4*e^4
+ 4*a^6*c^4*d^2*e^6)))^(1/2) + (x*(256*a*c^8*d^11*e^4 - 128*c^9*d^13*e^2 +
2944*a^6*c^3*d*e^14 + 21632*a^2*c^7*d^9*e^6 + 32256*a^3*c^6*d^7*e^8 + 4224*
a^4*c^5*d^5*e^10 - 3840*a^5*c^4*d^3*e^12))/(128*(a^4*e^8 + c^4*d^8 + 4*a*c^
3*d^6*e^2 + 4*a^3*c*d^2*e^6 + 6*a^2*c^2*d^4*e^4)))*((c^3*d^6*(-a^3*c^3)^(1/
2) - a^3*e^6*(-a^3*c^3)^(1/2) + 6*a^2*c^4*d^5*e + 6*a^4*c^2*d*e^5 - 20*a^3*
c^3*d^3*e^3 - 15*a*c^2*d^4*e^2*(-a^3*c^3)^(1/2) + 15*a^2*c*d^2*e^4*(-a^3*c^
3)^(1/2))/(256*(a^3*c^7*d^8 + a^7*c^3*e^8 + 4*a^4*c^6*d^6*e^2 + 6*a^5*c^5*d
^4*e^4 + 4*a^6*c^4*d^2*e^6)))^(1/2) + (16*c^6*d^9*e^3 - 960*a*c^5*d^7*e^5 +
 16*a^4*c^2*d*e^11 + 8288*a^2*c^4*d^5*e^7 - 3008*a^3*c^3*d^3*e^9)/(256*(a^3
*e^6 + c^3*d^6 + 3*a*c^2*d^4*e^2 + 3*a^2*c*d^2*e^4)))*((c^3*d^6*(-a^3*c^3)^
(1/2) - a^3*e^6*(-a^3*c^3)^(1/2) + 6*a^2*c^4*d^5*e + 6*a^4*c^2*d*e^5 - 20*a
^3*c^3*d^3*e^3 - 15*a*c^2*d^4*e^2*(-a^3*c^3)^(1/2) + 15*a^2*c*d^2*e^4*(-a^3
c^3)^(1/2))/(256(a^3*c^7*d^8 + a^7*c^3*e^8 + 4*a^4*c^6*d^6*e^2 + 6*a^5*c^
5*d^4*e^4 + 4*a^6*c^4*d^2*e^6)))^(1/2) - (x*(a^4*c*e^13 + 33*c^5*d^8*e^5 -
188*a*c^4*d^6*e^7 + 38*a^2*c^3*d^4*e^9 + 4*a^3*c^2*d^2*e^11))/(128*(a^4*e^8
 + c^4*d^8 + 4*a*c^3*d^6*e^2 + 4*a^3*c*d^2*e^6 + 6*a^2*c^2*d^4*e^4)))*((c^3
d^6(-a^3*c^3)^(1/2) - a^3*e^6*(-a^3*c^3)^(1/2) + 6*a^2*c^4*d^5*e + 6*a^4*
c^2*d*e^5 - 20*a^3*c^3*d^3*e^3 - 15*a*c^2*d^4*e^2*(-a^3*c^3)^(1/2) + 15*a^2
*c*d^2*e^4*(-a^3*c^3)^(1/2))/(256*(a^3*c^7*d^8 + a^7*c^3*e^8 + 4*a^4*c^6*d^
6*e^2 + 6*a^5*c^5*d^4*e^4 + 4*a^6*c^4*d^2*e^6)))^(1/2) + (((((28672*a^2*c^8
*d^10*e^4 - 4096*a*c^9*d^12*e^2 + 155648*a^3*c^7*d^8*e^6 + 253952*a^4*c^6*d
^6*e^8 + 176128*a^5*c^5*d^4*e^10 + 45056*a^6*c^4*d^2*e^12)/(256*(a^3*e^6 +
c^3*d^6 + 3*a*c^2*d^4*e^2 + 3*a^2*c*d^2*e^4)) + (x*((c^3*d^6*(-a^3*c^3)^(1/
2) - a^3*e^6*(-a^3*c^3)^(1/2) + 6*a^2*c^4*d^5*e + 6*a^4*c^2*d*e^5 - 20*a^3*
c^3*d^3*e^3 - 15*a*c^2*d^4*e^2*(-a^3*c^3)^(1/2) + 15*a^2*c*d^2*e^4*(-a^3*c^
3)^(1/2))/(256*(a^3*c^7*d^8 + a^7*c^3*e^8 + 4*a^4*c^6*d^6*e^2 + 6*a^5*c^5*d
^4*e^4 + 4*a^6*c^4*d^2*e^6)))^(1/2)*(65536*a^9*c^4*e^17 - 65536*a^2*c^11*d^
14*e^3 - 327680*a^3*c^10*d^12*e^5 - 589824*a^4*c^9*d^10*e^7 - 327680*a^5*c^
8*d^8*e^9 + 327680*a^6*c^7*d^6*e^11 + 589824*a^7*c^6*d^4*e^13 + 327680*a^8*
c^5*d^2*e^15))/(128*(a^4*e^8 + c^4*d^8 + 4*a*c^3*d^6*e^2 + 4*a^3*c*d^2*e^6
+ 6*a^2*c^2*d^4*e^4)))*((c^3*d^6*(-a^3*c^3)^(1/2) - a^3*e^6*(-a^3*c^3)^(1/2
) + 6*a^2*c^4*d^5*e + 6*a^4*c^2*d*e^5 - 20*a^3*c^3*d^3*e^3 - 15*a*c^2*d^4*e
^2*(-a^3*c^3)^(1/2) + 15*a^2*c*d^2*e^4*(-a^3*c^3)^(1/2))/(256*(a^3*c^7*d^8
+ a^7*c^3*e^8 + 4*a^4*c^6*d^6*e^2 + 6*a^5*c^5*d^4*e^4 + 4*a^6*c^4*d^2*e^6))
)^(1/2) - (x*(256*a*c^8*d^11*e^4 - 128*c^9*d^13*e^2 + 2944*a^6*c^3*d*e^14 +
 21632*a^2*c^7*d^9*e^6 + 32256*a^3*c^6*d^7*e^8 + 4224*a^4*c^5*d^5*e^10 - 38
40*a^5*c^4*d^3*e^12))/(128*(a^4*e^8 + c^4*d^8 + 4*a*c^3*d^6*e^2 + 4*a^3*c*d
^2*e^6 + 6*a^2*c^2*d^4*e^4)))*((c^3*d^6*(-a^3*c^3)^(1/2) - a^3*e^6*(-a^3*c^

3)^(1/2) + 6*a^2*c^4*d^5*e + 6*a^4*c^2*d*e^5 - 20*a^3*c^3*d^3*e^3 - 15*a*c^
2*d^4*e^2*(-a^3*c^3)^(1/2) + 15*a^2*c*d^2*e^4*(-a^3*c^3)^(1/2))/(256*(a^3*c
^7*d^8 + a^7*c^3*e^8 + 4*a^4*c^6*d^6*e^2 + 6*a^5*c^5*d^4*e^4 + 4*a^6*c^4*d^
2*e^6)))^(1/2) + (16*c^6*d^9*e^3 - 960*a*c^5*d^7*e^5 + 16*a^4*c^2*d*e^11 +
8288*a^2*c^4*d^5*e^7 - 3008*a^3*c^3*d^3*e^9)/(256*(a^3*e^6 + c^3*d^6 + 3*a*
c^2*d^4*e^2 + 3*a^2*c*d^2*e^4)))*((c^3*d^6*(-a^3*c^3)^(1/2) - a^3*e^6*(-a^3
*c^3)^(1/2) + 6*a^2*c^4*d^5*e + 6*a^4*c^2*d*e^5 - 20*a^3*c^3*d^3*e^3 - 15*a
*c^2*d^4*e^2*(-a^3*c^3)^(1/2) + 15*a^2*c*d^2*e^4*(-a^3*c^3)^(1/2))/(256*(a^
3*c^7*d^8 + a^7*c^3*e^8 + 4*a^4*c^6*d^6*e^2 + 6*a^5*c^5*d^4*e^4 + 4*a^6*c^4
*d^2*e^6)))^(1/2) + (x*(a^4*c*e^13 + 33*c^5*d^8*e^5 - 188*a*c^4*d^6*e^7 + 3
8*a^2*c^3*d^4*e^9 + 4*a^3*c^2*d^2*e^11))/(128*(a^4*e^8 + c^4*d^8 + 4*a*c^3*
d^6*e^2 + 4*a^3*c*d^2*e^6 + 6*a^2*c^2*d^4*e^4)))*((c^3*d^6*(-a^3*c^3)^(1/2)
 - a^3*e^6*(-a^3*c^3)^(1/2) + 6*a^2*c^4*d^5*e + 6*a^4*c^2*d*e^5 - 20*a^3*c^
3*d^3*e^3 - 15*a*c^2*d^4*e^2*(-a^3*c^3)^(1/2) + 15*a^2*c*d^2*e^4*(-a^3*c^3)
^(1/2))/(256*(a^3*c^7*d^8 + a^7*c^3*e^8 + 4*a^4*c^6*d^6*e^2 + 6*a^5*c^5*d^4
*e^4 + 4*a^6*c^4*d^2*e^6)))^(1/2)))*((c^3*d^6*(-a^3*c^3)^(1/2) - a^3*e^6*(-
a^3*c^3)^(1/2) + 6*a^2*c^4*d^5*e + 6*a^4*c^2*d*e^5 - 20*a^3*c^3*d^3*e^3 - 1
5*a*c^2*d^4*e^2*(-a^3*c^3)^(1/2) + 15*a^2*c*d^2*e^4*(-a^3*c^3)^(1/2))/(256*
(a^3*c^7*d^8 + a^7*c^3*e^8 + 4*a^4*c^6*d^6*e^2 + 6*a^5*c^5*d^4*e^4 + 4*a^6*
c^4*d^2*e^6)))^(1/2)*2i - ((d*x)/(4*(a*e^2 + c*d^2)) - (e*x^3)/(4*(a*e^2 +
c*d^2)))/(a + c*x^4) - (atan(((((((((c^6*d^9*e^3)/16 - (15*a*c^5*d^7*e^5)/4
+ (a^4*c^2*d*e^11)/16 + (259*a^2*c^4*d^5*e^7)/8 - (47*a^3*c^3*d^3*e^9)/4)/(
2*(a^3*e^6 + c^3*d^6 + 3*a*c^2*d^4*e^2 + 3*a^2*c*d^2*e^4)) + (((x*(256*a*c^
8*d^11*e^4 - 128*c^9*d^13*e^2 + 2944*a^6*c^3*d*e^14 + 21632*a^2*c^7*d^9*e^6
 + 32256*a^3*c^6*d^7*e^8 + 4224*a^4*c^5*d^5*e^10 - 3840*a^5*c^4*d^3*e^12))/
(256*(a^4*e^8 + c^4*d^8 + 4*a*c^3*d^6*e^2 + 4*a^3*c*d^2*e^6 + 6*a^2*c^2*d^4
*e^4)) + (((112*a^2*c^8*d^10*e^4 - 16*a*c^9*d^12*e^2 + 608*a^3*c^7*d^8*e^6
+ 992*a^4*c^6*d^6*e^8 + 688*a^5*c^5*d^4*e^10 + 176*a^6*c^4*d^2*e^12)/(2*(a^
3*e^6 + c^3*d^6 + 3*a*c^2*d^4*e^2 + 3*a^2*c*d^2*e^4)) - (x*(-d^3*e^3)^(1/2)
*(65536*a^9*c^4*e^17 - 65536*a^2*c^11*d^14*e^3 - 327680*a^3*c^10*d^12*e^5 -
 589824*a^4*c^9*d^10*e^7 - 327680*a^5*c^8*d^8*e^9 + 327680*a^6*c^7*d^6*e^11
 + 589824*a^7*c^6*d^4*e^13 + 327680*a^8*c^5*d^2*e^15))/(512*(a^2*e^4 + c^2*
d^4 + 2*a*c*d^2*e^2)*(a^4*e^8 + c^4*d^8 + 4*a*c^3*d^6*e^2 + 4*a^3*c*d^2*e^6
 + 6*a^2*c^2*d^4*e^4)))*(-d^3*e^3)^(1/2))/(2*(a^2*e^4 + c^2*d^4 + 2*a*c*d^2
e^2)))(-d^3*e^3)^(1/2))/(2*(a^2*e^4 + c^2*d^4 + 2*a*c*d^2*e^2)))*(-d^3*e^
3)^(1/2))/(2*(a^2*e^4 + c^2*d^4 + 2*a*c*d^2*e^2)) - (x*(a^4*c*e^13 + 33*c^5
*d^8*e^5 - 188*a*c^4*d^6*e^7 + 38*a^2*c^3*d^4*e^9 + 4*a^3*c^2*d^2*e^11))/(2
56*(a^4*e^8 + c^4*d^8 + 4*a*c^3*d^6*e^2 + 4*a^3*c*d^2*e^6 + 6*a^2*c^2*d^4*e
^4)))*(-d^3*e^3)^(1/2)*1i)/(a^2*e^4 + c^2*d^4 + 2*a*c*d^2*e^2) - ((((((c^6*
d^9*e^3)/16 - (15*a*c^5*d^7*e^5)/4 + (a^4*c^2*d*e^11)/16 + (259*a^2*c^4*d^5
*e^7)/8 - (47*a^3*c^3*d^3*e^9)/4)/(2*(a^3*e^6 + c^3*d^6 + 3*a*c^2*d^4*e^2 +
 3*a^2*c*d^2*e^4)) - (((x*(256*a*c^8*d^11*e^4 - 128*c^9*d^13*e^2 + 2944*a^6
*c^3*d*e^14 + 21632*a^2*c^7*d^9*e^6 + 32256*a^3*c^6*d^7*e^8 + 4224*a^4*c^5*
d^5*e^10 - 3840*a^5*c^4*d^3*e^12))/(256*(a^4*e^8 + c^4*d^8 + 4*a*c^3*d^6*e^
2 + 4*a^3*c*d^2*e^6 + 6*a^2*c^2*d^4*e^4)) - (((112*a^2*c^8*d^10*e^4 - 16*a*

c^9*d^12*e^2 + 608*a^3*c^7*d^8*e^6 + 992*a^4*c^6*d^6*e^8 + 688*a^5*c^5*d^4*e^10 + 176*a^6*c^4*d^2*e^12)/(2*(a^3*e^6 + c^3*d^6 + 3*a*c^2*d^4*e^2 + 3*a^2*c*d^2*e^4)) + (x*(-d^3*e^3)^(1/2)*(65536*a^9*c^4*e^17 - 65536*a^2*c^11*d^14*e^3 - 327680*a^3*c^10*d^12*e^5 - 589824*a^4*c^9*d^10*e^7 - 327680*a^5*c^8*d^8*e^9 + 327680*a^6*c^7*d^6*e^11 + 589824*a^7*c^6*d^4*e^13 + 327680*a^8*c^5*d^2*e^15))/(512*(a^2*e^4 + c^2*d^4 + 2*a*c*d^2*e^2)*(a^4*e^8 + c^4*d^8 + 4*a*c^3*d^6*e^2 + 4*a^3*c*d^2*e^6 + 6*a^2*c^2*d^4*e^4)))*(-d^3*e^3)^(1/2))/(2*(a^2*e^4 + c^2*d^4 + 2*a*c*d^2*e^2)))*(-d^3*e^3)^(1/2))/(2*(a^2*e^4 + c^2*d^4 + 2*a*c*d^2*e^2)))*(-d^3*e^3)^(1/2))/(2*(a^2*e^4 + c^2*d^4 + 2*a*c*d^2*e^2)) + (x*(a^4*c*e^13 + 33*c^5*d^8*e^5 - 188*a*c^4*d^6*e^7 + 38*a^2*c^3*d^4*e^9 + 4*a^3*c^2*d^2*e^11))/(256*(a^4*e^8 + c^4*d^8 + 4*a*c^3*d^6*e^2 + 4*a^3*c*d^2*e^6 + 6*a^2*c^2*d^4*e^4)))*(-d^3*e^3)^(1/2)*1i)/(a^2*e^4 + c^2*d^4 + 2*a*c*d^2*e^2))/(((5*c^2*d^4*e^6)/128 + (a*c*d^2*e^8)/128)/(a^3*e^6 + c^3*d^6 + 3*a*c^2*d^4*e^2 + 3*a^2*c*d^2*e^4) + ((((((c^6*d^9*e^3)/16 - (15*a*c^5*d^7*e^5)/4 + (a^4*c^2*d*e^11)/16 + (259*a^2*c^4*d^5*e^7)/8 - (47*a^3*c^3*d^3*e^9)/4)/(2*(a^3*e^6 + c^3*d^6 + 3*a*c^2*d^4*e^2 + 3*a^2*c*d^2*e^4)) + (((x*(256*a*c^8*d^11*e^4 - 128*c^9*d^13*e^2 + 2944*a^6*c^3*d*e^14 + 21632*a^2*c^7*d^9*e^6 + 32256*a^3*c^6*d^7*e^8 + 4224*a^4*c^5*d^5*e^10 - 3840*a^5*c^4*d^3*e^12))/(256*(a^4*e^8 + c^4*d^8 + 4*a*c^3*d^6*e^2 + 4*a^3*c*d^2*e^6 + 6*a^2*c^2*d^4*e^4)) + (((112*a^2*c^8*d^10*e^4 - 16*a*c^9*d^12*e^2 + 608*a^3*c^7*d^8*e^6 + 992*a^4*c^6*d^6*e^8 + 688*a^5*c^5*d^4*e^10 + 176*a^6*c^4*d^2*e^12)/(2*(a^3*e^6 + c^3*d^6 + 3*a*c^2*d^4*e^2 + 3*a^2*c*d^2*e^4)) - (x*(-d^3*e^3)^(1/2)*(65536*a^9*c^4*e^17 - 65536*a^2*c^11*d^14*e^3 - 327680*a^3*c^10*d^12*e^5 - 589824*a^4*c^9*d^10*e^7 - 327680*a^5*c^8*d^8*e^9 + 327680*a^6*c^7*d^6*e^11 + 589824*a^7*c^6*d^4*e^13 + 327680*a^8*c^5*d^2*e^15))/(512*(a^2*e^4 + c^2*d^4 + 2*a*c*d^2*e^2)*(a^4*e^8 + c^4*d^8 + 4*a*c^3*d^6*e^2 + 4*a^3*c*d^2*e^6 + 6*a^2*c^2*d^4*e^4)))*(-d^3*e^3)^(1/2))/(2*(a^2*e^4 + c^2*d^4 + 2*a*c*d^2*e^2)))*(-d^3*e^3)^(1/2))/(2*(a^2*e^4 + c^2*d^4 + 2*a*c*d^2*e^2)))*(-d^3*e^3)^(1/2))/(2*(a^2*e^4 + c^2*d^4 + 2*a*c*d^2*e^2)) - (x*(a^4*c*e^13 + 33*c^5*d^8*e^5 - 188*a*c^4*d^6*e^7 + 38*a^2*c^3*d^4*e^9 + 4*a^3*c^2*d^2*e^11))/(256*(a^4*e^8 + c^4*d^8 + 4*a*c^3*d^6*e^2 + 4*a^3*c*d^2*e^6 + 6*a^2*c^2*d^4*e^4)))*(-d^3*e^3)^(1/2))/(a^2*e^4 + c^2*d^4 + 2*a*c*d^2*e^2) + ((((((c^6*d^9*e^3)/16 - (15*a*c^5*d^7*e^5)/4 + (a^4*c^2*d*e^11)/16 + (259*a^2*c^4*d^5*e^7)/8 - (47*a^3*c^3*d^3*e^9)/4)/(2*(a^3*e^6 + c^3*d^6 + 3*a*c^2*d^4*e^2 + 3*a^2*c*d^2*e^4)) - (((x*(256*a*c^8*d^11*e^4 - 128*c^9*d^13*e^2 + 2944*a^6*c^3*d*e^14 + 21632*a^2*c^7*d^9*e^6 + 32256*a^3*c^6*d^7*e^8 + 4224*a^4*c^5*d^5*e^10 - 3840*a^5*c^4*d^3*e^12))/(256*(a^4*e^8 + c^4*d^8 + 4*a*c^3*d^6*e^2 + 4*a^3*c*d^2*e^6 + 6*a^2*c^2*d^4*e^4)) - (((112*a^2*c^8*d^10*e^4 - 16*a*c^9*d^12*e^2 + 608*a^3*c^7*d^8*e^6 + 992*a^4*c^6*d^6*e^8 + 688*a^5*c^5*d^4*e^10 + 176*a^6*c^4*d^2*e^12)/(2*(a^3*e^6 + c^3*d^6 + 3*a*c^2*d^4*e^2 + 3*a^2*c*d^2*e^4)) + (x*(-d^3*e^3)^(1/2)*(65536*a^9*c^4*e^17 - 65536*a^2*c^11*d^14*e^3 - 327680*a^3*c^10*d^12*e^5 - 589824*a^4*c^9*d^10*e^7 - 327680*a^5*c^8*d^8*e^9 + 327680*a^6*c^7*d^6*e^11 + 589824*a^7*c^6*d^4*e^13 + 327680*a^8*c^5*d^2*e^15))/(512*(a^2*e^4 + c^2*d^4 + 2*a*c*d^2*e^2)*(a^4*e^8 + c^4*d^8 + 4*a*c^3*d^6*e^2 + 4*a^3*c*d^2*e^6 + 6*a^2*c^2*d^4*e^4)))

```
*(-d^3*e^3)^(1/2))/(2*(a^2*e^4 + c^2*d^4 + 2*a*c*d^2*e^2)))*(-d^3*e^3)^(1/2
))/(2*(a^2*e^4 + c^2*d^4 + 2*a*c*d^2*e^2)))*(-d^3*e^3)^(1/2))/(2*(a^2*e^4 +
 c^2*d^4 + 2*a*c*d^2*e^2)) + (x*(a^4*c*e^13 + 33*c^5*d^8*e^5 - 188*a*c^4*d^
6*e^7 + 38*a^2*c^3*d^4*e^9 + 4*a^3*c^2*d^2*e^11))/(256*(a^4*e^8 + c^4*d^8 +
 4*a*c^3*d^6*e^2 + 4*a^3*c*d^2*e^6 + 6*a^2*c^2*d^4*e^4)))*(-d^3*e^3)^(1/2))
/(a^2*e^4 + c^2*d^4 + 2*a*c*d^2*e^2)))*(-d^3*e^3)^(1/2)*1i)/(a^2*e^4 + c^2*
d^4 + 2*a*c*d^2*e^2)
```

3.255 $\int \frac{x^2}{(d+ex^2)(a+cx^4)^2}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 685

$$
\begin{aligned}
\int \frac{x^2}{(d+ex^2)(a+cx^4)^2}\,dx &= \frac{x(ae+cdx^2)}{4a(cd^2+ae^2)(a+cx^4)} - \frac{\sqrt{d}e^{5/2}\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{(cd^2+ae^2)^2} \\
&+ \frac{\sqrt[4]{c}de\left(\sqrt{c}d-\sqrt{a}e\right)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}(cd^2+ae^2)^2} \\
&- \frac{\left(\sqrt{c}d+3\sqrt{a}e\right)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{8\sqrt{2}a^{5/4}\sqrt[4]{c}(cd^2+ae^2)} \\
&- \frac{\sqrt[4]{c}de\left(\sqrt{c}d-\sqrt{a}e\right)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}(cd^2+ae^2)^2} \\
&+ \frac{\left(\sqrt{c}d+3\sqrt{a}e\right)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{8\sqrt{2}a^{5/4}\sqrt[4]{c}(cd^2+ae^2)} \\
&+ \frac{\sqrt[4]{c}de\left(\sqrt{c}d+\sqrt{a}e\right)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)}{4\sqrt{2}a^{3/4}(cd^2+ae^2)^2} \\
&+ \frac{\left(\sqrt{c}d-3\sqrt{a}e\right)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)}{16\sqrt{2}a^{5/4}\sqrt[4]{c}(cd^2+ae^2)} \\
&- \frac{\sqrt[4]{c}de\left(\sqrt{c}d+\sqrt{a}e\right)\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)}{4\sqrt{2}a^{3/4}(cd^2+ae^2)^2} \\
&- \frac{\left(\sqrt{c}d-3\sqrt{a}e\right)\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)}{16\sqrt{2}a^{5/4}\sqrt[4]{c}(cd^2+ae^2)}
\end{aligned}
$$

```
[Out] 1/4*x*(c*d*x^2+a*e)/a/(a*e^2+c*d^2)/(c*x^4+a)+1/32*ln(-a^(1/4)*c^(1/4)*x*2^
(1/2)+a^(1/2)+x^2*c^(1/2))*(-3*e*a^(1/2)+d*c^(1/2))/a^(5/4)/c^(1/4)/(a*e^2+
c*d^2)*2^(1/2)-1/32*ln(a^(1/4)*c^(1/4)*x*2^(1/2)+a^(1/2)+x^2*c^(1/2))*(-3*e
*a^(1/2)+d*c^(1/2))/a^(5/4)/c^(1/4)/(a*e^2+c*d^2)*2^(1/2)-1/4*c^(1/4)*d*e*a
rctan(-1+c^(1/4)*x*2^(1/2)/a^(1/4))*(-e*a^(1/2)+d*c^(1/2))/a^(3/4)/(a*e^2+c
*d^2)^2*2^(1/2)-1/4*c^(1/4)*d*e*arctan(1+c^(1/4)*x*2^(1/2)/a^(1/4))*(-e*a^(
1/2)+d*c^(1/2))/a^(3/4)/(a*e^2+c*d^2)^2*2^(1/2)+1/8*c^(1/4)*d*e*ln(-a^(1/4)
*c^(1/4)*x*2^(1/2)+a^(1/2)+x^2*c^(1/2))*(e*a^(1/2)+d*c^(1/2))/a^(3/4)/(a*e^
2+c*d^2)^2*2^(1/2)-1/8*c^(1/4)*d*e*ln(a^(1/4)*c^(1/4)*x*2^(1/2)+a^(1/2)+x^2
*c^(1/2))*(e*a^(1/2)+d*c^(1/2))/a^(3/4)/(a*e^2+c*d^2)^2*2^(1/2)+1/16*arctan
(-1+c^(1/4)*x*2^(1/2)/a^(1/4))*(3*e*a^(1/2)+d*c^(1/2))/a^(5/4)/c^(1/4)/(a*e
^2+c*d^2)*2^(1/2)+1/16*arctan(1+c^(1/4)*x*2^(1/2)/a^(1/4))*(3*e*a^(1/2)+d*c
^(1/2))/a^(5/4)/c^(1/4)/(a*e^2+c*d^2)*2^(1/2)-e^(5/2)*arctan(x*e^(1/2)/d^(1
/2))*d^(1/2)/(a*e^2+c*d^2)^2
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 685, normalized size of antiderivative = 1.00, number of steps used = 23, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.455$, Rules

used = {1330, 1193, 1182, 1176, 631, 210, 1179, 642, 1185, 211}

$$\int \frac{x^2}{(d+ex^2)(a+cx^4)^2}\,dx = \frac{\sqrt[4]{c}de\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)(\sqrt{c}d-\sqrt{a}e)}{2\sqrt{2}a^{3/4}(ae^2+cd^2)^2} - \frac{\sqrt[4]{c}de\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)(\sqrt{c}d-\sqrt{a}e)}{2\sqrt{2}a^{3/4}(ae^2+cd^2)^2} - \frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)(3\sqrt{a}e+\sqrt{c}d)}{8\sqrt{2}a^{5/4}\sqrt[4]{c}(ae^2+cd^2)} + \frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)(3\sqrt{a}e+\sqrt{c}d)}{8\sqrt{2}a^{5/4}\sqrt[4]{c}(ae^2+cd^2)} + \frac{\sqrt[4]{c}de(\sqrt{a}e+\sqrt{c}d)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{4\sqrt{2}a^{3/4}(ae^2+cd^2)^2} - \frac{\sqrt[4]{c}de(\sqrt{a}e+\sqrt{c}d)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{4\sqrt{2}a^{3/4}(ae^2+cd^2)^2} + \frac{(\sqrt{c}d-3\sqrt{a}e)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{16\sqrt{2}a^{5/4}\sqrt[4]{c}(ae^2+cd^2)} - \frac{(\sqrt{c}d-3\sqrt{a}e)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{16\sqrt{2}a^{5/4}\sqrt[4]{c}(ae^2+cd^2)} - \frac{\sqrt{d}e^{5/2}\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{(ae^2+cd^2)^2} + \frac{x(ae+cdx^2)}{4a(a+cx^4)(ae^2+cd^2)}$$

```
[In] Int[x^2/((d + e*x^2)*(a + c*x^4)^2),x]
```

```
[Out] (x*(a*e + c*d*x^2))/(4*a*(c*d^2 + a*e^2)*(a + c*x^4)) - (Sqrt[d]*e^(5/2)*Ar
cTan[(Sqrt[e]*x)/Sqrt[d]])/(c*d^2 + a*e^2)^2 + (c^(1/4)*d*e*(Sqrt[c]*d - Sq
rt[a]*e)*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*(c*d^2
 + a*e^2)^2) - ((Sqrt[c]*d + 3*Sqrt[a]*e)*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^
(1/4)])/(8*Sqrt[2]*a^(5/4)*c^(1/4)*(c*d^2 + a*e^2)) - (c^(1/4)*d*e*(Sqrt[c]
*d - Sqrt[a]*e)*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)
*(c*d^2 + a*e^2)^2) + ((Sqrt[c]*d + 3*Sqrt[a]*e)*ArcTan[1 + (Sqrt[2]*c^(1/4
)*x)/a^(1/4)])/(8*Sqrt[2]*a^(5/4)*c^(1/4)*(c*d^2 + a*e^2)) + (c^(1/4)*d*e*(
Sqrt[c]*d + Sqrt[a]*e)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^
2])/(4*Sqrt[2]*a^(3/4)*(c*d^2 + a*e^2)^2) + ((Sqrt[c]*d - 3*Sqrt[a]*e)*Log[
Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(16*Sqrt[2]*a^(5/4)*c^(
1/4)*(c*d^2 + a*e^2)) - (c^(1/4)*d*e*(Sqrt[c]*d + Sqrt[a]*e)*Log[Sqrt[a] +
Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(4*Sqrt[2]*a^(3/4)*(c*d^2 + a*e^2
)^2) - ((Sqrt[c]*d - 3*Sqrt[a]*e)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x +
 Sqrt[c]*x^2])/(16*Sqrt[2]*a^(5/4)*c^(1/4)*(c*d^2 + a*e^2))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1182

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[(-a)
*c]
```

Rule 1185

```
Int[((d_) + (e_.)*(x_)^2)^(q_)/((a_) + (c_.)*(x_)^4), x_Symbol] :> Int[Expa
ndIntegrand[(d + e*x^2)^q/(a + c*x^4), x], x] /; FreeQ[{a, c, d, e}, x] &&
NeQ[c*d^2 + a*e^2, 0] && IntegerQ[q]
```

Rule 1193

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[(-x
)*(d + e*x^2)*((a + c*x^4)^(p + 1)/(4*a*(p + 1))), x] + Dist[1/(4*a*(p + 1)
), Int[Simp[d*(4*p + 5) + e*(4*p + 7)*x^2, x]*(a + c*x^4)^(p + 1), x], x] /
; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && IntegerQ
[2*p]
```

Rule 1330

```
Int[(((f_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^4)^(p_))/((d_.) + (e_.)*(x_)^2),
 x_Symbol] :> Dist[f^2/(c*d^2 + a*e^2), Int[(f*x)^(m - 2)*(a*e + c*d*x^2)*(
a + c*x^4)^p, x], x] - Dist[d*e*(f^2/(c*d^2 + a*e^2)), Int[(f*x)^(m - 2)*((
a + c*x^4)^(p + 1)/(d + e*x^2)), x], x] /; FreeQ[{a, c, d, e, f}, x] && LtQ
[p, -1] && GtQ[m, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\int \frac{ae+cdx^2}{(a+cx^4)^2}\,dx}{cd^2+ae^2} - \frac{(de)\int \frac{1}{(d+ex^2)(a+cx^4)}\,dx}{cd^2+ae^2} \\
&= \frac{x(ae+cdx^2)}{4a\,(cd^2+ae^2)\,(a+cx^4)} - \frac{\int \frac{-3ae-cdx^2}{a+cx^4}\,dx}{4a\,(cd^2+ae^2)} - \frac{(de)\int \left(\frac{e^2}{(cd^2+ae^2)(d+ex^2)} + \frac{c(d-ex^2)}{(cd^2+ae^2)(a+cx^4)}\right)dx}{cd^2+ae^2} \\
&= \frac{x(ae+cdx^2)}{4a\,(cd^2+ae^2)\,(a+cx^4)} - \frac{(cde)\int \frac{d-ex^2}{a+cx^4}\,dx}{(cd^2+ae^2)^2} - \frac{(de^3)\int \frac{1}{d+ex^2}\,dx}{(cd^2+ae^2)^2} \\
&\quad - \frac{\left(d-\frac{3\sqrt{a}e}{\sqrt{c}}\right)\int \frac{\sqrt{a}\sqrt{c}-cx^2}{a+cx^4}\,dx}{8a\,(cd^2+ae^2)} + \frac{\left(d+\frac{3\sqrt{a}e}{\sqrt{c}}\right)\int \frac{\sqrt{a}\sqrt{c}+cx^2}{a+cx^4}\,dx}{8a\,(cd^2+ae^2)}
\end{aligned}$$

$$= \frac{x(ae+cdx^2)}{4a(cd^2+ae^2)(a+cx^4)} - \frac{\sqrt{d}e^{5/2}\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{(cd^2+ae^2)^2} - \frac{\left(d\left(\frac{\sqrt{c}d}{\sqrt{a}}-e\right)e\right)\int \frac{\sqrt{a}\sqrt{c}+cx^2}{a+cx^4}dx}{2(cd^2+ae^2)^2} - \frac{\left(de\left(\frac{\sqrt{c}d}{\sqrt{a}}+e\right)\right)\int \frac{\sqrt{a}\sqrt{c}-cx^2}{a+cx^4}dx}{2(cd^2+ae^2)^2} + \frac{\left(\sqrt[4]{c}\left(d-\frac{3\sqrt{a}e}{\sqrt{c}}\right)\right)\int \frac{\frac{\sqrt{2}\sqrt[4]{a}}{\sqrt[4]{c}}+2x}{-\frac{\sqrt{a}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}-x^2}dx}{16\sqrt{2}a^{5/4}(cd^2+ae^2)} + \frac{\left(\sqrt[4]{c}\left(d-\frac{3\sqrt{a}e}{\sqrt{c}}\right)\right)\int \frac{\frac{\sqrt{2}\sqrt[4]{a}}{\sqrt[4]{c}}-2x}{-\frac{\sqrt{a}}{\sqrt{c}}+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}-x^2}dx}{16\sqrt{2}a^{5/4}(cd^2+ae^2)} + \frac{\left(d+\frac{3\sqrt{a}e}{\sqrt{c}}\right)\int \frac{1}{\frac{\sqrt{a}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}+x^2}dx}{16a(cd^2+ae^2)} + \frac{\left(d+\frac{3\sqrt{a}e}{\sqrt{c}}\right)\int \frac{1}{\frac{\sqrt{a}}{\sqrt{c}}+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}+x^2}dx}{16a(cd^2+ae^2)}$$

$$= \frac{x(ae+cdx^2)}{4a(cd^2+ae^2)(a+cx^4)} - \frac{\sqrt{d}e^{5/2}\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{(cd^2+ae^2)^2} + \frac{\sqrt[4]{c}\left(d-\frac{3\sqrt{a}e}{\sqrt{c}}\right)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)}{16\sqrt{2}a^{5/4}(cd^2+ae^2)} - \frac{\sqrt[4]{c}\left(d-\frac{3\sqrt{a}e}{\sqrt{c}}\right)\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)}{16\sqrt{2}a^{5/4}(cd^2+ae^2)} - \frac{\left(d\left(\frac{\sqrt{c}d}{\sqrt{a}}-e\right)e\right)\int \frac{1}{\frac{\sqrt{a}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}+x^2}dx}{4(cd^2+ae^2)^2} - \frac{\left(d\left(\frac{\sqrt{c}d}{\sqrt{a}}-e\right)e\right)\int \frac{1}{\frac{\sqrt{a}}{\sqrt{c}}+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}+x^2}dx}{4(cd^2+ae^2)^2} + \frac{\left(\sqrt[4]{c}de\left(\sqrt{c}d+\sqrt{a}e\right)\right)\int \frac{\frac{\sqrt{2}\sqrt[4]{a}}{\sqrt[4]{c}}+2x}{-\frac{\sqrt{a}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}-x^2}dx}{4\sqrt{2}a^{3/4}(cd^2+ae^2)^2} + \frac{\left(\sqrt[4]{c}de\left(\sqrt{c}d+\sqrt{a}e\right)\right)\int \frac{\frac{\sqrt{2}\sqrt[4]{a}}{\sqrt[4]{c}}-2x}{-\frac{\sqrt{a}}{\sqrt{c}}+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}-x^2}dx}{4\sqrt{2}a^{3/4}(cd^2+ae^2)^2} + \frac{\left(\sqrt[4]{c}\left(d+\frac{3\sqrt{a}e}{\sqrt{c}}\right)\right)\text{Subst}\left(\int \frac{1}{-1-x^2}dx, x, 1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{8\sqrt{2}a^{5/4}(cd^2+ae^2)} - \frac{\left(\sqrt[4]{c}\left(d+\frac{3\sqrt{a}e}{\sqrt{c}}\right)\right)\text{Subst}\left(\int \frac{1}{-1-x^2}dx, x, 1+\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{8\sqrt{2}a^{5/4}(cd^2+ae^2)}$$

$$= \frac{x(ae+cdx^2)}{4a\left(cd^2+ae^2\right)\left(a+cx^4\right)} - \frac{\sqrt{d}e^{5/2}\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\left(cd^2+ae^2\right)^2} - \frac{\sqrt[4]{c}\left(d+\frac{3\sqrt{a}e}{\sqrt{c}}\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{8\sqrt{2}a^{5/4}\left(cd^2+ae^2\right)} + \frac{\sqrt[4]{c}\left(d+\frac{3\sqrt{a}e}{\sqrt{c}}\right)\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{8\sqrt{2}a^{5/4}\left(cd^2+ae^2\right)} + \frac{\sqrt[4]{c}de\left(\sqrt{c}d+\sqrt{a}e\right)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)}{4\sqrt{2}a^{3/4}\left(cd^2+ae^2\right)^2} + \frac{\sqrt[4]{c}\left(d-\frac{3\sqrt{a}e}{\sqrt{c}}\right)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)}{16\sqrt{2}a^{5/4}\left(cd^2+ae^2\right)} - \frac{\sqrt[4]{c}de\left(\sqrt{c}d+\sqrt{a}e\right)\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)}{4\sqrt{2}a^{3/4}\left(cd^2+ae^2\right)^2} - \frac{\sqrt[4]{c}\left(d-\frac{3\sqrt{a}e}{\sqrt{c}}\right)\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)}{16\sqrt{2}a^{5/4}\left(cd^2+ae^2\right)} - \frac{\left(\sqrt[4]{c}de\left(\sqrt{c}d-\sqrt{a}e\right)\right)\text{Subst}\left(\int\frac{1}{-1-x^2}\,dx,x,1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}\left(cd^2+ae^2\right)^2} + \frac{\left(\sqrt[4]{c}de\left(\sqrt{c}d-\sqrt{a}e\right)\right)\text{Subst}\left(\int\frac{1}{-1-x^2}\,dx,x,1+\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}\left(cd^2+ae^2\right)^2}$$

$$= \frac{x(ae+cdx^2)}{4a\left(cd^2+ae^2\right)\left(a+cx^4\right)} - \frac{\sqrt{d}e^{5/2}\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\left(cd^2+ae^2\right)^2} + \frac{\sqrt[4]{c}de\left(\sqrt{c}d-\sqrt{a}e\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}\left(cd^2+ae^2\right)^2} - \frac{\sqrt[4]{c}\left(d+\frac{3\sqrt{a}e}{\sqrt{c}}\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{8\sqrt{2}a^{5/4}\left(cd^2+ae^2\right)} - \frac{\sqrt[4]{c}de\left(\sqrt{c}d-\sqrt{a}e\right)\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}\left(cd^2+ae^2\right)^2} + \frac{\sqrt[4]{c}\left(d+\frac{3\sqrt{a}e}{\sqrt{c}}\right)\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{8\sqrt{2}a^{5/4}\left(cd^2+ae^2\right)} + \frac{\sqrt[4]{c}de\left(\sqrt{c}d+\sqrt{a}e\right)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)}{4\sqrt{2}a^{3/4}\left(cd^2+ae^2\right)^2} + \frac{\sqrt[4]{c}\left(d-\frac{3\sqrt{a}e}{\sqrt{c}}\right)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)}{16\sqrt{2}a^{5/4}\left(cd^2+ae^2\right)} - \frac{\sqrt[4]{c}de\left(\sqrt{c}d+\sqrt{a}e\right)\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)}{4\sqrt{2}a^{3/4}\left(cd^2+ae^2\right)^2} - \frac{\sqrt[4]{c}\left(d-\frac{3\sqrt{a}e}{\sqrt{c}}\right)\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)}{16\sqrt{2}a^{5/4}\left(cd^2+ae^2\right)}$$

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 428, normalized size of antiderivative = 0.62

$$\int \frac{x^2}{(d+ex^2)\,(a+cx^4)^2}\,dx$$

$$= \frac{\frac{8(cd^2+ae^2)(aex+cdx^3)}{a(a+cx^4)} - 32\sqrt{d}e^{5/2}\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right) - \frac{2\sqrt{2}(c^{3/2}d^3-\sqrt{a}cd^2e+5a\sqrt{c}de^2+3a^{3/2}e^3)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{a^{5/4}\sqrt[4]{c}} + \frac{2\sqrt{2}(c^{3/}}{}}{}$$

```
[In] Integrate[x^2/((d + e*x^2)*(a + c*x^4)^2),x]
```

```
[Out] ((8*(c*d^2 + a*e^2)*(a*e*x + c*d*x^3))/(a*(a + c*x^4)) - 32*Sqrt[d]*e^(5/2)
*ArcTan[(Sqrt[e]*x)/Sqrt[d]] - (2*Sqrt[2]*(c^(3/2)*d^3 - Sqrt[a]*c*d^2*e +
5*a*Sqrt[c]*d*e^2 + 3*a^(3/2)*e^3)*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)])
/(a^(5/4)*c^(1/4)) + (2*Sqrt[2]*(c^(3/2)*d^3 - Sqrt[a]*c*d^2*e + 5*a*Sqrt[c
]*d*e^2 + 3*a^(3/2)*e^3)*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(a^(5/4)*
c^(1/4)) + (Sqrt[2]*(c^(3/2)*d^3 + Sqrt[a]*c*d^2*e + 5*a*Sqrt[c]*d*e^2 - 3*
a^(3/2)*e^3)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(a^(5/
4)*c^(1/4)) - (Sqrt[2]*(c^(3/2)*d^3 + Sqrt[a]*c*d^2*e + 5*a*Sqrt[c]*d*e^2 -
 3*a^(3/2)*e^3)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(a^
(5/4)*c^(1/4)))/(32*(c*d^2 + a*e^2)^2)
```

Maple [A] (verified)

Time = 0.44 (sec) , antiderivative size = 339, normalized size of antiderivative = 0.49

method	result
default	$\frac{\frac{\frac{cd(ae^2+cd^2)x^3}{4a}+\left(\frac{1}{4}ae^3+\frac{1}{4}cd^2e\right)x}{cx^4+a}+\frac{\frac{(3e^3a^2-acd^2e)\left(\frac{a}{c}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x^2+\left(\frac{a}{c}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{c}}}{x^2-\left(\frac{a}{c}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{c}}}\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{c}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{c}\right)^{\frac{1}{4}}}-1\right)\right)}{8a}}{4a}}{(ae^2+cd^2)^2}$
risch	Expression too large to display

```
[In] int(x^2/(e*x^2+d)/(c*x^4+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/(a*e^2+c*d^2)^2*((1/4*c*d*(a*e^2+c*d^2)/a*x^3+(1/4*a*e^3+1/4*c*d^2*e)*x)/
(c*x^4+a)+1/4/a*(1/8*(3*a^2*e^3-a*c*d^2*e)*(a/c)^(1/4)/a*2^(1/2)*(ln((x^2+(
a/c)^(1/4)*x*2^(1/2)+(a/c)^(1/2))/(x^2-(a/c)^(1/4)*x*2^(1/2)+(a/c)^(1/2)))+
2*arctan(2^(1/2)/(a/c)^(1/4)*x+1)+2*arctan(2^(1/2)/(a/c)^(1/4)*x-1))+1/8*(5
*a*c*d*e^2+c^2*d^3)/c/(a/c)^(1/4)*2^(1/2)*(ln((x^2-(a/c)^(1/4)*x*2^(1/2)+(a
/c)^(1/2))/(x^2+(a/c)^(1/4)*x*2^(1/2)+(a/c)^(1/2)))+2*arctan(2^(1/2)/(a/c)^
```

```
(1/4)*x+1)+2*arctan(2^(1/2)/(a/c)^(1/4)*x-1))))-e^3*d/(a*e^2+c*d^2)^2/(e*d)
^(1/2)*arctan(e*x/(e*d)^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 4877 vs. $2(513) = 1026$.

Time = 6.45 (sec) , antiderivative size = 9774, normalized size of antiderivative = 14.27

$$\int \frac{x^2}{(d+ex^2)(a+cx^4)^2}\,dx = \text{Too large to display}$$

```
[In] integrate(x^2/(e*x^2+d)/(c*x^4+a)^2,x, algorithm="fricas")
```

```
[Out] Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^2}{(d+ex^2)(a+cx^4)^2}\,dx = \text{Timed out}$$

```
[In] integrate(x**2/(e*x**2+d)/(c*x**4+a)**2,x)
```

```
[Out] Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^2}{(d+ex^2)(a+cx^4)^2}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(x^2/(e*x^2+d)/(c*x^4+a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 622, normalized size of antiderivative = 0.91

$$\int \frac{x^2}{(d+ex^2)(a+cx^4)^2}\,dx = -\frac{de^3\arctan\left(\frac{ex}{\sqrt{de}}\right)}{(c^2d^4+2\,acd^2e^2+a^2e^4)\sqrt{de}} - \frac{\left((ac^3)^{\frac{1}{4}}\,ac^2d^2e-3\,(ac^3)^{\frac{1}{4}}\,a^2ce^3-(ac^3)^{\frac{3}{4}}\,cd^3-5\,(ac^3)^{\frac{3}{4}}\,ade^2\right)\arctan\left(\frac{\sqrt{2}\left(2\,x+\sqrt{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\,\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{8\,\left(\sqrt{2}a^2c^4d^4+2\,\sqrt{2}a^3c^3d^2e^2+\sqrt{2}a^4c^2e^4\right)} - \frac{\left((ac^3)^{\frac{1}{4}}\,ac^2d^2e-3\,(ac^3)^{\frac{1}{4}}\,a^2ce^3-(ac^3)^{\frac{3}{4}}\,cd^3-5\,(ac^3)^{\frac{3}{4}}\,ade^2\right)\arctan\left(\frac{\sqrt{2}\left(2\,x-\sqrt{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\,\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{8\,\left(\sqrt{2}a^2c^4d^4+2\,\sqrt{2}a^3c^3d^2e^2+\sqrt{2}a^4c^2e^4\right)} - \frac{\left((ac^3)^{\frac{1}{4}}\,ac^2d^2e-3\,(ac^3)^{\frac{1}{4}}\,a^2ce^3+(ac^3)^{\frac{3}{4}}\,cd^3+5\,(ac^3)^{\frac{3}{4}}\,ade^2\right)\log\left(x^2+\sqrt{2}x\left(\frac{a}{c}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{c}}\right)}{16\,\left(\sqrt{2}a^2c^4d^4+2\,\sqrt{2}a^3c^3d^2e^2+\sqrt{2}a^4c^2e^4\right)} + \frac{\left((ac^3)^{\frac{1}{4}}\,ac^2d^2e-3\,(ac^3)^{\frac{1}{4}}\,a^2ce^3+(ac^3)^{\frac{3}{4}}\,cd^3+5\,(ac^3)^{\frac{3}{4}}\,ade^2\right)\log\left(x^2-\sqrt{2}x\left(\frac{a}{c}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{c}}\right)}{16\,\left(\sqrt{2}a^2c^4d^4+2\,\sqrt{2}a^3c^3d^2e^2+\sqrt{2}a^4c^2e^4\right)} + \frac{cdx^3+aex}{4\,(cx^4+a)(acd^2+a^2e^2)}$$

```
[In] integrate(x^2/(e*x^2+d)/(c*x^4+a)^2,x, algorithm="giac")
```

```
[Out] -d*e^3*arctan(e*x/sqrt(d*e))/((c^2*d^4 + 2*a*c*d^2*e^2 + a^2*e^4)*sqrt(d*e)
) - 1/8*((a*c^3)^(1/4)*a*c^2*d^2*e - 3*(a*c^3)^(1/4)*a^2*c*e^3 - (a*c^3)^(3
/4)*c*d^3 - 5*(a*c^3)^(3/4)*a*d*e^2)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(a/c
)^(1/4))/(a/c)^(1/4))/(sqrt(2)*a^2*c^4*d^4 + 2*sqrt(2)*a^3*c^3*d^2*e^2 + sq
rt(2)*a^4*c^2*e^4) - 1/8*((a*c^3)^(1/4)*a*c^2*d^2*e - 3*(a*c^3)^(1/4)*a^2*c
*e^3 - (a*c^3)^(3/4)*c*d^3 - 5*(a*c^3)^(3/4)*a*d*e^2)*arctan(1/2*sqrt(2)*(2
*x - sqrt(2)*(a/c)^(1/4))/(a/c)^(1/4))/(sqrt(2)*a^2*c^4*d^4 + 2*sqrt(2)*a^3
*c^3*d^2*e^2 + sqrt(2)*a^4*c^2*e^4) - 1/16*((a*c^3)^(1/4)*a*c^2*d^2*e - 3*(
a*c^3)^(1/4)*a^2*c*e^3 + (a*c^3)^(3/4)*c*d^3 + 5*(a*c^3)^(3/4)*a*d*e^2)*log
(x^2 + sqrt(2)*x*(a/c)^(1/4) + sqrt(a/c))/(sqrt(2)*a^2*c^4*d^4 + 2*sqrt(2)*
a^3*c^3*d^2*e^2 + sqrt(2)*a^4*c^2*e^4) + 1/16*((a*c^3)^(1/4)*a*c^2*d^2*e -
3*(a*c^3)^(1/4)*a^2*c*e^3 + (a*c^3)^(3/4)*c*d^3 + 5*(a*c^3)^(3/4)*a*d*e^2)*
log(x^2 - sqrt(2)*x*(a/c)^(1/4) + sqrt(a/c))/(sqrt(2)*a^2*c^4*d^4 + 2*sqrt(
2)*a^3*c^3*d^2*e^2 + sqrt(2)*a^4*c^2*e^4) + 1/4*(c*d*x^3 + a*e*x)/((c*x^4 +
 a)*(a*c*d^2 + a^2*e^2))
```

Mupad [B] (verification not implemented)

Time = 9.66 (sec) , antiderivative size = 17812, normalized size of antiderivative = 26.00

$$\int \frac{x^2}{(d+ex^2)(a+cx^4)^2}\,dx = \text{Too large to display}$$

```
[In] int(x^2/((a + c*x^4)^2*(d + e*x^2)),x)
[Out] ((e*x)/(4*(a*e^2 + c*d^2)) + (c*d*x^3)/(4*a*(a*e^2 + c*d^2)))/(a + c*x^4) +
 atan((((((53248*a^9*c^4*d*e^15 + 4096*a^3*c^10*d^13*e^3 + 73728*a^4*c^9*d
^11*e^5 + 307200*a^5*c^8*d^9*e^7 + 573440*a^6*c^7*d^7*e^9 + 552960*a^7*c^6*
d^5*e^11 + 270336*a^8*c^5*d^3*e^13)/(256*(a^6*e^8 + a^2*c^4*d^8 + 4*a^5*c*d
^2*e^6 + 4*a^3*c^3*d^6*e^2 + 6*a^4*c^2*d^4*e^4)) - (x*(-(c^3*d^6*(-a^5*c)^(
1/2) - 9*a^3*e^6*(-a^5*c)^(1/2) - 2*a^3*c^3*d^5*e - 4*a^4*c^2*d^3*e^3 + 30*
a^5*c*d*e^5 + 9*a*c^2*d^4*e^2*(-a^5*c)^(1/2) + 31*a^2*c*d^2*e^4*(-a^5*c)^(1
/2))/(256*(a^9*c*e^8 + a^5*c^5*d^8 + 4*a^6*c^4*d^6*e^2 + 6*a^7*c^3*d^4*e^4
+ 4*a^8*c^2*d^2*e^6)))^(1/2)*(65536*a^11*c^4*e^17 - 65536*a^4*c^11*d^14*e^3
 - 327680*a^5*c^10*d^12*e^5 - 589824*a^6*c^9*d^10*e^7 - 327680*a^7*c^8*d^8*
e^9 + 327680*a^8*c^7*d^6*e^11 + 589824*a^9*c^6*d^4*e^13 + 327680*a^10*c^5*d
^2*e^15))/(128*(a^6*e^8 + a^2*c^4*d^8 + 4*a^5*c*d^2*e^6 + 4*a^3*c^3*d^6*e^2
 + 6*a^4*c^2*d^4*e^4)))*(-(c^3*d^6*(-a^5*c)^(1/2) - 9*a^3*e^6*(-a^5*c)^(1/2
) - 2*a^3*c^3*d^5*e - 4*a^4*c^2*d^3*e^3 + 30*a^5*c*d*e^5 + 9*a*c^2*d^4*e^2*
(-a^5*c)^(1/2) + 31*a^2*c*d^2*e^4*(-a^5*c)^(1/2))/(256*(a^9*c*e^8 + a^5*c^5
*d^8 + 4*a^6*c^4*d^6*e^2 + 6*a^7*c^3*d^4*e^4 + 4*a^8*c^2*d^2*e^6)))^(1/2) +
 (x*(128*a*c^10*d^13*e^2 - 14208*a^7*c^4*d*e^14 + 768*a^2*c^9*d^11*e^4 + 39
68*a^3*c^8*d^9*e^6 + 27136*a^4*c^7*d^7*e^8 + 30592*a^5*c^6*d^5*e^10 - 7424*
a^6*c^5*d^3*e^12))/(128*(a^6*e^8 + a^2*c^4*d^8 + 4*a^5*c*d^2*e^6 + 4*a^3*c^
3*d^6*e^2 + 6*a^4*c^2*d^4*e^4)))*(-(c^3*d^6*(-a^5*c)^(1/2) - 9*a^3*e^6*(-a^
5*c)^(1/2) - 2*a^3*c^3*d^5*e - 4*a^4*c^2*d^3*e^3 + 30*a^5*c*d*e^5 + 9*a*c^2
*d^4*e^2*(-a^5*c)^(1/2) + 31*a^2*c*d^2*e^4*(-a^5*c)^(1/2))/(256*(a^9*c*e^8
+ a^5*c^5*d^8 + 4*a^6*c^4*d^6*e^2 + 6*a^7*c^3*d^4*e^4 + 4*a^8*c^2*d^2*e^6))
)^(1/2) + (16*c^9*d^12*e^2 + 208*a*c^8*d^10*e^4 + 672*a^2*c^7*d^8*e^6 + 928
*a^3*c^6*d^6*e^8 + 12880*a^4*c^5*d^4*e^10 + 12432*a^5*c^4*d^2*e^12)/(256*(a
^6*e^8 + a^2*c^4*d^8 + 4*a^5*c*d^2*e^6 + 4*a^3*c^3*d^6*e^2 + 6*a^4*c^2*d^4*
e^4)))*(-(c^3*d^6*(-a^5*c)^(1/2) - 9*a^3*e^6*(-a^5*c)^(1/2) - 2*a^3*c^3*d^5
*e - 4*a^4*c^2*d^3*e^3 + 30*a^5*c*d*e^5 + 9*a*c^2*d^4*e^2*(-a^5*c)^(1/2) +
31*a^2*c*d^2*e^4*(-a^5*c)^(1/2))/(256*(a^9*c*e^8 + a^5*c^5*d^8 + 4*a^6*c^4*
d^6*e^2 + 6*a^7*c^3*d^4*e^4 + 4*a^8*c^2*d^2*e^6)))^(1/2) - (x*(81*a^4*c^3*e
^13 + c^7*d^8*e^5 - 12*a*c^6*d^6*e^7 + 54*a^2*c^5*d^4*e^9 - 108*a^3*c^4*d^2
*e^11))/(128*(a^6*e^8 + a^2*c^4*d^8 + 4*a^5*c*d^2*e^6 + 4*a^3*c^3*d^6*e^2 +
 6*a^4*c^2*d^4*e^4)))*(-(c^3*d^6*(-a^5*c)^(1/2) - 9*a^3*e^6*(-a^5*c)^(1/2)
- 2*a^3*c^3*d^5*e - 4*a^4*c^2*d^3*e^3 + 30*a^5*c*d*e^5 + 9*a*c^2*d^4*e^2*(-
a^5*c)^(1/2) + 31*a^2*c*d^2*e^4*(-a^5*c)^(1/2))/(256*(a^9*c*e^8 + a^5*c^5*d
^8 + 4*a^6*c^4*d^6*e^2 + 6*a^7*c^3*d^4*e^4 + 4*a^8*c^2*d^2*e^6)))^(1/2)*1i
```

- (((((53248*a^9*c^4*d*e^15 + 4096*a^3*c^10*d^13*e^3 + 73728*a^4*c^9*d^11*e
^5 + 307200*a^5*c^8*d^9*e^7 + 573440*a^6*c^7*d^7*e^9 + 552960*a^7*c^6*d^5*e
^11 + 270336*a^8*c^5*d^3*e^13)/(256*(a^6*e^8 + a^2*c^4*d^8 + 4*a^5*c*d^2*e^
6 + 4*a^3*c^3*d^6*e^2 + 6*a^4*c^2*d^4*e^4)) + (x*(-(c^3*d^6*(-a^5*c)^(1/2)
- 9*a^3*e^6*(-a^5*c)^(1/2) - 2*a^3*c^3*d^5*e - 4*a^4*c^2*d^3*e^3 + 30*a^5*c
*d*e^5 + 9*a*c^2*d^4*e^2*(-a^5*c)^(1/2) + 31*a^2*c*d^2*e^4*(-a^5*c)^(1/2))/
(256*(a^9*c*e^8 + a^5*c^5*d^8 + 4*a^6*c^4*d^6*e^2 + 6*a^7*c^3*d^4*e^4 + 4*a
^8*c^2*d^2*e^6)))^(1/2)*(65536*a^11*c^4*e^17 - 65536*a^4*c^11*d^14*e^3 - 32
7680*a^5*c^10*d^12*e^5 - 589824*a^6*c^9*d^10*e^7 - 327680*a^7*c^8*d^8*e^9 +
 327680*a^8*c^7*d^6*e^11 + 589824*a^9*c^6*d^4*e^13 + 327680*a^10*c^5*d^2*e^
15))/(128*(a^6*e^8 + a^2*c^4*d^8 + 4*a^5*c*d^2*e^6 + 4*a^3*c^3*d^6*e^2 + 6*
a^4*c^2*d^4*e^4)))*(-(c^3*d^6*(-a^5*c)^(1/2) - 9*a^3*e^6*(-a^5*c)^(1/2) - 2
*a^3*c^3*d^5*e - 4*a^4*c^2*d^3*e^3 + 30*a^5*c*d*e^5 + 9*a*c^2*d^4*e^2*(-a^5
*c)^(1/2) + 31*a^2*c*d^2*e^4*(-a^5*c)^(1/2))/(256*(a^9*c*e^8 + a^5*c^5*d^8
+ 4*a^6*c^4*d^6*e^2 + 6*a^7*c^3*d^4*e^4 + 4*a^8*c^2*d^2*e^6)))^(1/2) - (x*(
128*a*c^10*d^13*e^2 - 14208*a^7*c^4*d*e^14 + 768*a^2*c^9*d^11*e^4 + 3968*a^
3*c^8*d^9*e^6 + 27136*a^4*c^7*d^7*e^8 + 30592*a^5*c^6*d^5*e^10 - 7424*a^6*c
^5*d^3*e^12))/(128*(a^6*e^8 + a^2*c^4*d^8 + 4*a^5*c*d^2*e^6 + 4*a^3*c^3*d^6
*e^2 + 6*a^4*c^2*d^4*e^4)))*(-(c^3*d^6*(-a^5*c)^(1/2) - 9*a^3*e^6*(-a^5*c)^
(1/2) - 2*a^3*c^3*d^5*e - 4*a^4*c^2*d^3*e^3 + 30*a^5*c*d*e^5 + 9*a*c^2*d^4*
e^2*(-a^5*c)^(1/2) + 31*a^2*c*d^2*e^4*(-a^5*c)^(1/2))/(256*(a^9*c*e^8 + a^5
*c^5*d^8 + 4*a^6*c^4*d^6*e^2 + 6*a^7*c^3*d^4*e^4 + 4*a^8*c^2*d^2*e^6)))^(1/
2) + (16*c^9*d^12*e^2 + 208*a*c^8*d^10*e^4 + 672*a^2*c^7*d^8*e^6 + 928*a^3*
c^6*d^6*e^8 + 12880*a^4*c^5*d^4*e^10 + 12432*a^5*c^4*d^2*e^12)/(256*(a^6*e^
8 + a^2*c^4*d^8 + 4*a^5*c*d^2*e^6 + 4*a^3*c^3*d^6*e^2 + 6*a^4*c^2*d^4*e^4))
)*(-(c^3*d^6*(-a^5*c)^(1/2) - 9*a^3*e^6*(-a^5*c)^(1/2) - 2*a^3*c^3*d^5*e -
4*a^4*c^2*d^3*e^3 + 30*a^5*c*d*e^5 + 9*a*c^2*d^4*e^2*(-a^5*c)^(1/2) + 31*a^
2*c*d^2*e^4*(-a^5*c)^(1/2))/(256*(a^9*c*e^8 + a^5*c^5*d^8 + 4*a^6*c^4*d^6*e
^2 + 6*a^7*c^3*d^4*e^4 + 4*a^8*c^2*d^2*e^6)))^(1/2) + (x*(81*a^4*c^3*e^13 +
 c^7*d^8*e^5 - 12*a*c^6*d^6*e^7 + 54*a^2*c^5*d^4*e^9 - 108*a^3*c^4*d^2*e^11
))/(128*(a^6*e^8 + a^2*c^4*d^8 + 4*a^5*c*d^2*e^6 + 4*a^3*c^3*d^6*e^2 + 6*a^
4*c^2*d^4*e^4)))*(-(c^3*d^6*(-a^5*c)^(1/2) - 9*a^3*e^6*(-a^5*c)^(1/2) - 2*a
^3*c^3*d^5*e - 4*a^4*c^2*d^3*e^3 + 30*a^5*c*d*e^5 + 9*a*c^2*d^4*e^2*(-a^5*c
)^(1/2) + 31*a^2*c*d^2*e^4*(-a^5*c)^(1/2))/(256*(a^9*c*e^8 + a^5*c^5*d^8 +
4*a^6*c^4*d^6*e^2 + 6*a^7*c^3*d^4*e^4 + 4*a^8*c^2*d^2*e^6)))^(1/2)*1i)/((((
((53248*a^9*c^4*d*e^15 + 4096*a^3*c^10*d^13*e^3 + 73728*a^4*c^9*d^11*e^5 +
307200*a^5*c^8*d^9*e^7 + 573440*a^6*c^7*d^7*e^9 + 552960*a^7*c^6*d^5*e^11 +
 270336*a^8*c^5*d^3*e^13)/(256*(a^6*e^8 + a^2*c^4*d^8 + 4*a^5*c*d^2*e^6 + 4
*a^3*c^3*d^6*e^2 + 6*a^4*c^2*d^4*e^4)) - (x*(-(c^3*d^6*(-a^5*c)^(1/2) - 9*a
^3*e^6*(-a^5*c)^(1/2) - 2*a^3*c^3*d^5*e - 4*a^4*c^2*d^3*e^3 + 30*a^5*c*d*e^
5 + 9*a*c^2*d^4*e^2*(-a^5*c)^(1/2) + 31*a^2*c*d^2*e^4*(-a^5*c)^(1/2))/(256*
(a^9*c*e^8 + a^5*c^5*d^8 + 4*a^6*c^4*d^6*e^2 + 6*a^7*c^3*d^4*e^4 + 4*a^8*c^
2*d^2*e^6)))^(1/2)*(65536*a^11*c^4*e^17 - 65536*a^4*c^11*d^14*e^3 - 327680*
a^5*c^10*d^12*e^5 - 589824*a^6*c^9*d^10*e^7 - 327680*a^7*c^8*d^8*e^9 + 3276
80*a^8*c^7*d^6*e^11 + 589824*a^9*c^6*d^4*e^13 + 327680*a^10*c^5*d^2*e^15))/

(128*(a^6*e^8 + a^2*c^4*d^8 + 4*a^5*c*d^2*e^6 + 4*a^3*c^3*d^6*e^2 + 6*a^4*c
^2*d^4*e^4)))*(-(c^3*d^6*(-a^5*c)^(1/2) - 9*a^3*e^6*(-a^5*c)^(1/2) - 2*a^3*
c^3*d^5*e - 4*a^4*c^2*d^3*e^3 + 30*a^5*c*d*e^5 + 9*a*c^2*d^4*e^2*(-a^5*c)^(
1/2) + 31*a^2*c*d^2*e^4*(-a^5*c)^(1/2))/(256*(a^9*c*e^8 + a^5*c^5*d^8 + 4*a
^6*c^4*d^6*e^2 + 6*a^7*c^3*d^4*e^4 + 4*a^8*c^2*d^2*e^6)))^(1/2) + (x*(128*a
*c^10*d^13*e^2 - 14208*a^7*c^4*d*e^14 + 768*a^2*c^9*d^11*e^4 + 3968*a^3*c^8
*d^9*e^6 + 27136*a^4*c^7*d^7*e^8 + 30592*a^5*c^6*d^5*e^10 - 7424*a^6*c^5*d^
3*e^12))/(128*(a^6*e^8 + a^2*c^4*d^8 + 4*a^5*c*d^2*e^6 + 4*a^3*c^3*d^6*e^2
+ 6*a^4*c^2*d^4*e^4)))*(-(c^3*d^6*(-a^5*c)^(1/2) - 9*a^3*e^6*(-a^5*c)^(1/2)
 - 2*a^3*c^3*d^5*e - 4*a^4*c^2*d^3*e^3 + 30*a^5*c*d*e^5 + 9*a*c^2*d^4*e^2*(
-a^5*c)^(1/2) + 31*a^2*c*d^2*e^4*(-a^5*c)^(1/2))/(256*(a^9*c*e^8 + a^5*c^5*
d^8 + 4*a^6*c^4*d^6*e^2 + 6*a^7*c^3*d^4*e^4 + 4*a^8*c^2*d^2*e^6)))^(1/2) +
(16*c^9*d^12*e^2 + 208*a*c^8*d^10*e^4 + 672*a^2*c^7*d^8*e^6 + 928*a^3*c^6*d
^6*e^8 + 12880*a^4*c^5*d^4*e^10 + 12432*a^5*c^4*d^2*e^12)/(256*(a^6*e^8 + a
^2*c^4*d^8 + 4*a^5*c*d^2*e^6 + 4*a^3*c^3*d^6*e^2 + 6*a^4*c^2*d^4*e^4)))*(-(
c^3*d^6*(-a^5*c)^(1/2) - 9*a^3*e^6*(-a^5*c)^(1/2) - 2*a^3*c^3*d^5*e - 4*a^4
*c^2*d^3*e^3 + 30*a^5*c*d*e^5 + 9*a*c^2*d^4*e^2*(-a^5*c)^(1/2) + 31*a^2*c*d
^2*e^4*(-a^5*c)^(1/2))/(256*(a^9*c*e^8 + a^5*c^5*d^8 + 4*a^6*c^4*d^6*e^2 +
6*a^7*c^3*d^4*e^4 + 4*a^8*c^2*d^2*e^6)))^(1/2) - (x*(81*a^4*c^3*e^13 + c^7*
d^8*e^5 - 12*a*c^6*d^6*e^7 + 54*a^2*c^5*d^4*e^9 - 108*a^3*c^4*d^2*e^11))/(1
28*(a^6*e^8 + a^2*c^4*d^8 + 4*a^5*c*d^2*e^6 + 4*a^3*c^3*d^6*e^2 + 6*a^4*c^2
*d^4*e^4)))*(-(c^3*d^6*(-a^5*c)^(1/2) - 9*a^3*e^6*(-a^5*c)^(1/2) - 2*a^3*c^
3*d^5*e - 4*a^4*c^2*d^3*e^3 + 30*a^5*c*d*e^5 + 9*a*c^2*d^4*e^2*(-a^5*c)^(1/
2) + 31*a^2*c*d^2*e^4*(-a^5*c)^(1/2))/(256*(a^9*c*e^8 + a^5*c^5*d^8 + 4*a^6
*c^4*d^6*e^2 + 6*a^7*c^3*d^4*e^4 + 4*a^8*c^2*d^2*e^6)))^(1/2) + (((((53248*
a^9*c^4*d*e^15 + 4096*a^3*c^10*d^13*e^3 + 73728*a^4*c^9*d^11*e^5 + 307200*a
^5*c^8*d^9*e^7 + 573440*a^6*c^7*d^7*e^9 + 552960*a^7*c^6*d^5*e^11 + 270336*
a^8*c^5*d^3*e^13)/(256*(a^6*e^8 + a^2*c^4*d^8 + 4*a^5*c*d^2*e^6 + 4*a^3*c^3
*d^6*e^2 + 6*a^4*c^2*d^4*e^4)) + (x*(-(c^3*d^6*(-a^5*c)^(1/2) - 9*a^3*e^6*(
-a^5*c)^(1/2) - 2*a^3*c^3*d^5*e - 4*a^4*c^2*d^3*e^3 + 30*a^5*c*d*e^5 + 9*a*
c^2*d^4*e^2*(-a^5*c)^(1/2) + 31*a^2*c*d^2*e^4*(-a^5*c)^(1/2))/(256*(a^9*c*e
^8 + a^5*c^5*d^8 + 4*a^6*c^4*d^6*e^2 + 6*a^7*c^3*d^4*e^4 + 4*a^8*c^2*d^2*e^
6)))^(1/2)*(65536*a^11*c^4*e^17 - 65536*a^4*c^11*d^14*e^3 - 327680*a^5*c^10
*d^12*e^5 - 589824*a^6*c^9*d^10*e^7 - 327680*a^7*c^8*d^8*e^9 + 327680*a^8*c
^7*d^6*e^11 + 589824*a^9*c^6*d^4*e^13 + 327680*a^10*c^5*d^2*e^15))/(128*(a^
6*e^8 + a^2*c^4*d^8 + 4*a^5*c*d^2*e^6 + 4*a^3*c^3*d^6*e^2 + 6*a^4*c^2*d^4*e
^4)))*(-(c^3*d^6*(-a^5*c)^(1/2) - 9*a^3*e^6*(-a^5*c)^(1/2) - 2*a^3*c^3*d^5*
e - 4*a^4*c^2*d^3*e^3 + 30*a^5*c*d*e^5 + 9*a*c^2*d^4*e^2*(-a^5*c)^(1/2) + 3
1*a^2*c*d^2*e^4*(-a^5*c)^(1/2))/(256*(a^9*c*e^8 + a^5*c^5*d^8 + 4*a^6*c^4*d
^6*e^2 + 6*a^7*c^3*d^4*e^4 + 4*a^8*c^2*d^2*e^6)))^(1/2) - (x*(128*a*c^10*d^
13*e^2 - 14208*a^7*c^4*d*e^14 + 768*a^2*c^9*d^11*e^4 + 3968*a^3*c^8*d^9*e^6
 + 27136*a^4*c^7*d^7*e^8 + 30592*a^5*c^6*d^5*e^10 - 7424*a^6*c^5*d^3*e^12))
/(128*(a^6*e^8 + a^2*c^4*d^8 + 4*a^5*c*d^2*e^6 + 4*a^3*c^3*d^6*e^2 + 6*a^4*
c^2*d^4*e^4)))*(-(c^3*d^6*(-a^5*c)^(1/2) - 9*a^3*e^6*(-a^5*c)^(1/2) - 2*a^3
*c^3*d^5*e - 4*a^4*c^2*d^3*e^3 + 30*a^5*c*d*e^5 + 9*a*c^2*d^4*e^2*(-a^5*c)^

(1/2) + 31*a^2*c*d^2*e^4*(-a^5*c)^(1/2))/(256*(a^9*c*e^8 + a^5*c^5*d^8 + 4*a^6*c^4*d^6*e^2 + 6*a^7*c^3*d^4*e^4 + 4*a^8*c^2*d^2*e^6)))^(1/2) + (16*c^9*d^12*e^2 + 208*a*c^8*d^10*e^4 + 672*a^2*c^7*d^8*e^6 + 928*a^3*c^6*d^6*e^8 + 12880*a^4*c^5*d^4*e^10 + 12432*a^5*c^4*d^2*e^12)/(256*(a^6*e^8 + a^2*c^4*d^8 + 4*a^5*c*d^2*e^6 + 4*a^3*c^3*d^6*e^2 + 6*a^4*c^2*d^4*e^4)))*(-(c^3*d^6*(-a^5*c)^(1/2) - 9*a^3*e^6*(-a^5*c)^(1/2) - 2*a^3*c^3*d^5*e - 4*a^4*c^2*d^3*e^3 + 30*a^5*c*d*e^5 + 9*a*c^2*d^4*e^2*(-a^5*c)^(1/2) + 31*a^2*c*d^2*e^4*(-a^5*c)^(1/2))/(256*(a^9*c*e^8 + a^5*c^5*d^8 + 4*a^6*c^4*d^6*e^2 + 6*a^7*c^3*d^4*e^4 + 4*a^8*c^2*d^2*e^6)))^(1/2) + (x*(81*a^4*c^3*e^13 + c^7*d^8*e^5 - 12*a*c^6*d^6*e^7 + 54*a^2*c^5*d^4*e^9 - 108*a^3*c^4*d^2*e^11))/(128*(a^6*e^8 + a^2*c^4*d^8 + 4*a^5*c*d^2*e^6 + 4*a^3*c^3*d^6*e^2 + 6*a^4*c^2*d^4*e^4)))*(-(c^3*d^6*(-a^5*c)^(1/2) - 9*a^3*e^6*(-a^5*c)^(1/2) - 2*a^3*c^3*d^5*e - 4*a^4*c^2*d^3*e^3 + 30*a^5*c*d*e^5 + 9*a*c^2*d^4*e^2*(-a^5*c)^(1/2) + 31*a^2*c*d^2*e^4*(-a^5*c)^(1/2))/(256*(a^9*c*e^8 + a^5*c^5*d^8 + 4*a^6*c^4*d^6*e^2 + 6*a^7*c^3*d^4*e^4 + 4*a^8*c^2*d^2*e^6)))^(1/2) + (5*c^5*d^5*e^7 + 54*a*c^4*d^3*e^9 + 81*a^2*c^3*d*e^11)/(128*(a^6*e^8 + a^2*c^4*d^8 + 4*a^5*c*d^2*e^6 + 4*a^3*c^3*d^6*e^2 + 6*a^4*c^2*d^4*e^4))))*(-(c^3*d^6*(-a^5*c)^(1/2) - 9*a^3*e^6*(-a^5*c)^(1/2) - 2*a^3*c^3*d^5*e - 4*a^4*c^2*d^3*e^3 + 30*a^5*c*d*e^5 + 9*a*c^2*d^4*e^2*(-a^5*c)^(1/2) + 31*a^2*c*d^2*e^4*(-a^5*c)^(1/2))/(256*(a^9*c*e^8 + a^5*c^5*d^8 + 4*a^6*c^4*d^6*e^2 + 6*a^7*c^3*d^4*e^4 + 4*a^8*c^2*d^2*e^6)))^(1/2)*2i + atan((((((53248*a^9*c^4*d*e^15 + 4096*a^3*c^10*d^13*e^3 + 73728*a^4*c^9*d^11*e^5 + 307200*a^5*c^8*d^9*e^7 + 573440*a^6*c^7*d^7*e^9 + 552960*a^7*c^6*d^5*e^11 + 270336*a^8*c^5*d^3*e^13)/(256*(a^6*e^8 + a^2*c^4*d^8 + 4*a^5*c*d^2*e^6 + 4*a^3*c^3*d^6*e^2 + 6*a^4*c^2*d^4*e^4)) - (x*((c^3*d^6*(-a^5*c)^(1/2) - 9*a^3*e^6*(-a^5*c)^(1/2) + 2*a^3*c^3*d^5*e + 4*a^4*c^2*d^3*e^3 - 30*a^5*c*d*e^5 + 9*a*c^2*d^4*e^2*(-a^5*c)^(1/2) + 31*a^2*c*d^2*e^4*(-a^5*c)^(1/2))/(256*(a^9*c*e^8 + a^5*c^5*d^8 + 4*a^6*c^4*d^6*e^2 + 6*a^7*c^3*d^4*e^4 + 4*a^8*c^2*d^2*e^6)))^(1/2)*(65536*a^11*c^4*e^17 - 65536*a^4*c^11*d^14*e^3 - 327680*a^5*c^10*d^12*e^5 - 589824*a^6*c^9*d^10*e^7 - 327680*a^7*c^8*d^8*e^9 + 327680*a^8*c^7*d^6*e^11 + 589824*a^9*c^6*d^4*e^13 + 327680*a^10*c^5*d^2*e^15))/(128*(a^6*e^8 + a^2*c^4*d^8 + 4*a^5*c*d^2*e^6 + 4*a^3*c^3*d^6*e^2 + 6*a^4*c^2*d^4*e^4)))*((c^3*d^6*(-a^5*c)^(1/2) - 9*a^3*e^6*(-a^5*c)^(1/2) + 2*a^3*c^3*d^5*e + 4*a^4*c^2*d^3*e^3 - 30*a^5*c*d*e^5 + 9*a*c^2*d^4*e^2*(-a^5*c)^(1/2) + 31*a^2*c*d^2*e^4*(-a^5*c)^(1/2))/(256*(a^9*c*e^8 + a^5*c^5*d^8 + 4*a^6*c^4*d^6*e^2 + 6*a^7*c^3*d^4*e^4 + 4*a^8*c^2*d^2*e^6)))^(1/2) + (x*(128*a*c^10*d^13*e^2 - 14208*a^7*c^4*d*e^14 + 768*a^2*c^9*d^11*e^4 + 3968*a^3*c^8*d^9*e^6 + 27136*a^4*c^7*d^7*e^8 + 30592*a^5*c^6*d^5*e^10 - 7424*a^6*c^5*d^3*e^12))/(128*(a^6*e^8 + a^2*c^4*d^8 + 4*a^5*c*d^2*e^6 + 4*a^3*c^3*d^6*e^2 + 6*a^4*c^2*d^4*e^4)))*((c^3*d^6*(-a^5*c)^(1/2) - 9*a^3*e^6*(-a^5*c)^(1/2) + 2*a^3*c^3*d^5*e + 4*a^4*c^2*d^3*e^3 - 30*a^5*c*d*e^5 + 9*a*c^2*d^4*e^2*(-a^5*c)^(1/2) + 31*a^2*c*d^2*e^4*(-a^5*c)^(1/2))/(256*(a^9*c*e^8 + a^5*c^5*d^8 + 4*a^6*c^4*d^6*e^2 + 6*a^7*c^3*d^4*e^4 + 4*a^8*c^2*d^2*e^6)))^(1/2) + (16*c^9*d^12*e^2 + 208*a*c^8*d^10*e^4 + 672*a^2*c^7*d^8*e^6 + 928*a^3*c^6*d^6*e^8 + 12880*a^4*c^5*d^4*e^10 + 12432*a^5*c^4*d^2*e^12)/(256*(a^6*e^8 + a^2*c^4*d^8 + 4*a^5*c*d^2*e^6 + 4*a^3*c

^3*d^6*e^2 + 6*a^4*c^2*d^4*e^4)))*((c^3*d^6*(-a^5*c)^(1/2) - 9*a^3*e^6*(-a^5*c)^(1/2) + 2*a^3*c^3*d^5*e + 4*a^4*c^2*d^3*e^3 - 30*a^5*c*d*e^5 + 9*a*c^2*d^4*e^2*(-a^5*c)^(1/2) + 31*a^2*c*d^2*e^4*(-a^5*c)^(1/2))/(256*(a^9*c*e^8 + a^5*c^5*d^8 + 4*a^6*c^4*d^6*e^2 + 6*a^7*c^3*d^4*e^4 + 4*a^8*c^2*d^2*e^6)))^(1/2) - (x*(81*a^4*c^3*e^13 + c^7*d^8*e^5 - 12*a*c^6*d^6*e^7 + 54*a^2*c^5*d^4*e^9 - 108*a^3*c^4*d^2*e^11))/(128*(a^6*e^8 + a^2*c^4*d^8 + 4*a^5*c*d^2*e^6 + 4*a^3*c^3*d^6*e^2 + 6*a^4*c^2*d^4*e^4)))*((c^3*d^6*(-a^5*c)^(1/2) - 9*a^3*e^6*(-a^5*c)^(1/2) + 2*a^3*c^3*d^5*e + 4*a^4*c^2*d^3*e^3 - 30*a^5*c*d*e^5 + 9*a*c^2*d^4*e^2*(-a^5*c)^(1/2) + 31*a^2*c*d^2*e^4*(-a^5*c)^(1/2))/(256*(a^9*c*e^8 + a^5*c^5*d^8 + 4*a^6*c^4*d^6*e^2 + 6*a^7*c^3*d^4*e^4 + 4*a^8*c^2*d^2*e^6)))^(1/2)*1i - (((((53248*a^9*c^4*d*e^15 + 4096*a^3*c^10*d^13*e^3 + 73728*a^4*c^9*d^11*e^5 + 307200*a^5*c^8*d^9*e^7 + 573440*a^6*c^7*d^7*e^9 + 552960*a^7*c^6*d^5*e^11 + 270336*a^8*c^5*d^3*e^13)/(256*(a^6*e^8 + a^2*c^4*d^8 + 4*a^5*c*d^2*e^6 + 4*a^3*c^3*d^6*e^2 + 6*a^4*c^2*d^4*e^4)) + (x*((c^3*d^6*(-a^5*c)^(1/2) - 9*a^3*e^6*(-a^5*c)^(1/2) + 2*a^3*c^3*d^5*e + 4*a^4*c^2*d^3*e^3 - 30*a^5*c*d*e^5 + 9*a*c^2*d^4*e^2*(-a^5*c)^(1/2) + 31*a^2*c*d^2*e^4*(-a^5*c)^(1/2))/(256*(a^9*c*e^8 + a^5*c^5*d^8 + 4*a^6*c^4*d^6*e^2 + 6*a^7*c^3*d^4*e^4 + 4*a^8*c^2*d^2*e^6)))^(1/2)*(65536*a^11*c^4*e^17 - 65536*a^4*c^11*d^14*e^3 - 327680*a^5*c^10*d^12*e^5 - 589824*a^6*c^9*d^10*e^7 - 327680*a^7*c^8*d^8*e^9 + 327680*a^8*c^7*d^6*e^11 + 589824*a^9*c^6*d^4*e^13 + 327680*a^10*c^5*d^2*e^15))/(128*(a^6*e^8 + a^2*c^4*d^8 + 4*a^5*c*d^2*e^6 + 4*a^3*c^3*d^6*e^2 + 6*a^4*c^2*d^4*e^4)))*((c^3*d^6*(-a^5*c)^(1/2) - 9*a^3*e^6*(-a^5*c)^(1/2) + 2*a^3*c^3*d^5*e + 4*a^4*c^2*d^3*e^3 - 30*a^5*c*d*e^5 + 9*a*c^2*d^4*e^2*(-a^5*c)^(1/2) + 31*a^2*c*d^2*e^4*(-a^5*c)^(1/2))/(256*(a^9*c*e^8 + a^5*c^5*d^8 + 4*a^6*c^4*d^6*e^2 + 6*a^7*c^3*d^4*e^4 + 4*a^8*c^2*d^2*e^6)))^(1/2) - (x*(128*a*c^10*d^13*e^2 - 14208*a^7*c^4*d*e^14 + 768*a^2*c^9*d^11*e^4 + 3968*a^3*c^8*d^9*e^6 + 27136*a^4*c^7*d^7*e^8 + 30592*a^5*c^6*d^5*e^10 - 7424*a^6*c^5*d^3*e^12))/(128*(a^6*e^8 + a^2*c^4*d^8 + 4*a^5*c*d^2*e^6 + 4*a^3*c^3*d^6*e^2 + 6*a^4*c^2*d^4*e^4)))*((c^3*d^6*(-a^5*c)^(1/2) - 9*a^3*e^6*(-a^5*c)^(1/2) + 2*a^3*c^3*d^5*e + 4*a^4*c^2*d^3*e^3 - 30*a^5*c*d*e^5 + 9*a*c^2*d^4*e^2*(-a^5*c)^(1/2) + 31*a^2*c*d^2*e^4*(-a^5*c)^(1/2))/(256*(a^9*c*e^8 + a^5*c^5*d^8 + 4*a^6*c^4*d^6*e^2 + 6*a^7*c^3*d^4*e^4 + 4*a^8*c^2*d^2*e^6)))^(1/2) + (16*c^9*d^12*e^2 + 208*a*c^8*d^10*e^4 + 672*a^2*c^7*d^8*e^6 + 928*a^3*c^6*d^6*e^8 + 12880*a^4*c^5*d^4*e^10 + 12432*a^5*c^4*d^2*e^12)/(256*(a^6*e^8 + a^2*c^4*d^8 + 4*a^5*c*d^2*e^6 + 4*a^3*c^3*d^6*e^2 + 6*a^4*c^2*d^4*e^4)))*((c^3*d^6*(-a^5*c)^(1/2) - 9*a^3*e^6*(-a^5*c)^(1/2) + 2*a^3*c^3*d^5*e + 4*a^4*c^2*d^3*e^3 - 30*a^5*c*d*e^5 + 9*a*c^2*d^4*e^2*(-a^5*c)^(1/2) + 31*a^2*c*d^2*e^4*(-a^5*c)^(1/2))/(256*(a^9*c*e^8 + a^5*c^5*d^8 + 4*a^6*c^4*d^6*e^2 + 6*a^7*c^3*d^4*e^4 + 4*a^8*c^2*d^2*e^6)))^(1/2) + (x*(81*a^4*c^3*e^13 + c^7*d^8*e^5 - 12*a*c^6*d^6*e^7 + 54*a^2*c^5*d^4*e^9 - 108*a^3*c^4*d^2*e^11))/(128*(a^6*e^8 + a^2*c^4*d^8 + 4*a^5*c*d^2*e^6 + 4*a^3*c^3*d^6*e^2 + 6*a^4*c^2*d^4*e^4)))*((c^3*d^6*(-a^5*c)^(1/2) - 9*a^3*e^6*(-a^5*c)^(1/2) + 2*a^3*c^3*d^5*e + 4*a^4*c^2*d^3*e^3 - 30*a^5*c*d*e^5 + 9*a*c^2*d^4*e^2*(-a^5*c)^(1/2) + 31*a^2*c*d^2*e^4*(-a^5*c)^(1/2))/(256*(a^9*c*e^8 + a^5*c^5*d^8 + 4*a^6*c^4*d^6*e^2 + 6*a^7*c^3*d^4*e^4 + 4*a^8*c^2*d^2*e

^6)))^(1/2)*1i)/((((((53248*a^9*c^4*d*e^15 + 4096*a^3*c^10*d^13*e^3 + 73728
*a^4*c^9*d^11*e^5 + 307200*a^5*c^8*d^9*e^7 + 573440*a^6*c^7*d^7*e^9 + 55296
0*a^7*c^6*d^5*e^11 + 270336*a^8*c^5*d^3*e^13)/(256*(a^6*e^8 + a^2*c^4*d^8 +
 4*a^5*c*d^2*e^6 + 4*a^3*c^3*d^6*e^2 + 6*a^4*c^2*d^4*e^4)) - (x*((c^3*d^6*(
-a^5*c)^(1/2) - 9*a^3*e^6*(-a^5*c)^(1/2) + 2*a^3*c^3*d^5*e + 4*a^4*c^2*d^3*
e^3 - 30*a^5*c*d*e^5 + 9*a*c^2*d^4*e^2*(-a^5*c)^(1/2) + 31*a^2*c*d^2*e^4*(-
a^5*c)^(1/2))/(256*(a^9*c*e^8 + a^5*c^5*d^8 + 4*a^6*c^4*d^6*e^2 + 6*a^7*c^3
*d^4*e^4 + 4*a^8*c^2*d^2*e^6)))^(1/2)*(65536*a^11*c^4*e^17 - 65536*a^4*c^11
*d^14*e^3 - 327680*a^5*c^10*d^12*e^5 - 589824*a^6*c^9*d^10*e^7 - 327680*a^7
*c^8*d^8*e^9 + 327680*a^8*c^7*d^6*e^11 + 589824*a^9*c^6*d^4*e^13 + 327680*a
^10*c^5*d^2*e^15))/(128*(a^6*e^8 + a^2*c^4*d^8 + 4*a^5*c*d^2*e^6 + 4*a^3*c^
3*d^6*e^2 + 6*a^4*c^2*d^4*e^4)))*((c^3*d^6*(-a^5*c)^(1/2) - 9*a^3*e^6*(-a^5
*c)^(1/2) + 2*a^3*c^3*d^5*e + 4*a^4*c^2*d^3*e^3 - 30*a^5*c*d*e^5 + 9*a*c^2*
d^4*e^2*(-a^5*c)^(1/2) + 31*a^2*c*d^2*e^4*(-a^5*c)^(1/2))/(256*(a^9*c*e^8 +
 a^5*c^5*d^8 + 4*a^6*c^4*d^6*e^2 + 6*a^7*c^3*d^4*e^4 + 4*a^8*c^2*d^2*e^6)))
^(1/2) + (x*(128*a*c^10*d^13*e^2 - 14208*a^7*c^4*d*e^14 + 768*a^2*c^9*d^11*
e^4 + 3968*a^3*c^8*d^9*e^6 + 27136*a^4*c^7*d^7*e^8 + 30592*a^5*c^6*d^5*e^10
 - 7424*a^6*c^5*d^3*e^12))/(128*(a^6*e^8 + a^2*c^4*d^8 + 4*a^5*c*d^2*e^6 +
4*a^3*c^3*d^6*e^2 + 6*a^4*c^2*d^4*e^4)))*((c^3*d^6*(-a^5*c)^(1/2) - 9*a^3*e
^6*(-a^5*c)^(1/2) + 2*a^3*c^3*d^5*e + 4*a^4*c^2*d^3*e^3 - 30*a^5*c*d*e^5 +
9*a*c^2*d^4*e^2*(-a^5*c)^(1/2) + 31*a^2*c*d^2*e^4*(-a^5*c)^(1/2))/(256*(a^9
*c*e^8 + a^5*c^5*d^8 + 4*a^6*c^4*d^6*e^2 + 6*a^7*c^3*d^4*e^4 + 4*a^8*c^2*d^
2*e^6)))^(1/2) + (16*c^9*d^12*e^2 + 208*a*c^8*d^10*e^4 + 672*a^2*c^7*d^8*e^
6 + 928*a^3*c^6*d^6*e^8 + 12880*a^4*c^5*d^4*e^10 + 12432*a^5*c^4*d^2*e^12)/
(256*(a^6*e^8 + a^2*c^4*d^8 + 4*a^5*c*d^2*e^6 + 4*a^3*c^3*d^6*e^2 + 6*a^4*c
^2*d^4*e^4)))*((c^3*d^6*(-a^5*c)^(1/2) - 9*a^3*e^6*(-a^5*c)^(1/2) + 2*a^3*c
^3*d^5*e + 4*a^4*c^2*d^3*e^3 - 30*a^5*c*d*e^5 + 9*a*c^2*d^4*e^2*(-a^5*c)^(1
/2) + 31*a^2*c*d^2*e^4*(-a^5*c)^(1/2))/(256*(a^9*c*e^8 + a^5*c^5*d^8 + 4*a^
6*c^4*d^6*e^2 + 6*a^7*c^3*d^4*e^4 + 4*a^8*c^2*d^2*e^6)))^(1/2) - (x*(81*a^4
*c^3*e^13 + c^7*d^8*e^5 - 12*a*c^6*d^6*e^7 + 54*a^2*c^5*d^4*e^9 - 108*a^3*c
^4*d^2*e^11))/(128*(a^6*e^8 + a^2*c^4*d^8 + 4*a^5*c*d^2*e^6 + 4*a^3*c^3*d^6
*e^2 + 6*a^4*c^2*d^4*e^4)))*((c^3*d^6*(-a^5*c)^(1/2) - 9*a^3*e^6*(-a^5*c)^(
1/2) + 2*a^3*c^3*d^5*e + 4*a^4*c^2*d^3*e^3 - 30*a^5*c*d*e^5 + 9*a*c^2*d^4*e
^2*(-a^5*c)^(1/2) + 31*a^2*c*d^2*e^4*(-a^5*c)^(1/2))/(256*(a^9*c*e^8 + a^5*
c^5*d^8 + 4*a^6*c^4*d^6*e^2 + 6*a^7*c^3*d^4*e^4 + 4*a^8*c^2*d^2*e^6)))^(1/2
) + (((((53248*a^9*c^4*d*e^15 + 4096*a^3*c^10*d^13*e^3 + 73728*a^4*c^9*d^11
*e^5 + 307200*a^5*c^8*d^9*e^7 + 573440*a^6*c^7*d^7*e^9 + 552960*a^7*c^6*d^5
*e^11 + 270336*a^8*c^5*d^3*e^13)/(256*(a^6*e^8 + a^2*c^4*d^8 + 4*a^5*c*d^2*
e^6 + 4*a^3*c^3*d^6*e^2 + 6*a^4*c^2*d^4*e^4)) + (x*((c^3*d^6*(-a^5*c)^(1/2)
 - 9*a^3*e^6*(-a^5*c)^(1/2) + 2*a^3*c^3*d^5*e + 4*a^4*c^2*d^3*e^3 - 30*a^5*
c*d*e^5 + 9*a*c^2*d^4*e^2*(-a^5*c)^(1/2) + 31*a^2*c*d^2*e^4*(-a^5*c)^(1/2))
/(256*(a^9*c*e^8 + a^5*c^5*d^8 + 4*a^6*c^4*d^6*e^2 + 6*a^7*c^3*d^4*e^4 + 4*
a^8*c^2*d^2*e^6)))^(1/2)*(65536*a^11*c^4*e^17 - 65536*a^4*c^11*d^14*e^3 - 3
27680*a^5*c^10*d^12*e^5 - 589824*a^6*c^9*d^10*e^7 - 327680*a^7*c^8*d^8*e^9
+ 327680*a^8*c^7*d^6*e^11 + 589824*a^9*c^6*d^4*e^13 + 327680*a^10*c^5*d^2*e

^15))/(128*(a^6*e^8 + a^2*c^4*d^8 + 4*a^5*c*d^2*e^6 + 4*a^3*c^3*d^6*e^2 + 6*a^4*c^2*d^4*e^4)))*((c^3*d^6*(-a^5*c)^(1/2) - 9*a^3*e^6*(-a^5*c)^(1/2) + 2*a^3*c^3*d^5*e + 4*a^4*c^2*d^3*e^3 - 30*a^5*c*d*e^5 + 9*a*c^2*d^4*e^2*(-a^5*c)^(1/2) + 31*a^2*c*d^2*e^4*(-a^5*c)^(1/2))/(256*(a^9*c*e^8 + a^5*c^5*d^8 + 4*a^6*c^4*d^6*e^2 + 6*a^7*c^3*d^4*e^4 + 4*a^8*c^2*d^2*e^6)))^(1/2) - (x*(128*a*c^10*d^13*e^2 - 14208*a^7*c^4*d*e^14 + 768*a^2*c^9*d^11*e^4 + 3968*a^3*c^8*d^9*e^6 + 27136*a^4*c^7*d^7*e^8 + 30592*a^5*c^6*d^5*e^10 - 7424*a^6*c^5*d^3*e^12))/(128*(a^6*e^8 + a^2*c^4*d^8 + 4*a^5*c*d^2*e^6 + 4*a^3*c^3*d^6*e^2 + 6*a^4*c^2*d^4*e^4)))*((c^3*d^6*(-a^5*c)^(1/2) - 9*a^3*e^6*(-a^5*c)^(1/2) + 2*a^3*c^3*d^5*e + 4*a^4*c^2*d^3*e^3 - 30*a^5*c*d*e^5 + 9*a*c^2*d^4*e^2*(-a^5*c)^(1/2) + 31*a^2*c*d^2*e^4*(-a^5*c)^(1/2))/(256*(a^9*c*e^8 + a^5*c^5*d^8 + 4*a^6*c^4*d^6*e^2 + 6*a^7*c^3*d^4*e^4 + 4*a^8*c^2*d^2*e^6)))^(1/2) + (16*c^9*d^12*e^2 + 208*a*c^8*d^10*e^4 + 672*a^2*c^7*d^8*e^6 + 928*a^3*c^6*d^6*e^8 + 12880*a^4*c^5*d^4*e^10 + 12432*a^5*c^4*d^2*e^12)/(256*(a^6*e^8 + a^2*c^4*d^8 + 4*a^5*c*d^2*e^6 + 4*a^3*c^3*d^6*e^2 + 6*a^4*c^2*d^4*e^4)))*((c^3*d^6*(-a^5*c)^(1/2) - 9*a^3*e^6*(-a^5*c)^(1/2) + 2*a^3*c^3*d^5*e + 4*a^4*c^2*d^3*e^3 - 30*a^5*c*d*e^5 + 9*a*c^2*d^4*e^2*(-a^5*c)^(1/2) + 31*a^2*c*d^2*e^4*(-a^5*c)^(1/2))/(256*(a^9*c*e^8 + a^5*c^5*d^8 + 4*a^6*c^4*d^6*e^2 + 6*a^7*c^3*d^4*e^4 + 4*a^8*c^2*d^2*e^6)))^(1/2) + (x*(81*a^4*c^3*e^13 + c^7*d^8*e^5 - 12*a*c^6*d^6*e^7 + 54*a^2*c^5*d^4*e^9 - 108*a^3*c^4*d^2*e^11))/(128*(a^6*e^8 + a^2*c^4*d^8 + 4*a^5*c*d^2*e^6 + 4*a^3*c^3*d^6*e^2 + 6*a^4*c^2*d^4*e^4)))*((c^3*d^6*(-a^5*c)^(1/2) - 9*a^3*e^6*(-a^5*c)^(1/2) + 2*a^3*c^3*d^5*e + 4*a^4*c^2*d^3*e^3 - 30*a^5*c*d*e^5 + 9*a*c^2*d^4*e^2*(-a^5*c)^(1/2) + 31*a^2*c*d^2*e^4*(-a^5*c)^(1/2))/(256*(a^9*c*e^8 + a^5*c^5*d^8 + 4*a^6*c^4*d^6*e^2 + 6*a^7*c^3*d^4*e^4 + 4*a^8*c^2*d^2*e^6)))^(1/2) + (5*c^5*d^5*e^7 + 54*a*c^4*d^3*e^9 + 81*a^2*c^3*d*e^11)/(128*(a^6*e^8 + a^2*c^4*d^8 + 4*a^5*c*d^2*e^6 + 4*a^3*c^3*d^6*e^2 + 6*a^4*c^2*d^4*e^4))))*((c^3*d^6*(-a^5*c)^(1/2) - 9*a^3*e^6*(-a^5*c)^(1/2) + 2*a^3*c^3*d^5*e + 4*a^4*c^2*d^3*e^3 - 30*a^5*c*d*e^5 + 9*a*c^2*d^4*e^2*(-a^5*c)^(1/2) + 31*a^2*c*d^2*e^4*(-a^5*c)^(1/2))/(256*(a^9*c*e^8 + a^5*c^5*d^8 + 4*a^6*c^4*d^6*e^2 + 6*a^7*c^3*d^4*e^4 + 4*a^8*c^2*d^2*e^6)))^(1/2)*2i - (atan((((-d*e^5)^(1/2)*((x*(81*a^4*c^3*e^13 + c^7*d^8*e^5 - 12*a*c^6*d^6*e^7 + 54*a^2*c^5*d^4*e^9 - 108*a^3*c^4*d^2*e^11))/(256*(a^6*e^8 + a^2*c^4*d^8 + 4*a^5*c*d^2*e^6 + 4*a^3*c^3*d^6*e^2 + 6*a^4*c^2*d^4*e^4)) - ((((c^9*d^12*e^2)/16 + (13*a*c^8*d^10*e^4)/16 + (21*a^2*c^7*d^8*e^6)/8 + (29*a^3*c^6*d^6*e^8)/8 + (805*a^4*c^5*d^4*e^10)/16 + (777*a^5*c^4*d^2*e^12)/16)/(2*(a^6*e^8 + a^2*c^4*d^8 + 4*a^5*c*d^2*e^6 + 4*a^3*c^3*d^6*e^2 + 6*a^4*c^2*d^4*e^4)) + ((((-d*e^5)^(1/2)*((208*a^9*c^4*d*e^15 + 16*a^3*c^10*d^13*e^3 + 288*a^4*c^9*d^11*e^5 + 1200*a^5*c^8*d^9*e^7 + 2240*a^6*c^7*d^7*e^9 + 2160*a^7*c^6*d^5*e^11 + 1056*a^8*c^5*d^3*e^13)/(2*(a^6*e^8 + a^2*c^4*d^8 + 4*a^5*c*d^2*e^6 + 4*a^3*c^3*d^6*e^2 + 6*a^4*c^2*d^4*e^4)) - (x*(-d*e^5)^(1/2)*(65536*a^11*c^4*e^17 - 65536*a^4*c^11*d^14*e^3 - 327680*a^5*c^10*d^12*e^5 - 589824*a^6*c^9*d^10*e^7 - 327680*a^7*c^8*d^8*e^9 + 327680*a^8*c^7*d^6*e^11 + 589824*a^9*c^6*d^4*e^13 + 327680*a^10*c^5*d^2*e^15))/(512*(a^2*e^4 + c^2*d^4 + 2*a*c*d^2*e^2)*(a^6*e^8 + a^2*c^4*d^8 + 4*a^5*c*d^2*e^6 + 4*a^3*c^3*d^6*e^2 + 6*a^4*c^2*d^4*e^4))))/(2*(a^2*e^4 +

c^2*d^4 + 2*a*c*d^2*e^2)) + (x*(128*a*c^10*d^13*e^2 - 14208*a^7*c^4*d*e^14 + 768*a^2*c^9*d^11*e^4 + 3968*a^3*c^8*d^9*e^6 + 27136*a^4*c^7*d^7*e^8 + 30592*a^5*c^6*d^5*e^10 - 7424*a^6*c^5*d^3*e^12))/(256*(a^6*e^8 + a^2*c^4*d^8 + 4*a^5*c*d^2*e^6 + 4*a^3*c^3*d^6*e^2 + 6*a^4*c^2*d^4*e^4)))*(-d*e^5)^(1/2))/(2*(a^2*e^4 + c^2*d^4 + 2*a*c*d^2*e^2)))*(-d*e^5)^(1/2))/(2*(a^2*e^4 + c^2*d^4 + 2*a*c*d^2*e^2)))*1i)/(a^2*e^4 + c^2*d^4 + 2*a*c*d^2*e^2) + ((-d*e^5)^(1/2)*((x*(81*a^4*c^3*e^13 + c^7*d^8*e^5 - 12*a*c^6*d^6*e^7 + 54*a^2*c^5*d^4*e^9 - 108*a^3*c^4*d^2*e^11))/(256*(a^6*e^8 + a^2*c^4*d^8 + 4*a^5*c*d^2*e^6 + 4*a^3*c^3*d^6*e^2 + 6*a^4*c^2*d^4*e^4)) + ((((c^9*d^12*e^2)/16 + (13*a*c^8*d^10*e^4)/16 + (21*a^2*c^7*d^8*e^6)/8 + (29*a^3*c^6*d^6*e^8)/8 + (805*a^4*c^5*d^4*e^10)/16 + (777*a^5*c^4*d^2*e^12)/16)/(2*(a^6*e^8 + a^2*c^4*d^8 + 4*a^5*c*d^2*e^6 + 4*a^3*c^3*d^6*e^2 + 6*a^4*c^2*d^4*e^4)) + ((((-d*e^5)^(1/2)*((208*a^9*c^4*d*e^15 + 16*a^3*c^10*d^13*e^3 + 288*a^4*c^9*d^11*e^5 + 1200*a^5*c^8*d^9*e^7 + 2240*a^6*c^7*d^7*e^9 + 2160*a^7*c^6*d^5*e^11 + 1056*a^8*c^5*d^3*e^13)/(2*(a^6*e^8 + a^2*c^4*d^8 + 4*a^5*c*d^2*e^6 + 4*a^3*c^3*d^6*e^2 + 6*a^4*c^2*d^4*e^4)) + (x*(-d*e^5)^(1/2)*(65536*a^11*c^4*e^17 - 65536*a^4*c^11*d^14*e^3 - 327680*a^5*c^10*d^12*e^5 - 589824*a^6*c^9*d^10*e^7 - 327680*a^7*c^8*d^8*e^9 + 327680*a^8*c^7*d^6*e^11 + 589824*a^9*c^6*d^4*e^13 + 327680*a^10*c^5*d^2*e^15))/(512*(a^2*e^4 + c^2*d^4 + 2*a*c*d^2*e^2)*(a^6*e^8 + a^2*c^4*d^8 + 4*a^5*c*d^2*e^6 + 4*a^3*c^3*d^6*e^2 + 6*a^4*c^2*d^4*e^4))))/(2*(a^2*e^4 + c^2*d^4 + 2*a*c*d^2*e^2)) - (x*(128*a*c^10*d^13*e^2 - 14208*a^7*c^4*d*e^14 + 768*a^2*c^9*d^11*e^4 + 3968*a^3*c^8*d^9*e^6 + 27136*a^4*c^7*d^7*e^8 + 30592*a^5*c^6*d^5*e^10 - 7424*a^6*c^5*d^3*e^12))/(256*(a^6*e^8 + a^2*c^4*d^8 + 4*a^5*c*d^2*e^6 + 4*a^3*c^3*d^6*e^2 + 6*a^4*c^2*d^4*e^4)))*(-d*e^5)^(1/2))/(2*(a^2*e^4 + c^2*d^4 + 2*a*c*d^2*e^2)))*(-d*e^5)^(1/2))/(2*(a^2*e^4 + c^2*d^4 + 2*a*c*d^2*e^2)))*1i)/(a^2*e^4 + c^2*d^4 + 2*a*c*d^2*e^2))/(((5*c^5*d^5*e^7)/128 + (27*a*c^4*d^3*e^9)/64 + (81*a^2*c^3*d*e^11)/128)/(a^6*e^8 + a^2*c^4*d^8 + 4*a^5*c*d^2*e^6 + 4*a^3*c^3*d^6*e^2 + 6*a^4*c^2*d^4*e^4) - ((-d*e^5)^(1/2)*((x*(81*a^4*c^3*e^13 + c^7*d^8*e^5 - 12*a*c^6*d^6*e^7 + 54*a^2*c^5*d^4*e^9 - 108*a^3*c^4*d^2*e^11))/(256*(a^6*e^8 + a^2*c^4*d^8 + 4*a^5*c*d^2*e^6 + 4*a^3*c^3*d^6*e^2 + 6*a^4*c^2*d^4*e^4)) - ((((c^9*d^12*e^2)/16 + (13*a*c^8*d^10*e^4)/16 + (21*a^2*c^7*d^8*e^6)/8 + (29*a^3*c^6*d^6*e^8)/8 + (805*a^4*c^5*d^4*e^10)/16 + (777*a^5*c^4*d^2*e^12)/16)/(2*(a^6*e^8 + a^2*c^4*d^8 + 4*a^5*c*d^2*e^6 + 4*a^3*c^3*d^6*e^2 + 6*a^4*c^2*d^4*e^4)) + ((((-d*e^5)^(1/2)*((208*a^9*c^4*d*e^15 + 16*a^3*c^10*d^13*e^3 + 288*a^4*c^9*d^11*e^5 + 1200*a^5*c^8*d^9*e^7 + 2240*a^6*c^7*d^7*e^9 + 2160*a^7*c^6*d^5*e^11 + 1056*a^8*c^5*d^3*e^13)/(2*(a^6*e^8 + a^2*c^4*d^8 + 4*a^5*c*d^2*e^6 + 4*a^3*c^3*d^6*e^2 + 6*a^4*c^2*d^4*e^4)) - (x*(-d*e^5)^(1/2)*(65536*a^11*c^4*e^17 - 65536*a^4*c^11*d^14*e^3 - 327680*a^5*c^10*d^12*e^5 - 589824*a^6*c^9*d^10*e^7 - 327680*a^7*c^8*d^8*e^9 + 327680*a^8*c^7*d^6*e^11 + 589824*a^9*c^6*d^4*e^13 + 327680*a^10*c^5*d^2*e^15))/(512*(a^2*e^4 + c^2*d^4 + 2*a*c*d^2*e^2)*(a^6*e^8 + a^2*c^4*d^8 + 4*a^5*c*d^2*e^6 + 4*a^3*c^3*d^6*e^2 + 6*a^4*c^2*d^4*e^4))))/(2*(a^2*e^4 + c^2*d^4 + 2*a*c*d^2*e^2)) + (x*(128*a*c^10*d^13*e^2 - 14208*a^7*c^4*d*e^14 + 768*a^2*c^9*d^11*e^4 + 3968*a^3*c^8*d^9*e^6 + 27136*a^4*c^7*d^7*e^8 + 30592*a^5*c^6*d^5*e^10 - 7424*a

^6*c^5*d^3*e^12))/(256*(a^6*e^8 + a^2*c^4*d^8 + 4*a^5*c*d^2*e^6 + 4*a^3*c^3
*d^6*e^2 + 6*a^4*c^2*d^4*e^4)))*(-d*e^5)^(1/2))/(2*(a^2*e^4 + c^2*d^4 + 2*a
*c*d^2*e^2)))*(-d*e^5)^(1/2))/(2*(a^2*e^4 + c^2*d^4 + 2*a*c*d^2*e^2))))/(a^
2*e^4 + c^2*d^4 + 2*a*c*d^2*e^2) + ((-d*e^5)^(1/2)*((x*(81*a^4*c^3*e^13 + c
^7*d^8*e^5 - 12*a*c^6*d^6*e^7 + 54*a^2*c^5*d^4*e^9 - 108*a^3*c^4*d^2*e^11))
/(256*(a^6*e^8 + a^2*c^4*d^8 + 4*a^5*c*d^2*e^6 + 4*a^3*c^3*d^6*e^2 + 6*a^4*
c^2*d^4*e^4)) + ((((c^9*d^12*e^2)/16 + (13*a*c^8*d^10*e^4)/16 + (21*a^2*c^7
*d^8*e^6)/8 + (29*a^3*c^6*d^6*e^8)/8 + (805*a^4*c^5*d^4*e^10)/16 + (777*a^5
*c^4*d^2*e^12)/16)/(2*(a^6*e^8 + a^2*c^4*d^8 + 4*a^5*c*d^2*e^6 + 4*a^3*c^3*
d^6*e^2 + 6*a^4*c^2*d^4*e^4)) + ((((-d*e^5)^(1/2)*((208*a^9*c^4*d*e^15 + 16
*a^3*c^10*d^13*e^3 + 288*a^4*c^9*d^11*e^5 + 1200*a^5*c^8*d^9*e^7 + 2240*a^6
*c^7*d^7*e^9 + 2160*a^7*c^6*d^5*e^11 + 1056*a^8*c^5*d^3*e^13)/(2*(a^6*e^8 +
a^2*c^4*d^8 + 4*a^5*c*d^2*e^6 + 4*a^3*c^3*d^6*e^2 + 6*a^4*c^2*d^4*e^4)) +
(x*(-d*e^5)^(1/2)*(65536*a^11*c^4*e^17 - 65536*a^4*c^11*d^14*e^3 - 327680*a
^5*c^10*d^12*e^5 - 589824*a^6*c^9*d^10*e^7 - 327680*a^7*c^8*d^8*e^9 + 32768
0*a^8*c^7*d^6*e^11 + 589824*a^9*c^6*d^4*e^13 + 327680*a^10*c^5*d^2*e^15))/(
512*(a^2*e^4 + c^2*d^4 + 2*a*c*d^2*e^2)*(a^6*e^8 + a^2*c^4*d^8 + 4*a^5*c*d^
2*e^6 + 4*a^3*c^3*d^6*e^2 + 6*a^4*c^2*d^4*e^4))))/(2*(a^2*e^4 + c^2*d^4 + 2
*a*c*d^2*e^2)) - (x*(128*a*c^10*d^13*e^2 - 14208*a^7*c^4*d*e^14 + 768*a^2*c
^9*d^11*e^4 + 3968*a^3*c^8*d^9*e^6 + 27136*a^4*c^7*d^7*e^8 + 30592*a^5*c^6*
d^5*e^10 - 7424*a^6*c^5*d^3*e^12))/(256*(a^6*e^8 + a^2*c^4*d^8 + 4*a^5*c*d^
2*e^6 + 4*a^3*c^3*d^6*e^2 + 6*a^4*c^2*d^4*e^4)))*(-d*e^5)^(1/2))/(2*(a^2*e^
4 + c^2*d^4 + 2*a*c*d^2*e^2)))*(-d*e^5)^(1/2))/(2*(a^2*e^4 + c^2*d^4 + 2*a*
c*d^2*e^2))))/(a^2*e^4 + c^2*d^4 + 2*a*c*d^2*e^2)))*(-d*e^5)^(1/2)*1i)/(a^2
*e^4 + c^2*d^4 + 2*a*c*d^2*e^2)

3.256 $\int \frac{1}{(d+ex^2)(a+cx^4)^2}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 689

$$\begin{aligned}\int \frac{1}{(d+ex^2)(a+cx^4)^2}\,dx &= \frac{cx(d-ex^2)}{4a(cd^2+ae^2)(a+cx^4)}+\frac{e^{7/2}\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{d}(cd^2+ae^2)^2}\\
&-\frac{\sqrt[4]{c}e^2(\sqrt{c}d-\sqrt{a}e)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}(cd^2+ae^2)^2}\\
&-\frac{\sqrt[4]{c}(3\sqrt{c}d-\sqrt{a}e)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{8\sqrt{2}a^{7/4}(cd^2+ae^2)}\\
&+\frac{\sqrt[4]{c}e^2(\sqrt{c}d-\sqrt{a}e)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}(cd^2+ae^2)^2}\\
&+\frac{\sqrt[4]{c}(3\sqrt{c}d-\sqrt{a}e)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{8\sqrt{2}a^{7/4}(cd^2+ae^2)}\\
&-\frac{\sqrt[4]{c}e^2(\sqrt{c}d+\sqrt{a}e)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)}{4\sqrt{2}a^{3/4}(cd^2+ae^2)^2}\\
&-\frac{\sqrt[4]{c}(3\sqrt{c}d+\sqrt{a}e)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)}{16\sqrt{2}a^{7/4}(cd^2+ae^2)}\\
&+\frac{\sqrt[4]{c}e^2(\sqrt{c}d+\sqrt{a}e)\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)}{4\sqrt{2}a^{3/4}(cd^2+ae^2)^2}\\
&+\frac{\sqrt[4]{c}(3\sqrt{c}d+\sqrt{a}e)\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)}{16\sqrt{2}a^{7/4}(cd^2+ae^2)}\end{aligned}$$

```
[Out] 1/4*c*x*(-e*x^2+d)/a/(a*e^2+c*d^2)/(c*x^4+a)+1/4*c^(1/4)*e^2*arctan(-1+c^(1
/4)*x*2^(1/2)/a^(1/4))*(-e*a^(1/2)+d*c^(1/2))/a^(3/4)/(a*e^2+c*d^2)^2*2^(1/
2)+1/4*c^(1/4)*e^2*arctan(1+c^(1/4)*x*2^(1/2)/a^(1/4))*(-e*a^(1/2)+d*c^(1/2
))/a^(3/4)/(a*e^2+c*d^2)^2*2^(1/2)-1/8*c^(1/4)*e^2*ln(-a^(1/4)*c^(1/4)*x*2^
(1/2)+a^(1/2)+x^2*c^(1/2))*(e*a^(1/2)+d*c^(1/2))/a^(3/4)/(a*e^2+c*d^2)^2*2^
(1/2)+1/8*c^(1/4)*e^2*ln(a^(1/4)*c^(1/4)*x*2^(1/2)+a^(1/2)+x^2*c^(1/2))*(e*
a^(1/2)+d*c^(1/2))/a^(3/4)/(a*e^2+c*d^2)^2*2^(1/2)+1/16*c^(1/4)*arctan(-1+c
^(1/4)*x*2^(1/2)/a^(1/4))*(-e*a^(1/2)+3*d*c^(1/2))/a^(7/4)/(a*e^2+c*d^2)*2^
(1/2)+1/16*c^(1/4)*arctan(1+c^(1/4)*x*2^(1/2)/a^(1/4))*(-e*a^(1/2)+3*d*c^(1
/2))/a^(7/4)/(a*e^2+c*d^2)*2^(1/2)-1/32*c^(1/4)*ln(-a^(1/4)*c^(1/4)*x*2^(1/
2)+a^(1/2)+x^2*c^(1/2))*(e*a^(1/2)+3*d*c^(1/2))/a^(7/4)/(a*e^2+c*d^2)*2^(1/
2)+1/32*c^(1/4)*ln(a^(1/4)*c^(1/4)*x*2^(1/2)+a^(1/2)+x^2*c^(1/2))*(e*a^(1/2
)+3*d*c^(1/2))/a^(7/4)/(a*e^2+c*d^2)*2^(1/2)+e^(7/2)*arctan(x*e^(1/2)/d^(1/
2))/(a*e^2+c*d^2)^2/d^(1/2)
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 689, normalized size of antiderivative = 1.00, number of steps used = 22, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.474$, Rules used

$= \{1253, 211, 1193, 1182, 1176, 631, 210, 1179, 642\}$

$$\int \frac{1}{(d+ex^2)(a+cx^4)^2}\,dx = -\frac{\sqrt[4]{c}e^2\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\left(\sqrt{c}d-\sqrt{a}e\right)}{2\sqrt{2}a^{3/4}\left(ae^2+cd^2\right)^2} + \frac{\sqrt[4]{c}e^2\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)\left(\sqrt{c}d-\sqrt{a}e\right)}{2\sqrt{2}a^{3/4}\left(ae^2+cd^2\right)^2} - \frac{\sqrt[4]{c}\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\left(3\sqrt{c}d-\sqrt{a}e\right)}{8\sqrt{2}a^{7/4}\left(ae^2+cd^2\right)} + \frac{\sqrt[4]{c}\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)\left(3\sqrt{c}d-\sqrt{a}e\right)}{8\sqrt{2}a^{7/4}\left(ae^2+cd^2\right)} - \frac{\sqrt[4]{c}e^2\left(\sqrt{a}e+\sqrt{c}d\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{4\sqrt{2}a^{3/4}\left(ae^2+cd^2\right)^2} + \frac{\sqrt[4]{c}e^2\left(\sqrt{a}e+\sqrt{c}d\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{4\sqrt{2}a^{3/4}\left(ae^2+cd^2\right)^2} - \frac{\sqrt[4]{c}\left(\sqrt{a}e+3\sqrt{c}d\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{16\sqrt{2}a^{7/4}\left(ae^2+cd^2\right)} + \frac{\sqrt[4]{c}\left(\sqrt{a}e+3\sqrt{c}d\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{16\sqrt{2}a^{7/4}\left(ae^2+cd^2\right)} + \frac{e^{7/2}\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{d}\left(ae^2+cd^2\right)^2} + \frac{cx(d-ex^2)}{4a\left(a+cx^4\right)\left(ae^2+cd^2\right)}$$

```
[In] Int[1/((d + e*x^2)*(a + c*x^4)^2),x]
```

```
[Out] (c*x*(d - e*x^2))/(4*a*(c*d^2 + a*e^2)*(a + c*x^4)) + (e^(7/2)*ArcTan[(Sqrt
[e]*x)/Sqrt[d]])/(Sqrt[d]*(c*d^2 + a*e^2)^2) - (c^(1/4)*e^2*(Sqrt[c]*d - Sq
rt[a]*e)*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*(c*d^2
 + a*e^2)^2) - (c^(1/4)*(3*Sqrt[c]*d - Sqrt[a]*e)*ArcTan[1 - (Sqrt[2]*c^(1/
4)*x)/a^(1/4)])/(8*Sqrt[2]*a^(7/4)*(c*d^2 + a*e^2)) + (c^(1/4)*e^2*(Sqrt[c]
*d - Sqrt[a]*e)*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)
*(c*d^2 + a*e^2)^2) + (c^(1/4)*(3*Sqrt[c]*d - Sqrt[a]*e)*ArcTan[1 + (Sqrt[2
]*c^(1/4)*x)/a^(1/4)])/(8*Sqrt[2]*a^(7/4)*(c*d^2 + a*e^2)) - (c^(1/4)*e^2*(
Sqrt[c]*d + Sqrt[a]*e)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^
2])/(4*Sqrt[2]*a^(3/4)*(c*d^2 + a*e^2)^2) - (c^(1/4)*(3*Sqrt[c]*d + Sqrt[a]
*e)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(16*Sqrt[2]*a^(
7/4)*(c*d^2 + a*e^2)) + (c^(1/4)*e^2*(Sqrt[c]*d + Sqrt[a]*e)*Log[Sqrt[a] +
Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(4*Sqrt[2]*a^(3/4)*(c*d^2 + a*e^2
)^2) + (c^(1/4)*(3*Sqrt[c]*d + Sqrt[a]*e)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(
1/4)*x + Sqrt[c]*x^2])/(16*Sqrt[2]*a^(7/4)*(c*d^2 + a*e^2))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1182

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[(-a)
*c]
```

Rule 1193

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[(-x
)*(d + e*x^2)*((a + c*x^4)^(p + 1)/(4*a*(p + 1))), x] + Dist[1/(4*a*(p + 1)
), Int[Simp[d*(4*p + 5) + e*(4*p + 7)*x^2, x]*(a + c*x^4)^(p + 1), x], x] /
; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && IntegerQ
[2*p]
```

Rule 1253

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (c_.)*(x_)^4)^(p_), x_Symbol] :> Int
[ExpandIntegrand[(d + e*x^2)^q*(a + c*x^4)^p, x], x] /; FreeQ[{a, c, d, e,
p, q}, x] && ((IntegerQ[p] && IntegerQ[q]) || IGtQ[p, 0])
```

Rubi steps

$$\text{integral} = \int \left(\frac{e^4}{\left(cd^2+ae^2\right)^2\left(d+ex^2\right)}+\frac{c\left(d-ex^2\right)}{\left(cd^2+ae^2\right)\left(a+cx^4\right)^2}-\frac{ce^2\left(-d+ex^2\right)}{\left(cd^2+ae^2\right)^2\left(a+cx^4\right)}\right)dx$$

$$= -\frac{\left(ce^2\right)\int\frac{-d+ex^2}{a+cx^4}\,dx}{\left(cd^2+ae^2\right)^2}+\frac{e^4\int\frac{1}{d+ex^2}\,dx}{\left(cd^2+ae^2\right)^2}+\frac{c\int\frac{d-ex^2}{\left(a+cx^4\right)^2}\,dx}{cd^2+ae^2}$$

$$= \frac{cx\left(d-ex^2\right)}{4a\left(cd^2+ae^2\right)\left(a+cx^4\right)}+\frac{e^{7/2}\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{d}\left(cd^2+ae^2\right)^2}+\frac{\left(\left(\frac{\sqrt{c}d}{\sqrt{a}}-e\right)e^2\right)\int\frac{\sqrt{a}\sqrt{c}+cx^2}{a+cx^4}\,dx}{2\left(cd^2+ae^2\right)^2}$$
$$+\frac{\left(e^2\left(\frac{\sqrt{c}d}{\sqrt{a}}+e\right)\right)\int\frac{\sqrt{a}\sqrt{c}-cx^2}{a+cx^4}\,dx}{2\left(cd^2+ae^2\right)^2}-\frac{c\int\frac{-3d+ex^2}{a+cx^4}\,dx}{4a\left(cd^2+ae^2\right)}$$

$$= \frac{cx\left(d-ex^2\right)}{4a\left(cd^2+ae^2\right)\left(a+cx^4\right)}+\frac{e^{7/2}\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{d}\left(cd^2+ae^2\right)^2}+\frac{\left(\left(\frac{\sqrt{c}d}{\sqrt{a}}-e\right)e^2\right)\int\frac{1}{\frac{\sqrt{a}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}+x^2}\,dx}{4\left(cd^2+ae^2\right)^2}$$
$$+\frac{\left(\left(\frac{\sqrt{c}d}{\sqrt{a}}-e\right)e^2\right)\int\frac{1}{\frac{\sqrt{a}}{\sqrt{c}}+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}+x^2}\,dx}{4\left(cd^2+ae^2\right)^2}-\frac{\left(\sqrt[4]{c}e^2\left(\sqrt{c}d+\sqrt{a}e\right)\right)\int\frac{\frac{\sqrt{2}\sqrt[4]{a}}{\sqrt[4]{c}}+2x}{-\frac{\sqrt{a}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}-x^2}\,dx}{4\sqrt{2}a^{3/4}\left(cd^2+ae^2\right)^2}$$
$$-\frac{\left(\sqrt[4]{c}e^2\left(\sqrt{c}d+\sqrt{a}e\right)\right)\int\frac{\frac{\sqrt{2}\sqrt[4]{a}}{\sqrt[4]{c}}-2x}{-\frac{\sqrt{a}}{\sqrt{c}}+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}-x^2}\,dx}{4\sqrt{2}a^{3/4}\left(cd^2+ae^2\right)^2}$$
$$+\frac{\left(\frac{3\sqrt{c}d}{\sqrt{a}}-e\right)\int\frac{\sqrt{a}\sqrt{c}+cx^2}{a+cx^4}\,dx}{8a\left(cd^2+ae^2\right)}+\frac{\left(\frac{3\sqrt{c}d}{\sqrt{a}}+e\right)\int\frac{\sqrt{a}\sqrt{c}-cx^2}{a+cx^4}\,dx}{8a\left(cd^2+ae^2\right)}$$

$$= \frac{cx(d-ex^2)}{4a\left(cd^2+ae^2\right)\left(a+cx^4\right)} + \frac{e^{7/2}\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{d}\left(cd^2+ae^2\right)^2}$$
$$- \frac{\sqrt[4]{c}e^2\left(\sqrt{c}d+\sqrt{a}e\right)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)}{4\sqrt{2}a^{3/4}\left(cd^2+ae^2\right)^2}$$
$$+ \frac{\sqrt[4]{c}e^2\left(\sqrt{c}d+\sqrt{a}e\right)\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)}{4\sqrt{2}a^{3/4}\left(cd^2+ae^2\right)^2}$$
$$+ \frac{\left(\sqrt[4]{c}e^2\left(\sqrt{c}d-\sqrt{a}e\right)\right)\mathrm{Subst}\left(\int\frac{1}{-1-x^2}\,dx, x, 1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}\left(cd^2+ae^2\right)^2}$$
$$- \frac{\left(\sqrt[4]{c}e^2\left(\sqrt{c}d-\sqrt{a}e\right)\right)\mathrm{Subst}\left(\int\frac{1}{-1-x^2}\,dx, x, 1+\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}\left(cd^2+ae^2\right)^2}$$
$$+ \frac{\left(\frac{3\sqrt{c}d}{\sqrt{a}}-e\right)\int\frac{1}{\frac{\sqrt{a}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}+x^2}\,dx}{16a\left(cd^2+ae^2\right)} + \frac{\left(\frac{3\sqrt{c}d}{\sqrt{a}}-e\right)\int\frac{1}{\frac{\sqrt{a}}{\sqrt{c}}+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}+x^2}\,dx}{16a\left(cd^2+ae^2\right)}$$
$$- \frac{\left(\sqrt[4]{c}\left(3\sqrt{c}d+\sqrt{a}e\right)\right)\int\frac{\frac{\sqrt{2}\sqrt[4]{a}}{\sqrt[4]{c}}+2x}{-\frac{\sqrt{a}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}-x^2}\,dx}{16\sqrt{2}a^{7/4}\left(cd^2+ae^2\right)}$$
$$- \frac{\left(\sqrt[4]{c}\left(3\sqrt{c}d+\sqrt{a}e\right)\right)\int\frac{\frac{\sqrt{2}\sqrt[4]{a}}{\sqrt[4]{c}}-2x}{-\frac{\sqrt{a}}{\sqrt{c}}+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}-x^2}\,dx}{16\sqrt{2}a^{7/4}\left(cd^2+ae^2\right)}$$

$$= \frac{cx(d-ex^2)}{4a\left(cd^2+ae^2\right)\left(a+cx^4\right)} + \frac{e^{7/2}\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{d}\left(cd^2+ae^2\right)^2}$$

$$- \frac{\sqrt[4]{c}e^2\left(\sqrt{c}d-\sqrt{a}e\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}\left(cd^2+ae^2\right)^2}$$

$$+ \frac{\sqrt[4]{c}e^2\left(\sqrt{c}d-\sqrt{a}e\right)\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}\left(cd^2+ae^2\right)^2}$$

$$- \frac{\sqrt[4]{c}e^2\left(\sqrt{c}d+\sqrt{a}e\right)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)}{4\sqrt{2}a^{3/4}\left(cd^2+ae^2\right)^2}$$

$$- \frac{\sqrt[4]{c}\left(3\sqrt{c}d+\sqrt{a}e\right)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)}{16\sqrt{2}a^{7/4}\left(cd^2+ae^2\right)}$$

$$+ \frac{\sqrt[4]{c}e^2\left(\sqrt{c}d+\sqrt{a}e\right)\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)}{4\sqrt{2}a^{3/4}\left(cd^2+ae^2\right)^2}$$

$$+ \frac{\sqrt[4]{c}\left(3\sqrt{c}d+\sqrt{a}e\right)\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)}{16\sqrt{2}a^{7/4}\left(cd^2+ae^2\right)}$$

$$+ \frac{\left(\sqrt[4]{c}\left(3\sqrt{c}d-\sqrt{a}e\right)\right)\text{Subst}\left(\int\frac{1}{-1-x^2}\,dx,x,1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{8\sqrt{2}a^{7/4}\left(cd^2+ae^2\right)}$$

$$- \frac{\left(\sqrt[4]{c}\left(3\sqrt{c}d-\sqrt{a}e\right)\right)\text{Subst}\left(\int\frac{1}{-1-x^2}\,dx,x,1+\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{8\sqrt{2}a^{7/4}\left(cd^2+ae^2\right)}$$

$$= \frac{cx(d-ex^2)}{4a\left(cd^2+ae^2\right)\left(a+cx^4\right)} + \frac{e^{7/2}\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{d}\left(cd^2+ae^2\right)^2}$$
$$- \frac{\sqrt[4]{c}e^2\left(\sqrt{c}d-\sqrt{a}e\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}\left(cd^2+ae^2\right)^2}$$
$$- \frac{\sqrt[4]{c}\left(3\sqrt{c}d-\sqrt{a}e\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{8\sqrt{2}a^{7/4}\left(cd^2+ae^2\right)}$$
$$+ \frac{\sqrt[4]{c}e^2\left(\sqrt{c}d-\sqrt{a}e\right)\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}\left(cd^2+ae^2\right)^2}$$
$$+ \frac{\sqrt[4]{c}\left(3\sqrt{c}d-\sqrt{a}e\right)\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{8\sqrt{2}a^{7/4}\left(cd^2+ae^2\right)}$$
$$- \frac{\sqrt[4]{c}e^2\left(\sqrt{c}d+\sqrt{a}e\right)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)}{4\sqrt{2}a^{3/4}\left(cd^2+ae^2\right)^2}$$
$$- \frac{\sqrt[4]{c}\left(3\sqrt{c}d+\sqrt{a}e\right)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)}{16\sqrt{2}a^{7/4}\left(cd^2+ae^2\right)}$$
$$+ \frac{\sqrt[4]{c}e^2\left(\sqrt{c}d+\sqrt{a}e\right)\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)}{4\sqrt{2}a^{3/4}\left(cd^2+ae^2\right)^2}$$
$$+ \frac{\sqrt[4]{c}\left(3\sqrt{c}d+\sqrt{a}e\right)\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)}{16\sqrt{2}a^{7/4}\left(cd^2+ae^2\right)}$$

Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 429, normalized size of antiderivative = 0.62

$$\int \frac{1}{\left(d+ex^2\right)\left(a+cx^4\right)^2}\,dx$$

$$= \frac{\frac{8c(cd^2+ae^2)x(d-ex^2)}{a(a+cx^4)} + \frac{32e^{7/2}\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{d}} + \frac{2\sqrt{2}\sqrt[4]{c}\left(-3c^{3/2}d^3+\sqrt{a}cd^2e-7a\sqrt{c}de^2+5a^{3/2}e^3\right)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{a^{7/4}} - \frac{2\sqrt{2}\sqrt[4]{c}(-3c^3}{}}{}$$

```
[In] Integrate[1/((d + e*x^2)*(a + c*x^4)^2),x]
```

```
[Out] ((8*c*(c*d^2 + a*e^2)*x*(d - e*x^2))/(a*(a + c*x^4)) + (32*e^(7/2)*ArcTan[(
Sqrt[e]*x)/Sqrt[d]])/Sqrt[d] + (2*Sqrt[2]*c^(1/4)*(-3*c^(3/2)*d^3 + Sqrt[a]
*c*d^2*e - 7*a*Sqrt[c]*d*e^2 + 5*a^(3/2)*e^3)*ArcTan[1 - (Sqrt[2]*c^(1/4)*x
)/a^(1/4)])/a^(7/4) - (2*Sqrt[2]*c^(1/4)*(-3*c^(3/2)*d^3 + Sqrt[a]*c*d^2*e
- 7*a*Sqrt[c]*d*e^2 + 5*a^(3/2)*e^3)*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)
```

```
])/a^(7/4) - (Sqrt[2]*c^(1/4)*(3*c^(3/2)*d^3 + Sqrt[a]*c*d^2*e + 7*a*Sqrt[c
]*d*e^2 + 5*a^(3/2)*e^3)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*
x^2])/a^(7/4) + (Sqrt[2]*c^(1/4)*(3*c^(3/2)*d^3 + Sqrt[a]*c*d^2*e + 7*a*Sqr
t[c]*d*e^2 + 5*a^(3/2)*e^3)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[
c]*x^2])/a^(7/4))/(32*(c*d^2 + a*e^2)^2)
```

Maple [A] (verified)

Time = 0.37 (sec) , antiderivative size = 334, normalized size of antiderivative = 0.48

method	result
default	$\frac{c\left(\frac{-\frac{e\left(a\,e^{2}+c\,d^{2}\right)x^{3}}{4a}+\frac{d\left(a\,e^{2}+c\,d^{2}\right)x}{4a}}{c\,x^{4}+a}+\frac{\frac{\left(7d\,e^{2}a+3d^{3}c\right)\left(\frac{a}{c}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x^{2}+\left(\frac{a}{c}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{c}}}{x^{2}-\left(\frac{a}{c}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{c}}}\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{c}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{c}\right)^{\frac{1}{4}}}-1\right)\right)}{8a}}{4a}\right.}{\left(a\,e^{2}+c\,d^{2}\right)^{2}}$
risch	Expression too large to display

```
[In] int(1/(e*x^2+d)/(c*x^4+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] c/(a*e^2+c*d^2)^2*((-1/4*e*(a*e^2+c*d^2)/a*x^3+1/4*d*(a*e^2+c*d^2)/a*x)/(c*
x^4+a)+1/4/a*(1/8*(7*a*d*e^2+3*c*d^3)*(a/c)^(1/4)/a*2^(1/2)*(ln((x^2+(a/c)^
(1/4)*x*2^(1/2)+(a/c)^(1/2))/(x^2-(a/c)^(1/4)*x*2^(1/2)+(a/c)^(1/2)))+2*arc
tan(2^(1/2)/(a/c)^(1/4)*x+1)+2*arctan(2^(1/2)/(a/c)^(1/4)*x-1))+1/8*(-5*a*e
^3-c*d^2*e)/c/(a/c)^(1/4)*2^(1/2)*(ln((x^2-(a/c)^(1/4)*x*2^(1/2)+(a/c)^(1/2
))/(x^2+(a/c)^(1/4)*x*2^(1/2)+(a/c)^(1/2)))+2*arctan(2^(1/2)/(a/c)^(1/4)*x+
1)+2*arctan(2^(1/2)/(a/c)^(1/4)*x-1))))+e^4/(a*e^2+c*d^2)^2/(e*d)^(1/2)*arc
tan(e*x/(e*d)^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 4934 vs. $2(518) = 1036$.

Time = 11.96 (sec) , antiderivative size = 9892, normalized size of antiderivative = 14.36

$$\int \frac{1}{(d+ex^2)\left(a+cx^4\right)^2}\,dx = \text{Too large to display}$$

```
[In] integrate(1/(e*x^2+d)/(c*x^4+a)^2,x, algorithm="fricas")
```

```
[Out] Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(d+ex^2)(a+cx^4)^2}\,dx = \text{Timed out}$$

```
[In] integrate(1/(e*x**2+d)/(c*x**4+a)**2,x)
```

```
[Out] Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{(d+ex^2)(a+cx^4)^2}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(1/(e*x^2+d)/(c*x^4+a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 621, normalized size of antiderivative = 0.90

$$\int \frac{1}{(d+ex^2)(a+cx^4)^2}\,dx = \frac{e^4\arctan\left(\frac{ex}{\sqrt{de}}\right)}{(c^2d^4+2\,acd^2e^2+a^2e^4)\sqrt{de}} + \frac{\left(3\,(ac^3)^{\frac{1}{4}}\,c^3d^3+7\,(ac^3)^{\frac{1}{4}}\,ac^2de^2-(ac^3)^{\frac{3}{4}}\,cd^2e-5\,(ac^3)^{\frac{3}{4}}\,ae^3\right)\arctan\left(\frac{\sqrt{2}\left(2\,x+\sqrt{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\,\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{8\,\left(\sqrt{2}a^2c^4d^4+2\,\sqrt{2}a^3c^3d^2e^2+\sqrt{2}a^4c^2e^4\right)} + \frac{\left(3\,(ac^3)^{\frac{1}{4}}\,c^3d^3+7\,(ac^3)^{\frac{1}{4}}\,ac^2de^2-(ac^3)^{\frac{3}{4}}\,cd^2e-5\,(ac^3)^{\frac{3}{4}}\,ae^3\right)\arctan\left(\frac{\sqrt{2}\left(2\,x-\sqrt{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\,\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{8\,\left(\sqrt{2}a^2c^4d^4+2\,\sqrt{2}a^3c^3d^2e^2+\sqrt{2}a^4c^2e^4\right)} + \frac{\left(3\,(ac^3)^{\frac{1}{4}}\,c^3d^3+7\,(ac^3)^{\frac{1}{4}}\,ac^2de^2+(ac^3)^{\frac{3}{4}}\,cd^2e+5\,(ac^3)^{\frac{3}{4}}\,ae^3\right)\log\left(x^2+\sqrt{2}x\left(\frac{a}{c}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{c}}\right)}{16\,\left(\sqrt{2}a^2c^4d^4+2\,\sqrt{2}a^3c^3d^2e^2+\sqrt{2}a^4c^2e^4\right)} - \frac{\left(3\,(ac^3)^{\frac{1}{4}}\,c^3d^3+7\,(ac^3)^{\frac{1}{4}}\,ac^2de^2+(ac^3)^{\frac{3}{4}}\,cd^2e+5\,(ac^3)^{\frac{3}{4}}\,ae^3\right)\log\left(x^2-\sqrt{2}x\left(\frac{a}{c}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{c}}\right)}{16\,\left(\sqrt{2}a^2c^4d^4+2\,\sqrt{2}a^3c^3d^2e^2+\sqrt{2}a^4c^2e^4\right)} - \frac{cex^3-cdx}{4\,(cx^4+a)(acd^2+a^2e^2)}$$

```
[In] integrate(1/(e*x^2+d)/(c*x^4+a)^2,x, algorithm="giac")
```

```
[Out] e^4*arctan(e*x/sqrt(d*e))/((c^2*d^4 + 2*a*c*d^2*e^2 + a^2*e^4)*sqrt(d*e)) +
 1/8*(3*(a*c^3)^(1/4)*c^3*d^3 + 7*(a*c^3)^(1/4)*a*c^2*d*e^2 - (a*c^3)^(3/4)
*c*d^2*e - 5*(a*c^3)^(3/4)*a*e^3)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(a/c)^(
1/4))/(a/c)^(1/4))/(sqrt(2)*a^2*c^4*d^4 + 2*sqrt(2)*a^3*c^3*d^2*e^2 + sqrt(
2)*a^4*c^2*e^4) + 1/8*(3*(a*c^3)^(1/4)*c^3*d^3 + 7*(a*c^3)^(1/4)*a*c^2*d*e^
2 - (a*c^3)^(3/4)*c*d^2*e - 5*(a*c^3)^(3/4)*a*e^3)*arctan(1/2*sqrt(2)*(2*x
- sqrt(2)*(a/c)^(1/4))/(a/c)^(1/4))/(sqrt(2)*a^2*c^4*d^4 + 2*sqrt(2)*a^3*c^
3*d^2*e^2 + sqrt(2)*a^4*c^2*e^4) + 1/16*(3*(a*c^3)^(1/4)*c^3*d^3 + 7*(a*c^3
)^(1/4)*a*c^2*d*e^2 + (a*c^3)^(3/4)*c*d^2*e + 5*(a*c^3)^(3/4)*a*e^3)*log(x^
2 + sqrt(2)*x*(a/c)^(1/4) + sqrt(a/c))/(sqrt(2)*a^2*c^4*d^4 + 2*sqrt(2)*a^3
*c^3*d^2*e^2 + sqrt(2)*a^4*c^2*e^4) - 1/16*(3*(a*c^3)^(1/4)*c^3*d^3 + 7*(a*
c^3)^(1/4)*a*c^2*d*e^2 + (a*c^3)^(3/4)*c*d^2*e + 5*(a*c^3)^(3/4)*a*e^3)*log
(x^2 - sqrt(2)*x*(a/c)^(1/4) + sqrt(a/c))/(sqrt(2)*a^2*c^4*d^4 + 2*sqrt(2)*
a^3*c^3*d^2*e^2 + sqrt(2)*a^4*c^2*e^4) - 1/4*(c*e*x^3 - c*d*x)/((c*x^4 + a)
*(a*c*d^2 + a^2*e^2))
```

Mupad [B] (verification not implemented)

Time = 9.26 (sec) , antiderivative size = 17945, normalized size of antiderivative = 26.04

$$\int \frac{1}{(d+ex^2)(a+cx^4)^2} dx = \text{Too large to display}$$

[In] int(1/((a + c*x^4)^2*(d + e*x^2)),x)

[Out] ((c*d*x)/(4*a*(a*e^2 + c*d^2)) - (c*e*x^3)/(4*a*(a*e^2 + c*d^2)))/(a + c*x^4) - atan(((((((65536*a^11*c^4*e^16 - 12288*a^4*c^11*d^14*e^2 - 57344*a^5*c^10*d^12*e^4 - 36864*a^6*c^9*d^10*e^6 + 245760*a^7*c^8*d^8*e^8 + 634880*a^8*c^7*d^6*e^10 + 663552*a^9*c^6*d^4*e^12 + 331776*a^10*c^5*d^2*e^14)/(256*(a^8*e^8 + a^4*c^4*d^8 + 4*a^7*c*d^2*e^6 + 4*a^5*c^3*d^6*e^2 + 6*a^6*c^2*d^4*e^4)) - (x*((9*c^3*d^6*(-a^7*c)^(1/2) - 25*a^3*e^6*(-a^7*c)^(1/2) + 6*a^4*c^3*d^5*e + 44*a^5*c^2*d^3*e^3 + 70*a^6*c*d*e^5 + 41*a*c^2*d^4*e^2*(-a^7*c)^(1/2) + 39*a^2*c*d^2*e^4*(-a^7*c)^(1/2))/(256*(a^11*e^8 + a^7*c^4*d^8 + 4*a^10*c*d^2*e^6 + 4*a^8*c^3*d^6*e^2 + 6*a^9*c^2*d^4*e^4)))^(1/2)*(65536*a^13*c^4*e^17 - 65536*a^6*c^11*d^14*e^3 - 327680*a^7*c^10*d^12*e^5 - 589824*a^8*c^9*d^10*e^7 - 327680*a^9*c^8*d^8*e^9 + 327680*a^10*c^7*d^6*e^11 + 589824*a^11*c^6*d^4*e^13 + 327680*a^12*c^5*d^2*e^15))/(128*(a^8*e^8 + a^4*c^4*d^8 + 4*a^7*c*d^2*e^6 + 4*a^5*c^3*d^6*e^2 + 6*a^6*c^2*d^4*e^4)))*((9*c^3*d^6*(-a^7*c)^(1/2) - 25*a^3*e^6*(-a^7*c)^(1/2) + 6*a^4*c^3*d^5*e + 44*a^5*c^2*d^3*e^3 + 70*a^6*c*d*e^5 + 41*a*c^2*d^4*e^2*(-a^7*c)^(1/2) + 39*a^2*c*d^2*e^4*(-a^7*c)^(1/2))/(256*(a^11*e^8 + a^7*c^4*d^8 + 4*a^10*c*d^2*e^6 + 4*a^8*c^3*d^6*e^2 + 6*a^9*c^2*d^4*e^4)))^(1/2) - (x*(1152*a^2*c^11*d^13*e^2 - 49024*a^8*c^5*d*e^14 + 7936*a^3*c^10*d^11*e^4 + 20352*a^4*c^9*d^9*e^6 + 8704*a^5*c^8*d^7*e^8 - 66688*a^6*c^7*d^5*e^10 - 110848*a^7*c^6*d^3*e^12))/(128*(a^8*e^8 + a^4*c^4*d^8 + 4*a^7*c*d^2*e^6 + 4*a^5*c^3*d^6*e^2 + 6*a^6*c^2*d^4*e^4)))*((9*c^3*d^6*(-a^7*c)^(1/2) - 25*a^3*e^6*(-a^7*c)^(1/2) + 6*a^4*c^3*d^5*e + 44*a^5*c^2*d^3*e^3 + 70*a^6*c*d*e^5 + 41*a*c^2*d^4*e^2*(-a^7*c)^(1/2) + 39*a^2*c*d^2*e^4*(-a^7*c)^(1/2))/(256*(a^11*e^8 + a^7*c^4*d^8 + 4*a^10*c*d^2*e^6 + 4*a^8*c^3*d^6*e^2 + 6*a^9*c^2*d^4*e^4)))^(1/2) - (720*a*c^10*d^11*e^3 + 20432*a^6*c^5*d*e^13 + 4880*a^2*c^9*d^9*e^5 + 12320*a^3*c^8*d^7*e^7 + 21024*a^4*c^7*d^5*e^9 + 33296*a^5*c^6*d^3*e^11)/(256*(a^8*e^8 + a^4*c^4*d^8 + 4*a^7*c*d^2*e^6 + 4*a^5*c^3*d^6*e^2 + 6*a^6*c^2*d^4*e^4)))*((9*c^3*d^6*(-a^7*c)^(1/2) - 25*a^3*e^6*(-a^7*c)^(1/2) + 6*a^4*c^3*d^5*e + 44*a^5*c^2*d^3*e^3 + 70*a^6*c*d*e^5 + 41*a*c^2*d^4*e^2*(-a^7*c)^(1/2) + 39*a^2*c*d^2*e^4*(-a^7*c)^(1/2))/(256*(a^11*e^8 + a^7*c^4*d^8 + 4*a^10*c*d^2*e^6 + 4*a^8*c^3*d^6*e^2 + 6*a^9*c^2*d^4*e^4)))^(1/2) - (x*(1425*a^4*c^5*e^13 + 81*c^9*d^8*e^5 + 612*a*c^8*d^6*e^7 + 1894*a^2*c^7*d^4*e^9 + 2532*a^3*c^6*d^2*e^11))/(128*(a^8*e^8 + a^4*c^4*d^8 + 4*a^7*c*d^2*e^6 + 4*a^5*c^3*d^6*e^2 + 6*a^6*c^2*d^4*e^4)))*((9*c^3*d^6*(-a^7*c)^(1/2) - 25*a^3*e^6*(-a^7*c)^(1/2) + 6*a^4*c^3*d^5*e + 44*a^5*c^2*d^3*e^3 + 70*a^6*c*d*e^5 + 41*a*c^2*d^4*e^2*(-a^7*c)^(1/2) + 39*a^2*c*d^2*e^4*(-a^7*c)^(1/2))/(256*(a^11*e^8 + a^7*c^4*d^8 + 4

*a^10*c*d^2*e^6 + 4*a^8*c^3*d^6*e^2 + 6*a^9*c^2*d^4*e^4)))^(1/2)*1i - (((((
65536*a^11*c^4*e^16 - 12288*a^4*c^11*d^14*e^2 - 57344*a^5*c^10*d^12*e^4 - 3
6864*a^6*c^9*d^10*e^6 + 245760*a^7*c^8*d^8*e^8 + 634880*a^8*c^7*d^6*e^10 +
663552*a^9*c^6*d^4*e^12 + 331776*a^10*c^5*d^2*e^14)/(256*(a^8*e^8 + a^4*c^4
*d^8 + 4*a^7*c*d^2*e^6 + 4*a^5*c^3*d^6*e^2 + 6*a^6*c^2*d^4*e^4)) + (x*((9*c
^3*d^6*(-a^7*c)^(1/2) - 25*a^3*e^6*(-a^7*c)^(1/2) + 6*a^4*c^3*d^5*e + 44*a^
5*c^2*d^3*e^3 + 70*a^6*c*d*e^5 + 41*a*c^2*d^4*e^2*(-a^7*c)^(1/2) + 39*a^2*c
*d^2*e^4*(-a^7*c)^(1/2))/(256*(a^11*e^8 + a^7*c^4*d^8 + 4*a^10*c*d^2*e^6 +
4*a^8*c^3*d^6*e^2 + 6*a^9*c^2*d^4*e^4)))^(1/2)*(65536*a^13*c^4*e^17 - 65536
*a^6*c^11*d^14*e^3 - 327680*a^7*c^10*d^12*e^5 - 589824*a^8*c^9*d^10*e^7 - 3
27680*a^9*c^8*d^8*e^9 + 327680*a^10*c^7*d^6*e^11 + 589824*a^11*c^6*d^4*e^13
 + 327680*a^12*c^5*d^2*e^15))/(128*(a^8*e^8 + a^4*c^4*d^8 + 4*a^7*c*d^2*e^6
 + 4*a^5*c^3*d^6*e^2 + 6*a^6*c^2*d^4*e^4)))*((9*c^3*d^6*(-a^7*c)^(1/2) - 25
*a^3*e^6*(-a^7*c)^(1/2) + 6*a^4*c^3*d^5*e + 44*a^5*c^2*d^3*e^3 + 70*a^6*c*d
*e^5 + 41*a*c^2*d^4*e^2*(-a^7*c)^(1/2) + 39*a^2*c*d^2*e^4*(-a^7*c)^(1/2))/(
256*(a^11*e^8 + a^7*c^4*d^8 + 4*a^10*c*d^2*e^6 + 4*a^8*c^3*d^6*e^2 + 6*a^9*
c^2*d^4*e^4)))^(1/2) + (x*(1152*a^2*c^11*d^13*e^2 - 49024*a^8*c^5*d*e^14 +
7936*a^3*c^10*d^11*e^4 + 20352*a^4*c^9*d^9*e^6 + 8704*a^5*c^8*d^7*e^8 - 666
88*a^6*c^7*d^5*e^10 - 110848*a^7*c^6*d^3*e^12))/(128*(a^8*e^8 + a^4*c^4*d^8
 + 4*a^7*c*d^2*e^6 + 4*a^5*c^3*d^6*e^2 + 6*a^6*c^2*d^4*e^4)))*((9*c^3*d^6*(
-a^7*c)^(1/2) - 25*a^3*e^6*(-a^7*c)^(1/2) + 6*a^4*c^3*d^5*e + 44*a^5*c^2*d^
3*e^3 + 70*a^6*c*d*e^5 + 41*a*c^2*d^4*e^2*(-a^7*c)^(1/2) + 39*a^2*c*d^2*e^4
*(-a^7*c)^(1/2))/(256*(a^11*e^8 + a^7*c^4*d^8 + 4*a^10*c*d^2*e^6 + 4*a^8*c^
3*d^6*e^2 + 6*a^9*c^2*d^4*e^4)))^(1/2) - (720*a*c^10*d^11*e^3 + 20432*a^6*c
^5*d*e^13 + 4880*a^2*c^9*d^9*e^5 + 12320*a^3*c^8*d^7*e^7 + 21024*a^4*c^7*d^
5*e^9 + 33296*a^5*c^6*d^3*e^11)/(256*(a^8*e^8 + a^4*c^4*d^8 + 4*a^7*c*d^2*e
^6 + 4*a^5*c^3*d^6*e^2 + 6*a^6*c^2*d^4*e^4)))*((9*c^3*d^6*(-a^7*c)^(1/2) -
25*a^3*e^6*(-a^7*c)^(1/2) + 6*a^4*c^3*d^5*e + 44*a^5*c^2*d^3*e^3 + 70*a^6*c
*d*e^5 + 41*a*c^2*d^4*e^2*(-a^7*c)^(1/2) + 39*a^2*c*d^2*e^4*(-a^7*c)^(1/2))
/(256*(a^11*e^8 + a^7*c^4*d^8 + 4*a^10*c*d^2*e^6 + 4*a^8*c^3*d^6*e^2 + 6*a^
9*c^2*d^4*e^4)))^(1/2) + (x*(1425*a^4*c^5*e^13 + 81*c^9*d^8*e^5 + 612*a*c^8
*d^6*e^7 + 1894*a^2*c^7*d^4*e^9 + 2532*a^3*c^6*d^2*e^11))/(128*(a^8*e^8 + a
^4*c^4*d^8 + 4*a^7*c*d^2*e^6 + 4*a^5*c^3*d^6*e^2 + 6*a^6*c^2*d^4*e^4)))*((9
*c^3*d^6*(-a^7*c)^(1/2) - 25*a^3*e^6*(-a^7*c)^(1/2) + 6*a^4*c^3*d^5*e + 44*
a^5*c^2*d^3*e^3 + 70*a^6*c*d*e^5 + 41*a*c^2*d^4*e^2*(-a^7*c)^(1/2) + 39*a^2
*c*d^2*e^4*(-a^7*c)^(1/2))/(256*(a^11*e^8 + a^7*c^4*d^8 + 4*a^10*c*d^2*e^6
 + 4*a^8*c^3*d^6*e^2 + 6*a^9*c^2*d^4*e^4)))^(1/2)*1i)/((125*a^2*c^5*e^12 + 8
1*c^7*d^4*e^8 + 270*a*c^6*d^2*e^10)/(128*(a^8*e^8 + a^4*c^4*d^8 + 4*a^7*c*d
^2*e^6 + 4*a^5*c^3*d^6*e^2 + 6*a^6*c^2*d^4*e^4)) + (((((65536*a^11*c^4*e^16
 - 12288*a^4*c^11*d^14*e^2 - 57344*a^5*c^10*d^12*e^4 - 36864*a^6*c^9*d^10*e
^6 + 245760*a^7*c^8*d^8*e^8 + 634880*a^8*c^7*d^6*e^10 + 663552*a^9*c^6*d^4*
e^12 + 331776*a^10*c^5*d^2*e^14)/(256*(a^8*e^8 + a^4*c^4*d^8 + 4*a^7*c*d^2*
e^6 + 4*a^5*c^3*d^6*e^2 + 6*a^6*c^2*d^4*e^4)) - (x*((9*c^3*d^6*(-a^7*c)^(1/
2) - 25*a^3*e^6*(-a^7*c)^(1/2) + 6*a^4*c^3*d^5*e + 44*a^5*c^2*d^3*e^3 + 70*
a^6*c*d*e^5 + 41*a*c^2*d^4*e^2*(-a^7*c)^(1/2) + 39*a^2*c*d^2*e^4*(-a^7*c)^(

1/2))/(256*(a^11*e^8 + a^7*c^4*d^8 + 4*a^10*c*d^2*e^6 + 4*a^8*c^3*d^6*e^2 + 6*a^9*c^2*d^4*e^4)))^(1/2)*(65536*a^13*c^4*e^17 - 65536*a^6*c^11*d^14*e^3 - 327680*a^7*c^10*d^12*e^5 - 589824*a^8*c^9*d^10*e^7 - 327680*a^9*c^8*d^8*e^9 + 327680*a^10*c^7*d^6*e^11 + 589824*a^11*c^6*d^4*e^13 + 327680*a^12*c^5*d^2*e^15))/(128*(a^8*e^8 + a^4*c^4*d^8 + 4*a^7*c*d^2*e^6 + 4*a^5*c^3*d^6*e^2 + 6*a^6*c^2*d^4*e^4)))*((9*c^3*d^6*(-a^7*c)^(1/2) - 25*a^3*e^6*(-a^7*c)^(1/2) + 6*a^4*c^3*d^5*e + 44*a^5*c^2*d^3*e^3 + 70*a^6*c*d*e^5 + 41*a*c^2*d^4*e^2*(-a^7*c)^(1/2) + 39*a^2*c*d^2*e^4*(-a^7*c)^(1/2))/(256*(a^11*e^8 + a^7*c^4*d^8 + 4*a^10*c*d^2*e^6 + 4*a^8*c^3*d^6*e^2 + 6*a^9*c^2*d^4*e^4)))^(1/2) - (x*(1152*a^2*c^11*d^13*e^2 - 49024*a^8*c^5*d*e^14 + 7936*a^3*c^10*d^11*e^4 + 20352*a^4*c^9*d^9*e^6 + 8704*a^5*c^8*d^7*e^8 - 66688*a^6*c^7*d^5*e^10 - 110848*a^7*c^6*d^3*e^12))/(128*(a^8*e^8 + a^4*c^4*d^8 + 4*a^7*c*d^2*e^6 + 4*a^5*c^3*d^6*e^2 + 6*a^6*c^2*d^4*e^4)))*((9*c^3*d^6*(-a^7*c)^(1/2) - 25*a^3*e^6*(-a^7*c)^(1/2) + 6*a^4*c^3*d^5*e + 44*a^5*c^2*d^3*e^3 + 70*a^6*c*d*e^5 + 41*a*c^2*d^4*e^2*(-a^7*c)^(1/2) + 39*a^2*c*d^2*e^4*(-a^7*c)^(1/2))/(256*(a^11*e^8 + a^7*c^4*d^8 + 4*a^10*c*d^2*e^6 + 4*a^8*c^3*d^6*e^2 + 6*a^9*c^2*d^4*e^4)))^(1/2) - (720*a*c^10*d^11*e^3 + 20432*a^6*c^5*d*e^13 + 4880*a^2*c^9*d^9*e^5 + 12320*a^3*c^8*d^7*e^7 + 21024*a^4*c^7*d^5*e^9 + 33296*a^5*c^6*d^3*e^11)/(256*(a^8*e^8 + a^4*c^4*d^8 + 4*a^7*c*d^2*e^6 + 4*a^5*c^3*d^6*e^2 + 6*a^6*c^2*d^4*e^4)))*((9*c^3*d^6*(-a^7*c)^(1/2) - 25*a^3*e^6*(-a^7*c)^(1/2) + 6*a^4*c^3*d^5*e + 44*a^5*c^2*d^3*e^3 + 70*a^6*c*d*e^5 + 41*a*c^2*d^4*e^2*(-a^7*c)^(1/2) + 39*a^2*c*d^2*e^4*(-a^7*c)^(1/2))/(256*(a^11*e^8 + a^7*c^4*d^8 + 4*a^10*c*d^2*e^6 + 4*a^8*c^3*d^6*e^2 + 6*a^9*c^2*d^4*e^4)))^(1/2) - (x*(1425*a^4*c^5*e^13 + 81*c^9*d^8*e^5 + 612*a*c^8*d^6*e^7 + 1894*a^2*c^7*d^4*e^9 + 2532*a^3*c^6*d^2*e^11))/(128*(a^8*e^8 + a^4*c^4*d^8 + 4*a^7*c*d^2*e^6 + 4*a^5*c^3*d^6*e^2 + 6*a^6*c^2*d^4*e^4)))*((9*c^3*d^6*(-a^7*c)^(1/2) - 25*a^3*e^6*(-a^7*c)^(1/2) + 6*a^4*c^3*d^5*e + 44*a^5*c^2*d^3*e^3 + 70*a^6*c*d*e^5 + 41*a*c^2*d^4*e^2*(-a^7*c)^(1/2) + 39*a^2*c*d^2*e^4*(-a^7*c)^(1/2))/(256*(a^11*e^8 + a^7*c^4*d^8 + 4*a^10*c*d^2*e^6 + 4*a^8*c^3*d^6*e^2 + 6*a^9*c^2*d^4*e^4)))^(1/2) + (((((65536*a^11*c^4*e^16 - 12288*a^4*c^11*d^14*e^2 - 57344*a^5*c^10*d^12*e^4 - 36864*a^6*c^9*d^10*e^6 + 245760*a^7*c^8*d^8*e^8 + 634880*a^8*c^7*d^6*e^10 + 663552*a^9*c^6*d^4*e^12 + 331776*a^10*c^5*d^2*e^14)/(256*(a^8*e^8 + a^4*c^4*d^8 + 4*a^7*c*d^2*e^6 + 4*a^5*c^3*d^6*e^2 + 6*a^6*c^2*d^4*e^4)) + (x*((9*c^3*d^6*(-a^7*c)^(1/2) - 25*a^3*e^6*(-a^7*c)^(1/2) + 6*a^4*c^3*d^5*e + 44*a^5*c^2*d^3*e^3 + 70*a^6*c*d*e^5 + 41*a*c^2*d^4*e^2*(-a^7*c)^(1/2) + 39*a^2*c*d^2*e^4*(-a^7*c)^(1/2))/(256*(a^11*e^8 + a^7*c^4*d^8 + 4*a^10*c*d^2*e^6 + 4*a^8*c^3*d^6*e^2 + 6*a^9*c^2*d^4*e^4)))^(1/2)*(65536*a^13*c^4*e^17 - 65536*a^6*c^11*d^14*e^3 - 327680*a^7*c^10*d^12*e^5 - 589824*a^8*c^9*d^10*e^7 - 327680*a^9*c^8*d^8*e^9 + 327680*a^10*c^7*d^6*e^11 + 589824*a^11*c^6*d^4*e^13 + 327680*a^12*c^5*d^2*e^15))/(128*(a^8*e^8 + a^4*c^4*d^8 + 4*a^7*c*d^2*e^6 + 4*a^5*c^3*d^6*e^2 + 6*a^6*c^2*d^4*e^4)))*((9*c^3*d^6*(-a^7*c)^(1/2) - 25*a^3*e^6*(-a^7*c)^(1/2) + 6*a^4*c^3*d^5*e + 44*a^5*c^2*d^3*e^3 + 70*a^6*c*d*e^5 + 41*a*c^2*d^4*e^2*(-a^7*c)^(1/2) + 39*a^2*c*d^2*e^4*(-a^7*c)^(1/2))/(256*(a^11*e^8 + a^7*c^4*d^8 + 4*a^10*c*d^2*e^6 + 4*a^8*c^3*d^6*e^2 + 6*a^9*c^2*d^4*e^4)))^(1/2) + (x*(1152*a^2*c^

11*d^13*e^2 - 49024*a^8*c^5*d*e^14 + 7936*a^3*c^10*d^11*e^4 + 20352*a^4*c^9
*d^9*e^6 + 8704*a^5*c^8*d^7*e^8 - 66688*a^6*c^7*d^5*e^10 - 110848*a^7*c^6*d
^3*e^12))/(128*(a^8*e^8 + a^4*c^4*d^8 + 4*a^7*c*d^2*e^6 + 4*a^5*c^3*d^6*e^2
 + 6*a^6*c^2*d^4*e^4)))*((9*c^3*d^6*(-a^7*c)^(1/2) - 25*a^3*e^6*(-a^7*c)^(1
/2) + 6*a^4*c^3*d^5*e + 44*a^5*c^2*d^3*e^3 + 70*a^6*c*d*e^5 + 41*a*c^2*d^4*
e^2*(-a^7*c)^(1/2) + 39*a^2*c*d^2*e^4*(-a^7*c)^(1/2))/(256*(a^11*e^8 + a^7*
c^4*d^8 + 4*a^10*c*d^2*e^6 + 4*a^8*c^3*d^6*e^2 + 6*a^9*c^2*d^4*e^4)))^(1/2)
 - (720*a*c^10*d^11*e^3 + 20432*a^6*c^5*d*e^13 + 4880*a^2*c^9*d^9*e^5 + 123
20*a^3*c^8*d^7*e^7 + 21024*a^4*c^7*d^5*e^9 + 33296*a^5*c^6*d^3*e^11)/(256*(
a^8*e^8 + a^4*c^4*d^8 + 4*a^7*c*d^2*e^6 + 4*a^5*c^3*d^6*e^2 + 6*a^6*c^2*d^4
e^4)))((9*c^3*d^6*(-a^7*c)^(1/2) - 25*a^3*e^6*(-a^7*c)^(1/2) + 6*a^4*c^3*
d^5*e + 44*a^5*c^2*d^3*e^3 + 70*a^6*c*d*e^5 + 41*a*c^2*d^4*e^2*(-a^7*c)^(1/
2) + 39*a^2*c*d^2*e^4*(-a^7*c)^(1/2))/(256*(a^11*e^8 + a^7*c^4*d^8 + 4*a^10
*c*d^2*e^6 + 4*a^8*c^3*d^6*e^2 + 6*a^9*c^2*d^4*e^4)))^(1/2) + (x*(1425*a^4*
c^5*e^13 + 81*c^9*d^8*e^5 + 612*a*c^8*d^6*e^7 + 1894*a^2*c^7*d^4*e^9 + 2532
*a^3*c^6*d^2*e^11))/(128*(a^8*e^8 + a^4*c^4*d^8 + 4*a^7*c*d^2*e^6 + 4*a^5*c
^3*d^6*e^2 + 6*a^6*c^2*d^4*e^4)))*((9*c^3*d^6*(-a^7*c)^(1/2) - 25*a^3*e^6*(
-a^7*c)^(1/2) + 6*a^4*c^3*d^5*e + 44*a^5*c^2*d^3*e^3 + 70*a^6*c*d*e^5 + 41*
a*c^2*d^4*e^2*(-a^7*c)^(1/2) + 39*a^2*c*d^2*e^4*(-a^7*c)^(1/2))/(256*(a^11*
e^8 + a^7*c^4*d^8 + 4*a^10*c*d^2*e^6 + 4*a^8*c^3*d^6*e^2 + 6*a^9*c^2*d^4*e^
4)))^(1/2)))*((9*c^3*d^6*(-a^7*c)^(1/2) - 25*a^3*e^6*(-a^7*c)^(1/2) + 6*a^4
*c^3*d^5*e + 44*a^5*c^2*d^3*e^3 + 70*a^6*c*d*e^5 + 41*a*c^2*d^4*e^2*(-a^7*c
)^(1/2) + 39*a^2*c*d^2*e^4*(-a^7*c)^(1/2))/(256*(a^11*e^8 + a^7*c^4*d^8 + 4
*a^10*c*d^2*e^6 + 4*a^8*c^3*d^6*e^2 + 6*a^9*c^2*d^4*e^4)))^(1/2)*2i - atan(
((((((65536*a^11*c^4*e^16 - 12288*a^4*c^11*d^14*e^2 - 57344*a^5*c^10*d^12*e
^4 - 36864*a^6*c^9*d^10*e^6 + 245760*a^7*c^8*d^8*e^8 + 634880*a^8*c^7*d^6*e
^10 + 663552*a^9*c^6*d^4*e^12 + 331776*a^10*c^5*d^2*e^14)/(256*(a^8*e^8 + a
^4*c^4*d^8 + 4*a^7*c*d^2*e^6 + 4*a^5*c^3*d^6*e^2 + 6*a^6*c^2*d^4*e^4)) - (x
*((25*a^3*e^6*(-a^7*c)^(1/2) - 9*c^3*d^6*(-a^7*c)^(1/2) + 6*a^4*c^3*d^5*e +
 44*a^5*c^2*d^3*e^3 + 70*a^6*c*d*e^5 - 41*a*c^2*d^4*e^2*(-a^7*c)^(1/2) - 39
*a^2*c*d^2*e^4*(-a^7*c)^(1/2))/(256*(a^11*e^8 + a^7*c^4*d^8 + 4*a^10*c*d^2*
e^6 + 4*a^8*c^3*d^6*e^2 + 6*a^9*c^2*d^4*e^4)))^(1/2)*(65536*a^13*c^4*e^17 -
 65536*a^6*c^11*d^14*e^3 - 327680*a^7*c^10*d^12*e^5 - 589824*a^8*c^9*d^10*e
^7 - 327680*a^9*c^8*d^8*e^9 + 327680*a^10*c^7*d^6*e^11 + 589824*a^11*c^6*d^
4*e^13 + 327680*a^12*c^5*d^2*e^15))/(128*(a^8*e^8 + a^4*c^4*d^8 + 4*a^7*c*d
^2*e^6 + 4*a^5*c^3*d^6*e^2 + 6*a^6*c^2*d^4*e^4)))*((25*a^3*e^6*(-a^7*c)^(1/
2) - 9*c^3*d^6*(-a^7*c)^(1/2) + 6*a^4*c^3*d^5*e + 44*a^5*c^2*d^3*e^3 + 70*a
^6*c*d*e^5 - 41*a*c^2*d^4*e^2*(-a^7*c)^(1/2) - 39*a^2*c*d^2*e^4*(-a^7*c)^(1
/2))/(256*(a^11*e^8 + a^7*c^4*d^8 + 4*a^10*c*d^2*e^6 + 4*a^8*c^3*d^6*e^2 +
6*a^9*c^2*d^4*e^4)))^(1/2) - (x*(1152*a^2*c^11*d^13*e^2 - 49024*a^8*c^5*d*e
^14 + 7936*a^3*c^10*d^11*e^4 + 20352*a^4*c^9*d^9*e^6 + 8704*a^5*c^8*d^7*e^8
 - 66688*a^6*c^7*d^5*e^10 - 110848*a^7*c^6*d^3*e^12))/(128*(a^8*e^8 + a^4*c
^4*d^8 + 4*a^7*c*d^2*e^6 + 4*a^5*c^3*d^6*e^2 + 6*a^6*c^2*d^4*e^4)))*((25*a^
3*e^6*(-a^7*c)^(1/2) - 9*c^3*d^6*(-a^7*c)^(1/2) + 6*a^4*c^3*d^5*e + 44*a^5*
c^2*d^3*e^3 + 70*a^6*c*d*e^5 - 41*a*c^2*d^4*e^2*(-a^7*c)^(1/2) - 39*a^2*c*d

^2*e^4*(-a^7*c)^(1/2))/(256*(a^11*e^8 + a^7*c^4*d^8 + 4*a^10*c*d^2*e^6 + 4*
a^8*c^3*d^6*e^2 + 6*a^9*c^2*d^4*e^4)))^(1/2) - (720*a*c^10*d^11*e^3 + 20432
*a^6*c^5*d*e^13 + 4880*a^2*c^9*d^9*e^5 + 12320*a^3*c^8*d^7*e^7 + 21024*a^4*
c^7*d^5*e^9 + 33296*a^5*c^6*d^3*e^11)/(256*(a^8*e^8 + a^4*c^4*d^8 + 4*a^7*c
*d^2*e^6 + 4*a^5*c^3*d^6*e^2 + 6*a^6*c^2*d^4*e^4)))*((25*a^3*e^6*(-a^7*c)^(
1/2) - 9*c^3*d^6*(-a^7*c)^(1/2) + 6*a^4*c^3*d^5*e + 44*a^5*c^2*d^3*e^3 + 70
*a^6*c*d*e^5 - 41*a*c^2*d^4*e^2*(-a^7*c)^(1/2) - 39*a^2*c*d^2*e^4*(-a^7*c)^
(1/2))/(256*(a^11*e^8 + a^7*c^4*d^8 + 4*a^10*c*d^2*e^6 + 4*a^8*c^3*d^6*e^2
+ 6*a^9*c^2*d^4*e^4)))^(1/2) - (x*(1425*a^4*c^5*e^13 + 81*c^9*d^8*e^5 + 612
*a*c^8*d^6*e^7 + 1894*a^2*c^7*d^4*e^9 + 2532*a^3*c^6*d^2*e^11))/(128*(a^8*e
^8 + a^4*c^4*d^8 + 4*a^7*c*d^2*e^6 + 4*a^5*c^3*d^6*e^2 + 6*a^6*c^2*d^4*e^4)
))*((25*a^3*e^6*(-a^7*c)^(1/2) - 9*c^3*d^6*(-a^7*c)^(1/2) + 6*a^4*c^3*d^5*e
 + 44*a^5*c^2*d^3*e^3 + 70*a^6*c*d*e^5 - 41*a*c^2*d^4*e^2*(-a^7*c)^(1/2) -
39*a^2*c*d^2*e^4*(-a^7*c)^(1/2))/(256*(a^11*e^8 + a^7*c^4*d^8 + 4*a^10*c*d^
2*e^6 + 4*a^8*c^3*d^6*e^2 + 6*a^9*c^2*d^4*e^4)))^(1/2)*1i - (((((65536*a^11
*c^4*e^16 - 12288*a^4*c^11*d^14*e^2 - 57344*a^5*c^10*d^12*e^4 - 36864*a^6*c
^9*d^10*e^6 + 245760*a^7*c^8*d^8*e^8 + 634880*a^8*c^7*d^6*e^10 + 663552*a^9
*c^6*d^4*e^12 + 331776*a^10*c^5*d^2*e^14)/(256*(a^8*e^8 + a^4*c^4*d^8 + 4*a
^7*c*d^2*e^6 + 4*a^5*c^3*d^6*e^2 + 6*a^6*c^2*d^4*e^4)) + (x*((25*a^3*e^6*(-
a^7*c)^(1/2) - 9*c^3*d^6*(-a^7*c)^(1/2) + 6*a^4*c^3*d^5*e + 44*a^5*c^2*d^3*
e^3 + 70*a^6*c*d*e^5 - 41*a*c^2*d^4*e^2*(-a^7*c)^(1/2) - 39*a^2*c*d^2*e^4*(
-a^7*c)^(1/2))/(256*(a^11*e^8 + a^7*c^4*d^8 + 4*a^10*c*d^2*e^6 + 4*a^8*c^3*
d^6*e^2 + 6*a^9*c^2*d^4*e^4)))^(1/2)*(65536*a^13*c^4*e^17 - 65536*a^6*c^11*
d^14*e^3 - 327680*a^7*c^10*d^12*e^5 - 589824*a^8*c^9*d^10*e^7 - 327680*a^9*
c^8*d^8*e^9 + 327680*a^10*c^7*d^6*e^11 + 589824*a^11*c^6*d^4*e^13 + 327680*
a^12*c^5*d^2*e^15))/(128*(a^8*e^8 + a^4*c^4*d^8 + 4*a^7*c*d^2*e^6 + 4*a^5*c
^3*d^6*e^2 + 6*a^6*c^2*d^4*e^4)))*((25*a^3*e^6*(-a^7*c)^(1/2) - 9*c^3*d^6*(
-a^7*c)^(1/2) + 6*a^4*c^3*d^5*e + 44*a^5*c^2*d^3*e^3 + 70*a^6*c*d*e^5 - 41*
a*c^2*d^4*e^2*(-a^7*c)^(1/2) - 39*a^2*c*d^2*e^4*(-a^7*c)^(1/2))/(256*(a^11*
e^8 + a^7*c^4*d^8 + 4*a^10*c*d^2*e^6 + 4*a^8*c^3*d^6*e^2 + 6*a^9*c^2*d^4*e^
4)))^(1/2) + (x*(1152*a^2*c^11*d^13*e^2 - 49024*a^8*c^5*d*e^14 + 7936*a^3*c
^10*d^11*e^4 + 20352*a^4*c^9*d^9*e^6 + 8704*a^5*c^8*d^7*e^8 - 66688*a^6*c^7
*d^5*e^10 - 110848*a^7*c^6*d^3*e^12))/(128*(a^8*e^8 + a^4*c^4*d^8 + 4*a^7*c
*d^2*e^6 + 4*a^5*c^3*d^6*e^2 + 6*a^6*c^2*d^4*e^4)))*((25*a^3*e^6*(-a^7*c)^(
1/2) - 9*c^3*d^6*(-a^7*c)^(1/2) + 6*a^4*c^3*d^5*e + 44*a^5*c^2*d^3*e^3 + 70
*a^6*c*d*e^5 - 41*a*c^2*d^4*e^2*(-a^7*c)^(1/2) - 39*a^2*c*d^2*e^4*(-a^7*c)^
(1/2))/(256*(a^11*e^8 + a^7*c^4*d^8 + 4*a^10*c*d^2*e^6 + 4*a^8*c^3*d^6*e^2
+ 6*a^9*c^2*d^4*e^4)))^(1/2) - (720*a*c^10*d^11*e^3 + 20432*a^6*c^5*d*e^13
+ 4880*a^2*c^9*d^9*e^5 + 12320*a^3*c^8*d^7*e^7 + 21024*a^4*c^7*d^5*e^9 + 33
296*a^5*c^6*d^3*e^11)/(256*(a^8*e^8 + a^4*c^4*d^8 + 4*a^7*c*d^2*e^6 + 4*a^5
*c^3*d^6*e^2 + 6*a^6*c^2*d^4*e^4)))*((25*a^3*e^6*(-a^7*c)^(1/2) - 9*c^3*d^6
*(-a^7*c)^(1/2) + 6*a^4*c^3*d^5*e + 44*a^5*c^2*d^3*e^3 + 70*a^6*c*d*e^5 - 4
1*a*c^2*d^4*e^2*(-a^7*c)^(1/2) - 39*a^2*c*d^2*e^4*(-a^7*c)^(1/2))/(256*(a^1
1*e^8 + a^7*c^4*d^8 + 4*a^10*c*d^2*e^6 + 4*a^8*c^3*d^6*e^2 + 6*a^9*c^2*d^4*
e^4)))^(1/2) + (x*(1425*a^4*c^5*e^13 + 81*c^9*d^8*e^5 + 612*a*c^8*d^6*e^7 +

1894*a^2*c^7*d^4*e^9 + 2532*a^3*c^6*d^2*e^11))/(128*(a^8*e^8 + a^4*c^4*d^8 + 4*a^7*c*d^2*e^6 + 4*a^5*c^3*d^6*e^2 + 6*a^6*c^2*d^4*e^4)))*((25*a^3*e^6*(-a^7*c)^(1/2) - 9*c^3*d^6*(-a^7*c)^(1/2) + 6*a^4*c^3*d^5*e + 44*a^5*c^2*d^3*e^3 + 70*a^6*c*d*e^5 - 41*a*c^2*d^4*e^2*(-a^7*c)^(1/2) - 39*a^2*c*d^2*e^4*(-a^7*c)^(1/2))/(256*(a^11*e^8 + a^7*c^4*d^8 + 4*a^10*c*d^2*e^6 + 4*a^8*c^3*d^6*e^2 + 6*a^9*c^2*d^4*e^4)))^(1/2)*1i)/((125*a^2*c^5*e^12 + 81*c^7*d^4*e^8 + 270*a*c^6*d^2*e^10)/(128*(a^8*e^8 + a^4*c^4*d^8 + 4*a^7*c*d^2*e^6 + 4*a^5*c^3*d^6*e^2 + 6*a^6*c^2*d^4*e^4)) + (((((65536*a^11*c^4*e^16 - 12288*a^4*c^11*d^14*e^2 - 57344*a^5*c^10*d^12*e^4 - 36864*a^6*c^9*d^10*e^6 + 245760*a^7*c^8*d^8*e^8 + 634880*a^8*c^7*d^6*e^10 + 663552*a^9*c^6*d^4*e^12 + 331776*a^10*c^5*d^2*e^14)/(256*(a^8*e^8 + a^4*c^4*d^8 + 4*a^7*c*d^2*e^6 + 4*a^5*c^3*d^6*e^2 + 6*a^6*c^2*d^4*e^4)) - (x*((25*a^3*e^6*(-a^7*c)^(1/2) - 9*c^3*d^6*(-a^7*c)^(1/2) + 6*a^4*c^3*d^5*e + 44*a^5*c^2*d^3*e^3 + 70*a^6*c*d*e^5 - 41*a*c^2*d^4*e^2*(-a^7*c)^(1/2) - 39*a^2*c*d^2*e^4*(-a^7*c)^(1/2))/(256*(a^11*e^8 + a^7*c^4*d^8 + 4*a^10*c*d^2*e^6 + 4*a^8*c^3*d^6*e^2 + 6*a^9*c^2*d^4*e^4)))^(1/2)*(65536*a^13*c^4*e^17 - 65536*a^6*c^11*d^14*e^3 - 327680*a^7*c^10*d^12*e^5 - 589824*a^8*c^9*d^10*e^7 - 327680*a^9*c^8*d^8*e^9 + 327680*a^10*c^7*d^6*e^11 + 589824*a^11*c^6*d^4*e^13 + 327680*a^12*c^5*d^2*e^15))/(128*(a^8*e^8 + a^4*c^4*d^8 + 4*a^7*c*d^2*e^6 + 4*a^5*c^3*d^6*e^2 + 6*a^6*c^2*d^4*e^4)))*((25*a^3*e^6*(-a^7*c)^(1/2) - 9*c^3*d^6*(-a^7*c)^(1/2) + 6*a^4*c^3*d^5*e + 44*a^5*c^2*d^3*e^3 + 70*a^6*c*d*e^5 - 41*a*c^2*d^4*e^2*(-a^7*c)^(1/2) - 39*a^2*c*d^2*e^4*(-a^7*c)^(1/2))/(256*(a^11*e^8 + a^7*c^4*d^8 + 4*a^10*c*d^2*e^6 + 4*a^8*c^3*d^6*e^2 + 6*a^9*c^2*d^4*e^4)))^(1/2) - (x*(1152*a^2*c^11*d^13*e^2 - 49024*a^8*c^5*d*e^14 + 7936*a^3*c^10*d^11*e^4 + 20352*a^4*c^9*d^9*e^6 + 8704*a^5*c^8*d^7*e^8 - 66688*a^6*c^7*d^5*e^10 - 110848*a^7*c^6*d^3*e^12))/(128*(a^8*e^8 + a^4*c^4*d^8 + 4*a^7*c*d^2*e^6 + 4*a^5*c^3*d^6*e^2 + 6*a^6*c^2*d^4*e^4)))*((25*a^3*e^6*(-a^7*c)^(1/2) - 9*c^3*d^6*(-a^7*c)^(1/2) + 6*a^4*c^3*d^5*e + 44*a^5*c^2*d^3*e^3 + 70*a^6*c*d*e^5 - 41*a*c^2*d^4*e^2*(-a^7*c)^(1/2) - 39*a^2*c*d^2*e^4*(-a^7*c)^(1/2))/(256*(a^11*e^8 + a^7*c^4*d^8 + 4*a^10*c*d^2*e^6 + 4*a^8*c^3*d^6*e^2 + 6*a^9*c^2*d^4*e^4)))^(1/2) - (720*a*c^10*d^11*e^3 + 20432*a^6*c^5*d*e^13 + 4880*a^2*c^9*d^9*e^5 + 12320*a^3*c^8*d^7*e^7 + 21024*a^4*c^7*d^5*e^9 + 33296*a^5*c^6*d^3*e^11)/(256*(a^8*e^8 + a^4*c^4*d^8 + 4*a^7*c*d^2*e^6 + 4*a^5*c^3*d^6*e^2 + 6*a^6*c^2*d^4*e^4)))*((25*a^3*e^6*(-a^7*c)^(1/2) - 9*c^3*d^6*(-a^7*c)^(1/2) + 6*a^4*c^3*d^5*e + 44*a^5*c^2*d^3*e^3 + 70*a^6*c*d*e^5 - 41*a*c^2*d^4*e^2*(-a^7*c)^(1/2) - 39*a^2*c*d^2*e^4*(-a^7*c)^(1/2))/(256*(a^11*e^8 + a^7*c^4*d^8 + 4*a^10*c*d^2*e^6 + 4*a^8*c^3*d^6*e^2 + 6*a^9*c^2*d^4*e^4)))^(1/2) - (x*(1425*a^4*c^5*e^13 + 81*c^9*d^8*e^5 + 612*a*c^8*d^6*e^7 + 1894*a^2*c^7*d^4*e^9 + 2532*a^3*c^6*d^2*e^11))/(128*(a^8*e^8 + a^4*c^4*d^8 + 4*a^7*c*d^2*e^6 + 4*a^5*c^3*d^6*e^2 + 6*a^6*c^2*d^4*e^4)))*((25*a^3*e^6*(-a^7*c)^(1/2) - 9*c^3*d^6*(-a^7*c)^(1/2) + 6*a^4*c^3*d^5*e + 44*a^5*c^2*d^3*e^3 + 70*a^6*c*d*e^5 - 41*a*c^2*d^4*e^2*(-a^7*c)^(1/2) - 39*a^2*c*d^2*e^4*(-a^7*c)^(1/2))/(256*(a^11*e^8 + a^7*c^4*d^8 + 4*a^10*c*d^2*e^6 + 4*a^8*c^3*d^6*e^2 + 6*a^9*c^2*d^4*e^4)))^(1/2) + (((((65536*a^11*c^4*e^16 - 12288*a^4*c^11*d^14*e^2 - 57344*a^5*c^10*d^12*e^4 - 36864*a^6*c^9*d^10*e^6 + 245760*a^7*c^8*d^8*e^8 +

634880*a^8*c^7*d^6*e^10 + 663552*a^9*c^6*d^4*e^12 + 331776*a^10*c^5*d^2*e^14)/(256*(a^8*e^8 + a^4*c^4*d^8 + 4*a^7*c*d^2*e^6 + 4*a^5*c^3*d^6*e^2 + 6*a^6*c^2*d^4*e^4)) + (x*((25*a^3*e^6*(-a^7*c)^(1/2) - 9*c^3*d^6*(-a^7*c)^(1/2) + 6*a^4*c^3*d^5*e + 44*a^5*c^2*d^3*e^3 + 70*a^6*c*d*e^5 - 41*a*c^2*d^4*e^2*(-a^7*c)^(1/2) - 39*a^2*c*d^2*e^4*(-a^7*c)^(1/2))/(256*(a^11*e^8 + a^7*c^4*d^8 + 4*a^10*c*d^2*e^6 + 4*a^8*c^3*d^6*e^2 + 6*a^9*c^2*d^4*e^4)))^(1/2)*(65536*a^13*c^4*e^17 - 65536*a^6*c^11*d^14*e^3 - 327680*a^7*c^10*d^12*e^5 - 589824*a^8*c^9*d^10*e^7 - 327680*a^9*c^8*d^8*e^9 + 327680*a^10*c^7*d^6*e^11 + 589824*a^11*c^6*d^4*e^13 + 327680*a^12*c^5*d^2*e^15))/(128*(a^8*e^8 + a^4*c^4*d^8 + 4*a^7*c*d^2*e^6 + 4*a^5*c^3*d^6*e^2 + 6*a^6*c^2*d^4*e^4)))*((25*a^3*e^6*(-a^7*c)^(1/2) - 9*c^3*d^6*(-a^7*c)^(1/2) + 6*a^4*c^3*d^5*e + 44*a^5*c^2*d^3*e^3 + 70*a^6*c*d*e^5 - 41*a*c^2*d^4*e^2*(-a^7*c)^(1/2) - 39*a^2*c*d^2*e^4*(-a^7*c)^(1/2))/(256*(a^11*e^8 + a^7*c^4*d^8 + 4*a^10*c*d^2*e^6 + 4*a^8*c^3*d^6*e^2 + 6*a^9*c^2*d^4*e^4)))^(1/2) + (x*(1152*a^2*c^11*d^13*e^2 - 49024*a^8*c^5*d*e^14 + 7936*a^3*c^10*d^11*e^4 + 20352*a^4*c^9*d^9*e^6 + 8704*a^5*c^8*d^7*e^8 - 66688*a^6*c^7*d^5*e^10 - 110848*a^7*c^6*d^3*e^12))/(128*(a^8*e^8 + a^4*c^4*d^8 + 4*a^7*c*d^2*e^6 + 4*a^5*c^3*d^6*e^2 + 6*a^6*c^2*d^4*e^4)))*((25*a^3*e^6*(-a^7*c)^(1/2) - 9*c^3*d^6*(-a^7*c)^(1/2) + 6*a^4*c^3*d^5*e + 44*a^5*c^2*d^3*e^3 + 70*a^6*c*d*e^5 - 41*a*c^2*d^4*e^2*(-a^7*c)^(1/2) - 39*a^2*c*d^2*e^4*(-a^7*c)^(1/2))/(256*(a^11*e^8 + a^7*c^4*d^8 + 4*a^10*c*d^2*e^6 + 4*a^8*c^3*d^6*e^2 + 6*a^9*c^2*d^4*e^4)))^(1/2) - (720*a*c^10*d^11*e^3 + 20432*a^6*c^5*d*e^13 + 4880*a^2*c^9*d^9*e^5 + 12320*a^3*c^8*d^7*e^7 + 21024*a^4*c^7*d^5*e^9 + 33296*a^5*c^6*d^3*e^11)/(256*(a^8*e^8 + a^4*c^4*d^8 + 4*a^7*c*d^2*e^6 + 4*a^5*c^3*d^6*e^2 + 6*a^6*c^2*d^4*e^4)))*((25*a^3*e^6*(-a^7*c)^(1/2) - 9*c^3*d^6*(-a^7*c)^(1/2) + 6*a^4*c^3*d^5*e + 44*a^5*c^2*d^3*e^3 + 70*a^6*c*d*e^5 - 41*a*c^2*d^4*e^2*(-a^7*c)^(1/2) - 39*a^2*c*d^2*e^4*(-a^7*c)^(1/2))/(256*(a^11*e^8 + a^7*c^4*d^8 + 4*a^10*c*d^2*e^6 + 4*a^8*c^3*d^6*e^2 + 6*a^9*c^2*d^4*e^4)))^(1/2) + (x*(1425*a^4*c^5*e^13 + 81*c^9*d^8*e^5 + 612*a*c^8*d^6*e^7 + 1894*a^2*c^7*d^4*e^9 + 2532*a^3*c^6*d^2*e^11))/(128*(a^8*e^8 + a^4*c^4*d^8 + 4*a^7*c*d^2*e^6 + 4*a^5*c^3*d^6*e^2 + 6*a^6*c^2*d^4*e^4)))*((25*a^3*e^6*(-a^7*c)^(1/2) - 9*c^3*d^6*(-a^7*c)^(1/2) + 6*a^4*c^3*d^5*e + 44*a^5*c^2*d^3*e^3 + 70*a^6*c*d*e^5 - 41*a*c^2*d^4*e^2*(-a^7*c)^(1/2) - 39*a^2*c*d^2*e^4*(-a^7*c)^(1/2))/(256*(a^11*e^8 + a^7*c^4*d^8 + 4*a^10*c*d^2*e^6 + 4*a^8*c^3*d^6*e^2 + 6*a^9*c^2*d^4*e^4)))^(1/2)))*((25*a^3*e^6*(-a^7*c)^(1/2) - 9*c^3*d^6*(-a^7*c)^(1/2) + 6*a^4*c^3*d^5*e + 44*a^5*c^2*d^3*e^3 + 70*a^6*c*d*e^5 - 41*a*c^2*d^4*e^2*(-a^7*c)^(1/2) - 39*a^2*c*d^2*e^4*(-a^7*c)^(1/2))/(256*(a^11*e^8 + a^7*c^4*d^8 + 4*a^10*c*d^2*e^6 + 4*a^8*c^3*d^6*e^2 + 6*a^9*c^2*d^4*e^4)))^(1/2)*2i + (atan(-(((((((45*a*c^10*d^11*e^3)/16 + (1277*a^6*c^5*d*e^13)/16 + (305*a^2*c^9*d^9*e^5)/16 + (385*a^3*c^8*d^7*e^7)/8 + (657*a^4*c^7*d^5*e^9)/8 + (2081*a^5*c^6*d^3*e^11)/16)/(2*(a^8*e^8 + a^4*c^4*d^8 + 4*a^7*c*d^2*e^6 + 4*a^5*c^3*d^6*e^2 + 6*a^6*c^2*d^4*e^4)) - (((((256*a^11*c^4*e^16 - 48*a^4*c^11*d^14*e^2 - 224*a^5*c^10*d^12*e^4 - 144*a^6*c^9*d^10*e^6 + 960*a^7*c^8*d^8*e^8 + 2480*a^8*c^7*d^6*e^10 + 2592*a^9*c^6*d^4*e^12 + 1296*a^10*c^5*d^2*e^14)/(2*(a^8*e^8 + a^4*c^4*d^8 + 4*a^7*c*d^2*e^6 + 4*a^5*c^3*d^6*e^2 + 6*a^6*c^2*d^4*e^4)) - (x

*(-d*e^7)^(1/2)*(65536*a^13*c^4*e^17 - 65536*a^6*c^11*d^14*e^3 - 327680*a^7
*c^10*d^12*e^5 - 589824*a^8*c^9*d^10*e^7 - 327680*a^9*c^8*d^8*e^9 + 327680*
a^10*c^7*d^6*e^11 + 589824*a^11*c^6*d^4*e^13 + 327680*a^12*c^5*d^2*e^15))/(
512*(c^2*d^5 + a^2*d*e^4 + 2*a*c*d^3*e^2)*(a^8*e^8 + a^4*c^4*d^8 + 4*a^7*c*
d^2*e^6 + 4*a^5*c^3*d^6*e^2 + 6*a^6*c^2*d^4*e^4)))*(-d*e^7)^(1/2))/(2*(c^2*
d^5 + a^2*d*e^4 + 2*a*c*d^3*e^2)) - (x*(1152*a^2*c^11*d^13*e^2 - 49024*a^8*
c^5*d*e^14 + 7936*a^3*c^10*d^11*e^4 + 20352*a^4*c^9*d^9*e^6 + 8704*a^5*c^8*
d^7*e^8 - 66688*a^6*c^7*d^5*e^10 - 110848*a^7*c^6*d^3*e^12))/(256*(a^8*e^8
+ a^4*c^4*d^8 + 4*a^7*c*d^2*e^6 + 4*a^5*c^3*d^6*e^2 + 6*a^6*c^2*d^4*e^4)))*
(-d*e^7)^(1/2))/(2*(c^2*d^5 + a^2*d*e^4 + 2*a*c*d^3*e^2)))*(-d*e^7)^(1/2))/
(2*(c^2*d^5 + a^2*d*e^4 + 2*a*c*d^3*e^2)) + (x*(1425*a^4*c^5*e^13 + 81*c^9*
d^8*e^5 + 612*a*c^8*d^6*e^7 + 1894*a^2*c^7*d^4*e^9 + 2532*a^3*c^6*d^2*e^11)
)/(256*(a^8*e^8 + a^4*c^4*d^8 + 4*a^7*c*d^2*e^6 + 4*a^5*c^3*d^6*e^2 + 6*a^6
*c^2*d^4*e^4)))*(-d*e^7)^(1/2)*1i)/(c^2*d^5 + a^2*d*e^4 + 2*a*c*d^3*e^2) -
((((((45*a*c^10*d^11*e^3)/16 + (1277*a^6*c^5*d*e^13)/16 + (305*a^2*c^9*d^9*
e^5)/16 + (385*a^3*c^8*d^7*e^7)/8 + (657*a^4*c^7*d^5*e^9)/8 + (2081*a^5*c^6
*d^3*e^11)/16)/(2*(a^8*e^8 + a^4*c^4*d^8 + 4*a^7*c*d^2*e^6 + 4*a^5*c^3*d^6*
e^2 + 6*a^6*c^2*d^4*e^4)) - (((((256*a^11*c^4*e^16 - 48*a^4*c^11*d^14*e^2 -
224*a^5*c^10*d^12*e^4 - 144*a^6*c^9*d^10*e^6 + 960*a^7*c^8*d^8*e^8 + 2480*
a^8*c^7*d^6*e^10 + 2592*a^9*c^6*d^4*e^12 + 1296*a^10*c^5*d^2*e^14)/(2*(a^8*
e^8 + a^4*c^4*d^8 + 4*a^7*c*d^2*e^6 + 4*a^5*c^3*d^6*e^2 + 6*a^6*c^2*d^4*e^4
)) + (x*(-d*e^7)^(1/2)*(65536*a^13*c^4*e^17 - 65536*a^6*c^11*d^14*e^3 - 327
680*a^7*c^10*d^12*e^5 - 589824*a^8*c^9*d^10*e^7 - 327680*a^9*c^8*d^8*e^9 +
327680*a^10*c^7*d^6*e^11 + 589824*a^11*c^6*d^4*e^13 + 327680*a^12*c^5*d^2*e
^15))/(512*(c^2*d^5 + a^2*d*e^4 + 2*a*c*d^3*e^2)*(a^8*e^8 + a^4*c^4*d^8 + 4
*a^7*c*d^2*e^6 + 4*a^5*c^3*d^6*e^2 + 6*a^6*c^2*d^4*e^4)))*(-d*e^7)^(1/2))/(
2*(c^2*d^5 + a^2*d*e^4 + 2*a*c*d^3*e^2)) + (x*(1152*a^2*c^11*d^13*e^2 - 490
24*a^8*c^5*d*e^14 + 7936*a^3*c^10*d^11*e^4 + 20352*a^4*c^9*d^9*e^6 + 8704*a
^5*c^8*d^7*e^8 - 66688*a^6*c^7*d^5*e^10 - 110848*a^7*c^6*d^3*e^12))/(256*(a
^8*e^8 + a^4*c^4*d^8 + 4*a^7*c*d^2*e^6 + 4*a^5*c^3*d^6*e^2 + 6*a^6*c^2*d^4*
e^4)))*(-d*e^7)^(1/2))/(2*(c^2*d^5 + a^2*d*e^4 + 2*a*c*d^3*e^2)))*(-d*e^7)^
(1/2))/(2*(c^2*d^5 + a^2*d*e^4 + 2*a*c*d^3*e^2)) - (x*(1425*a^4*c^5*e^13 +
81*c^9*d^8*e^5 + 612*a*c^8*d^6*e^7 + 1894*a^2*c^7*d^4*e^9 + 2532*a^3*c^6*d^
2*e^11))/(256*(a^8*e^8 + a^4*c^4*d^8 + 4*a^7*c*d^2*e^6 + 4*a^5*c^3*d^6*e^2
+ 6*a^6*c^2*d^4*e^4)))*(-d*e^7)^(1/2)*1i)/(c^2*d^5 + a^2*d*e^4 + 2*a*c*d^3*
e^2))/(((((((45*a*c^10*d^11*e^3)/16 + (1277*a^6*c^5*d*e^13)/16 + (305*a^2*c
^9*d^9*e^5)/16 + (385*a^3*c^8*d^7*e^7)/8 + (657*a^4*c^7*d^5*e^9)/8 + (2081*
a^5*c^6*d^3*e^11)/16)/(2*(a^8*e^8 + a^4*c^4*d^8 + 4*a^7*c*d^2*e^6 + 4*a^5*c
^3*d^6*e^2 + 6*a^6*c^2*d^4*e^4)) - (((((256*a^11*c^4*e^16 - 48*a^4*c^11*d^1
4*e^2 - 224*a^5*c^10*d^12*e^4 - 144*a^6*c^9*d^10*e^6 + 960*a^7*c^8*d^8*e^8
+ 2480*a^8*c^7*d^6*e^10 + 2592*a^9*c^6*d^4*e^12 + 1296*a^10*c^5*d^2*e^14)/(
2*(a^8*e^8 + a^4*c^4*d^8 + 4*a^7*c*d^2*e^6 + 4*a^5*c^3*d^6*e^2 + 6*a^6*c^2*
d^4*e^4)) - (x*(-d*e^7)^(1/2)*(65536*a^13*c^4*e^17 - 65536*a^6*c^11*d^14*e^
3 - 327680*a^7*c^10*d^12*e^5 - 589824*a^8*c^9*d^10*e^7 - 327680*a^9*c^8*d^8
*e^9 + 327680*a^10*c^7*d^6*e^11 + 589824*a^11*c^6*d^4*e^13 + 327680*a^12*c^

5*d^2*e^15))/(512*(c^2*d^5 + a^2*d*e^4 + 2*a*c*d^3*e^2)*(a^8*e^8 + a^4*c^4*d^8 + 4*a^7*c*d^2*e^6 + 4*a^5*c^3*d^6*e^2 + 6*a^6*c^2*d^4*e^4)))*(-d*e^7)^(1/2))/(2*(c^2*d^5 + a^2*d*e^4 + 2*a*c*d^3*e^2)) - (x*(1152*a^2*c^11*d^13*e^2 - 49024*a^8*c^5*d*e^14 + 7936*a^3*c^10*d^11*e^4 + 20352*a^4*c^9*d^9*e^6 + 8704*a^5*c^8*d^7*e^8 - 66688*a^6*c^7*d^5*e^10 - 110848*a^7*c^6*d^3*e^12))/(256*(a^8*e^8 + a^4*c^4*d^8 + 4*a^7*c*d^2*e^6 + 4*a^5*c^3*d^6*e^2 + 6*a^6*c^2*d^4*e^4)))*(-d*e^7)^(1/2))/(2*(c^2*d^5 + a^2*d*e^4 + 2*a*c*d^3*e^2)))*(-d*e^7)^(1/2))/(2*(c^2*d^5 + a^2*d*e^4 + 2*a*c*d^3*e^2)) + (x*(1425*a^4*c^5*e^13 + 81*c^9*d^8*e^5 + 612*a*c^8*d^6*e^7 + 1894*a^2*c^7*d^4*e^9 + 2532*a^3*c^6*d^2*e^11))/(256*(a^8*e^8 + a^4*c^4*d^8 + 4*a^7*c*d^2*e^6 + 4*a^5*c^3*d^6*e^2 + 6*a^6*c^2*d^4*e^4)))*(-d*e^7)^(1/2))/(c^2*d^5 + a^2*d*e^4 + 2*a*c*d^3*e^2) - ((125*a^2*c^5*e^12)/128 + (81*c^7*d^4*e^8)/128 + (135*a*c^6*d^2*e^10)/64)/(a^8*e^8 + a^4*c^4*d^8 + 4*a^7*c*d^2*e^6 + 4*a^5*c^3*d^6*e^2 + 6*a^6*c^2*d^4*e^4) + ((((((45*a*c^10*d^11*e^3)/16 + (1277*a^6*c^5*d*e^13)/16 + (305*a^2*c^9*d^9*e^5)/16 + (385*a^3*c^8*d^7*e^7)/8 + (657*a^4*c^7*d^5*e^9)/8 + (2081*a^5*c^6*d^3*e^11)/16)/(2*(a^8*e^8 + a^4*c^4*d^8 + 4*a^7*c*d^2*e^6 + 4*a^5*c^3*d^6*e^2 + 6*a^6*c^2*d^4*e^4)) - (((((256*a^11*c^4*e^16 - 48*a^4*c^11*d^14*e^2 - 224*a^5*c^10*d^12*e^4 - 144*a^6*c^9*d^10*e^6 + 960*a^7*c^8*d^8*e^8 + 2480*a^8*c^7*d^6*e^10 + 2592*a^9*c^6*d^4*e^12 + 1296*a^10*c^5*d^2*e^14)/(2*(a^8*e^8 + a^4*c^4*d^8 + 4*a^7*c*d^2*e^6 + 4*a^5*c^3*d^6*e^2 + 6*a^6*c^2*d^4*e^4)) + (x*(-d*e^7)^(1/2)*(65536*a^13*c^4*e^17 - 65536*a^6*c^11*d^14*e^3 - 327680*a^7*c^10*d^12*e^5 - 589824*a^8*c^9*d^10*e^7 - 327680*a^9*c^8*d^8*e^9 + 327680*a^10*c^7*d^6*e^11 + 589824*a^11*c^6*d^4*e^13 + 327680*a^12*c^5*d^2*e^15))/(512*(c^2*d^5 + a^2*d*e^4 + 2*a*c*d^3*e^2)*(a^8*e^8 + a^4*c^4*d^8 + 4*a^7*c*d^2*e^6 + 4*a^5*c^3*d^6*e^2 + 6*a^6*c^2*d^4*e^4)))*(-d*e^7)^(1/2))/(2*(c^2*d^5 + a^2*d*e^4 + 2*a*c*d^3*e^2)) + (x*(1152*a^2*c^11*d^13*e^2 - 49024*a^8*c^5*d*e^14 + 7936*a^3*c^10*d^11*e^4 + 20352*a^4*c^9*d^9*e^6 + 8704*a^5*c^8*d^7*e^8 - 66688*a^6*c^7*d^5*e^10 - 110848*a^7*c^6*d^3*e^12))/(256*(a^8*e^8 + a^4*c^4*d^8 + 4*a^7*c*d^2*e^6 + 4*a^5*c^3*d^6*e^2 + 6*a^6*c^2*d^4*e^4)))*(-d*e^7)^(1/2))/(2*(c^2*d^5 + a^2*d*e^4 + 2*a*c*d^3*e^2)))*(-d*e^7)^(1/2))/(2*(c^2*d^5 + a^2*d*e^4 + 2*a*c*d^3*e^2)) - (x*(1425*a^4*c^5*e^13 + 81*c^9*d^8*e^5 + 612*a*c^8*d^6*e^7 + 1894*a^2*c^7*d^4*e^9 + 2532*a^3*c^6*d^2*e^11))/(256*(a^8*e^8 + a^4*c^4*d^8 + 4*a^7*c*d^2*e^6 + 4*a^5*c^3*d^6*e^2 + 6*a^6*c^2*d^4*e^4)))*(-d*e^7)^(1/2))/(c^2*d^5 + a^2*d*e^4 + 2*a*c*d^3*e^2)))*(-d*e^7)^(1/2)*1i)/(c^2*d^5 + a^2*d*e^4 + 2*a*c*d^3*e^2)

3.257 $\int \frac{1}{x^2(d+ex^2)(a+cx^4)^2} dx$

Optimal result

Integrand size = 22, antiderivative size = 745

$$
\begin{aligned}
&\int \frac{1}{x^2 (d+ex^2) (a+cx^4)^2} \, dx \\
&= -\frac{1}{a^2dx} - \frac{cx(ae+cdx^2)}{4a^2 (cd^2+ae^2) (a+cx^4)} - \frac{e^{9/2} \arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{d^{3/2} (cd^2+ae^2)^2} \\
&+ \frac{c^{3/4}\left(\sqrt{c}d+3\sqrt{a}e\right) \arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{8\sqrt{2}a^{9/4} (cd^2+ae^2)} \\
&+ \frac{c^{3/4}\left(a^{3/2}e^3+\sqrt{c}d(cd^2+2ae^2)\right) \arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{9/4} (cd^2+ae^2)^2} \\
&- \frac{c^{3/4}\left(\sqrt{c}d+3\sqrt{a}e\right) \arctan\left(1+\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{8\sqrt{2}a^{9/4} (cd^2+ae^2)} \\
&- \frac{c^{3/4}\left(a^{3/2}e^3+\sqrt{c}d(cd^2+2ae^2)\right) \arctan\left(1+\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{9/4} (cd^2+ae^2)^2} \\
&- \frac{c^{3/4}\left(\sqrt{c}d-3\sqrt{a}e\right) \log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)}{16\sqrt{2}a^{9/4} (cd^2+ae^2)} \\
&+ \frac{c^{3/4}\left(a^{3/2}e^3-\sqrt{c}d(cd^2+2ae^2)\right) \log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)}{4\sqrt{2}a^{9/4} (cd^2+ae^2)^2} \\
&+ \frac{c^{3/4}\left(\sqrt{c}d-3\sqrt{a}e\right) \log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)}{16\sqrt{2}a^{9/4} (cd^2+ae^2)} \\
&- \frac{c^{3/4}\left(a^{3/2}e^3-\sqrt{c}d(cd^2+2ae^2)\right) \log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)}{4\sqrt{2}a^{9/4} (cd^2+ae^2)^2}
\end{aligned}
$$

```
[Out] -1/a^2/d/x-1/4*c*x*(c*d*x^2+a*e)/a^2/(a*e^2+c*d^2)/(c*x^4+a)-e^(9/2)*arctan
(x*e^(1/2)/d^(1/2))/d^(3/2)/(a*e^2+c*d^2)^2-1/32*c^(3/4)*ln(-a^(1/4)*c^(1/4
)*x*2^(1/2)+a^(1/2)+x^2*c^(1/2))*(-3*e*a^(1/2)+d*c^(1/2))/a^(9/4)/(a*e^2+c*
d^2)*2^(1/2)+1/32*c^(3/4)*ln(a^(1/4)*c^(1/4)*x*2^(1/2)+a^(1/2)+x^2*c^(1/2))
*(-3*e*a^(1/2)+d*c^(1/2))/a^(9/4)/(a*e^2+c*d^2)*2^(1/2)-1/16*c^(3/4)*arctan
(-1+c^(1/4)*x*2^(1/2)/a^(1/4))*(3*e*a^(1/2)+d*c^(1/2))/a^(9/4)/(a*e^2+c*d^2
)*2^(1/2)-1/16*c^(3/4)*arctan(1+c^(1/4)*x*2^(1/2)/a^(1/4))*(3*e*a^(1/2)+d*c
^(1/2))/a^(9/4)/(a*e^2+c*d^2)*2^(1/2)+1/8*c^(3/4)*ln(-a^(1/4)*c^(1/4)*x*2^(
1/2)+a^(1/2)+x^2*c^(1/2))*(a^(3/2)*e^3-d*(2*a*e^2+c*d^2)*c^(1/2))/a^(9/4)/(
a*e^2+c*d^2)^2*2^(1/2)-1/8*c^(3/4)*ln(a^(1/4)*c^(1/4)*x*2^(1/2)+a^(1/2)+x^2
*c^(1/2))*(a^(3/2)*e^3-d*(2*a*e^2+c*d^2)*c^(1/2))/a^(9/4)/(a*e^2+c*d^2)^2*2
^(1/2)-1/4*c^(3/4)*arctan(-1+c^(1/4)*x*2^(1/2)/a^(1/4))*(a^(3/2)*e^3+d*(2*a
```

```
*e^2+c*d^2)*c^(1/2))/a^(9/4)/(a*e^2+c*d^2)^2*2^(1/2)-1/4*c^(3/4)*arctan(1+c
^(1/4)*x*2^(1/2)/a^(1/4))*(a^(3/2)*e^3+d*(2*a*e^2+c*d^2)*c^(1/2))/a^(9/4)/(
a*e^2+c*d^2)^2*2^(1/2)
```

Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 745, normalized size of antiderivative = 1.00, number of steps used = 22, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.409, Rules used = {1350, 211, 1193, 1182, 1176, 631, 210, 1179, 642}

$$\begin{aligned}
&\int \frac{1}{x^2(d+ex^2)(a+cx^4)^2}dx \\
&= \frac{c^{3/4}\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)(3\sqrt{a}e+\sqrt{c}d)}{8\sqrt{2}a^{9/4}(ae^2+cd^2)} - \frac{c^{3/4}\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)(3\sqrt{a}e+\sqrt{c}d)}{8\sqrt{2}a^{9/4}(ae^2+cd^2)} \\
&+ \frac{c^{3/4}\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)(a^{3/2}e^3+\sqrt{c}d(2ae^2+cd^2))}{2\sqrt{2}a^{9/4}(ae^2+cd^2)^2} \\
&- \frac{c^{3/4}\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)(a^{3/2}e^3+\sqrt{c}d(2ae^2+cd^2))}{2\sqrt{2}a^{9/4}(ae^2+cd^2)^2} \\
&- \frac{c^{3/4}(\sqrt{c}d-3\sqrt{a}e)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{16\sqrt{2}a^{9/4}(ae^2+cd^2)} \\
&+ \frac{c^{3/4}(\sqrt{c}d-3\sqrt{a}e)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{16\sqrt{2}a^{9/4}(ae^2+cd^2)} \\
&+ \frac{c^{3/4}(a^{3/2}e^3-\sqrt{c}d(2ae^2+cd^2))\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{4\sqrt{2}a^{9/4}(ae^2+cd^2)^2} \\
&- \frac{c^{3/4}(a^{3/2}e^3-\sqrt{c}d(2ae^2+cd^2))\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{4\sqrt{2}a^{9/4}(ae^2+cd^2)^2} \\
&- \frac{cx(ae+cdx^2)}{4a^2(a+cx^4)(ae^2+cd^2)} - \frac{1}{a^2dx} - \frac{e^{9/2}\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{d^{3/2}(ae^2+cd^2)^2}
\end{aligned}$$

```
[In] Int[1/(x^2*(d + e*x^2)*(a + c*x^4)^2),x]
```

```
[Out] -(1/(a^2*d*x)) - (c*x*(a*e + c*d*x^2))/(4*a^2*(c*d^2 + a*e^2)*(a + c*x^4))
- (e^(9/2)*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(d^(3/2)*(c*d^2 + a*e^2)^2) + (c^(3
/4)*(Sqrt[c]*d + 3*Sqrt[a]*e)*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(8*S
qrt[2]*a^(9/4)*(c*d^2 + a*e^2)) + (c^(3/4)*(a^(3/2)*e^3 + Sqrt[c]*d*(c*d^2
+ 2*a*e^2))*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(9/4)*(c*
d^2 + a*e^2)^2) - (c^(3/4)*(Sqrt[c]*d + 3*Sqrt[a]*e)*ArcTan[1 + (Sqrt[2]*c^
(1/4)*x)/a^(1/4)])/(8*Sqrt[2]*a^(9/4)*(c*d^2 + a*e^2)) - (c^(3/4)*(a^(3/2)*
```

```
e^3 + Sqrt[c]*d*(c*d^2 + 2*a*e^2))*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)])
/(2*Sqrt[2]*a^(9/4)*(c*d^2 + a*e^2)^2) - (c^(3/4)*(Sqrt[c]*d - 3*Sqrt[a]*e)
*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(16*Sqrt[2]*a^(9/4
)*(c*d^2 + a*e^2)) + (c^(3/4)*(a^(3/2)*e^3 - Sqrt[c]*d*(c*d^2 + 2*a*e^2))*L
og[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(4*Sqrt[2]*a^(9/4)*(
c*d^2 + a*e^2)^2) + (c^(3/4)*(Sqrt[c]*d - 3*Sqrt[a]*e)*Log[Sqrt[a] + Sqrt[2
]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(16*Sqrt[2]*a^(9/4)*(c*d^2 + a*e^2)) -
(c^(3/4)*(a^(3/2)*e^3 - Sqrt[c]*d*(c*d^2 + 2*a*e^2))*Log[Sqrt[a] + Sqrt[2]*
a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(4*Sqrt[2]*a^(9/4)*(c*d^2 + a*e^2)^2)
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1182

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[(-a)
*c]
```

Rule 1193

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[(-x
)*(d + e*x^2)*((a + c*x^4)^(p + 1)/(4*a*(p + 1))), x] + Dist[1/(4*a*(p + 1)
), Int[Simp[d*(4*p + 5) + e*(4*p + 7)*x^2, x]*(a + c*x^4)^(p + 1), x], x] /
; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && IntegerQ
[2*p]
```

Rule 1350

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*(a + c*x^4)^p,
x], x] /; FreeQ[{a, c, d, e, f, m, p, q}, x] && (IGtQ[p, 0] || IGtQ[q, 0] |
| IntegersQ[m, q])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(\frac{1}{a^2dx^2} - \frac{e^5}{d\left(cd^2+ae^2\right)^2\left(d+ex^2\right)} - \frac{c(ae+cdx^2)}{a\left(cd^2+ae^2\right)\left(a+cx^4\right)^2} + \frac{c(-a^2e^3-cd(cd^2+2ae^2)\,x^2)}{a^2\left(cd^2+ae^2\right)^2\left(a+cx^4\right)}\right) dx \\
&= -\frac{1}{a^2dx} + \frac{c\int \frac{-a^2e^3-cd(cd^2+2ae^2)x^2}{a+cx^4}\,dx}{a^2\left(cd^2+ae^2\right)^2} - \frac{e^5\int \frac{1}{d+ex^2}\,dx}{d\left(cd^2+ae^2\right)^2} - \frac{c\int \frac{ae+cdx^2}{(a+cx^4)^2}\,dx}{a\left(cd^2+ae^2\right)} \\
&= -\frac{1}{a^2dx} - \frac{cx(ae+cdx^2)}{4a^2\left(cd^2+ae^2\right)\left(a+cx^4\right)} - \frac{e^{9/2}\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{d^{3/2}\left(cd^2+ae^2\right)^2} \\
&\quad + \frac{c\int \frac{-3ae-cdx^2}{a+cx^4}\,dx}{4a^2\left(cd^2+ae^2\right)} + \frac{\left(c\left(cd^3+2ade^2-\frac{a^{3/2}e^3}{\sqrt{c}}\right)\right)\int \frac{\sqrt{a}\sqrt{c}-cx^2}{a+cx^4}\,dx}{2a^2\left(cd^2+ae^2\right)^2} \\
&\quad - \frac{\left(c\left(cd^3+2ade^2+\frac{a^{3/2}e^3}{\sqrt{c}}\right)\right)\int \frac{\sqrt{a}\sqrt{c}+cx^2}{a+cx^4}\,dx}{2a^2\left(cd^2+ae^2\right)^2}
\end{aligned}$$

$$= -\frac{1}{a^2dx} - \frac{cx(ae+cdx^2)}{4a^2(cd^2+ae^2)(a+cx^4)} - \frac{e^{9/2}\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{d^{3/2}(cd^2+ae^2)^2}$$

$$+ \frac{\left(c\left(d-\frac{3\sqrt{a}e}{\sqrt{c}}\right)\right)\int\frac{\sqrt{a}\sqrt{c}-cx^2}{a+cx^4}\,dx}{8a^2(cd^2+ae^2)} - \frac{\left(c\left(d+\frac{3\sqrt{a}e}{\sqrt{c}}\right)\right)\int\frac{\sqrt{a}\sqrt{c}+cx^2}{a+cx^4}\,dx}{8a^2(cd^2+ae^2)}$$

$$- \frac{\left(c^{5/4}\left(cd^3+2ade^2-\frac{a^{3/2}e^3}{\sqrt{c}}\right)\right)\int\frac{\frac{\sqrt{2}\sqrt[4]{a}}{\sqrt[4]{c}}+2x}{-\frac{\sqrt{a}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}-x^2}\,dx}{4\sqrt{2}a^{9/4}(cd^2+ae^2)^2}$$

$$- \frac{\left(c^{5/4}\left(cd^3+2ade^2-\frac{a^{3/2}e^3}{\sqrt{c}}\right)\right)\int\frac{\frac{\sqrt{2}\sqrt[4]{a}}{\sqrt[4]{c}}-2x}{-\frac{\sqrt{a}}{\sqrt{c}}+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}-x^2}\,dx}{4\sqrt{2}a^{9/4}(cd^2+ae^2)^2}$$

$$- \frac{\left(c\left(cd^3+2ade^2+\frac{a^{3/2}e^3}{\sqrt{c}}\right)\right)\int\frac{1}{\frac{\sqrt{a}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}+x^2}\,dx}{4a^2(cd^2+ae^2)^2}$$

$$- \frac{\left(c\left(cd^3+2ade^2+\frac{a^{3/2}e^3}{\sqrt{c}}\right)\right)\int\frac{1}{\frac{\sqrt{a}}{\sqrt{c}}+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}+x^2}\,dx}{4a^2(cd^2+ae^2)^2}$$

$$= -\frac{1}{a^2dx} - \frac{cx(ae+cdx^2)}{4a^2(cd^2+ae^2)(a+cx^4)} - \frac{e^{9/2}\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{d^{3/2}(cd^2+ae^2)^2}$$
$$- \frac{c^{5/4}\left(cd^3+2ade^2-\frac{a^{3/2}e^3}{\sqrt{c}}\right)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)}{4\sqrt{2}a^{9/4}(cd^2+ae^2)^2}$$
$$+ \frac{c^{5/4}\left(cd^3+2ade^2-\frac{a^{3/2}e^3}{\sqrt{c}}\right)\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)}{4\sqrt{2}a^{9/4}(cd^2+ae^2)^2}$$
$$- \frac{\left(c^{5/4}\left(d-\frac{3\sqrt{a}e}{\sqrt{c}}\right)\right)\int\frac{\frac{\sqrt{2}\sqrt[4]{a}}{\sqrt[4]{c}}+2x}{-\frac{\sqrt{a}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}-x^2}\,dx}{16\sqrt{2}a^{9/4}(cd^2+ae^2)}$$
$$- \frac{\left(c^{5/4}\left(d-\frac{3\sqrt{a}e}{\sqrt{c}}\right)\right)\int\frac{\frac{\sqrt{2}\sqrt[4]{a}}{\sqrt[4]{c}}-2x}{-\frac{\sqrt{a}}{\sqrt{c}}+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}-x^2}\,dx}{16\sqrt{2}a^{9/4}(cd^2+ae^2)}$$
$$- \frac{\left(c\left(d+\frac{3\sqrt{a}e}{\sqrt{c}}\right)\right)\int\frac{1}{\frac{\sqrt{a}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}+x^2}\,dx}{16a^2(cd^2+ae^2)} - \frac{\left(c\left(d+\frac{3\sqrt{a}e}{\sqrt{c}}\right)\right)\int\frac{1}{\frac{\sqrt{a}}{\sqrt{c}}+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}+x^2}\,dx}{16a^2(cd^2+ae^2)}$$
$$- \frac{\left(c^{5/4}\left(cd^3+2ade^2+\frac{a^{3/2}e^3}{\sqrt{c}}\right)\right)\text{Subst}\left(\int\frac{1}{-1-x^2}\,dx,x,1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{9/4}(cd^2+ae^2)^2}$$
$$+ \frac{\left(c^{5/4}\left(cd^3+2ade^2+\frac{a^{3/2}e^3}{\sqrt{c}}\right)\right)\text{Subst}\left(\int\frac{1}{-1-x^2}\,dx,x,1+\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{9/4}(cd^2+ae^2)^2}$$

$$
\begin{aligned}
&= -\frac{1}{a^2dx} - \frac{cx(ae+cdx^2)}{4a^2\,(cd^2+ae^2)\,(a+cx^4)} - \frac{e^{9/2}\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{d^{3/2}\,(cd^2+ae^2)^2} \\
&+ \frac{c^{5/4}\left(cd^3+2ade^2+\frac{a^{3/2}e^3}{\sqrt{c}}\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{9/4}\,(cd^2+ae^2)^2} \\
&- \frac{c^{5/4}\left(cd^3+2ade^2+\frac{a^{3/2}e^3}{\sqrt{c}}\right)\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{9/4}\,(cd^2+ae^2)^2} \\
&- \frac{c^{5/4}\left(d-\frac{3\sqrt{a}e}{\sqrt{c}}\right)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)}{16\sqrt{2}a^{9/4}\,(cd^2+ae^2)} \\
&- \frac{c^{5/4}\left(cd^3+2ade^2-\frac{a^{3/2}e^3}{\sqrt{c}}\right)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)}{4\sqrt{2}a^{9/4}\,(cd^2+ae^2)^2} \\
&+ \frac{c^{5/4}\left(d-\frac{3\sqrt{a}e}{\sqrt{c}}\right)\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)}{16\sqrt{2}a^{9/4}\,(cd^2+ae^2)} \\
&+ \frac{c^{5/4}\left(cd^3+2ade^2-\frac{a^{3/2}e^3}{\sqrt{c}}\right)\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)}{4\sqrt{2}a^{9/4}\,(cd^2+ae^2)^2} \\
&- \frac{\left(c^{3/4}\left(\sqrt{c}d+3\sqrt{a}e\right)\right)\text{Subst}\left(\int\frac{1}{-1-x^2}\,dx,x,1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{8\sqrt{2}a^{9/4}\,(cd^2+ae^2)} \\
&+ \frac{\left(c^{3/4}\left(\sqrt{c}d+3\sqrt{a}e\right)\right)\text{Subst}\left(\int\frac{1}{-1-x^2}\,dx,x,1+\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{8\sqrt{2}a^{9/4}\,(cd^2+ae^2)}
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{1}{a^2dx} - \frac{cx(ae+cdx^2)}{4a^2(cd^2+ae^2)(a+cx^4)} - \frac{e^{9/2}\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{d^{3/2}(cd^2+ae^2)^2} \\
&+ \frac{c^{3/4}(\sqrt{c}d+3\sqrt{a}e)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{8\sqrt{2}a^{9/4}(cd^2+ae^2)} \\
&+ \frac{c^{5/4}\left(cd^3+2ade^2+\frac{a^{3/2}e^3}{\sqrt{c}}\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{9/4}(cd^2+ae^2)^2} \\
&- \frac{c^{3/4}(\sqrt{c}d+3\sqrt{a}e)\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{8\sqrt{2}a^{9/4}(cd^2+ae^2)} \\
&- \frac{c^{5/4}\left(cd^3+2ade^2+\frac{a^{3/2}e^3}{\sqrt{c}}\right)\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{9/4}(cd^2+ae^2)^2} \\
&- \frac{c^{5/4}\left(d-\frac{3\sqrt{a}e}{\sqrt{c}}\right)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)}{16\sqrt{2}a^{9/4}(cd^2+ae^2)} \\
&- \frac{c^{5/4}\left(cd^3+2ade^2-\frac{a^{3/2}e^3}{\sqrt{c}}\right)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)}{4\sqrt{2}a^{9/4}(cd^2+ae^2)^2} \\
&+ \frac{c^{5/4}\left(d-\frac{3\sqrt{a}e}{\sqrt{c}}\right)\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)}{16\sqrt{2}a^{9/4}(cd^2+ae^2)} \\
&+ \frac{c^{5/4}\left(cd^3+2ade^2-\frac{a^{3/2}e^3}{\sqrt{c}}\right)\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)}{4\sqrt{2}a^{9/4}(cd^2+ae^2)^2}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 499, normalized size of antiderivative = 0.67

$$\int \frac{1}{x^2(d+ex^2)(a+cx^4)^2}\,dx$$
$$= \frac{1}{32}\left(-\frac{32}{a^2dx}-\frac{8cx(ae+cdx^2)}{a^2(cd^2+ae^2)(a+cx^4)}-\frac{32e^{9/2}\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{d^{3/2}(cd^2+ae^2)^2}\right.$$
$$+\frac{2\sqrt{2}c^{3/4}\left(5c^{3/2}d^3+3\sqrt{a}cd^2e+9a\sqrt{c}de^2+7a^{3/2}e^3\right)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{a^{9/4}(cd^2+ae^2)^2}$$
$$-\frac{2\sqrt{2}c^{3/4}\left(5c^{3/2}d^3+3\sqrt{a}cd^2e+9a\sqrt{c}de^2+7a^{3/2}e^3\right)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{a^{9/4}(cd^2+ae^2)^2}$$
$$+\frac{\sqrt{2}c^{3/4}\left(-5c^{3/2}d^3+3\sqrt{a}cd^2e-9a\sqrt{c}de^2+7a^{3/2}e^3\right)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)}{a^{9/4}(cd^2+ae^2)^2}$$
$$\left.+\frac{\sqrt{2}c^{3/4}\left(5c^{3/2}d^3-3\sqrt{a}cd^2e+9a\sqrt{c}de^2-7a^{3/2}e^3\right)\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)}{a^{9/4}(cd^2+ae^2)^2}\right)$$

```
[In] Integrate[1/(x^2*(d + e*x^2)*(a + c*x^4)^2),x]
```

```
[Out] (-32/(a^2*d*x) - (8*c*x*(a*e + c*d*x^2))/(a^2*(c*d^2 + a*e^2)*(a + c*x^4))
- (32*e^(9/2)*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(d^(3/2)*(c*d^2 + a*e^2)^2) + (2
*Sqrt[2]*c^(3/4)*(5*c^(3/2)*d^3 + 3*Sqrt[a]*c*d^2*e + 9*a*Sqrt[c]*d*e^2 + 7
*a^(3/2)*e^3)*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(a^(9/4)*(c*d^2 + a*
e^2)^2) - (2*Sqrt[2]*c^(3/4)*(5*c^(3/2)*d^3 + 3*Sqrt[a]*c*d^2*e + 9*a*Sqrt[
c]*d*e^2 + 7*a^(3/2)*e^3)*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(a^(9/4)
*(c*d^2 + a*e^2)^2) + (Sqrt[2]*c^(3/4)*(-5*c^(3/2)*d^3 + 3*Sqrt[a]*c*d^2*e
- 9*a*Sqrt[c]*d*e^2 + 7*a^(3/2)*e^3)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*
x + Sqrt[c]*x^2])/(a^(9/4)*(c*d^2 + a*e^2)^2) + (Sqrt[2]*c^(3/4)*(5*c^(3/2)
*d^3 - 3*Sqrt[a]*c*d^2*e + 9*a*Sqrt[c]*d*e^2 - 7*a^(3/2)*e^3)*Log[Sqrt[a] +
 Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(a^(9/4)*(c*d^2 + a*e^2)^2))/32
```

Maple [A] (verified)

Time = 0.50 (sec) , antiderivative size = 355, normalized size of antiderivative = 0.48

method	result
default	$-\frac{1}{a^2 d x}-\frac{c\left(\frac{\left(\frac{1}{4}acd\,e^2+\frac{1}{4}c^2d^3\right)x^3+\left(\frac{1}{4}e^3a^2+\frac{1}{4}ac\,d^2e\right)x}{c\,x^4+a}+\frac{\left(7e^3a^2+3ac\,d^2e\right)\left(\frac{a}{c}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x^2+\left(\frac{a}{c}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{c}}}{x^2-\left(\frac{a}{c}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{c}}}\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{c}\right)^{\frac{1}{4}}}+1\right)+2\cdots\right.}{32a}\cdots\right)}{\left(a\,e^2+c\,d^2\cdots\right.}$
risch	Expression too large to display

```
[In] int(1/x^2/(e*x^2+d)/(c*x^4+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/a^2/d/x-c/(a*e^2+c*d^2)^2/a^2*(((1/4*a*c*d*e^2+1/4*c^2*d^3)*x^3+(1/4*e^3
*a^2+1/4*a*c*d^2*e)*x)/(c*x^4+a)+1/32*(7*a^2*e^3+3*a*c*d^2*e)*(a/c)^(1/4)/a
*2^(1/2)*(ln((x^2+(a/c)^(1/4)*x*2^(1/2)+(a/c)^(1/2))/(x^2-(a/c)^(1/4)*x*2^(
1/2)+(a/c)^(1/2)))+2*arctan(2^(1/2)/(a/c)^(1/4)*x+1)+2*arctan(2^(1/2)/(a/c)
^(1/4)*x-1))+1/32*(9*a*c*d*e^2+5*c^2*d^3)/c/(a/c)^(1/4)*2^(1/2)*(ln((x^2-(a
/c)^(1/4)*x*2^(1/2)+(a/c)^(1/2))/(x^2+(a/c)^(1/4)*x*2^(1/2)+(a/c)^(1/2)))+2
*arctan(2^(1/2)/(a/c)^(1/4)*x+1)+2*arctan(2^(1/2)/(a/c)^(1/4)*x-1)))-1/d*e^
5/(a*e^2+c*d^2)^2/(e*d)^(1/2)*arctan(e*x/(e*d)^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 5082 vs. 2(573) = 1146.

Time = 29.01 (sec) , antiderivative size = 10188, normalized size of antiderivative = 13.68

$$\int \frac{1}{x^2\left(d+ex^2\right)\left(a+cx^4\right)^2}\,dx = \text{Too large to display}$$

```
[In] integrate(1/x^2/(e*x^2+d)/(c*x^4+a)^2,x, algorithm="fricas")
```

```
[Out] Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{x^2\left(d+ex^2\right)\left(a+cx^4\right)^2}\,dx = \text{Timed out}$$

```
[In] integrate(1/x**2/(e*x**2+d)/(c*x**4+a)**2,x)
```

```
[Out] Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{x^2\left(d+ex^2\right)\left(a+cx^4\right)^2}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(1/x^2/(e*x^2+d)/(c*x^4+a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 659, normalized size of antiderivative = 0.88

$$\int \frac{1}{x^2\left(d+ex^2\right)\left(a+cx^4\right)^2}\,dx = -\frac{e^5\arctan\left(\frac{ex}{\sqrt{de}}\right)}{(c^2d^5+2\,acd^3e^2+a^2de^4)\sqrt{de}} - \frac{\left(3\left(ac^3\right)^{\frac{1}{4}}ac^2d^2e+7\left(ac^3\right)^{\frac{1}{4}}a^2ce^3+5\left(ac^3\right)^{\frac{3}{4}}cd^3+9\left(ac^3\right)^{\frac{3}{4}}ade^2\right)\arctan\left(\frac{\sqrt{2}\left(2\,x+\sqrt{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{8\left(\sqrt{2}a^3c^3d^4+2\,\sqrt{2}a^4c^2d^2e^2+\sqrt{2}a^5ce^4\right)} - \frac{\left(3\left(ac^3\right)^{\frac{1}{4}}ac^2d^2e+7\left(ac^3\right)^{\frac{1}{4}}a^2ce^3+5\left(ac^3\right)^{\frac{3}{4}}cd^3+9\left(ac^3\right)^{\frac{3}{4}}ade^2\right)\arctan\left(\frac{\sqrt{2}\left(2\,x-\sqrt{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{8\left(\sqrt{2}a^3c^3d^4+2\,\sqrt{2}a^4c^2d^2e^2+\sqrt{2}a^5ce^4\right)} - \frac{\left(3\left(ac^3\right)^{\frac{1}{4}}ac^2d^2e+7\left(ac^3\right)^{\frac{1}{4}}a^2ce^3-5\left(ac^3\right)^{\frac{3}{4}}cd^3-9\left(ac^3\right)^{\frac{3}{4}}ade^2\right)\log\left(x^2+\sqrt{2}x\left(\frac{a}{c}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{c}}\right)}{16\left(\sqrt{2}a^3c^3d^4+2\,\sqrt{2}a^4c^2d^2e^2+\sqrt{2}a^5ce^4\right)} + \frac{\left(3\left(ac^3\right)^{\frac{1}{4}}ac^2d^2e+7\left(ac^3\right)^{\frac{1}{4}}a^2ce^3-5\left(ac^3\right)^{\frac{3}{4}}cd^3-9\left(ac^3\right)^{\frac{3}{4}}ade^2\right)\log\left(x^2-\sqrt{2}x\left(\frac{a}{c}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{c}}\right)}{16\left(\sqrt{2}a^3c^3d^4+2\,\sqrt{2}a^4c^2d^2e^2+\sqrt{2}a^5ce^4\right)} - \frac{5\,c^2d^2x^4+4\,ace^2x^4+acdex^2+4\,acd^2+4\,a^2e^2}{4\,(a^2cd^3+a^3de^2)(cx^5+ax)}$$

```
[In] integrate(1/x^2/(e*x^2+d)/(c*x^4+a)^2,x, algorithm="giac")
```

```
[Out] -e^5*arctan(e*x/sqrt(d*e))/((c^2*d^5 + 2*a*c*d^3*e^2 + a^2*d*e^4)*sqrt(d*e)
) - 1/8*(3*(a*c^3)^(1/4)*a*c^2*d^2*e + 7*(a*c^3)^(1/4)*a^2*c*e^3 + 5*(a*c^3
)^(3/4)*c*d^3 + 9*(a*c^3)^(3/4)*a*d*e^2)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*
(a/c)^(1/4))/(a/c)^(1/4))/(sqrt(2)*a^3*c^3*d^4 + 2*sqrt(2)*a^4*c^2*d^2*e^2
```

```
+ sqrt(2)*a^5*c*e^4) - 1/8*(3*(a*c^3)^(1/4)*a*c^2*d^2*e + 7*(a*c^3)^(1/4)*a
^2*c*e^3 + 5*(a*c^3)^(3/4)*c*d^3 + 9*(a*c^3)^(3/4)*a*d*e^2)*arctan(1/2*sqrt
(2)*(2*x - sqrt(2)*(a/c)^(1/4))/(a/c)^(1/4))/(sqrt(2)*a^3*c^3*d^4 + 2*sqrt(
2)*a^4*c^2*d^2*e^2 + sqrt(2)*a^5*c*e^4) - 1/16*(3*(a*c^3)^(1/4)*a*c^2*d^2*e
 + 7*(a*c^3)^(1/4)*a^2*c*e^3 - 5*(a*c^3)^(3/4)*c*d^3 - 9*(a*c^3)^(3/4)*a*d*
e^2)*log(x^2 + sqrt(2)*x*(a/c)^(1/4) + sqrt(a/c))/(sqrt(2)*a^3*c^3*d^4 + 2*
sqrt(2)*a^4*c^2*d^2*e^2 + sqrt(2)*a^5*c*e^4) + 1/16*(3*(a*c^3)^(1/4)*a*c^2*
d^2*e + 7*(a*c^3)^(1/4)*a^2*c*e^3 - 5*(a*c^3)^(3/4)*c*d^3 - 9*(a*c^3)^(3/4)
*a*d*e^2)*log(x^2 - sqrt(2)*x*(a/c)^(1/4) + sqrt(a/c))/(sqrt(2)*a^3*c^3*d^4
 + 2*sqrt(2)*a^4*c^2*d^2*e^2 + sqrt(2)*a^5*c*e^4) - 1/4*(5*c^2*d^2*x^4 + 4*
a*c*e^2*x^4 + a*c*d*e*x^2 + 4*a*c*d^2 + 4*a^2*e^2)/((a^2*c*d^3 + a^3*d*e^2)
*(c*x^5 + a*x))
```

Mupad [B] (verification not implemented)

Time = 10.73 (sec) , antiderivative size = 24015, normalized size of antiderivative = 32.23

$$\int \frac{1}{x^2 (d + ex^2) (a + cx^4)^2} dx = \text{Too large to display}$$

```
[In] int(1/(x^2*(a + c*x^4)^2*(d + e*x^2)),x)
```

```
[Out] - (1/(a*d) + (c*e*x^2)/(4*a*(a*e^2 + c*d^2)) + (c*x^4*(4*a*e^2 + 5*c*d^2))/
(4*a^2*d*(a*e^2 + c*d^2)))/(a*x + c*x^5) - atan(((11875*a^5*c^10*d^15*e - a
^9*c^3*(72128*a^3*d*e^15 + 265655*c^3*d^7*e^9 - 76440*a*c^2*d^5*e^11 - 1785
85*a^2*c*d^3*e^13) + 68800*a^6*c^9*d^13*e^3 + 89403*a^7*c^8*d^11*e^5 - 1264
88*a^8*c^7*d^9*e^7)*(a^25*d^2*e^19*x*(-(49*a^3*e^6*(-a^9*c^3)^(1/2) - 25*c^
3*d^6*(-a^9*c^3)^(1/2) + 30*a^5*c^4*d^5*e + 126*a^7*c^2*d*e^5 + 124*a^6*c^3
*d^3*e^3 - 81*a*c^2*d^4*e^2*(-a^9*c^3)^(1/2) - 39*a^2*c*d^2*e^4*(-a^9*c^3)^
(1/2))/(a^13*e^8 + a^9*c^4*d^8 + 4*a^12*c*d^2*e^6 + 4*a^10*c^3*d^6*e^2 + 6*
a^11*c^2*d^4*e^4))^(5/2)*2i - a^15*c^2*e^17*x*(-(49*a^3*e^6*(-a^9*c^3)^(1/2
) - 25*c^3*d^6*(-a^9*c^3)^(1/2) + 30*a^5*c^4*d^5*e + 126*a^7*c^2*d*e^5 + 12
4*a^6*c^3*d^3*e^3 - 81*a*c^2*d^4*e^2*(-a^9*c^3)^(1/2) - 39*a^2*c*d^2*e^4*(-
a^9*c^3)^(1/2))/(a^13*e^8 + a^9*c^4*d^8 + 4*a^12*c*d^2*e^6 + 4*a^10*c^3*d^6
*e^2 + 6*a^11*c^2*d^4*e^4))^(1/2)*3136i - a^11*c^10*d^19*x*(-(49*a^3*e^6*(-
a^9*c^3)^(1/2) - 25*c^3*d^6*(-a^9*c^3)^(1/2) + 30*a^5*c^4*d^5*e + 126*a^7*c
^2*d*e^5 + 124*a^6*c^3*d^3*e^3 - 81*a*c^2*d^4*e^2*(-a^9*c^3)^(1/2) - 39*a^2
*c*d^2*e^4*(-a^9*c^3)^(1/2))/(a^13*e^8 + a^9*c^4*d^8 + 4*a^12*c*d^2*e^6 + 4
*a^10*c^3*d^6*e^2 + 6*a^11*c^2*d^4*e^4))^(3/2)*25i - a^16*c^9*d^20*e*x*(-(4
9*a^3*e^6*(-a^9*c^3)^(1/2) - 25*c^3*d^6*(-a^9*c^3)^(1/2) + 30*a^5*c^4*d^5*e
 + 126*a^7*c^2*d*e^5 + 124*a^6*c^3*d^3*e^3 - 81*a*c^2*d^4*e^2*(-a^9*c^3)^(1
/2) - 39*a^2*c*d^2*e^4*(-a^9*c^3)^(1/2))/(a^13*e^8 + a^9*c^4*d^8 + 4*a^12*c
*d^2*e^6 + 4*a^10*c^3*d^6*e^2 + 6*a^11*c^2*d^4*e^4))^(5/2)*2i + a^24*c*d^4*
e^17*x*(-(49*a^3*e^6*(-a^9*c^3)^(1/2) - 25*c^3*d^6*(-a^9*c^3)^(1/2) + 30*a^
5*c^4*d^5*e + 126*a^7*c^2*d*e^5 + 124*a^6*c^3*d^3*e^3 - 81*a*c^2*d^4*e^2*(-
```

a^9*c^3)^(1/2) - 39*a^2*c*d^2*e^4*(-a^9*c^3)^(1/2))/(a^13*e^8 + a^9*c^4*d^8 + 4*a^12*c*d^2*e^6 + 4*a^10*c^3*d^6*e^2 + 6*a^11*c^2*d^4*e^4))^(5/2)*14i + a^8*c^9*d^14*e^3*x*(-(49*a^3*e^6*(-a^9*c^3)^(1/2) - 25*c^3*d^6*(-a^9*c^3)^(1/2) + 30*a^5*c^4*d^5*e + 126*a^7*c^2*d*e^5 + 124*a^6*c^3*d^3*e^3 - 81*a*c^2*d^4*e^2*(-a^9*c^3)^(1/2) - 39*a^2*c*d^2*e^4*(-a^9*c^3)^(1/2))/(a^13*e^8 + a^9*c^4*d^8 + 4*a^12*c*d^2*e^6 + 4*a^10*c^3*d^6*e^2 + 6*a^11*c^2*d^4*e^4))^(1/2)*1250i + a^9*c^8*d^12*e^5*x*(-(49*a^3*e^6*(-a^9*c^3)^(1/2) - 25*c^3*d^6*(-a^9*c^3)^(1/2) + 30*a^5*c^4*d^5*e + 126*a^7*c^2*d*e^5 + 124*a^6*c^3*d^3*e^3 - 81*a*c^2*d^4*e^2*(-a^9*c^3)^(1/2) - 39*a^2*c*d^2*e^4*(-a^9*c^3)^(1/2))/(a^13*e^8 + a^9*c^4*d^8 + 4*a^12*c*d^2*e^6 + 4*a^10*c^3*d^6*e^2 + 6*a^11*c^2*d^4*e^4))^(1/2)*9900i + a^10*c^7*d^10*e^7*x*(-(49*a^3*e^6*(-a^9*c^3)^(1/2) - 25*c^3*d^6*(-a^9*c^3)^(1/2) + 30*a^5*c^4*d^5*e + 126*a^7*c^2*d*e^5 + 124*a^6*c^3*d^3*e^3 - 81*a*c^2*d^4*e^2*(-a^9*c^3)^(1/2) - 39*a^2*c*d^2*e^4*(-a^9*c^3)^(1/2))/(a^13*e^8 + a^9*c^4*d^8 + 4*a^12*c*d^2*e^6 + 4*a^10*c^3*d^6*e^2 + 6*a^11*c^2*d^4*e^4))^(1/2)*31902i + a^11*c^6*d^8*e^9*x*(-(49*a^3*e^6*(-a^9*c^3)^(1/2) - 25*c^3*d^6*(-a^9*c^3)^(1/2) + 30*a^5*c^4*d^5*e + 126*a^7*c^2*d*e^5 + 124*a^6*c^3*d^3*e^3 - 81*a*c^2*d^4*e^2*(-a^9*c^3)^(1/2) - 39*a^2*c*d^2*e^4*(-a^9*c^3)^(1/2))/(a^13*e^8 + a^9*c^4*d^8 + 4*a^12*c*d^2*e^6 + 4*a^10*c^3*d^6*e^2 + 6*a^11*c^2*d^4*e^4))^(1/2)*52008i + a^12*c^5*d^6*e^11*x*(-(49*a^3*e^6*(-a^9*c^3)^(1/2) - 25*c^3*d^6*(-a^9*c^3)^(1/2) + 30*a^5*c^4*d^5*e + 126*a^7*c^2*d*e^5 + 124*a^6*c^3*d^3*e^3 - 81*a*c^2*d^4*e^2*(-a^9*c^3)^(1/2) - 39*a^2*c*d^2*e^4*(-a^9*c^3)^(1/2))/(a^13*e^8 + a^9*c^4*d^8 + 4*a^12*c*d^2*e^6 + 4*a^10*c^3*d^6*e^2 + 6*a^11*c^2*d^4*e^4))^(1/2)*42238i + a^13*c^4*d^4*e^13*x*(-(49*a^3*e^6*(-a^9*c^3)^(1/2) - 25*c^3*d^6*(-a^9*c^3)^(1/2) + 30*a^5*c^4*d^5*e + 126*a^7*c^2*d*e^5 + 124*a^6*c^3*d^3*e^3 - 81*a*c^2*d^4*e^2*(-a^9*c^3)^(1/2) - 39*a^2*c*d^2*e^4*(-a^9*c^3)^(1/2))/(a^13*e^8 + a^9*c^4*d^8 + 4*a^12*c*d^2*e^6 + 4*a^10*c^3*d^6*e^2 + 6*a^11*c^2*d^4*e^4))^(1/2)*10924i - a^14*c^3*d^2*e^15*x*(-(49*a^3*e^6*(-a^9*c^3)^(1/2) - 25*c^3*d^6*(-a^9*c^3)^(1/2) + 30*a^5*c^4*d^5*e + 126*a^7*c^2*d*e^5 + 124*a^6*c^3*d^3*e^3 - 81*a*c^2*d^4*e^2*(-a^9*c^3)^(1/2) - 39*a^2*c*d^2*e^4*(-a^9*c^3)^(1/2))/(a^13*e^8 + a^9*c^4*d^8 + 4*a^12*c*d^2*e^6 + 4*a^10*c^3*d^6*e^2 + 6*a^11*c^2*d^4*e^4))^(1/2)*5694i - a^12*c^9*d^17*e^2*x*(-(49*a^3*e^6*(-a^9*c^3)^(1/2) - 25*c^3*d^6*(-a^9*c^3)^(1/2) + 30*a^5*c^4*d^5*e + 126*a^7*c^2*d*e^5 + 124*a^6*c^3*d^3*e^3 - 81*a*c^2*d^4*e^2*(-a^9*c^3)^(1/2) - 39*a^2*c*d^2*e^4*(-a^9*c^3)^(1/2))/(a^13*e^8 + a^9*c^4*d^8 + 4*a^12*c*d^2*e^6 + 4*a^10*c^3*d^6*e^2 + 6*a^11*c^2*d^4*e^4))^(3/2)*216i - a^13*c^8*d^15*e^4*x*(-(49*a^3*e^6*(-a^9*c^3)^(1/2) - 25*c^3*d^6*(-a^9*c^3)^(1/2) + 30*a^5*c^4*d^5*e + 126*a^7*c^2*d*e^5 + 124*a^6*c^3*d^3*e^3 - 81*a*c^2*d^4*e^2*(-a^9*c^3)^(1/2) - 39*a^2*c*d^2*e^4*(-a^9*c^3)^(1/2))/(a^13*e^8 + a^9*c^4*d^8 + 4*a^12*c*d^2*e^6 + 4*a^10*c^3*d^6*e^2 + 6*a^11*c^2*d^4*e^4))^(3/2)*700i - a^14*c^7*d^13*e^6*x*(-(49*a^3*e^6*(-a^9*c^3)^(1/2) - 25*c^3*d^6*(-a^9*c^3)^(1/2) + 30*a^5*c^4*d^5*e + 126*a^7*c^2*d*e^5 + 124*a^6*c^3*d^3*e^3 - 81*a*c^2*d^4*e^2*(-a^9*c^3)^(1/2) - 39*a^2*c*d^2*e^4*(-a^9*c^3)^(1/2))/(a^13*e^8 + a^9*c^4*d^8 + 4*a^12*c*d^2*e^6 + 4*a^10*c^3*d^6*e^2 + 6*a^11*c^2*d^4*e^4))^(3/2)*808i + a^15*c^6*d^11*e^8*x*(-(49*a^3*e^6*(-a^9*c^3)^(1/2) - 25*c^3*d^6*(-

a^9*c^3)^(1/2) + 30*a^5*c^4*d^5*e + 126*a^7*c^2*d*e^5 + 124*a^6*c^3*d^3*e^3 - 81*a*c^2*d^4*e^2*(-a^9*c^3)^(1/2) - 39*a^2*c*d^2*e^4*(-a^9*c^3)^(1/2))/(a^13*e^8 + a^9*c^4*d^8 + 4*a^12*c*d^2*e^6 + 4*a^10*c^3*d^6*e^2 + 6*a^11*c^2*d^4*e^4))^(3/2)*778i + a^16*c^5*d^9*e^10*x*(-(49*a^3*e^6*(-a^9*c^3)^(1/2) - 25*c^3*d^6*(-a^9*c^3)^(1/2) + 30*a^5*c^4*d^5*e + 126*a^7*c^2*d*e^5 + 124*a^6*c^3*d^3*e^3 - 81*a*c^2*d^4*e^2*(-a^9*c^3)^(1/2) - 39*a^2*c*d^2*e^4*(-a^9*c^3)^(1/2))/(a^13*e^8 + a^9*c^4*d^8 + 4*a^12*c*d^2*e^6 + 4*a^10*c^3*d^6*e^2 + 6*a^11*c^2*d^4*e^4))^(3/2)*3224i + a^17*c^4*d^7*e^12*x*(-(49*a^3*e^6*(-a^9*c^3)^(1/2) - 25*c^3*d^6*(-a^9*c^3)^(1/2) + 30*a^5*c^4*d^5*e + 126*a^7*c^2*d*e^5 + 124*a^6*c^3*d^3*e^3 - 81*a*c^2*d^4*e^2*(-a^9*c^3)^(1/2) - 39*a^2*c*d^2*e^4*(-a^9*c^3)^(1/2))/(a^13*e^8 + a^9*c^4*d^8 + 4*a^12*c*d^2*e^6 + 4*a^10*c^3*d^6*e^2 + 6*a^11*c^2*d^4*e^4))^(3/2)*3460i + a^18*c^3*d^5*e^14*x*(-(49*a^3*e^6*(-a^9*c^3)^(1/2) - 25*c^3*d^6*(-a^9*c^3)^(1/2) + 30*a^5*c^4*d^5*e + 126*a^7*c^2*d*e^5 + 124*a^6*c^3*d^3*e^3 - 81*a*c^2*d^4*e^2*(-a^9*c^3)^(1/2) - 39*a^2*c*d^2*e^4*(-a^9*c^3)^(1/2))/(a^13*e^8 + a^9*c^4*d^8 + 4*a^12*c*d^2*e^6 + 4*a^10*c^3*d^6*e^2 + 6*a^11*c^2*d^4*e^4))^(3/2)*1384i - a^19*c^2*d^3*e^16*x*(-(49*a^3*e^6*(-a^9*c^3)^(1/2) - 25*c^3*d^6*(-a^9*c^3)^(1/2) + 30*a^5*c^4*d^5*e + 126*a^7*c^2*d*e^5 + 124*a^6*c^3*d^3*e^3 - 81*a*c^2*d^4*e^2*(-a^9*c^3)^(1/2) - 39*a^2*c*d^2*e^4*(-a^9*c^3)^(1/2))/(a^13*e^8 + a^9*c^4*d^8 + 4*a^12*c*d^2*e^6 + 4*a^10*c^3*d^6*e^2 + 6*a^11*c^2*d^4*e^4))^(3/2)*57i - a^17*c^8*d^18*e^3*x*(-(49*a^3*e^6*(-a^9*c^3)^(1/2) - 25*c^3*d^6*(-a^9*c^3)^(1/2) + 30*a^5*c^4*d^5*e + 126*a^7*c^2*d*e^5 + 124*a^6*c^3*d^3*e^3 - 81*a*c^2*d^4*e^2*(-a^9*c^3)^(1/2) - 39*a^2*c*d^2*e^4*(-a^9*c^3)^(1/2))/(a^13*e^8 + a^9*c^4*d^8 + 4*a^12*c*d^2*e^6 + 4*a^10*c^3*d^6*e^2 + 6*a^11*c^2*d^4*e^4))^(5/2)*14i - a^18*c^7*d^16*e^5*x*(-(49*a^3*e^6*(-a^9*c^3)^(1/2) - 25*c^3*d^6*(-a^9*c^3)^(1/2) + 30*a^5*c^4*d^5*e + 126*a^7*c^2*d*e^5 + 124*a^6*c^3*d^3*e^3 - 81*a*c^2*d^4*e^2*(-a^9*c^3)^(1/2) - 39*a^2*c*d^2*e^4*(-a^9*c^3)^(1/2))/(a^13*e^8 + a^9*c^4*d^8 + 4*a^12*c*d^2*e^6 + 4*a^10*c^3*d^6*e^2 + 6*a^11*c^2*d^4*e^4))^(5/2)*40i - a^19*c^6*d^14*e^7*x*(-(49*a^3*e^6*(-a^9*c^3)^(1/2) - 25*c^3*d^6*(-a^9*c^3)^(1/2) + 30*a^5*c^4*d^5*e + 126*a^7*c^2*d*e^5 + 124*a^6*c^3*d^3*e^3 - 81*a*c^2*d^4*e^2*(-a^9*c^3)^(1/2) - 39*a^2*c*d^2*e^4*(-a^9*c^3)^(1/2))/(a^13*e^8 + a^9*c^4*d^8 + 4*a^12*c*d^2*e^6 + 4*a^10*c^3*d^6*e^2 + 6*a^11*c^2*d^4*e^4))^(5/2)*56i - a^20*c^5*d^12*e^9*x*(-(49*a^3*e^6*(-a^9*c^3)^(1/2) - 25*c^3*d^6*(-a^9*c^3)^(1/2) + 30*a^5*c^4*d^5*e + 126*a^7*c^2*d*e^5 + 124*a^6*c^3*d^3*e^3 - 81*a*c^2*d^4*e^2*(-a^9*c^3)^(1/2) - 39*a^2*c*d^2*e^4*(-a^9*c^3)^(1/2))/(a^13*e^8 + a^9*c^4*d^8 + 4*a^12*c*d^2*e^6 + 4*a^10*c^3*d^6*e^2 + 6*a^11*c^2*d^4*e^4))^(5/2)*28i + a^21*c^4*d^10*e^11*x*(-(49*a^3*e^6*(-a^9*c^3)^(1/2) - 25*c^3*d^6*(-a^9*c^3)^(1/2) + 30*a^5*c^4*d^5*e + 126*a^7*c^2*d*e^5 + 124*a^6*c^3*d^3*e^3 - 81*a*c^2*d^4*e^2*(-a^9*c^3)^(1/2) - 39*a^2*c*d^2*e^4*(-a^9*c^3)^(1/2))/(a^13*e^8 + a^9*c^4*d^8 + 4*a^12*c*d^2*e^6 + 4*a^10*c^3*d^6*e^2 + 6*a^11*c^2*d^4*e^4))^(5/2)*28i + a^22*c^3*d^8*e^13*x*(-(49*a^3*e^6*(-a^9*c^3)^(1/2) - 25*c^3*d^6*(-a^9*c^3)^(1/2) + 30*a^5*c^4*d^5*e + 126*a^7*c^2*d*e^5 + 124*a^6*c^3*d^3*e^3 - 81*a*c^2*d^4*e^2*(-a^9*c^3)^(1/2) - 39*a^2*c*d^2*e^4*(-a^9*c^3)^(1/2))/(a^13*e^8 + a^9*c^4*d^8 + 4*a^12*c*d^2*e^6 + 4*a^10*c^3*d^6*e^2 + 6*a^11*c^2*d

^4*e^4))^(5/2)*56i + a^23*c^2*d^6*e^15*x*(-(49*a^3*e^6*(-a^9*c^3)^(1/2) - 2
5*c^3*d^6*(-a^9*c^3)^(1/2) + 30*a^5*c^4*d^5*e + 126*a^7*c^2*d*e^5 + 124*a^6
*c^3*d^3*e^3 - 81*a*c^2*d^4*e^2*(-a^9*c^3)^(1/2) - 39*a^2*c*d^2*e^4*(-a^9*c
^3)^(1/2))/(a^13*e^8 + a^9*c^4*d^8 + 4*a^12*c*d^2*e^6 + 4*a^10*c^3*d^6*e^2
+ 6*a^11*c^2*d^4*e^4))^(5/2)*40i - a^20*c*d*e^18*x*(-(49*a^3*e^6*(-a^9*c^3)
^(1/2) - 25*c^3*d^6*(-a^9*c^3)^(1/2) + 30*a^5*c^4*d^5*e + 126*a^7*c^2*d*e^5
 + 124*a^6*c^3*d^3*e^3 - 81*a*c^2*d^4*e^2*(-a^9*c^3)^(1/2) - 39*a^2*c*d^2*e
^4*(-a^9*c^3)^(1/2))/(a^13*e^8 + a^9*c^4*d^8 + 4*a^12*c*d^2*e^6 + 4*a^10*c^
3*d^6*e^2 + 6*a^11*c^2*d^4*e^4))^(3/2)*128i) + (-a^9*c^3)^(1/2)*(3125*c^9*d
^16 + 21952*a^8*c*e^16 + 3000*a*c^8*d^14*e^2 - 77435*a^2*c^7*d^12*e^4 - 242
104*a^3*c^6*d^10*e^6 - 127665*a^4*c^5*d^8*e^8 + 240064*a^5*c^4*d^6*e^10 + 1
18199*a^6*c^3*d^4*e^12 - 130368*a^7*c^2*d^2*e^14)*(a^25*d^2*e^19*x*(-(49*a^
3*e^6*(-a^9*c^3)^(1/2) - 25*c^3*d^6*(-a^9*c^3)^(1/2) + 30*a^5*c^4*d^5*e + 1
26*a^7*c^2*d*e^5 + 124*a^6*c^3*d^3*e^3 - 81*a*c^2*d^4*e^2*(-a^9*c^3)^(1/2)
- 39*a^2*c*d^2*e^4*(-a^9*c^3)^(1/2))/(a^13*e^8 + a^9*c^4*d^8 + 4*a^12*c*d^2
*e^6 + 4*a^10*c^3*d^6*e^2 + 6*a^11*c^2*d^4*e^4))^(5/2)*2i - a^15*c^2*e^17*x
*(-(49*a^3*e^6*(-a^9*c^3)^(1/2) - 25*c^3*d^6*(-a^9*c^3)^(1/2) + 30*a^5*c^4*
d^5*e + 126*a^7*c^2*d*e^5 + 124*a^6*c^3*d^3*e^3 - 81*a*c^2*d^4*e^2*(-a^9*c^
3)^(1/2) - 39*a^2*c*d^2*e^4*(-a^9*c^3)^(1/2))/(a^13*e^8 + a^9*c^4*d^8 + 4*a
^12*c*d^2*e^6 + 4*a^10*c^3*d^6*e^2 + 6*a^11*c^2*d^4*e^4))^(1/2)*3136i - a^1
1*c^10*d^19*x*(-(49*a^3*e^6*(-a^9*c^3)^(1/2) - 25*c^3*d^6*(-a^9*c^3)^(1/2)
+ 30*a^5*c^4*d^5*e + 126*a^7*c^2*d*e^5 + 124*a^6*c^3*d^3*e^3 - 81*a*c^2*d^4
e^2(-a^9*c^3)^(1/2) - 39*a^2*c*d^2*e^4*(-a^9*c^3)^(1/2))/(a^13*e^8 + a^9*
c^4*d^8 + 4*a^12*c*d^2*e^6 + 4*a^10*c^3*d^6*e^2 + 6*a^11*c^2*d^4*e^4))^(3/2
)*25i - a^16*c^9*d^20*e*x*(-(49*a^3*e^6*(-a^9*c^3)^(1/2) - 25*c^3*d^6*(-a^9
*c^3)^(1/2) + 30*a^5*c^4*d^5*e + 126*a^7*c^2*d*e^5 + 124*a^6*c^3*d^3*e^3 -
81*a*c^2*d^4*e^2*(-a^9*c^3)^(1/2) - 39*a^2*c*d^2*e^4*(-a^9*c^3)^(1/2))/(a^1
3*e^8 + a^9*c^4*d^8 + 4*a^12*c*d^2*e^6 + 4*a^10*c^3*d^6*e^2 + 6*a^11*c^2*d^
4*e^4))^(5/2)*2i + a^24*c*d^4*e^17*x*(-(49*a^3*e^6*(-a^9*c^3)^(1/2) - 25*c^
3*d^6*(-a^9*c^3)^(1/2) + 30*a^5*c^4*d^5*e + 126*a^7*c^2*d*e^5 + 124*a^6*c^3
*d^3*e^3 - 81*a*c^2*d^4*e^2*(-a^9*c^3)^(1/2) - 39*a^2*c*d^2*e^4*(-a^9*c^3)^
(1/2))/(a^13*e^8 + a^9*c^4*d^8 + 4*a^12*c*d^2*e^6 + 4*a^10*c^3*d^6*e^2 + 6*
a^11*c^2*d^4*e^4))^(5/2)*14i + a^8*c^9*d^14*e^3*x*(-(49*a^3*e^6*(-a^9*c^3)^
(1/2) - 25*c^3*d^6*(-a^9*c^3)^(1/2) + 30*a^5*c^4*d^5*e + 126*a^7*c^2*d*e^5
+ 124*a^6*c^3*d^3*e^3 - 81*a*c^2*d^4*e^2*(-a^9*c^3)^(1/2) - 39*a^2*c*d^2*e^
4*(-a^9*c^3)^(1/2))/(a^13*e^8 + a^9*c^4*d^8 + 4*a^12*c*d^2*e^6 + 4*a^10*c^3
*d^6*e^2 + 6*a^11*c^2*d^4*e^4))^(1/2)*1250i + a^9*c^8*d^12*e^5*x*(-(49*a^3*
e^6*(-a^9*c^3)^(1/2) - 25*c^3*d^6*(-a^9*c^3)^(1/2) + 30*a^5*c^4*d^5*e + 126
*a^7*c^2*d*e^5 + 124*a^6*c^3*d^3*e^3 - 81*a*c^2*d^4*e^2*(-a^9*c^3)^(1/2) -
39*a^2*c*d^2*e^4*(-a^9*c^3)^(1/2))/(a^13*e^8 + a^9*c^4*d^8 + 4*a^12*c*d^2*e
^6 + 4*a^10*c^3*d^6*e^2 + 6*a^11*c^2*d^4*e^4))^(1/2)*9900i + a^10*c^7*d^10*
e^7*x*(-(49*a^3*e^6*(-a^9*c^3)^(1/2) - 25*c^3*d^6*(-a^9*c^3)^(1/2) + 30*a^5
*c^4*d^5*e + 126*a^7*c^2*d*e^5 + 124*a^6*c^3*d^3*e^3 - 81*a*c^2*d^4*e^2*(-a
^9*c^3)^(1/2) - 39*a^2*c*d^2*e^4*(-a^9*c^3)^(1/2))/(a^13*e^8 + a^9*c^4*d^8
+ 4*a^12*c*d^2*e^6 + 4*a^10*c^3*d^6*e^2 + 6*a^11*c^2*d^4*e^4))^(1/2)*31902i

+ a^11*c^6*d^8*e^9*x*(-(49*a^3*e^6*(-a^9*c^3)^(1/2) - 25*c^3*d^6*(-a^9*c^3
)^(1/2) + 30*a^5*c^4*d^5*e + 126*a^7*c^2*d*e^5 + 124*a^6*c^3*d^3*e^3 - 81*a
*c^2*d^4*e^2*(-a^9*c^3)^(1/2) - 39*a^2*c*d^2*e^4*(-a^9*c^3)^(1/2))/(a^13*e^
8 + a^9*c^4*d^8 + 4*a^12*c*d^2*e^6 + 4*a^10*c^3*d^6*e^2 + 6*a^11*c^2*d^4*e^
4))^(1/2)*52008i + a^12*c^5*d^6*e^11*x*(-(49*a^3*e^6*(-a^9*c^3)^(1/2) - 25*
c^3*d^6*(-a^9*c^3)^(1/2) + 30*a^5*c^4*d^5*e + 126*a^7*c^2*d*e^5 + 124*a^6*c
^3*d^3*e^3 - 81*a*c^2*d^4*e^2*(-a^9*c^3)^(1/2) - 39*a^2*c*d^2*e^4*(-a^9*c^3
)^(1/2))/(a^13*e^8 + a^9*c^4*d^8 + 4*a^12*c*d^2*e^6 + 4*a^10*c^3*d^6*e^2 +
6*a^11*c^2*d^4*e^4))^(1/2)*42238i + a^13*c^4*d^4*e^13*x*(-(49*a^3*e^6*(-a^9
*c^3)^(1/2) - 25*c^3*d^6*(-a^9*c^3)^(1/2) + 30*a^5*c^4*d^5*e + 126*a^7*c^2*
d*e^5 + 124*a^6*c^3*d^3*e^3 - 81*a*c^2*d^4*e^2*(-a^9*c^3)^(1/2) - 39*a^2*c*
d^2*e^4*(-a^9*c^3)^(1/2))/(a^13*e^8 + a^9*c^4*d^8 + 4*a^12*c*d^2*e^6 + 4*a^
10*c^3*d^6*e^2 + 6*a^11*c^2*d^4*e^4))^(1/2)*10924i - a^14*c^3*d^2*e^15*x*(-
(49*a^3*e^6*(-a^9*c^3)^(1/2) - 25*c^3*d^6*(-a^9*c^3)^(1/2) + 30*a^5*c^4*d^5
*e + 126*a^7*c^2*d*e^5 + 124*a^6*c^3*d^3*e^3 - 81*a*c^2*d^4*e^2*(-a^9*c^3)^
(1/2) - 39*a^2*c*d^2*e^4*(-a^9*c^3)^(1/2))/(a^13*e^8 + a^9*c^4*d^8 + 4*a^12
*c*d^2*e^6 + 4*a^10*c^3*d^6*e^2 + 6*a^11*c^2*d^4*e^4))^(1/2)*5694i - a^12*c
^9*d^17*e^2*x*(-(49*a^3*e^6*(-a^9*c^3)^(1/2) - 25*c^3*d^6*(-a^9*c^3)^(1/2)
+ 30*a^5*c^4*d^5*e + 126*a^7*c^2*d*e^5 + 124*a^6*c^3*d^3*e^3 - 81*a*c^2*d^4
e^2(-a^9*c^3)^(1/2) - 39*a^2*c*d^2*e^4*(-a^9*c^3)^(1/2))/(a^13*e^8 + a^9*
c^4*d^8 + 4*a^12*c*d^2*e^6 + 4*a^10*c^3*d^6*e^2 + 6*a^11*c^2*d^4*e^4))^(3/2
)*216i - a^13*c^8*d^15*e^4*x*(-(49*a^3*e^6*(-a^9*c^3)^(1/2) - 25*c^3*d^6*(-
a^9*c^3)^(1/2) + 30*a^5*c^4*d^5*e + 126*a^7*c^2*d*e^5 + 124*a^6*c^3*d^3*e^3
 - 81*a*c^2*d^4*e^2*(-a^9*c^3)^(1/2) - 39*a^2*c*d^2*e^4*(-a^9*c^3)^(1/2))/(
a^13*e^8 + a^9*c^4*d^8 + 4*a^12*c*d^2*e^6 + 4*a^10*c^3*d^6*e^2 + 6*a^11*c^2
*d^4*e^4))^(3/2)*700i - a^14*c^7*d^13*e^6*x*(-(49*a^3*e^6*(-a^9*c^3)^(1/2)
- 25*c^3*d^6*(-a^9*c^3)^(1/2) + 30*a^5*c^4*d^5*e + 126*a^7*c^2*d*e^5 + 124*
a^6*c^3*d^3*e^3 - 81*a*c^2*d^4*e^2*(-a^9*c^3)^(1/2) - 39*a^2*c*d^2*e^4*(-a^
9*c^3)^(1/2))/(a^13*e^8 + a^9*c^4*d^8 + 4*a^12*c*d^2*e^6 + 4*a^10*c^3*d^6*e
^2 + 6*a^11*c^2*d^4*e^4))^(3/2)*808i + a^15*c^6*d^11*e^8*x*(-(49*a^3*e^6*(-
a^9*c^3)^(1/2) - 25*c^3*d^6*(-a^9*c^3)^(1/2) + 30*a^5*c^4*d^5*e + 126*a^7*c
^2*d*e^5 + 124*a^6*c^3*d^3*e^3 - 81*a*c^2*d^4*e^2*(-a^9*c^3)^(1/2) - 39*a^2
*c*d^2*e^4*(-a^9*c^3)^(1/2))/(a^13*e^8 + a^9*c^4*d^8 + 4*a^12*c*d^2*e^6 + 4
*a^10*c^3*d^6*e^2 + 6*a^11*c^2*d^4*e^4))^(3/2)*778i + a^16*c^5*d^9*e^10*x*(
-(49*a^3*e^6*(-a^9*c^3)^(1/2) - 25*c^3*d^6*(-a^9*c^3)^(1/2) + 30*a^5*c^4*d^
5*e + 126*a^7*c^2*d*e^5 + 124*a^6*c^3*d^3*e^3 - 81*a*c^2*d^4*e^2*(-a^9*c^3)
^(1/2) - 39*a^2*c*d^2*e^4*(-a^9*c^3)^(1/2))/(a^13*e^8 + a^9*c^4*d^8 + 4*a^1
2*c*d^2*e^6 + 4*a^10*c^3*d^6*e^2 + 6*a^11*c^2*d^4*e^4))^(3/2)*3224i + a^17*
c^4*d^7*e^12*x*(-(49*a^3*e^6*(-a^9*c^3)^(1/2) - 25*c^3*d^6*(-a^9*c^3)^(1/2)
 + 30*a^5*c^4*d^5*e + 126*a^7*c^2*d*e^5 + 124*a^6*c^3*d^3*e^3 - 81*a*c^2*d^
4*e^2*(-a^9*c^3)^(1/2) - 39*a^2*c*d^2*e^4*(-a^9*c^3)^(1/2))/(a^13*e^8 + a^9
*c^4*d^8 + 4*a^12*c*d^2*e^6 + 4*a^10*c^3*d^6*e^2 + 6*a^11*c^2*d^4*e^4))^(3/
2)*3460i + a^18*c^3*d^5*e^14*x*(-(49*a^3*e^6*(-a^9*c^3)^(1/2) - 25*c^3*d^6*
(-a^9*c^3)^(1/2) + 30*a^5*c^4*d^5*e + 126*a^7*c^2*d*e^5 + 124*a^6*c^3*d^3*e
^3 - 81*a*c^2*d^4*e^2*(-a^9*c^3)^(1/2) - 39*a^2*c*d^2*e^4*(-a^9*c^3)^(1/2))

/(a^13*e^8 + a^9*c^4*d^8 + 4*a^12*c*d^2*e^6 + 4*a^10*c^3*d^6*e^2 + 6*a^11*c
^2*d^4*e^4))^(3/2)*1384i - a^19*c^2*d^3*e^16*x*(-(49*a^3*e^6*(-a^9*c^3)^(1/
2) - 25*c^3*d^6*(-a^9*c^3)^(1/2) + 30*a^5*c^4*d^5*e + 126*a^7*c^2*d*e^5 + 1
24*a^6*c^3*d^3*e^3 - 81*a*c^2*d^4*e^2*(-a^9*c^3)^(1/2) - 39*a^2*c*d^2*e^4*(
-a^9*c^3)^(1/2))/(a^13*e^8 + a^9*c^4*d^8 + 4*a^12*c*d^2*e^6 + 4*a^10*c^3*d^
6*e^2 + 6*a^11*c^2*d^4*e^4))^(3/2)*57i - a^17*c^8*d^18*e^3*x*(-(49*a^3*e^6*
(-a^9*c^3)^(1/2) - 25*c^3*d^6*(-a^9*c^3)^(1/2) + 30*a^5*c^4*d^5*e + 126*a^7
*c^2*d*e^5 + 124*a^6*c^3*d^3*e^3 - 81*a*c^2*d^4*e^2*(-a^9*c^3)^(1/2) - 39*a
^2*c*d^2*e^4*(-a^9*c^3)^(1/2))/(a^13*e^8 + a^9*c^4*d^8 + 4*a^12*c*d^2*e^6 +
4*a^10*c^3*d^6*e^2 + 6*a^11*c^2*d^4*e^4))^(5/2)*14i - a^18*c^7*d^16*e^5*x*
(-(49*a^3*e^6*(-a^9*c^3)^(1/2) - 25*c^3*d^6*(-a^9*c^3)^(1/2) + 30*a^5*c^4*d
^5*e + 126*a^7*c^2*d*e^5 + 124*a^6*c^3*d^3*e^3 - 81*a*c^2*d^4*e^2*(-a^9*c^3
)^(1/2) - 39*a^2*c*d^2*e^4*(-a^9*c^3)^(1/2))/(a^13*e^8 + a^9*c^4*d^8 + 4*a^
12*c*d^2*e^6 + 4*a^10*c^3*d^6*e^2 + 6*a^11*c^2*d^4*e^4))^(5/2)*40i - a^19*c
^6*d^14*e^7*x*(-(49*a^3*e^6*(-a^9*c^3)^(1/2) - 25*c^3*d^6*(-a^9*c^3)^(1/2)
+ 30*a^5*c^4*d^5*e + 126*a^7*c^2*d*e^5 + 124*a^6*c^3*d^3*e^3 - 81*a*c^2*d^4
e^2(-a^9*c^3)^(1/2) - 39*a^2*c*d^2*e^4*(-a^9*c^3)^(1/2))/(a^13*e^8 + a^9*
c^4*d^8 + 4*a^12*c*d^2*e^6 + 4*a^10*c^3*d^6*e^2 + 6*a^11*c^2*d^4*e^4))^(5/2
)*56i - a^20*c^5*d^12*e^9*x*(-(49*a^3*e^6*(-a^9*c^3)^(1/2) - 25*c^3*d^6*(-a
^9*c^3)^(1/2) + 30*a^5*c^4*d^5*e + 126*a^7*c^2*d*e^5 + 124*a^6*c^3*d^3*e^3
- 81*a*c^2*d^4*e^2*(-a^9*c^3)^(1/2) - 39*a^2*c*d^2*e^4*(-a^9*c^3)^(1/2))/(a
^13*e^8 + a^9*c^4*d^8 + 4*a^12*c*d^2*e^6 + 4*a^10*c^3*d^6*e^2 + 6*a^11*c^2*
d^4*e^4))^(5/2)*28i + a^21*c^4*d^10*e^11*x*(-(49*a^3*e^6*(-a^9*c^3)^(1/2) -
25*c^3*d^6*(-a^9*c^3)^(1/2) + 30*a^5*c^4*d^5*e + 126*a^7*c^2*d*e^5 + 124*a
^6*c^3*d^3*e^3 - 81*a*c^2*d^4*e^2*(-a^9*c^3)^(1/2) - 39*a^2*c*d^2*e^4*(-a^9
*c^3)^(1/2))/(a^13*e^8 + a^9*c^4*d^8 + 4*a^12*c*d^2*e^6 + 4*a^10*c^3*d^6*e^
2 + 6*a^11*c^2*d^4*e^4))^(5/2)*28i + a^22*c^3*d^8*e^13*x*(-(49*a^3*e^6*(-a^
9*c^3)^(1/2) - 25*c^3*d^6*(-a^9*c^3)^(1/2) + 30*a^5*c^4*d^5*e + 126*a^7*c^2
*d*e^5 + 124*a^6*c^3*d^3*e^3 - 81*a*c^2*d^4*e^2*(-a^9*c^3)^(1/2) - 39*a^2*c
*d^2*e^4*(-a^9*c^3)^(1/2))/(a^13*e^8 + a^9*c^4*d^8 + 4*a^12*c*d^2*e^6 + 4*a
^10*c^3*d^6*e^2 + 6*a^11*c^2*d^4*e^4))^(5/2)*56i + a^23*c^2*d^6*e^15*x*(-(4
9*a^3*e^6*(-a^9*c^3)^(1/2) - 25*c^3*d^6*(-a^9*c^3)^(1/2) + 30*a^5*c^4*d^5*e
+ 126*a^7*c^2*d*e^5 + 124*a^6*c^3*d^3*e^3 - 81*a*c^2*d^4*e^2*(-a^9*c^3)^(1
/2) - 39*a^2*c*d^2*e^4*(-a^9*c^3)^(1/2))/(a^13*e^8 + a^9*c^4*d^8 + 4*a^12*c
*d^2*e^6 + 4*a^10*c^3*d^6*e^2 + 6*a^11*c^2*d^4*e^4))^(5/2)*40i - a^20*c*d*e
^18*x*(-(49*a^3*e^6*(-a^9*c^3)^(1/2) - 25*c^3*d^6*(-a^9*c^3)^(1/2) + 30*a^5
*c^4*d^5*e + 126*a^7*c^2*d*e^5 + 124*a^6*c^3*d^3*e^3 - 81*a*c^2*d^4*e^2*(-a
^9*c^3)^(1/2) - 39*a^2*c*d^2*e^4*(-a^9*c^3)^(1/2))/(a^13*e^8 + a^9*c^4*d^8
+ 4*a^12*c*d^2*e^6 + 4*a^10*c^3*d^6*e^2 + 6*a^11*c^2*d^4*e^4))^(3/2)*128i))
/(9765625*a^9*c^21*d^32 + 481890304*a^25*c^5*e^32 + 159765625*a^10*c^20*d^3
0*e^2 + 1159031250*a^11*c^19*d^28*e^4 + 4879001250*a^12*c^18*d^26*e^6 + 130
43411775*a^13*c^17*d^24*e^8 + 22507897839*a^14*c^16*d^22*e^10 + 23209461788
*a^15*c^15*d^20*e^12 + 7790140604*a^16*c^14*d^18*e^14 - 15160518297*a^17*c^
13*d^16*e^16 - 24964288057*a^18*c^12*d^14*e^18 - 11511478798*a^19*c^11*d^12
*e^20 + 8613907074*a^20*c^10*d^10*e^22 + 11397074817*a^21*c^9*d^8*e^24 + 58

6708977*a^22*c^8*d^6*e^26 - 3576733440*a^23*c^7*d^4*e^28 - 521228288*a^24*c
^6*d^2*e^30))*(-(49*a^3*e^6*(-a^9*c^3)^(1/2) - 25*c^3*d^6*(-a^9*c^3)^(1/2)
+ 30*a^5*c^4*d^5*e + 126*a^7*c^2*d*e^5 + 124*a^6*c^3*d^3*e^3 - 81*a*c^2*d^4
e^2(-a^9*c^3)^(1/2) - 39*a^2*c*d^2*e^4*(-a^9*c^3)^(1/2))/(256*(a^13*e^8 +
a^9*c^4*d^8 + 4*a^12*c*d^2*e^6 + 4*a^10*c^3*d^6*e^2 + 6*a^11*c^2*d^4*e^4))
)^(1/2)*2i - atan(((11875*a^5*c^10*d^15*e - a^9*c^3*(72128*a^3*d*e^15 + 265
655*c^3*d^7*e^9 - 76440*a*c^2*d^5*e^11 - 178585*a^2*c*d^3*e^13) + 68800*a^6
*c^9*d^13*e^3 + 89403*a^7*c^8*d^11*e^5 - 126488*a^8*c^7*d^9*e^7)*(a^25*d^2*
e^19*x*(-(25*c^3*d^6*(-a^9*c^3)^(1/2) - 49*a^3*e^6*(-a^9*c^3)^(1/2) + 30*a^
5*c^4*d^5*e + 126*a^7*c^2*d*e^5 + 124*a^6*c^3*d^3*e^3 + 81*a*c^2*d^4*e^2*(-
a^9*c^3)^(1/2) + 39*a^2*c*d^2*e^4*(-a^9*c^3)^(1/2))/(a^13*e^8 + a^9*c^4*d^8
+ 4*a^12*c*d^2*e^6 + 4*a^10*c^3*d^6*e^2 + 6*a^11*c^2*d^4*e^4))^(5/2)*2i -
a^15*c^2*e^17*x*(-(25*c^3*d^6*(-a^9*c^3)^(1/2) - 49*a^3*e^6*(-a^9*c^3)^(1/2
) + 30*a^5*c^4*d^5*e + 126*a^7*c^2*d*e^5 + 124*a^6*c^3*d^3*e^3 + 81*a*c^2*d
^4*e^2*(-a^9*c^3)^(1/2) + 39*a^2*c*d^2*e^4*(-a^9*c^3)^(1/2))/(a^13*e^8 + a^
9*c^4*d^8 + 4*a^12*c*d^2*e^6 + 4*a^10*c^3*d^6*e^2 + 6*a^11*c^2*d^4*e^4))^(1
/2)*3136i - a^11*c^10*d^19*x*(-(25*c^3*d^6*(-a^9*c^3)^(1/2) - 49*a^3*e^6*(-
a^9*c^3)^(1/2) + 30*a^5*c^4*d^5*e + 126*a^7*c^2*d*e^5 + 124*a^6*c^3*d^3*e^3
+ 81*a*c^2*d^4*e^2*(-a^9*c^3)^(1/2) + 39*a^2*c*d^2*e^4*(-a^9*c^3)^(1/2))/(
a^13*e^8 + a^9*c^4*d^8 + 4*a^12*c*d^2*e^6 + 4*a^10*c^3*d^6*e^2 + 6*a^11*c^2
*d^4*e^4))^(3/2)*25i - a^16*c^9*d^20*e*x*(-(25*c^3*d^6*(-a^9*c^3)^(1/2) - 4
9*a^3*e^6*(-a^9*c^3)^(1/2) + 30*a^5*c^4*d^5*e + 126*a^7*c^2*d*e^5 + 124*a^6
*c^3*d^3*e^3 + 81*a*c^2*d^4*e^2*(-a^9*c^3)^(1/2) + 39*a^2*c*d^2*e^4*(-a^9*c
^3)^(1/2))/(a^13*e^8 + a^9*c^4*d^8 + 4*a^12*c*d^2*e^6 + 4*a^10*c^3*d^6*e^2
+ 6*a^11*c^2*d^4*e^4))^(5/2)*2i + a^24*c*d^4*e^17*x*(-(25*c^3*d^6*(-a^9*c^3
)^(1/2) - 49*a^3*e^6*(-a^9*c^3)^(1/2) + 30*a^5*c^4*d^5*e + 126*a^7*c^2*d*e^
5 + 124*a^6*c^3*d^3*e^3 + 81*a*c^2*d^4*e^2*(-a^9*c^3)^(1/2) + 39*a^2*c*d^2*
e^4*(-a^9*c^3)^(1/2))/(a^13*e^8 + a^9*c^4*d^8 + 4*a^12*c*d^2*e^6 + 4*a^10*c
^3*d^6*e^2 + 6*a^11*c^2*d^4*e^4))^(5/2)*14i + a^8*c^9*d^14*e^3*x*(-(25*c^3*
d^6*(-a^9*c^3)^(1/2) - 49*a^3*e^6*(-a^9*c^3)^(1/2) + 30*a^5*c^4*d^5*e + 126
*a^7*c^2*d*e^5 + 124*a^6*c^3*d^3*e^3 + 81*a*c^2*d^4*e^2*(-a^9*c^3)^(1/2) +
39*a^2*c*d^2*e^4*(-a^9*c^3)^(1/2))/(a^13*e^8 + a^9*c^4*d^8 + 4*a^12*c*d^2*e
^6 + 4*a^10*c^3*d^6*e^2 + 6*a^11*c^2*d^4*e^4))^(1/2)*1250i + a^9*c^8*d^12*e
^5*x*(-(25*c^3*d^6*(-a^9*c^3)^(1/2) - 49*a^3*e^6*(-a^9*c^3)^(1/2) + 30*a^5*
c^4*d^5*e + 126*a^7*c^2*d*e^5 + 124*a^6*c^3*d^3*e^3 + 81*a*c^2*d^4*e^2*(-a^
9*c^3)^(1/2) + 39*a^2*c*d^2*e^4*(-a^9*c^3)^(1/2))/(a^13*e^8 + a^9*c^4*d^8 +
4*a^12*c*d^2*e^6 + 4*a^10*c^3*d^6*e^2 + 6*a^11*c^2*d^4*e^4))^(1/2)*9900i +
a^10*c^7*d^10*e^7*x*(-(25*c^3*d^6*(-a^9*c^3)^(1/2) - 49*a^3*e^6*(-a^9*c^3)
^(1/2) + 30*a^5*c^4*d^5*e + 126*a^7*c^2*d*e^5 + 124*a^6*c^3*d^3*e^3 + 81*a*
c^2*d^4*e^2*(-a^9*c^3)^(1/2) + 39*a^2*c*d^2*e^4*(-a^9*c^3)^(1/2))/(a^13*e^8
+ a^9*c^4*d^8 + 4*a^12*c*d^2*e^6 + 4*a^10*c^3*d^6*e^2 + 6*a^11*c^2*d^4*e^4
))^(1/2)*31902i + a^11*c^6*d^8*e^9*x*(-(25*c^3*d^6*(-a^9*c^3)^(1/2) - 49*a^
3*e^6*(-a^9*c^3)^(1/2) + 30*a^5*c^4*d^5*e + 126*a^7*c^2*d*e^5 + 124*a^6*c^3
*d^3*e^3 + 81*a*c^2*d^4*e^2*(-a^9*c^3)^(1/2) + 39*a^2*c*d^2*e^4*(-a^9*c^3)^
(1/2))/(a^13*e^8 + a^9*c^4*d^8 + 4*a^12*c*d^2*e^6 + 4*a^10*c^3*d^6*e^2 + 6*

a^11*c^2*d^4*e^4))^(1/2)*52008i + a^12*c^5*d^6*e^11*x*(-(25*c^3*d^6*(-a^9*c^3)^(1/2) - 49*a^3*e^6*(-a^9*c^3)^(1/2) + 30*a^5*c^4*d^5*e + 126*a^7*c^2*d*e^5 + 124*a^6*c^3*d^3*e^3 + 81*a*c^2*d^4*e^2*(-a^9*c^3)^(1/2) + 39*a^2*c*d^2*e^4*(-a^9*c^3)^(1/2))/(a^13*e^8 + a^9*c^4*d^8 + 4*a^12*c*d^2*e^6 + 4*a^10*c^3*d^6*e^2 + 6*a^11*c^2*d^4*e^4))^(1/2)*42238i + a^13*c^4*d^4*e^13*x*(-(25*c^3*d^6*(-a^9*c^3)^(1/2) - 49*a^3*e^6*(-a^9*c^3)^(1/2) + 30*a^5*c^4*d^5*e + 126*a^7*c^2*d*e^5 + 124*a^6*c^3*d^3*e^3 + 81*a*c^2*d^4*e^2*(-a^9*c^3)^(1/2) + 39*a^2*c*d^2*e^4*(-a^9*c^3)^(1/2))/(a^13*e^8 + a^9*c^4*d^8 + 4*a^12*c*d^2*e^6 + 4*a^10*c^3*d^6*e^2 + 6*a^11*c^2*d^4*e^4))^(1/2)*10924i - a^14*c^3*d^2*e^15*x*(-(25*c^3*d^6*(-a^9*c^3)^(1/2) - 49*a^3*e^6*(-a^9*c^3)^(1/2) + 30*a^5*c^4*d^5*e + 126*a^7*c^2*d*e^5 + 124*a^6*c^3*d^3*e^3 + 81*a*c^2*d^4*e^2*(-a^9*c^3)^(1/2) + 39*a^2*c*d^2*e^4*(-a^9*c^3)^(1/2))/(a^13*e^8 + a^9*c^4*d^8 + 4*a^12*c*d^2*e^6 + 4*a^10*c^3*d^6*e^2 + 6*a^11*c^2*d^4*e^4))^(1/2)*5694i - a^12*c^9*d^17*e^2*x*(-(25*c^3*d^6*(-a^9*c^3)^(1/2) - 49*a^3*e^6*(-a^9*c^3)^(1/2) + 30*a^5*c^4*d^5*e + 126*a^7*c^2*d*e^5 + 124*a^6*c^3*d^3*e^3 + 81*a*c^2*d^4*e^2*(-a^9*c^3)^(1/2) + 39*a^2*c*d^2*e^4*(-a^9*c^3)^(1/2))/(a^13*e^8 + a^9*c^4*d^8 + 4*a^12*c*d^2*e^6 + 4*a^10*c^3*d^6*e^2 + 6*a^11*c^2*d^4*e^4))^(3/2)*216i - a^13*c^8*d^15*e^4*x*(-(25*c^3*d^6*(-a^9*c^3)^(1/2) - 49*a^3*e^6*(-a^9*c^3)^(1/2) + 30*a^5*c^4*d^5*e + 126*a^7*c^2*d*e^5 + 124*a^6*c^3*d^3*e^3 + 81*a*c^2*d^4*e^2*(-a^9*c^3)^(1/2) + 39*a^2*c*d^2*e^4*(-a^9*c^3)^(1/2))/(a^13*e^8 + a^9*c^4*d^8 + 4*a^12*c*d^2*e^6 + 4*a^10*c^3*d^6*e^2 + 6*a^11*c^2*d^4*e^4))^(3/2)*700i - a^14*c^7*d^13*e^6*x*(-(25*c^3*d^6*(-a^9*c^3)^(1/2) - 49*a^3*e^6*(-a^9*c^3)^(1/2) + 30*a^5*c^4*d^5*e + 126*a^7*c^2*d*e^5 + 124*a^6*c^3*d^3*e^3 + 81*a*c^2*d^4*e^2*(-a^9*c^3)^(1/2) + 39*a^2*c*d^2*e^4*(-a^9*c^3)^(1/2))/(a^13*e^8 + a^9*c^4*d^8 + 4*a^12*c*d^2*e^6 + 4*a^10*c^3*d^6*e^2 + 6*a^11*c^2*d^4*e^4))^(3/2)*808i + a^15*c^6*d^11*e^8*x*(-(25*c^3*d^6*(-a^9*c^3)^(1/2) - 49*a^3*e^6*(-a^9*c^3)^(1/2) + 30*a^5*c^4*d^5*e + 126*a^7*c^2*d*e^5 + 124*a^6*c^3*d^3*e^3 + 81*a*c^2*d^4*e^2*(-a^9*c^3)^(1/2) + 39*a^2*c*d^2*e^4*(-a^9*c^3)^(1/2))/(a^13*e^8 + a^9*c^4*d^8 + 4*a^12*c*d^2*e^6 + 4*a^10*c^3*d^6*e^2 + 6*a^11*c^2*d^4*e^4))^(3/2)*778i + a^16*c^5*d^9*e^10*x*(-(25*c^3*d^6*(-a^9*c^3)^(1/2) - 49*a^3*e^6*(-a^9*c^3)^(1/2) + 30*a^5*c^4*d^5*e + 126*a^7*c^2*d*e^5 + 124*a^6*c^3*d^3*e^3 + 81*a*c^2*d^4*e^2*(-a^9*c^3)^(1/2) + 39*a^2*c*d^2*e^4*(-a^9*c^3)^(1/2))/(a^13*e^8 + a^9*c^4*d^8 + 4*a^12*c*d^2*e^6 + 4*a^10*c^3*d^6*e^2 + 6*a^11*c^2*d^4*e^4))^(3/2)*3224i + a^17*c^4*d^7*e^12*x*(-(25*c^3*d^6*(-a^9*c^3)^(1/2) - 49*a^3*e^6*(-a^9*c^3)^(1/2) + 30*a^5*c^4*d^5*e + 126*a^7*c^2*d*e^5 + 124*a^6*c^3*d^3*e^3 + 81*a*c^2*d^4*e^2*(-a^9*c^3)^(1/2) + 39*a^2*c*d^2*e^4*(-a^9*c^3)^(1/2))/(a^13*e^8 + a^9*c^4*d^8 + 4*a^12*c*d^2*e^6 + 4*a^10*c^3*d^6*e^2 + 6*a^11*c^2*d^4*e^4))^(3/2)*3460i + a^18*c^3*d^5*e^14*x*(-(25*c^3*d^6*(-a^9*c^3)^(1/2) - 49*a^3*e^6*(-a^9*c^3)^(1/2) + 30*a^5*c^4*d^5*e + 126*a^7*c^2*d*e^5 + 124*a^6*c^3*d^3*e^3 + 81*a*c^2*d^4*e^2*(-a^9*c^3)^(1/2) + 39*a^2*c*d^2*e^4*(-a^9*c^3)^(1/2))/(a^13*e^8 + a^9*c^4*d^8 + 4*a^12*c*d^2*e^6 + 4*a^10*c^3*d^6*e^2 + 6*a^11*c^2*d^4*e^4))^(3/2)*1384i - a^19*c^2*d^3*e^16*x*(-(25*c^3*d^6*(-a^9*c^3)^(1/2) - 49*a^3*e^6*(-a^9*c^3)^(1/2) + 30*a^5*c^4*d^5*e + 126*a^7*c^2*d*e^5 + 124*a^6*c^3*d^3*e^3 + 81*a*c^2*d^4*e^2*(-a^9*c^3)^(1/2) + 39* a^2*c*d^2*e^4*(-a^9*c^3)^(1/2))/(a^13*e^8 + a^9*c^4*d^8 + 4*a^12*c*d^2*e^6 + 4*a^10*c^3*d^6*e^2 + 6*a^11*c^2*d^4*e^4))^(3/2)*57i - a^17*c^8*d^18*e^3*x *(-(25*c^3*d^6*(-a^9*c^3)^(1/2) - 49*a^3*e^6*(-a^9*c^3)^(1/2) + 30*a^5*c^4* d^5*e + 126*a^7*c^2*d*e^5 + 124*a^6*c^3*d^3*e^3 + 81*a*c^2*d^4*e^2*(-a^9*c^ 3)^(1/2) + 39*a^2*c*d^2*e^4*(-a^9*c^3)^(1/2))/(a^13*e^8 + a^9*c^4*d^8 + 4*a ^12*c*d^2*e^6 + 4*a^10*c^3*d^6*e^2 + 6*a^11*c^2*d^4*e^4))^(5/2)*14i - a^18* c^7*d^16*e^5*x*(-(25*c^3*d^6*(-a^9*c^3)^(1/2) - 49*a^3*e^6*(-a^9*c^3)^(1/2) + 30*a^5*c^4*d^5*e + 126*a^7*c^2*d*e^5 + 124*a^6*c^3*d^3*e^3 + 81*a*c^2*d^ 4*e^2*(-a^9*c^3)^(1/2) + 39*a^2*c*d^2*e^4*(-a^9*c^3)^(1/2))/(a^13*e^8 + a^9 *c^4*d^8 + 4*a^12*c*d^2*e^6 + 4*a^10*c^3*d^6*e^2 + 6*a^11*c^2*d^4*e^4))^(5/ 2)*40i - a^19*c^6*d^14*e^7*x*(-(25*c^3*d^6*(-a^9*c^3)^(1/2) - 49*a^3*e^6*(- a^9*c^3)^(1/2) + 30*a^5*c^4*d^5*e + 126*a^7*c^2*d*e^5 + 124*a^6*c^3*d^3*e^3 + 81*a*c^2*d^4*e^2*(-a^9*c^3)^(1/2) + 39*a^2*c*d^2*e^4*(-a^9*c^3)^(1/2))/(a^13*e^8 + a^9*c^4*d^8 + 4*a^12*c*d^2*e^6 + 4*a^10*c^3*d^6*e^2 + 6*a^11*c^2 *d^4*e^4))^(5/2)*56i - a^20*c^5*d^12*e^9*x*(-(25*c^3*d^6*(-a^9*c^3)^(1/2) - 49*a^3*e^6*(-a^9*c^3)^(1/2) + 30*a^5*c^4*d^5*e + 126*a^7*c^2*d*e^5 + 124*a ^6*c^3*d^3*e^3 + 81*a*c^2*d^4*e^2*(-a^9*c^3)^(1/2) + 39*a^2*c*d^2*e^4*(-a^9 *c^3)^(1/2))/(a^13*e^8 + a^9*c^4*d^8 + 4*a^12*c*d^2*e^6 + 4*a^10*c^3*d^6*e^ 2 + 6*a^11*c^2*d^4*e^4))^(5/2)*28i + a^21*c^4*d^10*e^11*x*(-(25*c^3*d^6*(-a ^9*c^3)^(1/2) - 49*a^3*e^6*(-a^9*c^3)^(1/2) + 30*a^5*c^4*d^5*e + 126*a^7*c^ 2*d*e^5 + 124*a^6*c^3*d^3*e^3 + 81*a*c^2*d^4*e^2*(-a^9*c^3)^(1/2) + 39*a^2* c*d^2*e^4*(-a^9*c^3)^(1/2))/(a^13*e^8 + a^9*c^4*d^8 + 4*a^12*c*d^2*e^6 + 4* a^10*c^3*d^6*e^2 + 6*a^11*c^2*d^4*e^4))^(5/2)*28i + a^22*c^3*d^8*e^13*x*(-(25*c^3*d^6*(-a^9*c^3)^(1/2) - 49*a^3*e^6*(-a^9*c^3)^(1/2) + 30*a^5*c^4*d^5* e + 126*a^7*c^2*d*e^5 + 124*a^6*c^3*d^3*e^3 + 81*a*c^2*d^4*e^2*(-a^9*c^3)^(1/2) + 39*a^2*c*d^2*e^4*(-a^9*c^3)^(1/2))/(a^13*e^8 + a^9*c^4*d^8 + 4*a^12* c*d^2*e^6 + 4*a^10*c^3*d^6*e^2 + 6*a^11*c^2*d^4*e^4))^(5/2)*56i + a^23*c^2* d^6*e^15*x*(-(25*c^3*d^6*(-a^9*c^3)^(1/2) - 49*a^3*e^6*(-a^9*c^3)^(1/2) + 3 0*a^5*c^4*d^5*e + 126*a^7*c^2*d*e^5 + 124*a^6*c^3*d^3*e^3 + 81*a*c^2*d^4*e^ 2*(-a^9*c^3)^(1/2) + 39*a^2*c*d^2*e^4*(-a^9*c^3)^(1/2))/(a^13*e^8 + a^9*c^4 *d^8 + 4*a^12*c*d^2*e^6 + 4*a^10*c^3*d^6*e^2 + 6*a^11*c^2*d^4*e^4))^(5/2)*4 0i - a^20*c*d*e^18*x*(-(25*c^3*d^6*(-a^9*c^3)^(1/2) - 49*a^3*e^6*(-a^9*c^3) ^(1/2) + 30*a^5*c^4*d^5*e + 126*a^7*c^2*d*e^5 + 124*a^6*c^3*d^3*e^3 + 81*a* c^2*d^4*e^2*(-a^9*c^3)^(1/2) + 39*a^2*c*d^2*e^4*(-a^9*c^3)^(1/2))/(a^13*e^8 + a^9*c^4*d^8 + 4*a^12*c*d^2*e^6 + 4*a^10*c^3*d^6*e^2 + 6*a^11*c^2*d^4*e^4))^(3/2)*128i) - (-a^9*c^3)^(1/2)*(3125*c^9*d^16 + 21952*a^8*c*e^16 + 3000* a*c^8*d^14*e^2 - 77435*a^2*c^7*d^12*e^4 - 242104*a^3*c^6*d^10*e^6 - 127665* a^4*c^5*d^8*e^8 + 240064*a^5*c^4*d^6*e^10 + 118199*a^6*c^3*d^4*e^12 - 13036 8*a^7*c^2*d^2*e^14)*(a^25*d^2*e^19*x*(-(25*c^3*d^6*(-a^9*c^3)^(1/2) - 49*a^ 3*e^6*(-a^9*c^3)^(1/2) + 30*a^5*c^4*d^5*e + 126*a^7*c^2*d*e^5 + 124*a^6*c^3 *d^3*e^3 + 81*a*c^2*d^4*e^2*(-a^9*c^3)^(1/2) + 39*a^2*c*d^2*e^4*(-a^9*c^3)^ (1/2))/(a^13*e^8 + a^9*c^4*d^8 + 4*a^12*c*d^2*e^6 + 4*a^10*c^3*d^6*e^2 + 6* a^11*c^2*d^4*e^4))^(5/2)*2i - a^15*c^2*e^17*x*(-(25*c^3*d^6*(-a^9*c^3)^(1/2) - 49*a^3*e^6*(-a^9*c^3)^(1/2) + 30*a^5*c^4*d^5*e + 126*a^7*c^2*d*e^5 + 12 4*a^6*c^3*d^3*e^3 + 81*a*c^2*d^4*e^2*(-a^9*c^3)^(1/2) + 39*a^2*c*d^2*e^4*(-

a^9*c^3)^(1/2))/(a^13*e^8 + a^9*c^4*d^8 + 4*a^12*c*d^2*e^6 + 4*a^10*c^3*d^6*e^2 + 6*a^11*c^2*d^4*e^4))^(1/2)*3136i - a^11*c^10*d^19*x*(-(25*c^3*d^6*(-a^9*c^3)^(1/2) - 49*a^3*e^6*(-a^9*c^3)^(1/2) + 30*a^5*c^4*d^5*e + 126*a^7*c^2*d*e^5 + 124*a^6*c^3*d^3*e^3 + 81*a*c^2*d^4*e^2*(-a^9*c^3)^(1/2) + 39*a^2*c*d^2*e^4*(-a^9*c^3)^(1/2))/(a^13*e^8 + a^9*c^4*d^8 + 4*a^12*c*d^2*e^6 + 4*a^10*c^3*d^6*e^2 + 6*a^11*c^2*d^4*e^4))^(3/2)*25i - a^16*c^9*d^20*e*x*(-(25*c^3*d^6*(-a^9*c^3)^(1/2) - 49*a^3*e^6*(-a^9*c^3)^(1/2) + 30*a^5*c^4*d^5*e + 126*a^7*c^2*d*e^5 + 124*a^6*c^3*d^3*e^3 + 81*a*c^2*d^4*e^2*(-a^9*c^3)^(1/2) + 39*a^2*c*d^2*e^4*(-a^9*c^3)^(1/2))/(a^13*e^8 + a^9*c^4*d^8 + 4*a^12*c*d^2*e^6 + 4*a^10*c^3*d^6*e^2 + 6*a^11*c^2*d^4*e^4))^(5/2)*2i + a^24*c*d^4*e^17*x*(-(25*c^3*d^6*(-a^9*c^3)^(1/2) - 49*a^3*e^6*(-a^9*c^3)^(1/2) + 30*a^5*c^4*d^5*e + 126*a^7*c^2*d*e^5 + 124*a^6*c^3*d^3*e^3 + 81*a*c^2*d^4*e^2*(-a^9*c^3)^(1/2) + 39*a^2*c*d^2*e^4*(-a^9*c^3)^(1/2))/(a^13*e^8 + a^9*c^4*d^8 + 4*a^12*c*d^2*e^6 + 4*a^10*c^3*d^6*e^2 + 6*a^11*c^2*d^4*e^4))^(5/2)*14i + a^8*c^9*d^14*e^3*x*(-(25*c^3*d^6*(-a^9*c^3)^(1/2) - 49*a^3*e^6*(-a^9*c^3)^(1/2) + 30*a^5*c^4*d^5*e + 126*a^7*c^2*d*e^5 + 124*a^6*c^3*d^3*e^3 + 81*a*c^2*d^4*e^2*(-a^9*c^3)^(1/2) + 39*a^2*c*d^2*e^4*(-a^9*c^3)^(1/2))/(a^13*e^8 + a^9*c^4*d^8 + 4*a^12*c*d^2*e^6 + 4*a^10*c^3*d^6*e^2 + 6*a^11*c^2*d^4*e^4))^(1/2)*1250i + a^9*c^8*d^12*e^5*x*(-(25*c^3*d^6*(-a^9*c^3)^(1/2) - 49*a^3*e^6*(-a^9*c^3)^(1/2) + 30*a^5*c^4*d^5*e + 126*a^7*c^2*d*e^5 + 124*a^6*c^3*d^3*e^3 + 81*a*c^2*d^4*e^2*(-a^9*c^3)^(1/2) + 39*a^2*c*d^2*e^4*(-a^9*c^3)^(1/2))/(a^13*e^8 + a^9*c^4*d^8 + 4*a^12*c*d^2*e^6 + 4*a^10*c^3*d^6*e^2 + 6*a^11*c^2*d^4*e^4))^(1/2)*9900i + a^10*c^7*d^10*e^7*x*(-(25*c^3*d^6*(-a^9*c^3)^(1/2) - 49*a^3*e^6*(-a^9*c^3)^(1/2) + 30*a^5*c^4*d^5*e + 126*a^7*c^2*d*e^5 + 124*a^6*c^3*d^3*e^3 + 81*a*c^2*d^4*e^2*(-a^9*c^3)^(1/2) + 39*a^2*c*d^2*e^4*(-a^9*c^3)^(1/2))/(a^13*e^8 + a^9*c^4*d^8 + 4*a^12*c*d^2*e^6 + 4*a^10*c^3*d^6*e^2 + 6*a^11*c^2*d^4*e^4))^(1/2)*31902i + a^11*c^6*d^8*e^9*x*(-(25*c^3*d^6*(-a^9*c^3)^(1/2) - 49*a^3*e^6*(-a^9*c^3)^(1/2) + 30*a^5*c^4*d^5*e + 126*a^7*c^2*d*e^5 + 124*a^6*c^3*d^3*e^3 + 81*a*c^2*d^4*e^2*(-a^9*c^3)^(1/2) + 39*a^2*c*d^2*e^4*(-a^9*c^3)^(1/2))/(a^13*e^8 + a^9*c^4*d^8 + 4*a^12*c*d^2*e^6 + 4*a^10*c^3*d^6*e^2 + 6*a^11*c^2*d^4*e^4))^(1/2)*52008i + a^12*c^5*d^6*e^11*x*(-(25*c^3*d^6*(-a^9*c^3)^(1/2) - 49*a^3*e^6*(-a^9*c^3)^(1/2) + 30*a^5*c^4*d^5*e + 126*a^7*c^2*d*e^5 + 124*a^6*c^3*d^3*e^3 + 81*a*c^2*d^4*e^2*(-a^9*c^3)^(1/2) + 39*a^2*c*d^2*e^4*(-a^9*c^3)^(1/2))/(a^13*e^8 + a^9*c^4*d^8 + 4*a^12*c*d^2*e^6 + 4*a^10*c^3*d^6*e^2 + 6*a^11*c^2*d^4*e^4))^(1/2)*42238i + a^13*c^4*d^4*e^13*x*(-(25*c^3*d^6*(-a^9*c^3)^(1/2) - 49*a^3*e^6*(-a^9*c^3)^(1/2) + 30*a^5*c^4*d^5*e + 126*a^7*c^2*d*e^5 + 124*a^6*c^3*d^3*e^3 + 81*a*c^2*d^4*e^2*(-a^9*c^3)^(1/2) + 39*a^2*c*d^2*e^4*(-a^9*c^3)^(1/2))/(a^13*e^8 + a^9*c^4*d^8 + 4*a^12*c*d^2*e^6 + 4*a^10*c^3*d^6*e^2 + 6*a^11*c^2*d^4*e^4))^(1/2)*10924i - a^14*c^3*d^2*e^15*x*(-(25*c^3*d^6*(-a^9*c^3)^(1/2) - 49*a^3*e^6*(-a^9*c^3)^(1/2) + 30*a^5*c^4*d^5*e + 126*a^7*c^2*d*e^5 + 124*a^6*c^3*d^3*e^3 + 81*a*c^2*d^4*e^2*(-a^9*c^3)^(1/2) + 39*a^2*c*d^2*e^4*(-a^9*c^3)^(1/2))/(a^13*e^8 + a^9*c^4*d^8 + 4*a^12*c*d^2*e^6 + 4*a^10*c^3*d^6*e^2 + 6*a^11*c^2*d^4*e^4))^(1/2)*5694i - a^12*c^9*d^17*e^2*x*(-(25*c^3*d^6*(-a^9*c^3)^(1/2) - 49*a^3*e^6*(-a^9*c^3)^(1/2) + 30*a^5*c^4*d^5*e + 126*a^7*c

^2*d*e^5 + 124*a^6*c^3*d^3*e^3 + 81*a*c^2*d^4*e^2*(-a^9*c^3)^(1/2) + 39*a^2
*c*d^2*e^4*(-a^9*c^3)^(1/2))/(a^13*e^8 + a^9*c^4*d^8 + 4*a^12*c*d^2*e^6 + 4
*a^10*c^3*d^6*e^2 + 6*a^11*c^2*d^4*e^4))^(3/2)*216i - a^13*c^8*d^15*e^4*x*(
-(25*c^3*d^6*(-a^9*c^3)^(1/2) - 49*a^3*e^6*(-a^9*c^3)^(1/2) + 30*a^5*c^4*d^
5*e + 126*a^7*c^2*d*e^5 + 124*a^6*c^3*d^3*e^3 + 81*a*c^2*d^4*e^2*(-a^9*c^3)
^(1/2) + 39*a^2*c*d^2*e^4*(-a^9*c^3)^(1/2))/(a^13*e^8 + a^9*c^4*d^8 + 4*a^1
2*c*d^2*e^6 + 4*a^10*c^3*d^6*e^2 + 6*a^11*c^2*d^4*e^4))^(3/2)*700i - a^14*c
^7*d^13*e^6*x*(-(25*c^3*d^6*(-a^9*c^3)^(1/2) - 49*a^3*e^6*(-a^9*c^3)^(1/2)
+ 30*a^5*c^4*d^5*e + 126*a^7*c^2*d*e^5 + 124*a^6*c^3*d^3*e^3 + 81*a*c^2*d^4
e^2(-a^9*c^3)^(1/2) + 39*a^2*c*d^2*e^4*(-a^9*c^3)^(1/2))/(a^13*e^8 + a^9*
c^4*d^8 + 4*a^12*c*d^2*e^6 + 4*a^10*c^3*d^6*e^2 + 6*a^11*c^2*d^4*e^4))^(3/2
)*808i + a^15*c^6*d^11*e^8*x*(-(25*c^3*d^6*(-a^9*c^3)^(1/2) - 49*a^3*e^6*(-
a^9*c^3)^(1/2) + 30*a^5*c^4*d^5*e + 126*a^7*c^2*d*e^5 + 124*a^6*c^3*d^3*e^3
 + 81*a*c^2*d^4*e^2*(-a^9*c^3)^(1/2) + 39*a^2*c*d^2*e^4*(-a^9*c^3)^(1/2))/(
a^13*e^8 + a^9*c^4*d^8 + 4*a^12*c*d^2*e^6 + 4*a^10*c^3*d^6*e^2 + 6*a^11*c^2
*d^4*e^4))^(3/2)*778i + a^16*c^5*d^9*e^10*x*(-(25*c^3*d^6*(-a^9*c^3)^(1/2)
- 49*a^3*e^6*(-a^9*c^3)^(1/2) + 30*a^5*c^4*d^5*e + 126*a^7*c^2*d*e^5 + 124*
a^6*c^3*d^3*e^3 + 81*a*c^2*d^4*e^2*(-a^9*c^3)^(1/2) + 39*a^2*c*d^2*e^4*(-a^
9*c^3)^(1/2))/(a^13*e^8 + a^9*c^4*d^8 + 4*a^12*c*d^2*e^6 + 4*a^10*c^3*d^6*e
^2 + 6*a^11*c^2*d^4*e^4))^(3/2)*3224i + a^17*c^4*d^7*e^12*x*(-(25*c^3*d^6*(
-a^9*c^3)^(1/2) - 49*a^3*e^6*(-a^9*c^3)^(1/2) + 30*a^5*c^4*d^5*e + 126*a^7*
c^2*d*e^5 + 124*a^6*c^3*d^3*e^3 + 81*a*c^2*d^4*e^2*(-a^9*c^3)^(1/2) + 39*a^
2*c*d^2*e^4*(-a^9*c^3)^(1/2))/(a^13*e^8 + a^9*c^4*d^8 + 4*a^12*c*d^2*e^6 +
4*a^10*c^3*d^6*e^2 + 6*a^11*c^2*d^4*e^4))^(3/2)*3460i + a^18*c^3*d^5*e^14*x
*(-(25*c^3*d^6*(-a^9*c^3)^(1/2) - 49*a^3*e^6*(-a^9*c^3)^(1/2) + 30*a^5*c^4*
d^5*e + 126*a^7*c^2*d*e^5 + 124*a^6*c^3*d^3*e^3 + 81*a*c^2*d^4*e^2*(-a^9*c^
3)^(1/2) + 39*a^2*c*d^2*e^4*(-a^9*c^3)^(1/2))/(a^13*e^8 + a^9*c^4*d^8 + 4*a
^12*c*d^2*e^6 + 4*a^10*c^3*d^6*e^2 + 6*a^11*c^2*d^4*e^4))^(3/2)*1384i - a^1
9*c^2*d^3*e^16*x*(-(25*c^3*d^6*(-a^9*c^3)^(1/2) - 49*a^3*e^6*(-a^9*c^3)^(1/
2) + 30*a^5*c^4*d^5*e + 126*a^7*c^2*d*e^5 + 124*a^6*c^3*d^3*e^3 + 81*a*c^2*
d^4*e^2*(-a^9*c^3)^(1/2) + 39*a^2*c*d^2*e^4*(-a^9*c^3)^(1/2))/(a^13*e^8 + a
^9*c^4*d^8 + 4*a^12*c*d^2*e^6 + 4*a^10*c^3*d^6*e^2 + 6*a^11*c^2*d^4*e^4))^(
3/2)*57i - a^17*c^8*d^18*e^3*x*(-(25*c^3*d^6*(-a^9*c^3)^(1/2) - 49*a^3*e^6*
(-a^9*c^3)^(1/2) + 30*a^5*c^4*d^5*e + 126*a^7*c^2*d*e^5 + 124*a^6*c^3*d^3*e
^3 + 81*a*c^2*d^4*e^2*(-a^9*c^3)^(1/2) + 39*a^2*c*d^2*e^4*(-a^9*c^3)^(1/2))
/(a^13*e^8 + a^9*c^4*d^8 + 4*a^12*c*d^2*e^6 + 4*a^10*c^3*d^6*e^2 + 6*a^11*c
^2*d^4*e^4))^(5/2)*14i - a^18*c^7*d^16*e^5*x*(-(25*c^3*d^6*(-a^9*c^3)^(1/2)
 - 49*a^3*e^6*(-a^9*c^3)^(1/2) + 30*a^5*c^4*d^5*e + 126*a^7*c^2*d*e^5 + 124
*a^6*c^3*d^3*e^3 + 81*a*c^2*d^4*e^2*(-a^9*c^3)^(1/2) + 39*a^2*c*d^2*e^4*(-a
^9*c^3)^(1/2))/(a^13*e^8 + a^9*c^4*d^8 + 4*a^12*c*d^2*e^6 + 4*a^10*c^3*d^6*
e^2 + 6*a^11*c^2*d^4*e^4))^(5/2)*40i - a^19*c^6*d^14*e^7*x*(-(25*c^3*d^6*(-
a^9*c^3)^(1/2) - 49*a^3*e^6*(-a^9*c^3)^(1/2) + 30*a^5*c^4*d^5*e + 126*a^7*c
^2*d*e^5 + 124*a^6*c^3*d^3*e^3 + 81*a*c^2*d^4*e^2*(-a^9*c^3)^(1/2) + 39*a^2
*c*d^2*e^4*(-a^9*c^3)^(1/2))/(a^13*e^8 + a^9*c^4*d^8 + 4*a^12*c*d^2*e^6 + 4
*a^10*c^3*d^6*e^2 + 6*a^11*c^2*d^4*e^4))^(5/2)*56i - a^20*c^5*d^12*e^9*x*(-

(25*c^3*d^6*(-a^9*c^3)^(1/2) - 49*a^3*e^6*(-a^9*c^3)^(1/2) + 30*a^5*c^4*d^5
*e + 126*a^7*c^2*d*e^5 + 124*a^6*c^3*d^3*e^3 + 81*a*c^2*d^4*e^2*(-a^9*c^3)^
(1/2) + 39*a^2*c*d^2*e^4*(-a^9*c^3)^(1/2))/(a^13*e^8 + a^9*c^4*d^8 + 4*a^12
*c*d^2*e^6 + 4*a^10*c^3*d^6*e^2 + 6*a^11*c^2*d^4*e^4))^(5/2)*28i + a^21*c^4
*d^10*e^11*x*(-(25*c^3*d^6*(-a^9*c^3)^(1/2) - 49*a^3*e^6*(-a^9*c^3)^(1/2) +
30*a^5*c^4*d^5*e + 126*a^7*c^2*d*e^5 + 124*a^6*c^3*d^3*e^3 + 81*a*c^2*d^4*
e^2*(-a^9*c^3)^(1/2) + 39*a^2*c*d^2*e^4*(-a^9*c^3)^(1/2))/(a^13*e^8 + a^9*c
^4*d^8 + 4*a^12*c*d^2*e^6 + 4*a^10*c^3*d^6*e^2 + 6*a^11*c^2*d^4*e^4))^(5/2)
*28i + a^22*c^3*d^8*e^13*x*(-(25*c^3*d^6*(-a^9*c^3)^(1/2) - 49*a^3*e^6*(-a^
9*c^3)^(1/2) + 30*a^5*c^4*d^5*e + 126*a^7*c^2*d*e^5 + 124*a^6*c^3*d^3*e^3 +
81*a*c^2*d^4*e^2*(-a^9*c^3)^(1/2) + 39*a^2*c*d^2*e^4*(-a^9*c^3)^(1/2))/(a^
13*e^8 + a^9*c^4*d^8 + 4*a^12*c*d^2*e^6 + 4*a^10*c^3*d^6*e^2 + 6*a^11*c^2*d
^4*e^4))^(5/2)*56i + a^23*c^2*d^6*e^15*x*(-(25*c^3*d^6*(-a^9*c^3)^(1/2) - 4
9*a^3*e^6*(-a^9*c^3)^(1/2) + 30*a^5*c^4*d^5*e + 126*a^7*c^2*d*e^5 + 124*a^6
*c^3*d^3*e^3 + 81*a*c^2*d^4*e^2*(-a^9*c^3)^(1/2) + 39*a^2*c*d^2*e^4*(-a^9*c
^3)^(1/2))/(a^13*e^8 + a^9*c^4*d^8 + 4*a^12*c*d^2*e^6 + 4*a^10*c^3*d^6*e^2
+ 6*a^11*c^2*d^4*e^4))^(5/2)*40i - a^20*c*d*e^18*x*(-(25*c^3*d^6*(-a^9*c^3)
^(1/2) - 49*a^3*e^6*(-a^9*c^3)^(1/2) + 30*a^5*c^4*d^5*e + 126*a^7*c^2*d*e^5
+ 124*a^6*c^3*d^3*e^3 + 81*a*c^2*d^4*e^2*(-a^9*c^3)^(1/2) + 39*a^2*c*d^2*e
^4*(-a^9*c^3)^(1/2))/(a^13*e^8 + a^9*c^4*d^8 + 4*a^12*c*d^2*e^6 + 4*a^10*c^
3*d^6*e^2 + 6*a^11*c^2*d^4*e^4))^(3/2)*128i))/(9765625*a^9*c^21*d^32 + 4818
90304*a^25*c^5*e^32 + 159765625*a^10*c^20*d^30*e^2 + 1159031250*a^11*c^19*d
^28*e^4 + 4879001250*a^12*c^18*d^26*e^6 + 13043411775*a^13*c^17*d^24*e^8 +
22507897839*a^14*c^16*d^22*e^10 + 23209461788*a^15*c^15*d^20*e^12 + 7790140
604*a^16*c^14*d^18*e^14 - 15160518297*a^17*c^13*d^16*e^16 - 24964288057*a^1
8*c^12*d^14*e^18 - 11511478798*a^19*c^11*d^12*e^20 + 8613907074*a^20*c^10*d
^10*e^22 + 11397074817*a^21*c^9*d^8*e^24 + 586708977*a^22*c^8*d^6*e^26 - 35
76733440*a^23*c^7*d^4*e^28 - 521228288*a^24*c^6*d^2*e^30))*(-(25*c^3*d^6*(-
a^9*c^3)^(1/2) - 49*a^3*e^6*(-a^9*c^3)^(1/2) + 30*a^5*c^4*d^5*e + 126*a^7*c
^2*d*e^5 + 124*a^6*c^3*d^3*e^3 + 81*a*c^2*d^4*e^2*(-a^9*c^3)^(1/2) + 39*a^2
*c*d^2*e^4*(-a^9*c^3)^(1/2))/(256*(a^13*e^8 + a^9*c^4*d^8 + 4*a^12*c*d^2*e^
6 + 4*a^10*c^3*d^6*e^2 + 6*a^11*c^2*d^4*e^4)))^(1/2)*2i - (atan((a^9*e^3*x*
(-d^3*e^9)^(5/2)*4096i - a^3*c^6*d^15*x*(-d^3*e^9)^(3/2)*26804i + c^9*d^24*
e^3*x*(-d^3*e^9)^(1/2)*625i - a^4*c^5*d^13*e^2*x*(-d^3*e^9)^(3/2)*24831i -
a^5*c^4*d^11*e^4*x*(-d^3*e^9)^(3/2)*8214i + a^6*c^3*d^9*e^6*x*(-d^3*e^9)^(3
/2)*13471i + a^7*c^2*d^7*e^8*x*(-d^3*e^9)^(3/2)*16128i + a^2*c^7*d^20*e^7*x
*(-d^3*e^9)^(1/2)*15951i + a*c^8*d^22*e^5*x*(-d^3*e^9)^(1/2)*4950i)/(4096*a
^9*d^8*e^25 + 625*c^9*d^26*e^7 + 4950*a*c^8*d^24*e^9 + 15951*a^2*c^7*d^22*e
^11 + 26804*a^3*c^6*d^20*e^13 + 24831*a^4*c^5*d^18*e^15 + 8214*a^5*c^4*d^16
*e^17 - 13471*a^6*c^3*d^14*e^19 - 16128*a^7*c^2*d^12*e^21))*(-d^3*e^9)^(1/2
)*1i)/(c^2*d^7 + a^2*d^3*e^4 + 2*a*c*d^5*e^2)

3.258 $\int \frac{1}{x^4(d+ex^2)(a+cx^4)^2} dx$

Optimal result

Integrand size = 22, antiderivative size = 751

$$\begin{aligned}
&\int \frac{1}{x^4(d+ex^2)(a+cx^4)^2} dx \\
&= -\frac{1}{3a^2dx^3}+\frac{e}{a^2d^2x}-\frac{c^2x(d-ex^2)}{4a^2(cd^2+ae^2)(a+cx^4)} \\
&+\frac{e^{11/2}\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{d^{5/2}(cd^2+ae^2)^2}+\frac{c^{5/4}(3\sqrt{c}d-\sqrt{a}e)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{8\sqrt{2}a^{11/4}(cd^2+ae^2)} \\
&+\frac{c^{5/4}(\sqrt{c}d-\sqrt{a}e)(cd^2+2ae^2)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{11/4}(cd^2+ae^2)^2} \\
&-\frac{c^{5/4}(3\sqrt{c}d-\sqrt{a}e)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{8\sqrt{2}a^{11/4}(cd^2+ae^2)} \\
&-\frac{c^{5/4}(\sqrt{c}d-\sqrt{a}e)(cd^2+2ae^2)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{11/4}(cd^2+ae^2)^2} \\
&+\frac{c^{5/4}(3\sqrt{c}d+\sqrt{a}e)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)}{16\sqrt{2}a^{11/4}(cd^2+ae^2)} \\
&+\frac{c^{5/4}(\sqrt{c}d+\sqrt{a}e)(cd^2+2ae^2)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)}{4\sqrt{2}a^{11/4}(cd^2+ae^2)^2} \\
&-\frac{c^{5/4}(3\sqrt{c}d+\sqrt{a}e)\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)}{16\sqrt{2}a^{11/4}(cd^2+ae^2)} \\
&-\frac{c^{5/4}(\sqrt{c}d+\sqrt{a}e)(cd^2+2ae^2)\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)}{4\sqrt{2}a^{11/4}(cd^2+ae^2)^2}
\end{aligned}$$

```
[Out] -1/3/a^2/d/x^3+e/a^2/d^2/x-1/4*c^2*x*(-e*x^2+d)/a^2/(a*e^2+c*d^2)/(c*x^4+a)
+e^(11/2)*arctan(x*e^(1/2)/d^(1/2))/d^(5/2)/(a*e^2+c*d^2)^2-1/4*c^(5/4)*(2*
a*e^2+c*d^2)*arctan(-1+c^(1/4)*x*2^(1/2)/a^(1/4))*(-e*a^(1/2)+d*c^(1/2))/a^
(11/4)/(a*e^2+c*d^2)^2*2^(1/2)-1/4*c^(5/4)*(2*a*e^2+c*d^2)*arctan(1+c^(1/4)
*x*2^(1/2)/a^(1/4))*(-e*a^(1/2)+d*c^(1/2))/a^(11/4)/(a*e^2+c*d^2)^2*2^(1/2)
+1/8*c^(5/4)*(2*a*e^2+c*d^2)*ln(-a^(1/4)*c^(1/4)*x*2^(1/2)+a^(1/2)+x^2*c^(1
/2))*(e*a^(1/2)+d*c^(1/2))/a^(11/4)/(a*e^2+c*d^2)^2*2^(1/2)-1/8*c^(5/4)*(2*
a*e^2+c*d^2)*ln(a^(1/4)*c^(1/4)*x*2^(1/2)+a^(1/2)+x^2*c^(1/2))*(e*a^(1/2)+d
*c^(1/2))/a^(11/4)/(a*e^2+c*d^2)^2*2^(1/2)-1/16*c^(5/4)*arctan(-1+c^(1/4)*x
*2^(1/2)/a^(1/4))*(-e*a^(1/2)+3*d*c^(1/2))/a^(11/4)/(a*e^2+c*d^2)*2^(1/2)-1
/16*c^(5/4)*arctan(1+c^(1/4)*x*2^(1/2)/a^(1/4))*(-e*a^(1/2)+3*d*c^(1/2))/a^
(11/4)/(a*e^2+c*d^2)*2^(1/2)+1/32*c^(5/4)*ln(-a^(1/4)*c^(1/4)*x*2^(1/2)+a^(
1/2)+x^2*c^(1/2))*(e*a^(1/2)+3*d*c^(1/2))/a^(11/4)/(a*e^2+c*d^2)*2^(1/2)-1/
32*c^(5/4)*ln(a^(1/4)*c^(1/4)*x*2^(1/2)+a^(1/2)+x^2*c^(1/2))*(e*a^(1/2)+3*d
*c^(1/2))/a^(11/4)/(a*e^2+c*d^2)*2^(1/2)
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 751, normalized size of antiderivative = 1.00, number of steps used = 22, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.409$, Rules used

$= \{1350, 211, 1193, 1182, 1176, 631, 210, 1179, 642\}$

$$\int \frac{1}{x^4 (d+ex^2) (a+cx^4)^2} \, dx$$
$$= \frac{c^{5/4} \arctan\left(1 - \frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right) (\sqrt{c}d - \sqrt{a}e) (2ae^2 + cd^2)}{2\sqrt{2}a^{11/4} (ae^2 + cd^2)^2}$$
$$+ \frac{c^{5/4} \arctan\left(1 - \frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right) (3\sqrt{c}d - \sqrt{a}e)}{8\sqrt{2}a^{11/4} (ae^2 + cd^2)}$$
$$- \frac{c^{5/4} \arctan\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}} + 1\right) (\sqrt{c}d - \sqrt{a}e) (2ae^2 + cd^2)}{2\sqrt{2}a^{11/4} (ae^2 + cd^2)^2}$$
$$- \frac{c^{5/4} \arctan\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}} + 1\right) (3\sqrt{c}d - \sqrt{a}e)}{8\sqrt{2}a^{11/4} (ae^2 + cd^2)}$$
$$+ \frac{c^{5/4}(\sqrt{a}e + \sqrt{c}d) (2ae^2 + cd^2) \log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x + \sqrt{a} + \sqrt{c}x^2\right)}{4\sqrt{2}a^{11/4} (ae^2 + cd^2)^2}$$
$$+ \frac{c^{5/4}(\sqrt{a}e + 3\sqrt{c}d) \log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x + \sqrt{a} + \sqrt{c}x^2\right)}{16\sqrt{2}a^{11/4} (ae^2 + cd^2)}$$
$$- \frac{c^{5/4}(\sqrt{a}e + \sqrt{c}d) (2ae^2 + cd^2) \log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x + \sqrt{a} + \sqrt{c}x^2\right)}{4\sqrt{2}a^{11/4} (ae^2 + cd^2)^2}$$
$$- \frac{c^{5/4}(\sqrt{a}e + 3\sqrt{c}d) \log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x + \sqrt{a} + \sqrt{c}x^2\right)}{16\sqrt{2}a^{11/4} (ae^2 + cd^2)}$$
$$- \frac{c^2x(d - ex^2)}{4a^2 (a + cx^4) (ae^2 + cd^2)} + \frac{e}{a^2d^2x} - \frac{1}{3a^2dx^3} + \frac{e^{11/2} \arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{d^{5/2} (ae^2 + cd^2)^2}$$

```
[In] Int[1/(x^4*(d + e*x^2)*(a + c*x^4)^2),x]
```

```
[Out] -1/3*1/(a^2*d*x^3) + e/(a^2*d^2*x) - (c^2*x*(d - e*x^2))/(4*a^2*(c*d^2 + a*
e^2)*(a + c*x^4)) + (e^(11/2)*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(d^(5/2)*(c*d^2
+ a*e^2)^2) + (c^(5/4)*(3*Sqrt[c]*d - Sqrt[a]*e)*ArcTan[1 - (Sqrt[2]*c^(1/4
)*x)/a^(1/4)])/(8*Sqrt[2]*a^(11/4)*(c*d^2 + a*e^2)) + (c^(5/4)*(Sqrt[c]*d -
 Sqrt[a]*e)*(c*d^2 + 2*a*e^2)*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(2*S
qrt[2]*a^(11/4)*(c*d^2 + a*e^2)^2) - (c^(5/4)*(3*Sqrt[c]*d - Sqrt[a]*e)*Arc
Tan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(8*Sqrt[2]*a^(11/4)*(c*d^2 + a*e^2))
- (c^(5/4)*(Sqrt[c]*d - Sqrt[a]*e)*(c*d^2 + 2*a*e^2)*ArcTan[1 + (Sqrt[2]*c^
(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(11/4)*(c*d^2 + a*e^2)^2) + (c^(5/4)*(3*Sqr
t[c]*d + Sqrt[a]*e)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])
/(16*Sqrt[2]*a^(11/4)*(c*d^2 + a*e^2)) + (c^(5/4)*(Sqrt[c]*d + Sqrt[a]*e)*(
c*d^2 + 2*a*e^2)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(4
```

```
*Sqrt[2]*a^(11/4)*(c*d^2 + a*e^2)^2) - (c^(5/4)*(3*Sqrt[c]*d + Sqrt[a]*e)*L
og[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(16*Sqrt[2]*a^(11/4)
*(c*d^2 + a*e^2)) - (c^(5/4)*(Sqrt[c]*d + Sqrt[a]*e)*(c*d^2 + 2*a*e^2)*Log[
Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(4*Sqrt[2]*a^(11/4)*(c*
d^2 + a*e^2)^2)
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1182

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
```

```
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[(-a)
*c]
```

Rule 1193

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[(-x
)*(d + e*x^2)*((a + c*x^4)^(p + 1)/(4*a*(p + 1))), x] + Dist[1/(4*a*(p + 1)
), Int[Simp[d*(4*p + 5) + e*(4*p + 7)*x^2, x]*(a + c*x^4)^(p + 1), x], x] /
; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && IntegerQ
[2*p]
```

Rule 1350

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*(a + c*x^4)^p,
x], x] /; FreeQ[{a, c, d, e, f, m, p, q}, x] && (IGtQ[p, 0] || IGtQ[q, 0] |
| IntegersQ[m, q])
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \left(\frac{1}{a^2dx^4}-\frac{e}{a^2d^2x^2}+\frac{e^6}{d^2\left(cd^2+ae^2\right)^2\left(d+ex^2\right)}-\frac{c^2(d-ex^2)}{a\left(cd^2+ae^2\right)\left(a+cx^4\right)^2}-\frac{c^2(cd^2+2ae^2)\left(d-ex^2\right)}{a^2\left(cd^2+ae^2\right)^2\left(a+cx^4\right)}\right)dx\\
&= -\frac{1}{3a^2dx^3}+\frac{e}{a^2d^2x}+\frac{e^6\int\frac{1}{d+ex^2}\,dx}{d^2\left(cd^2+ae^2\right)^2}-\frac{c^2\int\frac{d-ex^2}{\left(a+cx^4\right)^2}\,dx}{a\left(cd^2+ae^2\right)}-\frac{\left(c^2(cd^2+2ae^2)\right)\int\frac{d-ex^2}{a+cx^4}\,dx}{a^2\left(cd^2+ae^2\right)^2}\\
&= -\frac{1}{3a^2dx^3}+\frac{e}{a^2d^2x}-\frac{c^2x(d-ex^2)}{4a^2\left(cd^2+ae^2\right)\left(a+cx^4\right)}+\frac{e^{11/2}\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{d^{5/2}\left(cd^2+ae^2\right)^2}+\frac{c^2\int\frac{-3d+ex^2}{a+cx^4}\,dx}{4a^2\left(cd^2+ae^2\right)}\\
&\quad-\frac{\left(c\left(\frac{\sqrt{c}d}{\sqrt{a}}-e\right)\left(cd^2+2ae^2\right)\right)\int\frac{\sqrt{a}\sqrt{c}+cx^2}{a+cx^4}\,dx}{2a^2\left(cd^2+ae^2\right)^2}-\frac{\left(c\left(\frac{\sqrt{c}d}{\sqrt{a}}+e\right)\left(cd^2+2ae^2\right)\right)\int\frac{\sqrt{a}\sqrt{c}-cx^2}{a+cx^4}\,dx}{2a^2\left(cd^2+ae^2\right)^2}\end{aligned}$$

$$= -\frac{1}{3a^2dx^3} + \frac{e}{a^2d^2x} - \frac{c^2x(d-ex^2)}{4a^2(cd^2+ae^2)(a+cx^4)} + \frac{e^{11/2}\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{d^{5/2}(cd^2+ae^2)^2}$$

$$- \frac{\left(c\left(\frac{3\sqrt{c}d}{\sqrt{a}}-e\right)\right)\int \frac{\sqrt{a}\sqrt{c}+cx^2}{a+cx^4}\,dx}{8a^2(cd^2+ae^2)} - \frac{\left(c\left(\frac{3\sqrt{c}d}{\sqrt{a}}+e\right)\right)\int \frac{\sqrt{a}\sqrt{c}-cx^2}{a+cx^4}\,dx}{8a^2(cd^2+ae^2)}$$

$$- \frac{\left(c\left(\frac{\sqrt{c}d}{\sqrt{a}}-e\right)(cd^2+2ae^2)\right)\int \frac{1}{\frac{\sqrt{a}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}+x^2}\,dx}{4a^2(cd^2+ae^2)^2}$$

$$- \frac{\left(c\left(\frac{\sqrt{c}d}{\sqrt{a}}-e\right)(cd^2+2ae^2)\right)\int \frac{1}{\frac{\sqrt{a}}{\sqrt{c}}+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}+x^2}\,dx}{4a^2(cd^2+ae^2)^2}$$

$$+ \frac{\left(c^{5/4}\left(\frac{\sqrt{c}d}{\sqrt{a}}+e\right)(cd^2+2ae^2)\right)\int \frac{\frac{\sqrt{2}\sqrt[4]{a}}{\sqrt[4]{c}}+2x}{-\frac{\sqrt{a}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}-x^2}\,dx}{4\sqrt{2}a^{9/4}(cd^2+ae^2)^2}$$

$$+ \frac{\left(c^{5/4}\left(\frac{\sqrt{c}d}{\sqrt{a}}+e\right)(cd^2+2ae^2)\right)\int \frac{\frac{\sqrt{2}\sqrt[4]{a}}{\sqrt[4]{c}}-2x}{-\frac{\sqrt{a}}{\sqrt{c}}+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}-x^2}\,dx}{4\sqrt{2}a^{9/4}(cd^2+ae^2)^2}$$

$$= -\frac{1}{3a^2dx^3} + \frac{e}{a^2d^2x} - \frac{c^2x(d-ex^2)}{4a^2(cd^2+ae^2)(a+cx^4)} + \frac{e^{11/2}\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{d^{5/2}(cd^2+ae^2)^2}$$
$$+ \frac{c^{5/4}\left(\frac{\sqrt{c}d}{\sqrt{a}}+e\right)(cd^2+2ae^2)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)}{4\sqrt{2}a^{9/4}(cd^2+ae^2)^2}$$
$$- \frac{c^{5/4}\left(\frac{\sqrt{c}d}{\sqrt{a}}+e\right)(cd^2+2ae^2)\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)}{4\sqrt{2}a^{9/4}(cd^2+ae^2)^2}$$
$$- \frac{\left(c\left(\frac{3\sqrt{c}d}{\sqrt{a}}-e\right)\right)\int\frac{1}{\frac{\sqrt{a}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}+x^2}\,dx}{16a^2(cd^2+ae^2)} - \frac{\left(c\left(\frac{3\sqrt{c}d}{\sqrt{a}}-e\right)\right)\int\frac{1}{\frac{\sqrt{a}}{\sqrt{c}}+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}+x^2}\,dx}{16a^2(cd^2+ae^2)}$$
$$+ \frac{\left(c^{5/4}\left(\frac{3\sqrt{c}d}{\sqrt{a}}+e\right)\right)\int\frac{\frac{\sqrt{2}\sqrt[4]{a}}{\sqrt[4]{c}}+2x}{-\frac{\sqrt{a}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}-x^2}\,dx}{16\sqrt{2}a^{9/4}(cd^2+ae^2)} + \frac{\left(c^{5/4}\left(\frac{3\sqrt{c}d}{\sqrt{a}}+e\right)\right)\int\frac{\frac{\sqrt{2}\sqrt[4]{a}}{\sqrt[4]{c}}-2x}{-\frac{\sqrt{a}}{\sqrt{c}}+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}-x^2}\,dx}{16\sqrt{2}a^{9/4}(cd^2+ae^2)}$$
$$- \frac{\left(c^{5/4}\left(\frac{\sqrt{c}d}{\sqrt{a}}-e\right)(cd^2+2ae^2)\right)\text{Subst}\left(\int\frac{1}{-1-x^2}\,dx, x, 1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{9/4}(cd^2+ae^2)^2}$$
$$+ \frac{\left(c^{5/4}\left(\frac{\sqrt{c}d}{\sqrt{a}}-e\right)(cd^2+2ae^2)\right)\text{Subst}\left(\int\frac{1}{-1-x^2}\,dx, x, 1+\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{9/4}(cd^2+ae^2)^2}$$

$$= -\frac{1}{3a^2dx^3} + \frac{e}{a^2d^2x} - \frac{c^2x(d-ex^2)}{4a^2(cd^2+ae^2)(a+cx^4)} + \frac{e^{11/2}\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{d^{5/2}(cd^2+ae^2)^2}$$

$$+ \frac{c^{5/4}\left(\frac{\sqrt{c}d}{\sqrt{a}}-e\right)(cd^2+2ae^2)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{9/4}(cd^2+ae^2)^2}$$

$$- \frac{c^{5/4}\left(\frac{\sqrt{c}d}{\sqrt{a}}-e\right)(cd^2+2ae^2)\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{9/4}(cd^2+ae^2)^2}$$

$$+ \frac{c^{5/4}\left(\frac{3\sqrt{c}d}{\sqrt{a}}+e\right)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)}{16\sqrt{2}a^{9/4}(cd^2+ae^2)}$$

$$+ \frac{c^{5/4}\left(\frac{\sqrt{c}d}{\sqrt{a}}+e\right)(cd^2+2ae^2)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)}{4\sqrt{2}a^{9/4}(cd^2+ae^2)^2}$$

$$- \frac{c^{5/4}\left(\frac{3\sqrt{c}d}{\sqrt{a}}+e\right)\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)}{16\sqrt{2}a^{9/4}(cd^2+ae^2)}$$

$$- \frac{c^{5/4}\left(\frac{\sqrt{c}d}{\sqrt{a}}+e\right)(cd^2+2ae^2)\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)}{4\sqrt{2}a^{9/4}(cd^2+ae^2)^2}$$

$$- \frac{\left(c^{5/4}\left(\frac{3\sqrt{c}d}{\sqrt{a}}-e\right)\right)\text{Subst}\left(\int\frac{1}{-1-x^2}\,dx, x, 1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{8\sqrt{2}a^{9/4}(cd^2+ae^2)}$$

$$+ \frac{\left(c^{5/4}\left(\frac{3\sqrt{c}d}{\sqrt{a}}-e\right)\right)\text{Subst}\left(\int\frac{1}{-1-x^2}\,dx, x, 1+\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{8\sqrt{2}a^{9/4}(cd^2+ae^2)}$$

$$
\begin{aligned}
&= -\frac{1}{3a^2dx^3} + \frac{e}{a^2d^2x} - \frac{c^2x(d-ex^2)}{4a^2\left(cd^2+ae^2\right)\left(a+cx^4\right)} \\
&\quad + \frac{e^{11/2}\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{d^{5/2}\left(cd^2+ae^2\right)^2} + \frac{c^{5/4}\left(\frac{3\sqrt{c}d}{\sqrt{a}}-e\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{8\sqrt{2}a^{9/4}\left(cd^2+ae^2\right)} \\
&\quad + \frac{c^{5/4}\left(\frac{\sqrt{c}d}{\sqrt{a}}-e\right)\left(cd^2+2ae^2\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{9/4}\left(cd^2+ae^2\right)^2} \\
&\quad - \frac{c^{5/4}\left(\frac{3\sqrt{c}d}{\sqrt{a}}-e\right)\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{8\sqrt{2}a^{9/4}\left(cd^2+ae^2\right)} \\
&\quad - \frac{c^{5/4}\left(\frac{\sqrt{c}d}{\sqrt{a}}-e\right)\left(cd^2+2ae^2\right)\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{9/4}\left(cd^2+ae^2\right)^2} \\
&\quad + \frac{c^{5/4}\left(\frac{3\sqrt{c}d}{\sqrt{a}}+e\right)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)}{16\sqrt{2}a^{9/4}\left(cd^2+ae^2\right)} \\
&\quad + \frac{c^{5/4}\left(\frac{\sqrt{c}d}{\sqrt{a}}+e\right)\left(cd^2+2ae^2\right)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)}{4\sqrt{2}a^{9/4}\left(cd^2+ae^2\right)^2} \\
&\quad - \frac{c^{5/4}\left(\frac{3\sqrt{c}d}{\sqrt{a}}+e\right)\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)}{16\sqrt{2}a^{9/4}\left(cd^2+ae^2\right)} \\
&\quad - \frac{c^{5/4}\left(\frac{\sqrt{c}d}{\sqrt{a}}+e\right)\left(cd^2+2ae^2\right)\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)}{4\sqrt{2}a^{9/4}\left(cd^2+ae^2\right)^2}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.27 (sec) , antiderivative size = 513, normalized size of antiderivative = 0.68

$$\int \frac{1}{x^4 (d+ex^2) (a+cx^4)^2} \, dx$$
$$= \frac{1}{96}\left(-\frac{32}{a^2dx^3}+\frac{96e}{a^2d^2x}-\frac{24c^2x(d-ex^2)}{a^2(cd^2+ae^2)(a+cx^4)}+\frac{96e^{11/2}\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{d^{5/2}(cd^2+ae^2)^2}\right.$$
$$+\frac{6\sqrt{2}c^{5/4}\left(7c^{3/2}d^3-5\sqrt{a}cd^2e+11a\sqrt{c}de^2-9a^{3/2}e^3\right)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{a^{11/4}(cd^2+ae^2)^2}$$
$$+\frac{6\sqrt{2}c^{5/4}\left(-7c^{3/2}d^3+5\sqrt{a}cd^2e-11a\sqrt{c}de^2+9a^{3/2}e^3\right)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{a^{11/4}(cd^2+ae^2)^2}$$
$$+\frac{3\sqrt{2}c^{5/4}\left(7c^{3/2}d^3+5\sqrt{a}cd^2e+11a\sqrt{c}de^2+9a^{3/2}e^3\right)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)}{a^{11/4}(cd^2+ae^2)^2}$$
$$\left.-\frac{3\sqrt{2}c^{5/4}\left(7c^{3/2}d^3+5\sqrt{a}cd^2e+11a\sqrt{c}de^2+9a^{3/2}e^3\right)\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)}{a^{11/4}(cd^2+ae^2)^2}\right)$$

```
[In] Integrate[1/(x^4*(d + e*x^2)*(a + c*x^4)^2),x]
```

```
[Out] (-32/(a^2*d*x^3) + (96*e)/(a^2*d^2*x) - (24*c^2*x*(d - e*x^2))/(a^2*(c*d^2
+ a*e^2)*(a + c*x^4)) + (96*e^(11/2)*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(d^(5/2)*
(c*d^2 + a*e^2)^2) + (6*Sqrt[2]*c^(5/4)*(7*c^(3/2)*d^3 - 5*Sqrt[a]*c*d^2*e
+ 11*a*Sqrt[c]*d*e^2 - 9*a^(3/2)*e^3)*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4
)])/(a^(11/4)*(c*d^2 + a*e^2)^2) + (6*Sqrt[2]*c^(5/4)*(-7*c^(3/2)*d^3 + 5*S
qrt[a]*c*d^2*e - 11*a*Sqrt[c]*d*e^2 + 9*a^(3/2)*e^3)*ArcTan[1 + (Sqrt[2]*c^
(1/4)*x)/a^(1/4)])/(a^(11/4)*(c*d^2 + a*e^2)^2) + (3*Sqrt[2]*c^(5/4)*(7*c^(
3/2)*d^3 + 5*Sqrt[a]*c*d^2*e + 11*a*Sqrt[c]*d*e^2 + 9*a^(3/2)*e^3)*Log[Sqrt
[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(a^(11/4)*(c*d^2 + a*e^2)^2
) - (3*Sqrt[2]*c^(5/4)*(7*c^(3/2)*d^3 + 5*Sqrt[a]*c*d^2*e + 11*a*Sqrt[c]*d*
e^2 + 9*a^(3/2)*e^3)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2]
)/(a^(11/4)*(c*d^2 + a*e^2)^2))/96
```

Maple [A] (verified)

Time = 0.49 (sec) , antiderivative size = 355, normalized size of antiderivative = 0.47

method	result
default	$-\frac{1}{3a^2 d\,x^3}+\frac{e}{a^2 d^2 x}-\frac{c^2\left(\frac{\left(-\frac{1}{4}a\,e^3-\frac{1}{4}c\,d^2 e\right)x^3+\left(\frac{1}{4}d\,e^2 a+\frac{1}{4}d^3 c\right)x}{c\,x^4+a}+\frac{\left(11d\,e^2 a+7d^3 c\right)\left(\frac{a}{c}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x^2+\left(\frac{a}{c}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{c}}}{x^2-\left(\frac{a}{c}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{c}}}\right)+2\arctan\left(\ldots\right.\right.}{32a}\ldots\right)}{\ldots}$
risch	Expression too large to display

```
[In] int(1/x^4/(e*x^2+d)/(c*x^4+a)^2,x,method=_RETURNVERBOSE)
[Out] -1/3/a^2/d/x^3+e/a^2/d^2/x-c^2/(a*e^2+c*d^2)^2/a^2*(((-1/4*a*e^3-1/4*c*d^2*
e)*x^3+(1/4*d*e^2*a+1/4*d^3*c)*x)/(c*x^4+a)+1/32*(11*a*d*e^2+7*c*d^3)*(a/c)
^(1/4)/a*2^(1/2)*(ln((x^2+(a/c)^(1/4)*x*2^(1/2)+(a/c)^(1/2))/(x^2-(a/c)^(1/
4)*x*2^(1/2)+(a/c)^(1/2)))+2*arctan(2^(1/2)/(a/c)^(1/4)*x+1)+2*arctan(2^(1/
2)/(a/c)^(1/4)*x-1))+1/32*(-9*a*e^3-5*c*d^2*e)/c/(a/c)^(1/4)*2^(1/2)*(ln((x
^2-(a/c)^(1/4)*x*2^(1/2)+(a/c)^(1/2))/(x^2+(a/c)^(1/4)*x*2^(1/2)+(a/c)^(1/2
)))+2*arctan(2^(1/2)/(a/c)^(1/4)*x+1)+2*arctan(2^(1/2)/(a/c)^(1/4)*x-1)))+1
/d^2*e^6/(a*e^2+c*d^2)^2/(e*d)^(1/2)*arctan(e*x/(e*d)^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 5164 vs. 2(578) = 1156.

Time = 65.77 (sec) , antiderivative size = 10352, normalized size of antiderivative = 13.78

$$\int \frac{1}{x^4\left(d+ex^2\right)\left(a+cx^4\right)^2}\,dx = \text{Too large to display}$$

```
[In] integrate(1/x^4/(e*x^2+d)/(c*x^4+a)^2,x, algorithm="fricas")
[Out] Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{x^4\left(d+ex^2\right)\left(a+cx^4\right)^2}\,dx = \text{Timed out}$$

```
[In] integrate(1/x**4/(e*x**2+d)/(c*x**4+a)**2,x)
[Out] Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{x^4 (d + ex^2) (a + cx^4)^2} \, dx = \text{Exception raised: ValueError}$$

```
[In] integrate(1/x^4/(e*x^2+d)/(c*x^4+a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 645, normalized size of antiderivative = 0.86

$$\begin{aligned}\int \frac{1}{x^4 (d + ex^2) (a + cx^4)^2} \, dx = &\frac{e^6 \arctan\left(\frac{ex}{\sqrt{de}}\right)}{(c^2d^6 + 2\,acd^4e^2 + a^2d^2e^4)\sqrt{de}} \\ &- \frac{\left(7\,(ac^3)^{\frac{1}{4}}\,c^3d^3 + 11\,(ac^3)^{\frac{1}{4}}\,ac^2de^2 - 5\,(ac^3)^{\frac{3}{4}}\,cd^2e - 9\,(ac^3)^{\frac{3}{4}}\,ae^3\right)\arctan\left(\frac{\sqrt{2}\left(2\,x+\sqrt{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{8\left(\sqrt{2}a^3c^3d^4 + 2\,\sqrt{2}a^4c^2d^2e^2 + \sqrt{2}a^5ce^4\right)} \\ &- \frac{\left(7\,(ac^3)^{\frac{1}{4}}\,c^3d^3 + 11\,(ac^3)^{\frac{1}{4}}\,ac^2de^2 - 5\,(ac^3)^{\frac{3}{4}}\,cd^2e - 9\,(ac^3)^{\frac{3}{4}}\,ae^3\right)\arctan\left(\frac{\sqrt{2}\left(2\,x-\sqrt{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{8\left(\sqrt{2}a^3c^3d^4 + 2\,\sqrt{2}a^4c^2d^2e^2 + \sqrt{2}a^5ce^4\right)} \\ &- \frac{\left(7\,(ac^3)^{\frac{1}{4}}\,c^3d^3 + 11\,(ac^3)^{\frac{1}{4}}\,ac^2de^2 + 5\,(ac^3)^{\frac{3}{4}}\,cd^2e + 9\,(ac^3)^{\frac{3}{4}}\,ae^3\right)\log\left(x^2 + \sqrt{2}x\left(\frac{a}{c}\right)^{\frac{1}{4}} + \sqrt{\frac{a}{c}}\right)}{16\left(\sqrt{2}a^3c^3d^4 + 2\,\sqrt{2}a^4c^2d^2e^2 + \sqrt{2}a^5ce^4\right)} \\ &+ \frac{\left(7\,(ac^3)^{\frac{1}{4}}\,c^3d^3 + 11\,(ac^3)^{\frac{1}{4}}\,ac^2de^2 + 5\,(ac^3)^{\frac{3}{4}}\,cd^2e + 9\,(ac^3)^{\frac{3}{4}}\,ae^3\right)\log\left(x^2 - \sqrt{2}x\left(\frac{a}{c}\right)^{\frac{1}{4}} + \sqrt{\frac{a}{c}}\right)}{16\left(\sqrt{2}a^3c^3d^4 + 2\,\sqrt{2}a^4c^2d^2e^2 + \sqrt{2}a^5ce^4\right)} \\ &+ \frac{c^2ex^3 - c^2dx}{4\,(a^2cd^2 + a^3e^2)(cx^4 + a)} + \frac{3\,ex^2 - d}{3\,a^2d^2x^3}\end{aligned}$$

```
[In] integrate(1/x^4/(e*x^2+d)/(c*x^4+a)^2,x, algorithm="giac")
```

```
[Out] e^6*arctan(e*x/sqrt(d*e))/((c^2*d^6 + 2*a*c*d^4*e^2 + a^2*d^2*e^4)*sqrt(d*e
)) - 1/8*(7*(a*c^3)^(1/4)*c^3*d^3 + 11*(a*c^3)^(1/4)*a*c^2*d*e^2 - 5*(a*c^3
)^(3/4)*c*d^2*e - 9*(a*c^3)^(3/4)*a*e^3)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*
(a/c)^(1/4))/(a/c)^(1/4))/(sqrt(2)*a^3*c^3*d^4 + 2*sqrt(2)*a^4*c^2*d^2*e^2
```

```
+ sqrt(2)*a^5*c*e^4) - 1/8*(7*(a*c^3)^(1/4)*c^3*d^3 + 11*(a*c^3)^(1/4)*a*c^
2*d*e^2 - 5*(a*c^3)^(3/4)*c*d^2*e - 9*(a*c^3)^(3/4)*a*e^3)*arctan(1/2*sqrt(
2)*(2*x - sqrt(2)*(a/c)^(1/4))/(a/c)^(1/4))/(sqrt(2)*a^3*c^3*d^4 + 2*sqrt(2
)*a^4*c^2*d^2*e^2 + sqrt(2)*a^5*c*e^4) - 1/16*(7*(a*c^3)^(1/4)*c^3*d^3 + 11
*(a*c^3)^(1/4)*a*c^2*d*e^2 + 5*(a*c^3)^(3/4)*c*d^2*e + 9*(a*c^3)^(3/4)*a*e^
3)*log(x^2 + sqrt(2)*x*(a/c)^(1/4) + sqrt(a/c))/(sqrt(2)*a^3*c^3*d^4 + 2*sq
rt(2)*a^4*c^2*d^2*e^2 + sqrt(2)*a^5*c*e^4) + 1/16*(7*(a*c^3)^(1/4)*c^3*d^3
+ 11*(a*c^3)^(1/4)*a*c^2*d*e^2 + 5*(a*c^3)^(3/4)*c*d^2*e + 9*(a*c^3)^(3/4)*
a*e^3)*log(x^2 - sqrt(2)*x*(a/c)^(1/4) + sqrt(a/c))/(sqrt(2)*a^3*c^3*d^4 +
2*sqrt(2)*a^4*c^2*d^2*e^2 + sqrt(2)*a^5*c*e^4) + 1/4*(c^2*e*x^3 - c^2*d*x)/
((a^2*c*d^2 + a^3*e^2)*(c*x^4 + a)) + 1/3*(3*e*x^2 - d)/(a^2*d^2*x^3)
```

Mupad [B] (verification not implemented)

Time = 11.04 (sec) , antiderivative size = 20828, normalized size of antiderivative = 27.73

$$\int \frac{1}{x^4 (d+ex^2) (a+cx^4)^2} \, dx = \text{Too large to display}$$

```
[In] int(1/(x^4*(a + c*x^4)^2*(d + e*x^2)),x)
```

```
[Out] atan(((x*(4917248*a^10*c^18*d^36*e^5 + 50677760*a^11*c^17*d^34*e^7 + 230498
304*a^12*c^16*d^32*e^9 + 607559680*a^13*c^15*d^30*e^11 + 1026486272*a^14*c^
14*d^28*e^13 + 1166602240*a^15*c^13*d^26*e^15 + 923508736*a^16*c^12*d^24*e^
17 + 539500544*a^17*c^11*d^22*e^19 + 259409920*a^18*c^10*d^20*e^21 + 109709
312*a^19*c^9*d^18*e^23 + 34537472*a^20*c^8*d^16*e^25 + 5308416*a^21*c^7*d^1
4*e^27) - ((81*a^3*e^6*(-a^11*c^5)^(1/2) - 49*c^3*d^6*(-a^11*c^5)^(1/2) + 7
0*a^6*c^5*d^5*e + 198*a^8*c^3*d*e^5 + 236*a^7*c^4*d^3*e^3 - 129*a*c^2*d^4*e
^2*(-a^11*c^5)^(1/2) - 31*a^2*c*d^2*e^4*(-a^11*c^5)^(1/2))/(256*(a^15*e^8 +
 a^11*c^4*d^8 + 4*a^14*c*d^2*e^6 + 4*a^12*c^3*d^6*e^2 + 6*a^13*c^2*d^4*e^4)
))^(1/2)*((x*(1787297792*a^19*c^13*d^31*e^12 - 147587072*a^15*c^17*d^39*e^4
 - 698089472*a^16*c^16*d^37*e^6 - 1660157952*a^17*c^15*d^35*e^8 - 158806835
2*a^18*c^14*d^33*e^10 - 12845056*a^14*c^18*d^41*e^2 + 7839678464*a^20*c^12*
d^29*e^14 + 11879841792*a^21*c^11*d^27*e^16 + 10631249920*a^22*c^10*d^25*e^
18 + 6274940928*a^23*c^9*d^23*e^20 + 2652110848*a^24*c^8*d^21*e^22 + 891027
456*a^25*c^7*d^19*e^24 + 234881024*a^26*c^6*d^17*e^26 + 33554432*a^27*c^5*d
^15*e^28) + ((81*a^3*e^6*(-a^11*c^5)^(1/2) - 49*c^3*d^6*(-a^11*c^5)^(1/2) +
 70*a^6*c^5*d^5*e + 198*a^8*c^3*d*e^5 + 236*a^7*c^4*d^3*e^3 - 129*a*c^2*d^4
*e^2*(-a^11*c^5)^(1/2) - 31*a^2*c*d^2*e^4*(-a^11*c^5)^(1/2))/(256*(a^15*e^8
 + a^11*c^4*d^8 + 4*a^14*c*d^2*e^6 + 4*a^12*c^3*d^6*e^2 + 6*a^13*c^2*d^4*e^
4)))^(1/2)*(x*((81*a^3*e^6*(-a^11*c^5)^(1/2) - 49*c^3*d^6*(-a^11*c^5)^(1/2)
 + 70*a^6*c^5*d^5*e + 198*a^8*c^3*d*e^5 + 236*a^7*c^4*d^3*e^3 - 129*a*c^2*d
^4*e^2*(-a^11*c^5)^(1/2) - 31*a^2*c*d^2*e^4*(-a^11*c^5)^(1/2))/(256*(a^15*e
^8 + a^11*c^4*d^8 + 4*a^14*c*d^2*e^6 + 4*a^12*c^3*d^6*e^2 + 6*a^13*c^2*d^4*
e^4)))^(1/2)*(134217728*a^20*c^16*d^42*e^3 + 1342177280*a^21*c^15*d^40*e^5
```

+ 5905580032*a^22*c^14*d^38*e^7 + 14763950080*a^23*c^13*d^36*e^9 + 22145925
120*a^24*c^12*d^34*e^11 + 17716740096*a^25*c^11*d^32*e^13 - 17716740096*a^2
7*c^9*d^28*e^17 - 22145925120*a^28*c^8*d^26*e^19 - 14763950080*a^29*c^7*d^2
4*e^21 - 5905580032*a^30*c^6*d^22*e^23 - 1342177280*a^31*c^5*d^20*e^25 - 13
4217728*a^32*c^4*d^18*e^27) + 29360128*a^17*c^17*d^42*e^2 + 239075328*a^18*
c^16*d^40*e^4 + 708837376*a^19*c^15*d^38*e^6 + 465567744*a^20*c^14*d^36*e^8
 - 2726297600*a^21*c^13*d^34*e^10 - 9084862464*a^22*c^12*d^32*e^12 - 136147
10784*a^23*c^11*d^30*e^14 - 10745806848*a^24*c^10*d^28*e^16 - 2403336192*a^
25*c^9*d^26*e^18 + 3879731200*a^26*c^8*d^24*e^20 + 4517265408*a^27*c^7*d^22
*e^22 + 2294284288*a^28*c^6*d^20*e^24 + 603979776*a^29*c^5*d^18*e^26 + 6710
8864*a^30*c^4*d^16*e^28))*((81*a^3*e^6*(-a^11*c^5)^(1/2) - 49*c^3*d^6*(-a^1
1*c^5)^(1/2) + 70*a^6*c^5*d^5*e + 198*a^8*c^3*d*e^5 + 236*a^7*c^4*d^3*e^3 -
 129*a*c^2*d^4*e^2*(-a^11*c^5)^(1/2) - 31*a^2*c*d^2*e^4*(-a^11*c^5)^(1/2))/
(256*(a^15*e^8 + a^11*c^4*d^8 + 4*a^14*c*d^2*e^6 + 4*a^12*c^3*d^6*e^2 + 6*a
^13*c^2*d^4*e^4)))^(1/2) + 7225344*a^12*c^18*d^39*e^3 + 76972032*a^13*c^17*
d^37*e^5 + 367607808*a^14*c^16*d^35*e^7 + 1036910592*a^15*c^15*d^33*e^9 + 1
876983808*a^16*c^14*d^31*e^11 + 2115436544*a^17*c^13*d^29*e^13 + 1052803072
*a^18*c^12*d^27*e^15 - 848429056*a^19*c^11*d^25*e^17 - 2105458688*a^20*c^10
*d^23*e^19 - 1909030912*a^21*c^9*d^21*e^21 - 959037440*a^22*c^8*d^19*e^23 -
 262144000*a^23*c^7*d^17*e^25 - 30408704*a^24*c^6*d^15*e^27))*((81*a^3*e^6*
(-a^11*c^5)^(1/2) - 49*c^3*d^6*(-a^11*c^5)^(1/2) + 70*a^6*c^5*d^5*e + 198*a
^8*c^3*d*e^5 + 236*a^7*c^4*d^3*e^3 - 129*a*c^2*d^4*e^2*(-a^11*c^5)^(1/2) -
31*a^2*c*d^2*e^4*(-a^11*c^5)^(1/2))/(256*(a^15*e^8 + a^11*c^4*d^8 + 4*a^14*
c*d^2*e^6 + 4*a^12*c^3*d^6*e^2 + 6*a^13*c^2*d^4*e^4)))^(1/2)*1i + (x*(49172
48*a^10*c^18*d^36*e^5 + 50677760*a^11*c^17*d^34*e^7 + 230498304*a^12*c^16*d
^32*e^9 + 607559680*a^13*c^15*d^30*e^11 + 1026486272*a^14*c^14*d^28*e^13 +
1166602240*a^15*c^13*d^26*e^15 + 923508736*a^16*c^12*d^24*e^17 + 539500544*
a^17*c^11*d^22*e^19 + 259409920*a^18*c^10*d^20*e^21 + 109709312*a^19*c^9*d^
18*e^23 + 34537472*a^20*c^8*d^16*e^25 + 5308416*a^21*c^7*d^14*e^27) - ((81*
a^3*e^6*(-a^11*c^5)^(1/2) - 49*c^3*d^6*(-a^11*c^5)^(1/2) + 70*a^6*c^5*d^5*e
 + 198*a^8*c^3*d*e^5 + 236*a^7*c^4*d^3*e^3 - 129*a*c^2*d^4*e^2*(-a^11*c^5)^
(1/2) - 31*a^2*c*d^2*e^4*(-a^11*c^5)^(1/2))/(256*(a^15*e^8 + a^11*c^4*d^8 +
 4*a^14*c*d^2*e^6 + 4*a^12*c^3*d^6*e^2 + 6*a^13*c^2*d^4*e^4)))^(1/2)*((x*(1
787297792*a^19*c^13*d^31*e^12 - 147587072*a^15*c^17*d^39*e^4 - 698089472*a^
16*c^16*d^37*e^6 - 1660157952*a^17*c^15*d^35*e^8 - 1588068352*a^18*c^14*d^3
3*e^10 - 12845056*a^14*c^18*d^41*e^2 + 7839678464*a^20*c^12*d^29*e^14 + 118
79841792*a^21*c^11*d^27*e^16 + 10631249920*a^22*c^10*d^25*e^18 + 6274940928
*a^23*c^9*d^23*e^20 + 2652110848*a^24*c^8*d^21*e^22 + 891027456*a^25*c^7*d^
19*e^24 + 234881024*a^26*c^6*d^17*e^26 + 33554432*a^27*c^5*d^15*e^28) - ((8
1*a^3*e^6*(-a^11*c^5)^(1/2) - 49*c^3*d^6*(-a^11*c^5)^(1/2) + 70*a^6*c^5*d^5
*e + 198*a^8*c^3*d*e^5 + 236*a^7*c^4*d^3*e^3 - 129*a*c^2*d^4*e^2*(-a^11*c^5
)^(1/2) - 31*a^2*c*d^2*e^4*(-a^11*c^5)^(1/2))/(256*(a^15*e^8 + a^11*c^4*d^8
 + 4*a^14*c*d^2*e^6 + 4*a^12*c^3*d^6*e^2 + 6*a^13*c^2*d^4*e^4)))^(1/2)*(293
60128*a^17*c^17*d^42*e^2 - x*((81*a^3*e^6*(-a^11*c^5)^(1/2) - 49*c^3*d^6*(-
a^11*c^5)^(1/2) + 70*a^6*c^5*d^5*e + 198*a^8*c^3*d*e^5 + 236*a^7*c^4*d^3*e^

3 - 129*a*c^2*d^4*e^2*(-a^11*c^5)^(1/2) - 31*a^2*c*d^2*e^4*(-a^11*c^5)^(1/2
))/(256*(a^15*e^8 + a^11*c^4*d^8 + 4*a^14*c*d^2*e^6 + 4*a^12*c^3*d^6*e^2 +
6*a^13*c^2*d^4*e^4)))^(1/2)*(134217728*a^20*c^16*d^42*e^3 + 1342177280*a^21
*c^15*d^40*e^5 + 5905580032*a^22*c^14*d^38*e^7 + 14763950080*a^23*c^13*d^36
*e^9 + 22145925120*a^24*c^12*d^34*e^11 + 17716740096*a^25*c^11*d^32*e^13 -
17716740096*a^27*c^9*d^28*e^17 - 22145925120*a^28*c^8*d^26*e^19 - 147639500
80*a^29*c^7*d^24*e^21 - 5905580032*a^30*c^6*d^22*e^23 - 1342177280*a^31*c^5
*d^20*e^25 - 134217728*a^32*c^4*d^18*e^27) + 239075328*a^18*c^16*d^40*e^4 +
708837376*a^19*c^15*d^38*e^6 + 465567744*a^20*c^14*d^36*e^8 - 2726297600*a
^21*c^13*d^34*e^10 - 9084862464*a^22*c^12*d^32*e^12 - 13614710784*a^23*c^11
*d^30*e^14 - 10745806848*a^24*c^10*d^28*e^16 - 2403336192*a^25*c^9*d^26*e^1
8 + 3879731200*a^26*c^8*d^24*e^20 + 4517265408*a^27*c^7*d^22*e^22 + 2294284
288*a^28*c^6*d^20*e^24 + 603979776*a^29*c^5*d^18*e^26 + 67108864*a^30*c^4*d
^16*e^28))*((81*a^3*e^6*(-a^11*c^5)^(1/2) - 49*c^3*d^6*(-a^11*c^5)^(1/2) +
70*a^6*c^5*d^5*e + 198*a^8*c^3*d*e^5 + 236*a^7*c^4*d^3*e^3 - 129*a*c^2*d^4*
e^2*(-a^11*c^5)^(1/2) - 31*a^2*c*d^2*e^4*(-a^11*c^5)^(1/2))/(256*(a^15*e^8
+ a^11*c^4*d^8 + 4*a^14*c*d^2*e^6 + 4*a^12*c^3*d^6*e^2 + 6*a^13*c^2*d^4*e^4
)))^(1/2) - 7225344*a^12*c^18*d^39*e^3 - 76972032*a^13*c^17*d^37*e^5 - 3676
07808*a^14*c^16*d^35*e^7 - 1036910592*a^15*c^15*d^33*e^9 - 1876983808*a^16*
c^14*d^31*e^11 - 2115436544*a^17*c^13*d^29*e^13 - 1052803072*a^18*c^12*d^27
*e^15 + 848429056*a^19*c^11*d^25*e^17 + 2105458688*a^20*c^10*d^23*e^19 + 19
09030912*a^21*c^9*d^21*e^21 + 959037440*a^22*c^8*d^19*e^23 + 262144000*a^23
*c^7*d^17*e^25 + 30408704*a^24*c^6*d^15*e^27))*((81*a^3*e^6*(-a^11*c^5)^(1/
2) - 49*c^3*d^6*(-a^11*c^5)^(1/2) + 70*a^6*c^5*d^5*e + 198*a^8*c^3*d*e^5 +
236*a^7*c^4*d^3*e^3 - 129*a*c^2*d^4*e^2*(-a^11*c^5)^(1/2) - 31*a^2*c*d^2*e^
4*(-a^11*c^5)^(1/2))/(256*(a^15*e^8 + a^11*c^4*d^8 + 4*a^14*c*d^2*e^6 + 4*a
^12*c^3*d^6*e^2 + 6*a^13*c^2*d^4*e^4)))^(1/2)*1i)/((x*(4917248*a^10*c^18*d^
36*e^5 + 50677760*a^11*c^17*d^34*e^7 + 230498304*a^12*c^16*d^32*e^9 + 60755
9680*a^13*c^15*d^30*e^11 + 1026486272*a^14*c^14*d^28*e^13 + 1166602240*a^15
*c^13*d^26*e^15 + 923508736*a^16*c^12*d^24*e^17 + 539500544*a^17*c^11*d^22*
e^19 + 259409920*a^18*c^10*d^20*e^21 + 109709312*a^19*c^9*d^18*e^23 + 34537
472*a^20*c^8*d^16*e^25 + 5308416*a^21*c^7*d^14*e^27) - ((81*a^3*e^6*(-a^11*
c^5)^(1/2) - 49*c^3*d^6*(-a^11*c^5)^(1/2) + 70*a^6*c^5*d^5*e + 198*a^8*c^3*
d*e^5 + 236*a^7*c^4*d^3*e^3 - 129*a*c^2*d^4*e^2*(-a^11*c^5)^(1/2) - 31*a^2*
c*d^2*e^4*(-a^11*c^5)^(1/2))/(256*(a^15*e^8 + a^11*c^4*d^8 + 4*a^14*c*d^2*e
^6 + 4*a^12*c^3*d^6*e^2 + 6*a^13*c^2*d^4*e^4)))^(1/2)*((x*(1787297792*a^19*
c^13*d^31*e^12 - 147587072*a^15*c^17*d^39*e^4 - 698089472*a^16*c^16*d^37*e^
6 - 1660157952*a^17*c^15*d^35*e^8 - 1588068352*a^18*c^14*d^33*e^10 - 128450
56*a^14*c^18*d^41*e^2 + 7839678464*a^20*c^12*d^29*e^14 + 11879841792*a^21*c
^11*d^27*e^16 + 10631249920*a^22*c^10*d^25*e^18 + 6274940928*a^23*c^9*d^23*
e^20 + 2652110848*a^24*c^8*d^21*e^22 + 891027456*a^25*c^7*d^19*e^24 + 23488
1024*a^26*c^6*d^17*e^26 + 33554432*a^27*c^5*d^15*e^28) - ((81*a^3*e^6*(-a^1
1*c^5)^(1/2) - 49*c^3*d^6*(-a^11*c^5)^(1/2) + 70*a^6*c^5*d^5*e + 198*a^8*c^
3*d*e^5 + 236*a^7*c^4*d^3*e^3 - 129*a*c^2*d^4*e^2*(-a^11*c^5)^(1/2) - 31*a^
2*c*d^2*e^4*(-a^11*c^5)^(1/2))/(256*(a^15*e^8 + a^11*c^4*d^8 + 4*a^14*c*d^2

*e^6 + 4*a^12*c^3*d^6*e^2 + 6*a^13*c^2*d^4*e^4)))^(1/2)*(29360128*a^17*c^17
*d^42*e^2 - x*((81*a^3*e^6*(-a^11*c^5)^(1/2) - 49*c^3*d^6*(-a^11*c^5)^(1/2)
 + 70*a^6*c^5*d^5*e + 198*a^8*c^3*d*e^5 + 236*a^7*c^4*d^3*e^3 - 129*a*c^2*d
^4*e^2*(-a^11*c^5)^(1/2) - 31*a^2*c*d^2*e^4*(-a^11*c^5)^(1/2))/(256*(a^15*e
^8 + a^11*c^4*d^8 + 4*a^14*c*d^2*e^6 + 4*a^12*c^3*d^6*e^2 + 6*a^13*c^2*d^4*
e^4)))^(1/2)*(134217728*a^20*c^16*d^42*e^3 + 1342177280*a^21*c^15*d^40*e^5
+ 5905580032*a^22*c^14*d^38*e^7 + 14763950080*a^23*c^13*d^36*e^9 + 22145925
120*a^24*c^12*d^34*e^11 + 17716740096*a^25*c^11*d^32*e^13 - 17716740096*a^2
7*c^9*d^28*e^17 - 22145925120*a^28*c^8*d^26*e^19 - 14763950080*a^29*c^7*d^2
4*e^21 - 5905580032*a^30*c^6*d^22*e^23 - 1342177280*a^31*c^5*d^20*e^25 - 13
4217728*a^32*c^4*d^18*e^27) + 239075328*a^18*c^16*d^40*e^4 + 708837376*a^19
*c^15*d^38*e^6 + 465567744*a^20*c^14*d^36*e^8 - 2726297600*a^21*c^13*d^34*e
^10 - 9084862464*a^22*c^12*d^32*e^12 - 13614710784*a^23*c^11*d^30*e^14 - 10
745806848*a^24*c^10*d^28*e^16 - 2403336192*a^25*c^9*d^26*e^18 + 3879731200*
a^26*c^8*d^24*e^20 + 4517265408*a^27*c^7*d^22*e^22 + 2294284288*a^28*c^6*d^
20*e^24 + 603979776*a^29*c^5*d^18*e^26 + 67108864*a^30*c^4*d^16*e^28))*((81
*a^3*e^6*(-a^11*c^5)^(1/2) - 49*c^3*d^6*(-a^11*c^5)^(1/2) + 70*a^6*c^5*d^5*
e + 198*a^8*c^3*d*e^5 + 236*a^7*c^4*d^3*e^3 - 129*a*c^2*d^4*e^2*(-a^11*c^5)
^(1/2) - 31*a^2*c*d^2*e^4*(-a^11*c^5)^(1/2))/(256*(a^15*e^8 + a^11*c^4*d^8
+ 4*a^14*c*d^2*e^6 + 4*a^12*c^3*d^6*e^2 + 6*a^13*c^2*d^4*e^4)))^(1/2) - 722
5344*a^12*c^18*d^39*e^3 - 76972032*a^13*c^17*d^37*e^5 - 367607808*a^14*c^16
*d^35*e^7 - 1036910592*a^15*c^15*d^33*e^9 - 1876983808*a^16*c^14*d^31*e^11
- 2115436544*a^17*c^13*d^29*e^13 - 1052803072*a^18*c^12*d^27*e^15 + 8484290
56*a^19*c^11*d^25*e^17 + 2105458688*a^20*c^10*d^23*e^19 + 1909030912*a^21*c
^9*d^21*e^21 + 959037440*a^22*c^8*d^19*e^23 + 262144000*a^23*c^7*d^17*e^25
+ 30408704*a^24*c^6*d^15*e^27))*((81*a^3*e^6*(-a^11*c^5)^(1/2) - 49*c^3*d^6
*(-a^11*c^5)^(1/2) + 70*a^6*c^5*d^5*e + 198*a^8*c^3*d*e^5 + 236*a^7*c^4*d^3
*e^3 - 129*a*c^2*d^4*e^2*(-a^11*c^5)^(1/2) - 31*a^2*c*d^2*e^4*(-a^11*c^5)^(
1/2))/(256*(a^15*e^8 + a^11*c^4*d^8 + 4*a^14*c*d^2*e^6 + 4*a^12*c^3*d^6*e^2
 + 6*a^13*c^2*d^4*e^4)))^(1/2) - (x*(4917248*a^10*c^18*d^36*e^5 + 50677760*
a^11*c^17*d^34*e^7 + 230498304*a^12*c^16*d^32*e^9 + 607559680*a^13*c^15*d^3
0*e^11 + 1026486272*a^14*c^14*d^28*e^13 + 1166602240*a^15*c^13*d^26*e^15 +
923508736*a^16*c^12*d^24*e^17 + 539500544*a^17*c^11*d^22*e^19 + 259409920*a
^18*c^10*d^20*e^21 + 109709312*a^19*c^9*d^18*e^23 + 34537472*a^20*c^8*d^16*
e^25 + 5308416*a^21*c^7*d^14*e^27) - ((81*a^3*e^6*(-a^11*c^5)^(1/2) - 49*c^
3*d^6*(-a^11*c^5)^(1/2) + 70*a^6*c^5*d^5*e + 198*a^8*c^3*d*e^5 + 236*a^7*c^
4*d^3*e^3 - 129*a*c^2*d^4*e^2*(-a^11*c^5)^(1/2) - 31*a^2*c*d^2*e^4*(-a^11*c
^5)^(1/2))/(256*(a^15*e^8 + a^11*c^4*d^8 + 4*a^14*c*d^2*e^6 + 4*a^12*c^3*d^
6*e^2 + 6*a^13*c^2*d^4*e^4)))^(1/2)*((x*(1787297792*a^19*c^13*d^31*e^12 - 1
47587072*a^15*c^17*d^39*e^4 - 698089472*a^16*c^16*d^37*e^6 - 1660157952*a^1
7*c^15*d^35*e^8 - 1588068352*a^18*c^14*d^33*e^10 - 12845056*a^14*c^18*d^41*
e^2 + 7839678464*a^20*c^12*d^29*e^14 + 11879841792*a^21*c^11*d^27*e^16 + 10
631249920*a^22*c^10*d^25*e^18 + 6274940928*a^23*c^9*d^23*e^20 + 2652110848*
a^24*c^8*d^21*e^22 + 891027456*a^25*c^7*d^19*e^24 + 234881024*a^26*c^6*d^17
*e^26 + 33554432*a^27*c^5*d^15*e^28) + ((81*a^3*e^6*(-a^11*c^5)^(1/2) - 49*

c^3*d^6*(-a^11*c^5)^(1/2) + 70*a^6*c^5*d^5*e + 198*a^8*c^3*d*e^5 + 236*a^7*c^4*d^3*e^3 - 129*a*c^2*d^4*e^2*(-a^11*c^5)^(1/2) - 31*a^2*c*d^2*e^4*(-a^11*c^5)^(1/2))/(256*(a^15*e^8 + a^11*c^4*d^8 + 4*a^14*c*d^2*e^6 + 4*a^12*c^3*d^6*e^2 + 6*a^13*c^2*d^4*e^4)))^(1/2)*(x*((81*a^3*e^6*(-a^11*c^5)^(1/2) - 49*c^3*d^6*(-a^11*c^5)^(1/2) + 70*a^6*c^5*d^5*e + 198*a^8*c^3*d*e^5 + 236*a^7*c^4*d^3*e^3 - 129*a*c^2*d^4*e^2*(-a^11*c^5)^(1/2) - 31*a^2*c*d^2*e^4*(-a^11*c^5)^(1/2))/(256*(a^15*e^8 + a^11*c^4*d^8 + 4*a^14*c*d^2*e^6 + 4*a^12*c^3*d^6*e^2 + 6*a^13*c^2*d^4*e^4)))^(1/2)*(134217728*a^20*c^16*d^42*e^3 + 1342177280*a^21*c^15*d^40*e^5 + 5905580032*a^22*c^14*d^38*e^7 + 14763950080*a^23*c^13*d^36*e^9 + 22145925120*a^24*c^12*d^34*e^11 + 17716740096*a^25*c^11*d^32*e^13 - 17716740096*a^27*c^9*d^28*e^17 - 22145925120*a^28*c^8*d^26*e^19 - 14763950080*a^29*c^7*d^24*e^21 - 5905580032*a^30*c^6*d^22*e^23 - 1342177280*a^31*c^5*d^20*e^25 - 134217728*a^32*c^4*d^18*e^27) + 29360128*a^17*c^17*d^42*e^2 + 239075328*a^18*c^16*d^40*e^4 + 708837376*a^19*c^15*d^38*e^6 + 465567744*a^20*c^14*d^36*e^8 - 2726297600*a^21*c^13*d^34*e^10 - 9084862464*a^22*c^12*d^32*e^12 - 13614710784*a^23*c^11*d^30*e^14 - 10745806848*a^24*c^10*d^28*e^16 - 2403336192*a^25*c^9*d^26*e^18 + 3879731200*a^26*c^8*d^24*e^20 + 4517265408*a^27*c^7*d^22*e^22 + 2294284288*a^28*c^6*d^20*e^24 + 603979776*a^29*c^5*d^18*e^26 + 67108864*a^30*c^4*d^16*e^28))*((81*a^3*e^6*(-a^11*c^5)^(1/2) - 49*c^3*d^6*(-a^11*c^5)^(1/2) + 70*a^6*c^5*d^5*e + 198*a^8*c^3*d*e^5 + 236*a^7*c^4*d^3*e^3 - 129*a*c^2*d^4*e^2*(-a^11*c^5)^(1/2) - 31*a^2*c*d^2*e^4*(-a^11*c^5)^(1/2))/(256*(a^15*e^8 + a^11*c^4*d^8 + 4*a^14*c*d^2*e^6 + 4*a^12*c^3*d^6*e^2 + 6*a^13*c^2*d^4*e^4)))^(1/2) + 7225344*a^12*c^18*d^39*e^3 + 76972032*a^13*c^17*d^37*e^5 + 367607808*a^14*c^16*d^35*e^7 + 1036910592*a^15*c^15*d^33*e^9 + 1876983808*a^16*c^14*d^31*e^11 + 2115436544*a^17*c^13*d^29*e^13 + 1052803072*a^18*c^12*d^27*e^15 - 848429056*a^19*c^11*d^25*e^17 - 2105458688*a^20*c^10*d^23*e^19 - 1909030912*a^21*c^9*d^21*e^21 - 959037440*a^22*c^8*d^19*e^23 - 262144000*a^23*c^7*d^17*e^25 - 30408704*a^24*c^6*d^15*e^27))*((81*a^3*e^6*(-a^11*c^5)^(1/2) - 49*c^3*d^6*(-a^11*c^5)^(1/2) + 70*a^6*c^5*d^5*e + 198*a^8*c^3*d*e^5 + 236*a^7*c^4*d^3*e^3 - 129*a*c^2*d^4*e^2*(-a^11*c^5)^(1/2) - 31*a^2*c*d^2*e^4*(-a^11*c^5)^(1/2))/(256*(a^15*e^8 + a^11*c^4*d^8 + 4*a^14*c*d^2*e^6 + 4*a^12*c^3*d^6*e^2 + 6*a^13*c^2*d^4*e^4)))^(1/2) + 4917248*a^10*c^16*d^30*e^10 + 40843264*a^11*c^15*d^28*e^12 + 147507200*a^12*c^14*d^26*e^14 + 302962688*a^13*c^13*d^24*e^16 + 387512320*a^14*c^12*d^22*e^18 + 316418048*a^15*c^11*d^20*e^20 + 161224704*a^16*c^10*d^18*e^22 + 46909440*a^17*c^9*d^16*e^24 + 5971968*a^18*c^8*d^14*e^26))*((81*a^3*e^6*(-a^11*c^5)^(1/2) - 49*c^3*d^6*(-a^11*c^5)^(1/2) + 70*a^6*c^5*d^5*e + 198*a^8*c^3*d*e^5 + 236*a^7*c^4*d^3*e^3 - 129*a*c^2*d^4*e^2*(-a^11*c^5)^(1/2) - 31*a^2*c*d^2*e^4*(-a^11*c^5)^(1/2))/(256*(a^15*e^8 + a^11*c^4*d^8 + 4*a^14*c*d^2*e^6 + 4*a^12*c^3*d^6*e^2 + 6*a^13*c^2*d^4*e^4)))^(1/2)*2i - (1/(3*a*d) - (e*x^2)/(a*d^2) + (x^4*(7*c^2*d^2 + 4*a*c*e^2))/(12*a^2*d*(a*e^2 + c*d^2)) - (c*x^6*(4*a*e^3 + 5*c*d^2*e))/(4*a^2*d^2*(a*e^2 + c*d^2)))/(a*x^3 + c*x^7) + atan(((a^11*c^5*(156627*c^2*d^6*e^12 - 245952*a^2*d^2*e^16 + 324032*a*c*d^4*e^14) - 16807*a^5*c^13*d^18 + 46656*a^14*c^4*e^18 + 24696*a^6*c^12*d^16*e^2 + 455609*a^7*c^11*d^14*e^4 + 856936*a^8*c^10*d^12*e^6 - 2

7429*a^9*c^9*d^10*e^8 - 805344*a^10*c^8*d^8*e^10)*(a^13*c^11*d^21*x*((49*c^3*d^6*(-a^11*c^5)^(1/2) - 81*a^3*e^6*(-a^11*c^5)^(1/2) + 70*a^6*c^5*d^5*e + 198*a^8*c^3*d*e^5 + 236*a^7*c^4*d^3*e^3 + 129*a*c^2*d^4*e^2*(-a^11*c^5)^(1/2) + 31*a^2*c*d^2*e^4*(-a^11*c^5)^(1/2))/(a^15*e^8 + a^11*c^4*d^8 + 4*a^14*c*d^2*e^6 + 4*a^12*c^3*d^6*e^2 + 6*a^13*c^2*d^4*e^4))^(3/2)*49i + a^17*c^3*e^19*x*((49*c^3*d^6*(-a^11*c^5)^(1/2) - 81*a^3*e^6*(-a^11*c^5)^(1/2) + 70*a^6*c^5*d^5*e + 198*a^8*c^3*d*e^5 + 236*a^7*c^4*d^3*e^3 + 129*a*c^2*d^4*e^2*(-a^11*c^5)^(1/2) + 31*a^2*c*d^2*e^4*(-a^11*c^5)^(1/2))/(a^15*e^8 + a^11*c^4*d^8 + 4*a^14*c*d^2*e^6 + 4*a^12*c^3*d^6*e^2 + 6*a^13*c^2*d^4*e^4))^(1/2)*5184i + a^28*d^4*e^19*x*((49*c^3*d^6*(-a^11*c^5)^(1/2) - 81*a^3*e^6*(-a^11*c^5)^(1/2) + 70*a^6*c^5*d^5*e + 198*a^8*c^3*d*e^5 + 236*a^7*c^4*d^3*e^3 + 129*a*c^2*d^4*e^2*(-a^11*c^5)^(1/2) + 31*a^2*c*d^2*e^4*(-a^11*c^5)^(1/2))/(a^15*e^8 + a^11*c^4*d^8 + 4*a^14*c*d^2*e^6 + 4*a^12*c^3*d^6*e^2 + 6*a^13*c^2*d^4*e^4))^(5/2)*2i - a^19*c^9*d^22*e*x*((49*c^3*d^6*(-a^11*c^5)^(1/2) - 81*a^3*e^6*(-a^11*c^5)^(1/2) + 70*a^6*c^5*d^5*e + 198*a^8*c^3*d*e^5 + 236*a^7*c^4*d^3*e^3 + 129*a*c^2*d^4*e^2*(-a^11*c^5)^(1/2) + 31*a^2*c*d^2*e^4*(-a^11*c^5)^(1/2))/(a^15*e^8 + a^11*c^4*d^8 + 4*a^14*c*d^2*e^6 + 4*a^12*c^3*d^6*e^2 + 6*a^13*c^2*d^4*e^4))^(5/2)*2i + a^27*c*d^6*e^17*x*((49*c^3*d^6*(-a^11*c^5)^(1/2) - 81*a^3*e^6*(-a^11*c^5)^(1/2) + 70*a^6*c^5*d^5*e + 198*a^8*c^3*d*e^5 + 236*a^7*c^4*d^3*e^3 + 129*a*c^2*d^4*e^2*(-a^11*c^5)^(1/2) + 31*a^2*c*d^2*e^4*(-a^11*c^5)^(1/2))/(a^15*e^8 + a^11*c^4*d^8 + 4*a^14*c*d^2*e^6 + 4*a^12*c^3*d^6*e^2 + 6*a^13*c^2*d^4*e^4))^(5/2)*14i + a^9*c^11*d^16*e^3*x*((49*c^3*d^6*(-a^11*c^5)^(1/2) - 81*a^3*e^6*(-a^11*c^5)^(1/2) + 70*a^6*c^5*d^5*e + 198*a^8*c^3*d*e^5 + 236*a^7*c^4*d^3*e^3 + 129*a*c^2*d^4*e^2*(-a^11*c^5)^(1/2) + 31*a^2*c*d^2*e^4*(-a^11*c^5)^(1/2))/(a^15*e^8 + a^11*c^4*d^8 + 4*a^14*c*d^2*e^6 + 4*a^12*c^3*d^6*e^2 + 6*a^13*c^2*d^4*e^4))^(1/2)*4802i + a^10*c^10*d^14*e^5*x*((49*c^3*d^6*(-a^11*c^5)^(1/2) - 81*a^3*e^6*(-a^11*c^5)^(1/2) + 70*a^6*c^5*d^5*e + 198*a^8*c^3*d*e^5 + 236*a^7*c^4*d^3*e^3 + 129*a*c^2*d^4*e^2*(-a^11*c^5)^(1/2) + 31*a^2*c*d^2*e^4*(-a^11*c^5)^(1/2))/(a^15*e^8 + a^11*c^4*d^8 + 4*a^14*c*d^2*e^6 + 4*a^12*c^3*d^6*e^2 + 6*a^13*c^2*d^4*e^4))^(1/2)*35084i + a^11*c^9*d^12*e^7*x*((49*c^3*d^6*(-a^11*c^5)^(1/2) - 81*a^3*e^6*(-a^11*c^5)^(1/2) + 70*a^6*c^5*d^5*e + 198*a^8*c^3*d*e^5 + 236*a^7*c^4*d^3*e^3 + 129*a*c^2*d^4*e^2*(-a^11*c^5)^(1/2) + 31*a^2*c*d^2*e^4*(-a^11*c^5)^(1/2))/(a^15*e^8 + a^11*c^4*d^8 + 4*a^14*c*d^2*e^6 + 4*a^12*c^3*d^6*e^2 + 6*a^13*c^2*d^4*e^4))^(1/2)*105438i + a^12*c^8*d^10*e^9*x*((49*c^3*d^6*(-a^11*c^5)^(1/2) - 81*a^3*e^6*(-a^11*c^5)^(1/2) + 70*a^6*c^5*d^5*e + 198*a^8*c^3*d*e^5 + 236*a^7*c^4*d^3*e^3 + 129*a*c^2*d^4*e^2*(-a^11*c^5)^(1/2) + 31*a^2*c*d^2*e^4*(-a^11*c^5)^(1/2))/(a^15*e^8 + a^11*c^4*d^8 + 4*a^14*c*d^2*e^6 + 4*a^12*c^3*d^6*e^2 + 6*a^13*c^2*d^4*e^4))^(1/2)*166952i + a^13*c^7*d^8*e^11*x*((49*c^3*d^6*(-a^11*c^5)^(1/2) - 81*a^3*e^6*(-a^11*c^5)^(1/2) + 70*a^6*c^5*d^5*e + 198*a^8*c^3*d*e^5 + 236*a^7*c^4*d^3*e^3 + 129*a*c^2*d^4*e^2*(-a^11*c^5)^(1/2) + 31*a^2*c*d^2*e^4*(-a^11*c^5)^(1/2))/(a^15*e^8 + a^11*c^4*d^8 + 4*a^14*c*d^2*e^6 + 4*a^12*c^3*d^6*e^2 + 6*a^13*c^2*d^4*e^4))^(1/2)*150174i + a^14*c^6*d^6*e^13*x*((49*c^3*d^6*(-a^11*c^5)^(1/2) - 81*a^3*e^6*(-a^11*c^5)^(1/2) + 70*a^6*c^5*d^5*e + 198*a^8*c^3*d*e^5 + 236*a^7*

c^4*d^3*e^3 + 129*a*c^2*d^4*e^2*(-a^11*c^5)^(1/2) + 31*a^2*c*d^2*e^4*(-a^11
*c^5)^(1/2))/(a^15*e^8 + a^11*c^4*d^8 + 4*a^14*c*d^2*e^6 + 4*a^12*c^3*d^6*e
^2 + 6*a^13*c^2*d^4*e^4))^(1/2)*82444i + a^15*c^5*d^4*e^15*x*((49*c^3*d^6*(
-a^11*c^5)^(1/2) - 81*a^3*e^6*(-a^11*c^5)^(1/2) + 70*a^6*c^5*d^5*e + 198*a^
8*c^3*d*e^5 + 236*a^7*c^4*d^3*e^3 + 129*a*c^2*d^4*e^2*(-a^11*c^5)^(1/2) + 3
1*a^2*c*d^2*e^4*(-a^11*c^5)^(1/2))/(a^15*e^8 + a^11*c^4*d^8 + 4*a^14*c*d^2*
e^6 + 4*a^12*c^3*d^6*e^2 + 6*a^13*c^2*d^4*e^4))^(1/2)*37058i + a^16*c^4*d^2
*e^17*x*((49*c^3*d^6*(-a^11*c^5)^(1/2) - 81*a^3*e^6*(-a^11*c^5)^(1/2) + 70*
a^6*c^5*d^5*e + 198*a^8*c^3*d*e^5 + 236*a^7*c^4*d^3*e^3 + 129*a*c^2*d^4*e^2
*(-a^11*c^5)^(1/2) + 31*a^2*c*d^2*e^4*(-a^11*c^5)^(1/2))/(a^15*e^8 + a^11*c
^4*d^8 + 4*a^14*c*d^2*e^6 + 4*a^12*c^3*d^6*e^2 + 6*a^13*c^2*d^4*e^4))^(1/2)
*18176i + a^14*c^10*d^19*e^2*x*((49*c^3*d^6*(-a^11*c^5)^(1/2) - 81*a^3*e^6*
(-a^11*c^5)^(1/2) + 70*a^6*c^5*d^5*e + 198*a^8*c^3*d*e^5 + 236*a^7*c^4*d^3*
e^3 + 129*a*c^2*d^4*e^2*(-a^11*c^5)^(1/2) + 31*a^2*c*d^2*e^4*(-a^11*c^5)^(1
/2))/(a^15*e^8 + a^11*c^4*d^8 + 4*a^14*c*d^2*e^6 + 4*a^12*c^3*d^6*e^2 + 6*a
^13*c^2*d^4*e^4))^(3/2)*416i + a^15*c^9*d^17*e^4*x*((49*c^3*d^6*(-a^11*c^5)
^(1/2) - 81*a^3*e^6*(-a^11*c^5)^(1/2) + 70*a^6*c^5*d^5*e + 198*a^8*c^3*d*e^
5 + 236*a^7*c^4*d^3*e^3 + 129*a*c^2*d^4*e^2*(-a^11*c^5)^(1/2) + 31*a^2*c*d^
2*e^4*(-a^11*c^5)^(1/2))/(a^15*e^8 + a^11*c^4*d^8 + 4*a^14*c*d^2*e^6 + 4*a^
12*c^3*d^6*e^2 + 6*a^13*c^2*d^4*e^4))^(3/2)*1268i + a^16*c^8*d^15*e^6*x*((4
9*c^3*d^6*(-a^11*c^5)^(1/2) - 81*a^3*e^6*(-a^11*c^5)^(1/2) + 70*a^6*c^5*d^5
*e + 198*a^8*c^3*d*e^5 + 236*a^7*c^4*d^3*e^3 + 129*a*c^2*d^4*e^2*(-a^11*c^5
)^(1/2) + 31*a^2*c*d^2*e^4*(-a^11*c^5)^(1/2))/(a^15*e^8 + a^11*c^4*d^8 + 4*
a^14*c*d^2*e^6 + 4*a^12*c^3*d^6*e^2 + 6*a^13*c^2*d^4*e^4))^(3/2)*1232i - a^
17*c^7*d^13*e^8*x*((49*c^3*d^6*(-a^11*c^5)^(1/2) - 81*a^3*e^6*(-a^11*c^5)^(
1/2) + 70*a^6*c^5*d^5*e + 198*a^8*c^3*d*e^5 + 236*a^7*c^4*d^3*e^3 + 129*a*c
^2*d^4*e^2*(-a^11*c^5)^(1/2) + 31*a^2*c*d^2*e^4*(-a^11*c^5)^(1/2))/(a^15*e^
8 + a^11*c^4*d^8 + 4*a^14*c*d^2*e^6 + 4*a^12*c^3*d^6*e^2 + 6*a^13*c^2*d^4*e
^4))^(3/2)*1858i - a^18*c^6*d^11*e^10*x*((49*c^3*d^6*(-a^11*c^5)^(1/2) - 81
*a^3*e^6*(-a^11*c^5)^(1/2) + 70*a^6*c^5*d^5*e + 198*a^8*c^3*d*e^5 + 236*a^7
*c^4*d^3*e^3 + 129*a*c^2*d^4*e^2*(-a^11*c^5)^(1/2) + 31*a^2*c*d^2*e^4*(-a^1
1*c^5)^(1/2))/(a^15*e^8 + a^11*c^4*d^8 + 4*a^14*c*d^2*e^6 + 4*a^12*c^3*d^6*
e^2 + 6*a^13*c^2*d^4*e^4))^(3/2)*6208i - a^19*c^5*d^9*e^12*x*((49*c^3*d^6*(
-a^11*c^5)^(1/2) - 81*a^3*e^6*(-a^11*c^5)^(1/2) + 70*a^6*c^5*d^5*e + 198*a^
8*c^3*d*e^5 + 236*a^7*c^4*d^3*e^3 + 129*a*c^2*d^4*e^2*(-a^11*c^5)^(1/2) + 3
1*a^2*c*d^2*e^4*(-a^11*c^5)^(1/2))/(a^15*e^8 + a^11*c^4*d^8 + 4*a^14*c*d^2*
e^6 + 4*a^12*c^3*d^6*e^2 + 6*a^13*c^2*d^4*e^4))^(3/2)*6940i - a^20*c^4*d^7*
e^14*x*((49*c^3*d^6*(-a^11*c^5)^(1/2) - 81*a^3*e^6*(-a^11*c^5)^(1/2) + 70*a
^6*c^5*d^5*e + 198*a^8*c^3*d*e^5 + 236*a^7*c^4*d^3*e^3 + 129*a*c^2*d^4*e^2*
(-a^11*c^5)^(1/2) + 31*a^2*c*d^2*e^4*(-a^11*c^5)^(1/2))/(a^15*e^8 + a^11*c^
4*d^8 + 4*a^14*c*d^2*e^6 + 4*a^12*c^3*d^6*e^2 + 6*a^13*c^2*d^4*e^4))^(3/2)*
4016i - a^21*c^3*d^5*e^16*x*((49*c^3*d^6*(-a^11*c^5)^(1/2) - 81*a^3*e^6*(-a
^11*c^5)^(1/2) + 70*a^6*c^5*d^5*e + 198*a^8*c^3*d*e^5 + 236*a^7*c^4*d^3*e^3
 + 129*a*c^2*d^4*e^2*(-a^11*c^5)^(1/2) + 31*a^2*c*d^2*e^4*(-a^11*c^5)^(1/2)
)/(a^15*e^8 + a^11*c^4*d^8 + 4*a^14*c*d^2*e^6 + 4*a^12*c^3*d^6*e^2 + 6*a^13

*c^2*d^4*e^4))^(3/2)*1479i - a^22*c^2*d^3*e^18*x*((49*c^3*d^6*(-a^11*c^5)^(1/2) - 81*a^3*e^6*(-a^11*c^5)^(1/2) + 70*a^6*c^5*d^5*e + 198*a^8*c^3*d*e^5 + 236*a^7*c^4*d^3*e^3 + 129*a*c^2*d^4*e^2*(-a^11*c^5)^(1/2) + 31*a^2*c*d^2*e^4*(-a^11*c^5)^(1/2))/(a^15*e^8 + a^11*c^4*d^8 + 4*a^14*c*d^2*e^6 + 4*a^12*c^3*d^6*e^2 + 6*a^13*c^2*d^4*e^4))^(3/2)*512i - a^20*c^8*d^20*e^3*x*((49*c^3*d^6*(-a^11*c^5)^(1/2) - 81*a^3*e^6*(-a^11*c^5)^(1/2) + 70*a^6*c^5*d^5*e + 198*a^8*c^3*d*e^5 + 236*a^7*c^4*d^3*e^3 + 129*a*c^2*d^4*e^2*(-a^11*c^5)^(1/2) + 31*a^2*c*d^2*e^4*(-a^11*c^5)^(1/2))/(a^15*e^8 + a^11*c^4*d^8 + 4*a^14*c*d^2*e^6 + 4*a^12*c^3*d^6*e^2 + 6*a^13*c^2*d^4*e^4))^(5/2)*14i - a^21*c^7*d^18*e^5*x*((49*c^3*d^6*(-a^11*c^5)^(1/2) - 81*a^3*e^6*(-a^11*c^5)^(1/2) + 70*a^6*c^5*d^5*e + 198*a^8*c^3*d*e^5 + 236*a^7*c^4*d^3*e^3 + 129*a*c^2*d^4*e^2*(-a^11*c^5)^(1/2) + 31*a^2*c*d^2*e^4*(-a^11*c^5)^(1/2))/(a^15*e^8 + a^11*c^4*d^8 + 4*a^14*c*d^2*e^6 + 4*a^12*c^3*d^6*e^2 + 6*a^13*c^2*d^4*e^4))^(5/2)*40i - a^22*c^6*d^16*e^7*x*((49*c^3*d^6*(-a^11*c^5)^(1/2) - 81*a^3*e^6*(-a^11*c^5)^(1/2) + 70*a^6*c^5*d^5*e + 198*a^8*c^3*d*e^5 + 236*a^7*c^4*d^3*e^3 + 129*a*c^2*d^4*e^2*(-a^11*c^5)^(1/2) + 31*a^2*c*d^2*e^4*(-a^11*c^5)^(1/2))/(a^15*e^8 + a^11*c^4*d^8 + 4*a^14*c*d^2*e^6 + 4*a^12*c^3*d^6*e^2 + 6*a^13*c^2*d^4*e^4))^(5/2)*56i - a^23*c^5*d^14*e^9*x*((49*c^3*d^6*(-a^11*c^5)^(1/2) - 81*a^3*e^6*(-a^11*c^5)^(1/2) + 70*a^6*c^5*d^5*e + 198*a^8*c^3*d*e^5 + 236*a^7*c^4*d^3*e^3 + 129*a*c^2*d^4*e^2*(-a^11*c^5)^(1/2) + 31*a^2*c*d^2*e^4*(-a^11*c^5)^(1/2))/(a^15*e^8 + a^11*c^4*d^8 + 4*a^14*c*d^2*e^6 + 4*a^12*c^3*d^6*e^2 + 6*a^13*c^2*d^4*e^4))^(5/2)*28i + a^24*c^4*d^12*e^11*x*((49*c^3*d^6*(-a^11*c^5)^(1/2) - 81*a^3*e^6*(-a^11*c^5)^(1/2) + 70*a^6*c^5*d^5*e + 198*a^8*c^3*d*e^5 + 236*a^7*c^4*d^3*e^3 + 129*a*c^2*d^4*e^2*(-a^11*c^5)^(1/2) + 31*a^2*c*d^2*e^4*(-a^11*c^5)^(1/2))/(a^15*e^8 + a^11*c^4*d^8 + 4*a^14*c*d^2*e^6 + 4*a^12*c^3*d^6*e^2 + 6*a^13*c^2*d^4*e^4))^(5/2)*28i + a^25*c^3*d^10*e^13*x*((49*c^3*d^6*(-a^11*c^5)^(1/2) - 81*a^3*e^6*(-a^11*c^5)^(1/2) + 70*a^6*c^5*d^5*e + 198*a^8*c^3*d*e^5 + 236*a^7*c^4*d^3*e^3 + 129*a*c^2*d^4*e^2*(-a^11*c^5)^(1/2) + 31*a^2*c*d^2*e^4*(-a^11*c^5)^(1/2))/(a^15*e^8 + a^11*c^4*d^8 + 4*a^14*c*d^2*e^6 + 4*a^12*c^3*d^6*e^2 + 6*a^13*c^2*d^4*e^4))^(5/2)*56i + a^26*c^2*d^8*e^15*x*((49*c^3*d^6*(-a^11*c^5)^(1/2) - 81*a^3*e^6*(-a^11*c^5)^(1/2) + 70*a^6*c^5*d^5*e + 198*a^8*c^3*d*e^5 + 236*a^7*c^4*d^3*e^3 + 129*a*c^2*d^4*e^2*(-a^11*c^5)^(1/2) + 31*a^2*c*d^2*e^4*(-a^11*c^5)^(1/2))/(a^15*e^8 + a^11*c^4*d^8 + 4*a^14*c*d^2*e^6 + 4*a^12*c^3*d^6*e^2 + 6*a^13*c^2*d^4*e^4))^(5/2)*40i - a^23*c*d*e^20*x*((49*c^3*d^6*(-a^11*c^5)^(1/2) - 81*a^3*e^6*(-a^11*c^5)^(1/2) + 70*a^6*c^5*d^5*e + 198*a^8*c^3*d*e^5 + 236*a^7*c^4*d^3*e^3 + 129*a*c^2*d^4*e^2*(-a^11*c^5)^(1/2) + 31*a^2*c*d^2*e^4*(-a^11*c^5)^(1/2))/(a^15*e^8 + a^11*c^4*d^8 + 4*a^14*c*d^2*e^6 + 4*a^12*c^3*d^6*e^2 + 6*a^13*c^2*d^4*e^4))^(3/2)*128i) - (-a^11*c^5)^(1/2)*(69629*c^10*d^17*e + 286944*a*c^9*d^15*e^3 + 150336*a^8*c^2*d*e^17 + 110645*a^2*c^8*d^13*e^5 - 770024*a^3*c^7*d^11*e^7 - 606089*a^4*c^6*d^9*e^9 + 566984*a^5*c^5*d^7*e^11 + 157207*a^6*c^4*d^5*e^13 - 327104*a^7*c^3*d^3*e^15)*(a^13*c^11*d^21*x*((49*c^3*d^6*(-a^11*c^5)^(1/2) - 81*a^3*e^6*(-a^11*c^5)^(1/2) + 70*a^6*c^5*d^5*e + 198*a^8*c^3*d*e^5 + 236*a^7*c^4*d^3*e^3 + 129*a*c^2*d^4*e^2*(-a^11*c^5)^(1/2) + 31*a^2*c*d^2*e^4*(-a^11*c^5)^(1/2))/(a^15*e^8 + a^11

*c^4*d^8 + 4*a^14*c*d^2*e^6 + 4*a^12*c^3*d^6*e^2 + 6*a^13*c^2*d^4*e^4))^(3/
2)*49i + a^17*c^3*e^19*x*((49*c^3*d^6*(-a^11*c^5)^(1/2) - 81*a^3*e^6*(-a^11
*c^5)^(1/2) + 70*a^6*c^5*d^5*e + 198*a^8*c^3*d*e^5 + 236*a^7*c^4*d^3*e^3 +
129*a*c^2*d^4*e^2*(-a^11*c^5)^(1/2) + 31*a^2*c*d^2*e^4*(-a^11*c^5)^(1/2))/(
a^15*e^8 + a^11*c^4*d^8 + 4*a^14*c*d^2*e^6 + 4*a^12*c^3*d^6*e^2 + 6*a^13*c^
2*d^4*e^4))^(1/2)*5184i + a^28*d^4*e^19*x*((49*c^3*d^6*(-a^11*c^5)^(1/2) -
81*a^3*e^6*(-a^11*c^5)^(1/2) + 70*a^6*c^5*d^5*e + 198*a^8*c^3*d*e^5 + 236*a
^7*c^4*d^3*e^3 + 129*a*c^2*d^4*e^2*(-a^11*c^5)^(1/2) + 31*a^2*c*d^2*e^4*(-a
^11*c^5)^(1/2))/(a^15*e^8 + a^11*c^4*d^8 + 4*a^14*c*d^2*e^6 + 4*a^12*c^3*d^
6*e^2 + 6*a^13*c^2*d^4*e^4))^(5/2)*2i - a^19*c^9*d^22*e*x*((49*c^3*d^6*(-a^
11*c^5)^(1/2) - 81*a^3*e^6*(-a^11*c^5)^(1/2) + 70*a^6*c^5*d^5*e + 198*a^8*c
^3*d*e^5 + 236*a^7*c^4*d^3*e^3 + 129*a*c^2*d^4*e^2*(-a^11*c^5)^(1/2) + 31*a
^2*c*d^2*e^4*(-a^11*c^5)^(1/2))/(a^15*e^8 + a^11*c^4*d^8 + 4*a^14*c*d^2*e^6
 + 4*a^12*c^3*d^6*e^2 + 6*a^13*c^2*d^4*e^4))^(5/2)*2i + a^27*c*d^6*e^17*x*(
(49*c^3*d^6*(-a^11*c^5)^(1/2) - 81*a^3*e^6*(-a^11*c^5)^(1/2) + 70*a^6*c^5*d
^5*e + 198*a^8*c^3*d*e^5 + 236*a^7*c^4*d^3*e^3 + 129*a*c^2*d^4*e^2*(-a^11*c
^5)^(1/2) + 31*a^2*c*d^2*e^4*(-a^11*c^5)^(1/2))/(a^15*e^8 + a^11*c^4*d^8 +
4*a^14*c*d^2*e^6 + 4*a^12*c^3*d^6*e^2 + 6*a^13*c^2*d^4*e^4))^(5/2)*14i + a^
9*c^11*d^16*e^3*x*((49*c^3*d^6*(-a^11*c^5)^(1/2) - 81*a^3*e^6*(-a^11*c^5)^(
1/2) + 70*a^6*c^5*d^5*e + 198*a^8*c^3*d*e^5 + 236*a^7*c^4*d^3*e^3 + 129*a*c
^2*d^4*e^2*(-a^11*c^5)^(1/2) + 31*a^2*c*d^2*e^4*(-a^11*c^5)^(1/2))/(a^15*e^
8 + a^11*c^4*d^8 + 4*a^14*c*d^2*e^6 + 4*a^12*c^3*d^6*e^2 + 6*a^13*c^2*d^4*e
^4))^(1/2)*4802i + a^10*c^10*d^14*e^5*x*((49*c^3*d^6*(-a^11*c^5)^(1/2) - 81
*a^3*e^6*(-a^11*c^5)^(1/2) + 70*a^6*c^5*d^5*e + 198*a^8*c^3*d*e^5 + 236*a^7
*c^4*d^3*e^3 + 129*a*c^2*d^4*e^2*(-a^11*c^5)^(1/2) + 31*a^2*c*d^2*e^4*(-a^1
1*c^5)^(1/2))/(a^15*e^8 + a^11*c^4*d^8 + 4*a^14*c*d^2*e^6 + 4*a^12*c^3*d^6*
e^2 + 6*a^13*c^2*d^4*e^4))^(1/2)*35084i + a^11*c^9*d^12*e^7*x*((49*c^3*d^6*
(-a^11*c^5)^(1/2) - 81*a^3*e^6*(-a^11*c^5)^(1/2) + 70*a^6*c^5*d^5*e + 198*a
^8*c^3*d*e^5 + 236*a^7*c^4*d^3*e^3 + 129*a*c^2*d^4*e^2*(-a^11*c^5)^(1/2) +
31*a^2*c*d^2*e^4*(-a^11*c^5)^(1/2))/(a^15*e^8 + a^11*c^4*d^8 + 4*a^14*c*d^2
*e^6 + 4*a^12*c^3*d^6*e^2 + 6*a^13*c^2*d^4*e^4))^(1/2)*105438i + a^12*c^8*d
^10*e^9*x*((49*c^3*d^6*(-a^11*c^5)^(1/2) - 81*a^3*e^6*(-a^11*c^5)^(1/2) + 7
0*a^6*c^5*d^5*e + 198*a^8*c^3*d*e^5 + 236*a^7*c^4*d^3*e^3 + 129*a*c^2*d^4*e
^2*(-a^11*c^5)^(1/2) + 31*a^2*c*d^2*e^4*(-a^11*c^5)^(1/2))/(a^15*e^8 + a^11
*c^4*d^8 + 4*a^14*c*d^2*e^6 + 4*a^12*c^3*d^6*e^2 + 6*a^13*c^2*d^4*e^4))^(1/
2)*166952i + a^13*c^7*d^8*e^11*x*((49*c^3*d^6*(-a^11*c^5)^(1/2) - 81*a^3*e^
6*(-a^11*c^5)^(1/2) + 70*a^6*c^5*d^5*e + 198*a^8*c^3*d*e^5 + 236*a^7*c^4*d^
3*e^3 + 129*a*c^2*d^4*e^2*(-a^11*c^5)^(1/2) + 31*a^2*c*d^2*e^4*(-a^11*c^5)^
(1/2))/(a^15*e^8 + a^11*c^4*d^8 + 4*a^14*c*d^2*e^6 + 4*a^12*c^3*d^6*e^2 + 6
*a^13*c^2*d^4*e^4))^(1/2)*150174i + a^14*c^6*d^6*e^13*x*((49*c^3*d^6*(-a^11
*c^5)^(1/2) - 81*a^3*e^6*(-a^11*c^5)^(1/2) + 70*a^6*c^5*d^5*e + 198*a^8*c^3
*d*e^5 + 236*a^7*c^4*d^3*e^3 + 129*a*c^2*d^4*e^2*(-a^11*c^5)^(1/2) + 31*a^2
*c*d^2*e^4*(-a^11*c^5)^(1/2))/(a^15*e^8 + a^11*c^4*d^8 + 4*a^14*c*d^2*e^6 +
 4*a^12*c^3*d^6*e^2 + 6*a^13*c^2*d^4*e^4))^(1/2)*82444i + a^15*c^5*d^4*e^15
x((49*c^3*d^6*(-a^11*c^5)^(1/2) - 81*a^3*e^6*(-a^11*c^5)^(1/2) + 70*a^6*c

^5*d^5*e + 198*a^8*c^3*d*e^5 + 236*a^7*c^4*d^3*e^3 + 129*a*c^2*d^4*e^2*(-a^
11*c^5)^(1/2) + 31*a^2*c*d^2*e^4*(-a^11*c^5)^(1/2))/(a^15*e^8 + a^11*c^4*d^
8 + 4*a^14*c*d^2*e^6 + 4*a^12*c^3*d^6*e^2 + 6*a^13*c^2*d^4*e^4))^(1/2)*3705
8i + a^16*c^4*d^2*e^17*x*((49*c^3*d^6*(-a^11*c^5)^(1/2) - 81*a^3*e^6*(-a^11
*c^5)^(1/2) + 70*a^6*c^5*d^5*e + 198*a^8*c^3*d*e^5 + 236*a^7*c^4*d^3*e^3 +
129*a*c^2*d^4*e^2*(-a^11*c^5)^(1/2) + 31*a^2*c*d^2*e^4*(-a^11*c^5)^(1/2))/(
a^15*e^8 + a^11*c^4*d^8 + 4*a^14*c*d^2*e^6 + 4*a^12*c^3*d^6*e^2 + 6*a^13*c^
2*d^4*e^4))^(1/2)*18176i + a^14*c^10*d^19*e^2*x*((49*c^3*d^6*(-a^11*c^5)^(1
/2) - 81*a^3*e^6*(-a^11*c^5)^(1/2) + 70*a^6*c^5*d^5*e + 198*a^8*c^3*d*e^5 +
236*a^7*c^4*d^3*e^3 + 129*a*c^2*d^4*e^2*(-a^11*c^5)^(1/2) + 31*a^2*c*d^2*e
^4*(-a^11*c^5)^(1/2))/(a^15*e^8 + a^11*c^4*d^8 + 4*a^14*c*d^2*e^6 + 4*a^12*
c^3*d^6*e^2 + 6*a^13*c^2*d^4*e^4))^(3/2)*416i + a^15*c^9*d^17*e^4*x*((49*c^
3*d^6*(-a^11*c^5)^(1/2) - 81*a^3*e^6*(-a^11*c^5)^(1/2) + 70*a^6*c^5*d^5*e +
198*a^8*c^3*d*e^5 + 236*a^7*c^4*d^3*e^3 + 129*a*c^2*d^4*e^2*(-a^11*c^5)^(1
/2) + 31*a^2*c*d^2*e^4*(-a^11*c^5)^(1/2))/(a^15*e^8 + a^11*c^4*d^8 + 4*a^14
*c*d^2*e^6 + 4*a^12*c^3*d^6*e^2 + 6*a^13*c^2*d^4*e^4))^(3/2)*1268i + a^16*c
^8*d^15*e^6*x*((49*c^3*d^6*(-a^11*c^5)^(1/2) - 81*a^3*e^6*(-a^11*c^5)^(1/2)
+ 70*a^6*c^5*d^5*e + 198*a^8*c^3*d*e^5 + 236*a^7*c^4*d^3*e^3 + 129*a*c^2*d
^4*e^2*(-a^11*c^5)^(1/2) + 31*a^2*c*d^2*e^4*(-a^11*c^5)^(1/2))/(a^15*e^8 +
a^11*c^4*d^8 + 4*a^14*c*d^2*e^6 + 4*a^12*c^3*d^6*e^2 + 6*a^13*c^2*d^4*e^4))
^(3/2)*1232i - a^17*c^7*d^13*e^8*x*((49*c^3*d^6*(-a^11*c^5)^(1/2) - 81*a^3*
e^6*(-a^11*c^5)^(1/2) + 70*a^6*c^5*d^5*e + 198*a^8*c^3*d*e^5 + 236*a^7*c^4*
d^3*e^3 + 129*a*c^2*d^4*e^2*(-a^11*c^5)^(1/2) + 31*a^2*c*d^2*e^4*(-a^11*c^5
)^(1/2))/(a^15*e^8 + a^11*c^4*d^8 + 4*a^14*c*d^2*e^6 + 4*a^12*c^3*d^6*e^2 +
6*a^13*c^2*d^4*e^4))^(3/2)*1858i - a^18*c^6*d^11*e^10*x*((49*c^3*d^6*(-a^1
1*c^5)^(1/2) - 81*a^3*e^6*(-a^11*c^5)^(1/2) + 70*a^6*c^5*d^5*e + 198*a^8*c^
3*d*e^5 + 236*a^7*c^4*d^3*e^3 + 129*a*c^2*d^4*e^2*(-a^11*c^5)^(1/2) + 31*a^
2*c*d^2*e^4*(-a^11*c^5)^(1/2))/(a^15*e^8 + a^11*c^4*d^8 + 4*a^14*c*d^2*e^6
+ 4*a^12*c^3*d^6*e^2 + 6*a^13*c^2*d^4*e^4))^(3/2)*6208i - a^19*c^5*d^9*e^12
x((49*c^3*d^6*(-a^11*c^5)^(1/2) - 81*a^3*e^6*(-a^11*c^5)^(1/2) + 70*a^6*c
^5*d^5*e + 198*a^8*c^3*d*e^5 + 236*a^7*c^4*d^3*e^3 + 129*a*c^2*d^4*e^2*(-a^
11*c^5)^(1/2) + 31*a^2*c*d^2*e^4*(-a^11*c^5)^(1/2))/(a^15*e^8 + a^11*c^4*d^
8 + 4*a^14*c*d^2*e^6 + 4*a^12*c^3*d^6*e^2 + 6*a^13*c^2*d^4*e^4))^(3/2)*6940
i - a^20*c^4*d^7*e^14*x*((49*c^3*d^6*(-a^11*c^5)^(1/2) - 81*a^3*e^6*(-a^11*
c^5)^(1/2) + 70*a^6*c^5*d^5*e + 198*a^8*c^3*d*e^5 + 236*a^7*c^4*d^3*e^3 + 1
29*a*c^2*d^4*e^2*(-a^11*c^5)^(1/2) + 31*a^2*c*d^2*e^4*(-a^11*c^5)^(1/2))/(a
^15*e^8 + a^11*c^4*d^8 + 4*a^14*c*d^2*e^6 + 4*a^12*c^3*d^6*e^2 + 6*a^13*c^2
*d^4*e^4))^(3/2)*4016i - a^21*c^3*d^5*e^16*x*((49*c^3*d^6*(-a^11*c^5)^(1/2)
- 81*a^3*e^6*(-a^11*c^5)^(1/2) + 70*a^6*c^5*d^5*e + 198*a^8*c^3*d*e^5 + 23
6*a^7*c^4*d^3*e^3 + 129*a*c^2*d^4*e^2*(-a^11*c^5)^(1/2) + 31*a^2*c*d^2*e^4*
(-a^11*c^5)^(1/2))/(a^15*e^8 + a^11*c^4*d^8 + 4*a^14*c*d^2*e^6 + 4*a^12*c^3
*d^6*e^2 + 6*a^13*c^2*d^4*e^4))^(3/2)*1479i - a^22*c^2*d^3*e^18*x*((49*c^3*
d^6*(-a^11*c^5)^(1/2) - 81*a^3*e^6*(-a^11*c^5)^(1/2) + 70*a^6*c^5*d^5*e + 1
98*a^8*c^3*d*e^5 + 236*a^7*c^4*d^3*e^3 + 129*a*c^2*d^4*e^2*(-a^11*c^5)^(1/2
) + 31*a^2*c*d^2*e^4*(-a^11*c^5)^(1/2))/(a^15*e^8 + a^11*c^4*d^8 + 4*a^14*c

*d^2*e^6 + 4*a^12*c^3*d^6*e^2 + 6*a^13*c^2*d^4*e^4))^(3/2)*512i - a^20*c^8*
d^20*e^3*x*((49*c^3*d^6*(-a^11*c^5)^(1/2) - 81*a^3*e^6*(-a^11*c^5)^(1/2) +
70*a^6*c^5*d^5*e + 198*a^8*c^3*d*e^5 + 236*a^7*c^4*d^3*e^3 + 129*a*c^2*d^4*
e^2*(-a^11*c^5)^(1/2) + 31*a^2*c*d^2*e^4*(-a^11*c^5)^(1/2))/(a^15*e^8 + a^1
1*c^4*d^8 + 4*a^14*c*d^2*e^6 + 4*a^12*c^3*d^6*e^2 + 6*a^13*c^2*d^4*e^4))^(5
/2)*14i - a^21*c^7*d^18*e^5*x*((49*c^3*d^6*(-a^11*c^5)^(1/2) - 81*a^3*e^6*(
-a^11*c^5)^(1/2) + 70*a^6*c^5*d^5*e + 198*a^8*c^3*d*e^5 + 236*a^7*c^4*d^3*e
^3 + 129*a*c^2*d^4*e^2*(-a^11*c^5)^(1/2) + 31*a^2*c*d^2*e^4*(-a^11*c^5)^(1/
2))/(a^15*e^8 + a^11*c^4*d^8 + 4*a^14*c*d^2*e^6 + 4*a^12*c^3*d^6*e^2 + 6*a^
13*c^2*d^4*e^4))^(5/2)*40i - a^22*c^6*d^16*e^7*x*((49*c^3*d^6*(-a^11*c^5)^(
1/2) - 81*a^3*e^6*(-a^11*c^5)^(1/2) + 70*a^6*c^5*d^5*e + 198*a^8*c^3*d*e^5
+ 236*a^7*c^4*d^3*e^3 + 129*a*c^2*d^4*e^2*(-a^11*c^5)^(1/2) + 31*a^2*c*d^2*
e^4*(-a^11*c^5)^(1/2))/(a^15*e^8 + a^11*c^4*d^8 + 4*a^14*c*d^2*e^6 + 4*a^12
*c^3*d^6*e^2 + 6*a^13*c^2*d^4*e^4))^(5/2)*56i - a^23*c^5*d^14*e^9*x*((49*c^
3*d^6*(-a^11*c^5)^(1/2) - 81*a^3*e^6*(-a^11*c^5)^(1/2) + 70*a^6*c^5*d^5*e +
 198*a^8*c^3*d*e^5 + 236*a^7*c^4*d^3*e^3 + 129*a*c^2*d^4*e^2*(-a^11*c^5)^(1
/2) + 31*a^2*c*d^2*e^4*(-a^11*c^5)^(1/2))/(a^15*e^8 + a^11*c^4*d^8 + 4*a^14
*c*d^2*e^6 + 4*a^12*c^3*d^6*e^2 + 6*a^13*c^2*d^4*e^4))^(5/2)*28i + a^24*c^4
*d^12*e^11*x*((49*c^3*d^6*(-a^11*c^5)^(1/2) - 81*a^3*e^6*(-a^11*c^5)^(1/2)
+ 70*a^6*c^5*d^5*e + 198*a^8*c^3*d*e^5 + 236*a^7*c^4*d^3*e^3 + 129*a*c^2*d^
4*e^2*(-a^11*c^5)^(1/2) + 31*a^2*c*d^2*e^4*(-a^11*c^5)^(1/2))/(a^15*e^8 + a
^11*c^4*d^8 + 4*a^14*c*d^2*e^6 + 4*a^12*c^3*d^6*e^2 + 6*a^13*c^2*d^4*e^4))^
(5/2)*28i + a^25*c^3*d^10*e^13*x*((49*c^3*d^6*(-a^11*c^5)^(1/2) - 81*a^3*e^
6*(-a^11*c^5)^(1/2) + 70*a^6*c^5*d^5*e + 198*a^8*c^3*d*e^5 + 236*a^7*c^4*d^
3*e^3 + 129*a*c^2*d^4*e^2*(-a^11*c^5)^(1/2) + 31*a^2*c*d^2*e^4*(-a^11*c^5)^
(1/2))/(a^15*e^8 + a^11*c^4*d^8 + 4*a^14*c*d^2*e^6 + 4*a^12*c^3*d^6*e^2 + 6
*a^13*c^2*d^4*e^4))^(5/2)*56i + a^26*c^2*d^8*e^15*x*((49*c^3*d^6*(-a^11*c^5
)^(1/2) - 81*a^3*e^6*(-a^11*c^5)^(1/2) + 70*a^6*c^5*d^5*e + 198*a^8*c^3*d*e
^5 + 236*a^7*c^4*d^3*e^3 + 129*a*c^2*d^4*e^2*(-a^11*c^5)^(1/2) + 31*a^2*c*d
^2*e^4*(-a^11*c^5)^(1/2))/(a^15*e^8 + a^11*c^4*d^8 + 4*a^14*c*d^2*e^6 + 4*a
^12*c^3*d^6*e^2 + 6*a^13*c^2*d^4*e^4))^(5/2)*40i - a^23*c*d*e^20*x*((49*c^3
d^6(-a^11*c^5)^(1/2) - 81*a^3*e^6*(-a^11*c^5)^(1/2) + 70*a^6*c^5*d^5*e +
198*a^8*c^3*d*e^5 + 236*a^7*c^4*d^3*e^3 + 129*a*c^2*d^4*e^2*(-a^11*c^5)^(1/
2) + 31*a^2*c*d^2*e^4*(-a^11*c^5)^(1/2))/(a^15*e^8 + a^11*c^4*d^8 + 4*a^14*
c*d^2*e^6 + 4*a^12*c^3*d^6*e^2 + 6*a^13*c^2*d^4*e^4))^(3/2)*128i))/(2824752
49*a^10*c^26*d^36 + 2176782336*a^28*c^8*e^36 + 4018066297*a^11*c^25*d^34*e^
2 + 25254299042*a^12*c^24*d^32*e^4 + 91443453570*a^13*c^23*d^30*e^6 + 20709
3177767*a^14*c^22*d^28*e^8 + 292503608847*a^15*c^21*d^26*e^10 + 22503434162
8*a^16*c^20*d^24*e^12 + 22083537020*a^17*c^19*d^22*e^14 - 108969417553*a^18
*c^18*d^20*e^16 - 43670306041*a^19*c^17*d^18*e^18 + 58023955010*a^20*c^16*d
^16*e^20 + 18862267874*a^21*c^15*d^14*e^22 - 60676266279*a^22*c^14*d^12*e^2
4 - 33348619375*a^23*c^13*d^10*e^26 + 20433166080*a^24*c^12*d^8*e^28 + 9487
311616*a^25*c^11*d^6*e^30 - 7622553600*a^26*c^10*d^4*e^32 - 349360128*a^27*
c^9*d^2*e^34))*((49*c^3*d^6*(-a^11*c^5)^(1/2) - 81*a^3*e^6*(-a^11*c^5)^(1/2
) + 70*a^6*c^5*d^5*e + 198*a^8*c^3*d*e^5 + 236*a^7*c^4*d^3*e^3 + 129*a*c^2*

```
d^4*e^2*(-a^11*c^5)^(1/2) + 31*a^2*c*d^2*e^4*(-a^11*c^5)^(1/2))/(256*(a^15*
e^8 + a^11*c^4*d^8 + 4*a^14*c*d^2*e^6 + 4*a^12*c^3*d^6*e^2 + 6*a^13*c^2*d^4
*e^4)))^(1/2)*2i + (atan((a^11*e^3*x*(-d^5*e^11)^(5/2)*4096i - a^4*c^7*d^19
*x*(-d^5*e^11)^(3/2)*73519i + c^11*d^32*e^3*x*(-d^5*e^11)^(1/2)*2401i - a^5
*c^6*d^17*e^2*x*(-d^5*e^11)^(3/2)*34182i - a^6*c^5*d^15*e^4*x*(-d^5*e^11)^(
3/2)*15521i - a^7*c^4*d^13*e^6*x*(-d^5*e^11)^(3/2)*30208i - a^8*c^3*d^11*e^
8*x*(-d^5*e^11)^(3/2)*25344i + a^2*c^9*d^28*e^7*x*(-d^5*e^11)^(1/2)*52719i
+ a^3*c^8*d^26*e^9*x*(-d^5*e^11)^(1/2)*83476i + a*c^10*d^30*e^5*x*(-d^5*e^1
1)^(1/2)*17542i)/(4096*a^11*d^13*e^30 + 2401*c^11*d^35*e^8 + 17542*a*c^10*d
^33*e^10 + 52719*a^2*c^9*d^31*e^12 + 83476*a^3*c^8*d^29*e^14 + 73519*a^4*c^
7*d^27*e^16 + 34182*a^5*c^6*d^25*e^18 + 15521*a^6*c^5*d^23*e^20 + 30208*a^7
*c^4*d^21*e^22 + 25344*a^8*c^3*d^19*e^24))*(-d^5*e^11)^(1/2)*1i)/(c^2*d^9 +
 a^2*d^5*e^4 + 2*a*c*d^7*e^2)
```

3.259 $\int \frac{x^2}{(1+x^2)\sqrt{1+x^4}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 70

$$\int \frac{x^2}{(1+x^2)\sqrt{1+x^4}}\,dx = -\frac{\arctan\left(\frac{\sqrt{2}x}{\sqrt{1+x^4}}\right)}{2\sqrt{2}}+\frac{(1+x^2)\sqrt{\frac{1+x^4}{(1+x^2)^2}}\operatorname{EllipticF}\left(2\arctan(x),\frac{1}{2}\right)}{4\sqrt{1+x^4}}$$

```
[Out] -1/4*arctan(x*2^(1/2)/(x^4+1)^(1/2))*2^(1/2)+1/4*(x^2+1)*(cos(2*arctan(x))^
2)^(1/2)/cos(2*arctan(x))*EllipticF(sin(2*arctan(x)),1/2*2^(1/2))*((x^4+1)/
(x^2+1)^2)^(1/2)/(x^4+1)^(1/2)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {1332, 226, 1713, 209}

$$\int \frac{x^2}{(1+x^2)\sqrt{1+x^4}}\,dx = \frac{(x^2+1)\sqrt{\frac{x^4+1}{(x^2+1)^2}}\operatorname{EllipticF}\left(2\arctan(x),\frac{1}{2}\right)}{4\sqrt{x^4+1}}-\frac{\arctan\left(\frac{\sqrt{2}x}{\sqrt{x^4+1}}\right)}{2\sqrt{2}}$$

```
[In] Int[x^2/((1 + x^2)*Sqrt[1 + x^4]),x]
```

```
[Out] -1/2*ArcTan[(Sqrt[2]*x)/Sqrt[1 + x^4]]/Sqrt[2] + ((1 + x^2)*Sqrt[(1 + x^4)/
(1 + x^2)^2]*EllipticF[2*ArcTan[x], 1/2])/(4*Sqrt[1 + x^4])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 1332

```
Int[(x_)^2/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :>
Dist[d/(2*d*e), Int[1/Sqrt[a + c*x^4], x], x] - Dist[d/(2*d*e), Int[(d - e*
x^2)/((d + e*x^2)*Sqrt[a + c*x^4]), x], x] /; FreeQ[{a, c, d, e}, x] && NeQ
[c*d^2 + a*e^2, 0] && PosQ[c/a] && EqQ[c*d^2 - a*e^2, 0]
```

Rule 1713

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4])
, x_Symbol] :> Dist[A, Subst[Int[1/(d + 2*a*e*x^2), x], x, x/Sqrt[a + c*x^4
]], x] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[c*d^
2 - a*e^2, 0] && EqQ[B*d + A*e, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{2}\int \frac{1}{\sqrt{1+x^4}}\,dx - \frac{1}{2}\int \frac{1-x^2}{(1+x^2)\sqrt{1+x^4}}\,dx \\
&= \frac{(1+x^2)\sqrt{\frac{1+x^4}{(1+x^2)^2}}F\left(2\tan^{-1}(x)|\frac{1}{2}\right)}{4\sqrt{1+x^4}} - \frac{1}{2}\text{Subst}\left(\int \frac{1}{1+2x^2}\,dx, x, \frac{x}{\sqrt{1+x^4}}\right) \\
&= -\frac{\tan^{-1}\left(\frac{\sqrt{2}x}{\sqrt{1+x^4}}\right)}{2\sqrt{2}} + \frac{(1+x^2)\sqrt{\frac{1+x^4}{(1+x^2)^2}}F\left(2\tan^{-1}(x)|\frac{1}{2}\right)}{4\sqrt{1+x^4}}
\end{aligned}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.19 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.57

$$\int \frac{x^2}{(1+x^2)\sqrt{1+x^4}}\,dx = \sqrt[4]{-1}\left(-\text{EllipticF}\left(i\text{arcsinh}\left(\sqrt[4]{-1}x\right), -1\right) + \text{EllipticPi}\left(-i, i\text{arcsinh}\left(\sqrt[4]{-1}x\right), -1\right)\right)$$

```
[In] Integrate[x^2/((1 + x^2)*Sqrt[1 + x^4]),x]
[Out] (-1)^(1/4)*(-EllipticF[I*ArcSinh[(-1)^(1/4)*x], -1] + EllipticPi[-I, I*ArcS
inh[(-1)^(1/4)*x], -1])
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.06 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.57

method	result	size
default	$\frac{\sqrt{-i x^{2}+1}\,\sqrt{i x^{2}+1}\,F\left(x\left(\frac{\sqrt{2}}{2}+\frac{i\sqrt{2}}{2}\right),i\right)}{\left(\frac{\sqrt{2}}{2}+\frac{i\sqrt{2}}{2}\right)\sqrt{x^{4}+1}}+\frac{(-1)^{\frac{3}{4}}\sqrt{-i x^{2}+1}\,\sqrt{i x^{2}+1}\,\Pi\left((-1)^{\frac{1}{4}}x,i,-\sqrt{-i}\,(-1)^{\frac{3}{4}}\right)}{\sqrt{x^{4}+1}}$	110
elliptic	$\frac{\sqrt{-i x^{2}+1}\,\sqrt{i x^{2}+1}\,F\left(x\left(\frac{\sqrt{2}}{2}+\frac{i\sqrt{2}}{2}\right),i\right)}{\left(\frac{\sqrt{2}}{2}+\frac{i\sqrt{2}}{2}\right)\sqrt{x^{4}+1}}+\frac{(-1)^{\frac{3}{4}}\sqrt{-i x^{2}+1}\,\sqrt{i x^{2}+1}\,\Pi\left((-1)^{\frac{1}{4}}x,i,-\sqrt{-i}\,(-1)^{\frac{3}{4}}\right)}{\sqrt{x^{4}+1}}$	110

```
[In] int(x^2/(x^2+1)/(x^4+1)^(1/2),x,method=_RETURNVERBOSE)
[Out] 1/(1/2*2^(1/2)+1/2*I*2^(1/2))*(1-I*x^2)^(1/2)*(1+I*x^2)^(1/2)/(x^4+1)^(1/2)
*EllipticF(x*(1/2*2^(1/2)+1/2*I*2^(1/2)),I)+(-1)^(3/4)*(1-I*x^2)^(1/2)*(1+I
*x^2)^(1/2)/(x^4+1)^(1/2)*EllipticPi((-1)^(1/4)*x,I,(-I)^(1/2)/(-1)^(1/4))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.12 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.46

$$\int \frac{x^2}{(1+x^2)\sqrt{1+x^4}}\,dx = -\frac{1}{4}\sqrt{2}\arctan\left(\frac{\sqrt{2}x}{\sqrt{x^4+1}}\right) - \frac{1}{2}i\sqrt{i}F\left(\arcsin\left(\sqrt{i}x\right)\mid -1\right)$$

```
[In] integrate(x^2/(x^2+1)/(x^4+1)^(1/2),x, algorithm="fricas")
[Out] -1/4*sqrt(2)*arctan(sqrt(2)*x/sqrt(x^4 + 1)) - 1/2*I*sqrt(I)*elliptic_f(arc
sin(sqrt(I)*x), -1)
```

Sympy [F]

$$\int \frac{x^2}{(1+x^2)\sqrt{1+x^4}}\,dx = \int \frac{x^2}{(x^2+1)\sqrt{x^4+1}}\,dx$$

```
[In] integrate(x**2/(x**2+1)/(x**4+1)**(1/2),x)
[Out] Integral(x**2/((x**2 + 1)*sqrt(x**4 + 1)), x)
```

Maxima [F]

$$\int \frac{x^2}{(1+x^2)\sqrt{1+x^4}}\,dx = \int \frac{x^2}{\sqrt{x^4+1}(x^2+1)}\,dx$$

```
[In] integrate(x^2/(x^2+1)/(x^4+1)^(1/2),x, algorithm="maxima")
[Out] integrate(x^2/(sqrt(x^4 + 1)*(x^2 + 1)), x)
```

Giac [F]

$$\int \frac{x^2}{(1+x^2)\sqrt{1+x^4}}\,dx = \int \frac{x^2}{\sqrt{x^4+1}(x^2+1)}\,dx$$

```
[In] integrate(x^2/(x^2+1)/(x^4+1)^(1/2),x, algorithm="giac")
[Out] integrate(x^2/(sqrt(x^4 + 1)*(x^2 + 1)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{(1+x^2)\sqrt{1+x^4}}\,dx = \int \frac{x^2}{(x^2+1)\sqrt{x^4+1}}\,dx$$

```
[In] int(x^2/((x^2 + 1)*(x^4 + 1)^(1/2)),x)
[Out] int(x^2/((x^2 + 1)*(x^4 + 1)^(1/2)), x)
```

3.260 $\int \frac{x^2}{(1-x^2)\sqrt{1+x^4}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 70

$$\int \frac{x^2}{(1-x^2)\sqrt{1+x^4}}\,dx = \frac{\operatorname{arctanh}\left(\frac{\sqrt{2}x}{\sqrt{1+x^4}}\right)}{2\sqrt{2}} - \frac{(1+x^2)\sqrt{\frac{1+x^4}{(1+x^2)^2}}\,\text{EllipticF}\left(2\arctan(x),\frac{1}{2}\right)}{4\sqrt{1+x^4}}$$

```
[Out] 1/4*arctanh(x*2^(1/2)/(x^4+1)^(1/2))*2^(1/2)-1/4*(x^2+1)*(cos(2*arctan(x))^
2)^(1/2)/cos(2*arctan(x))*EllipticF(sin(2*arctan(x)),1/2*2^(1/2))*((x^4+1)/
(x^2+1)^2)^(1/2)/(x^4+1)^(1/2)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {1332, 226, 1713, 212}

$$\int \frac{x^2}{(1-x^2)\sqrt{1+x^4}}\,dx = \frac{\operatorname{arctanh}\left(\frac{\sqrt{2}x}{\sqrt{x^4+1}}\right)}{2\sqrt{2}} - \frac{(x^2+1)\sqrt{\frac{x^4+1}{(x^2+1)^2}}\,\text{EllipticF}\left(2\arctan(x),\frac{1}{2}\right)}{4\sqrt{x^4+1}}$$

```
[In] Int[x^2/((1 - x^2)*Sqrt[1 + x^4]),x]
```

```
[Out] ArcTanh[(Sqrt[2]*x)/Sqrt[1 + x^4]]/(2*Sqrt[2]) - ((1 + x^2)*Sqrt[(1 + x^4)/
(1 + x^2)^2]*EllipticF[2*ArcTan[x], 1/2])/(4*Sqrt[1 + x^4])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 1332

```
Int[(x_)^2/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :>
Dist[d/(2*d*e), Int[1/Sqrt[a + c*x^4], x], x] - Dist[d/(2*d*e), Int[(d - e*
x^2)/((d + e*x^2)*Sqrt[a + c*x^4]), x], x] /; FreeQ[{a, c, d, e}, x] && NeQ
[c*d^2 + a*e^2, 0] && PosQ[c/a] && EqQ[c*d^2 - a*e^2, 0]
```

Rule 1713

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4])
, x_Symbol] :> Dist[A, Subst[Int[1/(d + 2*a*e*x^2), x], x, x/Sqrt[a + c*x^4
]], x] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[c*d^
2 - a*e^2, 0] && EqQ[B*d + A*e, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\left(\frac{1}{2}\int \frac{1}{\sqrt{1+x^4}}\,dx\right) + \frac{1}{2}\int \frac{1+x^2}{(1-x^2)\sqrt{1+x^4}}\,dx \\
&= -\frac{(1+x^2)\sqrt{\frac{1+x^4}{(1+x^2)^2}}F\left(2\tan^{-1}(x)\middle|\frac{1}{2}\right)}{4\sqrt{1+x^4}} + \frac{1}{2}\text{Subst}\left(\int \frac{1}{1-2x^2}\,dx, x, \frac{x}{\sqrt{1+x^4}}\right) \\
&= \frac{\tanh^{-1}\left(\frac{\sqrt{2}x}{\sqrt{1+x^4}}\right)}{2\sqrt{2}} - \frac{(1+x^2)\sqrt{\frac{1+x^4}{(1+x^2)^2}}F\left(2\tan^{-1}(x)\middle|\frac{1}{2}\right)}{4\sqrt{1+x^4}}
\end{aligned}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.17 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.51

$$\int \frac{x^2}{(1-x^2)\sqrt{1+x^4}}\,dx = \sqrt[4]{-1}\left(\text{EllipticF}\left(i\text{arcsinh}\left(\sqrt[4]{-1}x\right), -1\right) - \text{EllipticPi}\left(i, \arcsin\left((-1)^{3/4}x\right), -1\right)\right)$$

```
[In] Integrate[x^2/((1 - x^2)*Sqrt[1 + x^4]),x]
[Out] (-1)^(1/4)*(EllipticF[I*ArcSinh[(-1)^(1/4)*x], -1] - EllipticPi[I, ArcSin[(
-1)^(3/4)*x], -1])
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.30 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.60

method	result	size
default	$-\frac{\sqrt{-i x^{2}+1}\,\sqrt{i x^{2}+1}\,F\left(x\left(\frac{\sqrt{2}}{2}+\frac{i\sqrt{2}}{2}\right),i\right)}{\left(\frac{\sqrt{2}}{2}+\frac{i\sqrt{2}}{2}\right)\sqrt{x^{4}+1}}-\frac{(-1)^{\frac{3}{4}}\sqrt{-i x^{2}+1}\,\sqrt{i x^{2}+1}\,\Pi\left((-1)^{\frac{1}{4}}x,-i,-\sqrt{-i}\,(-1)^{\frac{3}{4}}\right)}{\sqrt{x^{4}+1}}$	112
elliptic	$-\frac{\sqrt{-i x^{2}+1}\,\sqrt{i x^{2}+1}\,F\left(x\left(\frac{\sqrt{2}}{2}+\frac{i\sqrt{2}}{2}\right),i\right)}{\left(\frac{\sqrt{2}}{2}+\frac{i\sqrt{2}}{2}\right)\sqrt{x^{4}+1}}-\frac{(-1)^{\frac{3}{4}}\sqrt{-i x^{2}+1}\,\sqrt{i x^{2}+1}\,\Pi\left((-1)^{\frac{1}{4}}x,-i,-\sqrt{-i}\,(-1)^{\frac{3}{4}}\right)}{\sqrt{x^{4}+1}}$	112

```
[In] int(x^2/(-x^2+1)/(x^4+1)^(1/2),x,method=_RETURNVERBOSE)
[Out] -1/(1/2*2^(1/2)+1/2*I*2^(1/2))*(1-I*x^2)^(1/2)*(1+I*x^2)^(1/2)/(x^4+1)^(1/2
)*EllipticF(x*(1/2*2^(1/2)+1/2*I*2^(1/2)),I)-(-1)^(3/4)*(1-I*x^2)^(1/2)*(1+
I*x^2)^(1/2)/(x^4+1)^(1/2)*EllipticPi((-1)^(1/4)*x,-I,(-I)^(1/2)/(-1)^(1/4)
)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.11 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.80

$$\int \frac{x^2}{(1-x^2)\sqrt{1+x^4}}\,dx = \frac{1}{2}i\sqrt{i}F(\arcsin\left(\sqrt{i}x\right)\mid -1) + \frac{1}{8}\sqrt{2}\log\left(\frac{x^4+2\sqrt{2}\sqrt{x^4+1}x+2x^2+1}{x^4-2x^2+1}\right)$$

```
[In] integrate(x^2/(-x^2+1)/(x^4+1)^(1/2),x, algorithm="fricas")
[Out] 1/2*I*sqrt(I)*elliptic_f(arcsin(sqrt(I)*x), -1) + 1/8*sqrt(2)*log((x^4 + 2*
sqrt(2)*sqrt(x^4 + 1)*x + 2*x^2 + 1)/(x^4 - 2*x^2 + 1))
```

Sympy [F]

$$\int \frac{x^2}{(1-x^2)\sqrt{1+x^4}}\,dx = -\int \frac{x^2}{x^2\sqrt{x^4+1}-\sqrt{x^4+1}}\,dx$$

```
[In] integrate(x**2/(-x**2+1)/(x**4+1)**(1/2),x)
[Out] -Integral(x**2/(x**2*sqrt(x**4 + 1) - sqrt(x**4 + 1)), x)
```

Maxima [F]

$$\int \frac{x^2}{(1-x^2)\sqrt{1+x^4}}\,dx = \int -\frac{x^2}{\sqrt{x^4+1}(x^2-1)}\,dx$$

```
[In] integrate(x^2/(-x^2+1)/(x^4+1)^(1/2),x, algorithm="maxima")
[Out] -integrate(x^2/(sqrt(x^4 + 1)*(x^2 - 1)), x)
```

Giac [F]

$$\int \frac{x^2}{(1-x^2)\sqrt{1+x^4}}\,dx = \int -\frac{x^2}{\sqrt{x^4+1}(x^2-1)}\,dx$$

```
[In] integrate(x^2/(-x^2+1)/(x^4+1)^(1/2),x, algorithm="giac")
[Out] integrate(-x^2/(sqrt(x^4 + 1)*(x^2 - 1)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{(1-x^2)\sqrt{1+x^4}}\,dx = -\int \frac{x^2}{(x^2-1)\sqrt{x^4+1}}\,dx$$

```
[In] int(-x^2/((x^2 - 1)*(x^4 + 1)^(1/2)),x)
[Out] -int(x^2/((x^2 - 1)*(x^4 + 1)^(1/2)), x)
```

3.261 $\int \frac{x^2}{(1+x^2)\sqrt{1-x^4}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 99

$$\int \frac{x^2}{(1+x^2)\sqrt{1-x^4}}\,dx = -\frac{x(1-x^2)}{2\sqrt{1-x^4}} - \frac{\sqrt{1-x^2}\sqrt{1+x^2}E(\arcsin(x)|-1)}{2\sqrt{1-x^4}} + \frac{\sqrt{1-x^2}\sqrt{1+x^2}\,\text{EllipticF}(\arcsin(x),-1)}{\sqrt{1-x^4}}$$

```
[Out] -1/2*x*(-x^2+1)/(-x^4+1)^(1/2)-1/2*EllipticE(x,I)*(-x^2+1)^(1/2)*(x^2+1)^(1
/2)/(-x^4+1)^(1/2)+EllipticF(x,I)*(-x^2+1)^(1/2)*(x^2+1)^(1/2)/(-x^4+1)^(1/
2)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {1270, 482, 434, 435, 254, 227}

$$\int \frac{x^2}{(1+x^2)\sqrt{1-x^4}}\,dx = \frac{\sqrt{x^2+1}\sqrt{1-x^2}\,\text{EllipticF}(\arcsin(x),-1)}{\sqrt{1-x^4}} - \frac{\sqrt{x^2+1}\sqrt{1-x^2}E(\arcsin(x)|-1)}{2\sqrt{1-x^4}} - \frac{x(1-x^2)}{2\sqrt{1-x^4}}$$

```
[In] Int[x^2/((1 + x^2)*Sqrt[1 - x^4]),x]
```

```
[Out] -1/2*(x*(1 - x^2))/Sqrt[1 - x^4] - (Sqrt[1 - x^2]*Sqrt[1 + x^2]*EllipticE[A
rcSin[x], -1])/(2*Sqrt[1 - x^4]) + (Sqrt[1 - x^2]*Sqrt[1 + x^2]*EllipticF[A
rcSin[x], -1])/Sqrt[1 - x^4]
```

Rule 227

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[Rt[-b,
4]*(x/Rt[a, 4])], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rule 254

```
Int[((a1_.) + (b1_.)*(x_)^(n_))^(p_.)*((a2_.) + (b2_.)*(x_)^(n_))^(p_.), x_
Symbol] :> Int[(a1*a2 + b1*b2*x^(2*n))^p, x] /; FreeQ[{a1, b1, a2, b2, n, p
}, x] && EqQ[a2*b1 + a1*b2, 0] && (IntegerQ[p] || (GtQ[a1, 0] && GtQ[a2, 0]
))
```

Rule 434

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
b/d, Int[Sqrt[c + d*x^2]/Sqrt[a + b*x^2], x], x] - Dist[(b*c - a*d)/d, Int[
1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] && Po
sQ[d/c] && NegQ[b/a]
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 482

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*
((c + d*x^n)^(q + 1)/(n*(b*c - a*d)*(p + 1))), x] - Dist[e^n/(n*(b*c - a*d)
*(p + 1)), Int[(e*x)^(m - n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(m -
n + 1) + d*(m + n*(p + q + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e,
 q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && GeQ[n, m - n +
1] && GtQ[m - n + 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 1270

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (c_.)*(x_)^4)^(p_
), x_Symbol] :> Dist[(a + c*x^4)^FracPart[p]/((d + e*x^2)^FracPart[p]*(a/d
+ (c*x^2)/e)^FracPart[p]), Int[(f*x)^m*(d + e*x^2)^(q + p)*(a/d + (c/e)*x^2
)^p, x], x] /; FreeQ[{a, c, d, e, f, m, p, q}, x] && EqQ[c*d^2 + a*e^2, 0]
&&  !IntegerQ[p]
```

Rubi steps

$$\text{integral} = \frac{\left(\sqrt{1-x^2}\sqrt{1+x^2}\right)\int \frac{x^2}{\sqrt{1-x^2}(1+x^2)^{3/2}}\,dx}{\sqrt{1-x^4}}$$

$$= -\frac{x(1-x^2)}{2\sqrt{1-x^4}} + \frac{\left(\sqrt{1-x^2}\sqrt{1+x^2}\right)\int \frac{\sqrt{1-x^2}}{\sqrt{1+x^2}}\,dx}{2\sqrt{1-x^4}}$$

$$= -\frac{x(1-x^2)}{2\sqrt{1-x^4}} - \frac{\left(\sqrt{1-x^2}\sqrt{1+x^2}\right)\int \frac{\sqrt{1+x^2}}{\sqrt{1-x^2}}\,dx}{2\sqrt{1-x^4}} + \frac{\left(\sqrt{1-x^2}\sqrt{1+x^2}\right)\int \frac{1}{\sqrt{1-x^2}\sqrt{1+x^2}}\,dx}{\sqrt{1-x^4}}$$

$$= -\frac{x(1-x^2)}{2\sqrt{1-x^4}} - \frac{\sqrt{1-x^2}\sqrt{1+x^2}E(\sin^{-1}(x)|-1)}{2\sqrt{1-x^4}} + \frac{\left(\sqrt{1-x^2}\sqrt{1+x^2}\right)\int \frac{1}{\sqrt{1-x^4}}\,dx}{\sqrt{1-x^4}}$$

$$= -\frac{x(1-x^2)}{2\sqrt{1-x^4}} - \frac{\sqrt{1-x^2}\sqrt{1+x^2}E(\sin^{-1}(x)|-1)}{2\sqrt{1-x^4}} + \frac{\sqrt{1-x^2}\sqrt{1+x^2}F(\sin^{-1}(x)|-1)}{\sqrt{1-x^4}}$$

Mathematica [A] (verified)

Time = 10.13 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.46

$$\int \frac{x^2}{(1+x^2)\sqrt{1-x^4}}\,dx = \frac{1}{2}\left(-\frac{x}{\sqrt{1-x^4}} + \frac{x^3}{\sqrt{1-x^4}} - E(\arcsin(x)|-1) + 2\,\text{EllipticF}(\arcsin(x),-1)\right)$$

```
[In] Integrate[x^2/((1 + x^2)*Sqrt[1 - x^4]),x]
[Out] (-(x/Sqrt[1 - x^4]) + x^3/Sqrt[1 - x^4] - EllipticE[ArcSin[x], -1] + 2*Elli
pticF[ArcSin[x], -1])/2
```

Maple [A] (verified)

Time = 1.41 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.89

method	result	size
risch	$\frac{x(x^2-1)}{2\sqrt{-x^4+1}} + \frac{F(x,i)\sqrt{-x^2+1}\,\sqrt{x^2+1}}{2\sqrt{-x^4+1}} + \frac{\sqrt{-x^2+1}\,\sqrt{x^2+1}\,(F(x,i)-E(x,i))}{2\sqrt{-x^4+1}}$	88
default	$\frac{F(x,i)\sqrt{-x^2+1}\,\sqrt{x^2+1}}{2\sqrt{-x^4+1}} - \frac{(-x^2+1)x}{2\sqrt{(-x^2+1)(x^2+1)}} + \frac{\sqrt{-x^2+1}\,\sqrt{x^2+1}\,(F(x,i)-E(x,i))}{2\sqrt{-x^4+1}}$	96
elliptic	$\frac{F(x,i)\sqrt{-x^2+1}\,\sqrt{x^2+1}}{2\sqrt{-x^4+1}} - \frac{(-x^2+1)x}{2\sqrt{(-x^2+1)(x^2+1)}} + \frac{\sqrt{-x^2+1}\,\sqrt{x^2+1}\,(F(x,i)-E(x,i))}{2\sqrt{-x^4+1}}$	96

```
[In] int(x^2/(x^2+1)/(-x^4+1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*x*(x^2-1)/(-x^4+1)^(1/2)+1/2*EllipticF(x,I)*(-x^2+1)^(1/2)*(x^2+1)^(1/2
)/(-x^4+1)^(1/2)+1/2*(-x^2+1)^(1/2)*(x^2+1)^(1/2)/(-x^4+1)^(1/2)*(EllipticF
(x,I)-EllipticE(x,I))
```

Fricas [A] (verification not implemented)

none

Time = 0.08 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.42

$$\int \frac{x^2}{(1+x^2)\sqrt{1-x^4}}\,dx = -\frac{(x^2+1)E(\arcsin(x)\,|-1) - 2(x^2+1)F(\arcsin(x)\,|-1) + \sqrt{-x^4+1}x}{2(x^2+1)}$$

```
[In] integrate(x^2/(x^2+1)/(-x^4+1)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/2*((x^2 + 1)*elliptic_e(arcsin(x), -1) - 2*(x^2 + 1)*elliptic_f(arcsin(x
), -1) + sqrt(-x^4 + 1)*x)/(x^2 + 1)
```

Sympy [F]

$$\int \frac{x^2}{(1+x^2)\sqrt{1-x^4}}\,dx = \int \frac{x^2}{\sqrt{-(x-1)(x+1)(x^2+1)}(x^2+1)}\,dx$$

```
[In] integrate(x**2/(x**2+1)/(-x**4+1)**(1/2),x)
```

```
[Out] Integral(x**2/(sqrt(-(x - 1)*(x + 1)*(x**2 + 1))*(x**2 + 1)), x)
```

Maxima [F]

$$\int \frac{x^2}{(1+x^2)\sqrt{1-x^4}}\,dx = \int \frac{x^2}{\sqrt{-x^4+1}(x^2+1)}\,dx$$

```
[In] integrate(x^2/(x^2+1)/(-x^4+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^2/(sqrt(-x^4 + 1)*(x^2 + 1)), x)
```

Giac [F]

$$\int \frac{x^2}{(1+x^2)\sqrt{1-x^4}}\,dx = \int \frac{x^2}{\sqrt{-x^4+1}(x^2+1)}\,dx$$

```
[In] integrate(x^2/(x^2+1)/(-x^4+1)^(1/2),x, algorithm="giac")
[Out] integrate(x^2/(sqrt(-x^4 + 1)*(x^2 + 1)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{(1+x^2)\sqrt{1-x^4}}\,dx = \int \frac{x^2}{(x^2+1)\sqrt{1-x^4}}\,dx$$

```
[In] int(x^2/((x^2 + 1)*(1 - x^4)^(1/2)),x)
[Out] int(x^2/((x^2 + 1)*(1 - x^4)^(1/2)), x)
```

3.262 $\int \frac{x^2}{(1-x^2)\sqrt{1-x^4}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 61

$$\int \frac{x^2}{(1-x^2)\sqrt{1-x^4}}\,dx = \frac{x(1+x^2)}{2\sqrt{1-x^4}} - \frac{\sqrt{1-x^2}\sqrt{1+x^2}E(\arcsin(x)|-1)}{2\sqrt{1-x^4}}$$

```
[Out] 1/2*x*(x^2+1)/(-x^4+1)^(1/2)-1/2*EllipticE(x,I)*(-x^2+1)^(1/2)*(x^2+1)^(1/2
)/(-x^4+1)^(1/2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1270, 482, 435}

$$\int \frac{x^2}{(1-x^2)\sqrt{1-x^4}}\,dx = \frac{x(x^2+1)}{2\sqrt{1-x^4}} - \frac{\sqrt{1-x^2}\sqrt{x^2+1}E(\arcsin(x)|-1)}{2\sqrt{1-x^4}}$$

```
[In] Int[x^2/((1 - x^2)*Sqrt[1 - x^4]),x]
```

```
[Out] (x*(1 + x^2))/(2*Sqrt[1 - x^4]) - (Sqrt[1 - x^2]*Sqrt[1 + x^2]*EllipticE[Ar
cSin[x], -1])/(2*Sqrt[1 - x^4])
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 482

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*
((c + d*x^n)^(q + 1)/(n*(b*c - a*d)*(p + 1))), x] - Dist[e^n/(n*(b*c - a*d)
*(p + 1)), Int[(e*x)^(m - n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(m -
n + 1) + d*(m + n*(p + q + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e,
 q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && GeQ[n, m - n +
1] && GtQ[m - n + 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 1270

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (c_.)*(x_)^4)^(p_
), x_Symbol] :> Dist[(a + c*x^4)^FracPart[p]/((d + e*x^2)^FracPart[p]*(a/d
+ (c*x^2)/e)^FracPart[p]), Int[(f*x)^m*(d + e*x^2)^(q + p)*(a/d + (c/e)*x^2
)^p, x], x] /; FreeQ[{a, c, d, e, f, m, p, q}, x] && EqQ[c*d^2 + a*e^2, 0]
&&  !IntegerQ[p]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\left(\sqrt{1-x^2}\sqrt{1+x^2}\right)\int \frac{x^2}{(1-x^2)^{3/2}\sqrt{1+x^2}}\,dx}{\sqrt{1-x^4}}\\
&= \frac{x(1+x^2)}{2\sqrt{1-x^4}} - \frac{\left(\sqrt{1-x^2}\sqrt{1+x^2}\right)\int \frac{\sqrt{1+x^2}}{\sqrt{1-x^2}}\,dx}{2\sqrt{1-x^4}}\\
&= \frac{x(1+x^2)}{2\sqrt{1-x^4}} - \frac{\sqrt{1-x^2}\sqrt{1+x^2}E(\sin^{-1}(x)|-1)}{2\sqrt{1-x^4}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 10.15 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.61

$$\int \frac{x^2}{(1-x^2)\sqrt{1-x^4}}\,dx = \frac{x+x^3-\sqrt{1-x^4}E(\arcsin(x)|-1)}{2\sqrt{1-x^4}}$$

```
[In] Integrate[x^2/((1 - x^2)*Sqrt[1 - x^4]),x]
[Out] (x + x^3 - Sqrt[1 - x^4]*EllipticE[ArcSin[x], -1])/(2*Sqrt[1 - x^4])
```

Maple [A] (verified)

Time = 0.96 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.44

method	result	siz
risch	$\frac{x(x^2+1)}{2\sqrt{-x^4+1}}-\frac{F(x,i)\sqrt{-x^2+1}\,\sqrt{x^2+1}}{2\sqrt{-x^4+1}}+\frac{\sqrt{-x^2+1}\,\sqrt{x^2+1}\,(F(x,i)-E(x,i))}{2\sqrt{-x^4+1}}$	88
elliptic	$-\frac{(-x^2-1)x}{2\sqrt{(x^2-1)(-x^2-1)}}-\frac{F(x,i)\sqrt{-x^2+1}\,\sqrt{x^2+1}}{2\sqrt{-x^4+1}}+\frac{\sqrt{-x^2+1}\,\sqrt{x^2+1}\,(F(x,i)-E(x,i))}{2\sqrt{-x^4+1}}$	96
default	$-\frac{F(x,i)\sqrt{-x^2+1}\,\sqrt{x^2+1}}{2\sqrt{-x^4+1}}-\frac{-x^3+x^2-x+1}{4\sqrt{(x+1)(-x^3+x^2-x+1)}}+\frac{\sqrt{-x^2+1}\,\sqrt{x^2+1}\,(F(x,i)-E(x,i))}{2\sqrt{-x^4+1}}-\frac{-x^3-x^2-x-1}{4\sqrt{(x-1)(-x^3-x^2-x-1)}}$	14

```
[In] int(x^2/(-x^2+1)/(-x^4+1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*x*(x^2+1)/(-x^4+1)^(1/2)-1/2*EllipticF(x,I)*(-x^2+1)^(1/2)*(x^2+1)^(1/2
)/(-x^4+1)^(1/2)+1/2*(-x^2+1)^(1/2)*(x^2+1)^(1/2)/(-x^4+1)^(1/2)*(EllipticF
(x,I)-EllipticE(x,I))
```

Fricas [A] (verification not implemented)

none

Time = 0.09 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.51

$$\int \frac{x^2}{(1-x^2)\sqrt{1-x^4}}\,dx = -\frac{(x^2-1)E(\arcsin(x)\mid -1)+\sqrt{-x^4+1}x}{2(x^2-1)}$$

```
[In] integrate(x^2/(-x^2+1)/(-x^4+1)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/2*((x^2 - 1)*elliptic_e(arcsin(x), -1) + sqrt(-x^4 + 1)*x)/(x^2 - 1)
```

Sympy [F]

$$\int \frac{x^2}{(1-x^2)\sqrt{1-x^4}}\,dx = -\int \frac{x^2}{x^2\sqrt{1-x^4}-\sqrt{1-x^4}}\,dx$$

```
[In] integrate(x**2/(-x**2+1)/(-x**4+1)**(1/2),x)
```

```
[Out] -Integral(x**2/(x**2*sqrt(1 - x**4) - sqrt(1 - x**4)), x)
```

Maxima [F]

$$\int \frac{x^2}{(1-x^2)\sqrt{1-x^4}}\,dx = \int -\frac{x^2}{\sqrt{-x^4+1}(x^2-1)}\,dx$$

```
[In] integrate(x^2/(-x^2+1)/(-x^4+1)^(1/2),x, algorithm="maxima")
[Out] -integrate(x^2/(sqrt(-x^4 + 1)*(x^2 - 1)), x)
```

Giac [F]

$$\int \frac{x^2}{(1-x^2)\sqrt{1-x^4}}\,dx = \int -\frac{x^2}{\sqrt{-x^4+1}(x^2-1)}\,dx$$

```
[In] integrate(x^2/(-x^2+1)/(-x^4+1)^(1/2),x, algorithm="giac")
[Out] integrate(-x^2/(sqrt(-x^4 + 1)*(x^2 - 1)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{(1-x^2)\sqrt{1-x^4}}\,dx = -\int \frac{x^2}{(x^2-1)\sqrt{1-x^4}}\,dx$$

```
[In] int(-x^2/((x^2 - 1)*(1 - x^4)^(1/2)),x)
[Out] -int(x^2/((x^2 - 1)*(1 - x^4)^(1/2)), x)
```

3.263 $\int \frac{x^2}{(1+x^2)\sqrt{-1+x^4}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 113

$$\int \frac{x^2}{(1+x^2)\sqrt{-1+x^4}}\,dx = -\frac{x(1-x^2)}{2\sqrt{-1+x^4}} - \frac{\sqrt{1-x^2}\sqrt{1+x^2}\,E(\arcsin(x)|-1)}{2\sqrt{-1+x^4}} + \frac{\sqrt{-1+x^2}\sqrt{1+x^2}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{2}x}{\sqrt{-1+x^2}}\right),\frac{1}{2}\right)}{\sqrt{2}\sqrt{-1+x^4}}$$

```
[Out] -1/2*x*(-x^2+1)/(x^4-1)^(1/2)-1/2*EllipticE(x,I)*(-x^2+1)^(1/2)*(x^2+1)^(1/
2)/(x^4-1)^(1/2)+1/2*EllipticF(x*2^(1/2)/(x^2-1)^(1/2),1/2*2^(1/2))*(x^2-1)
^(1/2)*(x^2+1)^(1/2)*2^(1/2)/(x^4-1)^(1/2)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.350, Rules used = {1270, 482, 434, 438, 435, 259, 228}

$$\int \frac{x^2}{(1+x^2)\sqrt{-1+x^4}}\,dx = \frac{\sqrt{x^2-1}\sqrt{x^2+1}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{2}x}{\sqrt{x^2-1}}\right),\frac{1}{2}\right)}{\sqrt{2}\sqrt{x^4-1}} - \frac{\sqrt{x^2+1}\sqrt{1-x^2}\,E(\arcsin(x)|-1)}{2\sqrt{x^4-1}} - \frac{x(1-x^2)}{2\sqrt{x^4-1}}$$

```
[In] Int[x^2/((1 + x^2)*Sqrt[-1 + x^4]),x]
```

```
[Out] -1/2*(x*(1 - x^2))/Sqrt[-1 + x^4] - (Sqrt[1 - x^2]*Sqrt[1 + x^2]*EllipticE[
ArcSin[x], -1])/(2*Sqrt[-1 + x^4]) + (Sqrt[-1 + x^2]*Sqrt[1 + x^2]*Elliptic
F[ArcSin[(Sqrt[2]*x)/Sqrt[-1 + x^2]], 1/2])/(Sqrt[2]*Sqrt[-1 + x^4])
```

Rule 228

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[(-a)*b, 2]}, Sim
p[Sqrt[-a + q*x^2]*(Sqrt[(a + q*x^2)/q]/(Sqrt[2]*Sqrt[-a]*Sqrt[a + b*x^4]))
*EllipticF[ArcSin[x/Sqrt[(a + q*x^2)/(2*q)]], 1/2], x] /; IntegerQ[q]] /; F
reeQ[{a, b}, x] && LtQ[a, 0] && GtQ[b, 0]
```

Rule 259

```
Int[((a1_.) + (b1_.)*(x_)^(n_))^(p_)*((a2_.) + (b2_.)*(x_)^(n_))^(p_), x_Sy
mbol] :> Dist[(a1 + b1*x^n)^FracPart[p]*((a2 + b2*x^n)^FracPart[p]/(a1*a2 +
 b1*b2*x^(2*n))^FracPart[p]), Int[(a1*a2 + b1*b2*x^(2*n))^p, x], x] /; Free
Q[{a1, b1, a2, b2, n, p}, x] && EqQ[a2*b1 + a1*b2, 0] &&  !IntegerQ[p]
```

Rule 434

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
b/d, Int[Sqrt[c + d*x^2]/Sqrt[a + b*x^2], x], x] - Dist[(b*c - a*d)/d, Int[
1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] && Po
sQ[d/c] && NegQ[b/a]
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 438

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2], Int[Sqrt[a + b*x^2]/Sqrt[1 + (d/c)*x^2
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] &&  !GtQ[c, 0]
```

Rule 482

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*
((c + d*x^n)^(q + 1)/(n*(b*c - a*d)*(p + 1))), x] - Dist[e^n/(n*(b*c - a*d)
*(p + 1)), Int[(e*x)^(m - n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(m -
n + 1) + d*(m + n*(p + q + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e,
 q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && GeQ[n, m - n +
1] && GtQ[m - n + 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 1270

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (c_.)*(x_)^4)^(p_
), x_Symbol] :> Dist[(a + c*x^4)^FracPart[p]/((d + e*x^2)^FracPart[p]*(a/d
```

```
+ (c*x^2)/e)^FracPart[p]), Int[(f*x)^m*(d + e*x^2)^(q + p)*(a/d + (c/e)*x^2
)^p, x], x] /; FreeQ[{a, c, d, e, f, m, p, q}, x] && EqQ[c*d^2 + a*e^2, 0]
&&  !IntegerQ[p]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\left(\sqrt{-1+x^2}\sqrt{1+x^2}\right)\int \frac{x^2}{\sqrt{-1+x^2}(1+x^2)^{3/2}}\,dx}{\sqrt{-1+x^4}} \\
&= -\frac{x(1-x^2)}{2\sqrt{-1+x^4}} - \frac{\left(\sqrt{-1+x^2}\sqrt{1+x^2}\right)\int \frac{\sqrt{-1+x^2}}{\sqrt{1+x^2}}\,dx}{2\sqrt{-1+x^4}} \\
&= -\frac{x(1-x^2)}{2\sqrt{-1+x^4}} - \frac{\left(\sqrt{-1+x^2}\sqrt{1+x^2}\right)\int \frac{\sqrt{1+x^2}}{\sqrt{-1+x^2}}\,dx}{2\sqrt{-1+x^4}} + \frac{\left(\sqrt{-1+x^2}\sqrt{1+x^2}\right)\int \frac{1}{\sqrt{-1+x^2}\sqrt{1+x^2}}\,dx}{\sqrt{-1+x^4}} \\
&= -\frac{x(1-x^2)}{2\sqrt{-1+x^4}} - \frac{\left(\sqrt{1-x^2}\sqrt{1+x^2}\right)\int \frac{\sqrt{1+x^2}}{\sqrt{1-x^2}}\,dx}{2\sqrt{-1+x^4}} + \int \frac{1}{\sqrt{-1+x^4}}\,dx \\
&= -\frac{x(1-x^2)}{2\sqrt{-1+x^4}} - \frac{\sqrt{1-x^2}\sqrt{1+x^2}E(\sin^{-1}(x)|-1)}{2\sqrt{-1+x^4}} \\
&\quad + \frac{\sqrt{-1+x^2}\sqrt{1+x^2}F\left(\sin^{-1}\left(\frac{\sqrt{2}x}{\sqrt{-1+x^2}}\right)|\frac{1}{2}\right)}{\sqrt{2}\sqrt{-1+x^4}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 10.10 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.48

$$
\int \frac{x^2}{(1+x^2)\sqrt{-1+x^4}}\,dx = \frac{-x+x^3-\sqrt{1-x^4}E(\arcsin(x)|-1)+2\sqrt{1-x^4}\,\text{EllipticF}(\arcsin(x),-1)}{2\sqrt{-1+x^4}}
$$

```
[In] Integrate[x^2/((1 + x^2)*Sqrt[-1 + x^4]),x]
```

```
[Out] (-x + x^3 - Sqrt[1 - x^4]*EllipticE[ArcSin[x], -1] + 2*Sqrt[1 - x^4]*Ellipt
icF[ArcSin[x], -1])/(2*Sqrt[-1 + x^4])
```

Maple [A] (verified)

Time = 1.58 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.82

method	result	size
risch	$\frac{x(x^2-1)}{2\sqrt{x^4-1}}-\frac{i\sqrt{x^2+1}\,\sqrt{-x^2+1}\,F(ix,i)}{2\sqrt{x^4-1}}+\frac{i\sqrt{x^2+1}\,\sqrt{-x^2+1}\,(F(ix,i)-E(ix,i))}{2\sqrt{x^4-1}}$	93
default	$-\frac{i\sqrt{x^2+1}\,\sqrt{-x^2+1}\,F(ix,i)}{2\sqrt{x^4-1}}+\frac{(x^2-1)x}{2\sqrt{(x^2+1)(x^2-1)}}+\frac{i\sqrt{x^2+1}\,\sqrt{-x^2+1}\,(F(ix,i)-E(ix,i))}{2\sqrt{x^4-1}}$	99
elliptic	$-\frac{i\sqrt{x^2+1}\,\sqrt{-x^2+1}\,F(ix,i)}{2\sqrt{x^4-1}}+\frac{(x^2-1)x}{2\sqrt{(x^2+1)(x^2-1)}}+\frac{i\sqrt{x^2+1}\,\sqrt{-x^2+1}\,(F(ix,i)-E(ix,i))}{2\sqrt{x^4-1}}$	99

```
[In] int(x^2/(x^2+1)/(x^4-1)^(1/2),x,method=_RETURNVERBOSE)
[Out] 1/2*x*(x^2-1)/(x^4-1)^(1/2)-1/2*I*(x^2+1)^(1/2)*(-x^2+1)^(1/2)/(x^4-1)^(1/2
)*EllipticF(I*x,I)+1/2*I*(x^2+1)^(1/2)*(-x^2+1)^(1/2)/(x^4-1)^(1/2)*(Ellipt
icF(I*x,I)-EllipticE(I*x,I))
```

Fricas [A] (verification not implemented)

none

Time = 0.09 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.39

$$\int \frac{x^2}{(1+x^2)\sqrt{-1+x^4}}\,dx = \frac{(i\,x^2+i)E(\arcsin{(x)}\mid -1)-2\,(i\,x^2+i)F(\arcsin{(x)}\mid -1)+\sqrt{x^4-1}x}{2\,(x^2+1)}$$

```
[In] integrate(x^2/(x^2+1)/(x^4-1)^(1/2),x, algorithm="fricas")
[Out] 1/2*((I*x^2 + I)*elliptic_e(arcsin(x), -1) - 2*(I*x^2 + I)*elliptic_f(arcsi
n(x), -1) + sqrt(x^4 - 1)*x)/(x^2 + 1)
```

Sympy [F]

$$\int \frac{x^2}{(1+x^2)\sqrt{-1+x^4}}\,dx = \int \frac{x^2}{\sqrt{(x-1)\,(x+1)\,(x^2+1)}\,(x^2+1)}\,dx$$

```
[In] integrate(x**2/(x**2+1)/(x**4-1)**(1/2),x)
[Out] Integral(x**2/(sqrt((x - 1)*(x + 1)*(x**2 + 1))*(x**2 + 1)), x)
```

Maxima [F]

$$\int \frac{x^2}{(1+x^2)\sqrt{-1+x^4}}\,dx = \int \frac{x^2}{\sqrt{x^4-1}(x^2+1)}\,dx$$

```
[In] integrate(x^2/(x^2+1)/(x^4-1)^(1/2),x, algorithm="maxima")
[Out] integrate(x^2/(sqrt(x^4 - 1)*(x^2 + 1)), x)
```

Giac [F]

$$\int \frac{x^2}{(1+x^2)\sqrt{-1+x^4}}\,dx = \int \frac{x^2}{\sqrt{x^4-1}(x^2+1)}\,dx$$

```
[In] integrate(x^2/(x^2+1)/(x^4-1)^(1/2),x, algorithm="giac")
[Out] integrate(x^2/(sqrt(x^4 - 1)*(x^2 + 1)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{(1+x^2)\sqrt{-1+x^4}}\,dx = \int \frac{x^2}{(x^2+1)\,\sqrt{x^4-1}}\,dx$$

```
[In] int(x^2/((x^2 + 1)*(x^4 - 1)^(1/2)),x)
[Out] int(x^2/((x^2 + 1)*(x^4 - 1)^(1/2)), x)
```

3.264 $\int \frac{x^2}{(1-x^2)\sqrt{-1+x^4}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 57

$$\int \frac{x^2}{(1-x^2)\sqrt{-1+x^4}}\,dx = \frac{x(1+x^2)}{2\sqrt{-1+x^4}} - \frac{\sqrt{1-x^2}\sqrt{1+x^2}E(\arcsin(x)|-1)}{2\sqrt{-1+x^4}}$$

```
[Out] 1/2*x*(x^2+1)/(x^4-1)^(1/2)-1/2*EllipticE(x,I)*(-x^2+1)^(1/2)*(x^2+1)^(1/2)
/(x^4-1)^(1/2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {1270, 482, 437, 435}

$$\int \frac{x^2}{(1-x^2)\sqrt{-1+x^4}}\,dx = \frac{x(x^2+1)}{2\sqrt{x^4-1}} - \frac{\sqrt{1-x^2}\sqrt{x^2+1}E(\arcsin(x)|-1)}{2\sqrt{x^4-1}}$$

```
[In] Int[x^2/((1 - x^2)*Sqrt[-1 + x^4]),x]
```

```
[Out] (x*(1 + x^2))/(2*Sqrt[-1 + x^4]) - (Sqrt[1 - x^2]*Sqrt[1 + x^2]*EllipticE[A
rcSin[x], -1])/(2*Sqrt[-1 + x^4])
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 437

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
Sqrt[a + b*x^2]/Sqrt[1 + (b/a)*x^2], Int[Sqrt[1 + (b/a)*x^2]/Sqrt[c + d*x^2
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a, 0
]
```

Rule 482

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*
((c + d*x^n)^(q + 1)/(n*(b*c - a*d)*(p + 1))), x] - Dist[e^n/(n*(b*c - a*d)
*(p + 1)), Int[(e*x)^(m - n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(m -
n + 1) + d*(m + n*(p + q + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e,
 q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && GeQ[n, m - n +
1] && GtQ[m - n + 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 1270

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (c_.)*(x_)^4)^(p_
), x_Symbol] :> Dist[(a + c*x^4)^FracPart[p]/((d + e*x^2)^FracPart[p]*(a/d
+ (c*x^2)/e)^FracPart[p]), Int[(f*x)^m*(d + e*x^2)^(q + p)*(a/d + (c/e)*x^2
)^p, x], x] /; FreeQ[{a, c, d, e, f, m, p, q}, x] && EqQ[c*d^2 + a*e^2, 0]
&&  !IntegerQ[p]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\left(\sqrt{-1-x^2}\sqrt{1-x^2}\right)\int \frac{x^2}{\sqrt{-1-x^2}(1-x^2)^{3/2}}\,dx}{\sqrt{-1+x^4}}\\
&= \frac{x(1+x^2)}{2\sqrt{-1+x^4}} + \frac{\left(\sqrt{-1-x^2}\sqrt{1-x^2}\right)\int \frac{\sqrt{-1-x^2}}{\sqrt{1-x^2}}\,dx}{2\sqrt{-1+x^4}}\\
&= \frac{x(1+x^2)}{2\sqrt{-1+x^4}} + \frac{\left((-1-x^2)\sqrt{1-x^2}\right)\int \frac{\sqrt{1+x^2}}{\sqrt{1-x^2}}\,dx}{2\sqrt{1+x^2}\sqrt{-1+x^4}}\\
&= \frac{x(1+x^2)}{2\sqrt{-1+x^4}} - \frac{\sqrt{1-x^2}\sqrt{1+x^2}E(\sin^{-1}(x)|-1)}{2\sqrt{-1+x^4}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 10.14 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.61

$$\int \frac{x^2}{(1-x^2)\sqrt{-1+x^4}}\,dx = \frac{x+x^3-\sqrt{1-x^4}E(\arcsin(x)|-1)}{2\sqrt{-1+x^4}}$$

```
[In] Integrate[x^2/((1 - x^2)*Sqrt[-1 + x^4]),x]
[Out] (x + x^3 - Sqrt[1 - x^4]*EllipticE[ArcSin[x], -1])/(2*Sqrt[-1 + x^4])
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 92 vs. 2(45) = 90.

Time = 1.15 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.63

method	result	siz
risch	$\frac{x(x^2+1)}{2\sqrt{x^4-1}}+\frac{i\sqrt{x^2+1}\,\sqrt{-x^2+1}\,F(ix,i)}{2\sqrt{x^4-1}}+\frac{i\sqrt{x^2+1}\,\sqrt{-x^2+1}\,(F(ix,i)-E(ix,i))}{2\sqrt{x^4-1}}$	93
elliptic	$\frac{(x^2+1)x}{2\sqrt{(x^2+1)(x^2-1)}}+\frac{i\sqrt{x^2+1}\,\sqrt{-x^2+1}\,F(ix,i)}{2\sqrt{x^4-1}}+\frac{i\sqrt{x^2+1}\,\sqrt{-x^2+1}\,(F(ix,i)-E(ix,i))}{2\sqrt{x^4-1}}$	99
default	$\frac{i\sqrt{x^2+1}\,\sqrt{-x^2+1}\,F(ix,i)}{2\sqrt{x^4-1}}+\frac{x^3-x^2+x-1}{4\sqrt{(x+1)(x^3-x^2+x-1)}}+\frac{i\sqrt{x^2+1}\,\sqrt{-x^2+1}\,(F(ix,i)-E(ix,i))}{2\sqrt{x^4-1}}+\frac{x^3+x^2+x+1}{4\sqrt{(x-1)(x^3+x^2+x+1)}}$	13

```
[In] int(x^2/(-x^2+1)/(x^4-1)^(1/2),x,method=_RETURNVERBOSE)
[Out] 1/2*x*(x^2+1)/(x^4-1)^(1/2)+1/2*I*(x^2+1)^(1/2)*(-x^2+1)^(1/2)/(x^4-1)^(1/2
)*EllipticF(I*x,I)+1/2*I*(x^2+1)^(1/2)*(-x^2+1)^(1/2)/(x^4-1)^(1/2)*(Ellipt
icF(I*x,I)-EllipticE(I*x,I))
```

Fricas [A] (verification not implemented)

none

Time = 0.08 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.54

$$\int \frac{x^2}{(1-x^2)\sqrt{-1+x^4}}\,dx = \frac{(i\,x^2-i)E(\arcsin{(x)}\;|-1)+\sqrt{x^4-1}x}{2\,(x^2-1)}$$

```
[In] integrate(x^2/(-x^2+1)/(x^4-1)^(1/2),x, algorithm="fricas")
[Out] 1/2*((I*x^2 - I)*elliptic_e(arcsin(x), -1) + sqrt(x^4 - 1)*x)/(x^2 - 1)
```

Sympy [F]

$$\int \frac{x^2}{(1-x^2)\sqrt{-1+x^4}}\,dx = -\int \frac{x^2}{x^2\sqrt{x^4-1}-\sqrt{x^4-1}}\,dx$$

```
[In] integrate(x**2/(-x**2+1)/(x**4-1)**(1/2),x)
[Out] -Integral(x**2/(x**2*sqrt(x**4 - 1) - sqrt(x**4 - 1)), x)
```

Maxima [F]

$$\int \frac{x^2}{(1-x^2)\sqrt{-1+x^4}}\,dx = \int -\frac{x^2}{\sqrt{x^4-1}(x^2-1)}\,dx$$

```
[In] integrate(x^2/(-x^2+1)/(x^4-1)^(1/2),x, algorithm="maxima")
[Out] -integrate(x^2/(sqrt(x^4 - 1)*(x^2 - 1)), x)
```

Giac [F]

$$\int \frac{x^2}{(1-x^2)\sqrt{-1+x^4}}\,dx = \int -\frac{x^2}{\sqrt{x^4-1}(x^2-1)}\,dx$$

```
[In] integrate(x^2/(-x^2+1)/(x^4-1)^(1/2),x, algorithm="giac")
[Out] integrate(-x^2/(sqrt(x^4 - 1)*(x^2 - 1)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{(1-x^2)\sqrt{-1+x^4}}\,dx = -\int \frac{x^2}{(x^2-1)\sqrt{x^4-1}}\,dx$$

```
[In] int(-x^2/((x^2 - 1)*(x^4 - 1)^(1/2)),x)
[Out] -int(x^2/((x^2 - 1)*(x^4 - 1)^(1/2)), x)
```

3.265 $\int \frac{x^2}{(1+x^2)\sqrt{-1-x^4}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 74

$$\int \frac{x^2}{(1+x^2)\sqrt{-1-x^4}}\,dx = -\frac{\operatorname{arctanh}\left(\frac{\sqrt{2}x}{\sqrt{-1-x^4}}\right)}{2\sqrt{2}}+\frac{(1+x^2)\sqrt{\frac{1+x^4}{(1+x^2)^2}}\operatorname{EllipticF}\left(2\arctan(x),\frac{1}{2}\right)}{4\sqrt{-1-x^4}}$$

```
[Out] -1/4*arctanh(x*2^(1/2)/(-x^4-1)^(1/2))*2^(1/2)+1/4*(x^2+1)*(cos(2*arctan(x)
)^2)^(1/2)/cos(2*arctan(x))*EllipticF(sin(2*arctan(x)),1/2*2^(1/2))*((x^4+1
)/(x^2+1)^2)^(1/2)/(-x^4-1)^(1/2)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {1332, 226, 1713, 212}

$$\int \frac{x^2}{(1+x^2)\sqrt{-1-x^4}}\,dx = \frac{(x^2+1)\sqrt{\frac{x^4+1}{(x^2+1)^2}}\operatorname{EllipticF}\left(2\arctan(x),\frac{1}{2}\right)}{4\sqrt{-x^4-1}}-\frac{\operatorname{arctanh}\left(\frac{\sqrt{2}x}{\sqrt{-x^4-1}}\right)}{2\sqrt{2}}$$

```
[In] Int[x^2/((1 + x^2)*Sqrt[-1 - x^4]),x]
```

```
[Out] -1/2*ArcTanh[(Sqrt[2]*x)/Sqrt[-1 - x^4]]/Sqrt[2] + ((1 + x^2)*Sqrt[(1 + x^4
)/(1 + x^2)^2]*EllipticF[2*ArcTan[x], 1/2])/(4*Sqrt[-1 - x^4])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 1332

```
Int[(x_)^2/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :>
Dist[d/(2*d*e), Int[1/Sqrt[a + c*x^4], x], x] - Dist[d/(2*d*e), Int[(d - e*
x^2)/((d + e*x^2)*Sqrt[a + c*x^4]), x], x] /; FreeQ[{a, c, d, e}, x] && NeQ
[c*d^2 + a*e^2, 0] && PosQ[c/a] && EqQ[c*d^2 - a*e^2, 0]
```

Rule 1713

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4])
, x_Symbol] :> Dist[A, Subst[Int[1/(d + 2*a*e*x^2), x], x, x/Sqrt[a + c*x^4
]], x] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[c*d^
2 - a*e^2, 0] && EqQ[B*d + A*e, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{1}{2}\int \frac{1}{\sqrt{-1-x^4}}\,dx - \frac{1}{2}\int \frac{1-x^2}{(1+x^2)\sqrt{-1-x^4}}\,dx \\ &= \frac{(1+x^2)\sqrt{\frac{1+x^4}{(1+x^2)^2}}F\left(2\tan^{-1}(x)\middle|\frac{1}{2}\right)}{4\sqrt{-1-x^4}} - \frac{1}{2}\text{Subst}\left(\int \frac{1}{1-2x^2}\,dx, x, \frac{x}{\sqrt{-1-x^4}}\right) \\ &= -\frac{\tanh^{-1}\left(\frac{\sqrt{2}x}{\sqrt{-1-x^4}}\right)}{2\sqrt{2}} + \frac{(1+x^2)\sqrt{\frac{1+x^4}{(1+x^2)^2}}F\left(2\tan^{-1}(x)\middle|\frac{1}{2}\right)}{4\sqrt{-1-x^4}}\end{aligned}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.12 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.81

$$\int \frac{x^2}{(1+x^2)\sqrt{-1-x^4}}\,dx = \frac{\sqrt[4]{-1}\sqrt{1+x^4}\left(-\text{EllipticF}\left(i\text{arcsinh}\left(\sqrt[4]{-1}x\right),-1\right)+\text{EllipticPi}\left(-i,i\text{arcsinh}\left(\sqrt[4]{-1}x\right),-1\right)\right)}{\sqrt{-1-x^4}}$$

```
[In] Integrate[x^2/((1 + x^2)*Sqrt[-1 - x^4]),x]
[Out] ((-1)^(1/4)*Sqrt[1 + x^4]*(-EllipticF[I*ArcSinh[(-1)^(1/4)*x], -1] + Ellipt
icPi[-I, I*ArcSinh[(-1)^(1/4)*x], -1]))/Sqrt[-1 - x^4]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.40 (sec) , antiderivative size = 168, normalized size of antiderivative = 2.27

method	result
default	$\frac{\sqrt{i x^2+1}\,\sqrt{-i x^2+1}\,F\left(\left(\frac{\sqrt{2}}{2}-\frac{i\sqrt{2}}{2}\right)x,i\right)}{\left(\frac{\sqrt{2}}{2}-\frac{i\sqrt{2}}{2}\right)\sqrt{-x^4-1}}-\frac{i\sqrt{-i}\,\sqrt{i x^2+1}\,\sqrt{-i x^2+1}\,\Pi\left(\sqrt{-i}\,x,-i,\frac{(-1)^{\frac{1}{4}}}{\sqrt{-i}}\right)}{2\sqrt{-x^4-1}}-\frac{\sqrt{i x^2+1}\,\sqrt{-i x^2+1}\,\Pi\left(\sqrt{-i}\,x,-i,\frac{(-1 \ldots}{\sqrt{\ldots}}\right.}{2\sqrt{-i}\,\sqrt{-x^4-1}}$
elliptic	$\frac{\sqrt{i x^2+1}\,\sqrt{-i x^2+1}\,F\left(\left(\frac{\sqrt{2}}{2}-\frac{i\sqrt{2}}{2}\right)x,i\right)}{\left(\frac{\sqrt{2}}{2}-\frac{i\sqrt{2}}{2}\right)\sqrt{-x^4-1}}-\frac{i\sqrt{-i}\,\sqrt{i x^2+1}\,\sqrt{-i x^2+1}\,\Pi\left(\sqrt{-i}\,x,-i,\frac{(-1)^{\frac{1}{4}}}{\sqrt{-i}}\right)}{2\sqrt{-x^4-1}}-\frac{\sqrt{i x^2+1}\,\sqrt{-i x^2+1}\,\Pi\left(\sqrt{-i}\,x,-i,\frac{(-1 \ldots}{\sqrt{\ldots}}\right.}{2\sqrt{-i}\,\sqrt{-x^4-1}}$

```
[In] int(x^2/(x^2+1)/(-x^4-1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/(1/2*2^(1/2)-1/2*I*2^(1/2))*(1+I*x^2)^(1/2)*(1-I*x^2)^(1/2)/(-x^4-1)^(1/2
)*EllipticF((1/2*2^(1/2)-1/2*I*2^(1/2))*x,I)-1/2*I*(-I)^(1/2)*(1+I*x^2)^(1/
2)*(1-I*x^2)^(1/2)/(-x^4-1)^(1/2)*EllipticPi((-I)^(1/2)*x,-I,(-1)^(1/4)/(-I
)^(1/2))-1/2/(-I)^(1/2)*(1+I*x^2)^(1/2)*(1-I*x^2)^(1/2)/(-x^4-1)^(1/2)*Elli
pticPi((-I)^(1/2)*x,-I,(-1)^(1/4)/(-I)^(1/2))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.10 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.01

$$\int \frac{x^2}{(1+x^2)\sqrt{-1-x^4}}\,dx = -\frac{1}{2}\sqrt{i}F\left(\arcsin\left(\sqrt{i}x\right)\,|\,-1\right)-\frac{1}{8}\sqrt{2}\log\left(\frac{\sqrt{2}x+\sqrt{-x^4-1}}{x^2+1}\right)+\frac{1}{8}\sqrt{2}\log\left(-\frac{\sqrt{2}x-\sqrt{-x^4-1}}{x^2+1}\right)$$

```
[In] integrate(x^2/(x^2+1)/(-x^4-1)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/2*sqrt(I)*elliptic_f(arcsin(sqrt(I)*x), -1) - 1/8*sqrt(2)*log((sqrt(2)*x
 + sqrt(-x^4 - 1))/(x^2 + 1)) + 1/8*sqrt(2)*log(-(sqrt(2)*x - sqrt(-x^4 - 1
))/(x^2 + 1))
```

Sympy [F]

$$\int \frac{x^2}{(1+x^2)\sqrt{-1-x^4}}\,dx = \int \frac{x^2}{(x^2+1)\sqrt{-x^4-1}}\,dx$$

```
[In] integrate(x**2/(x**2+1)/(-x**4-1)**(1/2),x)
[Out] Integral(x**2/((x**2 + 1)*sqrt(-x**4 - 1)), x)
```

Maxima [F]

$$\int \frac{x^2}{(1+x^2)\sqrt{-1-x^4}}\,dx = \int \frac{x^2}{\sqrt{-x^4-1}(x^2+1)}\,dx$$

```
[In] integrate(x^2/(x^2+1)/(-x^4-1)^(1/2),x, algorithm="maxima")
[Out] integrate(x^2/(sqrt(-x^4 - 1)*(x^2 + 1)), x)
```

Giac [F]

$$\int \frac{x^2}{(1+x^2)\sqrt{-1-x^4}}\,dx = \int \frac{x^2}{\sqrt{-x^4-1}(x^2+1)}\,dx$$

```
[In] integrate(x^2/(x^2+1)/(-x^4-1)^(1/2),x, algorithm="giac")
[Out] integrate(x^2/(sqrt(-x^4 - 1)*(x^2 + 1)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{(1+x^2)\sqrt{-1-x^4}}\,dx = \int \frac{x^2}{(x^2+1)\,\sqrt{-x^4-1}}\,dx$$

```
[In] int(x^2/((x^2 + 1)*(- x^4 - 1)^(1/2)),x)
[Out] int(x^2/((x^2 + 1)*(- x^4 - 1)^(1/2)), x)
```

3.266 $\int \frac{x^2}{(1-x^2)\sqrt{-1-x^4}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 74

$$\int \frac{x^2}{(1-x^2)\sqrt{-1-x^4}}\,dx=\frac{\arctan\left(\frac{\sqrt{2}x}{\sqrt{-1-x^4}}\right)}{2\sqrt{2}}-\frac{(1+x^2)\sqrt{\frac{1+x^4}{(1+x^2)^2}}\,\text{EllipticF}\left(2\arctan(x),\frac{1}{2}\right)}{4\sqrt{-1-x^4}}$$

```
[Out] 1/4*arctan(x*2^(1/2)/(-x^4-1)^(1/2))*2^(1/2)-1/4*(x^2+1)*(cos(2*arctan(x))^
2)^(1/2)/cos(2*arctan(x))*EllipticF(sin(2*arctan(x)),1/2*2^(1/2))*((x^4+1)/
(x^2+1)^2)^(1/2)/(-x^4-1)^(1/2)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1332, 226, 1713, 209}

$$\int \frac{x^2}{(1-x^2)\sqrt{-1-x^4}}\,dx=\frac{\arctan\left(\frac{\sqrt{2}x}{\sqrt{-x^4-1}}\right)}{2\sqrt{2}}-\frac{(x^2+1)\sqrt{\frac{x^4+1}{(x^2+1)^2}}\,\text{EllipticF}\left(2\arctan(x),\frac{1}{2}\right)}{4\sqrt{-x^4-1}}$$

```
[In] Int[x^2/((1 - x^2)*Sqrt[-1 - x^4]),x]
```

```
[Out] ArcTan[(Sqrt[2]*x)/Sqrt[-1 - x^4]]/(2*Sqrt[2]) - ((1 + x^2)*Sqrt[(1 + x^4)/
(1 + x^2)^2]*EllipticF[2*ArcTan[x], 1/2])/(4*Sqrt[-1 - x^4])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 1332

```
Int[(x_)^2/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :>
Dist[d/(2*d*e), Int[1/Sqrt[a + c*x^4], x], x] - Dist[d/(2*d*e), Int[(d - e*
x^2)/((d + e*x^2)*Sqrt[a + c*x^4]), x], x] /; FreeQ[{a, c, d, e}, x] && NeQ
[c*d^2 + a*e^2, 0] && PosQ[c/a] && EqQ[c*d^2 - a*e^2, 0]
```

Rule 1713

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4])
, x_Symbol] :> Dist[A, Subst[Int[1/(d + 2*a*e*x^2), x], x, x/Sqrt[a + c*x^4
]], x] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[c*d^
2 - a*e^2, 0] && EqQ[B*d + A*e, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\left(\frac{1}{2}\int \frac{1}{\sqrt{-1-x^4}}\,dx\right)+\frac{1}{2}\int \frac{1+x^2}{(1-x^2)\sqrt{-1-x^4}}\,dx \\
&= -\frac{(1+x^2)\sqrt{\frac{1+x^4}{(1+x^2)^2}}F\left(2\tan^{-1}(x)\middle|\frac{1}{2}\right)}{4\sqrt{-1-x^4}}+\frac{1}{2}\text{Subst}\left(\int \frac{1}{1+2x^2}\,dx,x,\frac{x}{\sqrt{-1-x^4}}\right) \\
&= \frac{\tan^{-1}\left(\frac{\sqrt{2}x}{\sqrt{-1-x^4}}\right)}{2\sqrt{2}}-\frac{(1+x^2)\sqrt{\frac{1+x^4}{(1+x^2)^2}}F\left(2\tan^{-1}(x)\middle|\frac{1}{2}\right)}{4\sqrt{-1-x^4}}
\end{aligned}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.17 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.76

$$\int \frac{x^2}{(1-x^2)\sqrt{-1-x^4}}\,dx = \frac{\sqrt[4]{-1}\sqrt{1+x^4}\left(\text{EllipticF}\left(i\,\text{arcsinh}\left(\sqrt[4]{-1}x\right),-1\right)-\text{EllipticPi}\left(i,\arcsin\left((-1)^{3/4}x\right),-1\right)\right)}{\sqrt{-1-x^4}}$$

```
[In] Integrate[x^2/((1 - x^2)*Sqrt[-1 - x^4]),x]
[Out] ((-1)^(1/4)*Sqrt[1 + x^4]*(EllipticF[I*ArcSinh[(-1)^(1/4)*x], -1] - Ellipti
cPi[I, ArcSin[(-1)^(3/4)*x], -1]))/Sqrt[-1 - x^4]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.47 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.55

method	result	size
default	$-\frac{\sqrt{i x^2+1}\,\sqrt{-i x^2+1}\,F\left(\left(\frac{\sqrt{2}}{2}-\frac{i\sqrt{2}}{2}\right)x,i\right)}{\left(\frac{\sqrt{2}}{2}-\frac{i\sqrt{2}}{2}\right)\sqrt{-x^4-1}}+\frac{\sqrt{i x^2+1}\,\sqrt{-i x^2+1}\,\Pi\left(\sqrt{-i}\,x,i,\frac{(-1)^{\frac{1}{4}}}{\sqrt{-i}}\right)}{\sqrt{-i}\,\sqrt{-x^4-1}}$	115
elliptic	$-\frac{\sqrt{i x^2+1}\,\sqrt{-i x^2+1}\,F\left(\left(\frac{\sqrt{2}}{2}-\frac{i\sqrt{2}}{2}\right)x,i\right)}{\left(\frac{\sqrt{2}}{2}-\frac{i\sqrt{2}}{2}\right)\sqrt{-x^4-1}}+\frac{\sqrt{i x^2+1}\,\sqrt{-i x^2+1}\,\Pi\left(\sqrt{-i}\,x,i,\frac{(-1)^{\frac{1}{4}}}{\sqrt{-i}}\right)}{\sqrt{-i}\,\sqrt{-x^4-1}}$	115

```
[In] int(x^2/(-x^2+1)/(-x^4-1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/(1/2*2^(1/2)-1/2*I*2^(1/2))*(1+I*x^2)^(1/2)*(1-I*x^2)^(1/2)/(-x^4-1)^(1/
2)*EllipticF((1/2*2^(1/2)-1/2*I*2^(1/2))*x,I)+1/(-I)^(1/2)*(1+I*x^2)^(1/2)*
(1-I*x^2)^(1/2)/(-x^4-1)^(1/2)*EllipticPi((-I)^(1/2)*x,I,(-1)^(1/4)/(-I)^(1
/2))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.12 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.00

$$\int \frac{x^2}{(1-x^2)\sqrt{-1-x^4}}\,dx = \frac{1}{2}\sqrt{i}F\left(\arcsin\left(\sqrt{i}x\right)\mid -1\right) - \frac{1}{8}i\sqrt{2}\log\left(\frac{i\sqrt{2}x+\sqrt{-x^4-1}}{x^2-1}\right) + \frac{1}{8}i\sqrt{2}\log\left(\frac{-i\sqrt{2}x+\sqrt{-x^4-1}}{x^2-1}\right)$$

```
[In] integrate(x^2/(-x^2+1)/(-x^4-1)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2*sqrt(I)*elliptic_f(arcsin(sqrt(I)*x), -1) - 1/8*I*sqrt(2)*log((I*sqrt(2
)*x + sqrt(-x^4 - 1))/(x^2 - 1)) + 1/8*I*sqrt(2)*log((-I*sqrt(2)*x + sqrt(-
x^4 - 1))/(x^2 - 1))
```

Sympy [F]

$$\int \frac{x^2}{(1-x^2)\sqrt{-1-x^4}}\,dx = -\int \frac{x^2}{x^2\sqrt{-x^4-1}-\sqrt{-x^4-1}}\,dx$$

```
[In] integrate(x**2/(-x**2+1)/(-x**4-1)**(1/2),x)
[Out] -Integral(x**2/(x**2*sqrt(-x**4 - 1) - sqrt(-x**4 - 1)), x)
```

Maxima [F]

$$\int \frac{x^2}{(1-x^2)\sqrt{-1-x^4}}\,dx = \int -\frac{x^2}{\sqrt{-x^4-1}(x^2-1)}\,dx$$

```
[In] integrate(x^2/(-x^2+1)/(-x^4-1)^(1/2),x, algorithm="maxima")
[Out] -integrate(x^2/(sqrt(-x^4 - 1)*(x^2 - 1)), x)
```

Giac [F]

$$\int \frac{x^2}{(1-x^2)\sqrt{-1-x^4}}\,dx = \int -\frac{x^2}{\sqrt{-x^4-1}(x^2-1)}\,dx$$

```
[In] integrate(x^2/(-x^2+1)/(-x^4-1)^(1/2),x, algorithm="giac")
[Out] integrate(-x^2/(sqrt(-x^4 - 1)*(x^2 - 1)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{(1-x^2)\sqrt{-1-x^4}}\,dx = -\int \frac{x^2}{(x^2-1)\sqrt{-x^4-1}}\,dx$$

```
[In] int(-x^2/((x^2 - 1)*(- x^4 - 1)^(1/2)),x)
[Out] -int(x^2/((x^2 - 1)*(- x^4 - 1)^(1/2)), x)
```

3.267 $\int x^2\sqrt{c+dx^2}\sqrt{a^2+2abx^2+b^2x^4}\,dx$

Optimal result

Integrand size = 37, antiderivative size = 243

$$\int x^2\sqrt{c+dx^2}\sqrt{a^2+2abx^2+b^2x^4}\,dx = -\frac{c(bc-2ad)x\sqrt{c+dx^2}\sqrt{a^2+2abx^2+b^2x^4}}{16d^2(a+bx^2)} - \frac{(bc-2ad)x^3\sqrt{c+dx^2}\sqrt{a^2+2abx^2+b^2x^4}}{8d(a+bx^2)} + \frac{bx^3(c+dx^2)^{3/2}\sqrt{a^2+2abx^2+b^2x^4}}{6d(a+bx^2)} + \frac{c^2(bc-2ad)\sqrt{a^2+2abx^2+b^2x^4}\operatorname{arctanh}\left(\frac{\sqrt{d}x}{\sqrt{c+dx^2}}\right)}{16d^{5/2}(a+bx^2)}$$

```
[Out] 1/6*b*x^3*(d*x^2+c)^(3/2)*((b*x^2+a)^2)^(1/2)/d/(b*x^2+a)+1/16*c^2*(-2*a*d+
b*c)*arctanh(x*d^(1/2)/(d*x^2+c)^(1/2))*((b*x^2+a)^2)^(1/2)/d^(5/2)/(b*x^2+
a)-1/16*c*(-2*a*d+b*c)*x*(d*x^2+c)^(1/2)*((b*x^2+a)^2)^(1/2)/d^2/(b*x^2+a)-
1/8*(-2*a*d+b*c)*x^3*(d*x^2+c)^(1/2)*((b*x^2+a)^2)^(1/2)/d/(b*x^2+a)
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 243, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.162, Rules used

= {1264, 470, 285, 327, 223, 212}

$$\int x^2\sqrt{c+dx^2}\sqrt{a^2+2abx^2+b^2x^4}\,dx = \frac{c^2\sqrt{a^2+2abx^2+b^2x^4}(bc-2ad)\text{arctanh}\left(\frac{\sqrt{d}x}{\sqrt{c+dx^2}}\right)}{16d^{5/2}\left(a+bx^2\right)} - \frac{cx\sqrt{a^2+2abx^2+b^2x^4}\sqrt{c+dx^2}(bc-2ad)}{16d^2\left(a+bx^2\right)} + \frac{bx^3\sqrt{a^2+2abx^2+b^2x^4}\left(c+dx^2\right)^{3/2}}{6d\left(a+bx^2\right)} - \frac{x^3\sqrt{a^2+2abx^2+b^2x^4}\sqrt{c+dx^2}(bc-2ad)}{8d\left(a+bx^2\right)}$$

```
[In] Int[x^2*Sqrt[c + d*x^2]*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4],x]
```

```
[Out] -1/16*(c*(b*c - 2*a*d)*x*Sqrt[c + d*x^2]*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(
d^2*(a + b*x^2)) - ((b*c - 2*a*d)*x^3*Sqrt[c + d*x^2]*Sqrt[a^2 + 2*a*b*x^2
+ b^2*x^4])/(8*d*(a + b*x^2)) + (b*x^3*(c + d*x^2)^(3/2)*Sqrt[a^2 + 2*a*b*x
^2 + b^2*x^4])/(6*d*(a + b*x^2)) + (c^2*(b*c - 2*a*d)*Sqrt[a^2 + 2*a*b*x^2
+ b^2*x^4]*ArcTanh[(Sqrt[d]*x)/Sqrt[c + d*x^2]])/(16*d^(5/2)*(a + b*x^2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 285

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + n*p + 1))), x] + Dist[a*n*(p/(m + n*p + 1
)), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 1264

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_), x_Symbol] :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^Int
Part[p]*(b/2 + c*x^2)^(2*FracPart[p])), Int[(f*x)^m*(d + e*x^2)^q*(b/2 + c*
x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, f, m, p, q}, x] && EqQ[b^2 - 4*
a*c, 0] &&  !IntegerQ[p]
```

Rubi steps

$$\text{integral} = \frac{\sqrt{a^2+2abx^2+b^2x^4}\int x^2(ab+b^2x^2)\sqrt{c+dx^2}\,dx}{ab+b^2x^2}$$

$$= \frac{bx^3(c+dx^2)^{3/2}\sqrt{a^2+2abx^2+b^2x^4}}{6d(a+bx^2)} - \frac{\left(b(bc-2ad)\sqrt{a^2+2abx^2+b^2x^4}\right)\int x^2\sqrt{c+dx^2}\,dx}{2d(ab+b^2x^2)}$$

$$= -\frac{(bc-2ad)x^3\sqrt{c+dx^2}\sqrt{a^2+2abx^2+b^2x^4}}{8d(a+bx^2)} + \frac{bx^3(c+dx^2)^{3/2}\sqrt{a^2+2abx^2+b^2x^4}}{6d(a+bx^2)} - \frac{\left(bc(bc-2ad)\sqrt{a^2+2abx^2+b^2x^4}\right)\int \frac{x^2}{\sqrt{c+dx^2}}\,dx}{8d(ab+b^2x^2)}$$

$$= -\frac{c(bc-2ad)x\sqrt{c+dx^2}\sqrt{a^2+2abx^2+b^2x^4}}{16d^2(a+bx^2)} - \frac{(bc-2ad)x^3\sqrt{c+dx^2}\sqrt{a^2+2abx^2+b^2x^4}}{8d(a+bx^2)} + \frac{bx^3(c+dx^2)^{3/2}\sqrt{a^2+2abx^2+b^2x^4}}{6d(a+bx^2)} + \frac{\left(bc^2(bc-2ad)\sqrt{a^2+2abx^2+b^2x^4}\right)\int \frac{1}{\sqrt{c+dx^2}}\,dx}{16d^2(ab+b^2x^2)}$$

$$= -\frac{c(bc-2ad)x\sqrt{c+dx^2}\sqrt{a^2+2abx^2+b^2x^4}}{16d^2(a+bx^2)} - \frac{(bc-2ad)x^3\sqrt{c+dx^2}\sqrt{a^2+2abx^2+b^2x^4}}{8d(a+bx^2)} + \frac{bx^3(c+dx^2)^{3/2}\sqrt{a^2+2abx^2+b^2x^4}}{6d(a+bx^2)} + \frac{\left(bc^2(bc-2ad)\sqrt{a^2+2abx^2+b^2x^4}\right)\text{Subst}\left(\int \frac{1}{1-dx^2}\,dx, x, \frac{x}{\sqrt{c+dx^2}}\right)}{16d^2(ab+b^2x^2)}$$

$$= -\frac{c(bc-2ad)x\sqrt{c+dx^2}\sqrt{a^2+2abx^2+b^2x^4}}{16d^2(a+bx^2)} - \frac{(bc-2ad)x^3\sqrt{c+dx^2}\sqrt{a^2+2abx^2+b^2x^4}}{8d(a+bx^2)} + \frac{bx^3(c+dx^2)^{3/2}\sqrt{a^2+2abx^2+b^2x^4}}{6d(a+bx^2)} + \frac{c^2(bc-2ad)\sqrt{a^2+2abx^2+b^2x^4}\tanh^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c+dx^2}}\right)}{16d^{5/2}(a+bx^2)}$$

Mathematica [A] (verified)

Time = 0.48 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.53

$$\int x^2\sqrt{c+dx^2}\sqrt{a^2+2abx^2+b^2x^4}\,dx$$
$$=\frac{\sqrt{(a+bx^2)^2}\left(\sqrt{d}x\sqrt{c+dx^2}(6ad(c+2dx^2)+b(-3c^2+2cdx^2+8d^2x^4))+6c^2(bc-2ad)\text{arctanh}\left(\frac{\sqrt{d}x}{-\sqrt{c}+\sqrt{c+}}\right.\right.}{48d^{5/2}\,(a+bx^2)}$$

```
[In] Integrate[x^2*Sqrt[c + d*x^2]*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4],x]
```

```
[Out] (Sqrt[(a + b*x^2)^2]*(Sqrt[d]*x*Sqrt[c + d*x^2]*(6*a*d*(c + 2*d*x^2) + b*(-
3*c^2 + 2*c*d*x^2 + 8*d^2*x^4)) + 6*c^2*(b*c - 2*a*d)*ArcTanh[(Sqrt[d]*x)/(
-Sqrt[c] + Sqrt[c + d*x^2])]))/(48*d^(5/2)*(a + b*x^2))
```

Maple [A] (verified)

Time = 0.34 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.53

method	result
risch	$\frac{x\left(8b\,x^4d^2+12a\,d^2x^2+2bcd\,x^2+6acd-3b\,c^2\right)\sqrt{d\,x^2+c}\,\sqrt{\left(b\,x^2+a\right)^2}}{48d^2\left(b\,x^2+a\right)}-\frac{c^2\left(2da-bc\right)\ln\left(\sqrt{d}\,x+\sqrt{d\,x^2+c}\right)\sqrt{\left(b\,x^2+a\right)^2}}{16d^{\frac{5}{2}}\left(b\,x^2+a\right)}$
default	$\frac{\sqrt{\left(b\,x^2+a\right)^2}\,\left(8\left(d\,x^2+c\right)^{\frac{3}{2}}d^{\frac{3}{2}}b\,x^3+12\left(d\,x^2+c\right)^{\frac{3}{2}}d^{\frac{3}{2}}ax-6\left(d\,x^2+c\right)^{\frac{3}{2}}\sqrt{d}\,bcx-6\sqrt{d\,x^2+c}\,d^{\frac{3}{2}}acx+3\sqrt{d\,x^2+c}\,\sqrt{d}\,b\,c^2x-6\ln\left(\sqrt{d}\,x+\sqrt{d\,x^2}\right.\right.}{48\left(b\,x^2+a\right)d^{\frac{5}{2}}}$

```
[In] int(x^2*(d*x^2+c)^(1/2)*((b*x^2+a)^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/48*x*(8*b*d^2*x^4+12*a*d^2*x^2+2*b*c*d*x^2+6*a*c*d-3*b*c^2)*(d*x^2+c)^(1/
2)/d^2*((b*x^2+a)^2)^(1/2)/(b*x^2+a)-1/16*c^2*(2*a*d-b*c)/d^(5/2)*ln(d^(1/2
)*x+(d*x^2+c)^(1/2))*((b*x^2+a)^2)^(1/2)/(b*x^2+a)
```

Fricas [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 206, normalized size of antiderivative = 0.85

$$\int x^2\sqrt{c+dx^2}\sqrt{a^2+2abx^2+b^2x^4}\,dx$$
$$=\left[-\frac{3\,(bc^3-2\,ac^2d)\sqrt{d}\log\left(-2\,dx^2+2\,\sqrt{dx^2+c}\sqrt{d}x-c\right)-2\,(8\,bd^3x^5+2\,(bcd^2+6\,ad^3)x^3-3\,(bc^2d-2\,a}{96\,d^3}\right.$$
$$\left.-\frac{3\,(bc^3-2\,ac^2d)\sqrt{-d}\arctan\left(\frac{\sqrt{-d}x}{\sqrt{dx^2+c}}\right)-(8\,bd^3x^5+2\,(bcd^2+6\,ad^3)x^3-3\,(bc^2d-2\,acd^2)x)\sqrt{dx^2+c}}{48\,d^3}\right]$$

```
[In] integrate(x^2*(d*x^2+c)^(1/2)*((b*x^2+a)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/96*(3*(b*c^3 - 2*a*c^2*d)*sqrt(d)*log(-2*d*x^2 + 2*sqrt(d*x^2 + c)*sqrt
(d)*x - c) - 2*(8*b*d^3*x^5 + 2*(b*c*d^2 + 6*a*d^3)*x^3 - 3*(b*c^2*d - 2*a*
c*d^2)*x)*sqrt(d*x^2 + c))/d^3, -1/48*(3*(b*c^3 - 2*a*c^2*d)*sqrt(-d)*arcta
n(sqrt(-d)*x/sqrt(d*x^2 + c)) - (8*b*d^3*x^5 + 2*(b*c*d^2 + 6*a*d^3)*x^3 -
3*(b*c^2*d - 2*a*c*d^2)*x)*sqrt(d*x^2 + c))/d^3]
```

Sympy [F]

$$\int x^2\sqrt{c+dx^2}\sqrt{a^2+2abx^2+b^2x^4}\,dx=\int x^2\sqrt{c+dx^2}\sqrt{\left(a+bx^2\right)^2}\,dx$$

```
[In] integrate(x**2*(d*x**2+c)**(1/2)*((b*x**2+a)**2)**(1/2),x)
```

```
[Out] Integral(x**2*sqrt(c + d*x**2)*sqrt((a + b*x**2)**2), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.51

$$\begin{aligned}\int x^2\sqrt{c+dx^2}\sqrt{a^2+2abx^2+b^2x^4}\,dx&=\frac{\left(dx^2+c\right)^{\frac{3}{2}}bx^3}{6\,d}-\frac{\left(dx^2+c\right)^{\frac{3}{2}}bcx}{8\,d^2}+\frac{\sqrt{dx^2+c}bc^2x}{16\,d^2}\\&\quad+\frac{\left(dx^2+c\right)^{\frac{3}{2}}ax}{4\,d}-\frac{\sqrt{dx^2+c}acx}{8\,d}\\&\quad+\frac{bc^3\,\mathrm{arsinh}\left(\frac{dx}{\sqrt{cd}}\right)}{16\,d^{\frac{5}{2}}}-\frac{ac^2\,\mathrm{arsinh}\left(\frac{dx}{\sqrt{cd}}\right)}{8\,d^{\frac{3}{2}}}\end{aligned}$$

```
[In] integrate(x^2*(d*x^2+c)^(1/2)*((b*x^2+a)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/6*(d*x^2 + c)^(3/2)*b*x^3/d - 1/8*(d*x^2 + c)^(3/2)*b*c*x/d^2 + 1/16*sqrt
(d*x^2 + c)*b*c^2*x/d^2 + 1/4*(d*x^2 + c)^(3/2)*a*x/d - 1/8*sqrt(d*x^2 + c)
*a*c*x/d + 1/16*b*c^3*arcsinh(d*x/sqrt(c*d))/d^(5/2) - 1/8*a*c^2*arcsinh(d*
x/sqrt(c*d))/d^(3/2)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 156, normalized size of antiderivative = 0.64

$$\int x^2\sqrt{c+dx^2}\sqrt{a^2+2abx^2+b^2x^4}\,dx$$
$$=\frac{1}{48}\left(2\left(4\,bx^2\mathrm{sgn}\left(bx^2+a\right)+\frac{bcd^3\mathrm{sgn}(bx^2+a)+6\,ad^4\mathrm{sgn}(bx^2+a)}{d^4}\right)x^2-\frac{3\,(bc^2d^2\mathrm{sgn}(bx^2+a)-2\,acd^3\mathrm{sgn}}{d^4}\right.$$
$$-\frac{(bc^3\mathrm{sgn}(bx^2+a)-2\,ac^2d\mathrm{sgn}(bx^2+a))\log\left(\left|-\sqrt{d}x+\sqrt{dx^2+c}\right|\right)}{16\,d^{\frac{5}{2}}}$$

```
[In] integrate(x^2*(d*x^2+c)^(1/2)*((b*x^2+a)^2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/48*(2*(4*b*x^2*sgn(b*x^2 + a) + (b*c*d^3*sgn(b*x^2 + a) + 6*a*d^4*sgn(b*x
^2 + a))/d^4)*x^2 - 3*(b*c^2*d^2*sgn(b*x^2 + a) - 2*a*c*d^3*sgn(b*x^2 + a))
/d^4)*sqrt(d*x^2 + c)*x - 1/16*(b*c^3*sgn(b*x^2 + a) - 2*a*c^2*d*sgn(b*x^2
+ a))*log(abs(-sqrt(d)*x + sqrt(d*x^2 + c)))/d^(5/2)
```

Mupad [F(-1)]

Timed out.

$$\int x^2\sqrt{c+dx^2}\sqrt{a^2+2abx^2+b^2x^4}\,dx=\int x^2\,\sqrt{d\,x^2+c}\,\sqrt{{\left(b\,x^2+a\right)}^2}\,dx$$

```
[In] int(x^2*(c + d*x^2)^(1/2)*((a + b*x^2)^2)^(1/2),x)
```

```
[Out] int(x^2*(c + d*x^2)^(1/2)*((a + b*x^2)^2)^(1/2), x)
```

3.268 $\int x\sqrt{c+dx^2}\sqrt{a^2+2abx^2+b^2x^4}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 108

$$\int x\sqrt{c+dx^2}\sqrt{a^2+2abx^2+b^2x^4}\,dx = -\frac{(bc-ad)\left(c+dx^2\right)^{3/2}\sqrt{a^2+2abx^2+b^2x^4}}{3d^2\left(a+bx^2\right)} + \frac{b\left(c+dx^2\right)^{5/2}\sqrt{a^2+2abx^2+b^2x^4}}{5d^2\left(a+bx^2\right)}$$

```
[Out] -1/3*(-a*d+b*c)*(d*x^2+c)^(3/2)*((b*x^2+a)^2)^(1/2)/d^2/(b*x^2+a)+1/5*b*(d*
x^2+c)^(5/2)*((b*x^2+a)^2)^(1/2)/d^2/(b*x^2+a)
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.086, Rules used = {1261, 660, 45}

$$\int x\sqrt{c+dx^2}\sqrt{a^2+2abx^2+b^2x^4}\,dx = \frac{b\sqrt{a^2+2abx^2+b^2x^4}\left(c+dx^2\right)^{5/2}}{5d^2\left(a+bx^2\right)} - \frac{\sqrt{a^2+2abx^2+b^2x^4}\left(c+dx^2\right)^{3/2}(bc-ad)}{3d^2\left(a+bx^2\right)}$$

```
[In] Int[x*Sqrt[c + d*x^2]*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4],x]
```

```
[Out] -1/3*((b*c - a*d)*(c + d*x^2)^(3/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(d^2*(
a + b*x^2)) + (b*(c + d*x^2)^(5/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(5*d^2*
(a + b*x^2))
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 660

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sy
mbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*Fr
acPart[p])), Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[p] && NeQ[2*c*d - b*e,
0]
```

Rule 1261

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(
p_.), x_Symbol] :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x],
 x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{2}\text{Subst}\left(\int \sqrt{c+dx}\sqrt{a^2+2abx+b^2x^2}\,dx,x,x^2\right)\\
&= \frac{\sqrt{a^2+2abx^2+b^2x^4}\text{Subst}\left(\int \left(ab+b^2x\right)\sqrt{c+dx}\,dx,x,x^2\right)}{2\left(ab+b^2x^2\right)}\\
&= \frac{\sqrt{a^2+2abx^2+b^2x^4}\text{Subst}\left(\int \left(-\frac{b(bc-ad)\sqrt{c+dx}}{d}+\frac{b^2(c+dx)^{3/2}}{d}\right)dx,x,x^2\right)}{2\left(ab+b^2x^2\right)}\\
&= -\frac{(bc-ad)\left(c+dx^2\right)^{3/2}\sqrt{a^2+2abx^2+b^2x^4}}{3d^2\left(a+bx^2\right)}+\frac{b\left(c+dx^2\right)^{5/2}\sqrt{a^2+2abx^2+b^2x^4}}{5d^2\left(a+bx^2\right)}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.52

$$\int x\sqrt{c+dx^2}\sqrt{a^2+2abx^2+b^2x^4}\,dx=\frac{\sqrt{\left(a+bx^2\right)^2}\left(c+dx^2\right)^{3/2}\left(-2bc+5ad+3bdx^2\right)}{15d^2\left(a+bx^2\right)}$$

```
[In] Integrate[x*Sqrt[c + d*x^2]*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4],x]
[Out] (Sqrt[(a + b*x^2)^2]*(c + d*x^2)^(3/2)*(-2*b*c + 5*a*d + 3*b*d*x^2))/(15*d^
2*(a + b*x^2))
```

Maple [A] (verified)

Time = 0.30 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.47

method	result	size
gosper	$\frac{(dx^2+c)^{\frac{3}{2}}(3bdx^2+5da-2bc)\sqrt{(bx^2+a)^2}}{15d^2(bx^2+a)}$	51
default	$\frac{(dx^2+c)^{\frac{3}{2}}(3bdx^2+5da-2bc)\sqrt{(bx^2+a)^2}}{15d^2(bx^2+a)}$	51
risch	$\frac{\sqrt{(bx^2+a)^2}\,(3bx^4d^2+5a\,d^2x^2+bcd\,x^2+5acd-2b\,c^2)\sqrt{dx^2+c}}{15(bx^2+a)d^2}$	72

```
[In] int(x*(d*x^2+c)^(1/2)*((b*x^2+a)^2)^(1/2),x,method=_RETURNVERBOSE)
[Out] 1/15*(d*x^2+c)^(3/2)*(3*b*d*x^2+5*a*d-2*b*c)*((b*x^2+a)^2)^(1/2)/d^2/(b*x^2
+a)
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.46

$$\int x\sqrt{c+dx^2}\sqrt{a^2+2abx^2+b^2x^4}\,dx = \frac{(3\,bd^2x^4-2\,bc^2+5\,acd+(bcd+5\,ad^2)x^2)\sqrt{dx^2+c}}{15\,d^2}$$

```
[In] integrate(x*(d*x^2+c)^(1/2)*((b*x^2+a)^2)^(1/2),x, algorithm="fricas")
[Out] 1/15*(3*b*d^2*x^4 - 2*b*c^2 + 5*a*c*d + (b*c*d + 5*a*d^2)*x^2)*sqrt(d*x^2 +
 c)/d^2
```

Sympy [F]

$$\int x\sqrt{c+dx^2}\sqrt{a^2+2abx^2+b^2x^4}\,dx = \int x\sqrt{c+dx^2}\sqrt{(a+bx^2)^2}\,dx$$

```
[In] integrate(x*(d*x**2+c)**(1/2)*((b*x**2+a)**2)**(1/2),x)
[Out] Integral(x*sqrt(c + d*x**2)*sqrt((a + b*x**2)**2), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.46

$$\int x\sqrt{c+dx^2}\sqrt{a^2+2abx^2+b^2x^4}\,dx=\frac{(dx^2+c)^{\frac{3}{2}}bx^2}{5\,d}-\frac{2\,(dx^2+c)^{\frac{3}{2}}bc}{15\,d^2}+\frac{(dx^2+c)^{\frac{3}{2}}a}{3\,d}$$

```
[In] integrate(x*(d*x^2+c)^(1/2)*((b*x^2+a)^2)^(1/2),x, algorithm="maxima")
[Out] 1/5*(d*x^2 + c)^(3/2)*b*x^2/d - 2/15*(d*x^2 + c)^(3/2)*b*c/d^2 + 1/3*(d*x^2
 + c)^(3/2)*a/d
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.63

$$\int x\sqrt{c+dx^2}\sqrt{a^2+2abx^2+b^2x^4}\,dx$$
$$=\frac{3\,(dx^2+c)^{\frac{5}{2}}b\mathrm{sgn}(bx^2+a)-5\,(dx^2+c)^{\frac{3}{2}}bc\mathrm{sgn}(bx^2+a)+5\,(dx^2+c)^{\frac{3}{2}}ad\mathrm{sgn}(bx^2+a)}{15\,d^2}$$

```
[In] integrate(x*(d*x^2+c)^(1/2)*((b*x^2+a)^2)^(1/2),x, algorithm="giac")
[Out] 1/15*(3*(d*x^2 + c)^(5/2)*b*sgn(b*x^2 + a) - 5*(d*x^2 + c)^(3/2)*b*c*sgn(b*
x^2 + a) + 5*(d*x^2 + c)^(3/2)*a*d*sgn(b*x^2 + a))/d^2
```

Mupad [F(-1)]

Timed out.

$$\int x\sqrt{c+dx^2}\sqrt{a^2+2abx^2+b^2x^4}\,dx=\int x\,\sqrt{d\,x^2+c}\,\sqrt{{\left(b\,x^2+a\right)}^2}\,dx$$

```
[In] int(x*(c + d*x^2)^(1/2)*((a + b*x^2)^2)^(1/2),x)
[Out] int(x*(c + d*x^2)^(1/2)*((a + b*x^2)^2)^(1/2), x)
```

3.269 $\int \sqrt{c+dx^2}\sqrt{a^2+2abx^2+b^2x^4}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 178

$$\int \sqrt{c+dx^2}\sqrt{a^2+2abx^2+b^2x^4}\,dx = -\frac{(bc-4ad)x\sqrt{c+dx^2}\sqrt{a^2+2abx^2+b^2x^4}}{8d\,(a+bx^2)} + \frac{bx(c+dx^2)^{3/2}\sqrt{a^2+2abx^2+b^2x^4}}{4d\,(a+bx^2)} - \frac{c(bc-4ad)\sqrt{a^2+2abx^2+b^2x^4}\operatorname{arctanh}\left(\frac{\sqrt{d}x}{\sqrt{c+dx^2}}\right)}{8d^{3/2}\,(a+bx^2)}$$

```
[Out] 1/4*b*x*(d*x^2+c)^(3/2)*((b*x^2+a)^2)^(1/2)/d/(b*x^2+a)-1/8*c*(-4*a*d+b*c)*
arctanh(x*d^(1/2)/(d*x^2+c)^(1/2))*((b*x^2+a)^2)^(1/2)/d^(3/2)/(b*x^2+a)-1/
8*(-4*a*d+b*c)*x*(d*x^2+c)^(1/2)*((b*x^2+a)^2)^(1/2)/d/(b*x^2+a)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 178, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.147$, Rules used = {1162, 396, 201, 223, 212}

$$\int \sqrt{c+dx^2}\sqrt{a^2+2abx^2+b^2x^4}\,dx = -\frac{c\sqrt{a^2+2abx^2+b^2x^4}(bc-4ad)\operatorname{arctanh}\left(\frac{\sqrt{d}x}{\sqrt{c+dx^2}}\right)}{8d^{3/2}\,(a+bx^2)} + \frac{bx\sqrt{a^2+2abx^2+b^2x^4}(c+dx^2)^{3/2}}{4d\,(a+bx^2)} - \frac{x\sqrt{a^2+2abx^2+b^2x^4}\sqrt{c+dx^2}(bc-4ad)}{8d\,(a+bx^2)}$$

```
[In] Int[Sqrt[c + d*x^2]*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4],x]
```

```
[Out] -1/8*((b*c - 4*a*d)*x*Sqrt[c + d*x^2]*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(d*(
a + b*x^2)) + (b*x*(c + d*x^2)^(3/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(4*d*
(a + b*x^2)) - (c*(b*c - 4*a*d)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]*ArcTanh[(Sq
rt[d]*x)/Sqrt[c + d*x^2]])/(8*d^(3/2)*(a + b*x^2))
```

Rule 201

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Dist[a*n*(p/(n*p + 1)), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 396

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_),
x_Symbol] :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^
2)^(2*FracPart[p])), Int[(d + e*x^2)^q*(b/2 + c*x^2)^(2*p), x], x] /; FreeQ
[{a, b, c, d, e, p, q}, x] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[p]
```

Rubi steps

$$\text{integral} = \frac{\sqrt{a^2+2abx^2+b^2x^4}\int \left(ab+b^2x^2\right)\sqrt{c+dx^2}\,dx}{ab+b^2x^2}$$

$$= \frac{bx(c+dx^2)^{3/2}\sqrt{a^2+2abx^2+b^2x^4}}{4d\,(a+bx^2)} - \frac{\left(b(bc-4ad)\sqrt{a^2+2abx^2+b^2x^4}\right)\int \sqrt{c+dx^2}\,dx}{4d\,(ab+b^2x^2)}$$

$$= -\frac{(bc-4ad)x\sqrt{c+dx^2}\sqrt{a^2+2abx^2+b^2x^4}}{8d\,(a+bx^2)} + \frac{bx(c+dx^2)^{3/2}\sqrt{a^2+2abx^2+b^2x^4}}{4d\,(a+bx^2)} - \frac{\left(bc(bc-4ad)\sqrt{a^2+2abx^2+b^2x^4}\right)\int\frac{1}{\sqrt{c+dx^2}}\,dx}{8d\,(ab+b^2x^2)}$$

$$= -\frac{(bc-4ad)x\sqrt{c+dx^2}\sqrt{a^2+2abx^2+b^2x^4}}{8d\,(a+bx^2)} + \frac{bx(c+dx^2)^{3/2}\sqrt{a^2+2abx^2+b^2x^4}}{4d\,(a+bx^2)} - \frac{\left(bc(bc-4ad)\sqrt{a^2+2abx^2+b^2x^4}\right)\text{Subst}\left(\int\frac{1}{1-dx^2}\,dx,x,\frac{x}{\sqrt{c+dx^2}}\right)}{8d\,(ab+b^2x^2)}$$

$$= -\frac{(bc-4ad)x\sqrt{c+dx^2}\sqrt{a^2+2abx^2+b^2x^4}}{8d\,(a+bx^2)} + \frac{bx(c+dx^2)^{3/2}\sqrt{a^2+2abx^2+b^2x^4}}{4d\,(a+bx^2)} - \frac{c(bc-4ad)\sqrt{a^2+2abx^2+b^2x^4}\tanh^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c+dx^2}}\right)}{8d^{3/2}\,(a+bx^2)}$$

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.54

$$\int\sqrt{c+dx^2}\sqrt{a^2+2abx^2+b^2x^4}\,dx = \frac{\sqrt{(a+bx^2)^2}\left(\sqrt{d}x\sqrt{c+dx^2}(4ad+b(c+2dx^2))+c(bc-4ad)\log\left(-\sqrt{d}x+\sqrt{c+dx^2}\right)\right)}{8d^{3/2}\,(a+bx^2)}$$

```
[In] Integrate[Sqrt[c + d*x^2]*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4],x]
```

```
[Out] (Sqrt[(a + b*x^2)^2]*(Sqrt[d]*x*Sqrt[c + d*x^2]*(4*a*d + b*(c + 2*d*x^2)) +
 c*(b*c - 4*a*d)*Log[-(Sqrt[d]*x) + Sqrt[c + d*x^2]]))/(8*d^(3/2)*(a + b*x^
2))
```

Maple [A] (verified)

Time = 0.32 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.58

method	result	size
risch	$\frac{x(2bd\,x^2+4da+bc)\sqrt{d\,x^2+c}\,\sqrt{(b\,x^2+a)^2}}{8d(b\,x^2+a)}+\frac{c(4da-bc)\ln\left(\sqrt{d}\,x+\sqrt{d\,x^2+c}\right)\sqrt{(b\,x^2+a)^2}}{8d^{\frac{3}{2}}(b\,x^2+a)}$	103
default	$\frac{\sqrt{(b\,x^2+a)^2}\left(2\sqrt{d}\,(d\,x^2+c)^{\frac{3}{2}}bx+4d^{\frac{3}{2}}\sqrt{d\,x^2+c}\,ax-\sqrt{d}\,\sqrt{d\,x^2+c}\,bcx+4\ln\left(\sqrt{d}\,x+\sqrt{d\,x^2+c}\right)acd-\ln\left(\sqrt{d}\,x+\sqrt{d\,x^2+c}\right)b\,c^2\right)}{8(b\,x^2+a)d^{\frac{3}{2}}}$	119

```
[In] int((d*x^2+c)^(1/2)*((b*x^2+a)^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/8*x*(2*b*d*x^2+4*a*d+b*c)*(d*x^2+c)^(1/2)/d*((b*x^2+a)^2)^(1/2)/(b*x^2+a)
+1/8*c*(4*a*d-b*c)/d^(3/2)*ln(d^(1/2)*x+(d*x^2+c)^(1/2))*((b*x^2+a)^2)^(1/2
)/(b*x^2+a)
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 155, normalized size of antiderivative = 0.87

$$\int \sqrt{c+dx^2}\sqrt{a^2+2abx^2+b^2x^4}\,dx = \left[-\frac{(bc^2-4\,acd)\sqrt{d}\log\left(-2\,dx^2-2\,\sqrt{dx^2+c}\sqrt{d}x-c\right)-2\,(2\,bd^2x^3+(bcd+4\,ad^2)x)\sqrt{dx^2+c}}{16\,d^2}, \frac{(bc^2-4\,c}{}\right.$$

```
[In] integrate((d*x^2+c)^(1/2)*((b*x^2+a)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/16*((b*c^2 - 4*a*c*d)*sqrt(d)*log(-2*d*x^2 - 2*sqrt(d*x^2 + c)*sqrt(d)*
x - c) - 2*(2*b*d^2*x^3 + (b*c*d + 4*a*d^2)*x)*sqrt(d*x^2 + c))/d^2, 1/8*((
b*c^2 - 4*a*c*d)*sqrt(-d)*arctan(sqrt(-d)*x/sqrt(d*x^2 + c)) + (2*b*d^2*x^3
 + (b*c*d + 4*a*d^2)*x)*sqrt(d*x^2 + c))/d^2]
```

Sympy [F]

$$\int \sqrt{c+dx^2}\sqrt{a^2+2abx^2+b^2x^4}\,dx = \int \sqrt{c+dx^2}\sqrt{(a+bx^2)^2}\,dx$$

```
[In] integrate((d*x**2+c)**(1/2)*((b*x**2+a)**2)**(1/2),x)
```

```
[Out] Integral(sqrt(c + d*x**2)*sqrt((a + b*x**2)**2), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.46

$$\int \sqrt{c+dx^2}\sqrt{a^2+2abx^2+b^2x^4}\,dx = \frac{1}{2}\sqrt{dx^2+c}ax + \frac{(dx^2+c)^{\frac{3}{2}}bx}{4\,d} - \frac{\sqrt{dx^2+c}bcx}{8\,d} - \frac{bc^2\,\mathrm{arsinh}\left(\frac{dx}{\sqrt{cd}}\right)}{8\,d^{\frac{3}{2}}} + \frac{ac\,\mathrm{arsinh}\left(\frac{dx}{\sqrt{cd}}\right)}{2\,\sqrt{d}}$$

```
[In] integrate((d*x^2+c)^(1/2)*((b*x^2+a)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/2*sqrt(d*x^2 + c)*a*x + 1/4*(d*x^2 + c)^(3/2)*b*x/d - 1/8*sqrt(d*x^2 + c)
*b*c*x/d - 1/8*b*c^2*arcsinh(d*x/sqrt(c*d))/d^(3/2) + 1/2*a*c*arcsinh(d*x/s
qrt(c*d))/sqrt(d)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.61

$$\int \sqrt{c+dx^2}\sqrt{a^2+2abx^2+b^2x^4}\,dx = \frac{1}{8}\left(2\,bx^2\mathrm{sgn}\left(bx^2+a\right)+\frac{bcd\mathrm{sgn}(bx^2+a)+4\,ad^2\mathrm{sgn}(bx^2+a)}{d^2}\right)\sqrt{dx^2+c}x + \frac{(bc^2\mathrm{sgn}(bx^2+a)-4\,acd\mathrm{sgn}(bx^2+a))\log\left(\left|-\sqrt{d}x+\sqrt{dx^2+c}\right|\right)}{8\,d^{\frac{3}{2}}}$$

```
[In] integrate((d*x^2+c)^(1/2)*((b*x^2+a)^2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/8*(2*b*x^2*sgn(b*x^2 + a) + (b*c*d*sgn(b*x^2 + a) + 4*a*d^2*sgn(b*x^2 + a
))/d^2)*sqrt(d*x^2 + c)*x + 1/8*(b*c^2*sgn(b*x^2 + a) - 4*a*c*d*sgn(b*x^2 +
 a))*log(abs(-sqrt(d)*x + sqrt(d*x^2 + c)))/d^(3/2)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{c+dx^2}\sqrt{a^2+2abx^2+b^2x^4}\,dx = \int \sqrt{d\,x^2+c}\,\sqrt{{\left(b\,x^2+a\right)}^2}\,dx$$

```
[In] int((c + d*x^2)^(1/2)*((a + b*x^2)^2)^(1/2),x)
```

```
[Out] int((c + d*x^2)^(1/2)*((a + b*x^2)^2)^(1/2), x)
```

3.270 $\int \frac{\sqrt{c+dx^2}\sqrt{a^2+2abx^2+b^2x^4}}{x} \, dx$

Optimal result

Integrand size = 37, antiderivative size = 152

$$\int \frac{\sqrt{c+dx^2}\sqrt{a^2+2abx^2+b^2x^4}}{x} \, dx = \frac{a\sqrt{c+dx^2}\sqrt{a^2+2abx^2+b^2x^4}}{a+bx^2} + \frac{b(c+dx^2)^{3/2}\sqrt{a^2+2abx^2+b^2x^4}}{3d(a+bx^2)} - \frac{a\sqrt{c}\sqrt{a^2+2abx^2+b^2x^4}\,\text{arctanh}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{a+bx^2}$$

```
[Out] 1/3*b*(d*x^2+c)^(3/2)*((b*x^2+a)^2)^(1/2)/d/(b*x^2+a)-a*arctanh((d*x^2+c)^(
1/2)/c^(1/2))*c^(1/2)*((b*x^2+a)^2)^(1/2)/(b*x^2+a)+a*(d*x^2+c)^(1/2)*((b*x
^2+a)^2)^(1/2)/(b*x^2+a)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.162, Rules used = {1264, 457, 81, 52, 65, 214}

$$\int \frac{\sqrt{c+dx^2}\sqrt{a^2+2abx^2+b^2x^4}}{x} \, dx = -\frac{a\sqrt{c}\sqrt{a^2+2abx^2+b^2x^4}\,\text{arctanh}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{a+bx^2} + \frac{b\sqrt{a^2+2abx^2+b^2x^4}(c+dx^2)^{3/2}}{3d(a+bx^2)} + \frac{a\sqrt{a^2+2abx^2+b^2x^4}\sqrt{c+dx^2}}{a+bx^2}$$

```
[In] Int[(Sqrt[c + d*x^2]*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/x,x]
```

```
[Out] (a*Sqrt[c + d*x^2]*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(a + b*x^2) + (b*(c + d
*x^2)^(3/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(3*d*(a + b*x^2)) - (a*Sqrt[c]
*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]*ArcTanh[Sqrt[c + d*x^2]/Sqrt[c]])/(a + b*x
^2)
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 81

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p +
 2))), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1264

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_), x_Symbol] :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^Int
Part[p]*(b/2 + c*x^2)^(2*FracPart[p])), Int[(f*x)^m*(d + e*x^2)^q*(b/2 + c*
```

```
x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, f, m, p, q}, x] && EqQ[b^2 - 4*
a*c, 0] &&  !IntegerQ[p]
```

Rubi steps

$$\text{integral} = \frac{\sqrt{a^2+2abx^2+b^2x^4}\int \frac{(ab+b^2x^2)\sqrt{c+dx^2}}{x}\,dx}{ab+b^2x^2}$$

$$= \frac{\sqrt{a^2+2abx^2+b^2x^4}\text{Subst}\left(\int \frac{(ab+b^2x)\sqrt{c+dx}}{x}\,dx,x,x^2\right)}{2(ab+b^2x^2)}$$

$$= \frac{b(c+dx^2)^{3/2}\sqrt{a^2+2abx^2+b^2x^4}}{3d(a+bx^2)} + \frac{\left(ab\sqrt{a^2+2abx^2+b^2x^4}\right)\text{Subst}\left(\int \frac{\sqrt{c+dx}}{x}\,dx,x,x^2\right)}{2(ab+b^2x^2)}$$

$$= \frac{a\sqrt{c+dx^2}\sqrt{a^2+2abx^2+b^2x^4}}{a+bx^2} + \frac{b(c+dx^2)^{3/2}\sqrt{a^2+2abx^2+b^2x^4}}{3d(a+bx^2)} + \frac{\left(abc\sqrt{a^2+2abx^2+b^2x^4}\right)\text{Subst}\left(\int \frac{1}{x\sqrt{c+dx}}\,dx,x,x^2\right)}{2(ab+b^2x^2)}$$

$$= \frac{a\sqrt{c+dx^2}\sqrt{a^2+2abx^2+b^2x^4}}{a+bx^2} + \frac{b(c+dx^2)^{3/2}\sqrt{a^2+2abx^2+b^2x^4}}{3d(a+bx^2)} + \frac{\left(abc\sqrt{a^2+2abx^2+b^2x^4}\right)\text{Subst}\left(\int \frac{1}{-\frac{c}{d}+\frac{x^2}{d}}\,dx,x,\sqrt{c+dx^2}\right)}{d(ab+b^2x^2)}$$

$$= \frac{a\sqrt{c+dx^2}\sqrt{a^2+2abx^2+b^2x^4}}{a+bx^2} + \frac{b(c+dx^2)^{3/2}\sqrt{a^2+2abx^2+b^2x^4}}{3d(a+bx^2)} - \frac{a\sqrt{c}\sqrt{a^2+2abx^2+b^2x^4}\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{a+bx^2}$$

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.55

$$\int \frac{\sqrt{c+dx^2}\sqrt{a^2+2abx^2+b^2x^4}}{x}\,dx = \frac{\sqrt{(a+bx^2)^2}\left(\sqrt{c+dx^2}(3ad+b(c+dx^2))-3a\sqrt{c}d\,\text{arctanh}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)\right)}{3d(a+bx^2)}$$

```
[In] Integrate[(Sqrt[c + d*x^2]*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/x,x]
[Out] (Sqrt[(a + b*x^2)^2]*(Sqrt[c + d*x^2]*(3*a*d + b*(c + d*x^2)) - 3*a*Sqrt[c]
*d*ArcTanh[Sqrt[c + d*x^2]/Sqrt[c]]))/(3*d*(a + b*x^2))
```

Maple [A] (verified)

Time = 0.33 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.53

method	result	size
default	$-\frac{\sqrt{\left(b\,x^{2}+a\right)^{2}}\,\left(3\sqrt{c}\,\ln\left(\frac{2c+2\sqrt{c}\,\sqrt{d\,x^{2}+c}}{x}\right)a d-b\left(d\,x^{2}+c\right)^{\frac{3}{2}}-3\sqrt{d\,x^{2}+c}\,a d\right)}{3\left(b\,x^{2}+a\right)d}$	80

```
[In] int((d*x^2+c)^(1/2)*((b*x^2+a)^2)^(1/2)/x,x,method=_RETURNVERBOSE)
```

```
[Out] -1/3*((b*x^2+a)^2)^(1/2)*(3*c^(1/2)*ln(2*(c^(1/2)*(d*x^2+c)^(1/2)+c)/x)*a*d
-b*(d*x^2+c)^(3/2)-3*(d*x^2+c)^(1/2)*a*d)/(b*x^2+a)/d
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.81

$$\int \frac{\sqrt{c+dx^2}\sqrt{a^2+2abx^2+b^2x^4}}{x}\,dx = \left[\frac{3\,a\sqrt{c}d\log\left(-\frac{dx^2-2\sqrt{dx^2+c}\sqrt{c}+2\,c}{x^2}\right)+2\,(bdx^2+bc+3\,ad)\sqrt{dx^2+c}}{6\,d}, \frac{3\,a\sqrt{-c}d\arctan\left(\frac{\sqrt{-c}}{\sqrt{dx^2+c}}\right)+(bdx^2+\right.$$

```
[In] integrate((d*x^2+c)^(1/2)*((b*x^2+a)^2)^(1/2)/x,x, algorithm="fricas")
```

```
[Out] [1/6*(3*a*sqrt(c)*d*log(-(d*x^2 - 2*sqrt(d*x^2 + c)*sqrt(c) + 2*c)/x^2) + 2
*(b*d*x^2 + b*c + 3*a*d)*sqrt(d*x^2 + c))/d, 1/3*(3*a*sqrt(-c)*d*arctan(sqr
t(-c)/sqrt(d*x^2 + c)) + (b*d*x^2 + b*c + 3*a*d)*sqrt(d*x^2 + c))/d]
```

Sympy [F]

$$\int \frac{\sqrt{c+dx^2}\sqrt{a^2+2abx^2+b^2x^4}}{x}\,dx = \int \frac{\sqrt{c+dx^2}\sqrt{(a+bx^2)^2}}{x}\,dx$$

```
[In] integrate((d*x**2+c)**(1/2)*((b*x**2+a)**2)**(1/2)/x,x)
```

```
[Out] Integral(sqrt(c + d*x**2)*sqrt((a + b*x**2)**2)/x, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.30

$$\int \frac{\sqrt{c+dx^2}\sqrt{a^2+2abx^2+b^2x^4}}{x}\,dx = -a\sqrt{c}\operatorname{arsinh}\left(\frac{c}{\sqrt{cd}|x|}\right)+\sqrt{dx^2+c}a+\frac{(dx^2+c)^{\frac{3}{2}}b}{3\,d}$$

```
[In] integrate((d*x^2+c)^(1/2)*((b*x^2+a)^2)^(1/2)/x,x, algorithm="maxima")
[Out] -a*sqrt(c)*arcsinh(c/(sqrt(c*d)*abs(x))) + sqrt(d*x^2 + c)*a + 1/3*(d*x^2 +
 c)^(3/2)*b/d
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.55

$$\int \frac{\sqrt{c+dx^2}\sqrt{a^2+2abx^2+b^2x^4}}{x}\,dx = \frac{ac\arctan\left(\frac{\sqrt{dx^2+c}}{\sqrt{-c}}\right)\operatorname{sgn}(bx^2+a)}{\sqrt{-c}} + \frac{(dx^2+c)^{\frac{3}{2}}bd^2\operatorname{sgn}(bx^2+a)+3\sqrt{dx^2+c}ad^3\operatorname{sgn}(bx^2+a)}{3\,d^3}$$

```
[In] integrate((d*x^2+c)^(1/2)*((b*x^2+a)^2)^(1/2)/x,x, algorithm="giac")
[Out] a*c*arctan(sqrt(d*x^2 + c)/sqrt(-c))*sgn(b*x^2 + a)/sqrt(-c) + 1/3*((d*x^2
+ c)^(3/2)*b*d^2*sgn(b*x^2 + a) + 3*sqrt(d*x^2 + c)*a*d^3*sgn(b*x^2 + a))/d
^3
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{c+dx^2}\sqrt{a^2+2abx^2+b^2x^4}}{x}\,dx = \int \frac{\sqrt{d\,x^2+c}\,\sqrt{{\left(b\,x^2+a\right)}^2}}{x}\,dx$$

```
[In] int(((c + d*x^2)^(1/2)*((a + b*x^2)^2)^(1/2))/x,x)
[Out] int(((c + d*x^2)^(1/2)*((a + b*x^2)^2)^(1/2))/x, x)
```

3.271 $\int \frac{\sqrt{c+dx^2}\sqrt{a^2+2abx^2+b^2x^4}}{x^2}\,dx$

Optimal result

Integrand size = 37, antiderivative size = 177

$$\int \frac{\sqrt{c+dx^2}\sqrt{a^2+2abx^2+b^2x^4}}{x^2}\,dx=\frac{(bc+2ad)x\sqrt{c+dx^2}\sqrt{a^2+2abx^2+b^2x^4}}{2c\,(a+bx^2)}-\frac{a(c+dx^2)^{3/2}\sqrt{a^2+2abx^2+b^2x^4}}{cx\,(a+bx^2)}+\frac{(bc+2ad)\sqrt{a^2+2abx^2+b^2x^4}\,\text{arctanh}\left(\frac{\sqrt{d}x}{\sqrt{c+dx^2}}\right)}{2\sqrt{d}\,(a+bx^2)}$$

```
[Out] -a*(d*x^2+c)^(3/2)*((b*x^2+a)^2)^(1/2)/c/x/(b*x^2+a)+1/2*(2*a*d+b*c)*arctan
h(x*d^(1/2)/(d*x^2+c)^(1/2))*((b*x^2+a)^2)^(1/2)/(b*x^2+a)/d^(1/2)+1/2*(2*a
*d+b*c)*x*(d*x^2+c)^(1/2)*((b*x^2+a)^2)^(1/2)/c/(b*x^2+a)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 177, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.135, Rules used = {1264, 464, 201, 223, 212}

$$\int \frac{\sqrt{c+dx^2}\sqrt{a^2+2abx^2+b^2x^4}}{x^2}\,dx=\frac{\sqrt{a^2+2abx^2+b^2x^4}(2ad+bc)\text{arctanh}\left(\frac{\sqrt{d}x}{\sqrt{c+dx^2}}\right)}{2\sqrt{d}\,(a+bx^2)}-\frac{a\sqrt{a^2+2abx^2+b^2x^4}(c+dx^2)^{3/2}}{cx\,(a+bx^2)}+\frac{x\sqrt{a^2+2abx^2+b^2x^4}\sqrt{c+dx^2}(2ad+bc)}{2c\,(a+bx^2)}$$

```
[In] Int[(Sqrt[c + d*x^2]*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/x^2,x]
```

```
[Out] ((b*c + 2*a*d)*x*Sqrt[c + d*x^2]*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(2*c*(a +
 b*x^2)) - (a*(c + d*x^2)^(3/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(c*x*(a +
b*x^2)) + ((b*c + 2*a*d)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]*ArcTanh[(Sqrt[d]*x
)/Sqrt[c + d*x^2]])/(2*Sqrt[d]*(a + b*x^2))
```

Rule 201

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Dist[a*n*(p/(n*p + 1)), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 464

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 1264

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_), x_Symbol] :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^Int
Part[p]*(b/2 + c*x^2)^(2*FracPart[p])), Int[(f*x)^m*(d + e*x^2)^q*(b/2 + c*
x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, f, m, p, q}, x] && EqQ[b^2 - 4*
a*c, 0] &&  !IntegerQ[p]
```

Rubi steps

$$\text{integral} = \frac{\sqrt{a^2 + 2abx^2 + b^2x^4} \int \frac{(ab+b^2x^2)\sqrt{c+dx^2}}{x^2} \, dx}{ab + b^2x^2}$$

$$= -\frac{a(c+dx^2)^{3/2}\sqrt{a^2+2abx^2+b^2x^4}}{cx(a+bx^2)} + -\frac{\left((-b^2c-2abd)\sqrt{a^2+2abx^2+b^2x^4}\right)\int\sqrt{c+dx^2}\,dx}{c(ab+b^2x^2)}$$

$$= \frac{(bc+2ad)x\sqrt{c+dx^2}\sqrt{a^2+2abx^2+b^2x^4}}{2c(a+bx^2)} - \frac{a(c+dx^2)^{3/2}\sqrt{a^2+2abx^2+b^2x^4}}{cx(a+bx^2)} + -\frac{\left((-b^2c-2abd)\sqrt{a^2+2abx^2+b^2x^4}\right)\int\frac{1}{\sqrt{c+dx^2}}\,dx}{2(ab+b^2x^2)}$$

$$= \frac{(bc+2ad)x\sqrt{c+dx^2}\sqrt{a^2+2abx^2+b^2x^4}}{2c(a+bx^2)} - \frac{a(c+dx^2)^{3/2}\sqrt{a^2+2abx^2+b^2x^4}}{cx(a+bx^2)} + -\frac{\left((-b^2c-2abd)\sqrt{a^2+2abx^2+b^2x^4}\right)\mathrm{Subst}\left(\int\frac{1}{1-dx^2}\,dx, x, \frac{x}{\sqrt{c+dx^2}}\right)}{2(ab+b^2x^2)}$$

$$= \frac{(bc+2ad)x\sqrt{c+dx^2}\sqrt{a^2+2abx^2+b^2x^4}}{2c(a+bx^2)} - \frac{a(c+dx^2)^{3/2}\sqrt{a^2+2abx^2+b^2x^4}}{cx(a+bx^2)} + \frac{(bc+2ad)\sqrt{a^2+2abx^2+b^2x^4}\tanh^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c+dx^2}}\right)}{2\sqrt{d}(a+bx^2)}$$

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.57

$$\int\frac{\sqrt{c+dx^2}\sqrt{a^2+2abx^2+b^2x^4}}{x^2}\,dx = \frac{\sqrt{(a+bx^2)^2}\left(\sqrt{d}(-2a+bx^2)\sqrt{c+dx^2}+2(bc+2ad)x\,\mathrm{arctanh}\left(\frac{\sqrt{d}x}{-\sqrt{c}+\sqrt{c+dx^2}}\right)\right)}{2\sqrt{d}x(a+bx^2)}$$

```
[In] Integrate[(Sqrt[c + d*x^2]*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/x^2,x]
[Out] (Sqrt[(a + b*x^2)^2]*(Sqrt[d]*(-2*a + b*x^2)*Sqrt[c + d*x^2] + 2*(b*c + 2*a
*d)*x*ArcTanh[(Sqrt[d]*x)/(-Sqrt[c] + Sqrt[c + d*x^2])]))/(2*Sqrt[d]*x*(a +
 b*x^2))
```

Maple [A] (verified)

Time = 0.33 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.53

method	result
risch	$-\frac{\sqrt{d\,x^{2}+c}\,\left(-b\,x^{2}+2a\right)\sqrt{\left(b\,x^{2}+a\right)^{2}}}{2x\left(b\,x^{2}+a\right)}+\frac{\left(da+\frac{bc}{2}\right)\ln\left(\sqrt{d}\,x+\sqrt{d\,x^{2}+c}\right)\sqrt{\left(b\,x^{2}+a\right)^{2}}}{\sqrt{d}\,\left(b\,x^{2}+a\right)}$
default	$-\frac{\sqrt{\left(b\,x^{2}+a\right)^{2}}\,\left(-2\sqrt{d\,x^{2}+c}\,d^{\frac{3}{2}}\,a\,x^{2}-\sqrt{d\,x^{2}+c}\,\sqrt{d}\,bc\,x^{2}+2\left(d\,x^{2}+c\right)^{\frac{3}{2}}\sqrt{d}\,a-2\ln\left(\sqrt{d}\,x+\sqrt{d\,x^{2}+c}\right)acdx-\ln\left(\sqrt{d}\,x+\sqrt{d\,x^{2}+c}\right)b\,c^{2}x\right)}{2\left(b\,x^{2}+a\right)cx\sqrt{d}}$

```
[In] int((d*x^2+c)^(1/2)*((b*x^2+a)^2)^(1/2)/x^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*(d*x^2+c)^(1/2)*(-b*x^2+2*a)/x*((b*x^2+a)^2)^(1/2)/(b*x^2+a)+(d*a+1/2*
b*c)*ln(d^(1/2)*x+(d*x^2+c)^(1/2))/d^(1/2)*((b*x^2+a)^2)^(1/2)/(b*x^2+a)
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 134, normalized size of antiderivative = 0.76

$$\int \frac{\sqrt{c+dx^2}\sqrt{a^2+2abx^2+b^2x^4}}{x^2}\,dx = \left[\frac{(bc+2\,ad)\sqrt{d}x\log\left(-2\,dx^2-2\,\sqrt{dx^2+c}\sqrt{d}x-c\right)+2\,(bdx^2-2\,ad)\sqrt{dx^2+c}}{4\,dx}, -\frac{(bc+2\,ad)\sqrt{-d}x\arctan\left(\frac{\sqrt{-d}x}{\sqrt{dx^2+c}}\right)-(bdx^2-2\,ad)\sqrt{dx^2+c}}{2\,dx}\right]$$

```
[In] integrate((d*x^2+c)^(1/2)*((b*x^2+a)^2)^(1/2)/x^2,x, algorithm="fricas")
```

```
[Out] [1/4*((b*c + 2*a*d)*sqrt(d)*x*log(-2*d*x^2 - 2*sqrt(d*x^2 + c)*sqrt(d)*x -
c) + 2*(b*d*x^2 - 2*a*d)*sqrt(d*x^2 + c))/(d*x), -1/2*((b*c + 2*a*d)*sqrt(-
d)*x*arctan(sqrt(-d)*x/sqrt(d*x^2 + c)) - (b*d*x^2 - 2*a*d)*sqrt(d*x^2 + c)
)/(d*x)]
```

Sympy [F]

$$\int \frac{\sqrt{c+dx^2}\sqrt{a^2+2abx^2+b^2x^4}}{x^2}\,dx = \int \frac{\sqrt{c+dx^2}\sqrt{\left(a+bx^2\right)^2}}{x^2}\,dx$$

```
[In] integrate((d*x**2+c)**(1/2)*((b*x**2+a)**2)**(1/2)/x**2,x)
```

```
[Out] Integral(sqrt(c + d*x**2)*sqrt((a + b*x**2)**2)/x**2, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.33

$$\int \frac{\sqrt{c+dx^2}\sqrt{a^2+2abx^2+b^2x^4}}{x^2}\,dx = \frac{1}{2}\sqrt{dx^2+c}bx + \frac{bc\,\mathrm{arsinh}\left(\frac{dx}{\sqrt{cd}}\right)}{2\sqrt{d}} + a\sqrt{d}\,\mathrm{arsinh}\left(\frac{dx}{\sqrt{cd}}\right) - \frac{\sqrt{dx^2+c}a}{x}$$

```
[In] integrate((d*x^2+c)^(1/2)*((b*x^2+a)^2)^(1/2)/x^2,x, algorithm="maxima")
```

```
[Out] 1/2*sqrt(d*x^2 + c)*b*x + 1/2*b*c*arcsinh(d*x/sqrt(c*d))/sqrt(d) + a*sqrt(d
)*arcsinh(d*x/sqrt(c*d)) - sqrt(d*x^2 + c)*a/x
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.63

$$\int \frac{\sqrt{c+dx^2}\sqrt{a^2+2abx^2+b^2x^4}}{x^2}\,dx = \frac{1}{2}\sqrt{dx^2+c}bx\mathrm{sgn}\left(bx^2+a\right) + \frac{2\,ac\sqrt{d}\mathrm{sgn}(bx^2+a)}{\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^2-c} - \frac{(bc\mathrm{sgn}(bx^2+a)+2\,ad\mathrm{sgn}(bx^2+a))\log\left(\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^2\right)}{4\sqrt{d}}$$

```
[In] integrate((d*x^2+c)^(1/2)*((b*x^2+a)^2)^(1/2)/x^2,x, algorithm="giac")
```

```
[Out] 1/2*sqrt(d*x^2 + c)*b*x*sgn(b*x^2 + a) + 2*a*c*sqrt(d)*sgn(b*x^2 + a)/((sqr
t(d)*x - sqrt(d*x^2 + c))^2 - c) - 1/4*(b*c*sgn(b*x^2 + a) + 2*a*d*sgn(b*x^
2 + a))*log((sqrt(d)*x - sqrt(d*x^2 + c))^2)/sqrt(d)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{c+dx^2}\sqrt{a^2+2abx^2+b^2x^4}}{x^2}\,dx = \int \frac{\sqrt{d\,x^2+c}\,\sqrt{{\left(b\,x^2+a\right)}^2}}{x^2}\,dx$$

```
[In] int(((c + d*x^2)^(1/2)*((a + b*x^2)^2)^(1/2))/x^2,x)
[Out] int(((c + d*x^2)^(1/2)*((a + b*x^2)^2)^(1/2))/x^2, x)
```

3.272 $\int \frac{\sqrt{c+dx^2}\sqrt{a^2+2abx^2+b^2x^4}}{x^3}\,dx$

Optimal result

Integrand size = 37, antiderivative size = 177

$$\int \frac{\sqrt{c+dx^2}\sqrt{a^2+2abx^2+b^2x^4}}{x^3}\,dx = \frac{(2bc+ad)\sqrt{c+dx^2}\sqrt{a^2+2abx^2+b^2x^4}}{2c(a+bx^2)} - \frac{a(c+dx^2)^{3/2}\sqrt{a^2+2abx^2+b^2x^4}}{2cx^2(a+bx^2)} - \frac{(2bc+ad)\sqrt{a^2+2abx^2+b^2x^4}\operatorname{arctanh}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{2\sqrt{c}(a+bx^2)}$$

```
[Out] -1/2*a*(d*x^2+c)^(3/2)*((b*x^2+a)^2)^(1/2)/c/x^2/(b*x^2+a)-1/2*(a*d+2*b*c)*
arctanh((d*x^2+c)^(1/2)/c^(1/2))*((b*x^2+a)^2)^(1/2)/(b*x^2+a)/c^(1/2)+1/2*
(a*d+2*b*c)*(d*x^2+c)^(1/2)*((b*x^2+a)^2)^(1/2)/c/(b*x^2+a)
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 177, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.162, Rules used = {1264, 457, 79, 52, 65, 214}

$$\int \frac{\sqrt{c+dx^2}\sqrt{a^2+2abx^2+b^2x^4}}{x^3}\,dx = -\frac{\sqrt{a^2+2abx^2+b^2x^4}(ad+2bc)\operatorname{arctanh}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{2\sqrt{c}(a+bx^2)} - \frac{a\sqrt{a^2+2abx^2+b^2x^4}(c+dx^2)^{3/2}}{2cx^2(a+bx^2)} + \frac{\sqrt{a^2+2abx^2+b^2x^4}\sqrt{c+dx^2}(ad+2bc)}{2c(a+bx^2)}$$

```
[In] Int[(Sqrt[c + d*x^2]*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/x^3,x]
```

```
[Out] ((2*b*c + a*d)*Sqrt[c + d*x^2]*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(2*c*(a + b
*x^2)) - (a*(c + d*x^2)^(3/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(2*c*x^2*(a
+ b*x^2)) - ((2*b*c + a*d)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]*ArcTanh[Sqrt[c +
 d*x^2]/Sqrt[c]])/(2*Sqrt[c]*(a + b*x^2))
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 79

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/
(f*(p + 1)*(c*f - d*e))), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c
*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x]
, x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || I
ntegerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))
))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1264

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_), x_Symbol] :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^Int
```

```
Part[p]*(b/2 + c*x^2)^(2*FracPart[p])), Int[(f*x)^m*(d + e*x^2)^q*(b/2 + c*
x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, f, m, p, q}, x] && EqQ[b^2 - 4*
a*c, 0] &&  !IntegerQ[p]
```

Rubi steps

$$\text{integral}=\frac{\sqrt{a^2+2abx^2+b^2x^4}\int\frac{(ab+b^2x^2)\sqrt{c+dx^2}}{x^3}\,dx}{ab+b^2x^2}$$

$$=\frac{\sqrt{a^2+2abx^2+b^2x^4}\,\text{Subst}\left(\int\frac{(ab+b^2x)\sqrt{c+dx}}{x^2}\,dx,x,x^2\right)}{2\,(ab+b^2x^2)}$$

$$=-\frac{a(c+dx^2)^{3/2}\sqrt{a^2+2abx^2+b^2x^4}}{2cx^2\,(a+bx^2)}+\frac{\left(\left(b^2c+\frac{abd}{2}\right)\sqrt{a^2+2abx^2+b^2x^4}\right)\text{Subst}\left(\int\frac{\sqrt{c+dx}}{x}\,dx,x,x^2\right)}{2c\,(ab+b^2x^2)}$$

$$=\frac{(2bc+ad)\sqrt{c+dx^2}\sqrt{a^2+2abx^2+b^2x^4}}{2c\,(a+bx^2)}-\frac{a(c+dx^2)^{3/2}\sqrt{a^2+2abx^2+b^2x^4}}{2cx^2\,(a+bx^2)}+\frac{\left(\left(b^2c+\frac{abd}{2}\right)\sqrt{a^2+2abx^2+b^2x^4}\right)\text{Subst}\left(\int\frac{1}{x\sqrt{c+dx}}\,dx,x,x^2\right)}{2\,(ab+b^2x^2)}$$

$$=\frac{(2bc+ad)\sqrt{c+dx^2}\sqrt{a^2+2abx^2+b^2x^4}}{2c\,(a+bx^2)}-\frac{a(c+dx^2)^{3/2}\sqrt{a^2+2abx^2+b^2x^4}}{2cx^2\,(a+bx^2)}+\frac{\left(\left(b^2c+\frac{abd}{2}\right)\sqrt{a^2+2abx^2+b^2x^4}\right)\text{Subst}\left(\int\frac{1}{-\frac{c}{d}+\frac{x^2}{d}}\,dx,x,\sqrt{c+dx^2}\right)}{d\,(ab+b^2x^2)}$$

$$=\frac{(2bc+ad)\sqrt{c+dx^2}\sqrt{a^2+2abx^2+b^2x^4}}{2c\,(a+bx^2)}-\frac{a(c+dx^2)^{3/2}\sqrt{a^2+2abx^2+b^2x^4}}{2cx^2\,(a+bx^2)}-\frac{(2bc+ad)\sqrt{a^2+2abx^2+b^2x^4}\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{2\sqrt{c}\,(a+bx^2)}$$

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.51

$$\int \frac{\sqrt{c+dx^2}\sqrt{a^2+2abx^2+b^2x^4}}{x^3}\,dx$$
$$= -\frac{\sqrt{(a+bx^2)^2}\left(\sqrt{c}(a-2bx^2)\sqrt{c+dx^2}+(2bc+ad)x^2\operatorname{arctanh}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)\right)}{2\sqrt{c}x^2\,(a+bx^2)}$$

```
[In] Integrate[(Sqrt[c + d*x^2]*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/x^3,x]
[Out] -1/2*(Sqrt[(a + b*x^2)^2]*(Sqrt[c]*(a - 2*b*x^2)*Sqrt[c + d*x^2] + (2*b*c +
 a*d)*x^2*ArcTanh[Sqrt[c + d*x^2]/Sqrt[c]]))/(Sqrt[c]*x^2*(a + b*x^2))
```

Maple [A] (verified)

Time = 0.34 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.60

method	result	s
risch	$-\frac{a\sqrt{d\,x^2+c}\,\sqrt{(b\,x^2+a)^2}}{2x^2(b\,x^2+a)}+\frac{\left(b\sqrt{d\,x^2+c}-\frac{(da+2bc)\ln\left(\frac{2c+2\sqrt{c}\,\sqrt{d\,x^2+c}}{x}\right)}{2\sqrt{c}}\right)\sqrt{(b\,x^2+a)^2}}{b\,x^2+a}$	1
default	$-\frac{\sqrt{(b\,x^2+a)^2}\,\left(\sqrt{c}\,\ln\left(\frac{2c+2\sqrt{c}\,\sqrt{d\,x^2+c}}{x}\right)a d\,x^2+2c^{\frac{3}{2}}\,\ln\left(\frac{2c+2\sqrt{c}\,\sqrt{d\,x^2+c}}{x}\right)b\,x^2-\sqrt{d\,x^2+c}\,a d\,x^2-2\sqrt{d\,x^2+c}\,b c\,x^2+(d\,x^2+c)^{\frac{3}{2}}\,a\right)}{2(b\,x^2+a)c\,x^2}$	1

```
[In] int((d*x^2+c)^(1/2)*((b*x^2+a)^2)^(1/2)/x^3,x,method=_RETURNVERBOSE)
[Out] -1/2*a*(d*x^2+c)^(1/2)/x^2*((b*x^2+a)^2)^(1/2)/(b*x^2+a)+(b*(d*x^2+c)^(1/2)
-1/2*(a*d+2*b*c)/c^(1/2)*ln((2*c+2*c^(1/2)*(d*x^2+c)^(1/2))/x))*((b*x^2+a)^
2)^(1/2)/(b*x^2+a)
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 141, normalized size of antiderivative = 0.80

$$\int \frac{\sqrt{c+dx^2}\sqrt{a^2+2abx^2+b^2x^4}}{x^3}\,dx$$
$$= \left[\frac{(2\,bc+ad)\sqrt{c}x^2\log\left(-\frac{dx^2-2\,\sqrt{dx^2+c}\sqrt{c}+2\,c}{x^2}\right)+2\,(2\,bcx^2-ac)\sqrt{dx^2+c}}{4\,cx^2}, \frac{(2\,bc+ad)\sqrt{-c}x^2\arctan\left(\frac{\sqrt{-c}}{\sqrt{dx^2+c}}\right)}{2\,cx^2}\right.$$

```
[In] integrate((d*x^2+c)^(1/2)*((b*x^2+a)^2)^(1/2)/x^3,x, algorithm="fricas")
```

```
[Out] [1/4*((2*b*c + a*d)*sqrt(c)*x^2*log(-(d*x^2 - 2*sqrt(d*x^2 + c)*sqrt(c) + 2
*c)/x^2) + 2*(2*b*c*x^2 - a*c)*sqrt(d*x^2 + c))/(c*x^2), 1/2*((2*b*c + a*d)
*sqrt(-c)*x^2*arctan(sqrt(-c)/sqrt(d*x^2 + c)) + (2*b*c*x^2 - a*c)*sqrt(d*x
^2 + c))/(c*x^2)]
```

Sympy [F]

$$\int \frac{\sqrt{c+dx^2}\sqrt{a^2+2abx^2+b^2x^4}}{x^3}\,dx = \int \frac{\sqrt{c+dx^2}\sqrt{(a+bx^2)^2}}{x^3}\,dx$$

```
[In] integrate((d*x**2+c)**(1/2)*((b*x**2+a)**2)**(1/2)/x**3,x)
[Out] Integral(sqrt(c + d*x**2)*sqrt((a + b*x**2)**2)/x**3, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.47

$$\int \frac{\sqrt{c+dx^2}\sqrt{a^2+2abx^2+b^2x^4}}{x^3}\,dx = -b\sqrt{c}\operatorname{arsinh}\left(\frac{c}{\sqrt{cd}|x|}\right) - \frac{ad\operatorname{arsinh}\left(\frac{c}{\sqrt{cd}|x|}\right)}{2\sqrt{c}} + \sqrt{dx^2+c}b + \frac{\sqrt{dx^2+c}ad}{2c} - \frac{(dx^2+c)^{\frac{3}{2}}a}{2cx^2}$$

```
[In] integrate((d*x^2+c)^(1/2)*((b*x^2+a)^2)^(1/2)/x^3,x, algorithm="maxima")
[Out] -b*sqrt(c)*arcsinh(c/(sqrt(c*d)*abs(x))) - 1/2*a*d*arcsinh(c/(sqrt(c*d)*abs
(x)))/sqrt(c) + sqrt(d*x^2 + c)*b + 1/2*sqrt(d*x^2 + c)*a*d/c - 1/2*(d*x^2
+ c)^(3/2)*a/(c*x^2)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.56

$$\int \frac{\sqrt{c+dx^2}\sqrt{a^2+2abx^2+b^2x^4}}{x^3}\,dx = \frac{2\sqrt{dx^2+c}bd\operatorname{sgn}(bx^2+a) + \frac{(2bcd\operatorname{sgn}(bx^2+a)+ad^2\operatorname{sgn}(bx^2+a))\arctan\left(\frac{\sqrt{dx^2+c}}{\sqrt{-c}}\right)}{\sqrt{-c}} - \frac{\sqrt{dx^2+c}ad\operatorname{sgn}(bx^2+a)}{x^2}}{2d}$$

```
[In] integrate((d*x^2+c)^(1/2)*((b*x^2+a)^2)^(1/2)/x^3,x, algorithm="giac")
```

```
[Out] 1/2*(2*sqrt(d*x^2 + c)*b*d*sgn(b*x^2 + a) + (2*b*c*d*sgn(b*x^2 + a) + a*d^2
*sgn(b*x^2 + a))*arctan(sqrt(d*x^2 + c)/sqrt(-c))/sqrt(-c) - sqrt(d*x^2 + c
)*a*d*sgn(b*x^2 + a)/x^2)/d
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{c+dx^2}\sqrt{a^2+2abx^2+b^2x^4}}{x^3}\,dx=\int \frac{\sqrt{d\,x^2+c}\,\sqrt{{\left(b\,x^2+a\right)}^2}}{x^3}\,dx$$

```
[In] int(((c + d*x^2)^(1/2)*((a + b*x^2)^2)^(1/2))/x^3,x)
```

```
[Out] int(((c + d*x^2)^(1/2)*((a + b*x^2)^2)^(1/2))/x^3, x)
```

3.273 $\int x^3(d+ex^2)^2 (a+bx^2+cx^4)\, dx$

Optimal result

Integrand size = 25, antiderivative size = 78

$$\int x^3 \left(d+ex^2\right)^2 \left(a+bx^2+cx^4\right)\, dx = \frac{1}{4}ad^2x^4+\frac{1}{6}d(bd+2ae)x^6+\frac{1}{8}\left(cd^2+e(2bd+ae)\right)x^8 + \frac{1}{10}e(2cd+be)x^{10}+\frac{1}{12}ce^2x^{12}$$

```
[Out] 1/4*a*d^2*x^4+1/6*d*(2*a*e+b*d)*x^6+1/8*(c*d^2+e*(a*e+2*b*d))*x^8+1/10*e*(b
*e+2*c*d)*x^10+1/12*c*e^2*x^12
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.080$, Rules used = {1265, 785}

$$\int x^3 \left(d+ex^2\right)^2 \left(a+bx^2+cx^4\right)\, dx = \frac{1}{8}x^8\left(e(ae+2bd)+cd^2\right)+\frac{1}{6}dx^6(2ae+bd) + \frac{1}{4}ad^2x^4+\frac{1}{10}ex^{10}(be+2cd)+\frac{1}{12}ce^2x^{12}$$

```
[In] Int[x^3*(d + e*x^2)^2*(a + b*x^2 + c*x^4),x]
```

```
[Out] (a*d^2*x^4)/4 + (d*(b*d + 2*a*e)*x^6)/6 + ((c*d^2 + e*(2*b*d + a*e))*x^8)/8
 + (e*(2*c*d + b*e)*x^10)/10 + (c*e^2*x^12)/12
```

Rule 785

```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (
c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)*
(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^
```

```
2 - 4*a*c, 0] && IntegerQ[p] && (GtQ[p, 0] || (EqQ[a, 0] && IntegerQ[m]))
```

Rule 1265

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{1}{2}\text{Subst}\left(\int x(d+ex)^2\left(a+bx+cx^2\right)\,dx,x,x^2\right)\\ &= \frac{1}{2}\text{Subst}\left(\int \left(ad^2x+d(bd+2ae)x^2+\left(cd^2+e(2bd+ae)\right)x^3+e(2cd+be)x^4+ce^2x^5\right)\,dx,x,x^2\right)\\ &= \frac{1}{4}ad^2x^4+\frac{1}{6}d(bd+2ae)x^6+\frac{1}{8}\left(cd^2+e(2bd+ae)\right)x^8+\frac{1}{10}e(2cd+be)x^{10}+\frac{1}{12}ce^2x^{12}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.92

$$\int x^3\left(d+ex^2\right)^2\left(a+bx^2+cx^4\right)\,dx = \frac{1}{120}x^4\left(30ad^2+20d(bd+2ae)x^2+15\left(cd^2+e(2bd+ae)\right)x^4 + 12e(2cd+be)x^6+10ce^2x^8\right)$$

```
[In] Integrate[x^3*(d + e*x^2)^2*(a + b*x^2 + c*x^4),x]
[Out] (x^4*(30*a*d^2 + 20*d*(b*d + 2*a*e)*x^2 + 15*(c*d^2 + e*(2*b*d + a*e))*x^4
+ 12*e*(2*c*d + b*e)*x^6 + 10*c*e^2*x^8))/120
```

Maple [A] (verified)

Time = 0.36 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.94

method	result
default	$\frac{c e^2 x^{12}}{12}+\frac{(b e^2+2 d c e) x^{10}}{10}+\frac{(a e^2+2 b d e+c d^2) x^8}{8}+\frac{(2 e d a+b d^2) x^6}{6}+\frac{a d^2 x^4}{4}$
norman	$\frac{c e^2 x^{12}}{12}+\left(\frac{1}{10} b e^2+\frac{1}{5} d c e\right) x^{10}+\left(\frac{1}{8} a e^2+\frac{1}{4} b d e+\frac{1}{8} c d^2\right) x^8+\left(\frac{1}{3} e d a+\frac{1}{6} b d^2\right) x^6+\frac{a d^2 x^4}{4}$
gosper	$\frac{1}{12} c e^2 x^{12}+\frac{1}{10} x^{10} b e^2+\frac{1}{5} x^{10} d c e+\frac{1}{8} x^8 a e^2+\frac{1}{4} x^8 b d e+\frac{1}{8} x^8 c d^2+\frac{1}{3} x^6 e d a+\frac{1}{6} x^6 b d^2+\frac{1}{4} a d^2 x$
risch	$\frac{1}{12} c e^2 x^{12}+\frac{1}{10} x^{10} b e^2+\frac{1}{5} x^{10} d c e+\frac{1}{8} x^8 a e^2+\frac{1}{4} x^8 b d e+\frac{1}{8} x^8 c d^2+\frac{1}{3} x^6 e d a+\frac{1}{6} x^6 b d^2+\frac{1}{4} a d^2 x$
parallelrisch	$\frac{1}{12} c e^2 x^{12}+\frac{1}{10} x^{10} b e^2+\frac{1}{5} x^{10} d c e+\frac{1}{8} x^8 a e^2+\frac{1}{4} x^8 b d e+\frac{1}{8} x^8 c d^2+\frac{1}{3} x^6 e d a+\frac{1}{6} x^6 b d^2+\frac{1}{4} a d^2 x$

```
[In] int(x^3*(e*x^2+d)^2*(c*x^4+b*x^2+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/12*c*e^2*x^12+1/10*(b*e^2+2*c*d*e)*x^10+1/8*(a*e^2+2*b*d*e+c*d^2)*x^8+1/6
*(2*a*d*e+b*d^2)*x^6+1/4*a*d^2*x^4
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.92

$$\int x^3 (d+ex^2)^2 (a+bx^2+cx^4)\ dx = \frac{1}{12}ce^2x^{12}+\frac{1}{10}(2cde+be^2)x^{10}+\frac{1}{8}(cd^2+2bde+ae^2)x^8 + \frac{1}{4}ad^2x^4+\frac{1}{6}(bd^2+2ade)x^6$$

```
[In] integrate(x^3*(e*x^2+d)^2*(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] 1/12*c*e^2*x^12 + 1/10*(2*c*d*e + b*e^2)*x^10 + 1/8*(c*d^2 + 2*b*d*e + a*e^
2)*x^8 + 1/4*a*d^2*x^4 + 1/6*(b*d^2 + 2*a*d*e)*x^6
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.97

$$\int x^3 (d+ex^2)^2 (a+bx^2+cx^4)\ dx = \frac{ad^2x^4}{4}+\frac{ce^2x^{12}}{12}+x^{10}\left(\frac{be^2}{10}+\frac{cde}{5}\right) + x^8\left(\frac{ae^2}{8}+\frac{bde}{4}+\frac{cd^2}{8}\right)+x^6\left(\frac{ade}{3}+\frac{bd^2}{6}\right)$$

```
[In] integrate(x**3*(e*x**2+d)**2*(c*x**4+b*x**2+a),x)
```

```
[Out] a*d**2*x**4/4 + c*e**2*x**12/12 + x**10*(b*e**2/10 + c*d*e/5) + x**8*(a*e**
2/8 + b*d*e/4 + c*d**2/8) + x**6*(a*d*e/3 + b*d**2/6)
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.92

$$\int x^3 (d+ex^2)^2 (a+bx^2+cx^4) \, dx = \frac{1}{12} ce^2 x^{12} + \frac{1}{10} (2\,cde + be^2) x^{10} + \frac{1}{8} (cd^2 + 2\,bde + ae^2) x^8 + \frac{1}{4} ad^2 x^4 + \frac{1}{6} (bd^2 + 2\,ade) x^6$$

```
[In] integrate(x^3*(e*x^2+d)^2*(c*x^4+b*x^2+a),x, algorithm="maxima")
```

```
[Out] 1/12*c*e^2*x^12 + 1/10*(2*c*d*e + b*e^2)*x^10 + 1/8*(c*d^2 + 2*b*d*e + a*e^
2)*x^8 + 1/4*a*d^2*x^4 + 1/6*(b*d^2 + 2*a*d*e)*x^6
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.01

$$\int x^3 (d+ex^2)^2 (a+bx^2+cx^4) \, dx = \frac{1}{12} ce^2 x^{12} + \frac{1}{5} cdex^{10} + \frac{1}{10} be^2 x^{10} + \frac{1}{8} cd^2 x^8 + \frac{1}{4} bdex^8 + \frac{1}{8} ae^2 x^8 + \frac{1}{6} bd^2 x^6 + \frac{1}{3} adex^6 + \frac{1}{4} ad^2 x^4$$

```
[In] integrate(x^3*(e*x^2+d)^2*(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] 1/12*c*e^2*x^12 + 1/5*c*d*e*x^10 + 1/10*b*e^2*x^10 + 1/8*c*d^2*x^8 + 1/4*b*
d*e*x^8 + 1/8*a*e^2*x^8 + 1/6*b*d^2*x^6 + 1/3*a*d*e*x^6 + 1/4*a*d^2*x^4
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.94

$$\int x^3 (d+ex^2)^2 (a+bx^2+cx^4) \, dx = x^8 \left(\frac{c\,d^2}{8} + \frac{b\,d\,e}{4} + \frac{a\,e^2}{8}\right) + x^6 \left(\frac{b\,d^2}{6} + \frac{a\,e\,d}{3}\right) + x^{10} \left(\frac{b\,e^2}{10} + \frac{c\,d\,e}{5}\right) + \frac{a\,d^2\,x^4}{4} + \frac{c\,e^2\,x^{12}}{12}$$

```
[In] int(x^3*(d + e*x^2)^2*(a + b*x^2 + c*x^4),x)
```

```
[Out] x^8*((a*e^2)/8 + (c*d^2)/8 + (b*d*e)/4) + x^6*((b*d^2)/6 + (a*d*e)/3) + x^1
0*((b*e^2)/10 + (c*d*e)/5) + (a*d^2*x^4)/4 + (c*e^2*x^12)/12
```

3.274 $\int x^2(d+ex^2)^2\left(a+bx^2+cx^4\right)\,dx$

Optimal result

Integrand size = 25, antiderivative size = 78

$$\int x^2\left(d+ex^2\right)^2\left(a+bx^2+cx^4\right)\,dx=\frac{1}{3}ad^2x^3+\frac{1}{5}d(bd+2ae)x^5+\frac{1}{7}\left(cd^2+e(2bd+ae)\right)x^7+\frac{1}{9}e(2cd+be)x^9+\frac{1}{11}ce^2x^{11}$$

```
[Out] 1/3*a*d^2*x^3+1/5*d*(2*a*e+b*d)*x^5+1/7*(c*d^2+e*(a*e+2*b*d))*x^7+1/9*e*(b*
e+2*c*d)*x^9+1/11*c*e^2*x^11
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.040, Rules used = {1275}

$$\int x^2\left(d+ex^2\right)^2\left(a+bx^2+cx^4\right)\,dx=\frac{1}{7}x^7\left(e(ae+2bd)+cd^2\right)+\frac{1}{5}dx^5(2ae+bd)+\frac{1}{3}ad^2x^3+\frac{1}{9}ex^9(be+2cd)+\frac{1}{11}ce^2x^{11}$$

```
[In] Int[x^2*(d + e*x^2)^2*(a + b*x^2 + c*x^4),x]
```

```
[Out] (a*d^2*x^3)/3 + (d*(b*d + 2*a*e)*x^5)/5 + ((c*d^2 + e*(2*b*d + a*e))*x^7)/7
 + (e*(2*c*d + b*e)*x^9)/9 + (c*e^2*x^11)/11
```

Rule 1275

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*
(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && NeQ[
```

```
b^2 - 4*a*c, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rubi steps

$$\begin{aligned} \text{integral} &= \int \left(ad^2x^2 + d(bd + 2ae)x^4 + \left(cd^2 + e(2bd + ae)\right)x^6 + e(2cd + be)x^8 + ce^2x^{10}\right) dx \\ &= \frac{1}{3}ad^2x^3 + \frac{1}{5}d(bd + 2ae)x^5 + \frac{1}{7}\left(cd^2 + e(2bd + ae)\right)x^7 + \frac{1}{9}e(2cd + be)x^9 + \frac{1}{11}ce^2x^{11} \end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.00

$$\int x^2\left(d + ex^2\right)^2\left(a + bx^2 + cx^4\right) dx = \frac{1}{3}ad^2x^3 + \frac{1}{5}d(bd + 2ae)x^5 + \frac{1}{7}\left(cd^2 + 2bde + ae^2\right)x^7 + \frac{1}{9}e(2cd + be)x^9 + \frac{1}{11}ce^2x^{11}$$

```
[In] Integrate[x^2*(d + e*x^2)^2*(a + b*x^2 + c*x^4),x]
```

```
[Out] (a*d^2*x^3)/3 + (d*(b*d + 2*a*e)*x^5)/5 + ((c*d^2 + 2*b*d*e + a*e^2)*x^7)/7
 + (e*(2*c*d + b*e)*x^9)/9 + (c*e^2*x^11)/11
```

Maple [A] (verified)

Time = 0.35 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.94

method	result	s
default	$\frac{c\,e^2x^{11}}{11}+\frac{(b\,e^2+2dce)x^9}{9}+\frac{(a\,e^2+2bde+c\,d^2)x^7}{7}+\frac{(2eda+b\,d^2)x^5}{5}+\frac{a\,d^2x^3}{3}$	7
norman	$\frac{c\,e^2x^{11}}{11}+\left(\frac{1}{9}b\,e^2+\frac{2}{9}dce\right)x^9+\left(\frac{1}{7}a\,e^2+\frac{2}{7}bde+\frac{1}{7}c\,d^2\right)x^7+\left(\frac{2}{5}eda+\frac{1}{5}b\,d^2\right)x^5+\frac{a\,d^2x^3}{3}$	7
gosper	$\frac{1}{11}c\,e^2x^{11}+\frac{1}{9}x^9b\,e^2+\frac{2}{9}x^9dce+\frac{1}{7}x^7a\,e^2+\frac{2}{7}x^7bde+\frac{1}{7}x^7c\,d^2+\frac{2}{5}x^5eda+\frac{1}{5}x^5b\,d^2+\frac{1}{3}a\,d^2x^3$	8
risch	$\frac{1}{11}c\,e^2x^{11}+\frac{1}{9}x^9b\,e^2+\frac{2}{9}x^9dce+\frac{1}{7}x^7a\,e^2+\frac{2}{7}x^7bde+\frac{1}{7}x^7c\,d^2+\frac{2}{5}x^5eda+\frac{1}{5}x^5b\,d^2+\frac{1}{3}a\,d^2x^3$	8
parallelrisch	$\frac{1}{11}c\,e^2x^{11}+\frac{1}{9}x^9b\,e^2+\frac{2}{9}x^9dce+\frac{1}{7}x^7a\,e^2+\frac{2}{7}x^7bde+\frac{1}{7}x^7c\,d^2+\frac{2}{5}x^5eda+\frac{1}{5}x^5b\,d^2+\frac{1}{3}a\,d^2x^3$	8

```
[In] int(x^2*(e*x^2+d)^2*(c*x^4+b*x^2+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/11*c*e^2*x^11+1/9*(b*e^2+2*c*d*e)*x^9+1/7*(a*e^2+2*b*d*e+c*d^2)*x^7+1/5*(
2*a*d*e+b*d^2)*x^5+1/3*a*d^2*x^3
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.92

$$\int x^2 (d+ex^2)^2 \left(a+bx^2+cx^4\right) dx = \frac{1}{11} ce^2x^{11} + \frac{1}{9} \left(2\,cde + be^2\right)x^9 + \frac{1}{7} \left(cd^2 + 2\,bde + ae^2\right)x^7 + \frac{1}{3} ad^2x^3 + \frac{1}{5} \left(bd^2 + 2\,ade\right)x^5$$

```
[In] integrate(x^2*(e*x^2+d)^2*(c*x^4+b*x^2+a),x, algorithm="fricas")
[Out] 1/11*c*e^2*x^11 + 1/9*(2*c*d*e + b*e^2)*x^9 + 1/7*(c*d^2 + 2*b*d*e + a*e^2)
*x^7 + 1/3*a*d^2*x^3 + 1/5*(b*d^2 + 2*a*d*e)*x^5
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.05

$$\int x^2 (d+ex^2)^2 \left(a+bx^2+cx^4\right) dx = \frac{ad^2x^3}{3} + \frac{ce^2x^{11}}{11} + x^9 \left(\frac{be^2}{9} + \frac{2cde}{9}\right) + x^7 \left(\frac{ae^2}{7} + \frac{2bde}{7} + \frac{cd^2}{7}\right) + x^5 \cdot \left(\frac{2ade}{5} + \frac{bd^2}{5}\right)$$

```
[In] integrate(x**2*(e*x**2+d)**2*(c*x**4+b*x**2+a),x)
[Out] a*d**2*x**3/3 + c*e**2*x**11/11 + x**9*(b*e**2/9 + 2*c*d*e/9) + x**7*(a*e**
2/7 + 2*b*d*e/7 + c*d**2/7) + x**5*(2*a*d*e/5 + b*d**2/5)
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.92

$$\int x^2 (d+ex^2)^2 \left(a+bx^2+cx^4\right) dx = \frac{1}{11} ce^2x^{11} + \frac{1}{9} \left(2\,cde + be^2\right)x^9 + \frac{1}{7} \left(cd^2 + 2\,bde + ae^2\right)x^7 + \frac{1}{3} ad^2x^3 + \frac{1}{5} \left(bd^2 + 2\,ade\right)x^5$$

```
[In] integrate(x^2*(e*x^2+d)^2*(c*x^4+b*x^2+a),x, algorithm="maxima")
[Out] 1/11*c*e^2*x^11 + 1/9*(2*c*d*e + b*e^2)*x^9 + 1/7*(c*d^2 + 2*b*d*e + a*e^2)
*x^7 + 1/3*a*d^2*x^3 + 1/5*(b*d^2 + 2*a*d*e)*x^5
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.01

$$\int x^2(d+ex^2)^2\left(a+bx^2+cx^4\right)\,dx = \frac{1}{11}ce^2x^{11}+\frac{2}{9}cdex^9+\frac{1}{9}be^2x^9+\frac{1}{7}cd^2x^7+\frac{2}{7}bdex^7+\frac{1}{7}ae^2x^7+\frac{1}{5}bd^2x^5+\frac{2}{5}adex^5+\frac{1}{3}ad^2x^3$$

```
[In] integrate(x^2*(e*x^2+d)^2*(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] 1/11*c*e^2*x^11 + 2/9*c*d*e*x^9 + 1/9*b*e^2*x^9 + 1/7*c*d^2*x^7 + 2/7*b*d*e
*x^7 + 1/7*a*e^2*x^7 + 1/5*b*d^2*x^5 + 2/5*a*d*e*x^5 + 1/3*a*d^2*x^3
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.94

$$\int x^2(d+ex^2)^2\left(a+bx^2+cx^4\right)\,dx = x^7\left(\frac{c\,d^2}{7}+\frac{2\,b\,d\,e}{7}+\frac{a\,e^2}{7}\right)+x^5\left(\frac{b\,d^2}{5}+\frac{2\,a\,e\,d}{5}\right)+x^9\left(\frac{b\,e^2}{9}+\frac{2\,c\,d\,e}{9}\right)+\frac{a\,d^2\,x^3}{3}+\frac{c\,e^2\,x^{11}}{11}$$

```
[In] int(x^2*(d + e*x^2)^2*(a + b*x^2 + c*x^4),x)
```

```
[Out] x^7*((a*e^2)/7 + (c*d^2)/7 + (2*b*d*e)/7) + x^5*((b*d^2)/5 + (2*a*d*e)/5) +
 x^9*((b*e^2)/9 + (2*c*d*e)/9) + (a*d^2*x^3)/3 + (c*e^2*x^11)/11
```

3.275 $\int x(d+ex^2)^2(a+bx^2+cx^4)\,dx$

Optimal result

Integrand size = 23, antiderivative size = 75

$$\int x(d+ex^2)^2(a+bx^2+cx^4)\,dx=\frac{(cd^2-bde+ae^2)(d+ex^2)^3}{6e^3}-\frac{(2cd-be)(d+ex^2)^4}{8e^3}+\frac{c(d+ex^2)^5}{10e^3}$$

```
[Out] 1/6*(a*e^2-b*d*e+c*d^2)*(e*x^2+d)^3/e^3-1/8*(-b*e+2*c*d)*(e*x^2+d)^4/e^3+1/
10*c*(e*x^2+d)^5/e^3
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {1261, 712}

$$\int x(d+ex^2)^2(a+bx^2+cx^4)\,dx=\frac{(d+ex^2)^3(ae^2-bde+cd^2)}{6e^3}-\frac{(d+ex^2)^4(2cd-be)}{8e^3}+\frac{c(d+ex^2)^5}{10e^3}$$

```
[In] Int[x*(d + e*x^2)^2*(a + b*x^2 + c*x^4),x]
```

```
[Out] ((c*d^2 - b*d*e + a*e^2)*(d + e*x^2)^3)/(6*e^3) - ((2*c*d - b*e)*(d + e*x^2
)^4)/(8*e^3) + (c*(d + e*x^2)^5)/(10*e^3)
```

Rule 712

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /; F
```

```
reeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a
*e^2, 0] && NeQ[2*c*d - b*e, 0] && IntegerQ[p] && (GtQ[p, 0] || (EqQ[a, 0]
&& IntegerQ[m]))
```

Rule 1261

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(
p_.), x_Symbol] :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x],
 x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{2}\text{Subst}\left(\int (d+ex)^2\left(a+bx+cx^2\right)\,dx,x,x^2\right)\\
&= \frac{1}{2}\text{Subst}\left(\int\left(\frac{(cd^2-bde+ae^2)\,(d+ex)^2}{e^2}+\frac{(-2cd+be)(d+ex)^3}{e^2}+\frac{c(d+ex)^4}{e^2}\right)\,dx,x,x^2\right)\\
&= \frac{(cd^2-bde+ae^2)\,(d+ex^2)^3}{6e^3}-\frac{(2cd-be)\,(d+ex^2)^4}{8e^3}+\frac{c(d+ex^2)^5}{10e^3}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.96

$$\int x\left(d+ex^2\right)^2\left(a+bx^2+cx^4\right)\,dx=\frac{1}{120}x^2\left(60ad^2+30d(bd+2ae)x^2+20\left(cd^2+e(2bd+ae)\right)x^4+15e(2cd+be)x^6+12ce^2x^8\right)$$

```
[In] Integrate[x*(d + e*x^2)^2*(a + b*x^2 + c*x^4),x]
[Out] (x^2*(60*a*d^2 + 30*d*(b*d + 2*a*e)*x^2 + 20*(c*d^2 + e*(2*b*d + a*e))*x^4
+ 15*e*(2*c*d + b*e)*x^6 + 12*c*e^2*x^8))/120
```

Maple [A] (verified)

Time = 0.35 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.97

method	result
default	$\frac{c\,e^{2} x^{10}}{10}+\frac{\left(b\,e^{2}+2 d c e \right) x^{8}}{8}+\frac{\left(a\,e^{2}+2 b d e +c\,d^{2}\right) x^{6}}{6}+\frac{\left(2 e d a +b\,d^{2}\right) x^{4}}{4}+\frac{a\,d^{2} x^{2}}{2}$
norman	$\frac{c\,e^{2} x^{10}}{10}+\left(\frac{1}{8} b\,e^{2}+\frac{1}{4} d c e \right) x^{8}+\left(\frac{1}{6} a\,e^{2}+\frac{1}{3} b d e +\frac{1}{6} c\,d^{2}\right) x^{6}+\left(\frac{1}{2} e d a +\frac{1}{4} b\,d^{2}\right) x^{4}+\frac{a\,d^{2} x^{2}}{2}$
gosper	$\frac{1}{10} c\,e^{2} x^{10}+\frac{1}{8} x^{8} b\,e^{2}+\frac{1}{4} x^{8} d c e +\frac{1}{6} x^{6} a\,e^{2}+\frac{1}{3} x^{6} b d e +\frac{1}{6} x^{6} c\,d^{2}+\frac{1}{2} x^{4} e d a +\frac{1}{4} b\,x^{4} d^{2}+\frac{1}{2} a\,d^{2} x^{2}$
risch	$\frac{1}{10} c\,e^{2} x^{10}+\frac{1}{8} x^{8} b\,e^{2}+\frac{1}{4} x^{8} d c e +\frac{1}{6} x^{6} a\,e^{2}+\frac{1}{3} x^{6} b d e +\frac{1}{6} x^{6} c\,d^{2}+\frac{1}{2} x^{4} e d a +\frac{1}{4} b\,x^{4} d^{2}+\frac{1}{2} a\,d^{2} x^{2}$
parallelrisch	$\frac{1}{10} c\,e^{2} x^{10}+\frac{1}{8} x^{8} b\,e^{2}+\frac{1}{4} x^{8} d c e +\frac{1}{6} x^{6} a\,e^{2}+\frac{1}{3} x^{6} b d e +\frac{1}{6} x^{6} c\,d^{2}+\frac{1}{2} x^{4} e d a +\frac{1}{4} b\,x^{4} d^{2}+\frac{1}{2} a\,d^{2} x^{2}$

```
[In] int(x*(e*x^2+d)^2*(c*x^4+b*x^2+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/10*c*e^2*x^10+1/8*(b*e^2+2*c*d*e)*x^8+1/6*(a*e^2+2*b*d*e+c*d^2)*x^6+1/4*(
2*a*d*e+b*d^2)*x^4+1/2*a*d^2*x^2
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.96

$$\int x\left(d+ex^2\right)^2\left(a+bx^2+cx^4\right)\,dx=\frac{1}{10}\,ce^2x^{10}+\frac{1}{8}\,(2\,cde+be^2)x^8+\frac{1}{6}\,(cd^2+2\,bde+ae^2)x^6+\frac{1}{2}\,ad^2x^2+\frac{1}{4}\,(bd^2+2\,ade)x^4$$

```
[In] integrate(x*(e*x^2+d)^2*(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] 1/10*c*e^2*x^10 + 1/8*(2*c*d*e + b*e^2)*x^8 + 1/6*(c*d^2 + 2*b*d*e + a*e^2)
*x^6 + 1/2*a*d^2*x^2 + 1/4*(b*d^2 + 2*a*d*e)*x^4
```

Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.01

$$\int x\left(d+ex^2\right)^2\left(a+bx^2+cx^4\right)\,dx=\frac{ad^2x^2}{2}+\frac{ce^2x^{10}}{10}+x^8\left(\frac{be^2}{8}+\frac{cde}{4}\right)+x^6\left(\frac{ae^2}{6}+\frac{bde}{3}+\frac{cd^2}{6}\right)+x^4\left(\frac{ade}{2}+\frac{bd^2}{4}\right)$$

```
[In] integrate(x*(e*x**2+d)**2*(c*x**4+b*x**2+a),x)
```

```
[Out] a*d**2*x**2/2 + c*e**2*x**10/10 + x**8*(b*e**2/8 + c*d*e/4) + x**6*(a*e**2/
6 + b*d*e/3 + c*d**2/6) + x**4*(a*d*e/2 + b*d**2/4)
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.96

$$\int x(d+ex^2)^2(a+bx^2+cx^4)\,dx = \frac{1}{10}ce^2x^{10} + \frac{1}{8}(2\,cde+be^2)x^8 + \frac{1}{6}(cd^2+2\,bde+ae^2)x^6 + \frac{1}{2}ad^2x^2 + \frac{1}{4}(bd^2+2\,ade)x^4$$

```
[In] integrate(x*(e*x^2+d)^2*(c*x^4+b*x^2+a),x, algorithm="maxima")
[Out] 1/10*c*e^2*x^10 + 1/8*(2*c*d*e + b*e^2)*x^8 + 1/6*(c*d^2 + 2*b*d*e + a*e^2)
*x^6 + 1/2*a*d^2*x^2 + 1/4*(b*d^2 + 2*a*d*e)*x^4
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.05

$$\int x(d+ex^2)^2(a+bx^2+cx^4)\,dx = \frac{1}{10}ce^2x^{10} + \frac{1}{4}cdex^8 + \frac{1}{8}be^2x^8 + \frac{1}{6}cd^2x^6 + \frac{1}{3}bdex^6 + \frac{1}{6}ae^2x^6 + \frac{1}{4}bd^2x^4 + \frac{1}{2}adex^4 + \frac{1}{2}ad^2x^2$$

```
[In] integrate(x*(e*x^2+d)^2*(c*x^4+b*x^2+a),x, algorithm="giac")
[Out] 1/10*c*e^2*x^10 + 1/4*c*d*e*x^8 + 1/8*b*e^2*x^8 + 1/6*c*d^2*x^6 + 1/3*b*d*e
*x^6 + 1/6*a*e^2*x^6 + 1/4*b*d^2*x^4 + 1/2*a*d*e*x^4 + 1/2*a*d^2*x^2
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.97

$$\int x(d+ex^2)^2(a+bx^2+cx^4)\,dx = x^6\left(\frac{c\,d^2}{6}+\frac{b\,d\,e}{3}+\frac{a\,e^2}{6}\right) + x^4\left(\frac{b\,d^2}{4}+\frac{a\,e\,d}{2}\right) + x^8\left(\frac{b\,e^2}{8}+\frac{c\,d\,e}{4}\right) + \frac{a\,d^2\,x^2}{2} + \frac{c\,e^2\,x^{10}}{10}$$

```
[In] int(x*(d + e*x^2)^2*(a + b*x^2 + c*x^4),x)
[Out] x^6*((a*e^2)/6 + (c*d^2)/6 + (b*d*e)/3) + x^4*((b*d^2)/4 + (a*d*e)/2) + x^8
*((b*e^2)/8 + (c*d*e)/4) + (a*d^2*x^2)/2 + (c*e^2*x^10)/10
```

3.276 $\int (d+ex^2)^2 (a+bx^2+cx^4)\, dx$

Optimal result

Integrand size = 22, antiderivative size = 73

$$\int (d+ex^2)^2 (a+bx^2+cx^4)\, dx = ad^2x + \frac{1}{3}d(bd+2ae)x^3 + \frac{1}{5}\left(cd^2+e(2bd+ae)\right)x^5 + \frac{1}{7}e(2cd+be)x^7 + \frac{1}{9}ce^2x^9$$

```
[Out] a*d^2*x+1/3*d*(2*a*e+b*d)*x^3+1/5*(c*d^2+e*(a*e+2*b*d))*x^5+1/7*e*(b*e+2*c*
d)*x^7+1/9*c*e^2*x^9
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {1167}

$$\int (d+ex^2)^2 (a+bx^2+cx^4)\, dx = \frac{1}{5}x^5\left(e(ae+2bd)+cd^2\right) + \frac{1}{3}dx^3(2ae+bd) + ad^2x + \frac{1}{7}ex^7(be+2cd) + \frac{1}{9}ce^2x^9$$

```
[In] Int[(d + e*x^2)^2*(a + b*x^2 + c*x^4),x]
```

```
[Out] a*d^2*x + (d*(b*d + 2*a*e)*x^3)/3 + ((c*d^2 + e*(2*b*d + a*e))*x^5)/5 + (e*
(2*c*d + b*e)*x^7)/7 + (c*e^2*x^9)/9
```

Rule 1167

```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^q*(a + b*x^2 + c*x^4)^p, x],
x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e
```

```
+ a*e^2, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \left(ad^2+d(bd+2ae)x^2+\left(cd^2+e(2bd+ae)\right)x^4+e(2cd+be)x^6+ce^2x^8\right)\,dx\\ &= ad^2x+\frac{1}{3}d(bd+2ae)x^3+\frac{1}{5}\left(cd^2+e(2bd+ae)\right)x^5+\frac{1}{7}e(2cd+be)x^7+\frac{1}{9}ce^2x^9\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.00

$$\int \left(d+ex^2\right)^2\left(a+bx^2+cx^4\right)\,dx = ad^2x+\frac{1}{3}d(bd+2ae)x^3+\frac{1}{5}\left(cd^2+2bde+ae^2\right)x^5 +\frac{1}{7}e(2cd+be)x^7+\frac{1}{9}ce^2x^9$$

```
[In] Integrate[(d + e*x^2)^2*(a + b*x^2 + c*x^4),x]
[Out] a*d^2*x + (d*(b*d + 2*a*e)*x^3)/3 + ((c*d^2 + 2*b*d*e + a*e^2)*x^5)/5 + (e*
(2*c*d + b*e)*x^7)/7 + (c*e^2*x^9)/9
```

Maple [A] (verified)

Time = 0.29 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.96

method	result	size
default	$\frac{c\,e^2x^9}{9}+\frac{(b\,e^2+2dce)x^7}{7}+\frac{(a\,e^2+2bde+c\,d^2)x^5}{5}+\frac{(2eda+b\,d^2)x^3}{3}+a\,d^2x$	70
norman	$\frac{c\,e^2x^9}{9}+\left(\frac{1}{7}b\,e^2+\frac{2}{7}dce\right)x^7+\left(\frac{1}{5}a\,e^2+\frac{2}{5}bde+\frac{1}{5}c\,d^2\right)x^5+\left(\frac{2}{3}eda+\frac{1}{3}b\,d^2\right)x^3+a\,d^2x$	71
gosper	$\frac{1}{9}c\,e^2x^9+\frac{1}{7}x^7b\,e^2+\frac{2}{7}cde\,x^7+\frac{1}{5}x^5a\,e^2+\frac{2}{5}x^5bde+\frac{1}{5}x^5c\,d^2+\frac{2}{3}ade\,x^3+\frac{1}{3}x^3b\,d^2+a\,d^2x$	77
risch	$\frac{1}{9}c\,e^2x^9+\frac{1}{7}x^7b\,e^2+\frac{2}{7}cde\,x^7+\frac{1}{5}x^5a\,e^2+\frac{2}{5}x^5bde+\frac{1}{5}x^5c\,d^2+\frac{2}{3}ade\,x^3+\frac{1}{3}x^3b\,d^2+a\,d^2x$	77
parallelrisch	$\frac{1}{9}c\,e^2x^9+\frac{1}{7}x^7b\,e^2+\frac{2}{7}cde\,x^7+\frac{1}{5}x^5a\,e^2+\frac{2}{5}x^5bde+\frac{1}{5}x^5c\,d^2+\frac{2}{3}ade\,x^3+\frac{1}{3}x^3b\,d^2+a\,d^2x$	77

```
[In] int((e*x^2+d)^2*(c*x^4+b*x^2+a),x,method=_RETURNVERBOSE)
[Out] 1/9*c*e^2*x^9+1/7*(b*e^2+2*c*d*e)*x^7+1/5*(a*e^2+2*b*d*e+c*d^2)*x^5+1/3*(2*
a*d*e+b*d^2)*x^3+a*d^2*x
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.95

$$\int (d+ex^2)^2 (a+bx^2+cx^4) \, dx = \frac{1}{9}ce^2x^9 + \frac{1}{7}(2cde+be^2)x^7 + \frac{1}{5}(cd^2+2bde+ae^2)x^5 + ad^2x + \frac{1}{3}(bd^2+2ade)x^3$$

```
[In] integrate((e*x^2+d)^2*(c*x^4+b*x^2+a),x, algorithm="fricas")
[Out] 1/9*c*e^2*x^9 + 1/7*(2*c*d*e + b*e^2)*x^7 + 1/5*(c*d^2 + 2*b*d*e + a*e^2)*x
^5 + a*d^2*x + 1/3*(b*d^2 + 2*a*d*e)*x^3
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.07

$$\int (d+ex^2)^2 (a+bx^2+cx^4) \, dx = ad^2x + \frac{ce^2x^9}{9} + x^7\left(\frac{be^2}{7}+\frac{2cde}{7}\right) + x^5\left(\frac{ae^2}{5}+\frac{2bde}{5}+\frac{cd^2}{5}\right) + x^3 \cdot \left(\frac{2ade}{3}+\frac{bd^2}{3}\right)$$

```
[In] integrate((e*x**2+d)**2*(c*x**4+b*x**2+a),x)
[Out] a*d**2*x + c*e**2*x**9/9 + x**7*(b*e**2/7 + 2*c*d*e/7) + x**5*(a*e**2/5 + 2
*b*d*e/5 + c*d**2/5) + x**3*(2*a*d*e/3 + b*d**2/3)
```

Maxima [A] (verification not implemented)

none

Time = 0.18 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.95

$$\int (d+ex^2)^2 (a+bx^2+cx^4) \, dx = \frac{1}{9}ce^2x^9 + \frac{1}{7}(2cde+be^2)x^7 + \frac{1}{5}(cd^2+2bde+ae^2)x^5 + ad^2x + \frac{1}{3}(bd^2+2ade)x^3$$

```
[In] integrate((e*x^2+d)^2*(c*x^4+b*x^2+a),x, algorithm="maxima")
[Out] 1/9*c*e^2*x^9 + 1/7*(2*c*d*e + b*e^2)*x^7 + 1/5*(c*d^2 + 2*b*d*e + a*e^2)*x
^5 + a*d^2*x + 1/3*(b*d^2 + 2*a*d*e)*x^3
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.04

$$\int \left(d+ex^2\right)^2 \left(a+bx^2+cx^4\right) \, dx = \frac{1}{9} ce^2x^9 + \frac{2}{7} cdex^7 + \frac{1}{7} be^2x^7 + \frac{1}{5} cd^2x^5 + \frac{2}{5} bdex^5 + \frac{1}{5} ae^2x^5 + \frac{1}{3} bd^2x^3 + \frac{2}{3} adex^3 + ad^2x$$

```
[In] integrate((e*x^2+d)^2*(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] 1/9*c*e^2*x^9 + 2/7*c*d*e*x^7 + 1/7*b*e^2*x^7 + 1/5*c*d^2*x^5 + 2/5*b*d*e*x
^5 + 1/5*a*e^2*x^5 + 1/3*b*d^2*x^3 + 2/3*a*d*e*x^3 + a*d^2*x
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.96

$$\int \left(d+ex^2\right)^2 \left(a+bx^2+cx^4\right) \, dx = x^5 \left(\frac{c\,d^2}{5} + \frac{2\,b\,d\,e}{5} + \frac{a\,e^2}{5}\right) + x^3 \left(\frac{b\,d^2}{3} + \frac{2\,a\,e\,d}{3}\right) + x^7 \left(\frac{b\,e^2}{7} + \frac{2\,c\,d\,e}{7}\right) + \frac{c\,e^2\,x^9}{9} + a\,d^2\,x$$

```
[In] int((d + e*x^2)^2*(a + b*x^2 + c*x^4),x)
```

```
[Out] x^5*((a*e^2)/5 + (c*d^2)/5 + (2*b*d*e)/5) + x^3*((b*d^2)/3 + (2*a*d*e)/3) +
 x^7*((b*e^2)/7 + (2*c*d*e)/7) + (c*e^2*x^9)/9 + a*d^2*x
```

3.277 $\int \frac{(d+ex^2)^2(a+bx^2+cx^4)}{x}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 74

$$\int \frac{(d+ex^2)^2\,(a+bx^2+cx^4)}{x}\,dx = \frac{1}{2}d(bd+2ae)x^2+\frac{1}{4}\left(cd^2+e(2bd+ae)\right)x^4 + \frac{1}{6}e(2cd+be)x^6+\frac{1}{8}ce^2x^8+ad^2\log(x)$$

```
[Out] 1/2*d*(2*a*e+b*d)*x^2+1/4*(c*d^2+e*(a*e+2*b*d))*x^4+1/6*e*(b*e+2*c*d)*x^6+1
/8*c*e^2*x^8+a*d^2*ln(x)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.080, Rules used = {1265, 907}

$$\int \frac{(d+ex^2)^2\,(a+bx^2+cx^4)}{x}\,dx = \frac{1}{4}x^4\left(e(ae+2bd)+cd^2\right)+\frac{1}{2}dx^2(2ae+bd) + ad^2\log(x)+\frac{1}{6}ex^6(be+2cd)+\frac{1}{8}ce^2x^8$$

```
[In] Int[((d + e*x^2)^2*(a + b*x^2 + c*x^4))/x,x]
```

```
[Out] (d*(b*d + 2*a*e)*x^2)/2 + ((c*d^2 + e*(2*b*d + a*e))*x^4)/4 + (e*(2*c*d + b
*e)*x^6)/6 + (c*e^2*x^8)/8 + a*d^2*Log[x]
```

Rule 907

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g
```

```
*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ
[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && I
ntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ[m, 0] && ILtQ[n, 0])
)
```

Rule 1265

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{2}\text{Subst}\left(\int \frac{(d+ex)^2\,(a+bx+cx^2)}{x}\,dx, x, x^2\right) \\
&= \frac{1}{2}\text{Subst}\left(\int \left(d(bd+2ae)+\frac{ad^2}{x}+\left(cd^2+e(2bd+ae)\right)x+e(2cd+be)x^2+ce^2x^3\right)\,dx, x, x^2\right) \\
&= \frac{1}{2}d(bd+2ae)x^2+\frac{1}{4}\left(cd^2+e(2bd+ae)\right)x^4+\frac{1}{6}e(2cd+be)x^6+\frac{1}{8}ce^2x^8+ad^2\log(x)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.00

$$\int \frac{(d+ex^2)^2\,(a+bx^2+cx^4)}{x}\,dx = \frac{1}{2}d(bd+2ae)x^2+\frac{1}{4}\left(cd^2+2bde+ae^2\right)x^4 + \frac{1}{6}e(2cd+be)x^6+\frac{1}{8}ce^2x^8+ad^2\log(x)$$

```
[In] Integrate[((d + e*x^2)^2*(a + b*x^2 + c*x^4))/x,x]
[Out] (d*(b*d + 2*a*e)*x^2)/2 + ((c*d^2 + 2*b*d*e + a*e^2)*x^4)/4 + (e*(2*c*d + b
*e)*x^6)/6 + (c*e^2*x^8)/8 + a*d^2*Log[x]
```

Maple [A] (verified)

Time = 0.31 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.96

method	result	size
norman	$\left(\frac{1}{6}b\,e^{2}+\frac{1}{3}dce\right)x^{6}+\left(eda+\frac{1}{2}b\,d^{2}\right)x^{2}+\left(\frac{1}{4}a\,e^{2}+\frac{1}{2}bde+\frac{1}{4}c\,d^{2}\right)x^{4}+\frac{c\,e^{2}x^{8}}{8}+a\,d^{2}\ln\left(x\right)$	71
default	$\frac{c\,e^{2}x^{8}}{8}+\frac{b\,e^{2}x^{6}}{6}+\frac{cde\,x^{6}}{3}+\frac{a\,e^{2}x^{4}}{4}+\frac{eb\,x^{4}d}{2}+\frac{c\,d^{2}x^{4}}{4}+ade\,x^{2}+\frac{b\,d^{2}x^{2}}{2}+a\,d^{2}\ln\left(x\right)$	77
risch	$\frac{c\,e^{2}x^{8}}{8}+\frac{b\,e^{2}x^{6}}{6}+\frac{cde\,x^{6}}{3}+\frac{a\,e^{2}x^{4}}{4}+\frac{eb\,x^{4}d}{2}+\frac{c\,d^{2}x^{4}}{4}+ade\,x^{2}+\frac{b\,d^{2}x^{2}}{2}+a\,d^{2}\ln\left(x\right)$	77
parallelrisch	$\frac{c\,e^{2}x^{8}}{8}+\frac{b\,e^{2}x^{6}}{6}+\frac{cde\,x^{6}}{3}+\frac{a\,e^{2}x^{4}}{4}+\frac{eb\,x^{4}d}{2}+\frac{c\,d^{2}x^{4}}{4}+ade\,x^{2}+\frac{b\,d^{2}x^{2}}{2}+a\,d^{2}\ln\left(x\right)$	77

```
[In] int((e*x^2+d)^2*(c*x^4+b*x^2+a)/x,x,method=_RETURNVERBOSE)
[Out] (1/6*b*e^2+1/3*d*c*e)*x^6+(e*d*a+1/2*b*d^2)*x^2+(1/4*a*e^2+1/2*b*d*e+1/4*c*
d^2)*x^4+1/8*c*e^2*x^8+a*d^2*ln(x)
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.95

$$\int \frac{(d+ex^2)^2(a+bx^2+cx^4)}{x}\,dx = \frac{1}{8}\,ce^2x^8+\frac{1}{6}\,(2\,cde+be^2)x^6+\frac{1}{4}\,(cd^2+2\,bde+ae^2)x^4+ad^2\log\left(x\right)+\frac{1}{2}\,(bd^2+2\,ade)x^2$$

```
[In] integrate((e*x^2+d)^2*(c*x^4+b*x^2+a)/x,x, algorithm="fricas")
[Out] 1/8*c*e^2*x^8 + 1/6*(2*c*d*e + b*e^2)*x^6 + 1/4*(c*d^2 + 2*b*d*e + a*e^2)*x
^4 + a*d^2*log(x) + 1/2*(b*d^2 + 2*a*d*e)*x^2
```

Sympy [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.99

$$\int \frac{(d+ex^2)^2(a+bx^2+cx^4)}{x}\,dx = ad^2\log\left(x\right)+\frac{ce^2x^8}{8}+x^6\left(\frac{be^2}{6}+\frac{cde}{3}\right)+x^4\left(\frac{ae^2}{4}+\frac{bde}{2}+\frac{cd^2}{4}\right)+x^2\left(ade+\frac{bd^2}{2}\right)$$

```
[In] integrate((e*x**2+d)**2*(c*x**4+b*x**2+a)/x,x)
[Out] a*d**2*log(x) + c*e**2*x**8/8 + x**6*(b*e**2/6 + c*d*e/3) + x**4*(a*e**2/4
+ b*d*e/2 + c*d**2/4) + x**2*(a*d*e + b*d**2/2)
```

Maxima [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.99

$$\int \frac{(d+ex^2)^2(a+bx^2+cx^4)}{x}\,dx = \frac{1}{8}\,ce^2x^8 + \frac{1}{6}\,(2\,cde + be^2)x^6 + \frac{1}{4}\,(cd^2 + 2\,bde + ae^2)x^4 + \frac{1}{2}\,ad^2\log\left(x^2\right) + \frac{1}{2}\,(bd^2 + 2\,ade)x^2$$

```
[In] integrate((e*x^2+d)^2*(c*x^4+b*x^2+a)/x,x, algorithm="maxima")
```

```
[Out] 1/8*c*e^2*x^8 + 1/6*(2*c*d*e + b*e^2)*x^6 + 1/4*(c*d^2 + 2*b*d*e + a*e^2)*x
^4 + 1/2*a*d^2*log(x^2) + 1/2*(b*d^2 + 2*a*d*e)*x^2
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.07

$$\int \frac{(d+ex^2)^2(a+bx^2+cx^4)}{x}\,dx = \frac{1}{8}\,ce^2x^8 + \frac{1}{3}\,cdex^6 + \frac{1}{6}\,be^2x^6 + \frac{1}{4}\,cd^2x^4 + \frac{1}{2}\,bdex^4 + \frac{1}{4}\,ae^2x^4 + \frac{1}{2}\,bd^2x^2 + adex^2 + \frac{1}{2}\,ad^2\log\left(x^2\right)$$

```
[In] integrate((e*x^2+d)^2*(c*x^4+b*x^2+a)/x,x, algorithm="giac")
```

```
[Out] 1/8*c*e^2*x^8 + 1/3*c*d*e*x^6 + 1/6*b*e^2*x^6 + 1/4*c*d^2*x^4 + 1/2*b*d*e*x
^4 + 1/4*a*e^2*x^4 + 1/2*b*d^2*x^2 + a*d*e*x^2 + 1/2*a*d^2*log(x^2)
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.95

$$\int \frac{(d+ex^2)^2(a+bx^2+cx^4)}{x}\,dx = x^4\left(\frac{c\,d^2}{4} + \frac{b\,d\,e}{2} + \frac{a\,e^2}{4}\right) + x^2\left(\frac{b\,d^2}{2} + a\,e\,d\right) + x^6\left(\frac{b\,e^2}{6} + \frac{c\,d\,e}{3}\right) + \frac{c\,e^2\,x^8}{8} + a\,d^2\,\ln\left(x\right)$$

```
[In] int(((d + e*x^2)^2*(a + b*x^2 + c*x^4))/x,x)
```

```
[Out] x^4*((a*e^2)/4 + (c*d^2)/4 + (b*d*e)/2) + x^2*((b*d^2)/2 + a*d*e) + x^6*((b
*e^2)/6 + (c*d*e)/3) + (c*e^2*x^8)/8 + a*d^2*log(x)
```

3.278 $\int \frac{(d+ex^2)^2(a+bx^2+cx^4)}{x^2} dx$

Optimal result

Integrand size = 25, antiderivative size = 71

$$\int \frac{(d+ex^2)^2(a+bx^2+cx^4)}{x^2} dx = -\frac{ad^2}{x}+d(bd+2ae)x+\frac{1}{3}\left(cd^2+e(2bd+ae)\right)x^3+\frac{1}{5}e(2cd+be)x^5+\frac{1}{7}ce^2x^7$$

```
[Out] -a*d^2/x+d*(2*a*e+b*d)*x+1/3*(c*d^2+e*(a*e+2*b*d))*x^3+1/5*e*(b*e+2*c*d)*x^
5+1/7*c*e^2*x^7
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.040, Rules used = {1275}

$$\int \frac{(d+ex^2)^2(a+bx^2+cx^4)}{x^2} dx = \frac{1}{3}x^3\left(e(ae+2bd)+cd^2\right)+dx(2ae+bd)-\frac{ad^2}{x}+\frac{1}{5}ex^5(be+2cd)+\frac{1}{7}ce^2x^7$$

```
[In] Int[((d + e*x^2)^2*(a + b*x^2 + c*x^4))/x^2,x]
```

```
[Out] -((a*d^2)/x) + d*(b*d + 2*a*e)*x + ((c*d^2 + e*(2*b*d + a*e))*x^3)/3 + (e*(
2*c*d + b*e)*x^5)/5 + (c*e^2*x^7)/7
```

Rule 1275

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*
(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && NeQ[
```

```
b^2 - 4*a*c, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \left(d(bd+2ae)+\frac{ad^2}{x^2}+\left(cd^2+e(2bd+ae)\right)x^2+e(2cd+be)x^4+ce^2x^6\right)dx\\ &= -\frac{ad^2}{x}+d(bd+2ae)x+\frac{1}{3}\left(cd^2+e(2bd+ae)\right)x^3+\frac{1}{5}e(2cd+be)x^5+\frac{1}{7}ce^2x^7\end{aligned}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.00

$$\int \frac{(d+ex^2)^2(a+bx^2+cx^4)}{x^2}\,dx = -\frac{ad^2}{x}+d(bd+2ae)x+\frac{1}{3}\left(cd^2+2bde+ae^2\right)x^3+\frac{1}{5}e(2cd+be)x^5+\frac{1}{7}ce^2x^7$$

```
[In] Integrate[((d + e*x^2)^2*(a + b*x^2 + c*x^4))/x^2,x]
```

```
[Out] -((a*d^2)/x) + d*(b*d + 2*a*e)*x + ((c*d^2 + 2*b*d*e + a*e^2)*x^3)/3 + (e*(
2*c*d + b*e)*x^5)/5 + (c*e^2*x^7)/7
```

Maple [A] (verified)

Time = 0.30 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.04

method	result	size
norman	$\frac{\frac{c e^2 x^8}{7}+\left(\frac{1}{5} b e^2+\frac{2}{5} dce\right)x^6+\left(\frac{1}{3} a e^2+\frac{2}{3} bde+\frac{1}{3} c d^2\right)x^4+\left(2eda+b d^2\right)x^2-a d^2}{x}$	74
default	$\frac{c e^2 x^7}{7}+\frac{b e^2 x^5}{5}+\frac{2cde\, x^5}{5}+\frac{a e^2 x^3}{3}+\frac{2eb\, x^3 d}{3}+\frac{c d^2 x^3}{3}+2edax+b d^2 x-\frac{a d^2}{x}$	75
risch	$\frac{c e^2 x^7}{7}+\frac{b e^2 x^5}{5}+\frac{2cde\, x^5}{5}+\frac{a e^2 x^3}{3}+\frac{2eb\, x^3 d}{3}+\frac{c d^2 x^3}{3}+2edax+b d^2 x-\frac{a d^2}{x}$	75
gosper	$-\frac{-15c e^2 x^8-21b e^2 x^6-42cde\, x^6-35a e^2 x^4-70eb\, x^4 d-35c d^2 x^4-210ade\, x^2-105b d^2 x^2+105a d^2}{105x}$	82
parallelrisch	$\frac{15c e^2 x^8+21b e^2 x^6+42cde\, x^6+35a e^2 x^4+70eb\, x^4 d+35c d^2 x^4+210ade\, x^2+105b d^2 x^2-105a d^2}{105x}$	82

```
[In] int((e*x^2+d)^2*(c*x^4+b*x^2+a)/x^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/x*(1/7*c*e^2*x^8+(1/5*b*e^2+2/5*d*c*e)*x^6+(1/3*a*e^2+2/3*b*d*e+1/3*c*d^2
)*x^4+(2*a*d*e+b*d^2)*x^2-a*d^2)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.04

$$\int \frac{(d+ex^2)^2(a+bx^2+cx^4)}{x^2}\,dx = \frac{15\,ce^2x^8+21\,(2\,cde+be^2)x^6+35\,(cd^2+2\,bde+ae^2)x^4-105\,ad^2+105\,(bd^2+2\,ade)x^2}{105\,x}$$

```
[In] integrate((e*x^2+d)^2*(c*x^4+b*x^2+a)/x^2,x, algorithm="fricas")
```

```
[Out] 1/105*(15*c*e^2*x^8 + 21*(2*c*d*e + b*e^2)*x^6 + 35*(c*d^2 + 2*b*d*e + a*e^
2)*x^4 - 105*a*d^2 + 105*(b*d^2 + 2*a*d*e)*x^2)/x
```

Sympy [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.03

$$\int \frac{(d+ex^2)^2(a+bx^2+cx^4)}{x^2}\,dx = -\frac{ad^2}{x}+\frac{ce^2x^7}{7}+x^5\left(\frac{be^2}{5}+\frac{2cde}{5}\right)+x^3\left(\frac{ae^2}{3}+\frac{2bde}{3}+\frac{cd^2}{3}\right)+x\left(2ade+bd^2\right)$$

```
[In] integrate((e*x**2+d)**2*(c*x**4+b*x**2+a)/x**2,x)
```

```
[Out] -a*d**2/x + c*e**2*x**7/7 + x**5*(b*e**2/5 + 2*c*d*e/5) + x**3*(a*e**2/3 +
2*b*d*e/3 + c*d**2/3) + x*(2*a*d*e + b*d**2)
```

Maxima [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.97

$$\int \frac{(d+ex^2)^2(a+bx^2+cx^4)}{x^2}\,dx = \frac{1}{7}\,ce^2x^7+\frac{1}{5}\,(2\,cde+be^2)x^5+\frac{1}{3}\,(cd^2+2\,bde+ae^2)x^3-\frac{ad^2}{x}+\left(bd^2+2\,ade\right)x$$

```
[In] integrate((e*x^2+d)^2*(c*x^4+b*x^2+a)/x^2,x, algorithm="maxima")
```

```
[Out] 1/7*c*e^2*x^7 + 1/5*(2*c*d*e + b*e^2)*x^5 + 1/3*(c*d^2 + 2*b*d*e + a*e^2)*x
^3 - a*d^2/x + (b*d^2 + 2*a*d*e)*x
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.04

$$\int \frac{(d+ex^2)^2 (a+bx^2+cx^4)}{x^2} \, dx = \frac{1}{7} ce^2x^7 + \frac{2}{5} cdex^5 + \frac{1}{5} be^2x^5 + \frac{1}{3} cd^2x^3 + \frac{2}{3} bdex^3 + \frac{1}{3} ae^2x^3 + bd^2x + 2 adex - \frac{ad^2}{x}$$

```
[In] integrate((e*x^2+d)^2*(c*x^4+b*x^2+a)/x^2,x, algorithm="giac")
```

```
[Out] 1/7*c*e^2*x^7 + 2/5*c*d*e*x^5 + 1/5*b*e^2*x^5 + 1/3*c*d^2*x^3 + 2/3*b*d*e*x
^3 + 1/3*a*e^2*x^3 + b*d^2*x + 2*a*d*e*x - a*d^2/x
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.99

$$\int \frac{(d+ex^2)^2 (a+bx^2+cx^4)}{x^2} \, dx = x^3 \left(\frac{c\,d^2}{3} + \frac{2\,b\,d\,e}{3} + \frac{a\,e^2}{3}\right) + x\,\left(b\,d^2 + 2\,a\,e\,d\right) + x^5\,\left(\frac{b\,e^2}{5} + \frac{2\,c\,d\,e}{5}\right) - \frac{a\,d^2}{x} + \frac{c\,e^2\,x^7}{7}$$

```
[In] int(((d + e*x^2)^2*(a + b*x^2 + c*x^4))/x^2,x)
```

```
[Out] x^3*((a*e^2)/3 + (c*d^2)/3 + (2*b*d*e)/3) + x*(b*d^2 + 2*a*d*e) + x^5*((b*e
^2)/5 + (2*c*d*e)/5) - (a*d^2)/x + (c*e^2*x^7)/7
```

3.279 $\int \frac{(d+ex^2)^2(a+bx^2+cx^4)}{x^3}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 74

$$\int \frac{(d+ex^2)^2(a+bx^2+cx^4)}{x^3}\,dx = -\frac{ad^2}{2x^2}+\frac{1}{2}\left(cd^2+e(2bd+ae)\right)x^2 + \frac{1}{4}e(2cd+be)x^4+\frac{1}{6}ce^2x^6+d(bd+2ae)\log(x)$$

```
[Out] -1/2*a*d^2/x^2+1/2*(c*d^2+e*(a*e+2*b*d))*x^2+1/4*e*(b*e+2*c*d)*x^4+1/6*c*e^
2*x^6+d*(2*a*e+b*d)*ln(x)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.080, Rules used = {1265, 907}

$$\int \frac{(d+ex^2)^2(a+bx^2+cx^4)}{x^3}\,dx = \frac{1}{2}x^2\left(e(ae+2bd)+cd^2\right)+d\log(x)(2ae+bd) - \frac{ad^2}{2x^2}+\frac{1}{4}ex^4(be+2cd)+\frac{1}{6}ce^2x^6$$

```
[In] Int[((d + e*x^2)^2*(a + b*x^2 + c*x^4))/x^3,x]
```

```
[Out] -1/2*(a*d^2)/x^2 + ((c*d^2 + e*(2*b*d + a*e))*x^2)/2 + (e*(2*c*d + b*e)*x^4
)/4 + (c*e^2*x^6)/6 + d*(b*d + 2*a*e)*Log[x]
```

Rule 907

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g
```

```
*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ
[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && I
ntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ[m, 0] && ILtQ[n, 0])
)
```

Rule 1265

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{1}{2}\text{Subst}\left(\int \frac{(d+ex)^2\,(a+bx+cx^2)}{x^2}\,dx, x, x^2\right)\\ &= \frac{1}{2}\text{Subst}\left(\int \left(cd^2\left(1+\frac{e(2bd+ae)}{cd^2}\right)+\frac{ad^2}{x^2}+\frac{d(bd+2ae)}{x}+e(2cd+be)x+ce^2x^2\right)\,dx, x, x^2\right)\\ &= -\frac{ad^2}{2x^2}+\frac{1}{2}\left(cd^2+e(2bd+ae)\right)x^2+\frac{1}{4}e(2cd+be)x^4+\frac{1}{6}ce^2x^6+d(bd+2ae)\log(x)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.96

$$\int \frac{(d+ex^2)^2\,(a+bx^2+cx^4)}{x^3}\,dx = \frac{1}{12}\left(-\frac{6ad^2}{x^2}+6\left(cd^2+e(2bd+ae)\right)x^2+3e(2cd+be)x^4 + 2ce^2x^6+12d(bd+2ae)\log(x)\right)$$

```
[In] Integrate[((d + e*x^2)^2*(a + b*x^2 + c*x^4))/x^3,x]
[Out] ((-6*a*d^2)/x^2 + 6*(c*d^2 + e*(2*b*d + a*e))*x^2 + 3*e*(2*c*d + b*e)*x^4 +
 2*c*e^2*x^6 + 12*d*(b*d + 2*a*e)*Log[x])/12
```

Maple [A] (verified)

Time = 0.28 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.99

method	result	size
norman	$\frac{\left(\frac{1}{4} b\,e^{2}+\frac{1}{2} d c e\right) x^{6}+\left(\frac{1}{2} a\,e^{2}+b d e+\frac{1}{2} c\,d^{2}\right) x^{4}-\frac{a\,d^{2}}{2}+\frac{c\,e^{2} x^{8}}{6}}{x^{2}}+\left(2 e d a+b\,d^{2}\right) \ln \left(x \right)$	73
default	$\frac{c\,e^{2} x^{6}}{6}+\frac{b\,e^{2} x^{4}}{4}+\frac{d c\,x^{4} e}{2}+\frac{a\,e^{2} x^{2}}{2}+b d e\,x^{2}+\frac{c\,d^{2} x^{2}}{2}+d \left(2 a e+b d \right) \ln \left(x \right)-\frac{a\,d^{2}}{2 x^{2}}$	74
risch	$\frac{c\,e^{2} x^{6}}{6}+\frac{b\,e^{2} x^{4}}{4}+\frac{d c\,x^{4} e}{2}+\frac{a\,e^{2} x^{2}}{2}+b d e\,x^{2}+\frac{c\,d^{2} x^{2}}{2}-\frac{a\,d^{2}}{2 x^{2}}+2 \ln \left(x \right) a d e+\ln \left(x \right) b\,d^{2}$	76
parallelrisch	$\frac{2 c\,e^{2} x^{8}+3 b\,e^{2} x^{6}+6 c d e\,x^{6}+6 a\,e^{2} x^{4}+12 e b\,x^{4} d+6 c\,d^{2} x^{4}+24 \ln \left(x \right) x^{2} a d e+12 \ln \left(x \right) x^{2} b\,d^{2}-6 a\,d^{2}}{12 x^{2}}$	86

```
[In] int((e*x^2+d)^2*(c*x^4+b*x^2+a)/x^3,x,method=_RETURNVERBOSE)
```

```
[Out] ((1/4*b*e^2+1/2*d*c*e)*x^6+(1/2*a*e^2+b*d*e+1/2*c*d^2)*x^4-1/2*a*d^2+1/6*c*
e^2*x^8)/x^2+(2*a*d*e+b*d^2)*ln(x)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.03

$$\int \frac{(d+ex^2)^2 (a+bx^2+cx^4)}{x^3}\,dx = \frac{2\,ce^2x^8+3\,(2\,cde+be^2)x^6+6\,(cd^2+2\,bde+ae^2)x^4+12\,(bd^2+2\,ade)x^2\log(x)-6\,ad^2}{12\,x^2}$$

```
[In] integrate((e*x^2+d)^2*(c*x^4+b*x^2+a)/x^3,x, algorithm="fricas")
```

```
[Out] 1/12*(2*c*e^2*x^8 + 3*(2*c*d*e + b*e^2)*x^6 + 6*(c*d^2 + 2*b*d*e + a*e^2)*x
^4 + 12*(b*d^2 + 2*a*d*e)*x^2*log(x) - 6*a*d^2)/x^2
```

Sympy [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.96

$$\int \frac{(d+ex^2)^2 (a+bx^2+cx^4)}{x^3}\,dx = -\frac{ad^2}{2x^2}+\frac{ce^2x^6}{6}+d(2ae+bd)\log(x)+x^4\left(\frac{be^2}{4}+\frac{cde}{2}\right)+x^2\left(\frac{ae^2}{2}+bde+\frac{cd^2}{2}\right)$$

```
[In] integrate((e*x**2+d)**2*(c*x**4+b*x**2+a)/x**3,x)
```

```
[Out] -a*d**2/(2*x**2) + c*e**2*x**6/6 + d*(2*a*e + b*d)*log(x) + x**4*(b*e**2/4
+ c*d*e/2) + x**2*(a*e**2/2 + b*d*e + c*d**2/2)
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.99

$$\int \frac{(d+ex^2)^2 (a+bx^2+cx^4)}{x^3} dx = \frac{1}{6} ce^2x^6 + \frac{1}{4} (2 cde + be^2)x^4 + \frac{1}{2} (cd^2 + 2 bde + ae^2)x^2 + \frac{1}{2} (bd^2 + 2 ade) \log (x^2) - \frac{ad^2}{2 x^2}$$

```
[In] integrate((e*x^2+d)^2*(c*x^4+b*x^2+a)/x^3,x, algorithm="maxima")
```

```
[Out] 1/6*c*e^2*x^6 + 1/4*(2*c*d*e + b*e^2)*x^4 + 1/2*(c*d^2 + 2*b*d*e + a*e^2)*x
^2 + 1/2*(b*d^2 + 2*a*d*e)*log(x^2) - 1/2*a*d^2/x^2
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.30

$$\int \frac{(d+ex^2)^2 (a+bx^2+cx^4)}{x^3} dx = \frac{1}{6} ce^2x^6 + \frac{1}{2} cdex^4 + \frac{1}{4} be^2x^4 + \frac{1}{2} cd^2x^2 + bdex^2 + \frac{1}{2} ae^2x^2 + \frac{1}{2} (bd^2 + 2 ade) \log (x^2) - \frac{bd^2x^2 + 2 adex^2 + ad^2}{2 x^2}$$

```
[In] integrate((e*x^2+d)^2*(c*x^4+b*x^2+a)/x^3,x, algorithm="giac")
```

```
[Out] 1/6*c*e^2*x^6 + 1/2*c*d*e*x^4 + 1/4*b*e^2*x^4 + 1/2*c*d^2*x^2 + b*d*e*x^2 +
 1/2*a*e^2*x^2 + 1/2*(b*d^2 + 2*a*d*e)*log(x^2) - 1/2*(b*d^2*x^2 + 2*a*d*e*
x^2 + a*d^2)/x^2
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.95

$$\int \frac{(d+ex^2)^2 (a+bx^2+cx^4)}{x^3} dx = x^2 \left(\frac{c\,d^2}{2} + b\,d\,e + \frac{a\,e^2}{2}\right) + x^4 \left(\frac{b\,e^2}{4} + \frac{c\,d\,e}{2}\right) + \ln (x)\ \left(b\,d^2 + 2\,a\,e\,d\right) - \frac{a\,d^2}{2\,x^2} + \frac{c\,e^2\,x^6}{6}$$

```
[In] int(((d + e*x^2)^2*(a + b*x^2 + c*x^4))/x^3,x)
```

```
[Out] x^2*((a*e^2)/2 + (c*d^2)/2 + b*d*e) + x^4*((b*e^2)/4 + (c*d*e)/2) + log(x)*
(b*d^2 + 2*a*d*e) - (a*d^2)/(2*x^2) + (c*e^2*x^6)/6
```

3.280 $\int \frac{x^6(a+bx^2+cx^4)}{(d+ex^2)^2}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 168

$$\int \frac{x^6(a+bx^2+cx^4)}{(d+ex^2)^2}\,dx = -\frac{d(4cd^2-e(3bd-2ae))\,x}{e^5}+\frac{(3cd^2-e(2bd-ae))\,x^3}{3e^4} - \frac{(2cd-be)x^5}{5e^3}+\frac{cx^7}{7e^2}-\frac{d^2(cd^2-bde+ae^2)\,x}{2e^5\,(d+ex^2)} + \frac{d^{3/2}(9cd^2-e(7bd-5ae))\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{2e^{11/2}}$$

```
[Out] -d*(4*c*d^2-e*(-2*a*e+3*b*d))*x/e^5+1/3*(3*c*d^2-e*(-a*e+2*b*d))*x^3/e^4-1/
5*(-b*e+2*c*d)*x^5/e^3+1/7*c*x^7/e^2-1/2*d^2*(a*e^2-b*d*e+c*d^2)*x/e^5/(e*x
^2+d)+1/2*d^(3/2)*(9*c*d^2-e*(-5*a*e+7*b*d))*arctan(x*e^(1/2)/d^(1/2))/e^(1
1/2)
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 168, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {1271, 1824, 211}

$$\int \frac{x^6(a+bx^2+cx^4)}{(d+ex^2)^2}\,dx = \frac{d^{3/2}\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)(9cd^2-e(7bd-5ae))}{2e^{11/2}} - \frac{dx(4cd^2-e(3bd-2ae))}{e^5}+\frac{x^3(3cd^2-e(2bd-ae))}{3e^4} - \frac{d^2x(ae^2-bde+cd^2)}{2e^5\,(d+ex^2)}-\frac{x^5(2cd-be)}{5e^3}+\frac{cx^7}{7e^2}$$

```
[In] Int[(x^6*(a + b*x^2 + c*x^4))/(d + e*x^2)^2,x]
```

```
[Out] -((d*(4*c*d^2 - e*(3*b*d - 2*a*e))*x)/e^5) + ((3*c*d^2 - e*(2*b*d - a*e))*x
^3)/(3*e^4) - ((2*c*d - b*e)*x^5)/(5*e^3) + (c*x^7)/(7*e^2) - (d^2*(c*d^2 -
 b*d*e + a*e^2)*x)/(2*e^5*(d + e*x^2)) + (d^(3/2)*(9*c*d^2 - e*(7*b*d - 5*a
*e))*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(2*e^(11/2))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1271

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)
^4)^(p_.), x_Symbol] :> Simp[(-d)^(m/2 - 1)*(c*d^2 - b*d*e + a*e^2)^p*x*((d
 + e*x^2)^(q + 1)/(2*e^(2*p + m/2)*(q + 1))), x] + Dist[1/(2*e^(2*p + m/2)*
(q + 1)), Int[(d + e*x^2)^(q + 1)*ExpandToSum[Together[(1/(d + e*x^2))*(2*e
^(2*p + m/2)*(q + 1)*x^m*(a + b*x^2 + c*x^4)^p - (-d)^(m/2 - 1)*(c*d^2 - b*
d*e + a*e^2)^p*(d + e*(2*q + 3)*x^2))], x], x], x] /; FreeQ[{a, b, c, d, e}
, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p, 0] && ILtQ[q, -1] && IGtQ[m/2, 0]
```

Rule 1824

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[Pq*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\text{integral} = -\frac{d^2(cd^2-bde+ae^2)\,x}{2e^5\,(d+ex^2)} - \frac{\int \frac{-d^2(cd^2-bde+ae^2)+2de(cd^2-bde+ae^2)x^2-2e^2(cd^2-bde+ae^2)x^4+2e^3(cd-be)x^6-2ce^4x^8}{d+ex^2}\,dx}{2e^5}$$

$$= -\frac{d^2(cd^2-bde+ae^2)\,x}{2e^5\,(d+ex^2)} - \frac{\int \left(2d(4cd^2-e(3bd-2ae))-2e(3cd^2-e(2bd-ae))\,x^2+2e^2(2cd-be)x^4-2ce^3x^6+\frac{-9cd^4+7bd}{d+e}\right.}{2e^5}$$

$$= -\frac{d(4cd^2-e(3bd-2ae))\,x}{e^5} + \frac{(3cd^2-e(2bd-ae))\,x^3}{3e^4} - \frac{(2cd-be)x^5}{5e^3} + \frac{cx^7}{7e^2} - \frac{d^2(cd^2-bde+ae^2)\,x}{2e^5\,(d+ex^2)} + \frac{(d^2(9cd^2-e(7bd-5ae)))\int \frac{1}{d+ex^2}\,dx}{2e^5}$$

$$= -\frac{d(4cd^2-e(3bd-2ae))\,x}{e^5} + \frac{(3cd^2-e(2bd-ae))\,x^3}{3e^4} - \frac{(2cd-be)x^5}{5e^3} + \frac{cx^7}{7e^2} - \frac{d^2(cd^2-bde+ae^2)\,x}{2e^5\,(d+ex^2)} + \frac{d^{3/2}(9cd^2-e(7bd-5ae))\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{2e^{11/2}}$$

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 165, normalized size of antiderivative = 0.98

$$\int \frac{x^6(a+bx^2+cx^4)}{(d+ex^2)^2}\,dx = -\frac{d(4cd^2-3bde+2ae^2)\,x}{e^5}+\frac{(3cd^2-2bde+ae^2)\,x^3}{3e^4} + \frac{(-2cd+be)x^5}{5e^3}+\frac{cx^7}{7e^2}-\frac{(cd^4-bd^3e+ad^2e^2)\,x}{2e^5\,(d+ex^2)} + \frac{d^{3/2}(9cd^2-7bde+5ae^2)\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{2e^{11/2}}$$

```
[In] Integrate[(x^6*(a + b*x^2 + c*x^4))/(d + e*x^2)^2,x]
```

```
[Out] -((d*(4*c*d^2 - 3*b*d*e + 2*a*e^2)*x)/e^5) + ((3*c*d^2 - 2*b*d*e + a*e^2)*x
^3)/(3*e^4) + ((-2*c*d + b*e)*x^5)/(5*e^3) + (c*x^7)/(7*e^2) - ((c*d^4 - b*
d^3*e + a*d^2*e^2)*x)/(2*e^5*(d + e*x^2)) + (d^(3/2)*(9*c*d^2 - 7*b*d*e + 5
*a*e^2)*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(2*e^(11/2))
```

Maple [A] (verified)

Time = 0.35 (sec) , antiderivative size = 159, normalized size of antiderivative = 0.95

method	result
default	$-\frac{-\frac{1}{7}c\,x^7e^3-\frac{1}{5}b\,e^3x^5+\frac{2}{5}cd\,x^5e^2-\frac{1}{3}a\,e^3x^3+\frac{2}{3}bd\,e^2x^3-c\,d^2e\,x^3+2d\,e^2ax-3b\,d^2ex+4d^3cx}{e^5}+\frac{d^2\left(\frac{\left(-\frac{1}{2}a\,e^2+\frac{1}{2}bde-\frac{1}{2}c\,d^2\right)x}{e\,x^2+d}+\frac{(5a\,e^2-}{}\right.}{e^5}$
risch	$\frac{c\,x^7}{7e^2}+\frac{b\,x^5}{5e^2}-\frac{2cd\,x^5}{5e^3}+\frac{a\,x^3}{3e^2}-\frac{2bd\,x^3}{3e^3}+\frac{c\,d^2x^3}{e^4}-\frac{2dax}{e^3}+\frac{3b\,d^2x}{e^4}-\frac{4d^3cx}{e^5}+\frac{\left(-\frac{1}{2}e^2d^2a+\frac{1}{2}d^3eb-\frac{1}{2}d^4c\right)x}{e^5(e\,x^2+d)}+\frac{5\sqrt{-ed}\,d}{}$

```
[In] int(x^6*(c*x^4+b*x^2+a)/(e*x^2+d)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/e^5*(-1/7*c*x^7*e^3-1/5*b*e^3*x^5+2/5*c*d*x^5*e^2-1/3*a*e^3*x^3+2/3*b*d*
e^2*x^3-c*d^2*e*x^3+2*d*e^2*a*x-3*b*d^2*e*x+4*d^3*c*x)+d^2/e^5*((-1/2*a*e^2
+1/2*b*d*e-1/2*c*d^2)*x/(e*x^2+d)+1/2*(5*a*e^2-7*b*d*e+9*c*d^2)/(e*d)^(1/2)
*arctan(e*x/(e*d)^(1/2)))
```

Fricas [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 426, normalized size of antiderivative = 2.54

$$\int \frac{x^6(a+bx^2+cx^4)}{(d+ex^2)^2}\,dx$$
$$= \left[\frac{60\,ce^4x^9 - 12\,(9\,cde^3 - 7\,be^4)x^7 + 28\,(9\,cd^2e^2 - 7\,bde^3 + 5\,ae^4)x^5 - 140\,(9\,cd^3e - 7\,bd^2e^2 + 5\,ade^3)x^3 + 10}{4}\right.$$

```
[In] integrate(x^6*(c*x^4+b*x^2+a)/(e*x^2+d)^2,x, algorithm="fricas")
```

```
[Out] [1/420*(60*c*e^4*x^9 - 12*(9*c*d*e^3 - 7*b*e^4)*x^7 + 28*(9*c*d^2*e^2 - 7*b
*d*e^3 + 5*a*e^4)*x^5 - 140*(9*c*d^3*e - 7*b*d^2*e^2 + 5*a*d*e^3)*x^3 + 105
*(9*c*d^4 - 7*b*d^3*e + 5*a*d^2*e^2 + (9*c*d^3*e - 7*b*d^2*e^2 + 5*a*d*e^3)
*x^2)*sqrt(-d/e)*log((e*x^2 + 2*e*x*sqrt(-d/e) - d)/(e*x^2 + d)) - 210*(9*c
*d^4 - 7*b*d^3*e + 5*a*d^2*e^2)*x)/(e^6*x^2 + d*e^5), 1/210*(30*c*e^4*x^9 -
 6*(9*c*d*e^3 - 7*b*e^4)*x^7 + 14*(9*c*d^2*e^2 - 7*b*d*e^3 + 5*a*e^4)*x^5 -
 70*(9*c*d^3*e - 7*b*d^2*e^2 + 5*a*d*e^3)*x^3 + 105*(9*c*d^4 - 7*b*d^3*e +
5*a*d^2*e^2 + (9*c*d^3*e - 7*b*d^2*e^2 + 5*a*d*e^3)*x^2)*sqrt(d/e)*arctan(e
*x*sqrt(d/e)/d) - 105*(9*c*d^4 - 7*b*d^3*e + 5*a*d^2*e^2)*x)/(e^6*x^2 + d*e
^5)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 320 vs. 2(153) = 306.

Time = 0.65 (sec) , antiderivative size = 320, normalized size of antiderivative = 1.90

$$\int \frac{x^6(a+bx^2+cx^4)}{(d+ex^2)^2}\,dx$$
$$= \frac{cx^7}{7e^2} + x^5\left(\frac{b}{5e^2} - \frac{2cd}{5e^3}\right) + x^3\left(\frac{a}{3e^2} - \frac{2bd}{3e^3} + \frac{cd^2}{e^4}\right)$$
$$+ x\left(-\frac{2ad}{e^3} + \frac{3bd^2}{e^4} - \frac{4cd^3}{e^5}\right) + \frac{x(-ad^2e^2 + bd^3e - cd^4)}{2de^5 + 2e^6x^2}$$
$$- \frac{\sqrt{-\frac{d^3}{e^{11}}}\cdot(5ae^2 - 7bde + 9cd^2)\log\left(-\frac{e^5\sqrt{-\frac{d^3}{e^{11}}}\cdot(5ae^2-7bde+9cd^2)}{5ade^2-7bd^2e+9cd^3} + x\right)}{4}$$
$$+ \frac{\sqrt{-\frac{d^3}{e^{11}}}\cdot(5ae^2 - 7bde + 9cd^2)\log\left(\frac{e^5\sqrt{-\frac{d^3}{e^{11}}}\cdot(5ae^2-7bde+9cd^2)}{5ade^2-7bd^2e+9cd^3} + x\right)}{4}$$

```
[In] integrate(x**6*(c*x**4+b*x**2+a)/(e*x**2+d)**2,x)
```

```
[Out] c*x**7/(7*e**2) + x**5*(b/(5*e**2) - 2*c*d/(5*e**3)) + x**3*(a/(3*e**2) - 2
*b*d/(3*e**3) + c*d**2/e**4) + x*(-2*a*d/e**3 + 3*b*d**2/e**4 - 4*c*d**3/e*
*5) + x*(-a*d**2*e**2 + b*d**3*e - c*d**4)/(2*d*e**5 + 2*e**6*x**2) - sqrt(
-d**3/e**11)*(5*a*e**2 - 7*b*d*e + 9*c*d**2)*log(-e**5*sqrt(-d**3/e**11)*(5
*a*e**2 - 7*b*d*e + 9*c*d**2)/(5*a*d*e**2 - 7*b*d**2*e + 9*c*d**3) + x)/4 +
 sqrt(-d**3/e**11)*(5*a*e**2 - 7*b*d*e + 9*c*d**2)*log(e**5*sqrt(-d**3/e**1
1)*(5*a*e**2 - 7*b*d*e + 9*c*d**2)/(5*a*d*e**2 - 7*b*d**2*e + 9*c*d**3) + x
)/4
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^6(a+bx^2+cx^4)}{(d+ex^2)^2}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(x^6*(c*x^4+b*x^2+a)/(e*x^2+d)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [A] (verification not implemented)

none

Time = 0.42 (sec) , antiderivative size = 174, normalized size of antiderivative = 1.04

$$\int \frac{x^6(a+bx^2+cx^4)}{(d+ex^2)^2}\,dx = \frac{(9\,cd^4-7\,bd^3e+5\,ad^2e^2)\arctan\left(\frac{ex}{\sqrt{de}}\right)}{2\,\sqrt{de}e^5}-\frac{cd^4x-bd^3ex+ad^2e^2x}{2\,(ex^2+d)e^5}+\frac{15\,ce^{12}x^7-42\,cde^{11}x^5+21\,be^{12}x^5+105\,cd^2e^{10}x^3-70\,bde^{11}x^3+35\,ae^{12}x^3-420\,cd^3e^9x+315\,bd^2e^{10}x}{105\,e^{14}}$$

```
[In] integrate(x^6*(c*x^4+b*x^2+a)/(e*x^2+d)^2,x, algorithm="giac")
```

```
[Out] 1/2*(9*c*d^4 - 7*b*d^3*e + 5*a*d^2*e^2)*arctan(e*x/sqrt(d*e))/(sqrt(d*e)*e^
5) - 1/2*(c*d^4*x - b*d^3*e*x + a*d^2*e^2*x)/((e*x^2 + d)*e^5) + 1/105*(15*
c*e^12*x^7 - 42*c*d*e^11*x^5 + 21*b*e^12*x^5 + 105*c*d^2*e^10*x^3 - 70*b*d*
e^11*x^3 + 35*a*e^12*x^3 - 420*c*d^3*e^9*x + 315*b*d^2*e^10*x - 210*a*d*e^1
1*x)/e^14
```

Mupad [B] (verification not implemented)

Time = 7.70 (sec) , antiderivative size = 251, normalized size of antiderivative = 1.49

$$\int \frac{x^6(a+bx^2+cx^4)}{(d+ex^2)^2}\,dx = x^5\left(\frac{b}{5\,e^2}-\frac{2\,c\,d}{5\,e^3}\right)-x^3\left(\frac{c\,d^2}{3\,e^4}-\frac{a}{3\,e^2}+\frac{2\,d\,\left(\frac{b}{e^2}-\frac{2\,c\,d}{e^3}\right)}{3\,e}\right)+x\left(\frac{2\,d\,\left(\frac{c\,d^2}{e^4}-\frac{a}{e^2}+\frac{2\,d\,\left(\frac{b}{e^2}-\frac{2\,c\,d}{e^3}\right)}{e}\right)}{e}-\frac{d^2\,\left(\frac{b}{e^2}-\frac{2\,c\,d}{e^3}\right)}{e^2}\right)-\frac{x\,\left(\frac{c\,d^4}{2}-\frac{b\,d^3\,e}{2}+\frac{a\,d^2\,e^2}{2}\right)}{e^6\,x^2+d\,e^5}+\frac{c\,x^7}{7\,e^2}+\frac{d^{3/2}\,\mathrm{atan}\left(\frac{d^{3/2}\,\sqrt{e}\,x\,\left(9\,c\,d^2-7\,b\,d\,e+5\,a\,e^2\right)}{9\,c\,d^4-7\,b\,d^3\,e+5\,a\,d^2\,e^2}\right)\,\left(9\,c\,d^2-7\,b\,d\,e+5\,a\,e^2\right)}{2\,e^{11/2}}$$

```
[In] int((x^6*(a + b*x^2 + c*x^4))/(d + e*x^2)^2,x)
```

```
[Out] x^5*(b/(5*e^2) - (2*c*d)/(5*e^3)) - x^3*((c*d^2)/(3*e^4) - a/(3*e^2) + (2*d
*(b/e^2 - (2*c*d)/e^3))/(3*e)) + x*((2*d*((c*d^2)/e^4 - a/e^2 + (2*d*(b/e^2
 - (2*c*d)/e^3))/e))/e - (d^2*(b/e^2 - (2*c*d)/e^3))/e^2) - (x*((c*d^4)/2 +
 (a*d^2*e^2)/2 - (b*d^3*e)/2))/(d*e^5 + e^6*x^2) + (c*x^7)/(7*e^2) + (d^(3/
2)*atan((d^(3/2)*e^(1/2)*x*(5*a*e^2 + 9*c*d^2 - 7*b*d*e))/(9*c*d^4 + 5*a*d^
2*e^2 - 7*b*d^3*e))*(5*a*e^2 + 9*c*d^2 - 7*b*d*e))/(2*e^(11/2))
```

3.281 $\int \frac{x^4(a+bx^2+cx^4)}{(d+ex^2)^2} dx$

Optimal result

Integrand size = 25, antiderivative size = 135

$$\int \frac{x^4(a+bx^2+cx^4)}{(d+ex^2)^2} dx = \frac{(3cd^2-e(2bd-ae))x}{e^4} - \frac{(2cd-be)x^3}{3e^3} + \frac{cx^5}{5e^2} + \frac{d(cd^2-bde+ae^2)x}{2e^4(d+ex^2)} - \frac{\sqrt{d}(7cd^2-e(5bd-3ae))\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{2e^{9/2}}$$

```
[Out] (3*c*d^2-e*(-a*e+2*b*d))*x/e^4-1/3*(-b*e+2*c*d)*x^3/e^3+1/5*c*x^5/e^2+1/2*d
*(a*e^2-b*d*e+c*d^2)*x/e^4/(e*x^2+d)-1/2*(7*c*d^2-e*(-3*a*e+5*b*d))*arctan(
x*e^(1/2)/d^(1/2))*d^(1/2)/e^(9/2)
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {1271, 1824, 211}

$$\int \frac{x^4(a+bx^2+cx^4)}{(d+ex^2)^2} dx = -\frac{\sqrt{d}\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)(7cd^2-e(5bd-3ae))}{2e^{9/2}} + \frac{x(3cd^2-e(2bd-ae))}{e^4} + \frac{dx(ae^2-bde+cd^2)}{2e^4(d+ex^2)} - \frac{x^3(2cd-be)}{3e^3} + \frac{cx^5}{5e^2}$$

```
[In] Int[(x^4*(a + b*x^2 + c*x^4))/(d + e*x^2)^2,x]
```

```
[Out] ((3*c*d^2 - e*(2*b*d - a*e))*x)/e^4 - ((2*c*d - b*e)*x^3)/(3*e^3) + (c*x^5)
/(5*e^2) + (d*(c*d^2 - b*d*e + a*e^2)*x)/(2*e^4*(d + e*x^2)) - (Sqrt[d]*(7*
c*d^2 - e*(5*b*d - 3*a*e))*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(2*e^(9/2))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1271

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)
^4)^(p_.), x_Symbol] :> Simp[(-d)^(m/2 - 1)*(c*d^2 - b*d*e + a*e^2)^p*x*((d
 + e*x^2)^(q + 1)/(2*e^(2*p + m/2)*(q + 1))), x] + Dist[1/(2*e^(2*p + m/2)*
(q + 1)), Int[(d + e*x^2)^(q + 1)*ExpandToSum[Together[(1/(d + e*x^2))*(2*e
^(2*p + m/2)*(q + 1)*x^m*(a + b*x^2 + c*x^4)^p - (-d)^(m/2 - 1)*(c*d^2 - b*
d*e + a*e^2)^p*(d + e*(2*q + 3)*x^2))], x], x], x] /; FreeQ[{a, b, c, d, e}
, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p, 0] && ILtQ[q, -1] && IGtQ[m/2, 0]
```

Rule 1824

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[Pq*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{d(cd^2-bde+ae^2)\,x}{2e^4\,(d+ex^2)} - \frac{\int \frac{d(cd^2-bde+ae^2)-2e(cd^2-bde+ae^2)x^2+2e^2(cd-be)x^4-2ce^3x^6}{d+ex^2}\,dx}{2e^4}\\
&= \frac{d(cd^2-bde+ae^2)\,x}{2e^4\,(d+ex^2)}\\
&\quad - \frac{\int \left(-2(3cd^2-2bde+ae^2)+2e(2cd-be)x^2-2ce^2x^4+\frac{7cd^3-5bd^2e+3ade^2}{d+ex^2}\right)dx}{2e^4}\\
&= \frac{(3cd^2-e(2bd-ae))\,x}{e^4} - \frac{(2cd-be)x^3}{3e^3} + \frac{cx^5}{5e^2}\\
&\quad + \frac{d(cd^2-bde+ae^2)\,x}{2e^4\,(d+ex^2)} - \frac{(d(7cd^2-e(5bd-3ae)))\int \frac{1}{d+ex^2}\,dx}{2e^4}\\
&= \frac{(3cd^2-e(2bd-ae))\,x}{e^4} - \frac{(2cd-be)x^3}{3e^3} + \frac{cx^5}{5e^2}\\
&\quad + \frac{d(cd^2-bde+ae^2)\,x}{2e^4\,(d+ex^2)} - \frac{\sqrt{d}(7cd^2-e(5bd-3ae))\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{2e^{9/2}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 133, normalized size of antiderivative = 0.99

$$\int \frac{x^4(a+bx^2+cx^4)}{(d+ex^2)^2}\,dx = \frac{(3cd^2-2bde+ae^2)\,x}{e^4}+\frac{(-2cd+be)x^3}{3e^3}+\frac{cx^5}{5e^2}+\frac{(cd^3-bd^2e+ade^2)\,x}{2e^4\,(d+ex^2)}-\frac{\sqrt{d}(7cd^2-5bde+3ae^2)\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{2e^{9/2}}$$

```
[In] Integrate[(x^4*(a + b*x^2 + c*x^4))/(d + e*x^2)^2,x]
```

```
[Out] ((3*c*d^2 - 2*b*d*e + a*e^2)*x)/e^4 + ((-2*c*d + b*e)*x^3)/(3*e^3) + (c*x^5
)/(5*e^2) + ((c*d^3 - b*d^2*e + a*d*e^2)*x)/(2*e^4*(d + e*x^2)) - (Sqrt[d]*
(7*c*d^2 - 5*b*d*e + 3*a*e^2)*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(2*e^(9/2))
```

Maple [A] (verified)

Time = 0.35 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.90

method	result
default	$\frac{\frac{1}{5}c\,x^5e^2+\frac{1}{3}b\,e^2x^3-\frac{2}{3}dc\,x^3e+a\,e^2x-2bdex+3c\,d^2x}{e^4}-\frac{d\left(\frac{\left(-\frac{1}{2}a\,e^2+\frac{1}{2}bde-\frac{1}{2}c\,d^2\right)x}{e\,x^2+d}+\frac{\left(3a\,e^2-5bde+7c\,d^2\right)\arctan\left(\frac{ex}{\sqrt{ed}}\right)}{2\sqrt{ed}}\right)}{e^4}$
risch	$\frac{c\,x^5}{5e^2}+\frac{b\,x^3}{3e^2}-\frac{2dc\,x^3}{3e^3}+\frac{ax}{e^2}-\frac{2bdx}{e^3}+\frac{3c\,d^2x}{e^4}+\frac{\left(\frac{1}{2}d\,e^2a-\frac{1}{2}b\,d^2e+\frac{1}{2}d^3c\right)x}{e^4(e\,x^2+d)}+\frac{3\sqrt{-ed}\,\ln(-\sqrt{-ed}\,x-d)a}{4e^3}-\frac{5\sqrt{-ed}\,\ln(-\sqrt{}}{4e^4}$

```
[In] int(x^4*(c*x^4+b*x^2+a)/(e*x^2+d)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/e^4*(1/5*c*x^5*e^2+1/3*b*e^2*x^3-2/3*d*c*x^3*e+a*e^2*x-2*b*d*e*x+3*c*d^2*
x)-d/e^4*((-1/2*a*e^2+1/2*b*d*e-1/2*c*d^2)*x/(e*x^2+d)+1/2*(3*a*e^2-5*b*d*e
+7*c*d^2)/(e*d)^(1/2)*arctan(e*x/(e*d)^(1/2)))
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 350, normalized size of antiderivative = 2.59

$$\int \frac{x^4(a+bx^2+cx^4)}{(d+ex^2)^2}\,dx$$

$$= \left[\frac{12\,ce^3x^7-4\,(7\,cde^2-5\,be^3)x^5+20\,(7\,cd^2e-5\,bde^2+3\,ae^3)x^3+15\,(7\,cd^3-5\,bd^2e+3\,ade^2+(7\,cd^2e-}{60\,(e^5x^2+de^4)}\right.$$

```
[In] integrate(x^4*(c*x^4+b*x^2+a)/(e*x^2+d)^2,x, algorithm="fricas")
```

```
[Out] [1/60*(12*c*e^3*x^7 - 4*(7*c*d*e^2 - 5*b*e^3)*x^5 + 20*(7*c*d^2*e - 5*b*d*e
^2 + 3*a*e^3)*x^3 + 15*(7*c*d^3 - 5*b*d^2*e + 3*a*d*e^2 + (7*c*d^2*e - 5*b*
d*e^2 + 3*a*e^3)*x^2)*sqrt(-d/e)*log((e*x^2 - 2*e*x*sqrt(-d/e) - d)/(e*x^2
+ d)) + 30*(7*c*d^3 - 5*b*d^2*e + 3*a*d*e^2)*x)/(e^5*x^2 + d*e^4), 1/30*(6*
c*e^3*x^7 - 2*(7*c*d*e^2 - 5*b*e^3)*x^5 + 10*(7*c*d^2*e - 5*b*d*e^2 + 3*a*e
^3)*x^3 - 15*(7*c*d^3 - 5*b*d^2*e + 3*a*d*e^2 + (7*c*d^2*e - 5*b*d*e^2 + 3*
a*e^3)*x^2)*sqrt(d/e)*arctan(e*x*sqrt(d/e)/d) + 15*(7*c*d^3 - 5*b*d^2*e + 3
*a*d*e^2)*x)/(e^5*x^2 + d*e^4)]
```

Sympy [A] (verification not implemented)

Time = 0.62 (sec) , antiderivative size = 189, normalized size of antiderivative = 1.40

$$\int \frac{x^4(a+bx^2+cx^4)}{(d+ex^2)^2}\,dx = \frac{cx^5}{5e^2} + x^3\left(\frac{b}{3e^2} - \frac{2cd}{3e^3}\right) + x\left(\frac{a}{e^2} - \frac{2bd}{e^3} + \frac{3cd^2}{e^4}\right) + \frac{x(ade^2 - bd^2e + cd^3)}{2de^4 + 2e^5x^2} + \frac{\sqrt{-\frac{d}{e^9}}\cdot(3ae^2 - 5bde + 7cd^2)\log\left(-e^4\sqrt{-\frac{d}{e^9}} + x\right)}{4} - \frac{\sqrt{-\frac{d}{e^9}}\cdot(3ae^2 - 5bde + 7cd^2)\log\left(e^4\sqrt{-\frac{d}{e^9}} + x\right)}{4}$$

```
[In] integrate(x**4*(c*x**4+b*x**2+a)/(e*x**2+d)**2,x)
```

```
[Out] c*x**5/(5*e**2) + x**3*(b/(3*e**2) - 2*c*d/(3*e**3)) + x*(a/e**2 - 2*b*d/e*
*3 + 3*c*d**2/e**4) + x*(a*d*e**2 - b*d**2*e + c*d**3)/(2*d*e**4 + 2*e**5*x
**2) + sqrt(-d/e**9)*(3*a*e**2 - 5*b*d*e + 7*c*d**2)*log(-e**4*sqrt(-d/e**9
) + x)/4 - sqrt(-d/e**9)*(3*a*e**2 - 5*b*d*e + 7*c*d**2)*log(e**4*sqrt(-d/e
**9) + x)/4
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^4(a+bx^2+cx^4)}{(d+ex^2)^2}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(x^4*(c*x^4+b*x^2+a)/(e*x^2+d)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [A] (verification not implemented)

none

Time = 0.39 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.01

$$\int \frac{x^4(a+bx^2+cx^4)}{(d+ex^2)^2}\,dx = -\frac{(7\,cd^3-5\,bd^2e+3\,ade^2)\arctan\left(\frac{ex}{\sqrt{de}}\right)}{2\,\sqrt{de}e^4}+\frac{cd^3x-bd^2ex+ade^2x}{2\,(ex^2+d)e^4}+\frac{3\,ce^8x^5-10\,cde^7x^3+5\,be^8x^3+45\,cd^2e^6x-30\,bde^7x+15\,ae^8x}{15\,e^{10}}$$

```
[In] integrate(x^4*(c*x^4+b*x^2+a)/(e*x^2+d)^2,x, algorithm="giac")
```

```
[Out] -1/2*(7*c*d^3 - 5*b*d^2*e + 3*a*d*e^2)*arctan(e*x/sqrt(d*e))/(sqrt(d*e)*e^4
) + 1/2*(c*d^3*x - b*d^2*e*x + a*d*e^2*x)/((e*x^2 + d)*e^4) + 1/15*(3*c*e^8
*x^5 - 10*c*d*e^7*x^3 + 5*b*e^8*x^3 + 45*c*d^2*e^6*x - 30*b*d*e^7*x + 15*a*
e^8*x)/e^10
```

Mupad [B] (verification not implemented)

Time = 7.68 (sec) , antiderivative size = 179, normalized size of antiderivative = 1.33

$$\int \frac{x^4(a+bx^2+cx^4)}{(d+ex^2)^2}\,dx = x^3\left(\frac{b}{3\,e^2}-\frac{2\,c\,d}{3\,e^3}\right)-x\left(\frac{c\,d^2}{e^4}-\frac{a}{e^2}+\frac{2\,d\left(\frac{b}{e^2}-\frac{2\,c\,d}{e^3}\right)}{e}\right)+\frac{c\,x^5}{5\,e^2}+\frac{x\left(\frac{c\,d^3}{2}-\frac{b\,d^2\,e}{2}+\frac{a\,d\,e^2}{2}\right)}{e^5\,x^2+d\,e^4}-\frac{\sqrt{d}\,\mathrm{atan}\left(\frac{\sqrt{d}\,\sqrt{e}\,x\,(7\,c\,d^2-5\,b\,d\,e+3\,a\,e^2)}{7\,c\,d^3-5\,b\,d^2\,e+3\,a\,d\,e^2}\right)\,(7\,c\,d^2-5\,b\,d\,e+3\,a\,e^2)}{2\,e^{9/2}}$$

```
[In] int((x^4*(a + b*x^2 + c*x^4))/(d + e*x^2)^2,x)
```

```
[Out] x^3*(b/(3*e^2) - (2*c*d)/(3*e^3)) - x*((c*d^2)/e^4 - a/e^2 + (2*d*(b/e^2 -
(2*c*d)/e^3))/e) + (c*x^5)/(5*e^2) + (x*((c*d^3)/2 + (a*d*e^2)/2 - (b*d^2*e
)/2))/(d*e^4 + e^5*x^2) - (d^(1/2)*atan((d^(1/2)*e^(1/2)*x*(3*a*e^2 + 7*c*d
^2 - 5*b*d*e))/(7*c*d^3 + 3*a*d*e^2 - 5*b*d^2*e))*(3*a*e^2 + 7*c*d^2 - 5*b*
d*e))/(2*e^(9/2))
```

3.282 $\int \frac{x^2(a+bx^2+cx^4)}{(d+ex^2)^2}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 106

$$\int \frac{x^2(a+bx^2+cx^4)}{(d+ex^2)^2}\,dx = -\frac{(2cd-be)x}{e^3}+\frac{cx^3}{3e^2}-\frac{(cd^2-bde+ae^2)\,x}{2e^3\,(d+ex^2)}+\frac{(5cd^2-e(3bd-ae))\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{2\sqrt{d}e^{7/2}}$$

```
[Out] -(-b*e+2*c*d)*x/e^3+1/3*c*x^3/e^2-1/2*(a*e^2-b*d*e+c*d^2)*x/e^3/(e*x^2+d)+1
/2*(5*c*d^2-e*(-a*e+3*b*d))*arctan(x*e^(1/2)/d^(1/2))/e^(7/2)/d^(1/2)
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.120$, Rules used = {1271, 1167, 211}

$$\int \frac{x^2(a+bx^2+cx^4)}{(d+ex^2)^2}\,dx = \frac{\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)(5cd^2-e(3bd-ae))}{2\sqrt{d}e^{7/2}}-\frac{x(ae^2-bde+cd^2)}{2e^3\,(d+ex^2)}-\frac{x(2cd-be)}{e^3}+\frac{cx^3}{3e^2}$$

```
[In] Int[(x^2*(a + b*x^2 + c*x^4))/(d + e*x^2)^2,x]
```

```
[Out] -(((2*c*d - b*e)*x)/e^3) + (c*x^3)/(3*e^2) - ((c*d^2 - b*d*e + a*e^2)*x)/(2
*e^3*(d + e*x^2)) + ((5*c*d^2 - e*(3*b*d - a*e))*ArcTan[(Sqrt[e]*x)/Sqrt[d]
])/(2*Sqrt[d]*e^(7/2))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1167

```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^q*(a + b*x^2 + c*x^4)^p, x],
x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e
 + a*e^2, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rule 1271

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)
^4)^(p_.), x_Symbol] :> Simp[(-d)^(m/2 - 1)*(c*d^2 - b*d*e + a*e^2)^p*x*((d
 + e*x^2)^(q + 1)/(2*e^(2*p + m/2)*(q + 1))), x] + Dist[1/(2*e^(2*p + m/2)*
(q + 1)), Int[(d + e*x^2)^(q + 1)*ExpandToSum[Together[(1/(d + e*x^2))*(2*e
^(2*p + m/2)*(q + 1)*x^m*(a + b*x^2 + c*x^4)^p - (-d)^(m/2 - 1)*(c*d^2 - b*
d*e + a*e^2)^p*(d + e*(2*q + 3)*x^2))], x], x], x] /; FreeQ[{a, b, c, d, e}
, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p, 0] && ILtQ[q, -1] && IGtQ[m/2, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{(cd^2-bde+ae^2)\,x}{2e^3\,(d+ex^2)}-\frac{\int \frac{-cd^2+bde-ae^2+2e(cd-be)x^2-2ce^2x^4}{d+ex^2}\,dx}{2e^3}\\
&= -\frac{(cd^2-bde+ae^2)\,x}{2e^3\,(d+ex^2)}-\frac{\int \left(2(2cd-be)-2cex^2+\frac{-5cd^2+3bde-ae^2}{d+ex^2}\right)dx}{2e^3}\\
&= -\frac{(2cd-be)x}{e^3}+\frac{cx^3}{3e^2}-\frac{(cd^2-bde+ae^2)\,x}{2e^3\,(d+ex^2)}-\frac{(-5cd^2+e(3bd-ae))\int \frac{1}{d+ex^2}\,dx}{2e^3}\\
&= -\frac{(2cd-be)x}{e^3}+\frac{cx^3}{3e^2}-\frac{(cd^2-bde+ae^2)\,x}{2e^3\,(d+ex^2)}+\frac{(5cd^2-e(3bd-ae))\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{2\sqrt{d}e^{7/2}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.96

$$
\int \frac{x^2(a+bx^2+cx^4)}{(d+ex^2)^2}\,dx=\frac{(-2cd+be)x}{e^3}+\frac{cx^3}{3e^2}-\frac{(cd^2-bde+ae^2)\,x}{2e^3\,(d+ex^2)}+\frac{(5cd^2-3bde+ae^2)\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{2\sqrt{d}e^{7/2}}
$$

```
[In] Integrate[(x^2*(a + b*x^2 + c*x^4))/(d + e*x^2)^2,x]
```

```
[Out] ((-2*c*d + b*e)*x)/e^3 + (c*x^3)/(3*e^2) - ((c*d^2 - b*d*e + a*e^2)*x)/(2*e
^3*(d + e*x^2)) + ((5*c*d^2 - 3*b*d*e + a*e^2)*ArcTan[(Sqrt[e]*x)/Sqrt[d]])
/(2*Sqrt[d]*e^(7/2))
```

Maple [A] (verified)

Time = 0.37 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.85

method	result
default	$\frac{\frac{1}{3}c\,x^3e+bex-2cdx}{e^3}+\frac{\frac{\left(-\frac{1}{2}a\,e^2+\frac{1}{2}bde-\frac{1}{2}c\,d^2\right)x}{e\,x^2+d}+\frac{\left(a\,e^2-3bde+5c\,d^2\right)\arctan\left(\frac{ex}{\sqrt{ed}}\right)}{2\sqrt{ed}}}{e^3}$
risch	$\frac{c\,x^3}{3e^2}+\frac{bx}{e^2}-\frac{2cdx}{e^3}+\frac{\left(-\frac{1}{2}a\,e^2+\frac{1}{2}bde-\frac{1}{2}c\,d^2\right)x}{e^3\left(e\,x^2+d\right)}-\frac{\ln\left(ex+\sqrt{-ed}\right)a}{4e\sqrt{-ed}}+\frac{3\ln\left(ex+\sqrt{-ed}\right)bd}{4e^2\sqrt{-ed}}-\frac{5\ln\left(ex+\sqrt{-ed}\right)c\,d^2}{4e^3\sqrt{-ed}}+\frac{\ln(-ex+\sqrt{}}{4e\sqrt{-e}}$

```
[In] int(x^2*(c*x^4+b*x^2+a)/(e*x^2+d)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/e^3*(1/3*c*x^3*e+b*e*x-2*c*d*x)+1/e^3*((-1/2*a*e^2+1/2*b*d*e-1/2*c*d^2)*x
/(e*x^2+d)+1/2*(a*e^2-3*b*d*e+5*c*d^2)/(e*d)^(1/2)*arctan(e*x/(e*d)^(1/2)))
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 302, normalized size of antiderivative = 2.85

$$\int \frac{x^2(a+bx^2+cx^4)}{(d+ex^2)^2}\,dx = \left[\frac{4\,cde^3x^5-4\,(5\,cd^2e^2-3\,bde^3)x^3-3\,(5\,cd^3-3\,bd^2e+ade^2+(5\,cd^2e-3\,bde^2+ae^3)x^2)\sqrt{-de}\log\left(\frac{ex^2-2}{ex}\right.}{12\,(de^5x^2+d^2e^4)}\right.$$

```
[In] integrate(x^2*(c*x^4+b*x^2+a)/(e*x^2+d)^2,x, algorithm="fricas")
```

```
[Out] [1/12*(4*c*d*e^3*x^5 - 4*(5*c*d^2*e^2 - 3*b*d*e^3)*x^3 - 3*(5*c*d^3 - 3*b*d
^2*e + a*d*e^2 + (5*c*d^2*e - 3*b*d*e^2 + a*e^3)*x^2)*sqrt(-d*e)*log((e*x^2
 - 2*sqrt(-d*e)*x - d)/(e*x^2 + d)) - 6*(5*c*d^3*e - 3*b*d^2*e^2 + a*d*e^3)
*x)/(d*e^5*x^2 + d^2*e^4), 1/6*(2*c*d*e^3*x^5 - 2*(5*c*d^2*e^2 - 3*b*d*e^3)
*x^3 + 3*(5*c*d^3 - 3*b*d^2*e + a*d*e^2 + (5*c*d^2*e - 3*b*d*e^2 + a*e^3)*x
^2)*sqrt(d*e)*arctan(sqrt(d*e)*x/d) - 3*(5*c*d^3*e - 3*b*d^2*e^2 + a*d*e^3)
*x)/(d*e^5*x^2 + d^2*e^4)]
```

Sympy [A] (verification not implemented)

Time = 0.53 (sec) , antiderivative size = 162, normalized size of antiderivative = 1.53

$$\int \frac{x^2(a+bx^2+cx^4)}{(d+ex^2)^2}\,dx = \frac{cx^3}{3e^2} + x\left(\frac{b}{e^2} - \frac{2cd}{e^3}\right) + \frac{x(-ae^2+bde-cd^2)}{2de^3+2e^4x^2} - \frac{\sqrt{-\frac{1}{de^7}}(ae^2-3bde+5cd^2)\log\left(-de^3\sqrt{-\frac{1}{de^7}}+x\right)}{4} + \frac{\sqrt{-\frac{1}{de^7}}(ae^2-3bde+5cd^2)\log\left(de^3\sqrt{-\frac{1}{de^7}}+x\right)}{4}$$

```
[In] integrate(x**2*(c*x**4+b*x**2+a)/(e*x**2+d)**2,x)
```

```
[Out] c*x**3/(3*e**2) + x*(b/e**2 - 2*c*d/e**3) + x*(-a*e**2 + b*d*e - c*d**2)/(2
*d*e**3 + 2*e**4*x**2) - sqrt(-1/(d*e**7))*(a*e**2 - 3*b*d*e + 5*c*d**2)*lo
g(-d*e**3*sqrt(-1/(d*e**7)) + x)/4 + sqrt(-1/(d*e**7))*(a*e**2 - 3*b*d*e +
5*c*d**2)*log(d*e**3*sqrt(-1/(d*e**7)) + x)/4
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^2(a+bx^2+cx^4)}{(d+ex^2)^2}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(x^2*(c*x^4+b*x^2+a)/(e*x^2+d)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [A] (verification not implemented)

none

Time = 0.36 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.93

$$\int \frac{x^2(a+bx^2+cx^4)}{(d+ex^2)^2}\,dx = \frac{(5\,cd^2-3\,bde+ae^2)\arctan\left(\frac{ex}{\sqrt{de}}\right)}{2\,\sqrt{de}e^3} - \frac{cd^2x-bdex+ae^2x}{2\,(ex^2+d)e^3} + \frac{ce^4x^3-6\,cde^3x+3\,be^4x}{3\,e^6}$$

```
[In] integrate(x^2*(c*x^4+b*x^2+a)/(e*x^2+d)^2,x, algorithm="giac")
```

```
[Out] 1/2*(5*c*d^2 - 3*b*d*e + a*e^2)*arctan(e*x/sqrt(d*e))/(sqrt(d*e)*e^3) - 1/2
*(c*d^2*x - b*d*e*x + a*e^2*x)/((e*x^2 + d)*e^3) + 1/3*(c*e^4*x^3 - 6*c*d*e
^3*x + 3*b*e^4*x)/e^6
```

Mupad [B] (verification not implemented)

Time = 7.79 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.90

$$\int \frac{x^2(a+bx^2+cx^4)}{(d+ex^2)^2}\,dx = x\left(\frac{b}{e^2}-\frac{2\,c\,d}{e^3}\right)-\frac{x\left(\frac{c\,d^2}{2}-\frac{b\,d\,e}{2}+\frac{a\,e^2}{2}\right)}{e^4\,x^2+d\,e^3}+\frac{c\,x^3}{3\,e^2}+\frac{\mathrm{atan}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)\,\left(5\,c\,d^2-3\,b\,d\,e+a\,e^2\right)}{2\,\sqrt{d}\,e^{7/2}}$$

```
[In] int((x^2*(a + b*x^2 + c*x^4))/(d + e*x^2)^2,x)
```

```
[Out] x*(b/e^2 - (2*c*d)/e^3) - (x*((a*e^2)/2 + (c*d^2)/2 - (b*d*e)/2))/(d*e^3 +
e^4*x^2) + (c*x^3)/(3*e^2) + (atan((e^(1/2)*x)/d^(1/2))*(a*e^2 + 5*c*d^2 -
3*b*d*e))/(2*d^(1/2)*e^(7/2))
```

3.283 $\int \frac{a+bx^2+cx^4}{(d+ex^2)^2} dx$

Optimal result

Integrand size = 22, antiderivative size = 83

$$\int \frac{a+bx^2+cx^4}{(d+ex^2)^2} dx = \frac{cx}{e^2} + \frac{\left(a+\frac{d(cd-be)}{e^2}\right)x}{2d(d+ex^2)} - \frac{(3cd^2-e(bd+ae))\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{2d^{3/2}e^{5/2}}$$

```
[Out] c*x/e^2+1/2*(a+d*(-b*e+c*d)/e^2)*x/d/(e*x^2+d)-1/2*(3*c*d^2-e*(a*e+b*d))*ar
ctan(x*e^(1/2)/d^(1/2))/d^(3/2)/e^(5/2)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.05, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {1171, 396, 211}

$$\int \frac{a+bx^2+cx^4}{(d+ex^2)^2} dx = -\frac{\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)(3cd^2-e(ae+bd))}{2d^{3/2}e^{5/2}} + \frac{x(ae^2-bde+cd^2)}{2de^2(d+ex^2)} + \frac{cx}{e^2}$$

```
[In] Int[(a + b*x^2 + c*x^4)/(d + e*x^2)^2,x]
```

```
[Out] (c*x)/e^2 + ((c*d^2 - b*d*e + a*e^2)*x)/(2*d*e^2*(d + e*x^2)) - ((3*c*d^2 -
 e*(b*d + a*e))*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(2*d^(3/2)*e^(5/2))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 396

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 1171

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 + c*x^4)^p, d + e*x^2
, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], x
, 0]}, Simp[(-R)*x*((d + e*x^2)^(q + 1)/(2*d*(q + 1))), x] + Dist[1/(2*d*(q
 + 1)), Int[(d + e*x^2)^(q + 1)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3), x
], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
 - b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{(cd^2-bde+ae^2)\,x}{2de^2\,(d+ex^2)} - \frac{\int \frac{\frac{cd^2-e(bd+ae)}{e^2}-\frac{2cdx^2}{e}}{d+ex^2}\,dx}{2d}\\
&= \frac{cx}{e^2} + \frac{(cd^2-bde+ae^2)\,x}{2de^2\,(d+ex^2)} - \frac{(3cd^2-e(bd+ae))\int \frac{1}{d+ex^2}\,dx}{2de^2}\\
&= \frac{cx}{e^2} + \frac{(cd^2-bde+ae^2)\,x}{2de^2\,(d+ex^2)} - \frac{(3cd^2-e(bd+ae))\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{2d^{3/2}e^{5/2}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.06

$$\int \frac{a+bx^2+cx^4}{(d+ex^2)^2}\,dx = \frac{cx}{e^2} + \frac{(cd^2-bde+ae^2)\,x}{2de^2\,(d+ex^2)} - \frac{(3cd^2-bde-ae^2)\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{2d^{3/2}e^{5/2}}$$

```
[In] Integrate[(a + b*x^2 + c*x^4)/(d + e*x^2)^2,x]
```

```
[Out] (c*x)/e^2 + ((c*d^2 - b*d*e + a*e^2)*x)/(2*d*e^2*(d + e*x^2)) - ((3*c*d^2 -
 b*d*e - a*e^2)*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(2*d^(3/2)*e^(5/2))
```

Maple [A] (verified)

Time = 0.30 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.95

method	result
default	$\frac{cx}{e^2}+\frac{\frac{(a\,e^2-bde+c\,d^2)x}{2d(e\,x^2+d)}+\frac{(a\,e^2+bde-3c\,d^2)\arctan\left(\frac{ex}{\sqrt{ed}}\right)}{2d\sqrt{ed}}}{e^2}$
risch	$\frac{cx}{e^2}+\frac{(a\,e^2-bde+c\,d^2)x}{2d\,e^2(e\,x^2+d)}-\frac{\ln(ex+\sqrt{-ed})a}{4\sqrt{-ed}\,d}-\frac{\ln(ex+\sqrt{-ed})b}{4e\sqrt{-ed}}+\frac{3d\ln(ex+\sqrt{-ed})c}{4e^2\sqrt{-ed}}+\frac{\ln(-ex+\sqrt{-ed})a}{4\sqrt{-ed}\,d}+\frac{\ln(-ex+\sqrt{-ed})}{4e\sqrt{-ed}}$

```
[In] int((c*x^4+b*x^2+a)/(e*x^2+d)^2,x,method=_RETURNVERBOSE)
```

```
[Out] c*x/e^2+1/e^2*(1/2*(a*e^2-b*d*e+c*d^2)/d*x/(e*x^2+d)+1/2*(a*e^2+b*d*e-3*c*d
^2)/d/(e*d)^(1/2)*arctan(e*x/(e*d)^(1/2)))
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 268, normalized size of antiderivative = 3.23

$$\int \frac{a+bx^2+cx^4}{(d+ex^2)^2}\,dx = \left[\frac{4\,cd^2e^2x^3+(3\,cd^3-bd^2e-ade^2+(3\,cd^2e-bde^2-ae^3)x^2)\sqrt{-de}\log\left(\frac{ex^2-2\sqrt{-de}x-d}{ex^2+d}\right)+2\,(3\,cd^3e-bd^2}{4\,(d^2e^4x^2+d^3e^3)}\right.$$

```
[In] integrate((c*x^4+b*x^2+a)/(e*x^2+d)^2,x, algorithm="fricas")
```

```
[Out] [1/4*(4*c*d^2*e^2*x^3 + (3*c*d^3 - b*d^2*e - a*d*e^2 + (3*c*d^2*e - b*d*e^2
 - a*e^3)*x^2)*sqrt(-d*e)*log((e*x^2 - 2*sqrt(-d*e)*x - d)/(e*x^2 + d)) + 2
*(3*c*d^3*e - b*d^2*e^2 + a*d*e^3)*x)/(d^2*e^4*x^2 + d^3*e^3), 1/2*(2*c*d^2
*e^2*x^3 - (3*c*d^3 - b*d^2*e - a*d*e^2 + (3*c*d^2*e - b*d*e^2 - a*e^3)*x^2
)*sqrt(d*e)*arctan(sqrt(d*e)*x/d) + (3*c*d^3*e - b*d^2*e^2 + a*d*e^3)*x)/(d
^2*e^4*x^2 + d^3*e^3)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 153 vs. $2(75) = 150$.

Time = 0.42 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.84

$$\int \frac{a+bx^2+cx^4}{(d+ex^2)^2}\,dx = \frac{cx}{e^2} + \frac{x(ae^2-bde+cd^2)}{2d^2e^2+2de^3x^2} - \frac{\sqrt{-\frac{1}{d^3e^5}}(ae^2+bde-3cd^2)\log\left(-d^2e^2\sqrt{-\frac{1}{d^3e^5}}+x\right)}{4} + \frac{\sqrt{-\frac{1}{d^3e^5}}(ae^2+bde-3cd^2)\log\left(d^2e^2\sqrt{-\frac{1}{d^3e^5}}+x\right)}{4}$$

```
[In] integrate((c*x**4+b*x**2+a)/(e*x**2+d)**2,x)
```

```
[Out] c*x/e**2 + x*(a*e**2 - b*d*e + c*d**2)/(2*d**2*e**2 + 2*d*e**3*x**2) - sqrt
(-1/(d**3*e**5))*(a*e**2 + b*d*e - 3*c*d**2)*log(-d**2*e**2*sqrt(-1/(d**3*e
**5)) + x)/4 + sqrt(-1/(d**3*e**5))*(a*e**2 + b*d*e - 3*c*d**2)*log(d**2*e*
*2*sqrt(-1/(d**3*e**5)) + x)/4
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{a+bx^2+cx^4}{(d+ex^2)^2}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((c*x^4+b*x^2+a)/(e*x^2+d)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.00

$$\int \frac{a+bx^2+cx^4}{(d+ex^2)^2}\,dx = \frac{cx}{e^2} - \frac{(3\,cd^2-bde-ae^2)\arctan\left(\frac{ex}{\sqrt{de}}\right)}{2\,\sqrt{de}de^2} + \frac{cd^2x-bdex+ae^2x}{2\,(ex^2+d)de^2}$$

```
[In] integrate((c*x^4+b*x^2+a)/(e*x^2+d)^2,x, algorithm="giac")
```

```
[Out] c*x/e^2 - 1/2*(3*c*d^2 - b*d*e - a*e^2)*arctan(e*x/sqrt(d*e))/(sqrt(d*e)*d*
e^2) + 1/2*(c*d^2*x - b*d*e*x + a*e^2*x)/((e*x^2 + d)*d*e^2)
```

Mupad [B] (verification not implemented)

Time = 7.72 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.93

$$\int \frac{a+bx^2+cx^4}{(d+ex^2)^2}\,dx=\frac{c\,x}{e^2}+\frac{\mathrm{atan}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)\,\left(-3\,c\,d^2+b\,d\,e+a\,e^2\right)}{2\,d^{3/2}\,e^{5/2}}+\frac{x\,\left(c\,d^2-b\,d\,e+a\,e^2\right)}{2\,d\,\left(e^3\,x^2+d\,e^2\right)}$$

```
[In] int((a + b*x^2 + c*x^4)/(d + e*x^2)^2,x)
[Out] (c*x)/e^2 + (atan((e^(1/2)*x)/d^(1/2))*(a*e^2 - 3*c*d^2 + b*d*e))/(2*d^(3/2
)*e^(5/2)) + (x*(a*e^2 + c*d^2 - b*d*e))/(2*d*(d*e^2 + e^3*x^2))
```

3.284 $\int \frac{a+bx^2+cx^4}{x^2(d+ex^2)^2} dx$

Optimal result

Integrand size = 25, antiderivative size = 89

$$\int \frac{a+bx^2+cx^4}{x^2(d+ex^2)^2} dx = -\frac{a}{d^2x} - \frac{(cd^2-bde+ae^2)x}{2d^2e(d+ex^2)} + \frac{(cd^2+e(bd-3ae))\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{2d^{5/2}e^{3/2}}$$

```
[Out] -a/d^2/x-1/2*(a*e^2-b*d*e+c*d^2)*x/d^2/e/(e*x^2+d)+1/2*(c*d^2+e*(-3*a*e+b*d
))*arctan(x*e^(1/2)/d^(1/2))/d^(5/2)/e^(3/2)
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.97, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {1273, 464, 211}

$$\int \frac{a+bx^2+cx^4}{x^2(d+ex^2)^2} dx = \frac{\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)(e(bd-3ae)+cd^2)}{2d^{5/2}e^{3/2}} - \frac{x\left(\frac{c}{e}-\frac{bd-ae}{d^2}\right)}{2(d+ex^2)} - \frac{a}{d^2x}$$

```
[In] Int[(a + b*x^2 + c*x^4)/(x^2*(d + e*x^2)^2),x]
```

```
[Out] -(a/(d^2*x)) - ((c/e - (b*d - a*e)/d^2)*x)/(2*(d + e*x^2)) + ((c*d^2 + e*(b
*d - 3*a*e))*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(2*d^(5/2)*e^(3/2))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 464

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 1273

```
Int[(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^
4)^(p_.), x_Symbol] :> Simp[(-d)^(m/2 - 1)*(c*d^2 - b*d*e + a*e^2)^p*x*((d
+ e*x^2)^(q + 1)/(2*e^(2*p + m/2)*(q + 1))), x] + Dist[(-d)^(m/2 - 1)/(2*e^
(2*p)*(q + 1)), Int[x^m*(d + e*x^2)^(q + 1)*ExpandToSum[Together[(1/(d + e*
x^2))*(2*(-d)^(-m/2 + 1)*e^(2*p)*(q + 1)*(a + b*x^2 + c*x^4)^p - ((c*d^2 -
b*d*e + a*e^2)^p/(e^(m/2)*x^m))*(d + e*(2*q + 3)*x^2))], x], x], x] /; Free
Q[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p, 0] && ILtQ[q, -1] &
& ILtQ[m/2, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{\left(\frac{c}{e}-\frac{bd-ae}{d^2}\right)x}{2(d+ex^2)}-\frac{\int \frac{-2ade^2-e(cd^2+e(bd-ae))x^2}{x^2(d+ex^2)}\,dx}{2d^2e^2}\\
&= -\frac{a}{d^2x}-\frac{\left(\frac{c}{e}-\frac{bd-ae}{d^2}\right)x}{2(d+ex^2)}+\frac{1}{2}\left(\frac{c}{e}+\frac{bd-3ae}{d^2}\right)\int\frac{1}{d+ex^2}\,dx\\
&= -\frac{a}{d^2x}-\frac{\left(\frac{c}{e}-\frac{bd-ae}{d^2}\right)x}{2(d+ex^2)}+\frac{(cd^2+e(bd-3ae))\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{2d^{5/2}e^{3/2}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.00

$$\int \frac{a+bx^2+cx^4}{x^2(d+ex^2)^2}\,dx = -\frac{a}{d^2x}-\frac{(cd^2-bde+ae^2)\,x}{2d^2e\,(d+ex^2)}+\frac{(cd^2+bde-3ae^2)\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{2d^{5/2}e^{3/2}}$$

```
[In] Integrate[(a + b*x^2 + c*x^4)/(x^2*(d + e*x^2)^2),x]
```

```
[Out] -(a/(d^2*x)) - ((c*d^2 - b*d*e + a*e^2)*x)/(2*d^2*e*(d + e*x^2)) + ((c*d^2
+ b*d*e - 3*a*e^2)*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(2*d^(5/2)*e^(3/2))
```

Maple [A] (verified)

Time = 0.33 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.96

method	result
default	$-\frac{a}{d^2 x}-\frac{\frac{\left(a\,e^2-bde+c\,d^2\right)x}{2e\left(e\,x^2+d\right)}+\frac{\left(3a\,e^2-bde-c\,d^2\right)\arctan\left(\frac{ex}{\sqrt{ed}}\right)}{2e\sqrt{ed}}}{d^2}$
risch	$\frac{-\frac{\left(3a\,e^2-bde+c\,d^2\right)x^2}{2d^2e}-\frac{a}{d}}{x(e\,x^2+d)}+\frac{\left(\sum_{\_R=\text{RootOf}\left(d^5e^3\_Z^2+9a^2e^4-6abd\,e^3-6ac\,d^2e^2+b^2d^2e^2+2bc\,d^3e+c^2d^4\right)}\_R\ln\left(\left(3\_R^2d^5e^3+18a^2e^4-\right.\right.\right.}{4}$

[In] int((c*x^4+b*x^2+a)/x^2/(e*x^2+d)^2,x,method=_RETURNVERBOSE)

[Out] -a/d^2/x-1/d^2*(1/2*(a*e^2-b*d*e+c*d^2)/e*x/(e*x^2+d)+1/2*(3*a*e^2-b*d*e-c*d^2)/e/(e*d)^(1/2)*arctan(e*x/(e*d)^(1/2)))

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 267, normalized size of antiderivative = 3.00

$$\int \frac{a+bx^2+cx^4}{x^2\left(d+ex^2\right)^2}\,dx$$
$$=\left[-\frac{4\,ad^2e^2+2\,(cd^3e-bd^2e^2+3\,ade^3)x^2-((cd^2e+bde^2-3\,ae^3)x^3+(cd^3+bd^2e-3\,ade^2)x)\sqrt{-de}\log\left(\right.}{4\,(d^3e^3x^3+d^4e^2x)}\right.$$
$$-\frac{2\,ad^2e^2+(cd^3e-bd^2e^2+3\,ade^3)x^2-((cd^2e+bde^2-3\,ae^3)x^3+(cd^3+bd^2e-3\,ade^2)x)\sqrt{de}\arctan\left(\right.}{2\,(d^3e^3x^3+d^4e^2x)}$$

[In] integrate((c*x^4+b*x^2+a)/x^2/(e*x^2+d)^2,x, algorithm="fricas")

[Out] [-1/4*(4*a*d^2*e^2 + 2*(c*d^3*e - b*d^2*e^2 + 3*a*d*e^3)*x^2 - ((c*d^2*e + b*d*e^2 - 3*a*e^3)*x^3 + (c*d^3 + b*d^2*e - 3*a*d*e^2)*x)*sqrt(-d*e)*log((e*x^2 + 2*sqrt(-d*e)*x - d)/(e*x^2 + d)))/(d^3*e^3*x^3 + d^4*e^2*x), -1/2*(2*a*d^2*e^2 + (c*d^3*e - b*d^2*e^2 + 3*a*d*e^3)*x^2 - ((c*d^2*e + b*d*e^2 - 3*a*e^3)*x^3 + (c*d^3 + b*d^2*e - 3*a*d*e^2)*x)*sqrt(d*e)*arctan(sqrt(d*e)*x/d))/(d^3*e^3*x^3 + d^4*e^2*x)]

Sympy [A] (verification not implemented)

Time = 0.59 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.74

$$\int \frac{a+bx^2+cx^4}{x^2\left(d+ex^2\right)^2}\,dx = \frac{\sqrt{-\frac{1}{d^5e^3}}\cdot\left(3ae^2-bde-cd^2\right)\log\left(-d^3e\sqrt{-\frac{1}{d^5e^3}}+x\right)}{4} - \frac{\sqrt{-\frac{1}{d^5e^3}}\cdot\left(3ae^2-bde-cd^2\right)\log\left(d^3e\sqrt{-\frac{1}{d^5e^3}}+x\right)}{4} + \frac{-2ade+x^2(-3ae^2+bde-cd^2)}{2d^3ex+2d^2e^2x^3}$$

```
[In] integrate((c*x**4+b*x**2+a)/x**2/(e*x**2+d)**2,x)
```

```
[Out] sqrt(-1/(d**5*e**3))*(3*a*e**2 - b*d*e - c*d**2)*log(-d**3*e*sqrt(-1/(d**5*
e**3)) + x)/4 - sqrt(-1/(d**5*e**3))*(3*a*e**2 - b*d*e - c*d**2)*log(d**3*e
*sqrt(-1/(d**5*e**3)) + x)/4 + (-2*a*d*e + x**2*(-3*a*e**2 + b*d*e - c*d**2
))/(2*d**3*e*x + 2*d**2*e**2*x**3)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{a+bx^2+cx^4}{x^2\left(d+ex^2\right)^2}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((c*x^4+b*x^2+a)/x^2/(e*x^2+d)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [A] (verification not implemented)

none

Time = 0.36 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.00

$$\int \frac{a+bx^2+cx^4}{x^2\left(d+ex^2\right)^2}\,dx = \frac{\left(cd^2+bde-3\,ae^2\right)\arctan\left(\frac{ex}{\sqrt{de}}\right)}{2\,\sqrt{de}d^2e} - \frac{cd^2x^2-bdex^2+3\,ae^2x^2+2\,ade}{2\,(ex^3+dx)d^2e}$$

```
[In] integrate((c*x^4+b*x^2+a)/x^2/(e*x^2+d)^2,x, algorithm="giac")
```

```
[Out] 1/2*(c*d^2 + b*d*e - 3*a*e^2)*arctan(e*x/sqrt(d*e))/(sqrt(d*e)*d^2*e) - 1/2
*(c*d^2*x^2 - b*d*e*x^2 + 3*a*e^2*x^2 + 2*a*d*e)/((e*x^3 + d*x)*d^2*e)
```

Mupad [B] (verification not implemented)

Time = 7.63 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.91

$$\int \frac{a+bx^2+cx^4}{x^2\left(d+ex^2\right)^2}\,dx=\frac{\operatorname{atan}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)\,\left(c\,d^2+b\,d\,e-3\,a\,e^2\right)}{2\,d^{5/2}\,e^{3/2}}-\frac{\frac{a}{d}+\frac{x^2\,\left(c\,d^2-b\,d\,e+3\,a\,e^2\right)}{2\,d^2\,e}}{e\,x^3+d\,x}$$

```
[In] int((a + b*x^2 + c*x^4)/(x^2*(d + e*x^2)^2),x)
```

```
[Out] (atan((e^(1/2)*x)/d^(1/2))*(c*d^2 - 3*a*e^2 + b*d*e))/(2*d^(5/2)*e^(3/2)) -
 (a/d + (x^2*(3*a*e^2 + c*d^2 - b*d*e))/(2*d^2*e))/(d*x + e*x^3)
```

3.285 $\int \frac{a+bx^2+cx^4}{x^4(d+ex^2)^2} dx$

Optimal result

Integrand size = 25, antiderivative size = 106

$$\int \frac{a+bx^2+cx^4}{x^4(d+ex^2)^2} dx = -\frac{a}{3d^2x^3} - \frac{bd-2ae}{d^3x} + \frac{(cd^2-bde+ae^2)x}{2d^3(d+ex^2)} + \frac{(cd^2-e(3bd-5ae))\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{2d^{7/2}\sqrt{e}}$$

```
[Out] -1/3*a/d^2/x^3+(2*a*e-b*d)/d^3/x+1/2*(a*e^2-b*d*e+c*d^2)*x/d^3/(e*x^2+d)+1/
2*(c*d^2-e*(-5*a*e+3*b*d))*arctan(x*e^(1/2)/d^(1/2))/d^(7/2)/e^(1/2)
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {1273, 1275, 211}

$$\int \frac{a+bx^2+cx^4}{x^4(d+ex^2)^2} dx = \frac{\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)(cd^2-e(3bd-5ae))}{2d^{7/2}\sqrt{e}} + \frac{x(ae^2-bde+cd^2)}{2d^3(d+ex^2)} - \frac{bd-2ae}{d^3x} - \frac{a}{3d^2x^3}$$

```
[In] Int[(a + b*x^2 + c*x^4)/(x^4*(d + e*x^2)^2),x]
```

```
[Out] -1/3*a/(d^2*x^3) - (b*d - 2*a*e)/(d^3*x) + ((c*d^2 - b*d*e + a*e^2)*x)/(2*d
^3*(d + e*x^2)) + ((c*d^2 - e*(3*b*d - 5*a*e))*ArcTan[(Sqrt[e]*x)/Sqrt[d]])
/(2*d^(7/2)*Sqrt[e])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1273

```
Int[(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^
4)^(p_.), x_Symbol] :> Simp[(-d)^(m/2 - 1)*(c*d^2 - b*d*e + a*e^2)^p*x*((d
+ e*x^2)^(q + 1)/(2*e^(2*p + m/2)*(q + 1))), x] + Dist[(-d)^(m/2 - 1)/(2*e^
(2*p)*(q + 1)), Int[x^m*(d + e*x^2)^(q + 1)*ExpandToSum[Together[(1/(d + e*
x^2))*(2*(-d)^(-m/2 + 1)*e^(2*p)*(q + 1)*(a + b*x^2 + c*x^4)^p - ((c*d^2 -
b*d*e + a*e^2)^p/(e^(m/2)*x^m))*(d + e*(2*q + 3)*x^2))], x], x], x] /; Free
Q[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p, 0] && ILtQ[q, -1] &
& ILtQ[m/2, 0]
```

Rule 1275

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*
(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && NeQ[
b^2 - 4*a*c, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{(cd^2-bde+ae^2)\,x}{2d^3\,(d+ex^2)}+\frac{\int \frac{2ad^2e^2+2de^2(bd-ae)x^2+e^2(cd^2-bde+ae^2)x^4}{x^4(d+ex^2)}\,dx}{2d^3e^2}\\
&= \frac{(cd^2-bde+ae^2)\,x}{2d^3\,(d+ex^2)}+\frac{\int \left(\frac{2ade^2}{x^4}-\frac{2e^2(-bd+2ae)}{x^2}+\frac{e^2(cd^2-e(3bd-5ae))}{d+ex^2}\right)\,dx}{2d^3e^2}\\
&= -\frac{a}{3d^2x^3}-\frac{bd-2ae}{d^3x}+\frac{(cd^2-bde+ae^2)\,x}{2d^3\,(d+ex^2)}+\frac{(cd^2-e(3bd-5ae))\int \frac{1}{d+ex^2}\,dx}{2d^3}\\
&= -\frac{a}{3d^2x^3}-\frac{bd-2ae}{d^3x}+\frac{(cd^2-bde+ae^2)\,x}{2d^3\,(d+ex^2)}+\frac{(cd^2-e(3bd-5ae))\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{2d^{7/2}\sqrt{e}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.99

$$\int \frac{a+bx^2+cx^4}{x^4\,(d+ex^2)^2}\,dx = -\frac{a}{3d^2x^3}+\frac{-bd+2ae}{d^3x}+\frac{(cd^2-bde+ae^2)\,x}{2d^3\,(d+ex^2)}+\frac{(cd^2-3bde+5ae^2)\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{2d^{7/2}\sqrt{e}}$$

```
[In] Integrate[(a + b*x^2 + c*x^4)/(x^4*(d + e*x^2)^2),x]
[Out] -1/3*a/(d^2*x^3) + (-(b*d) + 2*a*e)/(d^3*x) + ((c*d^2 - b*d*e + a*e^2)*x)/(
2*d^3*(d + e*x^2)) + ((c*d^2 - 3*b*d*e + 5*a*e^2)*ArcTan[(Sqrt[e]*x)/Sqrt[d
]])/(2*d^(7/2)*Sqrt[e])
```

Maple [A] (verified)

Time = 0.34 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.89

method	result
default	$-\frac{a}{3d^2x^3}-\frac{-2ae+bd}{d^3x}+\frac{\frac{\left(\frac{1}{2}a\,e^2-\frac{1}{2}bde+\frac{1}{2}c\,d^2\right)x}{e\,x^2+d}+\frac{\left(5a\,e^2-3bde+c\,d^2\right)\arctan\left(\frac{ex}{\sqrt{ed}}\right)}{2\sqrt{ed}}}{d^3}$
risch	$\frac{\frac{\left(5a\,e^2-3bde+c\,d^2\right)x^4}{2d^3}+\frac{(5ae-3bd)x^2}{3d^2}-\frac{a}{3d}}{x^3(e\,x^2+d)}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(d^7\_Z^2e+25a^2e^4-30abd\,e^3+10ac\,d^2e^2+9b^2d^2e^2-6bc\,d^3e+c^2d^4\right)}\_R\ln\left(\left(3\_\right.\right.\right.}{}$

```
[In] int((c*x^4+b*x^2+a)/x^4/(e*x^2+d)^2,x,method=_RETURNVERBOSE)
[Out] -1/3*a/d^2/x^3-(-2*a*e+b*d)/d^3/x+1/d^3*((1/2*a*e^2-1/2*b*d*e+1/2*c*d^2)*x/
(e*x^2+d)+1/2*(5*a*e^2-3*b*d*e+c*d^2)/(e*d)^(1/2)*arctan(e*x/(e*d)^(1/2)))
```

Fricas [A] (verification not implemented)

none

Time = 0.35 (sec) , antiderivative size = 316, normalized size of antiderivative = 2.98

$$\int \frac{a+bx^2+cx^4}{x^4\left(d+ex^2\right)^2}\,dx$$
$$=\left[-\frac{4\,ad^3e-6\,(cd^3e-3\,bd^2e^2+5\,ade^3)x^4+4\,(3\,bd^3e-5\,ad^2e^2)x^2+3\,((cd^2e-3\,bde^2+5\,ae^3)x^5+(cd^3-\right.}{12\,(d^4e^2x^5+d^5ex^3)}$$
$$-\frac{2\,ad^3e-3\,(cd^3e-3\,bd^2e^2+5\,ade^3)x^4+2\,(3\,bd^3e-5\,ad^2e^2)x^2-3\,((cd^2e-3\,bde^2+5\,ae^3)x^5+(cd^3-}{6\,(d^4e^2x^5+d^5ex^3)}$$

```
[In] integrate((c*x^4+b*x^2+a)/x^4/(e*x^2+d)^2,x, algorithm="fricas")
[Out] [-1/12*(4*a*d^3*e - 6*(c*d^3*e - 3*b*d^2*e^2 + 5*a*d*e^3)*x^4 + 4*(3*b*d^3*
e - 5*a*d^2*e^2)*x^2 + 3*((c*d^2*e - 3*b*d*e^2 + 5*a*e^3)*x^5 + (c*d^3 - 3*
b*d^2*e + 5*a*d*e^2)*x^3)*sqrt(-d*e)*log((e*x^2 - 2*sqrt(-d*e)*x - d)/(e*x^
2 + d)))/(d^4*e^2*x^5 + d^5*e*x^3), -1/6*(2*a*d^3*e - 3*(c*d^3*e - 3*b*d^2*
e^2 + 5*a*d*e^3)*x^4 + 2*(3*b*d^3*e - 5*a*d^2*e^2)*x^2 - 3*((c*d^2*e - 3*b*
d*e^2 + 5*a*e^3)*x^5 + (c*d^3 - 3*b*d^2*e + 5*a*d*e^2)*x^3)*sqrt(d*e)*arcta
n(sqrt(d*e)*x/d))/(d^4*e^2*x^5 + d^5*e*x^3)]
```

Sympy [A] (verification not implemented)

Time = 0.77 (sec) , antiderivative size = 167, normalized size of antiderivative = 1.58

$$\int \frac{a+bx^2+cx^4}{x^4\left(d+ex^2\right)^2}\,dx = -\frac{\sqrt{-\frac{1}{d^7e}}\left(5ae^2-3bde+cd^2\right)\log\left(-d^4\sqrt{-\frac{1}{d^7e}}+x\right)}{4} + \frac{\sqrt{-\frac{1}{d^7e}}\left(5ae^2-3bde+cd^2\right)\log\left(d^4\sqrt{-\frac{1}{d^7e}}+x\right)}{4} + \frac{-2ad^2+x^4\cdot\left(15ae^2-9bde+3cd^2\right)+x^2\cdot\left(10ade-6bd^2\right)}{6d^4x^3+6d^3ex^5}$$

```
[In] integrate((c*x**4+b*x**2+a)/x**4/(e*x**2+d)**2,x)
```

```
[Out] -sqrt(-1/(d**7*e))*(5*a*e**2 - 3*b*d*e + c*d**2)*log(-d**4*sqrt(-1/(d**7*e)
) + x)/4 + sqrt(-1/(d**7*e))*(5*a*e**2 - 3*b*d*e + c*d**2)*log(d**4*sqrt(-1
/(d**7*e)) + x)/4 + (-2*a*d**2 + x**4*(15*a*e**2 - 9*b*d*e + 3*c*d**2) + x*
*2*(10*a*d*e - 6*b*d**2))/(6*d**4*x**3 + 6*d**3*e*x**5)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{a+bx^2+cx^4}{x^4\left(d+ex^2\right)^2}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((c*x^4+b*x^2+a)/x^4/(e*x^2+d)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.91

$$\int \frac{a+bx^2+cx^4}{x^4\left(d+ex^2\right)^2}\,dx = \frac{\left(cd^2-3\,bde+5\,ae^2\right)\arctan\left(\frac{ex}{\sqrt{de}}\right)}{2\,\sqrt{de}d^3} + \frac{cd^2x-bdex+ae^2x}{2\,(ex^2+d)d^3} - \frac{3\,bdx^2-6\,aex^2+ad}{3\,d^3x^3}$$

```
[In] integrate((c*x^4+b*x^2+a)/x^4/(e*x^2+d)^2,x, algorithm="giac")
```

```
[Out] 1/2*(c*d^2 - 3*b*d*e + 5*a*e^2)*arctan(e*x/sqrt(d*e))/(sqrt(d*e)*d^3) + 1/2
*(c*d^2*x - b*d*e*x + a*e^2*x)/((e*x^2 + d)*d^3) - 1/3*(3*b*d*x^2 - 6*a*e*x
^2 + a*d)/(d^3*x^3)
```

Mupad [B] (verification not implemented)

Time = 7.64 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.92

$$\int \frac{a+bx^2+cx^4}{x^4\left(d+ex^2\right)^2}\,dx=\frac{\frac{x^2\,\left(5\,a\,e-3\,b\,d\right)}{3\,d^2}-\frac{a}{3\,d}+\frac{x^4\,\left(c\,d^2-3\,b\,d\,e+5\,a\,e^2\right)}{2\,d^3}}{e\,x^5+d\,x^3}+\frac{\mathrm{atan}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)\,\left(c\,d^2-3\,b\,d\,e+5\,a\,e^2\right)}{2\,d^{7/2}\,\sqrt{e}}$$

```
[In] int((a + b*x^2 + c*x^4)/(x^4*(d + e*x^2)^2),x)
```

```
[Out] ((x^2*(5*a*e - 3*b*d))/(3*d^2) - a/(3*d) + (x^4*(5*a*e^2 + c*d^2 - 3*b*d*e)
)/(2*d^3))/(d*x^3 + e*x^5) + (atan((e^(1/2)*x)/d^(1/2))*(5*a*e^2 + c*d^2 -
3*b*d*e))/(2*d^(7/2)*e^(1/2))
```

3.286 $\int \frac{a+bx^2+cx^4}{x^6\left(d+ex^2\right)^2}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 136

$$\int \frac{a+bx^2+cx^4}{x^6\left(d+ex^2\right)^2}\,dx = -\frac{a}{5d^2x^5}-\frac{bd-2ae}{3d^3x^3}-\frac{cd^2-e(2bd-3ae)}{d^4x} -\frac{e(cd^2-bde+ae^2)\,x}{2d^4\left(d+ex^2\right)}-\frac{\sqrt{e}(3cd^2-e(5bd-7ae))\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{2d^{9/2}}$$

```
[Out] -1/5*a/d^2/x^5+1/3*(2*a*e-b*d)/d^3/x^3+(-c*d^2+e*(-3*a*e+2*b*d))/d^4/x-1/2*
e*(a*e^2-b*d*e+c*d^2)*x/d^4/(e*x^2+d)-1/2*(3*c*d^2-e*(-7*a*e+5*b*d))*arctan
(x*e^(1/2)/d^(1/2))*e^(1/2)/d^(9/2)
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {1273, 1816, 211}

$$\int \frac{a+bx^2+cx^4}{x^6\left(d+ex^2\right)^2}\,dx = -\frac{\sqrt{e}\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)(3cd^2-e(5bd-7ae))}{2d^{9/2}}-\frac{ex(ae^2-bde+cd^2)}{2d^4\left(d+ex^2\right)} -\frac{cd^2-e(2bd-3ae)}{d^4x}-\frac{bd-2ae}{3d^3x^3}-\frac{a}{5d^2x^5}$$

```
[In] Int[(a + b*x^2 + c*x^4)/(x^6*(d + e*x^2)^2),x]
```

```
[Out] -1/5*a/(d^2*x^5) - (b*d - 2*a*e)/(3*d^3*x^3) - (c*d^2 - e*(2*b*d - 3*a*e))/
(d^4*x) - (e*(c*d^2 - b*d*e + a*e^2)*x)/(2*d^4*(d + e*x^2)) - (Sqrt[e]*(3*c
*d^2 - e*(5*b*d - 7*a*e))*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(2*d^(9/2))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1273

```
Int[(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^
4)^(p_.), x_Symbol] :> Simp[(-d)^(m/2 - 1)*(c*d^2 - b*d*e + a*e^2)^p*x*((d
+ e*x^2)^(q + 1)/(2*e^(2*p + m/2)*(q + 1))), x] + Dist[(-d)^(m/2 - 1)/(2*e^
(2*p)*(q + 1)), Int[x^m*(d + e*x^2)^(q + 1)*ExpandToSum[Together[(1/(d + e*
x^2))*(2*(-d)^(-m/2 + 1)*e^(2*p)*(q + 1)*(a + b*x^2 + c*x^4)^p - ((c*d^2 -
b*d*e + a*e^2)^p/(e^(m/2)*x^m))*(d + e*(2*q + 3)*x^2))], x], x], x] /; Free
Q[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p, 0] && ILtQ[q, -1] &
& ILtQ[m/2, 0]
```

Rule 1816

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

integral

$$= -\frac{e(cd^2-bde+ae^2)\,x}{2d^4\,(d+ex^2)} - \frac{\int \frac{-2ad^3e^2-2d^2e^2(bd-ae)x^2-2de^2(cd^2-bde+ae^2)x^4+e^3(cd^2-bde+ae^2)x^6}{x^6(d+ex^2)}\,dx}{2d^4e^2}$$

$$= -\frac{e(cd^2-bde+ae^2)\,x}{2d^4\,(d+ex^2)} - \frac{\int \left(-\frac{2ad^2e^2}{x^6} - \frac{2de^2(bd-2ae)}{x^4} + \frac{2e^2(-cd^2+e(2bd-3ae))}{x^2} + \frac{e^3(3cd^2-e(5bd-7ae))}{d+ex^2}\right)dx}{2d^4e^2}$$

$$= -\frac{a}{5d^2x^5} - \frac{bd-2ae}{3d^3x^3} - \frac{cd^2-e(2bd-3ae)}{d^4x} - \frac{e(cd^2-bde+ae^2)\,x}{2d^4\,(d+ex^2)} - \frac{(e(3cd^2-e(5bd-7ae)))\int \frac{1}{d+ex^2}\,dx}{2d^4}$$

$$= -\frac{a}{5d^2x^5} - \frac{bd-2ae}{3d^3x^3} - \frac{cd^2-e(2bd-3ae)}{d^4x} - \frac{e(cd^2-bde+ae^2)\,x}{2d^4\,(d+ex^2)} - \frac{\sqrt{e}(3cd^2-e(5bd-7ae))\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{2d^{9/2}}$$

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 135, normalized size of antiderivative = 0.99

$$\int \frac{a+bx^2+cx^4}{x^6(d+ex^2)^2}\,dx = -\frac{a}{5d^2x^5}+\frac{-bd+2ae}{3d^3x^3}+\frac{-cd^2+2bde-3ae^2}{d^4x} - \frac{e(cd^2-bde+ae^2)\,x}{2d^4(d+ex^2)} - \frac{\sqrt{e}(3cd^2-5bde+7ae^2)\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{2d^{9/2}}$$

```
[In] Integrate[(a + b*x^2 + c*x^4)/(x^6*(d + e*x^2)^2),x]
```

```
[Out] -1/5*a/(d^2*x^5) + (-(b*d) + 2*a*e)/(3*d^3*x^3) + (-(c*d^2) + 2*b*d*e - 3*a
*e^2)/(d^4*x) - (e*(c*d^2 - b*d*e + a*e^2)*x)/(2*d^4*(d + e*x^2)) - (Sqrt[e
]*(3*c*d^2 - 5*b*d*e + 7*a*e^2)*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(2*d^(9/2))
```

Maple [A] (verified)

Time = 0.34 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.90

method	result
default	$-\frac{a}{5d^2x^5}-\frac{-2ae+bd}{3d^3x^3}-\frac{3a\,e^2-2bde+c\,d^2}{d^4x}-\frac{e\left(\frac{\left(\frac{1}{2}a\,e^2-\frac{1}{2}bde+\frac{1}{2}c\,d^2\right)x}{e\,x^2+d}+\frac{\left(7a\,e^2-5bde+3c\,d^2\right)\arctan\left(\frac{ex}{\sqrt{ed}}\right)}{2\sqrt{ed}}\right)}{d^4}$
risch	$\frac{-\frac{e\left(7a\,e^2-5bde+3c\,d^2\right)x^6}{2d^4}-\frac{\left(7a\,e^2-5bde+3c\,d^2\right)x^4}{3d^3}+\frac{(7ae-5bd)x^2}{15d^2}-\frac{a}{5d}}{x^5(e\,x^2+d)}+\left(\sum_{\_R=\mathrm{RootOf}\left(d^9\_Z^2+49a^2e^5-70abd\,e^4+42ac\,d^2e^3+25b^2d^2e^3-30\right.}\right.$

```
[In] int((c*x^4+b*x^2+a)/x^6/(e*x^2+d)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/5*a/d^2/x^5-1/3*(-2*a*e+b*d)/d^3/x^3-(3*a*e^2-2*b*d*e+c*d^2)/d^4/x-e/d^4
*((1/2*a*e^2-1/2*b*d*e+1/2*c*d^2)*x/(e*x^2+d)+1/2*(7*a*e^2-5*b*d*e+3*c*d^2)
/(e*d)^(1/2)*arctan(e*x/(e*d)^(1/2)))
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 360, normalized size of antiderivative = 2.65

$$\int \frac{a+bx^2+cx^4}{x^6 (d+ex^2)^2} dx$$
$$= \left[-\frac{30\,(3\,cd^2e-5\,bde^2+7\,ae^3)x^6+20\,(3\,cd^3-5\,bd^2e+7\,ade^2)x^4+12\,ad^3+4\,(5\,bd^3-7\,ad^2e)x^2-15\,((3}{60\,(d^4ex^7+d^5x^5)} \right.$$

$$-\frac{15\,(3\,cd^2e-5\,bde^2+7\,ae^3)x^6+10\,(3\,cd^3-5\,bd^2e+7\,ade^2)x^4+6\,ad^3+2\,(5\,bd^3-7\,ad^2e)x^2+15\,((3\,c}{30\,(d^4ex^7+d^5x^5)}$$

```
[In] integrate((c*x^4+b*x^2+a)/x^6/(e*x^2+d)^2,x, algorithm="fricas")
```

```
[Out] [-1/60*(30*(3*c*d^2*e - 5*b*d*e^2 + 7*a*e^3)*x^6 + 20*(3*c*d^3 - 5*b*d^2*e
+ 7*a*d*e^2)*x^4 + 12*a*d^3 + 4*(5*b*d^3 - 7*a*d^2*e)*x^2 - 15*((3*c*d^2*e
- 5*b*d*e^2 + 7*a*e^3)*x^7 + (3*c*d^3 - 5*b*d^2*e + 7*a*d*e^2)*x^5)*sqrt(-e
/d)*log((e*x^2 - 2*d*x*sqrt(-e/d) - d)/(e*x^2 + d)))/(d^4*e*x^7 + d^5*x^5),
 -1/30*(15*(3*c*d^2*e - 5*b*d*e^2 + 7*a*e^3)*x^6 + 10*(3*c*d^3 - 5*b*d^2*e
+ 7*a*d*e^2)*x^4 + 6*a*d^3 + 2*(5*b*d^3 - 7*a*d^2*e)*x^2 + 15*((3*c*d^2*e -
 5*b*d*e^2 + 7*a*e^3)*x^7 + (3*c*d^3 - 5*b*d^2*e + 7*a*d*e^2)*x^5)*sqrt(e/d
)*arctan(x*sqrt(e/d)))/(d^4*e*x^7 + d^5*x^5)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 284 vs. $2(126) = 252$.

Time = 1.05 (sec) , antiderivative size = 284, normalized size of antiderivative = 2.09

$$\int \frac{a+bx^2+cx^4}{x^6 (d+ex^2)^2} dx = \frac{\sqrt{-\frac{e}{d^9}}\cdot(7ae^2-5bde+3cd^2)\log\left(-\frac{d^5\sqrt{-\frac{e}{d^9}}\cdot(7ae^2-5bde+3cd^2)}{7ae^3-5bde^2+3cd^2e}+x\right)}{4}$$
$$-\frac{\sqrt{-\frac{e}{d^9}}\cdot(7ae^2-5bde+3cd^2)\log\left(\frac{d^5\sqrt{-\frac{e}{d^9}}\cdot(7ae^2-5bde+3cd^2)}{7ae^3-5bde^2+3cd^2e}+x\right)}{4}$$
$$+\frac{-6ad^3+x^6(-105ae^3+75bde^2-45cd^2e)+x^4(-70ade^2+50bd^2e-30cd^3)+x^2\cdot(14ad^2e-10bd^3)}{30d^5x^5+30d^4ex^7}$$

```
[In] integrate((c*x**4+b*x**2+a)/x**6/(e*x**2+d)**2,x)
```

```
[Out] sqrt(-e/d**9)*(7*a*e**2 - 5*b*d*e + 3*c*d**2)*log(-d**5*sqrt(-e/d**9)*(7*a*
e**2 - 5*b*d*e + 3*c*d**2)/(7*a*e**3 - 5*b*d*e**2 + 3*c*d**2*e) + x)/4 - sq
```

```
rt(-e/d**9)*(7*a*e**2 - 5*b*d*e + 3*c*d**2)*log(d**5*sqrt(-e/d**9)*(7*a*e**
2 - 5*b*d*e + 3*c*d**2)/(7*a*e**3 - 5*b*d*e**2 + 3*c*d**2*e) + x)/4 + (-6*a
*d**3 + x**6*(-105*a*e**3 + 75*b*d*e**2 - 45*c*d**2*e) + x**4*(-70*a*d*e**2
 + 50*b*d**2*e - 30*c*d**3) + x**2*(14*a*d**2*e - 10*b*d**3))/(30*d**5*x**5
 + 30*d**4*e*x**7)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{a+bx^2+cx^4}{x^6\left(d+ex^2\right)^2}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((c*x^4+b*x^2+a)/x^6/(e*x^2+d)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 135, normalized size of antiderivative = 0.99

$$\int \frac{a+bx^2+cx^4}{x^6\left(d+ex^2\right)^2}\,dx = -\frac{\left(3\,cd^2e-5\,bde^2+7\,ae^3\right)\arctan\left(\frac{ex}{\sqrt{de}}\right)}{2\,\sqrt{de}d^4}-\frac{cd^2ex-bde^2x+ae^3x}{2\,(ex^2+d)d^4}-\frac{15\,cd^2x^4-30\,bdex^4+45\,ae^2x^4+5\,bd^2x^2-10\,adex^2+3\,ad^2}{15\,d^4x^5}$$

```
[In] integrate((c*x^4+b*x^2+a)/x^6/(e*x^2+d)^2,x, algorithm="giac")
```

```
[Out] -1/2*(3*c*d^2*e - 5*b*d*e^2 + 7*a*e^3)*arctan(e*x/sqrt(d*e))/(sqrt(d*e)*d^4
) - 1/2*(c*d^2*e*x - b*d*e^2*x + a*e^3*x)/((e*x^2 + d)*d^4) - 1/15*(15*c*d^
2*x^4 - 30*b*d*e*x^4 + 45*a*e^2*x^4 + 5*b*d^2*x^2 - 10*a*d*e*x^2 + 3*a*d^2)
/(d^4*x^5)
```

Mupad [B] (verification not implemented)

Time = 7.70 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.94

$$\int \frac{a+bx^2+cx^4}{x^6\left(d+ex^2\right)^2}\,dx=-\frac{\frac{a}{5\,d}-\frac{x^2\,\left(7\,a\,e-5\,b\,d\right)}{15\,d^2}+\frac{x^4\,\left(3\,c\,d^2-5\,b\,d\,e+7\,a\,e^2\right)}{3\,d^3}+\frac{e\,x^6\,\left(3\,c\,d^2-5\,b\,d\,e+7\,a\,e^2\right)}{2\,d^4}}{e\,x^7+d\,x^5}-\frac{\sqrt{e}\,\mathrm{atan}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)\,\left(3\,c\,d^2-5\,b\,d\,e+7\,a\,e^2\right)}{2\,d^{9/2}}$$

```
[In] int((a + b*x^2 + c*x^4)/(x^6*(d + e*x^2)^2),x)
```

```
[Out] - (a/(5*d) - (x^2*(7*a*e - 5*b*d))/(15*d^2) + (x^4*(7*a*e^2 + 3*c*d^2 - 5*b
*d*e))/(3*d^3) + (e*x^6*(7*a*e^2 + 3*c*d^2 - 5*b*d*e))/(2*d^4))/(d*x^5 + e*
x^7) - (e^(1/2)*atan((e^(1/2)*x)/d^(1/2))*(7*a*e^2 + 3*c*d^2 - 5*b*d*e))/(2
*d^(9/2))
```

3.287 $\int \frac{a+bx^2+cx^4}{x^8(d+ex^2)^2}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 167

$$\int \frac{a+bx^2+cx^4}{x^8(d+ex^2)^2}\,dx = -\frac{a}{7d^2x^7}-\frac{bd-2ae}{5d^3x^5}-\frac{cd^2-e(2bd-3ae)}{3d^4x^3}+\frac{e(2cd^2-e(3bd-4ae))}{d^5x}+\frac{e^2(cd^2-bde+ae^2)\,x}{2d^5(d+ex^2)}+\frac{e^{3/2}(5cd^2-e(7bd-9ae))\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{2d^{11/2}}$$

```
[Out] -1/7*a/d^2/x^7+1/5*(2*a*e-b*d)/d^3/x^5+1/3*(-c*d^2+e*(-3*a*e+2*b*d))/d^4/x^
3+e*(2*c*d^2-e*(-4*a*e+3*b*d))/d^5/x+1/2*e^2*(a*e^2-b*d*e+c*d^2)*x/d^5/(e*x
^2+d)+1/2*e^(3/2)*(5*c*d^2-e*(-9*a*e+7*b*d))*arctan(x*e^(1/2)/d^(1/2))/d^(1
1/2)
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 167, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {1273, 1816, 211}

$$\int \frac{a+bx^2+cx^4}{x^8(d+ex^2)^2}\,dx = \frac{e^{3/2}\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)(5cd^2-e(7bd-9ae))}{2d^{11/2}}+\frac{e^2x(ae^2-bde+cd^2)}{2d^5(d+ex^2)}+\frac{e(2cd^2-e(3bd-4ae))}{d^5x}-\frac{cd^2-e(2bd-3ae)}{3d^4x^3}-\frac{bd-2ae}{5d^3x^5}-\frac{a}{7d^2x^7}$$

```
[In] Int[(a + b*x^2 + c*x^4)/(x^8*(d + e*x^2)^2),x]
```

```
[Out] -1/7*a/(d^2*x^7) - (b*d - 2*a*e)/(5*d^3*x^5) - (c*d^2 - e*(2*b*d - 3*a*e))/
(3*d^4*x^3) + (e*(2*c*d^2 - e*(3*b*d - 4*a*e)))/(d^5*x) + (e^2*(c*d^2 - b*d
```

```
*e + a*e^2)*x)/(2*d^5*(d + e*x^2)) + (e^(3/2)*(5*c*d^2 - e*(7*b*d - 9*a*e))
*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(2*d^(11/2))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1273

```
Int[(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^
4)^(p_.), x_Symbol] :> Simp[(-d)^(m/2 - 1)*(c*d^2 - b*d*e + a*e^2)^p*x*((d
+ e*x^2)^(q + 1)/(2*e^(2*p + m/2)*(q + 1))), x] + Dist[(-d)^(m/2 - 1)/(2*e^
(2*p)*(q + 1)), Int[x^m*(d + e*x^2)^(q + 1)*ExpandToSum[Together[(1/(d + e*
x^2))*(2*(-d)^(-m/2 + 1)*e^(2*p)*(q + 1)*(a + b*x^2 + c*x^4)^p - ((c*d^2 -
b*d*e + a*e^2)^p/(e^(m/2)*x^m))*(d + e*(2*q + 3)*x^2))], x], x], x] /; Free
Q[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p, 0] && ILtQ[q, -1] &
& ILtQ[m/2, 0]
```

Rule 1816

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{e^2(cd^2-bde+ae^2)\,x}{2d^5\,(d+ex^2)} + \frac{\int \frac{2ad^4e^2+2d^3e^2(bd-ae)x^2+2d^2e^2(cd^2-bde+ae^2)x^4-2de^3(cd^2-bde+ae^2)x^6+e^4(cd^2-bde+ae^2)x^8}{x^8(d+ex^2)}\,dx}{2d^5e^2}\\
&= \frac{e^2(cd^2-bde+ae^2)\,x}{2d^5\,(d+ex^2)} + \frac{\int \left(\frac{2ad^3e^2}{x^8}+\frac{2d^2e^2(bd-2ae)}{x^6}+\frac{2de^2(cd^2-e(2bd-3ae))}{x^4}+\frac{2e^3(-2cd^2+e(3bd-4ae))}{x^2}+\frac{e^4(5cd^2-e(7bd-9ae))}{d+ex^2}\right)dx}{2d^5e^2}\\
&= -\frac{a}{7d^2x^7}-\frac{bd-2ae}{5d^3x^5}-\frac{cd^2-e(2bd-3ae)}{3d^4x^3}+\frac{e(2cd^2-e(3bd-4ae))}{d^5x} + \frac{e^2(cd^2-bde+ae^2)\,x}{2d^5\,(d+ex^2)}+\frac{(e^2(5cd^2-e(7bd-9ae)))\int\frac{1}{d+ex^2}\,dx}{2d^5}\\
&= -\frac{a}{7d^2x^7}-\frac{bd-2ae}{5d^3x^5}-\frac{cd^2-e(2bd-3ae)}{3d^4x^3}+\frac{e(2cd^2-e(3bd-4ae))}{d^5x} + \frac{e^2(cd^2-bde+ae^2)\,x}{2d^5\,(d+ex^2)}+\frac{e^{3/2}(5cd^2-e(7bd-9ae))\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{2d^{11/2}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 166, normalized size of antiderivative = 0.99

$$\int \frac{a+bx^2+cx^4}{x^8 (d+ex^2)^2}\,dx = -\frac{a}{7d^2x^7}+\frac{-bd+2ae}{5d^3x^5}+\frac{-cd^2+2bde-3ae^2}{3d^4x^3}+\frac{e(2cd^2-3bde+4ae^2)}{d^5x} + \frac{e^2(cd^2-bde+ae^2)\,x}{2d^5\,(d+ex^2)}+\frac{e^{3/2}(5cd^2-7bde+9ae^2)\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{2d^{11/2}}$$

```
[In] Integrate[(a + b*x^2 + c*x^4)/(x^8*(d + e*x^2)^2),x]
```

```
[Out] -1/7*a/(d^2*x^7) + (-(b*d) + 2*a*e)/(5*d^3*x^5) + (-(c*d^2) + 2*b*d*e - 3*a
*e^2)/(3*d^4*x^3) + (e*(2*c*d^2 - 3*b*d*e + 4*a*e^2))/(d^5*x) + (e^2*(c*d^2
 - b*d*e + a*e^2)*x)/(2*d^5*(d + e*x^2)) + (e^(3/2)*(5*c*d^2 - 7*b*d*e + 9*
a*e^2)*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(2*d^(11/2))
```

Maple [A] (verified)

Time = 0.35 (sec) , antiderivative size = 149, normalized size of antiderivative = 0.89

method	result
default	$-\frac{a}{7d^2x^7}-\frac{-2ae+bd}{5d^3x^5}-\frac{3a\,e^2-2bde+c\,d^2}{3d^4x^3}+\frac{e(4a\,e^2-3bde+2c\,d^2)}{d^5x}+\frac{e^2\left(\frac{\left(\frac{1}{2}a\,e^2-\frac{1}{2}bde+\frac{1}{2}c\,d^2\right)x}{e\,x^2+d}+\frac{\left(9a\,e^2-7bde+5c\,d^2\right)\arctan\left(\frac{ex}{\sqrt{ed}}\right.}{2\sqrt{ed}}\right.}{d^5}$
risch	$\frac{\frac{e^2\left(9a\,e^2-7bde+5c\,d^2\right)x^8}{2d^5}+\frac{e\left(9a\,e^2-7bde+5c\,d^2\right)x^6}{3d^4}-\frac{\left(9a\,e^2-7bde+5c\,d^2\right)x^4}{15d^3}+\frac{(9ae-7bd)x^2}{35d^2}-\frac{a}{7d}}{x^7(e\,x^2+d)}+\frac{\left(\_R=\text{RootOf}\left(d^{11}\_Z^2+81a^2e^7-126abd\,e\right.\right.}{}$

```
[In] int((c*x^4+b*x^2+a)/x^8/(e*x^2+d)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/7*a/d^2/x^7-1/5*(-2*a*e+b*d)/d^3/x^5-1/3*(3*a*e^2-2*b*d*e+c*d^2)/d^4/x^3
+e*(4*a*e^2-3*b*d*e+2*c*d^2)/d^5/x+e^2/d^5*((1/2*a*e^2-1/2*b*d*e+1/2*c*d^2)
*x/(e*x^2+d)+1/2*(9*a*e^2-7*b*d*e+5*c*d^2)/(e*d)^(1/2)*arctan(e*x/(e*d)^(1/
2)))
```

Fricas [A] (verification not implemented)

none

Time = 0.34 (sec) , antiderivative size = 436, normalized size of antiderivative = 2.61

$$\int \frac{a+bx^2+cx^4}{x^8\left(d+ex^2\right)^2}\,dx$$
$$= \left[\frac{210\,(5\,cd^2e^2-7\,bde^3+9\,ae^4)x^8+140\,(5\,cd^3e-7\,bd^2e^2+9\,ade^3)x^6-60\,ad^4-28\,(5\,cd^4-7\,bd^3e+9\,ad\right.$$

```
[In] integrate((c*x^4+b*x^2+a)/x^8/(e*x^2+d)^2,x, algorithm="fricas")
```

```
[Out] [1/420*(210*(5*c*d^2*e^2 - 7*b*d*e^3 + 9*a*e^4)*x^8 + 140*(5*c*d^3*e - 7*b*
d^2*e^2 + 9*a*d*e^3)*x^6 - 60*a*d^4 - 28*(5*c*d^4 - 7*b*d^3*e + 9*a*d^2*e^2
)*x^4 - 12*(7*b*d^4 - 9*a*d^3*e)*x^2 + 105*((5*c*d^2*e^2 - 7*b*d*e^3 + 9*a*
e^4)*x^9 + (5*c*d^3*e - 7*b*d^2*e^2 + 9*a*d*e^3)*x^7)*sqrt(-e/d)*log((e*x^2
 + 2*d*x*sqrt(-e/d) - d)/(e*x^2 + d)))/(d^5*e*x^9 + d^6*x^7), 1/210*(105*(5
*c*d^2*e^2 - 7*b*d*e^3 + 9*a*e^4)*x^8 + 70*(5*c*d^3*e - 7*b*d^2*e^2 + 9*a*d
*e^3)*x^6 - 30*a*d^4 - 14*(5*c*d^4 - 7*b*d^3*e + 9*a*d^2*e^2)*x^4 - 6*(7*b*
d^4 - 9*a*d^3*e)*x^2 + 105*((5*c*d^2*e^2 - 7*b*d*e^3 + 9*a*e^4)*x^9 + (5*c*
d^3*e - 7*b*d^2*e^2 + 9*a*d*e^3)*x^7)*sqrt(e/d)*arctan(x*sqrt(e/d)))/(d^5*e
*x^9 + d^6*x^7)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 328 vs. $2(156) = 312$.

Time = 1.25 (sec) , antiderivative size = 328, normalized size of antiderivative = 1.96

$$\int \frac{a+bx^2+cx^4}{x^8\left(d+ex^2\right)^2}\,dx = -\frac{\sqrt{-\frac{e^3}{d^{11}}}\cdot(9ae^2-7bde+5cd^2)\log\left(-\frac{d^6\sqrt{-\frac{e^3}{d^{11}}}\cdot(9ae^2-7bde+5cd^2)}{9ae^4-7bde^3+5cd^2e^2}+x\right)}{4}$$
$$+\frac{\sqrt{-\frac{e^3}{d^{11}}}\cdot(9ae^2-7bde+5cd^2)\log\left(\frac{d^6\sqrt{-\frac{e^3}{d^{11}}}\cdot(9ae^2-7bde+5cd^2)}{9ae^4-7bde^3+5cd^2e^2}+x\right)}{4}$$
$$+\frac{-30ad^4+x^8\cdot(945ae^4-735bde^3+525cd^2e^2)+x^6\cdot(630ade^3-490bd^2e^2+350cd^3e)+x^4(-126ad^2e^2}{210d^6x^7+210d^5ex^9}$$

```
[In] integrate((c*x**4+b*x**2+a)/x**8/(e*x**2+d)**2,x)
```

```
[Out] -sqrt(-e**3/d**11)*(9*a*e**2 - 7*b*d*e + 5*c*d**2)*log(-d**6*sqrt(-e**3/d**
11)*(9*a*e**2 - 7*b*d*e + 5*c*d**2)/(9*a*e**4 - 7*b*d*e**3 + 5*c*d**2*e**2)
```

```
 + x)/4 + sqrt(-e**3/d**11)*(9*a*e**2 - 7*b*d*e + 5*c*d**2)*log(d**6*sqrt(-
e**3/d**11)*(9*a*e**2 - 7*b*d*e + 5*c*d**2)/(9*a*e**4 - 7*b*d*e**3 + 5*c*d*
*2*e**2) + x)/4 + (-30*a*d**4 + x**8*(945*a*e**4 - 735*b*d*e**3 + 525*c*d**
2*e**2) + x**6*(630*a*d*e**3 - 490*b*d**2*e**2 + 350*c*d**3*e) + x**4*(-126
*a*d**2*e**2 + 98*b*d**3*e - 70*c*d**4) + x**2*(54*a*d**3*e - 42*b*d**4))/(
210*d**6*x**7 + 210*d**5*e*x**9)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{a+bx^2+cx^4}{x^8\left(d+ex^2\right)^2}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((c*x^4+b*x^2+a)/x^8/(e*x^2+d)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 173, normalized size of antiderivative = 1.04

$$\int \frac{a+bx^2+cx^4}{x^8\left(d+ex^2\right)^2}\,dx = \frac{\left(5\,cd^2e^2-7\,bde^3+9\,ae^4\right)\arctan\left(\frac{ex}{\sqrt{de}}\right)}{2\,\sqrt{de}d^5} + \frac{cd^2e^2x-bde^3x+ae^4x}{2\,(ex^2+d)d^5} + \frac{210\,cd^2ex^6-315\,bde^2x^6+420\,ae^3x^6-35\,cd^3x^4+70\,bd^2ex^4-105\,ade^2x^4-21\,bd^3x^2+42\,ad^2ex^2-15\,a}{105\,d^5x^7}$$

```
[In] integrate((c*x^4+b*x^2+a)/x^8/(e*x^2+d)^2,x, algorithm="giac")
```

```
[Out] 1/2*(5*c*d^2*e^2 - 7*b*d*e^3 + 9*a*e^4)*arctan(e*x/sqrt(d*e))/(sqrt(d*e)*d^
5) + 1/2*(c*d^2*e^2*x - b*d*e^3*x + a*e^4*x)/((e*x^2 + d)*d^5) + 1/105*(210
*c*d^2*e*x^6 - 315*b*d*e^2*x^6 + 420*a*e^3*x^6 - 35*c*d^3*x^4 + 70*b*d^2*e*
x^4 - 105*a*d*e^2*x^4 - 21*b*d^3*x^2 + 42*a*d^2*e*x^2 - 15*a*d^3)/(d^5*x^7)
```

Mupad [B] (verification not implemented)

Time = 7.80 (sec) , antiderivative size = 156, normalized size of antiderivative = 0.93

$$\int \frac{a+bx^2+cx^4}{x^8\left(d+ex^2\right)^2}\,dx = \frac{\frac{x^2\,\left(9\,a\,e-7\,b\,d\right)}{35\,d^2}-\frac{a}{7\,d}-\frac{x^4\,\left(5\,c\,d^2-7\,b\,d\,e+9\,a\,e^2\right)}{15\,d^3}+\frac{e\,x^6\,\left(5\,c\,d^2-7\,b\,d\,e+9\,a\,e^2\right)}{3\,d^4}+\frac{e^2\,x^8\,\left(5\,c\,d^2-7\,b\,d\,e+9\,a\,e^2\right)}{2\,d^5}}{e\,x^9+d\,x^7} + \frac{e^{3/2}\,\mathrm{atan}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)\,\left(5\,c\,d^2-7\,b\,d\,e+9\,a\,e^2\right)}{2\,d^{11/2}}$$

```
[In] int((a + b*x^2 + c*x^4)/(x^8*(d + e*x^2)^2),x)
```

```
[Out] ((x^2*(9*a*e - 7*b*d))/(35*d^2) - a/(7*d) - (x^4*(9*a*e^2 + 5*c*d^2 - 7*b*d
*e))/(15*d^3) + (e*x^6*(9*a*e^2 + 5*c*d^2 - 7*b*d*e))/(3*d^4) + (e^2*x^8*(9
*a*e^2 + 5*c*d^2 - 7*b*d*e))/(2*d^5))/(d*x^7 + e*x^9) + (e^(3/2)*atan((e^(1
/2)*x)/d^(1/2))*(9*a*e^2 + 5*c*d^2 - 7*b*d*e))/(2*d^(11/2))
```

3.288 $\int \frac{x^6(a+bx^2+cx^4)}{(d+ex^2)^3}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 173

$$\int \frac{x^6(a+bx^2+cx^4)}{(d+ex^2)^3}\,dx = \frac{(6cd^2-e(3bd-ae))\,x}{e^5} - \frac{(3cd-be)x^3}{3e^4} + \frac{cx^5}{5e^3} - \frac{d^2(cd^2-bde+ae^2)\,x}{4e^5\,(d+ex^2)^2} + \frac{d(17cd^2-e(13bd-9ae))\,x}{8e^5\,(d+ex^2)} - \frac{\sqrt{d}(63cd^2-35bde+15ae^2)\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{8e^{11/2}}$$

```
[Out] (6*c*d^2-e*(-a*e+3*b*d))*x/e^5-1/3*(-b*e+3*c*d)*x^3/e^4+1/5*c*x^5/e^3-1/4*d
^2*(a*e^2-b*d*e+c*d^2)*x/e^5/(e*x^2+d)^2+1/8*d*(17*c*d^2-e*(-9*a*e+13*b*d))
*x/e^5/(e*x^2+d)-1/8*(15*a*e^2-35*b*d*e+63*c*d^2)*arctan(x*e^(1/2)/d^(1/2))
*d^(1/2)/e^(11/2)
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 173, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {1271, 1828, 1824, 211}

$$\int \frac{x^6(a+bx^2+cx^4)}{(d+ex^2)^3}\,dx = -\frac{\sqrt{d}\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)(15ae^2-35bde+63cd^2)}{8e^{11/2}} + \frac{dx(17cd^2-e(13bd-9ae))}{8e^5\,(d+ex^2)} + \frac{x(6cd^2-e(3bd-ae))}{e^5} - \frac{d^2x(ae^2-bde+cd^2)}{4e^5\,(d+ex^2)^2} - \frac{x^3(3cd-be)}{3e^4} + \frac{cx^5}{5e^3}$$

```
[In] Int[(x^6*(a + b*x^2 + c*x^4))/(d + e*x^2)^3,x]
```

```
[Out] ((6*c*d^2 - e*(3*b*d - a*e))*x)/e^5 - ((3*c*d - b*e)*x^3)/(3*e^4) + (c*x^5)
/(5*e^3) - (d^2*(c*d^2 - b*d*e + a*e^2)*x)/(4*e^5*(d + e*x^2)^2) + (d*(17*c
*d^2 - e*(13*b*d - 9*a*e))*x)/(8*e^5*(d + e*x^2)) - (Sqrt[d]*(63*c*d^2 - 35
*b*d*e + 15*a*e^2)*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(8*e^(11/2))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1271

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)
^4)^(p_.), x_Symbol] :> Simp[(-d)^(m/2 - 1)*(c*d^2 - b*d*e + a*e^2)^p*x*((d
 + e*x^2)^(q + 1)/(2*e^(2*p + m/2)*(q + 1))), x] + Dist[1/(2*e^(2*p + m/2)*
(q + 1)), Int[(d + e*x^2)^(q + 1)*ExpandToSum[Together[(1/(d + e*x^2))*(2*e
^(2*p + m/2)*(q + 1)*x^m*(a + b*x^2 + c*x^4)^p - (-d)^(m/2 - 1)*(c*d^2 - b*
d*e + a*e^2)^p*(d + e*(2*q + 3)*x^2))], x], x], x] /; FreeQ[{a, b, c, d, e}
, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p, 0] && ILtQ[q, -1] && IGtQ[m/2, 0]
```

Rule 1824

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[Pq*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 1828

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuot
ient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b
*f*x)*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] + Dist[1/(2*a*(p + 1)), Int
[(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Q + f*(2*p + 3), x], x], x]] /
; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]
```

Rubi steps

$$\text{integral} = -\frac{d^2(cd^2-bde+ae^2)\,x}{4e^5\,(d+ex^2)^2} - \frac{\int \frac{-d^2(cd^2-bde+ae^2)+4de(cd^2-bde+ae^2)x^2-4e^2(cd^2-bde+ae^2)x^4+4e^3(cd-be)x^6-4ce^4x^8}{(d+ex^2)^2}\,dx}{4e^5}$$

$$= -\frac{d^2(cd^2-bde+ae^2)\,x}{4e^5\,(d+ex^2)^2} + \frac{d(17cd^2-e(13bd-9ae))\,x}{8e^5\,(d+ex^2)} + \frac{\int \frac{-d^2(15cd^2-e(11bd-7ae))+8de(3cd^2-e(2bd-ae))x^2-8de^2(2cd-be)x^4+8cde^3x^6}{d+ex^2}\,dx}{8de^5}$$

$$= -\frac{d^2(cd^2-bde+ae^2)\,x}{4e^5\,(d+ex^2)^2}+\frac{d(17cd^2-e(13bd-9ae))\,x}{8e^5\,(d+ex^2)} + \frac{\int \left(8d(6cd^2-e(3bd-ae))-8de(3cd-be)x^2+8cde^2x^4+\frac{-63cd^4+35bd^3e-15ad^2e^2}{d+ex^2}\right)dx}{8de^5}$$

$$= \frac{(6cd^2-e(3bd-ae))\,x}{e^5}-\frac{(3cd-be)x^3}{3e^4}+\frac{cx^5}{5e^3}-\frac{d^2(cd^2-bde+ae^2)\,x}{4e^5\,(d+ex^2)^2} + \frac{d(17cd^2-e(13bd-9ae))\,x}{8e^5\,(d+ex^2)}+\frac{(-63cd^4+35bd^3e-15ad^2e^2)\int\frac{1}{d+ex^2}\,dx}{8de^5}$$

$$= \frac{(6cd^2-e(3bd-ae))\,x}{e^5}-\frac{(3cd-be)x^3}{3e^4}+\frac{cx^5}{5e^3}-\frac{d^2(cd^2-bde+ae^2)\,x}{4e^5\,(d+ex^2)^2} + \frac{d(17cd^2-e(13bd-9ae))\,x}{8e^5\,(d+ex^2)}-\frac{\sqrt{d}(63cd^2-5e(7bd-3ae))\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{8e^{11/2}}$$

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 170, normalized size of antiderivative = 0.98

$$\int\frac{x^6(a+bx^2+cx^4)}{(d+ex^2)^3}\,dx = \frac{(6cd^2+e(-3bd+ae))\,x}{e^5}+\frac{(-3cd+be)x^3}{3e^4}+\frac{cx^5}{5e^3} - \frac{(cd^4+d^2e(-bd+ae))\,x}{4e^5\,(d+ex^2)^2}+\frac{(17cd^3+de(-13bd+9ae))\,x}{8e^5\,(d+ex^2)} - \frac{\sqrt{d}(63cd^2+5e(-7bd+3ae))\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{8e^{11/2}}$$

```
[In] Integrate[(x^6*(a + b*x^2 + c*x^4))/(d + e*x^2)^3,x]
```

```
[Out] ((6*c*d^2 + e*(-3*b*d + a*e))*x)/e^5 + ((-3*c*d + b*e)*x^3)/(3*e^4) + (c*x^
5)/(5*e^3) - ((c*d^4 + d^2*e*(-(b*d) + a*e))*x)/(4*e^5*(d + e*x^2)^2) + ((1
7*c*d^3 + d*e*(-13*b*d + 9*a*e))*x)/(8*e^5*(d + e*x^2)) - (Sqrt[d]*(63*c*d^
2 + 5*e*(-7*b*d + 3*a*e))*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(8*e^(11/2))
```

Maple [A] (verified)

Time = 0.33 (sec) , antiderivative size = 151, normalized size of antiderivative = 0.87

method	result
default	$\frac{\frac{1}{5}c\,x^5e^2+\frac{1}{3}b\,e^2x^3-dc\,x^3e+a\,e^2x-3bdex+6c\,d^2x}{e^5}-\frac{d\left(\frac{\left(-\frac{9}{8}a\,e^3+\frac{13}{8}d\,e^2b-\frac{17}{8}c\,d^2e\right)x^3-\frac{d\left(7a\,e^2-11bde+15c\,d^2\right)x}{8}}{\left(e\,x^2+d\right)^2}+\frac{\left(15a\,e^2-35bde+63c\right.}{8\sqrt{e}}\right.}{e^5}$
risch	$\frac{c\,x^5}{5e^3}+\frac{b\,x^3}{3e^3}-\frac{dc\,x^3}{e^4}+\frac{ax}{e^3}-\frac{3bdx}{e^4}+\frac{6c\,d^2x}{e^5}+\frac{\left(\frac{9}{8}d\,e^3a-\frac{13}{8}e^2d^2b+\frac{17}{8}d^3ec\right)x^3+\frac{d^2\left(7a\,e^2-11bde+15c\,d^2\right)x}{8}}{e^5\left(e\,x^2+d\right)^2}+\frac{15\sqrt{-ed}\,\ln(-\sqrt{}}{16e^4}$

```
[In] int(x^6*(c*x^4+b*x^2+a)/(e*x^2+d)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/e^5*(1/5*c*x^5*e^2+1/3*b*e^2*x^3-d*c*x^3*e+a*e^2*x-3*b*d*e*x+6*c*d^2*x)-d
/e^5*(((-9/8*a*e^3+13/8*d*e^2*b-17/8*c*d^2*e)*x^3-1/8*d*(7*a*e^2-11*b*d*e+1
5*c*d^2)*x)/(e*x^2+d)^2+1/8*(15*a*e^2-35*b*d*e+63*c*d^2)/(e*d)^(1/2)*arctan
(e*x/(e*d)^(1/2)))
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 504, normalized size of antiderivative = 2.91

$$\int \frac{x^6(a+bx^2+cx^4)}{(d+ex^2)^3}\,dx$$
$$=\left[\frac{48\,ce^4x^9-16\,(9\,cde^3-5\,be^4)x^7+16\,(63\,cd^2e^2-35\,bde^3+15\,ae^4)x^5+50\,(63\,cd^3e-35\,bd^2e^2+15\,ade^3)}{}\right.$$

```
[In] integrate(x^6*(c*x^4+b*x^2+a)/(e*x^2+d)^3,x, algorithm="fricas")
```

```
[Out] [1/240*(48*c*e^4*x^9 - 16*(9*c*d*e^3 - 5*b*e^4)*x^7 + 16*(63*c*d^2*e^2 - 35
*b*d*e^3 + 15*a*e^4)*x^5 + 50*(63*c*d^3*e - 35*b*d^2*e^2 + 15*a*d*e^3)*x^3
+ 15*(63*c*d^4 - 35*b*d^3*e + 15*a*d^2*e^2 + (63*c*d^2*e^2 - 35*b*d*e^3 + 1
5*a*e^4)*x^4 + 2*(63*c*d^3*e - 35*b*d^2*e^2 + 15*a*d*e^3)*x^2)*sqrt(-d/e)*l
og((e*x^2 - 2*e*x*sqrt(-d/e) - d)/(e*x^2 + d)) + 30*(63*c*d^4 - 35*b*d^3*e
+ 15*a*d^2*e^2)*x)/(e^7*x^4 + 2*d*e^6*x^2 + d^2*e^5), 1/120*(24*c*e^4*x^9 -
 8*(9*c*d*e^3 - 5*b*e^4)*x^7 + 8*(63*c*d^2*e^2 - 35*b*d*e^3 + 15*a*e^4)*x^5
 + 25*(63*c*d^3*e - 35*b*d^2*e^2 + 15*a*d*e^3)*x^3 - 15*(63*c*d^4 - 35*b*d^
3*e + 15*a*d^2*e^2 + (63*c*d^2*e^2 - 35*b*d*e^3 + 15*a*e^4)*x^4 + 2*(63*c*d
^3*e - 35*b*d^2*e^2 + 15*a*d*e^3)*x^2)*sqrt(d/e)*arctan(e*x*sqrt(d/e)/d) +
15*(63*c*d^4 - 35*b*d^3*e + 15*a*d^2*e^2)*x)/(e^7*x^4 + 2*d*e^6*x^2 + d^2*e
^5)]
```

Sympy [A] (verification not implemented)

Time = 1.92 (sec) , antiderivative size = 235, normalized size of antiderivative = 1.36

$$\int \frac{x^6(a+bx^2+cx^4)}{(d+ex^2)^3}\,dx = \frac{cx^5}{5e^3} + x^3\left(\frac{b}{3e^3} - \frac{cd}{e^4}\right) + x\left(\frac{a}{e^3} - \frac{3bd}{e^4} + \frac{6cd^2}{e^5}\right) + \frac{\sqrt{-\frac{d}{e^{11}}}\cdot(15ae^2 - 35bde + 63cd^2)\log\left(-e^5\sqrt{-\frac{d}{e^{11}}} + x\right)}{16} - \frac{\sqrt{-\frac{d}{e^{11}}}\cdot(15ae^2 - 35bde + 63cd^2)\log\left(e^5\sqrt{-\frac{d}{e^{11}}} + x\right)}{16} + \frac{x^3\cdot(9ade^3 - 13bd^2e^2 + 17cd^3e) + x(7ad^2e^2 - 11bd^3e + 15cd^4)}{8d^2e^5 + 16de^6x^2 + 8e^7x^4}$$

```
[In] integrate(x**6*(c*x**4+b*x**2+a)/(e*x**2+d)**3,x)
```

```
[Out] c*x**5/(5*e**3) + x**3*(b/(3*e**3) - c*d/e**4) + x*(a/e**3 - 3*b*d/e**4 + 6
*c*d**2/e**5) + sqrt(-d/e**11)*(15*a*e**2 - 35*b*d*e + 63*c*d**2)*log(-e**5
*sqrt(-d/e**11) + x)/16 - sqrt(-d/e**11)*(15*a*e**2 - 35*b*d*e + 63*c*d**2)
*log(e**5*sqrt(-d/e**11) + x)/16 + (x**3*(9*a*d*e**3 - 13*b*d**2*e**2 + 17*
c*d**3*e) + x*(7*a*d**2*e**2 - 11*b*d**3*e + 15*c*d**4))/(8*d**2*e**5 + 16*
d*e**6*x**2 + 8*e**7*x**4)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^6(a+bx^2+cx^4)}{(d+ex^2)^3}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(x^6*(c*x^4+b*x^2+a)/(e*x^2+d)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 172, normalized size of antiderivative = 0.99

$$\int \frac{x^6(a+bx^2+cx^4)}{(d+ex^2)^3}\,dx$$
$$= -\frac{(63\,cd^3-35\,bd^2e+15\,ade^2)\arctan\left(\frac{ex}{\sqrt{de}}\right)}{8\,\sqrt{de}e^5} + \frac{17\,cd^3ex^3-13\,bd^2e^2x^3+9\,ade^3x^3+15\,cd^4x-11\,bd^3ex+7\,ad^2e^2x}{8\,(ex^2+d)^2e^5} + \frac{3\,ce^{12}x^5-15\,cde^{11}x^3+5\,be^{12}x^3+90\,cd^2e^{10}x-45\,bde^{11}x+15\,ae^{12}x}{15\,e^{15}}$$

```
[In] integrate(x^6*(c*x^4+b*x^2+a)/(e*x^2+d)^3,x, algorithm="giac")
```

```
[Out] -1/8*(63*c*d^3 - 35*b*d^2*e + 15*a*d*e^2)*arctan(e*x/sqrt(d*e))/(sqrt(d*e)*
e^5) + 1/8*(17*c*d^3*e*x^3 - 13*b*d^2*e^2*x^3 + 9*a*d*e^3*x^3 + 15*c*d^4*x
- 11*b*d^3*e*x + 7*a*d^2*e^2*x)/((e*x^2 + d)^2*e^5) + 1/15*(3*c*e^12*x^5 -
15*c*d*e^11*x^3 + 5*b*e^12*x^3 + 90*c*d^2*e^10*x - 45*b*d*e^11*x + 15*a*e^1
2*x)/e^15
```

Mupad [B] (verification not implemented)

Time = 7.75 (sec) , antiderivative size = 223, normalized size of antiderivative = 1.29

$$\int \frac{x^6(a+bx^2+cx^4)}{(d+ex^2)^3}\,dx$$
$$= x^3\left(\frac{b}{3\,e^3}-\frac{c\,d}{e^4}\right)-x\left(\frac{3\,c\,d^2}{e^5}-\frac{a}{e^3}+\frac{3\,d\,\left(\frac{b}{e^3}-\frac{3\,c\,d}{e^4}\right)}{e}\right) + \frac{\left(\frac{17\,c\,d^3\,e}{8}-\frac{13\,b\,d^2\,e^2}{8}+\frac{9\,a\,d\,e^3}{8}\right)\,x^3+\left(\frac{15\,c\,d^4}{8}-\frac{11\,b\,d^3\,e}{8}+\frac{7\,a\,d^2\,e^2}{8}\right)\,x}{d^2\,e^5+2\,d\,e^6\,x^2+e^7\,x^4}+\frac{c\,x^5}{5\,e^3} - \frac{\sqrt{d}\,\mathrm{atan}\left(\frac{\sqrt{d}\,\sqrt{e}\,x\,(63\,c\,d^2-35\,b\,d\,e+15\,a\,e^2)}{63\,c\,d^3-35\,b\,d^2\,e+15\,a\,d\,e^2}\right)\,(63\,c\,d^2-35\,b\,d\,e+15\,a\,e^2)}{8\,e^{11/2}}$$

```
[In] int((x^6*(a + b*x^2 + c*x^4))/(d + e*x^2)^3,x)
```

```
[Out] x^3*(b/(3*e^3) - (c*d)/e^4) - x*((3*c*d^2)/e^5 - a/e^3 + (3*d*(b/e^3 - (3*c
*d)/e^4))/e) + (x^3*((9*a*d*e^3)/8 - (13*b*d^2*e^2)/8 + (17*c*d^3*e)/8) + x
*((15*c*d^4)/8 + (7*a*d^2*e^2)/8 - (11*b*d^3*e)/8))/(d^2*e^5 + e^7*x^4 + 2*
d*e^6*x^2) + (c*x^5)/(5*e^3) - (d^(1/2)*atan((d^(1/2)*e^(1/2)*x*(15*a*e^2 +
 63*c*d^2 - 35*b*d*e))/(63*c*d^3 + 15*a*d*e^2 - 35*b*d^2*e))*(15*a*e^2 + 63
*c*d^2 - 35*b*d*e))/(8*e^(11/2))
```

3.289 $\int \frac{x^4(a+bx^2+cx^4)}{(d+ex^2)^3} dx$

Optimal result

Integrand size = 25, antiderivative size = 143

$$\int \frac{x^4(a+bx^2+cx^4)}{(d+ex^2)^3} dx = -\frac{(3cd-be)x}{e^4}+\frac{cx^3}{3e^3}+\frac{d(cd^2-bde+ae^2)\,x}{4e^4\,(d+ex^2)^2} - \frac{(13cd^2-e(9bd-5ae))\,x}{8e^4\,(d+ex^2)} + \frac{(35cd^2-3e(5bd-ae))\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{8\sqrt{d}e^{9/2}}$$

```
[Out] -(-b*e+3*c*d)*x/e^4+1/3*c*x^3/e^3+1/4*d*(a*e^2-b*d*e+c*d^2)*x/e^4/(e*x^2+d)
^2-1/8*(13*c*d^2-e*(-5*a*e+9*b*d))*x/e^4/(e*x^2+d)+1/8*(35*c*d^2-3*e*(-a*e+
5*b*d))*arctan(x*e^(1/2)/d^(1/2))/e^(9/2)/d^(1/2)
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {1271, 1828, 1167, 211}

$$\int \frac{x^4(a+bx^2+cx^4)}{(d+ex^2)^3} dx = \frac{\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)(35cd^2-3e(5bd-ae))}{8\sqrt{d}e^{9/2}} - \frac{x(13cd^2-e(9bd-5ae))}{8e^4\,(d+ex^2)} + \frac{dx(ae^2-bde+cd^2)}{4e^4\,(d+ex^2)^2} - \frac{x(3cd-be)}{e^4} + \frac{cx^3}{3e^3}$$

```
[In] Int[(x^4*(a + b*x^2 + c*x^4))/(d + e*x^2)^3,x]
```

```
[Out] -(((3*c*d - b*e)*x)/e^4) + (c*x^3)/(3*e^3) + (d*(c*d^2 - b*d*e + a*e^2)*x)/
(4*e^4*(d + e*x^2)^2) - ((13*c*d^2 - e*(9*b*d - 5*a*e))*x)/(8*e^4*(d + e*x^
2)) + ((35*c*d^2 - 3*e*(5*b*d - a*e))*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(8*Sqrt[
d]*e^(9/2))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1167

```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^q*(a + b*x^2 + c*x^4)^p, x],
x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e
 + a*e^2, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rule 1271

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)
^4)^(p_.), x_Symbol] :> Simp[(-d)^(m/2 - 1)*(c*d^2 - b*d*e + a*e^2)^p*x*((d
 + e*x^2)^(q + 1)/(2*e^(2*p + m/2)*(q + 1))), x] + Dist[1/(2*e^(2*p + m/2)*
(q + 1)), Int[(d + e*x^2)^(q + 1)*ExpandToSum[Together[(1/(d + e*x^2))*(2*e
^(2*p + m/2)*(q + 1)*x^m*(a + b*x^2 + c*x^4)^p - (-d)^(m/2 - 1)*(c*d^2 - b*
d*e + a*e^2)^p*(d + e*(2*q + 3)*x^2))], x], x], x] /; FreeQ[{a, b, c, d, e}
, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p, 0] && ILtQ[q, -1] && IGtQ[m/2, 0]
```

Rule 1828

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuot
ient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b
*f*x)*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] + Dist[1/(2*a*(p + 1)), Int
[(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Q + f*(2*p + 3), x], x], x]] /
; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]
```

Rubi steps

$$\text{integral} = \frac{d(cd^2-bde+ae^2)\,x}{4e^4\,(d+ex^2)^2} - \frac{\int \frac{d(cd^2-bde+ae^2)-4e(cd^2-bde+ae^2)x^2+4e^2(cd-be)x^4-4ce^3x^6}{(d+ex^2)^2}\,dx}{4e^4}$$

$$= \frac{d(cd^2-bde+ae^2)\,x}{4e^4\,(d+ex^2)^2} - \frac{(13cd^2-e(9bd-5ae))\,x}{8e^4\,(d+ex^2)} + \frac{\int \frac{d(11cd^2-e(7bd-3ae))-8de(2cd-be)x^2+8cde^2x^4}{d+ex^2}\,dx}{8de^4}$$

$$
\begin{aligned}
&= \frac{d(cd^2-bde+ae^2)\,x}{4e^4\,(d+ex^2)^2} - \frac{(13cd^2-e(9bd-5ae))\,x}{8e^4\,(d+ex^2)} \\
&\quad + \frac{\int \left(-8d(3cd-be)+8cdex^2+\frac{35cd^3-15bd^2e+3ade^2}{d+ex^2}\right)dx}{8de^4} \\
&= -\frac{(3cd-be)x}{e^4} + \frac{cx^3}{3e^3} + \frac{d(cd^2-bde+ae^2)\,x}{4e^4\,(d+ex^2)^2} \\
&\quad - \frac{(13cd^2-e(9bd-5ae))\,x}{8e^4\,(d+ex^2)} + \frac{(35cd^2-3e(5bd-ae))\int \frac{1}{d+ex^2}\,dx}{8e^4} \\
&= -\frac{(3cd-be)x}{e^4} + \frac{cx^3}{3e^3} + \frac{d(cd^2-bde+ae^2)\,x}{4e^4\,(d+ex^2)^2} \\
&\quad - \frac{(13cd^2-e(9bd-5ae))\,x}{8e^4\,(d+ex^2)} + \frac{(35cd^2-3e(5bd-ae))\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{8\sqrt{d}e^{9/2}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 141, normalized size of antiderivative = 0.99

$$
\begin{aligned}
\int \frac{x^4(a+bx^2+cx^4)}{(d+ex^2)^3}\,dx &= \frac{(-3cd+be)x}{e^4} + \frac{cx^3}{3e^3} + \frac{(cd^3-bd^2e+ade^2)\,x}{4e^4\,(d+ex^2)^2} \\
&\quad - \frac{(13cd^2-9bde+5ae^2)\,x}{8e^4\,(d+ex^2)} \\
&\quad + \frac{(35cd^2-15bde+3ae^2)\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{8\sqrt{d}e^{9/2}}
\end{aligned}
$$

```
[In] Integrate[(x^4*(a + b*x^2 + c*x^4))/(d + e*x^2)^3,x]
```

```
[Out] ((-3*c*d + b*e)*x)/e^4 + (c*x^3)/(3*e^3) + ((c*d^3 - b*d^2*e + a*d*e^2)*x)/
(4*e^4*(d + e*x^2)^2) - ((13*c*d^2 - 9*b*d*e + 5*a*e^2)*x)/(8*e^4*(d + e*x^
2)) + ((35*c*d^2 - 15*b*d*e + 3*a*e^2)*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(8*Sqrt
[d]*e^(9/2))
```

Maple [A] (verified)

Time = 0.33 (sec) , antiderivative size = 120, normalized size of antiderivative = 0.84

method	result
default	$\frac{\frac{1}{3}c\,x^3e+bex-3cdx}{e^4}+\frac{\frac{\left(-\frac{5}{8}a\,e^3+\frac{9}{8}d\,e^2b-\frac{13}{8}c\,d^2e\right)x^3-\frac{d\left(3a\,e^2-7bde+11c\,d^2\right)x}{8}}{\left(e\,x^2+d\right)^2}+\frac{\left(3a\,e^2-15bde+35c\,d^2\right)\arctan\left(\frac{ex}{\sqrt{ed}}\right)}{8\sqrt{ed}}}{e^4}$
risch	$\frac{c\,x^3}{3e^3}+\frac{bx}{e^3}-\frac{3cdx}{e^4}+\frac{\left(-\frac{5}{8}a\,e^3+\frac{9}{8}d\,e^2b-\frac{13}{8}c\,d^2e\right)x^3-\frac{d\left(3a\,e^2-7bde+11c\,d^2\right)x}{8}}{e^4\left(e\,x^2+d\right)^2}-\frac{3\ln\left(ex+\sqrt{-ed}\right)a}{16e^2\sqrt{-ed}}+\frac{15\ln\left(ex+\sqrt{-ed}\right)bd}{16e^3\sqrt{-ed}}-$ 35

[In] int(x^4*(c*x^4+b*x^2+a)/(e*x^2+d)^3,x,method=_RETURNVERBOSE)

[Out] 1/e^4*(1/3*c*x^3*e+b*e*x-3*c*d*x)+1/e^4*(((-5/8*a*e^3+9/8*d*e^2*b-13/8*c*d^2*e)*x^3-1/8*d*(3*a*e^2-7*b*d*e+11*c*d^2)*x)/(e*x^2+d)^2+1/8*(3*a*e^2-15*b*d*e+35*c*d^2)/(e*d)^(1/2)*arctan(e*x/(e*d)^(1/2)))

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 462, normalized size of antiderivative = 3.23

$$\int \frac{x^4(a+bx^2+cx^4)}{(d+ex^2)^3}\,dx = \left[\frac{16\,cde^4x^7-16\,(7\,cd^2e^3-3\,bde^4)x^5-10\,(35\,cd^3e^2-15\,bd^2e^3+3\,ade^4)x^3-3\,(35\,cd^4-15\,bd^3e+3\,ad^2e^2}{}\right.$$

[In] integrate(x^4*(c*x^4+b*x^2+a)/(e*x^2+d)^3,x, algorithm="fricas")

[Out] [1/48*(16*c*d*e^4*x^7 - 16*(7*c*d^2*e^3 - 3*b*d*e^4)*x^5 - 10*(35*c*d^3*e^2 - 15*b*d^2*e^3 + 3*a*d*e^4)*x^3 - 3*(35*c*d^4 - 15*b*d^3*e + 3*a*d^2*e^2 + (35*c*d^2*e^2 - 15*b*d*e^3 + 3*a*e^4)*x^4 + 2*(35*c*d^3*e - 15*b*d^2*e^2 + 3*a*d*e^3)*x^2)*sqrt(-d*e)*log((e*x^2 - 2*sqrt(-d*e)*x - d)/(e*x^2 + d)) - 6*(35*c*d^4*e - 15*b*d^3*e^2 + 3*a*d^2*e^3)*x)/(d*e^7*x^4 + 2*d^2*e^6*x^2 + d^3*e^5), 1/24*(8*c*d*e^4*x^7 - 8*(7*c*d^2*e^3 - 3*b*d*e^4)*x^5 - 5*(35*c*d^3*e^2 - 15*b*d^2*e^3 + 3*a*d*e^4)*x^3 + 3*(35*c*d^4 - 15*b*d^3*e + 3*a*d^2*e^2 + (35*c*d^2*e^2 - 15*b*d*e^3 + 3*a*e^4)*x^4 + 2*(35*c*d^3*e - 15*b*d^2*e^2 + 3*a*d*e^3)*x^2)*sqrt(d*e)*arctan(sqrt(d*e)*x/d) - 3*(35*c*d^4*e - 15*b*d^3*e^2 + 3*a*d^2*e^3)*x)/(d*e^7*x^4 + 2*d^2*e^6*x^2 + d^3*e^5)]

Sympy [A] (verification not implemented)

Time = 1.72 (sec) , antiderivative size = 212, normalized size of antiderivative = 1.48

$$\int \frac{x^4(a+bx^2+cx^4)}{(d+ex^2)^3}\,dx = \frac{cx^3}{3e^3} + x\left(\frac{b}{e^3} - \frac{3cd}{e^4}\right) - \frac{\sqrt{-\frac{1}{de^9}}\cdot(3ae^2-15bde+35cd^2)\log\left(-de^4\sqrt{-\frac{1}{de^9}}+x\right)}{16} + \frac{\sqrt{-\frac{1}{de^9}}\cdot(3ae^2-15bde+35cd^2)\log\left(de^4\sqrt{-\frac{1}{de^9}}+x\right)}{16} + \frac{x^3(-5ae^3+9bde^2-13cd^2e)+x(-3ade^2+7bd^2e-11cd^3)}{8d^2e^4+16de^5x^2+8e^6x^4}$$

```
[In] integrate(x**4*(c*x**4+b*x**2+a)/(e*x**2+d)**3,x)
```

```
[Out] c*x**3/(3*e**3) + x*(b/e**3 - 3*c*d/e**4) - sqrt(-1/(d*e**9))*(3*a*e**2 - 1
5*b*d*e + 35*c*d**2)*log(-d*e**4*sqrt(-1/(d*e**9)) + x)/16 + sqrt(-1/(d*e**
9))*(3*a*e**2 - 15*b*d*e + 35*c*d**2)*log(d*e**4*sqrt(-1/(d*e**9)) + x)/16
+ (x**3*(-5*a*e**3 + 9*b*d*e**2 - 13*c*d**2*e) + x*(-3*a*d*e**2 + 7*b*d**2*
e - 11*c*d**3))/(8*d**2*e**4 + 16*d*e**5*x**2 + 8*e**6*x**4)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^4(a+bx^2+cx^4)}{(d+ex^2)^3}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(x^4*(c*x^4+b*x^2+a)/(e*x^2+d)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 134, normalized size of antiderivative = 0.94

$$\int \frac{x^4(a+bx^2+cx^4)}{(d+ex^2)^3}\,dx = \frac{(35\,cd^2-15\,bde+3\,ae^2)\arctan\left(\frac{ex}{\sqrt{de}}\right)}{8\,\sqrt{de}e^4} - \frac{13\,cd^2ex^3-9\,bde^2x^3+5\,ae^3x^3+11\,cd^3x-7\,bd^2ex+3\,ade^2x}{8\,(ex^2+d)^2e^4} + \frac{ce^6x^3-9\,cde^5x+3\,be^6x}{3\,e^9}$$

```
[In] integrate(x^4*(c*x^4+b*x^2+a)/(e*x^2+d)^3,x, algorithm="giac")
```

```
[Out] 1/8*(35*c*d^2 - 15*b*d*e + 3*a*e^2)*arctan(e*x/sqrt(d*e))/(sqrt(d*e)*e^4) -
 1/8*(13*c*d^2*e*x^3 - 9*b*d*e^2*x^3 + 5*a*e^3*x^3 + 11*c*d^3*x - 7*b*d^2*e
*x + 3*a*d*e^2*x)/((e*x^2 + d)^2*e^4) + 1/3*(c*e^6*x^3 - 9*c*d*e^5*x + 3*b*
e^6*x)/e^9
```

Mupad [B] (verification not implemented)

Time = 7.71 (sec) , antiderivative size = 137, normalized size of antiderivative = 0.96

$$\int \frac{x^4(a+bx^2+cx^4)}{(d+ex^2)^3}\,dx = x\left(\frac{b}{e^3}-\frac{3\,c\,d}{e^4}\right) - \frac{\left(\frac{13\,c\,d^2\,e}{8}-\frac{9\,b\,d\,e^2}{8}+\frac{5\,a\,e^3}{8}\right)\,x^3+\left(\frac{11\,c\,d^3}{8}-\frac{7\,b\,d^2\,e}{8}+\frac{3\,a\,d\,e^2}{8}\right)\,x}{d^2\,e^4+2\,d\,e^5\,x^2+e^6\,x^4} + \frac{c\,x^3}{3\,e^3} + \frac{\mathrm{atan}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)\,\left(35\,c\,d^2-15\,b\,d\,e+3\,a\,e^2\right)}{8\,\sqrt{d}\,e^{9/2}}$$

```
[In] int((x^4*(a + b*x^2 + c*x^4))/(d + e*x^2)^3,x)
```

```
[Out] x*(b/e^3 - (3*c*d)/e^4) - (x*((11*c*d^3)/8 + (3*a*d*e^2)/8 - (7*b*d^2*e)/8)
 + x^3*((5*a*e^3)/8 - (9*b*d*e^2)/8 + (13*c*d^2*e)/8))/(d^2*e^4 + e^6*x^4 +
 2*d*e^5*x^2) + (c*x^3)/(3*e^3) + (atan((e^(1/2)*x)/d^(1/2))*(3*a*e^2 + 35*
c*d^2 - 15*b*d*e))/(8*d^(1/2)*e^(9/2))
```

3.290 $\int \frac{x^2(a+bx^2+cx^4)}{(d+ex^2)^3} dx$

Optimal result

Integrand size = 25, antiderivative size = 124

$$\int \frac{x^2(a+bx^2+cx^4)}{(d+ex^2)^3} dx = \frac{cx}{e^3} - \frac{(cd^2-bde+ae^2)\,x}{4e^3\,(d+ex^2)^2} + \frac{(9cd^2-e(5bd-ae))\,x}{8de^3\,(d+ex^2)} - \frac{(15cd^2-e(3bd+ae))\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{8d^{3/2}e^{7/2}}$$

```
[Out] c*x/e^3-1/4*(a*e^2-b*d*e+c*d^2)*x/e^3/(e*x^2+d)^2+1/8*(9*c*d^2-e*(-a*e+5*b*
d))*x/d/e^3/(e*x^2+d)-1/8*(15*c*d^2-e*(a*e+3*b*d))*arctan(x*e^(1/2)/d^(1/2)
)/d^(3/2)/e^(7/2)
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {1271, 1171, 396, 211}

$$\int \frac{x^2(a+bx^2+cx^4)}{(d+ex^2)^3} dx = -\frac{\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)(15cd^2-e(ae+3bd))}{8d^{3/2}e^{7/2}} + \frac{x(9cd^2-e(5bd-ae))}{8de^3\,(d+ex^2)} - \frac{x(ae^2-bde+cd^2)}{4e^3\,(d+ex^2)^2} + \frac{cx}{e^3}$$

```
[In] Int[(x^2*(a + b*x^2 + c*x^4))/(d + e*x^2)^3,x]
```

```
[Out] (c*x)/e^3 - ((c*d^2 - b*d*e + a*e^2)*x)/(4*e^3*(d + e*x^2)^2) + ((9*c*d^2 -
 e*(5*b*d - a*e))*x)/(8*d*e^3*(d + e*x^2)) - ((15*c*d^2 - e*(3*b*d + a*e))*
ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(8*d^(3/2)*e^(7/2))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 396

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 1171

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 + c*x^4)^p, d + e*x^2
, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], x
, 0]}, Simp[(-R)*x*((d + e*x^2)^(q + 1)/(2*d*(q + 1))), x] + Dist[1/(2*d*(q
 + 1)), Int[(d + e*x^2)^(q + 1)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3), x
], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
 - b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

Rule 1271

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)
^4)^(p_.), x_Symbol] :> Simp[(-d)^(m/2 - 1)*(c*d^2 - b*d*e + a*e^2)^p*x*((d
 + e*x^2)^(q + 1)/(2*e^(2*p + m/2)*(q + 1))), x] + Dist[1/(2*e^(2*p + m/2)*
(q + 1)), Int[(d + e*x^2)^(q + 1)*ExpandToSum[Together[(1/(d + e*x^2))*(2*e
^(2*p + m/2)*(q + 1)*x^m*(a + b*x^2 + c*x^4)^p - (-d)^(m/2 - 1)*(c*d^2 - b*
d*e + a*e^2)^p*(d + e*(2*q + 3)*x^2))], x], x], x] /; FreeQ[{a, b, c, d, e}
, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p, 0] && ILtQ[q, -1] && IGtQ[m/2, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{(cd^2-bde+ae^2)\,x}{4e^3\,(d+ex^2)^2}-\frac{\int \frac{-cd^2+bde-ae^2+4e(cd-be)x^2-4ce^2x^4}{(d+ex^2)^2}\,dx}{4e^3}\\
&= -\frac{(cd^2-bde+ae^2)\,x}{4e^3\,(d+ex^2)^2}+\frac{(9cd^2-e(5bd-ae))\,x}{8de^3\,(d+ex^2)}+\frac{\int \frac{-7cd^2+e(3bd+ae)+8cdex^2}{d+ex^2}\,dx}{8de^3}\\
&= \frac{cx}{e^3}-\frac{(cd^2-bde+ae^2)\,x}{4e^3\,(d+ex^2)^2}+\frac{(9cd^2-e(5bd-ae))\,x}{8de^3\,(d+ex^2)}-\frac{(15cd^2-e(3bd+ae))\int \frac{1}{d+ex^2}\,dx}{8de^3}\\
&= \frac{cx}{e^3}-\frac{(cd^2-bde+ae^2)\,x}{4e^3\,(d+ex^2)^2}+\frac{(9cd^2-e(5bd-ae))\,x}{8de^3\,(d+ex^2)}-\frac{(15cd^2-e(3bd+ae))\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{8d^{3/2}e^{7/2}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.98

$$\int \frac{x^2(a+bx^2+cx^4)}{(d+ex^2)^3}\,dx = \frac{cx}{e^3} - \frac{(cd^2-bde+ae^2)\,x}{4e^3\,(d+ex^2)^2} + \frac{(9cd^2-5bde+ae^2)\,x}{8de^3\,(d+ex^2)} - \frac{(15cd^2-3bde-ae^2)\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{8d^{3/2}e^{7/2}}$$

```
[In] Integrate[(x^2*(a + b*x^2 + c*x^4))/(d + e*x^2)^3,x]
```

```
[Out] (c*x)/e^3 - ((c*d^2 - b*d*e + a*e^2)*x)/(4*e^3*(d + e*x^2)^2) + ((9*c*d^2 -
 5*b*d*e + a*e^2)*x)/(8*d*e^3*(d + e*x^2)) - ((15*c*d^2 - 3*b*d*e - a*e^2)*
ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(8*d^(3/2)*e^(7/2))
```

Maple [A] (verified)

Time = 0.32 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.85

method	result
default	$\frac{cx}{e^3}+\frac{\frac{\frac{e\left(a\,e^2-5bde+9c\,d^2\right)x^3}{8d}+\left(-\frac{1}{8}a\,e^2-\frac{3}{8}bde+\frac{7}{8}c\,d^2\right)x}{\left(e\,x^2+d\right)^2}+\frac{\left(a\,e^2+3bde-15c\,d^2\right)\arctan\left(\frac{ex}{\sqrt{ed}}\right)}{8d\sqrt{ed}}}{e^3}$
risch	$\frac{cx}{e^3}+\frac{\frac{e\left(a\,e^2-5bde+9c\,d^2\right)x^3}{8d}+(-\frac{1}{8}a\,e^2-\frac{3}{8}bde+\frac{7}{8}c\,d^2)x}{e^3(e\,x^2+d)^2}-\frac{\ln(ex+\sqrt{-ed})a}{16e\sqrt{-ed}\,d}-\frac{3\ln(ex+\sqrt{-ed})b}{16e^2\sqrt{-ed}}+\frac{15d\ln(ex+\sqrt{-ed})c}{16e^3\sqrt{-ed}}+\frac{\ln(-ex}{16e\sqrt{}}$

```
[In] int(x^2*(c*x^4+b*x^2+a)/(e*x^2+d)^3,x,method=_RETURNVERBOSE)
```

```
[Out] c*x/e^3+1/e^3*((1/8*e*(a*e^2-5*b*d*e+9*c*d^2)/d*x^3+(-1/8*a*e^2-3/8*b*d*e+7
/8*c*d^2)*x)/(e*x^2+d)^2+1/8*(a*e^2+3*b*d*e-15*c*d^2)/d/(e*d)^(1/2)*arctan(
e*x/(e*d)^(1/2)))
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 421, normalized size of antiderivative = 3.40

$$\int \frac{x^2(a+bx^2+cx^4)}{(d+ex^2)^3}\,dx$$
$$= \left[\frac{16\,cd^2e^3x^5+2\,(25\,cd^3e^2-5\,bd^2e^3+ade^4)x^3+(15\,cd^4-3\,bd^3e-ad^2e^2+(15\,cd^2e^2-3\,bde^3-ae^4)x^4+2}{16\,(d^2e^6x^4+2\,d^3e^5x^2}\right.$$

```
[In] integrate(x^2*(c*x^4+b*x^2+a)/(e*x^2+d)^3,x, algorithm="fricas")
```

```
[Out] [1/16*(16*c*d^2*e^3*x^5 + 2*(25*c*d^3*e^2 - 5*b*d^2*e^3 + a*d*e^4)*x^3 + (1
5*c*d^4 - 3*b*d^3*e - a*d^2*e^2 + (15*c*d^2*e^2 - 3*b*d*e^3 - a*e^4)*x^4 +
2*(15*c*d^3*e - 3*b*d^2*e^2 - a*d*e^3)*x^2)*sqrt(-d*e)*log((e*x^2 - 2*sqrt(
-d*e)*x - d)/(e*x^2 + d)) + 2*(15*c*d^4*e - 3*b*d^3*e^2 - a*d^2*e^3)*x)/(d^
2*e^6*x^4 + 2*d^3*e^5*x^2 + d^4*e^4), 1/8*(8*c*d^2*e^3*x^5 + (25*c*d^3*e^2
- 5*b*d^2*e^3 + a*d*e^4)*x^3 - (15*c*d^4 - 3*b*d^3*e - a*d^2*e^2 + (15*c*d^
2*e^2 - 3*b*d*e^3 - a*e^4)*x^4 + 2*(15*c*d^3*e - 3*b*d^2*e^2 - a*d*e^3)*x^2
)*sqrt(d*e)*arctan(sqrt(d*e)*x/d) + (15*c*d^4*e - 3*b*d^3*e^2 - a*d^2*e^3)*
x)/(d^2*e^6*x^4 + 2*d^3*e^5*x^2 + d^4*e^4)]
```

Sympy [A] (verification not implemented)

Time = 1.29 (sec) , antiderivative size = 201, normalized size of antiderivative = 1.62

$$\int \frac{x^2(a+bx^2+cx^4)}{(d+ex^2)^3}\,dx = \frac{cx}{e^3} - \frac{\sqrt{-\frac{1}{d^3e^7}}\,(ae^2+3bde-15cd^2)\log\left(-d^2e^3\sqrt{-\frac{1}{d^3e^7}}+x\right)}{16} + \frac{\sqrt{-\frac{1}{d^3e^7}}\,(ae^2+3bde-15cd^2)\log\left(d^2e^3\sqrt{-\frac{1}{d^3e^7}}+x\right)}{16} + \frac{x^3(ae^3-5bde^2+9cd^2e)+x(-ade^2-3bd^2e+7cd^3)}{8d^3e^3+16d^2e^4x^2+8de^5x^4}$$

```
[In] integrate(x**2*(c*x**4+b*x**2+a)/(e*x**2+d)**3,x)
```

```
[Out] c*x/e**3 - sqrt(-1/(d**3*e**7))*(a*e**2 + 3*b*d*e - 15*c*d**2)*log(-d**2*e*
*3*sqrt(-1/(d**3*e**7)) + x)/16 + sqrt(-1/(d**3*e**7))*(a*e**2 + 3*b*d*e -
15*c*d**2)*log(d**2*e**3*sqrt(-1/(d**3*e**7)) + x)/16 + (x**3*(a*e**3 - 5*b
*d*e**2 + 9*c*d**2*e) + x*(-a*d*e**2 - 3*b*d**2*e + 7*c*d**3))/(8*d**3*e**3
 + 16*d**2*e**4*x**2 + 8*d*e**5*x**4)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^2(a+bx^2+cx^4)}{(d+ex^2)^3}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(x^2*(c*x^4+b*x^2+a)/(e*x^2+d)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 116, normalized size of antiderivative = 0.94

$$\int \frac{x^2(a+bx^2+cx^4)}{(d+ex^2)^3}\,dx = \frac{cx}{e^3} - \frac{(15\,cd^2-3\,bde-ae^2)\arctan\left(\frac{ex}{\sqrt{de}}\right)}{8\,\sqrt{de}de^3} + \frac{9\,cd^2ex^3-5\,bde^2x^3+ae^3x^3+7\,cd^3x-3\,bd^2ex-ade^2x}{8\,(ex^2+d)^2de^3}$$

[In] integrate(x^2*(c*x^4+b*x^2+a)/(e*x^2+d)^3,x, algorithm="giac")

[Out] c*x/e^3 - 1/8*(15*c*d^2 - 3*b*d*e - a*e^2)*arctan(e*x/sqrt(d*e))/(sqrt(d*e)*d*e^3) + 1/8*(9*c*d^2*e*x^3 - 5*b*d*e^2*x^3 + a*e^3*x^3 + 7*c*d^3*x - 3*b*d^2*e*x - a*d*e^2*x)/((e*x^2 + d)^2*d*e^3)

Mupad [B] (verification not implemented)

Time = 7.76 (sec) , antiderivative size = 118, normalized size of antiderivative = 0.95

$$\int \frac{x^2(a+bx^2+cx^4)}{(d+ex^2)^3}\,dx = \frac{c\,x}{e^3} - \frac{x\left(-\frac{7\,c\,d^2}{8}+\frac{3\,b\,d\,e}{8}+\frac{a\,e^2}{8}\right)-\frac{x^3\,(9\,c\,d^2\,e-5\,b\,d\,e^2+a\,e^3)}{8\,d}}{d^2\,e^3+2\,d\,e^4\,x^2+e^5\,x^4} + \frac{\mathrm{atan}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)\,(-15\,c\,d^2+3\,b\,d\,e+a\,e^2)}{8\,d^{3/2}\,e^{7/2}}$$

[In] int((x^2*(a + b*x^2 + c*x^4))/(d + e*x^2)^3,x)

[Out] (c*x)/e^3 - (x*((a*e^2)/8 - (7*c*d^2)/8 + (3*b*d*e)/8) - (x^3*(a*e^3 - 5*b*d*e^2 + 9*c*d^2*e))/(8*d))/(d^2*e^3 + e^5*x^4 + 2*d*e^4*x^2) + (atan((e^(1/2)*x)/d^(1/2))*(a*e^2 - 15*c*d^2 + 3*b*d*e))/(8*d^(3/2)*e^(7/2))

3.291 $\int \frac{a+bx^2+cx^4}{(d+ex^2)^3} dx$

Optimal result

Integrand size = 22, antiderivative size = 115

$$\int \frac{a+bx^2+cx^4}{(d+ex^2)^3} dx = \frac{\left(a+\frac{d(cd-be)}{e^2}\right)x}{4d(d+ex^2)^2} - \frac{(5cd^2-e(bd+3ae))x}{8d^2e^2(d+ex^2)} + \frac{(3cd^2+e(bd+3ae))\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{8d^{5/2}e^{5/2}}$$

```
[Out] 1/4*(a+d*(-b*e+c*d)/e^2)*x/d/(e*x^2+d)^2-1/8*(5*c*d^2-e*(3*a*e+b*d))*x/d^2/
e^2/(e*x^2+d)+1/8*(3*c*d^2+e*(3*a*e+b*d))*arctan(x*e^(1/2)/d^(1/2))/d^(5/2)
/e^(5/2)
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.03, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {1171, 393, 211}

$$\int \frac{a+bx^2+cx^4}{(d+ex^2)^3} dx = \frac{\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)(e(3ae+bd)+3cd^2)}{8d^{5/2}e^{5/2}} - \frac{x(5cd^2-e(3ae+bd))}{8d^2e^2(d+ex^2)} + \frac{x(ae^2-bde+cd^2)}{4de^2(d+ex^2)^2}$$

```
[In] Int[(a + b*x^2 + c*x^4)/(d + e*x^2)^3,x]
```

```
[Out] ((c*d^2 - b*d*e + a*e^2)*x)/(4*d*e^2*(d + e*x^2)^2) - ((5*c*d^2 - e*(b*d +
3*a*e))*x)/(8*d^2*e^2*(d + e*x^2)) + ((3*c*d^2 + e*(b*d + 3*a*e))*ArcTan[(S
qrt[e]*x)/Sqrt[d]])/(8*d^(5/2)*e^(5/2))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 393

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Dist[(a*d -
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; F
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n
 + p, 0])
```

Rule 1171

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 + c*x^4)^p, d + e*x^2
, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], x
, 0]}, Simp[(-R)*x*((d + e*x^2)^(q + 1)/(2*d*(q + 1))), x] + Dist[1/(2*d*(q
 + 1)), Int[(d + e*x^2)^(q + 1)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3), x
], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
 - b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{(cd^2-bde+ae^2)\,x}{4de^2\,(d+ex^2)^2}-\frac{\int \frac{-3a+\frac{d(cd-be)}{e^2}-\frac{4cdx^2}{e}}{(d+ex^2)^2}\,dx}{4d}\\
&= \frac{(cd^2-bde+ae^2)\,x}{4de^2\,(d+ex^2)^2}-\frac{(5cd^2-e(bd+3ae))\,x}{8d^2e^2\,(d+ex^2)}-\frac{\left(-\frac{4cd^2}{e}+e\left(-3a+\frac{d(cd-be)}{e^2}\right)\right)\int \frac{1}{d+ex^2}\,dx}{8d^2e}\\
&= \frac{(cd^2-bde+ae^2)\,x}{4de^2\,(d+ex^2)^2}-\frac{(5cd^2-e(bd+3ae))\,x}{8d^2e^2\,(d+ex^2)}+\frac{(3cd^2+e(bd+3ae))\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{8d^{5/2}e^{5/2}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.96

$$
\int \frac{a+bx^2+cx^4}{(d+ex^2)^3}\,dx = \frac{x(-cd^2(3d+5ex^2)+e(bd(-d+ex^2)+ae(5d+3ex^2)))}{8d^2e^2\,(d+ex^2)^2} + \frac{(3cd^2+e(bd+3ae))\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{8d^{5/2}e^{5/2}}
$$

```
[In] Integrate[(a + b*x^2 + c*x^4)/(d + e*x^2)^3,x]
```

```
[Out] (x*(-(c*d^2*(3*d + 5*e*x^2)) + e*(b*d*(-d + e*x^2) + a*e*(5*d + 3*e*x^2))))
/(8*d^2*e^2*(d + e*x^2)^2) + ((3*c*d^2 + e*(b*d + 3*a*e))*ArcTan[(Sqrt[e]*x
)/Sqrt[d]])/(8*d^(5/2)*e^(5/2))
```

Maple [A] (verified)

Time = 0.29 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.93

method	result
default	$\frac{\frac{\left(3a\,e^2+bde-5c\,d^2\right)x^3}{8d^2e}+\frac{\left(5a\,e^2-bde-3c\,d^2\right)x}{8d\,e^2}}{\left(e\,x^2+d\right)^2}+\frac{\left(3a\,e^2+bde+3c\,d^2\right)\arctan\left(\frac{ex}{\sqrt{ed}}\right)}{8e^2d^2\sqrt{ed}}$
risch	$\frac{\frac{\left(3a\,e^2+bde-5c\,d^2\right)x^3}{8d^2e}+\frac{\left(5a\,e^2-bde-3c\,d^2\right)x}{8d\,e^2}}{\left(e\,x^2+d\right)^2}-\frac{3\ln\left(ex+\sqrt{-ed}\right)a}{16\sqrt{-ed}\,d^2}-\frac{\ln\left(ex+\sqrt{-ed}\right)b}{16\sqrt{-ed}\,ed}-\frac{3\ln\left(ex+\sqrt{-ed}\right)c}{16\sqrt{-ed}\,e^2}+\frac{3\ln\left(-ex+\sqrt{-ed}\right)a}{16\sqrt{-ed}\,d^2}$

```
[In] int((c*x^4+b*x^2+a)/(e*x^2+d)^3,x,method=_RETURNVERBOSE)
```

```
[Out] (1/8*(3*a*e^2+b*d*e-5*c*d^2)/d^2/e*x^3+1/8*(5*a*e^2-b*d*e-3*c*d^2)/d/e^2*x)
/(e*x^2+d)^2+1/8*(3*a*e^2+b*d*e+3*c*d^2)/e^2/d^2/(e*d)^(1/2)*arctan(e*x/(e*
d)^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 391, normalized size of antiderivative = 3.40

$$\int \frac{a+bx^2+cx^4}{(d+ex^2)^3}\,dx$$
$$=\left[-\frac{2\,(5\,cd^3e^2-bd^2e^3-3\,ade^4)x^3+(3\,cd^4+bd^3e+3\,ad^2e^2+(3\,cd^2e^2+bde^3+3\,ae^4)x^4+2\,(3\,cd^3e+bd^2}{16\,(d^3e^5x^4+2\,d^4e^4x^2+d^5e^3)}\right.$$
$$-\frac{(5\,cd^3e^2-bd^2e^3-3\,ade^4)x^3-(3\,cd^4+bd^3e+3\,ad^2e^2+(3\,cd^2e^2+bde^3+3\,ae^4)x^4+2\,(3\,cd^3e+bd^2e^2}{8\,(d^3e^5x^4+2\,d^4e^4x^2+d^5e^3)}$$

```
[In] integrate((c*x^4+b*x^2+a)/(e*x^2+d)^3,x, algorithm="fricas")
```

```
[Out] [-1/16*(2*(5*c*d^3*e^2 - b*d^2*e^3 - 3*a*d*e^4)*x^3 + (3*c*d^4 + b*d^3*e +
3*a*d^2*e^2 + (3*c*d^2*e^2 + b*d*e^3 + 3*a*e^4)*x^4 + 2*(3*c*d^3*e + b*d^2*
e^2 + 3*a*d*e^3)*x^2)*sqrt(-d*e)*log((e*x^2 - 2*sqrt(-d*e)*x - d)/(e*x^2 +
d)) + 2*(3*c*d^4*e + b*d^3*e^2 - 5*a*d^2*e^3)*x)/(d^3*e^5*x^4 + 2*d^4*e^4*x
^2 + d^5*e^3), -1/8*((5*c*d^3*e^2 - b*d^2*e^3 - 3*a*d*e^4)*x^3 - (3*c*d^4 +
 b*d^3*e + 3*a*d^2*e^2 + (3*c*d^2*e^2 + b*d*e^3 + 3*a*e^4)*x^4 + 2*(3*c*d^3
```

```
*e + b*d^2*e^2 + 3*a*d*e^3)*x^2)*sqrt(d*e)*arctan(sqrt(d*e)*x/d) + (3*c*d^4
*e + b*d^3*e^2 - 5*a*d^2*e^3)*x)/(d^3*e^5*x^4 + 2*d^4*e^4*x^2 + d^5*e^3)]
```

Sympy [A] (verification not implemented)

Time = 0.77 (sec) , antiderivative size = 196, normalized size of antiderivative = 1.70

$$\int \frac{a+bx^2+cx^4}{(d+ex^2)^3}\,dx = -\frac{\sqrt{-\frac{1}{d^5e^5}}\cdot(3ae^2+bde+3cd^2)\log\left(-d^3e^2\sqrt{-\frac{1}{d^5e^5}}+x\right)}{16} + \frac{\sqrt{-\frac{1}{d^5e^5}}\cdot(3ae^2+bde+3cd^2)\log\left(d^3e^2\sqrt{-\frac{1}{d^5e^5}}+x\right)}{16} + \frac{x^3\cdot(3ae^3+bde^2-5cd^2e)+x(5ade^2-bd^2e-3cd^3)}{8d^4e^2+16d^3e^3x^2+8d^2e^4x^4}$$

```
[In] integrate((c*x**4+b*x**2+a)/(e*x**2+d)**3,x)
```

```
[Out] -sqrt(-1/(d**5*e**5))*(3*a*e**2 + b*d*e + 3*c*d**2)*log(-d**3*e**2*sqrt(-1/
(d**5*e**5)) + x)/16 + sqrt(-1/(d**5*e**5))*(3*a*e**2 + b*d*e + 3*c*d**2)*l
og(d**3*e**2*sqrt(-1/(d**5*e**5)) + x)/16 + (x**3*(3*a*e**3 + b*d*e**2 - 5*
c*d**2*e) + x*(5*a*d*e**2 - b*d**2*e - 3*c*d**3))/(8*d**4*e**2 + 16*d**3*e*
*3*x**2 + 8*d**2*e**4*x**4)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{a+bx^2+cx^4}{(d+ex^2)^3}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((c*x^4+b*x^2+a)/(e*x^2+d)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.95

$$\int \frac{a+bx^2+cx^4}{(d+ex^2)^3}\,dx = \frac{(3\,cd^2+bde+3\,ae^2)\arctan\left(\frac{ex}{\sqrt{de}}\right)}{8\,\sqrt{de}d^2e^2} - \frac{5\,cd^2ex^3-bde^2x^3-3\,ae^3x^3+3\,cd^3x+bd^2ex-5\,ade^2x}{8\,(ex^2+d)^2d^2e^2}$$

```
[In] integrate((c*x^4+b*x^2+a)/(e*x^2+d)^3,x, algorithm="giac")
```

```
[Out] 1/8*(3*c*d^2 + b*d*e + 3*a*e^2)*arctan(e*x/sqrt(d*e))/(sqrt(d*e)*d^2*e^2) -
 1/8*(5*c*d^2*e*x^3 - b*d*e^2*x^3 - 3*a*e^3*x^3 + 3*c*d^3*x + b*d^2*e*x - 5
*a*d*e^2*x)/((e*x^2 + d)^2*d^2*e^2)
```

Mupad [B] (verification not implemented)

Time = 7.71 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.97

$$\int \frac{a+bx^2+cx^4}{(d+ex^2)^3}\,dx = \frac{\mathrm{atan}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)\,\left(3\,c\,d^2+b\,d\,e+3\,a\,e^2\right)}{8\,d^{5/2}\,e^{5/2}} - \frac{\frac{x\,\left(3\,c\,d^2+b\,d\,e-5\,a\,e^2\right)}{8\,d\,e^2}-\frac{x^3\,\left(-5\,c\,d^2+b\,d\,e+3\,a\,e^2\right)}{8\,d^2\,e}}{d^2+2\,d\,e\,x^2+e^2\,x^4}$$

```
[In] int((a + b*x^2 + c*x^4)/(d + e*x^2)^3,x)
```

```
[Out] (atan((e^(1/2)*x)/d^(1/2))*(3*a*e^2 + 3*c*d^2 + b*d*e))/(8*d^(5/2)*e^(5/2))
 - ((x*(3*c*d^2 - 5*a*e^2 + b*d*e))/(8*d*e^2) - (x^3*(3*a*e^2 - 5*c*d^2 + b
*d*e))/(8*d^2*e))/(d^2 + e^2*x^4 + 2*d*e*x^2)
```

3.292 $\int \frac{a+bx^2+cx^4}{x^2(d+ex^2)^3}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 127

$$\int \frac{a+bx^2+cx^4}{x^2(d+ex^2)^3}\,dx = -\frac{a}{d^3x} - \frac{(cd^2-bde+ae^2)\,x}{4d^2e\,(d+ex^2)^2} + \frac{(cd^2+e(3bd-7ae))\,x}{8d^3e\,(d+ex^2)} + \frac{(cd^2+3e(bd-5ae))\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{8d^{7/2}e^{3/2}}$$

```
[Out] -a/d^3/x-1/4*(a*e^2-b*d*e+c*d^2)*x/d^2/e/(e*x^2+d)^2+1/8*(c*d^2+e*(-7*a*e+3
*b*d))*x/d^3/e/(e*x^2+d)+1/8*(c*d^2+3*e*(-5*a*e+b*d))*arctan(x*e^(1/2)/d^(1
/2))/d^(7/2)/e^(3/2)
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.98, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {1273, 467, 464, 211}

$$\int \frac{a+bx^2+cx^4}{x^2(d+ex^2)^3}\,dx = \frac{\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)(3e(bd-5ae)+cd^2)}{8d^{7/2}e^{3/2}} - \frac{x\left(\frac{c}{e}-\frac{bd-ae}{d^2}\right)}{4(d+ex^2)^2} + \frac{x(e(3bd-7ae)+cd^2)}{8d^3e\,(d+ex^2)} - \frac{a}{d^3x}$$

```
[In] Int[(a + b*x^2 + c*x^4)/(x^2*(d + e*x^2)^3),x]
```

```
[Out] -(a/(d^3*x)) - ((c/e - (b*d - a*e)/d^2)*x)/(4*(d + e*x^2)^2) + ((c*d^2 + e*
(3*b*d - 7*a*e))*x)/(8*d^3*e*(d + e*x^2)) + ((c*d^2 + 3*e*(b*d - 5*a*e))*Ar
cTan[(Sqrt[e]*x)/Sqrt[d]])/(8*d^(7/2)*e^(3/2))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 464

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 467

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[(-a)^(m/2 - 1)*(b*c - a*d)*x*((a + b*x^2)^(p + 1)/(2*b^(m/2 + 1)*(p
+ 1))), x] + Dist[1/(2*b^(m/2 + 1)*(p + 1)), Int[x^m*(a + b*x^2)^(p + 1)*Ex
pandToSum[2*b*(p + 1)*Together[(b^(m/2)*(c + d*x^2) - (-a)^(m/2 - 1)*(b*c -
 a*d)*x^(-m + 2))/(a + b*x^2)] - ((-a)^(m/2 - 1)*(b*c - a*d))/x^m, x], x],
x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && ILtQ[m/2
, 0] && (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

Rule 1273

```
Int[(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^
4)^(p_.), x_Symbol] :> Simp[(-d)^(m/2 - 1)*(c*d^2 - b*d*e + a*e^2)^p*x*((d
+ e*x^2)^(q + 1)/(2*e^(2*p + m/2)*(q + 1))), x] + Dist[(-d)^(m/2 - 1)/(2*e^
(2*p)*(q + 1)), Int[x^m*(d + e*x^2)^(q + 1)*ExpandToSum[Together[(1/(d + e*
x^2))*(2*(-d)^(-m/2 + 1)*e^(2*p)*(q + 1)*(a + b*x^2 + c*x^4)^p - ((c*d^2 -
b*d*e + a*e^2)^p/(e^(m/2)*x^m))*(d + e*(2*q + 3)*x^2))], x], x], x] /; Free
Q[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p, 0] && ILtQ[q, -1] &
& ILtQ[m/2, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{\left(\frac{c}{e}-\frac{bd-ae}{d^2}\right)x}{4\left(d+ex^2\right)^2}-\frac{\int \frac{-4ade^2-e\left(cd^2+3e(bd-ae)\right)x^2}{x^2\left(d+ex^2\right)^2}\,dx}{4d^2e^2}\\
&= -\frac{\left(\frac{c}{e}-\frac{bd-ae}{d^2}\right)x}{4\left(d+ex^2\right)^2}+\frac{\left(cd^2+e(3bd-7ae)\right)x}{8d^3e\left(d+ex^2\right)}+\frac{\int \frac{8ae^2+e\left(cd+e\left(3b-\frac{7ae}{d}\right)\right)x^2}{x^2\left(d+ex^2\right)}\,dx}{8d^2e^2}\\
&= -\frac{a}{d^3x}-\frac{\left(\frac{c}{e}-\frac{bd-ae}{d^2}\right)x}{4\left(d+ex^2\right)^2}+\frac{\left(cd^2+e(3bd-7ae)\right)x}{8d^3e\left(d+ex^2\right)}+\frac{\left(cd^2+3e(bd-5ae)\right)\int \frac{1}{d+ex^2}\,dx}{8d^3e}
\end{aligned}$$

$$= -\frac{a}{d^3x} - \frac{\left(\frac{c}{e} - \frac{bd-ae}{d^2}\right)x}{4(d+ex^2)^2} + \frac{(cd^2+e(3bd-7ae))x}{8d^3e(d+ex^2)} + \frac{(cd^2+3e(bd-5ae))\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{8d^{7/2}e^{3/2}}$$

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.98

$$\int \frac{a+bx^2+cx^4}{x^2(d+ex^2)^3}\,dx$$
$$= \frac{\frac{\sqrt{d}(-ae(8d^2+25dex^2+15e^2x^4)+dx^2(cd(-d+ex^2)+be(5d+3ex^2)))}{ex(d+ex^2)^2} + \frac{(cd^2+3e(bd-5ae))\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{e^{3/2}}}{8d^{7/2}}$$

```
[In] Integrate[(a + b*x^2 + c*x^4)/(x^2*(d + e*x^2)^3),x]
```

```
[Out] ((Sqrt[d]*(-(a*e*(8*d^2 + 25*d*e*x^2 + 15*e^2*x^4)) + d*x^2*(c*d*(-d + e*x^
2) + b*e*(5*d + 3*e*x^2))))/(e*x*(d + e*x^2)^2) + ((c*d^2 + 3*e*(b*d - 5*a*
e))*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/e^(3/2))/(8*d^(7/2))
```

Maple [A] (verified)

Time = 0.36 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.87

method	result
default	$-\frac{a}{d^3x} - \frac{\frac{\left(\frac{7}{8}a e^2-\frac{3}{8}bde-\frac{1}{8}c d^2\right)x^3+\frac{d\left(9a e^2-5bde+c d^2\right)x}{8e}}{\left(e x^2+d\right)^2}+\frac{\left(15a e^2-3bde-c d^2\right)\arctan\left(\frac{ex}{\sqrt{ed}}\right)}{8e\sqrt{ed}}}{d^3}$
risch	$\frac{-\frac{\left(15a e^2-3bde-c d^2\right)x^4}{8d^3}-\frac{\left(25a e^2-5bde+c d^2\right)x^2}{8d^2e}-\frac{a}{d}}{x(e x^2+d)^2} - \frac{15e\ln(-\sqrt{-ed}\,x-d)a}{16\sqrt{-ed}\,d^3} + \frac{3\ln(-\sqrt{-ed}\,x-d)b}{16\sqrt{-ed}\,d^2} + \frac{\ln(-\sqrt{-ed}\,x-d)c}{16\sqrt{-ed}\,ed} + \frac{15e\ln}{}$

```
[In] int((c*x^4+b*x^2+a)/x^2/(e*x^2+d)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -a/d^3/x-1/d^3*(((7/8*a*e^2-3/8*b*d*e-1/8*c*d^2)*x^3+1/8*d*(9*a*e^2-5*b*d*e
+c*d^2)/e*x)/(e*x^2+d)^2+1/8*(15*a*e^2-3*b*d*e-c*d^2)/e/(e*d)^(1/2)*arctan(
e*x/(e*d)^(1/2)))
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 421, normalized size of antiderivative = 3.31

$$\int \frac{a+bx^2+cx^4}{x^2\left(d+ex^2\right)^3}\,dx$$
$$= \left[-\frac{16\,ad^3e^2 - 2\,(cd^3e^2 + 3\,bd^2e^3 - 15\,ade^4)x^4 + 2\,(cd^4e - 5\,bd^3e^2 + 25\,ad^2e^3)x^2 - ((cd^2e^2 + 3\,bde^3 - 15\,a}{16\,(d^4e^4x^5 + 2\,d^5e^3}\right.$$
$$-\frac{8\,ad^3e^2 - (cd^3e^2 + 3\,bd^2e^3 - 15\,ade^4)x^4 + (cd^4e - 5\,bd^3e^2 + 25\,ad^2e^3)x^2 - ((cd^2e^2 + 3\,bde^3 - 15\,ae^4)x^5}{8\,(d^4e^4x^5 + 2\,d^5e^3x^3 + d^6}$$

```
[In] integrate((c*x^4+b*x^2+a)/x^2/(e*x^2+d)^3,x, algorithm="fricas")
```

```
[Out] [-1/16*(16*a*d^3*e^2 - 2*(c*d^3*e^2 + 3*b*d^2*e^3 - 15*a*d*e^4)*x^4 + 2*(c*
d^4*e - 5*b*d^3*e^2 + 25*a*d^2*e^3)*x^2 - ((c*d^2*e^2 + 3*b*d*e^3 - 15*a*e^
4)*x^5 + 2*(c*d^3*e + 3*b*d^2*e^2 - 15*a*d*e^3)*x^3 + (c*d^4 + 3*b*d^3*e -
15*a*d^2*e^2)*x)*sqrt(-d*e)*log((e*x^2 + 2*sqrt(-d*e)*x - d)/(e*x^2 + d)))/
(d^4*e^4*x^5 + 2*d^5*e^3*x^3 + d^6*e^2*x), -1/8*(8*a*d^3*e^2 - (c*d^3*e^2 +
 3*b*d^2*e^3 - 15*a*d*e^4)*x^4 + (c*d^4*e - 5*b*d^3*e^2 + 25*a*d^2*e^3)*x^2
 - ((c*d^2*e^2 + 3*b*d*e^3 - 15*a*e^4)*x^5 + 2*(c*d^3*e + 3*b*d^2*e^2 - 15*
a*d*e^3)*x^3 + (c*d^4 + 3*b*d^3*e - 15*a*d^2*e^2)*x)*sqrt(d*e)*arctan(sqrt(
d*e)*x/d))/(d^4*e^4*x^5 + 2*d^5*e^3*x^3 + d^6*e^2*x)]
```

Sympy [A] (verification not implemented)

Time = 1.03 (sec) , antiderivative size = 202, normalized size of antiderivative = 1.59

$$\int \frac{a+bx^2+cx^4}{x^2\left(d+ex^2\right)^3}\,dx = \frac{\sqrt{-\frac{1}{d^7e^3}}\cdot(15ae^2-3bde-cd^2)\log\left(-d^4e\sqrt{-\frac{1}{d^7e^3}}+x\right)}{16} - \frac{\sqrt{-\frac{1}{d^7e^3}}\cdot(15ae^2-3bde-cd^2)\log\left(d^4e\sqrt{-\frac{1}{d^7e^3}}+x\right)}{16} + \frac{-8ad^2e+x^4(-15ae^3+3bde^2+cd^2e)+x^2(-25ade^2+5bd^2e-cd^3)}{8d^5ex+16d^4e^2x^3+8d^3e^3x^5}$$

```
[In] integrate((c*x**4+b*x**2+a)/x**2/(e*x**2+d)**3,x)
```

```
[Out] sqrt(-1/(d**7*e**3))*(15*a*e**2 - 3*b*d*e - c*d**2)*log(-d**4*e*sqrt(-1/(d*
*7*e**3)) + x)/16 - sqrt(-1/(d**7*e**3))*(15*a*e**2 - 3*b*d*e - c*d**2)*log
(d**4*e*sqrt(-1/(d**7*e**3)) + x)/16 + (-8*a*d**2*e + x**4*(-15*a*e**3 + 3*
b*d*e**2 + c*d**2*e) + x**2*(-25*a*d*e**2 + 5*b*d**2*e - c*d**3))/(8*d**5*e
*x + 16*d**4*e**2*x**3 + 8*d**3*e**3*x**5)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{a+bx^2+cx^4}{x^2\left(d+ex^2\right)^3}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((c*x^4+b*x^2+a)/x^2/(e*x^2+d)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 118, normalized size of antiderivative = 0.93

$$\int \frac{a+bx^2+cx^4}{x^2\left(d+ex^2\right)^3}\,dx = \frac{\left(cd^2+3\,bde-15\,ae^2\right)\arctan\left(\frac{ex}{\sqrt{de}}\right)}{8\,\sqrt{de}d^3e} - \frac{a}{d^3x} + \frac{cd^2ex^3+3\,bde^2x^3-7\,ae^3x^3-cd^3x+5\,bd^2ex-9\,ade^2x}{8\,\left(ex^2+d\right)^2d^3e}$$

```
[In] integrate((c*x^4+b*x^2+a)/x^2/(e*x^2+d)^3,x, algorithm="giac")
```

```
[Out] 1/8*(c*d^2 + 3*b*d*e - 15*a*e^2)*arctan(e*x/sqrt(d*e))/(sqrt(d*e)*d^3*e) -
a/(d^3*x) + 1/8*(c*d^2*e*x^3 + 3*b*d*e^2*x^3 - 7*a*e^3*x^3 - c*d^3*x + 5*b*
d^2*e*x - 9*a*d*e^2*x)/((e*x^2 + d)^2*d^3*e)
```

Mupad [B] (verification not implemented)

Time = 7.70 (sec) , antiderivative size = 118, normalized size of antiderivative = 0.93

$$\int \frac{a+bx^2+cx^4}{x^2\left(d+ex^2\right)^3}\,dx = \frac{\mathrm{atan}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)\,\left(c\,d^2+3\,b\,d\,e-15\,a\,e^2\right)}{8\,d^{7/2}\,e^{3/2}} - \frac{\frac{a}{d}-\frac{x^4\,\left(c\,d^2+3\,b\,d\,e-15\,a\,e^2\right)}{8\,d^3}+\frac{x^2\,\left(c\,d^2-5\,b\,d\,e+25\,a\,e^2\right)}{8\,d^2\,e}}{d^2\,x+2\,d\,e\,x^3+e^2\,x^5}$$

```
[In] int((a + b*x^2 + c*x^4)/(x^2*(d + e*x^2)^3),x)
```

```
[Out] (atan((e^(1/2)*x)/d^(1/2))*(c*d^2 - 15*a*e^2 + 3*b*d*e))/(8*d^(7/2)*e^(3/2)
) - (a/d - (x^4*(c*d^2 - 15*a*e^2 + 3*b*d*e))/(8*d^3) + (x^2*(25*a*e^2 + c*
d^2 - 5*b*d*e))/(8*d^2*e))/(d^2*x + e^2*x^5 + 2*d*e*x^3)
```

3.293 $\int \frac{a+bx^2+cx^4}{x^4(d+ex^2)^3}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 142

$$\int \frac{a+bx^2+cx^4}{x^4(d+ex^2)^3}\,dx = -\frac{a}{3d^3x^3} - \frac{bd-3ae}{d^4x} + \frac{(cd^2-bde+ae^2)\,x}{4d^3(d+ex^2)^2} + \frac{(3cd^2-e(7bd-11ae))\,x}{8d^4(d+ex^2)} + \frac{(3cd^2-15bde+35ae^2)\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{8d^{9/2}\sqrt{e}}$$

```
[Out] -1/3*a/d^3/x^3+(3*a*e-b*d)/d^4/x+1/4*(a*e^2-b*d*e+c*d^2)*x/d^3/(e*x^2+d)^2+
1/8*(3*c*d^2-e*(-11*a*e+7*b*d))*x/d^4/(e*x^2+d)+1/8*(35*a*e^2-15*b*d*e+3*c*
d^2)*arctan(x*e^(1/2)/d^(1/2))/d^(9/2)/e^(1/2)
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {1273, 1275, 211}

$$\int \frac{a+bx^2+cx^4}{x^4(d+ex^2)^3}\,dx = \frac{\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)(35ae^2-15bde+3cd^2)}{8d^{9/2}\sqrt{e}} + \frac{x(3cd^2-e(7bd-11ae))}{8d^4(d+ex^2)} + \frac{x(ae^2-bde+cd^2)}{4d^3(d+ex^2)^2} - \frac{bd-3ae}{d^4x} - \frac{a}{3d^3x^3}$$

```
[In] Int[(a + b*x^2 + c*x^4)/(x^4*(d + e*x^2)^3),x]
```

```
[Out] -1/3*a/(d^3*x^3) - (b*d - 3*a*e)/(d^4*x) + ((c*d^2 - b*d*e + a*e^2)*x)/(4*d
^3*(d + e*x^2)^2) + ((3*c*d^2 - e*(7*b*d - 11*a*e))*x)/(8*d^4*(d + e*x^2))
```

```
+ ((3*c*d^2 - 15*b*d*e + 35*a*e^2)*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(8*d^(9/2)*
Sqrt[e])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1273

```
Int[(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^
4)^(p_.), x_Symbol] :> Simp[(-d)^(m/2 - 1)*(c*d^2 - b*d*e + a*e^2)^p*x*((d
+ e*x^2)^(q + 1)/(2*e^(2*p + m/2)*(q + 1))), x] + Dist[(-d)^(m/2 - 1)/(2*e^
(2*p)*(q + 1)), Int[x^m*(d + e*x^2)^(q + 1)*ExpandToSum[Together[(1/(d + e*
x^2))*(2*(-d)^(-m/2 + 1)*e^(2*p)*(q + 1)*(a + b*x^2 + c*x^4)^p - ((c*d^2 -
b*d*e + a*e^2)^p/(e^(m/2)*x^m))*(d + e*(2*q + 3)*x^2))], x], x], x] /; Free
Q[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p, 0] && ILtQ[q, -1] &
& ILtQ[m/2, 0]
```

Rule 1275

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*
(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && NeQ[
b^2 - 4*a*c, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{(cd^2-bde+ae^2)\,x}{4d^3\,(d+ex^2)^2}+\frac{\int \frac{4ad^2e^2+4de^2(bd-ae)x^2+3e^2(cd^2-bde+ae^2)x^4}{x^4(d+ex^2)^2}\,dx}{4d^3e^2}\\
&= \frac{(cd^2-bde+ae^2)\,x}{4d^3\,(d+ex^2)^2}+\frac{(3cd^2-e(7bd-11ae))\,x}{8d^4\,(d+ex^2)}\\
&\quad+\frac{\int \frac{8ad^4e^4+8d^3e^4(bd-2ae)x^2+d^2e^4(3cd^2-e(7bd-11ae))x^4}{x^4(d+ex^2)}\,dx}{8d^6e^4}\\
&= \frac{(cd^2-bde+ae^2)\,x}{4d^3\,(d+ex^2)^2}+\frac{(3cd^2-e(7bd-11ae))\,x}{8d^4\,(d+ex^2)}\\
&\quad+\frac{\int \left(\frac{8ad^3e^4}{x^4}+\frac{8d^2e^4(bd-3ae)}{x^2}+\frac{d^2e^4(3cd^2-15bde+35ae^2)}{d+ex^2}\right)dx}{8d^6e^4}\\
&= -\frac{a}{3d^3x^3}-\frac{bd-3ae}{d^4x}+\frac{(cd^2-bde+ae^2)\,x}{4d^3\,(d+ex^2)^2}\\
&\quad+\frac{(3cd^2-e(7bd-11ae))\,x}{8d^4\,(d+ex^2)}+\frac{(3cd^2-15bde+35ae^2)\int \frac{1}{d+ex^2}\,dx}{8d^4}
\end{aligned}
$$

$$= -\frac{a}{3d^3x^3} - \frac{bd-3ae}{d^4x} + \frac{(cd^2-bde+ae^2)\,x}{4d^3\,(d+ex^2)^2} + \frac{(3cd^2-e(7bd-11ae))\,x}{8d^4\,(d+ex^2)} + \frac{(3cd^2-15bde+35ae^2)\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{8d^{9/2}\sqrt{e}}$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 141, normalized size of antiderivative = 0.99

$$\int \frac{a+bx^2+cx^4}{x^4\,(d+ex^2)^3}\,dx = -\frac{a}{3d^3x^3} + \frac{-bd+3ae}{d^4x} + \frac{(cd^2-bde+ae^2)\,x}{4d^3\,(d+ex^2)^2} + \frac{(3cd^2-7bde+11ae^2)\,x}{8d^4\,(d+ex^2)} + \frac{(3cd^2-15bde+35ae^2)\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{8d^{9/2}\sqrt{e}}$$

```
[In] Integrate[(a + b*x^2 + c*x^4)/(x^4*(d + e*x^2)^3),x]
```

```
[Out] -1/3*a/(d^3*x^3) + (-(b*d) + 3*a*e)/(d^4*x) + ((c*d^2 - b*d*e + a*e^2)*x)/(
4*d^3*(d + e*x^2)^2) + ((3*c*d^2 - 7*b*d*e + 11*a*e^2)*x)/(8*d^4*(d + e*x^2
)) + ((3*c*d^2 - 15*b*d*e + 35*a*e^2)*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(8*d^(9/
2)*Sqrt[e])
```

Maple [A] (verified)

Time = 0.36 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.87

method	result
default	$-\frac{a}{3d^3x^3} - \frac{-3ae+bd}{d^4x} + \frac{\frac{\left(\frac{11}{8}a\,e^3-\frac{7}{8}d\,e^2b+\frac{3}{8}c\,d^2e\right)x^3+\frac{d\left(13a\,e^2-9bde+5c\,d^2\right)x}{8}}{\left(e\,x^2+d\right)^2}+\frac{\left(35a\,e^2-15bde+3c\,d^2\right)\arctan\left(\frac{ex}{\sqrt{ed}}\right)}{8\sqrt{ed}}}{d^4}$
risch	$\frac{\frac{e\left(35a\,e^2-15bde+3c\,d^2\right)x^6}{8d^4}+\frac{5\left(35a\,e^2-15bde+3c\,d^2\right)x^4}{24d^3}+\frac{(7ae-3bd)x^2}{3d^2}-\frac{a}{3d}}{x^3(e\,x^2+d)^2} - \frac{35\ln(-\sqrt{-ed}\,x+d)\,a\,e^2}{16\sqrt{-ed}\,d^4} + \frac{15\ln(-\sqrt{-ed}\,x+d)\,be}{16\sqrt{-ed}\,d^3} - \frac{3\ln}{}$

```
[In] int((c*x^4+b*x^2+a)/x^4/(e*x^2+d)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/3*a/d^3/x^3-(-3*a*e+b*d)/d^4/x+1/d^4*(((11/8*a*e^3-7/8*d*e^2*b+3/8*c*d^2
*e)*x^3+1/8*d*(13*a*e^2-9*b*d*e+5*c*d^2)*x)/(e*x^2+d)^2+1/8*(35*a*e^2-15*b*
d*e+3*c*d^2)/(e*d)^(1/2)*arctan(e*x/(e*d)^(1/2)))
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 476, normalized size of antiderivative = 3.35

$$\int \frac{a+bx^2+cx^4}{x^4\left(d+ex^2\right)^3}\,dx = \left[\frac{6\,(3\,cd^3e^2-15\,bd^2e^3+35\,ade^4)x^6-16\,ad^4e+10\,(3\,cd^4e-15\,bd^3e^2+35\,ad^2e^3)x^4-16\,(3\,bd^4e-7\,ad^3e^2)}{4}\right.$$

```
[In] integrate((c*x^4+b*x^2+a)/x^4/(e*x^2+d)^3,x, algorithm="fricas")
```

```
[Out] [1/48*(6*(3*c*d^3*e^2 - 15*b*d^2*e^3 + 35*a*d*e^4)*x^6 - 16*a*d^4*e + 10*(3
*c*d^4*e - 15*b*d^3*e^2 + 35*a*d^2*e^3)*x^4 - 16*(3*b*d^4*e - 7*a*d^3*e^2)*
x^2 - 3*((3*c*d^2*e^2 - 15*b*d*e^3 + 35*a*e^4)*x^7 + 2*(3*c*d^3*e - 15*b*d^
2*e^2 + 35*a*d*e^3)*x^5 + (3*c*d^4 - 15*b*d^3*e + 35*a*d^2*e^2)*x^3)*sqrt(-
d*e)*log((e*x^2 - 2*sqrt(-d*e)*x - d)/(e*x^2 + d)))/(d^5*e^3*x^7 + 2*d^6*e^
2*x^5 + d^7*e*x^3), 1/24*(3*(3*c*d^3*e^2 - 15*b*d^2*e^3 + 35*a*d*e^4)*x^6 -
 8*a*d^4*e + 5*(3*c*d^4*e - 15*b*d^3*e^2 + 35*a*d^2*e^3)*x^4 - 8*(3*b*d^4*e
 - 7*a*d^3*e^2)*x^2 + 3*((3*c*d^2*e^2 - 15*b*d*e^3 + 35*a*e^4)*x^7 + 2*(3*c
*d^3*e - 15*b*d^2*e^2 + 35*a*d*e^3)*x^5 + (3*c*d^4 - 15*b*d^3*e + 35*a*d^2*
e^2)*x^3)*sqrt(d*e)*arctan(sqrt(d*e)*x/d))/(d^5*e^3*x^7 + 2*d^6*e^2*x^5 + d
^7*e*x^3)]
```

Sympy [A] (verification not implemented)

Time = 1.35 (sec) , antiderivative size = 214, normalized size of antiderivative = 1.51

$$\int \frac{a+bx^2+cx^4}{x^4\left(d+ex^2\right)^3}\,dx = -\frac{\sqrt{-\frac{1}{d^9e}}(35ae^2-15bde+3cd^2)\log\left(-d^5\sqrt{-\frac{1}{d^9e}}+x\right)}{16} + \frac{\sqrt{-\frac{1}{d^9e}}(35ae^2-15bde+3cd^2)\log\left(d^5\sqrt{-\frac{1}{d^9e}}+x\right)}{16} + \frac{-8ad^3+x^6\cdot(105ae^3-45bde^2+9cd^2e)+x^4\cdot(175ade^2-75bd^2e+15cd^3)+x^2\cdot(56ad^2e-24bd^3)}{24d^6x^3+48d^5ex^5+24d^4e^2x^7}$$

```
[In] integrate((c*x**4+b*x**2+a)/x**4/(e*x**2+d)**3,x)
```

```
[Out] -sqrt(-1/(d**9*e))*(35*a*e**2 - 15*b*d*e + 3*c*d**2)*log(-d**5*sqrt(-1/(d**
9*e)) + x)/16 + sqrt(-1/(d**9*e))*(35*a*e**2 - 15*b*d*e + 3*c*d**2)*log(d**
5*sqrt(-1/(d**9*e)) + x)/16 + (-8*a*d**3 + x**6*(105*a*e**3 - 45*b*d*e**2 +
 9*c*d**2*e) + x**4*(175*a*d*e**2 - 75*b*d**2*e + 15*c*d**3) + x**2*(56*a*d
**2*e - 24*b*d**3))/(24*d**6*x**3 + 48*d**5*e*x**5 + 24*d**4*e**2*x**7)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{a+bx^2+cx^4}{x^4(d+ex^2)^3}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((c*x^4+b*x^2+a)/x^4/(e*x^2+d)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.92

$$\int \frac{a+bx^2+cx^4}{x^4(d+ex^2)^3}\,dx = \frac{(3\,cd^2-15\,bde+35\,ae^2)\arctan\left(\frac{ex}{\sqrt{de}}\right)}{8\,\sqrt{de}d^4} + \frac{3\,cd^2ex^3-7\,bde^2x^3+11\,ae^3x^3+5\,cd^3x-9\,bd^2ex+13\,ade^2x}{8\,(ex^2+d)^2d^4} - \frac{3\,bdx^2-9\,aex^2+ad}{3\,d^4x^3}$$

```
[In] integrate((c*x^4+b*x^2+a)/x^4/(e*x^2+d)^3,x, algorithm="giac")
```

```
[Out] 1/8*(3*c*d^2 - 15*b*d*e + 35*a*e^2)*arctan(e*x/sqrt(d*e))/(sqrt(d*e)*d^4) +
 1/8*(3*c*d^2*e*x^3 - 7*b*d*e^2*x^3 + 11*a*e^3*x^3 + 5*c*d^3*x - 9*b*d^2*e*
x + 13*a*d*e^2*x)/((e*x^2 + d)^2*d^4) - 1/3*(3*b*d*x^2 - 9*a*e*x^2 + a*d)/(
d^4*x^3)
```

Mupad [B] (verification not implemented)

Time = 7.74 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.97

$$\int \frac{a+bx^2+cx^4}{x^4(d+ex^2)^3}\,dx = \frac{\frac{x^2\,(7\,a\,e-3\,b\,d)}{3\,d^2}-\frac{a}{3\,d}+\frac{5\,x^4\,(3\,c\,d^2-15\,b\,d\,e+35\,a\,e^2)}{24\,d^3}+\frac{e\,x^6\,(3\,c\,d^2-15\,b\,d\,e+35\,a\,e^2)}{8\,d^4}}{d^2\,x^3+2\,d\,e\,x^5+e^2\,x^7} + \frac{\mathrm{atan}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)\,(3\,c\,d^2-15\,b\,d\,e+35\,a\,e^2)}{8\,d^{9/2}\,\sqrt{e}}$$

```
[In] int((a + b*x^2 + c*x^4)/(x^4*(d + e*x^2)^3),x)
```

```
[Out] ((x^2*(7*a*e - 3*b*d))/(3*d^2) - a/(3*d) + (5*x^4*(35*a*e^2 + 3*c*d^2 - 15*
b*d*e))/(24*d^3) + (e*x^6*(35*a*e^2 + 3*c*d^2 - 15*b*d*e))/(8*d^4))/(d^2*x^
3 + e^2*x^7 + 2*d*e*x^5) + (atan((e^(1/2)*x)/d^(1/2))*(35*a*e^2 + 3*c*d^2 -
 15*b*d*e))/(8*d^(9/2)*e^(1/2))
```

3.294 $\int \frac{a+bx^2+cx^4}{x^6(d+ex^2)^3}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 171

$$\int \frac{a+bx^2+cx^4}{x^6(d+ex^2)^3}\,dx = -\frac{a}{5d^3x^5}-\frac{bd-3ae}{3d^4x^3}-\frac{cd^2-3bde+6ae^2}{d^5x} -\frac{e(cd^2-bde+ae^2)\,x}{4d^4(d+ex^2)^2}-\frac{e(7cd^2-e(11bd-15ae))\,x}{8d^5(d+ex^2)} -\frac{\sqrt{e}(15cd^2-35bde+63ae^2)\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{8d^{11/2}}$$

```
[Out] -1/5*a/d^3/x^5+1/3*(3*a*e-b*d)/d^4/x^3+(-6*a*e^2+3*b*d*e-c*d^2)/d^5/x-1/4*e
*(a*e^2-b*d*e+c*d^2)*x/d^4/(e*x^2+d)^2-1/8*e*(7*c*d^2-e*(-15*a*e+11*b*d))*x
/d^5/(e*x^2+d)-1/8*(63*a*e^2-35*b*d*e+15*c*d^2)*arctan(x*e^(1/2)/d^(1/2))*e
^(1/2)/d^(11/2)
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 171, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {1273, 1819, 1816, 211}

$$\int \frac{a+bx^2+cx^4}{x^6(d+ex^2)^3}\,dx = -\frac{\sqrt{e}\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)(63ae^2-35bde+15cd^2)}{8d^{11/2}} -\frac{6ae^2-3bde+cd^2}{d^5x}-\frac{ex(7cd^2-e(11bd-15ae))}{8d^5(d+ex^2)} -\frac{ex(ae^2-bde+cd^2)}{4d^4(d+ex^2)^2}-\frac{bd-3ae}{3d^4x^3}-\frac{a}{5d^3x^5}$$

```
[In] Int[(a + b*x^2 + c*x^4)/(x^6*(d + e*x^2)^3),x]
```

```
[Out] -1/5*a/(d^3*x^5) - (b*d - 3*a*e)/(3*d^4*x^3) - (c*d^2 - 3*b*d*e + 6*a*e^2)/
(d^5*x) - (e*(c*d^2 - b*d*e + a*e^2)*x)/(4*d^4*(d + e*x^2)^2) - (e*(7*c*d^2
 - e*(11*b*d - 15*a*e))*x)/(8*d^5*(d + e*x^2)) - (Sqrt[e]*(15*c*d^2 - 35*b*
d*e + 63*a*e^2)*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(8*d^(11/2))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1273

```
Int[(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^
4)^(p_.), x_Symbol] :> Simp[(-d)^(m/2 - 1)*(c*d^2 - b*d*e + a*e^2)^p*x*((d
+ e*x^2)^(q + 1)/(2*e^(2*p + m/2)*(q + 1))), x] + Dist[(-d)^(m/2 - 1)/(2*e^
(2*p)*(q + 1)), Int[x^m*(d + e*x^2)^(q + 1)*ExpandToSum[Together[(1/(d + e*
x^2))*(2*(-d)^(-m/2 + 1)*e^(2*p)*(q + 1)*(a + b*x^2 + c*x^4)^p - ((c*d^2 -
b*d*e + a*e^2)^p/(e^(m/2)*x^m))*(d + e*(2*q + 3)*x^2))], x], x], x] /; Free
Q[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p, 0] && ILtQ[q, -1] &
& ILtQ[m/2, 0]
```

Rule 1816

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 1819

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b*f*x)*((a + b*x^2)^(p + 1)/(2*a*
b*(p + 1))), x] + Dist[1/(2*a*(p + 1)), Int[(c*x)^m*(a + b*x^2)^(p + 1)*Exp
andToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; Fr
eeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{e(cd^2-bde+ae^2)\,x}{4d^4\left(d+ex^2\right)^2}-\frac{\int \frac{-4ad^3e^2-4d^2e^2(bd-ae)x^2-4de^2\left(cd^2-bde+ae^2\right)x^4+3e^3\left(cd^2-bde+ae^2\right)x^6}{x^6\left(d+ex^2\right)^2}\,dx}{4d^4e^2}\\
&= -\frac{e(cd^2-bde+ae^2)\,x}{4d^4\left(d+ex^2\right)^2}-\frac{e(7cd^2-e(11bd-15ae))\,x}{8d^5\left(d+ex^2\right)}\\
&\quad+\frac{\int \frac{8ad^3e^2+8d^2e^2(bd-2ae)x^2+8de^2\left(cd^2-e(2bd-3ae)\right)x^4-e^3\left(7cd^2-e(11bd-15ae)\right)x^6}{x^6\left(d+ex^2\right)}\,dx}{8d^5e^2}
\end{aligned}$$

$$= -\frac{e(cd^2 - bde + ae^2)\,x}{4d^4\,(d + ex^2)^2} - \frac{e(7cd^2 - e(11bd - 15ae))\,x}{8d^5\,(d + ex^2)} + \frac{\int \left(\frac{8ad^2e^2}{x^6} + \frac{8de^2(bd-3ae)}{x^4} + \frac{8e^2(cd^2-3bde+6ae^2)}{x^2} - \frac{e^3(15cd^2-35bde+63ae^2)}{d+ex^2}\right) dx}{8d^5e^2}$$

$$= -\frac{a}{5d^3x^5} - \frac{bd - 3ae}{3d^4x^3} - \frac{cd^2 - 3bde + 6ae^2}{d^5x} - \frac{e(cd^2 - bde + ae^2)\,x}{4d^4\,(d + ex^2)^2} - \frac{e(7cd^2 - e(11bd - 15ae))\,x}{8d^5\,(d + ex^2)} - \frac{(e(15cd^2 - 35bde + 63ae^2)) \int \frac{1}{d+ex^2}\,dx}{8d^5}$$

$$= -\frac{a}{5d^3x^5} - \frac{bd - 3ae}{3d^4x^3} - \frac{cd^2 - 3bde + 6ae^2}{d^5x} - \frac{e(cd^2 - bde + ae^2)\,x}{4d^4\,(d + ex^2)^2} - \frac{e(7cd^2 - e(11bd - 15ae))\,x}{8d^5\,(d + ex^2)} - \frac{\sqrt{e}(15cd^2 - 35bde + 63ae^2)\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{8d^{11/2}}$$

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 173, normalized size of antiderivative = 1.01

$$\int \frac{a + bx^2 + cx^4}{x^6\,(d + ex^2)^3}\,dx = -\frac{a}{5d^3x^5} + \frac{-bd + 3ae}{3d^4x^3} + \frac{-cd^2 + 3bde - 6ae^2}{d^5x} - \frac{e(cd^2 - bde + ae^2)\,x}{4d^4\,(d + ex^2)^2} - \frac{(7cd^2e - 11bde^2 + 15ae^3)\,x}{8d^5\,(d + ex^2)} - \frac{\sqrt{e}(15cd^2 - 35bde + 63ae^2)\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{8d^{11/2}}$$

```
[In] Integrate[(a + b*x^2 + c*x^4)/(x^6*(d + e*x^2)^3),x]
```

```
[Out] -1/5*a/(d^3*x^5) + (-(b*d) + 3*a*e)/(3*d^4*x^3) + (-(c*d^2) + 3*b*d*e - 6*a
*e^2)/(d^5*x) - (e*(c*d^2 - b*d*e + a*e^2)*x)/(4*d^4*(d + e*x^2)^2) - ((7*c
*d^2*e - 11*b*d*e^2 + 15*a*e^3)*x)/(8*d^5*(d + e*x^2)) - (Sqrt[e]*(15*c*d^2
 - 35*b*d*e + 63*a*e^2)*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(8*d^(11/2))
```

Maple [A] (verified)

Time = 0.38 (sec) , antiderivative size = 151, normalized size of antiderivative = 0.88

method	result
default	$-\frac{a}{5d^3x^5}-\frac{-3ae+bd}{3d^4x^3}-\frac{6a\,e^2-3bde+c\,d^2}{d^5x}-\frac{e\left(\frac{\left(\frac{15}{8}a\,e^3-\frac{11}{8}d\,e^2b+\frac{7}{8}c\,d^2e\right)x^3+\frac{d\left(17a\,e^2-13bde+9c\,d^2\right)x}{8}}{\left(e\,x^2+d\right)^2}+\frac{\left(63a\,e^2-35bde+15c\,d^2\right)\operatorname{arcta}}{8\sqrt{ed}}\right.}{d^5}$
risch	$\frac{-\frac{e^2\left(63a\,e^2-35bde+15c\,d^2\right)x^8}{8d^5}-\frac{5e\left(63a\,e^2-35bde+15c\,d^2\right)x^6}{24d^4}-\frac{\left(63a\,e^2-35bde+15c\,d^2\right)x^4}{15d^3}+\frac{(9ae-5bd)x^2}{15d^2}-\frac{a}{5d}}{x^5\left(e\,x^2+d\right)^2}+\frac{63\sqrt{-ed}\,\ln\left(-ex+\sqrt{-ed}\right)a}{16d^6}$

[In] int((c*x^4+b*x^2+a)/x^6/(e*x^2+d)^3,x,method=_RETURNVERBOSE)

[Out] -1/5*a/d^3/x^5-1/3*(-3*a*e+b*d)/d^4/x^3-(6*a*e^2-3*b*d*e+c*d^2)/d^5/x-1/d^5 *e*(((15/8*a*e^3-11/8*d*e^2*b+7/8*c*d^2*e)*x^3+1/8*d*(17*a*e^2-13*b*d*e+9*c *d^2)*x)/(e*x^2+d)^2+1/8*(63*a*e^2-35*b*d*e+15*c*d^2)/(e*d)^(1/2)*arctan(e* x/(e*d)^(1/2)))

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 514, normalized size of antiderivative = 3.01

$$\int \frac{a+bx^2+cx^4}{x^6\left(d+ex^2\right)^3}\,dx$$

$$=\left[-\frac{30\,(15\,cd^2e^2-35\,bde^3+63\,ae^4)x^8+50\,(15\,cd^3e-35\,bd^2e^2+63\,ade^3)x^6+48\,ad^4+16\,(15\,cd^4-35\,bd^3e}{}\right.$$

$$-\frac{15\,(15\,cd^2e^2-35\,bde^3+63\,ae^4)x^8+25\,(15\,cd^3e-35\,bd^2e^2+63\,ade^3)x^6+24\,ad^4+8\,(15\,cd^4-35\,bd^3e+}{}$$

[In] integrate((c*x^4+b*x^2+a)/x^6/(e*x^2+d)^3,x, algorithm="fricas")

[Out] [-1/240*(30*(15*c*d^2*e^2 - 35*b*d*e^3 + 63*a*e^4)*x^8 + 50*(15*c*d^3*e - 35*b*d^2*e^2 + 63*a*d*e^3)*x^6 + 48*a*d^4 + 16*(15*c*d^4 - 35*b*d^3*e + 63*a *d^2*e^2)*x^4 + 16*(5*b*d^4 - 9*a*d^3*e)*x^2 - 15*((15*c*d^2*e^2 - 35*b*d*e ^3 + 63*a*e^4)*x^9 + 2*(15*c*d^3*e - 35*b*d^2*e^2 + 63*a*d*e^3)*x^7 + (15*c *d^4 - 35*b*d^3*e + 63*a*d^2*e^2)*x^5)*sqrt(-e/d)*log((e*x^2 - 2*d*x*sqrt(- e/d) - d)/(e*x^2 + d)))/(d^5*e^2*x^9 + 2*d^6*e*x^7 + d^7*x^5), -1/120*(15*(15*c*d^2*e^2 - 35*b*d*e^3 + 63*a*e^4)*x^8 + 25*(15*c*d^3*e - 35*b*d^2*e^2 + 63*a*d*e^3)*x^6 + 24*a*d^4 + 8*(15*c*d^4 - 35*b*d^3*e + 63*a*d^2*e^2)*x^4 + 8*(5*b*d^4 - 9*a*d^3*e)*x^2 + 15*((15*c*d^2*e^2 - 35*b*d*e^3 + 63*a*e^4)* x^9 + 2*(15*c*d^3*e - 35*b*d^2*e^2 + 63*a*d*e^3)*x^7 + (15*c*d^4 - 35*b*d^3 *e + 63*a*d^2*e^2)*x^5)*sqrt(e/d)*arctan(x*sqrt(e/d)))/(d^5*e^2*x^9 + 2*d^6 *e*x^7 + d^7*x^5)]

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 330 vs. $2(163) = 326$.

Time = 1.79 (sec) , antiderivative size = 330, normalized size of antiderivative = 1.93

$$\int \frac{a+bx^2+cx^4}{x^6(d+ex^2)^3}\,dx$$
$$= \frac{\sqrt{-\frac{e}{d^{11}}}\cdot(63ae^2-35bde+15cd^2)\log\left(-\frac{d^6\sqrt{-\frac{e}{d^{11}}}\cdot(63ae^2-35bde+15cd^2)}{63ae^3-35bde^2+15cd^2e}+x\right)}{16} - \frac{\sqrt{-\frac{e}{d^{11}}}\cdot(63ae^2-35bde+15cd^2)\log\left(\frac{d^6\sqrt{-\frac{e}{d^{11}}}\cdot(63ae^2-35bde+15cd^2)}{63ae^3-35bde^2+15cd^2e}+x\right)}{16} + \frac{-24ad^4+x^8(-945ae^4+525bde^3-225cd^2e^2)+x^6(-1575ade^3+875bd^2e^2-375cd^3e)+x^4(-504ad^2e}{120d^7x^5+240d^6ex^7+120d^5e^2x^9}$$

```
[In] integrate((c*x**4+b*x**2+a)/x**6/(e*x**2+d)**3,x)
```

```
[Out] sqrt(-e/d**11)*(63*a*e**2 - 35*b*d*e + 15*c*d**2)*log(-d**6*sqrt(-e/d**11)*
(63*a*e**2 - 35*b*d*e + 15*c*d**2)/(63*a*e**3 - 35*b*d*e**2 + 15*c*d**2*e)
+ x)/16 - sqrt(-e/d**11)*(63*a*e**2 - 35*b*d*e + 15*c*d**2)*log(d**6*sqrt(-
e/d**11)*(63*a*e**2 - 35*b*d*e + 15*c*d**2)/(63*a*e**3 - 35*b*d*e**2 + 15*c
*d**2*e) + x)/16 + (-24*a*d**4 + x**8*(-945*a*e**4 + 525*b*d*e**3 - 225*c*d
**2*e**2) + x**6*(-1575*a*d*e**3 + 875*b*d**2*e**2 - 375*c*d**3*e) + x**4*(
-504*a*d**2*e**2 + 280*b*d**3*e - 120*c*d**4) + x**2*(72*a*d**3*e - 40*b*d*
*4))/(120*d**7*x**5 + 240*d**6*e*x**7 + 120*d**5*e**2*x**9)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{a+bx^2+cx^4}{x^6(d+ex^2)^3}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((c*x^4+b*x^2+a)/x^6/(e*x^2+d)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 171, normalized size of antiderivative = 1.00

$$\int \frac{a+bx^2+cx^4}{x^6 (d+ex^2)^3}\,dx = -\frac{(15\,cd^2e - 35\,bde^2 + 63\,ae^3)\arctan\left(\frac{ex}{\sqrt{de}}\right)}{8\,\sqrt{de}d^5} - \frac{7\,cd^2e^2x^3 - 11\,bde^3x^3 + 15\,ae^4x^3 + 9\,cd^3ex - 13\,bd^2e^2x + 17\,ade^3x}{8\,(ex^2+d)^2d^5} - \frac{15\,cd^2x^4 - 45\,bdex^4 + 90\,ae^2x^4 + 5\,bd^2x^2 - 15\,adex^2 + 3\,ad^2}{15\,d^5x^5}$$

```
[In] integrate((c*x^4+b*x^2+a)/x^6/(e*x^2+d)^3,x, algorithm="giac")
```

```
[Out] -1/8*(15*c*d^2*e - 35*b*d*e^2 + 63*a*e^3)*arctan(e*x/sqrt(d*e))/(sqrt(d*e)*
d^5) - 1/8*(7*c*d^2*e^2*x^3 - 11*b*d*e^3*x^3 + 15*a*e^4*x^3 + 9*c*d^3*e*x -
 13*b*d^2*e^2*x + 17*a*d*e^3*x)/((e*x^2 + d)^2*d^5) - 1/15*(15*c*d^2*x^4 -
45*b*d*e*x^4 + 90*a*e^2*x^4 + 5*b*d^2*x^2 - 15*a*d*e*x^2 + 3*a*d^2)/(d^5*x^
5)
```

Mupad [B] (verification not implemented)

Time = 7.73 (sec) , antiderivative size = 168, normalized size of antiderivative = 0.98

$$\int \frac{a+bx^2+cx^4}{x^6 (d+ex^2)^3}\,dx = -\frac{\frac{a}{5\,d} - \frac{x^2\,(9\,a\,e-5\,b\,d)}{15\,d^2} + \frac{x^4\,(15\,c\,d^2-35\,b\,d\,e+63\,a\,e^2)}{15\,d^3} + \frac{5\,e\,x^6\,(15\,c\,d^2-35\,b\,d\,e+63\,a\,e^2)}{24\,d^4} + \frac{e^2\,x^8\,(15\,c\,d^2-35\,b\,d\,e+63\,a\,e^2)}{8\,d^5}}{d^2\,x^5 + 2\,d\,e\,x^7 + e^2\,x^9} - \frac{\sqrt{e}\,\mathrm{atan}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)\,(15\,c\,d^2 - 35\,b\,d\,e + 63\,a\,e^2)}{8\,d^{11/2}}$$

```
[In] int((a + b*x^2 + c*x^4)/(x^6*(d + e*x^2)^3),x)
```

```
[Out] - (a/(5*d) - (x^2*(9*a*e - 5*b*d))/(15*d^2) + (x^4*(63*a*e^2 + 15*c*d^2 - 3
5*b*d*e))/(15*d^3) + (5*e*x^6*(63*a*e^2 + 15*c*d^2 - 35*b*d*e))/(24*d^4) +
(e^2*x^8*(63*a*e^2 + 15*c*d^2 - 35*b*d*e))/(8*d^5))/(d^2*x^5 + e^2*x^9 + 2*
d*e*x^7) - (e^(1/2)*atan((e^(1/2)*x)/d^(1/2))*(63*a*e^2 + 15*c*d^2 - 35*b*d
*e))/(8*d^(11/2))
```

3.295 $\int \frac{x^9}{(d+ex^2)(a+bx^2+cx^4)}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 230

$$\begin{aligned}\int \frac{x^9}{(d+ex^2)(a+bx^2+cx^4)}\,dx &= -\frac{(cd+be)x^2}{2c^2e^2}+\frac{x^4}{4ce}-\frac{(b^4d-4ab^2cd+2a^2c^2d-ab^3e+3a^2bce)\,\text{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2c^3\sqrt{b^2-4ac}\,(cd^2-bde+ae^2)} \\ &\quad+\frac{d^4\log(d+ex^2)}{2e^3(cd^2-bde+ae^2)}-\frac{(b^3d-2abcd-ab^2e+a^2ce)\log(a+bx^2+cx^4)}{4c^3(cd^2-bde+ae^2)}\end{aligned}$$

```
[Out] -1/2*(b*e+c*d)*x^2/c^2/e^2+1/4*x^4/c/e+1/2*d^4*ln(e*x^2+d)/e^3/(a*e^2-b*d*e
+c*d^2)-1/4*(a^2*c*e-a*b^2*e-2*a*b*c*d+b^3*d)*ln(c*x^4+b*x^2+a)/c^3/(a*e^2-
b*d*e+c*d^2)-1/2*(3*a^2*b*c*e+2*a^2*c^2*d-a*b^3*e-4*a*b^2*c*d+b^4*d)*arctan
h((2*c*x^2+b)/(-4*a*c+b^2)^(1/2))/c^3/(a*e^2-b*d*e+c*d^2)/(-4*a*c+b^2)^(1/2
)
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 230, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {1265, 1642, 648, 632, 212, 642}

$$\int \frac{x^9}{(d+ex^2)(a+bx^2+cx^4)}\,dx$$
$$= -\frac{\operatorname{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)(3a^2bce+2a^2c^2d-ab^3e-4ab^2cd+b^4d)}{2c^3\sqrt{b^2-4ac}\,(ae^2-bde+cd^2)}$$
$$-\frac{(a^2ce-ab^2e-2abcd+b^3d)\log(a+bx^2+cx^4)}{4c^3(ae^2-bde+cd^2)}$$
$$+\frac{d^4\log(d+ex^2)}{2e^3(ae^2-bde+cd^2)}-\frac{x^2(be+cd)}{2c^2e^2}+\frac{x^4}{4ce}$$

```
[In] Int[x^9/((d + e*x^2)*(a + b*x^2 + c*x^4)),x]
```

```
[Out] -1/2*((c*d + b*e)*x^2)/(c^2*e^2) + x^4/(4*c*e) - ((b^4*d - 4*a*b^2*c*d + 2*
a^2*c^2*d - a*b^3*e + 3*a^2*b*c*e)*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]]
)/(2*c^3*Sqrt[b^2 - 4*a*c]*(c*d^2 - b*d*e + a*e^2)) + (d^4*Log[d + e*x^2])/
(2*e^3*(c*d^2 - b*d*e + a*e^2)) - ((b^3*d - 2*a*b*c*d - a*b^2*e + a^2*c*e)*
Log[a + b*x^2 + c*x^4])/(4*c^3*(c*d^2 - b*d*e + a*e^2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
```

```
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1265

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rule 1642

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{2}\text{Subst}\left(\int \frac{x^4}{(d+ex)(a+bx+cx^2)}\,dx, x, x^2\right) \\
&= \frac{1}{2}\text{Subst}\left(\int \left(\frac{-cd-be}{c^2e^2}+\frac{x}{ce}+\frac{d^4}{e^2(cd^2-bde+ae^2)(d+ex)}\right.\right. \\
&\qquad \left.\left.+\frac{-a(b^2d-acd-abe)-(b^3d-2abcd-ab^2e+a^2ce)x}{c^2(cd^2-bde+ae^2)(a+bx+cx^2)}\right)dx, x, x^2\right) \\
&= -\frac{(cd+be)x^2}{2c^2e^2}+\frac{x^4}{4ce}+\frac{d^4\log(d+ex^2)}{2e^3(cd^2-bde+ae^2)} \\
&\quad +\frac{\text{Subst}\left(\int \frac{-a(b^2d-acd-abe)-(b^3d-2abcd-ab^2e+a^2ce)x}{a+bx+cx^2}\,dx, x, x^2\right)}{2c^2(cd^2-bde+ae^2)} \\
&= -\frac{(cd+be)x^2}{2c^2e^2}+\frac{x^4}{4ce}+\frac{d^4\log(d+ex^2)}{2e^3(cd^2-bde+ae^2)} \\
&\quad -\frac{(b^3d-2abcd-ab^2e+a^2ce)\,\text{Subst}\left(\int \frac{b+2cx}{a+bx+cx^2}\,dx, x, x^2\right)}{4c^3(cd^2-bde+ae^2)} \\
&\quad +\frac{(b^4d-4ab^2cd+2a^2c^2d-ab^3e+3a^2bce)\,\text{Subst}\left(\int \frac{1}{a+bx+cx^2}\,dx, x, x^2\right)}{4c^3(cd^2-bde+ae^2)} \\
&= -\frac{(cd+be)x^2}{2c^2e^2}+\frac{x^4}{4ce}+\frac{d^4\log(d+ex^2)}{2e^3(cd^2-bde+ae^2)} \\
&\quad -\frac{(b^3d-2abcd-ab^2e+a^2ce)\log(a+bx^2+cx^4)}{4c^3(cd^2-bde+ae^2)} \\
&\quad -\frac{(b^4d-4ab^2cd+2a^2c^2d-ab^3e+3a^2bce)\,\text{Subst}\left(\int \frac{1}{b^2-4ac-x^2}\,dx, x, b+2cx^2\right)}{2c^3(cd^2-bde+ae^2)}
\end{aligned}$$

$$= -\frac{(cd+be)x^2}{2c^2e^2} + \frac{x^4}{4ce} - \frac{(b^4d - 4ab^2cd + 2a^2c^2d - ab^3e + 3a^2bce)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2c^3\sqrt{b^2-4ac}\,(cd^2-bde+ae^2)} + \frac{d^4\log(d+ex^2)}{2e^3(cd^2-bde+ae^2)} - \frac{(b^3d - 2abcd - ab^2e + a^2ce)\log(a+bx^2+cx^4)}{4c^3(cd^2-bde+ae^2)}$$

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 228, normalized size of antiderivative = 0.99

$$\int \frac{x^9}{(d+ex^2)(a+bx^2+cx^4)}\,dx = \frac{1}{4}\left(-\frac{2(cd+be)x^2}{c^2e^2} + \frac{x^4}{ce} - \frac{2(b^4d - 4ab^2cd + 2a^2c^2d - ab^3e + 3a^2bce)\arctan\left(\frac{b+2cx^2}{\sqrt{-b^2+4ac}}\right)}{c^3\sqrt{-b^2+4ac}\,(-cd^2+e(bd-ae))} + \frac{2d^4\log(d+ex^2)}{e^3(cd^2+e(-bd+ae))} + \frac{(-b^3d + 2abcd + ab^2e - a^2ce)\log(a+bx^2+cx^4)}{c^3(cd^2+e(-bd+ae))}\right)$$

```
[In] Integrate[x^9/((d + e*x^2)*(a + b*x^2 + c*x^4)),x]
```

```
[Out] ((-2*(c*d + b*e)*x^2)/(c^2*e^2) + x^4/(c*e) - (2*(b^4*d - 4*a*b^2*c*d + 2*a
^2*c^2*d - a*b^3*e + 3*a^2*b*c*e)*ArcTan[(b + 2*c*x^2)/Sqrt[-b^2 + 4*a*c]])
/(c^3*Sqrt[-b^2 + 4*a*c]*(-(c*d^2) + e*(b*d - a*e))) + (2*d^4*Log[d + e*x^2
])/(e^3*(c*d^2 + e*(-(b*d) + a*e))) + ((-(b^3*d) + 2*a*b*c*d + a*b^2*e - a^
2*c*e)*Log[a + b*x^2 + c*x^4])/(c^3*(c*d^2 + e*(-(b*d) + a*e))))/4
```

Maple [A] (verified)

Time = 0.51 (sec) , antiderivative size = 216, normalized size of antiderivative = 0.94

method	result
default	$\frac{(-c\,x^2e+be+cd)^2}{4e^3c^3} + \frac{\frac{(-a^2ce+a\,b^2e+2abcd-b^3d)\ln(c\,x^4+b\,x^2+a)}{2c} + \frac{2\left(a^2be+a^2cd-a\,b^2d-\frac{(-a^2ce+a\,b^2e+2abcd-b^3d)b}{2c}\right)\arctan\left(\frac{2c\,x^2+b}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}}}{2(a\,e^2-bde+c\,d^2)c^2}$
risch	$\frac{x^4}{4ce} - \frac{b\,x^2}{2e\,c^2} - \frac{d\,x^2}{2c\,e^2} + \frac{b^2}{4e\,c^3} + \frac{bd}{2e^2c^2} + \frac{d^2}{4c\,e^3} + \frac{d^4\ln(e\,x^2+d)}{2e^3(a\,e^2-bde+c\,d^2)} + \frac{\sum_{\_R=\text{RootOf}((4a^2c^2e^2-a\,b^2c\,e^2-4ab\,c^2de+4a\,c^3d^2+b^3cd}{}$

```
[In] int(x^9/(e*x^2+d)/(c*x^4+b*x^2+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*(-c*e*x^2+b*e+c*d)^2/e^3/c^3+1/2/(a*e^2-b*d*e+c*d^2)/c^2*(1/2*(-a^2*c*e
+a*b^2*e+2*a*b*c*d-b^3*d)/c*ln(c*x^4+b*x^2+a)+2*(a^2*b*e+a^2*c*d-a*b^2*d-1/
2*(-a^2*c*e+a*b^2*e+2*a*b*c*d-b^3*d)*b/c)/(4*a*c-b^2)^(1/2)*arctan((2*c*x^2
+b)/(4*a*c-b^2)^(1/2)))+1/2*d^4*ln(e*x^2+d)/e^3/(a*e^2-b*d*e+c*d^2)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{x^9}{(d+ex^2)\,(a+bx^2+cx^4)}\,dx = \text{Timed out}$$

```
[In] integrate(x^9/(e*x^2+d)/(c*x^4+b*x^2+a),x, algorithm="fricas")
[Out] Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^9}{(d+ex^2)\,(a+bx^2+cx^4)}\,dx = \text{Timed out}$$

```
[In] integrate(x**9/(e*x**2+d)/(c*x**4+b*x**2+a),x)
[Out] Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^9}{(d+ex^2)\,(a+bx^2+cx^4)}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(x^9/(e*x^2+d)/(c*x^4+b*x^2+a),x, algorithm="maxima")
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

Giac [A] (verification not implemented)

none

Time = 0.60 (sec) , antiderivative size = 233, normalized size of antiderivative = 1.01

$$\begin{aligned}\int \frac{x^9}{(d+ex^2)\,(a+bx^2+cx^4)}\,dx &= \frac{d^4\log\left(|ex^2+d|\right)}{2\,(cd^2e^3-bde^4+ae^5)} - \frac{(b^3d-2\,abcd-ab^2e+a^2ce)\log\left(cx^4+bx^2+a\right)}{4\,(c^4d^2-bc^3de+ac^3e^2)} \\ &\quad + \frac{(b^4d-4\,ab^2cd+2\,a^2c^2d-ab^3e+3\,a^2bce)\arctan\left(\frac{2\,cx^2+b}{\sqrt{-b^2+4\,ac}}\right)}{2\,(c^4d^2-bc^3de+ac^3e^2)\sqrt{-b^2+4\,ac}} + \frac{cex^4-2\,cdx^2-2\,bex^2}{4\,c^2e^2}\end{aligned}$$

```
[In] integrate(x^9/(e*x^2+d)/(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] 1/2*d^4*log(abs(e*x^2 + d))/(c*d^2*e^3 - b*d*e^4 + a*e^5) - 1/4*(b^3*d - 2*
a*b*c*d - a*b^2*e + a^2*c*e)*log(c*x^4 + b*x^2 + a)/(c^4*d^2 - b*c^3*d*e +
a*c^3*e^2) + 1/2*(b^4*d - 4*a*b^2*c*d + 2*a^2*c^2*d - a*b^3*e + 3*a^2*b*c*e
)*arctan((2*c*x^2 + b)/sqrt(-b^2 + 4*a*c))/((c^4*d^2 - b*c^3*d*e + a*c^3*e^
2)*sqrt(-b^2 + 4*a*c)) + 1/4*(c*e*x^4 - 2*c*d*x^2 - 2*b*e*x^2)/(c^2*e^2)
```

Mupad [B] (verification not implemented)

Time = 73.40 (sec) , antiderivative size = 7024, normalized size of antiderivative = 30.54

$$\int \frac{x^9}{(d+ex^2)(a+bx^2+cx^4)}\,dx = \text{Too large to display}$$

```
[In] int(x^9/((d + e*x^2)*(a + b*x^2 + c*x^4)),x)
```

```
[Out] (d^4*log(d + e*x^2))/(2*a*e^5 + 2*c*d^2*e^3 - 2*b*d*e^4) + (log((x^2*(a^7*e
^7 + b^7*d^7 - 2*a^3*b*c^3*d^7 - a^4*c^3*d^6*e - 2*a^6*c*d^2*e^5 + 7*a^2*b^
3*c^2*d^7 + 3*a^2*b^5*d^5*e^2 + 4*a^3*b^4*d^4*e^3 + 4*a^4*b^3*d^3*e^4 + 3*a
^5*b^2*d^2*e^5 + 2*a^5*c^2*d^4*e^3 - 5*a*b^5*c*d^7 + 2*a*b^6*d^6*e + 2*a^6*
b*d*e^6 - 8*a^2*b^4*c*d^6*e - 6*a^5*b*c*d^3*e^4 + 8*a^3*b^2*c^2*d^6*e - 9*a
^3*b^3*c*d^5*e^2 + 5*a^4*b*c^2*d^5*e^2 - 9*a^4*b^2*c*d^4*e^3))/(c^4*e^4) +
(a*d*(a^3*e^3 + b^3*d^3 - 2*a*b*c*d^3 + a*b^2*d^2*e + a^2*b*d*e^2 - a^2*c*d
^2*e)^2)/(c^4*e^4) + (((x^2*(4*a^2*c^6*d^8 + 6*a^4*b^4*e^8 + 18*a^6*c^2*e^8
 + 6*b^4*c^4*d^8 + 6*b^8*d^4*e^4 - 16*a*b^2*c^5*d^8 - 26*a^5*b^2*c*e^8 + 8*
a*b^7*d^3*e^5 + 8*a^3*b^5*d*e^7 - 2*b^5*c^3*d^7*e - 2*b^7*c*d^5*e^3 + 8*a^2
*b^6*d^2*e^6 - 20*a^3*c^5*d^6*e^2 + 40*a^4*c^4*d^4*e^4 - 36*a^5*c^3*d^2*e^6
 + 2*b^6*c^2*d^6*e^2 + 42*a^2*b^2*c^4*d^6*e^2 - 28*a^2*b^3*c^3*d^5*e^3 + 80
*a^2*b^4*c^2*d^4*e^4 - 64*a^3*b^2*c^3*d^4*e^4 + 80*a^3*b^3*c^2*d^3*e^5 + 48
*a^4*b^2*c^2*d^2*e^6 + 18*a*b^3*c^4*d^7*e - 40*a*b^6*c*d^4*e^4 - 26*a^2*b*c
^5*d^7*e - 32*a^4*b^3*c*d*e^7 + 12*a^5*b*c^2*d*e^7 - 16*a*b^4*c^3*d^6*e^2 +
 10*a*b^5*c^2*d^5*e^3 - 48*a^2*b^5*c*d^3*e^5 + 46*a^3*b*c^4*d^5*e^3 - 40*a^
3*b^4*c*d^2*e^6 - 48*a^4*b*c^3*d^3*e^5))/(c^4*e^4) + (((x^2*(8*a*b^8*e^9 +
8*b*c^8*d^9 + 8*b^9*d*e^8 + 120*a^5*c^4*e^9 - 72*a^2*b^6*c*e^9 - 8*b^2*c^7*
d^8*e - 8*b^8*c*d^2*e^7 + 212*a^3*b^4*c^2*e^9 - 240*a^4*b^2*c^3*e^9 - 112*a
^2*c^7*d^6*e^3 + 240*a^3*c^6*d^4*e^5 - 228*a^4*c^5*d^2*e^7 + 4*b^3*c^6*d^7*
e^2 - 24*b^4*c^5*d^6*e^3 + 32*b^5*c^4*d^5*e^4 - 24*b^6*c^3*d^4*e^5 + 4*b^7*
c^2*d^3*e^6 + 32*a*c^8*d^8*e - 56*a*b^7*c*d*e^8 - 428*a^2*b^2*c^5*d^4*e^5 +
 108*a^2*b^3*c^4*d^3*e^6 - 216*a^2*b^4*c^3*d^2*e^7 + 424*a^3*b^2*c^4*d^2*e^
7 - 16*a*b*c^7*d^7*e^2 + 8*a^4*b*c^4*d*e^8 + 88*a*b^2*c^6*d^6*e^3 - 116*a*b
^3*c^5*d^5*e^4 + 188*a*b^4*c^4*d^4*e^5 - 36*a*b^5*c^3*d^3*e^6 + 60*a*b^6*c^
2*d^2*e^7 + 40*a^2*b*c^6*d^5*e^4 + 100*a^2*b^5*c^2*d*e^8 - 72*a^3*b*c^5*d^3
*e^6 - 4*a^3*b^3*c^3*d*e^8))/(c^4*e^4) - (((x^2*(32*a*b^6*c^3*e^10 - 352*a^
4*c^6*e^10 + 128*a*c^9*d^6*e^4 + 32*b*c^9*d^7*e^3 + 32*b^7*c^3*d*e^9 - 256*
a^2*b^4*c^4*e^10 + 600*a^3*b^2*c^5*e^10 - 464*a^2*c^8*d^4*e^6 + 592*a^3*c^7
```

*d^2*e^8 - 64*b^2*c^8*d^6*e^4 + 56*b^3*c^7*d^5*e^5 - 48*b^4*c^6*d^4*e^6 + 56*b^5*c^5*d^3*e^7 - 64*b^6*c^4*d^2*e^8 - 688*a^2*b^2*c^6*d^2*e^8 - 192*a*b*c^8*d^5*e^5 - 224*a*b^5*c^4*d*e^9 - 72*a^3*b*c^6*d*e^9 + 272*a*b^2*c^7*d^4*e^6 - 200*a*b^3*c^6*d^3*e^7 + 360*a*b^4*c^5*d^2*e^8 + 136*a^2*b*c^7*d^3*e^7 + 424*a^2*b^3*c^5*d*e^9))/(c^4*e^4) + (32*a*d*(2*b^6*e^6 + 2*c^6*d^6 - 15*a^3*c^3*e^6 - 10*a*c^5*d^4*e^2 + 29*a^2*b^2*c^2*e^6 + 17*a^2*c^4*d^2*e^4 + 3*b^2*c^4*d^4*e^2 - b^3*c^3*d^3*e^3 + 3*b^4*c^2*d^2*e^4 - 14*a*b^4*c*e^6 - 2*b*c^5*d^5*e - 2*b^5*c*d*e^5 + 2*a*b*c^4*d^3*e^3 + 6*a*b^3*c^2*d*e^5 + a^2*b*c^3*d*e^5 - 13*a*b^2*c^3*d^2*e^4))/(c*e) - (8*e^2*(b^2*e^2 + c^2*d^2 - 3*a*c*e^2 - b*c*d*e)*(b^5*d + b^4*d*(b^2 - 4*a*c)^(1/2) - 4*a^3*c^2*e - a*b^4*e - 6*a*b^3*c*d - a*b^3*e*(b^2 - 4*a*c)^(1/2) + 8*a^2*b*c^2*d + 5*a^2*b^2*c*e + 2*a^2*c^2*d*(b^2 - 4*a*c)^(1/2) - 4*a*b^2*c*d*(b^2 - 4*a*c)^(1/2) + 3*a^2*b*c*e*(b^2 - 4*a*c)^(1/2))*(2*a*c^2*d^3 + a*b^2*e^3*x^2 + b*c^2*d^3*x^2 - 4*a^2*c*e^3*x^2 + b^3*d*e^2*x^2 + 2*a*b^2*d*e^2 - 6*a^2*c*d*e^2 + 4*a*c^2*d^2*e*x^2 - 2*b^2*c*d^2*e*x^2 - 2*a*b*c*d^2*e - 3*a*b*c*d*e^2*x^2))/(c*(4*a*c - b^2)*(a*e^2 + c*d^2 - b*d*e)))*(b^5*d + b^4*d*(b^2 - 4*a*c)^(1/2) - 4*a^3*c^2*e - a*b^4*e - 6*a*b^3*c*d - a*b^3*e*(b^2 - 4*a*c)^(1/2) + 8*a^2*b*c^2*d + 5*a^2*b^2*c*e + 2*a^2*c^2*d*(b^2 - 4*a*c)^(1/2) - 4*a*b^2*c*d*(b^2 - 4*a*c)^(1/2) + 3*a^2*b*c*e*(b^2 - 4*a*c)^(1/2)))/(4*c^3*(4*a*c - b^2)*(a*e^2 + c*d^2 - b*d*e)) + (4*a*d*(4*b^8*e^8 + 4*c^8*d^8 + 37*a^4*c^4*e^8 - 16*a*c^7*d^6*e^2 + 84*a^2*b^4*c^2*e^8 - 84*a^3*b^2*c^3*e^8 + 40*a^2*c^6*d^4*e^4 - 56*a^3*c^5*d^2*e^6 + 4*b^2*c^6*d^6*e^2 - 4*b^3*c^5*d^5*e^3 + 13*b^4*c^4*d^4*e^4 - 4*b^5*c^3*d^3*e^5 + 4*b^6*c^2*d^2*e^6 - 32*a*b^6*c*e^8 + 98*a^2*b^2*c^4*d^2*e^6 - 8*a*b^5*c^2*d*e^7 - 4*a^3*b*c^4*d*e^7 - 52*a*b^2*c^5*d^4*e^4 + 20*a*b^3*c^4*d^3*e^5 - 36*a*b^4*c^3*d^2*e^6 - 16*a^2*b*c^5*d^3*e^5 + 28*a^2*b^3*c^3*d*e^7))/(c^4*e^4))*(b^5*d + b^4*d*(b^2 - 4*a*c)^(1/2) - 4*a^3*c^2*e - a*b^4*e - 6*a*b^3*c*d - a*b^3*e*(b^2 - 4*a*c)^(1/2) + 8*a^2*b*c^2*d + 5*a^2*b^2*c*e + 2*a^2*c^2*d*(b^2 - 4*a*c)^(1/2) - 4*a*b^2*c*d*(b^2 - 4*a*c)^(1/2) + 3*a^2*b*c*e*(b^2 - 4*a*c)^(1/2)))/(4*c^3*(4*a*c - b^2)*(a*e^2 + c*d^2 - b*d*e)) + (4*a*d*(2*a^3*b^4*e^7 + 5*a^5*c^2*e^7 + 2*b^3*c^4*d^7 + 2*b^7*d^3*e^4 - 8*a^4*b^2*c*e^7 + 2*a*b^6*d^2*e^5 + 2*a^2*b^5*d*e^6 - 2*a^2*c^5*d^6*e + 6*a^3*c^4*d^4*e^3 - 9*a^4*c^3*d^2*e^5 + b^5*c^2*d^5*e^2 - 4*a*b*c^5*d^7 - a^2*b^2*c^3*d^4*e^3 + 20*a^2*b^3*c^2*d^3*e^4 + 12*a^3*b^2*c^2*d^2*e^5 + 2*a*b^2*c^4*d^6*e - 12*a*b^5*c*d^3*e^4 - 8*a^3*b^3*c*d*e^6 + 3*a^4*b*c^2*d*e^6 - 6*a*b^3*c^3*d^5*e^2 - a*b^4*c^2*d^4*e^3 + 10*a^2*b*c^4*d^5*e^2 - 10*a^2*b^4*c*d^2*e^5 - 12*a^3*b*c^3*d^3*e^4))/(c^4*e^4))*(b^5*d + b^4*d*(b^2 - 4*a*c)^(1/2) - 4*a^3*c^2*e - a*b^4*e - 6*a*b^3*c*d - a*b^3*e*(b^2 - 4*a*c)^(1/2) + 8*a^2*b*c^2*d + 5*a^2*b^2*c*e + 2*a^2*c^2*d*(b^2 - 4*a*c)^(1/2) - 4*a*b^2*c*d*(b^2 - 4*a*c)^(1/2) + 3*a^2*b*c*e*(b^2 - 4*a*c)^(1/2)))/(4*c^3*(4*a*c - b^2)*(a*e^2 + c*d^2 - b*d*e)))*(b^5*d + b^4*d*(b^2 - 4*a*c)^(1/2) - 4*a^3*c^2*e - a*b^4*e - 6*a*b^3*c*d - a*b^3*e*(b^2 - 4*a*c)^(1/2) + 8*a^2*b*c^2*d + 5*a^2*b^2*c*e + 2*a^2*c^2*d*(b^2 - 4*a*c)^(1/2) - 4*a*b^2*c*d*(b^2 - 4*a*c)^(1/2) + 3*a^2*b*c*e*(b^2 - 4*a*c)^(1/2)))/(4*(4*a*c^5*d^2 + 4*a^2*c^4*e^2 - b^2*c^4*d^2 - a*b^2*c^3*e^2 + b^3*c^3*d*e - 4*a*b*c^4*d*e)) - (log((x^2*(a^7*e^7 + b^7*d^7 - 2*a^3*b*c^3*d^7 - a^4*c^3*d^6* e - 2*a^6*c*d^2*e^5 + 7*a^2*b^3*c^2*d^7 + 3*a^2*b^5*d^5*e^2 + 4*a^3*b^4*d^4*e^3 + 4*a^4*b^3*d^3*e^4 + 3*a^5*b^2*d^2*e^5 + 2*a^5*c^2*d^4*e^3 - 5*a*b^5*c*d^7 + 2*a*b^6*d^6*e + 2*a^6*b*d*e^6 - 8*a^2*b^4*c*d^6*e - 6*a^5*b*c*d^3*e^4 + 8*a^3*b^2*c^2*d^6*e - 9*a^3*b^3*c*d^5*e^2 + 5*a^4*b*c^2*d^5*e^2 - 9*a^4*b^2*c*d^4*e^3))/(c^4*e^4) + (a*d*(a^3*e^3 + b^3*d^3 - 2*a*b*c*d^3 + a*b^2*d^2*e + a^2*b*d*e^2 - a^2*c*d^2*e)^2)/(c^4*e^4) - (((x^2*(4*a^2*c^6*d^8 + 6*a^4*b^4*e^8 + 18*a^6*c^2*e^8 + 6*b^4*c^4*d^8 + 6*b^8*d^4*e^4 - 16*a*b^2*c^5*d^8 - 26*a^5*b^2*c*e^8 + 8*a*b^7*d^3*e^5 + 8*a^3*b^5*d*e^7 - 2*b^5*c^3*d^7*e - 2*b^7*c*d^5*e^3 + 8*a^2*b^6*d^2*e^6 - 20*a^3*c^5*d^6*e^2 + 40*a^4*c^4*d^4*e^4 - 36*a^5*c^3*d^2*e^6 + 2*b^6*c^2*d^6*e^2 + 42*a^2*b^2*c^4*d^6*e^2 - 28*a^2*b^3*c^3*d^5*e^3 + 80*a^2*b^4*c^2*d^4*e^4 - 64*a^3*b^2*c^3*d^4*e^4 + 80*a^3*b^3*c^2*d^3*e^5 + 48*a^4*b^2*c^2*d^2*e^6 + 18*a*b^3*c^4*d^7*e - 40*a*b^6*c*d^4*e^4 - 26*a^2*b*c^5*d^7*e - 32*a^4*b^3*c*d*e^7 + 12*a^5*b*c^2*d*e^7 - 16*a*b^4*c^3*d^6*e^2 + 10*a*b^5*c^2*d^5*e^3 - 48*a^2*b^5*c*d^3*e^5 + 46*a^3*b*c^4*d^5*e^3 - 40*a^3*b^4*c*d^2*e^6 - 48*a^4*b*c^3*d^3*e^5))/(c^4*e^4) - (((x^2*(8*a*b^8*e^9 + 8*b*c^8*d^9 + 8*b^9*d*e^8 + 120*a^5*c^4*e^9 - 72*a^2*b^6*c*e^9 - 8*b^2*c^7*d^8*e - 8*b^8*c*d^2*e^7 + 212*a^3*b^4*c^2*e^9 - 240*a^4*b^2*c^3*e^9 - 112*a^2*c^7*d^6*e^3 + 240*a^3*c^6*d^4*e^5 - 228*a^4*c^5*d^2*e^7 + 4*b^3*c^6*d^7*e^2 - 24*b^4*c^5*d^6*e^3 + 32*b^5*c^4*d^5*e^4 - 24*b^6*c^3*d^4*e^5 + 4*b^7*c^2*d^3*e^6 + 32*a*c^8*d^8*e - 56*a*b^7*c*d*e^8 - 428*a^2*b^2*c^5*d^4*e^5 + 108*a^2*b^3*c^4*d^3*e^6 - 216*a^2*b^4*c^3*d^2*e^7 + 424*a^3*b^2*c^4*d^2*e^7 - 16*a*b*c^7*d^7*e^2 + 8*a^4*b*c^4*d*e^8 + 88*a*b^2*c^6*d^6*e^3 - 116*a*b^3*c^5*d^5*e^4 + 188*a*b^4*c^4*d^4*e^5 - 36*a*b^5*c^3*d^3*e^6 + 60*a*b^6*c^2*d^2*e^7 + 40*a^2*b*c^6*d^5*e^4 + 100*a^2*b^5*c^2*d*e^8 - 72*a^3*b*c^5*d^3*e^6 - 4*a^3*b^3*c^3*d*e^8))/(c^4*e^4) + (((x^2*(32*a*b^6*c^3*e^10 - 352*a^4*c^6*e^10 + 128*a*c^9*d^6*e^4 + 32*b*c^9*d^7*e^3 + 32*b^7*c^3*d*e^9 - 256*a^2*b^4*c^4*e^10 + 600*a^3*b^2*c^5*e^10 - 464*a^2*c^8*d^4*e^6 + 592*a^3*c^7*d^2*e^8 - 64*b^2*c^8*d^6*e^4 + 56*b^3*c^7*d^5*e^5 - 48*b^4*c^6*d^4*e^6 + 56*b^5*c^5*d^3*e^7 - 64*b^6*c^4*d^2*e^8 - 688*a^2*b^2*c^6*d^2*e^8 - 192*a*b*c^8*d^5*e^5 - 224*a*b^5*c^4*d*e^9 - 72*a^3*b*c^6*d*e^9 + 272*a*b^2*c^7*d^4*e^6 - 200*a*b^3*c^6*d^3*e^7 + 360*a*b^4*c^5*d^2*e^8 + 136*a^2*b*c^7*d^3*e^7 + 424*a^2*b^3*c^5*d*e^9))/(c^4*e^4) + (32*a*d*(2*b^6*e^6 + 2*c^6*d^6 - 15*a^3*c^3*e^6 - 10*a*c^5*d^4*e^2 + 29*a^2*b^2*c^2*e^6 + 17*a^2*c^4*d^2*e^4 + 3*b^2*c^4*d^4*e^2 - b^3*c^3*d^3*e^3 + 3*b^4*c^2*d^2*e^4 - 14*a*b^4*c*e^6 - 2*b*c^5*d^5*e - 2*b^5*c*d*e^5 + 2*a*b*c^4*d^3*e^3 + 6*a*b^3*c^2*d*e^5 + a^2*b*c^3*d*e^5 - 13*a*b^2*c^3*d^2*e^4))/(c*e) + (8*e^2*(b^2*e^2 + c^2*d^2 - 3*a*c*e^2 - b*c*d*e)*(b^4*d*(b^2 - 4*a*c)^(1/2) - b^5*d + 4*a^3*c^2*e + a*b^4*e + 6*a*b^3*c*d - a*b^3*e*(b^2 - 4*a*c)^(1/2) - 8*a^2*b*c^2*d - 5*a^2*b^2*c*e + 2*a^2*c^2*d*(b^2 - 4*a*c)^(1/2) - 4*a*b^2*c*d*(b^2 - 4*a*c)^(1/2) + 3*a^2*b*c*e*(b^2 - 4*a*c)^(1/2))*(2*a*c^2*d^3 + a*b^2*e^3*x^2 + b*c^2*d^3*x^2 - 4*a^2*c*e^3*x^2 + b^3*d*e^2*x^2 + 2*a*b^2*d*e^2 - 6*a^2*c*d*e^2 + 4*a*c^2*d^2*e*x^2 - 2*b^2*c*d^2*e*x^2 - 2*a*b*c*d^2*e - 3*a*b*c*d*e^2*x^2))/(c*(4*a*c - b^2)*(a*e^2 + c*d^2 - b*d*e)))*(b^4*d*(b^2 - 4*a*c)^(1/2) - b^5*d + 4*a^3*c^2*e + a*b^4*e + 6*a*b^3*c*d - a*b^3*e*(b^2 - 4*a*c)^(1/2) - 8*a^2*b*c^2*d - 5*a^2*b^2*c*e + 2*a^2*c^2*d*(b^2 -

4*a*c)^(1/2) - 4*a*b^2*c*d*(b^2 - 4*a*c)^(1/2) + 3*a^2*b*c*e*(b^2 - 4*a*c)^(1/2)))/(4*c^3*(4*a*c - b^2)*(a*e^2 + c*d^2 - b*d*e)) + (4*a*d*(4*b^8*e^8 + 4*c^8*d^8 + 37*a^4*c^4*e^8 - 16*a*c^7*d^6*e^2 + 84*a^2*b^4*c^2*e^8 - 84*a^3*b^2*c^3*e^8 + 40*a^2*c^6*d^4*e^4 - 56*a^3*c^5*d^2*e^6 + 4*b^2*c^6*d^6*e^2 - 4*b^3*c^5*d^5*e^3 + 13*b^4*c^4*d^4*e^4 - 4*b^5*c^3*d^3*e^5 + 4*b^6*c^2*d^2*e^6 - 32*a*b^6*c*e^8 + 98*a^2*b^2*c^4*d^2*e^6 - 8*a*b^5*c^2*d*e^7 - 4*a^3*b*c^4*d*e^7 - 52*a*b^2*c^5*d^4*e^4 + 20*a*b^3*c^4*d^3*e^5 - 36*a*b^4*c^3*d^2*e^6 - 16*a^2*b*c^5*d^3*e^5 + 28*a^2*b^3*c^3*d*e^7))/(c^4*e^4))*(b^4*d*(b^2 - 4*a*c)^(1/2) - b^5*d + 4*a^3*c^2*e + a*b^4*e + 6*a*b^3*c*d - a*b^3*e*(b^2 - 4*a*c)^(1/2) - 8*a^2*b*c^2*d - 5*a^2*b^2*c*e + 2*a^2*c^2*d*(b^2 - 4*a*c)^(1/2) - 4*a*b^2*c*d*(b^2 - 4*a*c)^(1/2) + 3*a^2*b*c*e*(b^2 - 4*a*c)^(1/2)))/(4*c^3*(4*a*c - b^2)*(a*e^2 + c*d^2 - b*d*e)) + (4*a*d*(2*a^3*b^4*e^7 + 5*a^5*c^2*e^7 + 2*b^3*c^4*d^7 + 2*b^7*d^3*e^4 - 8*a^4*b^2*c*e^7 + 2*a*b^6*d^2*e^5 + 2*a^2*b^5*d*e^6 - 2*a^2*c^5*d^6*e + 6*a^3*c^4*d^4*e^3 - 9*a^4*c^3*d^2*e^5 + b^5*c^2*d^5*e^2 - 4*a*b*c^5*d^7 - a^2*b^2*c^3*d^4*e^3 + 20*a^2*b^3*c^2*d^3*e^4 + 12*a^3*b^2*c^2*d^2*e^5 + 2*a*b^2*c^4*d^6*e - 12*a*b^5*c*d^3*e^4 - 8*a^3*b^3*c*d*e^6 + 3*a^4*b*c^2*d*e^6 - 6*a*b^3*c^3*d^5*e^2 - a*b^4*c^2*d^4*e^3 + 10*a^2*b*c^4*d^5*e^2 - 10*a^2*b^4*c*d^2*e^5 - 12*a^3*b*c^3*d^3*e^4))/(c^4*e^4))*(b^4*d*(b^2 - 4*a*c)^(1/2) - b^5*d + 4*a^3*c^2*e + a*b^4*e + 6*a*b^3*c*d - a*b^3*e*(b^2 - 4*a*c)^(1/2) - 8*a^2*b*c^2*d - 5*a^2*b^2*c*e + 2*a^2*c^2*d*(b^2 - 4*a*c)^(1/2) - 4*a*b^2*c*d*(b^2 - 4*a*c)^(1/2) + 3*a^2*b*c*e*(b^2 - 4*a*c)^(1/2)))/(4*c^3*(4*a*c - b^2)*(a*e^2 + c*d^2 - b*d*e)))*(b^4*d*(b^2 - 4*a*c)^(1/2) - b^5*d + 4*a^3*c^2*e + a*b^4*e + 6*a*b^3*c*d - a*b^3*e*(b^2 - 4*a*c)^(1/2) - 8*a^2*b*c^2*d - 5*a^2*b^2*c*e + 2*a^2*c^2*d*(b^2 - 4*a*c)^(1/2) - 4*a*b^2*c*d*(b^2 - 4*a*c)^(1/2) + 3*a^2*b*c*e*(b^2 - 4*a*c)^(1/2)))/(4*(4*a*c^5*d^2 + 4*a^2*c^4*e^2 - b^2*c^4*d^2 - a*b^2*c^3*e^2 + b^3*c^3*d*e - 4*a*b*c^4*d*e)) + x^4/(4*c*e) - (x^2*(b*e + c*d))/(2*c^2*e^2)

3.296 $\int \frac{x^7}{(d+ex^2)(a+bx^2+cx^4)}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 189

$$\int \frac{x^7}{(d+ex^2)(a+bx^2+cx^4)}\,dx = \frac{x^2}{2ce} + \frac{(b^3d-3abcd-ab^2e+2a^2ce)\,\text{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2c^2\sqrt{b^2-4ac}\,(cd^2-bde+ae^2)} - \frac{d^3\log(d+ex^2)}{2e^2(cd^2-bde+ae^2)} + \frac{(b^2d-acd-abe)\log(a+bx^2+cx^4)}{4c^2(cd^2-bde+ae^2)}$$

```
[Out] 1/2*x^2/c/e-1/2*d^3*ln(e*x^2+d)/e^2/(a*e^2-b*d*e+c*d^2)+1/4*(-a*b*e-a*c*d+b
^2*d)*ln(c*x^4+b*x^2+a)/c^2/(a*e^2-b*d*e+c*d^2)+1/2*(2*a^2*c*e-a*b^2*e-3*a*
b*c*d+b^3*d)*arctanh((2*c*x^2+b)/(-4*a*c+b^2)^(1/2))/c^2/(a*e^2-b*d*e+c*d^2
)/(-4*a*c+b^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 189, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {1265, 1642, 648, 632, 212, 642}

$$\int \frac{x^7}{(d+ex^2)(a+bx^2+cx^4)}\,dx = \frac{\text{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)(2a^2ce-ab^2e-3abcd+b^3d)}{2c^2\sqrt{b^2-4ac}\,(ae^2-bde+cd^2)} + \frac{(-abe-acd+b^2d)\log(a+bx^2+cx^4)}{4c^2(ae^2-bde+cd^2)} - \frac{d^3\log(d+ex^2)}{2e^2(ae^2-bde+cd^2)} + \frac{x^2}{2ce}$$

```
[In] Int[x^7/((d + e*x^2)*(a + b*x^2 + c*x^4)),x]
```

```
[Out] x^2/(2*c*e) + ((b^3*d - 3*a*b*c*d - a*b^2*e + 2*a^2*c*e)*ArcTanh[(b + 2*c*x
^2)/Sqrt[b^2 - 4*a*c]])/(2*c^2*Sqrt[b^2 - 4*a*c]*(c*d^2 - b*d*e + a*e^2)) -
 (d^3*Log[d + e*x^2])/(2*e^2*(c*d^2 - b*d*e + a*e^2)) + ((b^2*d - a*c*d - a
*b*e)*Log[a + b*x^2 + c*x^4])/(4*c^2*(c*d^2 - b*d*e + a*e^2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1265

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rule 1642

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\text{integral} = \frac{1}{2}\text{Subst}\left(\int \frac{x^3}{(d+ex)\,(a+bx+cx^2)}\,dx, x, x^2\right)$$
$$= \frac{1}{2}\text{Subst}\left(\int \left(\frac{1}{ce} - \frac{d^3}{e\,(cd^2-bde+ae^2)\,(d+ex)} + \frac{a(bd-ae)+(b^2d-acd-abe)\,x}{c\,(cd^2-bde+ae^2)\,(a+bx+cx^2)}\right) dx, x, x^2\right)$$
$$= \frac{x^2}{2ce} - \frac{d^3\log(d+ex^2)}{2e^2\,(cd^2-bde+ae^2)} + \frac{\text{Subst}\left(\int \frac{a(bd-ae)+(b^2d-acd-abe)x}{a+bx+cx^2}\,dx, x, x^2\right)}{2c\,(cd^2-bde+ae^2)}$$
$$= \frac{x^2}{2ce} - \frac{d^3\log(d+ex^2)}{2e^2\,(cd^2-bde+ae^2)} + \frac{(b^2d-acd-abe)\,\text{Subst}\left(\int \frac{b+2cx}{a+bx+cx^2}\,dx, x, x^2\right)}{4c^2\,(cd^2-bde+ae^2)} - \frac{(b^3d-3abcd-ab^2e+2a^2ce)\,\text{Subst}\left(\int \frac{1}{a+bx+cx^2}\,dx, x, x^2\right)}{4c^2\,(cd^2-bde+ae^2)}$$
$$= \frac{x^2}{2ce} - \frac{d^3\log(d+ex^2)}{2e^2\,(cd^2-bde+ae^2)} + \frac{(b^2d-acd-abe)\log(a+bx^2+cx^4)}{4c^2\,(cd^2-bde+ae^2)} + \frac{(b^3d-3abcd-ab^2e+2a^2ce)\,\text{Subst}\left(\int \frac{1}{b^2-4ac-x^2}\,dx, x, b+2cx^2\right)}{2c^2\,(cd^2-bde+ae^2)}$$
$$= \frac{x^2}{2ce} + \frac{(b^3d-3abcd-ab^2e+2a^2ce)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2c^2\sqrt{b^2-4ac}\,(cd^2-bde+ae^2)} - \frac{d^3\log(d+ex^2)}{2e^2\,(cd^2-bde+ae^2)} + \frac{(b^2d-acd-abe)\log(a+bx^2+cx^4)}{4c^2\,(cd^2-bde+ae^2)}$$

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 186, normalized size of antiderivative = 0.98

$$\int \frac{x^7}{(d+ex^2)\,(a+bx^2+cx^4)}\,dx = \frac{2e^2(b^3d-3abcd-ab^2e+2a^2ce)\arctan\left(\frac{b+2cx^2}{\sqrt{-b^2+4ac}}\right) + \sqrt{-b^2+4ac}(2c^2d^3\log(d+ex^2) + e(-2c(cd^2-bde}{4c^2\sqrt{-b^2+4ac}e^2\,(-cd^2+e(bd-ae))}$$

```
[In] Integrate[x^7/((d + e*x^2)*(a + b*x^2 + c*x^4)),x]
```

```
[Out] (2*e^2*(b^3*d - 3*a*b*c*d - a*b^2*e + 2*a^2*c*e)*ArcTan[(b + 2*c*x^2)/Sqrt[
-b^2 + 4*a*c]] + Sqrt[-b^2 + 4*a*c]*(2*c^2*d^3*Log[d + e*x^2] + e*(-2*c*(c*
d^2 - b*d*e + a*e^2)*x^2 + e*(-(b^2*d) + a*c*d + a*b*e)*Log[a + b*x^2 + c*x
^4])))/(4*c^2*Sqrt[-b^2 + 4*a*c]*e^2*(-(c*d^2) + e*(b*d - a*e)))
```

Maple [A] (verified)

Time = 0.56 (sec) , antiderivative size = 172, normalized size of antiderivative = 0.91

method	result	size
default	$\frac{x^2}{2ce}-\frac{\frac{\left(abe+acd-b^2d\right)\ln\left(c\,x^4+b\,x^2+a\right)}{2c}+\frac{2\left(e\,a^2-dab-\frac{\left(abe+acd-b^2d\right)b}{2c}\right)\arctan\left(\frac{2c\,x^2+b}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}}}{2(a\,e^2-bde+c\,d^2)c}-\frac{d^3\ln\left(e\,x^2+d\right)}{2e^2(a\,e^2-bde+c\,d^2)}$	172
risch	Expression too large to display	15081

```
[In] int(x^7/(e*x^2+d)/(c*x^4+b*x^2+a),x,method=_RETURNVERBOSE)
[Out] 1/2*x^2/c/e-1/2/(a*e^2-b*d*e+c*d^2)/c*(1/2*(a*b*e+a*c*d-b^2*d)/c*ln(c*x^4+b
*x^2+a)+2*(e*a^2-d*a*b-1/2*(a*b*e+a*c*d-b^2*d)*b/c)/(4*a*c-b^2)^(1/2)*arcta
n((2*c*x^2+b)/(4*a*c-b^2)^(1/2)))-1/2*d^3*ln(e*x^2+d)/e^2/(a*e^2-b*d*e+c*d^
2)
```

Fricas [A] (verification not implemented)

none

Time = 113.05 (sec) , antiderivative size = 616, normalized size of antiderivative = 3.26

$$\int\frac{x^7}{(d+ex^2)\left(a+bx^2+cx^4\right)}\,dx$$
$$=\left[-\frac{2\left(b^2c^2-4\,ac^3\right)d^3\log\left(ex^2+d\right)-2\left(\left(b^2c^2-4\,ac^3\right)d^2e-\left(b^3c-4\,abc^2\right)de^2+\left(ab^2c-4\,a^2c^2\right)e^3\right)x^2-\left(\left(b^3\right.\right.}{4\left(\left(b^2c^3\right.\right.}\right.$$
$$-\frac{2\left(b^2c^2-4\,ac^3\right)d^3\log\left(ex^2+d\right)-2\left(\left(b^2c^2-4\,ac^3\right)d^2e-\left(b^3c-4\,abc^2\right)de^2+\left(ab^2c-4\,a^2c^2\right)e^3\right)x^2-2\left(\left(b\right.\right.}{4\left(\left(b^2c^3-4\right.\right.}$$

```
[In] integrate(x^7/(e*x^2+d)/(c*x^4+b*x^2+a),x, algorithm="fricas")
[Out] [-1/4*(2*(b^2*c^2 - 4*a*c^3)*d^3*log(e*x^2 + d) - 2*((b^2*c^2 - 4*a*c^3)*d^
2*e - (b^3*c - 4*a*b*c^2)*d*e^2 + (a*b^2*c - 4*a^2*c^2)*e^3)*x^2 - ((b^3 -
3*a*b*c)*d*e^2 - (a*b^2 - 2*a^2*c)*e^3)*sqrt(b^2 - 4*a*c)*log((2*c^2*x^4 +
2*b*c*x^2 + b^2 - 2*a*c + (2*c*x^2 + b)*sqrt(b^2 - 4*a*c))/(c*x^4 + b*x^2 +
 a)) - ((b^4 - 5*a*b^2*c + 4*a^2*c^2)*d*e^2 - (a*b^3 - 4*a^2*b*c)*e^3)*log(
c*x^4 + b*x^2 + a))/((b^2*c^3 - 4*a*c^4)*d^2*e^2 - (b^3*c^2 - 4*a*b*c^3)*d*
e^3 + (a*b^2*c^2 - 4*a^2*c^3)*e^4), -1/4*(2*(b^2*c^2 - 4*a*c^3)*d^3*log(e*x
^2 + d) - 2*((b^2*c^2 - 4*a*c^3)*d^2*e - (b^3*c - 4*a*b*c^2)*d*e^2 + (a*b^2
*c - 4*a^2*c^2)*e^3)*x^2 - 2*((b^3 - 3*a*b*c)*d*e^2 - (a*b^2 - 2*a^2*c)*e^3
)*sqrt(-b^2 + 4*a*c)*arctan(-(2*c*x^2 + b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4*a*c)
) - ((b^4 - 5*a*b^2*c + 4*a^2*c^2)*d*e^2 - (a*b^3 - 4*a^2*b*c)*e^3)*log(c*x
^4 + b*x^2 + a))/((b^2*c^3 - 4*a*c^4)*d^2*e^2 - (b^3*c^2 - 4*a*b*c^3)*d*e^3
 + (a*b^2*c^2 - 4*a^2*c^3)*e^4)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^7}{(d+ex^2)(a+bx^2+cx^4)}\,dx = \text{Timed out}$$

```
[In] integrate(x**7/(e*x**2+d)/(c*x**4+b*x**2+a),x)
[Out] Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^7}{(d+ex^2)(a+bx^2+cx^4)}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(x^7/(e*x^2+d)/(c*x^4+b*x^2+a),x, algorithm="maxima")
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

Giac [A] (verification not implemented)

none

Time = 0.59 (sec) , antiderivative size = 194, normalized size of antiderivative = 1.03

$$\int \frac{x^7}{(d+ex^2)(a+bx^2+cx^4)}\,dx = -\frac{d^3\log(|ex^2+d|)}{2(cd^2e^2-bde^3+ae^4)} + \frac{(b^2d-acd-abe)\log(cx^4+bx^2+a)}{4(c^3d^2-bc^2de+ac^2e^2)} - \frac{(b^3d-3\,abcd-ab^2e+2\,a^2ce)\arctan\left(\frac{2\,cx^2+b}{\sqrt{-b^2+4\,ac}}\right)}{2(c^3d^2-bc^2de+ac^2e^2)\sqrt{-b^2+4\,ac}} + \frac{x^2}{2\,ce}$$

```
[In] integrate(x^7/(e*x^2+d)/(c*x^4+b*x^2+a),x, algorithm="giac")
[Out] -1/2*d^3*log(abs(e*x^2 + d))/(c*d^2*e^2 - b*d*e^3 + a*e^4) + 1/4*(b^2*d - a
*c*d - a*b*e)*log(c*x^4 + b*x^2 + a)/(c^3*d^2 - b*c^2*d*e + a*c^2*e^2) - 1/
2*(b^3*d - 3*a*b*c*d - a*b^2*e + 2*a^2*c*e)*arctan((2*c*x^2 + b)/sqrt(-b^2
+ 4*a*c))/((c^3*d^2 - b*c^2*d*e + a*c^2*e^2)*sqrt(-b^2 + 4*a*c)) + 1/2*x^2/
(c*e)
```

Mupad [B] (verification not implemented)

Time = 18.41 (sec) , antiderivative size = 2304, normalized size of antiderivative = 12.19

$$\int \frac{x^7}{(d+ex^2)(a+bx^2+cx^4)}\,dx = \text{Too large to display}$$

```
[In] int(x^7/((d + e*x^2)*(a + b*x^2 + c*x^4)),x)
```

```
[Out] x^2/(2*c*e) - (d^3*log(d + e*x^2))/(2*(a*e^4 + c*d^2*e^2 - b*d*e^3)) - (log
(a*e^5*x^2*(b^2 - 4*a*c)^(7/2) - 128*a^5*c^3*e^5 - 8*c^3*d^5*(b^2 - 4*a*c)^
(5/2) - 512*a^3*c^5*d^4*e + 8*b^2*c^3*d^5*(b^2 - 4*a*c)^(3/2) + 6*b^3*d^2*e
^3*(b^2 - 4*a*c)^(5/2) - 3*b^5*d^2*e^3*(b^2 - 4*a*c)^(3/2) + 32*a^4*b^2*c^2
*e^5 + 384*a^4*c^4*d^2*e^3 + 256*a^2*c^6*d^5*x^2 + 16*b^4*c^4*d^5*x^2 + 3*a
*d*e^4*(b^2 - 4*a*c)^(7/2) - 3*b*d^2*e^3*(b^2 - 4*a*c)^(7/2) - 3*c*d^3*e^2*
(b^2 - 4*a*c)^(7/2) - 16*a^2*b^3*c^3*d^3*e^2 + 48*a^2*b^4*c^2*d^2*e^3 - 288
*a^3*b^2*c^3*d^2*e^3 + 16*a^3*b^3*c^2*e^5*x^2 - 384*a^3*c^5*d^3*e^2*x^2 + 1
6*b^6*c^2*d^3*e^2*x^2 - 6*a*b^2*d*e^4*(b^2 - 4*a*c)^(5/2) + 3*a*b^4*d*e^4*(
b^2 - 4*a*c)^(3/2) + 8*b*c^2*d^4*e*(b^2 - 4*a*c)^(5/2) - 32*a*b^4*c^3*d^4*e
 + 192*a^4*b*c^3*d*e^4 - 2*b^2*c*d^3*e^2*(b^2 - 4*a*c)^(5/2) - 8*b^3*c^2*d^
4*e*(b^2 - 4*a*c)^(3/2) + 5*b^4*c*d^3*e^2*(b^2 - 4*a*c)^(3/2) - 2*a*b^2*e^5
*x^2*(b^2 - 4*a*c)^(5/2) + a*b^4*e^5*x^2*(b^2 - 4*a*c)^(3/2) + 16*b*c^4*d^5
*x^2*(b^2 - 4*a*c)^(3/2) - 3*c*d^2*e^3*x^2*(b^2 - 4*a*c)^(7/2) - 16*c^3*d^4
*e*x^2*(b^2 - 4*a*c)^(5/2) + 256*a^2*b^2*c^4*d^4*e + 64*a^3*b*c^4*d^3*e^2 -
 48*a^3*b^3*c^2*d*e^4 - 128*a*b^2*c^5*d^5*x^2 - 64*a^4*b*c^3*e^5*x^2 + 384*
a^4*c^4*d*e^4*x^2 - 32*b^5*c^3*d^4*e*x^2 + 480*a^2*b^2*c^4*d^3*e^2*x^2 + 48
*a^2*b^3*c^3*d^2*e^3*x^2 + 256*a*b^3*c^4*d^4*e*x^2 - 512*a^2*b*c^5*d^4*e*x^
2 + 8*b*c^2*d^3*e^2*x^2*(b^2 - 4*a*c)^(5/2) + 6*b^2*c*d^2*e^3*x^2*(b^2 - 4*
a*c)^(5/2) - 16*b^2*c^3*d^4*e*x^2*(b^2 - 4*a*c)^(3/2) - 3*b^4*c*d^2*e^3*x^2
*(b^2 - 4*a*c)^(3/2) - 160*a*b^4*c^3*d^3*e^2*x^2 - 192*a^3*b*c^4*d^2*e^3*x^
2 - 96*a^3*b^2*c^3*d*e^4*x^2 + 8*b^3*c^2*d^3*e^2*x^2*(b^2 - 4*a*c)^(3/2))*(
b^4*d - b^3*d*(b^2 - 4*a*c)^(1/2) + 4*a^2*c^2*d - a*b^3*e - 5*a*b^2*c*d + 4
*a^2*b*c*e + a*b^2*e*(b^2 - 4*a*c)^(1/2) - 2*a^2*c*e*(b^2 - 4*a*c)^(1/2) +
3*a*b*c*d*(b^2 - 4*a*c)^(1/2)))/(4*(4*a*c^4*d^2 + 4*a^2*c^3*e^2 - b^2*c^3*d
^2 - a*b^2*c^2*e^2 + b^3*c^2*d*e - 4*a*b*c^3*d*e)) - (log(8*c^3*d^5*(b^2 -
4*a*c)^(5/2) - 128*a^5*c^3*e^5 - a*e^5*x^2*(b^2 - 4*a*c)^(7/2) - 512*a^3*c^
5*d^4*e - 8*b^2*c^3*d^5*(b^2 - 4*a*c)^(3/2) - 6*b^3*d^2*e^3*(b^2 - 4*a*c)^(
5/2) + 3*b^5*d^2*e^3*(b^2 - 4*a*c)^(3/2) + 32*a^4*b^2*c^2*e^5 + 384*a^4*c^4
*d^2*e^3 + 256*a^2*c^6*d^5*x^2 + 16*b^4*c^4*d^5*x^2 - 3*a*d*e^4*(b^2 - 4*a*
c)^(7/2) + 3*b*d^2*e^3*(b^2 - 4*a*c)^(7/2) + 3*c*d^3*e^2*(b^2 - 4*a*c)^(7/2
) - 16*a^2*b^3*c^3*d^3*e^2 + 48*a^2*b^4*c^2*d^2*e^3 - 288*a^3*b^2*c^3*d^2*e
^3 + 16*a^3*b^3*c^2*e^5*x^2 - 384*a^3*c^5*d^3*e^2*x^2 + 16*b^6*c^2*d^3*e^2*
x^2 + 6*a*b^2*d*e^4*(b^2 - 4*a*c)^(5/2) - 3*a*b^4*d*e^4*(b^2 - 4*a*c)^(3/2)
 - 8*b*c^2*d^4*e*(b^2 - 4*a*c)^(5/2) - 32*a*b^4*c^3*d^4*e + 192*a^4*b*c^3*d
*e^4 + 2*b^2*c*d^3*e^2*(b^2 - 4*a*c)^(5/2) + 8*b^3*c^2*d^4*e*(b^2 - 4*a*c)^
```

(3/2) - 5*b^4*c*d^3*e^2*(b^2 - 4*a*c)^(3/2) + 2*a*b^2*e^5*x^2*(b^2 - 4*a*c)
^(5/2) - a*b^4*e^5*x^2*(b^2 - 4*a*c)^(3/2) - 16*b*c^4*d^5*x^2*(b^2 - 4*a*c)
^(3/2) + 3*c*d^2*e^3*x^2*(b^2 - 4*a*c)^(7/2) + 16*c^3*d^4*e*x^2*(b^2 - 4*a*
c)^(5/2) + 256*a^2*b^2*c^4*d^4*e + 64*a^3*b*c^4*d^3*e^2 - 48*a^3*b^3*c^2*d*
e^4 - 128*a*b^2*c^5*d^5*x^2 - 64*a^4*b*c^3*e^5*x^2 + 384*a^4*c^4*d*e^4*x^2
- 32*b^5*c^3*d^4*e*x^2 + 480*a^2*b^2*c^4*d^3*e^2*x^2 + 48*a^2*b^3*c^3*d^2*e
^3*x^2 + 256*a*b^3*c^4*d^4*e*x^2 - 512*a^2*b*c^5*d^4*e*x^2 - 8*b*c^2*d^3*e^
2*x^2*(b^2 - 4*a*c)^(5/2) - 6*b^2*c*d^2*e^3*x^2*(b^2 - 4*a*c)^(5/2) + 16*b^
2*c^3*d^4*e*x^2*(b^2 - 4*a*c)^(3/2) + 3*b^4*c*d^2*e^3*x^2*(b^2 - 4*a*c)^(3/
2) - 160*a*b^4*c^3*d^3*e^2*x^2 - 192*a^3*b*c^4*d^2*e^3*x^2 - 96*a^3*b^2*c^3
*d*e^4*x^2 - 8*b^3*c^2*d^3*e^2*x^2*(b^2 - 4*a*c)^(3/2))*(b^4*d + b^3*d*(b^2
 - 4*a*c)^(1/2) + 4*a^2*c^2*d - a*b^3*e - 5*a*b^2*c*d + 4*a^2*b*c*e - a*b^2
e(b^2 - 4*a*c)^(1/2) + 2*a^2*c*e*(b^2 - 4*a*c)^(1/2) - 3*a*b*c*d*(b^2 - 4
*a*c)^(1/2)))/(4*(4*a*c^4*d^2 + 4*a^2*c^3*e^2 - b^2*c^3*d^2 - a*b^2*c^2*e^2
 + b^3*c^2*d*e - 4*a*b*c^3*d*e))

3.297 $\int \frac{x^5}{(d+ex^2)(a+bx^2+cx^4)}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 158

$$\int \frac{x^5}{(d+ex^2)(a+bx^2+cx^4)}\,dx = -\frac{(b^2d-2acd-abe)\operatorname{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2c\sqrt{b^2-4ac}\,(cd^2-bde+ae^2)} + \frac{d^2\log(d+ex^2)}{2e(cd^2-bde+ae^2)} - \frac{(bd-ae)\log(a+bx^2+cx^4)}{4c(cd^2-bde+ae^2)}$$

```
[Out] 1/2*d^2*ln(e*x^2+d)/e/(a*e^2-b*d*e+c*d^2)-1/4*(-a*e+b*d)*ln(c*x^4+b*x^2+a)/
c/(a*e^2-b*d*e+c*d^2)-1/2*(-a*b*e-2*a*c*d+b^2*d)*arctanh((2*c*x^2+b)/(-4*a*
c+b^2)^(1/2))/c/(a*e^2-b*d*e+c*d^2)/(-4*a*c+b^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 158, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {1265, 1642, 648, 632, 212, 642}

$$\int \frac{x^5}{(d+ex^2)(a+bx^2+cx^4)}\,dx = -\frac{\operatorname{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)(-abe-2acd+b^2d)}{2c\sqrt{b^2-4ac}\,(ae^2-bde+cd^2)} + \frac{d^2\log(d+ex^2)}{2e(ae^2-bde+cd^2)} - \frac{(bd-ae)\log(a+bx^2+cx^4)}{4c(ae^2-bde+cd^2)}$$

```
[In] Int[x^5/((d + e*x^2)*(a + b*x^2 + c*x^4)),x]
```

```
[Out] -1/2*((b^2*d - 2*a*c*d - a*b*e)*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(
c*Sqrt[b^2 - 4*a*c]*(c*d^2 - b*d*e + a*e^2)) + (d^2*Log[d + e*x^2])/(2*e*(c
```

```
*d^2 - b*d*e + a*e^2)) - ((b*d - a*e)*Log[a + b*x^2 + c*x^4])/(4*c*(c*d^2 -
 b*d*e + a*e^2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1265

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rule 1642

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{1}{2}\text{Subst}\left(\int \frac{x^2}{(d+ex)\,(a+bx+cx^2)}\,dx, x, x^2\right)\\ &= \frac{1}{2}\text{Subst}\left(\int \left(\frac{d^2}{(cd^2-bde+ae^2)\,(d+ex)}+\frac{-ad-(bd-ae)x}{(cd^2-bde+ae^2)\,(a+bx+cx^2)}\right)\,dx, x, x^2\right)\end{aligned}$$

$$= \frac{d^2\log(d+ex^2)}{2e(cd^2-bde+ae^2)} + \frac{\text{Subst}\left(\int \frac{-ad-(bd-ae)x}{a+bx+cx^2}\,dx, x, x^2\right)}{2(cd^2-bde+ae^2)}$$

$$= \frac{d^2\log(d+ex^2)}{2e(cd^2-bde+ae^2)} - \frac{(bd-ae)\text{Subst}\left(\int \frac{b+2cx}{a+bx+cx^2}\,dx, x, x^2\right)}{4c(cd^2-bde+ae^2)} + \frac{(b^2d-2acd-abe)\,\text{Subst}\left(\int \frac{1}{a+bx+cx^2}\,dx, x, x^2\right)}{4c(cd^2-bde+ae^2)}$$

$$= \frac{d^2\log(d+ex^2)}{2e(cd^2-bde+ae^2)} - \frac{(bd-ae)\log(a+bx^2+cx^4)}{4c(cd^2-bde+ae^2)} - \frac{(b^2d-2acd-abe)\,\text{Subst}\left(\int \frac{1}{b^2-4ac-x^2}\,dx, x, b+2cx^2\right)}{2c(cd^2-bde+ae^2)}$$

$$= -\frac{(b^2d-2acd-abe)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2c\sqrt{b^2-4ac}\,(cd^2-bde+ae^2)} + \frac{d^2\log(d+ex^2)}{2e(cd^2-bde+ae^2)} - \frac{(bd-ae)\log(a+bx^2+cx^4)}{4c(cd^2-bde+ae^2)}$$

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 139, normalized size of antiderivative = 0.88

$$\int \frac{x^5}{(d+ex^2)(a+bx^2+cx^4)}\,dx = -\frac{2e(-b^2d+2acd+abe)\arctan\left(\frac{b+2cx^2}{\sqrt{-b^2+4ac}}\right)+\sqrt{-b^2+4ac}(-2cd^2\log(d+ex^2)+e(bd-ae)\log(a+bx^2}{4c\sqrt{-b^2+4ac}e\,(cd^2+e(-bd+ae))}$$

```
[In] Integrate[x^5/((d + e*x^2)*(a + b*x^2 + c*x^4)),x]
```

```
[Out] -1/4*(2*e*(-(b^2*d) + 2*a*c*d + a*b*e)*ArcTan[(b + 2*c*x^2)/Sqrt[-b^2 + 4*a
*c]] + Sqrt[-b^2 + 4*a*c]*(-2*c*d^2*Log[d + e*x^2] + e*(b*d - a*e)*Log[a +
b*x^2 + c*x^4]))/(c*Sqrt[-b^2 + 4*a*c]*e*(c*d^2 + e*(-(b*d) + a*e)))
```

Maple [A] (verified)

Time = 0.44 (sec) , antiderivative size = 137, normalized size of antiderivative = 0.87

method	result
default	$-\frac{\frac{(-ae+bd)\ln(c\,x^4+b\,x^2+a)}{2c}+\frac{2\left(da-\frac{(-ae+bd)b}{2c}\right)\arctan\left(\frac{2c\,x^2+b}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}}}{2(a\,e^2-bde+c\,d^2)}+\frac{d^2\ln(e\,x^2+d)}{2e(a\,e^2-bde+c\,d^2)}$
risch	$\frac{d^2\ln(e\,x^2+d)}{2e(a\,e^2-bde+c\,d^2)}+\frac{\left(\sum_{\_R=\text{RootOf}\left(\left(4a^2c^2e^2-a\,b^2c\,e^2-4ab\,c^2de+4a\,c^3d^2+b^3cde-b^2c^2d^2\right)\_Z^2+\left(-4a^2ce+a\,b^2e+4abcd-b^3d\right)\_Z+a^2\right)}\right.}{}$

```
[In] int(x^5/(e*x^2+d)/(c*x^4+b*x^2+a),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2/(a*e^2-b*d*e+c*d^2)*(1/2*(-a*e+b*d)/c*ln(c*x^4+b*x^2+a)+2*(d*a-1/2*(-a
*e+b*d)*b/c)/(4*a*c-b^2)^(1/2)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2)))+1/2*d
^2*ln(e*x^2+d)/e/(a*e^2-b*d*e+c*d^2)
```

Fricas [A] (verification not implemented)

none

Time = 32.44 (sec) , antiderivative size = 421, normalized size of antiderivative = 2.66

$$\int \frac{x^5}{(d+ex^2)(a+bx^2+cx^4)}\,dx$$
$$= \left[\frac{2\,(b^2c-4\,ac^2)d^2\log\left(ex^2+d\right)+(abe^2-(b^2-2\,ac)de)\sqrt{b^2-4\,ac}\log\left(\frac{2\,c^2x^4+2\,bcx^2+b^2-2\,ac+(2\,cx^2+b)\sqrt{b^2-4\,ac}}{cx^4+bx^2+a}\right)}{4\left((b^2c^2-4\,ac^3)d^2e-(b^3c-4\,abc^2)de^2+(ab^2c-4\,a^2c\right.}\right.$$

```
[In] integrate(x^5/(e*x^2+d)/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] [1/4*(2*(b^2*c - 4*a*c^2)*d^2*log(e*x^2 + d) + (a*b*e^2 - (b^2 - 2*a*c)*d*e
)*sqrt(b^2 - 4*a*c)*log((2*c^2*x^4 + 2*b*c*x^2 + b^2 - 2*a*c + (2*c*x^2 + b
)*sqrt(b^2 - 4*a*c))/(c*x^4 + b*x^2 + a)) - ((b^3 - 4*a*b*c)*d*e - (a*b^2 -
 4*a^2*c)*e^2)*log(c*x^4 + b*x^2 + a))/((b^2*c^2 - 4*a*c^3)*d^2*e - (b^3*c
- 4*a*b*c^2)*d*e^2 + (a*b^2*c - 4*a^2*c^2)*e^3), 1/4*(2*(b^2*c - 4*a*c^2)*d
^2*log(e*x^2 + d) + 2*(a*b*e^2 - (b^2 - 2*a*c)*d*e)*sqrt(-b^2 + 4*a*c)*arct
an(-(2*c*x^2 + b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4*a*c)) - ((b^3 - 4*a*b*c)*d*e
- (a*b^2 - 4*a^2*c)*e^2)*log(c*x^4 + b*x^2 + a))/((b^2*c^2 - 4*a*c^3)*d^2*e
 - (b^3*c - 4*a*b*c^2)*d*e^2 + (a*b^2*c - 4*a^2*c^2)*e^3)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^5}{(d+ex^2)(a+bx^2+cx^4)}\,dx = \text{Timed out}$$

```
[In] integrate(x**5/(e*x**2+d)/(c*x**4+b*x**2+a),x)
```

```
[Out] Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^5}{(d+ex^2)(a+bx^2+cx^4)}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(x^5/(e*x^2+d)/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

Giac [A] (verification not implemented)

none

Time = 0.58 (sec) , antiderivative size = 155, normalized size of antiderivative = 0.98

$$\int \frac{x^5}{(d+ex^2)(a+bx^2+cx^4)}\,dx = \frac{d^2\log(|ex^2+d|)}{2(cd^2e-bde^2+ae^3)} - \frac{(bd-ae)\log(cx^4+bx^2+a)}{4(c^2d^2-bcde+ace^2)} + \frac{(b^2d-2\,acd-abe)\arctan\left(\frac{2\,cx^2+b}{\sqrt{-b^2+4\,ac}}\right)}{2(c^2d^2-bcde+ace^2)\sqrt{-b^2+4\,ac}}$$

```
[In] integrate(x^5/(e*x^2+d)/(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] 1/2*d^2*log(abs(e*x^2 + d))/(c*d^2*e - b*d*e^2 + a*e^3) - 1/4*(b*d - a*e)*l
og(c*x^4 + b*x^2 + a)/(c^2*d^2 - b*c*d*e + a*c*e^2) + 1/2*(b^2*d - 2*a*c*d
- a*b*e)*arctan((2*c*x^2 + b)/sqrt(-b^2 + 4*a*c))/((c^2*d^2 - b*c*d*e + a*c
*e^2)*sqrt(-b^2 + 4*a*c))
```

Mupad [B] (verification not implemented)

Time = 15.42 (sec) , antiderivative size = 1853, normalized size of antiderivative = 11.73

$$\int \frac{x^5}{(d+ex^2)(a+bx^2+cx^4)}\,dx = \text{Too large to display}$$

```
[In] int(x^5/((d + e*x^2)*(a + b*x^2 + c*x^4)),x)
```

```
[Out] (d^2*log(d + e*x^2))/(2*a*e^3 - 2*b*d*e^2 + 2*c*d^2*e) + (log(4*a^2*e^4*(b^
2 - 4*a*c)^(5/2) + 8*c^2*d^4*(b^2 - 4*a*c)^(5/2) + 5*d^2*e^2*(b^2 - 4*a*c)^
(7/2) + 3*d*e^3*x^2*(b^2 - 4*a*c)^(7/2) - 16*a^3*b^3*c*e^4 + 64*a^4*b*c^2*e
^4 + 640*a^3*c^4*d^3*e - 384*a^4*c^3*d*e^3 - 4*a^2*b^2*e^4*(b^2 - 4*a*c)^(3
```

/2) - 8*b^2*c^2*d^4*(b^2 - 4*a*c)^(3/2) - 6*b^2*d^2*e^2*(b^2 - 4*a*c)^(5/2)
 + b^4*d^2*e^2*(b^2 - 4*a*c)^(3/2) - 256*a^2*c^5*d^4*x^2 - 128*a^4*c^3*e^4*
x^2 - 16*b^4*c^3*d^4*x^2 + 80*a^2*b^3*c^2*d^2*e^2 + 96*a^3*b^2*c^2*e^4*x^2
+ 640*a^3*c^4*d^2*e^2*x^2 + 4*b^3*c*d^3*e*(b^2 - 4*a*c)^(3/2) + 4*a*b*e^4*x
^2*(b^2 - 4*a*c)^(5/2) + 48*a*b^4*c^2*d^3*e - 16*a*b^5*c*d^2*e^2 - 4*a*b^3*
e^4*x^2*(b^2 - 4*a*c)^(3/2) - 16*b*c^3*d^4*x^2*(b^2 - 4*a*c)^(3/2) - 6*b^2*
d*e^3*x^2*(b^2 - 4*a*c)^(5/2) + 3*b^4*d*e^3*x^2*(b^2 - 4*a*c)^(3/2) + 20*c^
2*d^3*e*x^2*(b^2 - 4*a*c)^(5/2) - 352*a^2*b^2*c^3*d^3*e - 64*a^3*b*c^3*d^2*
e^2 + 96*a^3*b^2*c^2*d*e^3 + 128*a*b^2*c^4*d^4*x^2 - 16*a^2*b^4*c*e^4*x^2 +
 32*b^5*c^2*d^3*e*x^2 - 16*b^6*c*d^2*e^2*x^2 - 4*b*c*d^3*e*(b^2 - 4*a*c)^(5
/2) - 480*a^2*b^2*c^3*d^2*e^2*x^2 - 12*b*c*d^2*e^2*x^2*(b^2 - 4*a*c)^(5/2)
- 240*a*b^3*c^3*d^3*e*x^2 + 448*a^2*b*c^4*d^3*e*x^2 - 192*a^3*b*c^3*d*e^3*x
^2 + 12*b^2*c^2*d^3*e*x^2*(b^2 - 4*a*c)^(3/2) - 4*b^3*c*d^2*e^2*x^2*(b^2 -
4*a*c)^(3/2) + 144*a*b^4*c^2*d^2*e^2*x^2 + 48*a^2*b^3*c^2*d*e^3*x^2)*((b^3*
d)/4 + e*(a^2*c - (a*b^2)/4 + (a*b*(b^2 - 4*a*c)^(1/2))/4) - (b^2*d*(b^2 -
4*a*c)^(1/2))/4 + (a*c*d*(b^2 - 4*a*c)^(1/2))/2 - a*b*c*d))/(4*a*c^3*d^2 +
4*a^2*c^2*e^2 - b^2*c^2*d^2 + b^3*c*d*e - a*b^2*c*e^2 - 4*a*b*c^2*d*e) - (l
og(4*a^2*e^4*(b^2 - 4*a*c)^(5/2) + 8*c^2*d^4*(b^2 - 4*a*c)^(5/2) + 5*d^2*e^
2*(b^2 - 4*a*c)^(7/2) + 3*d*e^3*x^2*(b^2 - 4*a*c)^(7/2) + 16*a^3*b^3*c*e^4
- 64*a^4*b*c^2*e^4 - 640*a^3*c^4*d^3*e + 384*a^4*c^3*d*e^3 - 4*a^2*b^2*e^4*
(b^2 - 4*a*c)^(3/2) - 8*b^2*c^2*d^4*(b^2 - 4*a*c)^(3/2) - 6*b^2*d^2*e^2*(b^
2 - 4*a*c)^(5/2) + b^4*d^2*e^2*(b^2 - 4*a*c)^(3/2) + 256*a^2*c^5*d^4*x^2 +
128*a^4*c^3*e^4*x^2 + 16*b^4*c^3*d^4*x^2 - 80*a^2*b^3*c^2*d^2*e^2 - 96*a^3*
b^2*c^2*e^4*x^2 - 640*a^3*c^4*d^2*e^2*x^2 + 4*b^3*c*d^3*e*(b^2 - 4*a*c)^(3/
2) + 4*a*b*e^4*x^2*(b^2 - 4*a*c)^(5/2) - 48*a*b^4*c^2*d^3*e + 16*a*b^5*c*d^
2*e^2 - 4*a*b^3*e^4*x^2*(b^2 - 4*a*c)^(3/2) - 16*b*c^3*d^4*x^2*(b^2 - 4*a*c
)^(3/2) - 6*b^2*d*e^3*x^2*(b^2 - 4*a*c)^(5/2) + 3*b^4*d*e^3*x^2*(b^2 - 4*a*
c)^(3/2) + 20*c^2*d^3*e*x^2*(b^2 - 4*a*c)^(5/2) + 352*a^2*b^2*c^3*d^3*e + 6
4*a^3*b*c^3*d^2*e^2 - 96*a^3*b^2*c^2*d*e^3 - 128*a*b^2*c^4*d^4*x^2 + 16*a^2
*b^4*c*e^4*x^2 - 32*b^5*c^2*d^3*e*x^2 + 16*b^6*c*d^2*e^2*x^2 - 4*b*c*d^3*e*
(b^2 - 4*a*c)^(5/2) + 480*a^2*b^2*c^3*d^2*e^2*x^2 - 12*b*c*d^2*e^2*x^2*(b^2
 - 4*a*c)^(5/2) + 240*a*b^3*c^3*d^3*e*x^2 - 448*a^2*b*c^4*d^3*e*x^2 + 192*a
^3*b*c^3*d*e^3*x^2 + 12*b^2*c^2*d^3*e*x^2*(b^2 - 4*a*c)^(3/2) - 4*b^3*c*d^2
*e^2*x^2*(b^2 - 4*a*c)^(3/2) - 144*a*b^4*c^2*d^2*e^2*x^2 - 48*a^2*b^3*c^2*d
*e^3*x^2)*(e*((a*b^2)/4 - a^2*c + (a*b*(b^2 - 4*a*c)^(1/2))/4) - (b^3*d)/4
- (b^2*d*(b^2 - 4*a*c)^(1/2))/4 + (a*c*d*(b^2 - 4*a*c)^(1/2))/2 + a*b*c*d))
/(4*a*c^3*d^2 + 4*a^2*c^2*e^2 - b^2*c^2*d^2 + b^3*c*d*e - a*b^2*c*e^2 - 4*a
*b*c^2*d*e)

3.298 $\int \frac{x^3}{(d+ex^2)(a+bx^2+cx^4)}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 132

$$\int \frac{x^3}{(d+ex^2)(a+bx^2+cx^4)}\,dx = \frac{(bd-2ae)\text{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2\sqrt{b^2-4ac}\,(cd^2-bde+ae^2)} - \frac{d\log(d+ex^2)}{2(cd^2-bde+ae^2)} + \frac{d\log(a+bx^2+cx^4)}{4(cd^2-bde+ae^2)}$$

```
[Out] -1/2*d*ln(e*x^2+d)/(a*e^2-b*d*e+c*d^2)+1/4*d*ln(c*x^4+b*x^2+a)/(a*e^2-b*d*e
+c*d^2)+1/2*(-2*a*e+b*d)*arctanh((2*c*x^2+b)/(-4*a*c+b^2)^(1/2))/(a*e^2-b*d
*e+c*d^2)/(-4*a*c+b^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {1265, 814, 648, 632, 212, 642}

$$\int \frac{x^3}{(d+ex^2)(a+bx^2+cx^4)}\,dx = \frac{(bd-2ae)\text{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2\sqrt{b^2-4ac}\,(ae^2-bde+cd^2)} - \frac{d\log(d+ex^2)}{2(ae^2-bde+cd^2)} + \frac{d\log(a+bx^2+cx^4)}{4(ae^2-bde+cd^2)}$$

```
[In] Int[x^3/((d + e*x^2)*(a + b*x^2 + c*x^4)),x]
```

```
[Out] ((b*d - 2*a*e)*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(2*Sqrt[b^2 - 4*a*
c]*(c*d^2 - b*d*e + a*e^2)) - (d*Log[d + e*x^2])/(2*(c*d^2 - b*d*e + a*e^2)
) + (d*Log[a + b*x^2 + c*x^4])/(4*(c*d^2 - b*d*e + a*e^2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 814

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)/(a +
 b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 1265

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{1}{2}\text{Subst}\left(\int \frac{x}{(d+ex)\,(a+bx+cx^2)}\,dx, x, x^2\right)\\ &= \frac{1}{2}\text{Subst}\left(\int \left(-\frac{de}{(cd^2-bde+ae^2)\,(d+ex)}+\frac{ae+cdx}{(cd^2-bde+ae^2)\,(a+bx+cx^2)}\right)\,dx, x, x^2\right)\end{aligned}$$

$$= -\frac{d\log(d+ex^2)}{2(cd^2-bde+ae^2)} + \frac{\text{Subst}\left(\int \frac{ae+cdx}{a+bx+cx^2}\,dx, x, x^2\right)}{2(cd^2-bde+ae^2)}$$

$$= -\frac{d\log(d+ex^2)}{2(cd^2-bde+ae^2)} + \frac{d\text{Subst}\left(\int \frac{b+2cx}{a+bx+cx^2}\,dx, x, x^2\right)}{4(cd^2-bde+ae^2)} - \frac{(bd-2ae)\text{Subst}\left(\int \frac{1}{a+bx+cx^2}\,dx, x, x^2\right)}{4(cd^2-bde+ae^2)}$$

$$= -\frac{d\log(d+ex^2)}{2(cd^2-bde+ae^2)} + \frac{d\log(a+bx^2+cx^4)}{4(cd^2-bde+ae^2)} + \frac{(bd-2ae)\text{Subst}\left(\int \frac{1}{b^2-4ac-x^2}\,dx, x, b+2cx^2\right)}{2(cd^2-bde+ae^2)}$$

$$= \frac{(bd-2ae)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2\sqrt{b^2-4ac}\,(cd^2-bde+ae^2)} - \frac{d\log(d+ex^2)}{2(cd^2-bde+ae^2)} + \frac{d\log(a+bx^2+cx^4)}{4(cd^2-bde+ae^2)}$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.86

$$\int \frac{x^3}{(d+ex^2)(a+bx^2+cx^4)}\,dx = \frac{2(bd-2ae)\arctan\left(\frac{b+2cx^2}{\sqrt{-b^2+4ac}}\right) + \sqrt{-b^2+4ac}\,d(2\log(d+ex^2) - \log(a+bx^2+cx^4))}{4\sqrt{-b^2+4ac}\,(-cd^2+e(bd-ae))}$$

```
[In] Integrate[x^3/((d + e*x^2)*(a + b*x^2 + c*x^4)),x]
[Out] (2*(b*d - 2*a*e)*ArcTan[(b + 2*c*x^2)/Sqrt[-b^2 + 4*a*c]] + Sqrt[-b^2 + 4*a
*c]*d*(2*Log[d + e*x^2] - Log[a + b*x^2 + c*x^4]))/(4*Sqrt[-b^2 + 4*a*c]*(-
(c*d^2) + e*(b*d - a*e)))
```

Maple [A] (verified)

Time = 0.46 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.85

method	result	size
default	$\frac{\frac{d\ln(c\,x^4+b\,x^2+a)}{2}+\frac{2\left(ae-\frac{bd}{2}\right)\arctan\left(\frac{2c\,x^2+b}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}}}{2a\,e^2-2bde+2c\,d^2}-\frac{d\ln(e\,x^2+d)}{2(a\,e^2-bde+c\,d^2)}$	112
risch	Expression too large to display	4521

```
[In] int(x^3/(e*x^2+d)/(c*x^4+b*x^2+a),x,method=_RETURNVERBOSE)
[Out] 1/2/(a*e^2-b*d*e+c*d^2)*(1/2*d*ln(c*x^4+b*x^2+a)+2*(a*e-1/2*b*d)/(4*a*c-b^2
)^(1/2)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2)))-1/2*d*ln(e*x^2+d)/(a*e^2-b*d
*e+c*d^2)
```

Fricas [A] (verification not implemented)

none

Time = 11.07 (sec) , antiderivative size = 321, normalized size of antiderivative = 2.43

$$\int \frac{x^3}{(d+ex^2)(a+bx^2+cx^4)}\,dx$$
$$= \left[\frac{(b^2-4\,ac)d\log(cx^4+bx^2+a) - 2\,(b^2-4\,ac)d\log(ex^2+d) - \sqrt{b^2-4\,ac}(bd-2\,ae)\log\left(\frac{2\,c^2x^4+2\,bcx^2+b^2-}{cx}\right.}{4\,((b^2c-4\,ac^2)d^2-(b^3-4\,abc)de+(ab^2-4\,a^2c)e^2)}\right.$$

```
[In] integrate(x^3/(e*x^2+d)/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] [1/4*((b^2 - 4*a*c)*d*log(c*x^4 + b*x^2 + a) - 2*(b^2 - 4*a*c)*d*log(e*x^2
+ d) - sqrt(b^2 - 4*a*c)*(b*d - 2*a*e)*log((2*c^2*x^4 + 2*b*c*x^2 + b^2 - 2
*a*c - (2*c*x^2 + b)*sqrt(b^2 - 4*a*c))/(c*x^4 + b*x^2 + a)))/((b^2*c - 4*a
*c^2)*d^2 - (b^3 - 4*a*b*c)*d*e + (a*b^2 - 4*a^2*c)*e^2), 1/4*((b^2 - 4*a*c
)*d*log(c*x^4 + b*x^2 + a) - 2*(b^2 - 4*a*c)*d*log(e*x^2 + d) + 2*sqrt(-b^2
 + 4*a*c)*(b*d - 2*a*e)*arctan(-(2*c*x^2 + b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4*a
*c)))/((b^2*c - 4*a*c^2)*d^2 - (b^3 - 4*a*b*c)*d*e + (a*b^2 - 4*a^2*c)*e^2)
]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^3}{(d+ex^2)(a+bx^2+cx^4)}\,dx = \text{Timed out}$$

```
[In] integrate(x**3/(e*x**2+d)/(c*x**4+b*x**2+a),x)
```

```
[Out] Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^3}{(d+ex^2)(a+bx^2+cx^4)}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(x^3/(e*x^2+d)/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

Giac [A] (verification not implemented)

none

Time = 0.56 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.99

$$\int \frac{x^3}{(d+ex^2)(a+bx^2+cx^4)}\,dx = -\frac{de\log(|ex^2+d|)}{2(cd^2e-bde^2+ae^3)} + \frac{d\log(cx^4+bx^2+a)}{4(cd^2-bde+ae^2)} - \frac{(bd-2\,ae)\arctan\left(\frac{2\,cx^2+b}{\sqrt{-b^2+4\,ac}}\right)}{2(cd^2-bde+ae^2)\sqrt{-b^2+4\,ac}}$$

```
[In] integrate(x^3/(e*x^2+d)/(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] -1/2*d*e*log(abs(e*x^2 + d))/(c*d^2*e - b*d*e^2 + a*e^3) + 1/4*d*log(c*x^4
+ b*x^2 + a)/(c*d^2 - b*d*e + a*e^2) - 1/2*(b*d - 2*a*e)*arctan((2*c*x^2 +
b)/sqrt(-b^2 + 4*a*c))/((c*d^2 - b*d*e + a*e^2)*sqrt(-b^2 + 4*a*c))
```

Mupad [B] (verification not implemented)

Time = 14.13 (sec) , antiderivative size = 3704, normalized size of antiderivative = 28.06

$$\int \frac{x^3}{(d+ex^2)(a+bx^2+cx^4)}\,dx = \text{Too large to display}$$

```
[In] int(x^3/((d + e*x^2)*(a + b*x^2 + c*x^4)),x)
```

```
[Out] (log(76*d^3*e^3*(b^2 - 4*a*c)^(9/2) - 64*a^3*b^6*e^6 - 4608*a^3*c^6*d^6 + 5
12*a^6*c^3*e^6 - 320*a*b^4*c^4*d^6 + 512*a^4*b^4*c*e^6 - 64*a*b^8*d^2*e^4 -
 128*a^2*b^7*d*e^5 + 32*a^3*b^3*e^6*(b^2 - 4*a*c)^(3/2) - 48*b^3*c^3*d^6*(b
^2 - 4*a*c)^(3/2) - 68*b^2*d^3*e^3*(b^2 - 4*a*c)^(7/2) - 28*b^4*d^3*e^3*(b^
2 - 4*a*c)^(5/2) + 20*b^6*d^3*e^3*(b^2 - 4*a*c)^(3/2) + 4*a^2*e^6*x^2*(b^2
- 4*a*c)^(7/2) + 144*c^4*d^6*x^2*(b^2 - 4*a*c)^(5/2) + 39*d^2*e^4*x^2*(b^2
- 4*a*c)^(9/2) + 2432*a^2*b^2*c^5*d^6 - 1152*a^5*b^2*c^2*e^6 + 40448*a^4*c^
5*d^4*e^2 - 19968*a^5*c^4*d^2*e^4 - 64*a^2*b^7*e^6*x^2 - 64*b^5*c^4*d^6*x^2
 - 64*b^9*d^2*e^4*x^2 + 32*a^3*b*e^6*(b^2 - 4*a*c)^(5/2) + 48*b*c^3*d^6*(b^
2 - 4*a*c)^(5/2) + 40*a^2*d*e^5*(b^2 - 4*a*c)^(7/2) + 168*c^2*d^5*e*(b^2 -
4*a*c)^(7/2) + 40*a^2*b^2*e^6*x^2*(b^2 - 4*a*c)^(5/2) + 20*a^2*b^4*e^6*x^2*
(b^2 - 4*a*c)^(3/2) - 80*b^2*c^4*d^6*x^2*(b^2 - 4*a*c)^(3/2) + 155*b^2*d^2*
e^4*x^2*(b^2 - 4*a*c)^(7/2) - 155*b^4*d^2*e^4*x^2*(b^2 - 4*a*c)^(5/2) + 25*
b^6*d^2*e^4*x^2*(b^2 - 4*a*c)^(3/2) + 316*c^2*d^4*e^2*x^2*(b^2 - 4*a*c)^(7/
2) + 5120*a^2*b^4*c^3*d^4*e^2 - 4096*a^2*b^5*c^2*d^3*e^3 - 24448*a^3*b^2*c^
4*d^4*e^2 + 21760*a^3*b^3*c^3*d^3*e^3 - 9920*a^3*b^4*c^2*d^2*e^4 + 26240*a^
4*b^2*c^3*d^2*e^4 - 1600*a^4*b^3*c^2*e^6*x^2 + 38912*a^4*c^5*d^3*e^3*x^2 -
384*b^7*c^2*d^4*e^2*x^2 + 212*a*b*d^2*e^4*(b^2 - 4*a*c)^(7/2) - 176*b*c*d^4
*e^2*(b^2 - 4*a*c)^(7/2) + 256*a*b^5*c^3*d^5*e + 256*a*b^7*c*d^3*e^3 + 2560
```

*a^3*b*c^5*d^5*e + 1664*a^3*b^5*c*d*e^5 + 8704*a^5*b*c^3*d*e^5 - 128*a*b^8*
d*e^5*x^2 - 168*a*b^3*d^2*e^4*(b^2 - 4*a*c)^(5/2) + 20*a*b^5*d^2*e^4*(b^2 -
4*a*c)^(3/2) + 144*a^2*b^2*d*e^5*(b^2 - 4*a*c)^(5/2) - 56*a^2*b^4*d*e^5*(b
^2 - 4*a*c)^(3/2) - 272*b^2*c^2*d^5*e*(b^2 - 4*a*c)^(5/2) + 256*b^3*c*d^4*e
^2*(b^2 - 4*a*c)^(5/2) + 104*b^4*c^2*d^5*e*(b^2 - 4*a*c)^(3/2) - 80*b^5*c*d
^4*e^2*(b^2 - 4*a*c)^(3/2) - 384*a*b^6*c^2*d^4*e^2 - 1664*a^2*b^3*c^4*d^5*e
+ 1408*a^2*b^6*c*d^2*e^4 - 37888*a^4*b*c^4*d^3*e^3 - 6784*a^4*b^3*c^2*d*e^
5 + 448*a*b^3*c^5*d^6*x^2 - 768*a^2*b*c^6*d^6*x^2 + 576*a^3*b^5*c*e^6*x^2 +
1280*a^5*b*c^3*e^6*x^2 - 21504*a^3*c^6*d^5*e*x^2 - 5120*a^5*c^4*d*e^5*x^2
+ 256*b^6*c^3*d^5*e*x^2 + 256*b^8*c*d^3*e^3*x^2 - 26560*a^2*b^3*c^4*d^4*e^2
*x^2 + 25600*a^2*b^4*c^3*d^3*e^3*x^2 - 11264*a^2*b^5*c^2*d^2*e^4*x^2 - 5888
0*a^3*b^2*c^4*d^3*e^3*x^2 + 34880*a^3*b^3*c^3*d^2*e^4*x^2 + 80*a*b^3*d*e^5*
x^2*(b^2 - 4*a*c)^(5/2) - 40*a*b^5*d*e^5*x^2*(b^2 - 4*a*c)^(3/2) - 448*b*c*
d^3*e^3*x^2*(b^2 - 4*a*c)^(7/2) - 416*b*c^3*d^5*e*x^2*(b^2 - 4*a*c)^(5/2) -
3200*a*b^4*c^4*d^5*e*x^2 + 1472*a*b^7*c*d^2*e^4*x^2 + 1792*a^2*b^6*c*d*e^5
*x^2 + 192*b^3*c*d^3*e^3*x^2*(b^2 - 4*a*c)^(5/2) + 160*b^3*c^3*d^5*e*x^2*(b
^2 - 4*a*c)^(3/2) + 5504*a*b^5*c^3*d^4*e^2*x^2 - 4352*a*b^6*c^2*d^3*e^3*x^2
+ 14080*a^2*b^2*c^5*d^5*e*x^2 + 42752*a^3*b*c^5*d^4*e^2*x^2 - 8320*a^3*b^4
*c^2*d*e^5*x^2 - 37120*a^4*b*c^4*d^2*e^4*x^2 + 14080*a^4*b^2*c^3*d*e^5*x^2
+ 88*a*b*d*e^5*x^2*(b^2 - 4*a*c)^(7/2) + 168*b^2*c^2*d^4*e^2*x^2*(b^2 - 4*a
*c)^(5/2) - 100*b^4*c^2*d^4*e^2*x^2*(b^2 - 4*a*c)^(3/2))*(d*((b*(b^2 - 4*a*
c)^(1/2))/4 - a*c + b^2/4) - (a*e*(b^2 - 4*a*c)^(1/2))/2))/(a*b^2*e^2 - 4*a
*c^2*d^2 - 4*a^2*c*e^2 + b^2*c*d^2 - b^3*d*e + 4*a*b*c*d*e) - (log(76*d^3*e
^3*(b^2 - 4*a*c)^(9/2) + 64*a^3*b^6*e^6 + 4608*a^3*c^6*d^6 - 512*a^6*c^3*e^
6 + 320*a*b^4*c^4*d^6 - 512*a^4*b^4*c*e^6 + 64*a*b^8*d^2*e^4 + 128*a^2*b^7*
d*e^5 + 32*a^3*b^3*e^6*(b^2 - 4*a*c)^(3/2) - 48*b^3*c^3*d^6*(b^2 - 4*a*c)^(
3/2) - 68*b^2*d^3*e^3*(b^2 - 4*a*c)^(7/2) - 28*b^4*d^3*e^3*(b^2 - 4*a*c)^(5
/2) + 20*b^6*d^3*e^3*(b^2 - 4*a*c)^(3/2) + 4*a^2*e^6*x^2*(b^2 - 4*a*c)^(7/2
) + 144*c^4*d^6*x^2*(b^2 - 4*a*c)^(5/2) + 39*d^2*e^4*x^2*(b^2 - 4*a*c)^(9/2
) - 2432*a^2*b^2*c^5*d^6 + 1152*a^5*b^2*c^2*e^6 - 40448*a^4*c^5*d^4*e^2 + 1
9968*a^5*c^4*d^2*e^4 + 64*a^2*b^7*e^6*x^2 + 64*b^5*c^4*d^6*x^2 + 64*b^9*d^2
*e^4*x^2 + 32*a^3*b*e^6*(b^2 - 4*a*c)^(5/2) + 48*b*c^3*d^6*(b^2 - 4*a*c)^(5
/2) + 40*a^2*d*e^5*(b^2 - 4*a*c)^(7/2) + 168*c^2*d^5*e*(b^2 - 4*a*c)^(7/2)
+ 40*a^2*b^2*e^6*x^2*(b^2 - 4*a*c)^(5/2) + 20*a^2*b^4*e^6*x^2*(b^2 - 4*a*c)
^(3/2) - 80*b^2*c^4*d^6*x^2*(b^2 - 4*a*c)^(3/2) + 155*b^2*d^2*e^4*x^2*(b^2
- 4*a*c)^(7/2) - 155*b^4*d^2*e^4*x^2*(b^2 - 4*a*c)^(5/2) + 25*b^6*d^2*e^4*x
^2*(b^2 - 4*a*c)^(3/2) + 316*c^2*d^4*e^2*x^2*(b^2 - 4*a*c)^(7/2) - 5120*a^2
*b^4*c^3*d^4*e^2 + 4096*a^2*b^5*c^2*d^3*e^3 + 24448*a^3*b^2*c^4*d^4*e^2 - 2
1760*a^3*b^3*c^3*d^3*e^3 + 9920*a^3*b^4*c^2*d^2*e^4 - 26240*a^4*b^2*c^3*d^2
*e^4 + 1600*a^4*b^3*c^2*e^6*x^2 - 38912*a^4*c^5*d^3*e^3*x^2 + 384*b^7*c^2*d
^4*e^2*x^2 + 212*a*b*d^2*e^4*(b^2 - 4*a*c)^(7/2) - 176*b*c*d^4*e^2*(b^2 - 4
*a*c)^(7/2) - 256*a*b^5*c^3*d^5*e - 256*a*b^7*c*d^3*e^3 - 2560*a^3*b*c^5*d^
5*e - 1664*a^3*b^5*c*d*e^5 - 8704*a^5*b*c^3*d*e^5 + 128*a*b^8*d*e^5*x^2 - 1
68*a*b^3*d^2*e^4*(b^2 - 4*a*c)^(5/2) + 20*a*b^5*d^2*e^4*(b^2 - 4*a*c)^(3/2)
+ 144*a^2*b^2*d*e^5*(b^2 - 4*a*c)^(5/2) - 56*a^2*b^4*d*e^5*(b^2 - 4*a*c)^(

3/2) - 272*b^2*c^2*d^5*e*(b^2 - 4*a*c)^(5/2) + 256*b^3*c*d^4*e^2*(b^2 - 4*a
*c)^(5/2) + 104*b^4*c^2*d^5*e*(b^2 - 4*a*c)^(3/2) - 80*b^5*c*d^4*e^2*(b^2 -
 4*a*c)^(3/2) + 384*a*b^6*c^2*d^4*e^2 + 1664*a^2*b^3*c^4*d^5*e - 1408*a^2*b
^6*c*d^2*e^4 + 37888*a^4*b*c^4*d^3*e^3 + 6784*a^4*b^3*c^2*d*e^5 - 448*a*b^3
*c^5*d^6*x^2 + 768*a^2*b*c^6*d^6*x^2 - 576*a^3*b^5*c*e^6*x^2 - 1280*a^5*b*c
^3*e^6*x^2 + 21504*a^3*c^6*d^5*e*x^2 + 5120*a^5*c^4*d*e^5*x^2 - 256*b^6*c^3
*d^5*e*x^2 - 256*b^8*c*d^3*e^3*x^2 + 26560*a^2*b^3*c^4*d^4*e^2*x^2 - 25600*
a^2*b^4*c^3*d^3*e^3*x^2 + 11264*a^2*b^5*c^2*d^2*e^4*x^2 + 58880*a^3*b^2*c^4
*d^3*e^3*x^2 - 34880*a^3*b^3*c^3*d^2*e^4*x^2 + 80*a*b^3*d*e^5*x^2*(b^2 - 4*
a*c)^(5/2) - 40*a*b^5*d*e^5*x^2*(b^2 - 4*a*c)^(3/2) - 448*b*c*d^3*e^3*x^2*(
b^2 - 4*a*c)^(7/2) - 416*b*c^3*d^5*e*x^2*(b^2 - 4*a*c)^(5/2) + 3200*a*b^4*c
^4*d^5*e*x^2 - 1472*a*b^7*c*d^2*e^4*x^2 - 1792*a^2*b^6*c*d*e^5*x^2 + 192*b^
3*c*d^3*e^3*x^2*(b^2 - 4*a*c)^(5/2) + 160*b^3*c^3*d^5*e*x^2*(b^2 - 4*a*c)^(
3/2) - 5504*a*b^5*c^3*d^4*e^2*x^2 + 4352*a*b^6*c^2*d^3*e^3*x^2 - 14080*a^2*
b^2*c^5*d^5*e*x^2 - 42752*a^3*b*c^5*d^4*e^2*x^2 + 8320*a^3*b^4*c^2*d*e^5*x^
2 + 37120*a^4*b*c^4*d^2*e^4*x^2 - 14080*a^4*b^2*c^3*d*e^5*x^2 + 88*a*b*d*e^
5*x^2*(b^2 - 4*a*c)^(7/2) + 168*b^2*c^2*d^4*e^2*x^2*(b^2 - 4*a*c)^(5/2) - 1
00*b^4*c^2*d^4*e^2*x^2*(b^2 - 4*a*c)^(3/2))*(d*(a*c + (b*(b^2 - 4*a*c)^(1/2
))/4 - b^2/4) - (a*e*(b^2 - 4*a*c)^(1/2))/2))/(a*b^2*e^2 - 4*a*c^2*d^2 - 4*
a^2*c*e^2 + b^2*c*d^2 - b^3*d*e + 4*a*b*c*d*e) - (d*log(d + e*x^2))/(2*(a*e
^2 + c*d^2 - b*d*e))

3.299 $\int \frac{x}{(d+ex^2)(a+bx^2+cx^4)}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 133

$$\int \frac{x}{(d+ex^2)(a+bx^2+cx^4)}\,dx = -\frac{(2cd-be)\operatorname{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2\sqrt{b^2-4ac}\,(cd^2-bde+ae^2)} + \frac{e\log(d+ex^2)}{2(cd^2-bde+ae^2)} - \frac{e\log(a+bx^2+cx^4)}{4(cd^2-bde+ae^2)}$$

```
[Out] 1/2*e*ln(e*x^2+d)/(a*e^2-b*d*e+c*d^2)-1/4*e*ln(c*x^4+b*x^2+a)/(a*e^2-b*d*e+
c*d^2)-1/2*(-b*e+2*c*d)*arctanh((2*c*x^2+b)/(-4*a*c+b^2)^(1/2))/(a*e^2-b*d*
e+c*d^2)/(-4*a*c+b^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.280$, Rules used = {1261, 719, 31, 648, 632, 212, 642}

$$\int \frac{x}{(d+ex^2)(a+bx^2+cx^4)}\,dx = -\frac{(2cd-be)\operatorname{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2\sqrt{b^2-4ac}\,(ae^2-bde+cd^2)} + \frac{e\log(d+ex^2)}{2(ae^2-bde+cd^2)} - \frac{e\log(a+bx^2+cx^4)}{4(ae^2-bde+cd^2)}$$

```
[In] Int[x/((d + e*x^2)*(a + b*x^2 + c*x^4)),x]
```

```
[Out] -1/2*((2*c*d - b*e)*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(Sqrt[b^2 - 4
*a*c]*(c*d^2 - b*d*e + a*e^2)) + (e*Log[d + e*x^2])/(2*(c*d^2 - b*d*e + a*e
^2)) - (e*Log[a + b*x^2 + c*x^4])/(4*(c*d^2 - b*d*e + a*e^2))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 719

```
Int[1/(((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)), x_Symbol]
 :> Dist[e^2/(c*d^2 - b*d*e + a*e^2), Int[1/(d + e*x), x], x] + Dist[1/(c*d
^2 - b*d*e + a*e^2), Int[(c*d - b*e - c*e*x)/(a + b*x + c*x^2), x], x] /; F
reeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^
2, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 1261

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(
p_.), x_Symbol] :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x],
 x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]
```

Rubi steps

$$\text{integral} = \frac{1}{2}\text{Subst}\left(\int \frac{1}{(d+ex)\,(a+bx+cx^2)}\,dx,x,x^2\right)$$
$$= \frac{\text{Subst}\left(\int \frac{cd-be-cex}{a+bx+cx^2}\,dx,x,x^2\right)}{2\,(cd^2-bde+ae^2)} + \frac{e^2\text{Subst}\left(\int \frac{1}{d+ex}\,dx,x,x^2\right)}{2\,(cd^2-bde+ae^2)}$$
$$= \frac{e\log\left(d+ex^2\right)}{2\,(cd^2-bde+ae^2)} - \frac{e\text{Subst}\left(\int \frac{b+2cx}{a+bx+cx^2}\,dx,x,x^2\right)}{4\,(cd^2-bde+ae^2)} + \frac{(2cd-be)\text{Subst}\left(\int \frac{1}{a+bx+cx^2}\,dx,x,x^2\right)}{4\,(cd^2-bde+ae^2)}$$
$$= \frac{e\log\left(d+ex^2\right)}{2\,(cd^2-bde+ae^2)} - \frac{e\log\left(a+bx^2+cx^4\right)}{4\,(cd^2-bde+ae^2)} - \frac{(2cd-be)\text{Subst}\left(\int \frac{1}{b^2-4ac-x^2}\,dx,x,b+2cx^2\right)}{2\,(cd^2-bde+ae^2)}$$
$$= -\frac{(2cd-be)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2\sqrt{b^2-4ac}\,(cd^2-bde+ae^2)} + \frac{e\log\left(d+ex^2\right)}{2\,(cd^2-bde+ae^2)} - \frac{e\log\left(a+bx^2+cx^4\right)}{4\,(cd^2-bde+ae^2)}$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.84

$$\int \frac{x}{(d+ex^2)\,(a+bx^2+cx^4)}\,dx$$
$$= \frac{(-4cd+2be)\arctan\left(\frac{b+2cx^2}{\sqrt{-b^2+4ac}}\right)+\sqrt{-b^2+4ac}\,e(-2\log\left(d+ex^2\right)+\log\left(a+bx^2+cx^4\right))}{4\sqrt{-b^2+4ac}\,(-cd^2+e(bd-ae))}$$

```
[In] Integrate[x/((d + e*x^2)*(a + b*x^2 + c*x^4)),x]
[Out] ((-4*c*d + 2*b*e)*ArcTan[(b + 2*c*x^2)/Sqrt[-b^2 + 4*a*c]] + Sqrt[-b^2 + 4*
a*c]*e*(-2*Log[d + e*x^2] + Log[a + b*x^2 + c*x^4]))/(4*Sqrt[-b^2 + 4*a*c]*
(-(c*d^2) + e*(b*d - a*e)))
```

Maple [A] (verified)

Time = 0.36 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.85

method	result
default	$-\frac{\frac{e\ln\left(c\,x^4+b\,x^2+a\right)}{2}+\frac{2\left(\frac{be}{2}-cd\right)\arctan\left(\frac{2c\,x^2+b}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}}}{2(a\,e^2-bde+c\,d^2)}+\frac{e\ln\left(e\,x^2+d\right)}{2a\,e^2-2bde+2c\,d^2}$
risch	$\frac{e\ln\left(e\,x^2+d\right)}{2a\,e^2-2bde+2c\,d^2}+\frac{\left(\sum_{\_R=\text{RootOf}\left(\left(4a^2c\,e^2-a\,b^2e^2-4abcde+4a\,c^2d^2+b^3de-b^2c\,d^2\right)\_Z^2+\left(4ace-b^2e\right)\_Z+c\right)}\_R\ln\left(\left(\left(4a^2c\,e^3-a\,b^2e^3-\right.\right.\right.\right.}{}$

```
[In] int(x/(e*x^2+d)/(c*x^4+b*x^2+a),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2/(a*e^2-b*d*e+c*d^2)*(1/2*e*ln(c*x^4+b*x^2+a)+2*(1/2*b*e-c*d)/(4*a*c-b^
2)^(1/2)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2)))+1/2*e*ln(e*x^2+d)/(a*e^2-b*
d*e+c*d^2)
```

Fricas [A] (verification not implemented)

none

Time = 7.67 (sec) , antiderivative size = 321, normalized size of antiderivative = 2.41

$$\int \frac{x}{(d+ex^2)(a+bx^2+cx^4)}\,dx$$
$$= \Bigg[-\frac{(b^2-4\,ac)e\log\left(cx^4+bx^2+a\right)-2\,(b^2-4\,ac)e\log\left(ex^2+d\right)+\sqrt{b^2-4\,ac}(2\,cd-be)\log\left(\frac{2\,c^2x^4+2\,bcx^2+\cdots}{\cdots}\right)}{4\left((b^2c-4\,ac^2)d^2-(b^3-4\,abc)de+(ab^2-4\,a^2c)e^2\right)}$$
$$-\frac{(b^2-4\,ac)e\log\left(cx^4+bx^2+a\right)-2\,(b^2-4\,ac)e\log\left(ex^2+d\right)+2\,\sqrt{-b^2+4\,ac}(2\,cd-be)\arctan\left(-\frac{(2\,c\cdots}{\cdots}\right)}{4\left((b^2c-4\,ac^2)d^2-(b^3-4\,abc)de+(ab^2-4\,a^2c)e^2\right)}$$

```
[In] integrate(x/(e*x^2+d)/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] [-1/4*((b^2 - 4*a*c)*e*log(c*x^4 + b*x^2 + a) - 2*(b^2 - 4*a*c)*e*log(e*x^2
 + d) + sqrt(b^2 - 4*a*c)*(2*c*d - b*e)*log((2*c^2*x^4 + 2*b*c*x^2 + b^2 -
2*a*c + (2*c*x^2 + b)*sqrt(b^2 - 4*a*c))/(c*x^4 + b*x^2 + a)))/((b^2*c - 4*
a*c^2)*d^2 - (b^3 - 4*a*b*c)*d*e + (a*b^2 - 4*a^2*c)*e^2), -1/4*((b^2 - 4*a
*c)*e*log(c*x^4 + b*x^2 + a) - 2*(b^2 - 4*a*c)*e*log(e*x^2 + d) + 2*sqrt(-b
^2 + 4*a*c)*(2*c*d - b*e)*arctan(-(2*c*x^2 + b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4
*a*c)))/((b^2*c - 4*a*c^2)*d^2 - (b^3 - 4*a*b*c)*d*e + (a*b^2 - 4*a^2*c)*e^
2)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x}{(d+ex^2)(a+bx^2+cx^4)}\,dx = \text{Timed out}$$

```
[In] integrate(x/(e*x**2+d)/(c*x**4+b*x**2+a),x)
```

```
[Out] Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x}{(d+ex^2)(a+bx^2+cx^4)}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(x/(e*x^2+d)/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

Giac [A] (verification not implemented)

none

Time = 0.59 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.00

$$\int \frac{x}{(d+ex^2)(a+bx^2+cx^4)}\,dx = \frac{e^2\log(|ex^2+d|)}{2(cd^2e-bde^2+ae^3)} - \frac{e\log(cx^4+bx^2+a)}{4(cd^2-bde+ae^2)} + \frac{(2cd-be)\arctan\left(\frac{2cx^2+b}{\sqrt{-b^2+4ac}}\right)}{2(cd^2-bde+ae^2)\sqrt{-b^2+4ac}}$$

```
[In] integrate(x/(e*x^2+d)/(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] 1/2*e^2*log(abs(e*x^2 + d))/(c*d^2*e - b*d*e^2 + a*e^3) - 1/4*e*log(c*x^4 +
 b*x^2 + a)/(c*d^2 - b*d*e + a*e^2) + 1/2*(2*c*d - b*e)*arctan((2*c*x^2 + b
)/sqrt(-b^2 + 4*a*c))/((c*d^2 - b*d*e + a*e^2)*sqrt(-b^2 + 4*a*c))
```

Mupad [B] (verification not implemented)

Time = 13.23 (sec) , antiderivative size = 2434, normalized size of antiderivative = 18.30

$$\int \frac{x}{(d+ex^2)(a+bx^2+cx^4)}\,dx = \text{Too large to display}$$

```
[In] int(x/((d + e*x^2)*(a + b*x^2 + c*x^4)),x)
```

```
[Out] (e*log(d + e*x^2))/(2*a*e^2 + 2*c*d^2 - 2*b*d*e) - (log(36*a^4*c^3*e^5 - 4*
a*b^6*e^5 - 4*b^7*e^5*x^2 + 32*a^2*b^4*c*e^5 + 36*a^2*c^5*d^4*e - 4*a*c^6*d
^5*x^2 - 4*b^6*e^5*x^2*(b^2 - 4*a*c)^(1/2) - 73*a^3*b^2*c^2*e^5 - 184*a^3*c
^4*d^2*e^3 + b^2*c^5*d^5*x^2 - 4*a*b^5*e^5*(b^2 - 4*a*c)^(1/2) + 2*a*c^5*d^
5*(b^2 - 4*a*c)^(1/2) + 16*a*b^5*c*d*e^4 - 60*a^2*c^4*d^3*e^2*(b^2 - 4*a*c)
```

^(1/2) + 18*a^3*c^3*e^5*x^2*(b^2 - 4*a*c)^(1/2) + 146*a^2*b^2*c^3*d^2*e^3 - 101*a^2*b^3*c^2*e^5*x^2 + 120*a^2*c^5*d^3*e^2*x^2 + 19*b^4*c^3*d^3*e^2*x^2 - 25*b^5*c^2*d^2*e^3*x^2 - 9*a*b^2*c^4*d^4*e + 184*a^3*b*c^3*d*e^4 + 36*a*b^5*c*e^5*x^2 + 16*b^6*c*d*e^4*x^2 + 24*a^2*b^3*c*e^5*(b^2 - 4*a*c)^(1/2) - 33*a^3*b*c^2*e^5*(b^2 - 4*a*c)^(1/2) + 66*a^3*c^3*d*e^4*(b^2 - 4*a*c)^(1/2) + b*c^5*d^5*x^2*(b^2 - 4*a*c)^(1/2) + 18*a*b^3*c^3*d^3*e^2 - 25*a*b^4*c^2*d^2*e^3 - 72*a^2*b*c^4*d^3*e^2 - 110*a^2*b^3*c^2*d*e^4 + 84*a^3*b*c^3*e^5*x^2 - 132*a^3*c^4*d*e^4*x^2 - 7*b^3*c^4*d^4*e*x^2 + 28*a*b^4*c*e^5*x^2*(b^2 - 4*a*c)^(1/2) + 18*a*c^5*d^4*e*x^2*(b^2 - 4*a*c)^(1/2) + 16*b^5*c*d*e^4*x^2*(b^2 - 4*a*c)^(1/2) - 126*a*b^4*c^2*d*e^4*x^2 + 20*a*b^2*c^3*d^3*e^2*(b^2 - 4*a*c)^(1/2) - 25*a*b^3*c^2*d^2*e^3*(b^2 - 4*a*c)^(1/2) + 90*a^2*b*c^3*d^2*e^3*(b^2 - 4*a*c)^(1/2) - 78*a^2*b^2*c^2*d*e^4*(b^2 - 4*a*c)^(1/2) - 7*b^2*c^4*d^4*e*x^2*(b^2 - 4*a*c)^(1/2) - 106*a*b^2*c^4*d^3*e^2*x^2 + 168*a*b^3*c^3*d^2*e^3*x^2 - 272*a^2*b*c^4*d^2*e^3*x^2 + 281*a^2*b^2*c^3*d*e^4*x^2 - 5*a*b*c^4*d^4*e*(b^2 - 4*a*c)^(1/2) + 16*a*b^4*c*d*e^4*(b^2 - 4*a*c)^(1/2) - 53*a^2*b^2*c^2*e^5*x^2*(b^2 - 4*a*c)^(1/2) + 28*a*b*c^5*d^4*e*x^2 - 92*a^2*c^4*d^2*e^3*x^2*(b^2 - 4*a*c)^(1/2) + 19*b^3*c^3*d^3*e^2*x^2*(b^2 - 4*a*c)^(1/2) - 25*b^4*c^2*d^2*e^3*x^2*(b^2 - 4*a*c)^(1/2) + 118*a*b^2*c^3*d^2*e^3*x^2*(b^2 - 4*a*c)^(1/2) - 66*a*b*c^4*d^3*e^2*x^2*(b^2 - 4*a*c)^(1/2) - 94*a*b^3*c^2*d*e^4*x^2*(b^2 - 4*a*c)^(1/2) + 125*a^2*b*c^3*d*e^4*x^2*(b^2 - 4*a*c)^(1/2))*(e*((b*(b^2 - 4*a*c)^(1/2))/4 - a*c + b^2/4) - (c*d*(b^2 - 4*a*c)^(1/2))/2))/(a*b^2*e^2 - 4*a*c^2*d^2 - 4*a^2*c*e^2 + b^2*c*d^2 - b^3*d*e + 4*a*b*c*d*e) + (log(4*a*b^6*e^5 - 36*a^4*c^3*e^5 + 4*b^7*e^5*x^2 - 32*a^2*b^4*c*e^5 - 36*a^2*c^5*d^4*e + 4*a*c^6*d^5*x^2 - 4*b^6*e^5*x^2*(b^2 - 4*a*c)^(1/2) + 73*a^3*b^2*c^2*e^5 + 184*a^3*c^4*d^2*e^3 - b^2*c^5*d^5*x^2 - 4*a*b^5*e^5*(b^2 - 4*a*c)^(1/2) + 2*a*c^5*d^5*(b^2 - 4*a*c)^(1/2) - 16*a*b^5*c*d*e^4 - 60*a^2*c^4*d^3*e^2*(b^2 - 4*a*c)^(1/2) + 18*a^3*c^3*e^5*x^2*(b^2 - 4*a*c)^(1/2) - 146*a^2*b^2*c^3*d^2*e^3 + 101*a^2*b^3*c^2*e^5*x^2 - 120*a^2*c^5*d^3*e^2*x^2 - 19*b^4*c^3*d^3*e^2*x^2 + 25*b^5*c^2*d^2*e^3*x^2 + 9*a*b^2*c^4*d^4*e - 184*a^3*b*c^3*d*e^4 - 36*a*b^5*c*e^5*x^2 - 16*b^6*c*d*e^4*x^2 + 24*a^2*b^3*c*e^5*(b^2 - 4*a*c)^(1/2) - 33*a^3*b*c^2*e^5*(b^2 - 4*a*c)^(1/2) + 66*a^3*c^3*d*e^4*(b^2 - 4*a*c)^(1/2) + b*c^5*d^5*x^2*(b^2 - 4*a*c)^(1/2) - 18*a*b^3*c^3*d^3*e^2 + 25*a*b^4*c^2*d^2*e^3 + 72*a^2*b*c^4*d^3*e^2 + 110*a^2*b^3*c^2*d*e^4 - 84*a^3*b*c^3*e^5*x^2 + 132*a^3*c^4*d*e^4*x^2 + 7*b^3*c^4*d^4*e*x^2 + 28*a*b^4*c*e^5*x^2*(b^2 - 4*a*c)^(1/2) + 18*a*c^5*d^4*e*x^2*(b^2 - 4*a*c)^(1/2) + 16*b^5*c*d*e^4*x^2*(b^2 - 4*a*c)^(1/2) + 126*a*b^4*c^2*d*e^4*x^2 + 20*a*b^2*c^3*d^3*e^2*(b^2 - 4*a*c)^(1/2) - 25*a*b^3*c^2*d^2*e^3*(b^2 - 4*a*c)^(1/2) + 90*a^2*b*c^3*d^2*e^3*(b^2 - 4*a*c)^(1/2) - 78*a^2*b^2*c^2*d*e^4*(b^2 - 4*a*c)^(1/2) - 7*b^2*c^4*d^4*e*x^2*(b^2 - 4*a*c)^(1/2) + 106*a*b^2*c^4*d^3*e^2*x^2 - 168*a*b^3*c^3*d^2*e^3*x^2 + 272*a^2*b*c^4*d^2*e^3*x^2 - 281*a^2*b^2*c^3*d*e^4*x^2 - 5*a*b*c^4*d^4*e*(b^2 - 4*a*c)^(1/2) + 16*a*b^4*c*d*e^4*(b^2 - 4*a*c)^(1/2) - 53*a^2*b^2*c^2*e^5*x^2*(b^2 - 4*a*c)^(1/2) - 28*a*b*c^5*d^4*e*x^2 - 92*a^2*c^4*d^2*e^3*x^2*(b^2 - 4*a*c)^(1/2) + 19*b^3*c^3*d^3*e^2*x^2*(b^2 - 4*a*c)^(1/2) - 25*b^4*c^2*d^2*e^3*x^2*(b^2 - 4*a*c)^(1/2) + 118*a*b^2*c^3*d^2*e^3*x^2*(b^2 - 4*a*c)^(1/2) - 6

6*a*b*c^4*d^3*e^2*x^2*(b^2 - 4*a*c)^(1/2) - 94*a*b^3*c^2*d*e^4*x^2*(b^2 - 4*a*c)^(1/2) + 125*a^2*b*c^3*d*e^4*x^2*(b^2 - 4*a*c)^(1/2))*(e*(a*c + (b*(b^2 - 4*a*c)^(1/2))/4 - b^2/4) - (c*d*(b^2 - 4*a*c)^(1/2))/2))/(a*b^2*e^2 - 4*a*c^2*d^2 - 4*a^2*c*e^2 + b^2*c*d^2 - b^3*d*e + 4*a*b*c*d*e)

3.300 $\int \frac{1}{x(d+ex^2)(a+bx^2+cx^4)}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 167

$$\int \frac{1}{x(d+ex^2)(a+bx^2+cx^4)}\,dx = \frac{(bcd-b^2e+2ace)\operatorname{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2a\sqrt{b^2-4ac}\,(cd^2-bde+ae^2)}+\frac{\log(x)}{ad}-\frac{e^2\log(d+ex^2)}{2d(cd^2-bde+ae^2)}-\frac{(cd-be)\log(a+bx^2+cx^4)}{4a(cd^2-bde+ae^2)}$$

```
[Out] ln(x)/a/d-1/2*e^2*ln(e*x^2+d)/d/(a*e^2-b*d*e+c*d^2)-1/4*(-b*e+c*d)*ln(c*x^4
+b*x^2+a)/a/(a*e^2-b*d*e+c*d^2)+1/2*(2*a*c*e-b^2*e+b*c*d)*arctanh((2*c*x^2+
b)/(-4*a*c+b^2)^(1/2))/a/(a*e^2-b*d*e+c*d^2)/(-4*a*c+b^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 167, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {1265, 907, 648, 632, 212, 642}

$$\int \frac{1}{x(d+ex^2)(a+bx^2+cx^4)}\,dx = \frac{\operatorname{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)(2ace+b^2(-e)+bcd)}{2a\sqrt{b^2-4ac}\,(ae^2-bde+cd^2)}-\frac{e^2\log(d+ex^2)}{2d(ae^2-bde+cd^2)}-\frac{(cd-be)\log(a+bx^2+cx^4)}{4a(ae^2-bde+cd^2)}+\frac{\log(x)}{ad}$$

```
[In] Int[1/(x*(d + e*x^2)*(a + b*x^2 + c*x^4)),x]
```

```
[Out] ((b*c*d - b^2*e + 2*a*c*e)*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(2*a*S
qrt[b^2 - 4*a*c]*(c*d^2 - b*d*e + a*e^2)) + Log[x]/(a*d) - (e^2*Log[d + e*x
```

```
^2])/(2*d*(c*d^2 - b*d*e + a*e^2)) - ((c*d - b*e)*Log[a + b*x^2 + c*x^4])/(
4*a*(c*d^2 - b*d*e + a*e^2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 907

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g
*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ
[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && I
ntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ[m, 0] && ILtQ[n, 0])
)
```

Rule 1265

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{1}{2}\text{Subst}\left(\int \frac{1}{x(d+ex)\left(a+bx+cx^2\right)}\,dx,x,x^2\right)\\
&= \frac{1}{2}\text{Subst}\left(\int \left(\frac{1}{adx}-\frac{e^3}{d\left(cd^2-bde+ae^2\right)(d+ex)}+\frac{-bcd+b^2e-ace-c(cd-be)x}{a\left(cd^2-bde+ae^2\right)\left(a+bx+cx^2\right)}\right)dx,x,x^2\right)\\
&= \frac{\log(x)}{ad}-\frac{e^2\log\left(d+ex^2\right)}{2d\left(cd^2-bde+ae^2\right)}+\frac{\text{Subst}\left(\int \frac{-bcd+b^2e-ace-c(cd-be)x}{a+bx+cx^2}\,dx,x,x^2\right)}{2a\left(cd^2-bde+ae^2\right)}\\
&= \frac{\log(x)}{ad}-\frac{e^2\log\left(d+ex^2\right)}{2d\left(cd^2-bde+ae^2\right)}-\frac{(cd-be)\text{Subst}\left(\int \frac{b+2cx}{a+bx+cx^2}\,dx,x,x^2\right)}{4a\left(cd^2-bde+ae^2\right)}\\
&\quad-\frac{\left(bcd-b^2e+2ace\right)\text{Subst}\left(\int \frac{1}{a+bx+cx^2}\,dx,x,x^2\right)}{4a\left(cd^2-bde+ae^2\right)}\\
&= \frac{\log(x)}{ad}-\frac{e^2\log\left(d+ex^2\right)}{2d\left(cd^2-bde+ae^2\right)}-\frac{(cd-be)\log\left(a+bx^2+cx^4\right)}{4a\left(cd^2-bde+ae^2\right)}\\
&\quad+\frac{\left(bcd-b^2e+2ace\right)\text{Subst}\left(\int \frac{1}{b^2-4ac-x^2}\,dx,x,b+2cx^2\right)}{2a\left(cd^2-bde+ae^2\right)}\\
&= \frac{\left(bcd-b^2e+2ace\right)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2a\sqrt{b^2-4ac}\left(cd^2-bde+ae^2\right)}+\frac{\log(x)}{ad}\\
&\quad-\frac{e^2\log\left(d+ex^2\right)}{2d\left(cd^2-bde+ae^2\right)}-\frac{(cd-be)\log\left(a+bx^2+cx^4\right)}{4a\left(cd^2-bde+ae^2\right)}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 242, normalized size of antiderivative = 1.45

$$
\int \frac{1}{x\left(d+ex^2\right)\left(a+bx^2+cx^4\right)}\,dx
= \frac{4\sqrt{b^2-4ac}\left(cd^2+e(-bd+ae)\right)\log(x)-d\left(bcd+c\sqrt{b^2-4ac}d-b^2e+2ace-b\sqrt{b^2-4ac}e\right)\log\left(b-\sqrt{b^2 \cdots\right.}{4a\sqrt{b^2 \cdots}}
$$

```
[In] Integrate[1/(x*(d + e*x^2)*(a + b*x^2 + c*x^4)),x]
```

```
[Out] (4*Sqrt[b^2 - 4*a*c]*(c*d^2 + e*(-(b*d) + a*e))*Log[x] - d*(b*c*d + c*Sqrt[
b^2 - 4*a*c]*d - b^2*e + 2*a*c*e - b*Sqrt[b^2 - 4*a*c]*e)*Log[b - Sqrt[b^2
- 4*a*c] + 2*c*x^2] + d*(b*c*d - c*Sqrt[b^2 - 4*a*c]*d - b^2*e + 2*a*c*e +
b*Sqrt[b^2 - 4*a*c]*e)*Log[b + Sqrt[b^2 - 4*a*c] + 2*c*x^2] - 2*a*Sqrt[b^2
- 4*a*c]*e^2*Log[d + e*x^2])/(4*a*Sqrt[b^2 - 4*a*c]*d*(c*d^2 + e*(-(b*d) +
a*e)))
```

Maple [A] (verified)

Time = 0.42 (sec) , antiderivative size = 166, normalized size of antiderivative = 0.99

method	result
default	$\frac{\ln(x)}{ad}-\frac{\frac{\left(-ebc+c^{2}d\right)\ln\left(c\,x^{4}+b\,x^{2}+a\right)}{2c}+\frac{2\left(ace-b^{2}e+bcd-\frac{\left(-ebc+c^{2}d\right)b}{2c}\right)\arctan\left(\frac{2c\,x^{2}+b}{\sqrt{4ac-b^{2}}}\right)}{\sqrt{4ac-b^{2}}}}{2(a\,e^{2}-bde+c\,d^{2})a}-\frac{e^{2}\ln\left(e\,x^{2}+d\right)}{2d(a\,e^{2}-bde+c\,d^{2})}$
risch	$\frac{\ln(x)}{ad}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\left(4a^{3}c\,e^{2}-a^{2}b^{2}e^{2}-4a^{2}bcde+4a^{2}c^{2}d^{2}+a\,b^{3}de-a\,b^{2}c\,d^{2}\right)\_Z^{2}+\left(-4abce+4a\,c^{2}d+b^{3}e-b^{2}cd\right)\_Z+c^{2}\right)}\_R\ln\left(\left(\left(-12e^{4}\right.\right.\right.\right.}{}$

[In] int(1/x/(e*x^2+d)/(c*x^4+b*x^2+a),x,method=_RETURNVERBOSE)

[Out] ln(x)/a/d-1/2/(a*e^2-b*d*e+c*d^2)/a*(1/2*(-b*c*e+c^2*d)/c*ln(c*x^4+b*x^2+a)
+2*(a*c*e-b^2*e+b*c*d-1/2*(-b*c*e+c^2*d)*b/c)/(4*a*c-b^2)^(1/2)*arctan((2*c
*x^2+b)/(4*a*c-b^2)^(1/2)))-1/2*e^2*ln(e*x^2+d)/d/(a*e^2-b*d*e+c*d^2)

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{x\left(d+ex^2\right)\left(a+bx^2+cx^4\right)}\,dx = \text{Timed out}$$

[In] integrate(1/x/(e*x^2+d)/(c*x^4+b*x^2+a),x, algorithm="fricas")

[Out] Timed out

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{x\left(d+ex^2\right)\left(a+bx^2+cx^4\right)}\,dx = \text{Timed out}$$

[In] integrate(1/x/(e*x**2+d)/(c*x**4+b*x**2+a),x)

[Out] Timed out

Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{x\left(d+ex^2\right)\left(a+bx^2+cx^4\right)}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(1/x/(e*x^2+d)/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

Giac [A] (verification not implemented)

none

Time = 0.56 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.02

$$\int \frac{1}{x\left(d+ex^2\right)\left(a+bx^2+cx^4\right)}\,dx = -\frac{e^3\log\left(|ex^2+d|\right)}{2\left(cd^3e-bd^2e^2+ade^3\right)} - \frac{(cd-be)\log\left(cx^4+bx^2+a\right)}{4\left(acd^2-abde+a^2e^2\right)} - \frac{\left(bcd-b^2e+2\,ace\right)\arctan\left(\frac{2\,cx^2+b}{\sqrt{-b^2+4\,ac}}\right)}{2\left(acd^2-abde+a^2e^2\right)\sqrt{-b^2+4\,ac}} + \frac{\log\left(x^2\right)}{2\,ad}$$

```
[In] integrate(1/x/(e*x^2+d)/(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] -1/2*e^3*log(abs(e*x^2 + d))/(c*d^3*e - b*d^2*e^2 + a*d*e^3) - 1/4*(c*d - b
*e)*log(c*x^4 + b*x^2 + a)/(a*c*d^2 - a*b*d*e + a^2*e^2) - 1/2*(b*c*d - b^2
*e + 2*a*c*e)*arctan((2*c*x^2 + b)/sqrt(-b^2 + 4*a*c))/((a*c*d^2 - a*b*d*e
+ a^2*e^2)*sqrt(-b^2 + 4*a*c)) + 1/2*log(x^2)/(a*d)
```

Mupad [B] (verification not implemented)

Time = 19.84 (sec) , antiderivative size = 6285, normalized size of antiderivative = 37.63

$$\int \frac{1}{x\left(d+ex^2\right)\left(a+bx^2+cx^4\right)}\,dx = \text{Too large to display}$$

```
[In] int(1/(x*(d + e*x^2)*(a + b*x^2 + c*x^4)),x)
```

```
[Out] (log(256*a^4*e^8*(4*a*c - b^2)^4 - 80*c^4*d^8*(4*a*c - b^2)^4 - 61*d^4*e^4*
(4*a*c - b^2)^6 + 160*b^3*c^4*d^8*(b^2 - 4*a*c)^(5/2) + 16*b^5*c^4*d^8*(b^2
 - 4*a*c)^(3/2) - 184*b^3*d^4*e^4*(b^2 - 4*a*c)^(9/2) + 370*b^5*d^4*e^4*(b^
2 - 4*a*c)^(7/2) + 128*b^7*d^4*e^4*(b^2 - 4*a*c)^(5/2) + 5*b^9*d^4*e^4*(b^2
```

- 4*a*c)^(3/2) + 128*a^3*e^8*x^2*(b^2 - 4*a*c)^(9/2) + 160*c^5*d^8*x^2*(b^
2 - 4*a*c)^(7/2) - 256*a^4*b^2*e^8*(4*a*c - b^2)^3 + 32*b^2*c^4*d^8*(4*a*c
- b^2)^3 + 112*b^4*c^4*d^8*(4*a*c - b^2)^2 - 144*a^2*d^2*e^6*(4*a*c - b^2)^
5 + 544*b^2*d^4*e^4*(4*a*c - b^2)^5 + 382*b^4*d^4*e^4*(4*a*c - b^2)^4 - 152
*b^6*d^4*e^4*(4*a*c - b^2)^3 + 71*b^8*d^4*e^4*(4*a*c - b^2)^2 + 200*c^2*d^6
e^2(4*a*c - b^2)^5 - 13*d^3*e^5*x^2*(4*a*c - b^2)^6 + 512*a^4*b*e^8*(b^2
- 4*a*c)^(7/2) - 176*b*c^4*d^8*(b^2 - 4*a*c)^(7/2) - 26*a*d^3*e^5*(b^2 - 4*
a*c)^(11/2) + 352*a^3*d*e^7*(b^2 - 4*a*c)^(9/2) - 319*b*d^4*e^4*(b^2 - 4*a*
c)^(11/2) + 148*c*d^5*e^3*(b^2 - 4*a*c)^(11/2) + 168*c^3*d^7*e*(b^2 - 4*a*c
)^(9/2) - 768*a*b^3*d^3*e^5*(4*a*c - b^2)^4 - 368*a*b^5*d^3*e^5*(4*a*c - b^
2)^3 + 128*a^3*b^3*d*e^7*(4*a*c - b^2)^3 - 32*a*b^7*d^3*e^5*(4*a*c - b^2)^2
- 672*a^2*b^3*d^2*e^6*(b^2 - 4*a*c)^(7/2) - 272*a^2*b^5*d^2*e^6*(b^2 - 4*a
*c)^(5/2) + 408*b^3*c*d^5*e^3*(4*a*c - b^2)^4 + 256*b^3*c^3*d^7*e*(4*a*c -
b^2)^3 + 792*b^5*c*d^5*e^3*(4*a*c - b^2)^3 - 352*b^5*c^3*d^7*e*(4*a*c - b^2
)^2 - 248*b^7*c*d^5*e^3*(4*a*c - b^2)^2 - 328*b^3*c^2*d^6*e^2*(b^2 - 4*a*c)
^(7/2) + 1064*b^5*c^2*d^6*e^2*(b^2 - 4*a*c)^(5/2) + 40*b^7*c^2*d^6*e^2*(b^2
- 4*a*c)^(3/2) + 384*a^3*b*e^8*x^2*(4*a*c - b^2)^4 + 384*a^3*b^2*e^8*x^2*(
b^2 - 4*a*c)^(7/2) - 512*b*c^5*d^8*x^2*(4*a*c - b^2)^3 + 576*b^2*c^5*d^8*x^
2*(b^2 - 4*a*c)^(5/2) + 32*b^4*c^5*d^8*x^2*(b^2 - 4*a*c)^(3/2) - 176*a^2*d*
e^7*x^2*(4*a*c - b^2)^5 - 800*b^3*d^3*e^5*x^2*(b^2 - 4*a*c)^(9/2) + 158*b^5
*d^3*e^5*x^2*(b^2 - 4*a*c)^(7/2) + 56*b^7*d^3*e^5*x^2*(b^2 - 4*a*c)^(5/2) -
b^9*d^3*e^5*x^2*(b^2 - 4*a*c)^(3/2) - 336*c^4*d^7*e*x^2*(4*a*c - b^2)^4 +
400*c^3*d^6*e^2*x^2*(b^2 - 4*a*c)^(9/2) - 608*a^2*b^2*d^2*e^6*(4*a*c - b^2)
^4 + 560*a^2*b^4*d^2*e^6*(4*a*c - b^2)^3 - 1096*b^2*c^2*d^6*e^2*(4*a*c - b^
2)^4 - 872*b^4*c^2*d^6*e^2*(4*a*c - b^2)^3 + 424*b^6*c^2*d^6*e^2*(4*a*c - b
^2)^2 - 128*a^3*b^3*e^8*x^2*(4*a*c - b^2)^3 + 256*b^3*c^5*d^8*x^2*(4*a*c -
b^2)^2 + 584*b^2*d^3*e^5*x^2*(4*a*c - b^2)^5 - 410*b^4*d^3*e^5*x^2*(4*a*c -
b^2)^4 - 256*b^6*d^3*e^5*x^2*(4*a*c - b^2)^3 - 17*b^8*d^3*e^5*x^2*(4*a*c -
b^2)^2 + 296*c^2*d^5*e^3*x^2*(4*a*c - b^2)^5 + 336*a*b*d^3*e^5*(4*a*c - b^
2)^5 + 384*a^3*b*d*e^7*(4*a*c - b^2)^4 - 832*a*b^2*d^3*e^5*(b^2 - 4*a*c)^(9
/2) - 52*a*b^4*d^3*e^5*(b^2 - 4*a*c)^(7/2) + 144*a*b^6*d^3*e^5*(b^2 - 4*a*c
)^(5/2) - 2*a*b^8*d^3*e^5*(b^2 - 4*a*c)^(3/2) - 80*a^2*b*d^2*e^6*(b^2 - 4*a
*c)^(9/2) - 192*a^3*b^2*d*e^7*(b^2 - 4*a*c)^(7/2) + 96*a^3*b^4*d*e^7*(b^2 -
4*a*c)^(5/2) - 632*b*c*d^5*e^3*(4*a*c - b^2)^5 + 608*b*c^3*d^7*e*(4*a*c -
b^2)^4 - 776*b*c^2*d^6*e^2*(b^2 - 4*a*c)^(9/2) + 920*b^2*c*d^5*e^3*(b^2 - 4
*a*c)^(9/2) + 584*b^2*c^3*d^7*e*(b^2 - 4*a*c)^(7/2) - 384*b^4*c*d^5*e^3*(b^
2 - 4*a*c)^(7/2) - 712*b^4*c^3*d^7*e*(b^2 - 4*a*c)^(5/2) - 664*b^6*c*d^5*e^
3*(b^2 - 4*a*c)^(5/2) - 40*b^6*c^3*d^7*e*(b^2 - 4*a*c)^(3/2) - 20*b^8*c*d^5
e^3(b^2 - 4*a*c)^(3/2) + 72*a*d^2*e^6*x^2*(b^2 - 4*a*c)^(11/2) - 181*b*d^
3*e^5*x^2*(b^2 - 4*a*c)^(11/2) + 122*c*d^4*e^4*x^2*(b^2 - 4*a*c)^(11/2) + 3
68*a^2*b*d*e^7*x^2*(b^2 - 4*a*c)^(9/2) - 1552*b*c^4*d^7*e*x^2*(b^2 - 4*a*c)
^(7/2) - 3400*b^2*c^2*d^5*e^3*x^2*(4*a*c - b^2)^4 - 4800*b^3*c^3*d^6*e^2*x^
2*(4*a*c - b^2)^3 + 3448*b^4*c^2*d^5*e^3*x^2*(4*a*c - b^2)^3 + 928*b^5*c^3*
d^6*e^2*x^2*(4*a*c - b^2)^2 - 536*b^6*c^2*d^5*e^3*x^2*(4*a*c - b^2)^2 - 32*
a*b*d^2*e^6*x^2*(4*a*c - b^2)^5 - 344*a*b^2*d^2*e^6*x^2*(b^2 - 4*a*c)^(9/2)

- 616*a*b^4*d^2*e^6*x^2*(b^2 - 4*a*c)^(7/2) - 136*a*b^6*d^2*e^6*x^2*(b^2 - 4*a*c)^(5/2) - 160*a^2*b^3*d*e^7*x^2*(b^2 - 4*a*c)^(7/2) + 48*a^2*b^5*d*e^7*x^2*(b^2 - 4*a*c)^(5/2) - 760*b*c*d^4*e^4*x^2*(4*a*c - b^2)^5 - 1560*b*c^2*d^5*e^3*x^2*(b^2 - 4*a*c)^(9/2) + 1848*b^2*c*d^4*e^4*x^2*(b^2 - 4*a*c)^(9/2) - 2208*b^3*c^4*d^7*e*x^2*(b^2 - 4*a*c)^(5/2) + 1452*b^4*c*d^4*e^4*x^2*(b^2 - 4*a*c)^(7/2) - 80*b^5*c^4*d^7*e*x^2*(b^2 - 4*a*c)^(3/2) + 408*b^6*c*d^4*e^4*x^2*(b^2 - 4*a*c)^(5/2) + 10*b^8*c*d^4*e^4*x^2*(b^2 - 4*a*c)^(3/2) - 640*a*b^3*d^2*e^6*x^2*(4*a*c - b^2)^4 + 96*a^2*b^2*d*e^7*x^2*(4*a*c - b^2)^4 + 416*a*b^5*d^2*e^6*x^2*(4*a*c - b^2)^3 + 16*a^2*b^4*d*e^7*x^2*(4*a*c - b^2)^3 + 1952*b*c^3*d^6*e^2*x^2*(4*a*c - b^2)^4 + 2216*b^3*c*d^4*e^4*x^2*(4*a*c - b^2)^4 + 2720*b^2*c^4*d^7*e*x^2*(4*a*c - b^2)^3 - 712*b^5*c*d^4*e^4*x^2*(4*a*c - b^2)^3 - 784*b^4*c^4*d^7*e*x^2*(4*a*c - b^2)^2 + 152*b^7*c*d^4*e^4*x^2*(4*a*c - b^2)^2 + 4144*b^2*c^3*d^6*e^2*x^2*(b^2 - 4*a*c)^(7/2) - 4216*b^3*c^2*d^5*e^3*x^2*(b^2 - 4*a*c)^(7/2) + 3056*b^4*c^3*d^6*e^2*x^2*(b^2 - 4*a*c)^(5/2) - 1864*b^5*c^2*d^5*e^3*x^2*(b^2 - 4*a*c)^(5/2) + 80*b^6*c^3*d^6*e^2*x^2*(b^2 - 4*a*c)^(3/2) - 40*b^7*c^2*d^5*e^3*x^2*(b^2 - 4*a*c)^(3/2))*(d*((b^2*c)/4 - a*c^2 + (b*c*(b^2 - 4*a*c)^(1/2))/4) - (b^3*e)/4 - (b^2*e*(b^2 - 4*a*c)^(1/2))/4 + (a*c*e*(b^2 - 4*a*c)^(1/2))/2 + a*b*c*e))/(4*a^3*c*e^2 - a^2*b^2*e^2 + 4*a^2*c^2*d^2 + a*b^3*d*e - a*b^2*c*d^2 - 4*a^2*b*c*d*e) - (log(80*c^4*d^8*(4*a*c - b^2)^4 - 256*a^4*e^8*(4*a*c - b^2)^4 + 61*d^4*e^4*(4*a*c - b^2)^6 + 160*b^3*c^4*d^8*(b^2 - 4*a*c)^(5/2) + 16*b^5*c^4*d^8*(b^2 - 4*a*c)^(3/2) - 184*b^3*d^4*e^4*(b^2 - 4*a*c)^(9/2) + 370*b^5*d^4*e^4*(b^2 - 4*a*c)^(7/2) + 128*b^7*d^4*e^4*(b^2 - 4*a*c)^(5/2) + 5*b^9*d^4*e^4*(b^2 - 4*a*c)^(3/2) + 128*a^3*e^8*x^2*(b^2 - 4*a*c)^(9/2) + 160*c^5*d^8*x^2*(b^2 - 4*a*c)^(7/2) + 256*a^4*b^2*e^8*(4*a*c - b^2)^3 - 32*b^2*c^4*d^8*(4*a*c - b^2)^3 - 112*b^4*c^4*d^8*(4*a*c - b^2)^2 + 144*a^2*d^2*e^6*(4*a*c - b^2)^5 - 544*b^2*d^4*e^4*(4*a*c - b^2)^5 - 382*b^4*d^4*e^4*(4*a*c - b^2)^4 + 152*b^6*d^4*e^4*(4*a*c - b^2)^3 - 71*b^8*d^4*e^4*(4*a*c - b^2)^2 - 200*c^2*d^6*e^2*(4*a*c - b^2)^5 + 13*d^3*e^5*x^2*(4*a*c - b^2)^6 + 512*a^4*b*e^8*(b^2 - 4*a*c)^(7/2) - 176*b*c^4*d^8*(b^2 - 4*a*c)^(7/2) - 26*a*d^3*e^5*(b^2 - 4*a*c)^(11/2) + 352*a^3*d*e^7*(b^2 - 4*a*c)^(9/2) - 319*b*d^4*e^4*(b^2 - 4*a*c)^(11/2) + 148*c*d^5*e^3*(b^2 - 4*a*c)^(11/2) + 168*c^3*d^7*e*(b^2 - 4*a*c)^(9/2) + 768*a*b^3*d^3*e^5*(4*a*c - b^2)^4 + 368*a*b^5*d^3*e^5*(4*a*c - b^2)^3 - 128*a^3*b^3*d*e^7*(4*a*c - b^2)^3 + 32*a*b^7*d^3*e^5*(4*a*c - b^2)^2 - 672*a^2*b^3*d^2*e^6*(b^2 - 4*a*c)^(7/2) - 272*a^2*b^5*d^2*e^6*(b^2 - 4*a*c)^(5/2) - 408*b^3*c*d^5*e^3*(4*a*c - b^2)^4 - 256*b^3*c^3*d^7*e*(4*a*c - b^2)^3 - 792*b^5*c*d^5*e^3*(4*a*c - b^2)^3 + 352*b^5*c^3*d^7*e*(4*a*c - b^2)^2 + 248*b^7*c*d^5*e^3*(4*a*c - b^2)^2 - 328*b^3*c^2*d^6*e^2*(b^2 - 4*a*c)^(7/2) + 1064*b^5*c^2*d^6*e^2*(b^2 - 4*a*c)^(5/2) + 40*b^7*c^2*d^6*e^2*(b^2 - 4*a*c)^(3/2) - 384*a^3*b*e^8*x^2*(4*a*c - b^2)^4 + 384*a^3*b^2*e^8*x^2*(b^2 - 4*a*c)^(7/2) + 512*b*c^5*d^8*x^2*(4*a*c - b^2)^3 + 576*b^2*c^5*d^8*x^2*(b^2 - 4*a*c)^(5/2) + 32*b^4*c^5*d^8*x^2*(b^2 - 4*a*c)^(3/2) + 176*a^2*d*e^7*x^2*(4*a*c - b^2)^5 - 800*b^3*d^3*e^5*x^2*(b^2 - 4*a*c)^(9/2) + 158*b^5*d^3*e^5*x^2*(b^2 - 4*a*c)^(7/2) + 56*b^7*d^3*e^5*x^2*(b^2 - 4*a*c)^(5/2) - b^9*d^3*e^5*x^2*(b^2 - 4*a*c)^(3/2) + 336*c^4*d^7*e*x^2*(4*a*c - b

^2)^4 + 400*c^3*d^6*e^2*x^2*(b^2 - 4*a*c)^(9/2) + 608*a^2*b^2*d^2*e^6*(4*a*c - b^2)^4 - 560*a^2*b^4*d^2*e^6*(4*a*c - b^2)^3 + 1096*b^2*c^2*d^6*e^2*(4*a*c - b^2)^4 + 872*b^4*c^2*d^6*e^2*(4*a*c - b^2)^3 - 424*b^6*c^2*d^6*e^2*(4*a*c - b^2)^2 + 128*a^3*b^3*e^8*x^2*(4*a*c - b^2)^3 - 256*b^3*c^5*d^8*x^2*(4*a*c - b^2)^2 - 584*b^2*d^3*e^5*x^2*(4*a*c - b^2)^5 + 410*b^4*d^3*e^5*x^2*(4*a*c - b^2)^4 + 256*b^6*d^3*e^5*x^2*(4*a*c - b^2)^3 + 17*b^8*d^3*e^5*x^2*(4*a*c - b^2)^2 - 296*c^2*d^5*e^3*x^2*(4*a*c - b^2)^5 - 336*a*b*d^3*e^5*(4*a*c - b^2)^5 - 384*a^3*b*d*e^7*(4*a*c - b^2)^4 - 832*a*b^2*d^3*e^5*(b^2 - 4*a*c)^(9/2) - 52*a*b^4*d^3*e^5*(b^2 - 4*a*c)^(7/2) + 144*a*b^6*d^3*e^5*(b^2 - 4*a*c)^(5/2) - 2*a*b^8*d^3*e^5*(b^2 - 4*a*c)^(3/2) - 80*a^2*b*d^2*e^6*(b^2 - 4*a*c)^(9/2) - 192*a^3*b^2*d*e^7*(b^2 - 4*a*c)^(7/2) + 96*a^3*b^4*d*e^7*(b^2 - 4*a*c)^(5/2) + 632*b*c*d^5*e^3*(4*a*c - b^2)^5 - 608*b*c^3*d^7*e*(4*a*c - b^2)^4 - 776*b*c^2*d^6*e^2*(b^2 - 4*a*c)^(9/2) + 920*b^2*c*d^5*e^3*(b^2 - 4*a*c)^(9/2) + 584*b^2*c^3*d^7*e*(b^2 - 4*a*c)^(7/2) - 384*b^4*c*d^5*e^3*(b^2 - 4*a*c)^(7/2) - 712*b^4*c^3*d^7*e*(b^2 - 4*a*c)^(5/2) - 664*b^6*c*d^5*e^3*(b^2 - 4*a*c)^(5/2) - 40*b^6*c^3*d^7*e*(b^2 - 4*a*c)^(3/2) - 20*b^8*c*d^5*e^3*(b^2 - 4*a*c)^(3/2) + 72*a*d^2*e^6*x^2*(b^2 - 4*a*c)^(11/2) - 181*b*d^3*e^5*x^2*(b^2 - 4*a*c)^(11/2) + 122*c*d^4*e^4*x^2*(b^2 - 4*a*c)^(11/2) + 368*a^2*b*d*e^7*x^2*(b^2 - 4*a*c)^(9/2) - 1552*b*c^4*d^7*e*x^2*(b^2 - 4*a*c)^(7/2) + 3400*b^2*c^2*d^5*e^3*x^2*(4*a*c - b^2)^4 + 4800*b^3*c^3*d^6*e^2*x^2*(4*a*c - b^2)^3 - 3448*b^4*c^2*d^5*e^3*x^2*(4*a*c - b^2)^3 - 928*b^5*c^3*d^6*e^2*x^2*(4*a*c - b^2)^2 + 536*b^6*c^2*d^5*e^3*x^2*(4*a*c - b^2)^2 + 32*a*b*d^2*e^6*x^2*(4*a*c - b^2)^5 - 344*a*b^2*d^2*e^6*x^2*(b^2 - 4*a*c)^(9/2) - 616*a*b^4*d^2*e^6*x^2*(b^2 - 4*a*c)^(7/2) - 136*a*b^6*d^2*e^6*x^2*(b^2 - 4*a*c)^(5/2) - 160*a^2*b^3*d*e^7*x^2*(b^2 - 4*a*c)^(7/2) + 48*a^2*b^5*d*e^7*x^2*(b^2 - 4*a*c)^(5/2) + 760*b*c*d^4*e^4*x^2*(4*a*c - b^2)^5 - 1560*b*c^2*d^5*e^3*x^2*(b^2 - 4*a*c)^(9/2) + 1848*b^2*c*d^4*e^4*x^2*(b^2 - 4*a*c)^(9/2) - 2208*b^3*c^4*d^7*e*x^2*(b^2 - 4*a*c)^(5/2) + 1452*b^4*c*d^4*e^4*x^2*(b^2 - 4*a*c)^(7/2) - 80*b^5*c^4*d^7*e*x^2*(b^2 - 4*a*c)^(3/2) + 408*b^6*c*d^4*e^4*x^2*(b^2 - 4*a*c)^(5/2) + 10*b^8*c*d^4*e^4*x^2*(b^2 - 4*a*c)^(3/2) + 640*a*b^3*d^2*e^6*x^2*(4*a*c - b^2)^4 - 96*a^2*b^2*d*e^7*x^2*(4*a*c - b^2)^4 - 416*a*b^5*d^2*e^6*x^2*(4*a*c - b^2)^3 - 16*a^2*b^4*d*e^7*x^2*(4*a*c - b^2)^3 - 1952*b*c^3*d^6*e^2*x^2*(4*a*c - b^2)^4 - 2216*b^3*c*d^4*e^4*x^2*(4*a*c - b^2)^4 - 2720*b^2*c^4*d^7*e*x^2*(4*a*c - b^2)^3 + 712*b^5*c*d^4*e^4*x^2*(4*a*c - b^2)^3 + 784*b^4*c^4*d^7*e*x^2*(4*a*c - b^2)^2 - 152*b^7*c*d^4*e^4*x^2*(4*a*c - b^2)^2 + 4144*b^2*c^3*d^6*e^2*x^2*(b^2 - 4*a*c)^(7/2) - 4216*b^3*c^2*d^5*e^3*x^2*(b^2 - 4*a*c)^(7/2) + 3056*b^4*c^3*d^6*e^2*x^2*(b^2 - 4*a*c)^(5/2) - 1864*b^5*c^2*d^5*e^3*x^2*(b^2 - 4*a*c)^(5/2) + 80*b^6*c^3*d^6*e^2*x^2*(b^2 - 4*a*c)^(3/2) - 40*b^7*c^2*d^5*e^3*x^2*(b^2 - 4*a*c)^(3/2))*((b^3*e)/4 + d*(a*c^2 - (b^2*c)/4 + (b*c*(b^2 - 4*a*c)^(1/2))/4) - (b^2*e*(b^2 - 4*a*c)^(1/2))/4 + (a*c*e*(b^2 - 4*a*c)^(1/2))/2 - a*b*c*e))/(4*a^3*c*e^2 - a^2*b^2*e^2 + 4*a^2*c^2*d^2 + a*b^3*d*e - a*b^2*c*d^2 - 4*a^2*b*c*d*e) - (e^2*log(d + e*x^2))/(2*c*d^3 + 2*a*d*e^2 - 2*b*d^2*e) + log(x)/(a*d)

3.301 $\int \frac{1}{x^3(d+ex^2)(a+bx^2+cx^4)}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 205

$$\int \frac{1}{x^3(d+ex^2)(a+bx^2+cx^4)}\,dx = -\frac{1}{2adx^2} - \frac{(b^2cd-2ac^2d-b^3e+3abce)\,\text{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2a^2\sqrt{b^2-4ac}\,(cd^2-bde+ae^2)} - \frac{(bd+ae)\log(x)}{a^2d^2} + \frac{e^3\log(d+ex^2)}{2d^2(cd^2-bde+ae^2)} + \frac{(bcd-b^2e+ace)\log(a+bx^2+cx^4)}{4a^2(cd^2-bde+ae^2)}$$

```
[Out] -1/2/a/d/x^2-(a*e+b*d)*ln(x)/a^2/d^2+1/2*e^3*ln(e*x^2+d)/d^2/(a*e^2-b*d*e+c
*d^2)+1/4*(a*c*e-b^2*e+b*c*d)*ln(c*x^4+b*x^2+a)/a^2/(a*e^2-b*d*e+c*d^2)-1/2
*(3*a*b*c*e-2*a*c^2*d-b^3*e+b^2*c*d)*arctanh((2*c*x^2+b)/(-4*a*c+b^2)^(1/2)
)/a^2/(a*e^2-b*d*e+c*d^2)/(-4*a*c+b^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 205, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {1265, 907, 648, 632, 212, 642}

$$\int \frac{1}{x^3(d+ex^2)(a+bx^2+cx^4)}\,dx = -\frac{\text{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)(3abce-2ac^2d+b^3(-e)+b^2cd)}{2a^2\sqrt{b^2-4ac}\,(ae^2-bde+cd^2)} + \frac{(ace+b^2(-e)+bcd)\log(a+bx^2+cx^4)}{4a^2(ae^2-bde+cd^2)} - \frac{\log(x)(ae+bd)}{a^2d^2} + \frac{e^3\log(d+ex^2)}{2d^2(ae^2-bde+cd^2)} - \frac{1}{2adx^2}$$

```
[In] Int[1/(x^3*(d + e*x^2)*(a + b*x^2 + c*x^4)),x]
```

```
[Out] -1/2*1/(a*d*x^2) - ((b^2*c*d - 2*a*c^2*d - b^3*e + 3*a*b*c*e)*ArcTanh[(b +
2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(2*a^2*Sqrt[b^2 - 4*a*c]*(c*d^2 - b*d*e + a*e^
2)) - ((b*d + a*e)*Log[x])/(a^2*d^2) + (e^3*Log[d + e*x^2])/(2*d^2*(c*d^2 -
 b*d*e + a*e^2)) + ((b*c*d - b^2*e + a*c*e)*Log[a + b*x^2 + c*x^4])/(4*a^2*
(c*d^2 - b*d*e + a*e^2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 907

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g
*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ
[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && I
ntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ[m, 0] && ILtQ[n, 0])
)
```

Rule 1265

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
```

```
gerQ[(m - 1)/2]
```

Rubi steps

$$\text{integral} = \frac{1}{2}\text{Subst}\left(\int \frac{1}{x^2(d+ex)\left(a+bx+cx^2\right)}\,dx,x,x^2\right)$$

$$= \frac{1}{2}\text{Subst}\left(\int \left(\frac{1}{adx^2}+\frac{-bd-ae}{a^2d^2x}+\frac{e^4}{d^2\left(cd^2-bde+ae^2\right)(d+ex)}+\frac{b^2cd-ac^2d-b^3e+2abce+c(bcd-b^2e+ace)\,x}{a^2\left(cd^2-bde+ae^2\right)\left(a+bx+cx^2\right)}\right)dx,x,x^2\right)$$

$$= -\frac{1}{2adx^2}-\frac{(bd+ae)\log(x)}{a^2d^2}+\frac{e^3\log\left(d+ex^2\right)}{2d^2\left(cd^2-bde+ae^2\right)}+\frac{\text{Subst}\left(\int \frac{b^2cd-ac^2d-b^3e+2abce+c(bcd-b^2e+ace)x}{a+bx+cx^2}\,dx,x,x^2\right)}{2a^2\left(cd^2-bde+ae^2\right)}$$

$$= -\frac{1}{2adx^2}-\frac{(bd+ae)\log(x)}{a^2d^2}+\frac{e^3\log\left(d+ex^2\right)}{2d^2\left(cd^2-bde+ae^2\right)}+\frac{\left(bcd-b^2e+ace\right)\text{Subst}\left(\int \frac{b+2cx}{a+bx+cx^2}\,dx,x,x^2\right)}{4a^2\left(cd^2-bde+ae^2\right)}+\frac{\left(b^2cd-2ac^2d-b^3e+3abce\right)\text{Subst}\left(\int \frac{1}{a+bx+cx^2}\,dx,x,x^2\right)}{4a^2\left(cd^2-bde+ae^2\right)}$$

$$= -\frac{1}{2adx^2}-\frac{(bd+ae)\log(x)}{a^2d^2}+\frac{e^3\log\left(d+ex^2\right)}{2d^2\left(cd^2-bde+ae^2\right)}+\frac{\left(bcd-b^2e+ace\right)\log\left(a+bx^2+cx^4\right)}{4a^2\left(cd^2-bde+ae^2\right)}-\frac{\left(b^2cd-2ac^2d-b^3e+3abce\right)\text{Subst}\left(\int \frac{1}{b^2-4ac-x^2}\,dx,x,b+2cx^2\right)}{2a^2\left(cd^2-bde+ae^2\right)}$$

$$= -\frac{1}{2adx^2}-\frac{\left(b^2cd-2ac^2d-b^3e+3abce\right)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2a^2\sqrt{b^2-4ac}\left(cd^2-bde+ae^2\right)}-\frac{(bd+ae)\log(x)}{a^2d^2}+\frac{e^3\log\left(d+ex^2\right)}{2d^2\left(cd^2-bde+ae^2\right)}+\frac{\left(bcd-b^2e+ace\right)\log\left(a+bx^2+cx^4\right)}{4a^2\left(cd^2-bde+ae^2\right)}$$

Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 331, normalized size of antiderivative = 1.61

$$\int \frac{1}{x^3 (d+ex^2)(a+bx^2+cx^4)}\,dx = \frac{1}{4}\left(-\frac{2}{adx^2}-\frac{4(bd+ae)\log(x)}{a^2d^2} + \frac{\left(b^3e-bc\left(\sqrt{b^2-4ac}d+3ae\right)+ac\left(2cd-\sqrt{b^2-4ac}e\right)+b^2\left(-cd+\sqrt{b^2-4ac}e\right)\right)\log\left(b-\sqrt{b^2-4ac}+2\right.}{a^2\sqrt{b^2-4ac}\left(-cd^2+e(bd-ae)\right)} + \frac{\left(-b^3e+bc\left(-\sqrt{b^2-4ac}d+3ae\right)+b^2\left(cd+\sqrt{b^2-4ac}e\right)-ac\left(2cd+\sqrt{b^2-4ac}e\right)\right)\log\left(b+\sqrt{b^2-4ac}+\right.}{a^2\sqrt{b^2-4ac}\left(-cd^2+e(bd-ae)\right)} + \frac{2e^3\log(d+ex^2)}{cd^4+d^2e(-bd+ae)}\right)$$

```
[In] Integrate[1/(x^3*(d + e*x^2)*(a + b*x^2 + c*x^4)),x]
```

```
[Out] (-2/(a*d*x^2) - (4*(b*d + a*e)*Log[x])/(a^2*d^2) + ((b^3*e - b*c*(Sqrt[b^2
- 4*a*c]*d + 3*a*e) + a*c*(2*c*d - Sqrt[b^2 - 4*a*c]*e) + b^2*(-(c*d) + Sqr
t[b^2 - 4*a*c]*e))*Log[b - Sqrt[b^2 - 4*a*c] + 2*c*x^2])/(a^2*Sqrt[b^2 - 4*
a*c]*(-(c*d^2) + e*(b*d - a*e))) + ((-(b^3*e) + b*c*(-(Sqrt[b^2 - 4*a*c]*d)
 + 3*a*e) + b^2*(c*d + Sqrt[b^2 - 4*a*c]*e) - a*c*(2*c*d + Sqrt[b^2 - 4*a*c
]*e))*Log[b + Sqrt[b^2 - 4*a*c] + 2*c*x^2])/(a^2*Sqrt[b^2 - 4*a*c]*(-(c*d^2
) + e*(b*d - a*e))) + (2*e^3*Log[d + e*x^2])/(c*d^4 + d^2*e*(-(b*d) + a*e))
)/4
```

Maple [A] (verified)

Time = 0.50 (sec) , antiderivative size = 215, normalized size of antiderivative = 1.05

method	result
default	$-\frac{1}{2adx^2}+\frac{(-ae-bd)\ln(x)}{a^2d^2}+\frac{\frac{(ac^2e-b^2ce+bc^2d)\ln(cx^4+bx^2+a)}{2c}+\frac{2\left(2abce-ac^2d-b^3e+b^2cd-\frac{(ac^2e-b^2ce+bc^2d)b}{2c}\right)\arctan\left(\frac{2cx^2+b}{\sqrt{4ac-b^2}}\right.}{\sqrt{4ac-b^2}}}{2(ae^2-bde+cd^2)a^2}$
risch	$-\frac{1}{2adx^2}-\frac{e\ln(x)}{ad^2}-\frac{\ln(x)b}{a^2d}+\frac{e^3\ln(-ex^2-d)}{2d^2(ae^2-bde+cd^2)}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\left(4a^4ce^2-a^3b^2e^2-4a^3bcde+4a^3c^2d^2+a^2b^3de-a^2b^2cd^2\right)\_Z^2+\right.}\right.}{}$

```
[In] int(1/x^3/(e*x^2+d)/(c*x^4+b*x^2+a),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2/a/d/x^2+1/a^2/d^2*(-a*e-b*d)*ln(x)+1/2/(a*e^2-b*d*e+c*d^2)/a^2*(1/2*(a
*c^2*e-b^2*c*e+b*c^2*d)/c*ln(c*x^4+b*x^2+a)+2*(2*a*b*c*e-a*c^2*d-b^3*e+b^2*
c*d-1/2*(a*c^2*e-b^2*c*e+b*c^2*d)*b/c)/(4*a*c-b^2)^(1/2)*arctan((2*c*x^2+b)
/(4*a*c-b^2)^(1/2)))+1/2*e^3*ln(e*x^2+d)/d^2/(a*e^2-b*d*e+c*d^2)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{x^3(d+ex^2)(a+bx^2+cx^4)}\,dx = \text{Timed out}$$

```
[In] integrate(1/x^3/(e*x^2+d)/(c*x^4+b*x^2+a),x, algorithm="fricas")
[Out] Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{x^3(d+ex^2)(a+bx^2+cx^4)}\,dx = \text{Timed out}$$

```
[In] integrate(1/x**3/(e*x**2+d)/(c*x**4+b*x**2+a),x)
[Out] Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{x^3(d+ex^2)(a+bx^2+cx^4)}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(1/x^3/(e*x^2+d)/(c*x^4+b*x^2+a),x, algorithm="maxima")
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

Giac [A] (verification not implemented)

none

Time = 0.54 (sec) , antiderivative size = 232, normalized size of antiderivative = 1.13

$$\int \frac{1}{x^3(d+ex^2)(a+bx^2+cx^4)}\,dx = \frac{e^4\log(|ex^2+d|)}{2(cd^4e-bd^3e^2+ad^2e^3)} + \frac{(bcd-b^2e+ace)\log(cx^4+bx^2+a)}{4(a^2cd^2-a^2bde+a^3e^2)} + \frac{(b^2cd-2\,ac^2d-b^3e+3\,abce)\arctan\left(\frac{2\,cx^2+b}{\sqrt{-b^2+4\,ac}}\right)}{2(a^2cd^2-a^2bde+a^3e^2)\sqrt{-b^2+4\,ac}} - \frac{(bd+ae)\log(x^2)}{2\,a^2d^2} + \frac{bdx^2+aex^2-ad}{2\,a^2d^2x^2}$$

```
[In] integrate(1/x^3/(e*x^2+d)/(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] 1/2*e^4*log(abs(e*x^2 + d))/(c*d^4*e - b*d^3*e^2 + a*d^2*e^3) + 1/4*(b*c*d
- b^2*e + a*c*e)*log(c*x^4 + b*x^2 + a)/(a^2*c*d^2 - a^2*b*d*e + a^3*e^2) +
 1/2*(b^2*c*d - 2*a*c^2*d - b^3*e + 3*a*b*c*e)*arctan((2*c*x^2 + b)/sqrt(-b
^2 + 4*a*c))/((a^2*c*d^2 - a^2*b*d*e + a^3*e^2)*sqrt(-b^2 + 4*a*c)) - 1/2*(
b*d + a*e)*log(x^2)/(a^2*d^2) + 1/2*(b*d*x^2 + a*e*x^2 - a*d)/(a^2*d^2*x^2)
```

Mupad [B] (verification not implemented)

Time = 65.89 (sec) , antiderivative size = 5368, normalized size of antiderivative = 26.19

$$\int \frac{1}{x^3 (d+ex^2) (a+bx^2+cx^4)} \, dx = \text{Too large to display}$$

```
[In] int(1/(x^3*(d + e*x^2)*(a + b*x^2 + c*x^4)),x)
```

```
[Out] (log((((((4*c^2*e^2*(a*c^6*d^7 - 4*a^2*b^5*e^7 - 4*b^2*c^5*d^7 - 4*b^7*d^2*
e^5 + 28*a^3*b^3*c*e^7 - 48*a^4*b*c^2*e^7 + 8*b^3*c^4*d^6*e + 8*b^6*c*d^3*e
^4 - 16*a^2*c^5*d^5*e^2 + 16*a^3*c^4*d^3*e^4 - 4*b^4*c^3*d^5*e^2 - 4*b^5*c^
2*d^4*e^3 - 7*a*b^6*d*e^6 - 20*a*b*c^5*d^6*e + 56*a^2*b^2*c^3*d^3*e^4 - 76*
a^2*b^3*c^2*d^2*e^5 + 32*a*b^5*c*d^2*e^5 + 46*a^2*b^4*c*d*e^6 + 20*a*b^2*c^
4*d^5*e^2 + 6*a*b^3*c^3*d^4*e^3 - 44*a*b^4*c^2*d^3*e^4 + 22*a^2*b*c^4*d^4*e
^3 + 48*a^3*b*c^3*d^2*e^5 - 75*a^3*b^2*c^2*d*e^6))/(a^2*d^2) + (((16*c^2*e^
2*(a^3*b^4*e^7 + 16*a^5*c^2*e^7 + b^3*c^4*d^7 + b^7*d^3*e^4 - 8*a^4*b^2*c*e
^7 + 2*a*b^6*d^2*e^5 + 2*a^2*b^5*d*e^6 - 4*a^2*c^5*d^6*e - 4*b^4*c^3*d^6*e
- 4*b^6*c*d^4*e^3 + 20*a^3*c^4*d^4*e^3 - 32*a^4*c^3*d^2*e^5 + 6*b^5*c^2*d^5
*e^2 - a*b*c^5*d^7 - 52*a^2*b^2*c^3*d^4*e^3 + 45*a^2*b^3*c^2*d^3*e^4 + 48*a
^3*b^2*c^2*d^2*e^5 + 11*a*b^2*c^4*d^6*e - 12*a*b^5*c*d^3*e^4 - 15*a^3*b^3*c
*d*e^6 + 28*a^4*b*c^2*d*e^6 - 27*a*b^3*c^3*d^5*e^2 + 27*a*b^4*c^2*d^4*e^3 +
 27*a^2*b*c^4*d^5*e^2 - 18*a^2*b^4*c*d^2*e^5 - 52*a^3*b*c^3*d^3*e^4))/(a*d)
 + (8*c^2*e^2*x^2*(10*a*c^6*d^7 + a^2*b^5*e^7 + b^2*c^5*d^7 + b^7*d^2*e^5 -
 11*a^3*b^3*c*e^7 + 28*a^4*b*c^2*e^7 - 88*a^4*c^3*d*e^6 - 6*b^3*c^4*d^6*e -
 6*b^6*c*d^3*e^4 + 26*a^2*c^5*d^5*e^2 + 88*a^3*c^4*d^3*e^4 + 5*b^4*c^3*d^5*
e^2 + 5*b^5*c^2*d^4*e^3 + 12*a*b^6*d*e^6 - 3*a*b*c^5*d^6*e - 110*a^2*b^2*c^
3*d^3*e^4 + 155*a^2*b^3*c^2*d^2*e^5 - 28*a*b^5*c*d^2*e^5 - 93*a^2*b^4*c*d*e
^6 - 10*a*b^2*c^4*d^5*e^2 - 27*a*b^3*c^3*d^4*e^3 + 46*a*b^4*c^2*d^3*e^4 + 1
5*a^2*b*c^4*d^4*e^3 - 236*a^3*b*c^3*d^2*e^5 + 202*a^3*b^2*c^2*d*e^6))/(a*d)
 + (4*c^2*e^2*(a*b^2*e^3 + b*c^2*d^3 - 4*a^2*c*e^3 + b^3*d*e^2 + 4*a*c^2*d^
2*e - 2*b^2*c*d^2*e - 3*a*b*c*d*e^2)*(b^4*e + b^3*e*(b^2 - 4*a*c)^(1/2) + 4
*a^2*c^2*e - b^3*c*d + 4*a*b*c^2*d - 5*a*b^2*c*e + 2*a*c^2*d*(b^2 - 4*a*c)^
(1/2) - b^2*c*d*(b^2 - 4*a*c)^(1/2) - 3*a*b*c*e*(b^2 - 4*a*c)^(1/2))*(a*b^3
*d^2*e^2 + a^2*b^2*d*e^3 + 4*a^2*c^2*d^3*e - 10*a*c^3*d^4*x^2 - 12*a^3*c*e^
4*x^2 + 3*a^2*b^2*e^4*x^2 + 3*b^2*c^2*d^4*x^2 + 3*b^4*d^2*e^2*x^2 + a*b*c^2
*d^4 - 4*a^3*c*d*e^3 - 2*a*b^2*c*d^3*e - 14*a^2*c^2*d^2*e^2*x^2 - 3*a^2*b*c
*d^2*e^2 - 4*a*b^3*d*e^3*x^2 - 6*b^3*c*d^3*e*x^2 - 8*a*b^2*c*d^2*e^2*x^2 +
```

22*a*b*c^2*d^3*e*x^2 + 16*a^2*b*c*d*e^3*x^2))/(a^2*(4*a*c - b^2)*(a*e^2 + c*d^2 - b*d*e)))*(b^4*e + b^3*e*(b^2 - 4*a*c)^(1/2) + 4*a^2*c^2*e - b^3*c*d + 4*a*b*c^2*d - 5*a*b^2*c*e + 2*a*c^2*d*(b^2 - 4*a*c)^(1/2) - b^2*c*d*(b^2 - 4*a*c)^(1/2) - 3*a*b*c*e*(b^2 - 4*a*c)^(1/2)))/(4*a^2*(4*a*c - b^2)*(a*e^2 + c*d^2 - b*d*e)) - (4*c^2*e^2*x^2*(6*a*b^6*e^7 + 6*b*c^6*d^7 + 6*b^7*d*e^6 - 16*a^4*c^3*e^7 - 44*a^2*b^4*c*e^7 - 8*b^2*c^5*d^6*e - 8*b^6*c*d^2*e^5 + 84*a^3*b^2*c^2*e^7 + 30*a^2*c^5*d^4*e^3 - 2*b^3*c^4*d^5*e^2 + 8*b^4*c^3*d^4*e^3 - 2*b^5*c^2*d^3*e^4 + 11*a*c^6*d^6*e - 47*a*b^5*c*d*e^6 - 96*a^2*b^2*c^3*d^2*e^5 + 14*a*b*c^5*d^5*e^2 - 94*a^3*b*c^3*d*e^6 - 35*a*b^2*c^4*d^4*e^3 + 7*a*b^3*c^3*d^3*e^4 + 56*a*b^4*c^2*d^2*e^5 - 17*a^2*b*c^4*d^3*e^4 + 117*a^2*b^3*c^2*d*e^6))/(a^2*d^2))*(b^4*e + b^3*e*(b^2 - 4*a*c)^(1/2) + 4*a^2*c^2*e - b^3*c*d + 4*a*b*c^2*d - 5*a*b^2*c*e + 2*a*c^2*d*(b^2 - 4*a*c)^(1/2) - b^2*c*d*(b^2 - 4*a*c)^(1/2) - 3*a*b*c*e*(b^2 - 4*a*c)^(1/2)))/(4*a^2*(4*a*c - b^2)*(a*e^2 + c*d^2 - b*d*e)) + (4*c^2*e^2*x^2*(b^7*e^7 + c^7*d^7 - 6*a^3*b*c^3*e^7 + 2*a*c^6*d^5*e^2 - 4*a^3*c^4*d*e^6 + 14*a^2*b^3*c^2*e^7 + 6*a^2*c^5*d^3*e^4 + b^3*c^4*d^4*e^3 + b^4*c^3*d^3*e^4 - 7*a*b^5*c*e^7 + 2*a*b^4*c^2*d*e^6 - 6*a*b^2*c^4*d^3*e^4 + 3*a*b^3*c^3*d^2*e^5 - 9*a^2*b*c^4*d^2*e^5 - 5*a^2*b^2*c^3*d*e^6))/(a^3*d^3) + (4*c^2*e^2*(a*e + b*d)*(b^3*e^3 + c^3*d^3 - 3*a*b*c*e^3)^2)/(a^3*d^3))*(b^4*e + b^3*e*(b^2 - 4*a*c)^(1/2) + 4*a^2*c^2*e - b^3*c*d + 4*a*b*c^2*d - 5*a*b^2*c*e + 2*a*c^2*d*(b^2 - 4*a*c)^(1/2) - b^2*c*d*(b^2 - 4*a*c)^(1/2) - 3*a*b*c*e*(b^2 - 4*a*c)^(1/2)))/(4*a^2*(4*a*c - b^2)*(a*e^2 + c*d^2 - b*d*e)) - (2*c^5*e^5*x^2*(b^3*e^3 + c^3*d^3 - 3*a*b*c*e^3))/(a^3*d^3))*(b^4*e + b^3*e*(b^2 - 4*a*c)^(1/2) + 4*a^2*c^2*e - b^3*c*d + 4*a*b*c^2*d - 5*a*b^2*c*e + 2*a*c^2*d*(b^2 - 4*a*c)^(1/2) - b^2*c*d*(b^2 - 4*a*c)^(1/2) - 3*a*b*c*e*(b^2 - 4*a*c)^(1/2)))/(4*(4*a^4*c*e^2 - a^3*b^2*e^2 + 4*a^3*c^2*d^2 - a^2*b^2*c*d^2 + a^2*b^3*d*e - 4*a^3*b*c*d*e)) + (log((((((4*c^2*e^2*(a*c^6*d^7 - 4*a^2*b^5*e^7 - 4*b^2*c^5*d^7 - 4*b^7*d^2*e^5 + 28*a^3*b^3*c*e^7 - 48*a^4*b*c^2*e^7 + 8*b^3*c^4*d^6*e + 8*b^6*c*d^3*e^4 - 16*a^2*c^5*d^5*e^2 + 16*a^3*c^4*d^3*e^4 - 4*b^4*c^3*d^5*e^2 - 4*b^5*c^2*d^4*e^3 - 7*a*b^6*d*e^6 - 20*a*b*c^5*d^6*e + 56*a^2*b^2*c^3*d^3*e^4 - 76*a^2*b^3*c^2*d^2*e^5 + 32*a*b^5*c*d^2*e^5 + 46*a^2*b^4*c*d*e^6 + 20*a*b^2*c^4*d^5*e^2 + 6*a*b^3*c^3*d^4*e^3 - 44*a*b^4*c^2*d^3*e^4 + 22*a^2*b*c^4*d^4*e^3 + 48*a^3*b*c^3*d^2*e^5 - 75*a^3*b^2*c^2*d*e^6))/(a^2*d^2) + (((16*c^2*e^2*(a^3*b^4*e^7 + 16*a^5*c^2*e^7 + b^3*c^4*d^7 + b^7*d^3*e^4 - 8*a^4*b^2*c*e^7 + 2*a*b^6*d^2*e^5 + 2*a^2*b^5*d*e^6 - 4*a^2*c^5*d^6*e - 4*b^4*c^3*d^6*e - 4*b^6*c*d^4*e^3 + 20*a^3*c^4*d^4*e^3 - 32*a^4*c^3*d^2*e^5 + 6*b^5*c^2*d^5*e^2 - a*b*c^5*d^7 - 52*a^2*b^2*c^3*d^4*e^3 + 45*a^2*b^3*c^2*d^3*e^4 + 48*a^3*b^2*c^2*d^2*e^5 + 11*a*b^2*c^4*d^6*e - 12*a*b^5*c*d^3*e^4 - 15*a^3*b^3*c*d*e^6 + 28*a^4*b*c^2*d*e^6 - 27*a*b^3*c^3*d^5*e^2 + 27*a*b^4*c^2*d^4*e^3 + 27*a^2*b*c^4*d^5*e^2 - 18*a^2*b^4*c*d^2*e^5 - 52*a^3*b*c^3*d^3*e^4))/(a*d) + (8*c^2*e^2*x^2*(10*a*c^6*d^7 + a^2*b^5*e^7 + b^2*c^5*d^7 + b^7*d^2*e^5 - 11*a^3*b^3*c*e^7 + 28*a^4*b*c^2*e^7 - 88*a^4*c^3*d*e^6 - 6*b^3*c^4*d^6*e - 6*b^6*c*d^3*e^4 + 26*a^2*c^5*d^5*e^2 + 88*a^3*c^4*d^3*e^4 + 5*b^4*c^3*d^5*e^2 + 5*b^5*c^2*d^4*e^3 + 12*a*b^6*d*e^6 - 3*a*b*c^5*d^6*e - 110*a^2*b^2*c^3*d^3*e^4 + 155*a^2*b^3*c^2*d^2*e^5 - 28*a*b^5*c*d^2*e^5 - 93*a^2* b^4*c*d*e^6 - 10*a*b^2*c^4*d^5*e^2 - 27*a*b^3*c^3*d^4*e^3 + 46*a*b^4*c^2*d^3*e^4 + 15*a^2*b*c^4*d^4*e^3 - 236*a^3*b*c^3*d^2*e^5 + 202*a^3*b^2*c^2*d*e^6))/(a*d) + (4*c^2*e^2*(a*b^2*e^3 + b*c^2*d^3 - 4*a^2*c*e^3 + b^3*d*e^2 + 4*a*c^2*d^2*e - 2*b^2*c*d^2*e - 3*a*b*c*d*e^2)*(b^4*e - b^3*e*(b^2 - 4*a*c)^(1/2) + 4*a^2*c^2*e - b^3*c*d + 4*a*b*c^2*d - 5*a*b^2*c*e - 2*a*c^2*d*(b^2 - 4*a*c)^(1/2) + b^2*c*d*(b^2 - 4*a*c)^(1/2) + 3*a*b*c*e*(b^2 - 4*a*c)^(1/2))*(a*b^3*d^2*e^2 + a^2*b^2*d*e^3 + 4*a^2*c^2*d^3*e - 10*a*c^3*d^4*x^2 - 12*a^3*c*e^4*x^2 + 3*a^2*b^2*e^4*x^2 + 3*b^2*c^2*d^4*x^2 + 3*b^4*d^2*e^2*x^2 + a*b*c^2*d^4 - 4*a^3*c*d*e^3 - 2*a*b^2*c*d^3*e - 14*a^2*c^2*d^2*e^2*x^2 - 3*a^2*b*c*d^2*e^2 - 4*a*b^3*d*e^3*x^2 - 6*b^3*c*d^3*e*x^2 - 8*a*b^2*c*d^2*e^2*x^2 + 22*a*b*c^2*d^3*e*x^2 + 16*a^2*b*c*d*e^3*x^2))/(a^2*(4*a*c - b^2)*(a*e^2 + c*d^2 - b*d*e)))*(b^4*e - b^3*e*(b^2 - 4*a*c)^(1/2) + 4*a^2*c^2*e - b^3*c*d + 4*a*b*c^2*d - 5*a*b^2*c*e - 2*a*c^2*d*(b^2 - 4*a*c)^(1/2) + b^2*c*d*(b^2 - 4*a*c)^(1/2) + 3*a*b*c*e*(b^2 - 4*a*c)^(1/2)))/(4*a^2*(4*a*c - b^2)*(a*e^2 + c*d^2 - b*d*e)) - (4*c^2*e^2*x^2*(6*a*b^6*e^7 + 6*b*c^6*d^7 + 6*b^7*d*e^6 - 16*a^4*c^3*e^7 - 44*a^2*b^4*c*e^7 - 8*b^2*c^5*d^6*e - 8*b^6*c*d^2*e^5 + 84*a^3*b^2*c^2*e^7 + 30*a^2*c^5*d^4*e^3 - 2*b^3*c^4*d^5*e^2 + 8*b^4*c^3*d^4*e^3 - 2*b^5*c^2*d^3*e^4 + 11*a*c^6*d^6*e - 47*a*b^5*c*d*e^6 - 96*a^2*b^2*c^3*d^2*e^5 + 14*a*b*c^5*d^5*e^2 - 94*a^3*b*c^3*d*e^6 - 35*a*b^2*c^4*d^4*e^3 + 7*a*b^3*c^3*d^3*e^4 + 56*a*b^4*c^2*d^2*e^5 - 17*a^2*b*c^4*d^3*e^4 + 117*a^2*b^3*c^2*d*e^6))/(a^2*d^2))*(b^4*e - b^3*e*(b^2 - 4*a*c)^(1/2) + 4*a^2*c^2*e - b^3*c*d + 4*a*b*c^2*d - 5*a*b^2*c*e - 2*a*c^2*d*(b^2 - 4*a*c)^(1/2) + b^2*c*d*(b^2 - 4*a*c)^(1/2) + 3*a*b*c*e*(b^2 - 4*a*c)^(1/2)))/(4*a^2*(4*a*c - b^2)*(a*e^2 + c*d^2 - b*d*e)) + (4*c^2*e^2*x^2*(b^7*e^7 + c^7*d^7 - 6*a^3*b*c^3*e^7 + 2*a*c^6*d^5*e^2 - 4*a^3*c^4*d*e^6 + 14*a^2*b^3*c^2*e^7 + 6*a^2*c^5*d^3*e^4 + b^3*c^4*d^4*e^3 + b^4*c^3*d^3*e^4 - 7*a*b^5*c*e^7 + 2*a*b^4*c^2*d*e^6 - 6*a*b^2*c^4*d^3*e^4 + 3*a*b^3*c^3*d^2*e^5 - 9*a^2*b*c^4*d^2*e^5 - 5*a^2*b^2*c^3*d*e^6))/(a^3*d^3) + (4*c^2*e^2*(a*e + b*d)*(b^3*e^3 + c^3*d^3 - 3*a*b*c*e^3)^2)/(a^3*d^3))*(b^4*e - b^3*e*(b^2 - 4*a*c)^(1/2) + 4*a^2*c^2*e - b^3*c*d + 4*a*b*c^2*d - 5*a*b^2*c*e - 2*a*c^2*d*(b^2 - 4*a*c)^(1/2) + b^2*c*d*(b^2 - 4*a*c)^(1/2) + 3*a*b*c*e*(b^2 - 4*a*c)^(1/2)))/(4*a^2*(4*a*c - b^2)*(a*e^2 + c*d^2 - b*d*e)) - (2*c^5*e^5*x^2*(b^3*e^3 + c^3*d^3 - 3*a*b*c*e^3))/(a^3*d^3))*(b^4*e - b^3*e*(b^2 - 4*a*c)^(1/2) + 4*a^2*c^2*e - b^3*c*d + 4*a*b*c^2*d - 5*a*b^2*c*e - 2*a*c^2*d*(b^2 - 4*a*c)^(1/2) + b^2*c*d*(b^2 - 4*a*c)^(1/2) + 3*a*b*c*e*(b^2 - 4*a*c)^(1/2)))/(4*(4*a^4*c*e^2 - a^3*b^2*e^2 + 4*a^3*c^2*d^2 - a^2*b^2*c*d^2 + a^2*b^3*d*e - 4*a^3*b*c*d*e)) + (e^3*log(d + e*x^2))/(2*c*d^4 + 2*a*d^2*e^2 - 2*b*d^3*e) - 1/(2*a*d*x^2) - (log(x)*(a*e + b*d))/(a^2*d^2)

3.302 $\int \frac{1}{x^5(d+ex^2)(a+bx^2+cx^4)}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 268

$$\int \frac{1}{x^5(d+ex^2)(a+bx^2+cx^4)}\,dx = -\frac{1}{4adx^4}+\frac{bd+ae}{2a^2d^2x^2}+\frac{(b^3cd-3abc^2d-b^4e+4ab^2ce-2a^2c^2e)\,\text{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2a^3\sqrt{b^2-4ac}\,(cd^2-bde+ae^2)} + \frac{(b^2d^2+abde-a(cd^2-ae^2))\log(x)}{a^3d^3}-\frac{e^4\log(d+ex^2)}{2d^3(cd^2-bde+ae^2)} - \frac{(b^2cd-ac^2d-b^3e+2abce)\log(a+bx^2+cx^4)}{4a^3(cd^2-bde+ae^2)}$$

```
[Out] -1/4/a/d/x^4+1/2*(a*e+b*d)/a^2/d^2/x^2+(b^2*d^2+a*b*d*e-a*(-a*e^2+c*d^2))*l
n(x)/a^3/d^3-1/2*e^4*ln(e*x^2+d)/d^3/(a*e^2-b*d*e+c*d^2)-1/4*(2*a*b*c*e-a*c
^2*d-b^3*e+b^2*c*d)*ln(c*x^4+b*x^2+a)/a^3/(a*e^2-b*d*e+c*d^2)+1/2*(-2*a^2*c
^2*e+4*a*b^2*c*e-3*a*b*c^2*d-b^4*e+b^3*c*d)*arctanh((2*c*x^2+b)/(-4*a*c+b^2
)^(1/2))/a^3/(a*e^2-b*d*e+c*d^2)/(-4*a*c+b^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 268, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used

$= \{1265, 907, 648, 632, 212, 642\}$

$$\int \frac{1}{x^5 (d+ex^2)(a+bx^2+cx^4)}\,dx$$
$$= \frac{\log(x)\left(abde - a(cd^2-ae^2)+b^2d^2\right)}{a^3d^3} - \frac{\left(2abce - ac^2d + b^3(-e) + b^2cd\right)\log\left(a+bx^2+cx^4\right)}{4a^3\left(ae^2-bde+cd^2\right)} + \frac{ae+bd}{2a^2d^2x^2} + \frac{\operatorname{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)\left(-2a^2c^2e + 4ab^2ce - 3abc^2d + b^4(-e) + b^3cd\right)}{2a^3\sqrt{b^2-4ac}\left(ae^2-bde+cd^2\right)} - \frac{e^4\log\left(d+ex^2\right)}{2d^3\left(ae^2-bde+cd^2\right)} - \frac{1}{4adx^4}$$

```
[In] Int[1/(x^5*(d + e*x^2)*(a + b*x^2 + c*x^4)),x]
```

```
[Out] -1/4*1/(a*d*x^4) + (b*d + a*e)/(2*a^2*d^2*x^2) + ((b^3*c*d - 3*a*b*c^2*d -
b^4*e + 4*a*b^2*c*e - 2*a^2*c^2*e)*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]]
)/(2*a^3*Sqrt[b^2 - 4*a*c]*(c*d^2 - b*d*e + a*e^2)) + ((b^2*d^2 + a*b*d*e -
 a*(c*d^2 - a*e^2))*Log[x])/(a^3*d^3) - (e^4*Log[d + e*x^2])/(2*d^3*(c*d^2
- b*d*e + a*e^2)) - ((b^2*c*d - a*c^2*d - b^3*e + 2*a*b*c*e)*Log[a + b*x^2
+ c*x^4])/(4*a^3*(c*d^2 - b*d*e + a*e^2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
```

```
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 907

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g
*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ
[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && I
ntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ[m, 0] && ILtQ[n, 0])
)
```

Rule 1265

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{2}\text{Subst}\left(\int \frac{1}{x^3(d+ex)\,(a+bx+cx^2)}\,dx, x, x^2\right)\\
&= \frac{1}{2}\text{Subst}\left(\int \left(\frac{1}{adx^3}+\frac{-bd-ae}{a^2d^2x^2}+\frac{b^2d^2+abde-a(cd^2-ae^2)}{a^3d^3x}\right.\right.\\
&\quad -\frac{e^5}{d^3\,(cd^2-bde+ae^2)\,(d+ex)}\\
&\quad \left.\left.+\frac{-b^3cd+2abc^2d+b^4e-3ab^2ce+a^2c^2e-c(b^2cd-ac^2d-b^3e+2abce)\,x}{a^3\,(cd^2-bde+ae^2)\,(a+bx+cx^2)}\right)\,dx, x, x^2\right)\\
&= -\frac{1}{4adx^4}+\frac{bd+ae}{2a^2d^2x^2}+\frac{(b^2d^2+abde-a(cd^2-ae^2))\log(x)}{a^3d^3}-\frac{e^4\log(d+ex^2)}{2d^3\,(cd^2-bde+ae^2)}\\
&\quad +\frac{\text{Subst}\left(\int \frac{-b^3cd+2abc^2d+b^4e-3ab^2ce+a^2c^2e-c(b^2cd-ac^2d-b^3e+2abce)x}{a+bx+cx^2}\,dx, x, x^2\right)}{2a^3\,(cd^2-bde+ae^2)}\\
&= -\frac{1}{4adx^4}+\frac{bd+ae}{2a^2d^2x^2}+\frac{(b^2d^2+abde-a(cd^2-ae^2))\log(x)}{a^3d^3}-\frac{e^4\log(d+ex^2)}{2d^3\,(cd^2-bde+ae^2)}\\
&\quad -\frac{(b^2cd-ac^2d-b^3e+2abce)\,\text{Subst}\left(\int \frac{b+2cx}{a+bx+cx^2}\,dx, x, x^2\right)}{4a^3\,(cd^2-bde+ae^2)}\\
&\quad -\frac{(b^3cd-3abc^2d-b^4e+4ab^2ce-2a^2c^2e)\,\text{Subst}\left(\int \frac{1}{a+bx+cx^2}\,dx, x, x^2\right)}{4a^3\,(cd^2-bde+ae^2)}
\end{aligned}$$

$$= -\frac{1}{4adx^4} + \frac{bd+ae}{2a^2d^2x^2} + \frac{(b^2d^2+abde-a(cd^2-ae^2))\log(x)}{a^3d^3} - \frac{e^4\log(d+ex^2)}{2d^3(cd^2-bde+ae^2)} - \frac{(b^2cd-ac^2d-b^3e+2abce)\log(a+bx^2+cx^4)}{4a^3(cd^2-bde+ae^2)} + \frac{(b^3cd-3abc^2d-b^4e+4ab^2ce-2a^2c^2e)\operatorname{Subst}\left(\int \frac{1}{b^2-4ac-x^2}\,dx, x, b+2cx^2\right)}{2a^3(cd^2-bde+ae^2)}$$

$$= -\frac{1}{4adx^4} + \frac{bd+ae}{2a^2d^2x^2} + \frac{(b^3cd-3abc^2d-b^4e+4ab^2ce-2a^2c^2e)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2a^3\sqrt{b^2-4ac}\,(cd^2-bde+ae^2)} + \frac{(b^2d^2+abde-a(cd^2-ae^2))\log(x)}{a^3d^3} - \frac{e^4\log(d+ex^2)}{2d^3(cd^2-bde+ae^2)} - \frac{(b^2cd-ac^2d-b^3e+2abce)\log(a+bx^2+cx^4)}{4a^3(cd^2-bde+ae^2)}$$

Mathematica [A] (verified)

Time = 0.28 (sec) , antiderivative size = 426, normalized size of antiderivative = 1.59

$$\int \frac{1}{x^5(d+ex^2)(a+bx^2+cx^4)}\,dx$$

$$= \frac{1}{4}\left(-\frac{1}{adx^4} + \frac{2(bd+ae)}{a^2d^2x^2} + \frac{4(b^2d^2+abde+a(-cd^2+ae^2))\log(x)}{a^3d^3} - \frac{\left(b^4e+ac^2\left(\sqrt{b^2-4ac}d+2ae\right)-b^2c\left(\sqrt{b^2-4ac}d+4ae\right)+abc\left(3cd-2\sqrt{b^2-4ac}e\right)+b^3\left(-cd+\sqrt{b^2-\right.\right.}{a^3\sqrt{b^2-4ac}\,(-cd^2+e(bd-ae))} - \frac{\left(-b^4e+ac^2\left(\sqrt{b^2-4ac}d-2ae\right)+b^2c\left(-\sqrt{b^2-4ac}d+4ae\right)+b^3\left(cd+\sqrt{b^2-4ac}e\right)-abc\left(3cd+2\sqrt{b^2\right.\right.}{a^3\sqrt{b^2-4ac}\,(-cd^2+e(bd-ae))} - \frac{2e^4\log(d+ex^2)}{cd^5+d^3e(-bd+ae)}\right)$$

```
[In] Integrate[1/(x^5*(d + e*x^2)*(a + b*x^2 + c*x^4)),x]
```

```
[Out] (-(1/(a*d*x^4)) + (2*(b*d + a*e))/(a^2*d^2*x^2) + (4*(b^2*d^2 + a*b*d*e + a
*(-(c*d^2) + a*e^2))*Log[x])/(a^3*d^3) - ((b^4*e + a*c^2*(Sqrt[b^2 - 4*a*c]
*d + 2*a*e) - b^2*c*(Sqrt[b^2 - 4*a*c]*d + 4*a*e) + a*b*c*(3*c*d - 2*Sqrt[b
^2 - 4*a*c]*e) + b^3*(-(c*d) + Sqrt[b^2 - 4*a*c]*e))*Log[b - Sqrt[b^2 - 4*a
*c] + 2*c*x^2])/(a^3*Sqrt[b^2 - 4*a*c]*(-(c*d^2) + e*(b*d - a*e))) - ((-(b^
4*e) + a*c^2*(Sqrt[b^2 - 4*a*c]*d - 2*a*e) + b^2*c*(-(Sqrt[b^2 - 4*a*c]*d)
+ 4*a*e) + b^3*(c*d + Sqrt[b^2 - 4*a*c]*e) - a*b*c*(3*c*d + 2*Sqrt[b^2 - 4*
a*c]*e))*Log[b + Sqrt[b^2 - 4*a*c] + 2*c*x^2])/(a^3*Sqrt[b^2 - 4*a*c]*(-(c*
d^2) + e*(b*d - a*e))) - (2*e^4*Log[d + e*x^2])/(c*d^5 + d^3*e*(-(b*d) + a*
e)))/4
```

Maple [A] (verified)

Time = 0.54 (sec) , antiderivative size = 284, normalized size of antiderivative = 1.06

method	result
default	$-\frac{1}{4ad\,x^{4}}-\frac{-ae-bd}{2a^{2}d^{2}x^{2}}+\frac{\left(e^{2}a^{2}+abde-d^{2}ac+b^{2}d^{2}\right)\ln\left(x\right)}{d^{3}a^{3}}+\frac{\frac{\left(-2ab\,c^{2}e+a\,c^{3}d+b^{3}ce-b^{2}c^{2}d\right)\ln\left(c\,x^{4}+b\,x^{2}+a\right)}{2c}+\frac{2\left(a^{2}c^{2}e-3a\,b^{2}ce+2ab\right.}{}}{2(a\,e^{2}-bde-}$
risch	Expression too large to display

[In] int(1/x^5/(e*x^2+d)/(c*x^4+b*x^2+a),x,method=_RETURNVERBOSE)

[Out] -1/4/a/d/x^4-1/2*(-a*e-b*d)/a^2/d^2/x^2+(a^2*e^2+a*b*d*e-a*c*d^2+b^2*d^2)/d^3/a^3*ln(x)+1/2/(a*e^2-b*d*e+c*d^2)/a^3*(1/2*(-2*a*b*c^2*e+a*c^3*d+b^3*c*e-b^2*c^2*d)/c*ln(c*x^4+b*x^2+a)+2*(a^2*c^2*e-3*a*b^2*c*e+2*a*b*c^2*d+b^4*e-b^3*c*d-1/2*(-2*a*b*c^2*e+a*c^3*d+b^3*c*e-b^2*c^2*d)*b/c)/(4*a*c-b^2)^(1/2)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2)))-1/2*e^4*ln(e*x^2+d)/d^3/(a*e^2-b*d*e+c*d^2)

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{x^5\left(d+ex^2\right)\left(a+bx^2+cx^4\right)}\,dx=\text{Timed out}$$

[In] integrate(1/x^5/(e*x^2+d)/(c*x^4+b*x^2+a),x, algorithm="fricas")

[Out] Timed out

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{x^5\left(d+ex^2\right)\left(a+bx^2+cx^4\right)}\,dx=\text{Timed out}$$

[In] integrate(1/x**5/(e*x**2+d)/(c*x**4+b*x**2+a),x)

[Out] Timed out

Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{x^5 (d+ex^2) (a+bx^2+cx^4)} \, dx = \text{Exception raised: ValueError}$$

```
[In] integrate(1/x^5/(e*x^2+d)/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

Giac [A] (verification not implemented)

none

Time = 0.60 (sec) , antiderivative size = 327, normalized size of antiderivative = 1.22

$$\begin{aligned}
\int \frac{1}{x^5 (d+ex^2) (a+bx^2+cx^4)} \, dx &= -\frac{e^5 \log (|ex^2+d|)}{2 (cd^5 e - bd^4 e^2 + ad^3 e^3)} - \frac{(b^2 cd - ac^2 d - b^3 e + 2 abce) \log (cx^4+bx^2+a)}{4 (a^3 cd^2 - a^3 bde + a^4 e^2)} \\
&\quad - \frac{(b^3 cd - 3 abc^2 d - b^4 e + 4 ab^2 ce - 2 a^2 c^2 e) \arctan \left(\frac{2 cx^2+b}{\sqrt{-b^2+4 ac}} \right)}{2 (a^3 cd^2 - a^3 bde + a^4 e^2) \sqrt{-b^2+4 ac}} \\
&\quad + \frac{(b^2 d^2 - acd^2 + abde + a^2 e^2) \log (x^2)}{2 a^3 d^3} \\
&\quad - \frac{3 b^2 d^2 x^4 - 3 acd^2 x^4 + 3 abdex^4 + 3 a^2 e^2 x^4 - 2 abd^2 x^2 - 2 a^2 dex^2 + a^2 d^2}{4 a^3 d^3 x^4}
\end{aligned}$$

```
[In] integrate(1/x^5/(e*x^2+d)/(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] -1/2*e^5*log(abs(e*x^2 + d))/(c*d^5*e - b*d^4*e^2 + a*d^3*e^3) - 1/4*(b^2*c
*d - a*c^2*d - b^3*e + 2*a*b*c*e)*log(c*x^4 + b*x^2 + a)/(a^3*c*d^2 - a^3*b
*d*e + a^4*e^2) - 1/2*(b^3*c*d - 3*a*b*c^2*d - b^4*e + 4*a*b^2*c*e - 2*a^2*
c^2*e)*arctan((2*c*x^2 + b)/sqrt(-b^2 + 4*a*c))/((a^3*c*d^2 - a^3*b*d*e + a
^4*e^2)*sqrt(-b^2 + 4*a*c)) + 1/2*(b^2*d^2 - a*c*d^2 + a*b*d*e + a^2*e^2)*l
og(x^2)/(a^3*d^3) - 1/4*(3*b^2*d^2*x^4 - 3*a*c*d^2*x^4 + 3*a*b*d*e*x^4 + 3*
a^2*e^2*x^4 - 2*a*b*d^2*x^2 - 2*a^2*d*e*x^2 + a^2*d^2)/(a^3*d^3*x^4)
```

Mupad [B] (verification not implemented)

Time = 142.00 (sec) , antiderivative size = 10300, normalized size of antiderivative = 38.43

$$\int \frac{1}{x^5 (d+ex^2) (a+bx^2+cx^4)} dx = \text{Too large to display}$$

```
[In] int(1/(x^5*(d + e*x^2)*(a + b*x^2 + c*x^4)),x)
```

```
[Out] (log((c^8*e^8*(a^2*e^2 + b^2*d^2 - a*c*d^2 + a*b*d*e))/(a^6*d^6) - (c^9*e^9
*x^2)/(a^5*d^5) - (((c^5*e^5*(4*a^3*b^3*e^6 + 4*b^3*c^3*d^6 + 4*b^6*d^3*e^3
 + 8*a*b^5*d^2*e^4 + 8*a^2*b^4*d*e^5 + 4*a^2*c^4*d^5*e + 16*a^4*c^2*d*e^5 -
 19*a^3*c^3*d^3*e^3 - 4*a*b*c^4*d^6 - 12*a^4*b*c*e^6 + 36*a^2*b^2*c^2*d^3*e
^3 - 24*a*b^4*c*d^3*e^3 - 32*a^3*b^2*c*d*e^5 - 36*a^2*b^3*c*d^2*e^4 + 28*a^
3*b*c^2*d^2*e^4))/(a^6*d^6) - (((4*a^4*b^6*c^2*e^12 - 24*a^5*b^4*c^3*e^12 +
 36*a^6*b^2*c^4*e^12 - 4*a^3*c^9*d^8*e^4 + 64*a^4*c^8*d^6*e^6 - 144*a^5*c^7
*d^4*e^8 + 96*a^6*c^6*d^2*e^10 + 4*b^4*c^8*d^10*e^2 + 8*b^7*c^5*d^7*e^5 + 4
*b^10*c^2*d^4*e^8 + 64*a^2*b^3*c^7*d^7*e^5 - 8*a^2*b^4*c^6*d^6*e^6 - 8*a^2*
b^5*c^5*d^5*e^7 + 172*a^2*b^6*c^4*d^4*e^8 - 112*a^2*b^7*c^3*d^3*e^9 + 16*a^
2*b^8*c^2*d^2*e^10 - 72*a^3*b^2*c^7*d^6*e^6 + 56*a^3*b^3*c^6*d^5*e^7 - 312*
a^3*b^4*c^5*d^4*e^8 + 348*a^3*b^5*c^4*d^3*e^9 - 132*a^3*b^6*c^3*d^2*e^10 +
324*a^4*b^2*c^6*d^4*e^8 - 428*a^4*b^3*c^5*d^3*e^9 + 344*a^4*b^4*c^4*d^2*e^1
0 - 300*a^5*b^2*c^5*d^2*e^10 - 96*a^6*b*c^5*d*e^11 - 4*a*b^2*c^9*d^10*e^2 -
 4*a*b^3*c^8*d^9*e^3 - 48*a*b^5*c^6*d^7*e^5 + 8*a*b^6*c^5*d^6*e^6 - 44*a*b^
8*c^3*d^4*e^8 + 12*a*b^9*c^2*d^3*e^9 + 8*a^2*b*c^9*d^9*e^3 - 24*a^3*b*c^8*d
^7*e^5 + 12*a^3*b^7*c^2*d*e^11 - 88*a^4*b*c^7*d^5*e^7 - 88*a^4*b^5*c^3*d*e^
11 + 228*a^5*b*c^6*d^3*e^9 + 188*a^5*b^3*c^4*d*e^11)/(a^6*d^6) + (x^2*(32*a
^6*c^6*d*e^11 - 24*a^6*b*c^5*e^12 + 4*a^3*b^7*c^2*e^12 - 28*a^4*b^5*c^3*e^1
2 + 56*a^5*b^3*c^4*e^12 + 2*a^3*c^9*d^7*e^5 + 104*a^4*c^8*d^5*e^7 - 156*a^5
*c^7*d^3*e^9 + 4*b^3*c^9*d^10*e^2 + 4*b^6*c^6*d^7*e^5 + 4*b^7*c^5*d^6*e^6 +
 4*b^10*c^2*d^3*e^9 + 8*a^2*b^2*c^8*d^7*e^5 + 40*a^2*b^3*c^7*d^6*e^6 - 12*a
^2*b^5*c^5*d^4*e^8 + 180*a^2*b^6*c^4*d^3*e^9 - 116*a^2*b^7*c^3*d^2*e^10 - 9
2*a^3*b^2*c^7*d^5*e^7 + 84*a^3*b^3*c^6*d^4*e^8 - 350*a^3*b^4*c^5*d^3*e^9 +
388*a^3*b^5*c^4*d^2*e^10 + 348*a^4*b^2*c^6*d^3*e^9 - 524*a^4*b^3*c^5*d^2*e^
10 - 4*a*b^2*c^9*d^9*e^3 - 20*a*b^4*c^7*d^7*e^5 - 20*a*b^5*c^6*d^6*e^6 + 4*
a*b^6*c^5*d^5*e^7 - 44*a*b^8*c^3*d^3*e^9 + 12*a*b^9*c^2*d^2*e^10 + 8*a^2*b*
c^9*d^8*e^4 + 12*a^2*b^8*c^2*d*e^11 - 36*a^3*b*c^8*d^6*e^6 - 100*a^3*b^6*c^
3*d*e^11 - 132*a^4*b*c^7*d^4*e^8 + 264*a^4*b^4*c^4*d*e^11 + 264*a^5*b*c^6*d
^2*e^10 - 224*a^5*b^2*c^5*d*e^11))/(a^6*d^6) + (((192*a^6*b*c^4*e^11 - 256*
a^6*c^5*d*e^10 + 16*a^4*b^5*c^2*e^11 - 112*a^5*b^3*c^3*e^11 + 60*a^3*c^8*d^
7*e^4 - 320*a^4*c^7*d^5*e^6 + 480*a^5*c^6*d^3*e^8 + 16*b^4*c^7*d^9*e^2 - 32
*b^5*c^6*d^8*e^3 + 16*b^6*c^5*d^7*e^4 + 16*b^7*c^4*d^6*e^5 - 32*b^8*c^3*d^5
*e^6 + 16*b^9*c^2*d^4*e^7 + 16*a^2*b^2*c^7*d^7*e^4 + 120*a^2*b^3*c^6*d^6*e^
5 - 816*a^2*b^4*c^5*d^5*e^6 + 880*a^2*b^5*c^4*d^4*e^7 - 424*a^2*b^6*c^3*d^3
*e^8 + 56*a^2*b^7*c^2*d^2*e^9 + 832*a^3*b^2*c^6*d^5*e^6 - 1424*a^3*b^3*c^5*
```

d^4*e^7 + 1340*a^3*b^4*c^4*d^3*e^8 - 464*a^3*b^5*c^3*d^2*e^9 - 1512*a^4*b^2*c^5*d^3*e^8 + 1144*a^4*b^3*c^4*d^2*e^9 - 20*a*b^2*c^8*d^9*e^2 + 96*a*b^3*c^7*d^8*e^3 - 64*a*b^4*c^6*d^7*e^4 - 88*a*b^5*c^5*d^6*e^5 + 288*a*b^6*c^4*d^5*e^6 - 208*a*b^7*c^3*d^4*e^7 + 44*a*b^8*c^2*d^3*e^8 - 40*a^2*b*c^8*d^8*e^3 - 88*a^3*b*c^7*d^6*e^5 + 44*a^3*b^6*c^2*d*e^10 + 704*a^4*b*c^6*d^4*e^7 - 328*a^4*b^4*c^3*d*e^10 - 736*a^5*b*c^5*d^2*e^9 + 684*a^5*b^2*c^4*d*e^10)/(a^4*d^4) + (((256*a^6*c^4*e^10 + 16*a^4*b^4*c^2*e^10 - 128*a^5*b^2*c^3*e^10 - 192*a^3*c^7*d^6*e^4 + 448*a^4*c^6*d^4*e^6 - 512*a^5*c^5*d^2*e^8 + 16*b^4*c^6*d^8*e^2 - 64*b^5*c^5*d^7*e^3 + 96*b^6*c^4*d^6*e^4 - 64*b^7*c^3*d^5*e^5 + 16*b^8*c^2*d^4*e^6 + 768*a^2*b^2*c^6*d^6*e^4 - 1200*a^2*b^3*c^5*d^5*e^5 + 896*a^2*b^4*c^4*d^4*e^6 - 320*a^2*b^5*c^3*d^3*e^7 + 32*a^2*b^6*c^2*d^2*e^8 - 1392*a^3*b^2*c^5*d^4*e^6 + 1024*a^3*b^3*c^4*d^3*e^7 - 288*a^3*b^4*c^3*d^2*e^8 + 768*a^4*b^2*c^4*d^2*e^8 + 448*a^5*b*c^4*d*e^9 - 32*a*b^2*c^7*d^8*e^2 + 240*a*b^3*c^6*d^7*e^3 - 528*a*b^4*c^5*d^6*e^4 + 496*a*b^5*c^4*d^5*e^5 - 208*a*b^6*c^3*d^4*e^6 + 32*a*b^7*c^2*d^3*e^7 - 176*a^2*b*c^7*d^7*e^3 + 848*a^3*b*c^6*d^5*e^5 + 32*a^3*b^5*c^2*d*e^9 - 1024*a^4*b*c^5*d^3*e^7 - 240*a^4*b^3*c^3*d*e^9)/(a^2*d^2) + (8*c^2*e^2*x^2*(a^3*b^5*e^8 + b^3*c^5*d^8 + b^8*d^3*e^5 - 11*a^4*b^3*c*e^8 + 28*a^5*b*c^2*e^8 + 8*a*b^7*d^2*e^6 + 8*a^2*b^6*d*e^7 - 30*a^2*c^6*d^7*e - 24*a^5*c^3*d*e^7 - 6*b^4*c^4*d^7*e - 6*b^7*c*d^4*e^4 - 18*a^3*c^5*d^5*e^3 + 180*a^4*c^4*d^3*e^5 + 5*b^5*c^3*d^6*e^2 + 5*b^6*c^2*d^5*e^3 + 5*a*b*c^6*d^8 + 13*a^2*b^2*c^4*d^5*e^3 - 82*a^2*b^3*c^3*d^4*e^4 + 110*a^2*b^4*c^2*d^3*e^5 - 277*a^3*b^2*c^3*d^3*e^5 + 328*a^3*b^3*c^2*d^2*e^6 + 15*a*b^2*c^5*d^7*e - 17*a*b^6*c*d^3*e^5 - 57*a^3*b^4*c*d*e^7 - 27*a*b^3*c^4*d^6*e^2 - 24*a*b^4*c^3*d^5*e^3 + 40*a*b^5*c^2*d^4*e^4 + 67*a^2*b*c^5*d^6*e^2 - 92*a^2*b^5*c*d^2*e^6 + 72*a^3*b*c^4*d^4*e^4 - 352*a^4*b*c^3*d^2*e^6 + 106*a^4*b^2*c^2*d*e^7))/(a^2*d^2) - (4*c^2*e^2*(a*b^2*e^3 + b*c^2*d^3 - 4*a^2*c*e^3 + b^3*d*e^2 + 4*a*c^2*d^2*e - 2*b^2*c*d^2*e - 3*a*b*c*d*e^2)*(b^4*e*(b^2 - 4*a*c)^(1/2) - b^5*e + 4*a^2*c^3*d + b^4*c*d + 6*a*b^3*c*e - b^3*c*d*(b^2 - 4*a*c)^(1/2) - 5*a*b^2*c^2*d - 8*a^2*b*c^2*e + 2*a^2*c^2*e*(b^2 - 4*a*c)^(1/2) + 3*a*b*c^2*d*(b^2 - 4*a*c)^(1/2) - 4*a*b^2*c*e*(b^2 - 4*a*c)^(1/2))*(a*b^3*d^2*e^2 + a^2*b^2*d*e^3 + 4*a^2*c^2*d^3*e - 10*a*c^3*d^4*x^2 - 12*a^3*c*e^4*x^2 + 3*a^2*b^2*e^4*x^2 + 3*b^2*c^2*d^4*x^2 + 3*b^4*d^2*e^2*x^2 + a*b*c^2*d^4 - 4*a^3*c*d*e^3 - 2*a*b^2*c*d^3*e - 14*a^2*c^2*d^2*e^2*x^2 - 3*a^2*b*c*d^2*e^2 - 4*a*b^3*d*e^3*x^2 - 6*b^3*c*d^3*e*x^2 - 8*a*b^2*c*d^2*e^2*x^2 + 22*a*b*c^2*d^3*e*x^2 + 16*a^2*b*c*d*e^3*x^2))/(a^3*(4*a*c - b^2)*(a*e^2 + c*d^2 - b*d*e)))*(b^4*e*(b^2 - 4*a*c)^(1/2) - b^5*e + 4*a^2*c^3*d + b^4*c*d + 6*a*b^3*c*e - b^3*c*d*(b^2 - 4*a*c)^(1/2) - 5*a*b^2*c^2*d - 8*a^2*b*c^2*e + 2*a^2*c^2*e*(b^2 - 4*a*c)^(1/2) + 3*a*b*c^2*d*(b^2 - 4*a*c)^(1/2) - 4*a*b^2*c*e*(b^2 - 4*a*c)^(1/2)))/(4*a^3*(4*a*c - b^2)*(a*e^2 + c*d^2 - b*d*e)) + (4*c^2*e^2*x^2*(6*a^3*b^6*e^9 - 16*a^6*c^3*e^9 + 6*b^3*c^6*d^9 + 6*b^9*d^3*e^6 - 44*a^4*b^4*c*e^9 + 13*a*b^8*d^2*e^7 + 13*a^2*b^7*d*e^8 + 5*a^2*c^7*d^8*e - 8*b^4*c^5*d^8*e - 8*b^8*c*d^4*e^5 + 84*a^5*b^2*c^2*e^9 + 2*a^3*c^6*d^6*e^3 - 160*a^4*c^5*d^4*e^5 + 124*a^5*c^4*d^2*e^7 - 2*b^5*c^4*d^7*e^2 + 8*b^6*c^3*d^6*e^3 - 2*b^7*c^2*d^5*e^4 - 5*a*b*c^7*d^9 + 40*a^2*b^2*c^5*d^6*e^3 - 45*a^2*b^3*c^4*d^5*e^4 - 220*a^2*b^4*c^3*d^4

*e^5 + 316*a^2*b^5*c^2*d^3*e^6 + 264*a^3*b^2*c^4*d^4*e^5 - 546*a^3*b^3*c^3*
d^3*e^6 + 388*a^3*b^4*c^2*d^2*e^7 - 447*a^4*b^2*c^3*d^2*e^7 + 12*a*b^2*c^6*
d^8*e - 74*a*b^7*c*d^3*e^6 - 111*a^3*b^5*c*d*e^8 - 210*a^5*b*c^3*d*e^8 + 18
*a*b^3*c^5*d^7*e^2 - 43*a*b^4*c^4*d^6*e^3 + 19*a*b^5*c^3*d^5*e^4 + 72*a*b^6
*c^2*d^4*e^5 - 20*a^2*b*c^6*d^7*e^2 - 123*a^2*b^6*c*d^2*e^7 + 31*a^3*b*c^5*
d^5*e^4 + 328*a^4*b*c^4*d^3*e^6 + 290*a^4*b^3*c^2*d*e^8))/(a^4*d^4))*(b^4*e
*(b^2 - 4*a*c)^(1/2) - b^5*e + 4*a^2*c^3*d + b^4*c*d + 6*a*b^3*c*e - b^3*c*
d*(b^2 - 4*a*c)^(1/2) - 5*a*b^2*c^2*d - 8*a^2*b*c^2*e + 2*a^2*c^2*e*(b^2 -
4*a*c)^(1/2) + 3*a*b*c^2*d*(b^2 - 4*a*c)^(1/2) - 4*a*b^2*c*e*(b^2 - 4*a*c)^
(1/2)))/(4*a^3*(4*a*c - b^2)*(a*e^2 + c*d^2 - b*d*e)))*(b^4*e*(b^2 - 4*a*c)
^(1/2) - b^5*e + 4*a^2*c^3*d + b^4*c*d + 6*a*b^3*c*e - b^3*c*d*(b^2 - 4*a*c
)^(1/2) - 5*a*b^2*c^2*d - 8*a^2*b*c^2*e + 2*a^2*c^2*e*(b^2 - 4*a*c)^(1/2) +
3*a*b*c^2*d*(b^2 - 4*a*c)^(1/2) - 4*a*b^2*c*e*(b^2 - 4*a*c)^(1/2)))/(4*a^3
*(4*a*c - b^2)*(a*e^2 + c*d^2 - b*d*e)) + (2*c^5*e^5*x^2*(a^2*b^4*e^6 + 2*a
^4*c^2*e^6 + b^2*c^4*d^6 + b^6*d^2*e^4 - 4*a^3*b^2*c*e^6 + 3*a^2*c^4*d^4*e^
2 - 10*a^3*c^3*d^2*e^4 + 2*a*b^5*d*e^5 - a*b*c^4*d^5*e + 16*a^2*b^2*c^2*d^2
*e^4 - 7*a*b^4*c*d^2*e^4 - 11*a^2*b^3*c*d*e^5 + 13*a^3*b*c^2*d*e^5))/(a^6*d
^6))*(b^4*e*(b^2 - 4*a*c)^(1/2) - b^5*e + 4*a^2*c^3*d + b^4*c*d + 6*a*b^3*c
*e - b^3*c*d*(b^2 - 4*a*c)^(1/2) - 5*a*b^2*c^2*d - 8*a^2*b*c^2*e + 2*a^2*c^
2*e*(b^2 - 4*a*c)^(1/2) + 3*a*b*c^2*d*(b^2 - 4*a*c)^(1/2) - 4*a*b^2*c*e*(b^
2 - 4*a*c)^(1/2)))/(4*a^3*(4*a*c - b^2)*(a*e^2 + c*d^2 - b*d*e)))*(b^4*e*(b
^2 - 4*a*c)^(1/2) - b^5*e + 4*a^2*c^3*d + b^4*c*d + 6*a*b^3*c*e - b^3*c*d*(
b^2 - 4*a*c)^(1/2) - 5*a*b^2*c^2*d - 8*a^2*b*c^2*e + 2*a^2*c^2*e*(b^2 - 4*a
*c)^(1/2) + 3*a*b*c^2*d*(b^2 - 4*a*c)^(1/2) - 4*a*b^2*c*e*(b^2 - 4*a*c)^(1/
2)))/(4*(4*a^5*c*e^2 - a^4*b^2*e^2 + 4*a^4*c^2*d^2 - a^3*b^2*c*d^2 + a^3*b^
3*d*e - 4*a^4*b*c*d*e)) - (log((c^8*e^8*(a^2*e^2 + b^2*d^2 - a*c*d^2 + a*b*
d*e))/(a^6*d^6) - (c^9*e^9*x^2)/(a^5*d^5) + (((((4*a^4*b^6*c^2*e^12 - 24*a^
5*b^4*c^3*e^12 + 36*a^6*b^2*c^4*e^12 - 4*a^3*c^9*d^8*e^4 + 64*a^4*c^8*d^6*e
^6 - 144*a^5*c^7*d^4*e^8 + 96*a^6*c^6*d^2*e^10 + 4*b^4*c^8*d^10*e^2 + 8*b^7
*c^5*d^7*e^5 + 4*b^10*c^2*d^4*e^8 + 64*a^2*b^3*c^7*d^7*e^5 - 8*a^2*b^4*c^6*
d^6*e^6 - 8*a^2*b^5*c^5*d^5*e^7 + 172*a^2*b^6*c^4*d^4*e^8 - 112*a^2*b^7*c^3
*d^3*e^9 + 16*a^2*b^8*c^2*d^2*e^10 - 72*a^3*b^2*c^7*d^6*e^6 + 56*a^3*b^3*c^
6*d^5*e^7 - 312*a^3*b^4*c^5*d^4*e^8 + 348*a^3*b^5*c^4*d^3*e^9 - 132*a^3*b^6
*c^3*d^2*e^10 + 324*a^4*b^2*c^6*d^4*e^8 - 428*a^4*b^3*c^5*d^3*e^9 + 344*a^4
*b^4*c^4*d^2*e^10 - 300*a^5*b^2*c^5*d^2*e^10 - 96*a^6*b*c^5*d*e^11 - 4*a*b^
2*c^9*d^10*e^2 - 4*a*b^3*c^8*d^9*e^3 - 48*a*b^5*c^6*d^7*e^5 + 8*a*b^6*c^5*d
^6*e^6 - 44*a*b^8*c^3*d^4*e^8 + 12*a*b^9*c^2*d^3*e^9 + 8*a^2*b*c^9*d^9*e^3
- 24*a^3*b*c^8*d^7*e^5 + 12*a^3*b^7*c^2*d*e^11 - 88*a^4*b*c^7*d^5*e^7 - 88*
a^4*b^5*c^3*d*e^11 + 228*a^5*b*c^6*d^3*e^9 + 188*a^5*b^3*c^4*d*e^11)/(a^6*d
^6) + (x^2*(32*a^6*c^6*d*e^11 - 24*a^6*b*c^5*e^12 + 4*a^3*b^7*c^2*e^12 - 28
*a^4*b^5*c^3*e^12 + 56*a^5*b^3*c^4*e^12 + 2*a^3*c^9*d^7*e^5 + 104*a^4*c^8*d
^5*e^7 - 156*a^5*c^7*d^3*e^9 + 4*b^3*c^9*d^10*e^2 + 4*b^6*c^6*d^7*e^5 + 4*b
^7*c^5*d^6*e^6 + 4*b^10*c^2*d^3*e^9 + 8*a^2*b^2*c^8*d^7*e^5 + 40*a^2*b^3*c^
7*d^6*e^6 - 12*a^2*b^5*c^5*d^4*e^8 + 180*a^2*b^6*c^4*d^3*e^9 - 116*a^2*b^7*
c^3*d^2*e^10 - 92*a^3*b^2*c^7*d^5*e^7 + 84*a^3*b^3*c^6*d^4*e^8 - 350*a^3*b^

4*c^5*d^3*e^9 + 388*a^3*b^5*c^4*d^2*e^10 + 348*a^4*b^2*c^6*d^3*e^9 - 524*a^4*b^3*c^5*d^2*e^10 - 4*a*b^2*c^9*d^9*e^3 - 20*a*b^4*c^7*d^7*e^5 - 20*a*b^5*c^6*d^6*e^6 + 4*a*b^6*c^5*d^5*e^7 - 44*a*b^8*c^3*d^3*e^9 + 12*a*b^9*c^2*d^2*e^10 + 8*a^2*b*c^9*d^8*e^4 + 12*a^2*b^8*c^2*d*e^11 - 36*a^3*b*c^8*d^6*e^6 - 100*a^3*b^6*c^3*d*e^11 - 132*a^4*b*c^7*d^4*e^8 + 264*a^4*b^4*c^4*d*e^11 + 264*a^5*b*c^6*d^2*e^10 - 224*a^5*b^2*c^5*d*e^11))/(a^6*d^6) - (((192*a^6*b*c^4*e^11 - 256*a^6*c^5*d*e^10 + 16*a^4*b^5*c^2*e^11 - 112*a^5*b^3*c^3*e^11 + 60*a^3*c^8*d^7*e^4 - 320*a^4*c^7*d^5*e^6 + 480*a^5*c^6*d^3*e^8 + 16*b^4*c^7*d^9*e^2 - 32*b^5*c^6*d^8*e^3 + 16*b^6*c^5*d^7*e^4 + 16*b^7*c^4*d^6*e^5 - 32*b^8*c^3*d^5*e^6 + 16*b^9*c^2*d^4*e^7 + 16*a^2*b^2*c^7*d^7*e^4 + 120*a^2*b^3*c^6*d^6*e^5 - 816*a^2*b^4*c^5*d^5*e^6 + 880*a^2*b^5*c^4*d^4*e^7 - 424*a^2*b^6*c^3*d^3*e^8 + 56*a^2*b^7*c^2*d^2*e^9 + 832*a^3*b^2*c^6*d^5*e^6 - 1424*a^3*b^3*c^5*d^4*e^7 + 1340*a^3*b^4*c^4*d^3*e^8 - 464*a^3*b^5*c^3*d^2*e^9 - 1512*a^4*b^2*c^5*d^3*e^8 + 1144*a^4*b^3*c^4*d^2*e^9 - 20*a*b^2*c^8*d^9*e^2 + 96*a*b^3*c^7*d^8*e^3 - 64*a*b^4*c^6*d^7*e^4 - 88*a*b^5*c^5*d^6*e^5 + 288*a*b^6*c^4*d^5*e^6 - 208*a*b^7*c^3*d^4*e^7 + 44*a*b^8*c^2*d^3*e^8 - 40*a^2*b*c^8*d^8*e^3 - 88*a^3*b*c^7*d^6*e^5 + 44*a^3*b^6*c^2*d*e^10 + 704*a^4*b*c^6*d^4*e^7 - 328*a^4*b^4*c^3*d*e^10 - 736*a^5*b*c^5*d^2*e^9 + 684*a^5*b^2*c^4*d*e^10)/(a^4*d^4) - (((256*a^6*c^4*e^10 + 16*a^4*b^4*c^2*e^10 - 128*a^5*b^2*c^3*e^10 - 192*a^3*c^7*d^6*e^4 + 448*a^4*c^6*d^4*e^6 - 512*a^5*c^5*d^2*e^8 + 16*b^4*c^6*d^8*e^2 - 64*b^5*c^5*d^7*e^3 + 96*b^6*c^4*d^6*e^4 - 64*b^7*c^3*d^5*e^5 + 16*b^8*c^2*d^4*e^6 + 768*a^2*b^2*c^6*d^6*e^4 - 1200*a^2*b^3*c^5*d^5*e^5 + 896*a^2*b^4*c^4*d^4*e^6 - 320*a^2*b^5*c^3*d^3*e^7 + 32*a^2*b^6*c^2*d^2*e^8 - 1392*a^3*b^2*c^5*d^4*e^6 + 1024*a^3*b^3*c^4*d^3*e^7 - 288*a^3*b^4*c^3*d^2*e^8 + 768*a^4*b^2*c^4*d^2*e^8 + 448*a^5*b*c^4*d*e^9 - 32*a*b^2*c^7*d^8*e^2 + 240*a*b^3*c^6*d^7*e^3 - 528*a*b^4*c^5*d^6*e^4 + 496*a*b^5*c^4*d^5*e^5 - 208*a*b^6*c^3*d^4*e^6 + 32*a*b^7*c^2*d^3*e^7 - 176*a^2*b*c^7*d^7*e^3 + 848*a^3*b*c^6*d^5*e^5 + 32*a^3*b^5*c^2*d*e^9 - 1024*a^4*b*c^5*d^3*e^7 - 240*a^4*b^3*c^3*d*e^9)/(a^2*d^2) + (8*c^2*e^2*x^2*(a^3*b^5*e^8 + b^3*c^5*d^8 + b^8*d^3*e^5 - 11*a^4*b^3*c*e^8 + 28*a^5*b*c^2*e^8 + 8*a*b^7*d^2*e^6 + 8*a^2*b^6*d*e^7 - 30*a^2*c^6*d^7*e - 24*a^5*c^3*d*e^7 - 6*b^4*c^4*d^7*e - 6*b^7*c*d^4*e^4 - 18*a^3*c^5*d^5*e^3 + 180*a^4*c^4*d^3*e^5 + 5*b^5*c^3*d^6*e^2 + 5*b^6*c^2*d^5*e^3 + 5*a*b*c^6*d^8 + 13*a^2*b^2*c^4*d^5*e^3 - 82*a^2*b^3*c^3*d^4*e^4 + 110*a^2*b^4*c^2*d^3*e^5 - 277*a^3*b^2*c^3*d^3*e^5 + 328*a^3*b^3*c^2*d^2*e^6 + 15*a*b^2*c^5*d^7*e - 17*a*b^6*c*d^3*e^5 - 57*a^3*b^4*c*d*e^7 - 27*a*b^3*c^4*d^6*e^2 - 24*a*b^4*c^3*d^5*e^3 + 40*a*b^5*c^2*d^4*e^4 + 67*a^2*b*c^5*d^6*e^2 - 92*a^2*b^5*c*d^2*e^6 + 72*a^3*b*c^4*d^4*e^4 - 352*a^4*b*c^3*d^2*e^6 + 106*a^4*b^2*c^2*d*e^7))/(a^2*d^2) + (4*c^2*e^2*(a*b^2*e^3 + b*c^2*d^3 - 4*a^2*c*e^3 + b^3*d*e^2 + 4*a*c^2*d^2*e - 2*b^2*c*d^2*e - 3*a*b*c*d*e^2)*(b^5*e + b^4*e*(b^2 - 4*a*c)^(1/2) - 4*a^2*c^3*d - b^4*c*d - 6*a*b^3*c*e - b^3*c*d*(b^2 - 4*a*c)^(1/2) + 5*a*b^2*c^2*d + 8*a^2*b*c^2*e + 2*a^2*c^2*e*(b^2 - 4*a*c)^(1/2) + 3*a*b*c^2*d*(b^2 - 4*a*c)^(1/2) - 4*a*b^2*c*e*(b^2 - 4*a*c)^(1/2))*(a*b^3*d^2*e^2 + a^2*b^2*d*e^3 + 4*a^2*c^2*d^3*e - 10*a*c^3*d^4*x^2 - 12*a^3*c*e^4*x^2 + 3*a^2*b^2*e^4*x^2 + 3*b^2*c^2*d^4*x^2 + 3*b^4*d^2*e^2*x^2 + a*b*c^2*d^4 - 4*a^3*c*d*e^3 - 2*a*b^2*c*d

^3*e - 14*a^2*c^2*d^2*e^2*x^2 - 3*a^2*b*c*d^2*e^2 - 4*a*b^3*d*e^3*x^2 - 6*b
^3*c*d^3*e*x^2 - 8*a*b^2*c*d^2*e^2*x^2 + 22*a*b*c^2*d^3*e*x^2 + 16*a^2*b*c*
d*e^3*x^2))/(a^3*(4*a*c - b^2)*(a*e^2 + c*d^2 - b*d*e)))*(b^5*e + b^4*e*(b^
2 - 4*a*c)^(1/2) - 4*a^2*c^3*d - b^4*c*d - 6*a*b^3*c*e - b^3*c*d*(b^2 - 4*a
*c)^(1/2) + 5*a*b^2*c^2*d + 8*a^2*b*c^2*e + 2*a^2*c^2*e*(b^2 - 4*a*c)^(1/2)
 + 3*a*b*c^2*d*(b^2 - 4*a*c)^(1/2) - 4*a*b^2*c*e*(b^2 - 4*a*c)^(1/2)))/(4*a
^3*(4*a*c - b^2)*(a*e^2 + c*d^2 - b*d*e)) + (4*c^2*e^2*x^2*(6*a^3*b^6*e^9 -
 16*a^6*c^3*e^9 + 6*b^3*c^6*d^9 + 6*b^9*d^3*e^6 - 44*a^4*b^4*c*e^9 + 13*a*b
^8*d^2*e^7 + 13*a^2*b^7*d*e^8 + 5*a^2*c^7*d^8*e - 8*b^4*c^5*d^8*e - 8*b^8*c
*d^4*e^5 + 84*a^5*b^2*c^2*e^9 + 2*a^3*c^6*d^6*e^3 - 160*a^4*c^5*d^4*e^5 + 1
24*a^5*c^4*d^2*e^7 - 2*b^5*c^4*d^7*e^2 + 8*b^6*c^3*d^6*e^3 - 2*b^7*c^2*d^5*
e^4 - 5*a*b*c^7*d^9 + 40*a^2*b^2*c^5*d^6*e^3 - 45*a^2*b^3*c^4*d^5*e^4 - 220
*a^2*b^4*c^3*d^4*e^5 + 316*a^2*b^5*c^2*d^3*e^6 + 264*a^3*b^2*c^4*d^4*e^5 -
546*a^3*b^3*c^3*d^3*e^6 + 388*a^3*b^4*c^2*d^2*e^7 - 447*a^4*b^2*c^3*d^2*e^7
 + 12*a*b^2*c^6*d^8*e - 74*a*b^7*c*d^3*e^6 - 111*a^3*b^5*c*d*e^8 - 210*a^5*
b*c^3*d*e^8 + 18*a*b^3*c^5*d^7*e^2 - 43*a*b^4*c^4*d^6*e^3 + 19*a*b^5*c^3*d^
5*e^4 + 72*a*b^6*c^2*d^4*e^5 - 20*a^2*b*c^6*d^7*e^2 - 123*a^2*b^6*c*d^2*e^7
 + 31*a^3*b*c^5*d^5*e^4 + 328*a^4*b*c^4*d^3*e^6 + 290*a^4*b^3*c^2*d*e^8))/(
a^4*d^4))*(b^5*e + b^4*e*(b^2 - 4*a*c)^(1/2) - 4*a^2*c^3*d - b^4*c*d - 6*a*
b^3*c*e - b^3*c*d*(b^2 - 4*a*c)^(1/2) + 5*a*b^2*c^2*d + 8*a^2*b*c^2*e + 2*a
^2*c^2*e*(b^2 - 4*a*c)^(1/2) + 3*a*b*c^2*d*(b^2 - 4*a*c)^(1/2) - 4*a*b^2*c*
e*(b^2 - 4*a*c)^(1/2)))/(4*a^3*(4*a*c - b^2)*(a*e^2 + c*d^2 - b*d*e)))*(b^5
*e + b^4*e*(b^2 - 4*a*c)^(1/2) - 4*a^2*c^3*d - b^4*c*d - 6*a*b^3*c*e - b^3*
c*d*(b^2 - 4*a*c)^(1/2) + 5*a*b^2*c^2*d + 8*a^2*b*c^2*e + 2*a^2*c^2*e*(b^2
- 4*a*c)^(1/2) + 3*a*b*c^2*d*(b^2 - 4*a*c)^(1/2) - 4*a*b^2*c*e*(b^2 - 4*a*c
)^(1/2)))/(4*a^3*(4*a*c - b^2)*(a*e^2 + c*d^2 - b*d*e)) + (c^5*e^5*(4*a^3*b
^3*e^6 + 4*b^3*c^3*d^6 + 4*b^6*d^3*e^3 + 8*a*b^5*d^2*e^4 + 8*a^2*b^4*d*e^5
+ 4*a^2*c^4*d^5*e + 16*a^4*c^2*d*e^5 - 19*a^3*c^3*d^3*e^3 - 4*a*b*c^4*d^6 -
 12*a^4*b*c*e^6 + 36*a^2*b^2*c^2*d^3*e^3 - 24*a*b^4*c*d^3*e^3 - 32*a^3*b^2*
c*d*e^5 - 36*a^2*b^3*c*d^2*e^4 + 28*a^3*b*c^2*d^2*e^4))/(a^6*d^6) + (2*c^5*
e^5*x^2*(a^2*b^4*e^6 + 2*a^4*c^2*e^6 + b^2*c^4*d^6 + b^6*d^2*e^4 - 4*a^3*b^
2*c*e^6 + 3*a^2*c^4*d^4*e^2 - 10*a^3*c^3*d^2*e^4 + 2*a*b^5*d*e^5 - a*b*c^4*
d^5*e + 16*a^2*b^2*c^2*d^2*e^4 - 7*a*b^4*c*d^2*e^4 - 11*a^2*b^3*c*d*e^5 + 1
3*a^3*b*c^2*d*e^5))/(a^6*d^6))*(b^5*e + b^4*e*(b^2 - 4*a*c)^(1/2) - 4*a^2*c
^3*d - b^4*c*d - 6*a*b^3*c*e - b^3*c*d*(b^2 - 4*a*c)^(1/2) + 5*a*b^2*c^2*d
+ 8*a^2*b*c^2*e + 2*a^2*c^2*e*(b^2 - 4*a*c)^(1/2) + 3*a*b*c^2*d*(b^2 - 4*a*
c)^(1/2) - 4*a*b^2*c*e*(b^2 - 4*a*c)^(1/2)))/(4*a^3*(4*a*c - b^2)*(a*e^2 +
c*d^2 - b*d*e)))*(b^5*e + b^4*e*(b^2 - 4*a*c)^(1/2) - 4*a^2*c^3*d - b^4*c*d
 - 6*a*b^3*c*e - b^3*c*d*(b^2 - 4*a*c)^(1/2) + 5*a*b^2*c^2*d + 8*a^2*b*c^2*
e + 2*a^2*c^2*e*(b^2 - 4*a*c)^(1/2) + 3*a*b*c^2*d*(b^2 - 4*a*c)^(1/2) - 4*a
*b^2*c*e*(b^2 - 4*a*c)^(1/2)))/(4*(4*a^5*c*e^2 - a^4*b^2*e^2 + 4*a^4*c^2*d^
2 - a^3*b^2*c*d^2 + a^3*b^3*d*e - 4*a^4*b*c*d*e)) - (1/(4*a*d) - (x^2*(a*e
+ b*d))/(2*a^2*d^2))/x^4 - (e^4*log(d + e*x^2))/(2*(c*d^5 + a*d^3*e^2 - b*d
^4*e)) + (log(x)*(a^2*e^2 + b^2*d^2 - a*c*d^2 + a*b*d*e))/(a^3*d^3)

3.303 $\int \frac{x^8}{(d+ex^2)(a+bx^2+cx^4)}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 387

$$\begin{aligned}
&\int \frac{x^8}{(d+ex^2)(a+bx^2+cx^4)}\,dx\\
&=-\frac{(cd+be)x}{c^2e^2}+\frac{x^3}{3ce}\\
&\quad-\frac{\left(b^3d-2abcd-ab^2e+a^2ce-\frac{b^4d-4ab^2cd+2a^2c^2d-ab^3e+3a^2bce}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}c^{5/2}\sqrt{b-\sqrt{b^2-4ac}}\,(cd^2-bde+ae^2)}\\
&\quad-\frac{\left(b^3d-2abcd-ab^2e+a^2ce+\frac{b^4d-4ab^2cd+2a^2c^2d-ab^3e+3a^2bce}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{2}c^{5/2}\sqrt{b+\sqrt{b^2-4ac}}\,(cd^2-bde+ae^2)}\\
&\quad+\frac{d^{7/2}\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{e^{5/2}\,(cd^2-bde+ae^2)}
\end{aligned}$$

```
[Out] -(b*e+c*d)*x/c^2/e^2+1/3*x^3/c/e+d^(7/2)*arctan(x*e^(1/2)/d^(1/2))/e^(5/2)/
(a*e^2-b*d*e+c*d^2)-1/2*arctan(x*2^(1/2)*c^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/
2))*(b^3*d-2*a*b*c*d-a*b^2*e+a^2*c*e+(-3*a^2*b*c*e-2*a^2*c^2*d+a*b^3*e+4*a*
b^2*c*d-b^4*d)/(-4*a*c+b^2)^(1/2))/c^(5/2)/(a*e^2-b*d*e+c*d^2)*2^(1/2)/(b-(
-4*a*c+b^2)^(1/2))^(1/2)-1/2*arctan(x*2^(1/2)*c^(1/2)/(b+(-4*a*c+b^2)^(1/2)
)^(1/2))*(b^3*d-2*a*b*c*d-a*b^2*e+a^2*c*e+(3*a^2*b*c*e+2*a^2*c^2*d-a*b^3*e-
4*a*b^2*c*d+b^4*d)/(-4*a*c+b^2)^(1/2))/c^(5/2)/(a*e^2-b*d*e+c*d^2)*2^(1/2)/
(b+(-4*a*c+b^2)^(1/2))^(1/2)
```

Rubi [A] (verified)

Time = 2.49 (sec) , antiderivative size = 387, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {1301, 211, 1180}

$$\int \frac{x^8}{(d+ex^2)(a+bx^2+cx^4)}\,dx$$
$$= -\frac{\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)\left(-\frac{3a^2bce+2a^2c^2d-ab^3e-4ab^2cd+b^4d}{\sqrt{b^2-4ac}}+a^2ce-ab^2e-2abcd+b^3d\right)}{\sqrt{2}c^{5/2}\sqrt{b-\sqrt{b^2-4ac}}\,(ae^2-bde+cd^2)}$$
$$-\frac{\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)\left(\frac{3a^2bce+2a^2c^2d-ab^3e-4ab^2cd+b^4d}{\sqrt{b^2-4ac}}+a^2ce-ab^2e-2abcd+b^3d\right)}{\sqrt{2}c^{5/2}\sqrt{\sqrt{b^2-4ac}+b}\,(ae^2-bde+cd^2)}$$
$$+\frac{d^{7/2}\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{e^{5/2}(ae^2-bde+cd^2)}-\frac{x(be+cd)}{c^2e^2}+\frac{x^3}{3ce}$$

```
[In] Int[x^8/((d + e*x^2)*(a + b*x^2 + c*x^4)),x]
```

```
[Out] -(((c*d + b*e)*x)/(c^2*e^2)) + x^3/(3*c*e) - ((b^3*d - 2*a*b*c*d - a*b^2*e
+ a^2*c*e - (b^4*d - 4*a*b^2*c*d + 2*a^2*c^2*d - a*b^3*e + 3*a^2*b*c*e)/Sqr
t[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(S
qrt[2]*c^(5/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]*(c*d^2 - b*d*e + a*e^2)) - ((b^3
*d - 2*a*b*c*d - a*b^2*e + a^2*c*e + (b^4*d - 4*a*b^2*c*d + 2*a^2*c^2*d - a
*b^3*e + 3*a^2*b*c*e)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b
+ Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*c^(5/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]]*(c*d^2
- b*d*e + a*e^2)) + (d^(7/2)*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(e^(5/2)*(c*d^2 -
 b*d*e + a*e^2))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1301

```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.))/((a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*((d + e*x^2)^q/(a
```

```
+ b*x^2 + c*x^4)), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b^2 - 4
*a*c, 0] && IntegerQ[q] && IntegerQ[m]
```

Rubi steps

$$\text{integral} = \int \left(\frac{-cd-be}{c^2e^2} + \frac{x^2}{ce} + \frac{d^4}{e^2\left(cd^2-bde+ae^2\right)\left(d+ex^2\right)} + \frac{-a(b^2d-acd-abe)-\left(b^3d-2abcd-ab^2e+a^2ce\right)x^2}{c^2\left(cd^2-bde+ae^2\right)\left(a+bx^2+cx^4\right)}\right) dx$$

$$= -\frac{(cd+be)x}{c^2e^2} + \frac{x^3}{3ce} + \frac{\int \frac{-a\left(b^2d-acd-abe\right)+\left(-b^3d+2abcd+ab^2e-a^2ce\right)x^2}{a+bx^2+cx^4}\,dx}{c^2\left(cd^2-bde+ae^2\right)} + \frac{d^4\int \frac{1}{d+ex^2}\,dx}{e^2\left(cd^2-bde+ae^2\right)}$$

$$= -\frac{(cd+be)x}{c^2e^2} + \frac{x^3}{3ce} + \frac{d^{7/2}\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{e^{5/2}\left(cd^2-bde+ae^2\right)} - \frac{\left(b^3d-2abcd-ab^2e+a^2ce-\frac{b^4d-4ab^2cd+2a^2c^2d-ab^3e+3a^2bce}{\sqrt{b^2-4ac}}\right)\int \frac{1}{\frac{b}{2}-\frac{1}{2}\sqrt{b^2-4ac}+cx^2}\,dx}{2c^2\left(cd^2-bde+ae^2\right)} - \frac{\left(b^3d-2abcd-ab^2e+a^2ce+\frac{b^4d-4ab^2cd+2a^2c^2d-ab^3e+3a^2bce}{\sqrt{b^2-4ac}}\right)\int \frac{1}{\frac{b}{2}+\frac{1}{2}\sqrt{b^2-4ac}+cx^2}\,dx}{2c^2\left(cd^2-bde+ae^2\right)}$$

$$= -\frac{(cd+be)x}{c^2e^2} + \frac{x^3}{3ce} - \frac{\left(b^3d-2abcd-ab^2e+a^2ce-\frac{b^4d-4ab^2cd+2a^2c^2d-ab^3e+3a^2bce}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}c^{5/2}\sqrt{b-\sqrt{b^2-4ac}}\left(cd^2-bde+ae^2\right)} - \frac{\left(b^3d-2abcd-ab^2e+a^2ce+\frac{b^4d-4ab^2cd+2a^2c^2d-ab^3e+3a^2bce}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{2}c^{5/2}\sqrt{b+\sqrt{b^2-4ac}}\left(cd^2-bde+ae^2\right)} + \frac{d^{7/2}\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{e^{5/2}\left(cd^2-bde+ae^2\right)}$$

Mathematica [A] (verified)

Time = 0.41 (sec) , antiderivative size = 463, normalized size of antiderivative = 1.20

$$\int \frac{x^8}{(d+ex^2)(a+bx^2+cx^4)}\,dx = -\frac{(cd+be)x}{c^2e^2} + \frac{x^3}{3ce}$$
$$+ \frac{\left(-b^4d + b^3\left(\sqrt{b^2-4ac}d + ae\right) - abc\left(2\sqrt{b^2-4ac}d + 3ae\right) + ab^2\left(4cd - \sqrt{b^2-4ac}e\right) + a^2c\left(-2cd + \sqrt{b^2\right.\right.}{\sqrt{2}c^{5/2}\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}\,(-cd^2+e(bd-ae))}$$
$$+ \frac{\left(b^4d + b^3\left(\sqrt{b^2-4ac}d - ae\right) + abc\left(-2\sqrt{b^2-4ac}d + 3ae\right) + a^2c\left(2cd + \sqrt{b^2-4ac}e\right) - ab^2\left(4cd + \sqrt{b^2-\right.\right.}{\sqrt{2}c^{5/2}\sqrt{b^2-4ac}\sqrt{b+\sqrt{b^2-4ac}}\,(-cd^2+e(bd-ae))}$$
$$+ \frac{d^{7/2}\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{e^{5/2}\left(cd^2-bde+ae^2\right)}$$

```
[In] Integrate[x^8/((d + e*x^2)*(a + b*x^2 + c*x^4)),x]
```

```
[Out] -(((c*d + b*e)*x)/(c^2*e^2)) + x^3/(3*c*e) + ((-(b^4*d) + b^3*(Sqrt[b^2 - 4
*a*c]*d + a*e) - a*b*c*(2*Sqrt[b^2 - 4*a*c]*d + 3*a*e) + a*b^2*(4*c*d - Sqr
t[b^2 - 4*a*c]*e) + a^2*c*(-2*c*d + Sqrt[b^2 - 4*a*c]*e))*ArcTan[(Sqrt[2]*S
qrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*c^(5/2)*Sqrt[b^2 - 4*a*c]*
Sqrt[b - Sqrt[b^2 - 4*a*c]]*(-(c*d^2) + e*(b*d - a*e))) + ((b^4*d + b^3*(Sq
rt[b^2 - 4*a*c]*d - a*e) + a*b*c*(-2*Sqrt[b^2 - 4*a*c]*d + 3*a*e) + a^2*c*(
2*c*d + Sqrt[b^2 - 4*a*c]*e) - a*b^2*(4*c*d + Sqrt[b^2 - 4*a*c]*e))*ArcTan[
(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*c^(5/2)*Sqrt[b^2
 - 4*a*c]*Sqrt[b + Sqrt[b^2 - 4*a*c]]*(-(c*d^2) + e*(b*d - a*e))) + (d^(7/2
)*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(e^(5/2)*(c*d^2 - b*d*e + a*e^2))
```

Maple [A] (verified)

Time = 0.47 (sec) , antiderivative size = 413, normalized size of antiderivative = 1.07

method	result
default	$-\frac{-\frac{1}{3}c\,x^3e+bex+cdx}{e^2c^2} + \frac{\frac{\left(-a^2ce\sqrt{-4ac+b^2}+a\,b^2e\sqrt{-4ac+b^2}+2abcd\sqrt{-4ac+b^2}-b^3d\sqrt{-4ac+b^2}-3a^2bce-2a^2c^2d+a\,b^3e+4a\,b^2cd-d\,b^4\right)\sqrt{2}\ a}{2c\sqrt{-4ac+b^2}\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}}}{}$
risch	Expression too large to display

```
[In] int(x^8/(e*x^2+d)/(c*x^4+b*x^2+a),x,method=_RETURNVERBOSE)
```

```
[Out] -1/e^2/c^2*(-1/3*c*x^3*e+b*e*x+c*d*x)+4/(a*e^2-b*d*e+c*d^2)/c*(1/8*(-a^2*c*
e*(-4*a*c+b^2)^(1/2)+a*b^2*e*(-4*a*c+b^2)^(1/2)+2*a*b*c*d*(-4*a*c+b^2)^(1/2
)-b^3*d*(-4*a*c+b^2)^(1/2)-3*a^2*b*c*e-2*a^2*c^2*d+a*b^3*e+4*a*b^2*c*d-d*b^
```

```
4)/c/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x
*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))-1/8*(-a^2*c*e*(-4*a*c+b^2)^(1/2)
+a*b^2*e*(-4*a*c+b^2)^(1/2)+2*a*b*c*d*(-4*a*c+b^2)^(1/2)-b^3*d*(-4*a*c+b^2)
^(1/2)+3*a^2*b*c*e+2*a^2*c^2*d-a*b^3*e-4*a*b^2*c*d+d*b^4)/c/(-4*a*c+b^2)^(1
/2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(c*x*2^(1/2)/((-b+(-4*
a*c+b^2)^(1/2))*c)^(1/2)))+1/e^2*d^4/(a*e^2-b*d*e+c*d^2)/(e*d)^(1/2)*arctan
(e*x/(e*d)^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 12250 vs. $2(341) = 682$.

Time = 164.88 (sec) , antiderivative size = 24520, normalized size of antiderivative = 63.36

$$\int \frac{x^8}{(d+ex^2)(a+bx^2+cx^4)}\,dx = \text{Too large to display}$$

```
[In] integrate(x^8/(e*x^2+d)/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^8}{(d+ex^2)(a+bx^2+cx^4)}\,dx = \text{Timed out}$$

```
[In] integrate(x**8/(e*x**2+d)/(c*x**4+b*x**2+a),x)
```

```
[Out] Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^8}{(d+ex^2)(a+bx^2+cx^4)}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(x^8/(e*x^2+d)/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 12525 vs. $2(341) = 682$.

Time = 2.85 (sec) , antiderivative size = 12525, normalized size of antiderivative = 32.36

$$\int \frac{x^8}{(d+ex^2)(a+bx^2+cx^4)}\,dx = \text{Too large to display}$$

```
[In] integrate(x^8/(e*x^2+d)/(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] d^4*arctan(e*x/sqrt(d*e))/((c*d^2*e^2 - b*d*e^3 + a*e^4)*sqrt(d*e)) + 1/8*(
(2*b^7*c^8 - 16*a*b^5*c^9 + 36*a^2*b^3*c^10 - 16*a^3*b*c^11 - sqrt(2)*sqrt(
b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^7*c^6 + 8*sqrt(2)*sqrt(b^2 -
 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^5*c^7 + 2*sqrt(2)*sqrt(b^2 - 4*
a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^6*c^7 - 18*sqrt(2)*sqrt(b^2 - 4*a*c)
*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c^8 - 8*sqrt(2)*sqrt(b^2 - 4*a*c)*
sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^4*c^8 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(
b*c - sqrt(b^2 - 4*a*c)*c)*b^5*c^8 + 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c -
 sqrt(b^2 - 4*a*c)*c)*a^3*b*c^9 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sq
rt(b^2 - 4*a*c)*c)*a^2*b^2*c^9 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqr
t(b^2 - 4*a*c)*c)*a*b^3*c^9 - 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b
^2 - 4*a*c)*c)*a^2*b*c^10 - 2*(b^2 - 4*a*c)*b^5*c^8 + 8*(b^2 - 4*a*c)*a*b^3
*c^9 - 4*(b^2 - 4*a*c)*a^2*b*c^10)*d^5 - (4*b^8*c^7 - 30*a*b^6*c^8 + 58*a^2
*b^4*c^9 - 8*a^3*b^2*c^10 - 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2
 - 4*a*c)*c)*b^8*c^5 + 15*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4
*a*c)*c)*a*b^6*c^6 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*
c)*c)*b^7*c^6 - 29*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c
)*a^2*b^4*c^7 - 14*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c
)*a*b^5*c^7 - 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b
^6*c^7 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^
2*c^8 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^3
*c^8 + 7*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^4*c^
8 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^9 -
 4*(b^2 - 4*a*c)*b^6*c^7 + 14*(b^2 - 4*a*c)*a*b^4*c^8 - 2*(b^2 - 4*a*c)*a^2
*b^2*c^9)*d^4*e + (2*b^9*c^6 - 8*a*b^7*c^7 - 24*a^2*b^5*c^8 + 104*a^3*b^3*c
^9 - 32*a^4*b*c^10 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)
*c)*b^9*c^4 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a
*b^7*c^5 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^8*
c^5 + 12*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^5*
c^6 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^7*c^6 - 5
2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^3*c^7 - 2
4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^4*c^7 + 1
6*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*b*c^8 + 8*s
qrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^2*c^8 + 12*s
qrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c^8 - 4*sq
```

rt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b*c^9 - 2*(b^2
- 4*a*c)*b^7*c^6 + 24*(b^2 - 4*a*c)*a^2*b^3*c^8 - 8*(b^2 - 4*a*c)*a^3*b*c^9
)*d^3*e^2 - (6*a*b^8*c^6 - 42*a^2*b^6*c^7 + 68*a^3*b^4*c^8 + 16*a^4*b^2*c^9
- 3*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^8*c^4 +
21*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^6*c^5 +
6*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^7*c^5 - 34*
sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^4*c^6 - 18*
sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^5*c^6 - 3*s
qrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^6*c^6 - 8*sqrt
(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*b^2*c^7 - 4*sqrt(
2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^3*c^7 + 9*sqrt(2
)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^4*c^7 + 2*sqrt(2)
*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^2*c^8 - 6*(b^2 - 4
*a*c)*a*b^6*c^6 + 18*(b^2 - 4*a*c)*a^2*b^4*c^7 + 4*(b^2 - 4*a*c)*a^3*b^2*c^
8)*d^2*e^3 + (6*a^2*b^7*c^6 - 44*a^3*b^5*c^7 + 84*a^4*b^3*c^8 - 16*a^5*b*c^
9 - 3*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^7*c^4
+ 22*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^5*c^5
+ 6*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^6*c^5
- 42*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*b^3*c^6
- 20*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^4*c^6
- 3*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^5*c^6 +
8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^5*b*c^7 + 4*
sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*b^2*c^7 + 10*
sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^3*c^7 - 2*s
qrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*b*c^8 - 6*(b^2
- 4*a*c)*a^2*b^5*c^6 + 20*(b^2 - 4*a*c)*a^3*b^3*c^7 - 4*(b^2 - 4*a*c)*a^4*
b*c^8)*d*e^4 - (2*a^3*b^6*c^6 - 14*a^4*b^4*c^7 + 24*a^5*b^2*c^8 - sqrt(2)*s
qrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^6*c^4 + 7*sqrt(2)*sq
rt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*b^4*c^5 + 2*sqrt(2)*sqr
t(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^5*c^5 - 12*sqrt(2)*sqr
t(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^5*b^2*c^6 - 6*sqrt(2)*sqrt
(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*b^3*c^6 - sqrt(2)*sqrt(b^
2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^4*c^6 + 3*sqrt(2)*sqrt(b^2
- 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*b^2*c^7 - 2*(b^2 - 4*a*c)*a^3
*b^4*c^6 + 6*(b^2 - 4*a*c)*a^4*b^2*c^7)*e^5 - 2*(sqrt(2)*sqrt(b*c - sqrt(b^
2 - 4*a*c)*c)*a*b^6*c^4 - 9*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^4
*c^5 - 2*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^5*c^5 + 2*a*b^6*c^5 +
24*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^2*c^6 + 10*sqrt(2)*sqrt(b*
c - sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c^6 + sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)
*c)*a*b^4*c^6 - 18*a^2*b^4*c^6 - 16*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)
*a^4*c^7 - 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b*c^7 - 5*sqrt(2)*
sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^7 + 48*a^3*b^2*c^7 + 4*sqrt(2)*sq
rt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*c^8 - 32*a^4*c^8 - 2*(b^2 - 4*a*c)*a*b^4*
c^5 + 10*(b^2 - 4*a*c)*a^2*b^2*c^6 - 8*(b^2 - 4*a*c)*a^3*c^7)*d^3*abs(-c^3*
d^2 + b*c^2*d*e - a*c^2*e^2) + 2*(sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a

*b^7*c^3 - 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^5*c^4 - 2*sqrt(2
)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^6*c^4 + 2*a*b^7*c^4 + 16*sqrt(2)*sqrt
(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^3*c^5 + 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4
*a*c)*c)*a^2*b^4*c^5 + sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^5*c^5 -
16*a^2*b^5*c^5 - 4*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c^6 + 32
*a^3*b^3*c^6 - 2*(b^2 - 4*a*c)*a*b^5*c^4 + 8*(b^2 - 4*a*c)*a^2*b^3*c^5)*d^2
*e*abs(-c^3*d^2 + b*c^2*d*e - a*c^2*e^2) - 2*(2*sqrt(2)*sqrt(b*c - sqrt(b^2
 - 4*a*c)*c)*a^2*b^6*c^3 - 17*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b
^4*c^4 - 4*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^5*c^4 + 4*a^2*b^6*
c^4 + 40*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*b^2*c^5 + 18*sqrt(2)*s
qrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^3*c^5 + 2*sqrt(2)*sqrt(b*c - sqrt(b^2
- 4*a*c)*c)*a^2*b^4*c^5 - 34*a^3*b^4*c^5 - 16*sqrt(2)*sqrt(b*c - sqrt(b^2 -
 4*a*c)*c)*a^5*c^6 - 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*b*c^6 -
9*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^2*c^6 + 80*a^4*b^2*c^6 + 4*
sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*c^7 - 32*a^5*c^7 - 4*(b^2 - 4*a
*c)*a^2*b^4*c^4 + 18*(b^2 - 4*a*c)*a^3*b^2*c^5 - 8*(b^2 - 4*a*c)*a^4*c^6)*d
*e^2*abs(-c^3*d^2 + b*c^2*d*e - a*c^2*e^2) + 2*(sqrt(2)*sqrt(b*c - sqrt(b^2
 - 4*a*c)*c)*a^3*b^5*c^3 - 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*b^
3*c^4 - 2*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^4*c^4 + 2*a^3*b^5*c
^4 + 16*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^5*b*c^5 + 8*sqrt(2)*sqrt(
b*c - sqrt(b^2 - 4*a*c)*c)*a^4*b^2*c^5 + sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*
c)*c)*a^3*b^3*c^5 - 16*a^4*b^3*c^5 - 4*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)
*c)*a^4*b*c^6 + 32*a^5*b*c^6 - 2*(b^2 - 4*a*c)*a^3*b^3*c^4 + 8*(b^2 - 4*a*c
)*a^4*b*c^5)*e^3*abs(-c^3*d^2 + b*c^2*d*e - a*c^2*e^2) - (2*b^7*c^2 - 20*a*
b^5*c^3 + 64*a^2*b^3*c^4 - 64*a^3*b*c^5 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*
c - sqrt(b^2 - 4*a*c)*c)*b^7 + 10*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt
(b^2 - 4*a*c)*c)*a*b^5*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2
- 4*a*c)*c)*b^6*c - 32*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*
c)*c)*a^2*b^3*c^2 - 12*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*
c)*c)*a*b^4*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)
*b^5*c^2 + 32*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3
*b*c^3 + 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b
^2*c^3 + 6*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^3*
c^3 - 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b*c^4
 - 2*(b^2 - 4*a*c)*b^5*c^2 + 12*(b^2 - 4*a*c)*a*b^3*c^3 - 16*(b^2 - 4*a*c)*
a^2*b*c^4)*(c^3*d^2 - b*c^2*d*e + a*c^2*e^2)^2*d + (2*a*b^6*c^2 - 18*a^2*b^
4*c^3 + 48*a^3*b^2*c^4 - 32*a^4*c^5 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c -
sqrt(b^2 - 4*a*c)*c)*a*b^6 + 9*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^
2 - 4*a*c)*c)*a^2*b^4*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 -
 4*a*c)*c)*a*b^5*c - 24*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a
*c)*c)*a^3*b^2*c^2 - 10*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a
*c)*c)*a^2*b^3*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)
*c)*a*b^4*c^2 + 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c
)*a^4*c^3 + 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3
*b*c^3 + 5*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^

2*c^3 - 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*c^4 - 2*(b^2 - 4*a*c)*a*b^4*c^2 + 10*(b^2 - 4*a*c)*a^2*b^2*c^3 - 8*(b^2 - 4*a*c)*a^3*c^4)*(c^3*d^2 - b*c^2*d*e + a*c^2*e^2)^2*e)*arctan(2*sqrt(1/2)*x/sqrt((b*c^3*d^2 - b^2*c^2*d*e + a*b*c^2*e^2 + sqrt((b*c^3*d^2 - b^2*c^2*d*e + a*b*c^2*e^2)^2 - 4*(a*c^3*d^2 - a*b*c^2*d*e + a^2*c^2*e^2)*(c^4*d^2 - b*c^3*d*e + a*c^3*e^2)))/(c^4*d^2 - b*c^3*d*e + a*c^3*e^2)))/((a*b^4*c^7 - 8*a^2*b^2*c^8 - 2*a*b^3*c^8 + 16*a^3*c^9 + 8*a^2*b*c^9 + a*b^2*c^9 - 4*a^2*c^10)*d^4*abs(-c^3*d^2 + b*c^2*d*e - a*c^2*e^2)*abs(c) - 2*(a*b^5*c^6 - 8*a^2*b^3*c^7 - 2*a*b^4*c^7 + 16*a^3*b*c^8 + 8*a^2*b^2*c^8 + a*b^3*c^8 - 4*a^2*b*c^9)*d^3*e*abs(-c^3*d^2 + b*c^2*d*e - a*c^2*e^2)*abs(c) + (a*b^6*c^5 - 6*a^2*b^4*c^6 - 2*a*b^5*c^6 + 4*a^2*b^3*c^7 + a*b^4*c^7 + 32*a^4*c^8 + 16*a^3*b*c^8 - 2*a^2*b^2*c^8 - 8*a^3*c^9)*d^2*e^2*abs(-c^3*d^2 + b*c^2*d*e - a*c^2*e^2)*abs(c) - 2*(a^2*b^5*c^5 - 8*a^3*b^3*c^6 - 2*a^2*b^4*c^6 + 16*a^4*b*c^7 + 8*a^3*b^2*c^7 + a^2*b^3*c^7 - 4*a^3*b*c^8)*d*e^3*abs(-c^3*d^2 + b*c^2*d*e - a*c^2*e^2)*abs(c) + (a^3*b^4*c^5 - 8*a^4*b^2*c^6 - 2*a^3*b^3*c^6 + 16*a^5*c^7 + 8*a^4*b*c^7 + a^3*b^2*c^7 - 4*a^4*c^8)*e^4*abs(-c^3*d^2 + b*c^2*d*e - a*c^2*e^2)*abs(c)) - 1/8*((2*b^7*c^8 - 16*a*b^5*c^9 + 36*a^2*b^3*c^10 - 16*a^3*b*c^11 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^7*c^6 + 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^5*c^7 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^6*c^7 - 18*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c^8 - 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^4*c^8 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^5*c^8 + 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b*c^9 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^9 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c^9 - 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b*c^10 - 2*(b^2 - 4*a*c)*b^5*c^8 + 8*(b^2 - 4*a*c)*a*b^3*c^9 - 4*(b^2 - 4*a*c)*a^2*b*c^10)*d^5 - (4*b^8*c^7 - 30*a*b^6*c^8 + 58*a^2*b^4*c^9 - 8*a^3*b^2*c^10 - 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^8*c^5 + 15*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^6*c^6 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^7*c^6 - 29*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^4*c^7 - 14*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^5*c^7 - 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^6*c^7 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^2*c^8 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c^8 + 7*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^4*c^8 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^9 - 4*(b^2 - 4*a*c)*b^6*c^7 + 14*(b^2 - 4*a*c)*a*b^4*c^8 - 2*(b^2 - 4*a*c)*a^2*b^2*c^9)*d^4*e + (2*b^9*c^6 - 8*a*b^7*c^7 - 24*a^2*b^5*c^8 + 104*a^3*b^3*c^9 - 32*a^4*b*c^10 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^9*c^4 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^7*c^5 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^8*c^5 + 12*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^5*c^6 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt (b^2 - 4*a*c)*c)*b^7*c^6 - 52*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^3*c^7 - 24*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^4*c^7 + 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b*c^8 + 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^2*c^8 + 12*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c^8 - 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b*c^9 - 2*(b^2 - 4*a*c)*b^7*c^6 + 24*(b^2 - 4*a*c)*a^2*b^3*c^8 - 8*(b^2 - 4*a*c)*a^3*b*c^9)*d^3*e^2 - (6*a*b^8*c^6 - 42*a^2*b^6*c^7 + 68*a^3*b^4*c^8 + 16*a^4*b^2*c^9 - 3*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^8*c^4 + 21*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^6*c^5 + 6*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^7*c^5 - 34*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^4*c^6 - 18*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^5*c^6 - 3*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^6*c^6 - 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b^2*c^7 - 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^3*c^7 + 9*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^4*c^7 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^2*c^8 - 6*(b^2 - 4*a*c)*a*b^6*c^6 + 18*(b^2 - 4*a*c)*a^2*b^4*c^7 + 4*(b^2 - 4*a*c)*a^3*b^2*c^8)*d^2*e^3 + (6*a^2*b^7*c^6 - 44*a^3*b^5*c^7 + 84*a^4*b^3*c^8 - 16*a^5*b*c^9 - 3*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^7*c^4 + 22*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^5*c^5 + 6*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^6*c^5 - 42*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b^3*c^6 - 20*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^4*c^6 - 3*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^5*c^6 + 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^5*b*c^7 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b^2*c^7 + 10*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^3*c^7 - 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b*c^8 - 6*(b^2 - 4*a*c)*a^2*b^5*c^6 + 20*(b^2 - 4*a*c)*a^3*b^3*c^7 - 4*(b^2 - 4*a*c)*a^4*b*c^8)*d*e^4 - (2*a^3*b^6*c^6 - 14*a^4*b^4*c^7 + 24*a^5*b^2*c^8 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^6*c^4 + 7*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b^4*c^5 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^5*c^5 - 12*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^5*b^2*c^6 - 6*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b^3*c^6 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^4*c^6 + 3*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b^2*c^7 - 2*(b^2 - 4*a*c)*a^3*b^4*c^6 + 6*(b^2 - 4*a*c)*a^4*b^2*c^7)*e^5 + 2*(sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^6*c^4 - 9*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^4*c^5 - 2*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^5*c^5 - 2*a*b^6*c^5 + 24*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^2*c^6 + 10*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c^6 + sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^4*c^6 + 18*a^2*b^4*c^6 - 16*sqrt(2)*sqr

t(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*c^7 - 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*
c)*c)*a^3*b*c^7 - 5*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^7 - 4
8*a^3*b^2*c^7 + 4*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*c^8 + 32*a^4*
c^8 + 2*(b^2 - 4*a*c)*a*b^4*c^5 - 10*(b^2 - 4*a*c)*a^2*b^2*c^6 + 8*(b^2 - 4
*a*c)*a^3*c^7)*d^3*abs(-c^3*d^2 + b*c^2*d*e - a*c^2*e^2) - 2*(sqrt(2)*sqrt(
b*c + sqrt(b^2 - 4*a*c)*c)*a*b^7*c^3 - 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*
c)*c)*a^2*b^5*c^4 - 2*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^6*c^4 - 2
*a*b^7*c^4 + 16*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^3*c^5 + 8*sqr
t(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^4*c^5 + sqrt(2)*sqrt(b*c + sqrt(
b^2 - 4*a*c)*c)*a*b^5*c^5 + 16*a^2*b^5*c^5 - 4*sqrt(2)*sqrt(b*c + sqrt(b^2
- 4*a*c)*c)*a^2*b^3*c^6 - 32*a^3*b^3*c^6 + 2*(b^2 - 4*a*c)*a*b^5*c^4 - 8*(b
^2 - 4*a*c)*a^2*b^3*c^5)*d^2*e*abs(-c^3*d^2 + b*c^2*d*e - a*c^2*e^2) + 2*(2
*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^6*c^3 - 17*sqrt(2)*sqrt(b*c
+ sqrt(b^2 - 4*a*c)*c)*a^3*b^4*c^4 - 4*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)
*c)*a^2*b^5*c^4 - 4*a^2*b^6*c^4 + 40*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c
)*a^4*b^2*c^5 + 18*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^3*c^5 + 2*
sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^4*c^5 + 34*a^3*b^4*c^5 - 16*s
qrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^5*c^6 - 8*sqrt(2)*sqrt(b*c + sqrt(
b^2 - 4*a*c)*c)*a^4*b*c^6 - 9*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b
^2*c^6 - 80*a^4*b^2*c^6 + 4*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*c^7
 + 32*a^5*c^7 + 4*(b^2 - 4*a*c)*a^2*b^4*c^4 - 18*(b^2 - 4*a*c)*a^3*b^2*c^5
+ 8*(b^2 - 4*a*c)*a^4*c^6)*d*e^2*abs(-c^3*d^2 + b*c^2*d*e - a*c^2*e^2) - 2*
(sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^5*c^3 - 8*sqrt(2)*sqrt(b*c +
 sqrt(b^2 - 4*a*c)*c)*a^4*b^3*c^4 - 2*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*
c)*a^3*b^4*c^4 - 2*a^3*b^5*c^4 + 16*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)
*a^5*b*c^5 + 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b^2*c^5 + sqrt(2
)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^3*c^5 + 16*a^4*b^3*c^5 - 4*sqrt(2)*
sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b*c^6 - 32*a^5*b*c^6 + 2*(b^2 - 4*a*c)*
a^3*b^3*c^4 - 8*(b^2 - 4*a*c)*a^4*b*c^5)*e^3*abs(-c^3*d^2 + b*c^2*d*e - a*c
^2*e^2) - (2*b^7*c^2 - 20*a*b^5*c^3 + 64*a^2*b^3*c^4 - 64*a^3*b*c^5 - sqrt(
2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^7 + 10*sqrt(2)*sqrt(
b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^5*c + 2*sqrt(2)*sqrt(b^2 -
 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^6*c - 32*sqrt(2)*sqrt(b^2 - 4*a*c
)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c^2 - 12*sqrt(2)*sqrt(b^2 - 4*a*c
)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^4*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqr
t(b*c + sqrt(b^2 - 4*a*c)*c)*b^5*c^2 + 32*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*
c + sqrt(b^2 - 4*a*c)*c)*a^3*b*c^3 + 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c
+ sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^3 + 6*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c +
 sqrt(b^2 - 4*a*c)*c)*a*b^3*c^3 - 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sq
rt(b^2 - 4*a*c)*c)*a^2*b*c^4 - 2*(b^2 - 4*a*c)*b^5*c^2 + 12*(b^2 - 4*a*c)*a
*b^3*c^3 - 16*(b^2 - 4*a*c)*a^2*b*c^4)*(c^3*d^2 - b*c^2*d*e + a*c^2*e^2)^2*
d + (2*a*b^6*c^2 - 18*a^2*b^4*c^3 + 48*a^3*b^2*c^4 - 32*a^4*c^5 - sqrt(2)*s
qrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^6 + 9*sqrt(2)*sqrt(b^2
 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^4*c + 2*sqrt(2)*sqrt(b^2 -
4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^5*c - 24*sqrt(2)*sqrt(b^2 - 4*a*

```
c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^2*c^2 - 10*sqrt(2)*sqrt(b^2 - 4*a*
c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*
sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^4*c^2 + 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sq
rt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*c^3 + 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*
c + sqrt(b^2 - 4*a*c)*c)*a^3*b*c^3 + 5*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c +
 sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^3 - 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c +
sqrt(b^2 - 4*a*c)*c)*a^3*c^4 - 2*(b^2 - 4*a*c)*a*b^4*c^2 + 10*(b^2 - 4*a*c)
*a^2*b^2*c^3 - 8*(b^2 - 4*a*c)*a^3*c^4)*(c^3*d^2 - b*c^2*d*e + a*c^2*e^2)^2
*e)*arctan(2*sqrt(1/2)*x/sqrt((b*c^3*d^2 - b^2*c^2*d*e + a*b*c^2*e^2 - sqrt
((b*c^3*d^2 - b^2*c^2*d*e + a*b*c^2*e^2)^2 - 4*(a*c^3*d^2 - a*b*c^2*d*e + a
^2*c^2*e^2)*(c^4*d^2 - b*c^3*d*e + a*c^3*e^2)))/(c^4*d^2 - b*c^3*d*e + a*c^
3*e^2)))/((a*b^4*c^7 - 8*a^2*b^2*c^8 - 2*a*b^3*c^8 + 16*a^3*c^9 + 8*a^2*b*c
^9 + a*b^2*c^9 - 4*a^2*c^10)*d^4*abs(-c^3*d^2 + b*c^2*d*e - a*c^2*e^2)*abs(
c) - 2*(a*b^5*c^6 - 8*a^2*b^3*c^7 - 2*a*b^4*c^7 + 16*a^3*b*c^8 + 8*a^2*b^2*
c^8 + a*b^3*c^8 - 4*a^2*b*c^9)*d^3*e*abs(-c^3*d^2 + b*c^2*d*e - a*c^2*e^2)*
abs(c) + (a*b^6*c^5 - 6*a^2*b^4*c^6 - 2*a*b^5*c^6 + 4*a^2*b^3*c^7 + a*b^4*c
^7 + 32*a^4*c^8 + 16*a^3*b*c^8 - 2*a^2*b^2*c^8 - 8*a^3*c^9)*d^2*e^2*abs(-c^
3*d^2 + b*c^2*d*e - a*c^2*e^2)*abs(c) - 2*(a^2*b^5*c^5 - 8*a^3*b^3*c^6 - 2*
a^2*b^4*c^6 + 16*a^4*b*c^7 + 8*a^3*b^2*c^7 + a^2*b^3*c^7 - 4*a^3*b*c^8)*d*e
^3*abs(-c^3*d^2 + b*c^2*d*e - a*c^2*e^2)*abs(c) + (a^3*b^4*c^5 - 8*a^4*b^2*
c^6 - 2*a^3*b^3*c^6 + 16*a^5*c^7 + 8*a^4*b*c^7 + a^3*b^2*c^7 - 4*a^4*c^8)*e
^4*abs(-c^3*d^2 + b*c^2*d*e - a*c^2*e^2)*abs(c)) + 1/3*(c^2*e^2*x^3 - 3*c^2
*d*e*x - 3*b*c*e^2*x)/(c^3*e^3)
```

Mupad [B] (verification not implemented)

Time = 11.97 (sec) , antiderivative size = 41755, normalized size of antiderivative = 107.89

$$\int \frac{x^8}{(d+ex^2)(a+bx^2+cx^4)}\,dx = \text{Too large to display}$$

```
[In] int(x^8/((d + e*x^2)*(a + b*x^2 + c*x^4)),x)
```

```
[Out] atan(((((((192*a^3*c^8*d^6*e^5 + 384*a^4*c^7*d^4*e^7 + 192*a^5*c^6*d^2*e^9
- 48*a^2*b^2*c^7*d^6*e^5 + 96*a^2*b^3*c^6*d^5*e^6 - 48*a^2*b^4*c^5*d^4*e^7
+ 96*a^3*b^2*c^6*d^4*e^7 + 96*a^3*b^3*c^5*d^3*e^8 - 48*a^4*b^2*c^5*d^2*e^9
- 384*a^3*b*c^7*d^5*e^6 - 384*a^4*b*c^6*d^3*e^8)/(c^3*e^3) - (2*x*(-(b^9*d^
2 + a^2*b^7*e^2 + b^6*d^2*(-(4*a*c - b^2)^3)^(1/2) + 28*a^4*b*c^4*d^2 - 9*a
^3*b^5*c*e^2 - 20*a^5*b*c^3*e^2 - 2*a*b^8*d*e + 42*a^2*b^5*c^2*d^2 - 63*a^3
*b^3*c^3*d^2 + a^2*b^4*e^2*(-(4*a*c - b^2)^3)^(1/2) - a^3*c^3*d^2*(-(4*a*c
- b^2)^3)^(1/2) + 25*a^4*b^3*c^2*e^2 + a^4*c^2*e^2*(-(4*a*c - b^2)^3)^(1/2)
 - 11*a*b^7*c*d^2 - 16*a^5*c^4*d*e - 2*a*b^5*d*e*(-(4*a*c - b^2)^3)^(1/2) +
 20*a^2*b^6*c*d*e + 6*a^2*b^2*c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) - 5*a*b^4*c*
d^2*(-(4*a*c - b^2)^3)^(1/2) - 66*a^3*b^4*c^2*d*e + 76*a^4*b^2*c^3*d*e - 3*
a^3*b^2*c*e^2*(-(4*a*c - b^2)^3)^(1/2) + 8*a^2*b^3*c*d*e*(-(4*a*c - b^2)^3)
```

^(1/2) - 6*a^3*b*c^2*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^9*d^4 + 16*a^4*c^7*e^4 + b^4*c^7*d^4 - 8*a*b^2*c^8*d^4 - 2*b^5*c^6*d^3*e + a^2*b^4*c^5*e^4 - 8*a^3*b^2*c^6*e^4 + 32*a^3*c^8*d^2*e^2 + b^6*c^5*d^2*e^2 + 16*a*b^3*c^7*d^3*e - 2*a*b^5*c^5*d*e^3 - 32*a^2*b*c^8*d^3*e - 32*a^3*b*c^7*d*e^3 - 6*a*b^4*c^6*d^2*e^2 + 16*a^2*b^3*c^6*d*e^3)))^(1/2)*(128*a^4*b^2*c^6*e^12 - 16*a^3*b^4*c^5*e^12 - 256*a^5*c^7*e^12 + 256*a^2*c^10*d^6*e^6 + 256*a^3*c^9*d^4*e^8 - 256*a^4*c^8*d^2*e^10 - 16*b^3*c^9*d^7*e^5 + 64*b^4*c^8*d^6*e^6 - 96*b^5*c^7*d^5*e^7 + 64*b^6*c^6*d^4*e^8 - 16*b^7*c^5*d^3*e^9 + 256*a^2*b^2*c^8*d^4*e^8 + 144*a^2*b^3*c^7*d^3*e^9 - 96*a^2*b^4*c^6*d^2*e^10 + 192*a^3*b^2*c^7*d^2*e^10 + 64*a*b*c^10*d^7*e^5 + 320*a^4*b*c^7*d*e^11 - 320*a*b^2*c^9*d^6*e^6 + 528*a*b^3*c^8*d^5*e^7 - 336*a*b^4*c^7*d^4*e^8 + 48*a*b^5*c^6*d^3*e^9 + 16*a*b^6*c^5*d^2*e^10 - 576*a^2*b*c^9*d^5*e^7 + 16*a^2*b^5*c^5*d*e^11 - 320*a^3*b*c^8*d^3*e^9 - 144*a^3*b^3*c^6*d*e^11))/(c^3*e^3))*(-(b^9*d^2 + a^2*b^7*e^2 + b^6*d^2*(-(4*a*c - b^2)^3)^(1/2) + 28*a^4*b*c^4*d^2 - 9*a^3*b^5*c*e^2 - 20*a^5*b*c^3*e^2 - 2*a*b^8*d*e + 42*a^2*b^5*c^2*d^2 - 63*a^3*b^3*c^3*d^2 + a^2*b^4*e^2*(-(4*a*c - b^2)^3)^(1/2) - a^3*c^3*d^2*(-(4*a*c - b^2)^3)^(1/2) + 25*a^4*b^3*c^2*e^2 + a^4*c^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - 11*a*b^7*c*d^2 - 16*a^5*c^4*d*e - 2*a*b^5*d*e*(-(4*a*c - b^2)^3)^(1/2) + 20*a^2*b^6*c*d*e + 6*a^2*b^2*c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) - 5*a*b^4*c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 66*a^3*b^4*c^2*d*e + 76*a^4*b^2*c^3*d*e - 3*a^3*b^2*c*e^2*(-(4*a*c - b^2)^3)^(1/2) + 8*a^2*b^3*c*d*e*(-(4*a*c - b^2)^3)^(1/2) - 6*a^3*b*c^2*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^9*d^4 + 16*a^4*c^7*e^4 + b^4*c^7*d^4 - 8*a*b^2*c^8*d^4 - 2*b^5*c^6*d^3*e + a^2*b^4*c^5*e^4 - 8*a^3*b^2*c^6*e^4 + 32*a^3*c^8*d^2*e^2 + b^6*c^5*d^2*e^2 + 16*a*b^3*c^7*d^3*e - 2*a*b^5*c^5*d*e^3 - 32*a^2*b*c^8*d^3*e - 32*a^3*b*c^7*d*e^3 - 6*a*b^4*c^6*d^2*e^2 + 16*a^2*b^3*c^6*d*e^3)))^(1/2) + (2*x*(4*a^3*b^7*e^10 + 4*b^3*c^7*d^10 + 4*b^10*d^3*e^7 - 36*a^4*b^5*c*e^10 - 80*a^6*b*c^3*e^10 - 4*a*b^9*d^2*e^8 - 4*a^2*b^8*d*e^9 - 64*a^2*c^8*d^9*e - 56*a^6*c^4*d*e^9 - 8*b^4*c^6*d^9*e - 8*b^9*c*d^4*e^6 + 100*a^5*b^3*c^2*e^10 + 8*a^4*c^6*d^5*e^5 + 16*a^5*c^5*d^3*e^7 + 4*b^5*c^5*d^8*e^2 + 4*b^8*c^2*d^5*e^5 - 16*a*b*c^8*d^10 + 80*a^2*b^4*c^4*d^5*e^5 - 160*a^2*b^5*c^3*d^4*e^6 + 16*a^2*b^6*c^2*d^3*e^7 - 64*a^3*b^2*c^5*d^5*e^5 + 128*a^3*b^3*c^4*d^4*e^6 + 96*a^3*b^4*c^3*d^3*e^7 + 8*a^3*b^5*c^2*d^2*e^8 - 120*a^4*b^2*c^4*d^3*e^7 - 124*a^4*b^3*c^3*d^2*e^8 + 48*a*b^2*c^7*d^9*e - 24*a*b^8*c*d^3*e^7 + 48*a^3*b^6*c*d*e^9 - 28*a*b^3*c^6*d^8*e^2 - 32*a*b^6*c^3*d^5*e^5 + 64*a*b^7*c^2*d^4*e^6 + 48*a^2*b*c^7*d^8*e^2 + 20*a^2*b^7*c*d^2*e^8 - 16*a^4*b*c^5*d^4*e^6 - 184*a^4*b^4*c^2*d*e^9 + 96*a^5*b*c^4*d^2*e^8 + 240*a^5*b^2*c^3*d*e^9))/(c^3*e^3))*(-(b^9*d^2 + a^2*b^7*e^2 + b^6*d^2*(-(4*a*c - b^2)^3)^(1/2) + 28*a^4*b*c^4*d^2 - 9*a^3*b^5*c*e^2 - 20*a^5*b*c^3*e^2 - 2*a*b^8*d*e + 42*a^2*b^5*c^2*d^2 - 63*a^3*b^3*c^3*d^2 + a^2*b^4*e^2*(-(4*a*c - b^2)^3)^(1/2) - a^3*c^3*d^2*(-(4*a*c - b^2)^3)^(1/2) + 25*a^4*b^3*c^2*e^2 + a^4*c^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - 11*a*b^7*c*d^2 - 16*a^5*c^4*d*e - 2*a*b^5*d*e*(-(4*a*c - b^2)^3)^(1/2) + 20*a^2*b^6*c*d*e + 6*a^2*b^2*c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) - 5*a*b^4*c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 66*a^3*b^4*c^2*d*e + 76*a^4*b^2*c^3*d*e - 3*a^3*b^2*c*e^2*(-(4*a*c - b^2)^3)^(1/2) + 8*a^2*b^3*c*d*e*(-(4*a*c - b^2)^

3)^(1/2) - 6*a^3*b*c^2*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^9*d^4 + 16*a^4*c^7*e^4 + b^4*c^7*d^4 - 8*a*b^2*c^8*d^4 - 2*b^5*c^6*d^3*e + a^2*b^4*c^5*e^4 - 8*a^3*b^2*c^6*e^4 + 32*a^3*c^8*d^2*e^2 + b^6*c^5*d^2*e^2 + 16*a*b^3*c^7*d^3*e - 2*a*b^5*c^5*d*e^3 - 32*a^2*b*c^8*d^3*e - 32*a^3*b*c^7*d*e^3 - 6*a*b^4*c^6*d^2*e^2 + 16*a^2*b^3*c^6*d*e^3)))^(1/2) - (16*a^3*c^6*d^9 + 4*a*b^4*c^4*d^9 + 4*a*b^8*d^5*e^4 + 4*a^5*b^4*d*e^8 + 4*a^7*c^2*d*e^8 - 20*a^2*b^2*c^5*d^9 - 4*a^2*b^7*d^4*e^5 - 4*a^4*b^5*d^2*e^7 - 64*a^4*c^5*d^7*e^2 + 64*a^5*c^4*d^5*e^4 + 4*a^6*c^3*d^3*e^6 - 36*a^2*b^4*c^3*d^7*e^2 - 40*a^2*b^5*c^2*d^6*e^3 + 96*a^3*b^2*c^4*d^7*e^2 + 128*a^3*b^3*c^3*d^6*e^3 + 164*a^3*b^4*c^2*d^5*e^4 - 224*a^4*b^2*c^3*d^5*e^4 - 104*a^4*b^3*c^2*d^4*e^5 - 20*a^5*b^2*c^2*d^3*e^6 + 4*a*b^5*c^3*d^8*e + 4*a*b^7*c*d^6*e^3 + 64*a^3*b*c^5*d^8*e - 12*a^6*b^2*c*d*e^8 + 4*a*b^6*c^2*d^7*e^2 - 32*a^2*b^3*c^4*d^8*e - 44*a^2*b^6*c*d^5*e^4 + 36*a^3*b^5*c*d^4*e^5 - 128*a^4*b*c^4*d^6*e^3 + 8*a^4*b^4*c*d^3*e^6 + 88*a^5*b*c^3*d^4*e^5 + 8*a^5*b^3*c*d^2*e^7 + 4*a^6*b*c^2*d^2*e^7)/(c^3*e^3))*(-(b^9*d^2 + a^2*b^7*e^2 + b^6*d^2*(-(4*a*c - b^2)^3)^(1/2) + 28*a^4*b*c^4*d^2 - 9*a^3*b^5*c*e^2 - 20*a^5*b*c^3*e^2 - 2*a*b^8*d*e + 42*a^2*b^5*c^2*d^2 - 63*a^3*b^3*c^3*d^2 + a^2*b^4*e^2*(-(4*a*c - b^2)^3)^(1/2) - a^3*c^3*d^2*(-(4*a*c - b^2)^3)^(1/2) + 25*a^4*b^3*c^2*e^2 + a^4*c^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - 11*a*b^7*c*d^2 - 16*a^5*c^4*d*e - 2*a*b^5*d*e*(-(4*a*c - b^2)^3)^(1/2) + 20*a^2*b^6*c*d*e + 6*a^2*b^2*c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) - 5*a*b^4*c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 66*a^3*b^4*c^2*d*e + 76*a^4*b^2*c^3*d*e - 3*a^3*b^2*c*e^2*(-(4*a*c - b^2)^3)^(1/2) + 8*a^2*b^3*c*d*e*(-(4*a*c - b^2)^3)^(1/2) - 6*a^3*b*c^2*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^9*d^4 + 16*a^4*c^7*e^4 + b^4*c^7*d^4 - 8*a*b^2*c^8*d^4 - 2*b^5*c^6*d^3*e + a^2*b^4*c^5*e^4 - 8*a^3*b^2*c^6*e^4 + 32*a^3*c^8*d^2*e^2 + b^6*c^5*d^2*e^2 + 16*a*b^3*c^7*d^3*e - 2*a*b^5*c^5*d*e^3 - 32*a^2*b*c^8*d^3*e - 32*a^3*b*c^7*d*e^3 - 6*a*b^4*c^6*d^2*e^2 + 16*a^2*b^3*c^6*d*e^3)))^(1/2) + (2*x*(a^8*e^8 + b^8*d^8 + 2*a^4*c^4*d^8 + 20*a^2*b^4*c^2*d^8 - 16*a^3*b^2*c^3*d^8 - 8*a*b^6*c*d^8))/(c^3*e^3))*(-(b^9*d^2 + a^2*b^7*e^2 + b^6*d^2*(-(4*a*c - b^2)^3)^(1/2) + 28*a^4*b*c^4*d^2 - 9*a^3*b^5*c*e^2 - 20*a^5*b*c^3*e^2 - 2*a*b^8*d*e + 42*a^2*b^5*c^2*d^2 - 63*a^3*b^3*c^3*d^2 + a^2*b^4*e^2*(-(4*a*c - b^2)^3)^(1/2) - a^3*c^3*d^2*(-(4*a*c - b^2)^3)^(1/2) + 25*a^4*b^3*c^2*e^2 + a^4*c^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - 11*a*b^7*c*d^2 - 16*a^5*c^4*d*e - 2*a*b^5*d*e*(-(4*a*c - b^2)^3)^(1/2) + 20*a^2*b^6*c*d*e + 6*a^2*b^2*c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) - 5*a*b^4*c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 66*a^3*b^4*c^2*d*e + 76*a^4*b^2*c^3*d*e - 3*a^3*b^2*c*e^2*(-(4*a*c - b^2)^3)^(1/2) + 8*a^2*b^3*c*d*e*(-(4*a*c - b^2)^3)^(1/2) - 6*a^3*b*c^2*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^9*d^4 + 16*a^4*c^7*e^4 + b^4*c^7*d^4 - 8*a*b^2*c^8*d^4 - 2*b^5*c^6*d^3*e + a^2*b^4*c^5*e^4 - 8*a^3*b^2*c^6*e^4 + 32*a^3*c^8*d^2*e^2 + b^6*c^5*d^2*e^2 + 16*a*b^3*c^7*d^3*e - 2*a*b^5*c^5*d*e^3 - 32*a^2*b*c^8*d^3*e - 32*a^3*b*c^7*d*e^3 - 6*a*b^4*c^6*d^2*e^2 + 16*a^2*b^3*c^6*d*e^3)))^(1/2)*1i - (((((192*a^3*c^8*d^6*e^5 + 384*a^4*c^7*d^4*e^7 + 192*a^5*c^6*d^2*e^9 - 48*a^2*b^2*c^7*d^6*e^5 + 96*a^2*b^3*c^6*d^5*e^6 - 48*a^2*b^4*c^5*d^4*e^7 + 96*a^3*b^2*c^6*d^4*e^7 + 96*a^3*b^3*c^5*d^3*e^8 - 48*a^4*b^2*c^5*d^2*e^9 - 384*a^3*b*c^7*d^5*e^6 - 384*a^4*b*c^6*d^3*e^8)/(

c^3*e^3) + (2*x*(-(b^9*d^2 + a^2*b^7*e^2 + b^6*d^2*(-(4*a*c - b^2)^3)^(1/2)
 + 28*a^4*b*c^4*d^2 - 9*a^3*b^5*c*e^2 - 20*a^5*b*c^3*e^2 - 2*a*b^8*d*e + 42
*a^2*b^5*c^2*d^2 - 63*a^3*b^3*c^3*d^2 + a^2*b^4*e^2*(-(4*a*c - b^2)^3)^(1/2
) - a^3*c^3*d^2*(-(4*a*c - b^2)^3)^(1/2) + 25*a^4*b^3*c^2*e^2 + a^4*c^2*e^2
*(-(4*a*c - b^2)^3)^(1/2) - 11*a*b^7*c*d^2 - 16*a^5*c^4*d*e - 2*a*b^5*d*e*(
-(4*a*c - b^2)^3)^(1/2) + 20*a^2*b^6*c*d*e + 6*a^2*b^2*c^2*d^2*(-(4*a*c - b
^2)^3)^(1/2) - 5*a*b^4*c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 66*a^3*b^4*c^2*d*e
+ 76*a^4*b^2*c^3*d*e - 3*a^3*b^2*c*e^2*(-(4*a*c - b^2)^3)^(1/2) + 8*a^2*b^3
*c*d*e*(-(4*a*c - b^2)^3)^(1/2) - 6*a^3*b*c^2*d*e*(-(4*a*c - b^2)^3)^(1/2))
/(8*(16*a^2*c^9*d^4 + 16*a^4*c^7*e^4 + b^4*c^7*d^4 - 8*a*b^2*c^8*d^4 - 2*b^
5*c^6*d^3*e + a^2*b^4*c^5*e^4 - 8*a^3*b^2*c^6*e^4 + 32*a^3*c^8*d^2*e^2 + b^
6*c^5*d^2*e^2 + 16*a*b^3*c^7*d^3*e - 2*a*b^5*c^5*d*e^3 - 32*a^2*b*c^8*d^3*e
 - 32*a^3*b*c^7*d*e^3 - 6*a*b^4*c^6*d^2*e^2 + 16*a^2*b^3*c^6*d*e^3)))^(1/2)
*(128*a^4*b^2*c^6*e^12 - 16*a^3*b^4*c^5*e^12 - 256*a^5*c^7*e^12 + 256*a^2*c
^10*d^6*e^6 + 256*a^3*c^9*d^4*e^8 - 256*a^4*c^8*d^2*e^10 - 16*b^3*c^9*d^7*e
^5 + 64*b^4*c^8*d^6*e^6 - 96*b^5*c^7*d^5*e^7 + 64*b^6*c^6*d^4*e^8 - 16*b^7*
c^5*d^3*e^9 + 256*a^2*b^2*c^8*d^4*e^8 + 144*a^2*b^3*c^7*d^3*e^9 - 96*a^2*b^
4*c^6*d^2*e^10 + 192*a^3*b^2*c^7*d^2*e^10 + 64*a*b*c^10*d^7*e^5 + 320*a^4*b
*c^7*d*e^11 - 320*a*b^2*c^9*d^6*e^6 + 528*a*b^3*c^8*d^5*e^7 - 336*a*b^4*c^7
*d^4*e^8 + 48*a*b^5*c^6*d^3*e^9 + 16*a*b^6*c^5*d^2*e^10 - 576*a^2*b*c^9*d^5
*e^7 + 16*a^2*b^5*c^5*d*e^11 - 320*a^3*b*c^8*d^3*e^9 - 144*a^3*b^3*c^6*d*e^
11))/(c^3*e^3))*(-(b^9*d^2 + a^2*b^7*e^2 + b^6*d^2*(-(4*a*c - b^2)^3)^(1/2)
 + 28*a^4*b*c^4*d^2 - 9*a^3*b^5*c*e^2 - 20*a^5*b*c^3*e^2 - 2*a*b^8*d*e + 42
*a^2*b^5*c^2*d^2 - 63*a^3*b^3*c^3*d^2 + a^2*b^4*e^2*(-(4*a*c - b^2)^3)^(1/2
) - a^3*c^3*d^2*(-(4*a*c - b^2)^3)^(1/2) + 25*a^4*b^3*c^2*e^2 + a^4*c^2*e^2
*(-(4*a*c - b^2)^3)^(1/2) - 11*a*b^7*c*d^2 - 16*a^5*c^4*d*e - 2*a*b^5*d*e*(
-(4*a*c - b^2)^3)^(1/2) + 20*a^2*b^6*c*d*e + 6*a^2*b^2*c^2*d^2*(-(4*a*c - b
^2)^3)^(1/2) - 5*a*b^4*c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 66*a^3*b^4*c^2*d*e
+ 76*a^4*b^2*c^3*d*e - 3*a^3*b^2*c*e^2*(-(4*a*c - b^2)^3)^(1/2) + 8*a^2*b^3
*c*d*e*(-(4*a*c - b^2)^3)^(1/2) - 6*a^3*b*c^2*d*e*(-(4*a*c - b^2)^3)^(1/2))
/(8*(16*a^2*c^9*d^4 + 16*a^4*c^7*e^4 + b^4*c^7*d^4 - 8*a*b^2*c^8*d^4 - 2*b^
5*c^6*d^3*e + a^2*b^4*c^5*e^4 - 8*a^3*b^2*c^6*e^4 + 32*a^3*c^8*d^2*e^2 + b^
6*c^5*d^2*e^2 + 16*a*b^3*c^7*d^3*e - 2*a*b^5*c^5*d*e^3 - 32*a^2*b*c^8*d^3*e
 - 32*a^3*b*c^7*d*e^3 - 6*a*b^4*c^6*d^2*e^2 + 16*a^2*b^3*c^6*d*e^3)))^(1/2)
 - (2*x*(4*a^3*b^7*e^10 + 4*b^3*c^7*d^10 + 4*b^10*d^3*e^7 - 36*a^4*b^5*c*e^
10 - 80*a^6*b*c^3*e^10 - 4*a*b^9*d^2*e^8 - 4*a^2*b^8*d*e^9 - 64*a^2*c^8*d^9
*e - 56*a^6*c^4*d*e^9 - 8*b^4*c^6*d^9*e - 8*b^9*c*d^4*e^6 + 100*a^5*b^3*c^2
*e^10 + 8*a^4*c^6*d^5*e^5 + 16*a^5*c^5*d^3*e^7 + 4*b^5*c^5*d^8*e^2 + 4*b^8*
c^2*d^5*e^5 - 16*a*b*c^8*d^10 + 80*a^2*b^4*c^4*d^5*e^5 - 160*a^2*b^5*c^3*d^
4*e^6 + 16*a^2*b^6*c^2*d^3*e^7 - 64*a^3*b^2*c^5*d^5*e^5 + 128*a^3*b^3*c^4*d
^4*e^6 + 96*a^3*b^4*c^3*d^3*e^7 + 8*a^3*b^5*c^2*d^2*e^8 - 120*a^4*b^2*c^4*d
^3*e^7 - 124*a^4*b^3*c^3*d^2*e^8 + 48*a*b^2*c^7*d^9*e - 24*a*b^8*c*d^3*e^7
+ 48*a^3*b^6*c*d*e^9 - 28*a*b^3*c^6*d^8*e^2 - 32*a*b^6*c^3*d^5*e^5 + 64*a*b
^7*c^2*d^4*e^6 + 48*a^2*b*c^7*d^8*e^2 + 20*a^2*b^7*c*d^2*e^8 - 16*a^4*b*c^5
*d^4*e^6 - 184*a^4*b^4*c^2*d*e^9 + 96*a^5*b*c^4*d^2*e^8 + 240*a^5*b^2*c^3*d

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*e^9))/(c^3*e^3))*(-(b^9*d^2 + a^2*b^7*e^2 + b^6*d^2*(-(4*a*c - b^2)^3)^(1/
2) + 28*a^4*b*c^4*d^2 - 9*a^3*b^5*c*e^2 - 20*a^5*b*c^3*e^2 - 2*a*b^8*d*e +
42*a^2*b^5*c^2*d^2 - 63*a^3*b^3*c^3*d^2 + a^2*b^4*e^2*(-(4*a*c - b^2)^3)^(1
/2) - a^3*c^3*d^2*(-(4*a*c - b^2)^3)^(1/2) + 25*a^4*b^3*c^2*e^2 + a^4*c^2*e
^2*(-(4*a*c - b^2)^3)^(1/2) - 11*a*b^7*c*d^2 - 16*a^5*c^4*d*e - 2*a*b^5*d*e
*(-(4*a*c - b^2)^3)^(1/2) + 20*a^2*b^6*c*d*e + 6*a^2*b^2*c^2*d^2*(-(4*a*c -
 b^2)^3)^(1/2) - 5*a*b^4*c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 66*a^3*b^4*c^2*d*
e + 76*a^4*b^2*c^3*d*e - 3*a^3*b^2*c*e^2*(-(4*a*c - b^2)^3)^(1/2) + 8*a^2*b
^3*c*d*e*(-(4*a*c - b^2)^3)^(1/2) - 6*a^3*b*c^2*d*e*(-(4*a*c - b^2)^3)^(1/2
))/(8*(16*a^2*c^9*d^4 + 16*a^4*c^7*e^4 + b^4*c^7*d^4 - 8*a*b^2*c^8*d^4 - 2*
b^5*c^6*d^3*e + a^2*b^4*c^5*e^4 - 8*a^3*b^2*c^6*e^4 + 32*a^3*c^8*d^2*e^2 +
b^6*c^5*d^2*e^2 + 16*a*b^3*c^7*d^3*e - 2*a*b^5*c^5*d*e^3 - 32*a^2*b*c^8*d^3
*e - 32*a^3*b*c^7*d*e^3 - 6*a*b^4*c^6*d^2*e^2 + 16*a^2*b^3*c^6*d*e^3)))^(1/
2) - (16*a^3*c^6*d^9 + 4*a*b^4*c^4*d^9 + 4*a*b^8*d^5*e^4 + 4*a^5*b^4*d*e^8
+ 4*a^7*c^2*d*e^8 - 20*a^2*b^2*c^5*d^9 - 4*a^2*b^7*d^4*e^5 - 4*a^4*b^5*d^2*
e^7 - 64*a^4*c^5*d^7*e^2 + 64*a^5*c^4*d^5*e^4 + 4*a^6*c^3*d^3*e^6 - 36*a^2*
b^4*c^3*d^7*e^2 - 40*a^2*b^5*c^2*d^6*e^3 + 96*a^3*b^2*c^4*d^7*e^2 + 128*a^3
*b^3*c^3*d^6*e^3 + 164*a^3*b^4*c^2*d^5*e^4 - 224*a^4*b^2*c^3*d^5*e^4 - 104*
a^4*b^3*c^2*d^4*e^5 - 20*a^5*b^2*c^2*d^3*e^6 + 4*a*b^5*c^3*d^8*e + 4*a*b^7*
c*d^6*e^3 + 64*a^3*b*c^5*d^8*e - 12*a^6*b^2*c*d*e^8 + 4*a*b^6*c^2*d^7*e^2 -
 32*a^2*b^3*c^4*d^8*e - 44*a^2*b^6*c*d^5*e^4 + 36*a^3*b^5*c*d^4*e^5 - 128*a
^4*b*c^4*d^6*e^3 + 8*a^4*b^4*c*d^3*e^6 + 88*a^5*b*c^3*d^4*e^5 + 8*a^5*b^3*c
*d^2*e^7 + 4*a^6*b*c^2*d^2*e^7)/(c^3*e^3))*(-(b^9*d^2 + a^2*b^7*e^2 + b^6*d
^2*(-(4*a*c - b^2)^3)^(1/2) + 28*a^4*b*c^4*d^2 - 9*a^3*b^5*c*e^2 - 20*a^5*b
*c^3*e^2 - 2*a*b^8*d*e + 42*a^2*b^5*c^2*d^2 - 63*a^3*b^3*c^3*d^2 + a^2*b^4*
e^2*(-(4*a*c - b^2)^3)^(1/2) - a^3*c^3*d^2*(-(4*a*c - b^2)^3)^(1/2) + 25*a^
4*b^3*c^2*e^2 + a^4*c^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - 11*a*b^7*c*d^2 - 16*
a^5*c^4*d*e - 2*a*b^5*d*e*(-(4*a*c - b^2)^3)^(1/2) + 20*a^2*b^6*c*d*e + 6*a
^2*b^2*c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) - 5*a*b^4*c*d^2*(-(4*a*c - b^2)^3)^
(1/2) - 66*a^3*b^4*c^2*d*e + 76*a^4*b^2*c^3*d*e - 3*a^3*b^2*c*e^2*(-(4*a*c
- b^2)^3)^(1/2) + 8*a^2*b^3*c*d*e*(-(4*a*c - b^2)^3)^(1/2) - 6*a^3*b*c^2*d*
e*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^9*d^4 + 16*a^4*c^7*e^4 + b^4*c^7*d
^4 - 8*a*b^2*c^8*d^4 - 2*b^5*c^6*d^3*e + a^2*b^4*c^5*e^4 - 8*a^3*b^2*c^6*e^
4 + 32*a^3*c^8*d^2*e^2 + b^6*c^5*d^2*e^2 + 16*a*b^3*c^7*d^3*e - 2*a*b^5*c^5
*d*e^3 - 32*a^2*b*c^8*d^3*e - 32*a^3*b*c^7*d*e^3 - 6*a*b^4*c^6*d^2*e^2 + 16
*a^2*b^3*c^6*d*e^3)))^(1/2) - (2*x*(a^8*e^8 + b^8*d^8 + 2*a^4*c^4*d^8 + 20*
a^2*b^4*c^2*d^8 - 16*a^3*b^2*c^3*d^8 - 8*a*b^6*c*d^8))/(c^3*e^3))*(-(b^9*d^
2 + a^2*b^7*e^2 + b^6*d^2*(-(4*a*c - b^2)^3)^(1/2) + 28*a^4*b*c^4*d^2 - 9*a
^3*b^5*c*e^2 - 20*a^5*b*c^3*e^2 - 2*a*b^8*d*e + 42*a^2*b^5*c^2*d^2 - 63*a^3
*b^3*c^3*d^2 + a^2*b^4*e^2*(-(4*a*c - b^2)^3)^(1/2) - a^3*c^3*d^2*(-(4*a*c
- b^2)^3)^(1/2) + 25*a^4*b^3*c^2*e^2 + a^4*c^2*e^2*(-(4*a*c - b^2)^3)^(1/2)
 - 11*a*b^7*c*d^2 - 16*a^5*c^4*d*e - 2*a*b^5*d*e*(-(4*a*c - b^2)^3)^(1/2) +
 20*a^2*b^6*c*d*e + 6*a^2*b^2*c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) - 5*a*b^4*c*
d^2*(-(4*a*c - b^2)^3)^(1/2) - 66*a^3*b^4*c^2*d*e + 76*a^4*b^2*c^3*d*e - 3*
a^3*b^2*c*e^2*(-(4*a*c - b^2)^3)^(1/2) + 8*a^2*b^3*c*d*e*(-(4*a*c - b^2)^3)
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^(1/2) - 6*a^3*b*c^2*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^9*d^4 + 16*a^4*c^7*e^4 + b^4*c^7*d^4 - 8*a*b^2*c^8*d^4 - 2*b^5*c^6*d^3*e + a^2*b^4*c^5*e^4 - 8*a^3*b^2*c^6*e^4 + 32*a^3*c^8*d^2*e^2 + b^6*c^5*d^2*e^2 + 16*a*b^3*c^7*d^3*e - 2*a*b^5*c^5*d*e^3 - 32*a^2*b*c^8*d^3*e - 32*a^3*b*c^7*d*e^3 - 6*a*b^4*c^6*d^2*e^2 + 16*a^2*b^3*c^6*d*e^3)))^(1/2)*1i)/((((((192*a^3*c^8*d^6*e^5 + 384*a^4*c^7*d^4*e^7 + 192*a^5*c^6*d^2*e^9 - 48*a^2*b^2*c^7*d^6*e^5 + 96*a^2*b^3*c^6*d^5*e^6 - 48*a^2*b^4*c^5*d^4*e^7 + 96*a^3*b^2*c^6*d^4*e^7 + 96*a^3*b^3*c^5*d^3*e^8 - 48*a^4*b^2*c^5*d^2*e^9 - 384*a^3*b*c^7*d^5*e^6 - 384*a^4*b*c^6*d^3*e^8)/(c^3*e^3) - (2*x*(-(b^9*d^2 + a^2*b^7*e^2 + b^6*d^2*(-(4*a*c - b^2)^3)^(1/2) + 28*a^4*b*c^4*d^2 - 9*a^3*b^5*c*e^2 - 20*a^5*b*c^3*e^2 - 2*a*b^8*d*e + 42*a^2*b^5*c^2*d^2 - 63*a^3*b^3*c^3*d^2 + a^2*b^4*e^2*(-(4*a*c - b^2)^3)^(1/2) - a^3*c^3*d^2*(-(4*a*c - b^2)^3)^(1/2) + 25*a^4*b^3*c^2*e^2 + a^4*c^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - 11*a*b^7*c*d^2 - 16*a^5*c^4*d*e - 2*a*b^5*d*e*(-(4*a*c - b^2)^3)^(1/2) + 20*a^2*b^6*c*d*e + 6*a^2*b^2*c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) - 5*a*b^4*c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 66*a^3*b^4*c^2*d*e + 76*a^4*b^2*c^3*d*e - 3*a^3*b^2*c*e^2*(-(4*a*c - b^2)^3)^(1/2) + 8*a^2*b^3*c*d*e*(-(4*a*c - b^2)^3)^(1/2) - 6*a^3*b*c^2*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^9*d^4 + 16*a^4*c^7*e^4 + b^4*c^7*d^4 - 8*a*b^2*c^8*d^4 - 2*b^5*c^6*d^3*e + a^2*b^4*c^5*e^4 - 8*a^3*b^2*c^6*e^4 + 32*a^3*c^8*d^2*e^2 + b^6*c^5*d^2*e^2 + 16*a*b^3*c^7*d^3*e - 2*a*b^5*c^5*d*e^3 - 32*a^2*b*c^8*d^3*e - 32*a^3*b*c^7*d*e^3 - 6*a*b^4*c^6*d^2*e^2 + 16*a^2*b^3*c^6*d*e^3)))^(1/2)*(128*a^4*b^2*c^6*e^12 - 16*a^3*b^4*c^5*e^12 - 256*a^5*c^7*e^12 + 256*a^2*c^10*d^6*e^6 + 256*a^3*c^9*d^4*e^8 - 256*a^4*c^8*d^2*e^10 - 16*b^3*c^9*d^7*e^5 + 64*b^4*c^8*d^6*e^6 - 96*b^5*c^7*d^5*e^7 + 64*b^6*c^6*d^4*e^8 - 16*b^7*c^5*d^3*e^9 + 256*a^2*b^2*c^8*d^4*e^8 + 144*a^2*b^3*c^7*d^3*e^9 - 96*a^2*b^4*c^6*d^2*e^10 + 192*a^3*b^2*c^7*d^2*e^10 + 64*a*b*c^10*d^7*e^5 + 320*a^4*b*c^7*d*e^11 - 320*a*b^2*c^9*d^6*e^6 + 528*a*b^3*c^8*d^5*e^7 - 336*a*b^4*c^7*d^4*e^8 + 48*a*b^5*c^6*d^3*e^9 + 16*a*b^6*c^5*d^2*e^10 - 576*a^2*b*c^9*d^5*e^7 + 16*a^2*b^5*c^5*d*e^11 - 320*a^3*b*c^8*d^3*e^9 - 144*a^3*b^3*c^6*d*e^11))/(c^3*e^3))*(-(b^9*d^2 + a^2*b^7*e^2 + b^6*d^2*(-(4*a*c - b^2)^3)^(1/2) + 28*a^4*b*c^4*d^2 - 9*a^3*b^5*c*e^2 - 20*a^5*b*c^3*e^2 - 2*a*b^8*d*e + 42*a^2*b^5*c^2*d^2 - 63*a^3*b^3*c^3*d^2 + a^2*b^4*e^2*(-(4*a*c - b^2)^3)^(1/2) - a^3*c^3*d^2*(-(4*a*c - b^2)^3)^(1/2) + 25*a^4*b^3*c^2*e^2 + a^4*c^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - 11*a*b^7*c*d^2 - 16*a^5*c^4*d*e - 2*a*b^5*d*e*(-(4*a*c - b^2)^3)^(1/2) + 20*a^2*b^6*c*d*e + 6*a^2*b^2*c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) - 5*a*b^4*c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 66*a^3*b^4*c^2*d*e + 76*a^4*b^2*c^3*d*e - 3*a^3*b^2*c*e^2*(-(4*a*c - b^2)^3)^(1/2) + 8*a^2*b^3*c*d*e*(-(4*a*c - b^2)^3)^(1/2) - 6*a^3*b*c^2*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^9*d^4 + 16*a^4*c^7*e^4 + b^4*c^7*d^4 - 8*a*b^2*c^8*d^4 - 2*b^5*c^6*d^3*e + a^2*b^4*c^5*e^4 - 8*a^3*b^2*c^6*e^4 + 32*a^3*c^8*d^2*e^2 + b^6*c^5*d^2*e^2 + 16*a*b^3*c^7*d^3*e - 2*a*b^5*c^5*d*e^3 - 32*a^2*b*c^8*d^3*e - 32*a^3*b*c^7*d*e^3 - 6*a*b^4*c^6*d^2*e^2 + 16*a^2*b^3*c^6*d*e^3)))^(1/2) + (2*x*(4*a^3*b^7*e^10 + 4*b^3*c^7*d^10 + 4*b^10*d^3*e^7 - 36*a^4*b^5*c*e^10 - 80*a^6*b*c^3*e^10 - 4*a*b^9*d^2*e^8 - 4*a^2*b^8*d*e^9 - 64*a^2*c^8*d^9*e - 56*a^6*c^4*d*e^9 - 8*b^4*c^6*d^9*e - 8*b^9*c* d^4*e^6 + 100*a^5*b^3*c^2*e^10 + 8*a^4*c^6*d^5*e^5 + 16*a^5*c^5*d^3*e^7 + 4*b^5*c^5*d^8*e^2 + 4*b^8*c^2*d^5*e^5 - 16*a*b*c^8*d^10 + 80*a^2*b^4*c^4*d^5*e^5 - 160*a^2*b^5*c^3*d^4*e^6 + 16*a^2*b^6*c^2*d^3*e^7 - 64*a^3*b^2*c^5*d^5*e^5 + 128*a^3*b^3*c^4*d^4*e^6 + 96*a^3*b^4*c^3*d^3*e^7 + 8*a^3*b^5*c^2*d^2*e^8 - 120*a^4*b^2*c^4*d^3*e^7 - 124*a^4*b^3*c^3*d^2*e^8 + 48*a*b^2*c^7*d^9*e - 24*a*b^8*c*d^3*e^7 + 48*a^3*b^6*c*d*e^9 - 28*a*b^3*c^6*d^8*e^2 - 32*a*b^6*c^3*d^5*e^5 + 64*a*b^7*c^2*d^4*e^6 + 48*a^2*b*c^7*d^8*e^2 + 20*a^2*b^7*c*d^2*e^8 - 16*a^4*b*c^5*d^4*e^6 - 184*a^4*b^4*c^2*d*e^9 + 96*a^5*b*c^4*d^2*e^8 + 240*a^5*b^2*c^3*d*e^9))/(c^3*e^3))*(-(b^9*d^2 + a^2*b^7*e^2 + b^6*d^2*(-(4*a*c - b^2)^3)^(1/2) + 28*a^4*b*c^4*d^2 - 9*a^3*b^5*c*e^2 - 20*a^5*b*c^3*e^2 - 2*a*b^8*d*e + 42*a^2*b^5*c^2*d^2 - 63*a^3*b^3*c^3*d^2 + a^2*b^4*e^2*(-(4*a*c - b^2)^3)^(1/2) - a^3*c^3*d^2*(-(4*a*c - b^2)^3)^(1/2) + 25*a^4*b^3*c^2*e^2 + a^4*c^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - 11*a*b^7*c*d^2 - 16*a^5*c^4*d*e - 2*a*b^5*d*e*(-(4*a*c - b^2)^3)^(1/2) + 20*a^2*b^6*c*d*e + 6*a^2*b^2*c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) - 5*a*b^4*c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 66*a^3*b^4*c^2*d*e + 76*a^4*b^2*c^3*d*e - 3*a^3*b^2*c*e^2*(-(4*a*c - b^2)^3)^(1/2) + 8*a^2*b^3*c*d*e*(-(4*a*c - b^2)^3)^(1/2) - 6*a^3*b*c^2*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^9*d^4 + 16*a^4*c^7*e^4 + b^4*c^7*d^4 - 8*a*b^2*c^8*d^4 - 2*b^5*c^6*d^3*e + a^2*b^4*c^5*e^4 - 8*a^3*b^2*c^6*e^4 + 32*a^3*c^8*d^2*e^2 + b^6*c^5*d^2*e^2 + 16*a*b^3*c^7*d^3*e - 2*a*b^5*c^5*d*e^3 - 32*a^2*b*c^8*d^3*e - 32*a^3*b*c^7*d*e^3 - 6*a*b^4*c^6*d^2*e^2 + 16*a^2*b^3*c^6*d*e^3)))^(1/2) - (16*a^3*c^6*d^9 + 4*a*b^4*c^4*d^9 + 4*a*b^8*d^5*e^4 + 4*a^5*b^4*d*e^8 + 4*a^7*c^2*d*e^8 - 20*a^2*b^2*c^5*d^9 - 4*a^2*b^7*d^4*e^5 - 4*a^4*b^5*d^2*e^7 - 64*a^4*c^5*d^7*e^2 + 64*a^5*c^4*d^5*e^4 + 4*a^6*c^3*d^3*e^6 - 36*a^2*b^4*c^3*d^7*e^2 - 40*a^2*b^5*c^2*d^6*e^3 + 96*a^3*b^2*c^4*d^7*e^2 + 128*a^3*b^3*c^3*d^6*e^3 + 164*a^3*b^4*c^2*d^5*e^4 - 224*a^4*b^2*c^3*d^5*e^4 - 104*a^4*b^3*c^2*d^4*e^5 - 20*a^5*b^2*c^2*d^3*e^6 + 4*a*b^5*c^3*d^8*e + 4*a*b^7*c*d^6*e^3 + 64*a^3*b*c^5*d^8*e - 12*a^6*b^2*c*d*e^8 + 4*a*b^6*c^2*d^7*e^2 - 32*a^2*b^3*c^4*d^8*e - 44*a^2*b^6*c*d^5*e^4 + 36*a^3*b^5*c*d^4*e^5 - 128*a^4*b*c^4*d^6*e^3 + 8*a^4*b^4*c*d^3*e^6 + 88*a^5*b*c^3*d^4*e^5 + 8*a^5*b^3*c*d^2*e^7 + 4*a^6*b*c^2*d^2*e^7)/(c^3*e^3))*(-(b^9*d^2 + a^2*b^7*e^2 + b^6*d^2*(-(4*a*c - b^2)^3)^(1/2) + 28*a^4*b*c^4*d^2 - 9*a^3*b^5*c*e^2 - 20*a^5*b*c^3*e^2 - 2*a*b^8*d*e + 42*a^2*b^5*c^2*d^2 - 63*a^3*b^3*c^3*d^2 + a^2*b^4*e^2*(-(4*a*c - b^2)^3)^(1/2) - a^3*c^3*d^2*(-(4*a*c - b^2)^3)^(1/2) + 25*a^4*b^3*c^2*e^2 + a^4*c^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - 11*a*b^7*c*d^2 - 16*a^5*c^4*d*e - 2*a*b^5*d*e*(-(4*a*c - b^2)^3)^(1/2) + 20*a^2*b^6*c*d*e + 6*a^2*b^2*c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) - 5*a*b^4*c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 66*a^3*b^4*c^2*d*e + 76*a^4*b^2*c^3*d*e - 3*a^3*b^2*c*e^2*(-(4*a*c - b^2)^3)^(1/2) + 8*a^2*b^3*c*d*e*(-(4*a*c - b^2)^3)^(1/2) - 6*a^3*b*c^2*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^9*d^4 + 16*a^4*c^7*e^4 + b^4*c^7*d^4 - 8*a*b^2*c^8*d^4 - 2*b^5*c^6*d^3*e + a^2*b^4*c^5*e^4 - 8*a^3*b^2*c^6*e^4 + 32*a^3*c^8*d^2*e^2 + b^6*c^5*d^2*e^2 + 16*a*b^3*c^7*d^3*e - 2*a*b^5*c^5*d*e^3 - 32*a^2*b*c^8*d^3*e - 32*a^3*b*c^7*d*e^3 - 6*a*b^4*c^6*d^2*e^2 + 16*a^2*b^3*c^6*d*e^3)))^(1/2) + (2*x*(a^8*e^8 + b^8*d^8 + 2*a^4*c^4*d^8 + 20*a^2*b^4*c^2*d^8 - 16*a^3*b^2*c^3*d^8 - 8*a*b^6*c*d

^8))/(c^3*e^3))*(-(b^9*d^2 + a^2*b^7*e^2 + b^6*d^2*(-(4*a*c - b^2)^3)^(1/2)
 + 28*a^4*b*c^4*d^2 - 9*a^3*b^5*c*e^2 - 20*a^5*b*c^3*e^2 - 2*a*b^8*d*e + 42
*a^2*b^5*c^2*d^2 - 63*a^3*b^3*c^3*d^2 + a^2*b^4*e^2*(-(4*a*c - b^2)^3)^(1/2
) - a^3*c^3*d^2*(-(4*a*c - b^2)^3)^(1/2) + 25*a^4*b^3*c^2*e^2 + a^4*c^2*e^2
*(-(4*a*c - b^2)^3)^(1/2) - 11*a*b^7*c*d^2 - 16*a^5*c^4*d*e - 2*a*b^5*d*e*(
-(4*a*c - b^2)^3)^(1/2) + 20*a^2*b^6*c*d*e + 6*a^2*b^2*c^2*d^2*(-(4*a*c - b
^2)^3)^(1/2) - 5*a*b^4*c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 66*a^3*b^4*c^2*d*e
 + 76*a^4*b^2*c^3*d*e - 3*a^3*b^2*c*e^2*(-(4*a*c - b^2)^3)^(1/2) + 8*a^2*b^3
*c*d*e*(-(4*a*c - b^2)^3)^(1/2) - 6*a^3*b*c^2*d*e*(-(4*a*c - b^2)^3)^(1/2))
/(8*(16*a^2*c^9*d^4 + 16*a^4*c^7*e^4 + b^4*c^7*d^4 - 8*a*b^2*c^8*d^4 - 2*b^
5*c^6*d^3*e + a^2*b^4*c^5*e^4 - 8*a^3*b^2*c^6*e^4 + 32*a^3*c^8*d^2*e^2 + b^
6*c^5*d^2*e^2 + 16*a*b^3*c^7*d^3*e - 2*a*b^5*c^5*d*e^3 - 32*a^2*b*c^8*d^3*e
 - 32*a^3*b*c^7*d*e^3 - 6*a*b^4*c^6*d^2*e^2 + 16*a^2*b^3*c^6*d*e^3)))^(1/2)
 + (((((192*a^3*c^8*d^6*e^5 + 384*a^4*c^7*d^4*e^7 + 192*a^5*c^6*d^2*e^9 - 4
8*a^2*b^2*c^7*d^6*e^5 + 96*a^2*b^3*c^6*d^5*e^6 - 48*a^2*b^4*c^5*d^4*e^7 + 9
6*a^3*b^2*c^6*d^4*e^7 + 96*a^3*b^3*c^5*d^3*e^8 - 48*a^4*b^2*c^5*d^2*e^9 - 3
84*a^3*b*c^7*d^5*e^6 - 384*a^4*b*c^6*d^3*e^8)/(c^3*e^3) + (2*x*(-(b^9*d^2 +
 a^2*b^7*e^2 + b^6*d^2*(-(4*a*c - b^2)^3)^(1/2) + 28*a^4*b*c^4*d^2 - 9*a^3*
b^5*c*e^2 - 20*a^5*b*c^3*e^2 - 2*a*b^8*d*e + 42*a^2*b^5*c^2*d^2 - 63*a^3*b^
3*c^3*d^2 + a^2*b^4*e^2*(-(4*a*c - b^2)^3)^(1/2) - a^3*c^3*d^2*(-(4*a*c - b
^2)^3)^(1/2) + 25*a^4*b^3*c^2*e^2 + a^4*c^2*e^2*(-(4*a*c - b^2)^3)^(1/2) -
11*a*b^7*c*d^2 - 16*a^5*c^4*d*e - 2*a*b^5*d*e*(-(4*a*c - b^2)^3)^(1/2) + 20
*a^2*b^6*c*d*e + 6*a^2*b^2*c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) - 5*a*b^4*c*d^2
*(-(4*a*c - b^2)^3)^(1/2) - 66*a^3*b^4*c^2*d*e + 76*a^4*b^2*c^3*d*e - 3*a^3
*b^2*c*e^2*(-(4*a*c - b^2)^3)^(1/2) + 8*a^2*b^3*c*d*e*(-(4*a*c - b^2)^3)^(1
/2) - 6*a^3*b*c^2*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^9*d^4 + 16*a^4
*c^7*e^4 + b^4*c^7*d^4 - 8*a*b^2*c^8*d^4 - 2*b^5*c^6*d^3*e + a^2*b^4*c^5*e^
4 - 8*a^3*b^2*c^6*e^4 + 32*a^3*c^8*d^2*e^2 + b^6*c^5*d^2*e^2 + 16*a*b^3*c^7
*d^3*e - 2*a*b^5*c^5*d*e^3 - 32*a^2*b*c^8*d^3*e - 32*a^3*b*c^7*d*e^3 - 6*a*
b^4*c^6*d^2*e^2 + 16*a^2*b^3*c^6*d*e^3)))^(1/2)*(128*a^4*b^2*c^6*e^12 - 16*
a^3*b^4*c^5*e^12 - 256*a^5*c^7*e^12 + 256*a^2*c^10*d^6*e^6 + 256*a^3*c^9*d^
4*e^8 - 256*a^4*c^8*d^2*e^10 - 16*b^3*c^9*d^7*e^5 + 64*b^4*c^8*d^6*e^6 - 96
*b^5*c^7*d^5*e^7 + 64*b^6*c^6*d^4*e^8 - 16*b^7*c^5*d^3*e^9 + 256*a^2*b^2*c^
8*d^4*e^8 + 144*a^2*b^3*c^7*d^3*e^9 - 96*a^2*b^4*c^6*d^2*e^10 + 192*a^3*b^2
*c^7*d^2*e^10 + 64*a*b*c^10*d^7*e^5 + 320*a^4*b*c^7*d*e^11 - 320*a*b^2*c^9*
d^6*e^6 + 528*a*b^3*c^8*d^5*e^7 - 336*a*b^4*c^7*d^4*e^8 + 48*a*b^5*c^6*d^3*
e^9 + 16*a*b^6*c^5*d^2*e^10 - 576*a^2*b*c^9*d^5*e^7 + 16*a^2*b^5*c^5*d*e^11
 - 320*a^3*b*c^8*d^3*e^9 - 144*a^3*b^3*c^6*d*e^11))/(c^3*e^3))*(-(b^9*d^2 +
 a^2*b^7*e^2 + b^6*d^2*(-(4*a*c - b^2)^3)^(1/2) + 28*a^4*b*c^4*d^2 - 9*a^3*
b^5*c*e^2 - 20*a^5*b*c^3*e^2 - 2*a*b^8*d*e + 42*a^2*b^5*c^2*d^2 - 63*a^3*b^
3*c^3*d^2 + a^2*b^4*e^2*(-(4*a*c - b^2)^3)^(1/2) - a^3*c^3*d^2*(-(4*a*c - b
^2)^3)^(1/2) + 25*a^4*b^3*c^2*e^2 + a^4*c^2*e^2*(-(4*a*c - b^2)^3)^(1/2) -
11*a*b^7*c*d^2 - 16*a^5*c^4*d*e - 2*a*b^5*d*e*(-(4*a*c - b^2)^3)^(1/2) + 20
*a^2*b^6*c*d*e + 6*a^2*b^2*c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) - 5*a*b^4*c*d^2
*(-(4*a*c - b^2)^3)^(1/2) - 66*a^3*b^4*c^2*d*e + 76*a^4*b^2*c^3*d*e - 3*a^3

*b^2*c*e^2*(-(4*a*c - b^2)^3)^(1/2) + 8*a^2*b^3*c*d*e*(-(4*a*c - b^2)^3)^(1/2) - 6*a^3*b*c^2*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^9*d^4 + 16*a^4*c^7*e^4 + b^4*c^7*d^4 - 8*a*b^2*c^8*d^4 - 2*b^5*c^6*d^3*e + a^2*b^4*c^5*e^4 - 8*a^3*b^2*c^6*e^4 + 32*a^3*c^8*d^2*e^2 + b^6*c^5*d^2*e^2 + 16*a*b^3*c^7*d^3*e - 2*a*b^5*c^5*d*e^3 - 32*a^2*b*c^8*d^3*e - 32*a^3*b*c^7*d*e^3 - 6*a*b^4*c^6*d^2*e^2 + 16*a^2*b^3*c^6*d*e^3)))^(1/2) - (2*x*(4*a^3*b^7*e^10 + 4*b^3*c^7*d^10 + 4*b^10*d^3*e^7 - 36*a^4*b^5*c*e^10 - 80*a^6*b*c^3*e^10 - 4*a*b^9*d^2*e^8 - 4*a^2*b^8*d*e^9 - 64*a^2*c^8*d^9*e - 56*a^6*c^4*d*e^9 - 8*b^4*c^6*d^9*e - 8*b^9*c*d^4*e^6 + 100*a^5*b^3*c^2*e^10 + 8*a^4*c^6*d^5*e^5 + 16*a^5*c^5*d^3*e^7 + 4*b^5*c^5*d^8*e^2 + 4*b^8*c^2*d^5*e^5 - 16*a*b*c^8*d^10 + 80*a^2*b^4*c^4*d^5*e^5 - 160*a^2*b^5*c^3*d^4*e^6 + 16*a^2*b^6*c^2*d^3*e^7 - 64*a^3*b^2*c^5*d^5*e^5 + 128*a^3*b^3*c^4*d^4*e^6 + 96*a^3*b^4*c^3*d^3*e^7 + 8*a^3*b^5*c^2*d^2*e^8 - 120*a^4*b^2*c^4*d^3*e^7 - 124*a^4*b^3*c^3*d^2*e^8 + 48*a*b^2*c^7*d^9*e - 24*a*b^8*c*d^3*e^7 + 48*a^3*b^6*c*d*e^9 - 28*a*b^3*c^6*d^8*e^2 - 32*a*b^6*c^3*d^5*e^5 + 64*a*b^7*c^2*d^4*e^6 + 48*a^2*b*c^7*d^8*e^2 + 20*a^2*b^7*c*d^2*e^8 - 16*a^4*b*c^5*d^4*e^6 - 184*a^4*b^4*c^2*d*e^9 + 96*a^5*b*c^4*d^2*e^8 + 240*a^5*b^2*c^3*d*e^9))/(c^3*e^3))*(-(b^9*d^2 + a^2*b^7*e^2 + b^6*d^2*(-(4*a*c - b^2)^3)^(1/2) + 28*a^4*b*c^4*d^2 - 9*a^3*b^5*c*e^2 - 20*a^5*b*c^3*e^2 - 2*a*b^8*d*e + 42*a^2*b^5*c^2*d^2 - 63*a^3*b^3*c^3*d^2 + a^2*b^4*e^2*(-(4*a*c - b^2)^3)^(1/2) - a^3*c^3*d^2*(-(4*a*c - b^2)^3)^(1/2) + 25*a^4*b^3*c^2*e^2 + a^4*c^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - 11*a*b^7*c*d^2 - 16*a^5*c^4*d*e - 2*a*b^5*d*e*(-(4*a*c - b^2)^3)^(1/2) + 20*a^2*b^6*c*d*e + 6*a^2*b^2*c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) - 5*a*b^4*c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 66*a^3*b^4*c^2*d*e + 76*a^4*b^2*c^3*d*e - 3*a^3*b^2*c*e^2*(-(4*a*c - b^2)^3)^(1/2) + 8*a^2*b^3*c*d*e*(-(4*a*c - b^2)^3)^(1/2) - 6*a^3*b*c^2*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^9*d^4 + 16*a^4*c^7*e^4 + b^4*c^7*d^4 - 8*a*b^2*c^8*d^4 - 2*b^5*c^6*d^3*e + a^2*b^4*c^5*e^4 - 8*a^3*b^2*c^6*e^4 + 32*a^3*c^8*d^2*e^2 + b^6*c^5*d^2*e^2 + 16*a*b^3*c^7*d^3*e - 2*a*b^5*c^5*d*e^3 - 32*a^2*b*c^8*d^3*e - 32*a^3*b*c^7*d*e^3 - 6*a*b^4*c^6*d^2*e^2 + 16*a^2*b^3*c^6*d*e^3)))^(1/2) - (16*a^3*c^6*d^9 + 4*a*b^4*c^4*d^9 + 4*a*b^8*d^5*e^4 + 4*a^5*b^4*d*e^8 + 4*a^7*c^2*d*e^8 - 20*a^2*b^2*c^5*d^9 - 4*a^2*b^7*d^4*e^5 - 4*a^4*b^5*d^2*e^7 - 64*a^4*c^5*d^7*e^2 + 64*a^5*c^4*d^5*e^4 + 4*a^6*c^3*d^3*e^6 - 36*a^2*b^4*c^3*d^7*e^2 - 40*a^2*b^5*c^2*d^6*e^3 + 96*a^3*b^2*c^4*d^7*e^2 + 128*a^3*b^3*c^3*d^6*e^3 + 164*a^3*b^4*c^2*d^5*e^4 - 224*a^4*b^2*c^3*d^5*e^4 - 104*a^4*b^3*c^2*d^4*e^5 - 20*a^5*b^2*c^2*d^3*e^6 + 4*a*b^5*c^3*d^8*e + 4*a*b^7*c*d^6*e^3 + 64*a^3*b*c^5*d^8*e - 12*a^6*b^2*c*d*e^8 + 4*a*b^6*c^2*d^7*e^2 - 32*a^2*b^3*c^4*d^8*e - 44*a^2*b^6*c*d^5*e^4 + 36*a^3*b^5*c*d^4*e^5 - 128*a^4*b*c^4*d^6*e^3 + 8*a^4*b^4*c*d^3*e^6 + 88*a^5*b*c^3*d^4*e^5 + 8*a^5*b^3*c*d^2*e^7 + 4*a^6*b*c^2*d^2*e^7)/(c^3*e^3))*(-(b^9*d^2 + a^2*b^7*e^2 + b^6*d^2*(-(4*a*c - b^2)^3)^(1/2) + 28*a^4*b*c^4*d^2 - 9*a^3*b^5*c*e^2 - 20*a^5*b*c^3*e^2 - 2*a*b^8*d*e + 42*a^2*b^5*c^2*d^2 - 63*a^3*b^3*c^3*d^2 + a^2*b^4*e^2*(-(4*a*c - b^2)^3)^(1/2) - a^3*c^3*d^2*(-(4*a*c - b^2)^3)^(1/2) + 25*a^4*b^3*c^2*e^2 + a^4*c^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - 11*a*b^7*c*d^2 - 16*a^5*c^4*d*e - 2*a*b^5*d*e*(-(4*a*c - b^2)^3)^(1/2) + 20*a^2*b^6*c*d*e + 6*a^2*b^2*c^2*d^2*(-(4*a*c - b^

2)^3)^(1/2) - 5*a*b^4*c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 66*a^3*b^4*c^2*d*e + 76*a^4*b^2*c^3*d*e - 3*a^3*b^2*c*e^2*(-(4*a*c - b^2)^3)^(1/2) + 8*a^2*b^3*c*d*e*(-(4*a*c - b^2)^3)^(1/2) - 6*a^3*b*c^2*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^9*d^4 + 16*a^4*c^7*e^4 + b^4*c^7*d^4 - 8*a*b^2*c^8*d^4 - 2*b^5*c^6*d^3*e + a^2*b^4*c^5*e^4 - 8*a^3*b^2*c^6*e^4 + 32*a^3*c^8*d^2*e^2 + b^6*c^5*d^2*e^2 + 16*a*b^3*c^7*d^3*e - 2*a*b^5*c^5*d*e^3 - 32*a^2*b*c^8*d^3*e - 32*a^3*b*c^7*d*e^3 - 6*a*b^4*c^6*d^2*e^2 + 16*a^2*b^3*c^6*d*e^3)))^(1/2) - (2*x*(a^8*e^8 + b^8*d^8 + 2*a^4*c^4*d^8 + 20*a^2*b^4*c^2*d^8 - 16*a^3*b^2*c^3*d^8 - 8*a*b^6*c*d^8))/(c^3*e^3))*(-(b^9*d^2 + a^2*b^7*e^2 + b^6*d^2*(-(4*a*c - b^2)^3)^(1/2) + 28*a^4*b*c^4*d^2 - 9*a^3*b^5*c*e^2 - 20*a^5*b*c^3*e^2 - 2*a*b^8*d*e + 42*a^2*b^5*c^2*d^2 - 63*a^3*b^3*c^3*d^2 + a^2*b^4*e^2*(-(4*a*c - b^2)^3)^(1/2) - a^3*c^3*d^2*(-(4*a*c - b^2)^3)^(1/2) + 25*a^4*b^3*c^2*e^2 + a^4*c^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - 11*a*b^7*c*d^2 - 16*a^5*c^4*d*e - 2*a*b^5*d*e*(-(4*a*c - b^2)^3)^(1/2) + 20*a^2*b^6*c*d*e + 6*a^2*b^2*c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) - 5*a*b^4*c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 66*a^3*b^4*c^2*d*e + 76*a^4*b^2*c^3*d*e - 3*a^3*b^2*c*e^2*(-(4*a*c - b^2)^3)^(1/2) + 8*a^2*b^3*c*d*e*(-(4*a*c - b^2)^3)^(1/2) - 6*a^3*b*c^2*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^9*d^4 + 16*a^4*c^7*e^4 + b^4*c^7*d^4 - 8*a*b^2*c^8*d^4 - 2*b^5*c^6*d^3*e + a^2*b^4*c^5*e^4 - 8*a^3*b^2*c^6*e^4 + 32*a^3*c^8*d^2*e^2 + b^6*c^5*d^2*e^2 + 16*a*b^3*c^7*d^3*e - 2*a*b^5*c^5*d*e^3 - 32*a^2*b*c^8*d^3*e - 32*a^3*b*c^7*d*e^3 - 6*a*b^4*c^6*d^2*e^2 + 16*a^2*b^3*c^6*d*e^3)))^(1/2) + (2*(a^4*b^3*d^7 + a^7*d^4*e^3 + a^5*b^2*d^6*e + a^6*b*d^5*e^2 - 2*a^5*b*c*d^7 - a^6*c*d^6*e))/(c^3*e^3)))*(-(b^9*d^2 + a^2*b^7*e^2 + b^6*d^2*(-(4*a*c - b^2)^3)^(1/2) + 28*a^4*b*c^4*d^2 - 9*a^3*b^5*c*e^2 - 20*a^5*b*c^3*e^2 - 2*a*b^8*d*e + 42*a^2*b^5*c^2*d^2 - 63*a^3*b^3*c^3*d^2 + a^2*b^4*e^2*(-(4*a*c - b^2)^3)^(1/2) - a^3*c^3*d^2*(-(4*a*c - b^2)^3)^(1/2) + 25*a^4*b^3*c^2*e^2 + a^4*c^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - 11*a*b^7*c*d^2 - 16*a^5*c^4*d*e - 2*a*b^5*d*e*(-(4*a*c - b^2)^3)^(1/2) + 20*a^2*b^6*c*d*e + 6*a^2*b^2*c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) - 5*a*b^4*c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 66*a^3*b^4*c^2*d*e + 76*a^4*b^2*c^3*d*e - 3*a^3*b^2*c*e^2*(-(4*a*c - b^2)^3)^(1/2) + 8*a^2*b^3*c*d*e*(-(4*a*c - b^2)^3)^(1/2) - 6*a^3*b*c^2*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^9*d^4 + 16*a^4*c^7*e^4 + b^4*c^7*d^4 - 8*a*b^2*c^8*d^4 - 2*b^5*c^6*d^3*e + a^2*b^4*c^5*e^4 - 8*a^3*b^2*c^6*e^4 + 32*a^3*c^8*d^2*e^2 + b^6*c^5*d^2*e^2 + 16*a*b^3*c^7*d^3*e - 2*a*b^5*c^5*d*e^3 - 32*a^2*b*c^8*d^3*e - 32*a^3*b*c^7*d*e^3 - 6*a*b^4*c^6*d^2*e^2 + 16*a^2*b^3*c^6*d*e^3)))^(1/2)*2i + atan(((((((192*a^3*c^8*d^6*e^5 + 384*a^4*c^7*d^4*e^7 + 192*a^5*c^6*d^2*e^9 - 48*a^2*b^2*c^7*d^6*e^5 + 96*a^2*b^3*c^6*d^5*e^6 - 48*a^2*b^4*c^5*d^4*e^7 + 96*a^3*b^2*c^6*d^4*e^7 + 96*a^3*b^3*c^5*d^3*e^8 - 48*a^4*b^2*c^5*d^2*e^9 - 384*a^3*b*c^7*d^5*e^6 - 384*a^4*b*c^6*d^3*e^8)/(c^3*e^3) - (2*x*(-(b^9*d^2 + a^2*b^7*e^2 - b^6*d^2*(-(4*a*c - b^2)^3)^(1/2) + 28*a^4*b*c^4*d^2 - 9*a^3*b^5*c*e^2 - 20*a^5*b*c^3*e^2 - 2*a*b^8*d*e + 42*a^2*b^5*c^2*d^2 - 63*a^3*b^3*c^3*d^2 - a^2*b^4*e^2*(-(4*a*c - b^2)^3)^(1/2) + a^3*c^3*d^2*(-(4*a*c - b^2)^3)^(1/2) + 25*a^4*b^3*c^2*e^2 - a^4*c^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - 11*a*b^7*c*d^2 - 16*a^5*c^4*d*e + 2*a*b^5*d*e*(-(4*a*c - b^2)^3)^(1/2) + 20*a^2*b^6*c*d*e - 6*a^2*b^2 *c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) + 5*a*b^4*c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 66*a^3*b^4*c^2*d*e + 76*a^4*b^2*c^3*d*e + 3*a^3*b^2*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - 8*a^2*b^3*c*d*e*(-(4*a*c - b^2)^3)^(1/2) + 6*a^3*b*c^2*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^9*d^4 + 16*a^4*c^7*e^4 + b^4*c^7*d^4 - 8*a*b^2*c^8*d^4 - 2*b^5*c^6*d^3*e + a^2*b^4*c^5*e^4 - 8*a^3*b^2*c^6*e^4 + 32*a^3*c^8*d^2*e^2 + b^6*c^5*d^2*e^2 + 16*a*b^3*c^7*d^3*e - 2*a*b^5*c^5*d*e^3 - 32*a^2*b*c^8*d^3*e - 32*a^3*b*c^7*d*e^3 - 6*a*b^4*c^6*d^2*e^2 + 16*a^2*b^3*c^6*d*e^3)))^(1/2)*(128*a^4*b^2*c^6*e^12 - 16*a^3*b^4*c^5*e^12 - 256*a^5*c^7*e^12 + 256*a^2*c^10*d^6*e^6 + 256*a^3*c^9*d^4*e^8 - 256*a^4*c^8*d^2*e^10 - 16*b^3*c^9*d^7*e^5 + 64*b^4*c^8*d^6*e^6 - 96*b^5*c^7*d^5*e^7 + 64*b^6*c^6*d^4*e^8 - 16*b^7*c^5*d^3*e^9 + 256*a^2*b^2*c^8*d^4*e^8 + 144*a^2*b^3*c^7*d^3*e^9 - 96*a^2*b^4*c^6*d^2*e^10 + 192*a^3*b^2*c^7*d^2*e^10 + 64*a*b*c^10*d^7*e^5 + 320*a^4*b*c^7*d*e^11 - 320*a*b^2*c^9*d^6*e^6 + 528*a*b^3*c^8*d^5*e^7 - 336*a*b^4*c^7*d^4*e^8 + 48*a*b^5*c^6*d^3*e^9 + 16*a*b^6*c^5*d^2*e^10 - 576*a^2*b*c^9*d^5*e^7 + 16*a^2*b^5*c^5*d*e^11 - 320*a^3*b*c^8*d^3*e^9 - 144*a^3*b^3*c^6*d*e^11))/(c^3*e^3))*(-(b^9*d^2 + a^2*b^7*e^2 - b^6*d^2*(-(4*a*c - b^2)^3)^(1/2) + 28*a^4*b*c^4*d^2 - 9*a^3*b^5*c*e^2 - 20*a^5*b*c^3*e^2 - 2*a*b^8*d*e + 42*a^2*b^5*c^2*d^2 - 63*a^3*b^3*c^3*d^2 - a^2*b^4*e^2*(-(4*a*c - b^2)^3)^(1/2) + a^3*c^3*d^2*(-(4*a*c - b^2)^3)^(1/2) + 25*a^4*b^3*c^2*e^2 - a^4*c^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - 11*a*b^7*c*d^2 - 16*a^5*c^4*d*e + 2*a*b^5*d*e*(-(4*a*c - b^2)^3)^(1/2) + 20*a^2*b^6*c*d*e - 6*a^2*b^2*c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) + 5*a*b^4*c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 66*a^3*b^4*c^2*d*e + 76*a^4*b^2*c^3*d*e + 3*a^3*b^2*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - 8*a^2*b^3*c*d*e*(-(4*a*c - b^2)^3)^(1/2) + 6*a^3*b*c^2*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^9*d^4 + 16*a^4*c^7*e^4 + b^4*c^7*d^4 - 8*a*b^2*c^8*d^4 - 2*b^5*c^6*d^3*e + a^2*b^4*c^5*e^4 - 8*a^3*b^2*c^6*e^4 + 32*a^3*c^8*d^2*e^2 + b^6*c^5*d^2*e^2 + 16*a*b^3*c^7*d^3*e - 2*a*b^5*c^5*d*e^3 - 32*a^2*b*c^8*d^3*e - 32*a^3*b*c^7*d*e^3 - 6*a*b^4*c^6*d^2*e^2 + 16*a^2*b^3*c^6*d*e^3)))^(1/2) + (2*x*(4*a^3*b^7*e^10 + 4*b^3*c^7*d^10 + 4*b^10*d^3*e^7 - 36*a^4*b^5*c*e^10 - 80*a^6*b*c^3*e^10 - 4*a*b^9*d^2*e^8 - 4*a^2*b^8*d*e^9 - 64*a^2*c^8*d^9*e - 56*a^6*c^4*d*e^9 - 8*b^4*c^6*d^9*e - 8*b^9*c*d^4*e^6 + 100*a^5*b^3*c^2*e^10 + 8*a^4*c^6*d^5*e^5 + 16*a^5*c^5*d^3*e^7 + 4*b^5*c^5*d^8*e^2 + 4*b^8*c^2*d^5*e^5 - 16*a*b*c^8*d^10 + 80*a^2*b^4*c^4*d^5*e^5 - 160*a^2*b^5*c^3*d^4*e^6 + 16*a^2*b^6*c^2*d^3*e^7 - 64*a^3*b^2*c^5*d^5*e^5 + 128*a^3*b^3*c^4*d^4*e^6 + 96*a^3*b^4*c^3*d^3*e^7 + 8*a^3*b^5*c^2*d^2*e^8 - 120*a^4*b^2*c^4*d^3*e^7 - 124*a^4*b^3*c^3*d^2*e^8 + 48*a*b^2*c^7*d^9*e - 24*a*b^8*c*d^3*e^7 + 48*a^3*b^6*c*d*e^9 - 28*a*b^3*c^6*d^8*e^2 - 32*a*b^6*c^3*d^5*e^5 + 64*a*b^7*c^2*d^4*e^6 + 48*a^2*b*c^7*d^8*e^2 + 20*a^2*b^7*c*d^2*e^8 - 16*a^4*b*c^5*d^4*e^6 - 184*a^4*b^4*c^2*d*e^9 + 96*a^5*b*c^4*d^2*e^8 + 240*a^5*b^2*c^3*d*e^9))/(c^3*e^3))*(-(b^9*d^2 + a^2*b^7*e^2 - b^6*d^2*(-(4*a*c - b^2)^3)^(1/2) + 28*a^4*b*c^4*d^2 - 9*a^3*b^5*c*e^2 - 20*a^5*b*c^3*e^2 - 2*a*b^8*d*e + 42*a^2*b^5*c^2*d^2 - 63*a^3*b^3*c^3*d^2 - a^2*b^4*e^2*(-(4*a*c - b^2)^3)^(1/2) + a^3*c^3*d^2*(-(4*a*c - b^2)^3)^(1/2) + 25*a^4*b^3*c^2*e^2 - a^4*c^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - 11*a*b^7*c*d^2 - 16*a^5*c^4*d*e + 2*a*b^5*d*e*(-(4*a*c - b^2)^3)^(1/2) + 20*a^2*b^6*c*d*e - 6*a^2*b

^2*c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) + 5*a*b^4*c*d^2*(-(4*a*c - b^2)^3)^(1/2
) - 66*a^3*b^4*c^2*d*e + 76*a^4*b^2*c^3*d*e + 3*a^3*b^2*c*e^2*(-(4*a*c - b^
2)^3)^(1/2) - 8*a^2*b^3*c*d*e*(-(4*a*c - b^2)^3)^(1/2) + 6*a^3*b*c^2*d*e*(-
(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^9*d^4 + 16*a^4*c^7*e^4 + b^4*c^7*d^4 -
 8*a*b^2*c^8*d^4 - 2*b^5*c^6*d^3*e + a^2*b^4*c^5*e^4 - 8*a^3*b^2*c^6*e^4 +
32*a^3*c^8*d^2*e^2 + b^6*c^5*d^2*e^2 + 16*a*b^3*c^7*d^3*e - 2*a*b^5*c^5*d*e
^3 - 32*a^2*b*c^8*d^3*e - 32*a^3*b*c^7*d*e^3 - 6*a*b^4*c^6*d^2*e^2 + 16*a^2
*b^3*c^6*d*e^3)))^(1/2) - (16*a^3*c^6*d^9 + 4*a*b^4*c^4*d^9 + 4*a*b^8*d^5*e
^4 + 4*a^5*b^4*d*e^8 + 4*a^7*c^2*d*e^8 - 20*a^2*b^2*c^5*d^9 - 4*a^2*b^7*d^4
*e^5 - 4*a^4*b^5*d^2*e^7 - 64*a^4*c^5*d^7*e^2 + 64*a^5*c^4*d^5*e^4 + 4*a^6*
c^3*d^3*e^6 - 36*a^2*b^4*c^3*d^7*e^2 - 40*a^2*b^5*c^2*d^6*e^3 + 96*a^3*b^2*
c^4*d^7*e^2 + 128*a^3*b^3*c^3*d^6*e^3 + 164*a^3*b^4*c^2*d^5*e^4 - 224*a^4*b
^2*c^3*d^5*e^4 - 104*a^4*b^3*c^2*d^4*e^5 - 20*a^5*b^2*c^2*d^3*e^6 + 4*a*b^5
*c^3*d^8*e + 4*a*b^7*c*d^6*e^3 + 64*a^3*b*c^5*d^8*e - 12*a^6*b^2*c*d*e^8 +
4*a*b^6*c^2*d^7*e^2 - 32*a^2*b^3*c^4*d^8*e - 44*a^2*b^6*c*d^5*e^4 + 36*a^3*
b^5*c*d^4*e^5 - 128*a^4*b*c^4*d^6*e^3 + 8*a^4*b^4*c*d^3*e^6 + 88*a^5*b*c^3*
d^4*e^5 + 8*a^5*b^3*c*d^2*e^7 + 4*a^6*b*c^2*d^2*e^7)/(c^3*e^3))*(-(b^9*d^2
+ a^2*b^7*e^2 - b^6*d^2*(-(4*a*c - b^2)^3)^(1/2) + 28*a^4*b*c^4*d^2 - 9*a^3
*b^5*c*e^2 - 20*a^5*b*c^3*e^2 - 2*a*b^8*d*e + 42*a^2*b^5*c^2*d^2 - 63*a^3*b
^3*c^3*d^2 - a^2*b^4*e^2*(-(4*a*c - b^2)^3)^(1/2) + a^3*c^3*d^2*(-(4*a*c -
b^2)^3)^(1/2) + 25*a^4*b^3*c^2*e^2 - a^4*c^2*e^2*(-(4*a*c - b^2)^3)^(1/2) -
 11*a*b^7*c*d^2 - 16*a^5*c^4*d*e + 2*a*b^5*d*e*(-(4*a*c - b^2)^3)^(1/2) + 2
0*a^2*b^6*c*d*e - 6*a^2*b^2*c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) + 5*a*b^4*c*d^
2*(-(4*a*c - b^2)^3)^(1/2) - 66*a^3*b^4*c^2*d*e + 76*a^4*b^2*c^3*d*e + 3*a^
3*b^2*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - 8*a^2*b^3*c*d*e*(-(4*a*c - b^2)^3)^(
1/2) + 6*a^3*b*c^2*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^9*d^4 + 16*a^
4*c^7*e^4 + b^4*c^7*d^4 - 8*a*b^2*c^8*d^4 - 2*b^5*c^6*d^3*e + a^2*b^4*c^5*e
^4 - 8*a^3*b^2*c^6*e^4 + 32*a^3*c^8*d^2*e^2 + b^6*c^5*d^2*e^2 + 16*a*b^3*c^
7*d^3*e - 2*a*b^5*c^5*d*e^3 - 32*a^2*b*c^8*d^3*e - 32*a^3*b*c^7*d*e^3 - 6*a
*b^4*c^6*d^2*e^2 + 16*a^2*b^3*c^6*d*e^3)))^(1/2) + (2*x*(a^8*e^8 + b^8*d^8
+ 2*a^4*c^4*d^8 + 20*a^2*b^4*c^2*d^8 - 16*a^3*b^2*c^3*d^8 - 8*a*b^6*c*d^8))
/(c^3*e^3))*(-(b^9*d^2 + a^2*b^7*e^2 - b^6*d^2*(-(4*a*c - b^2)^3)^(1/2) + 2
8*a^4*b*c^4*d^2 - 9*a^3*b^5*c*e^2 - 20*a^5*b*c^3*e^2 - 2*a*b^8*d*e + 42*a^2
*b^5*c^2*d^2 - 63*a^3*b^3*c^3*d^2 - a^2*b^4*e^2*(-(4*a*c - b^2)^3)^(1/2) +
a^3*c^3*d^2*(-(4*a*c - b^2)^3)^(1/2) + 25*a^4*b^3*c^2*e^2 - a^4*c^2*e^2*(-(
4*a*c - b^2)^3)^(1/2) - 11*a*b^7*c*d^2 - 16*a^5*c^4*d*e + 2*a*b^5*d*e*(-(4*
a*c - b^2)^3)^(1/2) + 20*a^2*b^6*c*d*e - 6*a^2*b^2*c^2*d^2*(-(4*a*c - b^2)^
3)^(1/2) + 5*a*b^4*c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 66*a^3*b^4*c^2*d*e + 76
*a^4*b^2*c^3*d*e + 3*a^3*b^2*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - 8*a^2*b^3*c*d
e(-(4*a*c - b^2)^3)^(1/2) + 6*a^3*b*c^2*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*
(16*a^2*c^9*d^4 + 16*a^4*c^7*e^4 + b^4*c^7*d^4 - 8*a*b^2*c^8*d^4 - 2*b^5*c^
6*d^3*e + a^2*b^4*c^5*e^4 - 8*a^3*b^2*c^6*e^4 + 32*a^3*c^8*d^2*e^2 + b^6*c^
5*d^2*e^2 + 16*a*b^3*c^7*d^3*e - 2*a*b^5*c^5*d*e^3 - 32*a^2*b*c^8*d^3*e - 3
2*a^3*b*c^7*d*e^3 - 6*a*b^4*c^6*d^2*e^2 + 16*a^2*b^3*c^6*d*e^3)))^(1/2)*1i
- (((((192*a^3*c^8*d^6*e^5 + 384*a^4*c^7*d^4*e^7 + 192*a^5*c^6*d^2*e^9 - 48

*a^2*b^2*c^7*d^6*e^5 + 96*a^2*b^3*c^6*d^5*e^6 - 48*a^2*b^4*c^5*d^4*e^7 + 96*a^3*b^2*c^6*d^4*e^7 + 96*a^3*b^3*c^5*d^3*e^8 - 48*a^4*b^2*c^5*d^2*e^9 - 384*a^3*b*c^7*d^5*e^6 - 384*a^4*b*c^6*d^3*e^8)/(c^3*e^3) + (2*x*(-(b^9*d^2 + a^2*b^7*e^2 - b^6*d^2*(-(4*a*c - b^2)^3)^(1/2) + 28*a^4*b*c^4*d^2 - 9*a^3*b^5*c*e^2 - 20*a^5*b*c^3*e^2 - 2*a*b^8*d*e + 42*a^2*b^5*c^2*d^2 - 63*a^3*b^3*c^3*d^2 - a^2*b^4*e^2*(-(4*a*c - b^2)^3)^(1/2) + a^3*c^3*d^2*(-(4*a*c - b^2)^3)^(1/2) + 25*a^4*b^3*c^2*e^2 - a^4*c^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - 11*a*b^7*c*d^2 - 16*a^5*c^4*d*e + 2*a*b^5*d*e*(-(4*a*c - b^2)^3)^(1/2) + 20*a^2*b^6*c*d*e - 6*a^2*b^2*c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) + 5*a*b^4*c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 66*a^3*b^4*c^2*d*e + 76*a^4*b^2*c^3*d*e + 3*a^3*b^2*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - 8*a^2*b^3*c*d*e*(-(4*a*c - b^2)^3)^(1/2) + 6*a^3*b*c^2*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^9*d^4 + 16*a^4*c^7*e^4 + b^4*c^7*d^4 - 8*a*b^2*c^8*d^4 - 2*b^5*c^6*d^3*e + a^2*b^4*c^5*e^4 - 8*a^3*b^2*c^6*e^4 + 32*a^3*c^8*d^2*e^2 + b^6*c^5*d^2*e^2 + 16*a*b^3*c^7*d^3*e - 2*a*b^5*c^5*d*e^3 - 32*a^2*b*c^8*d^3*e - 32*a^3*b*c^7*d*e^3 - 6*a*b^4*c^6*d^2*e^2 + 16*a^2*b^3*c^6*d*e^3)))^(1/2)*(128*a^4*b^2*c^6*e^12 - 16*a^3*b^4*c^5*e^12 - 256*a^5*c^7*e^12 + 256*a^2*c^10*d^6*e^6 + 256*a^3*c^9*d^4*e^8 - 256*a^4*c^8*d^2*e^10 - 16*b^3*c^9*d^7*e^5 + 64*b^4*c^8*d^6*e^6 - 96*b^5*c^7*d^5*e^7 + 64*b^6*c^6*d^4*e^8 - 16*b^7*c^5*d^3*e^9 + 256*a^2*b^2*c^8*d^4*e^8 + 144*a^2*b^3*c^7*d^3*e^9 - 96*a^2*b^4*c^6*d^2*e^10 + 192*a^3*b^2*c^7*d^2*e^10 + 64*a*b*c^10*d^7*e^5 + 320*a^4*b*c^7*d*e^11 - 320*a*b^2*c^9*d^6*e^6 + 528*a*b^3*c^8*d^5*e^7 - 336*a*b^4*c^7*d^4*e^8 + 48*a*b^5*c^6*d^3*e^9 + 16*a*b^6*c^5*d^2*e^10 - 576*a^2*b*c^9*d^5*e^7 + 16*a^2*b^5*c^5*d*e^11 - 320*a^3*b*c^8*d^3*e^9 - 144*a^3*b^3*c^6*d*e^11))/(c^3*e^3))*(-(b^9*d^2 + a^2*b^7*e^2 - b^6*d^2*(-(4*a*c - b^2)^3)^(1/2) + 28*a^4*b*c^4*d^2 - 9*a^3*b^5*c*e^2 - 20*a^5*b*c^3*e^2 - 2*a*b^8*d*e + 42*a^2*b^5*c^2*d^2 - 63*a^3*b^3*c^3*d^2 - a^2*b^4*e^2*(-(4*a*c - b^2)^3)^(1/2) + a^3*c^3*d^2*(-(4*a*c - b^2)^3)^(1/2) + 25*a^4*b^3*c^2*e^2 - a^4*c^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - 11*a*b^7*c*d^2 - 16*a^5*c^4*d*e + 2*a*b^5*d*e*(-(4*a*c - b^2)^3)^(1/2) + 20*a^2*b^6*c*d*e - 6*a^2*b^2*c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) + 5*a*b^4*c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 66*a^3*b^4*c^2*d*e + 76*a^4*b^2*c^3*d*e + 3*a^3*b^2*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - 8*a^2*b^3*c*d*e*(-(4*a*c - b^2)^3)^(1/2) + 6*a^3*b*c^2*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^9*d^4 + 16*a^4*c^7*e^4 + b^4*c^7*d^4 - 8*a*b^2*c^8*d^4 - 2*b^5*c^6*d^3*e + a^2*b^4*c^5*e^4 - 8*a^3*b^2*c^6*e^4 + 32*a^3*c^8*d^2*e^2 + b^6*c^5*d^2*e^2 + 16*a*b^3*c^7*d^3*e - 2*a*b^5*c^5*d*e^3 - 32*a^2*b*c^8*d^3*e - 32*a^3*b*c^7*d*e^3 - 6*a*b^4*c^6*d^2*e^2 + 16*a^2*b^3*c^6*d*e^3)))^(1/2) - (2*x*(4*a^3*b^7*e^10 + 4*b^3*c^7*d^10 + 4*b^10*d^3*e^7 - 36*a^4*b^5*c*e^10 - 80*a^6*b*c^3*e^10 - 4*a*b^9*d^2*e^8 - 4*a^2*b^8*d*e^9 - 64*a^2*c^8*d^9*e - 56*a^6*c^4*d*e^9 - 8*b^4*c^6*d^9*e - 8*b^9*c*d^4*e^6 + 100*a^5*b^3*c^2*e^10 + 8*a^4*c^6*d^5*e^5 + 16*a^5*c^5*d^3*e^7 + 4*b^5*c^5*d^8*e^2 + 4*b^8*c^2*d^5*e^5 - 16*a*b*c^8*d^10 + 80*a^2*b^4*c^4*d^5*e^5 - 160*a^2*b^5*c^3*d^4*e^6 + 16*a^2*b^6*c^2*d^3*e^7 - 64*a^3*b^2*c^5*d^5*e^5 + 128*a^3*b^3*c^4*d^4*e^6 + 96*a^3*b^4*c^3*d^3*e^7 + 8*a^3*b^5*c^2*d^2*e^8 - 120*a^4*b^2*c^4*d^3*e^7 - 124*a^4*b^3*c^3*d^2*e^8 + 48*a*b^2*c^7*d^9*e - 24*a*b^8*c*d^3*e^7 + 48*a^3*b^6*c*d*e^9 - 28*a*b

^3*c^6*d^8*e^2 - 32*a*b^6*c^3*d^5*e^5 + 64*a*b^7*c^2*d^4*e^6 + 48*a^2*b*c^7
*d^8*e^2 + 20*a^2*b^7*c*d^2*e^8 - 16*a^4*b*c^5*d^4*e^6 - 184*a^4*b^4*c^2*d*
e^9 + 96*a^5*b*c^4*d^2*e^8 + 240*a^5*b^2*c^3*d*e^9))/(c^3*e^3))*(-(b^9*d^2
+ a^2*b^7*e^2 - b^6*d^2*(-(4*a*c - b^2)^3)^(1/2) + 28*a^4*b*c^4*d^2 - 9*a^3
*b^5*c*e^2 - 20*a^5*b*c^3*e^2 - 2*a*b^8*d*e + 42*a^2*b^5*c^2*d^2 - 63*a^3*b
^3*c^3*d^2 - a^2*b^4*e^2*(-(4*a*c - b^2)^3)^(1/2) + a^3*c^3*d^2*(-(4*a*c -
b^2)^3)^(1/2) + 25*a^4*b^3*c^2*e^2 - a^4*c^2*e^2*(-(4*a*c - b^2)^3)^(1/2) -
11*a*b^7*c*d^2 - 16*a^5*c^4*d*e + 2*a*b^5*d*e*(-(4*a*c - b^2)^3)^(1/2) + 2
0*a^2*b^6*c*d*e - 6*a^2*b^2*c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) + 5*a*b^4*c*d^
2*(-(4*a*c - b^2)^3)^(1/2) - 66*a^3*b^4*c^2*d*e + 76*a^4*b^2*c^3*d*e + 3*a^
3*b^2*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - 8*a^2*b^3*c*d*e*(-(4*a*c - b^2)^3)^(
1/2) + 6*a^3*b*c^2*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^9*d^4 + 16*a^
4*c^7*e^4 + b^4*c^7*d^4 - 8*a*b^2*c^8*d^4 - 2*b^5*c^6*d^3*e + a^2*b^4*c^5*e
^4 - 8*a^3*b^2*c^6*e^4 + 32*a^3*c^8*d^2*e^2 + b^6*c^5*d^2*e^2 + 16*a*b^3*c^
7*d^3*e - 2*a*b^5*c^5*d*e^3 - 32*a^2*b*c^8*d^3*e - 32*a^3*b*c^7*d*e^3 - 6*a
*b^4*c^6*d^2*e^2 + 16*a^2*b^3*c^6*d*e^3)))^(1/2) - (16*a^3*c^6*d^9 + 4*a*b^
4*c^4*d^9 + 4*a*b^8*d^5*e^4 + 4*a^5*b^4*d*e^8 + 4*a^7*c^2*d*e^8 - 20*a^2*b^
2*c^5*d^9 - 4*a^2*b^7*d^4*e^5 - 4*a^4*b^5*d^2*e^7 - 64*a^4*c^5*d^7*e^2 + 64
*a^5*c^4*d^5*e^4 + 4*a^6*c^3*d^3*e^6 - 36*a^2*b^4*c^3*d^7*e^2 - 40*a^2*b^5*
c^2*d^6*e^3 + 96*a^3*b^2*c^4*d^7*e^2 + 128*a^3*b^3*c^3*d^6*e^3 + 164*a^3*b^
4*c^2*d^5*e^4 - 224*a^4*b^2*c^3*d^5*e^4 - 104*a^4*b^3*c^2*d^4*e^5 - 20*a^5*
b^2*c^2*d^3*e^6 + 4*a*b^5*c^3*d^8*e + 4*a*b^7*c*d^6*e^3 + 64*a^3*b*c^5*d^8*
e - 12*a^6*b^2*c*d*e^8 + 4*a*b^6*c^2*d^7*e^2 - 32*a^2*b^3*c^4*d^8*e - 44*a^
2*b^6*c*d^5*e^4 + 36*a^3*b^5*c*d^4*e^5 - 128*a^4*b*c^4*d^6*e^3 + 8*a^4*b^4*
c*d^3*e^6 + 88*a^5*b*c^3*d^4*e^5 + 8*a^5*b^3*c*d^2*e^7 + 4*a^6*b*c^2*d^2*e^
7)/(c^3*e^3))*(-(b^9*d^2 + a^2*b^7*e^2 - b^6*d^2*(-(4*a*c - b^2)^3)^(1/2) +
28*a^4*b*c^4*d^2 - 9*a^3*b^5*c*e^2 - 20*a^5*b*c^3*e^2 - 2*a*b^8*d*e + 42*a
^2*b^5*c^2*d^2 - 63*a^3*b^3*c^3*d^2 - a^2*b^4*e^2*(-(4*a*c - b^2)^3)^(1/2)
+ a^3*c^3*d^2*(-(4*a*c - b^2)^3)^(1/2) + 25*a^4*b^3*c^2*e^2 - a^4*c^2*e^2*(
-(4*a*c - b^2)^3)^(1/2) - 11*a*b^7*c*d^2 - 16*a^5*c^4*d*e + 2*a*b^5*d*e*(-(
4*a*c - b^2)^3)^(1/2) + 20*a^2*b^6*c*d*e - 6*a^2*b^2*c^2*d^2*(-(4*a*c - b^2
)^3)^(1/2) + 5*a*b^4*c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 66*a^3*b^4*c^2*d*e +
76*a^4*b^2*c^3*d*e + 3*a^3*b^2*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - 8*a^2*b^3*c
*d*e*(-(4*a*c - b^2)^3)^(1/2) + 6*a^3*b*c^2*d*e*(-(4*a*c - b^2)^3)^(1/2))/(
8*(16*a^2*c^9*d^4 + 16*a^4*c^7*e^4 + b^4*c^7*d^4 - 8*a*b^2*c^8*d^4 - 2*b^5*
c^6*d^3*e + a^2*b^4*c^5*e^4 - 8*a^3*b^2*c^6*e^4 + 32*a^3*c^8*d^2*e^2 + b^6*
c^5*d^2*e^2 + 16*a*b^3*c^7*d^3*e - 2*a*b^5*c^5*d*e^3 - 32*a^2*b*c^8*d^3*e -
32*a^3*b*c^7*d*e^3 - 6*a*b^4*c^6*d^2*e^2 + 16*a^2*b^3*c^6*d*e^3)))^(1/2) -
(2*x*(a^8*e^8 + b^8*d^8 + 2*a^4*c^4*d^8 + 20*a^2*b^4*c^2*d^8 - 16*a^3*b^2*
c^3*d^8 - 8*a*b^6*c*d^8))/(c^3*e^3))*(-(b^9*d^2 + a^2*b^7*e^2 - b^6*d^2*(-(
4*a*c - b^2)^3)^(1/2) + 28*a^4*b*c^4*d^2 - 9*a^3*b^5*c*e^2 - 20*a^5*b*c^3*e
^2 - 2*a*b^8*d*e + 42*a^2*b^5*c^2*d^2 - 63*a^3*b^3*c^3*d^2 - a^2*b^4*e^2*(-
(4*a*c - b^2)^3)^(1/2) + a^3*c^3*d^2*(-(4*a*c - b^2)^3)^(1/2) + 25*a^4*b^3*
c^2*e^2 - a^4*c^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - 11*a*b^7*c*d^2 - 16*a^5*c^
4*d*e + 2*a*b^5*d*e*(-(4*a*c - b^2)^3)^(1/2) + 20*a^2*b^6*c*d*e - 6*a^2*b^2

*c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) + 5*a*b^4*c*d^2*(-(4*a*c - b^2)^3)^(1/2)
- 66*a^3*b^4*c^2*d*e + 76*a^4*b^2*c^3*d*e + 3*a^3*b^2*c*e^2*(-(4*a*c - b^2)
^3)^(1/2) - 8*a^2*b^3*c*d*e*(-(4*a*c - b^2)^3)^(1/2) + 6*a^3*b*c^2*d*e*(-(4
*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^9*d^4 + 16*a^4*c^7*e^4 + b^4*c^7*d^4 - 8
*a*b^2*c^8*d^4 - 2*b^5*c^6*d^3*e + a^2*b^4*c^5*e^4 - 8*a^3*b^2*c^6*e^4 + 32
*a^3*c^8*d^2*e^2 + b^6*c^5*d^2*e^2 + 16*a*b^3*c^7*d^3*e - 2*a*b^5*c^5*d*e^3
- 32*a^2*b*c^8*d^3*e - 32*a^3*b*c^7*d*e^3 - 6*a*b^4*c^6*d^2*e^2 + 16*a^2*b
^3*c^6*d*e^3)))^(1/2)*1i)/((((((192*a^3*c^8*d^6*e^5 + 384*a^4*c^7*d^4*e^7 +
192*a^5*c^6*d^2*e^9 - 48*a^2*b^2*c^7*d^6*e^5 + 96*a^2*b^3*c^6*d^5*e^6 - 48
*a^2*b^4*c^5*d^4*e^7 + 96*a^3*b^2*c^6*d^4*e^7 + 96*a^3*b^3*c^5*d^3*e^8 - 48
*a^4*b^2*c^5*d^2*e^9 - 384*a^3*b*c^7*d^5*e^6 - 384*a^4*b*c^6*d^3*e^8)/(c^3*
e^3) - (2*x*(-(b^9*d^2 + a^2*b^7*e^2 - b^6*d^2*(-(4*a*c - b^2)^3)^(1/2) + 2
8*a^4*b*c^4*d^2 - 9*a^3*b^5*c*e^2 - 20*a^5*b*c^3*e^2 - 2*a*b^8*d*e + 42*a^2
*b^5*c^2*d^2 - 63*a^3*b^3*c^3*d^2 - a^2*b^4*e^2*(-(4*a*c - b^2)^3)^(1/2) +
a^3*c^3*d^2*(-(4*a*c - b^2)^3)^(1/2) + 25*a^4*b^3*c^2*e^2 - a^4*c^2*e^2*(-(
4*a*c - b^2)^3)^(1/2) - 11*a*b^7*c*d^2 - 16*a^5*c^4*d*e + 2*a*b^5*d*e*(-(4*
a*c - b^2)^3)^(1/2) + 20*a^2*b^6*c*d*e - 6*a^2*b^2*c^2*d^2*(-(4*a*c - b^2)^
3)^(1/2) + 5*a*b^4*c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 66*a^3*b^4*c^2*d*e + 76
*a^4*b^2*c^3*d*e + 3*a^3*b^2*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - 8*a^2*b^3*c*d
e(-(4*a*c - b^2)^3)^(1/2) + 6*a^3*b*c^2*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*
(16*a^2*c^9*d^4 + 16*a^4*c^7*e^4 + b^4*c^7*d^4 - 8*a*b^2*c^8*d^4 - 2*b^5*c^
6*d^3*e + a^2*b^4*c^5*e^4 - 8*a^3*b^2*c^6*e^4 + 32*a^3*c^8*d^2*e^2 + b^6*c^
5*d^2*e^2 + 16*a*b^3*c^7*d^3*e - 2*a*b^5*c^5*d*e^3 - 32*a^2*b*c^8*d^3*e - 3
2*a^3*b*c^7*d*e^3 - 6*a*b^4*c^6*d^2*e^2 + 16*a^2*b^3*c^6*d*e^3)))^(1/2)*(12
8*a^4*b^2*c^6*e^12 - 16*a^3*b^4*c^5*e^12 - 256*a^5*c^7*e^12 + 256*a^2*c^10*
d^6*e^6 + 256*a^3*c^9*d^4*e^8 - 256*a^4*c^8*d^2*e^10 - 16*b^3*c^9*d^7*e^5 +
64*b^4*c^8*d^6*e^6 - 96*b^5*c^7*d^5*e^7 + 64*b^6*c^6*d^4*e^8 - 16*b^7*c^5*
d^3*e^9 + 256*a^2*b^2*c^8*d^4*e^8 + 144*a^2*b^3*c^7*d^3*e^9 - 96*a^2*b^4*c^
6*d^2*e^10 + 192*a^3*b^2*c^7*d^2*e^10 + 64*a*b*c^10*d^7*e^5 + 320*a^4*b*c^7
*d*e^11 - 320*a*b^2*c^9*d^6*e^6 + 528*a*b^3*c^8*d^5*e^7 - 336*a*b^4*c^7*d^4
*e^8 + 48*a*b^5*c^6*d^3*e^9 + 16*a*b^6*c^5*d^2*e^10 - 576*a^2*b*c^9*d^5*e^7
+ 16*a^2*b^5*c^5*d*e^11 - 320*a^3*b*c^8*d^3*e^9 - 144*a^3*b^3*c^6*d*e^11))
/(c^3*e^3))*(-(b^9*d^2 + a^2*b^7*e^2 - b^6*d^2*(-(4*a*c - b^2)^3)^(1/2) + 2
8*a^4*b*c^4*d^2 - 9*a^3*b^5*c*e^2 - 20*a^5*b*c^3*e^2 - 2*a*b^8*d*e + 42*a^2
*b^5*c^2*d^2 - 63*a^3*b^3*c^3*d^2 - a^2*b^4*e^2*(-(4*a*c - b^2)^3)^(1/2) +
a^3*c^3*d^2*(-(4*a*c - b^2)^3)^(1/2) + 25*a^4*b^3*c^2*e^2 - a^4*c^2*e^2*(-(
4*a*c - b^2)^3)^(1/2) - 11*a*b^7*c*d^2 - 16*a^5*c^4*d*e + 2*a*b^5*d*e*(-(4*
a*c - b^2)^3)^(1/2) + 20*a^2*b^6*c*d*e - 6*a^2*b^2*c^2*d^2*(-(4*a*c - b^2)^
3)^(1/2) + 5*a*b^4*c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 66*a^3*b^4*c^2*d*e + 76
*a^4*b^2*c^3*d*e + 3*a^3*b^2*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - 8*a^2*b^3*c*d
e(-(4*a*c - b^2)^3)^(1/2) + 6*a^3*b*c^2*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*
(16*a^2*c^9*d^4 + 16*a^4*c^7*e^4 + b^4*c^7*d^4 - 8*a*b^2*c^8*d^4 - 2*b^5*c^
6*d^3*e + a^2*b^4*c^5*e^4 - 8*a^3*b^2*c^6*e^4 + 32*a^3*c^8*d^2*e^2 + b^6*c^
5*d^2*e^2 + 16*a*b^3*c^7*d^3*e - 2*a*b^5*c^5*d*e^3 - 32*a^2*b*c^8*d^3*e - 3
2*a^3*b*c^7*d*e^3 - 6*a*b^4*c^6*d^2*e^2 + 16*a^2*b^3*c^6*d*e^3)))^(1/2) + (

2*x*(4*a^3*b^7*e^10 + 4*b^3*c^7*d^10 + 4*b^10*d^3*e^7 - 36*a^4*b^5*c*e^10 - 80*a^6*b*c^3*e^10 - 4*a*b^9*d^2*e^8 - 4*a^2*b^8*d*e^9 - 64*a^2*c^8*d^9*e - 56*a^6*c^4*d*e^9 - 8*b^4*c^6*d^9*e - 8*b^9*c*d^4*e^6 + 100*a^5*b^3*c^2*e^10 + 8*a^4*c^6*d^5*e^5 + 16*a^5*c^5*d^3*e^7 + 4*b^5*c^5*d^8*e^2 + 4*b^8*c^2*d^5*e^5 - 16*a*b*c^8*d^10 + 80*a^2*b^4*c^4*d^5*e^5 - 160*a^2*b^5*c^3*d^4*e^6 + 16*a^2*b^6*c^2*d^3*e^7 - 64*a^3*b^2*c^5*d^5*e^5 + 128*a^3*b^3*c^4*d^4*e^6 + 96*a^3*b^4*c^3*d^3*e^7 + 8*a^3*b^5*c^2*d^2*e^8 - 120*a^4*b^2*c^4*d^3*e^7 - 124*a^4*b^3*c^3*d^2*e^8 + 48*a*b^2*c^7*d^9*e - 24*a*b^8*c*d^3*e^7 + 48*a^3*b^6*c*d*e^9 - 28*a*b^3*c^6*d^8*e^2 - 32*a*b^6*c^3*d^5*e^5 + 64*a*b^7*c^2*d^4*e^6 + 48*a^2*b*c^7*d^8*e^2 + 20*a^2*b^7*c*d^2*e^8 - 16*a^4*b*c^5*d^4*e^6 - 184*a^4*b^4*c^2*d*e^9 + 96*a^5*b*c^4*d^2*e^8 + 240*a^5*b^2*c^3*d*e^9))/(c^3*e^3))*(-(b^9*d^2 + a^2*b^7*e^2 - b^6*d^2*(-(4*a*c - b^2)^3)^(1/2) + 28*a^4*b*c^4*d^2 - 9*a^3*b^5*c*e^2 - 20*a^5*b*c^3*e^2 - 2*a*b^8*d*e + 42*a^2*b^5*c^2*d^2 - 63*a^3*b^3*c^3*d^2 - a^2*b^4*e^2*(-(4*a*c - b^2)^3)^(1/2) + a^3*c^3*d^2*(-(4*a*c - b^2)^3)^(1/2) + 25*a^4*b^3*c^2*e^2 - a^4*c^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - 11*a*b^7*c*d^2 - 16*a^5*c^4*d*e + 2*a*b^5*d*e*(-(4*a*c - b^2)^3)^(1/2) + 20*a^2*b^6*c*d*e - 6*a^2*b^2*c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) + 5*a*b^4*c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 66*a^3*b^4*c^2*d*e + 76*a^4*b^2*c^3*d*e + 3*a^3*b^2*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - 8*a^2*b^3*c*d*e*(-(4*a*c - b^2)^3)^(1/2) + 6*a^3*b*c^2*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^9*d^4 + 16*a^4*c^7*e^4 + b^4*c^7*d^4 - 8*a*b^2*c^8*d^4 - 2*b^5*c^6*d^3*e + a^2*b^4*c^5*e^4 - 8*a^3*b^2*c^6*e^4 + 32*a^3*c^8*d^2*e^2 + b^6*c^5*d^2*e^2 + 16*a*b^3*c^7*d^3*e - 2*a*b^5*c^5*d*e^3 - 32*a^2*b*c^8*d^3*e - 32*a^3*b*c^7*d*e^3 - 6*a*b^4*c^6*d^2*e^2 + 16*a^2*b^3*c^6*d*e^3)))^(1/2) - (16*a^3*c^6*d^9 + 4*a*b^4*c^4*d^9 + 4*a*b^8*d^5*e^4 + 4*a^5*b^4*d*e^8 + 4*a^7*c^2*d*e^8 - 20*a^2*b^2*c^5*d^9 - 4*a^2*b^7*d^4*e^5 - 4*a^4*b^5*d^2*e^7 - 64*a^4*c^5*d^7*e^2 + 64*a^5*c^4*d^5*e^4 + 4*a^6*c^3*d^3*e^6 - 36*a^2*b^4*c^3*d^7*e^2 - 40*a^2*b^5*c^2*d^6*e^3 + 96*a^3*b^2*c^4*d^7*e^2 + 128*a^3*b^3*c^3*d^6*e^3 + 164*a^3*b^4*c^2*d^5*e^4 - 224*a^4*b^2*c^3*d^5*e^4 - 104*a^4*b^3*c^2*d^4*e^5 - 20*a^5*b^2*c^2*d^3*e^6 + 4*a*b^5*c^3*d^8*e + 4*a*b^7*c*d^6*e^3 + 64*a^3*b*c^5*d^8*e - 12*a^6*b^2*c*d*e^8 + 4*a*b^6*c^2*d^7*e^2 - 32*a^2*b^3*c^4*d^8*e - 44*a^2*b^6*c*d^5*e^4 + 36*a^3*b^5*c*d^4*e^5 - 128*a^4*b*c^4*d^6*e^3 + 8*a^4*b^4*c*d^3*e^6 + 88*a^5*b*c^3*d^4*e^5 + 8*a^5*b^3*c*d^2*e^7 + 4*a^6*b*c^2*d^2*e^7)/(c^3*e^3))*(-(b^9*d^2 + a^2*b^7*e^2 - b^6*d^2*(-(4*a*c - b^2)^3)^(1/2) + 28*a^4*b*c^4*d^2 - 9*a^3*b^5*c*e^2 - 20*a^5*b*c^3*e^2 - 2*a*b^8*d*e + 42*a^2*b^5*c^2*d^2 - 63*a^3*b^3*c^3*d^2 - a^2*b^4*e^2*(-(4*a*c - b^2)^3)^(1/2) + a^3*c^3*d^2*(-(4*a*c - b^2)^3)^(1/2) + 25*a^4*b^3*c^2*e^2 - a^4*c^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - 11*a*b^7*c*d^2 - 16*a^5*c^4*d*e + 2*a*b^5*d*e*(-(4*a*c - b^2)^3)^(1/2) + 20*a^2*b^6*c*d*e - 6*a^2*b^2*c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) + 5*a*b^4*c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 66*a^3*b^4*c^2*d*e + 76*a^4*b^2*c^3*d*e + 3*a^3*b^2*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - 8*a^2*b^3*c*d*e*(-(4*a*c - b^2)^3)^(1/2) + 6*a^3*b*c^2*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^9*d^4 + 16*a^4*c^7*e^4 + b^4*c^7*d^4 - 8*a*b^2*c^8*d^4 - 2*b^5*c^6*d^3*e + a^2*b^4*c^5*e^4 - 8*a^3*b^2*c^6*e^4 + 32*a^3*c^8*d^2*e^2 + b^6*c^5*d^2*e^2 + 16*a*b^3*c^7*d^3*e - 2*a*b^5*c^5*d*e

^3 - 32*a^2*b*c^8*d^3*e - 32*a^3*b*c^7*d*e^3 - 6*a*b^4*c^6*d^2*e^2 + 16*a^2
*b^3*c^6*d*e^3)))^(1/2) + (2*x*(a^8*e^8 + b^8*d^8 + 2*a^4*c^4*d^8 + 20*a^2*
b^4*c^2*d^8 - 16*a^3*b^2*c^3*d^8 - 8*a*b^6*c*d^8))/(c^3*e^3))*(-(b^9*d^2 +
a^2*b^7*e^2 - b^6*d^2*(-(4*a*c - b^2)^3)^(1/2) + 28*a^4*b*c^4*d^2 - 9*a^3*b
^5*c*e^2 - 20*a^5*b*c^3*e^2 - 2*a*b^8*d*e + 42*a^2*b^5*c^2*d^2 - 63*a^3*b^3
*c^3*d^2 - a^2*b^4*e^2*(-(4*a*c - b^2)^3)^(1/2) + a^3*c^3*d^2*(-(4*a*c - b^
2)^3)^(1/2) + 25*a^4*b^3*c^2*e^2 - a^4*c^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - 1
1*a*b^7*c*d^2 - 16*a^5*c^4*d*e + 2*a*b^5*d*e*(-(4*a*c - b^2)^3)^(1/2) + 20*
a^2*b^6*c*d*e - 6*a^2*b^2*c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) + 5*a*b^4*c*d^2*
(-(4*a*c - b^2)^3)^(1/2) - 66*a^3*b^4*c^2*d*e + 76*a^4*b^2*c^3*d*e + 3*a^3*
b^2*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - 8*a^2*b^3*c*d*e*(-(4*a*c - b^2)^3)^(1/
2) + 6*a^3*b*c^2*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^9*d^4 + 16*a^4*
c^7*e^4 + b^4*c^7*d^4 - 8*a*b^2*c^8*d^4 - 2*b^5*c^6*d^3*e + a^2*b^4*c^5*e^4
 - 8*a^3*b^2*c^6*e^4 + 32*a^3*c^8*d^2*e^2 + b^6*c^5*d^2*e^2 + 16*a*b^3*c^7*
d^3*e - 2*a*b^5*c^5*d*e^3 - 32*a^2*b*c^8*d^3*e - 32*a^3*b*c^7*d*e^3 - 6*a*b
^4*c^6*d^2*e^2 + 16*a^2*b^3*c^6*d*e^3)))^(1/2) + (((((192*a^3*c^8*d^6*e^5 +
 384*a^4*c^7*d^4*e^7 + 192*a^5*c^6*d^2*e^9 - 48*a^2*b^2*c^7*d^6*e^5 + 96*a^
2*b^3*c^6*d^5*e^6 - 48*a^2*b^4*c^5*d^4*e^7 + 96*a^3*b^2*c^6*d^4*e^7 + 96*a^
3*b^3*c^5*d^3*e^8 - 48*a^4*b^2*c^5*d^2*e^9 - 384*a^3*b*c^7*d^5*e^6 - 384*a^
4*b*c^6*d^3*e^8)/(c^3*e^3) + (2*x*(-(b^9*d^2 + a^2*b^7*e^2 - b^6*d^2*(-(4*a
*c - b^2)^3)^(1/2) + 28*a^4*b*c^4*d^2 - 9*a^3*b^5*c*e^2 - 20*a^5*b*c^3*e^2
- 2*a*b^8*d*e + 42*a^2*b^5*c^2*d^2 - 63*a^3*b^3*c^3*d^2 - a^2*b^4*e^2*(-(4*
a*c - b^2)^3)^(1/2) + a^3*c^3*d^2*(-(4*a*c - b^2)^3)^(1/2) + 25*a^4*b^3*c^2
*e^2 - a^4*c^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - 11*a*b^7*c*d^2 - 16*a^5*c^4*d
*e + 2*a*b^5*d*e*(-(4*a*c - b^2)^3)^(1/2) + 20*a^2*b^6*c*d*e - 6*a^2*b^2*c^
2*d^2*(-(4*a*c - b^2)^3)^(1/2) + 5*a*b^4*c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 6
6*a^3*b^4*c^2*d*e + 76*a^4*b^2*c^3*d*e + 3*a^3*b^2*c*e^2*(-(4*a*c - b^2)^3)
^(1/2) - 8*a^2*b^3*c*d*e*(-(4*a*c - b^2)^3)^(1/2) + 6*a^3*b*c^2*d*e*(-(4*a*
c - b^2)^3)^(1/2))/(8*(16*a^2*c^9*d^4 + 16*a^4*c^7*e^4 + b^4*c^7*d^4 - 8*a*
b^2*c^8*d^4 - 2*b^5*c^6*d^3*e + a^2*b^4*c^5*e^4 - 8*a^3*b^2*c^6*e^4 + 32*a^
3*c^8*d^2*e^2 + b^6*c^5*d^2*e^2 + 16*a*b^3*c^7*d^3*e - 2*a*b^5*c^5*d*e^3 -
32*a^2*b*c^8*d^3*e - 32*a^3*b*c^7*d*e^3 - 6*a*b^4*c^6*d^2*e^2 + 16*a^2*b^3*
c^6*d*e^3)))^(1/2)*(128*a^4*b^2*c^6*e^12 - 16*a^3*b^4*c^5*e^12 - 256*a^5*c^
7*e^12 + 256*a^2*c^10*d^6*e^6 + 256*a^3*c^9*d^4*e^8 - 256*a^4*c^8*d^2*e^10
- 16*b^3*c^9*d^7*e^5 + 64*b^4*c^8*d^6*e^6 - 96*b^5*c^7*d^5*e^7 + 64*b^6*c^6
*d^4*e^8 - 16*b^7*c^5*d^3*e^9 + 256*a^2*b^2*c^8*d^4*e^8 + 144*a^2*b^3*c^7*d
^3*e^9 - 96*a^2*b^4*c^6*d^2*e^10 + 192*a^3*b^2*c^7*d^2*e^10 + 64*a*b*c^10*d
^7*e^5 + 320*a^4*b*c^7*d*e^11 - 320*a*b^2*c^9*d^6*e^6 + 528*a*b^3*c^8*d^5*e
^7 - 336*a*b^4*c^7*d^4*e^8 + 48*a*b^5*c^6*d^3*e^9 + 16*a*b^6*c^5*d^2*e^10 -
 576*a^2*b*c^9*d^5*e^7 + 16*a^2*b^5*c^5*d*e^11 - 320*a^3*b*c^8*d^3*e^9 - 14
4*a^3*b^3*c^6*d*e^11))/(c^3*e^3))*(-(b^9*d^2 + a^2*b^7*e^2 - b^6*d^2*(-(4*a
*c - b^2)^3)^(1/2) + 28*a^4*b*c^4*d^2 - 9*a^3*b^5*c*e^2 - 20*a^5*b*c^3*e^2
- 2*a*b^8*d*e + 42*a^2*b^5*c^2*d^2 - 63*a^3*b^3*c^3*d^2 - a^2*b^4*e^2*(-(4*
a*c - b^2)^3)^(1/2) + a^3*c^3*d^2*(-(4*a*c - b^2)^3)^(1/2) + 25*a^4*b^3*c^2
*e^2 - a^4*c^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - 11*a*b^7*c*d^2 - 16*a^5*c^4*d

*e + 2*a*b^5*d*e*(-(4*a*c - b^2)^3)^(1/2) + 20*a^2*b^6*c*d*e - 6*a^2*b^2*c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) + 5*a*b^4*c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 66*a^3*b^4*c^2*d*e + 76*a^4*b^2*c^3*d*e + 3*a^3*b^2*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - 8*a^2*b^3*c*d*e*(-(4*a*c - b^2)^3)^(1/2) + 6*a^3*b*c^2*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^9*d^4 + 16*a^4*c^7*e^4 + b^4*c^7*d^4 - 8*a*b^2*c^8*d^4 - 2*b^5*c^6*d^3*e + a^2*b^4*c^5*e^4 - 8*a^3*b^2*c^6*e^4 + 32*a^3*c^8*d^2*e^2 + b^6*c^5*d^2*e^2 + 16*a*b^3*c^7*d^3*e - 2*a*b^5*c^5*d*e^3 - 32*a^2*b*c^8*d^3*e - 32*a^3*b*c^7*d*e^3 - 6*a*b^4*c^6*d^2*e^2 + 16*a^2*b^3*c^6*d*e^3)))^(1/2) - (2*x*(4*a^3*b^7*e^10 + 4*b^3*c^7*d^10 + 4*b^10*d^3*e^7 - 36*a^4*b^5*c*e^10 - 80*a^6*b*c^3*e^10 - 4*a*b^9*d^2*e^8 - 4*a^2*b^8*d*e^9 - 64*a^2*c^8*d^9*e - 56*a^6*c^4*d*e^9 - 8*b^4*c^6*d^9*e - 8*b^9*c*d^4*e^6 + 100*a^5*b^3*c^2*e^10 + 8*a^4*c^6*d^5*e^5 + 16*a^5*c^5*d^3*e^7 + 4*b^5*c^5*d^8*e^2 + 4*b^8*c^2*d^5*e^5 - 16*a*b*c^8*d^10 + 80*a^2*b^4*c^4*d^5*e^5 - 160*a^2*b^5*c^3*d^4*e^6 + 16*a^2*b^6*c^2*d^3*e^7 - 64*a^3*b^2*c^5*d^5*e^5 + 128*a^3*b^3*c^4*d^4*e^6 + 96*a^3*b^4*c^3*d^3*e^7 + 8*a^3*b^5*c^2*d^2*e^8 - 120*a^4*b^2*c^4*d^3*e^7 - 124*a^4*b^3*c^3*d^2*e^8 + 48*a*b^2*c^7*d^9*e - 24*a*b^8*c*d^3*e^7 + 48*a^3*b^6*c*d*e^9 - 28*a*b^3*c^6*d^8*e^2 - 32*a*b^6*c^3*d^5*e^5 + 64*a*b^7*c^2*d^4*e^6 + 48*a^2*b*c^7*d^8*e^2 + 20*a^2*b^7*c*d^2*e^8 - 16*a^4*b*c^5*d^4*e^6 - 184*a^4*b^4*c^2*d*e^9 + 96*a^5*b*c^4*d^2*e^8 + 240*a^5*b^2*c^3*d*e^9))/(c^3*e^3))*(-(b^9*d^2 + a^2*b^7*e^2 - b^6*d^2*(-(4*a*c - b^2)^3)^(1/2) + 28*a^4*b*c^4*d^2 - 9*a^3*b^5*c*e^2 - 20*a^5*b*c^3*e^2 - 2*a*b^8*d*e + 42*a^2*b^5*c^2*d^2 - 63*a^3*b^3*c^3*d^2 - a^2*b^4*e^2*(-(4*a*c - b^2)^3)^(1/2) + a^3*c^3*d^2*(-(4*a*c - b^2)^3)^(1/2) + 25*a^4*b^3*c^2*e^2 - a^4*c^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - 11*a*b^7*c*d^2 - 16*a^5*c^4*d*e + 2*a*b^5*d*e*(-(4*a*c - b^2)^3)^(1/2) + 20*a^2*b^6*c*d*e - 6*a^2*b^2*c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) + 5*a*b^4*c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 66*a^3*b^4*c^2*d*e + 76*a^4*b^2*c^3*d*e + 3*a^3*b^2*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - 8*a^2*b^3*c*d*e*(-(4*a*c - b^2)^3)^(1/2) + 6*a^3*b*c^2*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^9*d^4 + 16*a^4*c^7*e^4 + b^4*c^7*d^4 - 8*a*b^2*c^8*d^4 - 2*b^5*c^6*d^3*e + a^2*b^4*c^5*e^4 - 8*a^3*b^2*c^6*e^4 + 32*a^3*c^8*d^2*e^2 + b^6*c^5*d^2*e^2 + 16*a*b^3*c^7*d^3*e - 2*a*b^5*c^5*d*e^3 - 32*a^2*b*c^8*d^3*e - 32*a^3*b*c^7*d*e^3 - 6*a*b^4*c^6*d^2*e^2 + 16*a^2*b^3*c^6*d*e^3)))^(1/2) - (16*a^3*c^6*d^9 + 4*a*b^4*c^4*d^9 + 4*a*b^8*d^5*e^4 + 4*a^5*b^4*d*e^8 + 4*a^7*c^2*d*e^8 - 20*a^2*b^2*c^5*d^9 - 4*a^2*b^7*d^4*e^5 - 4*a^4*b^5*d^2*e^7 - 64*a^4*c^5*d^7*e^2 + 64*a^5*c^4*d^5*e^4 + 4*a^6*c^3*d^3*e^6 - 36*a^2*b^4*c^3*d^7*e^2 - 40*a^2*b^5*c^2*d^6*e^3 + 96*a^3*b^2*c^4*d^7*e^2 + 128*a^3*b^3*c^3*d^6*e^3 + 164*a^3*b^4*c^2*d^5*e^4 - 224*a^4*b^2*c^3*d^5*e^4 - 104*a^4*b^3*c^2*d^4*e^5 - 20*a^5*b^2*c^2*d^3*e^6 + 4*a*b^5*c^3*d^8*e + 4*a*b^7*c*d^6*e^3 + 64*a^3*b*c^5*d^8*e - 12*a^6*b^2*c*d*e^8 + 4*a*b^6*c^2*d^7*e^2 - 32*a^2*b^3*c^4*d^8*e - 44*a^2*b^6*c*d^5*e^4 + 36*a^3*b^5*c*d^4*e^5 - 128*a^4*b*c^4*d^6*e^3 + 8*a^4*b^4*c*d^3*e^6 + 88*a^5*b*c^3*d^4*e^5 + 8*a^5*b^3*c*d^2*e^7 + 4*a^6*b*c^2*d^2*e^7)/(c^3*e^3))*(-(b^9*d^2 + a^2*b^7*e^2 - b^6*d^2*(-(4*a*c - b^2)^3)^(1/2) + 28*a^4*b*c^4*d^2 - 9*a^3*b^5*c*e^2 - 20*a^5*b*c^3*e^2 - 2*a*b^8*d*e + 42*a^2*b^5*c^2*d^2 - 63*a^3*b^3*c^3*d^2 - a^2*b^4*e^2*(-(4*a*c - b^2)^3)^(1/2) + a^3*c^3*d^2*(-(4*a*c - b^2

)^3)^(1/2) + 25*a^4*b^3*c^2*e^2 - a^4*c^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - 11
*a*b^7*c*d^2 - 16*a^5*c^4*d*e + 2*a*b^5*d*e*(-(4*a*c - b^2)^3)^(1/2) + 20*a
^2*b^6*c*d*e - 6*a^2*b^2*c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) + 5*a*b^4*c*d^2*(
-(4*a*c - b^2)^3)^(1/2) - 66*a^3*b^4*c^2*d*e + 76*a^4*b^2*c^3*d*e + 3*a^3*b
^2*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - 8*a^2*b^3*c*d*e*(-(4*a*c - b^2)^3)^(1/2
) + 6*a^3*b*c^2*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^9*d^4 + 16*a^4*c
^7*e^4 + b^4*c^7*d^4 - 8*a*b^2*c^8*d^4 - 2*b^5*c^6*d^3*e + a^2*b^4*c^5*e^4
- 8*a^3*b^2*c^6*e^4 + 32*a^3*c^8*d^2*e^2 + b^6*c^5*d^2*e^2 + 16*a*b^3*c^7*d
^3*e - 2*a*b^5*c^5*d*e^3 - 32*a^2*b*c^8*d^3*e - 32*a^3*b*c^7*d*e^3 - 6*a*b^
4*c^6*d^2*e^2 + 16*a^2*b^3*c^6*d*e^3)))^(1/2) - (2*x*(a^8*e^8 + b^8*d^8 + 2
*a^4*c^4*d^8 + 20*a^2*b^4*c^2*d^8 - 16*a^3*b^2*c^3*d^8 - 8*a*b^6*c*d^8))/(c
^3*e^3))*(-(b^9*d^2 + a^2*b^7*e^2 - b^6*d^2*(-(4*a*c - b^2)^3)^(1/2) + 28*a
^4*b*c^4*d^2 - 9*a^3*b^5*c*e^2 - 20*a^5*b*c^3*e^2 - 2*a*b^8*d*e + 42*a^2*b^
5*c^2*d^2 - 63*a^3*b^3*c^3*d^2 - a^2*b^4*e^2*(-(4*a*c - b^2)^3)^(1/2) + a^3
*c^3*d^2*(-(4*a*c - b^2)^3)^(1/2) + 25*a^4*b^3*c^2*e^2 - a^4*c^2*e^2*(-(4*a
*c - b^2)^3)^(1/2) - 11*a*b^7*c*d^2 - 16*a^5*c^4*d*e + 2*a*b^5*d*e*(-(4*a*c
 - b^2)^3)^(1/2) + 20*a^2*b^6*c*d*e - 6*a^2*b^2*c^2*d^2*(-(4*a*c - b^2)^3)^
(1/2) + 5*a*b^4*c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 66*a^3*b^4*c^2*d*e + 76*a^
4*b^2*c^3*d*e + 3*a^3*b^2*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - 8*a^2*b^3*c*d*e*
(-(4*a*c - b^2)^3)^(1/2) + 6*a^3*b*c^2*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(16
*a^2*c^9*d^4 + 16*a^4*c^7*e^4 + b^4*c^7*d^4 - 8*a*b^2*c^8*d^4 - 2*b^5*c^6*d
^3*e + a^2*b^4*c^5*e^4 - 8*a^3*b^2*c^6*e^4 + 32*a^3*c^8*d^2*e^2 + b^6*c^5*d
^2*e^2 + 16*a*b^3*c^7*d^3*e - 2*a*b^5*c^5*d*e^3 - 32*a^2*b*c^8*d^3*e - 32*a
^3*b*c^7*d*e^3 - 6*a*b^4*c^6*d^2*e^2 + 16*a^2*b^3*c^6*d*e^3)))^(1/2) + (2*(
a^4*b^3*d^7 + a^7*d^4*e^3 + a^5*b^2*d^6*e + a^6*b*d^5*e^2 - 2*a^5*b*c*d^7 -
 a^6*c*d^6*e))/(c^3*e^3)))*(-(b^9*d^2 + a^2*b^7*e^2 - b^6*d^2*(-(4*a*c - b^
2)^3)^(1/2) + 28*a^4*b*c^4*d^2 - 9*a^3*b^5*c*e^2 - 20*a^5*b*c^3*e^2 - 2*a*b
^8*d*e + 42*a^2*b^5*c^2*d^2 - 63*a^3*b^3*c^3*d^2 - a^2*b^4*e^2*(-(4*a*c - b
^2)^3)^(1/2) + a^3*c^3*d^2*(-(4*a*c - b^2)^3)^(1/2) + 25*a^4*b^3*c^2*e^2 -
a^4*c^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - 11*a*b^7*c*d^2 - 16*a^5*c^4*d*e + 2*
a*b^5*d*e*(-(4*a*c - b^2)^3)^(1/2) + 20*a^2*b^6*c*d*e - 6*a^2*b^2*c^2*d^2*(
-(4*a*c - b^2)^3)^(1/2) + 5*a*b^4*c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 66*a^3*b
^4*c^2*d*e + 76*a^4*b^2*c^3*d*e + 3*a^3*b^2*c*e^2*(-(4*a*c - b^2)^3)^(1/2)
- 8*a^2*b^3*c*d*e*(-(4*a*c - b^2)^3)^(1/2) + 6*a^3*b*c^2*d*e*(-(4*a*c - b^2
)^3)^(1/2))/(8*(16*a^2*c^9*d^4 + 16*a^4*c^7*e^4 + b^4*c^7*d^4 - 8*a*b^2*c^8
*d^4 - 2*b^5*c^6*d^3*e + a^2*b^4*c^5*e^4 - 8*a^3*b^2*c^6*e^4 + 32*a^3*c^8*d
^2*e^2 + b^6*c^5*d^2*e^2 + 16*a*b^3*c^7*d^3*e - 2*a*b^5*c^5*d*e^3 - 32*a^2*
b*c^8*d^3*e - 32*a^3*b*c^7*d*e^3 - 6*a*b^4*c^6*d^2*e^2 + 16*a^2*b^3*c^6*d*e
^3)))^(1/2)*2i - (log(a^9*d^4*e^26 - b^9*d^13*e^17 + 2*a*b^8*d^12*e^18 - 2*
a^8*b*d^5*e^25 + 2*a^8*c*d^6*e^24 - a^2*b^7*d^11*e^19 + a^7*b^2*d^6*e^24 +
16*a^2*c^7*d^18*e^12 + 16*a^5*c^4*d^12*e^18 + a^7*c^2*d^8*e^22 + b^4*c^5*d^
18*e^12 + 16*a^2*c^7*x*(-d^7*e^5)^(5/2) + b^4*c^5*x*(-d^7*e^5)^(5/2) + a^9*
e^24*x*(-d^7*e^5)^(1/2) - 8*a*b^2*c^6*x*(-d^7*e^5)^(5/2) - 42*a^2*b^5*c^2*d
^13*e^17 + 63*a^3*b^3*c^3*d^13*e^17 + 66*a^3*b^4*c^2*d^12*e^18 - 76*a^4*b^2
*c^3*d^12*e^18 - 25*a^4*b^3*c^2*d^11*e^19 + a^2*b^7*e^12*x*(-d^7*e^5)^(3/2)

```
 + b^9*d^2*e^10*x*(-d^7*e^5)^(3/2) + 11*a*b^7*c*d^13*e^17 - 2*a^7*b*c*d^7*e
^23 - 8*a*b^2*c^6*d^18*e^12 - 20*a^2*b^6*c*d^12*e^18 + 9*a^3*b^5*c*d^11*e^1
9 - 28*a^4*b*c^4*d^13*e^17 + 20*a^5*b*c^3*d^11*e^19 + 25*a^4*b^3*c^2*e^12*x
*(-d^7*e^5)^(3/2) + a^7*b^2*d^2*e^22*x*(-d^7*e^5)^(1/2) + a^7*c^2*d^4*e^20*
x*(-d^7*e^5)^(1/2) - 2*a*b^8*d*e^11*x*(-d^7*e^5)^(3/2) - 2*a^8*b*d*e^23*x*(
-d^7*e^5)^(1/2) - 9*a^3*b^5*c*e^12*x*(-d^7*e^5)^(3/2) - 20*a^5*b*c^3*e^12*x
*(-d^7*e^5)^(3/2) - 16*a^5*c^4*d*e^11*x*(-d^7*e^5)^(3/2) + 2*a^8*c*d^2*e^22
*x*(-d^7*e^5)^(1/2) - 11*a*b^7*c*d^2*e^10*x*(-d^7*e^5)^(3/2) + 20*a^2*b^6*c
*d*e^11*x*(-d^7*e^5)^(3/2) - 2*a^7*b*c*d^3*e^21*x*(-d^7*e^5)^(1/2) - 66*a^3
*b^4*c^2*d*e^11*x*(-d^7*e^5)^(3/2) + 28*a^4*b*c^4*d^2*e^10*x*(-d^7*e^5)^(3/
2) + 76*a^4*b^2*c^3*d*e^11*x*(-d^7*e^5)^(3/2) + 42*a^2*b^5*c^2*d^2*e^10*x*(
-d^7*e^5)^(3/2) - 63*a^3*b^3*c^3*d^2*e^10*x*(-d^7*e^5)^(3/2))*(-d^7*e^5)^(1
/2))/(2*(a*e^7 + c*d^2*e^5 - b*d*e^6)) + (log(a^9*d^4*e^26 - b^9*d^13*e^17
+ 2*a*b^8*d^12*e^18 - 2*a^8*b*d^5*e^25 + 2*a^8*c*d^6*e^24 - a^2*b^7*d^11*e^
19 + a^7*b^2*d^6*e^24 + 16*a^2*c^7*d^18*e^12 + 16*a^5*c^4*d^12*e^18 + a^7*c
^2*d^8*e^22 + b^4*c^5*d^18*e^12 - 16*a^2*c^7*x*(-d^7*e^5)^(5/2) - b^4*c^5*x
*(-d^7*e^5)^(5/2) - a^9*e^24*x*(-d^7*e^5)^(1/2) + 8*a*b^2*c^6*x*(-d^7*e^5)^
(5/2) - 42*a^2*b^5*c^2*d^13*e^17 + 63*a^3*b^3*c^3*d^13*e^17 + 66*a^3*b^4*c^
2*d^12*e^18 - 76*a^4*b^2*c^3*d^12*e^18 - 25*a^4*b^3*c^2*d^11*e^19 - a^2*b^7
*e^12*x*(-d^7*e^5)^(3/2) - b^9*d^2*e^10*x*(-d^7*e^5)^(3/2) + 11*a*b^7*c*d^1
3*e^17 - 2*a^7*b*c*d^7*e^23 - 8*a*b^2*c^6*d^18*e^12 - 20*a^2*b^6*c*d^12*e^1
8 + 9*a^3*b^5*c*d^11*e^19 - 28*a^4*b*c^4*d^13*e^17 + 20*a^5*b*c^3*d^11*e^19
 - 25*a^4*b^3*c^2*e^12*x*(-d^7*e^5)^(3/2) - a^7*b^2*d^2*e^22*x*(-d^7*e^5)^(
1/2) - a^7*c^2*d^4*e^20*x*(-d^7*e^5)^(1/2) + 2*a*b^8*d*e^11*x*(-d^7*e^5)^(3
/2) + 2*a^8*b*d*e^23*x*(-d^7*e^5)^(1/2) + 9*a^3*b^5*c*e^12*x*(-d^7*e^5)^(3/
2) + 20*a^5*b*c^3*e^12*x*(-d^7*e^5)^(3/2) + 16*a^5*c^4*d*e^11*x*(-d^7*e^5)^
(3/2) - 2*a^8*c*d^2*e^22*x*(-d^7*e^5)^(1/2) + 11*a*b^7*c*d^2*e^10*x*(-d^7*e
^5)^(3/2) - 20*a^2*b^6*c*d*e^11*x*(-d^7*e^5)^(3/2) + 2*a^7*b*c*d^3*e^21*x*(
-d^7*e^5)^(1/2) + 66*a^3*b^4*c^2*d*e^11*x*(-d^7*e^5)^(3/2) - 28*a^4*b*c^4*d
^2*e^10*x*(-d^7*e^5)^(3/2) - 76*a^4*b^2*c^3*d*e^11*x*(-d^7*e^5)^(3/2) - 42*
a^2*b^5*c^2*d^2*e^10*x*(-d^7*e^5)^(3/2) + 63*a^3*b^3*c^3*d^2*e^10*x*(-d^7*e
^5)^(3/2))*(-d^7*e^5)^(1/2))/(2*a*e^7 + 2*c*d^2*e^5 - 2*b*d*e^6) + x^3/(3*c
*e) - (x*(b*e + c*d))/(c^2*e^2)
```

3.304 $\int \frac{x^6}{(d+ex^2)(a+bx^2+cx^4)}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 323

$$\int \frac{x^6}{(d+ex^2)(a+bx^2+cx^4)}\,dx = \frac{x}{ce}+\frac{\left(b^2d-acd-abe-\frac{b^3d-3abcd-ab^2e+2a^2ce}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}c^{3/2}\sqrt{b-\sqrt{b^2-4ac}}\,(cd^2-bde+ae^2)}+\frac{\left(b^2d-acd-abe+\frac{b^3d-3abcd-ab^2e+2a^2ce}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{2}c^{3/2}\sqrt{b+\sqrt{b^2-4ac}}\,(cd^2-bde+ae^2)}-\frac{d^{5/2}\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{e^{3/2}\,(cd^2-bde+ae^2)}$$

```
[Out] x/c/e-d^(5/2)*arctan(x*e^(1/2)/d^(1/2))/e^(3/2)/(a*e^2-b*d*e+c*d^2)+1/2*arc
tan(x*2^(1/2)*c^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2))*(b^2*d-a*c*d-a*b*e+(-2*
a^2*c*e+a*b^2*e+3*a*b*c*d-b^3*d)/(-4*a*c+b^2)^(1/2))/c^(3/2)/(a*e^2-b*d*e+c
*d^2)*2^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2)+1/2*arctan(x*2^(1/2)*c^(1/2)/(b+
(-4*a*c+b^2)^(1/2))^(1/2))*(b^2*d-a*c*d-a*b*e+(2*a^2*c*e-a*b^2*e-3*a*b*c*d+
b^3*d)/(-4*a*c+b^2)^(1/2))/c^(3/2)/(a*e^2-b*d*e+c*d^2)*2^(1/2)/(b+(-4*a*c+b
^2)^(1/2))^(1/2)
```

Rubi [A] (verified)

Time = 0.87 (sec) , antiderivative size = 323, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used

$= \{1301, 211, 1180\}$

$$\int \frac{x^6}{(d+ex^2)(a+bx^2+cx^4)}\,dx = \frac{\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)\left(-\frac{2a^2ce-ab^2e-3abcd+b^3d}{\sqrt{b^2-4ac}}-abe-acd+b^2d\right)}{\sqrt{2}c^{3/2}\sqrt{b-\sqrt{b^2-4ac}}\,(ae^2-bde+cd^2)} + \frac{\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)\left(\frac{2a^2ce-ab^2e-3abcd+b^3d}{\sqrt{b^2-4ac}}-abe-acd+b^2d\right)}{\sqrt{2}c^{3/2}\sqrt{\sqrt{b^2-4ac}+b}\,(ae^2-bde+cd^2)} - \frac{d^{5/2}\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{e^{3/2}(ae^2-bde+cd^2)} + \frac{x}{ce}$$

```
[In] Int[x^6/((d + e*x^2)*(a + b*x^2 + c*x^4)),x]
```

```
[Out] x/(c*e) + ((b^2*d - a*c*d - a*b*e - (b^3*d - 3*a*b*c*d - a*b^2*e + 2*a^2*c*
e)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]
]])/(Sqrt[2]*c^(3/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]*(c*d^2 - b*d*e + a*e^2)) +
 ((b^2*d - a*c*d - a*b*e + (b^3*d - 3*a*b*c*d - a*b^2*e + 2*a^2*c*e)/Sqrt[b
^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt
[2]*c^(3/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]]*(c*d^2 - b*d*e + a*e^2)) - (d^(5/2)
*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(e^(3/2)*(c*d^2 - b*d*e + a*e^2))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1301

```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.))/((a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*((d + e*x^2)^q/(a
+ b*x^2 + c*x^4)), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b^2 - 4
*a*c, 0] && IntegerQ[q] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(\frac{1}{ce} - \frac{d^3}{e\left(cd^2 - bde + ae^2\right)\left(d + ex^2\right)} + \frac{a(bd - ae) + \left(b^2d - acd - abe\right)x^2}{c\left(cd^2 - bde + ae^2\right)\left(a + bx^2 + cx^4\right)}\right) dx \\
&= \frac{x}{ce} + \frac{\int \frac{a(bd-ae)+\left(b^2d-acd-abe\right)x^2}{a+bx^2+cx^4}\,dx}{c\left(cd^2 - bde + ae^2\right)} - \frac{d^3 \int \frac{1}{d+ex^2}\,dx}{e\left(cd^2 - bde + ae^2\right)} \\
&= \frac{x}{ce} - \frac{d^{5/2}\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{e^{3/2}\left(cd^2 - bde + ae^2\right)} \\
&\quad + \frac{\left(b^2d - acd - abe - \frac{b^3d-3abcd-ab^2e+2a^2ce}{\sqrt{b^2-4ac}}\right)\int \frac{1}{\frac{b}{2}-\frac{1}{2}\sqrt{b^2-4ac}+cx^2}\,dx}{2c\left(cd^2 - bde + ae^2\right)} \\
&\quad + \frac{\left(b^2d - acd - abe + \frac{b^3d-3abcd-ab^2e+2a^2ce}{\sqrt{b^2-4ac}}\right)\int \frac{1}{\frac{b}{2}+\frac{1}{2}\sqrt{b^2-4ac}+cx^2}\,dx}{2c\left(cd^2 - bde + ae^2\right)} \\
&= \frac{x}{ce} + \frac{\left(b^2d - acd - abe - \frac{b^3d-3abcd-ab^2e+2a^2ce}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}c^{3/2}\sqrt{b - \sqrt{b^2-4ac}}\left(cd^2 - bde + ae^2\right)} \\
&\quad + \frac{\left(b^2d - acd - abe + \frac{b^3d-3abcd-ab^2e+2a^2ce}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{2}c^{3/2}\sqrt{b + \sqrt{b^2-4ac}}\left(cd^2 - bde + ae^2\right)} \\
&\quad - \frac{d^{5/2}\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{e^{3/2}\left(cd^2 - bde + ae^2\right)}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.34 (sec) , antiderivative size = 385, normalized size of antiderivative = 1.19

$$\begin{aligned}
\int \frac{x^6}{\left(d + ex^2\right)\left(a + bx^2 + cx^4\right)}\,dx &= \frac{x}{ce} \\
&+ \frac{\left(b^3d - b^2\left(\sqrt{b^2-4ac}d + ae\right) + ac\left(\sqrt{b^2-4ac}d + 2ae\right) + ab\left(-3cd + \sqrt{b^2-4ac}e\right)\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}c^{3/2}\sqrt{b^2-4ac}\sqrt{b - \sqrt{b^2-4ac}}\left(-cd^2 + e(bd - ae)\right)} \\
&+ \frac{\left(b^3d + b^2\left(\sqrt{b^2-4ac}d - ae\right) + ac\left(-\sqrt{b^2-4ac}d + 2ae\right) - ab\left(3cd + \sqrt{b^2-4ac}e\right)\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{2}c^{3/2}\sqrt{b^2-4ac}\sqrt{b + \sqrt{b^2-4ac}}\left(cd^2 + e(-bd + ae)\right)} \\
&- \frac{d^{5/2}\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{e^{3/2}\left(cd^2 - bde + ae^2\right)}
\end{aligned}$$

```
[In] Integrate[x^6/((d + e*x^2)*(a + b*x^2 + c*x^4)),x]
```

```
[Out] x/(c*e) + ((b^3*d - b^2*(Sqrt[b^2 - 4*a*c]*d + a*e) + a*c*(Sqrt[b^2 - 4*a*c
]*d + 2*a*e) + a*b*(-3*c*d + Sqrt[b^2 - 4*a*c]*e))*ArcTan[(Sqrt[2]*Sqrt[c]*
x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*c^(3/2)*Sqrt[b^2 - 4*a*c]*Sqrt[b
- Sqrt[b^2 - 4*a*c]]*(-(c*d^2) + e*(b*d - a*e))) + ((b^3*d + b^2*(Sqrt[b^2
- 4*a*c]*d - a*e) + a*c*(-(Sqrt[b^2 - 4*a*c]*d) + 2*a*e) - a*b*(3*c*d + Sqr
t[b^2 - 4*a*c]*e))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])
/(Sqrt[2]*c^(3/2)*Sqrt[b^2 - 4*a*c]*Sqrt[b + Sqrt[b^2 - 4*a*c]]*(c*d^2 + e*
(-(b*d) + a*e))) - (d^(5/2)*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(e^(3/2)*(c*d^2 -
b*d*e + a*e^2))
```

Maple [A] (verified)

Time = 0.49 (sec) , antiderivative size = 331, normalized size of antiderivative = 1.02

method	result
default	$\frac{x}{ce}+\frac{\frac{\left(-abe\sqrt{-4ac+b^2}-acd\sqrt{-4ac+b^2}+b^2d\sqrt{-4ac+b^2}+2a^2ce-a\,b^2e-3abcd+b^3d\right)\sqrt{2}\,\arctan\left(\frac{cx\sqrt{2}}{\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}}\right)}{2c\sqrt{-4ac+b^2}\,\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}}-\frac{\left(-abe\sqrt{-4ac+b^2}-acd\sqrt{\right.}{}}{}}{a\,e^2-bde+c\,d^2}$
risch	Expression too large to display

```
[In] int(x^6/(e*x^2+d)/(c*x^4+b*x^2+a),x,method=_RETURNVERBOSE)
```

```
[Out] x/c/e+4/(a*e^2-b*d*e+c*d^2)*(1/8*(-a*b*e*(-4*a*c+b^2)^(1/2)-a*c*d*(-4*a*c+b
^2)^(1/2)+b^2*d*(-4*a*c+b^2)^(1/2)+2*a^2*c*e-a*b^2*e-3*a*b*c*d+b^3*d)/c/(-4
*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)
/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))-1/8*(-a*b*e*(-4*a*c+b^2)^(1/2)-a*c*d*(-4
*a*c+b^2)^(1/2)+b^2*d*(-4*a*c+b^2)^(1/2)-2*a^2*c*e+a*b^2*e+3*a*b*c*d-b^3*d)
/c/(-4*a*c+b^2)^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(c*x
*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)))-1/e*d^3/(a*e^2-b*d*e+c*d^2)/(e
*d)^(1/2)*arctan(e*x/(e*d)^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 10064 vs. 2(281) = 562.

Time = 25.69 (sec) , antiderivative size = 20147, normalized size of antiderivative = 62.37

$$\int \frac{x^6}{(d+ex^2)\left(a+bx^2+cx^4\right)}\,dx=\text{Too large to display}$$

```
[In] integrate(x^6/(e*x^2+d)/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^6}{(d+ex^2)(a+bx^2+cx^4)} dx = \text{Timed out}$$

```
[In] integrate(x**6/(e*x**2+d)/(c*x**4+b*x**2+a),x)
```

```
[Out] Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^6}{(d+ex^2)(a+bx^2+cx^4)} dx = \text{Exception raised: ValueError}$$

```
[In] integrate(x^6/(e*x^2+d)/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 11046 vs. 2(281) = 562.

Time = 2.44 (sec) , antiderivative size = 11046, normalized size of antiderivative = 34.20

$$\int \frac{x^6}{(d+ex^2)(a+bx^2+cx^4)} dx = \text{Too large to display}$$

```
[In] integrate(x^6/(e*x^2+d)/(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] -d^3*arctan(e*x/sqrt(d*e))/((c*d^2*e - b*d*e^2 + a*e^3)*sqrt(d*e)) - 1/8*((
2*b^6*c^6 - 14*a*b^4*c^7 + 24*a^2*b^2*c^8 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(
b*c - sqrt(b^2 - 4*a*c)*c)*b^6*c^4 + 7*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c -
 sqrt(b^2 - 4*a*c)*c)*a*b^4*c^5 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sq
rt(b^2 - 4*a*c)*c)*b^5*c^5 - 12*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b
^2 - 4*a*c)*c)*a^2*b^2*c^6 - 6*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^
2 - 4*a*c)*c)*a*b^3*c^6 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4
*a*c)*c)*b^4*c^6 + 3*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)
*c)*a*b^2*c^7 - 2*(b^2 - 4*a*c)*b^4*c^6 + 6*(b^2 - 4*a*c)*a*b^2*c^7)*d^5 -
(4*b^7*c^5 - 26*a*b^5*c^6 + 36*a^2*b^3*c^7 + 16*a^3*b*c^8 - 2*sqrt(2)*sqrt(
b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^7*c^3 + 13*sqrt(2)*sqrt(b^2
```

- 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^5*c^4 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^6*c^4 - 18*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c^5 - 10*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^4*c^5 - 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^5*c^5 - 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b*c^6 - 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^6 + 5*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^3*c^6 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b*c^7 - 4*(b^2 - 4*a*c)*b^5*c^5 + 10*(b^2 - 4*a*c)*a*b^3*c^6 + 4*(b^2 - 4*a*c)*a^2*b*c^7)*d^4*e + (2*b^8*c^4 - 6*a*b^6*c^5 - 28*a^2*b^4*c^6 + 80*a^3*b^2*c^7 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^8*c^2 + 3*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^6*c^3 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^7*c^3 + 14*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^4*c^4 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^5*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^6*c^4 - 40*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^2*c^5 - 20*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c^5 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^4*c^5 + 10*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^6 - 2*(b^2 - 4*a*c)*b^6*c^4 - 2*(b^2 - 4*a*c)*a*b^4*c^5 + 20*(b^2 - 4*a*c)*a^2*b^2*c^6)*d^3*e^2 - (6*a*b^7*c^4 - 36*a^2*b^5*c^5 + 40*a^3*b^3*c^6 + 32*a^4*b*c^7 - 3*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^7*c^2 + 18*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^5*c^3 + 6*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^6*c^3 - 20*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^3*c^4 - 12*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^4*c^4 - 3*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^5*c^4 - 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*b*c^5 - 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^2*c^5 + 6*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c^5 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b*c^6 - 6*(b^2 - 4*a*c)*a*b^5*c^4 + 12*(b^2 - 4*a*c)*a^2*b^3*c^5 + 8*(b^2 - 4*a*c)*a^3*b*c^6)*d^2*e^3 + (6*a^2*b^6*c^4 - 38*a^3*b^4*c^5 + 56*a^4*b^2*c^6 - 3*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^6*c^2 + 19*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^4*c^3 + 6*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^5*c^3 - 28*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*b^2*c^4 - 14*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^3*c^4 - 3*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^4*c^4 + 7*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^2*c^5 - 6*(b^2 - 4*a*c)*a^2*b^4*c^4 + 14*(b^2 - 4*a*c)*a^3*b^2*c^5)*d*e^4 - (2*a^3*b^5*c^4 - 12*a^4*b^3*c^5 + 16*a^5*b*c^6 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^5*c^2 + 6*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*b^3*c^3 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 -

4*a*c)*c)*a^3*b^4*c^3 - 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^5*b*c^4 - 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*b^2*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^3*c^4 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*b*c^5 - 2*(b^2 - 4*a*c)*a^3*b^3*c^4 + 4*(b^2 - 4*a*c)*a^4*b*c^5)*e^5 - 2*(sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^5*c^3 - 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c^4 - 2*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^4*c^4 + 2*a*b^5*c^4 + 16*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b*c^5 + 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^5 + sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^3*c^5 - 16*a^2*b^3*c^5 - 4*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b*c^6 + 32*a^3*b*c^6 - 2*(b^2 - 4*a*c)*a*b^3*c^4 + 8*(b^2 - 4*a*c)*a^2*b*c^5)*d^3*abs(-c^2*d^2 + b*c*d*e - a*c*e^2) + 2*(sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^6*c^2 - 7*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^4*c^3 - 2*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^5*c^3 + 2*a*b^6*c^3 + 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^2*c^4 + 6*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c^4 + sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^4*c^4 - 14*a^2*b^4*c^4 + 16*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*c^5 + 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b*c^5 - 3*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^5 + 16*a^3*b^2*c^5 - 4*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*c^6 + 32*a^4*c^6 - 2*(b^2 - 4*a*c)*a*b^4*c^3 + 6*(b^2 - 4*a*c)*a^2*b^2*c^4 + 8*(b^2 - 4*a*c)*a^3*c^5)*d^2*e*abs(-c^2*d^2 + b*c*d*e - a*c*e^2) - 4*(sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^5*c^2 - 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^3*c^3 - 2*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^4*c^3 + 2*a^2*b^5*c^3 + 16*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*b*c^4 + 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^2*c^4 + sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c^4 - 16*a^3*b^3*c^4 - 4*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b*c^5 + 32*a^4*b*c^5 - 2*(b^2 - 4*a*c)*a^2*b^3*c^3 + 8*(b^2 - 4*a*c)*a^3*b*c^4)*d*e^2*abs(-c^2*d^2 + b*c*d*e - a*c*e^2) + 2*(sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^4*c^2 - 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*b^2*c^3 - 2*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^3*c^3 + 2*a^3*b^4*c^3 + 16*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^5*c^4 + 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*b*c^4 + sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^2*c^4 - 16*a^4*b^2*c^4 - 4*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*c^5 + 32*a^5*c^5 - 2*(b^2 - 4*a*c)*a^3*b^2*c^3 + 8*(b^2 - 4*a*c)*a^4*c^4)*e^3*abs(-c^2*d^2 + b*c*d*e - a*c*e^2) - (2*b^6*c^2 - 18*a*b^4*c^3 + 48*a^2*b^2*c^4 - 32*a^3*c^5 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^6 + 9*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^4*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^5*c - 24*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^2 - 10*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^3*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^4*c^2 + 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*c^3 + 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b*c^3 + 5*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^2*c^

3 - 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*c^4 - 2
*(b^2 - 4*a*c)*b^4*c^2 + 10*(b^2 - 4*a*c)*a*b^2*c^3 - 8*(b^2 - 4*a*c)*a^2*c
^4)*(c^2*d^2 - b*c*d*e + a*c*e^2)^2*d + (2*a*b^5*c^2 - 16*a^2*b^3*c^3 + 32*
a^3*b*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^5
 + 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c +
2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^4*c - 16*sq
rt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b*c^2 - 8*sqrt(
2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^2 - sqrt(2)*
sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^3*c^2 + 4*sqrt(2)*sqr
t(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b*c^3 - 2*(b^2 - 4*a*c)*
a*b^3*c^2 + 8*(b^2 - 4*a*c)*a^2*b*c^3)*(c^2*d^2 - b*c*d*e + a*c*e^2)^2*e)*a
rctan(2*sqrt(1/2)*x/sqrt((b*c^2*d^2 - b^2*c*d*e + a*b*c*e^2 + sqrt((b*c^2*d
^2 - b^2*c*d*e + a*b*c*e^2)^2 - 4*(a*c^2*d^2 - a*b*c*d*e + a^2*c*e^2)*(c^3*
d^2 - b*c^2*d*e + a*c^2*e^2)))/(c^3*d^2 - b*c^2*d*e + a*c^2*e^2)))/((a*b^4*
c^5 - 8*a^2*b^2*c^6 - 2*a*b^3*c^6 + 16*a^3*c^7 + 8*a^2*b*c^7 + a*b^2*c^7 -
4*a^2*c^8)*d^4*abs(-c^2*d^2 + b*c*d*e - a*c*e^2)*abs(c) - 2*(a*b^5*c^4 - 8*
a^2*b^3*c^5 - 2*a*b^4*c^5 + 16*a^3*b*c^6 + 8*a^2*b^2*c^6 + a*b^3*c^6 - 4*a^
2*b*c^7)*d^3*e*abs(-c^2*d^2 + b*c*d*e - a*c*e^2)*abs(c) + (a*b^6*c^3 - 6*a^
2*b^4*c^4 - 2*a*b^5*c^4 + 4*a^2*b^3*c^5 + a*b^4*c^5 + 32*a^4*c^6 + 16*a^3*b
*c^6 - 2*a^2*b^2*c^6 - 8*a^3*c^7)*d^2*e^2*abs(-c^2*d^2 + b*c*d*e - a*c*e^2)
abs(c) - 2(a^2*b^5*c^3 - 8*a^3*b^3*c^4 - 2*a^2*b^4*c^4 + 16*a^4*b*c^5 + 8
*a^3*b^2*c^5 + a^2*b^3*c^5 - 4*a^3*b*c^6)*d*e^3*abs(-c^2*d^2 + b*c*d*e - a*
c*e^2)*abs(c) + (a^3*b^4*c^3 - 8*a^4*b^2*c^4 - 2*a^3*b^3*c^4 + 16*a^5*c^5 +
 8*a^4*b*c^5 + a^3*b^2*c^5 - 4*a^4*c^6)*e^4*abs(-c^2*d^2 + b*c*d*e - a*c*e^
2)*abs(c)) - 1/8*((2*b^6*c^6 - 14*a*b^4*c^7 + 24*a^2*b^2*c^8 - sqrt(2)*sqrt
(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^6*c^4 + 7*sqrt(2)*sqrt(b^2
- 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^4*c^5 + 2*sqrt(2)*sqrt(b^2 - 4
*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^5*c^5 - 12*sqrt(2)*sqrt(b^2 - 4*a*c
)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^6 - 6*sqrt(2)*sqrt(b^2 - 4*a*c)
*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c^6 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt
(b*c + sqrt(b^2 - 4*a*c)*c)*b^4*c^6 + 3*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c
+ sqrt(b^2 - 4*a*c)*c)*a*b^2*c^7 - 2*(b^2 - 4*a*c)*b^4*c^6 + 6*(b^2 - 4*a*c
)*a*b^2*c^7)*d^5 - (4*b^7*c^5 - 26*a*b^5*c^6 + 36*a^2*b^3*c^7 + 16*a^3*b*c^
8 - 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^7*c^3 + 1
3*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^5*c^4 + 4*s
qrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^6*c^4 - 18*sqrt(
2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c^5 - 10*sqrt(
2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^4*c^5 - 2*sqrt(2)*
sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^5*c^5 - 8*sqrt(2)*sqrt(
b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b*c^6 - 4*sqrt(2)*sqrt(b^2
 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^6 + 5*sqrt(2)*sqrt(b^2
- 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c^6 + 2*sqrt(2)*sqrt(b^2 - 4
*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b*c^7 - 4*(b^2 - 4*a*c)*b^5*c^5 +
 10*(b^2 - 4*a*c)*a*b^3*c^6 + 4*(b^2 - 4*a*c)*a^2*b*c^7)*d^4*e + (2*b^8*c^4
 - 6*a*b^6*c^5 - 28*a^2*b^4*c^6 + 80*a^3*b^2*c^7 - sqrt(2)*sqrt(b^2 - 4*a*c

)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^8*c^2 + 3*sqrt(2)*sqrt(b^2 - 4*a*c)*sqr
t(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^6*c^3 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b
*c + sqrt(b^2 - 4*a*c)*c)*b^7*c^3 + 14*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c +
 sqrt(b^2 - 4*a*c)*c)*a^2*b^4*c^4 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c +
sqrt(b^2 - 4*a*c)*c)*a*b^5*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(
b^2 - 4*a*c)*c)*b^6*c^4 - 40*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2
- 4*a*c)*c)*a^3*b^2*c^5 - 20*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2
- 4*a*c)*c)*a^2*b^3*c^5 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4
*a*c)*c)*a*b^4*c^5 + 10*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a
*c)*c)*a^2*b^2*c^6 - 2*(b^2 - 4*a*c)*b^6*c^4 - 2*(b^2 - 4*a*c)*a*b^4*c^5 +
20*(b^2 - 4*a*c)*a^2*b^2*c^6)*d^3*e^2 - (6*a*b^7*c^4 - 36*a^2*b^5*c^5 + 40*
a^3*b^3*c^6 + 32*a^4*b*c^7 - 3*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^
2 - 4*a*c)*c)*a*b^7*c^2 + 18*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2
- 4*a*c)*c)*a^2*b^5*c^3 + 6*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 -
 4*a*c)*c)*a*b^6*c^3 - 20*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4
*a*c)*c)*a^3*b^3*c^4 - 12*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4
*a*c)*c)*a^2*b^4*c^4 - 3*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*
a*c)*c)*a*b^5*c^4 - 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*
c)*c)*a^4*b*c^5 - 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*
c)*a^3*b^2*c^5 + 6*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c
)*a^2*b^3*c^5 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)
*a^3*b*c^6 - 6*(b^2 - 4*a*c)*a*b^5*c^4 + 12*(b^2 - 4*a*c)*a^2*b^3*c^5 + 8*(
b^2 - 4*a*c)*a^3*b*c^6)*d^2*e^3 + (6*a^2*b^6*c^4 - 38*a^3*b^4*c^5 + 56*a^4*
b^2*c^6 - 3*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b
^6*c^2 + 19*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b
^4*c^3 + 6*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^
5*c^3 - 28*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b^
2*c^4 - 14*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^
3*c^4 - 3*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^4
*c^4 + 7*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^2*
c^5 - 6*(b^2 - 4*a*c)*a^2*b^4*c^4 + 14*(b^2 - 4*a*c)*a^3*b^2*c^5)*d*e^4 - (
2*a^3*b^5*c^4 - 12*a^4*b^3*c^5 + 16*a^5*b*c^6 - sqrt(2)*sqrt(b^2 - 4*a*c)*s
qrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^5*c^2 + 6*sqrt(2)*sqrt(b^2 - 4*a*c)*sq
rt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b^3*c^3 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqr
t(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^4*c^3 - 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt
(b*c + sqrt(b^2 - 4*a*c)*c)*a^5*b*c^4 - 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*
c + sqrt(b^2 - 4*a*c)*c)*a^4*b^2*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c +
 sqrt(b^2 - 4*a*c)*c)*a^3*b^3*c^4 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c +
sqrt(b^2 - 4*a*c)*c)*a^4*b*c^5 - 2*(b^2 - 4*a*c)*a^3*b^3*c^4 + 4*(b^2 - 4*a
*c)*a^4*b*c^5)*e^5 - 2*(sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^5*c^3 -
 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c^4 - 2*sqrt(2)*sqrt(b*c
 + sqrt(b^2 - 4*a*c)*c)*a*b^4*c^4 - 2*a*b^5*c^4 + 16*sqrt(2)*sqrt(b*c + sqr
t(b^2 - 4*a*c)*c)*a^3*b*c^5 + 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2
*b^2*c^5 + sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c^5 + 16*a^2*b^3*c
^5 - 4*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b*c^6 - 32*a^3*b*c^6 + 2

*(b^2 - 4*a*c)*a*b^3*c^4 - 8*(b^2 - 4*a*c)*a^2*b*c^5)*d^3*abs(-c^2*d^2 + b*c*d*e - a*c*e^2) + 2*(sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^6*c^2 - 7*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^4*c^3 - 2*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^5*c^3 - 2*a*b^6*c^3 + 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^2*c^4 + 6*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c^4 + sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^4*c^4 + 14*a^2*b^4*c^4 + 16*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*c^5 + 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b*c^5 - 3*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^5 - 16*a^3*b^2*c^5 - 4*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*c^6 - 32*a^4*c^6 + 2*(b^2 - 4*a*c)*a*b^4*c^3 - 6*(b^2 - 4*a*c)*a^2*b^2*c^4 - 8*(b^2 - 4*a*c)*a^3*c^5)*d^2*e*abs(-c^2*d^2 + b*c*d*e - a*c*e^2) - 4*(sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^5*c^2 - 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^3*c^3 - 2*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^4*c^3 - 2*a^2*b^5*c^3 + 16*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b*c^4 + 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^2*c^4 + sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c^4 + 16*a^3*b^3*c^4 - 4*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b*c^5 - 32*a^4*b*c^5 + 2*(b^2 - 4*a*c)*a^2*b^3*c^3 - 8*(b^2 - 4*a*c)*a^3*b*c^4)*d*e^2*abs(-c^2*d^2 + b*c*d*e - a*c*e^2) + 2*(sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^4*c^2 - 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b^2*c^3 - 2*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^3*c^3 - 2*a^3*b^4*c^3 + 16*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^5*c^4 + 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b*c^4 + sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^2*c^4 + 16*a^4*b^2*c^4 - 4*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*c^5 - 32*a^5*c^5 + 2*(b^2 - 4*a*c)*a^3*b^2*c^3 - 8*(b^2 - 4*a*c)*a^4*c^4)*e^3*abs(-c^2*d^2 + b*c*d*e - a*c*e^2) - (2*b^6*c^2 - 18*a*b^4*c^3 + 48*a^2*b^2*c^4 - 32*a^3*c^5 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^6 + 9*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^4*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^5*c - 24*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^2 - 10*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^4*c^2 + 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*c^3 + 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b*c^3 + 5*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^3 - 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*c^4 - 2*(b^2 - 4*a*c)*b^4*c^2 + 10*(b^2 - 4*a*c)*a*b^2*c^3 - 8*(b^2 - 4*a*c)*a^2*c^4)*(c^2*d^2 - b*c*d*e + a*c*e^2)^2*d + (2*a*b^5*c^2 - 16*a^2*b^3*c^3 + 32*a^3*b*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^5 + 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^4*c - 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b*c^2 - 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c^2 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b*c^3 - 2*(b^2 - 4*a*c)*a*b^3*c^2 + 8*(b^2 - 4*a*c)*a^2*b*c^3)*(c^2*d^2 - b*c*d*

```
e + a*c*e^2)^2*e)*arctan(2*sqrt(1/2)*x/sqrt((b*c^2*d^2 - b^2*c*d*e + a*b*c*
e^2 - sqrt((b*c^2*d^2 - b^2*c*d*e + a*b*c*e^2)^2 - 4*(a*c^2*d^2 - a*b*c*d*e
 + a^2*c*e^2)*(c^3*d^2 - b*c^2*d*e + a*c^2*e^2)))/(c^3*d^2 - b*c^2*d*e + a*
c^2*e^2)))/((a*b^4*c^5 - 8*a^2*b^2*c^6 - 2*a*b^3*c^6 + 16*a^3*c^7 + 8*a^2*b
*c^7 + a*b^2*c^7 - 4*a^2*c^8)*d^4*abs(-c^2*d^2 + b*c*d*e - a*c*e^2)*abs(c)
- 2*(a*b^5*c^4 - 8*a^2*b^3*c^5 - 2*a*b^4*c^5 + 16*a^3*b*c^6 + 8*a^2*b^2*c^6
 + a*b^3*c^6 - 4*a^2*b*c^7)*d^3*e*abs(-c^2*d^2 + b*c*d*e - a*c*e^2)*abs(c)
+ (a*b^6*c^3 - 6*a^2*b^4*c^4 - 2*a*b^5*c^4 + 4*a^2*b^3*c^5 + a*b^4*c^5 + 32
*a^4*c^6 + 16*a^3*b*c^6 - 2*a^2*b^2*c^6 - 8*a^3*c^7)*d^2*e^2*abs(-c^2*d^2 +
 b*c*d*e - a*c*e^2)*abs(c) - 2*(a^2*b^5*c^3 - 8*a^3*b^3*c^4 - 2*a^2*b^4*c^4
 + 16*a^4*b*c^5 + 8*a^3*b^2*c^5 + a^2*b^3*c^5 - 4*a^3*b*c^6)*d*e^3*abs(-c^2
*d^2 + b*c*d*e - a*c*e^2)*abs(c) + (a^3*b^4*c^3 - 8*a^4*b^2*c^4 - 2*a^3*b^3
*c^4 + 16*a^5*c^5 + 8*a^4*b*c^5 + a^3*b^2*c^5 - 4*a^4*c^6)*e^4*abs(-c^2*d^2
 + b*c*d*e - a*c*e^2)*abs(c)) + x/(c*e)
```

Mupad [B] (verification not implemented)

Time = 11.48 (sec) , antiderivative size = 33892, normalized size of antiderivative = 104.93

$$\int \frac{x^6}{(d+ex^2)(a+bx^2+cx^4)}\,dx = \text{Too large to display}$$

```
[In] int(x^6/((d + e*x^2)*(a + b*x^2 + c*x^4)),x)
```

```
[Out] x/(c*e) - atan(((((((64*a^5*c^4*d*e^8 + 64*a^3*c^6*d^5*e^4 + 128*a^4*c^5*d^
3*e^6 - 144*a^2*b^2*c^5*d^5*e^4 + 64*a^2*b^3*c^4*d^4*e^5 + 16*a^2*b^4*c^3*d
^3*e^6 - 96*a^3*b^2*c^4*d^3*e^6 + 16*a^3*b^3*c^3*d^2*e^7 - 16*a*b^3*c^5*d^6
*e^3 + 32*a*b^4*c^4*d^5*e^4 - 16*a*b^5*c^3*d^4*e^5 + 64*a^2*b*c^6*d^6*e^3 -
 64*a^4*b*c^4*d^2*e^7 - 16*a^4*b^2*c^3*d*e^8)/(c*e) - (2*x*(-(b^7*d^2 + a^2
*b^5*e^2 - b^4*d^2*(-(4*a*c - b^2)^3)^(1/2) - 20*a^3*b*c^3*d^2 - 7*a^3*b^3*
c*e^2 + 12*a^4*b*c^2*e^2 + a^3*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - 2*a*b^6*d*e
 + 25*a^2*b^3*c^2*d^2 - a^2*b^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - a^2*c^2*d^2*
(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c*d^2 + 16*a^4*c^3*d*e + 2*a*b^3*d*e*(-(
4*a*c - b^2)^3)^(1/2) + 16*a^2*b^4*c*d*e + 3*a*b^2*c*d^2*(-(4*a*c - b^2)^3)
^(1/2) - 36*a^3*b^2*c^2*d*e - 4*a^2*b*c*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(1
6*a^2*c^7*d^4 + 16*a^4*c^5*e^4 + b^4*c^5*d^4 - 8*a*b^2*c^6*d^4 - 2*b^5*c^4*
d^3*e + a^2*b^4*c^3*e^4 - 8*a^3*b^2*c^4*e^4 + 32*a^3*c^6*d^2*e^2 + b^6*c^3*
d^2*e^2 + 16*a*b^3*c^5*d^3*e - 2*a*b^5*c^3*d*e^3 - 32*a^2*b*c^6*d^3*e - 32*
a^3*b*c^5*d*e^3 - 6*a*b^4*c^4*d^2*e^2 + 16*a^2*b^3*c^4*d*e^3)))^(1/2)*(128*
a^4*b^2*c^4*e^10 - 16*a^3*b^4*c^3*e^10 - 256*a^5*c^5*e^10 + 256*a^2*c^8*d^6
*e^4 + 256*a^3*c^7*d^4*e^6 - 256*a^4*c^6*d^2*e^8 - 16*b^3*c^7*d^7*e^3 + 64*
b^4*c^6*d^6*e^4 - 96*b^5*c^5*d^5*e^5 + 64*b^6*c^4*d^4*e^6 - 16*b^7*c^3*d^3*
e^7 + 256*a^2*b^2*c^6*d^4*e^6 + 144*a^2*b^3*c^5*d^3*e^7 - 96*a^2*b^4*c^4*d^
2*e^8 + 192*a^3*b^2*c^5*d^2*e^8 + 64*a*b*c^8*d^7*e^3 + 320*a^4*b*c^5*d*e^9
- 320*a*b^2*c^7*d^6*e^4 + 528*a*b^3*c^6*d^5*e^5 - 336*a*b^4*c^5*d^4*e^6 + 4
```

8*a*b^5*c^4*d^3*e^7 + 16*a*b^6*c^3*d^2*e^8 - 576*a^2*b*c^7*d^5*e^5 + 16*a^2
*b^5*c^3*d*e^9 - 320*a^3*b*c^6*d^3*e^7 - 144*a^3*b^3*c^4*d*e^9))/(c*e))*(-(
b^7*d^2 + a^2*b^5*e^2 - b^4*d^2*(-(4*a*c - b^2)^3)^(1/2) - 20*a^3*b*c^3*d^2
 - 7*a^3*b^3*c*e^2 + 12*a^4*b*c^2*e^2 + a^3*c*e^2*(-(4*a*c - b^2)^3)^(1/2)
- 2*a*b^6*d*e + 25*a^2*b^3*c^2*d^2 - a^2*b^2*e^2*(-(4*a*c - b^2)^3)^(1/2) -
 a^2*c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c*d^2 + 16*a^4*c^3*d*e + 2*
a*b^3*d*e*(-(4*a*c - b^2)^3)^(1/2) + 16*a^2*b^4*c*d*e + 3*a*b^2*c*d^2*(-(4*
a*c - b^2)^3)^(1/2) - 36*a^3*b^2*c^2*d*e - 4*a^2*b*c*d*e*(-(4*a*c - b^2)^3)
^(1/2))/(8*(16*a^2*c^7*d^4 + 16*a^4*c^5*e^4 + b^4*c^5*d^4 - 8*a*b^2*c^6*d^4
 - 2*b^5*c^4*d^3*e + a^2*b^4*c^3*e^4 - 8*a^3*b^2*c^4*e^4 + 32*a^3*c^6*d^2*e
^2 + b^6*c^3*d^2*e^2 + 16*a*b^3*c^5*d^3*e - 2*a*b^5*c^3*d*e^3 - 32*a^2*b*c^
6*d^3*e - 32*a^3*b*c^5*d*e^3 - 6*a*b^4*c^4*d^2*e^2 + 16*a^2*b^3*c^4*d*e^3))
)^(1/2) + (2*x*(4*a^3*b^5*e^8 + 4*b^3*c^5*d^8 + 4*b^8*d^3*e^5 - 28*a^4*b^3*
c*e^8 + 48*a^5*b*c^2*e^8 - 4*a*b^7*d^2*e^6 - 4*a^2*b^6*d*e^7 - 64*a^2*c^6*d
^7*e + 56*a^5*c^3*d*e^7 - 8*b^4*c^4*d^7*e - 8*b^7*c*d^4*e^4 - 8*a^3*c^5*d^5
*e^3 - 16*a^4*c^4*d^3*e^5 + 4*b^5*c^3*d^6*e^2 + 4*b^6*c^2*d^5*e^3 - 16*a*b*
c^6*d^8 + 36*a^2*b^2*c^4*d^5*e^3 - 72*a^2*b^3*c^3*d^4*e^4 - 12*a^2*b^4*c^2*
d^3*e^5 + 64*a^3*b^2*c^3*d^3*e^5 + 28*a^3*b^3*c^2*d^2*e^6 + 48*a*b^2*c^5*d^
7*e - 16*a*b^6*c*d^3*e^5 + 40*a^3*b^4*c*d*e^7 - 28*a*b^3*c^4*d^6*e^2 - 24*a
*b^4*c^3*d^5*e^3 + 48*a*b^5*c^2*d^4*e^4 + 48*a^2*b*c^5*d^6*e^2 + 12*a^2*b^5
*c*d^2*e^6 + 16*a^3*b*c^4*d^4*e^4 - 64*a^4*b*c^3*d^2*e^6 - 108*a^4*b^2*c^2*
d*e^7))/(c*e))*(-(b^7*d^2 + a^2*b^5*e^2 - b^4*d^2*(-(4*a*c - b^2)^3)^(1/2)
- 20*a^3*b*c^3*d^2 - 7*a^3*b^3*c*e^2 + 12*a^4*b*c^2*e^2 + a^3*c*e^2*(-(4*a*
c - b^2)^3)^(1/2) - 2*a*b^6*d*e + 25*a^2*b^3*c^2*d^2 - a^2*b^2*e^2*(-(4*a*c
 - b^2)^3)^(1/2) - a^2*c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c*d^2 + 1
6*a^4*c^3*d*e + 2*a*b^3*d*e*(-(4*a*c - b^2)^3)^(1/2) + 16*a^2*b^4*c*d*e + 3
*a*b^2*c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 36*a^3*b^2*c^2*d*e - 4*a^2*b*c*d*e*
(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^7*d^4 + 16*a^4*c^5*e^4 + b^4*c^5*d^4
 - 8*a*b^2*c^6*d^4 - 2*b^5*c^4*d^3*e + a^2*b^4*c^3*e^4 - 8*a^3*b^2*c^4*e^4
+ 32*a^3*c^6*d^2*e^2 + b^6*c^3*d^2*e^2 + 16*a*b^3*c^5*d^3*e - 2*a*b^5*c^3*d
*e^3 - 32*a^2*b*c^6*d^3*e - 32*a^3*b*c^5*d*e^3 - 6*a*b^4*c^4*d^2*e^2 + 16*a
^2*b^3*c^4*d*e^3)))^(1/2) - (4*a*b^3*c^3*d^7 - 16*a^2*b*c^4*d^7 + 4*a*b^6*d
^4*e^3 + 4*a^4*b^3*d*e^6 + 48*a^3*c^4*d^6*e - 4*a^2*b^5*d^3*e^4 - 4*a^3*b^4
*d^2*e^5 - 60*a^4*c^3*d^4*e^3 + 4*a^5*c^2*d^2*e^5 - 8*a^5*b*c*d*e^6 - 32*a^
2*b^3*c^2*d^5*e^2 + 92*a^3*b^2*c^2*d^4*e^3 + 4*a*b^4*c^2*d^6*e + 4*a*b^5*c*
d^5*e^2 - 28*a^2*b^2*c^3*d^6*e - 36*a^2*b^4*c*d^4*e^3 + 64*a^3*b*c^3*d^5*e^
2 + 36*a^3*b^3*c*d^3*e^4 - 60*a^4*b*c^2*d^3*e^4 + 4*a^4*b^2*c*d^2*e^5)/(c*e
))*(-(b^7*d^2 + a^2*b^5*e^2 - b^4*d^2*(-(4*a*c - b^2)^3)^(1/2) - 20*a^3*b*c
^3*d^2 - 7*a^3*b^3*c*e^2 + 12*a^4*b*c^2*e^2 + a^3*c*e^2*(-(4*a*c - b^2)^3)^
(1/2) - 2*a*b^6*d*e + 25*a^2*b^3*c^2*d^2 - a^2*b^2*e^2*(-(4*a*c - b^2)^3)^(
1/2) - a^2*c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c*d^2 + 16*a^4*c^3*d*
e + 2*a*b^3*d*e*(-(4*a*c - b^2)^3)^(1/2) + 16*a^2*b^4*c*d*e + 3*a*b^2*c*d^2
*(-(4*a*c - b^2)^3)^(1/2) - 36*a^3*b^2*c^2*d*e - 4*a^2*b*c*d*e*(-(4*a*c - b
^2)^3)^(1/2))/(8*(16*a^2*c^7*d^4 + 16*a^4*c^5*e^4 + b^4*c^5*d^4 - 8*a*b^2*c
^6*d^4 - 2*b^5*c^4*d^3*e + a^2*b^4*c^3*e^4 - 8*a^3*b^2*c^4*e^4 + 32*a^3*c^6

*d^2*e^2 + b^6*c^3*d^2*e^2 + 16*a*b^3*c^5*d^3*e - 2*a*b^5*c^3*d*e^3 - 32*a^2*b*c^6*d^3*e - 32*a^3*b*c^5*d*e^3 - 6*a*b^4*c^4*d^2*e^2 + 16*a^2*b^3*c^4*d*e^3)))^(1/2) + (2*x*(a^6*e^6 + b^6*d^6 - 2*a^3*c^3*d^6 + 9*a^2*b^2*c^2*d^6 - 6*a*b^4*c*d^6))/(c*e))*(-(b^7*d^2 + a^2*b^5*e^2 - b^4*d^2*(-(4*a*c - b^2)^3)^(1/2) - 20*a^3*b*c^3*d^2 - 7*a^3*b^3*c*e^2 + 12*a^4*b*c^2*e^2 + a^3*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - 2*a*b^6*d*e + 25*a^2*b^3*c^2*d^2 - a^2*b^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - a^2*c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c*d^2 + 16*a^4*c^3*d*e + 2*a*b^3*d*e*(-(4*a*c - b^2)^3)^(1/2) + 16*a^2*b^4*c*d*e + 3*a*b^2*c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 36*a^3*b^2*c^2*d*e - 4*a^2*b*c*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^7*d^4 + 16*a^4*c^5*e^4 + b^4*c^5*d^4 - 8*a*b^2*c^6*d^4 - 2*b^5*c^4*d^3*e + a^2*b^4*c^3*e^4 - 8*a^3*b^2*c^4*e^4 + 32*a^3*c^6*d^2*e^2 + b^6*c^3*d^2*e^2 + 16*a*b^3*c^5*d^3*e - 2*a*b^5*c^3*d*e^3 - 32*a^2*b*c^6*d^3*e - 32*a^3*b*c^5*d*e^3 - 6*a*b^4*c^4*d^2*e^2 + 16*a^2*b^3*c^4*d*e^3)))^(1/2)*1i - (((((64*a^5*c^4*d*e^8 + 64*a^3*c^6*d^5*e^4 + 128*a^4*c^5*d^3*e^6 - 144*a^2*b^2*c^5*d^5*e^4 + 64*a^2*b^3*c^4*d^4*e^5 + 16*a^2*b^4*c^3*d^3*e^6 - 96*a^3*b^2*c^4*d^3*e^6 + 16*a^3*b^3*c^3*d^2*e^7 - 16*a*b^3*c^5*d^6*e^3 + 32*a*b^4*c^4*d^5*e^4 - 16*a*b^5*c^3*d^4*e^5 + 64*a^2*b*c^6*d^6*e^3 - 64*a^4*b*c^4*d^2*e^7 - 16*a^4*b^2*c^3*d*e^8)/(c*e) + (2*x*(-(b^7*d^2 + a^2*b^5*e^2 - b^4*d^2*(-(4*a*c - b^2)^3)^(1/2) - 20*a^3*b*c^3*d^2 - 7*a^3*b^3*c*e^2 + 12*a^4*b*c^2*e^2 + a^3*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - 2*a*b^6*d*e + 25*a^2*b^3*c^2*d^2 - a^2*b^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - a^2*c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c*d^2 + 16*a^4*c^3*d*e + 2*a*b^3*d*e*(-(4*a*c - b^2)^3)^(1/2) + 16*a^2*b^4*c*d*e + 3*a*b^2*c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 36*a^3*b^2*c^2*d*e - 4*a^2*b*c*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^7*d^4 + 16*a^4*c^5*e^4 + b^4*c^5*d^4 - 8*a*b^2*c^6*d^4 - 2*b^5*c^4*d^3*e + a^2*b^4*c^3*e^4 - 8*a^3*b^2*c^4*e^4 + 32*a^3*c^6*d^2*e^2 + b^6*c^3*d^2*e^2 + 16*a*b^3*c^5*d^3*e - 2*a*b^5*c^3*d*e^3 - 32*a^2*b*c^6*d^3*e - 32*a^3*b*c^5*d*e^3 - 6*a*b^4*c^4*d^2*e^2 + 16*a^2*b^3*c^4*d*e^3)))^(1/2)*(128*a^4*b^2*c^4*e^10 - 16*a^3*b^4*c^3*e^10 - 256*a^5*c^5*e^10 + 256*a^2*c^8*d^6*e^4 + 256*a^3*c^7*d^4*e^6 - 256*a^4*c^6*d^2*e^8 - 16*b^3*c^7*d^7*e^3 + 64*b^4*c^6*d^6*e^4 - 96*b^5*c^5*d^5*e^5 + 64*b^6*c^4*d^4*e^6 - 16*b^7*c^3*d^3*e^7 + 256*a^2*b^2*c^6*d^4*e^6 + 144*a^2*b^3*c^5*d^3*e^7 - 96*a^2*b^4*c^4*d^2*e^8 + 192*a^3*b^2*c^5*d^2*e^8 + 64*a*b*c^8*d^7*e^3 + 320*a^4*b*c^5*d*e^9 - 320*a*b^2*c^7*d^6*e^4 + 528*a*b^3*c^6*d^5*e^5 - 336*a*b^4*c^5*d^4*e^6 + 48*a*b^5*c^4*d^3*e^7 + 16*a*b^6*c^3*d^2*e^8 - 576*a^2*b*c^7*d^5*e^5 + 16*a^2*b^5*c^3*d*e^9 - 320*a^3*b*c^6*d^3*e^7 - 144*a^3*b^3*c^4*d*e^9))/(c*e))*(-(b^7*d^2 + a^2*b^5*e^2 - b^4*d^2*(-(4*a*c - b^2)^3)^(1/2) - 20*a^3*b*c^3*d^2 - 7*a^3*b^3*c*e^2 + 12*a^4*b*c^2*e^2 + a^3*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - 2*a*b^6*d*e + 25*a^2*b^3*c^2*d^2 - a^2*b^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - a^2*c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c*d^2 + 16*a^4*c^3*d*e + 2*a*b^3*d*e*(-(4*a*c - b^2)^3)^(1/2) + 16*a^2*b^4*c*d*e + 3*a*b^2*c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 36*a^3*b^2*c^2*d*e - 4*a^2*b*c*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^7*d^4 + 16*a^4*c^5*e^4 + b^4*c^5*d^4 - 8*a*b^2*c^6*d^4 - 2*b^5*c^4*d^3*e + a^2*b^4*c^3*e^4 - 8*a^3*b^2*c^4*e^4 + 32*a^3*c^6*d^2*e^2 + b^6*c^3*d^2*e^2 + 16*a*b^3*c^5*d^3*e - 2*a*b

^5*c^3*d*e^3 - 32*a^2*b*c^6*d^3*e - 32*a^3*b*c^5*d*e^3 - 6*a*b^4*c^4*d^2*e^
2 + 16*a^2*b^3*c^4*d*e^3)))^(1/2) - (2*x*(4*a^3*b^5*e^8 + 4*b^3*c^5*d^8 + 4
*b^8*d^3*e^5 - 28*a^4*b^3*c*e^8 + 48*a^5*b*c^2*e^8 - 4*a*b^7*d^2*e^6 - 4*a^
2*b^6*d*e^7 - 64*a^2*c^6*d^7*e + 56*a^5*c^3*d*e^7 - 8*b^4*c^4*d^7*e - 8*b^7
*c*d^4*e^4 - 8*a^3*c^5*d^5*e^3 - 16*a^4*c^4*d^3*e^5 + 4*b^5*c^3*d^6*e^2 + 4
*b^6*c^2*d^5*e^3 - 16*a*b*c^6*d^8 + 36*a^2*b^2*c^4*d^5*e^3 - 72*a^2*b^3*c^3
*d^4*e^4 - 12*a^2*b^4*c^2*d^3*e^5 + 64*a^3*b^2*c^3*d^3*e^5 + 28*a^3*b^3*c^2
*d^2*e^6 + 48*a*b^2*c^5*d^7*e - 16*a*b^6*c*d^3*e^5 + 40*a^3*b^4*c*d*e^7 - 2
8*a*b^3*c^4*d^6*e^2 - 24*a*b^4*c^3*d^5*e^3 + 48*a*b^5*c^2*d^4*e^4 + 48*a^2*
b*c^5*d^6*e^2 + 12*a^2*b^5*c*d^2*e^6 + 16*a^3*b*c^4*d^4*e^4 - 64*a^4*b*c^3*
d^2*e^6 - 108*a^4*b^2*c^2*d*e^7))/(c*e))*(-(b^7*d^2 + a^2*b^5*e^2 - b^4*d^2
*(-(4*a*c - b^2)^3)^(1/2) - 20*a^3*b*c^3*d^2 - 7*a^3*b^3*c*e^2 + 12*a^4*b*c
^2*e^2 + a^3*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - 2*a*b^6*d*e + 25*a^2*b^3*c^2*
d^2 - a^2*b^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - a^2*c^2*d^2*(-(4*a*c - b^2)^3)
^(1/2) - 9*a*b^5*c*d^2 + 16*a^4*c^3*d*e + 2*a*b^3*d*e*(-(4*a*c - b^2)^3)^(1
/2) + 16*a^2*b^4*c*d*e + 3*a*b^2*c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 36*a^3*b^
2*c^2*d*e - 4*a^2*b*c*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^7*d^4 + 16
*a^4*c^5*e^4 + b^4*c^5*d^4 - 8*a*b^2*c^6*d^4 - 2*b^5*c^4*d^3*e + a^2*b^4*c^
3*e^4 - 8*a^3*b^2*c^4*e^4 + 32*a^3*c^6*d^2*e^2 + b^6*c^3*d^2*e^2 + 16*a*b^3
*c^5*d^3*e - 2*a*b^5*c^3*d*e^3 - 32*a^2*b*c^6*d^3*e - 32*a^3*b*c^5*d*e^3 -
6*a*b^4*c^4*d^2*e^2 + 16*a^2*b^3*c^4*d*e^3)))^(1/2) - (4*a*b^3*c^3*d^7 - 16
*a^2*b*c^4*d^7 + 4*a*b^6*d^4*e^3 + 4*a^4*b^3*d*e^6 + 48*a^3*c^4*d^6*e - 4*a
^2*b^5*d^3*e^4 - 4*a^3*b^4*d^2*e^5 - 60*a^4*c^3*d^4*e^3 + 4*a^5*c^2*d^2*e^5
- 8*a^5*b*c*d*e^6 - 32*a^2*b^3*c^2*d^5*e^2 + 92*a^3*b^2*c^2*d^4*e^3 + 4*a*
b^4*c^2*d^6*e + 4*a*b^5*c*d^5*e^2 - 28*a^2*b^2*c^3*d^6*e - 36*a^2*b^4*c*d^4
*e^3 + 64*a^3*b*c^3*d^5*e^2 + 36*a^3*b^3*c*d^3*e^4 - 60*a^4*b*c^2*d^3*e^4 +
4*a^4*b^2*c*d^2*e^5)/(c*e))*(-(b^7*d^2 + a^2*b^5*e^2 - b^4*d^2*(-(4*a*c -
b^2)^3)^(1/2) - 20*a^3*b*c^3*d^2 - 7*a^3*b^3*c*e^2 + 12*a^4*b*c^2*e^2 + a^3
*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - 2*a*b^6*d*e + 25*a^2*b^3*c^2*d^2 - a^2*b^
2*e^2*(-(4*a*c - b^2)^3)^(1/2) - a^2*c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) - 9*a
*b^5*c*d^2 + 16*a^4*c^3*d*e + 2*a*b^3*d*e*(-(4*a*c - b^2)^3)^(1/2) + 16*a^2
*b^4*c*d*e + 3*a*b^2*c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 36*a^3*b^2*c^2*d*e -
4*a^2*b*c*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^7*d^4 + 16*a^4*c^5*e^4
+ b^4*c^5*d^4 - 8*a*b^2*c^6*d^4 - 2*b^5*c^4*d^3*e + a^2*b^4*c^3*e^4 - 8*a^
3*b^2*c^4*e^4 + 32*a^3*c^6*d^2*e^2 + b^6*c^3*d^2*e^2 + 16*a*b^3*c^5*d^3*e -
2*a*b^5*c^3*d*e^3 - 32*a^2*b*c^6*d^3*e - 32*a^3*b*c^5*d*e^3 - 6*a*b^4*c^4*
d^2*e^2 + 16*a^2*b^3*c^4*d*e^3)))^(1/2) - (2*x*(a^6*e^6 + b^6*d^6 - 2*a^3*c
^3*d^6 + 9*a^2*b^2*c^2*d^6 - 6*a*b^4*c*d^6))/(c*e))*(-(b^7*d^2 + a^2*b^5*e^
2 - b^4*d^2*(-(4*a*c - b^2)^3)^(1/2) - 20*a^3*b*c^3*d^2 - 7*a^3*b^3*c*e^2 +
12*a^4*b*c^2*e^2 + a^3*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - 2*a*b^6*d*e + 25*a
^2*b^3*c^2*d^2 - a^2*b^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - a^2*c^2*d^2*(-(4*a*
c - b^2)^3)^(1/2) - 9*a*b^5*c*d^2 + 16*a^4*c^3*d*e + 2*a*b^3*d*e*(-(4*a*c -
b^2)^3)^(1/2) + 16*a^2*b^4*c*d*e + 3*a*b^2*c*d^2*(-(4*a*c - b^2)^3)^(1/2)
- 36*a^3*b^2*c^2*d*e - 4*a^2*b*c*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c
^7*d^4 + 16*a^4*c^5*e^4 + b^4*c^5*d^4 - 8*a*b^2*c^6*d^4 - 2*b^5*c^4*d^3*e +

a^2*b^4*c^3*e^4 - 8*a^3*b^2*c^4*e^4 + 32*a^3*c^6*d^2*e^2 + b^6*c^3*d^2*e^2 + 16*a*b^3*c^5*d^3*e - 2*a*b^5*c^3*d*e^3 - 32*a^2*b*c^6*d^3*e - 32*a^3*b*c^5*d*e^3 - 6*a*b^4*c^4*d^2*e^2 + 16*a^2*b^3*c^4*d*e^3)))^(1/2)*1i)/((((((64*a^5*c^4*d*e^8 + 64*a^3*c^6*d^5*e^4 + 128*a^4*c^5*d^3*e^6 - 144*a^2*b^2*c^5*d^5*e^4 + 64*a^2*b^3*c^4*d^4*e^5 + 16*a^2*b^4*c^3*d^3*e^6 - 96*a^3*b^2*c^4*d^3*e^6 + 16*a^3*b^3*c^3*d^2*e^7 - 16*a*b^3*c^5*d^6*e^3 + 32*a*b^4*c^4*d^5*e^4 - 16*a*b^5*c^3*d^4*e^5 + 64*a^2*b*c^6*d^6*e^3 - 64*a^4*b*c^4*d^2*e^7 - 16*a^4*b^2*c^3*d*e^8)/(c*e) - (2*x*(-(b^7*d^2 + a^2*b^5*e^2 - b^4*d^2*(-(4*a*c - b^2)^3)^(1/2) - 20*a^3*b*c^3*d^2 - 7*a^3*b^3*c*e^2 + 12*a^4*b*c^2*e^2 + a^3*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - 2*a*b^6*d*e + 25*a^2*b^3*c^2*d^2 - a^2*b^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - a^2*c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c*d^2 + 16*a^4*c^3*d*e + 2*a*b^3*d*e*(-(4*a*c - b^2)^3)^(1/2) + 16*a^2*b^4*c*d*e + 3*a*b^2*c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 36*a^3*b^2*c^2*d*e - 4*a^2*b*c*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^7*d^4 + 16*a^4*c^5*e^4 + b^4*c^5*d^4 - 8*a*b^2*c^6*d^4 - 2*b^5*c^4*d^3*e + a^2*b^4*c^3*e^4 - 8*a^3*b^2*c^4*e^4 + 32*a^3*c^6*d^2*e^2 + b^6*c^3*d^2*e^2 + 16*a*b^3*c^5*d^3*e - 2*a*b^5*c^3*d*e^3 - 32*a^2*b*c^6*d^3*e - 32*a^3*b*c^5*d*e^3 - 6*a*b^4*c^4*d^2*e^2 + 16*a^2*b^3*c^4*d*e^3)))^(1/2)*(128*a^4*b^2*c^4*e^10 - 16*a^3*b^4*c^3*e^10 - 256*a^5*c^5*e^10 + 256*a^2*c^8*d^6*e^4 + 256*a^3*c^7*d^4*e^6 - 256*a^4*c^6*d^2*e^8 - 16*b^3*c^7*d^7*e^3 + 64*b^4*c^6*d^6*e^4 - 96*b^5*c^5*d^5*e^5 + 64*b^6*c^4*d^4*e^6 - 16*b^7*c^3*d^3*e^7 + 256*a^2*b^2*c^6*d^4*e^6 + 144*a^2*b^3*c^5*d^3*e^7 - 96*a^2*b^4*c^4*d^2*e^8 + 192*a^3*b^2*c^5*d^2*e^8 + 64*a*b*c^8*d^7*e^3 + 320*a^4*b*c^5*d*e^9 - 320*a*b^2*c^7*d^6*e^4 + 528*a*b^3*c^6*d^5*e^5 - 336*a*b^4*c^5*d^4*e^6 + 48*a*b^5*c^4*d^3*e^7 + 16*a*b^6*c^3*d^2*e^8 - 576*a^2*b*c^7*d^5*e^5 + 16*a^2*b^5*c^3*d*e^9 - 320*a^3*b*c^6*d^3*e^7 - 144*a^3*b^3*c^4*d*e^9))/(c*e))*(-(b^7*d^2 + a^2*b^5*e^2 - b^4*d^2*(-(4*a*c - b^2)^3)^(1/2) - 20*a^3*b*c^3*d^2 - 7*a^3*b^3*c*e^2 + 12*a^4*b*c^2*e^2 + a^3*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - 2*a*b^6*d*e + 25*a^2*b^3*c^2*d^2 - a^2*b^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - a^2*c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c*d^2 + 16*a^4*c^3*d*e + 2*a*b^3*d*e*(-(4*a*c - b^2)^3)^(1/2) + 16*a^2*b^4*c*d*e + 3*a*b^2*c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 36*a^3*b^2*c^2*d*e - 4*a^2*b*c*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^7*d^4 + 16*a^4*c^5*e^4 + b^4*c^5*d^4 - 8*a*b^2*c^6*d^4 - 2*b^5*c^4*d^3*e + a^2*b^4*c^3*e^4 - 8*a^3*b^2*c^4*e^4 + 32*a^3*c^6*d^2*e^2 + b^6*c^3*d^2*e^2 + 16*a*b^3*c^5*d^3*e - 2*a*b^5*c^3*d*e^3 - 32*a^2*b*c^6*d^3*e - 32*a^3*b*c^5*d*e^3 - 6*a*b^4*c^4*d^2*e^2 + 16*a^2*b^3*c^4*d*e^3)))^(1/2) + (2*x*(4*a^3*b^5*e^8 + 4*b^3*c^5*d^8 + 4*b^8*d^3*e^5 - 28*a^4*b^3*c*e^8 + 48*a^5*b*c^2*e^8 - 4*a*b^7*d^2*e^6 - 4*a^2*b^6*d*e^7 - 64*a^2*c^6*d^7*e + 56*a^5*c^3*d*e^7 - 8*b^4*c^4*d^7*e - 8*b^7*c*d^4*e^4 - 8*a^3*c^5*d^5*e^3 - 16*a^4*c^4*d^3*e^5 + 4*b^5*c^3*d^6*e^2 + 4*b^6*c^2*d^5*e^3 - 16*a*b*c^6*d^8 + 36*a^2*b^2*c^4*d^5*e^3 - 72*a^2*b^3*c^3*d^4*e^4 - 12*a^2*b^4*c^2*d^3*e^5 + 64*a^3*b^2*c^3*d^3*e^5 + 28*a^3*b^3*c^2*d^2*e^6 + 48*a*b^2*c^5*d^7*e - 16*a*b^6*c*d^3*e^5 + 40*a^3*b^4*c*d*e^7 - 28*a*b^3*c^4*d^6*e^2 - 24*a*b^4*c^3*d^5*e^3 + 48*a*b^5*c^2*d^4*e^4 + 48*a^2*b*c^5*d^6*e^2 + 12*a^2*b^5*c*d^2*e^6 + 16*a^3*b*c^4*d^4*e^4 - 64*a^4*b*c^3*d^2*e^6 - 108*a^4*b^2*c^2*d*e^7))/(c*e))*(-(b^7*d

^2 + a^2*b^5*e^2 - b^4*d^2*(-(4*a*c - b^2)^3)^(1/2) - 20*a^3*b*c^3*d^2 - 7*
a^3*b^3*c*e^2 + 12*a^4*b*c^2*e^2 + a^3*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - 2*a
*b^6*d*e + 25*a^2*b^3*c^2*d^2 - a^2*b^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - a^2*
c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c*d^2 + 16*a^4*c^3*d*e + 2*a*b^3
*d*e*(-(4*a*c - b^2)^3)^(1/2) + 16*a^2*b^4*c*d*e + 3*a*b^2*c*d^2*(-(4*a*c -
 b^2)^3)^(1/2) - 36*a^3*b^2*c^2*d*e - 4*a^2*b*c*d*e*(-(4*a*c - b^2)^3)^(1/2
))/(8*(16*a^2*c^7*d^4 + 16*a^4*c^5*e^4 + b^4*c^5*d^4 - 8*a*b^2*c^6*d^4 - 2*
b^5*c^4*d^3*e + a^2*b^4*c^3*e^4 - 8*a^3*b^2*c^4*e^4 + 32*a^3*c^6*d^2*e^2 +
b^6*c^3*d^2*e^2 + 16*a*b^3*c^5*d^3*e - 2*a*b^5*c^3*d*e^3 - 32*a^2*b*c^6*d^3
*e - 32*a^3*b*c^5*d*e^3 - 6*a*b^4*c^4*d^2*e^2 + 16*a^2*b^3*c^4*d*e^3)))^(1/
2) - (4*a*b^3*c^3*d^7 - 16*a^2*b*c^4*d^7 + 4*a*b^6*d^4*e^3 + 4*a^4*b^3*d*e^
6 + 48*a^3*c^4*d^6*e - 4*a^2*b^5*d^3*e^4 - 4*a^3*b^4*d^2*e^5 - 60*a^4*c^3*d
^4*e^3 + 4*a^5*c^2*d^2*e^5 - 8*a^5*b*c*d*e^6 - 32*a^2*b^3*c^2*d^5*e^2 + 92*
a^3*b^2*c^2*d^4*e^3 + 4*a*b^4*c^2*d^6*e + 4*a*b^5*c*d^5*e^2 - 28*a^2*b^2*c^
3*d^6*e - 36*a^2*b^4*c*d^4*e^3 + 64*a^3*b*c^3*d^5*e^2 + 36*a^3*b^3*c*d^3*e^
4 - 60*a^4*b*c^2*d^3*e^4 + 4*a^4*b^2*c*d^2*e^5)/(c*e))*(-(b^7*d^2 + a^2*b^5
*e^2 - b^4*d^2*(-(4*a*c - b^2)^3)^(1/2) - 20*a^3*b*c^3*d^2 - 7*a^3*b^3*c*e^
2 + 12*a^4*b*c^2*e^2 + a^3*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - 2*a*b^6*d*e + 2
5*a^2*b^3*c^2*d^2 - a^2*b^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - a^2*c^2*d^2*(-(4
*a*c - b^2)^3)^(1/2) - 9*a*b^5*c*d^2 + 16*a^4*c^3*d*e + 2*a*b^3*d*e*(-(4*a*
c - b^2)^3)^(1/2) + 16*a^2*b^4*c*d*e + 3*a*b^2*c*d^2*(-(4*a*c - b^2)^3)^(1/
2) - 36*a^3*b^2*c^2*d*e - 4*a^2*b*c*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^
2*c^7*d^4 + 16*a^4*c^5*e^4 + b^4*c^5*d^4 - 8*a*b^2*c^6*d^4 - 2*b^5*c^4*d^3*
e + a^2*b^4*c^3*e^4 - 8*a^3*b^2*c^4*e^4 + 32*a^3*c^6*d^2*e^2 + b^6*c^3*d^2*
e^2 + 16*a*b^3*c^5*d^3*e - 2*a*b^5*c^3*d*e^3 - 32*a^2*b*c^6*d^3*e - 32*a^3*
b*c^5*d*e^3 - 6*a*b^4*c^4*d^2*e^2 + 16*a^2*b^3*c^4*d*e^3)))^(1/2) + (2*x*(a
^6*e^6 + b^6*d^6 - 2*a^3*c^3*d^6 + 9*a^2*b^2*c^2*d^6 - 6*a*b^4*c*d^6))/(c*e
))*(-(b^7*d^2 + a^2*b^5*e^2 - b^4*d^2*(-(4*a*c - b^2)^3)^(1/2) - 20*a^3*b*c
^3*d^2 - 7*a^3*b^3*c*e^2 + 12*a^4*b*c^2*e^2 + a^3*c*e^2*(-(4*a*c - b^2)^3)^
(1/2) - 2*a*b^6*d*e + 25*a^2*b^3*c^2*d^2 - a^2*b^2*e^2*(-(4*a*c - b^2)^3)^(
1/2) - a^2*c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c*d^2 + 16*a^4*c^3*d*
e + 2*a*b^3*d*e*(-(4*a*c - b^2)^3)^(1/2) + 16*a^2*b^4*c*d*e + 3*a*b^2*c*d^2
*(-(4*a*c - b^2)^3)^(1/2) - 36*a^3*b^2*c^2*d*e - 4*a^2*b*c*d*e*(-(4*a*c - b
^2)^3)^(1/2))/(8*(16*a^2*c^7*d^4 + 16*a^4*c^5*e^4 + b^4*c^5*d^4 - 8*a*b^2*c
^6*d^4 - 2*b^5*c^4*d^3*e + a^2*b^4*c^3*e^4 - 8*a^3*b^2*c^4*e^4 + 32*a^3*c^6
*d^2*e^2 + b^6*c^3*d^2*e^2 + 16*a*b^3*c^5*d^3*e - 2*a*b^5*c^3*d*e^3 - 32*a^
2*b*c^6*d^3*e - 32*a^3*b*c^5*d*e^3 - 6*a*b^4*c^4*d^2*e^2 + 16*a^2*b^3*c^4*d
*e^3)))^(1/2) + (((((64*a^5*c^4*d*e^8 + 64*a^3*c^6*d^5*e^4 + 128*a^4*c^5*d^
3*e^6 - 144*a^2*b^2*c^5*d^5*e^4 + 64*a^2*b^3*c^4*d^4*e^5 + 16*a^2*b^4*c^3*d
^3*e^6 - 96*a^3*b^2*c^4*d^3*e^6 + 16*a^3*b^3*c^3*d^2*e^7 - 16*a*b^3*c^5*d^6
*e^3 + 32*a*b^4*c^4*d^5*e^4 - 16*a*b^5*c^3*d^4*e^5 + 64*a^2*b*c^6*d^6*e^3 -
 64*a^4*b*c^4*d^2*e^7 - 16*a^4*b^2*c^3*d*e^8)/(c*e) + (2*x*(-(b^7*d^2 + a^2
*b^5*e^2 - b^4*d^2*(-(4*a*c - b^2)^3)^(1/2) - 20*a^3*b*c^3*d^2 - 7*a^3*b^3*
c*e^2 + 12*a^4*b*c^2*e^2 + a^3*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - 2*a*b^6*d*e
 + 25*a^2*b^3*c^2*d^2 - a^2*b^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - a^2*c^2*d^2*

(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c*d^2 + 16*a^4*c^3*d*e + 2*a*b^3*d*e*(-(
4*a*c - b^2)^3)^(1/2) + 16*a^2*b^4*c*d*e + 3*a*b^2*c*d^2*(-(4*a*c - b^2)^3)
^(1/2) - 36*a^3*b^2*c^2*d*e - 4*a^2*b*c*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(1
6*a^2*c^7*d^4 + 16*a^4*c^5*e^4 + b^4*c^5*d^4 - 8*a*b^2*c^6*d^4 - 2*b^5*c^4*
d^3*e + a^2*b^4*c^3*e^4 - 8*a^3*b^2*c^4*e^4 + 32*a^3*c^6*d^2*e^2 + b^6*c^3*
d^2*e^2 + 16*a*b^3*c^5*d^3*e - 2*a*b^5*c^3*d*e^3 - 32*a^2*b*c^6*d^3*e - 32*
a^3*b*c^5*d*e^3 - 6*a*b^4*c^4*d^2*e^2 + 16*a^2*b^3*c^4*d*e^3)))^(1/2)*(128*
a^4*b^2*c^4*e^10 - 16*a^3*b^4*c^3*e^10 - 256*a^5*c^5*e^10 + 256*a^2*c^8*d^6
*e^4 + 256*a^3*c^7*d^4*e^6 - 256*a^4*c^6*d^2*e^8 - 16*b^3*c^7*d^7*e^3 + 64*
b^4*c^6*d^6*e^4 - 96*b^5*c^5*d^5*e^5 + 64*b^6*c^4*d^4*e^6 - 16*b^7*c^3*d^3*
e^7 + 256*a^2*b^2*c^6*d^4*e^6 + 144*a^2*b^3*c^5*d^3*e^7 - 96*a^2*b^4*c^4*d^
2*e^8 + 192*a^3*b^2*c^5*d^2*e^8 + 64*a*b*c^8*d^7*e^3 + 320*a^4*b*c^5*d*e^9
- 320*a*b^2*c^7*d^6*e^4 + 528*a*b^3*c^6*d^5*e^5 - 336*a*b^4*c^5*d^4*e^6 + 4
8*a*b^5*c^4*d^3*e^7 + 16*a*b^6*c^3*d^2*e^8 - 576*a^2*b*c^7*d^5*e^5 + 16*a^2
*b^5*c^3*d*e^9 - 320*a^3*b*c^6*d^3*e^7 - 144*a^3*b^3*c^4*d*e^9))/(c*e))*(-(
b^7*d^2 + a^2*b^5*e^2 - b^4*d^2*(-(4*a*c - b^2)^3)^(1/2) - 20*a^3*b*c^3*d^2
 - 7*a^3*b^3*c*e^2 + 12*a^4*b*c^2*e^2 + a^3*c*e^2*(-(4*a*c - b^2)^3)^(1/2)
- 2*a*b^6*d*e + 25*a^2*b^3*c^2*d^2 - a^2*b^2*e^2*(-(4*a*c - b^2)^3)^(1/2) -
 a^2*c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c*d^2 + 16*a^4*c^3*d*e + 2*
a*b^3*d*e*(-(4*a*c - b^2)^3)^(1/2) + 16*a^2*b^4*c*d*e + 3*a*b^2*c*d^2*(-(4*
a*c - b^2)^3)^(1/2) - 36*a^3*b^2*c^2*d*e - 4*a^2*b*c*d*e*(-(4*a*c - b^2)^3)
^(1/2))/(8*(16*a^2*c^7*d^4 + 16*a^4*c^5*e^4 + b^4*c^5*d^4 - 8*a*b^2*c^6*d^4
 - 2*b^5*c^4*d^3*e + a^2*b^4*c^3*e^4 - 8*a^3*b^2*c^4*e^4 + 32*a^3*c^6*d^2*e
^2 + b^6*c^3*d^2*e^2 + 16*a*b^3*c^5*d^3*e - 2*a*b^5*c^3*d*e^3 - 32*a^2*b*c^
6*d^3*e - 32*a^3*b*c^5*d*e^3 - 6*a*b^4*c^4*d^2*e^2 + 16*a^2*b^3*c^4*d*e^3))
)^(1/2) - (2*x*(4*a^3*b^5*e^8 + 4*b^3*c^5*d^8 + 4*b^8*d^3*e^5 - 28*a^4*b^3*
c*e^8 + 48*a^5*b*c^2*e^8 - 4*a*b^7*d^2*e^6 - 4*a^2*b^6*d*e^7 - 64*a^2*c^6*d
^7*e + 56*a^5*c^3*d*e^7 - 8*b^4*c^4*d^7*e - 8*b^7*c*d^4*e^4 - 8*a^3*c^5*d^5
*e^3 - 16*a^4*c^4*d^3*e^5 + 4*b^5*c^3*d^6*e^2 + 4*b^6*c^2*d^5*e^3 - 16*a*b*
c^6*d^8 + 36*a^2*b^2*c^4*d^5*e^3 - 72*a^2*b^3*c^3*d^4*e^4 - 12*a^2*b^4*c^2*
d^3*e^5 + 64*a^3*b^2*c^3*d^3*e^5 + 28*a^3*b^3*c^2*d^2*e^6 + 48*a*b^2*c^5*d^
7*e - 16*a*b^6*c*d^3*e^5 + 40*a^3*b^4*c*d*e^7 - 28*a*b^3*c^4*d^6*e^2 - 24*a
*b^4*c^3*d^5*e^3 + 48*a*b^5*c^2*d^4*e^4 + 48*a^2*b*c^5*d^6*e^2 + 12*a^2*b^5
*c*d^2*e^6 + 16*a^3*b*c^4*d^4*e^4 - 64*a^4*b*c^3*d^2*e^6 - 108*a^4*b^2*c^2*
d*e^7))/(c*e))*(-(b^7*d^2 + a^2*b^5*e^2 - b^4*d^2*(-(4*a*c - b^2)^3)^(1/2)
- 20*a^3*b*c^3*d^2 - 7*a^3*b^3*c*e^2 + 12*a^4*b*c^2*e^2 + a^3*c*e^2*(-(4*a*
c - b^2)^3)^(1/2) - 2*a*b^6*d*e + 25*a^2*b^3*c^2*d^2 - a^2*b^2*e^2*(-(4*a*c
 - b^2)^3)^(1/2) - a^2*c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c*d^2 + 1
6*a^4*c^3*d*e + 2*a*b^3*d*e*(-(4*a*c - b^2)^3)^(1/2) + 16*a^2*b^4*c*d*e + 3
*a*b^2*c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 36*a^3*b^2*c^2*d*e - 4*a^2*b*c*d*e*
(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^7*d^4 + 16*a^4*c^5*e^4 + b^4*c^5*d^4
 - 8*a*b^2*c^6*d^4 - 2*b^5*c^4*d^3*e + a^2*b^4*c^3*e^4 - 8*a^3*b^2*c^4*e^4
+ 32*a^3*c^6*d^2*e^2 + b^6*c^3*d^2*e^2 + 16*a*b^3*c^5*d^3*e - 2*a*b^5*c^3*d
*e^3 - 32*a^2*b*c^6*d^3*e - 32*a^3*b*c^5*d*e^3 - 6*a*b^4*c^4*d^2*e^2 + 16*a
^2*b^3*c^4*d*e^3)))^(1/2) - (4*a*b^3*c^3*d^7 - 16*a^2*b*c^4*d^7 + 4*a*b^6*d

^4*e^3 + 4*a^4*b^3*d*e^6 + 48*a^3*c^4*d^6*e - 4*a^2*b^5*d^3*e^4 - 4*a^3*b^4 *d^2*e^5 - 60*a^4*c^3*d^4*e^3 + 4*a^5*c^2*d^2*e^5 - 8*a^5*b*c*d*e^6 - 32*a^ 2*b^3*c^2*d^5*e^2 + 92*a^3*b^2*c^2*d^4*e^3 + 4*a*b^4*c^2*d^6*e + 4*a*b^5*c* d^5*e^2 - 28*a^2*b^2*c^3*d^6*e - 36*a^2*b^4*c*d^4*e^3 + 64*a^3*b*c^3*d^5*e^ 2 + 36*a^3*b^3*c*d^3*e^4 - 60*a^4*b*c^2*d^3*e^4 + 4*a^4*b^2*c*d^2*e^5)/(c*e))*(-(b^7*d^2 + a^2*b^5*e^2 - b^4*d^2*(-(4*a*c - b^2)^3)^(1/2) - 20*a^3*b*c ^3*d^2 - 7*a^3*b^3*c*e^2 + 12*a^4*b*c^2*e^2 + a^3*c*e^2*(-(4*a*c - b^2)^3)^ (1/2) - 2*a*b^6*d*e + 25*a^2*b^3*c^2*d^2 - a^2*b^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - a^2*c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c*d^2 + 16*a^4*c^3*d* e + 2*a*b^3*d*e*(-(4*a*c - b^2)^3)^(1/2) + 16*a^2*b^4*c*d*e + 3*a*b^2*c*d^2 *(-(4*a*c - b^2)^3)^(1/2) - 36*a^3*b^2*c^2*d*e - 4*a^2*b*c*d*e*(-(4*a*c - b ^2)^3)^(1/2))/(8*(16*a^2*c^7*d^4 + 16*a^4*c^5*e^4 + b^4*c^5*d^4 - 8*a*b^2*c ^6*d^4 - 2*b^5*c^4*d^3*e + a^2*b^4*c^3*e^4 - 8*a^3*b^2*c^4*e^4 + 32*a^3*c^6 *d^2*e^2 + b^6*c^3*d^2*e^2 + 16*a*b^3*c^5*d^3*e - 2*a*b^5*c^3*d*e^3 - 32*a^ 2*b*c^6*d^3*e - 32*a^3*b*c^5*d*e^3 - 6*a*b^4*c^4*d^2*e^2 + 16*a^2*b^3*c^4*d *e^3)))^(1/2) - (2*x*(a^6*e^6 + b^6*d^6 - 2*a^3*c^3*d^6 + 9*a^2*b^2*c^2*d^6 - 6*a*b^4*c*d^6))/(c*e))*(-(b^7*d^2 + a^2*b^5*e^2 - b^4*d^2*(-(4*a*c - b^2)^3)^(1/2) - 20*a^3*b*c^3*d^2 - 7*a^3*b^3*c*e^2 + 12*a^4*b*c^2*e^2 + a^3*c* e^2*(-(4*a*c - b^2)^3)^(1/2) - 2*a*b^6*d*e + 25*a^2*b^3*c^2*d^2 - a^2*b^2*e ^2*(-(4*a*c - b^2)^3)^(1/2) - a^2*c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^ 5*c*d^2 + 16*a^4*c^3*d*e + 2*a*b^3*d*e*(-(4*a*c - b^2)^3)^(1/2) + 16*a^2*b^ 4*c*d*e + 3*a*b^2*c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 36*a^3*b^2*c^2*d*e - 4*a ^2*b*c*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^7*d^4 + 16*a^4*c^5*e^4 + b^4*c^5*d^4 - 8*a*b^2*c^6*d^4 - 2*b^5*c^4*d^3*e + a^2*b^4*c^3*e^4 - 8*a^3*b ^2*c^4*e^4 + 32*a^3*c^6*d^2*e^2 + b^6*c^3*d^2*e^2 + 16*a*b^3*c^5*d^3*e - 2* a*b^5*c^3*d*e^3 - 32*a^2*b*c^6*d^3*e - 32*a^3*b*c^5*d*e^3 - 6*a*b^4*c^4*d^2 *e^2 + 16*a^2*b^3*c^4*d*e^3)))^(1/2) + (2*(a^3*b^2*d^5 - a^4*c*d^5 + a^5*d^ 3*e^2 + a^4*b*d^4*e))/(c*e)))*(-(b^7*d^2 + a^2*b^5*e^2 - b^4*d^2*(-(4*a*c - b^2)^3)^(1/2) - 20*a^3*b*c^3*d^2 - 7*a^3*b^3*c*e^2 + 12*a^4*b*c^2*e^2 + a^ 3*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - 2*a*b^6*d*e + 25*a^2*b^3*c^2*d^2 - a^2*b ^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - a^2*c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) - 9* a*b^5*c*d^2 + 16*a^4*c^3*d*e + 2*a*b^3*d*e*(-(4*a*c - b^2)^3)^(1/2) + 16*a^ 2*b^4*c*d*e + 3*a*b^2*c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 36*a^3*b^2*c^2*d*e - 4*a^2*b*c*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^7*d^4 + 16*a^4*c^5*e^ 4 + b^4*c^5*d^4 - 8*a*b^2*c^6*d^4 - 2*b^5*c^4*d^3*e + a^2*b^4*c^3*e^4 - 8*a ^3*b^2*c^4*e^4 + 32*a^3*c^6*d^2*e^2 + b^6*c^3*d^2*e^2 + 16*a*b^3*c^5*d^3*e - 2*a*b^5*c^3*d*e^3 - 32*a^2*b*c^6*d^3*e - 32*a^3*b*c^5*d*e^3 - 6*a*b^4*c^4 *d^2*e^2 + 16*a^2*b^3*c^4*d*e^3)))^(1/2)*2i - atan((((((64*a^5*c^4*d*e^8 + 64*a^3*c^6*d^5*e^4 + 128*a^4*c^5*d^3*e^6 - 144*a^2*b^2*c^5*d^5*e^4 + 64*a^ 2*b^3*c^4*d^4*e^5 + 16*a^2*b^4*c^3*d^3*e^6 - 96*a^3*b^2*c^4*d^3*e^6 + 16*a^ 3*b^3*c^3*d^2*e^7 - 16*a*b^3*c^5*d^6*e^3 + 32*a*b^4*c^4*d^5*e^4 - 16*a*b^5* c^3*d^4*e^5 + 64*a^2*b*c^6*d^6*e^3 - 64*a^4*b*c^4*d^2*e^7 - 16*a^4*b^2*c^3* d*e^8)/(c*e) - (2*x*(-(b^7*d^2 + a^2*b^5*e^2 + b^4*d^2*(-(4*a*c - b^2)^3)^(1/2) - 20*a^3*b*c^3*d^2 - 7*a^3*b^3*c*e^2 + 12*a^4*b*c^2*e^2 - a^3*c*e^2*(- (4*a*c - b^2)^3)^(1/2) - 2*a*b^6*d*e + 25*a^2*b^3*c^2*d^2 + a^2*b^2*e^2*(-(

4*a*c - b^2)^3)^(1/2) + a^2*c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c*d^2 + 16*a^4*c^3*d*e - 2*a*b^3*d*e*(-(4*a*c - b^2)^3)^(1/2) + 16*a^2*b^4*c*d*e - 3*a*b^2*c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 36*a^3*b^2*c^2*d*e + 4*a^2*b*c*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^7*d^4 + 16*a^4*c^5*e^4 + b^4*c^5*d^4 - 8*a*b^2*c^6*d^4 - 2*b^5*c^4*d^3*e + a^2*b^4*c^3*e^4 - 8*a^3*b^2*c^4*e^4 + 32*a^3*c^6*d^2*e^2 + b^6*c^3*d^2*e^2 + 16*a*b^3*c^5*d^3*e - 2*a*b^5*c^3*d*e^3 - 32*a^2*b*c^6*d^3*e - 32*a^3*b*c^5*d*e^3 - 6*a*b^4*c^4*d^2*e^2 + 16*a^2*b^3*c^4*d*e^3)))^(1/2)*(128*a^4*b^2*c^4*e^10 - 16*a^3*b^4*c^3*e^10 - 256*a^5*c^5*e^10 + 256*a^2*c^8*d^6*e^4 + 256*a^3*c^7*d^4*e^6 - 256*a^4*c^6*d^2*e^8 - 16*b^3*c^7*d^7*e^3 + 64*b^4*c^6*d^6*e^4 - 96*b^5*c^5*d^5*e^5 + 64*b^6*c^4*d^4*e^6 - 16*b^7*c^3*d^3*e^7 + 256*a^2*b^2*c^6*d^4*e^6 + 144*a^2*b^3*c^5*d^3*e^7 - 96*a^2*b^4*c^4*d^2*e^8 + 192*a^3*b^2*c^5*d^2*e^8 + 64*a*b*c^8*d^7*e^3 + 320*a^4*b*c^5*d*e^9 - 320*a*b^2*c^7*d^6*e^4 + 528*a*b^3*c^6*d^5*e^5 - 336*a*b^4*c^5*d^4*e^6 + 48*a*b^5*c^4*d^3*e^7 + 16*a*b^6*c^3*d^2*e^8 - 576*a^2*b*c^7*d^5*e^5 + 16*a^2*b^5*c^3*d*e^9 - 320*a^3*b*c^6*d^3*e^7 - 144*a^3*b^3*c^4*d*e^9))/(c*e))*(-(b^7*d^2 + a^2*b^5*e^2 + b^4*d^2*(-(4*a*c - b^2)^3)^(1/2) - 20*a^3*b*c^3*d^2 - 7*a^3*b^3*c*e^2 + 12*a^4*b*c^2*e^2 - a^3*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - 2*a*b^6*d*e + 25*a^2*b^3*c^2*d^2 + a^2*b^2*e^2*(-(4*a*c - b^2)^3)^(1/2) + a^2*c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c*d^2 + 16*a^4*c^3*d*e - 2*a*b^3*d*e*(-(4*a*c - b^2)^3)^(1/2) + 16*a^2*b^4*c*d*e - 3*a*b^2*c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 36*a^3*b^2*c^2*d*e + 4*a^2*b*c*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^7*d^4 + 16*a^4*c^5*e^4 + b^4*c^5*d^4 - 8*a*b^2*c^6*d^4 - 2*b^5*c^4*d^3*e + a^2*b^4*c^3*e^4 - 8*a^3*b^2*c^4*e^4 + 32*a^3*c^6*d^2*e^2 + b^6*c^3*d^2*e^2 + 16*a*b^3*c^5*d^3*e - 2*a*b^5*c^3*d*e^3 - 32*a^2*b*c^6*d^3*e - 32*a^3*b*c^5*d*e^3 - 6*a*b^4*c^4*d^2*e^2 + 16*a^2*b^3*c^4*d*e^3)))^(1/2) + (2*x*(4*a^3*b^5*e^8 + 4*b^3*c^5*d^8 + 4*b^8*d^3*e^5 - 28*a^4*b^3*c*e^8 + 48*a^5*b*c^2*e^8 - 4*a*b^7*d^2*e^6 - 4*a^2*b^6*d*e^7 - 64*a^2*c^6*d^7*e + 56*a^5*c^3*d*e^7 - 8*b^4*c^4*d^7*e - 8*b^7*c*d^4*e^4 - 8*a^3*c^5*d^5*e^3 - 16*a^4*c^4*d^3*e^5 + 4*b^5*c^3*d^6*e^2 + 4*b^6*c^2*d^5*e^3 - 16*a*b*c^6*d^8 + 36*a^2*b^2*c^4*d^5*e^3 - 72*a^2*b^3*c^3*d^4*e^4 - 12*a^2*b^4*c^2*d^3*e^5 + 64*a^3*b^2*c^3*d^3*e^5 + 28*a^3*b^3*c^2*d^2*e^6 + 48*a*b^2*c^5*d^7*e - 16*a*b^6*c*d^3*e^5 + 40*a^3*b^4*c*d*e^7 - 28*a*b^3*c^4*d^6*e^2 - 24*a*b^4*c^3*d^5*e^3 + 48*a*b^5*c^2*d^4*e^4 + 48*a^2*b*c^5*d^6*e^2 + 12*a^2*b^5*c*d^2*e^6 + 16*a^3*b*c^4*d^4*e^4 - 64*a^4*b*c^3*d^2*e^6 - 108*a^4*b^2*c^2*d*e^7))/(c*e))*(-(b^7*d^2 + a^2*b^5*e^2 + b^4*d^2*(-(4*a*c - b^2)^3)^(1/2) - 20*a^3*b*c^3*d^2 - 7*a^3*b^3*c*e^2 + 12*a^4*b*c^2*e^2 - a^3*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - 2*a*b^6*d*e + 25*a^2*b^3*c^2*d^2 + a^2*b^2*e^2*(-(4*a*c - b^2)^3)^(1/2) + a^2*c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c*d^2 + 16*a^4*c^3*d*e - 2*a*b^3*d*e*(-(4*a*c - b^2)^3)^(1/2) + 16*a^2*b^4*c*d*e - 3*a*b^2*c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 36*a^3*b^2*c^2*d*e + 4*a^2*b*c*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^7*d^4 + 16*a^4*c^5*e^4 + b^4*c^5*d^4 - 8*a*b^2*c^6*d^4 - 2*b^5*c^4*d^3*e + a^2*b^4*c^3*e^4 - 8*a^3*b^2*c^4*e^4 + 32*a^3*c^6*d^2*e^2 + b^6*c^3*d^2*e^2 + 16*a*b^3*c^5*d^3*e - 2*a*b^5*c^3*d*e^3 - 32*a^2*b*c^6*d^3*e - 32*a^3*b*c^5*d*e^3 - 6*a*b^4*c^4*d^2*e^2 + 16*a^2*b^3*c^4*d*e^3)))^(1/2) - (4*a*b^3*c^

3*d^7 - 16*a^2*b*c^4*d^7 + 4*a*b^6*d^4*e^3 + 4*a^4*b^3*d*e^6 + 48*a^3*c^4*d
^6*e - 4*a^2*b^5*d^3*e^4 - 4*a^3*b^4*d^2*e^5 - 60*a^4*c^3*d^4*e^3 + 4*a^5*c
^2*d^2*e^5 - 8*a^5*b*c*d*e^6 - 32*a^2*b^3*c^2*d^5*e^2 + 92*a^3*b^2*c^2*d^4*
e^3 + 4*a*b^4*c^2*d^6*e + 4*a*b^5*c*d^5*e^2 - 28*a^2*b^2*c^3*d^6*e - 36*a^2
*b^4*c*d^4*e^3 + 64*a^3*b*c^3*d^5*e^2 + 36*a^3*b^3*c*d^3*e^4 - 60*a^4*b*c^2
*d^3*e^4 + 4*a^4*b^2*c*d^2*e^5)/(c*e))*(-(b^7*d^2 + a^2*b^5*e^2 + b^4*d^2*(
-(4*a*c - b^2)^3)^(1/2) - 20*a^3*b*c^3*d^2 - 7*a^3*b^3*c*e^2 + 12*a^4*b*c^2
*e^2 - a^3*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - 2*a*b^6*d*e + 25*a^2*b^3*c^2*d^
2 + a^2*b^2*e^2*(-(4*a*c - b^2)^3)^(1/2) + a^2*c^2*d^2*(-(4*a*c - b^2)^3)^(
1/2) - 9*a*b^5*c*d^2 + 16*a^4*c^3*d*e - 2*a*b^3*d*e*(-(4*a*c - b^2)^3)^(1/2
) + 16*a^2*b^4*c*d*e - 3*a*b^2*c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 36*a^3*b^2*
c^2*d*e + 4*a^2*b*c*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^7*d^4 + 16*a
^4*c^5*e^4 + b^4*c^5*d^4 - 8*a*b^2*c^6*d^4 - 2*b^5*c^4*d^3*e + a^2*b^4*c^3*
e^4 - 8*a^3*b^2*c^4*e^4 + 32*a^3*c^6*d^2*e^2 + b^6*c^3*d^2*e^2 + 16*a*b^3*c
^5*d^3*e - 2*a*b^5*c^3*d*e^3 - 32*a^2*b*c^6*d^3*e - 32*a^3*b*c^5*d*e^3 - 6*
a*b^4*c^4*d^2*e^2 + 16*a^2*b^3*c^4*d*e^3)))^(1/2) + (2*x*(a^6*e^6 + b^6*d^6
 - 2*a^3*c^3*d^6 + 9*a^2*b^2*c^2*d^6 - 6*a*b^4*c*d^6))/(c*e))*(-(b^7*d^2 +
a^2*b^5*e^2 + b^4*d^2*(-(4*a*c - b^2)^3)^(1/2) - 20*a^3*b*c^3*d^2 - 7*a^3*b
^3*c*e^2 + 12*a^4*b*c^2*e^2 - a^3*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - 2*a*b^6*
d*e + 25*a^2*b^3*c^2*d^2 + a^2*b^2*e^2*(-(4*a*c - b^2)^3)^(1/2) + a^2*c^2*d
^2*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c*d^2 + 16*a^4*c^3*d*e - 2*a*b^3*d*e*
(-(4*a*c - b^2)^3)^(1/2) + 16*a^2*b^4*c*d*e - 3*a*b^2*c*d^2*(-(4*a*c - b^2)
^3)^(1/2) - 36*a^3*b^2*c^2*d*e + 4*a^2*b*c*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8
*(16*a^2*c^7*d^4 + 16*a^4*c^5*e^4 + b^4*c^5*d^4 - 8*a*b^2*c^6*d^4 - 2*b^5*c
^4*d^3*e + a^2*b^4*c^3*e^4 - 8*a^3*b^2*c^4*e^4 + 32*a^3*c^6*d^2*e^2 + b^6*c
^3*d^2*e^2 + 16*a*b^3*c^5*d^3*e - 2*a*b^5*c^3*d*e^3 - 32*a^2*b*c^6*d^3*e -
32*a^3*b*c^5*d*e^3 - 6*a*b^4*c^4*d^2*e^2 + 16*a^2*b^3*c^4*d*e^3)))^(1/2)*1i
 - (((((64*a^5*c^4*d*e^8 + 64*a^3*c^6*d^5*e^4 + 128*a^4*c^5*d^3*e^6 - 144*a
^2*b^2*c^5*d^5*e^4 + 64*a^2*b^3*c^4*d^4*e^5 + 16*a^2*b^4*c^3*d^3*e^6 - 96*a
^3*b^2*c^4*d^3*e^6 + 16*a^3*b^3*c^3*d^2*e^7 - 16*a*b^3*c^5*d^6*e^3 + 32*a*b
^4*c^4*d^5*e^4 - 16*a*b^5*c^3*d^4*e^5 + 64*a^2*b*c^6*d^6*e^3 - 64*a^4*b*c^4
*d^2*e^7 - 16*a^4*b^2*c^3*d*e^8)/(c*e) + (2*x*(-(b^7*d^2 + a^2*b^5*e^2 + b^
4*d^2*(-(4*a*c - b^2)^3)^(1/2) - 20*a^3*b*c^3*d^2 - 7*a^3*b^3*c*e^2 + 12*a^
4*b*c^2*e^2 - a^3*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - 2*a*b^6*d*e + 25*a^2*b^3
*c^2*d^2 + a^2*b^2*e^2*(-(4*a*c - b^2)^3)^(1/2) + a^2*c^2*d^2*(-(4*a*c - b^
2)^3)^(1/2) - 9*a*b^5*c*d^2 + 16*a^4*c^3*d*e - 2*a*b^3*d*e*(-(4*a*c - b^2)^
3)^(1/2) + 16*a^2*b^4*c*d*e - 3*a*b^2*c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 36*a
^3*b^2*c^2*d*e + 4*a^2*b*c*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^7*d^4
 + 16*a^4*c^5*e^4 + b^4*c^5*d^4 - 8*a*b^2*c^6*d^4 - 2*b^5*c^4*d^3*e + a^2*b
^4*c^3*e^4 - 8*a^3*b^2*c^4*e^4 + 32*a^3*c^6*d^2*e^2 + b^6*c^3*d^2*e^2 + 16*
a*b^3*c^5*d^3*e - 2*a*b^5*c^3*d*e^3 - 32*a^2*b*c^6*d^3*e - 32*a^3*b*c^5*d*e
^3 - 6*a*b^4*c^4*d^2*e^2 + 16*a^2*b^3*c^4*d*e^3)))^(1/2)*(128*a^4*b^2*c^4*e
^10 - 16*a^3*b^4*c^3*e^10 - 256*a^5*c^5*e^10 + 256*a^2*c^8*d^6*e^4 + 256*a^
3*c^7*d^4*e^6 - 256*a^4*c^6*d^2*e^8 - 16*b^3*c^7*d^7*e^3 + 64*b^4*c^6*d^6*e
^4 - 96*b^5*c^5*d^5*e^5 + 64*b^6*c^4*d^4*e^6 - 16*b^7*c^3*d^3*e^7 + 256*a^2

*b^2*c^6*d^4*e^6 + 144*a^2*b^3*c^5*d^3*e^7 - 96*a^2*b^4*c^4*d^2*e^8 + 192*a
^3*b^2*c^5*d^2*e^8 + 64*a*b*c^8*d^7*e^3 + 320*a^4*b*c^5*d*e^9 - 320*a*b^2*c
^7*d^6*e^4 + 528*a*b^3*c^6*d^5*e^5 - 336*a*b^4*c^5*d^4*e^6 + 48*a*b^5*c^4*d
^3*e^7 + 16*a*b^6*c^3*d^2*e^8 - 576*a^2*b*c^7*d^5*e^5 + 16*a^2*b^5*c^3*d*e^
9 - 320*a^3*b*c^6*d^3*e^7 - 144*a^3*b^3*c^4*d*e^9))/(c*e))*(-(b^7*d^2 + a^2
*b^5*e^2 + b^4*d^2*(-(4*a*c - b^2)^3)^(1/2) - 20*a^3*b*c^3*d^2 - 7*a^3*b^3*
c*e^2 + 12*a^4*b*c^2*e^2 - a^3*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - 2*a*b^6*d*e
 + 25*a^2*b^3*c^2*d^2 + a^2*b^2*e^2*(-(4*a*c - b^2)^3)^(1/2) + a^2*c^2*d^2*
(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c*d^2 + 16*a^4*c^3*d*e - 2*a*b^3*d*e*(-(
4*a*c - b^2)^3)^(1/2) + 16*a^2*b^4*c*d*e - 3*a*b^2*c*d^2*(-(4*a*c - b^2)^3)
^(1/2) - 36*a^3*b^2*c^2*d*e + 4*a^2*b*c*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(1
6*a^2*c^7*d^4 + 16*a^4*c^5*e^4 + b^4*c^5*d^4 - 8*a*b^2*c^6*d^4 - 2*b^5*c^4*
d^3*e + a^2*b^4*c^3*e^4 - 8*a^3*b^2*c^4*e^4 + 32*a^3*c^6*d^2*e^2 + b^6*c^3*
d^2*e^2 + 16*a*b^3*c^5*d^3*e - 2*a*b^5*c^3*d*e^3 - 32*a^2*b*c^6*d^3*e - 32*
a^3*b*c^5*d*e^3 - 6*a*b^4*c^4*d^2*e^2 + 16*a^2*b^3*c^4*d*e^3)))^(1/2) - (2*
x*(4*a^3*b^5*e^8 + 4*b^3*c^5*d^8 + 4*b^8*d^3*e^5 - 28*a^4*b^3*c*e^8 + 48*a^
5*b*c^2*e^8 - 4*a*b^7*d^2*e^6 - 4*a^2*b^6*d*e^7 - 64*a^2*c^6*d^7*e + 56*a^5
*c^3*d*e^7 - 8*b^4*c^4*d^7*e - 8*b^7*c*d^4*e^4 - 8*a^3*c^5*d^5*e^3 - 16*a^4
*c^4*d^3*e^5 + 4*b^5*c^3*d^6*e^2 + 4*b^6*c^2*d^5*e^3 - 16*a*b*c^6*d^8 + 36*
a^2*b^2*c^4*d^5*e^3 - 72*a^2*b^3*c^3*d^4*e^4 - 12*a^2*b^4*c^2*d^3*e^5 + 64*
a^3*b^2*c^3*d^3*e^5 + 28*a^3*b^3*c^2*d^2*e^6 + 48*a*b^2*c^5*d^7*e - 16*a*b^
6*c*d^3*e^5 + 40*a^3*b^4*c*d*e^7 - 28*a*b^3*c^4*d^6*e^2 - 24*a*b^4*c^3*d^5*
e^3 + 48*a*b^5*c^2*d^4*e^4 + 48*a^2*b*c^5*d^6*e^2 + 12*a^2*b^5*c*d^2*e^6 +
16*a^3*b*c^4*d^4*e^4 - 64*a^4*b*c^3*d^2*e^6 - 108*a^4*b^2*c^2*d*e^7))/(c*e)
)*(-(b^7*d^2 + a^2*b^5*e^2 + b^4*d^2*(-(4*a*c - b^2)^3)^(1/2) - 20*a^3*b*c^
3*d^2 - 7*a^3*b^3*c*e^2 + 12*a^4*b*c^2*e^2 - a^3*c*e^2*(-(4*a*c - b^2)^3)^(
1/2) - 2*a*b^6*d*e + 25*a^2*b^3*c^2*d^2 + a^2*b^2*e^2*(-(4*a*c - b^2)^3)^(1
/2) + a^2*c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c*d^2 + 16*a^4*c^3*d*e
 - 2*a*b^3*d*e*(-(4*a*c - b^2)^3)^(1/2) + 16*a^2*b^4*c*d*e - 3*a*b^2*c*d^2*
(-(4*a*c - b^2)^3)^(1/2) - 36*a^3*b^2*c^2*d*e + 4*a^2*b*c*d*e*(-(4*a*c - b^
2)^3)^(1/2))/(8*(16*a^2*c^7*d^4 + 16*a^4*c^5*e^4 + b^4*c^5*d^4 - 8*a*b^2*c^
6*d^4 - 2*b^5*c^4*d^3*e + a^2*b^4*c^3*e^4 - 8*a^3*b^2*c^4*e^4 + 32*a^3*c^6*
d^2*e^2 + b^6*c^3*d^2*e^2 + 16*a*b^3*c^5*d^3*e - 2*a*b^5*c^3*d*e^3 - 32*a^2
*b*c^6*d^3*e - 32*a^3*b*c^5*d*e^3 - 6*a*b^4*c^4*d^2*e^2 + 16*a^2*b^3*c^4*d*
e^3)))^(1/2) - (4*a*b^3*c^3*d^7 - 16*a^2*b*c^4*d^7 + 4*a*b^6*d^4*e^3 + 4*a^
4*b^3*d*e^6 + 48*a^3*c^4*d^6*e - 4*a^2*b^5*d^3*e^4 - 4*a^3*b^4*d^2*e^5 - 60
*a^4*c^3*d^4*e^3 + 4*a^5*c^2*d^2*e^5 - 8*a^5*b*c*d*e^6 - 32*a^2*b^3*c^2*d^5
*e^2 + 92*a^3*b^2*c^2*d^4*e^3 + 4*a*b^4*c^2*d^6*e + 4*a*b^5*c*d^5*e^2 - 28*
a^2*b^2*c^3*d^6*e - 36*a^2*b^4*c*d^4*e^3 + 64*a^3*b*c^3*d^5*e^2 + 36*a^3*b^
3*c*d^3*e^4 - 60*a^4*b*c^2*d^3*e^4 + 4*a^4*b^2*c*d^2*e^5)/(c*e))*(-(b^7*d^2
 + a^2*b^5*e^2 + b^4*d^2*(-(4*a*c - b^2)^3)^(1/2) - 20*a^3*b*c^3*d^2 - 7*a^
3*b^3*c*e^2 + 12*a^4*b*c^2*e^2 - a^3*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - 2*a*b
^6*d*e + 25*a^2*b^3*c^2*d^2 + a^2*b^2*e^2*(-(4*a*c - b^2)^3)^(1/2) + a^2*c^
2*d^2*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c*d^2 + 16*a^4*c^3*d*e - 2*a*b^3*d
e(-(4*a*c - b^2)^3)^(1/2) + 16*a^2*b^4*c*d*e - 3*a*b^2*c*d^2*(-(4*a*c - b

^2)^3)^(1/2) - 36*a^3*b^2*c^2*d*e + 4*a^2*b*c*d*e*(-(4*a*c - b^2)^3)^(1/2))
/(8*(16*a^2*c^7*d^4 + 16*a^4*c^5*e^4 + b^4*c^5*d^4 - 8*a*b^2*c^6*d^4 - 2*b^
5*c^4*d^3*e + a^2*b^4*c^3*e^4 - 8*a^3*b^2*c^4*e^4 + 32*a^3*c^6*d^2*e^2 + b^
6*c^3*d^2*e^2 + 16*a*b^3*c^5*d^3*e - 2*a*b^5*c^3*d*e^3 - 32*a^2*b*c^6*d^3*e
 - 32*a^3*b*c^5*d*e^3 - 6*a*b^4*c^4*d^2*e^2 + 16*a^2*b^3*c^4*d*e^3)))^(1/2)
 - (2*x*(a^6*e^6 + b^6*d^6 - 2*a^3*c^3*d^6 + 9*a^2*b^2*c^2*d^6 - 6*a*b^4*c*
d^6))/(c*e))*(-(b^7*d^2 + a^2*b^5*e^2 + b^4*d^2*(-(4*a*c - b^2)^3)^(1/2) -
20*a^3*b*c^3*d^2 - 7*a^3*b^3*c*e^2 + 12*a^4*b*c^2*e^2 - a^3*c*e^2*(-(4*a*c
- b^2)^3)^(1/2) - 2*a*b^6*d*e + 25*a^2*b^3*c^2*d^2 + a^2*b^2*e^2*(-(4*a*c -
 b^2)^3)^(1/2) + a^2*c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c*d^2 + 16*
a^4*c^3*d*e - 2*a*b^3*d*e*(-(4*a*c - b^2)^3)^(1/2) + 16*a^2*b^4*c*d*e - 3*a
*b^2*c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 36*a^3*b^2*c^2*d*e + 4*a^2*b*c*d*e*(-
(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^7*d^4 + 16*a^4*c^5*e^4 + b^4*c^5*d^4 -
 8*a*b^2*c^6*d^4 - 2*b^5*c^4*d^3*e + a^2*b^4*c^3*e^4 - 8*a^3*b^2*c^4*e^4 +
32*a^3*c^6*d^2*e^2 + b^6*c^3*d^2*e^2 + 16*a*b^3*c^5*d^3*e - 2*a*b^5*c^3*d*e
^3 - 32*a^2*b*c^6*d^3*e - 32*a^3*b*c^5*d*e^3 - 6*a*b^4*c^4*d^2*e^2 + 16*a^2
*b^3*c^4*d*e^3)))^(1/2)*1i)/((((((64*a^5*c^4*d*e^8 + 64*a^3*c^6*d^5*e^4 + 1
28*a^4*c^5*d^3*e^6 - 144*a^2*b^2*c^5*d^5*e^4 + 64*a^2*b^3*c^4*d^4*e^5 + 16*
a^2*b^4*c^3*d^3*e^6 - 96*a^3*b^2*c^4*d^3*e^6 + 16*a^3*b^3*c^3*d^2*e^7 - 16*
a*b^3*c^5*d^6*e^3 + 32*a*b^4*c^4*d^5*e^4 - 16*a*b^5*c^3*d^4*e^5 + 64*a^2*b*
c^6*d^6*e^3 - 64*a^4*b*c^4*d^2*e^7 - 16*a^4*b^2*c^3*d*e^8)/(c*e) - (2*x*(-(
b^7*d^2 + a^2*b^5*e^2 + b^4*d^2*(-(4*a*c - b^2)^3)^(1/2) - 20*a^3*b*c^3*d^2
 - 7*a^3*b^3*c*e^2 + 12*a^4*b*c^2*e^2 - a^3*c*e^2*(-(4*a*c - b^2)^3)^(1/2)
- 2*a*b^6*d*e + 25*a^2*b^3*c^2*d^2 + a^2*b^2*e^2*(-(4*a*c - b^2)^3)^(1/2) +
 a^2*c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c*d^2 + 16*a^4*c^3*d*e - 2*
a*b^3*d*e*(-(4*a*c - b^2)^3)^(1/2) + 16*a^2*b^4*c*d*e - 3*a*b^2*c*d^2*(-(4*
a*c - b^2)^3)^(1/2) - 36*a^3*b^2*c^2*d*e + 4*a^2*b*c*d*e*(-(4*a*c - b^2)^3)
^(1/2))/(8*(16*a^2*c^7*d^4 + 16*a^4*c^5*e^4 + b^4*c^5*d^4 - 8*a*b^2*c^6*d^4
 - 2*b^5*c^4*d^3*e + a^2*b^4*c^3*e^4 - 8*a^3*b^2*c^4*e^4 + 32*a^3*c^6*d^2*e
^2 + b^6*c^3*d^2*e^2 + 16*a*b^3*c^5*d^3*e - 2*a*b^5*c^3*d*e^3 - 32*a^2*b*c^
6*d^3*e - 32*a^3*b*c^5*d*e^3 - 6*a*b^4*c^4*d^2*e^2 + 16*a^2*b^3*c^4*d*e^3))
)^(1/2)*(128*a^4*b^2*c^4*e^10 - 16*a^3*b^4*c^3*e^10 - 256*a^5*c^5*e^10 + 25
6*a^2*c^8*d^6*e^4 + 256*a^3*c^7*d^4*e^6 - 256*a^4*c^6*d^2*e^8 - 16*b^3*c^7*
d^7*e^3 + 64*b^4*c^6*d^6*e^4 - 96*b^5*c^5*d^5*e^5 + 64*b^6*c^4*d^4*e^6 - 16
*b^7*c^3*d^3*e^7 + 256*a^2*b^2*c^6*d^4*e^6 + 144*a^2*b^3*c^5*d^3*e^7 - 96*a
^2*b^4*c^4*d^2*e^8 + 192*a^3*b^2*c^5*d^2*e^8 + 64*a*b*c^8*d^7*e^3 + 320*a^4
*b*c^5*d*e^9 - 320*a*b^2*c^7*d^6*e^4 + 528*a*b^3*c^6*d^5*e^5 - 336*a*b^4*c^
5*d^4*e^6 + 48*a*b^5*c^4*d^3*e^7 + 16*a*b^6*c^3*d^2*e^8 - 576*a^2*b*c^7*d^5
*e^5 + 16*a^2*b^5*c^3*d*e^9 - 320*a^3*b*c^6*d^3*e^7 - 144*a^3*b^3*c^4*d*e^9
))/(c*e))*(-(b^7*d^2 + a^2*b^5*e^2 + b^4*d^2*(-(4*a*c - b^2)^3)^(1/2) - 20*
a^3*b*c^3*d^2 - 7*a^3*b^3*c*e^2 + 12*a^4*b*c^2*e^2 - a^3*c*e^2*(-(4*a*c - b
^2)^3)^(1/2) - 2*a*b^6*d*e + 25*a^2*b^3*c^2*d^2 + a^2*b^2*e^2*(-(4*a*c - b^
2)^3)^(1/2) + a^2*c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c*d^2 + 16*a^4
*c^3*d*e - 2*a*b^3*d*e*(-(4*a*c - b^2)^3)^(1/2) + 16*a^2*b^4*c*d*e - 3*a*b^
2*c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 36*a^3*b^2*c^2*d*e + 4*a^2*b*c*d*e*(-(4*

a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^7*d^4 + 16*a^4*c^5*e^4 + b^4*c^5*d^4 - 8*a*b^2*c^6*d^4 - 2*b^5*c^4*d^3*e + a^2*b^4*c^3*e^4 - 8*a^3*b^2*c^4*e^4 + 32*a^3*c^6*d^2*e^2 + b^6*c^3*d^2*e^2 + 16*a*b^3*c^5*d^3*e - 2*a*b^5*c^3*d*e^3 - 32*a^2*b*c^6*d^3*e - 32*a^3*b*c^5*d*e^3 - 6*a*b^4*c^4*d^2*e^2 + 16*a^2*b^3*c^4*d*e^3)))^(1/2) + (2*x*(4*a^3*b^5*e^8 + 4*b^3*c^5*d^8 + 4*b^8*d^3*e^5 - 28*a^4*b^3*c*e^8 + 48*a^5*b*c^2*e^8 - 4*a*b^7*d^2*e^6 - 4*a^2*b^6*d*e^7 - 64*a^2*c^6*d^7*e + 56*a^5*c^3*d*e^7 - 8*b^4*c^4*d^7*e - 8*b^7*c*d^4*e^4 - 8*a^3*c^5*d^5*e^3 - 16*a^4*c^4*d^3*e^5 + 4*b^5*c^3*d^6*e^2 + 4*b^6*c^2*d^5*e^3 - 16*a*b*c^6*d^8 + 36*a^2*b^2*c^4*d^5*e^3 - 72*a^2*b^3*c^3*d^4*e^4 - 12*a^2*b^4*c^2*d^3*e^5 + 64*a^3*b^2*c^3*d^3*e^5 + 28*a^3*b^3*c^2*d^2*e^6 + 48*a*b^2*c^5*d^7*e - 16*a*b^6*c*d^3*e^5 + 40*a^3*b^4*c*d*e^7 - 28*a*b^3*c^4*d^6*e^2 - 24*a*b^4*c^3*d^5*e^3 + 48*a*b^5*c^2*d^4*e^4 + 48*a^2*b*c^5*d^6*e^2 + 12*a^2*b^5*c*d^2*e^6 + 16*a^3*b*c^4*d^4*e^4 - 64*a^4*b*c^3*d^2*e^6 - 108*a^4*b^2*c^2*d*e^7))/(c*e))*(-(b^7*d^2 + a^2*b^5*e^2 + b^4*d^2*(-(4*a*c - b^2)^3)^(1/2) - 20*a^3*b*c^3*d^2 - 7*a^3*b^3*c*e^2 + 12*a^4*b*c^2*e^2 - a^3*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - 2*a*b^6*d*e + 25*a^2*b^3*c^2*d^2 + a^2*b^2*e^2*(-(4*a*c - b^2)^3)^(1/2) + a^2*c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c*d^2 + 16*a^4*c^3*d*e - 2*a*b^3*d*e*(-(4*a*c - b^2)^3)^(1/2) + 16*a^2*b^4*c*d*e - 3*a*b^2*c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 36*a^3*b^2*c^2*d*e + 4*a^2*b*c*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^7*d^4 + 16*a^4*c^5*e^4 + b^4*c^5*d^4 - 8*a*b^2*c^6*d^4 - 2*b^5*c^4*d^3*e + a^2*b^4*c^3*e^4 - 8*a^3*b^2*c^4*e^4 + 32*a^3*c^6*d^2*e^2 + b^6*c^3*d^2*e^2 + 16*a*b^3*c^5*d^3*e - 2*a*b^5*c^3*d*e^3 - 32*a^2*b*c^6*d^3*e - 32*a^3*b*c^5*d*e^3 - 6*a*b^4*c^4*d^2*e^2 + 16*a^2*b^3*c^4*d*e^3)))^(1/2) - (4*a*b^3*c^3*d^7 - 16*a^2*b*c^4*d^7 + 4*a*b^6*d^4*e^3 + 4*a^4*b^3*d*e^6 + 48*a^3*c^4*d^6*e - 4*a^2*b^5*d^3*e^4 - 4*a^3*b^4*d^2*e^5 - 60*a^4*c^3*d^4*e^3 + 4*a^5*c^2*d^2*e^5 - 8*a^5*b*c*d*e^6 - 32*a^2*b^3*c^2*d^5*e^2 + 92*a^3*b^2*c^2*d^4*e^3 + 4*a*b^4*c^2*d^6*e + 4*a*b^5*c*d^5*e^2 - 28*a^2*b^2*c^3*d^6*e - 36*a^2*b^4*c*d^4*e^3 + 64*a^3*b*c^3*d^5*e^2 + 36*a^3*b^3*c*d^3*e^4 - 60*a^4*b*c^2*d^3*e^4 + 4*a^4*b^2*c*d^2*e^5)/(c*e))*(-(b^7*d^2 + a^2*b^5*e^2 + b^4*d^2*(-(4*a*c - b^2)^3)^(1/2) - 20*a^3*b*c^3*d^2 - 7*a^3*b^3*c*e^2 + 12*a^4*b*c^2*e^2 - a^3*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - 2*a*b^6*d*e + 25*a^2*b^3*c^2*d^2 + a^2*b^2*e^2*(-(4*a*c - b^2)^3)^(1/2) + a^2*c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c*d^2 + 16*a^4*c^3*d*e - 2*a*b^3*d*e*(-(4*a*c - b^2)^3)^(1/2) + 16*a^2*b^4*c*d*e - 3*a*b^2*c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 36*a^3*b^2*c^2*d*e + 4*a^2*b*c*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^7*d^4 + 16*a^4*c^5*e^4 + b^4*c^5*d^4 - 8*a*b^2*c^6*d^4 - 2*b^5*c^4*d^3*e + a^2*b^4*c^3*e^4 - 8*a^3*b^2*c^4*e^4 + 32*a^3*c^6*d^2*e^2 + b^6*c^3*d^2*e^2 + 16*a*b^3*c^5*d^3*e - 2*a*b^5*c^3*d*e^3 - 32*a^2*b*c^6*d^3*e - 32*a^3*b*c^5*d*e^3 - 6*a*b^4*c^4*d^2*e^2 + 16*a^2*b^3*c^4*d*e^3)))^(1/2) + (2*x*(a^6*e^6 + b^6*d^6 - 2*a^3*c^3*d^6 + 9*a^2*b^2*c^2*d^6 - 6*a*b^4*c*d^6))/(c*e))*(-(b^7*d^2 + a^2*b^5*e^2 + b^4*d^2*(-(4*a*c - b^2)^3)^(1/2) - 20*a^3*b*c^3*d^2 - 7*a^3*b^3*c*e^2 + 12*a^4*b*c^2*e^2 - a^3*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - 2*a*b^6*d*e + 25*a^2*b^3*c^2*d^2 + a^2*b^2*e^2*(-(4*a*c - b^2)^3)^(1/2) + a^2*c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c*d^2 + 16*a^4*c^3*d*e - 2*a*b^3*d*e*(-(4*a*c - b^2)^3)^(1/2

) + 16*a^2*b^4*c*d*e - 3*a*b^2*c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 36*a^3*b^2*c^2*d*e + 4*a^2*b*c*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^7*d^4 + 16*a^4*c^5*e^4 + b^4*c^5*d^4 - 8*a*b^2*c^6*d^4 - 2*b^5*c^4*d^3*e + a^2*b^4*c^3*e^4 - 8*a^3*b^2*c^4*e^4 + 32*a^3*c^6*d^2*e^2 + b^6*c^3*d^2*e^2 + 16*a*b^3*c^5*d^3*e - 2*a*b^5*c^3*d*e^3 - 32*a^2*b*c^6*d^3*e - 32*a^3*b*c^5*d*e^3 - 6*a*b^4*c^4*d^2*e^2 + 16*a^2*b^3*c^4*d*e^3)))^(1/2) + (((((64*a^5*c^4*d*e^8 + 64*a^3*c^6*d^5*e^4 + 128*a^4*c^5*d^3*e^6 - 144*a^2*b^2*c^5*d^5*e^4 + 64*a^2*b^3*c^4*d^4*e^5 + 16*a^2*b^4*c^3*d^3*e^6 - 96*a^3*b^2*c^4*d^3*e^6 + 16*a^3*b^3*c^3*d^2*e^7 - 16*a*b^3*c^5*d^6*e^3 + 32*a*b^4*c^4*d^5*e^4 - 16*a*b^5*c^3*d^4*e^5 + 64*a^2*b*c^6*d^6*e^3 - 64*a^4*b*c^4*d^2*e^7 - 16*a^4*b^2*c^3*d*e^8)/(c*e) + (2*x*(-(b^7*d^2 + a^2*b^5*e^2 + b^4*d^2*(-(4*a*c - b^2)^3)^(1/2) - 20*a^3*b*c^3*d^2 - 7*a^3*b^3*c*e^2 + 12*a^4*b*c^2*e^2 - a^3*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - 2*a*b^6*d*e + 25*a^2*b^3*c^2*d^2 + a^2*b^2*e^2*(-(4*a*c - b^2)^3)^(1/2) + a^2*c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c*d^2 + 16*a^4*c^3*d*e - 2*a*b^3*d*e*(-(4*a*c - b^2)^3)^(1/2) + 16*a^2*b^4*c*d*e - 3*a*b^2*c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 36*a^3*b^2*c^2*d*e + 4*a^2*b*c*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^7*d^4 + 16*a^4*c^5*e^4 + b^4*c^5*d^4 - 8*a*b^2*c^6*d^4 - 2*b^5*c^4*d^3*e + a^2*b^4*c^3*e^4 - 8*a^3*b^2*c^4*e^4 + 32*a^3*c^6*d^2*e^2 + b^6*c^3*d^2*e^2 + 16*a*b^3*c^5*d^3*e - 2*a*b^5*c^3*d*e^3 - 32*a^2*b*c^6*d^3*e - 32*a^3*b*c^5*d*e^3 - 6*a*b^4*c^4*d^2*e^2 + 16*a^2*b^3*c^4*d*e^3)))^(1/2)*(128*a^4*b^2*c^4*e^10 - 16*a^3*b^4*c^3*e^10 - 256*a^5*c^5*e^10 + 256*a^2*c^8*d^6*e^4 + 256*a^3*c^7*d^4*e^6 - 256*a^4*c^6*d^2*e^8 - 16*b^3*c^7*d^7*e^3 + 64*b^4*c^6*d^6*e^4 - 96*b^5*c^5*d^5*e^5 + 64*b^6*c^4*d^4*e^6 - 16*b^7*c^3*d^3*e^7 + 256*a^2*b^2*c^6*d^4*e^6 + 144*a^2*b^3*c^5*d^3*e^7 - 96*a^2*b^4*c^4*d^2*e^8 + 192*a^3*b^2*c^5*d^2*e^8 + 64*a*b*c^8*d^7*e^3 + 320*a^4*b*c^5*d*e^9 - 320*a*b^2*c^7*d^6*e^4 + 528*a*b^3*c^6*d^5*e^5 - 336*a*b^4*c^5*d^4*e^6 + 48*a*b^5*c^4*d^3*e^7 + 16*a*b^6*c^3*d^2*e^8 - 576*a^2*b*c^7*d^5*e^5 + 16*a^2*b^5*c^3*d*e^9 - 320*a^3*b*c^6*d^3*e^7 - 144*a^3*b^3*c^4*d*e^9))/(c*e))*(-(b^7*d^2 + a^2*b^5*e^2 + b^4*d^2*(-(4*a*c - b^2)^3)^(1/2) - 20*a^3*b*c^3*d^2 - 7*a^3*b^3*c*e^2 + 12*a^4*b*c^2*e^2 - a^3*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - 2*a*b^6*d*e + 25*a^2*b^3*c^2*d^2 + a^2*b^2*e^2*(-(4*a*c - b^2)^3)^(1/2) + a^2*c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c*d^2 + 16*a^4*c^3*d*e - 2*a*b^3*d*e*(-(4*a*c - b^2)^3)^(1/2) + 16*a^2*b^4*c*d*e - 3*a*b^2*c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 36*a^3*b^2*c^2*d*e + 4*a^2*b*c*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^7*d^4 + 16*a^4*c^5*e^4 + b^4*c^5*d^4 - 8*a*b^2*c^6*d^4 - 2*b^5*c^4*d^3*e + a^2*b^4*c^3*e^4 - 8*a^3*b^2*c^4*e^4 + 32*a^3*c^6*d^2*e^2 + b^6*c^3*d^2*e^2 + 16*a*b^3*c^5*d^3*e - 2*a*b^5*c^3*d*e^3 - 32*a^2*b*c^6*d^3*e - 32*a^3*b*c^5*d*e^3 - 6*a*b^4*c^4*d^2*e^2 + 16*a^2*b^3*c^4*d*e^3)))^(1/2) - (2*x*(4*a^3*b^5*e^8 + 4*b^3*c^5*d^8 + 4*b^8*d^3*e^5 - 28*a^4*b^3*c*e^8 + 48*a^5*b*c^2*e^8 - 4*a*b^7*d^2*e^6 - 4*a^2*b^6*d*e^7 - 64*a^2*c^6*d^7*e + 56*a^5*c^3*d*e^7 - 8*b^4*c^4*d^7*e - 8*b^7*c*d^4*e^4 - 8*a^3*c^5*d^5*e^3 - 16*a^4*c^4*d^3*e^5 + 4*b^5*c^3*d^6*e^2 + 4*b^6*c^2*d^5*e^3 - 16*a*b*c^6*d^8 + 36*a^2*b^2*c^4*d^5*e^3 - 72*a^2*b^3*c^3*d^4*e^4 - 12*a^2*b^4*c^2*d^3*e^5 + 64*a^3*b^2*c^3*d^3*e^5 + 28*a^3*b^3*c^2*d^2*e^6 + 48*a*b^2*c^5*d^7*e - 16*a*b^6*c*d^3*e^5 + 40*a^3*b^4*c

*d*e^7 - 28*a*b^3*c^4*d^6*e^2 - 24*a*b^4*c^3*d^5*e^3 + 48*a*b^5*c^2*d^4*e^4
 + 48*a^2*b*c^5*d^6*e^2 + 12*a^2*b^5*c*d^2*e^6 + 16*a^3*b*c^4*d^4*e^4 - 64*
a^4*b*c^3*d^2*e^6 - 108*a^4*b^2*c^2*d*e^7))/(c*e))*(-(b^7*d^2 + a^2*b^5*e^2
 + b^4*d^2*(-(4*a*c - b^2)^3)^(1/2) - 20*a^3*b*c^3*d^2 - 7*a^3*b^3*c*e^2 +
12*a^4*b*c^2*e^2 - a^3*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - 2*a*b^6*d*e + 25*a^
2*b^3*c^2*d^2 + a^2*b^2*e^2*(-(4*a*c - b^2)^3)^(1/2) + a^2*c^2*d^2*(-(4*a*c
 - b^2)^3)^(1/2) - 9*a*b^5*c*d^2 + 16*a^4*c^3*d*e - 2*a*b^3*d*e*(-(4*a*c -
b^2)^3)^(1/2) + 16*a^2*b^4*c*d*e - 3*a*b^2*c*d^2*(-(4*a*c - b^2)^3)^(1/2) -
 36*a^3*b^2*c^2*d*e + 4*a^2*b*c*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^
7*d^4 + 16*a^4*c^5*e^4 + b^4*c^5*d^4 - 8*a*b^2*c^6*d^4 - 2*b^5*c^4*d^3*e +
a^2*b^4*c^3*e^4 - 8*a^3*b^2*c^4*e^4 + 32*a^3*c^6*d^2*e^2 + b^6*c^3*d^2*e^2
+ 16*a*b^3*c^5*d^3*e - 2*a*b^5*c^3*d*e^3 - 32*a^2*b*c^6*d^3*e - 32*a^3*b*c^
5*d*e^3 - 6*a*b^4*c^4*d^2*e^2 + 16*a^2*b^3*c^4*d*e^3)))^(1/2) - (4*a*b^3*c^
3*d^7 - 16*a^2*b*c^4*d^7 + 4*a*b^6*d^4*e^3 + 4*a^4*b^3*d*e^6 + 48*a^3*c^4*d
^6*e - 4*a^2*b^5*d^3*e^4 - 4*a^3*b^4*d^2*e^5 - 60*a^4*c^3*d^4*e^3 + 4*a^5*c
^2*d^2*e^5 - 8*a^5*b*c*d*e^6 - 32*a^2*b^3*c^2*d^5*e^2 + 92*a^3*b^2*c^2*d^4*
e^3 + 4*a*b^4*c^2*d^6*e + 4*a*b^5*c*d^5*e^2 - 28*a^2*b^2*c^3*d^6*e - 36*a^2
*b^4*c*d^4*e^3 + 64*a^3*b*c^3*d^5*e^2 + 36*a^3*b^3*c*d^3*e^4 - 60*a^4*b*c^2
*d^3*e^4 + 4*a^4*b^2*c*d^2*e^5)/(c*e))*(-(b^7*d^2 + a^2*b^5*e^2 + b^4*d^2*(
-(4*a*c - b^2)^3)^(1/2) - 20*a^3*b*c^3*d^2 - 7*a^3*b^3*c*e^2 + 12*a^4*b*c^2
*e^2 - a^3*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - 2*a*b^6*d*e + 25*a^2*b^3*c^2*d^
2 + a^2*b^2*e^2*(-(4*a*c - b^2)^3)^(1/2) + a^2*c^2*d^2*(-(4*a*c - b^2)^3)^(
1/2) - 9*a*b^5*c*d^2 + 16*a^4*c^3*d*e - 2*a*b^3*d*e*(-(4*a*c - b^2)^3)^(1/2
) + 16*a^2*b^4*c*d*e - 3*a*b^2*c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 36*a^3*b^2*
c^2*d*e + 4*a^2*b*c*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^7*d^4 + 16*a
^4*c^5*e^4 + b^4*c^5*d^4 - 8*a*b^2*c^6*d^4 - 2*b^5*c^4*d^3*e + a^2*b^4*c^3*
e^4 - 8*a^3*b^2*c^4*e^4 + 32*a^3*c^6*d^2*e^2 + b^6*c^3*d^2*e^2 + 16*a*b^3*c
^5*d^3*e - 2*a*b^5*c^3*d*e^3 - 32*a^2*b*c^6*d^3*e - 32*a^3*b*c^5*d*e^3 - 6*
a*b^4*c^4*d^2*e^2 + 16*a^2*b^3*c^4*d*e^3)))^(1/2) - (2*x*(a^6*e^6 + b^6*d^6
 - 2*a^3*c^3*d^6 + 9*a^2*b^2*c^2*d^6 - 6*a*b^4*c*d^6))/(c*e))*(-(b^7*d^2 +
a^2*b^5*e^2 + b^4*d^2*(-(4*a*c - b^2)^3)^(1/2) - 20*a^3*b*c^3*d^2 - 7*a^3*b
^3*c*e^2 + 12*a^4*b*c^2*e^2 - a^3*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - 2*a*b^6*
d*e + 25*a^2*b^3*c^2*d^2 + a^2*b^2*e^2*(-(4*a*c - b^2)^3)^(1/2) + a^2*c^2*d
^2*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c*d^2 + 16*a^4*c^3*d*e - 2*a*b^3*d*e*
(-(4*a*c - b^2)^3)^(1/2) + 16*a^2*b^4*c*d*e - 3*a*b^2*c*d^2*(-(4*a*c - b^2)
^3)^(1/2) - 36*a^3*b^2*c^2*d*e + 4*a^2*b*c*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8
*(16*a^2*c^7*d^4 + 16*a^4*c^5*e^4 + b^4*c^5*d^4 - 8*a*b^2*c^6*d^4 - 2*b^5*c
^4*d^3*e + a^2*b^4*c^3*e^4 - 8*a^3*b^2*c^4*e^4 + 32*a^3*c^6*d^2*e^2 + b^6*c
^3*d^2*e^2 + 16*a*b^3*c^5*d^3*e - 2*a*b^5*c^3*d*e^3 - 32*a^2*b*c^6*d^3*e -
32*a^3*b*c^5*d*e^3 - 6*a*b^4*c^4*d^2*e^2 + 16*a^2*b^3*c^4*d*e^3)))^(1/2) +
(2*(a^3*b^2*d^5 - a^4*c*d^5 + a^5*d^3*e^2 + a^4*b*d^4*e))/(c*e)))*(-(b^7*d^
2 + a^2*b^5*e^2 + b^4*d^2*(-(4*a*c - b^2)^3)^(1/2) - 20*a^3*b*c^3*d^2 - 7*a
^3*b^3*c*e^2 + 12*a^4*b*c^2*e^2 - a^3*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - 2*a*
b^6*d*e + 25*a^2*b^3*c^2*d^2 + a^2*b^2*e^2*(-(4*a*c - b^2)^3)^(1/2) + a^2*c
^2*d^2*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c*d^2 + 16*a^4*c^3*d*e - 2*a*b^3*

d*e*(-(4*a*c - b^2)^3)^(1/2) + 16*a^2*b^4*c*d*e - 3*a*b^2*c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 36*a^3*b^2*c^2*d*e + 4*a^2*b*c*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^7*d^4 + 16*a^4*c^5*e^4 + b^4*c^5*d^4 - 8*a*b^2*c^6*d^4 - 2*b^5*c^4*d^3*e + a^2*b^4*c^3*e^4 - 8*a^3*b^2*c^4*e^4 + 32*a^3*c^6*d^2*e^2 + b^6*c^3*d^2*e^2 + 16*a*b^3*c^5*d^3*e - 2*a*b^5*c^3*d*e^3 - 32*a^2*b*c^6*d^3*e - 32*a^3*b*c^5*d*e^3 - 6*a*b^4*c^4*d^2*e^2 + 16*a^2*b^3*c^4*d*e^3)))^(1/2)*2i - (log(b^7*d^10*e^10 - a^7*d^3*e^17 - 2*a*b^6*d^9*e^11 + 2*a^6*b*d^4*e^16 - 2*a^6*c*d^5*e^15 + a^2*b^5*d^8*e^12 - a^5*b^2*d^5*e^15 - 16*a^2*c^5*d^13*e^7 + 16*a^4*c^3*d^9*e^11 - a^5*c^2*d^7*e^13 - b^4*c^3*d^13*e^7 + 16*a^2*c^5*x*(-d^5*e^3)^(5/2) + b^4*c^3*x*(-d^5*e^3)^(5/2) + a^7*e^16*x*(-d^5*e^3)^(1/2) - 8*a*b^2*c^4*x*(-d^5*e^3)^(5/2) + 25*a^2*b^3*c^2*d^10*e^10 - 36*a^3*b^2*c^2*d^9*e^11 + a^2*b^5*e^8*x*(-d^5*e^3)^(3/2) + b^7*d^2*e^6*x*(-d^5*e^3)^(3/2) - 9*a*b^5*c*d^10*e^10 + 2*a^5*b*c*d^6*e^14 + 8*a*b^2*c^4*d^13*e^7 + 16*a^2*b^4*c*d^9*e^11 - 20*a^3*b*c^3*d^10*e^10 - 7*a^3*b^3*c*d^8*e^12 + 12*a^4*b*c^2*d^8*e^12 + a^5*b^2*d^2*e^14*x*(-d^5*e^3)^(1/2) + a^5*c^2*d^4*e^12*x*(-d^5*e^3)^(1/2) - 2*a*b^6*d*e^7*x*(-d^5*e^3)^(3/2) - 2*a^6*b*d*e^15*x*(-d^5*e^3)^(1/2) - 7*a^3*b^3*c*e^8*x*(-d^5*e^3)^(3/2) + 12*a^4*b*c^2*e^8*x*(-d^5*e^3)^(3/2) + 16*a^4*c^3*d*e^7*x*(-d^5*e^3)^(3/2) + 2*a^6*c*d^2*e^14*x*(-d^5*e^3)^(1/2) - 9*a*b^5*c*d^2*e^6*x*(-d^5*e^3)^(3/2) + 16*a^2*b^4*c*d*e^7*x*(-d^5*e^3)^(3/2) - 2*a^5*b*c*d^3*e^13*x*(-d^5*e^3)^(1/2) - 20*a^3*b*c^3*d^2*e^6*x*(-d^5*e^3)^(3/2) - 36*a^3*b^2*c^2*d*e^7*x*(-d^5*e^3)^(3/2) + 25*a^2*b^3*c^2*d^2*e^6*x*(-d^5*e^3)^(3/2))*(-d^5*e^3)^(1/2))/(2*(a*e^5 + c*d^2*e^3 - b*d*e^4)) + (log(a^7*d^3*e^17 - b^7*d^10*e^10 + 2*a*b^6*d^9*e^11 - 2*a^6*b*d^4*e^16 + 2*a^6*c*d^5*e^15 - a^2*b^5*d^8*e^12 + a^5*b^2*d^5*e^15 + 16*a^2*c^5*d^13*e^7 - 16*a^4*c^3*d^9*e^11 + a^5*c^2*d^7*e^13 + b^4*c^3*d^13*e^7 + 16*a^2*c^5*x*(-d^5*e^3)^(5/2) + b^4*c^3*x*(-d^5*e^3)^(5/2) + a^7*e^16*x*(-d^5*e^3)^(1/2) - 8*a*b^2*c^4*x*(-d^5*e^3)^(5/2) - 25*a^2*b^3*c^2*d^10*e^10 + 36*a^3*b^2*c^2*d^9*e^11 + a^2*b^5*e^8*x*(-d^5*e^3)^(3/2) + b^7*d^2*e^6*x*(-d^5*e^3)^(3/2) + 9*a*b^5*c*d^10*e^10 - 2*a^5*b*c*d^6*e^14 - 8*a*b^2*c^4*d^13*e^7 - 16*a^2*b^4*c*d^9*e^11 + 20*a^3*b*c^3*d^10*e^10 + 7*a^3*b^3*c*d^8*e^12 - 12*a^4*b*c^2*d^8*e^12 + a^5*b^2*d^2*e^14*x*(-d^5*e^3)^(1/2) + a^5*c^2*d^4*e^12*x*(-d^5*e^3)^(1/2) - 2*a*b^6*d*e^7*x*(-d^5*e^3)^(3/2) - 2*a^6*b*d*e^15*x*(-d^5*e^3)^(1/2) - 7*a^3*b^3*c*e^8*x*(-d^5*e^3)^(3/2) + 12*a^4*b*c^2*e^8*x*(-d^5*e^3)^(3/2) + 16*a^4*c^3*d*e^7*x*(-d^5*e^3)^(3/2) + 2*a^6*c*d^2*e^14*x*(-d^5*e^3)^(1/2) - 9*a*b^5*c*d^2*e^6*x*(-d^5*e^3)^(3/2) + 16*a^2*b^4*c*d*e^7*x*(-d^5*e^3)^(3/2) - 2*a^5*b*c*d^3*e^13*x*(-d^5*e^3)^(1/2) - 20*a^3*b*c^3*d^2*e^6*x*(-d^5*e^3)^(3/2) - 36*a^3*b^2*c^2*d*e^7*x*(-d^5*e^3)^(3/2) + 25*a^2*b^3*c^2*d^2*e^6*x*(-d^5*e^3)^(3/2))*(-d^5*e^3)^(1/2))/(2*a*e^5 + 2*c*d^2*e^3 - 2*b*d*e^4)

3.305 $\int \frac{x^4}{(d+ex^2)(a+bx^2+cx^4)}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 280

$$\int \frac{x^4}{(d+ex^2)(a+bx^2+cx^4)}\,dx = -\frac{\left(bd-ae-\frac{b^2d-2acd-abe}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b-\sqrt{b^2-4ac}}\,(cd^2-bde+ae^2)} - \frac{\left(bd-ae+\frac{b^2d-2acd-abe}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b+\sqrt{b^2-4ac}}\,(cd^2-bde+ae^2)} + \frac{d^{3/2}\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{e}\,(cd^2-bde+ae^2)}$$

```
[Out] d^(3/2)*arctan(x*e^(1/2)/d^(1/2))/(a*e^2-b*d*e+c*d^2)/e^(1/2)-1/2*arctan(x*
2^(1/2)*c^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2))*(b*d-a*e+(a*b*e+2*a*c*d-b^2*d
)/(-4*a*c+b^2)^(1/2))/(a*e^2-b*d*e+c*d^2)*2^(1/2)/c^(1/2)/(b-(-4*a*c+b^2)^(
1/2))^(1/2)-1/2*arctan(x*2^(1/2)*c^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2))*(b*d
-a*e+(-a*b*e-2*a*c*d+b^2*d)/(-4*a*c+b^2)^(1/2))/(a*e^2-b*d*e+c*d^2)*2^(1/2)
/c^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2)
```

Rubi [A] (verified)

Time = 0.57 (sec) , antiderivative size = 280, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used

$= \{1301, 211, 1180\}$

$$\int \frac{x^4}{(d+ex^2)(a+bx^2+cx^4)}\,dx = -\frac{\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)\left(-\frac{-abe-2acd+b^2d}{\sqrt{b^2-4ac}}-ae+bd\right)}{\sqrt{2}\sqrt{c}\sqrt{b-\sqrt{b^2-4ac}}\,(ae^2-bde+cd^2)} - \frac{\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)\left(\frac{-abe-2acd+b^2d}{\sqrt{b^2-4ac}}-ae+bd\right)}{\sqrt{2}\sqrt{c}\sqrt{\sqrt{b^2-4ac}+b}\,(ae^2-bde+cd^2)} + \frac{d^{3/2}\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{e}\,(ae^2-bde+cd^2)}$$

```
[In] Int[x^4/((d + e*x^2)*(a + b*x^2 + c*x^4)),x]
```

```
[Out] -(((b*d - a*e - (b^2*d - 2*a*c*d - a*b*e)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2
]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*Sqrt[c]*Sqrt[b - Sqrt[b
^2 - 4*a*c]]*(c*d^2 - b*d*e + a*e^2))) - ((b*d - a*e + (b^2*d - 2*a*c*d - a
*b*e)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a
*c]]])/(Sqrt[2]*Sqrt[c]*Sqrt[b + Sqrt[b^2 - 4*a*c]]*(c*d^2 - b*d*e + a*e^2)
) + (d^(3/2)*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(Sqrt[e]*(c*d^2 - b*d*e + a*e^2))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1301

```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.))/((a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*((d + e*x^2)^q/(a
+ b*x^2 + c*x^4)), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b^2 - 4
*a*c, 0] && IntegerQ[q] && IntegerQ[m]
```

Rubi steps

$$\text{integral} = \int \left(\frac{d^2}{(cd^2-bde+ae^2)(d+ex^2)} + \frac{-ad-(bd-ae)x^2}{(cd^2-bde+ae^2)(a+bx^2+cx^4)}\right)dx$$

$$= \frac{\int \frac{-ad+(-bd+ae)x^2}{a+bx^2+cx^4}\,dx}{cd^2-bde+ae^2} + \frac{d^2\int \frac{1}{d+ex^2}\,dx}{cd^2-bde+ae^2}$$

$$= \frac{d^{3/2}\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{e}\,(cd^2-bde+ae^2)} - \frac{\left(bd-ae-\frac{b^2d-2acd-abe}{\sqrt{b^2-4ac}}\right)\int\frac{1}{\frac{b}{2}-\frac{1}{2}\sqrt{b^2-4ac}+cx^2}\,dx}{2\,(cd^2-bde+ae^2)} - \frac{\left(bd-ae+\frac{b^2d-2acd-abe}{\sqrt{b^2-4ac}}\right)\int\frac{1}{\frac{b}{2}+\frac{1}{2}\sqrt{b^2-4ac}+cx^2}\,dx}{2\,(cd^2-bde+ae^2)}$$

$$= -\frac{\left(bd-ae-\frac{b^2d-2acd-abe}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b-\sqrt{b^2-4ac}}\,(cd^2-bde+ae^2)} - \frac{\left(bd-ae+\frac{b^2d-2acd-abe}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b+\sqrt{b^2-4ac}}\,(cd^2-bde+ae^2)} + \frac{d^{3/2}\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{e}\,(cd^2-bde+ae^2)}$$

Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 323, normalized size of antiderivative = 1.15

$$\int \frac{x^4}{(d+ex^2)\,(a+bx^2+cx^4)}\,dx = \frac{\left(-b^2d+2acd+b\sqrt{b^2-4ac}d+abe-a\sqrt{b^2-4ac}e\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}\,(-cd^2+bde-ae^2)} + \frac{\left(b^2d-2acd+b\sqrt{b^2-4ac}d-abe-a\sqrt{b^2-4ac}e\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b^2-4ac}\sqrt{b+\sqrt{b^2-4ac}}\,(-cd^2+bde-ae^2)} + \frac{d^{3/2}\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{e}\,(cd^2-bde+ae^2)}$$

```
[In] Integrate[x^4/((d + e*x^2)*(a + b*x^2 + c*x^4)),x]
```

```
[Out] ((-(b^2*d) + 2*a*c*d + b*Sqrt[b^2 - 4*a*c]*d + a*b*e - a*Sqrt[b^2 - 4*a*c]*
e)*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*Sqrt[c
]*Sqrt[b^2 - 4*a*c]*Sqrt[b - Sqrt[b^2 - 4*a*c]]*(-(c*d^2) + b*d*e - a*e^2))
 + ((b^2*d - 2*a*c*d + b*Sqrt[b^2 - 4*a*c]*d - a*b*e - a*Sqrt[b^2 - 4*a*c]*
e)*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*Sqrt[c
]*Sqrt[b^2 - 4*a*c]*Sqrt[b + Sqrt[b^2 - 4*a*c]]*(-(c*d^2) + b*d*e - a*e^2))
 + (d^(3/2)*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(Sqrt[e]*(c*d^2 - b*d*e + a*e^2))
```

Maple [A] (verified)

Time = 0.43 (sec) , antiderivative size = 264, normalized size of antiderivative = 0.94

method	result
default	$\frac{4c\left(\frac{\left(ae\sqrt{-4ac+b^2}-bd\sqrt{-4ac+b^2}+abe+2acd-b^2d\right)\sqrt{2}\,\arctan\left(\frac{cx\sqrt{2}}{\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}}\right)}{8c\sqrt{-4ac+b^2}\,\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}}-\frac{\left(ae\sqrt{-4ac+b^2}-bd\sqrt{-4ac+b^2}-abe-2acd+b^2d\right)\sqrt{2}\,\text{arctanh}}{8c\sqrt{-4ac+b^2}\,\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)}}\right.}{a\,e^2-bde+c\,d^2}$
risch	Expression too large to display

```
[In] int(x^4/(e*x^2+d)/(c*x^4+b*x^2+a),x,method=_RETURNVERBOSE)
```

```
[Out] 4/(a*e^2-b*d*e+c*d^2)*c*(1/8*(a*e*(-4*a*c+b^2)^(1/2)-b*d*(-4*a*c+b^2)^(1/2)
+a*b*e+2*a*c*d-b^2*d)/c/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*
c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))-1/8*(a*e*(-4*
a*c+b^2)^(1/2)-b*d*(-4*a*c+b^2)^(1/2)-a*b*e-2*a*c*d+b^2*d)/c/(-4*a*c+b^2)^(
1/2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(c*x*2^(1/2)/((-b+(-4
*a*c+b^2)^(1/2))*c)^(1/2)))+d^2/(a*e^2-b*d*e+c*d^2)/(e*d)^(1/2)*arctan(e*x/
(e*d)^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 7767 vs. 2(236) = 472.

Time = 2.57 (sec) , antiderivative size = 15553, normalized size of antiderivative = 55.55

$$\int \frac{x^4}{(d+ex^2)(a+bx^2+cx^4)}\,dx = \text{Too large to display}$$

```
[In] integrate(x^4/(e*x^2+d)/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^4}{(d+ex^2)(a+bx^2+cx^4)}\,dx = \text{Timed out}$$

```
[In] integrate(x**4/(e*x**2+d)/(c*x**4+b*x**2+a),x)
```

```
[Out] Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^4}{(d+ex^2)(a+bx^2+cx^4)}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(x^4/(e*x^2+d)/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 8670 vs. $2(236) = 472$.

Time = 2.27 (sec) , antiderivative size = 8670, normalized size of antiderivative = 30.96

$$\int \frac{x^4}{(d+ex^2)(a+bx^2+cx^4)}\,dx = \text{Too large to display}$$

```
[In] integrate(x^4/(e*x^2+d)/(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] d^2*arctan(e*x/sqrt(d*e))/((c*d^2 - b*d*e + a*e^2)*sqrt(d*e)) + 1/8*((2*b^5
*c^4 - 12*a*b^3*c^5 + 16*a^2*b*c^6 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + s
qrt(b^2 - 4*a*c)*c)*b^5*c^2 + 6*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b
^2 - 4*a*c)*c)*a*b^3*c^3 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2
- 4*a*c)*c)*b^4*c^3 - 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a
*c)*c)*a^2*b*c^4 - 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)
*c)*a*b^2*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b
^3*c^4 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c^
5 - 2*(b^2 - 4*a*c)*b^3*c^4 + 4*(b^2 - 4*a*c)*a*b*c^5)*d^5 - (4*b^6*c^3 - 2
2*a*b^4*c^4 + 24*a^2*b^2*c^5 - 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(
b^2 - 4*a*c)*c)*b^6*c + 11*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 -
4*a*c)*c)*a*b^4*c^2 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a
*c)*c)*b^5*c^2 - 12*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*
c)*a^2*b^2*c^3 - 6*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c
)*a*b^3*c^3 - 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b
^4*c^3 + 3*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*
c^4 - 4*(b^2 - 4*a*c)*b^4*c^3 + 6*(b^2 - 4*a*c)*a*b^2*c^4)*d^4*e + (2*b^7*c
^2 - 4*a*b^5*c^3 - 24*a^2*b^3*c^4 + 32*a^3*b*c^5 - sqrt(2)*sqrt(b^2 - 4*a*c
)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^7 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*
c + sqrt(b^2 - 4*a*c)*c)*a*b^5*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + s
qrt(b^2 - 4*a*c)*c)*b^6*c + 12*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^
```

2 - 4*a*c)*c)*a^2*b^3*c^2 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^4*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^5*c^2 - 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b*c^3 - 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^3 - 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c^3 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b*c^4 - 2*(b^2 - 4*a*c)*b^5*c^2 - 4*(b^2 - 4*a*c)*a*b^3*c^3 + 8*(b^2 - 4*a*c)*a^2*b*c^4)*d^3*e^2 - (6*a*b^6*c^2 - 28*a^2*b^4*c^3 + 16*a^3*b^2*c^4 - 3*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^6 + 14*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^4*c + 6*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^5*c - 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^2*c^2 - 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c^2 - 3*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^4*c^2 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^3 - 6*(b^2 - 4*a*c)*a*b^4*c^2 + 4*(b^2 - 4*a*c)*a^2*b^2*c^3)*d^2*e^3 + (6*a^2*b^5*c^2 - 28*a^3*b^3*c^3 + 16*a^4*b*c^4 - 3*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^5 + 14*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^3*c + 6*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^4*c - 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b*c^2 - 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^2*c^2 - 3*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c^2 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b*c^3 - 6*(b^2 - 4*a*c)*a^2*b^3*c^2 + 4*(b^2 - 4*a*c)*a^3*b*c^3)*d*e^4 - (2*a^3*b^4*c^2 - 8*a^4*b^2*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^4 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b^2*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^3*c - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^2*c^2 - 2*(b^2 - 4*a*c)*a^3*b^2*c^2)*e^5 - 2*(sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^4*c^2 - 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^3 - 2*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c^3 - 2*a*b^4*c^3 + 16*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*c^4 + 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b*c^4 + sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^4 + 16*a^2*b^2*c^4 - 4*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*c^5 - 32*a^3*c^5 + 2*(b^2 - 4*a*c)*a*b^2*c^3 - 8*(b^2 - 4*a*c)*a^2*c^4)*d^3*abs(c*d^2 - b*d*e + a*e^2) + 2*(sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^5*c - 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c^2 - 2*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^4*c^2 - 2*a*b^5*c^2 + 16*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b*c^3 + 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^3 + sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c^3 + 16*a^2*b^3*c^3 - 4*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b*c^4 - 32*a^3*b*c^4 + 2*(b^2 - 4*a*c)*a*b^3*c^2 - 8*(b^2 - 4*a*c)*a^2*b*c^3)*d^2*e*abs(c*d^2 - b*d*e + a*e^2) - 2*(sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^4*c - 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^2*c^2 - 2*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c^2 - 2*a^2*b^4*c^2 + 16*sqrt(2)*sqrt(b*c + sqrt

(b^2 - 4*a*c)*c)*a^4*c^3 + 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b*
c^3 + sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^3 + 16*a^3*b^2*c^3
- 4*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*c^4 - 32*a^4*c^4 + 2*(b^2 -
4*a*c)*a^2*b^2*c^2 - 8*(b^2 - 4*a*c)*a^3*c^3)*d*e^2*abs(c*d^2 - b*d*e + a*
e^2) - (2*b^5*c^2 - 16*a*b^3*c^3 + 32*a^2*b*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)
*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^5 + 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c
+ sqrt(b^2 - 4*a*c)*c)*a*b^3*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sq
rt(b^2 - 4*a*c)*c)*b^4*c - 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2
- 4*a*c)*c)*a^2*b*c^2 - 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 -
4*a*c)*c)*a*b^2*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c
)*c)*b^3*c^2 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*
a*b*c^3 - 2*(b^2 - 4*a*c)*b^3*c^2 + 8*(b^2 - 4*a*c)*a*b*c^3)*(c*d^2 - b*d*e
+ a*e^2)^2*d + (2*a*b^4*c^2 - 16*a^2*b^2*c^3 + 32*a^3*c^4 - sqrt(2)*sqrt(b
^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^4 + 8*sqrt(2)*sqrt(b^2 - 4*
a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c
)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c - 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sq
rt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*c^2 - 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*
c + sqrt(b^2 - 4*a*c)*c)*a^2*b*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + s
qrt(b^2 - 4*a*c)*c)*a*b^2*c^2 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt
(b^2 - 4*a*c)*c)*a^2*c^3 - 2*(b^2 - 4*a*c)*a*b^2*c^2 + 8*(b^2 - 4*a*c)*a^2*
c^3)*(c*d^2 - b*d*e + a*e^2)^2*e)*arctan(2*sqrt(1/2)*x/sqrt((b*c*d^2 - b^2*
d*e + a*b*e^2 + sqrt((b*c*d^2 - b^2*d*e + a*b*e^2)^2 - 4*(a*c*d^2 - a*b*d*e
+ a^2*e^2)*(c^2*d^2 - b*c*d*e + a*c*e^2)))/(c^2*d^2 - b*c*d*e + a*c*e^2)))
/((a*b^4*c^3 - 8*a^2*b^2*c^4 - 2*a*b^3*c^4 + 16*a^3*c^5 + 8*a^2*b*c^5 + a*b
^2*c^5 - 4*a^2*c^6)*d^4*abs(c*d^2 - b*d*e + a*e^2)*abs(c) - 2*(a*b^5*c^2 -
8*a^2*b^3*c^3 - 2*a*b^4*c^3 + 16*a^3*b*c^4 + 8*a^2*b^2*c^4 + a*b^3*c^4 - 4*
a^2*b*c^5)*d^3*e*abs(c*d^2 - b*d*e + a*e^2)*abs(c) + (a*b^6*c - 6*a^2*b^4*c
^2 - 2*a*b^5*c^2 + 4*a^2*b^3*c^3 + a*b^4*c^3 + 32*a^4*c^4 + 16*a^3*b*c^4 -
2*a^2*b^2*c^4 - 8*a^3*c^5)*d^2*e^2*abs(c*d^2 - b*d*e + a*e^2)*abs(c) - 2*(a
^2*b^5*c - 8*a^3*b^3*c^2 - 2*a^2*b^4*c^2 + 16*a^4*b*c^3 + 8*a^3*b^2*c^3 + a
^2*b^3*c^3 - 4*a^3*b*c^4)*d*e^3*abs(c*d^2 - b*d*e + a*e^2)*abs(c) + (a^3*b^
4*c - 8*a^4*b^2*c^2 - 2*a^3*b^3*c^2 + 16*a^5*c^3 + 8*a^4*b*c^3 + a^3*b^2*c^
3 - 4*a^4*c^4)*e^4*abs(c*d^2 - b*d*e + a*e^2)*abs(c)) - 1/8*((2*b^5*c^4 - 1
2*a*b^3*c^5 + 16*a^2*b*c^6 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2
- 4*a*c)*c)*b^5*c^2 + 6*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a
*c)*c)*a*b^3*c^3 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)
*c)*b^4*c^3 - 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a
^2*b*c^4 - 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^
2*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^3*c^4 +
2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b*c^5 - 2*(b
^2 - 4*a*c)*b^3*c^4 + 4*(b^2 - 4*a*c)*a*b*c^5)*d^5 - (4*b^6*c^3 - 22*a*b^4*
c^4 + 24*a^2*b^2*c^5 - 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*
a*c)*c)*b^6*c + 11*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c
)*a*b^4*c^2 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b
^5*c^2 - 12*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b

^2*c^3 - 6*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^3*
c^3 - 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^4*c^3 +
 3*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^2*c^4 - 4*
(b^2 - 4*a*c)*b^4*c^3 + 6*(b^2 - 4*a*c)*a*b^2*c^4)*d^4*e + (2*b^7*c^2 - 4*a
*b^5*c^3 - 24*a^2*b^3*c^4 + 32*a^3*b*c^5 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b
*c - sqrt(b^2 - 4*a*c)*c)*b^7 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt
(b^2 - 4*a*c)*c)*a*b^5*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2
- 4*a*c)*c)*b^6*c + 12*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*
c)*c)*a^2*b^3*c^2 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c
)*c)*a*b^4*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*
b^5*c^2 - 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*
b*c^3 - 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^2
*c^3 - 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^3*c^
3 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b*c^4 -
 2*(b^2 - 4*a*c)*b^5*c^2 - 4*(b^2 - 4*a*c)*a*b^3*c^3 + 8*(b^2 - 4*a*c)*a^2*
b*c^4)*d^3*e^2 - (6*a*b^6*c^2 - 28*a^2*b^4*c^3 + 16*a^3*b^2*c^4 - 3*sqrt(2)
*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^6 + 14*sqrt(2)*sqrt(
b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^4*c + 6*sqrt(2)*sqrt(b^2
 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^5*c - 8*sqrt(2)*sqrt(b^2 - 4*
a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^2*c^2 - 4*sqrt(2)*sqrt(b^2 - 4*a
*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c^2 - 3*sqrt(2)*sqrt(b^2 - 4*a*
c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^4*c^2 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*
sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^3 - 6*(b^2 - 4*a*c)*a*b^4*c^2 + 4
*(b^2 - 4*a*c)*a^2*b^2*c^3)*d^2*e^3 + (6*a^2*b^5*c^2 - 28*a^3*b^3*c^3 + 16*
a^4*b*c^4 - 3*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2
*b^5 + 14*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^3
*c + 6*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^4*c
- 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*b*c^2 - 4
*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^2*c^2 - 3*
sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c^2 + 2*s
qrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b*c^3 - 6*(b^2
 - 4*a*c)*a^2*b^3*c^2 + 4*(b^2 - 4*a*c)*a^3*b*c^3)*d*e^4 - (2*a^3*b^4*c^2 -
 8*a^4*b^2*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*
a^3*b^4 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*b
^2*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^3*
c - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^2*c^2 -
 2*(b^2 - 4*a*c)*a^3*b^2*c^2)*e^5 + 2*(sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)
*c)*a*b^4*c^2 - 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^3 - 2*s
qrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^3*c^3 + 2*a*b^4*c^3 + 16*sqrt(2)
*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*c^4 + 8*sqrt(2)*sqrt(b*c - sqrt(b^2 -
4*a*c)*c)*a^2*b*c^4 + sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^2*c^4 - 1
6*a^2*b^2*c^4 - 4*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*c^5 + 32*a^3*
c^5 - 2*(b^2 - 4*a*c)*a*b^2*c^3 + 8*(b^2 - 4*a*c)*a^2*c^4)*d^3*abs(c*d^2 -
b*d*e + a*e^2) - 2*(sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^5*c - 8*sqr
t(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c^2 - 2*sqrt(2)*sqrt(b*c - sqr

t(b^2 - 4*a*c)*c)*a*b^4*c^2 + 2*a*b^5*c^2 + 16*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b*c^3 + 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^3 + sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^3*c^3 - 16*a^2*b^3*c^3 - 4*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b*c^4 + 32*a^3*b*c^4 - 2*(b^2 - 4*a*c)*a*b^3*c^2 + 8*(b^2 - 4*a*c)*a^2*b*c^3)*d^2*e*abs(c*d^2 - b*d*e + a*e^2) + 2*(sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^4*c - 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^2*c^2 - 2*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c^2 + 2*a^2*b^4*c^2 + 16*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*c^3 + 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b*c^3 + sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^3 - 16*a^3*b^2*c^3 - 4*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*c^4 + 32*a^4*c^4 - 2*(b^2 - 4*a*c)*a^2*b^2*c^2 + 8*(b^2 - 4*a*c)*a^3*c^3)*d*e^2*abs(c*d^2 - b*d*e + a*e^2) - (2*b^5*c^2 - 16*a*b^3*c^3 + 32*a^2*b*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^5 + 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^3*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^4*c - 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b*c^2 - 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^2*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^3*c^2 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b*c^3 - 2*(b^2 - 4*a*c)*b^3*c^2 + 8*(b^2 - 4*a*c)*a*b*c^3)*(c*d^2 - b*d*e + a*e^2)^2*d + (2*a*b^4*c^2 - 16*a^2*b^2*c^3 + 32*a^3*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^4 + 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^3*c - 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*c^2 - 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^2*c^2 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*c^3 - 2*(b^2 - 4*a*c)*a*b^2*c^2 + 8*(b^2 - 4*a*c)*a^2*c^3)*(c*d^2 - b*d*e + a*e^2)^2*e)*arctan(2*sqrt(1/2)*x/sqrt((b*c*d^2 - b^2*d*e + a*b*e^2 - sqrt((b*c*d^2 - b^2*d*e + a*b*e^2)^2 - 4*(a*c*d^2 - a*b*d*e + a^2*e^2)*(c^2*d^2 - b*c*d*e + a*c*e^2)))/(c^2*d^2 - b*c*d*e + a*c*e^2)))/((a*b^4*c^3 - 8*a^2*b^2*c^4 - 2*a*b^3*c^4 + 16*a^3*c^5 + 8*a^2*b*c^5 + a*b^2*c^5 - 4*a^2*c^6)*d^4*abs(c*d^2 - b*d*e + a*e^2)*abs(c) - 2*(a*b^5*c^2 - 8*a^2*b^3*c^3 - 2*a*b^4*c^3 + 16*a^3*b*c^4 + 8*a^2*b^2*c^4 + a*b^3*c^4 - 4*a^2*b*c^5)*d^3*e*abs(c*d^2 - b*d*e + a*e^2)*abs(c) + (a*b^6*c - 6*a^2*b^4*c^2 - 2*a*b^5*c^2 + 4*a^2*b^3*c^3 + a*b^4*c^3 + 32*a^4*c^4 + 16*a^3*b*c^4 - 2*a^2*b^2*c^4 - 8*a^3*c^5)*d^2*e^2*abs(c*d^2 - b*d*e + a*e^2)*abs(c) - 2*(a^2*b^5*c - 8*a^3*b^3*c^2 - 2*a^2*b^4*c^2 + 16*a^4*b*c^3 + 8*a^3*b^2*c^3 + a^2*b^3*c^3 - 4*a^3*b*c^4)*d*e^3*abs(c*d^2 - b*d*e + a*e^2)*abs(c) + (a^3*b^4*c - 8*a^4*b^2*c^2 - 2*a^3*b^3*c^2 + 16*a^5*c^3 + 8*a^4*b*c^3 + a^3*b^2*c^3 - 4*a^4*c^4)*e^4*abs(c*d^2 - b*d*e + a*e^2)*abs(c))

Mupad [B] (verification not implemented)

Time = 10.98 (sec) , antiderivative size = 25202, normalized size of antiderivative = 90.01

$$\int \frac{x^4}{(d+ex^2)(a+bx^2+cx^4)}\,dx = \text{Too large to display}$$

```
[In] int(x^4/((d + e*x^2)*(a + b*x^2 + c*x^4)),x)
```

```
[Out] atan((((-(b^5*d^2 + a^2*b^3*e^2 + a^2*e^2*(-(4*a*c - b^2)^3)^(1/2) + b^2*d^
2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*d^2 - 2*a*b^4*d*e - 7*a*b^3*c*d^2
 - a*c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a^3*b*c*e^2 - 16*a^3*c^2*d*e + 12*a
^2*b^2*c*d*e - 2*a*b*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^5*d^4 + 16*
a^4*c^3*e^4 + b^4*c^3*d^4 - 8*a*b^2*c^4*d^4 + a^2*b^4*c*e^4 - 2*b^5*c^2*d^3
*e + b^6*c*d^2*e^2 - 8*a^3*b^2*c^2*e^4 + 32*a^3*c^4*d^2*e^2 - 2*a*b^5*c*d*e
^3 + 16*a*b^3*c^3*d^3*e - 32*a^2*b*c^4*d^3*e - 32*a^3*b*c^3*d*e^3 - 6*a*b^4
*c^2*d^2*e^2 + 16*a^2*b^3*c^2*d*e^3)))^(1/2)*((x*(8*a^3*b^3*c*e^7 - 32*a^4*
b*c^2*e^7 - 112*a^4*c^3*d*e^6 + 8*b^3*c^4*d^6*e + 8*b^6*c*d^3*e^4 - 112*a^2
*c^5*d^5*e^2 + 32*a^3*c^4*d^3*e^4 - 8*b^4*c^3*d^5*e^2 - 8*b^5*c^2*d^4*e^3 -
 32*a*b*c^5*d^6*e - 48*a^2*b^2*c^3*d^3*e^4 + 8*a^2*b^3*c^2*d^2*e^5 - 8*a*b^
5*c*d^2*e^5 - 8*a^2*b^4*c*d*e^6 + 64*a*b^2*c^4*d^5*e^2 + 8*a*b^3*c^3*d^4*e^
3 - 16*a*b^4*c^2*d^3*e^4 + 64*a^2*b*c^4*d^4*e^3 + 64*a^3*b*c^3*d^2*e^5 + 64
*a^3*b^2*c^2*d*e^6) + (-(b^5*d^2 + a^2*b^3*e^2 + a^2*e^2*(-(4*a*c - b^2)^3)
^(1/2) + b^2*d^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*d^2 - 2*a*b^4*d*e
- 7*a*b^3*c*d^2 - a*c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a^3*b*c*e^2 - 16*a^3
*c^2*d*e + 12*a^2*b^2*c*d*e - 2*a*b*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^
2*c^5*d^4 + 16*a^4*c^3*e^4 + b^4*c^3*d^4 - 8*a*b^2*c^4*d^4 + a^2*b^4*c*e^4
- 2*b^5*c^2*d^3*e + b^6*c*d^2*e^2 - 8*a^3*b^2*c^2*e^4 + 32*a^3*c^4*d^2*e^2
- 2*a*b^5*c*d*e^3 + 16*a*b^3*c^3*d^3*e - 32*a^2*b*c^4*d^3*e - 32*a^3*b*c^3*
d*e^3 - 6*a*b^4*c^2*d^2*e^2 + 16*a^2*b^3*c^2*d*e^3)))^(1/2)*(64*a^2*c^6*d^6
*e^2 - x*(-(b^5*d^2 + a^2*b^3*e^2 + a^2*e^2*(-(4*a*c - b^2)^3)^(1/2) + b^2*
d^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*d^2 - 2*a*b^4*d*e - 7*a*b^3*c*d
^2 - a*c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a^3*b*c*e^2 - 16*a^3*c^2*d*e + 12
*a^2*b^2*c*d*e - 2*a*b*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^5*d^4 + 1
6*a^4*c^3*e^4 + b^4*c^3*d^4 - 8*a*b^2*c^4*d^4 + a^2*b^4*c*e^4 - 2*b^5*c^2*d
^3*e + b^6*c*d^2*e^2 - 8*a^3*b^2*c^2*e^4 + 32*a^3*c^4*d^2*e^2 - 2*a*b^5*c*d
*e^3 + 16*a*b^3*c^3*d^3*e - 32*a^2*b*c^4*d^3*e - 32*a^3*b*c^3*d*e^3 - 6*a*b
^4*c^2*d^2*e^2 + 16*a^2*b^3*c^2*d*e^3)))^(1/2)*(256*a^4*b^2*c^3*e^9 - 32*a^
3*b^4*c^2*e^9 - 512*a^5*c^4*e^9 + 512*a^2*c^7*d^6*e^3 + 512*a^3*c^6*d^4*e^5
 - 512*a^4*c^5*d^2*e^7 - 32*b^3*c^6*d^7*e^2 + 128*b^4*c^5*d^6*e^3 - 192*b^5
*c^4*d^5*e^4 + 128*b^6*c^3*d^4*e^5 - 32*b^7*c^2*d^3*e^6 + 512*a^2*b^2*c^5*d
^4*e^5 + 288*a^2*b^3*c^4*d^3*e^6 - 192*a^2*b^4*c^3*d^2*e^7 + 384*a^3*b^2*c^
4*d^2*e^7 + 128*a*b*c^7*d^7*e^2 + 640*a^4*b*c^4*d*e^8 - 640*a*b^2*c^6*d^6*e
^3 + 1056*a*b^3*c^5*d^5*e^4 - 672*a*b^4*c^4*d^4*e^5 + 96*a*b^5*c^3*d^3*e^6
+ 32*a*b^6*c^2*d^2*e^7 - 1152*a^2*b*c^6*d^5*e^4 + 32*a^2*b^5*c^2*d*e^8 - 64
```

0*a^3*b*c^5*d^3*e^6 - 288*a^3*b^3*c^3*d*e^8) + 128*a^3*c^5*d^4*e^4 + 64*a^4
*c^4*d^2*e^6 - 96*a^2*b^2*c^4*d^4*e^4 + 64*a^2*b^3*c^3*d^3*e^5 + 32*a^2*b^4
*c^2*d^2*e^6 - 144*a^3*b^2*c^3*d^2*e^6 + 64*a^4*b*c^3*d*e^7 - 16*a*b^2*c^5*
d^6*e^2 + 16*a*b^3*c^4*d^5*e^3 + 16*a*b^4*c^3*d^4*e^4 - 16*a*b^5*c^2*d^3*e^
5 - 64*a^2*b*c^5*d^5*e^3 - 16*a^3*b^3*c^2*d*e^7))*(-(b^5*d^2 + a^2*b^3*e^2
+ a^2*e^2*(-(4*a*c - b^2)^3)^(1/2) + b^2*d^2*(-(4*a*c - b^2)^3)^(1/2) + 12*
a^2*b*c^2*d^2 - 2*a*b^4*d*e - 7*a*b^3*c*d^2 - a*c*d^2*(-(4*a*c - b^2)^3)^(1
/2) - 4*a^3*b*c*e^2 - 16*a^3*c^2*d*e + 12*a^2*b^2*c*d*e - 2*a*b*d*e*(-(4*a*
c - b^2)^3)^(1/2))/(8*(16*a^2*c^5*d^4 + 16*a^4*c^3*e^4 + b^4*c^3*d^4 - 8*a*
b^2*c^4*d^4 + a^2*b^4*c*e^4 - 2*b^5*c^2*d^3*e + b^6*c*d^2*e^2 - 8*a^3*b^2*c
^2*e^4 + 32*a^3*c^4*d^2*e^2 - 2*a*b^5*c*d*e^3 + 16*a*b^3*c^3*d^3*e - 32*a^2
*b*c^4*d^3*e - 32*a^3*b*c^3*d*e^3 - 6*a*b^4*c^2*d^2*e^2 + 16*a^2*b^3*c^2*d*
e^3)))^(1/2) + 16*a^2*c^4*d^5*e + 4*a^4*c^2*d*e^5 - 60*a^3*c^3*d^3*e^3 + 24
*a^2*b^2*c^2*d^3*e^3 - 4*a*b^2*c^3*d^5*e - 4*a*b^4*c*d^3*e^3 - 4*a^3*b^2*c*
d*e^5 - 4*a*b^3*c^2*d^4*e^2 + 20*a^2*b*c^3*d^4*e^2 + 8*a^2*b^3*c*d^2*e^4 -
16*a^3*b*c^2*d^2*e^4) + x*(2*a^4*c*e^5 + 4*a^2*c^3*d^4*e + 2*b^4*c*d^4*e -
8*a*b^2*c^2*d^4*e))*(-(b^5*d^2 + a^2*b^3*e^2 + a^2*e^2*(-(4*a*c - b^2)^3)^(
1/2) + b^2*d^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*d^2 - 2*a*b^4*d*e -
7*a*b^3*c*d^2 - a*c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a^3*b*c*e^2 - 16*a^3*c
^2*d*e + 12*a^2*b^2*c*d*e - 2*a*b*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*
c^5*d^4 + 16*a^4*c^3*e^4 + b^4*c^3*d^4 - 8*a*b^2*c^4*d^4 + a^2*b^4*c*e^4 -
2*b^5*c^2*d^3*e + b^6*c*d^2*e^2 - 8*a^3*b^2*c^2*e^4 + 32*a^3*c^4*d^2*e^2 -
2*a*b^5*c*d*e^3 + 16*a*b^3*c^3*d^3*e - 32*a^2*b*c^4*d^3*e - 32*a^3*b*c^3*d*
e^3 - 6*a*b^4*c^2*d^2*e^2 + 16*a^2*b^3*c^2*d*e^3)))^(1/2)*1i + ((-(b^5*d^2
+ a^2*b^3*e^2 + a^2*e^2*(-(4*a*c - b^2)^3)^(1/2) + b^2*d^2*(-(4*a*c - b^2)^
3)^(1/2) + 12*a^2*b*c^2*d^2 - 2*a*b^4*d*e - 7*a*b^3*c*d^2 - a*c*d^2*(-(4*a*
c - b^2)^3)^(1/2) - 4*a^3*b*c*e^2 - 16*a^3*c^2*d*e + 12*a^2*b^2*c*d*e - 2*a
*b*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^5*d^4 + 16*a^4*c^3*e^4 + b^4*
c^3*d^4 - 8*a*b^2*c^4*d^4 + a^2*b^4*c*e^4 - 2*b^5*c^2*d^3*e + b^6*c*d^2*e^2
 - 8*a^3*b^2*c^2*e^4 + 32*a^3*c^4*d^2*e^2 - 2*a*b^5*c*d*e^3 + 16*a*b^3*c^3*
d^3*e - 32*a^2*b*c^4*d^3*e - 32*a^3*b*c^3*d*e^3 - 6*a*b^4*c^2*d^2*e^2 + 16*
a^2*b^3*c^2*d*e^3)))^(1/2)*((x*(8*a^3*b^3*c*e^7 - 32*a^4*b*c^2*e^7 - 112*a^
4*c^3*d*e^6 + 8*b^3*c^4*d^6*e + 8*b^6*c*d^3*e^4 - 112*a^2*c^5*d^5*e^2 + 32*
a^3*c^4*d^3*e^4 - 8*b^4*c^3*d^5*e^2 - 8*b^5*c^2*d^4*e^3 - 32*a*b*c^5*d^6*e
- 48*a^2*b^2*c^3*d^3*e^4 + 8*a^2*b^3*c^2*d^2*e^5 - 8*a*b^5*c*d^2*e^5 - 8*a^
2*b^4*c*d*e^6 + 64*a*b^2*c^4*d^5*e^2 + 8*a*b^3*c^3*d^4*e^3 - 16*a*b^4*c^2*d
^3*e^4 + 64*a^2*b*c^4*d^4*e^3 + 64*a^3*b*c^3*d^2*e^5 + 64*a^3*b^2*c^2*d*e^6
) - (-(b^5*d^2 + a^2*b^3*e^2 + a^2*e^2*(-(4*a*c - b^2)^3)^(1/2) + b^2*d^2*(
-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*d^2 - 2*a*b^4*d*e - 7*a*b^3*c*d^2 -
a*c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a^3*b*c*e^2 - 16*a^3*c^2*d*e + 12*a^2*
b^2*c*d*e - 2*a*b*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^5*d^4 + 16*a^4
*c^3*e^4 + b^4*c^3*d^4 - 8*a*b^2*c^4*d^4 + a^2*b^4*c*e^4 - 2*b^5*c^2*d^3*e
+ b^6*c*d^2*e^2 - 8*a^3*b^2*c^2*e^4 + 32*a^3*c^4*d^2*e^2 - 2*a*b^5*c*d*e^3
+ 16*a*b^3*c^3*d^3*e - 32*a^2*b*c^4*d^3*e - 32*a^3*b*c^3*d*e^3 - 6*a*b^4*c^
2*d^2*e^2 + 16*a^2*b^3*c^2*d*e^3)))^(1/2)*(x*(-(b^5*d^2 + a^2*b^3*e^2 + a^2

e^2(-(4*a*c - b^2)^3)^(1/2) + b^2*d^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b *c^2*d^2 - 2*a*b^4*d*e - 7*a*b^3*c*d^2 - a*c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a^3*b*c*e^2 - 16*a^3*c^2*d*e + 12*a^2*b^2*c*d*e - 2*a*b*d*e*(-(4*a*c - b ^2)^3)^(1/2))/(8*(16*a^2*c^5*d^4 + 16*a^4*c^3*e^4 + b^4*c^3*d^4 - 8*a*b^2*c ^4*d^4 + a^2*b^4*c*e^4 - 2*b^5*c^2*d^3*e + b^6*c*d^2*e^2 - 8*a^3*b^2*c^2*e^ 4 + 32*a^3*c^4*d^2*e^2 - 2*a*b^5*c*d*e^3 + 16*a*b^3*c^3*d^3*e - 32*a^2*b*c^ 4*d^3*e - 32*a^3*b*c^3*d*e^3 - 6*a*b^4*c^2*d^2*e^2 + 16*a^2*b^3*c^2*d*e^3)))^(1/2)*(256*a^4*b^2*c^3*e^9 - 32*a^3*b^4*c^2*e^9 - 512*a^5*c^4*e^9 + 512*a ^2*c^7*d^6*e^3 + 512*a^3*c^6*d^4*e^5 - 512*a^4*c^5*d^2*e^7 - 32*b^3*c^6*d^7 *e^2 + 128*b^4*c^5*d^6*e^3 - 192*b^5*c^4*d^5*e^4 + 128*b^6*c^3*d^4*e^5 - 32 *b^7*c^2*d^3*e^6 + 512*a^2*b^2*c^5*d^4*e^5 + 288*a^2*b^3*c^4*d^3*e^6 - 192* a^2*b^4*c^3*d^2*e^7 + 384*a^3*b^2*c^4*d^2*e^7 + 128*a*b*c^7*d^7*e^2 + 640*a ^4*b*c^4*d*e^8 - 640*a*b^2*c^6*d^6*e^3 + 1056*a*b^3*c^5*d^5*e^4 - 672*a*b^4 *c^4*d^4*e^5 + 96*a*b^5*c^3*d^3*e^6 + 32*a*b^6*c^2*d^2*e^7 - 1152*a^2*b*c^6 *d^5*e^4 + 32*a^2*b^5*c^2*d*e^8 - 640*a^3*b*c^5*d^3*e^6 - 288*a^3*b^3*c^3*d *e^8) + 64*a^2*c^6*d^6*e^2 + 128*a^3*c^5*d^4*e^4 + 64*a^4*c^4*d^2*e^6 - 96* a^2*b^2*c^4*d^4*e^4 + 64*a^2*b^3*c^3*d^3*e^5 + 32*a^2*b^4*c^2*d^2*e^6 - 144 *a^3*b^2*c^3*d^2*e^6 + 64*a^4*b*c^3*d*e^7 - 16*a*b^2*c^5*d^6*e^2 + 16*a*b^3 *c^4*d^5*e^3 + 16*a*b^4*c^3*d^4*e^4 - 16*a*b^5*c^2*d^3*e^5 - 64*a^2*b*c^5*d ^5*e^3 - 16*a^3*b^3*c^2*d*e^7))*(-(b^5*d^2 + a^2*b^3*e^2 + a^2*e^2*(-(4*a*c - b^2)^3)^(1/2) + b^2*d^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*d^2 - 2* a*b^4*d*e - 7*a*b^3*c*d^2 - a*c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a^3*b*c*e^ 2 - 16*a^3*c^2*d*e + 12*a^2*b^2*c*d*e - 2*a*b*d*e*(-(4*a*c - b^2)^3)^(1/2)) /(8*(16*a^2*c^5*d^4 + 16*a^4*c^3*e^4 + b^4*c^3*d^4 - 8*a*b^2*c^4*d^4 + a^2* b^4*c*e^4 - 2*b^5*c^2*d^3*e + b^6*c*d^2*e^2 - 8*a^3*b^2*c^2*e^4 + 32*a^3*c^ 4*d^2*e^2 - 2*a*b^5*c*d*e^3 + 16*a*b^3*c^3*d^3*e - 32*a^2*b*c^4*d^3*e - 32* a^3*b*c^3*d*e^3 - 6*a*b^4*c^2*d^2*e^2 + 16*a^2*b^3*c^2*d*e^3)))^(1/2) - 16* a^2*c^4*d^5*e - 4*a^4*c^2*d*e^5 + 60*a^3*c^3*d^3*e^3 - 24*a^2*b^2*c^2*d^3*e ^3 + 4*a*b^2*c^3*d^5*e + 4*a*b^4*c*d^3*e^3 + 4*a^3*b^2*c*d*e^5 + 4*a*b^3*c^ 2*d^4*e^2 - 20*a^2*b*c^3*d^4*e^2 - 8*a^2*b^3*c*d^2*e^4 + 16*a^3*b*c^2*d^2*e ^4) + x*(2*a^4*c*e^5 + 4*a^2*c^3*d^4*e + 2*b^4*c*d^4*e - 8*a*b^2*c^2*d^4*e))*(-(b^5*d^2 + a^2*b^3*e^2 + a^2*e^2*(-(4*a*c - b^2)^3)^(1/2) + b^2*d^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*d^2 - 2*a*b^4*d*e - 7*a*b^3*c*d^2 - a* c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a^3*b*c*e^2 - 16*a^3*c^2*d*e + 12*a^2*b^ 2*c*d*e - 2*a*b*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^5*d^4 + 16*a^4*c ^3*e^4 + b^4*c^3*d^4 - 8*a*b^2*c^4*d^4 + a^2*b^4*c*e^4 - 2*b^5*c^2*d^3*e + b^6*c*d^2*e^2 - 8*a^3*b^2*c^2*e^4 + 32*a^3*c^4*d^2*e^2 - 2*a*b^5*c*d*e^3 + 16*a*b^3*c^3*d^3*e - 32*a^2*b*c^4*d^3*e - 32*a^3*b*c^3*d*e^3 - 6*a*b^4*c^2* d^2*e^2 + 16*a^2*b^3*c^2*d*e^3)))^(1/2)*1i)/(((-(b^5*d^2 + a^2*b^3*e^2 + a^ 2*e^2*(-(4*a*c - b^2)^3)^(1/2) + b^2*d^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2* b*c^2*d^2 - 2*a*b^4*d*e - 7*a*b^3*c*d^2 - a*c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a^3*b*c*e^2 - 16*a^3*c^2*d*e + 12*a^2*b^2*c*d*e - 2*a*b*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^5*d^4 + 16*a^4*c^3*e^4 + b^4*c^3*d^4 - 8*a*b^2* c^4*d^4 + a^2*b^4*c*e^4 - 2*b^5*c^2*d^3*e + b^6*c*d^2*e^2 - 8*a^3*b^2*c^2*e ^4 + 32*a^3*c^4*d^2*e^2 - 2*a*b^5*c*d*e^3 + 16*a*b^3*c^3*d^3*e - 32*a^2*b*c

^4*d^3*e - 32*a^3*b*c^3*d*e^3 - 6*a*b^4*c^2*d^2*e^2 + 16*a^2*b^3*c^2*d*e^3)
))^(1/2)*((x*(8*a^3*b^3*c*e^7 - 32*a^4*b*c^2*e^7 - 112*a^4*c^3*d*e^6 + 8*b^
3*c^4*d^6*e + 8*b^6*c*d^3*e^4 - 112*a^2*c^5*d^5*e^2 + 32*a^3*c^4*d^3*e^4 -
8*b^4*c^3*d^5*e^2 - 8*b^5*c^2*d^4*e^3 - 32*a*b*c^5*d^6*e - 48*a^2*b^2*c^3*d
^3*e^4 + 8*a^2*b^3*c^2*d^2*e^5 - 8*a*b^5*c*d^2*e^5 - 8*a^2*b^4*c*d*e^6 + 64
*a*b^2*c^4*d^5*e^2 + 8*a*b^3*c^3*d^4*e^3 - 16*a*b^4*c^2*d^3*e^4 + 64*a^2*b*
c^4*d^4*e^3 + 64*a^3*b*c^3*d^2*e^5 + 64*a^3*b^2*c^2*d*e^6) + (-(b^5*d^2 + a
^2*b^3*e^2 + a^2*e^2*(-(4*a*c - b^2)^3)^(1/2) + b^2*d^2*(-(4*a*c - b^2)^3)^
(1/2) + 12*a^2*b*c^2*d^2 - 2*a*b^4*d*e - 7*a*b^3*c*d^2 - a*c*d^2*(-(4*a*c -
 b^2)^3)^(1/2) - 4*a^3*b*c*e^2 - 16*a^3*c^2*d*e + 12*a^2*b^2*c*d*e - 2*a*b*
d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^5*d^4 + 16*a^4*c^3*e^4 + b^4*c^3
*d^4 - 8*a*b^2*c^4*d^4 + a^2*b^4*c*e^4 - 2*b^5*c^2*d^3*e + b^6*c*d^2*e^2 -
8*a^3*b^2*c^2*e^4 + 32*a^3*c^4*d^2*e^2 - 2*a*b^5*c*d*e^3 + 16*a*b^3*c^3*d^3
*e - 32*a^2*b*c^4*d^3*e - 32*a^3*b*c^3*d*e^3 - 6*a*b^4*c^2*d^2*e^2 + 16*a^2
*b^3*c^2*d*e^3)))^(1/2)*(64*a^2*c^6*d^6*e^2 - x*(-(b^5*d^2 + a^2*b^3*e^2 +
a^2*e^2*(-(4*a*c - b^2)^3)^(1/2) + b^2*d^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^
2*b*c^2*d^2 - 2*a*b^4*d*e - 7*a*b^3*c*d^2 - a*c*d^2*(-(4*a*c - b^2)^3)^(1/2
) - 4*a^3*b*c*e^2 - 16*a^3*c^2*d*e + 12*a^2*b^2*c*d*e - 2*a*b*d*e*(-(4*a*c
- b^2)^3)^(1/2))/(8*(16*a^2*c^5*d^4 + 16*a^4*c^3*e^4 + b^4*c^3*d^4 - 8*a*b^
2*c^4*d^4 + a^2*b^4*c*e^4 - 2*b^5*c^2*d^3*e + b^6*c*d^2*e^2 - 8*a^3*b^2*c^2
*e^4 + 32*a^3*c^4*d^2*e^2 - 2*a*b^5*c*d*e^3 + 16*a*b^3*c^3*d^3*e - 32*a^2*b
*c^4*d^3*e - 32*a^3*b*c^3*d*e^3 - 6*a*b^4*c^2*d^2*e^2 + 16*a^2*b^3*c^2*d*e^
3)))^(1/2)*(256*a^4*b^2*c^3*e^9 - 32*a^3*b^4*c^2*e^9 - 512*a^5*c^4*e^9 + 51
2*a^2*c^7*d^6*e^3 + 512*a^3*c^6*d^4*e^5 - 512*a^4*c^5*d^2*e^7 - 32*b^3*c^6*
d^7*e^2 + 128*b^4*c^5*d^6*e^3 - 192*b^5*c^4*d^5*e^4 + 128*b^6*c^3*d^4*e^5 -
 32*b^7*c^2*d^3*e^6 + 512*a^2*b^2*c^5*d^4*e^5 + 288*a^2*b^3*c^4*d^3*e^6 - 1
92*a^2*b^4*c^3*d^2*e^7 + 384*a^3*b^2*c^4*d^2*e^7 + 128*a*b*c^7*d^7*e^2 + 64
0*a^4*b*c^4*d*e^8 - 640*a*b^2*c^6*d^6*e^3 + 1056*a*b^3*c^5*d^5*e^4 - 672*a*
b^4*c^4*d^4*e^5 + 96*a*b^5*c^3*d^3*e^6 + 32*a*b^6*c^2*d^2*e^7 - 1152*a^2*b*
c^6*d^5*e^4 + 32*a^2*b^5*c^2*d*e^8 - 640*a^3*b*c^5*d^3*e^6 - 288*a^3*b^3*c^
3*d*e^8) + 128*a^3*c^5*d^4*e^4 + 64*a^4*c^4*d^2*e^6 - 96*a^2*b^2*c^4*d^4*e^
4 + 64*a^2*b^3*c^3*d^3*e^5 + 32*a^2*b^4*c^2*d^2*e^6 - 144*a^3*b^2*c^3*d^2*e
^6 + 64*a^4*b*c^3*d*e^7 - 16*a*b^2*c^5*d^6*e^2 + 16*a*b^3*c^4*d^5*e^3 + 16*
a*b^4*c^3*d^4*e^4 - 16*a*b^5*c^2*d^3*e^5 - 64*a^2*b*c^5*d^5*e^3 - 16*a^3*b^
3*c^2*d*e^7))*(-(b^5*d^2 + a^2*b^3*e^2 + a^2*e^2*(-(4*a*c - b^2)^3)^(1/2) +
 b^2*d^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*d^2 - 2*a*b^4*d*e - 7*a*b^
3*c*d^2 - a*c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a^3*b*c*e^2 - 16*a^3*c^2*d*e
 + 12*a^2*b^2*c*d*e - 2*a*b*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^5*d^
4 + 16*a^4*c^3*e^4 + b^4*c^3*d^4 - 8*a*b^2*c^4*d^4 + a^2*b^4*c*e^4 - 2*b^5*
c^2*d^3*e + b^6*c*d^2*e^2 - 8*a^3*b^2*c^2*e^4 + 32*a^3*c^4*d^2*e^2 - 2*a*b^
5*c*d*e^3 + 16*a*b^3*c^3*d^3*e - 32*a^2*b*c^4*d^3*e - 32*a^3*b*c^3*d*e^3 -
6*a*b^4*c^2*d^2*e^2 + 16*a^2*b^3*c^2*d*e^3)))^(1/2) + 16*a^2*c^4*d^5*e + 4*
a^4*c^2*d*e^5 - 60*a^3*c^3*d^3*e^3 + 24*a^2*b^2*c^2*d^3*e^3 - 4*a*b^2*c^3*d
^5*e - 4*a*b^4*c*d^3*e^3 - 4*a^3*b^2*c*d*e^5 - 4*a*b^3*c^2*d^4*e^2 + 20*a^2
*b*c^3*d^4*e^2 + 8*a^2*b^3*c*d^2*e^4 - 16*a^3*b*c^2*d^2*e^4) + x*(2*a^4*c*e

^5 + 4*a^2*c^3*d^4*e + 2*b^4*c*d^4*e - 8*a*b^2*c^2*d^4*e))*(-(b^5*d^2 + a^2
*b^3*e^2 + a^2*e^2*(-(4*a*c - b^2)^3)^(1/2) + b^2*d^2*(-(4*a*c - b^2)^3)^(1
/2) + 12*a^2*b*c^2*d^2 - 2*a*b^4*d*e - 7*a*b^3*c*d^2 - a*c*d^2*(-(4*a*c - b
^2)^3)^(1/2) - 4*a^3*b*c*e^2 - 16*a^3*c^2*d*e + 12*a^2*b^2*c*d*e - 2*a*b*d*
e*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^5*d^4 + 16*a^4*c^3*e^4 + b^4*c^3*d
^4 - 8*a*b^2*c^4*d^4 + a^2*b^4*c*e^4 - 2*b^5*c^2*d^3*e + b^6*c*d^2*e^2 - 8*
a^3*b^2*c^2*e^4 + 32*a^3*c^4*d^2*e^2 - 2*a*b^5*c*d*e^3 + 16*a*b^3*c^3*d^3*e
 - 32*a^2*b*c^4*d^3*e - 32*a^3*b*c^3*d*e^3 - 6*a*b^4*c^2*d^2*e^2 + 16*a^2*b
^3*c^2*d*e^3)))^(1/2) - ((-(b^5*d^2 + a^2*b^3*e^2 + a^2*e^2*(-(4*a*c - b^2)
^3)^(1/2) + b^2*d^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*d^2 - 2*a*b^4*d
*e - 7*a*b^3*c*d^2 - a*c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a^3*b*c*e^2 - 16*
a^3*c^2*d*e + 12*a^2*b^2*c*d*e - 2*a*b*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(16
*a^2*c^5*d^4 + 16*a^4*c^3*e^4 + b^4*c^3*d^4 - 8*a*b^2*c^4*d^4 + a^2*b^4*c*e
^4 - 2*b^5*c^2*d^3*e + b^6*c*d^2*e^2 - 8*a^3*b^2*c^2*e^4 + 32*a^3*c^4*d^2*e
^2 - 2*a*b^5*c*d*e^3 + 16*a*b^3*c^3*d^3*e - 32*a^2*b*c^4*d^3*e - 32*a^3*b*c
^3*d*e^3 - 6*a*b^4*c^2*d^2*e^2 + 16*a^2*b^3*c^2*d*e^3)))^(1/2)*((x*(8*a^3*b
^3*c*e^7 - 32*a^4*b*c^2*e^7 - 112*a^4*c^3*d*e^6 + 8*b^3*c^4*d^6*e + 8*b^6*c
*d^3*e^4 - 112*a^2*c^5*d^5*e^2 + 32*a^3*c^4*d^3*e^4 - 8*b^4*c^3*d^5*e^2 - 8
*b^5*c^2*d^4*e^3 - 32*a*b*c^5*d^6*e - 48*a^2*b^2*c^3*d^3*e^4 + 8*a^2*b^3*c^
2*d^2*e^5 - 8*a*b^5*c*d^2*e^5 - 8*a^2*b^4*c*d*e^6 + 64*a*b^2*c^4*d^5*e^2 +
8*a*b^3*c^3*d^4*e^3 - 16*a*b^4*c^2*d^3*e^4 + 64*a^2*b*c^4*d^4*e^3 + 64*a^3*
b*c^3*d^2*e^5 + 64*a^3*b^2*c^2*d*e^6) - (-(b^5*d^2 + a^2*b^3*e^2 + a^2*e^2*
(-(4*a*c - b^2)^3)^(1/2) + b^2*d^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*
d^2 - 2*a*b^4*d*e - 7*a*b^3*c*d^2 - a*c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a^
3*b*c*e^2 - 16*a^3*c^2*d*e + 12*a^2*b^2*c*d*e - 2*a*b*d*e*(-(4*a*c - b^2)^3
)^(1/2))/(8*(16*a^2*c^5*d^4 + 16*a^4*c^3*e^4 + b^4*c^3*d^4 - 8*a*b^2*c^4*d^
4 + a^2*b^4*c*e^4 - 2*b^5*c^2*d^3*e + b^6*c*d^2*e^2 - 8*a^3*b^2*c^2*e^4 + 3
2*a^3*c^4*d^2*e^2 - 2*a*b^5*c*d*e^3 + 16*a*b^3*c^3*d^3*e - 32*a^2*b*c^4*d^3
*e - 32*a^3*b*c^3*d*e^3 - 6*a*b^4*c^2*d^2*e^2 + 16*a^2*b^3*c^2*d*e^3)))^(1/
2)*(x*(-(b^5*d^2 + a^2*b^3*e^2 + a^2*e^2*(-(4*a*c - b^2)^3)^(1/2) + b^2*d^2
*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*d^2 - 2*a*b^4*d*e - 7*a*b^3*c*d^2
- a*c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a^3*b*c*e^2 - 16*a^3*c^2*d*e + 12*a^
2*b^2*c*d*e - 2*a*b*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^5*d^4 + 16*a
^4*c^3*e^4 + b^4*c^3*d^4 - 8*a*b^2*c^4*d^4 + a^2*b^4*c*e^4 - 2*b^5*c^2*d^3*
e + b^6*c*d^2*e^2 - 8*a^3*b^2*c^2*e^4 + 32*a^3*c^4*d^2*e^2 - 2*a*b^5*c*d*e^
3 + 16*a*b^3*c^3*d^3*e - 32*a^2*b*c^4*d^3*e - 32*a^3*b*c^3*d*e^3 - 6*a*b^4*
c^2*d^2*e^2 + 16*a^2*b^3*c^2*d*e^3)))^(1/2)*(256*a^4*b^2*c^3*e^9 - 32*a^3*b
^4*c^2*e^9 - 512*a^5*c^4*e^9 + 512*a^2*c^7*d^6*e^3 + 512*a^3*c^6*d^4*e^5 -
512*a^4*c^5*d^2*e^7 - 32*b^3*c^6*d^7*e^2 + 128*b^4*c^5*d^6*e^3 - 192*b^5*c^
4*d^5*e^4 + 128*b^6*c^3*d^4*e^5 - 32*b^7*c^2*d^3*e^6 + 512*a^2*b^2*c^5*d^4*
e^5 + 288*a^2*b^3*c^4*d^3*e^6 - 192*a^2*b^4*c^3*d^2*e^7 + 384*a^3*b^2*c^4*d
^2*e^7 + 128*a*b*c^7*d^7*e^2 + 640*a^4*b*c^4*d*e^8 - 640*a*b^2*c^6*d^6*e^3
+ 1056*a*b^3*c^5*d^5*e^4 - 672*a*b^4*c^4*d^4*e^5 + 96*a*b^5*c^3*d^3*e^6 + 3
2*a*b^6*c^2*d^2*e^7 - 1152*a^2*b*c^6*d^5*e^4 + 32*a^2*b^5*c^2*d*e^8 - 640*a
^3*b*c^5*d^3*e^6 - 288*a^3*b^3*c^3*d*e^8) + 64*a^2*c^6*d^6*e^2 + 128*a^3*c^

5*d^4*e^4 + 64*a^4*c^4*d^2*e^6 - 96*a^2*b^2*c^4*d^4*e^4 + 64*a^2*b^3*c^3*d^3*e^5 + 32*a^2*b^4*c^2*d^2*e^6 - 144*a^3*b^2*c^3*d^2*e^6 + 64*a^4*b*c^3*d*e^7 - 16*a*b^2*c^5*d^6*e^2 + 16*a*b^3*c^4*d^5*e^3 + 16*a*b^4*c^3*d^4*e^4 - 16*a*b^5*c^2*d^3*e^5 - 64*a^2*b*c^5*d^5*e^3 - 16*a^3*b^3*c^2*d*e^7))*(-(b^5*d^2 + a^2*b^3*e^2 + a^2*e^2*(-(4*a*c - b^2)^3)^(1/2) + b^2*d^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*d^2 - 2*a*b^4*d*e - 7*a*b^3*c*d^2 - a*c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a^3*b*c*e^2 - 16*a^3*c^2*d*e + 12*a^2*b^2*c*d*e - 2*a*b*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^5*d^4 + 16*a^4*c^3*e^4 + b^4*c^3*d^4 - 8*a*b^2*c^4*d^4 + a^2*b^4*c*e^4 - 2*b^5*c^2*d^3*e + b^6*c*d^2*e^2 - 8*a^3*b^2*c^2*e^4 + 32*a^3*c^4*d^2*e^2 - 2*a*b^5*c*d*e^3 + 16*a*b^3*c^3*d^3*e - 32*a^2*b*c^4*d^3*e - 32*a^3*b*c^3*d*e^3 - 6*a*b^4*c^2*d^2*e^2 + 16*a^2*b^3*c^2*d*e^3)))^(1/2) - 16*a^2*c^4*d^5*e - 4*a^4*c^2*d*e^5 + 60*a^3*c^3*d^3*e^3 - 24*a^2*b^2*c^2*d^3*e^3 + 4*a*b^2*c^3*d^5*e + 4*a*b^4*c*d^3*e^3 + 4*a^3*b^2*c*d*e^5 + 4*a*b^3*c^2*d^4*e^2 - 20*a^2*b*c^3*d^4*e^2 - 8*a^2*b^3*c*d^2*e^4 + 16*a^3*b*c^2*d^2*e^4) + x*(2*a^4*c*e^5 + 4*a^2*c^3*d^4*e + 2*b^4*c*d^4*e - 8*a*b^2*c^2*d^4*e))*(-(b^5*d^2 + a^2*b^3*e^2 + a^2*e^2*(-(4*a*c - b^2)^3)^(1/2) + b^2*d^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*d^2 - 2*a*b^4*d*e - 7*a*b^3*c*d^2 - a*c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a^3*b*c*e^2 - 16*a^3*c^2*d*e + 12*a^2*b^2*c*d*e - 2*a*b*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^5*d^4 + 16*a^4*c^3*e^4 + b^4*c^3*d^4 - 8*a*b^2*c^4*d^4 + a^2*b^4*c*e^4 - 2*b^5*c^2*d^3*e + b^6*c*d^2*e^2 - 8*a^3*b^2*c^2*e^4 + 32*a^3*c^4*d^2*e^2 - 2*a*b^5*c*d*e^3 + 16*a*b^3*c^3*d^3*e - 32*a^2*b*c^4*d^3*e - 32*a^3*b*c^3*d*e^3 - 6*a*b^4*c^2*d^2*e^2 + 16*a^2*b^3*c^2*d*e^3)))^(1/2) + 2*a^3*c*d^2*e^2 + 2*a^2*b*c*d^3*e))*(-(b^5*d^2 + a^2*b^3*e^2 + a^2*e^2*(-(4*a*c - b^2)^3)^(1/2) + b^2*d^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*d^2 - 2*a*b^4*d*e - 7*a*b^3*c*d^2 - a*c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a^3*b*c*e^2 - 16*a^3*c^2*d*e + 12*a^2*b^2*c*d*e - 2*a*b*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^5*d^4 + 16*a^4*c^3*e^4 + b^4*c^3*d^4 - 8*a*b^2*c^4*d^4 + a^2*b^4*c*e^4 - 2*b^5*c^2*d^3*e + b^6*c*d^2*e^2 - 8*a^3*b^2*c^2*e^4 + 32*a^3*c^4*d^2*e^2 - 2*a*b^5*c*d*e^3 + 16*a*b^3*c^3*d^3*e - 32*a^2*b*c^4*d^3*e - 32*a^3*b*c^3*d*e^3 - 6*a*b^4*c^2*d^2*e^2 + 16*a^2*b^3*c^2*d*e^3)))^(1/2)*2i + atan((((-(b^5*d^2 + a^2*b^3*e^2 - a^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - b^2*d^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*d^2 - 2*a*b^4*d*e - 7*a*b^3*c*d^2 + a*c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a^3*b*c*e^2 - 16*a^3*c^2*d*e + 12*a^2*b^2*c*d*e + 2*a*b*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^5*d^4 + 16*a^4*c^3*e^4 + b^4*c^3*d^4 - 8*a*b^2*c^4*d^4 + a^2*b^4*c*e^4 - 2*b^5*c^2*d^3*e + b^6*c*d^2*e^2 - 8*a^3*b^2*c^2*e^4 + 32*a^3*c^4*d^2*e^2 - 2*a*b^5*c*d*e^3 + 16*a*b^3*c^3*d^3*e - 32*a^2*b*c^4*d^3*e - 32*a^3*b*c^3*d*e^3 - 6*a*b^4*c^2*d^2*e^2 + 16*a^2*b^3*c^2*d*e^3)))^(1/2)*((x*(8*a^3*b^3*c*e^7 - 32*a^4*b*c^2*e^7 - 112*a^4*c^3*d*e^6 + 8*b^3*c^4*d^6*e + 8*b^6*c*d^3*e^4 - 112*a^2*c^5*d^5*e^2 + 32*a^3*c^4*d^3*e^4 - 8*b^4*c^3*d^5*e^2 - 8*b^5*c^2*d^4*e^3 - 32*a*b*c^5*d^6*e - 48*a^2*b^2*c^3*d^3*e^4 + 8*a^2*b^3*c^2*d^2*e^5 - 8*a*b^5*c*d^2*e^5 - 8*a^2*b^4*c*d*e^6 + 64*a*b^2*c^4*d^5*e^2 + 8*a*b^3*c^3*d^4*e^3 - 16*a*b^4*c^2*d^3*e^4 + 64*a^2*b*c^4*d^4*e^3 + 64*a^3*b*c^3*d^2*e^5 + 64*a^3*b^2*c^2*d*e^6) + (-(b^5*d^2 + a^2*b^3*e^2 - a^2*e^2*(-(4*a*c -

b^2)^3)^(1/2) - b^2*d^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*d^2 - 2*a*b^4*d*e - 7*a*b^3*c*d^2 + a*c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a^3*b*c*e^2 - 16*a^3*c^2*d*e + 12*a^2*b^2*c*d*e + 2*a*b*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^5*d^4 + 16*a^4*c^3*e^4 + b^4*c^3*d^4 - 8*a*b^2*c^4*d^4 + a^2*b^4*c*e^4 - 2*b^5*c^2*d^3*e + b^6*c*d^2*e^2 - 8*a^3*b^2*c^2*e^4 + 32*a^3*c^4*d^2*e^2 - 2*a*b^5*c*d*e^3 + 16*a*b^3*c^3*d^3*e - 32*a^2*b*c^4*d^3*e - 32*a^3*b*c^3*d*e^3 - 6*a*b^4*c^2*d^2*e^2 + 16*a^2*b^3*c^2*d*e^3)))^(1/2)*(64*a^2*c^6*d^6*e^2 - x*(-(b^5*d^2 + a^2*b^3*e^2 - a^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - b^2*d^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*d^2 - 2*a*b^4*d*e - 7*a*b^3*c*d^2 + a*c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a^3*b*c*e^2 - 16*a^3*c^2*d*e + 12*a^2*b^2*c*d*e + 2*a*b*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^5*d^4 + 16*a^4*c^3*e^4 + b^4*c^3*d^4 - 8*a*b^2*c^4*d^4 + a^2*b^4*c*e^4 - 2*b^5*c^2*d^3*e + b^6*c*d^2*e^2 - 8*a^3*b^2*c^2*e^4 + 32*a^3*c^4*d^2*e^2 - 2*a*b^5*c*d*e^3 + 16*a*b^3*c^3*d^3*e - 32*a^2*b*c^4*d^3*e - 32*a^3*b*c^3*d*e^3 - 6*a*b^4*c^2*d^2*e^2 + 16*a^2*b^3*c^2*d*e^3)))^(1/2)*(256*a^4*b^2*c^3*e^9 - 32*a^3*b^4*c^2*e^9 - 512*a^5*c^4*e^9 + 512*a^2*c^7*d^6*e^3 + 512*a^3*c^6*d^4*e^5 - 512*a^4*c^5*d^2*e^7 - 32*b^3*c^6*d^7*e^2 + 128*b^4*c^5*d^6*e^3 - 192*b^5*c^4*d^5*e^4 + 128*b^6*c^3*d^4*e^5 - 32*b^7*c^2*d^3*e^6 + 512*a^2*b^2*c^5*d^4*e^5 + 288*a^2*b^3*c^4*d^3*e^6 - 192*a^2*b^4*c^3*d^2*e^7 + 384*a^3*b^2*c^4*d^2*e^7 + 128*a*b*c^7*d^7*e^2 + 640*a^4*b*c^4*d*e^8 - 640*a*b^2*c^6*d^6*e^3 + 1056*a*b^3*c^5*d^5*e^4 - 672*a*b^4*c^4*d^4*e^5 + 96*a*b^5*c^3*d^3*e^6 + 32*a*b^6*c^2*d^2*e^7 - 1152*a^2*b*c^6*d^5*e^4 + 32*a^2*b^5*c^2*d*e^8 - 640*a^3*b*c^5*d^3*e^6 - 288*a^3*b^3*c^3*d*e^8) + 128*a^3*c^5*d^4*e^4 + 64*a^4*c^4*d^2*e^6 - 96*a^2*b^2*c^4*d^4*e^4 + 64*a^2*b^3*c^3*d^3*e^5 + 32*a^2*b^4*c^2*d^2*e^6 - 144*a^3*b^2*c^3*d^2*e^6 + 64*a^4*b*c^3*d*e^7 - 16*a*b^2*c^5*d^6*e^2 + 16*a*b^3*c^4*d^5*e^3 + 16*a*b^4*c^3*d^4*e^4 - 16*a*b^5*c^2*d^3*e^5 - 64*a^2*b*c^5*d^5*e^3 - 16*a^3*b^3*c^2*d*e^7))*(-(b^5*d^2 + a^2*b^3*e^2 - a^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - b^2*d^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*d^2 - 2*a*b^4*d*e - 7*a*b^3*c*d^2 + a*c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a^3*b*c*e^2 - 16*a^3*c^2*d*e + 12*a^2*b^2*c*d*e + 2*a*b*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^5*d^4 + 16*a^4*c^3*e^4 + b^4*c^3*d^4 - 8*a*b^2*c^4*d^4 + a^2*b^4*c*e^4 - 2*b^5*c^2*d^3*e + b^6*c*d^2*e^2 - 8*a^3*b^2*c^2*e^4 + 32*a^3*c^4*d^2*e^2 - 2*a*b^5*c*d*e^3 + 16*a*b^3*c^3*d^3*e - 32*a^2*b*c^4*d^3*e - 32*a^3*b*c^3*d*e^3 - 6*a*b^4*c^2*d^2*e^2 + 16*a^2*b^3*c^2*d*e^3)))^(1/2) + 16*a^2*c^4*d^5*e + 4*a^4*c^2*d*e^5 - 60*a^3*c^3*d^3*e^3 + 24*a^2*b^2*c^2*d^3*e^3 - 4*a*b^2*c^3*d^5*e - 4*a*b^4*c*d^3*e^3 - 4*a^3*b^2*c*d*e^5 - 4*a*b^3*c^2*d^4*e^2 + 20*a^2*b*c^3*d^4*e^2 + 8*a^2*b^3*c*d^2*e^4 - 16*a^3*b*c^2*d^2*e^4) + x*(2*a^4*c*e^5 + 4*a^2*c^3*d^4*e + 2*b^4*c*d^4*e - 8*a*b^2*c^2*d^4*e))*(-(b^5*d^2 + a^2*b^3*e^2 - a^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - b^2*d^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*d^2 - 2*a*b^4*d*e - 7*a*b^3*c*d^2 + a*c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a^3*b*c*e^2 - 16*a^3*c^2*d*e + 12*a^2*b^2*c*d*e + 2*a*b*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^5*d^4 + 16*a^4*c^3*e^4 + b^4*c^3*d^4 - 8*a*b^2*c^4*d^4 + a^2*b^4*c*e^4 - 2*b^5*c^2*d^3*e + b^6*c*d^2*e^2 - 8*a^3*b^2*c^2*e^4 + 32*a^3*c^4*d^2*e^2 - 2*a*b^5*c*d*e^3 + 16*a*b^3*c^3*d^3*e - 32*a^2*b*c^4*d^3*e - 32*a^3*b *c^3*d*e^3 - 6*a*b^4*c^2*d^2*e^2 + 16*a^2*b^3*c^2*d*e^3)))^(1/2)*1i + ((-(b^5*d^2 + a^2*b^3*e^2 - a^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - b^2*d^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*d^2 - 2*a*b^4*d*e - 7*a*b^3*c*d^2 + a*c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a^3*b*c*e^2 - 16*a^3*c^2*d*e + 12*a^2*b^2*c*d*e + 2*a*b*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^5*d^4 + 16*a^4*c^3*e^4 + b^4*c^3*d^4 - 8*a*b^2*c^4*d^4 + a^2*b^4*c*e^4 - 2*b^5*c^2*d^3*e + b^6*c*d^2*e^2 - 8*a^3*b^2*c^2*e^4 + 32*a^3*c^4*d^2*e^2 - 2*a*b^5*c*d*e^3 + 16*a*b^3*c^3*d^3*e - 32*a^2*b*c^4*d^3*e - 32*a^3*b*c^3*d*e^3 - 6*a*b^4*c^2*d^2*e^2 + 16*a^2*b^3*c^2*d*e^3)))^(1/2)*((x*(8*a^3*b^3*c*e^7 - 32*a^4*b*c^2*e^7 - 112*a^4*c^3*d*e^6 + 8*b^3*c^4*d^6*e + 8*b^6*c*d^3*e^4 - 112*a^2*c^5*d^5*e^2 + 32*a^3*c^4*d^3*e^4 - 8*b^4*c^3*d^5*e^2 - 8*b^5*c^2*d^4*e^3 - 32*a*b*c^5*d^6*e - 48*a^2*b^2*c^3*d^3*e^4 + 8*a^2*b^3*c^2*d^2*e^5 - 8*a*b^5*c*d^2*e^5 - 8*a^2*b^4*c*d*e^6 + 64*a*b^2*c^4*d^5*e^2 + 8*a*b^3*c^3*d^4*e^3 - 16*a*b^4*c^2*d^3*e^4 + 64*a^2*b*c^4*d^4*e^3 + 64*a^3*b*c^3*d^2*e^5 + 64*a^3*b^2*c^2*d*e^6) - (-(b^5*d^2 + a^2*b^3*e^2 - a^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - b^2*d^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*d^2 - 2*a*b^4*d*e - 7*a*b^3*c*d^2 + a*c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a^3*b*c*e^2 - 16*a^3*c^2*d*e + 12*a^2*b^2*c*d*e + 2*a*b*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^5*d^4 + 16*a^4*c^3*e^4 + b^4*c^3*d^4 - 8*a*b^2*c^4*d^4 + a^2*b^4*c*e^4 - 2*b^5*c^2*d^3*e + b^6*c*d^2*e^2 - 8*a^3*b^2*c^2*e^4 + 32*a^3*c^4*d^2*e^2 - 2*a*b^5*c*d*e^3 + 16*a*b^3*c^3*d^3*e - 32*a^2*b*c^4*d^3*e - 32*a^3*b*c^3*d*e^3 - 6*a*b^4*c^2*d^2*e^2 + 16*a^2*b^3*c^2*d*e^3)))^(1/2)*(x*(-(b^5*d^2 + a^2*b^3*e^2 - a^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - b^2*d^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*d^2 - 2*a*b^4*d*e - 7*a*b^3*c*d^2 + a*c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a^3*b*c*e^2 - 16*a^3*c^2*d*e + 12*a^2*b^2*c*d*e + 2*a*b*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^5*d^4 + 16*a^4*c^3*e^4 + b^4*c^3*d^4 - 8*a*b^2*c^4*d^4 + a^2*b^4*c*e^4 - 2*b^5*c^2*d^3*e + b^6*c*d^2*e^2 - 8*a^3*b^2*c^2*e^4 + 32*a^3*c^4*d^2*e^2 - 2*a*b^5*c*d*e^3 + 16*a*b^3*c^3*d^3*e - 32*a^2*b*c^4*d^3*e - 32*a^3*b*c^3*d*e^3 - 6*a*b^4*c^2*d^2*e^2 + 16*a^2*b^3*c^2*d*e^3)))^(1/2)*(256*a^4*b^2*c^3*e^9 - 32*a^3*b^4*c^2*e^9 - 512*a^5*c^4*e^9 + 512*a^2*c^7*d^6*e^3 + 512*a^3*c^6*d^4*e^5 - 512*a^4*c^5*d^2*e^7 - 32*b^3*c^6*d^7*e^2 + 128*b^4*c^5*d^6*e^3 - 192*b^5*c^4*d^5*e^4 + 128*b^6*c^3*d^4*e^5 - 32*b^7*c^2*d^3*e^6 + 512*a^2*b^2*c^5*d^4*e^5 + 288*a^2*b^3*c^4*d^3*e^6 - 192*a^2*b^4*c^3*d^2*e^7 + 384*a^3*b^2*c^4*d^2*e^7 + 128*a*b*c^7*d^7*e^2 + 640*a^4*b*c^4*d*e^8 - 640*a*b^2*c^6*d^6*e^3 + 1056*a*b^3*c^5*d^5*e^4 - 672*a*b^4*c^4*d^4*e^5 + 96*a*b^5*c^3*d^3*e^6 + 32*a*b^6*c^2*d^2*e^7 - 1152*a^2*b*c^6*d^5*e^4 + 32*a^2*b^5*c^2*d*e^8 - 640*a^3*b*c^5*d^3*e^6 - 288*a^3*b^3*c^3*d*e^8) + 64*a^2*c^6*d^6*e^2 + 128*a^3*c^5*d^4*e^4 + 64*a^4*c^4*d^2*e^6 - 96*a^2*b^2*c^4*d^4*e^4 + 64*a^2*b^3*c^3*d^3*e^5 + 32*a^2*b^4*c^2*d^2*e^6 - 144*a^3*b^2*c^3*d^2*e^6 + 64*a^4*b*c^3*d*e^7 - 16*a*b^2*c^5*d^6*e^2 + 16*a*b^3*c^4*d^5*e^3 + 16*a*b^4*c^3*d^4*e^4 - 16*a*b^5*c^2*d^3*e^5 - 64*a^2*b*c^5*d^5*e^3 - 16*a^3*b^3*c^2*d*e^7))*(-(b^5*d^2 + a^2*b^3*e^2 - a^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - b^2*d^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*d^2 - 2*a*b^4*d*e - 7*a*b^3*c*d^2 + a*c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a^3*b*c*e^2 - 16*a^3*c^2*d*e + 12*a^2*b^2*c*d*e + 2*a*b*d*e*(-(4*a*c - b^2)^3)

^(1/2))/(8*(16*a^2*c^5*d^4 + 16*a^4*c^3*e^4 + b^4*c^3*d^4 - 8*a*b^2*c^4*d^4
 + a^2*b^4*c*e^4 - 2*b^5*c^2*d^3*e + b^6*c*d^2*e^2 - 8*a^3*b^2*c^2*e^4 + 32
*a^3*c^4*d^2*e^2 - 2*a*b^5*c*d*e^3 + 16*a*b^3*c^3*d^3*e - 32*a^2*b*c^4*d^3*
e - 32*a^3*b*c^3*d*e^3 - 6*a*b^4*c^2*d^2*e^2 + 16*a^2*b^3*c^2*d*e^3)))^(1/2
) - 16*a^2*c^4*d^5*e - 4*a^4*c^2*d*e^5 + 60*a^3*c^3*d^3*e^3 - 24*a^2*b^2*c^
2*d^3*e^3 + 4*a*b^2*c^3*d^5*e + 4*a*b^4*c*d^3*e^3 + 4*a^3*b^2*c*d*e^5 + 4*a
*b^3*c^2*d^4*e^2 - 20*a^2*b*c^3*d^4*e^2 - 8*a^2*b^3*c*d^2*e^4 + 16*a^3*b*c^
2*d^2*e^4) + x*(2*a^4*c*e^5 + 4*a^2*c^3*d^4*e + 2*b^4*c*d^4*e - 8*a*b^2*c^2
*d^4*e))*(-(b^5*d^2 + a^2*b^3*e^2 - a^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - b^2*
d^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*d^2 - 2*a*b^4*d*e - 7*a*b^3*c*d
^2 + a*c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a^3*b*c*e^2 - 16*a^3*c^2*d*e + 12
*a^2*b^2*c*d*e + 2*a*b*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^5*d^4 + 1
6*a^4*c^3*e^4 + b^4*c^3*d^4 - 8*a*b^2*c^4*d^4 + a^2*b^4*c*e^4 - 2*b^5*c^2*d
^3*e + b^6*c*d^2*e^2 - 8*a^3*b^2*c^2*e^4 + 32*a^3*c^4*d^2*e^2 - 2*a*b^5*c*d
*e^3 + 16*a*b^3*c^3*d^3*e - 32*a^2*b*c^4*d^3*e - 32*a^3*b*c^3*d*e^3 - 6*a*b
^4*c^2*d^2*e^2 + 16*a^2*b^3*c^2*d*e^3)))^(1/2)*1i)/(((-(b^5*d^2 + a^2*b^3*e
^2 - a^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - b^2*d^2*(-(4*a*c - b^2)^3)^(1/2) +
12*a^2*b*c^2*d^2 - 2*a*b^4*d*e - 7*a*b^3*c*d^2 + a*c*d^2*(-(4*a*c - b^2)^3)
^(1/2) - 4*a^3*b*c*e^2 - 16*a^3*c^2*d*e + 12*a^2*b^2*c*d*e + 2*a*b*d*e*(-(4
*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^5*d^4 + 16*a^4*c^3*e^4 + b^4*c^3*d^4 - 8
*a*b^2*c^4*d^4 + a^2*b^4*c*e^4 - 2*b^5*c^2*d^3*e + b^6*c*d^2*e^2 - 8*a^3*b^
2*c^2*e^4 + 32*a^3*c^4*d^2*e^2 - 2*a*b^5*c*d*e^3 + 16*a*b^3*c^3*d^3*e - 32*
a^2*b*c^4*d^3*e - 32*a^3*b*c^3*d*e^3 - 6*a*b^4*c^2*d^2*e^2 + 16*a^2*b^3*c^2
*d*e^3)))^(1/2)*((x*(8*a^3*b^3*c*e^7 - 32*a^4*b*c^2*e^7 - 112*a^4*c^3*d*e^6
 + 8*b^3*c^4*d^6*e + 8*b^6*c*d^3*e^4 - 112*a^2*c^5*d^5*e^2 + 32*a^3*c^4*d^3
*e^4 - 8*b^4*c^3*d^5*e^2 - 8*b^5*c^2*d^4*e^3 - 32*a*b*c^5*d^6*e - 48*a^2*b^
2*c^3*d^3*e^4 + 8*a^2*b^3*c^2*d^2*e^5 - 8*a*b^5*c*d^2*e^5 - 8*a^2*b^4*c*d*e
^6 + 64*a*b^2*c^4*d^5*e^2 + 8*a*b^3*c^3*d^4*e^3 - 16*a*b^4*c^2*d^3*e^4 + 64
*a^2*b*c^4*d^4*e^3 + 64*a^3*b*c^3*d^2*e^5 + 64*a^3*b^2*c^2*d*e^6) + (-(b^5*
d^2 + a^2*b^3*e^2 - a^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - b^2*d^2*(-(4*a*c - b
^2)^3)^(1/2) + 12*a^2*b*c^2*d^2 - 2*a*b^4*d*e - 7*a*b^3*c*d^2 + a*c*d^2*(-(
4*a*c - b^2)^3)^(1/2) - 4*a^3*b*c*e^2 - 16*a^3*c^2*d*e + 12*a^2*b^2*c*d*e +
 2*a*b*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^5*d^4 + 16*a^4*c^3*e^4 +
b^4*c^3*d^4 - 8*a*b^2*c^4*d^4 + a^2*b^4*c*e^4 - 2*b^5*c^2*d^3*e + b^6*c*d^2
*e^2 - 8*a^3*b^2*c^2*e^4 + 32*a^3*c^4*d^2*e^2 - 2*a*b^5*c*d*e^3 + 16*a*b^3*
c^3*d^3*e - 32*a^2*b*c^4*d^3*e - 32*a^3*b*c^3*d*e^3 - 6*a*b^4*c^2*d^2*e^2 +
 16*a^2*b^3*c^2*d*e^3)))^(1/2)*(64*a^2*c^6*d^6*e^2 - x*(-(b^5*d^2 + a^2*b^3
*e^2 - a^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - b^2*d^2*(-(4*a*c - b^2)^3)^(1/2)
+ 12*a^2*b*c^2*d^2 - 2*a*b^4*d*e - 7*a*b^3*c*d^2 + a*c*d^2*(-(4*a*c - b^2)^
3)^(1/2) - 4*a^3*b*c*e^2 - 16*a^3*c^2*d*e + 12*a^2*b^2*c*d*e + 2*a*b*d*e*(-
(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^5*d^4 + 16*a^4*c^3*e^4 + b^4*c^3*d^4 -
 8*a*b^2*c^4*d^4 + a^2*b^4*c*e^4 - 2*b^5*c^2*d^3*e + b^6*c*d^2*e^2 - 8*a^3*
b^2*c^2*e^4 + 32*a^3*c^4*d^2*e^2 - 2*a*b^5*c*d*e^3 + 16*a*b^3*c^3*d^3*e - 3
2*a^2*b*c^4*d^3*e - 32*a^3*b*c^3*d*e^3 - 6*a*b^4*c^2*d^2*e^2 + 16*a^2*b^3*c
^2*d*e^3)))^(1/2)*(256*a^4*b^2*c^3*e^9 - 32*a^3*b^4*c^2*e^9 - 512*a^5*c^4*e

^9 + 512*a^2*c^7*d^6*e^3 + 512*a^3*c^6*d^4*e^5 - 512*a^4*c^5*d^2*e^7 - 32*b
^3*c^6*d^7*e^2 + 128*b^4*c^5*d^6*e^3 - 192*b^5*c^4*d^5*e^4 + 128*b^6*c^3*d^
4*e^5 - 32*b^7*c^2*d^3*e^6 + 512*a^2*b^2*c^5*d^4*e^5 + 288*a^2*b^3*c^4*d^3*
e^6 - 192*a^2*b^4*c^3*d^2*e^7 + 384*a^3*b^2*c^4*d^2*e^7 + 128*a*b*c^7*d^7*e
^2 + 640*a^4*b*c^4*d*e^8 - 640*a*b^2*c^6*d^6*e^3 + 1056*a*b^3*c^5*d^5*e^4 -
 672*a*b^4*c^4*d^4*e^5 + 96*a*b^5*c^3*d^3*e^6 + 32*a*b^6*c^2*d^2*e^7 - 1152
*a^2*b*c^6*d^5*e^4 + 32*a^2*b^5*c^2*d*e^8 - 640*a^3*b*c^5*d^3*e^6 - 288*a^3
*b^3*c^3*d*e^8) + 128*a^3*c^5*d^4*e^4 + 64*a^4*c^4*d^2*e^6 - 96*a^2*b^2*c^4
*d^4*e^4 + 64*a^2*b^3*c^3*d^3*e^5 + 32*a^2*b^4*c^2*d^2*e^6 - 144*a^3*b^2*c^
3*d^2*e^6 + 64*a^4*b*c^3*d*e^7 - 16*a*b^2*c^5*d^6*e^2 + 16*a*b^3*c^4*d^5*e^
3 + 16*a*b^4*c^3*d^4*e^4 - 16*a*b^5*c^2*d^3*e^5 - 64*a^2*b*c^5*d^5*e^3 - 16
*a^3*b^3*c^2*d*e^7))*(-(b^5*d^2 + a^2*b^3*e^2 - a^2*e^2*(-(4*a*c - b^2)^3)^
(1/2) - b^2*d^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*d^2 - 2*a*b^4*d*e -
 7*a*b^3*c*d^2 + a*c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a^3*b*c*e^2 - 16*a^3*
c^2*d*e + 12*a^2*b^2*c*d*e + 2*a*b*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2
*c^5*d^4 + 16*a^4*c^3*e^4 + b^4*c^3*d^4 - 8*a*b^2*c^4*d^4 + a^2*b^4*c*e^4 -
 2*b^5*c^2*d^3*e + b^6*c*d^2*e^2 - 8*a^3*b^2*c^2*e^4 + 32*a^3*c^4*d^2*e^2 -
 2*a*b^5*c*d*e^3 + 16*a*b^3*c^3*d^3*e - 32*a^2*b*c^4*d^3*e - 32*a^3*b*c^3*d
*e^3 - 6*a*b^4*c^2*d^2*e^2 + 16*a^2*b^3*c^2*d*e^3)))^(1/2) + 16*a^2*c^4*d^5
*e + 4*a^4*c^2*d*e^5 - 60*a^3*c^3*d^3*e^3 + 24*a^2*b^2*c^2*d^3*e^3 - 4*a*b^
2*c^3*d^5*e - 4*a*b^4*c*d^3*e^3 - 4*a^3*b^2*c*d*e^5 - 4*a*b^3*c^2*d^4*e^2 +
 20*a^2*b*c^3*d^4*e^2 + 8*a^2*b^3*c*d^2*e^4 - 16*a^3*b*c^2*d^2*e^4) + x*(2*
a^4*c*e^5 + 4*a^2*c^3*d^4*e + 2*b^4*c*d^4*e - 8*a*b^2*c^2*d^4*e))*(-(b^5*d^
2 + a^2*b^3*e^2 - a^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - b^2*d^2*(-(4*a*c - b^2
)^3)^(1/2) + 12*a^2*b*c^2*d^2 - 2*a*b^4*d*e - 7*a*b^3*c*d^2 + a*c*d^2*(-(4*
a*c - b^2)^3)^(1/2) - 4*a^3*b*c*e^2 - 16*a^3*c^2*d*e + 12*a^2*b^2*c*d*e + 2
*a*b*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^5*d^4 + 16*a^4*c^3*e^4 + b^
4*c^3*d^4 - 8*a*b^2*c^4*d^4 + a^2*b^4*c*e^4 - 2*b^5*c^2*d^3*e + b^6*c*d^2*e
^2 - 8*a^3*b^2*c^2*e^4 + 32*a^3*c^4*d^2*e^2 - 2*a*b^5*c*d*e^3 + 16*a*b^3*c^
3*d^3*e - 32*a^2*b*c^4*d^3*e - 32*a^3*b*c^3*d*e^3 - 6*a*b^4*c^2*d^2*e^2 + 1
6*a^2*b^3*c^2*d*e^3)))^(1/2) - ((-(b^5*d^2 + a^2*b^3*e^2 - a^2*e^2*(-(4*a*c
 - b^2)^3)^(1/2) - b^2*d^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*d^2 - 2*
a*b^4*d*e - 7*a*b^3*c*d^2 + a*c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a^3*b*c*e^
2 - 16*a^3*c^2*d*e + 12*a^2*b^2*c*d*e + 2*a*b*d*e*(-(4*a*c - b^2)^3)^(1/2))
/(8*(16*a^2*c^5*d^4 + 16*a^4*c^3*e^4 + b^4*c^3*d^4 - 8*a*b^2*c^4*d^4 + a^2*
b^4*c*e^4 - 2*b^5*c^2*d^3*e + b^6*c*d^2*e^2 - 8*a^3*b^2*c^2*e^4 + 32*a^3*c^
4*d^2*e^2 - 2*a*b^5*c*d*e^3 + 16*a*b^3*c^3*d^3*e - 32*a^2*b*c^4*d^3*e - 32*
a^3*b*c^3*d*e^3 - 6*a*b^4*c^2*d^2*e^2 + 16*a^2*b^3*c^2*d*e^3)))^(1/2)*((x*(
8*a^3*b^3*c*e^7 - 32*a^4*b*c^2*e^7 - 112*a^4*c^3*d*e^6 + 8*b^3*c^4*d^6*e +
8*b^6*c*d^3*e^4 - 112*a^2*c^5*d^5*e^2 + 32*a^3*c^4*d^3*e^4 - 8*b^4*c^3*d^5*
e^2 - 8*b^5*c^2*d^4*e^3 - 32*a*b*c^5*d^6*e - 48*a^2*b^2*c^3*d^3*e^4 + 8*a^2
*b^3*c^2*d^2*e^5 - 8*a*b^5*c*d^2*e^5 - 8*a^2*b^4*c*d*e^6 + 64*a*b^2*c^4*d^5
*e^2 + 8*a*b^3*c^3*d^4*e^3 - 16*a*b^4*c^2*d^3*e^4 + 64*a^2*b*c^4*d^4*e^3 +
64*a^3*b*c^3*d^2*e^5 + 64*a^3*b^2*c^2*d*e^6) - (-(b^5*d^2 + a^2*b^3*e^2 - a
^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - b^2*d^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2

*b*c^2*d^2 - 2*a*b^4*d*e - 7*a*b^3*c*d^2 + a*c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a^3*b*c*e^2 - 16*a^3*c^2*d*e + 12*a^2*b^2*c*d*e + 2*a*b*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^5*d^4 + 16*a^4*c^3*e^4 + b^4*c^3*d^4 - 8*a*b^2*c^4*d^4 + a^2*b^4*c*e^4 - 2*b^5*c^2*d^3*e + b^6*c*d^2*e^2 - 8*a^3*b^2*c^2*e^4 + 32*a^3*c^4*d^2*e^2 - 2*a*b^5*c*d*e^3 + 16*a*b^3*c^3*d^3*e - 32*a^2*b*c^4*d^3*e - 32*a^3*b*c^3*d*e^3 - 6*a*b^4*c^2*d^2*e^2 + 16*a^2*b^3*c^2*d*e^3)))^(1/2)*(x*(-(b^5*d^2 + a^2*b^3*e^2 - a^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - b^2*d^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*d^2 - 2*a*b^4*d*e - 7*a*b^3*c*d^2 + a*c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a^3*b*c*e^2 - 16*a^3*c^2*d*e + 12*a^2*b^2*c*d*e + 2*a*b*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^5*d^4 + 16*a^4*c^3*e^4 + b^4*c^3*d^4 - 8*a*b^2*c^4*d^4 + a^2*b^4*c*e^4 - 2*b^5*c^2*d^3*e + b^6*c*d^2*e^2 - 8*a^3*b^2*c^2*e^4 + 32*a^3*c^4*d^2*e^2 - 2*a*b^5*c*d*e^3 + 16*a*b^3*c^3*d^3*e - 32*a^2*b*c^4*d^3*e - 32*a^3*b*c^3*d*e^3 - 6*a*b^4*c^2*d^2*e^2 + 16*a^2*b^3*c^2*d*e^3)))^(1/2)*(256*a^4*b^2*c^3*e^9 - 32*a^3*b^4*c^2*e^9 - 512*a^5*c^4*e^9 + 512*a^2*c^7*d^6*e^3 + 512*a^3*c^6*d^4*e^5 - 512*a^4*c^5*d^2*e^7 - 32*b^3*c^6*d^7*e^2 + 128*b^4*c^5*d^6*e^3 - 192*b^5*c^4*d^5*e^4 + 128*b^6*c^3*d^4*e^5 - 32*b^7*c^2*d^3*e^6 + 512*a^2*b^2*c^5*d^4*e^5 + 288*a^2*b^3*c^4*d^3*e^6 - 192*a^2*b^4*c^3*d^2*e^7 + 384*a^3*b^2*c^4*d^2*e^7 + 128*a*b*c^7*d^7*e^2 + 640*a^4*b*c^4*d*e^8 - 640*a*b^2*c^6*d^6*e^3 + 1056*a*b^3*c^5*d^5*e^4 - 672*a*b^4*c^4*d^4*e^5 + 96*a*b^5*c^3*d^3*e^6 + 32*a*b^6*c^2*d^2*e^7 - 1152*a^2*b*c^6*d^5*e^4 + 32*a^2*b^5*c^2*d*e^8 - 640*a^3*b*c^5*d^3*e^6 - 288*a^3*b^3*c^3*d*e^8) + 64*a^2*c^6*d^6*e^2 + 128*a^3*c^5*d^4*e^4 + 64*a^4*c^4*d^2*e^6 - 96*a^2*b^2*c^4*d^4*e^4 + 64*a^2*b^3*c^3*d^3*e^5 + 32*a^2*b^4*c^2*d^2*e^6 - 144*a^3*b^2*c^3*d^2*e^6 + 64*a^4*b*c^3*d*e^7 - 16*a*b^2*c^5*d^6*e^2 + 16*a*b^3*c^4*d^5*e^3 + 16*a*b^4*c^3*d^4*e^4 - 16*a*b^5*c^2*d^3*e^5 - 64*a^2*b*c^5*d^5*e^3 - 16*a^3*b^3*c^2*d*e^7))*(-(b^5*d^2 + a^2*b^3*e^2 - a^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - b^2*d^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*d^2 - 2*a*b^4*d*e - 7*a*b^3*c*d^2 + a*c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a^3*b*c*e^2 - 16*a^3*c^2*d*e + 12*a^2*b^2*c*d*e + 2*a*b*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^5*d^4 + 16*a^4*c^3*e^4 + b^4*c^3*d^4 - 8*a*b^2*c^4*d^4 + a^2*b^4*c*e^4 - 2*b^5*c^2*d^3*e + b^6*c*d^2*e^2 - 8*a^3*b^2*c^2*e^4 + 32*a^3*c^4*d^2*e^2 - 2*a*b^5*c*d*e^3 + 16*a*b^3*c^3*d^3*e - 32*a^2*b*c^4*d^3*e - 32*a^3*b*c^3*d*e^3 - 6*a*b^4*c^2*d^2*e^2 + 16*a^2*b^3*c^2*d*e^3)))^(1/2) - 16*a^2*c^4*d^5*e - 4*a^4*c^2*d*e^5 + 60*a^3*c^3*d^3*e^3 - 24*a^2*b^2*c^2*d^3*e^3 + 4*a*b^2*c^3*d^5*e + 4*a*b^4*c*d^3*e^3 + 4*a^3*b^2*c*d*e^5 + 4*a*b^3*c^2*d^4*e^2 - 20*a^2*b*c^3*d^4*e^2 - 8*a^2*b^3*c*d^2*e^4 + 16*a^3*b*c^2*d^2*e^4) + x*(2*a^4*c*e^5 + 4*a^2*c^3*d^4*e + 2*b^4*c*d^4*e - 8*a*b^2*c^2*d^4*e))*(-(b^5*d^2 + a^2*b^3*e^2 - a^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - b^2*d^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*d^2 - 2*a*b^4*d*e - 7*a*b^3*c*d^2 + a*c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a^3*b*c*e^2 - 16*a^3*c^2*d*e + 12*a^2*b^2*c*d*e + 2*a*b*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^5*d^4 + 16*a^4*c^3*e^4 + b^4*c^3*d^4 - 8*a*b^2*c^4*d^4 + a^2*b^4*c*e^4 - 2*b^5*c^2*d^3*e + b^6*c*d^2*e^2 - 8*a^3*b^2*c^2*e^4 + 32*a^3*c^4*d^2*e^2 - 2*a*b^5*c*d*e^3 + 16*a*b^3*c^3*d^3*e - 32*a^2*b*c^4*d^3*e - 32*a^3*b*c^3*d*e^3 - 6*a*b^4*c^2*d^2*e^2 + 16*a^2*b^3*c^2*d*e^3))

)^(1/2) + 2*a^3*c*d^2*e^2 + 2*a^2*b*c*d^3*e))*(-(b^5*d^2 + a^2*b^3*e^2 - a^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - b^2*d^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*d^2 - 2*a*b^4*d*e - 7*a*b^3*c*d^2 + a*c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a^3*b*c*e^2 - 16*a^3*c^2*d*e + 12*a^2*b^2*c*d*e + 2*a*b*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^5*d^4 + 16*a^4*c^3*e^4 + b^4*c^3*d^4 - 8*a*b^2*c^4*d^4 + a^2*b^4*c*e^4 - 2*b^5*c^2*d^3*e + b^6*c*d^2*e^2 - 8*a^3*b^2*c^2*e^4 + 32*a^3*c^4*d^2*e^2 - 2*a*b^5*c*d*e^3 + 16*a*b^3*c^3*d^3*e - 32*a^2*b*c^4*d^3*e - 32*a^3*b*c^3*d*e^3 - 6*a*b^4*c^2*d^2*e^2 + 16*a^2*b^3*c^2*d*e^3)))^(1/2)*2i - (log(a^5*d^2*e^8 - b^5*d^7*e^3 + 2*a*b^4*d^6*e^4 - 2*a^4*b*d^3*e^7 + 2*a^4*c*d^4*e^6 + b^4*c*d^8*e^2 + 16*a^2*c^3*x*(-d^3*e)^(5/2) + a^5*e^8*x*(-d^3*e)^(1/2) - a^2*b^3*d^5*e^5 + a^3*b^2*d^4*e^6 + 16*a^2*c^3*d^8*e^2 + 17*a^3*c^2*d^6*e^4 + b^4*c*x*(-d^3*e)^(5/2) + a^2*b^3*e^4*x*(-d^3*e)^(3/2) + b^5*d^2*e^2*x*(-d^3*e)^(3/2) + 7*a*b^3*c*d^7*e^3 + 2*a^3*b*c*d^5*e^5 - 8*a*b^2*c^2*x*(-d^3*e)^(5/2) - 8*a*b^2*c^2*d^8*e^2 - 12*a^2*b*c^2*d^7*e^3 - 12*a^2*b^2*c*d^6*e^4 - 2*a^3*b*c*e^4*x*(-d^3*e)^(3/2) - 2*a*b^4*d*e^3*x*(-d^3*e)^(3/2) - 2*a^4*b*d*e^7*x*(-d^3*e)^(1/2) - 17*a^3*c^2*d*e^3*x*(-d^3*e)^(3/2) + 2*a^4*c*d^2*e^6*x*(-d^3*e)^(1/2) + a^3*b^2*d^2*e^6*x*(-d^3*e)^(1/2) + 12*a^2*b*c^2*d^2*e^2*x*(-d^3*e)^(3/2) - 7*a*b^3*c*d^2*e^2*x*(-d^3*e)^(3/2) + 12*a^2*b^2*c*d*e^3*x*(-d^3*e)^(3/2))*(-d^3*e)^(1/2))/(2*(a*e^3 - b*d*e^2 + c*d^2*e)) + (log(a^5*d^2*e^8 - b^5*d^7*e^3 + 2*a*b^4*d^6*e^4 - 2*a^4*b*d^3*e^7 + 2*a^4*c*d^4*e^6 + b^4*c*d^8*e^2 - 16*a^2*c^3*x*(-d^3*e)^(5/2) - a^5*e^8*x*(-d^3*e)^(1/2) - a^2*b^3*d^5*e^5 + a^3*b^2*d^4*e^6 + 16*a^2*c^3*d^8*e^2 + 17*a^3*c^2*d^6*e^4 - b^4*c*x*(-d^3*e)^(5/2) - a^2*b^3*e^4*x*(-d^3*e)^(3/2) - b^5*d^2*e^2*x*(-d^3*e)^(3/2) + 7*a*b^3*c*d^7*e^3 + 2*a^3*b*c*d^5*e^5 + 8*a*b^2*c^2*x*(-d^3*e)^(5/2) - 8*a*b^2*c^2*d^8*e^2 - 12*a^2*b*c^2*d^7*e^3 - 12*a^2*b^2*c*d^6*e^4 + 2*a^3*b*c*e^4*x*(-d^3*e)^(3/2) + 2*a*b^4*d*e^3*x*(-d^3*e)^(3/2) + 2*a^4*b*d*e^7*x*(-d^3*e)^(1/2) + 17*a^3*c^2*d*e^3*x*(-d^3*e)^(3/2) - 2*a^4*c*d^2*e^6*x*(-d^3*e)^(1/2) - a^3*b^2*d^2*e^6*x*(-d^3*e)^(1/2) - 12*a^2*b*c^2*d^2*e^2*x*(-d^3*e)^(3/2) + 7*a*b^3*c*d^2*e^2*x*(-d^3*e)^(3/2) - 12*a^2*b^2*c*d*e^3*x*(-d^3*e)^(3/2))*(-d^3*e)^(1/2))/(2*a*e^3 - 2*b*d*e^2 + 2*c*d^2*e)

3.306 $\int \frac{x^2}{(d+ex^2)(a+bx^2+cx^4)}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 251

$$\int \frac{x^2}{(d+ex^2)(a+bx^2+cx^4)}\,dx = \frac{\sqrt{c}\left(d-\frac{bd-2ae}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{b-\sqrt{b^2-4ac}}\,(cd^2-bde+ae^2)} + \frac{\sqrt{c}\left(d+\frac{bd-2ae}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{b+\sqrt{b^2-4ac}}\,(cd^2-bde+ae^2)} - \frac{\sqrt{d}\sqrt{e}\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{cd^2-bde+ae^2}$$

```
[Out] -arctan(x*e^(1/2)/d^(1/2))*d^(1/2)*e^(1/2)/(a*e^2-b*d*e+c*d^2)+1/2*arctan(x
*2^(1/2)*c^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2))*c^(1/2)*(d+(2*a*e-b*d)/(-4*a
*c+b^2)^(1/2))/(a*e^2-b*d*e+c*d^2)*2^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2)+1/2
*arctan(x*2^(1/2)*c^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2))*c^(1/2)*(d+(-2*a*e+
b*d)/(-4*a*c+b^2)^(1/2))/(a*e^2-b*d*e+c*d^2)*2^(1/2)/(b+(-4*a*c+b^2)^(1/2))
^(1/2)
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 251, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used

= {1301, 211, 1180}

$$\int \frac{x^2}{(d+ex^2)(a+bx^2+cx^4)}\,dx = \frac{\sqrt{c}\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)\left(d-\frac{bd-2ae}{\sqrt{b^2-4ac}}\right)}{\sqrt{2}\sqrt{b-\sqrt{b^2-4ac}}\,(ae^2-bde+cd^2)} + \frac{\sqrt{c}\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)\left(\frac{bd-2ae}{\sqrt{b^2-4ac}}+d\right)}{\sqrt{2}\sqrt{\sqrt{b^2-4ac}+b}\,(ae^2-bde+cd^2)} - \frac{\sqrt{d}\sqrt{e}\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{ae^2-bde+cd^2}$$

```
[In] Int[x^2/((d + e*x^2)*(a + b*x^2 + c*x^4)),x]
```

```
[Out] (Sqrt[c]*(d - (b*d - 2*a*e)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/S
qrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*Sqrt[b - Sqrt[b^2 - 4*a*c]]*(c*d^2 -
b*d*e + a*e^2)) + (Sqrt[c]*(d + (b*d - 2*a*e)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sq
rt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*Sqrt[b + Sqrt[b^2 -
 4*a*c]]*(c*d^2 - b*d*e + a*e^2)) - (Sqrt[d]*Sqrt[e]*ArcTan[(Sqrt[e]*x)/Sqr
t[d]])/(c*d^2 - b*d*e + a*e^2)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1301

```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.))/((a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*((d + e*x^2)^q/(a
+ b*x^2 + c*x^4)), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b^2 - 4
*a*c, 0] && IntegerQ[q] && IntegerQ[m]
```

Rubi steps

$$\text{integral} = \int \left(-\frac{de}{(cd^2-bde+ae^2)(d+ex^2)} + \frac{ae+cdx^2}{(cd^2-bde+ae^2)(a+bx^2+cx^4)}\right) dx$$

$$= \frac{\int \frac{ae+cdx^2}{a+bx^2+cx^4}\,dx}{cd^2-bde+ae^2} - \frac{(de)\int \frac{1}{d+ex^2}\,dx}{cd^2-bde+ae^2}$$

$$= -\frac{\sqrt{d}\sqrt{e}\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{cd^2-bde+ae^2} + \frac{\left(c\left(d-\frac{bd-2ae}{\sqrt{b^2-4ac}}\right)\right)\int\frac{1}{\frac{b}{2}-\frac{1}{2}\sqrt{b^2-4ac}+cx^2}\,dx}{2\left(cd^2-bde+ae^2\right)}$$
$$+ \frac{\left(c\left(d+\frac{bd-2ae}{\sqrt{b^2-4ac}}\right)\right)\int\frac{1}{\frac{b}{2}+\frac{1}{2}\sqrt{b^2-4ac}+cx^2}\,dx}{2\left(cd^2-bde+ae^2\right)}$$

$$= \frac{\sqrt{c}\left(d-\frac{bd-2ae}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{b-\sqrt{b^2-4ac}}\left(cd^2-bde+ae^2\right)}$$
$$+ \frac{\sqrt{c}\left(d+\frac{bd-2ae}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{b+\sqrt{b^2-4ac}}\left(cd^2-bde+ae^2\right)} - \frac{\sqrt{d}\sqrt{e}\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{cd^2-bde+ae^2}$$

Mathematica [A] (verified)

Time = 0.32 (sec) , antiderivative size = 277, normalized size of antiderivative = 1.10

$$\int\frac{x^2}{(d+ex^2)(a+bx^2+cx^4)}\,dx = -\frac{\sqrt{c}\left(-bd+\sqrt{b^2-4ac}d+2ae\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}\left(-cd^2+bde-ae^2\right)}$$
$$- \frac{\sqrt{c}\left(bd+\sqrt{b^2-4ac}d-2ae\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{b^2-4ac}\sqrt{b+\sqrt{b^2-4ac}}\left(-cd^2+bde-ae^2\right)}$$
$$- \frac{\sqrt{d}\sqrt{e}\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{cd^2-bde+ae^2}$$

```
[In] Integrate[x^2/((d + e*x^2)*(a + b*x^2 + c*x^4)),x]
```

```
[Out] -((Sqrt[c]*(-(b*d) + Sqrt[b^2 - 4*a*c]*d + 2*a*e)*ArcTan[(Sqrt[2]*Sqrt[c]*x
)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*Sqrt[b^2 - 4*a*c]*Sqrt[b - Sqrt[b^
2 - 4*a*c]]*(-(c*d^2) + b*d*e - a*e^2))) - (Sqrt[c]*(b*d + Sqrt[b^2 - 4*a*c
]*d - 2*a*e)*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt
[2]*Sqrt[b^2 - 4*a*c]*Sqrt[b + Sqrt[b^2 - 4*a*c]]*(-(c*d^2) + b*d*e - a*e^2
)) - (Sqrt[d]*Sqrt[e]*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(c*d^2 - b*d*e + a*e^2)
```

Maple [A] (verified)

Time = 0.44 (sec) , antiderivative size = 213, normalized size of antiderivative = 0.85

method	result
default	$\frac{4c\left(\frac{\left(d\sqrt{-4ac+b^2}-2ae+bd\right)\sqrt{2}\,\arctan\left(\frac{cx\sqrt{2}}{\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}}\right)}{8\sqrt{-4ac+b^2}\,\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}}-\frac{\left(d\sqrt{-4ac+b^2}+2ae-bd\right)\sqrt{2}\,\mathrm{arctanh}\left(\frac{cx\sqrt{2}}{\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}}\right)}{8\sqrt{-4ac+b^2}\,\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}}\right)}{a\,e^2-bde+c\,d^2}-\frac{de\,\mathrm{arcta}}{(a\,e^2-bde}$
risch	Expression too large to display

[In] int(x^2/(e*x^2+d)/(c*x^4+b*x^2+a),x,method=_RETURNVERBOSE)

[Out] 4/(a*e^2-b*d*e+c*d^2)*c*(1/8*(d*(-4*a*c+b^2)^(1/2)-2*a*e+b*d)/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))-1/8*(d*(-4*a*c+b^2)^(1/2)+2*a*e-b*d)/(-4*a*c+b^2)^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(c*x*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)))-d*e/(a*e^2-b*d*e+c*d^2)/(e*d)^(1/2)*arctan(e*x/(e*d)^(1/2))

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 6125 vs. 2(209) = 418.

Time = 1.12 (sec) , antiderivative size = 12269, normalized size of antiderivative = 48.88

$$\int \frac{x^2}{(d+ex^2)(a+bx^2+cx^4)}\,dx = \text{Too large to display}$$

[In] integrate(x^2/(e*x^2+d)/(c*x^4+b*x^2+a),x, algorithm="fricas")

[Out] Too large to include

Sympy [F(-1)]

Timed out.

$$\int \frac{x^2}{(d+ex^2)(a+bx^2+cx^4)}\,dx = \text{Timed out}$$

[In] integrate(x**2/(e*x**2+d)/(c*x**4+b*x**2+a),x)

[Out] Timed out

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^2}{(d+ex^2)(a+bx^2+cx^4)}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(x^2/(e*x^2+d)/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 6936 vs. $2(209) = 418$.

Time = 1.78 (sec) , antiderivative size = 6936, normalized size of antiderivative = 27.63

$$\int \frac{x^2}{(d+ex^2)(a+bx^2+cx^4)}\,dx = \text{Too large to display}$$

```
[In] integrate(x^2/(e*x^2+d)/(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] -d*e*arctan(e*x/sqrt(d*e))/((c*d^2 - b*d*e + a*e^2)*sqrt(d*e)) - 1/8*((2*b^
4*c^4 - 8*a*b^2*c^5 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c
)*c)*b^4*c^2 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*
a*b^2*c^3 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^3
*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^2*c^4 -
2*(b^2 - 4*a*c)*b^2*c^4)*d^5 - 2*(2*b^5*c^3 - 6*a*b^3*c^4 - 8*a^2*b*c^5 - s
qrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^5*c + 3*sqrt(2)*
sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c^2 + 2*sqrt(2)*sqr
t(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^4*c^2 + 4*sqrt(2)*sqrt(b^2
 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b*c^3 + 2*sqrt(2)*sqrt(b^2 -
4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c
)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^3*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(
b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c^4 - 2*(b^2 - 4*a*c)*b^3*c^3 - 2*(b^2 - 4*a
*c)*a*b*c^4)*d^4*e + (2*b^6*c^2 + 4*a*b^4*c^3 - 48*a^2*b^2*c^4 - sqrt(2)*sq
rt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^6 - 2*sqrt(2)*sqrt(b^2 -
4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^4*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c
)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^5*c + 24*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt
(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^2 + 12*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt
(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c
+ sqrt(b^2 - 4*a*c)*c)*b^4*c^2 - 6*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqr
t(b^2 - 4*a*c)*c)*a*b^2*c^3 - 2*(b^2 - 4*a*c)*b^4*c^2 - 12*(b^2 - 4*a*c)*a*
```

b^2*c^3)*d^3*e^2 - 4*(2*a*b^5*c^2 - 6*a^2*b^3*c^3 - 8*a^3*b*c^4 - sqrt(2)*s
qrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^5 + 3*sqrt(2)*sqrt(b^2
 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c + 2*sqrt(2)*sqrt(b^2 -
4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^4*c + 4*sqrt(2)*sqrt(b^2 - 4*a*c
)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b*c^2 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*s
qrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt
(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c
+ sqrt(b^2 - 4*a*c)*c)*a^2*b*c^3 - 2*(b^2 - 4*a*c)*a*b^3*c^2 - 2*(b^2 - 4*a
*c)*a^2*b*c^3)*d^2*e^3 + 5*(2*a^2*b^4*c^2 - 8*a^3*b^2*c^3 - sqrt(2)*sqrt(b^
2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^4 + 4*sqrt(2)*sqrt(b^2 - 4
*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^2*c + 2*sqrt(2)*sqrt(b^2 - 4*a*
c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c - sqrt(2)*sqrt(b^2 - 4*a*c)*sq
rt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^2 - 2*(b^2 - 4*a*c)*a^2*b^2*c^2)*d*
e^4 - 2*(2*a^3*b^3*c^2 - 8*a^4*b*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c +
 sqrt(b^2 - 4*a*c)*c)*a^3*b^3 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt
(b^2 - 4*a*c)*c)*a^4*b*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2
- 4*a*c)*c)*a^3*b^2*c - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a
*c)*c)*a^3*b*c^2 - 2*(b^2 - 4*a*c)*a^3*b*c^2)*e^5 - 2*(sqrt(2)*sqrt(b*c + s
qrt(b^2 - 4*a*c)*c)*a*b^4*c - 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2
*b^2*c^2 - 2*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c^2 - 2*a*b^4*c^
2 + 16*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*c^3 + 8*sqrt(2)*sqrt(b*c
 + sqrt(b^2 - 4*a*c)*c)*a^2*b*c^3 + sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)
*a*b^2*c^3 + 16*a^2*b^2*c^3 - 4*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2
*c^4 - 32*a^3*c^4 + 2*(b^2 - 4*a*c)*a*b^2*c^2 - 8*(b^2 - 4*a*c)*a^2*c^3)*d^
2*e*abs(c*d^2 - b*d*e + a*e^2) + 2*(sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)
*a*b^5 - 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c - 2*sqrt(2)*sq
rt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^4*c - 2*a*b^5*c + 16*sqrt(2)*sqrt(b*c + s
qrt(b^2 - 4*a*c)*c)*a^3*b*c^2 + 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a
^2*b^2*c^2 + sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c^2 + 16*a^2*b^3
*c^2 - 4*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b*c^3 - 32*a^3*b*c^3 +
 2*(b^2 - 4*a*c)*a*b^3*c - 8*(b^2 - 4*a*c)*a^2*b*c^2)*d*e^2*abs(c*d^2 - b*d
*e + a*e^2) - 2*(sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^4 - 8*sqrt(2
)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^2*c - 2*sqrt(2)*sqrt(b*c + sqrt(b^2
 - 4*a*c)*c)*a^2*b^3*c - 2*a^2*b^4*c + 16*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a
*c)*c)*a^4*c^2 + 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b*c^2 + sqrt
(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^2 + 16*a^3*b^2*c^2 - 4*sqrt(2
)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*c^3 - 32*a^4*c^3 + 2*(b^2 - 4*a*c)*a^
2*b^2*c - 8*(b^2 - 4*a*c)*a^3*c^2)*e^3*abs(c*d^2 - b*d*e + a*e^2) - (2*b^4*
c^2 - 16*a*b^2*c^3 + 32*a^2*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt
(b^2 - 4*a*c)*c)*b^4 + 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*
a*c)*c)*a*b^2*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*
c)*b^3*c - 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2
*c^2 - 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c^2
- sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^2*c^2 + 4*sqr
t(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*c^3 - 2*(b^2 - 4*a

*c)*b^2*c^2 + 8*(b^2 - 4*a*c)*a*c^3)*(c*d^2 - b*d*e + a*e^2)^2*d)*arctan(2*
sqrt(1/2)*x/sqrt((b*c*d^2 - b^2*d*e + a*b*e^2 + sqrt((b*c*d^2 - b^2*d*e + a
*b*e^2)^2 - 4*(a*c*d^2 - a*b*d*e + a^2*e^2)*(c^2*d^2 - b*c*d*e + a*c*e^2)))
/(c^2*d^2 - b*c*d*e + a*c*e^2)))/((a*b^4*c^2 - 8*a^2*b^2*c^3 - 2*a*b^3*c^3
+ 16*a^3*c^4 + 8*a^2*b*c^4 + a*b^2*c^4 - 4*a^2*c^5)*d^4*abs(c*d^2 - b*d*e +
a*e^2)*abs(c) - 2*(a*b^5*c - 8*a^2*b^3*c^2 - 2*a*b^4*c^2 + 16*a^3*b*c^3 +
8*a^2*b^2*c^3 + a*b^3*c^3 - 4*a^2*b*c^4)*d^3*e*abs(c*d^2 - b*d*e + a*e^2)*a
bs(c) + (a*b^6 - 6*a^2*b^4*c - 2*a*b^5*c + 4*a^2*b^3*c^2 + a*b^4*c^2 + 32*a
^4*c^3 + 16*a^3*b*c^3 - 2*a^2*b^2*c^3 - 8*a^3*c^4)*d^2*e^2*abs(c*d^2 - b*d*
e + a*e^2)*abs(c) - 2*(a^2*b^5 - 8*a^3*b^3*c - 2*a^2*b^4*c + 16*a^4*b*c^2 +
8*a^3*b^2*c^2 + a^2*b^3*c^2 - 4*a^3*b*c^3)*d*e^3*abs(c*d^2 - b*d*e + a*e^2
)*abs(c) + (a^3*b^4 - 8*a^4*b^2*c - 2*a^3*b^3*c + 16*a^5*c^2 + 8*a^4*b*c^2
+ a^3*b^2*c^2 - 4*a^4*c^3)*e^4*abs(c*d^2 - b*d*e + a*e^2)*abs(c)) + 1/8*((2
*b^4*c^4 - 8*a*b^2*c^5 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*
a*c)*c)*b^4*c^2 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*
c)*a*b^2*c^3 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*
b^3*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^2*c^4
- 2*(b^2 - 4*a*c)*b^2*c^4)*d^5 - 2*(2*b^5*c^3 - 6*a*b^3*c^4 - 8*a^2*b*c^5
- sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^5*c + 3*sqrt(
2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^3*c^2 + 2*sqrt(2)*
sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^4*c^2 + 4*sqrt(2)*sqrt(
b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b*c^3 + 2*sqrt(2)*sqrt(b^2
- 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^2*c^3 - sqrt(2)*sqrt(b^2 - 4*
a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^3*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sq
rt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b*c^4 - 2*(b^2 - 4*a*c)*b^3*c^3 - 2*(b^2 -
4*a*c)*a*b*c^4)*d^4*e + (2*b^6*c^2 + 4*a*b^4*c^3 - 48*a^2*b^2*c^4 - sqrt(2)
*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^6 - 2*sqrt(2)*sqrt(b^2
- 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^4*c + 2*sqrt(2)*sqrt(b^2 - 4*
a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^5*c + 24*sqrt(2)*sqrt(b^2 - 4*a*c)*s
qrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^2 + 12*sqrt(2)*sqrt(b^2 - 4*a*c)*s
qrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^3*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b
*c - sqrt(b^2 - 4*a*c)*c)*b^4*c^2 - 6*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c -
sqrt(b^2 - 4*a*c)*c)*a*b^2*c^3 - 2*(b^2 - 4*a*c)*b^4*c^2 - 12*(b^2 - 4*a*c)
*a*b^2*c^3)*d^3*e^2 - 4*(2*a*b^5*c^2 - 6*a^2*b^3*c^3 - 8*a^3*b*c^4 - sqrt(2
)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^5 + 3*sqrt(2)*sqrt(
b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c + 2*sqrt(2)*sqrt(b^2
- 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^4*c + 4*sqrt(2)*sqrt(b^2 - 4*
a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b*c^2 + 2*sqrt(2)*sqrt(b^2 - 4*a*c
)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*s
qrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^3*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b
*c - sqrt(b^2 - 4*a*c)*c)*a^2*b*c^3 - 2*(b^2 - 4*a*c)*a*b^3*c^2 - 2*(b^2 -
4*a*c)*a^2*b*c^3)*d^2*e^3 + 5*(2*a^2*b^4*c^2 - 8*a^3*b^2*c^3 - sqrt(2)*sqrt
(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^4 + 4*sqrt(2)*sqrt(b^2
- 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^2*c + 2*sqrt(2)*sqrt(b^2 - 4
*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c - sqrt(2)*sqrt(b^2 - 4*a*c)

*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^2 - 2*(b^2 - 4*a*c)*a^2*b^2*c^2)
*d*e^4 - 2*(2*a^3*b^3*c^2 - 8*a^4*b*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*
c - sqrt(b^2 - 4*a*c)*c)*a^3*b^3 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - s
qrt(b^2 - 4*a*c)*c)*a^4*b*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b
^2 - 4*a*c)*c)*a^3*b^2*c - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 -
4*a*c)*c)*a^3*b*c^2 - 2*(b^2 - 4*a*c)*a^3*b*c^2)*e^5 + 2*(sqrt(2)*sqrt(b*c
- sqrt(b^2 - 4*a*c)*c)*a*b^4*c - 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*
a^2*b^2*c^2 - 2*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^3*c^2 + 2*a*b^4
*c^2 + 16*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*c^3 + 8*sqrt(2)*sqrt(
b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b*c^3 + sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)
*c)*a*b^2*c^3 - 16*a^2*b^2*c^3 - 4*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*
a^2*c^4 + 32*a^3*c^4 - 2*(b^2 - 4*a*c)*a*b^2*c^2 + 8*(b^2 - 4*a*c)*a^2*c^3)
*d^2*e*abs(c*d^2 - b*d*e + a*e^2) - 2*(sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)
*c)*a*b^5 - 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c - 2*sqrt(2)
*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^4*c + 2*a*b^5*c + 16*sqrt(2)*sqrt(b*c
- sqrt(b^2 - 4*a*c)*c)*a^3*b*c^2 + 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c
)*a^2*b^2*c^2 + sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^3*c^2 - 16*a^2*
b^3*c^2 - 4*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b*c^3 + 32*a^3*b*c^
3 - 2*(b^2 - 4*a*c)*a*b^3*c + 8*(b^2 - 4*a*c)*a^2*b*c^2)*d*e^2*abs(c*d^2 -
b*d*e + a*e^2) + 2*(sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^4 - 8*sqr
t(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^2*c - 2*sqrt(2)*sqrt(b*c - sqrt(
b^2 - 4*a*c)*c)*a^2*b^3*c + 2*a^2*b^4*c + 16*sqrt(2)*sqrt(b*c - sqrt(b^2 -
4*a*c)*c)*a^4*c^2 + 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b*c^2 + s
qrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^2 - 16*a^3*b^2*c^2 - 4*sqr
t(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*c^3 + 32*a^4*c^3 - 2*(b^2 - 4*a*c)
*a^2*b^2*c + 8*(b^2 - 4*a*c)*a^3*c^2)*e^3*abs(c*d^2 - b*d*e + a*e^2) - (2*b
^4*c^2 - 16*a*b^2*c^3 + 32*a^2*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - s
qrt(b^2 - 4*a*c)*c)*b^4 + 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 -
4*a*c)*c)*a*b^2*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*
c)*c)*b^3*c - 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*
a^2*c^2 - 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b*c
^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^2*c^2 + 4*
sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*c^3 - 2*(b^2 -
4*a*c)*b^2*c^2 + 8*(b^2 - 4*a*c)*a*c^3)*(c*d^2 - b*d*e + a*e^2)^2*d)*arctan
(2*sqrt(1/2)*x/sqrt((b*c*d^2 - b^2*d*e + a*b*e^2 - sqrt((b*c*d^2 - b^2*d*e
+ a*b*e^2)^2 - 4*(a*c*d^2 - a*b*d*e + a^2*e^2)*(c^2*d^2 - b*c*d*e + a*c*e^2
)))/(c^2*d^2 - b*c*d*e + a*c*e^2)))/((a*b^4*c^2 - 8*a^2*b^2*c^3 - 2*a*b^3*c
^3 + 16*a^3*c^4 + 8*a^2*b*c^4 + a*b^2*c^4 - 4*a^2*c^5)*d^4*abs(c*d^2 - b*d*
e + a*e^2)*abs(c) - 2*(a*b^5*c - 8*a^2*b^3*c^2 - 2*a*b^4*c^2 + 16*a^3*b*c^3
+ 8*a^2*b^2*c^3 + a*b^3*c^3 - 4*a^2*b*c^4)*d^3*e*abs(c*d^2 - b*d*e + a*e^2
)*abs(c) + (a*b^6 - 6*a^2*b^4*c - 2*a*b^5*c + 4*a^2*b^3*c^2 + a*b^4*c^2 + 3
2*a^4*c^3 + 16*a^3*b*c^3 - 2*a^2*b^2*c^3 - 8*a^3*c^4)*d^2*e^2*abs(c*d^2 - b
*d*e + a*e^2)*abs(c) - 2*(a^2*b^5 - 8*a^3*b^3*c - 2*a^2*b^4*c + 16*a^4*b*c^
2 + 8*a^3*b^2*c^2 + a^2*b^3*c^2 - 4*a^3*b*c^3)*d*e^3*abs(c*d^2 - b*d*e + a*
e^2)*abs(c) + (a^3*b^4 - 8*a^4*b^2*c - 2*a^3*b^3*c + 16*a^5*c^2 + 8*a^4*b*c

```
^2 + a^3*b^2*c^2 - 4*a^4*c^3)*e^4*abs(c*d^2 - b*d*e + a*e^2)*abs(c))
```

Mupad [B] (verification not implemented)

Time = 10.55 (sec) , antiderivative size = 19401, normalized size of antiderivative = 77.29

$$\int \frac{x^2}{(d+ex^2)(a+bx^2+cx^4)}\,dx = \text{Too large to display}$$

```
[In] int(x^2/((d + e*x^2)*(a + b*x^2 + c*x^4)),x)
```

```
[Out] (log(b^4*d^3*e^5 - a*b^3*d^2*e^6 + a*c^3*d^5*e^3 - b^3*c*d^4*e^4 + 2*a^2*c^
2*d^3*e^5 + a^3*c*d*e^7 + b^4*e^3*x*(-d*e)^(5/2) + a*b^3*e^5*x*(-d*e)^(3/2)
 + a^3*c*e^7*x*(-d*e)^(1/2) + 2*a*b*c^2*d^4*e^4 - 3*a*b^2*c*d^3*e^5 + 2*a^2
*b*c*d^2*e^6 + 2*a^2*c^2*e^3*x*(-d*e)^(5/2) - a*c^3*d*x*(-d*e)^(7/2) + b^3*
c*e*x*(-d*e)^(7/2) - 2*a*b*c^2*e*x*(-d*e)^(7/2) - 3*a*b^2*c*e^3*x*(-d*e)^(5
/2) - 2*a^2*b*c*e^5*x*(-d*e)^(3/2))*(-d*e)^(1/2))/(2*a*e^2 + 2*c*d^2 - 2*b*
d*e) - atan(((x*(2*a^2*c^3*e^5 - 4*a*c^4*d^2*e^3 + 2*b^2*c^3*d^2*e^3) - (-(
a*b^3*e^2 - a*e^2*(-(4*a*c - b^2)^3)^(1/2) + b^3*c*d^2 + c*d^2*(-(4*a*c - b
^2)^3)^(1/2) - 4*a*b*c^2*d^2 - 4*a^2*b*c*e^2 + 16*a^2*c^2*d*e - 4*a*b^2*c*d
*e)/(8*(a^2*b^4*e^4 + 16*a^2*c^4*d^4 + 16*a^4*c^2*e^4 + b^4*c^2*d^4 + b^6*d
^2*e^2 - 8*a*b^2*c^3*d^4 - 8*a^3*b^2*c*e^4 + 32*a^3*c^3*d^2*e^2 - 2*a*b^5*d
*e^3 - 2*b^5*c*d^3*e + 16*a*b^3*c^2*d^3*e - 6*a*b^4*c*d^2*e^2 - 32*a^2*b*c^
3*d^3*e + 16*a^2*b^3*c*d*e^3 - 32*a^3*b*c^2*d*e^3)))^(1/2)*((x*(32*a^3*b*c^
3*e^7 + 16*a*c^6*d^5*e^2 - 112*a^3*c^4*d*e^6 - 8*a^2*b^3*c^2*e^7 + 160*a^2*
c^5*d^3*e^4 - 8*b^2*c^5*d^5*e^2 + 8*b^3*c^4*d^4*e^3 + 8*b^4*c^3*d^3*e^4 - 8
*b^5*c^2*d^2*e^5 - 96*a*b^2*c^4*d^3*e^4 + 64*a*b^3*c^3*d^2*e^5 - 96*a^2*b*c
^4*d^2*e^5 + 24*a^2*b^2*c^3*d*e^6) + (-(a*b^3*e^2 - a*e^2*(-(4*a*c - b^2)^3
)^(1/2) + b^3*c*d^2 + c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a*b*c^2*d^2 - 4*a^
2*b*c*e^2 + 16*a^2*c^2*d*e - 4*a*b^2*c*d*e)/(8*(a^2*b^4*e^4 + 16*a^2*c^4*d^
4 + 16*a^4*c^2*e^4 + b^4*c^2*d^4 + b^6*d^2*e^2 - 8*a*b^2*c^3*d^4 - 8*a^3*b^
2*c*e^4 + 32*a^3*c^3*d^2*e^2 - 2*a*b^5*d*e^3 - 2*b^5*c*d^3*e + 16*a*b^3*c^2
*d^3*e - 6*a*b^4*c*d^2*e^2 - 32*a^2*b*c^3*d^3*e + 16*a^2*b^3*c*d*e^3 - 32*a
^3*b*c^2*d*e^3)))^(1/2)*(x*(-(a*b^3*e^2 - a*e^2*(-(4*a*c - b^2)^3)^(1/2) +
b^3*c*d^2 + c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a*b*c^2*d^2 - 4*a^2*b*c*e^2
+ 16*a^2*c^2*d*e - 4*a*b^2*c*d*e)/(8*(a^2*b^4*e^4 + 16*a^2*c^4*d^4 + 16*a^4
*c^2*e^4 + b^4*c^2*d^4 + b^6*d^2*e^2 - 8*a*b^2*c^3*d^4 - 8*a^3*b^2*c*e^4 +
32*a^3*c^3*d^2*e^2 - 2*a*b^5*d*e^3 - 2*b^5*c*d^3*e + 16*a*b^3*c^2*d^3*e - 6
*a*b^4*c*d^2*e^2 - 32*a^2*b*c^3*d^3*e + 16*a^2*b^3*c*d*e^3 - 32*a^3*b*c^2*d
*e^3)))^(1/2)*(256*a^4*b^2*c^3*e^9 - 32*a^3*b^4*c^2*e^9 - 512*a^5*c^4*e^9 +
 512*a^2*c^7*d^6*e^3 + 512*a^3*c^6*d^4*e^5 - 512*a^4*c^5*d^2*e^7 - 32*b^3*c
^6*d^7*e^2 + 128*b^4*c^5*d^6*e^3 - 192*b^5*c^4*d^5*e^4 + 128*b^6*c^3*d^4*e^
5 - 32*b^7*c^2*d^3*e^6 + 512*a^2*b^2*c^5*d^4*e^5 + 288*a^2*b^3*c^4*d^3*e^6
- 192*a^2*b^4*c^3*d^2*e^7 + 384*a^3*b^2*c^4*d^2*e^7 + 128*a*b*c^7*d^7*e^2 +
 640*a^4*b*c^4*d*e^8 - 640*a*b^2*c^6*d^6*e^3 + 1056*a*b^3*c^5*d^5*e^4 - 672
```

*a*b^4*c^4*d^4*e^5 + 96*a*b^5*c^3*d^3*e^6 + 32*a*b^6*c^2*d^2*e^7 - 1152*a^2*b*c^6*d^5*e^4 + 32*a^2*b^5*c^2*d*e^8 - 640*a^3*b*c^5*d^3*e^6 - 288*a^3*b^3*c^3*d*e^8) - 192*a^4*c^4*d*e^7 - 192*a^2*c^6*d^5*e^3 - 384*a^3*c^5*d^3*e^5 - 96*a^2*b^2*c^4*d^3*e^5 - 96*a^2*b^3*c^3*d^2*e^6 + 48*a*b^2*c^5*d^5*e^3 - 96*a*b^3*c^4*d^4*e^4 + 48*a*b^4*c^3*d^3*e^5 + 384*a^2*b*c^5*d^4*e^4 + 384*a^3*b*c^4*d^2*e^6 + 48*a^3*b^2*c^3*d*e^7))*(-(a*b^3*e^2 - a*e^2*(-(4*a*c - b^2)^3)^(1/2) + b^3*c*d^2 + c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a*b*c^2*d^2 - 4*a^2*b*c*e^2 + 16*a^2*c^2*d*e - 4*a*b^2*c*d*e)/(8*(a^2*b^4*e^4 + 16*a^2*c^4*d^4 + 16*a^4*c^2*e^4 + b^4*c^2*d^4 + b^6*d^2*e^2 - 8*a*b^2*c^3*d^4 - 8*a^3*b^2*c*e^4 + 32*a^3*c^3*d^2*e^2 - 2*a*b^5*d*e^3 - 2*b^5*c*d^3*e + 16*a*b^3*c^2*d^3*e - 6*a*b^4*c*d^2*e^2 - 32*a^2*b*c^3*d^3*e + 16*a^2*b^3*c*d*e^3 - 32*a^3*b*c^2*d*e^3)))^(1/2) - 4*a*c^5*d^4*e^2 - 52*a^2*c^4*d^2*e^4 + 8*a*b*c^4*d^3*e^3 - 4*a*b^3*c^2*d*e^5 + 20*a^2*b*c^3*d*e^5 + 8*a*b^2*c^3*d^2*e^4))*(-(a*b^3*e^2 - a*e^2*(-(4*a*c - b^2)^3)^(1/2) + b^3*c*d^2 + c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a*b*c^2*d^2 - 4*a^2*b*c*e^2 + 16*a^2*c^2*d*e - 4*a*b^2*c*d*e)/(8*(a^2*b^4*e^4 + 16*a^2*c^4*d^4 + 16*a^4*c^2*e^4 + b^4*c^2*d^4 + b^6*d^2*e^2 - 8*a*b^2*c^3*d^4 - 8*a^3*b^2*c*e^4 + 32*a^3*c^3*d^2*e^2 - 2*a*b^5*d*e^3 - 2*b^5*c*d^3*e + 16*a*b^3*c^2*d^3*e - 6*a*b^4*c*d^2*e^2 - 32*a^2*b*c^3*d^3*e + 16*a^2*b^3*c*d*e^3 - 32*a^3*b*c^2*d*e^3)))^(1/2)*1i + (x*(2*a^2*c^3*e^5 - 4*a*c^4*d^2*e^3 + 2*b^2*c^3*d^2*e^3) - (-(a*b^3*e^2 - a*e^2*(-(4*a*c - b^2)^3)^(1/2) + b^3*c*d^2 + c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a*b*c^2*d^2 - 4*a^2*b*c*e^2 + 16*a^2*c^2*d*e - 4*a*b^2*c*d*e)/(8*(a^2*b^4*e^4 + 16*a^2*c^4*d^4 + 16*a^4*c^2*e^4 + b^4*c^2*d^4 + b^6*d^2*e^2 - 8*a*b^2*c^3*d^4 - 8*a^3*b^2*c*e^4 + 32*a^3*c^3*d^2*e^2 - 2*a*b^5*d*e^3 - 2*b^5*c*d^3*e + 16*a*b^3*c^2*d^3*e - 6*a*b^4*c*d^2*e^2 - 32*a^2*b*c^3*d^3*e + 16*a^2*b^3*c*d*e^3 - 32*a^3*b*c^2*d*e^3)))^(1/2)*((x*(32*a^3*b*c^3*e^7 + 16*a*c^6*d^5*e^2 - 112*a^3*c^4*d*e^6 - 8*a^2*b^3*c^2*e^7 + 160*a^2*c^5*d^3*e^4 - 8*b^2*c^5*d^5*e^2 + 8*b^3*c^4*d^4*e^3 + 8*b^4*c^3*d^3*e^4 - 8*b^5*c^2*d^2*e^5 - 96*a*b^2*c^4*d^3*e^4 + 64*a*b^3*c^3*d^2*e^5 - 96*a^2*b*c^4*d^2*e^5 + 24*a^2*b^2*c^3*d*e^6) + (-(a*b^3*e^2 - a*e^2*(-(4*a*c - b^2)^3)^(1/2) + b^3*c*d^2 + c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a*b*c^2*d^2 - 4*a^2*b*c*e^2 + 16*a^2*c^2*d*e - 4*a*b^2*c*d*e)/(8*(a^2*b^4*e^4 + 16*a^2*c^4*d^4 + 16*a^4*c^2*e^4 + b^4*c^2*d^4 + b^6*d^2*e^2 - 8*a*b^2*c^3*d^4 - 8*a^3*b^2*c*e^4 + 32*a^3*c^3*d^2*e^2 - 2*a*b^5*d*e^3 - 2*b^5*c*d^3*e + 16*a*b^3*c^2*d^3*e - 6*a*b^4*c*d^2*e^2 - 32*a^2*b*c^3*d^3*e + 16*a^2*b^3*c*d*e^3 - 32*a^3*b*c^2*d*e^3)))^(1/2)*(x*(-(a*b^3*e^2 - a*e^2*(-(4*a*c - b^2)^3)^(1/2) + b^3*c*d^2 + c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a*b*c^2*d^2 - 4*a^2*b*c*e^2 + 16*a^2*c^2*d*e - 4*a*b^2*c*d*e)/(8*(a^2*b^4*e^4 + 16*a^2*c^4*d^4 + 16*a^4*c^2*e^4 + b^4*c^2*d^4 + b^6*d^2*e^2 - 8*a*b^2*c^3*d^4 - 8*a^3*b^2*c*e^4 + 32*a^3*c^3*d^2*e^2 - 2*a*b^5*d*e^3 - 2*b^5*c*d^3*e + 16*a*b^3*c^2*d^3*e - 6*a*b^4*c*d^2*e^2 - 32*a^2*b*c^3*d^3*e + 16*a^2*b^3*c*d*e^3 - 32*a^3*b*c^2*d*e^3)))^(1/2)*(256*a^4*b^2*c^3*e^9 - 32*a^3*b^4*c^2*e^9 - 512*a^5*c^4*e^9 + 512*a^2*c^7*d^6*e^3 + 512*a^3*c^6*d^4*e^5 - 512*a^4*c^5*d^2*e^7 - 32*b^3*c^6*d^7*e^2 + 128*b^4*c^5*d^6*e^3 - 192*b^5*c^4*d^5*e^4 + 128*b^6*c^3*d^4*e^5 - 32*b^7*c^2*d^3*e^6 + 512*a^2*b^2*c^5*d^4*e^5 + 288*a^2*b^3*c^4*d^3*e^6 - 192*a^2*b^4*c^3 *d^2*e^7 + 384*a^3*b^2*c^4*d^2*e^7 + 128*a*b*c^7*d^7*e^2 + 640*a^4*b*c^4*d*e^8 - 640*a*b^2*c^6*d^6*e^3 + 1056*a*b^3*c^5*d^5*e^4 - 672*a*b^4*c^4*d^4*e^5 + 96*a*b^5*c^3*d^3*e^6 + 32*a*b^6*c^2*d^2*e^7 - 1152*a^2*b*c^6*d^5*e^4 + 32*a^2*b^5*c^2*d*e^8 - 640*a^3*b*c^5*d^3*e^6 - 288*a^3*b^3*c^3*d*e^8) + 192*a^4*c^4*d*e^7 + 192*a^2*c^6*d^5*e^3 + 384*a^3*c^5*d^3*e^5 + 96*a^2*b^2*c^4*d^3*e^5 + 96*a^2*b^3*c^3*d^2*e^6 - 48*a*b^2*c^5*d^5*e^3 + 96*a*b^3*c^4*d^4*e^4 - 48*a*b^4*c^3*d^3*e^5 - 384*a^2*b*c^5*d^4*e^4 - 384*a^3*b*c^4*d^2*e^6 - 48*a^3*b^2*c^3*d*e^7))*(-(a*b^3*e^2 - a*e^2*(-(4*a*c - b^2)^3)^(1/2) + b^3*c*d^2 + c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a*b*c^2*d^2 - 4*a^2*b*c*e^2 + 16*a^2*c^2*d*e - 4*a*b^2*c*d*e)/(8*(a^2*b^4*e^4 + 16*a^2*c^4*d^4 + 16*a^4*c^2*e^4 + b^4*c^2*d^4 + b^6*d^2*e^2 - 8*a*b^2*c^3*d^4 - 8*a^3*b^2*c*e^4 + 32*a^3*c^3*d^2*e^2 - 2*a*b^5*d*e^3 - 2*b^5*c*d^3*e + 16*a*b^3*c^2*d^3*e - 6*a*b^4*c*d^2*e^2 - 32*a^2*b*c^3*d^3*e + 16*a^2*b^3*c*d*e^3 - 32*a^3*b*c^2*d*e^3)))^(1/2) + 4*a*c^5*d^4*e^2 + 52*a^2*c^4*d^2*e^4 - 8*a*b*c^4*d^3*e^3 + 4*a*b^3*c^2*d*e^5 - 20*a^2*b*c^3*d*e^5 - 8*a*b^2*c^3*d^2*e^4))*(-(a*b^3*e^2 - a*e^2*(-(4*a*c - b^2)^3)^(1/2) + b^3*c*d^2 + c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a*b*c^2*d^2 - 4*a^2*b*c*e^2 + 16*a^2*c^2*d*e - 4*a*b^2*c*d*e)/(8*(a^2*b^4*e^4 + 16*a^2*c^4*d^4 + 16*a^4*c^2*e^4 + b^4*c^2*d^4 + b^6*d^2*e^2 - 8*a*b^2*c^3*d^4 - 8*a^3*b^2*c*e^4 + 32*a^3*c^3*d^2*e^2 - 2*a*b^5*d*e^3 - 2*b^5*c*d^3*e + 16*a*b^3*c^2*d^3*e - 6*a*b^4*c*d^2*e^2 - 32*a^2*b*c^3*d^3*e + 16*a^2*b^3*c*d*e^3 - 32*a^3*b*c^2*d*e^3)))^(1/2)*1i)/((x*(2*a^2*c^3*e^5 - 4*a*c^4*d^2*e^3 + 2*b^2*c^3*d^2*e^3) - (-(a*b^3*e^2 - a*e^2*(-(4*a*c - b^2)^3)^(1/2) + b^3*c*d^2 + c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a*b*c^2*d^2 - 4*a^2*b*c*e^2 + 16*a^2*c^2*d*e - 4*a*b^2*c*d*e)/(8*(a^2*b^4*e^4 + 16*a^2*c^4*d^4 + 16*a^4*c^2*e^4 + b^4*c^2*d^4 + b^6*d^2*e^2 - 8*a*b^2*c^3*d^4 - 8*a^3*b^2*c*e^4 + 32*a^3*c^3*d^2*e^2 - 2*a*b^5*d*e^3 - 2*b^5*c*d^3*e + 16*a*b^3*c^2*d^3*e - 6*a*b^4*c*d^2*e^2 - 32*a^2*b*c^3*d^3*e + 16*a^2*b^3*c*d*e^3 - 32*a^3*b*c^2*d*e^3)))^(1/2)*((x*(32*a^3*b*c^3*e^7 + 16*a*c^6*d^5*e^2 - 112*a^3*c^4*d*e^6 - 8*a^2*b^3*c^2*e^7 + 160*a^2*c^5*d^3*e^4 - 8*b^2*c^5*d^5*e^2 + 8*b^3*c^4*d^4*e^3 + 8*b^4*c^3*d^3*e^4 - 8*b^5*c^2*d^2*e^5 - 96*a*b^2*c^4*d^3*e^4 + 64*a*b^3*c^3*d^2*e^5 - 96*a^2*b*c^4*d^2*e^5 + 24*a^2*b^2*c^3*d*e^6) + (-(a*b^3*e^2 - a*e^2*(-(4*a*c - b^2)^3)^(1/2) + b^3*c*d^2 + c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a*b*c^2*d^2 - 4*a^2*b*c*e^2 + 16*a^2*c^2*d*e - 4*a*b^2*c*d*e)/(8*(a^2*b^4*e^4 + 16*a^2*c^4*d^4 + 16*a^4*c^2*e^4 + b^4*c^2*d^4 + b^6*d^2*e^2 - 8*a*b^2*c^3*d^4 - 8*a^3*b^2*c*e^4 + 32*a^3*c^3*d^2*e^2 - 2*a*b^5*d*e^3 - 2*b^5*c*d^3*e + 16*a*b^3*c^2*d^3*e - 6*a*b^4*c*d^2*e^2 - 32*a^2*b*c^3*d^3*e + 16*a^2*b^3*c*d*e^3 - 32*a^3*b*c^2*d*e^3)))^(1/2)*(x*(-(a*b^3*e^2 - a*e^2*(-(4*a*c - b^2)^3)^(1/2) + b^3*c*d^2 + c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a*b*c^2*d^2 - 4*a^2*b*c*e^2 + 16*a^2*c^2*d*e - 4*a*b^2*c*d*e)/(8*(a^2*b^4*e^4 + 16*a^2*c^4*d^4 + 16*a^4*c^2*e^4 + b^4*c^2*d^4 + b^6*d^2*e^2 - 8*a*b^2*c^3*d^4 - 8*a^3*b^2*c*e^4 + 32*a^3*c^3*d^2*e^2 - 2*a*b^5*d*e^3 - 2*b^5*c*d^3*e + 16*a*b^3*c^2*d^3*e - 6*a*b^4*c*d^2*e^2 - 32*a^2*b*c^3*d^3*e + 16*a^2*b^3*c*d*e^3 - 32*a^3*b*c^2*d*e^3)))^(1/2)*(256*a^4*b^2*c^3*e^9 - 32*a^3*b^4*c^2*e^9 - 512*a^5*c^4*e^9 + 512*a^2*c^7*d^6*e^3 + 512*a^3*c^6*d^4*e^5 - 512*a^4*c^5*d^2*e^7 - 32*b^3*c^6*d^7*e^2 + 128*b^4*c^5*d^6*e^3 -

192*b^5*c^4*d^5*e^4 + 128*b^6*c^3*d^4*e^5 - 32*b^7*c^2*d^3*e^6 + 512*a^2*b^2*c^5*d^4*e^5 + 288*a^2*b^3*c^4*d^3*e^6 - 192*a^2*b^4*c^3*d^2*e^7 + 384*a^3*b^2*c^4*d^2*e^7 + 128*a*b*c^7*d^7*e^2 + 640*a^4*b*c^4*d*e^8 - 640*a*b^2*c^6*d^6*e^3 + 1056*a*b^3*c^5*d^5*e^4 - 672*a*b^4*c^4*d^4*e^5 + 96*a*b^5*c^3*d^3*e^6 + 32*a*b^6*c^2*d^2*e^7 - 1152*a^2*b*c^6*d^5*e^4 + 32*a^2*b^5*c^2*d*e^8 - 640*a^3*b*c^5*d^3*e^6 - 288*a^3*b^3*c^3*d*e^8) - 192*a^4*c^4*d*e^7 - 192*a^2*c^6*d^5*e^3 - 384*a^3*c^5*d^3*e^5 - 96*a^2*b^2*c^4*d^3*e^5 - 96*a^2*b^3*c^3*d^2*e^6 + 48*a*b^2*c^5*d^5*e^3 - 96*a*b^3*c^4*d^4*e^4 + 48*a*b^4*c^3*d^3*e^5 + 384*a^2*b*c^5*d^4*e^4 + 384*a^3*b*c^4*d^2*e^6 + 48*a^3*b^2*c^3*d*e^7))*(-(a*b^3*e^2 - a*e^2*(-(4*a*c - b^2)^3)^(1/2) + b^3*c*d^2 + c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a*b*c^2*d^2 - 4*a^2*b*c*e^2 + 16*a^2*c^2*d*e - 4*a*b^2*c*d*e)/(8*(a^2*b^4*e^4 + 16*a^2*c^4*d^4 + 16*a^4*c^2*e^4 + b^4*c^2*d^4 + b^6*d^2*e^2 - 8*a*b^2*c^3*d^4 - 8*a^3*b^2*c*e^4 + 32*a^3*c^3*d^2*e^2 - 2*a*b^5*d*e^3 - 2*b^5*c*d^3*e + 16*a*b^3*c^2*d^3*e - 6*a*b^4*c*d^2*e^2 - 32*a^2*b*c^3*d^3*e + 16*a^2*b^3*c*d*e^3 - 32*a^3*b*c^2*d*e^3)))^(1/2) - 4*a*c^5*d^4*e^2 - 52*a^2*c^4*d^2*e^4 + 8*a*b*c^4*d^3*e^3 - 4*a*b^3*c^2*d*e^5 + 20*a^2*b*c^3*d*e^5 + 8*a*b^2*c^3*d^2*e^4))*(-(a*b^3*e^2 - a*e^2*(-(4*a*c - b^2)^3)^(1/2) + b^3*c*d^2 + c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a*b*c^2*d^2 - 4*a^2*b*c*e^2 + 16*a^2*c^2*d*e - 4*a*b^2*c*d*e)/(8*(a^2*b^4*e^4 + 16*a^2*c^4*d^4 + 16*a^4*c^2*e^4 + b^4*c^2*d^4 + b^6*d^2*e^2 - 8*a*b^2*c^3*d^4 - 8*a^3*b^2*c*e^4 + 32*a^3*c^3*d^2*e^2 - 2*a*b^5*d*e^3 - 2*b^5*c*d^3*e + 16*a*b^3*c^2*d^3*e - 6*a*b^4*c*d^2*e^2 - 32*a^2*b*c^3*d^3*e + 16*a^2*b^3*c*d*e^3 - 32*a^3*b*c^2*d*e^3)))^(1/2) - (x*(2*a^2*c^3*e^5 - 4*a*c^4*d^2*e^3 + 2*b^2*c^3*d^2*e^3) - (-(a*b^3*e^2 - a*e^2*(-(4*a*c - b^2)^3)^(1/2) + b^3*c*d^2 + c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a*b*c^2*d^2 - 4*a^2*b*c*e^2 + 16*a^2*c^2*d*e - 4*a*b^2*c*d*e)/(8*(a^2*b^4*e^4 + 16*a^2*c^4*d^4 + 16*a^4*c^2*e^4 + b^4*c^2*d^4 + b^6*d^2*e^2 - 8*a*b^2*c^3*d^4 - 8*a^3*b^2*c*e^4 + 32*a^3*c^3*d^2*e^2 - 2*a*b^5*d*e^3 - 2*b^5*c*d^3*e + 16*a*b^3*c^2*d^3*e - 6*a*b^4*c*d^2*e^2 - 32*a^2*b*c^3*d^3*e + 16*a^2*b^3*c*d*e^3 - 32*a^3*b*c^2*d*e^3)))^(1/2)*((x*(32*a^3*b*c^3*e^7 + 16*a*c^6*d^5*e^2 - 112*a^3*c^4*d*e^6 - 8*a^2*b^3*c^2*e^7 + 160*a^2*c^5*d^3*e^4 - 8*b^2*c^5*d^5*e^2 + 8*b^3*c^4*d^4*e^3 + 8*b^4*c^3*d^3*e^4 - 8*b^5*c^2*d^2*e^5 - 96*a*b^2*c^4*d^3*e^4 + 64*a*b^3*c^3*d^2*e^5 - 96*a^2*b*c^4*d^2*e^5 + 24*a^2*b^2*c^3*d*e^6) + (-(a*b^3*e^2 - a*e^2*(-(4*a*c - b^2)^3)^(1/2) + b^3*c*d^2 + c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a*b*c^2*d^2 - 4*a^2*b*c*e^2 + 16*a^2*c^2*d*e - 4*a*b^2*c*d*e)/(8*(a^2*b^4*e^4 + 16*a^2*c^4*d^4 + 16*a^4*c^2*e^4 + b^4*c^2*d^4 + b^6*d^2*e^2 - 8*a*b^2*c^3*d^4 - 8*a^3*b^2*c*e^4 + 32*a^3*c^3*d^2*e^2 - 2*a*b^5*d*e^3 - 2*b^5*c*d^3*e + 16*a*b^3*c^2*d^3*e - 6*a*b^4*c*d^2*e^2 - 32*a^2*b*c^3*d^3*e + 16*a^2*b^3*c*d*e^3 - 32*a^3*b*c^2*d*e^3)))^(1/2)*(x*(-(a*b^3*e^2 - a*e^2*(-(4*a*c - b^2)^3)^(1/2) + b^3*c*d^2 + c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a*b*c^2*d^2 - 4*a^2*b*c*e^2 + 16*a^2*c^2*d*e - 4*a*b^2*c*d*e)/(8*(a^2*b^4*e^4 + 16*a^2*c^4*d^4 + 16*a^4*c^2*e^4 + b^4*c^2*d^4 + b^6*d^2*e^2 - 8*a*b^2*c^3*d^4 - 8*a^3*b^2*c*e^4 + 32*a^3*c^3*d^2*e^2 - 2*a*b^5*d*e^3 - 2*b^5*c*d^3*e + 16*a*b^3*c^2*d^3*e - 6*a*b^4*c*d^2*e^2 - 32*a^2*b*c^3*d^3*e + 16*a^2*b^3*c*d*e^3 - 32*a^3*b*c^2*d*e^3)))^(1/2)*(256*a^4*b^2*c^3*e^9 - 32*a^3*b^4*c^2*e^

9 - 512*a^5*c^4*e^9 + 512*a^2*c^7*d^6*e^3 + 512*a^3*c^6*d^4*e^5 - 512*a^4*c
^5*d^2*e^7 - 32*b^3*c^6*d^7*e^2 + 128*b^4*c^5*d^6*e^3 - 192*b^5*c^4*d^5*e^4
 + 128*b^6*c^3*d^4*e^5 - 32*b^7*c^2*d^3*e^6 + 512*a^2*b^2*c^5*d^4*e^5 + 288
*a^2*b^3*c^4*d^3*e^6 - 192*a^2*b^4*c^3*d^2*e^7 + 384*a^3*b^2*c^4*d^2*e^7 +
128*a*b*c^7*d^7*e^2 + 640*a^4*b*c^4*d*e^8 - 640*a*b^2*c^6*d^6*e^3 + 1056*a*
b^3*c^5*d^5*e^4 - 672*a*b^4*c^4*d^4*e^5 + 96*a*b^5*c^3*d^3*e^6 + 32*a*b^6*c
^2*d^2*e^7 - 1152*a^2*b*c^6*d^5*e^4 + 32*a^2*b^5*c^2*d*e^8 - 640*a^3*b*c^5*
d^3*e^6 - 288*a^3*b^3*c^3*d*e^8) + 192*a^4*c^4*d*e^7 + 192*a^2*c^6*d^5*e^3
+ 384*a^3*c^5*d^3*e^5 + 96*a^2*b^2*c^4*d^3*e^5 + 96*a^2*b^3*c^3*d^2*e^6 - 4
8*a*b^2*c^5*d^5*e^3 + 96*a*b^3*c^4*d^4*e^4 - 48*a*b^4*c^3*d^3*e^5 - 384*a^2
*b*c^5*d^4*e^4 - 384*a^3*b*c^4*d^2*e^6 - 48*a^3*b^2*c^3*d*e^7))*(-(a*b^3*e^
2 - a*e^2*(-(4*a*c - b^2)^3)^(1/2) + b^3*c*d^2 + c*d^2*(-(4*a*c - b^2)^3)^(
1/2) - 4*a*b*c^2*d^2 - 4*a^2*b*c*e^2 + 16*a^2*c^2*d*e - 4*a*b^2*c*d*e)/(8*(
a^2*b^4*e^4 + 16*a^2*c^4*d^4 + 16*a^4*c^2*e^4 + b^4*c^2*d^4 + b^6*d^2*e^2 -
 8*a*b^2*c^3*d^4 - 8*a^3*b^2*c*e^4 + 32*a^3*c^3*d^2*e^2 - 2*a*b^5*d*e^3 - 2
*b^5*c*d^3*e + 16*a*b^3*c^2*d^3*e - 6*a*b^4*c*d^2*e^2 - 32*a^2*b*c^3*d^3*e
+ 16*a^2*b^3*c*d*e^3 - 32*a^3*b*c^2*d*e^3)))^(1/2) + 4*a*c^5*d^4*e^2 + 52*a
^2*c^4*d^2*e^4 - 8*a*b*c^4*d^3*e^3 + 4*a*b^3*c^2*d*e^5 - 20*a^2*b*c^3*d*e^5
 - 8*a*b^2*c^3*d^2*e^4))*(-(a*b^3*e^2 - a*e^2*(-(4*a*c - b^2)^3)^(1/2) + b^
3*c*d^2 + c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a*b*c^2*d^2 - 4*a^2*b*c*e^2 +
16*a^2*c^2*d*e - 4*a*b^2*c*d*e)/(8*(a^2*b^4*e^4 + 16*a^2*c^4*d^4 + 16*a^4*c
^2*e^4 + b^4*c^2*d^4 + b^6*d^2*e^2 - 8*a*b^2*c^3*d^4 - 8*a^3*b^2*c*e^4 + 32
*a^3*c^3*d^2*e^2 - 2*a*b^5*d*e^3 - 2*b^5*c*d^3*e + 16*a*b^3*c^2*d^3*e - 6*a
*b^4*c*d^2*e^2 - 32*a^2*b*c^3*d^3*e + 16*a^2*b^3*c*d*e^3 - 32*a^3*b*c^2*d*e
^3)))^(1/2) + 2*a*c^3*d*e^3))*(-(a*b^3*e^2 - a*e^2*(-(4*a*c - b^2)^3)^(1/2)
 + b^3*c*d^2 + c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a*b*c^2*d^2 - 4*a^2*b*c*e
^2 + 16*a^2*c^2*d*e - 4*a*b^2*c*d*e)/(8*(a^2*b^4*e^4 + 16*a^2*c^4*d^4 + 16*
a^4*c^2*e^4 + b^4*c^2*d^4 + b^6*d^2*e^2 - 8*a*b^2*c^3*d^4 - 8*a^3*b^2*c*e^4
 + 32*a^3*c^3*d^2*e^2 - 2*a*b^5*d*e^3 - 2*b^5*c*d^3*e + 16*a*b^3*c^2*d^3*e
- 6*a*b^4*c*d^2*e^2 - 32*a^2*b*c^3*d^3*e + 16*a^2*b^3*c*d*e^3 - 32*a^3*b*c^
2*d*e^3)))^(1/2)*2i - (log(b^4*d^3*e^5 - a*b^3*d^2*e^6 + a*c^3*d^5*e^3 - b^
3*c*d^4*e^4 + 2*a^2*c^2*d^3*e^5 + a^3*c*d*e^7 - b^4*e^3*x*(-d*e)^(5/2) - a*
b^3*e^5*x*(-d*e)^(3/2) - a^3*c*e^7*x*(-d*e)^(1/2) + 2*a*b*c^2*d^4*e^4 - 3*a
*b^2*c*d^3*e^5 + 2*a^2*b*c*d^2*e^6 - 2*a^2*c^2*e^3*x*(-d*e)^(5/2) + a*c^3*d
x(-d*e)^(7/2) - b^3*c*e*x*(-d*e)^(7/2) + 2*a*b*c^2*e*x*(-d*e)^(7/2) + 3*a
*b^2*c*e^3*x*(-d*e)^(5/2) + 2*a^2*b*c*e^5*x*(-d*e)^(3/2))*(-d*e)^(1/2))/(2*
(a*e^2 + c*d^2 - b*d*e)) - atan(((x*(2*a^2*c^3*e^5 - 4*a*c^4*d^2*e^3 + 2*b^
2*c^3*d^2*e^3) - (-(a*b^3*e^2 + a*e^2*(-(4*a*c - b^2)^3)^(1/2) + b^3*c*d^2
- c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a*b*c^2*d^2 - 4*a^2*b*c*e^2 + 16*a^2*c
^2*d*e - 4*a*b^2*c*d*e)/(8*(a^2*b^4*e^4 + 16*a^2*c^4*d^4 + 16*a^4*c^2*e^4 +
 b^4*c^2*d^4 + b^6*d^2*e^2 - 8*a*b^2*c^3*d^4 - 8*a^3*b^2*c*e^4 + 32*a^3*c^3
*d^2*e^2 - 2*a*b^5*d*e^3 - 2*b^5*c*d^3*e + 16*a*b^3*c^2*d^3*e - 6*a*b^4*c*d
^2*e^2 - 32*a^2*b*c^3*d^3*e + 16*a^2*b^3*c*d*e^3 - 32*a^3*b*c^2*d*e^3)))^(1
/2)*((x*(32*a^3*b*c^3*e^7 + 16*a*c^6*d^5*e^2 - 112*a^3*c^4*d*e^6 - 8*a^2*b^
3*c^2*e^7 + 160*a^2*c^5*d^3*e^4 - 8*b^2*c^5*d^5*e^2 + 8*b^3*c^4*d^4*e^3 + 8

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*b^4*c^3*d^3*e^4 - 8*b^5*c^2*d^2*e^5 - 96*a*b^2*c^4*d^3*e^4 + 64*a*b^3*c^3*
d^2*e^5 - 96*a^2*b*c^4*d^2*e^5 + 24*a^2*b^2*c^3*d*e^6) + (-(a*b^3*e^2 + a*e
^2*(-(4*a*c - b^2)^3)^(1/2) + b^3*c*d^2 - c*d^2*(-(4*a*c - b^2)^3)^(1/2) -
4*a*b*c^2*d^2 - 4*a^2*b*c*e^2 + 16*a^2*c^2*d*e - 4*a*b^2*c*d*e)/(8*(a^2*b^4
*e^4 + 16*a^2*c^4*d^4 + 16*a^4*c^2*e^4 + b^4*c^2*d^4 + b^6*d^2*e^2 - 8*a*b^
2*c^3*d^4 - 8*a^3*b^2*c*e^4 + 32*a^3*c^3*d^2*e^2 - 2*a*b^5*d*e^3 - 2*b^5*c*
d^3*e + 16*a*b^3*c^2*d^3*e - 6*a*b^4*c*d^2*e^2 - 32*a^2*b*c^3*d^3*e + 16*a^
2*b^3*c*d*e^3 - 32*a^3*b*c^2*d*e^3)))^(1/2)*(x*(-(a*b^3*e^2 + a*e^2*(-(4*a*
c - b^2)^3)^(1/2) + b^3*c*d^2 - c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a*b*c^2*
d^2 - 4*a^2*b*c*e^2 + 16*a^2*c^2*d*e - 4*a*b^2*c*d*e)/(8*(a^2*b^4*e^4 + 16*
a^2*c^4*d^4 + 16*a^4*c^2*e^4 + b^4*c^2*d^4 + b^6*d^2*e^2 - 8*a*b^2*c^3*d^4
- 8*a^3*b^2*c*e^4 + 32*a^3*c^3*d^2*e^2 - 2*a*b^5*d*e^3 - 2*b^5*c*d^3*e + 16
*a*b^3*c^2*d^3*e - 6*a*b^4*c*d^2*e^2 - 32*a^2*b*c^3*d^3*e + 16*a^2*b^3*c*d*
e^3 - 32*a^3*b*c^2*d*e^3)))^(1/2)*(256*a^4*b^2*c^3*e^9 - 32*a^3*b^4*c^2*e^9
 - 512*a^5*c^4*e^9 + 512*a^2*c^7*d^6*e^3 + 512*a^3*c^6*d^4*e^5 - 512*a^4*c^
5*d^2*e^7 - 32*b^3*c^6*d^7*e^2 + 128*b^4*c^5*d^6*e^3 - 192*b^5*c^4*d^5*e^4
+ 128*b^6*c^3*d^4*e^5 - 32*b^7*c^2*d^3*e^6 + 512*a^2*b^2*c^5*d^4*e^5 + 288*
a^2*b^3*c^4*d^3*e^6 - 192*a^2*b^4*c^3*d^2*e^7 + 384*a^3*b^2*c^4*d^2*e^7 + 1
28*a*b*c^7*d^7*e^2 + 640*a^4*b*c^4*d*e^8 - 640*a*b^2*c^6*d^6*e^3 + 1056*a*b
^3*c^5*d^5*e^4 - 672*a*b^4*c^4*d^4*e^5 + 96*a*b^5*c^3*d^3*e^6 + 32*a*b^6*c^
2*d^2*e^7 - 1152*a^2*b*c^6*d^5*e^4 + 32*a^2*b^5*c^2*d*e^8 - 640*a^3*b*c^5*d
^3*e^6 - 288*a^3*b^3*c^3*d*e^8) - 192*a^4*c^4*d*e^7 - 192*a^2*c^6*d^5*e^3 -
 384*a^3*c^5*d^3*e^5 - 96*a^2*b^2*c^4*d^3*e^5 - 96*a^2*b^3*c^3*d^2*e^6 + 48
*a*b^2*c^5*d^5*e^3 - 96*a*b^3*c^4*d^4*e^4 + 48*a*b^4*c^3*d^3*e^5 + 384*a^2*
b*c^5*d^4*e^4 + 384*a^3*b*c^4*d^2*e^6 + 48*a^3*b^2*c^3*d*e^7))*(-(a*b^3*e^2
 + a*e^2*(-(4*a*c - b^2)^3)^(1/2) + b^3*c*d^2 - c*d^2*(-(4*a*c - b^2)^3)^(1
/2) - 4*a*b*c^2*d^2 - 4*a^2*b*c*e^2 + 16*a^2*c^2*d*e - 4*a*b^2*c*d*e)/(8*(a
^2*b^4*e^4 + 16*a^2*c^4*d^4 + 16*a^4*c^2*e^4 + b^4*c^2*d^4 + b^6*d^2*e^2 -
8*a*b^2*c^3*d^4 - 8*a^3*b^2*c*e^4 + 32*a^3*c^3*d^2*e^2 - 2*a*b^5*d*e^3 - 2*
b^5*c*d^3*e + 16*a*b^3*c^2*d^3*e - 6*a*b^4*c*d^2*e^2 - 32*a^2*b*c^3*d^3*e +
 16*a^2*b^3*c*d*e^3 - 32*a^3*b*c^2*d*e^3)))^(1/2) - 4*a*c^5*d^4*e^2 - 52*a^
2*c^4*d^2*e^4 + 8*a*b*c^4*d^3*e^3 - 4*a*b^3*c^2*d*e^5 + 20*a^2*b*c^3*d*e^5
+ 8*a*b^2*c^3*d^2*e^4))*(-(a*b^3*e^2 + a*e^2*(-(4*a*c - b^2)^3)^(1/2) + b^3
*c*d^2 - c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a*b*c^2*d^2 - 4*a^2*b*c*e^2 + 1
6*a^2*c^2*d*e - 4*a*b^2*c*d*e)/(8*(a^2*b^4*e^4 + 16*a^2*c^4*d^4 + 16*a^4*c^
2*e^4 + b^4*c^2*d^4 + b^6*d^2*e^2 - 8*a*b^2*c^3*d^4 - 8*a^3*b^2*c*e^4 + 32*
a^3*c^3*d^2*e^2 - 2*a*b^5*d*e^3 - 2*b^5*c*d^3*e + 16*a*b^3*c^2*d^3*e - 6*a*
b^4*c*d^2*e^2 - 32*a^2*b*c^3*d^3*e + 16*a^2*b^3*c*d*e^3 - 32*a^3*b*c^2*d*e^
3)))^(1/2)*1i + (x*(2*a^2*c^3*e^5 - 4*a*c^4*d^2*e^3 + 2*b^2*c^3*d^2*e^3) -
(-(a*b^3*e^2 + a*e^2*(-(4*a*c - b^2)^3)^(1/2) + b^3*c*d^2 - c*d^2*(-(4*a*c
- b^2)^3)^(1/2) - 4*a*b*c^2*d^2 - 4*a^2*b*c*e^2 + 16*a^2*c^2*d*e - 4*a*b^2*
c*d*e)/(8*(a^2*b^4*e^4 + 16*a^2*c^4*d^4 + 16*a^4*c^2*e^4 + b^4*c^2*d^4 + b^
6*d^2*e^2 - 8*a*b^2*c^3*d^4 - 8*a^3*b^2*c*e^4 + 32*a^3*c^3*d^2*e^2 - 2*a*b^
5*d*e^3 - 2*b^5*c*d^3*e + 16*a*b^3*c^2*d^3*e - 6*a*b^4*c*d^2*e^2 - 32*a^2*b
*c^3*d^3*e + 16*a^2*b^3*c*d*e^3 - 32*a^3*b*c^2*d*e^3)))^(1/2)*((x*(32*a^3*b
```

*c^3*e^7 + 16*a*c^6*d^5*e^2 - 112*a^3*c^4*d*e^6 - 8*a^2*b^3*c^2*e^7 + 160*a
^2*c^5*d^3*e^4 - 8*b^2*c^5*d^5*e^2 + 8*b^3*c^4*d^4*e^3 + 8*b^4*c^3*d^3*e^4
- 8*b^5*c^2*d^2*e^5 - 96*a*b^2*c^4*d^3*e^4 + 64*a*b^3*c^3*d^2*e^5 - 96*a^2*
b*c^4*d^2*e^5 + 24*a^2*b^2*c^3*d*e^6) + (-(a*b^3*e^2 + a*e^2*(-(4*a*c - b^2
)^3)^(1/2) + b^3*c*d^2 - c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a*b*c^2*d^2 - 4
*a^2*b*c*e^2 + 16*a^2*c^2*d*e - 4*a*b^2*c*d*e)/(8*(a^2*b^4*e^4 + 16*a^2*c^4
*d^4 + 16*a^4*c^2*e^4 + b^4*c^2*d^4 + b^6*d^2*e^2 - 8*a*b^2*c^3*d^4 - 8*a^3
*b^2*c*e^4 + 32*a^3*c^3*d^2*e^2 - 2*a*b^5*d*e^3 - 2*b^5*c*d^3*e + 16*a*b^3*
c^2*d^3*e - 6*a*b^4*c*d^2*e^2 - 32*a^2*b*c^3*d^3*e + 16*a^2*b^3*c*d*e^3 - 3
2*a^3*b*c^2*d*e^3)))^(1/2)*(x*(-(a*b^3*e^2 + a*e^2*(-(4*a*c - b^2)^3)^(1/2)
 + b^3*c*d^2 - c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a*b*c^2*d^2 - 4*a^2*b*c*e
^2 + 16*a^2*c^2*d*e - 4*a*b^2*c*d*e)/(8*(a^2*b^4*e^4 + 16*a^2*c^4*d^4 + 16*
a^4*c^2*e^4 + b^4*c^2*d^4 + b^6*d^2*e^2 - 8*a*b^2*c^3*d^4 - 8*a^3*b^2*c*e^4
 + 32*a^3*c^3*d^2*e^2 - 2*a*b^5*d*e^3 - 2*b^5*c*d^3*e + 16*a*b^3*c^2*d^3*e
- 6*a*b^4*c*d^2*e^2 - 32*a^2*b*c^3*d^3*e + 16*a^2*b^3*c*d*e^3 - 32*a^3*b*c^
2*d*e^3)))^(1/2)*(256*a^4*b^2*c^3*e^9 - 32*a^3*b^4*c^2*e^9 - 512*a^5*c^4*e^
9 + 512*a^2*c^7*d^6*e^3 + 512*a^3*c^6*d^4*e^5 - 512*a^4*c^5*d^2*e^7 - 32*b^
3*c^6*d^7*e^2 + 128*b^4*c^5*d^6*e^3 - 192*b^5*c^4*d^5*e^4 + 128*b^6*c^3*d^4
*e^5 - 32*b^7*c^2*d^3*e^6 + 512*a^2*b^2*c^5*d^4*e^5 + 288*a^2*b^3*c^4*d^3*e
^6 - 192*a^2*b^4*c^3*d^2*e^7 + 384*a^3*b^2*c^4*d^2*e^7 + 128*a*b*c^7*d^7*e^
2 + 640*a^4*b*c^4*d*e^8 - 640*a*b^2*c^6*d^6*e^3 + 1056*a*b^3*c^5*d^5*e^4 -
672*a*b^4*c^4*d^4*e^5 + 96*a*b^5*c^3*d^3*e^6 + 32*a*b^6*c^2*d^2*e^7 - 1152*
a^2*b*c^6*d^5*e^4 + 32*a^2*b^5*c^2*d*e^8 - 640*a^3*b*c^5*d^3*e^6 - 288*a^3*
b^3*c^3*d*e^8) + 192*a^4*c^4*d*e^7 + 192*a^2*c^6*d^5*e^3 + 384*a^3*c^5*d^3*
e^5 + 96*a^2*b^2*c^4*d^3*e^5 + 96*a^2*b^3*c^3*d^2*e^6 - 48*a*b^2*c^5*d^5*e^
3 + 96*a*b^3*c^4*d^4*e^4 - 48*a*b^4*c^3*d^3*e^5 - 384*a^2*b*c^5*d^4*e^4 - 3
84*a^3*b*c^4*d^2*e^6 - 48*a^3*b^2*c^3*d*e^7))*(-(a*b^3*e^2 + a*e^2*(-(4*a*c
 - b^2)^3)^(1/2) + b^3*c*d^2 - c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a*b*c^2*d
^2 - 4*a^2*b*c*e^2 + 16*a^2*c^2*d*e - 4*a*b^2*c*d*e)/(8*(a^2*b^4*e^4 + 16*a
^2*c^4*d^4 + 16*a^4*c^2*e^4 + b^4*c^2*d^4 + b^6*d^2*e^2 - 8*a*b^2*c^3*d^4 -
 8*a^3*b^2*c*e^4 + 32*a^3*c^3*d^2*e^2 - 2*a*b^5*d*e^3 - 2*b^5*c*d^3*e + 16*
a*b^3*c^2*d^3*e - 6*a*b^4*c*d^2*e^2 - 32*a^2*b*c^3*d^3*e + 16*a^2*b^3*c*d*e
^3 - 32*a^3*b*c^2*d*e^3)))^(1/2) + 4*a*c^5*d^4*e^2 + 52*a^2*c^4*d^2*e^4 - 8
*a*b*c^4*d^3*e^3 + 4*a*b^3*c^2*d*e^5 - 20*a^2*b*c^3*d*e^5 - 8*a*b^2*c^3*d^2
e^4))(-(a*b^3*e^2 + a*e^2*(-(4*a*c - b^2)^3)^(1/2) + b^3*c*d^2 - c*d^2*(-
(4*a*c - b^2)^3)^(1/2) - 4*a*b*c^2*d^2 - 4*a^2*b*c*e^2 + 16*a^2*c^2*d*e - 4
*a*b^2*c*d*e)/(8*(a^2*b^4*e^4 + 16*a^2*c^4*d^4 + 16*a^4*c^2*e^4 + b^4*c^2*d
^4 + b^6*d^2*e^2 - 8*a*b^2*c^3*d^4 - 8*a^3*b^2*c*e^4 + 32*a^3*c^3*d^2*e^2 -
 2*a*b^5*d*e^3 - 2*b^5*c*d^3*e + 16*a*b^3*c^2*d^3*e - 6*a*b^4*c*d^2*e^2 - 3
2*a^2*b*c^3*d^3*e + 16*a^2*b^3*c*d*e^3 - 32*a^3*b*c^2*d*e^3)))^(1/2)*1i)/((
x*(2*a^2*c^3*e^5 - 4*a*c^4*d^2*e^3 + 2*b^2*c^3*d^2*e^3) - (-(a*b^3*e^2 + a*
e^2*(-(4*a*c - b^2)^3)^(1/2) + b^3*c*d^2 - c*d^2*(-(4*a*c - b^2)^3)^(1/2) -
 4*a*b*c^2*d^2 - 4*a^2*b*c*e^2 + 16*a^2*c^2*d*e - 4*a*b^2*c*d*e)/(8*(a^2*b^
4*e^4 + 16*a^2*c^4*d^4 + 16*a^4*c^2*e^4 + b^4*c^2*d^4 + b^6*d^2*e^2 - 8*a*b
^2*c^3*d^4 - 8*a^3*b^2*c*e^4 + 32*a^3*c^3*d^2*e^2 - 2*a*b^5*d*e^3 - 2*b^5*c

*d^3*e + 16*a*b^3*c^2*d^3*e - 6*a*b^4*c*d^2*e^2 - 32*a^2*b*c^3*d^3*e + 16*a ^2*b^3*c*d*e^3 - 32*a^3*b*c^2*d*e^3)))^(1/2)*((x*(32*a^3*b*c^3*e^7 + 16*a*c ^6*d^5*e^2 - 112*a^3*c^4*d*e^6 - 8*a^2*b^3*c^2*e^7 + 160*a^2*c^5*d^3*e^4 - 8*b^2*c^5*d^5*e^2 + 8*b^3*c^4*d^4*e^3 + 8*b^4*c^3*d^3*e^4 - 8*b^5*c^2*d^2*e ^5 - 96*a*b^2*c^4*d^3*e^4 + 64*a*b^3*c^3*d^2*e^5 - 96*a^2*b*c^4*d^2*e^5 + 2 4*a^2*b^2*c^3*d*e^6) + (-(a*b^3*e^2 + a*e^2*(-(4*a*c - b^2)^3)^(1/2) + b^3* c*d^2 - c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a*b*c^2*d^2 - 4*a^2*b*c*e^2 + 16 *a^2*c^2*d*e - 4*a*b^2*c*d*e)/(8*(a^2*b^4*e^4 + 16*a^2*c^4*d^4 + 16*a^4*c^2 *e^4 + b^4*c^2*d^4 + b^6*d^2*e^2 - 8*a*b^2*c^3*d^4 - 8*a^3*b^2*c*e^4 + 32*a ^3*c^3*d^2*e^2 - 2*a*b^5*d*e^3 - 2*b^5*c*d^3*e + 16*a*b^3*c^2*d^3*e - 6*a*b ^4*c*d^2*e^2 - 32*a^2*b*c^3*d^3*e + 16*a^2*b^3*c*d*e^3 - 32*a^3*b*c^2*d*e^3)))^(1/2)*(x*(-(a*b^3*e^2 + a*e^2*(-(4*a*c - b^2)^3)^(1/2) + b^3*c*d^2 - c* d^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a*b*c^2*d^2 - 4*a^2*b*c*e^2 + 16*a^2*c^2*d *e - 4*a*b^2*c*d*e)/(8*(a^2*b^4*e^4 + 16*a^2*c^4*d^4 + 16*a^4*c^2*e^4 + b^4 *c^2*d^4 + b^6*d^2*e^2 - 8*a*b^2*c^3*d^4 - 8*a^3*b^2*c*e^4 + 32*a^3*c^3*d^2 *e^2 - 2*a*b^5*d*e^3 - 2*b^5*c*d^3*e + 16*a*b^3*c^2*d^3*e - 6*a*b^4*c*d^2*e ^2 - 32*a^2*b*c^3*d^3*e + 16*a^2*b^3*c*d*e^3 - 32*a^3*b*c^2*d*e^3)))^(1/2)* (256*a^4*b^2*c^3*e^9 - 32*a^3*b^4*c^2*e^9 - 512*a^5*c^4*e^9 + 512*a^2*c^7*d ^6*e^3 + 512*a^3*c^6*d^4*e^5 - 512*a^4*c^5*d^2*e^7 - 32*b^3*c^6*d^7*e^2 + 1 28*b^4*c^5*d^6*e^3 - 192*b^5*c^4*d^5*e^4 + 128*b^6*c^3*d^4*e^5 - 32*b^7*c^2 *d^3*e^6 + 512*a^2*b^2*c^5*d^4*e^5 + 288*a^2*b^3*c^4*d^3*e^6 - 192*a^2*b^4* c^3*d^2*e^7 + 384*a^3*b^2*c^4*d^2*e^7 + 128*a*b*c^7*d^7*e^2 + 640*a^4*b*c^4 *d*e^8 - 640*a*b^2*c^6*d^6*e^3 + 1056*a*b^3*c^5*d^5*e^4 - 672*a*b^4*c^4*d^4 *e^5 + 96*a*b^5*c^3*d^3*e^6 + 32*a*b^6*c^2*d^2*e^7 - 1152*a^2*b*c^6*d^5*e^4 + 32*a^2*b^5*c^2*d*e^8 - 640*a^3*b*c^5*d^3*e^6 - 288*a^3*b^3*c^3*d*e^8) - 192*a^4*c^4*d*e^7 - 192*a^2*c^6*d^5*e^3 - 384*a^3*c^5*d^3*e^5 - 96*a^2*b^2* c^4*d^3*e^5 - 96*a^2*b^3*c^3*d^2*e^6 + 48*a*b^2*c^5*d^5*e^3 - 96*a*b^3*c^4* d^4*e^4 + 48*a*b^4*c^3*d^3*e^5 + 384*a^2*b*c^5*d^4*e^4 + 384*a^3*b*c^4*d^2* e^6 + 48*a^3*b^2*c^3*d*e^7))*(-(a*b^3*e^2 + a*e^2*(-(4*a*c - b^2)^3)^(1/2) + b^3*c*d^2 - c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a*b*c^2*d^2 - 4*a^2*b*c*e^ 2 + 16*a^2*c^2*d*e - 4*a*b^2*c*d*e)/(8*(a^2*b^4*e^4 + 16*a^2*c^4*d^4 + 16*a ^4*c^2*e^4 + b^4*c^2*d^4 + b^6*d^2*e^2 - 8*a*b^2*c^3*d^4 - 8*a^3*b^2*c*e^4 + 32*a^3*c^3*d^2*e^2 - 2*a*b^5*d*e^3 - 2*b^5*c*d^3*e + 16*a*b^3*c^2*d^3*e - 6*a*b^4*c*d^2*e^2 - 32*a^2*b*c^3*d^3*e + 16*a^2*b^3*c*d*e^3 - 32*a^3*b*c^2 *d*e^3)))^(1/2) - 4*a*c^5*d^4*e^2 - 52*a^2*c^4*d^2*e^4 + 8*a*b*c^4*d^3*e^3 - 4*a*b^3*c^2*d*e^5 + 20*a^2*b*c^3*d*e^5 + 8*a*b^2*c^3*d^2*e^4))*(-(a*b^3*e ^2 + a*e^2*(-(4*a*c - b^2)^3)^(1/2) + b^3*c*d^2 - c*d^2*(-(4*a*c - b^2)^3)^ (1/2) - 4*a*b*c^2*d^2 - 4*a^2*b*c*e^2 + 16*a^2*c^2*d*e - 4*a*b^2*c*d*e)/(8* (a^2*b^4*e^4 + 16*a^2*c^4*d^4 + 16*a^4*c^2*e^4 + b^4*c^2*d^4 + b^6*d^2*e^2 - 8*a*b^2*c^3*d^4 - 8*a^3*b^2*c*e^4 + 32*a^3*c^3*d^2*e^2 - 2*a*b^5*d*e^3 - 2*b^5*c*d^3*e + 16*a*b^3*c^2*d^3*e - 6*a*b^4*c*d^2*e^2 - 32*a^2*b*c^3*d^3*e + 16*a^2*b^3*c*d*e^3 - 32*a^3*b*c^2*d*e^3)))^(1/2) - (x*(2*a^2*c^3*e^5 - 4 *a*c^4*d^2*e^3 + 2*b^2*c^3*d^2*e^3) - (-(a*b^3*e^2 + a*e^2*(-(4*a*c - b^2)^ 3)^(1/2) + b^3*c*d^2 - c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a*b*c^2*d^2 - 4*a ^2*b*c*e^2 + 16*a^2*c^2*d*e - 4*a*b^2*c*d*e)/(8*(a^2*b^4*e^4 + 16*a^2*c^4*d

^4 + 16*a^4*c^2*e^4 + b^4*c^2*d^4 + b^6*d^2*e^2 - 8*a*b^2*c^3*d^4 - 8*a^3*b
^2*c*e^4 + 32*a^3*c^3*d^2*e^2 - 2*a*b^5*d*e^3 - 2*b^5*c*d^3*e + 16*a*b^3*c^
2*d^3*e - 6*a*b^4*c*d^2*e^2 - 32*a^2*b*c^3*d^3*e + 16*a^2*b^3*c*d*e^3 - 32*
a^3*b*c^2*d*e^3)))^(1/2)*((x*(32*a^3*b*c^3*e^7 + 16*a*c^6*d^5*e^2 - 112*a^3
*c^4*d*e^6 - 8*a^2*b^3*c^2*e^7 + 160*a^2*c^5*d^3*e^4 - 8*b^2*c^5*d^5*e^2 +
8*b^3*c^4*d^4*e^3 + 8*b^4*c^3*d^3*e^4 - 8*b^5*c^2*d^2*e^5 - 96*a*b^2*c^4*d^
3*e^4 + 64*a*b^3*c^3*d^2*e^5 - 96*a^2*b*c^4*d^2*e^5 + 24*a^2*b^2*c^3*d*e^6)
 + (-(a*b^3*e^2 + a*e^2*(-(4*a*c - b^2)^3)^(1/2) + b^3*c*d^2 - c*d^2*(-(4*a
*c - b^2)^3)^(1/2) - 4*a*b*c^2*d^2 - 4*a^2*b*c*e^2 + 16*a^2*c^2*d*e - 4*a*b
^2*c*d*e)/(8*(a^2*b^4*e^4 + 16*a^2*c^4*d^4 + 16*a^4*c^2*e^4 + b^4*c^2*d^4 +
 b^6*d^2*e^2 - 8*a*b^2*c^3*d^4 - 8*a^3*b^2*c*e^4 + 32*a^3*c^3*d^2*e^2 - 2*a
*b^5*d*e^3 - 2*b^5*c*d^3*e + 16*a*b^3*c^2*d^3*e - 6*a*b^4*c*d^2*e^2 - 32*a^
2*b*c^3*d^3*e + 16*a^2*b^3*c*d*e^3 - 32*a^3*b*c^2*d*e^3)))^(1/2)*(x*(-(a*b^
3*e^2 + a*e^2*(-(4*a*c - b^2)^3)^(1/2) + b^3*c*d^2 - c*d^2*(-(4*a*c - b^2)^
3)^(1/2) - 4*a*b*c^2*d^2 - 4*a^2*b*c*e^2 + 16*a^2*c^2*d*e - 4*a*b^2*c*d*e)/
(8*(a^2*b^4*e^4 + 16*a^2*c^4*d^4 + 16*a^4*c^2*e^4 + b^4*c^2*d^4 + b^6*d^2*e
^2 - 8*a*b^2*c^3*d^4 - 8*a^3*b^2*c*e^4 + 32*a^3*c^3*d^2*e^2 - 2*a*b^5*d*e^3
 - 2*b^5*c*d^3*e + 16*a*b^3*c^2*d^3*e - 6*a*b^4*c*d^2*e^2 - 32*a^2*b*c^3*d^
3*e + 16*a^2*b^3*c*d*e^3 - 32*a^3*b*c^2*d*e^3)))^(1/2)*(256*a^4*b^2*c^3*e^9
 - 32*a^3*b^4*c^2*e^9 - 512*a^5*c^4*e^9 + 512*a^2*c^7*d^6*e^3 + 512*a^3*c^6
*d^4*e^5 - 512*a^4*c^5*d^2*e^7 - 32*b^3*c^6*d^7*e^2 + 128*b^4*c^5*d^6*e^3 -
 192*b^5*c^4*d^5*e^4 + 128*b^6*c^3*d^4*e^5 - 32*b^7*c^2*d^3*e^6 + 512*a^2*b
^2*c^5*d^4*e^5 + 288*a^2*b^3*c^4*d^3*e^6 - 192*a^2*b^4*c^3*d^2*e^7 + 384*a^
3*b^2*c^4*d^2*e^7 + 128*a*b*c^7*d^7*e^2 + 640*a^4*b*c^4*d*e^8 - 640*a*b^2*c
^6*d^6*e^3 + 1056*a*b^3*c^5*d^5*e^4 - 672*a*b^4*c^4*d^4*e^5 + 96*a*b^5*c^3*
d^3*e^6 + 32*a*b^6*c^2*d^2*e^7 - 1152*a^2*b*c^6*d^5*e^4 + 32*a^2*b^5*c^2*d*
e^8 - 640*a^3*b*c^5*d^3*e^6 - 288*a^3*b^3*c^3*d*e^8) + 192*a^4*c^4*d*e^7 +
192*a^2*c^6*d^5*e^3 + 384*a^3*c^5*d^3*e^5 + 96*a^2*b^2*c^4*d^3*e^5 + 96*a^2
*b^3*c^3*d^2*e^6 - 48*a*b^2*c^5*d^5*e^3 + 96*a*b^3*c^4*d^4*e^4 - 48*a*b^4*c
^3*d^3*e^5 - 384*a^2*b*c^5*d^4*e^4 - 384*a^3*b*c^4*d^2*e^6 - 48*a^3*b^2*c^3
*d*e^7))*(-(a*b^3*e^2 + a*e^2*(-(4*a*c - b^2)^3)^(1/2) + b^3*c*d^2 - c*d^2*
(-(4*a*c - b^2)^3)^(1/2) - 4*a*b*c^2*d^2 - 4*a^2*b*c*e^2 + 16*a^2*c^2*d*e -
 4*a*b^2*c*d*e)/(8*(a^2*b^4*e^4 + 16*a^2*c^4*d^4 + 16*a^4*c^2*e^4 + b^4*c^2
*d^4 + b^6*d^2*e^2 - 8*a*b^2*c^3*d^4 - 8*a^3*b^2*c*e^4 + 32*a^3*c^3*d^2*e^2
 - 2*a*b^5*d*e^3 - 2*b^5*c*d^3*e + 16*a*b^3*c^2*d^3*e - 6*a*b^4*c*d^2*e^2 -
 32*a^2*b*c^3*d^3*e + 16*a^2*b^3*c*d*e^3 - 32*a^3*b*c^2*d*e^3)))^(1/2) + 4*
a*c^5*d^4*e^2 + 52*a^2*c^4*d^2*e^4 - 8*a*b*c^4*d^3*e^3 + 4*a*b^3*c^2*d*e^5
 - 20*a^2*b*c^3*d*e^5 - 8*a*b^2*c^3*d^2*e^4))*(-(a*b^3*e^2 + a*e^2*(-(4*a*c
 - b^2)^3)^(1/2) + b^3*c*d^2 - c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a*b*c^2*d^
2 - 4*a^2*b*c*e^2 + 16*a^2*c^2*d*e - 4*a*b^2*c*d*e)/(8*(a^2*b^4*e^4 + 16*a^
2*c^4*d^4 + 16*a^4*c^2*e^4 + b^4*c^2*d^4 + b^6*d^2*e^2 - 8*a*b^2*c^3*d^4 -
8*a^3*b^2*c*e^4 + 32*a^3*c^3*d^2*e^2 - 2*a*b^5*d*e^3 - 2*b^5*c*d^3*e + 16*a
*b^3*c^2*d^3*e - 6*a*b^4*c*d^2*e^2 - 32*a^2*b*c^3*d^3*e + 16*a^2*b^3*c*d*e^
3 - 32*a^3*b*c^2*d*e^3)))^(1/2) + 2*a*c^3*d*e^3))*(-(a*b^3*e^2 + a*e^2*(-(4
*a*c - b^2)^3)^(1/2) + b^3*c*d^2 - c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a*b*c

```
^2*d^2 - 4*a^2*b*c*e^2 + 16*a^2*c^2*d*e - 4*a*b^2*c*d*e)/(8*(a^2*b^4*e^4 +
16*a^2*c^4*d^4 + 16*a^4*c^2*e^4 + b^4*c^2*d^4 + b^6*d^2*e^2 - 8*a*b^2*c^3*d
^4 - 8*a^3*b^2*c*e^4 + 32*a^3*c^3*d^2*e^2 - 2*a*b^5*d*e^3 - 2*b^5*c*d^3*e +
 16*a*b^3*c^2*d^3*e - 6*a*b^4*c*d^2*e^2 - 32*a^2*b*c^3*d^3*e + 16*a^2*b^3*c
*d*e^3 - 32*a^3*b*c^2*d*e^3)))^(1/2)*2i
```

3.307 $\int \frac{1}{(d+ex^2)(a+bx^2+cx^4)}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 254

$$\int \frac{1}{(d+ex^2)(a+bx^2+cx^4)}\,dx = -\frac{\sqrt{c}\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{b-\sqrt{b^2-4ac}}\,(cd^2-bde+ae^2)} - \frac{\sqrt{c}\left(e+\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{b+\sqrt{b^2-4ac}}\,(cd^2-bde+ae^2)} + \frac{e^{3/2}\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{d}\,(cd^2-bde+ae^2)}$$

```
[Out] e^(3/2)*arctan(x*e^(1/2)/d^(1/2))/(a*e^2-b*d*e+c*d^2)/d^(1/2)-1/2*arctan(x*
2^(1/2)*c^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2))*c^(1/2)*(e+(b*e-2*c*d)/(-4*a*
c+b^2)^(1/2))/(a*e^2-b*d*e+c*d^2)*2^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2)-1/2*
arctan(x*2^(1/2)*c^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2))*c^(1/2)*(e+(-b*e+2*c
*d)/(-4*a*c+b^2)^(1/2))/(a*e^2-b*d*e+c*d^2)*2^(1/2)/(b+(-4*a*c+b^2)^(1/2))^
(1/2)
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 254, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used

$= \{1184, 211, 1180\}$

$$\int \frac{1}{(d+ex^2)(a+bx^2+cx^4)}\,dx = -\frac{\sqrt{c}\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)}{\sqrt{2}\sqrt{b-\sqrt{b^2-4ac}}\,(ae^2-bde+cd^2)} - \frac{\sqrt{c}\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)\left(\frac{2cd-be}{\sqrt{b^2-4ac}}+e\right)}{\sqrt{2}\sqrt{\sqrt{b^2-4ac}+b}\,(ae^2-bde+cd^2)} + \frac{e^{3/2}\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{d}\,(ae^2-bde+cd^2)}$$

```
[In] Int[1/((d + e*x^2)*(a + b*x^2 + c*x^4)),x]
```

```
[Out] -((Sqrt[c]*(e - (2*c*d - b*e)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)
/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*Sqrt[b - Sqrt[b^2 - 4*a*c]]*(c*d^2
- b*d*e + a*e^2))) - (Sqrt[c]*(e + (2*c*d - b*e)/Sqrt[b^2 - 4*a*c])*ArcTan[
(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*Sqrt[b + Sqrt[b^
2 - 4*a*c]]*(c*d^2 - b*d*e + a*e^2)) + (e^(3/2)*ArcTan[(Sqrt[e]*x)/Sqrt[d]]
)/(Sqrt[d]*(c*d^2 - b*d*e + a*e^2))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1184

```
Int[((d_) + (e_.)*(x_)^2)^(q_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symb
ol] :> Int[ExpandIntegrand[(d + e*x^2)^q/(a + b*x^2 + c*x^4), x], x] /; Fre
eQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2,
 0] && IntegerQ[q]
```

Rubi steps

$$\text{integral} = \int \left(\frac{e^2}{(cd^2-bde+ae^2)(d+ex^2)} + \frac{cd-be-cex^2}{(cd^2-bde+ae^2)(a+bx^2+cx^4)}\right) dx$$

$$= \frac{\int \frac{cd-be-cex^2}{a+bx^2+cx^4}\,dx}{cd^2-bde+ae^2} + \frac{e^2\int \frac{1}{d+ex^2}\,dx}{cd^2-bde+ae^2}$$

$$= \frac{e^{3/2}\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{d}\,(cd^2-bde+ae^2)} - \frac{\left(c\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\right)\int \frac{1}{\frac{b}{2}-\frac{1}{2}\sqrt{b^2-4ac}+cx^2}\,dx}{2\,(cd^2-bde+ae^2)} - \frac{\left(c\left(e+\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\right)\int \frac{1}{\frac{b}{2}+\frac{1}{2}\sqrt{b^2-4ac}+cx^2}\,dx}{2\,(cd^2-bde+ae^2)}$$

$$= -\frac{\sqrt{c}\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{b-\sqrt{b^2-4ac}}\,(cd^2-bde+ae^2)} - \frac{\sqrt{c}\left(e+\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{b+\sqrt{b^2-4ac}}\,(cd^2-bde+ae^2)} + \frac{e^{3/2}\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{d}\,(cd^2-bde+ae^2)}$$

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 274, normalized size of antiderivative = 1.08

$$\int \frac{1}{(d+ex^2)(a+bx^2+cx^4)}\,dx = \frac{\sqrt{c}\left(-2cd+be+\sqrt{b^2-4ac}e\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}\,(-cd^2+bde-ae^2)} + \frac{\sqrt{c}\left(2cd-be+\sqrt{b^2-4ac}e\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{b^2-4ac}\sqrt{b+\sqrt{b^2-4ac}}\,(-cd^2+bde-ae^2)} + \frac{e^{3/2}\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{d}\,(cd^2-bde+ae^2)}$$

```
[In] Integrate[1/((d + e*x^2)*(a + b*x^2 + c*x^4)),x]
```

```
[Out] (Sqrt[c]*(-2*c*d + b*e + Sqrt[b^2 - 4*a*c]*e)*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sq
rt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*Sqrt[b^2 - 4*a*c]*Sqrt[b - Sqrt[b^2 -
4*a*c]]*(-(c*d^2) + b*d*e - a*e^2)) + (Sqrt[c]*(2*c*d - b*e + Sqrt[b^2 - 4*
a*c]*e)*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*S
qrt[b^2 - 4*a*c]*Sqrt[b + Sqrt[b^2 - 4*a*c]]*(-(c*d^2) + b*d*e - a*e^2)) +
(e^(3/2)*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(Sqrt[d]*(c*d^2 - b*d*e + a*e^2))
```

Maple [A] (verified)

Time = 0.34 (sec) , antiderivative size = 215, normalized size of antiderivative = 0.85

method	result
default	$\frac{4c\left(\frac{\left(be-2cd-e\sqrt{-4ac+b^2}\right)\sqrt{2}\,\arctan\left(\frac{cx\sqrt{2}}{\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}}\right)}{8\sqrt{-4ac+b^2}\,\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}}-\frac{\left(-e\sqrt{-4ac+b^2}-be+2cd\right)\sqrt{2}\,\operatorname{arctanh}\left(\frac{cx\sqrt{2}}{\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}}\right)}{8\sqrt{-4ac+b^2}\,\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}}\right)}{a\,e^2-bde+c\,d^2}+\frac{e^2\,\mathrm{arct}}{(a\,e^2-bd}$
risch	Expression too large to display

```
[In] int(1/(e*x^2+d)/(c*x^4+b*x^2+a),x,method=_RETURNVERBOSE)
```

```
[Out] 4/(a*e^2-b*d*e+c*d^2)*c*(1/8*(b*e-2*c*d-e*(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^
(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a
*c+b^2)^(1/2))*c)^(1/2))-1/8*(-e*(-4*a*c+b^2)^(1/2)-b*e+2*c*d)/(-4*a*c+b^2)
^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(c*x*2^(1/2)/((-b+(
-4*a*c+b^2)^(1/2))*c)^(1/2)))+e^2/(a*e^2-b*d*e+c*d^2)/(e*d)^(1/2)*arctan(e*
x/(e*d)^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 7995 vs. $2(210) = 420$.

Time = 13.24 (sec) , antiderivative size = 16013, normalized size of antiderivative = 63.04

$$\int \frac{1}{(d+ex^2)\,(a+bx^2+cx^4)}\,dx = \text{Too large to display}$$

```
[In] integrate(1/(e*x^2+d)/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(d+ex^2)\,(a+bx^2+cx^4)}\,dx = \text{Timed out}$$

```
[In] integrate(1/(e*x**2+d)/(c*x**4+b*x**2+a),x)
```

```
[Out] Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{(d+ex^2)(a+bx^2+cx^4)}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(1/(e*x^2+d)/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 7664 vs. 2(210) = 420.

Time = 1.78 (sec) , antiderivative size = 7664, normalized size of antiderivative = 30.17

$$\int \frac{1}{(d+ex^2)(a+bx^2+cx^4)}\,dx = \text{Too large to display}$$

```
[In] integrate(1/(e*x^2+d)/(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] e^2*arctan(e*x/sqrt(d*e))/((c*d^2 - b*d*e + a*e^2)*sqrt(d*e)) + 1/8*(2*(2*b
^3*c^5 - 8*a*b*c^6 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)
*c)*b^3*c^3 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a
*b*c^4 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^2*c^
4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b*c^5 - 2*(b^
2 - 4*a*c)*b*c^5)*d^5 - 5*(2*b^4*c^4 - 8*a*b^2*c^5 - sqrt(2)*sqrt(b^2 - 4*a
*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^4*c^2 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*s
qrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^3 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt
(b*c + sqrt(b^2 - 4*a*c)*c)*b^3*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c +
sqrt(b^2 - 4*a*c)*c)*b^2*c^4 - 2*(b^2 - 4*a*c)*b^2*c^4)*d^4*e + 4*(2*b^5*c^
3 - 6*a*b^3*c^4 - 8*a^2*b*c^5 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b
^2 - 4*a*c)*c)*b^5*c + 3*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*
a*c)*c)*a*b^3*c^2 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c
)*c)*b^4*c^2 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*
a^2*b*c^3 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b
^2*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^3*c^3
- sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c^4 - 2*(b^
2 - 4*a*c)*b^3*c^3 - 2*(b^2 - 4*a*c)*a*b*c^4)*d^3*e^2 - (2*b^6*c^2 + 4*a*b^
4*c^3 - 48*a^2*b^2*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*
a*c)*c)*b^6 - 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a
*b^4*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^5*c
```

+ 24*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^2
+ 12*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c^2 -
sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^4*c^2 - 6*sqrt(
2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^3 - 2*(b^2 - 4
*a*c)*b^4*c^2 - 12*(b^2 - 4*a*c)*a*b^2*c^3)*d^2*e^3 + 2*(2*a*b^5*c^2 - 6*a^
2*b^3*c^3 - 8*a^3*b*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4
*a*c)*c)*a*b^5 + 3*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c
)*a^2*b^3*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a
*b^4*c + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b*
c^2 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c
^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c^2 -
sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b*c^3 - 2*(b^
2 - 4*a*c)*a*b^3*c^2 - 2*(b^2 - 4*a*c)*a^2*b*c^3)*d*e^4 - (2*a^2*b^4*c^2 -
8*a^3*b^2*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a
^2*b^4 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^
2*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c
 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^2 -
2*(b^2 - 4*a*c)*a^2*b^2*c^2)*e^5 + 2*(sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*
c)*b^4*c^2 - 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^3 - 2*sqrt(2
)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^3*c^3 - 2*b^4*c^3 + 16*sqrt(2)*sqrt(b*c
 + sqrt(b^2 - 4*a*c)*c)*a^2*c^4 + 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)
*a*b*c^4 + sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^2*c^4 + 16*a*b^2*c^4 -
 4*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*c^5 - 32*a^2*c^5 + 2*(b^2 - 4*
a*c)*b^2*c^3 - 8*(b^2 - 4*a*c)*a*c^4)*d^3*abs(c*d^2 - b*d*e + a*e^2) - 4*(s
qrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^5*c - 8*sqrt(2)*sqrt(b*c + sqrt(b^
2 - 4*a*c)*c)*a*b^3*c^2 - 2*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^4*c^2
 - 2*b^5*c^2 + 16*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b*c^3 + 8*sqr
t(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^3 + sqrt(2)*sqrt(b*c + sqrt(b^
2 - 4*a*c)*c)*b^3*c^3 + 16*a*b^3*c^3 - 4*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*
c)*c)*a*b*c^4 - 32*a^2*b*c^4 + 2*(b^2 - 4*a*c)*b^3*c^2 - 8*(b^2 - 4*a*c)*a*
b*c^3)*d^2*e*abs(c*d^2 - b*d*e + a*e^2) + 2*(sqrt(2)*sqrt(b*c + sqrt(b^2 -
4*a*c)*c)*b^6 - 7*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^4*c - 2*sqrt(
2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^5*c - 2*b^6*c + 8*sqrt(2)*sqrt(b*c + s
qrt(b^2 - 4*a*c)*c)*a^2*b^2*c^2 + 6*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)
*a*b^3*c^2 + sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^4*c^2 + 14*a*b^4*c^2
 + 16*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*c^3 + 8*sqrt(2)*sqrt(b*c
+ sqrt(b^2 - 4*a*c)*c)*a^2*b*c^3 - 3*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c
)*a*b^2*c^3 - 16*a^2*b^2*c^3 - 4*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^
2*c^4 - 32*a^3*c^4 + 2*(b^2 - 4*a*c)*b^4*c - 6*(b^2 - 4*a*c)*a*b^2*c^2 - 8*
(b^2 - 4*a*c)*a^2*c^3)*d*e^2*abs(c*d^2 - b*d*e + a*e^2) - 2*(sqrt(2)*sqrt(b
*c + sqrt(b^2 - 4*a*c)*c)*a*b^5 - 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)
*a^2*b^3*c - 2*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^4*c - 2*a*b^5*c
+ 16*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b*c^2 + 8*sqrt(2)*sqrt(b*c
 + sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^2 + sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*
c)*a*b^3*c^2 + 16*a^2*b^3*c^2 - 4*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a

^2*b*c^3 - 32*a^3*b*c^3 + 2*(b^2 - 4*a*c)*a*b^3*c - 8*(b^2 - 4*a*c)*a^2*b*c ^2)*e^3*abs(c*d^2 - b*d*e + a*e^2) - (2*b^4*c^2 - 16*a*b^2*c^3 + 32*a^2*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^4 + 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c + 2*sqrt(2)*sqr t(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^3*c - 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*c^2 - 8*sqrt(2)*sqrt(b^2 - 4*a *c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqr t(b*c + sqrt(b^2 - 4*a*c)*c)*b^2*c^2 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*c^3 - 2*(b^2 - 4*a*c)*b^2*c^2 + 8*(b^2 - 4*a*c)*a *c^3)*(c*d^2 - b*d*e + a*e^2)^2*e)*arctan(2*sqrt(1/2)*x/sqrt((b*c*d^2 - b^2 *d*e + a*b*e^2 + sqrt((b*c*d^2 - b^2*d*e + a*b*e^2)^2 - 4*(a*c*d^2 - a*b*d* e + a^2*e^2)*(c^2*d^2 - b*c*d*e + a*c*e^2)))/(c^2*d^2 - b*c*d*e + a*c*e^2)))/((a*b^4*c^2 - 8*a^2*b^2*c^3 - 2*a*b^3*c^3 + 16*a^3*c^4 + 8*a^2*b*c^4 + a* b^2*c^4 - 4*a^2*c^5)*d^4*abs(c*d^2 - b*d*e + a*e^2)*abs(c) - 2*(a*b^5*c - 8 *a^2*b^3*c^2 - 2*a*b^4*c^2 + 16*a^3*b*c^3 + 8*a^2*b^2*c^3 + a*b^3*c^3 - 4*a ^2*b*c^4)*d^3*e*abs(c*d^2 - b*d*e + a*e^2)*abs(c) + (a*b^6 - 6*a^2*b^4*c - 2*a*b^5*c + 4*a^2*b^3*c^2 + a*b^4*c^2 + 32*a^4*c^3 + 16*a^3*b*c^3 - 2*a^2*b ^2*c^3 - 8*a^3*c^4)*d^2*e^2*abs(c*d^2 - b*d*e + a*e^2)*abs(c) - 2*(a^2*b^5 - 8*a^3*b^3*c - 2*a^2*b^4*c + 16*a^4*b*c^2 + 8*a^3*b^2*c^2 + a^2*b^3*c^2 - 4*a^3*b*c^3)*d*e^3*abs(c*d^2 - b*d*e + a*e^2)*abs(c) + (a^3*b^4 - 8*a^4*b^2 *c - 2*a^3*b^3*c + 16*a^5*c^2 + 8*a^4*b*c^2 + a^3*b^2*c^2 - 4*a^4*c^3)*e^4* abs(c*d^2 - b*d*e + a*e^2)*abs(c)) - 1/8*(2*(2*b^3*c^5 - 8*a*b*c^6 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^3*c^3 + 4*sqrt(2)*sqr t(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b*c^4 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^2*c^4 - sqrt(2)*sqrt(b^2 - 4*a* c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b*c^5 - 2*(b^2 - 4*a*c)*b*c^5)*d^5 - 5*(2*b^4*c^4 - 8*a*b^2*c^5 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4 *a*c)*c)*b^4*c^2 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c) *c)*a*b^2*c^3 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c) *b^3*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^2*c^ 4 - 2*(b^2 - 4*a*c)*b^2*c^4)*d^4*e + 4*(2*b^5*c^3 - 6*a*b^3*c^4 - 8*a^2*b*c ^5 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^5*c + 3*sq rt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^3*c^2 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^4*c^2 + 4*sqrt(2)*sq rt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b*c^3 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^2*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^3*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c) *sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b*c^4 - 2*(b^2 - 4*a*c)*b^3*c^3 - 2*(b^2 - 4*a*c)*a*b*c^4)*d^3*e^2 - (2*b^6*c^2 + 4*a*b^4*c^3 - 48*a^2*b^2*c^4 - sq rt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^6 - 2*sqrt(2)*sqr t(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^4*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^5*c + 24*sqrt(2)*sqrt(b^2 - 4*a *c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^2 + 12*sqrt(2)*sqrt(b^2 - 4*a *c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^3*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*s qrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^4*c^2 - 6*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b

*c - sqrt(b^2 - 4*a*c)*c)*a*b^2*c^3 - 2*(b^2 - 4*a*c)*b^4*c^2 - 12*(b^2 - 4
*a*c)*a*b^2*c^3)*d^2*e^3 + 2*(2*a*b^5*c^2 - 6*a^2*b^3*c^3 - 8*a^3*b*c^4 - s
qrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^5 + 3*sqrt(2)*
sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c + 2*sqrt(2)*sqr
t(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^4*c + 4*sqrt(2)*sqrt(b^2
 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b*c^2 + 2*sqrt(2)*sqrt(b^2 -
4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^2 - sqrt(2)*sqrt(b^2 - 4*a
*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^3*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*s
qrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b*c^3 - 2*(b^2 - 4*a*c)*a*b^3*c^2 - 2*(b
^2 - 4*a*c)*a^2*b*c^3)*d*e^4 - (2*a^2*b^4*c^2 - 8*a^3*b^2*c^3 - sqrt(2)*sqr
t(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^4 + 4*sqrt(2)*sqrt(b^2
 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^2*c + 2*sqrt(2)*sqrt(b^2 -
4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c - sqrt(2)*sqrt(b^2 - 4*a*c
)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^2 - 2*(b^2 - 4*a*c)*a^2*b^2*c^2
)*e^5 - 2*(sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^4*c^2 - 8*sqrt(2)*sqrt
(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^2*c^3 - 2*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a
*c)*c)*b^3*c^3 + 2*b^4*c^3 + 16*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2
*c^4 + 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b*c^4 + sqrt(2)*sqrt(b*c
 - sqrt(b^2 - 4*a*c)*c)*b^2*c^4 - 16*a*b^2*c^4 - 4*sqrt(2)*sqrt(b*c - sqrt(
b^2 - 4*a*c)*c)*a*c^5 + 32*a^2*c^5 - 2*(b^2 - 4*a*c)*b^2*c^3 + 8*(b^2 - 4*a
*c)*a*c^4)*d^3*abs(c*d^2 - b*d*e + a*e^2) + 4*(sqrt(2)*sqrt(b*c - sqrt(b^2
 - 4*a*c)*c)*b^5*c - 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^3*c^2 - 2
*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^4*c^2 + 2*b^5*c^2 + 16*sqrt(2)*s
qrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b*c^3 + 8*sqrt(2)*sqrt(b*c - sqrt(b^2 -
4*a*c)*c)*a*b^2*c^3 + sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^3*c^3 - 16*
a*b^3*c^3 - 4*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b*c^4 + 32*a^2*b*c^
4 - 2*(b^2 - 4*a*c)*b^3*c^2 + 8*(b^2 - 4*a*c)*a*b*c^3)*d^2*e*abs(c*d^2 - b*
d*e + a*e^2) - 2*(sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^6 - 7*sqrt(2)*s
qrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^4*c - 2*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*
a*c)*c)*b^5*c + 2*b^6*c + 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^2
*c^2 + 6*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^3*c^2 + sqrt(2)*sqrt(b
*c - sqrt(b^2 - 4*a*c)*c)*b^4*c^2 - 14*a*b^4*c^2 + 16*sqrt(2)*sqrt(b*c - sq
rt(b^2 - 4*a*c)*c)*a^3*c^3 + 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*
b*c^3 - 3*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^2*c^3 + 16*a^2*b^2*c^
3 - 4*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*c^4 + 32*a^3*c^4 - 2*(b^2
 - 4*a*c)*b^4*c + 6*(b^2 - 4*a*c)*a*b^2*c^2 + 8*(b^2 - 4*a*c)*a^2*c^3)*d*e^
2*abs(c*d^2 - b*d*e + a*e^2) + 2*(sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a
*b^5 - 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c - 2*sqrt(2)*sqrt
(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^4*c + 2*a*b^5*c + 16*sqrt(2)*sqrt(b*c - sqr
t(b^2 - 4*a*c)*c)*a^3*b*c^2 + 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2
*b^2*c^2 + sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^3*c^2 - 16*a^2*b^3*c
^2 - 4*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b*c^3 + 32*a^3*b*c^3 - 2
*(b^2 - 4*a*c)*a*b^3*c + 8*(b^2 - 4*a*c)*a^2*b*c^2)*e^3*abs(c*d^2 - b*d*e +
 a*e^2) - (2*b^4*c^2 - 16*a*b^2*c^3 + 32*a^2*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c
)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^4 + 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*

```
c - sqrt(b^2 - 4*a*c)*c)*a*b^2*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - s
qrt(b^2 - 4*a*c)*c)*b^3*c - 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^
2 - 4*a*c)*c)*a^2*c^2 - 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4
*a*c)*c)*a*b*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c
)*b^2*c^2 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*c
^3 - 2*(b^2 - 4*a*c)*b^2*c^2 + 8*(b^2 - 4*a*c)*a*c^3)*(c*d^2 - b*d*e + a*e^
2)^2*e)*arctan(2*sqrt(1/2)*x/sqrt((b*c*d^2 - b^2*d*e + a*b*e^2 - sqrt((b*c*
d^2 - b^2*d*e + a*b*e^2)^2 - 4*(a*c*d^2 - a*b*d*e + a^2*e^2)*(c^2*d^2 - b*c
*d*e + a*c*e^2)))/(c^2*d^2 - b*c*d*e + a*c*e^2)))/((a*b^4*c^2 - 8*a^2*b^2*c
^3 - 2*a*b^3*c^3 + 16*a^3*c^4 + 8*a^2*b*c^4 + a*b^2*c^4 - 4*a^2*c^5)*d^4*ab
s(c*d^2 - b*d*e + a*e^2)*abs(c) - 2*(a*b^5*c - 8*a^2*b^3*c^2 - 2*a*b^4*c^2
+ 16*a^3*b*c^3 + 8*a^2*b^2*c^3 + a*b^3*c^3 - 4*a^2*b*c^4)*d^3*e*abs(c*d^2 -
 b*d*e + a*e^2)*abs(c) + (a*b^6 - 6*a^2*b^4*c - 2*a*b^5*c + 4*a^2*b^3*c^2 +
 a*b^4*c^2 + 32*a^4*c^3 + 16*a^3*b*c^3 - 2*a^2*b^2*c^3 - 8*a^3*c^4)*d^2*e^2
*abs(c*d^2 - b*d*e + a*e^2)*abs(c) - 2*(a^2*b^5 - 8*a^3*b^3*c - 2*a^2*b^4*c
 + 16*a^4*b*c^2 + 8*a^3*b^2*c^2 + a^2*b^3*c^2 - 4*a^3*b*c^3)*d*e^3*abs(c*d^
2 - b*d*e + a*e^2)*abs(c) + (a^3*b^4 - 8*a^4*b^2*c - 2*a^3*b^3*c + 16*a^5*c
^2 + 8*a^4*b*c^2 + a^3*b^2*c^2 - 4*a^4*c^3)*e^4*abs(c*d^2 - b*d*e + a*e^2)*
abs(c))
```

Mupad [B] (verification not implemented)

Time = 11.03 (sec) , antiderivative size = 23640, normalized size of antiderivative = 93.07

$$\int \frac{1}{(d+ex^2)(a+bx^2+cx^4)}\,dx = \text{Too large to display}$$

```
[In] int(1/((d + e*x^2)*(a + b*x^2 + c*x^4)),x)
```

```
[Out] atan((((-(b^5*e^2 + b^3*c^2*d^2 + b^2*e^2*(-(4*a*c - b^2)^3)^(1/2) + c^2*d^
2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*e^2 - 2*b^4*c*d*e - 4*a*b*c^3*d^2
 - 7*a*b^3*c*e^2 - a*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - 16*a^2*c^3*d*e + 12*a
*b^2*c^2*d*e - 2*b*c*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^3*b^4*e^4 + 16*a^3
*c^4*d^4 + 16*a^5*c^2*e^4 + a*b^4*c^2*d^4 - 8*a^4*b^2*c*e^4 + a*b^6*d^2*e^2
 - 2*a^2*b^5*d*e^3 - 8*a^2*b^2*c^3*d^4 + 32*a^4*c^3*d^2*e^2 - 2*a*b^5*c*d^3
*e - 32*a^3*b*c^3*d^3*e + 16*a^3*b^3*c*d*e^3 - 32*a^4*b*c^2*d*e^3 + 16*a^2*
b^3*c^2*d^3*e - 6*a^2*b^4*c*d^2*e^2)))^(1/2)*((x*(16*b^5*c^2*e^7 + 16*c^7*d
^5*e^2 - 112*a*b^3*c^3*e^7 + 192*a^2*b*c^4*e^7 + 32*a*c^6*d^3*e^4 - 240*a^2
*c^5*d*e^6 - 32*b*c^6*d^4*e^3 - 32*b^4*c^3*d*e^6 + 16*b^2*c^5*d^3*e^4 + 16*
b^3*c^4*d^2*e^5 - 96*a*b*c^5*d^2*e^5 + 192*a*b^2*c^4*d*e^6) - (-(b^5*e^2 +
b^3*c^2*d^2 + b^2*e^2*(-(4*a*c - b^2)^3)^(1/2) + c^2*d^2*(-(4*a*c - b^2)^3)
^(1/2) + 12*a^2*b*c^2*e^2 - 2*b^4*c*d*e - 4*a*b*c^3*d^2 - 7*a*b^3*c*e^2 - a
*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - 16*a^2*c^3*d*e + 12*a*b^2*c^2*d*e - 2*b*c
*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^3*b^4*e^4 + 16*a^3*c^4*d^4 + 16*a^5*c^
2*e^4 + a*b^4*c^2*d^4 - 8*a^4*b^2*c*e^4 + a*b^6*d^2*e^2 - 2*a^2*b^5*d*e^3 -
```

8*a^2*b^2*c^3*d^4 + 32*a^4*c^3*d^2*e^2 - 2*a*b^5*c*d^3*e - 32*a^3*b*c^3*d^3*e + 16*a^3*b^3*c*d*e^3 - 32*a^4*b*c^2*d*e^3 + 16*a^2*b^3*c^2*d^3*e - 6*a^2*b^4*c*d^2*e^2)))^(1/2)*(x*(-(b^5*e^2 + b^3*c^2*d^2 + b^2*e^2*(-(4*a*c - b^2)^3)^(1/2) + c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*e^2 - 2*b^4*c*d*e - 4*a*b*c^3*d^2 - 7*a*b^3*c*e^2 - a*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - 16*a^2*c^3*d*e + 12*a*b^2*c^2*d*e - 2*b*c*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^3*b^4*e^4 + 16*a^3*c^4*d^4 + 16*a^5*c^2*e^4 + a*b^4*c^2*d^4 - 8*a^4*b^2*c*e^4 + a*b^6*d^2*e^2 - 2*a^2*b^5*d*e^3 - 8*a^2*b^2*c^3*d^4 + 32*a^4*c^3*d^2*e^2 - 2*a*b^5*c*d^3*e - 32*a^3*b*c^3*d^3*e + 16*a^3*b^3*c*d*e^3 - 32*a^4*b*c^2*d*e^3 + 16*a^2*b^3*c^2*d^3*e - 6*a^2*b^4*c*d^2*e^2)))^(1/2)*(256*a^4*b^2*c^3*e^9 - 32*a^3*b^4*c^2*e^9 - 512*a^5*c^4*e^9 + 512*a^2*c^7*d^6*e^3 + 512*a^3*c^6*d^4*e^5 - 512*a^4*c^5*d^2*e^7 - 32*b^3*c^6*d^7*e^2 + 128*b^4*c^5*d^6*e^3 - 192*b^5*c^4*d^5*e^4 + 128*b^6*c^3*d^4*e^5 - 32*b^7*c^2*d^3*e^6 + 512*a^2*b^2*c^5*d^4*e^5 + 288*a^2*b^3*c^4*d^3*e^6 - 192*a^2*b^4*c^3*d^2*e^7 + 384*a^3*b^2*c^4*d^2*e^7 + 128*a*b*c^7*d^7*e^2 + 640*a^4*b*c^4*d*e^8 - 640*a*b^2*c^6*d^6*e^3 + 1056*a*b^3*c^5*d^5*e^4 - 672*a*b^4*c^4*d^4*e^5 + 96*a*b^5*c^3*d^3*e^6 + 32*a*b^6*c^2*d^2*e^7 - 1152*a^2*b*c^6*d^5*e^4 + 32*a^2*b^5*c^2*d*e^8 - 640*a^3*b*c^5*d^3*e^6 - 288*a^3*b^3*c^3*d*e^8) - 256*a^4*c^4*e^8 + 64*a*c^7*d^6*e^2 - 16*a^2*b^4*c^2*e^8 + 128*a^3*b^2*c^3*e^8 - 128*a^2*c^6*d^4*e^4 - 448*a^3*c^5*d^2*e^6 - 16*b^2*c^6*d^6*e^2 + 64*b^3*c^5*d^5*e^3 - 96*b^4*c^4*d^4*e^4 + 64*b^5*c^3*d^3*e^5 - 16*b^6*c^2*d^2*e^6 + 240*a^2*b^2*c^4*d^2*e^6 - 256*a*b*c^6*d^5*e^3 + 32*a*b^5*c^2*d*e^7 + 384*a^3*b*c^4*d*e^7 + 416*a*b^2*c^5*d^4*e^4 - 288*a*b^3*c^4*d^3*e^5 + 32*a*b^4*c^3*d^2*e^6 + 128*a^2*b*c^5*d^3*e^5 - 224*a^2*b^3*c^3*d*e^7))*(-(b^5*e^2 + b^3*c^2*d^2 + b^2*e^2*(-(4*a*c - b^2)^3)^(1/2) + c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*e^2 - 2*b^4*c*d*e - 4*a*b*c^3*d^2 - 7*a*b^3*c*e^2 - a*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - 16*a^2*c^3*d*e + 12*a*b^2*c^2*d*e - 2*b*c*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^3*b^4*e^4 + 16*a^3*c^4*d^4 + 16*a^5*c^2*e^4 + a*b^4*c^2*d^4 - 8*a^4*b^2*c*e^4 + a*b^6*d^2*e^2 - 2*a^2*b^5*d*e^3 - 8*a^2*b^2*c^3*d^4 + 32*a^4*c^3*d^2*e^2 - 2*a*b^5*c*d^3*e - 32*a^3*b*c^3*d^3*e + 16*a^3*b^3*c*d*e^3 - 32*a^4*b*c^2*d*e^3 + 16*a^2*b^3*c^2*d^3*e - 6*a^2*b^4*c*d^2*e^2)))^(1/2) - 4*b^3*c^3*e^6 - 4*c^6*d^3*e^3 + 4*b*c^5*d^2*e^4 + 4*b^2*c^4*d*e^5 + 16*a*b*c^4*e^6 - 20*a*c^5*d*e^5) + 6*c^5*e^5*x)*(-(b^5*e^2 + b^3*c^2*d^2 + b^2*e^2*(-(4*a*c - b^2)^3)^(1/2) + c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*e^2 - 2*b^4*c*d*e - 4*a*b*c^3*d^2 - 7*a*b^3*c*e^2 - a*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - 16*a^2*c^3*d*e + 12*a*b^2*c^2*d*e - 2*b*c*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^3*b^4*e^4 + 16*a^3*c^4*d^4 + 16*a^5*c^2*e^4 + a*b^4*c^2*d^4 - 8*a^4*b^2*c*e^4 + a*b^6*d^2*e^2 - 2*a^2*b^5*d*e^3 - 8*a^2*b^2*c^3*d^4 + 32*a^4*c^3*d^2*e^2 - 2*a*b^5*c*d^3*e - 32*a^3*b*c^3*d^3*e + 16*a^3*b^3*c*d*e^3 - 32*a^4*b*c^2*d*e^3 + 16*a^2*b^3*c^2*d^3*e - 6*a^2*b^4*c*d^2*e^2)))^(1/2)*1i + ((-(b^5*e^2 + b^3*c^2*d^2 + b^2*e^2*(-(4*a*c - b^2)^3)^(1/2) + c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*e^2 - 2*b^4*c*d*e - 4*a*b*c^3*d^2 - 7*a*b^3*c*e^2 - a*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - 16*a^2*c^3*d*e + 12*a*b^2*c^2*d*e - 2*b*c*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^3*b^4*e^4 + 16*a^3*c^4*d^4 + 16*a^5*c^2*e^4 + a*b^4*c^2*d^4 - 8*a^4*b^

2*c*e^4 + a*b^6*d^2*e^2 - 2*a^2*b^5*d*e^3 - 8*a^2*b^2*c^3*d^4 + 32*a^4*c^3*
d^2*e^2 - 2*a*b^5*c*d^3*e - 32*a^3*b*c^3*d^3*e + 16*a^3*b^3*c*d*e^3 - 32*a^
4*b*c^2*d*e^3 + 16*a^2*b^3*c^2*d^3*e - 6*a^2*b^4*c*d^2*e^2)))^(1/2)*((x*(16
*b^5*c^2*e^7 + 16*c^7*d^5*e^2 - 112*a*b^3*c^3*e^7 + 192*a^2*b*c^4*e^7 + 32*
a*c^6*d^3*e^4 - 240*a^2*c^5*d*e^6 - 32*b*c^6*d^4*e^3 - 32*b^4*c^3*d*e^6 + 1
6*b^2*c^5*d^3*e^4 + 16*b^3*c^4*d^2*e^5 - 96*a*b*c^5*d^2*e^5 + 192*a*b^2*c^4
*d*e^6) - (-(b^5*e^2 + b^3*c^2*d^2 + b^2*e^2*(-(4*a*c - b^2)^3)^(1/2) + c^2
d^2(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*e^2 - 2*b^4*c*d*e - 4*a*b*c^3*
d^2 - 7*a*b^3*c*e^2 - a*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - 16*a^2*c^3*d*e + 1
2*a*b^2*c^2*d*e - 2*b*c*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^3*b^4*e^4 + 16*
a^3*c^4*d^4 + 16*a^5*c^2*e^4 + a*b^4*c^2*d^4 - 8*a^4*b^2*c*e^4 + a*b^6*d^2*
e^2 - 2*a^2*b^5*d*e^3 - 8*a^2*b^2*c^3*d^4 + 32*a^4*c^3*d^2*e^2 - 2*a*b^5*c*
d^3*e - 32*a^3*b*c^3*d^3*e + 16*a^3*b^3*c*d*e^3 - 32*a^4*b*c^2*d*e^3 + 16*a
^2*b^3*c^2*d^3*e - 6*a^2*b^4*c*d^2*e^2)))^(1/2)*(256*a^4*c^4*e^8 + x*(-(b^5
*e^2 + b^3*c^2*d^2 + b^2*e^2*(-(4*a*c - b^2)^3)^(1/2) + c^2*d^2*(-(4*a*c -
b^2)^3)^(1/2) + 12*a^2*b*c^2*e^2 - 2*b^4*c*d*e - 4*a*b*c^3*d^2 - 7*a*b^3*c*
e^2 - a*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - 16*a^2*c^3*d*e + 12*a*b^2*c^2*d*e
- 2*b*c*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^3*b^4*e^4 + 16*a^3*c^4*d^4 + 16
*a^5*c^2*e^4 + a*b^4*c^2*d^4 - 8*a^4*b^2*c*e^4 + a*b^6*d^2*e^2 - 2*a^2*b^5*
d*e^3 - 8*a^2*b^2*c^3*d^4 + 32*a^4*c^3*d^2*e^2 - 2*a*b^5*c*d^3*e - 32*a^3*b
*c^3*d^3*e + 16*a^3*b^3*c*d*e^3 - 32*a^4*b*c^2*d*e^3 + 16*a^2*b^3*c^2*d^3*e
- 6*a^2*b^4*c*d^2*e^2)))^(1/2)*(256*a^4*b^2*c^3*e^9 - 32*a^3*b^4*c^2*e^9 -
512*a^5*c^4*e^9 + 512*a^2*c^7*d^6*e^3 + 512*a^3*c^6*d^4*e^5 - 512*a^4*c^5*
d^2*e^7 - 32*b^3*c^6*d^7*e^2 + 128*b^4*c^5*d^6*e^3 - 192*b^5*c^4*d^5*e^4 +
128*b^6*c^3*d^4*e^5 - 32*b^7*c^2*d^3*e^6 + 512*a^2*b^2*c^5*d^4*e^5 + 288*a^
2*b^3*c^4*d^3*e^6 - 192*a^2*b^4*c^3*d^2*e^7 + 384*a^3*b^2*c^4*d^2*e^7 + 128
*a*b*c^7*d^7*e^2 + 640*a^4*b*c^4*d*e^8 - 640*a*b^2*c^6*d^6*e^3 + 1056*a*b^3
*c^5*d^5*e^4 - 672*a*b^4*c^4*d^4*e^5 + 96*a*b^5*c^3*d^3*e^6 + 32*a*b^6*c^2*
d^2*e^7 - 1152*a^2*b*c^6*d^5*e^4 + 32*a^2*b^5*c^2*d*e^8 - 640*a^3*b*c^5*d^3
*e^6 - 288*a^3*b^3*c^3*d*e^8) - 64*a*c^7*d^6*e^2 + 16*a^2*b^4*c^2*e^8 - 128
*a^3*b^2*c^3*e^8 + 128*a^2*c^6*d^4*e^4 + 448*a^3*c^5*d^2*e^6 + 16*b^2*c^6*d
^6*e^2 - 64*b^3*c^5*d^5*e^3 + 96*b^4*c^4*d^4*e^4 - 64*b^5*c^3*d^3*e^5 + 16*
b^6*c^2*d^2*e^6 - 240*a^2*b^2*c^4*d^2*e^6 + 256*a*b*c^6*d^5*e^3 - 32*a*b^5*
c^2*d*e^7 - 384*a^3*b*c^4*d*e^7 - 416*a*b^2*c^5*d^4*e^4 + 288*a*b^3*c^4*d^3
*e^5 - 32*a*b^4*c^3*d^2*e^6 - 128*a^2*b*c^5*d^3*e^5 + 224*a^2*b^3*c^3*d*e^7
))*(-(b^5*e^2 + b^3*c^2*d^2 + b^2*e^2*(-(4*a*c - b^2)^3)^(1/2) + c^2*d^2*(-
(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*e^2 - 2*b^4*c*d*e - 4*a*b*c^3*d^2 - 7
*a*b^3*c*e^2 - a*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - 16*a^2*c^3*d*e + 12*a*b^2
*c^2*d*e - 2*b*c*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^3*b^4*e^4 + 16*a^3*c^4
*d^4 + 16*a^5*c^2*e^4 + a*b^4*c^2*d^4 - 8*a^4*b^2*c*e^4 + a*b^6*d^2*e^2 - 2
*a^2*b^5*d*e^3 - 8*a^2*b^2*c^3*d^4 + 32*a^4*c^3*d^2*e^2 - 2*a*b^5*c*d^3*e -
32*a^3*b*c^3*d^3*e + 16*a^3*b^3*c*d*e^3 - 32*a^4*b*c^2*d*e^3 + 16*a^2*b^3*
c^2*d^3*e - 6*a^2*b^4*c*d^2*e^2)))^(1/2) + 4*b^3*c^3*e^6 + 4*c^6*d^3*e^3 -
4*b*c^5*d^2*e^4 - 4*b^2*c^4*d*e^5 - 16*a*b*c^4*e^6 + 20*a*c^5*d*e^5) + 6*c^
5*e^5*x)*(-(b^5*e^2 + b^3*c^2*d^2 + b^2*e^2*(-(4*a*c - b^2)^3)^(1/2) + c^2*

d^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*e^2 - 2*b^4*c*d*e - 4*a*b*c^3*d
^2 - 7*a*b^3*c*e^2 - a*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - 16*a^2*c^3*d*e + 12
*a*b^2*c^2*d*e - 2*b*c*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^3*b^4*e^4 + 16*a
^3*c^4*d^4 + 16*a^5*c^2*e^4 + a*b^4*c^2*d^4 - 8*a^4*b^2*c*e^4 + a*b^6*d^2*e
^2 - 2*a^2*b^5*d*e^3 - 8*a^2*b^2*c^3*d^4 + 32*a^4*c^3*d^2*e^2 - 2*a*b^5*c*d
^3*e - 32*a^3*b*c^3*d^3*e + 16*a^3*b^3*c*d*e^3 - 32*a^4*b*c^2*d*e^3 + 16*a^
2*b^3*c^2*d^3*e - 6*a^2*b^4*c*d^2*e^2)))^(1/2)*1i)/(((-(b^5*e^2 + b^3*c^2*d
^2 + b^2*e^2*(-(4*a*c - b^2)^3)^(1/2) + c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) +
12*a^2*b*c^2*e^2 - 2*b^4*c*d*e - 4*a*b*c^3*d^2 - 7*a*b^3*c*e^2 - a*c*e^2*(-
(4*a*c - b^2)^3)^(1/2) - 16*a^2*c^3*d*e + 12*a*b^2*c^2*d*e - 2*b*c*d*e*(-(4
*a*c - b^2)^3)^(1/2))/(8*(a^3*b^4*e^4 + 16*a^3*c^4*d^4 + 16*a^5*c^2*e^4 + a
*b^4*c^2*d^4 - 8*a^4*b^2*c*e^4 + a*b^6*d^2*e^2 - 2*a^2*b^5*d*e^3 - 8*a^2*b^
2*c^3*d^4 + 32*a^4*c^3*d^2*e^2 - 2*a*b^5*c*d^3*e - 32*a^3*b*c^3*d^3*e + 16*
a^3*b^3*c*d*e^3 - 32*a^4*b*c^2*d*e^3 + 16*a^2*b^3*c^2*d^3*e - 6*a^2*b^4*c*d
^2*e^2)))^(1/2)*((x*(16*b^5*c^2*e^7 + 16*c^7*d^5*e^2 - 112*a*b^3*c^3*e^7 +
192*a^2*b*c^4*e^7 + 32*a*c^6*d^3*e^4 - 240*a^2*c^5*d*e^6 - 32*b*c^6*d^4*e^3
 - 32*b^4*c^3*d*e^6 + 16*b^2*c^5*d^3*e^4 + 16*b^3*c^4*d^2*e^5 - 96*a*b*c^5*
d^2*e^5 + 192*a*b^2*c^4*d*e^6) - (-(b^5*e^2 + b^3*c^2*d^2 + b^2*e^2*(-(4*a*
c - b^2)^3)^(1/2) + c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*e^2 - 2
*b^4*c*d*e - 4*a*b*c^3*d^2 - 7*a*b^3*c*e^2 - a*c*e^2*(-(4*a*c - b^2)^3)^(1/
2) - 16*a^2*c^3*d*e + 12*a*b^2*c^2*d*e - 2*b*c*d*e*(-(4*a*c - b^2)^3)^(1/2)
)/(8*(a^3*b^4*e^4 + 16*a^3*c^4*d^4 + 16*a^5*c^2*e^4 + a*b^4*c^2*d^4 - 8*a^4
*b^2*c*e^4 + a*b^6*d^2*e^2 - 2*a^2*b^5*d*e^3 - 8*a^2*b^2*c^3*d^4 + 32*a^4*c
^3*d^2*e^2 - 2*a*b^5*c*d^3*e - 32*a^3*b*c^3*d^3*e + 16*a^3*b^3*c*d*e^3 - 32
*a^4*b*c^2*d*e^3 + 16*a^2*b^3*c^2*d^3*e - 6*a^2*b^4*c*d^2*e^2)))^(1/2)*(x*(
-(b^5*e^2 + b^3*c^2*d^2 + b^2*e^2*(-(4*a*c - b^2)^3)^(1/2) + c^2*d^2*(-(4*a
*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*e^2 - 2*b^4*c*d*e - 4*a*b*c^3*d^2 - 7*a*b
^3*c*e^2 - a*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - 16*a^2*c^3*d*e + 12*a*b^2*c^2
*d*e - 2*b*c*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^3*b^4*e^4 + 16*a^3*c^4*d^4
 + 16*a^5*c^2*e^4 + a*b^4*c^2*d^4 - 8*a^4*b^2*c*e^4 + a*b^6*d^2*e^2 - 2*a^2
*b^5*d*e^3 - 8*a^2*b^2*c^3*d^4 + 32*a^4*c^3*d^2*e^2 - 2*a*b^5*c*d^3*e - 32*
a^3*b*c^3*d^3*e + 16*a^3*b^3*c*d*e^3 - 32*a^4*b*c^2*d*e^3 + 16*a^2*b^3*c^2*
d^3*e - 6*a^2*b^4*c*d^2*e^2)))^(1/2)*(256*a^4*b^2*c^3*e^9 - 32*a^3*b^4*c^2*
e^9 - 512*a^5*c^4*e^9 + 512*a^2*c^7*d^6*e^3 + 512*a^3*c^6*d^4*e^5 - 512*a^4
*c^5*d^2*e^7 - 32*b^3*c^6*d^7*e^2 + 128*b^4*c^5*d^6*e^3 - 192*b^5*c^4*d^5*e
^4 + 128*b^6*c^3*d^4*e^5 - 32*b^7*c^2*d^3*e^6 + 512*a^2*b^2*c^5*d^4*e^5 + 2
88*a^2*b^3*c^4*d^3*e^6 - 192*a^2*b^4*c^3*d^2*e^7 + 384*a^3*b^2*c^4*d^2*e^7
+ 128*a*b*c^7*d^7*e^2 + 640*a^4*b*c^4*d*e^8 - 640*a*b^2*c^6*d^6*e^3 + 1056*
a*b^3*c^5*d^5*e^4 - 672*a*b^4*c^4*d^4*e^5 + 96*a*b^5*c^3*d^3*e^6 + 32*a*b^6
*c^2*d^2*e^7 - 1152*a^2*b*c^6*d^5*e^4 + 32*a^2*b^5*c^2*d*e^8 - 640*a^3*b*c^
5*d^3*e^6 - 288*a^3*b^3*c^3*d*e^8) - 256*a^4*c^4*e^8 + 64*a*c^7*d^6*e^2 - 1
6*a^2*b^4*c^2*e^8 + 128*a^3*b^2*c^3*e^8 - 128*a^2*c^6*d^4*e^4 - 448*a^3*c^5
*d^2*e^6 - 16*b^2*c^6*d^6*e^2 + 64*b^3*c^5*d^5*e^3 - 96*b^4*c^4*d^4*e^4 + 6
4*b^5*c^3*d^3*e^5 - 16*b^6*c^2*d^2*e^6 + 240*a^2*b^2*c^4*d^2*e^6 - 256*a*b*
c^6*d^5*e^3 + 32*a*b^5*c^2*d*e^7 + 384*a^3*b*c^4*d*e^7 + 416*a*b^2*c^5*d^4*

e^4 - 288*a*b^3*c^4*d^3*e^5 + 32*a*b^4*c^3*d^2*e^6 + 128*a^2*b*c^5*d^3*e^5 - 224*a^2*b^3*c^3*d*e^7))*(-(b^5*e^2 + b^3*c^2*d^2 + b^2*e^2*(-(4*a*c - b^2)^3)^(1/2) + c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*e^2 - 2*b^4*c* d*e - 4*a*b*c^3*d^2 - 7*a*b^3*c*e^2 - a*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - 16 *a^2*c^3*d*e + 12*a*b^2*c^2*d*e - 2*b*c*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(a ^3*b^4*e^4 + 16*a^3*c^4*d^4 + 16*a^5*c^2*e^4 + a*b^4*c^2*d^4 - 8*a^4*b^2*c* e^4 + a*b^6*d^2*e^2 - 2*a^2*b^5*d*e^3 - 8*a^2*b^2*c^3*d^4 + 32*a^4*c^3*d^2* e^2 - 2*a*b^5*c*d^3*e - 32*a^3*b*c^3*d^3*e + 16*a^3*b^3*c*d*e^3 - 32*a^4*b* c^2*d*e^3 + 16*a^2*b^3*c^2*d^3*e - 6*a^2*b^4*c*d^2*e^2)))^(1/2) - 4*b^3*c^3 *e^6 - 4*c^6*d^3*e^3 + 4*b*c^5*d^2*e^4 + 4*b^2*c^4*d*e^5 + 16*a*b*c^4*e^6 - 20*a*c^5*d*e^5) + 6*c^5*e^5*x)*(-(b^5*e^2 + b^3*c^2*d^2 + b^2*e^2*(-(4*a*c - b^2)^3)^(1/2) + c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*e^2 - 2* b^4*c*d*e - 4*a*b*c^3*d^2 - 7*a*b^3*c*e^2 - a*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - 16*a^2*c^3*d*e + 12*a*b^2*c^2*d*e - 2*b*c*d*e*(-(4*a*c - b^2)^3)^(1/2)) /(8*(a^3*b^4*e^4 + 16*a^3*c^4*d^4 + 16*a^5*c^2*e^4 + a*b^4*c^2*d^4 - 8*a^4* b^2*c*e^4 + a*b^6*d^2*e^2 - 2*a^2*b^5*d*e^3 - 8*a^2*b^2*c^3*d^4 + 32*a^4*c^ 3*d^2*e^2 - 2*a*b^5*c*d^3*e - 32*a^3*b*c^3*d^3*e + 16*a^3*b^3*c*d*e^3 - 32* a^4*b*c^2*d*e^3 + 16*a^2*b^3*c^2*d^3*e - 6*a^2*b^4*c*d^2*e^2)))^(1/2) - ((- (b^5*e^2 + b^3*c^2*d^2 + b^2*e^2*(-(4*a*c - b^2)^3)^(1/2) + c^2*d^2*(-(4*a* c - b^2)^3)^(1/2) + 12*a^2*b*c^2*e^2 - 2*b^4*c*d*e - 4*a*b*c^3*d^2 - 7*a*b^ 3*c*e^2 - a*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - 16*a^2*c^3*d*e + 12*a*b^2*c^2* d*e - 2*b*c*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^3*b^4*e^4 + 16*a^3*c^4*d^4 + 16*a^5*c^2*e^4 + a*b^4*c^2*d^4 - 8*a^4*b^2*c*e^4 + a*b^6*d^2*e^2 - 2*a^2* b^5*d*e^3 - 8*a^2*b^2*c^3*d^4 + 32*a^4*c^3*d^2*e^2 - 2*a*b^5*c*d^3*e - 32*a ^3*b*c^3*d^3*e + 16*a^3*b^3*c*d*e^3 - 32*a^4*b*c^2*d*e^3 + 16*a^2*b^3*c^2*d ^3*e - 6*a^2*b^4*c*d^2*e^2)))^(1/2)*((x*(16*b^5*c^2*e^7 + 16*c^7*d^5*e^2 - 112*a*b^3*c^3*e^7 + 192*a^2*b*c^4*e^7 + 32*a*c^6*d^3*e^4 - 240*a^2*c^5*d*e^ 6 - 32*b*c^6*d^4*e^3 - 32*b^4*c^3*d*e^6 + 16*b^2*c^5*d^3*e^4 + 16*b^3*c^4*d ^2*e^5 - 96*a*b*c^5*d^2*e^5 + 192*a*b^2*c^4*d*e^6) - (-(b^5*e^2 + b^3*c^2*d ^2 + b^2*e^2*(-(4*a*c - b^2)^3)^(1/2) + c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*e^2 - 2*b^4*c*d*e - 4*a*b*c^3*d^2 - 7*a*b^3*c*e^2 - a*c*e^2*(- (4*a*c - b^2)^3)^(1/2) - 16*a^2*c^3*d*e + 12*a*b^2*c^2*d*e - 2*b*c*d*e*(-(4 *a*c - b^2)^3)^(1/2))/(8*(a^3*b^4*e^4 + 16*a^3*c^4*d^4 + 16*a^5*c^2*e^4 + a *b^4*c^2*d^4 - 8*a^4*b^2*c*e^4 + a*b^6*d^2*e^2 - 2*a^2*b^5*d*e^3 - 8*a^2*b^ 2*c^3*d^4 + 32*a^4*c^3*d^2*e^2 - 2*a*b^5*c*d^3*e - 32*a^3*b*c^3*d^3*e + 16* a^3*b^3*c*d*e^3 - 32*a^4*b*c^2*d*e^3 + 16*a^2*b^3*c^2*d^3*e - 6*a^2*b^4*c*d ^2*e^2)))^(1/2)*(256*a^4*c^4*e^8 + x*(-(b^5*e^2 + b^3*c^2*d^2 + b^2*e^2*(-(4*a*c - b^2)^3)^(1/2) + c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*e^2 - 2*b^4*c*d*e - 4*a*b*c^3*d^2 - 7*a*b^3*c*e^2 - a*c*e^2*(-(4*a*c - b^2)^3) ^(1/2) - 16*a^2*c^3*d*e + 12*a*b^2*c^2*d*e - 2*b*c*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^3*b^4*e^4 + 16*a^3*c^4*d^4 + 16*a^5*c^2*e^4 + a*b^4*c^2*d^4 - 8 *a^4*b^2*c*e^4 + a*b^6*d^2*e^2 - 2*a^2*b^5*d*e^3 - 8*a^2*b^2*c^3*d^4 + 32*a ^4*c^3*d^2*e^2 - 2*a*b^5*c*d^3*e - 32*a^3*b*c^3*d^3*e + 16*a^3*b^3*c*d*e^3 - 32*a^4*b*c^2*d*e^3 + 16*a^2*b^3*c^2*d^3*e - 6*a^2*b^4*c*d^2*e^2)))^(1/2)* (256*a^4*b^2*c^3*e^9 - 32*a^3*b^4*c^2*e^9 - 512*a^5*c^4*e^9 + 512*a^2*c^7*d

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^6*e^3 + 512*a^3*c^6*d^4*e^5 - 512*a^4*c^5*d^2*e^7 - 32*b^3*c^6*d^7*e^2 + 1
28*b^4*c^5*d^6*e^3 - 192*b^5*c^4*d^5*e^4 + 128*b^6*c^3*d^4*e^5 - 32*b^7*c^2
*d^3*e^6 + 512*a^2*b^2*c^5*d^4*e^5 + 288*a^2*b^3*c^4*d^3*e^6 - 192*a^2*b^4*
c^3*d^2*e^7 + 384*a^3*b^2*c^4*d^2*e^7 + 128*a*b*c^7*d^7*e^2 + 640*a^4*b*c^4
*d*e^8 - 640*a*b^2*c^6*d^6*e^3 + 1056*a*b^3*c^5*d^5*e^4 - 672*a*b^4*c^4*d^4
*e^5 + 96*a*b^5*c^3*d^3*e^6 + 32*a*b^6*c^2*d^2*e^7 - 1152*a^2*b*c^6*d^5*e^4
 + 32*a^2*b^5*c^2*d*e^8 - 640*a^3*b*c^5*d^3*e^6 - 288*a^3*b^3*c^3*d*e^8) -
64*a*c^7*d^6*e^2 + 16*a^2*b^4*c^2*e^8 - 128*a^3*b^2*c^3*e^8 + 128*a^2*c^6*d
^4*e^4 + 448*a^3*c^5*d^2*e^6 + 16*b^2*c^6*d^6*e^2 - 64*b^3*c^5*d^5*e^3 + 96
*b^4*c^4*d^4*e^4 - 64*b^5*c^3*d^3*e^5 + 16*b^6*c^2*d^2*e^6 - 240*a^2*b^2*c^
4*d^2*e^6 + 256*a*b*c^6*d^5*e^3 - 32*a*b^5*c^2*d*e^7 - 384*a^3*b*c^4*d*e^7
- 416*a*b^2*c^5*d^4*e^4 + 288*a*b^3*c^4*d^3*e^5 - 32*a*b^4*c^3*d^2*e^6 - 12
8*a^2*b*c^5*d^3*e^5 + 224*a^2*b^3*c^3*d*e^7))*(-(b^5*e^2 + b^3*c^2*d^2 + b^
2*e^2*(-(4*a*c - b^2)^3)^(1/2) + c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*
b*c^2*e^2 - 2*b^4*c*d*e - 4*a*b*c^3*d^2 - 7*a*b^3*c*e^2 - a*c*e^2*(-(4*a*c
- b^2)^3)^(1/2) - 16*a^2*c^3*d*e + 12*a*b^2*c^2*d*e - 2*b*c*d*e*(-(4*a*c -
b^2)^3)^(1/2))/(8*(a^3*b^4*e^4 + 16*a^3*c^4*d^4 + 16*a^5*c^2*e^4 + a*b^4*c^
2*d^4 - 8*a^4*b^2*c*e^4 + a*b^6*d^2*e^2 - 2*a^2*b^5*d*e^3 - 8*a^2*b^2*c^3*d
^4 + 32*a^4*c^3*d^2*e^2 - 2*a*b^5*c*d^3*e - 32*a^3*b*c^3*d^3*e + 16*a^3*b^3
*c*d*e^3 - 32*a^4*b*c^2*d*e^3 + 16*a^2*b^3*c^2*d^3*e - 6*a^2*b^4*c*d^2*e^2)
))^(1/2) + 4*b^3*c^3*e^6 + 4*c^6*d^3*e^3 - 4*b*c^5*d^2*e^4 - 4*b^2*c^4*d*e^
5 - 16*a*b*c^4*e^6 + 20*a*c^5*d*e^5) + 6*c^5*e^5*x)*(-(b^5*e^2 + b^3*c^2*d^
2 + b^2*e^2*(-(4*a*c - b^2)^3)^(1/2) + c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) + 1
2*a^2*b*c^2*e^2 - 2*b^4*c*d*e - 4*a*b*c^3*d^2 - 7*a*b^3*c*e^2 - a*c*e^2*(-(
4*a*c - b^2)^3)^(1/2) - 16*a^2*c^3*d*e + 12*a*b^2*c^2*d*e - 2*b*c*d*e*(-(4*
a*c - b^2)^3)^(1/2))/(8*(a^3*b^4*e^4 + 16*a^3*c^4*d^4 + 16*a^5*c^2*e^4 + a*
b^4*c^2*d^4 - 8*a^4*b^2*c*e^4 + a*b^6*d^2*e^2 - 2*a^2*b^5*d*e^3 - 8*a^2*b^2
*c^3*d^4 + 32*a^4*c^3*d^2*e^2 - 2*a*b^5*c*d^3*e - 32*a^3*b*c^3*d^3*e + 16*a
^3*b^3*c*d*e^3 - 32*a^4*b*c^2*d*e^3 + 16*a^2*b^3*c^2*d^3*e - 6*a^2*b^4*c*d^
2*e^2)))^(1/2)))*(-(b^5*e^2 + b^3*c^2*d^2 + b^2*e^2*(-(4*a*c - b^2)^3)^(1/2
) + c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*e^2 - 2*b^4*c*d*e - 4*a
*b*c^3*d^2 - 7*a*b^3*c*e^2 - a*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - 16*a^2*c^3*
d*e + 12*a*b^2*c^2*d*e - 2*b*c*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^3*b^4*e^
4 + 16*a^3*c^4*d^4 + 16*a^5*c^2*e^4 + a*b^4*c^2*d^4 - 8*a^4*b^2*c*e^4 + a*b
^6*d^2*e^2 - 2*a^2*b^5*d*e^3 - 8*a^2*b^2*c^3*d^4 + 32*a^4*c^3*d^2*e^2 - 2*a
*b^5*c*d^3*e - 32*a^3*b*c^3*d^3*e + 16*a^3*b^3*c*d*e^3 - 32*a^4*b*c^2*d*e^3
 + 16*a^2*b^3*c^2*d^3*e - 6*a^2*b^4*c*d^2*e^2)))^(1/2)*2i + atan((((-(b^5*e
^2 + b^3*c^2*d^2 - b^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - c^2*d^2*(-(4*a*c - b^
2)^3)^(1/2) + 12*a^2*b*c^2*e^2 - 2*b^4*c*d*e - 4*a*b*c^3*d^2 - 7*a*b^3*c*e^
2 + a*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - 16*a^2*c^3*d*e + 12*a*b^2*c^2*d*e +
2*b*c*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^3*b^4*e^4 + 16*a^3*c^4*d^4 + 16*a
^5*c^2*e^4 + a*b^4*c^2*d^4 - 8*a^4*b^2*c*e^4 + a*b^6*d^2*e^2 - 2*a^2*b^5*d*
e^3 - 8*a^2*b^2*c^3*d^4 + 32*a^4*c^3*d^2*e^2 - 2*a*b^5*c*d^3*e - 32*a^3*b*c
^3*d^3*e + 16*a^3*b^3*c*d*e^3 - 32*a^4*b*c^2*d*e^3 + 16*a^2*b^3*c^2*d^3*e -
 6*a^2*b^4*c*d^2*e^2)))^(1/2)*((x*(16*b^5*c^2*e^7 + 16*c^7*d^5*e^2 - 112*a*
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b^3*c^3*e^7 + 192*a^2*b*c^4*e^7 + 32*a*c^6*d^3*e^4 - 240*a^2*c^5*d*e^6 - 32
*b*c^6*d^4*e^3 - 32*b^4*c^3*d*e^6 + 16*b^2*c^5*d^3*e^4 + 16*b^3*c^4*d^2*e^5
 - 96*a*b*c^5*d^2*e^5 + 192*a*b^2*c^4*d*e^6) - (-(b^5*e^2 + b^3*c^2*d^2 - b
^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2
*b*c^2*e^2 - 2*b^4*c*d*e - 4*a*b*c^3*d^2 - 7*a*b^3*c*e^2 + a*c*e^2*(-(4*a*c
 - b^2)^3)^(1/2) - 16*a^2*c^3*d*e + 12*a*b^2*c^2*d*e + 2*b*c*d*e*(-(4*a*c -
 b^2)^3)^(1/2))/(8*(a^3*b^4*e^4 + 16*a^3*c^4*d^4 + 16*a^5*c^2*e^4 + a*b^4*c
^2*d^4 - 8*a^4*b^2*c*e^4 + a*b^6*d^2*e^2 - 2*a^2*b^5*d*e^3 - 8*a^2*b^2*c^3*
d^4 + 32*a^4*c^3*d^2*e^2 - 2*a*b^5*c*d^3*e - 32*a^3*b*c^3*d^3*e + 16*a^3*b^
3*c*d*e^3 - 32*a^4*b*c^2*d*e^3 + 16*a^2*b^3*c^2*d^3*e - 6*a^2*b^4*c*d^2*e^2
)))^(1/2)*(x*(-(b^5*e^2 + b^3*c^2*d^2 - b^2*e^2*(-(4*a*c - b^2)^3)^(1/2) -
c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*e^2 - 2*b^4*c*d*e - 4*a*b*c
^3*d^2 - 7*a*b^3*c*e^2 + a*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - 16*a^2*c^3*d*e
+ 12*a*b^2*c^2*d*e + 2*b*c*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^3*b^4*e^4 +
16*a^3*c^4*d^4 + 16*a^5*c^2*e^4 + a*b^4*c^2*d^4 - 8*a^4*b^2*c*e^4 + a*b^6*d
^2*e^2 - 2*a^2*b^5*d*e^3 - 8*a^2*b^2*c^3*d^4 + 32*a^4*c^3*d^2*e^2 - 2*a*b^5
*c*d^3*e - 32*a^3*b*c^3*d^3*e + 16*a^3*b^3*c*d*e^3 - 32*a^4*b*c^2*d*e^3 + 1
6*a^2*b^3*c^2*d^3*e - 6*a^2*b^4*c*d^2*e^2)))^(1/2)*(256*a^4*b^2*c^3*e^9 - 3
2*a^3*b^4*c^2*e^9 - 512*a^5*c^4*e^9 + 512*a^2*c^7*d^6*e^3 + 512*a^3*c^6*d^4
*e^5 - 512*a^4*c^5*d^2*e^7 - 32*b^3*c^6*d^7*e^2 + 128*b^4*c^5*d^6*e^3 - 192
*b^5*c^4*d^5*e^4 + 128*b^6*c^3*d^4*e^5 - 32*b^7*c^2*d^3*e^6 + 512*a^2*b^2*c
^5*d^4*e^5 + 288*a^2*b^3*c^4*d^3*e^6 - 192*a^2*b^4*c^3*d^2*e^7 + 384*a^3*b^
2*c^4*d^2*e^7 + 128*a*b*c^7*d^7*e^2 + 640*a^4*b*c^4*d*e^8 - 640*a*b^2*c^6*d
^6*e^3 + 1056*a*b^3*c^5*d^5*e^4 - 672*a*b^4*c^4*d^4*e^5 + 96*a*b^5*c^3*d^3*
e^6 + 32*a*b^6*c^2*d^2*e^7 - 1152*a^2*b*c^6*d^5*e^4 + 32*a^2*b^5*c^2*d*e^8
- 640*a^3*b*c^5*d^3*e^6 - 288*a^3*b^3*c^3*d*e^8) - 256*a^4*c^4*e^8 + 64*a*c
^7*d^6*e^2 - 16*a^2*b^4*c^2*e^8 + 128*a^3*b^2*c^3*e^8 - 128*a^2*c^6*d^4*e^4
 - 448*a^3*c^5*d^2*e^6 - 16*b^2*c^6*d^6*e^2 + 64*b^3*c^5*d^5*e^3 - 96*b^4*c
^4*d^4*e^4 + 64*b^5*c^3*d^3*e^5 - 16*b^6*c^2*d^2*e^6 + 240*a^2*b^2*c^4*d^2*
e^6 - 256*a*b*c^6*d^5*e^3 + 32*a*b^5*c^2*d*e^7 + 384*a^3*b*c^4*d*e^7 + 416*
a*b^2*c^5*d^4*e^4 - 288*a*b^3*c^4*d^3*e^5 + 32*a*b^4*c^3*d^2*e^6 + 128*a^2*
b*c^5*d^3*e^5 - 224*a^2*b^3*c^3*d*e^7))*(-(b^5*e^2 + b^3*c^2*d^2 - b^2*e^2*
(-(4*a*c - b^2)^3)^(1/2) - c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*
e^2 - 2*b^4*c*d*e - 4*a*b*c^3*d^2 - 7*a*b^3*c*e^2 + a*c*e^2*(-(4*a*c - b^2)
^3)^(1/2) - 16*a^2*c^3*d*e + 12*a*b^2*c^2*d*e + 2*b*c*d*e*(-(4*a*c - b^2)^3
)^(1/2))/(8*(a^3*b^4*e^4 + 16*a^3*c^4*d^4 + 16*a^5*c^2*e^4 + a*b^4*c^2*d^4
- 8*a^4*b^2*c*e^4 + a*b^6*d^2*e^2 - 2*a^2*b^5*d*e^3 - 8*a^2*b^2*c^3*d^4 + 3
2*a^4*c^3*d^2*e^2 - 2*a*b^5*c*d^3*e - 32*a^3*b*c^3*d^3*e + 16*a^3*b^3*c*d*e
^3 - 32*a^4*b*c^2*d*e^3 + 16*a^2*b^3*c^2*d^3*e - 6*a^2*b^4*c*d^2*e^2)))^(1/
2) - 4*b^3*c^3*e^6 - 4*c^6*d^3*e^3 + 4*b*c^5*d^2*e^4 + 4*b^2*c^4*d*e^5 + 16
*a*b*c^4*e^6 - 20*a*c^5*d*e^5) + 6*c^5*e^5*x)*(-(b^5*e^2 + b^3*c^2*d^2 - b^
2*e^2*(-(4*a*c - b^2)^3)^(1/2) - c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*
b*c^2*e^2 - 2*b^4*c*d*e - 4*a*b*c^3*d^2 - 7*a*b^3*c*e^2 + a*c*e^2*(-(4*a*c
- b^2)^3)^(1/2) - 16*a^2*c^3*d*e + 12*a*b^2*c^2*d*e + 2*b*c*d*e*(-(4*a*c -
b^2)^3)^(1/2))/(8*(a^3*b^4*e^4 + 16*a^3*c^4*d^4 + 16*a^5*c^2*e^4 + a*b^4*c^

2*d^4 - 8*a^4*b^2*c*e^4 + a*b^6*d^2*e^2 - 2*a^2*b^5*d*e^3 - 8*a^2*b^2*c^3*d
^4 + 32*a^4*c^3*d^2*e^2 - 2*a*b^5*c*d^3*e - 32*a^3*b*c^3*d^3*e + 16*a^3*b^3
*c*d*e^3 - 32*a^4*b*c^2*d*e^3 + 16*a^2*b^3*c^2*d^3*e - 6*a^2*b^4*c*d^2*e^2)
))^(1/2)*1i + ((-(b^5*e^2 + b^3*c^2*d^2 - b^2*e^2*(-(4*a*c - b^2)^3)^(1/2)
- c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*e^2 - 2*b^4*c*d*e - 4*a*b
*c^3*d^2 - 7*a*b^3*c*e^2 + a*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - 16*a^2*c^3*d*
e + 12*a*b^2*c^2*d*e + 2*b*c*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^3*b^4*e^4
+ 16*a^3*c^4*d^4 + 16*a^5*c^2*e^4 + a*b^4*c^2*d^4 - 8*a^4*b^2*c*e^4 + a*b^6
*d^2*e^2 - 2*a^2*b^5*d*e^3 - 8*a^2*b^2*c^3*d^4 + 32*a^4*c^3*d^2*e^2 - 2*a*b
^5*c*d^3*e - 32*a^3*b*c^3*d^3*e + 16*a^3*b^3*c*d*e^3 - 32*a^4*b*c^2*d*e^3 +
16*a^2*b^3*c^2*d^3*e - 6*a^2*b^4*c*d^2*e^2)))^(1/2)*((x*(16*b^5*c^2*e^7 +
16*c^7*d^5*e^2 - 112*a*b^3*c^3*e^7 + 192*a^2*b*c^4*e^7 + 32*a*c^6*d^3*e^4 -
240*a^2*c^5*d*e^6 - 32*b*c^6*d^4*e^3 - 32*b^4*c^3*d*e^6 + 16*b^2*c^5*d^3*e
^4 + 16*b^3*c^4*d^2*e^5 - 96*a*b*c^5*d^2*e^5 + 192*a*b^2*c^4*d*e^6) - (-(b^
5*e^2 + b^3*c^2*d^2 - b^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - c^2*d^2*(-(4*a*c -
b^2)^3)^(1/2) + 12*a^2*b*c^2*e^2 - 2*b^4*c*d*e - 4*a*b*c^3*d^2 - 7*a*b^3*c
*e^2 + a*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - 16*a^2*c^3*d*e + 12*a*b^2*c^2*d*e
+ 2*b*c*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^3*b^4*e^4 + 16*a^3*c^4*d^4 + 1
6*a^5*c^2*e^4 + a*b^4*c^2*d^4 - 8*a^4*b^2*c*e^4 + a*b^6*d^2*e^2 - 2*a^2*b^5
*d*e^3 - 8*a^2*b^2*c^3*d^4 + 32*a^4*c^3*d^2*e^2 - 2*a*b^5*c*d^3*e - 32*a^3*
b*c^3*d^3*e + 16*a^3*b^3*c*d*e^3 - 32*a^4*b*c^2*d*e^3 + 16*a^2*b^3*c^2*d^3*
e - 6*a^2*b^4*c*d^2*e^2)))^(1/2)*(256*a^4*c^4*e^8 + x*(-(b^5*e^2 + b^3*c^2*
d^2 - b^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) +
12*a^2*b*c^2*e^2 - 2*b^4*c*d*e - 4*a*b*c^3*d^2 - 7*a*b^3*c*e^2 + a*c*e^2*(
-(4*a*c - b^2)^3)^(1/2) - 16*a^2*c^3*d*e + 12*a*b^2*c^2*d*e + 2*b*c*d*e*(-(
4*a*c - b^2)^3)^(1/2))/(8*(a^3*b^4*e^4 + 16*a^3*c^4*d^4 + 16*a^5*c^2*e^4 +
a*b^4*c^2*d^4 - 8*a^4*b^2*c*e^4 + a*b^6*d^2*e^2 - 2*a^2*b^5*d*e^3 - 8*a^2*b
^2*c^3*d^4 + 32*a^4*c^3*d^2*e^2 - 2*a*b^5*c*d^3*e - 32*a^3*b*c^3*d^3*e + 16
*a^3*b^3*c*d*e^3 - 32*a^4*b*c^2*d*e^3 + 16*a^2*b^3*c^2*d^3*e - 6*a^2*b^4*c*
d^2*e^2)))^(1/2)*(256*a^4*b^2*c^3*e^9 - 32*a^3*b^4*c^2*e^9 - 512*a^5*c^4*e^
9 + 512*a^2*c^7*d^6*e^3 + 512*a^3*c^6*d^4*e^5 - 512*a^4*c^5*d^2*e^7 - 32*b^
3*c^6*d^7*e^2 + 128*b^4*c^5*d^6*e^3 - 192*b^5*c^4*d^5*e^4 + 128*b^6*c^3*d^4
*e^5 - 32*b^7*c^2*d^3*e^6 + 512*a^2*b^2*c^5*d^4*e^5 + 288*a^2*b^3*c^4*d^3*e
^6 - 192*a^2*b^4*c^3*d^2*e^7 + 384*a^3*b^2*c^4*d^2*e^7 + 128*a*b*c^7*d^7*e^
2 + 640*a^4*b*c^4*d*e^8 - 640*a*b^2*c^6*d^6*e^3 + 1056*a*b^3*c^5*d^5*e^4 -
672*a*b^4*c^4*d^4*e^5 + 96*a*b^5*c^3*d^3*e^6 + 32*a*b^6*c^2*d^2*e^7 - 1152*
a^2*b*c^6*d^5*e^4 + 32*a^2*b^5*c^2*d*e^8 - 640*a^3*b*c^5*d^3*e^6 - 288*a^3*
b^3*c^3*d*e^8) - 64*a*c^7*d^6*e^2 + 16*a^2*b^4*c^2*e^8 - 128*a^3*b^2*c^3*e^
8 + 128*a^2*c^6*d^4*e^4 + 448*a^3*c^5*d^2*e^6 + 16*b^2*c^6*d^6*e^2 - 64*b^3
*c^5*d^5*e^3 + 96*b^4*c^4*d^4*e^4 - 64*b^5*c^3*d^3*e^5 + 16*b^6*c^2*d^2*e^6
- 240*a^2*b^2*c^4*d^2*e^6 + 256*a*b*c^6*d^5*e^3 - 32*a*b^5*c^2*d*e^7 - 384
*a^3*b*c^4*d*e^7 - 416*a*b^2*c^5*d^4*e^4 + 288*a*b^3*c^4*d^3*e^5 - 32*a*b^4
*c^3*d^2*e^6 - 128*a^2*b*c^5*d^3*e^5 + 224*a^2*b^3*c^3*d*e^7))*(-(b^5*e^2 +
b^3*c^2*d^2 - b^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - c^2*d^2*(-(4*a*c - b^2)^3
)^(1/2) + 12*a^2*b*c^2*e^2 - 2*b^4*c*d*e - 4*a*b*c^3*d^2 - 7*a*b^3*c*e^2 +

a*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - 16*a^2*c^3*d*e + 12*a*b^2*c^2*d*e + 2*b*c*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^3*b^4*e^4 + 16*a^3*c^4*d^4 + 16*a^5*c^2*e^4 + a*b^4*c^2*d^4 - 8*a^4*b^2*c*e^4 + a*b^6*d^2*e^2 - 2*a^2*b^5*d*e^3 - 8*a^2*b^2*c^3*d^4 + 32*a^4*c^3*d^2*e^2 - 2*a*b^5*c*d^3*e - 32*a^3*b*c^3*d^3*e + 16*a^3*b^3*c*d*e^3 - 32*a^4*b*c^2*d*e^3 + 16*a^2*b^3*c^2*d^3*e - 6*a^2*b^4*c*d^2*e^2)))^(1/2) + 4*b^3*c^3*e^6 + 4*c^6*d^3*e^3 - 4*b*c^5*d^2*e^4 - 4*b^2*c^4*d*e^5 - 16*a*b*c^4*e^6 + 20*a*c^5*d*e^5) + 6*c^5*e^5*x)*(-(b^5*e^2 + b^3*c^2*d^2 - b^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*e^2 - 2*b^4*c*d*e - 4*a*b*c^3*d^2 - 7*a*b^3*c*e^2 + a*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - 16*a^2*c^3*d*e + 12*a*b^2*c^2*d*e + 2*b*c*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^3*b^4*e^4 + 16*a^3*c^4*d^4 + 16*a^5*c^2*e^4 + a*b^4*c^2*d^4 - 8*a^4*b^2*c*e^4 + a*b^6*d^2*e^2 - 2*a^2*b^5*d*e^3 - 8*a^2*b^2*c^3*d^4 + 32*a^4*c^3*d^2*e^2 - 2*a*b^5*c*d^3*e - 32*a^3*b*c^3*d^3*e + 16*a^3*b^3*c*d*e^3 - 32*a^4*b*c^2*d*e^3 + 16*a^2*b^3*c^2*d^3*e - 6*a^2*b^4*c*d^2*e^2)))^(1/2)*1i)/(((-(b^5*e^2 + b^3*c^2*d^2 - b^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*e^2 - 2*b^4*c*d*e - 4*a*b*c^3*d^2 - 7*a*b^3*c*e^2 + a*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - 16*a^2*c^3*d*e + 12*a*b^2*c^2*d*e + 2*b*c*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^3*b^4*e^4 + 16*a^3*c^4*d^4 + 16*a^5*c^2*e^4 + a*b^4*c^2*d^4 - 8*a^4*b^2*c*e^4 + a*b^6*d^2*e^2 - 2*a^2*b^5*d*e^3 - 8*a^2*b^2*c^3*d^4 + 32*a^4*c^3*d^2*e^2 - 2*a*b^5*c*d^3*e - 32*a^3*b*c^3*d^3*e + 16*a^3*b^3*c*d*e^3 - 32*a^4*b*c^2*d*e^3 + 16*a^2*b^3*c^2*d^3*e - 6*a^2*b^4*c*d^2*e^2)))^(1/2)*((x*(16*b^5*c^2*e^7 + 16*c^7*d^5*e^2 - 112*a*b^3*c^3*e^7 + 192*a^2*b*c^4*e^7 + 32*a*c^6*d^3*e^4 - 240*a^2*c^5*d*e^6 - 32*b*c^6*d^4*e^3 - 32*b^4*c^3*d*e^6 + 16*b^2*c^5*d^3*e^4 + 16*b^3*c^4*d^2*e^5 - 96*a*b*c^5*d^2*e^5 + 192*a*b^2*c^4*d*e^6) - (-(b^5*e^2 + b^3*c^2*d^2 - b^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*e^2 - 2*b^4*c*d*e - 4*a*b*c^3*d^2 - 7*a*b^3*c*e^2 + a*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - 16*a^2*c^3*d*e + 12*a*b^2*c^2*d*e + 2*b*c*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^3*b^4*e^4 + 16*a^3*c^4*d^4 + 16*a^5*c^2*e^4 + a*b^4*c^2*d^4 - 8*a^4*b^2*c*e^4 + a*b^6*d^2*e^2 - 2*a^2*b^5*d*e^3 - 8*a^2*b^2*c^3*d^4 + 32*a^4*c^3*d^2*e^2 - 2*a*b^5*c*d^3*e - 32*a^3*b*c^3*d^3*e + 16*a^3*b^3*c*d*e^3 - 32*a^4*b*c^2*d*e^3 + 16*a^2*b^3*c^2*d^3*e - 6*a^2*b^4*c*d^2*e^2)))^(1/2)*(x*(-(b^5*e^2 + b^3*c^2*d^2 - b^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*e^2 - 2*b^4*c*d*e - 4*a*b*c^3*d^2 - 7*a*b^3*c*e^2 + a*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - 16*a^2*c^3*d*e + 12*a*b^2*c^2*d*e + 2*b*c*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^3*b^4*e^4 + 16*a^3*c^4*d^4 + 16*a^5*c^2*e^4 + a*b^4*c^2*d^4 - 8*a^4*b^2*c*e^4 + a*b^6*d^2*e^2 - 2*a^2*b^5*d*e^3 - 8*a^2*b^2*c^3*d^4 + 32*a^4*c^3*d^2*e^2 - 2*a*b^5*c*d^3*e - 32*a^3*b*c^3*d^3*e + 16*a^3*b^3*c*d*e^3 - 32*a^4*b*c^2*d*e^3 + 16*a^2*b^3*c^2*d^3*e - 6*a^2*b^4*c*d^2*e^2)))^(1/2)*(256*a^4*b^2*c^3*e^9 - 32*a^3*b^4*c^2*e^9 - 512*a^5*c^4*e^9 + 512*a^2*c^7*d^6*e^3 + 512*a^3*c^6*d^4*e^5 - 512*a^4*c^5*d^2*e^7 - 32*b^3*c^6*d^7*e^2 + 128*b^4*c^5*d^6*e^3 - 192*b^5*c^4*d^5*e^4 + 128*b^6*c^3*d^4*e^5 - 32*b^7*c^2*d^3*e^6 + 512*a^2*b^2*c^5*d^4*e^5 + 288*a^2*b^3*c^4*d^3*e^6 - 192*a^2*b^4*c^3*d^2*e^7 + 384*a^3*b^2*c^4*d^2*e^7 + 128*a*b*c^7*d

^7*e^2 + 640*a^4*b*c^4*d*e^8 - 640*a*b^2*c^6*d^6*e^3 + 1056*a*b^3*c^5*d^5*e
^4 - 672*a*b^4*c^4*d^4*e^5 + 96*a*b^5*c^3*d^3*e^6 + 32*a*b^6*c^2*d^2*e^7 -
1152*a^2*b*c^6*d^5*e^4 + 32*a^2*b^5*c^2*d*e^8 - 640*a^3*b*c^5*d^3*e^6 - 288
*a^3*b^3*c^3*d*e^8) - 256*a^4*c^4*e^8 + 64*a*c^7*d^6*e^2 - 16*a^2*b^4*c^2*e
^8 + 128*a^3*b^2*c^3*e^8 - 128*a^2*c^6*d^4*e^4 - 448*a^3*c^5*d^2*e^6 - 16*b
^2*c^6*d^6*e^2 + 64*b^3*c^5*d^5*e^3 - 96*b^4*c^4*d^4*e^4 + 64*b^5*c^3*d^3*e
^5 - 16*b^6*c^2*d^2*e^6 + 240*a^2*b^2*c^4*d^2*e^6 - 256*a*b*c^6*d^5*e^3 + 3
2*a*b^5*c^2*d*e^7 + 384*a^3*b*c^4*d*e^7 + 416*a*b^2*c^5*d^4*e^4 - 288*a*b^3
*c^4*d^3*e^5 + 32*a*b^4*c^3*d^2*e^6 + 128*a^2*b*c^5*d^3*e^5 - 224*a^2*b^3*c
^3*d*e^7))*(-(b^5*e^2 + b^3*c^2*d^2 - b^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - c^
2*d^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*e^2 - 2*b^4*c*d*e - 4*a*b*c^3
*d^2 - 7*a*b^3*c*e^2 + a*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - 16*a^2*c^3*d*e +
12*a*b^2*c^2*d*e + 2*b*c*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^3*b^4*e^4 + 16
*a^3*c^4*d^4 + 16*a^5*c^2*e^4 + a*b^4*c^2*d^4 - 8*a^4*b^2*c*e^4 + a*b^6*d^2
*e^2 - 2*a^2*b^5*d*e^3 - 8*a^2*b^2*c^3*d^4 + 32*a^4*c^3*d^2*e^2 - 2*a*b^5*c
*d^3*e - 32*a^3*b*c^3*d^3*e + 16*a^3*b^3*c*d*e^3 - 32*a^4*b*c^2*d*e^3 + 16*
a^2*b^3*c^2*d^3*e - 6*a^2*b^4*c*d^2*e^2)))^(1/2) - 4*b^3*c^3*e^6 - 4*c^6*d^
3*e^3 + 4*b*c^5*d^2*e^4 + 4*b^2*c^4*d*e^5 + 16*a*b*c^4*e^6 - 20*a*c^5*d*e^5
) + 6*c^5*e^5*x)*(-(b^5*e^2 + b^3*c^2*d^2 - b^2*e^2*(-(4*a*c - b^2)^3)^(1/2
) - c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*e^2 - 2*b^4*c*d*e - 4*a
*b*c^3*d^2 - 7*a*b^3*c*e^2 + a*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - 16*a^2*c^3*
d*e + 12*a*b^2*c^2*d*e + 2*b*c*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^3*b^4*e^
4 + 16*a^3*c^4*d^4 + 16*a^5*c^2*e^4 + a*b^4*c^2*d^4 - 8*a^4*b^2*c*e^4 + a*b
^6*d^2*e^2 - 2*a^2*b^5*d*e^3 - 8*a^2*b^2*c^3*d^4 + 32*a^4*c^3*d^2*e^2 - 2*a
*b^5*c*d^3*e - 32*a^3*b*c^3*d^3*e + 16*a^3*b^3*c*d*e^3 - 32*a^4*b*c^2*d*e^3
 + 16*a^2*b^3*c^2*d^3*e - 6*a^2*b^4*c*d^2*e^2)))^(1/2) - ((-(b^5*e^2 + b^3*
c^2*d^2 - b^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - c^2*d^2*(-(4*a*c - b^2)^3)^(1/
2) + 12*a^2*b*c^2*e^2 - 2*b^4*c*d*e - 4*a*b*c^3*d^2 - 7*a*b^3*c*e^2 + a*c*e
^2*(-(4*a*c - b^2)^3)^(1/2) - 16*a^2*c^3*d*e + 12*a*b^2*c^2*d*e + 2*b*c*d*e
*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^3*b^4*e^4 + 16*a^3*c^4*d^4 + 16*a^5*c^2*e^
4 + a*b^4*c^2*d^4 - 8*a^4*b^2*c*e^4 + a*b^6*d^2*e^2 - 2*a^2*b^5*d*e^3 - 8*a
^2*b^2*c^3*d^4 + 32*a^4*c^3*d^2*e^2 - 2*a*b^5*c*d^3*e - 32*a^3*b*c^3*d^3*e
+ 16*a^3*b^3*c*d*e^3 - 32*a^4*b*c^2*d*e^3 + 16*a^2*b^3*c^2*d^3*e - 6*a^2*b^
4*c*d^2*e^2)))^(1/2)*((x*(16*b^5*c^2*e^7 + 16*c^7*d^5*e^2 - 112*a*b^3*c^3*e
^7 + 192*a^2*b*c^4*e^7 + 32*a*c^6*d^3*e^4 - 240*a^2*c^5*d*e^6 - 32*b*c^6*d^
4*e^3 - 32*b^4*c^3*d*e^6 + 16*b^2*c^5*d^3*e^4 + 16*b^3*c^4*d^2*e^5 - 96*a*b
*c^5*d^2*e^5 + 192*a*b^2*c^4*d*e^6) - (-(b^5*e^2 + b^3*c^2*d^2 - b^2*e^2*(-
(4*a*c - b^2)^3)^(1/2) - c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*e^
2 - 2*b^4*c*d*e - 4*a*b*c^3*d^2 - 7*a*b^3*c*e^2 + a*c*e^2*(-(4*a*c - b^2)^3
)^(1/2) - 16*a^2*c^3*d*e + 12*a*b^2*c^2*d*e + 2*b*c*d*e*(-(4*a*c - b^2)^3)^
(1/2))/(8*(a^3*b^4*e^4 + 16*a^3*c^4*d^4 + 16*a^5*c^2*e^4 + a*b^4*c^2*d^4 -
8*a^4*b^2*c*e^4 + a*b^6*d^2*e^2 - 2*a^2*b^5*d*e^3 - 8*a^2*b^2*c^3*d^4 + 32*
a^4*c^3*d^2*e^2 - 2*a*b^5*c*d^3*e - 32*a^3*b*c^3*d^3*e + 16*a^3*b^3*c*d*e^3
 - 32*a^4*b*c^2*d*e^3 + 16*a^2*b^3*c^2*d^3*e - 6*a^2*b^4*c*d^2*e^2)))^(1/2)
*(256*a^4*c^4*e^8 + x*(-(b^5*e^2 + b^3*c^2*d^2 - b^2*e^2*(-(4*a*c - b^2)^3)

^(1/2) - c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*e^2 - 2*b^4*c*d*e - 4*a*b*c^3*d^2 - 7*a*b^3*c*e^2 + a*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - 16*a^2*c^3*d*e + 12*a*b^2*c^2*d*e + 2*b*c*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^3*b^4*e^4 + 16*a^3*c^4*d^4 + 16*a^5*c^2*e^4 + a*b^4*c^2*d^4 - 8*a^4*b^2*c*e^4 + a*b^6*d^2*e^2 - 2*a^2*b^5*d*e^3 - 8*a^2*b^2*c^3*d^4 + 32*a^4*c^3*d^2*e^2 - 2*a*b^5*c*d^3*e - 32*a^3*b*c^3*d^3*e + 16*a^3*b^3*c*d*e^3 - 32*a^4*b*c^2*d*e^3 + 16*a^2*b^3*c^2*d^3*e - 6*a^2*b^4*c*d^2*e^2)))^(1/2)*(256*a^4*b^2*c^3*e^9 - 32*a^3*b^4*c^2*e^9 - 512*a^5*c^4*e^9 + 512*a^2*c^7*d^6*e^3 + 512*a^3*c^6*d^4*e^5 - 512*a^4*c^5*d^2*e^7 - 32*b^3*c^6*d^7*e^2 + 128*b^4*c^5*d^6*e^3 - 192*b^5*c^4*d^5*e^4 + 128*b^6*c^3*d^4*e^5 - 32*b^7*c^2*d^3*e^6 + 512*a^2*b^2*c^5*d^4*e^5 + 288*a^2*b^3*c^4*d^3*e^6 - 192*a^2*b^4*c^3*d^2*e^7 + 384*a^3*b^2*c^4*d^2*e^7 + 128*a*b*c^7*d^7*e^2 + 640*a^4*b*c^4*d*e^8 - 640*a*b^2*c^6*d^6*e^3 + 1056*a*b^3*c^5*d^5*e^4 - 672*a*b^4*c^4*d^4*e^5 + 96*a*b^5*c^3*d^3*e^6 + 32*a*b^6*c^2*d^2*e^7 - 1152*a^2*b*c^6*d^5*e^4 + 32*a^2*b^5*c^2*d*e^8 - 640*a^3*b*c^5*d^3*e^6 - 288*a^3*b^3*c^3*d*e^8) - 64*a*c^7*d^6*e^2 + 16*a^2*b^4*c^2*e^8 - 128*a^3*b^2*c^3*e^8 + 128*a^2*c^6*d^4*e^4 + 448*a^3*c^5*d^2*e^6 + 16*b^2*c^6*d^6*e^2 - 64*b^3*c^5*d^5*e^3 + 96*b^4*c^4*d^4*e^4 - 64*b^5*c^3*d^3*e^5 + 16*b^6*c^2*d^2*e^6 - 240*a^2*b^2*c^4*d^2*e^6 + 256*a*b*c^6*d^5*e^3 - 32*a*b^5*c^2*d*e^7 - 384*a^3*b*c^4*d*e^7 - 416*a*b^2*c^5*d^4*e^4 + 288*a*b^3*c^4*d^3*e^5 - 32*a*b^4*c^3*d^2*e^6 - 128*a^2*b*c^5*d^3*e^5 + 224*a^2*b^3*c^3*d*e^7))*(-(b^5*e^2 + b^3*c^2*d^2 - b^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*e^2 - 2*b^4*c*d*e - 4*a*b*c^3*d^2 - 7*a*b^3*c*e^2 + a*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - 16*a^2*c^3*d*e + 12*a*b^2*c^2*d*e + 2*b*c*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^3*b^4*e^4 + 16*a^3*c^4*d^4 + 16*a^5*c^2*e^4 + a*b^4*c^2*d^4 - 8*a^4*b^2*c*e^4 + a*b^6*d^2*e^2 - 2*a^2*b^5*d*e^3 - 8*a^2*b^2*c^3*d^4 + 32*a^4*c^3*d^2*e^2 - 2*a*b^5*c*d^3*e - 32*a^3*b*c^3*d^3*e + 16*a^3*b^3*c*d*e^3 - 32*a^4*b*c^2*d*e^3 + 16*a^2*b^3*c^2*d^3*e - 6*a^2*b^4*c*d^2*e^2)))^(1/2) + 4*b^3*c^3*e^6 + 4*c^6*d^3*e^3 - 4*b*c^5*d^2*e^4 - 4*b^2*c^4*d*e^5 - 16*a*b*c^4*e^6 + 20*a*c^5*d*e^5) + 6*c^5*e^5*x)*(-(b^5*e^2 + b^3*c^2*d^2 - b^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*e^2 - 2*b^4*c*d*e - 4*a*b*c^3*d^2 - 7*a*b^3*c*e^2 + a*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - 16*a^2*c^3*d*e + 12*a*b^2*c^2*d*e + 2*b*c*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^3*b^4*e^4 + 16*a^3*c^4*d^4 + 16*a^5*c^2*e^4 + a*b^4*c^2*d^4 - 8*a^4*b^2*c*e^4 + a*b^6*d^2*e^2 - 2*a^2*b^5*d*e^3 - 8*a^2*b^2*c^3*d^4 + 32*a^4*c^3*d^2*e^2 - 2*a*b^5*c*d^3*e - 32*a^3*b*c^3*d^3*e + 16*a^3*b^3*c*d*e^3 - 32*a^4*b*c^2*d*e^3 + 16*a^2*b^3*c^2*d^3*e - 6*a^2*b^4*c*d^2*e^2)))^(1/2)))*(-(b^5*e^2 + b^3*c^2*d^2 - b^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*e^2 - 2*b^4*c*d*e - 4*a*b*c^3*d^2 - 7*a*b^3*c*e^2 + a*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - 16*a^2*c^3*d*e + 12*a*b^2*c^2*d*e + 2*b*c*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^3*b^4*e^4 + 16*a^3*c^4*d^4 + 16*a^5*c^2*e^4 + a*b^4*c^2*d^4 - 8*a^4*b^2*c*e^4 + a*b^6*d^2*e^2 - 2*a^2*b^5*d*e^3 - 8*a^2*b^2*c^3*d^4 + 32*a^4*c^3*d^2*e^2 - 2*a*b^5*c*d^3*e - 32*a^3*b*c^3*d^3*e + 16*a^3*b^3*c*d*e^3 - 32*a^4*b*c^2*d*e^3 + 16*a^2*b^3*c^2*d^3*e - 6*a^2*b^4*c*d^2*e^2)))^(1/2)*2i - (log(b^5*d*(-d*e^3)^(5/2) - b^

5*d^3*e^8*x + c^5*d^8*e^3*x + 2*a*c^4*d^5*(-d*e^3)^(3/2) - 16*a^3*c^2*e*(-d*e^3)^(5/2) - c^5*d^8*e*(-d*e^3)^(1/2) + b^2*c^3*d^5*(-d*e^3)^(3/2) - a*b^4*e*(-d*e^3)^(5/2) - 7*a*b^3*c*d*(-d*e^3)^(5/2) + 17*a^2*c^3*d^3*e^2*(-d*e^3)^(3/2) + a*b^4*d^2*e^9*x + 2*a*c^4*d^6*e^5*x - 2*b*c^4*d^7*e^4*x + 2*b^4*c*d^4*e^7*x + 12*a^2*b*c^2*d*(-d*e^3)^(5/2) + 8*a^2*b^2*c*e*(-d*e^3)^(5/2) + 17*a^2*c^3*d^4*e^7*x + 16*a^3*c^2*d^2*e^9*x + b^2*c^3*d^6*e^5*x - b^3*c^2*d^5*e^6*x - b^3*c^2*d^4*e*(-d*e^3)^(3/2) + 2*b^4*c*d^3*e^2*(-d*e^3)^(3/2) + 2*b*c^4*d^7*e^2*(-d*e^3)^(1/2) - 12*a*b^2*c^2*d^4*e^7*x - 12*a^2*b*c^2*d^3*e^8*x - 8*a^2*b^2*c*d^2*e^9*x - 12*a*b^2*c^2*d^3*e^2*(-d*e^3)^(3/2) + 2*a*b*c^3*d^5*e^6*x + 7*a*b^3*c*d^3*e^8*x + 2*a*b*c^3*d^4*e*(-d*e^3)^(3/2))*(-d*e^3)^(1/2))/(2*(c*d^3 + a*d*e^2 - b*d^2*e)) + (log(b^5*d*(-d*e^3)^(5/2) + b^5*d^3*e^8*x - c^5*d^8*e^3*x + 2*a*c^4*d^5*(-d*e^3)^(3/2) - 16*a^3*c^2*e*(-d*e^3)^(5/2) - c^5*d^8*e*(-d*e^3)^(1/2) + b^2*c^3*d^5*(-d*e^3)^(3/2) - a*b^4*e*(-d*e^3)^(5/2) - 7*a*b^3*c*d*(-d*e^3)^(5/2) + 17*a^2*c^3*d^3*e^2*(-d*e^3)^(3/2) - a*b^4*d^2*e^9*x - 2*a*c^4*d^6*e^5*x + 2*b*c^4*d^7*e^4*x - 2*b^4*c*d^4*e^7*x + 12*a^2*b*c^2*d*(-d*e^3)^(5/2) + 8*a^2*b^2*c*e*(-d*e^3)^(5/2) - 17*a^2*c^3*d^4*e^7*x - 16*a^3*c^2*d^2*e^9*x - b^2*c^3*d^6*e^5*x + b^3*c^2*d^5*e^6*x - b^3*c^2*d^4*e*(-d*e^3)^(3/2) + 2*b^4*c*d^3*e^2*(-d*e^3)^(3/2) + 2*b*c^4*d^7*e^2*(-d*e^3)^(1/2) + 12*a*b^2*c^2*d^4*e^7*x + 12*a^2*b*c^2*d^3*e^8*x + 8*a^2*b^2*c*d^2*e^9*x - 12*a*b^2*c^2*d^3*e^2*(-d*e^3)^(3/2) - 2*a*b*c^3*d^5*e^6*x - 7*a*b^3*c*d^3*e^8*x + 2*a*b*c^3*d^4*e*(-d*e^3)^(3/2))*(-d*e^3)^(1/2))/(2*c*d^3 + 2*a*d*e^2 - 2*b*d^2*e)

3.308 $\int \frac{1}{x^2(d+ex^2)(a+bx^2+cx^4)}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 298

$$\int \frac{1}{x^2\left(d+ex^2\right)\left(a+bx^2+cx^4\right)}\,dx = -\frac{1}{adx} - \frac{\sqrt{c}\left(cd-be+\frac{bcd-b^2e+2ace}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}a\sqrt{b-\sqrt{b^2-4ac}}\left(cd^2-bde+ae^2\right)} - \frac{\sqrt{c}\left(cd-be-\frac{bcd-b^2e+2ace}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{2}a\sqrt{b+\sqrt{b^2-4ac}}\left(cd^2-bde+ae^2\right)} - \frac{e^{5/2}\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{d^{3/2}\left(cd^2-bde+ae^2\right)}$$

```
[Out] -1/a/d/x-e^(5/2)*arctan(x*e^(1/2)/d^(1/2))/d^(3/2)/(a*e^2-b*d*e+c*d^2)-1/2*
arctan(x*2^(1/2)*c^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2))*c^(1/2)*(c*d-b*e+(2*
a*c*e-b^2*e+b*c*d)/(-4*a*c+b^2)^(1/2))/a/(a*e^2-b*d*e+c*d^2)*2^(1/2)/(b-(-4
*a*c+b^2)^(1/2))^(1/2)-1/2*arctan(x*2^(1/2)*c^(1/2)/(b+(-4*a*c+b^2)^(1/2))^
(1/2))*c^(1/2)*(c*d-b*e+(-2*a*c*e+b^2*e-b*c*d)/(-4*a*c+b^2)^(1/2))/a/(a*e^2
-b*d*e+c*d^2)*2^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2)
```

Rubi [A] (verified)

Time = 0.59 (sec) , antiderivative size = 298, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {1301, 211, 1180}

$$\int \frac{1}{x^2(d+ex^2)(a+bx^2+cx^4)}\,dx = -\frac{\sqrt{c}\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)\left(\frac{2ace+b^2(-e)+bcd}{\sqrt{b^2-4ac}}-be+cd\right)}{\sqrt{2}a\sqrt{b-\sqrt{b^2-4ac}}\,(ae^2-bde+cd^2)} - \frac{\sqrt{c}\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)\left(-\frac{2ace+b^2(-e)+bcd}{\sqrt{b^2-4ac}}-be+cd\right)}{\sqrt{2}a\sqrt{\sqrt{b^2-4ac}+b}\,(ae^2-bde+cd^2)} - \frac{e^{5/2}\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{d^{3/2}(ae^2-bde+cd^2)} - \frac{1}{adx}$$

```
[In] Int[1/(x^2*(d + e*x^2)*(a + b*x^2 + c*x^4)),x]
```

```
[Out] -(1/(a*d*x)) - (Sqrt[c]*(c*d - b*e + (b*c*d - b^2*e + 2*a*c*e)/Sqrt[b^2 - 4
*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*a*
Sqrt[b - Sqrt[b^2 - 4*a*c]]*(c*d^2 - b*d*e + a*e^2)) - (Sqrt[c]*(c*d - b*e
- (b*c*d - b^2*e + 2*a*c*e)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/S
qrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*a*Sqrt[b + Sqrt[b^2 - 4*a*c]]*(c*d^2
- b*d*e + a*e^2)) - (e^(5/2)*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(d^(3/2)*(c*d^2 -
 b*d*e + a*e^2))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1301

```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.))/((a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*((d + e*x^2)^q/(a
+ b*x^2 + c*x^4)), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b^2 - 4
*a*c, 0] && IntegerQ[q] && IntegerQ[m]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \left(\frac{1}{adx^2} - \frac{e^3}{d(cd^2-bde+ae^2)(d+ex^2)} + \frac{-bcd+b^2e-ace-c(cd-be)x^2}{a(cd^2-bde+ae^2)(a+bx^2+cx^4)}\right) dx \\
&= -\frac{1}{adx} + \frac{\int \frac{-bcd+b^2e-ace-c(cd-be)x^2}{a+bx^2+cx^4}\,dx}{a(cd^2-bde+ae^2)} - \frac{e^3\int \frac{1}{d+ex^2}\,dx}{d(cd^2-bde+ae^2)} \\
&= -\frac{1}{adx} - \frac{e^{5/2}\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{d^{3/2}(cd^2-bde+ae^2)} - \frac{\left(c\left(cd-be-\frac{bcd-b^2e+2ace}{\sqrt{b^2-4ac}}\right)\right)\int \frac{1}{\frac{b}{2}+\frac{1}{2}\sqrt{b^2-4ac}+cx^2}\,dx}{2a(cd^2-bde+ae^2)} \\
&\quad - \frac{\left(c\left(cd-be+\frac{bcd-b^2e+2ace}{\sqrt{b^2-4ac}}\right)\right)\int \frac{1}{\frac{b}{2}-\frac{1}{2}\sqrt{b^2-4ac}+cx^2}\,dx}{2a(cd^2-bde+ae^2)} \\
&= -\frac{1}{adx} - \frac{\sqrt{c}\left(cd-be+\frac{bcd-b^2e+2ace}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}a\sqrt{b-\sqrt{b^2-4ac}}(cd^2-bde+ae^2)} \\
&\quad - \frac{\sqrt{c}\left(cd-be-\frac{bcd-b^2e+2ace}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{2}a\sqrt{b+\sqrt{b^2-4ac}}(cd^2-bde+ae^2)} - \frac{e^{5/2}\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{d^{3/2}(cd^2-bde+ae^2)}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.26 (sec) , antiderivative size = 340, normalized size of antiderivative = 1.14

$$
\begin{aligned}
&\int \frac{1}{x^2(d+ex^2)(a+bx^2+cx^4)}\,dx \\
&= -\frac{1}{adx} - \frac{\sqrt{c}\left(bcd+c\sqrt{b^2-4ac}d-b^2e+2ace-b\sqrt{b^2-4ac}e\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}a\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}(cd^2+e(-bd+ae))} \\
&\quad + \frac{\sqrt{c}\left(bcd-c\sqrt{b^2-4ac}d-b^2e+2ace+b\sqrt{b^2-4ac}e\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{2}a\sqrt{b^2-4ac}\sqrt{b+\sqrt{b^2-4ac}}(cd^2+e(-bd+ae))} \\
&\quad - \frac{e^{5/2}\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{d^{3/2}(cd^2-bde+ae^2)}
\end{aligned}
$$

```
[In] Integrate[1/(x^2*(d + e*x^2)*(a + b*x^2 + c*x^4)),x]
```

```
[Out] -(1/(a*d*x)) - (Sqrt[c]*(b*c*d + c*Sqrt[b^2 - 4*a*c]*d - b^2*e + 2*a*c*e -
b*Sqrt[b^2 - 4*a*c]*e)*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c
]]])/(Sqrt[2]*a*Sqrt[b^2 - 4*a*c]*Sqrt[b - Sqrt[b^2 - 4*a*c]]*(c*d^2 + e*(-
(b*d) + a*e))) + (Sqrt[c]*(b*c*d - c*Sqrt[b^2 - 4*a*c]*d - b^2*e + 2*a*c*e
+ b*Sqrt[b^2 - 4*a*c]*e)*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a
*c]]])/(Sqrt[2]*a*Sqrt[b^2 - 4*a*c]*Sqrt[b + Sqrt[b^2 - 4*a*c]]*(c*d^2 + e*
(-(b*d) + a*e))) - (e^(5/2)*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(d^(3/2)*(c*d^2 -
b*d*e + a*e^2))
```

Maple [A] (verified)

Time = 0.46 (sec) , antiderivative size = 276, normalized size of antiderivative = 0.93

method	result
default	$\frac{4c\left(\frac{\left(be\sqrt{-4ac+b^2}-cd\sqrt{-4ac+b^2}+2ace-b^2e+bcd\right)\sqrt{2}\,\arctan\left(\frac{cx\sqrt{2}}{\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}}\right)}{8\sqrt{-4ac+b^2}\,\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}}-\frac{\left(be\sqrt{-4ac+b^2}-cd\sqrt{-4ac+b^2}-2ace+b^2e-bcd\right)\sqrt{2}\,\text{arcta}}{8\sqrt{-4ac+b^2}\,\sqrt{\left(-b+\sqrt{-4ac+b^2}\right.}}\right.}{(a\,e^2-bde+c\,d^2)a}$
risch	Expression too large to display

```
[In] int(1/x^2/(e*x^2+d)/(c*x^4+b*x^2+a),x,method=_RETURNVERBOSE)
```

```
[Out] 4/(a*e^2-b*d*e+c*d^2)/a*c*(1/8*(b*e*(-4*a*c+b^2)^(1/2)-c*d*(-4*a*c+b^2)^(1/
2)+2*a*c*e-b^2*e+b*c*d)/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*
c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))-1/8*(b*e*(-4*
a*c+b^2)^(1/2)-c*d*(-4*a*c+b^2)^(1/2)-2*a*c*e+b^2*e-b*c*d)/(-4*a*c+b^2)^(1/
2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(c*x*2^(1/2)/((-b+(-4*a
*c+b^2)^(1/2))*c)^(1/2)))-1/d*e^3/(a*e^2-b*d*e+c*d^2)/(e*d)^(1/2)*arctan(e*
x/(e*d)^(1/2))-1/a/d/x
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 10286 vs. 2(254) = 508.

Time = 166.67 (sec) , antiderivative size = 20595, normalized size of antiderivative = 69.11

$$\int \frac{1}{x^2\,(d+ex^2)\,(a+bx^2+cx^4)}\,dx = \text{Too large to display}$$

```
[In] integrate(1/x^2/(e*x^2+d)/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{x^2\,(d+ex^2)\,(a+bx^2+cx^4)}\,dx = \text{Timed out}$$

```
[In] integrate(1/x**2/(e*x**2+d)/(c*x**4+b*x**2+a),x)
```

```
[Out] Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{x^2 (d + ex^2) (a + bx^2 + cx^4)} dx = \text{Exception raised: ValueError}$$

```
[In] integrate(1/x^2/(e*x^2+d)/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 10072 vs. 2(254) = 508.

Time = 2.51 (sec) , antiderivative size = 10072, normalized size of antiderivative = 33.80

$$\int \frac{1}{x^2 (d + ex^2) (a + bx^2 + cx^4)} dx = \text{Too large to display}$$

```
[In] integrate(1/x^2/(e*x^2+d)/(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] -e^3*arctan(e*x/sqrt(d*e))/((c*d^3 - b*d^2*e + a*d*e^2)*sqrt(d*e)) - 1/8*((
2*a^2*b^4*c^5 - 8*a^3*b^2*c^6 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b
^2 - 4*a*c)*c)*a^2*b^4*c^3 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^
2 - 4*a*c)*c)*a^3*b^2*c^4 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2
 - 4*a*c)*c)*a^2*b^3*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 -
4*a*c)*c)*a^2*b^2*c^5 - 2*(b^2 - 4*a*c)*a^2*b^2*c^5)*d^5 - (6*a^2*b^5*c^4 -
 28*a^3*b^3*c^5 + 16*a^4*b*c^6 - 3*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqr
t(b^2 - 4*a*c)*c)*a^2*b^5*c^2 + 14*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqr
t(b^2 - 4*a*c)*c)*a^3*b^3*c^3 + 6*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt
(b^2 - 4*a*c)*c)*a^2*b^4*c^3 - 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(
b^2 - 4*a*c)*c)*a^4*b*c^4 - 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2
 - 4*a*c)*c)*a^3*b^2*c^4 - 3*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2
- 4*a*c)*c)*a^2*b^3*c^4 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 -
 4*a*c)*c)*a^3*b*c^5 - 6*(b^2 - 4*a*c)*a^2*b^3*c^4 + 4*(b^2 - 4*a*c)*a^3*b*
c^5)*d^4*e + (6*a^2*b^6*c^3 - 28*a^3*b^4*c^4 + 16*a^4*b^2*c^5 - 3*sqrt(2)*s
qrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^6*c + 14*sqrt(2)*sqr
t(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^4*c^2 + 6*sqrt(2)*sqrt
(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^5*c^2 - 8*sqrt(2)*sqrt(
b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b^2*c^3 - 4*sqrt(2)*sqrt(b
^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^3*c^3 - 3*sqrt(2)*sqrt(b^
2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^4*c^3 + 2*sqrt(2)*sqrt(b^2
```

- 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^2*c^4 - 6*(b^2 - 4*a*c)*a^2
*b^4*c^3 + 4*(b^2 - 4*a*c)*a^3*b^2*c^4)*d^3*e^2 - (2*a^2*b^7*c^2 - 4*a^3*b^
5*c^3 - 24*a^4*b^3*c^4 + 32*a^5*b*c^5 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c
+ sqrt(b^2 - 4*a*c)*c)*a^2*b^7 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqr
t(b^2 - 4*a*c)*c)*a^3*b^5*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b
^2 - 4*a*c)*c)*a^2*b^6*c + 12*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2
- 4*a*c)*c)*a^4*b^3*c^2 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2
- 4*a*c)*c)*a^3*b^4*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4
*a*c)*c)*a^2*b^5*c^2 - 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4
*a*c)*c)*a^5*b*c^3 - 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*
c)*c)*a^4*b^2*c^3 - 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c
)*c)*a^3*b^3*c^3 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)
*c)*a^4*b*c^4 - 2*(b^2 - 4*a*c)*a^2*b^5*c^2 - 4*(b^2 - 4*a*c)*a^3*b^3*c^3 +
8*(b^2 - 4*a*c)*a^4*b*c^4)*d^2*e^3 + (4*a^3*b^6*c^2 - 22*a^4*b^4*c^3 + 24*
a^5*b^2*c^4 - 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a
^3*b^6 + 11*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b
^4*c + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^5*
c - 12*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^5*b^2*c^
2 - 6*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b^3*c^2
- 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^4*c^2
+ 3*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b^2*c^3 -
4*(b^2 - 4*a*c)*a^3*b^4*c^2 + 6*(b^2 - 4*a*c)*a^4*b^2*c^3)*d*e^4 - (2*a^4*
b^5*c^2 - 12*a^5*b^3*c^3 + 16*a^6*b*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*
c + sqrt(b^2 - 4*a*c)*c)*a^4*b^5 + 6*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + s
qrt(b^2 - 4*a*c)*c)*a^5*b^3*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt
(b^2 - 4*a*c)*c)*a^4*b^4*c - 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^
2 - 4*a*c)*c)*a^6*b*c^2 - 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 -
4*a*c)*c)*a^5*b^2*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*
a*c)*c)*a^4*b^3*c^2 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a
*c)*c)*a^5*b*c^3 - 2*(b^2 - 4*a*c)*a^4*b^3*c^2 + 4*(b^2 - 4*a*c)*a^5*b*c^3)
e^5 + 2(sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^5*c^2 - 8*sqrt(2)*sqr
t(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c^3 - 2*sqrt(2)*sqrt(b*c + sqrt(b^2 -
4*a*c)*c)*a*b^4*c^3 - 2*a*b^5*c^3 + 16*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)
*c)*a^3*b*c^4 + 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^4 + sqr
t(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c^4 + 16*a^2*b^3*c^4 - 4*sqrt(2)
*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b*c^5 - 32*a^3*b*c^5 + 2*(b^2 - 4*a*c)
*a*b^3*c^3 - 8*(b^2 - 4*a*c)*a^2*b*c^4)*d^3*abs(a*c*d^2 - a*b*d*e + a^2*e^2
) - 2*(2*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^6*c - 17*sqrt(2)*sqrt(
b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^4*c^2 - 4*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*
a*c)*c)*a*b^5*c^2 - 4*a*b^6*c^2 + 40*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c
)*a^3*b^2*c^3 + 18*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c^3 + 2*
sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^4*c^3 + 34*a^2*b^4*c^3 - 16*sqr
t(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*c^4 - 8*sqrt(2)*sqrt(b*c + sqrt(b^
2 - 4*a*c)*c)*a^3*b*c^4 - 9*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^2
*c^4 - 80*a^3*b^2*c^4 + 4*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*c^5 +

32*a^4*c^5 + 4*(b^2 - 4*a*c)*a*b^4*c^2 - 18*(b^2 - 4*a*c)*a^2*b^2*c^3 + 8*
(b^2 - 4*a*c)*a^3*c^4)*d^2*e*abs(a*c*d^2 - a*b*d*e + a^2*e^2) + 2*(sqrt(2)*
sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^7 - 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a
*c)*c)*a^2*b^5*c - 2*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^6*c - 2*a*
b^7*c + 16*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^3*c^2 + 8*sqrt(2)*
sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^4*c^2 + sqrt(2)*sqrt(b*c + sqrt(b^2 -
4*a*c)*c)*a*b^5*c^2 + 16*a^2*b^5*c^2 - 4*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a
*c)*c)*a^2*b^3*c^3 - 32*a^3*b^3*c^3 + 2*(b^2 - 4*a*c)*a*b^5*c - 8*(b^2 - 4*
a*c)*a^2*b^3*c^2)*d*e^2*abs(a*c*d^2 - a*b*d*e + a^2*e^2) - 2*(sqrt(2)*sqrt(
b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^6 - 9*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)
*c)*a^3*b^4*c - 2*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^5*c - 2*a^2
*b^6*c + 24*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b^2*c^2 + 10*sqrt(2
)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^3*c^2 + sqrt(2)*sqrt(b*c + sqrt(b^2
- 4*a*c)*c)*a^2*b^4*c^2 + 18*a^3*b^4*c^2 - 16*sqrt(2)*sqrt(b*c + sqrt(b^2
- 4*a*c)*c)*a^5*c^3 - 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b*c^3 -
5*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^2*c^3 - 48*a^4*b^2*c^3 + 4
*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*c^4 + 32*a^5*c^4 + 2*(b^2 - 4*
a*c)*a^2*b^4*c - 10*(b^2 - 4*a*c)*a^3*b^2*c^2 + 8*(b^2 - 4*a*c)*a^4*c^3)*e^
3*abs(a*c*d^2 - a*b*d*e + a^2*e^2) + (2*b^4*c^3 - 16*a*b^2*c^4 + 32*a^2*c^5
- sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^4*c + 8*sqrt
(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^2 + 2*sqrt(2)
*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^3*c^2 - 16*sqrt(2)*sqr
t(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*c^3 - 8*sqrt(2)*sqrt(b^2
- 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c^3 - sqrt(2)*sqrt(b^2 - 4*a*
c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^2*c^3 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sq
rt(b*c + sqrt(b^2 - 4*a*c)*c)*a*c^4 - 2*(b^2 - 4*a*c)*b^2*c^3 + 8*(b^2 - 4*
a*c)*a*c^4)*(a*c*d^2 - a*b*d*e + a^2*e^2)^2*d - (2*b^5*c^2 - 16*a*b^3*c^3 +
32*a^2*b*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b
^5 + 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c +
2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^4*c - 16*sqrt
(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b*c^2 - 8*sqrt(2)
*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^2 - sqrt(2)*sqrt
(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^3*c^2 + 4*sqrt(2)*sqrt(b^2
- 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c^3 - 2*(b^2 - 4*a*c)*b^3*c^2
+ 8*(b^2 - 4*a*c)*a*b*c^3)*(a*c*d^2 - a*b*d*e + a^2*e^2)^2*e)*arctan(2*sqrt
(1/2)*x/sqrt((a*b*c*d^2 - a*b^2*d*e + a^2*b*e^2 + sqrt((a*b*c*d^2 - a*b^2*d
*e + a^2*b*e^2)^2 - 4*(a^2*c*d^2 - a^2*b*d*e + a^3*e^2)*(a*c^2*d^2 - a*b*c*
d*e + a^2*c*e^2)))/(a*c^2*d^2 - a*b*c*d*e + a^2*c*e^2)))/((a^3*b^4*c^2 - 8*
a^4*b^2*c^3 - 2*a^3*b^3*c^3 + 16*a^5*c^4 + 8*a^4*b*c^4 + a^3*b^2*c^4 - 4*a^
4*c^5)*d^4*abs(a*c*d^2 - a*b*d*e + a^2*e^2)*abs(c) - 2*(a^3*b^5*c - 8*a^4*b
^3*c^2 - 2*a^3*b^4*c^2 + 16*a^5*b*c^3 + 8*a^4*b^2*c^3 + a^3*b^3*c^3 - 4*a^4
*b*c^4)*d^3*e*abs(a*c*d^2 - a*b*d*e + a^2*e^2)*abs(c) + (a^3*b^6 - 6*a^4*b^
4*c - 2*a^3*b^5*c + 4*a^4*b^3*c^2 + a^3*b^4*c^2 + 32*a^6*c^3 + 16*a^5*b*c^3
- 2*a^4*b^2*c^3 - 8*a^5*c^4)*d^2*e^2*abs(a*c*d^2 - a*b*d*e + a^2*e^2)*abs(
c) - 2*(a^4*b^5 - 8*a^5*b^3*c - 2*a^4*b^4*c + 16*a^6*b*c^2 + 8*a^5*b^2*c^2

+ a^4*b^3*c^2 - 4*a^5*b*c^3)*d*e^3*abs(a*c*d^2 - a*b*d*e + a^2*e^2)*abs(c)
+ (a^5*b^4 - 8*a^6*b^2*c - 2*a^5*b^3*c + 16*a^7*c^2 + 8*a^6*b*c^2 + a^5*b^2
*c^2 - 4*a^6*c^3)*e^4*abs(a*c*d^2 - a*b*d*e + a^2*e^2)*abs(c)) - 1/8*((2*a^
2*b^4*c^5 - 8*a^3*b^2*c^6 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 -
4*a*c)*c)*a^2*b^4*c^3 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 -
4*a*c)*c)*a^3*b^2*c^4 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4
*a*c)*c)*a^2*b^3*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*
c)*c)*a^2*b^2*c^5 - 2*(b^2 - 4*a*c)*a^2*b^2*c^5)*d^5 - (6*a^2*b^5*c^4 - 28*
a^3*b^3*c^5 + 16*a^4*b*c^6 - 3*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^
2 - 4*a*c)*c)*a^2*b^5*c^2 + 14*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^
2 - 4*a*c)*c)*a^3*b^3*c^3 + 6*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2
- 4*a*c)*c)*a^2*b^4*c^3 - 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2
- 4*a*c)*c)*a^4*b*c^4 - 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4
*a*c)*c)*a^3*b^2*c^4 - 3*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*
a*c)*c)*a^2*b^3*c^4 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a
*c)*c)*a^3*b*c^5 - 6*(b^2 - 4*a*c)*a^2*b^3*c^4 + 4*(b^2 - 4*a*c)*a^3*b*c^5)
*d^4*e + (6*a^2*b^6*c^3 - 28*a^3*b^4*c^4 + 16*a^4*b^2*c^5 - 3*sqrt(2)*sqrt(
b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^6*c + 14*sqrt(2)*sqrt(b^
2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^4*c^2 + 6*sqrt(2)*sqrt(b^2
- 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^5*c^2 - 8*sqrt(2)*sqrt(b^2
- 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*b^2*c^3 - 4*sqrt(2)*sqrt(b^2 -
4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^3*c^3 - 3*sqrt(2)*sqrt(b^2 -
4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^4*c^3 + 2*sqrt(2)*sqrt(b^2 - 4
*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^2*c^4 - 6*(b^2 - 4*a*c)*a^2*b^4
c^3 + 4(b^2 - 4*a*c)*a^3*b^2*c^4)*d^3*e^2 - (2*a^2*b^7*c^2 - 4*a^3*b^5*c^
3 - 24*a^4*b^3*c^4 + 32*a^5*b*c^5 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sq
rt(b^2 - 4*a*c)*c)*a^2*b^7 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^
2 - 4*a*c)*c)*a^3*b^5*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 -
4*a*c)*c)*a^2*b^6*c + 12*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4
*a*c)*c)*a^4*b^3*c^2 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*
a*c)*c)*a^3*b^4*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c
)*c)*a^2*b^5*c^2 - 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c
)*c)*a^5*b*c^3 - 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c
)*a^4*b^2*c^3 - 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)
*a^3*b^3*c^3 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*
a^4*b*c^4 - 2*(b^2 - 4*a*c)*a^2*b^5*c^2 - 4*(b^2 - 4*a*c)*a^3*b^3*c^3 + 8*(
b^2 - 4*a*c)*a^4*b*c^4)*d^2*e^3 + (4*a^3*b^6*c^2 - 22*a^4*b^4*c^3 + 24*a^5*
b^2*c^4 - 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b
^6 + 11*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*b^4*c
+ 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^5*c -
12*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^5*b^2*c^2 -
6*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*b^3*c^2 - 2
*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^4*c^2 + 3*
sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*b^2*c^3 - 4*(
b^2 - 4*a*c)*a^3*b^4*c^2 + 6*(b^2 - 4*a*c)*a^4*b^2*c^3)*d*e^4 - (2*a^4*b^5*

c^2 - 12*a^5*b^3*c^3 + 16*a^6*b*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*b^5 + 6*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^5*b^3*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*b^4*c - 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^6*b*c^2 - 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^5*b^2*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*b^3*c^2 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^5*b*c^3 - 2*(b^2 - 4*a*c)*a^4*b^3*c^2 + 4*(b^2 - 4*a*c)*a^5*b*c^3)*e^5 + 2*(sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^5*c^2 - 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c^3 - 2*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^4*c^3 + 2*a*b^5*c^3 + 16*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b*c^4 + 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^4 + sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^3*c^4 - 16*a^2*b^3*c^4 - 4*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b*c^5 + 32*a^3*b*c^5 - 2*(b^2 - 4*a*c)*a*b^3*c^3 + 8*(b^2 - 4*a*c)*a^2*b*c^4)*d^3*abs(a*c*d^2 - a*b*d*e + a^2*e^2) - 2*(2*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^6*c - 17*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^4*c^2 - 4*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^5*c^2 + 4*a*b^6*c^2 + 40*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^2*c^3 + 18*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c^3 + 2*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^4*c^3 - 34*a^2*b^4*c^3 - 16*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*c^4 - 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b*c^4 - 9*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^4 + 80*a^3*b^2*c^4 + 4*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*c^5 - 32*a^4*c^5 - 4*(b^2 - 4*a*c)*a*b^4*c^2 + 18*(b^2 - 4*a*c)*a^2*b^2*c^3 - 8*(b^2 - 4*a*c)*a^3*c^4)*d^2*e*abs(a*c*d^2 - a*b*d*e + a^2*e^2) + 2*(sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^7 - 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^5*c - 2*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^6*c + 2*a*b^7*c + 16*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^3*c^2 + 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^4*c^2 + sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^5*c^2 - 16*a^2*b^5*c^2 - 4*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c^3 + 32*a^3*b^3*c^3 - 2*(b^2 - 4*a*c)*a*b^5*c + 8*(b^2 - 4*a*c)*a^2*b^3*c^2)*d*e^2*abs(a*c*d^2 - a*b*d*e + a^2*e^2) - 2*(sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^6 - 9*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^4*c - 2*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^5*c + 2*a^2*b^6*c + 24*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*b^2*c^2 + 10*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^3*c^2 + sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^4*c^2 - 18*a^3*b^4*c^2 - 16*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^5*c^3 - 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*b*c^3 - 5*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^2*c^3 + 48*a^4*b^2*c^3 + 4*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*c^4 - 32*a^5*c^4 - 2*(b^2 - 4*a*c)*a^2*b^4*c + 10*(b^2 - 4*a*c)*a^3*b^2*c^2 - 8*(b^2 - 4*a*c)*a^4*c^3)*e^3*abs(a*c*d^2 - a*b*d*e + a^2*e^2) + (2*b^4*c^3 - 16*a*b^2*c^4 + 32*a^2*c^5 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^4*c + 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^2*c^2 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^3*c^2 - 16*sqrt(2)*sqrt(b^

```
2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*c^3 - 8*sqrt(2)*sqrt(b^2 - 4
*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*s
qrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^2*c^3 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b
*c - sqrt(b^2 - 4*a*c)*c)*a*c^4 - 2*(b^2 - 4*a*c)*b^2*c^3 + 8*(b^2 - 4*a*c)
*a*c^4)*(a*c*d^2 - a*b*d*e + a^2*e^2)^2*d - (2*b^5*c^2 - 16*a*b^3*c^3 + 32*
a^2*b*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^5 +
 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^3*c + 2*sq
rt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^4*c - 16*sqrt(2)*
sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b*c^2 - 8*sqrt(2)*sqr
t(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^2*c^2 - sqrt(2)*sqrt(b^2
 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^3*c^2 + 4*sqrt(2)*sqrt(b^2 - 4*
a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b*c^3 - 2*(b^2 - 4*a*c)*b^3*c^2 + 8*
(b^2 - 4*a*c)*a*b*c^3)*(a*c*d^2 - a*b*d*e + a^2*e^2)^2*e)*arctan(2*sqrt(1/2
)*x/sqrt((a*b*c*d^2 - a*b^2*d*e + a^2*b*e^2 - sqrt((a*b*c*d^2 - a*b^2*d*e +
 a^2*b*e^2)^2 - 4*(a^2*c*d^2 - a^2*b*d*e + a^3*e^2)*(a*c^2*d^2 - a*b*c*d*e
+ a^2*c*e^2)))/(a*c^2*d^2 - a*b*c*d*e + a^2*c*e^2)))/((a^3*b^4*c^2 - 8*a^4*
b^2*c^3 - 2*a^3*b^3*c^3 + 16*a^5*c^4 + 8*a^4*b*c^4 + a^3*b^2*c^4 - 4*a^4*c^
5)*d^4*abs(a*c*d^2 - a*b*d*e + a^2*e^2)*abs(c) - 2*(a^3*b^5*c - 8*a^4*b^3*c
^2 - 2*a^3*b^4*c^2 + 16*a^5*b*c^3 + 8*a^4*b^2*c^3 + a^3*b^3*c^3 - 4*a^4*b*c
^4)*d^3*e*abs(a*c*d^2 - a*b*d*e + a^2*e^2)*abs(c) + (a^3*b^6 - 6*a^4*b^4*c
- 2*a^3*b^5*c + 4*a^4*b^3*c^2 + a^3*b^4*c^2 + 32*a^6*c^3 + 16*a^5*b*c^3 - 2
*a^4*b^2*c^3 - 8*a^5*c^4)*d^2*e^2*abs(a*c*d^2 - a*b*d*e + a^2*e^2)*abs(c) -
 2*(a^4*b^5 - 8*a^5*b^3*c - 2*a^4*b^4*c + 16*a^6*b*c^2 + 8*a^5*b^2*c^2 + a^
4*b^3*c^2 - 4*a^5*b*c^3)*d*e^3*abs(a*c*d^2 - a*b*d*e + a^2*e^2)*abs(c) + (a
^5*b^4 - 8*a^6*b^2*c - 2*a^5*b^3*c + 16*a^7*c^2 + 8*a^6*b*c^2 + a^5*b^2*c^2
 - 4*a^6*c^3)*e^4*abs(a*c*d^2 - a*b*d*e + a^2*e^2)*abs(c)) - 1/(a*d*x)
```

Mupad [B] (verification not implemented)

Time = 11.33 (sec) , antiderivative size = 33644, normalized size of antiderivative = 112.90

$$\int \frac{1}{x^2 (d+ex^2) (a+bx^2+cx^4)} \, dx = \text{Too large to display}$$

```
[In] int(1/(x^2*(d + e*x^2)*(a + b*x^2 + c*x^4)),x)
```

```
[Out] atan((((-(b^7*e^2 + b^5*c^2*d^2 + b^4*e^2*(-(4*a*c - b^2)^3)^(1/2) - 7*a*b^
3*c^3*d^2 + 12*a^2*b*c^4*d^2 - a*c^3*d^2*(-(4*a*c - b^2)^3)^(1/2) - 20*a^3*
b*c^3*e^2 - 2*b^6*c*d*e + 25*a^2*b^3*c^2*e^2 + a^2*c^2*e^2*(-(4*a*c - b^2)^
3)^(1/2) + b^2*c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c*e^2 + 16*a^3*c^
4*d*e + 16*a*b^4*c^2*d*e - 2*b^3*c*d*e*(-(4*a*c - b^2)^3)^(1/2) - 3*a*b^2*c
*e^2*(-(4*a*c - b^2)^3)^(1/2) - 36*a^2*b^2*c^3*d*e + 4*a*b*c^2*d*e*(-(4*a*c
 - b^2)^3)^(1/2))/(8*(a^5*b^4*e^4 + 16*a^5*c^4*d^4 + 16*a^7*c^2*e^4 - 8*a^6
*b^2*c*e^4 - 2*a^4*b^5*d*e^3 + a^3*b^4*c^2*d^4 - 8*a^4*b^2*c^3*d^4 + a^3*b^
6*d^2*e^2 + 32*a^6*c^3*d^2*e^2 - 2*a^3*b^5*c*d^3*e - 32*a^5*b*c^3*d^3*e + 1
```

6*a^5*b^3*c*d*e^3 - 32*a^6*b*c^2*d*e^3 + 16*a^4*b^3*c^2*d^3*e - 6*a^4*b^4*c
*d^2*e^2)))^(1/2)*(((-(b^7*e^2 + b^5*c^2*d^2 + b^4*e^2*(-(4*a*c - b^2)^3)^(
1/2) - 7*a*b^3*c^3*d^2 + 12*a^2*b*c^4*d^2 - a*c^3*d^2*(-(4*a*c - b^2)^3)^(1
/2) - 20*a^3*b*c^3*e^2 - 2*b^6*c*d*e + 25*a^2*b^3*c^2*e^2 + a^2*c^2*e^2*(-(
4*a*c - b^2)^3)^(1/2) + b^2*c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c*e^
2 + 16*a^3*c^4*d*e + 16*a*b^4*c^2*d*e - 2*b^3*c*d*e*(-(4*a*c - b^2)^3)^(1/2
) - 3*a*b^2*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - 36*a^2*b^2*c^3*d*e + 4*a*b*c^2
*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^5*b^4*e^4 + 16*a^5*c^4*d^4 + 16*a^7*c^
2*e^4 - 8*a^6*b^2*c*e^4 - 2*a^4*b^5*d*e^3 + a^3*b^4*c^2*d^4 - 8*a^4*b^2*c^3
*d^4 + a^3*b^6*d^2*e^2 + 32*a^6*c^3*d^2*e^2 - 2*a^3*b^5*c*d^3*e - 32*a^5*b*
c^3*d^3*e + 16*a^5*b^3*c*d*e^3 - 32*a^6*b*c^2*d*e^3 + 16*a^4*b^3*c^2*d^3*e
- 6*a^4*b^4*c*d^2*e^2)))^(1/2)*(192*a^10*c^7*d^14*e^3 - x*(-(b^7*e^2 + b^5*
c^2*d^2 + b^4*e^2*(-(4*a*c - b^2)^3)^(1/2) - 7*a*b^3*c^3*d^2 + 12*a^2*b*c^4
*d^2 - a*c^3*d^2*(-(4*a*c - b^2)^3)^(1/2) - 20*a^3*b*c^3*e^2 - 2*b^6*c*d*e
+ 25*a^2*b^3*c^2*e^2 + a^2*c^2*e^2*(-(4*a*c - b^2)^3)^(1/2) + b^2*c^2*d^2*(
-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c*e^2 + 16*a^3*c^4*d*e + 16*a*b^4*c^2*d*e
 - 2*b^3*c*d*e*(-(4*a*c - b^2)^3)^(1/2) - 3*a*b^2*c*e^2*(-(4*a*c - b^2)^3)^
(1/2) - 36*a^2*b^2*c^3*d*e + 4*a*b*c^2*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^
5*b^4*e^4 + 16*a^5*c^4*d^4 + 16*a^7*c^2*e^4 - 8*a^6*b^2*c*e^4 - 2*a^4*b^5*d
*e^3 + a^3*b^4*c^2*d^4 - 8*a^4*b^2*c^3*d^4 + a^3*b^6*d^2*e^2 + 32*a^6*c^3*d
^2*e^2 - 2*a^3*b^5*c*d^3*e - 32*a^5*b*c^3*d^3*e + 16*a^5*b^3*c*d*e^3 - 32*a
^6*b*c^2*d*e^3 + 16*a^4*b^3*c^2*d^3*e - 6*a^4*b^4*c*d^2*e^2)))^(1/2)*(512*a
^11*c^7*d^15*e^3 + 512*a^12*c^6*d^13*e^5 - 512*a^13*c^5*d^11*e^7 - 512*a^14
*c^4*d^9*e^9 - 32*a^9*b^3*c^6*d^16*e^2 + 128*a^9*b^4*c^5*d^15*e^3 - 192*a^9
*b^5*c^4*d^14*e^4 + 128*a^9*b^6*c^3*d^13*e^5 - 32*a^9*b^7*c^2*d^12*e^6 - 64
0*a^10*b^2*c^6*d^15*e^3 + 1056*a^10*b^3*c^5*d^14*e^4 - 672*a^10*b^4*c^4*d^1
3*e^5 + 96*a^10*b^5*c^3*d^12*e^6 + 32*a^10*b^6*c^2*d^11*e^7 + 512*a^11*b^2*
c^5*d^13*e^5 + 288*a^11*b^3*c^4*d^12*e^6 - 192*a^11*b^4*c^3*d^11*e^7 + 32*a
^11*b^5*c^2*d^10*e^8 + 384*a^12*b^2*c^4*d^11*e^7 - 288*a^12*b^3*c^3*d^10*e^
8 - 32*a^12*b^4*c^2*d^9*e^9 + 256*a^13*b^2*c^3*d^9*e^9 + 128*a^10*b*c^7*d^1
6*e^2 - 1152*a^11*b*c^6*d^14*e^4 - 640*a^12*b*c^5*d^12*e^6 + 640*a^13*b*c^4
*d^10*e^8) + 128*a^11*c^6*d^12*e^5 - 320*a^12*c^5*d^10*e^7 - 256*a^13*c^4*d
^8*e^9 - 16*a^8*b^3*c^6*d^15*e^2 + 64*a^8*b^4*c^5*d^14*e^3 - 96*a^8*b^5*c^4
*d^13*e^4 + 64*a^8*b^6*c^3*d^12*e^5 - 16*a^8*b^7*c^2*d^11*e^6 - 304*a^9*b^2
*c^6*d^14*e^3 + 512*a^9*b^3*c^5*d^13*e^4 - 352*a^9*b^4*c^4*d^12*e^5 + 64*a^
9*b^5*c^3*d^11*e^6 + 16*a^9*b^6*c^2*d^10*e^7 + 352*a^10*b^2*c^5*d^12*e^5 +
80*a^10*b^3*c^4*d^11*e^6 - 128*a^10*b^4*c^3*d^10*e^7 + 16*a^10*b^5*c^2*d^9*
e^8 + 336*a^11*b^2*c^4*d^10*e^7 - 128*a^11*b^3*c^3*d^9*e^8 - 16*a^11*b^4*c^
2*d^8*e^9 + 128*a^12*b^2*c^3*d^8*e^9 + 64*a^9*b*c^7*d^15*e^2 - 512*a^10*b*c
^6*d^13*e^4 - 320*a^11*b*c^5*d^11*e^6 + 256*a^12*b*c^4*d^9*e^8) + x*(112*a^
10*c^6*d^10*e^6 - 32*a^9*c^7*d^12*e^4 - 16*a^8*c^8*d^14*e^2 - 128*a^11*c^5*
d^8*e^8 + 8*a^7*b^2*c^7*d^14*e^2 - 16*a^7*b^3*c^6*d^13*e^3 + 8*a^7*b^4*c^5*
d^12*e^4 + 8*a^7*b^5*c^4*d^11*e^5 - 16*a^7*b^6*c^3*d^10*e^6 + 8*a^7*b^7*c^2
*d^9*e^7 - 72*a^8*b^3*c^5*d^11*e^5 + 128*a^8*b^4*c^4*d^10*e^6 - 72*a^8*b^5*
c^3*d^9*e^7 - 280*a^9*b^2*c^5*d^10*e^6 + 208*a^9*b^3*c^4*d^9*e^7 - 16*a^9*b

^4*c^3*d^8*e^8 + 8*a^9*b^5*c^2*d^7*e^9 + 96*a^10*b^2*c^4*d^8*e^8 - 56*a^10*
b^3*c^3*d^7*e^9 + 32*a^8*b*c^7*d^13*e^3 + 128*a^9*b*c^6*d^11*e^5 - 192*a^10
*b*c^5*d^9*e^7 + 96*a^11*b*c^4*d^7*e^9))*(-(b^7*e^2 + b^5*c^2*d^2 + b^4*e^2
*(-(4*a*c - b^2)^3)^(1/2) - 7*a*b^3*c^3*d^2 + 12*a^2*b*c^4*d^2 - a*c^3*d^2*
(-(4*a*c - b^2)^3)^(1/2) - 20*a^3*b*c^3*e^2 - 2*b^6*c*d*e + 25*a^2*b^3*c^2*
e^2 + a^2*c^2*e^2*(-(4*a*c - b^2)^3)^(1/2) + b^2*c^2*d^2*(-(4*a*c - b^2)^3)
^(1/2) - 9*a*b^5*c*e^2 + 16*a^3*c^4*d*e + 16*a*b^4*c^2*d*e - 2*b^3*c*d*e*(-
(4*a*c - b^2)^3)^(1/2) - 3*a*b^2*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - 36*a^2*b^
2*c^3*d*e + 4*a*b*c^2*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^5*b^4*e^4 + 16*a^
5*c^4*d^4 + 16*a^7*c^2*e^4 - 8*a^6*b^2*c*e^4 - 2*a^4*b^5*d*e^3 + a^3*b^4*c^
2*d^4 - 8*a^4*b^2*c^3*d^4 + a^3*b^6*d^2*e^2 + 32*a^6*c^3*d^2*e^2 - 2*a^3*b^
5*c*d^3*e - 32*a^5*b*c^3*d^3*e + 16*a^5*b^3*c*d*e^3 - 32*a^6*b*c^2*d*e^3 +
16*a^4*b^3*c^2*d^3*e - 6*a^4*b^4*c*d^2*e^2)))^(1/2) + 4*a^7*c^8*d^13*e^2 +
4*a^8*c^7*d^11*e^4 - 16*a^10*c^5*d^7*e^8 - 4*a^7*b^5*c^3*d^8*e^7 + 4*a^7*b^
6*c^2*d^7*e^8 + 24*a^8*b^3*c^4*d^8*e^7 - 28*a^8*b^4*c^3*d^7*e^8 + 52*a^9*b^
2*c^4*d^7*e^8 - 4*a^7*b*c^7*d^12*e^3 - 32*a^9*b*c^5*d^8*e^7) + x*(2*a^7*c^7
*d^9*e^5 - 4*a^8*c^6*d^7*e^7 + 2*a^7*b^2*c^5*d^7*e^7))*(-(b^7*e^2 + b^5*c^2
*d^2 + b^4*e^2*(-(4*a*c - b^2)^3)^(1/2) - 7*a*b^3*c^3*d^2 + 12*a^2*b*c^4*d^
2 - a*c^3*d^2*(-(4*a*c - b^2)^3)^(1/2) - 20*a^3*b*c^3*e^2 - 2*b^6*c*d*e + 2
5*a^2*b^3*c^2*e^2 + a^2*c^2*e^2*(-(4*a*c - b^2)^3)^(1/2) + b^2*c^2*d^2*(-(4
*a*c - b^2)^3)^(1/2) - 9*a*b^5*c*e^2 + 16*a^3*c^4*d*e + 16*a*b^4*c^2*d*e -
2*b^3*c*d*e*(-(4*a*c - b^2)^3)^(1/2) - 3*a*b^2*c*e^2*(-(4*a*c - b^2)^3)^(1/
2) - 36*a^2*b^2*c^3*d*e + 4*a*b*c^2*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^5*b
^4*e^4 + 16*a^5*c^4*d^4 + 16*a^7*c^2*e^4 - 8*a^6*b^2*c*e^4 - 2*a^4*b^5*d*e^
3 + a^3*b^4*c^2*d^4 - 8*a^4*b^2*c^3*d^4 + a^3*b^6*d^2*e^2 + 32*a^6*c^3*d^2*
e^2 - 2*a^3*b^5*c*d^3*e - 32*a^5*b*c^3*d^3*e + 16*a^5*b^3*c*d*e^3 - 32*a^6*
b*c^2*d*e^3 + 16*a^4*b^3*c^2*d^3*e - 6*a^4*b^4*c*d^2*e^2)))^(1/2)*1i - ((-(
b^7*e^2 + b^5*c^2*d^2 + b^4*e^2*(-(4*a*c - b^2)^3)^(1/2) - 7*a*b^3*c^3*d^2
+ 12*a^2*b*c^4*d^2 - a*c^3*d^2*(-(4*a*c - b^2)^3)^(1/2) - 20*a^3*b*c^3*e^2
- 2*b^6*c*d*e + 25*a^2*b^3*c^2*e^2 + a^2*c^2*e^2*(-(4*a*c - b^2)^3)^(1/2) +
b^2*c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c*e^2 + 16*a^3*c^4*d*e + 16
*a*b^4*c^2*d*e - 2*b^3*c*d*e*(-(4*a*c - b^2)^3)^(1/2) - 3*a*b^2*c*e^2*(-(4*
a*c - b^2)^3)^(1/2) - 36*a^2*b^2*c^3*d*e + 4*a*b*c^2*d*e*(-(4*a*c - b^2)^3)
^(1/2))/(8*(a^5*b^4*e^4 + 16*a^5*c^4*d^4 + 16*a^7*c^2*e^4 - 8*a^6*b^2*c*e^4
- 2*a^4*b^5*d*e^3 + a^3*b^4*c^2*d^4 - 8*a^4*b^2*c^3*d^4 + a^3*b^6*d^2*e^2
+ 32*a^6*c^3*d^2*e^2 - 2*a^3*b^5*c*d^3*e - 32*a^5*b*c^3*d^3*e + 16*a^5*b^3*
c*d*e^3 - 32*a^6*b*c^2*d*e^3 + 16*a^4*b^3*c^2*d^3*e - 6*a^4*b^4*c*d^2*e^2))
)^(1/2)*(((-(b^7*e^2 + b^5*c^2*d^2 + b^4*e^2*(-(4*a*c - b^2)^3)^(1/2) - 7*a
*b^3*c^3*d^2 + 12*a^2*b*c^4*d^2 - a*c^3*d^2*(-(4*a*c - b^2)^3)^(1/2) - 20*a
^3*b*c^3*e^2 - 2*b^6*c*d*e + 25*a^2*b^3*c^2*e^2 + a^2*c^2*e^2*(-(4*a*c - b^
2)^3)^(1/2) + b^2*c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c*e^2 + 16*a^3
*c^4*d*e + 16*a*b^4*c^2*d*e - 2*b^3*c*d*e*(-(4*a*c - b^2)^3)^(1/2) - 3*a*b^
2*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - 36*a^2*b^2*c^3*d*e + 4*a*b*c^2*d*e*(-(4*
a*c - b^2)^3)^(1/2))/(8*(a^5*b^4*e^4 + 16*a^5*c^4*d^4 + 16*a^7*c^2*e^4 - 8*
a^6*b^2*c*e^4 - 2*a^4*b^5*d*e^3 + a^3*b^4*c^2*d^4 - 8*a^4*b^2*c^3*d^4 + a^3

*b^6*d^2*e^2 + 32*a^6*c^3*d^2*e^2 - 2*a^3*b^5*c*d^3*e - 32*a^5*b*c^3*d^3*e + 16*a^5*b^3*c*d*e^3 - 32*a^6*b*c^2*d*e^3 + 16*a^4*b^3*c^2*d^3*e - 6*a^4*b^4*c*d^2*e^2)))^(1/2)*(x*(-(b^7*e^2 + b^5*c^2*d^2 + b^4*e^2*(-(4*a*c - b^2)^3)^(1/2) - 7*a*b^3*c^3*d^2 + 12*a^2*b*c^4*d^2 - a*c^3*d^2*(-(4*a*c - b^2)^3)^(1/2) - 20*a^3*b*c^3*e^2 - 2*b^6*c*d*e + 25*a^2*b^3*c^2*e^2 + a^2*c^2*e^2*(-(4*a*c - b^2)^3)^(1/2) + b^2*c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c*e^2 + 16*a^3*c^4*d*e + 16*a*b^4*c^2*d*e - 2*b^3*c*d*e*(-(4*a*c - b^2)^3)^(1/2) - 3*a*b^2*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - 36*a^2*b^2*c^3*d*e + 4*a*b*c^2*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^5*b^4*e^4 + 16*a^5*c^4*d^4 + 16*a^7*c^2*e^4 - 8*a^6*b^2*c*e^4 - 2*a^4*b^5*d*e^3 + a^3*b^4*c^2*d^4 - 8*a^4*b^2*c^3*d^4 + a^3*b^6*d^2*e^2 + 32*a^6*c^3*d^2*e^2 - 2*a^3*b^5*c*d^3*e - 32*a^5*b*c^3*d^3*e + 16*a^5*b^3*c*d*e^3 - 32*a^6*b*c^2*d*e^3 + 16*a^4*b^3*c^2*d^3*e - 6*a^4*b^4*c*d^2*e^2)))^(1/2)*(512*a^11*c^7*d^15*e^3 + 512*a^12*c^6*d^13*e^5 - 512*a^13*c^5*d^11*e^7 - 512*a^14*c^4*d^9*e^9 - 32*a^9*b^3*c^6*d^16*e^2 + 128*a^9*b^4*c^5*d^15*e^3 - 192*a^9*b^5*c^4*d^14*e^4 + 128*a^9*b^6*c^3*d^13*e^5 - 32*a^9*b^7*c^2*d^12*e^6 - 640*a^10*b^2*c^6*d^15*e^3 + 1056*a^10*b^3*c^5*d^14*e^4 - 672*a^10*b^4*c^4*d^13*e^5 + 96*a^10*b^5*c^3*d^12*e^6 + 32*a^10*b^6*c^2*d^11*e^7 + 512*a^11*b^2*c^5*d^13*e^5 + 288*a^11*b^3*c^4*d^12*e^6 - 192*a^11*b^4*c^3*d^11*e^7 + 32*a^11*b^5*c^2*d^10*e^8 + 384*a^12*b^2*c^4*d^11*e^7 - 288*a^12*b^3*c^3*d^10*e^8 - 32*a^12*b^4*c^2*d^9*e^9 + 256*a^13*b^2*c^3*d^9*e^9 + 128*a^10*b*c^7*d^16*e^2 - 1152*a^11*b*c^6*d^14*e^4 - 640*a^12*b*c^5*d^12*e^6 + 640*a^13*b*c^4*d^10*e^8) + 192*a^10*c^7*d^14*e^3 + 128*a^11*c^6*d^12*e^5 - 320*a^12*c^5*d^10*e^7 - 256*a^13*c^4*d^8*e^9 - 16*a^8*b^3*c^6*d^15*e^2 + 64*a^8*b^4*c^5*d^14*e^3 - 96*a^8*b^5*c^4*d^13*e^4 + 64*a^8*b^6*c^3*d^12*e^5 - 16*a^8*b^7*c^2*d^11*e^6 - 304*a^9*b^2*c^6*d^14*e^3 + 512*a^9*b^3*c^5*d^13*e^4 - 352*a^9*b^4*c^4*d^12*e^5 + 64*a^9*b^5*c^3*d^11*e^6 + 16*a^9*b^6*c^2*d^10*e^7 + 352*a^10*b^2*c^5*d^12*e^5 + 80*a^10*b^3*c^4*d^11*e^6 - 128*a^10*b^4*c^3*d^10*e^7 + 16*a^10*b^5*c^2*d^9*e^8 + 336*a^11*b^2*c^4*d^10*e^7 - 128*a^11*b^3*c^3*d^9*e^8 - 16*a^11*b^4*c^2*d^8*e^9 + 128*a^12*b^2*c^3*d^8*e^9 + 64*a^9*b*c^7*d^15*e^2 - 512*a^10*b*c^6*d^13*e^4 - 320*a^11*b*c^5*d^11*e^6 + 256*a^12*b*c^4*d^9*e^8) - x*(112*a^10*c^6*d^10*e^6 - 32*a^9*c^7*d^12*e^4 - 16*a^8*c^8*d^14*e^2 - 128*a^11*c^5*d^8*e^8 + 8*a^7*b^2*c^7*d^14*e^2 - 16*a^7*b^3*c^6*d^13*e^3 + 8*a^7*b^4*c^5*d^12*e^4 + 8*a^7*b^5*c^4*d^11*e^5 - 16*a^7*b^6*c^3*d^10*e^6 + 8*a^7*b^7*c^2*d^9*e^7 - 72*a^8*b^3*c^5*d^11*e^5 + 128*a^8*b^4*c^4*d^10*e^6 - 72*a^8*b^5*c^3*d^9*e^7 - 280*a^9*b^2*c^5*d^10*e^6 + 208*a^9*b^3*c^4*d^9*e^7 - 16*a^9*b^4*c^3*d^8*e^8 + 8*a^9*b^5*c^2*d^7*e^9 + 96*a^10*b^2*c^4*d^8*e^8 - 56*a^10*b^3*c^3*d^7*e^9 + 32*a^8*b*c^7*d^13*e^3 + 128*a^9*b*c^6*d^11*e^5 - 192*a^10*b*c^5*d^9*e^7 + 96*a^11*b*c^4*d^7*e^9))*(-(b^7*e^2 + b^5*c^2*d^2 + b^4*e^2*(-(4*a*c - b^2)^3)^(1/2) - 7*a*b^3*c^3*d^2 + 12*a^2*b*c^4*d^2 - a*c^3*d^2*(-(4*a*c - b^2)^3)^(1/2) - 20*a^3*b*c^3*e^2 - 2*b^6*c*d*e + 25*a^2*b^3*c^2*e^2 + a^2*c^2*e^2*(-(4*a*c - b^2)^3)^(1/2) + b^2*c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c*e^2 + 16*a^3*c^4*d*e + 16*a*b^4*c^2*d*e - 2*b^3*c*d*e*(-(4*a*c - b^2)^3)^(1/2) - 3*a*b^2*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - 36*a^2*b^2*c^3*d*e + 4*a*b*c^2*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^5*b^4*e^4 + 16*a^5*c^4*d^4

+ 16*a^7*c^2*e^4 - 8*a^6*b^2*c*e^4 - 2*a^4*b^5*d*e^3 + a^3*b^4*c^2*d^4 - 8*a^4*b^2*c^3*d^4 + a^3*b^6*d^2*e^2 + 32*a^6*c^3*d^2*e^2 - 2*a^3*b^5*c*d^3*e - 32*a^5*b*c^3*d^3*e + 16*a^5*b^3*c*d*e^3 - 32*a^6*b*c^2*d*e^3 + 16*a^4*b^3*c^2*d^3*e - 6*a^4*b^4*c*d^2*e^2)))^(1/2) + 4*a^7*c^8*d^13*e^2 + 4*a^8*c^7*d^11*e^4 - 16*a^10*c^5*d^7*e^8 - 4*a^7*b^5*c^3*d^8*e^7 + 4*a^7*b^6*c^2*d^7*e^8 + 24*a^8*b^3*c^4*d^8*e^7 - 28*a^8*b^4*c^3*d^7*e^8 + 52*a^9*b^2*c^4*d^7*e^8 - 4*a^7*b*c^7*d^12*e^3 - 32*a^9*b*c^5*d^8*e^7) - x*(2*a^7*c^7*d^9*e^5 - 4*a^8*c^6*d^7*e^7 + 2*a^7*b^2*c^5*d^7*e^7))*(-(b^7*e^2 + b^5*c^2*d^2 + b^4*e^2*(-(4*a*c - b^2)^3)^(1/2) - 7*a*b^3*c^3*d^2 + 12*a^2*b*c^4*d^2 - a*c^3*d^2*(-(4*a*c - b^2)^3)^(1/2) - 20*a^3*b*c^3*e^2 - 2*b^6*c*d*e + 25*a^2*b^3*c^2*e^2 + a^2*c^2*e^2*(-(4*a*c - b^2)^3)^(1/2) + b^2*c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c*e^2 + 16*a^3*c^4*d*e + 16*a*b^4*c^2*d*e - 2*b^3*c*d*e*(-(4*a*c - b^2)^3)^(1/2) - 3*a*b^2*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - 36*a^2*b^2*c^3*d*e + 4*a*b*c^2*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^5*b^4*e^4 + 16*a^5*c^4*d^4 + 16*a^7*c^2*e^4 - 8*a^6*b^2*c*e^4 - 2*a^4*b^5*d*e^3 + a^3*b^4*c^2*d^4 - 8*a^4*b^2*c^3*d^4 + a^3*b^6*d^2*e^2 + 32*a^6*c^3*d^2*e^2 - 2*a^3*b^5*c*d^3*e - 32*a^5*b*c^3*d^3*e + 16*a^5*b^3*c*d*e^3 - 32*a^6*b*c^2*d*e^3 + 16*a^4*b^3*c^2*d^3*e - 6*a^4*b^4*c*d^2*e^2)))^(1/2)*1i)/(((-(b^7*e^2 + b^5*c^2*d^2 + b^4*e^2*(-(4*a*c - b^2)^3)^(1/2) - 7*a*b^3*c^3*d^2 + 12*a^2*b*c^4*d^2 - a*c^3*d^2*(-(4*a*c - b^2)^3)^(1/2) - 20*a^3*b*c^3*e^2 - 2*b^6*c*d*e + 25*a^2*b^3*c^2*e^2 + a^2*c^2*e^2*(-(4*a*c - b^2)^3)^(1/2) + b^2*c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c*e^2 + 16*a^3*c^4*d*e + 16*a*b^4*c^2*d*e - 2*b^3*c*d*e*(-(4*a*c - b^2)^3)^(1/2) - 3*a*b^2*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - 36*a^2*b^2*c^3*d*e + 4*a*b*c^2*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^5*b^4*e^4 + 16*a^5*c^4*d^4 + 16*a^7*c^2*e^4 - 8*a^6*b^2*c*e^4 - 2*a^4*b^5*d*e^3 + a^3*b^4*c^2*d^4 - 8*a^4*b^2*c^3*d^4 + a^3*b^6*d^2*e^2 + 32*a^6*c^3*d^2*e^2 - 2*a^3*b^5*c*d^3*e - 32*a^5*b*c^3*d^3*e + 16*a^5*b^3*c*d*e^3 - 32*a^6*b*c^2*d*e^3 + 16*a^4*b^3*c^2*d^3*e - 6*a^4*b^4*c*d^2*e^2)))^(1/2)*(((-(b^7*e^2 + b^5*c^2*d^2 + b^4*e^2*(-(4*a*c - b^2)^3)^(1/2) - 7*a*b^3*c^3*d^2 + 12*a^2*b*c^4*d^2 - a*c^3*d^2*(-(4*a*c - b^2)^3)^(1/2) - 20*a^3*b*c^3*e^2 - 2*b^6*c*d*e + 25*a^2*b^3*c^2*e^2 + a^2*c^2*e^2*(-(4*a*c - b^2)^3)^(1/2) + b^2*c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c*e^2 + 16*a^3*c^4*d*e + 16*a*b^4*c^2*d*e - 2*b^3*c*d*e*(-(4*a*c - b^2)^3)^(1/2) - 3*a*b^2*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - 36*a^2*b^2*c^3*d*e + 4*a*b*c^2*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^5*b^4*e^4 + 16*a^5*c^4*d^4 + 16*a^7*c^2*e^4 - 8*a^6*b^2*c*e^4 - 2*a^4*b^5*d*e^3 + a^3*b^4*c^2*d^4 - 8*a^4*b^2*c^3*d^4 + a^3*b^6*d^2*e^2 + 32*a^6*c^3*d^2*e^2 - 2*a^3*b^5*c*d^3*e - 32*a^5*b*c^3*d^3*e + 16*a^5*b^3*c*d*e^3 - 32*a^6*b*c^2*d*e^3 + 16*a^4*b^3*c^2*d^3*e - 6*a^4*b^4*c*d^2*e^2)))^(1/2)*(192*a^10*c^7*d^14*e^3 - x*(-(b^7*e^2 + b^5*c^2*d^2 + b^4*e^2*(-(4*a*c - b^2)^3)^(1/2) - 7*a*b^3*c^3*d^2 + 12*a^2*b*c^4*d^2 - a*c^3*d^2*(-(4*a*c - b^2)^3)^(1/2) - 20*a^3*b*c^3*e^2 - 2*b^6*c*d*e + 25*a^2*b^3*c^2*e^2 + a^2*c^2*e^2*(-(4*a*c - b^2)^3)^(1/2) + b^2*c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c*e^2 + 16*a^3*c^4*d*e + 16*a*b^4*c^2*d*e - 2*b^3*c*d*e*(-(4*a*c - b^2)^3)^(1/2) - 3*a*b^2*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - 36*a^2*b^2*c^3*d*e + 4*a*b*c^2*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^5*b^4*e^4 + 16*a^5*c

^4*d^4 + 16*a^7*c^2*e^4 - 8*a^6*b^2*c*e^4 - 2*a^4*b^5*d*e^3 + a^3*b^4*c^2*d
^4 - 8*a^4*b^2*c^3*d^4 + a^3*b^6*d^2*e^2 + 32*a^6*c^3*d^2*e^2 - 2*a^3*b^5*c
*d^3*e - 32*a^5*b*c^3*d^3*e + 16*a^5*b^3*c*d*e^3 - 32*a^6*b*c^2*d*e^3 + 16*
a^4*b^3*c^2*d^3*e - 6*a^4*b^4*c*d^2*e^2)))^(1/2)*(512*a^11*c^7*d^15*e^3 + 5
12*a^12*c^6*d^13*e^5 - 512*a^13*c^5*d^11*e^7 - 512*a^14*c^4*d^9*e^9 - 32*a^
9*b^3*c^6*d^16*e^2 + 128*a^9*b^4*c^5*d^15*e^3 - 192*a^9*b^5*c^4*d^14*e^4 +
128*a^9*b^6*c^3*d^13*e^5 - 32*a^9*b^7*c^2*d^12*e^6 - 640*a^10*b^2*c^6*d^15*
e^3 + 1056*a^10*b^3*c^5*d^14*e^4 - 672*a^10*b^4*c^4*d^13*e^5 + 96*a^10*b^5*
c^3*d^12*e^6 + 32*a^10*b^6*c^2*d^11*e^7 + 512*a^11*b^2*c^5*d^13*e^5 + 288*a
^11*b^3*c^4*d^12*e^6 - 192*a^11*b^4*c^3*d^11*e^7 + 32*a^11*b^5*c^2*d^10*e^8
 + 384*a^12*b^2*c^4*d^11*e^7 - 288*a^12*b^3*c^3*d^10*e^8 - 32*a^12*b^4*c^2*
d^9*e^9 + 256*a^13*b^2*c^3*d^9*e^9 + 128*a^10*b*c^7*d^16*e^2 - 1152*a^11*b*
c^6*d^14*e^4 - 640*a^12*b*c^5*d^12*e^6 + 640*a^13*b*c^4*d^10*e^8) + 128*a^1
1*c^6*d^12*e^5 - 320*a^12*c^5*d^10*e^7 - 256*a^13*c^4*d^8*e^9 - 16*a^8*b^3*
c^6*d^15*e^2 + 64*a^8*b^4*c^5*d^14*e^3 - 96*a^8*b^5*c^4*d^13*e^4 + 64*a^8*b
^6*c^3*d^12*e^5 - 16*a^8*b^7*c^2*d^11*e^6 - 304*a^9*b^2*c^6*d^14*e^3 + 512*
a^9*b^3*c^5*d^13*e^4 - 352*a^9*b^4*c^4*d^12*e^5 + 64*a^9*b^5*c^3*d^11*e^6 +
 16*a^9*b^6*c^2*d^10*e^7 + 352*a^10*b^2*c^5*d^12*e^5 + 80*a^10*b^3*c^4*d^11
*e^6 - 128*a^10*b^4*c^3*d^10*e^7 + 16*a^10*b^5*c^2*d^9*e^8 + 336*a^11*b^2*c
^4*d^10*e^7 - 128*a^11*b^3*c^3*d^9*e^8 - 16*a^11*b^4*c^2*d^8*e^9 + 128*a^12
*b^2*c^3*d^8*e^9 + 64*a^9*b*c^7*d^15*e^2 - 512*a^10*b*c^6*d^13*e^4 - 320*a^
11*b*c^5*d^11*e^6 + 256*a^12*b*c^4*d^9*e^8) + x*(112*a^10*c^6*d^10*e^6 - 32
*a^9*c^7*d^12*e^4 - 16*a^8*c^8*d^14*e^2 - 128*a^11*c^5*d^8*e^8 + 8*a^7*b^2*
c^7*d^14*e^2 - 16*a^7*b^3*c^6*d^13*e^3 + 8*a^7*b^4*c^5*d^12*e^4 + 8*a^7*b^5
*c^4*d^11*e^5 - 16*a^7*b^6*c^3*d^10*e^6 + 8*a^7*b^7*c^2*d^9*e^7 - 72*a^8*b^
3*c^5*d^11*e^5 + 128*a^8*b^4*c^4*d^10*e^6 - 72*a^8*b^5*c^3*d^9*e^7 - 280*a^
9*b^2*c^5*d^10*e^6 + 208*a^9*b^3*c^4*d^9*e^7 - 16*a^9*b^4*c^3*d^8*e^8 + 8*a
^9*b^5*c^2*d^7*e^9 + 96*a^10*b^2*c^4*d^8*e^8 - 56*a^10*b^3*c^3*d^7*e^9 + 32
*a^8*b*c^7*d^13*e^3 + 128*a^9*b*c^6*d^11*e^5 - 192*a^10*b*c^5*d^9*e^7 + 96*
a^11*b*c^4*d^7*e^9))*(-(b^7*e^2 + b^5*c^2*d^2 + b^4*e^2*(-(4*a*c - b^2)^3)^
(1/2) - 7*a*b^3*c^3*d^2 + 12*a^2*b*c^4*d^2 - a*c^3*d^2*(-(4*a*c - b^2)^3)^(
1/2) - 20*a^3*b*c^3*e^2 - 2*b^6*c*d*e + 25*a^2*b^3*c^2*e^2 + a^2*c^2*e^2*(-
(4*a*c - b^2)^3)^(1/2) + b^2*c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c*e
^2 + 16*a^3*c^4*d*e + 16*a*b^4*c^2*d*e - 2*b^3*c*d*e*(-(4*a*c - b^2)^3)^(1/
2) - 3*a*b^2*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - 36*a^2*b^2*c^3*d*e + 4*a*b*c^
2*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^5*b^4*e^4 + 16*a^5*c^4*d^4 + 16*a^7*c
^2*e^4 - 8*a^6*b^2*c*e^4 - 2*a^4*b^5*d*e^3 + a^3*b^4*c^2*d^4 - 8*a^4*b^2*c^
3*d^4 + a^3*b^6*d^2*e^2 + 32*a^6*c^3*d^2*e^2 - 2*a^3*b^5*c*d^3*e - 32*a^5*b
*c^3*d^3*e + 16*a^5*b^3*c*d*e^3 - 32*a^6*b*c^2*d*e^3 + 16*a^4*b^3*c^2*d^3*e
 - 6*a^4*b^4*c*d^2*e^2)))^(1/2) + 4*a^7*c^8*d^13*e^2 + 4*a^8*c^7*d^11*e^4 -
 16*a^10*c^5*d^7*e^8 - 4*a^7*b^5*c^3*d^8*e^7 + 4*a^7*b^6*c^2*d^7*e^8 + 24*a
^8*b^3*c^4*d^8*e^7 - 28*a^8*b^4*c^3*d^7*e^8 + 52*a^9*b^2*c^4*d^7*e^8 - 4*a^
7*b*c^7*d^12*e^3 - 32*a^9*b*c^5*d^8*e^7) + x*(2*a^7*c^7*d^9*e^5 - 4*a^8*c^6
*d^7*e^7 + 2*a^7*b^2*c^5*d^7*e^7))*(-(b^7*e^2 + b^5*c^2*d^2 + b^4*e^2*(-(4*
a*c - b^2)^3)^(1/2) - 7*a*b^3*c^3*d^2 + 12*a^2*b*c^4*d^2 - a*c^3*d^2*(-(4*a

*c - b^2)^3)^(1/2) - 20*a^3*b*c^3*e^2 - 2*b^6*c*d*e + 25*a^2*b^3*c^2*e^2 + a^2*c^2*e^2*(-(4*a*c - b^2)^3)^(1/2) + b^2*c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c*e^2 + 16*a^3*c^4*d*e + 16*a*b^4*c^2*d*e - 2*b^3*c*d*e*(-(4*a*c - b^2)^3)^(1/2) - 3*a*b^2*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - 36*a^2*b^2*c^3*d*e + 4*a*b*c^2*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^5*b^4*e^4 + 16*a^5*c^4*d^4 + 16*a^7*c^2*e^4 - 8*a^6*b^2*c*e^4 - 2*a^4*b^5*d*e^3 + a^3*b^4*c^2*d^4 - 8*a^4*b^2*c^3*d^4 + a^3*b^6*d^2*e^2 + 32*a^6*c^3*d^2*e^2 - 2*a^3*b^5*c*d^3*e - 32*a^5*b*c^3*d^3*e + 16*a^5*b^3*c*d*e^3 - 32*a^6*b*c^2*d*e^3 + 16*a^4*b^3*c^2*d^3*e - 6*a^4*b^4*c*d^2*e^2)))^(1/2) + ((-(b^7*e^2 + b^5*c^2*d^2 + b^4*e^2*(-(4*a*c - b^2)^3)^(1/2) - 7*a*b^3*c^3*d^2 + 12*a^2*b*c^4*d^2 - a*c^3*d^2*(-(4*a*c - b^2)^3)^(1/2) - 20*a^3*b*c^3*e^2 - 2*b^6*c*d*e + 25*a^2*b^3*c^2*e^2 + a^2*c^2*e^2*(-(4*a*c - b^2)^3)^(1/2) + b^2*c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c*e^2 + 16*a^3*c^4*d*e + 16*a*b^4*c^2*d*e - 2*b^3*c*d*e*(-(4*a*c - b^2)^3)^(1/2) - 3*a*b^2*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - 36*a^2*b^2*c^3*d*e + 4*a*b*c^2*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^5*b^4*e^4 + 16*a^5*c^4*d^4 + 16*a^7*c^2*e^4 - 8*a^6*b^2*c*e^4 - 2*a^4*b^5*d*e^3 + a^3*b^4*c^2*d^4 - 8*a^4*b^2*c^3*d^4 + a^3*b^6*d^2*e^2 + 32*a^6*c^3*d^2*e^2 - 2*a^3*b^5*c*d^3*e - 32*a^5*b*c^3*d^3*e + 16*a^5*b^3*c*d*e^3 - 32*a^6*b*c^2*d*e^3 + 16*a^4*b^3*c^2*d^3*e - 6*a^4*b^4*c*d^2*e^2)))^(1/2)*(((-(b^7*e^2 + b^5*c^2*d^2 + b^4*e^2*(-(4*a*c - b^2)^3)^(1/2) - 7*a*b^3*c^3*d^2 + 12*a^2*b*c^4*d^2 - a*c^3*d^2*(-(4*a*c - b^2)^3)^(1/2) - 20*a^3*b*c^3*e^2 - 2*b^6*c*d*e + 25*a^2*b^3*c^2*e^2 + a^2*c^2*e^2*(-(4*a*c - b^2)^3)^(1/2) + b^2*c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c*e^2 + 16*a^3*c^4*d*e + 16*a*b^4*c^2*d*e - 2*b^3*c*d*e*(-(4*a*c - b^2)^3)^(1/2) - 3*a*b^2*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - 36*a^2*b^2*c^3*d*e + 4*a*b*c^2*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^5*b^4*e^4 + 16*a^5*c^4*d^4 + 16*a^7*c^2*e^4 - 8*a^6*b^2*c*e^4 - 2*a^4*b^5*d*e^3 + a^3*b^4*c^2*d^4 - 8*a^4*b^2*c^3*d^4 + a^3*b^6*d^2*e^2 + 32*a^6*c^3*d^2*e^2 - 2*a^3*b^5*c*d^3*e - 32*a^5*b*c^3*d^3*e + 16*a^5*b^3*c*d*e^3 - 32*a^6*b*c^2*d*e^3 + 16*a^4*b^3*c^2*d^3*e - 6*a^4*b^4*c*d^2*e^2)))^(1/2)*(x*(-(b^7*e^2 + b^5*c^2*d^2 + b^4*e^2*(-(4*a*c - b^2)^3)^(1/2) - 7*a*b^3*c^3*d^2 + 12*a^2*b*c^4*d^2 - a*c^3*d^2*(-(4*a*c - b^2)^3)^(1/2) - 20*a^3*b*c^3*e^2 - 2*b^6*c*d*e + 25*a^2*b^3*c^2*e^2 + a^2*c^2*e^2*(-(4*a*c - b^2)^3)^(1/2) + b^2*c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c*e^2 + 16*a^3*c^4*d*e + 16*a*b^4*c^2*d*e - 2*b^3*c*d*e*(-(4*a*c - b^2)^3)^(1/2) - 3*a*b^2*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - 36*a^2*b^2*c^3*d*e + 4*a*b*c^2*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^5*b^4*e^4 + 16*a^5*c^4*d^4 + 16*a^7*c^2*e^4 - 8*a^6*b^2*c*e^4 - 2*a^4*b^5*d*e^3 + a^3*b^4*c^2*d^4 - 8*a^4*b^2*c^3*d^4 + a^3*b^6*d^2*e^2 + 32*a^6*c^3*d^2*e^2 - 2*a^3*b^5*c*d^3*e - 32*a^5*b*c^3*d^3*e + 16*a^5*b^3*c*d*e^3 - 32*a^6*b*c^2*d*e^3 + 16*a^4*b^3*c^2*d^3*e - 6*a^4*b^4*c*d^2*e^2)))^(1/2)*(512*a^11*c^7*d^15*e^3 + 512*a^12*c^6*d^13*e^5 - 512*a^13*c^5*d^11*e^7 - 512*a^14*c^4*d^9*e^9 - 32*a^9*b^3*c^6*d^16*e^2 + 128*a^9*b^4*c^5*d^15*e^3 - 192*a^9*b^5*c^4*d^14*e^4 + 128*a^9*b^6*c^3*d^13*e^5 - 32*a^9*b^7*c^2*d^12*e^6 - 640*a^10*b^2*c^6*d^15*e^3 + 1056*a^10*b^3*c^5*d^14*e^4 - 672*a^10*b^4*c^4*d^13*e^5 + 96*a^10*b^5*c^3*d^12*e^6 + 32*a^10*b^6*c^2*d^11*e^7 + 512*a^11*b^2*c^5*d^13*e^5 + 288*a^11*b^3*c^4*d^12*e^6 - 192*a^11*b^4*c

^3*d^11*e^7 + 32*a^11*b^5*c^2*d^10*e^8 + 384*a^12*b^2*c^4*d^11*e^7 - 288*a^12*b^3*c^3*d^10*e^8 - 32*a^12*b^4*c^2*d^9*e^9 + 256*a^13*b^2*c^3*d^9*e^9 + 128*a^10*b*c^7*d^16*e^2 - 1152*a^11*b*c^6*d^14*e^4 - 640*a^12*b*c^5*d^12*e^6 + 640*a^13*b*c^4*d^10*e^8) + 192*a^10*c^7*d^14*e^3 + 128*a^11*c^6*d^12*e^5 - 320*a^12*c^5*d^10*e^7 - 256*a^13*c^4*d^8*e^9 - 16*a^8*b^3*c^6*d^15*e^2 + 64*a^8*b^4*c^5*d^14*e^3 - 96*a^8*b^5*c^4*d^13*e^4 + 64*a^8*b^6*c^3*d^12*e^5 - 16*a^8*b^7*c^2*d^11*e^6 - 304*a^9*b^2*c^6*d^14*e^3 + 512*a^9*b^3*c^5*d^13*e^4 - 352*a^9*b^4*c^4*d^12*e^5 + 64*a^9*b^5*c^3*d^11*e^6 + 16*a^9*b^6*c^2*d^10*e^7 + 352*a^10*b^2*c^5*d^12*e^5 + 80*a^10*b^3*c^4*d^11*e^6 - 128*a^10*b^4*c^3*d^10*e^7 + 16*a^10*b^5*c^2*d^9*e^8 + 336*a^11*b^2*c^4*d^10*e^7 - 128*a^11*b^3*c^3*d^9*e^8 - 16*a^11*b^4*c^2*d^8*e^9 + 128*a^12*b^2*c^3*d^8*e^9 + 64*a^9*b*c^7*d^15*e^2 - 512*a^10*b*c^6*d^13*e^4 - 320*a^11*b*c^5*d^11*e^6 + 256*a^12*b*c^4*d^9*e^8) - x*(112*a^10*c^6*d^10*e^6 - 32*a^9*c^7*d^12*e^4 - 16*a^8*c^8*d^14*e^2 - 128*a^11*c^5*d^8*e^8 + 8*a^7*b^2*c^7*d^14*e^2 - 16*a^7*b^3*c^6*d^13*e^3 + 8*a^7*b^4*c^5*d^12*e^4 + 8*a^7*b^5*c^4*d^11*e^5 - 16*a^7*b^6*c^3*d^10*e^6 + 8*a^7*b^7*c^2*d^9*e^7 - 72*a^8*b^3*c^5*d^11*e^5 + 128*a^8*b^4*c^4*d^10*e^6 - 72*a^8*b^5*c^3*d^9*e^7 - 280*a^9*b^2*c^5*d^10*e^6 + 208*a^9*b^3*c^4*d^9*e^7 - 16*a^9*b^4*c^3*d^8*e^8 + 8*a^9*b^5*c^2*d^7*e^9 + 96*a^10*b^2*c^4*d^8*e^8 - 56*a^10*b^3*c^3*d^7*e^9 + 32*a^8*b*c^7*d^13*e^3 + 128*a^9*b*c^6*d^11*e^5 - 192*a^10*b*c^5*d^9*e^7 + 96*a^11*b*c^4*d^7*e^9))*(-(b^7*e^2 + b^5*c^2*d^2 + b^4*e^2*(-(4*a*c - b^2)^3)^(1/2) - 7*a*b^3*c^3*d^2 + 12*a^2*b*c^4*d^2 - a*c^3*d^2*(-(4*a*c - b^2)^3)^(1/2) - 20*a^3*b*c^3*e^2 - 2*b^6*c*d*e + 25*a^2*b^3*c^2*e^2 + a^2*c^2*e^2*(-(4*a*c - b^2)^3)^(1/2) + b^2*c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c*e^2 + 16*a^3*c^4*d*e + 16*a*b^4*c^2*d*e - 2*b^3*c*d*e*(-(4*a*c - b^2)^3)^(1/2) - 3*a*b^2*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - 36*a^2*b^2*c^3*d*e + 4*a*b*c^2*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^5*b^4*e^4 + 16*a^5*c^4*d^4 + 16*a^7*c^2*e^4 - 8*a^6*b^2*c*e^4 - 2*a^4*b^5*d*e^3 + a^3*b^4*c^2*d^4 - 8*a^4*b^2*c^3*d^4 + a^3*b^6*d^2*e^2 + 32*a^6*c^3*d^2*e^2 - 2*a^3*b^5*c*d^3*e - 32*a^5*b*c^3*d^3*e + 16*a^5*b^3*c*d*e^3 - 32*a^6*b*c^2*d*e^3 + 16*a^4*b^3*c^2*d^3*e - 6*a^4*b^4*c*d^2*e^2)))^(1/2) + 4*a^7*c^8*d^13*e^2 + 4*a^8*c^7*d^11*e^4 - 16*a^10*c^5*d^7*e^8 - 4*a^7*b^5*c^3*d^8*e^7 + 4*a^7*b^6*c^2*d^7*e^8 + 24*a^8*b^3*c^4*d^8*e^7 - 28*a^8*b^4*c^3*d^7*e^8 + 52*a^9*b^2*c^4*d^7*e^8 - 4*a^7*b*c^7*d^12*e^3 - 32*a^9*b*c^5*d^8*e^7) - x*(2*a^7*c^7*d^9*e^5 - 4*a^8*c^6*d^7*e^7 + 2*a^7*b^2*c^5*d^7*e^7))*(-(b^7*e^2 + b^5*c^2*d^2 + b^4*e^2*(-(4*a*c - b^2)^3)^(1/2) - 7*a*b^3*c^3*d^2 + 12*a^2*b*c^4*d^2 - a*c^3*d^2*(-(4*a*c - b^2)^3)^(1/2) - 20*a^3*b*c^3*e^2 - 2*b^6*c*d*e + 25*a^2*b^3*c^2*e^2 + a^2*c^2*e^2*(-(4*a*c - b^2)^3)^(1/2) + b^2*c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c*e^2 + 16*a^3*c^4*d*e + 16*a*b^4*c^2*d*e - 2*b^3*c*d*e*(-(4*a*c - b^2)^3)^(1/2) - 3*a*b^2*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - 36*a^2*b^2*c^3*d*e + 4*a*b*c^2*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^5*b^4*e^4 + 16*a^5*c^4*d^4 + 16*a^7*c^2*e^4 - 8*a^6*b^2*c*e^4 - 2*a^4*b^5*d*e^3 + a^3*b^4*c^2*d^4 - 8*a^4*b^2*c^3*d^4 + a^3*b^6*d^2*e^2 + 32*a^6*c^3*d^2*e^2 - 2*a^3*b^5*c*d^3*e - 32*a^5*b*c^3*d^3*e + 16*a^5*b^3*c*d*e^3 - 32*a^6*b*c^2*d*e^3 + 16*a^4*b^3*c^2*d^3*e - 6*a^4*b^4*c*d^2*e^2)))^(1/2)))*(-(b^7*e^2 + b^5*c^2*d^2 + b^4*e^2*(-(4*

a*c - b^2)^3)^(1/2) - 7*a*b^3*c^3*d^2 + 12*a^2*b*c^4*d^2 - a*c^3*d^2*(-(4*a
*c - b^2)^3)^(1/2) - 20*a^3*b*c^3*e^2 - 2*b^6*c*d*e + 25*a^2*b^3*c^2*e^2 +
a^2*c^2*e^2*(-(4*a*c - b^2)^3)^(1/2) + b^2*c^2*d^2*(-(4*a*c - b^2)^3)^(1/2)
 - 9*a*b^5*c*e^2 + 16*a^3*c^4*d*e + 16*a*b^4*c^2*d*e - 2*b^3*c*d*e*(-(4*a*c
 - b^2)^3)^(1/2) - 3*a*b^2*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - 36*a^2*b^2*c^3*
d*e + 4*a*b*c^2*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^5*b^4*e^4 + 16*a^5*c^4*
d^4 + 16*a^7*c^2*e^4 - 8*a^6*b^2*c*e^4 - 2*a^4*b^5*d*e^3 + a^3*b^4*c^2*d^4
- 8*a^4*b^2*c^3*d^4 + a^3*b^6*d^2*e^2 + 32*a^6*c^3*d^2*e^2 - 2*a^3*b^5*c*d^
3*e - 32*a^5*b*c^3*d^3*e + 16*a^5*b^3*c*d*e^3 - 32*a^6*b*c^2*d*e^3 + 16*a^4
*b^3*c^2*d^3*e - 6*a^4*b^4*c*d^2*e^2)))^(1/2)*2i + atan((((-(b^7*e^2 + b^5*
c^2*d^2 - b^4*e^2*(-(4*a*c - b^2)^3)^(1/2) - 7*a*b^3*c^3*d^2 + 12*a^2*b*c^4
*d^2 + a*c^3*d^2*(-(4*a*c - b^2)^3)^(1/2) - 20*a^3*b*c^3*e^2 - 2*b^6*c*d*e
+ 25*a^2*b^3*c^2*e^2 - a^2*c^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - b^2*c^2*d^2*(
-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c*e^2 + 16*a^3*c^4*d*e + 16*a*b^4*c^2*d*e
 + 2*b^3*c*d*e*(-(4*a*c - b^2)^3)^(1/2) + 3*a*b^2*c*e^2*(-(4*a*c - b^2)^3)^
(1/2) - 36*a^2*b^2*c^3*d*e - 4*a*b*c^2*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^
5*b^4*e^4 + 16*a^5*c^4*d^4 + 16*a^7*c^2*e^4 - 8*a^6*b^2*c*e^4 - 2*a^4*b^5*d
*e^3 + a^3*b^4*c^2*d^4 - 8*a^4*b^2*c^3*d^4 + a^3*b^6*d^2*e^2 + 32*a^6*c^3*d
^2*e^2 - 2*a^3*b^5*c*d^3*e - 32*a^5*b*c^3*d^3*e + 16*a^5*b^3*c*d*e^3 - 32*a
^6*b*c^2*d*e^3 + 16*a^4*b^3*c^2*d^3*e - 6*a^4*b^4*c*d^2*e^2)))^(1/2)*(((-(b
^7*e^2 + b^5*c^2*d^2 - b^4*e^2*(-(4*a*c - b^2)^3)^(1/2) - 7*a*b^3*c^3*d^2 +
 12*a^2*b*c^4*d^2 + a*c^3*d^2*(-(4*a*c - b^2)^3)^(1/2) - 20*a^3*b*c^3*e^2 -
 2*b^6*c*d*e + 25*a^2*b^3*c^2*e^2 - a^2*c^2*e^2*(-(4*a*c - b^2)^3)^(1/2) -
b^2*c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c*e^2 + 16*a^3*c^4*d*e + 16*
a*b^4*c^2*d*e + 2*b^3*c*d*e*(-(4*a*c - b^2)^3)^(1/2) + 3*a*b^2*c*e^2*(-(4*a
*c - b^2)^3)^(1/2) - 36*a^2*b^2*c^3*d*e - 4*a*b*c^2*d*e*(-(4*a*c - b^2)^3)^
(1/2))/(8*(a^5*b^4*e^4 + 16*a^5*c^4*d^4 + 16*a^7*c^2*e^4 - 8*a^6*b^2*c*e^4
- 2*a^4*b^5*d*e^3 + a^3*b^4*c^2*d^4 - 8*a^4*b^2*c^3*d^4 + a^3*b^6*d^2*e^2 +
 32*a^6*c^3*d^2*e^2 - 2*a^3*b^5*c*d^3*e - 32*a^5*b*c^3*d^3*e + 16*a^5*b^3*c
*d*e^3 - 32*a^6*b*c^2*d*e^3 + 16*a^4*b^3*c^2*d^3*e - 6*a^4*b^4*c*d^2*e^2)))
^(1/2)*(192*a^10*c^7*d^14*e^3 - x*(-(b^7*e^2 + b^5*c^2*d^2 - b^4*e^2*(-(4*a
*c - b^2)^3)^(1/2) - 7*a*b^3*c^3*d^2 + 12*a^2*b*c^4*d^2 + a*c^3*d^2*(-(4*a*
c - b^2)^3)^(1/2) - 20*a^3*b*c^3*e^2 - 2*b^6*c*d*e + 25*a^2*b^3*c^2*e^2 - a
^2*c^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - b^2*c^2*d^2*(-(4*a*c - b^2)^3)^(1/2)
- 9*a*b^5*c*e^2 + 16*a^3*c^4*d*e + 16*a*b^4*c^2*d*e + 2*b^3*c*d*e*(-(4*a*c
- b^2)^3)^(1/2) + 3*a*b^2*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - 36*a^2*b^2*c^3*d
*e - 4*a*b*c^2*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^5*b^4*e^4 + 16*a^5*c^4*d
^4 + 16*a^7*c^2*e^4 - 8*a^6*b^2*c*e^4 - 2*a^4*b^5*d*e^3 + a^3*b^4*c^2*d^4 -
 8*a^4*b^2*c^3*d^4 + a^3*b^6*d^2*e^2 + 32*a^6*c^3*d^2*e^2 - 2*a^3*b^5*c*d^3
*e - 32*a^5*b*c^3*d^3*e + 16*a^5*b^3*c*d*e^3 - 32*a^6*b*c^2*d*e^3 + 16*a^4*
b^3*c^2*d^3*e - 6*a^4*b^4*c*d^2*e^2)))^(1/2)*(512*a^11*c^7*d^15*e^3 + 512*a
^12*c^6*d^13*e^5 - 512*a^13*c^5*d^11*e^7 - 512*a^14*c^4*d^9*e^9 - 32*a^9*b^
3*c^6*d^16*e^2 + 128*a^9*b^4*c^5*d^15*e^3 - 192*a^9*b^5*c^4*d^14*e^4 + 128*
a^9*b^6*c^3*d^13*e^5 - 32*a^9*b^7*c^2*d^12*e^6 - 640*a^10*b^2*c^6*d^15*e^3
+ 1056*a^10*b^3*c^5*d^14*e^4 - 672*a^10*b^4*c^4*d^13*e^5 + 96*a^10*b^5*c^3*

d^12*e^6 + 32*a^10*b^6*c^2*d^11*e^7 + 512*a^11*b^2*c^5*d^13*e^5 + 288*a^11*
b^3*c^4*d^12*e^6 - 192*a^11*b^4*c^3*d^11*e^7 + 32*a^11*b^5*c^2*d^10*e^8 + 3
84*a^12*b^2*c^4*d^11*e^7 - 288*a^12*b^3*c^3*d^10*e^8 - 32*a^12*b^4*c^2*d^9*
e^9 + 256*a^13*b^2*c^3*d^9*e^9 + 128*a^10*b*c^7*d^16*e^2 - 1152*a^11*b*c^6*
d^14*e^4 - 640*a^12*b*c^5*d^12*e^6 + 640*a^13*b*c^4*d^10*e^8) + 128*a^11*c^
6*d^12*e^5 - 320*a^12*c^5*d^10*e^7 - 256*a^13*c^4*d^8*e^9 - 16*a^8*b^3*c^6*
d^15*e^2 + 64*a^8*b^4*c^5*d^14*e^3 - 96*a^8*b^5*c^4*d^13*e^4 + 64*a^8*b^6*c
^3*d^12*e^5 - 16*a^8*b^7*c^2*d^11*e^6 - 304*a^9*b^2*c^6*d^14*e^3 + 512*a^9*
b^3*c^5*d^13*e^4 - 352*a^9*b^4*c^4*d^12*e^5 + 64*a^9*b^5*c^3*d^11*e^6 + 16*
a^9*b^6*c^2*d^10*e^7 + 352*a^10*b^2*c^5*d^12*e^5 + 80*a^10*b^3*c^4*d^11*e^6
 - 128*a^10*b^4*c^3*d^10*e^7 + 16*a^10*b^5*c^2*d^9*e^8 + 336*a^11*b^2*c^4*d
^10*e^7 - 128*a^11*b^3*c^3*d^9*e^8 - 16*a^11*b^4*c^2*d^8*e^9 + 128*a^12*b^2
*c^3*d^8*e^9 + 64*a^9*b*c^7*d^15*e^2 - 512*a^10*b*c^6*d^13*e^4 - 320*a^11*b
*c^5*d^11*e^6 + 256*a^12*b*c^4*d^9*e^8) + x*(112*a^10*c^6*d^10*e^6 - 32*a^9
*c^7*d^12*e^4 - 16*a^8*c^8*d^14*e^2 - 128*a^11*c^5*d^8*e^8 + 8*a^7*b^2*c^7*
d^14*e^2 - 16*a^7*b^3*c^6*d^13*e^3 + 8*a^7*b^4*c^5*d^12*e^4 + 8*a^7*b^5*c^4
*d^11*e^5 - 16*a^7*b^6*c^3*d^10*e^6 + 8*a^7*b^7*c^2*d^9*e^7 - 72*a^8*b^3*c^
5*d^11*e^5 + 128*a^8*b^4*c^4*d^10*e^6 - 72*a^8*b^5*c^3*d^9*e^7 - 280*a^9*b^
2*c^5*d^10*e^6 + 208*a^9*b^3*c^4*d^9*e^7 - 16*a^9*b^4*c^3*d^8*e^8 + 8*a^9*b
^5*c^2*d^7*e^9 + 96*a^10*b^2*c^4*d^8*e^8 - 56*a^10*b^3*c^3*d^7*e^9 + 32*a^8
*b*c^7*d^13*e^3 + 128*a^9*b*c^6*d^11*e^5 - 192*a^10*b*c^5*d^9*e^7 + 96*a^11
*b*c^4*d^7*e^9))*(-(b^7*e^2 + b^5*c^2*d^2 - b^4*e^2*(-(4*a*c - b^2)^3)^(1/2
) - 7*a*b^3*c^3*d^2 + 12*a^2*b*c^4*d^2 + a*c^3*d^2*(-(4*a*c - b^2)^3)^(1/2)
 - 20*a^3*b*c^3*e^2 - 2*b^6*c*d*e + 25*a^2*b^3*c^2*e^2 - a^2*c^2*e^2*(-(4*a
*c - b^2)^3)^(1/2) - b^2*c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c*e^2 +
 16*a^3*c^4*d*e + 16*a*b^4*c^2*d*e + 2*b^3*c*d*e*(-(4*a*c - b^2)^3)^(1/2) +
 3*a*b^2*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - 36*a^2*b^2*c^3*d*e - 4*a*b*c^2*d*
e*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^5*b^4*e^4 + 16*a^5*c^4*d^4 + 16*a^7*c^2*e
^4 - 8*a^6*b^2*c*e^4 - 2*a^4*b^5*d*e^3 + a^3*b^4*c^2*d^4 - 8*a^4*b^2*c^3*d^
4 + a^3*b^6*d^2*e^2 + 32*a^6*c^3*d^2*e^2 - 2*a^3*b^5*c*d^3*e - 32*a^5*b*c^3
*d^3*e + 16*a^5*b^3*c*d*e^3 - 32*a^6*b*c^2*d*e^3 + 16*a^4*b^3*c^2*d^3*e - 6
*a^4*b^4*c*d^2*e^2)))^(1/2) + 4*a^7*c^8*d^13*e^2 + 4*a^8*c^7*d^11*e^4 - 16*
a^10*c^5*d^7*e^8 - 4*a^7*b^5*c^3*d^8*e^7 + 4*a^7*b^6*c^2*d^7*e^8 + 24*a^8*b
^3*c^4*d^8*e^7 - 28*a^8*b^4*c^3*d^7*e^8 + 52*a^9*b^2*c^4*d^7*e^8 - 4*a^7*b*
c^7*d^12*e^3 - 32*a^9*b*c^5*d^8*e^7) + x*(2*a^7*c^7*d^9*e^5 - 4*a^8*c^6*d^7
*e^7 + 2*a^7*b^2*c^5*d^7*e^7))*(-(b^7*e^2 + b^5*c^2*d^2 - b^4*e^2*(-(4*a*c
 - b^2)^3)^(1/2) - 7*a*b^3*c^3*d^2 + 12*a^2*b*c^4*d^2 + a*c^3*d^2*(-(4*a*c -
 b^2)^3)^(1/2) - 20*a^3*b*c^3*e^2 - 2*b^6*c*d*e + 25*a^2*b^3*c^2*e^2 - a^2*
c^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - b^2*c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) - 9
*a*b^5*c*e^2 + 16*a^3*c^4*d*e + 16*a*b^4*c^2*d*e + 2*b^3*c*d*e*(-(4*a*c - b
^2)^3)^(1/2) + 3*a*b^2*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - 36*a^2*b^2*c^3*d*e
 - 4*a*b*c^2*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^5*b^4*e^4 + 16*a^5*c^4*d^4
 + 16*a^7*c^2*e^4 - 8*a^6*b^2*c*e^4 - 2*a^4*b^5*d*e^3 + a^3*b^4*c^2*d^4 - 8*
a^4*b^2*c^3*d^4 + a^3*b^6*d^2*e^2 + 32*a^6*c^3*d^2*e^2 - 2*a^3*b^5*c*d^3*e
 - 32*a^5*b*c^3*d^3*e + 16*a^5*b^3*c*d*e^3 - 32*a^6*b*c^2*d*e^3 + 16*a^4*b^3

*c^2*d^3*e - 6*a^4*b^4*c*d^2*e^2)))^(1/2)*1i - ((-(b^7*e^2 + b^5*c^2*d^2 - b^4*e^2*(-(4*a*c - b^2)^3)^(1/2) - 7*a*b^3*c^3*d^2 + 12*a^2*b*c^4*d^2 + a*c^3*d^2*(-(4*a*c - b^2)^3)^(1/2) - 20*a^3*b*c^3*e^2 - 2*b^6*c*d*e + 25*a^2*b^3*c^2*e^2 - a^2*c^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - b^2*c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c*e^2 + 16*a^3*c^4*d*e + 16*a*b^4*c^2*d*e + 2*b^3*c*d*e*(-(4*a*c - b^2)^3)^(1/2) + 3*a*b^2*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - 36*a^2*b^2*c^3*d*e - 4*a*b*c^2*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^5*b^4*e^4 + 16*a^5*c^4*d^4 + 16*a^7*c^2*e^4 - 8*a^6*b^2*c*e^4 - 2*a^4*b^5*d*e^3 + a^3*b^4*c^2*d^4 - 8*a^4*b^2*c^3*d^4 + a^3*b^6*d^2*e^2 + 32*a^6*c^3*d^2*e^2 - 2*a^3*b^5*c*d^3*e - 32*a^5*b*c^3*d^3*e + 16*a^5*b^3*c*d*e^3 - 32*a^6*b*c^2*d*e^3 + 16*a^4*b^3*c^2*d^3*e - 6*a^4*b^4*c*d^2*e^2)))^(1/2)*(((-(b^7*e^2 + b^5*c^2*d^2 - b^4*e^2*(-(4*a*c - b^2)^3)^(1/2) - 7*a*b^3*c^3*d^2 + 12*a^2*b*c^4*d^2 + a*c^3*d^2*(-(4*a*c - b^2)^3)^(1/2) - 20*a^3*b*c^3*e^2 - 2*b^6*c*d*e + 25*a^2*b^3*c^2*e^2 - a^2*c^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - b^2*c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c*e^2 + 16*a^3*c^4*d*e + 16*a*b^4*c^2*d*e + 2*b^3*c*d*e*(-(4*a*c - b^2)^3)^(1/2) + 3*a*b^2*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - 36*a^2*b^2*c^3*d*e - 4*a*b*c^2*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^5*b^4*e^4 + 16*a^5*c^4*d^4 + 16*a^7*c^2*e^4 - 8*a^6*b^2*c*e^4 - 2*a^4*b^5*d*e^3 + a^3*b^4*c^2*d^4 - 8*a^4*b^2*c^3*d^4 + a^3*b^6*d^2*e^2 + 32*a^6*c^3*d^2*e^2 - 2*a^3*b^5*c*d^3*e - 32*a^5*b*c^3*d^3*e + 16*a^5*b^3*c*d*e^3 - 32*a^6*b*c^2*d*e^3 + 16*a^4*b^3*c^2*d^3*e - 6*a^4*b^4*c*d^2*e^2)))^(1/2)*(x*(-(b^7*e^2 + b^5*c^2*d^2 - b^4*e^2*(-(4*a*c - b^2)^3)^(1/2) - 7*a*b^3*c^3*d^2 + 12*a^2*b*c^4*d^2 + a*c^3*d^2*(-(4*a*c - b^2)^3)^(1/2) - 20*a^3*b*c^3*e^2 - 2*b^6*c*d*e + 25*a^2*b^3*c^2*e^2 - a^2*c^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - b^2*c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c*e^2 + 16*a^3*c^4*d*e + 16*a*b^4*c^2*d*e + 2*b^3*c*d*e*(-(4*a*c - b^2)^3)^(1/2) + 3*a*b^2*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - 36*a^2*b^2*c^3*d*e - 4*a*b*c^2*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^5*b^4*e^4 + 16*a^5*c^4*d^4 + 16*a^7*c^2*e^4 - 8*a^6*b^2*c*e^4 - 2*a^4*b^5*d*e^3 + a^3*b^4*c^2*d^4 - 8*a^4*b^2*c^3*d^4 + a^3*b^6*d^2*e^2 + 32*a^6*c^3*d^2*e^2 - 2*a^3*b^5*c*d^3*e - 32*a^5*b*c^3*d^3*e + 16*a^5*b^3*c*d*e^3 - 32*a^6*b*c^2*d*e^3 + 16*a^4*b^3*c^2*d^3*e - 6*a^4*b^4*c*d^2*e^2)))^(1/2)*(512*a^11*c^7*d^15*e^3 + 512*a^12*c^6*d^13*e^5 - 512*a^13*c^5*d^11*e^7 - 512*a^14*c^4*d^9*e^9 - 32*a^9*b^3*c^6*d^16*e^2 + 128*a^9*b^4*c^5*d^15*e^3 - 192*a^9*b^5*c^4*d^14*e^4 + 128*a^9*b^6*c^3*d^13*e^5 - 32*a^9*b^7*c^2*d^12*e^6 - 640*a^10*b^2*c^6*d^15*e^3 + 1056*a^10*b^3*c^5*d^14*e^4 - 672*a^10*b^4*c^4*d^13*e^5 + 96*a^10*b^5*c^3*d^12*e^6 + 32*a^10*b^6*c^2*d^11*e^7 + 512*a^11*b^2*c^5*d^13*e^5 + 288*a^11*b^3*c^4*d^12*e^6 - 192*a^11*b^4*c^3*d^11*e^7 + 32*a^11*b^5*c^2*d^10*e^8 + 384*a^12*b^2*c^4*d^11*e^7 - 288*a^12*b^3*c^3*d^10*e^8 - 32*a^12*b^4*c^2*d^9*e^9 + 256*a^13*b^2*c^3*d^9*e^9 + 128*a^10*b*c^7*d^16*e^2 - 1152*a^11*b*c^6*d^14*e^4 - 640*a^12*b*c^5*d^12*e^6 + 640*a^13*b*c^4*d^10*e^8) + 192*a^10*c^7*d^14*e^3 + 128*a^11*c^6*d^12*e^5 - 320*a^12*c^5*d^10*e^7 - 256*a^13*c^4*d^8*e^9 - 16*a^8*b^3*c^6*d^15*e^2 + 64*a^8*b^4*c^5*d^14*e^3 - 96*a^8*b^5*c^4*d^13*e^4 + 64*a^8*b^6*c^3*d^12*e^5 - 16*a^8*b^7*c^2*d^11*e^6 - 304*a^9*b^2*c^6*d^14*e^3 + 512*a^9*b^3*c^5*d^13*e^4 - 352*a^9*b^4*c^4*d^12*e^5 + 64*a^9*b^5*c^3*d^11*e^6 + 16*a^9*b^6*c^

2*d^10*e^7 + 352*a^10*b^2*c^5*d^12*e^5 + 80*a^10*b^3*c^4*d^11*e^6 - 128*a^1
0*b^4*c^3*d^10*e^7 + 16*a^10*b^5*c^2*d^9*e^8 + 336*a^11*b^2*c^4*d^10*e^7 -
128*a^11*b^3*c^3*d^9*e^8 - 16*a^11*b^4*c^2*d^8*e^9 + 128*a^12*b^2*c^3*d^8*e
^9 + 64*a^9*b*c^7*d^15*e^2 - 512*a^10*b*c^6*d^13*e^4 - 320*a^11*b*c^5*d^11*
e^6 + 256*a^12*b*c^4*d^9*e^8) - x*(112*a^10*c^6*d^10*e^6 - 32*a^9*c^7*d^12*
e^4 - 16*a^8*c^8*d^14*e^2 - 128*a^11*c^5*d^8*e^8 + 8*a^7*b^2*c^7*d^14*e^2 -
 16*a^7*b^3*c^6*d^13*e^3 + 8*a^7*b^4*c^5*d^12*e^4 + 8*a^7*b^5*c^4*d^11*e^5
- 16*a^7*b^6*c^3*d^10*e^6 + 8*a^7*b^7*c^2*d^9*e^7 - 72*a^8*b^3*c^5*d^11*e^5
 + 128*a^8*b^4*c^4*d^10*e^6 - 72*a^8*b^5*c^3*d^9*e^7 - 280*a^9*b^2*c^5*d^10
*e^6 + 208*a^9*b^3*c^4*d^9*e^7 - 16*a^9*b^4*c^3*d^8*e^8 + 8*a^9*b^5*c^2*d^7
*e^9 + 96*a^10*b^2*c^4*d^8*e^8 - 56*a^10*b^3*c^3*d^7*e^9 + 32*a^8*b*c^7*d^1
3*e^3 + 128*a^9*b*c^6*d^11*e^5 - 192*a^10*b*c^5*d^9*e^7 + 96*a^11*b*c^4*d^7
e^9))(-(b^7*e^2 + b^5*c^2*d^2 - b^4*e^2*(-(4*a*c - b^2)^3)^(1/2) - 7*a*b^
3*c^3*d^2 + 12*a^2*b*c^4*d^2 + a*c^3*d^2*(-(4*a*c - b^2)^3)^(1/2) - 20*a^3*
b*c^3*e^2 - 2*b^6*c*d*e + 25*a^2*b^3*c^2*e^2 - a^2*c^2*e^2*(-(4*a*c - b^2)^
3)^(1/2) - b^2*c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c*e^2 + 16*a^3*c^
4*d*e + 16*a*b^4*c^2*d*e + 2*b^3*c*d*e*(-(4*a*c - b^2)^3)^(1/2) + 3*a*b^2*c
e^2(-(4*a*c - b^2)^3)^(1/2) - 36*a^2*b^2*c^3*d*e - 4*a*b*c^2*d*e*(-(4*a*c
 - b^2)^3)^(1/2))/(8*(a^5*b^4*e^4 + 16*a^5*c^4*d^4 + 16*a^7*c^2*e^4 - 8*a^6
*b^2*c*e^4 - 2*a^4*b^5*d*e^3 + a^3*b^4*c^2*d^4 - 8*a^4*b^2*c^3*d^4 + a^3*b^
6*d^2*e^2 + 32*a^6*c^3*d^2*e^2 - 2*a^3*b^5*c*d^3*e - 32*a^5*b*c^3*d^3*e + 1
6*a^5*b^3*c*d*e^3 - 32*a^6*b*c^2*d*e^3 + 16*a^4*b^3*c^2*d^3*e - 6*a^4*b^4*c
*d^2*e^2)))^(1/2) + 4*a^7*c^8*d^13*e^2 + 4*a^8*c^7*d^11*e^4 - 16*a^10*c^5*d
^7*e^8 - 4*a^7*b^5*c^3*d^8*e^7 + 4*a^7*b^6*c^2*d^7*e^8 + 24*a^8*b^3*c^4*d^8
*e^7 - 28*a^8*b^4*c^3*d^7*e^8 + 52*a^9*b^2*c^4*d^7*e^8 - 4*a^7*b*c^7*d^12*e
^3 - 32*a^9*b*c^5*d^8*e^7) - x*(2*a^7*c^7*d^9*e^5 - 4*a^8*c^6*d^7*e^7 + 2*a
^7*b^2*c^5*d^7*e^7))*(-(b^7*e^2 + b^5*c^2*d^2 - b^4*e^2*(-(4*a*c - b^2)^3)^
(1/2) - 7*a*b^3*c^3*d^2 + 12*a^2*b*c^4*d^2 + a*c^3*d^2*(-(4*a*c - b^2)^3)^(
1/2) - 20*a^3*b*c^3*e^2 - 2*b^6*c*d*e + 25*a^2*b^3*c^2*e^2 - a^2*c^2*e^2*(-
(4*a*c - b^2)^3)^(1/2) - b^2*c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c*e
^2 + 16*a^3*c^4*d*e + 16*a*b^4*c^2*d*e + 2*b^3*c*d*e*(-(4*a*c - b^2)^3)^(1/
2) + 3*a*b^2*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - 36*a^2*b^2*c^3*d*e - 4*a*b*c^
2*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^5*b^4*e^4 + 16*a^5*c^4*d^4 + 16*a^7*c
^2*e^4 - 8*a^6*b^2*c*e^4 - 2*a^4*b^5*d*e^3 + a^3*b^4*c^2*d^4 - 8*a^4*b^2*c^
3*d^4 + a^3*b^6*d^2*e^2 + 32*a^6*c^3*d^2*e^2 - 2*a^3*b^5*c*d^3*e - 32*a^5*b
*c^3*d^3*e + 16*a^5*b^3*c*d*e^3 - 32*a^6*b*c^2*d*e^3 + 16*a^4*b^3*c^2*d^3*e
 - 6*a^4*b^4*c*d^2*e^2)))^(1/2)*1i)/(((-(b^7*e^2 + b^5*c^2*d^2 - b^4*e^2*(-
(4*a*c - b^2)^3)^(1/2) - 7*a*b^3*c^3*d^2 + 12*a^2*b*c^4*d^2 + a*c^3*d^2*(-(
4*a*c - b^2)^3)^(1/2) - 20*a^3*b*c^3*e^2 - 2*b^6*c*d*e + 25*a^2*b^3*c^2*e^2
 - a^2*c^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - b^2*c^2*d^2*(-(4*a*c - b^2)^3)^(1
/2) - 9*a*b^5*c*e^2 + 16*a^3*c^4*d*e + 16*a*b^4*c^2*d*e + 2*b^3*c*d*e*(-(4*
a*c - b^2)^3)^(1/2) + 3*a*b^2*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - 36*a^2*b^2*c
^3*d*e - 4*a*b*c^2*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^5*b^4*e^4 + 16*a^5*c
^4*d^4 + 16*a^7*c^2*e^4 - 8*a^6*b^2*c*e^4 - 2*a^4*b^5*d*e^3 + a^3*b^4*c^2*d
^4 - 8*a^4*b^2*c^3*d^4 + a^3*b^6*d^2*e^2 + 32*a^6*c^3*d^2*e^2 - 2*a^3*b^5*c

*d^3*e - 32*a^5*b*c^3*d^3*e + 16*a^5*b^3*c*d*e^3 - 32*a^6*b*c^2*d*e^3 + 16*a^4*b^3*c^2*d^3*e - 6*a^4*b^4*c*d^2*e^2)))^(1/2)*(((-(b^7*e^2 + b^5*c^2*d^2 - b^4*e^2*(-(4*a*c - b^2)^3)^(1/2) - 7*a*b^3*c^3*d^2 + 12*a^2*b*c^4*d^2 + a*c^3*d^2*(-(4*a*c - b^2)^3)^(1/2) - 20*a^3*b*c^3*e^2 - 2*b^6*c*d*e + 25*a^2*b^3*c^2*e^2 - a^2*c^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - b^2*c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c*e^2 + 16*a^3*c^4*d*e + 16*a*b^4*c^2*d*e + 2*b^3*c*d*e*(-(4*a*c - b^2)^3)^(1/2) + 3*a*b^2*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - 36*a^2*b^2*c^3*d*e - 4*a*b*c^2*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^5*b^4*e^4 + 16*a^5*c^4*d^4 + 16*a^7*c^2*e^4 - 8*a^6*b^2*c*e^4 - 2*a^4*b^5*d*e^3 + a^3*b^4*c^2*d^4 - 8*a^4*b^2*c^3*d^4 + a^3*b^6*d^2*e^2 + 32*a^6*c^3*d^2*e^2 - 2*a^3*b^5*c*d^3*e - 32*a^5*b*c^3*d^3*e + 16*a^5*b^3*c*d*e^3 - 32*a^6*b*c^2*d*e^3 + 16*a^4*b^3*c^2*d^3*e - 6*a^4*b^4*c*d^2*e^2)))^(1/2)*(192*a^10*c^7*d^14*e^3 - x*(-(b^7*e^2 + b^5*c^2*d^2 - b^4*e^2*(-(4*a*c - b^2)^3)^(1/2) - 7*a*b^3*c^3*d^2 + 12*a^2*b*c^4*d^2 + a*c^3*d^2*(-(4*a*c - b^2)^3)^(1/2) - 20*a^3*b*c^3*e^2 - 2*b^6*c*d*e + 25*a^2*b^3*c^2*e^2 - a^2*c^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - b^2*c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c*e^2 + 16*a^3*c^4*d*e + 16*a*b^4*c^2*d*e + 2*b^3*c*d*e*(-(4*a*c - b^2)^3)^(1/2) + 3*a*b^2*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - 36*a^2*b^2*c^3*d*e - 4*a*b*c^2*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^5*b^4*e^4 + 16*a^5*c^4*d^4 + 16*a^7*c^2*e^4 - 8*a^6*b^2*c*e^4 - 2*a^4*b^5*d*e^3 + a^3*b^4*c^2*d^4 - 8*a^4*b^2*c^3*d^4 + a^3*b^6*d^2*e^2 + 32*a^6*c^3*d^2*e^2 - 2*a^3*b^5*c*d^3*e - 32*a^5*b*c^3*d^3*e + 16*a^5*b^3*c*d*e^3 - 32*a^6*b*c^2*d*e^3 + 16*a^4*b^3*c^2*d^3*e - 6*a^4*b^4*c*d^2*e^2)))^(1/2)*(512*a^11*c^7*d^15*e^3 + 512*a^12*c^6*d^13*e^5 - 512*a^13*c^5*d^11*e^7 - 512*a^14*c^4*d^9*e^9 - 32*a^9*b^3*c^6*d^16*e^2 + 128*a^9*b^4*c^5*d^15*e^3 - 192*a^9*b^5*c^4*d^14*e^4 + 128*a^9*b^6*c^3*d^13*e^5 - 32*a^9*b^7*c^2*d^12*e^6 - 640*a^10*b^2*c^6*d^15*e^3 + 1056*a^10*b^3*c^5*d^14*e^4 - 672*a^10*b^4*c^4*d^13*e^5 + 96*a^10*b^5*c^3*d^12*e^6 + 32*a^10*b^6*c^2*d^11*e^7 + 512*a^11*b^2*c^5*d^13*e^5 + 288*a^11*b^3*c^4*d^12*e^6 - 192*a^11*b^4*c^3*d^11*e^7 + 32*a^11*b^5*c^2*d^10*e^8 + 384*a^12*b^2*c^4*d^11*e^7 - 288*a^12*b^3*c^3*d^10*e^8 - 32*a^12*b^4*c^2*d^9*e^9 + 256*a^13*b^2*c^3*d^9*e^9 + 128*a^10*b*c^7*d^16*e^2 - 1152*a^11*b*c^6*d^14*e^4 - 640*a^12*b*c^5*d^12*e^6 + 640*a^13*b*c^4*d^10*e^8) + 128*a^11*c^6*d^12*e^5 - 320*a^12*c^5*d^10*e^7 - 256*a^13*c^4*d^8*e^9 - 16*a^8*b^3*c^6*d^15*e^2 + 64*a^8*b^4*c^5*d^14*e^3 - 96*a^8*b^5*c^4*d^13*e^4 + 64*a^8*b^6*c^3*d^12*e^5 - 16*a^8*b^7*c^2*d^11*e^6 - 304*a^9*b^2*c^6*d^14*e^3 + 512*a^9*b^3*c^5*d^13*e^4 - 352*a^9*b^4*c^4*d^12*e^5 + 64*a^9*b^5*c^3*d^11*e^6 + 16*a^9*b^6*c^2*d^10*e^7 + 352*a^10*b^2*c^5*d^12*e^5 + 80*a^10*b^3*c^4*d^11*e^6 - 128*a^10*b^4*c^3*d^10*e^7 + 16*a^10*b^5*c^2*d^9*e^8 + 336*a^11*b^2*c^4*d^10*e^7 - 128*a^11*b^3*c^3*d^9*e^8 - 16*a^11*b^4*c^2*d^8*e^9 + 128*a^12*b^2*c^3*d^8*e^9 + 64*a^9*b*c^7*d^15*e^2 - 512*a^10*b*c^6*d^13*e^4 - 320*a^11*b*c^5*d^11*e^6 + 256*a^12*b*c^4*d^9*e^8) + x*(112*a^10*c^6*d^10*e^6 - 32*a^9*c^7*d^12*e^4 - 16*a^8*c^8*d^14*e^2 - 128*a^11*c^5*d^8*e^8 + 8*a^7*b^2*c^7*d^14*e^2 - 16*a^7*b^3*c^6*d^13*e^3 + 8*a^7*b^4*c^5*d^12*e^4 + 8*a^7*b^5*c^4*d^11*e^5 - 16*a^7*b^6*c^3*d^10*e^6 + 8*a^7*b^7*c^2*d^9*e^7 - 72*a^8*b^3*c^5*d^11*e^5 + 128*a^8*b^4*c^4*d^10*e^6 - 72*a^8*b^5*c^3*d^9*e^7 - 280*a^9*b^2*c^5*d^10*e^6 + 208

*a^9*b^3*c^4*d^9*e^7 - 16*a^9*b^4*c^3*d^8*e^8 + 8*a^9*b^5*c^2*d^7*e^9 + 96*
a^10*b^2*c^4*d^8*e^8 - 56*a^10*b^3*c^3*d^7*e^9 + 32*a^8*b*c^7*d^13*e^3 + 12
8*a^9*b*c^6*d^11*e^5 - 192*a^10*b*c^5*d^9*e^7 + 96*a^11*b*c^4*d^7*e^9))*(-(
b^7*e^2 + b^5*c^2*d^2 - b^4*e^2*(-(4*a*c - b^2)^3)^(1/2) - 7*a*b^3*c^3*d^2
+ 12*a^2*b*c^4*d^2 + a*c^3*d^2*(-(4*a*c - b^2)^3)^(1/2) - 20*a^3*b*c^3*e^2
- 2*b^6*c*d*e + 25*a^2*b^3*c^2*e^2 - a^2*c^2*e^2*(-(4*a*c - b^2)^3)^(1/2) -
b^2*c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c*e^2 + 16*a^3*c^4*d*e + 16
*a*b^4*c^2*d*e + 2*b^3*c*d*e*(-(4*a*c - b^2)^3)^(1/2) + 3*a*b^2*c*e^2*(-(4*
a*c - b^2)^3)^(1/2) - 36*a^2*b^2*c^3*d*e - 4*a*b*c^2*d*e*(-(4*a*c - b^2)^3)
^(1/2))/(8*(a^5*b^4*e^4 + 16*a^5*c^4*d^4 + 16*a^7*c^2*e^4 - 8*a^6*b^2*c*e^4
- 2*a^4*b^5*d*e^3 + a^3*b^4*c^2*d^4 - 8*a^4*b^2*c^3*d^4 + a^3*b^6*d^2*e^2
+ 32*a^6*c^3*d^2*e^2 - 2*a^3*b^5*c*d^3*e - 32*a^5*b*c^3*d^3*e + 16*a^5*b^3*
c*d*e^3 - 32*a^6*b*c^2*d*e^3 + 16*a^4*b^3*c^2*d^3*e - 6*a^4*b^4*c*d^2*e^2))
)^(1/2) + 4*a^7*c^8*d^13*e^2 + 4*a^8*c^7*d^11*e^4 - 16*a^10*c^5*d^7*e^8 - 4
*a^7*b^5*c^3*d^8*e^7 + 4*a^7*b^6*c^2*d^7*e^8 + 24*a^8*b^3*c^4*d^8*e^7 - 28*
a^8*b^4*c^3*d^7*e^8 + 52*a^9*b^2*c^4*d^7*e^8 - 4*a^7*b*c^7*d^12*e^3 - 32*a^
9*b*c^5*d^8*e^7) + x*(2*a^7*c^7*d^9*e^5 - 4*a^8*c^6*d^7*e^7 + 2*a^7*b^2*c^5
*d^7*e^7))*(-(b^7*e^2 + b^5*c^2*d^2 - b^4*e^2*(-(4*a*c - b^2)^3)^(1/2) - 7*
a*b^3*c^3*d^2 + 12*a^2*b*c^4*d^2 + a*c^3*d^2*(-(4*a*c - b^2)^3)^(1/2) - 20*
a^3*b*c^3*e^2 - 2*b^6*c*d*e + 25*a^2*b^3*c^2*e^2 - a^2*c^2*e^2*(-(4*a*c - b
^2)^3)^(1/2) - b^2*c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c*e^2 + 16*a^
3*c^4*d*e + 16*a*b^4*c^2*d*e + 2*b^3*c*d*e*(-(4*a*c - b^2)^3)^(1/2) + 3*a*b
^2*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - 36*a^2*b^2*c^3*d*e - 4*a*b*c^2*d*e*(-(4
*a*c - b^2)^3)^(1/2))/(8*(a^5*b^4*e^4 + 16*a^5*c^4*d^4 + 16*a^7*c^2*e^4 - 8
*a^6*b^2*c*e^4 - 2*a^4*b^5*d*e^3 + a^3*b^4*c^2*d^4 - 8*a^4*b^2*c^3*d^4 + a^
3*b^6*d^2*e^2 + 32*a^6*c^3*d^2*e^2 - 2*a^3*b^5*c*d^3*e - 32*a^5*b*c^3*d^3*e
+ 16*a^5*b^3*c*d*e^3 - 32*a^6*b*c^2*d*e^3 + 16*a^4*b^3*c^2*d^3*e - 6*a^4*b
^4*c*d^2*e^2)))^(1/2) + ((-(b^7*e^2 + b^5*c^2*d^2 - b^4*e^2*(-(4*a*c - b^2)
^3)^(1/2) - 7*a*b^3*c^3*d^2 + 12*a^2*b*c^4*d^2 + a*c^3*d^2*(-(4*a*c - b^2)^
3)^(1/2) - 20*a^3*b*c^3*e^2 - 2*b^6*c*d*e + 25*a^2*b^3*c^2*e^2 - a^2*c^2*e^
2*(-(4*a*c - b^2)^3)^(1/2) - b^2*c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5
*c*e^2 + 16*a^3*c^4*d*e + 16*a*b^4*c^2*d*e + 2*b^3*c*d*e*(-(4*a*c - b^2)^3)
^(1/2) + 3*a*b^2*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - 36*a^2*b^2*c^3*d*e - 4*a*
b*c^2*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^5*b^4*e^4 + 16*a^5*c^4*d^4 + 16*a
^7*c^2*e^4 - 8*a^6*b^2*c*e^4 - 2*a^4*b^5*d*e^3 + a^3*b^4*c^2*d^4 - 8*a^4*b^
2*c^3*d^4 + a^3*b^6*d^2*e^2 + 32*a^6*c^3*d^2*e^2 - 2*a^3*b^5*c*d^3*e - 32*a
^5*b*c^3*d^3*e + 16*a^5*b^3*c*d*e^3 - 32*a^6*b*c^2*d*e^3 + 16*a^4*b^3*c^2*d
^3*e - 6*a^4*b^4*c*d^2*e^2)))^(1/2)*(((-(b^7*e^2 + b^5*c^2*d^2 - b^4*e^2*(-
(4*a*c - b^2)^3)^(1/2) - 7*a*b^3*c^3*d^2 + 12*a^2*b*c^4*d^2 + a*c^3*d^2*(-(
4*a*c - b^2)^3)^(1/2) - 20*a^3*b*c^3*e^2 - 2*b^6*c*d*e + 25*a^2*b^3*c^2*e^2
- a^2*c^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - b^2*c^2*d^2*(-(4*a*c - b^2)^3)^(1
/2) - 9*a*b^5*c*e^2 + 16*a^3*c^4*d*e + 16*a*b^4*c^2*d*e + 2*b^3*c*d*e*(-(4*
a*c - b^2)^3)^(1/2) + 3*a*b^2*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - 36*a^2*b^2*c
^3*d*e - 4*a*b*c^2*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^5*b^4*e^4 + 16*a^5*c
^4*d^4 + 16*a^7*c^2*e^4 - 8*a^6*b^2*c*e^4 - 2*a^4*b^5*d*e^3 + a^3*b^4*c^2*d

^4 - 8*a^4*b^2*c^3*d^4 + a^3*b^6*d^2*e^2 + 32*a^6*c^3*d^2*e^2 - 2*a^3*b^5*c
*d^3*e - 32*a^5*b*c^3*d^3*e + 16*a^5*b^3*c*d*e^3 - 32*a^6*b*c^2*d*e^3 + 16*
a^4*b^3*c^2*d^3*e - 6*a^4*b^4*c*d^2*e^2)))^(1/2)*(x*(-(b^7*e^2 + b^5*c^2*d^
2 - b^4*e^2*(-(4*a*c - b^2)^3)^(1/2) - 7*a*b^3*c^3*d^2 + 12*a^2*b*c^4*d^2 +
a*c^3*d^2*(-(4*a*c - b^2)^3)^(1/2) - 20*a^3*b*c^3*e^2 - 2*b^6*c*d*e + 25*a
^2*b^3*c^2*e^2 - a^2*c^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - b^2*c^2*d^2*(-(4*a*
c - b^2)^3)^(1/2) - 9*a*b^5*c*e^2 + 16*a^3*c^4*d*e + 16*a*b^4*c^2*d*e + 2*b
^3*c*d*e*(-(4*a*c - b^2)^3)^(1/2) + 3*a*b^2*c*e^2*(-(4*a*c - b^2)^3)^(1/2)
- 36*a^2*b^2*c^3*d*e - 4*a*b*c^2*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^5*b^4*
e^4 + 16*a^5*c^4*d^4 + 16*a^7*c^2*e^4 - 8*a^6*b^2*c*e^4 - 2*a^4*b^5*d*e^3 +
a^3*b^4*c^2*d^4 - 8*a^4*b^2*c^3*d^4 + a^3*b^6*d^2*e^2 + 32*a^6*c^3*d^2*e^2
- 2*a^3*b^5*c*d^3*e - 32*a^5*b*c^3*d^3*e + 16*a^5*b^3*c*d*e^3 - 32*a^6*b*c
^2*d*e^3 + 16*a^4*b^3*c^2*d^3*e - 6*a^4*b^4*c*d^2*e^2)))^(1/2)*(512*a^11*c^
7*d^15*e^3 + 512*a^12*c^6*d^13*e^5 - 512*a^13*c^5*d^11*e^7 - 512*a^14*c^4*d
^9*e^9 - 32*a^9*b^3*c^6*d^16*e^2 + 128*a^9*b^4*c^5*d^15*e^3 - 192*a^9*b^5*c
^4*d^14*e^4 + 128*a^9*b^6*c^3*d^13*e^5 - 32*a^9*b^7*c^2*d^12*e^6 - 640*a^10
*b^2*c^6*d^15*e^3 + 1056*a^10*b^3*c^5*d^14*e^4 - 672*a^10*b^4*c^4*d^13*e^5
+ 96*a^10*b^5*c^3*d^12*e^6 + 32*a^10*b^6*c^2*d^11*e^7 + 512*a^11*b^2*c^5*d^
13*e^5 + 288*a^11*b^3*c^4*d^12*e^6 - 192*a^11*b^4*c^3*d^11*e^7 + 32*a^11*b^
5*c^2*d^10*e^8 + 384*a^12*b^2*c^4*d^11*e^7 - 288*a^12*b^3*c^3*d^10*e^8 - 32
*a^12*b^4*c^2*d^9*e^9 + 256*a^13*b^2*c^3*d^9*e^9 + 128*a^10*b*c^7*d^16*e^2
- 1152*a^11*b*c^6*d^14*e^4 - 640*a^12*b*c^5*d^12*e^6 + 640*a^13*b*c^4*d^10*
e^8) + 192*a^10*c^7*d^14*e^3 + 128*a^11*c^6*d^12*e^5 - 320*a^12*c^5*d^10*e^
7 - 256*a^13*c^4*d^8*e^9 - 16*a^8*b^3*c^6*d^15*e^2 + 64*a^8*b^4*c^5*d^14*e^
3 - 96*a^8*b^5*c^4*d^13*e^4 + 64*a^8*b^6*c^3*d^12*e^5 - 16*a^8*b^7*c^2*d^11
*e^6 - 304*a^9*b^2*c^6*d^14*e^3 + 512*a^9*b^3*c^5*d^13*e^4 - 352*a^9*b^4*c^
4*d^12*e^5 + 64*a^9*b^5*c^3*d^11*e^6 + 16*a^9*b^6*c^2*d^10*e^7 + 352*a^10*b
^2*c^5*d^12*e^5 + 80*a^10*b^3*c^4*d^11*e^6 - 128*a^10*b^4*c^3*d^10*e^7 + 16
*a^10*b^5*c^2*d^9*e^8 + 336*a^11*b^2*c^4*d^10*e^7 - 128*a^11*b^3*c^3*d^9*e^
8 - 16*a^11*b^4*c^2*d^8*e^9 + 128*a^12*b^2*c^3*d^8*e^9 + 64*a^9*b*c^7*d^15*
e^2 - 512*a^10*b*c^6*d^13*e^4 - 320*a^11*b*c^5*d^11*e^6 + 256*a^12*b*c^4*d^
9*e^8) - x*(112*a^10*c^6*d^10*e^6 - 32*a^9*c^7*d^12*e^4 - 16*a^8*c^8*d^14*e
^2 - 128*a^11*c^5*d^8*e^8 + 8*a^7*b^2*c^7*d^14*e^2 - 16*a^7*b^3*c^6*d^13*e^
3 + 8*a^7*b^4*c^5*d^12*e^4 + 8*a^7*b^5*c^4*d^11*e^5 - 16*a^7*b^6*c^3*d^10*e
^6 + 8*a^7*b^7*c^2*d^9*e^7 - 72*a^8*b^3*c^5*d^11*e^5 + 128*a^8*b^4*c^4*d^10
*e^6 - 72*a^8*b^5*c^3*d^9*e^7 - 280*a^9*b^2*c^5*d^10*e^6 + 208*a^9*b^3*c^4*
d^9*e^7 - 16*a^9*b^4*c^3*d^8*e^8 + 8*a^9*b^5*c^2*d^7*e^9 + 96*a^10*b^2*c^4*
d^8*e^8 - 56*a^10*b^3*c^3*d^7*e^9 + 32*a^8*b*c^7*d^13*e^3 + 128*a^9*b*c^6*d
^11*e^5 - 192*a^10*b*c^5*d^9*e^7 + 96*a^11*b*c^4*d^7*e^9))*(-(b^7*e^2 + b^5
*c^2*d^2 - b^4*e^2*(-(4*a*c - b^2)^3)^(1/2) - 7*a*b^3*c^3*d^2 + 12*a^2*b*c^
4*d^2 + a*c^3*d^2*(-(4*a*c - b^2)^3)^(1/2) - 20*a^3*b*c^3*e^2 - 2*b^6*c*d*e
+ 25*a^2*b^3*c^2*e^2 - a^2*c^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - b^2*c^2*d^2*
(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c*e^2 + 16*a^3*c^4*d*e + 16*a*b^4*c^2*d*
e + 2*b^3*c*d*e*(-(4*a*c - b^2)^3)^(1/2) + 3*a*b^2*c*e^2*(-(4*a*c - b^2)^3)
^(1/2) - 36*a^2*b^2*c^3*d*e - 4*a*b*c^2*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(a

^5*b^4*e^4 + 16*a^5*c^4*d^4 + 16*a^7*c^2*e^4 - 8*a^6*b^2*c*e^4 - 2*a^4*b^5*
d*e^3 + a^3*b^4*c^2*d^4 - 8*a^4*b^2*c^3*d^4 + a^3*b^6*d^2*e^2 + 32*a^6*c^3*
d^2*e^2 - 2*a^3*b^5*c*d^3*e - 32*a^5*b*c^3*d^3*e + 16*a^5*b^3*c*d*e^3 - 32*
a^6*b*c^2*d*e^3 + 16*a^4*b^3*c^2*d^3*e - 6*a^4*b^4*c*d^2*e^2)))^(1/2) + 4*a
^7*c^8*d^13*e^2 + 4*a^8*c^7*d^11*e^4 - 16*a^10*c^5*d^7*e^8 - 4*a^7*b^5*c^3*
d^8*e^7 + 4*a^7*b^6*c^2*d^7*e^8 + 24*a^8*b^3*c^4*d^8*e^7 - 28*a^8*b^4*c^3*d
^7*e^8 + 52*a^9*b^2*c^4*d^7*e^8 - 4*a^7*b*c^7*d^12*e^3 - 32*a^9*b*c^5*d^8*e
^7) - x*(2*a^7*c^7*d^9*e^5 - 4*a^8*c^6*d^7*e^7 + 2*a^7*b^2*c^5*d^7*e^7))*(-
(b^7*e^2 + b^5*c^2*d^2 - b^4*e^2*(-(4*a*c - b^2)^3)^(1/2) - 7*a*b^3*c^3*d^2
 + 12*a^2*b*c^4*d^2 + a*c^3*d^2*(-(4*a*c - b^2)^3)^(1/2) - 20*a^3*b*c^3*e^2
 - 2*b^6*c*d*e + 25*a^2*b^3*c^2*e^2 - a^2*c^2*e^2*(-(4*a*c - b^2)^3)^(1/2)
- b^2*c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c*e^2 + 16*a^3*c^4*d*e + 1
6*a*b^4*c^2*d*e + 2*b^3*c*d*e*(-(4*a*c - b^2)^3)^(1/2) + 3*a*b^2*c*e^2*(-(4
*a*c - b^2)^3)^(1/2) - 36*a^2*b^2*c^3*d*e - 4*a*b*c^2*d*e*(-(4*a*c - b^2)^3
)^(1/2))/(8*(a^5*b^4*e^4 + 16*a^5*c^4*d^4 + 16*a^7*c^2*e^4 - 8*a^6*b^2*c*e^
4 - 2*a^4*b^5*d*e^3 + a^3*b^4*c^2*d^4 - 8*a^4*b^2*c^3*d^4 + a^3*b^6*d^2*e^2
 + 32*a^6*c^3*d^2*e^2 - 2*a^3*b^5*c*d^3*e - 32*a^5*b*c^3*d^3*e + 16*a^5*b^3
*c*d*e^3 - 32*a^6*b*c^2*d*e^3 + 16*a^4*b^3*c^2*d^3*e - 6*a^4*b^4*c*d^2*e^2)
))^(1/2)))*(-(b^7*e^2 + b^5*c^2*d^2 - b^4*e^2*(-(4*a*c - b^2)^3)^(1/2) - 7*
a*b^3*c^3*d^2 + 12*a^2*b*c^4*d^2 + a*c^3*d^2*(-(4*a*c - b^2)^3)^(1/2) - 20*
a^3*b*c^3*e^2 - 2*b^6*c*d*e + 25*a^2*b^3*c^2*e^2 - a^2*c^2*e^2*(-(4*a*c - b
^2)^3)^(1/2) - b^2*c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c*e^2 + 16*a^
3*c^4*d*e + 16*a*b^4*c^2*d*e + 2*b^3*c*d*e*(-(4*a*c - b^2)^3)^(1/2) + 3*a*b
^2*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - 36*a^2*b^2*c^3*d*e - 4*a*b*c^2*d*e*(-(4
*a*c - b^2)^3)^(1/2))/(8*(a^5*b^4*e^4 + 16*a^5*c^4*d^4 + 16*a^7*c^2*e^4 - 8
*a^6*b^2*c*e^4 - 2*a^4*b^5*d*e^3 + a^3*b^4*c^2*d^4 - 8*a^4*b^2*c^3*d^4 + a^
3*b^6*d^2*e^2 + 32*a^6*c^3*d^2*e^2 - 2*a^3*b^5*c*d^3*e - 32*a^5*b*c^3*d^3*e
 + 16*a^5*b^3*c*d*e^3 - 32*a^6*b*c^2*d*e^3 + 16*a^4*b^3*c^2*d^3*e - 6*a^4*b
^4*c*d^2*e^2)))^(1/2)*2i - (log(c^6*d^11*(-d^3*e^5)^(1/2) + b^6*d^6*e^9*x +
 c^6*d^12*e^3*x + b^5*c*d^3*(-d^3*e^5)^(3/2) - b^6*d^2*e*(-d^3*e^5)^(3/2) -
 a^2*b^4*e^3*(-d^3*e^5)^(3/2) - 16*a^4*c^2*e^3*(-d^3*e^5)^(3/2) - 7*a*b^3*c
^2*d^3*(-d^3*e^5)^(3/2) + 12*a^2*b*c^3*d^3*(-d^3*e^5)^(3/2) + 8*a^3*b^2*c*e
^3*(-d^3*e^5)^(3/2) + 16*a^3*c^3*d^2*e*(-d^3*e^5)^(3/2) + a*c^5*d^9*e^2*(-d
^3*e^5)^(1/2) + a*b^5*d^5*e^10*x + a*c^5*d^10*e^5*x - b*c^5*d^11*e^4*x - b^
5*c*d^7*e^8*x + a^2*b^4*d^4*e^11*x - 16*a^3*c^3*d^6*e^9*x + 16*a^4*c^2*d^4*
e^11*x - a*b^5*d*e^2*(-d^3*e^5)^(3/2) - b*c^5*d^10*e*(-d^3*e^5)^(1/2) - 24*
a^2*b^2*c^2*d^2*e*(-d^3*e^5)^(3/2) + 7*a*b^3*c^2*d^7*e^8*x - 12*a^2*b*c^3*d
^7*e^8*x - 8*a^2*b^3*c*d^5*e^10*x + 16*a^3*b*c^2*d^5*e^10*x - 8*a^3*b^2*c*d
^4*e^11*x + 9*a*b^4*c*d^2*e*(-d^3*e^5)^(3/2) + 24*a^2*b^2*c^2*d^6*e^9*x + 8
*a^2*b^3*c*d*e^2*(-d^3*e^5)^(3/2) - 16*a^3*b*c^2*d*e^2*(-d^3*e^5)^(3/2) - 9
*a*b^4*c*d^6*e^9*x)*(-d^3*e^5)^(1/2))/(2*(c*d^5 + a*d^3*e^2 - b*d^4*e)) + (
log(b^6*d^6*e^9*x - c^6*d^11*(-d^3*e^5)^(1/2) + c^6*d^12*e^3*x - b^5*c*d^3*
(-d^3*e^5)^(3/2) + b^6*d^2*e*(-d^3*e^5)^(3/2) + a^2*b^4*e^3*(-d^3*e^5)^(3/2
) + 16*a^4*c^2*e^3*(-d^3*e^5)^(3/2) + 7*a*b^3*c^2*d^3*(-d^3*e^5)^(3/2) - 12
*a^2*b*c^3*d^3*(-d^3*e^5)^(3/2) - 8*a^3*b^2*c*e^3*(-d^3*e^5)^(3/2) - 16*a^3

```
*c^3*d^2*e*(-d^3*e^5)^(3/2) - a*c^5*d^9*e^2*(-d^3*e^5)^(1/2) + a*b^5*d^5*e^
10*x + a*c^5*d^10*e^5*x - b*c^5*d^11*e^4*x - b^5*c*d^7*e^8*x + a^2*b^4*d^4*
e^11*x - 16*a^3*c^3*d^6*e^9*x + 16*a^4*c^2*d^4*e^11*x + a*b^5*d*e^2*(-d^3*e
^5)^(3/2) + b*c^5*d^10*e*(-d^3*e^5)^(1/2) + 24*a^2*b^2*c^2*d^2*e*(-d^3*e^5)
^(3/2) + 7*a*b^3*c^2*d^7*e^8*x - 12*a^2*b*c^3*d^7*e^8*x - 8*a^2*b^3*c*d^5*e
^10*x + 16*a^3*b*c^2*d^5*e^10*x - 8*a^3*b^2*c*d^4*e^11*x - 9*a*b^4*c*d^2*e*
(-d^3*e^5)^(3/2) + 24*a^2*b^2*c^2*d^6*e^9*x - 8*a^2*b^3*c*d*e^2*(-d^3*e^5)^
(3/2) + 16*a^3*b*c^2*d*e^2*(-d^3*e^5)^(3/2) - 9*a*b^4*c*d^6*e^9*x)*(-d^3*e^
5)^(1/2))/(2*c*d^5 + 2*a*d^3*e^2 - 2*b*d^4*e) - 1/(a*d*x)
```

3.309 $\int \frac{1}{x^4(d+ex^2)(a+bx^2+cx^4)}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 348

$$
\begin{aligned}
&\int \frac{1}{x^4\left(d+ex^2\right)\left(a+bx^2+cx^4\right)}\,dx\\
&=-\frac{1}{3adx^3}+\frac{bd+ae}{a^2d^2x}+\frac{\sqrt{c}\left(bcd-b^2e+ace+\frac{b^2cd-2ac^2d-b^3e+3abce}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}a^2\sqrt{b-\sqrt{b^2-4ac}}\left(cd^2-bde+ae^2\right)}\\
&+\frac{\sqrt{c}\left(bcd-b^2e+ace-\frac{b^2cd-2ac^2d-b^3e+3abce}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{2}a^2\sqrt{b+\sqrt{b^2-4ac}}\left(cd^2-bde+ae^2\right)}\\
&+\frac{e^{7/2}\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{d^{5/2}\left(cd^2-bde+ae^2\right)}
\end{aligned}
$$

```
[Out] -1/3/a/d/x^3+(a*e+b*d)/a^2/d^2/x+e^(7/2)*arctan(x*e^(1/2)/d^(1/2))/d^(5/2)/
(a*e^2-b*d*e+c*d^2)+1/2*arctan(x*2^(1/2)*c^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/
2))*c^(1/2)*(b*c*d-b^2*e+a*c*e+(3*a*b*c*e-2*a*c^2*d-b^3*e+b^2*c*d)/(-4*a*c+
b^2)^(1/2))/a^2/(a*e^2-b*d*e+c*d^2)*2^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2)+1/
2*arctan(x*2^(1/2)*c^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2))*c^(1/2)*(b*c*d-b^2
*e+a*c*e+(-3*a*b*c*e+2*a*c^2*d+b^3*e-b^2*c*d)/(-4*a*c+b^2)^(1/2))/a^2/(a*e^
2-b*d*e+c*d^2)*2^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2)
```

Rubi [A] (verified)

Time = 0.95 (sec) , antiderivative size = 348, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {1301, 211, 1180}

$$\int \frac{1}{x^4 (d+ex^2)(a+bx^2+cx^4)}\,dx = \frac{\sqrt{c}\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)\left(\frac{3abce-2ac^2d+b^3(-e)+b^2cd}{\sqrt{b^2-4ac}}+ace+b^2(-e)+bcd\right)}{\sqrt{2}a^2\sqrt{b-\sqrt{b^2-4ac}}\,(ae^2-bde+cd^2)} + \frac{\sqrt{c}\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)\left(-\frac{3abce-2ac^2d+b^3(-e)+b^2cd}{\sqrt{b^2-4ac}}+ace+b^2(-e)+bcd\right)}{\sqrt{2}a^2\sqrt{\sqrt{b^2-4ac}+b}\,(ae^2-bde+cd^2)} + \frac{ae+bd}{a^2d^2x} + \frac{e^{7/2}\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{d^{5/2}(ae^2-bde+cd^2)} - \frac{1}{3adx^3}$$

```
[In] Int[1/(x^4*(d + e*x^2)*(a + b*x^2 + c*x^4)),x]
```

```
[Out] -1/3*1/(a*d*x^3) + (b*d + a*e)/(a^2*d^2*x) + (Sqrt[c]*(b*c*d - b^2*e + a*c*
e + (b^2*c*d - 2*a*c^2*d - b^3*e + 3*a*b*c*e)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sq
rt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*a^2*Sqrt[b - Sqrt[b
^2 - 4*a*c]]*(c*d^2 - b*d*e + a*e^2)) + (Sqrt[c]*(b*c*d - b^2*e + a*c*e - (
b^2*c*d - 2*a*c^2*d - b^3*e + 3*a*b*c*e)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]
*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*a^2*Sqrt[b + Sqrt[b^2 -
4*a*c]]*(c*d^2 - b*d*e + a*e^2)) + (e^(7/2)*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(d
^(5/2)*(c*d^2 - b*d*e + a*e^2))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1301

```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.))/((a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*((d + e*x^2)^q/(a
+ b*x^2 + c*x^4)), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b^2 - 4
```

```
*a*c, 0] && IntegerQ[q] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(\frac{1}{adx^4} + \frac{-bd-ae}{a^2d^2x^2} + \frac{e^4}{d^2\left(cd^2-bde+ae^2\right)\left(d+ex^2\right)} + \frac{b^2cd-ac^2d-b^3e+2abce+c(bcd-b^2e+ace)\,x^2}{a^2\left(cd^2-bde+ae^2\right)\left(a+bx^2+cx^4\right)}\right) dx \\
&= -\frac{1}{3adx^3} + \frac{bd+ae}{a^2d^2x} + \frac{\int \frac{b^2cd-ac^2d-b^3e+2abce+c(bcd-b^2e+ace)x^2}{a+bx^2+cx^4}\,dx}{a^2\left(cd^2-bde+ae^2\right)} + \frac{e^4\int \frac{1}{d+ex^2}\,dx}{d^2\left(cd^2-bde+ae^2\right)} \\
&= -\frac{1}{3adx^3} + \frac{bd+ae}{a^2d^2x} + \frac{e^{7/2}\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{d^{5/2}\left(cd^2-bde+ae^2\right)} \\
&\quad + \frac{\left(c\left(bcd-b^2e+ace-\frac{b^2cd-2ac^2d-b^3e+3abce}{\sqrt{b^2-4ac}}\right)\right)\int \frac{1}{\frac{b}{2}+\frac{1}{2}\sqrt{b^2-4ac}+cx^2}\,dx}{2a^2\left(cd^2-bde+ae^2\right)} \\
&\quad + \frac{\left(c\left(bcd-b^2e+ace+\frac{b^2cd-2ac^2d-b^3e+3abce}{\sqrt{b^2-4ac}}\right)\right)\int \frac{1}{\frac{b}{2}-\frac{1}{2}\sqrt{b^2-4ac}+cx^2}\,dx}{2a^2\left(cd^2-bde+ae^2\right)} \\
&= -\frac{1}{3adx^3} + \frac{bd+ae}{a^2d^2x} + \frac{\sqrt{c}\left(bcd-b^2e+ace+\frac{b^2cd-2ac^2d-b^3e+3abce}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}a^2\sqrt{b-\sqrt{b^2-4ac}}\left(cd^2-bde+ae^2\right)} \\
&\quad + \frac{\sqrt{c}\left(bcd-b^2e+ace-\frac{b^2cd-2ac^2d-b^3e+3abce}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{2}a^2\sqrt{b+\sqrt{b^2-4ac}}\left(cd^2-bde+ae^2\right)} + \frac{e^{7/2}\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{d^{5/2}\left(cd^2-bde+ae^2\right)}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.36 (sec) , antiderivative size = 410, normalized size of antiderivative = 1.18

$$\begin{aligned}
\int \frac{1}{x^4\left(d+ex^2\right)\left(a+bx^2+cx^4\right)}\,dx &= -\frac{1}{3adx^3} + \frac{bd+ae}{a^2d^2x} \\
&\quad + \frac{\sqrt{c}\left(-b^3e+bc\left(\sqrt{b^2-4ac}d+3ae\right)+b^2\left(cd-\sqrt{b^2-4ac}e\right)+ac\left(-2cd+\sqrt{b^2-4ac}e\right)\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}a^2\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}\left(cd^2+e(-bd+ae)\right)} \\
&\quad + \frac{\sqrt{c}\left(b^3e+bc\left(\sqrt{b^2-4ac}d-3ae\right)-b^2\left(cd+\sqrt{b^2-4ac}e\right)+ac\left(2cd+\sqrt{b^2-4ac}e\right)\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{2}a^2\sqrt{b^2-4ac}\sqrt{b+\sqrt{b^2-4ac}}\left(cd^2+e(-bd+ae)\right)} \\
&\quad + \frac{e^{7/2}\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{d^{5/2}\left(cd^2-bde+ae^2\right)}
\end{aligned}$$

```
[In] Integrate[1/(x^4*(d + e*x^2)*(a + b*x^2 + c*x^4)),x]
```

```
[Out] -1/3*1/(a*d*x^3) + (b*d + a*e)/(a^2*d^2*x) + (Sqrt[c]*(-(b^3*e) + b*c*(Sqrt
[b^2 - 4*a*c]*d + 3*a*e) + b^2*(c*d - Sqrt[b^2 - 4*a*c]*e) + a*c*(-2*c*d +
Sqrt[b^2 - 4*a*c]*e))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]
]])/(Sqrt[2]*a^2*Sqrt[b^2 - 4*a*c]*Sqrt[b - Sqrt[b^2 - 4*a*c]]*(c*d^2 + e*(
-(b*d) + a*e))) + (Sqrt[c]*(b^3*e + b*c*(Sqrt[b^2 - 4*a*c]*d - 3*a*e) - b^2
*(c*d + Sqrt[b^2 - 4*a*c]*e) + a*c*(2*c*d + Sqrt[b^2 - 4*a*c]*e))*ArcTan[(S
qrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*a^2*Sqrt[b^2 - 4*a
*c]*Sqrt[b + Sqrt[b^2 - 4*a*c]]*(c*d^2 + e*(-(b*d) + a*e))) + (e^(7/2)*ArcT
an[(Sqrt[e]*x)/Sqrt[d]])/(d^(5/2)*(c*d^2 - b*d*e + a*e^2))
```

Maple [A] (verified)

Time = 0.49 (sec) , antiderivative size = 349, normalized size of antiderivative = 1.00

method	result
default	$-\frac{1}{3ad\,x^{3}}-\frac{-ae-bd}{x\,a^{2}d^{2}}+\frac{4c\left(\frac{\left(ace\sqrt{-4ac+b^{2}}-b^{2}e\sqrt{-4ac+b^{2}}+bcd\sqrt{-4ac+b^{2}}-3abce+2a\,c^{2}d+b^{3}e-b^{2}cd\right)\sqrt{2}\,\arctan\left(\frac{cx\sqrt{2}}{\sqrt{\left(b+\sqrt{-4ac+b^{2}}\right)c}}\right)}{8\sqrt{-4ac+b^{2}}\,\sqrt{\left(b+\sqrt{-4ac+b^{2}}\right)c}} \right.}{(a\,e^{2}-bd}$
risch	Expression too large to display

```
[In] int(1/x^4/(e*x^2+d)/(c*x^4+b*x^2+a),x,method=_RETURNVERBOSE)
```

```
[Out] -1/3/a/d/x^3-(-a*e-b*d)/x/a^2/d^2+4/(a*e^2-b*d*e+c*d^2)/a^2*c*(1/8*(a*c*e*(
-4*a*c+b^2)^(1/2)-b^2*e*(-4*a*c+b^2)^(1/2)+b*c*d*(-4*a*c+b^2)^(1/2)-3*a*b*c
*e+2*a*c^2*d+b^3*e-b^2*c*d)/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/
2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))-1/8*(a*c*
e*(-4*a*c+b^2)^(1/2)-b^2*e*(-4*a*c+b^2)^(1/2)+b*c*d*(-4*a*c+b^2)^(1/2)+3*a*
b*c*e-2*a*c^2*d-b^3*e+b^2*c*d)/(-4*a*c+b^2)^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2)
^(1/2))*c)^(1/2)*arctanh(c*x*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)))+1/
d^2*e^4/(a*e^2-b*d*e+c*d^2)/(e*d)^(1/2)*arctan(e*x/(e*d)^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 12482 vs. 2(304) = 608.

Time = 293.42 (sec) , antiderivative size = 24988, normalized size of antiderivative = 71.80

$$\int \frac{1}{x^4\,(d+ex^2)\,(a+bx^2+cx^4)}\,dx = \text{Too large to display}$$

```
[In] integrate(1/x^4/(e*x^2+d)/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{x^4 (d + ex^2) (a + bx^2 + cx^4)} dx = \text{Timed out}$$

```
[In] integrate(1/x**4/(e*x**2+d)/(c*x**4+b*x**2+a),x)
```

```
[Out] Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{x^4 (d + ex^2) (a + bx^2 + cx^4)} dx = \text{Exception raised: ValueError}$$

```
[In] integrate(1/x^4/(e*x^2+d)/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 12281 vs. $2(304) = 608$.

Time = 2.65 (sec) , antiderivative size = 12281, normalized size of antiderivative = 35.29

$$\int \frac{1}{x^4 (d + ex^2) (a + bx^2 + cx^4)} dx = \text{Too large to display}$$

```
[In] integrate(1/x^4/(e*x^2+d)/(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] e^4*arctan(e*x/sqrt(d*e))/((c*d^4 - b*d^3*e + a*d^2*e^2)*sqrt(d*e)) - 1/8*(
(2*a^4*b^5*c^5 - 12*a^5*b^3*c^6 + 16*a^6*b*c^7 - sqrt(2)*sqrt(b^2 - 4*a*c)*
sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b^5*c^3 + 6*sqrt(2)*sqrt(b^2 - 4*a*c)*s
qrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^5*b^3*c^4 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sq
rt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b^4*c^4 - 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqr
t(b*c + sqrt(b^2 - 4*a*c)*c)*a^6*b*c^5 - 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b
*c + sqrt(b^2 - 4*a*c)*c)*a^5*b^2*c^5 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c
+ sqrt(b^2 - 4*a*c)*c)*a^4*b^3*c^5 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c +
 sqrt(b^2 - 4*a*c)*c)*a^5*b*c^6 - 2*(b^2 - 4*a*c)*a^4*b^3*c^5 + 4*(b^2 - 4*
a*c)*a^5*b*c^6)*d^5 - (6*a^4*b^6*c^4 - 38*a^5*b^4*c^5 + 56*a^6*b^2*c^6 - 3*
sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b^6*c^2 + 19*
```

sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^5*b^4*c^3 + 6*s
qrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b^5*c^3 - 28*s
qrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^6*b^2*c^4 - 14*s
qrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^5*b^3*c^4 - 3*sq
rt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b^4*c^4 + 7*sqr
t(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^5*b^2*c^5 - 6*(b^2
 - 4*a*c)*a^4*b^4*c^4 + 14*(b^2 - 4*a*c)*a^5*b^2*c^5)*d^4*e + (6*a^4*b^7*c^
3 - 36*a^5*b^5*c^4 + 40*a^6*b^3*c^5 + 32*a^7*b*c^6 - 3*sqrt(2)*sqrt(b^2 - 4
*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b^7*c + 18*sqrt(2)*sqrt(b^2 - 4*a
*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^5*b^5*c^2 + 6*sqrt(2)*sqrt(b^2 - 4*a*
c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b^6*c^2 - 20*sqrt(2)*sqrt(b^2 - 4*a*
c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^6*b^3*c^3 - 12*sqrt(2)*sqrt(b^2 - 4*a*
c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^5*b^4*c^3 - 3*sqrt(2)*sqrt(b^2 - 4*a*c
)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b^5*c^3 - 16*sqrt(2)*sqrt(b^2 - 4*a*c
)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^7*b*c^4 - 8*sqrt(2)*sqrt(b^2 - 4*a*c)*s
qrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^6*b^2*c^4 + 6*sqrt(2)*sqrt(b^2 - 4*a*c)*sq
rt(b*c + sqrt(b^2 - 4*a*c)*c)*a^5*b^3*c^4 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqr
t(b*c + sqrt(b^2 - 4*a*c)*c)*a^6*b*c^5 - 6*(b^2 - 4*a*c)*a^4*b^5*c^3 + 12*(
b^2 - 4*a*c)*a^5*b^3*c^4 + 8*(b^2 - 4*a*c)*a^6*b*c^5)*d^3*e^2 - (2*a^4*b^8*
c^2 - 6*a^5*b^6*c^3 - 28*a^6*b^4*c^4 + 80*a^7*b^2*c^5 - sqrt(2)*sqrt(b^2 -
4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b^8 + 3*sqrt(2)*sqrt(b^2 - 4*a*c
)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^5*b^6*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*s
qrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b^7*c + 14*sqrt(2)*sqrt(b^2 - 4*a*c)*sqr
t(b*c + sqrt(b^2 - 4*a*c)*c)*a^6*b^4*c^2 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt
(b*c + sqrt(b^2 - 4*a*c)*c)*a^5*b^5*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*
c + sqrt(b^2 - 4*a*c)*c)*a^4*b^6*c^2 - 40*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*
c + sqrt(b^2 - 4*a*c)*c)*a^7*b^2*c^3 - 20*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*
c + sqrt(b^2 - 4*a*c)*c)*a^6*b^3*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c +
 sqrt(b^2 - 4*a*c)*c)*a^5*b^4*c^3 + 10*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c +
 sqrt(b^2 - 4*a*c)*c)*a^6*b^2*c^4 - 2*(b^2 - 4*a*c)*a^4*b^6*c^2 - 2*(b^2 -
4*a*c)*a^5*b^4*c^3 + 20*(b^2 - 4*a*c)*a^6*b^2*c^4)*d^2*e^3 + (4*a^5*b^7*c^2
 - 26*a^6*b^5*c^3 + 36*a^7*b^3*c^4 + 16*a^8*b*c^5 - 2*sqrt(2)*sqrt(b^2 - 4*
a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^5*b^7 + 13*sqrt(2)*sqrt(b^2 - 4*a*c)
*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^6*b^5*c + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sq
rt(b*c + sqrt(b^2 - 4*a*c)*c)*a^5*b^6*c - 18*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt
(b*c + sqrt(b^2 - 4*a*c)*c)*a^7*b^3*c^2 - 10*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt
(b*c + sqrt(b^2 - 4*a*c)*c)*a^6*b^4*c^2 - 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(
b*c + sqrt(b^2 - 4*a*c)*c)*a^5*b^5*c^2 - 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b
*c + sqrt(b^2 - 4*a*c)*c)*a^8*b*c^3 - 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c
+ sqrt(b^2 - 4*a*c)*c)*a^7*b^2*c^3 + 5*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c +
 sqrt(b^2 - 4*a*c)*c)*a^6*b^3*c^3 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c +
sqrt(b^2 - 4*a*c)*c)*a^7*b*c^4 - 4*(b^2 - 4*a*c)*a^5*b^5*c^2 + 10*(b^2 - 4*
a*c)*a^6*b^3*c^3 + 4*(b^2 - 4*a*c)*a^7*b*c^4)*d*e^4 - (2*a^6*b^6*c^2 - 14*a
^7*b^4*c^3 + 24*a^8*b^2*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2
 - 4*a*c)*c)*a^6*b^6 + 7*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*

a*c)*c)*a^7*b^4*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c
)*c)*a^6*b^5*c - 12*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*
c)*a^8*b^2*c^2 - 6*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c
)*a^7*b^3*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a
^6*b^4*c^2 + 3*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^
7*b^2*c^3 - 2*(b^2 - 4*a*c)*a^6*b^4*c^2 + 6*(b^2 - 4*a*c)*a^7*b^2*c^3)*e^5
 - 2*(sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^6*c^2 - 9*sqrt(2)*sqrt(b
*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^4*c^3 - 2*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a
*c)*c)*a^2*b^5*c^3 - 2*a^2*b^6*c^3 + 24*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c
)*c)*a^4*b^2*c^4 + 10*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^3*c^4 +
 sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^4*c^4 + 18*a^3*b^4*c^4 - 16*
sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^5*c^5 - 8*sqrt(2)*sqrt(b*c + sqrt
(b^2 - 4*a*c)*c)*a^4*b*c^5 - 5*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*
b^2*c^5 - 48*a^4*b^2*c^5 + 4*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*c^
6 + 32*a^5*c^6 + 2*(b^2 - 4*a*c)*a^2*b^4*c^3 - 10*(b^2 - 4*a*c)*a^3*b^2*c^4
 + 8*(b^2 - 4*a*c)*a^4*c^5)*d^3*abs(a^2*c*d^2 - a^2*b*d*e + a^3*e^2) + 2*(2
*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^7*c - 19*sqrt(2)*sqrt(b*c +
sqrt(b^2 - 4*a*c)*c)*a^3*b^5*c^2 - 4*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c
)*a^2*b^6*c^2 - 4*a^2*b^7*c^2 + 56*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*
a^4*b^3*c^3 + 22*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^4*c^3 + 2*sq
rt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^5*c^3 + 38*a^3*b^5*c^3 - 48*sqr
t(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^5*b*c^4 - 24*sqrt(2)*sqrt(b*c + sqrt
(b^2 - 4*a*c)*c)*a^4*b^2*c^4 - 11*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a
^3*b^3*c^4 - 112*a^4*b^3*c^4 + 12*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a
^4*b*c^5 + 96*a^5*b*c^5 + 4*(b^2 - 4*a*c)*a^2*b^5*c^2 - 22*(b^2 - 4*a*c)*a^
3*b^3*c^3 + 24*(b^2 - 4*a*c)*a^4*b*c^4)*d^2*e*abs(a^2*c*d^2 - a^2*b*d*e + a
^3*e^2) - 2*(sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^8 - 9*sqrt(2)*sq
rt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^6*c - 2*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4
*a*c)*c)*a^2*b^7*c - 2*a^2*b^8*c + 23*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*
c)*a^4*b^4*c^2 + 10*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^5*c^2 + s
qrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^6*c^2 + 18*a^3*b^6*c^2 - 8*sqr
t(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^5*b^2*c^3 - 6*sqrt(2)*sqrt(b*c + sqr
t(b^2 - 4*a*c)*c)*a^4*b^3*c^3 - 5*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a
^3*b^4*c^3 - 46*a^4*b^4*c^3 - 16*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^
6*c^4 - 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^5*b*c^4 + 3*sqrt(2)*sqr
t(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b^2*c^4 + 16*a^5*b^2*c^4 + 4*sqrt(2)*sqrt(
b*c + sqrt(b^2 - 4*a*c)*c)*a^5*c^5 + 32*a^6*c^5 + 2*(b^2 - 4*a*c)*a^2*b^6*c
 - 10*(b^2 - 4*a*c)*a^3*b^4*c^2 + 6*(b^2 - 4*a*c)*a^4*b^2*c^3 + 8*(b^2 - 4*
a*c)*a^5*c^4)*d*e^2*abs(a^2*c*d^2 - a^2*b*d*e + a^3*e^2) + 2*(sqrt(2)*sqrt(
b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^7 - 10*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c
)*c)*a^4*b^5*c - 2*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^6*c - 2*a^
3*b^7*c + 32*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^5*b^3*c^2 + 12*sqrt(
2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b^4*c^2 + sqrt(2)*sqrt(b*c + sqrt(b^
2 - 4*a*c)*c)*a^3*b^5*c^2 + 20*a^4*b^5*c^2 - 32*sqrt(2)*sqrt(b*c + sqrt(b^2
 - 4*a*c)*c)*a^6*b*c^3 - 16*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^5*b^2

```
*c^3 - 6*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b^3*c^3 - 64*a^5*b^3*c
^3 + 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^5*b*c^4 + 64*a^6*b*c^4 + 2
*(b^2 - 4*a*c)*a^3*b^5*c - 12*(b^2 - 4*a*c)*a^4*b^3*c^2 + 16*(b^2 - 4*a*c)*
a^5*b*c^3)*e^3*abs(a^2*c*d^2 - a^2*b*d*e + a^3*e^2) + (2*b^5*c^3 - 16*a*b^3
*c^4 + 32*a^2*b*c^5 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c
)*c)*b^5*c + 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*
b^3*c^2 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^4*c
^2 - 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b*c^3
 - 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^3 -
sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^3*c^3 + 4*sqrt(
2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c^4 - 2*(b^2 - 4*a
*c)*b^3*c^3 + 8*(b^2 - 4*a*c)*a*b*c^4)*(a^2*c*d^2 - a^2*b*d*e + a^3*e^2)^2*
d - (2*b^6*c^2 - 18*a*b^4*c^3 + 48*a^2*b^2*c^4 - 32*a^3*c^5 - sqrt(2)*sqrt(
b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^6 + 9*sqrt(2)*sqrt(b^2 - 4*a
*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^4*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*s
qrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^5*c - 24*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*
c + sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^2 - 10*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*
c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + s
qrt(b^2 - 4*a*c)*c)*b^4*c^2 + 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(
b^2 - 4*a*c)*c)*a^3*c^3 + 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 -
 4*a*c)*c)*a^2*b*c^3 + 5*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*
a*c)*c)*a*b^2*c^3 - 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c
)*c)*a^2*c^4 - 2*(b^2 - 4*a*c)*b^4*c^2 + 10*(b^2 - 4*a*c)*a*b^2*c^3 - 8*(b^
2 - 4*a*c)*a^2*c^4)*(a^2*c*d^2 - a^2*b*d*e + a^3*e^2)^2*e)*arctan(2*sqrt(1/
2)*x/sqrt((a^2*b*c*d^2 - a^2*b^2*d*e + a^3*b*e^2 + sqrt((a^2*b*c*d^2 - a^2*
b^2*d*e + a^3*b*e^2)^2 - 4*(a^3*c*d^2 - a^3*b*d*e + a^4*e^2)*(a^2*c^2*d^2 -
 a^2*b*c*d*e + a^3*c*e^2)))/(a^2*c^2*d^2 - a^2*b*c*d*e + a^3*c*e^2)))/((a^5
*b^4*c^2 - 8*a^6*b^2*c^3 - 2*a^5*b^3*c^3 + 16*a^7*c^4 + 8*a^6*b*c^4 + a^5*b
^2*c^4 - 4*a^6*c^5)*d^4*abs(a^2*c*d^2 - a^2*b*d*e + a^3*e^2)*abs(c) - 2*(a^
5*b^5*c - 8*a^6*b^3*c^2 - 2*a^5*b^4*c^2 + 16*a^7*b*c^3 + 8*a^6*b^2*c^3 + a^
5*b^3*c^3 - 4*a^6*b*c^4)*d^3*e*abs(a^2*c*d^2 - a^2*b*d*e + a^3*e^2)*abs(c)
+ (a^5*b^6 - 6*a^6*b^4*c - 2*a^5*b^5*c + 4*a^6*b^3*c^2 + a^5*b^4*c^2 + 32*a
^8*c^3 + 16*a^7*b*c^3 - 2*a^6*b^2*c^3 - 8*a^7*c^4)*d^2*e^2*abs(a^2*c*d^2 -
a^2*b*d*e + a^3*e^2)*abs(c) - 2*(a^6*b^5 - 8*a^7*b^3*c - 2*a^6*b^4*c + 16*a
^8*b*c^2 + 8*a^7*b^2*c^2 + a^6*b^3*c^2 - 4*a^7*b*c^3)*d*e^3*abs(a^2*c*d^2 -
 a^2*b*d*e + a^3*e^2)*abs(c) + (a^7*b^4 - 8*a^8*b^2*c - 2*a^7*b^3*c + 16*a^
9*c^2 + 8*a^8*b*c^2 + a^7*b^2*c^2 - 4*a^8*c^3)*e^4*abs(a^2*c*d^2 - a^2*b*d*
e + a^3*e^2)*abs(c)) - 1/8*((2*a^4*b^5*c^5 - 12*a^5*b^3*c^6 + 16*a^6*b*c^7
 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*b^5*c^3 + 6
*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^5*b^3*c^4 + 2*
sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*b^4*c^4 - 8*s
qrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^6*b*c^5 - 4*sqrt
(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^5*b^2*c^5 - sqrt(2)
*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*b^3*c^5 + 2*sqrt(2)*
sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^5*b*c^6 - 2*(b^2 - 4*a*
```

c)*a^4*b^3*c^5 + 4*(b^2 - 4*a*c)*a^5*b*c^6)*d^5 - (6*a^4*b^6*c^4 - 38*a^5*b^4*c^5 + 56*a^6*b^2*c^6 - 3*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*b^6*c^2 + 19*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^5*b^4*c^3 + 6*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*b^5*c^3 - 28*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^6*b^2*c^4 - 14*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^5*b^3*c^4 - 3*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*b^4*c^4 + 7*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^5*b^2*c^5 - 6*(b^2 - 4*a*c)*a^4*b^4*c^4 + 14*(b^2 - 4*a*c)*a^5*b^2*c^5)*d^4*e + (6*a^4*b^7*c^3 - 36*a^5*b^5*c^4 + 40*a^6*b^3*c^5 + 32*a^7*b*c^6 - 3*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*b^7*c + 18*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^5*b^5*c^2 + 6*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*b^6*c^2 - 20*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^6*b^3*c^3 - 12*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^5*b^4*c^3 - 3*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*b^5*c^3 - 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^7*b*c^4 - 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^6*b^2*c^4 + 6*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^5*b^3*c^4 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^6*b*c^5 - 6*(b^2 - 4*a*c)*a^4*b^5*c^3 + 12*(b^2 - 4*a*c)*a^5*b^3*c^4 + 8*(b^2 - 4*a*c)*a^6*b*c^5)*d^3*e^2 - (2*a^4*b^8*c^2 - 6*a^5*b^6*c^3 - 28*a^6*b^4*c^4 + 80*a^7*b^2*c^5 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*b^8 + 3*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^5*b^6*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*b^7*c + 14*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^6*b^4*c^2 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^5*b^5*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*b^6*c^2 - 40*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^7*b^2*c^3 - 20*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^6*b^3*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^5*b^4*c^3 + 10*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^6*b^2*c^4 - 2*(b^2 - 4*a*c)*a^4*b^6*c^2 - 2*(b^2 - 4*a*c)*a^5*b^4*c^3 + 20*(b^2 - 4*a*c)*a^6*b^2*c^4)*d^2*e^3 + (4*a^5*b^7*c^2 - 26*a^6*b^5*c^3 + 36*a^7*b^3*c^4 + 16*a^8*b*c^5 - 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^5*b^7 + 13*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^6*b^5*c + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^5*b^6*c - 18*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^7*b^3*c^2 - 10*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^6*b^4*c^2 - 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^5*b^5*c^2 - 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^8*b*c^3 - 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^7*b^2*c^3 + 5*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^6*b^3*c^3 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^7*b*c^4 - 4*(b^2 - 4*a*c)*a^5*b^5*c^2 + 10*(b^2 - 4*a*c)*a^6*b^3*c^3 + 4*(b^2 - 4*a*c)*a^7*b*c^4)*d *e^4 - (2*a^6*b^6*c^2 - 14*a^7*b^4*c^3 + 24*a^8*b^2*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^6*b^6 + 7*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^7*b^4*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^6*b^5*c - 12*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^8*b^2*c^2 - 6*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^7*b^3*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^6*b^4*c^2 + 3*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^7*b^2*c^3 - 2*(b^2 - 4*a*c)*a^6*b^4*c^2 + 6*(b^2 - 4*a*c)*a^7*b^2*c^3)*e^5 - 2*(sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^6*c^2 - 9*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^4*c^3 - 2*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^5*c^3 + 2*a^2*b^6*c^3 + 24*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*b^2*c^4 + 10*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^3*c^4 + sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^4*c^4 - 18*a^3*b^4*c^4 - 16*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^5*c^5 - 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*b*c^5 - 5*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^2*c^5 + 48*a^4*b^2*c^5 + 4*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*c^6 - 32*a^5*c^6 - 2*(b^2 - 4*a*c)*a^2*b^4*c^3 + 10*(b^2 - 4*a*c)*a^3*b^2*c^4 - 8*(b^2 - 4*a*c)*a^4*c^5)*d^3*abs(a^2*c*d^2 - a^2*b*d*e + a^3*e^2) + 2*(2*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^7*c - 19*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^5*c^2 - 4*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^6*c^2 + 4*a^2*b^7*c^2 + 56*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*b^3*c^3 + 22*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^4*c^3 + 2*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^5*c^3 - 38*a^3*b^5*c^3 - 48*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^5*b*c^4 - 24*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*b^2*c^4 - 11*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^3*c^4 + 112*a^4*b^3*c^4 + 12*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*b*c^5 - 96*a^5*b*c^5 - 4*(b^2 - 4*a*c)*a^2*b^5*c^2 + 22*(b^2 - 4*a*c)*a^3*b^3*c^3 - 24*(b^2 - 4*a*c)*a^4*b*c^4)*d^2*e*abs(a^2*c*d^2 - a^2*b*d*e + a^3*e^2) - 2*(sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^8 - 9*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^6*c - 2*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^7*c + 2*a^2*b^8*c + 23*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*b^4*c^2 + 10*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^5*c^2 + sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^6*c^2 - 18*a^3*b^6*c^2 - 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^5*b^2*c^3 - 6*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*b^3*c^3 - 5*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^4*c^3 + 46*a^4*b^4*c^3 - 16*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^6*c^4 - 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^5*b*c^4 + 3*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*b^2*c^4 - 16*a^5*b^2*c^4 + 4*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^5*c^5 - 32*a^6*c^5 - 2*(b^2 - 4*a*c)*a^2*b^6*c + 10*(b^2 - 4*a*c)*a^3*b^4*c^2 - 6*(b^2 - 4*a*c)*a^4*b^2*c^3 - 8*(b^2 - 4*a*c)*a^5*c^4)*d*e^2*abs(a^2*c*d^2 - a^2*b*d*e + a^3*e^2) + 2*(sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^7 - 10*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*b^5*c - 2*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^6*c + 2*a^3*b^7*c + 32*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^5*b^3*c^2 + 12*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*b^4*c^2

+ sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^5*c^2 - 20*a^4*b^5*c^2 - 32 *sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^6*b*c^3 - 16*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^5*b^2*c^3 - 6*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*b^3*c^3 + 64*a^5*b^3*c^3 + 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)* a^5*b*c^4 - 64*a^6*b*c^4 - 2*(b^2 - 4*a*c)*a^3*b^5*c + 12*(b^2 - 4*a*c)*a^4 *b^3*c^2 - 16*(b^2 - 4*a*c)*a^5*b*c^3)*e^3*abs(a^2*c*d^2 - a^2*b*d*e + a^3* e^2) + (2*b^5*c^3 - 16*a*b^3*c^4 + 32*a^2*b*c^5 - sqrt(2)*sqrt(b^2 - 4*a*c) *sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^5*c + 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b *c - sqrt(b^2 - 4*a*c)*c)*a*b^3*c^2 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^4*c^2 - 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sq rt(b^2 - 4*a*c)*c)*a^2*b*c^3 - 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^2*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^3*c^3 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a* c)*c)*a*b*c^4 - 2*(b^2 - 4*a*c)*b^3*c^3 + 8*(b^2 - 4*a*c)*a*b*c^4)*(a^2*c*d ^2 - a^2*b*d*e + a^3*e^2)^2*d - (2*b^6*c^2 - 18*a*b^4*c^3 + 48*a^2*b^2*c^4 - 32*a^3*c^5 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^ 6 + 9*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^4*c + 2 *sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^5*c - 24*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^2 - 10*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^3*c^2 - sqrt(2)*sq rt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^4*c^2 + 16*sqrt(2)*sqrt(b ^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*c^3 + 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b*c^3 + 5*sqrt(2)*sqrt(b^2 - 4*a *c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^2*c^3 - 4*sqrt(2)*sqrt(b^2 - 4*a*c) *sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*c^4 - 2*(b^2 - 4*a*c)*b^4*c^2 + 10*(b^ 2 - 4*a*c)*a*b^2*c^3 - 8*(b^2 - 4*a*c)*a^2*c^4)*(a^2*c*d^2 - a^2*b*d*e + a^ 3*e^2)^2*e)*arctan(2*sqrt(1/2)*x/sqrt((a^2*b*c*d^2 - a^2*b^2*d*e + a^3*b*e^ 2 - sqrt((a^2*b*c*d^2 - a^2*b^2*d*e + a^3*b*e^2)^2 - 4*(a^3*c*d^2 - a^3*b*d *e + a^4*e^2)*(a^2*c^2*d^2 - a^2*b*c*d*e + a^3*c*e^2)))/(a^2*c^2*d^2 - a^2* b*c*d*e + a^3*c*e^2)))/((a^5*b^4*c^2 - 8*a^6*b^2*c^3 - 2*a^5*b^3*c^3 + 16*a ^7*c^4 + 8*a^6*b*c^4 + a^5*b^2*c^4 - 4*a^6*c^5)*d^4*abs(a^2*c*d^2 - a^2*b*d *e + a^3*e^2)*abs(c) - 2*(a^5*b^5*c - 8*a^6*b^3*c^2 - 2*a^5*b^4*c^2 + 16*a^ 7*b*c^3 + 8*a^6*b^2*c^3 + a^5*b^3*c^3 - 4*a^6*b*c^4)*d^3*e*abs(a^2*c*d^2 - a^2*b*d*e + a^3*e^2)*abs(c) + (a^5*b^6 - 6*a^6*b^4*c - 2*a^5*b^5*c + 4*a^6* b^3*c^2 + a^5*b^4*c^2 + 32*a^8*c^3 + 16*a^7*b*c^3 - 2*a^6*b^2*c^3 - 8*a^7*c ^4)*d^2*e^2*abs(a^2*c*d^2 - a^2*b*d*e + a^3*e^2)*abs(c) - 2*(a^6*b^5 - 8*a^ 7*b^3*c - 2*a^6*b^4*c + 16*a^8*b*c^2 + 8*a^7*b^2*c^2 + a^6*b^3*c^2 - 4*a^7* b*c^3)*d*e^3*abs(a^2*c*d^2 - a^2*b*d*e + a^3*e^2)*abs(c) + (a^7*b^4 - 8*a^8 *b^2*c - 2*a^7*b^3*c + 16*a^9*c^2 + 8*a^8*b*c^2 + a^7*b^2*c^2 - 4*a^8*c^3)* e^4*abs(a^2*c*d^2 - a^2*b*d*e + a^3*e^2)*abs(c)) + 1/3*(3*b*d*x^2 + 3*a*e*x ^2 - a*d)/(a^2*d^2*x^3)

Mupad [B] (verification not implemented)

Time = 11.88 (sec) , antiderivative size = 42882, normalized size of antiderivative = 123.22

$$\int \frac{1}{x^4 (d+ex^2) (a+bx^2+cx^4)} \, dx = \text{Too large to display}$$

```
[In] int(1/(x^4*(d + e*x^2)*(a + b*x^2 + c*x^4)),x)
[Out] (log(c^9*d^27*e^6 - b^9*d^18*e^15 + 2*a*c^8*d^25*e^8 - 2*b*c^8*d^26*e^7 + 2
*b^8*c*d^19*e^14 + a^5*b^4*d^13*e^20 + a^2*c^7*d^23*e^10 + 16*a^4*c^5*d^19*
e^14 + 16*a^7*c^2*d^13*e^20 + b^2*c^7*d^25*e^8 - b^7*c^2*d^20*e^13 - 25*a^2
*b^3*c^4*d^20*e^13 + 66*a^2*b^4*c^3*d^19*e^14 - 42*a^2*b^5*c^2*d^18*e^15 -
76*a^3*b^2*c^4*d^19*e^14 + 63*a^3*b^3*c^3*d^18*e^15 - a^5*b^4*e^3*x*(-d^5*e
^7)^(5/2) + a^2*c^7*d^15*x*(-d^5*e^7)^(3/2) - 16*a^7*c^2*e^3*x*(-d^5*e^7)^(
5/2) - b^9*d^10*e^5*x*(-d^5*e^7)^(3/2) - c^9*d^24*e^3*x*(-d^5*e^7)^(1/2) -
2*a*b*c^7*d^24*e^9 + 11*a*b^7*c*d^18*e^15 + 9*a*b^5*c^3*d^20*e^13 - 20*a*b^
6*c^2*d^19*e^14 + 20*a^3*b*c^5*d^20*e^13 - 28*a^4*b*c^4*d^18*e^15 - 8*a^6*b
^2*c*d^13*e^20 + 16*a^4*c^5*d^11*e^4*x*(-d^5*e^7)^(3/2) - b^7*c^2*d^12*e^3*
x*(-d^5*e^7)^(3/2) - b^2*c^7*d^22*e^5*x*(-d^5*e^7)^(1/2) + 8*a^6*b^2*c*e^3*
x*(-d^5*e^7)^(5/2) - 2*a*c^8*d^22*e^5*x*(-d^5*e^7)^(1/2) + 2*b^8*c*d^11*e^4
*x*(-d^5*e^7)^(3/2) + 2*b*c^8*d^23*e^4*x*(-d^5*e^7)^(1/2) + 11*a*b^7*c*d^10
*e^5*x*(-d^5*e^7)^(3/2) + 2*a*b*c^7*d^21*e^6*x*(-d^5*e^7)^(1/2) + 9*a*b^5*c
^3*d^12*e^3*x*(-d^5*e^7)^(3/2) - 20*a*b^6*c^2*d^11*e^4*x*(-d^5*e^7)^(3/2) +
 20*a^3*b*c^5*d^12*e^3*x*(-d^5*e^7)^(3/2) - 28*a^4*b*c^4*d^10*e^5*x*(-d^5*e
^7)^(3/2) - 25*a^2*b^3*c^4*d^12*e^3*x*(-d^5*e^7)^(3/2) + 66*a^2*b^4*c^3*d^1
1*e^4*x*(-d^5*e^7)^(3/2) - 42*a^2*b^5*c^2*d^10*e^5*x*(-d^5*e^7)^(3/2) - 76*
a^3*b^2*c^4*d^11*e^4*x*(-d^5*e^7)^(3/2) + 63*a^3*b^3*c^3*d^10*e^5*x*(-d^5*e
^7)^(3/2))*(-d^5*e^7)^(1/2))/(2*c*d^7 + 2*a*d^5*e^2 - 2*b*d^6*e) - atan((((
-(b^9*e^2 + b^7*c^2*d^2 - b^6*e^2*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c^3*d^
2 - 20*a^3*b*c^5*d^2 + 28*a^4*b*c^4*e^2 - 2*b^8*c*d*e + 25*a^2*b^3*c^4*d^2
- a^2*c^4*d^2*(-(4*a*c - b^2)^3)^(1/2) + 42*a^2*b^5*c^2*e^2 - 63*a^3*b^3*c^
3*e^2 + a^3*c^3*e^2*(-(4*a*c - b^2)^3)^(1/2) - b^4*c^2*d^2*(-(4*a*c - b^2)^
3)^(1/2) - 11*a*b^7*c*e^2 - 16*a^4*c^5*d*e + 20*a*b^6*c^2*d*e + 2*b^5*c*d*e
*(-(4*a*c - b^2)^3)^(1/2) - 6*a^2*b^2*c^2*e^2*(-(4*a*c - b^2)^3)^(1/2) + 5*
a*b^4*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - 66*a^2*b^4*c^3*d*e + 76*a^3*b^2*c^4*
d*e + 3*a*b^2*c^3*d^2*(-(4*a*c - b^2)^3)^(1/2) - 8*a*b^3*c^2*d*e*(-(4*a*c -
 b^2)^3)^(1/2) + 6*a^2*b*c^3*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^7*b^4*e^4
+ 16*a^7*c^4*d^4 + 16*a^9*c^2*e^4 - 8*a^8*b^2*c*e^4 - 2*a^6*b^5*d*e^3 + a^5
*b^4*c^2*d^4 - 8*a^6*b^2*c^3*d^4 + a^5*b^6*d^2*e^2 + 32*a^8*c^3*d^2*e^2 - 2
*a^5*b^5*c*d^3*e - 32*a^7*b*c^3*d^3*e + 16*a^7*b^3*c*d*e^3 - 32*a^8*b*c^2*d
*e^3 + 16*a^6*b^3*c^2*d^3*e - 6*a^6*b^4*c*d^2*e^2)))^(1/2)*(((-(b^9*e^2 + b
^7*c^2*d^2 - b^6*e^2*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c^3*d^2 - 20*a^3*b*
c^5*d^2 + 28*a^4*b*c^4*e^2 - 2*b^8*c*d*e + 25*a^2*b^3*c^4*d^2 - a^2*c^4*d^2
*(-(4*a*c - b^2)^3)^(1/2) + 42*a^2*b^5*c^2*e^2 - 63*a^3*b^3*c^3*e^2 + a^3*c
```

^3*e^2*(-(4*a*c - b^2)^3)^(1/2) - b^4*c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) - 11
*a*b^7*c*e^2 - 16*a^4*c^5*d*e + 20*a*b^6*c^2*d*e + 2*b^5*c*d*e*(-(4*a*c - b
^2)^3)^(1/2) - 6*a^2*b^2*c^2*e^2*(-(4*a*c - b^2)^3)^(1/2) + 5*a*b^4*c*e^2*(
-(4*a*c - b^2)^3)^(1/2) - 66*a^2*b^4*c^3*d*e + 76*a^3*b^2*c^4*d*e + 3*a*b^2
*c^3*d^2*(-(4*a*c - b^2)^3)^(1/2) - 8*a*b^3*c^2*d*e*(-(4*a*c - b^2)^3)^(1/2
) + 6*a^2*b*c^3*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^7*b^4*e^4 + 16*a^7*c^4*
d^4 + 16*a^9*c^2*e^4 - 8*a^8*b^2*c*e^4 - 2*a^6*b^5*d*e^3 + a^5*b^4*c^2*d^4
- 8*a^6*b^2*c^3*d^4 + a^5*b^6*d^2*e^2 + 32*a^8*c^3*d^2*e^2 - 2*a^5*b^5*c*d^
3*e - 32*a^7*b*c^3*d^3*e + 16*a^7*b^3*c*d*e^3 - 32*a^8*b*c^2*d*e^3 + 16*a^6
*b^3*c^2*d^3*e - 6*a^6*b^4*c*d^2*e^2)))^(1/2)*(x*(-(b^9*e^2 + b^7*c^2*d^2 -
b^6*e^2*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c^3*d^2 - 20*a^3*b*c^5*d^2 + 28
*a^4*b*c^4*e^2 - 2*b^8*c*d*e + 25*a^2*b^3*c^4*d^2 - a^2*c^4*d^2*(-(4*a*c -
b^2)^3)^(1/2) + 42*a^2*b^5*c^2*e^2 - 63*a^3*b^3*c^3*e^2 + a^3*c^3*e^2*(-(4*
a*c - b^2)^3)^(1/2) - b^4*c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) - 11*a*b^7*c*e^2
- 16*a^4*c^5*d*e + 20*a*b^6*c^2*d*e + 2*b^5*c*d*e*(-(4*a*c - b^2)^3)^(1/2)
- 6*a^2*b^2*c^2*e^2*(-(4*a*c - b^2)^3)^(1/2) + 5*a*b^4*c*e^2*(-(4*a*c - b^
2)^3)^(1/2) - 66*a^2*b^4*c^3*d*e + 76*a^3*b^2*c^4*d*e + 3*a*b^2*c^3*d^2*(-(
4*a*c - b^2)^3)^(1/2) - 8*a*b^3*c^2*d*e*(-(4*a*c - b^2)^3)^(1/2) + 6*a^2*b*
c^3*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^7*b^4*e^4 + 16*a^7*c^4*d^4 + 16*a^9
*c^2*e^4 - 8*a^8*b^2*c*e^4 - 2*a^6*b^5*d*e^3 + a^5*b^4*c^2*d^4 - 8*a^6*b^2*
c^3*d^4 + a^5*b^6*d^2*e^2 + 32*a^8*c^3*d^2*e^2 - 2*a^5*b^5*c*d^3*e - 32*a^7
*b*c^3*d^3*e + 16*a^7*b^3*c*d*e^3 - 32*a^8*b*c^2*d*e^3 + 16*a^6*b^3*c^2*d^3
*e - 6*a^6*b^4*c*d^2*e^2)))^(1/2)*(512*a^20*c^7*d^24*e^3 + 512*a^21*c^6*d^2
2*e^5 - 512*a^22*c^5*d^20*e^7 - 512*a^23*c^4*d^18*e^9 - 32*a^18*b^3*c^6*d^2
5*e^2 + 128*a^18*b^4*c^5*d^24*e^3 - 192*a^18*b^5*c^4*d^23*e^4 + 128*a^18*b^
6*c^3*d^22*e^5 - 32*a^18*b^7*c^2*d^21*e^6 - 640*a^19*b^2*c^6*d^24*e^3 + 105
6*a^19*b^3*c^5*d^23*e^4 - 672*a^19*b^4*c^4*d^22*e^5 + 96*a^19*b^5*c^3*d^21*
e^6 + 32*a^19*b^6*c^2*d^20*e^7 + 512*a^20*b^2*c^5*d^22*e^5 + 288*a^20*b^3*c
^4*d^21*e^6 - 192*a^20*b^4*c^3*d^20*e^7 + 32*a^20*b^5*c^2*d^19*e^8 + 384*a^
21*b^2*c^4*d^20*e^7 - 288*a^21*b^3*c^3*d^19*e^8 - 32*a^21*b^4*c^2*d^18*e^9
+ 256*a^22*b^2*c^3*d^18*e^9 + 128*a^19*b*c^7*d^25*e^2 - 1152*a^20*b*c^6*d^2
3*e^4 - 640*a^21*b*c^5*d^21*e^6 + 640*a^22*b*c^4*d^19*e^8) - 64*a^18*c^8*d^
24*e^2 + 128*a^19*c^7*d^22*e^4 + 192*a^20*c^6*d^20*e^6 - 256*a^21*c^5*d^18*
e^8 - 256*a^22*c^4*d^16*e^10 - 16*a^16*b^4*c^6*d^24*e^2 + 64*a^16*b^5*c^5*d
^23*e^3 - 96*a^16*b^6*c^4*d^22*e^4 + 64*a^16*b^7*c^3*d^21*e^5 - 16*a^16*b^8
*c^2*d^20*e^6 + 80*a^17*b^2*c^7*d^24*e^2 - 368*a^17*b^3*c^6*d^23*e^3 + 608*
a^17*b^4*c^5*d^22*e^4 - 416*a^17*b^5*c^4*d^21*e^5 + 80*a^17*b^6*c^3*d^20*e^
6 + 16*a^17*b^7*c^2*d^19*e^7 - 928*a^18*b^2*c^6*d^22*e^4 + 640*a^18*b^3*c^5
*d^21*e^5 + 32*a^18*b^4*c^4*d^20*e^6 - 128*a^18*b^5*c^3*d^19*e^7 - 432*a^19
*b^2*c^5*d^20*e^6 + 304*a^19*b^3*c^4*d^19*e^7 - 16*a^19*b^4*c^3*d^18*e^8 +
16*a^19*b^5*c^2*d^17*e^9 + 128*a^20*b^2*c^4*d^18*e^8 - 128*a^20*b^3*c^3*d^1
7*e^9 - 16*a^20*b^4*c^2*d^16*e^10 + 128*a^21*b^2*c^3*d^16*e^10 + 448*a^18*b
*c^7*d^23*e^3 - 192*a^20*b*c^5*d^19*e^7 + 256*a^21*b*c^4*d^17*e^9) - x*(16*
a^16*c^9*d^23*e^2 + 32*a^17*c^8*d^21*e^4 - 112*a^18*c^7*d^19*e^6 - 128*a^20
*c^5*d^15*e^10 + 8*a^14*b^4*c^7*d^23*e^2 - 16*a^14*b^5*c^6*d^22*e^3 + 8*a^1

4*b^6*c^5*d^21*e^4 + 8*a^14*b^7*c^4*d^20*e^5 - 16*a^14*b^8*c^3*d^19*e^6 + 8*a^14*b^9*c^2*d^18*e^7 - 32*a^15*b^2*c^8*d^23*e^2 + 64*a^15*b^3*c^7*d^22*e^3 - 16*a^15*b^4*c^6*d^21*e^4 - 88*a^15*b^5*c^5*d^20*e^5 + 160*a^15*b^6*c^4*d^19*e^6 - 88*a^15*b^7*c^3*d^18*e^7 - 48*a^16*b^2*c^7*d^21*e^4 + 264*a^16*b^3*c^6*d^20*e^5 - 520*a^16*b^4*c^5*d^19*e^6 + 336*a^16*b^5*c^4*d^18*e^7 + 576*a^17*b^2*c^6*d^19*e^6 - 504*a^17*b^3*c^5*d^18*e^7 + 8*a^18*b^3*c^4*d^16*e^9 - 16*a^18*b^4*c^3*d^15*e^10 + 8*a^18*b^5*c^2*d^14*e^11 + 96*a^19*b^2*c^4*d^15*e^10 - 56*a^19*b^3*c^3*d^14*e^11 - 32*a^16*b*c^8*d^22*e^3 - 192*a^17*b*c^7*d^20*e^5 + 224*a^18*b*c^6*d^18*e^7 - 32*a^19*b*c^5*d^16*e^9 + 96*a^20*b*c^4*d^14*e^11))*(-(b^9*e^2 + b^7*c^2*d^2 - b^6*e^2*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c^3*d^2 - 20*a^3*b*c^5*d^2 + 28*a^4*b*c^4*e^2 - 2*b^8*c*d*e + 25*a^2*b^3*c^4*d^2 - a^2*c^4*d^2*(-(4*a*c - b^2)^3)^(1/2) + 42*a^2*b^5*c^2*e^2 - 63*a^3*b^3*c^3*e^2 + a^3*c^3*e^2*(-(4*a*c - b^2)^3)^(1/2) - b^4*c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) - 11*a*b^7*c*e^2 - 16*a^4*c^5*d*e + 20*a*b^6*c^2*d*e + 2*b^5*c*d*e*(-(4*a*c - b^2)^3)^(1/2) - 6*a^2*b^2*c^2*e^2*(-(4*a*c - b^2)^3)^(1/2) + 5*a*b^4*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - 66*a^2*b^4*c^3*d*e + 76*a^3*b^2*c^4*d*e + 3*a*b^2*c^3*d^2*(-(4*a*c - b^2)^3)^(1/2) - 8*a*b^3*c^2*d*e*(-(4*a*c - b^2)^3)^(1/2) + 6*a^2*b*c^3*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^7*b^4*e^4 + 16*a^7*c^4*d^4 + 16*a^9*c^2*e^4 - 8*a^8*b^2*c*e^4 - 2*a^6*b^5*d*e^3 + a^5*b^4*c^2*d^4 - 8*a^6*b^2*c^3*d^4 + a^5*b^6*d^2*e^2 + 32*a^8*c^3*d^2*e^2 - 2*a^5*b^5*c*d^3*e - 32*a^7*b*c^3*d^3*e + 16*a^7*b^3*c*d*e^3 - 32*a^8*b*c^2*d*e^3 + 16*a^6*b^3*c^2*d^3*e - 6*a^6*b^4*c*d^2*e^2)))^(1/2) - 4*a^15*c^9*d^21*e^3 - 4*a^16*c^8*d^19*e^5 + 48*a^18*c^6*d^15*e^9 - 4*a^14*b^2*c^8*d^21*e^3 - 4*a^14*b^7*c^3*d^16*e^8 + 4*a^14*b^8*c^2*d^15*e^9 + 36*a^15*b^5*c^4*d^16*e^8 - 44*a^15*b^6*c^3*d^15*e^9 + 4*a^15*b^7*c^2*d^14*e^10 - 100*a^16*b^3*c^5*d^16*e^8 + 160*a^16*b^4*c^4*d^15*e^9 - 32*a^16*b^5*c^3*d^14*e^10 - 204*a^17*b^2*c^5*d^15*e^9 + 76*a^17*b^3*c^4*d^14*e^10 + 4*a^14*b*c^9*d^22*e^2 + 8*a^15*b*c^8*d^20*e^4 + 80*a^17*b*c^6*d^16*e^8 - 48*a^18*b*c^5*d^14*e^10) - x*(2*a^14*c^9*d^18*e^5 + 4*a^16*c^7*d^14*e^9 + 2*a^14*b^4*c^5*d^14*e^9 - 8*a^15*b^2*c^6*d^14*e^9))*(-(b^9*e^2 + b^7*c^2*d^2 - b^6*e^2*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c^3*d^2 - 20*a^3*b*c^5*d^2 + 28*a^4*b*c^4*e^2 - 2*b^8*c*d*e + 25*a^2*b^3*c^4*d^2 - a^2*c^4*d^2*(-(4*a*c - b^2)^3)^(1/2) + 42*a^2*b^5*c^2*e^2 - 63*a^3*b^3*c^3*e^2 + a^3*c^3*e^2*(-(4*a*c - b^2)^3)^(1/2) - b^4*c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) - 11*a*b^7*c*e^2 - 16*a^4*c^5*d*e + 20*a*b^6*c^2*d*e + 2*b^5*c*d*e*(-(4*a*c - b^2)^3)^(1/2) - 6*a^2*b^2*c^2*e^2*(-(4*a*c - b^2)^3)^(1/2) + 5*a*b^4*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - 66*a^2*b^4*c^3*d*e + 76*a^3*b^2*c^4*d*e + 3*a*b^2*c^3*d^2*(-(4*a*c - b^2)^3)^(1/2) - 8*a*b^3*c^2*d*e*(-(4*a*c - b^2)^3)^(1/2) + 6*a^2*b*c^3*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^7*b^4*e^4 + 16*a^7*c^4*d^4 + 16*a^9*c^2*e^4 - 8*a^8*b^2*c*e^4 - 2*a^6*b^5*d*e^3 + a^5*b^4*c^2*d^4 - 8*a^6*b^2*c^3*d^4 + a^5*b^6*d^2*e^2 + 32*a^8*c^3*d^2*e^2 - 2*a^5*b^5*c*d^3*e - 32*a^7*b*c^3*d^3*e + 16*a^7*b^3*c*d*e^3 - 32*a^8*b*c^2*d*e^3 + 16*a^6*b^3*c^2*d^3*e - 6*a^6*b^4*c*d^2*e^2)))^(1/2)*1i + ((-(b^9*e^2 + b^7*c^2*d^2 - b^6*e^2*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c^3*d^2 - 20*a^3*b*c^5*d^2 + 28*a^4*b*c^4*e^2 - 2*b^8*c*d*e + 25*a^2*b^3*c^4*d^2 - a^2*c^4*d^2*(-(4*a*c - b^2)^3)^(1/

2) + 42*a^2*b^5*c^2*e^2 - 63*a^3*b^3*c^3*e^2 + a^3*c^3*e^2*(-(4*a*c - b^2)^
3)^(1/2) - b^4*c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) - 11*a*b^7*c*e^2 - 16*a^4*c
^5*d*e + 20*a*b^6*c^2*d*e + 2*b^5*c*d*e*(-(4*a*c - b^2)^3)^(1/2) - 6*a^2*b^
2*c^2*e^2*(-(4*a*c - b^2)^3)^(1/2) + 5*a*b^4*c*e^2*(-(4*a*c - b^2)^3)^(1/2)
 - 66*a^2*b^4*c^3*d*e + 76*a^3*b^2*c^4*d*e + 3*a*b^2*c^3*d^2*(-(4*a*c - b^2
)^3)^(1/2) - 8*a*b^3*c^2*d*e*(-(4*a*c - b^2)^3)^(1/2) + 6*a^2*b*c^3*d*e*(-(
4*a*c - b^2)^3)^(1/2))/(8*(a^7*b^4*e^4 + 16*a^7*c^4*d^4 + 16*a^9*c^2*e^4 -
8*a^8*b^2*c*e^4 - 2*a^6*b^5*d*e^3 + a^5*b^4*c^2*d^4 - 8*a^6*b^2*c^3*d^4 + a
^5*b^6*d^2*e^2 + 32*a^8*c^3*d^2*e^2 - 2*a^5*b^5*c*d^3*e - 32*a^7*b*c^3*d^3*
e + 16*a^7*b^3*c*d*e^3 - 32*a^8*b*c^2*d*e^3 + 16*a^6*b^3*c^2*d^3*e - 6*a^6*
b^4*c*d^2*e^2)))^(1/2)*(((-(b^9*e^2 + b^7*c^2*d^2 - b^6*e^2*(-(4*a*c - b^2)
^3)^(1/2) - 9*a*b^5*c^3*d^2 - 20*a^3*b*c^5*d^2 + 28*a^4*b*c^4*e^2 - 2*b^8*c
*d*e + 25*a^2*b^3*c^4*d^2 - a^2*c^4*d^2*(-(4*a*c - b^2)^3)^(1/2) + 42*a^2*b
^5*c^2*e^2 - 63*a^3*b^3*c^3*e^2 + a^3*c^3*e^2*(-(4*a*c - b^2)^3)^(1/2) - b^
4*c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) - 11*a*b^7*c*e^2 - 16*a^4*c^5*d*e + 20*a
*b^6*c^2*d*e + 2*b^5*c*d*e*(-(4*a*c - b^2)^3)^(1/2) - 6*a^2*b^2*c^2*e^2*(-(
4*a*c - b^2)^3)^(1/2) + 5*a*b^4*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - 66*a^2*b^4
*c^3*d*e + 76*a^3*b^2*c^4*d*e + 3*a*b^2*c^3*d^2*(-(4*a*c - b^2)^3)^(1/2) -
8*a*b^3*c^2*d*e*(-(4*a*c - b^2)^3)^(1/2) + 6*a^2*b*c^3*d*e*(-(4*a*c - b^2)^
3)^(1/2))/(8*(a^7*b^4*e^4 + 16*a^7*c^4*d^4 + 16*a^9*c^2*e^4 - 8*a^8*b^2*c*e
^4 - 2*a^6*b^5*d*e^3 + a^5*b^4*c^2*d^4 - 8*a^6*b^2*c^3*d^4 + a^5*b^6*d^2*e^
2 + 32*a^8*c^3*d^2*e^2 - 2*a^5*b^5*c*d^3*e - 32*a^7*b*c^3*d^3*e + 16*a^7*b^
3*c*d*e^3 - 32*a^8*b*c^2*d*e^3 + 16*a^6*b^3*c^2*d^3*e - 6*a^6*b^4*c*d^2*e^2
)))^(1/2)*(x*(-(b^9*e^2 + b^7*c^2*d^2 - b^6*e^2*(-(4*a*c - b^2)^3)^(1/2) -
9*a*b^5*c^3*d^2 - 20*a^3*b*c^5*d^2 + 28*a^4*b*c^4*e^2 - 2*b^8*c*d*e + 25*a^
2*b^3*c^4*d^2 - a^2*c^4*d^2*(-(4*a*c - b^2)^3)^(1/2) + 42*a^2*b^5*c^2*e^2 -
 63*a^3*b^3*c^3*e^2 + a^3*c^3*e^2*(-(4*a*c - b^2)^3)^(1/2) - b^4*c^2*d^2*(-
(4*a*c - b^2)^3)^(1/2) - 11*a*b^7*c*e^2 - 16*a^4*c^5*d*e + 20*a*b^6*c^2*d*e
 + 2*b^5*c*d*e*(-(4*a*c - b^2)^3)^(1/2) - 6*a^2*b^2*c^2*e^2*(-(4*a*c - b^2)
^3)^(1/2) + 5*a*b^4*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - 66*a^2*b^4*c^3*d*e + 7
6*a^3*b^2*c^4*d*e + 3*a*b^2*c^3*d^2*(-(4*a*c - b^2)^3)^(1/2) - 8*a*b^3*c^2*
d*e*(-(4*a*c - b^2)^3)^(1/2) + 6*a^2*b*c^3*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8
*(a^7*b^4*e^4 + 16*a^7*c^4*d^4 + 16*a^9*c^2*e^4 - 8*a^8*b^2*c*e^4 - 2*a^6*b
^5*d*e^3 + a^5*b^4*c^2*d^4 - 8*a^6*b^2*c^3*d^4 + a^5*b^6*d^2*e^2 + 32*a^8*c
^3*d^2*e^2 - 2*a^5*b^5*c*d^3*e - 32*a^7*b*c^3*d^3*e + 16*a^7*b^3*c*d*e^3 -
32*a^8*b*c^2*d*e^3 + 16*a^6*b^3*c^2*d^3*e - 6*a^6*b^4*c*d^2*e^2)))^(1/2)*(5
12*a^20*c^7*d^24*e^3 + 512*a^21*c^6*d^22*e^5 - 512*a^22*c^5*d^20*e^7 - 512*
a^23*c^4*d^18*e^9 - 32*a^18*b^3*c^6*d^25*e^2 + 128*a^18*b^4*c^5*d^24*e^3 -
192*a^18*b^5*c^4*d^23*e^4 + 128*a^18*b^6*c^3*d^22*e^5 - 32*a^18*b^7*c^2*d^2
1*e^6 - 640*a^19*b^2*c^6*d^24*e^3 + 1056*a^19*b^3*c^5*d^23*e^4 - 672*a^19*b
^4*c^4*d^22*e^5 + 96*a^19*b^5*c^3*d^21*e^6 + 32*a^19*b^6*c^2*d^20*e^7 + 512
*a^20*b^2*c^5*d^22*e^5 + 288*a^20*b^3*c^4*d^21*e^6 - 192*a^20*b^4*c^3*d^20*
e^7 + 32*a^20*b^5*c^2*d^19*e^8 + 384*a^21*b^2*c^4*d^20*e^7 - 288*a^21*b^3*c
^3*d^19*e^8 - 32*a^21*b^4*c^2*d^18*e^9 + 256*a^22*b^2*c^3*d^18*e^9 + 128*a^
19*b*c^7*d^25*e^2 - 1152*a^20*b*c^6*d^23*e^4 - 640*a^21*b*c^5*d^21*e^6 + 64

0*a^22*b*c^4*d^19*e^8) + 64*a^18*c^8*d^24*e^2 - 128*a^19*c^7*d^22*e^4 - 192
*a^20*c^6*d^20*e^6 + 256*a^21*c^5*d^18*e^8 + 256*a^22*c^4*d^16*e^10 + 16*a^
16*b^4*c^6*d^24*e^2 - 64*a^16*b^5*c^5*d^23*e^3 + 96*a^16*b^6*c^4*d^22*e^4 -
64*a^16*b^7*c^3*d^21*e^5 + 16*a^16*b^8*c^2*d^20*e^6 - 80*a^17*b^2*c^7*d^24
*e^2 + 368*a^17*b^3*c^6*d^23*e^3 - 608*a^17*b^4*c^5*d^22*e^4 + 416*a^17*b^5
*c^4*d^21*e^5 - 80*a^17*b^6*c^3*d^20*e^6 - 16*a^17*b^7*c^2*d^19*e^7 + 928*a
^18*b^2*c^6*d^22*e^4 - 640*a^18*b^3*c^5*d^21*e^5 - 32*a^18*b^4*c^4*d^20*e^6
+ 128*a^18*b^5*c^3*d^19*e^7 + 432*a^19*b^2*c^5*d^20*e^6 - 304*a^19*b^3*c^4
*d^19*e^7 + 16*a^19*b^4*c^3*d^18*e^8 - 16*a^19*b^5*c^2*d^17*e^9 - 128*a^20*
b^2*c^4*d^18*e^8 + 128*a^20*b^3*c^3*d^17*e^9 + 16*a^20*b^4*c^2*d^16*e^10 -
128*a^21*b^2*c^3*d^16*e^10 - 448*a^18*b*c^7*d^23*e^3 + 192*a^20*b*c^5*d^19*
e^7 - 256*a^21*b*c^4*d^17*e^9) - x*(16*a^16*c^9*d^23*e^2 + 32*a^17*c^8*d^21
*e^4 - 112*a^18*c^7*d^19*e^6 - 128*a^20*c^5*d^15*e^10 + 8*a^14*b^4*c^7*d^23
*e^2 - 16*a^14*b^5*c^6*d^22*e^3 + 8*a^14*b^6*c^5*d^21*e^4 + 8*a^14*b^7*c^4*
d^20*e^5 - 16*a^14*b^8*c^3*d^19*e^6 + 8*a^14*b^9*c^2*d^18*e^7 - 32*a^15*b^2
*c^8*d^23*e^2 + 64*a^15*b^3*c^7*d^22*e^3 - 16*a^15*b^4*c^6*d^21*e^4 - 88*a^
15*b^5*c^5*d^20*e^5 + 160*a^15*b^6*c^4*d^19*e^6 - 88*a^15*b^7*c^3*d^18*e^7
- 48*a^16*b^2*c^7*d^21*e^4 + 264*a^16*b^3*c^6*d^20*e^5 - 520*a^16*b^4*c^5*d
^19*e^6 + 336*a^16*b^5*c^4*d^18*e^7 + 576*a^17*b^2*c^6*d^19*e^6 - 504*a^17*
b^3*c^5*d^18*e^7 + 8*a^18*b^3*c^4*d^16*e^9 - 16*a^18*b^4*c^3*d^15*e^10 + 8*
a^18*b^5*c^2*d^14*e^11 + 96*a^19*b^2*c^4*d^15*e^10 - 56*a^19*b^3*c^3*d^14*e
^11 - 32*a^16*b*c^8*d^22*e^3 - 192*a^17*b*c^7*d^20*e^5 + 224*a^18*b*c^6*d^1
8*e^7 - 32*a^19*b*c^5*d^16*e^9 + 96*a^20*b*c^4*d^14*e^11))*(-(b^9*e^2 + b^7
*c^2*d^2 - b^6*e^2*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c^3*d^2 - 20*a^3*b*c^
5*d^2 + 28*a^4*b*c^4*e^2 - 2*b^8*c*d*e + 25*a^2*b^3*c^4*d^2 - a^2*c^4*d^2*(
-(4*a*c - b^2)^3)^(1/2) + 42*a^2*b^5*c^2*e^2 - 63*a^3*b^3*c^3*e^2 + a^3*c^3
e^2(-(4*a*c - b^2)^3)^(1/2) - b^4*c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) - 11*a
*b^7*c*e^2 - 16*a^4*c^5*d*e + 20*a*b^6*c^2*d*e + 2*b^5*c*d*e*(-(4*a*c - b^2
)^3)^(1/2) - 6*a^2*b^2*c^2*e^2*(-(4*a*c - b^2)^3)^(1/2) + 5*a*b^4*c*e^2*(-(
4*a*c - b^2)^3)^(1/2) - 66*a^2*b^4*c^3*d*e + 76*a^3*b^2*c^4*d*e + 3*a*b^2*c
^3*d^2*(-(4*a*c - b^2)^3)^(1/2) - 8*a*b^3*c^2*d*e*(-(4*a*c - b^2)^3)^(1/2)
+ 6*a^2*b*c^3*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^7*b^4*e^4 + 16*a^7*c^4*d^
4 + 16*a^9*c^2*e^4 - 8*a^8*b^2*c*e^4 - 2*a^6*b^5*d*e^3 + a^5*b^4*c^2*d^4 -
8*a^6*b^2*c^3*d^4 + a^5*b^6*d^2*e^2 + 32*a^8*c^3*d^2*e^2 - 2*a^5*b^5*c*d^3*
e - 32*a^7*b*c^3*d^3*e + 16*a^7*b^3*c*d*e^3 - 32*a^8*b*c^2*d*e^3 + 16*a^6*b
^3*c^2*d^3*e - 6*a^6*b^4*c*d^2*e^2)))^(1/2) + 4*a^15*c^9*d^21*e^3 + 4*a^16*
c^8*d^19*e^5 - 48*a^18*c^6*d^15*e^9 + 4*a^14*b^2*c^8*d^21*e^3 + 4*a^14*b^7*
c^3*d^16*e^8 - 4*a^14*b^8*c^2*d^15*e^9 - 36*a^15*b^5*c^4*d^16*e^8 + 44*a^15
*b^6*c^3*d^15*e^9 - 4*a^15*b^7*c^2*d^14*e^10 + 100*a^16*b^3*c^5*d^16*e^8 -
160*a^16*b^4*c^4*d^15*e^9 + 32*a^16*b^5*c^3*d^14*e^10 + 204*a^17*b^2*c^5*d^
15*e^9 - 76*a^17*b^3*c^4*d^14*e^10 - 4*a^14*b*c^9*d^22*e^2 - 8*a^15*b*c^8*d
^20*e^4 - 80*a^17*b*c^6*d^16*e^8 + 48*a^18*b*c^5*d^14*e^10) - x*(2*a^14*c^9
*d^18*e^5 + 4*a^16*c^7*d^14*e^9 + 2*a^14*b^4*c^5*d^14*e^9 - 8*a^15*b^2*c^6*
d^14*e^9))*(-(b^9*e^2 + b^7*c^2*d^2 - b^6*e^2*(-(4*a*c - b^2)^3)^(1/2) - 9*
a*b^5*c^3*d^2 - 20*a^3*b*c^5*d^2 + 28*a^4*b*c^4*e^2 - 2*b^8*c*d*e + 25*a^2*

b^3*c^4*d^2 - a^2*c^4*d^2*(-(4*a*c - b^2)^3)^(1/2) + 42*a^2*b^5*c^2*e^2 - 6
3*a^3*b^3*c^3*e^2 + a^3*c^3*e^2*(-(4*a*c - b^2)^3)^(1/2) - b^4*c^2*d^2*(-(4
*a*c - b^2)^3)^(1/2) - 11*a*b^7*c*e^2 - 16*a^4*c^5*d*e + 20*a*b^6*c^2*d*e +
 2*b^5*c*d*e*(-(4*a*c - b^2)^3)^(1/2) - 6*a^2*b^2*c^2*e^2*(-(4*a*c - b^2)^3
)^(1/2) + 5*a*b^4*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - 66*a^2*b^4*c^3*d*e + 76*
a^3*b^2*c^4*d*e + 3*a*b^2*c^3*d^2*(-(4*a*c - b^2)^3)^(1/2) - 8*a*b^3*c^2*d*
e*(-(4*a*c - b^2)^3)^(1/2) + 6*a^2*b*c^3*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(
a^7*b^4*e^4 + 16*a^7*c^4*d^4 + 16*a^9*c^2*e^4 - 8*a^8*b^2*c*e^4 - 2*a^6*b^5
*d*e^3 + a^5*b^4*c^2*d^4 - 8*a^6*b^2*c^3*d^4 + a^5*b^6*d^2*e^2 + 32*a^8*c^3
*d^2*e^2 - 2*a^5*b^5*c*d^3*e - 32*a^7*b*c^3*d^3*e + 16*a^7*b^3*c*d*e^3 - 32
*a^8*b*c^2*d*e^3 + 16*a^6*b^3*c^2*d^3*e - 6*a^6*b^4*c*d^2*e^2)))^(1/2)*1i)/
(((-(b^9*e^2 + b^7*c^2*d^2 - b^6*e^2*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c^3
*d^2 - 20*a^3*b*c^5*d^2 + 28*a^4*b*c^4*e^2 - 2*b^8*c*d*e + 25*a^2*b^3*c^4*d
^2 - a^2*c^4*d^2*(-(4*a*c - b^2)^3)^(1/2) + 42*a^2*b^5*c^2*e^2 - 63*a^3*b^3
*c^3*e^2 + a^3*c^3*e^2*(-(4*a*c - b^2)^3)^(1/2) - b^4*c^2*d^2*(-(4*a*c - b^
2)^3)^(1/2) - 11*a*b^7*c*e^2 - 16*a^4*c^5*d*e + 20*a*b^6*c^2*d*e + 2*b^5*c*
d*e*(-(4*a*c - b^2)^3)^(1/2) - 6*a^2*b^2*c^2*e^2*(-(4*a*c - b^2)^3)^(1/2) +
 5*a*b^4*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - 66*a^2*b^4*c^3*d*e + 76*a^3*b^2*c
^4*d*e + 3*a*b^2*c^3*d^2*(-(4*a*c - b^2)^3)^(1/2) - 8*a*b^3*c^2*d*e*(-(4*a*
c - b^2)^3)^(1/2) + 6*a^2*b*c^3*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^7*b^4*e
^4 + 16*a^7*c^4*d^4 + 16*a^9*c^2*e^4 - 8*a^8*b^2*c*e^4 - 2*a^6*b^5*d*e^3 +
a^5*b^4*c^2*d^4 - 8*a^6*b^2*c^3*d^4 + a^5*b^6*d^2*e^2 + 32*a^8*c^3*d^2*e^2
 - 2*a^5*b^5*c*d^3*e - 32*a^7*b*c^3*d^3*e + 16*a^7*b^3*c*d*e^3 - 32*a^8*b*c^
2*d*e^3 + 16*a^6*b^3*c^2*d^3*e - 6*a^6*b^4*c*d^2*e^2)))^(1/2)*(((-(b^9*e^2
 + b^7*c^2*d^2 - b^6*e^2*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c^3*d^2 - 20*a^3
*b*c^5*d^2 + 28*a^4*b*c^4*e^2 - 2*b^8*c*d*e + 25*a^2*b^3*c^4*d^2 - a^2*c^4*
d^2*(-(4*a*c - b^2)^3)^(1/2) + 42*a^2*b^5*c^2*e^2 - 63*a^3*b^3*c^3*e^2 + a^
3*c^3*e^2*(-(4*a*c - b^2)^3)^(1/2) - b^4*c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) -
 11*a*b^7*c*e^2 - 16*a^4*c^5*d*e + 20*a*b^6*c^2*d*e + 2*b^5*c*d*e*(-(4*a*c
 - b^2)^3)^(1/2) - 6*a^2*b^2*c^2*e^2*(-(4*a*c - b^2)^3)^(1/2) + 5*a*b^4*c*e^
2*(-(4*a*c - b^2)^3)^(1/2) - 66*a^2*b^4*c^3*d*e + 76*a^3*b^2*c^4*d*e + 3*a*
b^2*c^3*d^2*(-(4*a*c - b^2)^3)^(1/2) - 8*a*b^3*c^2*d*e*(-(4*a*c - b^2)^3)^(
1/2) + 6*a^2*b*c^3*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^7*b^4*e^4 + 16*a^7*c
^4*d^4 + 16*a^9*c^2*e^4 - 8*a^8*b^2*c*e^4 - 2*a^6*b^5*d*e^3 + a^5*b^4*c^2*d
^4 - 8*a^6*b^2*c^3*d^4 + a^5*b^6*d^2*e^2 + 32*a^8*c^3*d^2*e^2 - 2*a^5*b^5*c
*d^3*e - 32*a^7*b*c^3*d^3*e + 16*a^7*b^3*c*d*e^3 - 32*a^8*b*c^2*d*e^3 + 16*
a^6*b^3*c^2*d^3*e - 6*a^6*b^4*c*d^2*e^2)))^(1/2)*(x*(-(b^9*e^2 + b^7*c^2*d^
2 - b^6*e^2*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c^3*d^2 - 20*a^3*b*c^5*d^2 +
 28*a^4*b*c^4*e^2 - 2*b^8*c*d*e + 25*a^2*b^3*c^4*d^2 - a^2*c^4*d^2*(-(4*a*c
 - b^2)^3)^(1/2) + 42*a^2*b^5*c^2*e^2 - 63*a^3*b^3*c^3*e^2 + a^3*c^3*e^2*(-
(4*a*c - b^2)^3)^(1/2) - b^4*c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) - 11*a*b^7*c*
e^2 - 16*a^4*c^5*d*e + 20*a*b^6*c^2*d*e + 2*b^5*c*d*e*(-(4*a*c - b^2)^3)^(1
/2) - 6*a^2*b^2*c^2*e^2*(-(4*a*c - b^2)^3)^(1/2) + 5*a*b^4*c*e^2*(-(4*a*c -
 b^2)^3)^(1/2) - 66*a^2*b^4*c^3*d*e + 76*a^3*b^2*c^4*d*e + 3*a*b^2*c^3*d^2*
(-(4*a*c - b^2)^3)^(1/2) - 8*a*b^3*c^2*d*e*(-(4*a*c - b^2)^3)^(1/2) + 6*a^2

*b*c^3*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^7*b^4*e^4 + 16*a^7*c^4*d^4 + 16*a^9*c^2*e^4 - 8*a^8*b^2*c*e^4 - 2*a^6*b^5*d*e^3 + a^5*b^4*c^2*d^4 - 8*a^6*b^2*c^3*d^4 + a^5*b^6*d^2*e^2 + 32*a^8*c^3*d^2*e^2 - 2*a^5*b^5*c*d^3*e - 32*a^7*b*c^3*d^3*e + 16*a^7*b^3*c*d*e^3 - 32*a^8*b*c^2*d*e^3 + 16*a^6*b^3*c^2*d^3*e - 6*a^6*b^4*c*d^2*e^2)))^(1/2)*(512*a^20*c^7*d^24*e^3 + 512*a^21*c^6*d^22*e^5 - 512*a^22*c^5*d^20*e^7 - 512*a^23*c^4*d^18*e^9 - 32*a^18*b^3*c^6*d^25*e^2 + 128*a^18*b^4*c^5*d^24*e^3 - 192*a^18*b^5*c^4*d^23*e^4 + 128*a^18*b^6*c^3*d^22*e^5 - 32*a^18*b^7*c^2*d^21*e^6 - 640*a^19*b^2*c^6*d^24*e^3 + 1056*a^19*b^3*c^5*d^23*e^4 - 672*a^19*b^4*c^4*d^22*e^5 + 96*a^19*b^5*c^3*d^21*e^6 + 32*a^19*b^6*c^2*d^20*e^7 + 512*a^20*b^2*c^5*d^22*e^5 + 288*a^20*b^3*c^4*d^21*e^6 - 192*a^20*b^4*c^3*d^20*e^7 + 32*a^20*b^5*c^2*d^19*e^8 + 384*a^21*b^2*c^4*d^20*e^7 - 288*a^21*b^3*c^3*d^19*e^8 - 32*a^21*b^4*c^2*d^18*e^9 + 256*a^22*b^2*c^3*d^18*e^9 + 128*a^19*b*c^7*d^25*e^2 - 1152*a^20*b*c^6*d^23*e^4 - 640*a^21*b*c^5*d^21*e^6 + 640*a^22*b*c^4*d^19*e^8) + 64*a^18*c^8*d^24*e^2 - 128*a^19*c^7*d^22*e^4 - 192*a^20*c^6*d^20*e^6 + 256*a^21*c^5*d^18*e^8 + 256*a^22*c^4*d^16*e^10 + 16*a^16*b^4*c^6*d^24*e^2 - 64*a^16*b^5*c^5*d^23*e^3 + 96*a^16*b^6*c^4*d^22*e^4 - 64*a^16*b^7*c^3*d^21*e^5 + 16*a^16*b^8*c^2*d^20*e^6 - 80*a^17*b^2*c^7*d^24*e^2 + 368*a^17*b^3*c^6*d^23*e^3 - 608*a^17*b^4*c^5*d^22*e^4 + 416*a^17*b^5*c^4*d^21*e^5 - 80*a^17*b^6*c^3*d^20*e^6 - 16*a^17*b^7*c^2*d^19*e^7 + 928*a^18*b^2*c^6*d^22*e^4 - 640*a^18*b^3*c^5*d^21*e^5 - 32*a^18*b^4*c^4*d^20*e^6 + 128*a^18*b^5*c^3*d^19*e^7 + 432*a^19*b^2*c^5*d^20*e^6 - 304*a^19*b^3*c^4*d^19*e^7 + 16*a^19*b^4*c^3*d^18*e^8 - 16*a^19*b^5*c^2*d^17*e^9 - 128*a^20*b^2*c^4*d^18*e^8 + 128*a^20*b^3*c^3*d^17*e^9 + 16*a^20*b^4*c^2*d^16*e^10 - 128*a^21*b^2*c^3*d^16*e^10 - 448*a^18*b*c^7*d^23*e^3 + 192*a^20*b*c^5*d^19*e^7 - 256*a^21*b*c^4*d^17*e^9) - x*(16*a^16*c^9*d^23*e^2 + 32*a^17*c^8*d^21*e^4 - 112*a^18*c^7*d^19*e^6 - 128*a^20*c^5*d^15*e^10 + 8*a^14*b^4*c^7*d^23*e^2 - 16*a^14*b^5*c^6*d^22*e^3 + 8*a^14*b^6*c^5*d^21*e^4 + 8*a^14*b^7*c^4*d^20*e^5 - 16*a^14*b^8*c^3*d^19*e^6 + 8*a^14*b^9*c^2*d^18*e^7 - 32*a^15*b^2*c^8*d^23*e^2 + 64*a^15*b^3*c^7*d^22*e^3 - 16*a^15*b^4*c^6*d^21*e^4 - 88*a^15*b^5*c^5*d^20*e^5 + 160*a^15*b^6*c^4*d^19*e^6 - 88*a^15*b^7*c^3*d^18*e^7 - 48*a^16*b^2*c^7*d^21*e^4 + 264*a^16*b^3*c^6*d^20*e^5 - 520*a^16*b^4*c^5*d^19*e^6 + 336*a^16*b^5*c^4*d^18*e^7 + 576*a^17*b^2*c^6*d^19*e^6 - 504*a^17*b^3*c^5*d^18*e^7 + 8*a^18*b^3*c^4*d^16*e^9 - 16*a^18*b^4*c^3*d^15*e^10 + 8*a^18*b^5*c^2*d^14*e^11 + 96*a^19*b^2*c^4*d^15*e^10 - 56*a^19*b^3*c^3*d^14*e^11 - 32*a^16*b*c^8*d^22*e^3 - 192*a^17*b*c^7*d^20*e^5 + 224*a^18*b*c^6*d^18*e^7 - 32*a^19*b*c^5*d^16*e^9 + 96*a^20*b*c^4*d^14*e^11))*(-(b^9*e^2 + b^7*c^2*d^2 - b^6*e^2*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c^3*d^2 - 20*a^3*b*c^5*d^2 + 28*a^4*b*c^4*e^2 - 2*b^8*c*d*e + 25*a^2*b^3*c^4*d^2 - a^2*c^4*d^2*(-(4*a*c - b^2)^3)^(1/2) + 42*a^2*b^5*c^2*e^2 - 63*a^3*b^3*c^3*e^2 + a^3*c^3*e^2*(-(4*a*c - b^2)^3)^(1/2) - b^4*c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) - 11*a*b^7*c*e^2 - 16*a^4*c^5*d*e + 20*a*b^6*c^2*d*e + 2*b^5*c*d*e*(-(4*a*c - b^2)^3)^(1/2) - 6*a^2*b^2*c^2*e^2*(-(4*a*c - b^2)^3)^(1/2) + 5*a*b^4*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - 66*a^2*b^4*c^3*d*e + 76*a^3*b^2*c^4*d*e + 3*a*b^2*c^3*d^2*(-(4*a*c - b^2)^3)^(1/2) - 8*a*b^3*c^2*d*e*(-(4*a*c - b^2)^3)^(1/2) + 6*a^2*b*c^3*d*e*(-(4*a*c - b^2)^3)

^(1/2))/(8*(a^7*b^4*e^4 + 16*a^7*c^4*d^4 + 16*a^9*c^2*e^4 - 8*a^8*b^2*c*e^4 - 2*a^6*b^5*d*e^3 + a^5*b^4*c^2*d^4 - 8*a^6*b^2*c^3*d^4 + a^5*b^6*d^2*e^2 + 32*a^8*c^3*d^2*e^2 - 2*a^5*b^5*c*d^3*e - 32*a^7*b*c^3*d^3*e + 16*a^7*b^3*c*d*e^3 - 32*a^8*b*c^2*d*e^3 + 16*a^6*b^3*c^2*d^3*e - 6*a^6*b^4*c*d^2*e^2)))^(1/2) + 4*a^15*c^9*d^21*e^3 + 4*a^16*c^8*d^19*e^5 - 48*a^18*c^6*d^15*e^9 + 4*a^14*b^2*c^8*d^21*e^3 + 4*a^14*b^7*c^3*d^16*e^8 - 4*a^14*b^8*c^2*d^15*e^9 - 36*a^15*b^5*c^4*d^16*e^8 + 44*a^15*b^6*c^3*d^15*e^9 - 4*a^15*b^7*c^2*d^14*e^10 + 100*a^16*b^3*c^5*d^16*e^8 - 160*a^16*b^4*c^4*d^15*e^9 + 32*a^16*b^5*c^3*d^14*e^10 + 204*a^17*b^2*c^5*d^15*e^9 - 76*a^17*b^3*c^4*d^14*e^10 - 4*a^14*b*c^9*d^22*e^2 - 8*a^15*b*c^8*d^20*e^4 - 80*a^17*b*c^6*d^16*e^8 + 48*a^18*b*c^5*d^14*e^10) - x*(2*a^14*c^9*d^18*e^5 + 4*a^16*c^7*d^14*e^9 + 2*a^14*b^4*c^5*d^14*e^9 - 8*a^15*b^2*c^6*d^14*e^9))*(-(b^9*e^2 + b^7*c^2*d^2 - b^6*e^2*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c^3*d^2 - 20*a^3*b*c^5*d^2 + 28*a^4*b*c^4*e^2 - 2*b^8*c*d*e + 25*a^2*b^3*c^4*d^2 - a^2*c^4*d^2*(-(4*a*c - b^2)^3)^(1/2) + 42*a^2*b^5*c^2*e^2 - 63*a^3*b^3*c^3*e^2 + a^3*c^3*e^2*(-(4*a*c - b^2)^3)^(1/2) - b^4*c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) - 11*a*b^7*c*e^2 - 16*a^4*c^5*d*e + 20*a*b^6*c^2*d*e + 2*b^5*c*d*e*(-(4*a*c - b^2)^3)^(1/2) - 6*a^2*b^2*c^2*e^2*(-(4*a*c - b^2)^3)^(1/2) + 5*a*b^4*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - 66*a^2*b^4*c^3*d*e + 76*a^3*b^2*c^4*d*e + 3*a*b^2*c^3*d^2*(-(4*a*c - b^2)^3)^(1/2) - 8*a*b^3*c^2*d*e*(-(4*a*c - b^2)^3)^(1/2) + 6*a^2*b*c^3*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^7*b^4*e^4 + 16*a^7*c^4*d^4 + 16*a^9*c^2*e^4 - 8*a^8*b^2*c*e^4 - 2*a^6*b^5*d*e^3 + a^5*b^4*c^2*d^4 - 8*a^6*b^2*c^3*d^4 + a^5*b^6*d^2*e^2 + 32*a^8*c^3*d^2*e^2 - 2*a^5*b^5*c*d^3*e - 32*a^7*b*c^3*d^3*e + 16*a^7*b^3*c*d*e^3 - 32*a^8*b*c^2*d*e^3 + 16*a^6*b^3*c^2*d^3*e - 6*a^6*b^4*c*d^2*e^2)))^(1/2) - ((-(b^9*e^2 + b^7*c^2*d^2 - b^6*e^2*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c^3*d^2 - 20*a^3*b*c^5*d^2 + 28*a^4*b*c^4*e^2 - 2*b^8*c*d*e + 25*a^2*b^3*c^4*d^2 - a^2*c^4*d^2*(-(4*a*c - b^2)^3)^(1/2) + 42*a^2*b^5*c^2*e^2 - 63*a^3*b^3*c^3*e^2 + a^3*c^3*e^2*(-(4*a*c - b^2)^3)^(1/2) - b^4*c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) - 11*a*b^7*c*e^2 - 16*a^4*c^5*d*e + 20*a*b^6*c^2*d*e + 2*b^5*c*d*e*(-(4*a*c - b^2)^3)^(1/2) - 6*a^2*b^2*c^2*e^2*(-(4*a*c - b^2)^3)^(1/2) + 5*a*b^4*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - 66*a^2*b^4*c^3*d*e + 76*a^3*b^2*c^4*d*e + 3*a*b^2*c^3*d^2*(-(4*a*c - b^2)^3)^(1/2) - 8*a*b^3*c^2*d*e*(-(4*a*c - b^2)^3)^(1/2) + 6*a^2*b*c^3*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^7*b^4*e^4 + 16*a^7*c^4*d^4 + 16*a^9*c^2*e^4 - 8*a^8*b^2*c*e^4 - 2*a^6*b^5*d*e^3 + a^5*b^4*c^2*d^4 - 8*a^6*b^2*c^3*d^4 + a^5*b^6*d^2*e^2 + 32*a^8*c^3*d^2*e^2 - 2*a^5*b^5*c*d^3*e - 32*a^7*b*c^3*d^3*e + 16*a^7*b^3*c*d*e^3 - 32*a^8*b*c^2*d*e^3 + 16*a^6*b^3*c^2*d^3*e - 6*a^6*b^4*c*d^2*e^2)))^(1/2)*(((-(b^9*e^2 + b^7*c^2*d^2 - b^6*e^2*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c^3*d^2 - 20*a^3*b*c^5*d^2 + 28*a^4*b*c^4*e^2 - 2*b^8*c*d*e + 25*a^2*b^3*c^4*d^2 - a^2*c^4*d^2*(-(4*a*c - b^2)^3)^(1/2) + 42*a^2*b^5*c^2*e^2 - 63*a^3*b^3*c^3*e^2 + a^3*c^3*e^2*(-(4*a*c - b^2)^3)^(1/2) - b^4*c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) - 11*a*b^7*c*e^2 - 16*a^4*c^5*d*e + 20*a*b^6*c^2*d*e + 2*b^5*c*d*e*(-(4*a*c - b^2)^3)^(1/2) - 6*a^2*b^2*c^2*e^2*(-(4*a*c - b^2)^3)^(1/2) + 5*a*b^4*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - 66*a^2*b^4*c^3*d*e + 76*a^3*b^2*c^4*d*e + 3*a*b^2*c^3*d^2*(-(4*a*c - b^2)^3)^(1/2) -

8*a*b^3*c^2*d*e*(-(4*a*c - b^2)^3)^(1/2) + 6*a^2*b*c^3*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^7*b^4*e^4 + 16*a^7*c^4*d^4 + 16*a^9*c^2*e^4 - 8*a^8*b^2*c*e^4 - 2*a^6*b^5*d*e^3 + a^5*b^4*c^2*d^4 - 8*a^6*b^2*c^3*d^4 + a^5*b^6*d^2*e^2 + 32*a^8*c^3*d^2*e^2 - 2*a^5*b^5*c*d^3*e - 32*a^7*b*c^3*d^3*e + 16*a^7*b^3*c*d*e^3 - 32*a^8*b*c^2*d*e^3 + 16*a^6*b^3*c^2*d^3*e - 6*a^6*b^4*c*d^2*e^2)))^(1/2)*(x*(-(b^9*e^2 + b^7*c^2*d^2 - b^6*e^2*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c^3*d^2 - 20*a^3*b*c^5*d^2 + 28*a^4*b*c^4*e^2 - 2*b^8*c*d*e + 25*a^2*b^3*c^4*d^2 - a^2*c^4*d^2*(-(4*a*c - b^2)^3)^(1/2) + 42*a^2*b^5*c^2*e^2 - 63*a^3*b^3*c^3*e^2 + a^3*c^3*e^2*(-(4*a*c - b^2)^3)^(1/2) - b^4*c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) - 11*a*b^7*c*e^2 - 16*a^4*c^5*d*e + 20*a*b^6*c^2*d*e + 2*b^5*c*d*e*(-(4*a*c - b^2)^3)^(1/2) - 6*a^2*b^2*c^2*e^2*(-(4*a*c - b^2)^3)^(1/2) + 5*a*b^4*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - 66*a^2*b^4*c^3*d*e + 76*a^3*b^2*c^4*d*e + 3*a*b^2*c^3*d^2*(-(4*a*c - b^2)^3)^(1/2) - 8*a*b^3*c^2*d*e*(-(4*a*c - b^2)^3)^(1/2) + 6*a^2*b*c^3*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^7*b^4*e^4 + 16*a^7*c^4*d^4 + 16*a^9*c^2*e^4 - 8*a^8*b^2*c*e^4 - 2*a^6*b^5*d*e^3 + a^5*b^4*c^2*d^4 - 8*a^6*b^2*c^3*d^4 + a^5*b^6*d^2*e^2 + 32*a^8*c^3*d^2*e^2 - 2*a^5*b^5*c*d^3*e - 32*a^7*b*c^3*d^3*e + 16*a^7*b^3*c*d*e^3 - 32*a^8*b*c^2*d*e^3 + 16*a^6*b^3*c^2*d^3*e - 6*a^6*b^4*c*d^2*e^2)))^(1/2)*(512*a^20*c^7*d^24*e^3 + 512*a^21*c^6*d^22*e^5 - 512*a^22*c^5*d^20*e^7 - 512*a^23*c^4*d^18*e^9 - 32*a^18*b^3*c^6*d^25*e^2 + 128*a^18*b^4*c^5*d^24*e^3 - 192*a^18*b^5*c^4*d^23*e^4 + 128*a^18*b^6*c^3*d^22*e^5 - 32*a^18*b^7*c^2*d^21*e^6 - 640*a^19*b^2*c^6*d^24*e^3 + 1056*a^19*b^3*c^5*d^23*e^4 - 672*a^19*b^4*c^4*d^22*e^5 + 96*a^19*b^5*c^3*d^21*e^6 + 32*a^19*b^6*c^2*d^20*e^7 + 512*a^20*b^2*c^5*d^22*e^5 + 288*a^20*b^3*c^4*d^21*e^6 - 192*a^20*b^4*c^3*d^20*e^7 + 32*a^20*b^5*c^2*d^19*e^8 + 384*a^21*b^2*c^4*d^20*e^7 - 288*a^21*b^3*c^3*d^19*e^8 - 32*a^21*b^4*c^2*d^18*e^9 + 256*a^22*b^2*c^3*d^18*e^9 + 128*a^19*b*c^7*d^25*e^2 - 1152*a^20*b*c^6*d^23*e^4 - 640*a^21*b*c^5*d^21*e^6 + 640*a^22*b*c^4*d^19*e^8) - 64*a^18*c^8*d^24*e^2 + 128*a^19*c^7*d^22*e^4 + 192*a^20*c^6*d^20*e^6 - 256*a^21*c^5*d^18*e^8 - 256*a^22*c^4*d^16*e^10 - 16*a^16*b^4*c^6*d^24*e^2 + 64*a^16*b^5*c^5*d^23*e^3 - 96*a^16*b^6*c^4*d^22*e^4 + 64*a^16*b^7*c^3*d^21*e^5 - 16*a^16*b^8*c^2*d^20*e^6 + 80*a^17*b^2*c^7*d^24*e^2 - 368*a^17*b^3*c^6*d^23*e^3 + 608*a^17*b^4*c^5*d^22*e^4 - 416*a^17*b^5*c^4*d^21*e^5 + 80*a^17*b^6*c^3*d^20*e^6 + 16*a^17*b^7*c^2*d^19*e^7 - 928*a^18*b^2*c^6*d^22*e^4 + 640*a^18*b^3*c^5*d^21*e^5 + 32*a^18*b^4*c^4*d^20*e^6 - 128*a^18*b^5*c^3*d^19*e^7 - 432*a^19*b^2*c^5*d^20*e^6 + 304*a^19*b^3*c^4*d^19*e^7 - 16*a^19*b^4*c^3*d^18*e^8 + 16*a^19*b^5*c^2*d^17*e^9 + 128*a^20*b^2*c^4*d^18*e^8 - 128*a^20*b^3*c^3*d^17*e^9 - 16*a^20*b^4*c^2*d^16*e^10 + 128*a^21*b^2*c^3*d^16*e^10 + 448*a^18*b*c^7*d^23*e^3 - 192*a^20*b*c^5*d^19*e^7 + 256*a^21*b*c^4*d^17*e^9) - x*(16*a^16*c^9*d^23*e^2 + 32*a^17*c^8*d^21*e^4 - 112*a^18*c^7*d^19*e^6 - 128*a^20*c^5*d^15*e^10 + 8*a^14*b^4*c^7*d^23*e^2 - 16*a^14*b^5*c^6*d^22*e^3 + 8*a^14*b^6*c^5*d^21*e^4 + 8*a^14*b^7*c^4*d^20*e^5 - 16*a^14*b^8*c^3*d^19*e^6 + 8*a^14*b^9*c^2*d^18*e^7 - 32*a^15*b^2*c^8*d^23*e^2 + 64*a^15*b^3*c^7*d^22*e^3 - 16*a^15*b^4*c^6*d^21*e^4 - 88*a^15*b^5*c^5*d^20*e^5 + 160*a^15*b^6*c^4*d^19*e^6 - 88*a^15*b^7*c^3*d^18*e^7 - 48*a^16*b^2*c^7*d^21*e^4 + 264*a^16*b^3*c^6*d^20*e^5 - 520*a^16*b^4*c^5*d

^19*e^6 + 336*a^16*b^5*c^4*d^18*e^7 + 576*a^17*b^2*c^6*d^19*e^6 - 504*a^17*
b^3*c^5*d^18*e^7 + 8*a^18*b^3*c^4*d^16*e^9 - 16*a^18*b^4*c^3*d^15*e^10 + 8*
a^18*b^5*c^2*d^14*e^11 + 96*a^19*b^2*c^4*d^15*e^10 - 56*a^19*b^3*c^3*d^14*e
^11 - 32*a^16*b*c^8*d^22*e^3 - 192*a^17*b*c^7*d^20*e^5 + 224*a^18*b*c^6*d^1
8*e^7 - 32*a^19*b*c^5*d^16*e^9 + 96*a^20*b*c^4*d^14*e^11))*(-(b^9*e^2 + b^7
*c^2*d^2 - b^6*e^2*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c^3*d^2 - 20*a^3*b*c^
5*d^2 + 28*a^4*b*c^4*e^2 - 2*b^8*c*d*e + 25*a^2*b^3*c^4*d^2 - a^2*c^4*d^2*(
-(4*a*c - b^2)^3)^(1/2) + 42*a^2*b^5*c^2*e^2 - 63*a^3*b^3*c^3*e^2 + a^3*c^3
e^2(-(4*a*c - b^2)^3)^(1/2) - b^4*c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) - 11*a
*b^7*c*e^2 - 16*a^4*c^5*d*e + 20*a*b^6*c^2*d*e + 2*b^5*c*d*e*(-(4*a*c - b^2
)^3)^(1/2) - 6*a^2*b^2*c^2*e^2*(-(4*a*c - b^2)^3)^(1/2) + 5*a*b^4*c*e^2*(-(
4*a*c - b^2)^3)^(1/2) - 66*a^2*b^4*c^3*d*e + 76*a^3*b^2*c^4*d*e + 3*a*b^2*c
^3*d^2*(-(4*a*c - b^2)^3)^(1/2) - 8*a*b^3*c^2*d*e*(-(4*a*c - b^2)^3)^(1/2)
+ 6*a^2*b*c^3*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^7*b^4*e^4 + 16*a^7*c^4*d^
4 + 16*a^9*c^2*e^4 - 8*a^8*b^2*c*e^4 - 2*a^6*b^5*d*e^3 + a^5*b^4*c^2*d^4 -
8*a^6*b^2*c^3*d^4 + a^5*b^6*d^2*e^2 + 32*a^8*c^3*d^2*e^2 - 2*a^5*b^5*c*d^3*
e - 32*a^7*b*c^3*d^3*e + 16*a^7*b^3*c*d*e^3 - 32*a^8*b*c^2*d*e^3 + 16*a^6*b
^3*c^2*d^3*e - 6*a^6*b^4*c*d^2*e^2)))^(1/2) - 4*a^15*c^9*d^21*e^3 - 4*a^16*
c^8*d^19*e^5 + 48*a^18*c^6*d^15*e^9 - 4*a^14*b^2*c^8*d^21*e^3 - 4*a^14*b^7*
c^3*d^16*e^8 + 4*a^14*b^8*c^2*d^15*e^9 + 36*a^15*b^5*c^4*d^16*e^8 - 44*a^15
*b^6*c^3*d^15*e^9 + 4*a^15*b^7*c^2*d^14*e^10 - 100*a^16*b^3*c^5*d^16*e^8 +
160*a^16*b^4*c^4*d^15*e^9 - 32*a^16*b^5*c^3*d^14*e^10 - 204*a^17*b^2*c^5*d^
15*e^9 + 76*a^17*b^3*c^4*d^14*e^10 + 4*a^14*b*c^9*d^22*e^2 + 8*a^15*b*c^8*d
^20*e^4 + 80*a^17*b*c^6*d^16*e^8 - 48*a^18*b*c^5*d^14*e^10) - x*(2*a^14*c^9
*d^18*e^5 + 4*a^16*c^7*d^14*e^9 + 2*a^14*b^4*c^5*d^14*e^9 - 8*a^15*b^2*c^6*
d^14*e^9))*(-(b^9*e^2 + b^7*c^2*d^2 - b^6*e^2*(-(4*a*c - b^2)^3)^(1/2) - 9*
a*b^5*c^3*d^2 - 20*a^3*b*c^5*d^2 + 28*a^4*b*c^4*e^2 - 2*b^8*c*d*e + 25*a^2*
b^3*c^4*d^2 - a^2*c^4*d^2*(-(4*a*c - b^2)^3)^(1/2) + 42*a^2*b^5*c^2*e^2 - 6
3*a^3*b^3*c^3*e^2 + a^3*c^3*e^2*(-(4*a*c - b^2)^3)^(1/2) - b^4*c^2*d^2*(-(4
*a*c - b^2)^3)^(1/2) - 11*a*b^7*c*e^2 - 16*a^4*c^5*d*e + 20*a*b^6*c^2*d*e +
2*b^5*c*d*e*(-(4*a*c - b^2)^3)^(1/2) - 6*a^2*b^2*c^2*e^2*(-(4*a*c - b^2)^3
)^(1/2) + 5*a*b^4*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - 66*a^2*b^4*c^3*d*e + 76*
a^3*b^2*c^4*d*e + 3*a*b^2*c^3*d^2*(-(4*a*c - b^2)^3)^(1/2) - 8*a*b^3*c^2*d*
e*(-(4*a*c - b^2)^3)^(1/2) + 6*a^2*b*c^3*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(
a^7*b^4*e^4 + 16*a^7*c^4*d^4 + 16*a^9*c^2*e^4 - 8*a^8*b^2*c*e^4 - 2*a^6*b^5
*d*e^3 + a^5*b^4*c^2*d^4 - 8*a^6*b^2*c^3*d^4 + a^5*b^6*d^2*e^2 + 32*a^8*c^3
*d^2*e^2 - 2*a^5*b^5*c*d^3*e - 32*a^7*b*c^3*d^3*e + 16*a^7*b^3*c*d*e^3 - 32
*a^8*b*c^2*d*e^3 + 16*a^6*b^3*c^2*d^3*e - 6*a^6*b^4*c*d^2*e^2)))^(1/2) + 2*
a^14*c^8*d^14*e^8))*(-(b^9*e^2 + b^7*c^2*d^2 - b^6*e^2*(-(4*a*c - b^2)^3)^(
1/2) - 9*a*b^5*c^3*d^2 - 20*a^3*b*c^5*d^2 + 28*a^4*b*c^4*e^2 - 2*b^8*c*d*e
+ 25*a^2*b^3*c^4*d^2 - a^2*c^4*d^2*(-(4*a*c - b^2)^3)^(1/2) + 42*a^2*b^5*c^
2*e^2 - 63*a^3*b^3*c^3*e^2 + a^3*c^3*e^2*(-(4*a*c - b^2)^3)^(1/2) - b^4*c^2
d^2(-(4*a*c - b^2)^3)^(1/2) - 11*a*b^7*c*e^2 - 16*a^4*c^5*d*e + 20*a*b^6*
c^2*d*e + 2*b^5*c*d*e*(-(4*a*c - b^2)^3)^(1/2) - 6*a^2*b^2*c^2*e^2*(-(4*a*c
- b^2)^3)^(1/2) + 5*a*b^4*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - 66*a^2*b^4*c^3*

d*e + 76*a^3*b^2*c^4*d*e + 3*a*b^2*c^3*d^2*(-(4*a*c - b^2)^3)^(1/2) - 8*a*b
^3*c^2*d*e*(-(4*a*c - b^2)^3)^(1/2) + 6*a^2*b*c^3*d*e*(-(4*a*c - b^2)^3)^(1
/2))/(8*(a^7*b^4*e^4 + 16*a^7*c^4*d^4 + 16*a^9*c^2*e^4 - 8*a^8*b^2*c*e^4 -
2*a^6*b^5*d*e^3 + a^5*b^4*c^2*d^4 - 8*a^6*b^2*c^3*d^4 + a^5*b^6*d^2*e^2 + 3
2*a^8*c^3*d^2*e^2 - 2*a^5*b^5*c*d^3*e - 32*a^7*b*c^3*d^3*e + 16*a^7*b^3*c*d
*e^3 - 32*a^8*b*c^2*d*e^3 + 16*a^6*b^3*c^2*d^3*e - 6*a^6*b^4*c*d^2*e^2)))^(
1/2)*2i - (1/(3*a*d) - (x^2*(a*e + b*d))/(a^2*d^2))/x^3 - (log(c^9*d^27*e^6
 - b^9*d^18*e^15 + 2*a*c^8*d^25*e^8 - 2*b*c^8*d^26*e^7 + 2*b^8*c*d^19*e^14
+ a^5*b^4*d^13*e^20 + a^2*c^7*d^23*e^10 + 16*a^4*c^5*d^19*e^14 + 16*a^7*c^2
*d^13*e^20 + b^2*c^7*d^25*e^8 - b^7*c^2*d^20*e^13 - 25*a^2*b^3*c^4*d^20*e^1
3 + 66*a^2*b^4*c^3*d^19*e^14 - 42*a^2*b^5*c^2*d^18*e^15 - 76*a^3*b^2*c^4*d^
19*e^14 + 63*a^3*b^3*c^3*d^18*e^15 + a^5*b^4*e^3*x*(-d^5*e^7)^(5/2) - a^2*c
^7*d^15*x*(-d^5*e^7)^(3/2) + 16*a^7*c^2*e^3*x*(-d^5*e^7)^(5/2) + b^9*d^10*e
^5*x*(-d^5*e^7)^(3/2) + c^9*d^24*e^3*x*(-d^5*e^7)^(1/2) - 2*a*b*c^7*d^24*e^
9 + 11*a*b^7*c*d^18*e^15 + 9*a*b^5*c^3*d^20*e^13 - 20*a*b^6*c^2*d^19*e^14 +
 20*a^3*b*c^5*d^20*e^13 - 28*a^4*b*c^4*d^18*e^15 - 8*a^6*b^2*c*d^13*e^20 -
16*a^4*c^5*d^11*e^4*x*(-d^5*e^7)^(3/2) + b^7*c^2*d^12*e^3*x*(-d^5*e^7)^(3/2
) + b^2*c^7*d^22*e^5*x*(-d^5*e^7)^(1/2) - 8*a^6*b^2*c*e^3*x*(-d^5*e^7)^(5/2
) + 2*a*c^8*d^22*e^5*x*(-d^5*e^7)^(1/2) - 2*b^8*c*d^11*e^4*x*(-d^5*e^7)^(3/
2) - 2*b*c^8*d^23*e^4*x*(-d^5*e^7)^(1/2) - 11*a*b^7*c*d^10*e^5*x*(-d^5*e^7)
^(3/2) - 2*a*b*c^7*d^21*e^6*x*(-d^5*e^7)^(1/2) - 9*a*b^5*c^3*d^12*e^3*x*(-d
^5*e^7)^(3/2) + 20*a*b^6*c^2*d^11*e^4*x*(-d^5*e^7)^(3/2) - 20*a^3*b*c^5*d^1
2*e^3*x*(-d^5*e^7)^(3/2) + 28*a^4*b*c^4*d^10*e^5*x*(-d^5*e^7)^(3/2) + 25*a^
2*b^3*c^4*d^12*e^3*x*(-d^5*e^7)^(3/2) - 66*a^2*b^4*c^3*d^11*e^4*x*(-d^5*e^7
)^(3/2) + 42*a^2*b^5*c^2*d^10*e^5*x*(-d^5*e^7)^(3/2) + 76*a^3*b^2*c^4*d^11*
e^4*x*(-d^5*e^7)^(3/2) - 63*a^3*b^3*c^3*d^10*e^5*x*(-d^5*e^7)^(3/2))*(-d^5*
e^7)^(1/2))/(2*(c*d^7 + a*d^5*e^2 - b*d^6*e)) - atan((((-(b^9*e^2 + b^7*c^2
*d^2 + b^6*e^2*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c^3*d^2 - 20*a^3*b*c^5*d^
2 + 28*a^4*b*c^4*e^2 - 2*b^8*c*d*e + 25*a^2*b^3*c^4*d^2 + a^2*c^4*d^2*(-(4*
a*c - b^2)^3)^(1/2) + 42*a^2*b^5*c^2*e^2 - 63*a^3*b^3*c^3*e^2 - a^3*c^3*e^2
*(-(4*a*c - b^2)^3)^(1/2) + b^4*c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) - 11*a*b^7
*c*e^2 - 16*a^4*c^5*d*e + 20*a*b^6*c^2*d*e - 2*b^5*c*d*e*(-(4*a*c - b^2)^3)
^(1/2) + 6*a^2*b^2*c^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - 5*a*b^4*c*e^2*(-(4*a*
c - b^2)^3)^(1/2) - 66*a^2*b^4*c^3*d*e + 76*a^3*b^2*c^4*d*e - 3*a*b^2*c^3*d
^2*(-(4*a*c - b^2)^3)^(1/2) + 8*a*b^3*c^2*d*e*(-(4*a*c - b^2)^3)^(1/2) - 6*
a^2*b*c^3*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^7*b^4*e^4 + 16*a^7*c^4*d^4 +
16*a^9*c^2*e^4 - 8*a^8*b^2*c*e^4 - 2*a^6*b^5*d*e^3 + a^5*b^4*c^2*d^4 - 8*a^
6*b^2*c^3*d^4 + a^5*b^6*d^2*e^2 + 32*a^8*c^3*d^2*e^2 - 2*a^5*b^5*c*d^3*e -
32*a^7*b*c^3*d^3*e + 16*a^7*b^3*c*d*e^3 - 32*a^8*b*c^2*d*e^3 + 16*a^6*b^3*c
^2*d^3*e - 6*a^6*b^4*c*d^2*e^2)))^(1/2)*(((-(b^9*e^2 + b^7*c^2*d^2 + b^6*e^
2*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c^3*d^2 - 20*a^3*b*c^5*d^2 + 28*a^4*b*
c^4*e^2 - 2*b^8*c*d*e + 25*a^2*b^3*c^4*d^2 + a^2*c^4*d^2*(-(4*a*c - b^2)^3)
^(1/2) + 42*a^2*b^5*c^2*e^2 - 63*a^3*b^3*c^3*e^2 - a^3*c^3*e^2*(-(4*a*c - b
^2)^3)^(1/2) + b^4*c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) - 11*a*b^7*c*e^2 - 16*a
^4*c^5*d*e + 20*a*b^6*c^2*d*e - 2*b^5*c*d*e*(-(4*a*c - b^2)^3)^(1/2) + 6*a^

2*b^2*c^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - 5*a*b^4*c*e^2*(-(4*a*c - b^2)^3)^(
1/2) - 66*a^2*b^4*c^3*d*e + 76*a^3*b^2*c^4*d*e - 3*a*b^2*c^3*d^2*(-(4*a*c -
 b^2)^3)^(1/2) + 8*a*b^3*c^2*d*e*(-(4*a*c - b^2)^3)^(1/2) - 6*a^2*b*c^3*d*e
*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^7*b^4*e^4 + 16*a^7*c^4*d^4 + 16*a^9*c^2*e^
4 - 8*a^8*b^2*c*e^4 - 2*a^6*b^5*d*e^3 + a^5*b^4*c^2*d^4 - 8*a^6*b^2*c^3*d^4
 + a^5*b^6*d^2*e^2 + 32*a^8*c^3*d^2*e^2 - 2*a^5*b^5*c*d^3*e - 32*a^7*b*c^3*
d^3*e + 16*a^7*b^3*c*d*e^3 - 32*a^8*b*c^2*d*e^3 + 16*a^6*b^3*c^2*d^3*e - 6*
a^6*b^4*c*d^2*e^2)))^(1/2)*(x*(-(b^9*e^2 + b^7*c^2*d^2 + b^6*e^2*(-(4*a*c -
 b^2)^3)^(1/2) - 9*a*b^5*c^3*d^2 - 20*a^3*b*c^5*d^2 + 28*a^4*b*c^4*e^2 - 2*
b^8*c*d*e + 25*a^2*b^3*c^4*d^2 + a^2*c^4*d^2*(-(4*a*c - b^2)^3)^(1/2) + 42*
a^2*b^5*c^2*e^2 - 63*a^3*b^3*c^3*e^2 - a^3*c^3*e^2*(-(4*a*c - b^2)^3)^(1/2)
 + b^4*c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) - 11*a*b^7*c*e^2 - 16*a^4*c^5*d*e +
 20*a*b^6*c^2*d*e - 2*b^5*c*d*e*(-(4*a*c - b^2)^3)^(1/2) + 6*a^2*b^2*c^2*e^
2*(-(4*a*c - b^2)^3)^(1/2) - 5*a*b^4*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - 66*a^
2*b^4*c^3*d*e + 76*a^3*b^2*c^4*d*e - 3*a*b^2*c^3*d^2*(-(4*a*c - b^2)^3)^(1/
2) + 8*a*b^3*c^2*d*e*(-(4*a*c - b^2)^3)^(1/2) - 6*a^2*b*c^3*d*e*(-(4*a*c -
b^2)^3)^(1/2))/(8*(a^7*b^4*e^4 + 16*a^7*c^4*d^4 + 16*a^9*c^2*e^4 - 8*a^8*b^
2*c*e^4 - 2*a^6*b^5*d*e^3 + a^5*b^4*c^2*d^4 - 8*a^6*b^2*c^3*d^4 + a^5*b^6*d
^2*e^2 + 32*a^8*c^3*d^2*e^2 - 2*a^5*b^5*c*d^3*e - 32*a^7*b*c^3*d^3*e + 16*a
^7*b^3*c*d*e^3 - 32*a^8*b*c^2*d*e^3 + 16*a^6*b^3*c^2*d^3*e - 6*a^6*b^4*c*d^
2*e^2)))^(1/2)*(512*a^20*c^7*d^24*e^3 + 512*a^21*c^6*d^22*e^5 - 512*a^22*c^
5*d^20*e^7 - 512*a^23*c^4*d^18*e^9 - 32*a^18*b^3*c^6*d^25*e^2 + 128*a^18*b^
4*c^5*d^24*e^3 - 192*a^18*b^5*c^4*d^23*e^4 + 128*a^18*b^6*c^3*d^22*e^5 - 32
*a^18*b^7*c^2*d^21*e^6 - 640*a^19*b^2*c^6*d^24*e^3 + 1056*a^19*b^3*c^5*d^23
*e^4 - 672*a^19*b^4*c^4*d^22*e^5 + 96*a^19*b^5*c^3*d^21*e^6 + 32*a^19*b^6*c
^2*d^20*e^7 + 512*a^20*b^2*c^5*d^22*e^5 + 288*a^20*b^3*c^4*d^21*e^6 - 192*a
^20*b^4*c^3*d^20*e^7 + 32*a^20*b^5*c^2*d^19*e^8 + 384*a^21*b^2*c^4*d^20*e^7
 - 288*a^21*b^3*c^3*d^19*e^8 - 32*a^21*b^4*c^2*d^18*e^9 + 256*a^22*b^2*c^3*
d^18*e^9 + 128*a^19*b*c^7*d^25*e^2 - 1152*a^20*b*c^6*d^23*e^4 - 640*a^21*b*
c^5*d^21*e^6 + 640*a^22*b*c^4*d^19*e^8) - 64*a^18*c^8*d^24*e^2 + 128*a^19*c
^7*d^22*e^4 + 192*a^20*c^6*d^20*e^6 - 256*a^21*c^5*d^18*e^8 - 256*a^22*c^4*
d^16*e^10 - 16*a^16*b^4*c^6*d^24*e^2 + 64*a^16*b^5*c^5*d^23*e^3 - 96*a^16*b
^6*c^4*d^22*e^4 + 64*a^16*b^7*c^3*d^21*e^5 - 16*a^16*b^8*c^2*d^20*e^6 + 80*
a^17*b^2*c^7*d^24*e^2 - 368*a^17*b^3*c^6*d^23*e^3 + 608*a^17*b^4*c^5*d^22*e
^4 - 416*a^17*b^5*c^4*d^21*e^5 + 80*a^17*b^6*c^3*d^20*e^6 + 16*a^17*b^7*c^2
*d^19*e^7 - 928*a^18*b^2*c^6*d^22*e^4 + 640*a^18*b^3*c^5*d^21*e^5 + 32*a^18
*b^4*c^4*d^20*e^6 - 128*a^18*b^5*c^3*d^19*e^7 - 432*a^19*b^2*c^5*d^20*e^6 +
 304*a^19*b^3*c^4*d^19*e^7 - 16*a^19*b^4*c^3*d^18*e^8 + 16*a^19*b^5*c^2*d^1
7*e^9 + 128*a^20*b^2*c^4*d^18*e^8 - 128*a^20*b^3*c^3*d^17*e^9 - 16*a^20*b^4
*c^2*d^16*e^10 + 128*a^21*b^2*c^3*d^16*e^10 + 448*a^18*b*c^7*d^23*e^3 - 192
*a^20*b*c^5*d^19*e^7 + 256*a^21*b*c^4*d^17*e^9) - x*(16*a^16*c^9*d^23*e^2 +
 32*a^17*c^8*d^21*e^4 - 112*a^18*c^7*d^19*e^6 - 128*a^20*c^5*d^15*e^10 + 8*
a^14*b^4*c^7*d^23*e^2 - 16*a^14*b^5*c^6*d^22*e^3 + 8*a^14*b^6*c^5*d^21*e^4
+ 8*a^14*b^7*c^4*d^20*e^5 - 16*a^14*b^8*c^3*d^19*e^6 + 8*a^14*b^9*c^2*d^18*
e^7 - 32*a^15*b^2*c^8*d^23*e^2 + 64*a^15*b^3*c^7*d^22*e^3 - 16*a^15*b^4*c^6

*d^21*e^4 - 88*a^15*b^5*c^5*d^20*e^5 + 160*a^15*b^6*c^4*d^19*e^6 - 88*a^15*b^7*c^3*d^18*e^7 - 48*a^16*b^2*c^7*d^21*e^4 + 264*a^16*b^3*c^6*d^20*e^5 - 520*a^16*b^4*c^5*d^19*e^6 + 336*a^16*b^5*c^4*d^18*e^7 + 576*a^17*b^2*c^6*d^19*e^6 - 504*a^17*b^3*c^5*d^18*e^7 + 8*a^18*b^3*c^4*d^16*e^9 - 16*a^18*b^4*c^3*d^15*e^10 + 8*a^18*b^5*c^2*d^14*e^11 + 96*a^19*b^2*c^4*d^15*e^10 - 56*a^19*b^3*c^3*d^14*e^11 - 32*a^16*b*c^8*d^22*e^3 - 192*a^17*b*c^7*d^20*e^5 + 224*a^18*b*c^6*d^18*e^7 - 32*a^19*b*c^5*d^16*e^9 + 96*a^20*b*c^4*d^14*e^11)) *(-(b^9*e^2 + b^7*c^2*d^2 + b^6*e^2*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c^3*d^2 - 20*a^3*b*c^5*d^2 + 28*a^4*b*c^4*e^2 - 2*b^8*c*d*e + 25*a^2*b^3*c^4*d^2 + a^2*c^4*d^2*(-(4*a*c - b^2)^3)^(1/2) + 42*a^2*b^5*c^2*e^2 - 63*a^3*b^3*c^3*e^2 - a^3*c^3*e^2*(-(4*a*c - b^2)^3)^(1/2) + b^4*c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) - 11*a*b^7*c*e^2 - 16*a^4*c^5*d*e + 20*a*b^6*c^2*d*e - 2*b^5*c*d*e*(-(4*a*c - b^2)^3)^(1/2) + 6*a^2*b^2*c^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - 5*a*b^4*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - 66*a^2*b^4*c^3*d*e + 76*a^3*b^2*c^4*d*e - 3*a*b^2*c^3*d^2*(-(4*a*c - b^2)^3)^(1/2) + 8*a*b^3*c^2*d*e*(-(4*a*c - b^2)^3)^(1/2) - 6*a^2*b*c^3*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^7*b^4*e^4 + 16*a^7*c^4*d^4 + 16*a^9*c^2*e^4 - 8*a^8*b^2*c*e^4 - 2*a^6*b^5*d*e^3 + a^5*b^4*c^2*d^4 - 8*a^6*b^2*c^3*d^4 + a^5*b^6*d^2*e^2 + 32*a^8*c^3*d^2*e^2 - 2*a^5*b^5*c*d^3*e - 32*a^7*b*c^3*d^3*e + 16*a^7*b^3*c*d*e^3 - 32*a^8*b*c^2*d*e^3 + 16*a^6*b^3*c^2*d^3*e - 6*a^6*b^4*c*d^2*e^2)))^(1/2) - 4*a^15*c^9*d^21*e^3 - 4*a^16*c^8*d^19*e^5 + 48*a^18*c^6*d^15*e^9 - 4*a^14*b^2*c^8*d^21*e^3 - 4*a^14*b^7*c^3*d^16*e^8 + 4*a^14*b^8*c^2*d^15*e^9 + 36*a^15*b^5*c^4*d^16*e^8 - 44*a^15*b^6*c^3*d^15*e^9 + 4*a^15*b^7*c^2*d^14*e^10 - 100*a^16*b^3*c^5*d^16*e^8 + 160*a^16*b^4*c^4*d^15*e^9 - 32*a^16*b^5*c^3*d^14*e^10 - 204*a^17*b^2*c^5*d^15*e^9 + 76*a^17*b^3*c^4*d^14*e^10 + 4*a^14*b*c^9*d^22*e^2 + 8*a^15*b*c^8*d^20*e^4 + 80*a^17*b*c^6*d^16*e^8 - 48*a^18*b*c^5*d^14*e^10) - x*(2*a^14*c^9*d^18*e^5 + 4*a^16*c^7*d^14*e^9 + 2*a^14*b^4*c^5*d^14*e^9 - 8*a^15*b^2*c^6*d^14*e^9))*(-(b^9*e^2 + b^7*c^2*d^2 + b^6*e^2*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c^3*d^2 - 20*a^3*b*c^5*d^2 + 28*a^4*b*c^4*e^2 - 2*b^8*c*d*e + 25*a^2*b^3*c^4*d^2 + a^2*c^4*d^2*(-(4*a*c - b^2)^3)^(1/2) + 42*a^2*b^5*c^2*e^2 - 63*a^3*b^3*c^3*e^2 - a^3*c^3*e^2*(-(4*a*c - b^2)^3)^(1/2) + b^4*c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) - 11*a*b^7*c*e^2 - 16*a^4*c^5*d*e + 20*a*b^6*c^2*d*e - 2*b^5*c*d*e*(-(4*a*c - b^2)^3)^(1/2) + 6*a^2*b^2*c^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - 5*a*b^4*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - 66*a^2*b^4*c^3*d*e + 76*a^3*b^2*c^4*d*e - 3*a*b^2*c^3*d^2*(-(4*a*c - b^2)^3)^(1/2) + 8*a*b^3*c^2*d*e*(-(4*a*c - b^2)^3)^(1/2) - 6*a^2*b*c^3*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^7*b^4*e^4 + 16*a^7*c^4*d^4 + 16*a^9*c^2*e^4 - 8*a^8*b^2*c*e^4 - 2*a^6*b^5*d*e^3 + a^5*b^4*c^2*d^4 - 8*a^6*b^2*c^3*d^4 + a^5*b^6*d^2*e^2 + 32*a^8*c^3*d^2*e^2 - 2*a^5*b^5*c*d^3*e - 32*a^7*b*c^3*d^3*e + 16*a^7*b^3*c*d*e^3 - 32*a^8*b*c^2*d*e^3 + 16*a^6*b^3*c^2*d^3*e - 6*a^6*b^4*c*d^2*e^2)))^(1/2)*1i + ((-(b^9*e^2 + b^7*c^2*d^2 + b^6*e^2*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c^3*d^2 - 20*a^3*b*c^5*d^2 + 28*a^4*b*c^4*e^2 - 2*b^8*c*d*e + 25*a^2*b^3*c^4*d^2 + a^2*c^4*d^2*(-(4*a*c - b^2)^3)^(1/2) + 42*a^2*b^5*c^2*e^2 - 63*a^3*b^3*c^3*e^2 - a^3*c^3*e^2*(-(4*a*c - b^2)^3)^(1/2) + b^4*c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) - 11*a*b^7*c*e^2 - 16*a^4*c^5*d*e + 20*a*b^6*c

^2*d*e - 2*b^5*c*d*e*(-(4*a*c - b^2)^3)^(1/2) + 6*a^2*b^2*c^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - 5*a*b^4*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - 66*a^2*b^4*c^3*d*e + 76*a^3*b^2*c^4*d*e - 3*a*b^2*c^3*d^2*(-(4*a*c - b^2)^3)^(1/2) + 8*a*b^3*c^2*d*e*(-(4*a*c - b^2)^3)^(1/2) - 6*a^2*b*c^3*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^7*b^4*e^4 + 16*a^7*c^4*d^4 + 16*a^9*c^2*e^4 - 8*a^8*b^2*c*e^4 - 2*a^6*b^5*d*e^3 + a^5*b^4*c^2*d^4 - 8*a^6*b^2*c^3*d^4 + a^5*b^6*d^2*e^2 + 32*a^8*c^3*d^2*e^2 - 2*a^5*b^5*c*d^3*e - 32*a^7*b*c^3*d^3*e + 16*a^7*b^3*c*d*e^3 - 32*a^8*b*c^2*d*e^3 + 16*a^6*b^3*c^2*d^3*e - 6*a^6*b^4*c*d^2*e^2)))^(1/2)*(((-(b^9*e^2 + b^7*c^2*d^2 + b^6*e^2*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c^3*d^2 - 20*a^3*b*c^5*d^2 + 28*a^4*b*c^4*e^2 - 2*b^8*c*d*e + 25*a^2*b^3*c^4*d^2 + a^2*c^4*d^2*(-(4*a*c - b^2)^3)^(1/2) + 42*a^2*b^5*c^2*e^2 - 63*a^3*b^3*c^3*e^2 - a^3*c^3*e^2*(-(4*a*c - b^2)^3)^(1/2) + b^4*c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) - 11*a*b^7*c*e^2 - 16*a^4*c^5*d*e + 20*a*b^6*c^2*d*e - 2*b^5*c*d*e*(-(4*a*c - b^2)^3)^(1/2) + 6*a^2*b^2*c^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - 5*a*b^4*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - 66*a^2*b^4*c^3*d*e + 76*a^3*b^2*c^4*d*e - 3*a*b^2*c^3*d^2*(-(4*a*c - b^2)^3)^(1/2) + 8*a*b^3*c^2*d*e*(-(4*a*c - b^2)^3)^(1/2) - 6*a^2*b*c^3*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^7*b^4*e^4 + 16*a^7*c^4*d^4 + 16*a^9*c^2*e^4 - 8*a^8*b^2*c*e^4 - 2*a^6*b^5*d*e^3 + a^5*b^4*c^2*d^4 - 8*a^6*b^2*c^3*d^4 + a^5*b^6*d^2*e^2 + 32*a^8*c^3*d^2*e^2 - 2*a^5*b^5*c*d^3*e - 32*a^7*b*c^3*d^3*e + 16*a^7*b^3*c*d*e^3 - 32*a^8*b*c^2*d*e^3 + 16*a^6*b^3*c^2*d^3*e - 6*a^6*b^4*c*d^2*e^2)))^(1/2)*(x*(-(b^9*e^2 + b^7*c^2*d^2 + b^6*e^2*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c^3*d^2 - 20*a^3*b*c^5*d^2 + 28*a^4*b*c^4*e^2 - 2*b^8*c*d*e + 25*a^2*b^3*c^4*d^2 + a^2*c^4*d^2*(-(4*a*c - b^2)^3)^(1/2) + 42*a^2*b^5*c^2*e^2 - 63*a^3*b^3*c^3*e^2 - a^3*c^3*e^2*(-(4*a*c - b^2)^3)^(1/2) + b^4*c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) - 11*a*b^7*c*e^2 - 16*a^4*c^5*d*e + 20*a*b^6*c^2*d*e - 2*b^5*c*d*e*(-(4*a*c - b^2)^3)^(1/2) + 6*a^2*b^2*c^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - 5*a*b^4*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - 66*a^2*b^4*c^3*d*e + 76*a^3*b^2*c^4*d*e - 3*a*b^2*c^3*d^2*(-(4*a*c - b^2)^3)^(1/2) + 8*a*b^3*c^2*d*e*(-(4*a*c - b^2)^3)^(1/2) - 6*a^2*b*c^3*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^7*b^4*e^4 + 16*a^7*c^4*d^4 + 16*a^9*c^2*e^4 - 8*a^8*b^2*c*e^4 - 2*a^6*b^5*d*e^3 + a^5*b^4*c^2*d^4 - 8*a^6*b^2*c^3*d^4 + a^5*b^6*d^2*e^2 + 32*a^8*c^3*d^2*e^2 - 2*a^5*b^5*c*d^3*e - 32*a^7*b*c^3*d^3*e + 16*a^7*b^3*c*d*e^3 - 32*a^8*b*c^2*d*e^3 + 16*a^6*b^3*c^2*d^3*e - 6*a^6*b^4*c*d^2*e^2)))^(1/2)*(512*a^20*c^7*d^24*e^3 + 512*a^21*c^6*d^22*e^5 - 512*a^22*c^5*d^20*e^7 - 512*a^23*c^4*d^18*e^9 - 32*a^18*b^3*c^6*d^25*e^2 + 128*a^18*b^4*c^5*d^24*e^3 - 192*a^18*b^5*c^4*d^23*e^4 + 128*a^18*b^6*c^3*d^22*e^5 - 32*a^18*b^7*c^2*d^21*e^6 - 640*a^19*b^2*c^6*d^24*e^3 + 1056*a^19*b^3*c^5*d^23*e^4 - 672*a^19*b^4*c^4*d^22*e^5 + 96*a^19*b^5*c^3*d^21*e^6 + 32*a^19*b^6*c^2*d^20*e^7 + 512*a^20*b^2*c^5*d^22*e^5 + 288*a^20*b^3*c^4*d^21*e^6 - 192*a^20*b^4*c^3*d^20*e^7 + 32*a^20*b^5*c^2*d^19*e^8 + 384*a^21*b^2*c^4*d^20*e^7 - 288*a^21*b^3*c^3*d^19*e^8 - 32*a^21*b^4*c^2*d^18*e^9 + 256*a^22*b^2*c^3*d^18*e^9 + 128*a^19*b*c^7*d^25*e^2 - 1152*a^20*b*c^6*d^23*e^4 - 640*a^21*b*c^5*d^21*e^6 + 640*a^22*b*c^4*d^19*e^8) + 64*a^18*c^8*d^24*e^2 - 128*a^19*c^7*d^22*e^4 - 192*a^20*c^6*d^20*e^6 + 256*a^21*c^5*d^18*e^8 + 256*a^22*c^4*d^16*e^10 + 16*a^16*b^4*c^6*d^24*e^2

- 64*a^16*b^5*c^5*d^23*e^3 + 96*a^16*b^6*c^4*d^22*e^4 - 64*a^16*b^7*c^3*d^21*e^5 + 16*a^16*b^8*c^2*d^20*e^6 - 80*a^17*b^2*c^7*d^24*e^2 + 368*a^17*b^3*c^6*d^23*e^3 - 608*a^17*b^4*c^5*d^22*e^4 + 416*a^17*b^5*c^4*d^21*e^5 - 80*a^17*b^6*c^3*d^20*e^6 - 16*a^17*b^7*c^2*d^19*e^7 + 928*a^18*b^2*c^6*d^22*e^4 - 640*a^18*b^3*c^5*d^21*e^5 - 32*a^18*b^4*c^4*d^20*e^6 + 128*a^18*b^5*c^3*d^19*e^7 + 432*a^19*b^2*c^5*d^20*e^6 - 304*a^19*b^3*c^4*d^19*e^7 + 16*a^19*b^4*c^3*d^18*e^8 - 16*a^19*b^5*c^2*d^17*e^9 - 128*a^20*b^2*c^4*d^18*e^8 + 128*a^20*b^3*c^3*d^17*e^9 + 16*a^20*b^4*c^2*d^16*e^10 - 128*a^21*b^2*c^3*d^16*e^10 - 448*a^18*b*c^7*d^23*e^3 + 192*a^20*b*c^5*d^19*e^7 - 256*a^21*b*c^4*d^17*e^9) - x*(16*a^16*c^9*d^23*e^2 + 32*a^17*c^8*d^21*e^4 - 112*a^18*c^7*d^19*e^6 - 128*a^20*c^5*d^15*e^10 + 8*a^14*b^4*c^7*d^23*e^2 - 16*a^14*b^5*c^6*d^22*e^3 + 8*a^14*b^6*c^5*d^21*e^4 + 8*a^14*b^7*c^4*d^20*e^5 - 16*a^14*b^8*c^3*d^19*e^6 + 8*a^14*b^9*c^2*d^18*e^7 - 32*a^15*b^2*c^8*d^23*e^2 + 64*a^15*b^3*c^7*d^22*e^3 - 16*a^15*b^4*c^6*d^21*e^4 - 88*a^15*b^5*c^5*d^20*e^5 + 160*a^15*b^6*c^4*d^19*e^6 - 88*a^15*b^7*c^3*d^18*e^7 - 48*a^16*b^2*c^7*d^21*e^4 + 264*a^16*b^3*c^6*d^20*e^5 - 520*a^16*b^4*c^5*d^19*e^6 + 336*a^16*b^5*c^4*d^18*e^7 + 576*a^17*b^2*c^6*d^19*e^6 - 504*a^17*b^3*c^5*d^18*e^7 + 8*a^18*b^3*c^4*d^16*e^9 - 16*a^18*b^4*c^3*d^15*e^10 + 8*a^18*b^5*c^2*d^14*e^11 + 96*a^19*b^2*c^4*d^15*e^10 - 56*a^19*b^3*c^3*d^14*e^11 - 32*a^16*b*c^8*d^22*e^3 - 192*a^17*b*c^7*d^20*e^5 + 224*a^18*b*c^6*d^18*e^7 - 32*a^19*b*c^5*d^16*e^9 + 96*a^20*b*c^4*d^14*e^11))*(-(b^9*e^2 + b^7*c^2*d^2 + b^6*e^2*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c^3*d^2 - 20*a^3*b*c^5*d^2 + 28*a^4*b*c^4*e^2 - 2*b^8*c*d*e + 25*a^2*b^3*c^4*d^2 + a^2*c^4*d^2*(-(4*a*c - b^2)^3)^(1/2) + 42*a^2*b^5*c^2*e^2 - 63*a^3*b^3*c^3*e^2 - a^3*c^3*e^2*(-(4*a*c - b^2)^3)^(1/2) + b^4*c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) - 11*a*b^7*c*e^2 - 16*a^4*c^5*d*e + 20*a*b^6*c^2*d*e - 2*b^5*c*d*e*(-(4*a*c - b^2)^3)^(1/2) + 6*a^2*b^2*c^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - 5*a*b^4*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - 66*a^2*b^4*c^3*d*e + 76*a^3*b^2*c^4*d*e - 3*a*b^2*c^3*d^2*(-(4*a*c - b^2)^3)^(1/2) + 8*a*b^3*c^2*d*e*(-(4*a*c - b^2)^3)^(1/2) - 6*a^2*b*c^3*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^7*b^4*e^4 + 16*a^7*c^4*d^4 + 16*a^9*c^2*e^4 - 8*a^8*b^2*c*e^4 - 2*a^6*b^5*d*e^3 + a^5*b^4*c^2*d^4 - 8*a^6*b^2*c^3*d^4 + a^5*b^6*d^2*e^2 + 32*a^8*c^3*d^2*e^2 - 2*a^5*b^5*c*d^3*e - 32*a^7*b*c^3*d^3*e + 16*a^7*b^3*c*d*e^3 - 32*a^8*b*c^2*d*e^3 + 16*a^6*b^3*c^2*d^3*e - 6*a^6*b^4*c*d^2*e^2)))^(1/2) + 4*a^15*c^9*d^21*e^3 + 4*a^16*c^8*d^19*e^5 - 48*a^18*c^6*d^15*e^9 + 4*a^14*b^2*c^8*d^21*e^3 + 4*a^14*b^7*c^3*d^16*e^8 - 4*a^14*b^8*c^2*d^15*e^9 - 36*a^15*b^5*c^4*d^16*e^8 + 44*a^15*b^6*c^3*d^15*e^9 - 4*a^15*b^7*c^2*d^14*e^10 + 100*a^16*b^3*c^5*d^16*e^8 - 160*a^16*b^4*c^4*d^15*e^9 + 32*a^16*b^5*c^3*d^14*e^10 + 204*a^17*b^2*c^5*d^15*e^9 - 76*a^17*b^3*c^4*d^14*e^10 - 4*a^14*b*c^9*d^22*e^2 - 8*a^15*b*c^8*d^20*e^4 - 80*a^17*b*c^6*d^16*e^8 + 48*a^18*b*c^5*d^14*e^10) - x*(2*a^14*c^9*d^18*e^5 + 4*a^16*c^7*d^14*e^9 + 2*a^14*b^4*c^5*d^14*e^9 - 8*a^15*b^2*c^6*d^14*e^9))*(-(b^9*e^2 + b^7*c^2*d^2 + b^6*e^2*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c^3*d^2 - 20*a^3*b*c^5*d^2 + 28*a^4*b*c^4*e^2 - 2*b^8*c*d*e + 25*a^2*b^3*c^4*d^2 + a^2*c^4*d^2*(-(4*a*c - b^2)^3)^(1/2) + 42*a^2*b^5*c^2*e^2 - 63*a^3*b^3*c^3*e^2 - a^3*c^3*e^2*(-(4*a*c - b^2)^3)^(1/2) + b^4*c^2*d^2*(-(4*a*c - b^2)^3)^(1/2

) - 11*a*b^7*c*e^2 - 16*a^4*c^5*d*e + 20*a*b^6*c^2*d*e - 2*b^5*c*d*e*(-(4*a
*c - b^2)^3)^(1/2) + 6*a^2*b^2*c^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - 5*a*b^4*c
e^2(-(4*a*c - b^2)^3)^(1/2) - 66*a^2*b^4*c^3*d*e + 76*a^3*b^2*c^4*d*e - 3
*a*b^2*c^3*d^2*(-(4*a*c - b^2)^3)^(1/2) + 8*a*b^3*c^2*d*e*(-(4*a*c - b^2)^3
)^(1/2) - 6*a^2*b*c^3*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^7*b^4*e^4 + 16*a^
7*c^4*d^4 + 16*a^9*c^2*e^4 - 8*a^8*b^2*c*e^4 - 2*a^6*b^5*d*e^3 + a^5*b^4*c^
2*d^4 - 8*a^6*b^2*c^3*d^4 + a^5*b^6*d^2*e^2 + 32*a^8*c^3*d^2*e^2 - 2*a^5*b^
5*c*d^3*e - 32*a^7*b*c^3*d^3*e + 16*a^7*b^3*c*d*e^3 - 32*a^8*b*c^2*d*e^3 +
16*a^6*b^3*c^2*d^3*e - 6*a^6*b^4*c*d^2*e^2)))^(1/2)*1i)/(((-(b^9*e^2 + b^7*
c^2*d^2 + b^6*e^2*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c^3*d^2 - 20*a^3*b*c^5
*d^2 + 28*a^4*b*c^4*e^2 - 2*b^8*c*d*e + 25*a^2*b^3*c^4*d^2 + a^2*c^4*d^2*(-
(4*a*c - b^2)^3)^(1/2) + 42*a^2*b^5*c^2*e^2 - 63*a^3*b^3*c^3*e^2 - a^3*c^3*
e^2*(-(4*a*c - b^2)^3)^(1/2) + b^4*c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) - 11*a*
b^7*c*e^2 - 16*a^4*c^5*d*e + 20*a*b^6*c^2*d*e - 2*b^5*c*d*e*(-(4*a*c - b^2)
^3)^(1/2) + 6*a^2*b^2*c^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - 5*a*b^4*c*e^2*(-(4
*a*c - b^2)^3)^(1/2) - 66*a^2*b^4*c^3*d*e + 76*a^3*b^2*c^4*d*e - 3*a*b^2*c^
3*d^2*(-(4*a*c - b^2)^3)^(1/2) + 8*a*b^3*c^2*d*e*(-(4*a*c - b^2)^3)^(1/2) -
6*a^2*b*c^3*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^7*b^4*e^4 + 16*a^7*c^4*d^4
+ 16*a^9*c^2*e^4 - 8*a^8*b^2*c*e^4 - 2*a^6*b^5*d*e^3 + a^5*b^4*c^2*d^4 - 8
*a^6*b^2*c^3*d^4 + a^5*b^6*d^2*e^2 + 32*a^8*c^3*d^2*e^2 - 2*a^5*b^5*c*d^3*e
- 32*a^7*b*c^3*d^3*e + 16*a^7*b^3*c*d*e^3 - 32*a^8*b*c^2*d*e^3 + 16*a^6*b^
3*c^2*d^3*e - 6*a^6*b^4*c*d^2*e^2)))^(1/2)*(((-(b^9*e^2 + b^7*c^2*d^2 + b^6
e^2(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c^3*d^2 - 20*a^3*b*c^5*d^2 + 28*a^4
*b*c^4*e^2 - 2*b^8*c*d*e + 25*a^2*b^3*c^4*d^2 + a^2*c^4*d^2*(-(4*a*c - b^2)
^3)^(1/2) + 42*a^2*b^5*c^2*e^2 - 63*a^3*b^3*c^3*e^2 - a^3*c^3*e^2*(-(4*a*c
- b^2)^3)^(1/2) + b^4*c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) - 11*a*b^7*c*e^2 - 1
6*a^4*c^5*d*e + 20*a*b^6*c^2*d*e - 2*b^5*c*d*e*(-(4*a*c - b^2)^3)^(1/2) + 6
*a^2*b^2*c^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - 5*a*b^4*c*e^2*(-(4*a*c - b^2)^3
)^(1/2) - 66*a^2*b^4*c^3*d*e + 76*a^3*b^2*c^4*d*e - 3*a*b^2*c^3*d^2*(-(4*a*
c - b^2)^3)^(1/2) + 8*a*b^3*c^2*d*e*(-(4*a*c - b^2)^3)^(1/2) - 6*a^2*b*c^3*
d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^7*b^4*e^4 + 16*a^7*c^4*d^4 + 16*a^9*c^2
*e^4 - 8*a^8*b^2*c*e^4 - 2*a^6*b^5*d*e^3 + a^5*b^4*c^2*d^4 - 8*a^6*b^2*c^3*
d^4 + a^5*b^6*d^2*e^2 + 32*a^8*c^3*d^2*e^2 - 2*a^5*b^5*c*d^3*e - 32*a^7*b*c
^3*d^3*e + 16*a^7*b^3*c*d*e^3 - 32*a^8*b*c^2*d*e^3 + 16*a^6*b^3*c^2*d^3*e -
6*a^6*b^4*c*d^2*e^2)))^(1/2)*(x*(-(b^9*e^2 + b^7*c^2*d^2 + b^6*e^2*(-(4*a*
c - b^2)^3)^(1/2) - 9*a*b^5*c^3*d^2 - 20*a^3*b*c^5*d^2 + 28*a^4*b*c^4*e^2 -
2*b^8*c*d*e + 25*a^2*b^3*c^4*d^2 + a^2*c^4*d^2*(-(4*a*c - b^2)^3)^(1/2) +
42*a^2*b^5*c^2*e^2 - 63*a^3*b^3*c^3*e^2 - a^3*c^3*e^2*(-(4*a*c - b^2)^3)^(1
/2) + b^4*c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) - 11*a*b^7*c*e^2 - 16*a^4*c^5*d*
e + 20*a*b^6*c^2*d*e - 2*b^5*c*d*e*(-(4*a*c - b^2)^3)^(1/2) + 6*a^2*b^2*c^2
e^2(-(4*a*c - b^2)^3)^(1/2) - 5*a*b^4*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - 66
*a^2*b^4*c^3*d*e + 76*a^3*b^2*c^4*d*e - 3*a*b^2*c^3*d^2*(-(4*a*c - b^2)^3)^
(1/2) + 8*a*b^3*c^2*d*e*(-(4*a*c - b^2)^3)^(1/2) - 6*a^2*b*c^3*d*e*(-(4*a*c
- b^2)^3)^(1/2))/(8*(a^7*b^4*e^4 + 16*a^7*c^4*d^4 + 16*a^9*c^2*e^4 - 8*a^8
*b^2*c*e^4 - 2*a^6*b^5*d*e^3 + a^5*b^4*c^2*d^4 - 8*a^6*b^2*c^3*d^4 + a^5*b^

6*d^2*e^2 + 32*a^8*c^3*d^2*e^2 - 2*a^5*b^5*c*d^3*e - 32*a^7*b*c^3*d^3*e + 16*a^7*b^3*c*d*e^3 - 32*a^8*b*c^2*d*e^3 + 16*a^6*b^3*c^2*d^3*e - 6*a^6*b^4*c*d^2*e^2)))^(1/2)*(512*a^20*c^7*d^24*e^3 + 512*a^21*c^6*d^22*e^5 - 512*a^22*c^5*d^20*e^7 - 512*a^23*c^4*d^18*e^9 - 32*a^18*b^3*c^6*d^25*e^2 + 128*a^18*b^4*c^5*d^24*e^3 - 192*a^18*b^5*c^4*d^23*e^4 + 128*a^18*b^6*c^3*d^22*e^5 - 32*a^18*b^7*c^2*d^21*e^6 - 640*a^19*b^2*c^6*d^24*e^3 + 1056*a^19*b^3*c^5*d^23*e^4 - 672*a^19*b^4*c^4*d^22*e^5 + 96*a^19*b^5*c^3*d^21*e^6 + 32*a^19*b^6*c^2*d^20*e^7 + 512*a^20*b^2*c^5*d^22*e^5 + 288*a^20*b^3*c^4*d^21*e^6 - 192*a^20*b^4*c^3*d^20*e^7 + 32*a^20*b^5*c^2*d^19*e^8 + 384*a^21*b^2*c^4*d^20*e^7 - 288*a^21*b^3*c^3*d^19*e^8 - 32*a^21*b^4*c^2*d^18*e^9 + 256*a^22*b^2*c^3*d^18*e^9 + 128*a^19*b*c^7*d^25*e^2 - 1152*a^20*b*c^6*d^23*e^4 - 640*a^21*b*c^5*d^21*e^6 + 640*a^22*b*c^4*d^19*e^8) + 64*a^18*c^8*d^24*e^2 - 128*a^19*c^7*d^22*e^4 - 192*a^20*c^6*d^20*e^6 + 256*a^21*c^5*d^18*e^8 + 256*a^22*c^4*d^16*e^10 + 16*a^16*b^4*c^6*d^24*e^2 - 64*a^16*b^5*c^5*d^23*e^3 + 96*a^16*b^6*c^4*d^22*e^4 - 64*a^16*b^7*c^3*d^21*e^5 + 16*a^16*b^8*c^2*d^20*e^6 - 80*a^17*b^2*c^7*d^24*e^2 + 368*a^17*b^3*c^6*d^23*e^3 - 608*a^17*b^4*c^5*d^22*e^4 + 416*a^17*b^5*c^4*d^21*e^5 - 80*a^17*b^6*c^3*d^20*e^6 - 16*a^17*b^7*c^2*d^19*e^7 + 928*a^18*b^2*c^6*d^22*e^4 - 640*a^18*b^3*c^5*d^21*e^5 - 32*a^18*b^4*c^4*d^20*e^6 + 128*a^18*b^5*c^3*d^19*e^7 + 432*a^19*b^2*c^5*d^20*e^6 - 304*a^19*b^3*c^4*d^19*e^7 + 16*a^19*b^4*c^3*d^18*e^8 - 16*a^19*b^5*c^2*d^17*e^9 - 128*a^20*b^2*c^4*d^18*e^8 + 128*a^20*b^3*c^3*d^17*e^9 + 16*a^20*b^4*c^2*d^16*e^10 - 128*a^21*b^2*c^3*d^16*e^10 - 448*a^18*b*c^7*d^23*e^3 + 192*a^20*b*c^5*d^19*e^7 - 256*a^21*b*c^4*d^17*e^9) - x*(16*a^16*c^9*d^23*e^2 + 32*a^17*c^8*d^21*e^4 - 112*a^18*c^7*d^19*e^6 - 128*a^20*c^5*d^15*e^10 + 8*a^14*b^4*c^7*d^23*e^2 - 16*a^14*b^5*c^6*d^22*e^3 + 8*a^14*b^6*c^5*d^21*e^4 + 8*a^14*b^7*c^4*d^20*e^5 - 16*a^14*b^8*c^3*d^19*e^6 + 8*a^14*b^9*c^2*d^18*e^7 - 32*a^15*b^2*c^8*d^23*e^2 + 64*a^15*b^3*c^7*d^22*e^3 - 16*a^15*b^4*c^6*d^21*e^4 - 88*a^15*b^5*c^5*d^20*e^5 + 160*a^15*b^6*c^4*d^19*e^6 - 88*a^15*b^7*c^3*d^18*e^7 - 48*a^16*b^2*c^7*d^21*e^4 + 264*a^16*b^3*c^6*d^20*e^5 - 520*a^16*b^4*c^5*d^19*e^6 + 336*a^16*b^5*c^4*d^18*e^7 + 576*a^17*b^2*c^6*d^19*e^6 - 504*a^17*b^3*c^5*d^18*e^7 + 8*a^18*b^3*c^4*d^16*e^9 - 16*a^18*b^4*c^3*d^15*e^10 + 8*a^18*b^5*c^2*d^14*e^11 + 96*a^19*b^2*c^4*d^15*e^10 - 56*a^19*b^3*c^3*d^14*e^11 - 32*a^16*b*c^8*d^22*e^3 - 192*a^17*b*c^7*d^20*e^5 + 224*a^18*b*c^6*d^18*e^7 - 32*a^19*b*c^5*d^16*e^9 + 96*a^20*b*c^4*d^14*e^11))*(-(b^9*e^2 + b^7*c^2*d^2 + b^6*e^2*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c^3*d^2 - 20*a^3*b*c^5*d^2 + 28*a^4*b*c^4*e^2 - 2*b^8*c*d*e + 25*a^2*b^3*c^4*d^2 + a^2*c^4*d^2*(-(4*a*c - b^2)^3)^(1/2) + 42*a^2*b^5*c^2*e^2 - 63*a^3*b^3*c^3*e^2 - a^3*c^3*e^2*(-(4*a*c - b^2)^3)^(1/2) + b^4*c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) - 11*a*b^7*c*e^2 - 16*a^4*c^5*d*e + 20*a*b^6*c^2*d*e - 2*b^5*c*d*e*(-(4*a*c - b^2)^3)^(1/2) + 6*a^2*b^2*c^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - 5*a*b^4*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - 66*a^2*b^4*c^3*d*e + 76*a^3*b^2*c^4*d*e - 3*a*b^2*c^3*d^2*(-(4*a*c - b^2)^3)^(1/2) + 8*a*b^3*c^2*d*e*(-(4*a*c - b^2)^3)^(1/2) - 6*a^2*b*c^3*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^7*b^4*e^4 + 16*a^7*c^4*d^4 + 16*a^9*c^2*e^4 - 8*a^8*b^2*c*e^4 - 2*a^6*b^5*d*e^3 + a^5*b^4*c^2*d^4 - 8*a^6*b^2*c^3*d^4 + a^5*b^6*d^2*e^2 + 32*a^8*c^3*d^2*e^

2 - 2*a^5*b^5*c*d^3*e - 32*a^7*b*c^3*d^3*e + 16*a^7*b^3*c*d*e^3 - 32*a^8*b*
c^2*d*e^3 + 16*a^6*b^3*c^2*d^3*e - 6*a^6*b^4*c*d^2*e^2)))^(1/2) + 4*a^15*c^
9*d^21*e^3 + 4*a^16*c^8*d^19*e^5 - 48*a^18*c^6*d^15*e^9 + 4*a^14*b^2*c^8*d^
21*e^3 + 4*a^14*b^7*c^3*d^16*e^8 - 4*a^14*b^8*c^2*d^15*e^9 - 36*a^15*b^5*c^
4*d^16*e^8 + 44*a^15*b^6*c^3*d^15*e^9 - 4*a^15*b^7*c^2*d^14*e^10 + 100*a^16
*b^3*c^5*d^16*e^8 - 160*a^16*b^4*c^4*d^15*e^9 + 32*a^16*b^5*c^3*d^14*e^10 +
 204*a^17*b^2*c^5*d^15*e^9 - 76*a^17*b^3*c^4*d^14*e^10 - 4*a^14*b*c^9*d^22*
e^2 - 8*a^15*b*c^8*d^20*e^4 - 80*a^17*b*c^6*d^16*e^8 + 48*a^18*b*c^5*d^14*e
^10) - x*(2*a^14*c^9*d^18*e^5 + 4*a^16*c^7*d^14*e^9 + 2*a^14*b^4*c^5*d^14*e
^9 - 8*a^15*b^2*c^6*d^14*e^9))*(-(b^9*e^2 + b^7*c^2*d^2 + b^6*e^2*(-(4*a*c
 - b^2)^3)^(1/2) - 9*a*b^5*c^3*d^2 - 20*a^3*b*c^5*d^2 + 28*a^4*b*c^4*e^2 - 2
*b^8*c*d*e + 25*a^2*b^3*c^4*d^2 + a^2*c^4*d^2*(-(4*a*c - b^2)^3)^(1/2) + 42
*a^2*b^5*c^2*e^2 - 63*a^3*b^3*c^3*e^2 - a^3*c^3*e^2*(-(4*a*c - b^2)^3)^(1/2
) + b^4*c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) - 11*a*b^7*c*e^2 - 16*a^4*c^5*d*e
 + 20*a*b^6*c^2*d*e - 2*b^5*c*d*e*(-(4*a*c - b^2)^3)^(1/2) + 6*a^2*b^2*c^2*e
^2*(-(4*a*c - b^2)^3)^(1/2) - 5*a*b^4*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - 66*a
^2*b^4*c^3*d*e + 76*a^3*b^2*c^4*d*e - 3*a*b^2*c^3*d^2*(-(4*a*c - b^2)^3)^(1
/2) + 8*a*b^3*c^2*d*e*(-(4*a*c - b^2)^3)^(1/2) - 6*a^2*b*c^3*d*e*(-(4*a*c -
 b^2)^3)^(1/2))/(8*(a^7*b^4*e^4 + 16*a^7*c^4*d^4 + 16*a^9*c^2*e^4 - 8*a^8*b
^2*c*e^4 - 2*a^6*b^5*d*e^3 + a^5*b^4*c^2*d^4 - 8*a^6*b^2*c^3*d^4 + a^5*b^6*
d^2*e^2 + 32*a^8*c^3*d^2*e^2 - 2*a^5*b^5*c*d^3*e - 32*a^7*b*c^3*d^3*e + 16*
a^7*b^3*c*d*e^3 - 32*a^8*b*c^2*d*e^3 + 16*a^6*b^3*c^2*d^3*e - 6*a^6*b^4*c*d
^2*e^2)))^(1/2) - ((-(b^9*e^2 + b^7*c^2*d^2 + b^6*e^2*(-(4*a*c - b^2)^3)^(1
/2) - 9*a*b^5*c^3*d^2 - 20*a^3*b*c^5*d^2 + 28*a^4*b*c^4*e^2 - 2*b^8*c*d*e +
 25*a^2*b^3*c^4*d^2 + a^2*c^4*d^2*(-(4*a*c - b^2)^3)^(1/2) + 42*a^2*b^5*c^2
*e^2 - 63*a^3*b^3*c^3*e^2 - a^3*c^3*e^2*(-(4*a*c - b^2)^3)^(1/2) + b^4*c^2*
d^2*(-(4*a*c - b^2)^3)^(1/2) - 11*a*b^7*c*e^2 - 16*a^4*c^5*d*e + 20*a*b^6*c
^2*d*e - 2*b^5*c*d*e*(-(4*a*c - b^2)^3)^(1/2) + 6*a^2*b^2*c^2*e^2*(-(4*a*c
 - b^2)^3)^(1/2) - 5*a*b^4*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - 66*a^2*b^4*c^3*d
*e + 76*a^3*b^2*c^4*d*e - 3*a*b^2*c^3*d^2*(-(4*a*c - b^2)^3)^(1/2) + 8*a*b^
3*c^2*d*e*(-(4*a*c - b^2)^3)^(1/2) - 6*a^2*b*c^3*d*e*(-(4*a*c - b^2)^3)^(1/
2))/(8*(a^7*b^4*e^4 + 16*a^7*c^4*d^4 + 16*a^9*c^2*e^4 - 8*a^8*b^2*c*e^4 - 2
*a^6*b^5*d*e^3 + a^5*b^4*c^2*d^4 - 8*a^6*b^2*c^3*d^4 + a^5*b^6*d^2*e^2 + 32
*a^8*c^3*d^2*e^2 - 2*a^5*b^5*c*d^3*e - 32*a^7*b*c^3*d^3*e + 16*a^7*b^3*c*d*
e^3 - 32*a^8*b*c^2*d*e^3 + 16*a^6*b^3*c^2*d^3*e - 6*a^6*b^4*c*d^2*e^2)))^(1
/2)*(((-(b^9*e^2 + b^7*c^2*d^2 + b^6*e^2*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5
*c^3*d^2 - 20*a^3*b*c^5*d^2 + 28*a^4*b*c^4*e^2 - 2*b^8*c*d*e + 25*a^2*b^3*c
^4*d^2 + a^2*c^4*d^2*(-(4*a*c - b^2)^3)^(1/2) + 42*a^2*b^5*c^2*e^2 - 63*a^3
*b^3*c^3*e^2 - a^3*c^3*e^2*(-(4*a*c - b^2)^3)^(1/2) + b^4*c^2*d^2*(-(4*a*c
 - b^2)^3)^(1/2) - 11*a*b^7*c*e^2 - 16*a^4*c^5*d*e + 20*a*b^6*c^2*d*e - 2*b^
5*c*d*e*(-(4*a*c - b^2)^3)^(1/2) + 6*a^2*b^2*c^2*e^2*(-(4*a*c - b^2)^3)^(1/
2) - 5*a*b^4*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - 66*a^2*b^4*c^3*d*e + 76*a^3*b
^2*c^4*d*e - 3*a*b^2*c^3*d^2*(-(4*a*c - b^2)^3)^(1/2) + 8*a*b^3*c^2*d*e*(-(
4*a*c - b^2)^3)^(1/2) - 6*a^2*b*c^3*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^7*b
^4*e^4 + 16*a^7*c^4*d^4 + 16*a^9*c^2*e^4 - 8*a^8*b^2*c*e^4 - 2*a^6*b^5*d*e^

3 + a^5*b^4*c^2*d^4 - 8*a^6*b^2*c^3*d^4 + a^5*b^6*d^2*e^2 + 32*a^8*c^3*d^2*e^2 - 2*a^5*b^5*c*d^3*e - 32*a^7*b*c^3*d^3*e + 16*a^7*b^3*c*d*e^3 - 32*a^8*b*c^2*d*e^3 + 16*a^6*b^3*c^2*d^3*e - 6*a^6*b^4*c*d^2*e^2)))^(1/2)*(x*(-(b^9*e^2 + b^7*c^2*d^2 + b^6*e^2*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c^3*d^2 - 20*a^3*b*c^5*d^2 + 28*a^4*b*c^4*e^2 - 2*b^8*c*d*e + 25*a^2*b^3*c^4*d^2 + a^2*c^4*d^2*(-(4*a*c - b^2)^3)^(1/2) + 42*a^2*b^5*c^2*e^2 - 63*a^3*b^3*c^3*e^2 - a^3*c^3*e^2*(-(4*a*c - b^2)^3)^(1/2) + b^4*c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) - 11*a*b^7*c*e^2 - 16*a^4*c^5*d*e + 20*a*b^6*c^2*d*e - 2*b^5*c*d*e*(-(4*a*c - b^2)^3)^(1/2) + 6*a^2*b^2*c^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - 5*a*b^4*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - 66*a^2*b^4*c^3*d*e + 76*a^3*b^2*c^4*d*e - 3*a*b^2*c^3*d^2*(-(4*a*c - b^2)^3)^(1/2) + 8*a*b^3*c^2*d*e*(-(4*a*c - b^2)^3)^(1/2) - 6*a^2*b*c^3*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^7*b^4*e^4 + 16*a^7*c^4*d^4 + 16*a^9*c^2*e^4 - 8*a^8*b^2*c*e^4 - 2*a^6*b^5*d*e^3 + a^5*b^4*c^2*d^4 - 8*a^6*b^2*c^3*d^4 + a^5*b^6*d^2*e^2 + 32*a^8*c^3*d^2*e^2 - 2*a^5*b^5*c*d^3*e - 32*a^7*b*c^3*d^3*e + 16*a^7*b^3*c*d*e^3 - 32*a^8*b*c^2*d*e^3 + 16*a^6*b^3*c^2*d^3*e - 6*a^6*b^4*c*d^2*e^2)))^(1/2)*(512*a^20*c^7*d^24*e^3 + 512*a^21*c^6*d^22*e^5 - 512*a^22*c^5*d^20*e^7 - 512*a^23*c^4*d^18*e^9 - 32*a^18*b^3*c^6*d^25*e^2 + 128*a^18*b^4*c^5*d^24*e^3 - 192*a^18*b^5*c^4*d^23*e^4 + 128*a^18*b^6*c^3*d^22*e^5 - 32*a^18*b^7*c^2*d^21*e^6 - 640*a^19*b^2*c^6*d^24*e^3 + 1056*a^19*b^3*c^5*d^23*e^4 - 672*a^19*b^4*c^4*d^22*e^5 + 96*a^19*b^5*c^3*d^21*e^6 + 32*a^19*b^6*c^2*d^20*e^7 + 512*a^20*b^2*c^5*d^22*e^5 + 288*a^20*b^3*c^4*d^21*e^6 - 192*a^20*b^4*c^3*d^20*e^7 + 32*a^20*b^5*c^2*d^19*e^8 + 384*a^21*b^2*c^4*d^20*e^7 - 288*a^21*b^3*c^3*d^19*e^8 - 32*a^21*b^4*c^2*d^18*e^9 + 256*a^22*b^2*c^3*d^18*e^9 + 128*a^19*b*c^7*d^25*e^2 - 1152*a^20*b*c^6*d^23*e^4 - 640*a^21*b*c^5*d^21*e^6 + 640*a^22*b*c^4*d^19*e^8) - 64*a^18*c^8*d^24*e^2 + 128*a^19*c^7*d^22*e^4 + 192*a^20*c^6*d^20*e^6 - 256*a^21*c^5*d^18*e^8 - 256*a^22*c^4*d^16*e^10 - 16*a^16*b^4*c^6*d^24*e^2 + 64*a^16*b^5*c^5*d^23*e^3 - 96*a^16*b^6*c^4*d^22*e^4 + 64*a^16*b^7*c^3*d^21*e^5 - 16*a^16*b^8*c^2*d^20*e^6 + 80*a^17*b^2*c^7*d^24*e^2 - 368*a^17*b^3*c^6*d^23*e^3 + 608*a^17*b^4*c^5*d^22*e^4 - 416*a^17*b^5*c^4*d^21*e^5 + 80*a^17*b^6*c^3*d^20*e^6 + 16*a^17*b^7*c^2*d^19*e^7 - 928*a^18*b^2*c^6*d^22*e^4 + 640*a^18*b^3*c^5*d^21*e^5 + 32*a^18*b^4*c^4*d^20*e^6 - 128*a^18*b^5*c^3*d^19*e^7 - 432*a^19*b^2*c^5*d^20*e^6 + 304*a^19*b^3*c^4*d^19*e^7 - 16*a^19*b^4*c^3*d^18*e^8 + 16*a^19*b^5*c^2*d^17*e^9 + 128*a^20*b^2*c^4*d^18*e^8 - 128*a^20*b^3*c^3*d^17*e^9 - 16*a^20*b^4*c^2*d^16*e^10 + 128*a^21*b^2*c^3*d^16*e^10 + 448*a^18*b*c^7*d^23*e^3 - 192*a^20*b*c^5*d^19*e^7 + 256*a^21*b*c^4*d^17*e^9) - x*(16*a^16*c^9*d^23*e^2 + 32*a^17*c^8*d^21*e^4 - 112*a^18*c^7*d^19*e^6 - 128*a^20*c^5*d^15*e^10 + 8*a^14*b^4*c^7*d^23*e^2 - 16*a^14*b^5*c^6*d^22*e^3 + 8*a^14*b^6*c^5*d^21*e^4 + 8*a^14*b^7*c^4*d^20*e^5 - 16*a^14*b^8*c^3*d^19*e^6 + 8*a^14*b^9*c^2*d^18*e^7 - 32*a^15*b^2*c^8*d^23*e^2 + 64*a^15*b^3*c^7*d^22*e^3 - 16*a^15*b^4*c^6*d^21*e^4 - 88*a^15*b^5*c^5*d^20*e^5 + 160*a^15*b^6*c^4*d^19*e^6 - 88*a^15*b^7*c^3*d^18*e^7 - 48*a^16*b^2*c^7*d^21*e^4 + 264*a^16*b^3*c^6*d^20*e^5 - 520*a^16*b^4*c^5*d^19*e^6 + 336*a^16*b^5*c^4*d^18*e^7 + 576*a^17*b^2*c^6*d^19*e^6 - 504*a^17*b^3*c^5*d^18*e^7 + 8*a^18*b^3*c^4*d^16*e^9 - 16*a^18*b^4*c^3*d^15*e^10 + 8*a^18*b^5*c^2*d^14*e

^11 + 96*a^19*b^2*c^4*d^15*e^10 - 56*a^19*b^3*c^3*d^14*e^11 - 32*a^16*b*c^8 *d^22*e^3 - 192*a^17*b*c^7*d^20*e^5 + 224*a^18*b*c^6*d^18*e^7 - 32*a^19*b*c ^5*d^16*e^9 + 96*a^20*b*c^4*d^14*e^11))*(-(b^9*e^2 + b^7*c^2*d^2 + b^6*e^2* (-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c^3*d^2 - 20*a^3*b*c^5*d^2 + 28*a^4*b*c^ 4*e^2 - 2*b^8*c*d*e + 25*a^2*b^3*c^4*d^2 + a^2*c^4*d^2*(-(4*a*c - b^2)^3)^(1/2) + 42*a^2*b^5*c^2*e^2 - 63*a^3*b^3*c^3*e^2 - a^3*c^3*e^2*(-(4*a*c - b^2)^3)^(1/2) + b^4*c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) - 11*a*b^7*c*e^2 - 16*a^4 *c^5*d*e + 20*a*b^6*c^2*d*e - 2*b^5*c*d*e*(-(4*a*c - b^2)^3)^(1/2) + 6*a^2* b^2*c^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - 5*a*b^4*c*e^2*(-(4*a*c - b^2)^3)^(1/ 2) - 66*a^2*b^4*c^3*d*e + 76*a^3*b^2*c^4*d*e - 3*a*b^2*c^3*d^2*(-(4*a*c - b ^2)^3)^(1/2) + 8*a*b^3*c^2*d*e*(-(4*a*c - b^2)^3)^(1/2) - 6*a^2*b*c^3*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^7*b^4*e^4 + 16*a^7*c^4*d^4 + 16*a^9*c^2*e^4 - 8*a^8*b^2*c*e^4 - 2*a^6*b^5*d*e^3 + a^5*b^4*c^2*d^4 - 8*a^6*b^2*c^3*d^4 + a^5*b^6*d^2*e^2 + 32*a^8*c^3*d^2*e^2 - 2*a^5*b^5*c*d^3*e - 32*a^7*b*c^3*d^ 3*e + 16*a^7*b^3*c*d*e^3 - 32*a^8*b*c^2*d*e^3 + 16*a^6*b^3*c^2*d^3*e - 6*a^ 6*b^4*c*d^2*e^2)))^(1/2) - 4*a^15*c^9*d^21*e^3 - 4*a^16*c^8*d^19*e^5 + 48*a ^18*c^6*d^15*e^9 - 4*a^14*b^2*c^8*d^21*e^3 - 4*a^14*b^7*c^3*d^16*e^8 + 4*a^ 14*b^8*c^2*d^15*e^9 + 36*a^15*b^5*c^4*d^16*e^8 - 44*a^15*b^6*c^3*d^15*e^9 + 4*a^15*b^7*c^2*d^14*e^10 - 100*a^16*b^3*c^5*d^16*e^8 + 160*a^16*b^4*c^4*d^ 15*e^9 - 32*a^16*b^5*c^3*d^14*e^10 - 204*a^17*b^2*c^5*d^15*e^9 + 76*a^17*b^ 3*c^4*d^14*e^10 + 4*a^14*b*c^9*d^22*e^2 + 8*a^15*b*c^8*d^20*e^4 + 80*a^17*b *c^6*d^16*e^8 - 48*a^18*b*c^5*d^14*e^10) - x*(2*a^14*c^9*d^18*e^5 + 4*a^16* c^7*d^14*e^9 + 2*a^14*b^4*c^5*d^14*e^9 - 8*a^15*b^2*c^6*d^14*e^9))*(-(b^9*e ^2 + b^7*c^2*d^2 + b^6*e^2*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c^3*d^2 - 20* a^3*b*c^5*d^2 + 28*a^4*b*c^4*e^2 - 2*b^8*c*d*e + 25*a^2*b^3*c^4*d^2 + a^2*c ^4*d^2*(-(4*a*c - b^2)^3)^(1/2) + 42*a^2*b^5*c^2*e^2 - 63*a^3*b^3*c^3*e^2 - a^3*c^3*e^2*(-(4*a*c - b^2)^3)^(1/2) + b^4*c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) - 11*a*b^7*c*e^2 - 16*a^4*c^5*d*e + 20*a*b^6*c^2*d*e - 2*b^5*c*d*e*(-(4*a *c - b^2)^3)^(1/2) + 6*a^2*b^2*c^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - 5*a*b^4*c *e^2*(-(4*a*c - b^2)^3)^(1/2) - 66*a^2*b^4*c^3*d*e + 76*a^3*b^2*c^4*d*e - 3 *a*b^2*c^3*d^2*(-(4*a*c - b^2)^3)^(1/2) + 8*a*b^3*c^2*d*e*(-(4*a*c - b^2)^3)^(1/2) - 6*a^2*b*c^3*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^7*b^4*e^4 + 16*a^ 7*c^4*d^4 + 16*a^9*c^2*e^4 - 8*a^8*b^2*c*e^4 - 2*a^6*b^5*d*e^3 + a^5*b^4*c^ 2*d^4 - 8*a^6*b^2*c^3*d^4 + a^5*b^6*d^2*e^2 + 32*a^8*c^3*d^2*e^2 - 2*a^5*b^ 5*c*d^3*e - 32*a^7*b*c^3*d^3*e + 16*a^7*b^3*c*d*e^3 - 32*a^8*b*c^2*d*e^3 + 16*a^6*b^3*c^2*d^3*e - 6*a^6*b^4*c*d^2*e^2)))^(1/2) + 2*a^14*c^8*d^14*e^8)) *(-(b^9*e^2 + b^7*c^2*d^2 + b^6*e^2*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c^3* d^2 - 20*a^3*b*c^5*d^2 + 28*a^4*b*c^4*e^2 - 2*b^8*c*d*e + 25*a^2*b^3*c^4*d^ 2 + a^2*c^4*d^2*(-(4*a*c - b^2)^3)^(1/2) + 42*a^2*b^5*c^2*e^2 - 63*a^3*b^3* c^3*e^2 - a^3*c^3*e^2*(-(4*a*c - b^2)^3)^(1/2) + b^4*c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) - 11*a*b^7*c*e^2 - 16*a^4*c^5*d*e + 20*a*b^6*c^2*d*e - 2*b^5*c*d *e*(-(4*a*c - b^2)^3)^(1/2) + 6*a^2*b^2*c^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - 5*a*b^4*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - 66*a^2*b^4*c^3*d*e + 76*a^3*b^2*c^ 4*d*e - 3*a*b^2*c^3*d^2*(-(4*a*c - b^2)^3)^(1/2) + 8*a*b^3*c^2*d*e*(-(4*a*c - b^2)^3)^(1/2) - 6*a^2*b*c^3*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^7*b^4*e^

```
4 + 16*a^7*c^4*d^4 + 16*a^9*c^2*e^4 - 8*a^8*b^2*c*e^4 - 2*a^6*b^5*d*e^3 + a
^5*b^4*c^2*d^4 - 8*a^6*b^2*c^3*d^4 + a^5*b^6*d^2*e^2 + 32*a^8*c^3*d^2*e^2 -
 2*a^5*b^5*c*d^3*e - 32*a^7*b*c^3*d^3*e + 16*a^7*b^3*c*d*e^3 - 32*a^8*b*c^2
*d*e^3 + 16*a^6*b^3*c^2*d^3*e - 6*a^6*b^4*c*d^2*e^2)))^(1/2)*2i
```

3.310 $\int \frac{1}{\sqrt{fx}\left(d+ex^2\right)\left(a+bx^2+cx^4\right)}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 866

$$\int \frac{1}{\sqrt{fx}\,(d+ex^2)\,(a+bx^2+cx^4)}\,dx$$
$$= \frac{c^{3/4}\left(2cd-\left(b-\sqrt{b^2-4ac}\right)e\right)\arctan\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{fx}}{\sqrt[4]{-b-\sqrt{b^2-4ac}}\sqrt{f}}\right)}{\sqrt[4]{2}\sqrt{b^2-4ac}\left(-b-\sqrt{b^2-4ac}\right)^{3/4}(cd^2-bde+ae^2)\sqrt{f}}$$
$$-\frac{c^{3/4}\left(2cd-\left(b+\sqrt{b^2-4ac}\right)e\right)\arctan\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{fx}}{\sqrt[4]{-b+\sqrt{b^2-4ac}}\sqrt{f}}\right)}{\sqrt[4]{2}\sqrt{b^2-4ac}\left(-b+\sqrt{b^2-4ac}\right)^{3/4}(cd^2-bde+ae^2)\sqrt{f}}$$
$$-\frac{e^{7/4}\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{e}\sqrt{fx}}{\sqrt[4]{d}\sqrt{f}}\right)}{\sqrt{2}d^{3/4}(cd^2-bde+ae^2)\sqrt{f}}+\frac{e^{7/4}\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{e}\sqrt{fx}}{\sqrt[4]{d}\sqrt{f}}\right)}{\sqrt{2}d^{3/4}(cd^2-bde+ae^2)\sqrt{f}}$$
$$+\frac{c^{3/4}\left(2cd-\left(b-\sqrt{b^2-4ac}\right)e\right)\text{arctanh}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{fx}}{\sqrt[4]{-b-\sqrt{b^2-4ac}}\sqrt{f}}\right)}{\sqrt[4]{2}\sqrt{b^2-4ac}\left(-b-\sqrt{b^2-4ac}\right)^{3/4}(cd^2-bde+ae^2)\sqrt{f}}$$
$$-\frac{c^{3/4}\left(2cd-\left(b+\sqrt{b^2-4ac}\right)e\right)\text{arctanh}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{fx}}{\sqrt[4]{-b+\sqrt{b^2-4ac}}\sqrt{f}}\right)}{\sqrt[4]{2}\sqrt{b^2-4ac}\left(-b+\sqrt{b^2-4ac}\right)^{3/4}(cd^2-bde+ae^2)\sqrt{f}}$$
$$-\frac{e^{7/4}\log\left(\sqrt{d}\sqrt{f}+\sqrt{e}\sqrt{f}x-\sqrt{2}\sqrt[4]{d}\sqrt[4]{e}\sqrt{fx}\right)}{2\sqrt{2}d^{3/4}(cd^2-bde+ae^2)\sqrt{f}}$$
$$+\frac{e^{7/4}\log\left(\sqrt{d}\sqrt{f}+\sqrt{e}\sqrt{f}x+\sqrt{2}\sqrt[4]{d}\sqrt[4]{e}\sqrt{fx}\right)}{2\sqrt{2}d^{3/4}(cd^2-bde+ae^2)\sqrt{f}}$$

```
[Out] -1/2*e^(7/4)*arctan(1-e^(1/4)*2^(1/2)*(f*x)^(1/2)/d^(1/4)/f^(1/2))/d^(3/4)/
(a*e^2-b*d*e+c*d^2)*2^(1/2)/f^(1/2)+1/2*e^(7/4)*arctan(1+e^(1/4)*2^(1/2)*(f
*x)^(1/2)/d^(1/4)/f^(1/2))/d^(3/4)/(a*e^2-b*d*e+c*d^2)*2^(1/2)/f^(1/2)-1/4*
e^(7/4)*ln(d^(1/2)*f^(1/2)+x*e^(1/2)*f^(1/2)-d^(1/4)*e^(1/4)*2^(1/2)*(f*x)^
(1/2))/d^(3/4)/(a*e^2-b*d*e+c*d^2)*2^(1/2)/f^(1/2)+1/4*e^(7/4)*ln(d^(1/2)*f
^(1/2)+x*e^(1/2)*f^(1/2)+d^(1/4)*e^(1/4)*2^(1/2)*(f*x)^(1/2))/d^(3/4)/(a*e^
2-b*d*e+c*d^2)*2^(1/2)/f^(1/2)+1/2*c^(3/4)*arctan(2^(1/4)*c^(1/4)*(f*x)^(1/
2)/(-b-(-4*a*c+b^2)^(1/2))^(1/4)/f^(1/2))*(2*c*d-e*(b-(-4*a*c+b^2)^(1/2)))*
2^(3/4)/(a*e^2-b*d*e+c*d^2)/(-b-(-4*a*c+b^2)^(1/2))^(3/4)/(-4*a*c+b^2)^(1/2
)/f^(1/2)+1/2*c^(3/4)*arctanh(2^(1/4)*c^(1/4)*(f*x)^(1/2)/(-b-(-4*a*c+b^2)^
```

```
(1/2))^(1/4)/f^(1/2))*(2*c*d-e*(b-(-4*a*c+b^2)^(1/2)))*2^(3/4)/(a*e^2-b*d*e
+c*d^2)/(-b-(-4*a*c+b^2)^(1/2))^(3/4)/(-4*a*c+b^2)^(1/2)/f^(1/2)-1/2*c^(3/4
)*arctan(2^(1/4)*c^(1/4)*(f*x)^(1/2)/(-b+(-4*a*c+b^2)^(1/2))^(1/4)/f^(1/2))
*(2*c*d-e*(b+(-4*a*c+b^2)^(1/2)))*2^(3/4)/(a*e^2-b*d*e+c*d^2)/(-4*a*c+b^2)^
(1/2)/(-b+(-4*a*c+b^2)^(1/2))^(3/4)/f^(1/2)-1/2*c^(3/4)*arctanh(2^(1/4)*c^(
1/4)*(f*x)^(1/2)/(-b+(-4*a*c+b^2)^(1/2))^(1/4)/f^(1/2))*(2*c*d-e*(b+(-4*a*c
+b^2)^(1/2)))*2^(3/4)/(a*e^2-b*d*e+c*d^2)/(-4*a*c+b^2)^(1/2)/(-b+(-4*a*c+b^
2)^(1/2))^(3/4)/f^(1/2)
```

Rubi [A] (verified)

Time = 1.88 (sec) , antiderivative size = 866, normalized size of antiderivative = 1.00, number of steps used = 19, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.387, Rules used = {1283, 1438, 217, 1179, 642, 1176, 631, 210, 1436, 218, 214, 211}

$$\int \frac{1}{\sqrt{fx}\,(d+ex^2)\,(a+bx^2+cx^4)}\,dx$$
$$= -\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{e}\sqrt{fx}}{\sqrt[4]{d}\sqrt{f}}\right)e^{7/4}}{\sqrt{2}d^{3/4}\,(cd^2-bed+ae^2)\sqrt{f}}+\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{e}\sqrt{fx}}{\sqrt[4]{d}\sqrt{f}}+1\right)e^{7/4}}{\sqrt{2}d^{3/4}\,(cd^2-bed+ae^2)\sqrt{f}}$$
$$-\frac{\log\left(\sqrt{e}\sqrt{f}x+\sqrt{d}\sqrt{f}-\sqrt{2}\sqrt[4]{d}\sqrt[4]{e}\sqrt{fx}\right)e^{7/4}}{2\sqrt{2}d^{3/4}\,(cd^2-bed+ae^2)\sqrt{f}}$$
$$+\frac{\log\left(\sqrt{e}\sqrt{f}x+\sqrt{d}\sqrt{f}+\sqrt{2}\sqrt[4]{d}\sqrt[4]{e}\sqrt{fx}\right)e^{7/4}}{2\sqrt{2}d^{3/4}\,(cd^2-bed+ae^2)\sqrt{f}}$$
$$+\frac{c^{3/4}\left(2cd-\left(b-\sqrt{b^2-4ac}\right)e\right)\arctan\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{fx}}{\sqrt[4]{-b-\sqrt{b^2-4ac}}\sqrt{f}}\right)}{\sqrt[4]{2}\sqrt{b^2-4ac}\left(-b-\sqrt{b^2-4ac}\right)^{3/4}(cd^2-bed+ae^2)\sqrt{f}}$$
$$-\frac{c^{3/4}\left(2cd-\left(b+\sqrt{b^2-4ac}\right)e\right)\arctan\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{fx}}{\sqrt[4]{\sqrt{b^2-4ac}-b}\sqrt{f}}\right)}{\sqrt[4]{2}\sqrt{b^2-4ac}\left(\sqrt{b^2-4ac}-b\right)^{3/4}(cd^2-bed+ae^2)\sqrt{f}}$$
$$+\frac{c^{3/4}\left(2cd-\left(b-\sqrt{b^2-4ac}\right)e\right)\operatorname{arctanh}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{fx}}{\sqrt[4]{-b-\sqrt{b^2-4ac}}\sqrt{f}}\right)}{\sqrt[4]{2}\sqrt{b^2-4ac}\left(-b-\sqrt{b^2-4ac}\right)^{3/4}(cd^2-bed+ae^2)\sqrt{f}}$$
$$-\frac{c^{3/4}\left(2cd-\left(b+\sqrt{b^2-4ac}\right)e\right)\operatorname{arctanh}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{fx}}{\sqrt[4]{\sqrt{b^2-4ac}-b}\sqrt{f}}\right)}{\sqrt[4]{2}\sqrt{b^2-4ac}\left(\sqrt{b^2-4ac}-b\right)^{3/4}(cd^2-bed+ae^2)\sqrt{f}}$$

```
[In] Int[1/(Sqrt[f*x]*(d + e*x^2)*(a + b*x^2 + c*x^4)),x]
```

```
[Out] (c^(3/4)*(2*c*d - (b - Sqrt[b^2 - 4*a*c])*e)*ArcTan[(2^(1/4)*c^(1/4)*Sqrt[f
*x])/((-b - Sqrt[b^2 - 4*a*c])^(1/4)*Sqrt[f])])/(2^(1/4)*Sqrt[b^2 - 4*a*c]*
(-b - Sqrt[b^2 - 4*a*c])^(3/4)*(c*d^2 - b*d*e + a*e^2)*Sqrt[f]) - (c^(3/4)*
(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)*ArcTan[(2^(1/4)*c^(1/4)*Sqrt[f*x])/((-b
 + Sqrt[b^2 - 4*a*c])^(1/4)*Sqrt[f])])/(2^(1/4)*Sqrt[b^2 - 4*a*c]*(-b + Sqr
t[b^2 - 4*a*c])^(3/4)*(c*d^2 - b*d*e + a*e^2)*Sqrt[f]) - (e^(7/4)*ArcTan[1
- (Sqrt[2]*e^(1/4)*Sqrt[f*x])/(d^(1/4)*Sqrt[f])])/(Sqrt[2]*d^(3/4)*(c*d^2 -
 b*d*e + a*e^2)*Sqrt[f]) + (e^(7/4)*ArcTan[1 + (Sqrt[2]*e^(1/4)*Sqrt[f*x])/
(d^(1/4)*Sqrt[f])])/(Sqrt[2]*d^(3/4)*(c*d^2 - b*d*e + a*e^2)*Sqrt[f]) + (c^
(3/4)*(2*c*d - (b - Sqrt[b^2 - 4*a*c])*e)*ArcTanh[(2^(1/4)*c^(1/4)*Sqrt[f*x
])/((-b - Sqrt[b^2 - 4*a*c])^(1/4)*Sqrt[f])])/(2^(1/4)*Sqrt[b^2 - 4*a*c]*(-
b - Sqrt[b^2 - 4*a*c])^(3/4)*(c*d^2 - b*d*e + a*e^2)*Sqrt[f]) - (c^(3/4)*(2
*c*d - (b + Sqrt[b^2 - 4*a*c])*e)*ArcTanh[(2^(1/4)*c^(1/4)*Sqrt[f*x])/((-b
+ Sqrt[b^2 - 4*a*c])^(1/4)*Sqrt[f])])/(2^(1/4)*Sqrt[b^2 - 4*a*c]*(-b + Sqrt
[b^2 - 4*a*c])^(3/4)*(c*d^2 - b*d*e + a*e^2)*Sqrt[f]) - (e^(7/4)*Log[Sqrt[d
]*Sqrt[f] + Sqrt[e]*Sqrt[f]*x - Sqrt[2]*d^(1/4)*e^(1/4)*Sqrt[f*x]])/(2*Sqrt
[2]*d^(3/4)*(c*d^2 - b*d*e + a*e^2)*Sqrt[f]) + (e^(7/4)*Log[Sqrt[d]*Sqrt[f]
 + Sqrt[e]*Sqrt[f]*x + Sqrt[2]*d^(1/4)*e^(1/4)*Sqrt[f*x]])/(2*Sqrt[2]*d^(3/
4)*(c*d^2 - b*d*e + a*e^2)*Sqrt[f])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 218

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x], x]
+ Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a/b
, 0]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1283

```
Int[((f_.)*(x_))^(m_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c
_.)*(x_)^4)^(p_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k/f, Subst[I
nt[x^(k*(m + 1) - 1)*(d + e*(x^(2*k)/f^2))^q*(a + b*(x^(2*k)/f^k) + c*(x^(4
*k)/f^4))^p, x], x, (f*x)^(1/k)], x]] /; FreeQ[{a, b, c, d, e, f, p, q}, x]
 && NeQ[b^2 - 4*a*c, 0] && FractionQ[m] && IntegerQ[p]
```

Rule 1436

```
Int[((d_) + (e_.)*(x_)^(n_))/((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_)), x
_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q),
Int[1/(b/2 - q/2 + c*x^n), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(
b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[n2, 2*n]
```

```
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && (PosQ[b^2 - 4*a
*c] ||  !IGtQ[n/2, 0])
```

Rule 1438

```
Int[((d_) + (e_.)*(x_)^(n_))^(q_)/((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_
)), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^n)^q/(a + b*x^n + c*x^(2*n)),
 x], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c,
 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[q]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{2\text{Subst}\left(\int \frac{1}{\left(d+\frac{ex^4}{f^2}\right)\left(a+\frac{bx^4}{f^2}+\frac{cx^8}{f^4}\right)}\,dx, x, \sqrt{fx}\right)}{f} \\
&= \frac{2\text{Subst}\left(\int \left(\frac{e^2f^2}{(cd^2-bde+ae^2)(df^2+ex^4)}+\frac{cdf^4-bef^4-cef^2x^4}{(cd^2-bde+ae^2)(af^4+bf^2x^4+cx^8)}\right)dx, x, \sqrt{fx}\right)}{f} \\
&= \frac{2\text{Subst}\left(\int \frac{cdf^4-bef^4-cef^2x^4}{af^4+bf^2x^4+cx^8}\,dx, x, \sqrt{fx}\right)}{(cd^2-bde+ae^2)\,f}+\frac{(2e^2f)\,\text{Subst}\left(\int \frac{1}{df^2+ex^4}\,dx, x, \sqrt{fx}\right)}{cd^2-bde+ae^2} \\
&= \frac{e^2\text{Subst}\left(\int \frac{\sqrt{d}f-\sqrt{e}x^2}{df^2+ex^4}\,dx, x, \sqrt{fx}\right)}{\sqrt{d}\,(cd^2-bde+ae^2)}+\frac{e^2\text{Subst}\left(\int \frac{\sqrt{d}f+\sqrt{e}x^2}{df^2+ex^4}\,dx, x, \sqrt{fx}\right)}{\sqrt{d}\,(cd^2-bde+ae^2)} \\
&\quad -\frac{\left(c\left(2cd-\left(b-\sqrt{b^2-4ac}\right)e\right)f\right)\text{Subst}\left(\int \frac{1}{\frac{bf^2}{2}+\frac{1}{2}\sqrt{b^2-4ac}f^2+cx^4}\,dx, x, \sqrt{fx}\right)}{\sqrt{b^2-4ac}\,(cd^2-bde+ae^2)} \\
&\quad +\frac{\left(c\left(2cd-\left(b+\sqrt{b^2-4ac}\right)e\right)f\right)\text{Subst}\left(\int \frac{1}{\frac{bf^2}{2}-\frac{1}{2}\sqrt{b^2-4ac}f^2+cx^4}\,dx, x, \sqrt{fx}\right)}{\sqrt{b^2-4ac}\,(cd^2-bde+ae^2)}
\end{aligned}$$

$$= \frac{e^{3/2}\text{Subst}\left(\int \frac{1}{\frac{\sqrt{d}f}{\sqrt{e}}-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{f}x}{\sqrt[4]{e}}+x^2}dx,x,\sqrt{fx}\right)}{2\sqrt{d}\,(cd^2-bde+ae^2)}$$
$$+\frac{e^{3/2}\text{Subst}\left(\int \frac{1}{\frac{\sqrt{d}f}{\sqrt{e}}+\frac{\sqrt{2}\sqrt[4]{d}\sqrt{f}x}{\sqrt[4]{e}}+x^2}dx,x,\sqrt{fx}\right)}{2\sqrt{d}\,(cd^2-bde+ae^2)}$$
$$+\frac{\left(c\left(2cd-\left(b-\sqrt{b^2-4ac}\right)e\right)\right)\text{Subst}\left(\int \frac{1}{\sqrt{-b-\sqrt{b^2-4ac}}f-\sqrt{2}\sqrt{c}x^2}dx,x,\sqrt{fx}\right)}{\sqrt{b^2-4ac}\sqrt{-b-\sqrt{b^2-4ac}}\,(cd^2-bde+ae^2)}$$
$$+\frac{\left(c\left(2cd-\left(b-\sqrt{b^2-4ac}\right)e\right)\right)\text{Subst}\left(\int \frac{1}{\sqrt{-b-\sqrt{b^2-4ac}}f+\sqrt{2}\sqrt{c}x^2}dx,x,\sqrt{fx}\right)}{\sqrt{b^2-4ac}\sqrt{-b-\sqrt{b^2-4ac}}\,(cd^2-bde+ae^2)}$$
$$-\frac{\left(c\left(2cd-\left(b+\sqrt{b^2-4ac}\right)e\right)\right)\text{Subst}\left(\int \frac{1}{\sqrt{-b+\sqrt{b^2-4ac}}f-\sqrt{2}\sqrt{c}x^2}dx,x,\sqrt{fx}\right)}{\sqrt{b^2-4ac}\sqrt{-b+\sqrt{b^2-4ac}}\,(cd^2-bde+ae^2)}$$
$$-\frac{\left(c\left(2cd-\left(b+\sqrt{b^2-4ac}\right)e\right)\right)\text{Subst}\left(\int \frac{1}{\sqrt{-b+\sqrt{b^2-4ac}}f+\sqrt{2}\sqrt{c}x^2}dx,x,\sqrt{fx}\right)}{\sqrt{b^2-4ac}\sqrt{-b+\sqrt{b^2-4ac}}\,(cd^2-bde+ae^2)}$$
$$-\frac{e^{7/4}\text{Subst}\left(\int \frac{\frac{\sqrt{2}\sqrt[4]{d}\sqrt{f}}{\sqrt[4]{e}}+2x}{-\frac{\sqrt{d}f}{\sqrt{e}}-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{f}x}{\sqrt[4]{e}}-x^2}dx,x,\sqrt{fx}\right)}{2\sqrt{2}d^{3/4}\,(cd^2-bde+ae^2)\sqrt{f}}$$
$$-\frac{e^{7/4}\text{Subst}\left(\int \frac{\frac{\sqrt{2}\sqrt[4]{d}\sqrt{f}}{\sqrt[4]{e}}-2x}{-\frac{\sqrt{d}f}{\sqrt{e}}+\frac{\sqrt{2}\sqrt[4]{d}\sqrt{f}x}{\sqrt[4]{e}}-x^2}dx,x,\sqrt{fx}\right)}{2\sqrt{2}d^{3/4}\,(cd^2-bde+ae^2)\sqrt{f}}$$

$$
\begin{aligned}
&= \frac{c^{3/4}\left(2cd-\left(b-\sqrt{b^2-4ac}\right)e\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{fx}}{\sqrt[4]{-b-\sqrt{b^2-4ac}}\sqrt{f}}\right)}{\sqrt[4]{2}\sqrt{b^2-4ac}\left(-b-\sqrt{b^2-4ac}\right)^{3/4}\left(cd^2-bde+ae^2\right)\sqrt{f}} \\
&- \frac{c^{3/4}\left(2cd-\left(b+\sqrt{b^2-4ac}\right)e\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{fx}}{\sqrt[4]{-b+\sqrt{b^2-4ac}}\sqrt{f}}\right)}{\sqrt[4]{2}\sqrt{b^2-4ac}\left(-b+\sqrt{b^2-4ac}\right)^{3/4}\left(cd^2-bde+ae^2\right)\sqrt{f}} \\
&+ \frac{c^{3/4}\left(2cd-\left(b-\sqrt{b^2-4ac}\right)e\right)\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{fx}}{\sqrt[4]{-b-\sqrt{b^2-4ac}}\sqrt{f}}\right)}{\sqrt[4]{2}\sqrt{b^2-4ac}\left(-b-\sqrt{b^2-4ac}\right)^{3/4}\left(cd^2-bde+ae^2\right)\sqrt{f}} \\
&- \frac{c^{3/4}\left(2cd-\left(b+\sqrt{b^2-4ac}\right)e\right)\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{fx}}{\sqrt[4]{-b+\sqrt{b^2-4ac}}\sqrt{f}}\right)}{\sqrt[4]{2}\sqrt{b^2-4ac}\left(-b+\sqrt{b^2-4ac}\right)^{3/4}\left(cd^2-bde+ae^2\right)\sqrt{f}} \\
&- \frac{e^{7/4}\log\left(\sqrt{d}\sqrt{f}+\sqrt{e}\sqrt{f}x-\sqrt{2}\sqrt[4]{d}\sqrt[4]{e}\sqrt{fx}\right)}{2\sqrt{2}d^{3/4}\left(cd^2-bde+ae^2\right)\sqrt{f}} \\
&+ \frac{e^{7/4}\log\left(\sqrt{d}\sqrt{f}+\sqrt{e}\sqrt{f}x+\sqrt{2}\sqrt[4]{d}\sqrt[4]{e}\sqrt{fx}\right)}{2\sqrt{2}d^{3/4}\left(cd^2-bde+ae^2\right)\sqrt{f}} \\
&+ \frac{e^{7/4}\mathrm{Subst}\left(\int\frac{1}{-1-x^2}\,dx,x,1-\frac{\sqrt{2}\sqrt[4]{e}\sqrt{fx}}{\sqrt[4]{d}\sqrt{f}}\right)}{\sqrt{2}d^{3/4}\left(cd^2-bde+ae^2\right)\sqrt{f}} \\
&- \frac{e^{7/4}\mathrm{Subst}\left(\int\frac{1}{-1-x^2}\,dx,x,1+\frac{\sqrt{2}\sqrt[4]{e}\sqrt{fx}}{\sqrt[4]{d}\sqrt{f}}\right)}{\sqrt{2}d^{3/4}\left(cd^2-bde+ae^2\right)\sqrt{f}}
\end{aligned}
$$

$$= \frac{c^{3/4}\left(2cd-\left(b-\sqrt{b^2-4ac}\right)e\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{fx}}{\sqrt[4]{-b-\sqrt{b^2-4ac}}\sqrt{f}}\right)}{\sqrt[4]{2}\sqrt{b^2-4ac}\left(-b-\sqrt{b^2-4ac}\right)^{3/4}(cd^2-bde+ae^2)\sqrt{f}}$$

$$- \frac{c^{3/4}\left(2cd-\left(b+\sqrt{b^2-4ac}\right)e\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{fx}}{\sqrt[4]{-b+\sqrt{b^2-4ac}}\sqrt{f}}\right)}{\sqrt[4]{2}\sqrt{b^2-4ac}\left(-b+\sqrt{b^2-4ac}\right)^{3/4}(cd^2-bde+ae^2)\sqrt{f}}$$

$$- \frac{e^{7/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{e}\sqrt{fx}}{\sqrt[4]{d}\sqrt{f}}\right)}{\sqrt{2}d^{3/4}(cd^2-bde+ae^2)\sqrt{f}} + \frac{e^{7/4}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{e}\sqrt{fx}}{\sqrt[4]{d}\sqrt{f}}\right)}{\sqrt{2}d^{3/4}(cd^2-bde+ae^2)\sqrt{f}}$$

$$+ \frac{c^{3/4}\left(2cd-\left(b-\sqrt{b^2-4ac}\right)e\right)\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{fx}}{\sqrt[4]{-b-\sqrt{b^2-4ac}}\sqrt{f}}\right)}{\sqrt[4]{2}\sqrt{b^2-4ac}\left(-b-\sqrt{b^2-4ac}\right)^{3/4}(cd^2-bde+ae^2)\sqrt{f}}$$

$$- \frac{c^{3/4}\left(2cd-\left(b+\sqrt{b^2-4ac}\right)e\right)\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{fx}}{\sqrt[4]{-b+\sqrt{b^2-4ac}}\sqrt{f}}\right)}{\sqrt[4]{2}\sqrt{b^2-4ac}\left(-b+\sqrt{b^2-4ac}\right)^{3/4}(cd^2-bde+ae^2)\sqrt{f}}$$

$$- \frac{e^{7/4}\log\left(\sqrt{d}\sqrt{f}+\sqrt{e}\sqrt{f}x-\sqrt{2}\sqrt[4]{d}\sqrt[4]{e}\sqrt{fx}\right)}{2\sqrt{2}d^{3/4}(cd^2-bde+ae^2)\sqrt{f}}$$

$$+ \frac{e^{7/4}\log\left(\sqrt{d}\sqrt{f}+\sqrt{e}\sqrt{f}x+\sqrt{2}\sqrt[4]{d}\sqrt[4]{e}\sqrt{fx}\right)}{2\sqrt{2}d^{3/4}(cd^2-bde+ae^2)\sqrt{f}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.30 (sec) , antiderivative size = 215, normalized size of antiderivative = 0.25

$$\int \frac{1}{\sqrt{fx}\,(d+ex^2)\,(a+bx^2+cx^4)}\,dx =$$

$$-\frac{\sqrt{x}\left(\sqrt{2}e^{7/4}\left(\arctan\left(\frac{\sqrt{d}-\sqrt{e}x}{\sqrt{2}\sqrt[4]{d}\sqrt[4]{e}\sqrt{x}}\right)-\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{d}\sqrt[4]{e}\sqrt{x}}{\sqrt{d}+\sqrt{e}x}\right)\right)+d^{3/4}\text{RootSum}\left[a+b\#1^4+c\#1^8\&,-\right.\right.}{2d^{3/4}\left(cd^2+e(-bd+ae)\right)\sqrt{fx}}$$

```
[In] Integrate[1/(Sqrt[f*x]*(d + e*x^2)*(a + b*x^2 + c*x^4)),x]
```

```
[Out] -1/2*(Sqrt[x]*(Sqrt[2]*e^(7/4)*(ArcTan[(Sqrt[d] - Sqrt[e]*x)/(Sqrt[2]*d^(1/
4)*e^(1/4)*Sqrt[x])] - ArcTanh[(Sqrt[2]*d^(1/4)*e^(1/4)*Sqrt[x])/(Sqrt[d] +
```

```
Sqrt[e]*x)]) + d^(3/4)*RootSum[a + b*#1^4 + c*#1^8 & , (-(c*d*Log[Sqrt[x]
- #1]) + b*e*Log[Sqrt[x] - #1] + c*e*Log[Sqrt[x] - #1]*#1^4)/(b*#1^3 + 2*c*
#1^7) & ]))/(d^(3/4)*(c*d^2 + e*(-(b*d) + a*e))*Sqrt[f*x])
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.99 (sec) , antiderivative size = 264, normalized size of antiderivative = 0.30

method	result
derivativedivides	$2f^5\left(\frac{\sum_{\_R=\mathrm{RootOf}\left(c\_Z^8+bf^2\_Z^4+af^4\right)}\frac{\left(-\_R^4ce-bef^2+cdf^2\right)\ln\left(\sqrt{fx}-\_R\right)}{2\_R^7c+\_R^3bf^2}}{4f^4(ae^2-bde+cd^2)}+\frac{e^2\left(\frac{df^2}{e}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{fx+\left(\frac{df^2}{e}\right.}{fx-\left(\frac{df^2}{e}\right.}\right.\right.}{}\right.$
default	$\frac{\left(\frac{df^2}{e}\right)^{\frac{1}{4}}e^2\left(2\arctan\left(\frac{\sqrt{2}\sqrt{fx}+\left(\frac{df^2}{e}\right)^{\frac{1}{4}}}{\left(\frac{df^2}{e}\right)^{\frac{1}{4}}}\right)+\ln\left(\frac{fx+\left(\frac{df^2}{e}\right)^{\frac{1}{4}}\sqrt{fx}\sqrt{2}+\sqrt{\frac{df^2}{e}}}{fx-\left(\frac{df^2}{e}\right)^{\frac{1}{4}}\sqrt{fx}\sqrt{2}+\sqrt{\frac{df^2}{e}}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{fx}-\left(\frac{df^2}{e}\right)^{\frac{1}{4}}}{\left(\frac{df^2}{e}\right)^{\frac{1}{4}}}\right)\right)\sqrt{2}}{4f(ae^2-bde+cd^2)d}$
pseudoelliptic	$\frac{\left(\frac{df^2}{e}\right)^{\frac{1}{4}}e^2\left(2\arctan\left(\frac{\sqrt{2}\sqrt{fx}+\left(\frac{df^2}{e}\right)^{\frac{1}{4}}}{\left(\frac{df^2}{e}\right)^{\frac{1}{4}}}\right)+\ln\left(\frac{fx+\left(\frac{df^2}{e}\right)^{\frac{1}{4}}\sqrt{fx}\sqrt{2}+\sqrt{\frac{df^2}{e}}}{fx-\left(\frac{df^2}{e}\right)^{\frac{1}{4}}\sqrt{fx}\sqrt{2}+\sqrt{\frac{df^2}{e}}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{fx}-\left(\frac{df^2}{e}\right)^{\frac{1}{4}}}{\left(\frac{df^2}{e}\right)^{\frac{1}{4}}}\right)\right)\sqrt{2}}{4f(ae^2-bde+cd^2)d}$

```
[In] int(1/(e*x^2+d)/(c*x^4+b*x^2+a)/(f*x)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2*f^5*(1/4/f^4/(a*e^2-b*d*e+c*d^2)*sum((-_R^4*c*e-b*e*f^2+c*d*f^2)/(2*_R^7*
c+_R^3*b*f^2)*ln((f*x)^(1/2)-_R),_R=RootOf(_Z^8*c+_Z^4*b*f^2+a*f^4))+1/8*e^
2/f^6/(a*e^2-b*d*e+c*d^2)*(d*f^2/e)^(1/4)/d*2^(1/2)*(ln((f*x+(d*f^2/e)^(1/4
)*(f*x)^(1/2)*2^(1/2)+(d*f^2/e)^(1/2))/(f*x-(d*f^2/e)^(1/4)*(f*x)^(1/2)*2^(
1/2)+(d*f^2/e)^(1/2)))+2*arctan(2^(1/2)/(d*f^2/e)^(1/4)*(f*x)^(1/2)+1)+2*ar
ctan(2^(1/2)/(d*f^2/e)^(1/4)*(f*x)^(1/2)-1)))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{fx}\,(d+ex^2)\,(a+bx^2+cx^4)}\,dx = \text{Timed out}$$

```
[In] integrate(1/(e*x^2+d)/(c*x^4+b*x^2+a)/(f*x)^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]

$$\int \frac{1}{\sqrt{fx}\,(d+ex^2)\,(a+bx^2+cx^4)}\,dx = \int \frac{1}{\sqrt{fx}\,(d+ex^2)\,(a+bx^2+cx^4)}\,dx$$

```
[In] integrate(1/(e*x**2+d)/(c*x**4+b*x**2+a)/(f*x)**(1/2),x)
[Out] Integral(1/(sqrt(f*x)*(d + e*x**2)*(a + b*x**2 + c*x**4)), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{fx}\,(d+ex^2)\,(a+bx^2+cx^4)}\,dx = \int \frac{1}{(cx^4+bx^2+a)(ex^2+d)\sqrt{fx}}\,dx$$

```
[In] integrate(1/(e*x^2+d)/(c*x^4+b*x^2+a)/(f*x)^(1/2),x, algorithm="maxima")
[Out] -2*e^2*sqrt(x)/(c*d^3*sqrt(f) - b*d^2*e*sqrt(f) + a*d*e^2*sqrt(f)) + 1/4*(2
*sqrt(2)*e^2*arctan(1/2*sqrt(2)*(sqrt(2)*d^(1/4)*e^(1/4) + 2*sqrt(e)*sqrt(x
))/sqrt(sqrt(d)*sqrt(e)))/(sqrt(d)*sqrt(sqrt(d)*sqrt(e))) + 2*sqrt(2)*e^2*a
rctan(-1/2*sqrt(2)*(sqrt(2)*d^(1/4)*e^(1/4) - 2*sqrt(e)*sqrt(x))/sqrt(sqrt(
d)*sqrt(e)))/(sqrt(d)*sqrt(sqrt(d)*sqrt(e))) + sqrt(2)*e^(7/4)*log(sqrt(2)*
d^(1/4)*e^(1/4)*sqrt(x) + sqrt(e)*x + sqrt(d))/d^(3/4) - sqrt(2)*e^(7/4)*lo
g(-sqrt(2)*d^(1/4)*e^(1/4)*sqrt(x) + sqrt(e)*x + sqrt(d))/d^(3/4))/(c*d^2*s
qrt(f) - b*d*e*sqrt(f) + a*e^2*sqrt(f)) + 2*sqrt(x)/(a*d*sqrt(f)) + integra
te(-((c^2*d - b*c*e)*x^(7/2) + (b*c*d - b^2*e + a*c*e)*x^(3/2))/(a^3*e^2*sq
rt(f) + (a^2*c*e^2*sqrt(f) + (c^2*d^2*sqrt(f) - b*c*d*e*sqrt(f))*a)*x^4 + (
c*d^2*sqrt(f) - b*d*e*sqrt(f))*a^2 + (a^2*b*e^2*sqrt(f) + (b*c*d^2*sqrt(f)
- b^2*d*e*sqrt(f))*a)*x^2), x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{fx}\,(d+ex^2)\,(a+bx^2+cx^4)}\,dx = \text{Timed out}$$

```
[In] integrate(1/(e*x^2+d)/(c*x^4+b*x^2+a)/(f*x)^(1/2),x, algorithm="giac")
[Out] Timed out
```

Mupad [B] (verification not implemented)

Time = 12.01 (sec) , antiderivative size = 43112, normalized size of antiderivative = 49.78

$$\int \frac{1}{\sqrt{fx}\,(d+ex^2)\,(a+bx^2+cx^4)}\,dx = \text{Too large to display}$$

[In] int(1/((f*x)^(1/2)*(d + e*x^2)*(a + b*x^2 + c*x^4)),x)

[Out] symsum(log(-root(8388608*a^7*b*c^11*d^18*e*f^6*h^12 - 513802240*a^10*b^2*c^7*d^11*e^8*f^6*h^12 - 381681664*a^11*b^2*c^6*d^9*e^10*f^6*h^12 - 381681664*a^9*b^2*c^8*d^13*e^6*f^6*h^12 - 300941312*a^9*b^5*c^5*d^10*e^9*f^6*h^12 - 300941312*a^8*b^5*c^6*d^12*e^7*f^6*h^12 + 293601280*a^10*b^3*c^6*d^10*e^9*f^6*h^12 + 293601280*a^9*b^3*c^7*d^12*e^7*f^6*h^12 - 168820736*a^10*b^5*c^4*d^8*e^11*f^6*h^12 - 168820736*a^7*b^5*c^7*d^14*e^5*f^6*h^12 + 166068224*a^8*b^6*c^5*d^11*e^8*f^6*h^12 - 146800640*a^12*b^2*c^5*d^7*e^12*f^6*h^12 - 146800640*a^8*b^2*c^9*d^15*e^4*f^6*h^12 + 124780544*a^10*b^4*c^5*d^9*e^10*f^6*h^12 + 124780544*a^8*b^4*c^7*d^13*e^6*f^6*h^12 + 119275520*a^9*b^4*c^6*d^11*e^8*f^6*h^12 + 117440512*a^11*b^3*c^5*d^8*e^11*f^6*h^12 + 117440512*a^8*b^3*c^8*d^14*e^5*f^6*h^12 + 102760448*a^9*b^6*c^4*d^9*e^10*f^6*h^12 + 102760448*a^7*b^6*c^6*d^13*e^6*f^6*h^12 + 91750400*a^11*b^4*c^4*d^7*e^12*f^6*h^12 + 91750400*a^7*b^4*c^8*d^15*e^4*f^6*h^12 - 71065600*a^7*b^8*c^4*d^11*e^8*f^6*h^12 - 53444608*a^8*b^8*c^3*d^9*e^10*f^6*h^12 - 53444608*a^6*b^8*c^5*d^13*e^6*f^6*h^12 + 40370176*a^9*b^7*c^3*d^8*e^11*f^6*h^12 + 40370176*a^6*b^7*c^6*d^14*e^5*f^6*h^12 - 36700160*a^11*b^5*c^3*d^6*e^13*f^6*h^12 - 36700160*a^6*b^5*c^8*d^16*e^3*f^6*h^12 + 34078720*a^8*b^7*c^4*d^10*e^9*f^6*h^12 + 34078720*a^7*b^7*c^5*d^12*e^7*f^6*h^12 + 26214400*a^12*b^4*c^3*d^5*e^14*f^6*h^12 + 26214400*a^6*b^4*c^9*d^17*e^2*f^6*h^12 + 22118400*a^7*b^9*c^3*d^10*e^9*f^6*h^12 + 22118400*a^6*b^9*c^4*d^12*e^7*f^6*h^12 - 20971520*a^13*b^2*c^4*d^5*e^14*f^6*h^12 - 20971520*a^7*b^2*c^10*d^17*e^2*f^6*h^12 + 18350080*a^10*b^7*c^2*d^6*e^13*f^6*h^12 + 18350080*a^5*b^7*c^7*d^16*e^3*f^6*h^12 - 16629760*a^9*b^8*c^2*d^7*e^12*f^6*h^12 - 16629760*a^5*b^8*c^6*d^15*e^4*f^6*h^12 - 10485760*a^11*b^6*c^2*d^5*e^14*f^6*h^12 - 10485760*a^5*b^6*c^8*d^17*e^2*f^6*h^12 + 9175040*a^10*b^6*c^3*d^7*e^12*f^6*h^12 + 9175040*a^6*b^6*c^7*d^15*e^4*f^6*h^12 - 8388608*a^13*b^3*c^3*d^4*e^15*f^6*h^12 + 5619712*a^7*b^10*c^2*d^9*e^10*f^6*h^12 + 5619712*a^5*b^10*c^4*d^13*e^6*f^6*h^12 - 5570560*a^6*b^11*c^2*d^10*e^9*f^6*h^12 - 5570560*a^5*b^11*c^3*d^12*e^7*f^6*h^12 + 4358144*a^8*b^9*c^2*d^8*e^11*f^6*h^12 + 4358144*a^5*b^9*c^5*d^14*e^5*f^6*h^12 + 4259840*a^6*b^10*c^3*d^11*e^8*f^6*h^12 + 3899392*a^4*b^10*c^5*d^15*e^4*f^6*h^12 - 3440640*a^4*b^9*c^6*d^16*e^3*f^6*h^12 + 3145728*a^12*b^5*c^2*d^4*e^15*f^6*h^12 - 2523136*a^4*b^11*c^4*d^14*e^5*f^6*h^12 + 1802240*a^4*b^8*c^7*d^17*e^2*f^6*h^12 + 1556480*a^5*b^12*c^2*d^11*e^8*f^6*h^12 + 1048576*a^14*b^2*c^3*d^3*e^16*f^6*h^12 + 688128*a^4*b^12*c^3*d^13*e^6*f^6*h^12 - 393216*a^13*b^4*c^2*d^3*e^16*f^6*h^12 - 286720*a^3*b^12*c^4*d^15*e^4*f^6*h^12 + 229376*a^3*b^13*c^3*d^14*e^5*f^6*h^12 + 229376*a^3*b^11*c^5*d^16*e^3*f^6*h^12 +

163840*a^4*b^13*c^2*d^12*e^7*f^6*h^12 - 114688*a^3*b^14*c^2*d^13*e^6*f^6*h
^12 - 114688*a^3*b^10*c^6*d^17*e^2*f^6*h^12 + 293601280*a^11*b*c^7*d^10*e^9
*f^6*h^12 + 293601280*a^10*b*c^8*d^12*e^7*f^6*h^12 + 176160768*a^12*b*c^6*d
^8*e^11*f^6*h^12 + 176160768*a^9*b*c^9*d^14*e^5*f^6*h^12 + 58720256*a^13*b*
c^5*d^6*e^13*f^6*h^12 + 58720256*a^8*b*c^10*d^16*e^3*f^6*h^12 + 8388608*a^1
4*b*c^4*d^4*e^15*f^6*h^12 - 8388608*a^6*b^3*c^10*d^18*e*f^6*h^12 + 3899392*
a^8*b^10*c*d^7*e^12*f^6*h^12 - 3440640*a^9*b^9*c*d^6*e^13*f^6*h^12 + 314572
8*a^5*b^5*c^9*d^18*e*f^6*h^12 - 2523136*a^7*b^11*c*d^8*e^11*f^6*h^12 + 1802
240*a^10*b^8*c*d^5*e^14*f^6*h^12 + 688128*a^6*b^12*c*d^9*e^10*f^6*h^12 - 52
4288*a^11*b^7*c*d^4*e^15*f^6*h^12 - 524288*a^4*b^7*c^8*d^18*e*f^6*h^12 + 16
3840*a^5*b^13*c*d^10*e^9*f^6*h^12 - 163840*a^4*b^14*c*d^11*e^8*f^6*h^12 + 6
5536*a^12*b^6*c*d^3*e^16*f^6*h^12 + 32768*a^3*b^15*c*d^12*e^7*f^6*h^12 + 32
768*a^3*b^9*c^7*d^18*e*f^6*h^12 - 73400320*a^11*c^8*d^11*e^8*f^6*h^12 - 587
20256*a^12*c^7*d^9*e^10*f^6*h^12 - 58720256*a^10*c^9*d^13*e^6*f^6*h^12 - 29
360128*a^13*c^6*d^7*e^12*f^6*h^12 - 29360128*a^9*c^10*d^15*e^4*f^6*h^12 - 8
388608*a^14*c^5*d^5*e^14*f^6*h^12 - 8388608*a^8*c^11*d^17*e^2*f^6*h^12 - 10
48576*a^15*c^4*d^3*e^16*f^6*h^12 - 286720*a^7*b^12*d^7*e^12*f^6*h^12 + 2293
76*a^8*b^11*d^6*e^13*f^6*h^12 + 229376*a^6*b^13*d^8*e^11*f^6*h^12 - 114688*
a^9*b^10*d^5*e^14*f^6*h^12 - 114688*a^5*b^14*d^9*e^10*f^6*h^12 + 32768*a^10
*b^9*d^4*e^15*f^6*h^12 + 32768*a^4*b^15*d^10*e^9*f^6*h^12 - 4096*a^11*b^8*d
^3*e^16*f^6*h^12 - 4096*a^3*b^16*d^11*e^8*f^6*h^12 + 1048576*a^6*b^2*c^11*d
^19*f^6*h^12 - 393216*a^5*b^4*c^10*d^19*f^6*h^12 + 65536*a^4*b^6*c^9*d^19*f
^6*h^12 - 4096*a^3*b^8*c^8*d^19*f^6*h^12 - 1048576*a^7*c^12*d^19*f^6*h^12 +
262144*a^10*b*c^4*d*e^14*f^4*h^8 - 23552*a*b^6*c^8*d^14*e*f^4*h^8 - 16384*
a^7*b^7*c*d*e^14*f^4*h^8 - 3328*a*b^13*c*d^7*e^8*f^4*h^8 + 2429952*a^4*b^5*
c^6*d^9*e^6*f^4*h^8 - 1865728*a^6*b^3*c^6*d^7*e^8*f^4*h^8 - 1716224*a^4*b^4
*c^7*d^10*e^5*f^4*h^8 + 1605632*a^6*b^2*c^7*d^8*e^7*f^4*h^8 + 1584384*a^5*b
^5*c^5*d^7*e^8*f^4*h^8 + 1572864*a^5*b^2*c^8*d^10*e^5*f^4*h^8 - 1433600*a^5
*b^3*c^7*d^9*e^6*f^4*h^8 - 1261568*a^4*b^6*c^5*d^8*e^7*f^4*h^8 - 1124352*a^
3*b^4*c^8*d^12*e^3*f^4*h^8 - 1110016*a^7*b^3*c^5*d^5*e^10*f^4*h^8 + 1106176
*a^3*b^5*c^7*d^11*e^4*f^4*h^8 - 936960*a^5*b^6*c^4*d^6*e^9*f^4*h^8 - 838656
*a^2*b^7*c^6*d^11*e^4*f^4*h^8 - 795648*a^3*b^7*c^5*d^9*e^6*f^4*h^8 + 730880
*a^3*b^8*c^4*d^8*e^7*f^4*h^8 + 714752*a^2*b^6*c^7*d^12*e^3*f^4*h^8 + 686080
*a^7*b^4*c^4*d^4*e^11*f^4*h^8 + 641024*a^6*b^4*c^5*d^6*e^9*f^4*h^8 - 595968
*a^8*b^3*c^4*d^3*e^12*f^4*h^8 + 544768*a^3*b^3*c^9*d^13*e^2*f^4*h^8 + 51609
6*a^2*b^8*c^5*d^10*e^5*f^4*h^8 + 441856*a^6*b^5*c^4*d^5*e^10*f^4*h^8 + 3932
16*a^7*b^2*c^6*d^6*e^9*f^4*h^8 + 376832*a^4*b^2*c^9*d^12*e^3*f^4*h^8 - 3665
92*a^6*b^6*c^3*d^4*e^11*f^4*h^8 + 363520*a^4*b^8*c^3*d^6*e^9*f^4*h^8 - 3563
52*a^5*b^4*c^6*d^8*e^7*f^4*h^8 - 348672*a^2*b^5*c^8*d^13*e^2*f^4*h^8 - 3440
64*a^8*b^2*c^5*d^4*e^11*f^4*h^8 + 294912*a^8*b^4*c^3*d^2*e^13*f^4*h^8 + 210
944*a^4*b^3*c^8*d^11*e^4*f^4*h^8 - 198400*a^3*b^9*c^3*d^7*e^8*f^4*h^8 - 144
640*a^4*b^7*c^4*d^7*e^8*f^4*h^8 - 131072*a^9*b^2*c^4*d^2*e^13*f^4*h^8 - 131
072*a^7*b^6*c^2*d^2*e^13*f^4*h^8 - 129024*a^3*b^6*c^6*d^10*e^5*f^4*h^8 - 10
4448*a^2*b^10*c^3*d^8*e^7*f^4*h^8 + 96768*a^5*b^8*c^2*d^4*e^11*f^4*h^8 + 91
904*a^7*b^5*c^3*d^3*e^12*f^4*h^8 - 74240*a^4*b^9*c^2*d^5*e^10*f^4*h^8 - 716

80*a^2*b^9*c^4*d^9*e^6*f^4*h^8 + 58368*a^2*b^11*c^2*d^7*e^8*f^4*h^8 + 36864*a^5*b^7*c^3*d^5*e^10*f^4*h^8 - 35328*a^3*b^10*c^2*d^6*e^9*f^4*h^8 + 27136*a^6*b^7*c^2*d^3*e^12*f^4*h^8 + 909312*a^8*b*c^6*d^5*e^10*f^4*h^8 + 815104*a^9*b*c^5*d^3*e^12*f^4*h^8 - 651264*a^5*b*c^9*d^11*e^4*f^4*h^8 - 573440*a^6*b*c^8*d^9*e^6*f^4*h^8 - 262144*a^9*b^3*c^3*d*e^14*f^4*h^8 + 217088*a^7*b*c^7*d^7*e^8*f^4*h^8 + 211456*a*b^9*c^5*d^11*e^4*f^4*h^8 - 204800*a^4*b*c^10*d^13*e^2*f^4*h^8 - 172032*a*b^8*c^6*d^12*e^3*f^4*h^8 - 157696*a*b^10*c^4*d^10*e^5*f^4*h^8 - 131072*a^3*b^2*c^10*d^14*e*f^4*h^8 + 98304*a^8*b^5*c^2*d*e^14*f^4*h^8 + 92160*a^2*b^4*c^9*d^14*e*f^4*h^8 + 84992*a*b^7*c^7*d^13*e^2*f^4*h^8 + 64512*a*b^11*c^3*d^9*e^6*f^4*h^8 + 23552*a^6*b^8*c*d^2*e^13*f^4*h^8 + 18944*a^3*b^11*c*d^5*e^10*f^4*h^8 - 13312*a^4*b^10*c*d^4*e^11*f^4*h^8 - 9472*a^5*b^9*c*d^3*e^12*f^4*h^8 - 8192*a*b^12*c^2*d^8*e^7*f^4*h^8 - 6144*a^2*b^12*c*d^6*e^9*f^4*h^8 - 17920*b^11*c^4*d^11*e^4*f^4*h^8 + 14336*b^12*c^3*d^10*e^5*f^4*h^8 + 14336*b^10*c^5*d^12*e^3*f^4*h^8 - 7168*b^13*c^2*d^9*e^6*f^4*h^8 - 7168*b^9*c^6*d^13*e^2*f^4*h^8 - 425984*a^9*c^6*d^4*e^11*f^4*h^8 - 360448*a^8*c^7*d^6*e^9*f^4*h^8 - 262144*a^10*c^5*d^2*e^13*f^4*h^8 - 131072*a^7*c^8*d^8*e^7*f^4*h^8 + 98304*a^5*c^10*d^12*e^3*f^4*h^8 + 65536*a^6*c^9*d^10*e^5*f^4*h^8 - 1536*a^5*b^10*d^2*e^13*f^4*h^8 - 1536*a^2*b^13*d^5*e^10*f^4*h^8 + 768*a^4*b^11*d^3*e^12*f^4*h^8 + 768*a^3*b^12*d^4*e^11*f^4*h^8 + 65536*a^10*b^2*c^3*e^15*f^4*h^8 - 24576*a^9*b^4*c^2*e^15*f^4*h^8 - 10240*a^2*b^3*c^10*d^15*f^4*h^8 + 2048*b^14*c*d^8*e^7*f^4*h^8 + 2048*b^8*c^7*d^14*e*f^4*h^8 + 32768*a^4*c^11*d^14*e*f^4*h^8 + 1024*a^6*b^9*d*e^14*f^4*h^8 + 1024*a*b^14*d^6*e^9*f^4*h^8 + 4096*a^8*b^6*c*e^15*f^4*h^8 + 12288*a^3*b*c^11*d^15*f^4*h^8 + 2816*a*b^5*c^9*d^15*f^4*h^8 - 256*b^15*d^7*e^8*f^4*h^8 - 65536*a^11*c^4*e^15*f^4*h^8 - 256*b^7*c^8*d^15*f^4*h^8 - 256*a^7*b^8*e^15*f^4*h^8 - 896*a*b^8*c^2*d*e^10*f^2*h^4 + 192*a*b*c^9*d^8*e^3*f^2*h^4 + 11520*a^3*b^3*c^5*d^2*e^9*f^2*h^4 - 5856*a^2*b^5*c^4*d^2*e^9*f^2*h^4 - 5120*a^3*b^2*c^6*d^3*e^8*f^2*h^4 + 3200*a^2*b^4*c^5*d^3*e^8*f^2*h^4 - 640*a^2*b^3*c^6*d^4*e^7*f^2*h^4 - 96*a^2*b^2*c^7*d^5*e^6*f^2*h^4 - 10880*a^3*b^4*c^4*d*e^10*f^2*h^4 + 10240*a^4*b^2*c^5*d*e^10*f^2*h^4 - 7680*a^4*b*c^6*d^2*e^9*f^2*h^4 + 4672*a^2*b^6*c^3*d*e^10*f^2*h^4 + 1248*a*b^7*c^3*d^2*e^9*f^2*h^4 + 832*a^3*b*c^7*d^4*e^7*f^2*h^4 - 768*a*b^6*c^4*d^3*e^8*f^2*h^4 + 192*a^2*b*c^8*d^6*e^5*f^2*h^4 - 192*a*b^2*c^8*d^7*e^4*f^2*h^4 + 176*a*b^5*c^5*d^4*e^7*f^2*h^4 + 64*a*b^3*c^7*d^6*e^5*f^2*h^4 - 96*b^9*c^2*d^2*e^9*f^2*h^4 - 96*b^2*c^9*d^9*e^2*f^2*h^4 + 64*b^8*c^3*d^3*e^8*f^2*h^4 + 64*b^3*c^8*d^8*e^3*f^2*h^4 - 16*b^7*c^4*d^4*e^7*f^2*h^4 - 16*b^4*c^7*d^7*e^4*f^2*h^4 + 2032*a^4*c^7*d^3*e^8*f^2*h^4 - 96*a^2*c^9*d^7*e^4*f^2*h^4 - 64*a^3*c^8*d^5*e^6*f^2*h^4 - 4480*a^4*b^3*c^4*e^11*f^2*h^4 + 3696*a^3*b^5*c^3*e^11*f^2*h^4 - 1376*a^2*b^7*c^2*e^11*f^2*h^4 - 2048*a^5*c^6*d*e^10*f^2*h^4 - 64*a*c^10*d^9*e^2*f^2*h^4 + 1792*a^5*b*c^5*e^11*f^2*h^4 + 64*b^10*c*d*e^10*f^2*h^4 + 64*b*c^10*d^10*e*f^2*h^4 + 240*a*b^9*c*e^11*f^2*h^4 - 16*c^11*d^11*f^2*h^4 - 16*b^11*e^11*f^2*h^4 - c^7*e^7, h, k)*(root(8388608*a^7*b*c^11*d^18*e*f^6*h^12 - 513802240*a^10*b^2*c^7*d^11*e^8*f^6*h^12 - 381681664*a^11*b^2*c^6*d^9*e^10*f^6*h^12 - 381681664*a^9*b^2*c^8*d^13*e^6*f^6*h^12 - 300941312*a^9*b^5*c^5*d^10*e^9*f^6*h^12 - 300941312*a^8*b^5*c^6*d^12*e^7*f^6*h^12 + 293601280*a^10*b^3*c

^6*d^10*e^9*f^6*h^12 + 293601280*a^9*b^3*c^7*d^12*e^7*f^6*h^12 - 168820736*a^10*b^5*c^4*d^8*e^11*f^6*h^12 - 168820736*a^7*b^5*c^7*d^14*e^5*f^6*h^12 + 166068224*a^8*b^6*c^5*d^11*e^8*f^6*h^12 - 146800640*a^12*b^2*c^5*d^7*e^12*f^6*h^12 - 146800640*a^8*b^2*c^9*d^15*e^4*f^6*h^12 + 124780544*a^10*b^4*c^5*d^9*e^10*f^6*h^12 + 124780544*a^8*b^4*c^7*d^13*e^6*f^6*h^12 + 119275520*a^9*b^4*c^6*d^11*e^8*f^6*h^12 + 117440512*a^11*b^3*c^5*d^8*e^11*f^6*h^12 + 117440512*a^8*b^3*c^8*d^14*e^5*f^6*h^12 + 102760448*a^9*b^6*c^4*d^9*e^10*f^6*h^12 + 102760448*a^7*b^6*c^6*d^13*e^6*f^6*h^12 + 91750400*a^11*b^4*c^4*d^7*e^12*f^6*h^12 + 91750400*a^7*b^4*c^8*d^15*e^4*f^6*h^12 - 71065600*a^7*b^8*c^4*d^11*e^8*f^6*h^12 - 53444608*a^8*b^8*c^3*d^9*e^10*f^6*h^12 - 53444608*a^6*b^8*c^5*d^13*e^6*f^6*h^12 + 40370176*a^9*b^7*c^3*d^8*e^11*f^6*h^12 + 40370176*a^6*b^7*c^6*d^14*e^5*f^6*h^12 - 36700160*a^11*b^5*c^3*d^6*e^13*f^6*h^12 - 36700160*a^6*b^5*c^8*d^16*e^3*f^6*h^12 + 34078720*a^8*b^7*c^4*d^10*e^9*f^6*h^12 + 34078720*a^7*b^7*c^5*d^12*e^7*f^6*h^12 + 26214400*a^12*b^4*c^3*d^5*e^14*f^6*h^12 + 26214400*a^6*b^4*c^9*d^17*e^2*f^6*h^12 + 22118400*a^7*b^9*c^3*d^10*e^9*f^6*h^12 + 22118400*a^6*b^9*c^4*d^12*e^7*f^6*h^12 - 20971520*a^13*b^2*c^4*d^5*e^14*f^6*h^12 - 20971520*a^7*b^2*c^10*d^17*e^2*f^6*h^12 + 18350080*a^10*b^7*c^2*d^6*e^13*f^6*h^12 + 18350080*a^5*b^7*c^7*d^16*e^3*f^6*h^12 - 16629760*a^9*b^8*c^2*d^7*e^12*f^6*h^12 - 16629760*a^5*b^8*c^6*d^15*e^4*f^6*h^12 - 10485760*a^11*b^6*c^2*d^5*e^14*f^6*h^12 - 10485760*a^5*b^6*c^8*d^17*e^2*f^6*h^12 + 9175040*a^10*b^6*c^3*d^7*e^12*f^6*h^12 + 9175040*a^6*b^6*c^7*d^15*e^4*f^6*h^12 - 8388608*a^13*b^3*c^3*d^4*e^15*f^6*h^12 + 5619712*a^7*b^10*c^2*d^9*e^10*f^6*h^12 + 5619712*a^5*b^10*c^4*d^13*e^6*f^6*h^12 - 5570560*a^6*b^11*c^2*d^10*e^9*f^6*h^12 - 5570560*a^5*b^11*c^3*d^12*e^7*f^6*h^12 + 4358144*a^8*b^9*c^2*d^8*e^11*f^6*h^12 + 4358144*a^5*b^9*c^5*d^14*e^5*f^6*h^12 + 4259840*a^6*b^10*c^3*d^11*e^8*f^6*h^12 + 3899392*a^4*b^10*c^5*d^15*e^4*f^6*h^12 - 3440640*a^4*b^9*c^6*d^16*e^3*f^6*h^12 + 3145728*a^12*b^5*c^2*d^4*e^15*f^6*h^12 - 2523136*a^4*b^11*c^4*d^14*e^5*f^6*h^12 + 1802240*a^4*b^8*c^7*d^17*e^2*f^6*h^12 + 1556480*a^5*b^12*c^2*d^11*e^8*f^6*h^12 + 1048576*a^14*b^2*c^3*d^3*e^16*f^6*h^12 + 688128*a^4*b^12*c^3*d^13*e^6*f^6*h^12 - 393216*a^13*b^4*c^2*d^3*e^16*f^6*h^12 - 286720*a^3*b^12*c^4*d^15*e^4*f^6*h^12 + 229376*a^3*b^13*c^3*d^14*e^5*f^6*h^12 + 229376*a^3*b^11*c^5*d^16*e^3*f^6*h^12 + 163840*a^4*b^13*c^2*d^12*e^7*f^6*h^12 - 114688*a^3*b^14*c^2*d^13*e^6*f^6*h^12 - 114688*a^3*b^10*c^6*d^17*e^2*f^6*h^12 + 293601280*a^11*b*c^7*d^10*e^9*f^6*h^12 + 293601280*a^10*b*c^8*d^12*e^7*f^6*h^12 + 176160768*a^12*b*c^6*d^8*e^11*f^6*h^12 + 176160768*a^9*b*c^9*d^14*e^5*f^6*h^12 + 58720256*a^13*b*c^5*d^6*e^13*f^6*h^12 + 58720256*a^8*b*c^10*d^16*e^3*f^6*h^12 + 8388608*a^14*b*c^4*d^4*e^15*f^6*h^12 - 8388608*a^6*b^3*c^10*d^18*e*f^6*h^12 + 3899392*a^8*b^10*c*d^7*e^12*f^6*h^12 - 3440640*a^9*b^9*c*d^6*e^13*f^6*h^12 + 3145728*a^5*b^5*c^9*d^18*e*f^6*h^12 - 2523136*a^7*b^11*c*d^8*e^11*f^6*h^12 + 1802240*a^10*b^8*c*d^5*e^14*f^6*h^12 + 688128*a^6*b^12*c*d^9*e^10*f^6*h^12 - 524288*a^11*b^7*c*d^4*e^15*f^6*h^12 - 524288*a^4*b^7*c^8*d^18*e*f^6*h^12 + 163840*a^5*b^13*c*d^10*e^9*f^6*h^12 - 163840*a^4*b^14*c*d^11*e^8*f^6*h^12 + 65536*a^12*b^6*c*d^3*e^16*f^6*h^12 + 32768*a^3*b^15*c*d^12*e^7*f^6*h^12 + 32768*a^3*b^9*c^7*d^18*e*f^6*h^12 - 73400320*a^11*c^8*d^11*e^8*

f^6*h^12 - 58720256*a^12*c^7*d^9*e^10*f^6*h^12 - 58720256*a^10*c^9*d^13*e^6
*f^6*h^12 - 29360128*a^13*c^6*d^7*e^12*f^6*h^12 - 29360128*a^9*c^10*d^15*e^
4*f^6*h^12 - 8388608*a^14*c^5*d^5*e^14*f^6*h^12 - 8388608*a^8*c^11*d^17*e^2
*f^6*h^12 - 1048576*a^15*c^4*d^3*e^16*f^6*h^12 - 286720*a^7*b^12*d^7*e^12*f
^6*h^12 + 229376*a^8*b^11*d^6*e^13*f^6*h^12 + 229376*a^6*b^13*d^8*e^11*f^6*
h^12 - 114688*a^9*b^10*d^5*e^14*f^6*h^12 - 114688*a^5*b^14*d^9*e^10*f^6*h^1
2 + 32768*a^10*b^9*d^4*e^15*f^6*h^12 + 32768*a^4*b^15*d^10*e^9*f^6*h^12 - 4
096*a^11*b^8*d^3*e^16*f^6*h^12 - 4096*a^3*b^16*d^11*e^8*f^6*h^12 + 1048576*
a^6*b^2*c^11*d^19*f^6*h^12 - 393216*a^5*b^4*c^10*d^19*f^6*h^12 + 65536*a^4*
b^6*c^9*d^19*f^6*h^12 - 4096*a^3*b^8*c^8*d^19*f^6*h^12 - 1048576*a^7*c^12*d
^19*f^6*h^12 + 262144*a^10*b*c^4*d*e^14*f^4*h^8 - 23552*a*b^6*c^8*d^14*e*f^
4*h^8 - 16384*a^7*b^7*c*d*e^14*f^4*h^8 - 3328*a*b^13*c*d^7*e^8*f^4*h^8 + 24
29952*a^4*b^5*c^6*d^9*e^6*f^4*h^8 - 1865728*a^6*b^3*c^6*d^7*e^8*f^4*h^8 - 1
716224*a^4*b^4*c^7*d^10*e^5*f^4*h^8 + 1605632*a^6*b^2*c^7*d^8*e^7*f^4*h^8 +
 1584384*a^5*b^5*c^5*d^7*e^8*f^4*h^8 + 1572864*a^5*b^2*c^8*d^10*e^5*f^4*h^8
 - 1433600*a^5*b^3*c^7*d^9*e^6*f^4*h^8 - 1261568*a^4*b^6*c^5*d^8*e^7*f^4*h^
8 - 1124352*a^3*b^4*c^8*d^12*e^3*f^4*h^8 - 1110016*a^7*b^3*c^5*d^5*e^10*f^4
*h^8 + 1106176*a^3*b^5*c^7*d^11*e^4*f^4*h^8 - 936960*a^5*b^6*c^4*d^6*e^9*f^
4*h^8 - 838656*a^2*b^7*c^6*d^11*e^4*f^4*h^8 - 795648*a^3*b^7*c^5*d^9*e^6*f^
4*h^8 + 730880*a^3*b^8*c^4*d^8*e^7*f^4*h^8 + 714752*a^2*b^6*c^7*d^12*e^3*f^
4*h^8 + 686080*a^7*b^4*c^4*d^4*e^11*f^4*h^8 + 641024*a^6*b^4*c^5*d^6*e^9*f^
4*h^8 - 595968*a^8*b^3*c^4*d^3*e^12*f^4*h^8 + 544768*a^3*b^3*c^9*d^13*e^2*f
^4*h^8 + 516096*a^2*b^8*c^5*d^10*e^5*f^4*h^8 + 441856*a^6*b^5*c^4*d^5*e^10*
f^4*h^8 + 393216*a^7*b^2*c^6*d^6*e^9*f^4*h^8 + 376832*a^4*b^2*c^9*d^12*e^3*
f^4*h^8 - 366592*a^6*b^6*c^3*d^4*e^11*f^4*h^8 + 363520*a^4*b^8*c^3*d^6*e^9*
f^4*h^8 - 356352*a^5*b^4*c^6*d^8*e^7*f^4*h^8 - 348672*a^2*b^5*c^8*d^13*e^2*
f^4*h^8 - 344064*a^8*b^2*c^5*d^4*e^11*f^4*h^8 + 294912*a^8*b^4*c^3*d^2*e^13
*f^4*h^8 + 210944*a^4*b^3*c^8*d^11*e^4*f^4*h^8 - 198400*a^3*b^9*c^3*d^7*e^8
*f^4*h^8 - 144640*a^4*b^7*c^4*d^7*e^8*f^4*h^8 - 131072*a^9*b^2*c^4*d^2*e^13
*f^4*h^8 - 131072*a^7*b^6*c^2*d^2*e^13*f^4*h^8 - 129024*a^3*b^6*c^6*d^10*e^
5*f^4*h^8 - 104448*a^2*b^10*c^3*d^8*e^7*f^4*h^8 + 96768*a^5*b^8*c^2*d^4*e^1
1*f^4*h^8 + 91904*a^7*b^5*c^3*d^3*e^12*f^4*h^8 - 74240*a^4*b^9*c^2*d^5*e^10
*f^4*h^8 - 71680*a^2*b^9*c^4*d^9*e^6*f^4*h^8 + 58368*a^2*b^11*c^2*d^7*e^8*f
^4*h^8 + 36864*a^5*b^7*c^3*d^5*e^10*f^4*h^8 - 35328*a^3*b^10*c^2*d^6*e^9*f^
4*h^8 + 27136*a^6*b^7*c^2*d^3*e^12*f^4*h^8 + 909312*a^8*b*c^6*d^5*e^10*f^4*
h^8 + 815104*a^9*b*c^5*d^3*e^12*f^4*h^8 - 651264*a^5*b*c^9*d^11*e^4*f^4*h^8
 - 573440*a^6*b*c^8*d^9*e^6*f^4*h^8 - 262144*a^9*b^3*c^3*d*e^14*f^4*h^8 + 2
17088*a^7*b*c^7*d^7*e^8*f^4*h^8 + 211456*a*b^9*c^5*d^11*e^4*f^4*h^8 - 20480
0*a^4*b*c^10*d^13*e^2*f^4*h^8 - 172032*a*b^8*c^6*d^12*e^3*f^4*h^8 - 157696*
a*b^10*c^4*d^10*e^5*f^4*h^8 - 131072*a^3*b^2*c^10*d^14*e*f^4*h^8 + 98304*a^
8*b^5*c^2*d*e^14*f^4*h^8 + 92160*a^2*b^4*c^9*d^14*e*f^4*h^8 + 84992*a*b^7*c
^7*d^13*e^2*f^4*h^8 + 64512*a*b^11*c^3*d^9*e^6*f^4*h^8 + 23552*a^6*b^8*c*d^
2*e^13*f^4*h^8 + 18944*a^3*b^11*c*d^5*e^10*f^4*h^8 - 13312*a^4*b^10*c*d^4*e
^11*f^4*h^8 - 9472*a^5*b^9*c*d^3*e^12*f^4*h^8 - 8192*a*b^12*c^2*d^8*e^7*f^4
*h^8 - 6144*a^2*b^12*c*d^6*e^9*f^4*h^8 - 17920*b^11*c^4*d^11*e^4*f^4*h^8 +

14336*b^12*c^3*d^10*e^5*f^4*h^8 + 14336*b^10*c^5*d^12*e^3*f^4*h^8 - 7168*b^13*c^2*d^9*e^6*f^4*h^8 - 7168*b^9*c^6*d^13*e^2*f^4*h^8 - 425984*a^9*c^6*d^4*e^11*f^4*h^8 - 360448*a^8*c^7*d^6*e^9*f^4*h^8 - 262144*a^10*c^5*d^2*e^13*f^4*h^8 - 131072*a^7*c^8*d^8*e^7*f^4*h^8 + 98304*a^5*c^10*d^12*e^3*f^4*h^8 + 65536*a^6*c^9*d^10*e^5*f^4*h^8 - 1536*a^5*b^10*d^2*e^13*f^4*h^8 - 1536*a^2*b^13*d^5*e^10*f^4*h^8 + 768*a^4*b^11*d^3*e^12*f^4*h^8 + 768*a^3*b^12*d^4*e^11*f^4*h^8 + 65536*a^10*b^2*c^3*e^15*f^4*h^8 - 24576*a^9*b^4*c^2*e^15*f^4*h^8 - 10240*a^2*b^3*c^10*d^15*f^4*h^8 + 2048*b^14*c*d^8*e^7*f^4*h^8 + 2048*b^8*c^7*d^14*e*f^4*h^8 + 32768*a^4*c^11*d^14*e*f^4*h^8 + 1024*a^6*b^9*d*e^14*f^4*h^8 + 1024*a*b^14*d^6*e^9*f^4*h^8 + 4096*a^8*b^6*c*e^15*f^4*h^8 + 12288*a^3*b*c^11*d^15*f^4*h^8 + 2816*a*b^5*c^9*d^15*f^4*h^8 - 256*b^15*d^7*e^8*f^4*h^8 - 65536*a^11*c^4*e^15*f^4*h^8 - 256*b^7*c^8*d^15*f^4*h^8 - 256*a^7*b^8*e^15*f^4*h^8 - 896*a*b^8*c^2*d*e^10*f^2*h^4 + 192*a*b*c^9*d^8*e^3*f^2*h^4 + 11520*a^3*b^3*c^5*d^2*e^9*f^2*h^4 - 5856*a^2*b^5*c^4*d^2*e^9*f^2*h^4 - 5120*a^3*b^2*c^6*d^3*e^8*f^2*h^4 + 3200*a^2*b^4*c^5*d^3*e^8*f^2*h^4 - 640*a^2*b^3*c^6*d^4*e^7*f^2*h^4 - 96*a^2*b^2*c^7*d^5*e^6*f^2*h^4 - 10880*a^3*b^4*c^4*d*e^10*f^2*h^4 + 10240*a^4*b^2*c^5*d*e^10*f^2*h^4 - 7680*a^4*b*c^6*d^2*e^9*f^2*h^4 + 4672*a^2*b^6*c^3*d*e^10*f^2*h^4 + 1248*a*b^7*c^3*d^2*e^9*f^2*h^4 + 832*a^3*b*c^7*d^4*e^7*f^2*h^4 - 768*a*b^6*c^4*d^3*e^8*f^2*h^4 + 192*a^2*b*c^8*d^6*e^5*f^2*h^4 - 192*a*b^2*c^8*d^7*e^4*f^2*h^4 + 176*a*b^5*c^5*d^4*e^7*f^2*h^4 + 64*a*b^3*c^7*d^6*e^5*f^2*h^4 - 96*b^9*c^2*d^2*e^9*f^2*h^4 - 96*b^2*c^9*d^9*e^2*f^2*h^4 + 64*b^8*c^3*d^3*e^8*f^2*h^4 + 64*b^3*c^8*d^8*e^3*f^2*h^4 - 16*b^7*c^4*d^4*e^7*f^2*h^4 - 16*b^4*c^7*d^7*e^4*f^2*h^4 + 2032*a^4*c^7*d^3*e^8*f^2*h^4 - 96*a^2*c^9*d^7*e^4*f^2*h^4 - 64*a^3*c^8*d^5*e^6*f^2*h^4 - 4480*a^4*b^3*c^4*e^11*f^2*h^4 + 3696*a^3*b^5*c^3*e^11*f^2*h^4 - 1376*a^2*b^7*c^2*e^11*f^2*h^4 - 2048*a^5*c^6*d*e^10*f^2*h^4 - 64*a*c^10*d^9*e^2*f^2*h^4 + 1792*a^5*b*c^5*e^11*f^2*h^4 + 64*b^10*c*d*e^10*f^2*h^4 + 64*b*c^10*d^10*e*f^2*h^4 + 240*a*b^9*c*e^11*f^2*h^4 - 16*c^11*d^11*f^2*h^4 - 16*b^11*e^11*f^2*h^4 - c^7*e^7, h, k)*(root(8388608*a^7*b*c^11*d^18*e*f^6*h^12 - 513802240*a^10*b^2*c^7*d^11*e^8*f^6*h^12 - 381681664*a^11*b^2*c^6*d^9*e^10*f^6*h^12 - 381681664*a^9*b^2*c^8*d^13*e^6*f^6*h^12 - 300941312*a^9*b^5*c^5*d^10*e^9*f^6*h^12 - 300941312*a^8*b^5*c^6*d^12*e^7*f^6*h^12 + 293601280*a^10*b^3*c^6*d^10*e^9*f^6*h^12 + 293601280*a^9*b^3*c^7*d^12*e^7*f^6*h^12 - 168820736*a^10*b^5*c^4*d^8*e^11*f^6*h^12 - 168820736*a^7*b^5*c^7*d^14*e^5*f^6*h^12 + 166068224*a^8*b^6*c^5*d^11*e^8*f^6*h^12 - 146800640*a^12*b^2*c^5*d^7*e^12*f^6*h^12 - 146800640*a^8*b^2*c^9*d^15*e^4*f^6*h^12 + 124780544*a^10*b^4*c^5*d^9*e^10*f^6*h^12 + 124780544*a^8*b^4*c^7*d^13*e^6*f^6*h^12 + 119275520*a^9*b^4*c^6*d^11*e^8*f^6*h^12 + 117440512*a^11*b^3*c^5*d^8*e^11*f^6*h^12 + 117440512*a^8*b^3*c^8*d^14*e^5*f^6*h^12 + 102760448*a^9*b^6*c^4*d^9*e^10*f^6*h^12 + 102760448*a^7*b^6*c^6*d^13*e^6*f^6*h^12 + 91750400*a^11*b^4*c^4*d^7*e^12*f^6*h^12 + 91750400*a^7*b^4*c^8*d^15*e^4*f^6*h^12 - 71065600*a^7*b^8*c^4*d^11*e^8*f^6*h^12 - 53444608*a^8*b^8*c^3*d^9*e^10*f^6*h^12 - 53444608*a^6*b^8*c^5*d^13*e^6*f^6*h^12 + 40370176*a^9*b^7*c^3*d^8*e^11*f^6*h^12 + 40370176*a^6*b^7*c^6*d^14*e^5*f^6*h^12 - 36700160*a^11*b^5*c^3*d^6*e^13*f^6*h^12 - 36700160*a^6*b^5*c^8*d^16*e^3*f^6*h^12 + 34078720*a^8*b^7*

c^4*d^10*e^9*f^6*h^12 + 34078720*a^7*b^7*c^5*d^12*e^7*f^6*h^12 + 26214400*a
^12*b^4*c^3*d^5*e^14*f^6*h^12 + 26214400*a^6*b^4*c^9*d^17*e^2*f^6*h^12 + 22
118400*a^7*b^9*c^3*d^10*e^9*f^6*h^12 + 22118400*a^6*b^9*c^4*d^12*e^7*f^6*h^
12 - 20971520*a^13*b^2*c^4*d^5*e^14*f^6*h^12 - 20971520*a^7*b^2*c^10*d^17*e
^2*f^6*h^12 + 18350080*a^10*b^7*c^2*d^6*e^13*f^6*h^12 + 18350080*a^5*b^7*c^
7*d^16*e^3*f^6*h^12 - 16629760*a^9*b^8*c^2*d^7*e^12*f^6*h^12 - 16629760*a^5
*b^8*c^6*d^15*e^4*f^6*h^12 - 10485760*a^11*b^6*c^2*d^5*e^14*f^6*h^12 - 1048
5760*a^5*b^6*c^8*d^17*e^2*f^6*h^12 + 9175040*a^10*b^6*c^3*d^7*e^12*f^6*h^12
 + 9175040*a^6*b^6*c^7*d^15*e^4*f^6*h^12 - 8388608*a^13*b^3*c^3*d^4*e^15*f^
6*h^12 + 5619712*a^7*b^10*c^2*d^9*e^10*f^6*h^12 + 5619712*a^5*b^10*c^4*d^13
*e^6*f^6*h^12 - 5570560*a^6*b^11*c^2*d^10*e^9*f^6*h^12 - 5570560*a^5*b^11*c
^3*d^12*e^7*f^6*h^12 + 4358144*a^8*b^9*c^2*d^8*e^11*f^6*h^12 + 4358144*a^5*
b^9*c^5*d^14*e^5*f^6*h^12 + 4259840*a^6*b^10*c^3*d^11*e^8*f^6*h^12 + 389939
2*a^4*b^10*c^5*d^15*e^4*f^6*h^12 - 3440640*a^4*b^9*c^6*d^16*e^3*f^6*h^12 +
3145728*a^12*b^5*c^2*d^4*e^15*f^6*h^12 - 2523136*a^4*b^11*c^4*d^14*e^5*f^6*
h^12 + 1802240*a^4*b^8*c^7*d^17*e^2*f^6*h^12 + 1556480*a^5*b^12*c^2*d^11*e^
8*f^6*h^12 + 1048576*a^14*b^2*c^3*d^3*e^16*f^6*h^12 + 688128*a^4*b^12*c^3*d
^13*e^6*f^6*h^12 - 393216*a^13*b^4*c^2*d^3*e^16*f^6*h^12 - 286720*a^3*b^12*
c^4*d^15*e^4*f^6*h^12 + 229376*a^3*b^13*c^3*d^14*e^5*f^6*h^12 + 229376*a^3*
b^11*c^5*d^16*e^3*f^6*h^12 + 163840*a^4*b^13*c^2*d^12*e^7*f^6*h^12 - 114688
*a^3*b^14*c^2*d^13*e^6*f^6*h^12 - 114688*a^3*b^10*c^6*d^17*e^2*f^6*h^12 + 2
93601280*a^11*b*c^7*d^10*e^9*f^6*h^12 + 293601280*a^10*b*c^8*d^12*e^7*f^6*h
^12 + 176160768*a^12*b*c^6*d^8*e^11*f^6*h^12 + 176160768*a^9*b*c^9*d^14*e^5
*f^6*h^12 + 58720256*a^13*b*c^5*d^6*e^13*f^6*h^12 + 58720256*a^8*b*c^10*d^1
6*e^3*f^6*h^12 + 8388608*a^14*b*c^4*d^4*e^15*f^6*h^12 - 8388608*a^6*b^3*c^1
0*d^18*e*f^6*h^12 + 3899392*a^8*b^10*c*d^7*e^12*f^6*h^12 - 3440640*a^9*b^9*
c*d^6*e^13*f^6*h^12 + 3145728*a^5*b^5*c^9*d^18*e*f^6*h^12 - 2523136*a^7*b^1
1*c*d^8*e^11*f^6*h^12 + 1802240*a^10*b^8*c*d^5*e^14*f^6*h^12 + 688128*a^6*b
^12*c*d^9*e^10*f^6*h^12 - 524288*a^11*b^7*c*d^4*e^15*f^6*h^12 - 524288*a^4*
b^7*c^8*d^18*e*f^6*h^12 + 163840*a^5*b^13*c*d^10*e^9*f^6*h^12 - 163840*a^4*
b^14*c*d^11*e^8*f^6*h^12 + 65536*a^12*b^6*c*d^3*e^16*f^6*h^12 + 32768*a^3*b
^15*c*d^12*e^7*f^6*h^12 + 32768*a^3*b^9*c^7*d^18*e*f^6*h^12 - 73400320*a^11
*c^8*d^11*e^8*f^6*h^12 - 58720256*a^12*c^7*d^9*e^10*f^6*h^12 - 58720256*a^1
0*c^9*d^13*e^6*f^6*h^12 - 29360128*a^13*c^6*d^7*e^12*f^6*h^12 - 29360128*a^
9*c^10*d^15*e^4*f^6*h^12 - 8388608*a^14*c^5*d^5*e^14*f^6*h^12 - 8388608*a^8
*c^11*d^17*e^2*f^6*h^12 - 1048576*a^15*c^4*d^3*e^16*f^6*h^12 - 286720*a^7*b
^12*d^7*e^12*f^6*h^12 + 229376*a^8*b^11*d^6*e^13*f^6*h^12 + 229376*a^6*b^13
*d^8*e^11*f^6*h^12 - 114688*a^9*b^10*d^5*e^14*f^6*h^12 - 114688*a^5*b^14*d^
9*e^10*f^6*h^12 + 32768*a^10*b^9*d^4*e^15*f^6*h^12 + 32768*a^4*b^15*d^10*e^
9*f^6*h^12 - 4096*a^11*b^8*d^3*e^16*f^6*h^12 - 4096*a^3*b^16*d^11*e^8*f^6*h
^12 + 1048576*a^6*b^2*c^11*d^19*f^6*h^12 - 393216*a^5*b^4*c^10*d^19*f^6*h^1
2 + 65536*a^4*b^6*c^9*d^19*f^6*h^12 - 4096*a^3*b^8*c^8*d^19*f^6*h^12 - 1048
576*a^7*c^12*d^19*f^6*h^12 + 262144*a^10*b*c^4*d*e^14*f^4*h^8 - 23552*a*b^6
*c^8*d^14*e*f^4*h^8 - 16384*a^7*b^7*c*d*e^14*f^4*h^8 - 3328*a*b^13*c*d^7*e^
8*f^4*h^8 + 2429952*a^4*b^5*c^6*d^9*e^6*f^4*h^8 - 1865728*a^6*b^3*c^6*d^7*e

^8*f^4*h^8 - 1716224*a^4*b^4*c^7*d^10*e^5*f^4*h^8 + 1605632*a^6*b^2*c^7*d^8
*e^7*f^4*h^8 + 1584384*a^5*b^5*c^5*d^7*e^8*f^4*h^8 + 1572864*a^5*b^2*c^8*d^
10*e^5*f^4*h^8 - 1433600*a^5*b^3*c^7*d^9*e^6*f^4*h^8 - 1261568*a^4*b^6*c^5*
d^8*e^7*f^4*h^8 - 1124352*a^3*b^4*c^8*d^12*e^3*f^4*h^8 - 1110016*a^7*b^3*c^
5*d^5*e^10*f^4*h^8 + 1106176*a^3*b^5*c^7*d^11*e^4*f^4*h^8 - 936960*a^5*b^6*
c^4*d^6*e^9*f^4*h^8 - 838656*a^2*b^7*c^6*d^11*e^4*f^4*h^8 - 795648*a^3*b^7*
c^5*d^9*e^6*f^4*h^8 + 730880*a^3*b^8*c^4*d^8*e^7*f^4*h^8 + 714752*a^2*b^6*c
^7*d^12*e^3*f^4*h^8 + 686080*a^7*b^4*c^4*d^4*e^11*f^4*h^8 + 641024*a^6*b^4*
c^5*d^6*e^9*f^4*h^8 - 595968*a^8*b^3*c^4*d^3*e^12*f^4*h^8 + 544768*a^3*b^3*
c^9*d^13*e^2*f^4*h^8 + 516096*a^2*b^8*c^5*d^10*e^5*f^4*h^8 + 441856*a^6*b^5
*c^4*d^5*e^10*f^4*h^8 + 393216*a^7*b^2*c^6*d^6*e^9*f^4*h^8 + 376832*a^4*b^2
*c^9*d^12*e^3*f^4*h^8 - 366592*a^6*b^6*c^3*d^4*e^11*f^4*h^8 + 363520*a^4*b^
8*c^3*d^6*e^9*f^4*h^8 - 356352*a^5*b^4*c^6*d^8*e^7*f^4*h^8 - 348672*a^2*b^5
*c^8*d^13*e^2*f^4*h^8 - 344064*a^8*b^2*c^5*d^4*e^11*f^4*h^8 + 294912*a^8*b^
4*c^3*d^2*e^13*f^4*h^8 + 210944*a^4*b^3*c^8*d^11*e^4*f^4*h^8 - 198400*a^3*b
^9*c^3*d^7*e^8*f^4*h^8 - 144640*a^4*b^7*c^4*d^7*e^8*f^4*h^8 - 131072*a^9*b^
2*c^4*d^2*e^13*f^4*h^8 - 131072*a^7*b^6*c^2*d^2*e^13*f^4*h^8 - 129024*a^3*b
^6*c^6*d^10*e^5*f^4*h^8 - 104448*a^2*b^10*c^3*d^8*e^7*f^4*h^8 + 96768*a^5*b
^8*c^2*d^4*e^11*f^4*h^8 + 91904*a^7*b^5*c^3*d^3*e^12*f^4*h^8 - 74240*a^4*b^
9*c^2*d^5*e^10*f^4*h^8 - 71680*a^2*b^9*c^4*d^9*e^6*f^4*h^8 + 58368*a^2*b^11
*c^2*d^7*e^8*f^4*h^8 + 36864*a^5*b^7*c^3*d^5*e^10*f^4*h^8 - 35328*a^3*b^10*
c^2*d^6*e^9*f^4*h^8 + 27136*a^6*b^7*c^2*d^3*e^12*f^4*h^8 + 909312*a^8*b*c^6
*d^5*e^10*f^4*h^8 + 815104*a^9*b*c^5*d^3*e^12*f^4*h^8 - 651264*a^5*b*c^9*d^
11*e^4*f^4*h^8 - 573440*a^6*b*c^8*d^9*e^6*f^4*h^8 - 262144*a^9*b^3*c^3*d*e^
14*f^4*h^8 + 217088*a^7*b*c^7*d^7*e^8*f^4*h^8 + 211456*a*b^9*c^5*d^11*e^4*f
^4*h^8 - 204800*a^4*b*c^10*d^13*e^2*f^4*h^8 - 172032*a*b^8*c^6*d^12*e^3*f^4
*h^8 - 157696*a*b^10*c^4*d^10*e^5*f^4*h^8 - 131072*a^3*b^2*c^10*d^14*e*f^4*
h^8 + 98304*a^8*b^5*c^2*d*e^14*f^4*h^8 + 92160*a^2*b^4*c^9*d^14*e*f^4*h^8 +
 84992*a*b^7*c^7*d^13*e^2*f^4*h^8 + 64512*a*b^11*c^3*d^9*e^6*f^4*h^8 + 2355
2*a^6*b^8*c*d^2*e^13*f^4*h^8 + 18944*a^3*b^11*c*d^5*e^10*f^4*h^8 - 13312*a^
4*b^10*c*d^4*e^11*f^4*h^8 - 9472*a^5*b^9*c*d^3*e^12*f^4*h^8 - 8192*a*b^12*c
^2*d^8*e^7*f^4*h^8 - 6144*a^2*b^12*c*d^6*e^9*f^4*h^8 - 17920*b^11*c^4*d^11*
e^4*f^4*h^8 + 14336*b^12*c^3*d^10*e^5*f^4*h^8 + 14336*b^10*c^5*d^12*e^3*f^4
*h^8 - 7168*b^13*c^2*d^9*e^6*f^4*h^8 - 7168*b^9*c^6*d^13*e^2*f^4*h^8 - 4259
84*a^9*c^6*d^4*e^11*f^4*h^8 - 360448*a^8*c^7*d^6*e^9*f^4*h^8 - 262144*a^10*
c^5*d^2*e^13*f^4*h^8 - 131072*a^7*c^8*d^8*e^7*f^4*h^8 + 98304*a^5*c^10*d^12
*e^3*f^4*h^8 + 65536*a^6*c^9*d^10*e^5*f^4*h^8 - 1536*a^5*b^10*d^2*e^13*f^4*
h^8 - 1536*a^2*b^13*d^5*e^10*f^4*h^8 + 768*a^4*b^11*d^3*e^12*f^4*h^8 + 768*
a^3*b^12*d^4*e^11*f^4*h^8 + 65536*a^10*b^2*c^3*e^15*f^4*h^8 - 24576*a^9*b^4
*c^2*e^15*f^4*h^8 - 10240*a^2*b^3*c^10*d^15*f^4*h^8 + 2048*b^14*c*d^8*e^7*f
^4*h^8 + 2048*b^8*c^7*d^14*e*f^4*h^8 + 32768*a^4*c^11*d^14*e*f^4*h^8 + 1024
*a^6*b^9*d*e^14*f^4*h^8 + 1024*a*b^14*d^6*e^9*f^4*h^8 + 4096*a^8*b^6*c*e^15
*f^4*h^8 + 12288*a^3*b*c^11*d^15*f^4*h^8 + 2816*a*b^5*c^9*d^15*f^4*h^8 - 25
6*b^15*d^7*e^8*f^4*h^8 - 65536*a^11*c^4*e^15*f^4*h^8 - 256*b^7*c^8*d^15*f^4
*h^8 - 256*a^7*b^8*e^15*f^4*h^8 - 896*a*b^8*c^2*d*e^10*f^2*h^4 + 192*a*b*c^

9*d^8*e^3*f^2*h^4 + 11520*a^3*b^3*c^5*d^2*e^9*f^2*h^4 - 5856*a^2*b^5*c^4*d^2*e^9*f^2*h^4 - 5120*a^3*b^2*c^6*d^3*e^8*f^2*h^4 + 3200*a^2*b^4*c^5*d^3*e^8*f^2*h^4 - 640*a^2*b^3*c^6*d^4*e^7*f^2*h^4 - 96*a^2*b^2*c^7*d^5*e^6*f^2*h^4 - 10880*a^3*b^4*c^4*d*e^10*f^2*h^4 + 10240*a^4*b^2*c^5*d*e^10*f^2*h^4 - 7680*a^4*b*c^6*d^2*e^9*f^2*h^4 + 4672*a^2*b^6*c^3*d*e^10*f^2*h^4 + 1248*a*b^7*c^3*d^2*e^9*f^2*h^4 + 832*a^3*b*c^7*d^4*e^7*f^2*h^4 - 768*a*b^6*c^4*d^3*e^8*f^2*h^4 + 192*a^2*b*c^8*d^6*e^5*f^2*h^4 - 192*a*b^2*c^8*d^7*e^4*f^2*h^4 + 176*a*b^5*c^5*d^4*e^7*f^2*h^4 + 64*a*b^3*c^7*d^6*e^5*f^2*h^4 - 96*b^9*c^2*d^2*e^9*f^2*h^4 - 96*b^2*c^9*d^9*e^2*f^2*h^4 + 64*b^8*c^3*d^3*e^8*f^2*h^4 + 64*b^3*c^8*d^8*e^3*f^2*h^4 - 16*b^7*c^4*d^4*e^7*f^2*h^4 - 16*b^4*c^7*d^7*e^4*f^2*h^4 + 2032*a^4*c^7*d^3*e^8*f^2*h^4 - 96*a^2*c^9*d^7*e^4*f^2*h^4 - 64*a^3*c^8*d^5*e^6*f^2*h^4 - 4480*a^4*b^3*c^4*e^11*f^2*h^4 + 3696*a^3*b^5*c^3*e^11*f^2*h^4 - 1376*a^2*b^7*c^2*e^11*f^2*h^4 - 2048*a^5*c^6*d*e^10*f^2*h^4 - 64*a*c^10*d^9*e^2*f^2*h^4 + 1792*a^5*b*c^5*e^11*f^2*h^4 + 64*b^10*c*d*e^10*f^2*h^4 + 64*b*c^10*d^10*e*f^2*h^4 + 240*a*b^9*c*e^11*f^2*h^4 - 16*c^11*d^11*f^2*h^4 - 16*b^11*e^11*f^2*h^4 - c^7*e^7, h, k)^3*(root(8388608*a^7*b*c^11*d^18*e*f^6*h^12 - 513802240*a^10*b^2*c^7*d^11*e^8*f^6*h^12 - 381681664*a^11*b^2*c^6*d^9*e^10*f^6*h^12 - 381681664*a^9*b^2*c^8*d^13*e^6*f^6*h^12 - 300941312*a^9*b^5*c^5*d^10*e^9*f^6*h^12 - 300941312*a^8*b^5*c^6*d^12*e^7*f^6*h^12 + 293601280*a^10*b^3*c^6*d^10*e^9*f^6*h^12 + 293601280*a^9*b^3*c^7*d^12*e^7*f^6*h^12 - 168820736*a^10*b^5*c^4*d^8*e^11*f^6*h^12 - 168820736*a^7*b^5*c^7*d^14*e^5*f^6*h^12 + 166068224*a^8*b^6*c^5*d^11*e^8*f^6*h^12 - 146800640*a^12*b^2*c^5*d^7*e^12*f^6*h^12 - 146800640*a^8*b^2*c^9*d^15*e^4*f^6*h^12 + 124780544*a^10*b^4*c^5*d^9*e^10*f^6*h^12 + 124780544*a^8*b^4*c^7*d^13*e^6*f^6*h^12 + 119275520*a^9*b^4*c^6*d^11*e^8*f^6*h^12 + 117440512*a^11*b^3*c^5*d^8*e^11*f^6*h^12 + 117440512*a^8*b^3*c^8*d^14*e^5*f^6*h^12 + 102760448*a^9*b^6*c^4*d^9*e^10*f^6*h^12 + 102760448*a^7*b^6*c^6*d^13*e^6*f^6*h^12 + 91750400*a^11*b^4*c^4*d^7*e^12*f^6*h^12 + 91750400*a^7*b^4*c^8*d^15*e^4*f^6*h^12 - 71065600*a^7*b^8*c^4*d^11*e^8*f^6*h^12 - 53444608*a^8*b^8*c^3*d^9*e^10*f^6*h^12 - 53444608*a^6*b^8*c^5*d^13*e^6*f^6*h^12 + 40370176*a^9*b^7*c^3*d^8*e^11*f^6*h^12 + 40370176*a^6*b^7*c^6*d^14*e^5*f^6*h^12 - 36700160*a^11*b^5*c^3*d^6*e^13*f^6*h^12 - 36700160*a^6*b^5*c^8*d^16*e^3*f^6*h^12 + 34078720*a^8*b^7*c^4*d^10*e^9*f^6*h^12 + 34078720*a^7*b^7*c^5*d^12*e^7*f^6*h^12 + 26214400*a^12*b^4*c^3*d^5*e^14*f^6*h^12 + 26214400*a^6*b^4*c^9*d^17*e^2*f^6*h^12 + 22118400*a^7*b^9*c^3*d^10*e^9*f^6*h^12 + 22118400*a^6*b^9*c^4*d^12*e^7*f^6*h^12 - 20971520*a^13*b^2*c^4*d^5*e^14*f^6*h^12 - 20971520*a^7*b^2*c^10*d^17*e^2*f^6*h^12 + 18350080*a^10*b^7*c^2*d^6*e^13*f^6*h^12 + 18350080*a^5*b^7*c^7*d^16*e^3*f^6*h^12 - 16629760*a^9*b^8*c^2*d^7*e^12*f^6*h^12 - 16629760*a^5*b^8*c^6*d^15*e^4*f^6*h^12 - 10485760*a^11*b^6*c^2*d^5*e^14*f^6*h^12 - 10485760*a^5*b^6*c^8*d^17*e^2*f^6*h^12 + 9175040*a^10*b^6*c^3*d^7*e^12*f^6*h^12 + 9175040*a^6*b^6*c^7*d^15*e^4*f^6*h^12 - 8388608*a^13*b^3*c^3*d^4*e^15*f^6*h^12 + 5619712*a^7*b^10*c^2*d^9*e^10*f^6*h^12 + 5619712*a^5*b^10*c^4*d^13*e^6*f^6*h^12 - 5570560*a^6*b^11*c^2*d^10*e^9*f^6*h^12 - 5570560*a^5*b^11*c^3*d^12*e^7*f^6*h^12 + 4358144*a^8*b^9*c^2*d^8*e^11*f^6*h^12 + 4358144*a^5*b^9*c^5*d^14*e^5*f^6*h^12 + 4259840*a^6*b^10*c^3*d^11*e^8*f

^6*h^12 + 3899392*a^4*b^10*c^5*d^15*e^4*f^6*h^12 - 3440640*a^4*b^9*c^6*d^16
*e^3*f^6*h^12 + 3145728*a^12*b^5*c^2*d^4*e^15*f^6*h^12 - 2523136*a^4*b^11*c
^4*d^14*e^5*f^6*h^12 + 1802240*a^4*b^8*c^7*d^17*e^2*f^6*h^12 + 1556480*a^5*
b^12*c^2*d^11*e^8*f^6*h^12 + 1048576*a^14*b^2*c^3*d^3*e^16*f^6*h^12 + 68812
8*a^4*b^12*c^3*d^13*e^6*f^6*h^12 - 393216*a^13*b^4*c^2*d^3*e^16*f^6*h^12 -
286720*a^3*b^12*c^4*d^15*e^4*f^6*h^12 + 229376*a^3*b^13*c^3*d^14*e^5*f^6*h^
12 + 229376*a^3*b^11*c^5*d^16*e^3*f^6*h^12 + 163840*a^4*b^13*c^2*d^12*e^7*f
^6*h^12 - 114688*a^3*b^14*c^2*d^13*e^6*f^6*h^12 - 114688*a^3*b^10*c^6*d^17*
e^2*f^6*h^12 + 293601280*a^11*b*c^7*d^10*e^9*f^6*h^12 + 293601280*a^10*b*c^
8*d^12*e^7*f^6*h^12 + 176160768*a^12*b*c^6*d^8*e^11*f^6*h^12 + 176160768*a^
9*b*c^9*d^14*e^5*f^6*h^12 + 58720256*a^13*b*c^5*d^6*e^13*f^6*h^12 + 5872025
6*a^8*b*c^10*d^16*e^3*f^6*h^12 + 8388608*a^14*b*c^4*d^4*e^15*f^6*h^12 - 838
8608*a^6*b^3*c^10*d^18*e*f^6*h^12 + 3899392*a^8*b^10*c*d^7*e^12*f^6*h^12 -
3440640*a^9*b^9*c*d^6*e^13*f^6*h^12 + 3145728*a^5*b^5*c^9*d^18*e*f^6*h^12 -
2523136*a^7*b^11*c*d^8*e^11*f^6*h^12 + 1802240*a^10*b^8*c*d^5*e^14*f^6*h^1
2 + 688128*a^6*b^12*c*d^9*e^10*f^6*h^12 - 524288*a^11*b^7*c*d^4*e^15*f^6*h^
12 - 524288*a^4*b^7*c^8*d^18*e*f^6*h^12 + 163840*a^5*b^13*c*d^10*e^9*f^6*h^
12 - 163840*a^4*b^14*c*d^11*e^8*f^6*h^12 + 65536*a^12*b^6*c*d^3*e^16*f^6*h^
12 + 32768*a^3*b^15*c*d^12*e^7*f^6*h^12 + 32768*a^3*b^9*c^7*d^18*e*f^6*h^12
- 73400320*a^11*c^8*d^11*e^8*f^6*h^12 - 58720256*a^12*c^7*d^9*e^10*f^6*h^1
2 - 58720256*a^10*c^9*d^13*e^6*f^6*h^12 - 29360128*a^13*c^6*d^7*e^12*f^6*h^
12 - 29360128*a^9*c^10*d^15*e^4*f^6*h^12 - 8388608*a^14*c^5*d^5*e^14*f^6*h^
12 - 8388608*a^8*c^11*d^17*e^2*f^6*h^12 - 1048576*a^15*c^4*d^3*e^16*f^6*h^1
2 - 286720*a^7*b^12*d^7*e^12*f^6*h^12 + 229376*a^8*b^11*d^6*e^13*f^6*h^12 +
229376*a^6*b^13*d^8*e^11*f^6*h^12 - 114688*a^9*b^10*d^5*e^14*f^6*h^12 - 11
4688*a^5*b^14*d^9*e^10*f^6*h^12 + 32768*a^10*b^9*d^4*e^15*f^6*h^12 + 32768*
a^4*b^15*d^10*e^9*f^6*h^12 - 4096*a^11*b^8*d^3*e^16*f^6*h^12 - 4096*a^3*b^1
6*d^11*e^8*f^6*h^12 + 1048576*a^6*b^2*c^11*d^19*f^6*h^12 - 393216*a^5*b^4*c
^10*d^19*f^6*h^12 + 65536*a^4*b^6*c^9*d^19*f^6*h^12 - 4096*a^3*b^8*c^8*d^19
*f^6*h^12 - 1048576*a^7*c^12*d^19*f^6*h^12 + 262144*a^10*b*c^4*d*e^14*f^4*h
^8 - 23552*a*b^6*c^8*d^14*e*f^4*h^8 - 16384*a^7*b^7*c*d*e^14*f^4*h^8 - 3328
*a*b^13*c*d^7*e^8*f^4*h^8 + 2429952*a^4*b^5*c^6*d^9*e^6*f^4*h^8 - 1865728*a
^6*b^3*c^6*d^7*e^8*f^4*h^8 - 1716224*a^4*b^4*c^7*d^10*e^5*f^4*h^8 + 1605632
*a^6*b^2*c^7*d^8*e^7*f^4*h^8 + 1584384*a^5*b^5*c^5*d^7*e^8*f^4*h^8 + 157286
4*a^5*b^2*c^8*d^10*e^5*f^4*h^8 - 1433600*a^5*b^3*c^7*d^9*e^6*f^4*h^8 - 1261
568*a^4*b^6*c^5*d^8*e^7*f^4*h^8 - 1124352*a^3*b^4*c^8*d^12*e^3*f^4*h^8 - 11
10016*a^7*b^3*c^5*d^5*e^10*f^4*h^8 + 1106176*a^3*b^5*c^7*d^11*e^4*f^4*h^8 -
936960*a^5*b^6*c^4*d^6*e^9*f^4*h^8 - 838656*a^2*b^7*c^6*d^11*e^4*f^4*h^8 -
795648*a^3*b^7*c^5*d^9*e^6*f^4*h^8 + 730880*a^3*b^8*c^4*d^8*e^7*f^4*h^8 +
714752*a^2*b^6*c^7*d^12*e^3*f^4*h^8 + 686080*a^7*b^4*c^4*d^4*e^11*f^4*h^8 +
641024*a^6*b^4*c^5*d^6*e^9*f^4*h^8 - 595968*a^8*b^3*c^4*d^3*e^12*f^4*h^8 +
544768*a^3*b^3*c^9*d^13*e^2*f^4*h^8 + 516096*a^2*b^8*c^5*d^10*e^5*f^4*h^8
+ 441856*a^6*b^5*c^4*d^5*e^10*f^4*h^8 + 393216*a^7*b^2*c^6*d^6*e^9*f^4*h^8
+ 376832*a^4*b^2*c^9*d^12*e^3*f^4*h^8 - 366592*a^6*b^6*c^3*d^4*e^11*f^4*h^8
+ 363520*a^4*b^8*c^3*d^6*e^9*f^4*h^8 - 356352*a^5*b^4*c^6*d^8*e^7*f^4*h^8

- 348672*a^2*b^5*c^8*d^13*e^2*f^4*h^8 - 344064*a^8*b^2*c^5*d^4*e^11*f^4*h^8
 + 294912*a^8*b^4*c^3*d^2*e^13*f^4*h^8 + 210944*a^4*b^3*c^8*d^11*e^4*f^4*h^
8 - 198400*a^3*b^9*c^3*d^7*e^8*f^4*h^8 - 144640*a^4*b^7*c^4*d^7*e^8*f^4*h^8
 - 131072*a^9*b^2*c^4*d^2*e^13*f^4*h^8 - 131072*a^7*b^6*c^2*d^2*e^13*f^4*h^
8 - 129024*a^3*b^6*c^6*d^10*e^5*f^4*h^8 - 104448*a^2*b^10*c^3*d^8*e^7*f^4*h
^8 + 96768*a^5*b^8*c^2*d^4*e^11*f^4*h^8 + 91904*a^7*b^5*c^3*d^3*e^12*f^4*h^
8 - 74240*a^4*b^9*c^2*d^5*e^10*f^4*h^8 - 71680*a^2*b^9*c^4*d^9*e^6*f^4*h^8
+ 58368*a^2*b^11*c^2*d^7*e^8*f^4*h^8 + 36864*a^5*b^7*c^3*d^5*e^10*f^4*h^8 -
 35328*a^3*b^10*c^2*d^6*e^9*f^4*h^8 + 27136*a^6*b^7*c^2*d^3*e^12*f^4*h^8 +
909312*a^8*b*c^6*d^5*e^10*f^4*h^8 + 815104*a^9*b*c^5*d^3*e^12*f^4*h^8 - 651
264*a^5*b*c^9*d^11*e^4*f^4*h^8 - 573440*a^6*b*c^8*d^9*e^6*f^4*h^8 - 262144*
a^9*b^3*c^3*d*e^14*f^4*h^8 + 217088*a^7*b*c^7*d^7*e^8*f^4*h^8 + 211456*a*b^
9*c^5*d^11*e^4*f^4*h^8 - 204800*a^4*b*c^10*d^13*e^2*f^4*h^8 - 172032*a*b^8*
c^6*d^12*e^3*f^4*h^8 - 157696*a*b^10*c^4*d^10*e^5*f^4*h^8 - 131072*a^3*b^2*
c^10*d^14*e*f^4*h^8 + 98304*a^8*b^5*c^2*d*e^14*f^4*h^8 + 92160*a^2*b^4*c^9*
d^14*e*f^4*h^8 + 84992*a*b^7*c^7*d^13*e^2*f^4*h^8 + 64512*a*b^11*c^3*d^9*e^
6*f^4*h^8 + 23552*a^6*b^8*c*d^2*e^13*f^4*h^8 + 18944*a^3*b^11*c*d^5*e^10*f^
4*h^8 - 13312*a^4*b^10*c*d^4*e^11*f^4*h^8 - 9472*a^5*b^9*c*d^3*e^12*f^4*h^8
 - 8192*a*b^12*c^2*d^8*e^7*f^4*h^8 - 6144*a^2*b^12*c*d^6*e^9*f^4*h^8 - 1792
0*b^11*c^4*d^11*e^4*f^4*h^8 + 14336*b^12*c^3*d^10*e^5*f^4*h^8 + 14336*b^10*
c^5*d^12*e^3*f^4*h^8 - 7168*b^13*c^2*d^9*e^6*f^4*h^8 - 7168*b^9*c^6*d^13*e^
2*f^4*h^8 - 425984*a^9*c^6*d^4*e^11*f^4*h^8 - 360448*a^8*c^7*d^6*e^9*f^4*h^
8 - 262144*a^10*c^5*d^2*e^13*f^4*h^8 - 131072*a^7*c^8*d^8*e^7*f^4*h^8 + 983
04*a^5*c^10*d^12*e^3*f^4*h^8 + 65536*a^6*c^9*d^10*e^5*f^4*h^8 - 1536*a^5*b^
10*d^2*e^13*f^4*h^8 - 1536*a^2*b^13*d^5*e^10*f^4*h^8 + 768*a^4*b^11*d^3*e^1
2*f^4*h^8 + 768*a^3*b^12*d^4*e^11*f^4*h^8 + 65536*a^10*b^2*c^3*e^15*f^4*h^8
 - 24576*a^9*b^4*c^2*e^15*f^4*h^8 - 10240*a^2*b^3*c^10*d^15*f^4*h^8 + 2048*
b^14*c*d^8*e^7*f^4*h^8 + 2048*b^8*c^7*d^14*e*f^4*h^8 + 32768*a^4*c^11*d^14*
e*f^4*h^8 + 1024*a^6*b^9*d*e^14*f^4*h^8 + 1024*a*b^14*d^6*e^9*f^4*h^8 + 409
6*a^8*b^6*c*e^15*f^4*h^8 + 12288*a^3*b*c^11*d^15*f^4*h^8 + 2816*a*b^5*c^9*d
^15*f^4*h^8 - 256*b^15*d^7*e^8*f^4*h^8 - 65536*a^11*c^4*e^15*f^4*h^8 - 256*
b^7*c^8*d^15*f^4*h^8 - 256*a^7*b^8*e^15*f^4*h^8 - 896*a*b^8*c^2*d*e^10*f^2*
h^4 + 192*a*b*c^9*d^8*e^3*f^2*h^4 + 11520*a^3*b^3*c^5*d^2*e^9*f^2*h^4 - 585
6*a^2*b^5*c^4*d^2*e^9*f^2*h^4 - 5120*a^3*b^2*c^6*d^3*e^8*f^2*h^4 + 3200*a^2
*b^4*c^5*d^3*e^8*f^2*h^4 - 640*a^2*b^3*c^6*d^4*e^7*f^2*h^4 - 96*a^2*b^2*c^7
*d^5*e^6*f^2*h^4 - 10880*a^3*b^4*c^4*d*e^10*f^2*h^4 + 10240*a^4*b^2*c^5*d*e
^10*f^2*h^4 - 7680*a^4*b*c^6*d^2*e^9*f^2*h^4 + 4672*a^2*b^6*c^3*d*e^10*f^2*
h^4 + 1248*a*b^7*c^3*d^2*e^9*f^2*h^4 + 832*a^3*b*c^7*d^4*e^7*f^2*h^4 - 768*
a*b^6*c^4*d^3*e^8*f^2*h^4 + 192*a^2*b*c^8*d^6*e^5*f^2*h^4 - 192*a*b^2*c^8*d
^7*e^4*f^2*h^4 + 176*a*b^5*c^5*d^4*e^7*f^2*h^4 + 64*a*b^3*c^7*d^6*e^5*f^2*h
^4 - 96*b^9*c^2*d^2*e^9*f^2*h^4 - 96*b^2*c^9*d^9*e^2*f^2*h^4 + 64*b^8*c^3*d
^3*e^8*f^2*h^4 + 64*b^3*c^8*d^8*e^3*f^2*h^4 - 16*b^7*c^4*d^4*e^7*f^2*h^4 -
16*b^4*c^7*d^7*e^4*f^2*h^4 + 2032*a^4*c^7*d^3*e^8*f^2*h^4 - 96*a^2*c^9*d^7*
e^4*f^2*h^4 - 64*a^3*c^8*d^5*e^6*f^2*h^4 - 4480*a^4*b^3*c^4*e^11*f^2*h^4 +
3696*a^3*b^5*c^3*e^11*f^2*h^4 - 1376*a^2*b^7*c^2*e^11*f^2*h^4 - 2048*a^5*c^

6*d*e^10*f^2*h^4 - 64*a*c^10*d^9*e^2*f^2*h^4 + 1792*a^5*b*c^5*e^11*f^2*h^4
+ 64*b^10*c*d*e^10*f^2*h^4 + 64*b*c^10*d^10*e*f^2*h^4 + 240*a*b^9*c*e^11*f^
2*h^4 - 16*c^11*d^11*f^2*h^4 - 16*b^11*e^11*f^2*h^4 - c^7*e^7, h, k)*(root(
8388608*a^7*b*c^11*d^18*e*f^6*h^12 - 513802240*a^10*b^2*c^7*d^11*e^8*f^6*h^
12 - 381681664*a^11*b^2*c^6*d^9*e^10*f^6*h^12 - 381681664*a^9*b^2*c^8*d^13*
e^6*f^6*h^12 - 300941312*a^9*b^5*c^5*d^10*e^9*f^6*h^12 - 300941312*a^8*b^5*
c^6*d^12*e^7*f^6*h^12 + 293601280*a^10*b^3*c^6*d^10*e^9*f^6*h^12 + 29360128
0*a^9*b^3*c^7*d^12*e^7*f^6*h^12 - 168820736*a^10*b^5*c^4*d^8*e^11*f^6*h^12
- 168820736*a^7*b^5*c^7*d^14*e^5*f^6*h^12 + 166068224*a^8*b^6*c^5*d^11*e^8*
f^6*h^12 - 146800640*a^12*b^2*c^5*d^7*e^12*f^6*h^12 - 146800640*a^8*b^2*c^9
*d^15*e^4*f^6*h^12 + 124780544*a^10*b^4*c^5*d^9*e^10*f^6*h^12 + 124780544*a
^8*b^4*c^7*d^13*e^6*f^6*h^12 + 119275520*a^9*b^4*c^6*d^11*e^8*f^6*h^12 + 11
7440512*a^11*b^3*c^5*d^8*e^11*f^6*h^12 + 117440512*a^8*b^3*c^8*d^14*e^5*f^6
*h^12 + 102760448*a^9*b^6*c^4*d^9*e^10*f^6*h^12 + 102760448*a^7*b^6*c^6*d^1
3*e^6*f^6*h^12 + 91750400*a^11*b^4*c^4*d^7*e^12*f^6*h^12 + 91750400*a^7*b^4
*c^8*d^15*e^4*f^6*h^12 - 71065600*a^7*b^8*c^4*d^11*e^8*f^6*h^12 - 53444608*
a^8*b^8*c^3*d^9*e^10*f^6*h^12 - 53444608*a^6*b^8*c^5*d^13*e^6*f^6*h^12 + 40
370176*a^9*b^7*c^3*d^8*e^11*f^6*h^12 + 40370176*a^6*b^7*c^6*d^14*e^5*f^6*h^
12 - 36700160*a^11*b^5*c^3*d^6*e^13*f^6*h^12 - 36700160*a^6*b^5*c^8*d^16*e^
3*f^6*h^12 + 34078720*a^8*b^7*c^4*d^10*e^9*f^6*h^12 + 34078720*a^7*b^7*c^5*
d^12*e^7*f^6*h^12 + 26214400*a^12*b^4*c^3*d^5*e^14*f^6*h^12 + 26214400*a^6*
b^4*c^9*d^17*e^2*f^6*h^12 + 22118400*a^7*b^9*c^3*d^10*e^9*f^6*h^12 + 221184
00*a^6*b^9*c^4*d^12*e^7*f^6*h^12 - 20971520*a^13*b^2*c^4*d^5*e^14*f^6*h^12
- 20971520*a^7*b^2*c^10*d^17*e^2*f^6*h^12 + 18350080*a^10*b^7*c^2*d^6*e^13*
f^6*h^12 + 18350080*a^5*b^7*c^7*d^16*e^3*f^6*h^12 - 16629760*a^9*b^8*c^2*d^
7*e^12*f^6*h^12 - 16629760*a^5*b^8*c^6*d^15*e^4*f^6*h^12 - 10485760*a^11*b^
6*c^2*d^5*e^14*f^6*h^12 - 10485760*a^5*b^6*c^8*d^17*e^2*f^6*h^12 + 9175040*
a^10*b^6*c^3*d^7*e^12*f^6*h^12 + 9175040*a^6*b^6*c^7*d^15*e^4*f^6*h^12 - 83
88608*a^13*b^3*c^3*d^4*e^15*f^6*h^12 + 5619712*a^7*b^10*c^2*d^9*e^10*f^6*h^
12 + 5619712*a^5*b^10*c^4*d^13*e^6*f^6*h^12 - 5570560*a^6*b^11*c^2*d^10*e^9
*f^6*h^12 - 5570560*a^5*b^11*c^3*d^12*e^7*f^6*h^12 + 4358144*a^8*b^9*c^2*d^
8*e^11*f^6*h^12 + 4358144*a^5*b^9*c^5*d^14*e^5*f^6*h^12 + 4259840*a^6*b^10*
c^3*d^11*e^8*f^6*h^12 + 3899392*a^4*b^10*c^5*d^15*e^4*f^6*h^12 - 3440640*a^
4*b^9*c^6*d^16*e^3*f^6*h^12 + 3145728*a^12*b^5*c^2*d^4*e^15*f^6*h^12 - 2523
136*a^4*b^11*c^4*d^14*e^5*f^6*h^12 + 1802240*a^4*b^8*c^7*d^17*e^2*f^6*h^12
+ 1556480*a^5*b^12*c^2*d^11*e^8*f^6*h^12 + 1048576*a^14*b^2*c^3*d^3*e^16*f^
6*h^12 + 688128*a^4*b^12*c^3*d^13*e^6*f^6*h^12 - 393216*a^13*b^4*c^2*d^3*e^
16*f^6*h^12 - 286720*a^3*b^12*c^4*d^15*e^4*f^6*h^12 + 229376*a^3*b^13*c^3*d
^14*e^5*f^6*h^12 + 229376*a^3*b^11*c^5*d^16*e^3*f^6*h^12 + 163840*a^4*b^13*
c^2*d^12*e^7*f^6*h^12 - 114688*a^3*b^14*c^2*d^13*e^6*f^6*h^12 - 114688*a^3*
b^10*c^6*d^17*e^2*f^6*h^12 + 293601280*a^11*b*c^7*d^10*e^9*f^6*h^12 + 29360
1280*a^10*b*c^8*d^12*e^7*f^6*h^12 + 176160768*a^12*b*c^6*d^8*e^11*f^6*h^12
+ 176160768*a^9*b*c^9*d^14*e^5*f^6*h^12 + 58720256*a^13*b*c^5*d^6*e^13*f^6*
h^12 + 58720256*a^8*b*c^10*d^16*e^3*f^6*h^12 + 8388608*a^14*b*c^4*d^4*e^15*
f^6*h^12 - 8388608*a^6*b^3*c^10*d^18*e*f^6*h^12 + 3899392*a^8*b^10*c*d^7*e^

12*f^6*h^12 - 3440640*a^9*b^9*c*d^6*e^13*f^6*h^12 + 3145728*a^5*b^5*c^9*d^1
8*e*f^6*h^12 - 2523136*a^7*b^11*c*d^8*e^11*f^6*h^12 + 1802240*a^10*b^8*c*d^
5*e^14*f^6*h^12 + 688128*a^6*b^12*c*d^9*e^10*f^6*h^12 - 524288*a^11*b^7*c*d
^4*e^15*f^6*h^12 - 524288*a^4*b^7*c^8*d^18*e*f^6*h^12 + 163840*a^5*b^13*c*d
^10*e^9*f^6*h^12 - 163840*a^4*b^14*c*d^11*e^8*f^6*h^12 + 65536*a^12*b^6*c*d
^3*e^16*f^6*h^12 + 32768*a^3*b^15*c*d^12*e^7*f^6*h^12 + 32768*a^3*b^9*c^7*d
^18*e*f^6*h^12 - 73400320*a^11*c^8*d^11*e^8*f^6*h^12 - 58720256*a^12*c^7*d^
9*e^10*f^6*h^12 - 58720256*a^10*c^9*d^13*e^6*f^6*h^12 - 29360128*a^13*c^6*d
^7*e^12*f^6*h^12 - 29360128*a^9*c^10*d^15*e^4*f^6*h^12 - 8388608*a^14*c^5*d
^5*e^14*f^6*h^12 - 8388608*a^8*c^11*d^17*e^2*f^6*h^12 - 1048576*a^15*c^4*d^
3*e^16*f^6*h^12 - 286720*a^7*b^12*d^7*e^12*f^6*h^12 + 229376*a^8*b^11*d^6*e
^13*f^6*h^12 + 229376*a^6*b^13*d^8*e^11*f^6*h^12 - 114688*a^9*b^10*d^5*e^14
*f^6*h^12 - 114688*a^5*b^14*d^9*e^10*f^6*h^12 + 32768*a^10*b^9*d^4*e^15*f^6
*h^12 + 32768*a^4*b^15*d^10*e^9*f^6*h^12 - 4096*a^11*b^8*d^3*e^16*f^6*h^12
- 4096*a^3*b^16*d^11*e^8*f^6*h^12 + 1048576*a^6*b^2*c^11*d^19*f^6*h^12 - 39
3216*a^5*b^4*c^10*d^19*f^6*h^12 + 65536*a^4*b^6*c^9*d^19*f^6*h^12 - 4096*a^
3*b^8*c^8*d^19*f^6*h^12 - 1048576*a^7*c^12*d^19*f^6*h^12 + 262144*a^10*b*c^
4*d*e^14*f^4*h^8 - 23552*a*b^6*c^8*d^14*e*f^4*h^8 - 16384*a^7*b^7*c*d*e^14*
f^4*h^8 - 3328*a*b^13*c*d^7*e^8*f^4*h^8 + 2429952*a^4*b^5*c^6*d^9*e^6*f^4*h
^8 - 1865728*a^6*b^3*c^6*d^7*e^8*f^4*h^8 - 1716224*a^4*b^4*c^7*d^10*e^5*f^4
*h^8 + 1605632*a^6*b^2*c^7*d^8*e^7*f^4*h^8 + 1584384*a^5*b^5*c^5*d^7*e^8*f^
4*h^8 + 1572864*a^5*b^2*c^8*d^10*e^5*f^4*h^8 - 1433600*a^5*b^3*c^7*d^9*e^6*
f^4*h^8 - 1261568*a^4*b^6*c^5*d^8*e^7*f^4*h^8 - 1124352*a^3*b^4*c^8*d^12*e^
3*f^4*h^8 - 1110016*a^7*b^3*c^5*d^5*e^10*f^4*h^8 + 1106176*a^3*b^5*c^7*d^11
*e^4*f^4*h^8 - 936960*a^5*b^6*c^4*d^6*e^9*f^4*h^8 - 838656*a^2*b^7*c^6*d^11
*e^4*f^4*h^8 - 795648*a^3*b^7*c^5*d^9*e^6*f^4*h^8 + 730880*a^3*b^8*c^4*d^8*
e^7*f^4*h^8 + 714752*a^2*b^6*c^7*d^12*e^3*f^4*h^8 + 686080*a^7*b^4*c^4*d^4*
e^11*f^4*h^8 + 641024*a^6*b^4*c^5*d^6*e^9*f^4*h^8 - 595968*a^8*b^3*c^4*d^3*
e^12*f^4*h^8 + 544768*a^3*b^3*c^9*d^13*e^2*f^4*h^8 + 516096*a^2*b^8*c^5*d^1
0*e^5*f^4*h^8 + 441856*a^6*b^5*c^4*d^5*e^10*f^4*h^8 + 393216*a^7*b^2*c^6*d^
6*e^9*f^4*h^8 + 376832*a^4*b^2*c^9*d^12*e^3*f^4*h^8 - 366592*a^6*b^6*c^3*d^
4*e^11*f^4*h^8 + 363520*a^4*b^8*c^3*d^6*e^9*f^4*h^8 - 356352*a^5*b^4*c^6*d^
8*e^7*f^4*h^8 - 348672*a^2*b^5*c^8*d^13*e^2*f^4*h^8 - 344064*a^8*b^2*c^5*d^
4*e^11*f^4*h^8 + 294912*a^8*b^4*c^3*d^2*e^13*f^4*h^8 + 210944*a^4*b^3*c^8*d
^11*e^4*f^4*h^8 - 198400*a^3*b^9*c^3*d^7*e^8*f^4*h^8 - 144640*a^4*b^7*c^4*d
^7*e^8*f^4*h^8 - 131072*a^9*b^2*c^4*d^2*e^13*f^4*h^8 - 131072*a^7*b^6*c^2*d
^2*e^13*f^4*h^8 - 129024*a^3*b^6*c^6*d^10*e^5*f^4*h^8 - 104448*a^2*b^10*c^3
*d^8*e^7*f^4*h^8 + 96768*a^5*b^8*c^2*d^4*e^11*f^4*h^8 + 91904*a^7*b^5*c^3*d
^3*e^12*f^4*h^8 - 74240*a^4*b^9*c^2*d^5*e^10*f^4*h^8 - 71680*a^2*b^9*c^4*d^
9*e^6*f^4*h^8 + 58368*a^2*b^11*c^2*d^7*e^8*f^4*h^8 + 36864*a^5*b^7*c^3*d^5*
e^10*f^4*h^8 - 35328*a^3*b^10*c^2*d^6*e^9*f^4*h^8 + 27136*a^6*b^7*c^2*d^3*e
^12*f^4*h^8 + 909312*a^8*b*c^6*d^5*e^10*f^4*h^8 + 815104*a^9*b*c^5*d^3*e^12
*f^4*h^8 - 651264*a^5*b*c^9*d^11*e^4*f^4*h^8 - 573440*a^6*b*c^8*d^9*e^6*f^4
*h^8 - 262144*a^9*b^3*c^3*d*e^14*f^4*h^8 + 217088*a^7*b*c^7*d^7*e^8*f^4*h^8
 + 211456*a*b^9*c^5*d^11*e^4*f^4*h^8 - 204800*a^4*b*c^10*d^13*e^2*f^4*h^8 -

172032*a*b^8*c^6*d^12*e^3*f^4*h^8 - 157696*a*b^10*c^4*d^10*e^5*f^4*h^8 - 131072*a^3*b^2*c^10*d^14*e*f^4*h^8 + 98304*a^8*b^5*c^2*d*e^14*f^4*h^8 + 92160*a^2*b^4*c^9*d^14*e*f^4*h^8 + 84992*a*b^7*c^7*d^13*e^2*f^4*h^8 + 64512*a*b^11*c^3*d^9*e^6*f^4*h^8 + 23552*a^6*b^8*c*d^2*e^13*f^4*h^8 + 18944*a^3*b^11*c*d^5*e^10*f^4*h^8 - 13312*a^4*b^10*c*d^4*e^11*f^4*h^8 - 9472*a^5*b^9*c*d^3*e^12*f^4*h^8 - 8192*a*b^12*c^2*d^8*e^7*f^4*h^8 - 6144*a^2*b^12*c*d^6*e^9*f^4*h^8 - 17920*b^11*c^4*d^11*e^4*f^4*h^8 + 14336*b^12*c^3*d^10*e^5*f^4*h^8 + 14336*b^10*c^5*d^12*e^3*f^4*h^8 - 7168*b^13*c^2*d^9*e^6*f^4*h^8 - 7168*b^9*c^6*d^13*e^2*f^4*h^8 - 425984*a^9*c^6*d^4*e^11*f^4*h^8 - 360448*a^8*c^7*d^6*e^9*f^4*h^8 - 262144*a^10*c^5*d^2*e^13*f^4*h^8 - 131072*a^7*c^8*d^8*e^7*f^4*h^8 + 98304*a^5*c^10*d^12*e^3*f^4*h^8 + 65536*a^6*c^9*d^10*e^5*f^4*h^8 - 1536*a^5*b^10*d^2*e^13*f^4*h^8 - 1536*a^2*b^13*d^5*e^10*f^4*h^8 + 768*a^4*b^11*d^3*e^12*f^4*h^8 + 768*a^3*b^12*d^4*e^11*f^4*h^8 + 65536*a^10*b^2*c^3*e^15*f^4*h^8 - 24576*a^9*b^4*c^2*e^15*f^4*h^8 - 10240*a^2*b^3*c^10*d^15*f^4*h^8 + 2048*b^14*c*d^8*e^7*f^4*h^8 + 2048*b^8*c^7*d^14*e*f^4*h^8 + 32768*a^4*c^11*d^14*e*f^4*h^8 + 1024*a^6*b^9*d*e^14*f^4*h^8 + 1024*a*b^14*d^6*e^9*f^4*h^8 + 4096*a^8*b^6*c*e^15*f^4*h^8 + 12288*a^3*b*c^11*d^15*f^4*h^8 + 2816*a*b^5*c^9*d^15*f^4*h^8 - 256*b^15*d^7*e^8*f^4*h^8 - 65536*a^11*c^4*e^15*f^4*h^8 - 256*b^7*c^8*d^15*f^4*h^8 - 256*a^7*b^8*e^15*f^4*h^8 - 896*a*b^8*c^2*d*e^10*f^2*h^4 + 192*a*b*c^9*d^8*e^3*f^2*h^4 + 11520*a^3*b^3*c^5*d^2*e^9*f^2*h^4 - 5856*a^2*b^5*c^4*d^2*e^9*f^2*h^4 - 5120*a^3*b^2*c^6*d^3*e^8*f^2*h^4 + 3200*a^2*b^4*c^5*d^3*e^8*f^2*h^4 - 640*a^2*b^3*c^6*d^4*e^7*f^2*h^4 - 96*a^2*b^2*c^7*d^5*e^6*f^2*h^4 - 10880*a^3*b^4*c^4*d*e^10*f^2*h^4 + 10240*a^4*b^2*c^5*d*e^10*f^2*h^4 - 7680*a^4*b*c^6*d^2*e^9*f^2*h^4 + 4672*a^2*b^6*c^3*d*e^10*f^2*h^4 + 1248*a*b^7*c^3*d^2*e^9*f^2*h^4 + 832*a^3*b*c^7*d^4*e^7*f^2*h^4 - 768*a*b^6*c^4*d^3*e^8*f^2*h^4 + 192*a^2*b*c^8*d^6*e^5*f^2*h^4 - 192*a*b^2*c^8*d^7*e^4*f^2*h^4 + 176*a*b^5*c^5*d^4*e^7*f^2*h^4 + 64*a*b^3*c^7*d^6*e^5*f^2*h^4 - 96*b^9*c^2*d^2*e^9*f^2*h^4 - 96*b^2*c^9*d^9*e^2*f^2*h^4 + 64*b^8*c^3*d^3*e^8*f^2*h^4 + 64*b^3*c^8*d^8*e^3*f^2*h^4 - 16*b^7*c^4*d^4*e^7*f^2*h^4 - 16*b^4*c^7*d^7*e^4*f^2*h^4 + 2032*a^4*c^7*d^3*e^8*f^2*h^4 - 96*a^2*c^9*d^7*e^4*f^2*h^4 - 64*a^3*c^8*d^5*e^6*f^2*h^4 - 4480*a^4*b^3*c^4*e^11*f^2*h^4 + 3696*a^3*b^5*c^3*e^11*f^2*h^4 - 1376*a^2*b^7*c^2*e^11*f^2*h^4 - 2048*a^5*c^6*d*e^10*f^2*h^4 - 64*a*c^10*d^9*e^2*f^2*h^4 + 1792*a^5*b*c^5*e^11*f^2*h^4 + 64*b^10*c*d*e^10*f^2*h^4 + 64*b*c^10*d^10*e*f^2*h^4 + 240*a*b^9*c*e^11*f^2*h^4 - 16*c^11*d^11*f^2*h^4 - 16*b^11*e^11*f^2*h^4 - c^7*e^7, h, k)^3*(root(8388608*a^7*b*c^11*d^18*e*f^6*h^12 - 513802240*a^10*b^2*c^7*d^11*e^8*f^6*h^12 - 381681664*a^11*b^2*c^6*d^9*e^10*f^6*h^12 - 381681664*a^9*b^2*c^8*d^13*e^6*f^6*h^12 - 300941312*a^9*b^5*c^5*d^10*e^9*f^6*h^12 - 300941312*a^8*b^5*c^6*d^12*e^7*f^6*h^12 + 293601280*a^10*b^3*c^6*d^10*e^9*f^6*h^12 + 293601280*a^9*b^3*c^7*d^12*e^7*f^6*h^12 - 168820736*a^10*b^5*c^4*d^8*e^11*f^6*h^12 - 168820736*a^7*b^5*c^7*d^14*e^5*f^6*h^12 + 166068224*a^8*b^6*c^5*d^11*e^8*f^6*h^12 - 146800640*a^12*b^2*c^5*d^7*e^12*f^6*h^12 - 146800640*a^8*b^2*c^9*d^15*e^4*f^6*h^12 + 124780544*a^10*b^4*c^5*d^9*e^10*f^6*h^12 + 124780544*a^8*b^4*c^7*d^13*e^6*f^6*h^12 + 119275520*a^9*b^4*c^6*d^11*e^8*f^6*h^12 + 117440512*a^11*b^3*c^5*d^8*e^11*f^6*h^12 + 117440512*a^8*b^3*

c^8*d^14*e^5*f^6*h^12 + 102760448*a^9*b^6*c^4*d^9*e^10*f^6*h^12 + 102760448
*a^7*b^6*c^6*d^13*e^6*f^6*h^12 + 91750400*a^11*b^4*c^4*d^7*e^12*f^6*h^12 +
91750400*a^7*b^4*c^8*d^15*e^4*f^6*h^12 - 71065600*a^7*b^8*c^4*d^11*e^8*f^6*
h^12 - 53444608*a^8*b^8*c^3*d^9*e^10*f^6*h^12 - 53444608*a^6*b^8*c^5*d^13*e
^6*f^6*h^12 + 40370176*a^9*b^7*c^3*d^8*e^11*f^6*h^12 + 40370176*a^6*b^7*c^6
*d^14*e^5*f^6*h^12 - 36700160*a^11*b^5*c^3*d^6*e^13*f^6*h^12 - 36700160*a^6
*b^5*c^8*d^16*e^3*f^6*h^12 + 34078720*a^8*b^7*c^4*d^10*e^9*f^6*h^12 + 34078
720*a^7*b^7*c^5*d^12*e^7*f^6*h^12 + 26214400*a^12*b^4*c^3*d^5*e^14*f^6*h^12
 + 26214400*a^6*b^4*c^9*d^17*e^2*f^6*h^12 + 22118400*a^7*b^9*c^3*d^10*e^9*f
^6*h^12 + 22118400*a^6*b^9*c^4*d^12*e^7*f^6*h^12 - 20971520*a^13*b^2*c^4*d^
5*e^14*f^6*h^12 - 20971520*a^7*b^2*c^10*d^17*e^2*f^6*h^12 + 18350080*a^10*b
^7*c^2*d^6*e^13*f^6*h^12 + 18350080*a^5*b^7*c^7*d^16*e^3*f^6*h^12 - 1662976
0*a^9*b^8*c^2*d^7*e^12*f^6*h^12 - 16629760*a^5*b^8*c^6*d^15*e^4*f^6*h^12 -
10485760*a^11*b^6*c^2*d^5*e^14*f^6*h^12 - 10485760*a^5*b^6*c^8*d^17*e^2*f^6
*h^12 + 9175040*a^10*b^6*c^3*d^7*e^12*f^6*h^12 + 9175040*a^6*b^6*c^7*d^15*e
^4*f^6*h^12 - 8388608*a^13*b^3*c^3*d^4*e^15*f^6*h^12 + 5619712*a^7*b^10*c^2
*d^9*e^10*f^6*h^12 + 5619712*a^5*b^10*c^4*d^13*e^6*f^6*h^12 - 5570560*a^6*b
^11*c^2*d^10*e^9*f^6*h^12 - 5570560*a^5*b^11*c^3*d^12*e^7*f^6*h^12 + 435814
4*a^8*b^9*c^2*d^8*e^11*f^6*h^12 + 4358144*a^5*b^9*c^5*d^14*e^5*f^6*h^12 + 4
259840*a^6*b^10*c^3*d^11*e^8*f^6*h^12 + 3899392*a^4*b^10*c^5*d^15*e^4*f^6*h
^12 - 3440640*a^4*b^9*c^6*d^16*e^3*f^6*h^12 + 3145728*a^12*b^5*c^2*d^4*e^15
*f^6*h^12 - 2523136*a^4*b^11*c^4*d^14*e^5*f^6*h^12 + 1802240*a^4*b^8*c^7*d^
17*e^2*f^6*h^12 + 1556480*a^5*b^12*c^2*d^11*e^8*f^6*h^12 + 1048576*a^14*b^2
*c^3*d^3*e^16*f^6*h^12 + 688128*a^4*b^12*c^3*d^13*e^6*f^6*h^12 - 393216*a^1
3*b^4*c^2*d^3*e^16*f^6*h^12 - 286720*a^3*b^12*c^4*d^15*e^4*f^6*h^12 + 22937
6*a^3*b^13*c^3*d^14*e^5*f^6*h^12 + 229376*a^3*b^11*c^5*d^16*e^3*f^6*h^12 +
163840*a^4*b^13*c^2*d^12*e^7*f^6*h^12 - 114688*a^3*b^14*c^2*d^13*e^6*f^6*h^
12 - 114688*a^3*b^10*c^6*d^17*e^2*f^6*h^12 + 293601280*a^11*b*c^7*d^10*e^9*
f^6*h^12 + 293601280*a^10*b*c^8*d^12*e^7*f^6*h^12 + 176160768*a^12*b*c^6*d^
8*e^11*f^6*h^12 + 176160768*a^9*b*c^9*d^14*e^5*f^6*h^12 + 58720256*a^13*b*c
^5*d^6*e^13*f^6*h^12 + 58720256*a^8*b*c^10*d^16*e^3*f^6*h^12 + 8388608*a^14
*b*c^4*d^4*e^15*f^6*h^12 - 8388608*a^6*b^3*c^10*d^18*e*f^6*h^12 + 3899392*a
^8*b^10*c*d^7*e^12*f^6*h^12 - 3440640*a^9*b^9*c*d^6*e^13*f^6*h^12 + 3145728
*a^5*b^5*c^9*d^18*e*f^6*h^12 - 2523136*a^7*b^11*c*d^8*e^11*f^6*h^12 + 18022
40*a^10*b^8*c*d^5*e^14*f^6*h^12 + 688128*a^6*b^12*c*d^9*e^10*f^6*h^12 - 524
288*a^11*b^7*c*d^4*e^15*f^6*h^12 - 524288*a^4*b^7*c^8*d^18*e*f^6*h^12 + 163
840*a^5*b^13*c*d^10*e^9*f^6*h^12 - 163840*a^4*b^14*c*d^11*e^8*f^6*h^12 + 65
536*a^12*b^6*c*d^3*e^16*f^6*h^12 + 32768*a^3*b^15*c*d^12*e^7*f^6*h^12 + 327
68*a^3*b^9*c^7*d^18*e*f^6*h^12 - 73400320*a^11*c^8*d^11*e^8*f^6*h^12 - 5872
0256*a^12*c^7*d^9*e^10*f^6*h^12 - 58720256*a^10*c^9*d^13*e^6*f^6*h^12 - 293
60128*a^13*c^6*d^7*e^12*f^6*h^12 - 29360128*a^9*c^10*d^15*e^4*f^6*h^12 - 83
88608*a^14*c^5*d^5*e^14*f^6*h^12 - 8388608*a^8*c^11*d^17*e^2*f^6*h^12 - 104
8576*a^15*c^4*d^3*e^16*f^6*h^12 - 286720*a^7*b^12*d^7*e^12*f^6*h^12 + 22937
6*a^8*b^11*d^6*e^13*f^6*h^12 + 229376*a^6*b^13*d^8*e^11*f^6*h^12 - 114688*a
^9*b^10*d^5*e^14*f^6*h^12 - 114688*a^5*b^14*d^9*e^10*f^6*h^12 + 32768*a^10*

b^9*d^4*e^15*f^6*h^12 + 32768*a^4*b^15*d^10*e^9*f^6*h^12 - 4096*a^11*b^8*d^
3*e^16*f^6*h^12 - 4096*a^3*b^16*d^11*e^8*f^6*h^12 + 1048576*a^6*b^2*c^11*d^
19*f^6*h^12 - 393216*a^5*b^4*c^10*d^19*f^6*h^12 + 65536*a^4*b^6*c^9*d^19*f^
6*h^12 - 4096*a^3*b^8*c^8*d^19*f^6*h^12 - 1048576*a^7*c^12*d^19*f^6*h^12 +
262144*a^10*b*c^4*d*e^14*f^4*h^8 - 23552*a*b^6*c^8*d^14*e*f^4*h^8 - 16384*a
^7*b^7*c*d*e^14*f^4*h^8 - 3328*a*b^13*c*d^7*e^8*f^4*h^8 + 2429952*a^4*b^5*c
^6*d^9*e^6*f^4*h^8 - 1865728*a^6*b^3*c^6*d^7*e^8*f^4*h^8 - 1716224*a^4*b^4*
c^7*d^10*e^5*f^4*h^8 + 1605632*a^6*b^2*c^7*d^8*e^7*f^4*h^8 + 1584384*a^5*b^
5*c^5*d^7*e^8*f^4*h^8 + 1572864*a^5*b^2*c^8*d^10*e^5*f^4*h^8 - 1433600*a^5*
b^3*c^7*d^9*e^6*f^4*h^8 - 1261568*a^4*b^6*c^5*d^8*e^7*f^4*h^8 - 1124352*a^3
*b^4*c^8*d^12*e^3*f^4*h^8 - 1110016*a^7*b^3*c^5*d^5*e^10*f^4*h^8 + 1106176*
a^3*b^5*c^7*d^11*e^4*f^4*h^8 - 936960*a^5*b^6*c^4*d^6*e^9*f^4*h^8 - 838656*
a^2*b^7*c^6*d^11*e^4*f^4*h^8 - 795648*a^3*b^7*c^5*d^9*e^6*f^4*h^8 + 730880*
a^3*b^8*c^4*d^8*e^7*f^4*h^8 + 714752*a^2*b^6*c^7*d^12*e^3*f^4*h^8 + 686080*
a^7*b^4*c^4*d^4*e^11*f^4*h^8 + 641024*a^6*b^4*c^5*d^6*e^9*f^4*h^8 - 595968*
a^8*b^3*c^4*d^3*e^12*f^4*h^8 + 544768*a^3*b^3*c^9*d^13*e^2*f^4*h^8 + 516096
*a^2*b^8*c^5*d^10*e^5*f^4*h^8 + 441856*a^6*b^5*c^4*d^5*e^10*f^4*h^8 + 39321
6*a^7*b^2*c^6*d^6*e^9*f^4*h^8 + 376832*a^4*b^2*c^9*d^12*e^3*f^4*h^8 - 36659
2*a^6*b^6*c^3*d^4*e^11*f^4*h^8 + 363520*a^4*b^8*c^3*d^6*e^9*f^4*h^8 - 35635
2*a^5*b^4*c^6*d^8*e^7*f^4*h^8 - 348672*a^2*b^5*c^8*d^13*e^2*f^4*h^8 - 34406
4*a^8*b^2*c^5*d^4*e^11*f^4*h^8 + 294912*a^8*b^4*c^3*d^2*e^13*f^4*h^8 + 2109
44*a^4*b^3*c^8*d^11*e^4*f^4*h^8 - 198400*a^3*b^9*c^3*d^7*e^8*f^4*h^8 - 1446
40*a^4*b^7*c^4*d^7*e^8*f^4*h^8 - 131072*a^9*b^2*c^4*d^2*e^13*f^4*h^8 - 1310
72*a^7*b^6*c^2*d^2*e^13*f^4*h^8 - 129024*a^3*b^6*c^6*d^10*e^5*f^4*h^8 - 104
448*a^2*b^10*c^3*d^8*e^7*f^4*h^8 + 96768*a^5*b^8*c^2*d^4*e^11*f^4*h^8 + 919
04*a^7*b^5*c^3*d^3*e^12*f^4*h^8 - 74240*a^4*b^9*c^2*d^5*e^10*f^4*h^8 - 7168
0*a^2*b^9*c^4*d^9*e^6*f^4*h^8 + 58368*a^2*b^11*c^2*d^7*e^8*f^4*h^8 + 36864*
a^5*b^7*c^3*d^5*e^10*f^4*h^8 - 35328*a^3*b^10*c^2*d^6*e^9*f^4*h^8 + 27136*a
^6*b^7*c^2*d^3*e^12*f^4*h^8 + 909312*a^8*b*c^6*d^5*e^10*f^4*h^8 + 815104*a^
9*b*c^5*d^3*e^12*f^4*h^8 - 651264*a^5*b*c^9*d^11*e^4*f^4*h^8 - 573440*a^6*b
*c^8*d^9*e^6*f^4*h^8 - 262144*a^9*b^3*c^3*d*e^14*f^4*h^8 + 217088*a^7*b*c^7
*d^7*e^8*f^4*h^8 + 211456*a*b^9*c^5*d^11*e^4*f^4*h^8 - 204800*a^4*b*c^10*d^
13*e^2*f^4*h^8 - 172032*a*b^8*c^6*d^12*e^3*f^4*h^8 - 157696*a*b^10*c^4*d^10
*e^5*f^4*h^8 - 131072*a^3*b^2*c^10*d^14*e*f^4*h^8 + 98304*a^8*b^5*c^2*d*e^1
4*f^4*h^8 + 92160*a^2*b^4*c^9*d^14*e*f^4*h^8 + 84992*a*b^7*c^7*d^13*e^2*f^4
*h^8 + 64512*a*b^11*c^3*d^9*e^6*f^4*h^8 + 23552*a^6*b^8*c*d^2*e^13*f^4*h^8
+ 18944*a^3*b^11*c*d^5*e^10*f^4*h^8 - 13312*a^4*b^10*c*d^4*e^11*f^4*h^8 - 9
472*a^5*b^9*c*d^3*e^12*f^4*h^8 - 8192*a*b^12*c^2*d^8*e^7*f^4*h^8 - 6144*a^2
*b^12*c*d^6*e^9*f^4*h^8 - 17920*b^11*c^4*d^11*e^4*f^4*h^8 + 14336*b^12*c^3*
d^10*e^5*f^4*h^8 + 14336*b^10*c^5*d^12*e^3*f^4*h^8 - 7168*b^13*c^2*d^9*e^6*
f^4*h^8 - 7168*b^9*c^6*d^13*e^2*f^4*h^8 - 425984*a^9*c^6*d^4*e^11*f^4*h^8 -
360448*a^8*c^7*d^6*e^9*f^4*h^8 - 262144*a^10*c^5*d^2*e^13*f^4*h^8 - 131072
*a^7*c^8*d^8*e^7*f^4*h^8 + 98304*a^5*c^10*d^12*e^3*f^4*h^8 + 65536*a^6*c^9*
d^10*e^5*f^4*h^8 - 1536*a^5*b^10*d^2*e^13*f^4*h^8 - 1536*a^2*b^13*d^5*e^10*
f^4*h^8 + 768*a^4*b^11*d^3*e^12*f^4*h^8 + 768*a^3*b^12*d^4*e^11*f^4*h^8 + 6

5536*a^10*b^2*c^3*e^15*f^4*h^8 - 24576*a^9*b^4*c^2*e^15*f^4*h^8 - 10240*a^2
*b^3*c^10*d^15*f^4*h^8 + 2048*b^14*c*d^8*e^7*f^4*h^8 + 2048*b^8*c^7*d^14*e*
f^4*h^8 + 32768*a^4*c^11*d^14*e*f^4*h^8 + 1024*a^6*b^9*d*e^14*f^4*h^8 + 102
4*a*b^14*d^6*e^9*f^4*h^8 + 4096*a^8*b^6*c*e^15*f^4*h^8 + 12288*a^3*b*c^11*d
^15*f^4*h^8 + 2816*a*b^5*c^9*d^15*f^4*h^8 - 256*b^15*d^7*e^8*f^4*h^8 - 6553
6*a^11*c^4*e^15*f^4*h^8 - 256*b^7*c^8*d^15*f^4*h^8 - 256*a^7*b^8*e^15*f^4*h
^8 - 896*a*b^8*c^2*d*e^10*f^2*h^4 + 192*a*b*c^9*d^8*e^3*f^2*h^4 + 11520*a^3
*b^3*c^5*d^2*e^9*f^2*h^4 - 5856*a^2*b^5*c^4*d^2*e^9*f^2*h^4 - 5120*a^3*b^2*
c^6*d^3*e^8*f^2*h^4 + 3200*a^2*b^4*c^5*d^3*e^8*f^2*h^4 - 640*a^2*b^3*c^6*d^
4*e^7*f^2*h^4 - 96*a^2*b^2*c^7*d^5*e^6*f^2*h^4 - 10880*a^3*b^4*c^4*d*e^10*f
^2*h^4 + 10240*a^4*b^2*c^5*d*e^10*f^2*h^4 - 7680*a^4*b*c^6*d^2*e^9*f^2*h^4
+ 4672*a^2*b^6*c^3*d*e^10*f^2*h^4 + 1248*a*b^7*c^3*d^2*e^9*f^2*h^4 + 832*a^
3*b*c^7*d^4*e^7*f^2*h^4 - 768*a*b^6*c^4*d^3*e^8*f^2*h^4 + 192*a^2*b*c^8*d^6
*e^5*f^2*h^4 - 192*a*b^2*c^8*d^7*e^4*f^2*h^4 + 176*a*b^5*c^5*d^4*e^7*f^2*h^
4 + 64*a*b^3*c^7*d^6*e^5*f^2*h^4 - 96*b^9*c^2*d^2*e^9*f^2*h^4 - 96*b^2*c^9*
d^9*e^2*f^2*h^4 + 64*b^8*c^3*d^3*e^8*f^2*h^4 + 64*b^3*c^8*d^8*e^3*f^2*h^4 -
 16*b^7*c^4*d^4*e^7*f^2*h^4 - 16*b^4*c^7*d^7*e^4*f^2*h^4 + 2032*a^4*c^7*d^3
*e^8*f^2*h^4 - 96*a^2*c^9*d^7*e^4*f^2*h^4 - 64*a^3*c^8*d^5*e^6*f^2*h^4 - 44
80*a^4*b^3*c^4*e^11*f^2*h^4 + 3696*a^3*b^5*c^3*e^11*f^2*h^4 - 1376*a^2*b^7*
c^2*e^11*f^2*h^4 - 2048*a^5*c^6*d*e^10*f^2*h^4 - 64*a*c^10*d^9*e^2*f^2*h^4
+ 1792*a^5*b*c^5*e^11*f^2*h^4 + 64*b^10*c*d*e^10*f^2*h^4 + 64*b*c^10*d^10*e
*f^2*h^4 + 240*a*b^9*c*e^11*f^2*h^4 - 16*c^11*d^11*f^2*h^4 - 16*b^11*e^11*f
^2*h^4 - c^7*e^7, h, k)*(4697620480*a^9*c^11*d^7*e^13*f^55 - 1879048192*a^6
*c^14*d^13*e^7*f^55 - 2818572288*a^7*c^13*d^11*e^9*f^55 - 402653184*a^5*c^1
5*d^15*e^5*f^55 + 5637144576*a^10*c^10*d^5*e^15*f^55 + 2818572288*a^11*c^9*
d^3*e^17*f^55 + 536870912*a^12*c^8*d*e^19*f^55 + 2097152*a*b^7*c^12*d^16*e^
4*f^55 - 16777216*a*b^8*c^11*d^15*e^5*f^55 + 58720256*a*b^9*c^10*d^14*e^6*f
^55 - 117440512*a*b^10*c^9*d^13*e^7*f^55 + 146800640*a*b^11*c^8*d^12*e^8*f^
55 - 117440512*a*b^12*c^7*d^11*e^9*f^55 + 58720256*a*b^13*c^6*d^10*e^10*f^5
5 - 16777216*a*b^14*c^5*d^9*e^11*f^55 + 2097152*a*b^15*c^4*d^8*e^12*f^55 -
134217728*a^4*b*c^15*d^16*e^4*f^55 + 2147483648*a^5*b*c^14*d^14*e^6*f^55 +
10066329600*a^6*b*c^13*d^12*e^8*f^55 + 13421772800*a^7*b*c^12*d^10*e^10*f^5
5 + 671088640*a^8*b*c^11*d^8*e^12*f^55 + 2097152*a^8*b^8*c^4*d*e^19*f^55 -
12884901888*a^9*b*c^10*d^6*e^14*f^55 - 33554432*a^9*b^6*c^5*d*e^19*f^55 - 1
0603200512*a^10*b*c^9*d^4*e^16*f^55 + 201326592*a^10*b^4*c^6*d*e^19*f^55 -
2684354560*a^11*b*c^8*d^2*e^18*f^55 - 536870912*a^11*b^2*c^7*d*e^19*f^55 -
25165824*a^2*b^5*c^13*d^16*e^4*f^55 + 207618048*a^2*b^6*c^12*d^15*e^5*f^55
- 738197504*a^2*b^7*c^11*d^14*e^6*f^55 + 1468006400*a^2*b^8*c^10*d^13*e^7*f
^55 - 1761607680*a^2*b^9*c^9*d^12*e^8*f^55 + 1262485504*a^2*b^10*c^8*d^11*e
^9*f^55 - 469762048*a^2*b^11*c^7*d^10*e^10*f^55 + 25165824*a^2*b^12*c^6*d^9
*e^11*f^55 + 41943040*a^2*b^13*c^5*d^8*e^12*f^55 - 10485760*a^2*b^14*c^4*d^
7*e^13*f^55 + 100663296*a^3*b^3*c^14*d^16*e^4*f^55 - 880803840*a^3*b^4*c^13
*d^15*e^5*f^55 + 3221225472*a^3*b^5*c^12*d^14*e^6*f^55 - 6312427520*a^3*b^6
*c^11*d^13*e^7*f^55 + 6889144320*a^3*b^7*c^10*d^12*e^8*f^55 - 3548381184*a^
3*b^8*c^9*d^11*e^9*f^55 - 304087040*a^3*b^9*c^8*d^10*e^10*f^55 + 1371537408

*a^3*b^10*c^7*d^9*e^11*f^55 - 597688320*a^3*b^11*c^6*d^8*e^12*f^55 + 419430
40*a^3*b^12*c^5*d^7*e^13*f^55 + 18874368*a^3*b^13*c^4*d^6*e^14*f^55 + 13757
31712*a^4*b^2*c^14*d^15*e^5*f^55 - 5368709120*a^4*b^3*c^13*d^14*e^6*f^55 +
9982443520*a^4*b^4*c^12*d^13*e^7*f^55 - 7507804160*a^4*b^5*c^11*d^12*e^8*f^
55 - 3412066304*a^4*b^6*c^10*d^11*e^9*f^55 + 10955522048*a^4*b^7*c^9*d^10*e
^10*f^55 - 7748976640*a^4*b^8*c^8*d^9*e^11*f^55 + 1468006400*a^4*b^9*c^7*d^
8*e^12*f^55 + 618659840*a^4*b^10*c^6*d^7*e^13*f^55 - 218103808*a^4*b^11*c^5
*d^6*e^14*f^55 - 10485760*a^4*b^12*c^4*d^5*e^15*f^55 - 2348810240*a^5*b^2*c
^13*d^13*e^7*f^55 - 7549747200*a^5*b^3*c^12*d^12*e^8*f^55 + 24570232832*a^5
*b^4*c^11*d^11*e^9*f^55 - 27111981056*a^5*b^5*c^10*d^10*e^10*f^55 + 9638510
592*a^5*b^6*c^9*d^9*e^11*f^55 + 4854906880*a^5*b^7*c^8*d^8*e^12*f^55 - 4697
620480*a^5*b^8*c^7*d^7*e^13*f^55 + 742391808*a^5*b^9*c^6*d^6*e^14*f^55 + 16
7772160*a^5*b^10*c^5*d^5*e^15*f^55 - 10485760*a^5*b^11*c^4*d^4*e^16*f^55 -
18824036352*a^6*b^2*c^12*d^11*e^9*f^55 + 9395240960*a^6*b^3*c^11*d^10*e^10*
f^55 + 14596177920*a^6*b^4*c^10*d^9*e^11*f^55 - 22825402368*a^6*b^5*c^9*d^8
*e^12*f^55 + 10328473600*a^6*b^6*c^8*d^7*e^13*f^55 + 150994944*a^6*b^7*c^7*
d^6*e^14*f^55 - 1170210816*a^6*b^8*c^6*d^5*e^15*f^55 + 142606336*a^6*b^9*c^
5*d^4*e^16*f^55 + 18874368*a^6*b^10*c^4*d^3*e^17*f^55 - 24830279680*a^7*b^2
*c^11*d^9*e^11*f^55 + 20971520000*a^7*b^3*c^10*d^8*e^12*f^55 - 4487905280*a
^7*b^4*c^9*d^7*e^13*f^55 - 5972688896*a^7*b^5*c^8*d^6*e^14*f^55 + 455920844
8*a^7*b^6*c^7*d^5*e^15*f^55 - 538968064*a^7*b^7*c^6*d^4*e^16*f^55 - 2936012
80*a^7*b^8*c^5*d^3*e^17*f^55 - 10485760*a^7*b^9*c^4*d^2*e^18*f^55 - 6207569
920*a^8*b^2*c^10*d^7*e^13*f^55 + 13690208256*a^8*b^3*c^9*d^6*e^14*f^55 - 94
79127040*a^8*b^4*c^8*d^5*e^15*f^55 - 511705088*a^8*b^5*c^7*d^4*e^16*f^55 +
1667235840*a^8*b^6*c^6*d^3*e^17*f^55 + 167772160*a^8*b^7*c^5*d^2*e^18*f^55
+ 6241124352*a^9*b^2*c^9*d^5*e^15*f^55 + 6878658560*a^9*b^3*c^8*d^4*e^16*f^
55 - 3900702720*a^9*b^4*c^7*d^3*e^17*f^55 - 1006632960*a^9*b^5*c^6*d^2*e^18
*f^55 + 2181038080*a^10*b^2*c^8*d^3*e^17*f^55 + 2684354560*a^10*b^3*c^7*d^2
*e^18*f^55) + (f*x)^(1/2)*(268435456*a^11*c^8*e^19*f^54 + 1048576*a^7*b^8*c
^4*e^19*f^54 - 16777216*a^8*b^6*c^5*e^19*f^54 + 100663296*a^9*b^4*c^6*e^19*
f^54 - 268435456*a^10*b^2*c^7*e^19*f^54 - 134217728*a^4*c^15*d^14*e^5*f^54
- 402653184*a^5*c^14*d^12*e^7*f^54 - 268435456*a^6*c^13*d^10*e^9*f^54 + 536
870912*a^7*c^12*d^8*e^11*f^54 + 1476395008*a^8*c^11*d^6*e^13*f^54 + 1744830
464*a^9*c^10*d^4*e^15*f^54 + 1073741824*a^10*c^9*d^2*e^17*f^54 + 1048576*b^
7*c^12*d^15*e^4*f^54 - 8388608*b^8*c^11*d^14*e^5*f^54 + 29360128*b^9*c^10*d
^13*e^6*f^54 - 58720256*b^10*c^9*d^12*e^7*f^54 + 73400320*b^11*c^8*d^11*e^8
*f^54 - 58720256*b^12*c^7*d^10*e^9*f^54 + 29360128*b^13*c^6*d^9*e^10*f^54 -
8388608*b^14*c^5*d^8*e^11*f^54 + 1048576*b^15*c^4*d^7*e^12*f^54 - 10737418
24*a^10*b*c^8*d*e^18*f^54 - 11534336*a*b^5*c^13*d^15*e^4*f^54 + 96468992*a*
b^6*c^12*d^14*e^5*f^54 - 348127232*a*b^7*c^11*d^13*e^6*f^54 + 704643072*a*b
^8*c^10*d^12*e^7*f^54 - 866123776*a*b^9*c^9*d^11*e^8*f^54 + 645922816*a*b^1
0*c^8*d^10*e^9*f^54 - 264241152*a*b^11*c^7*d^9*e^10*f^54 + 33554432*a*b^12*
c^6*d^8*e^11*f^54 + 13631488*a*b^13*c^5*d^7*e^12*f^54 - 4194304*a*b^14*c^4*
d^6*e^13*f^54 - 50331648*a^3*b*c^15*d^15*e^4*f^54 + 838860800*a^4*b*c^14*d^
13*e^6*f^54 + 2667577344*a^5*b*c^13*d^11*e^8*f^54 + 2348810240*a^6*b*c^12*d

^9*e^10*f^54 - 4194304*a^6*b^9*c^4*d*e^18*f^54 - 889192448*a^7*b*c^11*d^7*e^12*f^54 + 67108864*a^7*b^7*c^5*d*e^18*f^54 - 3724541952*a^8*b*c^10*d^5*e^14*f^54 - 402653184*a^8*b^5*c^6*d*e^18*f^54 - 3338665984*a^9*b*c^9*d^3*e^16*f^54 + 1073741824*a^9*b^3*c^7*d*e^18*f^54 + 41943040*a^2*b^3*c^14*d^15*e^4*f^54 - 377487360*a^2*b^4*c^13*d^14*e^5*f^54 + 1428160512*a^2*b^5*c^12*d^13*e^6*f^54 - 2927624192*a^2*b^6*c^11*d^12*e^7*f^54 + 3435134976*a^2*b^7*c^10*d^11*e^8*f^54 - 2113929216*a^2*b^8*c^9*d^10*e^9*f^54 + 293601280*a^2*b^9*c^8*d^9*e^10*f^54 + 427819008*a^2*b^10*c^7*d^8*e^11*f^54 - 239075328*a^2*b^11*c^6*d^7*e^12*f^54 + 25165824*a^2*b^12*c^5*d^6*e^13*f^54 + 6291456*a^2*b^13*c^4*d^5*e^14*f^54 + 536870912*a^3*b^2*c^14*d^14*e^5*f^54 - 2231369728*a^3*b^3*c^13*d^13*e^6*f^54 + 4605345792*a^3*b^4*c^12*d^12*e^7*f^54 - 4530896896*a^3*b^5*c^11*d^11*e^8*f^54 + 528482304*a^3*b^6*c^10*d^10*e^9*f^54 + 3258974208*a^3*b^7*c^9*d^9*e^10*f^54 - 2993684480*a^3*b^8*c^8*d^8*e^11*f^54 + 812646400*a^3*b^9*c^7*d^7*e^12*f^54 + 144703488*a^3*b^10*c^6*d^6*e^13*f^54 - 77594624*a^3*b^11*c^5*d^5*e^14*f^54 - 3145728*a^3*b^12*c^4*d^4*e^15*f^54 - 1543503872*a^4*b^2*c^13*d^12*e^7*f^54 - 864026624*a^4*b^3*c^12*d^11*e^8*f^54 + 7029653504*a^4*b^4*c^11*d^10*e^9*f^54 - 9953083392*a^4*b^5*c^10*d^9*e^10*f^54 + 5167382528*a^4*b^6*c^9*d^8*e^11*f^54 + 592445440*a^4*b^7*c^8*d^7*e^12*f^54 - 1488977920*a^4*b^8*c^7*d^6*e^13*f^54 + 304087040*a^4*b^9*c^6*d^5*e^14*f^54 + 54525952*a^4*b^10*c^5*d^4*e^15*f^54 - 3145728*a^4*b^11*c^4*d^3*e^16*f^54 - 6442450944*a^5*b^2*c^12*d^10*e^9*f^54 + 5872025600*a^5*b^3*c^11*d^9*e^10*f^54 + 1459617792*a^5*b^4*c^10*d^8*e^11*f^54 - 6489636864*a^5*b^5*c^9*d^7*e^12*f^54 + 3837788160*a^5*b^6*c^8*d^6*e^13*f^54 - 150994944*a^5*b^7*c^7*d^5*e^14*f^54 - 396361728*a^5*b^8*c^6*d^4*e^15*f^54 + 38797312*a^5*b^9*c^5*d^3*e^16*f^54 + 6291456*a^5*b^10*c^4*d^2*e^17*f^54 - 6576668672*a^6*b^2*c^11*d^8*e^11*f^54 + 7642021888*a^6*b^3*c^10*d^7*e^12*f^54 - 2625634304*a^6*b^4*c^9*d^6*e^13*f^54 - 1809842176*a^6*b^5*c^8*d^5*e^14*f^54 + 1501560832*a^6*b^6*c^7*d^4*e^15*f^54 - 111149056*a^6*b^7*c^6*d^3*e^16*f^54 - 96468992*a^6*b^8*c^5*d^2*e^17*f^54 - 1610612736*a^7*b^2*c^10*d^6*e^13*f^54 + 4546625536*a^7*b^3*c^9*d^5*e^14*f^54 - 2810183680*a^7*b^4*c^8*d^4*e^15*f^54 - 376438784*a^7*b^5*c^7*d^3*e^16*f^54 + 536870912*a^7*b^6*c^6*d^2*e^17*f^54 + 1409286144*a^8*b^2*c^9*d^4*e^15*f^54 + 2441084928*a^8*b^3*c^8*d^3*e^16*f^54 - 1207959552*a^8*b^4*c^7*d^2*e^17*f^54 + 536870912*a^9*b^2*c^8*d^2*e^17*f^54)) + 8388608*a^7*c^9*e^16*f^53 - 131072*a^2*b^10*c^4*e^16*f^53 + 1966080*a^3*b^8*c^5*e^16*f^53 - 11141120*a^4*b^6*c^6*e^16*f^53 + 28835840*a^5*b^4*c^7*e^16*f^53 - 31457280*a^6*b^2*c^8*e^16*f^53 + 2097152*a^2*c^14*d^10*e^6*f^53 + 3145728*a^3*c^13*d^8*e^8*f^53 - 14680064*a^4*c^12*d^6*e^10*f^53 - 24641536*a^5*c^11*d^4*e^12*f^53 - 131072*b^2*c^14*d^12*e^4*f^53 + 655360*b^3*c^13*d^11*e^5*f^53 - 1310720*b^4*c^12*d^10*e^6*f^53 + 1310720*b^5*c^11*d^9*e^7*f^53 - 655360*b^6*c^10*d^8*e^8*f^53 + 262144*b^7*c^9*d^7*e^9*f^53 - 655360*b^8*c^8*d^6*e^10*f^53 + 1310720*b^9*c^7*d^5*e^11*f^53 - 1310720*b^10*c^6*d^4*e^12*f^53 + 655360*b^11*c^5*d^3*e^13*f^53 - 131072*b^12*c^4*d^2*e^14*f^53 + 524288*a*c^15*d^12*e^4*f^53 - 2621440*a*b*c^14*d^11*e^5*f^53 + 262144*a*b^11*c^4*d*e^15*f^53 + 27262976*a^6*b*c^9*d*e^15*f^53 + 4718592*a*b^2*c^13*d^10*e^6*f^53 - 3145728*a*b^3*c^12*d^9*e^7*f^53 - 524288*a*b^4*c^11*d^8 *e^8*f^53 + 131072*a*b^5*c^10*d^7*e^9*f^53 + 7208960*a*b^6*c^9*d^6*e^10*f^53 - 16252928*a*b^7*c^8*d^5*e^11*f^53 + 16515072*a*b^8*c^7*d^4*e^12*f^53 - 7733248*a*b^9*c^6*d^3*e^13*f^53 + 917504*a*b^10*c^5*d^2*e^14*f^53 - 8388608*a^2*b*c^13*d^9*e^7*f^53 - 3538944*a^2*b^9*c^5*d*e^15*f^53 - 15728640*a^3*b*c^12*d^7*e^9*f^53 + 16908288*a^3*b^7*c^6*d*e^15*f^53 + 60817408*a^4*b*c^11*d^5*e^11*f^53 - 30801920*a^4*b^5*c^7*d*e^15*f^53 + 98041856*a^5*b*c^10*d^3*e^13*f^53 + 5242880*a^5*b^3*c^8*d*e^15*f^53 + 11796480*a^2*b^2*c^12*d^8*e^8*f^53 - 786432*a^2*b^3*c^11*d^7*e^9*f^53 - 31719424*a^2*b^4*c^10*d^6*e^10*f^53 + 71958528*a^2*b^5*c^9*d^5*e^11*f^53 - 73269248*a^2*b^6*c^8*d^4*e^12*f^53 + 28835840*a^2*b^7*c^7*d^3*e^13*f^53 + 3145728*a^2*b^8*c^6*d^2*e^14*f^53 + 57147392*a^3*b^2*c^11*d^6*e^10*f^53 - 126877696*a^3*b^3*c^10*d^5*e^11*f^53 + 126877696*a^3*b^4*c^9*d^4*e^12*f^53 - 21102592*a^3*b^5*c^8*d^3*e^13*f^53 - 42336256*a^3*b^6*c^7*d^2*e^14*f^53 - 50462720*a^4*b^2*c^10*d^4*e^12*f^53 - 74317824*a^4*b^3*c^9*d^3*e^13*f^53 + 120586240*a^4*b^4*c^8*d^2*e^14*f^53 - 106954752*a^5*b^2*c^9*d^2*e^14*f^53) + (f*x)^(1/2)*(131072*b^11*c^4*e^15*f^52 + 131072*c^15*d^11*e^4*f^52 + 11272192*a^2*b^7*c^6*e^15*f^52 - 30277632*a^3*b^5*c^7*e^15*f^52 + 36700160*a^4*b^3*c^8*e^15*f^52 + 786432*a^2*c^13*d^7*e^8*f^52 + 524288*a^3*c^12*d^5*e^10*f^52 - 16646144*a^4*c^11*d^3*e^12*f^52 + 786432*b^2*c^13*d^9*e^6*f^52 - 524288*b^3*c^12*d^8*e^7*f^52 + 131072*b^4*c^11*d^7*e^8*f^52 + 131072*b^7*c^8*d^4*e^11*f^52 - 524288*b^8*c^7*d^3*e^12*f^52 + 786432*b^9*c^6*d^2*e^13*f^52 - 1966080*a*b^9*c^5*e^15*f^52 - 14680064*a^5*b*c^9*e^15*f^52 + 524288*a*c^14*d^9*e^6*f^52 + 16777216*a^5*c^10*d*e^14*f^52 - 524288*b*c^14*d^10*e^5*f^52 - 524288*b^10*c^5*d*e^14*f^52 - 1572864*a*b*c^13*d^8*e^7*f^52 + 7340032*a*b^8*c^6*d*e^14*f^52 + 1572864*a*b^2*c^12*d^7*e^8*f^52 - 524288*a*b^3*c^11*d^6*e^9*f^52 - 1441792*a*b^5*c^9*d^4*e^11*f^52 + 6291456*a*b^6*c^8*d^3*e^12*f^52 - 10223616*a*b^7*c^7*d^2*e^13*f^52 - 1572864*a^2*b*c^12*d^6*e^9*f^52 - 38273024*a^2*b^6*c^7*d*e^14*f^52 - 6815744*a^3*b*c^11*d^4*e^11*f^52 + 89128960*a^3*b^4*c^8*d*e^14*f^52 + 62914560*a^4*b*c^10*d^2*e^13*f^52 - 83886080*a^4*b^2*c^9*d*e^14*f^52 + 786432*a^2*b^2*c^11*d^5*e^10*f^52 + 5242880*a^2*b^3*c^10*d^4*e^11*f^52 - 26214400*a^2*b^4*c^9*d^3*e^12*f^52 + 47972352*a^2*b^5*c^8*d^2*e^13*f^52 + 41943040*a^3*b^2*c^10*d^3*e^12*f^52 - 94371840*a^3*b^3*c^9*d^2*e^13*f^52)) + 8192*b^3*c^9*e^12*f^51 + 8192*c^12*d^3*e^9*f^51 - 32768*a*b*c^10*e^12*f^51 + 40960*a*c^11*d*e^11*f^51 - 8192*b*c^11*d^2*e^10*f^51 - 8192*b^2*c^10*d*e^11*f^51) + 12288*c^11*e^11*f^50*(f*x)^(1/2)))*root(8388608*a^7*b*c^11*d^18*e*f^6*h^12 - 513802240*a^10*b^2*c^7*d^11*e^8*f^6*h^12 - 381681664*a^11*b^2*c^6*d^9*e^10*f^6*h^12 - 381681664*a^9*b^2*c^8*d^13*e^6*f^6*h^12 - 300941312*a^9*b^5*c^5*d^10*e^9*f^6*h^12 - 300941312*a^8*b^5*c^6*d^12*e^7*f^6*h^12 + 293601280*a^10*b^3*c^6*d^10*e^9*f^6*h^12 + 293601280*a^9*b^3*c^7*d^12*e^7*f^6*h^12 - 168820736*a^10*b^5*c^4*d^8*e^11*f^6*h^12 - 168820736*a^7*b^5*c^7*d^14*e^5*f^6*h^12 + 166068224*a^8*b^6*c^5*d^11*e^8*f^6*h^12 - 146800640*a^12*b^2*c^5*d^7*e^12*f^6*h^12 - 146800640*a^8*b^2*c^9*d^15*e^4*f^6*h^12 + 124780544*a^10*b^4*c^5*d^9*e^10*f^6*h^12 + 124780544*a^8*b^4*c^7*d^13*e^6*f^6*h^12 + 119275520*a^9*b^4*c^6*d^11*e^8*f^6*h^12 + 117440512*a^11*b^3*c^5*d^8*e^11*f^6*h^12 + 117440512*a^8*b^3*c^8*d^14*e^5*f^6*h^12 + 102760448*a^9*b^6*c^4 *d^9*e^10*f^6*h^12 + 102760448*a^7*b^6*c^6*d^13*e^6*f^6*h^12 + 91750400*a^11*b^4*c^4*d^7*e^12*f^6*h^12 + 91750400*a^7*b^4*c^8*d^15*e^4*f^6*h^12 - 71065600*a^7*b^8*c^4*d^11*e^8*f^6*h^12 - 53444608*a^8*b^8*c^3*d^9*e^10*f^6*h^12 - 53444608*a^6*b^8*c^5*d^13*e^6*f^6*h^12 + 40370176*a^9*b^7*c^3*d^8*e^11*f^6*h^12 + 40370176*a^6*b^7*c^6*d^14*e^5*f^6*h^12 - 36700160*a^11*b^5*c^3*d^6*e^13*f^6*h^12 - 36700160*a^6*b^5*c^8*d^16*e^3*f^6*h^12 + 34078720*a^8*b^7*c^4*d^10*e^9*f^6*h^12 + 34078720*a^7*b^7*c^5*d^12*e^7*f^6*h^12 + 26214400*a^12*b^4*c^3*d^5*e^14*f^6*h^12 + 26214400*a^6*b^4*c^9*d^17*e^2*f^6*h^12 + 22118400*a^7*b^9*c^3*d^10*e^9*f^6*h^12 + 22118400*a^6*b^9*c^4*d^12*e^7*f^6*h^12 - 20971520*a^13*b^2*c^4*d^5*e^14*f^6*h^12 - 20971520*a^7*b^2*c^10*d^17*e^2*f^6*h^12 + 18350080*a^10*b^7*c^2*d^6*e^13*f^6*h^12 + 18350080*a^5*b^7*c^7*d^16*e^3*f^6*h^12 - 16629760*a^9*b^8*c^2*d^7*e^12*f^6*h^12 - 16629760*a^5*b^8*c^6*d^15*e^4*f^6*h^12 - 10485760*a^11*b^6*c^2*d^5*e^14*f^6*h^12 - 10485760*a^5*b^6*c^8*d^17*e^2*f^6*h^12 + 9175040*a^10*b^6*c^3*d^7*e^12*f^6*h^12 + 9175040*a^6*b^6*c^7*d^15*e^4*f^6*h^12 - 8388608*a^13*b^3*c^3*d^4*e^15*f^6*h^12 + 5619712*a^7*b^10*c^2*d^9*e^10*f^6*h^12 + 5619712*a^5*b^10*c^4*d^13*e^6*f^6*h^12 - 5570560*a^6*b^11*c^2*d^10*e^9*f^6*h^12 - 5570560*a^5*b^11*c^3*d^12*e^7*f^6*h^12 + 4358144*a^8*b^9*c^2*d^8*e^11*f^6*h^12 + 4358144*a^5*b^9*c^5*d^14*e^5*f^6*h^12 + 4259840*a^6*b^10*c^3*d^11*e^8*f^6*h^12 + 3899392*a^4*b^10*c^5*d^15*e^4*f^6*h^12 - 3440640*a^4*b^9*c^6*d^16*e^3*f^6*h^12 + 3145728*a^12*b^5*c^2*d^4*e^15*f^6*h^12 - 2523136*a^4*b^11*c^4*d^14*e^5*f^6*h^12 + 1802240*a^4*b^8*c^7*d^17*e^2*f^6*h^12 + 1556480*a^5*b^12*c^2*d^11*e^8*f^6*h^12 + 1048576*a^14*b^2*c^3*d^3*e^16*f^6*h^12 + 688128*a^4*b^12*c^3*d^13*e^6*f^6*h^12 - 393216*a^13*b^4*c^2*d^3*e^16*f^6*h^12 - 286720*a^3*b^12*c^4*d^15*e^4*f^6*h^12 + 229376*a^3*b^13*c^3*d^14*e^5*f^6*h^12 + 229376*a^3*b^11*c^5*d^16*e^3*f^6*h^12 + 163840*a^4*b^13*c^2*d^12*e^7*f^6*h^12 - 114688*a^3*b^14*c^2*d^13*e^6*f^6*h^12 - 114688*a^3*b^10*c^6*d^17*e^2*f^6*h^12 + 293601280*a^11*b*c^7*d^10*e^9*f^6*h^12 + 293601280*a^10*b*c^8*d^12*e^7*f^6*h^12 + 176160768*a^12*b*c^6*d^8*e^11*f^6*h^12 + 176160768*a^9*b*c^9*d^14*e^5*f^6*h^12 + 58720256*a^13*b*c^5*d^6*e^13*f^6*h^12 + 58720256*a^8*b*c^10*d^16*e^3*f^6*h^12 + 8388608*a^14*b*c^4*d^4*e^15*f^6*h^12 - 8388608*a^6*b^3*c^10*d^18*e*f^6*h^12 + 3899392*a^8*b^10*c*d^7*e^12*f^6*h^12 - 3440640*a^9*b^9*c*d^6*e^13*f^6*h^12 + 3145728*a^5*b^5*c^9*d^18*e*f^6*h^12 - 2523136*a^7*b^11*c*d^8*e^11*f^6*h^12 + 1802240*a^10*b^8*c*d^5*e^14*f^6*h^12 + 688128*a^6*b^12*c*d^9*e^10*f^6*h^12 - 524288*a^11*b^7*c*d^4*e^15*f^6*h^12 - 524288*a^4*b^7*c^8*d^18*e*f^6*h^12 + 163840*a^5*b^13*c*d^10*e^9*f^6*h^12 - 163840*a^4*b^14*c*d^11*e^8*f^6*h^12 + 65536*a^12*b^6*c*d^3*e^16*f^6*h^12 + 32768*a^3*b^15*c*d^12*e^7*f^6*h^12 + 32768*a^3*b^9*c^7*d^18*e*f^6*h^12 - 73400320*a^11*c^8*d^11*e^8*f^6*h^12 - 58720256*a^12*c^7*d^9*e^10*f^6*h^12 - 58720256*a^10*c^9*d^13*e^6*f^6*h^12 - 29360128*a^13*c^6*d^7*e^12*f^6*h^12 - 29360128*a^9*c^10*d^15*e^4*f^6*h^12 - 8388608*a^14*c^5*d^5*e^14*f^6*h^12 - 8388608*a^8*c^11*d^17*e^2*f^6*h^12 - 1048576*a^15*c^4*d^3*e^16*f^6*h^12 - 286720*a^7*b^12*d^7*e^12*f^6*h^12 + 229376*a^8*b^11*d^6*e^13*f^6*h^12 + 229376*a^6*b^13*d^8*e^11*f^6*h^12 - 114688*a^9*b^10*d^5*e^14*f^6*h^12 - 114688*a^5*b^14*d^9*e^10*f^6*h^12 + 32768*a^10*b^9*d^4*e^15*f^6*h^12 + 32768*a^4*b^15*d^10*e

^9*f^6*h^12 - 4096*a^11*b^8*d^3*e^16*f^6*h^12 - 4096*a^3*b^16*d^11*e^8*f^6*
h^12 + 1048576*a^6*b^2*c^11*d^19*f^6*h^12 - 393216*a^5*b^4*c^10*d^19*f^6*h^
12 + 65536*a^4*b^6*c^9*d^19*f^6*h^12 - 4096*a^3*b^8*c^8*d^19*f^6*h^12 - 104
8576*a^7*c^12*d^19*f^6*h^12 + 262144*a^10*b*c^4*d*e^14*f^4*h^8 - 23552*a*b^
6*c^8*d^14*e*f^4*h^8 - 16384*a^7*b^7*c*d*e^14*f^4*h^8 - 3328*a*b^13*c*d^7*e
^8*f^4*h^8 + 2429952*a^4*b^5*c^6*d^9*e^6*f^4*h^8 - 1865728*a^6*b^3*c^6*d^7*
e^8*f^4*h^8 - 1716224*a^4*b^4*c^7*d^10*e^5*f^4*h^8 + 1605632*a^6*b^2*c^7*d^
8*e^7*f^4*h^8 + 1584384*a^5*b^5*c^5*d^7*e^8*f^4*h^8 + 1572864*a^5*b^2*c^8*d
^10*e^5*f^4*h^8 - 1433600*a^5*b^3*c^7*d^9*e^6*f^4*h^8 - 1261568*a^4*b^6*c^5
*d^8*e^7*f^4*h^8 - 1124352*a^3*b^4*c^8*d^12*e^3*f^4*h^8 - 1110016*a^7*b^3*c
^5*d^5*e^10*f^4*h^8 + 1106176*a^3*b^5*c^7*d^11*e^4*f^4*h^8 - 936960*a^5*b^6
*c^4*d^6*e^9*f^4*h^8 - 838656*a^2*b^7*c^6*d^11*e^4*f^4*h^8 - 795648*a^3*b^7
*c^5*d^9*e^6*f^4*h^8 + 730880*a^3*b^8*c^4*d^8*e^7*f^4*h^8 + 714752*a^2*b^6*
c^7*d^12*e^3*f^4*h^8 + 686080*a^7*b^4*c^4*d^4*e^11*f^4*h^8 + 641024*a^6*b^4
*c^5*d^6*e^9*f^4*h^8 - 595968*a^8*b^3*c^4*d^3*e^12*f^4*h^8 + 544768*a^3*b^3
*c^9*d^13*e^2*f^4*h^8 + 516096*a^2*b^8*c^5*d^10*e^5*f^4*h^8 + 441856*a^6*b^
5*c^4*d^5*e^10*f^4*h^8 + 393216*a^7*b^2*c^6*d^6*e^9*f^4*h^8 + 376832*a^4*b^
2*c^9*d^12*e^3*f^4*h^8 - 366592*a^6*b^6*c^3*d^4*e^11*f^4*h^8 + 363520*a^4*b
^8*c^3*d^6*e^9*f^4*h^8 - 356352*a^5*b^4*c^6*d^8*e^7*f^4*h^8 - 348672*a^2*b^
5*c^8*d^13*e^2*f^4*h^8 - 344064*a^8*b^2*c^5*d^4*e^11*f^4*h^8 + 294912*a^8*b
^4*c^3*d^2*e^13*f^4*h^8 + 210944*a^4*b^3*c^8*d^11*e^4*f^4*h^8 - 198400*a^3*
b^9*c^3*d^7*e^8*f^4*h^8 - 144640*a^4*b^7*c^4*d^7*e^8*f^4*h^8 - 131072*a^9*b
^2*c^4*d^2*e^13*f^4*h^8 - 131072*a^7*b^6*c^2*d^2*e^13*f^4*h^8 - 129024*a^3*
b^6*c^6*d^10*e^5*f^4*h^8 - 104448*a^2*b^10*c^3*d^8*e^7*f^4*h^8 + 96768*a^5*
b^8*c^2*d^4*e^11*f^4*h^8 + 91904*a^7*b^5*c^3*d^3*e^12*f^4*h^8 - 74240*a^4*b
^9*c^2*d^5*e^10*f^4*h^8 - 71680*a^2*b^9*c^4*d^9*e^6*f^4*h^8 + 58368*a^2*b^1
1*c^2*d^7*e^8*f^4*h^8 + 36864*a^5*b^7*c^3*d^5*e^10*f^4*h^8 - 35328*a^3*b^10
*c^2*d^6*e^9*f^4*h^8 + 27136*a^6*b^7*c^2*d^3*e^12*f^4*h^8 + 909312*a^8*b*c^
6*d^5*e^10*f^4*h^8 + 815104*a^9*b*c^5*d^3*e^12*f^4*h^8 - 651264*a^5*b*c^9*d
^11*e^4*f^4*h^8 - 573440*a^6*b*c^8*d^9*e^6*f^4*h^8 - 262144*a^9*b^3*c^3*d*e
^14*f^4*h^8 + 217088*a^7*b*c^7*d^7*e^8*f^4*h^8 + 211456*a*b^9*c^5*d^11*e^4*
f^4*h^8 - 204800*a^4*b*c^10*d^13*e^2*f^4*h^8 - 172032*a*b^8*c^6*d^12*e^3*f^
4*h^8 - 157696*a*b^10*c^4*d^10*e^5*f^4*h^8 - 131072*a^3*b^2*c^10*d^14*e*f^4
*h^8 + 98304*a^8*b^5*c^2*d*e^14*f^4*h^8 + 92160*a^2*b^4*c^9*d^14*e*f^4*h^8
+ 84992*a*b^7*c^7*d^13*e^2*f^4*h^8 + 64512*a*b^11*c^3*d^9*e^6*f^4*h^8 + 235
52*a^6*b^8*c*d^2*e^13*f^4*h^8 + 18944*a^3*b^11*c*d^5*e^10*f^4*h^8 - 13312*a
^4*b^10*c*d^4*e^11*f^4*h^8 - 9472*a^5*b^9*c*d^3*e^12*f^4*h^8 - 8192*a*b^12*
c^2*d^8*e^7*f^4*h^8 - 6144*a^2*b^12*c*d^6*e^9*f^4*h^8 - 17920*b^11*c^4*d^11
*e^4*f^4*h^8 + 14336*b^12*c^3*d^10*e^5*f^4*h^8 + 14336*b^10*c^5*d^12*e^3*f^
4*h^8 - 7168*b^13*c^2*d^9*e^6*f^4*h^8 - 7168*b^9*c^6*d^13*e^2*f^4*h^8 - 425
984*a^9*c^6*d^4*e^11*f^4*h^8 - 360448*a^8*c^7*d^6*e^9*f^4*h^8 - 262144*a^10
*c^5*d^2*e^13*f^4*h^8 - 131072*a^7*c^8*d^8*e^7*f^4*h^8 + 98304*a^5*c^10*d^1
2*e^3*f^4*h^8 + 65536*a^6*c^9*d^10*e^5*f^4*h^8 - 1536*a^5*b^10*d^2*e^13*f^4
*h^8 - 1536*a^2*b^13*d^5*e^10*f^4*h^8 + 768*a^4*b^11*d^3*e^12*f^4*h^8 + 768
*a^3*b^12*d^4*e^11*f^4*h^8 + 65536*a^10*b^2*c^3*e^15*f^4*h^8 - 24576*a^9*b^

```
4*c^2*e^15*f^4*h^8 - 10240*a^2*b^3*c^10*d^15*f^4*h^8 + 2048*b^14*c*d^8*e^7*
f^4*h^8 + 2048*b^8*c^7*d^14*e*f^4*h^8 + 32768*a^4*c^11*d^14*e*f^4*h^8 + 102
4*a^6*b^9*d*e^14*f^4*h^8 + 1024*a*b^14*d^6*e^9*f^4*h^8 + 4096*a^8*b^6*c*e^1
5*f^4*h^8 + 12288*a^3*b*c^11*d^15*f^4*h^8 + 2816*a*b^5*c^9*d^15*f^4*h^8 - 2
56*b^15*d^7*e^8*f^4*h^8 - 65536*a^11*c^4*e^15*f^4*h^8 - 256*b^7*c^8*d^15*f^
4*h^8 - 256*a^7*b^8*e^15*f^4*h^8 - 896*a*b^8*c^2*d*e^10*f^2*h^4 + 192*a*b*c
^9*d^8*e^3*f^2*h^4 + 11520*a^3*b^3*c^5*d^2*e^9*f^2*h^4 - 5856*a^2*b^5*c^4*d
^2*e^9*f^2*h^4 - 5120*a^3*b^2*c^6*d^3*e^8*f^2*h^4 + 3200*a^2*b^4*c^5*d^3*e^
8*f^2*h^4 - 640*a^2*b^3*c^6*d^4*e^7*f^2*h^4 - 96*a^2*b^2*c^7*d^5*e^6*f^2*h^
4 - 10880*a^3*b^4*c^4*d*e^10*f^2*h^4 + 10240*a^4*b^2*c^5*d*e^10*f^2*h^4 - 7
680*a^4*b*c^6*d^2*e^9*f^2*h^4 + 4672*a^2*b^6*c^3*d*e^10*f^2*h^4 + 1248*a*b^
7*c^3*d^2*e^9*f^2*h^4 + 832*a^3*b*c^7*d^4*e^7*f^2*h^4 - 768*a*b^6*c^4*d^3*e
^8*f^2*h^4 + 192*a^2*b*c^8*d^6*e^5*f^2*h^4 - 192*a*b^2*c^8*d^7*e^4*f^2*h^4
+ 176*a*b^5*c^5*d^4*e^7*f^2*h^4 + 64*a*b^3*c^7*d^6*e^5*f^2*h^4 - 96*b^9*c^2
*d^2*e^9*f^2*h^4 - 96*b^2*c^9*d^9*e^2*f^2*h^4 + 64*b^8*c^3*d^3*e^8*f^2*h^4
+ 64*b^3*c^8*d^8*e^3*f^2*h^4 - 16*b^7*c^4*d^4*e^7*f^2*h^4 - 16*b^4*c^7*d^7*
e^4*f^2*h^4 + 2032*a^4*c^7*d^3*e^8*f^2*h^4 - 96*a^2*c^9*d^7*e^4*f^2*h^4 - 6
4*a^3*c^8*d^5*e^6*f^2*h^4 - 4480*a^4*b^3*c^4*e^11*f^2*h^4 + 3696*a^3*b^5*c^
3*e^11*f^2*h^4 - 1376*a^2*b^7*c^2*e^11*f^2*h^4 - 2048*a^5*c^6*d*e^10*f^2*h^
4 - 64*a*c^10*d^9*e^2*f^2*h^4 + 1792*a^5*b*c^5*e^11*f^2*h^4 + 64*b^10*c*d*e
^10*f^2*h^4 + 64*b*c^10*d^10*e*f^2*h^4 + 240*a*b^9*c*e^11*f^2*h^4 - 16*c^11
*d^11*f^2*h^4 - 16*b^11*e^11*f^2*h^4 - c^7*e^7, h, k), k, 1, 12)
```

3.311 $\int \frac{x^5\sqrt{a+bx^2+cx^4}}{d+ex^2}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 272

$$\int \frac{x^5\sqrt{a+bx^2+cx^4}}{d+ex^2}\,dx = \frac{((2cd-be)(4cd+be)-2ce(2cd+be)x^2)\sqrt{a+bx^2+cx^4}}{16c^2e^3}+\frac{(a+bx^2+cx^4)^{3/2}}{6ce} - \frac{(16c^3d^3-b^3e^3-2bce^2(bd-2ae)-8c^2de(bd-ae))\operatorname{arctanh}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{32c^{5/2}e^4} + \frac{d^2\sqrt{cd^2-bde+ae^2}\operatorname{arctanh}\left(\frac{bd-2ae+(2cd-be)x^2}{2\sqrt{cd^2-bde+ae^2}\sqrt{a+bx^2+cx^4}}\right)}{2e^4}$$

```
[Out] 1/6*(c*x^4+b*x^2+a)^(3/2)/c/e-1/32*(16*c^3*d^3-b^3*e^3-2*b*c*e^2*(-2*a*e+b*
d)-8*c^2*d*e*(-a*e+b*d))*arctanh(1/2*(2*c*x^2+b)/c^(1/2)/(c*x^4+b*x^2+a)^(1
/2))/c^(5/2)/e^4+1/2*d^2*arctanh(1/2*(b*d-2*a*e+(-b*e+2*c*d)*x^2)/(a*e^2-b*
d*e+c*d^2)^(1/2)/(c*x^4+b*x^2+a)^(1/2))*(a*e^2-b*d*e+c*d^2)^(1/2)/e^4+1/16*
((-b*e+2*c*d)*(b*e+4*c*d)-2*c*e*(b*e+2*c*d)*x^2)*(c*x^4+b*x^2+a)^(1/2)/c^2/
e^3
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 272, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.241, Rules used

= {1265, 1667, 828, 857, 635, 212, 738}

$$\int \frac{x^5\sqrt{a+bx^2+cx^4}}{d+ex^2}\,dx$$
$$= -\frac{\operatorname{arctanh}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)(-8c^2de(bd-ae)-2bce^2(bd-2ae)-b^3e^3+16c^3d^3)}{32c^{5/2}e^4}$$
$$+\frac{d^2\sqrt{ae^2-bde+cd^2}\operatorname{arctanh}\left(\frac{-2ae+x^2(2cd-be)+bd}{2\sqrt{a+bx^2+cx^4}\sqrt{ae^2-bde+cd^2}}\right)}{2e^4}$$
$$+\frac{\sqrt{a+bx^2+cx^4}((2cd-be)(be+4cd)-2cex^2(be+2cd))}{16c^2e^3}+\frac{(a+bx^2+cx^4)^{3/2}}{6ce}$$

```
[In] Int[(x^5*Sqrt[a + b*x^2 + c*x^4])/(d + e*x^2),x]
```

```
[Out] (((2*c*d - b*e)*(4*c*d + b*e) - 2*c*e*(2*c*d + b*e)*x^2)*Sqrt[a + b*x^2 + c
*x^4])/(16*c^2*e^3) + (a + b*x^2 + c*x^4)^(3/2)/(6*c*e) - ((16*c^3*d^3 - b^
3*e^3 - 2*b*c*e^2*(b*d - 2*a*e) - 8*c^2*d*e*(b*d - a*e))*ArcTanh[(b + 2*c*x
^2)/(2*Sqrt[c]*Sqrt[a + b*x^2 + c*x^4])])/(32*c^(5/2)*e^4) + (d^2*Sqrt[c*d^
2 - b*d*e + a*e^2]*ArcTanh[(b*d - 2*a*e + (2*c*d - b*e)*x^2)/(2*Sqrt[c*d^2
- b*d*e + a*e^2]*Sqrt[a + b*x^2 + c*x^4])])/(2*e^4)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 635

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 828

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2)
 - g*(c*d + 2*c*d*p - b*e*p) + g*c*e*(m + 2*p + 1)*x)*((a + b*x + c*x^2)^p/
(c*e^2*(m + 2*p + 1)*(m + 2*p + 2))), x] - Dist[p/(c*e^2*(m + 2*p + 1)*(m +
 2*p + 2)), Int[(d + e*x)^m*(a + b*x + c*x^2)^(p - 1)*Simp[c*e*f*(b*d - 2*a
```

```
*e)*(m + 2*p + 2) + g*(a*e*(b*e - 2*c*d*m + b*e*m) + b*d*(b*e*p - c*d - 2*c
*d*p)) + (c*e*f*(2*c*d - b*e)*(m + 2*p + 2) + g*(b^2*e^2*(p + m + 1) - 2*c^
2*d^2*(1 + 2*p) - c*e*(b*d*(m - 2*p) + 2*a*e*(m + 2*p + 1))))*x, x], x], x]
 /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
- b*d*e + a*e^2, 0] && GtQ[p, 0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[
m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p]
|| IntegersQ[2*m, 2*p])
```

Rule 857

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1265

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rule 1667

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, S
imp[f*(d + e*x)^(m + q - 1)*((a + b*x + c*x^2)^(p + 1)/(c*e^(q - 1)*(m + q
+ 2*p + 1))), x] + Dist[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + b
*x + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1
)*(d + e*x)^q - f*(d + e*x)^(q - 2)*(b*d*e*(p + 1) + a*e^2*(m + q - 1) - c*
d^2*(m + q + 2*p + 1) - e*(2*c*d - b*e)*(m + q + p)*x), x], x], x] /; GtQ[q
, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, d, e, m, p}, x] && Poly
Q[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !(IGtQ
[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{1}{2}\text{Subst}\left(\int \frac{x^2\sqrt{a+bx+cx^2}}{d+ex}\,dx, x, x^2\right)\\ &= \frac{\left(a+bx^2+cx^4\right)^{3/2}}{6ce} + \frac{\text{Subst}\left(\int \frac{\left(-\frac{3}{2}bde-\frac{3}{2}e(2cd+be)x\right)\sqrt{a+bx+cx^2}}{d+ex}\,dx, x, x^2\right)}{6ce^2}\end{aligned}$$

$$= \frac{((2cd-be)(4cd+be)-2ce(2cd+be)x^2)\sqrt{a+bx^2+cx^4}}{16c^2e^3} + \frac{(a+bx^2+cx^4)^{3/2}}{6ce} - \frac{\text{Subst}\left(\int \frac{\frac{3}{4}de(2cd-be)(4bcd+b^2e-4ace)+\frac{3}{4}e(16c^3d^3-b^3e^3-2bce^2(bd-2ae)-8c^2de(bd-ae))x}{(d+ex)\sqrt{a+bx+cx^2}}dx, x, x^2\right)}{24c^2e^4}$$

$$= \frac{((2cd-be)(4cd+be)-2ce(2cd+be)x^2)\sqrt{a+bx^2+cx^4}}{16c^2e^3} + \frac{(a+bx^2+cx^4)^{3/2}}{6ce} + \frac{(d^2(cd^2-bde+ae^2))\,\text{Subst}\left(\int \frac{1}{(d+ex)\sqrt{a+bx+cx^2}}dx, x, x^2\right)}{2e^4} - \frac{(16c^3d^3-b^3e^3-2bce^2(bd-2ae)-8c^2de(bd-ae))\,\text{Subst}\left(\int \frac{1}{\sqrt{a+bx+cx^2}}dx, x, x^2\right)}{32c^2e^4}$$

$$= \frac{((2cd-be)(4cd+be)-2ce(2cd+be)x^2)\sqrt{a+bx^2+cx^4}}{16c^2e^3} + \frac{(a+bx^2+cx^4)^{3/2}}{6ce} - \frac{(d^2(cd^2-bde+ae^2))\,\text{Subst}\left(\int \frac{1}{4cd^2-4bde+4ae^2-x^2}dx, x, \frac{-bd+2ae-(2cd-be)x^2}{\sqrt{a+bx^2+cx^4}}\right)}{e^4} - \frac{(16c^3d^3-b^3e^3-2bce^2(bd-2ae)-8c^2de(bd-ae))\,\text{Subst}\left(\int \frac{1}{4c-x^2}dx, x, \frac{b+2cx^2}{\sqrt{a+bx^2+cx^4}}\right)}{16c^2e^4}$$

$$= \frac{((2cd-be)(4cd+be)-2ce(2cd+be)x^2)\sqrt{a+bx^2+cx^4}}{16c^2e^3} + \frac{(a+bx^2+cx^4)^{3/2}}{6ce} - \frac{(16c^3d^3-b^3e^3-2bce^2(bd-2ae)-8c^2de(bd-ae))\tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{32c^{5/2}e^4} + \frac{d^2\sqrt{cd^2-bde+ae^2}\tanh^{-1}\left(\frac{bd-2ae+(2cd-be)x^2}{2\sqrt{cd^2-bde+ae^2}\sqrt{a+bx^2+cx^4}}\right)}{2e^4}$$

Mathematica [A] (verified)

Time = 0.97 (sec) , antiderivative size = 259, normalized size of antiderivative = 0.95

$$\int \frac{x^5\sqrt{a+bx^2+cx^4}}{d+ex^2}\,dx = \frac{2\sqrt{c}e\sqrt{a+bx^2+cx^4}(-3b^2e^2+2ce(-3bd+4ae+bex^2)+4c^2(6d^2-3dex^2+2e^2x^4))+96c^{5/2}d^2\sqrt{-cd^2+}}{}$$

```
[In] Integrate[(x^5*Sqrt[a + b*x^2 + c*x^4])/(d + e*x^2),x]
```

```
[Out] (2*Sqrt[c]*e*Sqrt[a + b*x^2 + c*x^4]*(-3*b^2*e^2 + 2*c*e*(-3*b*d + 4*a*e +
b*e*x^2) + 4*c^2*(6*d^2 - 3*d*e*x^2 + 2*e^2*x^4)) + 96*c^(5/2)*d^2*Sqrt[-(c
*d^2) + b*d*e - a*e^2]*ArcTan[(Sqrt[c]*(d + e*x^2) - e*Sqrt[a + b*x^2 + c*x
^4])/Sqrt[-(c*d^2) + e*(b*d - a*e)]] + 3*(16*c^3*d^3 - b^3*e^3 - 2*b*c*e^2*
(b*d - 2*a*e) + 8*c^2*d*e*(-(b*d) + a*e))*Log[b + 2*c*x^2 - 2*Sqrt[c]*Sqrt[
a + b*x^2 + c*x^4]])/(96*c^(5/2)*e^4)
```

Maple [A] (verified)

Time = 0.57 (sec) , antiderivative size = 369, normalized size of antiderivative = 1.36

method	result
pseudoelliptic	$\frac{\left(\left(-3e^2d^2a+3d^3eb\right)c^{\frac{5}{2}}-3c^{\frac{7}{2}}d^4\right)\ln\left(\frac{2\sqrt{c\,x^4+b\,x^2+a}\,\sqrt{\frac{a\,e^2-bde+c\,d^2}{e^2}}\,e+\left(b\,x^2+2a\right)e-d\left(2c\,x^2+b\right)}{e\,x^2+d}\right)+e\sqrt{\frac{a\,e^2-bde+c\,d^2}{e^2}}\left(\left(\left(-\frac{3}{4}a\cdots\right.\right.\right.}{\cdots}$
risch	$\frac{\left(8e^2c^2x^4+2bc\,e^2x^2-12c^2de\,x^2+8e^2ac-3b^2e^2-6bcde+24c^2d^2\right)\sqrt{c\,x^4+b\,x^2+a}}{48c^2e^3}-\frac{8d^2\left(a\,e^2-bde+c\,d^2\right)c^2\ln\left(\frac{\frac{2a\,e^2-2bde+2c\,d^2}{e^2}\cdots}{\cdots}\right)}{\cdots}$
default	$\frac{\frac{\left(c\,x^4+b\,x^2+a\right)^{\frac{3}{2}}}{6c}-\frac{b\left(\frac{\left(2c\,x^2+b\right)\sqrt{c\,x^4+b\,x^2+a}}{4c}+\frac{\left(4ac-b^2\right)\ln\left(\frac{\frac{b}{2}+c\,x^2}{\sqrt{c}}+\sqrt{c\,x^4+b\,x^2+a}\right)}{8c^{\frac{3}{2}}}\right)}{4c}}{e}-\frac{d\left(\frac{\left(2c\,x^2+b\right)\sqrt{c\,x^4+b\,x^2+a}}{8c}+\frac{\left(4ac-b\cdots\right.}{\cdots}\right)}{e^2}\cdots$
elliptic	$\frac{-d\left(\frac{\left(2c\,x^2+b\right)\sqrt{c\,x^4+b\,x^2+a}}{4c}+\frac{\left(4ac-b^2\right)\ln\left(\frac{\frac{b}{2}+c\,x^2}{\sqrt{c}}+\sqrt{c\,x^4+b\,x^2+a}\right)}{8c^{\frac{3}{2}}}\right)+e\left(\frac{\left(c\,x^4+b\,x^2+a\right)^{\frac{3}{2}}}{3c}-\frac{b\left(\frac{\left(2c\,x^2+b\right)\sqrt{c\,x^4+b\,x^2+a}}{4c}+\frac{\left(4ac\cdots\right.}{\cdots}\right)}{\cdots}\cdots\right)}{2e^2}$

[In] int(x^5*(c*x^4+b*x^2+a)^(1/2)/(e*x^2+d),x,method=_RETURNVERBOSE)

[Out] 1/6/((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)/c^(5/2)*(((-3*a*d^2*e^2+3*b*d^3*e)*c^(5/2)-3*c^(7/2)*d^4)*ln((2*(c*x^4+b*x^2+a)^(1/2)*((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*e+(b*x^2+2*a)*e-d*(2*c*x^2+b))/(e*x^2+d))+e*((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*(((-3/4*a*b*c+3/16*b^3)*e^3-3/2*c*d*(a*c-1/4*b^2)*e^2+3/2*b*c^2*d^2*e-3*c^3*d^3)*ln((2*c*x^2+2*(c*x^4+b*x^2+a)^(1/2)*c^(1/2)+b)/c^(1/2))+e*((e^2*x^4-3/2*e*d*x^2+3*d^2)*c^(5/2)+e*(((1/4*b*x^2+a)*e-3/4*b*d)*c^(3/2)-3/8*b^2*e*c^(1/2)))*(c*x^4+b*x^2+a)^(1/2)+3/4*((a*b*c-1/4*b^3)*e^3+2*c*d*(a*c-1/4*b^2)*e^2-2*b*c^2*d^2*e+4*c^3*d^3)*ln(2)))/e^5

Fricas [A] (verification not implemented)

none

Time = 94.12 (sec) , antiderivative size = 1525, normalized size of antiderivative = 5.61

$$\int \frac{x^5\sqrt{a+bx^2+cx^4}}{d+ex^2}\,dx=\text{Too large to display}$$

```
[In] integrate(x^5*(c*x^4+b*x^2+a)^(1/2)/(e*x^2+d),x, algorithm="fricas")
[Out] [1/192*(48*sqrt(c*d^2 - b*d*e + a*e^2)*c^3*d^2*log(-((8*c^2*d^2 - 8*b*c*d*e
 + (b^2 + 4*a*c)*e^2)*x^4 - 8*a*b*d*e + 8*a^2*e^2 + (b^2 + 4*a*c)*d^2 + 2*(
4*b*c*d^2 + 4*a*b*e^2 - (3*b^2 + 4*a*c)*d*e)*x^2 + 4*sqrt(c*x^4 + b*x^2 + a
)*sqrt(c*d^2 - b*d*e + a*e^2)*((2*c*d - b*e)*x^2 + b*d - 2*a*e))/(e^2*x^4 +
 2*d*e*x^2 + d^2)) + 3*(16*c^3*d^3 - 8*b*c^2*d^2*e - 2*(b^2*c - 4*a*c^2)*d*
e^2 - (b^3 - 4*a*b*c)*e^3)*sqrt(c)*log(-8*c^2*x^4 - 8*b*c*x^2 - b^2 + 4*sqr
t(c*x^4 + b*x^2 + a)*(2*c*x^2 + b)*sqrt(c) - 4*a*c) + 4*(8*c^3*e^3*x^4 + 24
*c^3*d^2*e - 6*b*c^2*d*e^2 - (3*b^2*c - 8*a*c^2)*e^3 - 2*(6*c^3*d*e^2 - b*c
^2*e^3)*x^2)*sqrt(c*x^4 + b*x^2 + a))/(c^3*e^4), 1/192*(96*sqrt(-c*d^2 + b*
d*e - a*e^2)*c^3*d^2*arctan(-1/2*sqrt(c*x^4 + b*x^2 + a)*sqrt(-c*d^2 + b*d*
e - a*e^2)*((2*c*d - b*e)*x^2 + b*d - 2*a*e)/((c^2*d^2 - b*c*d*e + a*c*e^2)
*x^4 + a*c*d^2 - a*b*d*e + a^2*e^2 + (b*c*d^2 - b^2*d*e + a*b*e^2)*x^2)) +
3*(16*c^3*d^3 - 8*b*c^2*d^2*e - 2*(b^2*c - 4*a*c^2)*d*e^2 - (b^3 - 4*a*b*c)
*e^3)*sqrt(c)*log(-8*c^2*x^4 - 8*b*c*x^2 - b^2 + 4*sqrt(c*x^4 + b*x^2 + a)*
(2*c*x^2 + b)*sqrt(c) - 4*a*c) + 4*(8*c^3*e^3*x^4 + 24*c^3*d^2*e - 6*b*c^2*
d*e^2 - (3*b^2*c - 8*a*c^2)*e^3 - 2*(6*c^3*d*e^2 - b*c^2*e^3)*x^2)*sqrt(c*x
^4 + b*x^2 + a))/(c^3*e^4), 1/96*(24*sqrt(c*d^2 - b*d*e + a*e^2)*c^3*d^2*lo
g(-((8*c^2*d^2 - 8*b*c*d*e + (b^2 + 4*a*c)*e^2)*x^4 - 8*a*b*d*e + 8*a^2*e^2
 + (b^2 + 4*a*c)*d^2 + 2*(4*b*c*d^2 + 4*a*b*e^2 - (3*b^2 + 4*a*c)*d*e)*x^2
+ 4*sqrt(c*x^4 + b*x^2 + a)*sqrt(c*d^2 - b*d*e + a*e^2)*((2*c*d - b*e)*x^2
+ b*d - 2*a*e))/(e^2*x^4 + 2*d*e*x^2 + d^2)) + 3*(16*c^3*d^3 - 8*b*c^2*d^2*
e - 2*(b^2*c - 4*a*c^2)*d*e^2 - (b^3 - 4*a*b*c)*e^3)*sqrt(-c)*arctan(1/2*sq
rt(c*x^4 + b*x^2 + a)*(2*c*x^2 + b)*sqrt(-c)/(c^2*x^4 + b*c*x^2 + a*c)) + 2
*(8*c^3*e^3*x^4 + 24*c^3*d^2*e - 6*b*c^2*d*e^2 - (3*b^2*c - 8*a*c^2)*e^3 -
2*(6*c^3*d*e^2 - b*c^2*e^3)*x^2)*sqrt(c*x^4 + b*x^2 + a))/(c^3*e^4), 1/96*(
48*sqrt(-c*d^2 + b*d*e - a*e^2)*c^3*d^2*arctan(-1/2*sqrt(c*x^4 + b*x^2 + a)
*sqrt(-c*d^2 + b*d*e - a*e^2)*((2*c*d - b*e)*x^2 + b*d - 2*a*e)/((c^2*d^2 -
 b*c*d*e + a*c*e^2)*x^4 + a*c*d^2 - a*b*d*e + a^2*e^2 + (b*c*d^2 - b^2*d*e
+ a*b*e^2)*x^2)) + 3*(16*c^3*d^3 - 8*b*c^2*d^2*e - 2*(b^2*c - 4*a*c^2)*d*e^
2 - (b^3 - 4*a*b*c)*e^3)*sqrt(-c)*arctan(1/2*sqrt(c*x^4 + b*x^2 + a)*(2*c*x
^2 + b)*sqrt(-c)/(c^2*x^4 + b*c*x^2 + a*c)) + 2*(8*c^3*e^3*x^4 + 24*c^3*d^2
*e - 6*b*c^2*d*e^2 - (3*b^2*c - 8*a*c^2)*e^3 - 2*(6*c^3*d*e^2 - b*c^2*e^3)*
x^2)*sqrt(c*x^4 + b*x^2 + a))/(c^3*e^4)]
```

Sympy [F]

$$\int \frac{x^5\sqrt{a+bx^2+cx^4}}{d+ex^2}\,dx = \int \frac{x^5\sqrt{a+bx^2+cx^4}}{d+ex^2}\,dx$$

```
[In] integrate(x**5*(c*x**4+b*x**2+a)**(1/2)/(e*x**2+d),x)
```

```
[Out] Integral(x**5*sqrt(a + b*x**2 + c*x**4)/(d + e*x**2), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^5\sqrt{a+bx^2+cx^4}}{d+ex^2}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(x^5*(c*x^4+b*x^2+a)^(1/2)/(e*x^2+d),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [F(-2)]

Exception generated.

$$\int \frac{x^5\sqrt{a+bx^2+cx^4}}{d+ex^2}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(x^5*(c*x^4+b*x^2+a)^(1/2)/(e*x^2+d),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument Type
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^5\sqrt{a+bx^2+cx^4}}{d+ex^2}\,dx = \int \frac{x^5\,\sqrt{c\,x^4+b\,x^2+a}}{e\,x^2+d}\,dx$$

```
[In] int((x^5*(a + b*x^2 + c*x^4)^(1/2))/(d + e*x^2),x)
```

```
[Out] int((x^5*(a + b*x^2 + c*x^4)^(1/2))/(d + e*x^2), x)
```

3.312 $\int \frac{x^3\sqrt{a+bx^2+cx^4}}{d+ex^2}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 208

$$\int \frac{x^3\sqrt{a+bx^2+cx^4}}{d+ex^2}\,dx = -\frac{(4cd-be-2cex^2)\sqrt{a+bx^2+cx^4}}{8ce^2} + \frac{(8c^2d^2-b^2e^2-4ce(bd-ae))\,\mathrm{arctanh}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{16c^{3/2}e^3} - \frac{d\sqrt{cd^2-bde+ae^2}\,\mathrm{arctanh}\left(\frac{bd-2ae+(2cd-be)x^2}{2\sqrt{cd^2-bde+ae^2}\sqrt{a+bx^2+cx^4}}\right)}{2e^3}$$

```
[Out] 1/16*(8*c^2*d^2-b^2*e^2-4*c*e*(-a*e+b*d))*arctanh(1/2*(2*c*x^2+b)/c^(1/2)/(
c*x^4+b*x^2+a)^(1/2))/c^(3/2)/e^3-1/2*d*arctanh(1/2*(b*d-2*a*e+(-b*e+2*c*d)
*x^2)/(a*e^2-b*d*e+c*d^2)^(1/2)/(c*x^4+b*x^2+a)^(1/2))*(a*e^2-b*d*e+c*d^2)^
(1/2)/e^3-1/8*(-2*c*e*x^2-b*e+4*c*d)*(c*x^4+b*x^2+a)^(1/2)/c/e^2
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 208, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.207$, Rules used = {1265, 828, 857, 635, 212, 738}

$$\int \frac{x^3\sqrt{a+bx^2+cx^4}}{d+ex^2}\,dx = \frac{\mathrm{arctanh}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)(-4ce(bd-ae)-b^2e^2+8c^2d^2)}{16c^{3/2}e^3} - \frac{d\sqrt{ae^2-bde+cd^2}\,\mathrm{arctanh}\left(\frac{-2ae+x^2(2cd-be)+bd}{2\sqrt{a+bx^2+cx^4}\sqrt{ae^2-bde+cd^2}}\right)}{2e^3} - \frac{\sqrt{a+bx^2+cx^4}(-be+4cd-2cex^2)}{8ce^2}$$

```
[In] Int[(x^3*Sqrt[a + b*x^2 + c*x^4])/(d + e*x^2),x]
```

```
[Out] -1/8*((4*c*d - b*e - 2*c*e*x^2)*Sqrt[a + b*x^2 + c*x^4])/(c*e^2) + ((8*c^2*
d^2 - b^2*e^2 - 4*c*e*(b*d - a*e))*ArcTanh[(b + 2*c*x^2)/(2*Sqrt[c]*Sqrt[a
+ b*x^2 + c*x^4])])/(16*c^(3/2)*e^3) - (d*Sqrt[c*d^2 - b*d*e + a*e^2]*ArcTa
nh[(b*d - 2*a*e + (2*c*d - b*e)*x^2)/(2*Sqrt[c*d^2 - b*d*e + a*e^2]*Sqrt[a
+ b*x^2 + c*x^4])])/(2*e^3)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 635

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 828

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2)
 - g*(c*d + 2*c*d*p - b*e*p) + g*c*e*(m + 2*p + 1)*x)*((a + b*x + c*x^2)^p/
(c*e^2*(m + 2*p + 1)*(m + 2*p + 2))), x] - Dist[p/(c*e^2*(m + 2*p + 1)*(m +
 2*p + 2)), Int[(d + e*x)^m*(a + b*x + c*x^2)^(p - 1)*Simp[c*e*f*(b*d - 2*a
*e)*(m + 2*p + 2) + g*(a*e*(b*e - 2*c*d*m + b*e*m) + b*d*(b*e*p - c*d - 2*c
*d*p)) + (c*e*f*(2*c*d - b*e)*(m + 2*p + 2) + g*(b^2*e^2*(p + m + 1) - 2*c^
2*d^2*(1 + 2*p) - c*e*(b*d*(m - 2*p) + 2*a*e*(m + 2*p + 1))))*x, x], x], x]
 /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
- b*d*e + a*e^2, 0] && GtQ[p, 0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[
m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p]
|| IntegersQ[2*m, 2*p])
```

Rule 857

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
```

```
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1265

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{1}{2}\text{Subst}\left(\int \frac{x\sqrt{a+bx+cx^2}}{d+ex}\,dx, x, x^2\right)\\
&= -\frac{(4cd-be-2cex^2)\sqrt{a+bx^2+cx^4}}{8ce^2}\\
&\quad -\frac{\text{Subst}\left(\int \frac{-\frac{1}{2}d(4bcd-b^2e-4ace)-\frac{1}{2}(8c^2d^2-b^2e^2-4ce(bd-ae))x}{(d+ex)\sqrt{a+bx+cx^2}}\,dx, x, x^2\right)}{8ce^2}\\
&= -\frac{(4cd-be-2cex^2)\sqrt{a+bx^2+cx^4}}{8ce^2}\\
&\quad -\frac{(d(cd^2-bde+ae^2))\,\text{Subst}\left(\int \frac{1}{(d+ex)\sqrt{a+bx+cx^2}}\,dx, x, x^2\right)}{2e^3}\\
&\quad +\frac{(8c^2d^2-b^2e^2-4ce(bd-ae))\,\text{Subst}\left(\int \frac{1}{\sqrt{a+bx+cx^2}}\,dx, x, x^2\right)}{16ce^3}\\
&= -\frac{(4cd-be-2cex^2)\sqrt{a+bx^2+cx^4}}{8ce^2}\\
&\quad +\frac{(d(cd^2-bde+ae^2))\,\text{Subst}\left(\int \frac{1}{4cd^2-4bde+4ae^2-x^2}\,dx, x, \frac{-bd+2ae-(2cd-be)x^2}{\sqrt{a+bx^2+cx^4}}\right)}{e^3}\\
&\quad +\frac{(8c^2d^2-b^2e^2-4ce(bd-ae))\,\text{Subst}\left(\int \frac{1}{4c-x^2}\,dx, x, \frac{b+2cx^2}{\sqrt{a+bx^2+cx^4}}\right)}{8ce^3}\\
&= -\frac{(4cd-be-2cex^2)\sqrt{a+bx^2+cx^4}}{8ce^2}\\
&\quad +\frac{(8c^2d^2-b^2e^2-4ce(bd-ae))\tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{16c^{3/2}e^3}\\
&\quad -\frac{d\sqrt{cd^2-bde+ae^2}\tanh^{-1}\left(\frac{bd-2ae+(2cd-be)x^2}{2\sqrt{cd^2-bde+ae^2}\sqrt{a+bx^2+cx^4}}\right)}{2e^3}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.65 (sec) , antiderivative size = 199, normalized size of antiderivative = 0.96

$$\int \frac{x^3\sqrt{a+bx^2+cx^4}}{d+ex^2}\,dx$$
$$= \frac{2\sqrt{c}\left(e(-4cd+be+2cex^2)\sqrt{a+bx^2+cx^4}-8cd\sqrt{-cd^2+bde-ae^2}\arctan\left(\frac{\sqrt{c}(d+ex^2)-e\sqrt{a+bx^2+cx^4}}{\sqrt{-cd^2+bde-ae^2}}\right)\right)+(8}{16c^{3/2}e^3}$$

```
[In] Integrate[(x^3*Sqrt[a + b*x^2 + c*x^4])/(d + e*x^2),x]
```

```
[Out] (2*Sqrt[c]*(e*(-4*c*d + b*e + 2*c*e*x^2)*Sqrt[a + b*x^2 + c*x^4] - 8*c*d*Sq
rt[-(c*d^2) + b*d*e - a*e^2]*ArcTan[(Sqrt[c]*(d + e*x^2) - e*Sqrt[a + b*x^2
 + c*x^4])/Sqrt[-(c*d^2) + b*d*e - a*e^2]]) + (8*c^2*d^2 - b^2*e^2 + 4*c*e*
(-(b*d) + a*e))*ArcTanh[(b + 2*c*x^2)/(2*Sqrt[c]*Sqrt[a + b*x^2 + c*x^4])])
/(16*c^(3/2)*e^3)
```

Maple [A] (verified)

Time = 0.46 (sec) , antiderivative size = 286, normalized size of antiderivative = 1.38

method	result
pseudoelliptic	$\frac{(a e^2-bde+c d^2) d c^{\frac{5}{2}} \ln\left(\frac{2\sqrt{c x^4+b x^2+a}\sqrt{\frac{a e^2-bde+c d^2}{e^2}}\, e+(b x^2+2a)e-d(2c x^2+b)}{e x^2+d}\right)-\left(-\frac{\left(2c^2d^2+(a e^2-bde)c-\frac{b^2e^2}{4}\right)c\ln\left(\frac{2c}{}\right.}{2}\right.}{2\sqrt{\frac{a e^2-bde+}{e^2}}}$
risch	$\frac{(2c x^2 e+be-4cd)\sqrt{c x^4+b x^2+a}}{8c e^2}+\frac{\frac{(4e^2ac-b^2e^2-4bcde+8c^2d^2)\ln\left(\frac{\frac{b}{2}+c x^2}{\sqrt{c}}+\sqrt{c x^4+b x^2+a}\right)}{2e\sqrt{c}}+\frac{4d(a e^2-bde+c d^2)c\ln\left(\frac{2a e^2-2b}{e^2}\right.}{}}{}$
default	$\frac{\frac{(2c x^2+b)\sqrt{c x^4+b x^2+a}}{8c}+\frac{(4ac-b^2)\ln\left(\frac{\frac{b}{2}+c x^2}{\sqrt{c}}+\sqrt{c x^4+b x^2+a}\right)}{16c^{\frac{3}{2}}}}{e}-\frac{d\left(\sqrt{c\left(x^2+\frac{d}{e}\right)^2+\frac{(be-2cd)\left(x^2+\frac{d}{e}\right)}{e}+\frac{a e^2-bde+c d^2}{e^2}}+\frac{(be-2}{}\right.}{}$
elliptic	$\frac{\frac{(2c x^2+b)\sqrt{c x^4+b x^2+a}}{4c}+\frac{(4ac-b^2)\ln\left(\frac{\frac{b}{2}+c x^2}{\sqrt{c}}+\sqrt{c x^4+b x^2+a}\right)}{8c^{\frac{3}{2}}}}{2e}-\frac{d\left(\sqrt{c\left(x^2+\frac{d}{e}\right)^2+\frac{(be-2cd)\left(x^2+\frac{d}{e}\right)}{e}+\frac{a e^2-bde+c d^2}{e^2}}+\frac{(be-2}{}\right.}{}$

```
[In] int(x^3*(c*x^4+b*x^2+a)^(1/2)/(e*x^2+d),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2/((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*((a*e^2-b*d*e+c*d^2)*d*c^(5/2)*ln((2*(c
*x^4+b*x^2+a)^(1/2)*((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*e+(b*x^2+2*a)*e-d*(2*c*
x^2+b))/(e*x^2+d))-(-1/2*(2*c^2*d^2+(a*e^2-b*d*e)*c-1/4*b^2*e^2)*c*ln((2*c*
x^2+2*(c*x^4+b*x^2+a)^(1/2)*c^(1/2)+b)/c^(1/2))+e*c^(3/2)*((-1/2*e*x^2+d)*c
-1/4*b*e)*(c*x^4+b*x^2+a)^(1/2)+1/2*(2*c^2*d^2+(a*e^2-b*d*e)*c-1/4*b^2*e^2)
*ln(2)*c)*e*((a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/c^(5/2)/e^4
```

Fricas [A] (verification not implemented)

none

Time = 10.36 (sec) , antiderivative size = 1231, normalized size of antiderivative = 5.92

$$\int \frac{x^3\sqrt{a+bx^2+cx^4}}{d+ex^2}\,dx = \text{Too large to display}$$

```
[In] integrate(x^3*(c*x^4+b*x^2+a)^(1/2)/(e*x^2+d),x, algorithm="fricas")
```

```
[Out] [1/32*(8*sqrt(c*d^2 - b*d*e + a*e^2)*c^2*d*log(-((8*c^2*d^2 - 8*b*c*d*e + (
b^2 + 4*a*c)*e^2)*x^4 - 8*a*b*d*e + 8*a^2*e^2 + (b^2 + 4*a*c)*d^2 + 2*(4*b*
c*d^2 + 4*a*b*e^2 - (3*b^2 + 4*a*c)*d*e)*x^2 - 4*sqrt(c*x^4 + b*x^2 + a)*sq
rt(c*d^2 - b*d*e + a*e^2)*((2*c*d - b*e)*x^2 + b*d - 2*a*e))/(e^2*x^4 + 2*d
*e*x^2 + d^2)) + (8*c^2*d^2 - 4*b*c*d*e - (b^2 - 4*a*c)*e^2)*sqrt(c)*log(-8
*c^2*x^4 - 8*b*c*x^2 - b^2 - 4*sqrt(c*x^4 + b*x^2 + a)*(2*c*x^2 + b)*sqrt(c
) - 4*a*c) + 4*(2*c^2*e^2*x^2 - 4*c^2*d*e + b*c*e^2)*sqrt(c*x^4 + b*x^2 + a
))/(c^2*e^3), -1/32*(16*sqrt(-c*d^2 + b*d*e - a*e^2)*c^2*d*arctan(-1/2*sqrt
(c*x^4 + b*x^2 + a)*sqrt(-c*d^2 + b*d*e - a*e^2)*((2*c*d - b*e)*x^2 + b*d -
 2*a*e)/((c^2*d^2 - b*c*d*e + a*c*e^2)*x^4 + a*c*d^2 - a*b*d*e + a^2*e^2 +
(b*c*d^2 - b^2*d*e + a*b*e^2)*x^2)) - (8*c^2*d^2 - 4*b*c*d*e - (b^2 - 4*a*c
)*e^2)*sqrt(c)*log(-8*c^2*x^4 - 8*b*c*x^2 - b^2 - 4*sqrt(c*x^4 + b*x^2 + a)
*(2*c*x^2 + b)*sqrt(c) - 4*a*c) - 4*(2*c^2*e^2*x^2 - 4*c^2*d*e + b*c*e^2)*s
qrt(c*x^4 + b*x^2 + a))/(c^2*e^3), 1/16*(4*sqrt(c*d^2 - b*d*e + a*e^2)*c^2*
d*log(-((8*c^2*d^2 - 8*b*c*d*e + (b^2 + 4*a*c)*e^2)*x^4 - 8*a*b*d*e + 8*a^2
*e^2 + (b^2 + 4*a*c)*d^2 + 2*(4*b*c*d^2 + 4*a*b*e^2 - (3*b^2 + 4*a*c)*d*e)*
x^2 - 4*sqrt(c*x^4 + b*x^2 + a)*sqrt(c*d^2 - b*d*e + a*e^2)*((2*c*d - b*e)*
x^2 + b*d - 2*a*e))/(e^2*x^4 + 2*d*e*x^2 + d^2)) - (8*c^2*d^2 - 4*b*c*d*e -
 (b^2 - 4*a*c)*e^2)*sqrt(-c)*arctan(1/2*sqrt(c*x^4 + b*x^2 + a)*(2*c*x^2 +
b)*sqrt(-c)/(c^2*x^4 + b*c*x^2 + a*c)) + 2*(2*c^2*e^2*x^2 - 4*c^2*d*e + b*c
*e^2)*sqrt(c*x^4 + b*x^2 + a))/(c^2*e^3), -1/16*(8*sqrt(-c*d^2 + b*d*e - a*
e^2)*c^2*d*arctan(-1/2*sqrt(c*x^4 + b*x^2 + a)*sqrt(-c*d^2 + b*d*e - a*e^2)
*((2*c*d - b*e)*x^2 + b*d - 2*a*e)/((c^2*d^2 - b*c*d*e + a*c*e^2)*x^4 + a*c
*d^2 - a*b*d*e + a^2*e^2 + (b*c*d^2 - b^2*d*e + a*b*e^2)*x^2)) + (8*c^2*d^2
 - 4*b*c*d*e - (b^2 - 4*a*c)*e^2)*sqrt(-c)*arctan(1/2*sqrt(c*x^4 + b*x^2 +
a)*(2*c*x^2 + b)*sqrt(-c)/(c^2*x^4 + b*c*x^2 + a*c)) - 2*(2*c^2*e^2*x^2 - 4
*c^2*d*e + b*c*e^2)*sqrt(c*x^4 + b*x^2 + a))/(c^2*e^3)]
```

Sympy [F]

$$\int \frac{x^3\sqrt{a+bx^2+cx^4}}{d+ex^2}\,dx=\int \frac{x^3\sqrt{a+bx^2+cx^4}}{d+ex^2}\,dx$$

```
[In] integrate(x**3*(c*x**4+b*x**2+a)**(1/2)/(e*x**2+d),x)
```

```
[Out] Integral(x**3*sqrt(a + b*x**2 + c*x**4)/(d + e*x**2), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^3\sqrt{a+bx^2+cx^4}}{d+ex^2}\,dx=\text{Exception raised: ValueError}$$

```
[In] integrate(x^3*(c*x^4+b*x^2+a)^(1/2)/(e*x^2+d),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [F(-2)]

Exception generated.

$$\int \frac{x^3\sqrt{a+bx^2+cx^4}}{d+ex^2}\,dx=\text{Exception raised: TypeError}$$

```
[In] integrate(x^3*(c*x^4+b*x^2+a)^(1/2)/(e*x^2+d),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument Type
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3\sqrt{a+bx^2+cx^4}}{d+ex^2}\,dx=\int \frac{x^3\,\sqrt{c\,x^4+b\,x^2+a}}{e\,x^2+d}\,dx$$

```
[In] int((x^3*(a + b*x^2 + c*x^4)^(1/2))/(d + e*x^2),x)
```

```
[Out] int((x^3*(a + b*x^2 + c*x^4)^(1/2))/(d + e*x^2), x)
```

3.313 $\int \frac{x\sqrt{a+bx^2+cx^4}}{d+ex^2} dx$

Optimal result

Integrand size = 27, antiderivative size = 168

$$\int \frac{x\sqrt{a+bx^2+cx^4}}{d+ex^2}\,dx = \frac{\sqrt{a+bx^2+cx^4}}{2e} - \frac{(2cd-be)\operatorname{arctanh}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{4\sqrt{c}e^2} + \frac{\sqrt{cd^2-bde+ae^2}\operatorname{arctanh}\left(\frac{bd-2ae+(2cd-be)x^2}{2\sqrt{cd^2-bde+ae^2}\sqrt{a+bx^2+cx^4}}\right)}{2e^2}$$

```
[Out] -1/4*(-b*e+2*c*d)*arctanh(1/2*(2*c*x^2+b)/c^(1/2)/(c*x^4+b*x^2+a)^(1/2))/e^
2/c^(1/2)+1/2*arctanh(1/2*(b*d-2*a*e+(-b*e+2*c*d)*x^2)/(a*e^2-b*d*e+c*d^2)^
(1/2)/(c*x^4+b*x^2+a)^(1/2))*(a*e^2-b*d*e+c*d^2)^(1/2)/e^2+1/2*(c*x^4+b*x^2
+a)^(1/2)/e
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 168, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {1261, 748, 857, 635, 212, 738}

$$\int \frac{x\sqrt{a+bx^2+cx^4}}{d+ex^2}\,dx = \frac{\sqrt{ae^2-bde+cd^2}\operatorname{arctanh}\left(\frac{-2ae+x^2(2cd-be)+bd}{2\sqrt{a+bx^2+cx^4}\sqrt{ae^2-bde+cd^2}}\right)}{2e^2} - \frac{(2cd-be)\operatorname{arctanh}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{4\sqrt{c}e^2} + \frac{\sqrt{a+bx^2+cx^4}}{2e}$$

```
[In] Int[(x*Sqrt[a + b*x^2 + c*x^4])/(d + e*x^2),x]
```

```
[Out] Sqrt[a + b*x^2 + c*x^4]/(2*e) - ((2*c*d - b*e)*ArcTanh[(b + 2*c*x^2)/(2*Sqr
t[c]*Sqrt[a + b*x^2 + c*x^4])])/(4*Sqrt[c]*e^2) + (Sqrt[c*d^2 - b*d*e + a*e
```

```
^2]*ArcTanh[(b*d - 2*a*e + (2*c*d - b*e)*x^2)/(2*Sqrt[c*d^2 - b*d*e + a*e^2
]*Sqrt[a + b*x^2 + c*x^4])])/(2*e^2)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 635

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 748

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*((a + b*x + c*x^2)^p/(e*(m + 2*p + 1))), x
] - Dist[p/(e*(m + 2*p + 1)), Int[(d + e*x)^m*Simp[b*d - 2*a*e + (2*c*d - b
*e)*x, x]*(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e
, 0] && GtQ[p, 0] && NeQ[m + 2*p + 1, 0] && ( !RationalQ[m] || LtQ[m, 1]) &
&  !ILtQ[m + 2*p, 0] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 857

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1261

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(
p_.), x_Symbol] :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x],
 x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{1}{2}\text{Subst}\left(\int \frac{\sqrt{a+bx+cx^2}}{d+ex}\,dx, x, x^2\right) \\
&= \frac{\sqrt{a+bx^2+cx^4}}{2e} - \frac{\text{Subst}\left(\int \frac{bd-2ae+(2cd-be)x}{(d+ex)\sqrt{a+bx+cx^2}}\,dx, x, x^2\right)}{4e} \\
&= \frac{\sqrt{a+bx^2+cx^4}}{2e} - \frac{(2cd-be)\text{Subst}\left(\int \frac{1}{\sqrt{a+bx+cx^2}}\,dx, x, x^2\right)}{4e^2} \\
&\quad + \frac{\left(cd^2-bde+ae^2\right)\text{Subst}\left(\int \frac{1}{(d+ex)\sqrt{a+bx+cx^2}}\,dx, x, x^2\right)}{2e^2} \\
&= \frac{\sqrt{a+bx^2+cx^4}}{2e} - \frac{(2cd-be)\text{Subst}\left(\int \frac{1}{4c-x^2}\,dx, x, \frac{b+2cx^2}{\sqrt{a+bx^2+cx^4}}\right)}{2e^2} \\
&\quad - \frac{\left(cd^2-bde+ae^2\right)\text{Subst}\left(\int \frac{1}{4cd^2-4bde+4ae^2-x^2}\,dx, x, \frac{-bd+2ae-(2cd-be)x^2}{\sqrt{a+bx^2+cx^4}}\right)}{e^2} \\
&= \frac{\sqrt{a+bx^2+cx^4}}{2e} - \frac{(2cd-be)\tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{4\sqrt{c}e^2} \\
&\quad + \frac{\sqrt{cd^2-bde+ae^2}\tanh^{-1}\left(\frac{bd-2ae+(2cd-be)x^2}{2\sqrt{cd^2-bde+ae^2}\sqrt{a+bx^2+cx^4}}\right)}{2e^2}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.94 (sec) , antiderivative size = 160, normalized size of antiderivative = 0.95

$$
\int \frac{x\sqrt{a+bx^2+cx^4}}{d+ex^2}\,dx
= \frac{e\sqrt{a+bx^2+cx^4} + 2\sqrt{-cd^2+bde-ae^2}\arctan\left(\frac{\sqrt{-cd^2+bde-ae^2}x^2}{\sqrt{a}(d+ex^2)-d\sqrt{a+bx^2+cx^4}}\right) + \frac{(-2cd+be)\text{arctanh}\left(\frac{\sqrt{c}x^2}{-\sqrt{a}+\sqrt{a+bx^2+cx}}\right)}{\sqrt{c}}}{2e^2}
$$

```
[In] Integrate[(x*Sqrt[a + b*x^2 + c*x^4])/(d + e*x^2),x]
```

```
[Out] (e*Sqrt[a + b*x^2 + c*x^4] + 2*Sqrt[-(c*d^2) + b*d*e - a*e^2]*ArcTan[(Sqrt[
-(c*d^2) + b*d*e - a*e^2]*x^2)/(Sqrt[a]*(d + e*x^2) - d*Sqrt[a + b*x^2 + c*
x^4])] + ((-2*c*d + b*e)*ArcTanh[(Sqrt[c]*x^2)/(-Sqrt[a] + Sqrt[a + b*x^2 +
 c*x^4])])/Sqrt[c])/(2*e^2)
```

Maple [A] (verified)

Time = 0.40 (sec) , antiderivative size = 190, normalized size of antiderivative = 1.13

method	result
pseudoelliptic	$\frac{\sqrt{c x^4+b x^2+a}-\frac{(be-2cd)\left(\ln(2)-\ln\left(\frac{2c x^2+2\sqrt{c x^4+b x^2+a}\,\sqrt{c}+b}{\sqrt{c}}\right)\right)}{2e\sqrt{c}}-\frac{\left(a e^2-bde+c d^2\right)\ln\left(\frac{2\sqrt{c x^4+b x^2+a}\,\sqrt{\frac{a e^2-bde+c d^2}{e^2}}\,e+\left(b x^2 \cdots\right.}{e x^2+d}\right.}{\sqrt{\frac{a e^2-bde+c d^2}{e^2}}\,e^2}}{2e}$
risch	$\frac{\sqrt{c x^4+b x^2+a}}{2e}+\frac{\frac{(be-2cd)\ln\left(\frac{\frac{b}{2}+c x^2}{\sqrt{c}}+\sqrt{c x^4+b x^2+a}\right)}{2e\sqrt{c}}-\frac{\left(a e^2-bde+c d^2\right)\ln\left(\frac{\frac{2a e^2-2bde+2c d^2}{e^2}+\frac{(be-2cd)\left(x^2+\frac{d}{e}\right)}{e}+2\sqrt{\frac{a e^2-bde \cdots}{e^2}}\cdots}{x \cdots}\right.}{e^2\sqrt{\frac{a e^2-bde+c\,d \cdots}{e^2}}}}{2e}$
default	$\frac{\sqrt{c\left(x^2+\frac{d}{e}\right)^2+\frac{(be-2cd)\left(x^2+\frac{d}{e}\right)}{e}+\frac{a e^2-bde+c d^2}{e^2}}+\frac{(be-2cd)\ln\left(\frac{\frac{be-2cd}{2e}+c\left(x^2+\frac{d}{e}\right)}{\sqrt{c}}+\sqrt{c\left(x^2+\frac{d}{e}\right)^2+\frac{(be-2cd)\left(x^2+\frac{d}{e}\right)}{e}+\frac{a e^2-bde+c\,d \cdots}{e^2}}\right.}{2e\sqrt{c}}\cdots}{\cdots}$
elliptic	$\frac{\sqrt{c\left(x^2+\frac{d}{e}\right)^2+\frac{(be-2cd)\left(x^2+\frac{d}{e}\right)}{e}+\frac{a e^2-bde+c d^2}{e^2}}+\frac{(be-2cd)\ln\left(\frac{\frac{be-2cd}{2e}+c\left(x^2+\frac{d}{e}\right)}{\sqrt{c}}+\sqrt{c\left(x^2+\frac{d}{e}\right)^2+\frac{(be-2cd)\left(x^2+\frac{d}{e}\right)}{e}+\frac{a e^2-bde+c\,d \cdots}{e^2}}\right.}{2e\sqrt{c}}\cdots}{\cdots}$

```
[In] int(x*(c*x^4+b*x^2+a)^(1/2)/(e*x^2+d),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*((c*x^4+b*x^2+a)^(1/2)-1/2*(b*e-2*c*d)*(ln(2)-ln((2*c*x^2+2*(c*x^4+b*x^
2+a)^(1/2)*c^(1/2)+b)/c^(1/2)))/e/c^(1/2)-(a*e^2-b*d*e+c*d^2)/((a*e^2-b*d*e
+c*d^2)/e^2)^(1/2)*ln((2*(c*x^4+b*x^2+a)^(1/2)*((a*e^2-b*d*e+c*d^2)/e^2)^(1
/2)*e+(b*x^2+2*a)*e-d*(2*c*x^2+b))/(e*x^2+d))/e^2)/e
```

Fricas [A] (verification not implemented)

none

Time = 0.87 (sec) , antiderivative size = 1050, normalized size of antiderivative = 6.25

$$\int \frac{x\sqrt{a+bx^2+cx^4}}{d+ex^2}\,dx$$
$$=\left[\frac{4\sqrt{cx^4+bx^2+a}ce-(2\,cd-be)\sqrt{c}\log\left(-8\,c^2x^4-8\,bcx^2-b^2-4\sqrt{cx^4+bx^2+a}(2\,cx^2+b)\sqrt{c}-4\,ac\right)\cdots}{\cdots}\right.$$

```
[In] integrate(x*(c*x^4+b*x^2+a)^(1/2)/(e*x^2+d),x, algorithm="fricas")
```

```
[Out] [1/8*(4*sqrt(c*x^4 + b*x^2 + a)*c*e - (2*c*d - b*e)*sqrt(c)*log(-8*c^2*x^4
- 8*b*c*x^2 - b^2 - 4*sqrt(c*x^4 + b*x^2 + a)*(2*c*x^2 + b)*sqrt(c) - 4*a*c
```

```
) + 2*sqrt(c*d^2 - b*d*e + a*e^2)*c*log(-((8*c^2*d^2 - 8*b*c*d*e + (b^2 + 4
*a*c)*e^2)*x^4 - 8*a*b*d*e + 8*a^2*e^2 + (b^2 + 4*a*c)*d^2 + 2*(4*b*c*d^2 +
 4*a*b*e^2 - (3*b^2 + 4*a*c)*d*e)*x^2 + 4*sqrt(c*x^4 + b*x^2 + a)*sqrt(c*d^
2 - b*d*e + a*e^2)*((2*c*d - b*e)*x^2 + b*d - 2*a*e))/(e^2*x^4 + 2*d*e*x^2
+ d^2)))/(c*e^2), 1/4*(2*sqrt(c*x^4 + b*x^2 + a)*c*e + (2*c*d - b*e)*sqrt(-
c)*arctan(1/2*sqrt(c*x^4 + b*x^2 + a)*(2*c*x^2 + b)*sqrt(-c)/(c^2*x^4 + b*c
*x^2 + a*c)) + sqrt(c*d^2 - b*d*e + a*e^2)*c*log(-((8*c^2*d^2 - 8*b*c*d*e +
 (b^2 + 4*a*c)*e^2)*x^4 - 8*a*b*d*e + 8*a^2*e^2 + (b^2 + 4*a*c)*d^2 + 2*(4*
b*c*d^2 + 4*a*b*e^2 - (3*b^2 + 4*a*c)*d*e)*x^2 + 4*sqrt(c*x^4 + b*x^2 + a)*
sqrt(c*d^2 - b*d*e + a*e^2)*((2*c*d - b*e)*x^2 + b*d - 2*a*e))/(e^2*x^4 + 2
*d*e*x^2 + d^2)))/(c*e^2), 1/8*(4*sqrt(c*x^4 + b*x^2 + a)*c*e + 4*sqrt(-c*d
^2 + b*d*e - a*e^2)*c*arctan(-1/2*sqrt(c*x^4 + b*x^2 + a)*sqrt(-c*d^2 + b*d
*e - a*e^2)*((2*c*d - b*e)*x^2 + b*d - 2*a*e)/((c^2*d^2 - b*c*d*e + a*c*e^2
)*x^4 + a*c*d^2 - a*b*d*e + a^2*e^2 + (b*c*d^2 - b^2*d*e + a*b*e^2)*x^2)) -
 (2*c*d - b*e)*sqrt(c)*log(-8*c^2*x^4 - 8*b*c*x^2 - b^2 - 4*sqrt(c*x^4 + b*
x^2 + a)*(2*c*x^2 + b)*sqrt(c) - 4*a*c))/(c*e^2), 1/4*(2*sqrt(c*x^4 + b*x^2
 + a)*c*e + 2*sqrt(-c*d^2 + b*d*e - a*e^2)*c*arctan(-1/2*sqrt(c*x^4 + b*x^2
 + a)*sqrt(-c*d^2 + b*d*e - a*e^2)*((2*c*d - b*e)*x^2 + b*d - 2*a*e)/((c^2*
d^2 - b*c*d*e + a*c*e^2)*x^4 + a*c*d^2 - a*b*d*e + a^2*e^2 + (b*c*d^2 - b^2
*d*e + a*b*e^2)*x^2)) + (2*c*d - b*e)*sqrt(-c)*arctan(1/2*sqrt(c*x^4 + b*x^
2 + a)*(2*c*x^2 + b)*sqrt(-c)/(c^2*x^4 + b*c*x^2 + a*c)))/(c*e^2)]
```

Sympy [F]

$$\int \frac{x\sqrt{a+bx^2+cx^4}}{d+ex^2}\,dx=\int \frac{x\sqrt{a+bx^2+cx^4}}{d+ex^2}\,dx$$

```
[In] integrate(x*(c*x**4+b*x**2+a)**(1/2)/(e*x**2+d),x)
[Out] Integral(x*sqrt(a + b*x**2 + c*x**4)/(d + e*x**2), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x\sqrt{a+bx^2+cx^4}}{d+ex^2}\,dx=\text{Exception raised: ValueError}$$

```
[In] integrate(x*(c*x^4+b*x^2+a)^(1/2)/(e*x^2+d),x, algorithm="maxima")
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*e^2-b*d*e>0)', see `assume?` for
more de
```

Giac [F(-2)]

Exception generated.

$$\int \frac{x\sqrt{a+bx^2+cx^4}}{d+ex^2}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(x*(c*x^4+b*x^2+a)^(1/2)/(e*x^2+d),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument Type
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x\sqrt{a+bx^2+cx^4}}{d+ex^2}\,dx = \int \frac{x\,\sqrt{c\,x^4+b\,x^2+a}}{e\,x^2+d}\,dx$$

```
[In] int((x*(a + b*x^2 + c*x^4)^(1/2))/(d + e*x^2),x)
```

```
[Out] int((x*(a + b*x^2 + c*x^4)^(1/2))/(d + e*x^2), x)
```

3.314 $\int \frac{\sqrt{a+bx^2+cx^4}}{x(d+ex^2)}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 186

$$\int \frac{\sqrt{a+bx^2+cx^4}}{x(d+ex^2)}\,dx = -\frac{\sqrt{a}\,\text{arctanh}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{2d}+\frac{\sqrt{c}\,\text{arctanh}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{2e} - \frac{\sqrt{cd^2-bde+ae^2}\,\text{arctanh}\left(\frac{bd-2ae+(2cd-be)x^2}{2\sqrt{cd^2-bde+ae^2}\sqrt{a+bx^2+cx^4}}\right)}{2de}$$

```
[Out] -1/2*arctanh(1/2*(b*x^2+2*a)/a^(1/2)/(c*x^4+b*x^2+a)^(1/2))*a^(1/2)/d+1/2*a
rctanh(1/2*(2*c*x^2+b)/c^(1/2)/(c*x^4+b*x^2+a)^(1/2))*c^(1/2)/e-1/2*arctanh
(1/2*(b*d-2*a*e+(-b*e+2*c*d)*x^2)/(a*e^2-b*d*e+c*d^2)^(1/2)/(c*x^4+b*x^2+a)
^(1/2))*(a*e^2-b*d*e+c*d^2)^(1/2)/d/e
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 186, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.207, Rules used = {1265, 909, 738, 212, 857, 635}

$$\int \frac{\sqrt{a+bx^2+cx^4}}{x(d+ex^2)}\,dx = -\frac{\sqrt{ae^2-bde+cd^2}\,\text{arctanh}\left(\frac{-2ae+x^2(2cd-be)+bd}{2\sqrt{a+bx^2+cx^4}\sqrt{ae^2-bde+cd^2}}\right)}{2de} - \frac{\sqrt{a}\,\text{arctanh}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{2d}+\frac{\sqrt{c}\,\text{arctanh}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{2e}$$

```
[In] Int[Sqrt[a + b*x^2 + c*x^4]/(x*(d + e*x^2)),x]
```

```
[Out] -1/2*(Sqrt[a]*ArcTanh[(2*a + b*x^2)/(2*Sqrt[a]*Sqrt[a + b*x^2 + c*x^4])])/d
 + (Sqrt[c]*ArcTanh[(b + 2*c*x^2)/(2*Sqrt[c]*Sqrt[a + b*x^2 + c*x^4])])/(2*
```

```
e) - (Sqrt[c*d^2 - b*d*e + a*e^2]*ArcTanh[(b*d - 2*a*e + (2*c*d - b*e)*x^2)
/(2*Sqrt[c*d^2 - b*d*e + a*e^2]*Sqrt[a + b*x^2 + c*x^4])])/(2*d*e)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 635

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 857

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 909

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)/(((d_.) + (e_.)*(x_))*((f_.) +
 (g_.)*(x_))), x_Symbol] :> Dist[(c*d^2 - b*d*e + a*e^2)/(e*(e*f - d*g)), I
nt[(a + b*x + c*x^2)^(p - 1)/(d + e*x), x], x] - Dist[1/(e*(e*f - d*g)), In
t[Simp[c*d*f - b*e*f + a*e*g - c*(e*f - d*g)*x, x]*((a + b*x + c*x^2)^(p -
1)/(f + g*x)), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g,
0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && FractionQ[p]
&& GtQ[p, 0]
```

Rule 1265

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rubi steps

$$\text{integral} = \frac{1}{2}\text{Subst}\left(\int \frac{\sqrt{a+bx+cx^2}}{x(d+ex)}\,dx,x,x^2\right)$$
$$= -\frac{\text{Subst}\left(\int \frac{-bd+ae-cdx}{(d+ex)\sqrt{a+bx+cx^2}}\,dx,x,x^2\right)}{2d} + \frac{a\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx+cx^2}}\,dx,x,x^2\right)}{2d}$$
$$= -\frac{a\text{Subst}\left(\int \frac{1}{4a-x^2}\,dx,x,\frac{2a+bx^2}{\sqrt{a+bx^2+cx^4}}\right)}{d} + \frac{c\text{Subst}\left(\int \frac{1}{\sqrt{a+bx+cx^2}}\,dx,x,x^2\right)}{2e} - \frac{1}{2}\left(-b+\frac{cd}{e}+\frac{ae}{d}\right)\text{Subst}\left(\int \frac{1}{(d+ex)\sqrt{a+bx+cx^2}}\,dx,x,x^2\right)$$
$$= -\frac{\sqrt{a}\tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{2d} + \frac{c\text{Subst}\left(\int \frac{1}{4c-x^2}\,dx,x,\frac{b+2cx^2}{\sqrt{a+bx^2+cx^4}}\right)}{e} - \left(b-\frac{cd}{e}-\frac{ae}{d}\right)\text{Subst}\left(\int \frac{1}{4cd^2-4bde+4ae^2-x^2}\,dx,x,\frac{-bd+2ae-(2cd-be)x^2}{\sqrt{a+bx^2+cx^4}}\right)$$
$$= -\frac{\sqrt{a}\tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{2d} + \frac{\sqrt{c}\tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{2e} - \frac{\sqrt{cd^2-bde+ae^2}\tanh^{-1}\left(\frac{bd-2ae+(2cd-be)x^2}{2\sqrt{cd^2-bde+ae^2}\sqrt{a+bx^2+cx^4}}\right)}{2de}$$

Mathematica [A] (verified)

Time = 0.46 (sec) , antiderivative size = 174, normalized size of antiderivative = 0.94

$$\int \frac{\sqrt{a+bx^2+cx^4}}{x(d+ex^2)}\,dx = -\frac{2\sqrt{-cd^2+bde-ae^2}\arctan\left(\frac{\sqrt{c}(d+ex^2)-e\sqrt{a+bx^2+cx^4}}{\sqrt{-cd^2+bde-ae^2}}\right) - 2\sqrt{a}e\,\text{arctanh}\left(\frac{\sqrt{c}x^2-\sqrt{a+bx^2+cx^4}}{\sqrt{a}}\right) + \sqrt{c}d\log\left(e\left(b\right.\right.}{2de}$$

```
[In] Integrate[Sqrt[a + b*x^2 + c*x^4]/(x*(d + e*x^2)),x]
```

```
[Out] -1/2*(2*Sqrt[-(c*d^2) + b*d*e - a*e^2]*ArcTan[(Sqrt[c]*(d + e*x^2) - e*Sqrt
[a + b*x^2 + c*x^4])/Sqrt[-(c*d^2) + b*d*e - a*e^2]] - 2*Sqrt[a]*e*ArcTanh[
(Sqrt[c]*x^2 - Sqrt[a + b*x^2 + c*x^4])/Sqrt[a]] + Sqrt[c]*d*Log[e*(b + 2*c
*x^2 - 2*Sqrt[c]*Sqrt[a + b*x^2 + c*x^4])])/(d*e)
```

Maple [A] (verified)

Time = 0.45 (sec) , antiderivative size = 240, normalized size of antiderivative = 1.29

method	result
pseudoelliptic	$\frac{\left(\left(a\,e^{2}-bde\right)\sqrt{c}+c^{\frac{3}{2}}d^{2}\right)\ln\left(\frac{2\sqrt{c\,x^{4}+b\,x^{2}+a}\,\sqrt{\frac{a\,e^{2}-bde+c\,d^{2}}{e^{2}}}\,e+\left(b\,x^{2}+2a\right)e-d\left(2c\,x^{2}+b\right)}{e\,x^{2}+d}\right)-\sqrt{\frac{a\,e^{2}-bde+c\,d^{2}}{e^{2}}}\,e\left(\sqrt{c}\,\ln\left(\frac{2a+b\,x^{2}+\cdots}{\cdots}\right)\cdots\right)}{2\sqrt{\frac{a\,e^{2}-bde+c\,d^{2}}{e^{2}}}\,\sqrt{c}\,d\,e^{2}}$
elliptic	$\frac{\sqrt{c\,x^{4}+b\,x^{2}+a}+\frac{b\ln\left(\frac{\frac{b}{2}+c\,x^{2}}{\sqrt{c}}+\sqrt{c\,x^{4}+b\,x^{2}+a}\right)}{2\sqrt{c}}-\sqrt{a}\,\ln\left(\frac{2a+b\,x^{2}+2\sqrt{a}\,\sqrt{c\,x^{4}+b\,x^{2}+a}}{x^{2}}\right)}{2d}-\frac{\sqrt{c\left(x^{2}+\frac{d}{e}\right)^{2}+\frac{\left(be-2cd\right)\left(x^{2}+\frac{d}{e}\right)}{e}+\frac{a\,e^{2}\cdots}{\cdots}}}{\cdots}$
default	$\frac{\frac{\sqrt{c\,x^{4}+b\,x^{2}+a}}{2}+\frac{b\ln\left(\frac{\frac{b}{2}+c\,x^{2}}{\sqrt{c}}+\sqrt{c\,x^{4}+b\,x^{2}+a}\right)}{4\sqrt{c}}-\frac{\sqrt{a}\,\ln\left(\frac{2a+b\,x^{2}+2\sqrt{a}\,\sqrt{c\,x^{4}+b\,x^{2}+a}}{x^{2}}\right)}{2}}{d}-\frac{\sqrt{c\left(x^{2}+\frac{d}{e}\right)^{2}+\frac{\left(be-2cd\right)\left(x^{2}+\frac{d}{e}\right)}{e}+\frac{a\,e^{2}\cdots}{\cdots}}}{\cdots}$

```
[In] int((c*x^4+b*x^2+a)^(1/2)/x/(e*x^2+d),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2/((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*(((a*e^2-b*d*e)*c^(1/2)+c^(3/2)*d^2)*ln
((2*(c*x^4+b*x^2+a)^(1/2)*((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*e+(b*x^2+2*a)*e-d
*(2*c*x^2+b))/(e*x^2+d))-((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*e*(c^(1/2)*ln((2*a
+b*x^2+2*a^(1/2)*(c*x^4+b*x^2+a)^(1/2))/x^2)*e*a^(1/2)-ln((2*c*x^2+2*(c*x^4
+b*x^2+a)^(1/2)*c^(1/2)+b)/c^(1/2))*c*d+ln(2)*c*d))/c^(1/2)/d/e^2
```

Fricas [A] (verification not implemented)

none

Time = 42.12 (sec) , antiderivative size = 2367, normalized size of antiderivative = 12.73

$$\int \frac{\sqrt{a+bx^2+cx^4}}{x\left(d+ex^2\right)}\,dx = \text{Too large to display}$$

```
[In] integrate((c*x^4+b*x^2+a)^(1/2)/x/(e*x^2+d),x, algorithm="fricas")
```

```
[Out] [1/4*(sqrt(c)*d*log(-8*c^2*x^4 - 8*b*c*x^2 - b^2 - 4*sqrt(c*x^4 + b*x^2 + a
)*(2*c*x^2 + b)*sqrt(c) - 4*a*c) + sqrt(a)*e*log(-((b^2 + 4*a*c)*x^4 + 8*a*
b*x^2 - 4*sqrt(c*x^4 + b*x^2 + a)*(b*x^2 + 2*a)*sqrt(a) + 8*a^2)/x^4) + sqr
t(c*d^2 - b*d*e + a*e^2)*log(-((8*c^2*d^2 - 8*b*c*d*e + (b^2 + 4*a*c)*e^2)*
x^4 - 8*a*b*d*e + 8*a^2*e^2 + (b^2 + 4*a*c)*d^2 + 2*(4*b*c*d^2 + 4*a*b*e^2
- (3*b^2 + 4*a*c)*d*e)*x^2 - 4*sqrt(c*x^4 + b*x^2 + a)*sqrt(c*d^2 - b*d*e +
 a*e^2)*((2*c*d - b*e)*x^2 + b*d - 2*a*e))/(e^2*x^4 + 2*d*e*x^2 + d^2)))/(d
*e), -1/4*(2*sqrt(-c)*d*arctan(1/2*sqrt(c*x^4 + b*x^2 + a)*(2*c*x^2 + b)*sq
```

rt(-c)/(c^2*x^4 + b*c*x^2 + a*c)) - sqrt(a)*e*log(-((b^2 + 4*a*c)*x^4 + 8*a
*b*x^2 - 4*sqrt(c*x^4 + b*x^2 + a)*(b*x^2 + 2*a)*sqrt(a) + 8*a^2)/x^4) - sq
rt(c*d^2 - b*d*e + a*e^2)*log(-((8*c^2*d^2 - 8*b*c*d*e + (b^2 + 4*a*c)*e^2)
*x^4 - 8*a*b*d*e + 8*a^2*e^2 + (b^2 + 4*a*c)*d^2 + 2*(4*b*c*d^2 + 4*a*b*e^2
 - (3*b^2 + 4*a*c)*d*e)*x^2 - 4*sqrt(c*x^4 + b*x^2 + a)*sqrt(c*d^2 - b*d*e
+ a*e^2)*((2*c*d - b*e)*x^2 + b*d - 2*a*e))/(e^2*x^4 + 2*d*e*x^2 + d^2)))/(
d*e), 1/4*(sqrt(c)*d*log(-8*c^2*x^4 - 8*b*c*x^2 - b^2 - 4*sqrt(c*x^4 + b*x^
2 + a)*(2*c*x^2 + b)*sqrt(c) - 4*a*c) + sqrt(a)*e*log(-((b^2 + 4*a*c)*x^4 +
 8*a*b*x^2 - 4*sqrt(c*x^4 + b*x^2 + a)*(b*x^2 + 2*a)*sqrt(a) + 8*a^2)/x^4)
- 2*sqrt(-c*d^2 + b*d*e - a*e^2)*arctan(-1/2*sqrt(c*x^4 + b*x^2 + a)*sqrt(-
c*d^2 + b*d*e - a*e^2)*((2*c*d - b*e)*x^2 + b*d - 2*a*e)/((c^2*d^2 - b*c*d*
e + a*c*e^2)*x^4 + a*c*d^2 - a*b*d*e + a^2*e^2 + (b*c*d^2 - b^2*d*e + a*b*e
^2)*x^2)))/(d*e), -1/4*(2*sqrt(-c)*d*arctan(1/2*sqrt(c*x^4 + b*x^2 + a)*(2*
c*x^2 + b)*sqrt(-c)/(c^2*x^4 + b*c*x^2 + a*c)) - sqrt(a)*e*log(-((b^2 + 4*a
*c)*x^4 + 8*a*b*x^2 - 4*sqrt(c*x^4 + b*x^2 + a)*(b*x^2 + 2*a)*sqrt(a) + 8*a
^2)/x^4) + 2*sqrt(-c*d^2 + b*d*e - a*e^2)*arctan(-1/2*sqrt(c*x^4 + b*x^2 +
a)*sqrt(-c*d^2 + b*d*e - a*e^2)*((2*c*d - b*e)*x^2 + b*d - 2*a*e)/((c^2*d^2
 - b*c*d*e + a*c*e^2)*x^4 + a*c*d^2 - a*b*d*e + a^2*e^2 + (b*c*d^2 - b^2*d*
e + a*b*e^2)*x^2)))/(d*e), 1/4*(2*sqrt(-a)*e*arctan(1/2*sqrt(c*x^4 + b*x^2
+ a)*(b*x^2 + 2*a)*sqrt(-a)/(a*c*x^4 + a*b*x^2 + a^2)) + sqrt(c)*d*log(-8*c
^2*x^4 - 8*b*c*x^2 - b^2 - 4*sqrt(c*x^4 + b*x^2 + a)*(2*c*x^2 + b)*sqrt(c)
- 4*a*c) + sqrt(c*d^2 - b*d*e + a*e^2)*log(-((8*c^2*d^2 - 8*b*c*d*e + (b^2
+ 4*a*c)*e^2)*x^4 - 8*a*b*d*e + 8*a^2*e^2 + (b^2 + 4*a*c)*d^2 + 2*(4*b*c*d^
2 + 4*a*b*e^2 - (3*b^2 + 4*a*c)*d*e)*x^2 - 4*sqrt(c*x^4 + b*x^2 + a)*sqrt(c
*d^2 - b*d*e + a*e^2)*((2*c*d - b*e)*x^2 + b*d - 2*a*e))/(e^2*x^4 + 2*d*e*x
^2 + d^2)))/(d*e), 1/4*(2*sqrt(-a)*e*arctan(1/2*sqrt(c*x^4 + b*x^2 + a)*(b*
x^2 + 2*a)*sqrt(-a)/(a*c*x^4 + a*b*x^2 + a^2)) - 2*sqrt(-c)*d*arctan(1/2*sq
rt(c*x^4 + b*x^2 + a)*(2*c*x^2 + b)*sqrt(-c)/(c^2*x^4 + b*c*x^2 + a*c)) + s
qrt(c*d^2 - b*d*e + a*e^2)*log(-((8*c^2*d^2 - 8*b*c*d*e + (b^2 + 4*a*c)*e^2
)*x^4 - 8*a*b*d*e + 8*a^2*e^2 + (b^2 + 4*a*c)*d^2 + 2*(4*b*c*d^2 + 4*a*b*e^
2 - (3*b^2 + 4*a*c)*d*e)*x^2 - 4*sqrt(c*x^4 + b*x^2 + a)*sqrt(c*d^2 - b*d*e
 + a*e^2)*((2*c*d - b*e)*x^2 + b*d - 2*a*e))/(e^2*x^4 + 2*d*e*x^2 + d^2)))/
(d*e), 1/4*(2*sqrt(-a)*e*arctan(1/2*sqrt(c*x^4 + b*x^2 + a)*(b*x^2 + 2*a)*s
qrt(-a)/(a*c*x^4 + a*b*x^2 + a^2)) + sqrt(c)*d*log(-8*c^2*x^4 - 8*b*c*x^2 -
 b^2 - 4*sqrt(c*x^4 + b*x^2 + a)*(2*c*x^2 + b)*sqrt(c) - 4*a*c) - 2*sqrt(-c
*d^2 + b*d*e - a*e^2)*arctan(-1/2*sqrt(c*x^4 + b*x^2 + a)*sqrt(-c*d^2 + b*d
*e - a*e^2)*((2*c*d - b*e)*x^2 + b*d - 2*a*e)/((c^2*d^2 - b*c*d*e + a*c*e^2
)*x^4 + a*c*d^2 - a*b*d*e + a^2*e^2 + (b*c*d^2 - b^2*d*e + a*b*e^2)*x^2)))/
(d*e), 1/2*(sqrt(-a)*e*arctan(1/2*sqrt(c*x^4 + b*x^2 + a)*(b*x^2 + 2*a)*sqr
t(-a)/(a*c*x^4 + a*b*x^2 + a^2)) - sqrt(-c)*d*arctan(1/2*sqrt(c*x^4 + b*x^2
 + a)*(2*c*x^2 + b)*sqrt(-c)/(c^2*x^4 + b*c*x^2 + a*c)) - sqrt(-c*d^2 + b*d
*e - a*e^2)*arctan(-1/2*sqrt(c*x^4 + b*x^2 + a)*sqrt(-c*d^2 + b*d*e - a*e^2
)*((2*c*d - b*e)*x^2 + b*d - 2*a*e)/((c^2*d^2 - b*c*d*e + a*c*e^2)*x^4 + a*
c*d^2 - a*b*d*e + a^2*e^2 + (b*c*d^2 - b^2*d*e + a*b*e^2)*x^2)))/(d*e)]

Sympy [F]

$$\int \frac{\sqrt{a+bx^2+cx^4}}{x\,(d+ex^2)}\,dx = \int \frac{\sqrt{a+bx^2+cx^4}}{x\,(d+ex^2)}\,dx$$

```
[In] integrate((c*x**4+b*x**2+a)**(1/2)/x/(e*x**2+d),x)
[Out] Integral(sqrt(a + b*x**2 + c*x**4)/(x*(d + e*x**2)), x)
```

Maxima [F]

$$\int \frac{\sqrt{a+bx^2+cx^4}}{x\,(d+ex^2)}\,dx = \int \frac{\sqrt{cx^4+bx^2+a}}{(ex^2+d)x}\,dx$$

```
[In] integrate((c*x^4+b*x^2+a)^(1/2)/x/(e*x^2+d),x, algorithm="maxima")
[Out] integrate(sqrt(c*x^4 + b*x^2 + a)/((e*x^2 + d)*x), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\sqrt{a+bx^2+cx^4}}{x\,(d+ex^2)}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate((c*x^4+b*x^2+a)^(1/2)/x/(e*x^2+d),x, algorithm="giac")
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument Type
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx^2+cx^4}}{x\,(d+ex^2)}\,dx = \int \frac{\sqrt{c\,x^4+b\,x^2+a}}{x\,\left(e\,x^2+d\right)}\,dx$$

```
[In] int((a + b*x^2 + c*x^4)^(1/2)/(x*(d + e*x^2)),x)
[Out] int((a + b*x^2 + c*x^4)^(1/2)/(x*(d + e*x^2)), x)
```

3.315 $\int \frac{\sqrt{a+bx^2+cx^4}}{x^3(d+ex^2)}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 361

$$\int \frac{\sqrt{a+bx^2+cx^4}}{x^3(d+ex^2)}\,dx = -\frac{\sqrt{a+bx^2+cx^4}}{2dx^2} - \frac{b\,\text{arctanh}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{4\sqrt{a}d} + \frac{\sqrt{a}e\,\text{arctanh}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{2d^2} + \frac{\sqrt{c}\,\text{arctanh}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{2d} - \frac{be\,\text{arctanh}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{4\sqrt{c}d^2} - \frac{(2cd-be)\text{arctanh}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{4\sqrt{c}d^2} + \frac{\sqrt{cd^2-bde+ae^2}\,\text{arctanh}\left(\frac{bd-2ae+(2cd-be)x^2}{2\sqrt{cd^2-bde+ae^2}\sqrt{a+bx^2+cx^4}}\right)}{2d^2}$$

```
[Out] -1/4*b*arctanh(1/2*(b*x^2+2*a)/a^(1/2)/(c*x^4+b*x^2+a)^(1/2))/d/a^(1/2)+1/2
*e*arctanh(1/2*(b*x^2+2*a)/a^(1/2)/(c*x^4+b*x^2+a)^(1/2))*a^(1/2)/d^2-1/4*b
*e*arctanh(1/2*(2*c*x^2+b)/c^(1/2)/(c*x^4+b*x^2+a)^(1/2))/d^2/c^(1/2)-1/4*(
-b*e+2*c*d)*arctanh(1/2*(2*c*x^2+b)/c^(1/2)/(c*x^4+b*x^2+a)^(1/2))/d^2/c^(1
/2)+1/2*arctanh(1/2*(2*c*x^2+b)/c^(1/2)/(c*x^4+b*x^2+a)^(1/2))*c^(1/2)/d+1/
2*arctanh(1/2*(b*d-2*a*e+(-b*e+2*c*d)*x^2)/(a*e^2-b*d*e+c*d^2)^(1/2)/(c*x^4
+b*x^2+a)^(1/2))*(a*e^2-b*d*e+c*d^2)^(1/2)/d^2-1/2*(c*x^4+b*x^2+a)^(1/2)/d/
x^2
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 361, normalized size of antiderivative = 1.00, number of steps used = 21, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.276, Rules used = {1265, 974, 746, 857, 635, 212, 738, 748}

$$\int \frac{\sqrt{a+bx^2+cx^4}}{x^3(d+ex^2)}\,dx = \frac{\sqrt{ae^2-bde+cd^2}\,\text{arctanh}\left(\frac{-2ae+x^2(2cd-be)+bd}{2\sqrt{a+bx^2+cx^4}\sqrt{ae^2-bde+cd^2}}\right)}{2d^2} + \frac{\sqrt{a}e\,\text{arctanh}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{2d^2} - \frac{be\,\text{arctanh}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{4\sqrt{c}d^2} - \frac{(2cd-be)\text{arctanh}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{4\sqrt{c}d^2} - \frac{b\,\text{arctanh}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{4\sqrt{a}d} + \frac{\sqrt{c}\,\text{arctanh}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{2d} - \frac{\sqrt{a+bx^2+cx^4}}{2dx^2}$$

```
[In] Int[Sqrt[a + b*x^2 + c*x^4]/(x^3*(d + e*x^2)),x]
```

```
[Out] -1/2*Sqrt[a + b*x^2 + c*x^4]/(d*x^2) - (b*ArcTanh[(2*a + b*x^2)/(2*Sqrt[a]*
Sqrt[a + b*x^2 + c*x^4])])/(4*Sqrt[a]*d) + (Sqrt[a]*e*ArcTanh[(2*a + b*x^2)
/(2*Sqrt[a]*Sqrt[a + b*x^2 + c*x^4])])/(2*d^2) + (Sqrt[c]*ArcTanh[(b + 2*c*
x^2)/(2*Sqrt[c]*Sqrt[a + b*x^2 + c*x^4])])/(2*d) - (b*e*ArcTanh[(b + 2*c*x^
2)/(2*Sqrt[c]*Sqrt[a + b*x^2 + c*x^4])])/(4*Sqrt[c]*d^2) - ((2*c*d - b*e)*A
rcTanh[(b + 2*c*x^2)/(2*Sqrt[c]*Sqrt[a + b*x^2 + c*x^4])])/(4*Sqrt[c]*d^2)
+ (Sqrt[c*d^2 - b*d*e + a*e^2]*ArcTanh[(b*d - 2*a*e + (2*c*d - b*e)*x^2)/(2
*Sqrt[c*d^2 - b*d*e + a*e^2]*Sqrt[a + b*x^2 + c*x^4])])/(2*d^2)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 635

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 746

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*((a + b*x + c*x^2)^p/(e*(m + 1))), x] - Di
st[p/(e*(m + 1)), Int[(d + e*x)^(m + 1)*(b + 2*c*x)*(a + b*x + c*x^2)^(p -
1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*
d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && GtQ[p, 0] && (IntegerQ[p]
 || LtQ[m, -1]) && NeQ[m, -1] &&  !ILtQ[m + 2*p + 1, 0] && IntQuadraticQ[a,
 b, c, d, e, m, p, x]
```

Rule 748

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*((a + b*x + c*x^2)^p/(e*(m + 2*p + 1))), x
] - Dist[p/(e*(m + 2*p + 1)), Int[(d + e*x)^m*Simp[b*d - 2*a*e + (2*c*d - b
*e)*x, x]*(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e
, 0] && GtQ[p, 0] && NeQ[m + 2*p + 1, 0] && ( !RationalQ[m] || LtQ[m, 1]) &
&  !ILtQ[m + 2*p, 0] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 857

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 974

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g
*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ
[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && (
IntegerQ[p] || (ILtQ[m, 0] && ILtQ[n, 0])) &&  !(IGtQ[m, 0] || IGtQ[n, 0])
```

Rule 1265

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rubi steps

$$\text{integral} = \frac{1}{2}\text{Subst}\left(\int \frac{\sqrt{a+bx+cx^2}}{x^2(d+ex)}\,dx, x, x^2\right)$$

$$=\frac{1}{2}\text{Subst}\left(\int\left(\frac{\sqrt{a+bx+cx^2}}{dx^2}-\frac{e\sqrt{a+bx+cx^2}}{d^2x}+\frac{e^2\sqrt{a+bx+cx^2}}{d^2(d+ex)}\right)dx,x,x^2\right)$$

$$=\frac{\text{Subst}\left(\int\frac{\sqrt{a+bx+cx^2}}{x^2}dx,x,x^2\right)}{2d}-\frac{e\text{Subst}\left(\int\frac{\sqrt{a+bx+cx^2}}{x}dx,x,x^2\right)}{2d^2}+\frac{e^2\text{Subst}\left(\int\frac{\sqrt{a+bx+cx^2}}{d+ex}dx,x,x^2\right)}{2d^2}$$

$$=-\frac{\sqrt{a+bx^2+cx^4}}{2dx^2}+\frac{\text{Subst}\left(\int\frac{b+2cx}{x\sqrt{a+bx+cx^2}}dx,x,x^2\right)}{4d}+\frac{e\text{Subst}\left(\int\frac{-2a-bx}{x\sqrt{a+bx+cx^2}}dx,x,x^2\right)}{4d^2}-\frac{e\text{Subst}\left(\int\frac{bd-2ae+(2cd-be)x}{(d+ex)\sqrt{a+bx+cx^2}}dx,x,x^2\right)}{4d^2}$$

$$=-\frac{\sqrt{a+bx^2+cx^4}}{2dx^2}+\frac{b\text{Subst}\left(\int\frac{1}{x\sqrt{a+bx+cx^2}}dx,x,x^2\right)}{4d}+\frac{c\text{Subst}\left(\int\frac{1}{\sqrt{a+bx+cx^2}}dx,x,x^2\right)}{2d}-\frac{(ae)\text{Subst}\left(\int\frac{1}{x\sqrt{a+bx+cx^2}}dx,x,x^2\right)}{2d^2}-\frac{(be)\text{Subst}\left(\int\frac{1}{\sqrt{a+bx+cx^2}}dx,x,x^2\right)}{4d^2}-\frac{(2cd-be)\text{Subst}\left(\int\frac{1}{\sqrt{a+bx+cx^2}}dx,x,x^2\right)}{4d^2}-\frac{(e(bd-2ae)-d(2cd-be))\text{Subst}\left(\int\frac{1}{(d+ex)\sqrt{a+bx+cx^2}}dx,x,x^2\right)}{4d^2}$$

$$=-\frac{\sqrt{a+bx^2+cx^4}}{2dx^2}-\frac{b\text{Subst}\left(\int\frac{1}{4a-x^2}dx,x,\frac{2a+bx^2}{\sqrt{a+bx^2+cx^4}}\right)}{2d}+\frac{c\text{Subst}\left(\int\frac{1}{4c-x^2}dx,x,\frac{b+2cx^2}{\sqrt{a+bx^2+cx^4}}\right)}{d}+\frac{(ae)\text{Subst}\left(\int\frac{1}{4a-x^2}dx,x,\frac{2a+bx^2}{\sqrt{a+bx^2+cx^4}}\right)}{d^2}-\frac{(be)\text{Subst}\left(\int\frac{1}{4c-x^2}dx,x,\frac{b+2cx^2}{\sqrt{a+bx^2+cx^4}}\right)}{2d^2}-\frac{(2cd-be)\text{Subst}\left(\int\frac{1}{4c-x^2}dx,x,\frac{b+2cx^2}{\sqrt{a+bx^2+cx^4}}\right)}{2d^2}+\frac{(e(bd-2ae)-d(2cd-be))\text{Subst}\left(\int\frac{1}{4cd^2-4bde+4ae^2-x^2}dx,x,\frac{-bd+2ae-(2cd-be)x^2}{\sqrt{a+bx^2+cx^4}}\right)}{2d^2}$$

$$= -\frac{\sqrt{a+bx^2+cx^4}}{2dx^2} - \frac{b\tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{4\sqrt{a}d} + \frac{\sqrt{a}e\tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{2d^2} + \frac{\sqrt{c}\tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{2d} - \frac{be\tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{4\sqrt{c}d^2} - \frac{(2cd-be)\tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{4\sqrt{c}d^2} + \frac{\sqrt{cd^2-bde+ae^2}\tanh^{-1}\left(\frac{bd-2ae+(2cd-be)x^2}{2\sqrt{cd^2-bde+ae^2}\sqrt{a+bx^2+cx^4}}\right)}{2d^2}$$

Mathematica [A] (verified)

Time = 0.64 (sec) , antiderivative size = 158, normalized size of antiderivative = 0.44

$$\int \frac{\sqrt{a+bx^2+cx^4}}{x^3(d+ex^2)}\,dx = \frac{-\frac{d\sqrt{a+bx^2+cx^4}}{x^2} + 2\sqrt{-cd^2+bde-ae^2}\arctan\left(\frac{\sqrt{c}(d+ex^2)-e\sqrt{a+bx^2+cx^4}}{\sqrt{-cd^2+bde-ae^2}}\right) + \frac{(bd-2ae)\text{arctanh}\left(\frac{\sqrt{c}x^2-\sqrt{a+bx^2+cx^4}}{\sqrt{a}}\right)}{\sqrt{a}}}{2d^2}$$

```
[In] Integrate[Sqrt[a + b*x^2 + c*x^4]/(x^3*(d + e*x^2)),x]
```

```
[Out] (-((d*Sqrt[a + b*x^2 + c*x^4])/x^2) + 2*Sqrt[-(c*d^2) + b*d*e - a*e^2]*ArcT
an[(Sqrt[c]*(d + e*x^2) - e*Sqrt[a + b*x^2 + c*x^4])/Sqrt[-(c*d^2) + b*d*e
- a*e^2]] + ((b*d - 2*a*e)*ArcTanh[(Sqrt[c]*x^2 - Sqrt[a + b*x^2 + c*x^4])/
Sqrt[a]])/Sqrt[a])/(2*d^2)
```

Maple [A] (verified)

Time = 0.45 (sec) , antiderivative size = 225, normalized size of antiderivative = 0.62

method	result
pseudoelliptic	$-\frac{a x^2 (a e^2-bde+c d^2) \ln\left(\frac{2\sqrt{c x^4+b x^2+a}\sqrt{\frac{a e^2-bde+c d^2}{e^2}}e+\left(b x^2+2a\right)e-d\left(2c x^2+b\right)}{e x^2+d}\right)+e\left(\frac{x^2\left(b\sqrt{a}\,d-2a^{\frac{3}{2}}e\right)\ln\left(\frac{2a+b x^2+2\sqrt{a}}{\cdots}\right.}{2}\right.}{2\sqrt{\frac{a e^2-bde+c d^2}{e^2}}\,e a x^2 d^2}$
risch	$-\frac{\sqrt{c x^4+b x^2+a}}{2d x^2}-\frac{-\frac{(2ae-bd)\ln\left(\frac{2a+b x^2+2\sqrt{a}\sqrt{c x^4+b x^2+a}}{x^2}\right)}{2d\sqrt{a}}+\frac{\left(a e^2-bde+c d^2\right)\ln\left(\frac{\frac{2a e^2-2bde+2c d^2}{e^2}+\frac{(be-2cd)\left(x^2+\frac{d}{e}\right)}{e}+2\cdots}{\cdots}\right)}{de\sqrt{\frac{a e \cdots}{}}}}{2d}$
default	$\frac{-\frac{\left(c x^4+b x^2+a\right)^{\frac{3}{2}}}{2a x^2}+\frac{b\sqrt{c x^4+b x^2+a}}{2a}-\frac{b\ln\left(\frac{2a+b x^2+2\sqrt{a}\sqrt{c x^4+b x^2+a}}{x^2}\right)}{4\sqrt{a}}+\frac{c\sqrt{c x^4+b x^2+a}\,x^2}{2a}+\frac{\sqrt{c}\ln\left(\frac{\frac{b}{2}+c x^2}{\sqrt{c}}+\sqrt{c x^4+b x^2+a}\right)}{2}}{d}-\cdots$
elliptic	$\frac{-\frac{\left(c x^4+b x^2+a\right)^{\frac{3}{2}}}{a x^2}+\frac{b\left(\sqrt{c x^4+b x^2+a}+\frac{b\ln\left(\frac{\frac{b}{2}+c x^2}{\sqrt{c}}+\sqrt{c x^4+b x^2+a}\right)}{2\sqrt{c}}-\sqrt{a}\,\ln\left(\frac{2a+b x^2+2\sqrt{a}\sqrt{c x^4+b x^2+a}}{x^2}\right)\right)}{2a}+\frac{2c\left(\frac{\left(2c x^2+b\right)\sqrt{c x \cdots}}{4c}\right.}{\cdots}}{2d}$

```
[In] int((c*x^4+b*x^2+a)^(1/2)/x^3/(e*x^2+d),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2/((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*(a*x^2*(a*e^2-b*d*e+c*d^2)*ln((2*(c*x^
4+b*x^2+a)^(1/2)*((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*e+(b*x^2+2*a)*e-d*(2*c*x^2
+b))/(e*x^2+d))+e*(1/2*x^2*(b*a^(1/2)*d-2*a^(3/2)*e)*ln((2*a+b*x^2+2*a^(1/2
)*(c*x^4+b*x^2+a)^(1/2))/x^2)+a*d*(c*x^4+b*x^2+a)^(1/2))*((a*e^2-b*d*e+c*d^
2)/e^2)^(1/2))/e/a/x^2/d^2
```

Fricas [A] (verification not implemented)

none

Time = 0.48 (sec) , antiderivative size = 1094, normalized size of antiderivative = 3.03

$$\int \frac{\sqrt{a+bx^2+cx^4}}{x^3\,(d+ex^2)}\,dx$$
$$=\left[\frac{2\sqrt{cd^2-bde+ae^2}\,ax^2\log\left(-\frac{(8\,c^2d^2-8\,bcde+(b^2+4\,ac)e^2)x^4-8\,abde+8\,a^2e^2+(b^2+4\,ac)d^2+2\,(4\,bcd^2+4\,abe^2-(3\,b^2+4\,ac)de)x^2+4\cdots}{e^2x^4+2\,dex^2+d^2}\right.}{\cdots}\right.$$

```
[In] integrate((c*x^4+b*x^2+a)^(1/2)/x^3/(e*x^2+d),x, algorithm="fricas")
```

```
[Out] [1/8*(2*sqrt(c*d^2 - b*d*e + a*e^2)*a*x^2*log(-((8*c^2*d^2 - 8*b*c*d*e + (b
^2 + 4*a*c)*e^2)*x^4 - 8*a*b*d*e + 8*a^2*e^2 + (b^2 + 4*a*c)*d^2 + 2*(4*b*c
*d^2 + 4*a*b*e^2 - (3*b^2 + 4*a*c)*d*e)*x^2 + 4*sqrt(c*x^4 + b*x^2 + a)*sqr
t(c*d^2 - b*d*e + a*e^2)*((2*c*d - b*e)*x^2 + b*d - 2*a*e))/(e^2*x^4 + 2*d*
e*x^2 + d^2)) - (b*d - 2*a*e)*sqrt(a)*x^2*log(-((b^2 + 4*a*c)*x^4 + 8*a*b*x
^2 + 4*sqrt(c*x^4 + b*x^2 + a)*(b*x^2 + 2*a)*sqrt(a) + 8*a^2)/x^4) - 4*sqrt
(c*x^4 + b*x^2 + a)*a*d)/(a*d^2*x^2), 1/8*(4*sqrt(-c*d^2 + b*d*e - a*e^2)*a
*x^2*arctan(-1/2*sqrt(c*x^4 + b*x^2 + a)*sqrt(-c*d^2 + b*d*e - a*e^2)*((2*c
*d - b*e)*x^2 + b*d - 2*a*e)/((c^2*d^2 - b*c*d*e + a*c*e^2)*x^4 + a*c*d^2 -
 a*b*d*e + a^2*e^2 + (b*c*d^2 - b^2*d*e + a*b*e^2)*x^2)) - (b*d - 2*a*e)*sq
rt(a)*x^2*log(-((b^2 + 4*a*c)*x^4 + 8*a*b*x^2 + 4*sqrt(c*x^4 + b*x^2 + a)*(
b*x^2 + 2*a)*sqrt(a) + 8*a^2)/x^4) - 4*sqrt(c*x^4 + b*x^2 + a)*a*d)/(a*d^2*
x^2), 1/4*((b*d - 2*a*e)*sqrt(-a)*x^2*arctan(1/2*sqrt(c*x^4 + b*x^2 + a)*(b
*x^2 + 2*a)*sqrt(-a)/(a*c*x^4 + a*b*x^2 + a^2)) + sqrt(c*d^2 - b*d*e + a*e^
2)*a*x^2*log(-((8*c^2*d^2 - 8*b*c*d*e + (b^2 + 4*a*c)*e^2)*x^4 - 8*a*b*d*e
+ 8*a^2*e^2 + (b^2 + 4*a*c)*d^2 + 2*(4*b*c*d^2 + 4*a*b*e^2 - (3*b^2 + 4*a*c
)*d*e)*x^2 + 4*sqrt(c*x^4 + b*x^2 + a)*sqrt(c*d^2 - b*d*e + a*e^2)*((2*c*d
- b*e)*x^2 + b*d - 2*a*e))/(e^2*x^4 + 2*d*e*x^2 + d^2)) - 2*sqrt(c*x^4 + b*
x^2 + a)*a*d)/(a*d^2*x^2), 1/4*(2*sqrt(-c*d^2 + b*d*e - a*e^2)*a*x^2*arctan
(-1/2*sqrt(c*x^4 + b*x^2 + a)*sqrt(-c*d^2 + b*d*e - a*e^2)*((2*c*d - b*e)*x
^2 + b*d - 2*a*e)/((c^2*d^2 - b*c*d*e + a*c*e^2)*x^4 + a*c*d^2 - a*b*d*e +
a^2*e^2 + (b*c*d^2 - b^2*d*e + a*b*e^2)*x^2)) + (b*d - 2*a*e)*sqrt(-a)*x^2*
arctan(1/2*sqrt(c*x^4 + b*x^2 + a)*(b*x^2 + 2*a)*sqrt(-a)/(a*c*x^4 + a*b*x^
2 + a^2)) - 2*sqrt(c*x^4 + b*x^2 + a)*a*d)/(a*d^2*x^2)]
```

Sympy [F]

$$\int \frac{\sqrt{a+bx^2+cx^4}}{x^3\left(d+ex^2\right)}\,dx=\int \frac{\sqrt{a+bx^2+cx^4}}{x^3\left(d+ex^2\right)}\,dx$$

```
[In] integrate((c*x**4+b*x**2+a)**(1/2)/x**3/(e*x**2+d),x)
```

```
[Out] Integral(sqrt(a + b*x**2 + c*x**4)/(x**3*(d + e*x**2)), x)
```

Maxima [F]

$$\int \frac{\sqrt{a+bx^2+cx^4}}{x^3\left(d+ex^2\right)}\,dx = \int \frac{\sqrt{cx^4+bx^2+a}}{(ex^2+d)x^3}\,dx$$

```
[In] integrate((c*x^4+b*x^2+a)^(1/2)/x^3/(e*x^2+d),x, algorithm="maxima")
[Out] integrate(sqrt(c*x^4 + b*x^2 + a)/((e*x^2 + d)*x^3), x)
```

Giac [A] (verification not implemented)

none

Time = 0.34 (sec) , antiderivative size = 214, normalized size of antiderivative = 0.59

$$\int \frac{\sqrt{a+bx^2+cx^4}}{x^3\left(d+ex^2\right)}\,dx = \frac{\left(cd^2-bde+ae^2\right)\arctan\left(-\frac{\left(\sqrt{c}x^2-\sqrt{cx^4+bx^2+a}\right)e+\sqrt{c}d}{\sqrt{-cd^2+bde-ae^2}}\right)}{\sqrt{-cd^2+bde-ae^2}\,d^2} + \frac{(bd-2\,ae)\arctan\left(-\frac{\sqrt{c}x^2-\sqrt{cx^4+bx^2+a}}{\sqrt{-a}}\right)}{2\,\sqrt{-a}d^2} + \frac{\left(\sqrt{c}x^2-\sqrt{cx^4+bx^2+a}\right)b+2\,a\sqrt{c}}{2\left(\left(\sqrt{c}x^2-\sqrt{cx^4+bx^2+a}\right)^2-a\right)d}$$

```
[In] integrate((c*x^4+b*x^2+a)^(1/2)/x^3/(e*x^2+d),x, algorithm="giac")
[Out] (c*d^2 - b*d*e + a*e^2)*arctan(-((sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))*e
+ sqrt(c)*d)/sqrt(-c*d^2 + b*d*e - a*e^2))/(sqrt(-c*d^2 + b*d*e - a*e^2)*d^
2) + 1/2*(b*d - 2*a*e)*arctan(-(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))/sqrt
(-a))/(sqrt(-a)*d^2) + 1/2*((sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))*b + 2*a
*sqrt(c))/(((sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))^2 - a)*d)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx^2+cx^4}}{x^3\left(d+ex^2\right)}\,dx = \int \frac{\sqrt{c\,x^4+b\,x^2+a}}{x^3\,\left(e\,x^2+d\right)}\,dx$$

```
[In] int((a + b*x^2 + c*x^4)^(1/2)/(x^3*(d + e*x^2)),x)
[Out] int((a + b*x^2 + c*x^4)^(1/2)/(x^3*(d + e*x^2)), x)
```

3.316 $\int \frac{x^4\sqrt{1+2x^2+2x^4}}{3+2x^2}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 424

$$
\begin{aligned}
\int \frac{x^4\sqrt{1+2x^2+2x^4}}{3+2x^2}\,dx &= -\frac{1}{60}x(13-6x^2)\sqrt{1+2x^2+2x^4} \\
&+ \frac{109x\sqrt{1+2x^2+2x^4}}{60\sqrt{2}\left(1+\sqrt{2}x^2\right)} + \frac{3}{16}\sqrt{15}\arctan\left(\frac{\sqrt{\frac{5}{3}}x}{\sqrt{1+2x^2+2x^4}}\right) \\
&- \frac{109(1+\sqrt{2}x^2)\sqrt{\frac{1+2x^2+2x^4}{\left(1+\sqrt{2}x^2\right)^2}}E\left(2\arctan\left(\sqrt[4]{2}x\right)|\frac{1}{4}(2-\sqrt{2})\right)}{60\ 2^{3/4}\sqrt{1+2x^2+2x^4}} \\
&+ \frac{(-70+263\sqrt{2})\left(1+\sqrt{2}x^2\right)\sqrt{\frac{1+2x^2+2x^4}{\left(1+\sqrt{2}x^2\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\sqrt[4]{2}x\right),\frac{1}{4}(2-\sqrt{2})\right)}{60\ 2^{3/4}\left(-2+3\sqrt{2}\right)\sqrt{1+2x^2+2x^4}} \\
&+ \frac{15(3+\sqrt{2})\left(1+\sqrt{2}x^2\right)\sqrt{\frac{1+2x^2+2x^4}{\left(1+\sqrt{2}x^2\right)^2}}\operatorname{EllipticPi}\left(\frac{1}{24}(12-11\sqrt{2}),2\arctan\left(\sqrt[4]{2}x\right),\frac{1}{4}(2-\sqrt{2})\right)}{16\ 2^{3/4}\left(2-3\sqrt{2}\right)\sqrt{1+2x^2+2x^4}}
\end{aligned}
$$

```
[Out] 3/16*arctan(1/3*x*15^(1/2)/(2*x^4+2*x^2+1)^(1/2))*15^(1/2)-1/60*x*(-6*x^2+1
3)*(2*x^4+2*x^2+1)^(1/2)+109/120*x*(2*x^4+2*x^2+1)^(1/2)*2^(1/2)/(1+x^2*2^(
1/2))-109/120*(cos(2*arctan(2^(1/4)*x))^2)^(1/2)/cos(2*arctan(2^(1/4)*x))*E
llipticE(sin(2*arctan(2^(1/4)*x)),1/2*(2-2^(1/2))^(1/2))*(1+x^2*2^(1/2))*((
2*x^4+2*x^2+1)/(1+x^2*2^(1/2))^2)^(1/2)*2^(1/4)/(2*x^4+2*x^2+1)^(1/2)+15/32
*(cos(2*arctan(2^(1/4)*x))^2)^(1/2)/cos(2*arctan(2^(1/4)*x))*EllipticPi(sin
(2*arctan(2^(1/4)*x)),1/2-11/24*2^(1/2),1/2*(2-2^(1/2))^(1/2))*(3+2^(1/2))*
(1+x^2*2^(1/2))*((2*x^4+2*x^2+1)/(1+x^2*2^(1/2))^2)^(1/2)*2^(1/4)/(2-3*2^(1
/2))/(2*x^4+2*x^2+1)^(1/2)+1/120*(cos(2*arctan(2^(1/4)*x))^2)^(1/2)/cos(2*a
rctan(2^(1/4)*x))*EllipticF(sin(2*arctan(2^(1/4)*x)),1/2*(2-2^(1/2))^(1/2))
```

```
*(-70+263*2^(1/2))*(1+x^2*2^(1/2))*((2*x^4+2*x^2+1)/(1+x^2*2^(1/2))^2)^(1/2
)*2^(1/4)/(-2+3*2^(1/2))/(2*x^4+2*x^2+1)^(1/2)
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 619, normalized size of antiderivative = 1.46, number of steps used = 17, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.310, Rules used = {1349, 1105, 1211, 1117, 1209, 1130, 1222, 1230, 1720}

$$\int \frac{x^4\sqrt{1+2x^2+2x^4}}{3+2x^2}\,dx = \frac{3}{16}\sqrt{15}\arctan\left(\frac{\sqrt{\frac{5}{3}}x}{\sqrt{2x^4+2x^2+1}}\right)$$
$$+\frac{45\left(3+\sqrt{2}\right)\left(\sqrt{2}x^2+1\right)\sqrt{\frac{2x^4+2x^2+1}{\left(\sqrt{2}x^2+1\right)^2}}\text{EllipticF}\left(2\arctan\left(\sqrt[4]{2}x\right),\frac{1}{4}\left(2-\sqrt{2}\right)\right)}{112\sqrt[4]{2}\sqrt{2x^4+2x^2+1}}$$
$$-\frac{\left(1+\sqrt{2}\right)\left(\sqrt{2}x^2+1\right)\sqrt{\frac{2x^4+2x^2+1}{\left(\sqrt{2}x^2+1\right)^2}}\text{EllipticF}\left(2\arctan\left(\sqrt[4]{2}x\right),\frac{1}{4}\left(2-\sqrt{2}\right)\right)}{4\ 2^{3/4}\sqrt{2x^4+2x^2+1}}$$
$$-\frac{139\left(1-\sqrt{2}\right)\left(\sqrt{2}x^2+1\right)\sqrt{\frac{2x^4+2x^2+1}{\left(\sqrt{2}x^2+1\right)^2}}\text{EllipticF}\left(2\arctan\left(\sqrt[4]{2}x\right),\frac{1}{4}\left(2-\sqrt{2}\right)\right)}{240\sqrt[4]{2}\sqrt{2x^4+2x^2+1}}$$
$$-\frac{109\left(\sqrt{2}x^2+1\right)\sqrt{\frac{2x^4+2x^2+1}{\left(\sqrt{2}x^2+1\right)^2}}E\left(2\arctan\left(\sqrt[4]{2}x\right)|\frac{1}{4}\left(2-\sqrt{2}\right)\right)}{60\ 2^{3/4}\sqrt{2x^4+2x^2+1}}$$
$$-\frac{15\left(3+\sqrt{2}\right)^2\left(\sqrt{2}x^2+1\right)\sqrt{\frac{2x^4+2x^2+1}{\left(\sqrt{2}x^2+1\right)^2}}\text{EllipticPi}\left(\frac{1}{24}\left(12-11\sqrt{2}\right),2\arctan\left(\sqrt[4]{2}x\right),\frac{1}{4}\left(2-\sqrt{2}\right)\right)}{224\sqrt[4]{2}\sqrt{2x^4+2x^2+1}}$$
$$+\frac{1}{30}\left(3x^2+1\right)\sqrt{2x^4+2x^2+1}x+\frac{109\sqrt{2x^4+2x^2+1}x}{60\sqrt{2}\left(\sqrt{2}x^2+1\right)}-\frac{1}{4}\sqrt{2x^4+2x^2+1}x$$

```
[In] Int[(x^4*Sqrt[1 + 2*x^2 + 2*x^4])/(3 + 2*x^2),x]
```

```
[Out] -1/4*(x*Sqrt[1 + 2*x^2 + 2*x^4]) + (x*(1 + 3*x^2)*Sqrt[1 + 2*x^2 + 2*x^4])/
30 + (109*x*Sqrt[1 + 2*x^2 + 2*x^4])/(60*Sqrt[2]*(1 + Sqrt[2]*x^2)) + (3*Sq
rt[15]*ArcTan[(Sqrt[5/3]*x)/Sqrt[1 + 2*x^2 + 2*x^4]])/16 - (109*(1 + Sqrt[2
]*x^2)*Sqrt[(1 + 2*x^2 + 2*x^4)/(1 + Sqrt[2]*x^2)^2]*EllipticE[2*ArcTan[2^(
1/4)*x], (2 - Sqrt[2])/4])/(60*2^(3/4)*Sqrt[1 + 2*x^2 + 2*x^4]) - (139*(1 -
 Sqrt[2])*(1 + Sqrt[2]*x^2)*Sqrt[(1 + 2*x^2 + 2*x^4)/(1 + Sqrt[2]*x^2)^2]*E
llipticF[2*ArcTan[2^(1/4)*x], (2 - Sqrt[2])/4])/(240*2^(1/4)*Sqrt[1 + 2*x^2
 + 2*x^4]) - ((1 + Sqrt[2])*(1 + Sqrt[2]*x^2)*Sqrt[(1 + 2*x^2 + 2*x^4)/(1 +
 Sqrt[2]*x^2)^2]*EllipticF[2*ArcTan[2^(1/4)*x], (2 - Sqrt[2])/4])/(4*2^(3/4
)*Sqrt[1 + 2*x^2 + 2*x^4]) + (45*(3 + Sqrt[2])*(1 + Sqrt[2]*x^2)*Sqrt[(1 +
2*x^2 + 2*x^4)/(1 + Sqrt[2]*x^2)^2]*EllipticF[2*ArcTan[2^(1/4)*x], (2 - Sqr
```

```
t[2])/4])/(112*2^(1/4)*Sqrt[1 + 2*x^2 + 2*x^4]) - (15*(3 + Sqrt[2])^2*(1 +
Sqrt[2]*x^2)*Sqrt[(1 + 2*x^2 + 2*x^4)/(1 + Sqrt[2]*x^2)^2]*EllipticPi[(12 -
 11*Sqrt[2])/24, 2*ArcTan[2^(1/4)*x], (2 - Sqrt[2])/4])/(224*2^(1/4)*Sqrt[1
 + 2*x^2 + 2*x^4])
```

Rule 1105

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[x*((a + b*
x^2 + c*x^4)^p/(4*p + 1)), x] + Dist[2*(p/(4*p + 1)), Int[(2*a + b*x^2)*(a
+ b*x^2 + c*x^4)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c,
 0] && GtQ[p, 0] && IntegerQ[2*p]
```

Rule 1117

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1130

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Simp[d*(d*x)^(m - 1)*(a + b*x^2 + c*x^4)^p*((2*b*p + c*(m + 4*p - 1)*x^
2)/(c*(m + 4*p + 1)*(m + 4*p - 1))), x] - Dist[2*p*(d^2/(c*(m + 4*p + 1)*(m
 + 4*p - 1))), Int[(d*x)^(m - 2)*(a + b*x^2 + c*x^4)^(p - 1)*Simp[a*b*(m -
1) - (2*a*c*(m + 4*p - 1) - b^2*(m + 2*p - 1))*x^2, x], x], x] /; FreeQ[{a,
 b, c, d}, x] && NeQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && GtQ[m, 1] && IntegerQ[
2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1209

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

Rule 1211

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + b*x^2 + c*x^4
], x], x] - Dist[e/q, Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /; Ne
Q[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[
c/a]
```

Rule 1222

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Dist[-(e^2)^(-1), Int[(c*d - b*e - c*e*x^2)*(a + b*x^2 + c*x^4)^(p -
 1), x], x] + Dist[(c*d^2 - b*d*e + a*e^2)/e^2, Int[(a + b*x^2 + c*x^4)^(p
- 1)/(d + e*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0
] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IGtQ[p + 1/2, 0]
```

Rule 1230

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4]), x_S
ymbol] :> With[{q = Rt[c/a, 2]}, Dist[(c*d + a*e*q)/(c*d^2 - a*e^2), Int[1/
Sqrt[a + b*x^2 + c*x^4], x], x] - Dist[(a*e*(e + d*q))/(c*d^2 - a*e^2), Int
[(1 + q*x^2)/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]), x], x]] /; FreeQ[{a, b,
 c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[c/a]
```

Rule 1349

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*
(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, p, q}, x] && N
eQ[b^2 - 4*a*c, 0] && (IGtQ[p, 0] || IGtQ[q, 0] || IntegersQ[m, q])
```

Rule 1720

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4]), x_Symbol] :> With[{q = Rt[B/A, 2]}, Simp[(-(B*d - A*e))*(A
rcTan[Rt[-b + c*(d/e) + a*(e/d), 2]*(x/Sqrt[a + b*x^2 + c*x^4])]/(2*d*e*Rt[
-b + c*(d/e) + a*(e/d), 2])), x] + Simp[(B*d + A*e)*(A + B*x^2)*(Sqrt[A^2*(
(a + b*x^2 + c*x^4)/(a*(A + B*x^2)^2))]/(4*d*e*A*q*Sqrt[a + b*x^2 + c*x^4])
)*EllipticPi[Cancel[-(B*d - A*e)^2/(4*d*e*A*B)], 2*ArcTan[q*x], 1/2 - b*(A/
(4*a*B))], x]] /; FreeQ[{a, b, c, d, e, A, B}, x] && NeQ[b^2 - 4*a*c, 0] &&
 NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] && EqQ
[c*A^2 - a*B^2, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \left(-\frac{3}{4}\sqrt{1+2x^2+2x^4}+\frac{1}{2}x^2\sqrt{1+2x^2+2x^4}+\frac{9\sqrt{1+2x^2+2x^4}}{4(3+2x^2)}\right)dx\\ &= \frac{1}{2}\int x^2\sqrt{1+2x^2+2x^4}\,dx-\frac{3}{4}\int \sqrt{1+2x^2+2x^4}\,dx+\frac{9}{4}\int \frac{\sqrt{1+2x^2+2x^4}}{3+2x^2}\,dx\end{aligned}$$

$$= -\frac{1}{4}x\sqrt{1+2x^2+2x^4} + \frac{1}{30}x\left(1+3x^2\right)\sqrt{1+2x^2+2x^4}$$
$$-\frac{1}{60}\int \frac{2-4x^2}{\sqrt{1+2x^2+2x^4}}\,dx - \frac{1}{4}\int \frac{2+2x^2}{\sqrt{1+2x^2+2x^4}}\,dx$$
$$-\frac{9}{16}\int \frac{2-4x^2}{\sqrt{1+2x^2+2x^4}}\,dx + \frac{45}{8}\int \frac{1}{(3+2x^2)\sqrt{1+2x^2+2x^4}}\,dx$$

$$= -\frac{1}{4}x\sqrt{1+2x^2+2x^4} + \frac{1}{30}x\left(1+3x^2\right)\sqrt{1+2x^2+2x^4}$$
$$-\frac{\int \frac{1-\sqrt{2}x^2}{\sqrt{1+2x^2+2x^4}}\,dx}{15\sqrt{2}} + \frac{\int \frac{1-\sqrt{2}x^2}{\sqrt{1+2x^2+2x^4}}\,dx}{2\sqrt{2}} - \frac{9\int \frac{1-\sqrt{2}x^2}{\sqrt{1+2x^2+2x^4}}\,dx}{4\sqrt{2}}$$
$$-\frac{1}{30}\left(1-\sqrt{2}\right)\int \frac{1}{\sqrt{1+2x^2+2x^4}}\,dx - \frac{1}{8}\left(9\left(1-\sqrt{2}\right)\right)\int \frac{1}{\sqrt{1+2x^2+2x^4}}\,dx$$
$$-\frac{1}{4}\left(2+\sqrt{2}\right)\int \frac{1}{\sqrt{1+2x^2+2x^4}}\,dx + \frac{1}{56}\left(45\left(3+\sqrt{2}\right)\right)\int \frac{1}{\sqrt{1+2x^2+2x^4}}\,dx$$
$$-\frac{1}{56}\left(45\left(2+3\sqrt{2}\right)\right)\int \frac{1+\sqrt{2}x^2}{(3+2x^2)\sqrt{1+2x^2+2x^4}}\,dx$$

$$= -\frac{1}{4}x\sqrt{1+2x^2+2x^4} + \frac{1}{30}x\left(1+3x^2\right)\sqrt{1+2x^2+2x^4}$$
$$+\frac{109x\sqrt{1+2x^2+2x^4}}{60\sqrt{2}\,(1+\sqrt{2}x^2)} + \frac{3}{16}\sqrt{15}\tan^{-1}\left(\frac{\sqrt{\frac{5}{3}}x}{\sqrt{1+2x^2+2x^4}}\right)$$
$$-\frac{109(1+\sqrt{2}x^2)\sqrt{\frac{1+2x^2+2x^4}{\left(1+\sqrt{2}x^2\right)^2}}E\left(2\tan^{-1}\left(\sqrt[4]{2}x\right)|\frac{1}{4}(2-\sqrt{2})\right)}{60\ 2^{3/4}\sqrt{1+2x^2+2x^4}}$$
$$-\frac{139(1-\sqrt{2})\,(1+\sqrt{2}x^2)\sqrt{\frac{1+2x^2+2x^4}{\left(1+\sqrt{2}x^2\right)^2}}F\left(2\tan^{-1}\left(\sqrt[4]{2}x\right)|\frac{1}{4}(2-\sqrt{2})\right)}{240\sqrt[4]{2}\sqrt{1+2x^2+2x^4}}$$
$$-\frac{(1+\sqrt{2})\,(1+\sqrt{2}x^2)\sqrt{\frac{1+2x^2+2x^4}{\left(1+\sqrt{2}x^2\right)^2}}F\left(2\tan^{-1}\left(\sqrt[4]{2}x\right)|\frac{1}{4}(2-\sqrt{2})\right)}{4\ 2^{3/4}\sqrt{1+2x^2+2x^4}}$$
$$+\frac{45(3+\sqrt{2})\,(1+\sqrt{2}x^2)\sqrt{\frac{1+2x^2+2x^4}{\left(1+\sqrt{2}x^2\right)^2}}F\left(2\tan^{-1}\left(\sqrt[4]{2}x\right)|\frac{1}{4}(2-\sqrt{2})\right)}{112\sqrt[4]{2}\sqrt{1+2x^2+2x^4}}$$
$$-\frac{15(3+\sqrt{2})^2\,(1+\sqrt{2}x^2)\sqrt{\frac{1+2x^2+2x^4}{\left(1+\sqrt{2}x^2\right)^2}}\Pi\left(\frac{1}{24}(12-11\sqrt{2});2\tan^{-1}\left(\sqrt[4]{2}x\right)|\frac{1}{4}(2-\sqrt{2})\right)}{224\sqrt[4]{2}\sqrt{1+2x^2+2x^4}}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 5.77 (sec) , antiderivative size = 209, normalized size of antiderivative = 0.49

$$\int \frac{x^4\sqrt{1+2x^2+2x^4}}{3+2x^2}\,dx$$
$$= \frac{-52x-80x^3-56x^5+48x^7-218i\sqrt{1-i}\sqrt{1+(1-i)x^2}\sqrt{1+(1+i)x^2}E\left(i\text{arcsinh}\left(\sqrt{1-i}x\right)\middle|i\right)-(199}{}$$

```
[In] Integrate[(x^4*Sqrt[1 + 2*x^2 + 2*x^4])/(3 + 2*x^2),x]
```

```
[Out] (-52*x - 80*x^3 - 56*x^5 + 48*x^7 - (218*I)*Sqrt[1 - I]*Sqrt[1 + (1 - I)*x^
2]*Sqrt[1 + (1 + I)*x^2]*EllipticE[I*ArcSinh[Sqrt[1 - I]*x], I] - (199 - 41
7*I)*Sqrt[1 - I]*Sqrt[1 + (1 - I)*x^2]*Sqrt[1 + (1 + I)*x^2]*EllipticF[I*Ar
cSinh[Sqrt[1 - I]*x], I] + 225*(1 - I)^(3/2)*Sqrt[1 + (1 - I)*x^2]*Sqrt[1 +
 (1 + I)*x^2]*EllipticPi[1/3 + I/3, I*ArcSinh[Sqrt[1 - I]*x], I])/(240*Sqrt
[1 + 2*x^2 + 2*x^4])
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 4.10 (sec) , antiderivative size = 246, normalized size of antiderivative = 0.58

method	result
risch	$\frac{x(6x^2-13)\sqrt{2x^4+2x^2+1}}{60}-\frac{199\sqrt{1+(1-i)x^2}\,\sqrt{1+(1+i)x^2}\,F\left(x\sqrt{-1+i},\frac{\sqrt{2}}{2}+\frac{i\sqrt{2}}{2}\right)}{120\sqrt{-1+i}\,\sqrt{2x^4+2x^2+1}}+\frac{\left(-\frac{109}{120}+\frac{109i}{120}\right)\sqrt{1+(1-i)x^2}\,\sqrt{1+(1+i)x^2}\,(F}{\sqrt{-1+i}}$
elliptic	$\frac{x^3\sqrt{2x^4+2x^2+1}}{10}-\frac{13x\sqrt{2x^4+2x^2+1}}{60}-\frac{77\sqrt{-ix^2+x^2+1}\,\sqrt{ix^2+x^2+1}\,F\left(x\sqrt{-1+i},\frac{\sqrt{2}}{2}+\frac{i\sqrt{2}}{2}\right)}{30\sqrt{-1+i}\,\sqrt{2x^4+2x^2+1}}+\frac{109i\sqrt{-ix^2+x^2+1}\,\sqrt{ix^2+x^2+1}}{120\sqrt{-1+i}\,\sqrt{2x^4}}$
default	$-\frac{13x\sqrt{2x^4+2x^2+1}}{60}-\frac{8\sqrt{1+(1-i)x^2}\,\sqrt{1+(1+i)x^2}\,F\left(x\sqrt{-1+i},\frac{\sqrt{2}}{2}+\frac{i\sqrt{2}}{2}\right)}{15\sqrt{-1+i}\,\sqrt{2x^4+2x^2+1}}+\frac{\left(\frac{13}{60}-\frac{13i}{60}\right)\sqrt{1+(1-i)x^2}\,\sqrt{1+(1+i)x^2}\,\left(F\left(x\sqrt{-1+i}\right.\right.}{\sqrt{-1+i}\,\sqrt{2x^4+2x}}$

```
[In] int(x^4*(2*x^4+2*x^2+1)^(1/2)/(2*x^2+3),x,method=_RETURNVERBOSE)
```

```
[Out] 1/60*x*(6*x^2-13)*(2*x^4+2*x^2+1)^(1/2)-199/120/(-1+I)^(1/2)*(1+(1-I)*x^2)^
(1/2)*(1+(1+I)*x^2)^(1/2)/(2*x^4+2*x^2+1)^(1/2)*EllipticF(x*(-1+I)^(1/2),1/
2*2^(1/2)+1/2*I*2^(1/2))+(-109/120+109/120*I)/(-1+I)^(1/2)*(1+(1-I)*x^2)^(1
/2)*(1+(1+I)*x^2)^(1/2)/(2*x^4+2*x^2+1)^(1/2)*(EllipticF(x*(-1+I)^(1/2),1/2
*2^(1/2)+1/2*I*2^(1/2))-EllipticE(x*(-1+I)^(1/2),1/2*2^(1/2)+1/2*I*2^(1/2))
)+15/8/(-1+I)^(1/2)*(1-I*x^2+x^2)^(1/2)*(1+I*x^2+x^2)^(1/2)/(2*x^4+2*x^2+1)
^(1/2)*EllipticPi(x*(-1+I)^(1/2),1/3+1/3*I,(-1-I)^(1/2)/(-1+I)^(1/2))
```

Fricas [F]

$$\int \frac{x^4\sqrt{1+2x^2+2x^4}}{3+2x^2}\,dx = \int \frac{\sqrt{2\,x^4+2\,x^2+1}x^4}{2\,x^2+3}\,dx$$

```
[In] integrate(x^4*(2*x^4+2*x^2+1)^(1/2)/(2*x^2+3),x, algorithm="fricas")
[Out] integral(sqrt(2*x^4 + 2*x^2 + 1)*x^4/(2*x^2 + 3), x)
```

Sympy [F]

$$\int \frac{x^4\sqrt{1+2x^2+2x^4}}{3+2x^2}\,dx = \int \frac{x^4\sqrt{2x^4+2x^2+1}}{2x^2+3}\,dx$$

```
[In] integrate(x**4*(2*x**4+2*x**2+1)**(1/2)/(2*x**2+3),x)
[Out] Integral(x**4*sqrt(2*x**4 + 2*x**2 + 1)/(2*x**2 + 3), x)
```

Maxima [F]

$$\int \frac{x^4\sqrt{1+2x^2+2x^4}}{3+2x^2}\,dx = \int \frac{\sqrt{2\,x^4+2\,x^2+1}x^4}{2\,x^2+3}\,dx$$

```
[In] integrate(x^4*(2*x^4+2*x^2+1)^(1/2)/(2*x^2+3),x, algorithm="maxima")
[Out] integrate(sqrt(2*x^4 + 2*x^2 + 1)*x^4/(2*x^2 + 3), x)
```

Giac [F]

$$\int \frac{x^4\sqrt{1+2x^2+2x^4}}{3+2x^2}\,dx = \int \frac{\sqrt{2\,x^4+2\,x^2+1}x^4}{2\,x^2+3}\,dx$$

```
[In] integrate(x^4*(2*x^4+2*x^2+1)^(1/2)/(2*x^2+3),x, algorithm="giac")
[Out] integrate(sqrt(2*x^4 + 2*x^2 + 1)*x^4/(2*x^2 + 3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^4\sqrt{1+2x^2+2x^4}}{3+2x^2}\,dx = \int \frac{x^4\,\sqrt{2\,x^4+2\,x^2+1}}{2\,x^2+3}\,dx$$

```
[In] int((x^4*(2*x^2 + 2*x^4 + 1)^(1/2))/(2*x^2 + 3),x)
[Out] int((x^4*(2*x^2 + 2*x^4 + 1)^(1/2))/(2*x^2 + 3), x)
```

3.317 $\int \frac{x^2\sqrt{1+2x^2+2x^4}}{3+2x^2}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 417

$$\int \frac{x^2\sqrt{1+2x^2+2x^4}}{3+2x^2}\,dx = \frac{1}{6}x\sqrt{1+2x^2+2x^4}-\frac{7x\sqrt{1+2x^2+2x^4}}{6\sqrt{2}\,(1+\sqrt{2}x^2)}-\frac{1}{8}\sqrt{15}\arctan\left(\frac{\sqrt{\frac{5}{3}}x}{\sqrt{1+2x^2+2x^4}}\right)$$
$$+\frac{7(1+\sqrt{2}x^2)\sqrt{\frac{1+2x^2+2x^4}{(1+\sqrt{2}x^2)^2}}E\left(2\arctan\left(\sqrt[4]{2}x\right)|\frac{1}{4}(2-\sqrt{2})\right)}{6\,2^{3/4}\sqrt{1+2x^2+2x^4}}$$
$$-\frac{(-4+17\sqrt{2})\,(1+\sqrt{2}x^2)\sqrt{\frac{1+2x^2+2x^4}{(1+\sqrt{2}x^2)^2}}\,\text{EllipticF}\left(2\arctan\left(\sqrt[4]{2}x\right),\frac{1}{4}(2-\sqrt{2})\right)}{6\,2^{3/4}\,(-2+3\sqrt{2})\,\sqrt{1+2x^2+2x^4}}$$
$$-\frac{5(3+\sqrt{2})\,(1+\sqrt{2}x^2)\sqrt{\frac{1+2x^2+2x^4}{(1+\sqrt{2}x^2)^2}}\,\text{EllipticPi}\left(\frac{1}{24}(12-11\sqrt{2}),2\arctan\left(\sqrt[4]{2}x\right),\frac{1}{4}(2-\sqrt{2})\right)}{8\,2^{3/4}\,(2-3\sqrt{2})\,\sqrt{1+2x^2+2x^4}}$$

```
[Out] -1/8*arctan(1/3*x*15^(1/2)/(2*x^4+2*x^2+1)^(1/2))*15^(1/2)+1/6*x*(2*x^4+2*x
^2+1)^(1/2)-7/12*x*(2*x^4+2*x^2+1)^(1/2)*2^(1/2)/(1+x^2*2^(1/2))+7/12*(cos(
2*arctan(2^(1/4)*x))^2)^(1/2)/cos(2*arctan(2^(1/4)*x))*EllipticE(sin(2*arct
an(2^(1/4)*x)),1/2*(2-2^(1/2))^(1/2))*(1+x^2*2^(1/2))*((2*x^4+2*x^2+1)/(1+x
^2*2^(1/2))^2)^(1/2)*2^(1/4)/(2*x^4+2*x^2+1)^(1/2)-5/16*(cos(2*arctan(2^(1/
4)*x))^2)^(1/2)/cos(2*arctan(2^(1/4)*x))*EllipticPi(sin(2*arctan(2^(1/4)*x)
),1/2-11/24*2^(1/2),1/2*(2-2^(1/2))^(1/2))*(3+2^(1/2))*(1+x^2*2^(1/2))*((2*
x^4+2*x^2+1)/(1+x^2*2^(1/2))^2)^(1/2)*2^(1/4)/(2-3*2^(1/2))/(2*x^4+2*x^2+1)
^(1/2)-1/12*(cos(2*arctan(2^(1/4)*x))^2)^(1/2)/cos(2*arctan(2^(1/4)*x))*Ell
ipticF(sin(2*arctan(2^(1/4)*x)),1/2*(2-2^(1/2))^(1/2))*(-4+17*2^(1/2))*(1+x
```

```
^2*2^(1/2))*((2*x^4+2*x^2+1)/(1+x^2*2^(1/2))^2)^(1/2)*2^(1/4)/(-2+3*2^(1/2)
)/(2*x^4+2*x^2+1)^(1/2)
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 591, normalized size of antiderivative = 1.42, number of steps used = 13, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.276, Rules used = {1349, 1105, 1211, 1117, 1209, 1222, 1230, 1720}

$$\int \frac{x^2\sqrt{1+2x^2+2x^4}}{3+2x^2}\,dx = -\frac{1}{8}\sqrt{15}\arctan\left(\frac{\sqrt{\frac{5}{3}}x}{\sqrt{2x^4+2x^2+1}}\right) - \frac{15(3+\sqrt{2})\left(\sqrt{2}x^2+1\right)\sqrt{\frac{2x^4+2x^2+1}{\left(\sqrt{2}x^2+1\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\sqrt[4]{2}x\right),\frac{1}{4}(2-\sqrt{2})\right)}{56\sqrt[4]{2}\sqrt{2x^4+2x^2+1}} + \frac{(1+\sqrt{2})\left(\sqrt{2}x^2+1\right)\sqrt{\frac{2x^4+2x^2+1}{\left(\sqrt{2}x^2+1\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\sqrt[4]{2}x\right),\frac{1}{4}(2-\sqrt{2})\right)}{6\ 2^{3/4}\sqrt{2x^4+2x^2+1}} + \frac{3(1-\sqrt{2})\left(\sqrt{2}x^2+1\right)\sqrt{\frac{2x^4+2x^2+1}{\left(\sqrt{2}x^2+1\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\sqrt[4]{2}x\right),\frac{1}{4}(2-\sqrt{2})\right)}{8\sqrt[4]{2}\sqrt{2x^4+2x^2+1}} + \frac{7\left(\sqrt{2}x^2+1\right)\sqrt{\frac{2x^4+2x^2+1}{\left(\sqrt{2}x^2+1\right)^2}}E\left(2\arctan\left(\sqrt[4]{2}x\right)|\frac{1}{4}(2-\sqrt{2})\right)}{6\ 2^{3/4}\sqrt{2x^4+2x^2+1}} + \frac{5(3+\sqrt{2})^2\left(\sqrt{2}x^2+1\right)\sqrt{\frac{2x^4+2x^2+1}{\left(\sqrt{2}x^2+1\right)^2}}\,\text{EllipticPi}\left(\frac{1}{24}(12-11\sqrt{2}),2\arctan\left(\sqrt[4]{2}x\right),\frac{1}{4}(2-\sqrt{2})\right)}{112\sqrt[4]{2}\sqrt{2x^4+2x^2+1}} - \frac{7\sqrt{2x^4+2x^2+1}x}{6\sqrt{2}\left(\sqrt{2}x^2+1\right)} + \frac{1}{6}\sqrt{2x^4+2x^2+1}x$$

```
[In] Int[(x^2*Sqrt[1 + 2*x^2 + 2*x^4])/(3 + 2*x^2),x]
```

```
[Out] (x*Sqrt[1 + 2*x^2 + 2*x^4])/6 - (7*x*Sqrt[1 + 2*x^2 + 2*x^4])/(6*Sqrt[2]*(1
 + Sqrt[2]*x^2)) - (Sqrt[15]*ArcTan[(Sqrt[5/3]*x)/Sqrt[1 + 2*x^2 + 2*x^4]])
/8 + (7*(1 + Sqrt[2]*x^2)*Sqrt[(1 + 2*x^2 + 2*x^4)/(1 + Sqrt[2]*x^2)^2]*Ell
ipticE[2*ArcTan[2^(1/4)*x], (2 - Sqrt[2])/4])/(6*2^(3/4)*Sqrt[1 + 2*x^2 + 2
*x^4]) + (3*(1 - Sqrt[2])*(1 + Sqrt[2]*x^2)*Sqrt[(1 + 2*x^2 + 2*x^4)/(1 + S
qrt[2]*x^2)^2]*EllipticF[2*ArcTan[2^(1/4)*x], (2 - Sqrt[2])/4])/(8*2^(1/4)*
Sqrt[1 + 2*x^2 + 2*x^4]) + ((1 + Sqrt[2])*(1 + Sqrt[2]*x^2)*Sqrt[(1 + 2*x^2
 + 2*x^4)/(1 + Sqrt[2]*x^2)^2]*EllipticF[2*ArcTan[2^(1/4)*x], (2 - Sqrt[2])
/4])/(6*2^(3/4)*Sqrt[1 + 2*x^2 + 2*x^4]) - (15*(3 + Sqrt[2])*(1 + Sqrt[2]*x
^2)*Sqrt[(1 + 2*x^2 + 2*x^4)/(1 + Sqrt[2]*x^2)^2]*EllipticF[2*ArcTan[2^(1/4
)*x], (2 - Sqrt[2])/4])/(56*2^(1/4)*Sqrt[1 + 2*x^2 + 2*x^4]) + (5*(3 + Sqrt
```

```
[2])^2*(1 + Sqrt[2]*x^2)*Sqrt[(1 + 2*x^2 + 2*x^4)/(1 + Sqrt[2]*x^2)^2]*Elli
pticPi[(12 - 11*Sqrt[2])/24, 2*ArcTan[2^(1/4)*x], (2 - Sqrt[2])/4])/(112*2^
(1/4)*Sqrt[1 + 2*x^2 + 2*x^4])
```

Rule 1105

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[x*((a + b*
x^2 + c*x^4)^p/(4*p + 1)), x] + Dist[2*(p/(4*p + 1)), Int[(2*a + b*x^2)*(a
+ b*x^2 + c*x^4)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c,
 0] && GtQ[p, 0] && IntegerQ[2*p]
```

Rule 1117

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1209

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

Rule 1211

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + b*x^2 + c*x^4
], x], x] - Dist[e/q, Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /; Ne
Q[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[
c/a]
```

Rule 1222

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Dist[-(e^2)^(-1), Int[(c*d - b*e - c*e*x^2)*(a + b*x^2 + c*x^4)^(p -
 1), x], x] + Dist[(c*d^2 - b*d*e + a*e^2)/e^2, Int[(a + b*x^2 + c*x^4)^(p
- 1)/(d + e*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0
] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IGtQ[p + 1/2, 0]
```

Rule 1230

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4]), x_S
ymbol] :> With[{q = Rt[c/a, 2]}, Dist[(c*d + a*e*q)/(c*d^2 - a*e^2), Int[1/
```

```
Sqrt[a + b*x^2 + c*x^4], x], x] - Dist[(a*e*(e + d*q))/(c*d^2 - a*e^2), Int
[(1 + q*x^2)/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]), x], x]] /; FreeQ[{a, b,
 c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[c/a]
```

Rule 1349

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*
(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, p, q}, x] && N
eQ[b^2 - 4*a*c, 0] && (IGtQ[p, 0] || IGtQ[q, 0] || IntegersQ[m, q])
```

Rule 1720

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4]), x_Symbol] :> With[{q = Rt[B/A, 2]}, Simp[(-(B*d - A*e))*(A
rcTan[Rt[-b + c*(d/e) + a*(e/d), 2]*(x/Sqrt[a + b*x^2 + c*x^4])]/(2*d*e*Rt[
-b + c*(d/e) + a*(e/d), 2])), x] + Simp[(B*d + A*e)*(A + B*x^2)*(Sqrt[A^2*(
(a + b*x^2 + c*x^4)/(a*(A + B*x^2)^2))]/(4*d*e*A*q*Sqrt[a + b*x^2 + c*x^4])
)*EllipticPi[Cancel[-(B*d - A*e)^2/(4*d*e*A*B)], 2*ArcTan[q*x], 1/2 - b*(A/
(4*a*B))], x]] /; FreeQ[{a, b, c, d, e, A, B}, x] && NeQ[b^2 - 4*a*c, 0] &&
 NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] && EqQ
[c*A^2 - a*B^2, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \left(\frac{1}{2}\sqrt{1+2x^2+2x^4}-\frac{3\sqrt{1+2x^2+2x^4}}{2\,(3+2x^2)}\right)dx\\
&= \frac{1}{2}\int \sqrt{1+2x^2+2x^4}\,dx-\frac{3}{2}\int \frac{\sqrt{1+2x^2+2x^4}}{3+2x^2}\,dx\\
&= \frac{1}{6}x\sqrt{1+2x^2+2x^4}+\frac{1}{6}\int \frac{2+2x^2}{\sqrt{1+2x^2+2x^4}}\,dx\\
&\quad+\frac{3}{8}\int \frac{2-4x^2}{\sqrt{1+2x^2+2x^4}}\,dx-\frac{15}{4}\int \frac{1}{(3+2x^2)\sqrt{1+2x^2+2x^4}}\,dx\\
&= \frac{1}{6}x\sqrt{1+2x^2+2x^4}-\frac{\int \frac{1-\sqrt{2}x^2}{\sqrt{1+2x^2+2x^4}}\,dx}{3\sqrt{2}}+\frac{3\int \frac{1-\sqrt{2}x^2}{\sqrt{1+2x^2+2x^4}}\,dx}{2\sqrt{2}}\\
&\quad+\frac{1}{4}\left(3\left(1-\sqrt{2}\right)\right)\int \frac{1}{\sqrt{1+2x^2+2x^4}}\,dx+\frac{1}{6}\left(2+\sqrt{2}\right)\int \frac{1}{\sqrt{1+2x^2+2x^4}}\,dx\\
&\quad-\frac{1}{28}\left(15\left(3+\sqrt{2}\right)\right)\int \frac{1}{\sqrt{1+2x^2+2x^4}}\,dx\\
&\quad+\frac{1}{28}\left(15\left(2+3\sqrt{2}\right)\right)\int \frac{1+\sqrt{2}x^2}{(3+2x^2)\sqrt{1+2x^2+2x^4}}\,dx
\end{aligned}
$$

$$= \frac{1}{6}x\sqrt{1+2x^2+2x^4} - \frac{7x\sqrt{1+2x^2+2x^4}}{6\sqrt{2}\,(1+\sqrt{2}x^2)} - \frac{1}{8}\sqrt{15}\tan^{-1}\left(\frac{\sqrt{\frac{5}{3}}x}{\sqrt{1+2x^2+2x^4}}\right)$$
$$+ \frac{7\left(1+\sqrt{2}x^2\right)\sqrt{\frac{1+2x^2+2x^4}{\left(1+\sqrt{2}x^2\right)^2}}E\left(2\tan^{-1}\left(\sqrt[4]{2}x\right)|\frac{1}{4}\left(2-\sqrt{2}\right)\right)}{6\ 2^{3/4}\sqrt{1+2x^2+2x^4}}$$
$$+ \frac{3\left(1-\sqrt{2}\right)\left(1+\sqrt{2}x^2\right)\sqrt{\frac{1+2x^2+2x^4}{\left(1+\sqrt{2}x^2\right)^2}}F\left(2\tan^{-1}\left(\sqrt[4]{2}x\right)|\frac{1}{4}\left(2-\sqrt{2}\right)\right)}{8\sqrt[4]{2}\sqrt{1+2x^2+2x^4}}$$
$$+ \frac{\left(1+\sqrt{2}\right)\left(1+\sqrt{2}x^2\right)\sqrt{\frac{1+2x^2+2x^4}{\left(1+\sqrt{2}x^2\right)^2}}F\left(2\tan^{-1}\left(\sqrt[4]{2}x\right)|\frac{1}{4}\left(2-\sqrt{2}\right)\right)}{6\ 2^{3/4}\sqrt{1+2x^2+2x^4}}$$
$$- \frac{15\left(3+\sqrt{2}\right)\left(1+\sqrt{2}x^2\right)\sqrt{\frac{1+2x^2+2x^4}{\left(1+\sqrt{2}x^2\right)^2}}F\left(2\tan^{-1}\left(\sqrt[4]{2}x\right)|\frac{1}{4}\left(2-\sqrt{2}\right)\right)}{56\sqrt[4]{2}\sqrt{1+2x^2+2x^4}}$$
$$+ \frac{5\left(3+\sqrt{2}\right)^2\left(1+\sqrt{2}x^2\right)\sqrt{\frac{1+2x^2+2x^4}{\left(1+\sqrt{2}x^2\right)^2}}\Pi\left(\frac{1}{24}\left(12-11\sqrt{2}\right);2\tan^{-1}\left(\sqrt[4]{2}x\right)|\frac{1}{4}\left(2-\sqrt{2}\right)\right)}{112\sqrt[4]{2}\sqrt{1+2x^2+2x^4}}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 5.06 (sec) , antiderivative size = 204, normalized size of antiderivative = 0.49

$$\int \frac{x^2\sqrt{1+2x^2+2x^4}}{3+2x^2}\,dx$$
$$= \frac{4x+8x^3+8x^5+14i\sqrt{1-i}\sqrt{1+(1-i)x^2}\sqrt{1+(1+i)x^2}E\left(i\operatorname{arcsinh}\left(\sqrt{1-i}x\right)\middle|i\right)+(13-27i)\sqrt{1-i}}{}$$

```
[In] Integrate[(x^2*Sqrt[1 + 2*x^2 + 2*x^4])/(3 + 2*x^2),x]
```

```
[Out] (4*x + 8*x^3 + 8*x^5 + (14*I)*Sqrt[1 - I]*Sqrt[1 + (1 - I)*x^2]*Sqrt[1 + (1
 + I)*x^2]*EllipticE[I*ArcSinh[Sqrt[1 - I]*x], I] + (13 - 27*I)*Sqrt[1 - I]
*Sqrt[1 + (1 - I)*x^2]*Sqrt[1 + (1 + I)*x^2]*EllipticF[I*ArcSinh[Sqrt[1 - I
]*x], I] - 15*(1 - I)^(3/2)*Sqrt[1 + (1 - I)*x^2]*Sqrt[1 + (1 + I)*x^2]*Ell
ipticPi[1/3 + I/3, I*ArcSinh[Sqrt[1 - I]*x], I])/(24*Sqrt[1 + 2*x^2 + 2*x^4
])
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.37 (sec) , antiderivative size = 239, normalized size of antiderivative = 0.57

method	result
risch	$\frac{x\sqrt{2x^4+2x^2+1}}{6}+\frac{13\sqrt{1+(1-i)x^2}\,\sqrt{1+(1+i)x^2}\,F\left(x\sqrt{-1+i},\frac{\sqrt{2}}{2}+\frac{i\sqrt{2}}{2}\right)}{12\sqrt{-1+i}\,\sqrt{2x^4+2x^2+1}}+\frac{\left(\frac{7}{12}-\frac{7i}{12}\right)\sqrt{1+(1-i)x^2}\,\sqrt{1+(1+i)x^2}\,\left(F\left(x\sqrt{-1+i},\frac{\sqrt{2}}{2}\right.\right.}{\sqrt{-1+i}\,\sqrt{2x^4+2x^2+1}}$
elliptic	$\frac{x\sqrt{2x^4+2x^2+1}}{6}+\frac{5\sqrt{-ix^2+x^2+1}\,\sqrt{ix^2+x^2+1}\,F\left(x\sqrt{-1+i},\frac{\sqrt{2}}{2}+\frac{i\sqrt{2}}{2}\right)}{3\sqrt{-1+i}\,\sqrt{2x^4+2x^2+1}}-\frac{7i\sqrt{-ix^2+x^2+1}\,\sqrt{ix^2+x^2+1}\,F\left(x\sqrt{-1+i},\frac{\sqrt{2}}{2}+\frac{i\sqrt{2}}{2}\right)}{12\sqrt{-1+i}\,\sqrt{2x^4+2x^2+1}}-$
default	$\frac{x\sqrt{2x^4+2x^2+1}}{6}+\frac{\sqrt{1+(1-i)x^2}\,\sqrt{1+(1+i)x^2}\,F\left(x\sqrt{-1+i},\frac{\sqrt{2}}{2}+\frac{i\sqrt{2}}{2}\right)}{3\sqrt{-1+i}\,\sqrt{2x^4+2x^2+1}}+\frac{\left(-\frac{1}{6}+\frac{i}{6}\right)\sqrt{1+(1-i)x^2}\,\sqrt{1+(1+i)x^2}\,\left(F\left(x\sqrt{-1+i},\frac{\sqrt{2}}{2}+\right.\right.}{\sqrt{-1+i}\,\sqrt{2x^4+2x^2+1}}$

```
[In] int(x^2*(2*x^4+2*x^2+1)^(1/2)/(2*x^2+3),x,method=_RETURNVERBOSE)
```

```
[Out] 1/6*x*(2*x^4+2*x^2+1)^(1/2)+13/12/(-1+I)^(1/2)*(1+(1-I)*x^2)^(1/2)*(1+(1+I)
*x^2)^(1/2)/(2*x^4+2*x^2+1)^(1/2)*EllipticF(x*(-1+I)^(1/2),1/2*2^(1/2)+1/2*
I*2^(1/2))+(7/12-7/12*I)/(-1+I)^(1/2)*(1+(1-I)*x^2)^(1/2)*(1+(1+I)*x^2)^(1/
2)/(2*x^4+2*x^2+1)^(1/2)*(EllipticF(x*(-1+I)^(1/2),1/2*2^(1/2)+1/2*I*2^(1/2
))-EllipticE(x*(-1+I)^(1/2),1/2*2^(1/2)+1/2*I*2^(1/2)))-5/4/(-1+I)^(1/2)*(1
-I*x^2+x^2)^(1/2)*(1+I*x^2+x^2)^(1/2)/(2*x^4+2*x^2+1)^(1/2)*EllipticPi(x*(-
1+I)^(1/2),1/3+1/3*I,(-1-I)^(1/2)/(-1+I)^(1/2))
```

Fricas [F]

$$\int \frac{x^2\sqrt{1+2x^2+2x^4}}{3+2x^2}\,dx=\int \frac{\sqrt{2\,x^4+2\,x^2+1}x^2}{2\,x^2+3}\,dx$$

```
[In] integrate(x^2*(2*x^4+2*x^2+1)^(1/2)/(2*x^2+3),x, algorithm="fricas")
```

```
[Out] integral(sqrt(2*x^4 + 2*x^2 + 1)*x^2/(2*x^2 + 3), x)
```

Sympy [F]

$$\int \frac{x^2\sqrt{1+2x^2+2x^4}}{3+2x^2}\,dx=\int \frac{x^2\sqrt{2x^4+2x^2+1}}{2x^2+3}\,dx$$

```
[In] integrate(x**2*(2*x**4+2*x**2+1)**(1/2)/(2*x**2+3),x)
```

```
[Out] Integral(x**2*sqrt(2*x**4 + 2*x**2 + 1)/(2*x**2 + 3), x)
```

Maxima [F]

$$\int \frac{x^2\sqrt{1+2x^2+2x^4}}{3+2x^2}\,dx = \int \frac{\sqrt{2\,x^4+2\,x^2+1}x^2}{2\,x^2+3}\,dx$$

```
[In] integrate(x^2*(2*x^4+2*x^2+1)^(1/2)/(2*x^2+3),x, algorithm="maxima")
[Out] integrate(sqrt(2*x^4 + 2*x^2 + 1)*x^2/(2*x^2 + 3), x)
```

Giac [F]

$$\int \frac{x^2\sqrt{1+2x^2+2x^4}}{3+2x^2}\,dx = \int \frac{\sqrt{2\,x^4+2\,x^2+1}x^2}{2\,x^2+3}\,dx$$

```
[In] integrate(x^2*(2*x^4+2*x^2+1)^(1/2)/(2*x^2+3),x, algorithm="giac")
[Out] integrate(sqrt(2*x^4 + 2*x^2 + 1)*x^2/(2*x^2 + 3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2\sqrt{1+2x^2+2x^4}}{3+2x^2}\,dx = \int \frac{x^2\,\sqrt{2\,x^4+2\,x^2+1}}{2\,x^2+3}\,dx$$

```
[In] int((x^2*(2*x^2 + 2*x^4 + 1)^(1/2))/(2*x^2 + 3),x)
[Out] int((x^2*(2*x^2 + 2*x^4 + 1)^(1/2))/(2*x^2 + 3), x)
```

3.318 $\int \frac{\sqrt{1+2x^2+2x^4}}{3+2x^2}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 381

$$
\begin{aligned}
\int \frac{\sqrt{1+2x^2+2x^4}}{3+2x^2}\,dx &= \frac{x\sqrt{1+2x^2+2x^4}}{\sqrt{2}\left(1+\sqrt{2}x^2\right)}+\frac{1}{4}\sqrt{\frac{5}{3}}\arctan\left(\frac{\sqrt{\frac{5}{3}}x}{\sqrt{1+2x^2+2x^4}}\right) \\
&- \frac{\left(1+\sqrt{2}x^2\right)\sqrt{\frac{1+2x^2+2x^4}{\left(1+\sqrt{2}x^2\right)^2}}E\left(2\arctan\left(\sqrt[4]{2}x\right)|\frac{1}{4}\left(2-\sqrt{2}\right)\right)}{2^{3/4}\sqrt{1+2x^2+2x^4}} \\
&+ \frac{2^{3/4}\left(1+\sqrt{2}x^2\right)\sqrt{\frac{1+2x^2+2x^4}{\left(1+\sqrt{2}x^2\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\sqrt[4]{2}x\right),\frac{1}{4}\left(2-\sqrt{2}\right)\right)}{\left(-2+3\sqrt{2}\right)\sqrt{1+2x^2+2x^4}} \\
&+ \frac{5\left(3+\sqrt{2}\right)\left(1+\sqrt{2}x^2\right)\sqrt{\frac{1+2x^2+2x^4}{\left(1+\sqrt{2}x^2\right)^2}}\operatorname{EllipticPi}\left(\frac{1}{24}\left(12-11\sqrt{2}\right),2\arctan\left(\sqrt[4]{2}x\right),\frac{1}{4}\left(2-\sqrt{2}\right)\right)}{12\ 2^{3/4}\left(2-3\sqrt{2}\right)\sqrt{1+2x^2+2x^4}}
\end{aligned}
$$

```
[Out] 1/12*arctan(1/3*x*15^(1/2)/(2*x^4+2*x^2+1)^(1/2))*15^(1/2)+1/2*x*(2*x^4+2*x
^2+1)^(1/2)*2^(1/2)/(1+x^2*2^(1/2))-1/2*(cos(2*arctan(2^(1/4)*x))^2)^(1/2)/
cos(2*arctan(2^(1/4)*x))*EllipticE(sin(2*arctan(2^(1/4)*x)),1/2*(2-2^(1/2))
^(1/2))*(1+x^2*2^(1/2))*((2*x^4+2*x^2+1)/(1+x^2*2^(1/2))^2)^(1/2)*2^(1/4)/(
2*x^4+2*x^2+1)^(1/2)+5/24*(cos(2*arctan(2^(1/4)*x))^2)^(1/2)/cos(2*arctan(2
^(1/4)*x))*EllipticPi(sin(2*arctan(2^(1/4)*x)),1/2-11/24*2^(1/2),1/2*(2-2^(
1/2))^(1/2))*(3+2^(1/2))*(1+x^2*2^(1/2))*((2*x^4+2*x^2+1)/(1+x^2*2^(1/2))^2
)^(1/2)*2^(1/4)/(2-3*2^(1/2))/(2*x^4+2*x^2+1)^(1/2)+2^(3/4)*(cos(2*arctan(2
^(1/4)*x))^2)^(1/2)/cos(2*arctan(2^(1/4)*x))*EllipticF(sin(2*arctan(2^(1/4)
*x)),1/2*(2-2^(1/2))^(1/2))*(1+x^2*2^(1/2))*((2*x^4+2*x^2+1)/(1+x^2*2^(1/2)
)^2)^(1/2)/(-2+3*2^(1/2))/(2*x^4+2*x^2+1)^(1/2)
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 470, normalized size of antiderivative = 1.23, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {1222, 1211, 1117, 1209, 1230, 1720}

$$\begin{aligned}\int \frac{\sqrt{1+2x^2+2x^4}}{3+2x^2}\,dx &= \frac{1}{4}\sqrt{\frac{5}{3}}\arctan\left(\frac{\sqrt{\frac{5}{3}}x}{\sqrt{2x^4+2x^2+1}}\right)\\
&+\frac{5\left(3+\sqrt{2}\right)\left(\sqrt{2}x^2+1\right)\sqrt{\frac{2x^4+2x^2+1}{\left(\sqrt{2}x^2+1\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\sqrt[4]{2}x\right),\frac{1}{4}\left(2-\sqrt{2}\right)\right)}{28\sqrt[4]{2}\sqrt{2x^4+2x^2+1}}\\
&-\frac{\left(1-\sqrt{2}\right)\left(\sqrt{2}x^2+1\right)\sqrt{\frac{2x^4+2x^2+1}{\left(\sqrt{2}x^2+1\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\sqrt[4]{2}x\right),\frac{1}{4}\left(2-\sqrt{2}\right)\right)}{4\sqrt[4]{2}\sqrt{2x^4+2x^2+1}}\\
&-\frac{\left(\sqrt{2}x^2+1\right)\sqrt{\frac{2x^4+2x^2+1}{\left(\sqrt{2}x^2+1\right)^2}}E\left(2\arctan\left(\sqrt[4]{2}x\right)|\frac{1}{4}\left(2-\sqrt{2}\right)\right)}{2^{3/4}\sqrt{2x^4+2x^2+1}}\\
&-\frac{5\left(3+\sqrt{2}\right)^2\left(\sqrt{2}x^2+1\right)\sqrt{\frac{2x^4+2x^2+1}{\left(\sqrt{2}x^2+1\right)^2}}\operatorname{EllipticPi}\left(\frac{1}{24}\left(12-11\sqrt{2}\right),2\arctan\left(\sqrt[4]{2}x\right),\frac{1}{4}\left(2-\sqrt{2}\right)\right)}{168\sqrt[4]{2}\sqrt{2x^4+2x^2+1}}\\
&+\frac{\sqrt{2x^4+2x^2+1}x}{\sqrt{2}\left(\sqrt{2}x^2+1\right)}\end{aligned}$$

```
[In] Int[Sqrt[1 + 2*x^2 + 2*x^4]/(3 + 2*x^2),x]
```

```
[Out] (x*Sqrt[1 + 2*x^2 + 2*x^4])/(Sqrt[2]*(1 + Sqrt[2]*x^2)) + (Sqrt[5/3]*ArcTan
[(Sqrt[5/3]*x)/Sqrt[1 + 2*x^2 + 2*x^4]])/4 - ((1 + Sqrt[2]*x^2)*Sqrt[(1 + 2
*x^2 + 2*x^4)/(1 + Sqrt[2]*x^2)^2]*EllipticE[2*ArcTan[2^(1/4)*x], (2 - Sqrt
[2])/4])/(2^(3/4)*Sqrt[1 + 2*x^2 + 2*x^4]) - ((1 - Sqrt[2])*(1 + Sqrt[2]*x^
2)*Sqrt[(1 + 2*x^2 + 2*x^4)/(1 + Sqrt[2]*x^2)^2]*EllipticF[2*ArcTan[2^(1/4)
*x], (2 - Sqrt[2])/4])/(4*2^(1/4)*Sqrt[1 + 2*x^2 + 2*x^4]) + (5*(3 + Sqrt[2
])*(1 + Sqrt[2]*x^2)*Sqrt[(1 + 2*x^2 + 2*x^4)/(1 + Sqrt[2]*x^2)^2]*Elliptic
F[2*ArcTan[2^(1/4)*x], (2 - Sqrt[2])/4])/(28*2^(1/4)*Sqrt[1 + 2*x^2 + 2*x^4
]) - (5*(3 + Sqrt[2])^2*(1 + Sqrt[2]*x^2)*Sqrt[(1 + 2*x^2 + 2*x^4)/(1 + Sqr
t[2]*x^2)^2]*EllipticPi[(12 - 11*Sqrt[2])/24, 2*ArcTan[2^(1/4)*x], (2 - Sqr
t[2])/4])/(168*2^(1/4)*Sqrt[1 + 2*x^2 + 2*x^4])
```

Rule 1117

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1209

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

Rule 1211

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + b*x^2 + c*x^4
], x], x] - Dist[e/q, Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /; Ne
Q[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[
c/a]
```

Rule 1222

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Dist[-(e^2)^(-1), Int[(c*d - b*e - c*e*x^2)*(a + b*x^2 + c*x^4)^(p -
 1), x], x] + Dist[(c*d^2 - b*d*e + a*e^2)/e^2, Int[(a + b*x^2 + c*x^4)^(p
- 1)/(d + e*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0
] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IGtQ[p + 1/2, 0]
```

Rule 1230

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4]), x_S
ymbol] :> With[{q = Rt[c/a, 2]}, Dist[(c*d + a*e*q)/(c*d^2 - a*e^2), Int[1/
Sqrt[a + b*x^2 + c*x^4], x], x] - Dist[(a*e*(e + d*q))/(c*d^2 - a*e^2), Int
[(1 + q*x^2)/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]), x], x]] /; FreeQ[{a, b,
 c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[c/a]
```

Rule 1720

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4]), x_Symbol] :> With[{q = Rt[B/A, 2]}, Simp[(-(B*d - A*e))*(A
rcTan[Rt[-b + c*(d/e) + a*(e/d), 2]*(x/Sqrt[a + b*x^2 + c*x^4])]/(2*d*e*Rt[
-b + c*(d/e) + a*(e/d), 2])), x] + Simp[(B*d + A*e)*(A + B*x^2)*(Sqrt[A^2*(
(a + b*x^2 + c*x^4)/(a*(A + B*x^2)^2))]/(4*d*e*A*q*Sqrt[a + b*x^2 + c*x^4])
)*EllipticPi[Cancel[-(B*d - A*e)^2/(4*d*e*A*B)], 2*ArcTan[q*x], 1/2 - b*(A/
(4*a*B))], x]] /; FreeQ[{a, b, c, d, e, A, B}, x] && NeQ[b^2 - 4*a*c, 0] &&
 NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] && EqQ
[c*A^2 - a*B^2, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\left(\frac{1}{4}\int \frac{2-4x^2}{\sqrt{1+2x^2+2x^4}}\,dx\right)+\frac{5}{2}\int \frac{1}{(3+2x^2)\sqrt{1+2x^2+2x^4}}\,dx \\
&= -\frac{\int \frac{1-\sqrt{2}x^2}{\sqrt{1+2x^2+2x^4}}\,dx}{\sqrt{2}}-\frac{1}{2}\left(1-\sqrt{2}\right)\int \frac{1}{\sqrt{1+2x^2+2x^4}}\,dx \\
&\quad +\frac{1}{14}\left(5\left(3+\sqrt{2}\right)\right)\int \frac{1}{\sqrt{1+2x^2+2x^4}}\,dx \\
&\quad -\frac{1}{14}\left(5\left(2+3\sqrt{2}\right)\right)\int \frac{1+\sqrt{2}x^2}{(3+2x^2)\sqrt{1+2x^2+2x^4}}\,dx \\
&= \frac{x\sqrt{1+2x^2+2x^4}}{\sqrt{2}\left(1+\sqrt{2}x^2\right)}+\frac{1}{4}\sqrt{\frac{5}{3}}\tan^{-1}\left(\frac{\sqrt{\frac{5}{3}}x}{\sqrt{1+2x^2+2x^4}}\right) \\
&\quad -\frac{\left(1+\sqrt{2}x^2\right)\sqrt{\frac{1+2x^2+2x^4}{\left(1+\sqrt{2}x^2\right)^2}}E\left(2\tan^{-1}\left(\sqrt[4]{2}x\right)|\frac{1}{4}\left(2-\sqrt{2}\right)\right)}{2^{3/4}\sqrt{1+2x^2+2x^4}} \\
&\quad -\frac{\left(1-\sqrt{2}\right)\left(1+\sqrt{2}x^2\right)\sqrt{\frac{1+2x^2+2x^4}{\left(1+\sqrt{2}x^2\right)^2}}F\left(2\tan^{-1}\left(\sqrt[4]{2}x\right)|\frac{1}{4}\left(2-\sqrt{2}\right)\right)}{4\sqrt[4]{2}\sqrt{1+2x^2+2x^4}} \\
&\quad +\frac{5\left(3+\sqrt{2}\right)\left(1+\sqrt{2}x^2\right)\sqrt{\frac{1+2x^2+2x^4}{\left(1+\sqrt{2}x^2\right)^2}}F\left(2\tan^{-1}\left(\sqrt[4]{2}x\right)|\frac{1}{4}\left(2-\sqrt{2}\right)\right)}{28\sqrt[4]{2}\sqrt{1+2x^2+2x^4}} \\
&\quad -\frac{5\left(3+\sqrt{2}\right)^2\left(1+\sqrt{2}x^2\right)\sqrt{\frac{1+2x^2+2x^4}{\left(1+\sqrt{2}x^2\right)^2}}\Pi\left(\frac{1}{24}\left(12-11\sqrt{2}\right);2\tan^{-1}\left(\sqrt[4]{2}x\right)|\frac{1}{4}\left(2-\sqrt{2}\right)\right)}{168\sqrt[4]{2}\sqrt{1+2x^2+2x^4}}
\end{aligned}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 9.62 (sec) , antiderivative size = 127, normalized size of antiderivative = 0.33

$$\int \frac{\sqrt{1+2x^2+2x^4}}{3+2x^2}\,dx = -\frac{\sqrt{1+(1-i)x^2}\sqrt{1+(1+i)x^2}\left((3+3i)E\left(i\text{arcsinh}\left(\sqrt{1-i}x\right)\middle|i\right)-(3+6i)\,\text{EllipticF}\left(i\text{arcsinh}\left(\sqrt{1-\right.\right.}{6\sqrt{1-i}\sqrt{1+2x^2+2x^4}}$$

```
[In] Integrate[Sqrt[1 + 2*x^2 + 2*x^4]/(3 + 2*x^2),x]
```

```
[Out] -1/6*(Sqrt[1 + (1 - I)*x^2]*Sqrt[1 + (1 + I)*x^2]*((3 + 3*I)*EllipticE[I*Ar
cSinh[Sqrt[1 - I]*x], I] - (3 + 6*I)*EllipticF[I*ArcSinh[Sqrt[1 - I]*x], I]
 + (5*I)*EllipticPi[1/3 + I/3, I*ArcSinh[Sqrt[1 - I]*x], I]))/(Sqrt[1 - I]*
Sqrt[1 + 2*x^2 + 2*x^4])
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.62 (sec) , antiderivative size = 341, normalized size of antiderivative = 0.90

method	result
default	$-\frac{\sqrt{-i x^2+x^2+1}\,\sqrt{i x^2+x^2+1}\,F\left(x\sqrt{-1+i},\frac{\sqrt{2}}{2}+\frac{i\sqrt{2}}{2}\right)}{\sqrt{-1+i}\,\sqrt{2x^4+2x^2+1}}+\frac{i\sqrt{-i x^2+x^2+1}\,\sqrt{i x^2+x^2+1}\,F\left(x\sqrt{-1+i},\frac{\sqrt{2}}{2}+\frac{i\sqrt{2}}{2}\right)}{2\sqrt{-1+i}\,\sqrt{2x^4+2x^2+1}}+\frac{\sqrt{-i x^2+x^2+1}\,\sqrt{ix}}{2\sqrt{-}}$
elliptic	$-\frac{\sqrt{-i x^2+x^2+1}\,\sqrt{i x^2+x^2+1}\,F\left(x\sqrt{-1+i},\frac{\sqrt{2}}{2}+\frac{i\sqrt{2}}{2}\right)}{\sqrt{-1+i}\,\sqrt{2x^4+2x^2+1}}+\frac{i\sqrt{-i x^2+x^2+1}\,\sqrt{i x^2+x^2+1}\,F\left(x\sqrt{-1+i},\frac{\sqrt{2}}{2}+\frac{i\sqrt{2}}{2}\right)}{2\sqrt{-1+i}\,\sqrt{2x^4+2x^2+1}}+\frac{\sqrt{-i x^2+x^2+1}\,\sqrt{ix}}{2\sqrt{-}}$

```
[In] int((2*x^4+2*x^2+1)^(1/2)/(2*x^2+3),x,method=_RETURNVERBOSE)
```

```
[Out] -1/(-1+I)^(1/2)*(1-I*x^2+x^2)^(1/2)*(1+I*x^2+x^2)^(1/2)/(2*x^4+2*x^2+1)^(1/
2)*EllipticF(x*(-1+I)^(1/2),1/2*2^(1/2)+1/2*I*2^(1/2))+1/2*I/(-1+I)^(1/2)*(
1-I*x^2+x^2)^(1/2)*(1+I*x^2+x^2)^(1/2)/(2*x^4+2*x^2+1)^(1/2)*EllipticF(x*(-
1+I)^(1/2),1/2*2^(1/2)+1/2*I*2^(1/2))+1/2/(-1+I)^(1/2)*(1-I*x^2+x^2)^(1/2)*
(1+I*x^2+x^2)^(1/2)/(2*x^4+2*x^2+1)^(1/2)*EllipticE(x*(-1+I)^(1/2),1/2*2^(1
/2)+1/2*I*2^(1/2))-1/2*I/(-1+I)^(1/2)*(1-I*x^2+x^2)^(1/2)*(1+I*x^2+x^2)^(1/
2)/(2*x^4+2*x^2+1)^(1/2)*EllipticE(x*(-1+I)^(1/2),1/2*2^(1/2)+1/2*I*2^(1/2)
)+5/6/(-1+I)^(1/2)*(1-I*x^2+x^2)^(1/2)*(1+I*x^2+x^2)^(1/2)/(2*x^4+2*x^2+1)^
(1/2)*EllipticPi(x*(-1+I)^(1/2),1/3+1/3*I,(-1-I)^(1/2)/(-1+I)^(1/2))
```

Fricas [F]

$$\int \frac{\sqrt{1+2x^2+2x^4}}{3+2x^2}\,dx=\int \frac{\sqrt{2\,x^4+2\,x^2+1}}{2\,x^2+3}\,dx$$

```
[In] integrate((2*x^4+2*x^2+1)^(1/2)/(2*x^2+3),x, algorithm="fricas")
```

```
[Out] integral(sqrt(2*x^4 + 2*x^2 + 1)/(2*x^2 + 3), x)
```

Sympy [F]

$$\int \frac{\sqrt{1+2x^2+2x^4}}{3+2x^2}\,dx=\int \frac{\sqrt{2x^4+2x^2+1}}{2x^2+3}\,dx$$

```
[In] integrate((2*x**4+2*x**2+1)**(1/2)/(2*x**2+3),x)
```

```
[Out] Integral(sqrt(2*x**4 + 2*x**2 + 1)/(2*x**2 + 3), x)
```

Maxima [F]

$$\int \frac{\sqrt{1+2x^2+2x^4}}{3+2x^2}\,dx = \int \frac{\sqrt{2\,x^4+2\,x^2+1}}{2\,x^2+3}\,dx$$

```
[In] integrate((2*x^4+2*x^2+1)^(1/2)/(2*x^2+3),x, algorithm="maxima")
[Out] integrate(sqrt(2*x^4 + 2*x^2 + 1)/(2*x^2 + 3), x)
```

Giac [F]

$$\int \frac{\sqrt{1+2x^2+2x^4}}{3+2x^2}\,dx = \int \frac{\sqrt{2\,x^4+2\,x^2+1}}{2\,x^2+3}\,dx$$

```
[In] integrate((2*x^4+2*x^2+1)^(1/2)/(2*x^2+3),x, algorithm="giac")
[Out] integrate(sqrt(2*x^4 + 2*x^2 + 1)/(2*x^2 + 3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{1+2x^2+2x^4}}{3+2x^2}\,dx = \int \frac{\sqrt{2\,x^4+2\,x^2+1}}{2\,x^2+3}\,dx$$

```
[In] int((2*x^2 + 2*x^4 + 1)^(1/2)/(2*x^2 + 3),x)
[Out] int((2*x^2 + 2*x^4 + 1)^(1/2)/(2*x^2 + 3), x)
```

3.319 $\int \frac{\sqrt{1+2x^2+2x^4}}{x^2(3+2x^2)}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 399

$$\int \frac{\sqrt{1+2x^2+2x^4}}{x^2(3+2x^2)}\,dx = -\frac{\sqrt{1+2x^2+2x^4}}{3x}+\frac{\sqrt{2}x\sqrt{1+2x^2+2x^4}}{3\left(1+\sqrt{2}x^2\right)}-\frac{1}{6}\sqrt{\frac{5}{3}}\arctan\left(\frac{\sqrt{\frac{5}{3}}x}{\sqrt{1+2x^2+2x^4}}\right)-\frac{\sqrt[4]{2}(1+\sqrt{2}x^2)\sqrt{\frac{1+2x^2+2x^4}{\left(1+\sqrt{2}x^2\right)^2}}E\left(2\arctan\left(\sqrt[4]{2}x\right)|\frac{1}{4}(2-\sqrt{2})\right)}{3\sqrt{1+2x^2+2x^4}}+\frac{\left(3+\sqrt{2}\right)\left(1+\sqrt{2}x^2\right)\sqrt{\frac{1+2x^2+2x^4}{\left(1+\sqrt{2}x^2\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\sqrt[4]{2}x\right),\frac{1}{4}(2-\sqrt{2})\right)}{21\sqrt[4]{2}\sqrt{1+2x^2+2x^4}}+\frac{5\left(3+\sqrt{2}\right)^2\left(1+\sqrt{2}x^2\right)\sqrt{\frac{1+2x^2+2x^4}{\left(1+\sqrt{2}x^2\right)^2}}\,\text{EllipticPi}\left(\frac{1}{24}(12-11\sqrt{2}),2\arctan\left(\sqrt[4]{2}x\right),\frac{1}{4}(2-\sqrt{2})\right)}{252\sqrt[4]{2}\sqrt{1+2x^2+2x^4}}$$

```
[Out] -1/18*arctan(1/3*x*15^(1/2)/(2*x^4+2*x^2+1)^(1/2))*15^(1/2)-1/3*(2*x^4+2*x^
2+1)^(1/2)/x+1/3*x*(2*x^4+2*x^2+1)^(1/2)*2^(1/2)/(1+x^2*2^(1/2))-1/3*(cos(2
*arctan(2^(1/4)*x))^2)^(1/2)/cos(2*arctan(2^(1/4)*x))*EllipticE(sin(2*arcta
n(2^(1/4)*x)),1/2*(2-2^(1/2))^(1/2))*(1+x^2*2^(1/2))*((2*x^4+2*x^2+1)/(1+x^
2*2^(1/2))^2)^(1/2)*2^(1/4)/(2*x^4+2*x^2+1)^(1/2)+1/42*(cos(2*arctan(2^(1/4
)*x))^2)^(1/2)/cos(2*arctan(2^(1/4)*x))*EllipticF(sin(2*arctan(2^(1/4)*x)),
1/2*(2-2^(1/2))^(1/2))*(3+2^(1/2))*(1+x^2*2^(1/2))*((2*x^4+2*x^2+1)/(1+x^2*
2^(1/2))^2)^(1/2)*2^(3/4)/(2*x^4+2*x^2+1)^(1/2)+5/504*(cos(2*arctan(2^(1/4)
*x))^2)^(1/2)/cos(2*arctan(2^(1/4)*x))*EllipticPi(sin(2*arctan(2^(1/4)*x)),
1/2-11/24*2^(1/2),1/2*(2-2^(1/2))^(1/2))*(3+2^(1/2))^2*(1+x^2*2^(1/2))*((2*
x^4+2*x^2+1)/(1+x^2*2^(1/2))^2)^(1/2)*2^(3/4)/(2*x^4+2*x^2+1)^(1/2)
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 399, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.241, Rules used = {1325, 1295, 1211, 1117, 1209, 1230, 1720}

$$\int \frac{\sqrt{1+2x^2+2x^4}}{x^2(3+2x^2)}\,dx = -\frac{1}{6}\sqrt{\frac{5}{3}}\arctan\left(\frac{\sqrt{\frac{5}{3}}x}{\sqrt{2x^4+2x^2+1}}\right) + \frac{\left(3+\sqrt{2}\right)\left(\sqrt{2}x^2+1\right)\sqrt{\frac{2x^4+2x^2+1}{\left(\sqrt{2}x^2+1\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\sqrt[4]{2}x\right),\frac{1}{4}\left(2-\sqrt{2}\right)\right)}{21\sqrt[4]{2}\sqrt{2x^4+2x^2+1}} - \frac{\sqrt[4]{2}\left(\sqrt{2}x^2+1\right)\sqrt{\frac{2x^4+2x^2+1}{\left(\sqrt{2}x^2+1\right)^2}}E\left(2\arctan\left(\sqrt[4]{2}x\right)|\frac{1}{4}\left(2-\sqrt{2}\right)\right)}{3\sqrt{2x^4+2x^2+1}} + \frac{5\left(3+\sqrt{2}\right)^2\left(\sqrt{2}x^2+1\right)\sqrt{\frac{2x^4+2x^2+1}{\left(\sqrt{2}x^2+1\right)^2}}\,\text{EllipticPi}\left(\frac{1}{24}\left(12-11\sqrt{2}\right),2\arctan\left(\sqrt[4]{2}x\right),\frac{1}{4}\left(2-\sqrt{2}\right)\right)}{252\sqrt[4]{2}\sqrt{2x^4+2x^2+1}} + \frac{\sqrt{2}\sqrt{2x^4+2x^2+1}x}{3\left(\sqrt{2}x^2+1\right)} - \frac{\sqrt{2x^4+2x^2+1}}{3x}$$

```
[In] Int[Sqrt[1 + 2*x^2 + 2*x^4]/(x^2*(3 + 2*x^2)),x]
```

```
[Out] -1/3*Sqrt[1 + 2*x^2 + 2*x^4]/x + (Sqrt[2]*x*Sqrt[1 + 2*x^2 + 2*x^4])/(3*(1
+ Sqrt[2]*x^2)) - (Sqrt[5/3]*ArcTan[(Sqrt[5/3]*x)/Sqrt[1 + 2*x^2 + 2*x^4]])
/6 - (2^(1/4)*(1 + Sqrt[2]*x^2)*Sqrt[(1 + 2*x^2 + 2*x^4)/(1 + Sqrt[2]*x^2)^
2]*EllipticE[2*ArcTan[2^(1/4)*x], (2 - Sqrt[2])/4])/(3*Sqrt[1 + 2*x^2 + 2*x
^4]) + ((3 + Sqrt[2])*(1 + Sqrt[2]*x^2)*Sqrt[(1 + 2*x^2 + 2*x^4)/(1 + Sqrt[
2]*x^2)^2]*EllipticF[2*ArcTan[2^(1/4)*x], (2 - Sqrt[2])/4])/(21*2^(1/4)*Sqr
t[1 + 2*x^2 + 2*x^4]) + (5*(3 + Sqrt[2])^2*(1 + Sqrt[2]*x^2)*Sqrt[(1 + 2*x^
2 + 2*x^4)/(1 + Sqrt[2]*x^2)^2]*EllipticPi[(12 - 11*Sqrt[2])/24, 2*ArcTan[2
^(1/4)*x], (2 - Sqrt[2])/4])/(252*2^(1/4)*Sqrt[1 + 2*x^2 + 2*x^4])
```

Rule 1117

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1209

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
```

```
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

Rule 1211

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + b*x^2 + c*x^4
], x], x] - Dist[e/q, Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /; Ne
Q[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[
c/a]
```

Rule 1230

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4]), x_S
ymbol] :> With[{q = Rt[c/a, 2]}, Dist[(c*d + a*e*q)/(c*d^2 - a*e^2), Int[1/
Sqrt[a + b*x^2 + c*x^4], x], x] - Dist[(a*e*(e + d*q))/(c*d^2 - a*e^2), Int
[(1 + q*x^2)/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]), x], x]] /; FreeQ[{a, b,
 c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[c/a]
```

Rule 1295

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[d*(f*x)^(m + 1)*((a + b*x^2 + c*x^4)^(p + 1)
/(a*f*(m + 1))), x] + Dist[1/(a*f^2*(m + 1)), Int[(f*x)^(m + 2)*(a + b*x^2
+ c*x^4)^p*Simp[a*e*(m + 1) - b*d*(m + 2*p + 3) - c*d*(m + 4*p + 5)*x^2, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[m
, -1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1325

```
Int[(((f_.)*(x_))^(m_)*((a_.) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.))/((d_.)
+ (e_.)*(x_)^2), x_Symbol] :> Dist[1/(d*e), Int[(f*x)^m*(a*e + c*d*x^2)*(a
+ b*x^2 + c*x^4)^(p - 1), x], x] - Dist[(c*d^2 - b*d*e + a*e^2)/(d*e*f^2),
Int[(f*x)^(m + 2)*((a + b*x^2 + c*x^4)^(p - 1)/(d + e*x^2)), x], x] /; Free
Q[{a, b, c, d, e, f}, x] && NeQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && LtQ[m, 0]
```

Rule 1720

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4]), x_Symbol] :> With[{q = Rt[B/A, 2]}, Simp[(-(B*d - A*e))*(A
rcTan[Rt[-b + c*(d/e) + a*(e/d), 2]*(x/Sqrt[a + b*x^2 + c*x^4])]/(2*d*e*Rt[
-b + c*(d/e) + a*(e/d), 2])), x] + Simp[(B*d + A*e)*(A + B*x^2)*(Sqrt[A^2*(
(a + b*x^2 + c*x^4)/(a*(A + B*x^2)^2))]/(4*d*e*A*q*Sqrt[a + b*x^2 + c*x^4])
)*EllipticPi[Cancel[-(B*d - A*e)^2/(4*d*e*A*B)], 2*ArcTan[q*x], 1/2 - b*(A/
```

```
(4*a*B))], x]] /; FreeQ[{a, b, c, d, e, A, B}, x] && NeQ[b^2 - 4*a*c, 0] &&
 NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] && EqQ
[c*A^2 - a*B^2, 0]
```

Rubi steps

$$\text{integral} = \frac{1}{6}\int \frac{2+6x^2}{x^2\sqrt{1+2x^2+2x^4}}\,dx - \frac{5}{3}\int \frac{1}{(3+2x^2)\sqrt{1+2x^2+2x^4}}\,dx$$

$$= -\frac{\sqrt{1+2x^2+2x^4}}{3x} - \frac{1}{6}\int \frac{-6-4x^2}{\sqrt{1+2x^2+2x^4}}\,dx - \frac{1}{21}\left(5\left(3+\sqrt{2}\right)\right)\int \frac{1}{\sqrt{1+2x^2+2x^4}}\,dx + \frac{1}{21}\left(5\left(2+3\sqrt{2}\right)\right)\int \frac{1+\sqrt{2}x^2}{(3+2x^2)\sqrt{1+2x^2+2x^4}}\,dx$$

$$= -\frac{\sqrt{1+2x^2+2x^4}}{3x} - \frac{1}{6}\sqrt{\frac{5}{3}}\tan^{-1}\left(\frac{\sqrt{\frac{5}{3}}x}{\sqrt{1+2x^2+2x^4}}\right) - \frac{5(3+\sqrt{2})\left(1+\sqrt{2}x^2\right)\sqrt{\frac{1+2x^2+2x^4}{\left(1+\sqrt{2}x^2\right)^2}}F\left(2\tan^{-1}\left(\sqrt[4]{2}x\right)\middle|\frac{1}{4}(2-\sqrt{2})\right)}{42\sqrt[4]{2}\sqrt{1+2x^2+2x^4}} + \frac{5(3+\sqrt{2})^2\left(1+\sqrt{2}x^2\right)\sqrt{\frac{1+2x^2+2x^4}{\left(1+\sqrt{2}x^2\right)^2}}\Pi\left(\frac{1}{24}(12-11\sqrt{2});2\tan^{-1}\left(\sqrt[4]{2}x\right)\middle|\frac{1}{4}(2-\sqrt{2})\right)}{252\sqrt[4]{2}\sqrt{1+2x^2+2x^4}} - \frac{1}{3}\sqrt{2}\int \frac{1-\sqrt{2}x^2}{\sqrt{1+2x^2+2x^4}}\,dx - \frac{1}{3}\left(-3-\sqrt{2}\right)\int \frac{1}{\sqrt{1+2x^2+2x^4}}\,dx$$

$$= -\frac{\sqrt{1+2x^2+2x^4}}{3x} + \frac{\sqrt{2}x\sqrt{1+2x^2+2x^4}}{3\left(1+\sqrt{2}x^2\right)} - \frac{1}{6}\sqrt{\frac{5}{3}}\tan^{-1}\left(\frac{\sqrt{\frac{5}{3}}x}{\sqrt{1+2x^2+2x^4}}\right) - \frac{\sqrt[4]{2}\left(1+\sqrt{2}x^2\right)\sqrt{\frac{1+2x^2+2x^4}{\left(1+\sqrt{2}x^2\right)^2}}E\left(2\tan^{-1}\left(\sqrt[4]{2}x\right)\middle|\frac{1}{4}(2-\sqrt{2})\right)}{3\sqrt{1+2x^2+2x^4}} + \frac{(3+\sqrt{2})\left(1+\sqrt{2}x^2\right)\sqrt{\frac{1+2x^2+2x^4}{\left(1+\sqrt{2}x^2\right)^2}}F\left(2\tan^{-1}\left(\sqrt[4]{2}x\right)\middle|\frac{1}{4}(2-\sqrt{2})\right)}{21\sqrt[4]{2}\sqrt{1+2x^2+2x^4}} + \frac{5(3+\sqrt{2})^2\left(1+\sqrt{2}x^2\right)\sqrt{\frac{1+2x^2+2x^4}{\left(1+\sqrt{2}x^2\right)^2}}\Pi\left(\frac{1}{24}(12-11\sqrt{2});2\tan^{-1}\left(\sqrt[4]{2}x\right)\middle|\frac{1}{4}(2-\sqrt{2})\right)}{252\sqrt[4]{2}\sqrt{1+2x^2+2x^4}}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.18 (sec) , antiderivative size = 208, normalized size of antiderivative = 0.52

$$\int \frac{\sqrt{1+2x^2+2x^4}}{x^2(3+2x^2)}\,dx$$
$$= \frac{-6-12x^2-12x^4-6i\sqrt{1-i}x\sqrt{1+(1-i)x^2}\sqrt{1+(1+i)x^2}E\left(i\operatorname{arcsinh}\left(\sqrt{1-i}x\right)\middle|i\right)+(9-3i)\sqrt{1-i}x}{}$$

```
[In] Integrate[Sqrt[1 + 2*x^2 + 2*x^4]/(x^2*(3 + 2*x^2)),x]
```

```
[Out] (-6 - 12*x^2 - 12*x^4 - (6*I)*Sqrt[1 - I]*x*Sqrt[1 + (1 - I)*x^2]*Sqrt[1 +
(1 + I)*x^2]*EllipticE[I*ArcSinh[Sqrt[1 - I]*x], I] + (9 - 3*I)*Sqrt[1 - I]
*x*Sqrt[1 + (1 - I)*x^2]*Sqrt[1 + (1 + I)*x^2]*EllipticF[I*ArcSinh[Sqrt[1 -
 I]*x], I] - 5*(1 - I)^(3/2)*x*Sqrt[1 + (1 - I)*x^2]*Sqrt[1 + (1 + I)*x^2]*
EllipticPi[1/3 + I/3, I*ArcSinh[Sqrt[1 - I]*x], I])/(18*x*Sqrt[1 + 2*x^2 +
2*x^4])
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.90 (sec) , antiderivative size = 240, normalized size of antiderivative = 0.60

method	result
risch	$-\frac{\sqrt{2x^4+2x^2+1}}{3x}+\frac{\sqrt{1+(1-i)x^2}\,\sqrt{1+(1+i)x^2}\,F\left(x\sqrt{-1+i},\frac{\sqrt{2}}{2}+\frac{i\sqrt{2}}{2}\right)}{\sqrt{-1+i}\,\sqrt{2x^4+2x^2+1}}+\frac{\left(-\frac{1}{3}+\frac{i}{3}\right)\sqrt{1+(1-i)x^2}\,\sqrt{1+(1+i)x^2}\,\left(F\left(x\sqrt{-1+i},\frac{\sqrt{2}}{2}+\right.\right.}{\sqrt{-1+i}\,\sqrt{2x^4+2x^2+1}}$
elliptic	$-\frac{\sqrt{2x^4+2x^2+1}}{3x}+\frac{2\sqrt{-ix^2+x^2+1}\,\sqrt{ix^2+x^2+1}\,F\left(x\sqrt{-1+i},\frac{\sqrt{2}}{2}+\frac{i\sqrt{2}}{2}\right)}{3\sqrt{-1+i}\,\sqrt{2x^4+2x^2+1}}+\frac{i\sqrt{-ix^2+x^2+1}\,\sqrt{ix^2+x^2+1}\,F\left(x\sqrt{-1+i},\frac{\sqrt{2}}{2}+\frac{i\sqrt{2}}{2}\right)}{3\sqrt{-1+i}\,\sqrt{2x^4+2x^2+1}}+$
default	$-\frac{\sqrt{2x^4+2x^2+1}}{3x}+\frac{2\sqrt{1+(1-i)x^2}\,\sqrt{1+(1+i)x^2}\,F\left(x\sqrt{-1+i},\frac{\sqrt{2}}{2}+\frac{i\sqrt{2}}{2}\right)}{3\sqrt{-1+i}\,\sqrt{2x^4+2x^2+1}}+\frac{\left(-\frac{2}{3}+\frac{2i}{3}\right)\sqrt{1+(1-i)x^2}\,\sqrt{1+(1+i)x^2}\,\left(F\left(x\sqrt{-1+i},\frac{\sqrt{2}}{2}\right.\right.}{\sqrt{-1+i}\,\sqrt{2x^4+2x^2+1}}$

```
[In] int((2*x^4+2*x^2+1)^(1/2)/x^2/(2*x^2+3),x,method=_RETURNVERBOSE)
```

```
[Out] -1/3*(2*x^4+2*x^2+1)^(1/2)/x+1/(-1+I)^(1/2)*(1+(1-I)*x^2)^(1/2)*(1+(1+I)*x^
2)^(1/2)/(2*x^4+2*x^2+1)^(1/2)*EllipticF(x*(-1+I)^(1/2),1/2*2^(1/2)+1/2*I*2
^(1/2))+(-1/3+1/3*I)/(-1+I)^(1/2)*(1+(1-I)*x^2)^(1/2)*(1+(1+I)*x^2)^(1/2)/(
2*x^4+2*x^2+1)^(1/2)*(EllipticF(x*(-1+I)^(1/2),1/2*2^(1/2)+1/2*I*2^(1/2))-E
llipticE(x*(-1+I)^(1/2),1/2*2^(1/2)+1/2*I*2^(1/2)))-5/9/(-1+I)^(1/2)*(1-I*x
^2+x^2)^(1/2)*(1+I*x^2+x^2)^(1/2)/(2*x^4+2*x^2+1)^(1/2)*EllipticPi(x*(-1+I)
^(1/2),1/3+1/3*I,(-1-I)^(1/2)/(-1+I)^(1/2))
```

Fricas [F]

$$\int \frac{\sqrt{1+2x^2+2x^4}}{x^2(3+2x^2)}\,dx=\int \frac{\sqrt{2\,x^4+2\,x^2+1}}{(2\,x^2+3)x^2}\,dx$$

```
[In] integrate((2*x^4+2*x^2+1)^(1/2)/x^2/(2*x^2+3),x, algorithm="fricas")
[Out] integral(sqrt(2*x^4 + 2*x^2 + 1)/(2*x^4 + 3*x^2), x)
```

Sympy [F]

$$\int \frac{\sqrt{1+2x^2+2x^4}}{x^2(3+2x^2)}\,dx=\int \frac{\sqrt{2x^4+2x^2+1}}{x^2\cdot(2x^2+3)}\,dx$$

```
[In] integrate((2*x**4+2*x**2+1)**(1/2)/x**2/(2*x**2+3),x)
[Out] Integral(sqrt(2*x**4 + 2*x**2 + 1)/(x**2*(2*x**2 + 3)), x)
```

Maxima [F]

$$\int \frac{\sqrt{1+2x^2+2x^4}}{x^2(3+2x^2)}\,dx=\int \frac{\sqrt{2\,x^4+2\,x^2+1}}{(2\,x^2+3)x^2}\,dx$$

```
[In] integrate((2*x^4+2*x^2+1)^(1/2)/x^2/(2*x^2+3),x, algorithm="maxima")
[Out] integrate(sqrt(2*x^4 + 2*x^2 + 1)/((2*x^2 + 3)*x^2), x)
```

Giac [F]

$$\int \frac{\sqrt{1+2x^2+2x^4}}{x^2(3+2x^2)}\,dx=\int \frac{\sqrt{2\,x^4+2\,x^2+1}}{(2\,x^2+3)x^2}\,dx$$

```
[In] integrate((2*x^4+2*x^2+1)^(1/2)/x^2/(2*x^2+3),x, algorithm="giac")
[Out] integrate(sqrt(2*x^4 + 2*x^2 + 1)/((2*x^2 + 3)*x^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{1+2x^2+2x^4}}{x^2\,(3+2x^2)}\,dx = \int \frac{\sqrt{2\,x^4+2\,x^2+1}}{x^2\,\left(2\,x^2+3\right)}\,dx$$

```
[In] int((2*x^2 + 2*x^4 + 1)^(1/2)/(x^2*(2*x^2 + 3)),x)
[Out] int((2*x^2 + 2*x^4 + 1)^(1/2)/(x^2*(2*x^2 + 3)), x)
```

3.320 $\int \frac{\sqrt{1+2x^2+2x^4}}{x^4(3+2x^2)}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 360

$$\begin{aligned}\int \frac{\sqrt{1+2x^2+2x^4}}{x^4(3+2x^2)}\,dx = & -\frac{\sqrt{1+2x^2+2x^4}}{9x^3}+\frac{1}{9}\sqrt{\frac{5}{3}}\arctan\left(\frac{\sqrt{\frac{5}{3}}x}{\sqrt{1+2x^2+2x^4}}\right)\\ & -\frac{\left(1+\sqrt{2}x^2\right)\sqrt{\frac{1+2x^2+2x^4}{\left(1+\sqrt{2}x^2\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\sqrt[4]{2}x\right),\frac{1}{4}\left(2-\sqrt{2}\right)\right)}{9\sqrt[4]{2}\sqrt{1+2x^2+2x^4}}\\ & +\frac{5\left(3+\sqrt{2}\right)\left(1+\sqrt{2}x^2\right)\sqrt{\frac{1+2x^2+2x^4}{\left(1+\sqrt{2}x^2\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\sqrt[4]{2}x\right),\frac{1}{4}\left(2-\sqrt{2}\right)\right)}{63\sqrt[4]{2}\sqrt{1+2x^2+2x^4}}\\ & -\frac{5\left(3+\sqrt{2}\right)^2\left(1+\sqrt{2}x^2\right)\sqrt{\frac{1+2x^2+2x^4}{\left(1+\sqrt{2}x^2\right)^2}}\,\text{EllipticPi}\left(\frac{1}{24}\left(12-11\sqrt{2}\right),2\arctan\left(\sqrt[4]{2}x\right),\frac{1}{4}\left(2-\sqrt{2}\right)\right)}{378\sqrt[4]{2}\sqrt{1+2x^2+2x^4}}\end{aligned}$$

```
[Out] 1/27*arctan(1/3*x*15^(1/2)/(2*x^4+2*x^2+1)^(1/2))*15^(1/2)-1/9*(2*x^4+2*x^2
+1)^(1/2)/x^3-1/18*(cos(2*arctan(2^(1/4)*x))^2)^(1/2)/cos(2*arctan(2^(1/4)*
x))*EllipticF(sin(2*arctan(2^(1/4)*x)),1/2*(2-2^(1/2))^(1/2))*(1+x^2*2^(1/2
))*((2*x^4+2*x^2+1)/(1+x^2*2^(1/2))^2)^(1/2)*2^(3/4)/(2*x^4+2*x^2+1)^(1/2)+
5/126*(cos(2*arctan(2^(1/4)*x))^2)^(1/2)/cos(2*arctan(2^(1/4)*x))*EllipticF
(sin(2*arctan(2^(1/4)*x)),1/2*(2-2^(1/2))^(1/2))*(3+2^(1/2))*(1+x^2*2^(1/2)
)*((2*x^4+2*x^2+1)/(1+x^2*2^(1/2))^2)^(1/2)*2^(3/4)/(2*x^4+2*x^2+1)^(1/2)-5
/756*(cos(2*arctan(2^(1/4)*x))^2)^(1/2)/cos(2*arctan(2^(1/4)*x))*EllipticPi
(sin(2*arctan(2^(1/4)*x)),1/2-11/24*2^(1/2),1/2*(2-2^(1/2))^(1/2))*(3+2^(1/
2))^2*(1+x^2*2^(1/2))*((2*x^4+2*x^2+1)/(1+x^2*2^(1/2))^2)^(1/2)*2^(3/4)/(2*
x^4+2*x^2+1)^(1/2)
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 360, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.207, Rules used = {1323, 1295, 12, 1117, 1230, 1720}

$$\int \frac{\sqrt{1+2x^2+2x^4}}{x^4(3+2x^2)}\,dx = \frac{1}{9}\sqrt{\frac{5}{3}}\arctan\left(\frac{\sqrt{\frac{5}{3}}x}{\sqrt{2x^4+2x^2+1}}\right) + \frac{5(3+\sqrt{2})(\sqrt{2}x^2+1)\sqrt{\frac{2x^4+2x^2+1}{(\sqrt{2}x^2+1)^2}}\,\text{EllipticF}\left(2\arctan\left(\sqrt[4]{2}x\right),\frac{1}{4}(2-\sqrt{2})\right)}{63\sqrt[4]{2}\sqrt{2x^4+2x^2+1}} - \frac{(\sqrt{2}x^2+1)\sqrt{\frac{2x^4+2x^2+1}{(\sqrt{2}x^2+1)^2}}\,\text{EllipticF}\left(2\arctan\left(\sqrt[4]{2}x\right),\frac{1}{4}(2-\sqrt{2})\right)}{9\sqrt[4]{2}\sqrt{2x^4+2x^2+1}} - \frac{5(3+\sqrt{2})^2(\sqrt{2}x^2+1)\sqrt{\frac{2x^4+2x^2+1}{(\sqrt{2}x^2+1)^2}}\,\text{EllipticPi}\left(\frac{1}{24}(12-11\sqrt{2}),2\arctan\left(\sqrt[4]{2}x\right),\frac{1}{4}(2-\sqrt{2})\right)}{378\sqrt[4]{2}\sqrt{2x^4+2x^2+1}} - \frac{\sqrt{2x^4+2x^2+1}}{9x^3}$$

```
[In] Int[Sqrt[1 + 2*x^2 + 2*x^4]/(x^4*(3 + 2*x^2)),x]
```

```
[Out] -1/9*Sqrt[1 + 2*x^2 + 2*x^4]/x^3 + (Sqrt[5/3]*ArcTan[(Sqrt[5/3]*x)/Sqrt[1 +
 2*x^2 + 2*x^4]])/9 - ((1 + Sqrt[2]*x^2)*Sqrt[(1 + 2*x^2 + 2*x^4)/(1 + Sqrt
[2]*x^2)^2]*EllipticF[2*ArcTan[2^(1/4)*x], (2 - Sqrt[2])/4])/(9*2^(1/4)*Sqr
t[1 + 2*x^2 + 2*x^4]) + (5*(3 + Sqrt[2])*(1 + Sqrt[2]*x^2)*Sqrt[(1 + 2*x^2
+ 2*x^4)/(1 + Sqrt[2]*x^2)^2]*EllipticF[2*ArcTan[2^(1/4)*x], (2 - Sqrt[2])/
4])/(63*2^(1/4)*Sqrt[1 + 2*x^2 + 2*x^4]) - (5*(3 + Sqrt[2])^2*(1 + Sqrt[2]*
x^2)*Sqrt[(1 + 2*x^2 + 2*x^4)/(1 + Sqrt[2]*x^2)^2]*EllipticPi[(12 - 11*Sqrt
[2])/24, 2*ArcTan[2^(1/4)*x], (2 - Sqrt[2])/4])/(378*2^(1/4)*Sqrt[1 + 2*x^2
 + 2*x^4])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 1117

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1230

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4]), x_S
ymbol] :> With[{q = Rt[c/a, 2]}, Dist[(c*d + a*e*q)/(c*d^2 - a*e^2), Int[1/
Sqrt[a + b*x^2 + c*x^4], x], x] - Dist[(a*e*(e + d*q))/(c*d^2 - a*e^2), Int
[(1 + q*x^2)/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]), x], x]] /; FreeQ[{a, b,
 c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[c/a]
```

Rule 1295

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[d*(f*x)^(m + 1)*((a + b*x^2 + c*x^4)^(p + 1)
/(a*f*(m + 1))), x] + Dist[1/(a*f^2*(m + 1)), Int[(f*x)^(m + 2)*(a + b*x^2
+ c*x^4)^p*Simp[a*e*(m + 1) - b*d*(m + 2*p + 3) - c*d*(m + 4*p + 5)*x^2, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[m
, -1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1323

```
Int[(((f_.)*(x_))^(m_)*((a_.) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.))/((d_.)
+ (e_.)*(x_)^2), x_Symbol] :> Dist[1/d^2, Int[(f*x)^m*(a*d + (b*d - a*e)*x^
2)*(a + b*x^2 + c*x^4)^(p - 1), x], x] + Dist[(c*d^2 - b*d*e + a*e^2)/(d^2*
f^4), Int[(f*x)^(m + 4)*((a + b*x^2 + c*x^4)^(p - 1)/(d + e*x^2)), x], x] /
; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && LtQ[m
, -2]
```

Rule 1720

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4]), x_Symbol] :> With[{q = Rt[B/A, 2]}, Simp[(-(B*d - A*e))*(A
rcTan[Rt[-b + c*(d/e) + a*(e/d), 2]*(x/Sqrt[a + b*x^2 + c*x^4])]/(2*d*e*Rt[
-b + c*(d/e) + a*(e/d), 2])), x] + Simp[(B*d + A*e)*(A + B*x^2)*(Sqrt[A^2*(
(a + b*x^2 + c*x^4)/(a*(A + B*x^2)^2))]/(4*d*e*A*q*Sqrt[a + b*x^2 + c*x^4])
)*EllipticPi[Cancel[-(B*d - A*e)^2/(4*d*e*A*B)], 2*ArcTan[q*x], 1/2 - b*(A/
(4*a*B))], x]] /; FreeQ[{a, b, c, d, e, A, B}, x] && NeQ[b^2 - 4*a*c, 0] &&
 NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] && EqQ
[c*A^2 - a*B^2, 0]
```

Rubi steps

$$\text{integral} = \frac{1}{9}\int \frac{3+4x^2}{x^4\sqrt{1+2x^2+2x^4}}\,dx + \frac{10}{9}\int \frac{1}{(3+2x^2)\sqrt{1+2x^2+2x^4}}\,dx$$

$$= -\frac{\sqrt{1+2x^2+2x^4}}{9x^3} - \frac{1}{27}\int \frac{6}{\sqrt{1+2x^2+2x^4}}\,dx + \frac{1}{63}\left(10\left(3+\sqrt{2}\right)\right)\int \frac{1}{\sqrt{1+2x^2+2x^4}}\,dx - \frac{1}{63}\left(10\left(2+3\sqrt{2}\right)\right)\int \frac{1+\sqrt{2}x^2}{(3+2x^2)\sqrt{1+2x^2+2x^4}}\,dx$$

$$= -\frac{\sqrt{1+2x^2+2x^4}}{9x^3} + \frac{1}{9}\sqrt{\frac{5}{3}}\tan^{-1}\left(\frac{\sqrt{\frac{5}{3}}x}{\sqrt{1+2x^2+2x^4}}\right) + \frac{5\left(3+\sqrt{2}\right)\left(1+\sqrt{2}x^2\right)\sqrt{\frac{1+2x^2+2x^4}{\left(1+\sqrt{2}x^2\right)^2}}F\left(2\tan^{-1}\left(\sqrt[4]{2}x\right)|\frac{1}{4}\left(2-\sqrt{2}\right)\right)}{63\sqrt[4]{2}\sqrt{1+2x^2+2x^4}} - \frac{5\left(3+\sqrt{2}\right)^2\left(1+\sqrt{2}x^2\right)\sqrt{\frac{1+2x^2+2x^4}{\left(1+\sqrt{2}x^2\right)^2}}\Pi\left(\frac{1}{24}\left(12-11\sqrt{2}\right);2\tan^{-1}\left(\sqrt[4]{2}x\right)|\frac{1}{4}\left(2-\sqrt{2}\right)\right)}{378\sqrt[4]{2}\sqrt{1+2x^2+2x^4}} - \frac{2}{9}\int \frac{1}{\sqrt{1+2x^2+2x^4}}\,dx$$

$$= -\frac{\sqrt{1+2x^2+2x^4}}{9x^3} + \frac{1}{9}\sqrt{\frac{5}{3}}\tan^{-1}\left(\frac{\sqrt{\frac{5}{3}}x}{\sqrt{1+2x^2+2x^4}}\right) - \frac{\left(1+\sqrt{2}x^2\right)\sqrt{\frac{1+2x^2+2x^4}{\left(1+\sqrt{2}x^2\right)^2}}F\left(2\tan^{-1}\left(\sqrt[4]{2}x\right)|\frac{1}{4}\left(2-\sqrt{2}\right)\right)}{9\sqrt[4]{2}\sqrt{1+2x^2+2x^4}} + \frac{5\left(3+\sqrt{2}\right)\left(1+\sqrt{2}x^2\right)\sqrt{\frac{1+2x^2+2x^4}{\left(1+\sqrt{2}x^2\right)^2}}F\left(2\tan^{-1}\left(\sqrt[4]{2}x\right)|\frac{1}{4}\left(2-\sqrt{2}\right)\right)}{63\sqrt[4]{2}\sqrt{1+2x^2+2x^4}} - \frac{5\left(3+\sqrt{2}\right)^2\left(1+\sqrt{2}x^2\right)\sqrt{\frac{1+2x^2+2x^4}{\left(1+\sqrt{2}x^2\right)^2}}\Pi\left(\frac{1}{24}\left(12-11\sqrt{2}\right);2\tan^{-1}\left(\sqrt[4]{2}x\right)|\frac{1}{4}\left(2-\sqrt{2}\right)\right)}{378\sqrt[4]{2}\sqrt{1+2x^2+2x^4}}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.17 (sec) , antiderivative size = 154, normalized size of antiderivative = 0.43

$$\int \frac{\sqrt{1+2x^2+2x^4}}{x^4\left(3+2x^2\right)}\,dx = -\frac{3+6x^2+6x^4+3(1-i)^{3/2}x^3\sqrt{1+(1-i)x^2}\sqrt{1+(1+i)x^2}\,\text{EllipticF}\left(i\,\text{arcsinh}\left(\sqrt{1-i}x\right),i\right)-5(1-i}{27x^3\sqrt{1+2x^2+2x^4}}$$

```
[In] Integrate[Sqrt[1 + 2*x^2 + 2*x^4]/(x^4*(3 + 2*x^2)),x]
```

```
[Out] -1/27*(3 + 6*x^2 + 6*x^4 + 3*(1 - I)^(3/2)*x^3*Sqrt[1 + (1 - I)*x^2]*Sqrt[1
 + (1 + I)*x^2]*EllipticF[I*ArcSinh[Sqrt[1 - I]*x], I] - 5*(1 - I)^(3/2)*x^
3*Sqrt[1 + (1 - I)*x^2]*Sqrt[1 + (1 + I)*x^2]*EllipticPi[1/3 + I/3, I*ArcSi
nh[Sqrt[1 - I]*x], I])/(x^3*Sqrt[1 + 2*x^2 + 2*x^4])
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 3.19 (sec) , antiderivative size = 153, normalized size of antiderivative = 0.42

method	result
risch	$-\frac{\sqrt{2x^4+2x^2+1}}{9x^3}-\frac{2\sqrt{1+(1-i)x^2}\,\sqrt{1+(1+i)x^2}\,F\left(x\sqrt{-1+i},\frac{\sqrt{2}}{2}+\frac{i\sqrt{2}}{2}\right)}{9\sqrt{-1+i}\,\sqrt{2x^4+2x^2+1}}+\frac{10\sqrt{-ix^2+x^2+1}\,\sqrt{ix^2+x^2+1}\,\Pi\left(x\sqrt{-1+i},\frac{1}{3}+\frac{i}{3},\frac{\sqrt{-1\cdots}}{\sqrt{-1\cdots}}\cdots\right.}{27\sqrt{-1+i}\,\sqrt{2x^4+2x^2+1}}\cdots$
elliptic	$-\frac{\sqrt{2x^4+2x^2+1}}{9x^3}-\frac{2\sqrt{-ix^2+x^2+1}\,\sqrt{ix^2+x^2+1}\,F\left(x\sqrt{-1+i},\frac{\sqrt{2}}{2}+\frac{i\sqrt{2}}{2}\right)}{9\sqrt{-1+i}\,\sqrt{2x^4+2x^2+1}}+\frac{10\sqrt{-ix^2+x^2+1}\,\sqrt{ix^2+x^2+1}\,\Pi\left(x\sqrt{-1+i},\frac{1}{3}+\frac{i}{3},\frac{\sqrt{-1\cdots}}{\sqrt{-1\cdots}}\cdots\right.}{27\sqrt{-1+i}\,\sqrt{2x^4+2x^2+1}}\cdots$
default	$\frac{\left(\frac{2}{9}-\frac{2i}{9}\right)\sqrt{1+(1-i)x^2}\,\sqrt{1+(1+i)x^2}\,\left(F\left(x\sqrt{-1+i},\frac{\sqrt{2}}{2}+\frac{i\sqrt{2}}{2}\right)-E\left(x\sqrt{-1+i},\frac{\sqrt{2}}{2}+\frac{i\sqrt{2}}{2}\right)\right)}{\sqrt{-1+i}\,\sqrt{2x^4+2x^2+1}}-\frac{\sqrt{2x^4+2x^2+1}}{9x^3}-\frac{4\sqrt{-ix^2+x^2+1}\,\sqrt{ix^2\cdots}}{9\sqrt{-1\cdots}}\cdots$

```
[In] int((2*x^4+2*x^2+1)^(1/2)/x^4/(2*x^2+3),x,method=_RETURNVERBOSE)
```

```
[Out] -1/9*(2*x^4+2*x^2+1)^(1/2)/x^3-2/9/(-1+I)^(1/2)*(1+(1-I)*x^2)^(1/2)*(1+(1+I
)*x^2)^(1/2)/(2*x^4+2*x^2+1)^(1/2)*EllipticF(x*(-1+I)^(1/2),1/2*2^(1/2)+1/2
*I*2^(1/2))+10/27/(-1+I)^(1/2)*(1-I*x^2+x^2)^(1/2)*(1+I*x^2+x^2)^(1/2)/(2*x
^4+2*x^2+1)^(1/2)*EllipticPi(x*(-1+I)^(1/2),1/3+1/3*I,(-1-I)^(1/2)/(-1+I)^(
1/2))
```

Fricas [F]

$$\int \frac{\sqrt{1+2x^2+2x^4}}{x^4\,(3+2x^2)}\,dx=\int \frac{\sqrt{2\,x^4+2\,x^2+1}}{(2\,x^2+3)x^4}\,dx$$

```
[In] integrate((2*x^4+2*x^2+1)^(1/2)/x^4/(2*x^2+3),x, algorithm="fricas")
```

```
[Out] integral(sqrt(2*x^4 + 2*x^2 + 1)/(2*x^6 + 3*x^4), x)
```

Sympy [F]

$$\int \frac{\sqrt{1+2x^2+2x^4}}{x^4\,(3+2x^2)}\,dx = \int \frac{\sqrt{2x^4+2x^2+1}}{x^4\cdot(2x^2+3)}\,dx$$

```
[In] integrate((2*x**4+2*x**2+1)**(1/2)/x**4/(2*x**2+3),x)
[Out] Integral(sqrt(2*x**4 + 2*x**2 + 1)/(x**4*(2*x**2 + 3)), x)
```

Maxima [F]

$$\int \frac{\sqrt{1+2x^2+2x^4}}{x^4\,(3+2x^2)}\,dx = \int \frac{\sqrt{2\,x^4+2\,x^2+1}}{(2\,x^2+3)x^4}\,dx$$

```
[In] integrate((2*x^4+2*x^2+1)^(1/2)/x^4/(2*x^2+3),x, algorithm="maxima")
[Out] integrate(sqrt(2*x^4 + 2*x^2 + 1)/((2*x^2 + 3)*x^4), x)
```

Giac [F]

$$\int \frac{\sqrt{1+2x^2+2x^4}}{x^4\,(3+2x^2)}\,dx = \int \frac{\sqrt{2\,x^4+2\,x^2+1}}{(2\,x^2+3)x^4}\,dx$$

```
[In] integrate((2*x^4+2*x^2+1)^(1/2)/x^4/(2*x^2+3),x, algorithm="giac")
[Out] integrate(sqrt(2*x^4 + 2*x^2 + 1)/((2*x^2 + 3)*x^4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{1+2x^2+2x^4}}{x^4\,(3+2x^2)}\,dx = \int \frac{\sqrt{2\,x^4+2\,x^2+1}}{x^4\,\left(2\,x^2+3\right)}\,dx$$

```
[In] int((2*x^2 + 2*x^4 + 1)^(1/2)/(x^4*(2*x^2 + 3)),x)
[Out] int((2*x^2 + 2*x^4 + 1)^(1/2)/(x^4*(2*x^2 + 3)), x)
```

3.321 $\int \frac{\sqrt{1+2x^2+2x^4}}{x^6(3+2x^2)}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 546

$$\begin{aligned}\int \frac{\sqrt{1+2x^2+2x^4}}{x^6(3+2x^2)}\,dx &= -\frac{\sqrt{1+2x^2+2x^4}}{15x^5}+\frac{4\sqrt{1+2x^2+2x^4}}{135x^3}-\frac{4\sqrt{1+2x^2+2x^4}}{45x}\\
&+\frac{4\sqrt{2}x\sqrt{1+2x^2+2x^4}}{45\left(1+\sqrt{2}x^2\right)}-\frac{2}{27}\sqrt{\frac{5}{3}}\arctan\left(\frac{\sqrt{\frac{5}{3}}x}{\sqrt{1+2x^2+2x^4}}\right)\\
&-\frac{4\sqrt[4]{2}\left(1+\sqrt{2}x^2\right)\sqrt{\frac{1+2x^2+2x^4}{\left(1+\sqrt{2}x^2\right)^2}}E\left(2\arctan\left(\sqrt[4]{2}x\right)|\frac{1}{4}\left(2-\sqrt{2}\right)\right)}{45\sqrt{1+2x^2+2x^4}}\\
&+\frac{5\sqrt[4]{2}\left(5-3\sqrt{2}\right)\left(1+\sqrt{2}x^2\right)\sqrt{\frac{1+2x^2+2x^4}{\left(1+\sqrt{2}x^2\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\sqrt[4]{2}x\right),\frac{1}{4}\left(2-\sqrt{2}\right)\right)}{189\sqrt{1+2x^2+2x^4}}\\
&-\frac{\sqrt[4]{2}\left(19-2\sqrt{2}\right)\left(1+\sqrt{2}x^2\right)\sqrt{\frac{1+2x^2+2x^4}{\left(1+\sqrt{2}x^2\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\sqrt[4]{2}x\right),\frac{1}{4}\left(2-\sqrt{2}\right)\right)}{135\sqrt{1+2x^2+2x^4}}\\
&+\frac{5\left(3+\sqrt{2}\right)^2\left(1+\sqrt{2}x^2\right)\sqrt{\frac{1+2x^2+2x^4}{\left(1+\sqrt{2}x^2\right)^2}}\operatorname{EllipticPi}\left(\frac{1}{24}\left(12-11\sqrt{2}\right),2\arctan\left(\sqrt[4]{2}x\right),\frac{1}{4}\left(2-\sqrt{2}\right)\right)}{567\sqrt[4]{2}\sqrt{1+2x^2+2x^4}}\end{aligned}$$

```
[Out] -2/81*arctan(1/3*x*15^(1/2)/(2*x^4+2*x^2+1)^(1/2))*15^(1/2)-1/15*(2*x^4+2*x
^2+1)^(1/2)/x^5+4/135*(2*x^4+2*x^2+1)^(1/2)/x^3-4/45*(2*x^4+2*x^2+1)^(1/2)/
x+4/45*x*(2*x^4+2*x^2+1)^(1/2)*2^(1/2)/(1+x^2*2^(1/2))-4/45*(cos(2*arctan(2
^(1/4)*x))^2)^(1/2)/cos(2*arctan(2^(1/4)*x))*EllipticE(sin(2*arctan(2^(1/4)
*x)),1/2*(2-2^(1/2))^(1/2))*(1+x^2*2^(1/2))*((2*x^4+2*x^2+1)/(1+x^2*2^(1/2)
)^2)^(1/2)*2^(1/4)/(2*x^4+2*x^2+1)^(1/2)+5/189*2^(1/4)*(cos(2*arctan(2^(1/4
)*x))^2)^(1/2)/cos(2*arctan(2^(1/4)*x))*EllipticF(sin(2*arctan(2^(1/4)*x)),
```

```
1/2*(2-2^(1/2))^(1/2))*(5-3*2^(1/2))*(1+x^2*2^(1/2))*((2*x^4+2*x^2+1)/(1+x^
2*2^(1/2))^2)^(1/2)/(2*x^4+2*x^2+1)^(1/2)-1/135*2^(1/4)*(cos(2*arctan(2^(1/
4)*x))^2)^(1/2)/cos(2*arctan(2^(1/4)*x))*EllipticF(sin(2*arctan(2^(1/4)*x))
,1/2*(2-2^(1/2))^(1/2))*(19-2*2^(1/2))*(1+x^2*2^(1/2))*((2*x^4+2*x^2+1)/(1+
x^2*2^(1/2))^2)^(1/2)/(2*x^4+2*x^2+1)^(1/2)+5/1134*(cos(2*arctan(2^(1/4)*x)
)^2)^(1/2)/cos(2*arctan(2^(1/4)*x))*EllipticPi(sin(2*arctan(2^(1/4)*x)),1/2
-11/24*2^(1/2),1/2*(2-2^(1/2))^(1/2))*(3+2^(1/2))^2*(1+x^2*2^(1/2))*((2*x^4
+2*x^2+1)/(1+x^2*2^(1/2))^2)^(1/2)*2^(3/4)/(2*x^4+2*x^2+1)^(1/2)
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 546, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.310, Rules used = {1323, 1295, 1211, 1117, 1209, 1343, 1728, 1722, 1720}

$$\begin{aligned}\int \frac{\sqrt{1+2x^2+2x^4}}{x^6(3+2x^2)}\,dx &= -\frac{2}{27}\sqrt{\frac{5}{3}}\arctan\left(\frac{\sqrt{\frac{5}{3}}x}{\sqrt{2x^4+2x^2+1}}\right)\\
&-\frac{\sqrt[4]{2}\left(19-2\sqrt{2}\right)\left(\sqrt{2}x^2+1\right)\sqrt{\frac{2x^4+2x^2+1}{\left(\sqrt{2}x^2+1\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\sqrt[4]{2}x\right),\frac{1}{4}\left(2-\sqrt{2}\right)\right)}{135\sqrt{2x^4+2x^2+1}}\\
&+\frac{5\sqrt[4]{2}\left(5-3\sqrt{2}\right)\left(\sqrt{2}x^2+1\right)\sqrt{\frac{2x^4+2x^2+1}{\left(\sqrt{2}x^2+1\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\sqrt[4]{2}x\right),\frac{1}{4}\left(2-\sqrt{2}\right)\right)}{189\sqrt{2x^4+2x^2+1}}\\
&-\frac{4\sqrt[4]{2}\left(\sqrt{2}x^2+1\right)\sqrt{\frac{2x^4+2x^2+1}{\left(\sqrt{2}x^2+1\right)^2}}E\left(2\arctan\left(\sqrt[4]{2}x\right)|\frac{1}{4}\left(2-\sqrt{2}\right)\right)}{45\sqrt{2x^4+2x^2+1}}\\
&+\frac{5\left(3+\sqrt{2}\right)^2\left(\sqrt{2}x^2+1\right)\sqrt{\frac{2x^4+2x^2+1}{\left(\sqrt{2}x^2+1\right)^2}}\,\text{EllipticPi}\left(\frac{1}{24}\left(12-11\sqrt{2}\right),2\arctan\left(\sqrt[4]{2}x\right),\frac{1}{4}\left(2-\sqrt{2}\right)\right)}{567\sqrt[4]{2}\sqrt{2x^4+2x^2+1}}\\
&+\frac{4\sqrt{2}\sqrt{2x^4+2x^2+1}x}{45\left(\sqrt{2}x^2+1\right)}-\frac{4\sqrt{2x^4+2x^2+1}}{45x}-\frac{\sqrt{2x^4+2x^2+1}}{15x^5}+\frac{4\sqrt{2x^4+2x^2+1}}{135x^3}\end{aligned}$$

```
[In] Int[Sqrt[1 + 2*x^2 + 2*x^4]/(x^6*(3 + 2*x^2)),x]
```

```
[Out] -1/15*Sqrt[1 + 2*x^2 + 2*x^4]/x^5 + (4*Sqrt[1 + 2*x^2 + 2*x^4])/(135*x^3) -
 (4*Sqrt[1 + 2*x^2 + 2*x^4])/(45*x) + (4*Sqrt[2]*x*Sqrt[1 + 2*x^2 + 2*x^4])
/(45*(1 + Sqrt[2]*x^2)) - (2*Sqrt[5/3]*ArcTan[(Sqrt[5/3]*x)/Sqrt[1 + 2*x^2
+ 2*x^4]])/27 - (4*2^(1/4)*(1 + Sqrt[2]*x^2)*Sqrt[(1 + 2*x^2 + 2*x^4)/(1 +
Sqrt[2]*x^2)^2]*EllipticE[2*ArcTan[2^(1/4)*x], (2 - Sqrt[2])/4])/(45*Sqrt[1
 + 2*x^2 + 2*x^4]) + (5*2^(1/4)*(5 - 3*Sqrt[2])*(1 + Sqrt[2]*x^2)*Sqrt[(1 +
 2*x^2 + 2*x^4)/(1 + Sqrt[2]*x^2)^2]*EllipticF[2*ArcTan[2^(1/4)*x], (2 - Sq
rt[2])/4])/(189*Sqrt[1 + 2*x^2 + 2*x^4]) - (2^(1/4)*(19 - 2*Sqrt[2])*(1 + S
```

```
qrt[2]*x^2)*Sqrt[(1 + 2*x^2 + 2*x^4)/(1 + Sqrt[2]*x^2)^2]*EllipticF[2*ArcTa
n[2^(1/4)*x], (2 - Sqrt[2])/4])/(135*Sqrt[1 + 2*x^2 + 2*x^4]) + (5*(3 + Sqr
t[2])^2*(1 + Sqrt[2]*x^2)*Sqrt[(1 + 2*x^2 + 2*x^4)/(1 + Sqrt[2]*x^2)^2]*Ell
ipticPi[(12 - 11*Sqrt[2])/24, 2*ArcTan[2^(1/4)*x], (2 - Sqrt[2])/4])/(567*2
^(1/4)*Sqrt[1 + 2*x^2 + 2*x^4])
```

Rule 1117

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1209

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

Rule 1211

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + b*x^2 + c*x^4
], x], x] - Dist[e/q, Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /; Ne
Q[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[
c/a]
```

Rule 1295

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[d*(f*x)^(m + 1)*((a + b*x^2 + c*x^4)^(p + 1)
/(a*f*(m + 1))), x] + Dist[1/(a*f^2*(m + 1)), Int[(f*x)^(m + 2)*(a + b*x^2
+ c*x^4)^p*Simp[a*e*(m + 1) - b*d*(m + 2*p + 3) - c*d*(m + 4*p + 5)*x^2, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[m
, -1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1323

```
Int[(((f_.)*(x_))^(m_)*((a_.) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.))/((d_.)
+ (e_.)*(x_)^2), x_Symbol] :> Dist[1/d^2, Int[(f*x)^m*(a*d + (b*d - a*e)*x^
2)*(a + b*x^2 + c*x^4)^(p - 1), x], x] + Dist[(c*d^2 - b*d*e + a*e^2)/(d^2*
f^4), Int[(f*x)^(m + 4)*((a + b*x^2 + c*x^4)^(p - 1)/(d + e*x^2)), x], x] /
; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && LtQ[m
```

```
, -2]
```

Rule 1343

```
Int[(x_)^(m_)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^
4]), x_Symbol] :> Simp[x^(m + 1)*(Sqrt[a + b*x^2 + c*x^4]/(a*d*(m + 1))), x
] - Dist[1/(a*d*(m + 1)), Int[(x^(m + 2)/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^
4]))*Simp[a*e*(m + 1) + b*d*(m + 2) + (b*e*(m + 2) + c*d*(m + 3))*x^2 + c*e
*(m + 3)*x^4, x], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0
] && ILtQ[m/2, 0]
```

Rule 1720

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4]), x_Symbol] :> With[{q = Rt[B/A, 2]}, Simp[(-(B*d - A*e))*(A
rcTan[Rt[-b + c*(d/e) + a*(e/d), 2]*(x/Sqrt[a + b*x^2 + c*x^4])]/(2*d*e*Rt[
-b + c*(d/e) + a*(e/d), 2])), x] + Simp[(B*d + A*e)*(A + B*x^2)*(Sqrt[A^2*(
(a + b*x^2 + c*x^4)/(a*(A + B*x^2)^2))]/(4*d*e*A*q*Sqrt[a + b*x^2 + c*x^4])
)*EllipticPi[Cancel[-(B*d - A*e)^2/(4*d*e*A*B)], 2*ArcTan[q*x], 1/2 - b*(A/
(4*a*B))], x]] /; FreeQ[{a, b, c, d, e, A, B}, x] && NeQ[b^2 - 4*a*c, 0] &&
 NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] && EqQ
[c*A^2 - a*B^2, 0]
```

Rule 1722

```
Int[((A_.) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2
+ (c_.)*(x_)^4]), x_Symbol] :> With[{q = Rt[c/a, 2]}, Dist[(A*(c*d + a*e*q)
 - a*B*(e + d*q))/(c*d^2 - a*e^2), Int[1/Sqrt[a + b*x^2 + c*x^4], x], x] +
Dist[a*(B*d - A*e)*((e + d*q)/(c*d^2 - a*e^2)), Int[(1 + q*x^2)/((d + e*x^2
)*Sqrt[a + b*x^2 + c*x^4]), x], x]] /; FreeQ[{a, b, c, d, e, A, B}, x] && N
eQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0
] && PosQ[c/a] && NeQ[c*A^2 - a*B^2, 0]
```

Rule 1728

```
Int[(P4x_)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4])
, x_Symbol] :> With[{q = Rt[c/a, 2], A = Coeff[P4x, x, 0], B = Coeff[P4x, x
, 2], C = Coeff[P4x, x, 4]}, Dist[-C/(e*q), Int[(1 - q*x^2)/Sqrt[a + b*x^2
+ c*x^4], x], x] + Dist[1/(c*e), Int[(A*c*e + a*C*d*q + (B*c*e - C*(c*d - a
*e*q))*x^2)/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]), x], x]] /; FreeQ[{a, b,
c, d, e}, x] && PolyQ[P4x, x^2, 2] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*
d*e + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] &&  !GtQ[b^2 - 4*a*c,
 0]
```

Rubi steps

$$\text{integral} = \frac{1}{9}\int \frac{3+4x^2}{x^6\sqrt{1+2x^2+2x^4}}\,dx + \frac{10}{9}\int \frac{1}{x^2(3+2x^2)\sqrt{1+2x^2+2x^4}}\,dx$$

$$= -\frac{\sqrt{1+2x^2+2x^4}}{15x^5} - \frac{10\sqrt{1+2x^2+2x^4}}{27x} - \frac{1}{45}\int \frac{4+18x^2}{x^4\sqrt{1+2x^2+2x^4}}\,dx + \frac{10}{27}\int \frac{-2+6x^2+4x^4}{(3+2x^2)\sqrt{1+2x^2+2x^4}}\,dx$$

$$= -\frac{\sqrt{1+2x^2+2x^4}}{15x^5} + \frac{4\sqrt{1+2x^2+2x^4}}{135x^3} - \frac{10\sqrt{1+2x^2+2x^4}}{27x} + \frac{1}{135}\int \frac{-38+8x^2}{x^2\sqrt{1+2x^2+2x^4}}\,dx + \frac{5}{54}\int \frac{-8+12\sqrt{2}+\left(24-4\left(6-2\sqrt{2}\right)\right)x^2}{(3+2x^2)\sqrt{1+2x^2+2x^4}}\,dx - \frac{1}{27}\left(10\sqrt{2}\right)\int \frac{1-\sqrt{2}x^2}{\sqrt{1+2x^2+2x^4}}\,dx$$

$$= -\frac{\sqrt{1+2x^2+2x^4}}{15x^5} + \frac{4\sqrt{1+2x^2+2x^4}}{135x^3} - \frac{4\sqrt{1+2x^2+2x^4}}{45x} + \frac{10\sqrt{2}x\sqrt{1+2x^2+2x^4}}{27\left(1+\sqrt{2}x^2\right)} - \frac{10\sqrt[4]{2}\left(1+\sqrt{2}x^2\right)\sqrt{\frac{1+2x^2+2x^4}{\left(1+\sqrt{2}x^2\right)^2}}E\left(2\tan^{-1}\left(\sqrt[4]{2}x\right)\middle|\frac{1}{4}\left(2-\sqrt{2}\right)\right)}{27\sqrt{1+2x^2+2x^4}} - \frac{1}{135}\int \frac{-8+76x^2}{\sqrt{1+2x^2+2x^4}}\,dx - \frac{1}{189}\left(10\left(6-5\sqrt{2}\right)\right)\int \frac{1}{\sqrt{1+2x^2+2x^4}}\,dx + \frac{1}{189}\left(20\left(2+3\sqrt{2}\right)\right)\int \frac{1+\sqrt{2}x^2}{(3+2x^2)\sqrt{1+2x^2+2x^4}}\,dx$$

$$= -\frac{\sqrt{1+2x^2+2x^4}}{15x^5} + \frac{4\sqrt{1+2x^2+2x^4}}{135x^3} - \frac{4\sqrt{1+2x^2+2x^4}}{45x} + \frac{10\sqrt{2}x\sqrt{1+2x^2+2x^4}}{27\left(1+\sqrt{2}x^2\right)} - \frac{2}{27}\sqrt{\frac{5}{3}}\tan^{-1}\left(\frac{\sqrt{\frac{5}{3}}x}{\sqrt{1+2x^2+2x^4}}\right) - \frac{10\sqrt[4]{2}\left(1+\sqrt{2}x^2\right)\sqrt{\frac{1+2x^2+2x^4}{\left(1+\sqrt{2}x^2\right)^2}}E\left(2\tan^{-1}\left(\sqrt[4]{2}x\right)\middle|\frac{1}{4}\left(2-\sqrt{2}\right)\right)}{27\sqrt{1+2x^2+2x^4}} + \frac{5\sqrt[4]{2}\left(5-3\sqrt{2}\right)\left(1+\sqrt{2}x^2\right)\sqrt{\frac{1+2x^2+2x^4}{\left(1+\sqrt{2}x^2\right)^2}}F\left(2\tan^{-1}\left(\sqrt[4]{2}x\right)\middle|\frac{1}{4}\left(2-\sqrt{2}\right)\right)}{189\sqrt{1+2x^2+2x^4}} + \frac{5\left(3+\sqrt{2}\right)^2\left(1+\sqrt{2}x^2\right)\sqrt{\frac{1+2x^2+2x^4}{\left(1+\sqrt{2}x^2\right)^2}}\Pi\left(\frac{1}{24}\left(12-11\sqrt{2}\right);2\tan^{-1}\left(\sqrt[4]{2}x\right)\middle|\frac{1}{4}\left(2-\sqrt{2}\right)\right)}{567\sqrt[4]{2}\sqrt{1+2x^2+2x^4}} + \frac{1}{135}\left(38\sqrt{2}\right)\int \frac{1-\sqrt{2}x^2}{\sqrt{1+2x^2+2x^4}}\,dx + \frac{1}{135}\left(2\left(4-19\sqrt{2}\right)\right)\int \frac{1}{\sqrt{1+2x^2+2x^4}}\,dx$$

$$= -\frac{\sqrt{1+2x^2+2x^4}}{15x^5} + \frac{4\sqrt{1+2x^2+2x^4}}{135x^3} - \frac{4\sqrt{1+2x^2+2x^4}}{45x} + \frac{4\sqrt{2}x\sqrt{1+2x^2+2x^4}}{45\left(1+\sqrt{2}x^2\right)} - \frac{2}{27}\sqrt{\frac{5}{3}}\tan^{-1}\left(\frac{\sqrt{\frac{5}{3}}x}{\sqrt{1+2x^2+2x^4}}\right) - \frac{4\sqrt[4]{2}\left(1+\sqrt{2}x^2\right)\sqrt{\frac{1+2x^2+2x^4}{\left(1+\sqrt{2}x^2\right)^2}}E\left(2\tan^{-1}\left(\sqrt[4]{2}x\right)\middle|\frac{1}{4}\left(2-\sqrt{2}\right)\right)}{45\sqrt{1+2x^2+2x^4}} + \frac{5\sqrt[4]{2}\left(5-3\sqrt{2}\right)\left(1+\sqrt{2}x^2\right)\sqrt{\frac{1+2x^2+2x^4}{\left(1+\sqrt{2}x^2\right)^2}}F\left(2\tan^{-1}\left(\sqrt[4]{2}x\right)\middle|\frac{1}{4}\left(2-\sqrt{2}\right)\right)}{189\sqrt{1+2x^2+2x^4}} - \frac{\sqrt[4]{2}\left(19-2\sqrt{2}\right)\left(1+\sqrt{2}x^2\right)\sqrt{\frac{1+2x^2+2x^4}{\left(1+\sqrt{2}x^2\right)^2}}F\left(2\tan^{-1}\left(\sqrt[4]{2}x\right)\middle|\frac{1}{4}\left(2-\sqrt{2}\right)\right)}{135\sqrt{1+2x^2+2x^4}} + \frac{5\left(3+\sqrt{2}\right)^2\left(1+\sqrt{2}x^2\right)\sqrt{\frac{1+2x^2+2x^4}{\left(1+\sqrt{2}x^2\right)^2}}\Pi\left(\frac{1}{24}\left(12-11\sqrt{2}\right);2\tan^{-1}\left(\sqrt[4]{2}x\right)\middle|\frac{1}{4}\left(2-\sqrt{2}\right)\right)}{567\sqrt[4]{2}\sqrt{1+2x^2+2x^4}}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.22 (sec) , antiderivative size = 224, normalized size of antiderivative = 0.41

$$\int \frac{\sqrt{1+2x^2+2x^4}}{x^6\left(3+2x^2\right)}\,dx = -\frac{27+42x^2+66x^4+48x^6+72x^8+36i\sqrt{1-i}x^5\sqrt{1+(1-i)x^2}\sqrt{1+(1+i)x^2}E\left(i\,\mathrm{arcsinh}\left(\sqrt{1-i}x\right)\middle|\,i\right)}{}$$

```
[In] Integrate[Sqrt[1 + 2*x^2 + 2*x^4]/(x^6*(3 + 2*x^2)),x]
```

```
[Out] -1/405*(27 + 42*x^2 + 66*x^4 + 48*x^6 + 72*x^8 + (36*I)*Sqrt[1 - I]*x^5*Sqr
t[1 + (1 - I)*x^2]*Sqrt[1 + (1 + I)*x^2]*EllipticE[I*ArcSinh[Sqrt[1 - I]*x]
, I] - (12 + 24*I)*Sqrt[1 - I]*x^5*Sqrt[1 + (1 - I)*x^2]*Sqrt[1 + (1 + I)*x
^2]*EllipticF[I*ArcSinh[Sqrt[1 - I]*x], I] + 50*(1 - I)^(3/2)*x^5*Sqrt[1 +
(1 - I)*x^2]*Sqrt[1 + (1 + I)*x^2]*EllipticPi[1/3 + I/3, I*ArcSinh[Sqrt[1 -
 I]*x], I])/(x^5*Sqrt[1 + 2*x^2 + 2*x^4])
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 4.10 (sec) , antiderivative size = 263, normalized size of antiderivative = 0.48

method	result
risch	$-\frac{24x^8+16x^6+22x^4+14x^2+9}{135x^5\sqrt{2x^4+2x^2+1}}+\frac{8\sqrt{1+(1-i)x^2}\,\sqrt{1+(1+i)x^2}\,F\left(x\sqrt{-1+i},\frac{\sqrt{2}}{2}+\frac{i\sqrt{2}}{2}\right)}{135\sqrt{-1+i}\,\sqrt{2x^4+2x^2+1}}+$
elliptic	$-\frac{\sqrt{2x^4+2x^2+1}}{15x^5}+\frac{4\sqrt{2x^4+2x^2+1}}{135x^3}-\frac{4\sqrt{2x^4+2x^2+1}}{45x}-\frac{4\sqrt{-ix^2+x^2+1}\,\sqrt{ix^2+x^2+1}\,F\left(x\sqrt{-1+i},\frac{\sqrt{2}}{2}+\frac{i\sqrt{2}}{2}\right)}{135\sqrt{-1+i}\,\sqrt{2x^4+2x^2+1}}+$
default	$-\frac{\sqrt{2x^4+2x^2+1}}{15x^5}+\frac{4\sqrt{2x^4+2x^2+1}}{135x^3}-\frac{4\sqrt{2x^4+2x^2+1}}{45x}-\frac{4\sqrt{1+(1-i)x^2}\,\sqrt{1+(1+i)x^2}\,F\left(x\sqrt{-1+i},\frac{\sqrt{2}}{2}+\frac{i\sqrt{2}}{2}\right)}{45\sqrt{-1+i}\,\sqrt{2x^4+2x^2+1}}+$

```
[In] int((2*x^4+2*x^2+1)^(1/2)/x^6/(2*x^2+3),x,method=_RETURNVERBOSE)
```

```
[Out] -1/135*(24*x^8+16*x^6+22*x^4+14*x^2+9)/x^5/(2*x^4+2*x^2+1)^(1/2)+8/135/(-1+
I)^(1/2)*(1+(1-I)*x^2)^(1/2)*(1+(1+I)*x^2)^(1/2)/(2*x^4+2*x^2+1)^(1/2)*Elli
pticF(x*(-1+I)^(1/2),1/2*2^(1/2)+1/2*I*2^(1/2))+(-4/45+4/45*I)/(-1+I)^(1/2)
*(1+(1-I)*x^2)^(1/2)*(1+(1+I)*x^2)^(1/2)/(2*x^4+2*x^2+1)^(1/2)*(EllipticF(x
*(-1+I)^(1/2),1/2*2^(1/2)+1/2*I*2^(1/2))-EllipticE(x*(-1+I)^(1/2),1/2*2^(1/
2)+1/2*I*2^(1/2)))-20/81/(-1+I)^(1/2)*(1-I*x^2+x^2)^(1/2)*(1+I*x^2+x^2)^(1/
2)/(2*x^4+2*x^2+1)^(1/2)*EllipticPi(x*(-1+I)^(1/2),1/3+1/3*I,(-1-I)^(1/2)/(
-1+I)^(1/2))
```

Fricas [F]

$$\int \frac{\sqrt{1+2x^2+2x^4}}{x^6\,(3+2x^2)}\,dx=\int \frac{\sqrt{2\,x^4+2\,x^2+1}}{(2\,x^2+3)x^6}\,dx$$

```
[In] integrate((2*x^4+2*x^2+1)^(1/2)/x^6/(2*x^2+3),x, algorithm="fricas")
```

```
[Out] integral(sqrt(2*x^4 + 2*x^2 + 1)/(2*x^8 + 3*x^6), x)
```

Sympy [F]

$$\int \frac{\sqrt{1+2x^2+2x^4}}{x^6\,(3+2x^2)}\,dx=\int \frac{\sqrt{2x^4+2x^2+1}}{x^6\cdot(2x^2+3)}\,dx$$

```
[In] integrate((2*x**4+2*x**2+1)**(1/2)/x**6/(2*x**2+3),x)
```

```
[Out] Integral(sqrt(2*x**4 + 2*x**2 + 1)/(x**6*(2*x**2 + 3)), x)
```

Maxima [F]

$$\int \frac{\sqrt{1+2x^2+2x^4}}{x^6\,(3+2x^2)}\,dx = \int \frac{\sqrt{2\,x^4+2\,x^2+1}}{(2\,x^2+3)x^6}\,dx$$

```
[In] integrate((2*x^4+2*x^2+1)^(1/2)/x^6/(2*x^2+3),x, algorithm="maxima")
[Out] integrate(sqrt(2*x^4 + 2*x^2 + 1)/((2*x^2 + 3)*x^6), x)
```

Giac [F]

$$\int \frac{\sqrt{1+2x^2+2x^4}}{x^6\,(3+2x^2)}\,dx = \int \frac{\sqrt{2\,x^4+2\,x^2+1}}{(2\,x^2+3)x^6}\,dx$$

```
[In] integrate((2*x^4+2*x^2+1)^(1/2)/x^6/(2*x^2+3),x, algorithm="giac")
[Out] integrate(sqrt(2*x^4 + 2*x^2 + 1)/((2*x^2 + 3)*x^6), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{1+2x^2+2x^4}}{x^6\,(3+2x^2)}\,dx = \int \frac{\sqrt{2\,x^4+2\,x^2+1}}{x^6\,(2\,x^2+3)}\,dx$$

```
[In] int((2*x^2 + 2*x^4 + 1)^(1/2)/(x^6*(2*x^2 + 3)),x)
[Out] int((2*x^2 + 2*x^4 + 1)^(1/2)/(x^6*(2*x^2 + 3)), x)
```

3.322 $\int \frac{x^5\left(a+bx^2+cx^4\right)^{3/2}}{d+ex^2}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 482

$$\int \frac{x^5(a+bx^2+cx^4)^{3/2}}{d+ex^2}\,dx = \frac{(128c^4d^4+3b^4e^4-32c^3d^2e(5bd-4ae)+8bc^2de^2(2bd-3ae)+6b^2ce^3(bd-2c\cdots}{2\cdots}$$
$$+\frac{(16c^2d^2-6bcde-3b^2e^2-6ce(2cd+be)x^2)\left(a+bx^2+cx^4\right)^{3/2}}{96c^2e^3}+\frac{\left(a+bx^2+cx^4\right)^{5/2}}{10ce}$$
$$-\frac{(256c^5d^5+3b^5e^5+6b^3ce^4(bd-4ae)-384c^4d^3e(bd-ae)+96c^3de^2(bd-ae)^2+16bc^2e^3(b^2d^2-3abde+\cdots}{512c^{7/2}e^6}$$
$$+\frac{d^2\left(cd^2-bde+ae^2\right)^{3/2}\operatorname{arctanh}\left(\frac{bd-2ae+(2cd-be)x^2}{2\sqrt{cd^2-bde+ae^2}\sqrt{a+bx^2+cx^4}}\right)}{2e^6}$$

```
[Out] 1/96*(16*c^2*d^2-6*b*c*d*e-3*b^2*e^2-6*c*e*(b*e+2*c*d)*x^2)*(c*x^4+b*x^2+a)
^(3/2)/c^2/e^3+1/10*(c*x^4+b*x^2+a)^(5/2)/c/e-1/512*(256*c^5*d^5+3*b^5*e^5+
6*b^3*c*e^4*(-4*a*e+b*d)-384*c^4*d^3*e*(-a*e+b*d)+96*c^3*d*e^2*(-a*e+b*d)^2
+16*b*c^2*e^3*(3*a^2*e^2-3*a*b*d*e+b^2*d^2))*arctanh(1/2*(2*c*x^2+b)/c^(1/2
)/(c*x^4+b*x^2+a)^(1/2))/c^(7/2)/e^6+1/2*d^2*(a*e^2-b*d*e+c*d^2)^(3/2)*arct
anh(1/2*(b*d-2*a*e+(-b*e+2*c*d)*x^2)/(a*e^2-b*d*e+c*d^2)^(1/2)/(c*x^4+b*x^2
+a)^(1/2))/e^6+1/256*(128*c^4*d^4+3*b^4*e^4-32*c^3*d^2*e*(-4*a*e+5*b*d)+8*b
*c^2*d*e^2*(-3*a*e+2*b*d)+6*b^2*c*e^3*(-2*a*e+b*d)-2*c*e*(32*c^3*d^3-3*b^3*
e^3-8*c^2*d*e*(-3*a*e+2*b*d)-6*b*c*e^2*(-2*a*e+b*d))*x^2)*(c*x^4+b*x^2+a)^(
1/2)/c^3/e^5
```

Rubi [A] (verified)

Time = 0.69 (sec) , antiderivative size = 482, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.241$, Rules used = {1265, 1667, 828, 857, 635, 212, 738}

$$\int \frac{x^5(a+bx^2+cx^4)^{3/2}}{d+ex^2}\,dx =$$

$$-\frac{\operatorname{arctanh}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)(16bc^2e^3(3a^2e^2-3abde+b^2d^2)+6b^3ce^4(bd-4ae)-384c^4d^3e(bd-ae)+96c^3de^2}{512c^{7/2}e^6}$$

$$+\frac{d^2(ae^2-bde+cd^2)^{3/2}\operatorname{arctanh}\left(\frac{-2ae+x^2(2cd-be)+bd}{2\sqrt{a+bx^2+cx^4}\sqrt{ae^2-bde+cd^2}}\right)}{2e^6}$$

$$+\frac{(a+bx^2+cx^4)^{3/2}(-3b^2e^2-6cex^2(be+2cd)-6bcde+16c^2d^2)}{96c^2e^3}$$

$$+\frac{\sqrt{a+bx^2+cx^4}(-2cex^2(-8c^2de(2bd-3ae)-6bce^2(bd-2ae)-3b^3e^3+32c^3d^3)+6b^2ce^3(bd-2ae)-32c}{256c^3e^5}$$

$$+\frac{(a+bx^2+cx^4)^{5/2}}{10ce}$$

```
[In] Int[(x^5*(a + b*x^2 + c*x^4)^(3/2))/(d + e*x^2),x]
```

```
[Out] ((128*c^4*d^4 + 3*b^4*e^4 - 32*c^3*d^2*e*(5*b*d - 4*a*e) + 8*b*c^2*d*e^2*(2
*b*d - 3*a*e) + 6*b^2*c*e^3*(b*d - 2*a*e) - 2*c*e*(32*c^3*d^3 - 3*b^3*e^3 -
 8*c^2*d*e*(2*b*d - 3*a*e) - 6*b*c*e^2*(b*d - 2*a*e))*x^2)*Sqrt[a + b*x^2 +
 c*x^4])/(256*c^3*e^5) + ((16*c^2*d^2 - 6*b*c*d*e - 3*b^2*e^2 - 6*c*e*(2*c*
d + b*e)*x^2)*(a + b*x^2 + c*x^4)^(3/2))/(96*c^2*e^3) + (a + b*x^2 + c*x^4)
^(5/2)/(10*c*e) - ((256*c^5*d^5 + 3*b^5*e^5 + 6*b^3*c*e^4*(b*d - 4*a*e) - 3
84*c^4*d^3*e*(b*d - a*e) + 96*c^3*d*e^2*(b*d - a*e)^2 + 16*b*c^2*e^3*(b^2*d
^2 - 3*a*b*d*e + 3*a^2*e^2))*ArcTanh[(b + 2*c*x^2)/(2*Sqrt[c]*Sqrt[a + b*x^
2 + c*x^4])])/(512*c^(7/2)*e^6) + (d^2*(c*d^2 - b*d*e + a*e^2)^(3/2)*ArcTan
h[(b*d - 2*a*e + (2*c*d - b*e)*x^2)/(2*Sqrt[c*d^2 - b*d*e + a*e^2]*Sqrt[a +
 b*x^2 + c*x^4])])/(2*e^6)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 635

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 828

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2)
 - g*(c*d + 2*c*d*p - b*e*p) + g*c*e*(m + 2*p + 1)*x)*((a + b*x + c*x^2)^p/
(c*e^2*(m + 2*p + 1)*(m + 2*p + 2))), x] - Dist[p/(c*e^2*(m + 2*p + 1)*(m +
 2*p + 2)), Int[(d + e*x)^m*(a + b*x + c*x^2)^(p - 1)*Simp[c*e*f*(b*d - 2*a
*e)*(m + 2*p + 2) + g*(a*e*(b*e - 2*c*d*m + b*e*m) + b*d*(b*e*p - c*d - 2*c
*d*p)) + (c*e*f*(2*c*d - b*e)*(m + 2*p + 2) + g*(b^2*e^2*(p + m + 1) - 2*c^
2*d^2*(1 + 2*p) - c*e*(b*d*(m - 2*p) + 2*a*e*(m + 2*p + 1))))*x, x], x], x]
 /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
- b*d*e + a*e^2, 0] && GtQ[p, 0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[
m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p]
|| IntegersQ[2*m, 2*p])
```

Rule 857

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1265

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rule 1667

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, S
imp[f*(d + e*x)^(m + q - 1)*((a + b*x + c*x^2)^(p + 1)/(c*e^(q - 1)*(m + q
+ 2*p + 1))), x] + Dist[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + b
*x + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1
)*(d + e*x)^q - f*(d + e*x)^(q - 2)*(b*d*e*(p + 1) + a*e^2*(m + q - 1) - c*
d^2*(m + q + 2*p + 1) - e*(2*c*d - b*e)*(m + q + p)*x), x], x], x] /; GtQ[q
```

```
, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, d, e, m, p}, x] && Poly
Q[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !(IGtQ
[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rubi steps

$$\text{integral} = \frac{1}{2}\text{Subst}\left(\int \frac{x^2(a+bx+cx^2)^{3/2}}{d+ex}\,dx, x, x^2\right)$$

$$= \frac{(a+bx^2+cx^4)^{5/2}}{10ce} + \frac{\text{Subst}\left(\int \frac{\left(-\frac{5}{2}bde-\frac{5}{2}e(2cd+be)x\right)(a+bx+cx^2)^{3/2}}{d+ex}\,dx, x, x^2\right)}{10ce^2}$$

$$= \frac{(16c^2d^2-6bcde-3b^2e^2-6ce(2cd+be)x^2)(a+bx^2+cx^4)^{3/2}}{96c^2e^3} + \frac{(a+bx^2+cx^4)^{5/2}}{10ce}$$

$$- \frac{\text{Subst}\left(\int \frac{\left(-\frac{5}{4}de\left(6b^2cde+8ac^2de+3b^3e^2-4bc(4cd^2+3ae^2)\right)+\frac{5}{4}e\left(32c^3d^3-3b^3e^3-8c^2de(2bd-3ae)-6bce^2(bd-2ae)\right)x\right)\sqrt{a+bx+cx}}{d+ex}\right.}{80c^2e^4}$$

$$= \frac{(128c^4d^4+3b^4e^4-32c^3d^2e(5bd-4ae)+8bc^2de^2(2bd-3ae)+6b^2ce^3(bd-2ae)-2ce(32c^3d^3-3}{256c^3e^5}$$

$$+ \frac{(16c^2d^2-6bcde-3b^2e^2-6ce(2cd+be)x^2)(a+bx^2+cx^4)^{3/2}}{96c^2e^3}$$

$$+ \frac{(a+bx^2+cx^4)^{5/2}}{10ce}$$

$$+ \frac{\text{Subst}\left(\int \frac{-\frac{5}{8}de\left(6b^4cde^3+3b^5e^4+8b^3ce^2(2cd^2-3ae^2)-16b^2c^2de(10cd^2+3ae^2)-32ac^3de(4cd^2+5ae^2)+16bc^2(8c^2d^4+20acd^2e^2+3\right.}{(d+ex)\sqrt{a}}\right.}{320}$$

$$= \frac{(128c^4d^4+3b^4e^4-32c^3d^2e(5bd-4ae)+8bc^2de^2(2bd-3ae)+6b^2ce^3(bd-2ae)-2ce(32c^3d^3-3}{256c^3e^5}$$

$$+ \frac{(16c^2d^2-6bcde-3b^2e^2-6ce(2cd+be)x^2)(a+bx^2+cx^4)^{3/2}}{96c^2e^3}$$

$$+ \frac{(a+bx^2+cx^4)^{5/2}}{10ce} + \frac{\left(d^2(cd^2-bde+ae^2)^2\right)\text{Subst}\left(\int \frac{1}{(d+ex)\sqrt{a+bx+cx^2}}\,dx, x, x^2\right)}{2e^6}$$

$$- \frac{(256c^5d^5+3b^5e^5+6b^3ce^4(bd-4ae)-384c^4d^3e(bd-ae)+96c^3de^2(bd-ae)^2+16bc^2e^3(b^2d^2-}{512c^3e^6}$$

$$= \frac{(128c^4d^4 + 3b^4e^4 - 32c^3d^2e(5bd - 4ae) + 8bc^2de^2(2bd - 3ae) + 6b^2ce^3(bd - 2ae) - 2ce(32c^3d^3 - }{256c^3e^5}$$
$$+ \frac{(16c^2d^2 - 6bcde - 3b^2e^2 - 6ce(2cd + be)x^2)(a + bx^2 + cx^4)^{3/2}}{96c^2e^3}$$
$$+ \frac{(a + bx^2 + cx^4)^{5/2}}{10ce}$$
$$- \frac{\left(d^2(cd^2 - bde + ae^2)^2\right) \text{Subst}\left(\int \frac{1}{4cd^2-4bde+4ae^2-x^2} dx, x, \frac{-bd+2ae-(2cd-be)x^2}{\sqrt{a+bx^2+cx^4}}\right)}{e^6}$$
$$- \frac{(256c^5d^5 + 3b^5e^5 + 6b^3ce^4(bd - 4ae) - 384c^4d^3e(bd - ae) + 96c^3de^2(bd - ae)^2 + 16bc^2e^3(b^2d^2 - }{256c^3e^6}$$

$$= \frac{(128c^4d^4 + 3b^4e^4 - 32c^3d^2e(5bd - 4ae) + 8bc^2de^2(2bd - 3ae) + 6b^2ce^3(bd - 2ae) - 2ce(32c^3d^3 - }{256c^3e^5}$$
$$+ \frac{(16c^2d^2 - 6bcde - 3b^2e^2 - 6ce(2cd + be)x^2)(a + bx^2 + cx^4)^{3/2}}{96c^2e^3}$$
$$+ \frac{(a + bx^2 + cx^4)^{5/2}}{10ce}$$
$$- \frac{(256c^5d^5 + 3b^5e^5 + 6b^3ce^4(bd - 4ae) - 384c^4d^3e(bd - ae) + 96c^3de^2(bd - ae)^2 + 16bc^2e^3(b^2d^2 - }{512c^{7/2}e^6}$$
$$+ \frac{d^2(cd^2 - bde + ae^2)^{3/2} \tanh^{-1}\left(\frac{bd-2ae+(2cd-be)x^2}{2\sqrt{cd^2-bde+ae^2}\sqrt{a+bx^2+cx^4}}\right)}{2e^6}$$

Mathematica [A] (verified)

Time = 10.67 (sec) , antiderivative size = 545, normalized size of antiderivative = 1.13

$$\int \frac{x^5(a + bx^2 + cx^4)^{3/2}}{d + ex^2} dx = \frac{1280d^2(a + bx^2 + cx^4)^{3/2} - \frac{480de(b+2cx^2)(a+bx^2+cx^4)^{3/2}}{c} + \frac{768e^2(a+bx^2+cx^4)^{5/2}}{c} - \frac{90(}{}}{}$$

```
[In] Integrate[(x^5*(a + b*x^2 + c*x^4)^(3/2))/(d + e*x^2),x]
```

```
[Out] (1280*d^2*(a + b*x^2 + c*x^4)^(3/2) - (480*d*e*(b + 2*c*x^2)*(a + b*x^2 + c
*x^4)^(3/2))/c + (768*e^2*(a + b*x^2 + c*x^4)^(5/2))/c - (90*(b^2 - 4*a*c)*
d*e*(-2*Sqrt[c]*(b + 2*c*x^2)*Sqrt[a + b*x^2 + c*x^4] + (b^2 - 4*a*c)*ArcTa
nh[(b + 2*c*x^2)/(2*Sqrt[c]*Sqrt[a + b*x^2 + c*x^4])]))/c^(5/2) + (15*b*e^2
*(-16*(b + 2*c*x^2)*(a + b*x^2 + c*x^4)^(3/2) + 3*(b^2 - 4*a*c)*((2*(b + 2*
c*x^2)*Sqrt[a + b*x^2 + c*x^4])/c + ((-b^2 + 4*a*c)*ArcTanh[(b + 2*c*x^2)/(
2*Sqrt[c]*Sqrt[a + b*x^2 + c*x^4])])/c^(3/2))))/c^2 - (240*d^2*((2*c*d - b*
e)*(8*c^2*d^2 - b^2*e^2 + 4*c*e*(-2*b*d + 3*a*e))*ArcTanh[(b + 2*c*x^2)/(2*
```

```
Sqrt[c]*Sqrt[a + b*x^2 + c*x^4])] + 2*Sqrt[c]*(e*Sqrt[a + b*x^2 + c*x^4]*(-
(b^2*e^2) + 4*c^2*d*(-2*d + e*x^2) - 2*c*e*(-5*b*d + 4*a*e + b*e*x^2)) + 8*
c*(c*d^2 + e*(-(b*d) + a*e))^(3/2)*ArcTanh[(-(b*d) + 2*a*e - 2*c*d*x^2 + b*
e*x^2)/(2*Sqrt[c*d^2 + e*(-(b*d) + a*e)]*Sqrt[a + b*x^2 + c*x^4])])))/(c^(3
/2)*e^3))/(7680*e^3)
```

Maple [A] (verified)

Time = 0.64 (sec) , antiderivative size = 620, normalized size of antiderivative = 1.29

method	result
pseudoelliptic	$\frac{-5\left(e^{2}(ae-bd)^{2}c^{\frac{7}{2}}+(2e^{2}d^{2}a-2d^{3}eb)c^{\frac{9}{2}}+c^{\frac{11}{2}}d^{4}\right)d^{2}\ln\left(\frac{2\sqrt{c\,x^{4}+b\,x^{2}+a}\,\sqrt{\frac{a\,e^{2}-bde+c\,d^{2}}{e^{2}}}\,e+\left(b\,x^{2}+2a\right)e-d\left(2c\,x^{2}+b\right)}{e\,x^{2}+d}\right)+\left(\left(-\frac{15}{}\right.\right.}{}$
risch	$\frac{(384c^{4}e^{4}x^{8}+528b\,c^{3}e^{4}x^{6}-480c^{4}d\,e^{3}x^{6}+768a\,c^{3}e^{4}x^{4}+24b^{2}c^{2}e^{4}x^{4}-720b\,c^{3}d\,e^{3}x^{4}+640c^{4}d^{2}e^{2}x^{4}+168ab\,c^{2}e^{4}x^{2}-1200a\,c^{3}d\,e^{3}}{}$
default	Expression too large to display
elliptic	Expression too large to display

```
[In] int(x^5*(c*x^4+b*x^2+a)^(3/2)/(e*x^2+d),x,method=_RETURNVERBOSE)
```

```
[Out] 1/10/c^(7/2)/((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*(-5*(e^2*(a*e-b*d)^2*c^(7/2)+(
2*a*d^2*e^2-2*b*d^3*e)*c^(9/2)+c^(11/2)*d^4)*d^2*ln((2*(c*x^4+b*x^2+a)^(1/2
)*((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*e+(b*x^2+2*a)*e-d*(2*c*x^2+b))/(e*x^2+d))
+((-15/16*b*(a*c-1/4*b^2)^2*e^5-15/8*c*d*(a*c-1/4*b^2)^2*e^4+15/4*c^2*d^2*(
a*c-1/12*b^2)*b*e^3-15/2*(a*c+1/4*b^2)*c^3*d^3*e^2+15/2*b*c^4*d^4*e-5*c^5*d
^5)*ln((2*c*x^2+2*(c*x^4+b*x^2+a)^(1/2)*c^(1/2)+b)/c^(1/2))+e*(20/3*e*(3/10
*x^4*(11/16*b*x^2+a)*e^3-15/32*d*(3/5*b*x^2+a)*x^2*e^2+d^2*(7/16*b*x^2+a)*e
-15/16*b*d^3)*c^(7/2)+(5/3*d^2*e^2*x^4-5/2*d^3*e*x^2-5/4*d*e^3*x^6+e^4*x^8+
5*d^4)*c^(9/2)+(((1/16*b^2*x^4+a^2+7/16*a*b*x^2)*e^2-25/16*d*b*(1/10*b*x^2+
a)*e+5/8*b^2*d^2)*c^(5/2)-25/32*(((1/10*b*x^2+a)*e-3/10*b*d)*c^(3/2)-3/20*b
^2*e*c^(1/2))*e*b^2)*e^2)*(c*x^4+b*x^2+a)^(1/2)+15/16*(b*(a*c-1/4*b^2)^2*e^
5+2*c*d*(a*c-1/4*b^2)^2*e^4-4*c^2*d^2*(a*c-1/12*b^2)*b*e^3+(8*a*c^4+2*b^2*c
^3)*d^3*e^2-8*b*c^4*d^4*e+16/3*c^5*d^5)*ln(2))*e*((a*e^2-b*d*e+c*d^2)/e^2)^
(1/2))/e^7
```

Fricas [F(-1)]

Timed out.

$$\int \frac{x^5(a+bx^2+cx^4)^{3/2}}{d+ex^2}\,dx = \text{Timed out}$$

```
[In] integrate(x^5*(c*x^4+b*x^2+a)^(3/2)/(e*x^2+d),x, algorithm="fricas")
[Out] Timed out
```

Sympy [F]

$$\int \frac{x^5(a+bx^2+cx^4)^{3/2}}{d+ex^2}\,dx = \int \frac{x^5(a+bx^2+cx^4)^{\frac{3}{2}}}{d+ex^2}\,dx$$

```
[In] integrate(x**5*(c*x**4+b*x**2+a)**(3/2)/(e*x**2+d),x)
[Out] Integral(x**5*(a + b*x**2 + c*x**4)**(3/2)/(d + e*x**2), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^5(a+bx^2+cx^4)^{3/2}}{d+ex^2}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(x^5*(c*x^4+b*x^2+a)^(3/2)/(e*x^2+d),x, algorithm="maxima")
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [F(-2)]

Exception generated.

$$\int \frac{x^5(a+bx^2+cx^4)^{3/2}}{d+ex^2}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(x^5*(c*x^4+b*x^2+a)^(3/2)/(e*x^2+d),x, algorithm="giac")
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument Type
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^5(a+bx^2+cx^4)^{3/2}}{d+ex^2}\,dx = \int \frac{x^5\,(c\,x^4+b\,x^2+a)^{3/2}}{e\,x^2+d}\,dx$$

```
[In] int((x^5*(a + b*x^2 + c*x^4)^(3/2))/(d + e*x^2),x)
[Out] int((x^5*(a + b*x^2 + c*x^4)^(3/2))/(d + e*x^2), x)
```

3.323 $\int \frac{x^3(a+bx^2+cx^4)^{3/2}}{d+ex^2}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 360

$$\int \frac{x^3(a+bx^2+cx^4)^{3/2}}{d+ex^2}\,dx = -\frac{(64c^3d^3+3b^3e^3-16c^2de(5bd-4ae)+4bce^2(2bd-3ae)-2ce(16c^2d^2-3b^2e^2-4ce(2bd-3ae))x^2)\sqrt{a+}}{128c^2e^4}$$

$$-\frac{(8cd-3be-6cex^2)(a+bx^2+cx^4)^{3/2}}{48ce^2}$$

$$+\frac{(128c^4d^4+3b^4e^4+8b^2ce^3(bd-3ae)-192c^3d^2e(bd-ae)+48c^2e^2(bd-ae)^2)\,\text{arctanh}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{256c^{5/2}e^5}$$

$$-\frac{d(cd^2-bde+ae^2)^{3/2}\,\text{arctanh}\left(\frac{bd-2ae+(2cd-be)x^2}{2\sqrt{cd^2-bde+ae^2}\sqrt{a+bx^2+cx^4}}\right)}{2e^5}$$

```
[Out] -1/48*(-6*c*e*x^2-3*b*e+8*c*d)*(c*x^4+b*x^2+a)^(3/2)/c/e^2+1/256*(128*c^4*d
^4+3*b^4*e^4+8*b^2*c*e^3*(-3*a*e+b*d)-192*c^3*d^2*e*(-a*e+b*d)+48*c^2*e^2*(
-a*e+b*d)^2)*arctanh(1/2*(2*c*x^2+b)/c^(1/2)/(c*x^4+b*x^2+a)^(1/2))/c^(5/2)
/e^5-1/2*d*(a*e^2-b*d*e+c*d^2)^(3/2)*arctanh(1/2*(b*d-2*a*e+(-b*e+2*c*d)*x^
2)/(a*e^2-b*d*e+c*d^2)^(1/2)/(c*x^4+b*x^2+a)^(1/2))/e^5-1/128*(64*c^3*d^3+3
*b^3*e^3-16*c^2*d*e*(-4*a*e+5*b*d)+4*b*c*e^2*(-3*a*e+2*b*d)-2*c*e*(16*c^2*d
^2-3*b^2*e^2-4*c*e*(-3*a*e+2*b*d))*x^2)*(c*x^4+b*x^2+a)^(1/2)/c^2/e^4
```

Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 360, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.207$, Rules used = {1265, 828, 857, 635, 212, 738}

$$\int \frac{x^3(a+bx^2+cx^4)^{3/2}}{d+ex^2}\,dx = \frac{\operatorname{arctanh}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)(8b^2ce^3(bd-3ae)-192c^3d^2e(bd-ae)+48c^2e^2(bd-a}{256c^{5/2}e^5}$$
$$-\frac{d(ae^2-bde+cd^2)^{3/2}\operatorname{arctanh}\left(\frac{-2ae+x^2(2cd-be)+bd}{2\sqrt{a+bx^2+cx^4}\sqrt{ae^2-bde+cd^2}}\right)}{2e^5}$$
$$-\frac{\sqrt{a+bx^2+cx^4}(-2cex^2(-4ce(2bd-3ae)-3b^2e^2+16c^2d^2)-16c^2de(5bd-4ae)+4bce^2(2bd-3ae)+3b}{128c^2e^4}$$
$$-\frac{(a+bx^2+cx^4)^{3/2}(-3be+8cd-6cex^2)}{48ce^2}$$

```
[In] Int[(x^3*(a + b*x^2 + c*x^4)^(3/2))/(d + e*x^2),x]
```

```
[Out] -1/128*((64*c^3*d^3 + 3*b^3*e^3 - 16*c^2*d*e*(5*b*d - 4*a*e) + 4*b*c*e^2*(2
*b*d - 3*a*e) - 2*c*e*(16*c^2*d^2 - 3*b^2*e^2 - 4*c*e*(2*b*d - 3*a*e))*x^2)
*Sqrt[a + b*x^2 + c*x^4])/(c^2*e^4) - ((8*c*d - 3*b*e - 6*c*e*x^2)*(a + b*x
^2 + c*x^4)^(3/2))/(48*c*e^2) + ((128*c^4*d^4 + 3*b^4*e^4 + 8*b^2*c*e^3*(b*
d - 3*a*e) - 192*c^3*d^2*e*(b*d - a*e) + 48*c^2*e^2*(b*d - a*e)^2)*ArcTanh[
(b + 2*c*x^2)/(2*Sqrt[c]*Sqrt[a + b*x^2 + c*x^4])])/(256*c^(5/2)*e^5) - (d*
(c*d^2 - b*d*e + a*e^2)^(3/2)*ArcTanh[(b*d - 2*a*e + (2*c*d - b*e)*x^2)/(2*
Sqrt[c*d^2 - b*d*e + a*e^2]*Sqrt[a + b*x^2 + c*x^4])])/(2*e^5)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 635

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 828

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2)
 - g*(c*d + 2*c*d*p - b*e*p) + g*c*e*(m + 2*p + 1)*x)*((a + b*x + c*x^2)^p/
(c*e^2*(m + 2*p + 1)*(m + 2*p + 2))), x] - Dist[p/(c*e^2*(m + 2*p + 1)*(m +
 2*p + 2)), Int[(d + e*x)^m*(a + b*x + c*x^2)^(p - 1)*Simp[c*e*f*(b*d - 2*a
*e)*(m + 2*p + 2) + g*(a*e*(b*e - 2*c*d*m + b*e*m) + b*d*(b*e*p - c*d - 2*c
*d*p)) + (c*e*f*(2*c*d - b*e)*(m + 2*p + 2) + g*(b^2*e^2*(p + m + 1) - 2*c^
2*d^2*(1 + 2*p) - c*e*(b*d*(m - 2*p) + 2*a*e*(m + 2*p + 1))))*x, x], x], x]
 /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
- b*d*e + a*e^2, 0] && GtQ[p, 0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[
m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p]
|| IntegersQ[2*m, 2*p])
```

Rule 857

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1265

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rubi steps

$$\text{integral} = \frac{1}{2}\text{Subst}\left(\int \frac{x(a+bx+cx^2)^{3/2}}{d+ex}\,dx, x, x^2\right)$$

$$= -\frac{(8cd-3be-6cex^2)\,(a+bx^2+cx^4)^{3/2}}{48ce^2} - \frac{\text{Subst}\left(\int \frac{\left(\frac{1}{2}d\left(4ace-2b\left(4cd-\frac{3be}{2}\right)\right)-\frac{1}{2}(16c^2d^2-3b^2e^2-4ce(2bd-3ae))x\right)\sqrt{a+bx+cx^2}}{d+ex}\,dx, x, x^2\right)}{16ce^2}$$

$$= -\frac{(64c^3d^3+3b^3e^3-16c^2de(5bd-4ae)+4bce^2(2bd-3ae)-2ce(16c^2d^2-3b^2e^2-4ce(2bd-3ae)}{128c^2e^4}$$

$$-\frac{(8cd-3be-6cex^2)\left(a+bx^2+cx^4\right)^{3/2}}{48ce^2}$$

$$+\frac{\text{Subst}\left(\int\frac{-\frac{1}{4}d(4ce(bd-2ae)(8bcd-3b^2e-4ace)-(4bcd-b^2e-4ace)\left(16c^2d^2-3b^2e^2-4ce(2bd-3ae)\right))+\frac{1}{4}\left(128c^4d^4+3b^4e^4+8b^2ce^3(\right.}{(d+ex)\sqrt{a+bx+cx^2}}}{64c^2e^4}$$

$$= -\frac{(64c^3d^3+3b^3e^3-16c^2de(5bd-4ae)+4bce^2(2bd-3ae)-2ce(16c^2d^2-3b^2e^2-4ce(2bd-3ae)}{128c^2e^4}$$

$$-\frac{(8cd-3be-6cex^2)\left(a+bx^2+cx^4\right)^{3/2}}{48ce^2}$$

$$-\frac{\left(d\left(cd^2-bde+ae^2\right)^2\right)\text{Subst}\left(\int\frac{1}{(d+ex)\sqrt{a+bx+cx^2}}\,dx,x,x^2\right)}{2e^5}$$

$$+\frac{(128c^4d^4+3b^4e^4+8b^2ce^3(bd-3ae)-192c^3d^2e(bd-ae)+48c^2e^2(bd-ae)^2)\,\text{Subst}\left(\int\frac{1}{\sqrt{a+bx+cx^2}}\right.}{256c^2e^5}$$

$$= -\frac{(64c^3d^3+3b^3e^3-16c^2de(5bd-4ae)+4bce^2(2bd-3ae)-2ce(16c^2d^2-3b^2e^2-4ce(2bd-3ae)}{128c^2e^4}$$

$$-\frac{(8cd-3be-6cex^2)\left(a+bx^2+cx^4\right)^{3/2}}{48ce^2}$$

$$+\frac{\left(d\left(cd^2-bde+ae^2\right)^2\right)\text{Subst}\left(\int\frac{1}{4cd^2-4bde+4ae^2-x^2}\,dx,x,\frac{-bd+2ae-(2cd-be)x^2}{\sqrt{a+bx^2+cx^4}}\right)}{e^5}$$

$$+\frac{(128c^4d^4+3b^4e^4+8b^2ce^3(bd-3ae)-192c^3d^2e(bd-ae)+48c^2e^2(bd-ae)^2)\,\text{Subst}\left(\int\frac{1}{4c-x^2}\,dx,\right.}{128c^2e^5}$$

$$= -\frac{(64c^3d^3+3b^3e^3-16c^2de(5bd-4ae)+4bce^2(2bd-3ae)-2ce(16c^2d^2-3b^2e^2-4ce(2bd-3ae)}{128c^2e^4}$$

$$-\frac{(8cd-3be-6cex^2)\left(a+bx^2+cx^4\right)^{3/2}}{48ce^2}$$

$$+\frac{(128c^4d^4+3b^4e^4+8b^2ce^3(bd-3ae)-192c^3d^2e(bd-ae)+48c^2e^2(bd-ae)^2)\tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2}}\right.}{256c^{5/2}e^5}$$

$$-\frac{d\left(cd^2-bde+ae^2\right)^{3/2}\tanh^{-1}\left(\frac{bd-2ae+(2cd-be)x^2}{2\sqrt{cd^2-bde+ae^2}\sqrt{a+bx^2+cx^4}}\right)}{2e^5}$$

Mathematica [A] (verified)

Time = 1.94 (sec) , antiderivative size = 345, normalized size of antiderivative = 0.96

$$\int \frac{x^3(a+bx^2+cx^4)^{3/2}}{d+ex^2}\,dx = \frac{\frac{2e\sqrt{a+bx^2+cx^4}(-9b^3e^3+6bce^2(-4bd+10ae+bex^2)-16c^3(12d^3-6d^2ex^2+4de^2x^4-3e^3x^6)+8c^2e(ae(-32d}{c^2}}{}$$

[In] Integrate[(x^3*(a + b*x^2 + c*x^4)^(3/2))/(d + e*x^2),x]

[Out] ((2*e*Sqrt[a + b*x^2 + c*x^4]*(-9*b^3*e^3 + 6*b*c*e^2*(-4*b*d + 10*a*e + b*e*x^2) - 16*c^3*(12*d^3 - 6*d^2*e*x^2 + 4*d*e^2*x^4 - 3*e^3*x^6) + 8*c^2*e*(a*e*(-32*d + 15*e*x^2) + b*(30*d^2 - 14*d*e*x^2 + 9*e^2*x^4))))/c^2 - 768*d*Sqrt[-(c*d^2) + b*d*e - a*e^2]*(c*d^2 + e*(-(b*d) + a*e))*ArcTan[(Sqrt[c]*(d + e*x^2) - e*Sqrt[a + b*x^2 + c*x^4])/Sqrt[-(c*d^2) + e*(b*d - a*e)]] - (3*(128*c^4*d^4 + 3*b^4*e^4 + 8*b^2*c*e^3*(b*d - 3*a*e) - 192*c^3*d^2*e*(b*d - a*e) + 48*c^2*e^2*(b*d - a*e)^2)*Log[b + 2*c*x^2 - 2*Sqrt[c]*Sqrt[a + b*x^2 + c*x^4]])/c^(5/2))/(768*e^5)

Maple [A] (verified)

Time = 0.68 (sec) , antiderivative size = 494, normalized size of antiderivative = 1.37

method	result
pseudoelliptic	$\frac{\left(e^2(ae-bd)^2c^{\frac{7}{2}}+c^{\frac{9}{2}}d^2\left(2a\,e^2-2bde+c\,d^2\right)\right)d\ln\left(\frac{2\sqrt{c\,x^4+b\,x^2+a}\,\sqrt{\frac{a\,e^2-bde+c\,d^2}{e^2}}\,e+\left(b\,x^2+2a\right)e-d\left(2c\,x^2+b\right)}{e\,x^2+d}\right)-e\sqrt{\frac{a\,e^2-bde+}{e^2}}}{}$
risch	$\frac{(48c^3e^3x^6+72b\,c^2e^3x^4-64c^3d\,e^2x^4+120a\,c^2e^3x^2+6b^2c\,e^3x^2-112b\,c^2d\,e^2x^2+96c^3d^2e\,x^2+60abc\,e^3-256a\,c^2d\,e^2-9b^3e^3-24}{384c^2e^4}$
default	Expression too large to display
elliptic	Expression too large to display

[In] int(x^3*(c*x^4+b*x^2+a)^(3/2)/(e*x^2+d),x,method=_RETURNVERBOSE)

[Out] 1/2/c^(7/2)*((e^2*(a*e-b*d)^2*c^(7/2)+c^(9/2)*d^2*(2*a*e^2-2*b*d*e+c*d^2))*d*ln((2*(c*x^4+b*x^2+a)^(1/2)*((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*e+(b*x^2+2*a)*e-d*(2*c*x^2+b))/(e*x^2+d))-e*((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*(-3/8*((a*c-1/4*b^2)^2*e^4-2*c*d*(a*c-1/12*b^2)*b*e^3+(4*a*c^3+b^2*c^2)*d^2*e^2-4*b*c^3*e*d^3+8/3*c^4*d^4)*c*ln((2*c*x^2+2*(c*x^4+b*x^2+a)^(1/2)*c^(1/2)+b)/c^(1/2))+(4/3*e*(-15/32*(3/5*b*x^2+a)*x^2*e^2+d*(7/16*b*x^2+a)*e-15/16*b*d^2)*c^(

```
7/2)+3/64*c^(3/2)*b^3*e^3+c^(5/2)*(1/4*(-c^2*x^6-5/4*b*(1/10*b*x^2+a))*e^3+
1/8*d*(8/3*c^2*x^4+b^2)*e^2-1/2*d^2*e*c^2*x^2+c^2*d^3))*e*(c*x^4+b*x^2+a)^(
1/2)+3/8*((a*c-1/4*b^2)^2*e^4-2*c*d*(a*c-1/12*b^2)*b*e^3+(4*a*c^3+b^2*c^2)*
d^2*e^2-4*b*c^3*e*d^3+8/3*c^4*d^4)*ln(2)*c))/((a*e^2-b*d*e+c*d^2)/e^2)^(1/2
)/e^6
```

Fricas [F(-1)]

Timed out.

$$\int \frac{x^3(a+bx^2+cx^4)^{3/2}}{d+ex^2}\,dx = \text{Timed out}$$

```
[In] integrate(x^3*(c*x^4+b*x^2+a)^(3/2)/(e*x^2+d),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]

$$\int \frac{x^3(a+bx^2+cx^4)^{3/2}}{d+ex^2}\,dx = \int \frac{x^3(a+bx^2+cx^4)^{\frac{3}{2}}}{d+ex^2}\,dx$$

```
[In] integrate(x**3*(c*x**4+b*x**2+a)**(3/2)/(e*x**2+d),x)
```

```
[Out] Integral(x**3*(a + b*x**2 + c*x**4)**(3/2)/(d + e*x**2), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^3(a+bx^2+cx^4)^{3/2}}{d+ex^2}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(x^3*(c*x^4+b*x^2+a)^(3/2)/(e*x^2+d),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [F(-2)]

Exception generated.

$$\int \frac{x^3(a+bx^2+cx^4)^{3/2}}{d+ex^2}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(x^3*(c*x^4+b*x^2+a)^(3/2)/(e*x^2+d),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument Type
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3(a+bx^2+cx^4)^{3/2}}{d+ex^2}\,dx = \int \frac{x^3\,(c\,x^4+b\,x^2+a)^{3/2}}{e\,x^2+d}\,dx$$

```
[In] int((x^3*(a + b*x^2 + c*x^4)^(3/2))/(d + e*x^2),x)
```

```
[Out] int((x^3*(a + b*x^2 + c*x^4)^(3/2))/(d + e*x^2), x)
```

3.324 $\int \frac{x(a+bx^2+cx^4)^{3/2}}{d+ex^2} dx$

Optimal result

Integrand size = 27, antiderivative size = 269

$$\int \frac{x(a+bx^2+cx^4)^{3/2}}{d+ex^2} dx = \frac{(8c^2d^2+b^2e^2-2ce(5bd-4ae)-2ce(2cd-be)x^2)\sqrt{a+bx^2+cx^4}}{16ce^3} + \frac{(a+bx^2+cx^4)^{3/2}}{6e} - \frac{(2cd-be)(8c^2d^2-b^2e^2-4ce(2bd-3ae))\operatorname{arctanh}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{32c^{3/2}e^4} + \frac{(cd^2-bde+ae^2)^{3/2}\operatorname{arctanh}\left(\frac{bd-2ae+(2cd-be)x^2}{2\sqrt{cd^2-bde+ae^2}\sqrt{a+bx^2+cx^4}}\right)}{2e^4}$$

```
[Out] 1/6*(c*x^4+b*x^2+a)^(3/2)/e-1/32*(-b*e+2*c*d)*(8*c^2*d^2-b^2*e^2-4*c*e*(-3*
a*e+2*b*d))*arctanh(1/2*(2*c*x^2+b)/c^(1/2)/(c*x^4+b*x^2+a)^(1/2))/c^(3/2)/
e^4+1/2*(a*e^2-b*d*e+c*d^2)^(3/2)*arctanh(1/2*(b*d-2*a*e+(-b*e+2*c*d)*x^2)/
(a*e^2-b*d*e+c*d^2)^(1/2)/(c*x^4+b*x^2+a)^(1/2))/e^4+1/16*(8*c^2*d^2+b^2*e^
2-2*c*e*(-4*a*e+5*b*d)-2*c*e*(-b*e+2*c*d)*x^2)*(c*x^4+b*x^2+a)^(1/2)/c/e^3
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 269, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259, Rules used

$= \{1261, 748, 828, 857, 635, 212, 738\}$

$$\int \frac{x(a+bx^2+cx^4)^{3/2}}{d+ex^2}\,dx = -\frac{(2cd-be)\text{arctanh}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)(-4ce(2bd-3ae)-b^2e^2+8c^2d^2)}{32c^{3/2}e^4} + \frac{(ae^2-bde+cd^2)^{3/2}\,\text{arctanh}\left(\frac{-2ae+x^2(2cd-be)+bd}{2\sqrt{a+bx^2+cx^4}\sqrt{ae^2-bde+cd^2}}\right)}{2e^4} + \frac{\sqrt{a+bx^2+cx^4}(-2ce(5bd-4ae)+b^2e^2-2cex^2(2cd-be)+8c^2d^2)}{16ce^3} + \frac{(a+bx^2+cx^4)^{3/2}}{6e}$$

```
[In] Int[(x*(a + b*x^2 + c*x^4)^(3/2))/(d + e*x^2),x]
```

```
[Out] ((8*c^2*d^2 + b^2*e^2 - 2*c*e*(5*b*d - 4*a*e) - 2*c*e*(2*c*d - b*e)*x^2)*Sq
rt[a + b*x^2 + c*x^4])/(16*c*e^3) + (a + b*x^2 + c*x^4)^(3/2)/(6*e) - ((2*c
*d - b*e)*(8*c^2*d^2 - b^2*e^2 - 4*c*e*(2*b*d - 3*a*e))*ArcTanh[(b + 2*c*x^
2)/(2*Sqrt[c]*Sqrt[a + b*x^2 + c*x^4])])/(32*c^(3/2)*e^4) + ((c*d^2 - b*d*e
 + a*e^2)^(3/2)*ArcTanh[(b*d - 2*a*e + (2*c*d - b*e)*x^2)/(2*Sqrt[c*d^2 - b
*d*e + a*e^2]*Sqrt[a + b*x^2 + c*x^4])])/(2*e^4)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 635

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 748

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*((a + b*x + c*x^2)^p/(e*(m + 2*p + 1))), x
```

```
] - Dist[p/(e*(m + 2*p + 1)), Int[(d + e*x)^m*Simp[b*d - 2*a*e + (2*c*d - b
*e)*x, x]*(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e
, 0] && GtQ[p, 0] && NeQ[m + 2*p + 1, 0] && ( !RationalQ[m] || LtQ[m, 1]) &
&  !ILtQ[m + 2*p, 0] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 828

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2)
 - g*(c*d + 2*c*d*p - b*e*p) + g*c*e*(m + 2*p + 1)*x)*((a + b*x + c*x^2)^p/
(c*e^2*(m + 2*p + 1)*(m + 2*p + 2))), x] - Dist[p/(c*e^2*(m + 2*p + 1)*(m +
 2*p + 2)), Int[(d + e*x)^m*(a + b*x + c*x^2)^(p - 1)*Simp[c*e*f*(b*d - 2*a
*e)*(m + 2*p + 2) + g*(a*e*(b*e - 2*c*d*m + b*e*m) + b*d*(b*e*p - c*d - 2*c
*d*p)) + (c*e*f*(2*c*d - b*e)*(m + 2*p + 2) + g*(b^2*e^2*(p + m + 1) - 2*c^
2*d^2*(1 + 2*p) - c*e*(b*d*(m - 2*p) + 2*a*e*(m + 2*p + 1))))*x, x], x], x]
 /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
- b*d*e + a*e^2, 0] && GtQ[p, 0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[
m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p]
|| IntegersQ[2*m, 2*p])
```

Rule 857

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1261

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(
p_.), x_Symbol] :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x],
 x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]
```

Rubi steps

$$\text{integral} = \frac{1}{2}\text{Subst}\left(\int \frac{(a+bx+cx^2)^{3/2}}{d+ex}\,dx, x, x^2\right)$$

$$= \frac{(a+bx^2+cx^4)^{3/2}}{6e} - \frac{\text{Subst}\left(\int \frac{(bd-2ae+(2cd-be)x)\sqrt{a+bx+cx^2}}{d+ex}\,dx, x, x^2\right)}{4e}$$

$$
\begin{aligned}
&= \frac{(8c^2d^2+b^2e^2-2ce(5bd-4ae)-2ce(2cd-be)x^2)\sqrt{a+bx^2+cx^4}}{16ce^3}+\frac{(a+bx^2+cx^4)^{3/2}}{6e} \\
&+ \frac{\text{Subst}\left(\int \frac{\frac{1}{2}(4ce(bd-2ae)^2-d(2cd-be)(4bcd-b^2e-4ace))-\frac{1}{2}(2cd-be)(8c^2d^2-b^2e^2-4ce(2bd-3ae))x}{(d+ex)\sqrt{a+bx+cx^2}}\,dx,x,x^2\right)}{16ce^3} \\
&= \frac{(8c^2d^2+b^2e^2-2ce(5bd-4ae)-2ce(2cd-be)x^2)\sqrt{a+bx^2+cx^4}}{16ce^3} \\
&+ \frac{(a+bx^2+cx^4)^{3/2}}{6e}+\frac{(cd^2-bde+ae^2)^2\,\text{Subst}\left(\int \frac{1}{(d+ex)\sqrt{a+bx+cx^2}}\,dx,x,x^2\right)}{2e^4} \\
&- \frac{((2cd-be)(8c^2d^2-b^2e^2-4ce(2bd-3ae)))\,\text{Subst}\left(\int \frac{1}{\sqrt{a+bx+cx^2}}\,dx,x,x^2\right)}{32ce^4} \\
&= \frac{(8c^2d^2+b^2e^2-2ce(5bd-4ae)-2ce(2cd-be)x^2)\sqrt{a+bx^2+cx^4}}{16ce^3} \\
&+ \frac{(a+bx^2+cx^4)^{3/2}}{6e}-\frac{(cd^2-bde+ae^2)^2\,\text{Subst}\left(\int \frac{1}{4cd^2-4bde+4ae^2-x^2}\,dx,x,\frac{-bd+2ae-(2cd-be)x^2}{\sqrt{a+bx^2+cx^4}}\right)}{e^4} \\
&- \frac{((2cd-be)(8c^2d^2-b^2e^2-4ce(2bd-3ae)))\,\text{Subst}\left(\int \frac{1}{4c-x^2}\,dx,x,\frac{b+2cx^2}{\sqrt{a+bx^2+cx^4}}\right)}{16ce^4} \\
&= \frac{(8c^2d^2+b^2e^2-2ce(5bd-4ae)-2ce(2cd-be)x^2)\sqrt{a+bx^2+cx^4}}{16ce^3} \\
&+ \frac{(a+bx^2+cx^4)^{3/2}}{6e} \\
&- \frac{(2cd-be)(8c^2d^2-b^2e^2-4ce(2bd-3ae))\tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{32c^{3/2}e^4} \\
&+ \frac{(cd^2-bde+ae^2)^{3/2}\tanh^{-1}\left(\frac{bd-2ae+(2cd-be)x^2}{2\sqrt{cd^2-bde+ae^2}\sqrt{a+bx^2+cx^4}}\right)}{2e^4}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 1.60 (sec) , antiderivative size = 266, normalized size of antiderivative = 0.99

$$
\int \frac{x(a+bx^2+cx^4)^{3/2}}{d+ex^2}\,dx = \frac{\frac{e\sqrt{a+bx^2+cx^4}(3b^2e^2+2ce(-15bd+16ae+7bex^2)+4c^2(6d^2-3dex^2+2e^2x^4))}{c}+48\sqrt{-cd^2+e(bd-}
$$

```
[In] Integrate[(x*(a + b*x^2 + c*x^4)^(3/2))/(d + e*x^2),x]
[Out] ((e*Sqrt[a + b*x^2 + c*x^4]*(3*b^2*e^2 + 2*c*e*(-15*b*d + 16*a*e + 7*b*e*x^
2) + 4*c^2*(6*d^2 - 3*d*e*x^2 + 2*e^2*x^4)))/c + 48*Sqrt[-(c*d^2) + e*(b*d
 - a*e)]*(c*d^2 + e*(-(b*d) + a*e))*ArcTan[(Sqrt[-(c*d^2) + e*(b*d - a*e)]*x
^2)/(Sqrt[a]*(d + e*x^2) - d*Sqrt[a + b*x^2 + c*x^4])] - (3*(2*c*d - b*e)*(
8*c^2*d^2 - b^2*e^2 + 4*c*e*(-2*b*d + 3*a*e))*ArcTanh[(Sqrt[c]*x^2)/(-Sqrt[
a] + Sqrt[a + b*x^2 + c*x^4])])/c^(3/2))/(48*e^4)
```

Maple [A] (verified)

Time = 0.54 (sec) , antiderivative size = 361, normalized size of antiderivative = 1.34

method	result
pseudoelliptic	$-\frac{\left(e^{2}(ae-bd)^{2} c^{\frac{3}{2}}+2\left(e^{2}d^{2}a-d^{3}eb\right)c^{\frac{5}{2}}+c^{\frac{7}{2}}d^{4}\right)\ln\left(\frac{2\sqrt{c\,x^{4}+b\,x^{2}+a}\,\sqrt{\frac{a\,e^{2}-bde+c\,d^{2}}{e^{2}}}\,e+\left(b\,x^{2}+2a\right)e-d\left(2c\,x^{2}+b\right)}{e\,x^{2}+d}\right)}{\cdots}-\frac{4e\left(\frac{9(be-2cd)\cdots}{\cdots}\right.}{\cdots}$
risch	$\frac{\left(8e^{2}c^{2}x^{4}+14bc\,e^{2}x^{2}-12c^{2}de\,x^{2}+32e^{2}ac+3b^{2}e^{2}-30bcde+24c^{2}d^{2}\right)\sqrt{c\,x^{4}+b\,x^{2}+a}}{48c\,e^{3}}+\frac{\left(12abc\,e^{3}-24a\,c^{2}d\,e^{2}-b^{3}e^{3}-6b^{2}cd\,e^{2}+24b\cdots\right.}{2\cdots}$
default	$-\frac{\left(a^{2}e^{4}-2abd\,e^{3}+2ac\,d^{2}e^{2}+b^{2}d^{2}e^{2}-2bc\,d^{3}e+c^{2}d^{4}\right)\ln\left(\frac{\frac{2a\,e^{2}-2bde+2c\,d^{2}}{e^{2}}+\frac{(be-2cd)\left(x^{2}+\frac{d}{e}\right)}{e}+2\sqrt{\frac{a\,e^{2}-bde+c\,d^{2}}{e^{2}}}\,\sqrt{c\left(x^{2}+\frac{d}{e}\right)^{2}+\cdots}}{x^{2}+\frac{d}{e}}\right.}{2e^{5}\sqrt{\frac{a\,e^{2}-bde+c\,d^{2}}{e^{2}}}}$
elliptic	$-\frac{\left(a^{2}e^{4}-2abd\,e^{3}+2ac\,d^{2}e^{2}+b^{2}d^{2}e^{2}-2bc\,d^{3}e+c^{2}d^{4}\right)\ln\left(\frac{\frac{2a\,e^{2}-2bde+2c\,d^{2}}{e^{2}}+\frac{(be-2cd)\left(x^{2}+\frac{d}{e}\right)}{e}+2\sqrt{\frac{a\,e^{2}-bde+c\,d^{2}}{e^{2}}}\,\sqrt{c\left(x^{2}+\frac{d}{e}\right)^{2}+\cdots}}{x^{2}+\frac{d}{e}}\right.}{2e^{5}\sqrt{\frac{a\,e^{2}-bde+c\,d^{2}}{e^{2}}}}$

```
[In] int(x*(c*x^4+b*x^2+a)^(3/2)/(e*x^2+d),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*((e^2*(a*e-b*d)^2*c^(3/2)+2*(a*d^2*e^2-b*d^3*e)*c^(5/2)+c^(7/2)*d^4)*l
n((2*(c*x^4+b*x^2+a)^(1/2)*((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*e+(b*x^2+2*a)*e-
d*(2*c*x^2+b))/(e*x^2+d))-4/3*e*(9/16*(b*e-2*c*d)*((a*c-1/12*b^2)*e^2-2/3*b
*c*d*e+2/3*c^2*d^2)*ln((2*c*x^2+2*(c*x^4+b*x^2+a)^(1/2)*c^(1/2)+b)/c^(1/2))
+e*(e*((7/16*b*x^2+a)*e-15/16*b*d)*c^(3/2)+1/4*(e^2*x^4-3/2*e*d*x^2+3*d^2)*
```

```
c^(5/2)+3/32*b^2*e^2*c^(1/2))*(c*x^4+b*x^2+a)^(1/2)-9/16*(b*e-2*c*d)*((a*c-
1/12*b^2)*e^2-2/3*b*c*d*e+2/3*c^2*d^2)*ln(2))*((a*e^2-b*d*e+c*d^2)/e^2)^(1/
2))/((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)/c^(3/2)/e^5
```

Fricas [A] (verification not implemented)

none

Time = 111.85 (sec) , antiderivative size = 1589, normalized size of antiderivative = 5.91

$$\int \frac{x(a+bx^2+cx^4)^{3/2}}{d+ex^2}\,dx = \text{Too large to display}$$

```
[In] integrate(x*(c*x^4+b*x^2+a)^(3/2)/(e*x^2+d),x, algorithm="fricas")
```

```
[Out] [-1/192*(3*(16*c^3*d^3 - 24*b*c^2*d^2*e + 6*(b^2*c + 4*a*c^2)*d*e^2 + (b^3
- 12*a*b*c)*e^3)*sqrt(c)*log(-8*c^2*x^4 - 8*b*c*x^2 - b^2 - 4*sqrt(c*x^4 +
b*x^2 + a)*(2*c*x^2 + b)*sqrt(c) - 4*a*c) - 48*(c^3*d^2 - b*c^2*d*e + a*c^2
*e^2)*sqrt(c*d^2 - b*d*e + a*e^2)*log(-((8*c^2*d^2 - 8*b*c*d*e + (b^2 + 4*a
*c)*e^2)*x^4 - 8*a*b*d*e + 8*a^2*e^2 + (b^2 + 4*a*c)*d^2 + 2*(4*b*c*d^2 + 4
*a*b*e^2 - (3*b^2 + 4*a*c)*d*e)*x^2 + 4*sqrt(c*x^4 + b*x^2 + a)*sqrt(c*d^2
- b*d*e + a*e^2)*((2*c*d - b*e)*x^2 + b*d - 2*a*e))/(e^2*x^4 + 2*d*e*x^2 +
d^2)) - 4*(8*c^3*e^3*x^4 + 24*c^3*d^2*e - 30*b*c^2*d*e^2 + (3*b^2*c + 32*a*
c^2)*e^3 - 2*(6*c^3*d*e^2 - 7*b*c^2*e^3)*x^2)*sqrt(c*x^4 + b*x^2 + a))/(c^2
*e^4), 1/96*(3*(16*c^3*d^3 - 24*b*c^2*d^2*e + 6*(b^2*c + 4*a*c^2)*d*e^2 + (
b^3 - 12*a*b*c)*e^3)*sqrt(-c)*arctan(1/2*sqrt(c*x^4 + b*x^2 + a)*(2*c*x^2 +
 b)*sqrt(-c)/(c^2*x^4 + b*c*x^2 + a*c)) + 24*(c^3*d^2 - b*c^2*d*e + a*c^2*e
^2)*sqrt(c*d^2 - b*d*e + a*e^2)*log(-((8*c^2*d^2 - 8*b*c*d*e + (b^2 + 4*a*c
)*e^2)*x^4 - 8*a*b*d*e + 8*a^2*e^2 + (b^2 + 4*a*c)*d^2 + 2*(4*b*c*d^2 + 4*a
*b*e^2 - (3*b^2 + 4*a*c)*d*e)*x^2 + 4*sqrt(c*x^4 + b*x^2 + a)*sqrt(c*d^2 -
b*d*e + a*e^2)*((2*c*d - b*e)*x^2 + b*d - 2*a*e))/(e^2*x^4 + 2*d*e*x^2 + d^
2)) + 2*(8*c^3*e^3*x^4 + 24*c^3*d^2*e - 30*b*c^2*d*e^2 + (3*b^2*c + 32*a*c^
2)*e^3 - 2*(6*c^3*d*e^2 - 7*b*c^2*e^3)*x^2)*sqrt(c*x^4 + b*x^2 + a))/(c^2*e
^4), 1/192*(96*(c^3*d^2 - b*c^2*d*e + a*c^2*e^2)*sqrt(-c*d^2 + b*d*e - a*e^
2)*arctan(-1/2*sqrt(c*x^4 + b*x^2 + a)*sqrt(-c*d^2 + b*d*e - a*e^2)*((2*c*d
 - b*e)*x^2 + b*d - 2*a*e)/((c^2*d^2 - b*c*d*e + a*c*e^2)*x^4 + a*c*d^2 - a
*b*d*e + a^2*e^2 + (b*c*d^2 - b^2*d*e + a*b*e^2)*x^2)) - 3*(16*c^3*d^3 - 24
*b*c^2*d^2*e + 6*(b^2*c + 4*a*c^2)*d*e^2 + (b^3 - 12*a*b*c)*e^3)*sqrt(c)*lo
g(-8*c^2*x^4 - 8*b*c*x^2 - b^2 - 4*sqrt(c*x^4 + b*x^2 + a)*(2*c*x^2 + b)*sq
rt(c) - 4*a*c) + 4*(8*c^3*e^3*x^4 + 24*c^3*d^2*e - 30*b*c^2*d*e^2 + (3*b^2*
c + 32*a*c^2)*e^3 - 2*(6*c^3*d*e^2 - 7*b*c^2*e^3)*x^2)*sqrt(c*x^4 + b*x^2 +
 a))/(c^2*e^4), 1/96*(48*(c^3*d^2 - b*c^2*d*e + a*c^2*e^2)*sqrt(-c*d^2 + b*
d*e - a*e^2)*arctan(-1/2*sqrt(c*x^4 + b*x^2 + a)*sqrt(-c*d^2 + b*d*e - a*e^
2)*((2*c*d - b*e)*x^2 + b*d - 2*a*e)/((c^2*d^2 - b*c*d*e + a*c*e^2)*x^4 + a
*c*d^2 - a*b*d*e + a^2*e^2 + (b*c*d^2 - b^2*d*e + a*b*e^2)*x^2)) + 3*(16*c^
```

```
3*d^3 - 24*b*c^2*d^2*e + 6*(b^2*c + 4*a*c^2)*d*e^2 + (b^3 - 12*a*b*c)*e^3)*
sqrt(-c)*arctan(1/2*sqrt(c*x^4 + b*x^2 + a)*(2*c*x^2 + b)*sqrt(-c)/(c^2*x^4
 + b*c*x^2 + a*c)) + 2*(8*c^3*e^3*x^4 + 24*c^3*d^2*e - 30*b*c^2*d*e^2 + (3*
b^2*c + 32*a*c^2)*e^3 - 2*(6*c^3*d*e^2 - 7*b*c^2*e^3)*x^2)*sqrt(c*x^4 + b*x
^2 + a))/(c^2*e^4)]
```

Sympy [F]

$$\int \frac{x(a + bx^2 + cx^4)^{3/2}}{d + ex^2} \, dx = \int \frac{x(a + bx^2 + cx^4)^{\frac{3}{2}}}{d + ex^2} \, dx$$

```
[In] integrate(x*(c*x**4+b*x**2+a)**(3/2)/(e*x**2+d),x)
[Out] Integral(x*(a + b*x**2 + c*x**4)**(3/2)/(d + e*x**2), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x(a + bx^2 + cx^4)^{3/2}}{d + ex^2} \, dx = \text{Exception raised: ValueError}$$

```
[In] integrate(x*(c*x^4+b*x^2+a)^(3/2)/(e*x^2+d),x, algorithm="maxima")
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*e^2-b*d*e>0)', see `assume?` for
more de
```

Giac [F(-2)]

Exception generated.

$$\int \frac{x(a + bx^2 + cx^4)^{3/2}}{d + ex^2} \, dx = \text{Exception raised: TypeError}$$

```
[In] integrate(x*(c*x^4+b*x^2+a)^(3/2)/(e*x^2+d),x, algorithm="giac")
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument Type
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x(a+bx^2+cx^4)^{3/2}}{d+ex^2}\,dx = \int \frac{x\,(c\,x^4+b\,x^2+a)^{3/2}}{e\,x^2+d}\,dx$$

```
[In] int((x*(a + b*x^2 + c*x^4)^(3/2))/(d + e*x^2),x)
[Out] int((x*(a + b*x^2 + c*x^4)^(3/2))/(d + e*x^2), x)
```

3.325 $\int \frac{(a+bx^2+cx^4)^{3/2}}{x(d+ex^2)}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 350

$$\int \frac{(a+bx^2+cx^4)^{3/2}}{x(d+ex^2)}\,dx = \frac{a\sqrt{a+bx^2+cx^4}}{2d} - \frac{(4cd^2-e(5bd-4ae)-2cdex^2)\sqrt{a+bx^2+cx^4}}{8de^2} - \frac{a^{3/2}\operatorname{arctanh}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{2d} + \frac{ab\operatorname{arctanh}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{4\sqrt{c}d} + \frac{(8c^2d^3+be^2(3bd-4ae)-12cde(bd-ae))\operatorname{arctanh}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{16\sqrt{c}de^3} - \frac{(cd^2-bde+ae^2)^{3/2}\operatorname{arctanh}\left(\frac{bd-2ae+(2cd-be)x^2}{2\sqrt{cd^2-bde+ae^2}\sqrt{a+bx^2+cx^4}}\right)}{2de^3}$$

```
[Out] -1/2*a^(3/2)*arctanh(1/2*(b*x^2+2*a)/a^(1/2)/(c*x^4+b*x^2+a)^(1/2))/d-1/2*(
a*e^2-b*d*e+c*d^2)^(3/2)*arctanh(1/2*(b*d-2*a*e+(-b*e+2*c*d)*x^2)/(a*e^2-b*
d*e+c*d^2)^(1/2)/(c*x^4+b*x^2+a)^(1/2))/d/e^3+1/4*a*b*arctanh(1/2*(2*c*x^2+
b)/c^(1/2)/(c*x^4+b*x^2+a)^(1/2))/d/c^(1/2)+1/16*(8*c^2*d^3+b*e^2*(-4*a*e+3
*b*d)-12*c*d*e*(-a*e+b*d))*arctanh(1/2*(2*c*x^2+b)/c^(1/2)/(c*x^4+b*x^2+a)^
(1/2))/d/e^3/c^(1/2)+1/2*a*(c*x^4+b*x^2+a)^(1/2)/d-1/8*(4*c*d^2-e*(-4*a*e+5
*b*d)-2*c*d*e*x^2)*(c*x^4+b*x^2+a)^(1/2)/d/e^2
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 350, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.276, Rules used = {1265, 909, 748, 857, 635, 212, 738, 828}

$$\int \frac{(a+bx^2+cx^4)^{3/2}}{x(d+ex^2)}\,dx = -\frac{a^{3/2}\operatorname{arctanh}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{2d} + \frac{\operatorname{arctanh}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)(-12cde(bd-ae)+be^2(3bd-4ae)+8c^2d^3)}{16\sqrt{c}de^3} - \frac{(ae^2-bde+cd^2)^{3/2}\operatorname{arctanh}\left(\frac{-2ae+x^2(2cd-be)+bd}{2\sqrt{a+bx^2+cx^4}\sqrt{ae^2-bde+cd^2}}\right)}{2de^3} + \frac{ab\operatorname{arctanh}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{4\sqrt{c}d} - \frac{\sqrt{a+bx^2+cx^4}(-e(5bd-4ae)+4cd^2-2cdex^2)}{8de^2} + \frac{a\sqrt{a+bx^2+cx^4}}{2d}$$

```
[In] Int[(a + b*x^2 + c*x^4)^(3/2)/(x*(d + e*x^2)),x]
```

```
[Out] (a*Sqrt[a + b*x^2 + c*x^4])/(2*d) - ((4*c*d^2 - e*(5*b*d - 4*a*e) - 2*c*d*e
*x^2)*Sqrt[a + b*x^2 + c*x^4])/(8*d*e^2) - (a^(3/2)*ArcTanh[(2*a + b*x^2)/(
2*Sqrt[a]*Sqrt[a + b*x^2 + c*x^4])])/(2*d) + (a*b*ArcTanh[(b + 2*c*x^2)/(2*
Sqrt[c]*Sqrt[a + b*x^2 + c*x^4])])/(4*Sqrt[c]*d) + ((8*c^2*d^3 + b*e^2*(3*b
*d - 4*a*e) - 12*c*d*e*(b*d - a*e))*ArcTanh[(b + 2*c*x^2)/(2*Sqrt[c]*Sqrt[a
 + b*x^2 + c*x^4])])/(16*Sqrt[c]*d*e^3) - ((c*d^2 - b*d*e + a*e^2)^(3/2)*Ar
cTanh[(b*d - 2*a*e + (2*c*d - b*e)*x^2)/(2*Sqrt[c*d^2 - b*d*e + a*e^2]*Sqrt
[a + b*x^2 + c*x^4])])/(2*d*e^3)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 635

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
```

```
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 748

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*((a + b*x + c*x^2)^p/(e*(m + 2*p + 1))), x
] - Dist[p/(e*(m + 2*p + 1)), Int[(d + e*x)^m*Simp[b*d - 2*a*e + (2*c*d - b
*e)*x, x]*(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e
, 0] && GtQ[p, 0] && NeQ[m + 2*p + 1, 0] && ( !RationalQ[m] || LtQ[m, 1]) &
&  !ILtQ[m + 2*p, 0] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 828

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2)
 - g*(c*d + 2*c*d*p - b*e*p) + g*c*e*(m + 2*p + 1)*x)*((a + b*x + c*x^2)^p/
(c*e^2*(m + 2*p + 1)*(m + 2*p + 2))), x] - Dist[p/(c*e^2*(m + 2*p + 1)*(m +
 2*p + 2)), Int[(d + e*x)^m*(a + b*x + c*x^2)^(p - 1)*Simp[c*e*f*(b*d - 2*a
*e)*(m + 2*p + 2) + g*(a*e*(b*e - 2*c*d*m + b*e*m) + b*d*(b*e*p - c*d - 2*c
*d*p)) + (c*e*f*(2*c*d - b*e)*(m + 2*p + 2) + g*(b^2*e^2*(p + m + 1) - 2*c^
2*d^2*(1 + 2*p) - c*e*(b*d*(m - 2*p) + 2*a*e*(m + 2*p + 1))))*x, x], x], x]
 /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
- b*d*e + a*e^2, 0] && GtQ[p, 0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[
m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p]
|| IntegersQ[2*m, 2*p])
```

Rule 857

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 909

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)/(((d_.) + (e_.)*(x_))*((f_.) +
 (g_.)*(x_))), x_Symbol] :> Dist[(c*d^2 - b*d*e + a*e^2)/(e*(e*f - d*g)), I
nt[(a + b*x + c*x^2)^(p - 1)/(d + e*x), x], x] - Dist[1/(e*(e*f - d*g)), In
t[Simp[c*d*f - b*e*f + a*e*g - c*(e*f - d*g)*x, x]*((a + b*x + c*x^2)^(p -
1)/(f + g*x)), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g,
0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && FractionQ[p]
&& GtQ[p, 0]
```

Rule 1265

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rubi steps

$$\text{integral} = \frac{1}{2}\text{Subst}\left(\int \frac{(a+bx+cx^2)^{3/2}}{x(d+ex)}\,dx, x, x^2\right)$$

$$= -\frac{\text{Subst}\left(\int \frac{(-bd+ae-cdx)\sqrt{a+bx+cx^2}}{d+ex}\,dx, x, x^2\right)}{2d} + \frac{a\text{Subst}\left(\int \frac{\sqrt{a+bx+cx^2}}{x}\,dx, x, x^2\right)}{2d}$$

$$= \frac{a\sqrt{a+bx^2+cx^4}}{2d} - \frac{(4cd^2-e(5bd-4ae)-2cdex^2)\sqrt{a+bx^2+cx^4}}{8de^2} - \frac{a\text{Subst}\left(\int \frac{-2a-bx}{x\sqrt{a+bx+cx^2}}\,dx, x, x^2\right)}{4d} + \frac{\text{Subst}\left(\int \frac{\frac{1}{2}c(4bcd^3-5b^2d^2e-4acd^2e+12abde^2-8a^2e^3)+\frac{1}{2}c(8c^2d^3+be^2(3bd-4ae)-12cde(bd-ae))x}{(d+ex)\sqrt{a+bx+cx^2}}\,dx, x, x^2\right)}{8cde^2}$$

$$= \frac{a\sqrt{a+bx^2+cx^4}}{2d} - \frac{(4cd^2-e(5bd-4ae)-2cdex^2)\sqrt{a+bx^2+cx^4}}{8de^2} + \frac{a^2\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx+cx^2}}\,dx, x, x^2\right)}{2d} + \frac{(ab)\text{Subst}\left(\int \frac{1}{\sqrt{a+bx+cx^2}}\,dx, x, x^2\right)}{4d} - \frac{(cd^2-bde+ae^2)^2\,\text{Subst}\left(\int \frac{1}{(d+ex)\sqrt{a+bx+cx^2}}\,dx, x, x^2\right)}{2de^3} + \frac{(8c^2d^3+be^2(3bd-4ae)-12cde(bd-ae))\,\text{Subst}\left(\int \frac{1}{\sqrt{a+bx+cx^2}}\,dx, x, x^2\right)}{16de^3}$$

$$= \frac{a\sqrt{a+bx^2+cx^4}}{2d} - \frac{(4cd^2-e(5bd-4ae)-2cdex^2)\sqrt{a+bx^2+cx^4}}{8de^2} - \frac{a^2\text{Subst}\left(\int \frac{1}{4a-x^2}\,dx, x, \frac{2a+bx^2}{\sqrt{a+bx^2+cx^4}}\right)}{d} + \frac{(ab)\text{Subst}\left(\int \frac{1}{4c-x^2}\,dx, x, \frac{b+2cx^2}{\sqrt{a+bx^2+cx^4}}\right)}{2d} + \frac{(cd^2-bde+ae^2)^2\,\text{Subst}\left(\int \frac{1}{4cd^2-4bde+4ae^2-x^2}\,dx, x, \frac{-bd+2ae-(2cd-be)x^2}{\sqrt{a+bx^2+cx^4}}\right)}{de^3} + \frac{(8c^2d^3+be^2(3bd-4ae)-12cde(bd-ae))\,\text{Subst}\left(\int \frac{1}{4c-x^2}\,dx, x, \frac{b+2cx^2}{\sqrt{a+bx^2+cx^4}}\right)}{8de^3}$$

$$= \frac{a\sqrt{a+bx^2+cx^4}}{2d} - \frac{(4cd^2-e(5bd-4ae)-2cdex^2)\sqrt{a+bx^2+cx^4}}{8de^2} - \frac{a^{3/2}\tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{2d} + \frac{ab\tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{4\sqrt{c}d} + \frac{(8c^2d^3+be^2(3bd-4ae)-12cde(bd-ae))\tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{16\sqrt{c}de^3} - \frac{(cd^2-bde+ae^2)^{3/2}\tanh^{-1}\left(\frac{bd-2ae+(2cd-be)x^2}{2\sqrt{cd^2-bde+ae^2}\sqrt{a+bx^2+cx^4}}\right)}{2de^3}$$

Mathematica [A] (verified)

Time = 0.95 (sec) , antiderivative size = 258, normalized size of antiderivative = 0.74

$$\int \frac{(a+bx^2+cx^4)^{3/2}}{x(d+ex^2)}\,dx = \frac{(-4cd+5be+2cex^2)\sqrt{a+bx^2+cx^4}}{8e^2} - \frac{\sqrt{-cd^2+e(bd-ae)}(cd^2+e(-bd+ae))\arctan\left(\frac{\sqrt{c}(d+ex^2)-e\sqrt{a+bx^2+cx^4}}{\sqrt{-cd^2+e(bd-ae)}}\right)}{de^3} + \frac{a^{3/2}\text{arctanh}\left(\frac{\sqrt{c}x^2-\sqrt{a+bx^2+cx^4}}{\sqrt{a}}\right)}{d} - \frac{(8c^2d^2+3b^2e^2+12ce(-bd+ae))\log\left(b+2cx^2-2\sqrt{c}\sqrt{a+bx^2+cx^4}\right)}{16\sqrt{c}e^3}$$

```
[In] Integrate[(a + b*x^2 + c*x^4)^(3/2)/(x*(d + e*x^2)),x]
```

```
[Out] ((-4*c*d + 5*b*e + 2*c*e*x^2)*Sqrt[a + b*x^2 + c*x^4])/(8*e^2) - (Sqrt[-(c*
d^2) + e*(b*d - a*e)]*(c*d^2 + e*(-(b*d) + a*e))*ArcTan[(Sqrt[c]*(d + e*x^2
) - e*Sqrt[a + b*x^2 + c*x^4])/Sqrt[-(c*d^2) + e*(b*d - a*e)]])/(d*e^3) + (
a^(3/2)*ArcTanh[(Sqrt[c]*x^2 - Sqrt[a + b*x^2 + c*x^4])/Sqrt[a]])/d - ((8*c
^2*d^2 + 3*b^2*e^2 + 12*c*e*(-(b*d) + a*e))*Log[b + 2*c*x^2 - 2*Sqrt[c]*Sqr
t[a + b*x^2 + c*x^4]])/(16*Sqrt[c]*e^3)
```

Maple [A] (verified)

Time = 0.47 (sec) , antiderivative size = 360, normalized size of antiderivative = 1.03

method	result
pseudoelliptic	$\frac{\left(e^{2}(ae-bd)^{2}\sqrt{c}+2\left(e^{2}d^{2}a-d^{3}eb\right)c^{\frac{3}{2}}+c^{\frac{5}{2}}d^{4}\right)\ln\left(\frac{2\sqrt{c\,x^{4}+b\,x^{2}+a}\,\sqrt{\frac{a\,e^{2}-bde+c\,d^{2}}{e^{2}}}\,e+\left(b\,x^{2}+2a\right)e-d\left(2c\,x^{2}+b\right)}{e\,x^{2}+d}\right)+\frac{3e\left(d\left(\left(ac+\frac{b^{2}}{4}\right.\right.\right.}{}}{}$
elliptic	$-\frac{a^{\frac{3}{2}}\ln\left(\frac{2a+b\,x^{2}+2\sqrt{a}\,\sqrt{c\,x^{4}+b\,x^{2}+a}}{x^{2}}\right)}{2d}+\frac{\frac{b^{2}e^{2}\ln\left(\frac{\frac{b}{2}+c\,x^{2}}{\sqrt{c}}+\sqrt{c\,x^{4}+b\,x^{2}+a}\right)}{\sqrt{c}}+c^{\frac{3}{2}}d^{2}\ln\left(\frac{\frac{b}{2}+c\,x^{2}}{\sqrt{c}}+\sqrt{c\,x^{4}+b\,x^{2}+a}\right)+e^{2}c^{2}\left(\frac{x^{2}}{}\right.}{}$
default	Expression too large to display

```
[In] int((c*x^4+b*x^2+a)^(3/2)/x/(e*x^2+d),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2/((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)/c^(1/2)*((e^2*(a*e-b*d)^2*c^(1/2)+2*(a*
d^2*e^2-b*d^3*e)*c^(3/2)+c^(5/2)*d^4)*ln((2*(c*x^4+b*x^2+a)^(1/2)*((a*e^2-b
*d*e+c*d^2)/e^2)^(1/2)*e+(b*x^2+2*a)*e-d*(2*c*x^2+b))/(e*x^2+d))+3/2*e*(d*(
(a*c+1/4*b^2)*e^2-b*c*d*e+2/3*c^2*d^2)*ln((2*c*x^2+2*(c*x^4+b*x^2+a)^(1/2)*
c^(1/2)+b)/c^(1/2))-2/3*ln((2*a+b*x^2+2*a^(1/2)*(c*x^4+b*x^2+a)^(1/2))/x^2)
*a^(3/2)*e^3*c^(1/2)-(-5/6*e*(2/5*(e*x^2-2*d)*c^(3/2)+b*c^(1/2)*e)*(c*x^4+b
*x^2+a)^(1/2)+ln(2)*((a*c+1/4*b^2)*e^2-b*c*d*e+2/3*c^2*d^2))*d)*((a*e^2-b*d
*e+c*d^2)/e^2)^(1/2))/e^4/d
```

Fricas [F(-1)]

Timed out.

$$\int \frac{(a+bx^2+cx^4)^{3/2}}{x\,(d+ex^2)}\,dx = \text{Timed out}$$

```
[In] integrate((c*x^4+b*x^2+a)^(3/2)/x/(e*x^2+d),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]

$$\int \frac{(a+bx^2+cx^4)^{3/2}}{x(d+ex^2)}\,dx = \int \frac{(a+bx^2+cx^4)^{\frac{3}{2}}}{x(d+ex^2)}\,dx$$

```
[In] integrate((c*x**4+b*x**2+a)**(3/2)/x/(e*x**2+d),x)
[Out] Integral((a + b*x**2 + c*x**4)**(3/2)/(x*(d + e*x**2)), x)
```

Maxima [F]

$$\int \frac{(a+bx^2+cx^4)^{3/2}}{x(d+ex^2)}\,dx = \int \frac{(cx^4+bx^2+a)^{\frac{3}{2}}}{(ex^2+d)x}\,dx$$

```
[In] integrate((c*x^4+b*x^2+a)^(3/2)/x/(e*x^2+d),x, algorithm="maxima")
[Out] integrate((c*x^4 + b*x^2 + a)^(3/2)/((e*x^2 + d)*x), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(a+bx^2+cx^4)^{3/2}}{x(d+ex^2)}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate((c*x^4+b*x^2+a)^(3/2)/x/(e*x^2+d),x, algorithm="giac")
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument Type
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^2+cx^4)^{3/2}}{x(d+ex^2)}\,dx = \int \frac{(c\,x^4+b\,x^2+a)^{3/2}}{x\,(e\,x^2+d)}\,dx$$

```
[In] int((a + b*x^2 + c*x^4)^(3/2)/(x*(d + e*x^2)),x)
[Out] int((a + b*x^2 + c*x^4)^(3/2)/(x*(d + e*x^2)), x)
```

3.326 $\int \frac{(a+bx^2+cx^4)^{3/2}}{x^3(d+ex^2)}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 562

$$
\begin{aligned}
\int \frac{(a+bx^2+cx^4)^{3/2}}{x^3(d+ex^2)}\,dx &= \frac{3(3b+2cx^2)\sqrt{a+bx^2+cx^4}}{8d} \\
&- \frac{e(b^2+8ac+2bcx^2)\sqrt{a+bx^2+cx^4}}{16cd^2} \\
&+ \frac{(8c^2d^2+b^2e^2-2ce(5bd-4ae)-2ce(2cd-be)x^2)\sqrt{a+bx^2+cx^4}}{16cd^2e} \\
&- \frac{(a+bx^2+cx^4)^{3/2}}{2dx^2} - \frac{3\sqrt{a}b\operatorname{arctanh}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{4d} \\
&+ \frac{a^{3/2}e\operatorname{arctanh}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{2d^2} + \frac{3(b^2+4ac)\operatorname{arctanh}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{16\sqrt{c}d} \\
&+ \frac{b(b^2-12ac)e\operatorname{arctanh}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{32c^{3/2}d^2} \\
&- \frac{(2cd-be)(8c^2d^2-b^2e^2-4ce(2bd-3ae))\operatorname{arctanh}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{32c^{3/2}d^2e^2} \\
&+ \frac{(cd^2-bde+ae^2)^{3/2}\operatorname{arctanh}\left(\frac{bd-2ae+(2cd-be)x^2}{2\sqrt{cd^2-bde+ae^2}\sqrt{a+bx^2+cx^4}}\right)}{2d^2e^2}
\end{aligned}
$$

```
[Out] -1/2*(c*x^4+b*x^2+a)^(3/2)/d/x^2+1/2*a^(3/2)*e*arctanh(1/2*(b*x^2+2*a)/a^(1
/2)/(c*x^4+b*x^2+a)^(1/2))/d^2+1/32*b*(-12*a*c+b^2)*e*arctanh(1/2*(2*c*x^2+
b)/c^(1/2)/(c*x^4+b*x^2+a)^(1/2))/c^(3/2)/d^2-1/32*(-b*e+2*c*d)*(8*c^2*d^2-
b^2*e^2-4*c*e*(-3*a*e+2*b*d))*arctanh(1/2*(2*c*x^2+b)/c^(1/2)/(c*x^4+b*x^2+
a)^(1/2))/c^(3/2)/d^2/e^2+1/2*(a*e^2-b*d*e+c*d^2)^(3/2)*arctanh(1/2*(b*d-2*
a*e+(-b*e+2*c*d)*x^2)/(a*e^2-b*d*e+c*d^2)^(1/2)/(c*x^4+b*x^2+a)^(1/2))/d^2/
```

```
e^2-3/4*b*arctanh(1/2*(b*x^2+2*a)/a^(1/2)/(c*x^4+b*x^2+a)^(1/2))*a^(1/2)/d+
3/16*(4*a*c+b^2)*arctanh(1/2*(2*c*x^2+b)/c^(1/2)/(c*x^4+b*x^2+a)^(1/2))/d/c
^(1/2)+3/8*(2*c*x^2+3*b)*(c*x^4+b*x^2+a)^(1/2)/d-1/16*e*(2*b*c*x^2+8*a*c+b^
2)*(c*x^4+b*x^2+a)^(1/2)/c/d^2+1/16*(8*c^2*d^2+b^2*e^2-2*c*e*(-4*a*e+5*b*d)
-2*c*e*(-b*e+2*c*d)*x^2)*(c*x^4+b*x^2+a)^(1/2)/c/d^2/e
```

Rubi [A] (verified)

Time = 0.62 (sec) , antiderivative size = 562, normalized size of antiderivative = 1.00, number of steps used = 24, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.310$, Rules used = {1265, 974, 746, 828, 857, 635, 212, 738, 748}

$$\begin{aligned}\int \frac{(a+bx^2+cx^4)^{3/2}}{x^3(d+ex^2)}\,dx &= \frac{a^{3/2}e\text{arctanh}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{2d^2} \\ &+ \frac{be(b^2-12ac)\,\text{arctanh}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{32c^{3/2}d^2} \\ &- \frac{(2cd-be)\text{arctanh}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)(-4ce(2bd-3ae)-b^2e^2+8c^2d^2)}{32c^{3/2}d^2e^2} \\ &+ \frac{3(4ac+b^2)\,\text{arctanh}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{16\sqrt{c}d} \\ &+ \frac{(ae^2-bde+cd^2)^{3/2}\,\text{arctanh}\left(\frac{-2ae+x^2(2cd-be)+bd}{2\sqrt{a+bx^2+cx^4}\sqrt{ae^2-bde+cd^2}}\right)}{2d^2e^2} \\ &- \frac{3\sqrt{a}b\text{arctanh}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{4d} \\ &+ \frac{\sqrt{a+bx^2+cx^4}(-2ce(5bd-4ae)+b^2e^2-2cex^2(2cd-be)+8c^2d^2)}{16cd^2e} \\ &- \frac{e(8ac+b^2+2bcx^2)\sqrt{a+bx^2+cx^4}}{16cd^2} \\ &- \frac{(a+bx^2+cx^4)^{3/2}}{2dx^2}+\frac{3(3b+2cx^2)\sqrt{a+bx^2+cx^4}}{8d}\end{aligned}$$

```
[In] Int[(a + b*x^2 + c*x^4)^(3/2)/(x^3*(d + e*x^2)),x]
```

```
[Out] (3*(3*b + 2*c*x^2)*Sqrt[a + b*x^2 + c*x^4])/(8*d) - (e*(b^2 + 8*a*c + 2*b*c
*x^2)*Sqrt[a + b*x^2 + c*x^4])/(16*c*d^2) + ((8*c^2*d^2 + b^2*e^2 - 2*c*e*(
5*b*d - 4*a*e) - 2*c*e*(2*c*d - b*e)*x^2)*Sqrt[a + b*x^2 + c*x^4])/(16*c*d^
2*e) - (a + b*x^2 + c*x^4)^(3/2)/(2*d*x^2) - (3*Sqrt[a]*b*ArcTanh[(2*a + b*
x^2)/(2*Sqrt[a]*Sqrt[a + b*x^2 + c*x^4])])/(4*d) + (a^(3/2)*e*ArcTanh[(2*a
+ b*x^2)/(2*Sqrt[a]*Sqrt[a + b*x^2 + c*x^4])])/(2*d^2) + (3*(b^2 + 4*a*c)*A
rcTanh[(b + 2*c*x^2)/(2*Sqrt[c]*Sqrt[a + b*x^2 + c*x^4])])/(16*Sqrt[c]*d) +
 (b*(b^2 - 12*a*c)*e*ArcTanh[(b + 2*c*x^2)/(2*Sqrt[c]*Sqrt[a + b*x^2 + c*x^
```

```
4])])/(32*c^(3/2)*d^2) - ((2*c*d - b*e)*(8*c^2*d^2 - b^2*e^2 - 4*c*e*(2*b*d
 - 3*a*e))*ArcTanh[(b + 2*c*x^2)/(2*Sqrt[c]*Sqrt[a + b*x^2 + c*x^4])])/(32*
c^(3/2)*d^2*e^2) + ((c*d^2 - b*d*e + a*e^2)^(3/2)*ArcTanh[(b*d - 2*a*e + (2
*c*d - b*e)*x^2)/(2*Sqrt[c*d^2 - b*d*e + a*e^2]*Sqrt[a + b*x^2 + c*x^4])])/
(2*d^2*e^2)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 635

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 746

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*((a + b*x + c*x^2)^p/(e*(m + 1))), x] - Di
st[p/(e*(m + 1)), Int[(d + e*x)^(m + 1)*(b + 2*c*x)*(a + b*x + c*x^2)^(p -
1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*
d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && GtQ[p, 0] && (IntegerQ[p]
 || LtQ[m, -1]) && NeQ[m, -1] &&  !ILtQ[m + 2*p + 1, 0] && IntQuadraticQ[a,
 b, c, d, e, m, p, x]
```

Rule 748

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*((a + b*x + c*x^2)^p/(e*(m + 2*p + 1))), x
] - Dist[p/(e*(m + 2*p + 1)), Int[(d + e*x)^m*Simp[b*d - 2*a*e + (2*c*d - b
*e)*x, x]*(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e
, 0] && GtQ[p, 0] && NeQ[m + 2*p + 1, 0] && ( !RationalQ[m] || LtQ[m, 1]) &
&  !ILtQ[m + 2*p, 0] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 828

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2)
 - g*(c*d + 2*c*d*p - b*e*p) + g*c*e*(m + 2*p + 1)*x)*((a + b*x + c*x^2)^p/
(c*e^2*(m + 2*p + 1)*(m + 2*p + 2))), x] - Dist[p/(c*e^2*(m + 2*p + 1)*(m +
 2*p + 2)), Int[(d + e*x)^m*(a + b*x + c*x^2)^(p - 1)*Simp[c*e*f*(b*d - 2*a
*e)*(m + 2*p + 2) + g*(a*e*(b*e - 2*c*d*m + b*e*m) + b*d*(b*e*p - c*d - 2*c
*d*p)) + (c*e*f*(2*c*d - b*e)*(m + 2*p + 2) + g*(b^2*e^2*(p + m + 1) - 2*c^
2*d^2*(1 + 2*p) - c*e*(b*d*(m - 2*p) + 2*a*e*(m + 2*p + 1))))*x, x], x], x]
 /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
- b*d*e + a*e^2, 0] && GtQ[p, 0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[
m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p]
|| IntegersQ[2*m, 2*p])
```

Rule 857

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 974

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g
*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ
[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && (
IntegerQ[p] || (ILtQ[m, 0] && ILtQ[n, 0])) &&  !(IGtQ[m, 0] || IGtQ[n, 0])
```

Rule 1265

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rubi steps

$$\text{integral} = \frac{1}{2}\text{Subst}\left(\int \frac{(a+bx+cx^2)^{3/2}}{x^2(d+ex)}\,dx, x, x^2\right)$$

$$= \frac{1}{2}\text{Subst}\left(\int \left(\frac{(a+bx+cx^2)^{3/2}}{dx^2} - \frac{e(a+bx+cx^2)^{3/2}}{d^2x} + \frac{e^2(a+bx+cx^2)^{3/2}}{d^2(d+ex)}\right)\,dx, x, x^2\right)$$

$$= \frac{\text{Subst}\left(\int \frac{(a+bx+cx^2)^{3/2}}{x^2}\,dx,x,x^2\right)}{2d} - \frac{e\text{Subst}\left(\int \frac{(a+bx+cx^2)^{3/2}}{x}\,dx,x,x^2\right)}{2d^2} + \frac{e^2\text{Subst}\left(\int \frac{(a+bx+cx^2)^{3/2}}{d+ex}\,dx,x,x^2\right)}{2d^2}$$

$$= -\frac{(a+bx^2+cx^4)^{3/2}}{2dx^2} + \frac{3\text{Subst}\left(\int \frac{(b+2cx)\sqrt{a+bx+cx^2}}{x}\,dx,x,x^2\right)}{4d} + \frac{e\text{Subst}\left(\int \frac{(-2a-bx)\sqrt{a+bx+cx^2}}{x}\,dx,x,x^2\right)}{4d^2} - \frac{e\text{Subst}\left(\int \frac{(bd-2ae+(2cd-be)x)\sqrt{a+bx+cx^2}}{d+ex}\,dx,x,x^2\right)}{4d^2}$$

$$= \frac{3(3b+2cx^2)\sqrt{a+bx^2+cx^4}}{8d} - \frac{e(b^2+8ac+2bcx^2)\sqrt{a+bx^2+cx^4}}{16cd^2} + \frac{(8c^2d^2+b^2e^2-2ce(5bd-4ae)-2ce(2cd-be)x^2)\sqrt{a+bx^2+cx^4}}{16cd^2e} - \frac{(a+bx^2+cx^4)^{3/2}}{2dx^2} - \frac{3\text{Subst}\left(\int \frac{-4abc-c(b^2+4ac)x}{x\sqrt{a+bx+cx^2}}\,dx,x,x^2\right)}{16cd} + \frac{\text{Subst}\left(\int \frac{\frac{1}{2}\left(4ce(bd-2ae)^2-d(2cd-be)\left(4bcd-b^2e-4ace\right)\right)-\frac{1}{2}(2cd-be)\left(8c^2d^2-b^2e^2-4ce(2bd-3ae)\right)x}{(d+ex)\sqrt{a+bx+cx^2}}\,dx,x,x^2\right)}{16cd^2e} - \frac{e\text{Subst}\left(\int \frac{8a^2c-\frac{1}{2}b\left(b^2-12ac\right)x}{x\sqrt{a+bx+cx^2}}\,dx,x,x^2\right)}{16cd^2}$$

$$= \frac{3(3b+2cx^2)\sqrt{a+bx^2+cx^4}}{8d} - \frac{e(b^2+8ac+2bcx^2)\sqrt{a+bx^2+cx^4}}{16cd^2} + \frac{(8c^2d^2+b^2e^2-2ce(5bd-4ae)-2ce(2cd-be)x^2)\sqrt{a+bx^2+cx^4}}{16cd^2e} - \frac{(a+bx^2+cx^4)^{3/2}}{2dx^2} + \frac{(3ab)\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx+cx^2}}\,dx,x,x^2\right)}{4d} + \frac{(3(b^2+4ac))\,\text{Subst}\left(\int \frac{1}{\sqrt{a+bx+cx^2}}\,dx,x,x^2\right)}{16d} - \frac{(a^2e)\,\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx+cx^2}}\,dx,x,x^2\right)}{2d^2} + \frac{(b(b^2-12ac)\,e)\,\text{Subst}\left(\int \frac{1}{\sqrt{a+bx+cx^2}}\,dx,x,x^2\right)}{32cd^2} + \frac{(cd^2-bde+ae^2)^2\,\text{Subst}\left(\int \frac{1}{(d+ex)\sqrt{a+bx+cx^2}}\,dx,x,x^2\right)}{2d^2e^2} - \frac{((2cd-be)\,(8c^2d^2-b^2e^2-4ce(2bd-3ae)))\,\text{Subst}\left(\int \frac{1}{\sqrt{a+bx+cx^2}}\,dx,x,x^2\right)}{32cd^2e^2}$$

$$= \frac{3(3b+2cx^2)\sqrt{a+bx^2+cx^4}}{8d} - \frac{e(b^2+8ac+2bcx^2)\sqrt{a+bx^2+cx^4}}{16cd^2} + \frac{(8c^2d^2+b^2e^2-2ce(5bd-4ae)-2ce(2cd-be)x^2)\sqrt{a+bx^2+cx^4}}{16cd^2e} - \frac{(a+bx^2+cx^4)^{3/2}}{2dx^2} - \frac{(3ab)\text{Subst}\left(\int \frac{1}{4a-x^2}\,dx,x,\frac{2a+bx^2}{\sqrt{a+bx^2+cx^4}}\right)}{2d} + \frac{(3(b^2+4ac))\,\text{Subst}\left(\int \frac{1}{4c-x^2}\,dx,x,\frac{b+2cx^2}{\sqrt{a+bx^2+cx^4}}\right)}{8d} + \frac{(a^2e)\,\text{Subst}\left(\int \frac{1}{4a-x^2}\,dx,x,\frac{2a+bx^2}{\sqrt{a+bx^2+cx^4}}\right)}{d^2} + \frac{(b(b^2-12ac)\,e)\,\text{Subst}\left(\int \frac{1}{4c-x^2}\,dx,x,\frac{b+2cx^2}{\sqrt{a+bx^2+cx^4}}\right)}{16cd^2} - \frac{(cd^2-bde+ae^2)^2\,\text{Subst}\left(\int \frac{1}{4cd^2-4bde+4ae^2-x^2}\,dx,x,\frac{-bd+2ae-(2cd-be)x^2}{\sqrt{a+bx^2+cx^4}}\right)}{d^2e^2} - \frac{((2cd-be)\,(8c^2d^2-b^2e^2-4ce(2bd-3ae)))\,\text{Subst}\left(\int \frac{1}{4c-x^2}\,dx,x,\frac{b+2cx^2}{\sqrt{a+bx^2+cx^4}}\right)}{16cd^2e^2}$$

$$
\begin{aligned}
&= \frac{3(3b+2cx^2)\sqrt{a+bx^2+cx^4}}{8d} - \frac{e(b^2+8ac+2bcx^2)\sqrt{a+bx^2+cx^4}}{16cd^2} \\
&+ \frac{(8c^2d^2+b^2e^2-2ce(5bd-4ae)-2ce(2cd-be)x^2)\sqrt{a+bx^2+cx^4}}{16cd^2e} \\
&- \frac{(a+bx^2+cx^4)^{3/2}}{2dx^2} - \frac{3\sqrt{a}b\tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{4d} \\
&+ \frac{a^{3/2}e\tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{2d^2} + \frac{3(b^2+4ac)\tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{16\sqrt{c}d} \\
&+ \frac{b(b^2-12ac)\,e\tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{32c^{3/2}d^2} \\
&- \frac{(2cd-be)(8c^2d^2-b^2e^2-4ce(2bd-3ae))\tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{32c^{3/2}d^2e^2} \\
&+ \frac{(cd^2-bde+ae^2)^{3/2}\tanh^{-1}\left(\frac{bd-2ae+(2cd-be)x^2}{2\sqrt{cd^2-bde+ae^2}\sqrt{a+bx^2+cx^4}}\right)}{2d^2e^2}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 1.10 (sec) , antiderivative size = 230, normalized size of antiderivative = 0.41

$$
\int \frac{(a+bx^2+cx^4)^{3/2}}{x^3(d+ex^2)}\,dx = \frac{-\frac{4(-cd^2+e(bd-ae))^{3/2}\arctan\left(\frac{\sqrt{c}(d+ex^2)-e\sqrt{a+bx^2+cx^4}}{\sqrt{-cd^2+e(bd-ae)}}\right)}{e^2} + 2\sqrt{a}(-3bd+2ae)\text{arctanh}\left(\frac{-\sqrt{c}
$$

```
[In] Integrate[(a + b*x^2 + c*x^4)^(3/2)/(x^3*(d + e*x^2)),x]
```

```
[Out] ((-4*(-(c*d^2) + e*(b*d - a*e))^(3/2)*ArcTan[(Sqrt[c]*(d + e*x^2) - e*Sqrt[
a + b*x^2 + c*x^4])/Sqrt[-(c*d^2) + e*(b*d - a*e)]])/e^2 + 2*Sqrt[a]*(-3*b*
d + 2*a*e)*ArcTanh[(-(Sqrt[c]*x^2) + Sqrt[a + b*x^2 + c*x^4])/Sqrt[a]] + (d
*(2*e*(-(a*e) + c*d*x^2)*Sqrt[a + b*x^2 + c*x^4] + Sqrt[c]*d*(2*c*d - 3*b*e
)*x^2*Log[b + 2*c*x^2 - 2*Sqrt[c]*Sqrt[a + b*x^2 + c*x^4]]))/(e^2*x^2))/(4*
d^2)
```

Maple [A] (verified)

Time = 0.56 (sec) , antiderivative size = 344, normalized size of antiderivative = 0.61

method	result
pseudoelliptic	$-\frac{\left(e^{2}(ae-bd)^{2}\sqrt{c}+(2e^{2}d^{2}a-2d^{3}eb)c^{\frac{3}{2}}+c^{\frac{5}{2}}d^{4}\right)x^{2}\ln\left(\frac{2\sqrt{c\,x^{4}+b\,x^{2}+a}\,\sqrt{\frac{a\,e^{2}-bde+c\,d^{2}}{e^{2}}}\,e+\left(b\,x^{2}+2a\right)e-d\left(2c\,x^{2}+b\right)}{e\,x^{2}+d}\right)+e\left(-\frac{3(be}{}\right.}{}$
risch	$-\frac{a\sqrt{c\,x^{4}+b\,x^{2}+a}}{2d\,x^{2}}+\frac{\frac{\sqrt{a}\,(2ae-3bd)\ln\left(\frac{2a+b\,x^{2}+2\sqrt{a}\,\sqrt{c\,x^{4}+b\,x^{2}+a}}{x^{2}}\right)}{2d}+\frac{2cd\left(ec\left(\frac{\sqrt{c\,x^{4}+b\,x^{2}+a}}{2c}-\frac{b\ln\left(\frac{\frac{b}{2}+c\,x^{2}}{\sqrt{c}}+\sqrt{c\,x^{4}+b\,x^{2}+a}\right)}{4c^{\frac{3}{2}}}\right)\right.}{e^{2}}}{}$
elliptic	$\frac{a^{2}\left(-\frac{\sqrt{c\,x^{4}+b\,x^{2}+a}}{a\,x^{2}}+\frac{b\ln\left(\frac{2a+b\,x^{2}+2\sqrt{a}\,\sqrt{c\,x^{4}+b\,x^{2}+a}}{x^{2}}\right)}{2a^{\frac{3}{2}}}\right)}{2d}+\frac{\sqrt{a}\,(ae-2bd)\ln\left(\frac{2a+b\,x^{2}+2\sqrt{a}\,\sqrt{c\,x^{4}+b\,x^{2}+a}}{x^{2}}\right)}{2d^{2}}+\frac{c\left(\frac{2be\ln\left(\frac{b}{2}\right.}{}\right.}{}$
default	Expression too large to display

```
[In] int((c*x^4+b*x^2+a)^(3/2)/x^3/(e*x^2+d),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*((e^2*(a*e-b*d)^2*c^(1/2)+(2*a*d^2*e^2-2*b*d^3*e)*c^(3/2)+c^(5/2)*d^4)
*x^2*ln((2*(c*x^4+b*x^2+a)^(1/2)*((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*e+(b*x^2+2
*a)*e-d*(2*c*x^2+b))/(e*x^2+d))+e*(-3/2*(b*e-2/3*c*d)*c*d^2*x^2*ln((2*c*x^2
+2*(c*x^4+b*x^2+a)^(1/2)*c^(1/2)+b)/c^(1/2))+3/2*e^2*(b*a^(1/2)*d-2/3*a^(3/
2)*e)*c^(1/2)*x^2*ln((2*a+b*x^2+2*a^(1/2)*(c*x^4+b*x^2+a)^(1/2))/x^2)+(e*(-
c^(3/2)*d*x^2+a*c^(1/2)*e)*(c*x^4+b*x^2+a)^(1/2)+3/2*(b*e-2/3*c*d)*ln(2)*c*
d*x^2)*d)*((a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)/
c^(1/2)/e^3/x^2/d^2
```

Fricas [F(-1)]

Timed out.

$$\int \frac{(a+bx^2+cx^4)^{3/2}}{x^3(d+ex^2)}\,dx = \text{Timed out}$$

```
[In] integrate((c*x^4+b*x^2+a)^(3/2)/x^3/(e*x^2+d),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]

$$\int \frac{(a+bx^2+cx^4)^{3/2}}{x^3(d+ex^2)}\,dx = \int \frac{(a+bx^2+cx^4)^{\frac{3}{2}}}{x^3(d+ex^2)}\,dx$$

```
[In] integrate((c*x**4+b*x**2+a)**(3/2)/x**3/(e*x**2+d),x)
[Out] Integral((a + b*x**2 + c*x**4)**(3/2)/(x**3*(d + e*x**2)), x)
```

Maxima [F]

$$\int \frac{(a+bx^2+cx^4)^{3/2}}{x^3(d+ex^2)}\,dx = \int \frac{(cx^4+bx^2+a)^{\frac{3}{2}}}{(ex^2+d)x^3}\,dx$$

```
[In] integrate((c*x^4+b*x^2+a)^(3/2)/x^3/(e*x^2+d),x, algorithm="maxima")
[Out] integrate((c*x^4 + b*x^2 + a)^(3/2)/((e*x^2 + d)*x^3), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(a+bx^2+cx^4)^{3/2}}{x^3(d+ex^2)}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate((c*x^4+b*x^2+a)^(3/2)/x^3/(e*x^2+d),x, algorithm="giac")
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument Type
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^2+cx^4)^{3/2}}{x^3(d+ex^2)}\,dx = \int \frac{(c\,x^4+b\,x^2+a)^{3/2}}{x^3\,(e\,x^2+d)}\,dx$$

```
[In] int((a + b*x^2 + c*x^4)^(3/2)/(x^3*(d + e*x^2)),x)
[Out] int((a + b*x^2 + c*x^4)^(3/2)/(x^3*(d + e*x^2)), x)
```

3.327 $\int \frac{x^2(1+2x^2+2x^4)^{3/2}}{3-2x^2}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 463

$$\int \frac{x^2(1+2x^2+2x^4)^{3/2}}{3-2x^2}\,dx = -\frac{213}{140}x\sqrt{1+2x^2+2x^4}-\frac{27}{70}x^3\sqrt{1+2x^2+2x^4} -\frac{2211x\sqrt{1+2x^2+2x^4}}{140\sqrt{2}\,(1+\sqrt{2}x^2)}-\frac{1}{14}x(1+2x^2+2x^4)^{3/2}+\frac{17}{16}\sqrt{51}\text{arctanh}\left(\frac{\sqrt{\frac{17}{3}}x}{\sqrt{1+2x^2+2x^4}}\right) +\frac{2211(1+\sqrt{2}x^2)\sqrt{\frac{1+2x^2+2x^4}{(1+\sqrt{2}x^2)^2}}E\left(2\arctan\left(\sqrt[4]{2}x\right)|\frac{1}{4}(2-\sqrt{2})\right)}{140\ 2^{3/4}\sqrt{1+2x^2+2x^4}} -\frac{3(514+2717\sqrt{2})(1+\sqrt{2}x^2)\sqrt{\frac{1+2x^2+2x^4}{(1+\sqrt{2}x^2)^2}}\text{EllipticF}\left(2\arctan\left(\sqrt[4]{2}x\right),\frac{1}{4}(2-\sqrt{2})\right)}{140\ 2^{3/4}(2+3\sqrt{2})\sqrt{1+2x^2+2x^4}} -\frac{289(3-\sqrt{2})(1+\sqrt{2}x^2)\sqrt{\frac{1+2x^2+2x^4}{(1+\sqrt{2}x^2)^2}}\text{EllipticPi}\left(\frac{1}{24}(12+11\sqrt{2}),2\arctan\left(\sqrt[4]{2}x\right),\frac{1}{4}(2-\sqrt{2})\right)}{16\ 2^{3/4}(2+3\sqrt{2})\sqrt{1+2x^2+2x^4}}$$

```
[Out] -1/14*x*(2*x^4+2*x^2+1)^(3/2)+17/16*arctanh(1/3*x*51^(1/2)/(2*x^4+2*x^2+1)^
(1/2))*51^(1/2)-213/140*x*(2*x^4+2*x^2+1)^(1/2)-27/70*x^3*(2*x^4+2*x^2+1)^(
1/2)-2211/280*x*(2*x^4+2*x^2+1)^(1/2)*2^(1/2)/(1+x^2*2^(1/2))+2211/280*(cos
(2*arctan(2^(1/4)*x))^2)^(1/2)/cos(2*arctan(2^(1/4)*x))*EllipticE(sin(2*arc
tan(2^(1/4)*x)),1/2*(2-2^(1/2))^(1/2))*(1+x^2*2^(1/2))*((2*x^4+2*x^2+1)/(1+
x^2*2^(1/2))^2)^(1/2)*2^(1/4)/(2*x^4+2*x^2+1)^(1/2)-289/32*(cos(2*arctan(2^
(1/4)*x))^2)^(1/2)/cos(2*arctan(2^(1/4)*x))*EllipticPi(sin(2*arctan(2^(1/4)
*x)),1/2+11/24*2^(1/2),1/2*(2-2^(1/2))^(1/2))*(3-2^(1/2))*(1+x^2*2^(1/2))*(
(2*x^4+2*x^2+1)/(1+x^2*2^(1/2))^2)^(1/2)*2^(1/4)/(2+3*2^(1/2))/(2*x^4+2*x^2
+1)^(1/2)-3/280*(cos(2*arctan(2^(1/4)*x))^2)^(1/2)/cos(2*arctan(2^(1/4)*x))
```

```
*EllipticF(sin(2*arctan(2^(1/4)*x)),1/2*(2-2^(1/2))^(1/2))*(514+2717*2^(1/2
))*(1+x^2*2^(1/2))*((2*x^4+2*x^2+1)/(1+x^2*2^(1/2))^2)^(1/2)*2^(1/4)/(2+3*2
^(1/2))/(2*x^4+2*x^2+1)^(1/2)
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 875, normalized size of antiderivative = 1.89, number of steps used = 19, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.310$, Rules used = {1349, 1105, 1190, 1211, 1117, 1209, 1222, 1230, 1720}

$$\int \frac{x^2(1+2x^2+2x^4)^{3/2}}{3-2x^2}\,dx =$$

$$-\frac{1}{14}x\left(2x^4+2x^2+1\right)^{3/2}-\frac{3}{35}x\left(x^2+2\right)\sqrt{2x^4+2x^2+1}-\frac{3}{20}x\left(2x^2+9\right)\sqrt{2x^4+2x^2+1}$$
$$-\frac{6\sqrt{2}x\sqrt{2x^4+2x^2+1}}{35\left(\sqrt{2}x^2+1\right)}-\frac{309x\sqrt{2x^4+2x^2+1}}{20\sqrt{2}\left(\sqrt{2}x^2+1\right)}+\frac{17}{16}\sqrt{51}\text{arctanh}\left(\frac{\sqrt{\frac{17}{3}}x}{\sqrt{2x^4+2x^2+1}}\right)$$
$$+\frac{6\sqrt[4]{2}\left(\sqrt{2}x^2+1\right)\sqrt{\frac{2x^4+2x^2+1}{\left(\sqrt{2}x^2+1\right)^2}}E\left(2\arctan\left(\sqrt[4]{2}x\right)|\frac{1}{4}\left(2-\sqrt{2}\right)\right)}{35\sqrt{2x^4+2x^2+1}}$$
$$+\frac{309\left(\sqrt{2}x^2+1\right)\sqrt{\frac{2x^4+2x^2+1}{\left(\sqrt{2}x^2+1\right)^2}}E\left(2\arctan\left(\sqrt[4]{2}x\right)|\frac{1}{4}\left(2-\sqrt{2}\right)\right)}{20\ 2^{3/4}\sqrt{2x^4+2x^2+1}}$$
$$-\frac{3\left(9+8\sqrt{2}\right)\left(\sqrt{2}x^2+1\right)\sqrt{\frac{2x^4+2x^2+1}{\left(\sqrt{2}x^2+1\right)^2}}\text{EllipticF}\left(2\arctan\left(\sqrt[4]{2}x\right),\frac{1}{4}\left(2-\sqrt{2}\right)\right)}{20\ 2^{3/4}\sqrt{2x^4+2x^2+1}}$$
$$-\frac{3\left(3+2\sqrt{2}\right)\left(\sqrt{2}x^2+1\right)\sqrt{\frac{2x^4+2x^2+1}{\left(\sqrt{2}x^2+1\right)^2}}\text{EllipticF}\left(2\arctan\left(\sqrt[4]{2}x\right),\frac{1}{4}\left(2-\sqrt{2}\right)\right)}{70\sqrt[4]{2}\sqrt{2x^4+2x^2+1}}$$
$$-\frac{51\left(5+\sqrt{2}\right)\left(\sqrt{2}x^2+1\right)\sqrt{\frac{2x^4+2x^2+1}{\left(\sqrt{2}x^2+1\right)^2}}\text{EllipticF}\left(2\arctan\left(\sqrt[4]{2}x\right),\frac{1}{4}\left(2-\sqrt{2}\right)\right)}{16\sqrt[4]{2}\sqrt{2x^4+2x^2+1}}$$
$$+\frac{867\left(3-\sqrt{2}\right)\left(\sqrt{2}x^2+1\right)\sqrt{\frac{2x^4+2x^2+1}{\left(\sqrt{2}x^2+1\right)^2}}\text{EllipticF}\left(2\arctan\left(\sqrt[4]{2}x\right),\frac{1}{4}\left(2-\sqrt{2}\right)\right)}{112\sqrt[4]{2}\sqrt{2x^4+2x^2+1}}$$
$$-\frac{289\left(11-6\sqrt{2}\right)\left(\sqrt{2}x^2+1\right)\sqrt{\frac{2x^4+2x^2+1}{\left(\sqrt{2}x^2+1\right)^2}}\text{EllipticPi}\left(\frac{1}{24}\left(12+11\sqrt{2}\right),2\arctan\left(\sqrt[4]{2}x\right),\frac{1}{4}\left(2-\sqrt{2}\right)\right)}{224\sqrt[4]{2}\sqrt{2x^4+2x^2+1}}$$

```
[In] Int[(x^2*(1 + 2*x^2 + 2*x^4)^(3/2))/(3 - 2*x^2),x]
```

```
[Out] (-3*x*(2 + x^2)*Sqrt[1 + 2*x^2 + 2*x^4])/35 - (3*x*(9 + 2*x^2)*Sqrt[1 + 2*x
^2 + 2*x^4])/20 - (309*x*Sqrt[1 + 2*x^2 + 2*x^4])/(20*Sqrt[2]*(1 + Sqrt[2]*
```

```
x^2)) - (6*Sqrt[2]*x*Sqrt[1 + 2*x^2 + 2*x^4])/(35*(1 + Sqrt[2]*x^2)) - (x*(
1 + 2*x^2 + 2*x^4)^(3/2))/14 + (17*Sqrt[51]*ArcTanh[(Sqrt[17/3]*x)/Sqrt[1 +
 2*x^2 + 2*x^4]])/16 + (309*(1 + Sqrt[2]*x^2)*Sqrt[(1 + 2*x^2 + 2*x^4)/(1 +
 Sqrt[2]*x^2)^2]*EllipticE[2*ArcTan[2^(1/4)*x], (2 - Sqrt[2])/4])/(20*2^(3/
4)*Sqrt[1 + 2*x^2 + 2*x^4]) + (6*2^(1/4)*(1 + Sqrt[2]*x^2)*Sqrt[(1 + 2*x^2
+ 2*x^4)/(1 + Sqrt[2]*x^2)^2]*EllipticE[2*ArcTan[2^(1/4)*x], (2 - Sqrt[2])/
4])/(35*Sqrt[1 + 2*x^2 + 2*x^4]) + (867*(3 - Sqrt[2])*(1 + Sqrt[2]*x^2)*Sqr
t[(1 + 2*x^2 + 2*x^4)/(1 + Sqrt[2]*x^2)^2]*EllipticF[2*ArcTan[2^(1/4)*x], (
2 - Sqrt[2])/4])/(112*2^(1/4)*Sqrt[1 + 2*x^2 + 2*x^4]) - (51*(5 + Sqrt[2])*
(1 + Sqrt[2]*x^2)*Sqrt[(1 + 2*x^2 + 2*x^4)/(1 + Sqrt[2]*x^2)^2]*EllipticF[2
*ArcTan[2^(1/4)*x], (2 - Sqrt[2])/4])/(16*2^(1/4)*Sqrt[1 + 2*x^2 + 2*x^4])
- (3*(3 + 2*Sqrt[2])*(1 + Sqrt[2]*x^2)*Sqrt[(1 + 2*x^2 + 2*x^4)/(1 + Sqrt[2
]*x^2)^2]*EllipticF[2*ArcTan[2^(1/4)*x], (2 - Sqrt[2])/4])/(70*2^(1/4)*Sqrt
[1 + 2*x^2 + 2*x^4]) - (3*(9 + 8*Sqrt[2])*(1 + Sqrt[2]*x^2)*Sqrt[(1 + 2*x^2
 + 2*x^4)/(1 + Sqrt[2]*x^2)^2]*EllipticF[2*ArcTan[2^(1/4)*x], (2 - Sqrt[2])
/4])/(20*2^(3/4)*Sqrt[1 + 2*x^2 + 2*x^4]) - (289*(11 - 6*Sqrt[2])*(1 + Sqrt
[2]*x^2)*Sqrt[(1 + 2*x^2 + 2*x^4)/(1 + Sqrt[2]*x^2)^2]*EllipticPi[(12 + 11*
Sqrt[2])/24, 2*ArcTan[2^(1/4)*x], (2 - Sqrt[2])/4])/(224*2^(1/4)*Sqrt[1 + 2
*x^2 + 2*x^4])
```

Rule 1105

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[x*((a + b*
x^2 + c*x^4)^p/(4*p + 1)), x] + Dist[2*(p/(4*p + 1)), Int[(2*a + b*x^2)*(a
+ b*x^2 + c*x^4)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c,
 0] && GtQ[p, 0] && IntegerQ[2*p]
```

Rule 1117

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1190

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[x*(2*b*e*p + c*d*(4*p + 3) + c*e*(4*p + 1)*x^2)*((a + b*x^2 + c
*x^4)^p/(c*(4*p + 1)*(4*p + 3))), x] + Dist[2*(p/(c*(4*p + 1)*(4*p + 3))),
Int[Simp[2*a*c*d*(4*p + 3) - a*b*e + (2*a*c*e*(4*p + 1) + b*c*d*(4*p + 3) -
 b^2*e*(2*p + 1))*x^2, x]*(a + b*x^2 + c*x^4)^(p - 1), x], x] /; FreeQ[{a,
b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
GtQ[p, 0] && FractionQ[p] && IntegerQ[2*p]
```

Rule 1209

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
```

```
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

Rule 1211

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + b*x^2 + c*x^4
], x], x] - Dist[e/q, Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /; Ne
Q[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[
c/a]
```

Rule 1222

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Dist[-(e^2)^(-1), Int[(c*d - b*e - c*e*x^2)*(a + b*x^2 + c*x^4)^(p -
 1), x], x] + Dist[(c*d^2 - b*d*e + a*e^2)/e^2, Int[(a + b*x^2 + c*x^4)^(p
- 1)/(d + e*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0
] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IGtQ[p + 1/2, 0]
```

Rule 1230

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4]), x_S
ymbol] :> With[{q = Rt[c/a, 2]}, Dist[(c*d + a*e*q)/(c*d^2 - a*e^2), Int[1/
Sqrt[a + b*x^2 + c*x^4], x], x] - Dist[(a*e*(e + d*q))/(c*d^2 - a*e^2), Int
[(1 + q*x^2)/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]), x], x]] /; FreeQ[{a, b,
 c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[c/a]
```

Rule 1349

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*
(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, p, q}, x] && N
eQ[b^2 - 4*a*c, 0] && (IGtQ[p, 0] || IGtQ[q, 0] || IntegersQ[m, q])
```

Rule 1720

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4]), x_Symbol] :> With[{q = Rt[B/A, 2]}, Simp[(-(B*d - A*e))*(A
rcTan[Rt[-b + c*(d/e) + a*(e/d), 2]*(x/Sqrt[a + b*x^2 + c*x^4])]/(2*d*e*Rt[
-b + c*(d/e) + a*(e/d), 2])), x] + Simp[(B*d + A*e)*(A + B*x^2)*(Sqrt[A^2*(
(a + b*x^2 + c*x^4)/(a*(A + B*x^2)^2))]/(4*d*e*A*q*Sqrt[a + b*x^2 + c*x^4])
)*EllipticPi[Cancel[-(B*d - A*e)^2/(4*d*e*A*B)], 2*ArcTan[q*x], 1/2 - b*(A/
```

```
(4*a*B))], x]] /; FreeQ[{a, b, c, d, e, A, B}, x] && NeQ[b^2 - 4*a*c, 0] &&
 NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] && EqQ
[c*A^2 - a*B^2, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \left(-\frac{1}{2}\left(1+2x^2+2x^4\right)^{3/2}+\frac{3\left(1+2x^2+2x^4\right)^{3/2}}{2\left(3-2x^2\right)}\right)dx \\
&= -\left(\frac{1}{2}\int \left(1+2x^2+2x^4\right)^{3/2}\,dx\right)+\frac{3}{2}\int \frac{\left(1+2x^2+2x^4\right)^{3/2}}{3-2x^2}\,dx \\
&= -\frac{1}{14}x\left(1+2x^2+2x^4\right)^{3/2}-\frac{3}{14}\int \left(2+2x^2\right)\sqrt{1+2x^2+2x^4}\,dx \\
&\quad -\frac{3}{8}\int \left(10+4x^2\right)\sqrt{1+2x^2+2x^4}\,dx+\frac{51}{4}\int \frac{\sqrt{1+2x^2+2x^4}}{3-2x^2}\,dx \\
&= -\frac{3}{35}x\left(2+x^2\right)\sqrt{1+2x^2+2x^4}-\frac{3}{20}x\left(9+2x^2\right)\sqrt{1+2x^2+2x^4} \\
&\quad -\frac{1}{14}x\left(1+2x^2+2x^4\right)^{3/2}-\frac{1}{140}\int \frac{36+48x^2}{\sqrt{1+2x^2+2x^4}}\,dx-\frac{1}{80}\int \frac{192+216x^2}{\sqrt{1+2x^2+2x^4}}\,dx \\
&\quad -\frac{51}{16}\int \frac{10+4x^2}{\sqrt{1+2x^2+2x^4}}\,dx+\frac{867}{8}\int \frac{1}{\left(3-2x^2\right)\sqrt{1+2x^2+2x^4}}\,dx \\
&= -\frac{3}{35}x\left(2+x^2\right)\sqrt{1+2x^2+2x^4}-\frac{3}{20}x\left(9+2x^2\right)\sqrt{1+2x^2+2x^4} \\
&\quad -\frac{1}{14}x\left(1+2x^2+2x^4\right)^{3/2}+\frac{27\int \frac{1-\sqrt{2}x^2}{\sqrt{1+2x^2+2x^4}}\,dx}{10\sqrt{2}} \\
&\quad +\frac{51\int \frac{1-\sqrt{2}x^2}{\sqrt{1+2x^2+2x^4}}\,dx}{4\sqrt{2}}+\frac{1}{35}\left(6\sqrt{2}\right)\int \frac{1-\sqrt{2}x^2}{\sqrt{1+2x^2+2x^4}}\,dx \\
&\quad -\frac{1}{56}\left(867\left(2-3\sqrt{2}\right)\right)\int \frac{1+\sqrt{2}x^2}{\left(3-2x^2\right)\sqrt{1+2x^2+2x^4}}\,dx \\
&\quad +\frac{1}{56}\left(867\left(3-\sqrt{2}\right)\right)\int \frac{1}{\sqrt{1+2x^2+2x^4}}\,dx \\
&\quad -\frac{1}{8}\left(51\left(5+\sqrt{2}\right)\right)\int \frac{1}{\sqrt{1+2x^2+2x^4}}\,dx \\
&\quad -\frac{1}{35}\left(3\left(3+2\sqrt{2}\right)\right)\int \frac{1}{\sqrt{1+2x^2+2x^4}}\,dx \\
&\quad -\frac{1}{20}\left(3\left(16+9\sqrt{2}\right)\right)\int \frac{1}{\sqrt{1+2x^2+2x^4}}\,dx
\end{aligned}
$$

$$= -\frac{3}{35}x(2+x^2)\sqrt{1+2x^2+2x^4} - \frac{3}{20}x(9+2x^2)\sqrt{1+2x^2+2x^4}$$
$$- \frac{309x\sqrt{1+2x^2+2x^4}}{20\sqrt{2}\left(1+\sqrt{2}x^2\right)} - \frac{6\sqrt{2}x\sqrt{1+2x^2+2x^4}}{35\left(1+\sqrt{2}x^2\right)}$$
$$- \frac{1}{14}x(1+2x^2+2x^4)^{3/2} + \frac{17}{16}\sqrt{51}\tanh^{-1}\left(\frac{\sqrt{\frac{17}{3}}x}{\sqrt{1+2x^2+2x^4}}\right)$$
$$+ \frac{309(1+\sqrt{2}x^2)\sqrt{\frac{1+2x^2+2x^4}{\left(1+\sqrt{2}x^2\right)^2}}E\left(2\tan^{-1}\left(\sqrt[4]{2}x\right)|\frac{1}{4}(2-\sqrt{2})\right)}{20\ 2^{3/4}\sqrt{1+2x^2+2x^4}}$$
$$+ \frac{6\sqrt[4]{2}(1+\sqrt{2}x^2)\sqrt{\frac{1+2x^2+2x^4}{\left(1+\sqrt{2}x^2\right)^2}}E\left(2\tan^{-1}\left(\sqrt[4]{2}x\right)|\frac{1}{4}(2-\sqrt{2})\right)}{35\sqrt{1+2x^2+2x^4}}$$
$$+ \frac{867(3-\sqrt{2})\left(1+\sqrt{2}x^2\right)\sqrt{\frac{1+2x^2+2x^4}{\left(1+\sqrt{2}x^2\right)^2}}F\left(2\tan^{-1}\left(\sqrt[4]{2}x\right)|\frac{1}{4}(2-\sqrt{2})\right)}{112\sqrt[4]{2}\sqrt{1+2x^2+2x^4}}$$
$$- \frac{51(5+\sqrt{2})\left(1+\sqrt{2}x^2\right)\sqrt{\frac{1+2x^2+2x^4}{\left(1+\sqrt{2}x^2\right)^2}}F\left(2\tan^{-1}\left(\sqrt[4]{2}x\right)|\frac{1}{4}(2-\sqrt{2})\right)}{16\sqrt[4]{2}\sqrt{1+2x^2+2x^4}}$$
$$- \frac{3(3+2\sqrt{2})\left(1+\sqrt{2}x^2\right)\sqrt{\frac{1+2x^2+2x^4}{\left(1+\sqrt{2}x^2\right)^2}}F\left(2\tan^{-1}\left(\sqrt[4]{2}x\right)|\frac{1}{4}(2-\sqrt{2})\right)}{70\sqrt[4]{2}\sqrt{1+2x^2+2x^4}}$$
$$- \frac{3(9+8\sqrt{2})\left(1+\sqrt{2}x^2\right)\sqrt{\frac{1+2x^2+2x^4}{\left(1+\sqrt{2}x^2\right)^2}}F\left(2\tan^{-1}\left(\sqrt[4]{2}x\right)|\frac{1}{4}(2-\sqrt{2})\right)}{20\ 2^{3/4}\sqrt{1+2x^2+2x^4}}$$
$$- \frac{289(11-6\sqrt{2})\left(1+\sqrt{2}x^2\right)\sqrt{\frac{1+2x^2+2x^4}{\left(1+\sqrt{2}x^2\right)^2}}\Pi\left(\frac{1}{24}(12+11\sqrt{2});2\tan^{-1}\left(\sqrt[4]{2}x\right)|\frac{1}{4}(2-\sqrt{2})\right)}{224\sqrt[4]{2}\sqrt{1+2x^2+2x^4}}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 8.96 (sec) , antiderivative size = 214, normalized size of antiderivative = 0.46

$$\int \frac{x^2(1+2x^2+2x^4)^{3/2}}{3-2x^2}\,dx = \frac{-892x-2080x^3-2456x^5-752x^7-160x^9+4422i\sqrt{1-i}\sqrt{1+(1-i)x^2}\sqrt{}}{}$$

```
[In] Integrate[(x^2*(1 + 2*x^2 + 2*x^4)^(3/2))/(3 - 2*x^2),x]
```

```
[Out] (-892*x - 2080*x^3 - 2456*x^5 - 752*x^7 - 160*x^9 + (4422*I)*Sqrt[1 - I]*Sq
rt[1 + (1 - I)*x^2]*Sqrt[1 + (1 + I)*x^2]*EllipticE[I*ArcSinh[Sqrt[1 - I]*x
```

```
], I] - (9669 - 5247*I)*Sqrt[1 - I]*Sqrt[1 + (1 - I)*x^2]*Sqrt[1 + (1 + I)*
x^2]*EllipticF[I*ArcSinh[Sqrt[1 - I]*x], I] + 10115*(1 - I)^(3/2)*Sqrt[1 +
(1 - I)*x^2]*Sqrt[1 + (1 + I)*x^2]*EllipticPi[-1/3 - I/3, I*ArcSinh[Sqrt[1
- I]*x], I])/(560*Sqrt[1 + 2*x^2 + 2*x^4])
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 2.84 (sec) , antiderivative size = 251, normalized size of antiderivative = 0.54

method	result
risch	$-\frac{x(20x^4+74x^2+223)\sqrt{2x^4+2x^2+1}}{140}-\frac{9669\sqrt{1+(1-i)x^2}\,\sqrt{1+(1+i)x^2}\,F\left(x\sqrt{-1+i},\frac{\sqrt{2}}{2}+\frac{i\sqrt{2}}{2}\right)}{280\sqrt{-1+i}\,\sqrt{2x^4+2x^2+1}}+\frac{\left(\frac{2211}{280}-\frac{2211i}{280}\right)\sqrt{1+(1-i)x^2}\,\sqrt{1}}{}$
elliptic	$-\frac{x^5\sqrt{2x^4+2x^2+1}}{7}-\frac{37x^3\sqrt{2x^4+2x^2+1}}{70}-\frac{223x\sqrt{2x^4+2x^2+1}}{140}-\frac{3729\sqrt{-ix^2+x^2+1}\,\sqrt{ix^2+x^2+1}\,F\left(x\sqrt{-1+i},\frac{\sqrt{2}}{2}+\frac{i\sqrt{2}}{2}\right)}{140\sqrt{-1+i}\,\sqrt{2x^4+2x^2+1}}-\frac{22}{}$
default	$-\frac{x^5\sqrt{2x^4+2x^2+1}}{7}-\frac{37x^3\sqrt{2x^4+2x^2+1}}{70}-\frac{223x\sqrt{2x^4+2x^2+1}}{140}-\frac{9\sqrt{1+(1-i)x^2}\,\sqrt{1+(1+i)x^2}\,F\left(x\sqrt{-1+i},\frac{\sqrt{2}}{2}+\frac{i\sqrt{2}}{2}\right)}{35\sqrt{-1+i}\,\sqrt{2x^4+2x^2+1}}+\frac{\left(\frac{6}{35}-\right.}{}$

```
[In] int(x^2*(2*x^4+2*x^2+1)^(3/2)/(-2*x^2+3),x,method=_RETURNVERBOSE)
```

```
[Out] -1/140*x*(20*x^4+74*x^2+223)*(2*x^4+2*x^2+1)^(1/2)-9669/280/(-1+I)^(1/2)*(1
+(1-I)*x^2)^(1/2)*(1+(1+I)*x^2)^(1/2)/(2*x^4+2*x^2+1)^(1/2)*EllipticF(x*(-1
+I)^(1/2),1/2*2^(1/2)+1/2*I*2^(1/2))+(2211/280-2211/280*I)/(-1+I)^(1/2)*(1+
(1-I)*x^2)^(1/2)*(1+(1+I)*x^2)^(1/2)/(2*x^4+2*x^2+1)^(1/2)*(EllipticF(x*(-1
+I)^(1/2),1/2*2^(1/2)+1/2*I*2^(1/2))-EllipticE(x*(-1+I)^(1/2),1/2*2^(1/2)+1
/2*I*2^(1/2)))+289/8/(-1+I)^(1/2)*(1-I*x^2+x^2)^(1/2)*(1+I*x^2+x^2)^(1/2)/(
2*x^4+2*x^2+1)^(1/2)*EllipticPi(x*(-1+I)^(1/2),-1/3-1/3*I,(-1-I)^(1/2)/(-1+
I)^(1/2))
```

Fricas [F]

$$\int \frac{x^2(1+2x^2+2x^4)^{3/2}}{3-2x^2}\,dx=\int -\frac{(2\,x^4+2\,x^2+1)^{\frac{3}{2}}x^2}{2\,x^2-3}\,dx$$

```
[In] integrate(x^2*(2*x^4+2*x^2+1)^(3/2)/(-2*x^2+3),x, algorithm="fricas")
```

```
[Out] integral(-(2*x^6 + 2*x^4 + x^2)*sqrt(2*x^4 + 2*x^2 + 1)/(2*x^2 - 3), x)
```

Sympy [F]

$$\int \frac{x^2(1+2x^2+2x^4)^{3/2}}{3-2x^2}\,dx = -\int \frac{x^2\sqrt{2x^4+2x^2+1}}{2x^2-3}\,dx - \int \frac{2x^4\sqrt{2x^4+2x^2+1}}{2x^2-3}\,dx - \int \frac{2x^6\sqrt{2x^4+2x^2+1}}{2x^2-3}\,dx$$

```
[In] integrate(x**2*(2*x**4+2*x**2+1)**(3/2)/(-2*x**2+3),x)
[Out] -Integral(x**2*sqrt(2*x**4 + 2*x**2 + 1)/(2*x**2 - 3), x) - Integral(2*x**4
*sqrt(2*x**4 + 2*x**2 + 1)/(2*x**2 - 3), x) - Integral(2*x**6*sqrt(2*x**4 +
 2*x**2 + 1)/(2*x**2 - 3), x)
```

Maxima [F]

$$\int \frac{x^2(1+2x^2+2x^4)^{3/2}}{3-2x^2}\,dx = \int -\frac{(2\,x^4+2\,x^2+1)^{\frac{3}{2}}x^2}{2\,x^2-3}\,dx$$

```
[In] integrate(x^2*(2*x^4+2*x^2+1)^(3/2)/(-2*x^2+3),x, algorithm="maxima")
[Out] -integrate((2*x^4 + 2*x^2 + 1)^(3/2)*x^2/(2*x^2 - 3), x)
```

Giac [F]

$$\int \frac{x^2(1+2x^2+2x^4)^{3/2}}{3-2x^2}\,dx = \int -\frac{(2\,x^4+2\,x^2+1)^{\frac{3}{2}}x^2}{2\,x^2-3}\,dx$$

```
[In] integrate(x^2*(2*x^4+2*x^2+1)^(3/2)/(-2*x^2+3),x, algorithm="giac")
[Out] integrate(-(2*x^4 + 2*x^2 + 1)^(3/2)*x^2/(2*x^2 - 3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2(1+2x^2+2x^4)^{3/2}}{3-2x^2}\,dx = -\int \frac{x^2\,(2\,x^4+2\,x^2+1)^{3/2}}{2\,x^2-3}\,dx$$

```
[In] int(-(x^2*(2*x^2 + 2*x^4 + 1)^(3/2))/(2*x^2 - 3),x)
[Out] -int((x^2*(2*x^2 + 2*x^4 + 1)^(3/2))/(2*x^2 - 3), x)
```

3.328 $\int \frac{(1+2x^2+2x^4)^{3/2}}{3-2x^2}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 428

$$\int \frac{(1+2x^2+2x^4)^{3/2}}{3-2x^2}\,dx = -\frac{1}{10}x(9+2x^2)\sqrt{1+2x^2+2x^4} - \frac{103x\sqrt{1+2x^2+2x^4}}{10\sqrt{2}\,(1+\sqrt{2}x^2)} + \frac{17}{8}\sqrt{\frac{17}{3}}\text{arctanh}\left(\frac{\sqrt{\frac{17}{3}}x}{\sqrt{1+2x^2+2x^4}}\right) + \frac{103(1+\sqrt{2}x^2)\sqrt{\frac{1+2x^2+2x^4}{(1+\sqrt{2}x^2)^2}}E\left(2\arctan\left(\sqrt[4]{2}x\right)|\frac{1}{4}(2-\sqrt{2})\right)}{10\ 2^{3/4}\sqrt{1+2x^2+2x^4}} - \frac{(66+383\sqrt{2})\,(1+\sqrt{2}x^2)\sqrt{\frac{1+2x^2+2x^4}{(1+\sqrt{2}x^2)^2}}\,\text{EllipticF}\left(2\arctan\left(\sqrt[4]{2}x\right),\frac{1}{4}(2-\sqrt{2})\right)}{10\ 2^{3/4}\,(2+3\sqrt{2})\,\sqrt{1+2x^2+2x^4}} - \frac{289(3-\sqrt{2})\,(1+\sqrt{2}x^2)\sqrt{\frac{1+2x^2+2x^4}{(1+\sqrt{2}x^2)^2}}\,\text{EllipticPi}\left(\frac{1}{24}(12+11\sqrt{2}),2\arctan\left(\sqrt[4]{2}x\right),\frac{1}{4}(2-\sqrt{2})\right)}{24\ 2^{3/4}\,(2+3\sqrt{2})\,\sqrt{1+2x^2+2x^4}}$$

```
[Out] 17/24*arctanh(1/3*x*51^(1/2)/(2*x^4+2*x^2+1)^(1/2))*51^(1/2)-1/10*x*(2*x^2+
9)*(2*x^4+2*x^2+1)^(1/2)-103/20*x*(2*x^4+2*x^2+1)^(1/2)*2^(1/2)/(1+x^2*2^(1
/2))+103/20*(cos(2*arctan(2^(1/4)*x))^2)^(1/2)/cos(2*arctan(2^(1/4)*x))*Ell
ipticE(sin(2*arctan(2^(1/4)*x)),1/2*(2-2^(1/2))^(1/2))*(1+x^2*2^(1/2))*((2*
x^4+2*x^2+1)/(1+x^2*2^(1/2))^2)^(1/2)*2^(1/4)/(2*x^4+2*x^2+1)^(1/2)-289/48*
(cos(2*arctan(2^(1/4)*x))^2)^(1/2)/cos(2*arctan(2^(1/4)*x))*EllipticPi(sin(
2*arctan(2^(1/4)*x)),1/2+11/24*2^(1/2),1/2*(2-2^(1/2))^(1/2))*(3-2^(1/2))*(
1+x^2*2^(1/2))*((2*x^4+2*x^2+1)/(1+x^2*2^(1/2))^2)^(1/2)*2^(1/4)/(2+3*2^(1/
2))/(2*x^4+2*x^2+1)^(1/2)-1/20*(cos(2*arctan(2^(1/4)*x))^2)^(1/2)/cos(2*arc
tan(2^(1/4)*x))*EllipticF(sin(2*arctan(2^(1/4)*x)),1/2*(2-2^(1/2))^(1/2))*(
```

```
66+383*2^(1/2))*(1+x^2*2^(1/2))*((2*x^4+2*x^2+1)/(1+x^2*2^(1/2))^2)^(1/2)*2
^(1/4)/(2+3*2^(1/2))/(2*x^4+2*x^2+1)^(1/2)
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 602, normalized size of antiderivative = 1.41, number of steps used = 12, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.269, Rules used = {1222, 1190, 1211, 1117, 1209, 1230, 1720}

$$\int \frac{(1+2x^2+2x^4)^{3/2}}{3-2x^2}\,dx = -\frac{(9+8\sqrt{2})\left(\sqrt{2}x^2+1\right)\sqrt{\frac{2x^4+2x^2+1}{\left(\sqrt{2}x^2+1\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\sqrt[4]{2}x\right),\frac{1}{4}(2-\sqrt{2})\right)}{10\ 2^{3/4}\sqrt{2x^4+2x^2+1}}$$
$$-\frac{17(5+\sqrt{2})\left(\sqrt{2}x^2+1\right)\sqrt{\frac{2x^4+2x^2+1}{\left(\sqrt{2}x^2+1\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\sqrt[4]{2}x\right),\frac{1}{4}(2-\sqrt{2})\right)}{8\sqrt[4]{2}\sqrt{2x^4+2x^2+1}}$$
$$+\frac{289(3-\sqrt{2})\left(\sqrt{2}x^2+1\right)\sqrt{\frac{2x^4+2x^2+1}{\left(\sqrt{2}x^2+1\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\sqrt[4]{2}x\right),\frac{1}{4}(2-\sqrt{2})\right)}{56\sqrt[4]{2}\sqrt{2x^4+2x^2+1}}$$
$$+\frac{103\left(\sqrt{2}x^2+1\right)\sqrt{\frac{2x^4+2x^2+1}{\left(\sqrt{2}x^2+1\right)^2}}\,E\left(2\arctan\left(\sqrt[4]{2}x\right)|\frac{1}{4}(2-\sqrt{2})\right)}{10\ 2^{3/4}\sqrt{2x^4+2x^2+1}}$$
$$-\frac{289(11-6\sqrt{2})\left(\sqrt{2}x^2+1\right)\sqrt{\frac{2x^4+2x^2+1}{\left(\sqrt{2}x^2+1\right)^2}}\,\text{EllipticPi}\left(\frac{1}{24}(12+11\sqrt{2}),2\arctan\left(\sqrt[4]{2}x\right),\frac{1}{4}(2-\sqrt{2})\right)}{336\sqrt[4]{2}\sqrt{2x^4+2x^2+1}}$$
$$+\frac{17}{8}\sqrt{\frac{17}{3}}\text{arctanh}\left(\frac{\sqrt{\frac{17}{3}}x}{\sqrt{2x^4+2x^2+1}}\right)-\frac{1}{10}(2x^2+9)\sqrt{2x^4+2x^2+1}x-\frac{103\sqrt{2x^4+2x^2+1}x}{10\sqrt{2}\left(\sqrt{2}x^2+1\right)}$$

```
[In] Int[(1 + 2*x^2 + 2*x^4)^(3/2)/(3 - 2*x^2),x]
```

```
[Out] -1/10*(x*(9 + 2*x^2)*Sqrt[1 + 2*x^2 + 2*x^4]) - (103*x*Sqrt[1 + 2*x^2 + 2*x
^4])/(10*Sqrt[2]*(1 + Sqrt[2]*x^2)) + (17*Sqrt[17/3]*ArcTanh[(Sqrt[17/3]*x)
/Sqrt[1 + 2*x^2 + 2*x^4]])/8 + (103*(1 + Sqrt[2]*x^2)*Sqrt[(1 + 2*x^2 + 2*x
^4)/(1 + Sqrt[2]*x^2)^2]*EllipticE[2*ArcTan[2^(1/4)*x], (2 - Sqrt[2])/4])/(
10*2^(3/4)*Sqrt[1 + 2*x^2 + 2*x^4]) + (289*(3 - Sqrt[2])*(1 + Sqrt[2]*x^2)*
Sqrt[(1 + 2*x^2 + 2*x^4)/(1 + Sqrt[2]*x^2)^2]*EllipticF[2*ArcTan[2^(1/4)*x]
, (2 - Sqrt[2])/4])/(56*2^(1/4)*Sqrt[1 + 2*x^2 + 2*x^4]) - (17*(5 + Sqrt[2]
)*(1 + Sqrt[2]*x^2)*Sqrt[(1 + 2*x^2 + 2*x^4)/(1 + Sqrt[2]*x^2)^2]*EllipticF
[2*ArcTan[2^(1/4)*x], (2 - Sqrt[2])/4])/(8*2^(1/4)*Sqrt[1 + 2*x^2 + 2*x^4])
 - ((9 + 8*Sqrt[2])*(1 + Sqrt[2]*x^2)*Sqrt[(1 + 2*x^2 + 2*x^4)/(1 + Sqrt[2]
*x^2)^2]*EllipticF[2*ArcTan[2^(1/4)*x], (2 - Sqrt[2])/4])/(10*2^(3/4)*Sqrt[
```

```
1 + 2*x^2 + 2*x^4]) - (289*(11 - 6*Sqrt[2])*(1 + Sqrt[2]*x^2)*Sqrt[(1 + 2*x
^2 + 2*x^4)/(1 + Sqrt[2]*x^2)^2]*EllipticPi[(12 + 11*Sqrt[2])/24, 2*ArcTan[
2^(1/4)*x], (2 - Sqrt[2])/4])/(336*2^(1/4)*Sqrt[1 + 2*x^2 + 2*x^4])
```

Rule 1117

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1190

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[x*(2*b*e*p + c*d*(4*p + 3) + c*e*(4*p + 1)*x^2)*((a + b*x^2 + c
*x^4)^p/(c*(4*p + 1)*(4*p + 3))), x] + Dist[2*(p/(c*(4*p + 1)*(4*p + 3))),
Int[Simp[2*a*c*d*(4*p + 3) - a*b*e + (2*a*c*e*(4*p + 1) + b*c*d*(4*p + 3) -
 b^2*e*(2*p + 1))*x^2, x]*(a + b*x^2 + c*x^4)^(p - 1), x], x] /; FreeQ[{a,
b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
GtQ[p, 0] && FractionQ[p] && IntegerQ[2*p]
```

Rule 1209

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

Rule 1211

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + b*x^2 + c*x^4
], x], x] - Dist[e/q, Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /; Ne
Q[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[
c/a]
```

Rule 1222

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Dist[-(e^2)^(-1), Int[(c*d - b*e - c*e*x^2)*(a + b*x^2 + c*x^4)^(p -
 1), x], x] + Dist[(c*d^2 - b*d*e + a*e^2)/e^2, Int[(a + b*x^2 + c*x^4)^(p
- 1)/(d + e*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0
] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IGtQ[p + 1/2, 0]
```

Rule 1230

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4]), x_S
ymbol] :> With[{q = Rt[c/a, 2]}, Dist[(c*d + a*e*q)/(c*d^2 - a*e^2), Int[1/
Sqrt[a + b*x^2 + c*x^4], x], x] - Dist[(a*e*(e + d*q))/(c*d^2 - a*e^2), Int
[(1 + q*x^2)/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]), x], x]] /; FreeQ[{a, b,
 c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[c/a]
```

Rule 1720

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4]), x_Symbol] :> With[{q = Rt[B/A, 2]}, Simp[(-(B*d - A*e))*(A
rcTan[Rt[-b + c*(d/e) + a*(e/d), 2]*(x/Sqrt[a + b*x^2 + c*x^4])]/(2*d*e*Rt[
-b + c*(d/e) + a*(e/d), 2])), x] + Simp[(B*d + A*e)*(A + B*x^2)*(Sqrt[A^2*(
(a + b*x^2 + c*x^4)/(a*(A + B*x^2)^2))]/(4*d*e*A*q*Sqrt[a + b*x^2 + c*x^4])
)*EllipticPi[Cancel[-(B*d - A*e)^2/(4*d*e*A*B)], 2*ArcTan[q*x], 1/2 - b*(A/
(4*a*B))], x]] /; FreeQ[{a, b, c, d, e, A, B}, x] && NeQ[b^2 - 4*a*c, 0] &&
 NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] && EqQ
[c*A^2 - a*B^2, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\left(\frac{1}{4}\int \left(10+4x^2\right)\sqrt{1+2x^2+2x^4}\,dx\right)+\frac{17}{2}\int \frac{\sqrt{1+2x^2+2x^4}}{3-2x^2}\,dx\\
&= -\frac{1}{10}x\left(9+2x^2\right)\sqrt{1+2x^2+2x^4}-\frac{1}{120}\int \frac{192+216x^2}{\sqrt{1+2x^2+2x^4}}\,dx\\
&\quad -\frac{17}{8}\int \frac{10+4x^2}{\sqrt{1+2x^2+2x^4}}\,dx+\frac{289}{4}\int \frac{1}{\left(3-2x^2\right)\sqrt{1+2x^2+2x^4}}\,dx\\
&= -\frac{1}{10}x\left(9+2x^2\right)\sqrt{1+2x^2+2x^4}+\frac{9\int \frac{1-\sqrt{2}x^2}{\sqrt{1+2x^2+2x^4}}\,dx}{5\sqrt{2}}+\frac{17\int \frac{1-\sqrt{2}x^2}{\sqrt{1+2x^2+2x^4}}\,dx}{2\sqrt{2}}\\
&\quad -\frac{1}{28}\left(289\left(2-3\sqrt{2}\right)\right)\int \frac{1+\sqrt{2}x^2}{\left(3-2x^2\right)\sqrt{1+2x^2+2x^4}}\,dx\\
&\quad +\frac{1}{28}\left(289\left(3-\sqrt{2}\right)\right)\int \frac{1}{\sqrt{1+2x^2+2x^4}}\,dx\\
&\quad -\frac{1}{4}\left(17\left(5+\sqrt{2}\right)\right)\int \frac{1}{\sqrt{1+2x^2+2x^4}}\,dx-\frac{1}{10}\left(16+9\sqrt{2}\right)\int \frac{1}{\sqrt{1+2x^2+2x^4}}\,dx
\end{aligned}$$

$$= -\frac{1}{10}x(9+2x^2)\sqrt{1+2x^2+2x^4} - \frac{103x\sqrt{1+2x^2+2x^4}}{10\sqrt{2}\,(1+\sqrt{2}x^2)}$$
$$+ \frac{17}{8}\sqrt{\frac{17}{3}}\tanh^{-1}\left(\frac{\sqrt{\frac{17}{3}}x}{\sqrt{1+2x^2+2x^4}}\right)$$
$$+ \frac{103(1+\sqrt{2}x^2)\sqrt{\frac{1+2x^2+2x^4}{(1+\sqrt{2}x^2)^2}}E\left(2\tan^{-1}\left(\sqrt[4]{2}x\right)|\frac{1}{4}(2-\sqrt{2})\right)}{10\ 2^{3/4}\sqrt{1+2x^2+2x^4}}$$
$$+ \frac{289(3-\sqrt{2})\,(1+\sqrt{2}x^2)\sqrt{\frac{1+2x^2+2x^4}{(1+\sqrt{2}x^2)^2}}F\left(2\tan^{-1}\left(\sqrt[4]{2}x\right)|\frac{1}{4}(2-\sqrt{2})\right)}{56\sqrt[4]{2}\sqrt{1+2x^2+2x^4}}$$
$$- \frac{17(5+\sqrt{2})\,(1+\sqrt{2}x^2)\sqrt{\frac{1+2x^2+2x^4}{(1+\sqrt{2}x^2)^2}}F\left(2\tan^{-1}\left(\sqrt[4]{2}x\right)|\frac{1}{4}(2-\sqrt{2})\right)}{8\sqrt[4]{2}\sqrt{1+2x^2+2x^4}}$$
$$- \frac{(9+8\sqrt{2})\,(1+\sqrt{2}x^2)\sqrt{\frac{1+2x^2+2x^4}{(1+\sqrt{2}x^2)^2}}F\left(2\tan^{-1}\left(\sqrt[4]{2}x\right)|\frac{1}{4}(2-\sqrt{2})\right)}{10\ 2^{3/4}\sqrt{1+2x^2+2x^4}}$$
$$- \frac{289(11-6\sqrt{2})\,(1+\sqrt{2}x^2)\sqrt{\frac{1+2x^2+2x^4}{(1+\sqrt{2}x^2)^2}}\Pi\left(\frac{1}{24}(12+11\sqrt{2});2\tan^{-1}\left(\sqrt[4]{2}x\right)|\frac{1}{4}(2-\sqrt{2})\right)}{336\sqrt[4]{2}\sqrt{1+2x^2+2x^4}}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.20 (sec) , antiderivative size = 209, normalized size of antiderivative = 0.49

$$\int \frac{(1+2x^2+2x^4)^{3/2}}{3-2x^2}\,dx = \frac{-108x-240x^3-264x^5-48x^7+618i\sqrt{1-i}\sqrt{1+(1-i)x^2}\sqrt{1+(1+i)x^2}E(i}{}$$

```
[In] Integrate[(1 + 2*x^2 + 2*x^4)^(3/2)/(3 - 2*x^2),x]
```

```
[Out] (-108*x - 240*x^3 - 264*x^5 - 48*x^7 + (618*I)*Sqrt[1 - I]*Sqrt[1 + (1 - I)
*x^2]*Sqrt[1 + (1 + I)*x^2]*EllipticE[I*ArcSinh[Sqrt[1 - I]*x], I] - (1371
- 753*I)*Sqrt[1 - I]*Sqrt[1 + (1 - I)*x^2]*Sqrt[1 + (1 + I)*x^2]*EllipticF[
I*ArcSinh[Sqrt[1 - I]*x], I] + 1445*(1 - I)^(3/2)*Sqrt[1 + (1 - I)*x^2]*Sqr
t[1 + (1 + I)*x^2]*EllipticPi[-1/3 - I/3, I*ArcSinh[Sqrt[1 - I]*x], I])/(12
0*Sqrt[1 + 2*x^2 + 2*x^4])
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.35 (sec) , antiderivative size = 246, normalized size of antiderivative = 0.57

method	result
risch	$-\frac{x(2x^2+9)\sqrt{2x^4+2x^2+1}}{10}-\frac{457\sqrt{1+(1-i)x^2}\,\sqrt{1+(1+i)x^2}\,F\left(x\sqrt{-1+i},\frac{\sqrt{2}}{2}+\frac{i\sqrt{2}}{2}\right)}{20\sqrt{-1+i}\,\sqrt{2x^4+2x^2+1}}+\frac{\left(\frac{103}{20}-\frac{103i}{20}\right)\sqrt{1+(1-i)x^2}\,\sqrt{1+(1+i)x^2}\,(}{\sqrt{-1+}}$
default	$-\frac{x^3\sqrt{2x^4+2x^2+1}}{5}-\frac{9x\sqrt{2x^4+2x^2+1}}{10}-\frac{177\sqrt{-ix^2+x^2+1}\,\sqrt{ix^2+x^2+1}\,F\left(x\sqrt{-1+i},\frac{\sqrt{2}}{2}+\frac{i\sqrt{2}}{2}\right)}{10\sqrt{-1+i}\,\sqrt{2x^4+2x^2+1}}-\frac{103i\sqrt{-ix^2+x^2+1}\,\sqrt{ix^2+x}}{20\sqrt{-1+i}\,\sqrt{}}$
elliptic	$-\frac{x^3\sqrt{2x^4+2x^2+1}}{5}-\frac{9x\sqrt{2x^4+2x^2+1}}{10}-\frac{177\sqrt{-ix^2+x^2+1}\,\sqrt{ix^2+x^2+1}\,F\left(x\sqrt{-1+i},\frac{\sqrt{2}}{2}+\frac{i\sqrt{2}}{2}\right)}{10\sqrt{-1+i}\,\sqrt{2x^4+2x^2+1}}-\frac{103i\sqrt{-ix^2+x^2+1}\,\sqrt{ix^2+x}}{20\sqrt{-1+i}\,\sqrt{}}$

```
[In] int((2*x^4+2*x^2+1)^(3/2)/(-2*x^2+3),x,method=_RETURNVERBOSE)
```

```
[Out] -1/10*x*(2*x^2+9)*(2*x^4+2*x^2+1)^(1/2)-457/20/(-1+I)^(1/2)*(1+(1-I)*x^2)^(
1/2)*(1+(1+I)*x^2)^(1/2)/(2*x^4+2*x^2+1)^(1/2)*EllipticF(x*(-1+I)^(1/2),1/2
*2^(1/2)+1/2*I*2^(1/2))+(103/20-103/20*I)/(-1+I)^(1/2)*(1+(1-I)*x^2)^(1/2)*
(1+(1+I)*x^2)^(1/2)/(2*x^4+2*x^2+1)^(1/2)*(EllipticF(x*(-1+I)^(1/2),1/2*2^(
1/2)+1/2*I*2^(1/2))-EllipticE(x*(-1+I)^(1/2),1/2*2^(1/2)+1/2*I*2^(1/2)))+28
9/12/(-1+I)^(1/2)*(1-I*x^2+x^2)^(1/2)*(1+I*x^2+x^2)^(1/2)/(2*x^4+2*x^2+1)^(
1/2)*EllipticPi(x*(-1+I)^(1/2),-1/3-1/3*I,(-1-I)^(1/2)/(-1+I)^(1/2))
```

Fricas [F]

$$\int \frac{(1+2x^2+2x^4)^{3/2}}{3-2x^2}\,dx = \int -\frac{(2\,x^4+2\,x^2+1)^{\frac{3}{2}}}{2\,x^2-3}\,dx$$

```
[In] integrate((2*x^4+2*x^2+1)^(3/2)/(-2*x^2+3),x, algorithm="fricas")
```

```
[Out] integral(-(2*x^4 + 2*x^2 + 1)^(3/2)/(2*x^2 - 3), x)
```

Sympy [F]

$$\int \frac{(1+2x^2+2x^4)^{3/2}}{3-2x^2}\,dx = -\int \frac{\sqrt{2x^4+2x^2+1}}{2x^2-3}\,dx - \int \frac{2x^2\sqrt{2x^4+2x^2+1}}{2x^2-3}\,dx - \int \frac{2x^4\sqrt{2x^4+2x^2+1}}{2x^2-3}\,dx$$

```
[In] integrate((2*x**4+2*x**2+1)**(3/2)/(-2*x**2+3),x)
```

```
[Out] -Integral(sqrt(2*x**4 + 2*x**2 + 1)/(2*x**2 - 3), x) - Integral(2*x**2*sqrt
(2*x**4 + 2*x**2 + 1)/(2*x**2 - 3), x) - Integral(2*x**4*sqrt(2*x**4 + 2*x*
*2 + 1)/(2*x**2 - 3), x)
```

Maxima [F]

$$\int \frac{(1+2x^2+2x^4)^{3/2}}{3-2x^2}\,dx = \int -\frac{(2\,x^4+2\,x^2+1)^{\frac{3}{2}}}{2\,x^2-3}\,dx$$

```
[In] integrate((2*x^4+2*x^2+1)^(3/2)/(-2*x^2+3),x, algorithm="maxima")
[Out] -integrate((2*x^4 + 2*x^2 + 1)^(3/2)/(2*x^2 - 3), x)
```

Giac [F]

$$\int \frac{(1+2x^2+2x^4)^{3/2}}{3-2x^2}\,dx = \int -\frac{(2\,x^4+2\,x^2+1)^{\frac{3}{2}}}{2\,x^2-3}\,dx$$

```
[In] integrate((2*x^4+2*x^2+1)^(3/2)/(-2*x^2+3),x, algorithm="giac")
[Out] integrate(-(2*x^4 + 2*x^2 + 1)^(3/2)/(2*x^2 - 3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(1+2x^2+2x^4)^{3/2}}{3-2x^2}\,dx = -\int \frac{(2\,x^4+2\,x^2+1)^{3/2}}{2\,x^2-3}\,dx$$

```
[In] int(-(2*x^2 + 2*x^4 + 1)^(3/2)/(2*x^2 - 3),x)
[Out] -int((2*x^2 + 2*x^4 + 1)^(3/2)/(2*x^2 - 3), x)
```

3.329 $\int \frac{(1+2x^2+2x^4)^{3/2}}{x^2(3-2x^2)}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 722

$$\begin{aligned}
\int \frac{(1+2x^2+2x^4)^{3/2}}{x^2(3-2x^2)}\,dx &= -\frac{(1+x^2)\sqrt{1+2x^2+2x^4}}{3x}-\frac{17x\sqrt{1+2x^2+2x^4}}{3\sqrt{2}\left(1+\sqrt{2}x^2\right)} \\
&+\frac{\sqrt{2}x\sqrt{1+2x^2+2x^4}}{3\left(1+\sqrt{2}x^2\right)}+\frac{17}{12}\sqrt{\frac{17}{3}}\operatorname{arctanh}\left(\frac{\sqrt{\frac{17}{3}}x}{\sqrt{1+2x^2+2x^4}}\right) \\
&+\frac{17(1+\sqrt{2}x^2)\sqrt{\frac{1+2x^2+2x^4}{\left(1+\sqrt{2}x^2\right)^2}}E\left(2\arctan\left(\sqrt[4]{2}x\right)\middle|\frac{1}{4}(2-\sqrt{2})\right)}{3\,2^{3/4}\sqrt{1+2x^2+2x^4}} \\
&-\frac{\sqrt[4]{2}(1+\sqrt{2}x^2)\sqrt{\frac{1+2x^2+2x^4}{\left(1+\sqrt{2}x^2\right)^2}}E\left(2\arctan\left(\sqrt[4]{2}x\right)\middle|\frac{1}{4}(2-\sqrt{2})\right)}{3\sqrt{1+2x^2+2x^4}} \\
&+\frac{(1+\sqrt{2}x^2)\sqrt{\frac{1+2x^2+2x^4}{\left(1+\sqrt{2}x^2\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\sqrt[4]{2}x\right),\frac{1}{4}(2-\sqrt{2})\right)}{3\,2^{3/4}\sqrt{1+2x^2+2x^4}} \\
&+\frac{289(3-\sqrt{2})\left(1+\sqrt{2}x^2\right)\sqrt{\frac{1+2x^2+2x^4}{\left(1+\sqrt{2}x^2\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\sqrt[4]{2}x\right),\frac{1}{4}(2-\sqrt{2})\right)}{84\sqrt[4]{2}\sqrt{1+2x^2+2x^4}} \\
&-\frac{17(5+\sqrt{2})\left(1+\sqrt{2}x^2\right)\sqrt{\frac{1+2x^2+2x^4}{\left(1+\sqrt{2}x^2\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\sqrt[4]{2}x\right),\frac{1}{4}(2-\sqrt{2})\right)}{12\sqrt[4]{2}\sqrt{1+2x^2+2x^4}} \\
&-\frac{289(11-6\sqrt{2})\left(1+\sqrt{2}x^2\right)\sqrt{\frac{1+2x^2+2x^4}{\left(1+\sqrt{2}x^2\right)^2}}\operatorname{EllipticPi}\left(\frac{1}{24}(12+11\sqrt{2}),2\arctan\left(\sqrt[4]{2}x\right),\frac{1}{4}(2-\sqrt{2})\right)}{504\sqrt[4]{2}\sqrt{1+2x^2+2x^4}}
\end{aligned}$$

```
[Out] 17/36*arctanh(1/3*x*51^(1/2)/(2*x^4+2*x^2+1)^(1/2))*51^(1/2)-1/3*(x^2+1)*(2
*x^4+2*x^2+1)^(1/2)/x-5/2*x*(2*x^4+2*x^2+1)^(1/2)*2^(1/2)/(1+x^2*2^(1/2))+5
/2*(cos(2*arctan(2^(1/4)*x))^2)^(1/2)/cos(2*arctan(2^(1/4)*x))*EllipticE(si
n(2*arctan(2^(1/4)*x)),1/2*(2-2^(1/2))^(1/2))*(1+x^2*2^(1/2))*((2*x^4+2*x^2
+1)/(1+x^2*2^(1/2))^2)^(1/2)*2^(1/4)/(2*x^4+2*x^2+1)^(1/2)+1/6*(cos(2*arcta
n(2^(1/4)*x))^2)^(1/2)/cos(2*arctan(2^(1/4)*x))*EllipticF(sin(2*arctan(2^(1
/4)*x)),1/2*(2-2^(1/2))^(1/2))*(1+x^2*2^(1/2))*((2*x^4+2*x^2+1)/(1+x^2*2^(1
/2))^2)^(1/2)*2^(1/4)/(2*x^4+2*x^2+1)^(1/2)-289/1008*(cos(2*arctan(2^(1/4)*
x))^2)^(1/2)/cos(2*arctan(2^(1/4)*x))*EllipticPi(sin(2*arctan(2^(1/4)*x)),1
/2+11/24*2^(1/2),1/2*(2-2^(1/2))^(1/2))*(11-6*2^(1/2))*(1+x^2*2^(1/2))*((2*
x^4+2*x^2+1)/(1+x^2*2^(1/2))^2)^(1/2)*2^(3/4)/(2*x^4+2*x^2+1)^(1/2)+289/168
*(cos(2*arctan(2^(1/4)*x))^2)^(1/2)/cos(2*arctan(2^(1/4)*x))*EllipticF(sin(
2*arctan(2^(1/4)*x)),1/2*(2-2^(1/2))^(1/2))*(3-2^(1/2))*(1+x^2*2^(1/2))*((2
*x^4+2*x^2+1)/(1+x^2*2^(1/2))^2)^(1/2)*2^(3/4)/(2*x^4+2*x^2+1)^(1/2)-17/24*
(cos(2*arctan(2^(1/4)*x))^2)^(1/2)/cos(2*arctan(2^(1/4)*x))*EllipticF(sin(2
*arctan(2^(1/4)*x)),1/2*(2-2^(1/2))^(1/2))*(5+2^(1/2))*(1+x^2*2^(1/2))*((2*
x^4+2*x^2+1)/(1+x^2*2^(1/2))^2)^(1/2)*2^(3/4)/(2*x^4+2*x^2+1)^(1/2)
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 722, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.345$, Rules

used = {1325, 1285, 12, 1153, 1117, 1209, 1222, 1211, 1230, 1720}

$$\int \frac{(1+2x^2+2x^4)^{3/2}}{x^2(3-2x^2)}\,dx =$$
$$-\frac{17(5+\sqrt{2})\left(\sqrt{2}x^2+1\right)\sqrt{\frac{2x^4+2x^2+1}{\left(\sqrt{2}x^2+1\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\sqrt[4]{2}x\right),\frac{1}{4}(2-\sqrt{2})\right)}{12\sqrt[4]{2}\sqrt{2x^4+2x^2+1}}$$
$$+\frac{289(3-\sqrt{2})\left(\sqrt{2}x^2+1\right)\sqrt{\frac{2x^4+2x^2+1}{\left(\sqrt{2}x^2+1\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\sqrt[4]{2}x\right),\frac{1}{4}(2-\sqrt{2})\right)}{84\sqrt[4]{2}\sqrt{2x^4+2x^2+1}}$$
$$+\frac{\left(\sqrt{2}x^2+1\right)\sqrt{\frac{2x^4+2x^2+1}{\left(\sqrt{2}x^2+1\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\sqrt[4]{2}x\right),\frac{1}{4}(2-\sqrt{2})\right)}{3\ 2^{3/4}\sqrt{2x^4+2x^2+1}}$$
$$-\frac{\sqrt[4]{2}\left(\sqrt{2}x^2+1\right)\sqrt{\frac{2x^4+2x^2+1}{\left(\sqrt{2}x^2+1\right)^2}}\,E\left(2\arctan\left(\sqrt[4]{2}x\right)|\frac{1}{4}(2-\sqrt{2})\right)}{3\sqrt{2x^4+2x^2+1}}$$
$$+\frac{17\left(\sqrt{2}x^2+1\right)\sqrt{\frac{2x^4+2x^2+1}{\left(\sqrt{2}x^2+1\right)^2}}\,E\left(2\arctan\left(\sqrt[4]{2}x\right)|\frac{1}{4}(2-\sqrt{2})\right)}{3\ 2^{3/4}\sqrt{2x^4+2x^2+1}}$$
$$-\frac{289(11-6\sqrt{2})\left(\sqrt{2}x^2+1\right)\sqrt{\frac{2x^4+2x^2+1}{\left(\sqrt{2}x^2+1\right)^2}}\,\text{EllipticPi}\left(\frac{1}{24}(12+11\sqrt{2}),2\arctan\left(\sqrt[4]{2}x\right),\frac{1}{4}(2-\sqrt{2})\right)}{504\sqrt[4]{2}\sqrt{2x^4+2x^2+1}}$$
$$+\frac{17}{12}\sqrt{\frac{17}{3}}\text{arctanh}\left(\frac{\sqrt{\frac{17}{3}}x}{\sqrt{2x^4+2x^2+1}}\right)+\frac{\sqrt{2}\sqrt{2x^4+2x^2+1}x}{3\left(\sqrt{2}x^2+1\right)}-\frac{17\sqrt{2x^4+2x^2+1}x}{3\sqrt{2}\left(\sqrt{2}x^2+1\right)}-\frac{(x^2+1)\sqrt{2x^4+2x^2+1}}{3x}$$

```
[In] Int[(1 + 2*x^2 + 2*x^4)^(3/2)/(x^2*(3 - 2*x^2)),x]
```

```
[Out] -1/3*((1 + x^2)*Sqrt[1 + 2*x^2 + 2*x^4])/x - (17*x*Sqrt[1 + 2*x^2 + 2*x^4])
/(3*Sqrt[2]*(1 + Sqrt[2]*x^2)) + (Sqrt[2]*x*Sqrt[1 + 2*x^2 + 2*x^4])/(3*(1
+ Sqrt[2]*x^2)) + (17*Sqrt[17/3]*ArcTanh[(Sqrt[17/3]*x)/Sqrt[1 + 2*x^2 + 2*
x^4]])/12 + (17*(1 + Sqrt[2]*x^2)*Sqrt[(1 + 2*x^2 + 2*x^4)/(1 + Sqrt[2]*x^2
)^2]*EllipticE[2*ArcTan[2^(1/4)*x], (2 - Sqrt[2])/4])/(3*2^(3/4)*Sqrt[1 + 2
*x^2 + 2*x^4]) - (2^(1/4)*(1 + Sqrt[2]*x^2)*Sqrt[(1 + 2*x^2 + 2*x^4)/(1 + S
qrt[2]*x^2)^2]*EllipticE[2*ArcTan[2^(1/4)*x], (2 - Sqrt[2])/4])/(3*Sqrt[1 +
 2*x^2 + 2*x^4]) + ((1 + Sqrt[2]*x^2)*Sqrt[(1 + 2*x^2 + 2*x^4)/(1 + Sqrt[2]
*x^2)^2]*EllipticF[2*ArcTan[2^(1/4)*x], (2 - Sqrt[2])/4])/(3*2^(3/4)*Sqrt[1
 + 2*x^2 + 2*x^4]) + (289*(3 - Sqrt[2])*(1 + Sqrt[2]*x^2)*Sqrt[(1 + 2*x^2 +
 2*x^4)/(1 + Sqrt[2]*x^2)^2]*EllipticF[2*ArcTan[2^(1/4)*x], (2 - Sqrt[2])/4
])/(84*2^(1/4)*Sqrt[1 + 2*x^2 + 2*x^4]) - (17*(5 + Sqrt[2])*(1 + Sqrt[2]*x^
2)*Sqrt[(1 + 2*x^2 + 2*x^4)/(1 + Sqrt[2]*x^2)^2]*EllipticF[2*ArcTan[2^(1/4)
*x], (2 - Sqrt[2])/4])/(12*2^(1/4)*Sqrt[1 + 2*x^2 + 2*x^4]) - (289*(11 - 6*
Sqrt[2])*(1 + Sqrt[2]*x^2)*Sqrt[(1 + 2*x^2 + 2*x^4)/(1 + Sqrt[2]*x^2)^2]*El
```

```
lipticPi[(12 + 11*Sqrt[2])/24, 2*ArcTan[2^(1/4)*x], (2 - Sqrt[2])/4])/(504*
2^(1/4)*Sqrt[1 + 2*x^2 + 2*x^4])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 1117

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1153

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 2]}, Dist[1/q, Int[1/Sqrt[a + b*x^2 + c*x^4], x], x] - Dist[1/q, I
nt[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x]] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1209

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

Rule 1211

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + b*x^2 + c*x^4
], x], x] - Dist[e/q, Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /; Ne
Q[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[
c/a]
```

Rule 1222

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Dist[-(e^2)^(-1), Int[(c*d - b*e - c*e*x^2)*(a + b*x^2 + c*x^4)^(p -
 1), x], x] + Dist[(c*d^2 - b*d*e + a*e^2)/e^2, Int[(a + b*x^2 + c*x^4)^(p
- 1)/(d + e*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0
] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IGtQ[p + 1/2, 0]
```

Rule 1230

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4]), x_S
ymbol] :> With[{q = Rt[c/a, 2]}, Dist[(c*d + a*e*q)/(c*d^2 - a*e^2), Int[1/
Sqrt[a + b*x^2 + c*x^4], x], x] - Dist[(a*e*(e + d*q))/(c*d^2 - a*e^2), Int
[(1 + q*x^2)/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]), x], x]] /; FreeQ[{a, b,
 c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[c/a]
```

Rule 1285

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(a + b*x^2 + c*x^4)^p*((d*(m
+ 4*p + 3) + e*(m + 1)*x^2)/(f*(m + 1)*(m + 4*p + 3))), x] + Dist[2*(p/(f^2
*(m + 1)*(m + 4*p + 3))), Int[(f*x)^(m + 2)*(a + b*x^2 + c*x^4)^(p - 1)*Sim
p[2*a*e*(m + 1) - b*d*(m + 4*p + 3) + (b*e*(m + 1) - 2*c*d*(m + 4*p + 3))*x
^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b^2 - 4*a*c, 0] && Gt
Q[p, 0] && LtQ[m, -1] && m + 4*p + 3 != 0 && IntegerQ[2*p] && (IntegerQ[p]
|| IntegerQ[m])
```

Rule 1325

```
Int[(((f_.)*(x_))^(m_)*((a_.) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.))/((d_.)
+ (e_.)*(x_)^2), x_Symbol] :> Dist[1/(d*e), Int[(f*x)^m*(a*e + c*d*x^2)*(a
+ b*x^2 + c*x^4)^(p - 1), x], x] - Dist[(c*d^2 - b*d*e + a*e^2)/(d*e*f^2),
Int[(f*x)^(m + 2)*((a + b*x^2 + c*x^4)^(p - 1)/(d + e*x^2)), x], x] /; Free
Q[{a, b, c, d, e, f}, x] && NeQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && LtQ[m, 0]
```

Rule 1720

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4]), x_Symbol] :> With[{q = Rt[B/A, 2]}, Simp[(-(B*d - A*e))*(A
rcTan[Rt[-b + c*(d/e) + a*(e/d), 2]*(x/Sqrt[a + b*x^2 + c*x^4])]/(2*d*e*Rt[
-b + c*(d/e) + a*(e/d), 2])), x] + Simp[(B*d + A*e)*(A + B*x^2)*(Sqrt[A^2*(
(a + b*x^2 + c*x^4)/(a*(A + B*x^2)^2))]/(4*d*e*A*q*Sqrt[a + b*x^2 + c*x^4])
)*EllipticPi[Cancel[-(B*d - A*e)^2/(4*d*e*A*B)], 2*ArcTan[q*x], 1/2 - b*(A/
(4*a*B))], x]] /; FreeQ[{a, b, c, d, e, A, B}, x] && NeQ[b^2 - 4*a*c, 0] &&
 NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] && EqQ
[c*A^2 - a*B^2, 0]
```

Rubi steps

$$\text{integral} = -\left(\frac{1}{6}\int \frac{(-2+6x^2)\sqrt{1+2x^2+2x^4}}{x^2}\,dx\right)+\frac{17}{3}\int \frac{\sqrt{1+2x^2+2x^4}}{3-2x^2}\,dx$$

$$= -\frac{(1+x^2)\sqrt{1+2x^2+2x^4}}{3x} + \frac{1}{18}\int \frac{12x^2}{\sqrt{1+2x^2+2x^4}}\,dx - \frac{17}{12}\int \frac{10+4x^2}{\sqrt{1+2x^2+2x^4}}\,dx + \frac{289}{6}\int \frac{1}{(3-2x^2)\sqrt{1+2x^2+2x^4}}\,dx$$

$$= -\frac{(1+x^2)\sqrt{1+2x^2+2x^4}}{3x} + \frac{2}{3}\int \frac{x^2}{\sqrt{1+2x^2+2x^4}}\,dx + \frac{17\int \frac{1-\sqrt{2}x^2}{\sqrt{1+2x^2+2x^4}}\,dx}{3\sqrt{2}} - \frac{1}{42}\left(289\left(2-3\sqrt{2}\right)\right)\int \frac{1+\sqrt{2}x^2}{(3-2x^2)\sqrt{1+2x^2+2x^4}}\,dx + \frac{1}{42}\left(289\left(3-\sqrt{2}\right)\right)\int \frac{1}{\sqrt{1+2x^2+2x^4}}\,dx - \frac{1}{6}\left(17\left(5+\sqrt{2}\right)\right)\int \frac{1}{\sqrt{1+2x^2+2x^4}}\,dx$$

$$= -\frac{(1+x^2)\sqrt{1+2x^2+2x^4}}{3x} - \frac{17x\sqrt{1+2x^2+2x^4}}{3\sqrt{2}\,(1+\sqrt{2}x^2)} + \frac{17}{12}\sqrt{\frac{17}{3}}\tanh^{-1}\left(\frac{\sqrt{\frac{17}{3}}x}{\sqrt{1+2x^2+2x^4}}\right) + \frac{17(1+\sqrt{2}x^2)\sqrt{\frac{1+2x^2+2x^4}{\left(1+\sqrt{2}x^2\right)^2}}E\left(2\tan^{-1}\left(\sqrt[4]{2}x\right)|\frac{1}{4}(2-\sqrt{2})\right)}{3\ 2^{3/4}\sqrt{1+2x^2+2x^4}} + \frac{289(3-\sqrt{2})\,(1+\sqrt{2}x^2)\sqrt{\frac{1+2x^2+2x^4}{\left(1+\sqrt{2}x^2\right)^2}}F\left(2\tan^{-1}\left(\sqrt[4]{2}x\right)|\frac{1}{4}(2-\sqrt{2})\right)}{84\sqrt[4]{2}\sqrt{1+2x^2+2x^4}} - \frac{17(5+\sqrt{2})\,(1+\sqrt{2}x^2)\sqrt{\frac{1+2x^2+2x^4}{\left(1+\sqrt{2}x^2\right)^2}}F\left(2\tan^{-1}\left(\sqrt[4]{2}x\right)|\frac{1}{4}(2-\sqrt{2})\right)}{12\sqrt[4]{2}\sqrt{1+2x^2+2x^4}} - \frac{289(11-6\sqrt{2})\,(1+\sqrt{2}x^2)\sqrt{\frac{1+2x^2+2x^4}{\left(1+\sqrt{2}x^2\right)^2}}\Pi\left(\frac{1}{24}(12+11\sqrt{2})\,;2\tan^{-1}\left(\sqrt[4]{2}x\right)|\frac{1}{4}(2-\sqrt{2})\right)}{504\sqrt[4]{2}\sqrt{1+2x^2+2x^4}} + \frac{1}{3}\sqrt{2}\int \frac{1}{\sqrt{1+2x^2+2x^4}}\,dx - \frac{1}{3}\sqrt{2}\int \frac{1-\sqrt{2}x^2}{\sqrt{1+2x^2+2x^4}}\,dx$$

$$= -\frac{(1+x^2)\sqrt{1+2x^2+2x^4}}{3x} - \frac{17x\sqrt{1+2x^2+2x^4}}{3\sqrt{2}\,(1+\sqrt{2}x^2)}$$
$$+ \frac{\sqrt{2}x\sqrt{1+2x^2+2x^4}}{3\,(1+\sqrt{2}x^2)} + \frac{17}{12}\sqrt{\frac{17}{3}}\tanh^{-1}\left(\frac{\sqrt{\frac{17}{3}}x}{\sqrt{1+2x^2+2x^4}}\right)$$
$$+ \frac{17(1+\sqrt{2}x^2)\sqrt{\frac{1+2x^2+2x^4}{(1+\sqrt{2}x^2)^2}}E\left(2\tan^{-1}\left(\sqrt[4]{2}x\right)|\frac{1}{4}(2-\sqrt{2})\right)}{3\,2^{3/4}\sqrt{1+2x^2+2x^4}}$$
$$- \frac{\sqrt[4]{2}(1+\sqrt{2}x^2)\sqrt{\frac{1+2x^2+2x^4}{(1+\sqrt{2}x^2)^2}}E\left(2\tan^{-1}\left(\sqrt[4]{2}x\right)|\frac{1}{4}(2-\sqrt{2})\right)}{3\sqrt{1+2x^2+2x^4}}$$
$$+ \frac{(1+\sqrt{2}x^2)\sqrt{\frac{1+2x^2+2x^4}{(1+\sqrt{2}x^2)^2}}F\left(2\tan^{-1}\left(\sqrt[4]{2}x\right)|\frac{1}{4}(2-\sqrt{2})\right)}{3\,2^{3/4}\sqrt{1+2x^2+2x^4}}$$
$$+ \frac{289(3-\sqrt{2})\,(1+\sqrt{2}x^2)\sqrt{\frac{1+2x^2+2x^4}{(1+\sqrt{2}x^2)^2}}F\left(2\tan^{-1}\left(\sqrt[4]{2}x\right)|\frac{1}{4}(2-\sqrt{2})\right)}{84\sqrt[4]{2}\sqrt{1+2x^2+2x^4}}$$
$$- \frac{17(5+\sqrt{2})\,(1+\sqrt{2}x^2)\sqrt{\frac{1+2x^2+2x^4}{(1+\sqrt{2}x^2)^2}}F\left(2\tan^{-1}\left(\sqrt[4]{2}x\right)|\frac{1}{4}(2-\sqrt{2})\right)}{12\sqrt[4]{2}\sqrt{1+2x^2+2x^4}}$$
$$- \frac{289(11-6\sqrt{2})\,(1+\sqrt{2}x^2)\sqrt{\frac{1+2x^2+2x^4}{(1+\sqrt{2}x^2)^2}}\Pi\left(\frac{1}{24}(12+11\sqrt{2});2\tan^{-1}\left(\sqrt[4]{2}x\right)|\frac{1}{4}(2-\sqrt{2})\right)}{504\sqrt[4]{2}\sqrt{1+2x^2+2x^4}}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.24 (sec) , antiderivative size = 213, normalized size of antiderivative = 0.30

$$\int \frac{(1+2x^2+2x^4)^{3/2}}{x^2\,(3-2x^2)}\,dx = \frac{-12-36x^2-48x^4-24x^6+90i\sqrt{1-i}x\sqrt{1+(1-i)x^2}\sqrt{1+(1+i)x^2}E(i\text{arcs}}{}$$

```
[In] Integrate[(1 + 2*x^2 + 2*x^4)^(3/2)/(x^2*(3 - 2*x^2)),x]
```

```
[Out] (-12 - 36*x^2 - 48*x^4 - 24*x^6 + (90*I)*Sqrt[1 - I]*x*Sqrt[1 + (1 - I)*x^2
]*Sqrt[1 + (1 + I)*x^2]*EllipticE[I*ArcSinh[Sqrt[1 - I]*x], I] - (255 - 165
*I)*Sqrt[1 - I]*x*Sqrt[1 + (1 - I)*x^2]*Sqrt[1 + (1 + I)*x^2]*EllipticF[I*A
rcSinh[Sqrt[1 - I]*x], I] + 289*(1 - I)^(3/2)*x*Sqrt[1 + (1 - I)*x^2]*Sqrt[
1 + (1 + I)*x^2]*EllipticPi[-1/3 - I/3, I*ArcSinh[Sqrt[1 - I]*x], I])/(36*x
*Sqrt[1 + 2*x^2 + 2*x^4])
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 2.96 (sec) , antiderivative size = 258, normalized size of antiderivative = 0.36

method	result
risch	$-\frac{2x^6+4x^4+3x^2+1}{3x\sqrt{2x^4+2x^2+1}}-\frac{85\sqrt{1+(1-i)x^2}\,\sqrt{1+(1+i)x^2}\,F\left(x\sqrt{-1+i},\frac{\sqrt{2}}{2}+\frac{i\sqrt{2}}{2}\right)}{6\sqrt{-1+i}\,\sqrt{2x^4+2x^2+1}}+\frac{\left(\frac{5}{2}-\frac{5i}{2}\right)\sqrt{1+(1-i)x^2}\,\sqrt{1+(1+i)x^2}\,\left(F\left(x\sqrt{-1+i},\right.\right.}{\sqrt{-1+i}\,\sqrt{2x^4+2x}}$
elliptic	$-\frac{\sqrt{2x^4+2x^2+1}}{3x}-\frac{x\sqrt{2x^4+2x^2+1}}{3}-\frac{35\sqrt{-ix^2+x^2+1}\,\sqrt{ix^2+x^2+1}\,F\left(x\sqrt{-1+i},\frac{\sqrt{2}}{2}+\frac{i\sqrt{2}}{2}\right)}{3\sqrt{-1+i}\,\sqrt{2x^4+2x^2+1}}-\frac{5i\sqrt{-ix^2+x^2+1}\,\sqrt{ix^2+x^2+1}\,F\left(x\right.}{2\sqrt{-1+i}\,\sqrt{2x^4+2x}}$
default	$-\frac{\sqrt{2x^4+2x^2+1}}{3x}-\frac{x\sqrt{2x^4+2x^2+1}}{3}+\frac{16\sqrt{1+(1-i)x^2}\,\sqrt{1+(1+i)x^2}\,F\left(x\sqrt{-1+i},\frac{\sqrt{2}}{2}+\frac{i\sqrt{2}}{2}\right)}{15\sqrt{-1+i}\,\sqrt{2x^4+2x^2+1}}+\frac{\left(-\frac{14}{15}+\frac{14i}{15}\right)\sqrt{1+(1-i)x^2}\,\sqrt{1+(1-}{}$

```
[In] int((2*x^4+2*x^2+1)^(3/2)/x^2/(-2*x^2+3),x,method=_RETURNVERBOSE)
[Out] -1/3*(2*x^6+4*x^4+3*x^2+1)/x/(2*x^4+2*x^2+1)^(1/2)-85/6/(-1+I)^(1/2)*(1+(1-
I)*x^2)^(1/2)*(1+(1+I)*x^2)^(1/2)/(2*x^4+2*x^2+1)^(1/2)*EllipticF(x*(-1+I)^
(1/2),1/2*2^(1/2)+1/2*I*2^(1/2))+(5/2-5/2*I)/(-1+I)^(1/2)*(1+(1-I)*x^2)^(1/
2)*(1+(1+I)*x^2)^(1/2)/(2*x^4+2*x^2+1)^(1/2)*(EllipticF(x*(-1+I)^(1/2),1/2*
2^(1/2)+1/2*I*2^(1/2))-EllipticE(x*(-1+I)^(1/2),1/2*2^(1/2)+1/2*I*2^(1/2)))
+289/18/(-1+I)^(1/2)*(1-I*x^2+x^2)^(1/2)*(1+I*x^2+x^2)^(1/2)/(2*x^4+2*x^2+1
)^(1/2)*EllipticPi(x*(-1+I)^(1/2),-1/3-1/3*I,(-1-I)^(1/2)/(-1+I)^(1/2))
```

Fricas [F]

$$\int \frac{(1+2x^2+2x^4)^{3/2}}{x^2(3-2x^2)}\,dx=\int -\frac{(2\,x^4+2\,x^2+1)^{\frac{3}{2}}}{(2\,x^2-3)x^2}\,dx$$

```
[In] integrate((2*x^4+2*x^2+1)^(3/2)/x^2/(-2*x^2+3),x, algorithm="fricas")
[Out] integral(-(2*x^4 + 2*x^2 + 1)^(3/2)/(2*x^4 - 3*x^2), x)
```

Sympy [F]

$$\int \frac{(1+2x^2+2x^4)^{3/2}}{x^2(3-2x^2)}\,dx=-\int \frac{\sqrt{2x^4+2x^2+1}}{2x^4-3x^2}\,dx-\int \frac{2x^2\sqrt{2x^4+2x^2+1}}{2x^4-3x^2}\,dx-\int \frac{2x^4\sqrt{2x^4+2x^2+1}}{2x^4-3x^2}\,dx$$

```
[In] integrate((2*x**4+2*x**2+1)**(3/2)/x**2/(-2*x**2+3),x)
[Out] -Integral(sqrt(2*x**4 + 2*x**2 + 1)/(2*x**4 - 3*x**2), x) - Integral(2*x**2
*sqrt(2*x**4 + 2*x**2 + 1)/(2*x**4 - 3*x**2), x) - Integral(2*x**4*sqrt(2*x
**4 + 2*x**2 + 1)/(2*x**4 - 3*x**2), x)
```

Maxima [F]

$$\int \frac{(1+2x^2+2x^4)^{3/2}}{x^2(3-2x^2)}\,dx = \int -\frac{(2\,x^4+2\,x^2+1)^{\frac{3}{2}}}{(2\,x^2-3)x^2}\,dx$$

```
[In] integrate((2*x^4+2*x^2+1)^(3/2)/x^2/(-2*x^2+3),x, algorithm="maxima")
[Out] -integrate((2*x^4 + 2*x^2 + 1)^(3/2)/((2*x^2 - 3)*x^2), x)
```

Giac [F]

$$\int \frac{(1+2x^2+2x^4)^{3/2}}{x^2(3-2x^2)}\,dx = \int -\frac{(2\,x^4+2\,x^2+1)^{\frac{3}{2}}}{(2\,x^2-3)x^2}\,dx$$

```
[In] integrate((2*x^4+2*x^2+1)^(3/2)/x^2/(-2*x^2+3),x, algorithm="giac")
[Out] integrate(-(2*x^4 + 2*x^2 + 1)^(3/2)/((2*x^2 - 3)*x^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(1+2x^2+2x^4)^{3/2}}{x^2(3-2x^2)}\,dx = -\int \frac{(2\,x^4+2\,x^2+1)^{3/2}}{x^2\,(2\,x^2-3)}\,dx$$

```
[In] int(-(2*x^2 + 2*x^4 + 1)^(3/2)/(x^2*(2*x^2 - 3)),x)
[Out] -int((2*x^2 + 2*x^4 + 1)^(3/2)/(x^2*(2*x^2 - 3)), x)
```

3.330 $\int \frac{(1+2x^2+2x^4)^{3/2}}{x^4(3-2x^2)}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 625

$$\begin{aligned}
\int \frac{(1+2x^2+2x^4)^{3/2}}{x^4(3-2x^2)}\,dx &= -\frac{2\sqrt{1+2x^2+2x^4}}{x}-\frac{(1-8x^2)\sqrt{1+2x^2+2x^4}}{9x^3} \\
&+\frac{\sqrt{2}x\sqrt{1+2x^2+2x^4}}{9(1+\sqrt{2}x^2)}+\frac{17}{18}\sqrt{\frac{17}{3}}\operatorname{arctanh}\left(\frac{\sqrt{\frac{17}{3}}x}{\sqrt{1+2x^2+2x^4}}\right) \\
&-\frac{\sqrt[4]{2}(1+\sqrt{2}x^2)\sqrt{\frac{1+2x^2+2x^4}{(1+\sqrt{2}x^2)^2}}E\left(2\arctan\left(\sqrt[4]{2}x\right)|\frac{1}{4}(2-\sqrt{2})\right)}{9\sqrt{1+2x^2+2x^4}} \\
&+\frac{289(3-\sqrt{2})(1+\sqrt{2}x^2)\sqrt{\frac{1+2x^2+2x^4}{(1+\sqrt{2}x^2)^2}}\operatorname{EllipticF}\left(2\arctan\left(\sqrt[4]{2}x\right),\frac{1}{4}(2-\sqrt{2})\right)}{126\sqrt[4]{2}\sqrt{1+2x^2+2x^4}} \\
&-\frac{17(5+\sqrt{2})(1+\sqrt{2}x^2)\sqrt{\frac{1+2x^2+2x^4}{(1+\sqrt{2}x^2)^2}}\operatorname{EllipticF}\left(2\arctan\left(\sqrt[4]{2}x\right),\frac{1}{4}(2-\sqrt{2})\right)}{18\sqrt[4]{2}\sqrt{1+2x^2+2x^4}} \\
&+\frac{\sqrt[4]{2}(9+5\sqrt{2})(1+\sqrt{2}x^2)\sqrt{\frac{1+2x^2+2x^4}{(1+\sqrt{2}x^2)^2}}\operatorname{EllipticF}\left(2\arctan\left(\sqrt[4]{2}x\right),\frac{1}{4}(2-\sqrt{2})\right)}{9\sqrt{1+2x^2+2x^4}} \\
&-\frac{289(11-6\sqrt{2})(1+\sqrt{2}x^2)\sqrt{\frac{1+2x^2+2x^4}{(1+\sqrt{2}x^2)^2}}\operatorname{EllipticPi}\left(\frac{1}{24}(12+11\sqrt{2}),2\arctan\left(\sqrt[4]{2}x\right),\frac{1}{4}(2-\sqrt{2})\right)}{756\sqrt[4]{2}\sqrt{1+2x^2+2x^4}}
\end{aligned}$$

```
[Out] 17/54*arctanh(1/3*x*51^(1/2)/(2*x^4+2*x^2+1)^(1/2))*51^(1/2)-2*(2*x^4+2*x^2
+1)^(1/2)/x-1/9*(-8*x^2+1)*(2*x^4+2*x^2+1)^(1/2)/x^3+1/9*x*(2*x^4+2*x^2+1)^
(1/2)*2^(1/2)/(1+x^2*2^(1/2))-1/9*(cos(2*arctan(2^(1/4)*x))^2)^(1/2)/cos(2*
```

```
arctan(2^(1/4)*x))*EllipticE(sin(2*arctan(2^(1/4)*x)),1/2*(2-2^(1/2))^(1/2)
)*(1+x^2*2^(1/2))*((2*x^4+2*x^2+1)/(1+x^2*2^(1/2))^2)^(1/2)*2^(1/4)/(2*x^4+
2*x^2+1)^(1/2)-289/1512*(cos(2*arctan(2^(1/4)*x))^2)^(1/2)/cos(2*arctan(2^(
1/4)*x))*EllipticPi(sin(2*arctan(2^(1/4)*x)),1/2+11/24*2^(1/2),1/2*(2-2^(1/
2))^(1/2))*(11-6*2^(1/2))*(1+x^2*2^(1/2))*((2*x^4+2*x^2+1)/(1+x^2*2^(1/2))^
2)^(1/2)*2^(3/4)/(2*x^4+2*x^2+1)^(1/2)+289/252*(cos(2*arctan(2^(1/4)*x))^2)
^(1/2)/cos(2*arctan(2^(1/4)*x))*EllipticF(sin(2*arctan(2^(1/4)*x)),1/2*(2-2
^(1/2))^(1/2))*(3-2^(1/2))*(1+x^2*2^(1/2))*((2*x^4+2*x^2+1)/(1+x^2*2^(1/2))
^2)^(1/2)*2^(3/4)/(2*x^4+2*x^2+1)^(1/2)-17/36*(cos(2*arctan(2^(1/4)*x))^2)^
(1/2)/cos(2*arctan(2^(1/4)*x))*EllipticF(sin(2*arctan(2^(1/4)*x)),1/2*(2-2^
(1/2))^(1/2))*(5+2^(1/2))*(1+x^2*2^(1/2))*((2*x^4+2*x^2+1)/(1+x^2*2^(1/2))^
2)^(1/2)*2^(3/4)/(2*x^4+2*x^2+1)^(1/2)+1/9*2^(1/4)*(cos(2*arctan(2^(1/4)*x)
)^2)^(1/2)/cos(2*arctan(2^(1/4)*x))*EllipticF(sin(2*arctan(2^(1/4)*x)),1/2*
(2-2^(1/2))^(1/2))*(9+5*2^(1/2))*(1+x^2*2^(1/2))*((2*x^4+2*x^2+1)/(1+x^2*2^
(1/2))^2)^(1/2)/(2*x^4+2*x^2+1)^(1/2)
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 625, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.310$, Rules used = {1323, 1285, 1295, 1211, 1117, 1209, 1222, 1230, 1720}

$$\int \frac{(1+2x^2+2x^4)^{3/2}}{x^4(3-2x^2)}\,dx = \frac{\sqrt[4]{2}(9+5\sqrt{2})\left(\sqrt{2}x^2+1\right)\sqrt{\frac{2x^4+2x^2+1}{\left(\sqrt{2}x^2+1\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\sqrt[4]{2}x\right),\frac{1}{4}(2-\sqrt{2})\right)}{9\sqrt{2x^4+2x^2+1}}$$
$$-\frac{17(5+\sqrt{2})\left(\sqrt{2}x^2+1\right)\sqrt{\frac{2x^4+2x^2+1}{\left(\sqrt{2}x^2+1\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\sqrt[4]{2}x\right),\frac{1}{4}(2-\sqrt{2})\right)}{18\sqrt[4]{2}\sqrt{2x^4+2x^2+1}}$$
$$+\frac{289(3-\sqrt{2})\left(\sqrt{2}x^2+1\right)\sqrt{\frac{2x^4+2x^2+1}{\left(\sqrt{2}x^2+1\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\sqrt[4]{2}x\right),\frac{1}{4}(2-\sqrt{2})\right)}{126\sqrt[4]{2}\sqrt{2x^4+2x^2+1}}$$
$$-\frac{\sqrt[4]{2}\left(\sqrt{2}x^2+1\right)\sqrt{\frac{2x^4+2x^2+1}{\left(\sqrt{2}x^2+1\right)^2}}E\left(2\arctan\left(\sqrt[4]{2}x\right)|\frac{1}{4}(2-\sqrt{2})\right)}{9\sqrt{2x^4+2x^2+1}}$$
$$-\frac{289(11-6\sqrt{2})\left(\sqrt{2}x^2+1\right)\sqrt{\frac{2x^4+2x^2+1}{\left(\sqrt{2}x^2+1\right)^2}}\,\text{EllipticPi}\left(\frac{1}{24}(12+11\sqrt{2}),2\arctan\left(\sqrt[4]{2}x\right),\frac{1}{4}(2-\sqrt{2})\right)}{756\sqrt[4]{2}\sqrt{2x^4+2x^2+1}}$$
$$+\frac{17}{18}\sqrt{\frac{17}{3}}\text{arctanh}\left(\frac{\sqrt{\frac{17}{3}}x}{\sqrt{2x^4+2x^2+1}}\right)+\frac{\sqrt{2}\sqrt{2x^4+2x^2+1}x}{9\left(\sqrt{2}x^2+1\right)}$$
$$-\frac{2\sqrt{2x^4+2x^2+1}}{x}-\frac{(1-8x^2)\sqrt{2x^4+2x^2+1}}{9x^3}$$

```
[In] Int[(1 + 2*x^2 + 2*x^4)^(3/2)/(x^4*(3 - 2*x^2)),x]
```

```
[Out] (-2*Sqrt[1 + 2*x^2 + 2*x^4])/x - ((1 - 8*x^2)*Sqrt[1 + 2*x^2 + 2*x^4])/(9*x
^3) + (Sqrt[2]*x*Sqrt[1 + 2*x^2 + 2*x^4])/(9*(1 + Sqrt[2]*x^2)) + (17*Sqrt[
17/3]*ArcTanh[(Sqrt[17/3]*x)/Sqrt[1 + 2*x^2 + 2*x^4]])/18 - (2^(1/4)*(1 + S
qrt[2]*x^2)*Sqrt[(1 + 2*x^2 + 2*x^4)/(1 + Sqrt[2]*x^2)^2]*EllipticE[2*ArcTa
n[2^(1/4)*x], (2 - Sqrt[2])/4])/(9*Sqrt[1 + 2*x^2 + 2*x^4]) + (289*(3 - Sqr
t[2])*(1 + Sqrt[2]*x^2)*Sqrt[(1 + 2*x^2 + 2*x^4)/(1 + Sqrt[2]*x^2)^2]*Ellip
ticF[2*ArcTan[2^(1/4)*x], (2 - Sqrt[2])/4])/(126*2^(1/4)*Sqrt[1 + 2*x^2 + 2
*x^4]) - (17*(5 + Sqrt[2])*(1 + Sqrt[2]*x^2)*Sqrt[(1 + 2*x^2 + 2*x^4)/(1 +
Sqrt[2]*x^2)^2]*EllipticF[2*ArcTan[2^(1/4)*x], (2 - Sqrt[2])/4])/(18*2^(1/4
)*Sqrt[1 + 2*x^2 + 2*x^4]) + (2^(1/4)*(9 + 5*Sqrt[2])*(1 + Sqrt[2]*x^2)*Sqr
t[(1 + 2*x^2 + 2*x^4)/(1 + Sqrt[2]*x^2)^2]*EllipticF[2*ArcTan[2^(1/4)*x], (
2 - Sqrt[2])/4])/(9*Sqrt[1 + 2*x^2 + 2*x^4]) - (289*(11 - 6*Sqrt[2])*(1 + S
qrt[2]*x^2)*Sqrt[(1 + 2*x^2 + 2*x^4)/(1 + Sqrt[2]*x^2)^2]*EllipticPi[(12 +
11*Sqrt[2])/24, 2*ArcTan[2^(1/4)*x], (2 - Sqrt[2])/4])/(756*2^(1/4)*Sqrt[1
+ 2*x^2 + 2*x^4])
```

Rule 1117

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1209

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

Rule 1211

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + b*x^2 + c*x^4
], x], x] - Dist[e/q, Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /; Ne
Q[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[
c/a]
```

Rule 1222

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Dist[-(e^2)^(-1), Int[(c*d - b*e - c*e*x^2)*(a + b*x^2 + c*x^4)^(p -
 1), x], x] + Dist[(c*d^2 - b*d*e + a*e^2)/e^2, Int[(a + b*x^2 + c*x^4)^(p
```

```
- 1)/(d + e*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0
] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IGtQ[p + 1/2, 0]
```

Rule 1230

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4]), x_S
ymbol] :> With[{q = Rt[c/a, 2]}, Dist[(c*d + a*e*q)/(c*d^2 - a*e^2), Int[1/
Sqrt[a + b*x^2 + c*x^4], x], x] - Dist[(a*e*(e + d*q))/(c*d^2 - a*e^2), Int
[(1 + q*x^2)/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]), x], x]] /; FreeQ[{a, b,
 c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[c/a]
```

Rule 1285

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(a + b*x^2 + c*x^4)^p*((d*(m
+ 4*p + 3) + e*(m + 1)*x^2)/(f*(m + 1)*(m + 4*p + 3))), x] + Dist[2*(p/(f^2
*(m + 1)*(m + 4*p + 3))), Int[(f*x)^(m + 2)*(a + b*x^2 + c*x^4)^(p - 1)*Sim
p[2*a*e*(m + 1) - b*d*(m + 4*p + 3) + (b*e*(m + 1) - 2*c*d*(m + 4*p + 3))*x
^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b^2 - 4*a*c, 0] && Gt
Q[p, 0] && LtQ[m, -1] && m + 4*p + 3 != 0 && IntegerQ[2*p] && (IntegerQ[p]
|| IntegerQ[m])
```

Rule 1295

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[d*(f*x)^(m + 1)*((a + b*x^2 + c*x^4)^(p + 1)
/(a*f*(m + 1))), x] + Dist[1/(a*f^2*(m + 1)), Int[(f*x)^(m + 2)*(a + b*x^2
+ c*x^4)^p*Simp[a*e*(m + 1) - b*d*(m + 2*p + 3) - c*d*(m + 4*p + 5)*x^2, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[m
, -1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1323

```
Int[(((f_.)*(x_))^(m_)*((a_.) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.))/((d_.)
+ (e_.)*(x_)^2), x_Symbol] :> Dist[1/d^2, Int[(f*x)^m*(a*d + (b*d - a*e)*x^
2)*(a + b*x^2 + c*x^4)^(p - 1), x], x] + Dist[(c*d^2 - b*d*e + a*e^2)/(d^2*
f^4), Int[(f*x)^(m + 4)*((a + b*x^2 + c*x^4)^(p - 1)/(d + e*x^2)), x], x] /
; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && LtQ[m
, -2]
```

Rule 1720

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4]), x_Symbol] :> With[{q = Rt[B/A, 2]}, Simp[(-(B*d - A*e))*(A
rcTan[Rt[-b + c*(d/e) + a*(e/d), 2]*(x/Sqrt[a + b*x^2 + c*x^4])]/(2*d*e*Rt[
```

```
-b + c*(d/e) + a*(e/d), 2])), x] + Simp[(B*d + A*e)*(A + B*x^2)*(Sqrt[A^2*(
(a + b*x^2 + c*x^4)/(a*(A + B*x^2)^2))]/(4*d*e*A*q*Sqrt[a + b*x^2 + c*x^4])
)*EllipticPi[Cancel[-(B*d - A*e)^2/(4*d*e*A*B)], 2*ArcTan[q*x], 1/2 - b*(A/
(4*a*B))], x]] /; FreeQ[{a, b, c, d, e, A, B}, x] && NeQ[b^2 - 4*a*c, 0] &&
 NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] && EqQ
[c*A^2 - a*B^2, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{9}\int \frac{(3+8x^2)\sqrt{1+2x^2+2x^4}}{x^4}\,dx + \frac{34}{9}\int \frac{\sqrt{1+2x^2+2x^4}}{3-2x^2}\,dx \\
&= -\frac{(1-8x^2)\sqrt{1+2x^2+2x^4}}{9x^3} - \frac{1}{27}\int \frac{-54-60x^2}{x^2\sqrt{1+2x^2+2x^4}}\,dx \\
&\quad - \frac{17}{18}\int \frac{10+4x^2}{\sqrt{1+2x^2+2x^4}}\,dx + \frac{289}{9}\int \frac{1}{(3-2x^2)\sqrt{1+2x^2+2x^4}}\,dx \\
&= -\frac{2\sqrt{1+2x^2+2x^4}}{x} - \frac{(1-8x^2)\sqrt{1+2x^2+2x^4}}{9x^3} + \frac{1}{27}\int \frac{60+108x^2}{\sqrt{1+2x^2+2x^4}}\,dx \\
&\quad + \frac{1}{9}\left(17\sqrt{2}\right)\int \frac{1-\sqrt{2}x^2}{\sqrt{1+2x^2+2x^4}}\,dx - \frac{1}{63}\left(289\left(2-3\sqrt{2}\right)\right)\int \frac{1+\sqrt{2}x^2}{(3-2x^2)\sqrt{1+2x^2+2x^4}}\,dx \\
&\quad + \frac{1}{63}\left(289\left(3-\sqrt{2}\right)\right)\int \frac{1}{\sqrt{1+2x^2+2x^4}}\,dx - \frac{1}{9}\left(17\left(5+\sqrt{2}\right)\right)\int \frac{1}{\sqrt{1+2x^2+2x^4}}\,dx \\
&= -\frac{2\sqrt{1+2x^2+2x^4}}{x} - \frac{(1-8x^2)\sqrt{1+2x^2+2x^4}}{9x^3} \\
&\quad - \frac{17\sqrt{2}x\sqrt{1+2x^2+2x^4}}{9\left(1+\sqrt{2}x^2\right)} + \frac{17}{18}\sqrt{\frac{17}{3}}\tanh^{-1}\left(\frac{\sqrt{\frac{17}{3}}x}{\sqrt{1+2x^2+2x^4}}\right) \\
&\quad + \frac{17\sqrt[4]{2}(1+\sqrt{2}x^2)\sqrt{\frac{1+2x^2+2x^4}{\left(1+\sqrt{2}x^2\right)^2}}E\left(2\tan^{-1}\left(\sqrt[4]{2}x\right)|\frac{1}{4}(2-\sqrt{2})\right)}{9\sqrt{1+2x^2+2x^4}} \\
&\quad + \frac{289(3-\sqrt{2})\left(1+\sqrt{2}x^2\right)\sqrt{\frac{1+2x^2+2x^4}{\left(1+\sqrt{2}x^2\right)^2}}F\left(2\tan^{-1}\left(\sqrt[4]{2}x\right)|\frac{1}{4}(2-\sqrt{2})\right)}{126\sqrt[4]{2}\sqrt{1+2x^2+2x^4}} \\
&\quad - \frac{17(5+\sqrt{2})\left(1+\sqrt{2}x^2\right)\sqrt{\frac{1+2x^2+2x^4}{\left(1+\sqrt{2}x^2\right)^2}}F\left(2\tan^{-1}\left(\sqrt[4]{2}x\right)|\frac{1}{4}(2-\sqrt{2})\right)}{18\sqrt[4]{2}\sqrt{1+2x^2+2x^4}} \\
&\quad - \frac{289(11-6\sqrt{2})\left(1+\sqrt{2}x^2\right)\sqrt{\frac{1+2x^2+2x^4}{\left(1+\sqrt{2}x^2\right)^2}}\Pi\left(\frac{1}{24}(12+11\sqrt{2});2\tan^{-1}\left(\sqrt[4]{2}x\right)|\frac{1}{4}(2-\sqrt{2})\right)}{756\sqrt[4]{2}\sqrt{1+2x^2+2x^4}} \\
&\quad - \left(2\sqrt{2}\right)\int \frac{1-\sqrt{2}x^2}{\sqrt{1+2x^2+2x^4}}\,dx + \frac{1}{9}\left(2\left(10+9\sqrt{2}\right)\right)\int \frac{1}{\sqrt{1+2x^2+2x^4}}\,dx
\end{aligned}$$

$$= -\frac{2\sqrt{1+2x^2+2x^4}}{x} - \frac{(1-8x^2)\sqrt{1+2x^2+2x^4}}{9x^3} + \frac{\sqrt{2}x\sqrt{1+2x^2+2x^4}}{9\left(1+\sqrt{2}x^2\right)} + \frac{17}{18}\sqrt{\frac{17}{3}}\tanh^{-1}\left(\frac{\sqrt{\frac{17}{3}}x}{\sqrt{1+2x^2+2x^4}}\right) - \frac{\sqrt[4]{2}\left(1+\sqrt{2}x^2\right)\sqrt{\frac{1+2x^2+2x^4}{\left(1+\sqrt{2}x^2\right)^2}}E\left(2\tan^{-1}\left(\sqrt[4]{2}x\right)|\frac{1}{4}\left(2-\sqrt{2}\right)\right)}{9\sqrt{1+2x^2+2x^4}} + \frac{289\left(3-\sqrt{2}\right)\left(1+\sqrt{2}x^2\right)\sqrt{\frac{1+2x^2+2x^4}{\left(1+\sqrt{2}x^2\right)^2}}F\left(2\tan^{-1}\left(\sqrt[4]{2}x\right)|\frac{1}{4}\left(2-\sqrt{2}\right)\right)}{126\sqrt[4]{2}\sqrt{1+2x^2+2x^4}} - \frac{17\left(5+\sqrt{2}\right)\left(1+\sqrt{2}x^2\right)\sqrt{\frac{1+2x^2+2x^4}{\left(1+\sqrt{2}x^2\right)^2}}F\left(2\tan^{-1}\left(\sqrt[4]{2}x\right)|\frac{1}{4}\left(2-\sqrt{2}\right)\right)}{18\sqrt[4]{2}\sqrt{1+2x^2+2x^4}} + \frac{\sqrt[4]{2}\left(9+5\sqrt{2}\right)\left(1+\sqrt{2}x^2\right)\sqrt{\frac{1+2x^2+2x^4}{\left(1+\sqrt{2}x^2\right)^2}}F\left(2\tan^{-1}\left(\sqrt[4]{2}x\right)|\frac{1}{4}\left(2-\sqrt{2}\right)\right)}{9\sqrt{1+2x^2+2x^4}} - \frac{289\left(11-6\sqrt{2}\right)\left(1+\sqrt{2}x^2\right)\sqrt{\frac{1+2x^2+2x^4}{\left(1+\sqrt{2}x^2\right)^2}}\Pi\left(\frac{1}{24}\left(12+11\sqrt{2}\right);2\tan^{-1}\left(\sqrt[4]{2}x\right)|\frac{1}{4}\left(2-\sqrt{2}\right)\right)}{756\sqrt[4]{2}\sqrt{1+2x^2+2x^4}}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.25 (sec) , antiderivative size = 219, normalized size of antiderivative = 0.35

$$\int \frac{\left(1+2x^2+2x^4\right)^{3/2}}{x^4\left(3-2x^2\right)}\,dx = \frac{-6-72x^2-132x^4-120x^6-6i\sqrt{1-i}x^3\sqrt{1+(1-i)x^2}\sqrt{1+(1+i)x^2}E\left(i\mathrm{arc}\right.}{}$$

```
[In] Integrate[(1 + 2*x^2 + 2*x^4)^(3/2)/(x^4*(3 - 2*x^2)),x]
```

```
[Out] (-6 - 72*x^2 - 132*x^4 - 120*x^6 - (6*I)*Sqrt[1 - I]*x^3*Sqrt[1 + (1 - I)*x
^2]*Sqrt[1 + (1 + I)*x^2]*EllipticE[I*ArcSinh[Sqrt[1 - I]*x], I] - (195 - 2
01*I)*Sqrt[1 - I]*x^3*Sqrt[1 + (1 - I)*x^2]*Sqrt[1 + (1 + I)*x^2]*EllipticF
[I*ArcSinh[Sqrt[1 - I]*x], I] + 289*(1 - I)^(3/2)*x^3*Sqrt[1 + (1 - I)*x^2]
*Sqrt[1 + (1 + I)*x^2]*EllipticPi[-1/3 - I/3, I*ArcSinh[Sqrt[1 - I]*x], I])
/(54*x^3*Sqrt[1 + 2*x^2 + 2*x^4])
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 4.11 (sec) , antiderivative size = 258, normalized size of antiderivative = 0.41

method	result
risch	$-\frac{20x^6+22x^4+12x^2+1}{9x^3\sqrt{2x^4+2x^2+1}}+\frac{\left(-\frac{1}{9}+\frac{i}{9}\right)\sqrt{1+(1-i)x^2}\,\sqrt{1+(1+i)x^2}\,\left(F\left(x\sqrt{-1+i},\frac{\sqrt{2}}{2}+\frac{i\sqrt{2}}{2}\right)-E\left(x\sqrt{-1+i},\frac{\sqrt{2}}{2}+\frac{i\sqrt{2}}{2}\right)\right)}{\sqrt{-1+i}\,\sqrt{2x^4+2x^2+1}}-\frac{65\sqrt{1+(1-i \cdots}}{}$
elliptic	$-\frac{\sqrt{2x^4+2x^2+1}}{9x^3}-\frac{10\sqrt{2x^4+2x^2+1}}{9x}-\frac{22\sqrt{-ix^2+x^2+1}\,\sqrt{ix^2+x^2+1}\,F\left(x\sqrt{-1+i},\frac{\sqrt{2}}{2}+\frac{i\sqrt{2}}{2}\right)}{3\sqrt{-1+i}\,\sqrt{2x^4+2x^2+1}}+\frac{i\sqrt{-ix^2+x^2+1}\,\sqrt{ix^2+x^2+1}\,F\left(x \cdots\right.}{9\sqrt{-1+i}\,\sqrt{2x^4+2x \cdots}}$
default	$-\frac{10\sqrt{2x^4+2x^2+1}}{9x}+\frac{44\sqrt{1+(1-i)x^2}\,\sqrt{1+(1+i)x^2}\,F\left(x\sqrt{-1+i},\frac{\sqrt{2}}{2}+\frac{i\sqrt{2}}{2}\right)}{15\sqrt{-1+i}\,\sqrt{2x^4+2x^2+1}}+\frac{\left(-\frac{12}{5}+\frac{12i}{5}\right)\sqrt{1+(1-i)x^2}\,\sqrt{1+(1+i)x^2}\,\left(F\left(x\sqrt{-1+ \cdots}\right.\right.}{\sqrt{-1+i}\,\sqrt{2x^4+2x \cdots}}$

```
[In] int((2*x^4+2*x^2+1)^(3/2)/x^4/(-2*x^2+3),x,method=_RETURNVERBOSE)
[Out] -1/9*(20*x^6+22*x^4+12*x^2+1)/x^3/(2*x^4+2*x^2+1)^(1/2)+(-1/9+1/9*I)/(-1+I)
^(1/2)*(1+(1-I)*x^2)^(1/2)*(1+(1+I)*x^2)^(1/2)/(2*x^4+2*x^2+1)^(1/2)*(Ellip
ticF(x*(-1+I)^(1/2),1/2*2^(1/2)+1/2*I*2^(1/2))-EllipticE(x*(-1+I)^(1/2),1/2
*2^(1/2)+1/2*I*2^(1/2)))-65/9/(-1+I)^(1/2)*(1+(1-I)*x^2)^(1/2)*(1+(1+I)*x^2
)^(1/2)/(2*x^4+2*x^2+1)^(1/2)*EllipticF(x*(-1+I)^(1/2),1/2*2^(1/2)+1/2*I*2^
(1/2))+289/27/(-1+I)^(1/2)*(1-I*x^2+x^2)^(1/2)*(1+I*x^2+x^2)^(1/2)/(2*x^4+2
*x^2+1)^(1/2)*EllipticPi(x*(-1+I)^(1/2),-1/3-1/3*I,(-1-I)^(1/2)/(-1+I)^(1/2
))
```

Fricas [F]

$$\int \frac{(1+2x^2+2x^4)^{3/2}}{x^4\,(3-2x^2)}\,dx=\int -\frac{(2\,x^4+2\,x^2+1)^{\frac{3}{2}}}{(2\,x^2-3)x^4}\,dx$$

```
[In] integrate((2*x^4+2*x^2+1)^(3/2)/x^4/(-2*x^2+3),x, algorithm="fricas")
[Out] integral(-(2*x^4 + 2*x^2 + 1)^(3/2)/(2*x^6 - 3*x^4), x)
```

Sympy [F]

$$\int \frac{(1+2x^2+2x^4)^{3/2}}{x^4\,(3-2x^2)}\,dx=-\int \frac{\sqrt{2x^4+2x^2+1}}{2x^6-3x^4}\,dx-\int \frac{2x^2\sqrt{2x^4+2x^2+1}}{2x^6-3x^4}\,dx-\int \frac{2x^4\sqrt{2x^4+2x^2+1}}{2x^6-3x^4}\,dx$$

```
[In] integrate((2*x**4+2*x**2+1)**(3/2)/x**4/(-2*x**2+3),x)
```

```
[Out] -Integral(sqrt(2*x**4 + 2*x**2 + 1)/(2*x**6 - 3*x**4), x) - Integral(2*x**2
*sqrt(2*x**4 + 2*x**2 + 1)/(2*x**6 - 3*x**4), x) - Integral(2*x**4*sqrt(2*x
**4 + 2*x**2 + 1)/(2*x**6 - 3*x**4), x)
```

Maxima [F]

$$\int \frac{(1+2x^2+2x^4)^{3/2}}{x^4(3-2x^2)}\,dx = \int -\frac{(2\,x^4+2\,x^2+1)^{\frac{3}{2}}}{(2\,x^2-3)x^4}\,dx$$

```
[In] integrate((2*x^4+2*x^2+1)^(3/2)/x^4/(-2*x^2+3),x, algorithm="maxima")
[Out] -integrate((2*x^4 + 2*x^2 + 1)^(3/2)/((2*x^2 - 3)*x^4), x)
```

Giac [F]

$$\int \frac{(1+2x^2+2x^4)^{3/2}}{x^4(3-2x^2)}\,dx = \int -\frac{(2\,x^4+2\,x^2+1)^{\frac{3}{2}}}{(2\,x^2-3)x^4}\,dx$$

```
[In] integrate((2*x^4+2*x^2+1)^(3/2)/x^4/(-2*x^2+3),x, algorithm="giac")
[Out] integrate(-(2*x^4 + 2*x^2 + 1)^(3/2)/((2*x^2 - 3)*x^4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(1+2x^2+2x^4)^{3/2}}{x^4(3-2x^2)}\,dx = -\int \frac{(2\,x^4+2\,x^2+1)^{3/2}}{x^4\,(2\,x^2-3)}\,dx$$

```
[In] int(-(2*x^2 + 2*x^4 + 1)^(3/2)/(x^4*(2*x^2 - 3)),x)
[Out] -int((2*x^2 + 2*x^4 + 1)^(3/2)/(x^4*(2*x^2 - 3)), x)
```

3.331 $\int \frac{(1+2x^2+2x^4)^{3/2}}{x^6(3-2x^2)}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 553

$$
\begin{aligned}
\int \frac{(1+2x^2+2x^4)^{3/2}}{x^6(3-2x^2)}\,dx &= \frac{74\sqrt{1+2x^2+2x^4}}{135x^3} \\
&- \frac{262\sqrt{1+2x^2+2x^4}}{135x} - \frac{(3+40x^2)\sqrt{1+2x^2+2x^4}}{45x^5} \\
&+ \frac{262\sqrt{2}x\sqrt{1+2x^2+2x^4}}{135(1+\sqrt{2}x^2)} + \frac{17}{27}\sqrt{\frac{17}{3}}\operatorname{arctanh}\left(\frac{\sqrt{\frac{17}{3}}x}{\sqrt{1+2x^2+2x^4}}\right) \\
&- \frac{262\sqrt[4]{2}(1+\sqrt{2}x^2)\sqrt{\frac{1+2x^2+2x^4}{(1+\sqrt{2}x^2)^2}}E\left(2\arctan\left(\sqrt[4]{2}x\right)|\frac{1}{4}(2-\sqrt{2})\right)}{135\sqrt{1+2x^2+2x^4}} \\
&+ \frac{85\;2^{3/4}(3-\sqrt{2})(1+\sqrt{2}x^2)\sqrt{\frac{1+2x^2+2x^4}{(1+\sqrt{2}x^2)^2}}\operatorname{EllipticF}\left(2\arctan\left(\sqrt[4]{2}x\right),\frac{1}{4}(2-\sqrt{2})\right)}{189\sqrt{1+2x^2+2x^4}} \\
&+ \frac{2^{3/4}(37+23\sqrt{2})(1+\sqrt{2}x^2)\sqrt{\frac{1+2x^2+2x^4}{(1+\sqrt{2}x^2)^2}}\operatorname{EllipticF}\left(2\arctan\left(\sqrt[4]{2}x\right),\frac{1}{4}(2-\sqrt{2})\right)}{135\sqrt{1+2x^2+2x^4}} \\
&- \frac{289(11-6\sqrt{2})(1+\sqrt{2}x^2)\sqrt{\frac{1+2x^2+2x^4}{(1+\sqrt{2}x^2)^2}}\operatorname{EllipticPi}\left(\frac{1}{24}(12+11\sqrt{2}),2\arctan\left(\sqrt[4]{2}x\right),\frac{1}{4}(2-\sqrt{2})\right)}{1134\sqrt[4]{2}\sqrt{1+2x^2+2x^4}}
\end{aligned}
$$

```
[Out] 17/81*arctanh(1/3*x*51^(1/2)/(2*x^4+2*x^2+1)^(1/2))*51^(1/2)+74/135*(2*x^4+
2*x^2+1)^(1/2)/x^3-262/135*(2*x^4+2*x^2+1)^(1/2)/x-1/45*(40*x^2+3)*(2*x^4+2
*x^2+1)^(1/2)/x^5+262/135*x*(2*x^4+2*x^2+1)^(1/2)*2^(1/2)/(1+x^2*2^(1/2))-2
62/135*(cos(2*arctan(2^(1/4)*x))^2)^(1/2)/cos(2*arctan(2^(1/4)*x))*Elliptic
```

```
E(sin(2*arctan(2^(1/4)*x)),1/2*(2-2^(1/2))^(1/2))*(1+x^2*2^(1/2))*((2*x^4+2
*x^2+1)/(1+x^2*2^(1/2))^2)^(1/2)*2^(1/4)/(2*x^4+2*x^2+1)^(1/2)-289/2268*(co
s(2*arctan(2^(1/4)*x))^2)^(1/2)/cos(2*arctan(2^(1/4)*x))*EllipticPi(sin(2*a
rctan(2^(1/4)*x)),1/2+11/24*2^(1/2),1/2*(2-2^(1/2))^(1/2))*(11-6*2^(1/2))*(
1+x^2*2^(1/2))*((2*x^4+2*x^2+1)/(1+x^2*2^(1/2))^2)^(1/2)*2^(3/4)/(2*x^4+2*x
^2+1)^(1/2)+85/189*(cos(2*arctan(2^(1/4)*x))^2)^(1/2)/cos(2*arctan(2^(1/4)*
x))*EllipticF(sin(2*arctan(2^(1/4)*x)),1/2*(2-2^(1/2))^(1/2))*(3-2^(1/2))*(
1+x^2*2^(1/2))*((2*x^4+2*x^2+1)/(1+x^2*2^(1/2))^2)^(1/2)*2^(3/4)/(2*x^4+2*x
^2+1)^(1/2)+1/135*2^(3/4)*(cos(2*arctan(2^(1/4)*x))^2)^(1/2)/cos(2*arctan(2
^(1/4)*x))*EllipticF(sin(2*arctan(2^(1/4)*x)),1/2*(2-2^(1/2))^(1/2))*(37+23
*2^(1/2))*(1+x^2*2^(1/2))*((2*x^4+2*x^2+1)/(1+x^2*2^(1/2))^2)^(1/2)/(2*x^4+
2*x^2+1)^(1/2)
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 553, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.310$, Rules used = {1323, 1285, 1295, 1211, 1117, 1209, 1325, 1230, 1720}

$$\int \frac{(1+2x^2+2x^4)^{3/2}}{x^6(3-2x^2)}\,dx = \frac{2^{3/4}\left(37+23\sqrt{2}\right)\left(\sqrt{2}x^2+1\right)\sqrt{\frac{2x^4+2x^2+1}{\left(\sqrt{2}x^2+1\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\sqrt[4]{2}x\right),\frac{1}{4}\left(2-\sqrt{2}\right)\right)}{135\sqrt{2x^4+2x^2+1}}$$
$$+\frac{85\ 2^{3/4}\left(3-\sqrt{2}\right)\left(\sqrt{2}x^2+1\right)\sqrt{\frac{2x^4+2x^2+1}{\left(\sqrt{2}x^2+1\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\sqrt[4]{2}x\right),\frac{1}{4}\left(2-\sqrt{2}\right)\right)}{189\sqrt{2x^4+2x^2+1}}$$
$$-\frac{262\sqrt[4]{2}\left(\sqrt{2}x^2+1\right)\sqrt{\frac{2x^4+2x^2+1}{\left(\sqrt{2}x^2+1\right)^2}}\,E\left(2\arctan\left(\sqrt[4]{2}x\right)|\frac{1}{4}\left(2-\sqrt{2}\right)\right)}{135\sqrt{2x^4+2x^2+1}}$$
$$-\frac{289\left(11-6\sqrt{2}\right)\left(\sqrt{2}x^2+1\right)\sqrt{\frac{2x^4+2x^2+1}{\left(\sqrt{2}x^2+1\right)^2}}\,\text{EllipticPi}\left(\frac{1}{24}\left(12+11\sqrt{2}\right),2\arctan\left(\sqrt[4]{2}x\right),\frac{1}{4}\left(2-\sqrt{2}\right)\right)}{1134\sqrt[4]{2}\sqrt{2x^4+2x^2+1}}$$
$$+\frac{17}{27}\sqrt{\frac{17}{3}}\text{arctanh}\left(\frac{\sqrt{\frac{17}{3}}x}{\sqrt{2x^4+2x^2+1}}\right)+\frac{262\sqrt{2}\sqrt{2x^4+2x^2+1}x}{135\left(\sqrt{2}x^2+1\right)}-\frac{262\sqrt{2x^4+2x^2+1}}{135x}-\frac{(40x^2+3)\sqrt{2x^4+}}{45x^5}$$

```
[In] Int[(1 + 2*x^2 + 2*x^4)^(3/2)/(x^6*(3 - 2*x^2)),x]
```

```
[Out] (74*Sqrt[1 + 2*x^2 + 2*x^4])/(135*x^3) - (262*Sqrt[1 + 2*x^2 + 2*x^4])/(135
*x) - ((3 + 40*x^2)*Sqrt[1 + 2*x^2 + 2*x^4])/(45*x^5) + (262*Sqrt[2]*x*Sqrt
[1 + 2*x^2 + 2*x^4])/(135*(1 + Sqrt[2]*x^2)) + (17*Sqrt[17/3]*ArcTanh[(Sqrt
[17/3]*x)/Sqrt[1 + 2*x^2 + 2*x^4]])/27 - (262*2^(1/4)*(1 + Sqrt[2]*x^2)*Sqr
t[(1 + 2*x^2 + 2*x^4)/(1 + Sqrt[2]*x^2)^2]*EllipticE[2*ArcTan[2^(1/4)*x], (
2 - Sqrt[2])/4])/(135*Sqrt[1 + 2*x^2 + 2*x^4]) + (85*2^(3/4)*(3 - Sqrt[2])*
```

```
(1 + Sqrt[2]*x^2)*Sqrt[(1 + 2*x^2 + 2*x^4)/(1 + Sqrt[2]*x^2)^2]*EllipticF[2
*ArcTan[2^(1/4)*x], (2 - Sqrt[2])/4])/(189*Sqrt[1 + 2*x^2 + 2*x^4]) + (2^(3
/4)*(37 + 23*Sqrt[2])*(1 + Sqrt[2]*x^2)*Sqrt[(1 + 2*x^2 + 2*x^4)/(1 + Sqrt[
2]*x^2)^2]*EllipticF[2*ArcTan[2^(1/4)*x], (2 - Sqrt[2])/4])/(135*Sqrt[1 + 2
*x^2 + 2*x^4]) - (289*(11 - 6*Sqrt[2])*(1 + Sqrt[2]*x^2)*Sqrt[(1 + 2*x^2 +
2*x^4)/(1 + Sqrt[2]*x^2)^2]*EllipticPi[(12 + 11*Sqrt[2])/24, 2*ArcTan[2^(1/
4)*x], (2 - Sqrt[2])/4])/(1134*2^(1/4)*Sqrt[1 + 2*x^2 + 2*x^4])
```

Rule 1117

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1209

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

Rule 1211

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + b*x^2 + c*x^4
], x], x] - Dist[e/q, Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /; Ne
Q[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[
c/a]
```

Rule 1230

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4]), x_S
ymbol] :> With[{q = Rt[c/a, 2]}, Dist[(c*d + a*e*q)/(c*d^2 - a*e^2), Int[1/
Sqrt[a + b*x^2 + c*x^4], x], x] - Dist[(a*e*(e + d*q))/(c*d^2 - a*e^2), Int
[(1 + q*x^2)/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]), x], x]] /; FreeQ[{a, b,
 c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[c/a]
```

Rule 1285

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(a + b*x^2 + c*x^4)^p*((d*(m
+ 4*p + 3) + e*(m + 1)*x^2)/(f*(m + 1)*(m + 4*p + 3))), x] + Dist[2*(p/(f^2
*(m + 1)*(m + 4*p + 3))), Int[(f*x)^(m + 2)*(a + b*x^2 + c*x^4)^(p - 1)*Sim
```

```
p[2*a*e*(m + 1) - b*d*(m + 4*p + 3) + (b*e*(m + 1) - 2*c*d*(m + 4*p + 3))*x
^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b^2 - 4*a*c, 0] && Gt
Q[p, 0] && LtQ[m, -1] && m + 4*p + 3 != 0 && IntegerQ[2*p] && (IntegerQ[p]
|| IntegerQ[m])
```

Rule 1295

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[d*(f*x)^(m + 1)*((a + b*x^2 + c*x^4)^(p + 1)
/(a*f*(m + 1))), x] + Dist[1/(a*f^2*(m + 1)), Int[(f*x)^(m + 2)*(a + b*x^2
+ c*x^4)^p*Simp[a*e*(m + 1) - b*d*(m + 2*p + 3) - c*d*(m + 4*p + 5)*x^2, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[m
, -1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1323

```
Int[(((f_.)*(x_))^(m_)*((a_.) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.))/((d_.)
+ (e_.)*(x_)^2), x_Symbol] :> Dist[1/d^2, Int[(f*x)^m*(a*d + (b*d - a*e)*x^
2)*(a + b*x^2 + c*x^4)^(p - 1), x], x] + Dist[(c*d^2 - b*d*e + a*e^2)/(d^2*
f^4), Int[(f*x)^(m + 4)*((a + b*x^2 + c*x^4)^(p - 1)/(d + e*x^2)), x], x] /
; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && LtQ[m
, -2]
```

Rule 1325

```
Int[(((f_.)*(x_))^(m_)*((a_.) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.))/((d_.)
+ (e_.)*(x_)^2), x_Symbol] :> Dist[1/(d*e), Int[(f*x)^m*(a*e + c*d*x^2)*(a
+ b*x^2 + c*x^4)^(p - 1), x], x] - Dist[(c*d^2 - b*d*e + a*e^2)/(d*e*f^2),
Int[(f*x)^(m + 2)*((a + b*x^2 + c*x^4)^(p - 1)/(d + e*x^2)), x], x] /; Free
Q[{a, b, c, d, e, f}, x] && NeQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && LtQ[m, 0]
```

Rule 1720

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4]), x_Symbol] :> With[{q = Rt[B/A, 2]}, Simp[(-(B*d - A*e))*(A
rcTan[Rt[-b + c*(d/e) + a*(e/d), 2]*(x/Sqrt[a + b*x^2 + c*x^4])]/(2*d*e*Rt[
-b + c*(d/e) + a*(e/d), 2])), x] + Simp[(B*d + A*e)*(A + B*x^2)*(Sqrt[A^2*(
(a + b*x^2 + c*x^4)/(a*(A + B*x^2)^2))]/(4*d*e*A*q*Sqrt[a + b*x^2 + c*x^4])
)*EllipticPi[Cancel[-(B*d - A*e)^2/(4*d*e*A*B)], 2*ArcTan[q*x], 1/2 - b*(A/
(4*a*B))], x]] /; FreeQ[{a, b, c, d, e, A, B}, x] && NeQ[b^2 - 4*a*c, 0] &&
 NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] && EqQ
[c*A^2 - a*B^2, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{9}\int \frac{(3+8x^2)\sqrt{1+2x^2+2x^4}}{x^6}\,dx + \frac{34}{9}\int \frac{\sqrt{1+2x^2+2x^4}}{x^2(3-2x^2)}\,dx \\
&= -\frac{(3+40x^2)\sqrt{1+2x^2+2x^4}}{45x^5} + \frac{1}{45}\int \frac{-74-68x^2}{x^4\sqrt{1+2x^2+2x^4}}\,dx \\
&\quad - \frac{17}{27}\int \frac{-2+6x^2}{x^2\sqrt{1+2x^2+2x^4}}\,dx + \frac{578}{27}\int \frac{1}{(3-2x^2)\sqrt{1+2x^2+2x^4}}\,dx \\
&= \frac{74\sqrt{1+2x^2+2x^4}}{135x^3} - \frac{34\sqrt{1+2x^2+2x^4}}{27x} - \frac{(3+40x^2)\sqrt{1+2x^2+2x^4}}{45x^5} \\
&\quad - \frac{1}{135}\int \frac{-92-148x^2}{x^2\sqrt{1+2x^2+2x^4}}\,dx + \frac{17}{27}\int \frac{-6+4x^2}{\sqrt{1+2x^2+2x^4}}\,dx \\
&\quad - \frac{1}{189}\left(578\left(2-3\sqrt{2}\right)\right)\int \frac{1+\sqrt{2}x^2}{(3-2x^2)\sqrt{1+2x^2+2x^4}}\,dx \\
&\quad + \frac{1}{189}\left(578\left(3-\sqrt{2}\right)\right)\int \frac{1}{\sqrt{1+2x^2+2x^4}}\,dx \\
&= \frac{74\sqrt{1+2x^2+2x^4}}{135x^3} - \frac{262\sqrt{1+2x^2+2x^4}}{135x} \\
&\quad - \frac{(3+40x^2)\sqrt{1+2x^2+2x^4}}{45x^5} + \frac{17}{27}\sqrt{\frac{17}{3}}\tanh^{-1}\left(\frac{\sqrt{\frac{17}{3}}x}{\sqrt{1+2x^2+2x^4}}\right) \\
&\quad + \frac{289\left(3-\sqrt{2}\right)\left(1+\sqrt{2}x^2\right)\sqrt{\frac{1+2x^2+2x^4}{\left(1+\sqrt{2}x^2\right)^2}}F\left(2\tan^{-1}\left(\sqrt[4]{2}x\right)\middle|\frac{1}{4}\left(2-\sqrt{2}\right)\right)}{189\sqrt[4]{2}\sqrt{1+2x^2+2x^4}} \\
&\quad - \frac{289\left(11-6\sqrt{2}\right)\left(1+\sqrt{2}x^2\right)\sqrt{\frac{1+2x^2+2x^4}{\left(1+\sqrt{2}x^2\right)^2}}\Pi\left(\frac{1}{24}\left(12+11\sqrt{2}\right);2\tan^{-1}\left(\sqrt[4]{2}x\right)\middle|\frac{1}{4}\left(2-\sqrt{2}\right)\right)}{1134\sqrt[4]{2}\sqrt{1+2x^2+2x^4}} \\
&\quad + \frac{1}{135}\int \frac{148+184x^2}{\sqrt{1+2x^2+2x^4}}\,dx \\
&\quad - \frac{1}{27}\left(34\sqrt{2}\right)\int \frac{1-\sqrt{2}x^2}{\sqrt{1+2x^2+2x^4}}\,dx - \frac{1}{27}\left(34\left(3-\sqrt{2}\right)\right)\int \frac{1}{\sqrt{1+2x^2+2x^4}}\,dx
\end{aligned}$$

$$
\begin{aligned}
&= \frac{74\sqrt{1+2x^2+2x^4}}{135x^3} - \frac{262\sqrt{1+2x^2+2x^4}}{135x} - \frac{(3+40x^2)\sqrt{1+2x^2+2x^4}}{45x^5} \\
&\quad + \frac{34\sqrt{2}x\sqrt{1+2x^2+2x^4}}{27\left(1+\sqrt{2}x^2\right)} + \frac{17}{27}\sqrt{\frac{17}{3}}\tanh^{-1}\left(\frac{\sqrt{\frac{17}{3}}x}{\sqrt{1+2x^2+2x^4}}\right) \\
&\quad - \frac{34\sqrt[4]{2}\left(1+\sqrt{2}x^2\right)\sqrt{\frac{1+2x^2+2x^4}{\left(1+\sqrt{2}x^2\right)^2}}E\left(2\tan^{-1}\left(\sqrt[4]{2}x\right)|\frac{1}{4}\left(2-\sqrt{2}\right)\right)}{27\sqrt{1+2x^2+2x^4}} \\
&\quad + \frac{85\ 2^{3/4}\left(3-\sqrt{2}\right)\left(1+\sqrt{2}x^2\right)\sqrt{\frac{1+2x^2+2x^4}{\left(1+\sqrt{2}x^2\right)^2}}F\left(2\tan^{-1}\left(\sqrt[4]{2}x\right)|\frac{1}{4}\left(2-\sqrt{2}\right)\right)}{189\sqrt{1+2x^2+2x^4}} \\
&\quad - \frac{289\left(11-6\sqrt{2}\right)\left(1+\sqrt{2}x^2\right)\sqrt{\frac{1+2x^2+2x^4}{\left(1+\sqrt{2}x^2\right)^2}}\Pi\left(\frac{1}{24}\left(12+11\sqrt{2}\right);2\tan^{-1}\left(\sqrt[4]{2}x\right)|\frac{1}{4}\left(2-\sqrt{2}\right)\right)}{1134\sqrt[4]{2}\sqrt{1+2x^2+2x^4}} \\
&\quad - \frac{1}{135}\left(92\sqrt{2}\right)\int \frac{1-\sqrt{2}x^2}{\sqrt{1+2x^2+2x^4}}\,dx + \frac{1}{135}\left(4\left(37+23\sqrt{2}\right)\right)\int \frac{1}{\sqrt{1+2x^2+2x^4}}\,dx \\
&= \frac{74\sqrt{1+2x^2+2x^4}}{135x^3} - \frac{262\sqrt{1+2x^2+2x^4}}{135x} - \frac{(3+40x^2)\sqrt{1+2x^2+2x^4}}{45x^5} \\
&\quad + \frac{262\sqrt{2}x\sqrt{1+2x^2+2x^4}}{135\left(1+\sqrt{2}x^2\right)} + \frac{17}{27}\sqrt{\frac{17}{3}}\tanh^{-1}\left(\frac{\sqrt{\frac{17}{3}}x}{\sqrt{1+2x^2+2x^4}}\right) \\
&\quad - \frac{262\sqrt[4]{2}\left(1+\sqrt{2}x^2\right)\sqrt{\frac{1+2x^2+2x^4}{\left(1+\sqrt{2}x^2\right)^2}}E\left(2\tan^{-1}\left(\sqrt[4]{2}x\right)|\frac{1}{4}\left(2-\sqrt{2}\right)\right)}{135\sqrt{1+2x^2+2x^4}} \\
&\quad + \frac{85\ 2^{3/4}\left(3-\sqrt{2}\right)\left(1+\sqrt{2}x^2\right)\sqrt{\frac{1+2x^2+2x^4}{\left(1+\sqrt{2}x^2\right)^2}}F\left(2\tan^{-1}\left(\sqrt[4]{2}x\right)|\frac{1}{4}\left(2-\sqrt{2}\right)\right)}{189\sqrt{1+2x^2+2x^4}} \\
&\quad + \frac{2^{3/4}\left(37+23\sqrt{2}\right)\left(1+\sqrt{2}x^2\right)\sqrt{\frac{1+2x^2+2x^4}{\left(1+\sqrt{2}x^2\right)^2}}F\left(2\tan^{-1}\left(\sqrt[4]{2}x\right)|\frac{1}{4}\left(2-\sqrt{2}\right)\right)}{135\sqrt{1+2x^2+2x^4}} \\
&\quad - \frac{289\left(11-6\sqrt{2}\right)\left(1+\sqrt{2}x^2\right)\sqrt{\frac{1+2x^2+2x^4}{\left(1+\sqrt{2}x^2\right)^2}}\Pi\left(\frac{1}{24}\left(12+11\sqrt{2}\right);2\tan^{-1}\left(\sqrt[4]{2}x\right)|\frac{1}{4}\left(2-\sqrt{2}\right)\right)}{1134\sqrt[4]{2}\sqrt{1+2x^2+2x^4}}
\end{aligned}
$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.27 (sec) , antiderivative size = 224, normalized size of antiderivative = 0.41

$$\int \frac{(1+2x^2+2x^4)^{3/2}}{x^6(3-2x^2)}\,dx =$$
$$-\frac{27+192x^2+1116x^4+1848x^6+1572x^8+786i\sqrt{1-i}x^5\sqrt{1+(1-i)x^2}\sqrt{1+(1+i)x^2}E\left(i\text{arcsinh}\left(\sqrt{1-\right.\right.}{}$$

```
[In] Integrate[(1 + 2*x^2 + 2*x^4)^(3/2)/(x^6*(3 - 2*x^2)),x]
```

```
[Out] -1/405*(27 + 192*x^2 + 1116*x^4 + 1848*x^6 + 1572*x^8 + (786*I)*Sqrt[1 - I]
*x^5*Sqrt[1 + (1 - I)*x^2]*Sqrt[1 + (1 + I)*x^2]*EllipticE[I*ArcSinh[Sqrt[1
 - I]*x], I] + (543 - 1329*I)*Sqrt[1 - I]*x^5*Sqrt[1 + (1 - I)*x^2]*Sqrt[1
+ (1 + I)*x^2]*EllipticF[I*ArcSinh[Sqrt[1 - I]*x], I] - 1445*(1 - I)^(3/2)*
x^5*Sqrt[1 + (1 - I)*x^2]*Sqrt[1 + (1 + I)*x^2]*EllipticPi[-1/3 - I/3, I*Ar
cSinh[Sqrt[1 - I]*x], I])/(x^5*Sqrt[1 + 2*x^2 + 2*x^4])
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 5.11 (sec) , antiderivative size = 263, normalized size of antiderivative = 0.48

method	result
risch	$-\frac{524x^8+616x^6+372x^4+64x^2+9}{135x^5\sqrt{2x^4+2x^2+1}}-\frac{362\sqrt{1+(1-i)x^2}\sqrt{1+(1+i)x^2}\,F\left(x\sqrt{-1+i},\frac{\sqrt{2}}{2}+\frac{i\sqrt{2}}{2}\right)}{135\sqrt{-1+i}\sqrt{2x^4+2x^2+1}}+\frac{\left(-\frac{262}{135}+\frac{262i}{135}\right)\sqrt{1+(1-i)x^2}\sqrt{1+(1\cdots}}{}$
elliptic	$-\frac{\sqrt{2x^4+2x^2+1}}{15x^5}-\frac{46\sqrt{2x^4+2x^2+1}}{135x^3}-\frac{262\sqrt{2x^4+2x^2+1}}{135x}-\frac{208\sqrt{-ix^2+x^2+1}\sqrt{ix^2+x^2+1}\,F\left(x\sqrt{-1+i},\frac{\sqrt{2}}{2}+\frac{i\sqrt{2}}{2}\right)}{45\sqrt{-1+i}\sqrt{2x^4+2x^2+1}}+\frac{262i\sqrt{-i\cdots}}{}$
default	$-\frac{\sqrt{2x^4+2x^2+1}}{15x^5}-\frac{46\sqrt{2x^4+2x^2+1}}{135x^3}-\frac{262\sqrt{2x^4+2x^2+1}}{135x}+\frac{184\sqrt{1+(1-i)x^2}\sqrt{1+(1+i)x^2}\,F\left(x\sqrt{-1+i},\frac{\sqrt{2}}{2}+\frac{i\sqrt{2}}{2}\right)}{45\sqrt{-1+i}\sqrt{2x^4+2x^2+1}}+\frac{\left(-\frac{52}{15}+\frac{52}{15}\cdots\right.}{}$

```
[In] int((2*x^4+2*x^2+1)^(3/2)/x^6/(-2*x^2+3),x,method=_RETURNVERBOSE)
```

```
[Out] -1/135*(524*x^8+616*x^6+372*x^4+64*x^2+9)/x^5/(2*x^4+2*x^2+1)^(1/2)-362/135
/(-1+I)^(1/2)*(1+(1-I)*x^2)^(1/2)*(1+(1+I)*x^2)^(1/2)/(2*x^4+2*x^2+1)^(1/2)
*EllipticF(x*(-1+I)^(1/2),1/2*2^(1/2)+1/2*I*2^(1/2))+(-262/135+262/135*I)/(
-1+I)^(1/2)*(1+(1-I)*x^2)^(1/2)*(1+(1+I)*x^2)^(1/2)/(2*x^4+2*x^2+1)^(1/2)*(
EllipticF(x*(-1+I)^(1/2),1/2*2^(1/2)+1/2*I*2^(1/2))-EllipticE(x*(-1+I)^(1/2
),1/2*2^(1/2)+1/2*I*2^(1/2)))+578/81/(-1+I)^(1/2)*(1-I*x^2+x^2)^(1/2)*(1+I*
x^2+x^2)^(1/2)/(2*x^4+2*x^2+1)^(1/2)*EllipticPi(x*(-1+I)^(1/2),-1/3-1/3*I,(
-1-I)^(1/2)/(-1+I)^(1/2))
```

Fricas [F]

$$\int \frac{(1+2x^2+2x^4)^{3/2}}{x^6(3-2x^2)}\,dx = \int -\frac{(2\,x^4+2\,x^2+1)^{\frac{3}{2}}}{(2\,x^2-3)x^6}\,dx$$

```
[In] integrate((2*x^4+2*x^2+1)^(3/2)/x^6/(-2*x^2+3),x, algorithm="fricas")
[Out] integral(-(2*x^4 + 2*x^2 + 1)^(3/2)/(2*x^8 - 3*x^6), x)
```

Sympy [F]

$$\int \frac{(1+2x^2+2x^4)^{3/2}}{x^6(3-2x^2)}\,dx = -\int \frac{\sqrt{2x^4+2x^2+1}}{2x^8-3x^6}\,dx - \int \frac{2x^2\sqrt{2x^4+2x^2+1}}{2x^8-3x^6}\,dx - \int \frac{2x^4\sqrt{2x^4+2x^2+1}}{2x^8-3x^6}\,dx$$

```
[In] integrate((2*x**4+2*x**2+1)**(3/2)/x**6/(-2*x**2+3),x)
[Out] -Integral(sqrt(2*x**4 + 2*x**2 + 1)/(2*x**8 - 3*x**6), x) - Integral(2*x**2
*sqrt(2*x**4 + 2*x**2 + 1)/(2*x**8 - 3*x**6), x) - Integral(2*x**4*sqrt(2*x
**4 + 2*x**2 + 1)/(2*x**8 - 3*x**6), x)
```

Maxima [F]

$$\int \frac{(1+2x^2+2x^4)^{3/2}}{x^6(3-2x^2)}\,dx = \int -\frac{(2\,x^4+2\,x^2+1)^{\frac{3}{2}}}{(2\,x^2-3)x^6}\,dx$$

```
[In] integrate((2*x^4+2*x^2+1)^(3/2)/x^6/(-2*x^2+3),x, algorithm="maxima")
[Out] -integrate((2*x^4 + 2*x^2 + 1)^(3/2)/((2*x^2 - 3)*x^6), x)
```

Giac [F]

$$\int \frac{(1+2x^2+2x^4)^{3/2}}{x^6(3-2x^2)}\,dx = \int -\frac{(2\,x^4+2\,x^2+1)^{\frac{3}{2}}}{(2\,x^2-3)x^6}\,dx$$

```
[In] integrate((2*x^4+2*x^2+1)^(3/2)/x^6/(-2*x^2+3),x, algorithm="giac")
[Out] integrate(-(2*x^4 + 2*x^2 + 1)^(3/2)/((2*x^2 - 3)*x^6), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(1+2x^2+2x^4)^{3/2}}{x^6(3-2x^2)}\,dx = -\int \frac{(2\,x^4+2\,x^2+1)^{3/2}}{x^6\,(2\,x^2-3)}\,dx$$

```
[In] int(-(2*x^2 + 2*x^4 + 1)^(3/2)/(x^6*(2*x^2 - 3)),x)
[Out] -int((2*x^2 + 2*x^4 + 1)^(3/2)/(x^6*(2*x^2 - 3)), x)
```

3.332 $\int \frac{x^5}{(d+ex^2)\sqrt{a+bx^2+cx^4}}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 173

$$\int \frac{x^5}{(d+ex^2)\sqrt{a+bx^2+cx^4}}\,dx = \frac{\sqrt{a+bx^2+cx^4}}{2ce} - \frac{(2cd+be)\text{arctanh}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{4c^{3/2}e^2} + \frac{d^2\text{arctanh}\left(\frac{bd-2ae+(2cd-be)x^2}{2\sqrt{cd^2-bde+ae^2}\sqrt{a+bx^2+cx^4}}\right)}{2e^2\sqrt{cd^2-bde+ae^2}}$$

```
[Out] -1/4*(b*e+2*c*d)*arctanh(1/2*(2*c*x^2+b)/c^(1/2)/(c*x^4+b*x^2+a)^(1/2))/c^(
3/2)/e^2+1/2*d^2*arctanh(1/2*(b*d-2*a*e+(-b*e+2*c*d)*x^2)/(a*e^2-b*d*e+c*d^
2)^(1/2)/(c*x^4+b*x^2+a)^(1/2))/e^2/(a*e^2-b*d*e+c*d^2)^(1/2)+1/2*(c*x^4+b*
x^2+a)^(1/2)/c/e
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 173, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.207, Rules used = {1265, 1667, 857, 635, 212, 738}

$$\int \frac{x^5}{(d+ex^2)\sqrt{a+bx^2+cx^4}}\,dx = -\frac{(be+2cd)\text{arctanh}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{4c^{3/2}e^2} + \frac{d^2\text{arctanh}\left(\frac{-2ae+x^2(2cd-be)+bd}{2\sqrt{a+bx^2+cx^4}\sqrt{ae^2-bde+cd^2}}\right)}{2e^2\sqrt{ae^2-bde+cd^2}} + \frac{\sqrt{a+bx^2+cx^4}}{2ce}$$

```
[In] Int[x^5/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]),x]
```

```
[Out] Sqrt[a + b*x^2 + c*x^4]/(2*c*e) - ((2*c*d + b*e)*ArcTanh[(b + 2*c*x^2)/(2*S
qrt[c]*Sqrt[a + b*x^2 + c*x^4])])/(4*c^(3/2)*e^2) + (d^2*ArcTanh[(b*d - 2*a
*e + (2*c*d - b*e)*x^2)/(2*Sqrt[c*d^2 - b*d*e + a*e^2]*Sqrt[a + b*x^2 + c*x
^4])])/(2*e^2*Sqrt[c*d^2 - b*d*e + a*e^2])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 635

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 857

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1265

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rule 1667

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, S
imp[f*(d + e*x)^(m + q - 1)*((a + b*x + c*x^2)^(p + 1)/(c*e^(q - 1)*(m + q
+ 2*p + 1))), x] + Dist[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + b
*x + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1
)*(d + e*x)^q - f*(d + e*x)^(q - 2)*(b*d*e*(p + 1) + a*e^2*(m + q - 1) - c*
```

```
d^2*(m + q + 2*p + 1) - e*(2*c*d - b*e)*(m + q + p)*x), x], x], x] /; GtQ[q
, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, d, e, m, p}, x] && Poly
Q[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !(IGtQ
[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{2}\text{Subst}\left(\int \frac{x^2}{(d+ex)\sqrt{a+bx+cx^2}}\,dx, x, x^2\right)\\
&= \frac{\sqrt{a+bx^2+cx^4}}{2ce} + \frac{\text{Subst}\left(\int \frac{-\frac{1}{2}bde-\frac{1}{2}e(2cd+be)x}{(d+ex)\sqrt{a+bx+cx^2}}\,dx, x, x^2\right)}{2ce^2}\\
&= \frac{\sqrt{a+bx^2+cx^4}}{2ce} + \frac{d^2\text{Subst}\left(\int \frac{1}{(d+ex)\sqrt{a+bx+cx^2}}\,dx, x, x^2\right)}{2e^2}\\
&\quad - \frac{(2cd+be)\text{Subst}\left(\int \frac{1}{\sqrt{a+bx+cx^2}}\,dx, x, x^2\right)}{4ce^2}\\
&= \frac{\sqrt{a+bx^2+cx^4}}{2ce} - \frac{d^2\text{Subst}\left(\int \frac{1}{4cd^2-4bde+4ae^2-x^2}\,dx, x, \frac{-bd+2ae-(2cd-be)x^2}{\sqrt{a+bx^2+cx^4}}\right)}{e^2}\\
&\quad - \frac{(2cd+be)\text{Subst}\left(\int \frac{1}{4c-x^2}\,dx, x, \frac{b+2cx^2}{\sqrt{a+bx^2+cx^4}}\right)}{2ce^2}\\
&= \frac{\sqrt{a+bx^2+cx^4}}{2ce} - \frac{(2cd+be)\tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{4c^{3/2}e^2} + \frac{d^2\tanh^{-1}\left(\frac{bd-2ae+(2cd-be)x^2}{2\sqrt{cd^2-bde+ae^2}\sqrt{a+bx^2+cx^4}}\right)}{2e^2\sqrt{cd^2-bde+ae^2}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.78 (sec) , antiderivative size = 166, normalized size of antiderivative = 0.96

$$\int \frac{x^5}{(d+ex^2)\sqrt{a+bx^2+cx^4}}\,dx = \frac{2\sqrt{c}\left(e\sqrt{a+bx^2+cx^4} - \frac{2cd^2\arctan\left(\frac{\sqrt{c}(d+ex^2)-e\sqrt{a+bx^2+cx^4}}{\sqrt{-cd^2+bde-ae^2}}\right)}{\sqrt{-cd^2+bde-ae^2}}\right) - (2cd+be)\text{arctanh}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{4c^{3/2}e^2}$$

```
[In] Integrate[x^5/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]),x]
```

```
[Out] (2*Sqrt[c]*(e*Sqrt[a + b*x^2 + c*x^4] - (2*c*d^2*ArcTan[(Sqrt[c]*(d + e*x^2
) - e*Sqrt[a + b*x^2 + c*x^4])/Sqrt[-(c*d^2) + b*d*e - a*e^2]])/Sqrt[-(c*d^
2) + b*d*e - a*e^2]) - (2*c*d + b*e)*ArcTanh[(b + 2*c*x^2)/(2*Sqrt[c]*Sqrt[
a + b*x^2 + c*x^4])])/(4*c^(3/2)*e^2)
```

Maple [A] (verified)

Time = 0.44 (sec) , antiderivative size = 211, normalized size of antiderivative = 1.22

method	result
pseudoelliptic	$\frac{-\ln\left(\frac{2\sqrt{c\,x^4+b\,x^2+a}\,\sqrt{\frac{a\,e^2-bde+c\,d^2}{e^2}}\,e+\left(b\,x^2+2a\right)e-d\left(2c\,x^2+b\right)}{e\,x^2+d}\right)c^{\frac{3}{2}}d^2+\left(\left(-cd-\frac{be}{2}\right)\ln\left(\frac{2c\,x^2+2\sqrt{c\,x^4+b\,x^2+a}\,\sqrt{c}+b}{\sqrt{c}}\right)+\sqrt{c\,x^4\cdots}\right.}{2c^{\frac{3}{2}}\sqrt{\frac{a\,e^2-bde+c\,d^2}{e^2}}\,e^3}$
risch	$\frac{\sqrt{c\,x^4+b\,x^2+a}}{2ce}-\frac{\frac{(be+2cd)\ln\left(\frac{\frac{b}{2}+c\,x^2}{\sqrt{c}}+\sqrt{c\,x^4+b\,x^2+a}\right)}{2e\sqrt{c}}+\frac{c\,d^2\ln\left(\frac{\frac{2a\,e^2-2bde+2c\,d^2}{e^2}+\frac{(be-2cd)\left(x^2+\frac{d}{e}\right)}{e}+2\sqrt{\frac{a\,e^2-bde+c\,d^2}{e^2}}\sqrt{c\left(x^2\cdots\right.}}{x^2+\frac{d}{e}}\right.}{e^2\sqrt{\frac{a\,e^2-bde+c\,d^2}{e^2}}}}{2ce}$
default	$\frac{\frac{\sqrt{c\,x^4+b\,x^2+a}}{2c}-\frac{b\ln\left(\frac{\frac{b}{2}+c\,x^2}{\sqrt{c}}+\sqrt{c\,x^4+b\,x^2+a}\right)}{4c^{\frac{3}{2}}}}{e}-\frac{d\ln\left(\frac{\frac{b}{2}+c\,x^2}{\sqrt{c}}+\sqrt{c\,x^4+b\,x^2+a}\right)}{2e^2\sqrt{c}}-\frac{d^2\ln\left(\frac{\frac{2a\,e^2-2bde+2c\,d^2}{e^2}+\frac{(be-2cd)\left(x^2+\cdots\right.}{e}\cdots\right.}{\cdots}$
elliptic	$-\frac{d\ln\left(\frac{\frac{b}{2}+c\,x^2}{\sqrt{c}}+\sqrt{c\,x^4+b\,x^2+a}\right)}{2e^2\sqrt{c}}+\frac{\sqrt{c\,x^4+b\,x^2+a}}{2ce}-\frac{b\ln\left(\frac{\frac{b}{2}+c\,x^2}{\sqrt{c}}+\sqrt{c\,x^4+b\,x^2+a}\right)}{4e\,c^{\frac{3}{2}}}-\frac{d^2\ln\left(\frac{\frac{2a\,e^2-2bde+2c\,d^2}{e^2}+\frac{(be-2c\cdots}{\cdots}}{\cdots}\right.}{\cdots}$

```
[In] int(x^5/(e*x^2+d)/(c*x^4+b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2/c^(3/2)*(-ln((2*(c*x^4+b*x^2+a)^(1/2)*((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*e
+(b*x^2+2*a)*e-d*(2*c*x^2+b))/(e*x^2+d))*c^(3/2)*d^2+((-c*d-1/2*b*e)*ln((2*
c*x^2+2*(c*x^4+b*x^2+a)^(1/2)*c^(1/2)+b)/c^(1/2))+(c*x^4+b*x^2+a)^(1/2)*c^(
1/2)*e+1/2*ln(2)*(b*e+2*c*d))*e*((a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/((a*e^2-b*
d*e+c*d^2)/e^2)^(1/2)/e^3
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 319 vs. 2(149) = 298.

Time = 12.72 (sec) , antiderivative size = 1364, normalized size of antiderivative = 7.88

$$\int \frac{x^5}{(d+ex^2)\sqrt{a+bx^2+cx^4}}\,dx = \text{Too large to display}$$

```
[In] integrate(x^5/(e*x^2+d)/(c*x^4+b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/8*(2*sqrt(c*d^2 - b*d*e + a*e^2)*c^2*d^2*log(-((8*c^2*d^2 - 8*b*c*d*e +
(b^2 + 4*a*c)*e^2)*x^4 - 8*a*b*d*e + 8*a^2*e^2 + (b^2 + 4*a*c)*d^2 + 2*(4*b
*c*d^2 + 4*a*b*e^2 - (3*b^2 + 4*a*c)*d*e)*x^2 + 4*sqrt(c*x^4 + b*x^2 + a)*s
qrt(c*d^2 - b*d*e + a*e^2)*((2*c*d - b*e)*x^2 + b*d - 2*a*e))/(e^2*x^4 + 2*
d*e*x^2 + d^2)) + (2*c^2*d^3 - b*c*d^2*e + a*b*e^3 - (b^2 - 2*a*c)*d*e^2)*s
```

```
qrt(c)*log(-8*c^2*x^4 - 8*b*c*x^2 - b^2 + 4*sqrt(c*x^4 + b*x^2 + a)*(2*c*x^
2 + b)*sqrt(c) - 4*a*c) + 4*(c^2*d^2*e - b*c*d*e^2 + a*c*e^3)*sqrt(c*x^4 +
b*x^2 + a))/(c^3*d^2*e^2 - b*c^2*d*e^3 + a*c^2*e^4), 1/8*(4*sqrt(-c*d^2 + b
*d*e - a*e^2)*c^2*d^2*arctan(-1/2*sqrt(c*x^4 + b*x^2 + a)*sqrt(-c*d^2 + b*d
*e - a*e^2)*((2*c*d - b*e)*x^2 + b*d - 2*a*e)/((c^2*d^2 - b*c*d*e + a*c*e^2
)*x^4 + a*c*d^2 - a*b*d*e + a^2*e^2 + (b*c*d^2 - b^2*d*e + a*b*e^2)*x^2)) +
 (2*c^2*d^3 - b*c*d^2*e + a*b*e^3 - (b^2 - 2*a*c)*d*e^2)*sqrt(c)*log(-8*c^2
*x^4 - 8*b*c*x^2 - b^2 + 4*sqrt(c*x^4 + b*x^2 + a)*(2*c*x^2 + b)*sqrt(c) -
4*a*c) + 4*(c^2*d^2*e - b*c*d*e^2 + a*c*e^3)*sqrt(c*x^4 + b*x^2 + a))/(c^3*
d^2*e^2 - b*c^2*d*e^3 + a*c^2*e^4), 1/4*(sqrt(c*d^2 - b*d*e + a*e^2)*c^2*d^
2*log(-((8*c^2*d^2 - 8*b*c*d*e + (b^2 + 4*a*c)*e^2)*x^4 - 8*a*b*d*e + 8*a^2
*e^2 + (b^2 + 4*a*c)*d^2 + 2*(4*b*c*d^2 + 4*a*b*e^2 - (3*b^2 + 4*a*c)*d*e)*
x^2 + 4*sqrt(c*x^4 + b*x^2 + a)*sqrt(c*d^2 - b*d*e + a*e^2)*((2*c*d - b*e)*
x^2 + b*d - 2*a*e))/(e^2*x^4 + 2*d*e*x^2 + d^2)) + (2*c^2*d^3 - b*c*d^2*e +
 a*b*e^3 - (b^2 - 2*a*c)*d*e^2)*sqrt(-c)*arctan(1/2*sqrt(c*x^4 + b*x^2 + a)
*(2*c*x^2 + b)*sqrt(-c)/(c^2*x^4 + b*c*x^2 + a*c)) + 2*(c^2*d^2*e - b*c*d*e
^2 + a*c*e^3)*sqrt(c*x^4 + b*x^2 + a))/(c^3*d^2*e^2 - b*c^2*d*e^3 + a*c^2*e
^4), 1/4*(2*sqrt(-c*d^2 + b*d*e - a*e^2)*c^2*d^2*arctan(-1/2*sqrt(c*x^4 + b
*x^2 + a)*sqrt(-c*d^2 + b*d*e - a*e^2)*((2*c*d - b*e)*x^2 + b*d - 2*a*e)/((
c^2*d^2 - b*c*d*e + a*c*e^2)*x^4 + a*c*d^2 - a*b*d*e + a^2*e^2 + (b*c*d^2 -
 b^2*d*e + a*b*e^2)*x^2)) + (2*c^2*d^3 - b*c*d^2*e + a*b*e^3 - (b^2 - 2*a*c
)*d*e^2)*sqrt(-c)*arctan(1/2*sqrt(c*x^4 + b*x^2 + a)*(2*c*x^2 + b)*sqrt(-c)
/(c^2*x^4 + b*c*x^2 + a*c)) + 2*(c^2*d^2*e - b*c*d*e^2 + a*c*e^3)*sqrt(c*x^
4 + b*x^2 + a))/(c^3*d^2*e^2 - b*c^2*d*e^3 + a*c^2*e^4)]
```

Sympy [F]

$$\int \frac{x^5}{(d+ex^2)\sqrt{a+bx^2+cx^4}}\,dx = \int \frac{x^5}{(d+ex^2)\sqrt{a+bx^2+cx^4}}\,dx$$

```
[In] integrate(x**5/(e*x**2+d)/(c*x**4+b*x**2+a)**(1/2),x)
[Out] Integral(x**5/((d + e*x**2)*sqrt(a + b*x**2 + c*x**4)), x)
```

Maxima [F]

$$\int \frac{x^5}{(d+ex^2)\sqrt{a+bx^2+cx^4}}\,dx = \int \frac{x^5}{\sqrt{cx^4+bx^2+a}(ex^2+d)}\,dx$$

```
[In] integrate(x^5/(e*x^2+d)/(c*x^4+b*x^2+a)^(1/2),x, algorithm="maxima")
[Out] integrate(x^5/(sqrt(c*x^4 + b*x^2 + a)*(e*x^2 + d)), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{x^5}{(d+ex^2)\sqrt{a+bx^2+cx^4}}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(x^5/(e*x^2+d)/(c*x^4+b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument Type
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^5}{(d+ex^2)\sqrt{a+bx^2+cx^4}}\,dx = \int \frac{x^5}{(e\,x^2+d)\,\sqrt{c\,x^4+b\,x^2+a}}\,dx$$

```
[In] int(x^5/((d + e*x^2)*(a + b*x^2 + c*x^4)^(1/2)),x)
```

```
[Out] int(x^5/((d + e*x^2)*(a + b*x^2 + c*x^4)^(1/2)), x)
```

3.333 $\int \frac{x^3}{(d+ex^2)\sqrt{a+bx^2+cx^4}}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 137

$$\int \frac{x^3}{(d+ex^2)\sqrt{a+bx^2+cx^4}}\,dx = \frac{\operatorname{arctanh}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{2\sqrt{c}e} - \frac{d\operatorname{arctanh}\left(\frac{bd-2ae+(2cd-be)x^2}{2\sqrt{cd^2-bde+ae^2}\sqrt{a+bx^2+cx^4}}\right)}{2e\sqrt{cd^2-bde+ae^2}}$$

```
[Out] 1/2*arctanh(1/2*(2*c*x^2+b)/c^(1/2)/(c*x^4+b*x^2+a)^(1/2))/e/c^(1/2)-1/2*d*
arctanh(1/2*(b*d-2*a*e+(-b*e+2*c*d)*x^2)/(a*e^2-b*d*e+c*d^2)^(1/2)/(c*x^4+b
*x^2+a)^(1/2))/e/(a*e^2-b*d*e+c*d^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.172, Rules used = {1265, 857, 635, 212, 738}

$$\int \frac{x^3}{(d+ex^2)\sqrt{a+bx^2+cx^4}}\,dx = \frac{\operatorname{arctanh}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{2\sqrt{c}e} - \frac{d\operatorname{arctanh}\left(\frac{-2ae+x^2(2cd-be)+bd}{2\sqrt{a+bx^2+cx^4}\sqrt{ae^2-bde+cd^2}}\right)}{2e\sqrt{ae^2-bde+cd^2}}$$

```
[In] Int[x^3/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]),x]
```

```
[Out] ArcTanh[(b + 2*c*x^2)/(2*Sqrt[c]*Sqrt[a + b*x^2 + c*x^4])]/(2*Sqrt[c]*e) -
(d*ArcTanh[(b*d - 2*a*e + (2*c*d - b*e)*x^2)/(2*Sqrt[c*d^2 - b*d*e + a*e^2]
*Sqrt[a + b*x^2 + c*x^4])])/(2*e*Sqrt[c*d^2 - b*d*e + a*e^2])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 635

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 857

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1265

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{2}\text{Subst}\left(\int \frac{x}{(d+ex)\sqrt{a+bx+cx^2}}\,dx, x, x^2\right)\\
&= \frac{\text{Subst}\left(\int \frac{1}{\sqrt{a+bx+cx^2}}\,dx, x, x^2\right)}{2e} - \frac{d\text{Subst}\left(\int \frac{1}{(d+ex)\sqrt{a+bx+cx^2}}\,dx, x, x^2\right)}{2e}\\
&= \frac{\text{Subst}\left(\int \frac{1}{4c-x^2}\,dx, x, \frac{b+2cx^2}{\sqrt{a+bx^2+cx^4}}\right)}{e} + \frac{d\text{Subst}\left(\int \frac{1}{4cd^2-4bde+4ae^2-x^2}\,dx, x, \frac{-bd+2ae-(2cd-be)x^2}{\sqrt{a+bx^2+cx^4}}\right)}{e}\\
&= \frac{\tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{2\sqrt{c}e} - \frac{d\tanh^{-1}\left(\frac{bd-2ae+(2cd-be)x^2}{2\sqrt{cd^2-bde+ae^2}\sqrt{a+bx^2+cx^4}}\right)}{2e\sqrt{cd^2-bde+ae^2}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.42 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.07

$$\int \frac{x^3}{(d+ex^2)\sqrt{a+bx^2+cx^4}}\,dx$$
$$= -\frac{\frac{2d\sqrt{-cd^2+bde-ae^2}\arctan\left(\frac{\sqrt{c}(d+ex^2)-e\sqrt{a+bx^2+cx^4}}{\sqrt{-cd^2+e(bd-ae)}}\right)}{cd^2+e(-bd+ae)}+\frac{\log\left(e\left(b+2cx^2-2\sqrt{c}\sqrt{a+bx^2+cx^4}\right)\right)}{\sqrt{c}}}{2e}$$

```
[In] Integrate[x^3/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]),x]
```

```
[Out] -1/2*((2*d*Sqrt[-(c*d^2) + b*d*e - a*e^2]*ArcTan[(Sqrt[c]*(d + e*x^2) - e*S
qrt[a + b*x^2 + c*x^4])/Sqrt[-(c*d^2) + e*(b*d - a*e)]])/(c*d^2 + e*(-(b*d)
 + a*e)) + Log[e*(b + 2*c*x^2 - 2*Sqrt[c]*Sqrt[a + b*x^2 + c*x^4])]/Sqrt[c]
)/e
```

Maple [A] (verified)

Time = 0.36 (sec) , antiderivative size = 171, normalized size of antiderivative = 1.25

method	result
pseudoelliptic	$\frac{d\ln\left(\frac{2\sqrt{c\,x^4+b\,x^2+a}\,\sqrt{\frac{a\,e^2-bde+c\,d^2}{e^2}}\,e+\left(b\,x^2+2a\right)e-d\left(2c\,x^2+b\right)}{e\,x^2+d}\right)\sqrt{c}+\sqrt{\frac{a\,e^2-bde+c\,d^2}{e^2}}\,e\left(-\ln(2)+\ln\left(\frac{2c\,x^2+2\sqrt{c\,x^4+b\,x^2+a}\,\cdots}{\sqrt{c}}\right.\right.}{2\sqrt{c}\,\sqrt{\frac{a\,e^2-bde+c\,d^2}{e^2}}\,e^2}$
default	$\frac{\ln\left(\frac{\frac{b}{2}+c\,x^2}{\sqrt{c}}+\sqrt{c\,x^4+b\,x^2+a}\right)}{2e\sqrt{c}}+\frac{d\ln\left(\frac{\frac{2a\,e^2-2bde+2c\,d^2}{e^2}+\frac{(be-2cd)\left(x^2+\frac{d}{e}\right)}{e}+2\sqrt{\frac{a\,e^2-bde+c\,d^2}{e^2}}\,\sqrt{c\left(x^2+\frac{d}{e}\right)^2+\frac{(be-2cd)\left(x^2+\frac{d}{e}\right)}{e}\cdots}}{x^2+\frac{d}{e}}\right.}{2e^2\sqrt{\frac{a\,e^2-bde+c\,d^2}{e^2}}}$
elliptic	$\frac{\ln\left(\frac{\frac{b}{2}+c\,x^2}{\sqrt{c}}+\sqrt{c\,x^4+b\,x^2+a}\right)}{2e\sqrt{c}}+\frac{d\ln\left(\frac{\frac{2a\,e^2-2bde+2c\,d^2}{e^2}+\frac{(be-2cd)\left(x^2+\frac{d}{e}\right)}{e}+2\sqrt{\frac{a\,e^2-bde+c\,d^2}{e^2}}\,\sqrt{c\left(x^2+\frac{d}{e}\right)^2+\frac{(be-2cd)\left(x^2+\frac{d}{e}\right)}{e}\cdots}}{x^2+\frac{d}{e}}\right.}{2e^2\sqrt{\frac{a\,e^2-bde+c\,d^2}{e^2}}}$

```
[In] int(x^3/(e*x^2+d)/(c*x^4+b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2/c^(1/2)*(d*ln((2*(c*x^4+b*x^2+a)^(1/2)*((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*
e+(b*x^2+2*a)*e-d*(2*c*x^2+b))/(e*x^2+d))*c^(1/2)+((a*e^2-b*d*e+c*d^2)/e^2)
^(1/2)*e*(-ln(2)+ln((2*c*x^2+2*(c*x^4+b*x^2+a)^(1/2)*c^(1/2)+b)/c^(1/2))))/
((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)/e^2
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 248 vs. $2(117) = 234$.

Time = 0.96 (sec) , antiderivative size = 1084, normalized size of antiderivative = 7.91

$$\int \frac{x^3}{(d+ex^2)\sqrt{a+bx^2+cx^4}}\,dx$$

$$= \left[\frac{\sqrt{cd^2-bde+ae^2}cd\log\left(-\frac{(8c^2d^2-8bcde+(b^2+4ac)e^2)x^4-8abde+8a^2e^2+(b^2+4ac)d^2+2(4bcd^2+4abe^2-(3b^2+4ac)de)x^2-4\sqrt{c}}{e^2x^4+2dex^2+d^2}\right.}{} \right.$$

$$-\frac{2\sqrt{-cd^2+bde-ae^2}cd\arctan\left(-\frac{\sqrt{cx^4+bx^2+a}\sqrt{-cd^2+bde-ae^2}((2cd-be)x^2+bd-2ae)}{2((c^2d^2-bcde+ace^2)x^4+acd^2-abde+a^2e^2+(bcd^2-b^2de+abe^2)x^2)}\right)-(cd^2-bde+ae^2}{4(c^2d^2e-bcde^2+ace^3)}$$

$$-\frac{\sqrt{-cd^2+bde-ae^2}cd\arctan\left(-\frac{\sqrt{cx^4+bx^2+a}\sqrt{-cd^2+bde-ae^2}((2cd-be)x^2+bd-2ae)}{2((c^2d^2-bcde+ace^2)x^4+acd^2-abde+a^2e^2+(bcd^2-b^2de+abe^2)x^2)}\right)+(cd^2-bde+ae^2)}{2(c^2d^2e-bcde^2+ace^3)}$$

```
[In] integrate(x^3/(e*x^2+d)/(c*x^4+b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*(sqrt(c*d^2 - b*d*e + a*e^2)*c*d*log(-((8*c^2*d^2 - 8*b*c*d*e + (b^2 +
 4*a*c)*e^2)*x^4 - 8*a*b*d*e + 8*a^2*e^2 + (b^2 + 4*a*c)*d^2 + 2*(4*b*c*d^2
 + 4*a*b*e^2 - (3*b^2 + 4*a*c)*d*e)*x^2 - 4*sqrt(c*x^4 + b*x^2 + a)*sqrt(c*
d^2 - b*d*e + a*e^2)*((2*c*d - b*e)*x^2 + b*d - 2*a*e))/(e^2*x^4 + 2*d*e*x^
2 + d^2)) + (c*d^2 - b*d*e + a*e^2)*sqrt(c)*log(-8*c^2*x^4 - 8*b*c*x^2 - b^
2 - 4*sqrt(c*x^4 + b*x^2 + a)*(2*c*x^2 + b)*sqrt(c) - 4*a*c))/(c^2*d^2*e -
b*c*d*e^2 + a*c*e^3), -1/4*(2*sqrt(-c*d^2 + b*d*e - a*e^2)*c*d*arctan(-1/2*
sqrt(c*x^4 + b*x^2 + a)*sqrt(-c*d^2 + b*d*e - a*e^2)*((2*c*d - b*e)*x^2 + b
*d - 2*a*e)/((c^2*d^2 - b*c*d*e + a*c*e^2)*x^4 + a*c*d^2 - a*b*d*e + a^2*e^
2 + (b*c*d^2 - b^2*d*e + a*b*e^2)*x^2)) - (c*d^2 - b*d*e + a*e^2)*sqrt(c)*l
og(-8*c^2*x^4 - 8*b*c*x^2 - b^2 - 4*sqrt(c*x^4 + b*x^2 + a)*(2*c*x^2 + b)*s
qrt(c) - 4*a*c))/(c^2*d^2*e - b*c*d*e^2 + a*c*e^3), 1/4*(sqrt(c*d^2 - b*d*e
 + a*e^2)*c*d*log(-((8*c^2*d^2 - 8*b*c*d*e + (b^2 + 4*a*c)*e^2)*x^4 - 8*a*b
*d*e + 8*a^2*e^2 + (b^2 + 4*a*c)*d^2 + 2*(4*b*c*d^2 + 4*a*b*e^2 - (3*b^2 +
4*a*c)*d*e)*x^2 - 4*sqrt(c*x^4 + b*x^2 + a)*sqrt(c*d^2 - b*d*e + a*e^2)*((2
*c*d - b*e)*x^2 + b*d - 2*a*e))/(e^2*x^4 + 2*d*e*x^2 + d^2)) - 2*(c*d^2 - b
*d*e + a*e^2)*sqrt(-c)*arctan(1/2*sqrt(c*x^4 + b*x^2 + a)*(2*c*x^2 + b)*sqr
t(-c)/(c^2*x^4 + b*c*x^2 + a*c)))/(c^2*d^2*e - b*c*d*e^2 + a*c*e^3), -1/2*(
sqrt(-c*d^2 + b*d*e - a*e^2)*c*d*arctan(-1/2*sqrt(c*x^4 + b*x^2 + a)*sqrt(-
c*d^2 + b*d*e - a*e^2)*((2*c*d - b*e)*x^2 + b*d - 2*a*e)/((c^2*d^2 - b*c*d*
e + a*c*e^2)*x^4 + a*c*d^2 - a*b*d*e + a^2*e^2 + (b*c*d^2 - b^2*d*e + a*b*e
^2)*x^2)) + (c*d^2 - b*d*e + a*e^2)*sqrt(-c)*arctan(1/2*sqrt(c*x^4 + b*x^2
+ a)*(2*c*x^2 + b)*sqrt(-c)/(c^2*x^4 + b*c*x^2 + a*c)))/(c^2*d^2*e - b*c*d*
e^2 + a*c*e^3)]
```

Sympy [F]

$$\int \frac{x^3}{(d+ex^2)\sqrt{a+bx^2+cx^4}}\,dx = \int \frac{x^3}{(d+ex^2)\sqrt{a+bx^2+cx^4}}\,dx$$

```
[In] integrate(x**3/(e*x**2+d)/(c*x**4+b*x**2+a)**(1/2),x)
[Out] Integral(x**3/((d + e*x**2)*sqrt(a + b*x**2 + c*x**4)), x)
```

Maxima [F]

$$\int \frac{x^3}{(d+ex^2)\sqrt{a+bx^2+cx^4}}\,dx = \int \frac{x^3}{\sqrt{cx^4+bx^2+a}(ex^2+d)}\,dx$$

```
[In] integrate(x^3/(e*x^2+d)/(c*x^4+b*x^2+a)^(1/2),x, algorithm="maxima")
[Out] integrate(x^3/(sqrt(c*x^4 + b*x^2 + a)*(e*x^2 + d)), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{x^3}{(d+ex^2)\sqrt{a+bx^2+cx^4}}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(x^3/(e*x^2+d)/(c*x^4+b*x^2+a)^(1/2),x, algorithm="giac")
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument Type
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3}{(d+ex^2)\sqrt{a+bx^2+cx^4}}\,dx = \int \frac{x^3}{(e\,x^2+d)\,\sqrt{c\,x^4+b\,x^2+a}}\,dx$$

```
[In] int(x^3/((d + e*x^2)*(a + b*x^2 + c*x^4)^(1/2)),x)
[Out] int(x^3/((d + e*x^2)*(a + b*x^2 + c*x^4)^(1/2)), x)
```

3.334 $\int \frac{x}{(d+ex^2)\sqrt{a+bx^2+cx^4}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 86

$$\int \frac{x}{(d+ex^2)\sqrt{a+bx^2+cx^4}}\,dx=\frac{\operatorname{arctanh}\left(\frac{bd-2ae+(2cd-be)x^2}{2\sqrt{cd^2-bde+ae^2}\sqrt{a+bx^2+cx^4}}\right)}{2\sqrt{cd^2-bde+ae^2}}$$

```
[Out] 1/2*arctanh(1/2*(b*d-2*a*e+(-b*e+2*c*d)*x^2)/(a*e^2-b*d*e+c*d^2)^(1/2)/(c*x
^4+b*x^2+a)^(1/2))/(a*e^2-b*d*e+c*d^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {1261, 738, 212}

$$\int \frac{x}{(d+ex^2)\sqrt{a+bx^2+cx^4}}\,dx=\frac{\operatorname{arctanh}\left(\frac{-2ae+x^2(2cd-be)+bd}{2\sqrt{a+bx^2+cx^4}\sqrt{ae^2-bde+cd^2}}\right)}{2\sqrt{ae^2-bde+cd^2}}$$

```
[In] Int[x/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]),x]
```

```
[Out] ArcTanh[(b*d - 2*a*e + (2*c*d - b*e)*x^2)/(2*Sqrt[c*d^2 - b*d*e + a*e^2]*Sq
rt[a + b*x^2 + c*x^4])]/(2*Sqrt[c*d^2 - b*d*e + a*e^2])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 1261

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(
p_.), x_Symbol] :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x],
 x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{2}\text{Subst}\left(\int \frac{1}{(d+ex)\sqrt{a+bx+cx^2}}\,dx, x, x^2\right)\\
&= -\text{Subst}\left(\int \frac{1}{4cd^2-4bde+4ae^2-x^2}\,dx, x, \frac{-bd+2ae-(2cd-be)x^2}{\sqrt{a+bx^2+cx^4}}\right)\\
&= -\frac{\tanh^{-1}\left(\frac{-bd+2ae-(2cd-be)x^2}{2\sqrt{cd^2-bde+ae^2}\sqrt{a+bx^2+cx^4}}\right)}{2\sqrt{cd^2-bde+ae^2}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.28 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.12

$$\int \frac{x}{(d+ex^2)\sqrt{a+bx^2+cx^4}}\,dx = \frac{\sqrt{-cd^2+bde-ae^2}\arctan\left(\frac{\sqrt{c}(d+ex^2)-e\sqrt{a+bx^2+cx^4}}{\sqrt{-cd^2+e(bd-ae)}}\right)}{cd^2+e(-bd+ae)}$$

```
[In] Integrate[x/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]),x]
[Out] (Sqrt[-(c*d^2) + b*d*e - a*e^2]*ArcTan[(Sqrt[c]*(d + e*x^2) - e*Sqrt[a + b*
x^2 + c*x^4])/Sqrt[-(c*d^2) + e*(b*d - a*e)]])/(c*d^2 + e*(-(b*d) + a*e))
```

Maple [A] (verified)

Time = 0.35 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.17

method	result	size
pseudoelliptic	$-\frac{\ln\left(\frac{2\sqrt{c\,x^4+b\,x^2+a}\,\sqrt{\frac{a\,e^2-bde+c\,d^2}{e^2}}\,e+\left(b\,x^2+2a\right)e-d\left(2c\,x^2+b\right)}{e\,x^2+d}\right)}{2\sqrt{\frac{a\,e^2-bde+c\,d^2}{e^2}}\,e}$	101
default	$-\frac{\ln\left(\frac{\frac{2a\,e^2-2bde+2c\,d^2}{e^2}+\frac{(be-2cd)\left(x^2+\frac{d}{e}\right)}{e}+2\sqrt{\frac{a\,e^2-bde+c\,d^2}{e^2}}\,\sqrt{c\left(x^2+\frac{d}{e}\right)^2+\frac{(be-2cd)\left(x^2+\frac{d}{e}\right)}{e}+\frac{a\,e^2-bde+c\,d^2}{e^2}}}{x^2+\frac{d}{e}}\right)}{2e\sqrt{\frac{a\,e^2-bde+c\,d^2}{e^2}}}$	165
elliptic	$-\frac{\ln\left(\frac{\frac{2a\,e^2-2bde+2c\,d^2}{e^2}+\frac{(be-2cd)\left(x^2+\frac{d}{e}\right)}{e}+2\sqrt{\frac{a\,e^2-bde+c\,d^2}{e^2}}\,\sqrt{c\left(x^2+\frac{d}{e}\right)^2+\frac{(be-2cd)\left(x^2+\frac{d}{e}\right)}{e}+\frac{a\,e^2-bde+c\,d^2}{e^2}}}{x^2+\frac{d}{e}}\right)}{2e\sqrt{\frac{a\,e^2-bde+c\,d^2}{e^2}}}$	165

```
[In] int(x/(e*x^2+d)/(c*x^4+b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2/((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*ln((2*(c*x^4+b*x^2+a)^(1/2)*((a*e^2-b*
d*e+c*d^2)/e^2)^(1/2)*e+(b*x^2+2*a)*e-d*(2*c*x^2+b))/(e*x^2+d))/e
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 166 vs. 2(76) = 152.

Time = 0.34 (sec) , antiderivative size = 357, normalized size of antiderivative = 4.15

$$\int \frac{x}{(d+ex^2)\sqrt{a+bx^2+cx^4}}\,dx$$
$$= \left[\frac{\log\left(-\frac{(8\,c^2d^2-8\,bcde+(b^2+4\,ac)e^2)x^4-8\,abde+8\,a^2e^2+(b^2+4\,ac)d^2+2\,(4\,bcd^2+4\,abe^2-(3\,b^2+4\,ac)de)x^2+4\,\sqrt{cx^4+bx^2+a}\sqrt{cd^2-bde+ae^2}\,(}{e^2x^4+2\,dex^2+d^2}\right.}{4\,\sqrt{cd^2-bde+ae^2}}\right.$$

```
[In] integrate(x/(e*x^2+d)/(c*x^4+b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*log(-((8*c^2*d^2 - 8*b*c*d*e + (b^2 + 4*a*c)*e^2)*x^4 - 8*a*b*d*e + 8*
a^2*e^2 + (b^2 + 4*a*c)*d^2 + 2*(4*b*c*d^2 + 4*a*b*e^2 - (3*b^2 + 4*a*c)*d*
e)*x^2 + 4*sqrt(c*x^4 + b*x^2 + a)*sqrt(c*d^2 - b*d*e + a*e^2)*((2*c*d - b*
e)*x^2 + b*d - 2*a*e))/(e^2*x^4 + 2*d*e*x^2 + d^2))/sqrt(c*d^2 - b*d*e + a*
e^2), 1/2*sqrt(-c*d^2 + b*d*e - a*e^2)*arctan(-1/2*sqrt(c*x^4 + b*x^2 + a)*
sqrt(-c*d^2 + b*d*e - a*e^2)*((2*c*d - b*e)*x^2 + b*d - 2*a*e)/((c^2*d^2 -
b*c*d*e + a*c*e^2)*x^4 + a*c*d^2 - a*b*d*e + a^2*e^2 + (b*c*d^2 - b^2*d*e +
 a*b*e^2)*x^2))/(c*d^2 - b*d*e + a*e^2)]
```

Sympy [F]

$$\int \frac{x}{(d+ex^2)\sqrt{a+bx^2+cx^4}}\,dx=\int \frac{x}{(d+ex^2)\sqrt{a+bx^2+cx^4}}\,dx$$

```
[In] integrate(x/(e*x**2+d)/(c*x**4+b*x**2+a)**(1/2),x)
[Out] Integral(x/((d + e*x**2)*sqrt(a + b*x**2 + c*x**4)), x)
```

Maxima [F]

$$\int \frac{x}{(d+ex^2)\sqrt{a+bx^2+cx^4}}\,dx=\int \frac{x}{\sqrt{cx^4+bx^2+a}(ex^2+d)}\,dx$$

```
[In] integrate(x/(e*x^2+d)/(c*x^4+b*x^2+a)^(1/2),x, algorithm="maxima")
[Out] integrate(x/(sqrt(c*x^4 + b*x^2 + a)*(e*x^2 + d)), x)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.86

$$\int \frac{x}{(d+ex^2)\sqrt{a+bx^2+cx^4}}\,dx=\frac{\arctan\left(-\frac{\left(\sqrt{c}x^2-\sqrt{cx^4+bx^2+a}\right)e+\sqrt{c}d}{\sqrt{-cd^2+bde-ae^2}}\right)}{\sqrt{-cd^2+bde-ae^2}}$$

```
[In] integrate(x/(e*x^2+d)/(c*x^4+b*x^2+a)^(1/2),x, algorithm="giac")
[Out] arctan(-((sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))*e + sqrt(c)*d)/sqrt(-c*d^2
 + b*d*e - a*e^2))/sqrt(-c*d^2 + b*d*e - a*e^2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x}{(d+ex^2)\sqrt{a+bx^2+cx^4}}\,dx=\int \frac{x}{(e\,x^2+d)\,\sqrt{c\,x^4+b\,x^2+a}}\,dx$$

```
[In] int(x/((d + e*x^2)*(a + b*x^2 + c*x^4)^(1/2)),x)
[Out] int(x/((d + e*x^2)*(a + b*x^2 + c*x^4)^(1/2)), x)
```

3.335 $\int \frac{1}{x(d+ex^2)\sqrt{a+bx^2+cx^4}}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 138

$$\int \frac{1}{x(d+ex^2)\sqrt{a+bx^2+cx^4}}\,dx = -\frac{\operatorname{arctanh}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{2\sqrt{a}d} - \frac{e\operatorname{arctanh}\left(\frac{bd-2ae+(2cd-be)x^2}{2\sqrt{cd^2-bde+ae^2}\sqrt{a+bx^2+cx^4}}\right)}{2d\sqrt{cd^2-bde+ae^2}}$$

```
[Out] -1/2*arctanh(1/2*(b*x^2+2*a)/a^(1/2)/(c*x^4+b*x^2+a)^(1/2))/d/a^(1/2)-1/2*e
*arctanh(1/2*(b*d-2*a*e+(-b*e+2*c*d)*x^2)/(a*e^2-b*d*e+c*d^2)^(1/2)/(c*x^4+
b*x^2+a)^(1/2))/d/(a*e^2-b*d*e+c*d^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.138, Rules used = {1265, 974, 738, 212}

$$\int \frac{1}{x(d+ex^2)\sqrt{a+bx^2+cx^4}}\,dx = -\frac{e\operatorname{arctanh}\left(\frac{-2ae+x^2(2cd-be)+bd}{2\sqrt{a+bx^2+cx^4}\sqrt{ae^2-bde+cd^2}}\right)}{2d\sqrt{ae^2-bde+cd^2}} - \frac{\operatorname{arctanh}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{2\sqrt{a}d}$$

```
[In] Int[1/(x*(d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]),x]
```

```
[Out] -1/2*ArcTanh[(2*a + b*x^2)/(2*Sqrt[a]*Sqrt[a + b*x^2 + c*x^4])]/(Sqrt[a]*d)
 - (e*ArcTanh[(b*d - 2*a*e + (2*c*d - b*e)*x^2)/(2*Sqrt[c*d^2 - b*d*e + a*e
^2]*Sqrt[a + b*x^2 + c*x^4])])/(2*d*Sqrt[c*d^2 - b*d*e + a*e^2])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 974

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g
*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ
[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && (
IntegerQ[p] || (ILtQ[m, 0] && ILtQ[n, 0])) &&  !(IGtQ[m, 0] || IGtQ[n, 0])
```

Rule 1265

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{1}{2}\text{Subst}\left(\int \frac{1}{x(d+ex)\sqrt{a+bx+cx^2}}\,dx, x, x^2\right) \\
&= \frac{1}{2}\text{Subst}\left(\int \left(\frac{1}{dx\sqrt{a+bx+cx^2}} - \frac{e}{d(d+ex)\sqrt{a+bx+cx^2}}\right)\,dx, x, x^2\right) \\
&= \frac{\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx+cx^2}}\,dx, x, x^2\right)}{2d} - \frac{e\text{Subst}\left(\int \frac{1}{(d+ex)\sqrt{a+bx+cx^2}}\,dx, x, x^2\right)}{2d} \\
&= -\frac{\text{Subst}\left(\int \frac{1}{4a-x^2}\,dx, x, \frac{2a+bx^2}{\sqrt{a+bx^2+cx^4}}\right)}{d} + \frac{e\text{Subst}\left(\int \frac{1}{4cd^2-4bde+4ae^2-x^2}\,dx, x, \frac{-bd+2ae-(2cd-be)x^2}{\sqrt{a+bx^2+cx^4}}\right)}{d} \\
&= -\frac{\tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{2\sqrt{a}d} - \frac{e\tanh^{-1}\left(\frac{bd-2ae+(2cd-be)x^2}{2\sqrt{cd^2-bde+ae^2}\sqrt{a+bx^2+cx^4}}\right)}{2d\sqrt{cd^2-bde+ae^2}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.48 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.04

$$\int \frac{1}{x\left(d+ex^2\right)\sqrt{a+bx^2+cx^4}}\,dx = \frac{-\frac{e\sqrt{-cd^2+bde-ae^2}\arctan\left(\frac{\sqrt{c}\left(d+ex^2\right)-e\sqrt{a+bx^2+cx^4}}{\sqrt{-cd^2+e(bd-ae)}}\right)}{cd^2+e(-bd+ae)}+\frac{\operatorname{arctanh}\left(\frac{\sqrt{c}x^2-\sqrt{a+bx^2+cx^4}}{\sqrt{a}}\right)}{\sqrt{a}}}{d}$$

```
[In] Integrate[1/(x*(d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]),x]
```

```
[Out] (-((e*Sqrt[-(c*d^2) + b*d*e - a*e^2]*ArcTan[(Sqrt[c]*(d + e*x^2) - e*Sqrt[a
 + b*x^2 + c*x^4])/Sqrt[-(c*d^2) + e*(b*d - a*e)]])/(c*d^2 + e*(-(b*d) + a*
e))) + ArcTanh[(Sqrt[c]*x^2 - Sqrt[a + b*x^2 + c*x^4])/Sqrt[a]]/Sqrt[a])/d
```

Maple [A] (verified)

Time = 0.40 (sec) , antiderivative size = 166, normalized size of antiderivative = 1.20

method	result
pseudoelliptic	$\frac{\ln\left(\frac{2\sqrt{c\,x^4+b\,x^2+a}\,\sqrt{\frac{a\,e^2-bde+c\,d^2}{e^2}}\,e+\left(b\,x^2+2a\right)e-d\left(2c\,x^2+b\right)}{e\,x^2+d}\right)\sqrt{a}-\ln\left(\frac{2a+b\,x^2+2\sqrt{a}\,\sqrt{c\,x^4+b\,x^2+a}}{x^2}\right)\sqrt{\frac{a\,e^2-bde+c\,d^2}{e^2}}}{2d\sqrt{a}\,\sqrt{\frac{a\,e^2-bde+c\,d^2}{e^2}}}$
default	$-\frac{\ln\left(\frac{2a+b\,x^2+2\sqrt{a}\,\sqrt{c\,x^4+b\,x^2+a}}{x^2}\right)}{2d\sqrt{a}}+\frac{\ln\left(\frac{\frac{2a\,e^2-2bde+2c\,d^2}{e^2}+\frac{(be-2cd)\left(x^2+\frac{d}{e}\right)}{e}+2\sqrt{\frac{a\,e^2-bde+c\,d^2}{e^2}}\,\sqrt{c\left(x^2+\frac{d}{e}\right)^2+\frac{(be-2cd)\left(x^2+\right.}{e}}}{x^2+\frac{d}{e}}\right.}{2d\sqrt{\frac{a\,e^2-bde+c\,d^2}{e^2}}}$
elliptic	$-\frac{\ln\left(\frac{2a+b\,x^2+2\sqrt{a}\,\sqrt{c\,x^4+b\,x^2+a}}{x^2}\right)}{2d\sqrt{a}}+\frac{\ln\left(\frac{\frac{2a\,e^2-2bde+2c\,d^2}{e^2}+\frac{(be-2cd)\left(x^2+\frac{d}{e}\right)}{e}+2\sqrt{\frac{a\,e^2-bde+c\,d^2}{e^2}}\,\sqrt{c\left(x^2+\frac{d}{e}\right)^2+\frac{(be-2cd)\left(x^2+\right.}{e}}}{x^2+\frac{d}{e}}\right.}{2d\sqrt{\frac{a\,e^2-bde+c\,d^2}{e^2}}}$

```
[In] int(1/x/(e*x^2+d)/(c*x^4+b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*(ln((2*(c*x^4+b*x^2+a)^(1/2)*((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*e+(b*x^2+2
*a)*e-d*(2*c*x^2+b))/(e*x^2+d))*a^(1/2)-ln((2*a+b*x^2+2*a^(1/2)*(c*x^4+b*x^
2+a)^(1/2))/x^2)*((a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/d/a^(1/2)/((a*e^2-b*d*e+c
*d^2)/e^2)^(1/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 249 vs. $2(118) = 236$.

Time = 0.46 (sec) , antiderivative size = 1097, normalized size of antiderivative = 7.95

$$\int \frac{1}{x\left(d+ex^2\right)\sqrt{a+bx^2+cx^4}}\,dx$$

$$= \left[\frac{\sqrt{cd^2-bde+ae^2}ae\log\left(-\frac{(8\,c^2d^2-8\,bcde+(b^2+4\,ac)e^2)x^4-8\,abde+8\,a^2e^2+(b^2+4\,ac)d^2+2\,(4\,bcd^2+4\,abe^2-(3\,b^2+4\,ac)de)x^2-4}{e^2x^4+2\,dex^2+d^2}\right.}{4\,(acd^3} \right.$$

$$-\frac{2\sqrt{-cd^2+bde-ae^2}ae\arctan\left(-\frac{\sqrt{cx^4+bx^2+a}\sqrt{-cd^2+bde-ae^2}((2\,cd-be)x^2+bd-2\,ae)}{2\,((c^2d^2-bcde+ace^2)x^4+acd^2-abde+a^2e^2+(bcd^2-b^2de+abe^2)x^2)}\right)-(cd^2-bde+a}{4\,(acd^3-abd^2e+a^2de^2)}$$

$$-\frac{\sqrt{-cd^2+bde-ae^2}ae\arctan\left(-\frac{\sqrt{cx^4+bx^2+a}\sqrt{-cd^2+bde-ae^2}((2\,cd-be)x^2+bd-2\,ae)}{2\,((c^2d^2-bcde+ace^2)x^4+acd^2-abde+a^2e^2+(bcd^2-b^2de+abe^2)x^2)}\right)-(cd^2-bde+ae^2}{2\,(acd^3-abd^2e+a^2de^2)}$$

```
[In] integrate(1/x/(e*x^2+d)/(c*x^4+b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*(sqrt(c*d^2 - b*d*e + a*e^2)*a*e*log(-((8*c^2*d^2 - 8*b*c*d*e + (b^2 +
 4*a*c)*e^2)*x^4 - 8*a*b*d*e + 8*a^2*e^2 + (b^2 + 4*a*c)*d^2 + 2*(4*b*c*d^2
 + 4*a*b*e^2 - (3*b^2 + 4*a*c)*d*e)*x^2 - 4*sqrt(c*x^4 + b*x^2 + a)*sqrt(c*
d^2 - b*d*e + a*e^2)*((2*c*d - b*e)*x^2 + b*d - 2*a*e))/(e^2*x^4 + 2*d*e*x^
2 + d^2)) + (c*d^2 - b*d*e + a*e^2)*sqrt(a)*log(-((b^2 + 4*a*c)*x^4 + 8*a*b
*x^2 - 4*sqrt(c*x^4 + b*x^2 + a)*(b*x^2 + 2*a)*sqrt(a) + 8*a^2)/x^4))/(a*c*
d^3 - a*b*d^2*e + a^2*d*e^2), -1/4*(2*sqrt(-c*d^2 + b*d*e - a*e^2)*a*e*arct
an(-1/2*sqrt(c*x^4 + b*x^2 + a)*sqrt(-c*d^2 + b*d*e - a*e^2)*((2*c*d - b*e)
*x^2 + b*d - 2*a*e)/((c^2*d^2 - b*c*d*e + a*c*e^2)*x^4 + a*c*d^2 - a*b*d*e
+ a^2*e^2 + (b*c*d^2 - b^2*d*e + a*b*e^2)*x^2)) - (c*d^2 - b*d*e + a*e^2)*s
qrt(a)*log(-((b^2 + 4*a*c)*x^4 + 8*a*b*x^2 - 4*sqrt(c*x^4 + b*x^2 + a)*(b*x
^2 + 2*a)*sqrt(a) + 8*a^2)/x^4))/(a*c*d^3 - a*b*d^2*e + a^2*d*e^2), 1/4*(sq
rt(c*d^2 - b*d*e + a*e^2)*a*e*log(-((8*c^2*d^2 - 8*b*c*d*e + (b^2 + 4*a*c)*
e^2)*x^4 - 8*a*b*d*e + 8*a^2*e^2 + (b^2 + 4*a*c)*d^2 + 2*(4*b*c*d^2 + 4*a*b
*e^2 - (3*b^2 + 4*a*c)*d*e)*x^2 - 4*sqrt(c*x^4 + b*x^2 + a)*sqrt(c*d^2 - b*
d*e + a*e^2)*((2*c*d - b*e)*x^2 + b*d - 2*a*e))/(e^2*x^4 + 2*d*e*x^2 + d^2)
) + 2*(c*d^2 - b*d*e + a*e^2)*sqrt(-a)*arctan(1/2*sqrt(c*x^4 + b*x^2 + a)*(
b*x^2 + 2*a)*sqrt(-a)/(a*c*x^4 + a*b*x^2 + a^2)))/(a*c*d^3 - a*b*d^2*e + a^
2*d*e^2), -1/2*(sqrt(-c*d^2 + b*d*e - a*e^2)*a*e*arctan(-1/2*sqrt(c*x^4 + b
*x^2 + a)*sqrt(-c*d^2 + b*d*e - a*e^2)*((2*c*d - b*e)*x^2 + b*d - 2*a*e)/((
c^2*d^2 - b*c*d*e + a*c*e^2)*x^4 + a*c*d^2 - a*b*d*e + a^2*e^2 + (b*c*d^2 -
 b^2*d*e + a*b*e^2)*x^2)) - (c*d^2 - b*d*e + a*e^2)*sqrt(-a)*arctan(1/2*sqr
t(c*x^4 + b*x^2 + a)*(b*x^2 + 2*a)*sqrt(-a)/(a*c*x^4 + a*b*x^2 + a^2)))/(a*
c*d^3 - a*b*d^2*e + a^2*d*e^2)]
```

Sympy [F]

$$\int \frac{1}{x\left(d+ex^2\right)\sqrt{a+bx^2+cx^4}}\,dx = \int \frac{1}{x\left(d+ex^2\right)\sqrt{a+bx^2+cx^4}}\,dx$$

```
[In] integrate(1/x/(e*x**2+d)/(c*x**4+b*x**2+a)**(1/2),x)
[Out] Integral(1/(x*(d + e*x**2)*sqrt(a + b*x**2 + c*x**4)), x)
```

Maxima [F]

$$\int \frac{1}{x\left(d+ex^2\right)\sqrt{a+bx^2+cx^4}}\,dx = \int \frac{1}{\sqrt{cx^4+bx^2+a}(ex^2+d)x}\,dx$$

```
[In] integrate(1/x/(e*x^2+d)/(c*x^4+b*x^2+a)^(1/2),x, algorithm="maxima")
[Out] integrate(1/(sqrt(c*x^4 + b*x^2 + a)*(e*x^2 + d)*x), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{1}{x\left(d+ex^2\right)\sqrt{a+bx^2+cx^4}}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(1/x/(e*x^2+d)/(c*x^4+b*x^2+a)^(1/2),x, algorithm="giac")
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument Type
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x\left(d+ex^2\right)\sqrt{a+bx^2+cx^4}}\,dx = \int \frac{1}{x\,\left(e\,x^2+d\right)\,\sqrt{c\,x^4+b\,x^2+a}}\,dx$$

```
[In] int(1/(x*(d + e*x^2)*(a + b*x^2 + c*x^4)^(1/2)),x)
[Out] int(1/(x*(d + e*x^2)*(a + b*x^2 + c*x^4)^(1/2)), x)
```

3.336 $\int \frac{1}{x^3(d+ex^2)\sqrt{a+bx^2+cx^4}}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 218

$$\int \frac{1}{x^3\left(d+ex^2\right)\sqrt{a+bx^2+cx^4}}\,dx = -\frac{\sqrt{a+bx^2+cx^4}}{2adx^2}+\frac{b\operatorname{arctanh}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{4a^{3/2}d} + \frac{e\operatorname{arctanh}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{2\sqrt{a}d^2} + \frac{e^2\operatorname{arctanh}\left(\frac{bd-2ae+(2cd-be)x^2}{2\sqrt{cd^2-bde+ae^2}\sqrt{a+bx^2+cx^4}}\right)}{2d^2\sqrt{cd^2-bde+ae^2}}$$

```
[Out] 1/4*b*arctanh(1/2*(b*x^2+2*a)/a^(1/2)/(c*x^4+b*x^2+a)^(1/2))/a^(3/2)/d+1/2*
e*arctanh(1/2*(b*x^2+2*a)/a^(1/2)/(c*x^4+b*x^2+a)^(1/2))/d^2/a^(1/2)+1/2*e^
2*arctanh(1/2*(b*d-2*a*e+(-b*e+2*c*d)*x^2)/(a*e^2-b*d*e+c*d^2)^(1/2)/(c*x^4
+b*x^2+a)^(1/2))/d^2/(a*e^2-b*d*e+c*d^2)^(1/2)-1/2*(c*x^4+b*x^2+a)^(1/2)/a/
d/x^2
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 218, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.172, Rules used

= {1265, 974, 744, 738, 212}

$$\int \frac{1}{x^3\left(d+ex^2\right)\sqrt{a+bx^2+cx^4}}\,dx=\frac{b\text{arctanh}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{4a^{3/2}d}+\frac{e^2\text{arctanh}\left(\frac{-2ae+x^2(2cd-be)+bd}{2\sqrt{a+bx^2+cx^4}\sqrt{ae^2-bde+cd^2}}\right)}{2d^2\sqrt{ae^2-bde+cd^2}}+\frac{e\text{arctanh}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{2\sqrt{a}d^2}-\frac{\sqrt{a+bx^2+cx^4}}{2adx^2}$$

```
[In] Int[1/(x^3*(d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]),x]
```

```
[Out] -1/2*Sqrt[a + b*x^2 + c*x^4]/(a*d*x^2) + (b*ArcTanh[(2*a + b*x^2)/(2*Sqrt[a
]*Sqrt[a + b*x^2 + c*x^4])])/(4*a^(3/2)*d) + (e*ArcTanh[(2*a + b*x^2)/(2*Sq
rt[a]*Sqrt[a + b*x^2 + c*x^4])])/(2*Sqrt[a]*d^2) + (e^2*ArcTanh[(b*d - 2*a*
e + (2*c*d - b*e)*x^2)/(2*Sqrt[c*d^2 - b*d*e + a*e^2]*Sqrt[a + b*x^2 + c*x^
4])])/(2*d^2*Sqrt[c*d^2 - b*d*e + a*e^2])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 744

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m + 1)*((a + b*x + c*x^2)^(p + 1)/((m + 1)*(c*d
^2 - b*d*e + a*e^2))), x] + Dist[(2*c*d - b*e)/(2*(c*d^2 - b*d*e + a*e^2)),
 Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e,
 m, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2
*c*d - b*e, 0] && EqQ[m + 2*p + 3, 0]
```

Rule 974

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g
*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ
[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && (
```

```
IntegerQ[p] || (ILtQ[m, 0] && ILtQ[n, 0])) &&  !(IGtQ[m, 0] || IGtQ[n, 0])
```

Rule 1265

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rubi steps

$$\text{integral} = \frac{1}{2}\text{Subst}\left(\int \frac{1}{x^2(d+ex)\sqrt{a+bx+cx^2}}\,dx, x, x^2\right)$$

$$= \frac{1}{2}\text{Subst}\left(\int \left(\frac{1}{dx^2\sqrt{a+bx+cx^2}} - \frac{e}{d^2x\sqrt{a+bx+cx^2}} + \frac{e^2}{d^2(d+ex)\sqrt{a+bx+cx^2}}\right)dx, x, x^2\right)$$

$$= \frac{\text{Subst}\left(\int \frac{1}{x^2\sqrt{a+bx+cx^2}}\,dx, x, x^2\right)}{2d} - \frac{e\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx+cx^2}}\,dx, x, x^2\right)}{2d^2} + \frac{e^2\text{Subst}\left(\int \frac{1}{(d+ex)\sqrt{a+bx+cx^2}}\,dx, x, x^2\right)}{2d^2}$$

$$= -\frac{\sqrt{a+bx^2+cx^4}}{2adx^2} - \frac{b\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx+cx^2}}\,dx, x, x^2\right)}{4ad} + \frac{e\text{Subst}\left(\int \frac{1}{4a-x^2}\,dx, x, \frac{2a+bx^2}{\sqrt{a+bx^2+cx^4}}\right)}{d^2} - \frac{e^2\text{Subst}\left(\int \frac{1}{4cd^2-4bde+4ae^2-x^2}\,dx, x, \frac{-bd+2ae-(2cd-be)x^2}{\sqrt{a+bx^2+cx^4}}\right)}{d^2}$$

$$= -\frac{\sqrt{a+bx^2+cx^4}}{2adx^2} + \frac{e\tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{2\sqrt{a}d^2} + \frac{e^2\tanh^{-1}\left(\frac{bd-2ae+(2cd-be)x^2}{2\sqrt{cd^2-bde+ae^2}\sqrt{a+bx^2+cx^4}}\right)}{2d^2\sqrt{cd^2-bde+ae^2}} + \frac{b\text{Subst}\left(\int \frac{1}{4a-x^2}\,dx, x, \frac{2a+bx^2}{\sqrt{a+bx^2+cx^4}}\right)}{2ad}$$

$$= -\frac{\sqrt{a+bx^2+cx^4}}{2adx^2} + \frac{b\tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{4a^{3/2}d} + \frac{e\tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{2\sqrt{a}d^2} + \frac{e^2\tanh^{-1}\left(\frac{bd-2ae+(2cd-be)x^2}{2\sqrt{cd^2-bde+ae^2}\sqrt{a+bx^2+cx^4}}\right)}{2d^2\sqrt{cd^2-bde+ae^2}}$$

Mathematica [A] (verified)

Time = 1.01 (sec) , antiderivative size = 209, normalized size of antiderivative = 0.96

$$\int \frac{1}{x^3 (d+ex^2) \sqrt{a+bx^2+cx^4}} \, dx$$

$$= \frac{-\frac{2d\sqrt{a+bx^2+cx^4}}{ax^2} + \frac{4e^2\sqrt{-cd^2+bde-ae^2}\arctan\left(\frac{\sqrt{-cd^2+bde-ae^2}x^2}{\sqrt{a}(d+ex^2)-d\sqrt{a+bx^2+cx^4}}\right)}{cd^2+e(-bd+ae)} + \frac{(bd+2ae)\log(x^2)}{a^{3/2}} - \frac{(bd+2ae)\log\left(ad^2\left(2a+bx^2-2\sqrt{a}\sqrt{a+\dots}\right.\right.}{a^{3/2}}}{4d^2}$$

[In] Integrate[1/(x^3*(d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]),x]

[Out] ((-2*d*Sqrt[a + b*x^2 + c*x^4])/(a*x^2) + (4*e^2*Sqrt[-(c*d^2) + b*d*e - a*e^2]*ArcTan[(Sqrt[-(c*d^2) + b*d*e - a*e^2]*x^2)/(Sqrt[a]*(d + e*x^2) - d*Sqrt[a + b*x^2 + c*x^4])]])/(c*d^2 + e*(-(b*d) + a*e)) + ((b*d + 2*a*e)*Log[x^2])/a^(3/2) - ((b*d + 2*a*e)*Log[a*d^2*(2*a + b*x^2 - 2*Sqrt[a]*Sqrt[a + b*x^2 + c*x^4])])/a^(3/2))/(4*d^2)

Maple [A] (verified)

Time = 0.46 (sec) , antiderivative size = 206, normalized size of antiderivative = 0.94

method	result
pseudoelliptic	$\frac{-\ln\left(\frac{2\sqrt{c\,x^4+b\,x^2+a}\sqrt{\frac{a\,e^2-bde+c\,d^2}{e^2}}\,e+\left(b\,x^2+2a\right)e-d\left(2c\,x^2+b\right)}{e\,x^2+d}\right)a^{\frac{3}{2}}e\,x^2+\sqrt{\frac{a\,e^2-bde+c\,d^2}{e^2}}\left(x^2\left(ae+\frac{bd}{2}\right)\ln\left(\frac{2a+b\,x^2+2\sqrt{a}\,\sqrt{c\dots}}{x^2}\right.\right.}{2a^{\frac{3}{2}}\sqrt{\frac{a\,e^2-bde+c\,d^2}{e^2}}\,d^2x^2}$
risch	$-\frac{\sqrt{c\,x^4+b\,x^2+a}}{2ad\,x^2}-\frac{-\frac{(2ae+bd)\ln\left(\frac{2a+b\,x^2+2\sqrt{a}\,\sqrt{c\,x^4+b\,x^2+a}}{x^2}\right)}{2d\sqrt{a}}+\frac{ae\ln\left(\frac{\frac{2a\,e^2-2bde+2c\,d^2}{e^2}+\frac{(be-2cd)\left(x^2+\frac{d}{e}\right)}{e}+2\sqrt{\frac{a\,e^2-bde+c\,d\dots}{e^2}}\dots}{x^2+\frac{d}{e}}\right.}{d\sqrt{\frac{a\,e^2-bde+\dots}{e^2}}}}{2da}$
default	$\frac{-\frac{\sqrt{c\,x^4+b\,x^2+a}}{2a\,x^2}+\frac{b\ln\left(\frac{2a+b\,x^2+2\sqrt{a}\,\sqrt{c\,x^4+b\,x^2+a}}{x^2}\right)}{4a^{\frac{3}{2}}}}{d}+\frac{e\ln\left(\frac{2a+b\,x^2+2\sqrt{a}\,\sqrt{c\,x^4+b\,x^2+a}}{x^2}\right)}{2d^2\sqrt{a}}-\frac{e\ln\left(\frac{\frac{2a\,e^2-2bde+2c\,d^2}{e^2}+\frac{(be-2c\dots}{\dots}}{\dots}\right.}{\dots}$
elliptic	$-\frac{\sqrt{c\,x^4+b\,x^2+a}}{2ad\,x^2}+\frac{b\ln\left(\frac{2a+b\,x^2+2\sqrt{a}\,\sqrt{c\,x^4+b\,x^2+a}}{x^2}\right)}{4d\,a^{\frac{3}{2}}}+\frac{e\ln\left(\frac{2a+b\,x^2+2\sqrt{a}\,\sqrt{c\,x^4+b\,x^2+a}}{x^2}\right)}{2d^2\sqrt{a}}-\frac{e\ln\left(\frac{\frac{2a\,e^2-2bde+2c\,d^2}{e^2}+\frac{(b\dots}{\dots}}{\dots}\right.}{\dots}$

[In] int(1/x^3/(e*x^2+d)/(c*x^4+b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)

[Out] 1/2/a^(3/2)*(-ln((2*(c*x^4+b*x^2+a)^(1/2)*((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*e+(b*x^2+2*a)*e-d*(2*c*x^2+b))/(e*x^2+d))*a^(3/2)*e*x^2+((a*e^2-b*d*e+c*d^2)

```
/e^2)^(1/2)*(x^2*(a*e+1/2*b*d)*ln((2*a+b*x^2+2*a^(1/2)*(c*x^4+b*x^2+a)^(1/2
))/x^2)-(c*x^4+b*x^2+a)^(1/2)*a^(1/2)*d))/((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)/d
^2/x^2
```

Fricas [A] (verification not implemented)

none

Time = 0.68 (sec) , antiderivative size = 1414, normalized size of antiderivative = 6.49

$$\int \frac{1}{x^3 (d+ex^2) \sqrt{a+bx^2+cx^4}} \, dx = \text{Too large to display}$$

```
[In] integrate(1/x^3/(e*x^2+d)/(c*x^4+b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/8*(2*sqrt(c*d^2 - b*d*e + a*e^2)*a^2*e^2*x^2*log(-((8*c^2*d^2 - 8*b*c*d*
e + (b^2 + 4*a*c)*e^2)*x^4 - 8*a*b*d*e + 8*a^2*e^2 + (b^2 + 4*a*c)*d^2 + 2*
(4*b*c*d^2 + 4*a*b*e^2 - (3*b^2 + 4*a*c)*d*e)*x^2 + 4*sqrt(c*x^4 + b*x^2 +
a)*sqrt(c*d^2 - b*d*e + a*e^2)*((2*c*d - b*e)*x^2 + b*d - 2*a*e))/(e^2*x^4
+ 2*d*e*x^2 + d^2)) + (b*c*d^3 - a*b*d*e^2 + 2*a^2*e^3 - (b^2 - 2*a*c)*d^2*
e)*sqrt(a)*x^2*log(-((b^2 + 4*a*c)*x^4 + 8*a*b*x^2 + 4*sqrt(c*x^4 + b*x^2 +
 a)*(b*x^2 + 2*a)*sqrt(a) + 8*a^2)/x^4) - 4*(a*c*d^3 - a*b*d^2*e + a^2*d*e^
2)*sqrt(c*x^4 + b*x^2 + a))/((a^2*c*d^4 - a^2*b*d^3*e + a^3*d^2*e^2)*x^2),
1/8*(4*sqrt(-c*d^2 + b*d*e - a*e^2)*a^2*e^2*x^2*arctan(-1/2*sqrt(c*x^4 + b*
x^2 + a)*sqrt(-c*d^2 + b*d*e - a*e^2)*((2*c*d - b*e)*x^2 + b*d - 2*a*e)/((c
^2*d^2 - b*c*d*e + a*c*e^2)*x^4 + a*c*d^2 - a*b*d*e + a^2*e^2 + (b*c*d^2 -
b^2*d*e + a*b*e^2)*x^2)) + (b*c*d^3 - a*b*d*e^2 + 2*a^2*e^3 - (b^2 - 2*a*c)
*d^2*e)*sqrt(a)*x^2*log(-((b^2 + 4*a*c)*x^4 + 8*a*b*x^2 + 4*sqrt(c*x^4 + b*
x^2 + a)*(b*x^2 + 2*a)*sqrt(a) + 8*a^2)/x^4) - 4*(a*c*d^3 - a*b*d^2*e + a^2
*d*e^2)*sqrt(c*x^4 + b*x^2 + a))/((a^2*c*d^4 - a^2*b*d^3*e + a^3*d^2*e^2)*x
^2), 1/4*(sqrt(c*d^2 - b*d*e + a*e^2)*a^2*e^2*x^2*log(-((8*c^2*d^2 - 8*b*c*
d*e + (b^2 + 4*a*c)*e^2)*x^4 - 8*a*b*d*e + 8*a^2*e^2 + (b^2 + 4*a*c)*d^2 +
2*(4*b*c*d^2 + 4*a*b*e^2 - (3*b^2 + 4*a*c)*d*e)*x^2 + 4*sqrt(c*x^4 + b*x^2
+ a)*sqrt(c*d^2 - b*d*e + a*e^2)*((2*c*d - b*e)*x^2 + b*d - 2*a*e))/(e^2*x^
4 + 2*d*e*x^2 + d^2)) - (b*c*d^3 - a*b*d*e^2 + 2*a^2*e^3 - (b^2 - 2*a*c)*d^
2*e)*sqrt(-a)*x^2*arctan(1/2*sqrt(c*x^4 + b*x^2 + a)*(b*x^2 + 2*a)*sqrt(-a)
/(a*c*x^4 + a*b*x^2 + a^2)) - 2*(a*c*d^3 - a*b*d^2*e + a^2*d*e^2)*sqrt(c*x^
4 + b*x^2 + a))/((a^2*c*d^4 - a^2*b*d^3*e + a^3*d^2*e^2)*x^2), 1/4*(2*sqrt(
-c*d^2 + b*d*e - a*e^2)*a^2*e^2*x^2*arctan(-1/2*sqrt(c*x^4 + b*x^2 + a)*sqr
t(-c*d^2 + b*d*e - a*e^2)*((2*c*d - b*e)*x^2 + b*d - 2*a*e)/((c^2*d^2 - b*c
*d*e + a*c*e^2)*x^4 + a*c*d^2 - a*b*d*e + a^2*e^2 + (b*c*d^2 - b^2*d*e + a*
b*e^2)*x^2)) - (b*c*d^3 - a*b*d*e^2 + 2*a^2*e^3 - (b^2 - 2*a*c)*d^2*e)*sqrt
(-a)*x^2*arctan(1/2*sqrt(c*x^4 + b*x^2 + a)*(b*x^2 + 2*a)*sqrt(-a)/(a*c*x^4
 + a*b*x^2 + a^2)) - 2*(a*c*d^3 - a*b*d^2*e + a^2*d*e^2)*sqrt(c*x^4 + b*x^2
 + a))/((a^2*c*d^4 - a^2*b*d^3*e + a^3*d^2*e^2)*x^2)]
```

Sympy [F]

$$\int \frac{1}{x^3 (d+ex^2) \sqrt{a+bx^2+cx^4}}\, dx = \int \frac{1}{x^3 (d+ex^2) \sqrt{a+bx^2+cx^4}}\, dx$$

```
[In] integrate(1/x**3/(e*x**2+d)/(c*x**4+b*x**2+a)**(1/2),x)
[Out] Integral(1/(x**3*(d + e*x**2)*sqrt(a + b*x**2 + c*x**4)), x)
```

Maxima [F]

$$\int \frac{1}{x^3 (d+ex^2) \sqrt{a+bx^2+cx^4}}\, dx = \int \frac{1}{\sqrt{cx^4+bx^2+a}(ex^2+d)x^3}\, dx$$

```
[In] integrate(1/x^3/(e*x^2+d)/(c*x^4+b*x^2+a)^(1/2),x, algorithm="maxima")
[Out] integrate(1/(sqrt(c*x^4 + b*x^2 + a)*(e*x^2 + d)*x^3), x)
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 207, normalized size of antiderivative = 0.95

$$\int \frac{1}{x^3 (d+ex^2) \sqrt{a+bx^2+cx^4}}\, dx = \frac{e^2 \arctan\left(-\frac{\left(\sqrt{c}x^2-\sqrt{cx^4+bx^2+a}\right)e+\sqrt{c}d}{\sqrt{-cd^2+bde-ae^2}}\right)}{\sqrt{-cd^2+bde-ae^2}d^2} - \frac{(bd+2\,ae)\arctan\left(-\frac{\sqrt{c}x^2-\sqrt{cx^4+bx^2+a}}{\sqrt{-a}}\right)}{2\sqrt{-a}ad^2} + \frac{\left(\sqrt{c}x^2-\sqrt{cx^4+bx^2+a}\right)b+2\,a\sqrt{c}}{2\left(\left(\sqrt{c}x^2-\sqrt{cx^4+bx^2+a}\right)^2-a\right)ad}$$

```
[In] integrate(1/x^3/(e*x^2+d)/(c*x^4+b*x^2+a)^(1/2),x, algorithm="giac")
[Out] e^2*arctan(-((sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))*e + sqrt(c)*d)/sqrt(-c
*d^2 + b*d*e - a*e^2))/(sqrt(-c*d^2 + b*d*e - a*e^2)*d^2) - 1/2*(b*d + 2*a*
e)*arctan(-(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))/sqrt(-a))/(sqrt(-a)*a*d^
2) + 1/2*((sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))*b + 2*a*sqrt(c))/(((sqrt(
c)*x^2 - sqrt(c*x^4 + b*x^2 + a))^2 - a)*a*d)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^3 (d+ex^2)\sqrt{a+bx^2+cx^4}}\,dx = \int \frac{1}{x^3 \left(e\,x^2+d\right)\,\sqrt{c\,x^4+b\,x^2+a}}\,dx$$

```
[In] int(1/(x^3*(d + e*x^2)*(a + b*x^2 + c*x^4)^(1/2)),x)
[Out] int(1/(x^3*(d + e*x^2)*(a + b*x^2 + c*x^4)^(1/2)), x)
```

3.337 $\int \frac{x^4}{(3+2x^2)\sqrt{1+2x^2+2x^4}}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 418

$$\begin{aligned}\int \frac{x^4}{(3+2x^2)\sqrt{1+2x^2+2x^4}}\,dx &= \frac{x\sqrt{1+2x^2+2x^4}}{2\sqrt{2}\,(1+\sqrt{2}x^2)} - \frac{3\sqrt{\frac{3}{10}}(3-\sqrt{2})\arctan\left(\frac{\sqrt{\frac{5}{3}}x}{\sqrt{1+2x^2+2x^4}}\right)}{4\,(2-3\sqrt{2})}\\ &- \frac{(1+\sqrt{2}x^2)\sqrt{\frac{1+2x^2+2x^4}{(1+\sqrt{2}x^2)^2}}E\left(2\arctan\left(\sqrt[4]{2}x\right)|\frac{1}{4}(2-\sqrt{2})\right)}{2\,2^{3/4}\sqrt{1+2x^2+2x^4}}\\ &+ \frac{(1-3\sqrt{2})\,(1+\sqrt{2}x^2)\sqrt{\frac{1+2x^2+2x^4}{(1+\sqrt{2}x^2)^2}}\,\text{EllipticF}\left(2\arctan\left(\sqrt[4]{2}x\right),\frac{1}{4}(2-\sqrt{2})\right)}{2\,2^{3/4}\,(2-3\sqrt{2})\sqrt{1+2x^2+2x^4}}\\ &+ \frac{3(3+\sqrt{2})\,(1+\sqrt{2}x^2)\sqrt{\frac{1+2x^2+2x^4}{(1+\sqrt{2}x^2)^2}}\,\text{EllipticPi}\left(\frac{1}{24}(12-11\sqrt{2}),2\arctan\left(\sqrt[4]{2}x\right),\frac{1}{4}(2-\sqrt{2})\right)}{8\,2^{3/4}\,(2-3\sqrt{2})\sqrt{1+2x^2+2x^4}}\end{aligned}$$

```
[Out] -3/40*arctan(1/3*x*15^(1/2)/(2*x^4+2*x^2+1)^(1/2))*30^(1/2)*(3-2^(1/2))/(2-
3*2^(1/2))+1/4*x*(2*x^4+2*x^2+1)^(1/2)*2^(1/2)/(1+x^2*2^(1/2))-1/4*(cos(2*a
rctan(2^(1/4)*x))^2)^(1/2)/cos(2*arctan(2^(1/4)*x))*EllipticE(sin(2*arctan(
2^(1/4)*x)),1/2*(2-2^(1/2))^(1/2))*(1+x^2*2^(1/2))*((2*x^4+2*x^2+1)/(1+x^2*
2^(1/2))^2)^(1/2)*2^(1/4)/(2*x^4+2*x^2+1)^(1/2)+1/4*(cos(2*arctan(2^(1/4)*x
))^2)^(1/2)/cos(2*arctan(2^(1/4)*x))*EllipticF(sin(2*arctan(2^(1/4)*x)),1/2
*(2-2^(1/2))^(1/2))*(1-3*2^(1/2))*(1+x^2*2^(1/2))*((2*x^4+2*x^2+1)/(1+x^2*2
^(1/2))^2)^(1/2)*2^(1/4)/(2-3*2^(1/2))/(2*x^4+2*x^2+1)^(1/2)+3/16*(cos(2*ar
ctan(2^(1/4)*x))^2)^(1/2)/cos(2*arctan(2^(1/4)*x))*EllipticPi(sin(2*arctan(
2^(1/4)*x)),1/2-11/24*2^(1/2),1/2*(2-2^(1/2))^(1/2))*(3+2^(1/2))*(1+x^2*2^(
1/2))*((2*x^4+2*x^2+1)/(1+x^2*2^(1/2))^2)^(1/2)*2^(1/4)/(2-3*2^(1/2))/(2*x^
4+2*x^2+1)^(1/2)
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 418, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.138, Rules used = {1339, 1117, 1209, 1720}

$$\int \frac{x^4}{(3+2x^2)\sqrt{1+2x^2+2x^4}}\,dx = -\frac{3\sqrt{\frac{3}{10}}(3-\sqrt{2})\arctan\left(\frac{\sqrt{\frac{5}{3}}x}{\sqrt{2x^4+2x^2+1}}\right)}{4(2-3\sqrt{2})} + \frac{(1-3\sqrt{2})(\sqrt{2}x^2+1)\sqrt{\frac{2x^4+2x^2+1}{(\sqrt{2}x^2+1)^2}}\,\text{EllipticF}\left(2\arctan\left(\sqrt[4]{2}x\right),\frac{1}{4}(2-\sqrt{2})\right)}{2\,2^{3/4}(2-3\sqrt{2})\sqrt{2x^4+2x^2+1}} - \frac{(\sqrt{2}x^2+1)\sqrt{\frac{2x^4+2x^2+1}{(\sqrt{2}x^2+1)^2}}E\left(2\arctan\left(\sqrt[4]{2}x\right)|\frac{1}{4}(2-\sqrt{2})\right)}{2\,2^{3/4}\sqrt{2x^4+2x^2+1}} + \frac{3(3+\sqrt{2})(\sqrt{2}x^2+1)\sqrt{\frac{2x^4+2x^2+1}{(\sqrt{2}x^2+1)^2}}\,\text{EllipticPi}\left(\frac{1}{24}(12-11\sqrt{2}),2\arctan\left(\sqrt[4]{2}x\right),\frac{1}{4}(2-\sqrt{2})\right)}{8\,2^{3/4}(2-3\sqrt{2})\sqrt{2x^4+2x^2+1}} + \frac{\sqrt{2x^4+2x^2+1}x}{2\sqrt{2}(\sqrt{2}x^2+1)}$$

```
[In] Int[x^4/((3 + 2*x^2)*Sqrt[1 + 2*x^2 + 2*x^4]),x]
```

```
[Out] (x*Sqrt[1 + 2*x^2 + 2*x^4])/(2*Sqrt[2]*(1 + Sqrt[2]*x^2)) - (3*Sqrt[3/10]*(
3 - Sqrt[2])*ArcTan[(Sqrt[5/3]*x)/Sqrt[1 + 2*x^2 + 2*x^4]])/(4*(2 - 3*Sqrt[
2])) - ((1 + Sqrt[2]*x^2)*Sqrt[(1 + 2*x^2 + 2*x^4)/(1 + Sqrt[2]*x^2)^2]*Ell
ipticE[2*ArcTan[2^(1/4)*x], (2 - Sqrt[2])/4])/(2*2^(3/4)*Sqrt[1 + 2*x^2 + 2
*x^4]) + ((1 - 3*Sqrt[2])*(1 + Sqrt[2]*x^2)*Sqrt[(1 + 2*x^2 + 2*x^4)/(1 + S
qrt[2]*x^2)^2]*EllipticF[2*ArcTan[2^(1/4)*x], (2 - Sqrt[2])/4])/(2*2^(3/4)*
(2 - 3*Sqrt[2])*Sqrt[1 + 2*x^2 + 2*x^4]) + (3*(3 + Sqrt[2])*(1 + Sqrt[2]*x^
2)*Sqrt[(1 + 2*x^2 + 2*x^4)/(1 + Sqrt[2]*x^2)^2]*EllipticPi[(12 - 11*Sqrt[2
])/24, 2*ArcTan[2^(1/4)*x], (2 - Sqrt[2])/4])/(8*2^(3/4)*(2 - 3*Sqrt[2])*Sq
rt[1 + 2*x^2 + 2*x^4])
```

Rule 1117

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1209

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
```

```
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

Rule 1339

```
Int[(x_)^4/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4])
, x_Symbol] :> With[{q = Rt[c/a, 2]}, Dist[-(2*c*d - a*e*q)/(c*e*(e - d*q))
, Int[1/Sqrt[a + b*x^2 + c*x^4], x], x] + (-Dist[1/(e*q), Int[(1 - q*x^2)/S
qrt[a + b*x^2 + c*x^4], x], x] + Dist[d^2/(e*(e - d*q)), Int[(1 + q*x^2)/((
d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]), x], x])] /; FreeQ[{a, b, c, d, e}, x]
&& NeQ[b^2 - 4*a*c, 0] && PosQ[c/a] && NeQ[c*d^2 - a*e^2, 0]
```

Rule 1720

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4]), x_Symbol] :> With[{q = Rt[B/A, 2]}, Simp[(-(B*d - A*e))*(A
rcTan[Rt[-b + c*(d/e) + a*(e/d), 2]*(x/Sqrt[a + b*x^2 + c*x^4])]/(2*d*e*Rt[
-b + c*(d/e) + a*(e/d), 2])), x] + Simp[(B*d + A*e)*(A + B*x^2)*(Sqrt[A^2*(
(a + b*x^2 + c*x^4)/(a*(A + B*x^2)^2))]/(4*d*e*A*q*Sqrt[a + b*x^2 + c*x^4])
)*EllipticPi[Cancel[-(B*d - A*e)^2/(4*d*e*A*B)], 2*ArcTan[q*x], 1/2 - b*(A/
(4*a*B))], x]] /; FreeQ[{a, b, c, d, e, A, B}, x] && NeQ[b^2 - 4*a*c, 0] &&
 NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] && EqQ
[c*A^2 - a*B^2, 0]
```

Rubi steps

$$\text{integral} = -\frac{\int \frac{1-\sqrt{2}x^2}{\sqrt{1+2x^2+2x^4}}\,dx}{2\sqrt{2}} + \frac{9\int \frac{1+\sqrt{2}x^2}{(3+2x^2)\sqrt{1+2x^2+2x^4}}\,dx}{2\left(2-3\sqrt{2}\right)} + \frac{\left(-12+2\sqrt{2}\right)\int \frac{1}{\sqrt{1+2x^2+2x^4}}\,dx}{4\left(2-3\sqrt{2}\right)}$$

$$= \frac{x\sqrt{1+2x^2+2x^4}}{2\sqrt{2}\left(1+\sqrt{2}x^2\right)} - \frac{3\sqrt{\frac{3}{10}}\left(3-\sqrt{2}\right)\tan^{-1}\left(\frac{\sqrt{\frac{5}{3}}x}{\sqrt{1+2x^2+2x^4}}\right)}{4\left(2-3\sqrt{2}\right)} - \frac{\left(1+\sqrt{2}x^2\right)\sqrt{\frac{1+2x^2+2x^4}{\left(1+\sqrt{2}x^2\right)^2}}E\left(2\tan^{-1}\left(\sqrt[4]{2}x\right)|\frac{1}{4}\left(2-\sqrt{2}\right)\right)}{2\ 2^{3/4}\sqrt{1+2x^2+2x^4}} + \frac{\left(1-3\sqrt{2}\right)\left(1+\sqrt{2}x^2\right)\sqrt{\frac{1+2x^2+2x^4}{\left(1+\sqrt{2}x^2\right)^2}}F\left(2\tan^{-1}\left(\sqrt[4]{2}x\right)|\frac{1}{4}\left(2-\sqrt{2}\right)\right)}{2\ 2^{3/4}\left(2-3\sqrt{2}\right)\sqrt{1+2x^2+2x^4}} + \frac{3\left(3+\sqrt{2}\right)\left(1+\sqrt{2}x^2\right)\sqrt{\frac{1+2x^2+2x^4}{\left(1+\sqrt{2}x^2\right)^2}}\Pi\left(\frac{1}{24}\left(12-11\sqrt{2}\right);2\tan^{-1}\left(\sqrt[4]{2}x\right)|\frac{1}{4}\left(2-\sqrt{2}\right)\right)}{8\ 2^{3/4}\left(2-3\sqrt{2}\right)\sqrt{1+2x^2+2x^4}}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.21 (sec) , antiderivative size = 127, normalized size of antiderivative = 0.30

$$\int \frac{x^4}{(3+2x^2)\sqrt{1+2x^2+2x^4}}\,dx = -\frac{\sqrt{1+(1-i)x^2}\sqrt{1+(1+i)x^2}\left((1+i)E\left(i\text{arcsinh}\left(\sqrt{1-i}x\right)\middle|i\right)-(1+4i)\,\text{EllipticF}\left(i\text{arcsinh}\left(\sqrt{1-i}x\right.\right.\right.}{4\sqrt{1-i}\sqrt{1+2x^2+2x^4}}$$

```
[In] Integrate[x^4/((3 + 2*x^2)*Sqrt[1 + 2*x^2 + 2*x^4]),x]
```

```
[Out] -1/4*(Sqrt[1 + (1 - I)*x^2]*Sqrt[1 + (1 + I)*x^2]*((1 + I)*EllipticE[I*ArcS
inh[Sqrt[1 - I]*x], I] - (1 + 4*I)*EllipticF[I*ArcSinh[Sqrt[1 - I]*x], I] +
 (3*I)*EllipticPi[1/3 + I/3, I*ArcSinh[Sqrt[1 - I]*x], I]))/(Sqrt[1 - I]*Sq
rt[1 + 2*x^2 + 2*x^4])
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.89 (sec) , antiderivative size = 222, normalized size of antiderivative = 0.53

method	result
default	$-\frac{3\sqrt{1+(1-i)x^2}\,\sqrt{1+(1+i)x^2}\,F\left(x\sqrt{-1+i},\frac{\sqrt{2}}{2}+\frac{i\sqrt{2}}{2}\right)}{4\sqrt{-1+i}\,\sqrt{2x^4+2x^2+1}}+\frac{\left(-\frac{1}{4}+\frac{i}{4}\right)\sqrt{1+(1-i)x^2}\,\sqrt{1+(1+i)x^2}\,\left(F\left(x\sqrt{-1+i},\frac{\sqrt{2}}{2}+\frac{i\sqrt{2}}{2}\right)-E\left(x\sqrt{-\right.\right.}{\sqrt{-1+i}\,\sqrt{2x^4+2x^2+1}}$
elliptic	$-\frac{\sqrt{-ix^2+x^2+1}\,\sqrt{ix^2+x^2+1}\,F\left(x\sqrt{-1+i},\frac{\sqrt{2}}{2}+\frac{i\sqrt{2}}{2}\right)}{\sqrt{-1+i}\,\sqrt{2x^4+2x^2+1}}+\frac{i\sqrt{-ix^2+x^2+1}\,\sqrt{ix^2+x^2+1}\,F\left(x\sqrt{-1+i},\frac{\sqrt{2}}{2}+\frac{i\sqrt{2}}{2}\right)}{4\sqrt{-1+i}\,\sqrt{2x^4+2x^2+1}}+\frac{\sqrt{-ix^2+x^2+1}\,\sqrt{}}{4\sqrt{}}$

```
[In] int(x^4/(2*x^2+3)/(2*x^4+2*x^2+1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -3/4/(-1+I)^(1/2)*(1+(1-I)*x^2)^(1/2)*(1+(1+I)*x^2)^(1/2)/(2*x^4+2*x^2+1)^(
1/2)*EllipticF(x*(-1+I)^(1/2),1/2*2^(1/2)+1/2*I*2^(1/2))+(-1/4+1/4*I)/(-1+I
)^(1/2)*(1+(1-I)*x^2)^(1/2)*(1+(1+I)*x^2)^(1/2)/(2*x^4+2*x^2+1)^(1/2)*(Elli
pticF(x*(-1+I)^(1/2),1/2*2^(1/2)+1/2*I*2^(1/2))-EllipticE(x*(-1+I)^(1/2),1/
2*2^(1/2)+1/2*I*2^(1/2)))+3/4/(-1+I)^(1/2)*(1-I*x^2+x^2)^(1/2)*(1+I*x^2+x^2
)^(1/2)/(2*x^4+2*x^2+1)^(1/2)*EllipticPi(x*(-1+I)^(1/2),1/3+1/3*I,(-1-I)^(1
/2)/(-1+I)^(1/2))
```

Fricas [F]

$$\int \frac{x^4}{(3+2x^2)\sqrt{1+2x^2+2x^4}}\,dx = \int \frac{x^4}{\sqrt{2\,x^4+2\,x^2+1}(2\,x^2+3)}\,dx$$

```
[In] integrate(x^4/(2*x^2+3)/(2*x^4+2*x^2+1)^(1/2),x, algorithm="fricas")
[Out] integral(sqrt(2*x^4 + 2*x^2 + 1)*x^4/(4*x^6 + 10*x^4 + 8*x^2 + 3), x)
```

Sympy [F]

$$\int \frac{x^4}{(3+2x^2)\sqrt{1+2x^2+2x^4}}\,dx = \int \frac{x^4}{(2x^2+3)\sqrt{2x^4+2x^2+1}}\,dx$$

```
[In] integrate(x**4/(2*x**2+3)/(2*x**4+2*x**2+1)**(1/2),x)
[Out] Integral(x**4/((2*x**2 + 3)*sqrt(2*x**4 + 2*x**2 + 1)), x)
```

Maxima [F]

$$\int \frac{x^4}{(3+2x^2)\sqrt{1+2x^2+2x^4}}\,dx = \int \frac{x^4}{\sqrt{2\,x^4+2\,x^2+1}(2\,x^2+3)}\,dx$$

```
[In] integrate(x^4/(2*x^2+3)/(2*x^4+2*x^2+1)^(1/2),x, algorithm="maxima")
[Out] integrate(x^4/(sqrt(2*x^4 + 2*x^2 + 1)*(2*x^2 + 3)), x)
```

Giac [F]

$$\int \frac{x^4}{(3+2x^2)\sqrt{1+2x^2+2x^4}}\,dx = \int \frac{x^4}{\sqrt{2\,x^4+2\,x^2+1}(2\,x^2+3)}\,dx$$

```
[In] integrate(x^4/(2*x^2+3)/(2*x^4+2*x^2+1)^(1/2),x, algorithm="giac")
[Out] integrate(x^4/(sqrt(2*x^4 + 2*x^2 + 1)*(2*x^2 + 3)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^4}{(3+2x^2)\sqrt{1+2x^2+2x^4}}\,dx = \int \frac{x^4}{(2\,x^2+3)\,\sqrt{2\,x^4+2\,x^2+1}}\,dx$$

```
[In] int(x^4/((2*x^2 + 3)*(2*x^2 + 2*x^4 + 1)^(1/2)),x)
[Out] int(x^4/((2*x^2 + 3)*(2*x^2 + 2*x^4 + 1)^(1/2)), x)
```

3.338 $\int \frac{x^2}{(3+2x^2)\sqrt{1+2x^2+2x^4}}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 247

$$\int \frac{x^2}{(3+2x^2)\sqrt{1+2x^2+2x^4}}\,dx = -\frac{1}{4}\sqrt{\frac{3}{5}}\arctan\left(\frac{\sqrt{\frac{5}{3}}x}{\sqrt{1+2x^2+2x^4}}\right) - \frac{\left(3+\sqrt{2}\right)\left(1+\sqrt{2}x^2\right)\sqrt{\frac{1+2x^2+2x^4}{\left(1+\sqrt{2}x^2\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\sqrt[4]{2}x\right),\frac{1}{4}\left(2-\sqrt{2}\right)\right)}{14\,2^{3/4}\sqrt{1+2x^2+2x^4}} + \frac{\left(3+\sqrt{2}\right)^2\left(1+\sqrt{2}x^2\right)\sqrt{\frac{1+2x^2+2x^4}{\left(1+\sqrt{2}x^2\right)^2}}\,\text{EllipticPi}\left(\frac{1}{24}\left(12-11\sqrt{2}\right),2\arctan\left(\sqrt[4]{2}x\right),\frac{1}{4}\left(2-\sqrt{2}\right)\right)}{56\sqrt[4]{2}\sqrt{1+2x^2+2x^4}}$$

```
[Out] -1/20*arctan(1/3*x*15^(1/2)/(2*x^4+2*x^2+1)^(1/2))*15^(1/2)-1/28*(cos(2*arc
tan(2^(1/4)*x))^2)^(1/2)/cos(2*arctan(2^(1/4)*x))*EllipticF(sin(2*arctan(2^
(1/4)*x)),1/2*(2-2^(1/2))^(1/2))*(3+2^(1/2))*(1+x^2*2^(1/2))*((2*x^4+2*x^2+
1)/(1+x^2*2^(1/2))^2)^(1/2)*2^(1/4)/(2*x^4+2*x^2+1)^(1/2)+1/112*(cos(2*arct
an(2^(1/4)*x))^2)^(1/2)/cos(2*arctan(2^(1/4)*x))*EllipticPi(sin(2*arctan(2^
(1/4)*x)),1/2-11/24*2^(1/2),1/2*(2-2^(1/2))^(1/2))*(3+2^(1/2))^2*(1+x^2*2^(
1/2))*((2*x^4+2*x^2+1)/(1+x^2*2^(1/2))^2)^(1/2)*2^(3/4)/(2*x^4+2*x^2+1)^(1/
2)
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 247, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {1333, 1117, 1720}

$$\int \frac{x^2}{(3+2x^2)\sqrt{1+2x^2+2x^4}}\,dx = -\frac{1}{4}\sqrt{\frac{3}{5}}\arctan\left(\frac{\sqrt{\frac{5}{3}}x}{\sqrt{2x^4+2x^2+1}}\right) - \frac{\left(3+\sqrt{2}\right)\left(\sqrt{2}x^2+1\right)\sqrt{\frac{2x^4+2x^2+1}{\left(\sqrt{2}x^2+1\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\sqrt[4]{2}x\right),\frac{1}{4}\left(2-\sqrt{2}\right)\right)}{14\,2^{3/4}\sqrt{2x^4+2x^2+1}} + \frac{\left(3+\sqrt{2}\right)^2\left(\sqrt{2}x^2+1\right)\sqrt{\frac{2x^4+2x^2+1}{\left(\sqrt{2}x^2+1\right)^2}}\,\text{EllipticPi}\left(\frac{1}{24}\left(12-11\sqrt{2}\right),2\arctan\left(\sqrt[4]{2}x\right),\frac{1}{4}\left(2-\sqrt{2}\right)\right)}{56\sqrt[4]{2}\sqrt{2x^4+2x^2+1}}$$

```
[In] Int[x^2/((3 + 2*x^2)*Sqrt[1 + 2*x^2 + 2*x^4]),x]
```

```
[Out] -1/4*(Sqrt[3/5]*ArcTan[(Sqrt[5/3]*x)/Sqrt[1 + 2*x^2 + 2*x^4]]) - ((3 + Sqrt
[2])*(1 + Sqrt[2]*x^2)*Sqrt[(1 + 2*x^2 + 2*x^4)/(1 + Sqrt[2]*x^2)^2]*Ellipt
icF[2*ArcTan[2^(1/4)*x], (2 - Sqrt[2])/4])/(14*2^(3/4)*Sqrt[1 + 2*x^2 + 2*x
^4]) + ((3 + Sqrt[2])^2*(1 + Sqrt[2]*x^2)*Sqrt[(1 + 2*x^2 + 2*x^4)/(1 + Sqr
t[2]*x^2)^2]*EllipticPi[(12 - 11*Sqrt[2])/24, 2*ArcTan[2^(1/4)*x], (2 - Sqr
t[2])/4])/(56*2^(1/4)*Sqrt[1 + 2*x^2 + 2*x^4])
```

Rule 1117

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1333

```
Int[(x_)^2/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4])
, x_Symbol] :> With[{q = Rt[c/a, 2]}, Dist[(-a)*((e + d*q)/(c*d^2 - a*e^2))
, Int[1/Sqrt[a + b*x^2 + c*x^4], x], x] + Dist[a*d*((e + d*q)/(c*d^2 - a*e^
2)), Int[(1 + q*x^2)/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]), x], x]] /; Free
Q[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2,
0] && PosQ[c/a] && NeQ[c*d^2 - a*e^2, 0]
```

Rule 1720

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4]), x_Symbol] :> With[{q = Rt[B/A, 2]}, Simp[(-(B*d - A*e))*(A
rcTan[Rt[-b + c*(d/e) + a*(e/d), 2]*(x/Sqrt[a + b*x^2 + c*x^4])]/(2*d*e*Rt[
```

```
-b + c*(d/e) + a*(e/d), 2])), x] + Simp[(B*d + A*e)*(A + B*x^2)*(Sqrt[A^2*(
(a + b*x^2 + c*x^4)/(a*(A + B*x^2)^2))]/(4*d*e*A*q*Sqrt[a + b*x^2 + c*x^4])
)*EllipticPi[Cancel[-(B*d - A*e)^2/(4*d*e*A*B)], 2*ArcTan[q*x], 1/2 - b*(A/
(4*a*B))], x]] /; FreeQ[{a, b, c, d, e, A, B}, x] && NeQ[b^2 - 4*a*c, 0] &&
 NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] && EqQ
[c*A^2 - a*B^2, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\left(\frac{1}{14}\left(2+3\sqrt{2}\right)\int\frac{1}{\sqrt{1+2x^2+2x^4}}\,dx\right) \\
&\quad+\frac{1}{14}\left(3\left(2+3\sqrt{2}\right)\right)\int\frac{1+\sqrt{2}x^2}{(3+2x^2)\sqrt{1+2x^2+2x^4}}\,dx \\
&= -\frac{1}{4}\sqrt{\frac{3}{5}}\tan^{-1}\left(\frac{\sqrt{\frac{5}{3}}x}{\sqrt{1+2x^2+2x^4}}\right) \\
&\quad-\frac{\left(3+\sqrt{2}\right)\left(1+\sqrt{2}x^2\right)\sqrt{\frac{1+2x^2+2x^4}{\left(1+\sqrt{2}x^2\right)^2}}F\left(2\tan^{-1}\left(\sqrt[4]{2}x\right)|\frac{1}{4}\left(2-\sqrt{2}\right)\right)}{14\ 2^{3/4}\sqrt{1+2x^2+2x^4}} \\
&\quad+\frac{\left(3+\sqrt{2}\right)^2\left(1+\sqrt{2}x^2\right)\sqrt{\frac{1+2x^2+2x^4}{\left(1+\sqrt{2}x^2\right)^2}}\Pi\left(\frac{1}{24}\left(12-11\sqrt{2}\right);2\tan^{-1}\left(\sqrt[4]{2}x\right)|\frac{1}{4}\left(2-\sqrt{2}\right)\right)}{56\sqrt[4]{2}\sqrt{1+2x^2+2x^4}}
\end{aligned}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.12 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.40

$$\int\frac{x^2}{(3+2x^2)\sqrt{1+2x^2+2x^4}}\,dx = \frac{(1-i)^{3/2}\sqrt{1+(1-i)x^2}\sqrt{1+(1+i)x^2}\left(\text{EllipticF}\left(i\text{arcsinh}\left(\sqrt{1-i}x\right),i\right)-\text{EllipticPi}\left(\frac{1}{3}+\frac{i}{3},i\text{arcsinh}\left(\sqrt{\right.\right.\right.}{4\sqrt{1+2x^2+2x^4}}$$

```
[In] Integrate[x^2/((3 + 2*x^2)*Sqrt[1 + 2*x^2 + 2*x^4]),x]
[Out] ((1 - I)^(3/2)*Sqrt[1 + (1 - I)*x^2]*Sqrt[1 + (1 + I)*x^2]*(EllipticF[I*Arc
Sinh[Sqrt[1 - I]*x], I] - EllipticPi[1/3 + I/3, I*ArcSinh[Sqrt[1 - I]*x], I
]))/(4*Sqrt[1 + 2*x^2 + 2*x^4])
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.75 (sec) , antiderivative size = 134, normalized size of antiderivative = 0.54

method	result	size
default	$\frac{\sqrt{1+(1-i)x^2}\,\sqrt{1+(1+i)x^2}\,F\left(x\sqrt{-1+i},\frac{\sqrt{2}}{2}+\frac{i\sqrt{2}}{2}\right)}{2\sqrt{-1+i}\,\sqrt{2x^4+2x^2+1}}-\frac{\sqrt{-ix^2+x^2+1}\,\sqrt{ix^2+x^2+1}\,\Pi\left(x\sqrt{-1+i},\frac{1}{3}+\frac{i}{3},\frac{\sqrt{-1-i}}{\sqrt{-1+i}}\right)}{2\sqrt{-1+i}\,\sqrt{2x^4+2x^2+1}}$	134
elliptic	$\frac{\sqrt{-ix^2+x^2+1}\,\sqrt{ix^2+x^2+1}\,F\left(x\sqrt{-1+i},\frac{\sqrt{2}}{2}+\frac{i\sqrt{2}}{2}\right)}{2\sqrt{-1+i}\,\sqrt{2x^4+2x^2+1}}-\frac{\sqrt{-ix^2+x^2+1}\,\sqrt{ix^2+x^2+1}\,\Pi\left(x\sqrt{-1+i},\frac{1}{3}+\frac{i}{3},\frac{\sqrt{-1-i}}{\sqrt{-1+i}}\right)}{2\sqrt{-1+i}\,\sqrt{2x^4+2x^2+1}}$	138

```
[In] int(x^2/(2*x^2+3)/(2*x^4+2*x^2+1)^(1/2),x,method=_RETURNVERBOSE)
[Out] 1/2/(-1+I)^(1/2)*(1+(1-I)*x^2)^(1/2)*(1+(1+I)*x^2)^(1/2)/(2*x^4+2*x^2+1)^(1
/2)*EllipticF(x*(-1+I)^(1/2),1/2*2^(1/2)+1/2*I*2^(1/2))-1/2/(-1+I)^(1/2)*(1
-I*x^2+x^2)^(1/2)*(1+I*x^2+x^2)^(1/2)/(2*x^4+2*x^2+1)^(1/2)*EllipticPi(x*(-
1+I)^(1/2),1/3+1/3*I,(-1-I)^(1/2)/(-1+I)^(1/2))
```

Fricas [F]

$$\int \frac{x^2}{(3+2x^2)\sqrt{1+2x^2+2x^4}}\,dx=\int \frac{x^2}{\sqrt{2\,x^4+2\,x^2+1}(2\,x^2+3)}\,dx$$

```
[In] integrate(x^2/(2*x^2+3)/(2*x^4+2*x^2+1)^(1/2),x, algorithm="fricas")
[Out] integral(sqrt(2*x^4 + 2*x^2 + 1)*x^2/(4*x^6 + 10*x^4 + 8*x^2 + 3), x)
```

Sympy [F]

$$\int \frac{x^2}{(3+2x^2)\sqrt{1+2x^2+2x^4}}\,dx=\int \frac{x^2}{(2x^2+3)\sqrt{2x^4+2x^2+1}}\,dx$$

```
[In] integrate(x**2/(2*x**2+3)/(2*x**4+2*x**2+1)**(1/2),x)
[Out] Integral(x**2/((2*x**2 + 3)*sqrt(2*x**4 + 2*x**2 + 1)), x)
```

Maxima [F]

$$\int \frac{x^2}{(3+2x^2)\sqrt{1+2x^2+2x^4}}\,dx = \int \frac{x^2}{\sqrt{2\,x^4+2\,x^2+1}(2\,x^2+3)}\,dx$$

```
[In] integrate(x^2/(2*x^2+3)/(2*x^4+2*x^2+1)^(1/2),x, algorithm="maxima")
[Out] integrate(x^2/(sqrt(2*x^4 + 2*x^2 + 1)*(2*x^2 + 3)), x)
```

Giac [F]

$$\int \frac{x^2}{(3+2x^2)\sqrt{1+2x^2+2x^4}}\,dx = \int \frac{x^2}{\sqrt{2\,x^4+2\,x^2+1}(2\,x^2+3)}\,dx$$

```
[In] integrate(x^2/(2*x^2+3)/(2*x^4+2*x^2+1)^(1/2),x, algorithm="giac")
[Out] integrate(x^2/(sqrt(2*x^4 + 2*x^2 + 1)*(2*x^2 + 3)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{(3+2x^2)\sqrt{1+2x^2+2x^4}}\,dx = \int \frac{x^2}{(2\,x^2+3)\,\sqrt{2\,x^4+2\,x^2+1}}\,dx$$

```
[In] int(x^2/((2*x^2 + 3)*(2*x^2 + 2*x^4 + 1)^(1/2)),x)
[Out] int(x^2/((2*x^2 + 3)*(2*x^2 + 2*x^4 + 1)^(1/2)), x)
```

3.339 $\int \frac{1}{(3+2x^2)\sqrt{1+2x^2+2x^4}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 245

$$\int \frac{1}{(3+2x^2)\sqrt{1+2x^2+2x^4}}\,dx = \frac{\arctan\left(\frac{\sqrt{\frac{5}{3}}x}{\sqrt{1+2x^2+2x^4}}\right)}{2\sqrt{15}} + \frac{\left(3+\sqrt{2}\right)\left(1+\sqrt{2}x^2\right)\sqrt{\frac{1+2x^2+2x^4}{\left(1+\sqrt{2}x^2\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\sqrt[4]{2}x\right),\frac{1}{4}\left(2-\sqrt{2}\right)\right)}{14\sqrt[4]{2}\sqrt{1+2x^2+2x^4}} - \frac{\left(3+\sqrt{2}\right)^2\left(1+\sqrt{2}x^2\right)\sqrt{\frac{1+2x^2+2x^4}{\left(1+\sqrt{2}x^2\right)^2}}\,\text{EllipticPi}\left(\frac{1}{24}\left(12-11\sqrt{2}\right),2\arctan\left(\sqrt[4]{2}x\right),\frac{1}{4}\left(2-\sqrt{2}\right)\right)}{84\sqrt[4]{2}\sqrt{1+2x^2+2x^4}}$$

```
[Out] 1/30*arctan(1/3*x*15^(1/2)/(2*x^4+2*x^2+1)^(1/2))*15^(1/2)+1/28*(cos(2*arct
an(2^(1/4)*x))^2)^(1/2)/cos(2*arctan(2^(1/4)*x))*EllipticF(sin(2*arctan(2^(
1/4)*x)),1/2*(2-2^(1/2))^(1/2))*(3+2^(1/2))*(1+x^2*2^(1/2))*((2*x^4+2*x^2+1
)/(1+x^2*2^(1/2))^2)^(1/2)*2^(3/4)/(2*x^4+2*x^2+1)^(1/2)-1/168*(cos(2*arcta
n(2^(1/4)*x))^2)^(1/2)/cos(2*arctan(2^(1/4)*x))*EllipticPi(sin(2*arctan(2^(
1/4)*x)),1/2-11/24*2^(1/2),1/2*(2-2^(1/2))^(1/2))*(3+2^(1/2))^2*(1+x^2*2^(1
/2))*((2*x^4+2*x^2+1)/(1+x^2*2^(1/2))^2)^(1/2)*2^(3/4)/(2*x^4+2*x^2+1)^(1/2
)
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 245, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {1230, 1117, 1720}

$$\int \frac{1}{(3+2x^2)\sqrt{1+2x^2+2x^4}}\,dx = \frac{\arctan\left(\frac{\sqrt{\frac{5}{3}}x}{\sqrt{2x^4+2x^2+1}}\right)}{2\sqrt{15}} + \frac{(3+\sqrt{2})\left(\sqrt{2}x^2+1\right)\sqrt{\frac{2x^4+2x^2+1}{\left(\sqrt{2}x^2+1\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\sqrt[4]{2}x\right),\frac{1}{4}(2-\sqrt{2})\right)}{14\sqrt[4]{2}\sqrt{2x^4+2x^2+1}} - \frac{(3+\sqrt{2})^2\left(\sqrt{2}x^2+1\right)\sqrt{\frac{2x^4+2x^2+1}{\left(\sqrt{2}x^2+1\right)^2}}\,\text{EllipticPi}\left(\frac{1}{24}(12-11\sqrt{2}),2\arctan\left(\sqrt[4]{2}x\right),\frac{1}{4}(2-\sqrt{2})\right)}{84\sqrt[4]{2}\sqrt{2x^4+2x^2+1}}$$

```
[In] Int[1/((3 + 2*x^2)*Sqrt[1 + 2*x^2 + 2*x^4]),x]
```

```
[Out] ArcTan[(Sqrt[5/3]*x)/Sqrt[1 + 2*x^2 + 2*x^4]]/(2*Sqrt[15]) + ((3 + Sqrt[2])
*(1 + Sqrt[2]*x^2)*Sqrt[(1 + 2*x^2 + 2*x^4)/(1 + Sqrt[2]*x^2)^2]*EllipticF[
2*ArcTan[2^(1/4)*x], (2 - Sqrt[2])/4])/(14*2^(1/4)*Sqrt[1 + 2*x^2 + 2*x^4])
 - ((3 + Sqrt[2])^2*(1 + Sqrt[2]*x^2)*Sqrt[(1 + 2*x^2 + 2*x^4)/(1 + Sqrt[2]
*x^2)^2]*EllipticPi[(12 - 11*Sqrt[2])/24, 2*ArcTan[2^(1/4)*x], (2 - Sqrt[2]
)/4])/(84*2^(1/4)*Sqrt[1 + 2*x^2 + 2*x^4])
```

Rule 1117

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1230

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4]), x_S
ymbol] :> With[{q = Rt[c/a, 2]}, Dist[(c*d + a*e*q)/(c*d^2 - a*e^2), Int[1/
Sqrt[a + b*x^2 + c*x^4], x], x] - Dist[(a*e*(e + d*q))/(c*d^2 - a*e^2), Int
[(1 + q*x^2)/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]), x], x]] /; FreeQ[{a, b,
 c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[c/a]
```

Rule 1720

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4]), x_Symbol] :> With[{q = Rt[B/A, 2]}, Simp[(-(B*d - A*e))*(A
rcTan[Rt[-b + c*(d/e) + a*(e/d), 2]*(x/Sqrt[a + b*x^2 + c*x^4])]/(2*d*e*Rt[
```

```
-b + c*(d/e) + a*(e/d), 2])), x] + Simp[(B*d + A*e)*(A + B*x^2)*(Sqrt[A^2*(
(a + b*x^2 + c*x^4)/(a*(A + B*x^2)^2))]/(4*d*e*A*q*Sqrt[a + b*x^2 + c*x^4])
)*EllipticPi[Cancel[-(B*d - A*e)^2/(4*d*e*A*B)], 2*ArcTan[q*x], 1/2 - b*(A/
(4*a*B))], x]] /; FreeQ[{a, b, c, d, e, A, B}, x] && NeQ[b^2 - 4*a*c, 0] &&
 NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] && EqQ
[c*A^2 - a*B^2, 0]
```

Rubi steps

$$\text{integral} = \frac{1}{7}\left(3+\sqrt{2}\right)\int \frac{1}{\sqrt{1+2x^2+2x^4}}\,dx-\frac{1}{7}\left(2+3\sqrt{2}\right)\int \frac{1+\sqrt{2}x^2}{(3+2x^2)\sqrt{1+2x^2+2x^4}}\,dx$$

$$=\frac{\tan^{-1}\left(\frac{\sqrt{\frac{5}{3}}x}{\sqrt{1+2x^2+2x^4}}\right)}{2\sqrt{15}}+\frac{\left(3+\sqrt{2}\right)\left(1+\sqrt{2}x^2\right)\sqrt{\frac{1+2x^2+2x^4}{\left(1+\sqrt{2}x^2\right)^2}}F\left(2\tan^{-1}\left(\sqrt[4]{2}x\right)|\frac{1}{4}(2-\sqrt{2})\right)}{14\sqrt[4]{2}\sqrt{1+2x^2+2x^4}}-\frac{\left(3+\sqrt{2}\right)^2\left(1+\sqrt{2}x^2\right)\sqrt{\frac{1+2x^2+2x^4}{\left(1+\sqrt{2}x^2\right)^2}}\Pi\left(\frac{1}{24}(12-11\sqrt{2});2\tan^{-1}\left(\sqrt[4]{2}x\right)|\frac{1}{4}(2-\sqrt{2})\right)}{84\sqrt[4]{2}\sqrt{1+2x^2+2x^4}}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.07 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.33

$$\int \frac{1}{(3+2x^2)\sqrt{1+2x^2+2x^4}}\,dx = -\frac{i\sqrt{1+(1-i)x^2}\sqrt{1+(1+i)x^2}\,\text{EllipticPi}\left(\frac{1}{3}+\frac{i}{3},i\text{arcsinh}\left(\sqrt{1-i}x\right),i\right)}{3\sqrt{1-i}\sqrt{1+2x^2+2x^4}}$$

```
[In] Integrate[1/((3 + 2*x^2)*Sqrt[1 + 2*x^2 + 2*x^4]),x]
[Out] ((-1/3*I)*Sqrt[1 + (1 - I)*x^2]*Sqrt[1 + (1 + I)*x^2]*EllipticPi[1/3 + I/3,
 I*ArcSinh[Sqrt[1 - I]*x], I])/(Sqrt[1 - I]*Sqrt[1 + 2*x^2 + 2*x^4])
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.64 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.29

method	result	size
default	$\frac{\sqrt{-i x^2+x^2+1}\,\sqrt{i x^2+x^2+1}\,\Pi\left(x\sqrt{-1+i},\frac{1}{3}+\frac{i}{3},\frac{\sqrt{-1-i}}{\sqrt{-1+i}}\right)}{3\sqrt{-1+i}\,\sqrt{2x^4+2x^2+1}}$	70
elliptic	$\frac{\sqrt{-i x^2+x^2+1}\,\sqrt{i x^2+x^2+1}\,\Pi\left(x\sqrt{-1+i},\frac{1}{3}+\frac{i}{3},\frac{\sqrt{-1-i}}{\sqrt{-1+i}}\right)}{3\sqrt{-1+i}\,\sqrt{2x^4+2x^2+1}}$	70

```
[In] int(1/(2*x^2+3)/(2*x^4+2*x^2+1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3/(-1+I)^(1/2)*(1-I*x^2+x^2)^(1/2)*(1+I*x^2+x^2)^(1/2)/(2*x^4+2*x^2+1)^(1
/2)*EllipticPi(x*(-1+I)^(1/2),1/3+1/3*I,(-1-I)^(1/2)/(-1+I)^(1/2))
```

Fricas [F]

$$\int \frac{1}{(3+2x^2)\sqrt{1+2x^2+2x^4}}\,dx = \int \frac{1}{\sqrt{2\,x^4+2\,x^2+1}(2\,x^2+3)}\,dx$$

```
[In] integrate(1/(2*x^2+3)/(2*x^4+2*x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(2*x^4 + 2*x^2 + 1)/(4*x^6 + 10*x^4 + 8*x^2 + 3), x)
```

Sympy [F]

$$\int \frac{1}{(3+2x^2)\sqrt{1+2x^2+2x^4}}\,dx = \int \frac{1}{(2x^2+3)\sqrt{2x^4+2x^2+1}}\,dx$$

```
[In] integrate(1/(2*x**2+3)/(2*x**4+2*x**2+1)**(1/2),x)
```

```
[Out] Integral(1/((2*x**2 + 3)*sqrt(2*x**4 + 2*x**2 + 1)), x)
```

Maxima [F]

$$\int \frac{1}{(3+2x^2)\sqrt{1+2x^2+2x^4}}\,dx = \int \frac{1}{\sqrt{2\,x^4+2\,x^2+1}(2\,x^2+3)}\,dx$$

```
[In] integrate(1/(2*x^2+3)/(2*x^4+2*x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(2*x^4 + 2*x^2 + 1)*(2*x^2 + 3)), x)
```

Giac [F]

$$\int \frac{1}{(3+2x^2)\sqrt{1+2x^2+2x^4}}\,dx = \int \frac{1}{\sqrt{2\,x^4+2\,x^2+1}(2\,x^2+3)}\,dx$$

```
[In] integrate(1/(2*x^2+3)/(2*x^4+2*x^2+1)^(1/2),x, algorithm="giac")
[Out] integrate(1/(sqrt(2*x^4 + 2*x^2 + 1)*(2*x^2 + 3)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(3+2x^2)\sqrt{1+2x^2+2x^4}}\,dx = \int \frac{1}{(2\,x^2+3)\,\sqrt{2\,x^4+2\,x^2+1}}\,dx$$

```
[In] int(1/((2*x^2 + 3)*(2*x^2 + 2*x^4 + 1)^(1/2)),x)
[Out] int(1/((2*x^2 + 3)*(2*x^2 + 2*x^4 + 1)^(1/2)), x)
```

3.340 $\int \frac{1}{x^2(3+2x^2)\sqrt{1+2x^2+2x^4}}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 399

$$\int \frac{1}{x^2(3+2x^2)\sqrt{1+2x^2+2x^4}}\,dx = -\frac{\sqrt{1+2x^2+2x^4}}{3x}+\frac{\sqrt{2}x\sqrt{1+2x^2+2x^4}}{3\left(1+\sqrt{2}x^2\right)} - \frac{\arctan\left(\frac{\sqrt{\frac{5}{3}}x}{\sqrt{1+2x^2+2x^4}}\right)}{3\sqrt{15}} - \frac{\sqrt[4]{2}\left(1+\sqrt{2}x^2\right)\sqrt{\frac{1+2x^2+2x^4}{\left(1+\sqrt{2}x^2\right)^2}}E\left(2\arctan\left(\sqrt[4]{2}x\right)|\frac{1}{4}\left(2-\sqrt{2}\right)\right)}{3\sqrt{1+2x^2+2x^4}} + \frac{\left(5-3\sqrt{2}\right)\left(1+\sqrt{2}x^2\right)\sqrt{\frac{1+2x^2+2x^4}{\left(1+\sqrt{2}x^2\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\sqrt[4]{2}x\right),\frac{1}{4}\left(2-\sqrt{2}\right)\right)}{21\ 2^{3/4}\sqrt{1+2x^2+2x^4}} + \frac{\left(3+\sqrt{2}\right)^2\left(1+\sqrt{2}x^2\right)\sqrt{\frac{1+2x^2+2x^4}{\left(1+\sqrt{2}x^2\right)^2}}\operatorname{EllipticPi}\left(\frac{1}{24}\left(12-11\sqrt{2}\right),2\arctan\left(\sqrt[4]{2}x\right),\frac{1}{4}\left(2-\sqrt{2}\right)\right)}{126\sqrt[4]{2}\sqrt{1+2x^2+2x^4}}$$

```
[Out] -1/45*arctan(1/3*x*15^(1/2)/(2*x^4+2*x^2+1)^(1/2))*15^(1/2)-1/3*(2*x^4+2*x^
2+1)^(1/2)/x+1/3*x*(2*x^4+2*x^2+1)^(1/2)*2^(1/2)/(1+x^2*2^(1/2))-1/3*(cos(2
*arctan(2^(1/4)*x))^2)^(1/2)/cos(2*arctan(2^(1/4)*x))*EllipticE(sin(2*arcta
n(2^(1/4)*x)),1/2*(2-2^(1/2))^(1/2))*(1+x^2*2^(1/2))*((2*x^4+2*x^2+1)/(1+x^
2*2^(1/2))^2)^(1/2)*2^(1/4)/(2*x^4+2*x^2+1)^(1/2)+1/42*2^(1/4)*(cos(2*arcta
n(2^(1/4)*x))^2)^(1/2)/cos(2*arctan(2^(1/4)*x))*EllipticF(sin(2*arctan(2^(1
/4)*x)),1/2*(2-2^(1/2))^(1/2))*(5-3*2^(1/2))*(1+x^2*2^(1/2))*((2*x^4+2*x^2+
1)/(1+x^2*2^(1/2))^2)^(1/2)/(2*x^4+2*x^2+1)^(1/2)+1/252*(cos(2*arctan(2^(1/
4)*x))^2)^(1/2)/cos(2*arctan(2^(1/4)*x))*EllipticPi(sin(2*arctan(2^(1/4)*x)
),1/2-11/24*2^(1/2),1/2*(2-2^(1/2))^(1/2))*(3+2^(1/2))^2*(1+x^2*2^(1/2))*((
2*x^4+2*x^2+1)/(1+x^2*2^(1/2))^2)^(1/2)*2^(3/4)/(2*x^4+2*x^2+1)^(1/2)
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 399, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.207, Rules used = {1343, 1728, 1209, 1722, 1117, 1720}

$$\int \frac{1}{x^2(3+2x^2)\sqrt{1+2x^2+2x^4}}\,dx = -\frac{\arctan\left(\frac{\sqrt{\frac{5}{3}}x}{\sqrt{2x^4+2x^2+1}}\right)}{3\sqrt{15}} + \frac{(5-3\sqrt{2})(\sqrt{2}x^2+1)\sqrt{\frac{2x^4+2x^2+1}{(\sqrt{2}x^2+1)^2}}\,\text{EllipticF}\left(2\arctan\left(\sqrt[4]{2}x\right),\frac{1}{4}(2-\sqrt{2})\right)}{21\, 2^{3/4}\sqrt{2x^4+2x^2+1}} - \frac{\sqrt[4]{2}(\sqrt{2}x^2+1)\sqrt{\frac{2x^4+2x^2+1}{(\sqrt{2}x^2+1)^2}}\,E\left(2\arctan\left(\sqrt[4]{2}x\right)|\frac{1}{4}(2-\sqrt{2})\right)}{3\sqrt{2x^4+2x^2+1}} + \frac{(3+\sqrt{2})^2(\sqrt{2}x^2+1)\sqrt{\frac{2x^4+2x^2+1}{(\sqrt{2}x^2+1)^2}}\,\text{EllipticPi}\left(\frac{1}{24}(12-11\sqrt{2}),2\arctan\left(\sqrt[4]{2}x\right),\frac{1}{4}(2-\sqrt{2})\right)}{126\sqrt[4]{2}\sqrt{2x^4+2x^2+1}} + \frac{\sqrt{2}\sqrt{2x^4+2x^2+1}x}{3(\sqrt{2}x^2+1)} - \frac{\sqrt{2x^4+2x^2+1}}{3x}$$

```
[In] Int[1/(x^2*(3 + 2*x^2)*Sqrt[1 + 2*x^2 + 2*x^4]),x]
```

```
[Out] -1/3*Sqrt[1 + 2*x^2 + 2*x^4]/x + (Sqrt[2]*x*Sqrt[1 + 2*x^2 + 2*x^4])/(3*(1
+ Sqrt[2]*x^2)) - ArcTan[(Sqrt[5/3]*x)/Sqrt[1 + 2*x^2 + 2*x^4]]/(3*Sqrt[15]
) - (2^(1/4)*(1 + Sqrt[2]*x^2)*Sqrt[(1 + 2*x^2 + 2*x^4)/(1 + Sqrt[2]*x^2)^2
]*EllipticE[2*ArcTan[2^(1/4)*x], (2 - Sqrt[2])/4])/(3*Sqrt[1 + 2*x^2 + 2*x^
4]) + ((5 - 3*Sqrt[2])*(1 + Sqrt[2]*x^2)*Sqrt[(1 + 2*x^2 + 2*x^4)/(1 + Sqrt
[2]*x^2)^2]*EllipticF[2*ArcTan[2^(1/4)*x], (2 - Sqrt[2])/4])/(21*2^(3/4)*Sq
rt[1 + 2*x^2 + 2*x^4]) + ((3 + Sqrt[2])^2*(1 + Sqrt[2]*x^2)*Sqrt[(1 + 2*x^2
 + 2*x^4)/(1 + Sqrt[2]*x^2)^2]*EllipticPi[(12 - 11*Sqrt[2])/24, 2*ArcTan[2^
(1/4)*x], (2 - Sqrt[2])/4])/(126*2^(1/4)*Sqrt[1 + 2*x^2 + 2*x^4])
```

Rule 1117

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1209

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
```

```
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

Rule 1343

```
Int[(x_)^(m_)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^
4]), x_Symbol] :> Simp[x^(m + 1)*(Sqrt[a + b*x^2 + c*x^4]/(a*d*(m + 1))), x
] - Dist[1/(a*d*(m + 1)), Int[(x^(m + 2)/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^
4]))*Simp[a*e*(m + 1) + b*d*(m + 2) + (b*e*(m + 2) + c*d*(m + 3))*x^2 + c*e
*(m + 3)*x^4, x], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0
] && ILtQ[m/2, 0]
```

Rule 1720

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4]), x_Symbol] :> With[{q = Rt[B/A, 2]}, Simp[(-(B*d - A*e))*(A
rcTan[Rt[-b + c*(d/e) + a*(e/d), 2]*(x/Sqrt[a + b*x^2 + c*x^4])]/(2*d*e*Rt[
-b + c*(d/e) + a*(e/d), 2])), x] + Simp[(B*d + A*e)*(A + B*x^2)*(Sqrt[A^2*(
(a + b*x^2 + c*x^4)/(a*(A + B*x^2)^2))]/(4*d*e*A*q*Sqrt[a + b*x^2 + c*x^4])
)*EllipticPi[Cancel[-(B*d - A*e)^2/(4*d*e*A*B)], 2*ArcTan[q*x], 1/2 - b*(A/
(4*a*B))], x]] /; FreeQ[{a, b, c, d, e, A, B}, x] && NeQ[b^2 - 4*a*c, 0] &&
 NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] && EqQ
[c*A^2 - a*B^2, 0]
```

Rule 1722

```
Int[((A_.) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2
+ (c_.)*(x_)^4]), x_Symbol] :> With[{q = Rt[c/a, 2]}, Dist[(A*(c*d + a*e*q)
 - a*B*(e + d*q))/(c*d^2 - a*e^2), Int[1/Sqrt[a + b*x^2 + c*x^4], x], x] +
Dist[a*(B*d - A*e)*((e + d*q)/(c*d^2 - a*e^2)), Int[(1 + q*x^2)/((d + e*x^2
)*Sqrt[a + b*x^2 + c*x^4]), x], x]] /; FreeQ[{a, b, c, d, e, A, B}, x] && N
eQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0
] && PosQ[c/a] && NeQ[c*A^2 - a*B^2, 0]
```

Rule 1728

```
Int[(P4x_)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4])
, x_Symbol] :> With[{q = Rt[c/a, 2], A = Coeff[P4x, x, 0], B = Coeff[P4x, x
, 2], C = Coeff[P4x, x, 4]}, Dist[-C/(e*q), Int[(1 - q*x^2)/Sqrt[a + b*x^2
+ c*x^4], x], x] + Dist[1/(c*e), Int[(A*c*e + a*C*d*q + (B*c*e - C*(c*d - a
*e*q))*x^2)/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]), x], x]] /; FreeQ[{a, b,
c, d, e}, x] && PolyQ[P4x, x^2, 2] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*
d*e + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] &&  !GtQ[b^2 - 4*a*c,
 0]
```

Rubi steps

$$\text{integral} = -\frac{\sqrt{1+2x^2+2x^4}}{3x}+\frac{1}{3}\int \frac{-2+6x^2+4x^4}{(3+2x^2)\sqrt{1+2x^2+2x^4}}\,dx$$

$$= -\frac{\sqrt{1+2x^2+2x^4}}{3x}+\frac{1}{12}\int \frac{-8+12\sqrt{2}+\left(24-4\left(6-2\sqrt{2}\right)\right)x^2}{(3+2x^2)\sqrt{1+2x^2+2x^4}}\,dx - \frac{1}{3}\sqrt{2}\int \frac{1-\sqrt{2}x^2}{\sqrt{1+2x^2+2x^4}}\,dx$$

$$= -\frac{\sqrt{1+2x^2+2x^4}}{3x}+\frac{\sqrt{2}x\sqrt{1+2x^2+2x^4}}{3\left(1+\sqrt{2}x^2\right)} - \frac{\sqrt[4]{2}\left(1+\sqrt{2}x^2\right)\sqrt{\frac{1+2x^2+2x^4}{\left(1+\sqrt{2}x^2\right)^2}}E\left(2\tan^{-1}\left(\sqrt[4]{2}x\right)\middle|\frac{1}{4}\left(2-\sqrt{2}\right)\right)}{3\sqrt{1+2x^2+2x^4}} + \frac{1}{21}\left(2\left(2+3\sqrt{2}\right)\right)\int \frac{1+\sqrt{2}x^2}{(3+2x^2)\sqrt{1+2x^2+2x^4}}\,dx + \frac{1}{21}\left(-6+5\sqrt{2}\right)\int \frac{1}{\sqrt{1+2x^2+2x^4}}\,dx$$

$$= -\frac{\sqrt{1+2x^2+2x^4}}{3x}+\frac{\sqrt{2}x\sqrt{1+2x^2+2x^4}}{3\left(1+\sqrt{2}x^2\right)} - \frac{\tan^{-1}\left(\frac{\sqrt{\frac{5}{3}}x}{\sqrt{1+2x^2+2x^4}}\right)}{3\sqrt{15}} - \frac{\sqrt[4]{2}\left(1+\sqrt{2}x^2\right)\sqrt{\frac{1+2x^2+2x^4}{\left(1+\sqrt{2}x^2\right)^2}}E\left(2\tan^{-1}\left(\sqrt[4]{2}x\right)\middle|\frac{1}{4}\left(2-\sqrt{2}\right)\right)}{3\sqrt{1+2x^2+2x^4}} + \frac{\left(5-3\sqrt{2}\right)\left(1+\sqrt{2}x^2\right)\sqrt{\frac{1+2x^2+2x^4}{\left(1+\sqrt{2}x^2\right)^2}}F\left(2\tan^{-1}\left(\sqrt[4]{2}x\right)\middle|\frac{1}{4}\left(2-\sqrt{2}\right)\right)}{21\ 2^{3/4}\sqrt{1+2x^2+2x^4}} + \frac{\left(3+\sqrt{2}\right)^2\left(1+\sqrt{2}x^2\right)\sqrt{\frac{1+2x^2+2x^4}{\left(1+\sqrt{2}x^2\right)^2}}\Pi\left(\frac{1}{24}\left(12-11\sqrt{2}\right);2\tan^{-1}\left(\sqrt[4]{2}x\right)\middle|\frac{1}{4}\left(2-\sqrt{2}\right)\right)}{126\sqrt[4]{2}\sqrt{1+2x^2+2x^4}}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.18 (sec) , antiderivative size = 147, normalized size of antiderivative = 0.37

$$\int \frac{1}{x^2(3+2x^2)\sqrt{1+2x^2+2x^4}}\,dx = -\frac{i\left(-3i(1+2x^2+2x^4)+\sqrt{1-i}x\sqrt{1+(1-i)x^2}\sqrt{1+(1+i)x^2}\left(3E\left(i\text{arcsinh}\left(\sqrt{1-i}x\right)\middle|i\right)-3\,\text{Ellipti}\right.\right.}{9x\sqrt{1+2x^2+2x^4}}$$

```
[In] Integrate[1/(x^2*(3 + 2*x^2)*Sqrt[1 + 2*x^2 + 2*x^4]),x]
```

```
[Out] ((-1/9*I)*((-3*I)*(1 + 2*x^2 + 2*x^4) + Sqrt[1 - I]*x*Sqrt[1 + (1 - I)*x^2]
*Sqrt[1 + (1 + I)*x^2]*(3*EllipticE[I*ArcSinh[Sqrt[1 - I]*x], I] - 3*Ellipt
icF[I*ArcSinh[Sqrt[1 - I]*x], I] - (1 + I)*EllipticPi[1/3 + I/3, I*ArcSinh[
Sqrt[1 - I]*x], I])))/(x*Sqrt[1 + 2*x^2 + 2*x^4])
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.93 (sec) , antiderivative size = 178, normalized size of antiderivative = 0.45

method	result
default	$-\frac{\sqrt{2x^4+2x^2+1}}{3x}+\frac{\left(-\frac{1}{3}+\frac{i}{3}\right)\sqrt{1+(1-i)x^2}\,\sqrt{1+(1+i)x^2}\,\left(F\left(x\sqrt{-1+i},\frac{\sqrt{2}}{2}+\frac{i\sqrt{2}}{2}\right)-E\left(x\sqrt{-1+i},\frac{\sqrt{2}}{2}+\frac{i\sqrt{2}}{2}\right)\right)}{\sqrt{-1+i}\,\sqrt{2x^4+2x^2+1}}-\frac{2\sqrt{-ix^2+x^2+1}\,\sqrt{i\cdots}}{9\sqrt{\cdots}}$
risch	$-\frac{\sqrt{2x^4+2x^2+1}}{3x}+\frac{\left(-\frac{1}{3}+\frac{i}{3}\right)\sqrt{1+(1-i)x^2}\,\sqrt{1+(1+i)x^2}\,\left(F\left(x\sqrt{-1+i},\frac{\sqrt{2}}{2}+\frac{i\sqrt{2}}{2}\right)-E\left(x\sqrt{-1+i},\frac{\sqrt{2}}{2}+\frac{i\sqrt{2}}{2}\right)\right)}{\sqrt{-1+i}\,\sqrt{2x^4+2x^2+1}}-\frac{2\sqrt{-ix^2+x^2+1}\,\sqrt{i\cdots}}{9\sqrt{\cdots}}$
elliptic	$-\frac{\sqrt{2x^4+2x^2+1}}{3x}-\frac{\sqrt{-ix^2+x^2+1}\,\sqrt{ix^2+x^2+1}\,F\left(x\sqrt{-1+i},\frac{\sqrt{2}}{2}+\frac{i\sqrt{2}}{2}\right)}{3\sqrt{-1+i}\,\sqrt{2x^4+2x^2+1}}+\frac{i\sqrt{-ix^2+x^2+1}\,\sqrt{ix^2+x^2+1}\,F\left(x\sqrt{-1+i},\frac{\sqrt{2}}{2}+\frac{i\sqrt{2}}{2}\right)}{3\sqrt{-1+i}\,\sqrt{2x^4+2x^2+1}}+\cdots$

```
[In] int(1/x^2/(2*x^2+3)/(2*x^4+2*x^2+1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/3*(2*x^4+2*x^2+1)^(1/2)/x+(-1/3+1/3*I)/(-1+I)^(1/2)*(1+(1-I)*x^2)^(1/2)*
(1+(1+I)*x^2)^(1/2)/(2*x^4+2*x^2+1)^(1/2)*(EllipticF(x*(-1+I)^(1/2),1/2*2^(
1/2)+1/2*I*2^(1/2))-EllipticE(x*(-1+I)^(1/2),1/2*2^(1/2)+1/2*I*2^(1/2)))-2/
9/(-1+I)^(1/2)*(1-I*x^2+x^2)^(1/2)*(1+I*x^2+x^2)^(1/2)/(2*x^4+2*x^2+1)^(1/2
)*EllipticPi(x*(-1+I)^(1/2),1/3+1/3*I,(-1-I)^(1/2)/(-1+I)^(1/2))
```

Fricas [F]

$$\int \frac{1}{x^2\left(3+2x^2\right)\sqrt{1+2x^2+2x^4}}\,dx=\int \frac{1}{\sqrt{2\,x^4+2\,x^2+1}(2\,x^2+3)x^2}\,dx$$

```
[In] integrate(1/x^2/(2*x^2+3)/(2*x^4+2*x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(2*x^4 + 2*x^2 + 1)/(4*x^8 + 10*x^6 + 8*x^4 + 3*x^2), x)
```

Sympy [F]

$$\int \frac{1}{x^2 (3+2x^2) \sqrt{1+2x^2+2x^4}} \, dx = \int \frac{1}{x^2 \cdot (2x^2+3) \sqrt{2x^4+2x^2+1}} \, dx$$

```
[In] integrate(1/x**2/(2*x**2+3)/(2*x**4+2*x**2+1)**(1/2),x)
[Out] Integral(1/(x**2*(2*x**2 + 3)*sqrt(2*x**4 + 2*x**2 + 1)), x)
```

Maxima [F]

$$\int \frac{1}{x^2 (3+2x^2) \sqrt{1+2x^2+2x^4}} \, dx = \int \frac{1}{\sqrt{2\,x^4+2\,x^2+1}(2\,x^2+3)x^2} \, dx$$

```
[In] integrate(1/x^2/(2*x^2+3)/(2*x^4+2*x^2+1)^(1/2),x, algorithm="maxima")
[Out] integrate(1/(sqrt(2*x^4 + 2*x^2 + 1)*(2*x^2 + 3)*x^2), x)
```

Giac [F]

$$\int \frac{1}{x^2 (3+2x^2) \sqrt{1+2x^2+2x^4}} \, dx = \int \frac{1}{\sqrt{2\,x^4+2\,x^2+1}(2\,x^2+3)x^2} \, dx$$

```
[In] integrate(1/x^2/(2*x^2+3)/(2*x^4+2*x^2+1)^(1/2),x, algorithm="giac")
[Out] integrate(1/(sqrt(2*x^4 + 2*x^2 + 1)*(2*x^2 + 3)*x^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^2 (3+2x^2) \sqrt{1+2x^2+2x^4}} \, dx = \int \frac{1}{x^2\,\left(2\,x^2+3\right)\,\sqrt{2\,x^4+2\,x^2+1}} \, dx$$

```
[In] int(1/(x^2*(2*x^2 + 3)*(2*x^2 + 2*x^4 + 1)^(1/2)),x)
[Out] int(1/(x^2*(2*x^2 + 3)*(2*x^2 + 2*x^4 + 1)^(1/2)), x)
```

3.341 $\int \frac{1}{x^4(3+2x^2)\sqrt{1+2x^2+2x^4}}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 422

$$\int \frac{1}{x^4(3+2x^2)\sqrt{1+2x^2+2x^4}}\,dx = -\frac{\sqrt{1+2x^2+2x^4}}{9x^3}+\frac{2\sqrt{1+2x^2+2x^4}}{3x}-\frac{2\sqrt{2}x\sqrt{1+2x^2+2x^4}}{3\left(1+\sqrt{2}x^2\right)}+\frac{2\arctan\left(\frac{\sqrt{\frac{5}{3}}x}{\sqrt{1+2x^2+2x^4}}\right)}{9\sqrt{15}}$$
$$+\frac{2\sqrt[4]{2}\left(1+\sqrt{2}x^2\right)\sqrt{\frac{1+2x^2+2x^4}{\left(1+\sqrt{2}x^2\right)^2}}E\left(2\arctan\left(\sqrt[4]{2}x\right)|\frac{1}{4}\left(2-\sqrt{2}\right)\right)}{3\sqrt{1+2x^2+2x^4}}$$
$$-\frac{\left(1+19\sqrt{2}\right)\left(1+\sqrt{2}x^2\right)\sqrt{\frac{1+2x^2+2x^4}{\left(1+\sqrt{2}x^2\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\sqrt[4]{2}x\right),\frac{1}{4}\left(2-\sqrt{2}\right)\right)}{63\sqrt[4]{2}\sqrt{1+2x^2+2x^4}}$$
$$-\frac{\left(3+\sqrt{2}\right)^2\left(1+\sqrt{2}x^2\right)\sqrt{\frac{1+2x^2+2x^4}{\left(1+\sqrt{2}x^2\right)^2}}\operatorname{EllipticPi}\left(\frac{1}{24}\left(12-11\sqrt{2}\right),2\arctan\left(\sqrt[4]{2}x\right),\frac{1}{4}\left(2-\sqrt{2}\right)\right)}{189\sqrt[4]{2}\sqrt{1+2x^2+2x^4}}$$

```
[Out] 2/135*arctan(1/3*x*15^(1/2)/(2*x^4+2*x^2+1)^(1/2))*15^(1/2)-1/9*(2*x^4+2*x^
2+1)^(1/2)/x^3+2/3*(2*x^4+2*x^2+1)^(1/2)/x-2/3*x*(2*x^4+2*x^2+1)^(1/2)*2^(1
/2)/(1+x^2*2^(1/2))+2/3*(cos(2*arctan(2^(1/4)*x))^2)^(1/2)/cos(2*arctan(2^(
1/4)*x))*EllipticE(sin(2*arctan(2^(1/4)*x)),1/2*(2-2^(1/2))^(1/2))*(1+x^2*2
^(1/2))*((2*x^4+2*x^2+1)/(1+x^2*2^(1/2))^2)^(1/2)*2^(1/4)/(2*x^4+2*x^2+1)^(
1/2)-1/378*(cos(2*arctan(2^(1/4)*x))^2)^(1/2)/cos(2*arctan(2^(1/4)*x))*Elli
pticPi(sin(2*arctan(2^(1/4)*x)),1/2-11/24*2^(1/2),1/2*(2-2^(1/2))^(1/2))*(3
+2^(1/2))^2*(1+x^2*2^(1/2))*((2*x^4+2*x^2+1)/(1+x^2*2^(1/2))^2)^(1/2)*2^(3/
4)/(2*x^4+2*x^2+1)^(1/2)-1/126*(cos(2*arctan(2^(1/4)*x))^2)^(1/2)/cos(2*arc
tan(2^(1/4)*x))*EllipticF(sin(2*arctan(2^(1/4)*x)),1/2*(2-2^(1/2))^(1/2))*(
```

```
1+19*2^(1/2))*(1+x^2*2^(1/2))*((2*x^4+2*x^2+1)/(1+x^2*2^(1/2))^2)^(1/2)*2^(
3/4)/(2*x^4+2*x^2+1)^(1/2)
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 422, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.241$, Rules used = {1343, 1697, 1728, 1209, 1722, 1117, 1720}

$$\begin{aligned}\int \frac{1}{x^4(3+2x^2)\sqrt{1+2x^2+2x^4}}\,dx &= \frac{2\arctan\left(\frac{\sqrt{\frac{5}{3}}x}{\sqrt{2x^4+2x^2+1}}\right)}{9\sqrt{15}} \\ &- \frac{\left(1+19\sqrt{2}\right)\left(\sqrt{2}x^2+1\right)\sqrt{\frac{2x^4+2x^2+1}{\left(\sqrt{2}x^2+1\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\sqrt[4]{2}x\right),\frac{1}{4}\left(2-\sqrt{2}\right)\right)}{63\sqrt[4]{2}\sqrt{2x^4+2x^2+1}} \\ &+ \frac{2\sqrt[4]{2}\left(\sqrt{2}x^2+1\right)\sqrt{\frac{2x^4+2x^2+1}{\left(\sqrt{2}x^2+1\right)^2}}\,E\left(2\arctan\left(\sqrt[4]{2}x\right)|\frac{1}{4}\left(2-\sqrt{2}\right)\right)}{3\sqrt{2x^4+2x^2+1}} \\ &- \frac{\left(3+\sqrt{2}\right)^2\left(\sqrt{2}x^2+1\right)\sqrt{\frac{2x^4+2x^2+1}{\left(\sqrt{2}x^2+1\right)^2}}\,\text{EllipticPi}\left(\frac{1}{24}\left(12-11\sqrt{2}\right),2\arctan\left(\sqrt[4]{2}x\right),\frac{1}{4}\left(2-\sqrt{2}\right)\right)}{189\sqrt[4]{2}\sqrt{2x^4+2x^2+1}} \\ &- \frac{2\sqrt{2}\sqrt{2x^4+2x^2+1}x}{3\left(\sqrt{2}x^2+1\right)} + \frac{2\sqrt{2x^4+2x^2+1}}{3x} - \frac{\sqrt{2x^4+2x^2+1}}{9x^3}\end{aligned}$$

```
[In] Int[1/(x^4*(3 + 2*x^2)*Sqrt[1 + 2*x^2 + 2*x^4]),x]
```

```
[Out] -1/9*Sqrt[1 + 2*x^2 + 2*x^4]/x^3 + (2*Sqrt[1 + 2*x^2 + 2*x^4])/(3*x) - (2*S
qrt[2]*x*Sqrt[1 + 2*x^2 + 2*x^4])/(3*(1 + Sqrt[2]*x^2)) + (2*ArcTan[(Sqrt[5
/3]*x)/Sqrt[1 + 2*x^2 + 2*x^4]])/(9*Sqrt[15]) + (2*2^(1/4)*(1 + Sqrt[2]*x^2
)*Sqrt[(1 + 2*x^2 + 2*x^4)/(1 + Sqrt[2]*x^2)^2]*EllipticE[2*ArcTan[2^(1/4)*
x], (2 - Sqrt[2])/4])/(3*Sqrt[1 + 2*x^2 + 2*x^4]) - ((1 + 19*Sqrt[2])*(1 +
Sqrt[2]*x^2)*Sqrt[(1 + 2*x^2 + 2*x^4)/(1 + Sqrt[2]*x^2)^2]*EllipticF[2*ArcT
an[2^(1/4)*x], (2 - Sqrt[2])/4])/(63*2^(1/4)*Sqrt[1 + 2*x^2 + 2*x^4]) - ((3
 + Sqrt[2])^2*(1 + Sqrt[2]*x^2)*Sqrt[(1 + 2*x^2 + 2*x^4)/(1 + Sqrt[2]*x^2)^
2]*EllipticPi[(12 - 11*Sqrt[2])/24, 2*ArcTan[2^(1/4)*x], (2 - Sqrt[2])/4])/
(189*2^(1/4)*Sqrt[1 + 2*x^2 + 2*x^4])
```

Rule 1117

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1209

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

Rule 1343

```
Int[(x_)^(m_)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^
4]), x_Symbol] :> Simp[x^(m + 1)*(Sqrt[a + b*x^2 + c*x^4]/(a*d*(m + 1))), x
] - Dist[1/(a*d*(m + 1)), Int[(x^(m + 2)/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^
4]))*Simp[a*e*(m + 1) + b*d*(m + 2) + (b*e*(m + 2) + c*d*(m + 3))*x^2 + c*e
*(m + 3)*x^4, x], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0
] && ILtQ[m/2, 0]
```

Rule 1697

```
Int[((Px_)*(x_)^(m_))/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_
.)*(x_)^4]), x_Symbol] :> With[{A = Coeff[Px, x, 0], B = Coeff[Px, x, 2], C
 = Coeff[Px, x, 4]}, Simp[A*x^(m + 1)*(Sqrt[a + b*x^2 + c*x^4]/(a*d*(m + 1)
)), x] + Dist[1/(a*d*(m + 1)), Int[(x^(m + 2)/((d + e*x^2)*Sqrt[a + b*x^2 +
 c*x^4]))*Simp[a*B*d*(m + 1) - A*(a*e*(m + 1) + b*d*(m + 2)) + (a*C*d*(m +
1) - A*(b*e*(m + 2) + c*d*(m + 3)))*x^2 - A*c*e*(m + 3)*x^4, x], x], x]] /;
 FreeQ[{a, b, c, d, e}, x] && PolyQ[Px, x^2, 2] && NeQ[b^2 - 4*a*c, 0] && I
LtQ[m/2, 0]
```

Rule 1720

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4]), x_Symbol] :> With[{q = Rt[B/A, 2]}, Simp[(-(B*d - A*e))*(A
rcTan[Rt[-b + c*(d/e) + a*(e/d), 2]*(x/Sqrt[a + b*x^2 + c*x^4])]/(2*d*e*Rt[
-b + c*(d/e) + a*(e/d), 2])), x] + Simp[(B*d + A*e)*(A + B*x^2)*(Sqrt[A^2*(
(a + b*x^2 + c*x^4)/(a*(A + B*x^2)^2))]/(4*d*e*A*q*Sqrt[a + b*x^2 + c*x^4])
)*EllipticPi[Cancel[-(B*d - A*e)^2/(4*d*e*A*B)], 2*ArcTan[q*x], 1/2 - b*(A/
(4*a*B))], x]] /; FreeQ[{a, b, c, d, e, A, B}, x] && NeQ[b^2 - 4*a*c, 0] &&
 NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] && EqQ
[c*A^2 - a*B^2, 0]
```

Rule 1722

```
Int[((A_.) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2
+ (c_.)*(x_)^4]), x_Symbol] :> With[{q = Rt[c/a, 2]}, Dist[(A*(c*d + a*e*q)
 - a*B*(e + d*q))/(c*d^2 - a*e^2), Int[1/Sqrt[a + b*x^2 + c*x^4], x], x] +
```

```
Dist[a*(B*d - A*e)*((e + d*q)/(c*d^2 - a*e^2)), Int[(1 + q*x^2)/((d + e*x^2
)*Sqrt[a + b*x^2 + c*x^4]), x], x]] /; FreeQ[{a, b, c, d, e, A, B}, x] && N
eQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0
] && PosQ[c/a] && NeQ[c*A^2 - a*B^2, 0]
```

Rule 1728

```
Int[(P4x_)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4])
, x_Symbol] :> With[{q = Rt[c/a, 2], A = Coeff[P4x, x, 0], B = Coeff[P4x, x
, 2], C = Coeff[P4x, x, 4]}, Dist[-C/(e*q), Int[(1 - q*x^2)/Sqrt[a + b*x^2
+ c*x^4], x], x] + Dist[1/(c*e), Int[(A*c*e + a*C*d*q + (B*c*e - C*(c*d - a
*e*q))*x^2)/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]), x], x]] /; FreeQ[{a, b,
c, d, e}, x] && PolyQ[P4x, x^2, 2] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*
d*e + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] &&  !GtQ[b^2 - 4*a*c,
 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{\sqrt{1+2x^2+2x^4}}{9x^3}+\frac{1}{9}\int\frac{-18-14x^2-4x^4}{x^2\left(3+2x^2\right)\sqrt{1+2x^2+2x^4}}\,dx\\
&= -\frac{\sqrt{1+2x^2+2x^4}}{9x^3}+\frac{2\sqrt{1+2x^2+2x^4}}{3x}-\frac{1}{27}\int\frac{6+120x^2+72x^4}{\left(3+2x^2\right)\sqrt{1+2x^2+2x^4}}\,dx\\
&= -\frac{\sqrt{1+2x^2+2x^4}}{9x^3}+\frac{2\sqrt{1+2x^2+2x^4}}{3x}\\
&\quad-\frac{1}{108}\int\frac{24+216\sqrt{2}+\left(480-72\left(6-2\sqrt{2}\right)\right)x^2}{\left(3+2x^2\right)\sqrt{1+2x^2+2x^4}}\,dx\\
&\quad+\frac{1}{3}\left(2\sqrt{2}\right)\int\frac{1-\sqrt{2}x^2}{\sqrt{1+2x^2+2x^4}}\,dx\\
&= -\frac{\sqrt{1+2x^2+2x^4}}{9x^3}+\frac{2\sqrt{1+2x^2+2x^4}}{3x}-\frac{2\sqrt{2}x\sqrt{1+2x^2+2x^4}}{3\left(1+\sqrt{2}x^2\right)}\\
&\quad+\frac{2\sqrt[4]{2}\left(1+\sqrt{2}x^2\right)\sqrt{\frac{1+2x^2+2x^4}{\left(1+\sqrt{2}x^2\right)^2}}E\left(2\tan^{-1}\left(\sqrt[4]{2}x\right)\middle|\frac{1}{4}\left(2-\sqrt{2}\right)\right)}{3\sqrt{1+2x^2+2x^4}}\\
&\quad-\frac{1}{63}\left(4\left(2+3\sqrt{2}\right)\right)\int\frac{1+\sqrt{2}x^2}{\left(3+2x^2\right)\sqrt{1+2x^2+2x^4}}\,dx\\
&\quad-\frac{1}{63}\left(2\left(1+19\sqrt{2}\right)\right)\int\frac{1}{\sqrt{1+2x^2+2x^4}}\,dx
\end{aligned}$$

$$= -\frac{\sqrt{1+2x^2+2x^4}}{9x^3} + \frac{2\sqrt{1+2x^2+2x^4}}{3x} - \frac{2\sqrt{2}x\sqrt{1+2x^2+2x^4}}{3\left(1+\sqrt{2}x^2\right)} + \frac{2\tan^{-1}\left(\frac{\sqrt{\frac{5}{3}}x}{\sqrt{1+2x^2+2x^4}}\right)}{9\sqrt{15}} + \frac{2\sqrt[4]{2}(1+\sqrt{2}x^2)\sqrt{\frac{1+2x^2+2x^4}{\left(1+\sqrt{2}x^2\right)^2}}E\left(2\tan^{-1}\left(\sqrt[4]{2}x\right)|\frac{1}{4}(2-\sqrt{2})\right)}{3\sqrt{1+2x^2+2x^4}} - \frac{\left(1+19\sqrt{2}\right)\left(1+\sqrt{2}x^2\right)\sqrt{\frac{1+2x^2+2x^4}{\left(1+\sqrt{2}x^2\right)^2}}F\left(2\tan^{-1}\left(\sqrt[4]{2}x\right)|\frac{1}{4}(2-\sqrt{2})\right)}{63\sqrt[4]{2}\sqrt{1+2x^2+2x^4}} - \frac{\left(3+\sqrt{2}\right)^2\left(1+\sqrt{2}x^2\right)\sqrt{\frac{1+2x^2+2x^4}{\left(1+\sqrt{2}x^2\right)^2}}\Pi\left(\frac{1}{24}\left(12-11\sqrt{2}\right);2\tan^{-1}\left(\sqrt[4]{2}x\right)|\frac{1}{4}(2-\sqrt{2})\right)}{189\sqrt[4]{2}\sqrt{1+2x^2+2x^4}}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.18 (sec) , antiderivative size = 219, normalized size of antiderivative = 0.52

$$\int \frac{1}{x^4\left(3+2x^2\right)\sqrt{1+2x^2+2x^4}}\,dx = \frac{-3+12x^2+30x^4+36x^6+18i\sqrt{1-i}x^3\sqrt{1+(1-i)x^2}\sqrt{1+(1+i)x^2}E\left(i\operatorname{arcsinh}\left(\sqrt{1-i}x\right)\middle|i\right)-(3+1}{}$$

```
[In] Integrate[1/(x^4*(3 + 2*x^2)*Sqrt[1 + 2*x^2 + 2*x^4]),x]
```

```
[Out] (-3 + 12*x^2 + 30*x^4 + 36*x^6 + (18*I)*Sqrt[1 - I]*x^3*Sqrt[1 + (1 - I)*x^
2]*Sqrt[1 + (1 + I)*x^2]*EllipticE[I*ArcSinh[Sqrt[1 - I]*x], I] - (3 + 15*I
)*Sqrt[1 - I]*x^3*Sqrt[1 + (1 - I)*x^2]*Sqrt[1 + (1 + I)*x^2]*EllipticF[I*A
rcSinh[Sqrt[1 - I]*x], I] + 2*(1 - I)^(3/2)*x^3*Sqrt[1 + (1 - I)*x^2]*Sqrt[
1 + (1 + I)*x^2]*EllipticPi[1/3 + I/3, I*ArcSinh[Sqrt[1 - I]*x], I])/(27*x^
3*Sqrt[1 + 2*x^2 + 2*x^4])
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 3.46 (sec) , antiderivative size = 258, normalized size of antiderivative = 0.61

method	result
risch	$\frac{12x^6+10x^4+4x^2-1}{9x^3\sqrt{2x^4+2x^2+1}}-\frac{2\sqrt{1+(1-i)x^2}\,\sqrt{1+(1+i)x^2}\,F\left(x\sqrt{-1+i},\frac{\sqrt{2}}{2}+\frac{i\sqrt{2}}{2}\right)}{9\sqrt{-1+i}\,\sqrt{2x^4+2x^2+1}}+\frac{\left(\frac{2}{3}-\frac{2i}{3}\right)\sqrt{1+(1-i)x^2}\,\sqrt{1+(1+i)x^2}\,\left(F\left(x\sqrt{-1+i},\right.\right.}{\sqrt{-1+i}\,\sqrt{2x^4+2x}}$
default	$\frac{2\sqrt{2x^4+2x^2+1}}{3x}+\frac{\left(\frac{2}{3}-\frac{2i}{3}\right)\sqrt{1+(1-i)x^2}\,\sqrt{1+(1+i)x^2}\,\left(F\left(x\sqrt{-1+i},\frac{\sqrt{2}}{2}+\frac{i\sqrt{2}}{2}\right)-E\left(x\sqrt{-1+i},\frac{\sqrt{2}}{2}+\frac{i\sqrt{2}}{2}\right)\right)}{\sqrt{-1+i}\,\sqrt{2x^4+2x^2+1}}-\frac{\sqrt{2x^4+2x^2+1}}{9x^3}-$
elliptic	$-\frac{\sqrt{2x^4+2x^2+1}}{9x^3}+\frac{2\sqrt{2x^4+2x^2+1}}{3x}+\frac{4\sqrt{-ix^2+x^2+1}\,\sqrt{ix^2+x^2+1}\,F\left(x\sqrt{-1+i},\frac{\sqrt{2}}{2}+\frac{i\sqrt{2}}{2}\right)}{9\sqrt{-1+i}\,\sqrt{2x^4+2x^2+1}}-\frac{2i\sqrt{-ix^2+x^2+1}\,\sqrt{ix^2+x^2+1}\,F\left(\right.}{3\sqrt{-1+i}\,\sqrt{2x^4+2x}}$

```
[In] int(1/x^4/(2*x^2+3)/(2*x^4+2*x^2+1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/9*(12*x^6+10*x^4+4*x^2-1)/x^3/(2*x^4+2*x^2+1)^(1/2)-2/9/(-1+I)^(1/2)*(1+(
1-I)*x^2)^(1/2)*(1+(1+I)*x^2)^(1/2)/(2*x^4+2*x^2+1)^(1/2)*EllipticF(x*(-1+I
)^(1/2),1/2*2^(1/2)+1/2*I*2^(1/2))+(2/3-2/3*I)/(-1+I)^(1/2)*(1+(1-I)*x^2)^(
1/2)*(1+(1+I)*x^2)^(1/2)/(2*x^4+2*x^2+1)^(1/2)*(EllipticF(x*(-1+I)^(1/2),1/
2*2^(1/2)+1/2*I*2^(1/2))-EllipticE(x*(-1+I)^(1/2),1/2*2^(1/2)+1/2*I*2^(1/2)
))+4/27/(-1+I)^(1/2)*(1-I*x^2+x^2)^(1/2)*(1+I*x^2+x^2)^(1/2)/(2*x^4+2*x^2+1
)^(1/2)*EllipticPi(x*(-1+I)^(1/2),1/3+1/3*I,(-1-I)^(1/2)/(-1+I)^(1/2))
```

Fricas [F]

$$\int \frac{1}{x^4\left(3+2x^2\right)\sqrt{1+2x^2+2x^4}}\,dx=\int \frac{1}{\sqrt{2\,x^4+2\,x^2+1}(2\,x^2+3)x^4}\,dx$$

```
[In] integrate(1/x^4/(2*x^2+3)/(2*x^4+2*x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(2*x^4 + 2*x^2 + 1)/(4*x^10 + 10*x^8 + 8*x^6 + 3*x^4), x)
```

Sympy [F]

$$\int \frac{1}{x^4\left(3+2x^2\right)\sqrt{1+2x^2+2x^4}}\,dx=\int \frac{1}{x^4\cdot\left(2x^2+3\right)\sqrt{2x^4+2x^2+1}}\,dx$$

```
[In] integrate(1/x**4/(2*x**2+3)/(2*x**4+2*x**2+1)**(1/2),x)
```

```
[Out] Integral(1/(x**4*(2*x**2 + 3)*sqrt(2*x**4 + 2*x**2 + 1)), x)
```

Maxima [F]

$$\int \frac{1}{x^4 (3+2x^2)\sqrt{1+2x^2+2x^4}}\,dx = \int \frac{1}{\sqrt{2\,x^4+2\,x^2+1}(2\,x^2+3)x^4}\,dx$$

```
[In] integrate(1/x^4/(2*x^2+3)/(2*x^4+2*x^2+1)^(1/2),x, algorithm="maxima")
[Out] integrate(1/(sqrt(2*x^4 + 2*x^2 + 1)*(2*x^2 + 3)*x^4), x)
```

Giac [F]

$$\int \frac{1}{x^4 (3+2x^2)\sqrt{1+2x^2+2x^4}}\,dx = \int \frac{1}{\sqrt{2\,x^4+2\,x^2+1}(2\,x^2+3)x^4}\,dx$$

```
[In] integrate(1/x^4/(2*x^2+3)/(2*x^4+2*x^2+1)^(1/2),x, algorithm="giac")
[Out] integrate(1/(sqrt(2*x^4 + 2*x^2 + 1)*(2*x^2 + 3)*x^4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^4 (3+2x^2)\sqrt{1+2x^2+2x^4}}\,dx = \int \frac{1}{x^4\,\left(2\,x^2+3\right)\,\sqrt{2\,x^4+2\,x^2+1}}\,dx$$

```
[In] int(1/(x^4*(2*x^2 + 3)*(2*x^2 + 2*x^4 + 1)^(1/2)),x)
[Out] int(1/(x^4*(2*x^2 + 3)*(2*x^2 + 2*x^4 + 1)^(1/2)), x)
```

3.342 $\int \frac{x^7}{(d+ex^2)(a+bx^2+cx^4)^{3/2}}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 236

$$\int \frac{x^7}{(d+ex^2)(a+bx^2+cx^4)^{3/2}}\,dx = \frac{a(b^2d-2acd-abe)+(b^3d-3abcd-ab^2e+2a^2ce)\,x^2}{c\,(b^2-4ac)\,(cd^2-bde+ae^2)\sqrt{a+bx^2+cx^4}} + \frac{\operatorname{arctanh}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{2c^{3/2}e} - \frac{d^3\operatorname{arctanh}\left(\frac{bd-2ae+(2cd-be)x^2}{2\sqrt{cd^2-bde+ae^2}\sqrt{a+bx^2+cx^4}}\right)}{2e\,(cd^2-bde+ae^2)^{3/2}}$$

```
[Out] 1/2*arctanh(1/2*(2*c*x^2+b)/c^(1/2)/(c*x^4+b*x^2+a)^(1/2))/c^(3/2)/e-1/2*d^
3*arctanh(1/2*(b*d-2*a*e+(-b*e+2*c*d)*x^2)/(a*e^2-b*d*e+c*d^2)^(1/2)/(c*x^4
+b*x^2+a)^(1/2))/e/(a*e^2-b*d*e+c*d^2)^(3/2)+(a*(-a*b*e-2*a*c*d+b^2*d)+(2*a
^2*c*e-a*b^2*e-3*a*b*c*d+b^3*d)*x^2)/c/(-4*a*c+b^2)/(a*e^2-b*d*e+c*d^2)/(c*
x^4+b*x^2+a)^(1/2)
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 236, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.207, Rules used = {1265, 1660, 857, 635, 212, 738}

$$\int \frac{x^7}{(d+ex^2)(a+bx^2+cx^4)^{3/2}}\,dx = \frac{x^2(2a^2ce-ab^2e-3abcd+b^3d)+a(-abe-2acd+b^2d)}{c\,(b^2-4ac)\sqrt{a+bx^2+cx^4}\,(ae^2-bde+cd^2)} + \frac{\operatorname{arctanh}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{2c^{3/2}e} - \frac{d^3\operatorname{arctanh}\left(\frac{-2ae+x^2(2cd-be)+bd}{2\sqrt{a+bx^2+cx^4}\sqrt{ae^2-bde+cd^2}}\right)}{2e\,(ae^2-bde+cd^2)^{3/2}}$$

```
[In] Int[x^7/((d + e*x^2)*(a + b*x^2 + c*x^4)^(3/2)),x]
```

```
[Out] (a*(b^2*d - 2*a*c*d - a*b*e) + (b^3*d - 3*a*b*c*d - a*b^2*e + 2*a^2*c*e)*x^
2)/(c*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)*Sqrt[a + b*x^2 + c*x^4]) + ArcT
anh[(b + 2*c*x^2)/(2*Sqrt[c]*Sqrt[a + b*x^2 + c*x^4])]/(2*c^(3/2)*e) - (d^3
*ArcTanh[(b*d - 2*a*e + (2*c*d - b*e)*x^2)/(2*Sqrt[c*d^2 - b*d*e + a*e^2]*S
qrt[a + b*x^2 + c*x^4])])/(2*e*(c*d^2 - b*d*e + a*e^2)^(3/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 635

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 857

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1265

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rule 1660

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{Q = PolynomialQuotient[(d + e*x)^m*Pq, a + b*x + c*x
^2, x], f = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x],
x, 0], g = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x], x
, 1]}, Simp[(b*f - 2*a*g + (2*c*f - b*g)*x)*((a + b*x + c*x^2)^(p + 1)/((p
```

```
+ 1)*(b^2 - 4*a*c))), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[(d + e*x)^m*
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[((p + 1)*(b^2 - 4*a*c)*Q)/(d + e*x)^m
 - ((2*p + 3)*(2*c*f - b*g))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, b, c, d,
 e}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2
, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{2}\text{Subst}\left(\int \frac{x^3}{(d+ex)\left(a+bx+cx^2\right)^{3/2}}\,dx, x, x^2\right)\\
&= \frac{a(b^2d-2acd-abe)+\left(b^3d-3abcd-ab^2e+2a^2ce\right)x^2}{c\left(b^2-4ac\right)\left(cd^2-bde+ae^2\right)\sqrt{a+bx^2+cx^4}}\\
&\quad-\frac{\text{Subst}\left(\int \frac{\frac{\left(b^2-4ac\right)d(bd-ae)}{2c\left(cd^2-bde+ae^2\right)}-\frac{\left(b^2-4ac\right)x}{2c}}{(d+ex)\sqrt{a+bx+cx^2}}\,dx, x, x^2\right)}{b^2-4ac}\\
&= \frac{a(b^2d-2acd-abe)+\left(b^3d-3abcd-ab^2e+2a^2ce\right)x^2}{c\left(b^2-4ac\right)\left(cd^2-bde+ae^2\right)\sqrt{a+bx^2+cx^4}}\\
&\quad+\frac{\text{Subst}\left(\int \frac{1}{\sqrt{a+bx+cx^2}}\,dx, x, x^2\right)}{2ce}-\frac{d^3\text{Subst}\left(\int \frac{1}{(d+ex)\sqrt{a+bx+cx^2}}\,dx, x, x^2\right)}{2e\left(cd^2-bde+ae^2\right)}\\
&= \frac{a(b^2d-2acd-abe)+\left(b^3d-3abcd-ab^2e+2a^2ce\right)x^2}{c\left(b^2-4ac\right)\left(cd^2-bde+ae^2\right)\sqrt{a+bx^2+cx^4}}\\
&\quad+\frac{\text{Subst}\left(\int \frac{1}{4c-x^2}\,dx, x, \frac{b+2cx^2}{\sqrt{a+bx^2+cx^4}}\right)}{ce}\\
&\quad+\frac{d^3\text{Subst}\left(\int \frac{1}{4cd^2-4bde+4ae^2-x^2}\,dx, x, \frac{-bd+2ae-(2cd-be)x^2}{\sqrt{a+bx^2+cx^4}}\right)}{e\left(cd^2-bde+ae^2\right)}\\
&= \frac{a(b^2d-2acd-abe)+\left(b^3d-3abcd-ab^2e+2a^2ce\right)x^2}{c\left(b^2-4ac\right)\left(cd^2-bde+ae^2\right)\sqrt{a+bx^2+cx^4}}\\
&\quad+\frac{\tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{2c^{3/2}e}-\frac{d^3\tanh^{-1}\left(\frac{bd-2ae+(2cd-be)x^2}{2\sqrt{cd^2-bde+ae^2}\sqrt{a+bx^2+cx^4}}\right)}{2e\left(cd^2-bde+ae^2\right)^{3/2}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.94 (sec) , antiderivative size = 251, normalized size of antiderivative = 1.06

$$\int \frac{x^7}{(d+ex^2)\left(a+bx^2+cx^4\right)^{3/2}}\,dx = \frac{-b^3dx^2+ab(-bd+3cdx^2+bex^2)+a^2(be+2c(d-ex^2))}{c\left(-b^2+4ac\right)\left(cd^2+e(-bd+ae)\right)\sqrt{a+bx^2+cx^4}} - \frac{d^3\sqrt{-cd^2+bde-ae^2}\arctan\left(\frac{\sqrt{c}(d+ex^2)-e\sqrt{a+bx^2+cx^4}}{\sqrt{-cd^2+e(bd-ae)}}\right)}{e\left(cd^2+e(-bd+ae)\right)^2} - \frac{\log\left(ce\left(b+2cx^2-2\sqrt{c}\sqrt{a+bx^2+cx^4}\right)\right)}{2c^{3/2}e}$$

```
[In] Integrate[x^7/((d + e*x^2)*(a + b*x^2 + c*x^4)^(3/2)),x]
```

```
[Out] (-(b^3*d*x^2) + a*b*(-(b*d) + 3*c*d*x^2 + b*e*x^2) + a^2*(b*e + 2*c*(d - e*
x^2)))/(c*(-b^2 + 4*a*c)*(c*d^2 + e*(-(b*d) + a*e))*Sqrt[a + b*x^2 + c*x^4]
) - (d^3*Sqrt[-(c*d^2) + b*d*e - a*e^2]*ArcTan[(Sqrt[c]*(d + e*x^2) - e*Sqr
t[a + b*x^2 + c*x^4])/Sqrt[-(c*d^2) + e*(b*d - a*e)]])/(e*(c*d^2 + e*(-(b*d
) + a*e))^2) - Log[c*e*(b + 2*c*x^2 - 2*Sqrt[c]*Sqrt[a + b*x^2 + c*x^4])]/(
2*c^(3/2)*e)
```

Maple [A] (verified)

Time = 0.59 (sec) , antiderivative size = 365, normalized size of antiderivative = 1.55

method	result
pseudoelliptic	$\frac{\sqrt{c\,x^4+b\,x^2+a}\left(ac-\frac{b^2}{4}\right)d^3c^{\frac{5}{2}}\ln\left(\frac{2\sqrt{c\,x^4+b\,x^2+a}\,\sqrt{\frac{a\,e^2-bde+c\,d^2}{e^2}}\,e+\left(b\,x^2+2a\right)e-d\left(2c\,x^2+b\right)}{e\,x^2+d}\right)+e\sqrt{\frac{a\,e^2-bde+c\,d^2}{e^2}}\left(\sqrt{c\,x^4+b\,x^2 \cdots}\right.}{2\sqrt{\frac{a\,e^2-b \cdots}{\cdots}}}$
elliptic	$\frac{\ln\left(\frac{\frac{b}{2}+c\,x^2}{\sqrt{c}}+\sqrt{c\,x^4+b\,x^2+a}\right)}{2e\,c^{\frac{3}{2}}}-\frac{2c\,d^3\ln\left(\frac{\frac{2a\,e^2-2bde+2c\,d^2}{e^2}+\frac{(be-2cd)\left(x^2+\frac{d}{e}\right)}{e}+2\sqrt{\frac{a\,e^2-bde+c\,d^2}{e^2}}\,\sqrt{c\left(x^2+\frac{d}{e}\right)^2+\frac{(be-2cd)\left(x^2+\cdots\right.}{e}\cdots}}{x^2+\frac{d}{e}}\right)}{e^2\left(e\sqrt{-4ac+b^2}-be+2cd\right)\left(e\sqrt{-4ac+b^2}+be-2cd\right)\sqrt{\frac{a\,e^2-bde+c\,d^2}{e^2}}\cdots}$
default	$\frac{-\frac{x^2}{2c\sqrt{c\,x^4+b\,x^2+a}}-\frac{b\left(-\frac{1}{c\sqrt{c\,x^4+b\,x^2+a}}-\frac{b\left(2c\,x^2+b\right)}{c\left(4ac-b^2\right)\sqrt{c\,x^4+b\,x^2+a}}\right)}{4c}+\frac{\ln\left(\frac{\frac{b}{2}+c\,x^2}{\sqrt{c}}+\sqrt{c\,x^4+b\,x^2+a}\right)}{2c^{\frac{3}{2}}}}{e}+\frac{d^2\left(2c\,x^2+b\right)}{e^3\sqrt{c\,x^4+b\,x^2+a}\,(4ac-b\cdots}$

```
[In] int(x^7/(e*x^2+d)/(c*x^4+b*x^2+a)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2/((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)/(c*x^4+b*x^2+a)^(1/2)*((c*x^4+b*x^2+a)^
(1/2)*(a*c-1/4*b^2)*d^3*c^(5/2)*ln((2*(c*x^4+b*x^2+a)^(1/2)*((a*e^2-b*d*e+c
```

```
*d^2)/e^2)^(1/2)*e+(b*x^2+2*a)*e-d*(2*c*x^2+b))/(e*x^2+d))+e*((a*e^2-b*d*e+
c*d^2)/e^2)^(1/2)*((c*x^4+b*x^2+a)^(1/2)*(a*e^2-b*d*e+c*d^2)*c*(a*c-1/4*b^2
)*ln((2*c*x^2+2*(c*x^4+b*x^2+a)^(1/2)*c^(1/2)+b)/c^(1/2))-ln(2)*(a*e^2-b*d*
e+c*d^2)*c*(a*c-1/4*b^2)*(c*x^4+b*x^2+a)^(1/2)+e*((-a*e*x^2+d*(3/2*b*x^2+a)
)*a*c+1/2*b*(b*x^2+a)*(a*e-b*d))*c^(3/2)))/e^2/(a*e^2-b*d*e+c*d^2)/(a*c-1/4
*b^2)/c^(5/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1202 vs. $2(214) = 428$.

Time = 37.45 (sec) , antiderivative size = 4901, normalized size of antiderivative = 20.77

$$\int \frac{x^7}{(d+ex^2)\left(a+bx^2+cx^4\right)^{3/2}}\,dx = \text{Too large to display}$$

```
[In] integrate(x^7/(e*x^2+d)/(c*x^4+b*x^2+a)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/4*(((a*b^2*c^2 - 4*a^2*c^3)*d^4 - 2*(a*b^3*c - 4*a^2*b*c^2)*d^3*e + (a*b
^4 - 2*a^2*b^2*c - 8*a^3*c^2)*d^2*e^2 - 2*(a^2*b^3 - 4*a^3*b*c)*d*e^3 + (a^
3*b^2 - 4*a^4*c)*e^4 + ((b^2*c^3 - 4*a*c^4)*d^4 - 2*(b^3*c^2 - 4*a*b*c^3)*d
^3*e + (b^4*c - 2*a*b^2*c^2 - 8*a^2*c^3)*d^2*e^2 - 2*(a*b^3*c - 4*a^2*b*c^2
)*d*e^3 + (a^2*b^2*c - 4*a^3*c^2)*e^4)*x^4 + ((b^3*c^2 - 4*a*b*c^3)*d^4 - 2
*(b^4*c - 4*a*b^2*c^2)*d^3*e + (b^5 - 2*a*b^3*c - 8*a^2*b*c^2)*d^2*e^2 - 2*
(a*b^4 - 4*a^2*b^2*c)*d*e^3 + (a^2*b^3 - 4*a^3*b*c)*e^4)*x^2)*sqrt(c)*log(-
8*c^2*x^4 - 8*b*c*x^2 - b^2 - 4*sqrt(c*x^4 + b*x^2 + a)*(2*c*x^2 + b)*sqrt(
c) - 4*a*c) + ((b^2*c^3 - 4*a*c^4)*d^3*x^4 + (b^3*c^2 - 4*a*b*c^3)*d^3*x^2
+ (a*b^2*c^2 - 4*a^2*c^3)*d^3)*sqrt(c*d^2 - b*d*e + a*e^2)*log(-((8*c^2*d^2
 - 8*b*c*d*e + (b^2 + 4*a*c)*e^2)*x^4 - 8*a*b*d*e + 8*a^2*e^2 + (b^2 + 4*a*
c)*d^2 + 2*(4*b*c*d^2 + 4*a*b*e^2 - (3*b^2 + 4*a*c)*d*e)*x^2 - 4*sqrt(c*x^4
 + b*x^2 + a)*sqrt(c*d^2 - b*d*e + a*e^2)*((2*c*d - b*e)*x^2 + b*d - 2*a*e)
)/(e^2*x^4 + 2*d*e*x^2 + d^2)) - 4*(a^3*b*c*e^4 - (a*b^2*c^2 - 2*a^2*c^3)*d
^3*e + (a*b^3*c - a^2*b*c^2)*d^2*e^2 - 2*(a^2*b^2*c - a^3*c^2)*d*e^3 - ((b^
3*c^2 - 3*a*b*c^3)*d^3*e - (b^4*c - 2*a*b^2*c^2 - 2*a^2*c^3)*d^2*e^2 + (2*a
*b^3*c - 5*a^2*b*c^2)*d*e^3 - (a^2*b^2*c - 2*a^3*c^2)*e^4)*x^2)*sqrt(c*x^4
+ b*x^2 + a))/((a*b^2*c^4 - 4*a^2*c^5)*d^4*e - 2*(a*b^3*c^3 - 4*a^2*b*c^4)*
d^3*e^2 + (a*b^4*c^2 - 2*a^2*b^2*c^3 - 8*a^3*c^4)*d^2*e^3 - 2*(a^2*b^3*c^2
- 4*a^3*b*c^3)*d*e^4 + (a^3*b^2*c^2 - 4*a^4*c^3)*e^5 + ((b^2*c^5 - 4*a*c^6)
*d^4*e - 2*(b^3*c^4 - 4*a*b*c^5)*d^3*e^2 + (b^4*c^3 - 2*a*b^2*c^4 - 8*a^2*c
^5)*d^2*e^3 - 2*(a*b^3*c^3 - 4*a^2*b*c^4)*d*e^4 + (a^2*b^2*c^3 - 4*a^3*c^4)
*e^5)*x^4 + ((b^3*c^4 - 4*a*b*c^5)*d^4*e - 2*(b^4*c^3 - 4*a*b^2*c^4)*d^3*e^
2 + (b^5*c^2 - 2*a*b^3*c^3 - 8*a^2*b*c^4)*d^2*e^3 - 2*(a*b^4*c^2 - 4*a^2*b^
2*c^3)*d*e^4 + (a^2*b^3*c^2 - 4*a^3*b*c^3)*e^5)*x^2), -1/4*(2*((b^2*c^3 - 4
*a*c^4)*d^3*x^4 + (b^3*c^2 - 4*a*b*c^3)*d^3*x^2 + (a*b^2*c^2 - 4*a^2*c^3)*d
^3)*sqrt(-c*d^2 + b*d*e - a*e^2)*arctan(-1/2*sqrt(c*x^4 + b*x^2 + a)*sqrt(-
```

c*d^2 + b*d*e - a*e^2)*((2*c*d - b*e)*x^2 + b*d - 2*a*e)/((c^2*d^2 - b*c*d*e + a*c*e^2)*x^4 + a*c*d^2 - a*b*d*e + a^2*e^2 + (b*c*d^2 - b^2*d*e + a*b*e^2)*x^2)) - ((a*b^2*c^2 - 4*a^2*c^3)*d^4 - 2*(a*b^3*c - 4*a^2*b*c^2)*d^3*e + (a*b^4 - 2*a^2*b^2*c - 8*a^3*c^2)*d^2*e^2 - 2*(a^2*b^3 - 4*a^3*b*c)*d*e^3 + (a^3*b^2 - 4*a^4*c)*e^4 + ((b^2*c^3 - 4*a*c^4)*d^4 - 2*(b^3*c^2 - 4*a*b*c^3)*d^3*e + (b^4*c - 2*a*b^2*c^2 - 8*a^2*c^3)*d^2*e^2 - 2*(a*b^3*c - 4*a^2*b*c^2)*d*e^3 + (a^2*b^2*c - 4*a^3*c^2)*e^4)*x^4 + ((b^3*c^2 - 4*a*b*c^3)*d^4 - 2*(b^4*c - 4*a*b^2*c^2)*d^3*e + (b^5 - 2*a*b^3*c - 8*a^2*b*c^2)*d^2*e^2 - 2*(a*b^4 - 4*a^2*b^2*c)*d*e^3 + (a^2*b^3 - 4*a^3*b*c)*e^4)*x^2)*sqrt(c)*log(-8*c^2*x^4 - 8*b*c*x^2 - b^2 - 4*sqrt(c*x^4 + b*x^2 + a)*(2*c*x^2 + b)*sqrt(c) - 4*a*c) + 4*(a^3*b*c*e^4 - (a*b^2*c^2 - 2*a^2*c^3)*d^3*e + (a*b^3*c - a^2*b*c^2)*d^2*e^2 - 2*(a^2*b^2*c - a^3*c^2)*d*e^3 - ((b^3*c^2 - 3*a*b*c^3)*d^3*e - (b^4*c - 2*a*b^2*c^2 - 2*a^2*c^3)*d^2*e^2 + (2*a*b^3*c - 5*a^2*b*c^2)*d*e^3 - (a^2*b^2*c - 2*a^3*c^2)*e^4)*x^2)*sqrt(c*x^4 + b*x^2 + a))/((a*b^2*c^4 - 4*a^2*c^5)*d^4*e - 2*(a*b^3*c^3 - 4*a^2*b*c^4)*d^3*e^2 + (a*b^4*c^2 - 2*a^2*b^2*c^3 - 8*a^3*c^4)*d^2*e^3 - 2*(a^2*b^3*c^2 - 4*a^3*b*c^3)*d*e^4 + (a^3*b^2*c^2 - 4*a^4*c^3)*e^5 + ((b^2*c^5 - 4*a*c^6)*d^4*e - 2*(b^3*c^4 - 4*a*b*c^5)*d^3*e^2 + (b^4*c^3 - 2*a*b^2*c^4 - 8*a^2*c^5)*d^2*e^3 - 2*(a*b^3*c^3 - 4*a^2*b*c^4)*d*e^4 + (a^2*b^2*c^3 - 4*a^3*c^4)*e^5)*x^4 + ((b^3*c^4 - 4*a*b*c^5)*d^4*e - 2*(b^4*c^3 - 4*a*b^2*c^4)*d^3*e^2 + (b^5*c^2 - 2*a*b^3*c^3 - 8*a^2*b*c^4)*d^2*e^3 - 2*(a*b^4*c^2 - 4*a^2*b^2*c^3)*d*e^4 + (a^2*b^3*c^2 - 4*a^3*b*c^3)*e^5)*x^2), -1/4*(2*((a*b^2*c^2 - 4*a^2*c^3)*d^4 - 2*(a*b^3*c - 4*a^2*b*c^2)*d^3*e + (a*b^4 - 2*a^2*b^2*c - 8*a^3*c^2)*d^2*e^2 - 2*(a^2*b^3 - 4*a^3*b*c)*d*e^3 + (a^3*b^2 - 4*a^4*c)*e^4 + ((b^2*c^3 - 4*a*c^4)*d^4 - 2*(b^3*c^2 - 4*a*b*c^3)*d^3*e + (b^4*c - 2*a*b^2*c^2 - 8*a^2*c^3)*d^2*e^2 - 2*(a*b^3*c - 4*a^2*b*c^2)*d*e^3 + (a^2*b^2*c - 4*a^3*c^2)*e^4)*x^4 + ((b^3*c^2 - 4*a*b*c^3)*d^4 - 2*(b^4*c - 4*a*b^2*c^2)*d^3*e + (b^5 - 2*a*b^3*c - 8*a^2*b*c^2)*d^2*e^2 - 2*(a*b^4 - 4*a^2*b^2*c)*d*e^3 + (a^2*b^3 - 4*a^3*b*c)*e^4)*x^2)*sqrt(-c)*arctan(1/2*sqrt(c*x^4 + b*x^2 + a)*(2*c*x^2 + b)*sqrt(-c)/(c^2*x^4 + b*c*x^2 + a*c)) - ((b^2*c^3 - 4*a*c^4)*d^3*x^4 + (b^3*c^2 - 4*a*b*c^3)*d^3*x^2 + (a*b^2*c^2 - 4*a^2*c^3)*d^3)*sqrt(c*d^2 - b*d*e + a*e^2)*log(-((8*c^2*d^2 - 8*b*c*d*e + (b^2 + 4*a*c)*e^2)*x^4 - 8*a*b*d*e + 8*a^2*e^2 + (b^2 + 4*a*c)*d^2 + 2*(4*b*c*d^2 + 4*a*b*e^2 - (3*b^2 + 4*a*c)*d*e)*x^2 - 4*sqrt(c*x^4 + b*x^2 + a)*sqrt(c*d^2 - b*d*e + a*e^2)*((2*c*d - b*e)*x^2 + b*d - 2*a*e))/(e^2*x^4 + 2*d*e*x^2 + d^2)) + 4*(a^3*b*c*e^4 - (a*b^2*c^2 - 2*a^2*c^3)*d^3*e + (a*b^3*c - a^2*b*c^2)*d^2*e^2 - 2*(a^2*b^2*c - a^3*c^2)*d*e^3 - ((b^3*c^2 - 3*a*b*c^3)*d^3*e - (b^4*c - 2*a*b^2*c^2 - 2*a^2*c^3)*d^2*e^2 + (2*a*b^3*c - 5*a^2*b*c^2)*d*e^3 - (a^2*b^2*c - 2*a^3*c^2)*e^4)*x^2)*sqrt(c*x^4 + b*x^2 + a))/((a*b^2*c^4 - 4*a^2*c^5)*d^4*e - 2*(a*b^3*c^3 - 4*a^2*b*c^4)*d^3*e^2 + (a*b^4*c^2 - 2*a^2*b^2*c^3 - 8*a^3*c^4)*d^2*e^3 - 2*(a^2*b^3*c^2 - 4*a^3*b*c^3)*d*e^4 + (a^3*b^2*c^2 - 4*a^4*c^3)*e^5 + ((b^2*c^5 - 4*a*c^6)*d^4*e - 2*(b^3*c^4 - 4*a*b*c^5)*d^3*e^2 + (b^4*c^3 - 2*a*b^2*c^4 - 8*a^2*c^5)*d^2*e^3 - 2*(a*b^3*c^3 - 4*a^2*b*c^4)*d*e^4 + (a^2*b^2*c^3 - 4*a^3*c^4)*e^5)*x^4 + ((b^3*c^4 - 4*a*b*c^5)*d^4*e - 2*(b^4*c^3 - 4*a*b^2*c^4)*d^3*e^2 + (b^5*c^2 - 2*a*b^3*c^3 - 8*a^2*b*c

```
^4)*d^2*e^3 - 2*(a*b^4*c^2 - 4*a^2*b^2*c^3)*d*e^4 + (a^2*b^3*c^2 - 4*a^3*b*
c^3)*e^5)*x^2), -1/2*(((b^2*c^3 - 4*a*c^4)*d^3*x^4 + (b^3*c^2 - 4*a*b*c^3)*
d^3*x^2 + (a*b^2*c^2 - 4*a^2*c^3)*d^3)*sqrt(-c*d^2 + b*d*e - a*e^2)*arctan(
-1/2*sqrt(c*x^4 + b*x^2 + a)*sqrt(-c*d^2 + b*d*e - a*e^2)*((2*c*d - b*e)*x^
2 + b*d - 2*a*e)/((c^2*d^2 - b*c*d*e + a*c*e^2)*x^4 + a*c*d^2 - a*b*d*e + a
^2*e^2 + (b*c*d^2 - b^2*d*e + a*b*e^2)*x^2)) + ((a*b^2*c^2 - 4*a^2*c^3)*d^4
 - 2*(a*b^3*c - 4*a^2*b*c^2)*d^3*e + (a*b^4 - 2*a^2*b^2*c - 8*a^3*c^2)*d^2*
e^2 - 2*(a^2*b^3 - 4*a^3*b*c)*d*e^3 + (a^3*b^2 - 4*a^4*c)*e^4 + ((b^2*c^3 -
 4*a*c^4)*d^4 - 2*(b^3*c^2 - 4*a*b*c^3)*d^3*e + (b^4*c - 2*a*b^2*c^2 - 8*a^
2*c^3)*d^2*e^2 - 2*(a*b^3*c - 4*a^2*b*c^2)*d*e^3 + (a^2*b^2*c - 4*a^3*c^2)*
e^4)*x^4 + ((b^3*c^2 - 4*a*b*c^3)*d^4 - 2*(b^4*c - 4*a*b^2*c^2)*d^3*e + (b^
5 - 2*a*b^3*c - 8*a^2*b*c^2)*d^2*e^2 - 2*(a*b^4 - 4*a^2*b^2*c)*d*e^3 + (a^2
*b^3 - 4*a^3*b*c)*e^4)*x^2)*sqrt(-c)*arctan(1/2*sqrt(c*x^4 + b*x^2 + a)*(2*
c*x^2 + b)*sqrt(-c)/(c^2*x^4 + b*c*x^2 + a*c)) + 2*(a^3*b*c*e^4 - (a*b^2*c^
2 - 2*a^2*c^3)*d^3*e + (a*b^3*c - a^2*b*c^2)*d^2*e^2 - 2*(a^2*b^2*c - a^3*c
^2)*d*e^3 - ((b^3*c^2 - 3*a*b*c^3)*d^3*e - (b^4*c - 2*a*b^2*c^2 - 2*a^2*c^3
)*d^2*e^2 + (2*a*b^3*c - 5*a^2*b*c^2)*d*e^3 - (a^2*b^2*c - 2*a^3*c^2)*e^4)*
x^2)*sqrt(c*x^4 + b*x^2 + a))/((a*b^2*c^4 - 4*a^2*c^5)*d^4*e - 2*(a*b^3*c^3
 - 4*a^2*b*c^4)*d^3*e^2 + (a*b^4*c^2 - 2*a^2*b^2*c^3 - 8*a^3*c^4)*d^2*e^3 -
 2*(a^2*b^3*c^2 - 4*a^3*b*c^3)*d*e^4 + (a^3*b^2*c^2 - 4*a^4*c^3)*e^5 + ((b^
2*c^5 - 4*a*c^6)*d^4*e - 2*(b^3*c^4 - 4*a*b*c^5)*d^3*e^2 + (b^4*c^3 - 2*a*b
^2*c^4 - 8*a^2*c^5)*d^2*e^3 - 2*(a*b^3*c^3 - 4*a^2*b*c^4)*d*e^4 + (a^2*b^2*
c^3 - 4*a^3*c^4)*e^5)*x^4 + ((b^3*c^4 - 4*a*b*c^5)*d^4*e - 2*(b^4*c^3 - 4*a
*b^2*c^4)*d^3*e^2 + (b^5*c^2 - 2*a*b^3*c^3 - 8*a^2*b*c^4)*d^2*e^3 - 2*(a*b^
4*c^2 - 4*a^2*b^2*c^3)*d*e^4 + (a^2*b^3*c^2 - 4*a^3*b*c^3)*e^5)*x^2)]
```

Sympy [F]

$$\int \frac{x^7}{(d+ex^2)\,(a+bx^2+cx^4)^{3/2}}\,dx = \int \frac{x^7}{(d+ex^2)\,(a+bx^2+cx^4)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(x**7/(e*x**2+d)/(c*x**4+b*x**2+a)**(3/2),x)
[Out] Integral(x**7/((d + e*x**2)*(a + b*x**2 + c*x**4)**(3/2)), x)
```

Maxima [F]

$$\int \frac{x^7}{(d+ex^2)\,(a+bx^2+cx^4)^{3/2}}\,dx = \int \frac{x^7}{(cx^4+bx^2+a)^{\frac{3}{2}}(ex^2+d)}\,dx$$

```
[In] integrate(x^7/(e*x^2+d)/(c*x^4+b*x^2+a)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(x^7/((c*x^4 + b*x^2 + a)^(3/2)*(e*x^2 + d)), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{x^7}{(d+ex^2)\,(a+bx^2+cx^4)^{3/2}}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(x^7/(e*x^2+d)/(c*x^4+b*x^2+a)^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument Type
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^7}{(d+ex^2)\,(a+bx^2+cx^4)^{3/2}}\,dx = \int \frac{x^7}{(e\,x^2+d)\,\left(c\,x^4+b\,x^2+a\right)^{3/2}}\,dx$$

```
[In] int(x^7/((d + e*x^2)*(a + b*x^2 + c*x^4)^(3/2)),x)
```

```
[Out] int(x^7/((d + e*x^2)*(a + b*x^2 + c*x^4)^(3/2)), x)
```

3.343 $\int \frac{x^5}{(d+ex^2)(a+bx^2+cx^4)^{3/2}}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 167

$$\int \frac{x^5}{(d+ex^2)(a+bx^2+cx^4)^{3/2}}\,dx = -\frac{a(bd-2ae)+(b^2d-2acd-abe)\,x^2}{(b^2-4ac)\,(cd^2-bde+ae^2)\,\sqrt{a+bx^2+cx^4}} + \frac{d^2\operatorname{arctanh}\left(\frac{bd-2ae+(2cd-be)x^2}{2\sqrt{cd^2-bde+ae^2}\sqrt{a+bx^2+cx^4}}\right)}{2\,(cd^2-bde+ae^2)^{3/2}}$$

```
[Out] 1/2*d^2*arctanh(1/2*(b*d-2*a*e+(-b*e+2*c*d)*x^2)/(a*e^2-b*d*e+c*d^2)^(1/2)/
(c*x^4+b*x^2+a)^(1/2))/(a*e^2-b*d*e+c*d^2)^(3/2)+(-a*(-2*a*e+b*d)-(-a*b*e-2
*a*c*d+b^2*d)*x^2)/(-4*a*c+b^2)/(a*e^2-b*d*e+c*d^2)/(c*x^4+b*x^2+a)^(1/2)
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 167, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.172, Rules used = {1265, 1660, 12, 738, 212}

$$\int \frac{x^5}{(d+ex^2)(a+bx^2+cx^4)^{3/2}}\,dx = \frac{d^2\operatorname{arctanh}\left(\frac{-2ae+x^2(2cd-be)+bd}{2\sqrt{a+bx^2+cx^4}\sqrt{ae^2-bde+cd^2}}\right)}{2\,(ae^2-bde+cd^2)^{3/2}} - \frac{x^2(-abe-2acd+b^2d)+a(bd-2ae)}{(b^2-4ac)\sqrt{a+bx^2+cx^4}\,(ae^2-bde+cd^2)}$$

```
[In] Int[x^5/((d + e*x^2)*(a + b*x^2 + c*x^4)^(3/2)),x]
```

```
[Out] -((a*(b*d - 2*a*e) + (b^2*d - 2*a*c*d - a*b*e)*x^2)/((b^2 - 4*a*c)*(c*d^2 -
 b*d*e + a*e^2)*Sqrt[a + b*x^2 + c*x^4])) + (d^2*ArcTanh[(b*d - 2*a*e + (2*
```

```
c*d - b*e)*x^2)/(2*Sqrt[c*d^2 - b*d*e + a*e^2]*Sqrt[a + b*x^2 + c*x^4])])/(
2*(c*d^2 - b*d*e + a*e^2)^(3/2))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 1265

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rule 1660

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{Q = PolynomialQuotient[(d + e*x)^m*Pq, a + b*x + c*x
^2, x], f = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x],
x, 0], g = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x], x
, 1]}, Simp[(b*f - 2*a*g + (2*c*f - b*g)*x)*((a + b*x + c*x^2)^(p + 1)/((p
+ 1)*(b^2 - 4*a*c))), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[(d + e*x)^m*
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[((p + 1)*(b^2 - 4*a*c)*Q)/(d + e*x)^m
 - ((2*p + 3)*(2*c*f - b*g))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, b, c, d,
 e}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2
, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rubi steps

$$\text{integral} = \frac{1}{2}\text{Subst}\left(\int \frac{x^2}{(d+ex)\,(a+bx+cx^2)^{3/2}}\,dx, x, x^2\right)$$

$$= -\frac{a(bd-2ae)+(b^2d-2acd-abe)\,x^2}{(b^2-4ac)\,(cd^2-bde+ae^2)\,\sqrt{a+bx^2+cx^4}} - \frac{\text{Subst}\left(\int -\frac{(b^2-4ac)d^2}{2(cd^2-bde+ae^2)(d+ex)\sqrt{a+bx+cx^2}}\,dx, x, x^2\right)}{b^2-4ac}$$

$$= -\frac{a(bd-2ae)+(b^2d-2acd-abe)\,x^2}{(b^2-4ac)\,(cd^2-bde+ae^2)\,\sqrt{a+bx^2+cx^4}} + \frac{d^2\text{Subst}\left(\int \frac{1}{(d+ex)\sqrt{a+bx+cx^2}}\,dx, x, x^2\right)}{2\,(cd^2-bde+ae^2)}$$

$$= -\frac{a(bd-2ae)+(b^2d-2acd-abe)\,x^2}{(b^2-4ac)\,(cd^2-bde+ae^2)\,\sqrt{a+bx^2+cx^4}} - \frac{d^2\text{Subst}\left(\int \frac{1}{4cd^2-4bde+4ae^2-x^2}\,dx, x, \frac{-bd+2ae-(2cd-be)x^2}{\sqrt{a+bx^2+cx^4}}\right)}{cd^2-bde+ae^2}$$

$$= -\frac{a(bd-2ae)+(b^2d-2acd-abe)\,x^2}{(b^2-4ac)\,(cd^2-bde+ae^2)\,\sqrt{a+bx^2+cx^4}} + \frac{d^2\tanh^{-1}\left(\frac{bd-2ae+(2cd-be)x^2}{2\sqrt{cd^2-bde+ae^2}\sqrt{a+bx^2+cx^4}}\right)}{2\,(cd^2-bde+ae^2)^{3/2}}$$

Mathematica [A] (verified)

Time = 0.64 (sec) , antiderivative size = 180, normalized size of antiderivative = 1.08

$$\int \frac{x^5}{(d+ex^2)\,(a+bx^2+cx^4)^{3/2}}\,dx = \frac{-2a^2e+b^2dx^2-2acdx^2+ab(d-ex^2)}{(b^2-4ac)\,(-cd^2+e(bd-ae))\,\sqrt{a+bx^2+cx^4}} + \frac{d^2\sqrt{-cd^2+bde-ae^2}\,\arctan\left(\frac{\sqrt{c}(d+ex^2)-e\sqrt{a+bx^2+cx^4}}{\sqrt{-cd^2+e(bd-ae)}}\right)}{(cd^2+e(-bd+ae))^2}$$

```
[In] Integrate[x^5/((d + e*x^2)*(a + b*x^2 + c*x^4)^(3/2)),x]
```

```
[Out] (-2*a^2*e + b^2*d*x^2 - 2*a*c*d*x^2 + a*b*(d - e*x^2))/((b^2 - 4*a*c)*(-(c*
d^2) + e*(b*d - a*e))*Sqrt[a + b*x^2 + c*x^4]) + (d^2*Sqrt[-(c*d^2) + b*d*e
 - a*e^2]*ArcTan[(Sqrt[c]*(d + e*x^2) - e*Sqrt[a + b*x^2 + c*x^4])/Sqrt[-(c
*d^2) + e*(b*d - a*e)]])/(c*d^2 + e*(-(b*d) + a*e))^2
```

Maple [A] (verified)

Time = 0.44 (sec) , antiderivative size = 229, normalized size of antiderivative = 1.37

method	result
pseudoelliptic	$-\frac{\sqrt{c x^4+b x^2+a}\, d^2 \left(ac-\frac{b^2}{4}\right) \ln\left(\frac{2\sqrt{c x^4+b x^2+a}\sqrt{\frac{a e^2-bde+c d^2}{e^2}}\, e+\left(b x^2+2a\right)e-d\left(2c x^2+b\right)}{e x^2+d}\right)+\left(a\left(\frac{b x^2}{2}+a\right)e-\frac{d\left(\left(-2c x^2+b\right)a \cdots\right.}{2}\cdots\right.}{2\sqrt{c x^4+b x^2+a}\sqrt{\frac{a e^2-bde+c d^2}{e^2}}\, e\left(a e^2-bde+c d^2\right)\left(ac-\frac{b^2}{4}\right)}$
elliptic	$\frac{2c d^2 \ln\left(\frac{\frac{2a e^2-2bde+2c d^2}{e^2}+\frac{(be-2cd)\left(x^2+\frac{d}{e}\right)}{e}+2\sqrt{\frac{a e^2-bde+c d^2}{e^2}}\sqrt{c\left(x^2+\frac{d}{e}\right)^2+\frac{(be-2cd)\left(x^2+\frac{d}{e}\right)}{e}+\frac{a e^2-bde+c d^2}{e^2}}}{x^2+\frac{d}{e}}\right)}{\left(e\sqrt{-4ac+b^2}-be+2cd\right)\left(e\sqrt{-4ac+b^2}+be-2cd\right)e\sqrt{\frac{a e^2-bde+c d^2}{e^2}}}+\frac{\left(b+\sqrt{-4\cdots}\right.}{\cdots}$
default	$-\frac{b x^2+2a}{e\sqrt{c x^4+b x^2+a}\,(4ac-b^2)}-\frac{d(2c x^2+b)}{e^2\sqrt{c x^4+b x^2+a}\,(4ac-b^2)}+\frac{d^2\left(-\frac{2c\sqrt{c\left(x^2-\frac{-b+\sqrt{-4ac+b^2}}{2c}\right)^2+\sqrt{-4ac+b^2}\left(x^2-\frac{-b+\sqrt{-4a\cdots}}{2c}\right)\cdots}}{\left(e\sqrt{-4ac+b^2}-be+2cd\right)\left(-4ac+b^2\right)\left(x^2-\frac{-b+\sqrt{-4ac+b\cdots}}{2c}\right)\cdots}\cdots\right.}{\cdots}$

```
[In] int(x^5/(e*x^2+d)/(c*x^4+b*x^2+a)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*((c*x^4+b*x^2+a)^(1/2)*d^2*(a*c-1/4*b^2)*ln((2*(c*x^4+b*x^2+a)^(1/2)*(
(a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*e+(b*x^2+2*a)*e-d*(2*c*x^2+b))/(e*x^2+d))+(a
*(1/2*b*x^2+a)*e-1/2*d*((-2*c*x^2+b)*a+b^2*x^2))*e*((a*e^2-b*d*e+c*d^2)/e^2
)^(1/2))/(c*x^4+b*x^2+a)^(1/2)/((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)/e/(a*e^2-b*d
*e+c*d^2)/(a*c-1/4*b^2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 669 vs. $2(155) = 310$.

Time = 0.54 (sec) , antiderivative size = 1381, normalized size of antiderivative = 8.27

$$\int \frac{x^5}{(d+ex^2)\,(a+bx^2+cx^4)^{3/2}}\,dx = \text{Too large to display}$$

```
[In] integrate(x^5/(e*x^2+d)/(c*x^4+b*x^2+a)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/4*(((b^2*c - 4*a*c^2)*d^2*x^4 + (b^3 - 4*a*b*c)*d^2*x^2 + (a*b^2 - 4*a^2
*c)*d^2)*sqrt(c*d^2 - b*d*e + a*e^2)*log(-((8*c^2*d^2 - 8*b*c*d*e + (b^2 +
4*a*c)*e^2)*x^4 - 8*a*b*d*e + 8*a^2*e^2 + (b^2 + 4*a*c)*d^2 + 2*(4*b*c*d^2
+ 4*a*b*e^2 - (3*b^2 + 4*a*c)*d*e)*x^2 + 4*sqrt(c*x^4 + b*x^2 + a)*sqrt(c*d
^2 - b*d*e + a*e^2)*((2*c*d - b*e)*x^2 + b*d - 2*a*e))/(e^2*x^4 + 2*d*e*x^2
 + d^2)) - 4*(a*b*c*d^3 + 3*a^2*b*d*e^2 - 2*a^3*e^3 - (a*b^2 + 2*a^2*c)*d^2
*e - (a^2*b*e^3 - (b^2*c - 2*a*c^2)*d^3 + (b^3 - a*b*c)*d^2*e - 2*(a*b^2 -
a^2*c)*d*e^2)*x^2)*sqrt(c*x^4 + b*x^2 + a))/((a*b^2*c^2 - 4*a^2*c^3)*d^4 -
2*(a*b^3*c - 4*a^2*b*c^2)*d^3*e + (a*b^4 - 2*a^2*b^2*c - 8*a^3*c^2)*d^2*e^2
 - 2*(a^2*b^3 - 4*a^3*b*c)*d*e^3 + (a^3*b^2 - 4*a^4*c)*e^4 + ((b^2*c^3 - 4*
```

```
a*c^4)*d^4 - 2*(b^3*c^2 - 4*a*b*c^3)*d^3*e + (b^4*c - 2*a*b^2*c^2 - 8*a^2*c
^3)*d^2*e^2 - 2*(a*b^3*c - 4*a^2*b*c^2)*d*e^3 + (a^2*b^2*c - 4*a^3*c^2)*e^4
)*x^4 + ((b^3*c^2 - 4*a*b*c^3)*d^4 - 2*(b^4*c - 4*a*b^2*c^2)*d^3*e + (b^5 -
 2*a*b^3*c - 8*a^2*b*c^2)*d^2*e^2 - 2*(a*b^4 - 4*a^2*b^2*c)*d*e^3 + (a^2*b^
3 - 4*a^3*b*c)*e^4)*x^2), 1/2*(((b^2*c - 4*a*c^2)*d^2*x^4 + (b^3 - 4*a*b*c)
*d^2*x^2 + (a*b^2 - 4*a^2*c)*d^2)*sqrt(-c*d^2 + b*d*e - a*e^2)*arctan(-1/2*
sqrt(c*x^4 + b*x^2 + a)*sqrt(-c*d^2 + b*d*e - a*e^2)*((2*c*d - b*e)*x^2 + b
*d - 2*a*e)/((c^2*d^2 - b*c*d*e + a*c*e^2)*x^4 + a*c*d^2 - a*b*d*e + a^2*e^
2 + (b*c*d^2 - b^2*d*e + a*b*e^2)*x^2)) - 2*(a*b*c*d^3 + 3*a^2*b*d*e^2 - 2*
a^3*e^3 - (a*b^2 + 2*a^2*c)*d^2*e - (a^2*b*e^3 - (b^2*c - 2*a*c^2)*d^3 + (b
^3 - a*b*c)*d^2*e - 2*(a*b^2 - a^2*c)*d*e^2)*x^2)*sqrt(c*x^4 + b*x^2 + a))/
((a*b^2*c^2 - 4*a^2*c^3)*d^4 - 2*(a*b^3*c - 4*a^2*b*c^2)*d^3*e + (a*b^4 - 2
*a^2*b^2*c - 8*a^3*c^2)*d^2*e^2 - 2*(a^2*b^3 - 4*a^3*b*c)*d*e^3 + (a^3*b^2
- 4*a^4*c)*e^4 + ((b^2*c^3 - 4*a*c^4)*d^4 - 2*(b^3*c^2 - 4*a*b*c^3)*d^3*e +
 (b^4*c - 2*a*b^2*c^2 - 8*a^2*c^3)*d^2*e^2 - 2*(a*b^3*c - 4*a^2*b*c^2)*d*e^
3 + (a^2*b^2*c - 4*a^3*c^2)*e^4)*x^4 + ((b^3*c^2 - 4*a*b*c^3)*d^4 - 2*(b^4*
c - 4*a*b^2*c^2)*d^3*e + (b^5 - 2*a*b^3*c - 8*a^2*b*c^2)*d^2*e^2 - 2*(a*b^4
 - 4*a^2*b^2*c)*d*e^3 + (a^2*b^3 - 4*a^3*b*c)*e^4)*x^2)]
```

Sympy [F]

$$\int \frac{x^5}{(d+ex^2)(a+bx^2+cx^4)^{3/2}}\,dx = \int \frac{x^5}{(d+ex^2)(a+bx^2+cx^4)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(x**5/(e*x**2+d)/(c*x**4+b*x**2+a)**(3/2),x)
[Out] Integral(x**5/((d + e*x**2)*(a + b*x**2 + c*x**4)**(3/2)), x)
```

Maxima [F]

$$\int \frac{x^5}{(d+ex^2)(a+bx^2+cx^4)^{3/2}}\,dx = \int \frac{x^5}{(cx^4+bx^2+a)^{\frac{3}{2}}(ex^2+d)}\,dx$$

```
[In] integrate(x^5/(e*x^2+d)/(c*x^4+b*x^2+a)^(3/2),x, algorithm="maxima")
[Out] integrate(x^5/((c*x^4 + b*x^2 + a)^(3/2)*(e*x^2 + d)), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 468 vs. 2(155) = 310.

Time = 0.31 (sec) , antiderivative size = 468, normalized size of antiderivative = 2.80

$$\int \frac{x^5}{(d+ex^2)(a+bx^2+cx^4)^{3/2}}\,dx = \frac{d^2\arctan\left(-\frac{\left(\sqrt{c}x^2-\sqrt{cx^4+bx^2+a}\right)e+\sqrt{c}d}{\sqrt{-cd^2+bde-ae^2}}\right)}{(cd^2-bde+ae^2)\sqrt{-cd^2+bde-ae^2}} - \frac{\frac{(b^2cd^3-2\,ac^2d^3-b^3d^2e+abcd^2e+2\,ab^2de^2-2\,a^2cde^2-a^2be^3)x^2}{b^2c^2d^4-4\,ac^3d^4-2\,b^3cd^3e+8\,abc^2d^3e+b^4d^2e^2-2\,ab^2cd^2e^2-8\,a^2c^2d^2e^2-2\,ab^3de^3+8\,a^2bcde^3+a^2b^2e^4-4\,a^3ce^4}+\frac{}{b^2c^2d^4-4\,ac^3d^4-2\,b^3cd^3e+8\,ab}}{\sqrt{cx^4+bx^2+a}}$$

```
[In] integrate(x^5/(e*x^2+d)/(c*x^4+b*x^2+a)^(3/2),x, algorithm="giac")
```

```
[Out] d^2*arctan(-((sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))*e + sqrt(c)*d)/sqrt(-c
*d^2 + b*d*e - a*e^2))/((c*d^2 - b*d*e + a*e^2)*sqrt(-c*d^2 + b*d*e - a*e^2
)) - ((b^2*c*d^3 - 2*a*c^2*d^3 - b^3*d^2*e + a*b*c*d^2*e + 2*a*b^2*d*e^2 -
2*a^2*c*d*e^2 - a^2*b*e^3)*x^2/(b^2*c^2*d^4 - 4*a*c^3*d^4 - 2*b^3*c*d^3*e +
 8*a*b*c^2*d^3*e + b^4*d^2*e^2 - 2*a*b^2*c*d^2*e^2 - 8*a^2*c^2*d^2*e^2 - 2*
a*b^3*d*e^3 + 8*a^2*b*c*d*e^3 + a^2*b^2*e^4 - 4*a^3*c*e^4) + (a*b*c*d^3 - a
*b^2*d^2*e - 2*a^2*c*d^2*e + 3*a^2*b*d*e^2 - 2*a^3*e^3)/(b^2*c^2*d^4 - 4*a*
c^3*d^4 - 2*b^3*c*d^3*e + 8*a*b*c^2*d^3*e + b^4*d^2*e^2 - 2*a*b^2*c*d^2*e^2
 - 8*a^2*c^2*d^2*e^2 - 2*a*b^3*d*e^3 + 8*a^2*b*c*d*e^3 + a^2*b^2*e^4 - 4*a^
3*c*e^4))/sqrt(c*x^4 + b*x^2 + a)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^5}{(d+ex^2)(a+bx^2+cx^4)^{3/2}}\,dx = \int \frac{x^5}{(e\,x^2+d)\,\left(c\,x^4+b\,x^2+a\right)^{3/2}}\,dx$$

```
[In] int(x^5/((d + e*x^2)*(a + b*x^2 + c*x^4)^(3/2)),x)
```

```
[Out] int(x^5/((d + e*x^2)*(a + b*x^2 + c*x^4)^(3/2)), x)
```

3.344 $\int \frac{x^3}{(d+ex^2)(a+bx^2+cx^4)^{3/2}}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 159

$$\int \frac{x^3}{(d+ex^2)(a+bx^2+cx^4)^{3/2}}\,dx = \frac{a(2cd-be)+c(bd-2ae)x^2}{(b^2-4ac)(cd^2-bde+ae^2)\sqrt{a+bx^2+cx^4}} - \frac{d e\,\text{arctanh}\left(\frac{bd-2ae+(2cd-be)x^2}{2\sqrt{cd^2-bde+ae^2}\sqrt{a+bx^2+cx^4}}\right)}{2(cd^2-bde+ae^2)^{3/2}}$$

```
[Out] -1/2*d*e*arctanh(1/2*(b*d-2*a*e+(-b*e+2*c*d)*x^2)/(a*e^2-b*d*e+c*d^2)^(1/2)
/(c*x^4+b*x^2+a)^(1/2))/(a*e^2-b*d*e+c*d^2)^(3/2)+(a*(-b*e+2*c*d)+c*(-2*a*e
+b*d)*x^2)/(-4*a*c+b^2)/(a*e^2-b*d*e+c*d^2)/(c*x^4+b*x^2+a)^(1/2)
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 159, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.172, Rules used = {1265, 836, 12, 738, 212}

$$\int \frac{x^3}{(d+ex^2)(a+bx^2+cx^4)^{3/2}}\,dx = \frac{cx^2(bd-2ae)+a(2cd-be)}{(b^2-4ac)\sqrt{a+bx^2+cx^4}\,(ae^2-bde+cd^2)} - \frac{d e\,\text{arctanh}\left(\frac{-2ae+x^2(2cd-be)+bd}{2\sqrt{a+bx^2+cx^4}\sqrt{ae^2-bde+cd^2}}\right)}{2(ae^2-bde+cd^2)^{3/2}}$$

```
[In] Int[x^3/((d + e*x^2)*(a + b*x^2 + c*x^4)^(3/2)),x]
```

```
[Out] (a*(2*c*d - b*e) + c*(b*d - 2*a*e)*x^2)/((b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e
^2)*Sqrt[a + b*x^2 + c*x^4]) - (d*e*ArcTanh[(b*d - 2*a*e + (2*c*d - b*e)*x^
```

```
2)/(2*Sqrt[c*d^2 - b*d*e + a*e^2]*Sqrt[a + b*x^2 + c*x^4])])/(2*(c*d^2 - b*
d*e + a*e^2)^(3/2))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 836

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(f*(b*c*d - b^2*e + 2
*a*c*e) - a*g*(2*c*d - b*e) + c*(f*(2*c*d - b*e) - g*(b*d - 2*a*e))*x)*((a
+ b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2))), x]
 + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^m*
(a + b*x + c*x^2)^(p + 1)*Simp[f*(b*c*d*e*(2*p - m + 2) + b^2*e^2*(p + m +
2) - 2*c^2*d^2*(2*p + 3) - 2*a*c*e^2*(m + 2*p + 3)) - g*(a*e*(b*e - 2*c*d*m
 + b*e*m) - b*d*(3*c*d - b*e + 2*c*d*p - b*e*p)) + c*e*(g*(b*d - 2*a*e) - f
*(2*c*d - b*e))*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1
] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 1265

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rubi steps

$$\text{integral} = \frac{1}{2}\text{Subst}\left(\int \frac{x}{(d+ex)\,(a+bx+cx^2)^{3/2}}\,dx, x, x^2\right)$$

$$= \frac{a(2cd-be)+c(bd-2ae)x^2}{(b^2-4ac)(cd^2-bde+ae^2)\sqrt{a+bx^2+cx^4}} - \frac{\text{Subst}\left(\int \frac{(b^2-4ac)de}{2(d+ex)\sqrt{a+bx+cx^2}}\,dx, x, x^2\right)}{(b^2-4ac)(cd^2-bde+ae^2)}$$

$$= \frac{a(2cd-be)+c(bd-2ae)x^2}{(b^2-4ac)(cd^2-bde+ae^2)\sqrt{a+bx^2+cx^4}} - \frac{(de)\text{Subst}\left(\int \frac{1}{(d+ex)\sqrt{a+bx+cx^2}}\,dx, x, x^2\right)}{2(cd^2-bde+ae^2)}$$

$$= \frac{a(2cd-be)+c(bd-2ae)x^2}{(b^2-4ac)(cd^2-bde+ae^2)\sqrt{a+bx^2+cx^4}} + \frac{(de)\text{Subst}\left(\int \frac{1}{4cd^2-4bde+4ae^2-x^2}\,dx, x, \frac{-bd+2ae-(2cd-be)x^2}{\sqrt{a+bx^2+cx^4}}\right)}{cd^2-bde+ae^2}$$

$$= \frac{a(2cd-be)+c(bd-2ae)x^2}{(b^2-4ac)(cd^2-bde+ae^2)\sqrt{a+bx^2+cx^4}} - \frac{de\tanh^{-1}\left(\frac{bd-2ae+(2cd-be)x^2}{2\sqrt{cd^2-bde+ae^2}\sqrt{a+bx^2+cx^4}}\right)}{2(cd^2-bde+ae^2)^{3/2}}$$

Mathematica [A] (verified)

Time = 0.70 (sec) , antiderivative size = 178, normalized size of antiderivative = 1.12

$$\int \frac{x^3}{(d+ex^2)(a+bx^2+cx^4)^{3/2}}\,dx = \frac{\frac{(cd^2+e(-bd+ae))(-abe+bcdx^2+2ac(d-ex^2))}{\sqrt{a+bx^2+cx^4}} + (-b^2+4ac)\,de\sqrt{-cd^2+e(bd-\cdots}}{(b^2-4ac)(cd^2+e(-bd+ae))^2}$$

```
[In] Integrate[x^3/((d + e*x^2)*(a + b*x^2 + c*x^4)^(3/2)),x]
```

```
[Out] (((c*d^2 + e*(-(b*d) + a*e))*(-(a*b*e) + b*c*d*x^2 + 2*a*c*(d - e*x^2)))/Sq
rt[a + b*x^2 + c*x^4] + (-b^2 + 4*a*c)*d*e*Sqrt[-(c*d^2) + e*(b*d - a*e)]*A
rcTan[(Sqrt[c]*(d + e*x^2) - e*Sqrt[a + b*x^2 + c*x^4])/Sqrt[-(c*d^2) + e*(
b*d - a*e)]])/((b^2 - 4*a*c)*(c*d^2 + e*(-(b*d) + a*e))^2)
```

Maple [A] (verified)

Time = 0.43 (sec) , antiderivative size = 216, normalized size of antiderivative = 1.36

method	result
pseudoelliptic	$\frac{\frac{d\ln\left(\frac{2\sqrt{c x^4+b x^2+a}\sqrt{\frac{a e^2-bde+c d^2}{e^2}}\, e+\left(b x^2+2a\right)e-d\left(2c x^2+b\right)}{e x^2+d}\right)\left(4ac-b^2\right)\sqrt{c x^4+b x^2+a}}{2}+\left(a\left(2c x^2+b\right)e-2\left(\frac{b x^2}{2}+a\right)cd\right)\sqrt{\frac{a e^2 \cdots}{}}}{4\sqrt{c x^4+b x^2+a}\sqrt{\frac{a e^2-bde+c d^2}{e^2}}\left(a e^2-bde+c d^2\right)\left(ac-\frac{b^2}{4}\right)}$
elliptic	$-\frac{2cd\ln\left(\frac{\frac{2a e^2-2bde+2c d^2}{e^2}+\frac{(be-2cd)\left(x^2+\frac{d}{e}\right)}{e}+2\sqrt{\frac{a e^2-bde+c d^2}{e^2}}\sqrt{c\left(x^2+\frac{d}{e}\right)^2+\frac{(be-2cd)\left(x^2+\frac{d}{e}\right)}{e}+\frac{a e^2-bde+c d^2}{e^2}}}{x^2+\frac{d}{e}}\right)}{\left(e\sqrt{-4ac+b^2}-be+2cd\right)\left(e\sqrt{-4ac+b^2}+be-2cd\right)\sqrt{\frac{a e^2-bde+c d^2}{e^2}}}-\frac{\left(-b+\sqrt{\cdots}\right.}{\cdots}$
default	$\frac{2c x^2+b}{e\sqrt{c x^4+b x^2+a}\,(4ac-b^2)}-\frac{d\left(-\frac{2c\sqrt{c\left(x^2-\frac{-b+\sqrt{-4ac+b^2}}{2c}\right)^2+\sqrt{-4ac+b^2}\left(x^2-\frac{-b+\sqrt{-4ac+b^2}}{2c}\right)}}{\left(e\sqrt{-4ac+b^2}-be+2cd\right)\left(-4ac+b^2\right)\left(x^2-\frac{-b+\sqrt{-4ac+b^2}}{2c}\right)}+\frac{2c\sqrt{c\left(x^2+\frac{b+\sqrt{-4ac+b^2}}{2c}\right)^2 \cdots}}{\left(e\sqrt{-4ac+b^2}+be-2cd\right)\left(\cdots\right.}\right.}{\cdots}$

[In] int(x^3/(e*x^2+d)/(c*x^4+b*x^2+a)^(3/2),x,method=_RETURNVERBOSE)

[Out] 1/4/(c*x^4+b*x^2+a)^(1/2)*(1/2*d*ln((2*(c*x^4+b*x^2+a)^(1/2)*((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*e+(b*x^2+2*a)*e-d*(2*c*x^2+b))/(e*x^2+d))*(4*a*c-b^2)*(c*x^4+b*x^2+a)^(1/2)+(a*(2*c*x^2+b)*e-2*(1/2*b*x^2+a)*c*d)*((a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)/(a*e^2-b*d*e+c*d^2)/(a*c-1/4*b^2)

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 653 vs. 2(147) = 294.

Time = 0.58 (sec) , antiderivative size = 1349, normalized size of antiderivative = 8.48

$$\int \frac{x^3}{(d+ex^2)\left(a+bx^2+cx^4\right)^{3/2}}\,dx = \text{Too large to display}$$

[In] integrate(x^3/(e*x^2+d)/(c*x^4+b*x^2+a)^(3/2),x, algorithm="fricas")

[Out] [1/4*(((b^2*c - 4*a*c^2)*d*e*x^4 + (b^3 - 4*a*b*c)*d*e*x^2 + (a*b^2 - 4*a^2*c)*d*e)*sqrt(c*d^2 - b*d*e + a*e^2)*log(-((8*c^2*d^2 - 8*b*c*d*e + (b^2 + 4*a*c)*e^2)*x^4 - 8*a*b*d*e + 8*a^2*e^2 + (b^2 + 4*a*c)*d^2 + 2*(4*b*c*d^2 + 4*a*b*e^2 - (3*b^2 + 4*a*c)*d*e)*x^2 - 4*sqrt(c*x^4 + b*x^2 + a)*sqrt(c*d^2 - b*d*e + a*e^2)*((2*c*d - b*e)*x^2 + b*d - 2*a*e))/(e^2*x^4 + 2*d*e*x^2 + d^2)) + 4*(2*a*c^2*d^3 - 3*a*b*c*d^2*e - a^2*b*e^3 + (a*b^2 + 2*a^2*c)*d

```
*e^2 + (b*c^2*d^3 + 3*a*b*c*d*e^2 - 2*a^2*c*e^3 - (b^2*c + 2*a*c^2)*d^2*e)*
x^2)*sqrt(c*x^4 + b*x^2 + a))/((a*b^2*c^2 - 4*a^2*c^3)*d^4 - 2*(a*b^3*c - 4
*a^2*b*c^2)*d^3*e + (a*b^4 - 2*a^2*b^2*c - 8*a^3*c^2)*d^2*e^2 - 2*(a^2*b^3
- 4*a^3*b*c)*d*e^3 + (a^3*b^2 - 4*a^4*c)*e^4 + ((b^2*c^3 - 4*a*c^4)*d^4 - 2
*(b^3*c^2 - 4*a*b*c^3)*d^3*e + (b^4*c - 2*a*b^2*c^2 - 8*a^2*c^3)*d^2*e^2 -
2*(a*b^3*c - 4*a^2*b*c^2)*d*e^3 + (a^2*b^2*c - 4*a^3*c^2)*e^4)*x^4 + ((b^3*
c^2 - 4*a*b*c^3)*d^4 - 2*(b^4*c - 4*a*b^2*c^2)*d^3*e + (b^5 - 2*a*b^3*c - 8
*a^2*b*c^2)*d^2*e^2 - 2*(a*b^4 - 4*a^2*b^2*c)*d*e^3 + (a^2*b^3 - 4*a^3*b*c)
*e^4)*x^2), -1/2*(((b^2*c - 4*a*c^2)*d*e*x^4 + (b^3 - 4*a*b*c)*d*e*x^2 + (a
*b^2 - 4*a^2*c)*d*e)*sqrt(-c*d^2 + b*d*e - a*e^2)*arctan(-1/2*sqrt(c*x^4 +
b*x^2 + a)*sqrt(-c*d^2 + b*d*e - a*e^2)*((2*c*d - b*e)*x^2 + b*d - 2*a*e)/(
(c^2*d^2 - b*c*d*e + a*c*e^2)*x^4 + a*c*d^2 - a*b*d*e + a^2*e^2 + (b*c*d^2
- b^2*d*e + a*b*e^2)*x^2)) - 2*(2*a*c^2*d^3 - 3*a*b*c*d^2*e - a^2*b*e^3 + (
a*b^2 + 2*a^2*c)*d*e^2 + (b*c^2*d^3 + 3*a*b*c*d*e^2 - 2*a^2*c*e^3 - (b^2*c
+ 2*a*c^2)*d^2*e)*x^2)*sqrt(c*x^4 + b*x^2 + a))/((a*b^2*c^2 - 4*a^2*c^3)*d^
4 - 2*(a*b^3*c - 4*a^2*b*c^2)*d^3*e + (a*b^4 - 2*a^2*b^2*c - 8*a^3*c^2)*d^2
*e^2 - 2*(a^2*b^3 - 4*a^3*b*c)*d*e^3 + (a^3*b^2 - 4*a^4*c)*e^4 + ((b^2*c^3
- 4*a*c^4)*d^4 - 2*(b^3*c^2 - 4*a*b*c^3)*d^3*e + (b^4*c - 2*a*b^2*c^2 - 8*a
^2*c^3)*d^2*e^2 - 2*(a*b^3*c - 4*a^2*b*c^2)*d*e^3 + (a^2*b^2*c - 4*a^3*c^2)
*e^4)*x^4 + ((b^3*c^2 - 4*a*b*c^3)*d^4 - 2*(b^4*c - 4*a*b^2*c^2)*d^3*e + (b
^5 - 2*a*b^3*c - 8*a^2*b*c^2)*d^2*e^2 - 2*(a*b^4 - 4*a^2*b^2*c)*d*e^3 + (a^
2*b^3 - 4*a^3*b*c)*e^4)*x^2)]
```

Sympy [F]

$$\int \frac{x^3}{(d+ex^2)\,(a+bx^2+cx^4)^{3/2}}\,dx = \int \frac{x^3}{(d+ex^2)\,(a+bx^2+cx^4)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(x**3/(e*x**2+d)/(c*x**4+b*x**2+a)**(3/2),x)
[Out] Integral(x**3/((d + e*x**2)*(a + b*x**2 + c*x**4)**(3/2)), x)
```

Maxima [F]

$$\int \frac{x^3}{(d+ex^2)\,(a+bx^2+cx^4)^{3/2}}\,dx = \int \frac{x^3}{(cx^4+bx^2+a)^{\frac{3}{2}}(ex^2+d)}\,dx$$

```
[In] integrate(x^3/(e*x^2+d)/(c*x^4+b*x^2+a)^(3/2),x, algorithm="maxima")
[Out] integrate(x^3/((c*x^4 + b*x^2 + a)^(3/2)*(e*x^2 + d)), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 451 vs. $2(147) = 294$.

Time = 0.29 (sec) , antiderivative size = 451, normalized size of antiderivative = 2.84

$$\int \frac{x^3}{(d+ex^2)(a+bx^2+cx^4)^{3/2}}\,dx = -\frac{de\arctan\left(-\frac{\left(\sqrt{c}x^2-\sqrt{cx^4+bx^2+a}\right)e+\sqrt{c}d}{\sqrt{-cd^2+bde-ae^2}}\right)}{(cd^2-bde+ae^2)\sqrt{-cd^2+bde-ae^2}}$$
$$+\frac{\frac{(bc^2d^3-b^2cd^2e-2\,ac^2d^2e+3\,abcde^2-2\,a^2ce^3)x^2}{b^2c^2d^4-4\,ac^3d^4-2\,b^3cd^3e+8\,abc^2d^3e+b^4d^2e^2-2\,ab^2cd^2e^2-8\,a^2c^2d^2e^2-2\,ab^3de^3+8\,a^2bcde^3+a^2b^2e^4-4\,a^3ce^4}+\frac{}{b^2c^2d^4-4\,ac^3d^4-2\,b^3cd^3e+8\,abc}}{\sqrt{cx^4+bx^2+a}}$$

```
[In] integrate(x^3/(e*x^2+d)/(c*x^4+b*x^2+a)^(3/2),x, algorithm="giac")
```

```
[Out] -d*e*arctan(-((sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))*e + sqrt(c)*d)/sqrt(-
c*d^2 + b*d*e - a*e^2))/((c*d^2 - b*d*e + a*e^2)*sqrt(-c*d^2 + b*d*e - a*e^
2)) + ((b*c^2*d^3 - b^2*c*d^2*e - 2*a*c^2*d^2*e + 3*a*b*c*d*e^2 - 2*a^2*c*e
^3)*x^2/(b^2*c^2*d^4 - 4*a*c^3*d^4 - 2*b^3*c*d^3*e + 8*a*b*c^2*d^3*e + b^4*
d^2*e^2 - 2*a*b^2*c*d^2*e^2 - 8*a^2*c^2*d^2*e^2 - 2*a*b^3*d*e^3 + 8*a^2*b*c
*d*e^3 + a^2*b^2*e^4 - 4*a^3*c*e^4) + (2*a*c^2*d^3 - 3*a*b*c*d^2*e + a*b^2*
d*e^2 + 2*a^2*c*d*e^2 - a^2*b*e^3)/(b^2*c^2*d^4 - 4*a*c^3*d^4 - 2*b^3*c*d^3
*e + 8*a*b*c^2*d^3*e + b^4*d^2*e^2 - 2*a*b^2*c*d^2*e^2 - 8*a^2*c^2*d^2*e^2
- 2*a*b^3*d*e^3 + 8*a^2*b*c*d*e^3 + a^2*b^2*e^4 - 4*a^3*c*e^4))/sqrt(c*x^4
+ b*x^2 + a)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3}{(d+ex^2)(a+bx^2+cx^4)^{3/2}}\,dx = \int \frac{x^3}{(e\,x^2+d)\,\left(c\,x^4+b\,x^2+a\right)^{3/2}}\,dx$$

```
[In] int(x^3/((d + e*x^2)*(a + b*x^2 + c*x^4)^(3/2)),x)
```

```
[Out] int(x^3/((d + e*x^2)*(a + b*x^2 + c*x^4)^(3/2)), x)
```

3.345 $\int \frac{x}{(d+ex^2)(a+bx^2+cx^4)^{3/2}} \, dx$

Optimal result

Integrand size = 27, antiderivative size = 166

$$\int \frac{x}{(d+ex^2)(a+bx^2+cx^4)^{3/2}} \, dx = -\frac{bcd-b^2e+2ace+c(2cd-be)x^2}{(b^2-4ac)(cd^2-bde+ae^2)\sqrt{a+bx^2+cx^4}} + \frac{e^2\text{arctanh}\left(\frac{bd-2ae+(2cd-be)x^2}{2\sqrt{cd^2-bde+ae^2}\sqrt{a+bx^2+cx^4}}\right)}{2(cd^2-bde+ae^2)^{3/2}}$$

```
[Out] 1/2*e^2*arctanh(1/2*(b*d-2*a*e+(-b*e+2*c*d)*x^2)/(a*e^2-b*d*e+c*d^2)^(1/2)/
(c*x^4+b*x^2+a)^(1/2))/(a*e^2-b*d*e+c*d^2)^(3/2)+(-b*c*d+b^2*e-2*a*c*e-c*(-
b*e+2*c*d)*x^2)/(-4*a*c+b^2)/(a*e^2-b*d*e+c*d^2)/(c*x^4+b*x^2+a)^(1/2)
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 166, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$, Rules used = {1261, 754, 12, 738, 212}

$$\int \frac{x}{(d+ex^2)(a+bx^2+cx^4)^{3/2}} \, dx = \frac{e^2\text{arctanh}\left(\frac{-2ae+x^2(2cd-be)+bd}{2\sqrt{a+bx^2+cx^4}\sqrt{ae^2-bde+cd^2}}\right)}{2(ae^2-bde+cd^2)^{3/2}} - \frac{2ace+b^2(-e)+cx^2(2cd-be)+bcd}{(b^2-4ac)\sqrt{a+bx^2+cx^4}(ae^2-bde+cd^2)}$$

```
[In] Int[x/((d + e*x^2)*(a + b*x^2 + c*x^4)^(3/2)),x]
```

```
[Out] -((b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e)*x^2)/((b^2 - 4*a*c)*(c*d^2 - b
*d*e + a*e^2)*Sqrt[a + b*x^2 + c*x^4])) + (e^2*ArcTanh[(b*d - 2*a*e + (2*c*
```

```
d - b*e)*x^2)/(2*Sqrt[c*d^2 - b*d*e + a*e^2]*Sqrt[a + b*x^2 + c*x^4])])/(2*
(c*d^2 - b*d*e + a*e^2)^(3/2))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 754

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*(b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e)
*x)*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^
2))), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d +
 e*x)^m*Simp[b*c*d*e*(2*p - m + 2) + b^2*e^2*(m + p + 2) - 2*c^2*d^2*(2*p +
 3) - 2*a*c*e^2*(m + 2*p + 3) - c*e*(2*c*d - b*e)*(m + 2*p + 4)*x, x]*(a +
b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4
*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[p,
-1] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 1261

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(
p_.), x_Symbol] :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x],
 x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]
```

Rubi steps

$$\text{integral} = \frac{1}{2}\text{Subst}\left(\int \frac{1}{(d+ex)\left(a+bx+cx^2\right)^{3/2}}\,dx, x, x^2\right)$$

$$= -\frac{bcd-b^2e+2ace+c(2cd-be)x^2}{\left(b^2-4ac\right)\left(cd^2-bde+ae^2\right)\sqrt{a+bx^2+cx^4}} - \frac{\text{Subst}\left(\int -\frac{\left(b^2-4ac\right)e^2}{2(d+ex)\sqrt{a+bx+cx^2}}\,dx, x, x^2\right)}{\left(b^2-4ac\right)\left(cd^2-bde+ae^2\right)}$$

$$= -\frac{bcd-b^2e+2ace+c(2cd-be)x^2}{(b^2-4ac)(cd^2-bde+ae^2)\sqrt{a+bx^2+cx^4}}+\frac{e^2\text{Subst}\left(\int \frac{1}{(d+ex)\sqrt{a+bx+cx^2}}\,dx,x,x^2\right)}{2(cd^2-bde+ae^2)}$$

$$= -\frac{bcd-b^2e+2ace+c(2cd-be)x^2}{(b^2-4ac)(cd^2-bde+ae^2)\sqrt{a+bx^2+cx^4}} - \frac{e^2\text{Subst}\left(\int \frac{1}{4cd^2-4bde+4ae^2-x^2}\,dx,x,\frac{-bd+2ae-(2cd-be)x^2}{\sqrt{a+bx^2+cx^4}}\right)}{cd^2-bde+ae^2}$$

$$= -\frac{bcd-b^2e+2ace+c(2cd-be)x^2}{(b^2-4ac)(cd^2-bde+ae^2)\sqrt{a+bx^2+cx^4}}+\frac{e^2\tanh^{-1}\left(\frac{bd-2ae+(2cd-be)x^2}{2\sqrt{cd^2-bde+ae^2}\sqrt{a+bx^2+cx^4}}\right)}{2(cd^2-bde+ae^2)^{3/2}}$$

Mathematica [A] (verified)

Time = 0.90 (sec) , antiderivative size = 182, normalized size of antiderivative = 1.10

$$\int \frac{x}{(d+ex^2)(a+bx^2+cx^4)^{3/2}}\,dx = \frac{-b^2e+2c(ae+cdx^2)+bc(d-ex^2)}{(b^2-4ac)(-cd^2+e(bd-ae))\sqrt{a+bx^2+cx^4}} + \frac{e^2\sqrt{-cd^2+e(bd-ae)}\arctan\left(\frac{\sqrt{-cd^2+e(bd-ae)}x^2}{\sqrt{a}(d+ex^2)-d\sqrt{a+bx^2+cx^4}}\right)}{(cd^2+e(-bd+ae))^2}$$

```
[In] Integrate[x/((d + e*x^2)*(a + b*x^2 + c*x^4)^(3/2)),x]
```

```
[Out] (-(b^2*e) + 2*c*(a*e + c*d*x^2) + b*c*(d - e*x^2))/((b^2 - 4*a*c)*(-(c*d^2)
 + e*(b*d - a*e))*Sqrt[a + b*x^2 + c*x^4]) + (e^2*Sqrt[-(c*d^2) + e*(b*d -
a*e)]*ArcTan[(Sqrt[-(c*d^2) + e*(b*d - a*e)]*x^2)/(Sqrt[a]*(d + e*x^2) - d*
Sqrt[a + b*x^2 + c*x^4])])/(c*d^2 + e*(-(b*d) + a*e))^2
```

Maple [A] (verified)

Time = 0.40 (sec) , antiderivative size = 223, normalized size of antiderivative = 1.34

method	result
pseudoelliptic	$\frac{-e\sqrt{c\,x^4+b\,x^2+a}\,\left(ac-\frac{b^2}{4}\right)\ln\left(\frac{2\sqrt{c\,x^4+b\,x^2+a}\,\sqrt{\frac{a\,e^2-bde+c\,d^2}{e^2}}\,e+\left(b\,x^2+2a\right)e-d\left(2c\,x^2+b\right)}{e\,x^2+d}\right)+\left(c^2d\,x^2+\left(\left(-\frac{b\,x^2}{2}+a\right)e+\frac{bd}{2}\right)c-\right.}{2\sqrt{c\,x^4+b\,x^2+a}\,\sqrt{\frac{a\,e^2-bde+c\,d^2}{e^2}}\,(a\,e^2-bde+c\,d^2)\left(ac-\frac{b^2}{4}\right)}$
default	$-\frac{2c\sqrt{c\left(x^2-\frac{-b+\sqrt{-4ac+b^2}}{2c}\right)^2+\sqrt{-4ac+b^2}\,\left(x^2-\frac{-b+\sqrt{-4ac+b^2}}{2c}\right)}}{\left(e\sqrt{-4ac+b^2}-be+2cd\right)(-4ac+b^2)\left(x^2-\frac{-b+\sqrt{-4ac+b^2}}{2c}\right)}+\frac{2c\sqrt{c\left(x^2+\frac{b+\sqrt{-4ac+b^2}}{2c}\right)^2-\sqrt{-4ac+b^2}\,\left(x^2+\frac{b+\sqrt{}}{}\right.}}{\left(e\sqrt{-4ac+b^2}+be-2cd\right)(-4ac+b^2)\left(x^2+\frac{b+\sqrt{-4a}}{2c}\right.}$
elliptic	$-\frac{2c\sqrt{c\left(x^2-\frac{-b+\sqrt{-4ac+b^2}}{2c}\right)^2+\sqrt{-4ac+b^2}\,\left(x^2-\frac{-b+\sqrt{-4ac+b^2}}{2c}\right)}}{\left(e\sqrt{-4ac+b^2}-be+2cd\right)(-4ac+b^2)\left(x^2-\frac{-b+\sqrt{-4ac+b^2}}{2c}\right)}+\frac{2c\sqrt{c\left(x^2+\frac{b+\sqrt{-4ac+b^2}}{2c}\right)^2-\sqrt{-4ac+b^2}\,\left(x^2+\frac{b+\sqrt{}}{}\right.}}{\left(e\sqrt{-4ac+b^2}+be-2cd\right)(-4ac+b^2)\left(x^2+\frac{b+\sqrt{-4a}}{2c}\right.}$

```
[In] int(x/(e*x^2+d)/(c*x^4+b*x^2+a)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*(-e*(c*x^4+b*x^2+a)^(1/2)*(a*c-1/4*b^2)*ln((2*(c*x^4+b*x^2+a)^(1/2)*((a
*e^2-b*d*e+c*d^2)/e^2)^(1/2)*e+(b*x^2+2*a)*e-d*(2*c*x^2+b))/(e*x^2+d))+(c^2
*d*x^2+((-1/2*b*x^2+a)*e+1/2*b*d)*c-1/2*b^2*e)*((a*e^2-b*d*e+c*d^2)/e^2)^(1
/2))/(c*x^4+b*x^2+a)^(1/2)/((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)/(a*e^2-b*d*e+c*d
^2)/(a*c-1/4*b^2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 668 vs. 2(154) = 308.

Time = 0.56 (sec) , antiderivative size = 1379, normalized size of antiderivative = 8.31

$$\int \frac{x}{(d+ex^2)\left(a+bx^2+cx^4\right)^{3/2}}\,dx = \text{Too large to display}$$

```
[In] integrate(x/(e*x^2+d)/(c*x^4+b*x^2+a)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/4*(((b^2*c - 4*a*c^2)*e^2*x^4 + (b^3 - 4*a*b*c)*e^2*x^2 + (a*b^2 - 4*a^2
*c)*e^2)*sqrt(c*d^2 - b*d*e + a*e^2)*log(-((8*c^2*d^2 - 8*b*c*d*e + (b^2 +
4*a*c)*e^2)*x^4 - 8*a*b*d*e + 8*a^2*e^2 + (b^2 + 4*a*c)*d^2 + 2*(4*b*c*d^2
+ 4*a*b*e^2 - (3*b^2 + 4*a*c)*d*e)*x^2 + 4*sqrt(c*x^4 + b*x^2 + a)*sqrt(c*d
^2 - b*d*e + a*e^2)*((2*c*d - b*e)*x^2 + b*d - 2*a*e))/(e^2*x^4 + 2*d*e*x^2
 + d^2)) - 4*(b*c^2*d^3 - 2*(b^2*c - a*c^2)*d^2*e + (b^3 - a*b*c)*d*e^2 - (
a*b^2 - 2*a^2*c)*e^3 + (2*c^3*d^3 - 3*b*c^2*d^2*e - a*b*c*e^3 + (b^2*c + 2*
a*c^2)*d*e^2)*x^2)*sqrt(c*x^4 + b*x^2 + a))/((a*b^2*c^2 - 4*a^2*c^3)*d^4 -
2*(a*b^3*c - 4*a^2*b*c^2)*d^3*e + (a*b^4 - 2*a^2*b^2*c - 8*a^3*c^2)*d^2*e^2
```

```
 - 2*(a^2*b^3 - 4*a^3*b*c)*d*e^3 + (a^3*b^2 - 4*a^4*c)*e^4 + ((b^2*c^3 - 4*
a*c^4)*d^4 - 2*(b^3*c^2 - 4*a*b*c^3)*d^3*e + (b^4*c - 2*a*b^2*c^2 - 8*a^2*c
^3)*d^2*e^2 - 2*(a*b^3*c - 4*a^2*b*c^2)*d*e^3 + (a^2*b^2*c - 4*a^3*c^2)*e^4
)*x^4 + ((b^3*c^2 - 4*a*b*c^3)*d^4 - 2*(b^4*c - 4*a*b^2*c^2)*d^3*e + (b^5 -
 2*a*b^3*c - 8*a^2*b*c^2)*d^2*e^2 - 2*(a*b^4 - 4*a^2*b^2*c)*d*e^3 + (a^2*b^
3 - 4*a^3*b*c)*e^4)*x^2), 1/2*(((b^2*c - 4*a*c^2)*e^2*x^4 + (b^3 - 4*a*b*c)
*e^2*x^2 + (a*b^2 - 4*a^2*c)*e^2)*sqrt(-c*d^2 + b*d*e - a*e^2)*arctan(-1/2*
sqrt(c*x^4 + b*x^2 + a)*sqrt(-c*d^2 + b*d*e - a*e^2)*((2*c*d - b*e)*x^2 + b
*d - 2*a*e)/((c^2*d^2 - b*c*d*e + a*c*e^2)*x^4 + a*c*d^2 - a*b*d*e + a^2*e^
2 + (b*c*d^2 - b^2*d*e + a*b*e^2)*x^2)) - 2*(b*c^2*d^3 - 2*(b^2*c - a*c^2)*
d^2*e + (b^3 - a*b*c)*d*e^2 - (a*b^2 - 2*a^2*c)*e^3 + (2*c^3*d^3 - 3*b*c^2*
d^2*e - a*b*c*e^3 + (b^2*c + 2*a*c^2)*d*e^2)*x^2)*sqrt(c*x^4 + b*x^2 + a))/
((a*b^2*c^2 - 4*a^2*c^3)*d^4 - 2*(a*b^3*c - 4*a^2*b*c^2)*d^3*e + (a*b^4 - 2
*a^2*b^2*c - 8*a^3*c^2)*d^2*e^2 - 2*(a^2*b^3 - 4*a^3*b*c)*d*e^3 + (a^3*b^2
- 4*a^4*c)*e^4 + ((b^2*c^3 - 4*a*c^4)*d^4 - 2*(b^3*c^2 - 4*a*b*c^3)*d^3*e +
 (b^4*c - 2*a*b^2*c^2 - 8*a^2*c^3)*d^2*e^2 - 2*(a*b^3*c - 4*a^2*b*c^2)*d*e^
3 + (a^2*b^2*c - 4*a^3*c^2)*e^4)*x^4 + ((b^3*c^2 - 4*a*b*c^3)*d^4 - 2*(b^4*
c - 4*a*b^2*c^2)*d^3*e + (b^5 - 2*a*b^3*c - 8*a^2*b*c^2)*d^2*e^2 - 2*(a*b^4
 - 4*a^2*b^2*c)*d*e^3 + (a^2*b^3 - 4*a^3*b*c)*e^4)*x^2)]
```

Sympy [F]

$$\int \frac{x}{(d+ex^2)(a+bx^2+cx^4)^{3/2}}\,dx = \int \frac{x}{(d+ex^2)(a+bx^2+cx^4)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(x/(e*x**2+d)/(c*x**4+b*x**2+a)**(3/2),x)
[Out] Integral(x/((d + e*x**2)*(a + b*x**2 + c*x**4)**(3/2)), x)
```

Maxima [F]

$$\int \frac{x}{(d+ex^2)(a+bx^2+cx^4)^{3/2}}\,dx = \int \frac{x}{(cx^4+bx^2+a)^{\frac{3}{2}}(ex^2+d)}\,dx$$

```
[In] integrate(x/(e*x^2+d)/(c*x^4+b*x^2+a)^(3/2),x, algorithm="maxima")
[Out] integrate(x/((c*x^4 + b*x^2 + a)^(3/2)*(e*x^2 + d)), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 468 vs. $2(154) = 308$.

Time = 0.28 (sec) , antiderivative size = 468, normalized size of antiderivative = 2.82

$$\int \frac{x}{(d+ex^2)\left(a+bx^2+cx^4\right)^{3/2}}\,dx = \frac{e^2\arctan\left(-\frac{\left(\sqrt{c}x^2-\sqrt{cx^4+bx^2+a}\right)e+\sqrt{c}d}{\sqrt{-cd^2+bde-ae^2}}\right)}{\left(cd^2-bde+ae^2\right)\sqrt{-cd^2+bde-ae^2}}$$
$$-\frac{\frac{\left(2\,c^3d^3-3\,bc^2d^2e+b^2cde^2+2\,ac^2de^2-abce^3\right)x^2}{b^2c^2d^4-4\,ac^3d^4-2\,b^3cd^3e+8\,abc^2d^3e+b^4d^2e^2-2\,ab^2cd^2e^2-8\,a^2c^2d^2e^2-2\,ab^3de^3+8\,a^2bcde^3+a^2b^2e^4-4\,a^3ce^4}+\frac{bc^2d^3\cdots}{b^2c^2d^4-4\,ac^3d^4-2\,b^3cd^3e+8\,abc\cdots}}{\sqrt{cx^4+bx^2+a}}$$

```
[In] integrate(x/(e*x^2+d)/(c*x^4+b*x^2+a)^(3/2),x, algorithm="giac")
```

```
[Out] e^2*arctan(-((sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))*e + sqrt(c)*d)/sqrt(-c
*d^2 + b*d*e - a*e^2))/((c*d^2 - b*d*e + a*e^2)*sqrt(-c*d^2 + b*d*e - a*e^2
)) - ((2*c^3*d^3 - 3*b*c^2*d^2*e + b^2*c*d*e^2 + 2*a*c^2*d*e^2 - a*b*c*e^3)
*x^2/(b^2*c^2*d^4 - 4*a*c^3*d^4 - 2*b^3*c*d^3*e + 8*a*b*c^2*d^3*e + b^4*d^2
*e^2 - 2*a*b^2*c*d^2*e^2 - 8*a^2*c^2*d^2*e^2 - 2*a*b^3*d*e^3 + 8*a^2*b*c*d*
e^3 + a^2*b^2*e^4 - 4*a^3*c*e^4) + (b*c^2*d^3 - 2*b^2*c*d^2*e + 2*a*c^2*d^2
*e + b^3*d*e^2 - a*b*c*d*e^2 - a*b^2*e^3 + 2*a^2*c*e^3)/(b^2*c^2*d^4 - 4*a*
c^3*d^4 - 2*b^3*c*d^3*e + 8*a*b*c^2*d^3*e + b^4*d^2*e^2 - 2*a*b^2*c*d^2*e^2
 - 8*a^2*c^2*d^2*e^2 - 2*a*b^3*d*e^3 + 8*a^2*b*c*d*e^3 + a^2*b^2*e^4 - 4*a^
3*c*e^4))/sqrt(c*x^4 + b*x^2 + a)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x}{(d+ex^2)\left(a+bx^2+cx^4\right)^{3/2}}\,dx = \int \frac{x}{\left(e\,x^2+d\right)\,{\left(c\,x^4+b\,x^2+a\right)}^{3/2}}\,dx$$

```
[In] int(x/((d + e*x^2)*(a + b*x^2 + c*x^4)^(3/2)),x)
```

```
[Out] int(x/((d + e*x^2)*(a + b*x^2 + c*x^4)^(3/2)), x)
```

3.346 $\int \frac{1}{x(d+ex^2)(a+bx^2+cx^4)^{3/2}}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 266

$$\int \frac{1}{x(d+ex^2)(a+bx^2+cx^4)^{3/2}}\,dx = \frac{b^2-2ac+bcx^2}{a(b^2-4ac)\,d\sqrt{a+bx^2+cx^4}} + \frac{e(bcd-b^2e+2ace+c(2cd-be)x^2)}{(b^2-4ac)\,d\,(cd^2-bde+ae^2)\sqrt{a+bx^2+cx^4}} - \frac{\operatorname{arctanh}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{2a^{3/2}d} - \frac{e^3\operatorname{arctanh}\left(\frac{bd-2ae+(2cd-be)x^2}{2\sqrt{cd^2-bde+ae^2}\sqrt{a+bx^2+cx^4}}\right)}{2d\,(cd^2-bde+ae^2)^{3/2}}$$

```
[Out] -1/2*arctanh(1/2*(b*x^2+2*a)/a^(1/2)/(c*x^4+b*x^2+a)^(1/2))/a^(3/2)/d-1/2*e
^3*arctanh(1/2*(b*d-2*a*e+(-b*e+2*c*d)*x^2)/(a*e^2-b*d*e+c*d^2)^(1/2)/(c*x^
4+b*x^2+a)^(1/2))/d/(a*e^2-b*d*e+c*d^2)^(3/2)+(b*c*x^2-2*a*c+b^2)/a/(-4*a*c
+b^2)/d/(c*x^4+b*x^2+a)^(1/2)+e*(b*c*d-b^2*e+2*a*c*e+c*(-b*e+2*c*d)*x^2)/(-
4*a*c+b^2)/d/(a*e^2-b*d*e+c*d^2)/(c*x^4+b*x^2+a)^(1/2)
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 266, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.207, Rules used

= {1265, 974, 754, 12, 738, 212}

$$\int \frac{1}{x\left(d+ex^2\right)\left(a+bx^2+cx^4\right)^{3/2}}\,dx = -\frac{\operatorname{arctanh}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{2a^{3/2}d} - \frac{e^3\operatorname{arctanh}\left(\frac{-2ae+x^2(2cd-be)+bd}{2\sqrt{a+bx^2+cx^4}\sqrt{ae^2-bde+cd^2}}\right)}{2d\left(ae^2-bde+cd^2\right)^{3/2}} + \frac{e(2ace+b^2(-e)+cx^2(2cd-be)+bcd)}{d\left(b^2-4ac\right)\sqrt{a+bx^2+cx^4}\left(ae^2-bde+cd^2\right)} + \frac{-2ac+b^2+bcx^2}{ad\left(b^2-4ac\right)\sqrt{a+bx^2+cx^4}}$$

```
[In] Int[1/(x*(d + e*x^2)*(a + b*x^2 + c*x^4)^(3/2)),x]
```

```
[Out] (b^2 - 2*a*c + b*c*x^2)/(a*(b^2 - 4*a*c)*d*Sqrt[a + b*x^2 + c*x^4]) + (e*(b
*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e)*x^2))/((b^2 - 4*a*c)*d*(c*d^2 - b*
d*e + a*e^2)*Sqrt[a + b*x^2 + c*x^4]) - ArcTanh[(2*a + b*x^2)/(2*Sqrt[a]*Sq
rt[a + b*x^2 + c*x^4])]/(2*a^(3/2)*d) - (e^3*ArcTanh[(b*d - 2*a*e + (2*c*d
- b*e)*x^2)/(2*Sqrt[c*d^2 - b*d*e + a*e^2]*Sqrt[a + b*x^2 + c*x^4])])/(2*d*
(c*d^2 - b*d*e + a*e^2)^(3/2))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 754

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*(b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e)
*x)*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^
2))), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d +
 e*x)^m*Simp[b*c*d*e*(2*p - m + 2) + b^2*e^2*(m + p + 2) - 2*c^2*d^2*(2*p +
 3) - 2*a*c*e^2*(m + 2*p + 3) - c*e*(2*c*d - b*e)*(m + 2*p + 4)*x, x]*(a +
b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4
```

```
*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[p,
-1] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 974

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g
*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ
[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && (
IntegerQ[p] || (ILtQ[m, 0] && ILtQ[n, 0])) &&  !(IGtQ[m, 0] || IGtQ[n, 0])
```

Rule 1265

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{2}\text{Subst}\left(\int \frac{1}{x(d+ex)\left(a+bx+cx^2\right)^{3/2}}\,dx,x,x^2\right)\\
&= \frac{1}{2}\text{Subst}\left(\int \left(\frac{1}{dx\left(a+bx+cx^2\right)^{3/2}}-\frac{e}{d(d+ex)\left(a+bx+cx^2\right)^{3/2}}\right)\,dx,x,x^2\right)\\
&= \frac{\text{Subst}\left(\int \frac{1}{x(a+bx+cx^2)^{3/2}}\,dx,x,x^2\right)}{2d}-\frac{e\text{Subst}\left(\int \frac{1}{(d+ex)(a+bx+cx^2)^{3/2}}\,dx,x,x^2\right)}{2d}\\
&= \frac{b^2-2ac+bcx^2}{a\left(b^2-4ac\right)d\sqrt{a+bx^2+cx^4}}+\frac{e(bcd-b^2e+2ace+c(2cd-be)x^2)}{\left(b^2-4ac\right)d\left(cd^2-bde+ae^2\right)\sqrt{a+bx^2+cx^4}}\\
&\quad -\frac{\text{Subst}\left(\int \frac{-\frac{b^2}{2}+2ac}{x\sqrt{a+bx+cx^2}}\,dx,x,x^2\right)}{a\left(b^2-4ac\right)d}+\frac{e\text{Subst}\left(\int -\frac{(b^2-4ac)e^2}{2(d+ex)\sqrt{a+bx+cx^2}}\,dx,x,x^2\right)}{\left(b^2-4ac\right)d\left(cd^2-bde+ae^2\right)}\\
&= \frac{b^2-2ac+bcx^2}{a\left(b^2-4ac\right)d\sqrt{a+bx^2+cx^4}}+\frac{e(bcd-b^2e+2ace+c(2cd-be)x^2)}{\left(b^2-4ac\right)d\left(cd^2-bde+ae^2\right)\sqrt{a+bx^2+cx^4}}\\
&\quad +\frac{\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx+cx^2}}\,dx,x,x^2\right)}{2ad}-\frac{e^3\text{Subst}\left(\int \frac{1}{(d+ex)\sqrt{a+bx+cx^2}}\,dx,x,x^2\right)}{2d\left(cd^2-bde+ae^2\right)}\\
&= \frac{b^2-2ac+bcx^2}{a\left(b^2-4ac\right)d\sqrt{a+bx^2+cx^4}}+\frac{e(bcd-b^2e+2ace+c(2cd-be)x^2)}{\left(b^2-4ac\right)d\left(cd^2-bde+ae^2\right)\sqrt{a+bx^2+cx^4}}\\
&\quad -\frac{\text{Subst}\left(\int \frac{1}{4a-x^2}\,dx,x,\frac{2a+bx^2}{\sqrt{a+bx^2+cx^4}}\right)}{ad}+\frac{e^3\text{Subst}\left(\int \frac{1}{4cd^2-4bde+4ae^2-x^2}\,dx,x,\frac{-bd+2ae-(2cd-be)x^2}{\sqrt{a+bx^2+cx^4}}\right)}{d\left(cd^2-bde+ae^2\right)}
\end{aligned}$$

$$= \frac{b^2 - 2ac + bcx^2}{a(b^2 - 4ac)\, d\sqrt{a + bx^2 + cx^4}} + \frac{e(bcd - b^2e + 2ace + c(2cd - be)x^2)}{(b^2 - 4ac)\, d\,(cd^2 - bde + ae^2)\sqrt{a + bx^2 + cx^4}} - \frac{\tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{2a^{3/2}d} - \frac{e^3\tanh^{-1}\left(\frac{bd-2ae+(2cd-be)x^2}{2\sqrt{cd^2-bde+ae^2}\sqrt{a+bx^2+cx^4}}\right)}{2d\,(cd^2 - bde + ae^2)^{3/2}}$$

Mathematica [A] (verified)

Time = 1.14 (sec) , antiderivative size = 247, normalized size of antiderivative = 0.93

$$\int \frac{1}{x\,(d + ex^2)\,(a + bx^2 + cx^4)^{3/2}}\,dx = \frac{b^3e - bc(3ae + cdx^2) + 2ac^2(d - ex^2) + b^2c(-d + ex^2)}{a\,(-b^2 + 4ac)\,(cd^2 + e(-bd + ae))\sqrt{a + bx^2 + cx^4}} - \frac{e^3\sqrt{-cd^2 + e(bd - ae)}\arctan\left(\frac{\sqrt{c}(d+ex^2)-e\sqrt{a+bx^2+cx^4}}{\sqrt{-cd^2+e(bd-ae)}}\right)}{d\,(cd^2 + e(-bd + ae))^2} + \frac{\operatorname{arctanh}\left(\frac{\sqrt{c}x^2-\sqrt{a+bx^2+cx^4}}{\sqrt{a}}\right)}{a^{3/2}d}$$

```
[In] Integrate[1/(x*(d + e*x^2)*(a + b*x^2 + c*x^4)^(3/2)),x]
```

```
[Out] (b^3*e - b*c*(3*a*e + c*d*x^2) + 2*a*c^2*(d - e*x^2) + b^2*c*(-d + e*x^2))/
(a*(-b^2 + 4*a*c)*(c*d^2 + e*(-(b*d) + a*e))*Sqrt[a + b*x^2 + c*x^4]) - (e^
3*Sqrt[-(c*d^2) + e*(b*d - a*e)]*ArcTan[(Sqrt[c]*(d + e*x^2) - e*Sqrt[a + b
*x^2 + c*x^4])/Sqrt[-(c*d^2) + e*(b*d - a*e)]])/(d*(c*d^2 + e*(-(b*d) + a*e
))^2) + ArcTanh[(Sqrt[c]*x^2 - Sqrt[a + b*x^2 + c*x^4])/Sqrt[a]]/(a^(3/2)*d
)
```

Maple [A] (verified)

Time = 0.68 (sec) , antiderivative size = 285, normalized size of antiderivative = 1.07

method	result
pseudoelliptic	$\frac{\frac{1}{a\sqrt{c\,x^4+b\,x^2+a}}-\frac{b\left(2c\,x^2+b\right)}{a\left(4ac-b^2\right)\sqrt{c\,x^4+b\,x^2+a}}-\frac{\ln\left(\frac{2a+b\,x^2+2\sqrt{a}\,\sqrt{c\,x^4+b\,x^2+a}}{x^2}\right)}{a^{\frac{3}{2}}}+\frac{-\frac{e^2}{\sqrt{c\,x^4+b\,x^2+a}}+\frac{(be-2cd)e\left(2c\,x^2+b\right)}{\sqrt{c\,x^4+b\,x^2+a}\left(4ac-b^2\right)}+\frac{e^2\ln}{}}{}}{2d}$
default	$\frac{\frac{1}{2a\sqrt{c\,x^4+b\,x^2+a}}-\frac{b\left(2c\,x^2+b\right)}{2a\left(4ac-b^2\right)\sqrt{c\,x^4+b\,x^2+a}}-\frac{\ln\left(\frac{2a+b\,x^2+2\sqrt{a}\,\sqrt{c\,x^4+b\,x^2+a}}{x^2}\right)}{2a^{\frac{3}{2}}}}{d}-\frac{e\left(-\frac{2c\sqrt{c\left(x^2-\frac{-b+\sqrt{-4ac+b^2}}{2c}\right)^2+\sqrt{-4ac}}}{\left(e\sqrt{-4ac+b^2}-be+2cd\right)\left(-4ac+b^2\right)}\right.}{}$
elliptic	$\frac{2c\ln\left(\frac{2a+b\,x^2+2\sqrt{a}\,\sqrt{c\,x^4+b\,x^2+a}}{x^2}\right)}{d\left(-b+\sqrt{-4ac+b^2}\right)\left(b+\sqrt{-4ac+b^2}\right)\sqrt{a}}-\frac{2c\,e^2\ln\left(\frac{\frac{2a\,e^2-2bde+2c\,d^2}{e^2}+\frac{(be-2cd)\left(x^2+\frac{d}{e}\right)}{e}+2\sqrt{\frac{a\,e^2-bde+c\,d^2}{e^2}}\,\sqrt{c\left(x^2+\frac{d}{e}\right)^2+}}{x^2+\frac{d}{e}}\right.}{\left(e\sqrt{-4ac+b^2}-be+2cd\right)\left(e\sqrt{-4ac+b^2}+be-2cd\right)d\sqrt{\frac{a\,e^2}{}}}$

```
[In] int(1/x/(e*x^2+d)/(c*x^4+b*x^2+a)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2/d*(1/a/(c*x^4+b*x^2+a)^(1/2)-b/a*(2*c*x^2+b)/(4*a*c-b^2)/(c*x^4+b*x^2+a
)^(1/2)-1/a^(3/2)*ln((2*a+b*x^2+2*a^(1/2)*(c*x^4+b*x^2+a)^(1/2))/x^2)+1/(a*
e^2-b*d*e+c*d^2)*(-e^2/(c*x^4+b*x^2+a)^(1/2)+(b*e-2*c*d)*e*(2*c*x^2+b)/(c*x
^4+b*x^2+a)^(1/2)/(4*a*c-b^2)+e^2/((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*ln((2*(c*
x^4+b*x^2+a)^(1/2)*((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*e+(b*x^2+2*a)*e-d*(2*c*x
^2+b))/(e*x^2+d))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1202 vs. 2(242) = 484.

Time = 2.44 (sec) , antiderivative size = 4909, normalized size of antiderivative = 18.45

$$\int \frac{1}{x\left(d+ex^2\right)\left(a+bx^2+cx^4\right)^{3/2}}\,dx = \text{Too large to display}$$

```
[In] integrate(1/x/(e*x^2+d)/(c*x^4+b*x^2+a)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/4*(((a^2*b^2*c - 4*a^3*c^2)*e^3*x^4 + (a^2*b^3 - 4*a^3*b*c)*e^3*x^2 + (a
^3*b^2 - 4*a^4*c)*e^3)*sqrt(c*d^2 - b*d*e + a*e^2)*log(-((8*c^2*d^2 - 8*b*c
*d*e + (b^2 + 4*a*c)*e^2)*x^4 - 8*a*b*d*e + 8*a^2*e^2 + (b^2 + 4*a*c)*d^2 +
 2*(4*b*c*d^2 + 4*a*b*e^2 - (3*b^2 + 4*a*c)*d*e)*x^2 - 4*sqrt(c*x^4 + b*x^2
 + a)*sqrt(c*d^2 - b*d*e + a*e^2)*((2*c*d - b*e)*x^2 + b*d - 2*a*e))/(e^2*x
^4 + 2*d*e*x^2 + d^2)) + ((a*b^2*c^2 - 4*a^2*c^3)*d^4 - 2*(a*b^3*c - 4*a^2*
b*c^2)*d^3*e + (a*b^4 - 2*a^2*b^2*c - 8*a^3*c^2)*d^2*e^2 - 2*(a^2*b^3 - 4*a
^3*b*c)*d*e^3 + (a^3*b^2 - 4*a^4*c)*e^4 + ((b^2*c^3 - 4*a*c^4)*d^4 - 2*(b^3
```

*c^2 - 4*a*b*c^3)*d^3*e + (b^4*c - 2*a*b^2*c^2 - 8*a^2*c^3)*d^2*e^2 - 2*(a*
b^3*c - 4*a^2*b*c^2)*d*e^3 + (a^2*b^2*c - 4*a^3*c^2)*e^4)*x^4 + ((b^3*c^2 -
 4*a*b*c^3)*d^4 - 2*(b^4*c - 4*a*b^2*c^2)*d^3*e + (b^5 - 2*a*b^3*c - 8*a^2*
b*c^2)*d^2*e^2 - 2*(a*b^4 - 4*a^2*b^2*c)*d*e^3 + (a^2*b^3 - 4*a^3*b*c)*e^4)
*x^2)*sqrt(a)*log(-((b^2 + 4*a*c)*x^4 + 8*a*b*x^2 - 4*sqrt(c*x^4 + b*x^2 +
a)*(b*x^2 + 2*a)*sqrt(a) + 8*a^2)/x^4) + 4*((a*b^2*c^2 - 2*a^2*c^3)*d^4 - (
2*a*b^3*c - 5*a^2*b*c^2)*d^3*e + (a*b^4 - 2*a^2*b^2*c - 2*a^3*c^2)*d^2*e^2
- (a^2*b^3 - 3*a^3*b*c)*d*e^3 + (a*b*c^3*d^4 - 2*(a*b^2*c^2 - a^2*c^3)*d^3*
e + (a*b^3*c - a^2*b*c^2)*d^2*e^2 - (a^2*b^2*c - 2*a^3*c^2)*d*e^3)*x^2)*sqr
t(c*x^4 + b*x^2 + a))/((a^3*b^2*c^2 - 4*a^4*c^3)*d^5 - 2*(a^3*b^3*c - 4*a^4
*b*c^2)*d^4*e + (a^3*b^4 - 2*a^4*b^2*c - 8*a^5*c^2)*d^3*e^2 - 2*(a^4*b^3 -
4*a^5*b*c)*d^2*e^3 + (a^5*b^2 - 4*a^6*c)*d*e^4 + ((a^2*b^2*c^3 - 4*a^3*c^4)
d^5 - 2(a^2*b^3*c^2 - 4*a^3*b*c^3)*d^4*e + (a^2*b^4*c - 2*a^3*b^2*c^2 - 8
*a^4*c^3)*d^3*e^2 - 2*(a^3*b^3*c - 4*a^4*b*c^2)*d^2*e^3 + (a^4*b^2*c - 4*a^
5*c^2)*d*e^4)*x^4 + ((a^2*b^3*c^2 - 4*a^3*b*c^3)*d^5 - 2*(a^2*b^4*c - 4*a^3
*b^2*c^2)*d^4*e + (a^2*b^5 - 2*a^3*b^3*c - 8*a^4*b*c^2)*d^3*e^2 - 2*(a^3*b^
4 - 4*a^4*b^2*c)*d^2*e^3 + (a^4*b^3 - 4*a^5*b*c)*d*e^4)*x^2), -1/4*(2*((a^2
*b^2*c - 4*a^3*c^2)*e^3*x^4 + (a^2*b^3 - 4*a^3*b*c)*e^3*x^2 + (a^3*b^2 - 4*
a^4*c)*e^3)*sqrt(-c*d^2 + b*d*e - a*e^2)*arctan(-1/2*sqrt(c*x^4 + b*x^2 + a
)*sqrt(-c*d^2 + b*d*e - a*e^2)*((2*c*d - b*e)*x^2 + b*d - 2*a*e)/((c^2*d^2
- b*c*d*e + a*c*e^2)*x^4 + a*c*d^2 - a*b*d*e + a^2*e^2 + (b*c*d^2 - b^2*d*e
 + a*b*e^2)*x^2)) - ((a*b^2*c^2 - 4*a^2*c^3)*d^4 - 2*(a*b^3*c - 4*a^2*b*c^2
)*d^3*e + (a*b^4 - 2*a^2*b^2*c - 8*a^3*c^2)*d^2*e^2 - 2*(a^2*b^3 - 4*a^3*b*
c)*d*e^3 + (a^3*b^2 - 4*a^4*c)*e^4 + ((b^2*c^3 - 4*a*c^4)*d^4 - 2*(b^3*c^2
- 4*a*b*c^3)*d^3*e + (b^4*c - 2*a*b^2*c^2 - 8*a^2*c^3)*d^2*e^2 - 2*(a*b^3*c
 - 4*a^2*b*c^2)*d*e^3 + (a^2*b^2*c - 4*a^3*c^2)*e^4)*x^4 + ((b^3*c^2 - 4*a*
b*c^3)*d^4 - 2*(b^4*c - 4*a*b^2*c^2)*d^3*e + (b^5 - 2*a*b^3*c - 8*a^2*b*c^2
)*d^2*e^2 - 2*(a*b^4 - 4*a^2*b^2*c)*d*e^3 + (a^2*b^3 - 4*a^3*b*c)*e^4)*x^2)
*sqrt(a)*log(-((b^2 + 4*a*c)*x^4 + 8*a*b*x^2 - 4*sqrt(c*x^4 + b*x^2 + a)*(b
*x^2 + 2*a)*sqrt(a) + 8*a^2)/x^4) - 4*((a*b^2*c^2 - 2*a^2*c^3)*d^4 - (2*a*b
^3*c - 5*a^2*b*c^2)*d^3*e + (a*b^4 - 2*a^2*b^2*c - 2*a^3*c^2)*d^2*e^2 - (a^
2*b^3 - 3*a^3*b*c)*d*e^3 + (a*b*c^3*d^4 - 2*(a*b^2*c^2 - a^2*c^3)*d^3*e + (
a*b^3*c - a^2*b*c^2)*d^2*e^2 - (a^2*b^2*c - 2*a^3*c^2)*d*e^3)*x^2)*sqrt(c*x
^4 + b*x^2 + a))/((a^3*b^2*c^2 - 4*a^4*c^3)*d^5 - 2*(a^3*b^3*c - 4*a^4*b*c^
2)*d^4*e + (a^3*b^4 - 2*a^4*b^2*c - 8*a^5*c^2)*d^3*e^2 - 2*(a^4*b^3 - 4*a^5
*b*c)*d^2*e^3 + (a^5*b^2 - 4*a^6*c)*d*e^4 + ((a^2*b^2*c^3 - 4*a^3*c^4)*d^5
- 2*(a^2*b^3*c^2 - 4*a^3*b*c^3)*d^4*e + (a^2*b^4*c - 2*a^3*b^2*c^2 - 8*a^4*
c^3)*d^3*e^2 - 2*(a^3*b^3*c - 4*a^4*b*c^2)*d^2*e^3 + (a^4*b^2*c - 4*a^5*c^2
)*d*e^4)*x^4 + ((a^2*b^3*c^2 - 4*a^3*b*c^3)*d^5 - 2*(a^2*b^4*c - 4*a^3*b^2*
c^2)*d^4*e + (a^2*b^5 - 2*a^3*b^3*c - 8*a^4*b*c^2)*d^3*e^2 - 2*(a^3*b^4 - 4
*a^4*b^2*c)*d^2*e^3 + (a^4*b^3 - 4*a^5*b*c)*d*e^4)*x^2), 1/4*(2*((a*b^2*c^2
 - 4*a^2*c^3)*d^4 - 2*(a*b^3*c - 4*a^2*b*c^2)*d^3*e + (a*b^4 - 2*a^2*b^2*c
- 8*a^3*c^2)*d^2*e^2 - 2*(a^2*b^3 - 4*a^3*b*c)*d*e^3 + (a^3*b^2 - 4*a^4*c)*
e^4 + ((b^2*c^3 - 4*a*c^4)*d^4 - 2*(b^3*c^2 - 4*a*b*c^3)*d^3*e + (b^4*c - 2
*a*b^2*c^2 - 8*a^2*c^3)*d^2*e^2 - 2*(a*b^3*c - 4*a^2*b*c^2)*d*e^3 + (a^2*b^

2*c - 4*a^3*c^2)*e^4)*x^4 + ((b^3*c^2 - 4*a*b*c^3)*d^4 - 2*(b^4*c - 4*a*b^2
*c^2)*d^3*e + (b^5 - 2*a*b^3*c - 8*a^2*b*c^2)*d^2*e^2 - 2*(a*b^4 - 4*a^2*b^
2*c)*d*e^3 + (a^2*b^3 - 4*a^3*b*c)*e^4)*x^2)*sqrt(-a)*arctan(1/2*sqrt(c*x^4
 + b*x^2 + a)*(b*x^2 + 2*a)*sqrt(-a)/(a*c*x^4 + a*b*x^2 + a^2)) + ((a^2*b^2
*c - 4*a^3*c^2)*e^3*x^4 + (a^2*b^3 - 4*a^3*b*c)*e^3*x^2 + (a^3*b^2 - 4*a^4*
c)*e^3)*sqrt(c*d^2 - b*d*e + a*e^2)*log(-((8*c^2*d^2 - 8*b*c*d*e + (b^2 + 4
*a*c)*e^2)*x^4 - 8*a*b*d*e + 8*a^2*e^2 + (b^2 + 4*a*c)*d^2 + 2*(4*b*c*d^2 +
 4*a*b*e^2 - (3*b^2 + 4*a*c)*d*e)*x^2 - 4*sqrt(c*x^4 + b*x^2 + a)*sqrt(c*d^
2 - b*d*e + a*e^2)*((2*c*d - b*e)*x^2 + b*d - 2*a*e))/(e^2*x^4 + 2*d*e*x^2
+ d^2)) + 4*((a*b^2*c^2 - 2*a^2*c^3)*d^4 - (2*a*b^3*c - 5*a^2*b*c^2)*d^3*e
+ (a*b^4 - 2*a^2*b^2*c - 2*a^3*c^2)*d^2*e^2 - (a^2*b^3 - 3*a^3*b*c)*d*e^3 +
 (a*b*c^3*d^4 - 2*(a*b^2*c^2 - a^2*c^3)*d^3*e + (a*b^3*c - a^2*b*c^2)*d^2*e
^2 - (a^2*b^2*c - 2*a^3*c^2)*d*e^3)*x^2)*sqrt(c*x^4 + b*x^2 + a))/((a^3*b^2
*c^2 - 4*a^4*c^3)*d^5 - 2*(a^3*b^3*c - 4*a^4*b*c^2)*d^4*e + (a^3*b^4 - 2*a^
4*b^2*c - 8*a^5*c^2)*d^3*e^2 - 2*(a^4*b^3 - 4*a^5*b*c)*d^2*e^3 + (a^5*b^2 -
 4*a^6*c)*d*e^4 + ((a^2*b^2*c^3 - 4*a^3*c^4)*d^5 - 2*(a^2*b^3*c^2 - 4*a^3*b
*c^3)*d^4*e + (a^2*b^4*c - 2*a^3*b^2*c^2 - 8*a^4*c^3)*d^3*e^2 - 2*(a^3*b^3*
c - 4*a^4*b*c^2)*d^2*e^3 + (a^4*b^2*c - 4*a^5*c^2)*d*e^4)*x^4 + ((a^2*b^3*c
^2 - 4*a^3*b*c^3)*d^5 - 2*(a^2*b^4*c - 4*a^3*b^2*c^2)*d^4*e + (a^2*b^5 - 2*
a^3*b^3*c - 8*a^4*b*c^2)*d^3*e^2 - 2*(a^3*b^4 - 4*a^4*b^2*c)*d^2*e^3 + (a^4
*b^3 - 4*a^5*b*c)*d*e^4)*x^2), -1/2*(((a^2*b^2*c - 4*a^3*c^2)*e^3*x^4 + (a^
2*b^3 - 4*a^3*b*c)*e^3*x^2 + (a^3*b^2 - 4*a^4*c)*e^3)*sqrt(-c*d^2 + b*d*e -
 a*e^2)*arctan(-1/2*sqrt(c*x^4 + b*x^2 + a)*sqrt(-c*d^2 + b*d*e - a*e^2)*((
2*c*d - b*e)*x^2 + b*d - 2*a*e)/((c^2*d^2 - b*c*d*e + a*c*e^2)*x^4 + a*c*d^
2 - a*b*d*e + a^2*e^2 + (b*c*d^2 - b^2*d*e + a*b*e^2)*x^2)) - ((a*b^2*c^2 -
 4*a^2*c^3)*d^4 - 2*(a*b^3*c - 4*a^2*b*c^2)*d^3*e + (a*b^4 - 2*a^2*b^2*c -
8*a^3*c^2)*d^2*e^2 - 2*(a^2*b^3 - 4*a^3*b*c)*d*e^3 + (a^3*b^2 - 4*a^4*c)*e^
4 + ((b^2*c^3 - 4*a*c^4)*d^4 - 2*(b^3*c^2 - 4*a*b*c^3)*d^3*e + (b^4*c - 2*a
*b^2*c^2 - 8*a^2*c^3)*d^2*e^2 - 2*(a*b^3*c - 4*a^2*b*c^2)*d*e^3 + (a^2*b^2*
c - 4*a^3*c^2)*e^4)*x^4 + ((b^3*c^2 - 4*a*b*c^3)*d^4 - 2*(b^4*c - 4*a*b^2*c
^2)*d^3*e + (b^5 - 2*a*b^3*c - 8*a^2*b*c^2)*d^2*e^2 - 2*(a*b^4 - 4*a^2*b^2*
c)*d*e^3 + (a^2*b^3 - 4*a^3*b*c)*e^4)*x^2)*sqrt(-a)*arctan(1/2*sqrt(c*x^4 +
 b*x^2 + a)*(b*x^2 + 2*a)*sqrt(-a)/(a*c*x^4 + a*b*x^2 + a^2)) - 2*((a*b^2*c
^2 - 2*a^2*c^3)*d^4 - (2*a*b^3*c - 5*a^2*b*c^2)*d^3*e + (a*b^4 - 2*a^2*b^2*
c - 2*a^3*c^2)*d^2*e^2 - (a^2*b^3 - 3*a^3*b*c)*d*e^3 + (a*b*c^3*d^4 - 2*(a*
b^2*c^2 - a^2*c^3)*d^3*e + (a*b^3*c - a^2*b*c^2)*d^2*e^2 - (a^2*b^2*c - 2*a
^3*c^2)*d*e^3)*x^2)*sqrt(c*x^4 + b*x^2 + a))/((a^3*b^2*c^2 - 4*a^4*c^3)*d^5
 - 2*(a^3*b^3*c - 4*a^4*b*c^2)*d^4*e + (a^3*b^4 - 2*a^4*b^2*c - 8*a^5*c^2)*
d^3*e^2 - 2*(a^4*b^3 - 4*a^5*b*c)*d^2*e^3 + (a^5*b^2 - 4*a^6*c)*d*e^4 + ((a
^2*b^2*c^3 - 4*a^3*c^4)*d^5 - 2*(a^2*b^3*c^2 - 4*a^3*b*c^3)*d^4*e + (a^2*b^
4*c - 2*a^3*b^2*c^2 - 8*a^4*c^3)*d^3*e^2 - 2*(a^3*b^3*c - 4*a^4*b*c^2)*d^2*
e^3 + (a^4*b^2*c - 4*a^5*c^2)*d*e^4)*x^4 + ((a^2*b^3*c^2 - 4*a^3*b*c^3)*d^5
 - 2*(a^2*b^4*c - 4*a^3*b^2*c^2)*d^4*e + (a^2*b^5 - 2*a^3*b^3*c - 8*a^4*b*c
^2)*d^3*e^2 - 2*(a^3*b^4 - 4*a^4*b^2*c)*d^2*e^3 + (a^4*b^3 - 4*a^5*b*c)*d*e
^4)*x^2)]

Sympy [F]

$$\int \frac{1}{x\,(d+ex^2)\,(a+bx^2+cx^4)^{3/2}}\,dx = \int \frac{1}{x\,(d+ex^2)\,(a+bx^2+cx^4)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/x/(e*x**2+d)/(c*x**4+b*x**2+a)**(3/2),x)
[Out] Integral(1/(x*(d + e*x**2)*(a + b*x**2 + c*x**4)**(3/2)), x)
```

Maxima [F]

$$\int \frac{1}{x\,(d+ex^2)\,(a+bx^2+cx^4)^{3/2}}\,dx = \int \frac{1}{(cx^4+bx^2+a)^{\frac{3}{2}}(ex^2+d)x}\,dx$$

```
[In] integrate(1/x/(e*x^2+d)/(c*x^4+b*x^2+a)^(3/2),x, algorithm="maxima")
[Out] integrate(1/((c*x^4 + b*x^2 + a)^(3/2)*(e*x^2 + d)*x), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{1}{x\,(d+ex^2)\,(a+bx^2+cx^4)^{3/2}}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(1/x/(e*x^2+d)/(c*x^4+b*x^2+a)^(3/2),x, algorithm="giac")
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument Type
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x\,(d+ex^2)\,(a+bx^2+cx^4)^{3/2}}\,dx = \int \frac{1}{x\,(e\,x^2+d)\,(c\,x^4+b\,x^2+a)^{3/2}}\,dx$$

```
[In] int(1/(x*(d + e*x^2)*(a + b*x^2 + c*x^4)^(3/2)),x)
[Out] int(1/(x*(d + e*x^2)*(a + b*x^2 + c*x^4)^(3/2)), x)
```

3.347 $\int \frac{1}{x^3(d+ex^2)(a+bx^2+cx^4)^{3/2}}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 419

$$
\begin{aligned}
\int \frac{1}{x^3 (d+ex^2) (a+bx^2+cx^4)^{3/2}}\,dx = &-\frac{e(b^2-2ac+bcx^2)}{a(b^2-4ac)\,d^2\sqrt{a+bx^2+cx^4}} \\
&+\frac{b^2-2ac+bcx^2}{a(b^2-4ac)\,dx^2\sqrt{a+bx^2+cx^4}}-\frac{e^2(bcd-b^2e+2ace+c(2cd-be)x^2)}{(b^2-4ac)\,d^2(cd^2-bde+ae^2)\sqrt{a+bx^2+cx^4}} \\
&-\frac{(3b^2-8ac)\sqrt{a+bx^2+cx^4}}{2a^2(b^2-4ac)\,dx^2}+\frac{3b\,\text{arctanh}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{4a^{5/2}d} \\
&+\frac{e\,\text{arctanh}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{2a^{3/2}d^2}+\frac{e^4\,\text{arctanh}\left(\frac{bd-2ae+(2cd-be)x^2}{2\sqrt{cd^2-bde+ae^2}\sqrt{a+bx^2+cx^4}}\right)}{2d^2(cd^2-bde+ae^2)^{3/2}}
\end{aligned}
$$

```
[Out] 3/4*b*arctanh(1/2*(b*x^2+2*a)/a^(1/2)/(c*x^4+b*x^2+a)^(1/2))/a^(5/2)/d+1/2*
e*arctanh(1/2*(b*x^2+2*a)/a^(1/2)/(c*x^4+b*x^2+a)^(1/2))/a^(3/2)/d^2+1/2*e^
4*arctanh(1/2*(b*d-2*a*e+(-b*e+2*c*d)*x^2)/(a*e^2-b*d*e+c*d^2)^(1/2)/(c*x^4
+b*x^2+a)^(1/2))/d^2/(a*e^2-b*d*e+c*d^2)^(3/2)-e*(b*c*x^2-2*a*c+b^2)/a/(-4*
a*c+b^2)/d^2/(c*x^4+b*x^2+a)^(1/2)+(b*c*x^2-2*a*c+b^2)/a/(-4*a*c+b^2)/d/x^2
/(c*x^4+b*x^2+a)^(1/2)-e^2*(b*c*d-b^2*e+2*a*c*e+c*(-b*e+2*c*d)*x^2)/(-4*a*c
+b^2)/d^2/(a*e^2-b*d*e+c*d^2)/(c*x^4+b*x^2+a)^(1/2)-1/2*(-8*a*c+3*b^2)*(c*x
^4+b*x^2+a)^(1/2)/a^2/(-4*a*c+b^2)/d/x^2
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 419, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.241$, Rules used = {1265, 974, 754, 820, 738, 212, 12}

$$\int \frac{1}{x^3 (d+ex^2) (a+bx^2+cx^4)^{3/2}} \, dx = \frac{e \text{arctanh}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{2a^{3/2}d^2} + \frac{3b \text{arctanh}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{4a^{5/2}d} - \frac{(3b^2-8ac)\sqrt{a+bx^2+cx^4}}{2a^2dx^2(b^2-4ac)} + \frac{e^4 \text{arctanh}\left(\frac{-2ae+x^2(2cd-be)+bd}{2\sqrt{a+bx^2+cx^4}\sqrt{ae^2-bde+cd^2}}\right)}{2d^2(ae^2-bde+cd^2)^{3/2}} - \frac{e^2(2ace+b^2(-e)+cx^2(2cd-be)+bcd)}{d^2(b^2-4ac)\sqrt{a+bx^2+cx^4}(ae^2-bde+cd^2)} - \frac{e(-2ac+b^2+bcx^2)}{ad^2(b^2-4ac)\sqrt{a+bx^2+cx^4}} + \frac{-2ac+b^2+bcx^2}{adx^2(b^2-4ac)\sqrt{a+bx^2+cx^4}}$$

```
[In] Int[1/(x^3*(d + e*x^2)*(a + b*x^2 + c*x^4)^(3/2)),x]
```

```
[Out] -((e*(b^2 - 2*a*c + b*c*x^2))/(a*(b^2 - 4*a*c)*d^2*Sqrt[a + b*x^2 + c*x^4])
) + (b^2 - 2*a*c + b*c*x^2)/(a*(b^2 - 4*a*c)*d*x^2*Sqrt[a + b*x^2 + c*x^4])
 - (e^2*(b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e)*x^2))/((b^2 - 4*a*c)*d^2
*(c*d^2 - b*d*e + a*e^2)*Sqrt[a + b*x^2 + c*x^4]) - ((3*b^2 - 8*a*c)*Sqrt[a
 + b*x^2 + c*x^4])/(2*a^2*(b^2 - 4*a*c)*d*x^2) + (3*b*ArcTanh[(2*a + b*x^2)
/(2*Sqrt[a]*Sqrt[a + b*x^2 + c*x^4])])/(4*a^(5/2)*d) + (e*ArcTanh[(2*a + b*
x^2)/(2*Sqrt[a]*Sqrt[a + b*x^2 + c*x^4])])/(2*a^(3/2)*d^2) + (e^4*ArcTanh[(
b*d - 2*a*e + (2*c*d - b*e)*x^2)/(2*Sqrt[c*d^2 - b*d*e + a*e^2]*Sqrt[a + b*
x^2 + c*x^4])])/(2*d^2*(c*d^2 - b*d*e + a*e^2)^(3/2))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
```

```
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 754

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*(b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e)
*x)*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^
2))), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d +
 e*x)^m*Simp[b*c*d*e*(2*p - m + 2) + b^2*e^2*(m + p + 2) - 2*c^2*d^2*(2*p +
 3) - 2*a*c*e^2*(m + 2*p + 3) - c*e*(2*c*d - b*e)*(m + 2*p + 4)*x, x]*(a +
b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4
*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[p,
-1] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 820

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a +
 b*x + c*x^2)^(p + 1)/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2))), x] - Dist[(b*(e
*f + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(
m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[Simplify[m
+ 2*p + 3], 0]
```

Rule 974

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g
*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ
[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && (
IntegerQ[p] || (ILtQ[m, 0] && ILtQ[n, 0])) &&  !(IGtQ[m, 0] || IGtQ[n, 0])
```

Rule 1265

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rubi steps

$$\text{integral} = \frac{1}{2}\text{Subst}\left(\int \frac{1}{x^2(d+ex)\,(a+bx+cx^2)^{3/2}}\,dx, x, x^2\right)$$

$$=\frac{1}{2}\text{Subst}\left(\int\left(\frac{1}{dx^2(a+bx+cx^2)^{3/2}}-\frac{e}{d^2x(a+bx+cx^2)^{3/2}}+\frac{e^2}{d^2(d+ex)(a+bx+cx^2)^{3/2}}\right)dx,x,x^2\right)$$

$$=\frac{\text{Subst}\left(\int\frac{1}{x^2(a+bx+cx^2)^{3/2}}dx,x,x^2\right)}{2d}-\frac{e\text{Subst}\left(\int\frac{1}{x(a+bx+cx^2)^{3/2}}dx,x,x^2\right)}{2d^2}+\frac{e^2\text{Subst}\left(\int\frac{1}{(d+ex)(a+bx+cx^2)^{3/2}}dx,x,x^2\right)}{2d^2}$$

$$=-\frac{e(b^2-2ac+bcx^2)}{a(b^2-4ac)d^2\sqrt{a+bx^2+cx^4}}+\frac{b^2-2ac+bcx^2}{a(b^2-4ac)dx^2\sqrt{a+bx^2+cx^4}}-\frac{e^2(bcd-b^2e+2ace+c(2cd-be)x^2)}{(b^2-4ac)d^2(cd^2-bde+ae^2)\sqrt{a+bx^2+cx^4}}-\frac{\text{Subst}\left(\int\frac{\frac{1}{2}(-3b^2+8ac)-bcx}{x^2\sqrt{a+bx+cx^2}}dx,x,x^2\right)}{a(b^2-4ac)d}+\frac{e\text{Subst}\left(\int\frac{-\frac{b^2}{2}+2ac}{x\sqrt{a+bx+cx^2}}dx,x,x^2\right)}{a(b^2-4ac)d^2}-\frac{e^2\text{Subst}\left(\int-\frac{(b^2-4ac)e^2}{2(d+ex)\sqrt{a+bx+cx^2}}dx,x,x^2\right)}{(b^2-4ac)d^2(cd^2-bde+ae^2)}$$

$$=-\frac{e(b^2-2ac+bcx^2)}{a(b^2-4ac)d^2\sqrt{a+bx^2+cx^4}}+\frac{b^2-2ac+bcx^2}{a(b^2-4ac)dx^2\sqrt{a+bx^2+cx^4}}-\frac{e^2(bcd-b^2e+2ace+c(2cd-be)x^2)}{(b^2-4ac)d^2(cd^2-bde+ae^2)\sqrt{a+bx^2+cx^4}}-\frac{(3b^2-8ac)\sqrt{a+bx^2+cx^4}}{2a^2(b^2-4ac)dx^2}-\frac{(3b)\text{Subst}\left(\int\frac{1}{x\sqrt{a+bx+cx^2}}dx,x,x^2\right)}{4a^2d}-\frac{e\text{Subst}\left(\int\frac{1}{x\sqrt{a+bx+cx^2}}dx,x,x^2\right)}{2ad^2}+\frac{e^4\text{Subst}\left(\int\frac{1}{(d+ex)\sqrt{a+bx+cx^2}}dx,x,x^2\right)}{2d^2(cd^2-bde+ae^2)}$$

$$=-\frac{e(b^2-2ac+bcx^2)}{a(b^2-4ac)d^2\sqrt{a+bx^2+cx^4}}+\frac{b^2-2ac+bcx^2}{a(b^2-4ac)dx^2\sqrt{a+bx^2+cx^4}}-\frac{e^2(bcd-b^2e+2ace+c(2cd-be)x^2)}{(b^2-4ac)d^2(cd^2-bde+ae^2)\sqrt{a+bx^2+cx^4}}-\frac{(3b^2-8ac)\sqrt{a+bx^2+cx^4}}{2a^2(b^2-4ac)dx^2}+\frac{(3b)\text{Subst}\left(\int\frac{1}{4a-x^2}dx,x,\frac{2a+bx^2}{\sqrt{a+bx^2+cx^4}}\right)}{2a^2d}+\frac{e\text{Subst}\left(\int\frac{1}{4a-x^2}dx,x,\frac{2a+bx^2}{\sqrt{a+bx^2+cx^4}}\right)}{ad^2}-\frac{e^4\text{Subst}\left(\int\frac{1}{4cd^2-4bde+4ae^2-x^2}dx,x,\frac{-bd+2ae-(2cd-be)x^2}{\sqrt{a+bx^2+cx^4}}\right)}{d^2(cd^2-bde+ae^2)}$$

$$
\begin{aligned}
&= -\frac{e(b^2-2ac+bcx^2)}{a\left(b^2-4ac\right)d^2\sqrt{a+bx^2+cx^4}} + \frac{b^2-2ac+bcx^2}{a\left(b^2-4ac\right)dx^2\sqrt{a+bx^2+cx^4}} \\
&\quad - \frac{e^2(bcd-b^2e+2ace+c(2cd-be)x^2)}{\left(b^2-4ac\right)d^2\left(cd^2-bde+ae^2\right)\sqrt{a+bx^2+cx^4}} \\
&\quad - \frac{\left(3b^2-8ac\right)\sqrt{a+bx^2+cx^4}}{2a^2\left(b^2-4ac\right)dx^2} + \frac{3b\tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{4a^{5/2}d} \\
&\quad + \frac{e\tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{2a^{3/2}d^2} + \frac{e^4\tanh^{-1}\left(\frac{bd-2ae+(2cd-be)x^2}{2\sqrt{cd^2-bde+ae^2}\sqrt{a+bx^2+cx^4}}\right)}{2d^2\left(cd^2-bde+ae^2\right)^{3/2}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 1.68 (sec) , antiderivative size = 363, normalized size of antiderivative = 0.87

$$
\int \frac{1}{x^3\left(d+ex^2\right)\left(a+bx^2+cx^4\right)^{3/2}}\,dx =
$$

$$
-\frac{\frac{d\left(4a^3ce^2+3b^2d(-cd+be)x^2\left(b+cx^2\right)+a^2\left(-b^2e^2+4bce\left(-d+ex^2\right)+4c^2\left(d^2+dex^2+e^2x^4\right)\right)+a\left(8c^3d^2x^4+b^3e\left(d-ex^2\right)+10bc^2dx^2\left(d-ex^2\right)-b^2c(d^2+\right.\right.}{a^2(b^2-4ac)(-cd^2+e(bd-ae))x^2\sqrt{a+bx^2+cx^4}}}{2d^2}
$$

```
[In] Integrate[1/(x^3*(d + e*x^2)*(a + b*x^2 + c*x^4)^(3/2)),x]
```

```
[Out] -1/2*((d*(4*a^3*c*e^2 + 3*b^2*d*(-(c*d) + b*e)*x^2*(b + c*x^2) + a^2*(-(b^2
*e^2) + 4*b*c*e*(-d + e*x^2) + 4*c^2*(d^2 + d*e*x^2 + e^2*x^4)) + a*(8*c^3*
d^2*x^4 + b^3*e*(d - e*x^2) + 10*b*c^2*d*x^2*(d - e*x^2) - b^2*c*(d^2 + 12*
d*e*x^2 + e^2*x^4))))/(a^2*(b^2 - 4*a*c)*(-(c*d^2) + e*(b*d - a*e))*x^2*Sqr
t[a + b*x^2 + c*x^4]) - (2*e^4*Sqrt[-(c*d^2) + b*d*e - a*e^2]*ArcTan[(Sqrt[
c]*(d + e*x^2) - e*Sqrt[a + b*x^2 + c*x^4])/Sqrt[-(c*d^2) + e*(b*d - a*e)]]
)/(c*d^2 + e*(-(b*d) + a*e))^2 + ((3*b*d + 2*a*e)*ArcTanh[(Sqrt[c]*x^2 - Sq
rt[a + b*x^2 + c*x^4])/Sqrt[a]])/a^(5/2))/d^2
```

Maple [A] (verified)

Time = 0.94 (sec) , antiderivative size = 413, normalized size of antiderivative = 0.99

method	result
risch	$-\frac{\sqrt{c x^4+b x^2+a}}{2a^2 d x^2}-\frac{-\frac{(2ae+3bd)\ln\left(\frac{2a+b x^2+2\sqrt{a}\sqrt{c x^4+b x^2+a}}{x^2}\right)}{2d\sqrt{a}}+\frac{a^2 e^3\ln\left(\frac{\frac{2a e^2-2bde+2c d^2}{e^2}+\frac{(be-2cd)\left(x^2+\frac{d}{e}\right)}{e}+2\sqrt{\frac{a e^2-bde-\cdots}{e^2}}\cdots}{x^2\cdots}\right)}{d\left(a e^2-bde+c d^2\right)\sqrt{\cdots}}\cdots}{\cdots}$
pseudoelliptic	$\frac{\frac{\sqrt{c x^4+b x^2+a}\, e^3 x^2\left(a^{\frac{5}{2}} b^2-4a^{\frac{7}{2}} c\right)\ln\left(\frac{2\sqrt{c x^4+b x^2+a}\sqrt{\frac{a e^2-bde+c d^2}{e^2}}\, e+\left(b x^2+2a\right)e-d\left(2c x^2+b\right)}{e x^2+d}\right)}{4}+\left(\left(a e^2-bde+c d^2\right)\left(ae+\frac{3bd}{2}\cdots\right)\cdots\right)}{\cdots}$
elliptic	$-\frac{2c\left(-\frac{\sqrt{c x^4+b x^2+a}}{a x^2}+\frac{b\ln\left(\frac{2a+b x^2+2\sqrt{a}\sqrt{c x^4+b x^2+a}}{x^2}\right)}{2a^{\frac{3}{2}}}\right)}{d\left(-b+\sqrt{-4ac+b^2}\right)\left(b+\sqrt{-4ac+b^2}\right)}+\frac{8c^2(ae+bd)\ln\left(\frac{2a+b x^2+2\sqrt{a}\sqrt{c x^4+b x^2+a}}{x^2}\right)}{d^2\left(-b+\sqrt{-4ac+b^2}\right)^2\left(b+\sqrt{-4ac+b^2}\right)^2\sqrt{a}}+\frac{2c e^3\ln\left(\cdots\right)}{\cdots}$
default	$\frac{-\frac{1}{2a x^2\sqrt{c x^4+b x^2+a}}-\frac{3b\left(\frac{1}{a\sqrt{c x^4+b x^2+a}}-\frac{b\left(2c x^2+b\right)}{a\left(4ac-b^2\right)\sqrt{c x^4+b x^2+a}}-\frac{\ln\left(\frac{2a+b x^2+2\sqrt{a}\sqrt{c x^4+b x^2+a}}{x^2}\right)}{a^{\frac{3}{2}}}\right)}{4a}-\frac{2c\left(2c x^2+b\right)}{a\left(4ac-b^2\right)\sqrt{c x^4+b x^2\cdots}}\cdots}{d}$

```
[In] int(1/x^3/(e*x^2+d)/(c*x^4+b*x^2+a)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2/a^2/d*(c*x^4+b*x^2+a)^(1/2)/x^2-1/2/a^2/d*(-1/2*(2*a*e+3*b*d)/d/a^(1/2
)*ln((2*a+b*x^2+2*a^(1/2)*(c*x^4+b*x^2+a)^(1/2))/x^2)+a^2*e^3/d/(a*e^2-b*d*
e+c*d^2)/((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*ln((2*(a*e^2-b*d*e+c*d^2)/e^2+(b*e
-2*c*d)/e*(x^2+1/e*d)+2*((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*(c*(x^2+1/e*d)^2+(b
*e-2*c*d)/e*(x^2+1/e*d)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/(x^2+1/e*d))-2*d/(a
*e^2-b*d*e+c*d^2)*(-(a*c^2*e-b^2*c*e+b*c^2*d)/(c*x^4+b*x^2+a)^(1/2)*(b*x^2+
2*a)/(4*a*c-b^2)+(2*a*b*c*e-a*c^2*d-b^3*e+b^2*c*d)*(2*c*x^2+b)/(4*a*c-b^2)/
(c*x^4+b*x^2+a)^(1/2)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1597 vs. $2(379) = 758$.

Time = 5.17 (sec) , antiderivative size = 6486, normalized size of antiderivative = 15.48

$$\int \frac{1}{x^3\left(d+ex^2\right)\left(a+bx^2+cx^4\right)^{3/2}}\,dx = \text{Too large to display}$$

```
[In] integrate(1/x^3/(e*x^2+d)/(c*x^4+b*x^2+a)^(3/2),x, algorithm="fricas")
```

```
[Out] Too large to include
```

Sympy [F]

$$\int \frac{1}{x^3 (d+ex^2) (a+bx^2+cx^4)^{3/2}} \, dx = \int \frac{1}{x^3 (d+ex^2) (a+bx^2+cx^4)^{\frac{3}{2}}} \, dx$$

```
[In] integrate(1/x**3/(e*x**2+d)/(c*x**4+b*x**2+a)**(3/2),x)
[Out] Integral(1/(x**3*(d + e*x**2)*(a + b*x**2 + c*x**4)**(3/2)), x)
```

Maxima [F]

$$\int \frac{1}{x^3 (d+ex^2) (a+bx^2+cx^4)^{3/2}} \, dx = \int \frac{1}{(cx^4+bx^2+a)^{\frac{3}{2}} (ex^2+d) x^3} \, dx$$

```
[In] integrate(1/x^3/(e*x^2+d)/(c*x^4+b*x^2+a)^(3/2),x, algorithm="maxima")
[Out] integrate(1/((c*x^4 + b*x^2 + a)^(3/2)*(e*x^2 + d)*x^3), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 777 vs. $2(379) = 758$.

Time = 0.48 (sec) , antiderivative size = 777, normalized size of antiderivative = 1.85

$$\int \frac{1}{x^3 (d+ex^2) (a+bx^2+cx^4)^{3/2}} \, dx = \frac{e^4 \arctan\left(-\frac{\left(\sqrt{c}x^2-\sqrt{cx^4+bx^2+a}\right)e+\sqrt{c}d}{\sqrt{-cd^2+bde-ae^2}}\right)}{(cd^4-bd^3e+ad^2e^2)\sqrt{-cd^2+bde-ae^2}} - \frac{\frac{(a^2b^2c^3d^3-2\,a^3c^4d^3-2\,a^2b^3c^2d^2e+5\,a^3bc^3d^2e+a^2b^4cde^2-2\,a^3b^2c^2de^2-2\,a^4c^3de^2-a^3b^3ce^3+3\,a^4bc^2e^3)x^2}{a^4b^2c^2d^4-4\,a^5c^3d^4-2\,a^4b^3cd^3e+8\,a^5bc^2d^3e+a^4b^4d^2e^2-2\,a^5b^2cd^2e^2-8\,a^6c^2d^2e^2-2\,a^5b^3de^3+8\,a^6bcde^3+a^6b^2e^4-4\,a^7ce^4} + \frac{a^2b^3c^2d^3-3\,a^3bc^3 \cdots}{a^4b^2c^2d^4-4\,a^5c^3d \cdots}}{\sqrt{cx^4+bx^2+a}} - \frac{(3\,bd+2\,ae)\arctan\left(-\frac{\sqrt{c}x^2-\sqrt{cx^4+bx^2+a}}{\sqrt{-a}}\right)}{2\,\sqrt{-a}a^2d^2} + \frac{(\sqrt{c}x^2-\sqrt{cx^4+bx^2+a})b+2\,a\sqrt{c}}{2\left(\left(\sqrt{c}x^2-\sqrt{cx^4+bx^2+a}\right)^2-a\right)a^2d}$$

```
[In] integrate(1/x^3/(e*x^2+d)/(c*x^4+b*x^2+a)^(3/2),x, algorithm="giac")
[Out] e^4*arctan(-((sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))*e + sqrt(c)*d)/sqrt(-c
*d^2 + b*d*e - a*e^2))/((c*d^4 - b*d^3*e + a*d^2*e^2)*sqrt(-c*d^2 + b*d*e -
 a*e^2)) - ((a^2*b^2*c^3*d^3 - 2*a^3*c^4*d^3 - 2*a^2*b^3*c^2*d^2*e + 5*a^3*
b*c^3*d^2*e + a^2*b^4*c*d*e^2 - 2*a^3*b^2*c^2*d*e^2 - 2*a^4*c^3*d*e^2 - a^3
*b^3*c*e^3 + 3*a^4*b*c^2*e^3)*x^2/(a^4*b^2*c^2*d^4 - 4*a^5*c^3*d^4 - 2*a^4*
b^3*c*d^3*e + 8*a^5*b*c^2*d^3*e + a^4*b^4*d^2*e^2 - 2*a^5*b^2*c*d^2*e^2 - 8
*a^6*c^2*d^2*e^2 - 2*a^5*b^3*d*e^3 + 8*a^6*b*c*d*e^3 + a^6*b^2*e^4 - 4*a^7*
```

```
c*e^4) + (a^2*b^3*c^2*d^3 - 3*a^3*b*c^3*d^3 - 2*a^2*b^4*c*d^2*e + 7*a^3*b^2
*c^2*d^2*e - 2*a^4*c^3*d^2*e + a^2*b^5*d*e^2 - 3*a^3*b^3*c*d*e^2 - a^4*b*c^
2*d*e^2 - a^3*b^4*e^3 + 4*a^4*b^2*c*e^3 - 2*a^5*c^2*e^3)/(a^4*b^2*c^2*d^4 -
 4*a^5*c^3*d^4 - 2*a^4*b^3*c*d^3*e + 8*a^5*b*c^2*d^3*e + a^4*b^4*d^2*e^2 -
2*a^5*b^2*c*d^2*e^2 - 8*a^6*c^2*d^2*e^2 - 2*a^5*b^3*d*e^3 + 8*a^6*b*c*d*e^3
 + a^6*b^2*e^4 - 4*a^7*c*e^4))/sqrt(c*x^4 + b*x^2 + a) - 1/2*(3*b*d + 2*a*e
)*arctan(-(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))/sqrt(-a))/(sqrt(-a)*a^2*d
^2) + 1/2*((sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))*b + 2*a*sqrt(c))/(((sqrt
(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))^2 - a)*a^2*d)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^3\left(d+ex^2\right)\left(a+bx^2+cx^4\right)^{3/2}}\,dx=\int \frac{1}{x^3\,\left(e\,x^2+d\right)\,\left(c\,x^4+b\,x^2+a\right)^{3/2}}\,dx$$

```
[In] int(1/(x^3*(d + e*x^2)*(a + b*x^2 + c*x^4)^(3/2)),x)
[Out] int(1/(x^3*(d + e*x^2)*(a + b*x^2 + c*x^4)^(3/2)), x)
```

3.348 $\int \frac{x^8}{(3+2x^2)(1+2x^2+2x^4)^{3/2}} \, dx$

Optimal result

Integrand size = 29, antiderivative size = 449

$$\int \frac{x^8}{(3+2x^2)(1+2x^2+2x^4)^{3/2}} \, dx = \frac{x^3(1-2x^2)}{20\sqrt{1+2x^2+2x^4}} + \frac{1}{20}x\sqrt{1+2x^2+2x^4} + \frac{x\sqrt{1+2x^2+2x^4}}{10\sqrt{2}\left(1+\sqrt{2}x^2\right)} + \frac{27}{80}\sqrt{\frac{3}{5}}\arctan\left(\frac{\sqrt{\frac{5}{3}}x}{\sqrt{1+2x^2+2x^4}}\right) - \frac{\left(1+\sqrt{2}x^2\right)\sqrt{\frac{1+2x^2+2x^4}{\left(1+\sqrt{2}x^2\right)^2}}E\left(2\arctan\left(\sqrt[4]{2}x\right)|\frac{1}{4}\left(2-\sqrt{2}\right)\right)}{10\ 2^{3/4}\sqrt{1+2x^2+2x^4}} + \frac{\left(-2+7\sqrt{2}\right)\left(1+\sqrt{2}x^2\right)\sqrt{\frac{1+2x^2+2x^4}{\left(1+\sqrt{2}x^2\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\sqrt[4]{2}x\right),\frac{1}{4}\left(2-\sqrt{2}\right)\right)}{8\ 2^{3/4}\left(-2+3\sqrt{2}\right)\sqrt{1+2x^2+2x^4}} + \frac{27\left(3+\sqrt{2}\right)\left(1+\sqrt{2}x^2\right)\sqrt{\frac{1+2x^2+2x^4}{\left(1+\sqrt{2}x^2\right)^2}}\operatorname{EllipticPi}\left(\frac{1}{24}\left(12-11\sqrt{2}\right),2\arctan\left(\sqrt[4]{2}x\right),\frac{1}{4}\left(2-\sqrt{2}\right)\right)}{80\ 2^{3/4}\left(2-3\sqrt{2}\right)\sqrt{1+2x^2+2x^4}}$$

```
[Out] 27/400*arctan(1/3*x*15^(1/2)/(2*x^4+2*x^2+1)^(1/2))*15^(1/2)+1/20*x^3*(-2*x
^2+1)/(2*x^4+2*x^2+1)^(1/2)+1/20*x*(2*x^4+2*x^2+1)^(1/2)+1/20*x*(2*x^4+2*x^
2+1)^(1/2)*2^(1/2)/(1+x^2*2^(1/2))-1/20*(cos(2*arctan(2^(1/4)*x))^2)^(1/2)/
cos(2*arctan(2^(1/4)*x))*EllipticE(sin(2*arctan(2^(1/4)*x)),1/2*(2-2^(1/2))
^(1/2))*(1+x^2*2^(1/2))*((2*x^4+2*x^2+1)/(1+x^2*2^(1/2))^2)^(1/2)*2^(1/4)/(
2*x^4+2*x^2+1)^(1/2)+27/160*(cos(2*arctan(2^(1/4)*x))^2)^(1/2)/cos(2*arctan
(2^(1/4)*x))*EllipticPi(sin(2*arctan(2^(1/4)*x)),1/2-11/24*2^(1/2),1/2*(2-2
^(1/2))^(1/2))*(3+2^(1/2))*(1+x^2*2^(1/2))*((2*x^4+2*x^2+1)/(1+x^2*2^(1/2))
^2)^(1/2)*2^(1/4)/(2-3*2^(1/2))/(2*x^4+2*x^2+1)^(1/2)+1/16*(cos(2*arctan(2^
(1/4)*x))^2)^(1/2)/cos(2*arctan(2^(1/4)*x))*EllipticF(sin(2*arctan(2^(1/4)*
```

```
x)),1/2*(2-2^(1/2))^(1/2))*(-2+7*2^(1/2))*(1+x^2*2^(1/2))*((2*x^4+2*x^2+1)/
(1+x^2*2^(1/2))^2)^(1/2)*2^(1/4)/(-2+3*2^(1/2))/(2*x^4+2*x^2+1)^(1/2)
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 566, normalized size of antiderivative = 1.26, number of steps used = 10, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.276$, Rules used = {1327, 1289, 1293, 1211, 1117, 1209, 1339, 1720}

$$\int \frac{x^8}{(3+2x^2)(1+2x^2+2x^4)^{3/2}}\,dx = -\frac{27\sqrt{\frac{3}{10}}(3-\sqrt{2})\arctan\left(\frac{\sqrt{\frac{5}{3}}x}{\sqrt{2x^4+2x^2+1}}\right)}{40(2-3\sqrt{2})} - \frac{(7+\sqrt{2})(\sqrt{2}x^2+1)\sqrt{\frac{2x^4+2x^2+1}{(\sqrt{2}x^2+1)^2}}\,\text{EllipticF}\left(2\arctan\left(\sqrt[4]{2}x\right),\frac{1}{4}(2-\sqrt{2})\right)}{40\,2^{3/4}\sqrt{2x^4+2x^2+1}} + \frac{9(1-3\sqrt{2})(\sqrt{2}x^2+1)\sqrt{\frac{2x^4+2x^2+1}{(\sqrt{2}x^2+1)^2}}\,\text{EllipticF}\left(2\arctan\left(\sqrt[4]{2}x\right),\frac{1}{4}(2-\sqrt{2})\right)}{20\,2^{3/4}(2-3\sqrt{2})\sqrt{2x^4+2x^2+1}} - \frac{(\sqrt{2}x^2+1)\sqrt{\frac{2x^4+2x^2+1}{(\sqrt{2}x^2+1)^2}}E\left(2\arctan\left(\sqrt[4]{2}x\right)|\frac{1}{4}(2-\sqrt{2})\right)}{10\,2^{3/4}\sqrt{2x^4+2x^2+1}} + \frac{27(3+\sqrt{2})(\sqrt{2}x^2+1)\sqrt{\frac{2x^4+2x^2+1}{(\sqrt{2}x^2+1)^2}}\,\text{EllipticPi}\left(\frac{1}{24}(12-11\sqrt{2}),2\arctan\left(\sqrt[4]{2}x\right),\frac{1}{4}(2-\sqrt{2})\right)}{80\,2^{3/4}(2-3\sqrt{2})\sqrt{2x^4+2x^2+1}} + \frac{\sqrt{2x^4+2x^2+1}x}{10\sqrt{2}(\sqrt{2}x^2+1)} + \frac{1}{20}\sqrt{2x^4+2x^2+1}x + \frac{(1-2x^2)x^3}{20\sqrt{2x^4+2x^2+1}}$$

```
[In] Int[x^8/((3 + 2*x^2)*(1 + 2*x^2 + 2*x^4)^(3/2)),x]
```

```
[Out] (x^3*(1 - 2*x^2))/(20*Sqrt[1 + 2*x^2 + 2*x^4]) + (x*Sqrt[1 + 2*x^2 + 2*x^4]
)/20 + (x*Sqrt[1 + 2*x^2 + 2*x^4])/(10*Sqrt[2]*(1 + Sqrt[2]*x^2)) - (27*Sqr
t[3/10]*(3 - Sqrt[2])*ArcTan[(Sqrt[5/3]*x)/Sqrt[1 + 2*x^2 + 2*x^4]])/(40*(2
 - 3*Sqrt[2])) - ((1 + Sqrt[2]*x^2)*Sqrt[(1 + 2*x^2 + 2*x^4)/(1 + Sqrt[2]*x
^2)^2]*EllipticE[2*ArcTan[2^(1/4)*x], (2 - Sqrt[2])/4])/(10*2^(3/4)*Sqrt[1
+ 2*x^2 + 2*x^4]) + (9*(1 - 3*Sqrt[2])*(1 + Sqrt[2]*x^2)*Sqrt[(1 + 2*x^2 +
2*x^4)/(1 + Sqrt[2]*x^2)^2]*EllipticF[2*ArcTan[2^(1/4)*x], (2 - Sqrt[2])/4]
)/(20*2^(3/4)*(2 - 3*Sqrt[2])*Sqrt[1 + 2*x^2 + 2*x^4]) - ((7 + Sqrt[2])*(1
+ Sqrt[2]*x^2)*Sqrt[(1 + 2*x^2 + 2*x^4)/(1 + Sqrt[2]*x^2)^2]*EllipticF[2*Ar
cTan[2^(1/4)*x], (2 - Sqrt[2])/4])/(40*2^(3/4)*Sqrt[1 + 2*x^2 + 2*x^4]) + (
27*(3 + Sqrt[2])*(1 + Sqrt[2]*x^2)*Sqrt[(1 + 2*x^2 + 2*x^4)/(1 + Sqrt[2]*x^
2)^2]*EllipticPi[(12 - 11*Sqrt[2])/24, 2*ArcTan[2^(1/4)*x], (2 - Sqrt[2])/4
])/(80*2^(3/4)*(2 - 3*Sqrt[2])*Sqrt[1 + 2*x^2 + 2*x^4])
```

Rule 1117

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1209

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

Rule 1211

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + b*x^2 + c*x^4
], x], x] - Dist[e/q, Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /; Ne
Q[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[
c/a]
```

Rule 1289

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_.), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(a + b*x^2 + c*x^4)^(p + 1)
*((b*d - 2*a*e - (b*e - 2*c*d)*x^2)/(2*(p + 1)*(b^2 - 4*a*c))), x] - Dist[f
^2/(2*(p + 1)*(b^2 - 4*a*c)), Int[(f*x)^(m - 2)*(a + b*x^2 + c*x^4)^(p + 1)
*Simp[(m - 1)*(b*d - 2*a*e) - (4*p + 4 + m + 1)*(b*e - 2*c*d)*x^2, x], x],
x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] &&
GtQ[m, 1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1293

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[e*f*(f*x)^(m - 1)*((a + b*x^2 + c*x^4)^(p +
1)/(c*(m + 4*p + 3))), x] - Dist[f^2/(c*(m + 4*p + 3)), Int[(f*x)^(m - 2)*(
a + b*x^2 + c*x^4)^p*Simp[a*e*(m - 1) + (b*e*(m + 2*p + 1) - c*d*(m + 4*p +
 3))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c,
 0] && GtQ[m, 1] && NeQ[m + 4*p + 3, 0] && IntegerQ[2*p] && (IntegerQ[p] ||
 IntegerQ[m])
```

Rule 1327

```
Int[(((f_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_))/((d_.)
+ (e_.)*(x_)^2), x_Symbol] :> Dist[-f^4/(c*d^2 - b*d*e + a*e^2), Int[(f*x)^
(m - 4)*(a*d + (b*d - a*e)*x^2)*(a + b*x^2 + c*x^4)^p, x], x] + Dist[d^2*(f
^4/(c*d^2 - b*d*e + a*e^2)), Int[(f*x)^(m - 4)*((a + b*x^2 + c*x^4)^(p + 1)
/(d + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b^2 - 4*a*c, 0
] && LtQ[p, -1] && GtQ[m, 2]
```

Rule 1339

```
Int[(x_)^4/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4])
, x_Symbol] :> With[{q = Rt[c/a, 2]}, Dist[-(2*c*d - a*e*q)/(c*e*(e - d*q))
, Int[1/Sqrt[a + b*x^2 + c*x^4], x], x] + (-Dist[1/(e*q), Int[(1 - q*x^2)/S
qrt[a + b*x^2 + c*x^4], x], x] + Dist[d^2/(e*(e - d*q)), Int[(1 + q*x^2)/((
d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]), x], x])] /; FreeQ[{a, b, c, d, e}, x]
&& NeQ[b^2 - 4*a*c, 0] && PosQ[c/a] && NeQ[c*d^2 - a*e^2, 0]
```

Rule 1720

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4]), x_Symbol] :> With[{q = Rt[B/A, 2]}, Simp[(-(B*d - A*e))*(A
rcTan[Rt[-b + c*(d/e) + a*(e/d), 2]*(x/Sqrt[a + b*x^2 + c*x^4])]/(2*d*e*Rt[
-b + c*(d/e) + a*(e/d), 2])), x] + Simp[(B*d + A*e)*(A + B*x^2)*(Sqrt[A^2*(
(a + b*x^2 + c*x^4)/(a*(A + B*x^2)^2))]/(4*d*e*A*q*Sqrt[a + b*x^2 + c*x^4])
)*EllipticPi[Cancel[-(B*d - A*e)^2/(4*d*e*A*B)], 2*ArcTan[q*x], 1/2 - b*(A/
(4*a*B))], x]] /; FreeQ[{a, b, c, d, e, A, B}, x] && NeQ[b^2 - 4*a*c, 0] &&
 NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] && EqQ
[c*A^2 - a*B^2, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\left(\frac{1}{10}\int \frac{x^4(3+4x^2)}{(1+2x^2+2x^4)^{3/2}}\,dx\right)+\frac{9}{10}\int \frac{x^4}{(3+2x^2)\sqrt{1+2x^2+2x^4}}\,dx \\
&= \frac{x^3(1-2x^2)}{20\sqrt{1+2x^2+2x^4}}+\frac{1}{40}\int \frac{x^2(-6+12x^2)}{\sqrt{1+2x^2+2x^4}}\,dx-\frac{9\int \frac{1-\sqrt{2}x^2}{\sqrt{1+2x^2+2x^4}}\,dx}{20\sqrt{2}} \\
&\quad+\frac{81\int \frac{1+\sqrt{2}x^2}{(3+2x^2)\sqrt{1+2x^2+2x^4}}\,dx}{20\left(2-3\sqrt{2}\right)}+\frac{\left(9\left(-12+2\sqrt{2}\right)\right)\int \frac{1}{\sqrt{1+2x^2+2x^4}}\,dx}{40\left(2-3\sqrt{2}\right)}
\end{aligned}$$

$$= \frac{x^3(1-2x^2)}{20\sqrt{1+2x^2+2x^4}} + \frac{1}{20}x\sqrt{1+2x^2+2x^4}$$
$$+ \frac{9x\sqrt{1+2x^2+2x^4}}{20\sqrt{2}\left(1+\sqrt{2}x^2\right)} - \frac{27\sqrt{\frac{3}{10}}(3-\sqrt{2})\tan^{-1}\left(\frac{\sqrt{\frac{5}{3}}x}{\sqrt{1+2x^2+2x^4}}\right)}{40\left(2-3\sqrt{2}\right)}$$
$$- \frac{9\left(1+\sqrt{2}x^2\right)\sqrt{\frac{1+2x^2+2x^4}{\left(1+\sqrt{2}x^2\right)^2}}E\left(2\tan^{-1}\left(\sqrt[4]{2}x\right)|\frac{1}{4}\left(2-\sqrt{2}\right)\right)}{20\ 2^{3/4}\sqrt{1+2x^2+2x^4}}$$
$$+ \frac{9\left(1-3\sqrt{2}\right)\left(1+\sqrt{2}x^2\right)\sqrt{\frac{1+2x^2+2x^4}{\left(1+\sqrt{2}x^2\right)^2}}F\left(2\tan^{-1}\left(\sqrt[4]{2}x\right)|\frac{1}{4}\left(2-\sqrt{2}\right)\right)}{20\ 2^{3/4}\left(2-3\sqrt{2}\right)\sqrt{1+2x^2+2x^4}}$$
$$+ \frac{27\left(3+\sqrt{2}\right)\left(1+\sqrt{2}x^2\right)\sqrt{\frac{1+2x^2+2x^4}{\left(1+\sqrt{2}x^2\right)^2}}\Pi\left(\frac{1}{24}\left(12-11\sqrt{2}\right);2\tan^{-1}\left(\sqrt[4]{2}x\right)|\frac{1}{4}\left(2-\sqrt{2}\right)\right)}{80\ 2^{3/4}\left(2-3\sqrt{2}\right)\sqrt{1+2x^2+2x^4}}$$
$$- \frac{1}{240}\int \frac{12+84x^2}{\sqrt{1+2x^2+2x^4}}\,dx$$
$$= \frac{x^3(1-2x^2)}{20\sqrt{1+2x^2+2x^4}} + \frac{1}{20}x\sqrt{1+2x^2+2x^4}$$
$$+ \frac{9x\sqrt{1+2x^2+2x^4}}{20\sqrt{2}\left(1+\sqrt{2}x^2\right)} - \frac{27\sqrt{\frac{3}{10}}(3-\sqrt{2})\tan^{-1}\left(\frac{\sqrt{\frac{5}{3}}x}{\sqrt{1+2x^2+2x^4}}\right)}{40\left(2-3\sqrt{2}\right)}$$
$$- \frac{9\left(1+\sqrt{2}x^2\right)\sqrt{\frac{1+2x^2+2x^4}{\left(1+\sqrt{2}x^2\right)^2}}E\left(2\tan^{-1}\left(\sqrt[4]{2}x\right)|\frac{1}{4}\left(2-\sqrt{2}\right)\right)}{20\ 2^{3/4}\sqrt{1+2x^2+2x^4}}$$
$$+ \frac{9\left(1-3\sqrt{2}\right)\left(1+\sqrt{2}x^2\right)\sqrt{\frac{1+2x^2+2x^4}{\left(1+\sqrt{2}x^2\right)^2}}F\left(2\tan^{-1}\left(\sqrt[4]{2}x\right)|\frac{1}{4}\left(2-\sqrt{2}\right)\right)}{20\ 2^{3/4}\left(2-3\sqrt{2}\right)\sqrt{1+2x^2+2x^4}}$$
$$+ \frac{27\left(3+\sqrt{2}\right)\left(1+\sqrt{2}x^2\right)\sqrt{\frac{1+2x^2+2x^4}{\left(1+\sqrt{2}x^2\right)^2}}\Pi\left(\frac{1}{24}\left(12-11\sqrt{2}\right);2\tan^{-1}\left(\sqrt[4]{2}x\right)|\frac{1}{4}\left(2-\sqrt{2}\right)\right)}{80\ 2^{3/4}\left(2-3\sqrt{2}\right)\sqrt{1+2x^2+2x^4}}$$
$$+ \frac{7\int \frac{1-\sqrt{2}x^2}{\sqrt{1+2x^2+2x^4}}\,dx}{20\sqrt{2}} - \frac{1}{40}\left(2+7\sqrt{2}\right)\int \frac{1}{\sqrt{1+2x^2+2x^4}}\,dx$$

$$= \frac{x^3(1-2x^2)}{20\sqrt{1+2x^2+2x^4}} + \frac{1}{20}x\sqrt{1+2x^2+2x^4} + \frac{x\sqrt{1+2x^2+2x^4}}{10\sqrt{2}\left(1+\sqrt{2}x^2\right)} - \frac{27\sqrt{\frac{3}{10}}(3-\sqrt{2})\tan^{-1}\left(\frac{\sqrt{\frac{5}{3}}x}{\sqrt{1+2x^2+2x^4}}\right)}{40\left(2-3\sqrt{2}\right)} - \frac{\left(1+\sqrt{2}x^2\right)\sqrt{\frac{1+2x^2+2x^4}{\left(1+\sqrt{2}x^2\right)^2}}E\left(2\tan^{-1}\left(\sqrt[4]{2}x\right)|\frac{1}{4}(2-\sqrt{2})\right)}{10\ 2^{3/4}\sqrt{1+2x^2+2x^4}} + \frac{9(1-3\sqrt{2})\left(1+\sqrt{2}x^2\right)\sqrt{\frac{1+2x^2+2x^4}{\left(1+\sqrt{2}x^2\right)^2}}F\left(2\tan^{-1}\left(\sqrt[4]{2}x\right)|\frac{1}{4}(2-\sqrt{2})\right)}{20\ 2^{3/4}\left(2-3\sqrt{2}\right)\sqrt{1+2x^2+2x^4}} - \frac{(7+\sqrt{2})\left(1+\sqrt{2}x^2\right)\sqrt{\frac{1+2x^2+2x^4}{\left(1+\sqrt{2}x^2\right)^2}}F\left(2\tan^{-1}\left(\sqrt[4]{2}x\right)|\frac{1}{4}(2-\sqrt{2})\right)}{40\ 2^{3/4}\sqrt{1+2x^2+2x^4}} + \frac{27(3+\sqrt{2})\left(1+\sqrt{2}x^2\right)\sqrt{\frac{1+2x^2+2x^4}{\left(1+\sqrt{2}x^2\right)^2}}\Pi\left(\frac{1}{24}(12-11\sqrt{2});2\tan^{-1}\left(\sqrt[4]{2}x\right)|\frac{1}{4}(2-\sqrt{2})\right)}{80\ 2^{3/4}\left(2-3\sqrt{2}\right)\sqrt{1+2x^2+2x^4}}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.27 (sec) , antiderivative size = 199, normalized size of antiderivative = 0.44

$$\int \frac{x^8}{(3+2x^2)(1+2x^2+2x^4)^{3/2}}\,dx = \frac{4x+12x^3-4i\sqrt{1-i}\sqrt{1+(1-i)x^2}\sqrt{1+(1+i)x^2}E\left(i\text{arcsinh}\left(\sqrt{1-\right.\right.}{}$$

```
[In] Integrate[x^8/((3 + 2*x^2)*(1 + 2*x^2 + 2*x^4)^(3/2)),x]
```

```
[Out] (4*x + 12*x^3 - (4*I)*Sqrt[1 - I]*Sqrt[1 + (1 - I)*x^2]*Sqrt[1 + (1 + I)*x^
2]*EllipticE[I*ArcSinh[Sqrt[1 - I]*x], I] - (29 - 33*I)*Sqrt[1 - I]*Sqrt[1
+ (1 - I)*x^2]*Sqrt[1 + (1 + I)*x^2]*EllipticF[I*ArcSinh[Sqrt[1 - I]*x], I]
 + 27*(1 - I)^(3/2)*Sqrt[1 + (1 - I)*x^2]*Sqrt[1 + (1 + I)*x^2]*EllipticPi[
1/3 + I/3, I*ArcSinh[Sqrt[1 - I]*x], I])/(80*Sqrt[1 + 2*x^2 + 2*x^4])
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 4.94 (sec) , antiderivative size = 246, normalized size of antiderivative = 0.55

method	result
risch	$\frac{x(3x^2+1)}{20\sqrt{2x^4+2x^2+1}}-\frac{29\sqrt{1+(1-i)x^2}\,\sqrt{1+(1+i)x^2}\,F\left(x\sqrt{-1+i},\frac{\sqrt{2}}{2}+\frac{i\sqrt{2}}{2}\right)}{40\sqrt{-1+i}\,\sqrt{2x^4+2x^2+1}}+\frac{\left(-\frac{1}{20}+\frac{i}{20}\right)\sqrt{1+(1-i)x^2}\,\sqrt{1+(1+i)x^2}\,\left(F\left(x\sqrt{-1+i}\right.\right.}{\sqrt{-1+i}\,\sqrt{2x^4+2x}}$
elliptic	$-\frac{4\left(-\frac{3}{80}x^3-\frac{1}{80}x\right)}{\sqrt{2x^4+2x^2+1}}-\frac{31\sqrt{-ix^2+x^2+1}\,\sqrt{ix^2+x^2+1}\,F\left(x\sqrt{-1+i},\frac{\sqrt{2}}{2}+\frac{i\sqrt{2}}{2}\right)}{40\sqrt{-1+i}\,\sqrt{2x^4+2x^2+1}}+\frac{i\sqrt{-ix^2+x^2+1}\,\sqrt{ix^2+x^2+1}\,F\left(x\sqrt{-1+i},\frac{\sqrt{2}}{2}+\frac{i\sqrt{2}}{2}\right.}{20\sqrt{-1+i}\,\sqrt{2x^4+2x^2+1}}$
default	$\frac{27x^3}{16\sqrt{2x^4+2x^2+1}}-\frac{11\sqrt{1+(1-i)x^2}\,\sqrt{1+(1+i)x^2}\,F\left(x\sqrt{-1+i},\frac{\sqrt{2}}{2}+\frac{i\sqrt{2}}{2}\right)}{4\sqrt{-1+i}\,\sqrt{2x^4+2x^2+1}}+\frac{\left(\frac{47}{32}-\frac{47i}{32}\right)\sqrt{1+(1-i)x^2}\,\sqrt{1+(1+i)x^2}\,\left(F\left(x\sqrt{-1+i},\right.\right.}{\sqrt{-1+i}\,\sqrt{2x^4+2x^2}}$

```
[In] int(x^8/(2*x^2+3)/(2*x^4+2*x^2+1)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/20*x*(3*x^2+1)/(2*x^4+2*x^2+1)^(1/2)-29/40/(-1+I)^(1/2)*(1+(1-I)*x^2)^(1/
2)*(1+(1+I)*x^2)^(1/2)/(2*x^4+2*x^2+1)^(1/2)*EllipticF(x*(-1+I)^(1/2),1/2*2
^(1/2)+1/2*I*2^(1/2))+(-1/20+1/20*I)/(-1+I)^(1/2)*(1+(1-I)*x^2)^(1/2)*(1+(1
+I)*x^2)^(1/2)/(2*x^4+2*x^2+1)^(1/2)*(EllipticF(x*(-1+I)^(1/2),1/2*2^(1/2)+
1/2*I*2^(1/2))-EllipticE(x*(-1+I)^(1/2),1/2*2^(1/2)+1/2*I*2^(1/2)))+27/40/(
-1+I)^(1/2)*(1-I*x^2+x^2)^(1/2)*(1+I*x^2+x^2)^(1/2)/(2*x^4+2*x^2+1)^(1/2)*E
llipticPi(x*(-1+I)^(1/2),1/3+1/3*I,(-1-I)^(1/2)/(-1+I)^(1/2))
```

Fricas [F]

$$\int \frac{x^8}{(3+2x^2)\left(1+2x^2+2x^4\right)^{3/2}}\,dx=\int \frac{x^8}{\left(2\,x^4+2\,x^2+1\right)^{\frac{3}{2}}\left(2\,x^2+3\right)}\,dx$$

```
[In] integrate(x^8/(2*x^2+3)/(2*x^4+2*x^2+1)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(2*x^4 + 2*x^2 + 1)*x^8/(8*x^10 + 28*x^8 + 40*x^6 + 32*x^4 + 1
4*x^2 + 3), x)
```

Sympy [F]

$$\int \frac{x^8}{(3+2x^2)\left(1+2x^2+2x^4\right)^{3/2}}\,dx=\int \frac{x^8}{\left(2x^2+3\right)\left(2x^4+2x^2+1\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(x**8/(2*x**2+3)/(2*x**4+2*x**2+1)**(3/2),x)
```

```
[Out] Integral(x**8/((2*x**2 + 3)*(2*x**4 + 2*x**2 + 1)**(3/2)), x)
```

Maxima [F]

$$\int \frac{x^8}{(3+2x^2)(1+2x^2+2x^4)^{3/2}}\,dx = \int \frac{x^8}{(2\,x^4+2\,x^2+1)^{\frac{3}{2}}(2\,x^2+3)}\,dx$$

```
[In] integrate(x^8/(2*x^2+3)/(2*x^4+2*x^2+1)^(3/2),x, algorithm="maxima")
[Out] integrate(x^8/((2*x^4 + 2*x^2 + 1)^(3/2)*(2*x^2 + 3)), x)
```

Giac [F]

$$\int \frac{x^8}{(3+2x^2)(1+2x^2+2x^4)^{3/2}}\,dx = \int \frac{x^8}{(2\,x^4+2\,x^2+1)^{\frac{3}{2}}(2\,x^2+3)}\,dx$$

```
[In] integrate(x^8/(2*x^2+3)/(2*x^4+2*x^2+1)^(3/2),x, algorithm="giac")
[Out] integrate(x^8/((2*x^4 + 2*x^2 + 1)^(3/2)*(2*x^2 + 3)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^8}{(3+2x^2)(1+2x^2+2x^4)^{3/2}}\,dx = \int \frac{x^8}{(2\,x^2+3)\,(2\,x^4+2\,x^2+1)^{3/2}}\,dx$$

```
[In] int(x^8/((2*x^2 + 3)*(2*x^2 + 2*x^4 + 1)^(3/2)),x)
[Out] int(x^8/((2*x^2 + 3)*(2*x^2 + 2*x^4 + 1)^(3/2)), x)
```

3.349 $\int \frac{x^6}{(3+2x^2)(1+2x^2+2x^4)^{3/2}} dx$

Optimal result

Integrand size = 29, antiderivative size = 423

$$\int \frac{x^6}{(3+2x^2)(1+2x^2+2x^4)^{3/2}} dx = \frac{x(1-2x^2)}{20\sqrt{1+2x^2+2x^4}} + \frac{x\sqrt{1+2x^2+2x^4}}{10\sqrt{2}\left(1+\sqrt{2}x^2\right)} - \frac{9}{40}\sqrt{\frac{3}{5}}\arctan\left(\frac{\sqrt{\frac{5}{3}}x}{\sqrt{1+2x^2+2x^4}}\right) - \frac{\left(1+\sqrt{2}x^2\right)\sqrt{\frac{1+2x^2+2x^4}{\left(1+\sqrt{2}x^2\right)^2}}E\left(2\arctan\left(\sqrt[4]{2}x\right)\middle|\frac{1}{4}\left(2-\sqrt{2}\right)\right)}{10\ 2^{3/4}\sqrt{1+2x^2+2x^4}} - \frac{\left(\sqrt[4]{2}+2^{3/4}\right)\left(1+\sqrt{2}x^2\right)\sqrt{\frac{1+2x^2+2x^4}{\left(1+\sqrt{2}x^2\right)^2}}\text{EllipticF}\left(2\arctan\left(\sqrt[4]{2}x\right),\frac{1}{4}\left(2-\sqrt{2}\right)\right)}{8\left(-2+3\sqrt{2}\right)\sqrt{1+2x^2+2x^4}} - \frac{9\left(3+\sqrt{2}\right)\left(1+\sqrt{2}x^2\right)\sqrt{\frac{1+2x^2+2x^4}{\left(1+\sqrt{2}x^2\right)^2}}\text{EllipticPi}\left(\frac{1}{24}\left(12-11\sqrt{2}\right),2\arctan\left(\sqrt[4]{2}x\right),\frac{1}{4}\left(2-\sqrt{2}\right)\right)}{40\ 2^{3/4}\left(2-3\sqrt{2}\right)\sqrt{1+2x^2+2x^4}}$$

```
[Out] -9/200*arctan(1/3*x*15^(1/2)/(2*x^4+2*x^2+1)^(1/2))*15^(1/2)+1/20*x*(-2*x^2
+1)/(2*x^4+2*x^2+1)^(1/2)+1/20*x*(2*x^4+2*x^2+1)^(1/2)*2^(1/2)/(1+x^2*2^(1/
2))-1/20*(cos(2*arctan(2^(1/4)*x))^2)^(1/2)/cos(2*arctan(2^(1/4)*x))*Ellipt
icE(sin(2*arctan(2^(1/4)*x)),1/2*(2-2^(1/2))^(1/2))*(1+x^2*2^(1/2))*((2*x^4
+2*x^2+1)/(1+x^2*2^(1/2))^2)^(1/2)*2^(1/4)/(2*x^4+2*x^2+1)^(1/2)-9/80*(cos(
2*arctan(2^(1/4)*x))^2)^(1/2)/cos(2*arctan(2^(1/4)*x))*EllipticPi(sin(2*arc
tan(2^(1/4)*x)),1/2-11/24*2^(1/2),1/2*(2-2^(1/2))^(1/2))*(3+2^(1/2))*(1+x^2
*2^(1/2))*((2*x^4+2*x^2+1)/(1+x^2*2^(1/2))^2)^(1/2)*2^(1/4)/(2-3*2^(1/2))/(
2*x^4+2*x^2+1)^(1/2)-1/8*(2^(1/4)+2^(3/4))*(cos(2*arctan(2^(1/4)*x))^2)^(1/
2)/cos(2*arctan(2^(1/4)*x))*EllipticF(sin(2*arctan(2^(1/4)*x)),1/2*(2-2^(1/
```

```
2))^(1/2))*(1+x^2*2^(1/2))*((2*x^4+2*x^2+1)/(1+x^2*2^(1/2))^2)^(1/2)/(-2+3*
2^(1/2))/(2*x^4+2*x^2+1)^(1/2)
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 503, normalized size of antiderivative = 1.19, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.241, Rules used = {1327, 1289, 1211, 1117, 1209, 1333, 1720}

$$\int \frac{x^6}{(3+2x^2)(1+2x^2+2x^4)^{3/2}}\,dx = -\frac{9}{40}\sqrt{\frac{3}{5}}\arctan\left(\frac{\sqrt{\frac{5}{3}}x}{\sqrt{2x^4+2x^2+1}}\right) - \frac{9(3+\sqrt{2})(\sqrt{2}x^2+1)\sqrt{\frac{2x^4+2x^2+1}{(\sqrt{2}x^2+1)^2}}\text{EllipticF}\left(2\arctan\left(\sqrt[4]{2}x\right),\frac{1}{4}(2-\sqrt{2})\right)}{140\,2^{3/4}\sqrt{2x^4+2x^2+1}} - \frac{(1-\sqrt{2})(\sqrt{2}x^2+1)\sqrt{\frac{2x^4+2x^2+1}{(\sqrt{2}x^2+1)^2}}\text{EllipticF}\left(2\arctan\left(\sqrt[4]{2}x\right),\frac{1}{4}(2-\sqrt{2})\right)}{40\sqrt[4]{2}\sqrt{2x^4+2x^2+1}} - \frac{(\sqrt{2}x^2+1)\sqrt{\frac{2x^4+2x^2+1}{(\sqrt{2}x^2+1)^2}}E\left(2\arctan\left(\sqrt[4]{2}x\right)|\frac{1}{4}(2-\sqrt{2})\right)}{10\,2^{3/4}\sqrt{2x^4+2x^2+1}} + \frac{9(3+\sqrt{2})^2(\sqrt{2}x^2+1)\sqrt{\frac{2x^4+2x^2+1}{(\sqrt{2}x^2+1)^2}}\text{EllipticPi}\left(\frac{1}{24}(12-11\sqrt{2}),2\arctan\left(\sqrt[4]{2}x\right),\frac{1}{4}(2-\sqrt{2})\right)}{560\sqrt[4]{2}\sqrt{2x^4+2x^2+1}} + \frac{\sqrt{2x^4+2x^2+1}x}{10\sqrt{2}(\sqrt{2}x^2+1)} + \frac{(1-2x^2)x}{20\sqrt{2x^4+2x^2+1}}$$

```
[In] Int[x^6/((3 + 2*x^2)*(1 + 2*x^2 + 2*x^4)^(3/2)),x]
```

```
[Out] (x*(1 - 2*x^2))/(20*Sqrt[1 + 2*x^2 + 2*x^4]) + (x*Sqrt[1 + 2*x^2 + 2*x^4])/
(10*Sqrt[2]*(1 + Sqrt[2]*x^2)) - (9*Sqrt[3/5]*ArcTan[(Sqrt[5/3]*x)/Sqrt[1 +
 2*x^2 + 2*x^4]])/40 - ((1 + Sqrt[2]*x^2)*Sqrt[(1 + 2*x^2 + 2*x^4)/(1 + Sqr
t[2]*x^2)^2]*EllipticE[2*ArcTan[2^(1/4)*x], (2 - Sqrt[2])/4])/(10*2^(3/4)*S
qrt[1 + 2*x^2 + 2*x^4]) - ((1 - Sqrt[2])*(1 + Sqrt[2]*x^2)*Sqrt[(1 + 2*x^2
+ 2*x^4)/(1 + Sqrt[2]*x^2)^2]*EllipticF[2*ArcTan[2^(1/4)*x], (2 - Sqrt[2])/
4])/(40*2^(1/4)*Sqrt[1 + 2*x^2 + 2*x^4]) - (9*(3 + Sqrt[2])*(1 + Sqrt[2]*x^
2)*Sqrt[(1 + 2*x^2 + 2*x^4)/(1 + Sqrt[2]*x^2)^2]*EllipticF[2*ArcTan[2^(1/4)
*x], (2 - Sqrt[2])/4])/(140*2^(3/4)*Sqrt[1 + 2*x^2 + 2*x^4]) + (9*(3 + Sqrt
[2])^2*(1 + Sqrt[2]*x^2)*Sqrt[(1 + 2*x^2 + 2*x^4)/(1 + Sqrt[2]*x^2)^2]*Elli
pticPi[(12 - 11*Sqrt[2])/24, 2*ArcTan[2^(1/4)*x], (2 - Sqrt[2])/4])/(560*2^
(1/4)*Sqrt[1 + 2*x^2 + 2*x^4])
```

Rule 1117

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1209

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

Rule 1211

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + b*x^2 + c*x^4
], x], x] - Dist[e/q, Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /; Ne
Q[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[
c/a]
```

Rule 1289

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_.), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(a + b*x^2 + c*x^4)^(p + 1)
*((b*d - 2*a*e - (b*e - 2*c*d)*x^2)/(2*(p + 1)*(b^2 - 4*a*c))), x] - Dist[f
^2/(2*(p + 1)*(b^2 - 4*a*c)), Int[(f*x)^(m - 2)*(a + b*x^2 + c*x^4)^(p + 1)
*Simp[(m - 1)*(b*d - 2*a*e) - (4*p + 4 + m + 1)*(b*e - 2*c*d)*x^2, x], x],
x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] &&
GtQ[m, 1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1327

```
Int[(((f_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_))/((d_.)
+ (e_.)*(x_)^2), x_Symbol] :> Dist[-f^4/(c*d^2 - b*d*e + a*e^2), Int[(f*x)^
(m - 4)*(a*d + (b*d - a*e)*x^2)*(a + b*x^2 + c*x^4)^p, x], x] + Dist[d^2*(f
^4/(c*d^2 - b*d*e + a*e^2)), Int[(f*x)^(m - 4)*((a + b*x^2 + c*x^4)^(p + 1)
/(d + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b^2 - 4*a*c, 0
] && LtQ[p, -1] && GtQ[m, 2]
```

Rule 1333

```
Int[(x_)^2/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4])
, x_Symbol] :> With[{q = Rt[c/a, 2]}, Dist[(-a)*((e + d*q)/(c*d^2 - a*e^2))
, Int[1/Sqrt[a + b*x^2 + c*x^4], x], x] + Dist[a*d*((e + d*q)/(c*d^2 - a*e^
```

```
2)), Int[(1 + q*x^2)/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]), x], x]] /; Free
Q[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2,
0] && PosQ[c/a] && NeQ[c*d^2 - a*e^2, 0]
```

Rule 1720

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4]), x_Symbol] :> With[{q = Rt[B/A, 2]}, Simp[(-(B*d - A*e))*(A
rcTan[Rt[-b + c*(d/e) + a*(e/d), 2]*(x/Sqrt[a + b*x^2 + c*x^4])]/(2*d*e*Rt[
-b + c*(d/e) + a*(e/d), 2])), x] + Simp[(B*d + A*e)*(A + B*x^2)*(Sqrt[A^2*(
(a + b*x^2 + c*x^4)/(a*(A + B*x^2)^2))]/(4*d*e*A*q*Sqrt[a + b*x^2 + c*x^4])
)*EllipticPi[Cancel[-(B*d - A*e)^2/(4*d*e*A*B)], 2*ArcTan[q*x], 1/2 - b*(A/
(4*a*B))], x]] /; FreeQ[{a, b, c, d, e, A, B}, x] && NeQ[b^2 - 4*a*c, 0] &&
 NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] && EqQ
[c*A^2 - a*B^2, 0]
```

Rubi steps

$$\text{integral} = -\left(\frac{1}{10}\int \frac{x^2(3+4x^2)}{(1+2x^2+2x^4)^{3/2}}\,dx\right) + \frac{9}{10}\int \frac{x^2}{(3+2x^2)\sqrt{1+2x^2+2x^4}}\,dx$$

$$= \frac{x(1-2x^2)}{20\sqrt{1+2x^2+2x^4}} + \frac{1}{40}\int \frac{-2+4x^2}{\sqrt{1+2x^2+2x^4}}\,dx - \frac{1}{140}\left(9\left(2+3\sqrt{2}\right)\right)\int \frac{1}{\sqrt{1+2x^2+2x^4}}\,dx + \frac{1}{140}\left(27\left(2+3\sqrt{2}\right)\right)\int \frac{1+\sqrt{2}x^2}{(3+2x^2)\sqrt{1+2x^2+2x^4}}\,dx$$

$$= \frac{x(1-2x^2)}{20\sqrt{1+2x^2+2x^4}} - \frac{9}{40}\sqrt{\frac{3}{5}}\tan^{-1}\left(\frac{\sqrt{\frac{5}{3}}x}{\sqrt{1+2x^2+2x^4}}\right) - \frac{9(3+\sqrt{2})\,(1+\sqrt{2}x^2)\sqrt{\frac{1+2x^2+2x^4}{\left(1+\sqrt{2}x^2\right)^2}}F\left(2\tan^{-1}\left(\sqrt[4]{2}x\right)|\frac{1}{4}(2-\sqrt{2})\right)}{140\ 2^{3/4}\sqrt{1+2x^2+2x^4}} + \frac{9(3+\sqrt{2})^2\,(1+\sqrt{2}x^2)\sqrt{\frac{1+2x^2+2x^4}{\left(1+\sqrt{2}x^2\right)^2}}\Pi\left(\frac{1}{24}(12-11\sqrt{2});2\tan^{-1}\left(\sqrt[4]{2}x\right)|\frac{1}{4}(2-\sqrt{2})\right)}{560\sqrt[4]{2}\sqrt{1+2x^2+2x^4}} - \frac{\int \frac{1-\sqrt{2}x^2}{\sqrt{1+2x^2+2x^4}}\,dx}{10\sqrt{2}} + \frac{1}{20}\left(-1+\sqrt{2}\right)\int \frac{1}{\sqrt{1+2x^2+2x^4}}\,dx$$

$$= \frac{x(1-2x^2)}{20\sqrt{1+2x^2+2x^4}} + \frac{x\sqrt{1+2x^2+2x^4}}{10\sqrt{2}\,(1+\sqrt{2}x^2)} - \frac{9}{40}\sqrt{\frac{3}{5}}\tan^{-1}\left(\frac{\sqrt{\frac{5}{3}}x}{\sqrt{1+2x^2+2x^4}}\right)$$
$$- \frac{(1+\sqrt{2}x^2)\sqrt{\frac{1+2x^2+2x^4}{\left(1+\sqrt{2}x^2\right)^2}}E\left(2\tan^{-1}\left(\sqrt[4]{2}x\right)|\frac{1}{4}(2-\sqrt{2})\right)}{10\ 2^{3/4}\sqrt{1+2x^2+2x^4}}$$
$$- \frac{(1-\sqrt{2})\,(1+\sqrt{2}x^2)\sqrt{\frac{1+2x^2+2x^4}{\left(1+\sqrt{2}x^2\right)^2}}F\left(2\tan^{-1}\left(\sqrt[4]{2}x\right)|\frac{1}{4}(2-\sqrt{2})\right)}{40\sqrt[4]{2}\sqrt{1+2x^2+2x^4}}$$
$$- \frac{9(3+\sqrt{2})\,(1+\sqrt{2}x^2)\sqrt{\frac{1+2x^2+2x^4}{\left(1+\sqrt{2}x^2\right)^2}}F\left(2\tan^{-1}\left(\sqrt[4]{2}x\right)|\frac{1}{4}(2-\sqrt{2})\right)}{140\ 2^{3/4}\sqrt{1+2x^2+2x^4}}$$
$$+ \frac{9(3+\sqrt{2})^2\,(1+\sqrt{2}x^2)\sqrt{\frac{1+2x^2+2x^4}{\left(1+\sqrt{2}x^2\right)^2}}\Pi\left(\frac{1}{24}(12-11\sqrt{2});2\tan^{-1}\left(\sqrt[4]{2}x\right)|\frac{1}{4}(2-\sqrt{2})\right)}{560\sqrt[4]{2}\sqrt{1+2x^2+2x^4}}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 9.68 (sec) , antiderivative size = 199, normalized size of antiderivative = 0.47

$$\int \frac{x^6}{(3+2x^2)\,(1+2x^2+2x^4)^{3/2}}\,dx = \frac{2x-4x^3-2i\sqrt{1-i}\sqrt{1+(1-i)x^2}\sqrt{1+(1+i)x^2}E\big(i\text{arcsinh}\big(\sqrt{1-}}{}$$

```
[In] Integrate[x^6/((3 + 2*x^2)*(1 + 2*x^2 + 2*x^4)^(3/2)),x]
```

```
[Out] (2*x - 4*x^3 - (2*I)*Sqrt[1 - I]*Sqrt[1 + (1 - I)*x^2]*Sqrt[1 + (1 + I)*x^2
]*EllipticE[I*ArcSinh[Sqrt[1 - I]*x], I] + (8 - 6*I)*Sqrt[1 - I]*Sqrt[1 + (
1 - I)*x^2]*Sqrt[1 + (1 + I)*x^2]*EllipticF[I*ArcSinh[Sqrt[1 - I]*x], I] -
9*(1 - I)^(3/2)*Sqrt[1 + (1 - I)*x^2]*Sqrt[1 + (1 + I)*x^2]*EllipticPi[1/3
+ I/3, I*ArcSinh[Sqrt[1 - I]*x], I])/(40*Sqrt[1 + 2*x^2 + 2*x^4])
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 2.52 (sec) , antiderivative size = 246, normalized size of antiderivative = 0.58

method	result
risch	$-\frac{x(2x^2-1)}{20\sqrt{2x^4+2x^2+1}}+\frac{2\sqrt{1+(1-i)x^2}\,\sqrt{1+(1+i)x^2}\,F\left(x\sqrt{-1+i},\frac{\sqrt{2}}{2}+\frac{i\sqrt{2}}{2}\right)}{5\sqrt{-1+i}\,\sqrt{2x^4+2x^2+1}}+\frac{\left(-\frac{1}{20}+\frac{i}{20}\right)\sqrt{1+(1-i)x^2}\,\sqrt{1+(1+i)x^2}\,\left(F\left(x\sqrt{-1+i},\right.\right.}{\sqrt{-1+i}\,\sqrt{2x^4+2x^2}}$
elliptic	$-\frac{4\left(\frac{1}{40}x^3-\frac{1}{80}x\right)}{\sqrt{2x^4+2x^2+1}}+\frac{7\sqrt{-ix^2+x^2+1}\,\sqrt{ix^2+x^2+1}\,F\left(x\sqrt{-1+i},\frac{\sqrt{2}}{2}+\frac{i\sqrt{2}}{2}\right)}{20\sqrt{-1+i}\,\sqrt{2x^4+2x^2+1}}+\frac{i\sqrt{-ix^2+x^2+1}\,\sqrt{ix^2+x^2+1}\,F\left(x\sqrt{-1+i},\frac{\sqrt{2}}{2}+\frac{i\sqrt{2}}{2}\right)}{20\sqrt{-1+i}\,\sqrt{2x^4+2x^2+1}}+$
default	$-\frac{9x^3}{8\sqrt{2x^4+2x^2+1}}+\frac{7\sqrt{1+(1-i)x^2}\,\sqrt{1+(1+i)x^2}\,F\left(x\sqrt{-1+i},\frac{\sqrt{2}}{2}+\frac{i\sqrt{2}}{2}\right)}{4\sqrt{-1+i}\,\sqrt{2x^4+2x^2+1}}+\frac{\left(-\frac{17}{16}+\frac{17i}{16}\right)\sqrt{1+(1-i)x^2}\,\sqrt{1+(1+i)x^2}\,\left(F\left(x\sqrt{-1+i},\right.\right.}{\sqrt{-1+i}\,\sqrt{2x^4+2x^2}}$

```
[In] int(x^6/(2*x^2+3)/(2*x^4+2*x^2+1)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/20*x*(2*x^2-1)/(2*x^4+2*x^2+1)^(1/2)+2/5/(-1+I)^(1/2)*(1+(1-I)*x^2)^(1/2
)*(1+(1+I)*x^2)^(1/2)/(2*x^4+2*x^2+1)^(1/2)*EllipticF(x*(-1+I)^(1/2),1/2*2^
(1/2)+1/2*I*2^(1/2))+(-1/20+1/20*I)/(-1+I)^(1/2)*(1+(1-I)*x^2)^(1/2)*(1+(1+
I)*x^2)^(1/2)/(2*x^4+2*x^2+1)^(1/2)*(EllipticF(x*(-1+I)^(1/2),1/2*2^(1/2)+1
/2*I*2^(1/2))-EllipticE(x*(-1+I)^(1/2),1/2*2^(1/2)+1/2*I*2^(1/2)))-9/20/(-1
+I)^(1/2)*(1-I*x^2+x^2)^(1/2)*(1+I*x^2+x^2)^(1/2)/(2*x^4+2*x^2+1)^(1/2)*Ell
ipticPi(x*(-1+I)^(1/2),1/3+1/3*I,(-1-I)^(1/2)/(-1+I)^(1/2))
```

Fricas [F]

$$\int \frac{x^6}{(3+2x^2)\left(1+2x^2+2x^4\right)^{3/2}}\,dx=\int \frac{x^6}{\left(2\,x^4+2\,x^2+1\right)^{\frac{3}{2}}\left(2\,x^2+3\right)}\,dx$$

```
[In] integrate(x^6/(2*x^2+3)/(2*x^4+2*x^2+1)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(2*x^4 + 2*x^2 + 1)*x^6/(8*x^10 + 28*x^8 + 40*x^6 + 32*x^4 + 1
4*x^2 + 3), x)
```

Sympy [F]

$$\int \frac{x^6}{(3+2x^2)\left(1+2x^2+2x^4\right)^{3/2}}\,dx=\int \frac{x^6}{\left(2x^2+3\right)\left(2x^4+2x^2+1\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(x**6/(2*x**2+3)/(2*x**4+2*x**2+1)**(3/2),x)
```

```
[Out] Integral(x**6/((2*x**2 + 3)*(2*x**4 + 2*x**2 + 1)**(3/2)), x)
```

Maxima [F]

$$\int \frac{x^6}{(3+2x^2)\left(1+2x^2+2x^4\right)^{3/2}}\,dx=\int \frac{x^6}{\left(2\,x^4+2\,x^2+1\right)^{\frac{3}{2}}(2\,x^2+3)}\,dx$$

```
[In] integrate(x^6/(2*x^2+3)/(2*x^4+2*x^2+1)^(3/2),x, algorithm="maxima")
[Out] integrate(x^6/((2*x^4 + 2*x^2 + 1)^(3/2)*(2*x^2 + 3)), x)
```

Giac [F]

$$\int \frac{x^6}{(3+2x^2)\left(1+2x^2+2x^4\right)^{3/2}}\,dx=\int \frac{x^6}{\left(2\,x^4+2\,x^2+1\right)^{\frac{3}{2}}(2\,x^2+3)}\,dx$$

```
[In] integrate(x^6/(2*x^2+3)/(2*x^4+2*x^2+1)^(3/2),x, algorithm="giac")
[Out] integrate(x^6/((2*x^4 + 2*x^2 + 1)^(3/2)*(2*x^2 + 3)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^6}{(3+2x^2)\left(1+2x^2+2x^4\right)^{3/2}}\,dx=\int \frac{x^6}{(2\,x^2+3)\,\left(2\,x^4+2\,x^2+1\right)^{3/2}}\,dx$$

```
[In] int(x^6/((2*x^2 + 3)*(2*x^2 + 2*x^4 + 1)^(3/2)),x)
[Out] int(x^6/((2*x^2 + 3)*(2*x^2 + 2*x^4 + 1)^(3/2)), x)
```

3.350 $\int \frac{x^4}{(3+2x^2)(1+2x^2+2x^4)^{3/2}}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 422

$$\int \frac{x^4}{(3+2x^2)(1+2x^2+2x^4)^{3/2}}\,dx = -\frac{x(2+x^2)}{10\sqrt{1+2x^2+2x^4}} + \frac{x\sqrt{1+2x^2+2x^4}}{10\sqrt{2}\left(1+\sqrt{2}x^2\right)} + \frac{3}{20}\sqrt{\frac{3}{5}}\arctan\left(\frac{\sqrt{\frac{5}{3}}x}{\sqrt{1+2x^2+2x^4}}\right) - \frac{\left(1+\sqrt{2}x^2\right)\sqrt{\frac{1+2x^2+2x^4}{\left(1+\sqrt{2}x^2\right)^2}}E\left(2\arctan\left(\sqrt[4]{2}x\right)\left|\frac{1}{4}\left(2-\sqrt{2}\right)\right.\right)}{10\ 2^{3/4}\sqrt{1+2x^2+2x^4}} + \frac{\left(2+\sqrt{2}\right)\left(1+\sqrt{2}x^2\right)\sqrt{\frac{1+2x^2+2x^4}{\left(1+\sqrt{2}x^2\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\sqrt[4]{2}x\right),\frac{1}{4}\left(2-\sqrt{2}\right)\right)}{4\ 2^{3/4}\left(-2+3\sqrt{2}\right)\sqrt{1+2x^2+2x^4}} + \frac{3\left(3+\sqrt{2}\right)\left(1+\sqrt{2}x^2\right)\sqrt{\frac{1+2x^2+2x^4}{\left(1+\sqrt{2}x^2\right)^2}}\operatorname{EllipticPi}\left(\frac{1}{24}\left(12-11\sqrt{2}\right),2\arctan\left(\sqrt[4]{2}x\right),\frac{1}{4}\left(2-\sqrt{2}\right)\right)}{20\ 2^{3/4}\left(2-3\sqrt{2}\right)\sqrt{1+2x^2+2x^4}}$$

```
[Out] 3/100*arctan(1/3*x*15^(1/2)/(2*x^4+2*x^2+1)^(1/2))*15^(1/2)-1/10*x*(x^2+2)/
(2*x^4+2*x^2+1)^(1/2)+1/20*x*(2*x^4+2*x^2+1)^(1/2)*2^(1/2)/(1+x^2*2^(1/2))-
1/20*(cos(2*arctan(2^(1/4)*x))^2)^(1/2)/cos(2*arctan(2^(1/4)*x))*EllipticE(
sin(2*arctan(2^(1/4)*x)),1/2*(2-2^(1/2))^(1/2))*(1+x^2*2^(1/2))*((2*x^4+2*x
^2+1)/(1+x^2*2^(1/2))^2)^(1/2)*2^(1/4)/(2*x^4+2*x^2+1)^(1/2)+3/40*(cos(2*ar
ctan(2^(1/4)*x))^2)^(1/2)/cos(2*arctan(2^(1/4)*x))*EllipticPi(sin(2*arctan(
2^(1/4)*x)),1/2-11/24*2^(1/2),1/2*(2-2^(1/2))^(1/2))*(3+2^(1/2))*(1+x^2*2^(
1/2))*((2*x^4+2*x^2+1)/(1+x^2*2^(1/2))^2)^(1/2)*2^(1/4)/(2-3*2^(1/2))/(2*x^
4+2*x^2+1)^(1/2)+1/8*(cos(2*arctan(2^(1/4)*x))^2)^(1/2)/cos(2*arctan(2^(1/4
)*x))*EllipticF(sin(2*arctan(2^(1/4)*x)),1/2*(2-2^(1/2))^(1/2))*(2+2^(1/2))
```

```
*(1+x^2*2^(1/2))*((2*x^4+2*x^2+1)/(1+x^2*2^(1/2))^2)^(1/2)*2^(1/4)/(-2+3*2^
(1/2))/(2*x^4+2*x^2+1)^(1/2)
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 501, normalized size of antiderivative = 1.19, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.241$, Rules used = {1327, 1192, 1211, 1117, 1209, 1230, 1720}

$$\int \frac{x^4}{(3+2x^2)(1+2x^2+2x^4)^{3/2}}\,dx = \frac{3}{20}\sqrt{\frac{3}{5}}\arctan\left(\frac{\sqrt{\frac{5}{3}}x}{\sqrt{2x^4+2x^2+1}}\right)$$
$$+\frac{9(3+\sqrt{2})(\sqrt{2}x^2+1)\sqrt{\frac{2x^4+2x^2+1}{(\sqrt{2}x^2+1)^2}}\operatorname{EllipticF}\left(2\arctan\left(\sqrt[4]{2}x\right),\frac{1}{4}(2-\sqrt{2})\right)}{140\sqrt[4]{2}\sqrt{2x^4+2x^2+1}}$$
$$+\frac{(1-\sqrt{2})(\sqrt{2}x^2+1)\sqrt{\frac{2x^4+2x^2+1}{(\sqrt{2}x^2+1)^2}}\operatorname{EllipticF}\left(2\arctan\left(\sqrt[4]{2}x\right),\frac{1}{4}(2-\sqrt{2})\right)}{20\ 2^{3/4}\sqrt{2x^4+2x^2+1}}$$
$$-\frac{(\sqrt{2}x^2+1)\sqrt{\frac{2x^4+2x^2+1}{(\sqrt{2}x^2+1)^2}}E\left(2\arctan\left(\sqrt[4]{2}x\right)|\frac{1}{4}(2-\sqrt{2})\right)}{10\ 2^{3/4}\sqrt{2x^4+2x^2+1}}$$
$$-\frac{3(3+\sqrt{2})^2(\sqrt{2}x^2+1)\sqrt{\frac{2x^4+2x^2+1}{(\sqrt{2}x^2+1)^2}}\operatorname{EllipticPi}\left(\frac{1}{24}(12-11\sqrt{2}),2\arctan\left(\sqrt[4]{2}x\right),\frac{1}{4}(2-\sqrt{2})\right)}{280\sqrt[4]{2}\sqrt{2x^4+2x^2+1}}$$
$$+\frac{\sqrt{2x^4+2x^2+1}x}{10\sqrt{2}(\sqrt{2}x^2+1)}-\frac{(x^2+2)x}{10\sqrt{2x^4+2x^2+1}}$$

```
[In] Int[x^4/((3 + 2*x^2)*(1 + 2*x^2 + 2*x^4)^(3/2)),x]
```

```
[Out] -1/10*(x*(2 + x^2))/Sqrt[1 + 2*x^2 + 2*x^4] + (x*Sqrt[1 + 2*x^2 + 2*x^4])/(
10*Sqrt[2]*(1 + Sqrt[2]*x^2)) + (3*Sqrt[3/5]*ArcTan[(Sqrt[5/3]*x)/Sqrt[1 +
2*x^2 + 2*x^4]])/20 - ((1 + Sqrt[2]*x^2)*Sqrt[(1 + 2*x^2 + 2*x^4)/(1 + Sqrt
[2]*x^2)^2]*EllipticE[2*ArcTan[2^(1/4)*x], (2 - Sqrt[2])/4])/(10*2^(3/4)*Sq
rt[1 + 2*x^2 + 2*x^4]) + ((1 - Sqrt[2])*(1 + Sqrt[2]*x^2)*Sqrt[(1 + 2*x^2 +
 2*x^4)/(1 + Sqrt[2]*x^2)^2]*EllipticF[2*ArcTan[2^(1/4)*x], (2 - Sqrt[2])/4
])/(20*2^(3/4)*Sqrt[1 + 2*x^2 + 2*x^4]) + (9*(3 + Sqrt[2])*(1 + Sqrt[2]*x^2
)*Sqrt[(1 + 2*x^2 + 2*x^4)/(1 + Sqrt[2]*x^2)^2]*EllipticF[2*ArcTan[2^(1/4)*
x], (2 - Sqrt[2])/4])/(140*2^(1/4)*Sqrt[1 + 2*x^2 + 2*x^4]) - (3*(3 + Sqrt[
2])^2*(1 + Sqrt[2]*x^2)*Sqrt[(1 + 2*x^2 + 2*x^4)/(1 + Sqrt[2]*x^2)^2]*Ellip
ticPi[(12 - 11*Sqrt[2])/24, 2*ArcTan[2^(1/4)*x], (2 - Sqrt[2])/4])/(280*2^(
1/4)*Sqrt[1 + 2*x^2 + 2*x^4])
```

Rule 1117

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1192

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[x*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)*((a + b*x^2 +
 c*x^4)^(p + 1)/(2*a*(p + 1)*(b^2 - 4*a*c))), x] + Dist[1/(2*a*(p + 1)*(b^2
 - 4*a*c)), Int[Simp[(2*p + 3)*d*b^2 - a*b*e - 2*a*c*d*(4*p + 5) + (4*p + 7
)*(d*b - 2*a*e)*c*x^2, x]*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; FreeQ[{a,
b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
LtQ[p, -1] && IntegerQ[2*p]
```

Rule 1209

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

Rule 1211

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + b*x^2 + c*x^4
], x], x] - Dist[e/q, Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /; Ne
Q[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[
c/a]
```

Rule 1230

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4]), x_S
ymbol] :> With[{q = Rt[c/a, 2]}, Dist[(c*d + a*e*q)/(c*d^2 - a*e^2), Int[1/
Sqrt[a + b*x^2 + c*x^4], x], x] - Dist[(a*e*(e + d*q))/(c*d^2 - a*e^2), Int
[(1 + q*x^2)/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]), x], x]] /; FreeQ[{a, b,
 c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[c/a]
```

Rule 1327

```
Int[(((f_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_))/((d_.)
+ (e_.)*(x_)^2), x_Symbol] :> Dist[-f^4/(c*d^2 - b*d*e + a*e^2), Int[(f*x)^
(m - 4)*(a*d + (b*d - a*e)*x^2)*(a + b*x^2 + c*x^4)^p, x], x] + Dist[d^2*(f
```

```
^4/(c*d^2 - b*d*e + a*e^2)), Int[(f*x)^(m - 4)*((a + b*x^2 + c*x^4)^(p + 1)
/(d + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b^2 - 4*a*c, 0
] && LtQ[p, -1] && GtQ[m, 2]
```

Rule 1720

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4]), x_Symbol] :> With[{q = Rt[B/A, 2]}, Simp[(-(B*d - A*e))*(A
rcTan[Rt[-b + c*(d/e) + a*(e/d), 2]*(x/Sqrt[a + b*x^2 + c*x^4])]/(2*d*e*Rt[
-b + c*(d/e) + a*(e/d), 2])), x] + Simp[(B*d + A*e)*(A + B*x^2)*(Sqrt[A^2*(
(a + b*x^2 + c*x^4)/(a*(A + B*x^2)^2))]/(4*d*e*A*q*Sqrt[a + b*x^2 + c*x^4])
)*EllipticPi[Cancel[-(B*d - A*e)^2/(4*d*e*A*B)], 2*ArcTan[q*x], 1/2 - b*(A/
(4*a*B))], x]] /; FreeQ[{a, b, c, d, e, A, B}, x] && NeQ[b^2 - 4*a*c, 0] &&
 NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] && EqQ
[c*A^2 - a*B^2, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\left(\frac{1}{10}\int\frac{3+4x^2}{\left(1+2x^2+2x^4\right)^{3/2}}\,dx\right)+\frac{9}{10}\int\frac{1}{\left(3+2x^2\right)\sqrt{1+2x^2+2x^4}}\,dx\\
&= -\frac{x\left(2+x^2\right)}{10\sqrt{1+2x^2+2x^4}}-\frac{1}{40}\int\frac{4-4x^2}{\sqrt{1+2x^2+2x^4}}\,dx\\
&\quad+\frac{1}{70}\left(9\left(3+\sqrt{2}\right)\right)\int\frac{1}{\sqrt{1+2x^2+2x^4}}\,dx\\
&\quad-\frac{1}{70}\left(9\left(2+3\sqrt{2}\right)\right)\int\frac{1+\sqrt{2}x^2}{\left(3+2x^2\right)\sqrt{1+2x^2+2x^4}}\,dx\\
&= -\frac{x\left(2+x^2\right)}{10\sqrt{1+2x^2+2x^4}}+\frac{3}{20}\sqrt{\frac{3}{5}}\tan^{-1}\left(\frac{\sqrt{\frac{5}{3}}x}{\sqrt{1+2x^2+2x^4}}\right)\\
&\quad+\frac{9\left(3+\sqrt{2}\right)\left(1+\sqrt{2}x^2\right)\sqrt{\frac{1+2x^2+2x^4}{\left(1+\sqrt{2}x^2\right)^2}}F\left(2\tan^{-1}\left(\sqrt[4]{2}x\right)\middle|\frac{1}{4}\left(2-\sqrt{2}\right)\right)}{140\sqrt[4]{2}\sqrt{1+2x^2+2x^4}}\\
&\quad-\frac{3\left(3+\sqrt{2}\right)^2\left(1+\sqrt{2}x^2\right)\sqrt{\frac{1+2x^2+2x^4}{\left(1+\sqrt{2}x^2\right)^2}}\Pi\left(\frac{1}{24}\left(12-11\sqrt{2}\right);2\tan^{-1}\left(\sqrt[4]{2}x\right)\middle|\frac{1}{4}\left(2-\sqrt{2}\right)\right)}{280\sqrt[4]{2}\sqrt{1+2x^2+2x^4}}\\
&\quad-\frac{\int\frac{1-\sqrt{2}x^2}{\sqrt{1+2x^2+2x^4}}\,dx}{10\sqrt{2}}-\frac{1}{20}\left(2-\sqrt{2}\right)\int\frac{1}{\sqrt{1+2x^2+2x^4}}\,dx
\end{aligned}
$$

$$= -\frac{x(2+x^2)}{10\sqrt{1+2x^2+2x^4}} + \frac{x\sqrt{1+2x^2+2x^4}}{10\sqrt{2}\left(1+\sqrt{2}x^2\right)} + \frac{3}{20}\sqrt{\frac{3}{5}}\tan^{-1}\left(\frac{\sqrt{\frac{5}{3}}x}{\sqrt{1+2x^2+2x^4}}\right)$$
$$-\frac{\left(1+\sqrt{2}x^2\right)\sqrt{\frac{1+2x^2+2x^4}{\left(1+\sqrt{2}x^2\right)^2}}E\left(2\tan^{-1}\left(\sqrt[4]{2}x\right)|\frac{1}{4}\left(2-\sqrt{2}\right)\right)}{10\,2^{3/4}\sqrt{1+2x^2+2x^4}}$$
$$+\frac{\left(1-\sqrt{2}\right)\left(1+\sqrt{2}x^2\right)\sqrt{\frac{1+2x^2+2x^4}{\left(1+\sqrt{2}x^2\right)^2}}F\left(2\tan^{-1}\left(\sqrt[4]{2}x\right)|\frac{1}{4}\left(2-\sqrt{2}\right)\right)}{20\,2^{3/4}\sqrt{1+2x^2+2x^4}}$$
$$+\frac{9\left(3+\sqrt{2}\right)\left(1+\sqrt{2}x^2\right)\sqrt{\frac{1+2x^2+2x^4}{\left(1+\sqrt{2}x^2\right)^2}}F\left(2\tan^{-1}\left(\sqrt[4]{2}x\right)|\frac{1}{4}\left(2-\sqrt{2}\right)\right)}{140\sqrt[4]{2}\sqrt{1+2x^2+2x^4}}$$
$$-\frac{3\left(3+\sqrt{2}\right)^2\left(1+\sqrt{2}x^2\right)\sqrt{\frac{1+2x^2+2x^4}{\left(1+\sqrt{2}x^2\right)^2}}\Pi\left(\frac{1}{24}\left(12-11\sqrt{2}\right);2\tan^{-1}\left(\sqrt[4]{2}x\right)|\frac{1}{4}\left(2-\sqrt{2}\right)\right)}{280\sqrt[4]{2}\sqrt{1+2x^2+2x^4}}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 7.10 (sec) , antiderivative size = 199, normalized size of antiderivative = 0.47

$$\int \frac{x^4}{(3+2x^2)\left(1+2x^2+2x^4\right)^{3/2}}\,dx =$$
$$-\frac{4x+2x^3+i\sqrt{1-i}\sqrt{1+(1-i)x^2}\sqrt{1+(1+i)x^2}E\left(i\operatorname{arcsinh}\left(\sqrt{1-i}x\right)\middle|i\right)+(1-2i)\sqrt{1-i}\sqrt{1+(1-i)}}{}$$

```
[In] Integrate[x^4/((3 + 2*x^2)*(1 + 2*x^2 + 2*x^4)^(3/2)),x]
```

```
[Out] -1/20*(4*x + 2*x^3 + I*Sqrt[1 - I]*Sqrt[1 + (1 - I)*x^2]*Sqrt[1 + (1 + I)*x
^2]*EllipticE[I*ArcSinh[Sqrt[1 - I]*x], I] + (1 - 2*I)*Sqrt[1 - I]*Sqrt[1 +
 (1 - I)*x^2]*Sqrt[1 + (1 + I)*x^2]*EllipticF[I*ArcSinh[Sqrt[1 - I]*x], I]
- 3*(1 - I)^(3/2)*Sqrt[1 + (1 - I)*x^2]*Sqrt[1 + (1 + I)*x^2]*EllipticPi[1/
3 + I/3, I*ArcSinh[Sqrt[1 - I]*x], I])/Sqrt[1 + 2*x^2 + 2*x^4]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.65 (sec) , antiderivative size = 244, normalized size of antiderivative = 0.58

method	result
risch	$-\frac{x(x^2+2)}{10\sqrt{2x^4+2x^2+1}}-\frac{\sqrt{1+(1-i)x^2}\,\sqrt{1+(1+i)x^2}\,F\left(x\sqrt{-1+i},\frac{\sqrt{2}}{2}+\frac{i\sqrt{2}}{2}\right)}{10\sqrt{-1+i}\,\sqrt{2x^4+2x^2+1}}+\frac{\left(-\frac{1}{20}+\frac{i}{20}\right)\sqrt{1+(1-i)x^2}\,\sqrt{1+(1+i)x^2}\,\left(F\left(x\sqrt{-1+i}\right.\right.}{\sqrt{-1+i}\,\sqrt{2x^4+2x}}$
elliptic	$-\frac{4\left(\frac{1}{40}x^3+\frac{1}{20}x\right)}{\sqrt{2x^4+2x^2+1}}-\frac{3\sqrt{-ix^2+x^2+1}\,\sqrt{ix^2+x^2+1}\,F\left(x\sqrt{-1+i},\frac{\sqrt{2}}{2}+\frac{i\sqrt{2}}{2}\right)}{20\sqrt{-1+i}\,\sqrt{2x^4+2x^2+1}}+\frac{i\sqrt{-ix^2+x^2+1}\,\sqrt{ix^2+x^2+1}\,F\left(x\sqrt{-1+i},\frac{\sqrt{2}}{2}+\frac{i\sqrt{2}}{2}\right)}{20\sqrt{-1+i}\,\sqrt{2x^4+2x^2+1}}$
default	$\frac{3x^3}{4\sqrt{2x^4+2x^2+1}}-\frac{\sqrt{1+(1-i)x^2}\,\sqrt{1+(1+i)x^2}\,F\left(x\sqrt{-1+i},\frac{\sqrt{2}}{2}+\frac{i\sqrt{2}}{2}\right)}{\sqrt{-1+i}\,\sqrt{2x^4+2x^2+1}}+\frac{\left(\frac{5}{8}-\frac{5i}{8}\right)\sqrt{1+(1-i)x^2}\,\sqrt{1+(1+i)x^2}\,\left(F\left(x\sqrt{-1+i},\frac{\sqrt{2}}{2}+\right.\right.}{\sqrt{-1+i}\,\sqrt{2x^4+2x^2+1}}$

```
[In] int(x^4/(2*x^2+3)/(2*x^4+2*x^2+1)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/10*x*(x^2+2)/(2*x^4+2*x^2+1)^(1/2)-1/10/(-1+I)^(1/2)*(1+(1-I)*x^2)^(1/2)
*(1+(1+I)*x^2)^(1/2)/(2*x^4+2*x^2+1)^(1/2)*EllipticF(x*(-1+I)^(1/2),1/2*2^(
1/2)+1/2*I*2^(1/2))+(-1/20+1/20*I)/(-1+I)^(1/2)*(1+(1-I)*x^2)^(1/2)*(1+(1+I
)*x^2)^(1/2)/(2*x^4+2*x^2+1)^(1/2)*(EllipticF(x*(-1+I)^(1/2),1/2*2^(1/2)+1/
2*I*2^(1/2))-EllipticE(x*(-1+I)^(1/2),1/2*2^(1/2)+1/2*I*2^(1/2)))+3/10/(-1+
I)^(1/2)*(1-I*x^2+x^2)^(1/2)*(1+I*x^2+x^2)^(1/2)/(2*x^4+2*x^2+1)^(1/2)*Elli
pticPi(x*(-1+I)^(1/2),1/3+1/3*I,(-1-I)^(1/2)/(-1+I)^(1/2))
```

Fricas [F]

$$\int \frac{x^4}{(3+2x^2)\left(1+2x^2+2x^4\right)^{3/2}}\,dx=\int \frac{x^4}{\left(2\,x^4+2\,x^2+1\right)^{\frac{3}{2}}\left(2\,x^2+3\right)}\,dx$$

```
[In] integrate(x^4/(2*x^2+3)/(2*x^4+2*x^2+1)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(2*x^4 + 2*x^2 + 1)*x^4/(8*x^10 + 28*x^8 + 40*x^6 + 32*x^4 + 1
4*x^2 + 3), x)
```

Sympy [F]

$$\int \frac{x^4}{(3+2x^2)\left(1+2x^2+2x^4\right)^{3/2}}\,dx=\int \frac{x^4}{(2x^2+3)\left(2x^4+2x^2+1\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(x**4/(2*x**2+3)/(2*x**4+2*x**2+1)**(3/2),x)
```

```
[Out] Integral(x**4/((2*x**2 + 3)*(2*x**4 + 2*x**2 + 1)**(3/2)), x)
```

Maxima [F]

$$\int \frac{x^4}{(3+2x^2)\,(1+2x^2+2x^4)^{3/2}}\,dx = \int \frac{x^4}{(2\,x^4+2\,x^2+1)^{\frac{3}{2}}(2\,x^2+3)}\,dx$$

```
[In] integrate(x^4/(2*x^2+3)/(2*x^4+2*x^2+1)^(3/2),x, algorithm="maxima")
[Out] integrate(x^4/((2*x^4 + 2*x^2 + 1)^(3/2)*(2*x^2 + 3)), x)
```

Giac [F]

$$\int \frac{x^4}{(3+2x^2)\,(1+2x^2+2x^4)^{3/2}}\,dx = \int \frac{x^4}{(2\,x^4+2\,x^2+1)^{\frac{3}{2}}(2\,x^2+3)}\,dx$$

```
[In] integrate(x^4/(2*x^2+3)/(2*x^4+2*x^2+1)^(3/2),x, algorithm="giac")
[Out] integrate(x^4/((2*x^4 + 2*x^2 + 1)^(3/2)*(2*x^2 + 3)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^4}{(3+2x^2)\,(1+2x^2+2x^4)^{3/2}}\,dx = \int \frac{x^4}{(2\,x^2+3)\,\left(2\,x^4+2\,x^2+1\right)^{3/2}}\,dx$$

```
[In] int(x^4/((2*x^2 + 3)*(2*x^2 + 2*x^4 + 1)^(3/2)),x)
[Out] int(x^4/((2*x^2 + 3)*(2*x^2 + 2*x^4 + 1)^(3/2)), x)
```

3.351 $\int \frac{x^2}{(3+2x^2)\left(1+2x^2+2x^4\right)^{3/2}} \, dx$

Optimal result

Integrand size = 29, antiderivative size = 423

$$\begin{aligned}
\int \frac{x^2}{(3+2x^2)\left(1+2x^2+2x^4\right)^{3/2}} \, dx &= \frac{x(3+4x^2)}{10\sqrt{1+2x^2+2x^4}} \\
&- \frac{\sqrt{2}x\sqrt{1+2x^2+2x^4}}{5\left(1+\sqrt{2}x^2\right)} - \frac{1}{10}\sqrt{\frac{3}{5}}\arctan\left(\frac{\sqrt{\frac{5}{3}}x}{\sqrt{1+2x^2+2x^4}}\right) \\
&+ \frac{\sqrt[4]{2}\left(1+\sqrt{2}x^2\right)\sqrt{\frac{1+2x^2+2x^4}{\left(1+\sqrt{2}x^2\right)^2}}E\left(2\arctan\left(\sqrt[4]{2}x\right)|\frac{1}{4}\left(2-\sqrt{2}\right)\right)}{5\sqrt{1+2x^2+2x^4}} \\
&- \frac{\left(\sqrt[4]{2}+2^{3/4}\right)\left(1+\sqrt{2}x^2\right)\sqrt{\frac{1+2x^2+2x^4}{\left(1+\sqrt{2}x^2\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\sqrt[4]{2}x\right),\frac{1}{4}\left(2-\sqrt{2}\right)\right)}{4\left(-2+3\sqrt{2}\right)\sqrt{1+2x^2+2x^4}} \\
&- \frac{\left(3+\sqrt{2}\right)\left(1+\sqrt{2}x^2\right)\sqrt{\frac{1+2x^2+2x^4}{\left(1+\sqrt{2}x^2\right)^2}}\operatorname{EllipticPi}\left(\frac{1}{24}\left(12-11\sqrt{2}\right),2\arctan\left(\sqrt[4]{2}x\right),\frac{1}{4}\left(2-\sqrt{2}\right)\right)}{10\,2^{3/4}\left(2-3\sqrt{2}\right)\sqrt{1+2x^2+2x^4}}
\end{aligned}$$

```
[Out] -1/50*arctan(1/3*x*15^(1/2)/(2*x^4+2*x^2+1)^(1/2))*15^(1/2)+1/10*x*(4*x^2+3
)/(2*x^4+2*x^2+1)^(1/2)-1/5*x*(2*x^4+2*x^2+1)^(1/2)*2^(1/2)/(1+x^2*2^(1/2))
+1/5*(cos(2*arctan(2^(1/4)*x))^2)^(1/2)/cos(2*arctan(2^(1/4)*x))*EllipticE(
sin(2*arctan(2^(1/4)*x)),1/2*(2-2^(1/2))^(1/2))*(1+x^2*2^(1/2))*((2*x^4+2*x
^2+1)/(1+x^2*2^(1/2))^2)^(1/2)*2^(1/4)/(2*x^4+2*x^2+1)^(1/2)-1/20*(cos(2*ar
ctan(2^(1/4)*x))^2)^(1/2)/cos(2*arctan(2^(1/4)*x))*EllipticPi(sin(2*arctan(
2^(1/4)*x)),1/2-11/24*2^(1/2),1/2*(2-2^(1/2))^(1/2))*(3+2^(1/2))*(1+x^2*2^(
1/2))*((2*x^4+2*x^2+1)/(1+x^2*2^(1/2))^2)^(1/2)*2^(1/4)/(2-3*2^(1/2))/(2*x^
4+2*x^2+1)^(1/2)-1/4*(2^(1/4)+2^(3/4))*(cos(2*arctan(2^(1/4)*x))^2)^(1/2)/c
os(2*arctan(2^(1/4)*x))*EllipticF(sin(2*arctan(2^(1/4)*x)),1/2*(2-2^(1/2))^
```

```
(1/2))*(1+x^2*2^(1/2))*((2*x^4+2*x^2+1)/(1+x^2*2^(1/2))^2)^(1/2)/(-2+3*2^(1
/2))/(2*x^4+2*x^2+1)^(1/2)
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 503, normalized size of antiderivative = 1.19, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.241, Rules used = {1329, 1192, 1211, 1117, 1209, 1230, 1720}

$$\int \frac{x^2}{(3+2x^2)(1+2x^2+2x^4)^{3/2}}\,dx = -\frac{1}{10}\sqrt{\frac{3}{5}}\arctan\left(\frac{\sqrt{\frac{5}{3}}x}{\sqrt{2x^4+2x^2+1}}\right) - \frac{(1+2\sqrt{2})(\sqrt{2}x^2+1)\sqrt{\frac{2x^4+2x^2+1}{(\sqrt{2}x^2+1)^2}}\,\text{EllipticF}\left(2\arctan\left(\sqrt[4]{2}x\right),\frac{1}{4}(2-\sqrt{2})\right)}{20\sqrt[4]{2}\sqrt{2x^4+2x^2+1}} - \frac{3(3+\sqrt{2})(\sqrt{2}x^2+1)\sqrt{\frac{2x^4+2x^2+1}{(\sqrt{2}x^2+1)^2}}\,\text{EllipticF}\left(2\arctan\left(\sqrt[4]{2}x\right),\frac{1}{4}(2-\sqrt{2})\right)}{70\sqrt[4]{2}\sqrt{2x^4+2x^2+1}} + \frac{\sqrt[4]{2}(\sqrt{2}x^2+1)\sqrt{\frac{2x^4+2x^2+1}{(\sqrt{2}x^2+1)^2}}E\left(2\arctan\left(\sqrt[4]{2}x\right)|\frac{1}{4}(2-\sqrt{2})\right)}{5\sqrt{2x^4+2x^2+1}} + \frac{(3+\sqrt{2})^2(\sqrt{2}x^2+1)\sqrt{\frac{2x^4+2x^2+1}{(\sqrt{2}x^2+1)^2}}\,\text{EllipticPi}\left(\frac{1}{24}(12-11\sqrt{2}),2\arctan\left(\sqrt[4]{2}x\right),\frac{1}{4}(2-\sqrt{2})\right)}{140\sqrt[4]{2}\sqrt{2x^4+2x^2+1}} - \frac{\sqrt{2}\sqrt{2x^4+2x^2+1}x}{5(\sqrt{2}x^2+1)} + \frac{(4x^2+3)x}{10\sqrt{2x^4+2x^2+1}}$$

```
[In] Int[x^2/((3 + 2*x^2)*(1 + 2*x^2 + 2*x^4)^(3/2)),x]
```

```
[Out] (x*(3 + 4*x^2))/(10*Sqrt[1 + 2*x^2 + 2*x^4]) - (Sqrt[2]*x*Sqrt[1 + 2*x^2 +
2*x^4])/(5*(1 + Sqrt[2]*x^2)) - (Sqrt[3/5]*ArcTan[(Sqrt[5/3]*x)/Sqrt[1 + 2*
x^2 + 2*x^4]])/10 + (2^(1/4)*(1 + Sqrt[2]*x^2)*Sqrt[(1 + 2*x^2 + 2*x^4)/(1
+ Sqrt[2]*x^2)^2]*EllipticE[2*ArcTan[2^(1/4)*x], (2 - Sqrt[2])/4])/(5*Sqrt[
1 + 2*x^2 + 2*x^4]) - (3*(3 + Sqrt[2])*(1 + Sqrt[2]*x^2)*Sqrt[(1 + 2*x^2 +
2*x^4)/(1 + Sqrt[2]*x^2)^2]*EllipticF[2*ArcTan[2^(1/4)*x], (2 - Sqrt[2])/4]
)/(70*2^(1/4)*Sqrt[1 + 2*x^2 + 2*x^4]) - ((1 + 2*Sqrt[2])*(1 + Sqrt[2]*x^2)
*Sqrt[(1 + 2*x^2 + 2*x^4)/(1 + Sqrt[2]*x^2)^2]*EllipticF[2*ArcTan[2^(1/4)*x
], (2 - Sqrt[2])/4])/(20*2^(1/4)*Sqrt[1 + 2*x^2 + 2*x^4]) + ((3 + Sqrt[2])^
2*(1 + Sqrt[2]*x^2)*Sqrt[(1 + 2*x^2 + 2*x^4)/(1 + Sqrt[2]*x^2)^2]*EllipticP
i[(12 - 11*Sqrt[2])/24, 2*ArcTan[2^(1/4)*x], (2 - Sqrt[2])/4])/(140*2^(1/4)
*Sqrt[1 + 2*x^2 + 2*x^4])
```

Rule 1117

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1192

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[x*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)*((a + b*x^2 +
 c*x^4)^(p + 1)/(2*a*(p + 1)*(b^2 - 4*a*c))), x] + Dist[1/(2*a*(p + 1)*(b^2
 - 4*a*c)), Int[Simp[(2*p + 3)*d*b^2 - a*b*e - 2*a*c*d*(4*p + 5) + (4*p + 7
)*(d*b - 2*a*e)*c*x^2, x]*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; FreeQ[{a,
b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
LtQ[p, -1] && IntegerQ[2*p]
```

Rule 1209

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

Rule 1211

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + b*x^2 + c*x^4
], x], x] - Dist[e/q, Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /; Ne
Q[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[
c/a]
```

Rule 1230

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4]), x_S
ymbol] :> With[{q = Rt[c/a, 2]}, Dist[(c*d + a*e*q)/(c*d^2 - a*e^2), Int[1/
Sqrt[a + b*x^2 + c*x^4], x], x] - Dist[(a*e*(e + d*q))/(c*d^2 - a*e^2), Int
[(1 + q*x^2)/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]), x], x]] /; FreeQ[{a, b,
 c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[c/a]
```

Rule 1329

```
Int[(((f_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_))/((d_.)
+ (e_.)*(x_)^2), x_Symbol] :> Dist[f^2/(c*d^2 - b*d*e + a*e^2), Int[(f*x)^(
m - 2)*(a*e + c*d*x^2)*(a + b*x^2 + c*x^4)^p, x], x] - Dist[d*e*(f^2/(c*d^2
```

```
 - b*d*e + a*e^2)), Int[(f*x)^(m - 2)*((a + b*x^2 + c*x^4)^(p + 1)/(d + e*x
^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[
p, -1] && GtQ[m, 0]
```

Rule 1720

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4]), x_Symbol] :> With[{q = Rt[B/A, 2]}, Simp[(-(B*d - A*e))*(A
rcTan[Rt[-b + c*(d/e) + a*(e/d), 2]*(x/Sqrt[a + b*x^2 + c*x^4])]/(2*d*e*Rt[
-b + c*(d/e) + a*(e/d), 2])), x] + Simp[(B*d + A*e)*(A + B*x^2)*(Sqrt[A^2*(
(a + b*x^2 + c*x^4)/(a*(A + B*x^2)^2))]/(4*d*e*A*q*Sqrt[a + b*x^2 + c*x^4])
)*EllipticPi[Cancel[-(B*d - A*e)^2/(4*d*e*A*B)], 2*ArcTan[q*x], 1/2 - b*(A/
(4*a*B))], x]] /; FreeQ[{a, b, c, d, e, A, B}, x] && NeQ[b^2 - 4*a*c, 0] &&
 NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] && EqQ
[c*A^2 - a*B^2, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{10}\int \frac{2+6x^2}{(1+2x^2+2x^4)^{3/2}}\,dx-\frac{3}{5}\int \frac{1}{(3+2x^2)\sqrt{1+2x^2+2x^4}}\,dx\\
&= \frac{x(3+4x^2)}{10\sqrt{1+2x^2+2x^4}}+\frac{1}{40}\int \frac{-4-16x^2}{\sqrt{1+2x^2+2x^4}}\,dx\\
&\quad -\frac{1}{35}\left(3\left(3+\sqrt{2}\right)\right)\int \frac{1}{\sqrt{1+2x^2+2x^4}}\,dx\\
&\quad +\frac{1}{35}\left(3\left(2+3\sqrt{2}\right)\right)\int \frac{1+\sqrt{2}x^2}{(3+2x^2)\sqrt{1+2x^2+2x^4}}\,dx\\
&= \frac{x(3+4x^2)}{10\sqrt{1+2x^2+2x^4}}-\frac{1}{10}\sqrt{\frac{3}{5}}\tan^{-1}\left(\frac{\sqrt{\frac{5}{3}}x}{\sqrt{1+2x^2+2x^4}}\right)\\
&\quad -\frac{3\left(3+\sqrt{2}\right)\left(1+\sqrt{2}x^2\right)\sqrt{\frac{1+2x^2+2x^4}{\left(1+\sqrt{2}x^2\right)^2}}F\left(2\tan^{-1}\left(\sqrt[4]{2}x\right)\middle|\frac{1}{4}\left(2-\sqrt{2}\right)\right)}{70\sqrt[4]{2}\sqrt{1+2x^2+2x^4}}\\
&\quad +\frac{\left(3+\sqrt{2}\right)^2\left(1+\sqrt{2}x^2\right)\sqrt{\frac{1+2x^2+2x^4}{\left(1+\sqrt{2}x^2\right)^2}}\Pi\left(\frac{1}{24}\left(12-11\sqrt{2}\right);2\tan^{-1}\left(\sqrt[4]{2}x\right)\middle|\frac{1}{4}\left(2-\sqrt{2}\right)\right)}{140\sqrt[4]{2}\sqrt{1+2x^2+2x^4}}\\
&\quad +\frac{1}{5}\sqrt{2}\int \frac{1-\sqrt{2}x^2}{\sqrt{1+2x^2+2x^4}}\,dx+\frac{1}{10}\left(-1-2\sqrt{2}\right)\int \frac{1}{\sqrt{1+2x^2+2x^4}}\,dx
\end{aligned}$$

$$= \frac{x(3+4x^2)}{10\sqrt{1+2x^2+2x^4}} - \frac{\sqrt{2}x\sqrt{1+2x^2+2x^4}}{5\left(1+\sqrt{2}x^2\right)} - \frac{1}{10}\sqrt{\frac{3}{5}}\tan^{-1}\left(\frac{\sqrt{\frac{5}{3}}x}{\sqrt{1+2x^2+2x^4}}\right)$$
$$+ \frac{\sqrt[4]{2}(1+\sqrt{2}x^2)\sqrt{\frac{1+2x^2+2x^4}{\left(1+\sqrt{2}x^2\right)^2}}E\left(2\tan^{-1}\left(\sqrt[4]{2}x\right)|\frac{1}{4}(2-\sqrt{2})\right)}{5\sqrt{1+2x^2+2x^4}}$$
$$- \frac{3(3+\sqrt{2})\left(1+\sqrt{2}x^2\right)\sqrt{\frac{1+2x^2+2x^4}{\left(1+\sqrt{2}x^2\right)^2}}F\left(2\tan^{-1}\left(\sqrt[4]{2}x\right)|\frac{1}{4}(2-\sqrt{2})\right)}{70\sqrt[4]{2}\sqrt{1+2x^2+2x^4}}$$
$$- \frac{(1+2\sqrt{2})\left(1+\sqrt{2}x^2\right)\sqrt{\frac{1+2x^2+2x^4}{\left(1+\sqrt{2}x^2\right)^2}}F\left(2\tan^{-1}\left(\sqrt[4]{2}x\right)|\frac{1}{4}(2-\sqrt{2})\right)}{20\sqrt[4]{2}\sqrt{1+2x^2+2x^4}}$$
$$+ \frac{\left(3+\sqrt{2}\right)^2\left(1+\sqrt{2}x^2\right)\sqrt{\frac{1+2x^2+2x^4}{\left(1+\sqrt{2}x^2\right)^2}}\Pi\left(\frac{1}{24}(12-11\sqrt{2});2\tan^{-1}\left(\sqrt[4]{2}x\right)|\frac{1}{4}(2-\sqrt{2})\right)}{140\sqrt[4]{2}\sqrt{1+2x^2+2x^4}}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 7.00 (sec) , antiderivative size = 199, normalized size of antiderivative = 0.47

$$\int \frac{x^2}{(3+2x^2)(1+2x^2+2x^4)^{3/2}}\,dx = \frac{6x+8x^3+4i\sqrt{1-i}\sqrt{1+(1-i)x^2}\sqrt{1+(1+i)x^2}E\left(i\text{arcsinh}\left(\sqrt{1-\right.\right.}}{}$$

```
[In] Integrate[x^2/((3 + 2*x^2)*(1 + 2*x^2 + 2*x^4)^(3/2)),x]
```

```
[Out] (6*x + 8*x^3 + (4*I)*Sqrt[1 - I]*Sqrt[1 + (1 - I)*x^2]*Sqrt[1 + (1 + I)*x^2
]*EllipticE[I*ArcSinh[Sqrt[1 - I]*x], I] - (1 + 3*I)*Sqrt[1 - I]*Sqrt[1 + (
1 - I)*x^2]*Sqrt[1 + (1 + I)*x^2]*EllipticF[I*ArcSinh[Sqrt[1 - I]*x], I] -
2*(1 - I)^(3/2)*Sqrt[1 + (1 - I)*x^2]*Sqrt[1 + (1 + I)*x^2]*EllipticPi[1/3
+ I/3, I*ArcSinh[Sqrt[1 - I]*x], I])/(20*Sqrt[1 + 2*x^2 + 2*x^4])
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.27 (sec) , antiderivative size = 246, normalized size of antiderivative = 0.58

method	result
risch	$\frac{x(4x^2+3)}{10\sqrt{2x^4+2x^2+1}}-\frac{\sqrt{1+(1-i)x^2}\,\sqrt{1+(1+i)x^2}\,F\left(x\sqrt{-1+i},\frac{\sqrt{2}}{2}+\frac{i\sqrt{2}}{2}\right)}{10\sqrt{-1+i}\,\sqrt{2x^4+2x^2+1}}+\frac{\left(\frac{1}{5}-\frac{i}{5}\right)\sqrt{1+(1-i)x^2}\,\sqrt{1+(1+i)x^2}\,\left(F\left(x\sqrt{-1+i},\frac{\sqrt{2}}{2}+\frac{i\sqrt{2}}{2}\right)\right.}{\sqrt{-1+i}\,\sqrt{2x^4+2x^2+1}}$
elliptic	$-\frac{4\left(-\frac{1}{10}x^3-\frac{3}{40}x\right)}{\sqrt{2x^4+2x^2+1}}+\frac{\sqrt{-ix^2+x^2+1}\,\sqrt{ix^2+x^2+1}\,F\left(x\sqrt{-1+i},\frac{\sqrt{2}}{2}+\frac{i\sqrt{2}}{2}\right)}{10\sqrt{-1+i}\,\sqrt{2x^4+2x^2+1}}-\frac{i\sqrt{-ix^2+x^2+1}\,\sqrt{ix^2+x^2+1}\,F\left(x\sqrt{-1+i},\frac{\sqrt{2}}{2}+\frac{i\sqrt{2}}{2}\right)}{5\sqrt{-1+i}\,\sqrt{2x^4+2x^2+1}}-$
default	$-\frac{x^3}{2\sqrt{2x^4+2x^2+1}}+\frac{\sqrt{1+(1-i)x^2}\,\sqrt{1+(1+i)x^2}\,F\left(x\sqrt{-1+i},\frac{\sqrt{2}}{2}+\frac{i\sqrt{2}}{2}\right)}{2\sqrt{-1+i}\,\sqrt{2x^4+2x^2+1}}+\frac{\left(-\frac{1}{4}+\frac{i}{4}\right)\sqrt{1+(1-i)x^2}\,\sqrt{1+(1+i)x^2}\,\left(F\left(x\sqrt{-1+i},\frac{\sqrt{2}}{2}+\right.\right.}{\sqrt{-1+i}\,\sqrt{2x^4+2x^2+1}}$

```
[In] int(x^2/(2*x^2+3)/(2*x^4+2*x^2+1)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/10*x*(4*x^2+3)/(2*x^4+2*x^2+1)^(1/2)-1/10/(-1+I)^(1/2)*(1+(1-I)*x^2)^(1/2
)*(1+(1+I)*x^2)^(1/2)/(2*x^4+2*x^2+1)^(1/2)*EllipticF(x*(-1+I)^(1/2),1/2*2^
(1/2)+1/2*I*2^(1/2))+(1/5-1/5*I)/(-1+I)^(1/2)*(1+(1-I)*x^2)^(1/2)*(1+(1+I)*
x^2)^(1/2)/(2*x^4+2*x^2+1)^(1/2)*(EllipticF(x*(-1+I)^(1/2),1/2*2^(1/2)+1/2*
I*2^(1/2))-EllipticE(x*(-1+I)^(1/2),1/2*2^(1/2)+1/2*I*2^(1/2)))-1/5/(-1+I)^
(1/2)*(1-I*x^2+x^2)^(1/2)*(1+I*x^2+x^2)^(1/2)/(2*x^4+2*x^2+1)^(1/2)*Ellipti
cPi(x*(-1+I)^(1/2),1/3+1/3*I,(-1-I)^(1/2)/(-1+I)^(1/2))
```

Fricas [F]

$$\int \frac{x^2}{(3+2x^2)\left(1+2x^2+2x^4\right)^{3/2}}\,dx=\int \frac{x^2}{\left(2\,x^4+2\,x^2+1\right)^{\frac{3}{2}}\left(2\,x^2+3\right)}\,dx$$

```
[In] integrate(x^2/(2*x^2+3)/(2*x^4+2*x^2+1)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(2*x^4 + 2*x^2 + 1)*x^2/(8*x^10 + 28*x^8 + 40*x^6 + 32*x^4 + 1
4*x^2 + 3), x)
```

Sympy [F]

$$\int \frac{x^2}{(3+2x^2)\left(1+2x^2+2x^4\right)^{3/2}}\,dx=\int \frac{x^2}{\left(2x^2+3\right)\left(2x^4+2x^2+1\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(x**2/(2*x**2+3)/(2*x**4+2*x**2+1)**(3/2),x)
```

```
[Out] Integral(x**2/((2*x**2 + 3)*(2*x**4 + 2*x**2 + 1)**(3/2)), x)
```

Maxima [F]

$$\int \frac{x^2}{(3+2x^2)\,(1+2x^2+2x^4)^{3/2}}\,dx = \int \frac{x^2}{(2\,x^4+2\,x^2+1)^{\frac{3}{2}}(2\,x^2+3)}\,dx$$

```
[In] integrate(x^2/(2*x^2+3)/(2*x^4+2*x^2+1)^(3/2),x, algorithm="maxima")
[Out] integrate(x^2/((2*x^4 + 2*x^2 + 1)^(3/2)*(2*x^2 + 3)), x)
```

Giac [F]

$$\int \frac{x^2}{(3+2x^2)\,(1+2x^2+2x^4)^{3/2}}\,dx = \int \frac{x^2}{(2\,x^4+2\,x^2+1)^{\frac{3}{2}}(2\,x^2+3)}\,dx$$

```
[In] integrate(x^2/(2*x^2+3)/(2*x^4+2*x^2+1)^(3/2),x, algorithm="giac")
[Out] integrate(x^2/((2*x^4 + 2*x^2 + 1)^(3/2)*(2*x^2 + 3)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{(3+2x^2)\,(1+2x^2+2x^4)^{3/2}}\,dx = \int \frac{x^2}{(2\,x^2+3)\,\left(2\,x^4+2\,x^2+1\right)^{3/2}}\,dx$$

```
[In] int(x^2/((2*x^2 + 3)*(2*x^2 + 2*x^4 + 1)^(3/2)),x)
[Out] int(x^2/((2*x^2 + 3)*(2*x^2 + 2*x^4 + 1)^(3/2)), x)
```

3.352 $\int \frac{1}{(3+2x^2)(1+2x^2+2x^4)^{3/2}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 422

$$\int \frac{1}{(3+2x^2)(1+2x^2+2x^4)^{3/2}}\,dx = -\frac{x(1+3x^2)}{5\sqrt{1+2x^2+2x^4}}+\frac{3x\sqrt{1+2x^2+2x^4}}{5\sqrt{2}\,(1+\sqrt{2}x^2)} + \frac{\arctan\left(\frac{\sqrt{\frac{5}{3}}x}{\sqrt{1+2x^2+2x^4}}\right)}{5\sqrt{15}} - \frac{3(1+\sqrt{2}x^2)\sqrt{\frac{1+2x^2+2x^4}{(1+\sqrt{2}x^2)^2}}E\left(2\arctan\left(\sqrt[4]{2}x\right)\middle|\frac{1}{4}(2-\sqrt{2})\right)}{5\;2^{3/4}\sqrt{1+2x^2+2x^4}} + \frac{(2+\sqrt{2})\,(1+\sqrt{2}x^2)\sqrt{\frac{1+2x^2+2x^4}{(1+\sqrt{2}x^2)^2}}\,\text{EllipticF}\left(2\arctan\left(\sqrt[4]{2}x\right),\frac{1}{4}(2-\sqrt{2})\right)}{2\;2^{3/4}\,(-2+3\sqrt{2})\sqrt{1+2x^2+2x^4}} + \frac{(3+\sqrt{2})\,(1+\sqrt{2}x^2)\sqrt{\frac{1+2x^2+2x^4}{(1+\sqrt{2}x^2)^2}}\,\text{EllipticPi}\left(\frac{1}{24}(12-11\sqrt{2}),2\arctan\left(\sqrt[4]{2}x\right),\frac{1}{4}(2-\sqrt{2})\right)}{15\;2^{3/4}\,(2-3\sqrt{2})\sqrt{1+2x^2+2x^4}}$$

```
[Out] 1/75*arctan(1/3*x*15^(1/2)/(2*x^4+2*x^2+1)^(1/2))*15^(1/2)-1/5*x*(3*x^2+1)/
(2*x^4+2*x^2+1)^(1/2)+3/10*x*(2*x^4+2*x^2+1)^(1/2)*2^(1/2)/(1+x^2*2^(1/2))-
3/10*(cos(2*arctan(2^(1/4)*x))^2)^(1/2)/cos(2*arctan(2^(1/4)*x))*EllipticE(
sin(2*arctan(2^(1/4)*x)),1/2*(2-2^(1/2))^(1/2))*(1+x^2*2^(1/2))*((2*x^4+2*x
^2+1)/(1+x^2*2^(1/2))^2)^(1/2)*2^(1/4)/(2*x^4+2*x^2+1)^(1/2)+1/30*(cos(2*ar
ctan(2^(1/4)*x))^2)^(1/2)/cos(2*arctan(2^(1/4)*x))*EllipticPi(sin(2*arctan(
2^(1/4)*x)),1/2-11/24*2^(1/2),1/2*(2-2^(1/2))^(1/2))*(3+2^(1/2))*(1+x^2*2^(
1/2))*((2*x^4+2*x^2+1)/(1+x^2*2^(1/2))^2)^(1/2)*2^(1/4)/(2-3*2^(1/2))/(2*x^
4+2*x^2+1)^(1/2)+1/4*(cos(2*arctan(2^(1/4)*x))^2)^(1/2)/cos(2*arctan(2^(1/4
)*x))*EllipticF(sin(2*arctan(2^(1/4)*x)),1/2*(2-2^(1/2))^(1/2))*(2+2^(1/2))
*(1+x^2*2^(1/2))*((2*x^4+2*x^2+1)/(1+x^2*2^(1/2))^2)^(1/2)*2^(1/4)/(-2+3*2^
(1/2))/(2*x^4+2*x^2+1)^(1/2)
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 501, normalized size of antiderivative = 1.19, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.269, Rules used = {1235, 1192, 1211, 1117, 1209, 1230, 1720}

$$\int \frac{1}{(3+2x^2)(1+2x^2+2x^4)^{3/2}}\,dx = \frac{\arctan\left(\frac{\sqrt{\frac{5}{3}}x}{\sqrt{2x^4+2x^2+1}}\right)}{5\sqrt{15}}$$
$$+\frac{(3+2\sqrt{2})\left(\sqrt{2}x^2+1\right)\sqrt{\frac{2x^4+2x^2+1}{\left(\sqrt{2}x^2+1\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\sqrt[4]{2}x\right),\frac{1}{4}(2-\sqrt{2})\right)}{10\ 2^{3/4}\sqrt{2x^4+2x^2+1}}$$
$$+\frac{(3+\sqrt{2})\left(\sqrt{2}x^2+1\right)\sqrt{\frac{2x^4+2x^2+1}{\left(\sqrt{2}x^2+1\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\sqrt[4]{2}x\right),\frac{1}{4}(2-\sqrt{2})\right)}{35\sqrt[4]{2}\sqrt{2x^4+2x^2+1}}$$
$$-\frac{3\left(\sqrt{2}x^2+1\right)\sqrt{\frac{2x^4+2x^2+1}{\left(\sqrt{2}x^2+1\right)^2}}E\left(2\arctan\left(\sqrt[4]{2}x\right)|\frac{1}{4}(2-\sqrt{2})\right)}{5\ 2^{3/4}\sqrt{2x^4+2x^2+1}}$$
$$-\frac{(3+\sqrt{2})^2\left(\sqrt{2}x^2+1\right)\sqrt{\frac{2x^4+2x^2+1}{\left(\sqrt{2}x^2+1\right)^2}}\,\text{EllipticPi}\left(\frac{1}{24}(12-11\sqrt{2}),2\arctan\left(\sqrt[4]{2}x\right),\frac{1}{4}(2-\sqrt{2})\right)}{210\sqrt[4]{2}\sqrt{2x^4+2x^2+1}}$$
$$+\frac{3\sqrt{2x^4+2x^2+1}x}{5\sqrt{2}\left(\sqrt{2}x^2+1\right)}-\frac{(3x^2+1)\,x}{5\sqrt{2x^4+2x^2+1}}$$

```
[In] Int[1/((3 + 2*x^2)*(1 + 2*x^2 + 2*x^4)^(3/2)),x]
```

```
[Out] -1/5*(x*(1 + 3*x^2))/Sqrt[1 + 2*x^2 + 2*x^4] + (3*x*Sqrt[1 + 2*x^2 + 2*x^4]
)/(5*Sqrt[2]*(1 + Sqrt[2]*x^2)) + ArcTan[(Sqrt[5/3]*x)/Sqrt[1 + 2*x^2 + 2*x
^4]]/(5*Sqrt[15]) - (3*(1 + Sqrt[2]*x^2)*Sqrt[(1 + 2*x^2 + 2*x^4)/(1 + Sqrt
[2]*x^2)^2]*EllipticE[2*ArcTan[2^(1/4)*x], (2 - Sqrt[2])/4])/(5*2^(3/4)*Sqr
t[1 + 2*x^2 + 2*x^4]) + ((3 + Sqrt[2])*(1 + Sqrt[2]*x^2)*Sqrt[(1 + 2*x^2 +
2*x^4)/(1 + Sqrt[2]*x^2)^2]*EllipticF[2*ArcTan[2^(1/4)*x], (2 - Sqrt[2])/4]
)/(35*2^(1/4)*Sqrt[1 + 2*x^2 + 2*x^4]) + ((3 + 2*Sqrt[2])*(1 + Sqrt[2]*x^2)
*Sqrt[(1 + 2*x^2 + 2*x^4)/(1 + Sqrt[2]*x^2)^2]*EllipticF[2*ArcTan[2^(1/4)*x
], (2 - Sqrt[2])/4])/(10*2^(3/4)*Sqrt[1 + 2*x^2 + 2*x^4]) - ((3 + Sqrt[2])^
2*(1 + Sqrt[2]*x^2)*Sqrt[(1 + 2*x^2 + 2*x^4)/(1 + Sqrt[2]*x^2)^2]*EllipticP
i[(12 - 11*Sqrt[2])/24, 2*ArcTan[2^(1/4)*x], (2 - Sqrt[2])/4])/(210*2^(1/4)
*Sqrt[1 + 2*x^2 + 2*x^4])
```

Rule 1117

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
```

```
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1192

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[x*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)*((a + b*x^2 +
 c*x^4)^(p + 1)/(2*a*(p + 1)*(b^2 - 4*a*c))), x] + Dist[1/(2*a*(p + 1)*(b^2
 - 4*a*c)), Int[Simp[(2*p + 3)*d*b^2 - a*b*e - 2*a*c*d*(4*p + 5) + (4*p + 7
)*(d*b - 2*a*e)*c*x^2, x]*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; FreeQ[{a,
b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
LtQ[p, -1] && IntegerQ[2*p]
```

Rule 1209

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

Rule 1211

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + b*x^2 + c*x^4
], x], x] - Dist[e/q, Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /; Ne
Q[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[
c/a]
```

Rule 1230

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4]), x_S
ymbol] :> With[{q = Rt[c/a, 2]}, Dist[(c*d + a*e*q)/(c*d^2 - a*e^2), Int[1/
Sqrt[a + b*x^2 + c*x^4], x], x] - Dist[(a*e*(e + d*q))/(c*d^2 - a*e^2), Int
[(1 + q*x^2)/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]), x], x]] /; FreeQ[{a, b,
 c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[c/a]
```

Rule 1235

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Dist[1/(c*d^2 - b*d*e + a*e^2), Int[(c*d - b*e - c*e*x^2)*(a + b*x^2
 + c*x^4)^p, x], x] + Dist[e^2/(c*d^2 - b*d*e + a*e^2), Int[(a + b*x^2 + c*
x^4)^(p + 1)/(d + e*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && ILtQ[p + 1/2, 0]
```

Rule 1720

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4]), x_Symbol] :> With[{q = Rt[B/A, 2]}, Simp[(-(B*d - A*e))*(A
rcTan[Rt[-b + c*(d/e) + a*(e/d), 2]*(x/Sqrt[a + b*x^2 + c*x^4])]/(2*d*e*Rt[
-b + c*(d/e) + a*(e/d), 2])), x] + Simp[(B*d + A*e)*(A + B*x^2)*(Sqrt[A^2*(
(a + b*x^2 + c*x^4)/(a*(A + B*x^2)^2))]/(4*d*e*A*q*Sqrt[a + b*x^2 + c*x^4])
)*EllipticPi[Cancel[-(B*d - A*e)^2/(4*d*e*A*B)], 2*ArcTan[q*x], 1/2 - b*(A/
(4*a*B))], x]] /; FreeQ[{a, b, c, d, e, A, B}, x] && NeQ[b^2 - 4*a*c, 0] &&
 NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] && EqQ
[c*A^2 - a*B^2, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{10}\int \frac{2-4x^2}{\left(1+2x^2+2x^4\right)^{3/2}}\,dx+\frac{2}{5}\int \frac{1}{\left(3+2x^2\right)\sqrt{1+2x^2+2x^4}}\,dx\\
&= -\frac{x\left(1+3x^2\right)}{5\sqrt{1+2x^2+2x^4}}+\frac{1}{40}\int \frac{16+24x^2}{\sqrt{1+2x^2+2x^4}}\,dx\\
&\quad+\frac{1}{35}\left(2\left(3+\sqrt{2}\right)\right)\int \frac{1}{\sqrt{1+2x^2+2x^4}}\,dx\\
&\quad-\frac{1}{35}\left(2\left(2+3\sqrt{2}\right)\right)\int \frac{1+\sqrt{2}x^2}{\left(3+2x^2\right)\sqrt{1+2x^2+2x^4}}\,dx\\
&= -\frac{x\left(1+3x^2\right)}{5\sqrt{1+2x^2+2x^4}}+\frac{\tan^{-1}\left(\frac{\sqrt{\frac{5}{3}}x}{\sqrt{1+2x^2+2x^4}}\right)}{5\sqrt{15}}\\
&\quad+\frac{\left(3+\sqrt{2}\right)\left(1+\sqrt{2}x^2\right)\sqrt{\frac{1+2x^2+2x^4}{\left(1+\sqrt{2}x^2\right)^2}}F\left(2\tan^{-1}\left(\sqrt[4]{2}x\right)|\frac{1}{4}\left(2-\sqrt{2}\right)\right)}{35\sqrt[4]{2}\sqrt{1+2x^2+2x^4}}\\
&\quad-\frac{\left(3+\sqrt{2}\right)^2\left(1+\sqrt{2}x^2\right)\sqrt{\frac{1+2x^2+2x^4}{\left(1+\sqrt{2}x^2\right)^2}}\Pi\left(\frac{1}{24}\left(12-11\sqrt{2}\right);2\tan^{-1}\left(\sqrt[4]{2}x\right)|\frac{1}{4}\left(2-\sqrt{2}\right)\right)}{210\sqrt[4]{2}\sqrt{1+2x^2+2x^4}}\\
&\quad-\frac{3\int \frac{1-\sqrt{2}x^2}{\sqrt{1+2x^2+2x^4}}\,dx}{5\sqrt{2}}+\frac{1}{10}\left(4+3\sqrt{2}\right)\int \frac{1}{\sqrt{1+2x^2+2x^4}}\,dx
\end{aligned}$$

$$= -\frac{x(1+3x^2)}{5\sqrt{1+2x^2+2x^4}} + \frac{3x\sqrt{1+2x^2+2x^4}}{5\sqrt{2}\left(1+\sqrt{2}x^2\right)} + \frac{\tan^{-1}\left(\frac{\sqrt{\frac{5}{3}}x}{\sqrt{1+2x^2+2x^4}}\right)}{5\sqrt{15}}$$
$$- \frac{3\left(1+\sqrt{2}x^2\right)\sqrt{\frac{1+2x^2+2x^4}{\left(1+\sqrt{2}x^2\right)^2}}E\left(2\tan^{-1}\left(\sqrt[4]{2}x\right)|\frac{1}{4}\left(2-\sqrt{2}\right)\right)}{5\ 2^{3/4}\sqrt{1+2x^2+2x^4}}$$
$$+ \frac{\left(3+\sqrt{2}\right)\left(1+\sqrt{2}x^2\right)\sqrt{\frac{1+2x^2+2x^4}{\left(1+\sqrt{2}x^2\right)^2}}F\left(2\tan^{-1}\left(\sqrt[4]{2}x\right)|\frac{1}{4}\left(2-\sqrt{2}\right)\right)}{35\sqrt[4]{2}\sqrt{1+2x^2+2x^4}}$$
$$+ \frac{\left(3+2\sqrt{2}\right)\left(1+\sqrt{2}x^2\right)\sqrt{\frac{1+2x^2+2x^4}{\left(1+\sqrt{2}x^2\right)^2}}F\left(2\tan^{-1}\left(\sqrt[4]{2}x\right)|\frac{1}{4}\left(2-\sqrt{2}\right)\right)}{10\ 2^{3/4}\sqrt{1+2x^2+2x^4}}$$
$$- \frac{\left(3+\sqrt{2}\right)^2\left(1+\sqrt{2}x^2\right)\sqrt{\frac{1+2x^2+2x^4}{\left(1+\sqrt{2}x^2\right)^2}}\Pi\left(\frac{1}{24}\left(12-11\sqrt{2}\right);2\tan^{-1}\left(\sqrt[4]{2}x\right)|\frac{1}{4}\left(2-\sqrt{2}\right)\right)}{210\sqrt[4]{2}\sqrt{1+2x^2+2x^4}}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.15 (sec) , antiderivative size = 199, normalized size of antiderivative = 0.47

$$\int \frac{1}{(3+2x^2)\left(1+2x^2+2x^4\right)^{3/2}}\,dx = \frac{-6x-18x^3-9i\sqrt{1-i}\sqrt{1+(1-i)x^2}\sqrt{1+(1+i)x^2}E\left(i\text{arcsinh}\left(\sqrt{1-\right.\right.}{}$$

```
[In] Integrate[1/((3 + 2*x^2)*(1 + 2*x^2 + 2*x^4)^(3/2)),x]
```

```
[Out] (-6*x - 18*x^3 - (9*I)*Sqrt[1 - I]*Sqrt[1 + (1 - I)*x^2]*Sqrt[1 + (1 + I)*x
^2]*EllipticE[I*ArcSinh[Sqrt[1 - I]*x], I] + (6 + 3*I)*Sqrt[1 - I]*Sqrt[1 +
 (1 - I)*x^2]*Sqrt[1 + (1 + I)*x^2]*EllipticF[I*ArcSinh[Sqrt[1 - I]*x], I]
+ 2*(1 - I)^(3/2)*Sqrt[1 + (1 - I)*x^2]*Sqrt[1 + (1 + I)*x^2]*EllipticPi[1/
3 + I/3, I*ArcSinh[Sqrt[1 - I]*x], I])/(30*Sqrt[1 + 2*x^2 + 2*x^4])
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.32 (sec) , antiderivative size = 246, normalized size of antiderivative = 0.58

method	result
risch	$-\frac{x(3x^2+1)}{5\sqrt{2x^4+2x^2+1}}+\frac{2\sqrt{1+(1-i)x^2}\,\sqrt{1+(1+i)x^2}\,F\left(x\sqrt{-1+i},\frac{\sqrt{2}}{2}+\frac{i\sqrt{2}}{2}\right)}{5\sqrt{-1+i}\,\sqrt{2x^4+2x^2+1}}+\frac{\left(-\frac{3}{10}+\frac{3i}{10}\right)\sqrt{1+(1-i)x^2}\,\sqrt{1+(1+i)x^2}\,\left(F\left(x\sqrt{-1+i}\right.\right.}{\sqrt{-1+i}\,\sqrt{2x^4+2x}}$
default	$-\frac{4\left(\frac{3}{20}x^3+\frac{1}{20}x\right)}{\sqrt{2x^4+2x^2+1}}+\frac{\sqrt{-ix^2+x^2+1}\,\sqrt{ix^2+x^2+1}\,F\left(x\sqrt{-1+i},\frac{\sqrt{2}}{2}+\frac{i\sqrt{2}}{2}\right)}{10\sqrt{-1+i}\,\sqrt{2x^4+2x^2+1}}+\frac{3i\sqrt{-ix^2+x^2+1}\,\sqrt{ix^2+x^2+1}\,F\left(x\sqrt{-1+i},\frac{\sqrt{2}}{2}+\frac{i\sqrt{2}}{2}\right)}{10\sqrt{-1+i}\,\sqrt{2x^4+2x^2+1}}$
elliptic	$-\frac{4\left(\frac{3}{20}x^3+\frac{1}{20}x\right)}{\sqrt{2x^4+2x^2+1}}+\frac{\sqrt{-ix^2+x^2+1}\,\sqrt{ix^2+x^2+1}\,F\left(x\sqrt{-1+i},\frac{\sqrt{2}}{2}+\frac{i\sqrt{2}}{2}\right)}{10\sqrt{-1+i}\,\sqrt{2x^4+2x^2+1}}+\frac{3i\sqrt{-ix^2+x^2+1}\,\sqrt{ix^2+x^2+1}\,F\left(x\sqrt{-1+i},\frac{\sqrt{2}}{2}+\frac{i\sqrt{2}}{2}\right)}{10\sqrt{-1+i}\,\sqrt{2x^4+2x^2+1}}$

```
[In] int(1/(2*x^2+3)/(2*x^4+2*x^2+1)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/5*x*(3*x^2+1)/(2*x^4+2*x^2+1)^(1/2)+2/5/(-1+I)^(1/2)*(1+(1-I)*x^2)^(1/2)
*(1+(1+I)*x^2)^(1/2)/(2*x^4+2*x^2+1)^(1/2)*EllipticF(x*(-1+I)^(1/2),1/2*2^(
1/2)+1/2*I*2^(1/2))+(-3/10+3/10*I)/(-1+I)^(1/2)*(1+(1-I)*x^2)^(1/2)*(1+(1+I
)*x^2)^(1/2)/(2*x^4+2*x^2+1)^(1/2)*(EllipticF(x*(-1+I)^(1/2),1/2*2^(1/2)+1/
2*I*2^(1/2))-EllipticE(x*(-1+I)^(1/2),1/2*2^(1/2)+1/2*I*2^(1/2)))+2/15/(-1+
I)^(1/2)*(1-I*x^2+x^2)^(1/2)*(1+I*x^2+x^2)^(1/2)/(2*x^4+2*x^2+1)^(1/2)*Elli
pticPi(x*(-1+I)^(1/2),1/3+1/3*I,(-1-I)^(1/2)/(-1+I)^(1/2))
```

Fricas [F]

$$\int \frac{1}{(3+2x^2)\left(1+2x^2+2x^4\right)^{3/2}}\,dx=\int \frac{1}{\left(2\,x^4+2\,x^2+1\right)^{\frac{3}{2}}\left(2\,x^2+3\right)}\,dx$$

```
[In] integrate(1/(2*x^2+3)/(2*x^4+2*x^2+1)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(2*x^4 + 2*x^2 + 1)/(8*x^10 + 28*x^8 + 40*x^6 + 32*x^4 + 14*x^
2 + 3), x)
```

Sympy [F]

$$\int \frac{1}{(3+2x^2)\left(1+2x^2+2x^4\right)^{3/2}}\,dx=\int \frac{1}{\left(2x^2+3\right)\left(2x^4+2x^2+1\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/(2*x**2+3)/(2*x**4+2*x**2+1)**(3/2),x)
```

```
[Out] Integral(1/((2*x**2 + 3)*(2*x**4 + 2*x**2 + 1)**(3/2)), x)
```

Maxima [F]

$$\int \frac{1}{(3+2x^2)(1+2x^2+2x^4)^{3/2}}\,dx = \int \frac{1}{(2\,x^4+2\,x^2+1)^{\frac{3}{2}}(2\,x^2+3)}\,dx$$

```
[In] integrate(1/(2*x^2+3)/(2*x^4+2*x^2+1)^(3/2),x, algorithm="maxima")
[Out] integrate(1/((2*x^4 + 2*x^2 + 1)^(3/2)*(2*x^2 + 3)), x)
```

Giac [F]

$$\int \frac{1}{(3+2x^2)(1+2x^2+2x^4)^{3/2}}\,dx = \int \frac{1}{(2\,x^4+2\,x^2+1)^{\frac{3}{2}}(2\,x^2+3)}\,dx$$

```
[In] integrate(1/(2*x^2+3)/(2*x^4+2*x^2+1)^(3/2),x, algorithm="giac")
[Out] integrate(1/((2*x^4 + 2*x^2 + 1)^(3/2)*(2*x^2 + 3)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(3+2x^2)(1+2x^2+2x^4)^{3/2}}\,dx = \int \frac{1}{(2\,x^2+3)\,\left(2\,x^4+2\,x^2+1\right)^{3/2}}\,dx$$

```
[In] int(1/((2*x^2 + 3)*(2*x^2 + 2*x^4 + 1)^(3/2)),x)
[Out] int(1/((2*x^2 + 3)*(2*x^2 + 2*x^4 + 1)^(3/2)), x)
```

3.353 $\int \frac{1}{x^2(3+2x^2)(1+2x^2+2x^4)^{3/2}}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 468

$$\int \frac{1}{x^2\left(3+2x^2\right)\left(1+2x^2+2x^4\right)^{3/2}}\,dx = -\frac{x}{3\sqrt{1+2x^2+2x^4}}+\frac{2x(1+3x^2)}{15\sqrt{1+2x^2+2x^4}} -\frac{\sqrt{1+2x^2+2x^4}}{3x}+\frac{2\sqrt{2}x\sqrt{1+2x^2+2x^4}}{15\left(1+\sqrt{2}x^2\right)}-\frac{2\arctan\left(\frac{\sqrt{\frac{5}{3}}x}{\sqrt{1+2x^2+2x^4}}\right)}{15\sqrt{15}} -\frac{2\sqrt[4]{2}(1+\sqrt{2}x^2)\sqrt{\frac{1+2x^2+2x^4}{\left(1+\sqrt{2}x^2\right)^2}}E\left(2\arctan\left(\sqrt[4]{2}x\right)\left|\frac{1}{4}(2-\sqrt{2})\right.\right)}{15\sqrt{1+2x^2+2x^4}} +\frac{\left(-7+3\sqrt{2}\right)\left(1+\sqrt{2}x^2\right)\sqrt{\frac{1+2x^2+2x^4}{\left(1+\sqrt{2}x^2\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\sqrt[4]{2}x\right),\frac{1}{4}(2-\sqrt{2})\right)}{3\,2^{3/4}\left(-2+3\sqrt{2}\right)\sqrt{1+2x^2+2x^4}} -\frac{\sqrt[4]{2}(3+\sqrt{2})\left(1+\sqrt{2}x^2\right)\sqrt{\frac{1+2x^2+2x^4}{\left(1+\sqrt{2}x^2\right)^2}}\operatorname{EllipticPi}\left(\frac{1}{24}(12-11\sqrt{2}),2\arctan\left(\sqrt[4]{2}x\right),\frac{1}{4}(2-\sqrt{2})\right)}{45\left(2-3\sqrt{2}\right)\sqrt{1+2x^2+2x^4}}$$

```
[Out] -2/225*arctan(1/3*x*15^(1/2)/(2*x^4+2*x^2+1)^(1/2))*15^(1/2)-1/3*x/(2*x^4+2
*x^2+1)^(1/2)+2/15*x*(3*x^2+1)/(2*x^4+2*x^2+1)^(1/2)-1/3*(2*x^4+2*x^2+1)^(1
/2)/x+2/15*x*(2*x^4+2*x^2+1)^(1/2)*2^(1/2)/(1+x^2*2^(1/2))-2/15*(cos(2*arct
an(2^(1/4)*x))^2)^(1/2)/cos(2*arctan(2^(1/4)*x))*EllipticE(sin(2*arctan(2^(
1/4)*x)),1/2*(2-2^(1/2))^(1/2))*(1+x^2*2^(1/2))*((2*x^4+2*x^2+1)/(1+x^2*2^(
1/2))^2)^(1/2)*2^(1/4)/(2*x^4+2*x^2+1)^(1/2)-1/45*(cos(2*arctan(2^(1/4)*x))
^2)^(1/2)/cos(2*arctan(2^(1/4)*x))*EllipticPi(sin(2*arctan(2^(1/4)*x)),1/2-
11/24*2^(1/2),1/2*(2-2^(1/2))^(1/2))*(3+2^(1/2))*(1+x^2*2^(1/2))*((2*x^4+2*
x^2+1)/(1+x^2*2^(1/2))^2)^(1/2)*2^(1/4)/(2-3*2^(1/2))/(2*x^4+2*x^2+1)^(1/2)
+1/6*(cos(2*arctan(2^(1/4)*x))^2)^(1/2)/cos(2*arctan(2^(1/4)*x))*EllipticF(
```

```
sin(2*arctan(2^(1/4)*x)),1/2*(2-2^(1/2))^(1/2))*(-7+3*2^(1/2))*(1+x^2*2^(1/
2))*((2*x^4+2*x^2+1)/(1+x^2*2^(1/2))^2)^(1/2)*2^(1/4)/(-2+3*2^(1/2))/(2*x^4
+2*x^2+1)^(1/2)
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 644, normalized size of antiderivative = 1.38, number of steps used = 15, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.345$, Rules used = {1349, 1135, 1295, 1211, 1117, 1209, 1235, 1192, 1230, 1720}

$$\int \frac{1}{x^2(3+2x^2)(1+2x^2+2x^4)^{3/2}}\,dx = -\frac{2\arctan\left(\frac{\sqrt{\frac{5}{3}}x}{\sqrt{2x^4+2x^2+1}}\right)}{15\sqrt{15}}$$
$$-\frac{(3+2\sqrt{2})\left(\sqrt{2}x^2+1\right)\sqrt{\frac{2x^4+2x^2+1}{\left(\sqrt{2}x^2+1\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\sqrt[4]{2}x\right),\frac{1}{4}(2-\sqrt{2})\right)}{15\ 2^{3/4}\sqrt{2x^4+2x^2+1}}$$
$$-\frac{2^{3/4}(3+\sqrt{2})\left(\sqrt{2}x^2+1\right)\sqrt{\frac{2x^4+2x^2+1}{\left(\sqrt{2}x^2+1\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\sqrt[4]{2}x\right),\frac{1}{4}(2-\sqrt{2})\right)}{105\sqrt{2x^4+2x^2+1}}$$
$$-\frac{(1-\sqrt{2})\left(\sqrt{2}x^2+1\right)\sqrt{\frac{2x^4+2x^2+1}{\left(\sqrt{2}x^2+1\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\sqrt[4]{2}x\right),\frac{1}{4}(2-\sqrt{2})\right)}{6\sqrt[4]{2}\sqrt{2x^4+2x^2+1}}$$
$$-\frac{2\sqrt[4]{2}\left(\sqrt{2}x^2+1\right)\sqrt{\frac{2x^4+2x^2+1}{\left(\sqrt{2}x^2+1\right)^2}}\,E\left(2\arctan\left(\sqrt[4]{2}x\right)|\frac{1}{4}(2-\sqrt{2})\right)}{15\sqrt{2x^4+2x^2+1}}$$
$$+\frac{(3+\sqrt{2})^2\left(\sqrt{2}x^2+1\right)\sqrt{\frac{2x^4+2x^2+1}{\left(\sqrt{2}x^2+1\right)^2}}\,\text{EllipticPi}\left(\frac{1}{24}(12-11\sqrt{2}),2\arctan\left(\sqrt[4]{2}x\right),\frac{1}{4}(2-\sqrt{2})\right)}{315\sqrt[4]{2}\sqrt{2x^4+2x^2+1}}$$
$$+\frac{2\sqrt{2}\sqrt{2x^4+2x^2+1}x}{15\left(\sqrt{2}x^2+1\right)}+\frac{2(3x^2+1)x}{15\sqrt{2x^4+2x^2+1}}-\frac{x}{3\sqrt{2x^4+2x^2+1}}-\frac{\sqrt{2x^4+2x^2+1}}{3x}$$

```
[In] Int[1/(x^2*(3 + 2*x^2)*(1 + 2*x^2 + 2*x^4)^(3/2)),x]
```

```
[Out] -1/3*x/Sqrt[1 + 2*x^2 + 2*x^4] + (2*x*(1 + 3*x^2))/(15*Sqrt[1 + 2*x^2 + 2*x
^4]) - Sqrt[1 + 2*x^2 + 2*x^4]/(3*x) + (2*Sqrt[2]*x*Sqrt[1 + 2*x^2 + 2*x^4]
)/(15*(1 + Sqrt[2]*x^2)) - (2*ArcTan[(Sqrt[5/3]*x)/Sqrt[1 + 2*x^2 + 2*x^4]]
)/(15*Sqrt[15]) - (2*2^(1/4)*(1 + Sqrt[2]*x^2)*Sqrt[(1 + 2*x^2 + 2*x^4)/(1
+ Sqrt[2]*x^2)^2]*EllipticE[2*ArcTan[2^(1/4)*x], (2 - Sqrt[2])/4])/(15*Sqrt
[1 + 2*x^2 + 2*x^4]) - ((1 - Sqrt[2])*(1 + Sqrt[2]*x^2)*Sqrt[(1 + 2*x^2 + 2
*x^4)/(1 + Sqrt[2]*x^2)^2]*EllipticF[2*ArcTan[2^(1/4)*x], (2 - Sqrt[2])/4])
/(6*2^(1/4)*Sqrt[1 + 2*x^2 + 2*x^4]) - (2^(3/4)*(3 + Sqrt[2])*(1 + Sqrt[2]*
x^2)*Sqrt[(1 + 2*x^2 + 2*x^4)/(1 + Sqrt[2]*x^2)^2]*EllipticF[2*ArcTan[2^(1/
```

```
4)*x], (2 - Sqrt[2])/4])/(105*Sqrt[1 + 2*x^2 + 2*x^4]) - ((3 + 2*Sqrt[2])*(
1 + Sqrt[2]*x^2)*Sqrt[(1 + 2*x^2 + 2*x^4)/(1 + Sqrt[2]*x^2)^2]*EllipticF[2*
ArcTan[2^(1/4)*x], (2 - Sqrt[2])/4])/(15*2^(3/4)*Sqrt[1 + 2*x^2 + 2*x^4]) +
 ((3 + Sqrt[2])^2*(1 + Sqrt[2]*x^2)*Sqrt[(1 + 2*x^2 + 2*x^4)/(1 + Sqrt[2]*x
^2)^2]*EllipticPi[(12 - 11*Sqrt[2])/24, 2*ArcTan[2^(1/4)*x], (2 - Sqrt[2])/
4])/(315*2^(1/4)*Sqrt[1 + 2*x^2 + 2*x^4])
```

Rule 1117

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1135

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Simp[(-(d*x)^(m + 1))*(b^2 - 2*a*c + b*c*x^2)*((a + b*x^2 + c*x^4)^(p +
 1)/(2*a*d*(p + 1)*(b^2 - 4*a*c))), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c))
, Int[(d*x)^m*(a + b*x^2 + c*x^4)^(p + 1)*Simp[b^2*(m + 2*p + 3) - 2*a*c*(m
 + 4*p + 5) + b*c*(m + 4*p + 7)*x^2, x], x], x] /; FreeQ[{a, b, c, d, m}, x
] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && IntegerQ[2*p] && (IntegerQ[p] ||
IntegerQ[m])
```

Rule 1192

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[x*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)*((a + b*x^2 +
 c*x^4)^(p + 1)/(2*a*(p + 1)*(b^2 - 4*a*c))), x] + Dist[1/(2*a*(p + 1)*(b^2
 - 4*a*c)), Int[Simp[(2*p + 3)*d*b^2 - a*b*e - 2*a*c*d*(4*p + 5) + (4*p + 7
)*(d*b - 2*a*e)*c*x^2, x]*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; FreeQ[{a,
b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
LtQ[p, -1] && IntegerQ[2*p]
```

Rule 1209

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

Rule 1211

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + b*x^2 + c*x^4
```

```
], x], x] - Dist[e/q, Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /; Ne
Q[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[
c/a]
```

Rule 1230

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4]), x_S
ymbol] :> With[{q = Rt[c/a, 2]}, Dist[(c*d + a*e*q)/(c*d^2 - a*e^2), Int[1/
Sqrt[a + b*x^2 + c*x^4], x], x] - Dist[(a*e*(e + d*q))/(c*d^2 - a*e^2), Int
[(1 + q*x^2)/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]), x], x]] /; FreeQ[{a, b,
 c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[c/a]
```

Rule 1235

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Dist[1/(c*d^2 - b*d*e + a*e^2), Int[(c*d - b*e - c*e*x^2)*(a + b*x^2
 + c*x^4)^p, x], x] + Dist[e^2/(c*d^2 - b*d*e + a*e^2), Int[(a + b*x^2 + c*
x^4)^(p + 1)/(d + e*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && ILtQ[p + 1/2, 0]
```

Rule 1295

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[d*(f*x)^(m + 1)*((a + b*x^2 + c*x^4)^(p + 1)
/(a*f*(m + 1))), x] + Dist[1/(a*f^2*(m + 1)), Int[(f*x)^(m + 2)*(a + b*x^2
+ c*x^4)^p*Simp[a*e*(m + 1) - b*d*(m + 2*p + 3) - c*d*(m + 4*p + 5)*x^2, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[m
, -1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1349

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*
(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, p, q}, x] && N
eQ[b^2 - 4*a*c, 0] && (IGtQ[p, 0] || IGtQ[q, 0] || IntegersQ[m, q])
```

Rule 1720

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4]), x_Symbol] :> With[{q = Rt[B/A, 2]}, Simp[(-(B*d - A*e))*(A
rcTan[Rt[-b + c*(d/e) + a*(e/d), 2]*(x/Sqrt[a + b*x^2 + c*x^4])]/(2*d*e*Rt[
-b + c*(d/e) + a*(e/d), 2])), x] + Simp[(B*d + A*e)*(A + B*x^2)*(Sqrt[A^2*(
(a + b*x^2 + c*x^4)/(a*(A + B*x^2)^2))]/(4*d*e*A*q*Sqrt[a + b*x^2 + c*x^4])
)*EllipticPi[Cancel[-(B*d - A*e)^2/(4*d*e*A*B)], 2*ArcTan[q*x], 1/2 - b*(A/
(4*a*B))], x]] /; FreeQ[{a, b, c, d, e, A, B}, x] && NeQ[b^2 - 4*a*c, 0] &&
```

```
NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] && EqQ
[c*A^2 - a*B^2, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(\frac{1}{3x^2\left(1+2x^2+2x^4\right)^{3/2}}-\frac{2}{3\left(3+2x^2\right)\left(1+2x^2+2x^4\right)^{3/2}}\right)dx\\
&= \frac{1}{3}\int \frac{1}{x^2\left(1+2x^2+2x^4\right)^{3/2}}\,dx-\frac{2}{3}\int \frac{1}{\left(3+2x^2\right)\left(1+2x^2+2x^4\right)^{3/2}}\,dx\\
&= -\frac{x}{3\sqrt{1+2x^2+2x^4}}-\frac{1}{15}\int \frac{2-4x^2}{\left(1+2x^2+2x^4\right)^{3/2}}\,dx\\
&\quad+\frac{1}{12}\int \frac{4-4x^2}{x^2\sqrt{1+2x^2+2x^4}}\,dx-\frac{4}{15}\int \frac{1}{\left(3+2x^2\right)\sqrt{1+2x^2+2x^4}}\,dx\\
&= -\frac{x}{3\sqrt{1+2x^2+2x^4}}+\frac{2x(1+3x^2)}{15\sqrt{1+2x^2+2x^4}}-\frac{\sqrt{1+2x^2+2x^4}}{3x}\\
&\quad-\frac{1}{60}\int \frac{16+24x^2}{\sqrt{1+2x^2+2x^4}}\,dx-\frac{1}{12}\int \frac{4-8x^2}{\sqrt{1+2x^2+2x^4}}\,dx\\
&\quad-\frac{1}{105}\left(4\left(3+\sqrt{2}\right)\right)\int \frac{1}{\sqrt{1+2x^2+2x^4}}\,dx\\
&\quad+\frac{1}{105}\left(4\left(2+3\sqrt{2}\right)\right)\int \frac{1+\sqrt{2}x^2}{\left(3+2x^2\right)\sqrt{1+2x^2+2x^4}}\,dx\\
&= -\frac{x}{3\sqrt{1+2x^2+2x^4}}+\frac{2x(1+3x^2)}{15\sqrt{1+2x^2+2x^4}}-\frac{\sqrt{1+2x^2+2x^4}}{3x}-\frac{2\tan^{-1}\left(\frac{\sqrt{\frac{5}{3}}x}{\sqrt{1+2x^2+2x^4}}\right)}{15\sqrt{15}}\\
&\quad-\frac{2^{3/4}\left(3+\sqrt{2}\right)\left(1+\sqrt{2}x^2\right)\sqrt{\frac{1+2x^2+2x^4}{\left(1+\sqrt{2}x^2\right)^2}}F\left(2\tan^{-1}\left(\sqrt[4]{2}x\right)\middle|\frac{1}{4}\left(2-\sqrt{2}\right)\right)}{105\sqrt{1+2x^2+2x^4}}\\
&\quad+\frac{\left(3+\sqrt{2}\right)^2\left(1+\sqrt{2}x^2\right)\sqrt{\frac{1+2x^2+2x^4}{\left(1+\sqrt{2}x^2\right)^2}}\Pi\left(\frac{1}{24}\left(12-11\sqrt{2}\right);2\tan^{-1}\left(\sqrt[4]{2}x\right)\middle|\frac{1}{4}\left(2-\sqrt{2}\right)\right)}{315\sqrt[4]{2}\sqrt{1+2x^2+2x^4}}\\
&\quad+\frac{1}{5}\sqrt{2}\int \frac{1-\sqrt{2}x^2}{\sqrt{1+2x^2+2x^4}}\,dx-\frac{1}{3}\sqrt{2}\int \frac{1-\sqrt{2}x^2}{\sqrt{1+2x^2+2x^4}}\,dx\\
&\quad-\frac{1}{3}\left(1-\sqrt{2}\right)\int \frac{1}{\sqrt{1+2x^2+2x^4}}\,dx-\frac{1}{15}\left(4+3\sqrt{2}\right)\int \frac{1}{\sqrt{1+2x^2+2x^4}}\,dx
\end{aligned}$$

$$
\begin{aligned}
&= -\frac{x}{3\sqrt{1+2x^2+2x^4}} + \frac{2x(1+3x^2)}{15\sqrt{1+2x^2+2x^4}} - \frac{\sqrt{1+2x^2+2x^4}}{3x} \\
&+ \frac{2\sqrt{2}x\sqrt{1+2x^2+2x^4}}{15\left(1+\sqrt{2}x^2\right)} - \frac{2\tan^{-1}\left(\frac{\sqrt{\frac{5}{3}}x}{\sqrt{1+2x^2+2x^4}}\right)}{15\sqrt{15}} \\
&- \frac{2\sqrt[4]{2}(1+\sqrt{2}x^2)\sqrt{\frac{1+2x^2+2x^4}{\left(1+\sqrt{2}x^2\right)^2}}E\left(2\tan^{-1}\left(\sqrt[4]{2}x\right)|\frac{1}{4}(2-\sqrt{2})\right)}{15\sqrt{1+2x^2+2x^4}} \\
&- \frac{\left(1-\sqrt{2}\right)\left(1+\sqrt{2}x^2\right)\sqrt{\frac{1+2x^2+2x^4}{\left(1+\sqrt{2}x^2\right)^2}}F\left(2\tan^{-1}\left(\sqrt[4]{2}x\right)|\frac{1}{4}(2-\sqrt{2})\right)}{6\sqrt[4]{2}\sqrt{1+2x^2+2x^4}} \\
&- \frac{2^{3/4}(3+\sqrt{2})\left(1+\sqrt{2}x^2\right)\sqrt{\frac{1+2x^2+2x^4}{\left(1+\sqrt{2}x^2\right)^2}}F\left(2\tan^{-1}\left(\sqrt[4]{2}x\right)|\frac{1}{4}(2-\sqrt{2})\right)}{105\sqrt{1+2x^2+2x^4}} \\
&- \frac{\left(3+2\sqrt{2}\right)\left(1+\sqrt{2}x^2\right)\sqrt{\frac{1+2x^2+2x^4}{\left(1+\sqrt{2}x^2\right)^2}}F\left(2\tan^{-1}\left(\sqrt[4]{2}x\right)|\frac{1}{4}(2-\sqrt{2})\right)}{15\ 2^{3/4}\sqrt{1+2x^2+2x^4}} \\
&+ \frac{\left(3+\sqrt{2}\right)^2\left(1+\sqrt{2}x^2\right)\sqrt{\frac{1+2x^2+2x^4}{\left(1+\sqrt{2}x^2\right)^2}}\Pi\left(\frac{1}{24}(12-11\sqrt{2});2\tan^{-1}\left(\sqrt[4]{2}x\right)|\frac{1}{4}(2-\sqrt{2})\right)}{315\sqrt[4]{2}\sqrt{1+2x^2+2x^4}}
\end{aligned}
$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.21 (sec) , antiderivative size = 211, normalized size of antiderivative = 0.45

$$
\int \frac{1}{x^2\left(3+2x^2\right)\left(1+2x^2+2x^4\right)^{3/2}}\,dx = \frac{-12i\sqrt{1-i}x\sqrt{1+(1-i)x^2}\sqrt{1+(1+i)x^2}E\left(i\text{arcsinh}\left(\sqrt{1-i}x\right)\middle|i\right)
$$

```
[In] Integrate[1/(x^2*(3 + 2*x^2)*(1 + 2*x^2 + 2*x^4)^(3/2)),x]
```

```
[Out] ((-12*I)*Sqrt[1 - I]*x*Sqrt[1 + (1 - I)*x^2]*Sqrt[1 + (1 + I)*x^2]*Elliptic
E[I*ArcSinh[Sqrt[1 - I]*x], I] - (27 - 39*I)*Sqrt[1 - I]*x*Sqrt[1 + (1 - I)
*x^2]*Sqrt[1 + (1 + I)*x^2]*EllipticF[I*ArcSinh[Sqrt[1 - I]*x], I] - 2*(15
+ 39*x^2 + 12*x^4 + 2*(1 - I)^(3/2)*x*Sqrt[1 + (1 - I)*x^2]*Sqrt[1 + (1 + I
)*x^2]*EllipticPi[1/3 + I/3, I*ArcSinh[Sqrt[1 - I]*x], I]))/(90*x*Sqrt[1 +
2*x^2 + 2*x^4])
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.98 (sec) , antiderivative size = 253, normalized size of antiderivative = 0.54

method	result
risch	$-\frac{4x^4+13x^2+5}{15x\sqrt{2x^4+2x^2+1}}-\frac{3\sqrt{1+(1-i)x^2}\,\sqrt{1+(1+i)x^2}\,F\left(x\sqrt{-1+i},\frac{\sqrt{2}}{2}+\frac{i\sqrt{2}}{2}\right)}{5\sqrt{-1+i}\,\sqrt{2x^4+2x^2+1}}+\frac{\left(-\frac{2}{15}+\frac{2i}{15}\right)\sqrt{1+(1-i)x^2}\,\sqrt{1+(1+i)x^2}\,\left(F\left(x\sqrt{-1\right.\right.}{\sqrt{-1+i}\,\sqrt{2x^4+}}$
elliptic	$-\frac{4\left(-\frac{1}{10}x^3+\frac{1}{20}x\right)}{\sqrt{2x^4+2x^2+1}}-\frac{\sqrt{2x^4+2x^2+1}}{3x}-\frac{11\sqrt{-ix^2+x^2+1}\,\sqrt{ix^2+x^2+1}\,F\left(x\sqrt{-1+i},\frac{\sqrt{2}}{2}+\frac{i\sqrt{2}}{2}\right)}{15\sqrt{-1+i}\,\sqrt{2x^4+2x^2+1}}+\frac{2i\sqrt{-ix^2+x^2+1}\,\sqrt{ix^2+x^2+1}\,F}{15\sqrt{-1+i}\,\sqrt{2x^4+}}$
default	$-\frac{x}{3\sqrt{2x^4+2x^2+1}}-\frac{\sqrt{2x^4+2x^2+1}}{3x}-\frac{\sqrt{1+(1-i)x^2}\,\sqrt{1+(1+i)x^2}\,F\left(x\sqrt{-1+i},\frac{\sqrt{2}}{2}+\frac{i\sqrt{2}}{2}\right)}{3\sqrt{-1+i}\,\sqrt{2x^4+2x^2+1}}+\frac{\left(-\frac{1}{3}+\frac{i}{3}\right)\sqrt{1+(1-i)x^2}\,\sqrt{1+(1+i)}}{\sqrt{}}$

```
[In] int(1/x^2/(2*x^2+3)/(2*x^4+2*x^2+1)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/15*(4*x^4+13*x^2+5)/x/(2*x^4+2*x^2+1)^(1/2)-3/5/(-1+I)^(1/2)*(1+(1-I)*x^
2)^(1/2)*(1+(1+I)*x^2)^(1/2)/(2*x^4+2*x^2+1)^(1/2)*EllipticF(x*(-1+I)^(1/2)
,1/2*2^(1/2)+1/2*I*2^(1/2))+(-2/15+2/15*I)/(-1+I)^(1/2)*(1+(1-I)*x^2)^(1/2)
*(1+(1+I)*x^2)^(1/2)/(2*x^4+2*x^2+1)^(1/2)*(EllipticF(x*(-1+I)^(1/2),1/2*2^
(1/2)+1/2*I*2^(1/2))-EllipticE(x*(-1+I)^(1/2),1/2*2^(1/2)+1/2*I*2^(1/2)))-4
/45/(-1+I)^(1/2)*(1-I*x^2+x^2)^(1/2)*(1+I*x^2+x^2)^(1/2)/(2*x^4+2*x^2+1)^(1
/2)*EllipticPi(x*(-1+I)^(1/2),1/3+1/3*I,(-1-I)^(1/2)/(-1+I)^(1/2))
```

Fricas [F]

$$\int \frac{1}{x^2\left(3+2x^2\right)\left(1+2x^2+2x^4\right)^{3/2}}\,dx=\int \frac{1}{\left(2\,x^4+2\,x^2+1\right)^{\frac{3}{2}}\left(2\,x^2+3\right)x^2}\,dx$$

```
[In] integrate(1/x^2/(2*x^2+3)/(2*x^4+2*x^2+1)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(2*x^4 + 2*x^2 + 1)/(8*x^12 + 28*x^10 + 40*x^8 + 32*x^6 + 14*x
^4 + 3*x^2), x)
```

Sympy [F]

$$\int \frac{1}{x^2\left(3+2x^2\right)\left(1+2x^2+2x^4\right)^{3/2}}\,dx=\int \frac{1}{x^2\cdot\left(2x^2+3\right)\left(2x^4+2x^2+1\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/x**2/(2*x**2+3)/(2*x**4+2*x**2+1)**(3/2),x)
```

```
[Out] Integral(1/(x**2*(2*x**2 + 3)*(2*x**4 + 2*x**2 + 1)**(3/2)), x)
```

Maxima [F]

$$\int \frac{1}{x^2\left(3+2x^2\right)\left(1+2x^2+2x^4\right)^{3/2}}\,dx = \int \frac{1}{\left(2\,x^4+2\,x^2+1\right)^{\frac{3}{2}}\left(2\,x^2+3\right)x^2}\,dx$$

```
[In] integrate(1/x^2/(2*x^2+3)/(2*x^4+2*x^2+1)^(3/2),x, algorithm="maxima")
[Out] integrate(1/((2*x^4 + 2*x^2 + 1)^(3/2)*(2*x^2 + 3)*x^2), x)
```

Giac [F]

$$\int \frac{1}{x^2\left(3+2x^2\right)\left(1+2x^2+2x^4\right)^{3/2}}\,dx = \int \frac{1}{\left(2\,x^4+2\,x^2+1\right)^{\frac{3}{2}}\left(2\,x^2+3\right)x^2}\,dx$$

```
[In] integrate(1/x^2/(2*x^2+3)/(2*x^4+2*x^2+1)^(3/2),x, algorithm="giac")
[Out] integrate(1/((2*x^4 + 2*x^2 + 1)^(3/2)*(2*x^2 + 3)*x^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^2\left(3+2x^2\right)\left(1+2x^2+2x^4\right)^{3/2}}\,dx = \int \frac{1}{x^2\,\left(2\,x^2+3\right)\,{\left(2\,x^4+2\,x^2+1\right)}^{3/2}}\,dx$$

```
[In] int(1/(x^2*(2*x^2 + 3)*(2*x^2 + 2*x^4 + 1)^(3/2)),x)
[Out] int(1/(x^2*(2*x^2 + 3)*(2*x^2 + 2*x^4 + 1)^(3/2)), x)
```

3.354 $\int \frac{x^7\sqrt{d+ex^2}}{a+bx^2+cx^4}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 406

$$\int \frac{x^7\sqrt{d+ex^2}}{a+bx^2+cx^4}\,dx=\frac{(b^2-ac)\sqrt{d+ex^2}}{c^3}-\frac{(cd+be)\left(d+ex^2\right)^{3/2}}{3c^2e^2}+\frac{\left(d+ex^2\right)^{5/2}}{5ce^2}-\frac{\left(b^2cd-ac^2d-b^3e+2abce-\frac{b^3cd-3abc^2d-b^4e+4ab^2ce-2a^2c^2e}{\sqrt{b^2-4ac}}\right)\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex^2}}{\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}}\right)}{\sqrt{2}c^{7/2}\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}}-\frac{\left(b^2cd-ac^2d-b^3e+2abce+\frac{b^3cd-3abc^2d-b^4e+4ab^2ce-2a^2c^2e}{\sqrt{b^2-4ac}}\right)\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex^2}}{\sqrt{2cd-\left(b+\sqrt{b^2-4ac}\right)e}}\right)}{\sqrt{2}c^{7/2}\sqrt{2cd-\left(b+\sqrt{b^2-4ac}\right)e}}$$

```
[Out] -1/3*(b*e+c*d)*(e*x^2+d)^(3/2)/c^2/e^2+1/5*(e*x^2+d)^(5/2)/c/e^2+(-a*c+b^2)
*(e*x^2+d)^(1/2)/c^3-1/2*arctanh(2^(1/2)*c^(1/2)*(e*x^2+d)^(1/2)/(2*c*d-e*(
b-(-4*a*c+b^2)^(1/2)))^(1/2))*(b^2*c*d-a*c^2*d-b^3*e+2*a*b*c*e+(2*a^2*c^2*e
-4*a*b^2*c*e+3*a*b*c^2*d+b^4*e-b^3*c*d)/(-4*a*c+b^2)^(1/2))/c^(7/2)*2^(1/2)
/(2*c*d-e*(b-(-4*a*c+b^2)^(1/2)))^(1/2)-1/2*arctanh(2^(1/2)*c^(1/2)*(e*x^2+
d)^(1/2)/(2*c*d-e*(b+(-4*a*c+b^2)^(1/2)))^(1/2))*(b^2*c*d-a*c^2*d-b^3*e+2*a
*b*c*e+(-2*a^2*c^2*e+4*a*b^2*c*e-3*a*b*c^2*d-b^4*e+b^3*c*d)/(-4*a*c+b^2)^(1
/2))/c^(7/2)*2^(1/2)/(2*c*d-e*(b+(-4*a*c+b^2)^(1/2)))^(1/2)
```

Rubi [A] (verified)

Time = 5.95 (sec) , antiderivative size = 406, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.172$, Rules used = {1265, 911, 1301, 1180, 214}

$$\int \frac{x^7\sqrt{d+ex^2}}{a+bx^2+cx^4}\,dx = -\frac{\left(-\frac{-2a^2c^2e+4ab^2ce-3abc^2d+b^4(-e)+b^3cd}{\sqrt{b^2-4ac}}+2abce-ac^2d+b^3(-e)+b^2cd\right)\text{arctanh}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex^2}}{\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}\right)}{\sqrt{2}c^{7/2}\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}} - \frac{\left(\frac{-2a^2c^2e+4ab^2ce-3abc^2d+b^4(-e)+b^3cd}{\sqrt{b^2-4ac}}+2abce-ac^2d+b^3(-e)+b^2cd\right)\text{arctanh}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex^2}}{\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}\right)}{\sqrt{2}c^{7/2}\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}} + \frac{(b^2-ac)\sqrt{d+ex^2}}{c^3} - \frac{(d+ex^2)^{3/2}(be+cd)}{3c^2e^2} + \frac{(d+ex^2)^{5/2}}{5ce^2}$$

```
[In] Int[(x^7*Sqrt[d + e*x^2])/(a + b*x^2 + c*x^4),x]
```

```
[Out] ((b^2 - a*c)*Sqrt[d + e*x^2])/c^3 - ((c*d + b*e)*(d + e*x^2)^(3/2))/(3*c^2*
e^2) + (d + e*x^2)^(5/2)/(5*c*e^2) - ((b^2*c*d - a*c^2*d - b^3*e + 2*a*b*c*
e - (b^3*c*d - 3*a*b*c^2*d - b^4*e + 4*a*b^2*c*e - 2*a^2*c^2*e)/Sqrt[b^2 -
4*a*c])*ArcTanh[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x^2])/Sqrt[2*c*d - (b - Sqrt[b^
2 - 4*a*c])*e]])/(Sqrt[2]*c^(7/2)*Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e])
- ((b^2*c*d - a*c^2*d - b^3*e + 2*a*b*c*e + (b^3*c*d - 3*a*b*c^2*d - b^4*e
+ 4*a*b^2*c*e - 2*a^2*c^2*e)/Sqrt[b^2 - 4*a*c])*ArcTanh[(Sqrt[2]*Sqrt[c]*Sq
rt[d + e*x^2])/Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]])/(Sqrt[2]*c^(7/2)*S
qrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 911

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> With[{q = Denominator[m]}, Dist[q/e, S
ubst[Int[x^(q*(m + 1) - 1)*((e*f - d*g)/e + g*(x^q/e))^n*((c*d^2 - b*d*e +
a*e^2)/e^2 - (2*c*d - b*e)*(x^q/e^2) + c*(x^(2*q)/e^2))^p, x], x, (d + e*x)
^(1/q)], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegersQ[n, p] && Fra
```

```
ctionQ[m]
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1265

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rule 1301

```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.))/((a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*((d + e*x^2)^q/(a
+ b*x^2 + c*x^4)), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b^2 - 4
*a*c, 0] && IntegerQ[q] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{2}\text{Subst}\left(\int \frac{x^3\sqrt{d+ex}}{a+bx+cx^2}\,dx,x,x^2\right)\\
&= \frac{\text{Subst}\left(\int \frac{x^2\left(-\frac{d}{e}+\frac{x^2}{e}\right)^3}{\frac{cd^2-bde+ae^2}{e^2}-\frac{(2cd-be)x^2}{e^2}+\frac{cx^4}{e^2}}\,dx,x,\sqrt{d+ex^2}\right)}{e}\\
&= \frac{\text{Subst}\left(\int \left(\frac{(b^2-ac)e}{c^3}-\frac{(cd+be)x^2}{c^2e}+\frac{x^4}{ce}-\frac{(b^2-ac)(cd^2-bde+ae^2)-(b^2cd-ac^2d-b^3e+2abce)x^2}{c^3e\left(\frac{cd^2-bde+ae^2}{e^2}-\frac{(2cd-be)x^2}{e^2}+\frac{cx^4}{e^2}\right)}\right)dx,x,\sqrt{d+ex^2}\right)}{e}\\
&= \frac{(b^2-ac)\sqrt{d+ex^2}}{c^3}-\frac{(cd+be)(d+ex^2)^{3/2}}{3c^2e^2}+\frac{(d+ex^2)^{5/2}}{5ce^2}\\
&\quad-\frac{\text{Subst}\left(\int \frac{(b^2-ac)(cd^2-bde+ae^2)+(-b^2cd+ac^2d+b^3e-2abce)x^2}{\frac{cd^2-bde+ae^2}{e^2}-\frac{(2cd-be)x^2}{e^2}+\frac{cx^4}{e^2}}\,dx,x,\sqrt{d+ex^2}\right)}{c^3e^2}
\end{aligned}$$

$$= \frac{(b^2-ac)\sqrt{d+ex^2}}{c^3} - \frac{(cd+be)(d+ex^2)^{3/2}}{3c^2e^2} + \frac{(d+ex^2)^{5/2}}{5ce^2}$$
$$+ \frac{\left(b^2cd-ac^2d-b^3e+2abce-\frac{b^3cd-3abc^2d-b^4e+4ab^2ce-2a^2c^2e}{\sqrt{b^2-4ac}}\right)\text{Subst}\left(\int \frac{1}{-\frac{\sqrt{b^2-4ac}}{2e}-\frac{2cd-be}{2e^2}+\frac{cx^2}{e^2}}\,dx,x,\sqrt{d\right.}{2c^3e^2}$$
$$+ \frac{\left(b^2cd-ac^2d-b^3e+2abce+\frac{b^3cd-3abc^2d-b^4e+4ab^2ce-2a^2c^2e}{\sqrt{b^2-4ac}}\right)\text{Subst}\left(\int \frac{1}{\frac{\sqrt{b^2-4ac}}{2e}-\frac{2cd-be}{2e^2}+\frac{cx^2}{e^2}}\,dx,x,\sqrt{d\right.}{2c^3e^2}$$
$$= \frac{(b^2-ac)\sqrt{d+ex^2}}{c^3} - \frac{(cd+be)(d+ex^2)^{3/2}}{3c^2e^2} + \frac{(d+ex^2)^{5/2}}{5ce^2}$$
$$- \frac{\left(b^2cd-ac^2d-b^3e+2abce-\frac{b^3cd-3abc^2d-b^4e+4ab^2ce-2a^2c^2e}{\sqrt{b^2-4ac}}\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex^2}}{\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}}\right)}{\sqrt{2}c^{7/2}\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}}$$
$$- \frac{\left(b^2cd-ac^2d-b^3e+2abce+\frac{b^3cd-3abc^2d-b^4e+4ab^2ce-2a^2c^2e}{\sqrt{b^2-4ac}}\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex^2}}{\sqrt{2cd-\left(b+\sqrt{b^2-4ac}\right)e}}\right)}{\sqrt{2}c^{7/2}\sqrt{2cd-\left(b+\sqrt{b^2-4ac}\right)e}}$$

Mathematica [A] (verified)

Time = 1.30 (sec) , antiderivative size = 475, normalized size of antiderivative = 1.17

$$\int \frac{x^7\sqrt{d+ex^2}}{a+bx^2+cx^4}\,dx$$

$$= \frac{\frac{2\sqrt{c}\sqrt{d+ex^2}(15b^2e^2+c^2(-2d^2+dex^2+3e^2x^4)-5ce(3ae+b(d+ex^2)))}{e^2} - \frac{15\sqrt{2}\left(-b^4e+ac^2\left(\sqrt{b^2-4ac}d-2ae\right)+b^2c\left(-\sqrt{b^2-4ac}d+4ae\right)+b^3\left(cd+\right.\right.}{\sqrt{b^2-4ac}\sqrt{-2cd+}}}{}$$

```
[In] Integrate[(x^7*Sqrt[d + e*x^2])/(a + b*x^2 + c*x^4),x]
```

```
[Out] ((2*Sqrt[c]*Sqrt[d + e*x^2]*(15*b^2*e^2 + c^2*(-2*d^2 + d*e*x^2 + 3*e^2*x^4
) - 5*c*e*(3*a*e + b*(d + e*x^2))))/e^2 - (15*Sqrt[2]*(-(b^4*e) + a*c^2*(Sq
rt[b^2 - 4*a*c]*d - 2*a*e) + b^2*c*(-(Sqrt[b^2 - 4*a*c]*d) + 4*a*e) + b^3*(
c*d + Sqrt[b^2 - 4*a*c]*e) - a*b*c*(3*c*d + 2*Sqrt[b^2 - 4*a*c]*e))*ArcTan[
(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x^2])/Sqrt[-2*c*d + b*e - Sqrt[b^2 - 4*a*c]*e]]
)/(Sqrt[b^2 - 4*a*c]*Sqrt[-2*c*d + (b - Sqrt[b^2 - 4*a*c])*e]) - (15*Sqrt[2
]*(b^4*e + a*c^2*(Sqrt[b^2 - 4*a*c]*d + 2*a*e) - b^2*c*(Sqrt[b^2 - 4*a*c]*d
 + 4*a*e) + a*b*c*(3*c*d - 2*Sqrt[b^2 - 4*a*c]*e) + b^3*(-(c*d) + Sqrt[b^2
- 4*a*c]*e))*ArcTan[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x^2])/Sqrt[-2*c*d + (b + Sq
rt[b^2 - 4*a*c])*e]])/(Sqrt[b^2 - 4*a*c]*Sqrt[-2*c*d + (b + Sqrt[b^2 - 4*a*
c])*e]))/(30*c^(7/2))
```

Maple [A] (verified)

Time = 2.39 (sec) , antiderivative size = 473, normalized size of antiderivative = 1.17

method	result
risch	$-\frac{(-3e^2c^2x^4+5bce^2x^2-c^2dex^2+15e^2ac-15b^2e^2+5bcde+2c^2d^2)\sqrt{ex^2+d}}{15e^2c^3}-\frac{\sqrt{2}\left(\sqrt{\left(be-2cd+\sqrt{-4e^2\left(ac-\frac{b^2}{4}\right)}\right)c}\left(\left(-\cdots\right.\right.\right.}{\cdots}$
pseudoelliptic	$-\frac{\left(\left(\left(abc-\frac{1}{2}b^3\right)e-\frac{dc\left(ac-b^2\right)}{2}\right)\sqrt{-4e^2\left(ac-\frac{b^2}{4}\right)}+e\left(\left(-2ab^2c+\frac{1}{2}b^4+a^2c^2\right)e+\frac{3cdb\left(ac-\frac{b^2}{3}\right)}{2}\right)\right)e^2\sqrt{2}\sqrt{\left(be-2cd+\sqrt{-4e\cdots}\right.}}{\cdots}$
default	$\frac{\frac{x^2\left(ex^2+d\right)^{\frac{3}{2}}}{5e}-\frac{2d\left(ex^2+d\right)^{\frac{3}{2}}}{15e^2}}{c}-\frac{b\left(ex^2+d\right)^{\frac{3}{2}}}{3c^2e}+\frac{-\left(ac-b^2\right)\sqrt{ex^2+d}+\frac{\left(-2a^2c^2e^2+4ab^2ce^2-3abc^2de-b^4e^2+b^3cde+2\sqrt{-e^2\left(4\cdots\right.}\right.}{\cdots}}{\cdots}$

```
[In] int(x^7*(e*x^2+d)^(1/2)/(c*x^4+b*x^2+a),x,method=_RETURNVERBOSE)
```

```
[Out] -1/15*(-3*c^2*e^2*x^4+5*b*c*e^2*x^2-c^2*d*e*x^2+15*a*c*e^2-15*b^2*e^2+5*b*c
*d*e+2*c^2*d^2)*(e*x^2+d)^(1/2)/e^2/c^3-1/c^3/((b*e-2*c*d+(-4*e^2*(a*c-1/4*
b^2))^(1/2))*c)^(1/2)*2^(1/2)/((-b*e+2*c*d+(-4*e^2*(a*c-1/4*b^2))^(1/2))*c)
^(1/2)/(-4*e^2*(a*c-1/4*b^2))^(1/2)*(((b*e-2*c*d+(-4*e^2*(a*c-1/4*b^2))^(1/
2))*c)^(1/2)*((-1/2*a*c^2*d+b*(a*e+1/2*b*d)*c-1/2*b^3*e)*(-4*e^2*(a*c-1/4*b
^2))^(1/2)+e*(a*(a*e+3/2*b*d)*c^2+(-2*a*b^2*e-1/2*b^3*d)*c+1/2*b^4*e))*arct
anh(c*(e*x^2+d)^(1/2)*2^(1/2)/((-b*e+2*c*d+(-4*e^2*(a*c-1/4*b^2))^(1/2))*c)
^(1/2))+((-b*e+2*c*d+(-4*e^2*(a*c-1/4*b^2))^(1/2))*c)^(1/2)*((1/2*a*c^2*d+(
-a*b*e-1/2*b^2*d)*c+1/2*b^3*e)*(-4*e^2*(a*c-1/4*b^2))^(1/2)+e*(a*(a*e+3/2*b
*d)*c^2+(-2*a*b^2*e-1/2*b^3*d)*c+1/2*b^4*e))*arctan(c*(e*x^2+d)^(1/2)*2^(1/
2)/((b*e-2*c*d+(-4*e^2*(a*c-1/4*b^2))^(1/2))*c)^(1/2)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 5829 vs. 2(356) = 712.

Time = 257.63 (sec) , antiderivative size = 5829, normalized size of antiderivative = 14.36

$$\int \frac{x^7\sqrt{d+ex^2}}{a+bx^2+cx^4}\,dx = \text{Too large to display}$$

```
[In] integrate(x^7*(e*x^2+d)^(1/2)/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] Too large to include
```

Sympy [F]

$$\int \frac{x^7\sqrt{d+ex^2}}{a+bx^2+cx^4}\,dx = \int \frac{x^7\sqrt{d+ex^2}}{a+bx^2+cx^4}\,dx$$

[In] integrate(x**7*(e*x**2+d)**(1/2)/(c*x**4+b*x**2+a),x)

[Out] Integral(x**7*sqrt(d + e*x**2)/(a + b*x**2 + c*x**4), x)

Maxima [F]

$$\int \frac{x^7\sqrt{d+ex^2}}{a+bx^2+cx^4}\,dx = \int \frac{\sqrt{ex^2+d}x^7}{cx^4+bx^2+a}\,dx$$

[In] integrate(x^7*(e*x^2+d)^(1/2)/(c*x^4+b*x^2+a),x, algorithm="maxima")

[Out] integrate(sqrt(e*x^2 + d)*x^7/(c*x^4 + b*x^2 + a), x)

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 959 vs. $2(356) = 712$.

Time = 0.33 (sec) , antiderivative size = 959, normalized size of antiderivative = 2.36

$$\int \frac{x^7\sqrt{d+ex^2}}{a+bx^2+cx^4}\,dx =$$

$$-\frac{(((b^4c-5\,ab^2c^2+4\,a^2c^3)d-(b^5-6\,ab^3c+8\,a^2bc^2)e)c^2e^2+2\,(b^3c^4-3\,abc^5)d^2e-(3\,b^4c^3-11\,ab^2c^4+4}{}$$

$$+\frac{(((b^4c-5\,ab^2c^2+4\,a^2c^3)d-(b^5-6\,ab^3c+8\,a^2bc^2)e)c^2e^2+2\,(b^3c^4-3\,abc^5)d^2e-(3\,b^4c^3-11\,ab^2c^4+4}{}$$

$$+\frac{3\,(ex^2+d)^{\frac{5}{2}}c^4e^8-5\,(ex^2+d)^{\frac{3}{2}}c^4de^8-5\,(ex^2+d)^{\frac{3}{2}}bc^3e^9+15\,\sqrt{ex^2+d}b^2c^2e^{10}-15\,\sqrt{ex^2+d}ac^3e^{10}}{15\,c^5e^{10}}$$

[In] integrate(x^7*(e*x^2+d)^(1/2)/(c*x^4+b*x^2+a),x, algorithm="giac")

[Out] -(((b^4*c - 5*a*b^2*c^2 + 4*a^2*c^3)*d - (b^5 - 6*a*b^3*c + 8*a^2*b*c^2)*e)
*c^2*e^2 + 2*(b^3*c^4 - 3*a*b*c^5)*d^2*e - (3*b^4*c^3 - 11*a*b^2*c^4 + 4*a^

```
2*c^5)*d*e^2 + (b^5*c^2 - 4*a*b^3*c^3 + 2*a^2*b*c^4)*e^3 - 2*((b^2*c^3 - a*
c^4)*sqrt(b^2 - 4*a*c)*d^2 - (b^3*c^2 - a*b*c^3)*sqrt(b^2 - 4*a*c)*d*e + (a
*b^2*c^2 - a^2*c^3)*sqrt(b^2 - 4*a*c)*e^2)*abs(c)*abs(e))*arctan(2*sqrt(1/2
)*sqrt(e*x^2 + d)/sqrt(-(2*c^6*d*e^12 - b*c^5*e^13 + sqrt(-4*(c^6*d^2*e^12
- b*c^5*d*e^13 + a*c^5*e^14)*c^6*e^12 + (2*c^6*d*e^12 - b*c^5*e^13)^2))/(c^
6*e^12)))/((2*sqrt(b^2 - 4*a*c)*c^4*d - (b^2*c^3 - 4*a*c^4 + sqrt(b^2 - 4*a
*c)*b*c^3)*e)*sqrt(-4*c^2*d + 2*(b*c - sqrt(b^2 - 4*a*c)*c)*e)*c^2*abs(e))
+ (((b^4*c - 5*a*b^2*c^2 + 4*a^2*c^3)*d - (b^5 - 6*a*b^3*c + 8*a^2*b*c^2)*e
)*c^2*e^2 + 2*(b^3*c^4 - 3*a*b*c^5)*d^2*e - (3*b^4*c^3 - 11*a*b^2*c^4 + 4*a
^2*c^5)*d*e^2 + (b^5*c^2 - 4*a*b^3*c^3 + 2*a^2*b*c^4)*e^3 + 2*((b^2*c^3 - a
*c^4)*sqrt(b^2 - 4*a*c)*d^2 - (b^3*c^2 - a*b*c^3)*sqrt(b^2 - 4*a*c)*d*e + (
a*b^2*c^2 - a^2*c^3)*sqrt(b^2 - 4*a*c)*e^2)*abs(c)*abs(e))*arctan(2*sqrt(1/
2)*sqrt(e*x^2 + d)/sqrt(-(2*c^6*d*e^12 - b*c^5*e^13 - sqrt(-4*(c^6*d^2*e^12
 - b*c^5*d*e^13 + a*c^5*e^14)*c^6*e^12 + (2*c^6*d*e^12 - b*c^5*e^13)^2))/(c
^6*e^12)))/((2*sqrt(b^2 - 4*a*c)*c^4*d + (b^2*c^3 - 4*a*c^4 - sqrt(b^2 - 4*
a*c)*b*c^3)*e)*sqrt(-4*c^2*d + 2*(b*c + sqrt(b^2 - 4*a*c)*c)*e)*c^2*abs(e))
 + 1/15*(3*(e*x^2 + d)^(5/2)*c^4*e^8 - 5*(e*x^2 + d)^(3/2)*c^4*d*e^8 - 5*(e
*x^2 + d)^(3/2)*b*c^3*e^9 + 15*sqrt(e*x^2 + d)*b^2*c^2*e^10 - 15*sqrt(e*x^2
 + d)*a*c^3*e^10)/(c^5*e^10)
```

Mupad [B] (verification not implemented)

Time = 8.90 (sec) , antiderivative size = 11195, normalized size of antiderivative = 27.57

$$\int \frac{x^7\sqrt{d+ex^2}}{a+bx^2+cx^4}\,dx = \text{Too large to display}$$

```
[In] int((x^7*(d + e*x^2)^(1/2))/(a + b*x^2 + c*x^4),x)
```

```
[Out] (d + e*x^2)^(1/2)*((3*d^2)/(c*e^2) - (a*e^4 + c*d^2*e^2 - b*d*e^3)/(c^2*e^4
) + (((3*d)/(c*e^2) + (b*e^3 - 2*c*d*e^2)/(c^2*e^4))*(b*e^3 - 2*c*d*e^2))/(
c*e^2)) - (d + e*x^2)^(3/2)*(d/(c*e^2) + (b*e^3 - 2*c*d*e^2)/(3*c^2*e^4)) +
 atan(((((16*a^3*c^6*e^4 + 4*a*b^4*c^4*e^4 - 4*b^5*c^4*d*e^3 - 20*a^2*b^2*c
^5*e^4 + 16*a^2*c^7*d^2*e^2 + 4*b^4*c^5*d^2*e^2 + 20*a*b^3*c^5*d*e^3 - 16*a
^2*b*c^6*d*e^3 - 20*a*b^2*c^6*d^2*e^2)/c^5 - (2*(d + e*x^2)^(1/2)*(-(b^9*e
- 8*a^4*c^5*d - b^6*e*(-(4*a*c - b^2)^3)^(1/2) - b^8*c*d - 33*a^2*b^4*c^3*d
 + 38*a^3*b^2*c^4*d + 42*a^2*b^5*c^2*e - 63*a^3*b^3*c^3*e + a^3*c^3*e*(-(4*
a*c - b^2)^3)^(1/2) - 11*a*b^7*c*e + 10*a*b^6*c^2*d + 28*a^4*b*c^4*e + b^5*
c*d*(-(4*a*c - b^2)^3)^(1/2) + 5*a*b^4*c*e*(-(4*a*c - b^2)^3)^(1/2) - 4*a*b
^3*c^2*d*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b*c^3*d*(-(4*a*c - b^2)^3)^(1/2)
- 6*a^2*b^2*c^2*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^9 + b^4*c^7 - 8*a*
b^2*c^8)))^(1/2)*(4*b^3*c^7*e^3 - 8*b^2*c^8*d*e^2 - 16*a*b*c^8*e^3 + 32*a*c
^9*d*e^2))/c^5)*(-(b^9*e - 8*a^4*c^5*d - b^6*e*(-(4*a*c - b^2)^3)^(1/2) - b
^8*c*d - 33*a^2*b^4*c^3*d + 38*a^3*b^2*c^4*d + 42*a^2*b^5*c^2*e - 63*a^3*b^
3*c^3*e + a^3*c^3*e*(-(4*a*c - b^2)^3)^(1/2) - 11*a*b^7*c*e + 10*a*b^6*c^2*
```

d + 28*a^4*b*c^4*e + b^5*c*d*(-(4*a*c - b^2)^3)^(1/2) + 5*a*b^4*c*e*(-(4*a*c - b^2)^3)^(1/2) - 4*a*b^3*c^2*d*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b*c^3*d*(-(4*a*c - b^2)^3)^(1/2) - 6*a^2*b^2*c^2*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^9 + b^4*c^7 - 8*a*b^2*c^8)))^(1/2) - (2*(d + e*x^2)^(1/2)*(b^8*e^4 + 2*a^4*c^4*e^4 + 20*a^2*b^4*c^2*e^4 - 16*a^3*b^2*c^3*e^4 - 2*a^3*c^5*d^2*e^2 + b^6*c^2*d^2*e^2 - 8*a*b^6*c*e^4 - 2*b^7*c*d*e^3 + 9*a^2*b^2*c^4*d^2*e^2 + 14*a*b^5*c^2*d*e^3 + 14*a^3*b*c^4*d*e^3 - 6*a*b^4*c^3*d^2*e^2 - 28*a^2*b^3*c^3*d*e^3))/c^5)*(-(b^9*e - 8*a^4*c^5*d - b^6*e*(-(4*a*c - b^2)^3)^(1/2) - b^8*c*d - 33*a^2*b^4*c^3*d + 38*a^3*b^2*c^4*d + 42*a^2*b^5*c^2*e - 63*a^3*b^3*c^3*e + a^3*c^3*e*(-(4*a*c - b^2)^3)^(1/2) - 11*a*b^7*c*e + 10*a*b^6*c^2*d + 28*a^4*b*c^4*e + b^5*c*d*(-(4*a*c - b^2)^3)^(1/2) + 5*a*b^4*c*e*(-(4*a*c - b^2)^3)^(1/2) - 4*a*b^3*c^2*d*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b*c^3*d*(-(4*a*c - b^2)^3)^(1/2) - 6*a^2*b^2*c^2*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^9 + b^4*c^7 - 8*a*b^2*c^8)))^(1/2)*1i - (((16*a^3*c^6*e^4 + 4*a*b^4*c^4*e^4 - 4*b^5*c^4*d*e^3 - 20*a^2*b^2*c^5*e^4 + 16*a^2*c^7*d^2*e^2 + 4*b^4*c^5*d^2*e^2 + 20*a*b^3*c^5*d*e^3 - 16*a^2*b*c^6*d*e^3 - 20*a*b^2*c^6*d^2*e^2)/c^5 + (2*(d + e*x^2)^(1/2)*(-(b^9*e - 8*a^4*c^5*d - b^6*e*(-(4*a*c - b^2)^3)^(1/2) - b^8*c*d - 33*a^2*b^4*c^3*d + 38*a^3*b^2*c^4*d + 42*a^2*b^5*c^2*e - 63*a^3*b^3*c^3*e + a^3*c^3*e*(-(4*a*c - b^2)^3)^(1/2) - 11*a*b^7*c*e + 10*a*b^6*c^2*d + 28*a^4*b*c^4*e + b^5*c*d*(-(4*a*c - b^2)^3)^(1/2) + 5*a*b^4*c*e*(-(4*a*c - b^2)^3)^(1/2) - 4*a*b^3*c^2*d*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b*c^3*d*(-(4*a*c - b^2)^3)^(1/2) - 6*a^2*b^2*c^2*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^9 + b^4*c^7 - 8*a*b^2*c^8)))^(1/2)*(4*b^3*c^7*e^3 - 8*b^2*c^8*d*e^2 - 16*a*b*c^8*e^3 + 32*a*c^9*d*e^2))/c^5)*(-(b^9*e - 8*a^4*c^5*d - b^6*e*(-(4*a*c - b^2)^3)^(1/2) - b^8*c*d - 33*a^2*b^4*c^3*d + 38*a^3*b^2*c^4*d + 42*a^2*b^5*c^2*e - 63*a^3*b^3*c^3*e + a^3*c^3*e*(-(4*a*c - b^2)^3)^(1/2) - 11*a*b^7*c*e + 10*a*b^6*c^2*d + 28*a^4*b*c^4*e + b^5*c*d*(-(4*a*c - b^2)^3)^(1/2) + 5*a*b^4*c*e*(-(4*a*c - b^2)^3)^(1/2) - 4*a*b^3*c^2*d*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b*c^3*d*(-(4*a*c - b^2)^3)^(1/2) - 6*a^2*b^2*c^2*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^9 + b^4*c^7 - 8*a*b^2*c^8)))^(1/2) + (2*(d + e*x^2)^(1/2)*(b^8*e^4 + 2*a^4*c^4*e^4 + 20*a^2*b^4*c^2*e^4 - 16*a^3*b^2*c^3*e^4 - 2*a^3*c^5*d^2*e^2 + b^6*c^2*d^2*e^2 - 8*a*b^6*c*e^4 - 2*b^7*c*d*e^3 + 9*a^2*b^2*c^4*d^2*e^2 + 14*a*b^5*c^2*d*e^3 + 14*a^3*b*c^4*d*e^3 - 6*a*b^4*c^3*d^2*e^2 - 28*a^2*b^3*c^3*d*e^3))/c^5)*(-(b^9*e - 8*a^4*c^5*d - b^6*e*(-(4*a*c - b^2)^3)^(1/2) - b^8*c*d - 33*a^2*b^4*c^3*d + 38*a^3*b^2*c^4*d + 42*a^2*b^5*c^2*e - 63*a^3*b^3*c^3*e + a^3*c^3*e*(-(4*a*c - b^2)^3)^(1/2) - 11*a*b^7*c*e + 10*a*b^6*c^2*d + 28*a^4*b*c^4*e + b^5*c*d*(-(4*a*c - b^2)^3)^(1/2) + 5*a*b^4*c*e*(-(4*a*c - b^2)^3)^(1/2) - 4*a*b^3*c^2*d*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b*c^3*d*(-(4*a*c - b^2)^3)^(1/2) - 6*a^2*b^2*c^2*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^9 + b^4*c^7 - 8*a*b^2*c^8)))^(1/2)*1i)/((((16*a^3*c^6*e^4 + 4*a*b^4*c^4*e^4 - 4*b^5*c^4*d*e^3 - 20*a^2*b^2*c^5*e^4 + 16*a^2*c^7*d^2*e^2 + 4*b^4*c^5*d^2*e^2 + 20*a*b^3*c^5*d*e^3 - 16*a^2*b*c^6*d*e^3 - 20*a*b^2*c^6*d^2*e^2)/c^5 - (2*(d + e*x^2)^(1/2)*(-(b^9*e - 8*a^4*c^5*d - b^6*e*(-(4*a*c - b^2)^3)^(1/2) - b^8*c*d - 33*a^2*b^4*c^3*d + 38*a^3*b^2*c^4*d + 42*a^2*b^5*c^2*e - 63*a^3*b^3*c^3*e + a^

3*c^3*e*(-(4*a*c - b^2)^3)^(1/2) - 11*a*b^7*c*e + 10*a*b^6*c^2*d + 28*a^4*b
*c^4*e + b^5*c*d*(-(4*a*c - b^2)^3)^(1/2) + 5*a*b^4*c*e*(-(4*a*c - b^2)^3)^
(1/2) - 4*a*b^3*c^2*d*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b*c^3*d*(-(4*a*c - b
^2)^3)^(1/2) - 6*a^2*b^2*c^2*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^9 + b
^4*c^7 - 8*a*b^2*c^8)))^(1/2)*(4*b^3*c^7*e^3 - 8*b^2*c^8*d*e^2 - 16*a*b*c^8
*e^3 + 32*a*c^9*d*e^2))/c^5)*(-(b^9*e - 8*a^4*c^5*d - b^6*e*(-(4*a*c - b^2)
^3)^(1/2) - b^8*c*d - 33*a^2*b^4*c^3*d + 38*a^3*b^2*c^4*d + 42*a^2*b^5*c^2*
e - 63*a^3*b^3*c^3*e + a^3*c^3*e*(-(4*a*c - b^2)^3)^(1/2) - 11*a*b^7*c*e +
10*a*b^6*c^2*d + 28*a^4*b*c^4*e + b^5*c*d*(-(4*a*c - b^2)^3)^(1/2) + 5*a*b^
4*c*e*(-(4*a*c - b^2)^3)^(1/2) - 4*a*b^3*c^2*d*(-(4*a*c - b^2)^3)^(1/2) + 3
*a^2*b*c^3*d*(-(4*a*c - b^2)^3)^(1/2) - 6*a^2*b^2*c^2*e*(-(4*a*c - b^2)^3)^
(1/2))/(8*(16*a^2*c^9 + b^4*c^7 - 8*a*b^2*c^8)))^(1/2) - (2*(d + e*x^2)^(1/
2)*(b^8*e^4 + 2*a^4*c^4*e^4 + 20*a^2*b^4*c^2*e^4 - 16*a^3*b^2*c^3*e^4 - 2*a
^3*c^5*d^2*e^2 + b^6*c^2*d^2*e^2 - 8*a*b^6*c*e^4 - 2*b^7*c*d*e^3 + 9*a^2*b^
2*c^4*d^2*e^2 + 14*a*b^5*c^2*d*e^3 + 14*a^3*b*c^4*d*e^3 - 6*a*b^4*c^3*d^2*e
^2 - 28*a^2*b^3*c^3*d*e^3))/c^5)*(-(b^9*e - 8*a^4*c^5*d - b^6*e*(-(4*a*c -
b^2)^3)^(1/2) - b^8*c*d - 33*a^2*b^4*c^3*d + 38*a^3*b^2*c^4*d + 42*a^2*b^5*
c^2*e - 63*a^3*b^3*c^3*e + a^3*c^3*e*(-(4*a*c - b^2)^3)^(1/2) - 11*a*b^7*c*
e + 10*a*b^6*c^2*d + 28*a^4*b*c^4*e + b^5*c*d*(-(4*a*c - b^2)^3)^(1/2) + 5*
a*b^4*c*e*(-(4*a*c - b^2)^3)^(1/2) - 4*a*b^3*c^2*d*(-(4*a*c - b^2)^3)^(1/2)
 + 3*a^2*b*c^3*d*(-(4*a*c - b^2)^3)^(1/2) - 6*a^2*b^2*c^2*e*(-(4*a*c - b^2)
^3)^(1/2))/(8*(16*a^2*c^9 + b^4*c^7 - 8*a*b^2*c^8)))^(1/2) + (((16*a^3*c^6*
e^4 + 4*a*b^4*c^4*e^4 - 4*b^5*c^4*d*e^3 - 20*a^2*b^2*c^5*e^4 + 16*a^2*c^7*d
^2*e^2 + 4*b^4*c^5*d^2*e^2 + 20*a*b^3*c^5*d*e^3 - 16*a^2*b*c^6*d*e^3 - 20*a
*b^2*c^6*d^2*e^2)/c^5 + (2*(d + e*x^2)^(1/2)*(-(b^9*e - 8*a^4*c^5*d - b^6*e
*(-(4*a*c - b^2)^3)^(1/2) - b^8*c*d - 33*a^2*b^4*c^3*d + 38*a^3*b^2*c^4*d +
 42*a^2*b^5*c^2*e - 63*a^3*b^3*c^3*e + a^3*c^3*e*(-(4*a*c - b^2)^3)^(1/2) -
 11*a*b^7*c*e + 10*a*b^6*c^2*d + 28*a^4*b*c^4*e + b^5*c*d*(-(4*a*c - b^2)^3
)^(1/2) + 5*a*b^4*c*e*(-(4*a*c - b^2)^3)^(1/2) - 4*a*b^3*c^2*d*(-(4*a*c - b
^2)^3)^(1/2) + 3*a^2*b*c^3*d*(-(4*a*c - b^2)^3)^(1/2) - 6*a^2*b^2*c^2*e*(-(
4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^9 + b^4*c^7 - 8*a*b^2*c^8)))^(1/2)*(4*b
^3*c^7*e^3 - 8*b^2*c^8*d*e^2 - 16*a*b*c^8*e^3 + 32*a*c^9*d*e^2))/c^5)*(-(b^
9*e - 8*a^4*c^5*d - b^6*e*(-(4*a*c - b^2)^3)^(1/2) - b^8*c*d - 33*a^2*b^4*c
^3*d + 38*a^3*b^2*c^4*d + 42*a^2*b^5*c^2*e - 63*a^3*b^3*c^3*e + a^3*c^3*e*(
-(4*a*c - b^2)^3)^(1/2) - 11*a*b^7*c*e + 10*a*b^6*c^2*d + 28*a^4*b*c^4*e +
b^5*c*d*(-(4*a*c - b^2)^3)^(1/2) + 5*a*b^4*c*e*(-(4*a*c - b^2)^3)^(1/2) - 4
*a*b^3*c^2*d*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b*c^3*d*(-(4*a*c - b^2)^3)^(1
/2) - 6*a^2*b^2*c^2*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^9 + b^4*c^7 -
8*a*b^2*c^8)))^(1/2) + (2*(d + e*x^2)^(1/2)*(b^8*e^4 + 2*a^4*c^4*e^4 + 20*a
^2*b^4*c^2*e^4 - 16*a^3*b^2*c^3*e^4 - 2*a^3*c^5*d^2*e^2 + b^6*c^2*d^2*e^2 -
 8*a*b^6*c*e^4 - 2*b^7*c*d*e^3 + 9*a^2*b^2*c^4*d^2*e^2 + 14*a*b^5*c^2*d*e^3
 + 14*a^3*b*c^4*d*e^3 - 6*a*b^4*c^3*d^2*e^2 - 28*a^2*b^3*c^3*d*e^3))/c^5)*(
-(b^9*e - 8*a^4*c^5*d - b^6*e*(-(4*a*c - b^2)^3)^(1/2) - b^8*c*d - 33*a^2*b
^4*c^3*d + 38*a^3*b^2*c^4*d + 42*a^2*b^5*c^2*e - 63*a^3*b^3*c^3*e + a^3*c^3
e(-(4*a*c - b^2)^3)^(1/2) - 11*a*b^7*c*e + 10*a*b^6*c^2*d + 28*a^4*b*c^4*

e + b^5*c*d*(-(4*a*c - b^2)^3)^(1/2) + 5*a*b^4*c*e*(-(4*a*c - b^2)^3)^(1/2)
 - 4*a*b^3*c^2*d*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b*c^3*d*(-(4*a*c - b^2)^3
)^(1/2) - 6*a^2*b^2*c^2*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^9 + b^4*c^
7 - 8*a*b^2*c^8)))^(1/2) - (2*(a^4*b^3*e^5 - a^3*b^4*d*e^4 + a^5*c^2*d*e^4
+ a^4*c^3*d^3*e^2 - 2*a^5*b*c*e^5 - a^3*b^2*c^2*d^3*e^2 + a^4*b^2*c*d*e^4 +
 2*a^3*b^3*c*d^2*e^3 - 3*a^4*b*c^2*d^2*e^3))/c^5))*(-(b^9*e - 8*a^4*c^5*d -
 b^6*e*(-(4*a*c - b^2)^3)^(1/2) - b^8*c*d - 33*a^2*b^4*c^3*d + 38*a^3*b^2*c
^4*d + 42*a^2*b^5*c^2*e - 63*a^3*b^3*c^3*e + a^3*c^3*e*(-(4*a*c - b^2)^3)^(
1/2) - 11*a*b^7*c*e + 10*a*b^6*c^2*d + 28*a^4*b*c^4*e + b^5*c*d*(-(4*a*c -
b^2)^3)^(1/2) + 5*a*b^4*c*e*(-(4*a*c - b^2)^3)^(1/2) - 4*a*b^3*c^2*d*(-(4*a
*c - b^2)^3)^(1/2) + 3*a^2*b*c^3*d*(-(4*a*c - b^2)^3)^(1/2) - 6*a^2*b^2*c^2
e(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^9 + b^4*c^7 - 8*a*b^2*c^8)))^(1/2
)*2i + atan(((((16*a^3*c^6*e^4 + 4*a*b^4*c^4*e^4 - 4*b^5*c^4*d*e^3 - 20*a^2
*b^2*c^5*e^4 + 16*a^2*c^7*d^2*e^2 + 4*b^4*c^5*d^2*e^2 + 20*a*b^3*c^5*d*e^3
- 16*a^2*b*c^6*d*e^3 - 20*a*b^2*c^6*d^2*e^2)/c^5 - (2*(d + e*x^2)^(1/2)*((8
*a^4*c^5*d - b^9*e - b^6*e*(-(4*a*c - b^2)^3)^(1/2) + b^8*c*d + 33*a^2*b^4*
c^3*d - 38*a^3*b^2*c^4*d - 42*a^2*b^5*c^2*e + 63*a^3*b^3*c^3*e + a^3*c^3*e*
(-(4*a*c - b^2)^3)^(1/2) + 11*a*b^7*c*e - 10*a*b^6*c^2*d - 28*a^4*b*c^4*e +
 b^5*c*d*(-(4*a*c - b^2)^3)^(1/2) + 5*a*b^4*c*e*(-(4*a*c - b^2)^3)^(1/2) -
4*a*b^3*c^2*d*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b*c^3*d*(-(4*a*c - b^2)^3)^(
1/2) - 6*a^2*b^2*c^2*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^9 + b^4*c^7 -
 8*a*b^2*c^8)))^(1/2)*(4*b^3*c^7*e^3 - 8*b^2*c^8*d*e^2 - 16*a*b*c^8*e^3 + 3
2*a*c^9*d*e^2))/c^5)*((8*a^4*c^5*d - b^9*e - b^6*e*(-(4*a*c - b^2)^3)^(1/2)
 + b^8*c*d + 33*a^2*b^4*c^3*d - 38*a^3*b^2*c^4*d - 42*a^2*b^5*c^2*e + 63*a^
3*b^3*c^3*e + a^3*c^3*e*(-(4*a*c - b^2)^3)^(1/2) + 11*a*b^7*c*e - 10*a*b^6*
c^2*d - 28*a^4*b*c^4*e + b^5*c*d*(-(4*a*c - b^2)^3)^(1/2) + 5*a*b^4*c*e*(-(
4*a*c - b^2)^3)^(1/2) - 4*a*b^3*c^2*d*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b*c^
3*d*(-(4*a*c - b^2)^3)^(1/2) - 6*a^2*b^2*c^2*e*(-(4*a*c - b^2)^3)^(1/2))/(8
*(16*a^2*c^9 + b^4*c^7 - 8*a*b^2*c^8)))^(1/2) - (2*(d + e*x^2)^(1/2)*(b^8*e
^4 + 2*a^4*c^4*e^4 + 20*a^2*b^4*c^2*e^4 - 16*a^3*b^2*c^3*e^4 - 2*a^3*c^5*d^
2*e^2 + b^6*c^2*d^2*e^2 - 8*a*b^6*c*e^4 - 2*b^7*c*d*e^3 + 9*a^2*b^2*c^4*d^2
*e^2 + 14*a*b^5*c^2*d*e^3 + 14*a^3*b*c^4*d*e^3 - 6*a*b^4*c^3*d^2*e^2 - 28*a
^2*b^3*c^3*d*e^3))/c^5)*((8*a^4*c^5*d - b^9*e - b^6*e*(-(4*a*c - b^2)^3)^(1
/2) + b^8*c*d + 33*a^2*b^4*c^3*d - 38*a^3*b^2*c^4*d - 42*a^2*b^5*c^2*e + 63
*a^3*b^3*c^3*e + a^3*c^3*e*(-(4*a*c - b^2)^3)^(1/2) + 11*a*b^7*c*e - 10*a*b
^6*c^2*d - 28*a^4*b*c^4*e + b^5*c*d*(-(4*a*c - b^2)^3)^(1/2) + 5*a*b^4*c*e*
(-(4*a*c - b^2)^3)^(1/2) - 4*a*b^3*c^2*d*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b
*c^3*d*(-(4*a*c - b^2)^3)^(1/2) - 6*a^2*b^2*c^2*e*(-(4*a*c - b^2)^3)^(1/2))
/(8*(16*a^2*c^9 + b^4*c^7 - 8*a*b^2*c^8)))^(1/2)*1i - (((16*a^3*c^6*e^4 + 4
*a*b^4*c^4*e^4 - 4*b^5*c^4*d*e^3 - 20*a^2*b^2*c^5*e^4 + 16*a^2*c^7*d^2*e^2
+ 4*b^4*c^5*d^2*e^2 + 20*a*b^3*c^5*d*e^3 - 16*a^2*b*c^6*d*e^3 - 20*a*b^2*c^
6*d^2*e^2)/c^5 + (2*(d + e*x^2)^(1/2)*((8*a^4*c^5*d - b^9*e - b^6*e*(-(4*a*
c - b^2)^3)^(1/2) + b^8*c*d + 33*a^2*b^4*c^3*d - 38*a^3*b^2*c^4*d - 42*a^2*
b^5*c^2*e + 63*a^3*b^3*c^3*e + a^3*c^3*e*(-(4*a*c - b^2)^3)^(1/2) + 11*a*b^
7*c*e - 10*a*b^6*c^2*d - 28*a^4*b*c^4*e + b^5*c*d*(-(4*a*c - b^2)^3)^(1/2)

+ 5*a*b^4*c*e*(-(4*a*c - b^2)^3)^(1/2) - 4*a*b^3*c^2*d*(-(4*a*c - b^2)^3)^(
1/2) + 3*a^2*b*c^3*d*(-(4*a*c - b^2)^3)^(1/2) - 6*a^2*b^2*c^2*e*(-(4*a*c -
b^2)^3)^(1/2))/(8*(16*a^2*c^9 + b^4*c^7 - 8*a*b^2*c^8)))^(1/2)*(4*b^3*c^7*e
^3 - 8*b^2*c^8*d*e^2 - 16*a*b*c^8*e^3 + 32*a*c^9*d*e^2))/c^5)*((8*a^4*c^5*d
 - b^9*e - b^6*e*(-(4*a*c - b^2)^3)^(1/2) + b^8*c*d + 33*a^2*b^4*c^3*d - 38
*a^3*b^2*c^4*d - 42*a^2*b^5*c^2*e + 63*a^3*b^3*c^3*e + a^3*c^3*e*(-(4*a*c -
 b^2)^3)^(1/2) + 11*a*b^7*c*e - 10*a*b^6*c^2*d - 28*a^4*b*c^4*e + b^5*c*d*(
-(4*a*c - b^2)^3)^(1/2) + 5*a*b^4*c*e*(-(4*a*c - b^2)^3)^(1/2) - 4*a*b^3*c^
2*d*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b*c^3*d*(-(4*a*c - b^2)^3)^(1/2) - 6*a
^2*b^2*c^2*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^9 + b^4*c^7 - 8*a*b^2*c
^8)))^(1/2) + (2*(d + e*x^2)^(1/2)*(b^8*e^4 + 2*a^4*c^4*e^4 + 20*a^2*b^4*c^
2*e^4 - 16*a^3*b^2*c^3*e^4 - 2*a^3*c^5*d^2*e^2 + b^6*c^2*d^2*e^2 - 8*a*b^6*
c*e^4 - 2*b^7*c*d*e^3 + 9*a^2*b^2*c^4*d^2*e^2 + 14*a*b^5*c^2*d*e^3 + 14*a^3
*b*c^4*d*e^3 - 6*a*b^4*c^3*d^2*e^2 - 28*a^2*b^3*c^3*d*e^3))/c^5)*((8*a^4*c^
5*d - b^9*e - b^6*e*(-(4*a*c - b^2)^3)^(1/2) + b^8*c*d + 33*a^2*b^4*c^3*d -
 38*a^3*b^2*c^4*d - 42*a^2*b^5*c^2*e + 63*a^3*b^3*c^3*e + a^3*c^3*e*(-(4*a*
c - b^2)^3)^(1/2) + 11*a*b^7*c*e - 10*a*b^6*c^2*d - 28*a^4*b*c^4*e + b^5*c*
d*(-(4*a*c - b^2)^3)^(1/2) + 5*a*b^4*c*e*(-(4*a*c - b^2)^3)^(1/2) - 4*a*b^3
*c^2*d*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b*c^3*d*(-(4*a*c - b^2)^3)^(1/2) -
6*a^2*b^2*c^2*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^9 + b^4*c^7 - 8*a*b^
2*c^8)))^(1/2)*1i)/((((16*a^3*c^6*e^4 + 4*a*b^4*c^4*e^4 - 4*b^5*c^4*d*e^3 -
 20*a^2*b^2*c^5*e^4 + 16*a^2*c^7*d^2*e^2 + 4*b^4*c^5*d^2*e^2 + 20*a*b^3*c^5
*d*e^3 - 16*a^2*b*c^6*d*e^3 - 20*a*b^2*c^6*d^2*e^2)/c^5 - (2*(d + e*x^2)^(1
/2)*((8*a^4*c^5*d - b^9*e - b^6*e*(-(4*a*c - b^2)^3)^(1/2) + b^8*c*d + 33*a
^2*b^4*c^3*d - 38*a^3*b^2*c^4*d - 42*a^2*b^5*c^2*e + 63*a^3*b^3*c^3*e + a^3
*c^3*e*(-(4*a*c - b^2)^3)^(1/2) + 11*a*b^7*c*e - 10*a*b^6*c^2*d - 28*a^4*b*
c^4*e + b^5*c*d*(-(4*a*c - b^2)^3)^(1/2) + 5*a*b^4*c*e*(-(4*a*c - b^2)^3)^(
1/2) - 4*a*b^3*c^2*d*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b*c^3*d*(-(4*a*c - b^
2)^3)^(1/2) - 6*a^2*b^2*c^2*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^9 + b^
4*c^7 - 8*a*b^2*c^8)))^(1/2)*(4*b^3*c^7*e^3 - 8*b^2*c^8*d*e^2 - 16*a*b*c^8*
e^3 + 32*a*c^9*d*e^2))/c^5)*((8*a^4*c^5*d - b^9*e - b^6*e*(-(4*a*c - b^2)^3
)^(1/2) + b^8*c*d + 33*a^2*b^4*c^3*d - 38*a^3*b^2*c^4*d - 42*a^2*b^5*c^2*e
+ 63*a^3*b^3*c^3*e + a^3*c^3*e*(-(4*a*c - b^2)^3)^(1/2) + 11*a*b^7*c*e - 10
*a*b^6*c^2*d - 28*a^4*b*c^4*e + b^5*c*d*(-(4*a*c - b^2)^3)^(1/2) + 5*a*b^4*
c*e*(-(4*a*c - b^2)^3)^(1/2) - 4*a*b^3*c^2*d*(-(4*a*c - b^2)^3)^(1/2) + 3*a
^2*b*c^3*d*(-(4*a*c - b^2)^3)^(1/2) - 6*a^2*b^2*c^2*e*(-(4*a*c - b^2)^3)^(1
/2))/(8*(16*a^2*c^9 + b^4*c^7 - 8*a*b^2*c^8)))^(1/2) - (2*(d + e*x^2)^(1/2)
*(b^8*e^4 + 2*a^4*c^4*e^4 + 20*a^2*b^4*c^2*e^4 - 16*a^3*b^2*c^3*e^4 - 2*a^3
*c^5*d^2*e^2 + b^6*c^2*d^2*e^2 - 8*a*b^6*c*e^4 - 2*b^7*c*d*e^3 + 9*a^2*b^2*
c^4*d^2*e^2 + 14*a*b^5*c^2*d*e^3 + 14*a^3*b*c^4*d*e^3 - 6*a*b^4*c^3*d^2*e^2
 - 28*a^2*b^3*c^3*d*e^3))/c^5)*((8*a^4*c^5*d - b^9*e - b^6*e*(-(4*a*c - b^2
)^3)^(1/2) + b^8*c*d + 33*a^2*b^4*c^3*d - 38*a^3*b^2*c^4*d - 42*a^2*b^5*c^2
*e + 63*a^3*b^3*c^3*e + a^3*c^3*e*(-(4*a*c - b^2)^3)^(1/2) + 11*a*b^7*c*e -
 10*a*b^6*c^2*d - 28*a^4*b*c^4*e + b^5*c*d*(-(4*a*c - b^2)^3)^(1/2) + 5*a*b
^4*c*e*(-(4*a*c - b^2)^3)^(1/2) - 4*a*b^3*c^2*d*(-(4*a*c - b^2)^3)^(1/2) +

3*a^2*b*c^3*d*(-(4*a*c - b^2)^3)^(1/2) - 6*a^2*b^2*c^2*e*(-(4*a*c - b^2)^3)
^(1/2))/(8*(16*a^2*c^9 + b^4*c^7 - 8*a*b^2*c^8)))^(1/2) + (((16*a^3*c^6*e^4
 + 4*a*b^4*c^4*e^4 - 4*b^5*c^4*d*e^3 - 20*a^2*b^2*c^5*e^4 + 16*a^2*c^7*d^2*
e^2 + 4*b^4*c^5*d^2*e^2 + 20*a*b^3*c^5*d*e^3 - 16*a^2*b*c^6*d*e^3 - 20*a*b^
2*c^6*d^2*e^2)/c^5 + (2*(d + e*x^2)^(1/2)*((8*a^4*c^5*d - b^9*e - b^6*e*(-(
4*a*c - b^2)^3)^(1/2) + b^8*c*d + 33*a^2*b^4*c^3*d - 38*a^3*b^2*c^4*d - 42*
a^2*b^5*c^2*e + 63*a^3*b^3*c^3*e + a^3*c^3*e*(-(4*a*c - b^2)^3)^(1/2) + 11*
a*b^7*c*e - 10*a*b^6*c^2*d - 28*a^4*b*c^4*e + b^5*c*d*(-(4*a*c - b^2)^3)^(1
/2) + 5*a*b^4*c*e*(-(4*a*c - b^2)^3)^(1/2) - 4*a*b^3*c^2*d*(-(4*a*c - b^2)^
3)^(1/2) + 3*a^2*b*c^3*d*(-(4*a*c - b^2)^3)^(1/2) - 6*a^2*b^2*c^2*e*(-(4*a*
c - b^2)^3)^(1/2))/(8*(16*a^2*c^9 + b^4*c^7 - 8*a*b^2*c^8)))^(1/2)*(4*b^3*c
^7*e^3 - 8*b^2*c^8*d*e^2 - 16*a*b*c^8*e^3 + 32*a*c^9*d*e^2))/c^5)*((8*a^4*c
^5*d - b^9*e - b^6*e*(-(4*a*c - b^2)^3)^(1/2) + b^8*c*d + 33*a^2*b^4*c^3*d
 - 38*a^3*b^2*c^4*d - 42*a^2*b^5*c^2*e + 63*a^3*b^3*c^3*e + a^3*c^3*e*(-(4*a
*c - b^2)^3)^(1/2) + 11*a*b^7*c*e - 10*a*b^6*c^2*d - 28*a^4*b*c^4*e + b^5*c
d(-(4*a*c - b^2)^3)^(1/2) + 5*a*b^4*c*e*(-(4*a*c - b^2)^3)^(1/2) - 4*a*b^
3*c^2*d*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b*c^3*d*(-(4*a*c - b^2)^3)^(1/2) -
 6*a^2*b^2*c^2*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^9 + b^4*c^7 - 8*a*b
^2*c^8)))^(1/2) + (2*(d + e*x^2)^(1/2)*(b^8*e^4 + 2*a^4*c^4*e^4 + 20*a^2*b^
4*c^2*e^4 - 16*a^3*b^2*c^3*e^4 - 2*a^3*c^5*d^2*e^2 + b^6*c^2*d^2*e^2 - 8*a*
b^6*c*e^4 - 2*b^7*c*d*e^3 + 9*a^2*b^2*c^4*d^2*e^2 + 14*a*b^5*c^2*d*e^3 + 14
*a^3*b*c^4*d*e^3 - 6*a*b^4*c^3*d^2*e^2 - 28*a^2*b^3*c^3*d*e^3))/c^5)*((8*a^
4*c^5*d - b^9*e - b^6*e*(-(4*a*c - b^2)^3)^(1/2) + b^8*c*d + 33*a^2*b^4*c^3
*d - 38*a^3*b^2*c^4*d - 42*a^2*b^5*c^2*e + 63*a^3*b^3*c^3*e + a^3*c^3*e*(-(
4*a*c - b^2)^3)^(1/2) + 11*a*b^7*c*e - 10*a*b^6*c^2*d - 28*a^4*b*c^4*e + b^
5*c*d*(-(4*a*c - b^2)^3)^(1/2) + 5*a*b^4*c*e*(-(4*a*c - b^2)^3)^(1/2) - 4*a
*b^3*c^2*d*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b*c^3*d*(-(4*a*c - b^2)^3)^(1/2
) - 6*a^2*b^2*c^2*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^9 + b^4*c^7 - 8*
a*b^2*c^8)))^(1/2) - (2*(a^4*b^3*e^5 - a^3*b^4*d*e^4 + a^5*c^2*d*e^4 + a^4*
c^3*d^3*e^2 - 2*a^5*b*c*e^5 - a^3*b^2*c^2*d^3*e^2 + a^4*b^2*c*d*e^4 + 2*a^3
*b^3*c*d^2*e^3 - 3*a^4*b*c^2*d^2*e^3))/c^5))*((8*a^4*c^5*d - b^9*e - b^6*e*
(-(4*a*c - b^2)^3)^(1/2) + b^8*c*d + 33*a^2*b^4*c^3*d - 38*a^3*b^2*c^4*d -
42*a^2*b^5*c^2*e + 63*a^3*b^3*c^3*e + a^3*c^3*e*(-(4*a*c - b^2)^3)^(1/2) +
11*a*b^7*c*e - 10*a*b^6*c^2*d - 28*a^4*b*c^4*e + b^5*c*d*(-(4*a*c - b^2)^3)
^(1/2) + 5*a*b^4*c*e*(-(4*a*c - b^2)^3)^(1/2) - 4*a*b^3*c^2*d*(-(4*a*c - b^
2)^3)^(1/2) + 3*a^2*b*c^3*d*(-(4*a*c - b^2)^3)^(1/2) - 6*a^2*b^2*c^2*e*(-(4
*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^9 + b^4*c^7 - 8*a*b^2*c^8)))^(1/2)*2i +
(d + e*x^2)^(5/2)/(5*c*e^2)

3.355 $\int \frac{x^5\sqrt{d+ex^2}}{a+bx^2+cx^4}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 324

$$\int \frac{x^5\sqrt{d+ex^2}}{a+bx^2+cx^4}\,dx = -\frac{b\sqrt{d+ex^2}}{c^2}+\frac{(d+ex^2)^{3/2}}{3ce} + \frac{\left(bcd-b^2e+ace-\frac{b^2cd-2ac^2d-b^3e+3abce}{\sqrt{b^2-4ac}}\right)\text{arctanh}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex^2}}{\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}}\right)}{\sqrt{2}c^{5/2}\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}} + \frac{\left(bcd-b^2e+ace+\frac{b^2cd-2ac^2d-b^3e+3abce}{\sqrt{b^2-4ac}}\right)\text{arctanh}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex^2}}{\sqrt{2cd-\left(b+\sqrt{b^2-4ac}\right)e}}\right)}{\sqrt{2}c^{5/2}\sqrt{2cd-\left(b+\sqrt{b^2-4ac}\right)e}}$$

```
[Out] 1/3*(e*x^2+d)^(3/2)/c/e-b*(e*x^2+d)^(1/2)/c^2+1/2*arctanh(2^(1/2)*c^(1/2)*(
e*x^2+d)^(1/2)/(2*c*d-e*(b-(-4*a*c+b^2)^(1/2)))^(1/2))*(b*c*d-b^2*e+a*c*e+(
-3*a*b*c*e+2*a*c^2*d+b^3*e-b^2*c*d)/(-4*a*c+b^2)^(1/2))/c^(5/2)*2^(1/2)/(2*
c*d-e*(b-(-4*a*c+b^2)^(1/2)))^(1/2)+1/2*arctanh(2^(1/2)*c^(1/2)*(e*x^2+d)^(
1/2)/(2*c*d-e*(b+(-4*a*c+b^2)^(1/2)))^(1/2))*(b*c*d-b^2*e+a*c*e+(3*a*b*c*e-
2*a*c^2*d-b^3*e+b^2*c*d)/(-4*a*c+b^2)^(1/2))/c^(5/2)*2^(1/2)/(2*c*d-e*(b+(-
4*a*c+b^2)^(1/2)))^(1/2)
```

Rubi [A] (verified)

Time = 2.26 (sec) , antiderivative size = 324, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.172, Rules used = {1265, 911, 1301, 1180, 214}

$$\int \frac{x^5\sqrt{d+ex^2}}{a+bx^2+cx^4}\,dx$$
$$=\frac{\left(-\frac{3abce-2ac^2d+b^3(-e)+b^2cd}{\sqrt{b^2-4ac}}+ace+b^2(-e)+bcd\right)\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex^2}}{\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}\right)}{\sqrt{2}c^{5/2}\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}$$
$$+\frac{\left(\frac{3abce-2ac^2d+b^3(-e)+b^2cd}{\sqrt{b^2-4ac}}+ace+b^2(-e)+bcd\right)\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex^2}}{\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}\right)}{\sqrt{2}c^{5/2}\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}$$
$$-\frac{b\sqrt{d+ex^2}}{c^2}+\frac{\left(d+ex^2\right)^{3/2}}{3ce}$$

```
[In] Int[(x^5*Sqrt[d + e*x^2])/(a + b*x^2 + c*x^4),x]
```

```
[Out] -((b*Sqrt[d + e*x^2])/c^2) + (d + e*x^2)^(3/2)/(3*c*e) + ((b*c*d - b^2*e +
a*c*e - (b^2*c*d - 2*a*c^2*d - b^3*e + 3*a*b*c*e)/Sqrt[b^2 - 4*a*c])*ArcTan
h[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x^2])/Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]
])/(Sqrt[2]*c^(5/2)*Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]) + ((b*c*d - b^
2*e + a*c*e + (b^2*c*d - 2*a*c^2*d - b^3*e + 3*a*b*c*e)/Sqrt[b^2 - 4*a*c])*
ArcTanh[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x^2])/Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*
c])*e]])/(Sqrt[2]*c^(5/2)*Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 911

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> With[{q = Denominator[m]}, Dist[q/e, S
ubst[Int[x^(q*(m + 1) - 1)*((e*f - d*g)/e + g*(x^q/e))^n*((c*d^2 - b*d*e +
a*e^2)/e^2 - (2*c*d - b*e)*(x^q/e^2) + c*(x^(2*q)/e^2))^p, x], x, (d + e*x)
^(1/q)], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegersQ[n, p] && Fra
ctionQ[m]
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1265

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rule 1301

```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.))/((a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*((d + e*x^2)^q/(a
+ b*x^2 + c*x^4)), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b^2 - 4
*a*c, 0] && IntegerQ[q] && IntegerQ[m]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{1}{2}\text{Subst}\left(\int \frac{x^2\sqrt{d+ex}}{a+bx+cx^2}\,dx,x,x^2\right)\\
&= \frac{\text{Subst}\left(\int \frac{x^2\left(-\frac{d}{e}+\frac{x^2}{e}\right)^2}{\frac{cd^2-bde+ae^2}{e^2}-\frac{(2cd-be)x^2}{e^2}+\frac{cx^4}{e^2}}\,dx,x,\sqrt{d+ex^2}\right)}{e}\\
&= \frac{\text{Subst}\left(\int \left(-\frac{be}{c^2}+\frac{x^2}{c}+\frac{b(cd^2-bde+ae^2)-(bcd-b^2e+ace)x^2}{c^2e\left(\frac{cd^2-bde+ae^2}{e^2}-\frac{(2cd-be)x^2}{e^2}+\frac{cx^4}{e^2}\right)}\right)dx,x,\sqrt{d+ex^2}\right)}{e}\\
&= -\frac{b\sqrt{d+ex^2}}{c^2}+\frac{(d+ex^2)^{3/2}}{3ce}+\frac{\text{Subst}\left(\int \frac{b(cd^2-bde+ae^2)+(-bcd+b^2e-ace)x^2}{\frac{cd^2-bde+ae^2}{e^2}-\frac{(2cd-be)x^2}{e^2}+\frac{cx^4}{e^2}}\,dx,x,\sqrt{d+ex^2}\right)}{c^2e^2}\\
&= -\frac{b\sqrt{d+ex^2}}{c^2}+\frac{(d+ex^2)^{3/2}}{3ce}\\
&\quad -\frac{\left(bcd-b^2e+ace-\frac{b^2cd-2ac^2d-b^3e+3abce}{\sqrt{b^2-4ac}}\right)\text{Subst}\left(\int \frac{1}{-\frac{\sqrt{b^2-4ac}}{2e}-\frac{2cd-be}{2e^2}+\frac{cx^2}{e^2}}\,dx,x,\sqrt{d+ex^2}\right)}{2c^2e^2}\\
&\quad -\frac{\left(bcd-b^2e+ace+\frac{b^2cd-2ac^2d-b^3e+3abce}{\sqrt{b^2-4ac}}\right)\text{Subst}\left(\int \frac{1}{\frac{\sqrt{b^2-4ac}}{2e}-\frac{2cd-be}{2e^2}+\frac{cx^2}{e^2}}\,dx,x,\sqrt{d+ex^2}\right)}{2c^2e^2}
\end{aligned}
$$

$$= -\frac{b\sqrt{d+ex^2}}{c^2} + \frac{(d+ex^2)^{3/2}}{3ce}$$
$$+ \frac{\left(bcd - b^2e + ace - \frac{b^2cd-2ac^2d-b^3e+3abce}{\sqrt{b^2-4ac}}\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex^2}}{\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}}\right)}{\sqrt{2}c^{5/2}\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}}$$
$$+ \frac{\left(bcd - b^2e + ace + \frac{b^2cd-2ac^2d-b^3e+3abce}{\sqrt{b^2-4ac}}\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex^2}}{\sqrt{2cd-\left(b+\sqrt{b^2-4ac}\right)e}}\right)}{\sqrt{2}c^{5/2}\sqrt{2cd-\left(b+\sqrt{b^2-4ac}\right)e}}$$

Mathematica [A] (verified)

Time = 0.98 (sec) , antiderivative size = 383, normalized size of antiderivative = 1.18

$$\int \frac{x^5\sqrt{d+ex^2}}{a+bx^2+cx^4}\,dx$$
$$= \frac{\frac{2\sqrt{c}\sqrt{d+ex^2}\left(-3be+c(d+ex^2)\right)}{e} + \frac{3\sqrt{2}\left(-b^3e+bc\left(-\sqrt{b^2-4ac}d+3ae\right)+b^2\left(cd+\sqrt{b^2-4ac}e\right)-ac\left(2cd+\sqrt{b^2-4ac}e\right)\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex^2}}{\sqrt{-2cd+be-\sqrt{b^2-4ac}\ldots}}\right.}{\sqrt{b^2-4ac}\sqrt{-2cd+\left(b-\sqrt{b^2-4ac}\right)e}}}{6c^{5/2}}$$

```
[In] Integrate[(x^5*Sqrt[d + e*x^2])/(a + b*x^2 + c*x^4),x]
```

```
[Out] ((2*Sqrt[c]*Sqrt[d + e*x^2]*(-3*b*e + c*(d + e*x^2)))/e + (3*Sqrt[2]*(-(b^3
*e) + b*c*(-(Sqrt[b^2 - 4*a*c]*d) + 3*a*e) + b^2*(c*d + Sqrt[b^2 - 4*a*c]*e
) - a*c*(2*c*d + Sqrt[b^2 - 4*a*c]*e))*ArcTan[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x
^2])/Sqrt[-2*c*d + b*e - Sqrt[b^2 - 4*a*c]*e]])/(Sqrt[b^2 - 4*a*c]*Sqrt[-2*
c*d + (b - Sqrt[b^2 - 4*a*c])*e]) + (3*Sqrt[2]*(b^3*e - b*c*(Sqrt[b^2 - 4*a
*c]*d + 3*a*e) + a*c*(2*c*d - Sqrt[b^2 - 4*a*c]*e) + b^2*(-(c*d) + Sqrt[b^2
 - 4*a*c]*e))*ArcTan[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x^2])/Sqrt[-2*c*d + (b + S
qrt[b^2 - 4*a*c])*e]])/(Sqrt[b^2 - 4*a*c]*Sqrt[-2*c*d + (b + Sqrt[b^2 - 4*a
*c])*e]))/(6*c^(5/2))
```

Maple [A] (verified)

Time = 0.87 (sec) , antiderivative size = 396, normalized size of antiderivative = 1.22

method	result
risch	$-\frac{(-c x^2 e+3be-cd)\sqrt{e x^2+d}}{3e c^2} - \frac{\sqrt{2}\left(-\left(((ae+bd)c-b^2e)\sqrt{-4e^2\left(ac-\frac{b^2}{4}\right)}+2a c^2 de+(-3ab e^2-b^2de)c+b^3e^2\right)\sqrt{\left(be-2\cdots\right.}\right.}{\cdots}$
default	$\frac{(e x^2+d)^{\frac{3}{2}}}{3ce} - \frac{b\sqrt{e x^2+d}+\frac{\left(3ab e^2c-2a c^2de-b^3e^2+b^2cde+\sqrt{-e^2(4ac-b^2)}\,ace-\sqrt{-e^2(4ac-b^2)}\,b^2e+\sqrt{-e^2(4ac-b^2)}\,bcd\right)\sqrt{2}\,\mathrm{arct}\cdots}{2\sqrt{-e^2(4ac-b^2)}\sqrt{\left(be-2cd+\sqrt{-e^2(4ac-b^2)}\right)c}\cdots}}{\cdots}$
pseudoelliptic	$-\frac{-\left(((ac-b^2)e+bcd)\sqrt{-4e^2\left(ac-\frac{b^2}{4}\right)}+(-3abc+b^3)e^2+d(2a c^2-b^2c)e\right)e\sqrt{2}\sqrt{\left(be-2cd+\sqrt{-4e^2\left(ac-\frac{b^2}{4}\right)}\right)c}\,\mathrm{arctanh}\left(\cdots\right.}{\cdots}$

```
[In] int(x^5*(e*x^2+d)^(1/2)/(c*x^4+b*x^2+a),x,method=_RETURNVERBOSE)
```

```
[Out] -1/3*(-c*e*x^2+3*b*e-c*d)*(e*x^2+d)^(1/2)/e/c^2-1/2/c^2/((b*e-2*c*d+(-4*e^2
*(a*c-1/4*b^2))^(1/2))*c)^(1/2)*2^(1/2)*(-(((a*e+b*d)*c-b^2*e)*(-4*e^2*(a*c
-1/4*b^2))^(1/2)+2*a*c^2*d*e+(-3*a*b*e^2-b^2*d*e)*c+b^3*e^2)*((b*e-2*c*d+(-
4*e^2*(a*c-1/4*b^2))^(1/2))*c)^(1/2)*arctanh(c*(e*x^2+d)^(1/2)*2^(1/2)/((-b
*e+2*c*d+(-4*e^2*(a*c-1/4*b^2))^(1/2))*c)^(1/2))+arctan(c*(e*x^2+d)^(1/2)*2
^(1/2)/((b*e-2*c*d+(-4*e^2*(a*c-1/4*b^2))^(1/2))*c)^(1/2))*((-b*e+2*c*d+(-4
*e^2*(a*c-1/4*b^2))^(1/2))*c)^(1/2)*(((a*e+b*d)*c-b^2*e)*(-4*e^2*(a*c-1/4*b
^2))^(1/2)-2*a*c^2*d*e+(3*a*b*e^2+b^2*d*e)*c-b^3*e^2))/((-b*e+2*c*d+(-4*e^2
*(a*c-1/4*b^2))^(1/2))*c)^(1/2)/(-4*e^2*(a*c-1/4*b^2))^(1/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 4182 vs. $2(280) = 560$.

Time = 102.89 (sec) , antiderivative size = 4182, normalized size of antiderivative = 12.91

$$\int \frac{x^5\sqrt{d+ex^2}}{a+bx^2+cx^4}\,dx = \text{Too large to display}$$

```
[In] integrate(x^5*(e*x^2+d)^(1/2)/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] -1/12*(3*sqrt(1/2)*c^2*e*sqrt(((b^4*c - 4*a*b^2*c^2 + 2*a^2*c^3)*d - (b^5 -
 5*a*b^3*c + 5*a^2*b*c^2)*e + (b^2*c^5 - 4*a*c^6)*sqrt(((b^6*c^2 - 4*a*b^4*
```

c^3 + 4*a^2*b^2*c^4)*d^2 - 2*(b^7*c - 5*a*b^5*c^2 + 7*a^2*b^3*c^3 - 2*a^3*b*c^4)*d*e + (b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)*e^2)/(b^2*c^10 - 4*a*c^11)))/(b^2*c^5 - 4*a*c^6))*log(-(2*(a^2*b^4*c - 2*a^3*b^2*c^2)*d^2 - 2*(a^2*b^5 - 2*a^3*b^3*c - a^4*b*c^2)*d*e + 2*(a^3*b^4 - 3*a^4*b^2*c + a^5*c^2)*e^2 + ((a^2*b^4*c - 2*a^3*b^2*c^2)*d*e - (a^2*b^5 - 3*a^3*b^3*c + a^4*b*c^2)*e^2)*x^2 + 2*sqrt(1/2)*sqrt(e*x^2 + d)*((b^7*c - 7*a*b^5*c^2 + 14*a^2*b^3*c^3 - 8*a^3*b*c^4)*d - (b^8 - 8*a*b^6*c + 20*a^2*b^4*c^2 - 17*a^3*b^2*c^3 + 4*a^4*c^4)*e - (b^5*c^5 - 7*a*b^3*c^6 + 12*a^2*b*c^7)*sqrt(((b^6*c^2 - 4*a*b^4*c^3 + 4*a^2*b^2*c^4)*d^2 - 2*(b^7*c - 5*a*b^5*c^2 + 7*a^2*b^3*c^3 - 2*a^3*b*c^4)*d*e + (b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)*e^2)/(b^2*c^10 - 4*a*c^11)))*sqrt(((b^4*c - 4*a*b^2*c^2 + 2*a^2*c^3)*d - (b^5 - 5*a*b^3*c + 5*a^2*b*c^2)*e + (b^2*c^5 - 4*a*c^6)*sqrt(((b^6*c^2 - 4*a*b^4*c^3 + 4*a^2*b^2*c^4)*d^2 - 2*(b^7*c - 5*a*b^5*c^2 + 7*a^2*b^3*c^3 - 2*a^3*b*c^4)*d*e + (b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)*e^2)/(b^2*c^10 - 4*a*c^11)))/(b^2*c^5 - 4*a*c^6)) - ((a^2*b^2*c^5 - 4*a^3*c^6)*e*x^2 + 2*(a^2*b^2*c^5 - 4*a^3*c^6)*d)*sqrt(((b^6*c^2 - 4*a*b^4*c^3 + 4*a^2*b^2*c^4)*d^2 - 2*(b^7*c - 5*a*b^5*c^2 + 7*a^2*b^3*c^3 - 2*a^3*b*c^4)*d*e + (b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)*e^2)/(b^2*c^10 - 4*a*c^11)))/x^2) - 3*sqrt(1/2)*c^2*e*sqrt(((b^4*c - 4*a*b^2*c^2 + 2*a^2*c^3)*d - (b^5 - 5*a*b^3*c + 5*a^2*b*c^2)*e + (b^2*c^5 - 4*a*c^6)*sqrt(((b^6*c^2 - 4*a*b^4*c^3 + 4*a^2*b^2*c^4)*d^2 - 2*(b^7*c - 5*a*b^5*c^2 + 7*a^2*b^3*c^3 - 2*a^3*b*c^4)*d*e + (b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)*e^2)/(b^2*c^10 - 4*a*c^11)))/(b^2*c^5 - 4*a*c^6))*log(-(2*(a^2*b^4*c - 2*a^3*b^2*c^2)*d^2 - 2*(a^2*b^5 - 2*a^3*b^3*c - a^4*b*c^2)*d*e + 2*(a^3*b^4 - 3*a^4*b^2*c + a^5*c^2)*e^2 + ((a^2*b^4*c - 2*a^3*b^2*c^2)*d*e - (a^2*b^5 - 3*a^3*b^3*c + a^4*b*c^2)*e^2)*x^2 - 2*sqrt(1/2)*sqrt(e*x^2 + d)*((b^7*c - 7*a*b^5*c^2 + 14*a^2*b^3*c^3 - 8*a^3*b*c^4)*d - (b^8 - 8*a*b^6*c + 20*a^2*b^4*c^2 - 17*a^3*b^2*c^3 + 4*a^4*c^4)*e - (b^5*c^5 - 7*a*b^3*c^6 + 12*a^2*b*c^7)*sqrt(((b^6*c^2 - 4*a*b^4*c^3 + 4*a^2*b^2*c^4)*d^2 - 2*(b^7*c - 5*a*b^5*c^2 + 7*a^2*b^3*c^3 - 2*a^3*b*c^4)*d*e + (b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)*e^2)/(b^2*c^10 - 4*a*c^11)))*sqrt(((b^4*c - 4*a*b^2*c^2 + 2*a^2*c^3)*d - (b^5 - 5*a*b^3*c + 5*a^2*b*c^2)*e + (b^2*c^5 - 4*a*c^6)*sqrt(((b^6*c^2 - 4*a*b^4*c^3 + 4*a^2*b^2*c^4)*d^2 - 2*(b^7*c - 5*a*b^5*c^2 + 7*a^2*b^3*c^3 - 2*a^3*b*c^4)*d*e + (b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)*e^2)/(b^2*c^10 - 4*a*c^11)))/(b^2*c^5 - 4*a*c^6)) - ((a^2*b^2*c^5 - 4*a^3*c^6)*e*x^2 + 2*(a^2*b^2*c^5 - 4*a^3*c^6)*d)*sqrt(((b^6*c^2 - 4*a*b^4*c^3 + 4*a^2*b^2*c^4)*d^2 - 2*(b^7*c - 5*a*b^5*c^2 + 7*a^2*b^3*c^3 - 2*a^3*b*c^4)*d*e + (b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)*e^2)/(b^2*c^10 - 4*a*c^11)))/x^2) + 3*sqrt(1/2)*c^2*e*sqrt(((b^4*c - 4*a*b^2*c^2 + 2*a^2*c^3)*d - (b^5 - 5*a*b^3*c + 5*a^2*b*c^2)*e - (b^2*c^5 - 4*a*c^6)*sqrt(((b^6*c^2 - 4*a*b^4*c^3 + 4*a^2*b^2*c^4)*d^2 - 2*(b^7*c - 5*a*b^5*c^2 + 7*a^2*b^3*c^3 - 2*a^3*b*c^4)*d*e + (b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)*e^2)/(b^2*c^10 - 4*a*c^11)))/(b^2*c^5 - 4*a*c^6))*log(-(2*(a^2*b^4*c - 2*a^3*b^2*c^2)*d^2 - 2*(a^2*b^5 - 2*a^3*b^3*c - a^4*b*c^2)*d*e +

2*(a^3*b^4 - 3*a^4*b^2*c + a^5*c^2)*e^2 + ((a^2*b^4*c - 2*a^3*b^2*c^2)*d*e - (a^2*b^5 - 3*a^3*b^3*c + a^4*b*c^2)*e^2)*x^2 + 2*sqrt(1/2)*sqrt(e*x^2 + d)*((b^7*c - 7*a*b^5*c^2 + 14*a^2*b^3*c^3 - 8*a^3*b*c^4)*d - (b^8 - 8*a*b^6*c + 20*a^2*b^4*c^2 - 17*a^3*b^2*c^3 + 4*a^4*c^4)*e + (b^5*c^5 - 7*a*b^3*c^6 + 12*a^2*b*c^7)*sqrt(((b^6*c^2 - 4*a*b^4*c^3 + 4*a^2*b^2*c^4)*d^2 - 2*(b^7*c - 5*a*b^5*c^2 + 7*a^2*b^3*c^3 - 2*a^3*b*c^4)*d*e + (b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)*e^2)/(b^2*c^10 - 4*a*c^11)))*sqrt(((b^4*c - 4*a*b^2*c^2 + 2*a^2*c^3)*d - (b^5 - 5*a*b^3*c + 5*a^2*b*c^2)*e - (b^2*c^5 - 4*a*c^6)*sqrt(((b^6*c^2 - 4*a*b^4*c^3 + 4*a^2*b^2*c^4)*d^2 - 2*(b^7*c - 5*a*b^5*c^2 + 7*a^2*b^3*c^3 - 2*a^3*b*c^4)*d*e + (b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)*e^2)/(b^2*c^10 - 4*a*c^11)))/(b^2*c^5 - 4*a*c^6)) + ((a^2*b^2*c^5 - 4*a^3*c^6)*e*x^2 + 2*(a^2*b^2*c^5 - 4*a^3*c^6)*d)*sqrt(((b^6*c^2 - 4*a*b^4*c^3 + 4*a^2*b^2*c^4)*d^2 - 2*(b^7*c - 5*a*b^5*c^2 + 7*a^2*b^3*c^3 - 2*a^3*b*c^4)*d*e + (b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)*e^2)/(b^2*c^10 - 4*a*c^11)))/x^2) - 3*sqrt(1/2)*c^2*e*sqrt(((b^4*c - 4*a*b^2*c^2 + 2*a^2*c^3)*d - (b^5 - 5*a*b^3*c + 5*a^2*b*c^2)*e - (b^2*c^5 - 4*a*c^6)*sqrt(((b^6*c^2 - 4*a*b^4*c^3 + 4*a^2*b^2*c^4)*d^2 - 2*(b^7*c - 5*a*b^5*c^2 + 7*a^2*b^3*c^3 - 2*a^3*b*c^4)*d*e + (b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)*e^2)/(b^2*c^10 - 4*a*c^11)))/(b^2*c^5 - 4*a*c^6))*log(-(2*(a^2*b^4*c - 2*a^3*b^2*c^2)*d^2 - 2*(a^2*b^5 - 2*a^3*b^3*c - a^4*b*c^2)*d*e + 2*(a^3*b^4 - 3*a^4*b^2*c + a^5*c^2)*e^2 + ((a^2*b^4*c - 2*a^3*b^2*c^2)*d*e - (a^2*b^5 - 3*a^3*b^3*c + a^4*b*c^2)*e^2)*x^2 - 2*sqrt(1/2)*sqrt(e*x^2 + d)*((b^7*c - 7*a*b^5*c^2 + 14*a^2*b^3*c^3 - 8*a^3*b*c^4)*d - (b^8 - 8*a*b^6*c + 20*a^2*b^4*c^2 - 17*a^3*b^2*c^3 + 4*a^4*c^4)*e + (b^5*c^5 - 7*a*b^3*c^6 + 12*a^2*b*c^7)*sqrt(((b^6*c^2 - 4*a*b^4*c^3 + 4*a^2*b^2*c^4)*d^2 - 2*(b^7*c - 5*a*b^5*c^2 + 7*a^2*b^3*c^3 - 2*a^3*b*c^4)*d*e + (b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)*e^2)/(b^2*c^10 - 4*a*c^11)))*sqrt(((b^4*c - 4*a*b^2*c^2 + 2*a^2*c^3)*d - (b^5 - 5*a*b^3*c + 5*a^2*b*c^2)*e - (b^2*c^5 - 4*a*c^6)*sqrt(((b^6*c^2 - 4*a*b^4*c^3 + 4*a^2*b^2*c^4)*d^2 - 2*(b^7*c - 5*a*b^5*c^2 + 7*a^2*b^3*c^3 - 2*a^3*b*c^4)*d*e + (b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)*e^2)/(b^2*c^10 - 4*a*c^11)))/(b^2*c^5 - 4*a*c^6)) + ((a^2*b^2*c^5 - 4*a^3*c^6)*e*x^2 + 2*(a^2*b^2*c^5 - 4*a^3*c^6)*d)*sqrt(((b^6*c^2 - 4*a*b^4*c^3 + 4*a^2*b^2*c^4)*d^2 - 2*(b^7*c - 5*a*b^5*c^2 + 7*a^2*b^3*c^3 - 2*a^3*b*c^4)*d*e + (b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)*e^2)/(b^2*c^10 - 4*a*c^11)))/x^2) - 4*(c*e*x^2 + c*d - 3*b*e)*sqrt(e*x^2 + d))/(c^2*e)

Sympy [F]

$$\int \frac{x^5\sqrt{d+ex^2}}{a+bx^2+cx^4}\,dx = \int \frac{x^5\sqrt{d+ex^2}}{a+bx^2+cx^4}\,dx$$

[In] integrate(x**5*(e*x**2+d)**(1/2)/(c*x**4+b*x**2+a),x)

[Out] Integral(x**5*sqrt(d + e*x**2)/(a + b*x**2 + c*x**4), x)

Maxima [F]

$$\int \frac{x^5\sqrt{d+ex^2}}{a+bx^2+cx^4}\,dx = \int \frac{\sqrt{ex^2+d}x^5}{cx^4+bx^2+a}\,dx$$

[In] integrate(x^5*(e*x^2+d)^(1/2)/(c*x^4+b*x^2+a),x, algorithm="maxima")

[Out] integrate(sqrt(e*x^2 + d)*x^5/(c*x^4 + b*x^2 + a), x)

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 776 vs. 2(280) = 560.

Time = 0.32 (sec) , antiderivative size = 776, normalized size of antiderivative = 2.40

$$\int \frac{x^5\sqrt{d+ex^2}}{a+bx^2+cx^4}\,dx$$

$$= \frac{(((b^3c-4\,abc^2)d-(b^4-5\,ab^2c+4\,a^2c^2)e)c^2e^2+2\,(b^2c^4-2\,ac^5)d^2e-(3\,b^3c^3-8\,abc^4)de^2+(b^4c^2-3\,ab^2c}{(2\,\sqrt{b^2-4\,ac}c^3d-\left(b^2c^2-4\,a\right.}$$

$$-\frac{(((b^3c-4\,abc^2)d-(b^4-5\,ab^2c+4\,a^2c^2)e)c^2e^2+2\,(b^2c^4-2\,ac^5)d^2e-(3\,b^3c^3-8\,abc^4)de^2+(b^4c^2-3\,a}{(2\,\sqrt{b^2-4\,ac}c^3d+\left(b^2c^2-\right.}$$

$$+\frac{(ex^2+d)^{\frac{3}{2}}c^2e^2-3\,\sqrt{ex^2+d}bce^3}{3\,c^3e^3}$$

[In] integrate(x^5*(e*x^2+d)^(1/2)/(c*x^4+b*x^2+a),x, algorithm="giac")

[Out] (((b^3*c - 4*a*b*c^2)*d - (b^4 - 5*a*b^2*c + 4*a^2*c^2)*e)*c^2*e^2 + 2*(b^2
*c^4 - 2*a*c^5)*d^2*e - (3*b^3*c^3 - 8*a*b*c^4)*d*e^2 + (b^4*c^2 - 3*a*b^2*

```
c^3)*e^3 - 2*(sqrt(b^2 - 4*a*c)*b*c^3*d^2 - sqrt(b^2 - 4*a*c)*b^2*c^2*d*e +
 sqrt(b^2 - 4*a*c)*a*b*c^2*e^2)*abs(c)*abs(e))*arctan(2*sqrt(1/2)*sqrt(e*x^
2 + d)/sqrt(-(2*c^4*d*e^4 - b*c^3*e^5 + sqrt(-4*(c^4*d^2*e^4 - b*c^3*d*e^5
+ a*c^3*e^6)*c^4*e^4 + (2*c^4*d*e^4 - b*c^3*e^5)^2))/(c^4*e^4)))/((2*sqrt(b
^2 - 4*a*c)*c^3*d - (b^2*c^2 - 4*a*c^3 + sqrt(b^2 - 4*a*c)*b*c^2)*e)*sqrt(-
4*c^2*d + 2*(b*c - sqrt(b^2 - 4*a*c)*c)*e)*c^2*abs(e)) - (((b^3*c - 4*a*b*c
^2)*d - (b^4 - 5*a*b^2*c + 4*a^2*c^2)*e)*c^2*e^2 + 2*(b^2*c^4 - 2*a*c^5)*d^
2*e - (3*b^3*c^3 - 8*a*b*c^4)*d*e^2 + (b^4*c^2 - 3*a*b^2*c^3)*e^3 + 2*(sqrt
(b^2 - 4*a*c)*b*c^3*d^2 - sqrt(b^2 - 4*a*c)*b^2*c^2*d*e + sqrt(b^2 - 4*a*c)
*a*b*c^2*e^2)*abs(c)*abs(e))*arctan(2*sqrt(1/2)*sqrt(e*x^2 + d)/sqrt(-(2*c^
4*d*e^4 - b*c^3*e^5 - sqrt(-4*(c^4*d^2*e^4 - b*c^3*d*e^5 + a*c^3*e^6)*c^4*e
^4 + (2*c^4*d*e^4 - b*c^3*e^5)^2))/(c^4*e^4)))/((2*sqrt(b^2 - 4*a*c)*c^3*d
+ (b^2*c^2 - 4*a*c^3 - sqrt(b^2 - 4*a*c)*b*c^2)*e)*sqrt(-4*c^2*d + 2*(b*c +
 sqrt(b^2 - 4*a*c)*c)*e)*c^2*abs(e)) + 1/3*((e*x^2 + d)^(3/2)*c^2*e^2 - 3*s
qrt(e*x^2 + d)*b*c*e^3)/(c^3*e^3)
```

Mupad [B] (verification not implemented)

Time = 8.68 (sec) , antiderivative size = 8222, normalized size of antiderivative = 25.38

$$\int \frac{x^5\sqrt{d+ex^2}}{a+bx^2+cx^4}\,dx = \text{Too large to display}$$

```
[In] int((x^5*(d + e*x^2)^(1/2))/(a + b*x^2 + c*x^4),x)
```

```
[Out] (d + e*x^2)^(3/2)/(3*c*e) - atan(((((4*a*b^3*c^3*e^4 - 16*a^2*b*c^4*e^4 - 4
*b^4*c^3*d*e^3 + 4*b^3*c^4*d^2*e^2 - 16*a*b*c^5*d^2*e^2 + 16*a*b^2*c^4*d*e^
3)/c^3 - (2*(d + e*x^2)^(1/2)*(-(b^7*e + 8*a^3*c^4*d + b^4*e*(-(4*a*c - b^2
)^3)^(1/2) - b^6*c*d - 18*a^2*b^2*c^3*d + 25*a^2*b^3*c^2*e + a^2*c^2*e*(-(4
*a*c - b^2)^3)^(1/2) - 9*a*b^5*c*e + 8*a*b^4*c^2*d - 20*a^3*b*c^3*e - b^3*c
*d*(-(4*a*c - b^2)^3)^(1/2) + 2*a*b*c^2*d*(-(4*a*c - b^2)^3)^(1/2) - 3*a*b^
2*c*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^7 + b^4*c^5 - 8*a*b^2*c^6)))^(
1/2)*(4*b^3*c^5*e^3 - 8*b^2*c^6*d*e^2 - 16*a*b*c^6*e^3 + 32*a*c^7*d*e^2))/c
^3)*(-(b^7*e + 8*a^3*c^4*d + b^4*e*(-(4*a*c - b^2)^3)^(1/2) - b^6*c*d - 18*
a^2*b^2*c^3*d + 25*a^2*b^3*c^2*e + a^2*c^2*e*(-(4*a*c - b^2)^3)^(1/2) - 9*a
*b^5*c*e + 8*a*b^4*c^2*d - 20*a^3*b*c^3*e - b^3*c*d*(-(4*a*c - b^2)^3)^(1/2
) + 2*a*b*c^2*d*(-(4*a*c - b^2)^3)^(1/2) - 3*a*b^2*c*e*(-(4*a*c - b^2)^3)^(
1/2))/(8*(16*a^2*c^7 + b^4*c^5 - 8*a*b^2*c^6)))^(1/2) - (2*(d + e*x^2)^(1/2
)*(b^6*e^4 - 2*a^3*c^3*e^4 + 9*a^2*b^2*c^2*e^4 + 2*a^2*c^4*d^2*e^2 + b^4*c^
2*d^2*e^2 - 6*a*b^4*c*e^4 - 2*b^5*c*d*e^3 + 10*a*b^3*c^2*d*e^3 - 10*a^2*b*c
^3*d*e^3 - 4*a*b^2*c^3*d^2*e^2))/c^3)*(-(b^7*e + 8*a^3*c^4*d + b^4*e*(-(4*a
*c - b^2)^3)^(1/2) - b^6*c*d - 18*a^2*b^2*c^3*d + 25*a^2*b^3*c^2*e + a^2*c^
2*e*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c*e + 8*a*b^4*c^2*d - 20*a^3*b*c^3*e
 - b^3*c*d*(-(4*a*c - b^2)^3)^(1/2) + 2*a*b*c^2*d*(-(4*a*c - b^2)^3)^(1/2)
- 3*a*b^2*c*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^7 + b^4*c^5 - 8*a*b^2*
```

c^6)))^(1/2)*1i - (((4*a*b^3*c^3*e^4 - 16*a^2*b*c^4*e^4 - 4*b^4*c^3*d*e^3 + 4*b^3*c^4*d^2*e^2 - 16*a*b*c^5*d^2*e^2 + 16*a*b^2*c^4*d*e^3)/c^3 + (2*(d + e*x^2)^(1/2)*(-(b^7*e + 8*a^3*c^4*d + b^4*e*(-(4*a*c - b^2)^3)^(1/2) - b^6*c*d - 18*a^2*b^2*c^3*d + 25*a^2*b^3*c^2*e + a^2*c^2*e*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c*e + 8*a*b^4*c^2*d - 20*a^3*b*c^3*e - b^3*c*d*(-(4*a*c - b^2)^3)^(1/2) + 2*a*b*c^2*d*(-(4*a*c - b^2)^3)^(1/2) - 3*a*b^2*c*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^7 + b^4*c^5 - 8*a*b^2*c^6)))^(1/2)*(4*b^3*c^5*e^3 - 8*b^2*c^6*d*e^2 - 16*a*b*c^6*e^3 + 32*a*c^7*d*e^2))/c^3)*(-(b^7*e + 8*a^3*c^4*d + b^4*e*(-(4*a*c - b^2)^3)^(1/2) - b^6*c*d - 18*a^2*b^2*c^3*d + 25*a^2*b^3*c^2*e + a^2*c^2*e*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c*e + 8*a*b^4*c^2*d - 20*a^3*b*c^3*e - b^3*c*d*(-(4*a*c - b^2)^3)^(1/2) + 2*a*b*c^2*d*(-(4*a*c - b^2)^3)^(1/2) - 3*a*b^2*c*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^7 + b^4*c^5 - 8*a*b^2*c^6)))^(1/2) + (2*(d + e*x^2)^(1/2)*(b^6*e^4 - 2*a^3*c^3*e^4 + 9*a^2*b^2*c^2*e^4 + 2*a^2*c^4*d^2*e^2 + b^4*c^2*d^2*e^2 - 6*a*b^4*c*e^4 - 2*b^5*c*d*e^3 + 10*a*b^3*c^2*d*e^3 - 10*a^2*b*c^3*d*e^3 - 4*a*b^2*c^3*d^2*e^2))/c^3)*(-(b^7*e + 8*a^3*c^4*d + b^4*e*(-(4*a*c - b^2)^3)^(1/2) - b^6*c*d - 18*a^2*b^2*c^3*d + 25*a^2*b^3*c^2*e + a^2*c^2*e*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c*e + 8*a*b^4*c^2*d - 20*a^3*b*c^3*e - b^3*c*d*(-(4*a*c - b^2)^3)^(1/2) + 2*a*b*c^2*d*(-(4*a*c - b^2)^3)^(1/2) - 3*a*b^2*c*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^7 + b^4*c^5 - 8*a*b^2*c^6)))^(1/2)*1i) /((2*(a^4*c*e^5 - a^3*b^2*e^5 + a^2*b^3*d*e^4 + a^3*c^2*d^2*e^3 + a^2*b*c^2*d^3*e^2 - 2*a^2*b^2*c*d^2*e^3))/c^3 + (((4*a*b^3*c^3*e^4 - 16*a^2*b*c^4*e^4 - 4*b^4*c^3*d*e^3 + 4*b^3*c^4*d^2*e^2 - 16*a*b*c^5*d^2*e^2 + 16*a*b^2*c^4*d*e^3)/c^3 - (2*(d + e*x^2)^(1/2)*(-(b^7*e + 8*a^3*c^4*d + b^4*e*(-(4*a*c - b^2)^3)^(1/2) - b^6*c*d - 18*a^2*b^2*c^3*d + 25*a^2*b^3*c^2*e + a^2*c^2*e*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c*e + 8*a*b^4*c^2*d - 20*a^3*b*c^3*e - b^3*c*d*(-(4*a*c - b^2)^3)^(1/2) + 2*a*b*c^2*d*(-(4*a*c - b^2)^3)^(1/2) - 3*a*b^2*c*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^7 + b^4*c^5 - 8*a*b^2*c^6)))^(1/2)*(4*b^3*c^5*e^3 - 8*b^2*c^6*d*e^2 - 16*a*b*c^6*e^3 + 32*a*c^7*d*e^2))/c^3)*(-(b^7*e + 8*a^3*c^4*d + b^4*e*(-(4*a*c - b^2)^3)^(1/2) - b^6*c*d - 18*a^2*b^2*c^3*d + 25*a^2*b^3*c^2*e + a^2*c^2*e*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c*e + 8*a*b^4*c^2*d - 20*a^3*b*c^3*e - b^3*c*d*(-(4*a*c - b^2)^3)^(1/2) + 2*a*b*c^2*d*(-(4*a*c - b^2)^3)^(1/2) - 3*a*b^2*c*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^7 + b^4*c^5 - 8*a*b^2*c^6)))^(1/2) - (2*(d + e*x^2)^(1/2)*(b^6*e^4 - 2*a^3*c^3*e^4 + 9*a^2*b^2*c^2*e^4 + 2*a^2*c^4*d^2*e^2 + b^4*c^2*d^2*e^2 - 6*a*b^4*c*e^4 - 2*b^5*c*d*e^3 + 10*a*b^3*c^2*d*e^3 - 10*a^2*b*c^3*d*e^3 - 4*a*b^2*c^3*d^2*e^2))/c^3)*(-(b^7*e + 8*a^3*c^4*d + b^4*e*(-(4*a*c - b^2)^3)^(1/2) - b^6*c*d - 18*a^2*b^2*c^3*d + 25*a^2*b^3*c^2*e + a^2*c^2*e*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c*e + 8*a*b^4*c^2*d - 20*a^3*b*c^3*e - b^3*c*d*(-(4*a*c - b^2)^3)^(1/2) + 2*a*b*c^2*d*(-(4*a*c - b^2)^3)^(1/2) - 3*a*b^2*c*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^7 + b^4*c^5 - 8*a*b^2*c^6)))^(1/2) + (((4*a*b^3*c^3*e^4 - 16*a^2*b*c^4*e^4 - 4*b^4*c^3*d*e^3 + 4*b^3*c^4*d^2*e^2 - 16*a*b*c^5*d^2*e^2 + 16*a*b^2*c^4*d*e^3)/c^3 + (2*(d + e*x^2)^(1/2)*(-(b^7*e + 8*a^3*c^4*d + b^4*e*(-(4*a*c - b^2)^3)^(1/2) - b^6*c*d - 18*a^2*b^2*c^3*d + 25*a^2*b^3*c^2*e + a^2*c^2*e*(-(4*a*c - b^2)^3)

^(1/2) - 9*a*b^5*c*e + 8*a*b^4*c^2*d - 20*a^3*b*c^3*e - b^3*c*d*(-(4*a*c - b^2)^3)^(1/2) + 2*a*b*c^2*d*(-(4*a*c - b^2)^3)^(1/2) - 3*a*b^2*c*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^7 + b^4*c^5 - 8*a*b^2*c^6)))^(1/2)*(4*b^3*c^5*e^3 - 8*b^2*c^6*d*e^2 - 16*a*b*c^6*e^3 + 32*a*c^7*d*e^2))/c^3)*(-(b^7*e + 8*a^3*c^4*d + b^4*e*(-(4*a*c - b^2)^3)^(1/2) - b^6*c*d - 18*a^2*b^2*c^3*d + 25*a^2*b^3*c^2*e + a^2*c^2*e*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c*e + 8*a*b^4*c^2*d - 20*a^3*b*c^3*e - b^3*c*d*(-(4*a*c - b^2)^3)^(1/2) + 2*a*b*c^2*d*(-(4*a*c - b^2)^3)^(1/2) - 3*a*b^2*c*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^7 + b^4*c^5 - 8*a*b^2*c^6)))^(1/2) + (2*(d + e*x^2)^(1/2)*(b^6*e^4 - 2*a^3*c^3*e^4 + 9*a^2*b^2*c^2*e^4 + 2*a^2*c^4*d^2*e^2 + b^4*c^2*d^2*e^2 - 6*a*b^4*c*e^4 - 2*b^5*c*d*e^3 + 10*a*b^3*c^2*d*e^3 - 10*a^2*b*c^3*d*e^3 - 4*a*b^2*c^3*d^2*e^2))/c^3)*(-(b^7*e + 8*a^3*c^4*d + b^4*e*(-(4*a*c - b^2)^3)^(1/2) - b^6*c*d - 18*a^2*b^2*c^3*d + 25*a^2*b^3*c^2*e + a^2*c^2*e*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c*e + 8*a*b^4*c^2*d - 20*a^3*b*c^3*e - b^3*c*d*(-(4*a*c - b^2)^3)^(1/2) + 2*a*b*c^2*d*(-(4*a*c - b^2)^3)^(1/2) - 3*a*b^2*c*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^7 + b^4*c^5 - 8*a*b^2*c^6)))^(1/2))) *(-(b^7*e + 8*a^3*c^4*d + b^4*e*(-(4*a*c - b^2)^3)^(1/2) - b^6*c*d - 18*a^2*b^2*c^3*d + 25*a^2*b^3*c^2*e + a^2*c^2*e*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c*e + 8*a*b^4*c^2*d - 20*a^3*b*c^3*e - b^3*c*d*(-(4*a*c - b^2)^3)^(1/2) + 2*a*b*c^2*d*(-(4*a*c - b^2)^3)^(1/2) - 3*a*b^2*c*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^7 + b^4*c^5 - 8*a*b^2*c^6)))^(1/2)*2i - atan(((((4*a*b^3*c^3*e^4 - 16*a^2*b*c^4*e^4 - 4*b^4*c^3*d*e^3 + 4*b^3*c^4*d^2*e^2 - 16*a*b*c^5*d^2*e^2 + 16*a*b^2*c^4*d*e^3)/c^3 - (2*(d + e*x^2)^(1/2)*((b^4*e*(-(4*a*c - b^2)^3)^(1/2) - 8*a^3*c^4*d - b^7*e + b^6*c*d + 18*a^2*b^2*c^3*d - 25*a^2*b^3*c^2*e + a^2*c^2*e*(-(4*a*c - b^2)^3)^(1/2) + 9*a*b^5*c*e - 8*a*b^4*c^2*d + 20*a^3*b*c^3*e - b^3*c*d*(-(4*a*c - b^2)^3)^(1/2) + 2*a*b*c^2*d*(-(4*a*c - b^2)^3)^(1/2) - 3*a*b^2*c*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^7 + b^4*c^5 - 8*a*b^2*c^6)))^(1/2)*(4*b^3*c^5*e^3 - 8*b^2*c^6*d*e^2 - 16*a*b*c^6*e^3 + 32*a*c^7*d*e^2))/c^3)*((b^4*e*(-(4*a*c - b^2)^3)^(1/2) - 8*a^3*c^4*d - b^7*e + b^6*c*d + 18*a^2*b^2*c^3*d - 25*a^2*b^3*c^2*e + a^2*c^2*e*(-(4*a*c - b^2)^3)^(1/2) + 9*a*b^5*c*e - 8*a*b^4*c^2*d + 20*a^3*b*c^3*e - b^3*c*d*(-(4*a*c - b^2)^3)^(1/2) + 2*a*b*c^2*d*(-(4*a*c - b^2)^3)^(1/2) - 3*a*b^2*c*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^7 + b^4*c^5 - 8*a*b^2*c^6)))^(1/2) - (2*(d + e*x^2)^(1/2)*(b^6*e^4 - 2*a^3*c^3*e^4 + 9*a^2*b^2*c^2*e^4 + 2*a^2*c^4*d^2*e^2 + b^4*c^2*d^2*e^2 - 6*a*b^4*c*e^4 - 2*b^5*c*d*e^3 + 10*a*b^3*c^2*d*e^3 - 10*a^2*b*c^3*d*e^3 - 4*a*b^2*c^3*d^2*e^2))/c^3)*((b^4*e*(-(4*a*c - b^2)^3)^(1/2) - 8*a^3*c^4*d - b^7*e + b^6*c*d + 18*a^2*b^2*c^3*d - 25*a^2*b^3*c^2*e + a^2*c^2*e*(-(4*a*c - b^2)^3)^(1/2) + 9*a*b^5*c*e - 8*a*b^4*c^2*d + 20*a^3*b*c^3*e - b^3*c*d*(-(4*a*c - b^2)^3)^(1/2) + 2*a*b*c^2*d*(-(4*a*c - b^2)^3)^(1/2) - 3*a*b^2*c*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^7 + b^4*c^5 - 8*a*b^2*c^6)))^(1/2)*1i - (((4*a*b^3*c^3*e^4 - 16*a^2*b*c^4*e^4 - 4*b^4*c^3*d*e^3 + 4*b^3*c^4*d^2*e^2 - 16*a*b*c^5*d^2*e^2 + 16*a*b^2*c^4*d*e^3)/c^3 + (2*(d + e*x^2)^(1/2)*((b^4*e*(-(4*a*c - b^2)^3)^(1/2) - 8*a^3*c^4*d - b^7*e + b^6*c*d + 18*a^2*b^2*c^3*d - 25*a^2*b^3*c^2*e + a^2*c^2*e*(-(4*a*c - b^2)^3)^(1/2) + 9*a*b^5*c*e - 8*a*b^4*c^2*d + 20*a^3*b*c^3*e

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 - b^3*c*d*(-(4*a*c - b^2)^3)^(1/2) + 2*a*b*c^2*d*(-(4*a*c - b^2)^3)^(1/2)
- 3*a*b^2*c*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^7 + b^4*c^5 - 8*a*b^2*
c^6)))^(1/2)*(4*b^3*c^5*e^3 - 8*b^2*c^6*d*e^2 - 16*a*b*c^6*e^3 + 32*a*c^7*d
*e^2))/c^3)*((b^4*e*(-(4*a*c - b^2)^3)^(1/2) - 8*a^3*c^4*d - b^7*e + b^6*c*
d + 18*a^2*b^2*c^3*d - 25*a^2*b^3*c^2*e + a^2*c^2*e*(-(4*a*c - b^2)^3)^(1/2
) + 9*a*b^5*c*e - 8*a*b^4*c^2*d + 20*a^3*b*c^3*e - b^3*c*d*(-(4*a*c - b^2)^
3)^(1/2) + 2*a*b*c^2*d*(-(4*a*c - b^2)^3)^(1/2) - 3*a*b^2*c*e*(-(4*a*c - b^
2)^3)^(1/2))/(8*(16*a^2*c^7 + b^4*c^5 - 8*a*b^2*c^6)))^(1/2) + (2*(d + e*x^
2)^(1/2)*(b^6*e^4 - 2*a^3*c^3*e^4 + 9*a^2*b^2*c^2*e^4 + 2*a^2*c^4*d^2*e^2 +
 b^4*c^2*d^2*e^2 - 6*a*b^4*c*e^4 - 2*b^5*c*d*e^3 + 10*a*b^3*c^2*d*e^3 - 10*
a^2*b*c^3*d*e^3 - 4*a*b^2*c^3*d^2*e^2))/c^3)*((b^4*e*(-(4*a*c - b^2)^3)^(1/
2) - 8*a^3*c^4*d - b^7*e + b^6*c*d + 18*a^2*b^2*c^3*d - 25*a^2*b^3*c^2*e +
a^2*c^2*e*(-(4*a*c - b^2)^3)^(1/2) + 9*a*b^5*c*e - 8*a*b^4*c^2*d + 20*a^3*b
*c^3*e - b^3*c*d*(-(4*a*c - b^2)^3)^(1/2) + 2*a*b*c^2*d*(-(4*a*c - b^2)^3)^
(1/2) - 3*a*b^2*c*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^7 + b^4*c^5 - 8*
a*b^2*c^6)))^(1/2)*1i)/((2*(a^4*c*e^5 - a^3*b^2*e^5 + a^2*b^3*d*e^4 + a^3*c
^2*d^2*e^3 + a^2*b*c^2*d^3*e^2 - 2*a^2*b^2*c*d^2*e^3))/c^3 + (((4*a*b^3*c^3
*e^4 - 16*a^2*b*c^4*e^4 - 4*b^4*c^3*d*e^3 + 4*b^3*c^4*d^2*e^2 - 16*a*b*c^5*
d^2*e^2 + 16*a*b^2*c^4*d*e^3)/c^3 - (2*(d + e*x^2)^(1/2)*((b^4*e*(-(4*a*c -
 b^2)^3)^(1/2) - 8*a^3*c^4*d - b^7*e + b^6*c*d + 18*a^2*b^2*c^3*d - 25*a^2*
b^3*c^2*e + a^2*c^2*e*(-(4*a*c - b^2)^3)^(1/2) + 9*a*b^5*c*e - 8*a*b^4*c^2*
d + 20*a^3*b*c^3*e - b^3*c*d*(-(4*a*c - b^2)^3)^(1/2) + 2*a*b*c^2*d*(-(4*a*
c - b^2)^3)^(1/2) - 3*a*b^2*c*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^7 +
b^4*c^5 - 8*a*b^2*c^6)))^(1/2)*(4*b^3*c^5*e^3 - 8*b^2*c^6*d*e^2 - 16*a*b*c^
6*e^3 + 32*a*c^7*d*e^2))/c^3)*((b^4*e*(-(4*a*c - b^2)^3)^(1/2) - 8*a^3*c^4*
d - b^7*e + b^6*c*d + 18*a^2*b^2*c^3*d - 25*a^2*b^3*c^2*e + a^2*c^2*e*(-(4*
a*c - b^2)^3)^(1/2) + 9*a*b^5*c*e - 8*a*b^4*c^2*d + 20*a^3*b*c^3*e - b^3*c*
d*(-(4*a*c - b^2)^3)^(1/2) + 2*a*b*c^2*d*(-(4*a*c - b^2)^3)^(1/2) - 3*a*b^2
*c*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^7 + b^4*c^5 - 8*a*b^2*c^6)))^(1
/2) - (2*(d + e*x^2)^(1/2)*(b^6*e^4 - 2*a^3*c^3*e^4 + 9*a^2*b^2*c^2*e^4 + 2
*a^2*c^4*d^2*e^2 + b^4*c^2*d^2*e^2 - 6*a*b^4*c*e^4 - 2*b^5*c*d*e^3 + 10*a*b
^3*c^2*d*e^3 - 10*a^2*b*c^3*d*e^3 - 4*a*b^2*c^3*d^2*e^2))/c^3)*((b^4*e*(-(4
*a*c - b^2)^3)^(1/2) - 8*a^3*c^4*d - b^7*e + b^6*c*d + 18*a^2*b^2*c^3*d - 2
5*a^2*b^3*c^2*e + a^2*c^2*e*(-(4*a*c - b^2)^3)^(1/2) + 9*a*b^5*c*e - 8*a*b^
4*c^2*d + 20*a^3*b*c^3*e - b^3*c*d*(-(4*a*c - b^2)^3)^(1/2) + 2*a*b*c^2*d*(
-(4*a*c - b^2)^3)^(1/2) - 3*a*b^2*c*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*
c^7 + b^4*c^5 - 8*a*b^2*c^6)))^(1/2) + (((4*a*b^3*c^3*e^4 - 16*a^2*b*c^4*e^
4 - 4*b^4*c^3*d*e^3 + 4*b^3*c^4*d^2*e^2 - 16*a*b*c^5*d^2*e^2 + 16*a*b^2*c^4
*d*e^3)/c^3 + (2*(d + e*x^2)^(1/2)*((b^4*e*(-(4*a*c - b^2)^3)^(1/2) - 8*a^3
*c^4*d - b^7*e + b^6*c*d + 18*a^2*b^2*c^3*d - 25*a^2*b^3*c^2*e + a^2*c^2*e*
(-(4*a*c - b^2)^3)^(1/2) + 9*a*b^5*c*e - 8*a*b^4*c^2*d + 20*a^3*b*c^3*e - b
^3*c*d*(-(4*a*c - b^2)^3)^(1/2) + 2*a*b*c^2*d*(-(4*a*c - b^2)^3)^(1/2) - 3*
a*b^2*c*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^7 + b^4*c^5 - 8*a*b^2*c^6)
))^(1/2)*(4*b^3*c^5*e^3 - 8*b^2*c^6*d*e^2 - 16*a*b*c^6*e^3 + 32*a*c^7*d*e^2
))/c^3)*((b^4*e*(-(4*a*c - b^2)^3)^(1/2) - 8*a^3*c^4*d - b^7*e + b^6*c*d +
```

```
18*a^2*b^2*c^3*d - 25*a^2*b^3*c^2*e + a^2*c^2*e*(-(4*a*c - b^2)^3)^(1/2) +
9*a*b^5*c*e - 8*a*b^4*c^2*d + 20*a^3*b*c^3*e - b^3*c*d*(-(4*a*c - b^2)^3)^(
1/2) + 2*a*b*c^2*d*(-(4*a*c - b^2)^3)^(1/2) - 3*a*b^2*c*e*(-(4*a*c - b^2)^3
)^(1/2))/(8*(16*a^2*c^7 + b^4*c^5 - 8*a*b^2*c^6)))^(1/2) + (2*(d + e*x^2)^(
1/2)*(b^6*e^4 - 2*a^3*c^3*e^4 + 9*a^2*b^2*c^2*e^4 + 2*a^2*c^4*d^2*e^2 + b^4
*c^2*d^2*e^2 - 6*a*b^4*c*e^4 - 2*b^5*c*d*e^3 + 10*a*b^3*c^2*d*e^3 - 10*a^2*
b*c^3*d*e^3 - 4*a*b^2*c^3*d^2*e^2))/c^3)*((b^4*e*(-(4*a*c - b^2)^3)^(1/2) -
 8*a^3*c^4*d - b^7*e + b^6*c*d + 18*a^2*b^2*c^3*d - 25*a^2*b^3*c^2*e + a^2*
c^2*e*(-(4*a*c - b^2)^3)^(1/2) + 9*a*b^5*c*e - 8*a*b^4*c^2*d + 20*a^3*b*c^3
*e - b^3*c*d*(-(4*a*c - b^2)^3)^(1/2) + 2*a*b*c^2*d*(-(4*a*c - b^2)^3)^(1/2
) - 3*a*b^2*c*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^7 + b^4*c^5 - 8*a*b^
2*c^6)))^(1/2)))*((b^4*e*(-(4*a*c - b^2)^3)^(1/2) - 8*a^3*c^4*d - b^7*e + b
^6*c*d + 18*a^2*b^2*c^3*d - 25*a^2*b^3*c^2*e + a^2*c^2*e*(-(4*a*c - b^2)^3)
^(1/2) + 9*a*b^5*c*e - 8*a*b^4*c^2*d + 20*a^3*b*c^3*e - b^3*c*d*(-(4*a*c -
b^2)^3)^(1/2) + 2*a*b*c^2*d*(-(4*a*c - b^2)^3)^(1/2) - 3*a*b^2*c*e*(-(4*a*c
 - b^2)^3)^(1/2))/(8*(16*a^2*c^7 + b^4*c^5 - 8*a*b^2*c^6)))^(1/2)*2i - ((2*
d)/(c*e) + (b*e^2 - 2*c*d*e)/(c^2*e^2))*(d + e*x^2)^(1/2)
```

3.356 $\int \frac{x^3\sqrt{d+ex^2}}{a+bx^2+cx^4}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 292

$$\int \frac{x^3\sqrt{d+ex^2}}{a+bx^2+cx^4}\,dx = \frac{\sqrt{d+ex^2}}{c}+\frac{\left(bcd-b^2e+2ace-\sqrt{b^2-4ac}(cd-be)\right)\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex^2}}{\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}}\right)}{\sqrt{2}c^{3/2}\sqrt{b^2-4ac}\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}} - \frac{\left(bcd-b^2e+2ace+\sqrt{b^2-4ac}(cd-be)\right)\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex^2}}{\sqrt{2cd-\left(b+\sqrt{b^2-4ac}\right)e}}\right)}{\sqrt{2}c^{3/2}\sqrt{b^2-4ac}\sqrt{2cd-\left(b+\sqrt{b^2-4ac}\right)e}}$$

```
[Out] (e*x^2+d)^(1/2)/c+1/2*arctanh(2^(1/2)*c^(1/2)*(e*x^2+d)^(1/2)/(2*c*d-e*(b-(
-4*a*c+b^2)^(1/2)))^(1/2))*(b*c*d-b^2*e+2*a*c*e-(-b*e+c*d)*(-4*a*c+b^2)^(1/
2))/c^(3/2)*2^(1/2)/(-4*a*c+b^2)^(1/2)/(2*c*d-e*(b-(-4*a*c+b^2)^(1/2)))^(1/
2)-1/2*arctanh(2^(1/2)*c^(1/2)*(e*x^2+d)^(1/2)/(2*c*d-e*(b+(-4*a*c+b^2)^(1/
2)))^(1/2))*(b*c*d-b^2*e+2*a*c*e+(-b*e+c*d)*(-4*a*c+b^2)^(1/2))/c^(3/2)*2^(
1/2)/(-4*a*c+b^2)^(1/2)/(2*c*d-e*(b+(-4*a*c+b^2)^(1/2)))^(1/2)
```

Rubi [A] (verified)

Time = 2.39 (sec) , antiderivative size = 292, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.172, Rules used = {1265, 838, 840, 1180, 214}

$$\int \frac{x^3\sqrt{d+ex^2}}{a+bx^2+cx^4}\,dx$$
$$= \frac{\left(-\sqrt{b^2-4ac}(cd-be)+2ace+b^2(-e)+bcd\right)\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex^2}}{\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}\right)}{\sqrt{2}c^{3/2}\sqrt{b^2-4ac}\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}$$
$$-\frac{\left(\sqrt{b^2-4ac}(cd-be)+2ace+b^2(-e)+bcd\right)\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex^2}}{\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}\right)}{\sqrt{2}c^{3/2}\sqrt{b^2-4ac}\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}$$
$$+\frac{\sqrt{d+ex^2}}{c}$$

```
[In] Int[(x^3*Sqrt[d + e*x^2])/(a + b*x^2 + c*x^4),x]
```

```
[Out] Sqrt[d + e*x^2]/c + ((b*c*d - b^2*e + 2*a*c*e - Sqrt[b^2 - 4*a*c]*(c*d - b*
e))*ArcTanh[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x^2])/Sqrt[2*c*d - (b - Sqrt[b^2 -
4*a*c])*e]])/(Sqrt[2]*c^(3/2)*Sqrt[b^2 - 4*a*c]*Sqrt[2*c*d - (b - Sqrt[b^2
- 4*a*c])*e]) - ((b*c*d - b^2*e + 2*a*c*e + Sqrt[b^2 - 4*a*c]*(c*d - b*e))*
ArcTanh[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x^2])/Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*
c])*e]])/(Sqrt[2]*c^(3/2)*Sqrt[b^2 - 4*a*c]*Sqrt[2*c*d - (b + Sqrt[b^2 - 4*
a*c])*e])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 838

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Simp[g*((d + e*x)^m/(c*m)), x] + Dist[1/c, Int[
(d + e*x)^(m - 1)*(Simp[c*d*f - a*e*g + (g*c*d - b*e*g + c*e*f)*x, x]/(a +
b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*c
, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && FractionQ[m] && GtQ[m, 0]
```

Rule 840

```
Int[((f_.) + (g_.)*(x_))/(Sqrt[(d_.) + (e_.)*(x_)]*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)), x_Symbol] :> Dist[2, Subst[Int[(e*f - d*g + g*x^2)/(c*d^2 - b
```

```
*d*e + a*e^2 - (2*c*d - b*e)*x^2 + c*x^4), x], x, Sqrt[d + e*x]], x] /; Fre
eQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e +
a*e^2, 0]
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1265

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{2}\text{Subst}\left(\int \frac{x\sqrt{d+ex}}{a+bx+cx^2}\,dx, x, x^2\right)\\
&= \frac{\sqrt{d+ex^2}}{c} + \frac{\text{Subst}\left(\int \frac{-ae+(cd-be)x}{\sqrt{d+ex}(a+bx+cx^2)}\,dx, x, x^2\right)}{2c}\\
&= \frac{\sqrt{d+ex^2}}{c} + \frac{\text{Subst}\left(\int \frac{-ae^2-d(cd-be)+(cd-be)x^2}{cd^2-bde+ae^2+(-2cd+be)x^2+cx^4}\,dx, x, \sqrt{d+ex^2}\right)}{c}\\
&= \frac{\sqrt{d+ex^2}}{c}\\
&\quad - \frac{\left(bcd-b^2e+2ace-\sqrt{b^2-4ac}(cd-be)\right)\text{Subst}\left(\int \frac{1}{-\frac{1}{2}\sqrt{b^2-4ac}e+\frac{1}{2}(-2cd+be)+cx^2}\,dx, x, \sqrt{d+ex^2}\right)}{2c\sqrt{b^2-4ac}}\\
&\quad + \frac{\left(bcd-b^2e+2ace+\sqrt{b^2-4ac}(cd-be)\right)\text{Subst}\left(\int \frac{1}{\frac{1}{2}\sqrt{b^2-4ac}e+\frac{1}{2}(-2cd+be)+cx^2}\,dx, x, \sqrt{d+ex^2}\right)}{2c\sqrt{b^2-4ac}}
\end{aligned}$$

$$
= \frac{\sqrt{d+ex^2}}{c} + \frac{\left(bcd - b^2e + 2ace - \sqrt{b^2-4ac}(cd-be)\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex^2}}{\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}}\right)}{\sqrt{2}c^{3/2}\sqrt{b^2-4ac}\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}}
- \frac{\left(bcd - b^2e + 2ace + \sqrt{b^2-4ac}(cd-be)\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex^2}}{\sqrt{2cd-\left(b+\sqrt{b^2-4ac}\right)e}}\right)}{\sqrt{2}c^{3/2}\sqrt{b^2-4ac}\sqrt{2cd-\left(b+\sqrt{b^2-4ac}\right)e}}
$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.95 (sec) , antiderivative size = 350, normalized size of antiderivative = 1.20

$$
\int \frac{x^3\sqrt{d+ex^2}}{a+bx^2+cx^4}\,dx
$$

$$
= \frac{2\sqrt{c}\sqrt{d+ex^2} - \frac{\left(-ibcd - c\sqrt{-b^2+4ac}d + ib^2e - 2iace + b\sqrt{-b^2+4ac}e\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex^2}}{\sqrt{-2cd+be-i\sqrt{-b^2+4ac}e}}\right)}{\sqrt{-\frac{b^2}{2}+2ac}\sqrt{-2cd+\left(b-i\sqrt{-b^2+4ac}\right)e}} - \frac{\left(ibcd - c\sqrt{-b^2+4ac}d - ib^2e + 2\cdots\right.}{\sqrt{-\frac{b^2}{2}\cdots}}}{2c^{3/2}}
$$

```
[In] Integrate[(x^3*Sqrt[d + e*x^2])/(a + b*x^2 + c*x^4),x]
```

```
[Out] (2*Sqrt[c]*Sqrt[d + e*x^2] - (((-I)*b*c*d - c*Sqrt[-b^2 + 4*a*c]*d + I*b^2*
e - (2*I)*a*c*e + b*Sqrt[-b^2 + 4*a*c]*e)*ArcTan[(Sqrt[2]*Sqrt[c]*Sqrt[d +
e*x^2])/Sqrt[-2*c*d + b*e - I*Sqrt[-b^2 + 4*a*c]*e]])/(Sqrt[-1/2*b^2 + 2*a*
c]*Sqrt[-2*c*d + (b - I*Sqrt[-b^2 + 4*a*c])*e]) - ((I*b*c*d - c*Sqrt[-b^2 +
 4*a*c]*d - I*b^2*e + (2*I)*a*c*e + b*Sqrt[-b^2 + 4*a*c]*e)*ArcTan[(Sqrt[2]
*Sqrt[c]*Sqrt[d + e*x^2])/Sqrt[-2*c*d + b*e + I*Sqrt[-b^2 + 4*a*c]*e]])/(Sq
rt[-1/2*b^2 + 2*a*c]*Sqrt[-2*c*d + (b + I*Sqrt[-b^2 + 4*a*c])*e]))/(2*c^(3/
2))
```

Maple [A] (verified)

Time = 0.61 (sec) , antiderivative size = 316, normalized size of antiderivative = 1.08

method	result
risch	$\frac{\sqrt{e x^2+d}}{c}-\frac{\sqrt{2}\left(-\frac{\left(2e^2ac-b^2e^2+bcde+\sqrt{-e^2\left(4ac-b^2\right)}\,be-\sqrt{-e^2\left(4ac-b^2\right)}\,cd\right)\operatorname{arctanh}\left(\frac{c\sqrt{e x^2+d}\,\sqrt{2}}{\sqrt{\left(-be+2cd+\sqrt{-e^2\left(4ac-b^2\right)}\right)c}}\right)}{\sqrt{\left(-be+2cd+\sqrt{-e^2\left(4ac-b^2\right)}\right)c}}+\frac{\left(-2\right.}{}\right.}{2c\sqrt{-e^2(4ac-b^2)}}$
default	$\frac{\sqrt{e x^2+d}-\frac{\left(-2e^2ac+b^2e^2-bcde+\sqrt{-e^2\left(4ac-b^2\right)}\,be-\sqrt{-e^2\left(4ac-b^2\right)}\,cd\right)\sqrt{2}\,\arctan\left(\frac{c\sqrt{e x^2+d}\,\sqrt{2}}{\sqrt{\left(be-2cd+\sqrt{-e^2\left(4ac-b^2\right)}\right)c}}\right)}{2\sqrt{-e^2\left(4ac-b^2\right)}\,\sqrt{\left(be-2cd+\sqrt{-e^2\left(4ac-b^2\right)}\right)c}}+\frac{\left(2e^2ac-b^2e^2\right.}{}}{c}$
pseudoelliptic	$\frac{\sqrt{e x^2+d}-\frac{\left(-2e^2ac+b^2e^2-bcde+\sqrt{-e^2\left(4ac-b^2\right)}\,be-\sqrt{-e^2\left(4ac-b^2\right)}\,cd\right)\sqrt{2}\,\arctan\left(\frac{c\sqrt{e x^2+d}\,\sqrt{2}}{\sqrt{\left(be-2cd+\sqrt{-e^2\left(4ac-b^2\right)}\right)c}}\right)}{2\sqrt{-e^2\left(4ac-b^2\right)}\,\sqrt{\left(be-2cd+\sqrt{-e^2\left(4ac-b^2\right)}\right)c}}+\frac{\left(2e^2ac-b^2e^2\right.}{}}{c}$

```
[In] int(x^3*(e*x^2+d)^(1/2)/(c*x^4+b*x^2+a),x,method=_RETURNVERBOSE)
```

```
[Out] (e*x^2+d)^(1/2)/c-1/2*2^(1/2)/c/(-e^2*(4*a*c-b^2))^(1/2)*(-(2*e^2*a*c-b^2*e
^2+b*c*d*e+(-e^2*(4*a*c-b^2))^(1/2)*b*e-(-e^2*(4*a*c-b^2))^(1/2)*c*d)/((-b*
e+2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2)*arctanh(c*(e*x^2+d)^(1/2)*2^(1/2
)/((-b*e+2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2))+(-2*e^2*a*c+b^2*e^2-b*c*
d*e+(-e^2*(4*a*c-b^2))^(1/2)*b*e-(-e^2*(4*a*c-b^2))^(1/2)*c*d)/((b*e-2*c*d+
(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2)*arctan(c*(e*x^2+d)^(1/2)*2^(1/2)/((b*e-2
*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2435 vs. $2(247) = 494$.

Time = 29.97 (sec) , antiderivative size = 2435, normalized size of antiderivative = 8.34

$$\int \frac{x^3\sqrt{d+ex^2}}{a+bx^2+cx^4}\,dx = \text{Too large to display}$$

```
[In] integrate(x^3*(e*x^2+d)^(1/2)/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] 1/4*(sqrt(1/2)*c*sqrt(((b^2*c - 2*a*c^2)*d - (b^3 - 3*a*b*c)*e + (b^2*c^3 -
 4*a*c^4)*sqrt((b^2*c^2*d^2 - 2*(b^3*c - a*b*c^2)*d*e + (b^4 - 2*a*b^2*c +
a^2*c^2)*e^2)/(b^2*c^6 - 4*a*c^7)))/(b^2*c^3 - 4*a*c^4))*log((2*a*b^2*c*d^2
 - 2*a*b^3*d*e + 2*(a^2*b^2 - a^3*c)*e^2 + (a*b^2*c*d*e - (a*b^3 - a^2*b*c)
*e^2)*x^2 + 2*sqrt(1/2)*sqrt(e*x^2 + d)*((b^4*c - 4*a*b^2*c^2)*d - (b^5 - 5
*a*b^3*c + 4*a^2*b*c^2)*e - (b^4*c^3 - 6*a*b^2*c^4 + 8*a^2*c^5)*sqrt((b^2*c
```

^2*d^2 - 2*(b^3*c - a*b*c^2)*d*e + (b^4 - 2*a*b^2*c + a^2*c^2)*e^2)/(b^2*c^6 - 4*a*c^7)))*sqrt(((b^2*c - 2*a*c^2)*d - (b^3 - 3*a*b*c)*e + (b^2*c^3 - 4*a*c^4)*sqrt((b^2*c^2*d^2 - 2*(b^3*c - a*b*c^2)*d*e + (b^4 - 2*a*b^2*c + a^2*c^2)*e^2)/(b^2*c^6 - 4*a*c^7)))/(b^2*c^3 - 4*a*c^4)) - ((a*b^2*c^3 - 4*a^2*c^4)*e*x^2 + 2*(a*b^2*c^3 - 4*a^2*c^4)*d)*sqrt((b^2*c^2*d^2 - 2*(b^3*c - a*b*c^2)*d*e + (b^4 - 2*a*b^2*c + a^2*c^2)*e^2)/(b^2*c^6 - 4*a*c^7)))/x^2) - sqrt(1/2)*c*sqrt(((b^2*c - 2*a*c^2)*d - (b^3 - 3*a*b*c)*e + (b^2*c^3 - 4*a*c^4)*sqrt((b^2*c^2*d^2 - 2*(b^3*c - a*b*c^2)*d*e + (b^4 - 2*a*b^2*c + a^2*c^2)*e^2)/(b^2*c^6 - 4*a*c^7)))/(b^2*c^3 - 4*a*c^4))*log((2*a*b^2*c*d^2 - 2*a*b^3*d*e + 2*(a^2*b^2 - a^3*c)*e^2 + (a*b^2*c*d*e - (a*b^3 - a^2*b*c)*e^2)*x^2 - 2*sqrt(1/2)*sqrt(e*x^2 + d)*((b^4*c - 4*a*b^2*c^2)*d - (b^5 - 5*a*b^3*c + 4*a^2*b*c^2)*e - (b^4*c^3 - 6*a*b^2*c^4 + 8*a^2*c^5)*sqrt((b^2*c^2*d^2 - 2*(b^3*c - a*b*c^2)*d*e + (b^4 - 2*a*b^2*c + a^2*c^2)*e^2)/(b^2*c^6 - 4*a*c^7)))*sqrt(((b^2*c - 2*a*c^2)*d - (b^3 - 3*a*b*c)*e + (b^2*c^3 - 4*a*c^4)*sqrt((b^2*c^2*d^2 - 2*(b^3*c - a*b*c^2)*d*e + (b^4 - 2*a*b^2*c + a^2*c^2)*e^2)/(b^2*c^6 - 4*a*c^7)))/(b^2*c^3 - 4*a*c^4)) - ((a*b^2*c^3 - 4*a^2*c^4)*e*x^2 + 2*(a*b^2*c^3 - 4*a^2*c^4)*d)*sqrt((b^2*c^2*d^2 - 2*(b^3*c - a*b*c^2)*d*e + (b^4 - 2*a*b^2*c + a^2*c^2)*e^2)/(b^2*c^6 - 4*a*c^7)))/x^2) + sqrt(1/2)*c*sqrt(((b^2*c - 2*a*c^2)*d - (b^3 - 3*a*b*c)*e - (b^2*c^3 - 4*a*c^4)*sqrt((b^2*c^2*d^2 - 2*(b^3*c - a*b*c^2)*d*e + (b^4 - 2*a*b^2*c + a^2*c^2)*e^2)/(b^2*c^6 - 4*a*c^7)))/(b^2*c^3 - 4*a*c^4))*log((2*a*b^2*c*d^2 - 2*a*b^3*d*e + 2*(a^2*b^2 - a^3*c)*e^2 + (a*b^2*c*d*e - (a*b^3 - a^2*b*c)*e^2)*x^2 + 2*sqrt(1/2)*sqrt(e*x^2 + d)*((b^4*c - 4*a*b^2*c^2)*d - (b^5 - 5*a*b^3*c + 4*a^2*b*c^2)*e + (b^4*c^3 - 6*a*b^2*c^4 + 8*a^2*c^5)*sqrt((b^2*c^2*d^2 - 2*(b^3*c - a*b*c^2)*d*e + (b^4 - 2*a*b^2*c + a^2*c^2)*e^2)/(b^2*c^6 - 4*a*c^7)))*sqrt(((b^2*c - 2*a*c^2)*d - (b^3 - 3*a*b*c)*e - (b^2*c^3 - 4*a*c^4)*sqrt((b^2*c^2*d^2 - 2*(b^3*c - a*b*c^2)*d*e + (b^4 - 2*a*b^2*c + a^2*c^2)*e^2)/(b^2*c^6 - 4*a*c^7)))/(b^2*c^3 - 4*a*c^4)) + ((a*b^2*c^3 - 4*a^2*c^4)*e*x^2 + 2*(a*b^2*c^3 - 4*a^2*c^4)*d)*sqrt((b^2*c^2*d^2 - 2*(b^3*c - a*b*c^2)*d*e + (b^4 - 2*a*b^2*c + a^2*c^2)*e^2)/(b^2*c^6 - 4*a*c^7)))/x^2) - sqrt(1/2)*c*sqrt(((b^2*c - 2*a*c^2)*d - (b^3 - 3*a*b*c)*e - (b^2*c^3 - 4*a*c^4)*sqrt((b^2*c^2*d^2 - 2*(b^3*c - a*b*c^2)*d*e + (b^4 - 2*a*b^2*c + a^2*c^2)*e^2)/(b^2*c^6 - 4*a*c^7)))/(b^2*c^3 - 4*a*c^4))*log((2*a*b^2*c*d^2 - 2*a*b^3*d*e + 2*(a^2*b^2 - a^3*c)*e^2 + (a*b^2*c*d*e - (a*b^3 - a^2*b*c)*e^2)*x^2 - 2*sqrt(1/2)*sqrt(e*x^2 + d)*((b^4*c - 4*a*b^2*c^2)*d - (b^5 - 5*a*b^3*c + 4*a^2*b*c^2)*e + (b^4*c^3 - 6*a*b^2*c^4 + 8*a^2*c^5)*sqrt((b^2*c^2*d^2 - 2*(b^3*c - a*b*c^2)*d*e + (b^4 - 2*a*b^2*c + a^2*c^2)*e^2)/(b^2*c^6 - 4*a*c^7)))*sqrt(((b^2*c - 2*a*c^2)*d - (b^3 - 3*a*b*c)*e - (b^2*c^3 - 4*a*c^4)*sqrt((b^2*c^2*d^2 - 2*(b^3*c - a*b*c^2)*d*e + (b^4 - 2*a*b^2*c + a^2*c^2)*e^2)/(b^2*c^6 - 4*a*c^7)))/(b^2*c^3 - 4*a*c^4)) + ((a*b^2*c^3 - 4*a^2*c^4)*e*x^2 + 2*(a*b^2*c^3 - 4*a^2*c^4)*d)*sqrt((b^2*c^2*d^2 - 2*(b^3*c - a*b*c^2)*d*e + (b^4 - 2*a*b^2*c + a^2*c^2)*e^2)/(b^2*c^6 - 4*a*c^7)))/x^2) + 4*sqrt(e*x^2 + d))/c

Sympy [F]

$$\int \frac{x^3\sqrt{d+ex^2}}{a+bx^2+cx^4}\,dx = \int \frac{x^3\sqrt{d+ex^2}}{a+bx^2+cx^4}\,dx$$

```
[In] integrate(x**3*(e*x**2+d)**(1/2)/(c*x**4+b*x**2+a),x)
[Out] Integral(x**3*sqrt(d + e*x**2)/(a + b*x**2 + c*x**4), x)
```

Maxima [F]

$$\int \frac{x^3\sqrt{d+ex^2}}{a+bx^2+cx^4}\,dx = \int \frac{\sqrt{ex^2+d}x^3}{cx^4+bx^2+a}\,dx$$

```
[In] integrate(x^3*(e*x^2+d)^(1/2)/(c*x^4+b*x^2+a),x, algorithm="maxima")
[Out] integrate(sqrt(e*x^2 + d)*x^3/(c*x^4 + b*x^2 + a), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 626 vs. $2(247) = 494$.

Time = 0.33 (sec) , antiderivative size = 626, normalized size of antiderivative = 2.14

$$\int \frac{x^3\sqrt{d+ex^2}}{a+bx^2+cx^4}\,dx = \frac{\sqrt{ex^2+d}}{c}$$

$$-\frac{\left(2\,bc^4d^2e+\left(\left(b^2c-4\,ac^2\right)d-\left(b^3-4\,abc\right)e\right)c^2e^2-\left(3\,b^2c^3-4\,ac^4\right)de^2+\left(b^3c^2-2\,abc^3\right)e^3-2\left(\sqrt{b^2-4\,ac}\right.\right.}{\left(2\sqrt{b^2-4\,ac}c^2d-\left(b^2c-4\,ac^2+\sqrt{b^2-4\,ac}b\right.\right.}$$

$$+\frac{\left(2\,bc^4d^2e+\left(\left(b^2c-4\,ac^2\right)d-\left(b^3-4\,abc\right)e\right)c^2e^2-\left(3\,b^2c^3-4\,ac^4\right)de^2+\left(b^3c^2-2\,abc^3\right)e^3+2\left(\sqrt{b^2-4\,ac}\right.\right.}{\left(2\sqrt{b^2-4\,ac}c^2d+\left(b^2c-4\,ac^2-\sqrt{b^2-4\,ac}b\right.\right.}$$

```
[In] integrate(x^3*(e*x^2+d)^(1/2)/(c*x^4+b*x^2+a),x, algorithm="giac")
[Out] sqrt(e*x^2 + d)/c - (2*b*c^4*d^2*e + ((b^2*c - 4*a*c^2)*d - (b^3 - 4*a*b*c)
*e)*c^2*e^2 - (3*b^2*c^3 - 4*a*c^4)*d*e^2 + (b^3*c^2 - 2*a*b*c^3)*e^3 - 2*(
sqrt(b^2 - 4*a*c)*c^3*d^2 - sqrt(b^2 - 4*a*c)*b*c^2*d*e + sqrt(b^2 - 4*a*c)
```

```
*a*c^2*e^2)*abs(c)*abs(e))*arctan(2*sqrt(1/2)*sqrt(e*x^2 + d)/sqrt(-(2*c^2*
d - b*c*e + sqrt(-4*(c^2*d^2 - b*c*d*e + a*c*e^2)*c^2 + (2*c^2*d - b*c*e)^2
))/c^2))/((2*sqrt(b^2 - 4*a*c)*c^2*d - (b^2*c - 4*a*c^2 + sqrt(b^2 - 4*a*c)
*b*c)*e)*sqrt(-4*c^2*d + 2*(b*c - sqrt(b^2 - 4*a*c)*c)*e)*c^2*abs(e)) + (2*
b*c^4*d^2*e + ((b^2*c - 4*a*c^2)*d - (b^3 - 4*a*b*c)*e)*c^2*e^2 - (3*b^2*c^
3 - 4*a*c^4)*d*e^2 + (b^3*c^2 - 2*a*b*c^3)*e^3 + 2*(sqrt(b^2 - 4*a*c)*c^3*d
^2 - sqrt(b^2 - 4*a*c)*b*c^2*d*e + sqrt(b^2 - 4*a*c)*a*c^2*e^2)*abs(c)*abs(
e))*arctan(2*sqrt(1/2)*sqrt(e*x^2 + d)/sqrt(-(2*c^2*d - b*c*e - sqrt(-4*(c^
2*d^2 - b*c*d*e + a*c*e^2)*c^2 + (2*c^2*d - b*c*e)^2))/c^2))/((2*sqrt(b^2 -
 4*a*c)*c^2*d + (b^2*c - 4*a*c^2 - sqrt(b^2 - 4*a*c)*b*c)*e)*sqrt(-4*c^2*d
+ 2*(b*c + sqrt(b^2 - 4*a*c)*c)*e)*c^2*abs(e))
```

Mupad [B] (verification not implemented)

Time = 9.19 (sec) , antiderivative size = 5705, normalized size of antiderivative = 19.54

$$\int \frac{x^3\sqrt{d+ex^2}}{a+bx^2+cx^4}\,dx = \text{Too large to display}$$

```
[In] int((x^3*(d + e*x^2)^(1/2))/(a + b*x^2 + c*x^4),x)
```

```
[Out] (d + e*x^2)^(1/2)/c - atan(((((16*a^2*c^3*e^4 - 4*a*b^2*c^2*e^4 + 16*a*c^4*
d^2*e^2 + 4*b^3*c^2*d*e^3 - 4*b^2*c^3*d^2*e^2 - 16*a*b*c^3*d*e^3)/c - (2*(d
 + e*x^2)^(1/2)*((8*a^2*c^3*d - b^5*e - b^2*e*(-(4*a*c - b^2)^3)^(1/2) + b^
4*c*d + 7*a*b^3*c*e + a*c*e*(-(4*a*c - b^2)^3)^(1/2) + b*c*d*(-(4*a*c - b^2
)^3)^(1/2) - 6*a*b^2*c^2*d - 12*a^2*b*c^2*e)/(8*(16*a^2*c^5 + b^4*c^3 - 8*a
*b^2*c^4)))^(1/2)*(4*b^3*c^3*e^3 - 8*b^2*c^4*d*e^2 - 16*a*b*c^4*e^3 + 32*a*
c^5*d*e^2))/c)*((8*a^2*c^3*d - b^5*e - b^2*e*(-(4*a*c - b^2)^3)^(1/2) + b^4
*c*d + 7*a*b^3*c*e + a*c*e*(-(4*a*c - b^2)^3)^(1/2) + b*c*d*(-(4*a*c - b^2)
^3)^(1/2) - 6*a*b^2*c^2*d - 12*a^2*b*c^2*e)/(8*(16*a^2*c^5 + b^4*c^3 - 8*a*
b^2*c^4)))^(1/2) - (2*(d + e*x^2)^(1/2)*(b^4*e^4 + 2*a^2*c^2*e^4 - 2*a*c^3*
d^2*e^2 + b^2*c^2*d^2*e^2 - 4*a*b^2*c*e^4 - 2*b^3*c*d*e^3 + 6*a*b*c^2*d*e^3
))/c)*((8*a^2*c^3*d - b^5*e - b^2*e*(-(4*a*c - b^2)^3)^(1/2) + b^4*c*d + 7*
a*b^3*c*e + a*c*e*(-(4*a*c - b^2)^3)^(1/2) + b*c*d*(-(4*a*c - b^2)^3)^(1/2)
 - 6*a*b^2*c^2*d - 12*a^2*b*c^2*e)/(8*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4))
)^(1/2)*1i - (((16*a^2*c^3*e^4 - 4*a*b^2*c^2*e^4 + 16*a*c^4*d^2*e^2 + 4*b^3
*c^2*d*e^3 - 4*b^2*c^3*d^2*e^2 - 16*a*b*c^3*d*e^3)/c + (2*(d + e*x^2)^(1/2)
*((8*a^2*c^3*d - b^5*e - b^2*e*(-(4*a*c - b^2)^3)^(1/2) + b^4*c*d + 7*a*b^3
*c*e + a*c*e*(-(4*a*c - b^2)^3)^(1/2) + b*c*d*(-(4*a*c - b^2)^3)^(1/2) - 6*
a*b^2*c^2*d - 12*a^2*b*c^2*e)/(8*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/
2)*(4*b^3*c^3*e^3 - 8*b^2*c^4*d*e^2 - 16*a*b*c^4*e^3 + 32*a*c^5*d*e^2))/c)*
((8*a^2*c^3*d - b^5*e - b^2*e*(-(4*a*c - b^2)^3)^(1/2) + b^4*c*d + 7*a*b^3*
c*e + a*c*e*(-(4*a*c - b^2)^3)^(1/2) + b*c*d*(-(4*a*c - b^2)^3)^(1/2) - 6*a
*b^2*c^2*d - 12*a^2*b*c^2*e)/(8*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2
) + (2*(d + e*x^2)^(1/2)*(b^4*e^4 + 2*a^2*c^2*e^4 - 2*a*c^3*d^2*e^2 + b^2*c
```

^2*d^2*e^2 - 4*a*b^2*c*e^4 - 2*b^3*c*d*e^3 + 6*a*b*c^2*d*e^3))/c)*((8*a^2*c
^3*d - b^5*e - b^2*e*(-(4*a*c - b^2)^3)^(1/2) + b^4*c*d + 7*a*b^3*c*e + a*c
e(-(4*a*c - b^2)^3)^(1/2) + b*c*d*(-(4*a*c - b^2)^3)^(1/2) - 6*a*b^2*c^2*
d - 12*a^2*b*c^2*e)/(8*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2)*1i)/(((
(16*a^2*c^3*e^4 - 4*a*b^2*c^2*e^4 + 16*a*c^4*d^2*e^2 + 4*b^3*c^2*d*e^3 - 4*
b^2*c^3*d^2*e^2 - 16*a*b*c^3*d*e^3)/c - (2*(d + e*x^2)^(1/2)*((8*a^2*c^3*d
- b^5*e - b^2*e*(-(4*a*c - b^2)^3)^(1/2) + b^4*c*d + 7*a*b^3*c*e + a*c*e*(-
(4*a*c - b^2)^3)^(1/2) + b*c*d*(-(4*a*c - b^2)^3)^(1/2) - 6*a*b^2*c^2*d - 1
2*a^2*b*c^2*e)/(8*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2)*(4*b^3*c^3*e
^3 - 8*b^2*c^4*d*e^2 - 16*a*b*c^4*e^3 + 32*a*c^5*d*e^2))/c)*((8*a^2*c^3*d -
 b^5*e - b^2*e*(-(4*a*c - b^2)^3)^(1/2) + b^4*c*d + 7*a*b^3*c*e + a*c*e*(-(
4*a*c - b^2)^3)^(1/2) + b*c*d*(-(4*a*c - b^2)^3)^(1/2) - 6*a*b^2*c^2*d - 12
*a^2*b*c^2*e)/(8*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2) - (2*(d + e*x
^2)^(1/2)*(b^4*e^4 + 2*a^2*c^2*e^4 - 2*a*c^3*d^2*e^2 + b^2*c^2*d^2*e^2 - 4*
a*b^2*c*e^4 - 2*b^3*c*d*e^3 + 6*a*b*c^2*d*e^3))/c)*((8*a^2*c^3*d - b^5*e -
b^2*e*(-(4*a*c - b^2)^3)^(1/2) + b^4*c*d + 7*a*b^3*c*e + a*c*e*(-(4*a*c - b
^2)^3)^(1/2) + b*c*d*(-(4*a*c - b^2)^3)^(1/2) - 6*a*b^2*c^2*d - 12*a^2*b*c^
2*e)/(8*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2) - (2*(a*c^2*d^3*e^2 -
a^2*b*e^5 + a*b^2*d*e^4 + a^2*c*d*e^4 - 2*a*b*c*d^2*e^3))/c + (((16*a^2*c^3
*e^4 - 4*a*b^2*c^2*e^4 + 16*a*c^4*d^2*e^2 + 4*b^3*c^2*d*e^3 - 4*b^2*c^3*d^2
*e^2 - 16*a*b*c^3*d*e^3)/c + (2*(d + e*x^2)^(1/2)*((8*a^2*c^3*d - b^5*e - b
^2*e*(-(4*a*c - b^2)^3)^(1/2) + b^4*c*d + 7*a*b^3*c*e + a*c*e*(-(4*a*c - b^
2)^3)^(1/2) + b*c*d*(-(4*a*c - b^2)^3)^(1/2) - 6*a*b^2*c^2*d - 12*a^2*b*c^2
e)/(8(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2)*(4*b^3*c^3*e^3 - 8*b^2*
c^4*d*e^2 - 16*a*b*c^4*e^3 + 32*a*c^5*d*e^2))/c)*((8*a^2*c^3*d - b^5*e - b^
2*e*(-(4*a*c - b^2)^3)^(1/2) + b^4*c*d + 7*a*b^3*c*e + a*c*e*(-(4*a*c - b^2
)^3)^(1/2) + b*c*d*(-(4*a*c - b^2)^3)^(1/2) - 6*a*b^2*c^2*d - 12*a^2*b*c^2*
e)/(8*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2) + (2*(d + e*x^2)^(1/2)*(
b^4*e^4 + 2*a^2*c^2*e^4 - 2*a*c^3*d^2*e^2 + b^2*c^2*d^2*e^2 - 4*a*b^2*c*e^4
 - 2*b^3*c*d*e^3 + 6*a*b*c^2*d*e^3))/c)*((8*a^2*c^3*d - b^5*e - b^2*e*(-(4*
a*c - b^2)^3)^(1/2) + b^4*c*d + 7*a*b^3*c*e + a*c*e*(-(4*a*c - b^2)^3)^(1/2
) + b*c*d*(-(4*a*c - b^2)^3)^(1/2) - 6*a*b^2*c^2*d - 12*a^2*b*c^2*e)/(8*(16
*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2)))*((8*a^2*c^3*d - b^5*e - b^2*e*(
-(4*a*c - b^2)^3)^(1/2) + b^4*c*d + 7*a*b^3*c*e + a*c*e*(-(4*a*c - b^2)^3)^
(1/2) + b*c*d*(-(4*a*c - b^2)^3)^(1/2) - 6*a*b^2*c^2*d - 12*a^2*b*c^2*e)/(8
*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2)*2i - atan(((((16*a^2*c^3*e^4
- 4*a*b^2*c^2*e^4 + 16*a*c^4*d^2*e^2 + 4*b^3*c^2*d*e^3 - 4*b^2*c^3*d^2*e^2
- 16*a*b*c^3*d*e^3)/c - (2*(d + e*x^2)^(1/2)*(-(b^5*e - 8*a^2*c^3*d - b^2*e
*(-(4*a*c - b^2)^3)^(1/2) - b^4*c*d - 7*a*b^3*c*e + a*c*e*(-(4*a*c - b^2)^3
)^(1/2) + b*c*d*(-(4*a*c - b^2)^3)^(1/2) + 6*a*b^2*c^2*d + 12*a^2*b*c^2*e)/
(8*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2)*(4*b^3*c^3*e^3 - 8*b^2*c^4*
d*e^2 - 16*a*b*c^4*e^3 + 32*a*c^5*d*e^2))/c)*(-(b^5*e - 8*a^2*c^3*d - b^2*e
*(-(4*a*c - b^2)^3)^(1/2) - b^4*c*d - 7*a*b^3*c*e + a*c*e*(-(4*a*c - b^2)^3
)^(1/2) + b*c*d*(-(4*a*c - b^2)^3)^(1/2) + 6*a*b^2*c^2*d + 12*a^2*b*c^2*e)/
(8*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2) - (2*(d + e*x^2)^(1/2)*(b^4

*e^4 + 2*a^2*c^2*e^4 - 2*a*c^3*d^2*e^2 + b^2*c^2*d^2*e^2 - 4*a*b^2*c*e^4 - 2*b^3*c*d*e^3 + 6*a*b*c^2*d*e^3))/c)*(-(b^5*e - 8*a^2*c^3*d - b^2*e*(-(4*a*c - b^2)^3)^(1/2) - b^4*c*d - 7*a*b^3*c*e + a*c*e*(-(4*a*c - b^2)^3)^(1/2) + b*c*d*(-(4*a*c - b^2)^3)^(1/2) + 6*a*b^2*c^2*d + 12*a^2*b*c^2*e)/(8*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2)*1i - (((16*a^2*c^3*e^4 - 4*a*b^2*c^2*e^4 + 16*a*c^4*d^2*e^2 + 4*b^3*c^2*d*e^3 - 4*b^2*c^3*d^2*e^2 - 16*a*b*c^3*d*e^3)/c + (2*(d + e*x^2)^(1/2)*(-(b^5*e - 8*a^2*c^3*d - b^2*e*(-(4*a*c - b^2)^3)^(1/2) - b^4*c*d - 7*a*b^3*c*e + a*c*e*(-(4*a*c - b^2)^3)^(1/2) + b*c*d*(-(4*a*c - b^2)^3)^(1/2) + 6*a*b^2*c^2*d + 12*a^2*b*c^2*e)/(8*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2)*(4*b^3*c^3*e^3 - 8*b^2*c^4*d*e^2 - 16*a*b*c^4*e^3 + 32*a*c^5*d*e^2))/c)*(-(b^5*e - 8*a^2*c^3*d - b^2*e*(-(4*a*c - b^2)^3)^(1/2) - b^4*c*d - 7*a*b^3*c*e + a*c*e*(-(4*a*c - b^2)^3)^(1/2) + b*c*d*(-(4*a*c - b^2)^3)^(1/2) + 6*a*b^2*c^2*d + 12*a^2*b*c^2*e)/(8*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2) + (2*(d + e*x^2)^(1/2)*(b^4*e^4 + 2*a^2*c^2*e^4 - 2*a*c^3*d^2*e^2 + b^2*c^2*d^2*e^2 - 4*a*b^2*c*e^4 - 2*b^3*c*d*e^3 + 6*a*b*c^2*d*e^3))/c)*(-(b^5*e - 8*a^2*c^3*d - b^2*e*(-(4*a*c - b^2)^3)^(1/2) - b^4*c*d - 7*a*b^3*c*e + a*c*e*(-(4*a*c - b^2)^3)^(1/2) + b*c*d*(-(4*a*c - b^2)^3)^(1/2) + 6*a*b^2*c^2*d + 12*a^2*b*c^2*e)/(8*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2)*1i)/((((16*a^2*c^3*e^4 - 4*a*b^2*c^2*e^4 + 16*a*c^4*d^2*e^2 + 4*b^3*c^2*d*e^3 - 4*b^2*c^3*d^2*e^2 - 16*a*b*c^3*d*e^3)/c - (2*(d + e*x^2)^(1/2)*(-(b^5*e - 8*a^2*c^3*d - b^2*e*(-(4*a*c - b^2)^3)^(1/2) - b^4*c*d - 7*a*b^3*c*e + a*c*e*(-(4*a*c - b^2)^3)^(1/2) + b*c*d*(-(4*a*c - b^2)^3)^(1/2) + 6*a*b^2*c^2*d + 12*a^2*b*c^2*e)/(8*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2)*(4*b^3*c^3*e^3 - 8*b^2*c^4*d*e^2 - 16*a*b*c^4*e^3 + 32*a*c^5*d*e^2))/c)*(-(b^5*e - 8*a^2*c^3*d - b^2*e*(-(4*a*c - b^2)^3)^(1/2) - b^4*c*d - 7*a*b^3*c*e + a*c*e*(-(4*a*c - b^2)^3)^(1/2) + b*c*d*(-(4*a*c - b^2)^3)^(1/2) + 6*a*b^2*c^2*d + 12*a^2*b*c^2*e)/(8*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2) - (2*(d + e*x^2)^(1/2)*(b^4*e^4 + 2*a^2*c^2*e^4 - 2*a*c^3*d^2*e^2 + b^2*c^2*d^2*e^2 - 4*a*b^2*c*e^4 - 2*b^3*c*d*e^3 + 6*a*b*c^2*d*e^3))/c)*(-(b^5*e - 8*a^2*c^3*d - b^2*e*(-(4*a*c - b^2)^3)^(1/2) - b^4*c*d - 7*a*b^3*c*e + a*c*e*(-(4*a*c - b^2)^3)^(1/2) + b*c*d*(-(4*a*c - b^2)^3)^(1/2) + 6*a*b^2*c^2*d + 12*a^2*b*c^2*e)/(8*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2) - (2*(a*c^2*d^3*e^2 - a^2*b*e^5 + a*b^2*d*e^4 + a^2*c*d*e^4 - 2*a*b*c*d^2*e^3))/c + (((16*a^2*c^3*e^4 - 4*a*b^2*c^2*e^4 + 16*a*c^4*d^2*e^2 + 4*b^3*c^2*d*e^3 - 4*b^2*c^3*d^2*e^2 - 16*a*b*c^3*d*e^3)/c + (2*(d + e*x^2)^(1/2)*(-(b^5*e - 8*a^2*c^3*d - b^2*e*(-(4*a*c - b^2)^3)^(1/2) - b^4*c*d - 7*a*b^3*c*e + a*c*e*(-(4*a*c - b^2)^3)^(1/2) + b*c*d*(-(4*a*c - b^2)^3)^(1/2) + 6*a*b^2*c^2*d + 12*a^2*b*c^2*e)/(8*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2)*(4*b^3*c^3*e^3 - 8*b^2*c^4*d*e^2 - 16*a*b*c^4*e^3 + 32*a*c^5*d*e^2))/c)*(-(b^5*e - 8*a^2*c^3*d - b^2*e*(-(4*a*c - b^2)^3)^(1/2) - b^4*c*d - 7*a*b^3*c*e + a*c*e*(-(4*a*c - b^2)^3)^(1/2) + b*c*d*(-(4*a*c - b^2)^3)^(1/2) + 6*a*b^2*c^2*d + 12*a^2*b*c^2*e)/(8*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2) + (2*(d + e*x^2)^(1/2)*(b^4*e^4 + 2*a^2*c^2*e^4 - 2*a*c^3*d^2*e^2 + b^2*c^2*d^2*e^2 - 4*a*b^2*c*e^4 - 2*b^3*c*d*e^3 + 6*a*b*c^2*d*e^3))/c)*(-(b^5*e - 8*a^2*c^3*d - b^2*e*(-(4*a*c - b^2)^3)^(1/2) - b^4*c*d - 7*

```
a*b^3*c*e + a*c*e*(-(4*a*c - b^2)^3)^(1/2) + b*c*d*(-(4*a*c - b^2)^3)^(1/2)
 + 6*a*b^2*c^2*d + 12*a^2*b*c^2*e)/(8*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4))
)^(1/2)))*(-(b^5*e - 8*a^2*c^3*d - b^2*e*(-(4*a*c - b^2)^3)^(1/2) - b^4*c*d
 - 7*a*b^3*c*e + a*c*e*(-(4*a*c - b^2)^3)^(1/2) + b*c*d*(-(4*a*c - b^2)^3)^
(1/2) + 6*a*b^2*c^2*d + 12*a^2*b*c^2*e)/(8*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*
c^4)))^(1/2)*2i
```

3.357 $\int \frac{x\sqrt{d+ex^2}}{a+bx^2+cx^4}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 202

$$\int \frac{x\sqrt{d+ex^2}}{a+bx^2+cx^4}\,dx = -\frac{\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}\,\text{arctanh}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex^2}}{\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b^2-4ac}} + \frac{\sqrt{2cd-\left(b+\sqrt{b^2-4ac}\right)e}\,\text{arctanh}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex^2}}{\sqrt{2cd-\left(b+\sqrt{b^2-4ac}\right)e}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b^2-4ac}}$$

```
[Out] -1/2*arctanh(2^(1/2)*c^(1/2)*(e*x^2+d)^(1/2)/(2*c*d-e*(b-(-4*a*c+b^2)^(1/2)
))^(1/2))*(2*c*d-e*(b-(-4*a*c+b^2)^(1/2)))^(1/2)*2^(1/2)/c^(1/2)/(-4*a*c+b^
2)^(1/2)+1/2*arctanh(2^(1/2)*c^(1/2)*(e*x^2+d)^(1/2)/(2*c*d-e*(b+(-4*a*c+b^
2)^(1/2)))^(1/2))*(2*c*d-e*(b+(-4*a*c+b^2)^(1/2)))^(1/2)*2^(1/2)/c^(1/2)/(-
4*a*c+b^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 202, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used

$= \{1261, 713, 1144, 214\}$

$$\int \frac{x\sqrt{d+ex^2}}{a+bx^2+cx^4}\,dx = \frac{\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex^2}}{\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b^2-4ac}} - \frac{\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex^2}}{\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b^2-4ac}}$$

```
[In] Int[(x*Sqrt[d + e*x^2])/(a + b*x^2 + c*x^4),x]
```

```
[Out] -((Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]*ArcTanh[(Sqrt[2]*Sqrt[c]*Sqrt[d
+ e*x^2])/Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]])/(Sqrt[2]*Sqrt[c]*Sqrt[b
^2 - 4*a*c])) + (Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]*ArcTanh[(Sqrt[2]*S
qrt[c]*Sqrt[d + e*x^2])/Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]])/(Sqrt[2]*
Sqrt[c]*Sqrt[b^2 - 4*a*c])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 713

```
Int[Sqrt[(d_.) + (e_.)*(x_)]/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol]
 :> Dist[2*e, Subst[Int[x^2/(c*d^2 - b*d*e + a*e^2 - (2*c*d - b*e)*x^2 + c*
x^4), x], x, Sqrt[d + e*x]], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4
*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 1144

```
Int[((d_.)*(x_))^(m_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> Wi
th[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(d^2/2)*(b/q + 1), Int[(d*x)^(m - 2)/(b/2
 + q/2 + c*x^2), x], x] - Dist[(d^2/2)*(b/q - 1), Int[(d*x)^(m - 2)/(b/2 -
q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - 4*a*c, 0] && G
eQ[m, 2]
```

Rule 1261

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(
p_.), x_Symbol] :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x],
 x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]
```

Rubi steps

$$\text{integral} = \frac{1}{2}\text{Subst}\left(\int \frac{\sqrt{d+ex}}{a+bx+cx^2}\,dx, x, x^2\right)$$
$$= e\text{Subst}\left(\int \frac{x^2}{cd^2-bde+ae^2+(-2cd+be)x^2+cx^4}\,dx, x, \sqrt{d+ex^2}\right)$$
$$= -\left(\frac{1}{2}\left(-e - \frac{2cd-be}{\sqrt{b^2-4ac}}\right)\text{Subst}\left(\int \frac{1}{-\frac{1}{2}\sqrt{b^2-4ac}e+\frac{1}{2}(-2cd+be)+cx^2}\,dx, x, \sqrt{d+ex^2}\right)\right) + \frac{1}{2}\left(e - \frac{2cd-be}{\sqrt{b^2-4ac}}\right)\text{Subst}\left(\int \frac{1}{\frac{1}{2}\sqrt{b^2-4ac}e+\frac{1}{2}(-2cd+be)+cx^2}\,dx, x, \sqrt{d+ex^2}\right)$$
$$= -\frac{\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex^2}}{\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b^2-4ac}} + \frac{\sqrt{2cd-\left(b+\sqrt{b^2-4ac}\right)e}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex^2}}{\sqrt{2cd-\left(b+\sqrt{b^2-4ac}\right)e}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b^2-4ac}}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.59 (sec) , antiderivative size = 256, normalized size of antiderivative = 1.27

$$\int \frac{x\sqrt{d+ex^2}}{a+bx^2+cx^4}\,dx = \frac{\frac{\left(-2icd+\left(ib+\sqrt{-b^2+4ac}\right)e\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex^2}}{\sqrt{-2cd+be-i\sqrt{-b^2+4ac}e}}\right)}{\sqrt{-2cd+\left(b-i\sqrt{-b^2+4ac}\right)e}} + \frac{\left(2icd+\left(-ib+\sqrt{-b^2+4ac}\right)e\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex^2}}{\sqrt{-2cd+be+i\sqrt{-b^2+4ac}e}}\right)}{\sqrt{-2cd+\left(b+i\sqrt{-b^2+4ac}\right)e}}}{\sqrt{2}\sqrt{c}\sqrt{-b^2+4ac}}$$

```
[In] Integrate[(x*Sqrt[d + e*x^2])/(a + b*x^2 + c*x^4),x]
[Out] ((((-2*I)*c*d + (I*b + Sqrt[-b^2 + 4*a*c])*e)*ArcTan[(Sqrt[2]*Sqrt[c]*Sqrt[
d + e*x^2])/Sqrt[-2*c*d + b*e - I*Sqrt[-b^2 + 4*a*c]*e]])/Sqrt[-2*c*d + (b
```

```
- I*Sqrt[-b^2 + 4*a*c])*e] + (((2*I)*c*d + ((-I)*b + Sqrt[-b^2 + 4*a*c])*e)
*ArcTan[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x^2])/Sqrt[-2*c*d + b*e + I*Sqrt[-b^2 +
 4*a*c]*e]])/Sqrt[-2*c*d + (b + I*Sqrt[-b^2 + 4*a*c])*e])/(Sqrt[2]*Sqrt[c]*
Sqrt[-b^2 + 4*a*c])
```

Maple [A] (verified)

Time = 0.39 (sec) , antiderivative size = 227, normalized size of antiderivative = 1.12

method	result
default	$\frac{e\sqrt{2}\left(\frac{\left(be-2cd+\sqrt{-e^{2}\left(4ac-b^{2}\right)}\right)\arctan\left(\frac{c\sqrt{e\,x^{2}+d}\,\sqrt{2}}{\sqrt{\left(be-2cd+\sqrt{-e^{2}\left(4ac-b^{2}\right)}\right)c}}\right)}{\sqrt{\left(be-2cd+\sqrt{-e^{2}\left(4ac-b^{2}\right)}\right)c}}-\frac{\left(-be+2cd+\sqrt{-e^{2}\left(4ac-b^{2}\right)}\right)\operatorname{arctanh}\left(\frac{c\sqrt{e\,x^{2}\cdots}}{\sqrt{\left(-be+2cd+\sqrt{-\cdots}\right.}}\right.}{\sqrt{\left(-be+2cd+\sqrt{-e^{2}\left(4ac-b^{2}\right)}\right)c}}\right.}{2\sqrt{-e^{2}\left(4ac-b^{2}\right)}}$
pseudoelliptic	$\frac{e\sqrt{2}\left(\frac{\left(be-2cd+\sqrt{-e^{2}\left(4ac-b^{2}\right)}\right)\arctan\left(\frac{c\sqrt{e\,x^{2}+d}\,\sqrt{2}}{\sqrt{\left(be-2cd+\sqrt{-e^{2}\left(4ac-b^{2}\right)}\right)c}}\right)}{\sqrt{\left(be-2cd+\sqrt{-e^{2}\left(4ac-b^{2}\right)}\right)c}}-\frac{\left(-be+2cd+\sqrt{-e^{2}\left(4ac-b^{2}\right)}\right)\operatorname{arctanh}\left(\frac{c\sqrt{e\,x^{2}\cdots}}{\sqrt{\left(-be+2cd+\sqrt{-\cdots}\right.}}\right.}{\sqrt{\left(-be+2cd+\sqrt{-e^{2}\left(4ac-b^{2}\right)}\right)c}}\right.}{2\sqrt{-e^{2}\left(4ac-b^{2}\right)}}$

```
[In] int(x*(e*x^2+d)^(1/2)/(c*x^4+b*x^2+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*e*2^(1/2)/(-e^2*(4*a*c-b^2))^(1/2)*((b*e-2*c*d+(-e^2*(4*a*c-b^2))^(1/2)
)/((b*e-2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2)*arctan(c*(e*x^2+d)^(1/2)*2
^(1/2)/((b*e-2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2))-(-b*e+2*c*d+(-e^2*(4
*a*c-b^2))^(1/2))/((-b*e+2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2)*arctanh(c
*(e*x^2+d)^(1/2)*2^(1/2)/((-b*e+2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1085 vs. $2(163) = 326$.

Time = 6.08 (sec) , antiderivative size = 1085, normalized size of antiderivative = 5.37

$$\int \frac{x\sqrt{d+ex^2}}{a+bx^2+cx^4}\,dx =$$

$$-\frac{1}{4}\sqrt{\frac{1}{2}}\sqrt{\frac{2\,cd-be+(b^2c-4\,ac^2)\sqrt{\frac{e^2}{b^2c^2-4\,ac^3}}}{b^2c-4\,ac^2}}\log\left(\frac{be^2x^2+2\,bde-2\,ae^2+2\,\sqrt{\frac{1}{2}}\sqrt{ex^2+d}\Big((b^2-4\,ac)e\right.$$

$$+\frac{1}{4}\sqrt{\frac{1}{2}}\sqrt{\frac{2\,cd-be+(b^2c-4\,ac^2)\sqrt{\frac{e^2}{b^2c^2-4\,ac^3}}}{b^2c-4\,ac^2}}\log\left(\frac{be^2x^2+2\,bde-2\,ae^2-2\,\sqrt{\frac{1}{2}}\sqrt{ex^2+d}\Big((b^2-4\,ac)e\right.$$

$$-\frac{1}{4}\sqrt{\frac{1}{2}}\sqrt{\frac{2\,cd-be-(b^2c-4\,ac^2)\sqrt{\frac{e^2}{b^2c^2-4\,ac^3}}}{b^2c-4\,ac^2}}\log\left(\frac{be^2x^2+2\,bde-2\,ae^2+2\,\sqrt{\frac{1}{2}}\sqrt{ex^2+d}\Big((b^2-4\,ac)e\right.$$

$$+\frac{1}{4}\sqrt{\frac{1}{2}}\sqrt{\frac{2\,cd-be-(b^2c-4\,ac^2)\sqrt{\frac{e^2}{b^2c^2-4\,ac^3}}}{b^2c-4\,ac^2}}\log\left(\frac{be^2x^2+2\,bde-2\,ae^2-2\,\sqrt{\frac{1}{2}}\sqrt{ex^2+d}\Big((b^2-4\,ac)e\right.$$

```
[In] integrate(x*(e*x^2+d)^(1/2)/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] -1/4*sqrt(1/2)*sqrt((2*c*d - b*e + (b^2*c - 4*a*c^2)*sqrt(e^2/(b^2*c^2 - 4*
a*c^3)))/(b^2*c - 4*a*c^2))*log((b*e^2*x^2 + 2*b*d*e - 2*a*e^2 + 2*sqrt(1/2
)*sqrt(e*x^2 + d)*((b^2 - 4*a*c)*e + (b^3*c - 4*a*b*c^2)*sqrt(e^2/(b^2*c^2
- 4*a*c^3)))*sqrt((2*c*d - b*e + (b^2*c - 4*a*c^2)*sqrt(e^2/(b^2*c^2 - 4*a*
c^3)))/(b^2*c - 4*a*c^2)) + ((b^2*c - 4*a*c^2)*e*x^2 + 2*(b^2*c - 4*a*c^2)*
d)*sqrt(e^2/(b^2*c^2 - 4*a*c^3)))/x^2) + 1/4*sqrt(1/2)*sqrt((2*c*d - b*e +
(b^2*c - 4*a*c^2)*sqrt(e^2/(b^2*c^2 - 4*a*c^3)))/(b^2*c - 4*a*c^2))*log((b*
e^2*x^2 + 2*b*d*e - 2*a*e^2 - 2*sqrt(1/2)*sqrt(e*x^2 + d)*((b^2 - 4*a*c)*e
+ (b^3*c - 4*a*b*c^2)*sqrt(e^2/(b^2*c^2 - 4*a*c^3)))*sqrt((2*c*d - b*e + (b
^2*c - 4*a*c^2)*sqrt(e^2/(b^2*c^2 - 4*a*c^3)))/(b^2*c - 4*a*c^2)) + ((b^2*c
 - 4*a*c^2)*e*x^2 + 2*(b^2*c - 4*a*c^2)*d)*sqrt(e^2/(b^2*c^2 - 4*a*c^3)))/x
^2) - 1/4*sqrt(1/2)*sqrt((2*c*d - b*e - (b^2*c - 4*a*c^2)*sqrt(e^2/(b^2*c^2
 - 4*a*c^3)))/(b^2*c - 4*a*c^2))*log((b*e^2*x^2 + 2*b*d*e - 2*a*e^2 + 2*sqr
t(1/2)*sqrt(e*x^2 + d)*((b^2 - 4*a*c)*e - (b^3*c - 4*a*b*c^2)*sqrt(e^2/(b^2
```

```
*c^2 - 4*a*c^3)))*sqrt((2*c*d - b*e - (b^2*c - 4*a*c^2)*sqrt(e^2/(b^2*c^2 -
 4*a*c^3)))/(b^2*c - 4*a*c^2)) - ((b^2*c - 4*a*c^2)*e*x^2 + 2*(b^2*c - 4*a*
c^2)*d)*sqrt(e^2/(b^2*c^2 - 4*a*c^3)))/x^2) + 1/4*sqrt(1/2)*sqrt((2*c*d - b
*e - (b^2*c - 4*a*c^2)*sqrt(e^2/(b^2*c^2 - 4*a*c^3)))/(b^2*c - 4*a*c^2))*lo
g((b*e^2*x^2 + 2*b*d*e - 2*a*e^2 - 2*sqrt(1/2)*sqrt(e*x^2 + d)*((b^2 - 4*a*
c)*e - (b^3*c - 4*a*b*c^2)*sqrt(e^2/(b^2*c^2 - 4*a*c^3)))*sqrt((2*c*d - b*e
 - (b^2*c - 4*a*c^2)*sqrt(e^2/(b^2*c^2 - 4*a*c^3)))/(b^2*c - 4*a*c^2)) - ((
b^2*c - 4*a*c^2)*e*x^2 + 2*(b^2*c - 4*a*c^2)*d)*sqrt(e^2/(b^2*c^2 - 4*a*c^3
)))/x^2)
```

Sympy [F]

$$\int \frac{x\sqrt{d+ex^2}}{a+bx^2+cx^4}\,dx=\int \frac{x\sqrt{d+ex^2}}{a+bx^2+cx^4}\,dx$$

```
[In] integrate(x*(e*x**2+d)**(1/2)/(c*x**4+b*x**2+a),x)
[Out] Integral(x*sqrt(d + e*x**2)/(a + b*x**2 + c*x**4), x)
```

Maxima [F]

$$\int \frac{x\sqrt{d+ex^2}}{a+bx^2+cx^4}\,dx=\int \frac{\sqrt{ex^2+d}x}{cx^4+bx^2+a}\,dx$$

```
[In] integrate(x*(e*x^2+d)^(1/2)/(c*x^4+b*x^2+a),x, algorithm="maxima")
[Out] integrate(sqrt(e*x^2 + d)*x/(c*x^4 + b*x^2 + a), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 449 vs. $2(163) = 326$.

Time = 0.31 (sec) , antiderivative size = 449, normalized size of antiderivative = 2.22

$$\int \frac{x\sqrt{d+ex^2}}{a+bx^2+cx^4}\,dx$$

$$=\frac{\left(\sqrt{-4\,c^2d+2\,\left(bc-\sqrt{b^2-4\,ac}c\right)e}(b^2-4\,ac)e^3-(4\,c^2d^2e-4\,bcde^2+b^2e^3)\sqrt{-4\,c^2d+2\,\left(bc-\sqrt{b^2-4\,ac}\right.}\right.}{8\left(\sqrt{b^2-4\,ac}c^2d^2-\sqrt{b^2-4\,ac}bcde+\sqrt{b^2-4\,ac}ace^2\right)|c||e}$$

$$-\frac{\left(\sqrt{-4\,c^2d+2\,\left(bc+\sqrt{b^2-4\,ac}c\right)e}(b^2-4\,ac)e^3-(4\,c^2d^2e-4\,bcde^2+b^2e^3)\sqrt{-4\,c^2d+2\,\left(bc+\sqrt{b^2-4\right.}\right.}}{8\left(\sqrt{b^2-4\,ac}c^2d^2-\sqrt{b^2-4\,ac}bcde+\sqrt{b^2-4\,ac}ace^2\right)|c}$$

```
[In] integrate(x*(e*x^2+d)^(1/2)/(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] 1/8*(sqrt(-4*c^2*d + 2*(b*c - sqrt(b^2 - 4*a*c)*c)*e)*(b^2 - 4*a*c)*e^3 - (
4*c^2*d^2*e - 4*b*c*d*e^2 + b^2*e^3)*sqrt(-4*c^2*d + 2*(b*c - sqrt(b^2 - 4*
a*c)*c)*e))*arctan(2*sqrt(1/2)*sqrt(e*x^2 + d)/sqrt(-(2*c*d - b*e + sqrt((2
*c*d - b*e)^2 - 4*(c*d^2 - b*d*e + a*e^2)*c))/c))/((sqrt(b^2 - 4*a*c)*c^2*d
^2 - sqrt(b^2 - 4*a*c)*b*c*d*e + sqrt(b^2 - 4*a*c)*a*c*e^2)*abs(c)*abs(e))
- 1/8*(sqrt(-4*c^2*d + 2*(b*c + sqrt(b^2 - 4*a*c)*c)*e)*(b^2 - 4*a*c)*e^3 -
 (4*c^2*d^2*e - 4*b*c*d*e^2 + b^2*e^3)*sqrt(-4*c^2*d + 2*(b*c + sqrt(b^2 -
4*a*c)*c)*e))*arctan(2*sqrt(1/2)*sqrt(e*x^2 + d)/sqrt(-(2*c*d - b*e - sqrt(
(2*c*d - b*e)^2 - 4*(c*d^2 - b*d*e + a*e^2)*c))/c))/((sqrt(b^2 - 4*a*c)*c^2
*d^2 - sqrt(b^2 - 4*a*c)*b*c*d*e + sqrt(b^2 - 4*a*c)*a*c*e^2)*abs(c)*abs(e)
)
```

Mupad [B] (verification not implemented)

Time = 8.60 (sec) , antiderivative size = 717, normalized size of antiderivative = 3.55

$$\int \frac{x\sqrt{d+ex^2}}{a+bx^2+cx^4}\,dx = -2\,\mathrm{atanh}\left(\frac{2\left(\sqrt{e\,x^2+d}\,(-2\,b^2\,c\,e^4+4\,b\,c^2\,d\,e^3-4\,c^3\,d^2\,e^2+4\,a\,c^2\,e^4)+\frac{\sqrt{e\,x^2+d}\,(8\,b^3\,c^2\,e^3-16\,d\,b^2\,c^3\,e^2-32\,a\,b\,c^3}{}\right.}{2\,c^2\,d^2\,e^3-2\,b}\right. -2\,\mathrm{atanh}\left(\frac{2\left(\sqrt{e\,x^2+d}\,(-2\,b^2\,c\,e^4+4\,b\,c^2\,d\,e^3-4\,c^3\,d^2\,e^2+4\,a\,c^2\,e^4)-\frac{\sqrt{e\,x^2+d}\,(8\,b^3\,c^2\,e^3-16\,d\,b^2\,c^3\,e^2-32\,a\,b\,c^3}{}\right.}{2\,c^2\,d^2\,e^3-2\,b\,c}\right.$$

```
[In] int((x*(d + e*x^2)^(1/2))/(a + b*x^2 + c*x^4),x)
```

```
[Out] - 2*atanh((2*((d + e*x^2)^(1/2)*(4*a*c^2*e^4 - 2*b^2*c*e^4 - 4*c^3*d^2*e^2
+ 4*b*c^2*d*e^3) + ((d + e*x^2)^(1/2)*(8*b^3*c^2*e^3 - 16*b^2*c^3*d*e^2 - 3
2*a*b*c^3*e^3 + 64*a*c^4*d*e^2)*(b^3*e + e*(-(4*a*c - b^2)^3)^(1/2) + 8*a*c
^2*d - 2*b^2*c*d - 4*a*b*c*e))/(8*(b^4*c + 16*a^2*c^3 - 8*a*b^2*c^2)))*(-(b
^3*e + e*(-(4*a*c - b^2)^3)^(1/2) + 8*a*c^2*d - 2*b^2*c*d - 4*a*b*c*e)/(8*(
b^4*c + 16*a^2*c^3 - 8*a*b^2*c^2)))^(1/2))/(2*c^2*d^2*e^3 + 2*a*c*e^5 - 2*b
*c*d*e^4))*(-(b^3*e + e*(-(4*a*c - b^2)^3)^(1/2) + 8*a*c^2*d - 2*b^2*c*d -
4*a*b*c*e)/(8*(b^4*c + 16*a^2*c^3 - 8*a*b^2*c^2)))^(1/2) - 2*atanh((2*((d +
 e*x^2)^(1/2)*(4*a*c^2*e^4 - 2*b^2*c*e^4 - 4*c^3*d^2*e^2 + 4*b*c^2*d*e^3) -
 ((d + e*x^2)^(1/2)*(8*b^3*c^2*e^3 - 16*b^2*c^3*d*e^2 - 32*a*b*c^3*e^3 + 64
*a*c^4*d*e^2)*(e*(-(4*a*c - b^2)^3)^(1/2) - b^3*e - 8*a*c^2*d + 2*b^2*c*d +
```

```
 4*a*b*c*e))/(8*(b^4*c + 16*a^2*c^3 - 8*a*b^2*c^2)))*((e*(-(4*a*c - b^2)^3)
^(1/2) - b^3*e - 8*a*c^2*d + 2*b^2*c*d + 4*a*b*c*e)/(8*(b^4*c + 16*a^2*c^3
- 8*a*b^2*c^2)))^(1/2))/(2*c^2*d^2*e^3 + 2*a*c*e^5 - 2*b*c*d*e^4))*((e*(-(4
*a*c - b^2)^3)^(1/2) - b^3*e - 8*a*c^2*d + 2*b^2*c*d + 4*a*b*c*e)/(8*(b^4*c
 + 16*a^2*c^3 - 8*a*b^2*c^2)))^(1/2)
```

3.358 $\int \frac{\sqrt{d+ex^2}}{x(a+bx^2+cx^4)}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 281

$$\int \frac{\sqrt{d+ex^2}}{x\left(a+bx^2+cx^4\right)}\,dx = -\frac{\sqrt{d}\operatorname{arctanh}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)}{a} + \frac{\sqrt{c}\left(bd+\sqrt{b^2-4ac}d-2ae\right)\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex^2}}{\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}}\right)}{\sqrt{2}a\sqrt{b^2-4ac}\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}} - \frac{\sqrt{c}\left(bd-\sqrt{b^2-4ac}d-2ae\right)\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex^2}}{\sqrt{2cd-\left(b+\sqrt{b^2-4ac}\right)e}}\right)}{\sqrt{2}a\sqrt{b^2-4ac}\sqrt{2cd-\left(b+\sqrt{b^2-4ac}\right)e}}$$

```
[Out] -arctanh((e*x^2+d)^(1/2)/d^(1/2))*d^(1/2)/a+1/2*arctanh(2^(1/2)*c^(1/2)*(e*
x^2+d)^(1/2)/(2*c*d-e*(b-(-4*a*c+b^2)^(1/2)))^(1/2))*c^(1/2)*(b*d-2*a*e+d*(
-4*a*c+b^2)^(1/2))/a*2^(1/2)/(-4*a*c+b^2)^(1/2)/(2*c*d-e*(b-(-4*a*c+b^2)^(1
/2)))^(1/2)-1/2*arctanh(2^(1/2)*c^(1/2)*(e*x^2+d)^(1/2)/(2*c*d-e*(b+(-4*a*c
+b^2)^(1/2)))^(1/2))*c^(1/2)*(b*d-2*a*e-d*(-4*a*c+b^2)^(1/2))/a*2^(1/2)/(-4
*a*c+b^2)^(1/2)/(2*c*d-e*(b+(-4*a*c+b^2)^(1/2)))^(1/2)
```

Rubi [A] (verified)

Time = 0.80 (sec) , antiderivative size = 281, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.207$, Rules used = {1265, 911, 1301, 212, 1180, 214}

$$\int \frac{\sqrt{d+ex^2}}{x\left(a+bx^2+cx^4\right)}\,dx = \frac{\sqrt{c}\left(d\sqrt{b^2-4ac}-2ae+bd\right)\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex^2}}{\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}\right)}{\sqrt{2}a\sqrt{b^2-4ac}\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}} - \frac{\sqrt{c}\left(-d\sqrt{b^2-4ac}-2ae+bd\right)\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex^2}}{\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}\right)}{\sqrt{2}a\sqrt{b^2-4ac}\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}} - \frac{\sqrt{d}\operatorname{arctanh}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)}{a}$$

```
[In] Int[Sqrt[d + e*x^2]/(x*(a + b*x^2 + c*x^4)),x]
```

```
[Out] -((Sqrt[d]*ArcTanh[Sqrt[d + e*x^2]/Sqrt[d]])/a) + (Sqrt[c]*(b*d + Sqrt[b^2
- 4*a*c]*d - 2*a*e)*ArcTanh[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x^2])/Sqrt[2*c*d -
(b - Sqrt[b^2 - 4*a*c])*e]])/(Sqrt[2]*a*Sqrt[b^2 - 4*a*c]*Sqrt[2*c*d - (b -
 Sqrt[b^2 - 4*a*c])*e]) - (Sqrt[c]*(b*d - Sqrt[b^2 - 4*a*c]*d - 2*a*e)*ArcT
anh[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x^2])/Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*
e]])/(Sqrt[2]*a*Sqrt[b^2 - 4*a*c]*Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 911

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> With[{q = Denominator[m]}, Dist[q/e, S
ubst[Int[x^(q*(m + 1) - 1)*((e*f - d*g)/e + g*(x^q/e))^n*((c*d^2 - b*d*e +
a*e^2)/e^2 - (2*c*d - b*e)*(x^q/e^2) + c*(x^(2*q)/e^2))^p, x], x, (d + e*x)
^(1/q)], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegersQ[n, p] && Fra
```

```
ctionQ[m]
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1265

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rule 1301

```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.))/((a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*((d + e*x^2)^q/(a
+ b*x^2 + c*x^4)), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b^2 - 4
*a*c, 0] && IntegerQ[q] && IntegerQ[m]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{1}{2}\text{Subst}\left(\int \frac{\sqrt{d+ex}}{x\left(a+bx+cx^2\right)}\,dx, x, x^2\right)\\
&= \frac{\text{Subst}\left(\int \frac{x^2}{\left(-\frac{d}{e}+\frac{x^2}{e}\right)\left(\frac{cd^2-bde+ae^2}{e^2}-\frac{(2cd-be)x^2}{e^2}+\frac{cx^4}{e^2}\right)}\,dx, x, \sqrt{d+ex^2}\right)}{e}\\
&= \frac{\text{Subst}\left(\int \left(-\frac{de}{a(d-x^2)}+\frac{e(cd^2-bde+ae^2-cdx^2)}{a(cd^2-bde+ae^2-(2cd-be)x^2+cx^4)}\right)\,dx, x, \sqrt{d+ex^2}\right)}{e}\\
&= \frac{\text{Subst}\left(\int \frac{cd^2-bde+ae^2-cdx^2}{cd^2-bde+ae^2+(-2cd+be)x^2+cx^4}\,dx, x, \sqrt{d+ex^2}\right)}{a} - \frac{d\text{Subst}\left(\int \frac{1}{d-x^2}\,dx, x, \sqrt{d+ex^2}\right)}{a}\\
&= -\frac{\sqrt{d}\tanh^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)}{a}\\
&\quad + \frac{\left(c\left(bd-\sqrt{b^2-4ac}\,d-2ae\right)\right)\text{Subst}\left(\int \frac{1}{\frac{1}{2}\sqrt{b^2-4ac}\,e+\frac{1}{2}(-2cd+be)+cx^2}\,dx, x, \sqrt{d+ex^2}\right)}{2a\sqrt{b^2-4ac}}\\
&\quad - \frac{\left(c\left(bd+\sqrt{b^2-4ac}\,d-2ae\right)\right)\text{Subst}\left(\int \frac{1}{-\frac{1}{2}\sqrt{b^2-4ac}\,e+\frac{1}{2}(-2cd+be)+cx^2}\,dx, x, \sqrt{d+ex^2}\right)}{2a\sqrt{b^2-4ac}}
\end{aligned}
$$

$$=-\frac{\sqrt{d}\tanh^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)}{a}+\frac{\sqrt{c}\left(bd+\sqrt{b^2-4ac}d-2ae\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex^2}}{\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}}\right)}{\sqrt{2}a\sqrt{b^2-4ac}\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}}$$
$$-\frac{\sqrt{c}\left(bd-\sqrt{b^2-4ac}d-2ae\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex^2}}{\sqrt{2cd-\left(b+\sqrt{b^2-4ac}\right)e}}\right)}{\sqrt{2}a\sqrt{b^2-4ac}\sqrt{2cd-\left(b+\sqrt{b^2-4ac}\right)e}}$$

Mathematica [A] (verified)

Time = 0.64 (sec) , antiderivative size = 274, normalized size of antiderivative = 0.98

$$\int\frac{\sqrt{d+ex^2}}{x\left(a+bx^2+cx^4\right)}\,dx=$$
$$-\frac{\frac{\sqrt{2}\sqrt{c}\left(bd+\sqrt{b^2-4ac}d-2ae\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex^2}}{\sqrt{-2cd+be-\sqrt{b^2-4ac}e}}\right)}{\sqrt{b^2-4ac}\sqrt{-2cd+\left(b-\sqrt{b^2-4ac}\right)e}}+\frac{\sqrt{2}\sqrt{c}\left(-bd+\sqrt{b^2-4ac}d+2ae\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex^2}}{\sqrt{-2cd+\left(b+\sqrt{b^2-4ac}\right)e}}\right)}{\sqrt{b^2-4ac}\sqrt{-2cd+\left(b+\sqrt{b^2-4ac}\right)e}}+2\sqrt{d}\text{a}}{2a}$$

```
[In] Integrate[Sqrt[d + e*x^2]/(x*(a + b*x^2 + c*x^4)),x]
```

```
[Out] -1/2*((Sqrt[2]*Sqrt[c]*(b*d + Sqrt[b^2 - 4*a*c]*d - 2*a*e)*ArcTan[(Sqrt[2]*
Sqrt[c]*Sqrt[d + e*x^2])/Sqrt[-2*c*d + b*e - Sqrt[b^2 - 4*a*c]*e]])/(Sqrt[b
^2 - 4*a*c]*Sqrt[-2*c*d + (b - Sqrt[b^2 - 4*a*c])*e]) + (Sqrt[2]*Sqrt[c]*(-
(b*d) + Sqrt[b^2 - 4*a*c]*d + 2*a*e)*ArcTan[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x^2
])/Sqrt[-2*c*d + (b + Sqrt[b^2 - 4*a*c])*e]])/(Sqrt[b^2 - 4*a*c]*Sqrt[-2*c*
d + (b + Sqrt[b^2 - 4*a*c])*e]) + 2*Sqrt[d]*ArcTanh[Sqrt[d + e*x^2]/Sqrt[d]
])/a
```

Maple [A] (verified)

Time = 0.46 (sec) , antiderivative size = 358, normalized size of antiderivative = 1.27

method	result
pseudoelliptic	$-\frac{\sqrt{2}\sqrt{\left(be-2cd+\sqrt{-4e^2\left(ac-\frac{b^2}{4}\right)}\right)c}\,c\left(a e^2-\frac{bde}{2}-\frac{\sqrt{-4e^2\left(ac-\frac{b^2}{4}\right)}\,d}{2}\right)\operatorname{arctanh}\left(\frac{c\sqrt{e x^2+d}\,\sqrt{2}}{\sqrt{\left(-be+2cd+\sqrt{-4e^2\left(ac-\frac{b^2}{4}\right)}\right)c}}\right)+\sqrt{(\cdots}}{\sqrt{\left(-b\cdots}}$
default	$\frac{\sqrt{e x^2+d}-\sqrt{d}\,\ln\left(\frac{2d+2\sqrt{d}\,\sqrt{e x^2+d}}{x}\right)}{a}-\frac{\sqrt{2}\sqrt{\left(be-2cd+\sqrt{-4e^2\left(ac-\frac{b^2}{4}\right)}\right)c}\,c\left(a e^2-\frac{bde}{2}-\frac{\sqrt{-4e^2\left(ac-\frac{b^2}{4}\right)}\,d}{2}\right)\operatorname{arctanh}\left(\cdots\right.}{\cdots}$

```
[In] int((e*x^2+d)^(1/2)/x/(c*x^4+b*x^2+a),x,method=_RETURNVERBOSE)
```

```
[Out] -(2^(1/2)*((b*e-2*c*d+(-4*e^2*(a*c-1/4*b^2))^(1/2))*c)^(1/2)*c*(a*e^2-1/2*b
*d*e-1/2*(-4*e^2*(a*c-1/4*b^2))^(1/2)*d)*arctanh(c*(e*x^2+d)^(1/2)*2^(1/2)/
((-b*e+2*c*d+(-4*e^2*(a*c-1/4*b^2))^(1/2))*c)^(1/2))+((-b*e+2*c*d+(-4*e^2*(
a*c-1/4*b^2))^(1/2))*c)^(1/2)*(2^(1/2)*c*(1/2*(-4*e^2*(a*c-1/4*b^2))^(1/2)*
d+e*(a*e-1/2*b*d))*arctan(c*(e*x^2+d)^(1/2)*2^(1/2)/((b*e-2*c*d+(-4*e^2*(a*
c-1/4*b^2))^(1/2))*c)^(1/2))+d^(1/2)*arctanh((e*x^2+d)^(1/2)/d^(1/2))*(-4*e
^2*(a*c-1/4*b^2))^(1/2)*((b*e-2*c*d+(-4*e^2*(a*c-1/4*b^2))^(1/2))*c)^(1/2))
)/((-b*e+2*c*d+(-4*e^2*(a*c-1/4*b^2))^(1/2))*c)^(1/2)/((b*e-2*c*d+(-4*e^2*(
a*c-1/4*b^2))^(1/2))*c)^(1/2)/(-4*e^2*(a*c-1/4*b^2))^(1/2)/a
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1557 vs. $2(232) = 464$.

Time = 49.74 (sec) , antiderivative size = 3126, normalized size of antiderivative = 11.12

$$\int \frac{\sqrt{d+ex^2}}{x\left(a+bx^2+cx^4\right)}\,dx = \text{Too large to display}$$

```
[In] integrate((e*x^2+d)^(1/2)/x/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] [-1/4*(sqrt(1/2)*a*sqrt(-(a*b*e - (b^2 - 2*a*c)*d + (a^2*b^2 - 4*a^3*c)*sqr
t((b^2*d^2 - 2*a*b*d*e + a^2*e^2)/(a^4*b^2 - 4*a^5*c)))/(a^2*b^2 - 4*a^3*c)
)*log(-(2*b^2*d^2 - 4*a*b*d*e + 2*a^2*e^2 + (b^2*d*e - a*b*e^2)*x^2 + 4*sqr
t(1/2)*(a^3*b^2 - 4*a^4*c)*sqrt(e*x^2 + d)*sqrt((b^2*d^2 - 2*a*b*d*e + a^2*
e^2)/(a^4*b^2 - 4*a^5*c))*sqrt(-(a*b*e - (b^2 - 2*a*c)*d + (a^2*b^2 - 4*a^3
*c)*sqrt((b^2*d^2 - 2*a*b*d*e + a^2*e^2)/(a^4*b^2 - 4*a^5*c)))/(a^2*b^2 - 4
*a^3*c)) - ((a^2*b^2 - 4*a^3*c)*e*x^2 + 2*(a^2*b^2 - 4*a^3*c)*d)*sqrt((b^2*
```

d^2 - 2*a*b*d*e + a^2*e^2)/(a^4*b^2 - 4*a^5*c)))/x^2) - sqrt(1/2)*a*sqrt(-(
a*b*e - (b^2 - 2*a*c)*d + (a^2*b^2 - 4*a^3*c)*sqrt((b^2*d^2 - 2*a*b*d*e + a
^2*e^2)/(a^4*b^2 - 4*a^5*c)))/(a^2*b^2 - 4*a^3*c))*log(-(2*b^2*d^2 - 4*a*b*
d*e + 2*a^2*e^2 + (b^2*d*e - a*b*e^2)*x^2 - 4*sqrt(1/2)*(a^3*b^2 - 4*a^4*c)
*sqrt(e*x^2 + d)*sqrt((b^2*d^2 - 2*a*b*d*e + a^2*e^2)/(a^4*b^2 - 4*a^5*c))*
sqrt(-(a*b*e - (b^2 - 2*a*c)*d + (a^2*b^2 - 4*a^3*c)*sqrt((b^2*d^2 - 2*a*b*
d*e + a^2*e^2)/(a^4*b^2 - 4*a^5*c)))/(a^2*b^2 - 4*a^3*c)) - ((a^2*b^2 - 4*a
^3*c)*e*x^2 + 2*(a^2*b^2 - 4*a^3*c)*d)*sqrt((b^2*d^2 - 2*a*b*d*e + a^2*e^2)
/(a^4*b^2 - 4*a^5*c)))/x^2) - sqrt(1/2)*a*sqrt(-(a*b*e - (b^2 - 2*a*c)*d -
(a^2*b^2 - 4*a^3*c)*sqrt((b^2*d^2 - 2*a*b*d*e + a^2*e^2)/(a^4*b^2 - 4*a^5*c
)))/(a^2*b^2 - 4*a^3*c))*log(-(2*b^2*d^2 - 4*a*b*d*e + 2*a^2*e^2 + (b^2*d*e
 - a*b*e^2)*x^2 + 4*sqrt(1/2)*(a^3*b^2 - 4*a^4*c)*sqrt(e*x^2 + d)*sqrt((b^2
*d^2 - 2*a*b*d*e + a^2*e^2)/(a^4*b^2 - 4*a^5*c))*sqrt(-(a*b*e - (b^2 - 2*a*
c)*d - (a^2*b^2 - 4*a^3*c)*sqrt((b^2*d^2 - 2*a*b*d*e + a^2*e^2)/(a^4*b^2 -
4*a^5*c)))/(a^2*b^2 - 4*a^3*c)) + ((a^2*b^2 - 4*a^3*c)*e*x^2 + 2*(a^2*b^2 -
 4*a^3*c)*d)*sqrt((b^2*d^2 - 2*a*b*d*e + a^2*e^2)/(a^4*b^2 - 4*a^5*c)))/x^2
) + sqrt(1/2)*a*sqrt(-(a*b*e - (b^2 - 2*a*c)*d - (a^2*b^2 - 4*a^3*c)*sqrt((
b^2*d^2 - 2*a*b*d*e + a^2*e^2)/(a^4*b^2 - 4*a^5*c)))/(a^2*b^2 - 4*a^3*c))*l
og(-(2*b^2*d^2 - 4*a*b*d*e + 2*a^2*e^2 + (b^2*d*e - a*b*e^2)*x^2 - 4*sqrt(1
/2)*(a^3*b^2 - 4*a^4*c)*sqrt(e*x^2 + d)*sqrt((b^2*d^2 - 2*a*b*d*e + a^2*e^2
)/(a^4*b^2 - 4*a^5*c))*sqrt(-(a*b*e - (b^2 - 2*a*c)*d - (a^2*b^2 - 4*a^3*c)
*sqrt((b^2*d^2 - 2*a*b*d*e + a^2*e^2)/(a^4*b^2 - 4*a^5*c)))/(a^2*b^2 - 4*a^
3*c)) + ((a^2*b^2 - 4*a^3*c)*e*x^2 + 2*(a^2*b^2 - 4*a^3*c)*d)*sqrt((b^2*d^2
 - 2*a*b*d*e + a^2*e^2)/(a^4*b^2 - 4*a^5*c)))/x^2) - 2*sqrt(d)*log(-(e*x^2
- 2*sqrt(e*x^2 + d)*sqrt(d) + 2*d)/x^2))/a, -1/4*(sqrt(1/2)*a*sqrt(-(a*b*e
- (b^2 - 2*a*c)*d + (a^2*b^2 - 4*a^3*c)*sqrt((b^2*d^2 - 2*a*b*d*e + a^2*e^2
)/(a^4*b^2 - 4*a^5*c)))/(a^2*b^2 - 4*a^3*c))*log(-(2*b^2*d^2 - 4*a*b*d*e +
2*a^2*e^2 + (b^2*d*e - a*b*e^2)*x^2 + 4*sqrt(1/2)*(a^3*b^2 - 4*a^4*c)*sqrt(
e*x^2 + d)*sqrt((b^2*d^2 - 2*a*b*d*e + a^2*e^2)/(a^4*b^2 - 4*a^5*c))*sqrt(-
(a*b*e - (b^2 - 2*a*c)*d + (a^2*b^2 - 4*a^3*c)*sqrt((b^2*d^2 - 2*a*b*d*e +
a^2*e^2)/(a^4*b^2 - 4*a^5*c)))/(a^2*b^2 - 4*a^3*c)) - ((a^2*b^2 - 4*a^3*c)*
e*x^2 + 2*(a^2*b^2 - 4*a^3*c)*d)*sqrt((b^2*d^2 - 2*a*b*d*e + a^2*e^2)/(a^4*
b^2 - 4*a^5*c)))/x^2) - sqrt(1/2)*a*sqrt(-(a*b*e - (b^2 - 2*a*c)*d + (a^2*b
^2 - 4*a^3*c)*sqrt((b^2*d^2 - 2*a*b*d*e + a^2*e^2)/(a^4*b^2 - 4*a^5*c)))/(a
^2*b^2 - 4*a^3*c))*log(-(2*b^2*d^2 - 4*a*b*d*e + 2*a^2*e^2 + (b^2*d*e - a*b
*e^2)*x^2 - 4*sqrt(1/2)*(a^3*b^2 - 4*a^4*c)*sqrt(e*x^2 + d)*sqrt((b^2*d^2 -
 2*a*b*d*e + a^2*e^2)/(a^4*b^2 - 4*a^5*c))*sqrt(-(a*b*e - (b^2 - 2*a*c)*d +
 (a^2*b^2 - 4*a^3*c)*sqrt((b^2*d^2 - 2*a*b*d*e + a^2*e^2)/(a^4*b^2 - 4*a^5*
c)))/(a^2*b^2 - 4*a^3*c)) - ((a^2*b^2 - 4*a^3*c)*e*x^2 + 2*(a^2*b^2 - 4*a^3
*c)*d)*sqrt((b^2*d^2 - 2*a*b*d*e + a^2*e^2)/(a^4*b^2 - 4*a^5*c)))/x^2) - sq
rt(1/2)*a*sqrt(-(a*b*e - (b^2 - 2*a*c)*d - (a^2*b^2 - 4*a^3*c)*sqrt((b^2*d^
2 - 2*a*b*d*e + a^2*e^2)/(a^4*b^2 - 4*a^5*c)))/(a^2*b^2 - 4*a^3*c))*log(-(2
*b^2*d^2 - 4*a*b*d*e + 2*a^2*e^2 + (b^2*d*e - a*b*e^2)*x^2 + 4*sqrt(1/2)*(a
^3*b^2 - 4*a^4*c)*sqrt(e*x^2 + d)*sqrt((b^2*d^2 - 2*a*b*d*e + a^2*e^2)/(a^4
*b^2 - 4*a^5*c))*sqrt(-(a*b*e - (b^2 - 2*a*c)*d - (a^2*b^2 - 4*a^3*c)*sqrt(

```
(b^2*d^2 - 2*a*b*d*e + a^2*e^2)/(a^4*b^2 - 4*a^5*c)))/(a^2*b^2 - 4*a^3*c))
+ ((a^2*b^2 - 4*a^3*c)*e*x^2 + 2*(a^2*b^2 - 4*a^3*c)*d)*sqrt((b^2*d^2 - 2*a
*b*d*e + a^2*e^2)/(a^4*b^2 - 4*a^5*c)))/x^2) + sqrt(1/2)*a*sqrt(-(a*b*e - (
b^2 - 2*a*c)*d - (a^2*b^2 - 4*a^3*c)*sqrt((b^2*d^2 - 2*a*b*d*e + a^2*e^2)/(
a^4*b^2 - 4*a^5*c)))/(a^2*b^2 - 4*a^3*c))*log(-(2*b^2*d^2 - 4*a*b*d*e + 2*a
^2*e^2 + (b^2*d*e - a*b*e^2)*x^2 - 4*sqrt(1/2)*(a^3*b^2 - 4*a^4*c)*sqrt(e*x
^2 + d)*sqrt((b^2*d^2 - 2*a*b*d*e + a^2*e^2)/(a^4*b^2 - 4*a^5*c))*sqrt(-(a*
b*e - (b^2 - 2*a*c)*d - (a^2*b^2 - 4*a^3*c)*sqrt((b^2*d^2 - 2*a*b*d*e + a^2
*e^2)/(a^4*b^2 - 4*a^5*c)))/(a^2*b^2 - 4*a^3*c)) + ((a^2*b^2 - 4*a^3*c)*e*x
^2 + 2*(a^2*b^2 - 4*a^3*c)*d)*sqrt((b^2*d^2 - 2*a*b*d*e + a^2*e^2)/(a^4*b^2
 - 4*a^5*c)))/x^2) - 4*sqrt(-d)*arctan(sqrt(-d)/sqrt(e*x^2 + d)))/a]
```

Sympy [F]

$$\int \frac{\sqrt{d+ex^2}}{x\left(a+bx^2+cx^4\right)}\,dx=\int \frac{\sqrt{d+ex^2}}{x\left(a+bx^2+cx^4\right)}\,dx$$

```
[In] integrate((e*x**2+d)**(1/2)/x/(c*x**4+b*x**2+a),x)
[Out] Integral(sqrt(d + e*x**2)/(x*(a + b*x**2 + c*x**4)), x)
```

Maxima [F]

$$\int \frac{\sqrt{d+ex^2}}{x\left(a+bx^2+cx^4\right)}\,dx=\int \frac{\sqrt{ex^2+d}}{(cx^4+bx^2+a)x}\,dx$$

```
[In] integrate((e*x^2+d)^(1/2)/x/(c*x^4+b*x^2+a),x, algorithm="maxima")
[Out] integrate(sqrt(e*x^2 + d)/((c*x^4 + b*x^2 + a)*x), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 724 vs. $2(232) = 464$.

Time = 0.31 (sec) , antiderivative size = 724, normalized size of antiderivative = 2.58

$$\int \frac{\sqrt{d+ex^2}}{x\,(a+bx^2+cx^4)}\,dx = \frac{d\arctan\left(\frac{\sqrt{ex^2+d}}{\sqrt{-d}}\right)}{a\sqrt{-d}}$$

$$-\frac{\left(\sqrt{-4\,c^2d+2\,\left(bc-\sqrt{b^2-4\,ac}c\right)e}(b^2-4\,ac)a^2de^2-2\,\left(\sqrt{b^2-4\,ac}acd^2-\sqrt{b^2-4\,ac}abde+\sqrt{b^2-4\,ac}\right.\right.}{}$$

$$+\frac{\left(\sqrt{-4\,c^2d+2\,\left(bc+\sqrt{b^2-4\,ac}c\right)e}(b^2-4\,ac)a^2de^2+2\,\left(\sqrt{b^2-4\,ac}acd^2-\sqrt{b^2-4\,ac}abde+\sqrt{b^2-4\,ac}\right.\right.}{}$$

```
[In] integrate((e*x^2+d)^(1/2)/x/(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] d*arctan(sqrt(e*x^2 + d)/sqrt(-d))/(a*sqrt(-d)) - 1/8*(sqrt(-4*c^2*d + 2*(b
*c - sqrt(b^2 - 4*a*c)*c)*e)*(b^2 - 4*a*c)*a^2*d*e^2 - 2*(sqrt(b^2 - 4*a*c)
*a*c*d^2 - sqrt(b^2 - 4*a*c)*a*b*d*e + sqrt(b^2 - 4*a*c)*a^2*e^2)*sqrt(-4*c
^2*d + 2*(b*c - sqrt(b^2 - 4*a*c)*c)*e)*abs(a)*abs(e) - (2*a^2*b*c*d^2*e +
2*a^3*b*e^3 - (a^2*b^2 + 4*a^3*c)*d*e^2)*sqrt(-4*c^2*d + 2*(b*c - sqrt(b^2
- 4*a*c)*c)*e))*arctan(2*sqrt(1/2)*sqrt(e*x^2 + d)/sqrt(-(2*a*c*d - a*b*e +
 sqrt(-4*(a*c*d^2 - a*b*d*e + a^2*e^2)*a*c + (2*a*c*d - a*b*e)^2))/(a*c)))/
((sqrt(b^2 - 4*a*c)*a^2*c*d^2 - sqrt(b^2 - 4*a*c)*a^2*b*d*e + sqrt(b^2 - 4*
a*c)*a^3*e^2)*abs(a)*abs(c)*abs(e)) + 1/8*(sqrt(-4*c^2*d + 2*(b*c + sqrt(b^
2 - 4*a*c)*c)*e)*(b^2 - 4*a*c)*a^2*d*e^2 + 2*(sqrt(b^2 - 4*a*c)*a*c*d^2 - s
qrt(b^2 - 4*a*c)*a*b*d*e + sqrt(b^2 - 4*a*c)*a^2*e^2)*sqrt(-4*c^2*d + 2*(b*
c + sqrt(b^2 - 4*a*c)*c)*e)*abs(a)*abs(e) - (2*a^2*b*c*d^2*e + 2*a^3*b*e^3
- (a^2*b^2 + 4*a^3*c)*d*e^2)*sqrt(-4*c^2*d + 2*(b*c + sqrt(b^2 - 4*a*c)*c)*
e))*arctan(2*sqrt(1/2)*sqrt(e*x^2 + d)/sqrt(-(2*a*c*d - a*b*e - sqrt(-4*(a*
c*d^2 - a*b*d*e + a^2*e^2)*a*c + (2*a*c*d - a*b*e)^2))/(a*c)))/((sqrt(b^2 -
 4*a*c)*a^2*c*d^2 - sqrt(b^2 - 4*a*c)*a^2*b*d*e + sqrt(b^2 - 4*a*c)*a^3*e^2
)*abs(a)*abs(c)*abs(e))
```

Mupad [B] (verification not implemented)

Time = 11.49 (sec) , antiderivative size = 10964, normalized size of antiderivative = 39.02

$$\int \frac{\sqrt{d+ex^2}}{x\,(a+bx^2+cx^4)}\,dx = \text{Too large to display}$$

```
[In] int((d + e*x^2)^(1/2)/(x*(a + b*x^2 + c*x^4)),x)
```

[Out] atan((((d + e*x^2)^(1/2)*(2*a^2*c^3*e^12 + 6*c^5*d^4*e^8 - 8*b*c^4*d^3*e^9 + 4*b^2*c^3*d^2*e^10 - 4*a*b*c^3*d*e^11) + ((b^4*d + 8*a^2*c^2*d - a*b^3*e + a*e*(-(4*a*c - b^2)^3)^(1/2) - b*d*(-(4*a*c - b^2)^3)^(1/2) - 6*a*b^2*c*d + 4*a^2*b*c*e)/(8*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)))^(1/2)*((((b^4*d + 8*a^2*c^2*d - a*b^3*e + a*e*(-(4*a*c - b^2)^3)^(1/2) - b*d*(-(4*a*c - b^2)^3)^(1/2) - 6*a*b^2*c*d + 4*a^2*b*c*e)/(8*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)))^(1/2)*((d + e*x^2)^(1/2)*((b^4*d + 8*a^2*c^2*d - a*b^3*e + a*e*(-(4*a*c - b^2)^3)^(1/2) - b*d*(-(4*a*c - b^2)^3)^(1/2) - 6*a*b^2*c*d + 4*a^2*b*c*e)/(8*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)))^(1/2)*(512*a^5*c^4*e^10 + 32*a^3*b^4*c^2*e^10 - 256*a^4*b^2*c^3*e^10 + 768*a^4*c^5*d^2*e^8 + 64*a^2*b^4*c^3*d^2*e^8 - 448*a^3*b^2*c^4*d^2*e^8 - 896*a^4*b*c^4*d*e^9 - 64*a^2*b^5*c^2*d*e^9 + 480*a^3*b^3*c^3*d*e^9) - 192*a^4*c^4*d*e^10 - 192*a^3*c^5*d^3*e^8 + 48*a^2*b^2*c^4*d^3*e^8 - 48*a^2*b^3*c^3*d^2*e^9 + 192*a^3*b*c^4*d^2*e^9 + 48*a^3*b^2*c^3*d*e^10) - (d + e*x^2)^(1/2)*(32*a^3*b*c^3*e^11 + 48*a^3*c^4*d*e^10 - 8*a^2*b^3*c^2*e^11 + 144*a^2*c^5*d^3*e^8 + 16*b^4*c^3*d^3*e^8 - 16*b^5*c^2*d^2*e^9 + 16*a*b^4*c^2*d*e^10 - 96*a*b^2*c^4*d^3*e^8 + 96*a*b^3*c^3*d^2*e^9 - 144*a^2*b*c^4*d^2*e^9 - 72*a^2*b^2*c^3*d*e^10))*((b^4*d + 8*a^2*c^2*d - a*b^3*e + a*e*(-(4*a*c - b^2)^3)^(1/2) - b*d*(-(4*a*c - b^2)^3)^(1/2) - 6*a*b^2*c*d + 4*a^2*b*c*e)/(8*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)))^(1/2) + 12*a*c^5*d^4*e^8 + 12*a^2*c^4*d^2*e^10 - 4*b^2*c^4*d^4*e^8 + 4*b^4*c^2*d^2*e^10 + 8*a*b*c^4*d^3*e^9 - 4*a*b^3*c^2*d*e^11 + 20*a^2*b*c^3*d*e^11 - 24*a*b^2*c^3*d^2*e^10))*((b^4*d + 8*a^2*c^2*d - a*b^3*e + a*e*(-(4*a*c - b^2)^3)^(1/2) - b*d*(-(4*a*c - b^2)^3)^(1/2) - 6*a*b^2*c*d + 4*a^2*b*c*e)/(8*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)))^(1/2)*1i + ((d + e*x^2)^(1/2)*(2*a^2*c^3*e^12 + 6*c^5*d^4*e^8 - 8*b*c^4*d^3*e^9 + 4*b^2*c^3*d^2*e^10 - 4*a*b*c^3*d*e^11) - ((b^4*d + 8*a^2*c^2*d - a*b^3*e + a*e*(-(4*a*c - b^2)^3)^(1/2) - b*d*(-(4*a*c - b^2)^3)^(1/2) - 6*a*b^2*c*d + 4*a^2*b*c*e)/(8*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)))^(1/2)*(12*a*c^5*d^4*e^8 - (((b^4*d + 8*a^2*c^2*d - a*b^3*e + a*e*(-(4*a*c - b^2)^3)^(1/2) - b*d*(-(4*a*c - b^2)^3)^(1/2) - 6*a*b^2*c*d + 4*a^2*b*c*e)/(8*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)))^(1/2)*((d + e*x^2)^(1/2)*((b^4*d + 8*a^2*c^2*d - a*b^3*e + a*e*(-(4*a*c - b^2)^3)^(1/2) - b*d*(-(4*a*c - b^2)^3)^(1/2) - 6*a*b^2*c*d + 4*a^2*b*c*e)/(8*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)))^(1/2)*(512*a^5*c^4*e^10 + 32*a^3*b^4*c^2*e^10 - 256*a^4*b^2*c^3*e^10 + 768*a^4*c^5*d^2*e^8 + 64*a^2*b^4*c^3*d^2*e^8 - 448*a^3*b^2*c^4*d^2*e^8 - 896*a^4*b*c^4*d*e^9 - 64*a^2*b^5*c^2*d*e^9 + 480*a^3*b^3*c^3*d*e^9) + 192*a^4*c^4*d*e^10 + 192*a^3*c^5*d^3*e^8 - 48*a^2*b^2*c^4*d^3*e^8 + 48*a^2*b^3*c^3*d^2*e^9 - 192*a^3*b*c^4*d^2*e^9 - 48*a^3*b^2*c^3*d*e^10) - (d + e*x^2)^(1/2)*(32*a^3*b*c^3*e^11 + 48*a^3*c^4*d*e^10 - 8*a^2*b^3*c^2*e^11 + 144*a^2*c^5*d^3*e^8 + 16*b^4*c^3*d^3*e^8 - 16*b^5*c^2*d^2*e^9 + 16*a*b^4*c^2*d*e^10 - 96*a*b^2*c^4*d^3*e^8 + 96*a*b^3*c^3*d^2*e^9 - 144*a^2*b*c^4*d^2*e^9 - 72*a^2*b^2*c^3*d*e^10))*((b^4*d + 8*a^2*c^2*d - a*b^3*e + a*e*(-(4*a*c - b^2)^3)^(1/2) - b*d*(-(4*a*c - b^2)^3)^(1/2) - 6*a*b^2*c*d + 4*a^2*b*c*e)/(8*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)))^(1/2) + 12*a^2*c^4*d^2*e^10 - 4*b^2*c^4*d^4*e^8 + 4*b^4*c^2*d^2*e^10 + 8*a*b*c^4*d^3*e^9 - 4*a*b^3*c^2*d*e^11 + 20*a^2*b*c^3*d*e^11 - 24*a*b^2*c^3*d^2*e^10

))*((b^4*d + 8*a^2*c^2*d - a*b^3*e + a*e*(-(4*a*c - b^2)^3)^(1/2) - b*d*(-(
4*a*c - b^2)^3)^(1/2) - 6*a*b^2*c*d + 4*a^2*b*c*e)/(8*(a^2*b^4 + 16*a^4*c^2
 - 8*a^3*b^2*c)))^(1/2)*1i)/(((d + e*x^2)^(1/2)*(2*a^2*c^3*e^12 + 6*c^5*d^4
*e^8 - 8*b*c^4*d^3*e^9 + 4*b^2*c^3*d^2*e^10 - 4*a*b*c^3*d*e^11) - ((b^4*d +
 8*a^2*c^2*d - a*b^3*e + a*e*(-(4*a*c - b^2)^3)^(1/2) - b*d*(-(4*a*c - b^2)
^3)^(1/2) - 6*a*b^2*c*d + 4*a^2*b*c*e)/(8*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2
c)))^(1/2)(12*a*c^5*d^4*e^8 - (((b^4*d + 8*a^2*c^2*d - a*b^3*e + a*e*(-(4
*a*c - b^2)^3)^(1/2) - b*d*(-(4*a*c - b^2)^3)^(1/2) - 6*a*b^2*c*d + 4*a^2*b
*c*e)/(8*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)))^(1/2)*((d + e*x^2)^(1/2)*((
b^4*d + 8*a^2*c^2*d - a*b^3*e + a*e*(-(4*a*c - b^2)^3)^(1/2) - b*d*(-(4*a*c
 - b^2)^3)^(1/2) - 6*a*b^2*c*d + 4*a^2*b*c*e)/(8*(a^2*b^4 + 16*a^4*c^2 - 8*
a^3*b^2*c)))^(1/2)*(512*a^5*c^4*e^10 + 32*a^3*b^4*c^2*e^10 - 256*a^4*b^2*c^
3*e^10 + 768*a^4*c^5*d^2*e^8 + 64*a^2*b^4*c^3*d^2*e^8 - 448*a^3*b^2*c^4*d^2
*e^8 - 896*a^4*b*c^4*d*e^9 - 64*a^2*b^5*c^2*d*e^9 + 480*a^3*b^3*c^3*d*e^9)
+ 192*a^4*c^4*d*e^10 + 192*a^3*c^5*d^3*e^8 - 48*a^2*b^2*c^4*d^3*e^8 + 48*a^
2*b^3*c^3*d^2*e^9 - 192*a^3*b*c^4*d^2*e^9 - 48*a^3*b^2*c^3*d*e^10) - (d + e
x^2)^(1/2)(32*a^3*b*c^3*e^11 + 48*a^3*c^4*d*e^10 - 8*a^2*b^3*c^2*e^11 + 1
44*a^2*c^5*d^3*e^8 + 16*b^4*c^3*d^3*e^8 - 16*b^5*c^2*d^2*e^9 + 16*a*b^4*c^2
*d*e^10 - 96*a*b^2*c^4*d^3*e^8 + 96*a*b^3*c^3*d^2*e^9 - 144*a^2*b*c^4*d^2*e
^9 - 72*a^2*b^2*c^3*d*e^10))*((b^4*d + 8*a^2*c^2*d - a*b^3*e + a*e*(-(4*a*c
 - b^2)^3)^(1/2) - b*d*(-(4*a*c - b^2)^3)^(1/2) - 6*a*b^2*c*d + 4*a^2*b*c*e
)/(8*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)))^(1/2) + 12*a^2*c^4*d^2*e^10 - 4
*b^2*c^4*d^4*e^8 + 4*b^4*c^2*d^2*e^10 + 8*a*b*c^4*d^3*e^9 - 4*a*b^3*c^2*d*e
^11 + 20*a^2*b*c^3*d*e^11 - 24*a*b^2*c^3*d^2*e^10))*((b^4*d + 8*a^2*c^2*d -
 a*b^3*e + a*e*(-(4*a*c - b^2)^3)^(1/2) - b*d*(-(4*a*c - b^2)^3)^(1/2) - 6*
a*b^2*c*d + 4*a^2*b*c*e)/(8*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)))^(1/2) -
((d + e*x^2)^(1/2)*(2*a^2*c^3*e^12 + 6*c^5*d^4*e^8 - 8*b*c^4*d^3*e^9 + 4*b^
2*c^3*d^2*e^10 - 4*a*b*c^3*d*e^11) + ((b^4*d + 8*a^2*c^2*d - a*b^3*e + a*e*
(-(4*a*c - b^2)^3)^(1/2) - b*d*(-(4*a*c - b^2)^3)^(1/2) - 6*a*b^2*c*d + 4*a
^2*b*c*e)/(8*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)))^(1/2)*((((b^4*d + 8*a^2
*c^2*d - a*b^3*e + a*e*(-(4*a*c - b^2)^3)^(1/2) - b*d*(-(4*a*c - b^2)^3)^(1
/2) - 6*a*b^2*c*d + 4*a^2*b*c*e)/(8*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)))^
(1/2)*((d + e*x^2)^(1/2)*((b^4*d + 8*a^2*c^2*d - a*b^3*e + a*e*(-(4*a*c - b
^2)^3)^(1/2) - b*d*(-(4*a*c - b^2)^3)^(1/2) - 6*a*b^2*c*d + 4*a^2*b*c*e)/(8
*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)))^(1/2)*(512*a^5*c^4*e^10 + 32*a^3*b^
4*c^2*e^10 - 256*a^4*b^2*c^3*e^10 + 768*a^4*c^5*d^2*e^8 + 64*a^2*b^4*c^3*d^
2*e^8 - 448*a^3*b^2*c^4*d^2*e^8 - 896*a^4*b*c^4*d*e^9 - 64*a^2*b^5*c^2*d*e^
9 + 480*a^3*b^3*c^3*d*e^9) - 192*a^4*c^4*d*e^10 - 192*a^3*c^5*d^3*e^8 + 48*
a^2*b^2*c^4*d^3*e^8 - 48*a^2*b^3*c^3*d^2*e^9 + 192*a^3*b*c^4*d^2*e^9 + 48*a
^3*b^2*c^3*d*e^10) - (d + e*x^2)^(1/2)*(32*a^3*b*c^3*e^11 + 48*a^3*c^4*d*e^
10 - 8*a^2*b^3*c^2*e^11 + 144*a^2*c^5*d^3*e^8 + 16*b^4*c^3*d^3*e^8 - 16*b^5
*c^2*d^2*e^9 + 16*a*b^4*c^2*d*e^10 - 96*a*b^2*c^4*d^3*e^8 + 96*a*b^3*c^3*d^
2*e^9 - 144*a^2*b*c^4*d^2*e^9 - 72*a^2*b^2*c^3*d*e^10))*((b^4*d + 8*a^2*c^2
*d - a*b^3*e + a*e*(-(4*a*c - b^2)^3)^(1/2) - b*d*(-(4*a*c - b^2)^3)^(1/2)
- 6*a*b^2*c*d + 4*a^2*b*c*e)/(8*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)))^(1/2

) + 12*a*c^5*d^4*e^8 + 12*a^2*c^4*d^2*e^10 - 4*b^2*c^4*d^4*e^8 + 4*b^4*c^2*d^2*e^10 + 8*a*b*c^4*d^3*e^9 - 4*a*b^3*c^2*d*e^11 + 20*a^2*b*c^3*d*e^11 - 24*a*b^2*c^3*d^2*e^10))*((b^4*d + 8*a^2*c^2*d - a*b^3*e + a*e*(-(4*a*c - b^2)^3)^(1/2) - b*d*(-(4*a*c - b^2)^3)^(1/2) - 6*a*b^2*c*d + 4*a^2*b*c*e)/(8*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)))^(1/2) + 2*c^4*d^3*e^10 - 2*b*c^3*d^2*e^11 + 2*a*c^3*d*e^12))*((b^4*d + 8*a^2*c^2*d - a*b^3*e + a*e*(-(4*a*c - b^2)^3)^(1/2) - b*d*(-(4*a*c - b^2)^3)^(1/2) - 6*a*b^2*c*d + 4*a^2*b*c*e)/(8*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)))^(1/2)*2i + atan((((d + e*x^2)^(1/2)*(2*a^2*c^3*e^12 + 6*c^5*d^4*e^8 - 8*b*c^4*d^3*e^9 + 4*b^2*c^3*d^2*e^10 - 4*a*b*c^3*d*e^11) + ((b^4*d + 8*a^2*c^2*d - a*b^3*e - a*e*(-(4*a*c - b^2)^3)^(1/2) + b*d*(-(4*a*c - b^2)^3)^(1/2) - 6*a*b^2*c*d + 4*a^2*b*c*e)/(8*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)))^(1/2)*((((b^4*d + 8*a^2*c^2*d - a*b^3*e - a*e*(-(4*a*c - b^2)^3)^(1/2) + b*d*(-(4*a*c - b^2)^3)^(1/2) - 6*a*b^2*c*d + 4*a^2*b*c*e)/(8*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)))^(1/2)*((d + e*x^2)^(1/2)*((b^4*d + 8*a^2*c^2*d - a*b^3*e - a*e*(-(4*a*c - b^2)^3)^(1/2) + b*d*(-(4*a*c - b^2)^3)^(1/2) - 6*a*b^2*c*d + 4*a^2*b*c*e)/(8*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)))^(1/2)*(512*a^5*c^4*e^10 + 32*a^3*b^4*c^2*e^10 - 256*a^4*b^2*c^3*e^10 + 768*a^4*c^5*d^2*e^8 + 64*a^2*b^4*c^3*d^2*e^8 - 448*a^3*b^2*c^4*d^2*e^8 - 896*a^4*b*c^4*d*e^9 - 64*a^2*b^5*c^2*d*e^9 + 480*a^3*b^3*c^3*d*e^9) - 192*a^4*c^4*d*e^10 - 192*a^3*c^5*d^3*e^8 + 48*a^2*b^2*c^4*d^3*e^8 - 48*a^2*b^3*c^3*d^2*e^9 + 192*a^3*b*c^4*d^2*e^9 + 48*a^3*b^2*c^3*d*e^10) - (d + e*x^2)^(1/2)*(32*a^3*b*c^3*e^11 + 48*a^3*c^4*d*e^10 - 8*a^2*b^3*c^2*e^11 + 144*a^2*c^5*d^3*e^8 + 16*b^4*c^3*d^3*e^8 - 16*b^5*c^2*d^2*e^9 + 16*a*b^4*c^2*d*e^10 - 96*a*b^2*c^4*d^3*e^8 + 96*a*b^3*c^3*d^2*e^9 - 144*a^2*b*c^4*d^2*e^9 - 72*a^2*b^2*c^3*d*e^10))*((b^4*d + 8*a^2*c^2*d - a*b^3*e - a*e*(-(4*a*c - b^2)^3)^(1/2) + b*d*(-(4*a*c - b^2)^3)^(1/2) - 6*a*b^2*c*d + 4*a^2*b*c*e)/(8*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)))^(1/2) + 12*a*c^5*d^4*e^8 + 12*a^2*c^4*d^2*e^10 - 4*b^2*c^4*d^4*e^8 + 4*b^4*c^2*d^2*e^10 + 8*a*b*c^4*d^3*e^9 - 4*a*b^3*c^2*d*e^11 + 20*a^2*b*c^3*d*e^11 - 24*a*b^2*c^3*d^2*e^10))*((b^4*d + 8*a^2*c^2*d - a*b^3*e - a*e*(-(4*a*c - b^2)^3)^(1/2) + b*d*(-(4*a*c - b^2)^3)^(1/2) - 6*a*b^2*c*d + 4*a^2*b*c*e)/(8*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)))^(1/2)*1i + ((d + e*x^2)^(1/2)*(2*a^2*c^3*e^12 + 6*c^5*d^4*e^8 - 8*b*c^4*d^3*e^9 + 4*b^2*c^3*d^2*e^10 - 4*a*b*c^3*d*e^11) - ((b^4*d + 8*a^2*c^2*d - a*b^3*e - a*e*(-(4*a*c - b^2)^3)^(1/2) + b*d*(-(4*a*c - b^2)^3)^(1/2) - 6*a*b^2*c*d + 4*a^2*b*c*e)/(8*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)))^(1/2)*(12*a*c^5*d^4*e^8 - (((b^4*d + 8*a^2*c^2*d - a*b^3*e - a*e*(-(4*a*c - b^2)^3)^(1/2) + b*d*(-(4*a*c - b^2)^3)^(1/2) - 6*a*b^2*c*d + 4*a^2*b*c*e)/(8*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)))^(1/2)*((d + e*x^2)^(1/2)*((b^4*d + 8*a^2*c^2*d - a*b^3*e - a*e*(-(4*a*c - b^2)^3)^(1/2) + b*d*(-(4*a*c - b^2)^3)^(1/2) - 6*a*b^2*c*d + 4*a^2*b*c*e)/(8*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)))^(1/2)*(512*a^5*c^4*e^10 + 32*a^3*b^4*c^2*e^10 - 256*a^4*b^2*c^3*e^10 + 768*a^4*c^5*d^2*e^8 + 64*a^2*b^4*c^3*d^2*e^8 - 448*a^3*b^2*c^4*d^2*e^8 - 896*a^4*b*c^4*d*e^9 - 64*a^2*b^5*c^2*d*e^9 + 480*a^3*b^3*c^3*d*e^9) + 192*a^4*c^4*d*e^10 + 192*a^3*c^5*d^3*e^8 - 48*a^2*b^2*c^4*d^3*e^8 + 48*a^2*b^3*c^3*d^2*e^9 - 192*a^3*b*c^4*d^2*e^9 - 48*a^3*b^2*c^3*d*e^10) - (d +

e*x^2)^(1/2)*(32*a^3*b*c^3*e^11 + 48*a^3*c^4*d*e^10 - 8*a^2*b^3*c^2*e^11 + 144*a^2*c^5*d^3*e^8 + 16*b^4*c^3*d^3*e^8 - 16*b^5*c^2*d^2*e^9 + 16*a*b^4*c^2*d*e^10 - 96*a*b^2*c^4*d^3*e^8 + 96*a*b^3*c^3*d^2*e^9 - 144*a^2*b*c^4*d^2*e^9 - 72*a^2*b^2*c^3*d*e^10))*((b^4*d + 8*a^2*c^2*d - a*b^3*e - a*e*(-(4*a*c - b^2)^3)^(1/2) + b*d*(-(4*a*c - b^2)^3)^(1/2) - 6*a*b^2*c*d + 4*a^2*b*c*e)/(8*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)))^(1/2) + 12*a^2*c^4*d^2*e^10 - 4*b^2*c^4*d^4*e^8 + 4*b^4*c^2*d^2*e^10 + 8*a*b*c^4*d^3*e^9 - 4*a*b^3*c^2*d*e^11 + 20*a^2*b*c^3*d*e^11 - 24*a*b^2*c^3*d^2*e^10))*((b^4*d + 8*a^2*c^2*d - a*b^3*e - a*e*(-(4*a*c - b^2)^3)^(1/2) + b*d*(-(4*a*c - b^2)^3)^(1/2) - 6*a*b^2*c*d + 4*a^2*b*c*e)/(8*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)))^(1/2)*1i)/(((d + e*x^2)^(1/2)*(2*a^2*c^3*e^12 + 6*c^5*d^4*e^8 - 8*b*c^4*d^3*e^9 + 4*b^2*c^3*d^2*e^10 - 4*a*b*c^3*d*e^11) - ((b^4*d + 8*a^2*c^2*d - a*b^3*e - a*e*(-(4*a*c - b^2)^3)^(1/2) + b*d*(-(4*a*c - b^2)^3)^(1/2) - 6*a*b^2*c*d + 4*a^2*b*c*e)/(8*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)))^(1/2)*(12*a*c^5*d^4*e^8 - (((b^4*d + 8*a^2*c^2*d - a*b^3*e - a*e*(-(4*a*c - b^2)^3)^(1/2) + b*d*(-(4*a*c - b^2)^3)^(1/2) - 6*a*b^2*c*d + 4*a^2*b*c*e)/(8*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)))^(1/2)*((d + e*x^2)^(1/2)*((b^4*d + 8*a^2*c^2*d - a*b^3*e - a*e*(-(4*a*c - b^2)^3)^(1/2) + b*d*(-(4*a*c - b^2)^3)^(1/2) - 6*a*b^2*c*d + 4*a^2*b*c*e)/(8*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)))^(1/2)*(512*a^5*c^4*e^10 + 32*a^3*b^4*c^2*e^10 - 256*a^4*b^2*c^3*e^10 + 768*a^4*c^5*d^2*e^8 + 64*a^2*b^4*c^3*d^2*e^8 - 448*a^3*b^2*c^4*d^2*e^8 - 896*a^4*b*c^4*d*e^9 - 64*a^2*b^5*c^2*d*e^9 + 480*a^3*b^3*c^3*d*e^9) + 192*a^4*c^4*d*e^10 + 192*a^3*c^5*d^3*e^8 - 48*a^2*b^2*c^4*d^3*e^8 + 48*a^2*b^3*c^3*d^2*e^9 - 192*a^3*b*c^4*d^2*e^9 - 48*a^3*b^2*c^3*d*e^10) - (d + e*x^2)^(1/2)*(32*a^3*b*c^3*e^11 + 48*a^3*c^4*d*e^10 - 8*a^2*b^3*c^2*e^11 + 144*a^2*c^5*d^3*e^8 + 16*b^4*c^3*d^3*e^8 - 16*b^5*c^2*d^2*e^9 + 16*a*b^4*c^2*d*e^10 - 96*a*b^2*c^4*d^3*e^8 + 96*a*b^3*c^3*d^2*e^9 - 144*a^2*b*c^4*d^2*e^9 - 72*a^2*b^2*c^3*d*e^10))*((b^4*d + 8*a^2*c^2*d - a*b^3*e - a*e*(-(4*a*c - b^2)^3)^(1/2) + b*d*(-(4*a*c - b^2)^3)^(1/2) - 6*a*b^2*c*d + 4*a^2*b*c*e)/(8*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)))^(1/2) + 12*a^2*c^4*d^2*e^10 - 4*b^2*c^4*d^4*e^8 + 4*b^4*c^2*d^2*e^10 + 8*a*b*c^4*d^3*e^9 - 4*a*b^3*c^2*d*e^11 + 20*a^2*b*c^3*d*e^11 - 24*a*b^2*c^3*d^2*e^10))*((b^4*d + 8*a^2*c^2*d - a*b^3*e - a*e*(-(4*a*c - b^2)^3)^(1/2) + b*d*(-(4*a*c - b^2)^3)^(1/2) - 6*a*b^2*c*d + 4*a^2*b*c*e)/(8*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)))^(1/2) - ((d + e*x^2)^(1/2)*(2*a^2*c^3*e^12 + 6*c^5*d^4*e^8 - 8*b*c^4*d^3*e^9 + 4*b^2*c^3*d^2*e^10 - 4*a*b*c^3*d*e^11) + ((b^4*d + 8*a^2*c^2*d - a*b^3*e - a*e*(-(4*a*c - b^2)^3)^(1/2) + b*d*(-(4*a*c - b^2)^3)^(1/2) - 6*a*b^2*c*d + 4*a^2*b*c*e)/(8*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)))^(1/2)*((((b^4*d + 8*a^2*c^2*d - a*b^3*e - a*e*(-(4*a*c - b^2)^3)^(1/2) + b*d*(-(4*a*c - b^2)^3)^(1/2) - 6*a*b^2*c*d + 4*a^2*b*c*e)/(8*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)))^(1/2)*((d + e*x^2)^(1/2)*((b^4*d + 8*a^2*c^2*d - a*b^3*e - a*e*(-(4*a*c - b^2)^3)^(1/2) + b*d*(-(4*a*c - b^2)^3)^(1/2) - 6*a*b^2*c*d + 4*a^2*b*c*e)/(8*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)))^(1/2)*(512*a^5*c^4*e^10 + 32*a^3*b^4*c^2*e^10 - 256*a^4*b^2*c^3*e^10 + 768*a^4*c^5*d^2*e^8 + 64*a^2*b^4*c^3*d^2*e^8 - 448*a^3*b^2*c^4*d^2*e^8 - 896*a^4*b*c^4*d*e^9 - 64*a^2*b^5*c^2*d*e^9 + 480*a^3*b^3*c^3*d*e^9

) - 192*a^4*c^4*d*e^10 - 192*a^3*c^5*d^3*e^8 + 48*a^2*b^2*c^4*d^3*e^8 - 48*a^2*b^3*c^3*d^2*e^9 + 192*a^3*b*c^4*d^2*e^9 + 48*a^3*b^2*c^3*d*e^10) - (d + e*x^2)^(1/2)*(32*a^3*b*c^3*e^11 + 48*a^3*c^4*d*e^10 - 8*a^2*b^3*c^2*e^11 + 144*a^2*c^5*d^3*e^8 + 16*b^4*c^3*d^3*e^8 - 16*b^5*c^2*d^2*e^9 + 16*a*b^4*c^2*d*e^10 - 96*a*b^2*c^4*d^3*e^8 + 96*a*b^3*c^3*d^2*e^9 - 144*a^2*b*c^4*d^2*e^9 - 72*a^2*b^2*c^3*d*e^10))*((b^4*d + 8*a^2*c^2*d - a*b^3*e - a*e*(-(4*a*c - b^2)^3)^(1/2) + b*d*(-(4*a*c - b^2)^3)^(1/2) - 6*a*b^2*c*d + 4*a^2*b*c*e)/(8*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)))^(1/2) + 12*a*c^5*d^4*e^8 + 12*a^2*c^4*d^2*e^10 - 4*b^2*c^4*d^4*e^8 + 4*b^4*c^2*d^2*e^10 + 8*a*b*c^4*d^3*e^9 - 4*a*b^3*c^2*d*e^11 + 20*a^2*b*c^3*d*e^11 - 24*a*b^2*c^3*d^2*e^10))*((b^4*d + 8*a^2*c^2*d - a*b^3*e - a*e*(-(4*a*c - b^2)^3)^(1/2) + b*d*(-(4*a*c - b^2)^3)^(1/2) - 6*a*b^2*c*d + 4*a^2*b*c*e)/(8*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)))^(1/2) + 2*c^4*d^3*e^10 - 2*b*c^3*d^2*e^11 + 2*a*c^3*d*e^12))*((b^4*d + 8*a^2*c^2*d - a*b^3*e - a*e*(-(4*a*c - b^2)^3)^(1/2) + b*d*(-(4*a*c - b^2)^3)^(1/2) - 6*a*b^2*c*d + 4*a^2*b*c*e)/(8*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)))^(1/2)*2i - (d^(1/2)*atanh((20*c^4*d^(5/2)*e^10*(d + e*x^2)^(1/2))/(20*c^4*d^3*e^10 - 12*b*c^3*d^2*e^11 + (18*c^5*d^5*e^8)/a + 2*a*c^3*d*e^12 + (6*b^2*c^3*d^3*e^10)/a + (2*b^3*c^2*d^2*e^11)/a - (4*b^2*c^4*d^5*e^8)/a^2 + (6*b^3*c^3*d^4*e^9)/a^2 - (2*b^4*c^2*d^3*e^10)/a^2 - (28*b*c^4*d^4*e^9)/a) + (18*c^5*d^(9/2)*e^8*(d + e*x^2)^(1/2))/(18*c^5*d^5*e^8 + 20*a*c^4*d^3*e^10 + 2*a^2*c^3*d*e^12 - 28*b*c^4*d^4*e^9 + 6*b^2*c^3*d^3*e^10 + 2*b^3*c^2*d^2*e^11 - (4*b^2*c^4*d^5*e^8)/a + (6*b^3*c^3*d^4*e^9)/a - (2*b^4*c^2*d^3*e^10)/a - 12*a*b*c^3*d^2*e^11) - (28*b*c^4*d^(7/2)*e^9*(d + e*x^2)^(1/2))/(18*c^5*d^5*e^8 + 20*a*c^4*d^3*e^10 + 2*a^2*c^3*d*e^12 - 28*b*c^4*d^4*e^9 + 6*b^2*c^3*d^3*e^10 + 2*b^3*c^2*d^2*e^11 - (4*b^2*c^4*d^5*e^8)/a + (6*b^3*c^3*d^4*e^9)/a - (2*b^4*c^2*d^3*e^10)/a - 12*a*b*c^3*d^2*e^11) + (2*b^3*c^2*d^(3/2)*e^11*(d + e*x^2)^(1/2))/(18*c^5*d^5*e^8 + 20*a*c^4*d^3*e^10 + 2*a^2*c^3*d*e^12 - 28*b*c^4*d^4*e^9 + 6*b^2*c^3*d^3*e^10 + 2*b^3*c^2*d^2*e^11 - (4*b^2*c^4*d^5*e^8)/a + (6*b^3*c^3*d^4*e^9)/a - (2*b^4*c^2*d^3*e^10)/a - 12*a*b*c^3*d^2*e^11) + (6*b^2*c^3*d^(5/2)*e^10*(d + e*x^2)^(1/2))/(18*c^5*d^5*e^8 + 20*a*c^4*d^3*e^10 + 2*a^2*c^3*d*e^12 - 28*b*c^4*d^4*e^9 + 6*b^2*c^3*d^3*e^10 + 2*b^3*c^2*d^2*e^11 - (4*b^2*c^4*d^5*e^8)/a + (6*b^3*c^3*d^4*e^9)/a - (2*b^4*c^2*d^3*e^10)/a - 12*a*b*c^3*d^2*e^11) - (2*b^4*c^2*d^(5/2)*e^10*(d + e*x^2)^(1/2))/(18*a*c^5*d^5*e^8 + 2*a^3*c^3*d*e^12 + 20*a^2*c^4*d^3*e^10 - 4*b^2*c^4*d^5*e^8 + 6*b^3*c^3*d^4*e^9 - 2*b^4*c^2*d^3*e^10 - 28*a*b*c^4*d^4*e^9 + 6*a*b^2*c^3*d^3*e^10 + 2*a*b^3*c^2*d^2*e^11 - 12*a^2*b*c^3*d^2*e^11) + (6*b^3*c^3*d^(7/2)*e^9*(d + e*x^2)^(1/2))/(18*a*c^5*d^5*e^8 + 2*a^3*c^3*d*e^12 + 20*a^2*c^4*d^3*e^10 - 4*b^2*c^4*d^5*e^8 + 6*b^3*c^3*d^4*e^9 - 2*b^4*c^2*d^3*e^10 - 28*a*b*c^4*d^4*e^9 + 6*a*b^2*c^3*d^3*e^10 + 2*a*b^3*c^2*d^2*e^11 - 12*a^2*b*c^3*d^2*e^11) - (4*b^2*c^4*d^(9/2)*e^8*(d + e*x^2)^(1/2))/(18*a*c^5*d^5*e^8 + 2*a^3*c^3*d*e^12 + 20*a^2*c^4*d^3*e^10 - 4*b^2*c^4*d^5*e^8 + 6*b^3*c^3*d^4*e^9 - 2*b^4*c^2*d^3*e^10 - 28*a*b*c^4*d^4*e^9 + 6*a*b^2*c^3*d^3*e^10 + 2*a*b^3*c^2*d^2*e^11 - 12*a^2*b*c^3*d^2*e^11) + (2*a*c^3*d^(1/2)*e^12*(d + e*x^2)^(1/2))/(20*c^4*d^3*e^10 - 12*b*c^3*d^2*e^11 + (18*c^5*d^5*e^8)/a + 2*a*c^3*d*e^12 + (6*b^2*c^3*d^3*e^10)/a + (2*b^3

```
*c^2*d^2*e^11)/a - (4*b^2*c^4*d^5*e^8)/a^2 + (6*b^3*c^3*d^4*e^9)/a^2 - (2*b
^4*c^2*d^3*e^10)/a^2 - (28*b*c^4*d^4*e^9)/a) - (12*b*c^3*d^(3/2)*e^11*(d +
e*x^2)^(1/2))/(20*c^4*d^3*e^10 - 12*b*c^3*d^2*e^11 + (18*c^5*d^5*e^8)/a + 2
*a*c^3*d*e^12 + (6*b^2*c^3*d^3*e^10)/a + (2*b^3*c^2*d^2*e^11)/a - (4*b^2*c^
4*d^5*e^8)/a^2 + (6*b^3*c^3*d^4*e^9)/a^2 - (2*b^4*c^2*d^3*e^10)/a^2 - (28*b
*c^4*d^4*e^9)/a)))/a
```

3.359 $\int \frac{\sqrt{d+ex^2}}{x^3(a+bx^2+cx^4)}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 382

$$\begin{aligned}\int \frac{\sqrt{d+ex^2}}{x^3\left(a+bx^2+cx^4\right)}\,dx &= -\frac{\sqrt{d+ex^2}}{2ax^2}+\frac{e\mathrm{arctanh}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)}{2a\sqrt{d}}+\frac{(bd-ae)\mathrm{arctanh}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)}{a^2\sqrt{d}}\\ &\quad-\frac{\sqrt{c}\left(b^2d-2acd-abe+\sqrt{b^2-4ac}(bd-ae)\right)\mathrm{arctanh}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex^2}}{\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}}\right)}{\sqrt{2}a^2\sqrt{b^2-4ac}\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}}\\ &\quad+\frac{\sqrt{c}\left(b^2d-b\left(\sqrt{b^2-4ac}d+ae\right)-a\left(2cd-\sqrt{b^2-4ac}e\right)\right)\mathrm{arctanh}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex^2}}{\sqrt{2cd-\left(b+\sqrt{b^2-4ac}\right)e}}\right)}{\sqrt{2}a^2\sqrt{b^2-4ac}\sqrt{2cd-\left(b+\sqrt{b^2-4ac}\right)e}}\end{aligned}$$

```
[Out] 1/2*e*arctanh((e*x^2+d)^(1/2)/d^(1/2))/a/d^(1/2)+(-a*e+b*d)*arctanh((e*x^2+
d)^(1/2)/d^(1/2))/a^2/d^(1/2)-1/2*(e*x^2+d)^(1/2)/a/x^2-1/2*arctanh(2^(1/2)
*c^(1/2)*(e*x^2+d)^(1/2)/(2*c*d-e*(b-(-4*a*c+b^2)^(1/2)))^(1/2))*c^(1/2)*(b
^2*d-2*a*c*d-a*b*e+(-a*e+b*d)*(-4*a*c+b^2)^(1/2))/a^2*2^(1/2)/(-4*a*c+b^2)^
(1/2)/(2*c*d-e*(b-(-4*a*c+b^2)^(1/2)))^(1/2)+1/2*arctanh(2^(1/2)*c^(1/2)*(e
*x^2+d)^(1/2)/(2*c*d-e*(b+(-4*a*c+b^2)^(1/2)))^(1/2))*c^(1/2)*(b^2*d-2*a*c*
d-a*b*e-(-a*e+b*d)*(-4*a*c+b^2)^(1/2))/a^2*2^(1/2)/(-4*a*c+b^2)^(1/2)/(2*c*
d-e*(b+(-4*a*c+b^2)^(1/2)))^(1/2)
```

Rubi [A] (verified)

Time = 2.73 (sec) , antiderivative size = 370, normalized size of antiderivative = 0.97, number of steps used = 10, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.241, Rules used = {1265, 911, 1301, 205, 212, 1180, 214}

$$\int \frac{\sqrt{d+ex^2}}{x^3\left(a+bx^2+cx^4\right)}\,dx$$
$$= -\frac{\sqrt{c}\left(\sqrt{b^2-4ac}(bd-ae)-abe-2acd+b^2d\right)\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex^2}}{\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}\right)}{\sqrt{2}a^2\sqrt{b^2-4ac}\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}$$
$$+\frac{\sqrt{c}\left(-\sqrt{b^2-4ac}(bd-ae)-abe-2acd+b^2d\right)\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex^2}}{\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}\right)}{\sqrt{2}a^2\sqrt{b^2-4ac}\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}$$
$$+\frac{(bd-ae)\operatorname{arctanh}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)}{a^2\sqrt{d}}+\frac{e\operatorname{arctanh}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)}{2a\sqrt{d}}-\frac{\sqrt{d+ex^2}}{2ax^2}$$

```
[In] Int[Sqrt[d + e*x^2]/(x^3*(a + b*x^2 + c*x^4)),x]
```

```
[Out] -1/2*Sqrt[d + e*x^2]/(a*x^2) + (e*ArcTanh[Sqrt[d + e*x^2]/Sqrt[d]])/(2*a*Sq
rt[d]) + ((b*d - a*e)*ArcTanh[Sqrt[d + e*x^2]/Sqrt[d]])/(a^2*Sqrt[d]) - (Sq
rt[c]*(b^2*d - 2*a*c*d - a*b*e + Sqrt[b^2 - 4*a*c]*(b*d - a*e))*ArcTanh[(Sq
rt[2]*Sqrt[c]*Sqrt[d + e*x^2])/Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]])/(S
qrt[2]*a^2*Sqrt[b^2 - 4*a*c]*Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]) + (Sq
rt[c]*(b^2*d - 2*a*c*d - a*b*e - Sqrt[b^2 - 4*a*c]*(b*d - a*e))*ArcTanh[(Sq
rt[2]*Sqrt[c]*Sqrt[d + e*x^2])/Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]])/(S
qrt[2]*a^2*Sqrt[b^2 - 4*a*c]*Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integ
erQ[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denom
inator[p + 1/n] < Denominator[p])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 911

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> With[{q = Denominator[m]}, Dist[q/e, S
ubst[Int[x^(q*(m + 1) - 1)*((e*f - d*g)/e + g*(x^q/e))^n*((c*d^2 - b*d*e +
a*e^2)/e^2 - (2*c*d - b*e)*(x^q/e^2) + c*(x^(2*q)/e^2))^p, x], x, (d + e*x)
^(1/q)], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegersQ[n, p] && Fra
ctionQ[m]
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1265

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rule 1301

```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.))/((a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*((d + e*x^2)^q/(a
+ b*x^2 + c*x^4)), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b^2 - 4
*a*c, 0] && IntegerQ[q] && IntegerQ[m]
```

Rubi steps

$$\text{integral} = \frac{1}{2}\text{Subst}\left(\int \frac{\sqrt{d+ex}}{x^2\left(a+bx+cx^2\right)}\,dx, x, x^2\right)$$

$$= \frac{\text{Subst}\left(\int \frac{x^2}{\left(-\frac{d}{e}+\frac{x^2}{e}\right)^2\left(\frac{cd^2-bde+ae^2}{e^2}-\frac{(2cd-be)x^2}{e^2}+\frac{cx^4}{e^2}\right)}\,dx, x, \sqrt{d+ex^2}\right)}{e}$$

$$= \frac{\text{Subst}\left(\int \left(\frac{de^2}{a(d-x^2)^2} - \frac{e(-bd+ae)}{a^2(d-x^2)} + \frac{e(-b(cd^2-bde+ae^2)+c(bd-ae)x^2)}{a^2(cd^2-bde+ae^2-(2cd-be)x^2+cx^4)}\right) dx, x, \sqrt{d+ex^2}\right)}{e}$$

$$= \frac{\text{Subst}\left(\int \frac{-b(cd^2-bde+ae^2)+c(bd-ae)x^2}{cd^2-bde+ae^2+(-2cd+be)x^2+cx^4} dx, x, \sqrt{d+ex^2}\right)}{a^2} + \frac{(de)\text{Subst}\left(\int \frac{1}{(d-x^2)^2} dx, x, \sqrt{d+ex^2}\right)}{a} + \frac{(bd-ae)\text{Subst}\left(\int \frac{1}{d-x^2} dx, x, \sqrt{d+ex^2}\right)}{a^2}$$

$$= -\frac{\sqrt{d+ex^2}}{2ax^2} + \frac{(bd-ae)\tanh^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)}{a^2\sqrt{d}} + \frac{e\text{Subst}\left(\int \frac{1}{d-x^2} dx, x, \sqrt{d+ex^2}\right)}{2a} - \frac{\left(c\left(b^2d-2acd-abe-\sqrt{b^2-4ac}(bd-ae)\right)\right)\text{Subst}\left(\int \frac{1}{\frac{1}{2}\sqrt{b^2-4ac}e+\frac{1}{2}(-2cd+be)+cx^2} dx, x, \sqrt{d+ex^2}\right)}{2a^2\sqrt{b^2-4ac}} + \frac{\left(c\left(b^2d-2acd-abe+\sqrt{b^2-4ac}(bd-ae)\right)\right)\text{Subst}\left(\int \frac{1}{-\frac{1}{2}\sqrt{b^2-4ac}e+\frac{1}{2}(-2cd+be)+cx^2} dx, x, \sqrt{d+ex^2}\right)}{2a^2\sqrt{b^2-4ac}}$$

$$= -\frac{\sqrt{d+ex^2}}{2ax^2} + \frac{e\tanh^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)}{2a\sqrt{d}} + \frac{(bd-ae)\tanh^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)}{a^2\sqrt{d}} - \frac{\sqrt{c}\left(b^2d-2acd-abe+\sqrt{b^2-4ac}(bd-ae)\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex^2}}{\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}}\right)}{\sqrt{2}a^2\sqrt{b^2-4ac}\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}} + \frac{\sqrt{c}\left(b^2d-2acd-abe-\sqrt{b^2-4ac}(bd-ae)\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex^2}}{\sqrt{2cd-\left(b+\sqrt{b^2-4ac}\right)e}}\right)}{\sqrt{2}a^2\sqrt{b^2-4ac}\sqrt{2cd-\left(b+\sqrt{b^2-4ac}\right)e}}$$

Mathematica [A] (verified)

Time = 1.05 (sec) , antiderivative size = 348, normalized size of antiderivative = 0.91

$$\int \frac{\sqrt{d+ex^2}}{x^3(a+bx^2+cx^4)} dx$$

$$= \frac{-\frac{a\sqrt{d+ex^2}}{x^2} + \frac{\sqrt{2}\sqrt{c}\left(b^2d-2acd+b\sqrt{b^2-4ac}d-abe-a\sqrt{b^2-4ac}e\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex^2}}{\sqrt{-2cd+be-\sqrt{b^2-4ac}e}}\right)}{\sqrt{b^2-4ac}\sqrt{-2cd+\left(b-\sqrt{b^2-4ac}\right)e}} + \frac{\sqrt{2}\sqrt{c}\left(-b^2d+2acd+b\sqrt{b^2-4ac}d+abe-a\right.}{\sqrt{b^2-4ac}\sqrt{-2c}}}{2a^2}$$

```
[In] Integrate[Sqrt[d + e*x^2]/(x^3*(a + b*x^2 + c*x^4)),x]
```

```
[Out] (-((a*Sqrt[d + e*x^2])/x^2) + (Sqrt[2]*Sqrt[c]*(b^2*d - 2*a*c*d + b*Sqrt[b^
2 - 4*a*c]*d - a*b*e - a*Sqrt[b^2 - 4*a*c]*e)*ArcTan[(Sqrt[2]*Sqrt[c]*Sqrt[
d + e*x^2])/Sqrt[-2*c*d + b*e - Sqrt[b^2 - 4*a*c]*e]])/(Sqrt[b^2 - 4*a*c]*S
qrt[-2*c*d + (b - Sqrt[b^2 - 4*a*c])*e]) + (Sqrt[2]*Sqrt[c]*(-(b^2*d) + 2*a
*c*d + b*Sqrt[b^2 - 4*a*c]*d + a*b*e - a*Sqrt[b^2 - 4*a*c]*e)*ArcTan[(Sqrt[
2]*Sqrt[c]*Sqrt[d + e*x^2])/Sqrt[-2*c*d + (b + Sqrt[b^2 - 4*a*c])*e]])/(Sqr
t[b^2 - 4*a*c]*Sqrt[-2*c*d + (b + Sqrt[b^2 - 4*a*c])*e]) + ((2*b*d - a*e)*A
rcTanh[Sqrt[d + e*x^2]/Sqrt[d]])/Sqrt[d])/(2*a^2)
```

Maple [A] (verified)

Time = 0.82 (sec) , antiderivative size = 364, normalized size of antiderivative = 0.95

method	result
risch	$-\frac{\sqrt{e\,x^2+d}}{2a\,x^2}-\frac{-\frac{(-ae+2bd)\ln\left(\frac{2d+2\sqrt{d}\,\sqrt{e\,x^2+d}}{x}\right)}{a\sqrt{d}}+\frac{c\sqrt{2}\left(-\frac{\left(ab\,e^2+2acde-b^2de+\sqrt{-e^2\left(4ac-b^2\right)}\,ae-\sqrt{-e^2\left(4ac-b^2\right)}\,bd\right)\operatorname{arctanh}}{\sqrt{\left(-be+2cd+\sqrt{-e^2\left(4ac-b^2\right)}\right)c}}\right.}{}}{}$
pseudoelliptic	$\frac{\sqrt{2}\sqrt{\left(be-2cd+\sqrt{-4e^2\left(ac-\frac{b^2}{4}\right)}\right)c}\,c\,x^2\left(\frac{\left(ea\sqrt{d}-d^{\frac{3}{2}}b\right)\sqrt{-4e^2\left(ac-\frac{b^2}{4}\right)}}{2}+e\left(\left(ac-\frac{b^2}{2}\right)d^{\frac{3}{2}}+\frac{bea\sqrt{d}}{2}\right)\right)\operatorname{arctanh}\left(\frac{c}{\sqrt{\left(-be+2cd\right.}}\right.}{}$
default	$\frac{-\frac{\left(e\,x^2+d\right)^{\frac{3}{2}}}{2d\,x^2}+\frac{e\left(\sqrt{e\,x^2+d}-\sqrt{d}\,\ln\left(\frac{2d+2\sqrt{d}\,\sqrt{e\,x^2+d}}{x}\right)\right)}{2d}}{a}-\frac{b\left(\sqrt{e\,x^2+d}-\sqrt{d}\,\ln\left(\frac{2d+2\sqrt{d}\,\sqrt{e\,x^2+d}}{x}\right)\right)}{a^2}-\frac{-\sqrt{2}\sqrt{\left(be-2cd+\right.}}{}$

```
[In] int((e*x^2+d)^(1/2)/x^3/(c*x^4+b*x^2+a),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*(e*x^2+d)^(1/2)/a/x^2-1/2/a*(-(-a*e+2*b*d)/a/d^(1/2)*ln((2*d+2*d^(1/2)
*(e*x^2+d)^(1/2))/x)+1/a*c*2^(1/2)/(-e^2*(4*a*c-b^2))^(1/2)*(-(a*b*e^2+2*a*
c*d*e-b^2*d*e+(-e^2*(4*a*c-b^2))^(1/2)*a*e-(-e^2*(4*a*c-b^2))^(1/2)*b*d)/((
-b*e+2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2)*arctanh(c*(e*x^2+d)^(1/2)*2^(
1/2)/((-b*e+2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2))+(-a*b*e^2-2*a*c*d*e+b
^2*d*e+(-e^2*(4*a*c-b^2))^(1/2)*a*e-(-e^2*(4*a*c-b^2))^(1/2)*b*d)/((b*e-2*c
*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2)*arctan(c*(e*x^2+d)^(1/2)*2^(1/2)/((b*
e-2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3290 vs. 2(309) = 618.

Time = 260.18 (sec) , antiderivative size = 6592, normalized size of antiderivative = 17.26

$$\int \frac{\sqrt{d+ex^2}}{x^3\left(a+bx^2+cx^4\right)}\,dx = \text{Too large to display}$$

```
[In] integrate((e*x^2+d)^(1/2)/x^3/(c*x^4+b*x^2+a),x, algorithm="fricas")
[Out] Too large to include
```

Sympy [F]

$$\int \frac{\sqrt{d+ex^2}}{x^3\left(a+bx^2+cx^4\right)}\,dx = \int \frac{\sqrt{d+ex^2}}{x^3\left(a+bx^2+cx^4\right)}\,dx$$

```
[In] integrate((e*x**2+d)**(1/2)/x**3/(c*x**4+b*x**2+a),x)
[Out] Integral(sqrt(d + e*x**2)/(x**3*(a + b*x**2 + c*x**4)), x)
```

Maxima [F]

$$\int \frac{\sqrt{d+ex^2}}{x^3\left(a+bx^2+cx^4\right)}\,dx = \int \frac{\sqrt{ex^2+d}}{(cx^4+bx^2+a)x^3}\,dx$$

```
[In] integrate((e*x^2+d)^(1/2)/x^3/(c*x^4+b*x^2+a),x, algorithm="maxima")
[Out] integrate(sqrt(e*x^2 + d)/((c*x^4 + b*x^2 + a)*x^3), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 784 vs. 2(309) = 618.

Time = 0.33 (sec) , antiderivative size = 784, normalized size of antiderivative = 2.05

$$\int \frac{\sqrt{d+ex^2}}{x^3\,(a+bx^2+cx^4)}\,dx = -\frac{(2\,bd-ae)\arctan\left(\frac{\sqrt{ex^2+d}}{\sqrt{-d}}\right)}{2\,a^2\sqrt{-d}}$$

$$+\frac{\left(\sqrt{-4\,c^2d+2\,(bc-\sqrt{b^2-4\,ac}c)e}((b^3-4\,abc)d-(ab^2-4\,a^2c)e)e^2-2\,(\sqrt{b^2-4\,ac}bcd^2-\sqrt{b^2-4\,ac}\right.}{}$$

$$-\frac{\left(\sqrt{-4\,c^2d+2\,(bc+\sqrt{b^2-4\,ac}c)e}((b^3-4\,abc)d-(ab^2-4\,a^2c)e)e^2+2\,(\sqrt{b^2-4\,ac}bcd^2-\sqrt{b^2-4\,ac}\right.}{}$$

$$-\frac{\sqrt{ex^2+d}}{2\,ax^2}$$

```
[In] integrate((e*x^2+d)^(1/2)/x^3/(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] -1/2*(2*b*d - a*e)*arctan(sqrt(e*x^2 + d)/sqrt(-d))/(a^2*sqrt(-d)) + 1/8*(s
qrt(-4*c^2*d + 2*(b*c - sqrt(b^2 - 4*a*c)*c)*e)*((b^3 - 4*a*b*c)*d - (a*b^2
 - 4*a^2*c)*e)*e^2 - 2*(sqrt(b^2 - 4*a*c)*b*c*d^2 - sqrt(b^2 - 4*a*c)*b^2*d
*e + sqrt(b^2 - 4*a*c)*a*b*e^2)*sqrt(-4*c^2*d + 2*(b*c - sqrt(b^2 - 4*a*c)*
c)*e)*abs(e) + (b^3*d*e^2 - a*b^2*e^3 - 2*(b^2*c - 2*a*c^2)*d^2*e)*sqrt(-4*
c^2*d + 2*(b*c - sqrt(b^2 - 4*a*c)*c)*e))*arctan(2*sqrt(1/2)*sqrt(e*x^2 + d
)/sqrt(-(2*a^2*c*d - a^2*b*e + sqrt(-4*(a^2*c*d^2 - a^2*b*d*e + a^3*e^2)*a^
2*c + (2*a^2*c*d - a^2*b*e)^2))/(a^2*c)))/((sqrt(b^2 - 4*a*c)*a^2*c*d^2 - s
qrt(b^2 - 4*a*c)*a^2*b*d*e + sqrt(b^2 - 4*a*c)*a^3*e^2)*abs(c)*abs(e)) - 1/
8*(sqrt(-4*c^2*d + 2*(b*c + sqrt(b^2 - 4*a*c)*c)*e)*((b^3 - 4*a*b*c)*d - (a
*b^2 - 4*a^2*c)*e)*e^2 + 2*(sqrt(b^2 - 4*a*c)*b*c*d^2 - sqrt(b^2 - 4*a*c)*b
^2*d*e + sqrt(b^2 - 4*a*c)*a*b*e^2)*sqrt(-4*c^2*d + 2*(b*c + sqrt(b^2 - 4*a
*c)*c)*e)*abs(e) + (b^3*d*e^2 - a*b^2*e^3 - 2*(b^2*c - 2*a*c^2)*d^2*e)*sqrt
(-4*c^2*d + 2*(b*c + sqrt(b^2 - 4*a*c)*c)*e))*arctan(2*sqrt(1/2)*sqrt(e*x^2
 + d)/sqrt(-(2*a^2*c*d - a^2*b*e - sqrt(-4*(a^2*c*d^2 - a^2*b*d*e + a^3*e^2
)*a^2*c + (2*a^2*c*d - a^2*b*e)^2))/(a^2*c)))/((sqrt(b^2 - 4*a*c)*a^2*c*d^2
 - sqrt(b^2 - 4*a*c)*a^2*b*d*e + sqrt(b^2 - 4*a*c)*a^3*e^2)*abs(c)*abs(e))
- 1/2*sqrt(e*x^2 + d)/(a*x^2)
```

Mupad [B] (verification not implemented)

Time = 10.83 (sec) , antiderivative size = 19959, normalized size of antiderivative = 52.25

$$\int \frac{\sqrt{d+ex^2}}{x^3\,(a+bx^2+cx^4)}\,dx = \text{Too large to display}$$

[In] int((d + e*x^2)^(1/2)/(x^3*(a + b*x^2 + c*x^4)),x)

[Out] (atan((((a*e - 2*b*d)*(((d + e*x^2)^(1/2)*(6*a^4*c^5*e^12 + 4*a^2*c^7*d^4*e^8 + 6*a^3*c^6*d^2*e^10 + 4*b^4*c^5*d^4*e^8 + 21*a^2*b^2*c^5*d^2*e^10 - 18*a^3*b*c^5*d*e^11 - 8*a*b^2*c^6*d^4*e^8 - 12*a*b^3*c^5*d^3*e^9))/(2*a^4) - (((16*a^5*b*c^4*e^12 + 20*a^5*c^5*d*e^11 + a^3*b^5*c^2*e^12 - 8*a^4*b^3*c^3*e^12 + 20*a^4*c^6*d^3*e^9 + 40*a^2*b^3*c^5*d^4*e^8 - 20*a^2*b^4*c^4*d^3*e^9 - 27*a^2*b^5*c^3*d^2*e^10 - 20*a^3*b^2*c^5*d^3*e^9 + 84*a^3*b^3*c^4*d^2*e^10 - 8*a*b^5*c^4*d^4*e^8 + 6*a*b^6*c^3*d^3*e^9 + 2*a*b^7*c^2*d^2*e^10 - 3*a^2*b^6*c^2*d*e^11 - 32*a^3*b*c^6*d^4*e^8 + 28*a^3*b^4*c^3*d*e^11 - 36*a^4*b*c^5*d^2*e^10 - 68*a^4*b^2*c^4*d*e^11)/a^4 - ((a*e - 2*b*d)*(((d + e*x^2)^(1/2)*(240*a^6*b*c^4*e^11 + 64*a^6*c^5*d*e^10 + 20*a^4*b^5*c^2*e^11 - 140*a^5*b^3*c^3*e^11 + 160*a^5*c^6*d^3*e^8 - 32*a^2*b^6*c^3*d^3*e^8 + 32*a^2*b^7*c^2*d^2*e^9 + 224*a^3*b^4*c^4*d^3*e^8 - 208*a^3*b^5*c^3*d^2*e^9 - 432*a^4*b^2*c^5*d^3*e^8 + 272*a^4*b^3*c^4*d^2*e^9 - 48*a^3*b^6*c^2*d*e^10 + 348*a^4*b^4*c^3*d*e^10 + 224*a^5*b*c^5*d^2*e^9 - 648*a^5*b^2*c^4*d*e^10))/(2*a^4) - ((a*e - 2*b*d)*((128*a^8*c^4*e^11 + 8*a^6*b^4*c^2*e^11 - 64*a^7*b^2*c^3*e^11 + 128*a^7*c^5*d^2*e^9 + 32*a^5*b^3*c^4*d^3*e^8 - 24*a^5*b^4*c^3*d^2*e^9 + 64*a^6*b^2*c^4*d^2*e^9 - 256*a^7*b*c^4*d*e^10 - 8*a^5*b^5*c^2*d*e^10 - 128*a^6*b*c^5*d^3*e^8 + 96*a^6*b^3*c^3*d*e^10)/a^4 - ((d + e*x^2)^(1/2)*(a*e - 2*b*d)*(1024*a^9*c^4*e^10 + 64*a^7*b^4*c^2*e^10 - 512*a^8*b^2*c^3*e^10 + 1536*a^8*c^5*d^2*e^8 + 128*a^6*b^4*c^3*d^2*e^8 - 896*a^7*b^2*c^4*d^2*e^8 - 1792*a^8*b*c^4*d*e^9 - 128*a^6*b^5*c^2*d*e^9 + 960*a^7*b^3*c^3*d*e^9))/(8*a^6*d^(1/2))))/(4*a^2*d^(1/2))))/(4*a^2*d^(1/2)))*(a*e - 2*b*d))/(4*a^2*d^(1/2)))*1i)/(4*a^2*d^(1/2)) + ((a*e - 2*b*d)*(((d + e*x^2)^(1/2)*(6*a^4*c^5*e^12 + 4*a^2*c^7*d^4*e^8 + 6*a^3*c^6*d^2*e^10 + 4*b^4*c^5*d^4*e^8 + 21*a^2*b^2*c^5*d^2*e^10 - 18*a^3*b*c^5*d*e^11 - 8*a*b^2*c^6*d^4*e^8 - 12*a*b^3*c^5*d^3*e^9))/(2*a^4) + (((16*a^5*b*c^4*e^12 + 20*a^5*c^5*d*e^11 + a^3*b^5*c^2*e^12 - 8*a^4*b^3*c^3*e^12 + 20*a^4*c^6*d^3*e^9 + 40*a^2*b^3*c^5*d^4*e^8 - 20*a^2*b^4*c^4*d^3*e^9 - 27*a^2*b^5*c^3*d^2*e^10 - 20*a^3*b^2*c^5*d^3*e^9 + 84*a^3*b^3*c^4*d^2*e^10 - 8*a*b^5*c^4*d^4*e^8 + 6*a*b^6*c^3*d^3*e^9 + 2*a*b^7*c^2*d^2*e^10 - 3*a^2*b^6*c^2*d*e^11 - 32*a^3*b*c^6*d^4*e^8 + 28*a^3*b^4*c^3*d*e^11 - 36*a^4*b*c^5*d^2*e^10 - 68*a^4*b^2*c^4*d*e^11)/a^4 + ((a*e - 2*b*d)*(((d + e*x^2)^(1/2)*(240*a^6*b*c^4*e^11 + 64*a^6*c^5*d*e^10 + 20*a^4*b^5*c^2*e^11 - 140*a^5*b^3*c^3*e^11 + 160*a^5*c^6*d^3*e^8 - 32*a^2*b^6*c^3*d^3*e^8 + 32*a^2*b^7*c^2*d^2*e^9 + 224*a^3*b^4*c^4*d^3*e^8 - 208*a^3*b^5*c^3*d^2*e^9 - 432*a^4*b^2*c^5*d^3*e^8 + 272*a^4*b^3*c^4*d^2*e^9 - 48*a^3*b^6*c^2*d*e^10 + 348*a^4*b^4*c^3*d*e^10 + 224*a^5*b*c^5*d^2*e^9 - 648*a^5*b^2*c

^4*d*e^10))/(2*a^4) + ((a*e - 2*b*d)*((128*a^8*c^4*e^11 + 8*a^6*b^4*c^2*e^1
1 - 64*a^7*b^2*c^3*e^11 + 128*a^7*c^5*d^2*e^9 + 32*a^5*b^3*c^4*d^3*e^8 - 24
*a^5*b^4*c^3*d^2*e^9 + 64*a^6*b^2*c^4*d^2*e^9 - 256*a^7*b*c^4*d*e^10 - 8*a^
5*b^5*c^2*d*e^10 - 128*a^6*b*c^5*d^3*e^8 + 96*a^6*b^3*c^3*d*e^10)/a^4 + ((d
 + e*x^2)^(1/2)*(a*e - 2*b*d)*(1024*a^9*c^4*e^10 + 64*a^7*b^4*c^2*e^10 - 51
2*a^8*b^2*c^3*e^10 + 1536*a^8*c^5*d^2*e^8 + 128*a^6*b^4*c^3*d^2*e^8 - 896*a
^7*b^2*c^4*d^2*e^8 - 1792*a^8*b*c^4*d*e^9 - 128*a^6*b^5*c^2*d*e^9 + 960*a^7
*b^3*c^3*d*e^9))/(8*a^6*d^(1/2))))/(4*a^2*d^(1/2))))/(4*a^2*d^(1/2)))*(a*e
- 2*b*d))/(4*a^2*d^(1/2)))*1i)/(4*a^2*d^(1/2)))/(((a^3*c^5*e^13)/2 + a*c^7*
d^4*e^9 - 2*b*c^7*d^5*e^8 + (3*a^2*c^6*d^2*e^11)/2 + 2*b^2*c^6*d^4*e^9 - 4*
a*b*c^6*d^3*e^10 - (3*a^2*b*c^5*d*e^12)/2 + a*b^2*c^5*d^2*e^11)/a^4 - ((a*e
 - 2*b*d)*(((d + e*x^2)^(1/2)*(6*a^4*c^5*e^12 + 4*a^2*c^7*d^4*e^8 + 6*a^3*c
^6*d^2*e^10 + 4*b^4*c^5*d^4*e^8 + 21*a^2*b^2*c^5*d^2*e^10 - 18*a^3*b*c^5*d*
e^11 - 8*a*b^2*c^6*d^4*e^8 - 12*a*b^3*c^5*d^3*e^9))/(2*a^4) - (((16*a^5*b*c
^4*e^12 + 20*a^5*c^5*d*e^11 + a^3*b^5*c^2*e^12 - 8*a^4*b^3*c^3*e^12 + 20*a^
4*c^6*d^3*e^9 + 40*a^2*b^3*c^5*d^4*e^8 - 20*a^2*b^4*c^4*d^3*e^9 - 27*a^2*b^
5*c^3*d^2*e^10 - 20*a^3*b^2*c^5*d^3*e^9 + 84*a^3*b^3*c^4*d^2*e^10 - 8*a*b^5
*c^4*d^4*e^8 + 6*a*b^6*c^3*d^3*e^9 + 2*a*b^7*c^2*d^2*e^10 - 3*a^2*b^6*c^2*d
*e^11 - 32*a^3*b*c^6*d^4*e^8 + 28*a^3*b^4*c^3*d*e^11 - 36*a^4*b*c^5*d^2*e^1
0 - 68*a^4*b^2*c^4*d*e^11)/a^4 - ((a*e - 2*b*d)*(((d + e*x^2)^(1/2)*(240*a^
6*b*c^4*e^11 + 64*a^6*c^5*d*e^10 + 20*a^4*b^5*c^2*e^11 - 140*a^5*b^3*c^3*e^
11 + 160*a^5*c^6*d^3*e^8 - 32*a^2*b^6*c^3*d^3*e^8 + 32*a^2*b^7*c^2*d^2*e^9
+ 224*a^3*b^4*c^4*d^3*e^8 - 208*a^3*b^5*c^3*d^2*e^9 - 432*a^4*b^2*c^5*d^3*e
^8 + 272*a^4*b^3*c^4*d^2*e^9 - 48*a^3*b^6*c^2*d*e^10 + 348*a^4*b^4*c^3*d*e^
10 + 224*a^5*b*c^5*d^2*e^9 - 648*a^5*b^2*c^4*d*e^10))/(2*a^4) - ((a*e - 2*b
d)((128*a^8*c^4*e^11 + 8*a^6*b^4*c^2*e^11 - 64*a^7*b^2*c^3*e^11 + 128*a^7
*c^5*d^2*e^9 + 32*a^5*b^3*c^4*d^3*e^8 - 24*a^5*b^4*c^3*d^2*e^9 + 64*a^6*b^2
*c^4*d^2*e^9 - 256*a^7*b*c^4*d*e^10 - 8*a^5*b^5*c^2*d*e^10 - 128*a^6*b*c^5*
d^3*e^8 + 96*a^6*b^3*c^3*d*e^10)/a^4 - ((d + e*x^2)^(1/2)*(a*e - 2*b*d)*(10
24*a^9*c^4*e^10 + 64*a^7*b^4*c^2*e^10 - 512*a^8*b^2*c^3*e^10 + 1536*a^8*c^5
*d^2*e^8 + 128*a^6*b^4*c^3*d^2*e^8 - 896*a^7*b^2*c^4*d^2*e^8 - 1792*a^8*b*c
^4*d*e^9 - 128*a^6*b^5*c^2*d*e^9 + 960*a^7*b^3*c^3*d*e^9))/(8*a^6*d^(1/2)))
)/(4*a^2*d^(1/2))))/(4*a^2*d^(1/2)))*(a*e - 2*b*d))/(4*a^2*d^(1/2))))/(4*a^
2*d^(1/2)) + ((a*e - 2*b*d)*(((d + e*x^2)^(1/2)*(6*a^4*c^5*e^12 + 4*a^2*c^7
*d^4*e^8 + 6*a^3*c^6*d^2*e^10 + 4*b^4*c^5*d^4*e^8 + 21*a^2*b^2*c^5*d^2*e^10
 - 18*a^3*b*c^5*d*e^11 - 8*a*b^2*c^6*d^4*e^8 - 12*a*b^3*c^5*d^3*e^9))/(2*a^
4) + (((16*a^5*b*c^4*e^12 + 20*a^5*c^5*d*e^11 + a^3*b^5*c^2*e^12 - 8*a^4*b^
3*c^3*e^12 + 20*a^4*c^6*d^3*e^9 + 40*a^2*b^3*c^5*d^4*e^8 - 20*a^2*b^4*c^4*d
^3*e^9 - 27*a^2*b^5*c^3*d^2*e^10 - 20*a^3*b^2*c^5*d^3*e^9 + 84*a^3*b^3*c^4*
d^2*e^10 - 8*a*b^5*c^4*d^4*e^8 + 6*a*b^6*c^3*d^3*e^9 + 2*a*b^7*c^2*d^2*e^10
 - 3*a^2*b^6*c^2*d*e^11 - 32*a^3*b*c^6*d^4*e^8 + 28*a^3*b^4*c^3*d*e^11 - 36
*a^4*b*c^5*d^2*e^10 - 68*a^4*b^2*c^4*d*e^11)/a^4 + ((a*e - 2*b*d)*(((d + e*
x^2)^(1/2)*(240*a^6*b*c^4*e^11 + 64*a^6*c^5*d*e^10 + 20*a^4*b^5*c^2*e^11 -
140*a^5*b^3*c^3*e^11 + 160*a^5*c^6*d^3*e^8 - 32*a^2*b^6*c^3*d^3*e^8 + 32*a^
2*b^7*c^2*d^2*e^9 + 224*a^3*b^4*c^4*d^3*e^8 - 208*a^3*b^5*c^3*d^2*e^9 - 432

*a^4*b^2*c^5*d^3*e^8 + 272*a^4*b^3*c^4*d^2*e^9 - 48*a^3*b^6*c^2*d*e^10 + 348*a^4*b^4*c^3*d*e^10 + 224*a^5*b*c^5*d^2*e^9 - 648*a^5*b^2*c^4*d*e^10))/(2*a^4) + ((a*e - 2*b*d)*((128*a^8*c^4*e^11 + 8*a^6*b^4*c^2*e^11 - 64*a^7*b^2*c^3*e^11 + 128*a^7*c^5*d^2*e^9 + 32*a^5*b^3*c^4*d^3*e^8 - 24*a^5*b^4*c^3*d^2*e^9 + 64*a^6*b^2*c^4*d^2*e^9 - 256*a^7*b*c^4*d*e^10 - 8*a^5*b^5*c^2*d*e^10 - 128*a^6*b*c^5*d^3*e^8 + 96*a^6*b^3*c^3*d*e^10)/a^4 + ((d + e*x^2)^(1/2)*(a*e - 2*b*d)*(1024*a^9*c^4*e^10 + 64*a^7*b^4*c^2*e^10 - 512*a^8*b^2*c^3*e^10 + 1536*a^8*c^5*d^2*e^8 + 128*a^6*b^4*c^3*d^2*e^8 - 896*a^7*b^2*c^4*d^2*e^8 - 1792*a^8*b*c^4*d*e^9 - 128*a^6*b^5*c^2*d*e^9 + 960*a^7*b^3*c^3*d*e^9))/(8*a^6*d^(1/2))))/(4*a^2*d^(1/2))))/(4*a^2*d^(1/2)))*(a*e - 2*b*d))/(4*a^2*d^(1/2))))/(4*a^2*d^(1/2))))*(a*e - 2*b*d)*1i)/(2*a^2*d^(1/2)) - atan((((64*a^5*b*c^4*e^12 + 80*a^5*c^5*d*e^11 + 4*a^3*b^5*c^2*e^12 - 32*a^4*b^3*c^3*e^12 + 80*a^4*c^6*d^3*e^9 + 160*a^2*b^3*c^5*d^4*e^8 - 80*a^2*b^4*c^4*d^3*e^9 - 108*a^2*b^5*c^3*d^2*e^10 - 80*a^3*b^2*c^5*d^3*e^9 + 336*a^3*b^3*c^4*d^2*e^10 - 32*a*b^5*c^4*d^4*e^8 + 24*a*b^6*c^3*d^3*e^9 + 8*a*b^7*c^2*d^2*e^10 - 12*a^2*b^6*c^2*d*e^11 - 128*a^3*b*c^6*d^4*e^8 + 112*a^3*b^4*c^3*d*e^11 - 144*a^4*b*c^5*d^2*e^10 - 272*a^4*b^2*c^4*d*e^11)/(4*a^4) + (((512*a^8*c^4*e^11 + 32*a^6*b^4*c^2*e^11 - 256*a^7*b^2*c^3*e^11 + 512*a^7*c^5*d^2*e^9 + 128*a^5*b^3*c^4*d^3*e^8 - 96*a^5*b^4*c^3*d^2*e^9 + 256*a^6*b^2*c^4*d^2*e^9 - 1024*a^7*b*c^4*d*e^10 - 32*a^5*b^5*c^2*d*e^10 - 512*a^6*b*c^5*d^3*e^8 + 384*a^6*b^3*c^3*d*e^10)/(4*a^4) - ((d + e*x^2)^(1/2)*(-(8*a^3*c^3*d - b^6*d - b^3*d*(-(4*a*c - b^2)^3)^(1/2) + a*b^5*e - 18*a^2*b^2*c^2*d + 8*a*b^4*c*d + a*b^2*e*(-(4*a*c - b^2)^3)^(1/2) - 7*a^2*b^3*c*e + 12*a^3*b*c^2*e - a^2*c*e*(-(4*a*c - b^2)^3)^(1/2) + 2*a*b*c*d*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^4*b^4 + 16*a^6*c^2 - 8*a^5*b^2*c)))^(1/2)*(1024*a^9*c^4*e^10 + 64*a^7*b^4*c^2*e^10 - 512*a^8*b^2*c^3*e^10 + 1536*a^8*c^5*d^2*e^8 + 128*a^6*b^4*c^3*d^2*e^8 - 896*a^7*b^2*c^4*d^2*e^8 - 1792*a^8*b*c^4*d*e^9 - 128*a^6*b^5*c^2*d*e^9 + 960*a^7*b^3*c^3*d*e^9))/(2*a^4))*(-(8*a^3*c^3*d - b^6*d - b^3*d*(-(4*a*c - b^2)^3)^(1/2) + a*b^5*e - 18*a^2*b^2*c^2*d + 8*a*b^4*c*d + a*b^2*e*(-(4*a*c - b^2)^3)^(1/2) - 7*a^2*b^3*c*e + 12*a^3*b*c^2*e - a^2*c*e*(-(4*a*c - b^2)^3)^(1/2) + 2*a*b*c*d*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^4*b^4 + 16*a^6*c^2 - 8*a^5*b^2*c)))^(1/2) - ((d + e*x^2)^(1/2)*(240*a^6*b*c^4*e^11 + 64*a^6*c^5*d*e^10 + 20*a^4*b^5*c^2*e^11 - 140*a^5*b^3*c^3*e^11 + 160*a^5*c^6*d^3*e^8 - 32*a^2*b^6*c^3*d^3*e^8 + 32*a^2*b^7*c^2*d^2*e^9 + 224*a^3*b^4*c^4*d^3*e^8 - 208*a^3*b^5*c^3*d^2*e^9 - 432*a^4*b^2*c^5*d^3*e^8 + 272*a^4*b^3*c^4*d^2*e^9 - 48*a^3*b^6*c^2*d*e^10 + 348*a^4*b^4*c^3*d*e^10 + 224*a^5*b*c^5*d^2*e^9 - 648*a^5*b^2*c^4*d*e^10))/(2*a^4))*(-(8*a^3*c^3*d - b^6*d - b^3*d*(-(4*a*c - b^2)^3)^(1/2) + a*b^5*e - 18*a^2*b^2*c^2*d + 8*a*b^4*c*d + a*b^2*e*(-(4*a*c - b^2)^3)^(1/2) - 7*a^2*b^3*c*e + 12*a^3*b*c^2*e - a^2*c*e*(-(4*a*c - b^2)^3)^(1/2) + 2*a*b*c*d*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^4*b^4 + 16*a^6*c^2 - 8*a^5*b^2*c)))^(1/2))*(-(8*a^3*c^3*d - b^6*d - b^3*d*(-(4*a*c - b^2)^3)^(1/2) + a*b^5*e - 18*a^2*b^2*c^2*d + 8*a*b^4*c*d + a*b^2*e*(-(4*a*c - b^2)^3)^(1/2) - 7*a^2*b^3*c*e + 12*a^3*b*c^2*e - a^2*c*e*(-(4*a*c - b^2)^3)^(1/2) + 2*a*b*c*d*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^4*b^4 + 16*a^6*c^2 - 8*a^5*b^2*c)))^(1/2) - ((d + e*x^2)^(1/2)*(6*a^4*c^5*e^12 + 4*a^2*c^7*d^4*e^8 +

6*a^3*c^6*d^2*e^10 + 4*b^4*c^5*d^4*e^8 + 21*a^2*b^2*c^5*d^2*e^10 - 18*a^3*b*c^5*d*e^11 - 8*a*b^2*c^6*d^4*e^8 - 12*a*b^3*c^5*d^3*e^9))/(2*a^4))*(-(8*a^3*c^3*d - b^6*d - b^3*d*(-(4*a*c - b^2)^3)^(1/2) + a*b^5*e - 18*a^2*b^2*c^2*d + 8*a*b^4*c*d + a*b^2*e*(-(4*a*c - b^2)^3)^(1/2) - 7*a^2*b^3*c*e + 12*a^3*b*c^2*e - a^2*c*e*(-(4*a*c - b^2)^3)^(1/2) + 2*a*b*c*d*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^4*b^4 + 16*a^6*c^2 - 8*a^5*b^2*c)))^(1/2)*1i - (((64*a^5*b*c^4*e^12 + 80*a^5*c^5*d*e^11 + 4*a^3*b^5*c^2*e^12 - 32*a^4*b^3*c^3*e^12 + 80*a^4*c^6*d^3*e^9 + 160*a^2*b^3*c^5*d^4*e^8 - 80*a^2*b^4*c^4*d^3*e^9 - 108*a^2*b^5*c^3*d^2*e^10 - 80*a^3*b^2*c^5*d^3*e^9 + 336*a^3*b^3*c^4*d^2*e^10 - 32*a*b^5*c^4*d^4*e^8 + 24*a*b^6*c^3*d^3*e^9 + 8*a*b^7*c^2*d^2*e^10 - 12*a^2*b^6*c^2*d*e^11 - 128*a^3*b*c^6*d^4*e^8 + 112*a^3*b^4*c^3*d*e^11 - 144*a^4*b*c^5*d^2*e^10 - 272*a^4*b^2*c^4*d*e^11)/(4*a^4) + (((512*a^8*c^4*e^11 + 32*a^6*b^4*c^2*e^11 - 256*a^7*b^2*c^3*e^11 + 512*a^7*c^5*d^2*e^9 + 128*a^5*b^3*c^4*d^3*e^8 - 96*a^5*b^4*c^3*d^2*e^9 + 256*a^6*b^2*c^4*d^2*e^9 - 1024*a^7*b*c^4*d*e^10 - 32*a^5*b^5*c^2*d*e^10 - 512*a^6*b*c^5*d^3*e^8 + 384*a^6*b^3*c^3*d*e^10)/(4*a^4) + ((d + e*x^2)^(1/2)*(-(8*a^3*c^3*d - b^6*d - b^3*d*(-(4*a*c - b^2)^3)^(1/2) + a*b^5*e - 18*a^2*b^2*c^2*d + 8*a*b^4*c*d + a*b^2*e*(-(4*a*c - b^2)^3)^(1/2) - 7*a^2*b^3*c*e + 12*a^3*b*c^2*e - a^2*c*e*(-(4*a*c - b^2)^3)^(1/2) + 2*a*b*c*d*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^4*b^4 + 16*a^6*c^2 - 8*a^5*b^2*c)))^(1/2)*(1024*a^9*c^4*e^10 + 64*a^7*b^4*c^2*e^10 - 512*a^8*b^2*c^3*e^10 + 1536*a^8*c^5*d^2*e^8 + 128*a^6*b^4*c^3*d^2*e^8 - 896*a^7*b^2*c^4*d^2*e^8 - 1792*a^8*b*c^4*d*e^9 - 128*a^6*b^5*c^2*d*e^9 + 960*a^7*b^3*c^3*d*e^9))/(2*a^4))*(-(8*a^3*c^3*d - b^6*d - b^3*d*(-(4*a*c - b^2)^3)^(1/2) + a*b^5*e - 18*a^2*b^2*c^2*d + 8*a*b^4*c*d + a*b^2*e*(-(4*a*c - b^2)^3)^(1/2) - 7*a^2*b^3*c*e + 12*a^3*b*c^2*e - a^2*c*e*(-(4*a*c - b^2)^3)^(1/2) + 2*a*b*c*d*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^4*b^4 + 16*a^6*c^2 - 8*a^5*b^2*c)))^(1/2) + ((d + e*x^2)^(1/2)*(240*a^6*b*c^4*e^11 + 64*a^6*c^5*d*e^10 + 20*a^4*b^5*c^2*e^11 - 140*a^5*b^3*c^3*e^11 + 160*a^5*c^6*d^3*e^8 - 32*a^2*b^6*c^3*d^3*e^8 + 32*a^2*b^7*c^2*d^2*e^9 + 224*a^3*b^4*c^4*d^3*e^8 - 208*a^3*b^5*c^3*d^2*e^9 - 432*a^4*b^2*c^5*d^3*e^8 + 272*a^4*b^3*c^4*d^2*e^9 - 48*a^3*b^6*c^2*d*e^10 + 348*a^4*b^4*c^3*d*e^10 + 224*a^5*b*c^5*d^2*e^9 - 648*a^5*b^2*c^4*d*e^10))/(2*a^4))*(-(8*a^3*c^3*d - b^6*d - b^3*d*(-(4*a*c - b^2)^3)^(1/2) + a*b^5*e - 18*a^2*b^2*c^2*d + 8*a*b^4*c*d + a*b^2*e*(-(4*a*c - b^2)^3)^(1/2) - 7*a^2*b^3*c*e + 12*a^3*b*c^2*e - a^2*c*e*(-(4*a*c - b^2)^3)^(1/2) + 2*a*b*c*d*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^4*b^4 + 16*a^6*c^2 - 8*a^5*b^2*c)))^(1/2))*(-(8*a^3*c^3*d - b^6*d - b^3*d*(-(4*a*c - b^2)^3)^(1/2) + a*b^5*e - 18*a^2*b^2*c^2*d + 8*a*b^4*c*d + a*b^2*e*(-(4*a*c - b^2)^3)^(1/2) - 7*a^2*b^3*c*e + 12*a^3*b*c^2*e - a^2*c*e*(-(4*a*c - b^2)^3)^(1/2) + 2*a*b*c*d*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^4*b^4 + 16*a^6*c^2 - 8*a^5*b^2*c)))^(1/2) + ((d + e*x^2)^(1/2)*(6*a^4*c^5*e^12 + 4*a^2*c^7*d^4*e^8 + 6*a^3*c^6*d^2*e^10 + 4*b^4*c^5*d^4*e^8 + 21*a^2*b^2*c^5*d^2*e^10 - 18*a^3*b*c^5*d*e^11 - 8*a*b^2*c^6*d^4*e^8 - 12*a*b^3*c^5*d^3*e^9))/(2*a^4))*(-(8*a^3*c^3*d - b^6*d - b^3*d*(-(4*a*c - b^2)^3)^(1/2) + a*b^5*e - 18*a^2*b^2*c^2*d + 8*a*b^4*c*d + a*b^2*e*(-(4*a*c - b^2)^3)^(1/2) - 7*a^2*b^3*c*e + 12*a^3*b*c^2*e - a^2*c*e*(-(4*a*c - b^2)^3)^(1/2) + 2*a*b*c*d*(-(4*a*c - b^2)^3)^(1/2))/(8

*(a^4*b^4 + 16*a^6*c^2 - 8*a^5*b^2*c)))^(1/2)*1i)/((a^3*c^5*e^13 + 2*a*c^7*
d^4*e^9 - 4*b*c^7*d^5*e^8 + 3*a^2*c^6*d^2*e^11 + 4*b^2*c^6*d^4*e^9 - 8*a*b*
c^6*d^3*e^10 - 3*a^2*b*c^5*d*e^12 + 2*a*b^2*c^5*d^2*e^11)/(2*a^4) + (((64*a
^5*b*c^4*e^12 + 80*a^5*c^5*d*e^11 + 4*a^3*b^5*c^2*e^12 - 32*a^4*b^3*c^3*e^1
2 + 80*a^4*c^6*d^3*e^9 + 160*a^2*b^3*c^5*d^4*e^8 - 80*a^2*b^4*c^4*d^3*e^9 -
108*a^2*b^5*c^3*d^2*e^10 - 80*a^3*b^2*c^5*d^3*e^9 + 336*a^3*b^3*c^4*d^2*e^
10 - 32*a*b^5*c^4*d^4*e^8 + 24*a*b^6*c^3*d^3*e^9 + 8*a*b^7*c^2*d^2*e^10 - 1
2*a^2*b^6*c^2*d*e^11 - 128*a^3*b*c^6*d^4*e^8 + 112*a^3*b^4*c^3*d*e^11 - 144
*a^4*b*c^5*d^2*e^10 - 272*a^4*b^2*c^4*d*e^11)/(4*a^4) + (((512*a^8*c^4*e^11
+ 32*a^6*b^4*c^2*e^11 - 256*a^7*b^2*c^3*e^11 + 512*a^7*c^5*d^2*e^9 + 128*a
^5*b^3*c^4*d^3*e^8 - 96*a^5*b^4*c^3*d^2*e^9 + 256*a^6*b^2*c^4*d^2*e^9 - 102
4*a^7*b*c^4*d*e^10 - 32*a^5*b^5*c^2*d*e^10 - 512*a^6*b*c^5*d^3*e^8 + 384*a^
6*b^3*c^3*d*e^10)/(4*a^4) - ((d + e*x^2)^(1/2)*(-(8*a^3*c^3*d - b^6*d - b^3
d(-(4*a*c - b^2)^3)^(1/2) + a*b^5*e - 18*a^2*b^2*c^2*d + 8*a*b^4*c*d + a*
b^2*e*(-(4*a*c - b^2)^3)^(1/2) - 7*a^2*b^3*c*e + 12*a^3*b*c^2*e - a^2*c*e*(
-(4*a*c - b^2)^3)^(1/2) + 2*a*b*c*d*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^4*b^4 +
16*a^6*c^2 - 8*a^5*b^2*c)))^(1/2)*(1024*a^9*c^4*e^10 + 64*a^7*b^4*c^2*e^10
- 512*a^8*b^2*c^3*e^10 + 1536*a^8*c^5*d^2*e^8 + 128*a^6*b^4*c^3*d^2*e^8 -
896*a^7*b^2*c^4*d^2*e^8 - 1792*a^8*b*c^4*d*e^9 - 128*a^6*b^5*c^2*d*e^9 + 96
0*a^7*b^3*c^3*d*e^9))/(2*a^4))*(-(8*a^3*c^3*d - b^6*d - b^3*d*(-(4*a*c - b^
2)^3)^(1/2) + a*b^5*e - 18*a^2*b^2*c^2*d + 8*a*b^4*c*d + a*b^2*e*(-(4*a*c -
b^2)^3)^(1/2) - 7*a^2*b^3*c*e + 12*a^3*b*c^2*e - a^2*c*e*(-(4*a*c - b^2)^3
)^(1/2) + 2*a*b*c*d*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^4*b^4 + 16*a^6*c^2 - 8*
a^5*b^2*c)))^(1/2) - ((d + e*x^2)^(1/2)*(240*a^6*b*c^4*e^11 + 64*a^6*c^5*d*
e^10 + 20*a^4*b^5*c^2*e^11 - 140*a^5*b^3*c^3*e^11 + 160*a^5*c^6*d^3*e^8 - 3
2*a^2*b^6*c^3*d^3*e^8 + 32*a^2*b^7*c^2*d^2*e^9 + 224*a^3*b^4*c^4*d^3*e^8 -
208*a^3*b^5*c^3*d^2*e^9 - 432*a^4*b^2*c^5*d^3*e^8 + 272*a^4*b^3*c^4*d^2*e^9
- 48*a^3*b^6*c^2*d*e^10 + 348*a^4*b^4*c^3*d*e^10 + 224*a^5*b*c^5*d^2*e^9 -
648*a^5*b^2*c^4*d*e^10))/(2*a^4))*(-(8*a^3*c^3*d - b^6*d - b^3*d*(-(4*a*c
- b^2)^3)^(1/2) + a*b^5*e - 18*a^2*b^2*c^2*d + 8*a*b^4*c*d + a*b^2*e*(-(4*a
*c - b^2)^3)^(1/2) - 7*a^2*b^3*c*e + 12*a^3*b*c^2*e - a^2*c*e*(-(4*a*c - b^
2)^3)^(1/2) + 2*a*b*c*d*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^4*b^4 + 16*a^6*c^2
- 8*a^5*b^2*c)))^(1/2))*(-(8*a^3*c^3*d - b^6*d - b^3*d*(-(4*a*c - b^2)^3)^(
1/2) + a*b^5*e - 18*a^2*b^2*c^2*d + 8*a*b^4*c*d + a*b^2*e*(-(4*a*c - b^2)^3
)^(1/2) - 7*a^2*b^3*c*e + 12*a^3*b*c^2*e - a^2*c*e*(-(4*a*c - b^2)^3)^(1/2)
+ 2*a*b*c*d*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^4*b^4 + 16*a^6*c^2 - 8*a^5*b^2
*c)))^(1/2) - ((d + e*x^2)^(1/2)*(6*a^4*c^5*e^12 + 4*a^2*c^7*d^4*e^8 + 6*a^
3*c^6*d^2*e^10 + 4*b^4*c^5*d^4*e^8 + 21*a^2*b^2*c^5*d^2*e^10 - 18*a^3*b*c^5
*d*e^11 - 8*a*b^2*c^6*d^4*e^8 - 12*a*b^3*c^5*d^3*e^9))/(2*a^4))*(-(8*a^3*c^
3*d - b^6*d - b^3*d*(-(4*a*c - b^2)^3)^(1/2) + a*b^5*e - 18*a^2*b^2*c^2*d +
8*a*b^4*c*d + a*b^2*e*(-(4*a*c - b^2)^3)^(1/2) - 7*a^2*b^3*c*e + 12*a^3*b*
c^2*e - a^2*c*e*(-(4*a*c - b^2)^3)^(1/2) + 2*a*b*c*d*(-(4*a*c - b^2)^3)^(1/
2))/(8*(a^4*b^4 + 16*a^6*c^2 - 8*a^5*b^2*c)))^(1/2) + (((64*a^5*b*c^4*e^12
+ 80*a^5*c^5*d*e^11 + 4*a^3*b^5*c^2*e^12 - 32*a^4*b^3*c^3*e^12 + 80*a^4*c^6
*d^3*e^9 + 160*a^2*b^3*c^5*d^4*e^8 - 80*a^2*b^4*c^4*d^3*e^9 - 108*a^2*b^5*c

^3*d^2*e^10 - 80*a^3*b^2*c^5*d^3*e^9 + 336*a^3*b^3*c^4*d^2*e^10 - 32*a*b^5*
c^4*d^4*e^8 + 24*a*b^6*c^3*d^3*e^9 + 8*a*b^7*c^2*d^2*e^10 - 12*a^2*b^6*c^2*
d*e^11 - 128*a^3*b*c^6*d^4*e^8 + 112*a^3*b^4*c^3*d*e^11 - 144*a^4*b*c^5*d^2
*e^10 - 272*a^4*b^2*c^4*d*e^11)/(4*a^4) + (((512*a^8*c^4*e^11 + 32*a^6*b^4*
c^2*e^11 - 256*a^7*b^2*c^3*e^11 + 512*a^7*c^5*d^2*e^9 + 128*a^5*b^3*c^4*d^3
*e^8 - 96*a^5*b^4*c^3*d^2*e^9 + 256*a^6*b^2*c^4*d^2*e^9 - 1024*a^7*b*c^4*d*
e^10 - 32*a^5*b^5*c^2*d*e^10 - 512*a^6*b*c^5*d^3*e^8 + 384*a^6*b^3*c^3*d*e^
10)/(4*a^4) + ((d + e*x^2)^(1/2)*(-(8*a^3*c^3*d - b^6*d - b^3*d*(-(4*a*c -
b^2)^3)^(1/2) + a*b^5*e - 18*a^2*b^2*c^2*d + 8*a*b^4*c*d + a*b^2*e*(-(4*a*c
 - b^2)^3)^(1/2) - 7*a^2*b^3*c*e + 12*a^3*b*c^2*e - a^2*c*e*(-(4*a*c - b^2)
^3)^(1/2) + 2*a*b*c*d*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^4*b^4 + 16*a^6*c^2 -
8*a^5*b^2*c)))^(1/2)*(1024*a^9*c^4*e^10 + 64*a^7*b^4*c^2*e^10 - 512*a^8*b^2
*c^3*e^10 + 1536*a^8*c^5*d^2*e^8 + 128*a^6*b^4*c^3*d^2*e^8 - 896*a^7*b^2*c^
4*d^2*e^8 - 1792*a^8*b*c^4*d*e^9 - 128*a^6*b^5*c^2*d*e^9 + 960*a^7*b^3*c^3*
d*e^9))/(2*a^4))*(-(8*a^3*c^3*d - b^6*d - b^3*d*(-(4*a*c - b^2)^3)^(1/2) +
a*b^5*e - 18*a^2*b^2*c^2*d + 8*a*b^4*c*d + a*b^2*e*(-(4*a*c - b^2)^3)^(1/2)
 - 7*a^2*b^3*c*e + 12*a^3*b*c^2*e - a^2*c*e*(-(4*a*c - b^2)^3)^(1/2) + 2*a*
b*c*d*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^4*b^4 + 16*a^6*c^2 - 8*a^5*b^2*c)))^(
1/2) + ((d + e*x^2)^(1/2)*(240*a^6*b*c^4*e^11 + 64*a^6*c^5*d*e^10 + 20*a^4*
b^5*c^2*e^11 - 140*a^5*b^3*c^3*e^11 + 160*a^5*c^6*d^3*e^8 - 32*a^2*b^6*c^3*
d^3*e^8 + 32*a^2*b^7*c^2*d^2*e^9 + 224*a^3*b^4*c^4*d^3*e^8 - 208*a^3*b^5*c^
3*d^2*e^9 - 432*a^4*b^2*c^5*d^3*e^8 + 272*a^4*b^3*c^4*d^2*e^9 - 48*a^3*b^6*
c^2*d*e^10 + 348*a^4*b^4*c^3*d*e^10 + 224*a^5*b*c^5*d^2*e^9 - 648*a^5*b^2*c
^4*d*e^10))/(2*a^4))*(-(8*a^3*c^3*d - b^6*d - b^3*d*(-(4*a*c - b^2)^3)^(1/2
) + a*b^5*e - 18*a^2*b^2*c^2*d + 8*a*b^4*c*d + a*b^2*e*(-(4*a*c - b^2)^3)^(
1/2) - 7*a^2*b^3*c*e + 12*a^3*b*c^2*e - a^2*c*e*(-(4*a*c - b^2)^3)^(1/2) +
2*a*b*c*d*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^4*b^4 + 16*a^6*c^2 - 8*a^5*b^2*c)
))^(1/2))*(-(8*a^3*c^3*d - b^6*d - b^3*d*(-(4*a*c - b^2)^3)^(1/2) + a*b^5*e
 - 18*a^2*b^2*c^2*d + 8*a*b^4*c*d + a*b^2*e*(-(4*a*c - b^2)^3)^(1/2) - 7*a^
2*b^3*c*e + 12*a^3*b*c^2*e - a^2*c*e*(-(4*a*c - b^2)^3)^(1/2) + 2*a*b*c*d*(
-(4*a*c - b^2)^3)^(1/2))/(8*(a^4*b^4 + 16*a^6*c^2 - 8*a^5*b^2*c)))^(1/2) +
((d + e*x^2)^(1/2)*(6*a^4*c^5*e^12 + 4*a^2*c^7*d^4*e^8 + 6*a^3*c^6*d^2*e^10
 + 4*b^4*c^5*d^4*e^8 + 21*a^2*b^2*c^5*d^2*e^10 - 18*a^3*b*c^5*d*e^11 - 8*a*
b^2*c^6*d^4*e^8 - 12*a*b^3*c^5*d^3*e^9))/(2*a^4))*(-(8*a^3*c^3*d - b^6*d -
b^3*d*(-(4*a*c - b^2)^3)^(1/2) + a*b^5*e - 18*a^2*b^2*c^2*d + 8*a*b^4*c*d +
 a*b^2*e*(-(4*a*c - b^2)^3)^(1/2) - 7*a^2*b^3*c*e + 12*a^3*b*c^2*e - a^2*c*
e*(-(4*a*c - b^2)^3)^(1/2) + 2*a*b*c*d*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^4*b^
4 + 16*a^6*c^2 - 8*a^5*b^2*c)))^(1/2)))*(-(8*a^3*c^3*d - b^6*d - b^3*d*(-(4
*a*c - b^2)^3)^(1/2) + a*b^5*e - 18*a^2*b^2*c^2*d + 8*a*b^4*c*d + a*b^2*e*(
-(4*a*c - b^2)^3)^(1/2) - 7*a^2*b^3*c*e + 12*a^3*b*c^2*e - a^2*c*e*(-(4*a*c
 - b^2)^3)^(1/2) + 2*a*b*c*d*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^4*b^4 + 16*a^6
*c^2 - 8*a^5*b^2*c)))^(1/2)*2i - (d + e*x^2)^(1/2)/(2*a*x^2) - atan(((((64*
a^5*b*c^4*e^12 + 80*a^5*c^5*d*e^11 + 4*a^3*b^5*c^2*e^12 - 32*a^4*b^3*c^3*e^
12 + 80*a^4*c^6*d^3*e^9 + 160*a^2*b^3*c^5*d^4*e^8 - 80*a^2*b^4*c^4*d^3*e^9
- 108*a^2*b^5*c^3*d^2*e^10 - 80*a^3*b^2*c^5*d^3*e^9 + 336*a^3*b^3*c^4*d^2*e

^10 - 32*a*b^5*c^4*d^4*e^8 + 24*a*b^6*c^3*d^3*e^9 + 8*a*b^7*c^2*d^2*e^10 - 12*a^2*b^6*c^2*d*e^11 - 128*a^3*b*c^6*d^4*e^8 + 112*a^3*b^4*c^3*d*e^11 - 144*a^4*b*c^5*d^2*e^10 - 272*a^4*b^2*c^4*d*e^11)/(4*a^4) + (((512*a^8*c^4*e^11 + 32*a^6*b^4*c^2*e^11 - 256*a^7*b^2*c^3*e^11 + 512*a^7*c^5*d^2*e^9 + 128*a^5*b^3*c^4*d^3*e^8 - 96*a^5*b^4*c^3*d^2*e^9 + 256*a^6*b^2*c^4*d^2*e^9 - 1024*a^7*b*c^4*d*e^10 - 32*a^5*b^5*c^2*d*e^10 - 512*a^6*b*c^5*d^3*e^8 + 384*a^6*b^3*c^3*d*e^10)/(4*a^4) - ((d + e*x^2)^(1/2)*(-(8*a^3*c^3*d - b^6*d + b^3*d*(-(4*a*c - b^2)^3)^(1/2) + a*b^5*e - 18*a^2*b^2*c^2*d + 8*a*b^4*c*d - a*b^2*e*(-(4*a*c - b^2)^3)^(1/2) - 7*a^2*b^3*c*e + 12*a^3*b*c^2*e + a^2*c*e*(-(4*a*c - b^2)^3)^(1/2) - 2*a*b*c*d*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^4*b^4 + 16*a^6*c^2 - 8*a^5*b^2*c)))^(1/2)*(1024*a^9*c^4*e^10 + 64*a^7*b^4*c^2*e^10 - 512*a^8*b^2*c^3*e^10 + 1536*a^8*c^5*d^2*e^8 + 128*a^6*b^4*c^3*d^2*e^8 - 896*a^7*b^2*c^4*d^2*e^8 - 1792*a^8*b*c^4*d*e^9 - 128*a^6*b^5*c^2*d*e^9 + 960*a^7*b^3*c^3*d*e^9))/(2*a^4))*(-(8*a^3*c^3*d - b^6*d + b^3*d*(-(4*a*c - b^2)^3)^(1/2) + a*b^5*e - 18*a^2*b^2*c^2*d + 8*a*b^4*c*d - a*b^2*e*(-(4*a*c - b^2)^3)^(1/2) - 7*a^2*b^3*c*e + 12*a^3*b*c^2*e + a^2*c*e*(-(4*a*c - b^2)^3)^(1/2) - 2*a*b*c*d*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^4*b^4 + 16*a^6*c^2 - 8*a^5*b^2*c)))^(1/2) - ((d + e*x^2)^(1/2)*(240*a^6*b*c^4*e^11 + 64*a^6*c^5*d*e^10 + 20*a^4*b^5*c^2*e^11 - 140*a^5*b^3*c^3*e^11 + 160*a^5*c^6*d^3*e^8 - 32*a^2*b^6*c^3*d^3*e^8 + 32*a^2*b^7*c^2*d^2*e^9 + 224*a^3*b^4*c^4*d^3*e^8 - 208*a^3*b^5*c^3*d^2*e^9 - 432*a^4*b^2*c^5*d^3*e^8 + 272*a^4*b^3*c^4*d^2*e^9 - 48*a^3*b^6*c^2*d*e^10 + 348*a^4*b^4*c^3*d*e^10 + 224*a^5*b*c^5*d^2*e^9 - 648*a^5*b^2*c^4*d*e^10))/(2*a^4))*(-(8*a^3*c^3*d - b^6*d + b^3*d*(-(4*a*c - b^2)^3)^(1/2) + a*b^5*e - 18*a^2*b^2*c^2*d + 8*a*b^4*c*d - a*b^2*e*(-(4*a*c - b^2)^3)^(1/2) - 7*a^2*b^3*c*e + 12*a^3*b*c^2*e + a^2*c*e*(-(4*a*c - b^2)^3)^(1/2) - 2*a*b*c*d*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^4*b^4 + 16*a^6*c^2 - 8*a^5*b^2*c)))^(1/2))*(-(8*a^3*c^3*d - b^6*d + b^3*d*(-(4*a*c - b^2)^3)^(1/2) + a*b^5*e - 18*a^2*b^2*c^2*d + 8*a*b^4*c*d - a*b^2*e*(-(4*a*c - b^2)^3)^(1/2) - 7*a^2*b^3*c*e + 12*a^3*b*c^2*e + a^2*c*e*(-(4*a*c - b^2)^3)^(1/2) - 2*a*b*c*d*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^4*b^4 + 16*a^6*c^2 - 8*a^5*b^2*c)))^(1/2) - ((d + e*x^2)^(1/2)*(6*a^4*c^5*e^12 + 4*a^2*c^7*d^4*e^8 + 6*a^3*c^6*d^2*e^10 + 4*b^4*c^5*d^4*e^8 + 21*a^2*b^2*c^5*d^2*e^10 - 18*a^3*b*c^5*d*e^11 - 8*a*b^2*c^6*d^4*e^8 - 12*a*b^3*c^5*d^3*e^9))/(2*a^4))*(-(8*a^3*c^3*d - b^6*d + b^3*d*(-(4*a*c - b^2)^3)^(1/2) + a*b^5*e - 18*a^2*b^2*c^2*d + 8*a*b^4*c*d - a*b^2*e*(-(4*a*c - b^2)^3)^(1/2) - 7*a^2*b^3*c*e + 12*a^3*b*c^2*e + a^2*c*e*(-(4*a*c - b^2)^3)^(1/2) - 2*a*b*c*d*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^4*b^4 + 16*a^6*c^2 - 8*a^5*b^2*c)))^(1/2)*1i - (((64*a^5*b*c^4*e^12 + 80*a^5*c^5*d*e^11 + 4*a^3*b^5*c^2*e^12 - 32*a^4*b^3*c^3*e^12 + 80*a^4*c^6*d^3*e^9 + 160*a^2*b^3*c^5*d^4*e^8 - 80*a^2*b^4*c^4*d^3*e^9 - 108*a^2*b^5*c^3*d^2*e^10 - 80*a^3*b^2*c^5*d^3*e^9 + 336*a^3*b^3*c^4*d^2*e^10 - 32*a*b^5*c^4*d^4*e^8 + 24*a*b^6*c^3*d^3*e^9 + 8*a*b^7*c^2*d^2*e^10 - 12*a^2*b^6*c^2*d*e^11 - 128*a^3*b*c^6*d^4*e^8 + 112*a^3*b^4*c^3*d*e^11 - 144*a^4*b*c^5*d^2*e^10 - 272*a^4*b^2*c^4*d*e^11)/(4*a^4) + (((512*a^8*c^4*e^11 + 32*a^6*b^4*c^2*e^11 - 256*a^7*b^2*c^3*e^11 + 512*a^7*c^5*d^2*e^9 + 128*a^5*b^3*c^4*d^3*e^8 - 96*a^5*b^4*c^3*d^2*e^9 + 256*a^6*b^2*c^4*d^2*e^9 - 1024*a^7*b*c^

4*d*e^10 - 32*a^5*b^5*c^2*d*e^10 - 512*a^6*b*c^5*d^3*e^8 + 384*a^6*b^3*c^3*d*e^10)/(4*a^4) + ((d + e*x^2)^(1/2)*(-(8*a^3*c^3*d - b^6*d + b^3*d*(-(4*a*c - b^2)^3)^(1/2) + a*b^5*e - 18*a^2*b^2*c^2*d + 8*a*b^4*c*d - a*b^2*e*(-(4*a*c - b^2)^3)^(1/2) - 7*a^2*b^3*c*e + 12*a^3*b*c^2*e + a^2*c*e*(-(4*a*c - b^2)^3)^(1/2) - 2*a*b*c*d*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^4*b^4 + 16*a^6*c^2 - 8*a^5*b^2*c)))^(1/2)*(1024*a^9*c^4*e^10 + 64*a^7*b^4*c^2*e^10 - 512*a^8*b^2*c^3*e^10 + 1536*a^8*c^5*d^2*e^8 + 128*a^6*b^4*c^3*d^2*e^8 - 896*a^7*b^2*c^4*d^2*e^8 - 1792*a^8*b*c^4*d*e^9 - 128*a^6*b^5*c^2*d*e^9 + 960*a^7*b^3*c^3*d*e^9))/(2*a^4))*(-(8*a^3*c^3*d - b^6*d + b^3*d*(-(4*a*c - b^2)^3)^(1/2) + a*b^5*e - 18*a^2*b^2*c^2*d + 8*a*b^4*c*d - a*b^2*e*(-(4*a*c - b^2)^3)^(1/2) - 7*a^2*b^3*c*e + 12*a^3*b*c^2*e + a^2*c*e*(-(4*a*c - b^2)^3)^(1/2) - 2*a*b*c*d*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^4*b^4 + 16*a^6*c^2 - 8*a^5*b^2*c)))^(1/2) + ((d + e*x^2)^(1/2)*(240*a^6*b*c^4*e^11 + 64*a^6*c^5*d*e^10 + 20*a^4*b^5*c^2*e^11 - 140*a^5*b^3*c^3*e^11 + 160*a^5*c^6*d^3*e^8 - 32*a^2*b^6*c^3*d^3*e^8 + 32*a^2*b^7*c^2*d^2*e^9 + 224*a^3*b^4*c^4*d^3*e^8 - 208*a^3*b^5*c^3*d^2*e^9 - 432*a^4*b^2*c^5*d^3*e^8 + 272*a^4*b^3*c^4*d^2*e^9 - 48*a^3*b^6*c^2*d*e^10 + 348*a^4*b^4*c^3*d*e^10 + 224*a^5*b*c^5*d^2*e^9 - 648*a^5*b^2*c^4*d*e^10))/(2*a^4))*(-(8*a^3*c^3*d - b^6*d + b^3*d*(-(4*a*c - b^2)^3)^(1/2) + a*b^5*e - 18*a^2*b^2*c^2*d + 8*a*b^4*c*d - a*b^2*e*(-(4*a*c - b^2)^3)^(1/2) - 7*a^2*b^3*c*e + 12*a^3*b*c^2*e + a^2*c*e*(-(4*a*c - b^2)^3)^(1/2) - 2*a*b*c*d*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^4*b^4 + 16*a^6*c^2 - 8*a^5*b^2*c)))^(1/2))*(-(8*a^3*c^3*d - b^6*d + b^3*d*(-(4*a*c - b^2)^3)^(1/2) + a*b^5*e - 18*a^2*b^2*c^2*d + 8*a*b^4*c*d - a*b^2*e*(-(4*a*c - b^2)^3)^(1/2) - 7*a^2*b^3*c*e + 12*a^3*b*c^2*e + a^2*c*e*(-(4*a*c - b^2)^3)^(1/2) - 2*a*b*c*d*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^4*b^4 + 16*a^6*c^2 - 8*a^5*b^2*c)))^(1/2) + ((d + e*x^2)^(1/2)*(6*a^4*c^5*e^12 + 4*a^2*c^7*d^4*e^8 + 6*a^3*c^6*d^2*e^10 + 4*b^4*c^5*d^4*e^8 + 21*a^2*b^2*c^5*d^2*e^10 - 18*a^3*b*c^5*d*e^11 - 8*a*b^2*c^6*d^4*e^8 - 12*a*b^3*c^5*d^3*e^9))/(2*a^4))*(-(8*a^3*c^3*d - b^6*d + b^3*d*(-(4*a*c - b^2)^3)^(1/2) + a*b^5*e - 18*a^2*b^2*c^2*d + 8*a*b^4*c*d - a*b^2*e*(-(4*a*c - b^2)^3)^(1/2) - 7*a^2*b^3*c*e + 12*a^3*b*c^2*e + a^2*c*e*(-(4*a*c - b^2)^3)^(1/2) - 2*a*b*c*d*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^4*b^4 + 16*a^6*c^2 - 8*a^5*b^2*c)))^(1/2)*1i)/((a^3*c^5*e^13 + 2*a*c^7*d^4*e^9 - 4*b*c^7*d^5*e^8 + 3*a^2*c^6*d^2*e^11 + 4*b^2*c^6*d^4*e^9 - 8*a*b*c^6*d^3*e^10 - 3*a^2*b*c^5*d*e^12 + 2*a*b^2*c^5*d^2*e^11)/(2*a^4) + (((64*a^5*b*c^4*e^12 + 80*a^5*c^5*d*e^11 + 4*a^3*b^5*c^2*e^12 - 32*a^4*b^3*c^3*e^12 + 80*a^4*c^6*d^3*e^9 + 160*a^2*b^3*c^5*d^4*e^8 - 80*a^2*b^4*c^4*d^3*e^9 - 108*a^2*b^5*c^3*d^2*e^10 - 80*a^3*b^2*c^5*d^3*e^9 + 336*a^3*b^3*c^4*d^2*e^10 - 32*a*b^5*c^4*d^4*e^8 + 24*a*b^6*c^3*d^3*e^9 + 8*a*b^7*c^2*d^2*e^10 - 12*a^2*b^6*c^2*d*e^11 - 128*a^3*b*c^6*d^4*e^8 + 112*a^3*b^4*c^3*d*e^11 - 144*a^4*b*c^5*d^2*e^10 - 272*a^4*b^2*c^4*d*e^11)/(4*a^4) + (((512*a^8*c^4*e^11 + 32*a^6*b^4*c^2*e^11 - 256*a^7*b^2*c^3*e^11 + 512*a^7*c^5*d^2*e^9 + 128*a^5*b^3*c^4*d^3*e^8 - 96*a^5*b^4*c^3*d^2*e^9 + 256*a^6*b^2*c^4*d^2*e^9 - 1024*a^7*b*c^4*d*e^10 - 32*a^5*b^5*c^2*d*e^10 - 512*a^6*b*c^5*d^3*e^8 + 384*a^6*b^3*c^3*d*e^10)/(4*a^4) - ((d + e*x^2)^(1/2)*(-(8*a^3*c^3*d - b^6*d + b^3*d*(-(4*a*c - b^2)^3)^(1/2) + a*b^5*e - 18*a^2*b^2*c^2*d + 8*a*b^4*c*d - a*b^2*

e*(-(4*a*c - b^2)^3)^(1/2) - 7*a^2*b^3*c*e + 12*a^3*b*c^2*e + a^2*c*e*(-(4*
a*c - b^2)^3)^(1/2) - 2*a*b*c*d*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^4*b^4 + 16*
a^6*c^2 - 8*a^5*b^2*c)))^(1/2)*(1024*a^9*c^4*e^10 + 64*a^7*b^4*c^2*e^10 - 5
12*a^8*b^2*c^3*e^10 + 1536*a^8*c^5*d^2*e^8 + 128*a^6*b^4*c^3*d^2*e^8 - 896*
a^7*b^2*c^4*d^2*e^8 - 1792*a^8*b*c^4*d*e^9 - 128*a^6*b^5*c^2*d*e^9 + 960*a^
7*b^3*c^3*d*e^9))/(2*a^4))*(-(8*a^3*c^3*d - b^6*d + b^3*d*(-(4*a*c - b^2)^3
)^(1/2) + a*b^5*e - 18*a^2*b^2*c^2*d + 8*a*b^4*c*d - a*b^2*e*(-(4*a*c - b^2
)^3)^(1/2) - 7*a^2*b^3*c*e + 12*a^3*b*c^2*e + a^2*c*e*(-(4*a*c - b^2)^3)^(1
/2) - 2*a*b*c*d*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^4*b^4 + 16*a^6*c^2 - 8*a^5*
b^2*c)))^(1/2) - ((d + e*x^2)^(1/2)*(240*a^6*b*c^4*e^11 + 64*a^6*c^5*d*e^10
 + 20*a^4*b^5*c^2*e^11 - 140*a^5*b^3*c^3*e^11 + 160*a^5*c^6*d^3*e^8 - 32*a^
2*b^6*c^3*d^3*e^8 + 32*a^2*b^7*c^2*d^2*e^9 + 224*a^3*b^4*c^4*d^3*e^8 - 208*
a^3*b^5*c^3*d^2*e^9 - 432*a^4*b^2*c^5*d^3*e^8 + 272*a^4*b^3*c^4*d^2*e^9 - 4
8*a^3*b^6*c^2*d*e^10 + 348*a^4*b^4*c^3*d*e^10 + 224*a^5*b*c^5*d^2*e^9 - 648
*a^5*b^2*c^4*d*e^10))/(2*a^4))*(-(8*a^3*c^3*d - b^6*d + b^3*d*(-(4*a*c - b^
2)^3)^(1/2) + a*b^5*e - 18*a^2*b^2*c^2*d + 8*a*b^4*c*d - a*b^2*e*(-(4*a*c -
 b^2)^3)^(1/2) - 7*a^2*b^3*c*e + 12*a^3*b*c^2*e + a^2*c*e*(-(4*a*c - b^2)^3
)^(1/2) - 2*a*b*c*d*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^4*b^4 + 16*a^6*c^2 - 8*
a^5*b^2*c)))^(1/2))*(-(8*a^3*c^3*d - b^6*d + b^3*d*(-(4*a*c - b^2)^3)^(1/2)
 + a*b^5*e - 18*a^2*b^2*c^2*d + 8*a*b^4*c*d - a*b^2*e*(-(4*a*c - b^2)^3)^(1
/2) - 7*a^2*b^3*c*e + 12*a^3*b*c^2*e + a^2*c*e*(-(4*a*c - b^2)^3)^(1/2) - 2
*a*b*c*d*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^4*b^4 + 16*a^6*c^2 - 8*a^5*b^2*c))
)^(1/2) - ((d + e*x^2)^(1/2)*(6*a^4*c^5*e^12 + 4*a^2*c^7*d^4*e^8 + 6*a^3*c^
6*d^2*e^10 + 4*b^4*c^5*d^4*e^8 + 21*a^2*b^2*c^5*d^2*e^10 - 18*a^3*b*c^5*d*e
^11 - 8*a*b^2*c^6*d^4*e^8 - 12*a*b^3*c^5*d^3*e^9))/(2*a^4))*(-(8*a^3*c^3*d
 - b^6*d + b^3*d*(-(4*a*c - b^2)^3)^(1/2) + a*b^5*e - 18*a^2*b^2*c^2*d + 8*a
*b^4*c*d - a*b^2*e*(-(4*a*c - b^2)^3)^(1/2) - 7*a^2*b^3*c*e + 12*a^3*b*c^2*
e + a^2*c*e*(-(4*a*c - b^2)^3)^(1/2) - 2*a*b*c*d*(-(4*a*c - b^2)^3)^(1/2))/
(8*(a^4*b^4 + 16*a^6*c^2 - 8*a^5*b^2*c)))^(1/2) + (((64*a^5*b*c^4*e^12 + 80
*a^5*c^5*d*e^11 + 4*a^3*b^5*c^2*e^12 - 32*a^4*b^3*c^3*e^12 + 80*a^4*c^6*d^3
*e^9 + 160*a^2*b^3*c^5*d^4*e^8 - 80*a^2*b^4*c^4*d^3*e^9 - 108*a^2*b^5*c^3*d
^2*e^10 - 80*a^3*b^2*c^5*d^3*e^9 + 336*a^3*b^3*c^4*d^2*e^10 - 32*a*b^5*c^4*
d^4*e^8 + 24*a*b^6*c^3*d^3*e^9 + 8*a*b^7*c^2*d^2*e^10 - 12*a^2*b^6*c^2*d*e^
11 - 128*a^3*b*c^6*d^4*e^8 + 112*a^3*b^4*c^3*d*e^11 - 144*a^4*b*c^5*d^2*e^1
0 - 272*a^4*b^2*c^4*d*e^11)/(4*a^4) + (((512*a^8*c^4*e^11 + 32*a^6*b^4*c^2*
e^11 - 256*a^7*b^2*c^3*e^11 + 512*a^7*c^5*d^2*e^9 + 128*a^5*b^3*c^4*d^3*e^8
 - 96*a^5*b^4*c^3*d^2*e^9 + 256*a^6*b^2*c^4*d^2*e^9 - 1024*a^7*b*c^4*d*e^10
 - 32*a^5*b^5*c^2*d*e^10 - 512*a^6*b*c^5*d^3*e^8 + 384*a^6*b^3*c^3*d*e^10)/
(4*a^4) + ((d + e*x^2)^(1/2)*(-(8*a^3*c^3*d - b^6*d + b^3*d*(-(4*a*c - b^2)
^3)^(1/2) + a*b^5*e - 18*a^2*b^2*c^2*d + 8*a*b^4*c*d - a*b^2*e*(-(4*a*c - b
^2)^3)^(1/2) - 7*a^2*b^3*c*e + 12*a^3*b*c^2*e + a^2*c*e*(-(4*a*c - b^2)^3)^
(1/2) - 2*a*b*c*d*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^4*b^4 + 16*a^6*c^2 - 8*a^
5*b^2*c)))^(1/2)*(1024*a^9*c^4*e^10 + 64*a^7*b^4*c^2*e^10 - 512*a^8*b^2*c^3
*e^10 + 1536*a^8*c^5*d^2*e^8 + 128*a^6*b^4*c^3*d^2*e^8 - 896*a^7*b^2*c^4*d^
2*e^8 - 1792*a^8*b*c^4*d*e^9 - 128*a^6*b^5*c^2*d*e^9 + 960*a^7*b^3*c^3*d*e^

9))/(2*a^4))*(-(8*a^3*c^3*d - b^6*d + b^3*d*(-(4*a*c - b^2)^3)^(1/2) + a*b^5*e - 18*a^2*b^2*c^2*d + 8*a*b^4*c*d - a*b^2*e*(-(4*a*c - b^2)^3)^(1/2) - 7*a^2*b^3*c*e + 12*a^3*b*c^2*e + a^2*c*e*(-(4*a*c - b^2)^3)^(1/2) - 2*a*b*c*d*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^4*b^4 + 16*a^6*c^2 - 8*a^5*b^2*c)))^(1/2) + ((d + e*x^2)^(1/2)*(240*a^6*b*c^4*e^11 + 64*a^6*c^5*d*e^10 + 20*a^4*b^5*c^2*e^11 - 140*a^5*b^3*c^3*e^11 + 160*a^5*c^6*d^3*e^8 - 32*a^2*b^6*c^3*d^3*e^8 + 32*a^2*b^7*c^2*d^2*e^9 + 224*a^3*b^4*c^4*d^3*e^8 - 208*a^3*b^5*c^3*d^2*e^9 - 432*a^4*b^2*c^5*d^3*e^8 + 272*a^4*b^3*c^4*d^2*e^9 - 48*a^3*b^6*c^2*d*e^10 + 348*a^4*b^4*c^3*d*e^10 + 224*a^5*b*c^5*d^2*e^9 - 648*a^5*b^2*c^4*d*e^10))/(2*a^4))*(-(8*a^3*c^3*d - b^6*d + b^3*d*(-(4*a*c - b^2)^3)^(1/2) + a*b^5*e - 18*a^2*b^2*c^2*d + 8*a*b^4*c*d - a*b^2*e*(-(4*a*c - b^2)^3)^(1/2) - 7*a^2*b^3*c*e + 12*a^3*b*c^2*e + a^2*c*e*(-(4*a*c - b^2)^3)^(1/2) - 2*a*b*c*d*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^4*b^4 + 16*a^6*c^2 - 8*a^5*b^2*c)))^(1/2))*(-(8*a^3*c^3*d - b^6*d + b^3*d*(-(4*a*c - b^2)^3)^(1/2) + a*b^5*e - 18*a^2*b^2*c^2*d + 8*a*b^4*c*d - a*b^2*e*(-(4*a*c - b^2)^3)^(1/2) - 7*a^2*b^3*c*e + 12*a^3*b*c^2*e + a^2*c*e*(-(4*a*c - b^2)^3)^(1/2) - 2*a*b*c*d*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^4*b^4 + 16*a^6*c^2 - 8*a^5*b^2*c)))^(1/2) + ((d + e*x^2)^(1/2)*(6*a^4*c^5*e^12 + 4*a^2*c^7*d^4*e^8 + 6*a^3*c^6*d^2*e^10 + 4*b^4*c^5*d^4*e^8 + 21*a^2*b^2*c^5*d^2*e^10 - 18*a^3*b*c^5*d*e^11 - 8*a*b^2*c^6*d^4*e^8 - 12*a*b^3*c^5*d^3*e^9))/(2*a^4))*(-(8*a^3*c^3*d - b^6*d + b^3*d*(-(4*a*c - b^2)^3)^(1/2) + a*b^5*e - 18*a^2*b^2*c^2*d + 8*a*b^4*c*d - a*b^2*e*(-(4*a*c - b^2)^3)^(1/2) - 7*a^2*b^3*c*e + 12*a^3*b*c^2*e + a^2*c*e*(-(4*a*c - b^2)^3)^(1/2) - 2*a*b*c*d*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^4*b^4 + 16*a^6*c^2 - 8*a^5*b^2*c)))^(1/2)))*(-(8*a^3*c^3*d - b^6*d + b^3*d*(-(4*a*c - b^2)^3)^(1/2) + a*b^5*e - 18*a^2*b^2*c^2*d + 8*a*b^4*c*d - a*b^2*e*(-(4*a*c - b^2)^3)^(1/2) - 7*a^2*b^3*c*e + 12*a^3*b*c^2*e + a^2*c*e*(-(4*a*c - b^2)^3)^(1/2) - 2*a*b*c*d*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^4*b^4 + 16*a^6*c^2 - 8*a^5*b^2*c)))^(1/2)*2i

3.360 $\int \frac{\sqrt{d+ex^2}}{x^5(a+bx^2+cx^4)}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 552

$$\int \frac{\sqrt{d+ex^2}}{x^5(a+bx^2+cx^4)}\,dx = -\frac{\sqrt{d+ex^2}}{4ax^4}+\frac{3e\sqrt{d+ex^2}}{8adx^2}+\frac{(bd-ae)\sqrt{d+ex^2}}{2a^2dx^2}-\frac{3e^2\operatorname{arctanh}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)}{8ad^{3/2}}-\frac{e(bd-ae)\operatorname{arctanh}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)}{2a^2d^{3/2}}-\frac{(b^2d-acd-abe)\operatorname{arctanh}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)}{a^3\sqrt{d}}+\frac{\sqrt{c}\left(b^3d-ac\left(\sqrt{b^2-4ac}d-2ae\right)+b^2\left(\sqrt{b^2-4ac}d-ae\right)-ab\left(3cd+\sqrt{b^2-4ac}e\right)\right)\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d-}}{\sqrt{2cd-\left(b-\sqrt{b\right.}}}\right.}{\sqrt{2}a^3\sqrt{b^2-4ac}\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}}-\frac{\sqrt{c}\left(b^3d-b^2\left(\sqrt{b^2-4ac}d+ae\right)+ac\left(\sqrt{b^2-4ac}d+2ae\right)-ab\left(3cd-\sqrt{b^2-4ac}e\right)\right)\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d-}}{\sqrt{2cd-\left(b+\sqrt{b\right.}}}\right.}{\sqrt{2}a^3\sqrt{b^2-4ac}\sqrt{2cd-\left(b+\sqrt{b^2-4ac}\right)e}}$$

```
[Out] -3/8*e^2*arctanh((e*x^2+d)^(1/2)/d^(1/2))/a/d^(3/2)-1/2*e*(-a*e+b*d)*arctan
h((e*x^2+d)^(1/2)/d^(1/2))/a^2/d^(3/2)-(-a*b*e-a*c*d+b^2*d)*arctanh((e*x^2+
d)^(1/2)/d^(1/2))/a^3/d^(1/2)-1/4*(e*x^2+d)^(1/2)/a/x^4+3/8*e*(e*x^2+d)^(1/
2)/a/d/x^2+1/2*(-a*e+b*d)*(e*x^2+d)^(1/2)/a^2/d/x^2+1/2*arctanh(2^(1/2)*c^(
1/2)*(e*x^2+d)^(1/2)/(2*c*d-e*(b-(-4*a*c+b^2)^(1/2)))^(1/2))*c^(1/2)*(b^3*d
-a*c*(-2*a*e+d*(-4*a*c+b^2)^(1/2))+b^2*(-a*e+d*(-4*a*c+b^2)^(1/2))-a*b*(3*c
*d+e*(-4*a*c+b^2)^(1/2)))/a^3*2^(1/2)/(-4*a*c+b^2)^(1/2)/(2*c*d-e*(b-(-4*a*
c+b^2)^(1/2)))^(1/2)-1/2*arctanh(2^(1/2)*c^(1/2)*(e*x^2+d)^(1/2)/(2*c*d-e*(
```

```
b+(-4*a*c+b^2)^(1/2)))^(1/2))*c^(1/2)*(b^3*d-b^2*(a*e+d*(-4*a*c+b^2)^(1/2))
+a*c*(2*a*e+d*(-4*a*c+b^2)^(1/2))-a*b*(3*c*d-e*(-4*a*c+b^2)^(1/2)))/a^3*2^(
1/2)/(-4*a*c+b^2)^(1/2)/(2*c*d-e*(b+(-4*a*c+b^2)^(1/2)))^(1/2)
```

Rubi [A] (verified)

Time = 2.69 (sec) , antiderivative size = 552, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.241, Rules used = {1265, 911, 1301, 205, 212, 1180, 214}

$$\int \frac{\sqrt{d+ex^2}}{x^5(a+bx^2+cx^4)}\,dx = -\frac{\text{arctanh}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)(-abe-acd+b^2d)}{a^3\sqrt{d}}$$
$$+\frac{\sqrt{c}\left(b^2\left(d\sqrt{b^2-4ac}-ae\right)-ab\left(e\sqrt{b^2-4ac}+3cd\right)-ac\left(d\sqrt{b^2-4ac}-2ae\right)+b^3d\right)\text{arctanh}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{}}{\sqrt{2cd-e(b-}}\right.}{\sqrt{2}a^3\sqrt{b^2-4ac}\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}$$
$$-\frac{\sqrt{c}\left(-b^2\left(d\sqrt{b^2-4ac}+ae\right)-ab\left(3cd-e\sqrt{b^2-4ac}\right)+ac\left(d\sqrt{b^2-4ac}+2ae\right)+b^3d\right)\text{arctanh}\left(\frac{\sqrt{2}\sqrt{}}{\sqrt{2cd-e(}}\right.}{\sqrt{2}a^3\sqrt{b^2-4ac}\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}$$
$$-\frac{e(bd-ae)\text{arctanh}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)}{2a^2d^{3/2}}+\frac{\sqrt{d+ex^2}(bd-ae)}{2a^2dx^2}$$
$$-\frac{3e^2\text{arctanh}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)}{8ad^{3/2}}+\frac{3e\sqrt{d+ex^2}}{8adx^2}-\frac{\sqrt{d+ex^2}}{4ax^4}$$

```
[In] Int[Sqrt[d + e*x^2]/(x^5*(a + b*x^2 + c*x^4)),x]
```

```
[Out] -1/4*Sqrt[d + e*x^2]/(a*x^4) + (3*e*Sqrt[d + e*x^2])/(8*a*d*x^2) + ((b*d -
a*e)*Sqrt[d + e*x^2])/(2*a^2*d*x^2) - (3*e^2*ArcTanh[Sqrt[d + e*x^2]/Sqrt[d
]])/(8*a*d^(3/2)) - (e*(b*d - a*e)*ArcTanh[Sqrt[d + e*x^2]/Sqrt[d]])/(2*a^2
*d^(3/2)) - ((b^2*d - a*c*d - a*b*e)*ArcTanh[Sqrt[d + e*x^2]/Sqrt[d]])/(a^3
*Sqrt[d]) + (Sqrt[c]*(b^3*d - a*c*(Sqrt[b^2 - 4*a*c]*d - 2*a*e) + b^2*(Sqrt
[b^2 - 4*a*c]*d - a*e) - a*b*(3*c*d + Sqrt[b^2 - 4*a*c]*e))*ArcTanh[(Sqrt[2
]*Sqrt[c]*Sqrt[d + e*x^2])/Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]])/(Sqrt[
2]*a^3*Sqrt[b^2 - 4*a*c]*Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]) - (Sqrt[c
]*(b^3*d - b^2*(Sqrt[b^2 - 4*a*c]*d + a*e) + a*c*(Sqrt[b^2 - 4*a*c]*d + 2*a
*e) - a*b*(3*c*d - Sqrt[b^2 - 4*a*c]*e))*ArcTanh[(Sqrt[2]*Sqrt[c]*Sqrt[d +
e*x^2])/Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]])/(Sqrt[2]*a^3*Sqrt[b^2 - 4
*a*c]*Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integ
erQ[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denom
inator[p + 1/n] < Denominator[p])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 911

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> With[{q = Denominator[m]}, Dist[q/e, S
ubst[Int[x^(q*(m + 1) - 1)*((e*f - d*g)/e + g*(x^q/e))^n*((c*d^2 - b*d*e +
a*e^2)/e^2 - (2*c*d - b*e)*(x^q/e^2) + c*(x^(2*q)/e^2))^p, x], x, (d + e*x)
^(1/q)], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegersQ[n, p] && Fra
ctionQ[m]
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1265

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rule 1301

```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.))/((a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*((d + e*x^2)^q/(a
```

```
+ b*x^2 + c*x^4)), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b^2 - 4
*a*c, 0] && IntegerQ[q] && IntegerQ[m]
```

Rubi steps

$$\text{integral} = \frac{1}{2}\text{Subst}\left(\int \frac{\sqrt{d+ex}}{x^3\left(a+bx+cx^2\right)}\,dx, x, x^2\right)$$

$$= \frac{\text{Subst}\left(\int \frac{x^2}{\left(-\frac{d}{e}+\frac{x^2}{e}\right)^3\left(\frac{cd^2-bde+ae^2}{e^2}-\frac{(2cd-be)x^2}{e^2}+\frac{cx^4}{e^2}\right)}\,dx, x, \sqrt{d+ex^2}\right)}{e}$$

$$= \frac{\text{Subst}\left(\int \left(-\frac{de^3}{a(d-x^2)^3}+\frac{e^2(-bd+ae)}{a^2(d-x^2)^2}+\frac{e(-b^2d+acd+abe)}{a^3(d-x^2)}+\frac{e\left(\left(b^2-ac\right)\left(cd^2-bde+ae^2\right)-c\left(b^2d-acd-abe\right)x^2\right)}{a^3\left(cd^2-bde+ae^2-(2cd-be)x^2+cx^4\right)}\right)\,dx, x, \cdots}{e}$$

$$= \frac{\text{Subst}\left(\int \frac{\left(b^2-ac\right)\left(cd^2-bde+ae^2\right)-c\left(b^2d-acd-abe\right)x^2}{cd^2-bde+ae^2+(-2cd+be)x^2+cx^4}\,dx, x, \sqrt{d+ex^2}\right)}{a^3}$$
$$-\frac{\left(de^2\right)\text{Subst}\left(\int \frac{1}{(d-x^2)^3}\,dx, x, \sqrt{d+ex^2}\right)}{a}$$
$$-\frac{(e(bd-ae))\text{Subst}\left(\int \frac{1}{(d-x^2)^2}\,dx, x, \sqrt{d+ex^2}\right)}{a^2}$$
$$-\frac{\left(b^2d-acd-abe\right)\text{Subst}\left(\int \frac{1}{d-x^2}\,dx, x, \sqrt{d+ex^2}\right)}{a^3}$$

$$= -\frac{\sqrt{d+ex^2}}{4ax^4}+\frac{(bd-ae)\sqrt{d+ex^2}}{2a^2dx^2}-\frac{\left(b^2d-acd-abe\right)\tanh^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)}{a^3\sqrt{d}}$$
$$-\frac{\left(3e^2\right)\text{Subst}\left(\int \frac{1}{(d-x^2)^2}\,dx, x, \sqrt{d+ex^2}\right)}{4a}$$
$$-\frac{(e(bd-ae))\text{Subst}\left(\int \frac{1}{d-x^2}\,dx, x, \sqrt{d+ex^2}\right)}{2a^2d}$$
$$+\frac{\left(c\left(b^3d-b^2\left(\sqrt{b^2-4ac}\,d+ae\right)+ac\left(\sqrt{b^2-4ac}\,d+2ae\right)-ab\left(3cd-\sqrt{b^2-4ac}\,e\right)\right)\right)\text{Subst}\left(\int \frac{1}{\frac{1}{2}\cdots}\cdots\right)}{2a^3\sqrt{b^2-4ac}}$$
$$-\frac{\left(c\left(b^3d-ac\left(\sqrt{b^2-4ac}\,d-2ae\right)+b^2\left(\sqrt{b^2-4ac}\,d-ae\right)-ab\left(3cd+\sqrt{b^2-4ac}\,e\right)\right)\right)\text{Subst}\left(\int \frac{\cdots}{-\cdots}\cdots\right)}{2a^3\sqrt{b^2-4ac}}$$

$$= -\frac{\sqrt{d+ex^2}}{4ax^4} + \frac{3e\sqrt{d+ex^2}}{8adx^2} + \frac{(bd-ae)\sqrt{d+ex^2}}{2a^2dx^2} - \frac{e(bd-ae)\tanh^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)}{2a^2d^{3/2}} - \frac{(b^2d-acd-abe)\tanh^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)}{a^3\sqrt{d}} + \frac{\sqrt{c}\left(b^3d-ac\left(\sqrt{b^2-4ac}d-2ae\right)+b^2\left(\sqrt{b^2-4ac}d-ae\right)-ab\left(3cd+\sqrt{b^2-4ac}e\right)\right)\tanh^{-1}\left(\frac{}{\sqrt{2c}}\right.}{\sqrt{2}a^3\sqrt{b^2-4ac}\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}} - \frac{\sqrt{c}\left(b^3d-b^2\left(\sqrt{b^2-4ac}d+ae\right)+ac\left(\sqrt{b^2-4ac}d+2ae\right)-ab\left(3cd-\sqrt{b^2-4ac}e\right)\right)\tanh^{-1}\left(\frac{}{\sqrt{2c}}\right.}{\sqrt{2}a^3\sqrt{b^2-4ac}\sqrt{2cd-\left(b+\sqrt{b^2-4ac}\right)e}} - \frac{(3e^2)\operatorname{Subst}\left(\int\frac{1}{d-x^2}\,dx,x,\sqrt{d+ex^2}\right)}{8ad}$$

$$= -\frac{\sqrt{d+ex^2}}{4ax^4} + \frac{3e\sqrt{d+ex^2}}{8adx^2} + \frac{(bd-ae)\sqrt{d+ex^2}}{2a^2dx^2} - \frac{3e^2\tanh^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)}{8ad^{3/2}} - \frac{e(bd-ae)\tanh^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)}{2a^2d^{3/2}} - \frac{(b^2d-acd-abe)\tanh^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)}{a^3\sqrt{d}} + \frac{\sqrt{c}\left(b^3d-ac\left(\sqrt{b^2-4ac}d-2ae\right)+b^2\left(\sqrt{b^2-4ac}d-ae\right)-ab\left(3cd+\sqrt{b^2-4ac}e\right)\right)\tanh^{-1}\left(\frac{}{\sqrt{2c}}\right.}{\sqrt{2}a^3\sqrt{b^2-4ac}\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}} - \frac{\sqrt{c}\left(b^3d-b^2\left(\sqrt{b^2-4ac}d+ae\right)+ac\left(\sqrt{b^2-4ac}d+2ae\right)-ab\left(3cd-\sqrt{b^2-4ac}e\right)\right)\tanh^{-1}\left(\frac{}{\sqrt{2c}}\right.}{\sqrt{2}a^3\sqrt{b^2-4ac}\sqrt{2cd-\left(b+\sqrt{b^2-4ac}\right)e}}$$

Mathematica [A] (verified)

Time = 1.70 (sec) , antiderivative size = 445, normalized size of antiderivative = 0.81

$$\int\frac{\sqrt{d+ex^2}}{x^5\left(a+bx^2+cx^4\right)}\,dx$$

$$= \frac{\frac{a\sqrt{d+ex^2}\left(4bdx^2-a\left(2d+ex^2\right)\right)}{dx^4} + \frac{4\sqrt{2}\sqrt{c}\left(-b^3d+ac\left(\sqrt{b^2-4ac}d-2ae\right)+b^2\left(-\sqrt{b^2-4ac}d+ae\right)+ab\left(3cd+\sqrt{b^2-4ac}e\right)\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex^2}}{\sqrt{-2cd+be-\sqrt{b^2-4}}}\right.}{\sqrt{b^2-4ac}\sqrt{-2cd+\left(b-\sqrt{b^2-4ac}\right)e}}}{}$$

[In] Integrate[Sqrt[d + e*x^2]/(x^5*(a + b*x^2 + c*x^4)),x]

[Out] ((a*Sqrt[d + e*x^2]*(4*b*d*x^2 - a*(2*d + e*x^2)))/(d*x^4) + (4*Sqrt[2]*Sqrt[c]*(-(b^3*d) + a*c*(Sqrt[b^2 - 4*a*c]*d - 2*a*e) + b^2*(-(Sqrt[b^2 - 4*a*c]*d) + a*e) + a*b*(3*c*d + Sqrt[b^2 - 4*a*c]*e))*ArcTan[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x^2])/Sqrt[-2*c*d + b*e - Sqrt[b^2 - 4*a*c]*e]])/(Sqrt[b^2 - 4*a*c]*Sqrt[-2*c*d + (b - Sqrt[b^2 - 4*a*c])*e]) + (4*Sqrt[2]*Sqrt[c]*(b^3*d - b^2*(Sqrt[b^2 - 4*a*c]*d + a*e) + a*c*(Sqrt[b^2 - 4*a*c]*d + 2*a*e) + a*b*(-3*c*d + Sqrt[b^2 - 4*a*c]*e))*ArcTan[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x^2])/Sqrt[-2*c*d + (b + Sqrt[b^2 - 4*a*c])*e]])/(Sqrt[b^2 - 4*a*c]*Sqrt[-2*c*d + (b + Sqrt[b^2 - 4*a*c])*e]) + ((-8*b^2*d^2 + 4*a*b*d*e + a*(8*c*d^2 + a*e^2))*ArcTanh[Sqrt[d + e*x^2]/Sqrt[d]])/d^(3/2))/(8*a^3)

Maple [A] (verified)

Time = 1.15 (sec) , antiderivative size = 463, normalized size of antiderivative = 0.84

method	result
risch	$-\frac{\sqrt{e\,x^2+d}\,(ae\,x^2-4bd\,x^2+2da)}{8a^2x^4d}-\frac{-\frac{\left(e^2a^2+4abde+8d^2ac-8b^2d^2\right)\ln\left(\frac{2d+2\sqrt{d}\,\sqrt{e\,x^2+d}}{x}\right)}{a\sqrt{d}}-\frac{8d\sqrt{2}\,c\left(\left(\frac{(-acd-b(ae-bd))\sqrt{-4e^2 \cdots}}{2}\right.\right.}{\cdots}}{\cdots}$
pseudoelliptic	$-\frac{-8\sqrt{2}\,c\sqrt{\left(be-2cd+\sqrt{-4e^2\left(ac-\frac{b^2}{4}\right)}\right)c}\,x^4\left(\frac{\left(-a\,d^{\frac{3}{2}}\,be-d^{\frac{5}{2}}\left(ac-b^2\right)\right)\sqrt{-4e^2\left(ac-\frac{b^2}{4}\right)}}{2}+e\left(ae\left(ac-\frac{b^2}{2}\right)d^{\frac{3}{2}}-\frac{3d^{\frac{5}{2}}\,b\left(ac-\frac{b^2}{3}\cdots\right.}{2}\right.\right.}{\cdots}$
default	$\frac{-\frac{\left(e\,x^2+d\right)^{\frac{3}{2}}}{4d\,x^4}-\frac{e\left(-\frac{\left(e\,x^2+d\right)^{\frac{3}{2}}}{2d\,x^2}+\frac{e\left(\sqrt{e\,x^2+d}-\sqrt{d}\,\ln\left(\frac{2d+2\sqrt{d}\,\sqrt{e\,x^2+d}}{x}\right)\right)}{2d}\right)}{4d}}{a}-\frac{b\left(-\frac{\left(e\,x^2+d\right)^{\frac{3}{2}}}{2d\,x^2}+\frac{e\left(\sqrt{e\,x^2+d}-\sqrt{d}\,\ln\left(\frac{2d+2\sqrt{d}\,\sqrt{\cdots}}{x}\right.\right.}{2d}\cdots\right.}{a^2}$

[In] int((e*x^2+d)^(1/2)/x^5/(c*x^4+b*x^2+a),x,method=_RETURNVERBOSE)

[Out] -1/8*(e*x^2+d)^(1/2)*(a*e*x^2-4*b*d*x^2+2*a*d)/a^2/x^4/d-1/8/a^2/d*(-(a^2*e^2+4*a*b*d*e+8*a*c*d^2-8*b^2*d^2)/a/d^(1/2)*ln((2*d+2*d^(1/2)*(e*x^2+d)^(1/2))/x)-8*d/a/((b*e-2*c*d+(-4*e^2*(a*c-1/4*b^2))^(1/2))*c)^(1/2)*2^(1/2)*c/((-b*e+2*c*d+(-4*e^2*(a*c-1/4*b^2))^(1/2))*c)^(1/2)*((1/2*(-a*c*d-b*(a*e-b*d))*(-4*e^2*(a*c-1/4*b^2))^(1/2)+e*(a*(a*e-3/2*b*d)*c-1/2*b^2*(a*e-b*d)))*((b*e-2*c*d+(-4*e^2*(a*c-1/4*b^2))^(1/2))*c)^(1/2)*arctanh(c*(e*x^2+d)^(1/2)*2^(1/2)/((-b*e+2*c*d+(-4*e^2*(a*c-1/4*b^2))^(1/2))*c)^(1/2))+arctan(c*(e*x^2+d)^(1/2)*2^(1/2)/((b*e-2*c*d+(-4*e^2*(a*c-1/4*b^2))^(1/2))*c)^(1/2))*((-b*e+2*c*d+(-4*e^2*(a*c-1/4*b^2))^(1/2))*c)^(1/2)*(1/2*(a*c*d+b*(a*e-b*d))*(-

```
4*e^2*(a*c-1/4*b^2))^(1/2)+e*(a*(a*e-3/2*b*d)*c-1/2*b^2*(a*e-b*d))))/(-4*e^
2*(a*c-1/4*b^2))^(1/2))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt{d+ex^2}}{x^5\left(a+bx^2+cx^4\right)}\,dx=\text{Timed out}$$

```
[In] integrate((e*x^2+d)^(1/2)/x^5/(c*x^4+b*x^2+a),x, algorithm="fricas")
[Out] Timed out
```

Sympy [F]

$$\int \frac{\sqrt{d+ex^2}}{x^5\left(a+bx^2+cx^4\right)}\,dx=\int \frac{\sqrt{d+ex^2}}{x^5\left(a+bx^2+cx^4\right)}\,dx$$

```
[In] integrate((e*x**2+d)**(1/2)/x**5/(c*x**4+b*x**2+a),x)
[Out] Integral(sqrt(d + e*x**2)/(x**5*(a + b*x**2 + c*x**4)), x)
```

Maxima [F]

$$\int \frac{\sqrt{d+ex^2}}{x^5\left(a+bx^2+cx^4\right)}\,dx=\int \frac{\sqrt{ex^2+d}}{(cx^4+bx^2+a)x^5}\,dx$$

```
[In] integrate((e*x^2+d)^(1/2)/x^5/(c*x^4+b*x^2+a),x, algorithm="maxima")
[Out] integrate(sqrt(e*x^2 + d)/((c*x^4 + b*x^2 + a)*x^5), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1057 vs. $2(468) = 936$.

Time = 0.35 (sec) , antiderivative size = 1057, normalized size of antiderivative = 1.91

$$\int \frac{\sqrt{d+ex^2}}{x^5\left(a+bx^2+cx^4\right)}\,dx =$$

$$-\frac{\left(\sqrt{-4\,c^2d+2\,\left(bc-\sqrt{b^2-4\,ac}c\right)e}\left((b^4-5\,ab^2c+4\,a^2c^2)d-(ab^3-4\,a^2bc)e\right)a^2e^2-2\,\left((ab^2c-a^2c^2)\sqrt{\right.\right.}{}$$

$$+\frac{\left(\sqrt{-4\,c^2d+2\,\left(bc+\sqrt{b^2-4\,ac}c\right)e}\left((b^4-5\,ab^2c+4\,a^2c^2)d-(ab^3-4\,a^2bc)e\right)a^2e^2+2\,\left((ab^2c-a^2c^2)\sqrt{\right.\right.}{}$$

$$+\frac{(8\,b^2d^2-8\,acd^2-4\,abde-a^2e^2)\arctan\left(\frac{\sqrt{ex^2+d}}{\sqrt{-d}}\right)}{8\,a^3\sqrt{-d}d}$$

$$+\frac{4\,(ex^2+d)^{\frac{3}{2}}bde-4\,\sqrt{ex^2+d}bd^2e-(ex^2+d)^{\frac{3}{2}}ae^2-\sqrt{ex^2+d}ade^2}{8\,a^2de^2x^4}$$

```
[In] integrate((e*x^2+d)^(1/2)/x^5/(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] -1/8*(sqrt(-4*c^2*d + 2*(b*c - sqrt(b^2 - 4*a*c)*c)*e)*((b^4 - 5*a*b^2*c +
4*a^2*c^2)*d - (a*b^3 - 4*a^2*b*c)*e)*a^2*e^2 - 2*((a*b^2*c - a^2*c^2)*sqrt
(b^2 - 4*a*c)*d^2 - (a*b^3 - a^2*b*c)*sqrt(b^2 - 4*a*c)*d*e + (a^2*b^2 - a^
3*c)*sqrt(b^2 - 4*a*c)*e^2)*sqrt(-4*c^2*d + 2*(b*c - sqrt(b^2 - 4*a*c)*c)*e
)*abs(a)*abs(e) - (2*(a^2*b^3*c - 3*a^3*b*c^2)*d^2*e - (a^2*b^4 - a^3*b^2*c
 - 4*a^4*c^2)*d*e^2 + (a^3*b^3 - 2*a^4*b*c)*e^3)*sqrt(-4*c^2*d + 2*(b*c - s
qrt(b^2 - 4*a*c)*c)*e))*arctan(2*sqrt(1/2)*sqrt(e*x^2 + d)/sqrt(-(2*a^3*c*d
 - a^3*b*e + sqrt(-4*(a^3*c*d^2 - a^3*b*d*e + a^4*e^2)*a^3*c + (2*a^3*c*d -
 a^3*b*e)^2))/(a^3*c)))/((sqrt(b^2 - 4*a*c)*a^4*c*d^2 - sqrt(b^2 - 4*a*c)*a
^4*b*d*e + sqrt(b^2 - 4*a*c)*a^5*e^2)*abs(a)*abs(c)*abs(e)) + 1/8*(sqrt(-4*
c^2*d + 2*(b*c + sqrt(b^2 - 4*a*c)*c)*e)*((b^4 - 5*a*b^2*c + 4*a^2*c^2)*d -
 (a*b^3 - 4*a^2*b*c)*e)*a^2*e^2 + 2*((a*b^2*c - a^2*c^2)*sqrt(b^2 - 4*a*c)*
d^2 - (a*b^3 - a^2*b*c)*sqrt(b^2 - 4*a*c)*d*e + (a^2*b^2 - a^3*c)*sqrt(b^2
- 4*a*c)*e^2)*sqrt(-4*c^2*d + 2*(b*c + sqrt(b^2 - 4*a*c)*c)*e)*abs(a)*abs(e
) - (2*(a^2*b^3*c - 3*a^3*b*c^2)*d^2*e - (a^2*b^4 - a^3*b^2*c - 4*a^4*c^2)*
d*e^2 + (a^3*b^3 - 2*a^4*b*c)*e^3)*sqrt(-4*c^2*d + 2*(b*c + sqrt(b^2 - 4*a*
c)*c)*e))*arctan(2*sqrt(1/2)*sqrt(e*x^2 + d)/sqrt(-(2*a^3*c*d - a^3*b*e - s
qrt(-4*(a^3*c*d^2 - a^3*b*d*e + a^4*e^2)*a^3*c + (2*a^3*c*d - a^3*b*e)^2))/
(a^3*c)))/((sqrt(b^2 - 4*a*c)*a^4*c*d^2 - sqrt(b^2 - 4*a*c)*a^4*b*d*e + sqr
t(b^2 - 4*a*c)*a^5*e^2)*abs(a)*abs(c)*abs(e)) + 1/8*(8*b^2*d^2 - 8*a*c*d^2
- 4*a*b*d*e - a^2*e^2)*arctan(sqrt(e*x^2 + d)/sqrt(-d))/(a^3*sqrt(-d)*d) +
1/8*(4*(e*x^2 + d)^(3/2)*b*d*e - 4*sqrt(e*x^2 + d)*b*d^2*e - (e*x^2 + d)^(3
/2)*a*e^2 - sqrt(e*x^2 + d)*a*d*e^2)/(a^2*d*e^2*x^4)
```

Mupad [B] (verification not implemented)

Time = 11.97 (sec) , antiderivative size = 33925, normalized size of antiderivative = 61.46

$$\int \frac{\sqrt{d+ex^2}}{x^5\left(a+bx^2+cx^4\right)}\,dx=\text{Too large to display}$$

[In] int((d + e*x^2)^(1/2)/(x^5*(a + b*x^2 + c*x^4)),x)

[Out] atan((((((2048*a^12*c^4*d*e^12 + 12288*a^10*c^6*d^5*e^8 + 14336*a^11*c^5*d^3*e^10 + 2048*a^8*b^4*c^4*d^5*e^8 - 1536*a^8*b^5*c^3*d^4*e^9 - 512*a^8*b^6*c^2*d^3*e^10 - 11264*a^9*b^2*c^5*d^5*e^8 + 7168*a^9*b^3*c^4*d^4*e^9 + 6272*a^9*b^4*c^3*d^3*e^10 + 384*a^9*b^5*c^2*d^2*e^11 - 20480*a^10*b^2*c^4*d^3*e^10 - 3072*a^10*b^3*c^3*d^2*e^11 - 4096*a^10*b*c^5*d^4*e^9 + 128*a^10*b^4*c^2*d*e^12 + 6144*a^11*b*c^4*d^2*e^11 - 1024*a^11*b^2*c^3*d*e^12)/(64*a^8*d^2) - ((d + e*x^2)^(1/2)*((b^8*d + 8*a^4*c^4*d - b^5*d*(-(4*a*c - b^2)^3)^(1/2) - a*b^7*e + 33*a^2*b^4*c^2*d - 38*a^3*b^2*c^3*d - 25*a^3*b^3*c^2*e + a^3*c^2*e*(-(4*a*c - b^2)^3)^(1/2) - 10*a*b^6*c*d + a*b^4*e*(-(4*a*c - b^2)^3)^(1/2) + 9*a^2*b^5*c*e + 20*a^4*b*c^3*e + 4*a*b^3*c*d*(-(4*a*c - b^2)^3)^(1/2) - 3*a^2*b*c^2*d*(-(4*a*c - b^2)^3)^(1/2) - 3*a^2*b^2*c*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^6*b^4 + 16*a^8*c^2 - 8*a^7*b^2*c)))^(1/2)*(24576*a^12*c^5*d^4*e^8 + 16384*a^13*c^4*d^2*e^10 + 2048*a^10*b^4*c^3*d^4*e^8 - 2048*a^10*b^5*c^2*d^3*e^9 - 14336*a^11*b^2*c^4*d^4*e^8 + 15360*a^11*b^3*c^3*d^3*e^9 + 1024*a^11*b^4*c^2*d^2*e^10 - 8192*a^12*b^2*c^3*d^2*e^10 - 28672*a^12*b*c^4*d^3*e^9))/(32*a^8*d^2))*((b^8*d + 8*a^4*c^4*d - b^5*d*(-(4*a*c - b^2)^3)^(1/2) - a*b^7*e + 33*a^2*b^4*c^2*d - 38*a^3*b^2*c^3*d - 25*a^3*b^3*c^2*e + a^3*c^2*e*(-(4*a*c - b^2)^3)^(1/2) - 10*a*b^6*c*d + a*b^4*e*(-(4*a*c - b^2)^3)^(1/2) + 9*a^2*b^5*c*e + 20*a^4*b*c^3*e + 4*a*b^3*c*d*(-(4*a*c - b^2)^3)^(1/2) - 3*a^2*b*c^2*d*(-(4*a*c - b^2)^3)^(1/2) - 3*a^2*b^2*c*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^6*b^4 + 16*a^8*c^2 - 8*a^7*b^2*c)))^(1/2) + ((d + e*x^2)^(1/2)*(32*a^10*c^5*d*e^12 - 48*a^10*b*c^4*e^13 - 4*a^8*b^5*c^2*e^13 + 28*a^9*b^3*c^3*e^13 + 4608*a^8*c^7*d^5*e^8 + 2048*a^9*c^6*d^3*e^10 + 512*a^4*b^8*c^3*d^5*e^8 - 512*a^4*b^9*c^2*d^4*e^9 - 4608*a^5*b^6*c^4*d^5*e^8 + 4352*a^5*b^7*c^3*d^4*e^9 + 768*a^5*b^8*c^2*d^3*e^10 + 14080*a^6*b^4*c^5*d^5*e^8 - 11264*a^6*b^5*c^4*d^4*e^9 - 6912*a^6*b^6*c^3*d^3*e^10 - 256*a^6*b^7*c^2*d^2*e^11 - 16384*a^7*b^2*c^6*d^5*e^8 + 7168*a^7*b^3*c^5*d^4*e^9 + 19776*a^7*b^4*c^4*d^3*e^10 + 2272*a^7*b^5*c^3*d^2*e^11 - 18048*a^8*b^2*c^5*d^3*e^10 - 6144*a^8*b^3*c^4*d^2*e^11 - 32*a^7*b^6*c^2*d*e^12 + 3584*a^8*b*c^6*d^4*e^9 + 228*a^8*b^4*c^3*d*e^12 + 4608*a^9*b*c^5*d^2*e^11 - 408*a^9*b^2*c^4*d*e^12))/(32*a^8*d^2))*((b^8*d + 8*a^4*c^4*d - b^5*d*(-(4*a*c - b^2)^3)^(1/2) - a*b^7*e + 33*a^2*b^4*c^2*d - 38*a^3*b^2*c^3*d - 25*a^3*b^3*c^2*e + a^3*c^2*e*(-(4*a*c - b^2)^3)^(1/2) - 10*a*b^6*c*d + a*b^4*e*(-(4*a*c - b^2)^3)^(1/2) + 9*a^2*b^5*c*e + 20*a^4*b*c^3*e + 4*a*b^3*c*d*(-(4*a*c - b^2)^3)^(1/2) - 3*a^2*b*c^2*d*(-(4*a*c - b^2)^3)^(1/2) - 3*a^2*b^2*c*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^6*b^4 + 16*a^8*c^2 - 8*a^7*b^2*c)))^(1/2) + (16*a^9*c^5*e^14

- 4*a^6*b^6*c^2*e^14 + 28*a^7*b^4*c^3*e^14 - 52*a^8*b^2*c^4*e^14 - 768*a^6*
c^8*d^6*e^8 - 768*a^7*c^7*d^4*e^10 + 16*a^8*c^6*d^2*e^12 - 512*a^2*b^8*c^4*
d^6*e^8 + 384*a^2*b^9*c^3*d^5*e^9 + 128*a^2*b^10*c^2*d^4*e^10 + 3840*a^3*b^
6*c^5*d^6*e^8 - 2048*a^3*b^7*c^4*d^5*e^9 - 2208*a^3*b^8*c^3*d^4*e^10 - 224*
a^3*b^9*c^2*d^3*e^11 - 8704*a^4*b^4*c^6*d^6*e^8 + 896*a^4*b^5*c^5*d^5*e^9 +
10752*a^4*b^6*c^4*d^4*e^10 + 2688*a^4*b^7*c^3*d^3*e^11 + 96*a^4*b^8*c^2*d^
2*e^12 + 6400*a^5*b^2*c^7*d^6*e^8 + 5632*a^5*b^3*c^6*d^5*e^9 - 18144*a^5*b^
4*c^5*d^4*e^10 - 10464*a^5*b^5*c^4*d^3*e^11 - 836*a^5*b^6*c^3*d^2*e^12 + 93
44*a^6*b^2*c^6*d^4*e^10 + 14592*a^6*b^3*c^5*d^3*e^11 + 2236*a^6*b^4*c^4*d^2
*e^12 - 1716*a^7*b^2*c^5*d^2*e^12 - 528*a^8*b*c^5*d*e^13 + 4*a^5*b^7*c^2*d*
e^13 - 4352*a^6*b*c^7*d^5*e^9 - 92*a^6*b^5*c^3*d*e^13 - 5632*a^7*b*c^6*d^3*
e^11 + 436*a^7*b^3*c^4*d*e^13)/(64*a^8*d^2))*((b^8*d + 8*a^4*c^4*d - b^5*d*
(-(4*a*c - b^2)^3)^(1/2) - a*b^7*e + 33*a^2*b^4*c^2*d - 38*a^3*b^2*c^3*d -
25*a^3*b^3*c^2*e + a^3*c^2*e*(-(4*a*c - b^2)^3)^(1/2) - 10*a*b^6*c*d + a*b^
4*e*(-(4*a*c - b^2)^3)^(1/2) + 9*a^2*b^5*c*e + 20*a^4*b*c^3*e + 4*a*b^3*c*d
*(-(4*a*c - b^2)^3)^(1/2) - 3*a^2*b*c^2*d*(-(4*a*c - b^2)^3)^(1/2) - 3*a^2*
b^2*c*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^6*b^4 + 16*a^8*c^2 - 8*a^7*b^2*c)))
^(1/2) - ((d + e*x^2)^(1/2)*(a^6*b^2*c^5*e^14 - 2*a^7*c^6*e^14 + 192*a^4*c^
9*d^6*e^8 + 32*a^5*c^8*d^4*e^10 + 34*a^6*c^7*d^2*e^12 + 64*b^8*c^5*d^6*e^8
+ 704*a^2*b^4*c^7*d^6*e^8 + 960*a^2*b^5*c^6*d^5*e^9 + 192*a^2*b^6*c^5*d^4*e
^10 - 512*a^3*b^2*c^8*d^6*e^8 - 1280*a^3*b^3*c^7*d^5*e^9 - 752*a^3*b^4*c^6*
d^4*e^10 - 56*a^3*b^5*c^5*d^3*e^11 + 704*a^4*b^2*c^7*d^4*e^10 + 128*a^4*b^3
*c^6*d^3*e^11 - 15*a^4*b^4*c^5*d^2*e^12 + 60*a^5*b^2*c^6*d^2*e^12 - 10*a^6*
b*c^6*d*e^13 - 384*a*b^6*c^6*d^6*e^8 - 192*a*b^7*c^5*d^5*e^9 + 384*a^4*b*c^
8*d^5*e^9 - 144*a^5*b*c^7*d^3*e^11 + 6*a^5*b^3*c^5*d*e^13))/(32*a^8*d^2))*(
(b^8*d + 8*a^4*c^4*d - b^5*d*(-(4*a*c - b^2)^3)^(1/2) - a*b^7*e + 33*a^2*b^
4*c^2*d - 38*a^3*b^2*c^3*d - 25*a^3*b^3*c^2*e + a^3*c^2*e*(-(4*a*c - b^2)^3
)^(1/2) - 10*a*b^6*c*d + a*b^4*e*(-(4*a*c - b^2)^3)^(1/2) + 9*a^2*b^5*c*e +
20*a^4*b*c^3*e + 4*a*b^3*c*d*(-(4*a*c - b^2)^3)^(1/2) - 3*a^2*b*c^2*d*(-(4
*a*c - b^2)^3)^(1/2) - 3*a^2*b^2*c*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^6*b^4
+ 16*a^8*c^2 - 8*a^7*b^2*c)))^(1/2)*1i - (((((2048*a^12*c^4*d*e^12 + 12288*
a^10*c^6*d^5*e^8 + 14336*a^11*c^5*d^3*e^10 + 2048*a^8*b^4*c^4*d^5*e^8 - 153
6*a^8*b^5*c^3*d^4*e^9 - 512*a^8*b^6*c^2*d^3*e^10 - 11264*a^9*b^2*c^5*d^5*e^
8 + 7168*a^9*b^3*c^4*d^4*e^9 + 6272*a^9*b^4*c^3*d^3*e^10 + 384*a^9*b^5*c^2*
d^2*e^11 - 20480*a^10*b^2*c^4*d^3*e^10 - 3072*a^10*b^3*c^3*d^2*e^11 - 4096*
a^10*b*c^5*d^4*e^9 + 128*a^10*b^4*c^2*d*e^12 + 6144*a^11*b*c^4*d^2*e^11 - 1
024*a^11*b^2*c^3*d*e^12)/(64*a^8*d^2) + ((d + e*x^2)^(1/2)*((b^8*d + 8*a^4*
c^4*d - b^5*d*(-(4*a*c - b^2)^3)^(1/2) - a*b^7*e + 33*a^2*b^4*c^2*d - 38*a^
3*b^2*c^3*d - 25*a^3*b^3*c^2*e + a^3*c^2*e*(-(4*a*c - b^2)^3)^(1/2) - 10*a*
b^6*c*d + a*b^4*e*(-(4*a*c - b^2)^3)^(1/2) + 9*a^2*b^5*c*e + 20*a^4*b*c^3*e
+ 4*a*b^3*c*d*(-(4*a*c - b^2)^3)^(1/2) - 3*a^2*b*c^2*d*(-(4*a*c - b^2)^3)^
(1/2) - 3*a^2*b^2*c*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^6*b^4 + 16*a^8*c^2 -
8*a^7*b^2*c)))^(1/2)*(24576*a^12*c^5*d^4*e^8 + 16384*a^13*c^4*d^2*e^10 + 20
48*a^10*b^4*c^3*d^4*e^8 - 2048*a^10*b^5*c^2*d^3*e^9 - 14336*a^11*b^2*c^4*d^
4*e^8 + 15360*a^11*b^3*c^3*d^3*e^9 + 1024*a^11*b^4*c^2*d^2*e^10 - 8192*a^12

*b^2*c^3*d^2*e^10 - 28672*a^12*b*c^4*d^3*e^9))/(32*a^8*d^2))*((b^8*d + 8*a^
4*c^4*d - b^5*d*(-(4*a*c - b^2)^3)^(1/2) - a*b^7*e + 33*a^2*b^4*c^2*d - 38*
a^3*b^2*c^3*d - 25*a^3*b^3*c^2*e + a^3*c^2*e*(-(4*a*c - b^2)^3)^(1/2) - 10*
a*b^6*c*d + a*b^4*e*(-(4*a*c - b^2)^3)^(1/2) + 9*a^2*b^5*c*e + 20*a^4*b*c^3
*e + 4*a*b^3*c*d*(-(4*a*c - b^2)^3)^(1/2) - 3*a^2*b*c^2*d*(-(4*a*c - b^2)^3
)^(1/2) - 3*a^2*b^2*c*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^6*b^4 + 16*a^8*c^2
- 8*a^7*b^2*c)))^(1/2) - ((d + e*x^2)^(1/2)*(32*a^10*c^5*d*e^12 - 48*a^10*b
*c^4*e^13 - 4*a^8*b^5*c^2*e^13 + 28*a^9*b^3*c^3*e^13 + 4608*a^8*c^7*d^5*e^8
+ 2048*a^9*c^6*d^3*e^10 + 512*a^4*b^8*c^3*d^5*e^8 - 512*a^4*b^9*c^2*d^4*e^
9 - 4608*a^5*b^6*c^4*d^5*e^8 + 4352*a^5*b^7*c^3*d^4*e^9 + 768*a^5*b^8*c^2*d
^3*e^10 + 14080*a^6*b^4*c^5*d^5*e^8 - 11264*a^6*b^5*c^4*d^4*e^9 - 6912*a^6*
b^6*c^3*d^3*e^10 - 256*a^6*b^7*c^2*d^2*e^11 - 16384*a^7*b^2*c^6*d^5*e^8 + 7
168*a^7*b^3*c^5*d^4*e^9 + 19776*a^7*b^4*c^4*d^3*e^10 + 2272*a^7*b^5*c^3*d^2
*e^11 - 18048*a^8*b^2*c^5*d^3*e^10 - 6144*a^8*b^3*c^4*d^2*e^11 - 32*a^7*b^6
*c^2*d*e^12 + 3584*a^8*b*c^6*d^4*e^9 + 228*a^8*b^4*c^3*d*e^12 + 4608*a^9*b*
c^5*d^2*e^11 - 408*a^9*b^2*c^4*d*e^12))/(32*a^8*d^2))*((b^8*d + 8*a^4*c^4*d
- b^5*d*(-(4*a*c - b^2)^3)^(1/2) - a*b^7*e + 33*a^2*b^4*c^2*d - 38*a^3*b^2
*c^3*d - 25*a^3*b^3*c^2*e + a^3*c^2*e*(-(4*a*c - b^2)^3)^(1/2) - 10*a*b^6*c
*d + a*b^4*e*(-(4*a*c - b^2)^3)^(1/2) + 9*a^2*b^5*c*e + 20*a^4*b*c^3*e + 4*
a*b^3*c*d*(-(4*a*c - b^2)^3)^(1/2) - 3*a^2*b*c^2*d*(-(4*a*c - b^2)^3)^(1/2)
- 3*a^2*b^2*c*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^6*b^4 + 16*a^8*c^2 - 8*a^7
*b^2*c)))^(1/2) + (16*a^9*c^5*e^14 - 4*a^6*b^6*c^2*e^14 + 28*a^7*b^4*c^3*e^
14 - 52*a^8*b^2*c^4*e^14 - 768*a^6*c^8*d^6*e^8 - 768*a^7*c^7*d^4*e^10 + 16*
a^8*c^6*d^2*e^12 - 512*a^2*b^8*c^4*d^6*e^8 + 384*a^2*b^9*c^3*d^5*e^9 + 128*
a^2*b^10*c^2*d^4*e^10 + 3840*a^3*b^6*c^5*d^6*e^8 - 2048*a^3*b^7*c^4*d^5*e^9
- 2208*a^3*b^8*c^3*d^4*e^10 - 224*a^3*b^9*c^2*d^3*e^11 - 8704*a^4*b^4*c^6*
d^6*e^8 + 896*a^4*b^5*c^5*d^5*e^9 + 10752*a^4*b^6*c^4*d^4*e^10 + 2688*a^4*b
^7*c^3*d^3*e^11 + 96*a^4*b^8*c^2*d^2*e^12 + 6400*a^5*b^2*c^7*d^6*e^8 + 5632
*a^5*b^3*c^6*d^5*e^9 - 18144*a^5*b^4*c^5*d^4*e^10 - 10464*a^5*b^5*c^4*d^3*e
^11 - 836*a^5*b^6*c^3*d^2*e^12 + 9344*a^6*b^2*c^6*d^4*e^10 + 14592*a^6*b^3*
c^5*d^3*e^11 + 2236*a^6*b^4*c^4*d^2*e^12 - 1716*a^7*b^2*c^5*d^2*e^12 - 528*
a^8*b*c^5*d*e^13 + 4*a^5*b^7*c^2*d*e^13 - 4352*a^6*b*c^7*d^5*e^9 - 92*a^6*b
^5*c^3*d*e^13 - 5632*a^7*b*c^6*d^3*e^11 + 436*a^7*b^3*c^4*d*e^13)/(64*a^8*d
^2))*((b^8*d + 8*a^4*c^4*d - b^5*d*(-(4*a*c - b^2)^3)^(1/2) - a*b^7*e + 33*
a^2*b^4*c^2*d - 38*a^3*b^2*c^3*d - 25*a^3*b^3*c^2*e + a^3*c^2*e*(-(4*a*c -
b^2)^3)^(1/2) - 10*a*b^6*c*d + a*b^4*e*(-(4*a*c - b^2)^3)^(1/2) + 9*a^2*b^5
*c*e + 20*a^4*b*c^3*e + 4*a*b^3*c*d*(-(4*a*c - b^2)^3)^(1/2) - 3*a^2*b*c^2*
d*(-(4*a*c - b^2)^3)^(1/2) - 3*a^2*b^2*c*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^
6*b^4 + 16*a^8*c^2 - 8*a^7*b^2*c)))^(1/2) + ((d + e*x^2)^(1/2)*(a^6*b^2*c^5
*e^14 - 2*a^7*c^6*e^14 + 192*a^4*c^9*d^6*e^8 + 32*a^5*c^8*d^4*e^10 + 34*a^6
*c^7*d^2*e^12 + 64*b^8*c^5*d^6*e^8 + 704*a^2*b^4*c^7*d^6*e^8 + 960*a^2*b^5*
c^6*d^5*e^9 + 192*a^2*b^6*c^5*d^4*e^10 - 512*a^3*b^2*c^8*d^6*e^8 - 1280*a^3
*b^3*c^7*d^5*e^9 - 752*a^3*b^4*c^6*d^4*e^10 - 56*a^3*b^5*c^5*d^3*e^11 + 704
*a^4*b^2*c^7*d^4*e^10 + 128*a^4*b^3*c^6*d^3*e^11 - 15*a^4*b^4*c^5*d^2*e^12
+ 60*a^5*b^2*c^6*d^2*e^12 - 10*a^6*b*c^6*d*e^13 - 384*a*b^6*c^6*d^6*e^8 - 1

92*a*b^7*c^5*d^5*e^9 + 384*a^4*b*c^8*d^5*e^9 - 144*a^5*b*c^7*d^3*e^11 + 6*a
^5*b^3*c^5*d*e^13))/(32*a^8*d^2))*((b^8*d + 8*a^4*c^4*d - b^5*d*(-(4*a*c -
b^2)^3)^(1/2) - a*b^7*e + 33*a^2*b^4*c^2*d - 38*a^3*b^2*c^3*d - 25*a^3*b^3*
c^2*e + a^3*c^2*e*(-(4*a*c - b^2)^3)^(1/2) - 10*a*b^6*c*d + a*b^4*e*(-(4*a*
c - b^2)^3)^(1/2) + 9*a^2*b^5*c*e + 20*a^4*b*c^3*e + 4*a*b^3*c*d*(-(4*a*c -
 b^2)^3)^(1/2) - 3*a^2*b*c^2*d*(-(4*a*c - b^2)^3)^(1/2) - 3*a^2*b^2*c*e*(-(
4*a*c - b^2)^3)^(1/2))/(8*(a^6*b^4 + 16*a^8*c^2 - 8*a^7*b^2*c)))^(1/2)*1i)/
((((((2048*a^12*c^4*d*e^12 + 12288*a^10*c^6*d^5*e^8 + 14336*a^11*c^5*d^3*e^
10 + 2048*a^8*b^4*c^4*d^5*e^8 - 1536*a^8*b^5*c^3*d^4*e^9 - 512*a^8*b^6*c^2*
d^3*e^10 - 11264*a^9*b^2*c^5*d^5*e^8 + 7168*a^9*b^3*c^4*d^4*e^9 + 6272*a^9*
b^4*c^3*d^3*e^10 + 384*a^9*b^5*c^2*d^2*e^11 - 20480*a^10*b^2*c^4*d^3*e^10 -
 3072*a^10*b^3*c^3*d^2*e^11 - 4096*a^10*b*c^5*d^4*e^9 + 128*a^10*b^4*c^2*d*
e^12 + 6144*a^11*b*c^4*d^2*e^11 - 1024*a^11*b^2*c^3*d*e^12)/(64*a^8*d^2) -
((d + e*x^2)^(1/2)*((b^8*d + 8*a^4*c^4*d - b^5*d*(-(4*a*c - b^2)^3)^(1/2) -
 a*b^7*e + 33*a^2*b^4*c^2*d - 38*a^3*b^2*c^3*d - 25*a^3*b^3*c^2*e + a^3*c^2
e(-(4*a*c - b^2)^3)^(1/2) - 10*a*b^6*c*d + a*b^4*e*(-(4*a*c - b^2)^3)^(1/
2) + 9*a^2*b^5*c*e + 20*a^4*b*c^3*e + 4*a*b^3*c*d*(-(4*a*c - b^2)^3)^(1/2)
- 3*a^2*b*c^2*d*(-(4*a*c - b^2)^3)^(1/2) - 3*a^2*b^2*c*e*(-(4*a*c - b^2)^3)
^(1/2))/(8*(a^6*b^4 + 16*a^8*c^2 - 8*a^7*b^2*c)))^(1/2)*(24576*a^12*c^5*d^4
*e^8 + 16384*a^13*c^4*d^2*e^10 + 2048*a^10*b^4*c^3*d^4*e^8 - 2048*a^10*b^5*
c^2*d^3*e^9 - 14336*a^11*b^2*c^4*d^4*e^8 + 15360*a^11*b^3*c^3*d^3*e^9 + 102
4*a^11*b^4*c^2*d^2*e^10 - 8192*a^12*b^2*c^3*d^2*e^10 - 28672*a^12*b*c^4*d^3
*e^9))/(32*a^8*d^2))*((b^8*d + 8*a^4*c^4*d - b^5*d*(-(4*a*c - b^2)^3)^(1/2)
 - a*b^7*e + 33*a^2*b^4*c^2*d - 38*a^3*b^2*c^3*d - 25*a^3*b^3*c^2*e + a^3*c
^2*e*(-(4*a*c - b^2)^3)^(1/2) - 10*a*b^6*c*d + a*b^4*e*(-(4*a*c - b^2)^3)^(
1/2) + 9*a^2*b^5*c*e + 20*a^4*b*c^3*e + 4*a*b^3*c*d*(-(4*a*c - b^2)^3)^(1/2
) - 3*a^2*b*c^2*d*(-(4*a*c - b^2)^3)^(1/2) - 3*a^2*b^2*c*e*(-(4*a*c - b^2)^
3)^(1/2))/(8*(a^6*b^4 + 16*a^8*c^2 - 8*a^7*b^2*c)))^(1/2) + ((d + e*x^2)^(1
/2)*(32*a^10*c^5*d*e^12 - 48*a^10*b*c^4*e^13 - 4*a^8*b^5*c^2*e^13 + 28*a^9*
b^3*c^3*e^13 + 4608*a^8*c^7*d^5*e^8 + 2048*a^9*c^6*d^3*e^10 + 512*a^4*b^8*c
^3*d^5*e^8 - 512*a^4*b^9*c^2*d^4*e^9 - 4608*a^5*b^6*c^4*d^5*e^8 + 4352*a^5*
b^7*c^3*d^4*e^9 + 768*a^5*b^8*c^2*d^3*e^10 + 14080*a^6*b^4*c^5*d^5*e^8 - 11
264*a^6*b^5*c^4*d^4*e^9 - 6912*a^6*b^6*c^3*d^3*e^10 - 256*a^6*b^7*c^2*d^2*e
^11 - 16384*a^7*b^2*c^6*d^5*e^8 + 7168*a^7*b^3*c^5*d^4*e^9 + 19776*a^7*b^4*
c^4*d^3*e^10 + 2272*a^7*b^5*c^3*d^2*e^11 - 18048*a^8*b^2*c^5*d^3*e^10 - 614
4*a^8*b^3*c^4*d^2*e^11 - 32*a^7*b^6*c^2*d*e^12 + 3584*a^8*b*c^6*d^4*e^9 + 2
28*a^8*b^4*c^3*d*e^12 + 4608*a^9*b*c^5*d^2*e^11 - 408*a^9*b^2*c^4*d*e^12))/
(32*a^8*d^2))*((b^8*d + 8*a^4*c^4*d - b^5*d*(-(4*a*c - b^2)^3)^(1/2) - a*b^
7*e + 33*a^2*b^4*c^2*d - 38*a^3*b^2*c^3*d - 25*a^3*b^3*c^2*e + a^3*c^2*e*(-
(4*a*c - b^2)^3)^(1/2) - 10*a*b^6*c*d + a*b^4*e*(-(4*a*c - b^2)^3)^(1/2) +
9*a^2*b^5*c*e + 20*a^4*b*c^3*e + 4*a*b^3*c*d*(-(4*a*c - b^2)^3)^(1/2) - 3*a
^2*b*c^2*d*(-(4*a*c - b^2)^3)^(1/2) - 3*a^2*b^2*c*e*(-(4*a*c - b^2)^3)^(1/2
))/(8*(a^6*b^4 + 16*a^8*c^2 - 8*a^7*b^2*c)))^(1/2) + (16*a^9*c^5*e^14 - 4*a
^6*b^6*c^2*e^14 + 28*a^7*b^4*c^3*e^14 - 52*a^8*b^2*c^4*e^14 - 768*a^6*c^8*d
^6*e^8 - 768*a^7*c^7*d^4*e^10 + 16*a^8*c^6*d^2*e^12 - 512*a^2*b^8*c^4*d^6*e

^8 + 384*a^2*b^9*c^3*d^5*e^9 + 128*a^2*b^10*c^2*d^4*e^10 + 3840*a^3*b^6*c^5
*d^6*e^8 - 2048*a^3*b^7*c^4*d^5*e^9 - 2208*a^3*b^8*c^3*d^4*e^10 - 224*a^3*b
^9*c^2*d^3*e^11 - 8704*a^4*b^4*c^6*d^6*e^8 + 896*a^4*b^5*c^5*d^5*e^9 + 1075
2*a^4*b^6*c^4*d^4*e^10 + 2688*a^4*b^7*c^3*d^3*e^11 + 96*a^4*b^8*c^2*d^2*e^1
2 + 6400*a^5*b^2*c^7*d^6*e^8 + 5632*a^5*b^3*c^6*d^5*e^9 - 18144*a^5*b^4*c^5
*d^4*e^10 - 10464*a^5*b^5*c^4*d^3*e^11 - 836*a^5*b^6*c^3*d^2*e^12 + 9344*a^
6*b^2*c^6*d^4*e^10 + 14592*a^6*b^3*c^5*d^3*e^11 + 2236*a^6*b^4*c^4*d^2*e^12
 - 1716*a^7*b^2*c^5*d^2*e^12 - 528*a^8*b*c^5*d*e^13 + 4*a^5*b^7*c^2*d*e^13
- 4352*a^6*b*c^7*d^5*e^9 - 92*a^6*b^5*c^3*d*e^13 - 5632*a^7*b*c^6*d^3*e^11
+ 436*a^7*b^3*c^4*d*e^13)/(64*a^8*d^2))*((b^8*d + 8*a^4*c^4*d - b^5*d*(-(4*
a*c - b^2)^3)^(1/2) - a*b^7*e + 33*a^2*b^4*c^2*d - 38*a^3*b^2*c^3*d - 25*a^
3*b^3*c^2*e + a^3*c^2*e*(-(4*a*c - b^2)^3)^(1/2) - 10*a*b^6*c*d + a*b^4*e*(
-(4*a*c - b^2)^3)^(1/2) + 9*a^2*b^5*c*e + 20*a^4*b*c^3*e + 4*a*b^3*c*d*(-(4
*a*c - b^2)^3)^(1/2) - 3*a^2*b*c^2*d*(-(4*a*c - b^2)^3)^(1/2) - 3*a^2*b^2*c
e(-(4*a*c - b^2)^3)^(1/2))/(8*(a^6*b^4 + 16*a^8*c^2 - 8*a^7*b^2*c)))^(1/2
) - ((d + e*x^2)^(1/2)*(a^6*b^2*c^5*e^14 - 2*a^7*c^6*e^14 + 192*a^4*c^9*d^6
*e^8 + 32*a^5*c^8*d^4*e^10 + 34*a^6*c^7*d^2*e^12 + 64*b^8*c^5*d^6*e^8 + 704
*a^2*b^4*c^7*d^6*e^8 + 960*a^2*b^5*c^6*d^5*e^9 + 192*a^2*b^6*c^5*d^4*e^10 -
 512*a^3*b^2*c^8*d^6*e^8 - 1280*a^3*b^3*c^7*d^5*e^9 - 752*a^3*b^4*c^6*d^4*e
^10 - 56*a^3*b^5*c^5*d^3*e^11 + 704*a^4*b^2*c^7*d^4*e^10 + 128*a^4*b^3*c^6*
d^3*e^11 - 15*a^4*b^4*c^5*d^2*e^12 + 60*a^5*b^2*c^6*d^2*e^12 - 10*a^6*b*c^6
*d*e^13 - 384*a*b^6*c^6*d^6*e^8 - 192*a*b^7*c^5*d^5*e^9 + 384*a^4*b*c^8*d^5
*e^9 - 144*a^5*b*c^7*d^3*e^11 + 6*a^5*b^3*c^5*d*e^13))/(32*a^8*d^2))*((b^8*
d + 8*a^4*c^4*d - b^5*d*(-(4*a*c - b^2)^3)^(1/2) - a*b^7*e + 33*a^2*b^4*c^2
*d - 38*a^3*b^2*c^3*d - 25*a^3*b^3*c^2*e + a^3*c^2*e*(-(4*a*c - b^2)^3)^(1/
2) - 10*a*b^6*c*d + a*b^4*e*(-(4*a*c - b^2)^3)^(1/2) + 9*a^2*b^5*c*e + 20*a
^4*b*c^3*e + 4*a*b^3*c*d*(-(4*a*c - b^2)^3)^(1/2) - 3*a^2*b*c^2*d*(-(4*a*c
- b^2)^3)^(1/2) - 3*a^2*b^2*c*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^6*b^4 + 16*
a^8*c^2 - 8*a^7*b^2*c)))^(1/2) + (((((2048*a^12*c^4*d*e^12 + 12288*a^10*c^6
*d^5*e^8 + 14336*a^11*c^5*d^3*e^10 + 2048*a^8*b^4*c^4*d^5*e^8 - 1536*a^8*b^
5*c^3*d^4*e^9 - 512*a^8*b^6*c^2*d^3*e^10 - 11264*a^9*b^2*c^5*d^5*e^8 + 7168
*a^9*b^3*c^4*d^4*e^9 + 6272*a^9*b^4*c^3*d^3*e^10 + 384*a^9*b^5*c^2*d^2*e^11
 - 20480*a^10*b^2*c^4*d^3*e^10 - 3072*a^10*b^3*c^3*d^2*e^11 - 4096*a^10*b*c
^5*d^4*e^9 + 128*a^10*b^4*c^2*d*e^12 + 6144*a^11*b*c^4*d^2*e^11 - 1024*a^11
*b^2*c^3*d*e^12)/(64*a^8*d^2) + ((d + e*x^2)^(1/2)*((b^8*d + 8*a^4*c^4*d -
b^5*d*(-(4*a*c - b^2)^3)^(1/2) - a*b^7*e + 33*a^2*b^4*c^2*d - 38*a^3*b^2*c^
3*d - 25*a^3*b^3*c^2*e + a^3*c^2*e*(-(4*a*c - b^2)^3)^(1/2) - 10*a*b^6*c*d
+ a*b^4*e*(-(4*a*c - b^2)^3)^(1/2) + 9*a^2*b^5*c*e + 20*a^4*b*c^3*e + 4*a*b
^3*c*d*(-(4*a*c - b^2)^3)^(1/2) - 3*a^2*b*c^2*d*(-(4*a*c - b^2)^3)^(1/2) -
3*a^2*b^2*c*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^6*b^4 + 16*a^8*c^2 - 8*a^7*b^
2*c)))^(1/2)*(24576*a^12*c^5*d^4*e^8 + 16384*a^13*c^4*d^2*e^10 + 2048*a^10*
b^4*c^3*d^4*e^8 - 2048*a^10*b^5*c^2*d^3*e^9 - 14336*a^11*b^2*c^4*d^4*e^8 +
15360*a^11*b^3*c^3*d^3*e^9 + 1024*a^11*b^4*c^2*d^2*e^10 - 8192*a^12*b^2*c^3
*d^2*e^10 - 28672*a^12*b*c^4*d^3*e^9))/(32*a^8*d^2))*((b^8*d + 8*a^4*c^4*d
- b^5*d*(-(4*a*c - b^2)^3)^(1/2) - a*b^7*e + 33*a^2*b^4*c^2*d - 38*a^3*b^2*

c^3*d - 25*a^3*b^3*c^2*e + a^3*c^2*e*(-(4*a*c - b^2)^3)^(1/2) - 10*a*b^6*c*d + a*b^4*e*(-(4*a*c - b^2)^3)^(1/2) + 9*a^2*b^5*c*e + 20*a^4*b*c^3*e + 4*a*b^3*c*d*(-(4*a*c - b^2)^3)^(1/2) - 3*a^2*b*c^2*d*(-(4*a*c - b^2)^3)^(1/2) - 3*a^2*b^2*c*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^6*b^4 + 16*a^8*c^2 - 8*a^7*b^2*c)))^(1/2) - ((d + e*x^2)^(1/2)*(32*a^10*c^5*d*e^12 - 48*a^10*b*c^4*e^13 - 4*a^8*b^5*c^2*e^13 + 28*a^9*b^3*c^3*e^13 + 4608*a^8*c^7*d^5*e^8 + 2048*a^9*c^6*d^3*e^10 + 512*a^4*b^8*c^3*d^5*e^8 - 512*a^4*b^9*c^2*d^4*e^9 - 4608*a^5*b^6*c^4*d^5*e^8 + 4352*a^5*b^7*c^3*d^4*e^9 + 768*a^5*b^8*c^2*d^3*e^10 + 14080*a^6*b^4*c^5*d^5*e^8 - 11264*a^6*b^5*c^4*d^4*e^9 - 6912*a^6*b^6*c^3*d^3*e^10 - 256*a^6*b^7*c^2*d^2*e^11 - 16384*a^7*b^2*c^6*d^5*e^8 + 7168*a^7*b^3*c^5*d^4*e^9 + 19776*a^7*b^4*c^4*d^3*e^10 + 2272*a^7*b^5*c^3*d^2*e^11 - 18048*a^8*b^2*c^5*d^3*e^10 - 6144*a^8*b^3*c^4*d^2*e^11 - 32*a^7*b^6*c^2*d*e^12 + 3584*a^8*b*c^6*d^4*e^9 + 228*a^8*b^4*c^3*d*e^12 + 4608*a^9*b*c^5*d^2*e^11 - 408*a^9*b^2*c^4*d*e^12))/(32*a^8*d^2))*((b^8*d + 8*a^4*c^4*d - b^5*d*(-(4*a*c - b^2)^3)^(1/2) - a*b^7*e + 33*a^2*b^4*c^2*d - 38*a^3*b^2*c^3*d - 25*a^3*b^3*c^2*e + a^3*c^2*e*(-(4*a*c - b^2)^3)^(1/2) - 10*a*b^6*c*d + a*b^4*e*(-(4*a*c - b^2)^3)^(1/2) + 9*a^2*b^5*c*e + 20*a^4*b*c^3*e + 4*a*b^3*c*d*(-(4*a*c - b^2)^3)^(1/2) - 3*a^2*b*c^2*d*(-(4*a*c - b^2)^3)^(1/2) - 3*a^2*b^2*c*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^6*b^4 + 16*a^8*c^2 - 8*a^7*b^2*c)))^(1/2) + (16*a^9*c^5*e^14 - 4*a^6*b^6*c^2*e^14 + 28*a^7*b^4*c^3*e^14 - 52*a^8*b^2*c^4*e^14 - 768*a^6*c^8*d^6*e^8 - 768*a^7*c^7*d^4*e^10 + 16*a^8*c^6*d^2*e^12 - 512*a^2*b^8*c^4*d^6*e^8 + 384*a^2*b^9*c^3*d^5*e^9 + 128*a^2*b^10*c^2*d^4*e^10 + 3840*a^3*b^6*c^5*d^6*e^8 - 2048*a^3*b^7*c^4*d^5*e^9 - 2208*a^3*b^8*c^3*d^4*e^10 - 224*a^3*b^9*c^2*d^3*e^11 - 8704*a^4*b^4*c^6*d^6*e^8 + 896*a^4*b^5*c^5*d^5*e^9 + 10752*a^4*b^6*c^4*d^4*e^10 + 2688*a^4*b^7*c^3*d^3*e^11 + 96*a^4*b^8*c^2*d^2*e^12 + 6400*a^5*b^2*c^7*d^6*e^8 + 5632*a^5*b^3*c^6*d^5*e^9 - 18144*a^5*b^4*c^5*d^4*e^10 - 10464*a^5*b^5*c^4*d^3*e^11 - 836*a^5*b^6*c^3*d^2*e^12 + 9344*a^6*b^2*c^6*d^4*e^10 + 14592*a^6*b^3*c^5*d^3*e^11 + 2236*a^6*b^4*c^4*d^2*e^12 - 1716*a^7*b^2*c^5*d^2*e^12 - 528*a^8*b*c^5*d*e^13 + 4*a^5*b^7*c^2*d*e^13 - 4352*a^6*b*c^7*d^5*e^9 - 92*a^6*b^5*c^3*d*e^13 - 5632*a^7*b*c^6*d^3*e^11 + 436*a^7*b^3*c^4*d*e^13)/(64*a^8*d^2))*((b^8*d + 8*a^4*c^4*d - b^5*d*(-(4*a*c - b^2)^3)^(1/2) - a*b^7*e + 33*a^2*b^4*c^2*d - 38*a^3*b^2*c^3*d - 25*a^3*b^3*c^2*e + a^3*c^2*e*(-(4*a*c - b^2)^3)^(1/2) - 10*a*b^6*c*d + a*b^4*e*(-(4*a*c - b^2)^3)^(1/2) + 9*a^2*b^5*c*e + 20*a^4*b*c^3*e + 4*a*b^3*c*d*(-(4*a*c - b^2)^3)^(1/2) - 3*a^2*b*c^2*d*(-(4*a*c - b^2)^3)^(1/2) - 3*a^2*b^2*c*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^6*b^4 + 16*a^8*c^2 - 8*a^7*b^2*c)))^(1/2) + ((d + e*x^2)^(1/2)*(a^6*b^2*c^5*e^14 - 2*a^7*c^6*e^14 + 192*a^4*c^9*d^6*e^8 + 32*a^5*c^8*d^4*e^10 + 34*a^6*c^7*d^2*e^12 + 64*b^8*c^5*d^6*e^8 + 704*a^2*b^4*c^7*d^6*e^8 + 960*a^2*b^5*c^6*d^5*e^9 + 192*a^2*b^6*c^5*d^4*e^10 - 512*a^3*b^2*c^8*d^6*e^8 - 1280*a^3*b^3*c^7*d^5*e^9 - 752*a^3*b^4*c^6*d^4*e^10 - 56*a^3*b^5*c^5*d^3*e^11 + 704*a^4*b^2*c^7*d^4*e^10 + 128*a^4*b^3*c^6*d^3*e^11 - 15*a^4*b^4*c^5*d^2*e^12 + 60*a^5*b^2*c^6*d^2*e^12 - 10*a^6*b*c^6*d*e^13 - 384*a*b^6*c^6*d^6*e^8 - 192*a*b^7*c^5*d^5*e^9 + 384*a^4*b*c^8*d^5*e^9 - 144*a^5*b*c^7*d^3*e^11 + 6*a^5*b^3*c^5*d*e^13))/(32*a^8*d^2))*((b^8*d + 8*a^4*c^4*d - b^5*d*(-(4*a*c - b^2)^3)^

(1/2) - a*b^7*e + 33*a^2*b^4*c^2*d - 38*a^3*b^2*c^3*d - 25*a^3*b^3*c^2*e + a^3*c^2*e*(-(4*a*c - b^2)^3)^(1/2) - 10*a*b^6*c*d + a*b^4*e*(-(4*a*c - b^2)^3)^(1/2) + 9*a^2*b^5*c*e + 20*a^4*b*c^3*e + 4*a*b^3*c*d*(-(4*a*c - b^2)^3)^(1/2) - 3*a^2*b*c^2*d*(-(4*a*c - b^2)^3)^(1/2) - 3*a^2*b^2*c*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^6*b^4 + 16*a^8*c^2 - 8*a^7*b^2*c)))^(1/2) + (7*a^5*c^7*d*e^14 + 56*a^3*c^9*d^5*e^10 + 63*a^4*c^8*d^3*e^12 - 64*b^4*c^8*d^7*e^8 + 64*b^5*c^7*d^6*e^9 + 64*a^2*b^2*c^8*d^5*e^10 + 224*a^2*b^3*c^7*d^4*e^11 - 112*a^3*b^2*c^7*d^3*e^12 + 64*a*b^2*c^9*d^7*e^8 + 64*a*b^3*c^8*d^6*e^9 - 192*a*b^4*c^7*d^5*e^10 - 96*a^2*b*c^9*d^6*e^9 - 136*a^3*b*c^8*d^4*e^11 + 9*a^4*b*c^7*d^2*e^13)/(32*a^8*d^2)))*((b^8*d + 8*a^4*c^4*d - b^5*d*(-(4*a*c - b^2)^3)^(1/2) - a*b^7*e + 33*a^2*b^4*c^2*d - 38*a^3*b^2*c^3*d - 25*a^3*b^3*c^2*e + a^3*c^2*e*(-(4*a*c - b^2)^3)^(1/2) - 10*a*b^6*c*d + a*b^4*e*(-(4*a*c - b^2)^3)^(1/2) + 9*a^2*b^5*c*e + 20*a^4*b*c^3*e + 4*a*b^3*c*d*(-(4*a*c - b^2)^3)^(1/2) - 3*a^2*b*c^2*d*(-(4*a*c - b^2)^3)^(1/2) - 3*a^2*b^2*c*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^6*b^4 + 16*a^8*c^2 - 8*a^7*b^2*c)))^(1/2)*2i + atan((((((2048*a^12*c^4*d*e^12 + 12288*a^10*c^6*d^5*e^8 + 14336*a^11*c^5*d^3*e^10 + 2048*a^8*b^4*c^4*d^5*e^8 - 1536*a^8*b^5*c^3*d^4*e^9 - 512*a^8*b^6*c^2*d^3*e^10 - 11264*a^9*b^2*c^5*d^5*e^8 + 7168*a^9*b^3*c^4*d^4*e^9 + 6272*a^9*b^4*c^3*d^3*e^10 + 384*a^9*b^5*c^2*d^2*e^11 - 20480*a^10*b^2*c^4*d^3*e^10 - 3072*a^10*b^3*c^3*d^2*e^11 - 4096*a^10*b*c^5*d^4*e^9 + 128*a^10*b^4*c^2*d*e^12 + 6144*a^11*b*c^4*d^2*e^11 - 1024*a^11*b^2*c^3*d*e^12)/(64*a^8*d^2) - ((d + e*x^2)^(1/2)*((b^8*d + 8*a^4*c^4*d + b^5*d*(-(4*a*c - b^2)^3)^(1/2) - a*b^7*e + 33*a^2*b^4*c^2*d - 38*a^3*b^2*c^3*d - 25*a^3*b^3*c^2*e - a^3*c^2*e*(-(4*a*c - b^2)^3)^(1/2) - 10*a*b^6*c*d - a*b^4*e*(-(4*a*c - b^2)^3)^(1/2) + 9*a^2*b^5*c*e + 20*a^4*b*c^3*e - 4*a*b^3*c*d*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b*c^2*d*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b^2*c*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^6*b^4 + 16*a^8*c^2 - 8*a^7*b^2*c)))^(1/2)*(24576*a^12*c^5*d^4*e^8 + 16384*a^13*c^4*d^2*e^10 + 2048*a^10*b^4*c^3*d^4*e^8 - 2048*a^10*b^5*c^2*d^3*e^9 - 14336*a^11*b^2*c^4*d^4*e^8 + 15360*a^11*b^3*c^3*d^3*e^9 + 1024*a^11*b^4*c^2*d^2*e^10 - 8192*a^12*b^2*c^3*d^2*e^10 - 28672*a^12*b*c^4*d^3*e^9))/(32*a^8*d^2))*((b^8*d + 8*a^4*c^4*d + b^5*d*(-(4*a*c - b^2)^3)^(1/2) - a*b^7*e + 33*a^2*b^4*c^2*d - 38*a^3*b^2*c^3*d - 25*a^3*b^3*c^2*e - a^3*c^2*e*(-(4*a*c - b^2)^3)^(1/2) - 10*a*b^6*c*d - a*b^4*e*(-(4*a*c - b^2)^3)^(1/2) + 9*a^2*b^5*c*e + 20*a^4*b*c^3*e - 4*a*b^3*c*d*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b*c^2*d*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b^2*c*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^6*b^4 + 16*a^8*c^2 - 8*a^7*b^2*c)))^(1/2) + ((d + e*x^2)^(1/2)*(32*a^10*c^5*d*e^12 - 48*a^10*b*c^4*e^13 - 4*a^8*b^5*c^2*e^13 + 28*a^9*b^3*c^3*e^13 + 4608*a^8*c^7*d^5*e^8 + 2048*a^9*c^6*d^3*e^10 + 512*a^4*b^8*c^3*d^5*e^8 - 512*a^4*b^9*c^2*d^4*e^9 - 4608*a^5*b^6*c^4*d^5*e^8 + 4352*a^5*b^7*c^3*d^4*e^9 + 768*a^5*b^8*c^2*d^3*e^10 + 14080*a^6*b^4*c^5*d^5*e^8 - 11264*a^6*b^5*c^4*d^4*e^9 - 6912*a^6*b^6*c^3*d^3*e^10 - 256*a^6*b^7*c^2*d^2*e^11 - 16384*a^7*b^2*c^6*d^5*e^8 + 7168*a^7*b^3*c^5*d^4*e^9 + 19776*a^7*b^4*c^4*d^3*e^10 + 2272*a^7*b^5*c^3*d^2*e^11 - 18048*a^8*b^2*c^5*d^3*e^10 - 6144*a^8*b^3*c^4*d^2*e^11 - 32*a^7*b^6*c^2*d*e^12 + 3584*a^8*b*c^6*d^4*e^9 + 228*a^8*b^4*c^3*d*e^12 + 4608*a^9*b*c^5*d^2*e^11 - 408*a^9*b^2*c^4*d*e^1

2))/(32*a^8*d^2))*((b^8*d + 8*a^4*c^4*d + b^5*d*(-(4*a*c - b^2)^3)^(1/2) - a*b^7*e + 33*a^2*b^4*c^2*d - 38*a^3*b^2*c^3*d - 25*a^3*b^3*c^2*e - a^3*c^2*e*(-(4*a*c - b^2)^3)^(1/2) - 10*a*b^6*c*d - a*b^4*e*(-(4*a*c - b^2)^3)^(1/2) + 9*a^2*b^5*c*e + 20*a^4*b*c^3*e - 4*a*b^3*c*d*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b*c^2*d*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b^2*c*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^6*b^4 + 16*a^8*c^2 - 8*a^7*b^2*c)))^(1/2) + (16*a^9*c^5*e^14 - 4*a^6*b^6*c^2*e^14 + 28*a^7*b^4*c^3*e^14 - 52*a^8*b^2*c^4*e^14 - 768*a^6*c^8*d^6*e^8 - 768*a^7*c^7*d^4*e^10 + 16*a^8*c^6*d^2*e^12 - 512*a^2*b^8*c^4*d^6*e^8 + 384*a^2*b^9*c^3*d^5*e^9 + 128*a^2*b^10*c^2*d^4*e^10 + 3840*a^3*b^6*c^5*d^6*e^8 - 2048*a^3*b^7*c^4*d^5*e^9 - 2208*a^3*b^8*c^3*d^4*e^10 - 224*a^3*b^9*c^2*d^3*e^11 - 8704*a^4*b^4*c^6*d^6*e^8 + 896*a^4*b^5*c^5*d^5*e^9 + 10752*a^4*b^6*c^4*d^4*e^10 + 2688*a^4*b^7*c^3*d^3*e^11 + 96*a^4*b^8*c^2*d^2*e^12 + 6400*a^5*b^2*c^7*d^6*e^8 + 5632*a^5*b^3*c^6*d^5*e^9 - 18144*a^5*b^4*c^5*d^4*e^10 - 10464*a^5*b^5*c^4*d^3*e^11 - 836*a^5*b^6*c^3*d^2*e^12 + 9344*a^6*b^2*c^6*d^4*e^10 + 14592*a^6*b^3*c^5*d^3*e^11 + 2236*a^6*b^4*c^4*d^2*e^12 - 1716*a^7*b^2*c^5*d^2*e^12 - 528*a^8*b*c^5*d*e^13 + 4*a^5*b^7*c^2*d*e^13 - 4352*a^6*b*c^7*d^5*e^9 - 92*a^6*b^5*c^3*d*e^13 - 5632*a^7*b*c^6*d^3*e^11 + 436*a^7*b^3*c^4*d*e^13)/(64*a^8*d^2))*((b^8*d + 8*a^4*c^4*d + b^5*d*(-(4*a*c - b^2)^3)^(1/2) - a*b^7*e + 33*a^2*b^4*c^2*d - 38*a^3*b^2*c^3*d - 25*a^3*b^3*c^2*e - a^3*c^2*e*(-(4*a*c - b^2)^3)^(1/2) - 10*a*b^6*c*d - a*b^4*e*(-(4*a*c - b^2)^3)^(1/2) + 9*a^2*b^5*c*e + 20*a^4*b*c^3*e - 4*a*b^3*c*d*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b*c^2*d*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b^2*c*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^6*b^4 + 16*a^8*c^2 - 8*a^7*b^2*c)))^(1/2) - ((d + e*x^2)^(1/2)*(a^6*b^2*c^5*e^14 - 2*a^7*c^6*e^14 + 192*a^4*c^9*d^6*e^8 + 32*a^5*c^8*d^4*e^10 + 34*a^6*c^7*d^2*e^12 + 64*b^8*c^5*d^6*e^8 + 704*a^2*b^4*c^7*d^6*e^8 + 960*a^2*b^5*c^6*d^5*e^9 + 192*a^2*b^6*c^5*d^4*e^10 - 512*a^3*b^2*c^8*d^6*e^8 - 1280*a^3*b^3*c^7*d^5*e^9 - 752*a^3*b^4*c^6*d^4*e^10 - 56*a^3*b^5*c^5*d^3*e^11 + 704*a^4*b^2*c^7*d^4*e^10 + 128*a^4*b^3*c^6*d^3*e^11 - 15*a^4*b^4*c^5*d^2*e^12 + 60*a^5*b^2*c^6*d^2*e^12 - 10*a^6*b*c^6*d*e^13 - 384*a*b^6*c^6*d^6*e^8 - 192*a*b^7*c^5*d^5*e^9 + 384*a^4*b*c^8*d^5*e^9 - 144*a^5*b*c^7*d^3*e^11 + 6*a^5*b^3*c^5*d*e^13))/(32*a^8*d^2))*((b^8*d + 8*a^4*c^4*d + b^5*d*(-(4*a*c - b^2)^3)^(1/2) - a*b^7*e + 33*a^2*b^4*c^2*d - 38*a^3*b^2*c^3*d - 25*a^3*b^3*c^2*e - a^3*c^2*e*(-(4*a*c - b^2)^3)^(1/2) - 10*a*b^6*c*d - a*b^4*e*(-(4*a*c - b^2)^3)^(1/2) + 9*a^2*b^5*c*e + 20*a^4*b*c^3*e - 4*a*b^3*c*d*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b*c^2*d*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b^2*c*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^6*b^4 + 16*a^8*c^2 - 8*a^7*b^2*c)))^(1/2)*1i - (((((2048*a^12*c^4*d*e^12 + 12288*a^10*c^6*d^5*e^8 + 14336*a^11*c^5*d^3*e^10 + 2048*a^8*b^4*c^4*d^5*e^8 - 1536*a^8*b^5*c^3*d^4*e^9 - 512*a^8*b^6*c^2*d^3*e^10 - 11264*a^9*b^2*c^5*d^5*e^8 + 7168*a^9*b^3*c^4*d^4*e^9 + 6272*a^9*b^4*c^3*d^3*e^10 + 384*a^9*b^5*c^2*d^2*e^11 - 20480*a^10*b^2*c^4*d^3*e^10 - 3072*a^10*b^3*c^3*d^2*e^11 - 4096*a^10*b*c^5*d^4*e^9 + 128*a^10*b^4*c^2*d*e^12 + 6144*a^11*b*c^4*d^2*e^11 - 1024*a^11*b^2*c^3*d*e^12)/(64*a^8*d^2) + ((d + e*x^2)^(1/2)*((b^8*d + 8*a^4*c^4*d + b^5*d*(-(4*a*c - b^2)^3)^(1/2) - a*b^7*e + 33*a^2*b^4*c^2*d - 38*a^3*b^2*c^3*d - 25*a^3*b^3*c^2*e - a^3*c^2*e*(-(4*a*c - b^2)^3)^(1/2) - 10*a*b

^6*c*d - a*b^4*e*(-(4*a*c - b^2)^3)^(1/2) + 9*a^2*b^5*c*e + 20*a^4*b*c^3*e
- 4*a*b^3*c*d*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b*c^2*d*(-(4*a*c - b^2)^3)^(
1/2) + 3*a^2*b^2*c*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^6*b^4 + 16*a^8*c^2 - 8
*a^7*b^2*c)))^(1/2)*(24576*a^12*c^5*d^4*e^8 + 16384*a^13*c^4*d^2*e^10 + 204
8*a^10*b^4*c^3*d^4*e^8 - 2048*a^10*b^5*c^2*d^3*e^9 - 14336*a^11*b^2*c^4*d^4
*e^8 + 15360*a^11*b^3*c^3*d^3*e^9 + 1024*a^11*b^4*c^2*d^2*e^10 - 8192*a^12*
b^2*c^3*d^2*e^10 - 28672*a^12*b*c^4*d^3*e^9))/(32*a^8*d^2))*((b^8*d + 8*a^4
*c^4*d + b^5*d*(-(4*a*c - b^2)^3)^(1/2) - a*b^7*e + 33*a^2*b^4*c^2*d - 38*a
^3*b^2*c^3*d - 25*a^3*b^3*c^2*e - a^3*c^2*e*(-(4*a*c - b^2)^3)^(1/2) - 10*a
*b^6*c*d - a*b^4*e*(-(4*a*c - b^2)^3)^(1/2) + 9*a^2*b^5*c*e + 20*a^4*b*c^3*
e - 4*a*b^3*c*d*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b*c^2*d*(-(4*a*c - b^2)^3)
^(1/2) + 3*a^2*b^2*c*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^6*b^4 + 16*a^8*c^2 -
 8*a^7*b^2*c)))^(1/2) - ((d + e*x^2)^(1/2)*(32*a^10*c^5*d*e^12 - 48*a^10*b*
c^4*e^13 - 4*a^8*b^5*c^2*e^13 + 28*a^9*b^3*c^3*e^13 + 4608*a^8*c^7*d^5*e^8
+ 2048*a^9*c^6*d^3*e^10 + 512*a^4*b^8*c^3*d^5*e^8 - 512*a^4*b^9*c^2*d^4*e^9
 - 4608*a^5*b^6*c^4*d^5*e^8 + 4352*a^5*b^7*c^3*d^4*e^9 + 768*a^5*b^8*c^2*d^
3*e^10 + 14080*a^6*b^4*c^5*d^5*e^8 - 11264*a^6*b^5*c^4*d^4*e^9 - 6912*a^6*b
^6*c^3*d^3*e^10 - 256*a^6*b^7*c^2*d^2*e^11 - 16384*a^7*b^2*c^6*d^5*e^8 + 71
68*a^7*b^3*c^5*d^4*e^9 + 19776*a^7*b^4*c^4*d^3*e^10 + 2272*a^7*b^5*c^3*d^2*
e^11 - 18048*a^8*b^2*c^5*d^3*e^10 - 6144*a^8*b^3*c^4*d^2*e^11 - 32*a^7*b^6*
c^2*d*e^12 + 3584*a^8*b*c^6*d^4*e^9 + 228*a^8*b^4*c^3*d*e^12 + 4608*a^9*b*c
^5*d^2*e^11 - 408*a^9*b^2*c^4*d*e^12))/(32*a^8*d^2))*((b^8*d + 8*a^4*c^4*d
+ b^5*d*(-(4*a*c - b^2)^3)^(1/2) - a*b^7*e + 33*a^2*b^4*c^2*d - 38*a^3*b^2*
c^3*d - 25*a^3*b^3*c^2*e - a^3*c^2*e*(-(4*a*c - b^2)^3)^(1/2) - 10*a*b^6*c*
d - a*b^4*e*(-(4*a*c - b^2)^3)^(1/2) + 9*a^2*b^5*c*e + 20*a^4*b*c^3*e - 4*a
*b^3*c*d*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b*c^2*d*(-(4*a*c - b^2)^3)^(1/2)
+ 3*a^2*b^2*c*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^6*b^4 + 16*a^8*c^2 - 8*a^7*
b^2*c)))^(1/2) + (16*a^9*c^5*e^14 - 4*a^6*b^6*c^2*e^14 + 28*a^7*b^4*c^3*e^1
4 - 52*a^8*b^2*c^4*e^14 - 768*a^6*c^8*d^6*e^8 - 768*a^7*c^7*d^4*e^10 + 16*a
^8*c^6*d^2*e^12 - 512*a^2*b^8*c^4*d^6*e^8 + 384*a^2*b^9*c^3*d^5*e^9 + 128*a
^2*b^10*c^2*d^4*e^10 + 3840*a^3*b^6*c^5*d^6*e^8 - 2048*a^3*b^7*c^4*d^5*e^9
- 2208*a^3*b^8*c^3*d^4*e^10 - 224*a^3*b^9*c^2*d^3*e^11 - 8704*a^4*b^4*c^6*d
^6*e^8 + 896*a^4*b^5*c^5*d^5*e^9 + 10752*a^4*b^6*c^4*d^4*e^10 + 2688*a^4*b^
7*c^3*d^3*e^11 + 96*a^4*b^8*c^2*d^2*e^12 + 6400*a^5*b^2*c^7*d^6*e^8 + 5632*
a^5*b^3*c^6*d^5*e^9 - 18144*a^5*b^4*c^5*d^4*e^10 - 10464*a^5*b^5*c^4*d^3*e^
11 - 836*a^5*b^6*c^3*d^2*e^12 + 9344*a^6*b^2*c^6*d^4*e^10 + 14592*a^6*b^3*c
^5*d^3*e^11 + 2236*a^6*b^4*c^4*d^2*e^12 - 1716*a^7*b^2*c^5*d^2*e^12 - 528*a
^8*b*c^5*d*e^13 + 4*a^5*b^7*c^2*d*e^13 - 4352*a^6*b*c^7*d^5*e^9 - 92*a^6*b^
5*c^3*d*e^13 - 5632*a^7*b*c^6*d^3*e^11 + 436*a^7*b^3*c^4*d*e^13)/(64*a^8*d^
2))*((b^8*d + 8*a^4*c^4*d + b^5*d*(-(4*a*c - b^2)^3)^(1/2) - a*b^7*e + 33*a
^2*b^4*c^2*d - 38*a^3*b^2*c^3*d - 25*a^3*b^3*c^2*e - a^3*c^2*e*(-(4*a*c - b
^2)^3)^(1/2) - 10*a*b^6*c*d - a*b^4*e*(-(4*a*c - b^2)^3)^(1/2) + 9*a^2*b^5*
c*e + 20*a^4*b*c^3*e - 4*a*b^3*c*d*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b*c^2*d
*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b^2*c*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^6
*b^4 + 16*a^8*c^2 - 8*a^7*b^2*c)))^(1/2) + ((d + e*x^2)^(1/2)*(a^6*b^2*c^5*

e^14 - 2*a^7*c^6*e^14 + 192*a^4*c^9*d^6*e^8 + 32*a^5*c^8*d^4*e^10 + 34*a^6*c^7*d^2*e^12 + 64*b^8*c^5*d^6*e^8 + 704*a^2*b^4*c^7*d^6*e^8 + 960*a^2*b^5*c^6*d^5*e^9 + 192*a^2*b^6*c^5*d^4*e^10 - 512*a^3*b^2*c^8*d^6*e^8 - 1280*a^3*b^3*c^7*d^5*e^9 - 752*a^3*b^4*c^6*d^4*e^10 - 56*a^3*b^5*c^5*d^3*e^11 + 704*a^4*b^2*c^7*d^4*e^10 + 128*a^4*b^3*c^6*d^3*e^11 - 15*a^4*b^4*c^5*d^2*e^12 + 60*a^5*b^2*c^6*d^2*e^12 - 10*a^6*b*c^6*d*e^13 - 384*a*b^6*c^6*d^6*e^8 - 192*a*b^7*c^5*d^5*e^9 + 384*a^4*b*c^8*d^5*e^9 - 144*a^5*b*c^7*d^3*e^11 + 6*a^5*b^3*c^5*d*e^13))/(32*a^8*d^2))*((b^8*d + 8*a^4*c^4*d + b^5*d*(-(4*a*c - b^2)^3)^(1/2) - a*b^7*e + 33*a^2*b^4*c^2*d - 38*a^3*b^2*c^3*d - 25*a^3*b^3*c^2*e - a^3*c^2*e*(-(4*a*c - b^2)^3)^(1/2) - 10*a*b^6*c*d - a*b^4*e*(-(4*a*c - b^2)^3)^(1/2) + 9*a^2*b^5*c*e + 20*a^4*b*c^3*e - 4*a*b^3*c*d*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b*c^2*d*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b^2*c*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^6*b^4 + 16*a^8*c^2 - 8*a^7*b^2*c)))^(1/2)*1i)/((((((2048*a^12*c^4*d*e^12 + 12288*a^10*c^6*d^5*e^8 + 14336*a^11*c^5*d^3*e^10 + 2048*a^8*b^4*c^4*d^5*e^8 - 1536*a^8*b^5*c^3*d^4*e^9 - 512*a^8*b^6*c^2*d^3*e^10 - 11264*a^9*b^2*c^5*d^5*e^8 + 7168*a^9*b^3*c^4*d^4*e^9 + 6272*a^9*b^4*c^3*d^3*e^10 + 384*a^9*b^5*c^2*d^2*e^11 - 20480*a^10*b^2*c^4*d^3*e^10 - 3072*a^10*b^3*c^3*d^2*e^11 - 4096*a^10*b*c^5*d^4*e^9 + 128*a^10*b^4*c^2*d*e^12 + 6144*a^11*b*c^4*d^2*e^11 - 1024*a^11*b^2*c^3*d*e^12)/(64*a^8*d^2) - ((d + e*x^2)^(1/2)*((b^8*d + 8*a^4*c^4*d + b^5*d*(-(4*a*c - b^2)^3)^(1/2) - a*b^7*e + 33*a^2*b^4*c^2*d - 38*a^3*b^2*c^3*d - 25*a^3*b^3*c^2*e - a^3*c^2*e*(-(4*a*c - b^2)^3)^(1/2) - 10*a*b^6*c*d - a*b^4*e*(-(4*a*c - b^2)^3)^(1/2) + 9*a^2*b^5*c*e + 20*a^4*b*c^3*e - 4*a*b^3*c*d*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b*c^2*d*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b^2*c*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^6*b^4 + 16*a^8*c^2 - 8*a^7*b^2*c)))^(1/2)*(24576*a^12*c^5*d^4*e^8 + 16384*a^13*c^4*d^2*e^10 + 2048*a^10*b^4*c^3*d^4*e^8 - 2048*a^10*b^5*c^2*d^3*e^9 - 14336*a^11*b^2*c^4*d^4*e^8 + 15360*a^11*b^3*c^3*d^3*e^9 + 1024*a^11*b^4*c^2*d^2*e^10 - 8192*a^12*b^2*c^3*d^2*e^10 - 28672*a^12*b*c^4*d^3*e^9))/(32*a^8*d^2))*((b^8*d + 8*a^4*c^4*d + b^5*d*(-(4*a*c - b^2)^3)^(1/2) - a*b^7*e + 33*a^2*b^4*c^2*d - 38*a^3*b^2*c^3*d - 25*a^3*b^3*c^2*e - a^3*c^2*e*(-(4*a*c - b^2)^3)^(1/2) - 10*a*b^6*c*d - a*b^4*e*(-(4*a*c - b^2)^3)^(1/2) + 9*a^2*b^5*c*e + 20*a^4*b*c^3*e - 4*a*b^3*c*d*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b*c^2*d*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b^2*c*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^6*b^4 + 16*a^8*c^2 - 8*a^7*b^2*c)))^(1/2) + ((d + e*x^2)^(1/2)*(32*a^10*c^5*d*e^12 - 48*a^10*b*c^4*e^13 - 4*a^8*b^5*c^2*e^13 + 28*a^9*b^3*c^3*e^13 + 4608*a^8*c^7*d^5*e^8 + 2048*a^9*c^6*d^3*e^10 + 512*a^4*b^8*c^3*d^5*e^8 - 512*a^4*b^9*c^2*d^4*e^9 - 4608*a^5*b^6*c^4*d^5*e^8 + 4352*a^5*b^7*c^3*d^4*e^9 + 768*a^5*b^8*c^2*d^3*e^10 + 14080*a^6*b^4*c^5*d^5*e^8 - 11264*a^6*b^5*c^4*d^4*e^9 - 6912*a^6*b^6*c^3*d^3*e^10 - 256*a^6*b^7*c^2*d^2*e^11 - 16384*a^7*b^2*c^6*d^5*e^8 + 7168*a^7*b^3*c^5*d^4*e^9 + 19776*a^7*b^4*c^4*d^3*e^10 + 2272*a^7*b^5*c^3*d^2*e^11 - 18048*a^8*b^2*c^5*d^3*e^10 - 6144*a^8*b^3*c^4*d^2*e^11 - 32*a^7*b^6*c^2*d*e^12 + 3584*a^8*b*c^6*d^4*e^9 + 228*a^8*b^4*c^3*d*e^12 + 4608*a^9*b*c^5*d^2*e^11 - 408*a^9*b^2*c^4*d*e^12))/(32*a^8*d^2))*((b^8*d + 8*a^4*c^4*d + b^5*d*(-(4*a*c - b^2)^3)^(1/2) - a*b^7*e + 33*a^2*b^4*c^2*d - 38*a^3*b^2*c^3*d - 25*a^3*b^3*c^2*e - a^3*c^2*e*(-(

4*a*c - b^2)^3)^(1/2) - 10*a*b^6*c*d - a*b^4*e*(-(4*a*c - b^2)^3)^(1/2) + 9
*a^2*b^5*c*e + 20*a^4*b*c^3*e - 4*a*b^3*c*d*(-(4*a*c - b^2)^3)^(1/2) + 3*a^
2*b*c^2*d*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b^2*c*e*(-(4*a*c - b^2)^3)^(1/2)
)/(8*(a^6*b^4 + 16*a^8*c^2 - 8*a^7*b^2*c)))^(1/2) + (16*a^9*c^5*e^14 - 4*a^
6*b^6*c^2*e^14 + 28*a^7*b^4*c^3*e^14 - 52*a^8*b^2*c^4*e^14 - 768*a^6*c^8*d^
6*e^8 - 768*a^7*c^7*d^4*e^10 + 16*a^8*c^6*d^2*e^12 - 512*a^2*b^8*c^4*d^6*e^
8 + 384*a^2*b^9*c^3*d^5*e^9 + 128*a^2*b^10*c^2*d^4*e^10 + 3840*a^3*b^6*c^5*
d^6*e^8 - 2048*a^3*b^7*c^4*d^5*e^9 - 2208*a^3*b^8*c^3*d^4*e^10 - 224*a^3*b^
9*c^2*d^3*e^11 - 8704*a^4*b^4*c^6*d^6*e^8 + 896*a^4*b^5*c^5*d^5*e^9 + 10752
*a^4*b^6*c^4*d^4*e^10 + 2688*a^4*b^7*c^3*d^3*e^11 + 96*a^4*b^8*c^2*d^2*e^12
 + 6400*a^5*b^2*c^7*d^6*e^8 + 5632*a^5*b^3*c^6*d^5*e^9 - 18144*a^5*b^4*c^5*
d^4*e^10 - 10464*a^5*b^5*c^4*d^3*e^11 - 836*a^5*b^6*c^3*d^2*e^12 + 9344*a^6
*b^2*c^6*d^4*e^10 + 14592*a^6*b^3*c^5*d^3*e^11 + 2236*a^6*b^4*c^4*d^2*e^12
- 1716*a^7*b^2*c^5*d^2*e^12 - 528*a^8*b*c^5*d*e^13 + 4*a^5*b^7*c^2*d*e^13 -
 4352*a^6*b*c^7*d^5*e^9 - 92*a^6*b^5*c^3*d*e^13 - 5632*a^7*b*c^6*d^3*e^11 +
 436*a^7*b^3*c^4*d*e^13)/(64*a^8*d^2))*((b^8*d + 8*a^4*c^4*d + b^5*d*(-(4*a
*c - b^2)^3)^(1/2) - a*b^7*e + 33*a^2*b^4*c^2*d - 38*a^3*b^2*c^3*d - 25*a^3
*b^3*c^2*e - a^3*c^2*e*(-(4*a*c - b^2)^3)^(1/2) - 10*a*b^6*c*d - a*b^4*e*(-
(4*a*c - b^2)^3)^(1/2) + 9*a^2*b^5*c*e + 20*a^4*b*c^3*e - 4*a*b^3*c*d*(-(4*
a*c - b^2)^3)^(1/2) + 3*a^2*b*c^2*d*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b^2*c*
e*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^6*b^4 + 16*a^8*c^2 - 8*a^7*b^2*c)))^(1/2)
 - ((d + e*x^2)^(1/2)*(a^6*b^2*c^5*e^14 - 2*a^7*c^6*e^14 + 192*a^4*c^9*d^6*
e^8 + 32*a^5*c^8*d^4*e^10 + 34*a^6*c^7*d^2*e^12 + 64*b^8*c^5*d^6*e^8 + 704*
a^2*b^4*c^7*d^6*e^8 + 960*a^2*b^5*c^6*d^5*e^9 + 192*a^2*b^6*c^5*d^4*e^10 -
512*a^3*b^2*c^8*d^6*e^8 - 1280*a^3*b^3*c^7*d^5*e^9 - 752*a^3*b^4*c^6*d^4*e^
10 - 56*a^3*b^5*c^5*d^3*e^11 + 704*a^4*b^2*c^7*d^4*e^10 + 128*a^4*b^3*c^6*d
^3*e^11 - 15*a^4*b^4*c^5*d^2*e^12 + 60*a^5*b^2*c^6*d^2*e^12 - 10*a^6*b*c^6*
d*e^13 - 384*a*b^6*c^6*d^6*e^8 - 192*a*b^7*c^5*d^5*e^9 + 384*a^4*b*c^8*d^5*
e^9 - 144*a^5*b*c^7*d^3*e^11 + 6*a^5*b^3*c^5*d*e^13))/(32*a^8*d^2))*((b^8*d
 + 8*a^4*c^4*d + b^5*d*(-(4*a*c - b^2)^3)^(1/2) - a*b^7*e + 33*a^2*b^4*c^2*
d - 38*a^3*b^2*c^3*d - 25*a^3*b^3*c^2*e - a^3*c^2*e*(-(4*a*c - b^2)^3)^(1/2
) - 10*a*b^6*c*d - a*b^4*e*(-(4*a*c - b^2)^3)^(1/2) + 9*a^2*b^5*c*e + 20*a^
4*b*c^3*e - 4*a*b^3*c*d*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b*c^2*d*(-(4*a*c -
 b^2)^3)^(1/2) + 3*a^2*b^2*c*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^6*b^4 + 16*a
^8*c^2 - 8*a^7*b^2*c)))^(1/2) + (((((2048*a^12*c^4*d*e^12 + 12288*a^10*c^6*
d^5*e^8 + 14336*a^11*c^5*d^3*e^10 + 2048*a^8*b^4*c^4*d^5*e^8 - 1536*a^8*b^5
*c^3*d^4*e^9 - 512*a^8*b^6*c^2*d^3*e^10 - 11264*a^9*b^2*c^5*d^5*e^8 + 7168*
a^9*b^3*c^4*d^4*e^9 + 6272*a^9*b^4*c^3*d^3*e^10 + 384*a^9*b^5*c^2*d^2*e^11
- 20480*a^10*b^2*c^4*d^3*e^10 - 3072*a^10*b^3*c^3*d^2*e^11 - 4096*a^10*b*c^
5*d^4*e^9 + 128*a^10*b^4*c^2*d*e^12 + 6144*a^11*b*c^4*d^2*e^11 - 1024*a^11*
b^2*c^3*d*e^12)/(64*a^8*d^2) + ((d + e*x^2)^(1/2)*((b^8*d + 8*a^4*c^4*d + b
^5*d*(-(4*a*c - b^2)^3)^(1/2) - a*b^7*e + 33*a^2*b^4*c^2*d - 38*a^3*b^2*c^3
*d - 25*a^3*b^3*c^2*e - a^3*c^2*e*(-(4*a*c - b^2)^3)^(1/2) - 10*a*b^6*c*d -
 a*b^4*e*(-(4*a*c - b^2)^3)^(1/2) + 9*a^2*b^5*c*e + 20*a^4*b*c^3*e - 4*a*b^
3*c*d*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b*c^2*d*(-(4*a*c - b^2)^3)^(1/2) + 3

*a^2*b^2*c*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^6*b^4 + 16*a^8*c^2 - 8*a^7*b^2*c)))^(1/2)*(24576*a^12*c^5*d^4*e^8 + 16384*a^13*c^4*d^2*e^10 + 2048*a^10*b^4*c^3*d^4*e^8 - 2048*a^10*b^5*c^2*d^3*e^9 - 14336*a^11*b^2*c^4*d^4*e^8 + 15360*a^11*b^3*c^3*d^3*e^9 + 1024*a^11*b^4*c^2*d^2*e^10 - 8192*a^12*b^2*c^3*d^2*e^10 - 28672*a^12*b*c^4*d^3*e^9))/(32*a^8*d^2))*((b^8*d + 8*a^4*c^4*d + b^5*d*(-(4*a*c - b^2)^3)^(1/2) - a*b^7*e + 33*a^2*b^4*c^2*d - 38*a^3*b^2*c^3*d - 25*a^3*b^3*c^2*e - a^3*c^2*e*(-(4*a*c - b^2)^3)^(1/2) - 10*a*b^6*c*d - a*b^4*e*(-(4*a*c - b^2)^3)^(1/2) + 9*a^2*b^5*c*e + 20*a^4*b*c^3*e - 4*a*b^3*c*d*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b*c^2*d*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b^2*c*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^6*b^4 + 16*a^8*c^2 - 8*a^7*b^2*c)))^(1/2) - ((d + e*x^2)^(1/2)*(32*a^10*c^5*d*e^12 - 48*a^10*b*c^4*e^13 - 4*a^8*b^5*c^2*e^13 + 28*a^9*b^3*c^3*e^13 + 4608*a^8*c^7*d^5*e^8 + 2048*a^9*c^6*d^3*e^10 + 512*a^4*b^8*c^3*d^5*e^8 - 512*a^4*b^9*c^2*d^4*e^9 - 4608*a^5*b^6*c^4*d^5*e^8 + 4352*a^5*b^7*c^3*d^4*e^9 + 768*a^5*b^8*c^2*d^3*e^10 + 14080*a^6*b^4*c^5*d^5*e^8 - 11264*a^6*b^5*c^4*d^4*e^9 - 6912*a^6*b^6*c^3*d^3*e^10 - 256*a^6*b^7*c^2*d^2*e^11 - 16384*a^7*b^2*c^6*d^5*e^8 + 7168*a^7*b^3*c^5*d^4*e^9 + 19776*a^7*b^4*c^4*d^3*e^10 + 2272*a^7*b^5*c^3*d^2*e^11 - 18048*a^8*b^2*c^5*d^3*e^10 - 6144*a^8*b^3*c^4*d^2*e^11 - 32*a^7*b^6*c^2*d*e^12 + 3584*a^8*b*c^6*d^4*e^9 + 228*a^8*b^4*c^3*d*e^12 + 4608*a^9*b*c^5*d^2*e^11 - 408*a^9*b^2*c^4*d*e^12))/(32*a^8*d^2))*((b^8*d + 8*a^4*c^4*d + b^5*d*(-(4*a*c - b^2)^3)^(1/2) - a*b^7*e + 33*a^2*b^4*c^2*d - 38*a^3*b^2*c^3*d - 25*a^3*b^3*c^2*e - a^3*c^2*e*(-(4*a*c - b^2)^3)^(1/2) - 10*a*b^6*c*d - a*b^4*e*(-(4*a*c - b^2)^3)^(1/2) + 9*a^2*b^5*c*e + 20*a^4*b*c^3*e - 4*a*b^3*c*d*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b*c^2*d*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b^2*c*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^6*b^4 + 16*a^8*c^2 - 8*a^7*b^2*c)))^(1/2) + (16*a^9*c^5*e^14 - 4*a^6*b^6*c^2*e^14 + 28*a^7*b^4*c^3*e^14 - 52*a^8*b^2*c^4*e^14 - 768*a^6*c^8*d^6*e^8 - 768*a^7*c^7*d^4*e^10 + 16*a^8*c^6*d^2*e^12 - 512*a^2*b^8*c^4*d^6*e^8 + 384*a^2*b^9*c^3*d^5*e^9 + 128*a^2*b^10*c^2*d^4*e^10 + 3840*a^3*b^6*c^5*d^6*e^8 - 2048*a^3*b^7*c^4*d^5*e^9 - 2208*a^3*b^8*c^3*d^4*e^10 - 224*a^3*b^9*c^2*d^3*e^11 - 8704*a^4*b^4*c^6*d^6*e^8 + 896*a^4*b^5*c^5*d^5*e^9 + 10752*a^4*b^6*c^4*d^4*e^10 + 2688*a^4*b^7*c^3*d^3*e^11 + 96*a^4*b^8*c^2*d^2*e^12 + 6400*a^5*b^2*c^7*d^6*e^8 + 5632*a^5*b^3*c^6*d^5*e^9 - 18144*a^5*b^4*c^5*d^4*e^10 - 10464*a^5*b^5*c^4*d^3*e^11 - 836*a^5*b^6*c^3*d^2*e^12 + 9344*a^6*b^2*c^6*d^4*e^10 + 14592*a^6*b^3*c^5*d^3*e^11 + 2236*a^6*b^4*c^4*d^2*e^12 - 1716*a^7*b^2*c^5*d^2*e^12 - 528*a^8*b*c^5*d*e^13 + 4*a^5*b^7*c^2*d*e^13 - 4352*a^6*b*c^7*d^5*e^9 - 92*a^6*b^5*c^3*d*e^13 - 5632*a^7*b*c^6*d^3*e^11 + 436*a^7*b^3*c^4*d*e^13)/(64*a^8*d^2))*((b^8*d + 8*a^4*c^4*d + b^5*d*(-(4*a*c - b^2)^3)^(1/2) - a*b^7*e + 33*a^2*b^4*c^2*d - 38*a^3*b^2*c^3*d - 25*a^3*b^3*c^2*e - a^3*c^2*e*(-(4*a*c - b^2)^3)^(1/2) - 10*a*b^6*c*d - a*b^4*e*(-(4*a*c - b^2)^3)^(1/2) + 9*a^2*b^5*c*e + 20*a^4*b*c^3*e - 4*a*b^3*c*d*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b*c^2*d*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b^2*c*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^6*b^4 + 16*a^8*c^2 - 8*a^7*b^2*c)))^(1/2) + ((d + e*x^2)^(1/2)*(a^6*b^2*c^5*e^14 - 2*a^7*c^6*e^14 + 192*a^4*c^9*d^6*e^8 + 32*a^5*c^8*d^4*e^10 + 34*a^6*c^7*d^2*e^12 + 64*b^8*c^5*d^6*e^8 + 704*a^2*b^4*c^7*d^6*e^8 + 960*a^2*b^5*c^6*d^5*e

^9 + 192*a^2*b^6*c^5*d^4*e^10 - 512*a^3*b^2*c^8*d^6*e^8 - 1280*a^3*b^3*c^7*
d^5*e^9 - 752*a^3*b^4*c^6*d^4*e^10 - 56*a^3*b^5*c^5*d^3*e^11 + 704*a^4*b^2*
c^7*d^4*e^10 + 128*a^4*b^3*c^6*d^3*e^11 - 15*a^4*b^4*c^5*d^2*e^12 + 60*a^5*
b^2*c^6*d^2*e^12 - 10*a^6*b*c^6*d*e^13 - 384*a*b^6*c^6*d^6*e^8 - 192*a*b^7*
c^5*d^5*e^9 + 384*a^4*b*c^8*d^5*e^9 - 144*a^5*b*c^7*d^3*e^11 + 6*a^5*b^3*c^
5*d*e^13))/(32*a^8*d^2))*((b^8*d + 8*a^4*c^4*d + b^5*d*(-(4*a*c - b^2)^3)^(
1/2) - a*b^7*e + 33*a^2*b^4*c^2*d - 38*a^3*b^2*c^3*d - 25*a^3*b^3*c^2*e - a
^3*c^2*e*(-(4*a*c - b^2)^3)^(1/2) - 10*a*b^6*c*d - a*b^4*e*(-(4*a*c - b^2)^
3)^(1/2) + 9*a^2*b^5*c*e + 20*a^4*b*c^3*e - 4*a*b^3*c*d*(-(4*a*c - b^2)^3)^
(1/2) + 3*a^2*b*c^2*d*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b^2*c*e*(-(4*a*c - b
^2)^3)^(1/2))/(8*(a^6*b^4 + 16*a^8*c^2 - 8*a^7*b^2*c)))^(1/2) + (7*a^5*c^7*
d*e^14 + 56*a^3*c^9*d^5*e^10 + 63*a^4*c^8*d^3*e^12 - 64*b^4*c^8*d^7*e^8 + 6
4*b^5*c^7*d^6*e^9 + 64*a^2*b^2*c^8*d^5*e^10 + 224*a^2*b^3*c^7*d^4*e^11 - 11
2*a^3*b^2*c^7*d^3*e^12 + 64*a*b^2*c^9*d^7*e^8 + 64*a*b^3*c^8*d^6*e^9 - 192*
a*b^4*c^7*d^5*e^10 - 96*a^2*b*c^9*d^6*e^9 - 136*a^3*b*c^8*d^4*e^11 + 9*a^4*
b*c^7*d^2*e^13)/(32*a^8*d^2)))*((b^8*d + 8*a^4*c^4*d + b^5*d*(-(4*a*c - b^2
)^3)^(1/2) - a*b^7*e + 33*a^2*b^4*c^2*d - 38*a^3*b^2*c^3*d - 25*a^3*b^3*c^2
*e - a^3*c^2*e*(-(4*a*c - b^2)^3)^(1/2) - 10*a*b^6*c*d - a*b^4*e*(-(4*a*c -
b^2)^3)^(1/2) + 9*a^2*b^5*c*e + 20*a^4*b*c^3*e - 4*a*b^3*c*d*(-(4*a*c - b^
2)^3)^(1/2) + 3*a^2*b*c^2*d*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b^2*c*e*(-(4*a
c - b^2)^3)^(1/2))/(8(a^6*b^4 + 16*a^8*c^2 - 8*a^7*b^2*c)))^(1/2)*2i - ((
(d + e*x^2)^(1/2)*(a*e^2 + 4*b*d*e))/(8*a^2) + ((d + e*x^2)^(3/2)*(a*e^2 -
4*b*d*e))/(8*a^2*d))/((d + e*x^2)^2 - 2*d*(d + e*x^2) + d^2) + (atan(-(((((
d + e*x^2)^(1/2)*(a^6*b^2*c^5*e^14 - 2*a^7*c^6*e^14 + 192*a^4*c^9*d^6*e^8 +
32*a^5*c^8*d^4*e^10 + 34*a^6*c^7*d^2*e^12 + 64*b^8*c^5*d^6*e^8 + 704*a^2*b
^4*c^7*d^6*e^8 + 960*a^2*b^5*c^6*d^5*e^9 + 192*a^2*b^6*c^5*d^4*e^10 - 512*a
^3*b^2*c^8*d^6*e^8 - 1280*a^3*b^3*c^7*d^5*e^9 - 752*a^3*b^4*c^6*d^4*e^10 -
56*a^3*b^5*c^5*d^3*e^11 + 704*a^4*b^2*c^7*d^4*e^10 + 128*a^4*b^3*c^6*d^3*e^
11 - 15*a^4*b^4*c^5*d^2*e^12 + 60*a^5*b^2*c^6*d^2*e^12 - 10*a^6*b*c^6*d*e^1
3 - 384*a*b^6*c^6*d^6*e^8 - 192*a*b^7*c^5*d^5*e^9 + 384*a^4*b*c^8*d^5*e^9 -
144*a^5*b*c^7*d^3*e^11 + 6*a^5*b^3*c^5*d*e^13))/(32*a^8*d^2) - ((((a^9*c^5
*e^14)/4 - (a^6*b^6*c^2*e^14)/16 + (7*a^7*b^4*c^3*e^14)/16 - (13*a^8*b^2*c^
4*e^14)/16 - 12*a^6*c^8*d^6*e^8 - 12*a^7*c^7*d^4*e^10 + (a^8*c^6*d^2*e^12)/
4 - 8*a^2*b^8*c^4*d^6*e^8 + 6*a^2*b^9*c^3*d^5*e^9 + 2*a^2*b^10*c^2*d^4*e^10
+ 60*a^3*b^6*c^5*d^6*e^8 - 32*a^3*b^7*c^4*d^5*e^9 - (69*a^3*b^8*c^3*d^4*e^
10)/2 - (7*a^3*b^9*c^2*d^3*e^11)/2 - 136*a^4*b^4*c^6*d^6*e^8 + 14*a^4*b^5*c
^5*d^5*e^9 + 168*a^4*b^6*c^4*d^4*e^10 + 42*a^4*b^7*c^3*d^3*e^11 + (3*a^4*b^
8*c^2*d^2*e^12)/2 + 100*a^5*b^2*c^7*d^6*e^8 + 88*a^5*b^3*c^6*d^5*e^9 - (567
*a^5*b^4*c^5*d^4*e^10)/2 - (327*a^5*b^5*c^4*d^3*e^11)/2 - (209*a^5*b^6*c^3*
d^2*e^12)/16 + 146*a^6*b^2*c^6*d^4*e^10 + 228*a^6*b^3*c^5*d^3*e^11 + (559*a
^6*b^4*c^4*d^2*e^12)/16 - (429*a^7*b^2*c^5*d^2*e^12)/16 - (33*a^8*b*c^5*d*e
^13)/4 + (a^5*b^7*c^2*d*e^13)/16 - 68*a^6*b*c^7*d^5*e^9 - (23*a^6*b^5*c^3*d
*e^13)/16 - 88*a^7*b*c^6*d^3*e^11 + (109*a^7*b^3*c^4*d*e^13)/16)/(a^8*d^2)
+ (((((32*a^12*c^4*d*e^12 + 192*a^10*c^6*d^5*e^8 + 224*a^11*c^5*d^3*e^10 +
32*a^8*b^4*c^4*d^5*e^8 - 24*a^8*b^5*c^3*d^4*e^9 - 8*a^8*b^6*c^2*d^3*e^10 -

176*a^9*b^2*c^5*d^5*e^8 + 112*a^9*b^3*c^4*d^4*e^9 + 98*a^9*b^4*c^3*d^3*e^10 + 6*a^9*b^5*c^2*d^2*e^11 - 320*a^10*b^2*c^4*d^3*e^10 - 48*a^10*b^3*c^3*d^2*e^11 - 64*a^10*b*c^5*d^4*e^9 + 2*a^10*b^4*c^2*d*e^12 + 96*a^11*b*c^4*d^2*e^11 - 16*a^11*b^2*c^3*d*e^12)/(a^8*d^2) - ((d + e*x^2)^(1/2)*(a^2*e^2 - 8*b^2*d^2 + 8*a*c*d^2 + 4*a*b*d*e)*(24576*a^12*c^5*d^4*e^8 + 16384*a^13*c^4*d^2*e^10 + 2048*a^10*b^4*c^3*d^4*e^8 - 2048*a^10*b^5*c^2*d^3*e^9 - 14336*a^11*b^2*c^4*d^4*e^8 + 15360*a^11*b^3*c^3*d^3*e^9 + 1024*a^11*b^4*c^2*d^2*e^10 - 8192*a^12*b^2*c^3*d^2*e^10 - 28672*a^12*b*c^4*d^3*e^9))/(512*a^11*d^2*(d^3)^(1/2)))*(a^2*e^2 - 8*b^2*d^2 + 8*a*c*d^2 + 4*a*b*d*e))/(16*a^3*(d^3)^(1/2)) + ((d + e*x^2)^(1/2)*(32*a^10*c^5*d*e^12 - 48*a^10*b*c^4*e^13 - 4*a^8*b^5*c^2*e^13 + 28*a^9*b^3*c^3*e^13 + 4608*a^8*c^7*d^5*e^8 + 2048*a^9*c^6*d^3*e^10 + 512*a^4*b^8*c^3*d^5*e^8 - 512*a^4*b^9*c^2*d^4*e^9 - 4608*a^5*b^6*c^4*d^5*e^8 + 4352*a^5*b^7*c^3*d^4*e^9 + 768*a^5*b^8*c^2*d^3*e^10 + 14080*a^6*b^4*c^5*d^5*e^8 - 11264*a^6*b^5*c^4*d^4*e^9 - 6912*a^6*b^6*c^3*d^3*e^10 - 256*a^6*b^7*c^2*d^2*e^11 - 16384*a^7*b^2*c^6*d^5*e^8 + 7168*a^7*b^3*c^5*d^4*e^9 + 19776*a^7*b^4*c^4*d^3*e^10 + 2272*a^7*b^5*c^3*d^2*e^11 - 18048*a^8*b^2*c^5*d^3*e^10 - 6144*a^8*b^3*c^4*d^2*e^11 - 32*a^7*b^6*c^2*d*e^12 + 3584*a^8*b*c^6*d^4*e^9 + 228*a^8*b^4*c^3*d*e^12 + 4608*a^9*b*c^5*d^2*e^11 - 408*a^9*b^2*c^4*d*e^12))/(32*a^8*d^2))*(a^2*e^2 - 8*b^2*d^2 + 8*a*c*d^2 + 4*a*b*d*e))/(16*a^3*(d^3)^(1/2)))*(a^2*e^2 - 8*b^2*d^2 + 8*a*c*d^2 + 4*a*b*d*e))/(16*a^3*(d^3)^(1/2)))*(a^2*e^2 - 8*b^2*d^2 + 8*a*c*d^2 + 4*a*b*d*e)*1i)/(16*a^3*(d^3)^(1/2)) + ((((d + e*x^2)^(1/2)*(a^6*b^2*c^5*e^14 - 2*a^7*c^6*e^14 + 192*a^4*c^9*d^6*e^8 + 32*a^5*c^8*d^4*e^10 + 34*a^6*c^7*d^2*e^12 + 64*b^8*c^5*d^6*e^8 + 704*a^2*b^4*c^7*d^6*e^8 + 960*a^2*b^5*c^6*d^5*e^9 + 192*a^2*b^6*c^5*d^4*e^10 - 512*a^3*b^2*c^8*d^6*e^8 - 1280*a^3*b^3*c^7*d^5*e^9 - 752*a^3*b^4*c^6*d^4*e^10 - 56*a^3*b^5*c^5*d^3*e^11 + 704*a^4*b^2*c^7*d^4*e^10 + 128*a^4*b^3*c^6*d^3*e^11 - 15*a^4*b^4*c^5*d^2*e^12 + 60*a^5*b^2*c^6*d^2*e^12 - 10*a^6*b*c^6*d*e^13 - 384*a*b^6*c^6*d^6*e^8 - 192*a*b^7*c^5*d^5*e^9 + 384*a^4*b*c^8*d^5*e^9 - 144*a^5*b*c^7*d^3*e^11 + 6*a^5*b^3*c^5*d*e^13))/(32*a^8*d^2) + ((((a^9*c^5*e^14)/4 - (a^6*b^6*c^2*e^14)/16 + (7*a^7*b^4*c^3*e^14)/16 - (13*a^8*b^2*c^4*e^14)/16 - 12*a^6*c^8*d^6*e^8 - 12*a^7*c^7*d^4*e^10 + (a^8*c^6*d^2*e^12)/4 - 8*a^2*b^8*c^4*d^6*e^8 + 6*a^2*b^9*c^3*d^5*e^9 + 2*a^2*b^10*c^2*d^4*e^10 + 60*a^3*b^6*c^5*d^6*e^8 - 32*a^3*b^7*c^4*d^5*e^9 - (69*a^3*b^8*c^3*d^4*e^10)/2 - (7*a^3*b^9*c^2*d^3*e^11)/2 - 136*a^4*b^4*c^6*d^6*e^8 + 14*a^4*b^5*c^5*d^5*e^9 + 168*a^4*b^6*c^4*d^4*e^10 + 42*a^4*b^7*c^3*d^3*e^11 + (3*a^4*b^8*c^2*d^2*e^12)/2 + 100*a^5*b^2*c^7*d^6*e^8 + 88*a^5*b^3*c^6*d^5*e^9 - (567*a^5*b^4*c^5*d^4*e^10)/2 - (327*a^5*b^5*c^4*d^3*e^11)/2 - (209*a^5*b^6*c^3*d^2*e^12)/16 + 146*a^6*b^2*c^6*d^4*e^10 + 228*a^6*b^3*c^5*d^3*e^11 + (559*a^6*b^4*c^4*d^2*e^12)/16 - (429*a^7*b^2*c^5*d^2*e^12)/16 - (33*a^8*b*c^5*d*e^13)/4 + (a^5*b^7*c^2*d*e^13)/16 - 68*a^6*b*c^7*d^5*e^9 - (23*a^6*b^5*c^3*d*e^13)/16 - 88*a^7*b*c^6*d^3*e^11 + (109*a^7*b^3*c^4*d*e^13)/16)/(a^8*d^2) + (((((32*a^12*c^4*d*e^12 + 192*a^10*c^6*d^5*e^8 + 224*a^11*c^5*d^3*e^10 + 32*a^8*b^4*c^4*d^5*e^8 - 24*a^8*b^5*c^3*d^4*e^9 - 8*a^8*b^6*c^2*d^3*e^10 - 176*a^9*b^2*c^5*d^5*e^8 + 112*a^9*b^3*c^4*d^4*e^9 + 98*a^9*b^4*c^3*d^3*e^10 + 6*a^9*b^5*c^2*d^2*e^11 - 320*a^10*b^2*c^4*d^3*e

^10 - 48*a^10*b^3*c^3*d^2*e^11 - 64*a^10*b*c^5*d^4*e^9 + 2*a^10*b^4*c^2*d*e
^12 + 96*a^11*b*c^4*d^2*e^11 - 16*a^11*b^2*c^3*d*e^12)/(a^8*d^2) + ((d + e*
x^2)^(1/2)*(a^2*e^2 - 8*b^2*d^2 + 8*a*c*d^2 + 4*a*b*d*e)*(24576*a^12*c^5*d^
4*e^8 + 16384*a^13*c^4*d^2*e^10 + 2048*a^10*b^4*c^3*d^4*e^8 - 2048*a^10*b^5
*c^2*d^3*e^9 - 14336*a^11*b^2*c^4*d^4*e^8 + 15360*a^11*b^3*c^3*d^3*e^9 + 10
24*a^11*b^4*c^2*d^2*e^10 - 8192*a^12*b^2*c^3*d^2*e^10 - 28672*a^12*b*c^4*d^
3*e^9))/(512*a^11*d^2*(d^3)^(1/2)))*(a^2*e^2 - 8*b^2*d^2 + 8*a*c*d^2 + 4*a*
b*d*e))/(16*a^3*(d^3)^(1/2)) - ((d + e*x^2)^(1/2)*(32*a^10*c^5*d*e^12 - 48*
a^10*b*c^4*e^13 - 4*a^8*b^5*c^2*e^13 + 28*a^9*b^3*c^3*e^13 + 4608*a^8*c^7*d
^5*e^8 + 2048*a^9*c^6*d^3*e^10 + 512*a^4*b^8*c^3*d^5*e^8 - 512*a^4*b^9*c^2*
d^4*e^9 - 4608*a^5*b^6*c^4*d^5*e^8 + 4352*a^5*b^7*c^3*d^4*e^9 + 768*a^5*b^8
*c^2*d^3*e^10 + 14080*a^6*b^4*c^5*d^5*e^8 - 11264*a^6*b^5*c^4*d^4*e^9 - 691
2*a^6*b^6*c^3*d^3*e^10 - 256*a^6*b^7*c^2*d^2*e^11 - 16384*a^7*b^2*c^6*d^5*e
^8 + 7168*a^7*b^3*c^5*d^4*e^9 + 19776*a^7*b^4*c^4*d^3*e^10 + 2272*a^7*b^5*c
^3*d^2*e^11 - 18048*a^8*b^2*c^5*d^3*e^10 - 6144*a^8*b^3*c^4*d^2*e^11 - 32*a
^7*b^6*c^2*d*e^12 + 3584*a^8*b*c^6*d^4*e^9 + 228*a^8*b^4*c^3*d*e^12 + 4608*
a^9*b*c^5*d^2*e^11 - 408*a^9*b^2*c^4*d*e^12))/(32*a^8*d^2))*(a^2*e^2 - 8*b^
2*d^2 + 8*a*c*d^2 + 4*a*b*d*e))/(16*a^3*(d^3)^(1/2)))*(a^2*e^2 - 8*b^2*d^2
+ 8*a*c*d^2 + 4*a*b*d*e))/(16*a^3*(d^3)^(1/2)))*(a^2*e^2 - 8*b^2*d^2 + 8*a*
c*d^2 + 4*a*b*d*e)*1i)/(16*a^3*(d^3)^(1/2)))/(((7*a^5*c^7*d*e^14)/32 + (7*a
^3*c^9*d^5*e^10)/4 + (63*a^4*c^8*d^3*e^12)/32 - 2*b^4*c^8*d^7*e^8 + 2*b^5*c
^7*d^6*e^9 + 2*a^2*b^2*c^8*d^5*e^10 + 7*a^2*b^3*c^7*d^4*e^11 - (7*a^3*b^2*c
^7*d^3*e^12)/2 + 2*a*b^2*c^9*d^7*e^8 + 2*a*b^3*c^8*d^6*e^9 - 6*a*b^4*c^7*d^
5*e^10 - 3*a^2*b*c^9*d^6*e^9 - (17*a^3*b*c^8*d^4*e^11)/4 + (9*a^4*b*c^7*d^2
*e^13)/32)/(a^8*d^2) - ((((d + e*x^2)^(1/2)*(a^6*b^2*c^5*e^14 - 2*a^7*c^6*e
^14 + 192*a^4*c^9*d^6*e^8 + 32*a^5*c^8*d^4*e^10 + 34*a^6*c^7*d^2*e^12 + 64*
b^8*c^5*d^6*e^8 + 704*a^2*b^4*c^7*d^6*e^8 + 960*a^2*b^5*c^6*d^5*e^9 + 192*a
^2*b^6*c^5*d^4*e^10 - 512*a^3*b^2*c^8*d^6*e^8 - 1280*a^3*b^3*c^7*d^5*e^9 -
752*a^3*b^4*c^6*d^4*e^10 - 56*a^3*b^5*c^5*d^3*e^11 + 704*a^4*b^2*c^7*d^4*e^
10 + 128*a^4*b^3*c^6*d^3*e^11 - 15*a^4*b^4*c^5*d^2*e^12 + 60*a^5*b^2*c^6*d^
2*e^12 - 10*a^6*b*c^6*d*e^13 - 384*a*b^6*c^6*d^6*e^8 - 192*a*b^7*c^5*d^5*e^
9 + 384*a^4*b*c^8*d^5*e^9 - 144*a^5*b*c^7*d^3*e^11 + 6*a^5*b^3*c^5*d*e^13))
/(32*a^8*d^2) - ((((a^9*c^5*e^14)/4 - (a^6*b^6*c^2*e^14)/16 + (7*a^7*b^4*c^
3*e^14)/16 - (13*a^8*b^2*c^4*e^14)/16 - 12*a^6*c^8*d^6*e^8 - 12*a^7*c^7*d^4
*e^10 + (a^8*c^6*d^2*e^12)/4 - 8*a^2*b^8*c^4*d^6*e^8 + 6*a^2*b^9*c^3*d^5*e^
9 + 2*a^2*b^10*c^2*d^4*e^10 + 60*a^3*b^6*c^5*d^6*e^8 - 32*a^3*b^7*c^4*d^5*e
^9 - (69*a^3*b^8*c^3*d^4*e^10)/2 - (7*a^3*b^9*c^2*d^3*e^11)/2 - 136*a^4*b^4
*c^6*d^6*e^8 + 14*a^4*b^5*c^5*d^5*e^9 + 168*a^4*b^6*c^4*d^4*e^10 + 42*a^4*b
^7*c^3*d^3*e^11 + (3*a^4*b^8*c^2*d^2*e^12)/2 + 100*a^5*b^2*c^7*d^6*e^8 + 88
*a^5*b^3*c^6*d^5*e^9 - (567*a^5*b^4*c^5*d^4*e^10)/2 - (327*a^5*b^5*c^4*d^3*
e^11)/2 - (209*a^5*b^6*c^3*d^2*e^12)/16 + 146*a^6*b^2*c^6*d^4*e^10 + 228*a^
6*b^3*c^5*d^3*e^11 + (559*a^6*b^4*c^4*d^2*e^12)/16 - (429*a^7*b^2*c^5*d^2*e
^12)/16 - (33*a^8*b*c^5*d*e^13)/4 + (a^5*b^7*c^2*d*e^13)/16 - 68*a^6*b*c^7*
d^5*e^9 - (23*a^6*b^5*c^3*d*e^13)/16 - 88*a^7*b*c^6*d^3*e^11 + (109*a^7*b^3
*c^4*d*e^13)/16)/(a^8*d^2) + (((((32*a^12*c^4*d*e^12 + 192*a^10*c^6*d^5*e^8

+ 224*a^11*c^5*d^3*e^10 + 32*a^8*b^4*c^4*d^5*e^8 - 24*a^8*b^5*c^3*d^4*e^9
- 8*a^8*b^6*c^2*d^3*e^10 - 176*a^9*b^2*c^5*d^5*e^8 + 112*a^9*b^3*c^4*d^4*e^
9 + 98*a^9*b^4*c^3*d^3*e^10 + 6*a^9*b^5*c^2*d^2*e^11 - 320*a^10*b^2*c^4*d^3
*e^10 - 48*a^10*b^3*c^3*d^2*e^11 - 64*a^10*b*c^5*d^4*e^9 + 2*a^10*b^4*c^2*d
*e^12 + 96*a^11*b*c^4*d^2*e^11 - 16*a^11*b^2*c^3*d*e^12)/(a^8*d^2) - ((d +
e*x^2)^(1/2)*(a^2*e^2 - 8*b^2*d^2 + 8*a*c*d^2 + 4*a*b*d*e)*(24576*a^12*c^5*
d^4*e^8 + 16384*a^13*c^4*d^2*e^10 + 2048*a^10*b^4*c^3*d^4*e^8 - 2048*a^10*b
^5*c^2*d^3*e^9 - 14336*a^11*b^2*c^4*d^4*e^8 + 15360*a^11*b^3*c^3*d^3*e^9 +
1024*a^11*b^4*c^2*d^2*e^10 - 8192*a^12*b^2*c^3*d^2*e^10 - 28672*a^12*b*c^4*
d^3*e^9))/(512*a^11*d^2*(d^3)^(1/2)))*(a^2*e^2 - 8*b^2*d^2 + 8*a*c*d^2 + 4*
a*b*d*e))/(16*a^3*(d^3)^(1/2)) + ((d + e*x^2)^(1/2)*(32*a^10*c^5*d*e^12 - 4
8*a^10*b*c^4*e^13 - 4*a^8*b^5*c^2*e^13 + 28*a^9*b^3*c^3*e^13 + 4608*a^8*c^7
*d^5*e^8 + 2048*a^9*c^6*d^3*e^10 + 512*a^4*b^8*c^3*d^5*e^8 - 512*a^4*b^9*c^
2*d^4*e^9 - 4608*a^5*b^6*c^4*d^5*e^8 + 4352*a^5*b^7*c^3*d^4*e^9 + 768*a^5*b
^8*c^2*d^3*e^10 + 14080*a^6*b^4*c^5*d^5*e^8 - 11264*a^6*b^5*c^4*d^4*e^9 - 6
912*a^6*b^6*c^3*d^3*e^10 - 256*a^6*b^7*c^2*d^2*e^11 - 16384*a^7*b^2*c^6*d^5
*e^8 + 7168*a^7*b^3*c^5*d^4*e^9 + 19776*a^7*b^4*c^4*d^3*e^10 + 2272*a^7*b^5
*c^3*d^2*e^11 - 18048*a^8*b^2*c^5*d^3*e^10 - 6144*a^8*b^3*c^4*d^2*e^11 - 32
*a^7*b^6*c^2*d*e^12 + 3584*a^8*b*c^6*d^4*e^9 + 228*a^8*b^4*c^3*d*e^12 + 460
8*a^9*b*c^5*d^2*e^11 - 408*a^9*b^2*c^4*d*e^12))/(32*a^8*d^2))*(a^2*e^2 - 8*
b^2*d^2 + 8*a*c*d^2 + 4*a*b*d*e))/(16*a^3*(d^3)^(1/2)))*(a^2*e^2 - 8*b^2*d^
2 + 8*a*c*d^2 + 4*a*b*d*e))/(16*a^3*(d^3)^(1/2)))*(a^2*e^2 - 8*b^2*d^2 + 8*
a*c*d^2 + 4*a*b*d*e))/(16*a^3*(d^3)^(1/2)) + ((((d + e*x^2)^(1/2)*(a^6*b^2*
c^5*e^14 - 2*a^7*c^6*e^14 + 192*a^4*c^9*d^6*e^8 + 32*a^5*c^8*d^4*e^10 + 34*
a^6*c^7*d^2*e^12 + 64*b^8*c^5*d^6*e^8 + 704*a^2*b^4*c^7*d^6*e^8 + 960*a^2*b
^5*c^6*d^5*e^9 + 192*a^2*b^6*c^5*d^4*e^10 - 512*a^3*b^2*c^8*d^6*e^8 - 1280*
a^3*b^3*c^7*d^5*e^9 - 752*a^3*b^4*c^6*d^4*e^10 - 56*a^3*b^5*c^5*d^3*e^11 +
704*a^4*b^2*c^7*d^4*e^10 + 128*a^4*b^3*c^6*d^3*e^11 - 15*a^4*b^4*c^5*d^2*e^
12 + 60*a^5*b^2*c^6*d^2*e^12 - 10*a^6*b*c^6*d*e^13 - 384*a*b^6*c^6*d^6*e^8
- 192*a*b^7*c^5*d^5*e^9 + 384*a^4*b*c^8*d^5*e^9 - 144*a^5*b*c^7*d^3*e^11 +
6*a^5*b^3*c^5*d*e^13))/(32*a^8*d^2) + ((((a^9*c^5*e^14)/4 - (a^6*b^6*c^2*e^
14)/16 + (7*a^7*b^4*c^3*e^14)/16 - (13*a^8*b^2*c^4*e^14)/16 - 12*a^6*c^8*d^
6*e^8 - 12*a^7*c^7*d^4*e^10 + (a^8*c^6*d^2*e^12)/4 - 8*a^2*b^8*c^4*d^6*e^8
+ 6*a^2*b^9*c^3*d^5*e^9 + 2*a^2*b^10*c^2*d^4*e^10 + 60*a^3*b^6*c^5*d^6*e^8
- 32*a^3*b^7*c^4*d^5*e^9 - (69*a^3*b^8*c^3*d^4*e^10)/2 - (7*a^3*b^9*c^2*d^3
*e^11)/2 - 136*a^4*b^4*c^6*d^6*e^8 + 14*a^4*b^5*c^5*d^5*e^9 + 168*a^4*b^6*c
^4*d^4*e^10 + 42*a^4*b^7*c^3*d^3*e^11 + (3*a^4*b^8*c^2*d^2*e^12)/2 + 100*a^
5*b^2*c^7*d^6*e^8 + 88*a^5*b^3*c^6*d^5*e^9 - (567*a^5*b^4*c^5*d^4*e^10)/2 -
(327*a^5*b^5*c^4*d^3*e^11)/2 - (209*a^5*b^6*c^3*d^2*e^12)/16 + 146*a^6*b^2
*c^6*d^4*e^10 + 228*a^6*b^3*c^5*d^3*e^11 + (559*a^6*b^4*c^4*d^2*e^12)/16 -
(429*a^7*b^2*c^5*d^2*e^12)/16 - (33*a^8*b*c^5*d*e^13)/4 + (a^5*b^7*c^2*d*e^
13)/16 - 68*a^6*b*c^7*d^5*e^9 - (23*a^6*b^5*c^3*d*e^13)/16 - 88*a^7*b*c^6*d
^3*e^11 + (109*a^7*b^3*c^4*d*e^13)/16)/(a^8*d^2) + (((((32*a^12*c^4*d*e^12
+ 192*a^10*c^6*d^5*e^8 + 224*a^11*c^5*d^3*e^10 + 32*a^8*b^4*c^4*d^5*e^8 - 2
4*a^8*b^5*c^3*d^4*e^9 - 8*a^8*b^6*c^2*d^3*e^10 - 176*a^9*b^2*c^5*d^5*e^8 +

112*a^9*b^3*c^4*d^4*e^9 + 98*a^9*b^4*c^3*d^3*e^10 + 6*a^9*b^5*c^2*d^2*e^11 - 320*a^10*b^2*c^4*d^3*e^10 - 48*a^10*b^3*c^3*d^2*e^11 - 64*a^10*b*c^5*d^4*e^9 + 2*a^10*b^4*c^2*d*e^12 + 96*a^11*b*c^4*d^2*e^11 - 16*a^11*b^2*c^3*d*e^12)/(a^8*d^2) + ((d + e*x^2)^(1/2)*(a^2*e^2 - 8*b^2*d^2 + 8*a*c*d^2 + 4*a*b*d*e)*(24576*a^12*c^5*d^4*e^8 + 16384*a^13*c^4*d^2*e^10 + 2048*a^10*b^4*c^3*d^4*e^8 - 2048*a^10*b^5*c^2*d^3*e^9 - 14336*a^11*b^2*c^4*d^4*e^8 + 15360*a^11*b^3*c^3*d^3*e^9 + 1024*a^11*b^4*c^2*d^2*e^10 - 8192*a^12*b^2*c^3*d^2*e^10 - 28672*a^12*b*c^4*d^3*e^9))/(512*a^11*d^2*(d^3)^(1/2)))*(a^2*e^2 - 8*b^2*d^2 + 8*a*c*d^2 + 4*a*b*d*e))/(16*a^3*(d^3)^(1/2)) - ((d + e*x^2)^(1/2)*(32*a^10*c^5*d*e^12 - 48*a^10*b*c^4*e^13 - 4*a^8*b^5*c^2*e^13 + 28*a^9*b^3*c^3*e^13 + 4608*a^8*c^7*d^5*e^8 + 2048*a^9*c^6*d^3*e^10 + 512*a^4*b^8*c^3*d^5*e^8 - 512*a^4*b^9*c^2*d^4*e^9 - 4608*a^5*b^6*c^4*d^5*e^8 + 4352*a^5*b^7*c^3*d^4*e^9 + 768*a^5*b^8*c^2*d^3*e^10 + 14080*a^6*b^4*c^5*d^5*e^8 - 11264*a^6*b^5*c^4*d^4*e^9 - 6912*a^6*b^6*c^3*d^3*e^10 - 256*a^6*b^7*c^2*d^2*e^11 - 16384*a^7*b^2*c^6*d^5*e^8 + 7168*a^7*b^3*c^5*d^4*e^9 + 19776*a^7*b^4*c^4*d^3*e^10 + 2272*a^7*b^5*c^3*d^2*e^11 - 18048*a^8*b^2*c^5*d^3*e^10 - 6144*a^8*b^3*c^4*d^2*e^11 - 32*a^7*b^6*c^2*d*e^12 + 3584*a^8*b*c^6*d^4*e^9 + 228*a^8*b^4*c^3*d*e^12 + 4608*a^9*b*c^5*d^2*e^11 - 408*a^9*b^2*c^4*d*e^12))/(32*a^8*d^2))*(a^2*e^2 - 8*b^2*d^2 + 8*a*c*d^2 + 4*a*b*d*e))/(16*a^3*(d^3)^(1/2)))*(a^2*e^2 - 8*b^2*d^2 + 8*a*c*d^2 + 4*a*b*d*e))/(16*a^3*(d^3)^(1/2)))*(a^2*e^2 - 8*b^2*d^2 + 8*a*c*d^2 + 4*a*b*d*e))/(16*a^3*(d^3)^(1/2))))*(a^2*e^2 - 8*b^2*d^2 + 8*a*c*d^2 + 4*a*b*d*e)*1i)/(8*a^3*(d^3)^(1/2))

3.361 $\int \frac{x^4\sqrt{d+ex^2}}{a+bx^2+cx^4}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 390

$$\int \frac{x^4\sqrt{d+ex^2}}{a+bx^2+cx^4}\,dx = \frac{x\sqrt{d+ex^2}}{2c} - \frac{\left(bcd-b^2e+ace-\frac{b^2cd-2ac^2d-b^3e+3abce}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}x}{\sqrt{b-\sqrt{b^2-4ac}}\sqrt{d+ex^2}}\right)}{c^2\sqrt{b-\sqrt{b^2-4ac}}\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}} - \frac{\left(bcd-b^2e+ace+\frac{b^2cd-2ac^2d-b^3e+3abce}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2cd-\left(b+\sqrt{b^2-4ac}\right)e}x}{\sqrt{b+\sqrt{b^2-4ac}}\sqrt{d+ex^2}}\right)}{c^2\sqrt{b+\sqrt{b^2-4ac}}\sqrt{2cd-\left(b+\sqrt{b^2-4ac}\right)e}} + \frac{(cd-2be)\text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{2c^2\sqrt{e}}$$

```
[Out] 1/2*(-2*b*e+c*d)*arctanh(x*e^(1/2)/(e*x^2+d)^(1/2))/c^2/e^(1/2)+1/2*x*(e*x^
2+d)^(1/2)/c-arctan(x*(2*c*d-e*(b-(-4*a*c+b^2)^(1/2)))^(1/2)/(e*x^2+d)^(1/2
)/(b-(-4*a*c+b^2)^(1/2))^(1/2))*(b*c*d-b^2*e+a*c*e+(-3*a*b*c*e+2*a*c^2*d+b^
3*e-b^2*c*d)/(-4*a*c+b^2)^(1/2))/c^2/(2*c*d-e*(b-(-4*a*c+b^2)^(1/2)))^(1/2)
/(b-(-4*a*c+b^2)^(1/2))^(1/2)-arctan(x*(2*c*d-e*(b+(-4*a*c+b^2)^(1/2)))^(1/
2)/(e*x^2+d)^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2))*(b*c*d-b^2*e+a*c*e+(3*a*b*
c*e-2*a*c^2*d-b^3*e+b^2*c*d)/(-4*a*c+b^2)^(1/2))/c^2/(b+(-4*a*c+b^2)^(1/2))
^(1/2)/(2*c*d-e*(b+(-4*a*c+b^2)^(1/2)))^(1/2)
```

Rubi [A] (verified)

Time = 1.82 (sec) , antiderivative size = 390, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.241, Rules used = {1305, 396, 223, 212, 1706, 385, 211}

$$\int \frac{x^4\sqrt{d+ex^2}}{a+bx^2+cx^4}\,dx$$
$$=-\frac{\left(-\frac{3abce-2ac^2d+b^3(-e)+b^2cd}{\sqrt{b^2-4ac}}+ace+b^2(-e)+bcd\right)\arctan\left(\frac{x\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}{\sqrt{b-\sqrt{b^2-4ac}}\sqrt{d+ex^2}}\right)}{c^2\sqrt{b-\sqrt{b^2-4ac}}\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}$$
$$-\frac{\left(\frac{3abce-2ac^2d+b^3(-e)+b^2cd}{\sqrt{b^2-4ac}}+ace+b^2(-e)+bcd\right)\arctan\left(\frac{x\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}{\sqrt{\sqrt{b^2-4ac}+b}\sqrt{d+ex^2}}\right)}{c^2\sqrt{\sqrt{b^2-4ac}+b}\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}$$
$$+\frac{\text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)(cd-2be)}{2c^2\sqrt{e}}+\frac{x\sqrt{d+ex^2}}{2c}$$

```
[In] Int[(x^4*Sqrt[d + e*x^2])/(a + b*x^2 + c*x^4),x]
```

```
[Out] (x*Sqrt[d + e*x^2])/(2*c) - ((b*c*d - b^2*e + a*c*e - (b^2*c*d - 2*a*c^2*d
- b^3*e + 3*a*b*c*e)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2*c*d - (b - Sqrt[b^2
- 4*a*c])*e]*x)/(Sqrt[b - Sqrt[b^2 - 4*a*c]]*Sqrt[d + e*x^2])])/(c^2*Sqrt[b
 - Sqrt[b^2 - 4*a*c]]*Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]) - ((b*c*d -
b^2*e + a*c*e + (b^2*c*d - 2*a*c^2*d - b^3*e + 3*a*b*c*e)/Sqrt[b^2 - 4*a*c]
)*ArcTan[(Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]*x)/(Sqrt[b + Sqrt[b^2 - 4
*a*c]]*Sqrt[d + e*x^2])])/(c^2*Sqrt[b + Sqrt[b^2 - 4*a*c]]*Sqrt[2*c*d - (b
+ Sqrt[b^2 - 4*a*c])*e]) + ((c*d - 2*b*e)*ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^
2]])/(2*c^2*Sqrt[e])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 396

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 1305

```
Int[(((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x_)^2)^(q_))/((a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4), x_Symbol] :> Dist[f^4/c^2, Int[(f*x)^(m - 4)*(c*d - b*e + c
*e*x^2)*(d + e*x^2)^(q - 1), x], x] - Dist[f^4/c^2, Int[(f*x)^(m - 4)*(d +
e*x^2)^(q - 1)*(Simp[a*(c*d - b*e) + (b*c*d - b^2*e + a*c*e)*x^2, x]/(a + b
*x^2 + c*x^4)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b^2 - 4*a*c,
0] &&  !IntegerQ[q] && GtQ[q, 0] && GtQ[m, 3]
```

Rule 1706

```
Int[(Px_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^
(p_.), x_Symbol] :> Int[ExpandIntegrand[Px*(d + e*x^2)^q*(a + b*x^2 + c*x^4
)^p, x], x] /; FreeQ[{a, b, c, d, e, q}, x] && PolyQ[Px, x^2] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[p]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\int \frac{cd-be+cex^2}{\sqrt{d+ex^2}}\,dx}{c^2} - \frac{\int \frac{a(cd-be)+(bcd-b^2e+ace)x^2}{\sqrt{d+ex^2}(a+bx^2+cx^4)}\,dx}{c^2} \\
&= \frac{x\sqrt{d+ex^2}}{2c} \\
&\quad - \frac{\int \left(\frac{bcd-b^2e+ace+\frac{-b^2cd+2ac^2d+b^3e-3abce}{\sqrt{b^2-4ac}}}{\left(b-\sqrt{b^2-4ac}+2cx^2\right)\sqrt{d+ex^2}} + \frac{bcd-b^2e+ace-\frac{-b^2cd+2ac^2d+b^3e-3abce}{\sqrt{b^2-4ac}}}{\left(b+\sqrt{b^2-4ac}+2cx^2\right)\sqrt{d+ex^2}}\right)dx}{c^2} \\
&\quad + \frac{(cd-2be)\int \frac{1}{\sqrt{d+ex^2}}\,dx}{2c^2}
\end{aligned}
$$

$$= \frac{x\sqrt{d+ex^2}}{2c} + \frac{(cd-2be)\text{Subst}\left(\int \frac{1}{1-ex^2}\,dx, x, \frac{x}{\sqrt{d+ex^2}}\right)}{2c^2} - \frac{\left(bcd-b^2e+ace-\frac{b^2cd-2ac^2d-b^3e+3abce}{\sqrt{b^2-4ac}}\right)\int \frac{1}{\left(b-\sqrt{b^2-4ac}+2cx^2\right)\sqrt{d+ex^2}}\,dx}{c^2} - \frac{\left(bcd-b^2e+ace+\frac{b^2cd-2ac^2d-b^3e+3abce}{\sqrt{b^2-4ac}}\right)\int \frac{1}{\left(b+\sqrt{b^2-4ac}+2cx^2\right)\sqrt{d+ex^2}}\,dx}{c^2}$$

$$= \frac{x\sqrt{d+ex^2}}{2c} + \frac{(cd-2be)\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{2c^2\sqrt{e}} - \frac{\left(bcd-b^2e+ace-\frac{b^2cd-2ac^2d-b^3e+3abce}{\sqrt{b^2-4ac}}\right)\text{Subst}\left(\int \frac{1}{b-\sqrt{b^2-4ac}-\left(-2cd+\left(b-\sqrt{b^2-4ac}\right)e\right)x^2}\,dx, x, \frac{x}{\sqrt{d+ex^2}}\right)}{c^2} - \frac{\left(bcd-b^2e+ace+\frac{b^2cd-2ac^2d-b^3e+3abce}{\sqrt{b^2-4ac}}\right)\text{Subst}\left(\int \frac{1}{b+\sqrt{b^2-4ac}-\left(-2cd+\left(b+\sqrt{b^2-4ac}\right)e\right)x^2}\,dx, x, \frac{x}{\sqrt{d+ex^2}}\right)}{c^2}$$

$$= \frac{x\sqrt{d+ex^2}}{2c} - \frac{\left(bcd-b^2e+ace-\frac{b^2cd-2ac^2d-b^3e+3abce}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}\,x}{\sqrt{b-\sqrt{b^2-4ac}}\sqrt{d+ex^2}}\right)}{c^2\sqrt{b-\sqrt{b^2-4ac}}\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}} - \frac{\left(bcd-b^2e+ace+\frac{b^2cd-2ac^2d-b^3e+3abce}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2cd-\left(b+\sqrt{b^2-4ac}\right)e}\,x}{\sqrt{b+\sqrt{b^2-4ac}}\sqrt{d+ex^2}}\right)}{c^2\sqrt{b+\sqrt{b^2-4ac}}\sqrt{2cd-\left(b+\sqrt{b^2-4ac}\right)e}} + \frac{(cd-2be)\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{2c^2\sqrt{e}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 1.01 (sec) , antiderivative size = 786, normalized size of antiderivative = 2.02

$$\int \frac{x^4\sqrt{d+ex^2}}{a+bx^2+cx^4}\,dx$$

$$= \frac{2cx\sqrt{d+ex^2} + \frac{4(cd-2be)\text{arctanh}\left(\frac{\sqrt{e}x}{-\sqrt{d}+\sqrt{d+ex^2}}\right)}{\sqrt{e}} + \text{RootSum}\Big[ae^4 + 4bde^2\#1^2 - 4ae^3\#1^2 + 16cd^2\#1^4 - 8bde}{}$$

```
[In] Integrate[(x^4*Sqrt[d + e*x^2])/(a + b*x^2 + c*x^4),x]
```

[Out] (2*c*x*Sqrt[d + e*x^2] + (4*(c*d - 2*b*e)*ArcTanh[(Sqrt[e]*x)/(-Sqrt[d] + Sqrt[d + e*x^2])])/Sqrt[e] + RootSum[a*e^4 + 4*b*d*e^2*#1^2 - 4*a*e^3*#1^2 + 16*c*d^2*#1^4 - 8*b*d*e*#1^4 + 6*a*e^2*#1^4 + 4*b*d*#1^6 - 4*a*e*#1^6 + a*#1^8 & , (a*c*d*e^3*Log[x] - a*b*e^4*Log[x] - a*c*d*e^3*Log[-Sqrt[d] + Sqrt[d + e*x^2] - x*#1] + a*b*e^4*Log[-Sqrt[d] + Sqrt[d + e*x^2] - x*#1] + 4*b*c*d^2*e*Log[x]*#1^2 - 4*b^2*d*e^2*Log[x]*#1^2 + a*c*d*e^2*Log[x]*#1^2 + 3*a*b*e^3*Log[x]*#1^2 - 4*b*c*d^2*e*Log[-Sqrt[d] + Sqrt[d + e*x^2] - x*#1]*#1^2 + 4*b^2*d*e^2*Log[-Sqrt[d] + Sqrt[d + e*x^2] - x*#1]*#1^2 - a*c*d*e^2*Log[-Sqrt[d] + Sqrt[d + e*x^2] - x*#1]*#1^2 - 3*a*b*e^3*Log[-Sqrt[d] + Sqrt[d + e*x^2] - x*#1]*#1^2 - 4*b*c*d^2*Log[x]*#1^4 + 4*b^2*d*e*Log[x]*#1^4 - a*c*d*e*Log[x]*#1^4 - 3*a*b*e^2*Log[x]*#1^4 + 4*b*c*d^2*Log[-Sqrt[d] + Sqrt[d + e*x^2] - x*#1]*#1^4 - 4*b^2*d*e*Log[-Sqrt[d] + Sqrt[d + e*x^2] - x*#1]*#1^4 + a*c*d*e*Log[-Sqrt[d] + Sqrt[d + e*x^2] - x*#1]*#1^4 + 3*a*b*e^2*Log[-Sqrt[d] + Sqrt[d + e*x^2] - x*#1]*#1^4 - a*c*d*Log[x]*#1^6 + a*b*e*Log[x]*#1^6 + a*c*d*Log[-Sqrt[d] + Sqrt[d + e*x^2] - x*#1]*#1^6 - a*b*e*Log[-Sqrt[d] + Sqrt[d + e*x^2] - x*#1]*#1^6)/(b*d*e^2*#1 - a*e^3*#1 + 8*c*d^2*#1^3 - 4*b*d*e*#1^3 + 3*a*e^2*#1^3 + 3*b*d*#1^5 - 3*a*e*#1^5 + a*#1^7) &])/(4*c^2)

Maple [A] (verified)

Time = 2.62 (sec) , antiderivative size = 360, normalized size of antiderivative = 0.92

method	result
risch	$\frac{x\sqrt{e x^2+d}}{2c}-\frac{\frac{(2be-cd)\ln\left(x\sqrt{e}+\sqrt{e x^2+d}\right)}{c\sqrt{e}}+\frac{a\sqrt{2}\left(\frac{\left(2acde-b^2de+bcd^2+\sqrt{-d^2\left(4ac-b^2\right)}\,be-\sqrt{-d^2\left(4ac-b^2\right)}\,cd\right)\arctan\left(\frac{\cdots}{x\sqrt{\left(-2a\cdots\right.}}\right.}{\sqrt{\left(-2ae+bd+\sqrt{-d^2\left(4ac-b^2\right)}\right)a}}\cdots\right.}{\cdots}}{\cdots}$
default	$\frac{\frac{x\sqrt{e x^2+d}}{2}+\frac{d\ln\left(x\sqrt{e}+\sqrt{e x^2+d}\right)}{2\sqrt{e}}}{c}-\frac{a\sqrt{2}\sqrt{\left(-2ae+bd+\sqrt{-4d^2\left(ac-\frac{b^2}{4}\right)}\right)a}\left(\frac{(-be+cd)\sqrt{-4d^2\left(ac-\frac{b^2}{4}\right)}}{2}+d\left(ace-\frac{b(be-cd)}{2}\right)\cdots\right.}{\cdots}$
pseudoelliptic	$-\frac{a\sqrt{2}\left(\frac{\left(cd\sqrt{e}-e^{\frac{3}{2}}b\right)\sqrt{-4d^2\left(ac-\frac{b^2}{4}\right)}}{2}+\left(\left(ac-\frac{b^2}{2}\right)e^{\frac{3}{2}}+\frac{b\sqrt{e}\,cd}{2}\right)d\right)\sqrt{\left(-2ae+bd+\sqrt{-4d^2\left(ac-\frac{b^2}{4}\right)}\right)a}\,\operatorname{arctanh}\left(\frac{\cdots}{x\sqrt{\left(2ae-\cdots\right.}}\right.}{\cdots}$

[In] int(x^4*(e*x^2+d)^(1/2)/(c*x^4+b*x^2+a),x,method=_RETURNVERBOSE)

[Out] 1/2*x*(e*x^2+d)^(1/2)/c-1/2/c*((2*b*e-c*d)/c*ln(x*e^(1/2)+(e*x^2+d)^(1/2))/e^(1/2)+1/c*a*2^(1/2)/(-d^2*(4*a*c-b^2))^(1/2)*((2*a*c*d*e-b^2*d*e+b*c*d^2+

```
(-d^2*(4*a*c-b^2))^(1/2)*b*e-(-d^2*(4*a*c-b^2))^(1/2)*c*d)/((-2*a*e+b*d+(-d
^2*(4*a*c-b^2))^(1/2))*a)^(1/2)*arctan(a/x*(e*x^2+d)^(1/2)*2^(1/2)/((-2*a*e
+b*d+(-d^2*(4*a*c-b^2))^(1/2))*a)^(1/2))-(-2*a*c*d*e+b^2*d*e-b*c*d^2+(-d^2*
(4*a*c-b^2))^(1/2)*b*e-(-d^2*(4*a*c-b^2))^(1/2)*c*d)/((2*a*e-b*d+(-d^2*(4*a
*c-b^2))^(1/2))*a)^(1/2)*arctanh(a/x*(e*x^2+d)^(1/2)*2^(1/2)/((2*a*e-b*d+(-
d^2*(4*a*c-b^2))^(1/2))*a)^(1/2)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3263 vs. $2(338) = 676$.

Time = 14.87 (sec) , antiderivative size = 6534, normalized size of antiderivative = 16.75

$$\int \frac{x^4\sqrt{d+ex^2}}{a+bx^2+cx^4}\,dx = \text{Too large to display}$$

```
[In] integrate(x^4*(e*x^2+d)^(1/2)/(c*x^4+b*x^2+a),x, algorithm="fricas")
[Out] Too large to include
```

Sympy [F]

$$\int \frac{x^4\sqrt{d+ex^2}}{a+bx^2+cx^4}\,dx = \int \frac{x^4\sqrt{d+ex^2}}{a+bx^2+cx^4}\,dx$$

```
[In] integrate(x**4*(e*x**2+d)**(1/2)/(c*x**4+b*x**2+a),x)
[Out] Integral(x**4*sqrt(d + e*x**2)/(a + b*x**2 + c*x**4), x)
```

Maxima [F]

$$\int \frac{x^4\sqrt{d+ex^2}}{a+bx^2+cx^4}\,dx = \int \frac{\sqrt{ex^2+d}x^4}{cx^4+bx^2+a}\,dx$$

```
[In] integrate(x^4*(e*x^2+d)^(1/2)/(c*x^4+b*x^2+a),x, algorithm="maxima")
[Out] integrate(sqrt(e*x^2 + d)*x^4/(c*x^4 + b*x^2 + a), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{x^4\sqrt{d+ex^2}}{a+bx^2+cx^4}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(x^4*(e*x^2+d)^(1/2)/(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:index.cc index_m i_lex_is_greater Err
or: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^4\sqrt{d+ex^2}}{a+bx^2+cx^4}\,dx = \int \frac{x^4\,\sqrt{e\,x^2+d}}{c\,x^4+b\,x^2+a}\,dx$$

```
[In] int((x^4*(d + e*x^2)^(1/2))/(a + b*x^2 + c*x^4),x)
```

```
[Out] int((x^4*(d + e*x^2)^(1/2))/(a + b*x^2 + c*x^4), x)
```

3.362 $\int \frac{x^2\sqrt{d+ex^2}}{a+bx^2+cx^4}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 324

$$\int \frac{x^2\sqrt{d+ex^2}}{a+bx^2+cx^4}\,dx = \frac{\left(cd-be-\frac{bcd-b^2e+2ace}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}x}{\sqrt{b-\sqrt{b^2-4ac}}\sqrt{d+ex^2}}\right)}{c\sqrt{b-\sqrt{b^2-4ac}}\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}} + \frac{\left(cd-be+\frac{bcd-b^2e+2ace}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2cd-\left(b+\sqrt{b^2-4ac}\right)e}x}{\sqrt{b+\sqrt{b^2-4ac}}\sqrt{d+ex^2}}\right)}{c\sqrt{b+\sqrt{b^2-4ac}}\sqrt{2cd-\left(b+\sqrt{b^2-4ac}\right)e}} + \frac{\sqrt{e}\,\text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{c}$$

```
[Out] arctanh(x*e^(1/2)/(e*x^2+d)^(1/2))*e^(1/2)/c+arctan(x*(2*c*d-e*(b-(-4*a*c+b
^2)^(1/2)))^(1/2)/(e*x^2+d)^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2))*(c*d-b*e+(-
2*a*c*e+b^2*e-b*c*d)/(-4*a*c+b^2)^(1/2))/c/(2*c*d-e*(b-(-4*a*c+b^2)^(1/2)))
^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2)+arctan(x*(2*c*d-e*(b+(-4*a*c+b^2)^(1/2)
))^(1/2)/(e*x^2+d)^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2))*(c*d-b*e+(2*a*c*e-b^
2*e+b*c*d)/(-4*a*c+b^2)^(1/2))/c/(b+(-4*a*c+b^2)^(1/2))^(1/2)/(2*c*d-e*(b+(
-4*a*c+b^2)^(1/2)))^(1/2)
```

Rubi [A] (verified)

Time = 1.04 (sec) , antiderivative size = 324, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.207, Rules used = {1307, 223, 212, 1706, 385, 211}

$$\int \frac{x^2\sqrt{d+ex^2}}{a+bx^2+cx^4}\,dx = \frac{\left(-\frac{2ace+b^2(-e)+bcd}{\sqrt{b^2-4ac}}-be+cd\right)\arctan\left(\frac{x\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}{\sqrt{b-\sqrt{b^2-4ac}}\sqrt{d+ex^2}}\right)}{c\sqrt{b-\sqrt{b^2-4ac}}\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}} + \frac{\left(\frac{2ace+b^2(-e)+bcd}{\sqrt{b^2-4ac}}-be+cd\right)\arctan\left(\frac{x\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}{\sqrt{\sqrt{b^2-4ac}+b}\sqrt{d+ex^2}}\right)}{c\sqrt{\sqrt{b^2-4ac}+b}\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}} + \frac{\sqrt{e}\,\text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{c}$$

```
[In] Int[(x^2*Sqrt[d + e*x^2])/(a + b*x^2 + c*x^4),x]
```

```
[Out] ((c*d - b*e - (b*c*d - b^2*e + 2*a*c*e)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2*c
*d - (b - Sqrt[b^2 - 4*a*c])*e]*x)/(Sqrt[b - Sqrt[b^2 - 4*a*c]]*Sqrt[d + e*
x^2])])/(c*Sqrt[b - Sqrt[b^2 - 4*a*c]]*Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])
*e]) + ((c*d - b*e + (b*c*d - b^2*e + 2*a*c*e)/Sqrt[b^2 - 4*a*c])*ArcTan[(S
qrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]*x)/(Sqrt[b + Sqrt[b^2 - 4*a*c]]*Sqrt
[d + e*x^2])])/(c*Sqrt[b + Sqrt[b^2 - 4*a*c]]*Sqrt[2*c*d - (b + Sqrt[b^2 -
4*a*c])*e]) + (Sqrt[e]*ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]])/c
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 1307

```
Int[(((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x_)^2)^(q_))/((a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4), x_Symbol] :> Dist[e*(f^2/c), Int[(f*x)^(m - 2)*(d + e*x^2)^
(q - 1), x], x] - Dist[f^2/c, Int[(f*x)^(m - 2)*(d + e*x^2)^(q - 1)*(Simp[a
*e - (c*d - b*e)*x^2, x]/(a + b*x^2 + c*x^4)), x], x] /; FreeQ[{a, b, c, d,
 e, f}, x] && NeQ[b^2 - 4*a*c, 0] &&  !IntegerQ[q] && GtQ[q, 0] && GtQ[m, 1
] && LeQ[m, 3]
```

Rule 1706

```
Int[(Px_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^
(p_.), x_Symbol] :> Int[ExpandIntegrand[Px*(d + e*x^2)^q*(a + b*x^2 + c*x^4
)^p, x], x] /; FreeQ[{a, b, c, d, e, q}, x] && PolyQ[Px, x^2] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[p]
```

Rubi steps

$$\text{integral} = -\frac{\int \frac{ae-(cd-be)x^2}{\sqrt{d+ex^2}(a+bx^2+cx^4)}\,dx}{c} + \frac{e\int \frac{1}{\sqrt{d+ex^2}}\,dx}{c}$$

$$= -\frac{\int \left(\frac{-cd+be+\frac{bcd-b^2e+2ace}{\sqrt{b^2-4ac}}}{\left(b-\sqrt{b^2-4ac}+2cx^2\right)\sqrt{d+ex^2}} + \frac{-cd+be-\frac{bcd-b^2e+2ace}{\sqrt{b^2-4ac}}}{\left(b+\sqrt{b^2-4ac}+2cx^2\right)\sqrt{d+ex^2}}\right)dx}{c} + \frac{e\text{Subst}\left(\int \frac{1}{1-ex^2}\,dx, x, \frac{x}{\sqrt{d+ex^2}}\right)}{c}$$

$$= \frac{\sqrt{e}\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{c} + \frac{\left(cd-be+\frac{bcd-b^2e+2ace}{\sqrt{b^2-4ac}}\right)\int \frac{1}{\left(b+\sqrt{b^2-4ac}+2cx^2\right)\sqrt{d+ex^2}}\,dx}{c} - \frac{\left(-cd+be+\frac{bcd-b^2e+2ace}{\sqrt{b^2-4ac}}\right)\int \frac{1}{\left(b-\sqrt{b^2-4ac}+2cx^2\right)\sqrt{d+ex^2}}\,dx}{c}$$

$$= \frac{\sqrt{e}\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{c} + \frac{\left(cd-be+\frac{bcd-b^2e+2ace}{\sqrt{b^2-4ac}}\right)\text{Subst}\left(\int \frac{1}{b+\sqrt{b^2-4ac}-\left(-2cd+\left(b+\sqrt{b^2-4ac}\right)e\right)x^2}\,dx, x, \frac{x}{\sqrt{d+ex^2}}\right)}{c} - \frac{\left(-cd+be+\frac{bcd-b^2e+2ace}{\sqrt{b^2-4ac}}\right)\text{Subst}\left(\int \frac{1}{b-\sqrt{b^2-4ac}-\left(-2cd+\left(b-\sqrt{b^2-4ac}\right)e\right)x^2}\,dx, x, \frac{x}{\sqrt{d+ex^2}}\right)}{c}$$

$$= \frac{\left(cd-be-\frac{bcd-b^2e+2ace}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}x}{\sqrt{b-\sqrt{b^2-4ac}}\sqrt{d+ex^2}}\right)}{c\sqrt{b-\sqrt{b^2-4ac}}\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}} + \frac{\left(cd-be+\frac{bcd-b^2e+2ace}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2cd-\left(b+\sqrt{b^2-4ac}\right)e}x}{\sqrt{b+\sqrt{b^2-4ac}}\sqrt{d+ex^2}}\right)}{c\sqrt{b+\sqrt{b^2-4ac}}\sqrt{2cd-\left(b+\sqrt{b^2-4ac}\right)e}} + \frac{\sqrt{e}\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{c}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 7468 vs. $2(324) = 648$.

Time = 16.20 (sec) , antiderivative size = 7468, normalized size of antiderivative = 23.05

$$\int \frac{x^2\sqrt{d+ex^2}}{a+bx^2+cx^4}\,dx = \text{Result too large to show}$$

```
[In] Integrate[(x^2*Sqrt[d + e*x^2])/(a + b*x^2 + c*x^4),x]
[Out] Result too large to show
```

Maple [A] (verified)

Time = 0.40 (sec) , antiderivative size = 368, normalized size of antiderivative = 1.14

method	result
default	$-\frac{a\sqrt{\left(-2ae+bd+\sqrt{-4d^2\left(ac-\frac{b^2}{4}\right)}\right)a}\,\sqrt{2}\left(bde-2c\,d^2+\sqrt{-4d^2\left(ac-\frac{b^2}{4}\right)}\,e\right)\operatorname{arctanh}\left(\frac{a\sqrt{e\,x^2+d}\,\sqrt{2}}{x\sqrt{\left(2ae-bd+\sqrt{-4d^2\left(ac-\frac{b^2}{4}\right)}\right)a}}\right)-}{2\sqrt{}}$
pseudoelliptic	$-\frac{a\sqrt{\left(-2ae+bd+\sqrt{-4d^2\left(ac-\frac{b^2}{4}\right)}\right)a}\,\sqrt{2}\left(bde-2c\,d^2+\sqrt{-4d^2\left(ac-\frac{b^2}{4}\right)}\,e\right)\operatorname{arctanh}\left(\frac{a\sqrt{e\,x^2+d}\,\sqrt{2}}{x\sqrt{\left(2ae-bd+\sqrt{-4d^2\left(ac-\frac{b^2}{4}\right)}\right)a}}\right)-}{2\sqrt{}}$

```
[In] int(x^2*(e*x^2+d)^(1/2)/(c*x^4+b*x^2+a),x,method=_RETURNVERBOSE)
[Out] -1/2/((-2*a*e+b*d+(-4*d^2*(a*c-1/4*b^2))^(1/2))*a)^(1/2)*(a*((-2*a*e+b*d+(-
4*d^2*(a*c-1/4*b^2))^(1/2))*a)^(1/2)*2^(1/2)*(b*d*e-2*c*d^2+(-4*d^2*(a*c-1/
4*b^2))^(1/2)*e)*arctanh(a/x*(e*x^2+d)^(1/2)*2^(1/2)/((2*a*e-b*d+(-4*d^2*(a
```

```
*c-1/4*b^2))^(1/2))*a)^(1/2))-(a*2^(1/2)*(-b*d*e+2*c*d^2+(-4*d^2*(a*c-1/4*b
^2))^(1/2)*e)*arctan(a/x*(e*x^2+d)^(1/2)*2^(1/2)/((-2*a*e+b*d+(-4*d^2*(a*c-
1/4*b^2))^(1/2))*a)^(1/2))+2*(-4*d^2*(a*c-1/4*b^2))^(1/2)*arctanh((e*x^2+d)
^(1/2)/x/e^(1/2))*((-2*a*e+b*d+(-4*d^2*(a*c-1/4*b^2))^(1/2))*a)^(1/2)*e^(1/
2))*((2*a*e-b*d+(-4*d^2*(a*c-1/4*b^2))^(1/2))*a)^(1/2))/((2*a*e-b*d+(-4*d^2
*(a*c-1/4*b^2))^(1/2))*a)^(1/2)/(-4*d^2*(a*c-1/4*b^2))^(1/2)/c
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1626 vs. $2(278) = 556$.

Time = 2.50 (sec) , antiderivative size = 3260, normalized size of antiderivative = 10.06

$$\int \frac{x^2\sqrt{d+ex^2}}{a+bx^2+cx^4}\,dx = \text{Too large to display}$$

```
[In] integrate(x^2*(e*x^2+d)^(1/2)/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] [-1/4*(sqrt(1/2)*c*sqrt(-(b*c*d - (b^2 - 2*a*c)*e + (b^2*c^2 - 4*a*c^3)*sqr
t((c^2*d^2 - 2*b*c*d*e + b^2*e^2)/(b^2*c^4 - 4*a*c^5)))/(b^2*c^2 - 4*a*c^3)
)*log(-((b^2*c^2 - 4*a*c^3)*d*x^2*sqrt((c^2*d^2 - 2*b*c*d*e + b^2*e^2)/(b^2
*c^4 - 4*a*c^5)) + 2*a*c*d^2 - 2*a*b*d*e - (b*c*d^2 + 4*a*b*e^2 - (b^2 + 4*
a*c)*d*e)*x^2 + 2*sqrt(1/2)*sqrt(e*x^2 + d)*((b^3*c^2 - 4*a*b*c^3)*x*sqrt((
c^2*d^2 - 2*b*c*d*e + b^2*e^2)/(b^2*c^4 - 4*a*c^5)) - ((b^2*c - 4*a*c^2)*d
- (b^3 - 4*a*b*c)*e)*x)*sqrt(-(b*c*d - (b^2 - 2*a*c)*e + (b^2*c^2 - 4*a*c^3
)*sqrt((c^2*d^2 - 2*b*c*d*e + b^2*e^2)/(b^2*c^4 - 4*a*c^5)))/(b^2*c^2 - 4*a
*c^3)))/x^2) - sqrt(1/2)*c*sqrt(-(b*c*d - (b^2 - 2*a*c)*e + (b^2*c^2 - 4*a*
c^3)*sqrt((c^2*d^2 - 2*b*c*d*e + b^2*e^2)/(b^2*c^4 - 4*a*c^5)))/(b^2*c^2 -
4*a*c^3))*log(-((b^2*c^2 - 4*a*c^3)*d*x^2*sqrt((c^2*d^2 - 2*b*c*d*e + b^2*e
^2)/(b^2*c^4 - 4*a*c^5)) + 2*a*c*d^2 - 2*a*b*d*e - (b*c*d^2 + 4*a*b*e^2 - (
b^2 + 4*a*c)*d*e)*x^2 - 2*sqrt(1/2)*sqrt(e*x^2 + d)*((b^3*c^2 - 4*a*b*c^3)*
x*sqrt((c^2*d^2 - 2*b*c*d*e + b^2*e^2)/(b^2*c^4 - 4*a*c^5)) - ((b^2*c - 4*a
*c^2)*d - (b^3 - 4*a*b*c)*e)*x)*sqrt(-(b*c*d - (b^2 - 2*a*c)*e + (b^2*c^2 -
 4*a*c^3)*sqrt((c^2*d^2 - 2*b*c*d*e + b^2*e^2)/(b^2*c^4 - 4*a*c^5)))/(b^2*c
^2 - 4*a*c^3)))/x^2) + sqrt(1/2)*c*sqrt(-(b*c*d - (b^2 - 2*a*c)*e - (b^2*c^
2 - 4*a*c^3)*sqrt((c^2*d^2 - 2*b*c*d*e + b^2*e^2)/(b^2*c^4 - 4*a*c^5)))/(b^
2*c^2 - 4*a*c^3))*log(((b^2*c^2 - 4*a*c^3)*d*x^2*sqrt((c^2*d^2 - 2*b*c*d*e
+ b^2*e^2)/(b^2*c^4 - 4*a*c^5)) - 2*a*c*d^2 + 2*a*b*d*e + (b*c*d^2 + 4*a*b*
e^2 - (b^2 + 4*a*c)*d*e)*x^2 + 2*sqrt(1/2)*sqrt(e*x^2 + d)*((b^3*c^2 - 4*a*
b*c^3)*x*sqrt((c^2*d^2 - 2*b*c*d*e + b^2*e^2)/(b^2*c^4 - 4*a*c^5)) + ((b^2*
c - 4*a*c^2)*d - (b^3 - 4*a*b*c)*e)*x)*sqrt(-(b*c*d - (b^2 - 2*a*c)*e - (b^
2*c^2 - 4*a*c^3)*sqrt((c^2*d^2 - 2*b*c*d*e + b^2*e^2)/(b^2*c^4 - 4*a*c^5)))
/(b^2*c^2 - 4*a*c^3)))/x^2) - sqrt(1/2)*c*sqrt(-(b*c*d - (b^2 - 2*a*c)*e -
(b^2*c^2 - 4*a*c^3)*sqrt((c^2*d^2 - 2*b*c*d*e + b^2*e^2)/(b^2*c^4 - 4*a*c^5
)))/(b^2*c^2 - 4*a*c^3))*log(((b^2*c^2 - 4*a*c^3)*d*x^2*sqrt((c^2*d^2 - 2*b
```

```
*c*d*e + b^2*e^2)/(b^2*c^4 - 4*a*c^5)) - 2*a*c*d^2 + 2*a*b*d*e + (b*c*d^2 +
 4*a*b*e^2 - (b^2 + 4*a*c)*d*e)*x^2 - 2*sqrt(1/2)*sqrt(e*x^2 + d)*((b^3*c^2
 - 4*a*b*c^3)*x*sqrt((c^2*d^2 - 2*b*c*d*e + b^2*e^2)/(b^2*c^4 - 4*a*c^5)) +
 ((b^2*c - 4*a*c^2)*d - (b^3 - 4*a*b*c)*e)*x)*sqrt(-(b*c*d - (b^2 - 2*a*c)*
e - (b^2*c^2 - 4*a*c^3)*sqrt((c^2*d^2 - 2*b*c*d*e + b^2*e^2)/(b^2*c^4 - 4*a
*c^5)))/(b^2*c^2 - 4*a*c^3)))/x^2) - 2*sqrt(e)*log(-2*e*x^2 - 2*sqrt(e*x^2
+ d)*sqrt(e)*x - d))/c, -1/4*(sqrt(1/2)*c*sqrt(-(b*c*d - (b^2 - 2*a*c)*e +
(b^2*c^2 - 4*a*c^3)*sqrt((c^2*d^2 - 2*b*c*d*e + b^2*e^2)/(b^2*c^4 - 4*a*c^5
)))/(b^2*c^2 - 4*a*c^3))*log(-((b^2*c^2 - 4*a*c^3)*d*x^2*sqrt((c^2*d^2 - 2*
b*c*d*e + b^2*e^2)/(b^2*c^4 - 4*a*c^5)) + 2*a*c*d^2 - 2*a*b*d*e - (b*c*d^2
+ 4*a*b*e^2 - (b^2 + 4*a*c)*d*e)*x^2 + 2*sqrt(1/2)*sqrt(e*x^2 + d)*((b^3*c^
2 - 4*a*b*c^3)*x*sqrt((c^2*d^2 - 2*b*c*d*e + b^2*e^2)/(b^2*c^4 - 4*a*c^5))
- ((b^2*c - 4*a*c^2)*d - (b^3 - 4*a*b*c)*e)*x)*sqrt(-(b*c*d - (b^2 - 2*a*c)
*e + (b^2*c^2 - 4*a*c^3)*sqrt((c^2*d^2 - 2*b*c*d*e + b^2*e^2)/(b^2*c^4 - 4*
a*c^5)))/(b^2*c^2 - 4*a*c^3)))/x^2) - sqrt(1/2)*c*sqrt(-(b*c*d - (b^2 - 2*a
*c)*e + (b^2*c^2 - 4*a*c^3)*sqrt((c^2*d^2 - 2*b*c*d*e + b^2*e^2)/(b^2*c^4 -
 4*a*c^5)))/(b^2*c^2 - 4*a*c^3))*log(-((b^2*c^2 - 4*a*c^3)*d*x^2*sqrt((c^2*
d^2 - 2*b*c*d*e + b^2*e^2)/(b^2*c^4 - 4*a*c^5)) + 2*a*c*d^2 - 2*a*b*d*e - (
b*c*d^2 + 4*a*b*e^2 - (b^2 + 4*a*c)*d*e)*x^2 - 2*sqrt(1/2)*sqrt(e*x^2 + d)*
((b^3*c^2 - 4*a*b*c^3)*x*sqrt((c^2*d^2 - 2*b*c*d*e + b^2*e^2)/(b^2*c^4 - 4*
a*c^5)) - ((b^2*c - 4*a*c^2)*d - (b^3 - 4*a*b*c)*e)*x)*sqrt(-(b*c*d - (b^2
- 2*a*c)*e + (b^2*c^2 - 4*a*c^3)*sqrt((c^2*d^2 - 2*b*c*d*e + b^2*e^2)/(b^2*
c^4 - 4*a*c^5)))/(b^2*c^2 - 4*a*c^3)))/x^2) + sqrt(1/2)*c*sqrt(-(b*c*d - (b
^2 - 2*a*c)*e - (b^2*c^2 - 4*a*c^3)*sqrt((c^2*d^2 - 2*b*c*d*e + b^2*e^2)/(b
^2*c^4 - 4*a*c^5)))/(b^2*c^2 - 4*a*c^3))*log(((b^2*c^2 - 4*a*c^3)*d*x^2*sqr
t((c^2*d^2 - 2*b*c*d*e + b^2*e^2)/(b^2*c^4 - 4*a*c^5)) - 2*a*c*d^2 + 2*a*b*
d*e + (b*c*d^2 + 4*a*b*e^2 - (b^2 + 4*a*c)*d*e)*x^2 + 2*sqrt(1/2)*sqrt(e*x^
2 + d)*((b^3*c^2 - 4*a*b*c^3)*x*sqrt((c^2*d^2 - 2*b*c*d*e + b^2*e^2)/(b^2*c
^4 - 4*a*c^5)) + ((b^2*c - 4*a*c^2)*d - (b^3 - 4*a*b*c)*e)*x)*sqrt(-(b*c*d
- (b^2 - 2*a*c)*e - (b^2*c^2 - 4*a*c^3)*sqrt((c^2*d^2 - 2*b*c*d*e + b^2*e^2
)/(b^2*c^4 - 4*a*c^5)))/(b^2*c^2 - 4*a*c^3)))/x^2) - sqrt(1/2)*c*sqrt(-(b*c
*d - (b^2 - 2*a*c)*e - (b^2*c^2 - 4*a*c^3)*sqrt((c^2*d^2 - 2*b*c*d*e + b^2*
e^2)/(b^2*c^4 - 4*a*c^5)))/(b^2*c^2 - 4*a*c^3))*log(((b^2*c^2 - 4*a*c^3)*d*
x^2*sqrt((c^2*d^2 - 2*b*c*d*e + b^2*e^2)/(b^2*c^4 - 4*a*c^5)) - 2*a*c*d^2 +
 2*a*b*d*e + (b*c*d^2 + 4*a*b*e^2 - (b^2 + 4*a*c)*d*e)*x^2 - 2*sqrt(1/2)*sq
rt(e*x^2 + d)*((b^3*c^2 - 4*a*b*c^3)*x*sqrt((c^2*d^2 - 2*b*c*d*e + b^2*e^2)
/(b^2*c^4 - 4*a*c^5)) + ((b^2*c - 4*a*c^2)*d - (b^3 - 4*a*b*c)*e)*x)*sqrt(-
(b*c*d - (b^2 - 2*a*c)*e - (b^2*c^2 - 4*a*c^3)*sqrt((c^2*d^2 - 2*b*c*d*e +
b^2*e^2)/(b^2*c^4 - 4*a*c^5)))/(b^2*c^2 - 4*a*c^3)))/x^2) + 4*sqrt(-e)*arct
an(sqrt(-e)*x/sqrt(e*x^2 + d)))/c]
```

Sympy [F]

$$\int \frac{x^2\sqrt{d+ex^2}}{a+bx^2+cx^4}\,dx = \int \frac{x^2\sqrt{d+ex^2}}{a+bx^2+cx^4}\,dx$$

```
[In] integrate(x**2*(e*x**2+d)**(1/2)/(c*x**4+b*x**2+a),x)
[Out] Integral(x**2*sqrt(d + e*x**2)/(a + b*x**2 + c*x**4), x)
```

Maxima [F]

$$\int \frac{x^2\sqrt{d+ex^2}}{a+bx^2+cx^4}\,dx = \int \frac{\sqrt{ex^2+d}x^2}{cx^4+bx^2+a}\,dx$$

```
[In] integrate(x^2*(e*x^2+d)^(1/2)/(c*x^4+b*x^2+a),x, algorithm="maxima")
[Out] integrate(sqrt(e*x^2 + d)*x^2/(c*x^4 + b*x^2 + a), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{x^2\sqrt{d+ex^2}}{a+bx^2+cx^4}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(x^2*(e*x^2+d)^(1/2)/(c*x^4+b*x^2+a),x, algorithm="giac")
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:index.cc index_m i_lex_is_greater Err
or: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2\sqrt{d+ex^2}}{a+bx^2+cx^4}\,dx = \int \frac{x^2\,\sqrt{e\,x^2+d}}{c\,x^4+b\,x^2+a}\,dx$$

```
[In] int((x^2*(d + e*x^2)^(1/2))/(a + b*x^2 + c*x^4),x)
[Out] int((x^2*(d + e*x^2)^(1/2))/(a + b*x^2 + c*x^4), x)
```

3.363 $\int \frac{\sqrt{d+ex^2}}{a+bx^2+cx^4} \, dx$

Optimal result

Integrand size = 26, antiderivative size = 240

$$\int \frac{\sqrt{d+ex^2}}{a+bx^2+cx^4} \, dx = \frac{\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}\arctan\left(\frac{\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}x}{\sqrt{b-\sqrt{b^2-4ac}}\sqrt{d+ex^2}}\right)}{\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}} - \frac{\sqrt{2cd-\left(b+\sqrt{b^2-4ac}\right)e}\arctan\left(\frac{\sqrt{2cd-\left(b+\sqrt{b^2-4ac}\right)e}x}{\sqrt{b+\sqrt{b^2-4ac}}\sqrt{d+ex^2}}\right)}{\sqrt{b^2-4ac}\sqrt{b+\sqrt{b^2-4ac}}}$$

```
[Out] arctan(x*(2*c*d-e*(b-(-4*a*c+b^2)^(1/2)))^(1/2)/(e*x^2+d)^(1/2)/(b-(-4*a*c+
b^2)^(1/2))^(1/2))*(2*c*d-e*(b-(-4*a*c+b^2)^(1/2)))^(1/2)/(-4*a*c+b^2)^(1/2
)/(b-(-4*a*c+b^2)^(1/2))^(1/2)-arctan(x*(2*c*d-e*(b+(-4*a*c+b^2)^(1/2)))^(1
/2)/(e*x^2+d)^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2))*(2*c*d-e*(b+(-4*a*c+b^2)^
(1/2)))^(1/2)/(-4*a*c+b^2)^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2)
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 240, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used

$= \{1188, 399, 223, 212, 385, 211\}$

$$\int \frac{\sqrt{d+ex^2}}{a+bx^2+cx^4}\,dx = \frac{\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}\arctan\left(\frac{x\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}{\sqrt{b-\sqrt{b^2-4ac}}\sqrt{d+ex^2}}\right)}{\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}} - \frac{\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}\arctan\left(\frac{x\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}{\sqrt{\sqrt{b^2-4ac}+b}\sqrt{d+ex^2}}\right)}{\sqrt{b^2-4ac}\sqrt{\sqrt{b^2-4ac}+b}}$$

```
[In] Int[Sqrt[d + e*x^2]/(a + b*x^2 + c*x^4),x]
```

```
[Out] (Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]*ArcTan[(Sqrt[2*c*d - (b - Sqrt[b^2
 - 4*a*c])*e]*x)/(Sqrt[b - Sqrt[b^2 - 4*a*c]]*Sqrt[d + e*x^2])])/(Sqrt[b^2
- 4*a*c]*Sqrt[b - Sqrt[b^2 - 4*a*c]]) - (Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c
])*e]*ArcTan[(Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]*x)/(Sqrt[b + Sqrt[b^2
 - 4*a*c]]*Sqrt[d + e*x^2])])/(Sqrt[b^2 - 4*a*c]*Sqrt[b + Sqrt[b^2 - 4*a*c]
])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 399

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Di
st[b/d, Int[(a + b*x^n)^(p - 1), x], x] - Dist[(b*c - a*d)/d, Int[(a + b*x^
n)^(p - 1)/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, p}, x] && NeQ[b*c - a*
```

```
d, 0] && EqQ[n*(p - 1) + 1, 0] && IntegerQ[n]
```

Rule 1188

```
Int[((d_) + (e_.)*(x_)^2)^(q_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symb
ol] :> With[{r = Rt[b^2 - 4*a*c, 2]}, Dist[2*(c/r), Int[(d + e*x^2)^q/(b -
r + 2*c*x^2), x], x] - Dist[2*(c/r), Int[(d + e*x^2)^q/(b + r + 2*c*x^2), x
], x]] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
- b*d*e + a*e^2, 0] &&  !IntegerQ[q]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{(2c)\int \frac{\sqrt{d+ex^2}}{b-\sqrt{b^2-4ac}+2cx^2}\,dx}{\sqrt{b^2-4ac}} - \frac{(2c)\int \frac{\sqrt{d+ex^2}}{b+\sqrt{b^2-4ac}+2cx^2}\,dx}{\sqrt{b^2-4ac}} \\
&= \frac{\left(2cd-\left(b-\sqrt{b^2-4ac}\right)e\right)\int \frac{1}{\left(b-\sqrt{b^2-4ac}+2cx^2\right)\sqrt{d+ex^2}}\,dx}{\sqrt{b^2-4ac}} \\
&\quad + \frac{\left(-2cd+\left(b+\sqrt{b^2-4ac}\right)e\right)\int \frac{1}{\left(b+\sqrt{b^2-4ac}+2cx^2\right)\sqrt{d+ex^2}}\,dx}{\sqrt{b^2-4ac}} \\
&= \frac{\left(2cd-\left(b-\sqrt{b^2-4ac}\right)e\right)\text{Subst}\left(\int \frac{1}{b-\sqrt{b^2-4ac}-\left(-2cd+\left(b-\sqrt{b^2-4ac}\right)e\right)x^2}\,dx, x, \frac{x}{\sqrt{d+ex^2}}\right)}{\sqrt{b^2-4ac}} \\
&\quad + \frac{\left(-2cd+\left(b+\sqrt{b^2-4ac}\right)e\right)\text{Subst}\left(\int \frac{1}{b+\sqrt{b^2-4ac}-\left(-2cd+\left(b+\sqrt{b^2-4ac}\right)e\right)x^2}\,dx, x, \frac{x}{\sqrt{d+ex^2}}\right)}{\sqrt{b^2-4ac}} \\
&= \frac{\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}\,\tan^{-1}\left(\frac{\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}\,x}{\sqrt{b-\sqrt{b^2-4ac}}\sqrt{d+ex^2}}\right)}{\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}} \\
&\quad - \frac{\sqrt{2cd-\left(b+\sqrt{b^2-4ac}\right)e}\,\tan^{-1}\left(\frac{\sqrt{2cd-\left(b+\sqrt{b^2-4ac}\right)e}\,x}{\sqrt{b+\sqrt{b^2-4ac}}\sqrt{d+ex^2}}\right)}{\sqrt{b^2-4ac}\sqrt{b+\sqrt{b^2-4ac}}}
\end{aligned}
$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 2607 vs. 2(240) = 480.

Time = 13.42 (sec) , antiderivative size = 2607, normalized size of antiderivative = 10.86

$$\int \frac{\sqrt{d+ex^2}}{a+bx^2+cx^4}\,dx = \text{Result too large to show}$$

```
[In] Integrate[Sqrt[d + e*x^2]/(a + b*x^2 + c*x^4),x]
```

```
[Out] (Sqrt[-((b + Sqrt[b^2 - 4*a*c])/c)]*(2*c*d + (-b + Sqrt[b^2 - 4*a*c])*e)*Sq
rt[2*d - ((b + Sqrt[b^2 - 4*a*c])*e)/c]*Log[-(Sqrt[2]*Sqrt[(-b + Sqrt[b^2 -
 4*a*c])/c]) + 2*x] - Sqrt[-((b + Sqrt[b^2 - 4*a*c])/c)]*(2*c*d + (-b + Sqr
t[b^2 - 4*a*c])*e)*Sqrt[2*d - ((b + Sqrt[b^2 - 4*a*c])*e)/c]*Log[Sqrt[2]*Sq
rt[(-b + Sqrt[b^2 - 4*a*c])/c] + 2*x] - 2*c*Sqrt[(-b + Sqrt[b^2 - 4*a*c])/c
]*d*Sqrt[(2*c*d - b*e + Sqrt[b^2 - 4*a*c]*e)/c]*Log[-(Sqrt[2]*Sqrt[-((b + S
qrt[b^2 - 4*a*c])/c)]) + 2*x] + b*Sqrt[(-b + Sqrt[b^2 - 4*a*c])/c]*e*Sqrt[(
2*c*d - b*e + Sqrt[b^2 - 4*a*c]*e)/c]*Log[-(Sqrt[2]*Sqrt[-((b + Sqrt[b^2 -
4*a*c])/c)]) + 2*x] + Sqrt[b^2 - 4*a*c]*Sqrt[(-b + Sqrt[b^2 - 4*a*c])/c]*e*
Sqrt[(2*c*d - b*e + Sqrt[b^2 - 4*a*c]*e)/c]*Log[-(Sqrt[2]*Sqrt[-((b + Sqrt[
b^2 - 4*a*c])/c)]) + 2*x] + 2*c*Sqrt[(-b + Sqrt[b^2 - 4*a*c])/c]*d*Sqrt[(2*
c*d - b*e + Sqrt[b^2 - 4*a*c]*e)/c]*Log[Sqrt[2]*Sqrt[-((b + Sqrt[b^2 - 4*a*
c])/c)] + 2*x] - b*Sqrt[(-b + Sqrt[b^2 - 4*a*c])/c]*e*Sqrt[(2*c*d - b*e + S
qrt[b^2 - 4*a*c]*e)/c]*Log[Sqrt[2]*Sqrt[-((b + Sqrt[b^2 - 4*a*c])/c)] + 2*x
] - Sqrt[b^2 - 4*a*c]*Sqrt[(-b + Sqrt[b^2 - 4*a*c])/c]*e*Sqrt[(2*c*d - b*e
+ Sqrt[b^2 - 4*a*c]*e)/c]*Log[Sqrt[2]*Sqrt[-((b + Sqrt[b^2 - 4*a*c])/c)] +
2*x] + 2*c*Sqrt[-((b + Sqrt[b^2 - 4*a*c])/c)]*d*Sqrt[2*d - ((b + Sqrt[b^2 -
 4*a*c])*e)/c]*Log[4*d - 2*Sqrt[2]*Sqrt[(-b + Sqrt[b^2 - 4*a*c])/c]*e*x + 2
*Sqrt[4*d + (2*(-b + Sqrt[b^2 - 4*a*c])*e)/c]*Sqrt[d + e*x^2]] - b*Sqrt[-((
b + Sqrt[b^2 - 4*a*c])/c)]*e*Sqrt[2*d - ((b + Sqrt[b^2 - 4*a*c])*e)/c]*Log[
4*d - 2*Sqrt[2]*Sqrt[(-b + Sqrt[b^2 - 4*a*c])/c]*e*x + 2*Sqrt[4*d + (2*(-b
+ Sqrt[b^2 - 4*a*c])*e)/c]*Sqrt[d + e*x^2]] + Sqrt[b^2 - 4*a*c]*Sqrt[-((b +
 Sqrt[b^2 - 4*a*c])/c)]*e*Sqrt[2*d - ((b + Sqrt[b^2 - 4*a*c])*e)/c]*Log[4*d
 - 2*Sqrt[2]*Sqrt[(-b + Sqrt[b^2 - 4*a*c])/c]*e*x + 2*Sqrt[4*d + (2*(-b + S
qrt[b^2 - 4*a*c])*e)/c]*Sqrt[d + e*x^2]] - 2*c*Sqrt[-((b + Sqrt[b^2 - 4*a*c
])/c)]*d*Sqrt[2*d - ((b + Sqrt[b^2 - 4*a*c])*e)/c]*Log[4*d + 2*Sqrt[2]*Sqrt
[(-b + Sqrt[b^2 - 4*a*c])/c]*e*x + 2*Sqrt[4*d + (2*(-b + Sqrt[b^2 - 4*a*c])
*e)/c]*Sqrt[d + e*x^2]] + b*Sqrt[-((b + Sqrt[b^2 - 4*a*c])/c)]*e*Sqrt[2*d -
 ((b + Sqrt[b^2 - 4*a*c])*e)/c]*Log[4*d + 2*Sqrt[2]*Sqrt[(-b + Sqrt[b^2 - 4
*a*c])/c]*e*x + 2*Sqrt[4*d + (2*(-b + Sqrt[b^2 - 4*a*c])*e)/c]*Sqrt[d + e*x
^2]] - Sqrt[b^2 - 4*a*c]*Sqrt[-((b + Sqrt[b^2 - 4*a*c])/c)]*e*Sqrt[2*d - ((
b + Sqrt[b^2 - 4*a*c])*e)/c]*Log[4*d + 2*Sqrt[2]*Sqrt[(-b + Sqrt[b^2 - 4*a*
c])/c]*e*x + 2*Sqrt[4*d + (2*(-b + Sqrt[b^2 - 4*a*c])*e)/c]*Sqrt[d + e*x^2]
] - 2*c*Sqrt[(-b + Sqrt[b^2 - 4*a*c])/c]*d*Sqrt[(2*c*d - b*e + Sqrt[b^2 - 4
*a*c]*e)/c]*Log[4*d - 2*Sqrt[2]*Sqrt[-((b + Sqrt[b^2 - 4*a*c])/c)]*e*x + 2*
```

```
Sqrt[4*d - (2*(b + Sqrt[b^2 - 4*a*c])*e)/c]*Sqrt[d + e*x^2]] + b*Sqrt[(-b +
 Sqrt[b^2 - 4*a*c])/c]*e*Sqrt[(2*c*d - b*e + Sqrt[b^2 - 4*a*c]*e)/c]*Log[4*
d - 2*Sqrt[2]*Sqrt[-((b + Sqrt[b^2 - 4*a*c])/c)]*e*x + 2*Sqrt[4*d - (2*(b +
 Sqrt[b^2 - 4*a*c])*e)/c]*Sqrt[d + e*x^2]] + Sqrt[b^2 - 4*a*c]*Sqrt[(-b + S
qrt[b^2 - 4*a*c])/c]*e*Sqrt[(2*c*d - b*e + Sqrt[b^2 - 4*a*c]*e)/c]*Log[4*d
- 2*Sqrt[2]*Sqrt[-((b + Sqrt[b^2 - 4*a*c])/c)]*e*x + 2*Sqrt[4*d - (2*(b + S
qrt[b^2 - 4*a*c])*e)/c]*Sqrt[d + e*x^2]] + 2*c*Sqrt[(-b + Sqrt[b^2 - 4*a*c]
)/c]*d*Sqrt[(2*c*d - b*e + Sqrt[b^2 - 4*a*c]*e)/c]*Log[4*d + 2*Sqrt[2]*Sqrt
[-((b + Sqrt[b^2 - 4*a*c])/c)]*e*x + 2*Sqrt[4*d - (2*(b + Sqrt[b^2 - 4*a*c]
)*e)/c]*Sqrt[d + e*x^2]] - b*Sqrt[(-b + Sqrt[b^2 - 4*a*c])/c]*e*Sqrt[(2*c*d
 - b*e + Sqrt[b^2 - 4*a*c]*e)/c]*Log[4*d + 2*Sqrt[2]*Sqrt[-((b + Sqrt[b^2 -
 4*a*c])/c)]*e*x + 2*Sqrt[4*d - (2*(b + Sqrt[b^2 - 4*a*c])*e)/c]*Sqrt[d + e
*x^2]] - Sqrt[b^2 - 4*a*c]*Sqrt[(-b + Sqrt[b^2 - 4*a*c])/c]*e*Sqrt[(2*c*d -
 b*e + Sqrt[b^2 - 4*a*c]*e)/c]*Log[4*d + 2*Sqrt[2]*Sqrt[-((b + Sqrt[b^2 - 4
*a*c])/c)]*e*x + 2*Sqrt[4*d - (2*(b + Sqrt[b^2 - 4*a*c])*e)/c]*Sqrt[d + e*x
^2]])/(2*c*Sqrt[b^2 - 4*a*c]*Sqrt[(-b + Sqrt[b^2 - 4*a*c])/c]*Sqrt[-((b + S
qrt[b^2 - 4*a*c])/c)]*Sqrt[(2*c*d - b*e + Sqrt[b^2 - 4*a*c]*e)/c]*Sqrt[2*d
- ((b + Sqrt[b^2 - 4*a*c])*e)/c])
```

Maple [A] (verified)

Time = 0.36 (sec) , antiderivative size = 233, normalized size of antiderivative = 0.97

method	result
default	$-\frac{d\sqrt{2}\left(\frac{\left(-2ae+bd+\sqrt{-d^{2}\left(4ac-b^{2}\right)}\right)\arctan\left(\frac{a\sqrt{e\,x^{2}+d}\,\sqrt{2}}{x\sqrt{\left(-2ae+bd+\sqrt{-d^{2}\left(4ac-b^{2}\right)}\right)a}}\right)}{\sqrt{\left(-2ae+bd+\sqrt{-d^{2}\left(4ac-b^{2}\right)}\right)a}}-\frac{\left(2ae-bd+\sqrt{-d^{2}\left(4ac-b^{2}\right)}\right)\operatorname{arctanh}\left(\frac{\cdots}{x\sqrt{\left(2ae\cdots\right.}}\right.}{\sqrt{\left(2ae-bd+\sqrt{-d^{2}\left(4ac-\cdots\right.}\right.}}\right.}{2\sqrt{-d^{2}\left(4ac-b^{2}\right)}}$
pseudoelliptic	$-\frac{d\sqrt{2}\left(\frac{\left(-2ae+bd+\sqrt{-d^{2}\left(4ac-b^{2}\right)}\right)\arctan\left(\frac{a\sqrt{e\,x^{2}+d}\,\sqrt{2}}{x\sqrt{\left(-2ae+bd+\sqrt{-d^{2}\left(4ac-b^{2}\right)}\right)a}}\right)}{\sqrt{\left(-2ae+bd+\sqrt{-d^{2}\left(4ac-b^{2}\right)}\right)a}}-\frac{\left(2ae-bd+\sqrt{-d^{2}\left(4ac-b^{2}\right)}\right)\operatorname{arctanh}\left(\frac{\cdots}{x\sqrt{\left(2ae\cdots\right.}}\right.}{\sqrt{\left(2ae-bd+\sqrt{-d^{2}\left(4ac-\cdots\right.}\right.}}\right.}{2\sqrt{-d^{2}\left(4ac-b^{2}\right)}}$

```
[In] int((e*x^2+d)^(1/2)/(c*x^4+b*x^2+a),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*d*2^(1/2)/(-d^2*(4*a*c-b^2))^(1/2)*((-2*a*e+b*d+(-d^2*(4*a*c-b^2))^(1/
2))/((-2*a*e+b*d+(-d^2*(4*a*c-b^2))^(1/2))*a)^(1/2)*arctan(a/x*(e*x^2+d)^(1
/2)*2^(1/2)/((-2*a*e+b*d+(-d^2*(4*a*c-b^2))^(1/2))*a)^(1/2))-(2*a*e-b*d+(-d
^2*(4*a*c-b^2))^(1/2))/((2*a*e-b*d+(-d^2*(4*a*c-b^2))^(1/2))*a)^(1/2)*arcta
nh(a/x*(e*x^2+d)^(1/2)*2^(1/2)/((2*a*e-b*d+(-d^2*(4*a*c-b^2))^(1/2))*a)^(1/
2)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 985 vs. 2(200) = 400.

Time = 0.83 (sec) , antiderivative size = 985, normalized size of antiderivative = 4.10

$$\int \frac{\sqrt{d+ex^2}}{a+bx^2+cx^4}\,dx$$

$$= \frac{1}{4}\sqrt{\frac{1}{2}}\sqrt{-\frac{bd-2\,ae+(ab^2-4\,a^2c)\sqrt{\frac{d^2}{a^2b^2-4\,a^3c}}}{ab^2-4\,a^2c}}\log\left(-\frac{(ab^2-4\,a^2c)d\sqrt{\frac{d^2}{a^2b^2-4\,a^3c}}x^2+4\,\sqrt{\frac{1}{2}}(a^2b^2-4\,a^3c)\sqrt{\cdots}}{\cdots}\right.$$

$$-\frac{1}{4}\sqrt{\frac{1}{2}}\sqrt{-\frac{bd-2\,ae+(ab^2-4\,a^2c)\sqrt{\frac{d^2}{a^2b^2-4\,a^3c}}}{ab^2-4\,a^2c}}\log\left(-\frac{(ab^2-4\,a^2c)d\sqrt{\frac{d^2}{a^2b^2-4\,a^3c}}x^2-4\,\sqrt{\frac{1}{2}}(a^2b^2-4\,a^3c)\cdots}{\cdots}\right.$$

$$+\frac{1}{4}\sqrt{\frac{1}{2}}\sqrt{-\frac{bd-2\,ae-(ab^2-4\,a^2c)\sqrt{\frac{d^2}{a^2b^2-4\,a^3c}}}{ab^2-4\,a^2c}}\log\left(\frac{(ab^2-4\,a^2c)d\sqrt{\frac{d^2}{a^2b^2-4\,a^3c}}x^2+4\,\sqrt{\frac{1}{2}}(a^2b^2-4\,a^3c)\sqrt{\cdots}}{\cdots}\right.$$

$$-\frac{1}{4}\sqrt{\frac{1}{2}}\sqrt{-\frac{bd-2\,ae-(ab^2-4\,a^2c)\sqrt{\frac{d^2}{a^2b^2-4\,a^3c}}}{ab^2-4\,a^2c}}\log\left(\frac{(ab^2-4\,a^2c)d\sqrt{\frac{d^2}{a^2b^2-4\,a^3c}}x^2-4\,\sqrt{\frac{1}{2}}(a^2b^2-4\,a^3c)\sqrt{\cdots}}{\cdots}\right.$$

```
[In] integrate((e*x^2+d)^(1/2)/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] 1/4*sqrt(1/2)*sqrt(-(b*d - 2*a*e + (a*b^2 - 4*a^2*c)*sqrt(d^2/(a^2*b^2 - 4*
a^3*c)))/(a*b^2 - 4*a^2*c))*log(-((a*b^2 - 4*a^2*c)*d*sqrt(d^2/(a^2*b^2 - 4
*a^3*c))*x^2 + 4*sqrt(1/2)*(a^2*b^2 - 4*a^3*c)*sqrt(e*x^2 + d)*sqrt(d^2/(a^
2*b^2 - 4*a^3*c))*x*sqrt(-(b*d - 2*a*e + (a*b^2 - 4*a^2*c)*sqrt(d^2/(a^2*b^
2 - 4*a^3*c)))/(a*b^2 - 4*a^2*c)) - 2*a*d^2 + (b*d^2 - 4*a*d*e)*x^2)/x^2) -
 1/4*sqrt(1/2)*sqrt(-(b*d - 2*a*e + (a*b^2 - 4*a^2*c)*sqrt(d^2/(a^2*b^2 - 4
*a^3*c)))/(a*b^2 - 4*a^2*c))*log(-((a*b^2 - 4*a^2*c)*d*sqrt(d^2/(a^2*b^2 -
4*a^3*c))*x^2 - 4*sqrt(1/2)*(a^2*b^2 - 4*a^3*c)*sqrt(e*x^2 + d)*sqrt(d^2/(a
^2*b^2 - 4*a^3*c))*x*sqrt(-(b*d - 2*a*e + (a*b^2 - 4*a^2*c)*sqrt(d^2/(a^2*b
^2 - 4*a^3*c)))/(a*b^2 - 4*a^2*c)) - 2*a*d^2 + (b*d^2 - 4*a*d*e)*x^2)/x^2)
+ 1/4*sqrt(1/2)*sqrt(-(b*d - 2*a*e - (a*b^2 - 4*a^2*c)*sqrt(d^2/(a^2*b^2 -
4*a^3*c)))/(a*b^2 - 4*a^2*c))*log(((a*b^2 - 4*a^2*c)*d*sqrt(d^2/(a^2*b^2 -
4*a^3*c))*x^2 + 4*sqrt(1/2)*(a^2*b^2 - 4*a^3*c)*sqrt(e*x^2 + d)*sqrt(d^2/(a
^2*b^2 - 4*a^3*c))*x*sqrt(-(b*d - 2*a*e - (a*b^2 - 4*a^2*c)*sqrt(d^2/(a^2*b
```

```
^2 - 4*a^3*c)))/(a*b^2 - 4*a^2*c)) + 2*a*d^2 - (b*d^2 - 4*a*d*e)*x^2)/x^2)
 - 1/4*sqrt(1/2)*sqrt(-(b*d - 2*a*e - (a*b^2 - 4*a^2*c)*sqrt(d^2/(a^2*b^2 -
4*a^3*c)))/(a*b^2 - 4*a^2*c))*log(((a*b^2 - 4*a^2*c)*d*sqrt(d^2/(a^2*b^2 -
4*a^3*c))*x^2 - 4*sqrt(1/2)*(a^2*b^2 - 4*a^3*c)*sqrt(e*x^2 + d)*sqrt(d^2/(a
^2*b^2 - 4*a^3*c))*x*sqrt(-(b*d - 2*a*e - (a*b^2 - 4*a^2*c)*sqrt(d^2/(a^2*b
^2 - 4*a^3*c)))/(a*b^2 - 4*a^2*c)) + 2*a*d^2 - (b*d^2 - 4*a*d*e)*x^2)/x^2)
```

Sympy [F]

$$\int \frac{\sqrt{d+ex^2}}{a+bx^2+cx^4}\,dx = \int \frac{\sqrt{d+ex^2}}{a+bx^2+cx^4}\,dx$$

```
[In] integrate((e*x**2+d)**(1/2)/(c*x**4+b*x**2+a),x)
[Out] Integral(sqrt(d + e*x**2)/(a + b*x**2 + c*x**4), x)
```

Maxima [F]

$$\int \frac{\sqrt{d+ex^2}}{a+bx^2+cx^4}\,dx = \int \frac{\sqrt{ex^2+d}}{cx^4+bx^2+a}\,dx$$

```
[In] integrate((e*x^2+d)^(1/2)/(c*x^4+b*x^2+a),x, algorithm="maxima")
[Out] integrate(sqrt(e*x^2 + d)/(c*x^4 + b*x^2 + a), x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{\sqrt{d+ex^2}}{a+bx^2+cx^4}\,dx = \text{Timed out}$$

```
[In] integrate((e*x^2+d)^(1/2)/(c*x^4+b*x^2+a),x, algorithm="giac")
[Out] Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{d+ex^2}}{a+bx^2+cx^4}\,dx=\int \frac{\sqrt{e\,x^2+d}}{c\,x^4+b\,x^2+a}\,dx$$

```
[In] int((d + e*x^2)^(1/2)/(a + b*x^2 + c*x^4),x)
[Out] int((d + e*x^2)^(1/2)/(a + b*x^2 + c*x^4), x)
```

3.364 $\int \frac{\sqrt{d+ex^2}}{x^2(a+bx^2+cx^4)}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 291

$$\int \frac{\sqrt{d+ex^2}}{x^2(a+bx^2+cx^4)}\,dx = -\frac{\sqrt{d+ex^2}}{ax} - \frac{c\left(d+\frac{bd-2ae}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}\,x}{\sqrt{b-\sqrt{b^2-4ac}}\sqrt{d+ex^2}}\right)}{a\sqrt{b-\sqrt{b^2-4ac}}\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}} - \frac{c\left(d-\frac{bd-2ae}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2cd-\left(b+\sqrt{b^2-4ac}\right)e}\,x}{\sqrt{b+\sqrt{b^2-4ac}}\sqrt{d+ex^2}}\right)}{a\sqrt{b+\sqrt{b^2-4ac}}\sqrt{2cd-\left(b+\sqrt{b^2-4ac}\right)e}}$$

```
[Out] -(e*x^2+d)^(1/2)/a/x-c*arctan(x*(2*c*d-e*(b-(-4*a*c+b^2)^(1/2)))^(1/2)/(e*x
^2+d)^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2))*(d+(-2*a*e+b*d)/(-4*a*c+b^2)^(1/2
))/a/(2*c*d-e*(b-(-4*a*c+b^2)^(1/2)))^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2)-c*
arctan(x*(2*c*d-e*(b+(-4*a*c+b^2)^(1/2)))^(1/2)/(e*x^2+d)^(1/2)/(b+(-4*a*c+
b^2)^(1/2))^(1/2))*(d+(2*a*e-b*d)/(-4*a*c+b^2)^(1/2))/a/(b+(-4*a*c+b^2)^(1/
2))^(1/2)/(2*c*d-e*(b+(-4*a*c+b^2)^(1/2)))^(1/2)
```

Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 291, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.172$, Rules used

$= \{1309, 270, 1706, 385, 211\}$

$$\int \frac{\sqrt{d+ex^2}}{x^2\left(a+bx^2+cx^4\right)}\,dx = -\frac{c\left(\frac{bd-2ae}{\sqrt{b^2-4ac}}+d\right)\arctan\left(\frac{x\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}{\sqrt{b-\sqrt{b^2-4ac}}\sqrt{d+ex^2}}\right)}{a\sqrt{b-\sqrt{b^2-4ac}}\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}} - \frac{c\left(d-\frac{bd-2ae}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{x\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}{\sqrt{\sqrt{b^2-4ac}+b}\sqrt{d+ex^2}}\right)}{a\sqrt{\sqrt{b^2-4ac}+b}\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}} - \frac{\sqrt{d+ex^2}}{ax}$$

```
[In] Int[Sqrt[d + e*x^2]/(x^2*(a + b*x^2 + c*x^4)),x]
```

```
[Out] -(Sqrt[d + e*x^2]/(a*x)) - (c*(d + (b*d - 2*a*e)/Sqrt[b^2 - 4*a*c])*ArcTan[
(Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]*x)/(Sqrt[b - Sqrt[b^2 - 4*a*c]]*Sq
rt[d + e*x^2])])/(a*Sqrt[b - Sqrt[b^2 - 4*a*c]]*Sqrt[2*c*d - (b - Sqrt[b^2
- 4*a*c])*e]) - (c*(d - (b*d - 2*a*e)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2*c*d
 - (b + Sqrt[b^2 - 4*a*c])*e]*x)/(Sqrt[b + Sqrt[b^2 - 4*a*c]]*Sqrt[d + e*x^
2])])/(a*Sqrt[b + Sqrt[b^2 - 4*a*c]]*Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e
])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 1309

```
Int[(((f_.)*(x_))^(m_)*((d_.) + (e_.)*(x_)^2)^(q_))/((a_) + (b_.)*(x_)^2 +
(c_.)*(x_)^4), x_Symbol] :> Dist[d/a, Int[(f*x)^m*(d + e*x^2)^(q - 1), x],
x] - Dist[1/(a*f^2), Int[(f*x)^(m + 2)*(d + e*x^2)^(q - 1)*(Simp[b*d - a*e
+ c*d*x^2, x]/(a + b*x^2 + c*x^4)), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
&& NeQ[b^2 - 4*a*c, 0] &&  !IntegerQ[q] && GtQ[q, 0] && LtQ[m, 0]
```

Rule 1706

```
Int[(Px_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^
(p_.), x_Symbol] :> Int[ExpandIntegrand[Px*(d + e*x^2)^q*(a + b*x^2 + c*x^4
)^p, x], x] /; FreeQ[{a, b, c, d, e, q}, x] && PolyQ[Px, x^2] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[p]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{\int \frac{bd-ae+cdx^2}{\sqrt{d+ex^2}(a+bx^2+cx^4)}\,dx}{a}+\frac{d\int \frac{1}{x^2\sqrt{d+ex^2}}\,dx}{a}\\
&= -\frac{\sqrt{d+ex^2}}{ax}-\frac{\int \left(\frac{cd+\frac{c(bd-2ae)}{\sqrt{b^2-4ac}}}{\left(b-\sqrt{b^2-4ac}+2cx^2\right)\sqrt{d+ex^2}}+\frac{cd-\frac{c(bd-2ae)}{\sqrt{b^2-4ac}}}{\left(b+\sqrt{b^2-4ac}+2cx^2\right)\sqrt{d+ex^2}}\right)dx}{a}\\
&= -\frac{\sqrt{d+ex^2}}{ax}-\frac{\left(c\left(d-\frac{bd-2ae}{\sqrt{b^2-4ac}}\right)\right)\int \frac{1}{\left(b+\sqrt{b^2-4ac}+2cx^2\right)\sqrt{d+ex^2}}\,dx}{a}\\
&\quad -\frac{\left(c\left(d+\frac{bd-2ae}{\sqrt{b^2-4ac}}\right)\right)\int \frac{1}{\left(b-\sqrt{b^2-4ac}+2cx^2\right)\sqrt{d+ex^2}}\,dx}{a}\\
&= -\frac{\sqrt{d+ex^2}}{ax}-\frac{\left(c\left(d-\frac{bd-2ae}{\sqrt{b^2-4ac}}\right)\right)\text{Subst}\left(\int \frac{1}{b+\sqrt{b^2-4ac}-\left(-2cd+\left(b+\sqrt{b^2-4ac}\right)e\right)x^2}\,dx,x,\frac{x}{\sqrt{d+ex^2}}\right)}{a}\\
&\quad -\frac{\left(c\left(d+\frac{bd-2ae}{\sqrt{b^2-4ac}}\right)\right)\text{Subst}\left(\int \frac{1}{b-\sqrt{b^2-4ac}-\left(-2cd+\left(b-\sqrt{b^2-4ac}\right)e\right)x^2}\,dx,x,\frac{x}{\sqrt{d+ex^2}}\right)}{a}\\
&= -\frac{\sqrt{d+ex^2}}{ax}-\frac{c\left(d+\frac{bd-2ae}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}\,x}{\sqrt{b-\sqrt{b^2-4ac}}\sqrt{d+ex^2}}\right)}{a\sqrt{b-\sqrt{b^2-4ac}}\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}}\\
&\quad -\frac{c\left(d-\frac{bd-2ae}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2cd-\left(b+\sqrt{b^2-4ac}\right)e}\,x}{\sqrt{b+\sqrt{b^2-4ac}}\sqrt{d+ex^2}}\right)}{a\sqrt{b+\sqrt{b^2-4ac}}\sqrt{2cd-\left(b+\sqrt{b^2-4ac}\right)e}}
\end{aligned}
$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.73 (sec) , antiderivative size = 626, normalized size of antiderivative = 2.15

$$\int \frac{\sqrt{d+ex^2}}{x^2(a+bx^2+cx^4)}\,dx = -\frac{\sqrt{d+ex^2}}{ax} + \frac{\text{RootSum}\left[ae^4+4bde^2\#1^2-4ae^3\#1^2+16cd^2\#1^4-8bde\#1^4+6ae^2\#1^4+4bd\#1^6-4ae\#1^6+a\#1^8\&\right.}{}$$

```
[In] Integrate[Sqrt[d + e*x^2]/(x^2*(a + b*x^2 + c*x^4)),x]
```

```
[Out] -(Sqrt[d + e*x^2]/(a*x)) + RootSum[a*e^4 + 4*b*d*e^2*#1^2 - 4*a*e^3*#1^2 +
16*c*d^2*#1^4 - 8*b*d*e*#1^4 + 6*a*e^2*#1^4 + 4*b*d*#1^6 - 4*a*e*#1^6 + a*#
1^8 & , (b*d*e^3*Log[x] - a*e^4*Log[x] - b*d*e^3*Log[-Sqrt[d] + Sqrt[d + e*
x^2] - x*#1] + a*e^4*Log[-Sqrt[d] + Sqrt[d + e*x^2] - x*#1] + 4*c*d^2*e*Log
[x]*#1^2 - 3*b*d*e^2*Log[x]*#1^2 + 3*a*e^3*Log[x]*#1^2 - 4*c*d^2*e*Log[-Sqr
t[d] + Sqrt[d + e*x^2] - x*#1]*#1^2 + 3*b*d*e^2*Log[-Sqrt[d] + Sqrt[d + e*x
^2] - x*#1]*#1^2 - 3*a*e^3*Log[-Sqrt[d] + Sqrt[d + e*x^2] - x*#1]*#1^2 - 4*
c*d^2*Log[x]*#1^4 + 3*b*d*e*Log[x]*#1^4 - 3*a*e^2*Log[x]*#1^4 + 4*c*d^2*Log
[-Sqrt[d] + Sqrt[d + e*x^2] - x*#1]*#1^4 - 3*b*d*e*Log[-Sqrt[d] + Sqrt[d +
e*x^2] - x*#1]*#1^4 + 3*a*e^2*Log[-Sqrt[d] + Sqrt[d + e*x^2] - x*#1]*#1^4 -
 b*d*Log[x]*#1^6 + a*e*Log[x]*#1^6 + b*d*Log[-Sqrt[d] + Sqrt[d + e*x^2] - x
*#1]*#1^6 - a*e*Log[-Sqrt[d] + Sqrt[d + e*x^2] - x*#1]*#1^6)/(b*d*e^2*#1 -
a*e^3*#1 + 8*c*d^2*#1^3 - 4*b*d*e*#1^3 + 3*a*e^2*#1^3 + 3*b*d*#1^5 - 3*a*e*
#1^5 + a*#1^7) & ]/(4*a)
```

Maple [A] (verified)

Time = 0.77 (sec) , antiderivative size = 326, normalized size of antiderivative = 1.12

method	result
risch	$-\frac{\sqrt{e x^2+d}}{a x}-\frac{\sqrt{2}\left(\frac{\left(abde+2d^2ac-b^2d^2+\sqrt{-d^2\left(4ac-b^2\right)}\,ae-\sqrt{-d^2\left(4ac-b^2\right)}\,bd\right)\arctan\left(\frac{a\sqrt{e x^2+d}\,\sqrt{2}}{x\sqrt{\left(-2ae+bd+\sqrt{-d^2\left(4ac-b^2\right)}\right)a}}\right)}{\sqrt{\left(-2ae+bd+\sqrt{-d^2\left(4ac-b^2\right)}\right)a}}-\right.}{2a\sqrt{-d^2(4a}}$
pseudoelliptic	$\frac{-\frac{\sqrt{e x^2+d}}{x}-\frac{\left(abde+2d^2ac-b^2d^2+\sqrt{-d^2\left(4ac-b^2\right)}\,ae-\sqrt{-d^2\left(4ac-b^2\right)}\,bd\right)\sqrt{2}\arctan\left(\frac{a\sqrt{e x^2+d}\,\sqrt{2}}{x\sqrt{\left(-2ae+bd+\sqrt{-d^2\left(4ac-b^2\right)}\right)a}}\right)}{2\sqrt{-d^2\left(4ac-b^2\right)}\sqrt{\left(-2ae+bd+\sqrt{-d^2\left(4ac-b^2\right)}\right)a}}+\frac{\left(-abd\right.}{}}{a}$
default	$\frac{-\frac{\left(e x^2+d\right)^{\frac{3}{2}}}{dx}+\frac{2e\left(\frac{x\sqrt{e x^2+d}}{2}+\frac{d\ln\left(x\sqrt{e}+\sqrt{e x^2+d}\right)}{2\sqrt{e}}\right)}{d}}{a}+\frac{\sqrt{\left(-2ae+bd+\sqrt{-4d^2\left(ac-\frac{b^2}{4}\right)}\right)a}\,\sqrt{2}\left(\left(ae-bd\right)\sqrt{-4d^2\left(ac-\frac{b^2}{4}\right)}\right.}{}$

[In] int((e*x^2+d)^(1/2)/x^2/(c*x^4+b*x^2+a),x,method=_RETURNVERBOSE)

[Out] -(e*x^2+d)^(1/2)/a/x-1/2*2^(1/2)/a/(-d^2*(4*a*c-b^2))^(1/2)*((a*b*d*e+2*d^2*a*c-b^2*d^2+(-d^2*(4*a*c-b^2))^(1/2)*a*e-(-d^2*(4*a*c-b^2))^(1/2)*b*d)/((-2*a*e+b*d+(-d^2*(4*a*c-b^2))^(1/2))*a)^(1/2)*arctan(a/x*(e*x^2+d)^(1/2)*2^(1/2)/((-2*a*e+b*d+(-d^2*(4*a*c-b^2))^(1/2))*a)^(1/2))-(-a*b*d*e-2*d^2*a*c+b^2*d^2+(-d^2*(4*a*c-b^2))^(1/2)*a*e-(-d^2*(4*a*c-b^2))^(1/2)*b*d)/((2*a*e-b*d+(-d^2*(4*a*c-b^2))^(1/2))*a)^(1/2)*arctanh(a/x*(e*x^2+d)^(1/2)*2^(1/2)/((2*a*e-b*d+(-d^2*(4*a*c-b^2))^(1/2))*a)^(1/2)))

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2402 vs. $2(249) = 498$.

Time = 1.63 (sec) , antiderivative size = 2402, normalized size of antiderivative = 8.25

$$\int \frac{\sqrt{d+ex^2}}{x^2\left(a+bx^2+cx^4\right)}\,dx = \text{Too large to display}$$

[In] integrate((e*x^2+d)^(1/2)/x^2/(c*x^4+b*x^2+a),x, algorithm="fricas")

[Out] -1/4*(sqrt(1/2)*a*x*sqrt(-((b^3 - 3*a*b*c)*d - (a*b^2 - 2*a^2*c)*e + (a^3*b^2 - 4*a^4*c)*sqrt((a^2*b^2*e^2 + (b^4 - 2*a*b^2*c + a^2*c^2)*d^2 - 2*(a*b^3 - a^2*b*c)*d*e)/(a^6*b^2 - 4*a^7*c)))/(a^3*b^2 - 4*a^4*c))*log((2*a^2*b*c*d*e + (a^3*b^2*c - 4*a^4*c^2)*d*x^2*sqrt((a^2*b^2*e^2 + (b^4 - 2*a*b^2*c + a^2*c^2)*d^2 - 2*(a*b^3 - a^2*b*c)*d*e)/(a^6*b^2 - 4*a^7*c)) - 2*(a*b^2*c - a^2*c^2)*d^2 + (4*a^2*b*c*e^2 + (b^3*c - a*b*c^2)*d^2 - (5*a*b^2*c - 4*a^

```
2*c^2)*d*e)*x^2 + 2*sqrt(1/2)*sqrt(e*x^2 + d)*((a^4*b^3 - 4*a^5*b*c)*x*sqrt
((a^2*b^2*e^2 + (b^4 - 2*a*b^2*c + a^2*c^2)*d^2 - 2*(a*b^3 - a^2*b*c)*d*e)/
(a^6*b^2 - 4*a^7*c)) - ((a*b^4 - 5*a^2*b^2*c + 4*a^3*c^2)*d - (a^2*b^3 - 4*
a^3*b*c)*e)*x)*sqrt(-((b^3 - 3*a*b*c)*d - (a*b^2 - 2*a^2*c)*e + (a^3*b^2 -
4*a^4*c)*sqrt((a^2*b^2*e^2 + (b^4 - 2*a*b^2*c + a^2*c^2)*d^2 - 2*(a*b^3 - a
^2*b*c)*d*e)/(a^6*b^2 - 4*a^7*c)))/(a^3*b^2 - 4*a^4*c)))/x^2) - sqrt(1/2)*a
*x*sqrt(-((b^3 - 3*a*b*c)*d - (a*b^2 - 2*a^2*c)*e + (a^3*b^2 - 4*a^4*c)*sqr
t((a^2*b^2*e^2 + (b^4 - 2*a*b^2*c + a^2*c^2)*d^2 - 2*(a*b^3 - a^2*b*c)*d*e)
/(a^6*b^2 - 4*a^7*c)))/(a^3*b^2 - 4*a^4*c))*log((2*a^2*b*c*d*e + (a^3*b^2*c
 - 4*a^4*c^2)*d*x^2*sqrt((a^2*b^2*e^2 + (b^4 - 2*a*b^2*c + a^2*c^2)*d^2 - 2
*(a*b^3 - a^2*b*c)*d*e)/(a^6*b^2 - 4*a^7*c)) - 2*(a*b^2*c - a^2*c^2)*d^2 +
(4*a^2*b*c*e^2 + (b^3*c - a*b*c^2)*d^2 - (5*a*b^2*c - 4*a^2*c^2)*d*e)*x^2 -
 2*sqrt(1/2)*sqrt(e*x^2 + d)*((a^4*b^3 - 4*a^5*b*c)*x*sqrt((a^2*b^2*e^2 + (
b^4 - 2*a*b^2*c + a^2*c^2)*d^2 - 2*(a*b^3 - a^2*b*c)*d*e)/(a^6*b^2 - 4*a^7*
c)) - ((a*b^4 - 5*a^2*b^2*c + 4*a^3*c^2)*d - (a^2*b^3 - 4*a^3*b*c)*e)*x)*sq
rt(-((b^3 - 3*a*b*c)*d - (a*b^2 - 2*a^2*c)*e + (a^3*b^2 - 4*a^4*c)*sqrt((a^
2*b^2*e^2 + (b^4 - 2*a*b^2*c + a^2*c^2)*d^2 - 2*(a*b^3 - a^2*b*c)*d*e)/(a^6
*b^2 - 4*a^7*c)))/(a^3*b^2 - 4*a^4*c)))/x^2) - sqrt(1/2)*a*x*sqrt(-((b^3 -
3*a*b*c)*d - (a*b^2 - 2*a^2*c)*e - (a^3*b^2 - 4*a^4*c)*sqrt((a^2*b^2*e^2 +
(b^4 - 2*a*b^2*c + a^2*c^2)*d^2 - 2*(a*b^3 - a^2*b*c)*d*e)/(a^6*b^2 - 4*a^7
*c)))/(a^3*b^2 - 4*a^4*c))*log((2*a^2*b*c*d*e - (a^3*b^2*c - 4*a^4*c^2)*d*x
^2*sqrt((a^2*b^2*e^2 + (b^4 - 2*a*b^2*c + a^2*c^2)*d^2 - 2*(a*b^3 - a^2*b*c
)*d*e)/(a^6*b^2 - 4*a^7*c)) - 2*(a*b^2*c - a^2*c^2)*d^2 + (4*a^2*b*c*e^2 +
(b^3*c - a*b*c^2)*d^2 - (5*a*b^2*c - 4*a^2*c^2)*d*e)*x^2 + 2*sqrt(1/2)*sqrt
(e*x^2 + d)*((a^4*b^3 - 4*a^5*b*c)*x*sqrt((a^2*b^2*e^2 + (b^4 - 2*a*b^2*c +
 a^2*c^2)*d^2 - 2*(a*b^3 - a^2*b*c)*d*e)/(a^6*b^2 - 4*a^7*c)) + ((a*b^4 - 5
*a^2*b^2*c + 4*a^3*c^2)*d - (a^2*b^3 - 4*a^3*b*c)*e)*x)*sqrt(-((b^3 - 3*a*b
*c)*d - (a*b^2 - 2*a^2*c)*e - (a^3*b^2 - 4*a^4*c)*sqrt((a^2*b^2*e^2 + (b^4
- 2*a*b^2*c + a^2*c^2)*d^2 - 2*(a*b^3 - a^2*b*c)*d*e)/(a^6*b^2 - 4*a^7*c)))
/(a^3*b^2 - 4*a^4*c)))/x^2) + sqrt(1/2)*a*x*sqrt(-((b^3 - 3*a*b*c)*d - (a*b
^2 - 2*a^2*c)*e - (a^3*b^2 - 4*a^4*c)*sqrt((a^2*b^2*e^2 + (b^4 - 2*a*b^2*c
+ a^2*c^2)*d^2 - 2*(a*b^3 - a^2*b*c)*d*e)/(a^6*b^2 - 4*a^7*c)))/(a^3*b^2 -
4*a^4*c))*log((2*a^2*b*c*d*e - (a^3*b^2*c - 4*a^4*c^2)*d*x^2*sqrt((a^2*b^2*
e^2 + (b^4 - 2*a*b^2*c + a^2*c^2)*d^2 - 2*(a*b^3 - a^2*b*c)*d*e)/(a^6*b^2 -
 4*a^7*c)) - 2*(a*b^2*c - a^2*c^2)*d^2 + (4*a^2*b*c*e^2 + (b^3*c - a*b*c^2)
*d^2 - (5*a*b^2*c - 4*a^2*c^2)*d*e)*x^2 - 2*sqrt(1/2)*sqrt(e*x^2 + d)*((a^4
*b^3 - 4*a^5*b*c)*x*sqrt((a^2*b^2*e^2 + (b^4 - 2*a*b^2*c + a^2*c^2)*d^2 - 2
*(a*b^3 - a^2*b*c)*d*e)/(a^6*b^2 - 4*a^7*c)) + ((a*b^4 - 5*a^2*b^2*c + 4*a^
3*c^2)*d - (a^2*b^3 - 4*a^3*b*c)*e)*x)*sqrt(-((b^3 - 3*a*b*c)*d - (a*b^2 -
2*a^2*c)*e - (a^3*b^2 - 4*a^4*c)*sqrt((a^2*b^2*e^2 + (b^4 - 2*a*b^2*c + a^2
*c^2)*d^2 - 2*(a*b^3 - a^2*b*c)*d*e)/(a^6*b^2 - 4*a^7*c)))/(a^3*b^2 - 4*a^4
*c)))/x^2) + 4*sqrt(e*x^2 + d))/(a*x)
```

Sympy [F]

$$\int \frac{\sqrt{d+ex^2}}{x^2(a+bx^2+cx^4)}\,dx = \int \frac{\sqrt{d+ex^2}}{x^2(a+bx^2+cx^4)}\,dx$$

```
[In] integrate((e*x**2+d)**(1/2)/x**2/(c*x**4+b*x**2+a),x)
[Out] Integral(sqrt(d + e*x**2)/(x**2*(a + b*x**2 + c*x**4)), x)
```

Maxima [F]

$$\int \frac{\sqrt{d+ex^2}}{x^2(a+bx^2+cx^4)}\,dx = \int \frac{\sqrt{ex^2+d}}{(cx^4+bx^2+a)x^2}\,dx$$

```
[In] integrate((e*x^2+d)^(1/2)/x^2/(c*x^4+b*x^2+a),x, algorithm="maxima")
[Out] integrate(sqrt(e*x^2 + d)/((c*x^4 + b*x^2 + a)*x^2), x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{\sqrt{d+ex^2}}{x^2(a+bx^2+cx^4)}\,dx = \text{Timed out}$$

```
[In] integrate((e*x^2+d)^(1/2)/x^2/(c*x^4+b*x^2+a),x, algorithm="giac")
[Out] Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{d+ex^2}}{x^2(a+bx^2+cx^4)}\,dx = \int \frac{\sqrt{e\,x^2+d}}{x^2\,\left(c\,x^4+b\,x^2+a\right)}\,dx$$

```
[In] int((d + e*x^2)^(1/2)/(x^2*(a + b*x^2 + c*x^4)),x)
[Out] int((d + e*x^2)^(1/2)/(x^2*(a + b*x^2 + c*x^4)), x)
```

3.365 $\int \frac{\sqrt{d+ex^2}}{x^4(a+bx^2+cx^4)}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 373

$$\int \frac{\sqrt{d+ex^2}}{x^4(a+bx^2+cx^4)}\,dx = -\frac{\sqrt{d+ex^2}}{3ax^3}+\frac{2e\sqrt{d+ex^2}}{3adx}+\frac{(bd-ae)\sqrt{d+ex^2}}{a^2dx} + \frac{c\left(bd-ae+\frac{b^2d-2acd-abe}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}\,x}{\sqrt{b-\sqrt{b^2-4ac}}\sqrt{d+ex^2}}\right)}{a^2\sqrt{b-\sqrt{b^2-4ac}}\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}} + \frac{c\left(bd-ae-\frac{b^2d-2acd-abe}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2cd-\left(b+\sqrt{b^2-4ac}\right)e}\,x}{\sqrt{b+\sqrt{b^2-4ac}}\sqrt{d+ex^2}}\right)}{a^2\sqrt{b+\sqrt{b^2-4ac}}\sqrt{2cd-\left(b+\sqrt{b^2-4ac}\right)e}}$$

```
[Out] -1/3*(e*x^2+d)^(1/2)/a/x^3+2/3*e*(e*x^2+d)^(1/2)/a/d/x+(-a*e+b*d)*(e*x^2+d)
^(1/2)/a^2/d/x+c*arctan(x*(2*c*d-e*(b-(-4*a*c+b^2)^(1/2)))^(1/2)/(e*x^2+d)^
(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2))*(b*d-a*e+(-a*b*e-2*a*c*d+b^2*d)/(-4*a*c
+b^2)^(1/2))/a^2/(2*c*d-e*(b-(-4*a*c+b^2)^(1/2)))^(1/2)/(b-(-4*a*c+b^2)^(1/
2))^(1/2)+c*arctan(x*(2*c*d-e*(b+(-4*a*c+b^2)^(1/2)))^(1/2)/(e*x^2+d)^(1/2)
/(b+(-4*a*c+b^2)^(1/2))^(1/2))*(b*d-a*e+(a*b*e+2*a*c*d-b^2*d)/(-4*a*c+b^2)^
(1/2))/a^2/(b+(-4*a*c+b^2)^(1/2))^(1/2)/(2*c*d-e*(b+(-4*a*c+b^2)^(1/2)))^(1
/2)
```

Rubi [A] (verified)

Time = 1.78 (sec) , antiderivative size = 373, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.241, Rules used = {1309, 277, 270, 6860, 1706, 385, 211}

$$\int \frac{\sqrt{d+ex^2}}{x^4(a+bx^2+cx^4)}\,dx = \frac{c\left(\frac{-abe-2acd+b^2d}{\sqrt{b^2-4ac}}-ae+bd\right)\arctan\left(\frac{x\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}{\sqrt{b-\sqrt{b^2-4ac}}\sqrt{d+ex^2}}\right)}{a^2\sqrt{b-\sqrt{b^2-4ac}}\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}} + \frac{c\left(-\frac{-abe-2acd+b^2d}{\sqrt{b^2-4ac}}-ae+bd\right)\arctan\left(\frac{x\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}{\sqrt{\sqrt{b^2-4ac}+b}\sqrt{d+ex^2}}\right)}{a^2\sqrt{\sqrt{b^2-4ac}+b}\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}} + \frac{\sqrt{d+ex^2}(bd-ae)}{a^2dx} + \frac{2e\sqrt{d+ex^2}}{3adx} - \frac{\sqrt{d+ex^2}}{3ax^3}$$

```
[In] Int[Sqrt[d + e*x^2]/(x^4*(a + b*x^2 + c*x^4)),x]
```

```
[Out] -1/3*Sqrt[d + e*x^2]/(a*x^3) + (2*e*Sqrt[d + e*x^2])/(3*a*d*x) + ((b*d - a*
e)*Sqrt[d + e*x^2])/(a^2*d*x) + (c*(b*d - a*e + (b^2*d - 2*a*c*d - a*b*e)/S
qrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]*x)/(Sqrt[
b - Sqrt[b^2 - 4*a*c]]*Sqrt[d + e*x^2])])/(a^2*Sqrt[b - Sqrt[b^2 - 4*a*c]]*
Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]) + (c*(b*d - a*e - (b^2*d - 2*a*c*d
 - a*b*e)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e
]*x)/(Sqrt[b + Sqrt[b^2 - 4*a*c]]*Sqrt[d + e*x^2])])/(a^2*Sqrt[b + Sqrt[b^2
 - 4*a*c]]*Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 277

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x^(m + 1)*((
a + b*x^n)^(p + 1)/(a*(m + 1))), x] - Dist[b*((m + n*(p + 1) + 1)/(a*(m + 1
))), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 1309

```
Int[(((f_.)*(x_))^(m_)*((d_.) + (e_.)*(x_)^2)^(q_))/((a_) + (b_.)*(x_)^2 +
(c_.)*(x_)^4), x_Symbol] :> Dist[d/a, Int[(f*x)^m*(d + e*x^2)^(q - 1), x],
x] - Dist[1/(a*f^2), Int[(f*x)^(m + 2)*(d + e*x^2)^(q - 1)*(Simp[b*d - a*e
+ c*d*x^2, x]/(a + b*x^2 + c*x^4)), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
&& NeQ[b^2 - 4*a*c, 0] &&  !IntegerQ[q] && GtQ[q, 0] && LtQ[m, 0]
```

Rule 1706

```
Int[(Px_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^
(p_.), x_Symbol] :> Int[ExpandIntegrand[Px*(d + e*x^2)^q*(a + b*x^2 + c*x^4
)^p, x], x] /; FreeQ[{a, b, c, d, e, q}, x] && PolyQ[Px, x^2] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[p]
```

Rule 6860

```
Int[(u_)/((a_.) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.)), x_Symbol] :> With[
{v = RationalFunctionExpand[u/(a + b*x^n + c*x^(2*n)), x]}, Int[v, x] /; Su
mQ[v]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && IGtQ[n, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{\int \frac{bd-ae+cdx^2}{x^2\sqrt{d+ex^2}(a+bx^2+cx^4)}\,dx}{a}+\frac{d\int \frac{1}{x^4\sqrt{d+ex^2}}\,dx}{a}\\
&= -\frac{\sqrt{d+ex^2}}{3ax^3}-\frac{\int \left(\frac{bd-ae}{ax^2\sqrt{d+ex^2}}+\frac{-b^2d+acd+abe-c(bd-ae)x^2}{a\sqrt{d+ex^2}(a+bx^2+cx^4)}\right)dx}{a}-\frac{(2e)\int \frac{1}{x^2\sqrt{d+ex^2}}\,dx}{3a}\\
&= -\frac{\sqrt{d+ex^2}}{3ax^3}+\frac{2e\sqrt{d+ex^2}}{3adx}-\frac{\int \frac{-b^2d+acd+abe-c(bd-ae)x^2}{\sqrt{d+ex^2}(a+bx^2+cx^4)}\,dx}{a^2}-\frac{(bd-ae)\int \frac{1}{x^2\sqrt{d+ex^2}}\,dx}{a^2}\\
&= -\frac{\sqrt{d+ex^2}}{3ax^3}+\frac{2e\sqrt{d+ex^2}}{3adx}+\frac{(bd-ae)\sqrt{d+ex^2}}{a^2dx}\\
&\quad -\frac{\int \left(\frac{-c(bd-ae)-\frac{c\left(b^2d-2acd-abe\right)}{\sqrt{b^2-4ac}}}{\left(b-\sqrt{b^2-4ac}+2cx^2\right)\sqrt{d+ex^2}}+\frac{-c(bd-ae)+\frac{c\left(b^2d-2acd-abe\right)}{\sqrt{b^2-4ac}}}{\left(b+\sqrt{b^2-4ac}+2cx^2\right)\sqrt{d+ex^2}}\right)dx}{a^2}
\end{aligned}
$$

$$= -\frac{\sqrt{d+ex^2}}{3ax^3} + \frac{2e\sqrt{d+ex^2}}{3adx} + \frac{(bd-ae)\sqrt{d+ex^2}}{a^2dx} + \frac{\left(c\left(bd-ae-\frac{b^2d-2acd-abe}{\sqrt{b^2-4ac}}\right)\right)\int \frac{1}{\left(b+\sqrt{b^2-4ac}+2cx^2\right)\sqrt{d+ex^2}}dx}{a^2} + \frac{\left(c\left(bd-ae+\frac{b^2d-2acd-abe}{\sqrt{b^2-4ac}}\right)\right)\int \frac{1}{\left(b-\sqrt{b^2-4ac}+2cx^2\right)\sqrt{d+ex^2}}dx}{a^2}$$

$$= -\frac{\sqrt{d+ex^2}}{3ax^3} + \frac{2e\sqrt{d+ex^2}}{3adx} + \frac{(bd-ae)\sqrt{d+ex^2}}{a^2dx} + \frac{\left(c\left(bd-ae-\frac{b^2d-2acd-abe}{\sqrt{b^2-4ac}}\right)\right)\text{Subst}\left(\int \frac{1}{b+\sqrt{b^2-4ac}-\left(-2cd+\left(b+\sqrt{b^2-4ac}\right)e\right)x^2}dx, x, \frac{x}{\sqrt{d+ex^2}}\right)}{a^2} + \frac{\left(c\left(bd-ae+\frac{b^2d-2acd-abe}{\sqrt{b^2-4ac}}\right)\right)\text{Subst}\left(\int \frac{1}{b-\sqrt{b^2-4ac}-\left(-2cd+\left(b-\sqrt{b^2-4ac}\right)e\right)x^2}dx, x, \frac{x}{\sqrt{d+ex^2}}\right)}{a^2}$$

$$= -\frac{\sqrt{d+ex^2}}{3ax^3} + \frac{2e\sqrt{d+ex^2}}{3adx} + \frac{(bd-ae)\sqrt{d+ex^2}}{a^2dx} + \frac{c\left(bd-ae+\frac{b^2d-2acd-abe}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}x}{\sqrt{b-\sqrt{b^2-4ac}}\sqrt{d+ex^2}}\right)}{a^2\sqrt{b-\sqrt{b^2-4ac}}\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}} + \frac{c\left(bd-ae-\frac{b^2d-2acd-abe}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2cd-\left(b+\sqrt{b^2-4ac}\right)e}x}{\sqrt{b+\sqrt{b^2-4ac}}\sqrt{d+ex^2}}\right)}{a^2\sqrt{b+\sqrt{b^2-4ac}}\sqrt{2cd-\left(b+\sqrt{b^2-4ac}\right)e}}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 7477 vs. 2(373) = 746.

Time = 16.38 (sec) , antiderivative size = 7477, normalized size of antiderivative = 20.05

$$\int \frac{\sqrt{d+ex^2}}{x^4\left(a+bx^2+cx^4\right)}dx = \text{Result too large to show}$$

```
[In] Integrate[Sqrt[d + e*x^2]/(x^4*(a + b*x^2 + c*x^4)),x]
[Out] Result too large to show
```

Maple [A] (verified)

Time = 0.91 (sec) , antiderivative size = 398, normalized size of antiderivative = 1.07

method	result
risch	$-\frac{\sqrt{e\,x^2+d}\,(ae\,x^2-3bd\,x^2+da)}{3a^2x^3d}-\frac{\sqrt{2}\left(\left(\frac{((be+cd)a-b^2d)\sqrt{-4d^2\left(ac-\frac{b^2}{4}\right)}}{2}+d\left(a^2ce-\frac{b(be+3cd)a}{2}+\frac{b^3d}{2}\right)\right)\sqrt{\left(-2ae+bd+\sqrt{-\right.}}}{}$
pseudoelliptic	$-\frac{\sqrt{\left(-2ae+bd+\sqrt{-4d^2\left(ac-\frac{b^2}{4}\right)}\right)a}\left(\frac{\left(d\left(ac-b^2\right)+abe\right)\sqrt{-4d^2\left(ac-\frac{b^2}{4}\right)}}{2}+d\left(\frac{\left(-3abc+b^3\right)d}{2}+ae\left(ac-\frac{b^2}{2}\right)\right)\right)\sqrt{2}\,d\,x^3\operatorname{arctanh}}{}$
default	$-\frac{\left(e\,x^2+d\right)^{\frac{3}{2}}}{3ad\,x^3}-\frac{b\left(-\frac{\left(e\,x^2+d\right)^{\frac{3}{2}}}{dx}+\frac{2e\left(\frac{x\sqrt{e\,x^2+d}}{2}+\frac{d\ln\left(x\sqrt{e}+\sqrt{e\,x^2+d}\right)}{2\sqrt{e}}\right)}{d}\right)}{a^2}-\frac{\sqrt{2}\sqrt{\left(-2ae+bd+\sqrt{-4d^2\left(ac-\frac{b^2}{4}\right)}\right)a}\left(\frac{(d(}{}\right.}{}$

```
[In] int((e*x^2+d)^(1/2)/x^4/(c*x^4+b*x^2+a),x,method=_RETURNVERBOSE)
```

```
[Out] -1/3*(e*x^2+d)^(1/2)*(a*e*x^2-3*b*d*x^2+a*d)/a^2/x^3/d-1/a^2/((2*a*e-b*d+(-
4*d^2*(a*c-1/4*b^2))^(1/2))*a)^(1/2)*2^(1/2)/((-2*a*e+b*d+(-4*d^2*(a*c-1/4*
b^2))^(1/2))*a)^(1/2)*((1/2*((b*e+c*d)*a-b^2*d)*(-4*d^2*(a*c-1/4*b^2))^(1/2
)+d*(a^2*c*e-1/2*b*(b*e+3*c*d)*a+1/2*b^3*d))*((-2*a*e+b*d+(-4*d^2*(a*c-1/4*
b^2))^(1/2))*a)^(1/2)*arctanh(a/x*(e*x^2+d)^(1/2)*2^(1/2)/((2*a*e-b*d+(-4*d
^2*(a*c-1/4*b^2))^(1/2))*a)^(1/2))+((2*a*e-b*d+(-4*d^2*(a*c-1/4*b^2))^(1/2)
)*a)^(1/2)*arctan(a/x*(e*x^2+d)^(1/2)*2^(1/2)/((-2*a*e+b*d+(-4*d^2*(a*c-1/4
*b^2))^(1/2))*a)^(1/2))*(1/2*((-b*e-c*d)*a+b^2*d)*(-4*d^2*(a*c-1/4*b^2))^(1
/2)+d*(a^2*c*e-1/2*b*(b*e+3*c*d)*a+1/2*b^3*d)))/(-4*d^2*(a*c-1/4*b^2))^(1/2
)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 4095 vs. 2(323) = 646.

Time = 6.51 (sec) , antiderivative size = 4095, normalized size of antiderivative = 10.98

$$\int \frac{\sqrt{d+ex^2}}{x^4\left(a+bx^2+cx^4\right)}\,dx = \text{Too large to display}$$

```
[In] integrate((e*x^2+d)^(1/2)/x^4/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

[Out] 1/12*(3*sqrt(1/2)*a^2*d*x^3*sqrt(-((b^5 - 5*a*b^3*c + 5*a^2*b*c^2)*d - (a*b^4 - 4*a^2*b^2*c + 2*a^3*c^2)*e - (a^5*b^2 - 4*a^6*c)*sqrt(((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)*d^2 - 2*(a*b^7 - 5*a^2*b^5*c + 7*a^3*b^3*c^2 - 2*a^4*b*c^3)*d*e + (a^2*b^6 - 4*a^3*b^4*c + 4*a^4*b^2*c^2)*e^2)/(a^10*b^2 - 4*a^11*c)))/(a^5*b^2 - 4*a^6*c))*log(((a^5*b^2*c^2 - 4*a^6*c^3)*d*x^2*sqrt(((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)*d^2 - 2*(a*b^7 - 5*a^2*b^5*c + 7*a^3*b^3*c^2 - 2*a^4*b*c^3)*d*e + (a^2*b^6 - 4*a^3*b^4*c + 4*a^4*b^2*c^2)*e^2)/(a^10*b^2 - 4*a^11*c)) + 2*(a*b^4*c^2 - 3*a^2*b^2*c^3 + a^3*c^4)*d^2 - 2*(a^2*b^3*c^2 - 2*a^3*b*c^3)*d*e - ((b^5*c^2 - 3*a*b^3*c^3 + a^2*b*c^4)*d^2 - (5*a*b^4*c^2 - 14*a^2*b^2*c^3 + 4*a^3*c^4)*d*e + 4*(a^2*b^3*c^2 - 2*a^3*b*c^3)*e^2)*x^2 + 2*sqrt(1/2)*sqrt(e*x^2 + d)*((a^6*b^4 - 6*a^7*b^2*c + 8*a^8*c^2)*x*sqrt(((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)*d^2 - 2*(a*b^7 - 5*a^2*b^5*c + 7*a^3*b^3*c^2 - 2*a^4*b*c^3)*d*e + (a^2*b^6 - 4*a^3*b^4*c + 4*a^4*b^2*c^2)*e^2)/(a^10*b^2 - 4*a^11*c)) + ((a*b^7 - 7*a^2*b^5*c + 13*a^3*b^3*c^2 - 4*a^4*b*c^3)*d - (a^2*b^6 - 6*a^3*b^4*c + 8*a^4*b^2*c^2)*e)*x)*sqrt(-((b^5 - 5*a*b^3*c + 5*a^2*b*c^2)*d - (a*b^4 - 4*a^2*b^2*c + 2*a^3*c^2)*e - (a^5*b^2 - 4*a^6*c)*sqrt(((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)*d^2 - 2*(a*b^7 - 5*a^2*b^5*c + 7*a^3*b^3*c^2 - 2*a^4*b*c^3)*d*e + (a^2*b^6 - 4*a^3*b^4*c + 4*a^4*b^2*c^2)*e^2)/(a^10*b^2 - 4*a^11*c)))/(a^5*b^2 - 4*a^6*c)))/x^2) - 3*sqrt(1/2)*a^2*d*x^3*sqrt(-((b^5 - 5*a*b^3*c + 5*a^2*b*c^2)*d - (a*b^4 - 4*a^2*b^2*c + 2*a^3*c^2)*e - (a^5*b^2 - 4*a^6*c)*sqrt(((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)*d^2 - 2*(a*b^7 - 5*a^2*b^5*c + 7*a^3*b^3*c^2 - 2*a^4*b*c^3)*d*e + (a^2*b^6 - 4*a^3*b^4*c + 4*a^4*b^2*c^2)*e^2)/(a^10*b^2 - 4*a^11*c)))/(a^5*b^2 - 4*a^6*c))*log(((a^5*b^2*c^2 - 4*a^6*c^3)*d*x^2*sqrt(((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)*d^2 - 2*(a*b^7 - 5*a^2*b^5*c + 7*a^3*b^3*c^2 - 2*a^4*b*c^3)*d*e + (a^2*b^6 - 4*a^3*b^4*c + 4*a^4*b^2*c^2)*e^2)/(a^10*b^2 - 4*a^11*c)) + 2*(a*b^4*c^2 - 3*a^2*b^2*c^3 + a^3*c^4)*d^2 - 2*(a^2*b^3*c^2 - 2*a^3*b*c^3)*d*e - ((b^5*c^2 - 3*a*b^3*c^3 + a^2*b*c^4)*d^2 - (5*a*b^4*c^2 - 14*a^2*b^2*c^3 + 4*a^3*c^4)*d*e + 4*(a^2*b^3*c^2 - 2*a^3*b*c^3)*e^2)*x^2 - 2*sqrt(1/2)*sqrt(e*x^2 + d)*((a^6*b^4 - 6*a^7*b^2*c + 8*a^8*c^2)*x*sqrt(((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)*d^2 - 2*(a*b^7 - 5*a^2*b^5*c + 7*a^3*b^3*c^2 - 2*a^4*b*c^3)*d*e + (a^2*b^6 - 4*a^3*b^4*c + 4*a^4*b^2*c^2)*e^2)/(a^10*b^2 - 4*a^11*c)) + ((a*b^7 - 7*a^2*b^5*c + 13*a^3*b^3*c^2 - 4*a^4*b*c^3)*d - (a^2*b^6 - 6*a^3*b^4*c + 8*a^4*b^2*c^2)*e)*x)*sqrt(-((b^5 - 5*a*b^3*c + 5*a^2*b*c^2)*d - (a*b^4 - 4*a^2*b^2*c + 2*a^3*c^2)*e - (a^5*b^2 - 4*a^6*c)*sqrt(((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)*d^2 - 2*(a*b^7 - 5*a^2*b^5*c + 7*a^3*b^3*c^2 - 2*a^4*b*c^3)*d*e + (a^2*b^6 - 4*a^3*b^4*c + 4*a^4*b^2*c^2)*e^2)/(a^10*b^2 - 4*a^11*c)))/(a^5*b^2 - 4*a^6*c)))/x^2) + 3*sqrt(1/2)*a^2*d*x^3*sqrt(-((b^5 - 5*a*b^3*c + 5*a^2*b*c^2)*d - (a*b^4 - 4*a^2*b^2*c + 2*a^3*c^2)*e + (a^5*b^2 - 4*a^6*c)*sqrt(((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)*d^2 - 2*(a*b^7 - 5*a^2*b^5*c + 7*a^3*b^3*c^2 - 2*a^4*b*c^3)*d*e + (a^2*b^6 - 4*a^3*b^4*c + 4*a^4*b^2*c^2)*e^2)/(a^10*b^2 - 4*a^11*c)))/(a^5*b^2 - 4*a^6*c))*log(-((a^

5*b^2*c^2 - 4*a^6*c^3)*d*x^2*sqrt(((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^
3*b^2*c^3 + a^4*c^4)*d^2 - 2*(a*b^7 - 5*a^2*b^5*c + 7*a^3*b^3*c^2 - 2*a^4*b
*c^3)*d*e + (a^2*b^6 - 4*a^3*b^4*c + 4*a^4*b^2*c^2)*e^2)/(a^10*b^2 - 4*a^11
c)) - 2(a*b^4*c^2 - 3*a^2*b^2*c^3 + a^3*c^4)*d^2 + 2*(a^2*b^3*c^2 - 2*a^3
*b*c^3)*d*e + ((b^5*c^2 - 3*a*b^3*c^3 + a^2*b*c^4)*d^2 - (5*a*b^4*c^2 - 14*
a^2*b^2*c^3 + 4*a^3*c^4)*d*e + 4*(a^2*b^3*c^2 - 2*a^3*b*c^3)*e^2)*x^2 + 2*s
qrt(1/2)*sqrt(e*x^2 + d)*((a^6*b^4 - 6*a^7*b^2*c + 8*a^8*c^2)*x*sqrt(((b^8
- 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)*d^2 - 2*(a*b^7 - 5*
a^2*b^5*c + 7*a^3*b^3*c^2 - 2*a^4*b*c^3)*d*e + (a^2*b^6 - 4*a^3*b^4*c + 4*a
^4*b^2*c^2)*e^2)/(a^10*b^2 - 4*a^11*c)) - ((a*b^7 - 7*a^2*b^5*c + 13*a^3*b^
3*c^2 - 4*a^4*b*c^3)*d - (a^2*b^6 - 6*a^3*b^4*c + 8*a^4*b^2*c^2)*e)*x)*sqrt
(-((b^5 - 5*a*b^3*c + 5*a^2*b*c^2)*d - (a*b^4 - 4*a^2*b^2*c + 2*a^3*c^2)*e
+ (a^5*b^2 - 4*a^6*c)*sqrt(((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c
^3 + a^4*c^4)*d^2 - 2*(a*b^7 - 5*a^2*b^5*c + 7*a^3*b^3*c^2 - 2*a^4*b*c^3)*d
*e + (a^2*b^6 - 4*a^3*b^4*c + 4*a^4*b^2*c^2)*e^2)/(a^10*b^2 - 4*a^11*c)))/(
a^5*b^2 - 4*a^6*c)))/x^2) - 3*sqrt(1/2)*a^2*d*x^3*sqrt(-((b^5 - 5*a*b^3*c +
5*a^2*b*c^2)*d - (a*b^4 - 4*a^2*b^2*c + 2*a^3*c^2)*e + (a^5*b^2 - 4*a^6*c)
*sqrt(((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)*d^2 - 2
*(a*b^7 - 5*a^2*b^5*c + 7*a^3*b^3*c^2 - 2*a^4*b*c^3)*d*e + (a^2*b^6 - 4*a^3
*b^4*c + 4*a^4*b^2*c^2)*e^2)/(a^10*b^2 - 4*a^11*c)))/(a^5*b^2 - 4*a^6*c))*l
og(-((a^5*b^2*c^2 - 4*a^6*c^3)*d*x^2*sqrt(((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^
2 - 6*a^3*b^2*c^3 + a^4*c^4)*d^2 - 2*(a*b^7 - 5*a^2*b^5*c + 7*a^3*b^3*c^2 -
2*a^4*b*c^3)*d*e + (a^2*b^6 - 4*a^3*b^4*c + 4*a^4*b^2*c^2)*e^2)/(a^10*b^2
- 4*a^11*c)) - 2*(a*b^4*c^2 - 3*a^2*b^2*c^3 + a^3*c^4)*d^2 + 2*(a^2*b^3*c^2
- 2*a^3*b*c^3)*d*e + ((b^5*c^2 - 3*a*b^3*c^3 + a^2*b*c^4)*d^2 - (5*a*b^4*c
^2 - 14*a^2*b^2*c^3 + 4*a^3*c^4)*d*e + 4*(a^2*b^3*c^2 - 2*a^3*b*c^3)*e^2)*x
^2 - 2*sqrt(1/2)*sqrt(e*x^2 + d)*((a^6*b^4 - 6*a^7*b^2*c + 8*a^8*c^2)*x*sqr
t(((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)*d^2 - 2*(a*
b^7 - 5*a^2*b^5*c + 7*a^3*b^3*c^2 - 2*a^4*b*c^3)*d*e + (a^2*b^6 - 4*a^3*b^4
*c + 4*a^4*b^2*c^2)*e^2)/(a^10*b^2 - 4*a^11*c)) - ((a*b^7 - 7*a^2*b^5*c + 1
3*a^3*b^3*c^2 - 4*a^4*b*c^3)*d - (a^2*b^6 - 6*a^3*b^4*c + 8*a^4*b^2*c^2)*e)
*x)*sqrt(-((b^5 - 5*a*b^3*c + 5*a^2*b*c^2)*d - (a*b^4 - 4*a^2*b^2*c + 2*a^3
*c^2)*e + (a^5*b^2 - 4*a^6*c)*sqrt(((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a
^3*b^2*c^3 + a^4*c^4)*d^2 - 2*(a*b^7 - 5*a^2*b^5*c + 7*a^3*b^3*c^2 - 2*a^4*
b*c^3)*d*e + (a^2*b^6 - 4*a^3*b^4*c + 4*a^4*b^2*c^2)*e^2)/(a^10*b^2 - 4*a^1
1*c)))/(a^5*b^2 - 4*a^6*c)))/x^2) + 4*((3*b*d - a*e)*x^2 - a*d)*sqrt(e*x^2
+ d))/(a^2*d*x^3)

Sympy [F]

$$\int \frac{\sqrt{d+ex^2}}{x^4(a+bx^2+cx^4)}\,dx = \int \frac{\sqrt{d+ex^2}}{x^4(a+bx^2+cx^4)}\,dx$$

```
[In] integrate((e*x**2+d)**(1/2)/x**4/(c*x**4+b*x**2+a),x)
[Out] Integral(sqrt(d + e*x**2)/(x**4*(a + b*x**2 + c*x**4)), x)
```

Maxima [F]

$$\int \frac{\sqrt{d+ex^2}}{x^4(a+bx^2+cx^4)}\,dx = \int \frac{\sqrt{ex^2+d}}{(cx^4+bx^2+a)x^4}\,dx$$

```
[In] integrate((e*x^2+d)^(1/2)/x^4/(c*x^4+b*x^2+a),x, algorithm="maxima")
[Out] integrate(sqrt(e*x^2 + d)/((c*x^4 + b*x^2 + a)*x^4), x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{\sqrt{d+ex^2}}{x^4(a+bx^2+cx^4)}\,dx = \text{Timed out}$$

```
[In] integrate((e*x^2+d)^(1/2)/x^4/(c*x^4+b*x^2+a),x, algorithm="giac")
[Out] Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{d+ex^2}}{x^4(a+bx^2+cx^4)}\,dx = \int \frac{\sqrt{e\,x^2+d}}{x^4\,(c\,x^4+b\,x^2+a)}\,dx$$

```
[In] int((d + e*x^2)^(1/2)/(x^4*(a + b*x^2 + c*x^4)),x)
[Out] int((d + e*x^2)^(1/2)/(x^4*(a + b*x^2 + c*x^4)), x)
```

3.366 $\int \frac{\sqrt{d+ex^2}}{x^6(a+bx^2+cx^4)}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 512

$$\int \frac{\sqrt{d+ex^2}}{x^6(a+bx^2+cx^4)}\,dx = -\frac{\sqrt{d+ex^2}}{5ax^5}+\frac{4e\sqrt{d+ex^2}}{15adx^3}+\frac{(bd-ae)\sqrt{d+ex^2}}{3a^2dx^3}-\frac{8e^2\sqrt{d+ex^2}}{15ad^2x} -\frac{2e(bd-ae)\sqrt{d+ex^2}}{3a^2d^2x}-\frac{(b^2d-acd-abe)\sqrt{d+ex^2}}{a^3dx} -\frac{c\left(b^2d-acd-abe+\frac{b^3d-3abcd-ab^2e+2a^2ce}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}x}{\sqrt{b-\sqrt{b^2-4ac}}\sqrt{d+ex^2}}\right)}{a^3\sqrt{b-\sqrt{b^2-4ac}}\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}} -\frac{c\left(b^2d-acd-abe-\frac{b^3d-3abcd-ab^2e+2a^2ce}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2cd-\left(b+\sqrt{b^2-4ac}\right)e}x}{\sqrt{b+\sqrt{b^2-4ac}}\sqrt{d+ex^2}}\right)}{a^3\sqrt{b+\sqrt{b^2-4ac}}\sqrt{2cd-\left(b+\sqrt{b^2-4ac}\right)e}}$$

```
[Out] -1/5*(e*x^2+d)^(1/2)/a/x^5+4/15*e*(e*x^2+d)^(1/2)/a/d/x^3+1/3*(-a*e+b*d)*(e
*x^2+d)^(1/2)/a^2/d/x^3-8/15*e^2*(e*x^2+d)^(1/2)/a/d^2/x-2/3*e*(-a*e+b*d)*(
e*x^2+d)^(1/2)/a^2/d^2/x-(-a*b*e-a*c*d+b^2*d)*(e*x^2+d)^(1/2)/a^3/d/x-c*arc
tan(x*(2*c*d-e*(b-(-4*a*c+b^2)^(1/2)))^(1/2)/(e*x^2+d)^(1/2)/(b-(-4*a*c+b^2
)^(1/2))^(1/2))*(b^2*d-a*c*d-a*b*e+(2*a^2*c*e-a*b^2*e-3*a*b*c*d+b^3*d)/(-4*
a*c+b^2)^(1/2))/a^3/(2*c*d-e*(b-(-4*a*c+b^2)^(1/2)))^(1/2)/(b-(-4*a*c+b^2)^
(1/2))^(1/2)-c*arctan(x*(2*c*d-e*(b+(-4*a*c+b^2)^(1/2)))^(1/2)/(e*x^2+d)^(1
/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2))*(b^2*d-a*c*d-a*b*e+(-2*a^2*c*e+a*b^2*e+3*
a*b*c*d-b^3*d)/(-4*a*c+b^2)^(1/2))/a^3/(b+(-4*a*c+b^2)^(1/2))^(1/2)/(2*c*d-
e*(b+(-4*a*c+b^2)^(1/2)))^(1/2)
```

Rubi [A] (verified)

Time = 3.50 (sec) , antiderivative size = 512, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.241, Rules used = {1309, 277, 270, 6860, 1706, 385, 211}

$$\begin{aligned}&\int \frac{\sqrt{d+ex^2}}{x^6(a+bx^2+cx^4)}dx\\ &=-\frac{\sqrt{d+ex^2}(-abe-acd+b^2d)}{a^3dx}-\frac{2e\sqrt{d+ex^2}(bd-ae)}{3a^2d^2x}+\frac{\sqrt{d+ex^2}(bd-ae)}{3a^2dx^3}\\ &-\frac{c\left(\frac{2a^2ce-ab^2e-3abcd+b^3d}{\sqrt{b^2-4ac}}-abe-acd+b^2d\right)\arctan\left(\frac{x\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}{\sqrt{b-\sqrt{b^2-4ac}}\sqrt{d+ex^2}}\right)}{a^3\sqrt{b-\sqrt{b^2-4ac}}\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}\\ &-\frac{c\left(-\frac{2a^2ce-ab^2e-3abcd+b^3d}{\sqrt{b^2-4ac}}-abe-acd+b^2d\right)\arctan\left(\frac{x\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}{\sqrt{\sqrt{b^2-4ac}+b}\sqrt{d+ex^2}}\right)}{a^3\sqrt{\sqrt{b^2-4ac}+b}\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}\\ &-\frac{8e^2\sqrt{d+ex^2}}{15ad^2x}-\frac{\sqrt{d+ex^2}}{5ax^5}+\frac{4e\sqrt{d+ex^2}}{15adx^3}\end{aligned}$$

```
[In] Int[Sqrt[d + e*x^2]/(x^6*(a + b*x^2 + c*x^4)),x]
```

```
[Out] -1/5*Sqrt[d + e*x^2]/(a*x^5) + (4*e*Sqrt[d + e*x^2])/(15*a*d*x^3) + ((b*d -
 a*e)*Sqrt[d + e*x^2])/(3*a^2*d*x^3) - (8*e^2*Sqrt[d + e*x^2])/(15*a*d^2*x)
 - (2*e*(b*d - a*e)*Sqrt[d + e*x^2])/(3*a^2*d^2*x) - ((b^2*d - a*c*d - a*b*
e)*Sqrt[d + e*x^2])/(a^3*d*x) - (c*(b^2*d - a*c*d - a*b*e + (b^3*d - 3*a*b*
c*d - a*b^2*e + 2*a^2*c*e)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2*c*d - (b - Sqr
t[b^2 - 4*a*c])*e]*x)/(Sqrt[b - Sqrt[b^2 - 4*a*c]]*Sqrt[d + e*x^2])])/(a^3*
Sqrt[b - Sqrt[b^2 - 4*a*c]]*Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]) - (c*(
b^2*d - a*c*d - a*b*e - (b^3*d - 3*a*b*c*d - a*b^2*e + 2*a^2*c*e)/Sqrt[b^2
- 4*a*c])*ArcTan[(Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]*x)/(Sqrt[b + Sqrt
[b^2 - 4*a*c]]*Sqrt[d + e*x^2])])/(a^3*Sqrt[b + Sqrt[b^2 - 4*a*c]]*Sqrt[2*c
*d - (b + Sqrt[b^2 - 4*a*c])*e])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
```

```
p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 277

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x^(m + 1)*((
a + b*x^n)^(p + 1)/(a*(m + 1))), x] - Dist[b*((m + n*(p + 1) + 1)/(a*(m + 1
))), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 1309

```
Int[(((f_.)*(x_))^(m_)*((d_.) + (e_.)*(x_)^2)^(q_))/((a_) + (b_.)*(x_)^2 +
(c_.)*(x_)^4), x_Symbol] :> Dist[d/a, Int[(f*x)^m*(d + e*x^2)^(q - 1), x],
x] - Dist[1/(a*f^2), Int[(f*x)^(m + 2)*(d + e*x^2)^(q - 1)*(Simp[b*d - a*e
+ c*d*x^2, x]/(a + b*x^2 + c*x^4)), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
&& NeQ[b^2 - 4*a*c, 0] &&  !IntegerQ[q] && GtQ[q, 0] && LtQ[m, 0]
```

Rule 1706

```
Int[(Px_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^
(p_.), x_Symbol] :> Int[ExpandIntegrand[Px*(d + e*x^2)^q*(a + b*x^2 + c*x^4
)^p, x], x] /; FreeQ[{a, b, c, d, e, q}, x] && PolyQ[Px, x^2] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[p]
```

Rule 6860

```
Int[(u_)/((a_.) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.)), x_Symbol] :> With[
{v = RationalFunctionExpand[u/(a + b*x^n + c*x^(2*n)), x]}, Int[v, x] /; Su
mQ[v]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{\int \frac{bd-ae+cdx^2}{x^4\sqrt{d+ex^2}(a+bx^2+cx^4)}\,dx}{a}+\frac{d\int \frac{1}{x^6\sqrt{d+ex^2}}\,dx}{a}\\
&= -\frac{\sqrt{d+ex^2}}{5ax^5}-\frac{\int \left(\frac{bd-ae}{ax^4\sqrt{d+ex^2}}+\frac{-b^2d+acd+abe}{a^2x^2\sqrt{d+ex^2}}+\frac{b^3d-2abcd-ab^2e+a^2ce+c(b^2d-acd-abe)x^2}{a^2\sqrt{d+ex^2}(a+bx^2+cx^4)}\right)dx}{a}\\
&\quad -\frac{(4e)\int \frac{1}{x^4\sqrt{d+ex^2}}\,dx}{5a}
\end{aligned}$$

$$= -\frac{\sqrt{d+ex^2}}{5ax^5} + \frac{4e\sqrt{d+ex^2}}{15adx^3} - \frac{\int \frac{b^3d-2abcd-ab^2e+a^2ce+c(b^2d-acd-abe)x^2}{\sqrt{d+ex^2}(a+bx^2+cx^4)}\,dx}{a^3} + \frac{(8e^2)\int \frac{1}{x^2\sqrt{d+ex^2}}\,dx}{15ad} - \frac{(bd-ae)\int \frac{1}{x^4\sqrt{d+ex^2}}\,dx}{a^2} + \frac{(b^2d-acd-abe)\int \frac{1}{x^2\sqrt{d+ex^2}}\,dx}{a^3}$$

$$= -\frac{\sqrt{d+ex^2}}{5ax^5} + \frac{4e\sqrt{d+ex^2}}{15adx^3} + \frac{(bd-ae)\sqrt{d+ex^2}}{3a^2dx^3} - \frac{8e^2\sqrt{d+ex^2}}{15ad^2x} - \frac{(b^2d-acd-abe)\sqrt{d+ex^2}}{a^3dx} - \frac{\int \left(\frac{c(b^2d-acd-abe)+\frac{c(b^3d-3abcd-ab^2e+2a^2ce)}{\sqrt{b^2-4ac}}}{\left(b-\sqrt{b^2-4ac}+2cx^2\right)\sqrt{d+ex^2}} + \frac{c(b^2d-acd-abe)-\frac{c(b^3d-3abcd-ab^2e+2a^2ce)}{\sqrt{b^2-4ac}}}{\left(b+\sqrt{b^2-4ac}+2cx^2\right)\sqrt{d+ex^2}}\right)dx}{a^3} + \frac{(2e(bd-ae))\int \frac{1}{x^2\sqrt{d+ex^2}}\,dx}{3a^2d}$$

$$= -\frac{\sqrt{d+ex^2}}{5ax^5} + \frac{4e\sqrt{d+ex^2}}{15adx^3} + \frac{(bd-ae)\sqrt{d+ex^2}}{3a^2dx^3} - \frac{8e^2\sqrt{d+ex^2}}{15ad^2x} - \frac{2e(bd-ae)\sqrt{d+ex^2}}{3a^2d^2x} - \frac{(b^2d-acd-abe)\sqrt{d+ex^2}}{a^3dx} - \frac{\left(c\left(b^2d-acd-abe-\frac{b^3d-3abcd-ab^2e+2a^2ce}{\sqrt{b^2-4ac}}\right)\right)\int \frac{1}{\left(b+\sqrt{b^2-4ac}+2cx^2\right)\sqrt{d+ex^2}}\,dx}{a^3} - \frac{\left(c\left(b^2d-acd-abe+\frac{b^3d-3abcd-ab^2e+2a^2ce}{\sqrt{b^2-4ac}}\right)\right)\int \frac{1}{\left(b-\sqrt{b^2-4ac}+2cx^2\right)\sqrt{d+ex^2}}\,dx}{a^3}$$

$$= -\frac{\sqrt{d+ex^2}}{5ax^5} + \frac{4e\sqrt{d+ex^2}}{15adx^3} + \frac{(bd-ae)\sqrt{d+ex^2}}{3a^2dx^3} - \frac{8e^2\sqrt{d+ex^2}}{15ad^2x} - \frac{2e(bd-ae)\sqrt{d+ex^2}}{3a^2d^2x} - \frac{(b^2d-acd-abe)\sqrt{d+ex^2}}{a^3dx} - \frac{\left(c\left(b^2d-acd-abe-\frac{b^3d-3abcd-ab^2e+2a^2ce}{\sqrt{b^2-4ac}}\right)\right)\text{Subst}\left(\int \frac{1}{b+\sqrt{b^2-4ac}-\left(-2cd+\left(b+\sqrt{b^2-4ac}\right)e\right)x^2}\,dx, x, \frac{x}{\sqrt{d-}}\right.}{a^3} - \frac{\left(c\left(b^2d-acd-abe+\frac{b^3d-3abcd-ab^2e+2a^2ce}{\sqrt{b^2-4ac}}\right)\right)\text{Subst}\left(\int \frac{1}{b-\sqrt{b^2-4ac}-\left(-2cd+\left(b-\sqrt{b^2-4ac}\right)e\right)x^2}\,dx, x, \frac{x}{\sqrt{d-}}\right.}{a^3}$$

$$= -\frac{\sqrt{d+ex^2}}{5ax^5} + \frac{4e\sqrt{d+ex^2}}{15adx^3} + \frac{(bd-ae)\sqrt{d+ex^2}}{3a^2dx^3} - \frac{8e^2\sqrt{d+ex^2}}{15ad^2x} - \frac{2e(bd-ae)\sqrt{d+ex^2}}{3a^2d^2x} - \frac{(b^2d-acd-abe)\sqrt{d+ex^2}}{a^3dx} - \frac{c\left(b^2d-acd-abe+\frac{b^3d-3abcd-ab^2e+2a^2ce}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}x}{\sqrt{b-\sqrt{b^2-4ac}}\sqrt{d+ex^2}}\right)}{a^3\sqrt{b-\sqrt{b^2-4ac}}\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}} - \frac{c\left(b^2d-acd-abe-\frac{b^3d-3abcd-ab^2e+2a^2ce}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2cd-\left(b+\sqrt{b^2-4ac}\right)e}x}{\sqrt{b+\sqrt{b^2-4ac}}\sqrt{d+ex^2}}\right)}{a^3\sqrt{b+\sqrt{b^2-4ac}}\sqrt{2cd-\left(b+\sqrt{b^2-4ac}\right)e}}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 10511 vs. $2(512) = 1024$.

Time = 16.52 (sec) , antiderivative size = 10511, normalized size of antiderivative = 20.53

$$\int \frac{\sqrt{d+ex^2}}{x^6(a+bx^2+cx^4)}\,dx = \text{Result too large to show}$$

```
[In] Integrate[Sqrt[d + e*x^2]/(x^6*(a + b*x^2 + c*x^4)),x]
[Out] Result too large to show
```

Maple [A] (verified)

Time = 1.21 (sec) , antiderivative size = 500, normalized size of antiderivative = 0.98

method	result
risch	$-\frac{\sqrt{e\,x^2+d}\,(-2a^2e^2x^4-5abde\,x^4-15ac\,d^2x^4+15b^2d^2x^4+a^2de\,x^2-5ab\,d^2x^2+3a^2d^2)}{15d^2a^3x^5}+\frac{\sqrt{2}\left(-\left((a^2ce+(-b^2e-2bcd)a+\cdots\right.\right.}{\cdots}$ $\frac{5\sqrt{\left(-2ae+bd+\sqrt{-4d^2\left(ac-\frac{b^2}{4}\right)}\right)a}\left(\left(\left((-2abc+b^3)d+ae(ac-b^2)\right)\sqrt{-4d^2\left(ac-\frac{b^2}{4}\right)}+(-2a^2c^2+4a\,b^2c-b^4)d^2+e(-3cb\,a^2+a\,b^3)d\right)\cdots\right.}{2}$
pseudoelliptic	$-\frac{\cdots}{\cdots}$
default	$\frac{-\frac{(e\,x^2+d)^{\frac{3}{2}}}{5d\,x^5}+\frac{2e(e\,x^2+d)^{\frac{3}{2}}}{15d^2x^3}}{a}+\frac{b(e\,x^2+d)^{\frac{3}{2}}}{3a^2d\,x^3}+\frac{(-ac+b^2)\left(-\frac{(e\,x^2+d)^{\frac{3}{2}}}{dx}+\frac{2e\left(\frac{x\sqrt{e\,x^2+d}}{2}+\frac{d\ln\left(x\sqrt{e}+\sqrt{e\,x^2+d}\right)}{2\sqrt{e}}\right)}{d}\right)}{a^3}-\frac{\left(\left(\cdots\right.\right.}{\cdots}$

```
[In] int((e*x^2+d)^(1/2)/x^6/(c*x^4+b*x^2+a),x,method=_RETURNVERBOSE)
```

```
[Out] -1/15*(e*x^2+d)^(1/2)*(-2*a^2*e^2*x^4-5*a*b*d*e*x^4-15*a*c*d^2*x^4+15*b^2*d
^2*x^4+a^2*d*e*x^2-5*a*b*d^2*x^2+3*a^2*d^2)/d^2/a^3/x^5+1/2/a^3/((2*a*e-b*d
+(-4*d^2*(a*c-1/4*b^2))^(1/2))*a)^(1/2)*2^(1/2)/((-2*a*e+b*d+(-4*d^2*(a*c-1
/4*b^2))^(1/2))*a)^(1/2)*(-((a^2*c*e+(-b^2*e-2*b*c*d)*a+b^3*d)*(-4*d^2*(a*c
-1/4*b^2))^(1/2)+(-3*b*c*d*e-2*c^2*d^2)*a^2+b^2*d*(b*e+4*c*d)*a-b^4*d^2)*((
-2*a*e+b*d+(-4*d^2*(a*c-1/4*b^2))^(1/2))*a)^(1/2)*arctanh(a/x*(e*x^2+d)^(1/
2)*2^(1/2)/((2*a*e-b*d+(-4*d^2*(a*c-1/4*b^2))^(1/2))*a)^(1/2))+((2*a*e-b*d+
(-4*d^2*(a*c-1/4*b^2))^(1/2))*a)^(1/2)*arctan(a/x*(e*x^2+d)^(1/2)*2^(1/2)/(
(-2*a*e+b*d+(-4*d^2*(a*c-1/4*b^2))^(1/2))*a)^(1/2))*((a^2*c*e+(-b^2*e-2*b*c
*d)*a+b^3*d)*(-4*d^2*(a*c-1/4*b^2))^(1/2)+(3*b*c*d*e+2*c^2*d^2)*a^2+(-b^3*d
*e-4*b^2*c*d^2)*a+b^4*d^2))/(-4*d^2*(a*c-1/4*b^2))^(1/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 5773 vs. $2(450) = 900$.

Time = 15.78 (sec) , antiderivative size = 5773, normalized size of antiderivative = 11.28

$$\int \frac{\sqrt{d+ex^2}}{x^6(a+bx^2+cx^4)}\,dx = \text{Too large to display}$$

```
[In] integrate((e*x^2+d)^(1/2)/x^6/(c*x^4+b*x^2+a),x, algorithm="fricas")
[Out] Too large to include
```

Sympy [F]

$$\int \frac{\sqrt{d+ex^2}}{x^6(a+bx^2+cx^4)}\,dx = \int \frac{\sqrt{d+ex^2}}{x^6(a+bx^2+cx^4)}\,dx$$

```
[In] integrate((e*x**2+d)**(1/2)/x**6/(c*x**4+b*x**2+a),x)
[Out] Integral(sqrt(d + e*x**2)/(x**6*(a + b*x**2 + c*x**4)), x)
```

Maxima [F]

$$\int \frac{\sqrt{d+ex^2}}{x^6(a+bx^2+cx^4)}\,dx = \int \frac{\sqrt{ex^2+d}}{(cx^4+bx^2+a)x^6}\,dx$$

```
[In] integrate((e*x^2+d)^(1/2)/x^6/(c*x^4+b*x^2+a),x, algorithm="maxima")
[Out] integrate(sqrt(e*x^2 + d)/((c*x^4 + b*x^2 + a)*x^6), x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{\sqrt{d+ex^2}}{x^6(a+bx^2+cx^4)}\,dx = \text{Timed out}$$

```
[In] integrate((e*x^2+d)^(1/2)/x^6/(c*x^4+b*x^2+a),x, algorithm="giac")
[Out] Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{d+ex^2}}{x^6\,(a+bx^2+cx^4)}\,dx = \int \frac{\sqrt{e\,x^2+d}}{x^6\,\left(c\,x^4+b\,x^2+a\right)}\,dx$$

```
[In] int((d + e*x^2)^(1/2)/(x^6*(a + b*x^2 + c*x^4)),x)
[Out] int((d + e*x^2)^(1/2)/(x^6*(a + b*x^2 + c*x^4)), x)
```

3.367 $\int \frac{x^3\left(d+ex^2\right)^{3/2}}{a+bx^2+cx^4}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 460

$$\int \frac{x^3(d+ex^2)^{3/2}}{a+bx^2+cx^4}\,dx = \frac{(cd-be)\sqrt{d+ex^2}}{c^2} + \frac{(d+ex^2)^{3/2}}{3c}$$

$$+ \frac{\left(b^3e^2 - b^2e\left(2cd+\sqrt{b^2-4ac}e\right) + c\left(a\sqrt{b^2-4ac}e^2 - cd\left(\sqrt{b^2-4ac}d-4ae\right)\right) + bc\left(cd^2+e\left(2\sqrt{b^2-4ac}d-3\right.\right.\right.}{\sqrt{2}c^{5/2}\sqrt{b^2-4ac}\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}}$$

$$- \frac{\left(b^3e^2 - b^2e\left(2cd-\sqrt{b^2-4ac}e\right) + bc\left(cd^2-e\left(2\sqrt{b^2-4ac}d+3ae\right)\right) - c\left(a\sqrt{b^2-4ac}e^2 - cd\left(\sqrt{b^2-4ac}d+\right.\right.\right.}{\sqrt{2}c^{5/2}\sqrt{b^2-4ac}\sqrt{2cd-\left(b+\sqrt{b^2-4ac}\right)e}}$$

```
[Out] 1/3*(e*x^2+d)^(3/2)/c+(-b*e+c*d)*(e*x^2+d)^(1/2)/c^2+1/2*arctanh(2^(1/2)*c^
(1/2)*(e*x^2+d)^(1/2)/(2*c*d-e*(b-(-4*a*c+b^2)^(1/2)))^(1/2))*(b^3*e^2-b^2*
e*(2*c*d+e*(-4*a*c+b^2)^(1/2))+c*(a*e^2*(-4*a*c+b^2)^(1/2)-c*d*(-4*a*e+d*(-
4*a*c+b^2)^(1/2)))+b*c*(c*d^2+e*(-3*a*e+2*d*(-4*a*c+b^2)^(1/2))))/c^(5/2)*2
^(1/2)/(-4*a*c+b^2)^(1/2)/(2*c*d-e*(b-(-4*a*c+b^2)^(1/2)))^(1/2)-1/2*arctan
h(2^(1/2)*c^(1/2)*(e*x^2+d)^(1/2)/(2*c*d-e*(b+(-4*a*c+b^2)^(1/2)))^(1/2))*(
b^3*e^2-b^2*e*(2*c*d-e*(-4*a*c+b^2)^(1/2))-c*(a*e^2*(-4*a*c+b^2)^(1/2)-c*d*
(4*a*e+d*(-4*a*c+b^2)^(1/2)))+b*c*(c*d^2-e*(3*a*e+2*d*(-4*a*c+b^2)^(1/2))))
/c^(5/2)*2^(1/2)/(-4*a*c+b^2)^(1/2)/(2*c*d-e*(b+(-4*a*c+b^2)^(1/2)))^(1/2)
```

Rubi [A] (verified)

Time = 3.11 (sec) , antiderivative size = 460, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.172, Rules used = {1265, 838, 840, 1180, 214}

$$\int \frac{x^3(d+ex^2)^{3/2}}{a+bx^2+cx^4}\,dx = \frac{\left(bc\left(e\left(2d\sqrt{b^2-4ac}-3ae\right)+cd^2\right)+c\left(ae^2\sqrt{b^2-4ac}-cd\left(d\sqrt{b^2-4ac}-4ae\right)\right)-b^2\right.}{\sqrt{2}c^{5/2}\sqrt{b^2-4ac}\sqrt{2cd-e\left(b-\sqrt{\right.}}}$$

$$-\frac{\left(bc\left(cd^2-e\left(2d\sqrt{b^2-4ac}+3ae\right)\right)-c\left(ae^2\sqrt{b^2-4ac}-cd\left(d\sqrt{b^2-4ac}+4ae\right)\right)-b^2e\left(2cd-e\sqrt{b^2-4ac}\right)\right.}{\sqrt{2}c^{5/2}\sqrt{b^2-4ac}\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}$$

$$+\frac{\sqrt{d+ex^2}(cd-be)}{c^2}+\frac{(d+ex^2)^{3/2}}{3c}$$

```
[In] Int[(x^3*(d + e*x^2)^(3/2))/(a + b*x^2 + c*x^4),x]
```

```
[Out] ((c*d - b*e)*Sqrt[d + e*x^2])/c^2 + (d + e*x^2)^(3/2)/(3*c) + ((b^3*e^2 - b
^2*e*(2*c*d + Sqrt[b^2 - 4*a*c]*e) + c*(a*Sqrt[b^2 - 4*a*c]*e^2 - c*d*(Sqrt
[b^2 - 4*a*c]*d - 4*a*e)) + b*c*(c*d^2 + e*(2*Sqrt[b^2 - 4*a*c]*d - 3*a*e))
)*ArcTanh[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x^2])/Sqrt[2*c*d - (b - Sqrt[b^2 - 4*
a*c])*e]])/(Sqrt[2]*c^(5/2)*Sqrt[b^2 - 4*a*c]*Sqrt[2*c*d - (b - Sqrt[b^2 -
4*a*c])*e]) - ((b^3*e^2 - b^2*e*(2*c*d - Sqrt[b^2 - 4*a*c]*e) + b*c*(c*d^2
- e*(2*Sqrt[b^2 - 4*a*c]*d + 3*a*e)) - c*(a*Sqrt[b^2 - 4*a*c]*e^2 - c*d*(Sq
rt[b^2 - 4*a*c]*d + 4*a*e)))*ArcTanh[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x^2])/Sqrt
[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]])/(Sqrt[2]*c^(5/2)*Sqrt[b^2 - 4*a*c]*Sq
rt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 838

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Simp[g*((d + e*x)^m/(c*m)), x] + Dist[1/c, Int[
(d + e*x)^(m - 1)*(Simp[c*d*f - a*e*g + (g*c*d - b*e*g + c*e*f)*x, x]/(a +
b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*c
, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && FractionQ[m] && GtQ[m, 0]
```

Rule 840

```
Int[((f_.) + (g_.)*(x_))/(Sqrt[(d_.) + (e_.)*(x_)]*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)), x_Symbol] :> Dist[2, Subst[Int[(e*f - d*g + g*x^2)/(c*d^2 - b
*d*e + a*e^2 - (2*c*d - b*e)*x^2 + c*x^4), x], x, Sqrt[d + e*x]], x] /; Fre
eQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e +
a*e^2, 0]
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1265

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rubi steps

$$\text{integral} = \frac{1}{2}\text{Subst}\left(\int \frac{x(d+ex)^{3/2}}{a+bx+cx^2}\,dx, x, x^2\right)$$

$$= \frac{(d+ex^2)^{3/2}}{3c} + \frac{\text{Subst}\left(\int \frac{\sqrt{d+ex}(-ae+(cd-be)x)}{a+bx+cx^2}\,dx, x, x^2\right)}{2c}$$

$$= \frac{(cd-be)\sqrt{d+ex^2}}{c^2} + \frac{(d+ex^2)^{3/2}}{3c} + \frac{\text{Subst}\left(\int \frac{-ae(2cd-be)+(c^2d^2+b^2e^2-ce(2bd+ae))x}{\sqrt{d+ex}(a+bx+cx^2)}\,dx, x, x^2\right)}{2c^2}$$

$$= \frac{(cd-be)\sqrt{d+ex^2}}{c^2} + \frac{(d+ex^2)^{3/2}}{3c} + \frac{\text{Subst}\left(\int \frac{-ae^2(2cd-be)-d(c^2d^2+b^2e^2-ce(2bd+ae))+(c^2d^2+b^2e^2-ce(2bd+ae))x^2}{cd^2-bde+ae^2+(-2cd+be)x^2+cx^4}\,dx, x, \sqrt{d+ex^2}\right)}{c^2}$$

$$= \frac{(cd-be)\sqrt{d+ex^2}}{c^2} + \frac{(d+ex^2)^{3/2}}{3c} - \frac{\left(b^3e^2-b^2e\left(2cd+\sqrt{b^2-4ac}e\right)+c\left(a\sqrt{b^2-4ac}e^2-cd\left(\sqrt{b^2-4ac}d-4ae\right)\right)+bc\left(cd^2+e\left(2\sqrt{b^2\right.\right.\right.}{2c^2\sqrt{b^2-4ac}} + \frac{\left(b^3e^2-b^2e\left(2cd-\sqrt{b^2-4ac}e\right)+bc\left(cd^2-e\left(2\sqrt{b^2-4ac}d+3ae\right)\right)-c\left(a\sqrt{b^2-4ac}e^2-cd\left(\sqrt{b^2\right.\right.\right.}{2c^2\sqrt{b^2-4ac}}$$

$$= \frac{(cd-be)\sqrt{d+ex^2}}{c^2} + \frac{(d+ex^2)^{3/2}}{3c}$$

$$+ \frac{\left(b^3e^2 - b^2e\left(2cd+\sqrt{b^2-4ac}e\right) + c\left(a\sqrt{b^2-4ac}e^2 - cd\left(\sqrt{b^2-4ac}d - 4ae\right)\right) + bc\left(cd^2 + e\left(2\sqrt{b\right.\right.\right.}{\sqrt{2}c^{5/2}\sqrt{b^2-4ac}\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}}$$

$$- \frac{\left(b^3e^2 - b^2e\left(2cd-\sqrt{b^2-4ac}e\right) + bc\left(cd^2 - e\left(2\sqrt{b^2-4ac}d + 3ae\right)\right) - c\left(a\sqrt{b^2-4ac}e^2 - cd\left(\sqrt{b\right.\right.\right.}{\sqrt{2}c^{5/2}\sqrt{b^2-4ac}\sqrt{2cd-\left(b+\sqrt{b^2-4ac}\right)e}}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 1.76 (sec) , antiderivative size = 501, normalized size of antiderivative = 1.09

$$\int \frac{x^3(d+ex^2)^{3/2}}{a+bx^2+cx^4}\,dx = \frac{2\sqrt{c}\sqrt{d+ex^2}(4cd-3be+cex^2) + \frac{3\left(ib^3e^2+b^2e\left(-2icd+\sqrt{-b^2+4ac}e\right)+ibc\left(cd^2+e\left(2i\sqrt{-b^2+4ac}d-3\right.\right.\right.}{\sqrt{-\frac{b^2}{2}+\right.}}}{}$$

```
[In] Integrate[(x^3*(d + e*x^2)^(3/2))/(a + b*x^2 + c*x^4),x]
```

```
[Out] (2*Sqrt[c]*Sqrt[d + e*x^2]*(4*c*d - 3*b*e + c*e*x^2) + (3*(I*b^3*e^2 + b^2*
e*((-2*I)*c*d + Sqrt[-b^2 + 4*a*c]*e) + I*b*c*(c*d^2 + e*((2*I)*Sqrt[-b^2 +
 4*a*c]*d - 3*a*e)) + c*(-(a*Sqrt[-b^2 + 4*a*c]*e^2) + c*d*(Sqrt[-b^2 + 4*a
*c]*d + (4*I)*a*e)))*ArcTan[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x^2])/Sqrt[-2*c*d +
 b*e - I*Sqrt[-b^2 + 4*a*c]*e]])/(Sqrt[-1/2*b^2 + 2*a*c]*Sqrt[-2*c*d + (b -
 I*Sqrt[-b^2 + 4*a*c])*e]) + (3*((-I)*b^3*e^2 + b^2*e*((2*I)*c*d + Sqrt[-b^
2 + 4*a*c]*e) + b*c*((-I)*c*d^2 + e*(-2*Sqrt[-b^2 + 4*a*c]*d + (3*I)*a*e))
+ c*(-(a*Sqrt[-b^2 + 4*a*c]*e^2) + c*d*(Sqrt[-b^2 + 4*a*c]*d - (4*I)*a*e)))
*ArcTan[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x^2])/Sqrt[-2*c*d + b*e + I*Sqrt[-b^2 +
 4*a*c]*e]])/(Sqrt[-1/2*b^2 + 2*a*c]*Sqrt[-2*c*d + (b + I*Sqrt[-b^2 + 4*a*c
])*e]))/(6*c^(5/2))
```

Maple [A] (verified)

Time = 0.85 (sec) , antiderivative size = 447, normalized size of antiderivative = 0.97

method	result
risch	$-\frac{(-c x^2 e+3be-4cd)\sqrt{e x^2+d}}{3c^2}-\frac{\sqrt{2}\left(-\left((-c^2d^2+(a e^2+2bde)c-b^2e^2)\sqrt{-4e^2\left(ac-\frac{b^2}{4}\right)}+(4d e^2a+b d^2e)c^2+(-3e^3ab-2b\right.\right.}{}$
default	$-\frac{-\left(((ac-b^2)e^2+2bcde-c^2d^2)\sqrt{-4e^2\left(ac-\frac{b^2}{4}\right)}+(-3abc+b^3)e^3+2d(2a c^2-b^2c)e^2+b c^2d^2e\right)\sqrt{2}\sqrt{\left(be-2cd+\sqrt{-4e^2\left(ac-\right.}\right.}}{}$
pseudoelliptic	$-\frac{-\left(((ac-b^2)e^2+2bcde-c^2d^2)\sqrt{-4e^2\left(ac-\frac{b^2}{4}\right)}+(-3abc+b^3)e^3+2d(2a c^2-b^2c)e^2+b c^2d^2e\right)\sqrt{2}\sqrt{\left(be-2cd+\sqrt{-4e^2\left(ac-\right.}\right.}}{}$

```
[In] int(x^3*(e*x^2+d)^(3/2)/(c*x^4+b*x^2+a),x,method=_RETURNVERBOSE)
[Out] -1/3*(-c*e*x^2+3*b*e-4*c*d)*(e*x^2+d)^(1/2)/c^2-1/2/c^2/((b*e-2*c*d+(-4*e^2
*(a*c-1/4*b^2))^(1/2))*c)^(1/2)*2^(1/2)*(-((-c^2*d^2+(a*e^2+2*b*d*e)*c-b^2*
e^2)*(-4*e^2*(a*c-1/4*b^2))^(1/2)+(4*a*d*e^2+b*d^2*e)*c^2+(-3*a*b*e^3-2*b^2
*d*e^2)*c+b^3*e^3)*((b*e-2*c*d+(-4*e^2*(a*c-1/4*b^2))^(1/2))*c)^(1/2)*arcta
nh(c*(e*x^2+d)^(1/2)*2^(1/2)/((-b*e+2*c*d+(-4*e^2*(a*c-1/4*b^2))^(1/2))*c)^
(1/2))+((-c^2*d^2+(a*e^2+2*b*d*e)*c-b^2*e^2)*(-4*e^2*(a*c-1/4*b^2))^(1/2)+(
-4*a*d*e^2-b*d^2*e)*c^2+(3*a*b*e^3+2*b^2*d*e^2)*c-b^3*e^3)*arctan(c*(e*x^2+
d)^(1/2)*2^(1/2)/((b*e-2*c*d+(-4*e^2*(a*c-1/4*b^2))^(1/2))*c)^(1/2))*((-b*e
+2*c*d+(-4*e^2*(a*c-1/4*b^2))^(1/2))*c)^(1/2))/((-b*e+2*c*d+(-4*e^2*(a*c-1/
4*b^2))^(1/2))*c)^(1/2)/(-4*e^2*(a*c-1/4*b^2))^(1/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 8200 vs. $2(400) = 800$.

Time = 249.64 (sec) , antiderivative size = 8200, normalized size of antiderivative = 17.83

$$\int \frac{x^3(d+ex^2)^{3/2}}{a+bx^2+cx^4}\,dx = \text{Too large to display}$$

```
[In] integrate(x^3*(e*x^2+d)^(3/2)/(c*x^4+b*x^2+a),x, algorithm="fricas")
[Out] Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^3(d+ex^2)^{3/2}}{a+bx^2+cx^4}\,dx = \text{Timed out}$$

```
[In] integrate(x**3*(e*x**2+d)**(3/2)/(c*x**4+b*x**2+a),x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{x^3(d+ex^2)^{3/2}}{a+bx^2+cx^4}\,dx = \int \frac{(ex^2+d)^{\frac{3}{2}}x^3}{cx^4+bx^2+a}\,dx$$

```
[In] integrate(x^3*(e*x^2+d)^(3/2)/(c*x^4+b*x^2+a),x, algorithm="maxima")
[Out] integrate((e*x^2 + d)^(3/2)*x^3/(c*x^4 + b*x^2 + a), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 867 vs. $2(400) = 800$.

Time = 0.39 (sec) , antiderivative size = 867, normalized size of antiderivative = 1.88

$$\int \frac{x^3(d+ex^2)^{3/2}}{a+bx^2+cx^4}\,dx = \frac{(ex^2+d)^{\frac{3}{2}}c^2 + 3\sqrt{ex^2+d}c^2d - 3\sqrt{ex^2+d}bce}{3c^3}$$

$$- \frac{(2bc^5d^3e + ((b^2c^2-4ac^3)d^2 - 2(b^3c-4abc^2)de + (b^4-5ab^2c+4a^2c^2)e^2)c^2e^2 - (5b^2c^4-8ac^5)d^2e^2 +}{(}$$

$$+ \frac{(2bc^5d^3e + ((b^2c^2-4ac^3)d^2 - 2(b^3c-4abc^2)de + (b^4-5ab^2c+4a^2c^2)e^2)c^2e^2 - (5b^2c^4-8ac^5)d^2e^2 + 2}{(2}$$

```
[In] integrate(x^3*(e*x^2+d)^(3/2)/(c*x^4+b*x^2+a),x, algorithm="giac")
[Out] 1/3*((e*x^2 + d)^(3/2)*c^2 + 3*sqrt(e*x^2 + d)*c^2*d - 3*sqrt(e*x^2 + d)*b*
c*e)/c^3 - (2*b*c^5*d^3*e + ((b^2*c^2 - 4*a*c^3)*d^2 - 2*(b^3*c - 4*a*b*c^2
)*d*e + (b^4 - 5*a*b^2*c + 4*a^2*c^2)*e^2)*c^2*e^2 - (5*b^2*c^4 - 8*a*c^5)*
```

```
d^2*e^2 + 2*(2*b^3*c^3 - 5*a*b*c^4)*d*e^3 - (b^4*c^2 - 3*a*b^2*c^3)*e^4 - 2
*(sqrt(b^2 - 4*a*c)*c^4*d^3 - 2*sqrt(b^2 - 4*a*c)*b*c^3*d^2*e - sqrt(b^2 -
4*a*c)*a*b*c^2*e^3 + (b^2*c^2 + a*c^3)*sqrt(b^2 - 4*a*c)*d*e^2)*abs(c)*abs(
e))*arctan(2*sqrt(1/2)*sqrt(e*x^2 + d)/sqrt(-(2*c^4*d - b*c^3*e + sqrt(-4*(
c^4*d^2 - b*c^3*d*e + a*c^3*e^2)*c^4 + (2*c^4*d - b*c^3*e)^2))/c^4))/((2*sq
rt(b^2 - 4*a*c)*c^3*d - (b^2*c^2 - 4*a*c^3 + sqrt(b^2 - 4*a*c)*b*c^2)*e)*sq
rt(-4*c^2*d + 2*(b*c - sqrt(b^2 - 4*a*c)*c)*e)*c^2*abs(e)) + (2*b*c^5*d^3*e
 + ((b^2*c^2 - 4*a*c^3)*d^2 - 2*(b^3*c - 4*a*b*c^2)*d*e + (b^4 - 5*a*b^2*c
+ 4*a^2*c^2)*e^2)*c^2*e^2 - (5*b^2*c^4 - 8*a*c^5)*d^2*e^2 + 2*(2*b^3*c^3 -
5*a*b*c^4)*d*e^3 - (b^4*c^2 - 3*a*b^2*c^3)*e^4 + 2*(sqrt(b^2 - 4*a*c)*c^4*d
^3 - 2*sqrt(b^2 - 4*a*c)*b*c^3*d^2*e - sqrt(b^2 - 4*a*c)*a*b*c^2*e^3 + (b^2
*c^2 + a*c^3)*sqrt(b^2 - 4*a*c)*d*e^2)*abs(c)*abs(e))*arctan(2*sqrt(1/2)*sq
rt(e*x^2 + d)/sqrt(-(2*c^4*d - b*c^3*e - sqrt(-4*(c^4*d^2 - b*c^3*d*e + a*c
^3*e^2)*c^4 + (2*c^4*d - b*c^3*e)^2))/c^4))/((2*sqrt(b^2 - 4*a*c)*c^3*d + (
b^2*c^2 - 4*a*c^3 - sqrt(b^2 - 4*a*c)*b*c^2)*e)*sqrt(-4*c^2*d + 2*(b*c + sq
rt(b^2 - 4*a*c)*c)*e)*c^2*abs(e))
```

Mupad [B] (verification not implemented)

Time = 9.38 (sec) , antiderivative size = 16951, normalized size of antiderivative = 36.85

$$\int \frac{x^3(d+ex^2)^{3/2}}{a+bx^2+cx^4}\,dx = \text{Too large to display}$$

```
[In] int((x^3*(d + e*x^2)^(3/2))/(a + b*x^2 + c*x^4),x)
```

```
[Out] (d + e*x^2)^(3/2)/(3*c) - atan(((((4*a*b^3*c^3*e^5 - 16*a^2*b*c^4*e^5 + 16*
a*c^6*d^3*e^2 + 16*a^2*c^5*d*e^4 - 4*b^4*c^3*d*e^4 - 4*b^2*c^5*d^3*e^2 + 8*
b^3*c^4*d^2*e^3 - 32*a*b*c^5*d^2*e^3 + 12*a*b^2*c^4*d*e^4)/c^3 - (2*(d + e*
x^2)^(1/2)*(-(((4*b^7*e^3 - 32*a^2*c^5*d^3 - 4*b^4*c^3*d^3 + 24*a*b^2*c^4*d
^3 - 80*a^3*b*c^3*e^3 + 96*a^3*c^4*d*e^2 + 12*b^5*c^2*d^2*e + 100*a^2*b^3*c
^2*e^3 - 36*a*b^5*c*e^3 - 12*b^6*c*d*e^2 - 84*a*b^3*c^3*d^2*e + 96*a*b^4*c^
2*d*e^2 + 144*a^2*b*c^4*d^2*e - 216*a^2*b^2*c^3*d*e^2)^2/4 - (256*a^2*c^7 +
 16*b^4*c^5 - 128*a*b^2*c^6)*(a^5*e^6 + a^2*c^3*d^6 + 3*a^4*c*d^2*e^4 - a^2
*b^3*d^3*e^3 + 3*a^3*b^2*d^2*e^4 + 3*a^3*c^2*d^4*e^2 - 3*a^4*b*d*e^5 - 3*a^
2*b*c^2*d^5*e - 6*a^3*b*c*d^3*e^3 + 3*a^2*b^2*c*d^4*e^2))^(1/2) + 2*b^7*e^3
 - 16*a^2*c^5*d^3 - 2*b^4*c^3*d^3 + 12*a*b^2*c^4*d^3 - 40*a^3*b*c^3*e^3 + 4
8*a^3*c^4*d*e^2 + 6*b^5*c^2*d^2*e + 50*a^2*b^3*c^2*e^3 - 18*a*b^5*c*e^3 - 6
*b^6*c*d*e^2 - 42*a*b^3*c^3*d^2*e + 48*a*b^4*c^2*d*e^2 + 72*a^2*b*c^4*d^2*e
 - 108*a^2*b^2*c^3*d*e^2)/(16*(16*a^2*c^7 + b^4*c^5 - 8*a*b^2*c^6)))^(1/2)*
(4*b^3*c^5*e^3 - 8*b^2*c^6*d*e^2 - 16*a*b*c^6*e^3 + 32*a*c^7*d*e^2))/c^3)*(
-(((4*b^7*e^3 - 32*a^2*c^5*d^3 - 4*b^4*c^3*d^3 + 24*a*b^2*c^4*d^3 - 80*a^3*
b*c^3*e^3 + 96*a^3*c^4*d*e^2 + 12*b^5*c^2*d^2*e + 100*a^2*b^3*c^2*e^3 - 36*
a*b^5*c*e^3 - 12*b^6*c*d*e^2 - 84*a*b^3*c^3*d^2*e + 96*a*b^4*c^2*d*e^2 + 14
4*a^2*b*c^4*d^2*e - 216*a^2*b^2*c^3*d*e^2)^2/4 - (256*a^2*c^7 + 16*b^4*c^5
```

- 128*a*b^2*c^6)*(a^5*e^6 + a^2*c^3*d^6 + 3*a^4*c*d^2*e^4 - a^2*b^3*d^3*e^3 + 3*a^3*b^2*d^2*e^4 + 3*a^3*c^2*d^4*e^2 - 3*a^4*b*d*e^5 - 3*a^2*b*c^2*d^5*e - 6*a^3*b*c*d^3*e^3 + 3*a^2*b^2*c*d^4*e^2))^(1/2) + 2*b^7*e^3 - 16*a^2*c^5*d^3 - 2*b^4*c^3*d^3 + 12*a*b^2*c^4*d^3 - 40*a^3*b*c^3*e^3 + 48*a^3*c^4*d*e^2 + 6*b^5*c^2*d^2*e + 50*a^2*b^3*c^2*e^3 - 18*a*b^5*c*e^3 - 6*b^6*c*d*e^2 - 42*a*b^3*c^3*d^2*e + 48*a*b^4*c^2*d*e^2 + 72*a^2*b*c^4*d^2*e - 108*a^2*b^2*c^3*d*e^2)/(16*(16*a^2*c^7 + b^4*c^5 - 8*a*b^2*c^6)))^(1/2) - (2*(d + e*x^2)^(1/2)*(b^6*e^6 - 2*a^3*c^3*e^6 - 2*a*c^5*d^4*e^2 + 9*a^2*b^2*c^2*e^6 + 12*a^2*c^4*d^2*e^4 + b^2*c^4*d^4*e^2 - 4*b^3*c^3*d^3*e^3 + 6*b^4*c^2*d^2*e^4 - 6*a*b^4*c*e^6 - 4*b^5*c*d*e^5 + 12*a*b*c^4*d^3*e^3 + 20*a*b^3*c^2*d*e^5 - 20*a^2*b*c^3*d*e^5 - 24*a*b^2*c^3*d^2*e^4))/c^3)*(-(((4*b^7*e^3 - 32*a^2*c^5*d^3 - 4*b^4*c^3*d^3 + 24*a*b^2*c^4*d^3 - 80*a^3*b*c^3*e^3 + 96*a^3*c^4*d*e^2 + 12*b^5*c^2*d^2*e + 100*a^2*b^3*c^2*e^3 - 36*a*b^5*c*e^3 - 12*b^6*c*d*e^2 - 84*a*b^3*c^3*d^2*e + 96*a*b^4*c^2*d*e^2 + 144*a^2*b*c^4*d^2*e - 216*a^2*b^2*c^3*d*e^2)^2/4 - (256*a^2*c^7 + 16*b^4*c^5 - 128*a*b^2*c^6)*(a^5*e^6 + a^2*c^3*d^6 + 3*a^4*c*d^2*e^4 - a^2*b^3*d^3*e^3 + 3*a^3*b^2*d^2*e^4 + 3*a^3*c^2*d^4*e^2 - 3*a^4*b*d*e^5 - 3*a^2*b*c^2*d^5*e - 6*a^3*b*c*d^3*e^3 + 3*a^2*b^2*c*d^4*e^2))^(1/2) + 2*b^7*e^3 - 16*a^2*c^5*d^3 - 2*b^4*c^3*d^3 + 12*a*b^2*c^4*d^3 - 40*a^3*b*c^3*e^3 + 48*a^3*c^4*d*e^2 + 6*b^5*c^2*d^2*e + 50*a^2*b^3*c^2*e^3 - 18*a*b^5*c*e^3 - 6*b^6*c*d*e^2 - 42*a*b^3*c^3*d^2*e + 48*a*b^4*c^2*d*e^2 + 72*a^2*b*c^4*d^2*e - 108*a^2*b^2*c^3*d*e^2)/(16*(16*a^2*c^7 + b^4*c^5 - 8*a*b^2*c^6)))^(1/2)*1i - (((4*a*b^3*c^3*e^5 - 16*a^2*b*c^4*e^5 + 16*a*c^6*d^3*e^2 + 16*a^2*c^5*d*e^4 - 4*b^4*c^3*d*e^4 - 4*b^2*c^5*d^3*e^2 + 8*b^3*c^4*d^2*e^3 - 32*a*b*c^5*d^2*e^3 + 12*a*b^2*c^4*d*e^4)/c^3 + (2*(d + e*x^2)^(1/2)*(-(((4*b^7*e^3 - 32*a^2*c^5*d^3 - 4*b^4*c^3*d^3 + 24*a*b^2*c^4*d^3 - 80*a^3*b*c^3*e^3 + 96*a^3*c^4*d*e^2 + 12*b^5*c^2*d^2*e + 100*a^2*b^3*c^2*e^3 - 36*a*b^5*c*e^3 - 12*b^6*c*d*e^2 - 84*a*b^3*c^3*d^2*e + 96*a*b^4*c^2*d*e^2 + 144*a^2*b*c^4*d^2*e - 216*a^2*b^2*c^3*d*e^2)^2/4 - (256*a^2*c^7 + 16*b^4*c^5 - 128*a*b^2*c^6)*(a^5*e^6 + a^2*c^3*d^6 + 3*a^4*c*d^2*e^4 - a^2*b^3*d^3*e^3 + 3*a^3*b^2*d^2*e^4 + 3*a^3*c^2*d^4*e^2 - 3*a^4*b*d*e^5 - 3*a^2*b*c^2*d^5*e - 6*a^3*b*c*d^3*e^3 + 3*a^2*b^2*c*d^4*e^2))^(1/2) + 2*b^7*e^3 - 16*a^2*c^5*d^3 - 2*b^4*c^3*d^3 + 12*a*b^2*c^4*d^3 - 40*a^3*b*c^3*e^3 + 48*a^3*c^4*d*e^2 + 6*b^5*c^2*d^2*e + 50*a^2*b^3*c^2*e^3 - 18*a*b^5*c*e^3 - 6*b^6*c*d*e^2 - 42*a*b^3*c^3*d^2*e + 48*a*b^4*c^2*d*e^2 + 72*a^2*b*c^4*d^2*e - 108*a^2*b^2*c^3*d*e^2)/(16*(16*a^2*c^7 + b^4*c^5 - 8*a*b^2*c^6)))^(1/2)*(4*b^3*c^5*e^3 - 8*b^2*c^6*d*e^2 - 16*a*b*c^6*e^3 + 32*a*c^7*d*e^2))/c^3)*(-(((4*b^7*e^3 - 32*a^2*c^5*d^3 - 4*b^4*c^3*d^3 + 24*a*b^2*c^4*d^3 - 80*a^3*b*c^3*e^3 + 96*a^3*c^4*d*e^2 + 12*b^5*c^2*d^2*e + 100*a^2*b^3*c^2*e^3 - 36*a*b^5*c*e^3 - 12*b^6*c*d*e^2 - 84*a*b^3*c^3*d^2*e + 96*a*b^4*c^2*d*e^2 + 144*a^2*b*c^4*d^2*e - 216*a^2*b^2*c^3*d*e^2)^2/4 - (256*a^2*c^7 + 16*b^4*c^5 - 128*a*b^2*c^6)*(a^5*e^6 + a^2*c^3*d^6 + 3*a^4*c*d^2*e^4 - a^2*b^3*d^3*e^3 + 3*a^3*b^2*d^2*e^4 + 3*a^3*c^2*d^4*e^2 - 3*a^4*b*d*e^5 - 3*a^2*b*c^2*d^5*e - 6*a^3*b*c*d^3*e^3 + 3*a^2*b^2*c*d^4*e^2))^(1/2) + 2*b^7*e^3 - 16*a^2*c^5*d^3 - 2*b^4*c^3*d^3 + 12*a*b^2*c^4*d^3 - 40*a^3*b*c^3*e^3 + 48*a^3*c^4*d*e^2 + 6*b^5*c^2*d^2*e + 50*a^2*b^3*c^2*e^3 - 18*a*b^5*c*e^3

- 6*b^6*c*d*e^2 - 42*a*b^3*c^3*d^2*e + 48*a*b^4*c^2*d*e^2 + 72*a^2*b*c^4*d^2*e - 108*a^2*b^2*c^3*d*e^2)/(16*(16*a^2*c^7 + b^4*c^5 - 8*a*b^2*c^6)))^(1/2) + (2*(d + e*x^2)^(1/2)*(b^6*e^6 - 2*a^3*c^3*e^6 - 2*a*c^5*d^4*e^2 + 9*a^2*b^2*c^2*e^6 + 12*a^2*c^4*d^2*e^4 + b^2*c^4*d^4*e^2 - 4*b^3*c^3*d^3*e^3 + 6*b^4*c^2*d^2*e^4 - 6*a*b^4*c*e^6 - 4*b^5*c*d*e^5 + 12*a*b*c^4*d^3*e^3 + 20*a*b^3*c^2*d*e^5 - 20*a^2*b*c^3*d*e^5 - 24*a*b^2*c^3*d^2*e^4))/c^3)*(-(((4*b^7*e^3 - 32*a^2*c^5*d^3 - 4*b^4*c^3*d^3 + 24*a*b^2*c^4*d^3 - 80*a^3*b*c^3*e^3 + 96*a^3*c^4*d*e^2 + 12*b^5*c^2*d^2*e + 100*a^2*b^3*c^2*e^3 - 36*a*b^5*c*e^3 - 12*b^6*c*d*e^2 - 84*a*b^3*c^3*d^2*e + 96*a*b^4*c^2*d*e^2 + 144*a^2*b*c^4*d^2*e - 216*a^2*b^2*c^3*d*e^2)^2/4 - (256*a^2*c^7 + 16*b^4*c^5 - 128*a*b^2*c^6)*(a^5*e^6 + a^2*c^3*d^6 + 3*a^4*c*d^2*e^4 - a^2*b^3*d^3*e^3 + 3*a^3*b^2*d^2*e^4 + 3*a^3*c^2*d^4*e^2 - 3*a^4*b*d*e^5 - 3*a^2*b*c^2*d^5*e - 6*a^3*b*c*d^3*e^3 + 3*a^2*b^2*c*d^4*e^2))^(1/2) + 2*b^7*e^3 - 16*a^2*c^5*d^3 - 2*b^4*c^3*d^3 + 12*a*b^2*c^4*d^3 - 40*a^3*b*c^3*e^3 + 48*a^3*c^4*d*e^2 + 6*b^5*c^2*d^2*e + 50*a^2*b^3*c^2*e^3 - 18*a*b^5*c*e^3 - 6*b^6*c*d*e^2 - 42*a*b^3*c^3*d^2*e + 48*a*b^4*c^2*d*e^2 + 72*a^2*b*c^4*d^2*e - 108*a^2*b^2*c^3*d*e^2)/(16*(16*a^2*c^7 + b^4*c^5 - 8*a*b^2*c^6)))^(1/2)*1i)/((((4*a*b^3*c^3*e^5 - 16*a^2*b*c^4*e^5 + 16*a*c^6*d^3*e^2 + 16*a^2*c^5*d*e^4 - 4*b^4*c^3*d*e^4 - 4*b^2*c^5*d^3*e^2 + 8*b^3*c^4*d^2*e^3 - 32*a*b*c^5*d^2*e^3 + 12*a*b^2*c^4*d*e^4)/c^3 - (2*(d + e*x^2)^(1/2)*(-(((4*b^7*e^3 - 32*a^2*c^5*d^3 - 4*b^4*c^3*d^3 + 24*a*b^2*c^4*d^3 - 80*a^3*b*c^3*e^3 + 96*a^3*c^4*d*e^2 + 12*b^5*c^2*d^2*e + 100*a^2*b^3*c^2*e^3 - 36*a*b^5*c*e^3 - 12*b^6*c*d*e^2 - 84*a*b^3*c^3*d^2*e + 96*a*b^4*c^2*d*e^2 + 144*a^2*b*c^4*d^2*e - 216*a^2*b^2*c^3*d*e^2)^2/4 - (256*a^2*c^7 + 16*b^4*c^5 - 128*a*b^2*c^6)*(a^5*e^6 + a^2*c^3*d^6 + 3*a^4*c*d^2*e^4 - a^2*b^3*d^3*e^3 + 3*a^3*b^2*d^2*e^4 + 3*a^3*c^2*d^4*e^2 - 3*a^4*b*d*e^5 - 3*a^2*b*c^2*d^5*e - 6*a^3*b*c*d^3*e^3 + 3*a^2*b^2*c*d^4*e^2))^(1/2) + 2*b^7*e^3 - 16*a^2*c^5*d^3 - 2*b^4*c^3*d^3 + 12*a*b^2*c^4*d^3 - 40*a^3*b*c^3*e^3 + 48*a^3*c^4*d*e^2 + 6*b^5*c^2*d^2*e + 50*a^2*b^3*c^2*e^3 - 18*a*b^5*c*e^3 - 6*b^6*c*d*e^2 - 42*a*b^3*c^3*d^2*e + 48*a*b^4*c^2*d*e^2 + 72*a^2*b*c^4*d^2*e - 108*a^2*b^2*c^3*d*e^2)/(16*(16*a^2*c^7 + b^4*c^5 - 8*a*b^2*c^6)))^(1/2)*(4*b^3*c^5*e^3 - 8*b^2*c^6*d*e^2 - 16*a*b*c^6*e^3 + 32*a*c^7*d*e^2))/c^3)*(-(((4*b^7*e^3 - 32*a^2*c^5*d^3 - 4*b^4*c^3*d^3 + 24*a*b^2*c^4*d^3 - 80*a^3*b*c^3*e^3 + 96*a^3*c^4*d*e^2 + 12*b^5*c^2*d^2*e + 100*a^2*b^3*c^2*e^3 - 36*a*b^5*c*e^3 - 12*b^6*c*d*e^2 - 84*a*b^3*c^3*d^2*e + 96*a*b^4*c^2*d*e^2 + 144*a^2*b*c^4*d^2*e - 216*a^2*b^2*c^3*d*e^2)^2/4 - (256*a^2*c^7 + 16*b^4*c^5 - 128*a*b^2*c^6)*(a^5*e^6 + a^2*c^3*d^6 + 3*a^4*c*d^2*e^4 - a^2*b^3*d^3*e^3 + 3*a^3*b^2*d^2*e^4 + 3*a^3*c^2*d^4*e^2 - 3*a^4*b*d*e^5 - 3*a^2*b*c^2*d^5*e - 6*a^3*b*c*d^3*e^3 + 3*a^2*b^2*c*d^4*e^2))^(1/2) + 2*b^7*e^3 - 16*a^2*c^5*d^3 - 2*b^4*c^3*d^3 + 12*a*b^2*c^4*d^3 - 40*a^3*b*c^3*e^3 + 48*a^3*c^4*d*e^2 + 6*b^5*c^2*d^2*e + 50*a^2*b^3*c^2*e^3 - 18*a*b^5*c*e^3 - 6*b^6*c*d*e^2 - 42*a*b^3*c^3*d^2*e + 48*a*b^4*c^2*d*e^2 + 72*a^2*b*c^4*d^2*e - 108*a^2*b^2*c^3*d*e^2)/(16*(16*a^2*c^7 + b^4*c^5 - 8*a*b^2*c^6)))^(1/2) - (2*(d + e*x^2)^(1/2)*(b^6*e^6 - 2*a^3*c^3*e^6 - 2*a*c^5*d^4*e^2 + 9*a^2*b^2*c^2*e^6 + 12*a^2*c^4*d^2*e^4 + b^2*c^4*d^4*e^2 - 4*b^3*c^3*d^3*e^3 + 6*b^4*c^2*d^2*e^4 - 6*a*b^4*c*e^6 - 4*b^5*c*d*e^5 + 12*a*b*c

^4*d^3*e^3 + 20*a*b^3*c^2*d*e^5 - 20*a^2*b*c^3*d*e^5 - 24*a*b^2*c^3*d^2*e^4
))/c^3)*(-(((4*b^7*e^3 - 32*a^2*c^5*d^3 - 4*b^4*c^3*d^3 + 24*a*b^2*c^4*d^3
- 80*a^3*b*c^3*e^3 + 96*a^3*c^4*d*e^2 + 12*b^5*c^2*d^2*e + 100*a^2*b^3*c^2*
e^3 - 36*a*b^5*c*e^3 - 12*b^6*c*d*e^2 - 84*a*b^3*c^3*d^2*e + 96*a*b^4*c^2*d
*e^2 + 144*a^2*b*c^4*d^2*e - 216*a^2*b^2*c^3*d*e^2)^2/4 - (256*a^2*c^7 + 16
*b^4*c^5 - 128*a*b^2*c^6)*(a^5*e^6 + a^2*c^3*d^6 + 3*a^4*c*d^2*e^4 - a^2*b^
3*d^3*e^3 + 3*a^3*b^2*d^2*e^4 + 3*a^3*c^2*d^4*e^2 - 3*a^4*b*d*e^5 - 3*a^2*b
*c^2*d^5*e - 6*a^3*b*c*d^3*e^3 + 3*a^2*b^2*c*d^4*e^2))^(1/2) + 2*b^7*e^3 -
16*a^2*c^5*d^3 - 2*b^4*c^3*d^3 + 12*a*b^2*c^4*d^3 - 40*a^3*b*c^3*e^3 + 48*a
^3*c^4*d*e^2 + 6*b^5*c^2*d^2*e + 50*a^2*b^3*c^2*e^3 - 18*a*b^5*c*e^3 - 6*b^
6*c*d*e^2 - 42*a*b^3*c^3*d^2*e + 48*a*b^4*c^2*d*e^2 + 72*a^2*b*c^4*d^2*e -
108*a^2*b^2*c^3*d*e^2)/(16*(16*a^2*c^7 + b^4*c^5 - 8*a*b^2*c^6)))^(1/2) + (
((4*a*b^3*c^3*e^5 - 16*a^2*b*c^4*e^5 + 16*a*c^6*d^3*e^2 + 16*a^2*c^5*d*e^4
- 4*b^4*c^3*d*e^4 - 4*b^2*c^5*d^3*e^2 + 8*b^3*c^4*d^2*e^3 - 32*a*b*c^5*d^2*
e^3 + 12*a*b^2*c^4*d*e^4)/c^3 + (2*(d + e*x^2)^(1/2)*(-(((4*b^7*e^3 - 32*a^
2*c^5*d^3 - 4*b^4*c^3*d^3 + 24*a*b^2*c^4*d^3 - 80*a^3*b*c^3*e^3 + 96*a^3*c^
4*d*e^2 + 12*b^5*c^2*d^2*e + 100*a^2*b^3*c^2*e^3 - 36*a*b^5*c*e^3 - 12*b^6*
c*d*e^2 - 84*a*b^3*c^3*d^2*e + 96*a*b^4*c^2*d*e^2 + 144*a^2*b*c^4*d^2*e - 2
16*a^2*b^2*c^3*d*e^2)^2/4 - (256*a^2*c^7 + 16*b^4*c^5 - 128*a*b^2*c^6)*(a^5
*e^6 + a^2*c^3*d^6 + 3*a^4*c*d^2*e^4 - a^2*b^3*d^3*e^3 + 3*a^3*b^2*d^2*e^4
+ 3*a^3*c^2*d^4*e^2 - 3*a^4*b*d*e^5 - 3*a^2*b*c^2*d^5*e - 6*a^3*b*c*d^3*e^3
 + 3*a^2*b^2*c*d^4*e^2))^(1/2) + 2*b^7*e^3 - 16*a^2*c^5*d^3 - 2*b^4*c^3*d^3
 + 12*a*b^2*c^4*d^3 - 40*a^3*b*c^3*e^3 + 48*a^3*c^4*d*e^2 + 6*b^5*c^2*d^2*e
 + 50*a^2*b^3*c^2*e^3 - 18*a*b^5*c*e^3 - 6*b^6*c*d*e^2 - 42*a*b^3*c^3*d^2*e
 + 48*a*b^4*c^2*d*e^2 + 72*a^2*b*c^4*d^2*e - 108*a^2*b^2*c^3*d*e^2)/(16*(16
*a^2*c^7 + b^4*c^5 - 8*a*b^2*c^6)))^(1/2)*(4*b^3*c^5*e^3 - 8*b^2*c^6*d*e^2
- 16*a*b*c^6*e^3 + 32*a*c^7*d*e^2))/c^3)*(-(((4*b^7*e^3 - 32*a^2*c^5*d^3 -
4*b^4*c^3*d^3 + 24*a*b^2*c^4*d^3 - 80*a^3*b*c^3*e^3 + 96*a^3*c^4*d*e^2 + 12
*b^5*c^2*d^2*e + 100*a^2*b^3*c^2*e^3 - 36*a*b^5*c*e^3 - 12*b^6*c*d*e^2 - 84
*a*b^3*c^3*d^2*e + 96*a*b^4*c^2*d*e^2 + 144*a^2*b*c^4*d^2*e - 216*a^2*b^2*c
^3*d*e^2)^2/4 - (256*a^2*c^7 + 16*b^4*c^5 - 128*a*b^2*c^6)*(a^5*e^6 + a^2*c
^3*d^6 + 3*a^4*c*d^2*e^4 - a^2*b^3*d^3*e^3 + 3*a^3*b^2*d^2*e^4 + 3*a^3*c^2*
d^4*e^2 - 3*a^4*b*d*e^5 - 3*a^2*b*c^2*d^5*e - 6*a^3*b*c*d^3*e^3 + 3*a^2*b^2
*c*d^4*e^2))^(1/2) + 2*b^7*e^3 - 16*a^2*c^5*d^3 - 2*b^4*c^3*d^3 + 12*a*b^2*
c^4*d^3 - 40*a^3*b*c^3*e^3 + 48*a^3*c^4*d*e^2 + 6*b^5*c^2*d^2*e + 50*a^2*b^
3*c^2*e^3 - 18*a*b^5*c*e^3 - 6*b^6*c*d*e^2 - 42*a*b^3*c^3*d^2*e + 48*a*b^4*
c^2*d*e^2 + 72*a^2*b*c^4*d^2*e - 108*a^2*b^2*c^3*d*e^2)/(16*(16*a^2*c^7 + b
^4*c^5 - 8*a*b^2*c^6)))^(1/2) + (2*(d + e*x^2)^(1/2)*(b^6*e^6 - 2*a^3*c^3*e
^6 - 2*a*c^5*d^4*e^2 + 9*a^2*b^2*c^2*e^6 + 12*a^2*c^4*d^2*e^4 + b^2*c^4*d^4
*e^2 - 4*b^3*c^3*d^3*e^3 + 6*b^4*c^2*d^2*e^4 - 6*a*b^4*c*e^6 - 4*b^5*c*d*e^
5 + 12*a*b*c^4*d^3*e^3 + 20*a*b^3*c^2*d*e^5 - 20*a^2*b*c^3*d*e^5 - 24*a*b^2
*c^3*d^2*e^4))/c^3)*(-(((4*b^7*e^3 - 32*a^2*c^5*d^3 - 4*b^4*c^3*d^3 + 24*a*
b^2*c^4*d^3 - 80*a^3*b*c^3*e^3 + 96*a^3*c^4*d*e^2 + 12*b^5*c^2*d^2*e + 100*
a^2*b^3*c^2*e^3 - 36*a*b^5*c*e^3 - 12*b^6*c*d*e^2 - 84*a*b^3*c^3*d^2*e + 96
*a*b^4*c^2*d*e^2 + 144*a^2*b*c^4*d^2*e - 216*a^2*b^2*c^3*d*e^2)^2/4 - (256*

a^2*c^7 + 16*b^4*c^5 - 128*a*b^2*c^6)*(a^5*e^6 + a^2*c^3*d^6 + 3*a^4*c*d^2*e^4 - a^2*b^3*d^3*e^3 + 3*a^3*b^2*d^2*e^4 + 3*a^3*c^2*d^4*e^2 - 3*a^4*b*d*e^5 - 3*a^2*b*c^2*d^5*e - 6*a^3*b*c*d^3*e^3 + 3*a^2*b^2*c*d^4*e^2))^(1/2) + 2*b^7*e^3 - 16*a^2*c^5*d^3 - 2*b^4*c^3*d^3 + 12*a*b^2*c^4*d^3 - 40*a^3*b*c^3*e^3 + 48*a^3*c^4*d*e^2 + 6*b^5*c^2*d^2*e + 50*a^2*b^3*c^2*e^3 - 18*a*b^5*c*e^3 - 6*b^6*c*d*e^2 - 42*a*b^3*c^3*d^2*e + 48*a*b^4*c^2*d*e^2 + 72*a^2*b*c^4*d^2*e - 108*a^2*b^2*c^3*d*e^2)/(16*(16*a^2*c^7 + b^4*c^5 - 8*a*b^2*c^6)))^(1/2) + (2*(a^4*c*e^8 - a^3*b^2*e^8 - a*b^4*d^2*e^6 + 2*a^2*b^3*d*e^7 - a*c^4*d^6*e^2 - a^2*c^3*d^4*e^4 + a^3*c^2*d^2*e^6 + 4*a*b*c^3*d^5*e^3 + 4*a*b^3*c*d^3*e^5 - 6*a*b^2*c^2*d^4*e^4 + 4*a^2*b*c^2*d^3*e^5 - 5*a^2*b^2*c*d^2*e^6))/c^3))*(-(((4*b^7*e^3 - 32*a^2*c^5*d^3 - 4*b^4*c^3*d^3 + 24*a*b^2*c^4*d^3 - 80*a^3*b*c^3*e^3 + 96*a^3*c^4*d*e^2 + 12*b^5*c^2*d^2*e + 100*a^2*b^3*c^2*e^3 - 36*a*b^5*c*e^3 - 12*b^6*c*d*e^2 - 84*a*b^3*c^3*d^2*e + 96*a*b^4*c^2*d*e^2 + 144*a^2*b*c^4*d^2*e - 216*a^2*b^2*c^3*d*e^2)^2/4 - (256*a^2*c^7 + 16*b^4*c^5 - 128*a*b^2*c^6)*(a^5*e^6 + a^2*c^3*d^6 + 3*a^4*c*d^2*e^4 - a^2*b^3*d^3*e^3 + 3*a^3*b^2*d^2*e^4 + 3*a^3*c^2*d^4*e^2 - 3*a^4*b*d*e^5 - 3*a^2*b*c^2*d^5*e - 6*a^3*b*c*d^3*e^3 + 3*a^2*b^2*c*d^4*e^2))^(1/2) + 2*b^7*e^3 - 16*a^2*c^5*d^3 - 2*b^4*c^3*d^3 + 12*a*b^2*c^4*d^3 - 40*a^3*b*c^3*e^3 + 48*a^3*c^4*d*e^2 + 6*b^5*c^2*d^2*e + 50*a^2*b^3*c^2*e^3 - 18*a*b^5*c*e^3 - 6*b^6*c*d*e^2 - 42*a*b^3*c^3*d^2*e + 48*a*b^4*c^2*d*e^2 + 72*a^2*b*c^4*d^2*e - 108*a^2*b^2*c^3*d*e^2)/(16*(16*a^2*c^7 + b^4*c^5 - 8*a*b^2*c^6)))^(1/2)*2i - (d + e*x^2)^(1/2)*(d/c + (b*e - 2*c*d)/c^2) - atan(((((4*a*b^3*c^3*e^5 - 16*a^2*b*c^4*e^5 + 16*a*c^6*d^3*e^2 + 16*a^2*c^5*d*e^4 - 4*b^4*c^3*d*e^4 - 4*b^2*c^5*d^3*e^2 + 8*b^3*c^4*d^2*e^3 - 32*a*b*c^5*d^2*e^3 + 12*a*b^2*c^4*d*e^4)/c^3 - (2*(d + e*x^2)^(1/2)*((((4*b^7*e^3 - 32*a^2*c^5*d^3 - 4*b^4*c^3*d^3 + 24*a*b^2*c^4*d^3 - 80*a^3*b*c^3*e^3 + 96*a^3*c^4*d*e^2 + 12*b^5*c^2*d^2*e + 100*a^2*b^3*c^2*e^3 - 36*a*b^5*c*e^3 - 12*b^6*c*d*e^2 - 84*a*b^3*c^3*d^2*e + 96*a*b^4*c^2*d*e^2 + 144*a^2*b*c^4*d^2*e - 216*a^2*b^2*c^3*d*e^2)^2/4 - (256*a^2*c^7 + 16*b^4*c^5 - 128*a*b^2*c^6)*(a^5*e^6 + a^2*c^3*d^6 + 3*a^4*c*d^2*e^4 - a^2*b^3*d^3*e^3 + 3*a^3*b^2*d^2*e^4 + 3*a^3*c^2*d^4*e^2 - 3*a^4*b*d*e^5 - 3*a^2*b*c^2*d^5*e - 6*a^3*b*c*d^3*e^3 + 3*a^2*b^2*c*d^4*e^2))^(1/2) - 2*b^7*e^3 + 16*a^2*c^5*d^3 + 2*b^4*c^3*d^3 - 12*a*b^2*c^4*d^3 + 40*a^3*b*c^3*e^3 - 48*a^3*c^4*d*e^2 - 6*b^5*c^2*d^2*e - 50*a^2*b^3*c^2*e^3 + 18*a*b^5*c*e^3 + 6*b^6*c*d*e^2 + 42*a*b^3*c^3*d^2*e - 48*a*b^4*c^2*d*e^2 - 72*a^2*b*c^4*d^2*e + 108*a^2*b^2*c^3*d*e^2)/(16*(16*a^2*c^7 + b^4*c^5 - 8*a*b^2*c^6)))^(1/2)*(4*b^3*c^5*e^3 - 8*b^2*c^6*d*e^2 - 16*a*b*c^6*e^3 + 32*a*c^7*d*e^2))/c^3)*((((4*b^7*e^3 - 32*a^2*c^5*d^3 - 4*b^4*c^3*d^3 + 24*a*b^2*c^4*d^3 - 80*a^3*b*c^3*e^3 + 96*a^3*c^4*d*e^2 + 12*b^5*c^2*d^2*e + 100*a^2*b^3*c^2*e^3 - 36*a*b^5*c*e^3 - 12*b^6*c*d*e^2 - 84*a*b^3*c^3*d^2*e + 96*a*b^4*c^2*d*e^2 + 144*a^2*b*c^4*d^2*e - 216*a^2*b^2*c^3*d*e^2)^2/4 - (256*a^2*c^7 + 16*b^4*c^5 - 128*a*b^2*c^6)*(a^5*e^6 + a^2*c^3*d^6 + 3*a^4*c*d^2*e^4 - a^2*b^3*d^3*e^3 + 3*a^3*b^2*d^2*e^4 + 3*a^3*c^2*d^4*e^2 - 3*a^4*b*d*e^5 - 3*a^2*b*c^2*d^5*e - 6*a^3*b*c*d^3*e^3 + 3*a^2*b^2*c*d^4*e^2))^(1/2) - 2*b^7*e^3 + 16*a^2*c^5*d^3 + 2*b^4*c^3*d^3 - 12*a*b^2*c^4*d^3 + 40*a^3*b*c^3*e^3 - 48*a^3*c^4*d*e^2 - 6*b^5*c^2*d^2*e - 50*a^2*b^3*c^2*e^3 + 18*a *b^5*c*e^3 + 6*b^6*c*d*e^2 + 42*a*b^3*c^3*d^2*e - 48*a*b^4*c^2*d*e^2 - 72*a ^2*b*c^4*d^2*e + 108*a^2*b^2*c^3*d*e^2)/(16*(16*a^2*c^7 + b^4*c^5 - 8*a*b^2 *c^6)))^(1/2) - (2*(d + e*x^2)^(1/2)*(b^6*e^6 - 2*a^3*c^3*e^6 - 2*a*c^5*d^4 *e^2 + 9*a^2*b^2*c^2*e^6 + 12*a^2*c^4*d^2*e^4 + b^2*c^4*d^4*e^2 - 4*b^3*c^3 *d^3*e^3 + 6*b^4*c^2*d^2*e^4 - 6*a*b^4*c*e^6 - 4*b^5*c*d*e^5 + 12*a*b*c^4*d ^3*e^3 + 20*a*b^3*c^2*d*e^5 - 20*a^2*b*c^3*d*e^5 - 24*a*b^2*c^3*d^2*e^4))/c ^3)*((((4*b^7*e^3 - 32*a^2*c^5*d^3 - 4*b^4*c^3*d^3 + 24*a*b^2*c^4*d^3 - 80* a^3*b*c^3*e^3 + 96*a^3*c^4*d*e^2 + 12*b^5*c^2*d^2*e + 100*a^2*b^3*c^2*e^3 - 36*a*b^5*c*e^3 - 12*b^6*c*d*e^2 - 84*a*b^3*c^3*d^2*e + 96*a*b^4*c^2*d*e^2 + 144*a^2*b*c^4*d^2*e - 216*a^2*b^2*c^3*d*e^2)^2/4 - (256*a^2*c^7 + 16*b^4* c^5 - 128*a*b^2*c^6)*(a^5*e^6 + a^2*c^3*d^6 + 3*a^4*c*d^2*e^4 - a^2*b^3*d^3 *e^3 + 3*a^3*b^2*d^2*e^4 + 3*a^3*c^2*d^4*e^2 - 3*a^4*b*d*e^5 - 3*a^2*b*c^2* d^5*e - 6*a^3*b*c*d^3*e^3 + 3*a^2*b^2*c*d^4*e^2))^(1/2) - 2*b^7*e^3 + 16*a^ 2*c^5*d^3 + 2*b^4*c^3*d^3 - 12*a*b^2*c^4*d^3 + 40*a^3*b*c^3*e^3 - 48*a^3*c^ 4*d*e^2 - 6*b^5*c^2*d^2*e - 50*a^2*b^3*c^2*e^3 + 18*a*b^5*c*e^3 + 6*b^6*c*d *e^2 + 42*a*b^3*c^3*d^2*e - 48*a*b^4*c^2*d*e^2 - 72*a^2*b*c^4*d^2*e + 108*a ^2*b^2*c^3*d*e^2)/(16*(16*a^2*c^7 + b^4*c^5 - 8*a*b^2*c^6)))^(1/2)*1i - (((4*a*b^3*c^3*e^5 - 16*a^2*b*c^4*e^5 + 16*a*c^6*d^3*e^2 + 16*a^2*c^5*d*e^4 - 4*b^4*c^3*d*e^4 - 4*b^2*c^5*d^3*e^2 + 8*b^3*c^4*d^2*e^3 - 32*a*b*c^5*d^2*e^ 3 + 12*a*b^2*c^4*d*e^4)/c^3 + (2*(d + e*x^2)^(1/2)*((((4*b^7*e^3 - 32*a^2*c ^5*d^3 - 4*b^4*c^3*d^3 + 24*a*b^2*c^4*d^3 - 80*a^3*b*c^3*e^3 + 96*a^3*c^4*d *e^2 + 12*b^5*c^2*d^2*e + 100*a^2*b^3*c^2*e^3 - 36*a*b^5*c*e^3 - 12*b^6*c*d *e^2 - 84*a*b^3*c^3*d^2*e + 96*a*b^4*c^2*d*e^2 + 144*a^2*b*c^4*d^2*e - 216* a^2*b^2*c^3*d*e^2)^2/4 - (256*a^2*c^7 + 16*b^4*c^5 - 128*a*b^2*c^6)*(a^5*e^ 6 + a^2*c^3*d^6 + 3*a^4*c*d^2*e^4 - a^2*b^3*d^3*e^3 + 3*a^3*b^2*d^2*e^4 + 3 *a^3*c^2*d^4*e^2 - 3*a^4*b*d*e^5 - 3*a^2*b*c^2*d^5*e - 6*a^3*b*c*d^3*e^3 + 3*a^2*b^2*c*d^4*e^2))^(1/2) - 2*b^7*e^3 + 16*a^2*c^5*d^3 + 2*b^4*c^3*d^3 - 12*a*b^2*c^4*d^3 + 40*a^3*b*c^3*e^3 - 48*a^3*c^4*d*e^2 - 6*b^5*c^2*d^2*e - 50*a^2*b^3*c^2*e^3 + 18*a*b^5*c*e^3 + 6*b^6*c*d*e^2 + 42*a*b^3*c^3*d^2*e - 48*a*b^4*c^2*d*e^2 - 72*a^2*b*c^4*d^2*e + 108*a^2*b^2*c^3*d*e^2)/(16*(16*a^ 2*c^7 + b^4*c^5 - 8*a*b^2*c^6)))^(1/2)*(4*b^3*c^5*e^3 - 8*b^2*c^6*d*e^2 - 1 6*a*b*c^6*e^3 + 32*a*c^7*d*e^2))/c^3)*((((4*b^7*e^3 - 32*a^2*c^5*d^3 - 4*b^ 4*c^3*d^3 + 24*a*b^2*c^4*d^3 - 80*a^3*b*c^3*e^3 + 96*a^3*c^4*d*e^2 + 12*b^5 *c^2*d^2*e + 100*a^2*b^3*c^2*e^3 - 36*a*b^5*c*e^3 - 12*b^6*c*d*e^2 - 84*a*b ^3*c^3*d^2*e + 96*a*b^4*c^2*d*e^2 + 144*a^2*b*c^4*d^2*e - 216*a^2*b^2*c^3*d *e^2)^2/4 - (256*a^2*c^7 + 16*b^4*c^5 - 128*a*b^2*c^6)*(a^5*e^6 + a^2*c^3*d ^6 + 3*a^4*c*d^2*e^4 - a^2*b^3*d^3*e^3 + 3*a^3*b^2*d^2*e^4 + 3*a^3*c^2*d^4* e^2 - 3*a^4*b*d*e^5 - 3*a^2*b*c^2*d^5*e - 6*a^3*b*c*d^3*e^3 + 3*a^2*b^2*c*d ^4*e^2))^(1/2) - 2*b^7*e^3 + 16*a^2*c^5*d^3 + 2*b^4*c^3*d^3 - 12*a*b^2*c^4* d^3 + 40*a^3*b*c^3*e^3 - 48*a^3*c^4*d*e^2 - 6*b^5*c^2*d^2*e - 50*a^2*b^3*c^ 2*e^3 + 18*a*b^5*c*e^3 + 6*b^6*c*d*e^2 + 42*a*b^3*c^3*d^2*e - 48*a*b^4*c^2* d*e^2 - 72*a^2*b*c^4*d^2*e + 108*a^2*b^2*c^3*d*e^2)/(16*(16*a^2*c^7 + b^4*c ^5 - 8*a*b^2*c^6)))^(1/2) + (2*(d + e*x^2)^(1/2)*(b^6*e^6 - 2*a^3*c^3*e^6 - 2*a*c^5*d^4*e^2 + 9*a^2*b^2*c^2*e^6 + 12*a^2*c^4*d^2*e^4 + b^2*c^4*d^4*e^2 - 4*b^3*c^3*d^3*e^3 + 6*b^4*c^2*d^2*e^4 - 6*a*b^4*c*e^6 - 4*b^5*c*d*e^5 +

12*a*b*c^4*d^3*e^3 + 20*a*b^3*c^2*d*e^5 - 20*a^2*b*c^3*d*e^5 - 24*a*b^2*c^3
*d^2*e^4))/c^3)*((((4*b^7*e^3 - 32*a^2*c^5*d^3 - 4*b^4*c^3*d^3 + 24*a*b^2*c
^4*d^3 - 80*a^3*b*c^3*e^3 + 96*a^3*c^4*d*e^2 + 12*b^5*c^2*d^2*e + 100*a^2*b
^3*c^2*e^3 - 36*a*b^5*c*e^3 - 12*b^6*c*d*e^2 - 84*a*b^3*c^3*d^2*e + 96*a*b^
4*c^2*d*e^2 + 144*a^2*b*c^4*d^2*e - 216*a^2*b^2*c^3*d*e^2)^2/4 - (256*a^2*c
^7 + 16*b^4*c^5 - 128*a*b^2*c^6)*(a^5*e^6 + a^2*c^3*d^6 + 3*a^4*c*d^2*e^4 -
a^2*b^3*d^3*e^3 + 3*a^3*b^2*d^2*e^4 + 3*a^3*c^2*d^4*e^2 - 3*a^4*b*d*e^5 -
3*a^2*b*c^2*d^5*e - 6*a^3*b*c*d^3*e^3 + 3*a^2*b^2*c*d^4*e^2))^(1/2) - 2*b^7
*e^3 + 16*a^2*c^5*d^3 + 2*b^4*c^3*d^3 - 12*a*b^2*c^4*d^3 + 40*a^3*b*c^3*e^3
- 48*a^3*c^4*d*e^2 - 6*b^5*c^2*d^2*e - 50*a^2*b^3*c^2*e^3 + 18*a*b^5*c*e^3
+ 6*b^6*c*d*e^2 + 42*a*b^3*c^3*d^2*e - 48*a*b^4*c^2*d*e^2 - 72*a^2*b*c^4*d
^2*e + 108*a^2*b^2*c^3*d*e^2)/(16*(16*a^2*c^7 + b^4*c^5 - 8*a*b^2*c^6)))^(1
/2)*1i)/((((4*a*b^3*c^3*e^5 - 16*a^2*b*c^4*e^5 + 16*a*c^6*d^3*e^2 + 16*a^2*
c^5*d*e^4 - 4*b^4*c^3*d*e^4 - 4*b^2*c^5*d^3*e^2 + 8*b^3*c^4*d^2*e^3 - 32*a*
b*c^5*d^2*e^3 + 12*a*b^2*c^4*d*e^4)/c^3 - (2*(d + e*x^2)^(1/2)*((((4*b^7*e^
3 - 32*a^2*c^5*d^3 - 4*b^4*c^3*d^3 + 24*a*b^2*c^4*d^3 - 80*a^3*b*c^3*e^3 +
96*a^3*c^4*d*e^2 + 12*b^5*c^2*d^2*e + 100*a^2*b^3*c^2*e^3 - 36*a*b^5*c*e^3
- 12*b^6*c*d*e^2 - 84*a*b^3*c^3*d^2*e + 96*a*b^4*c^2*d*e^2 + 144*a^2*b*c^4*
d^2*e - 216*a^2*b^2*c^3*d*e^2)^2/4 - (256*a^2*c^7 + 16*b^4*c^5 - 128*a*b^2*
c^6)*(a^5*e^6 + a^2*c^3*d^6 + 3*a^4*c*d^2*e^4 - a^2*b^3*d^3*e^3 + 3*a^3*b^2
*d^2*e^4 + 3*a^3*c^2*d^4*e^2 - 3*a^4*b*d*e^5 - 3*a^2*b*c^2*d^5*e - 6*a^3*b*
c*d^3*e^3 + 3*a^2*b^2*c*d^4*e^2))^(1/2) - 2*b^7*e^3 + 16*a^2*c^5*d^3 + 2*b^
4*c^3*d^3 - 12*a*b^2*c^4*d^3 + 40*a^3*b*c^3*e^3 - 48*a^3*c^4*d*e^2 - 6*b^5*
c^2*d^2*e - 50*a^2*b^3*c^2*e^3 + 18*a*b^5*c*e^3 + 6*b^6*c*d*e^2 + 42*a*b^3*
c^3*d^2*e - 48*a*b^4*c^2*d*e^2 - 72*a^2*b*c^4*d^2*e + 108*a^2*b^2*c^3*d*e^2
)/(16*(16*a^2*c^7 + b^4*c^5 - 8*a*b^2*c^6)))^(1/2)*(4*b^3*c^5*e^3 - 8*b^2*c
^6*d*e^2 - 16*a*b*c^6*e^3 + 32*a*c^7*d*e^2))/c^3)*((((4*b^7*e^3 - 32*a^2*c^
5*d^3 - 4*b^4*c^3*d^3 + 24*a*b^2*c^4*d^3 - 80*a^3*b*c^3*e^3 + 96*a^3*c^4*d*
e^2 + 12*b^5*c^2*d^2*e + 100*a^2*b^3*c^2*e^3 - 36*a*b^5*c*e^3 - 12*b^6*c*d*
e^2 - 84*a*b^3*c^3*d^2*e + 96*a*b^4*c^2*d*e^2 + 144*a^2*b*c^4*d^2*e - 216*a
^2*b^2*c^3*d*e^2)^2/4 - (256*a^2*c^7 + 16*b^4*c^5 - 128*a*b^2*c^6)*(a^5*e^6
+ a^2*c^3*d^6 + 3*a^4*c*d^2*e^4 - a^2*b^3*d^3*e^3 + 3*a^3*b^2*d^2*e^4 + 3*
a^3*c^2*d^4*e^2 - 3*a^4*b*d*e^5 - 3*a^2*b*c^2*d^5*e - 6*a^3*b*c*d^3*e^3 + 3
*a^2*b^2*c*d^4*e^2))^(1/2) - 2*b^7*e^3 + 16*a^2*c^5*d^3 + 2*b^4*c^3*d^3 - 1
2*a*b^2*c^4*d^3 + 40*a^3*b*c^3*e^3 - 48*a^3*c^4*d*e^2 - 6*b^5*c^2*d^2*e - 5
0*a^2*b^3*c^2*e^3 + 18*a*b^5*c*e^3 + 6*b^6*c*d*e^2 + 42*a*b^3*c^3*d^2*e - 4
8*a*b^4*c^2*d*e^2 - 72*a^2*b*c^4*d^2*e + 108*a^2*b^2*c^3*d*e^2)/(16*(16*a^2
*c^7 + b^4*c^5 - 8*a*b^2*c^6)))^(1/2) - (2*(d + e*x^2)^(1/2)*(b^6*e^6 - 2*a
^3*c^3*e^6 - 2*a*c^5*d^4*e^2 + 9*a^2*b^2*c^2*e^6 + 12*a^2*c^4*d^2*e^4 + b^2
*c^4*d^4*e^2 - 4*b^3*c^3*d^3*e^3 + 6*b^4*c^2*d^2*e^4 - 6*a*b^4*c*e^6 - 4*b^
5*c*d*e^5 + 12*a*b*c^4*d^3*e^3 + 20*a*b^3*c^2*d*e^5 - 20*a^2*b*c^3*d*e^5 -
24*a*b^2*c^3*d^2*e^4))/c^3)*((((4*b^7*e^3 - 32*a^2*c^5*d^3 - 4*b^4*c^3*d^3
+ 24*a*b^2*c^4*d^3 - 80*a^3*b*c^3*e^3 + 96*a^3*c^4*d*e^2 + 12*b^5*c^2*d^2*e
+ 100*a^2*b^3*c^2*e^3 - 36*a*b^5*c*e^3 - 12*b^6*c*d*e^2 - 84*a*b^3*c^3*d^2
*e + 96*a*b^4*c^2*d*e^2 + 144*a^2*b*c^4*d^2*e - 216*a^2*b^2*c^3*d*e^2)^2/4

- (256*a^2*c^7 + 16*b^4*c^5 - 128*a*b^2*c^6)*(a^5*e^6 + a^2*c^3*d^6 + 3*a^4*c*d^2*e^4 - a^2*b^3*d^3*e^3 + 3*a^3*b^2*d^2*e^4 + 3*a^3*c^2*d^4*e^2 - 3*a^4*b*d*e^5 - 3*a^2*b*c^2*d^5*e - 6*a^3*b*c*d^3*e^3 + 3*a^2*b^2*c*d^4*e^2))^(1/2) - 2*b^7*e^3 + 16*a^2*c^5*d^3 + 2*b^4*c^3*d^3 - 12*a*b^2*c^4*d^3 + 40*a^3*b*c^3*e^3 - 48*a^3*c^4*d*e^2 - 6*b^5*c^2*d^2*e - 50*a^2*b^3*c^2*e^3 + 18*a*b^5*c*e^3 + 6*b^6*c*d*e^2 + 42*a*b^3*c^3*d^2*e - 48*a*b^4*c^2*d*e^2 - 72*a^2*b*c^4*d^2*e + 108*a^2*b^2*c^3*d*e^2)/(16*(16*a^2*c^7 + b^4*c^5 - 8*a*b^2*c^6)))^(1/2) + (((4*a*b^3*c^3*e^5 - 16*a^2*b*c^4*e^5 + 16*a*c^6*d^3*e^2 + 16*a^2*c^5*d*e^4 - 4*b^4*c^3*d*e^4 - 4*b^2*c^5*d^3*e^2 + 8*b^3*c^4*d^2*e^3 - 32*a*b*c^5*d^2*e^3 + 12*a*b^2*c^4*d*e^4)/c^3 + (2*(d + e*x^2)^(1/2)*((((4*b^7*e^3 - 32*a^2*c^5*d^3 - 4*b^4*c^3*d^3 + 24*a*b^2*c^4*d^3 - 80*a^3*b*c^3*e^3 + 96*a^3*c^4*d*e^2 + 12*b^5*c^2*d^2*e + 100*a^2*b^3*c^2*e^3 - 36*a*b^5*c*e^3 - 12*b^6*c*d*e^2 - 84*a*b^3*c^3*d^2*e + 96*a*b^4*c^2*d*e^2 + 144*a^2*b*c^4*d^2*e - 216*a^2*b^2*c^3*d*e^2)^2/4 - (256*a^2*c^7 + 16*b^4*c^5 - 128*a*b^2*c^6)*(a^5*e^6 + a^2*c^3*d^6 + 3*a^4*c*d^2*e^4 - a^2*b^3*d^3*e^3 + 3*a^3*b^2*d^2*e^4 + 3*a^3*c^2*d^4*e^2 - 3*a^4*b*d*e^5 - 3*a^2*b*c^2*d^5*e - 6*a^3*b*c*d^3*e^3 + 3*a^2*b^2*c*d^4*e^2))^(1/2) - 2*b^7*e^3 + 16*a^2*c^5*d^3 + 2*b^4*c^3*d^3 - 12*a*b^2*c^4*d^3 + 40*a^3*b*c^3*e^3 - 48*a^3*c^4*d*e^2 - 6*b^5*c^2*d^2*e - 50*a^2*b^3*c^2*e^3 + 18*a*b^5*c*e^3 + 6*b^6*c*d*e^2 + 42*a*b^3*c^3*d^2*e - 48*a*b^4*c^2*d*e^2 - 72*a^2*b*c^4*d^2*e + 108*a^2*b^2*c^3*d*e^2)/(16*(16*a^2*c^7 + b^4*c^5 - 8*a*b^2*c^6)))^(1/2)*(4*b^3*c^5*e^3 - 8*b^2*c^6*d*e^2 - 16*a*b*c^6*e^3 + 32*a*c^7*d*e^2))/c^3)*((((4*b^7*e^3 - 32*a^2*c^5*d^3 - 4*b^4*c^3*d^3 + 24*a*b^2*c^4*d^3 - 80*a^3*b*c^3*e^3 + 96*a^3*c^4*d*e^2 + 12*b^5*c^2*d^2*e + 100*a^2*b^3*c^2*e^3 - 36*a*b^5*c*e^3 - 12*b^6*c*d*e^2 - 84*a*b^3*c^3*d^2*e + 96*a*b^4*c^2*d*e^2 + 144*a^2*b*c^4*d^2*e - 216*a^2*b^2*c^3*d*e^2)^2/4 - (256*a^2*c^7 + 16*b^4*c^5 - 128*a*b^2*c^6)*(a^5*e^6 + a^2*c^3*d^6 + 3*a^4*c*d^2*e^4 - a^2*b^3*d^3*e^3 + 3*a^3*b^2*d^2*e^4 + 3*a^3*c^2*d^4*e^2 - 3*a^4*b*d*e^5 - 3*a^2*b*c^2*d^5*e - 6*a^3*b*c*d^3*e^3 + 3*a^2*b^2*c*d^4*e^2))^(1/2) - 2*b^7*e^3 + 16*a^2*c^5*d^3 + 2*b^4*c^3*d^3 - 12*a*b^2*c^4*d^3 + 40*a^3*b*c^3*e^3 - 48*a^3*c^4*d*e^2 - 6*b^5*c^2*d^2*e - 50*a^2*b^3*c^2*e^3 + 18*a*b^5*c*e^3 + 6*b^6*c*d*e^2 + 42*a*b^3*c^3*d^2*e - 48*a*b^4*c^2*d*e^2 - 72*a^2*b*c^4*d^2*e + 108*a^2*b^2*c^3*d*e^2)/(16*(16*a^2*c^7 + b^4*c^5 - 8*a*b^2*c^6)))^(1/2) + (2*(d + e*x^2)^(1/2)*(b^6*e^6 - 2*a^3*c^3*e^6 - 2*a*c^5*d^4*e^2 + 9*a^2*b^2*c^2*e^6 + 12*a^2*c^4*d^2*e^4 + b^2*c^4*d^4*e^2 - 4*b^3*c^3*d^3*e^3 + 6*b^4*c^2*d^2*e^4 - 6*a*b^4*c*e^6 - 4*b^5*c*d*e^5 + 12*a*b*c^4*d^3*e^3 + 20*a*b^3*c^2*d*e^5 - 20*a^2*b*c^3*d*e^5 - 24*a*b^2*c^3*d^2*e^4))/c^3)*((((4*b^7*e^3 - 32*a^2*c^5*d^3 - 4*b^4*c^3*d^3 + 24*a*b^2*c^4*d^3 - 80*a^3*b*c^3*e^3 + 96*a^3*c^4*d*e^2 + 12*b^5*c^2*d^2*e + 100*a^2*b^3*c^2*e^3 - 36*a*b^5*c*e^3 - 12*b^6*c*d*e^2 - 84*a*b^3*c^3*d^2*e + 96*a*b^4*c^2*d*e^2 + 144*a^2*b*c^4*d^2*e - 216*a^2*b^2*c^3*d*e^2)^2/4 - (256*a^2*c^7 + 16*b^4*c^5 - 128*a*b^2*c^6)*(a^5*e^6 + a^2*c^3*d^6 + 3*a^4*c*d^2*e^4 - a^2*b^3*d^3*e^3 + 3*a^3*b^2*d^2*e^4 + 3*a^3*c^2*d^4*e^2 - 3*a^4*b*d*e^5 - 3*a^2*b*c^2*d^5*e - 6*a^3*b*c*d^3*e^3 + 3*a^2*b^2*c*d^4*e^2))^(1/2) - 2*b^7*e^3 + 16*a^2*c^5*d^3 + 2*b^4*c^3*d^3 - 12*a*b^2*c^4*d^3 + 40*a^3*b*c^3*e^3 - 48*a^3*c^4*d*e^2 - 6*b^5*c^2*d^2*e - 50*a^2*b^3*c^2

```
*e^3 + 18*a*b^5*c*e^3 + 6*b^6*c*d*e^2 + 42*a*b^3*c^3*d^2*e - 48*a*b^4*c^2*d
*e^2 - 72*a^2*b*c^4*d^2*e + 108*a^2*b^2*c^3*d*e^2)/(16*(16*a^2*c^7 + b^4*c^
5 - 8*a*b^2*c^6)))^(1/2) + (2*(a^4*c*e^8 - a^3*b^2*e^8 - a*b^4*d^2*e^6 + 2*
a^2*b^3*d*e^7 - a*c^4*d^6*e^2 - a^2*c^3*d^4*e^4 + a^3*c^2*d^2*e^6 + 4*a*b*c
^3*d^5*e^3 + 4*a*b^3*c*d^3*e^5 - 6*a*b^2*c^2*d^4*e^4 + 4*a^2*b*c^2*d^3*e^5
- 5*a^2*b^2*c*d^2*e^6))/c^3))*((((4*b^7*e^3 - 32*a^2*c^5*d^3 - 4*b^4*c^3*d^
3 + 24*a*b^2*c^4*d^3 - 80*a^3*b*c^3*e^3 + 96*a^3*c^4*d*e^2 + 12*b^5*c^2*d^2
*e + 100*a^2*b^3*c^2*e^3 - 36*a*b^5*c*e^3 - 12*b^6*c*d*e^2 - 84*a*b^3*c^3*d
^2*e + 96*a*b^4*c^2*d*e^2 + 144*a^2*b*c^4*d^2*e - 216*a^2*b^2*c^3*d*e^2)^2/
4 - (256*a^2*c^7 + 16*b^4*c^5 - 128*a*b^2*c^6)*(a^5*e^6 + a^2*c^3*d^6 + 3*a
^4*c*d^2*e^4 - a^2*b^3*d^3*e^3 + 3*a^3*b^2*d^2*e^4 + 3*a^3*c^2*d^4*e^2 - 3*
a^4*b*d*e^5 - 3*a^2*b*c^2*d^5*e - 6*a^3*b*c*d^3*e^3 + 3*a^2*b^2*c*d^4*e^2))
^(1/2) - 2*b^7*e^3 + 16*a^2*c^5*d^3 + 2*b^4*c^3*d^3 - 12*a*b^2*c^4*d^3 + 40
*a^3*b*c^3*e^3 - 48*a^3*c^4*d*e^2 - 6*b^5*c^2*d^2*e - 50*a^2*b^3*c^2*e^3 +
18*a*b^5*c*e^3 + 6*b^6*c*d*e^2 + 42*a*b^3*c^3*d^2*e - 48*a*b^4*c^2*d*e^2 -
72*a^2*b*c^4*d^2*e + 108*a^2*b^2*c^3*d*e^2)/(16*(16*a^2*c^7 + b^4*c^5 - 8*a
*b^2*c^6)))^(1/2)*2i
```

3.368 $\int \frac{x(d+ex^2)^{3/2}}{a+bx^2+cx^4} dx$

Optimal result

Integrand size = 27, antiderivative size = 327

$$\int \frac{x(d+ex^2)^{3/2}}{a+bx^2+cx^4} dx = \frac{e\sqrt{d+ex^2}}{c} - \frac{\left(2c^2d^2+b\left(b-\sqrt{b^2-4ac}\right)e^2-2ce\left(bd-\sqrt{b^2-4ac}d+ae\right)\right)\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex^2}}{\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}}\right)}{\sqrt{2}c^{3/2}\sqrt{b^2-4ac}\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}} + \frac{\left(2c^2d^2+b\left(b+\sqrt{b^2-4ac}\right)e^2-2ce\left(bd+\sqrt{b^2-4ac}d+ae\right)\right)\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex^2}}{\sqrt{2cd-\left(b+\sqrt{b^2-4ac}\right)e}}\right)}{\sqrt{2}c^{3/2}\sqrt{b^2-4ac}\sqrt{2cd-\left(b+\sqrt{b^2-4ac}\right)e}}$$

```
[Out] e*(e*x^2+d)^(1/2)/c-1/2*arctanh(2^(1/2)*c^(1/2)*(e*x^2+d)^(1/2)/(2*c*d-e*(b
-(-4*a*c+b^2)^(1/2)))^(1/2))*(2*c^2*d^2+b*e^2*(b-(-4*a*c+b^2)^(1/2))-2*c*e*
(b*d+a*e-d*(-4*a*c+b^2)^(1/2)))/c^(3/2)*2^(1/2)/(-4*a*c+b^2)^(1/2)/(2*c*d-e
*(b-(-4*a*c+b^2)^(1/2)))^(1/2)+1/2*arctanh(2^(1/2)*c^(1/2)*(e*x^2+d)^(1/2)/
(2*c*d-e*(b+(-4*a*c+b^2)^(1/2)))^(1/2))*(2*c^2*d^2+b*e^2*(b+(-4*a*c+b^2)^(1
/2))-2*c*e*(b*d+a*e+d*(-4*a*c+b^2)^(1/2)))/c^(3/2)*2^(1/2)/(-4*a*c+b^2)^(1/
2)/(2*c*d-e*(b+(-4*a*c+b^2)^(1/2)))^(1/2)
```

Rubi [A] (verified)

Time = 0.81 (sec) , antiderivative size = 327, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$, Rules used = {1261, 717, 840, 1180, 214}

$$\int \frac{x(d+ex^2)^{3/2}}{a+bx^2+cx^4}\,dx =$$
$$-\frac{\left(-2ce\left(-d\sqrt{b^2-4ac}+ae+bd\right)+be^2\left(b-\sqrt{b^2-4ac}\right)+2c^2d^2\right)\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex^2}}{\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}\right)}{\sqrt{2}c^{3/2}\sqrt{b^2-4ac}\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}$$
$$+\frac{\left(-2ce\left(d\sqrt{b^2-4ac}+ae+bd\right)+be^2\left(\sqrt{b^2-4ac}+b\right)+2c^2d^2\right)\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex^2}}{\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}\right)}{\sqrt{2}c^{3/2}\sqrt{b^2-4ac}\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}$$
$$+\frac{e\sqrt{d+ex^2}}{c}$$

```
[In] Int[(x*(d + e*x^2)^(3/2))/(a + b*x^2 + c*x^4),x]
```

```
[Out] (e*Sqrt[d + e*x^2])/c - ((2*c^2*d^2 + b*(b - Sqrt[b^2 - 4*a*c])*e^2 - 2*c*e
*(b*d - Sqrt[b^2 - 4*a*c]*d + a*e))*ArcTanh[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x^2
])/Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]])/(Sqrt[2]*c^(3/2)*Sqrt[b^2 - 4*
a*c]*Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]) + ((2*c^2*d^2 + b*(b + Sqrt[b
^2 - 4*a*c])*e^2 - 2*c*e*(b*d + Sqrt[b^2 - 4*a*c]*d + a*e))*ArcTanh[(Sqrt[2
]*Sqrt[c]*Sqrt[d + e*x^2])/Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]])/(Sqrt[
2]*c^(3/2)*Sqrt[b^2 - 4*a*c]*Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 717

```
Int[((d_.) + (e_.)*(x_))^(m_)/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol
] :> Simp[e*((d + e*x)^(m - 1)/(c*(m - 1))), x] + Dist[1/c, Int[(d + e*x)^(
m - 2)*(Simp[c*d^2 - a*e^2 + e*(2*c*d - b*e)*x, x]/(a + b*x + c*x^2)), x],
x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e
 + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && GtQ[m, 1]
```

Rule 840

```
Int[((f_.) + (g_.)*(x_))/(Sqrt[(d_.) + (e_.)*(x_)]*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)), x_Symbol] :> Dist[2, Subst[Int[(e*f - d*g + g*x^2)/(c*d^2 - b
*d*e + a*e^2 - (2*c*d - b*e)*x^2 + c*x^4), x], x, Sqrt[d + e*x]], x] /; Fre
eQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e +
a*e^2, 0]
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1261

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(
p_.), x_Symbol] :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x],
 x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]
```

Rubi steps

$$\text{integral} = \frac{1}{2}\text{Subst}\left(\int \frac{(d+ex)^{3/2}}{a+bx+cx^2}\,dx, x, x^2\right)$$

$$= \frac{e\sqrt{d+ex^2}}{c} + \frac{\text{Subst}\left(\int \frac{cd^2-ae^2+e(2cd-be)x}{\sqrt{d+ex}(a+bx+cx^2)}\,dx, x, x^2\right)}{2c}$$

$$= \frac{e\sqrt{d+ex^2}}{c} + \frac{\text{Subst}\left(\int \frac{-de(2cd-be)+e(cd^2-ae^2)+e(2cd-be)x^2}{cd^2-bde+ae^2+(-2cd+be)x^2+cx^4}\,dx, x, \sqrt{d+ex^2}\right)}{c}$$

$$= \frac{e\sqrt{d+ex^2}}{c} + \frac{\left(2c^2d^2+b\left(b-\sqrt{b^2-4ac}\right)e^2-2ce\left(bd-\sqrt{b^2-4ac}\,d+ae\right)\right)\text{Subst}\left(\int \frac{1}{-\frac{1}{2}\sqrt{b^2-4ac}\,e+\frac{1}{2}(-2cd+be)+cx^2}\right.}{2c\sqrt{b^2-4ac}} - \frac{\left(2c^2d^2+b\left(b+\sqrt{b^2-4ac}\right)e^2-2ce\left(bd+\sqrt{b^2-4ac}\,d+ae\right)\right)\text{Subst}\left(\int \frac{1}{\frac{1}{2}\sqrt{b^2-4ac}\,e+\frac{1}{2}(-2cd+be)+cx^2}\right.}{2c\sqrt{b^2-4ac}}$$

$$= \frac{e\sqrt{d+ex^2}}{c} - \frac{\left(2c^2d^2+b\left(b-\sqrt{b^2-4ac}\right)e^2-2ce\left(bd-\sqrt{b^2-4ac}\,d+ae\right)\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex^2}}{\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}}\right)}{\sqrt{2}c^{3/2}\sqrt{b^2-4ac}\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}} + \frac{\left(2c^2d^2+b\left(b+\sqrt{b^2-4ac}\right)e^2-2ce\left(bd+\sqrt{b^2-4ac}\,d+ae\right)\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex^2}}{\sqrt{2cd-\left(b+\sqrt{b^2-4ac}\right)e}}\right)}{\sqrt{2}c^{3/2}\sqrt{b^2-4ac}\sqrt{2cd-\left(b+\sqrt{b^2-4ac}\right)e}}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 1.20 (sec) , antiderivative size = 373, normalized size of antiderivative = 1.14

$$\int \frac{x(d+ex^2)^{3/2}}{a+bx^2+cx^4}\,dx = \frac{2\sqrt{c}e\sqrt{d+ex^2} + \frac{\left(-2ic^2d^2-b\left(ib+\sqrt{-b^2+4ac}\right)e^2+2ce\left(ibd+\sqrt{-b^2+4ac}\,d+iae\right)\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex^2}}{\sqrt{-2cd+be-i\sqrt{-b^2+4a}}}\right.}{\sqrt{-\frac{b^2}{2}+2ac}\sqrt{-2cd+\left(b-i\sqrt{-b^2+4ac}\right)e}}}{2c^{3/2}}$$

```
[In] Integrate[(x*(d + e*x^2)^(3/2))/(a + b*x^2 + c*x^4),x]
```

```
[Out] (2*Sqrt[c]*e*Sqrt[d + e*x^2] + (((-2*I)*c^2*d^2 - b*(I*b + Sqrt[-b^2 + 4*a*
c])*e^2 + 2*c*e*(I*b*d + Sqrt[-b^2 + 4*a*c]*d + I*a*e))*ArcTan[(Sqrt[2]*Sqr
t[c]*Sqrt[d + e*x^2])/Sqrt[-2*c*d + b*e - I*Sqrt[-b^2 + 4*a*c]*e]])/(Sqrt[-
1/2*b^2 + 2*a*c]*Sqrt[-2*c*d + (b - I*Sqrt[-b^2 + 4*a*c])*e]) + (((2*I)*c^2
*d^2 - b*((-I)*b + Sqrt[-b^2 + 4*a*c])*e^2 + 2*c*e*((-I)*b*d + Sqrt[-b^2 +
4*a*c]*d - I*a*e))*ArcTan[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x^2])/Sqrt[-2*c*d + b
*e + I*Sqrt[-b^2 + 4*a*c]*e]])/(Sqrt[-1/2*b^2 + 2*a*c]*Sqrt[-2*c*d + (b + I
*Sqrt[-b^2 + 4*a*c])*e]))/(2*c^(3/2))
```

Maple [A] (verified)

Time = 0.57 (sec) , antiderivative size = 335, normalized size of antiderivative = 1.02

method	result
risch	$\frac{e\sqrt{e\,x^2+d}}{c} - \frac{e\sqrt{2}\left(-\frac{\left(2e^2ac-b^2e^2+2bcde-2c^2d^2+\sqrt{-e^2\left(4ac-b^2\right)}\,be-2\sqrt{-e^2\left(4ac-b^2\right)}\,cd\right)\operatorname{arctanh}\left(\frac{c\sqrt{e\,x^2+d}\,\sqrt{2}}{\sqrt{\left(-be+2cd+\sqrt{-e^2\left(4ac-b\right.}\right.}}\right.}{\sqrt{\left(-be+2cd+\sqrt{-e^2\left(4ac-b^2\right)}\right)c}}\right.}{2c\sqrt{}}$
default	$\frac{e\left(\sqrt{e\,x^2+d}-\frac{\left(-2e^2ac+b^2e^2-2bcde+2c^2d^2+\sqrt{-e^2\left(4ac-b^2\right)}\,be-2\sqrt{-e^2\left(4ac-b^2\right)}\,cd\right)\sqrt{2}\,\arctan\left(\frac{c\sqrt{e\,x^2+d}\,\sqrt{2}}{\sqrt{\left(be-2cd+\sqrt{-e^2\left(4ac-b^2\right)}\right)c}}\right)}{2\sqrt{-e^2\left(4ac-b^2\right)}\,\sqrt{\left(be-2cd+\sqrt{-e^2\left(4ac-b^2\right)}\right)c}}\right.}{c}$
pseudoelliptic	$\frac{e\left(\sqrt{e\,x^2+d}-\frac{\left(-2e^2ac+b^2e^2-2bcde+2c^2d^2+\sqrt{-e^2\left(4ac-b^2\right)}\,be-2\sqrt{-e^2\left(4ac-b^2\right)}\,cd\right)\sqrt{2}\,\arctan\left(\frac{c\sqrt{e\,x^2+d}\,\sqrt{2}}{\sqrt{\left(be-2cd+\sqrt{-e^2\left(4ac-b^2\right)}\right)c}}\right)}{2\sqrt{-e^2\left(4ac-b^2\right)}\,\sqrt{\left(be-2cd+\sqrt{-e^2\left(4ac-b^2\right)}\right)c}}\right.}{c}$

```
[In] int(x*(e*x^2+d)^(3/2)/(c*x^4+b*x^2+a),x,method=_RETURNVERBOSE)
```

```
[Out] e*(e*x^2+d)^(1/2)/c-1/2/c*e*2^(1/2)/(-e^2*(4*a*c-b^2))^(1/2)*(-(2*e^2*a*c-b
^2*e^2+2*b*c*d*e-2*c^2*d^2+(-e^2*(4*a*c-b^2))^(1/2)*b*e-2*(-e^2*(4*a*c-b^2)
)^(1/2)*c*d)/((-b*e+2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2)*arctanh(c*(e*x
^2+d)^(1/2)*2^(1/2)/((-b*e+2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2))+(-2*e^
2*a*c+b^2*e^2-2*b*c*d*e+2*c^2*d^2+(-e^2*(4*a*c-b^2))^(1/2)*b*e-2*(-e^2*(4*a
*c-b^2))^(1/2)*c*d)/((b*e-2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2)*arctan(c
*(e*x^2+d)^(1/2)*2^(1/2)/((b*e-2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 4444 vs. $2(278) = 556$.

Time = 65.87 (sec) , antiderivative size = 4444, normalized size of antiderivative = 13.59

$$\int \frac{x(d+ex^2)^{3/2}}{a+bx^2+cx^4}\,dx = \text{Too large to display}$$

```
[In] integrate(x*(e*x^2+d)^(3/2)/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] -1/4*(sqrt(1/2)*c*sqrt((2*c^3*d^3 - 3*b*c^2*d^2*e + 3*(b^2*c - 2*a*c^2)*d*e
^2 - (b^3 - 3*a*b*c)*e^3 + (b^2*c^3 - 4*a*c^4)*sqrt((9*c^4*d^4*e^2 - 18*b*c
^3*d^3*e^3 + 3*(5*b^2*c^2 - 2*a*c^3)*d^2*e^4 - 6*(b^3*c - a*b*c^2)*d*e^5 +
(b^4 - 2*a*b^2*c + a^2*c^2)*e^6)/(b^2*c^6 - 4*a*c^7)))/(b^2*c^3 - 4*a*c^4))
```

```
*log(-(6*b*c^3*d^5*e - 6*(2*b^2*c^2 + a*c^3)*d^4*e^2 + 8*(b^3*c + 2*a*b*c^2
)*d^3*e^3 - 2*(b^4 + 6*a*b^2*c + 2*a^2*c^2)*d^2*e^4 + 2*(2*a*b^3 + a^2*b*c)
*d*e^5 - 2*(a^2*b^2 - a^3*c)*e^6 + (3*b*c^3*d^4*e^2 - 6*b^2*c^2*d^3*e^3 + 2
*(2*b^3*c + a*b*c^2)*d^2*e^4 - (b^4 + 2*a*b^2*c)*d*e^5 + (a*b^3 - a^2*b*c)*
e^6)*x^2 + 2*sqrt(1/2)*(3*(b^2*c^3 - 4*a*c^4)*d^3*e - 6*(b^3*c^2 - 4*a*b*c^
3)*d^2*e^2 + (4*b^4*c - 17*a*b^2*c^2 + 4*a^2*c^3)*d*e^3 - (b^5 - 5*a*b^3*c
+ 4*a^2*b*c^2)*e^4 + ((b^3*c^4 - 4*a*b*c^5)*d - (b^4*c^3 - 6*a*b^2*c^4 + 8*
a^2*c^5)*e)*sqrt((9*c^4*d^4*e^2 - 18*b*c^3*d^3*e^3 + 3*(5*b^2*c^2 - 2*a*c^3
)*d^2*e^4 - 6*(b^3*c - a*b*c^2)*d*e^5 + (b^4 - 2*a*b^2*c + a^2*c^2)*e^6)/(b
^2*c^6 - 4*a*c^7)))*sqrt(e*x^2 + d)*sqrt((2*c^3*d^3 - 3*b*c^2*d^2*e + 3*(b^
2*c - 2*a*c^2)*d*e^2 - (b^3 - 3*a*b*c)*e^3 + (b^2*c^3 - 4*a*c^4)*sqrt((9*c^
4*d^4*e^2 - 18*b*c^3*d^3*e^3 + 3*(5*b^2*c^2 - 2*a*c^3)*d^2*e^4 - 6*(b^3*c -
 a*b*c^2)*d*e^5 + (b^4 - 2*a*b^2*c + a^2*c^2)*e^6)/(b^2*c^6 - 4*a*c^7)))/(b
^2*c^3 - 4*a*c^4)) + (2*(b^2*c^4 - 4*a*c^5)*d^3 - 2*(b^3*c^3 - 4*a*b*c^4)*d
^2*e + 2*(a*b^2*c^3 - 4*a^2*c^4)*d*e^2 + ((b^2*c^4 - 4*a*c^5)*d^2*e - (b^3*
c^3 - 4*a*b*c^4)*d*e^2 + (a*b^2*c^3 - 4*a^2*c^4)*e^3)*x^2)*sqrt((9*c^4*d^4*
e^2 - 18*b*c^3*d^3*e^3 + 3*(5*b^2*c^2 - 2*a*c^3)*d^2*e^4 - 6*(b^3*c - a*b*c
^2)*d*e^5 + (b^4 - 2*a*b^2*c + a^2*c^2)*e^6)/(b^2*c^6 - 4*a*c^7)))/x^2) - s
qrt(1/2)*c*sqrt((2*c^3*d^3 - 3*b*c^2*d^2*e + 3*(b^2*c - 2*a*c^2)*d*e^2 - (b
^3 - 3*a*b*c)*e^3 + (b^2*c^3 - 4*a*c^4)*sqrt((9*c^4*d^4*e^2 - 18*b*c^3*d^3*
e^3 + 3*(5*b^2*c^2 - 2*a*c^3)*d^2*e^4 - 6*(b^3*c - a*b*c^2)*d*e^5 + (b^4 -
2*a*b^2*c + a^2*c^2)*e^6)/(b^2*c^6 - 4*a*c^7)))/(b^2*c^3 - 4*a*c^4))*log(-(
6*b*c^3*d^5*e - 6*(2*b^2*c^2 + a*c^3)*d^4*e^2 + 8*(b^3*c + 2*a*b*c^2)*d^3*e
^3 - 2*(b^4 + 6*a*b^2*c + 2*a^2*c^2)*d^2*e^4 + 2*(2*a*b^3 + a^2*b*c)*d*e^5
- 2*(a^2*b^2 - a^3*c)*e^6 + (3*b*c^3*d^4*e^2 - 6*b^2*c^2*d^3*e^3 + 2*(2*b^3
*c + a*b*c^2)*d^2*e^4 - (b^4 + 2*a*b^2*c)*d*e^5 + (a*b^3 - a^2*b*c)*e^6)*x^
2 - 2*sqrt(1/2)*(3*(b^2*c^3 - 4*a*c^4)*d^3*e - 6*(b^3*c^2 - 4*a*b*c^3)*d^2*
e^2 + (4*b^4*c - 17*a*b^2*c^2 + 4*a^2*c^3)*d*e^3 - (b^5 - 5*a*b^3*c + 4*a^2
*b*c^2)*e^4 + ((b^3*c^4 - 4*a*b*c^5)*d - (b^4*c^3 - 6*a*b^2*c^4 + 8*a^2*c^5
)*e)*sqrt((9*c^4*d^4*e^2 - 18*b*c^3*d^3*e^3 + 3*(5*b^2*c^2 - 2*a*c^3)*d^2*e
^4 - 6*(b^3*c - a*b*c^2)*d*e^5 + (b^4 - 2*a*b^2*c + a^2*c^2)*e^6)/(b^2*c^6
- 4*a*c^7)))*sqrt(e*x^2 + d)*sqrt((2*c^3*d^3 - 3*b*c^2*d^2*e + 3*(b^2*c - 2
*a*c^2)*d*e^2 - (b^3 - 3*a*b*c)*e^3 + (b^2*c^3 - 4*a*c^4)*sqrt((9*c^4*d^4*e
^2 - 18*b*c^3*d^3*e^3 + 3*(5*b^2*c^2 - 2*a*c^3)*d^2*e^4 - 6*(b^3*c - a*b*c^
2)*d*e^5 + (b^4 - 2*a*b^2*c + a^2*c^2)*e^6)/(b^2*c^6 - 4*a*c^7)))/(b^2*c^3
- 4*a*c^4)) + (2*(b^2*c^4 - 4*a*c^5)*d^3 - 2*(b^3*c^3 - 4*a*b*c^4)*d^2*e +
2*(a*b^2*c^3 - 4*a^2*c^4)*d*e^2 + ((b^2*c^4 - 4*a*c^5)*d^2*e - (b^3*c^3 - 4
*a*b*c^4)*d*e^2 + (a*b^2*c^3 - 4*a^2*c^4)*e^3)*x^2)*sqrt((9*c^4*d^4*e^2 - 1
8*b*c^3*d^3*e^3 + 3*(5*b^2*c^2 - 2*a*c^3)*d^2*e^4 - 6*(b^3*c - a*b*c^2)*d*e
^5 + (b^4 - 2*a*b^2*c + a^2*c^2)*e^6)/(b^2*c^6 - 4*a*c^7)))/x^2) + sqrt(1/2
)*c*sqrt((2*c^3*d^3 - 3*b*c^2*d^2*e + 3*(b^2*c - 2*a*c^2)*d*e^2 - (b^3 - 3*
a*b*c)*e^3 - (b^2*c^3 - 4*a*c^4)*sqrt((9*c^4*d^4*e^2 - 18*b*c^3*d^3*e^3 + 3
*(5*b^2*c^2 - 2*a*c^3)*d^2*e^4 - 6*(b^3*c - a*b*c^2)*d*e^5 + (b^4 - 2*a*b^2
*c + a^2*c^2)*e^6)/(b^2*c^6 - 4*a*c^7)))/(b^2*c^3 - 4*a*c^4))*log(-(6*b*c^3
*d^5*e - 6*(2*b^2*c^2 + a*c^3)*d^4*e^2 + 8*(b^3*c + 2*a*b*c^2)*d^3*e^3 - 2*
```

(b^4 + 6*a*b^2*c + 2*a^2*c^2)*d^2*e^4 + 2*(2*a*b^3 + a^2*b*c)*d*e^5 - 2*(a^
2*b^2 - a^3*c)*e^6 + (3*b*c^3*d^4*e^2 - 6*b^2*c^2*d^3*e^3 + 2*(2*b^3*c + a*
b*c^2)*d^2*e^4 - (b^4 + 2*a*b^2*c)*d*e^5 + (a*b^3 - a^2*b*c)*e^6)*x^2 + 2*s
qrt(1/2)*(3*(b^2*c^3 - 4*a*c^4)*d^3*e - 6*(b^3*c^2 - 4*a*b*c^3)*d^2*e^2 + (
4*b^4*c - 17*a*b^2*c^2 + 4*a^2*c^3)*d*e^3 - (b^5 - 5*a*b^3*c + 4*a^2*b*c^2)
*e^4 - ((b^3*c^4 - 4*a*b*c^5)*d - (b^4*c^3 - 6*a*b^2*c^4 + 8*a^2*c^5)*e)*sq
rt((9*c^4*d^4*e^2 - 18*b*c^3*d^3*e^3 + 3*(5*b^2*c^2 - 2*a*c^3)*d^2*e^4 - 6*
(b^3*c - a*b*c^2)*d*e^5 + (b^4 - 2*a*b^2*c + a^2*c^2)*e^6)/(b^2*c^6 - 4*a*c
^7)))*sqrt(e*x^2 + d)*sqrt((2*c^3*d^3 - 3*b*c^2*d^2*e + 3*(b^2*c - 2*a*c^2)
*d*e^2 - (b^3 - 3*a*b*c)*e^3 - (b^2*c^3 - 4*a*c^4)*sqrt((9*c^4*d^4*e^2 - 18
*b*c^3*d^3*e^3 + 3*(5*b^2*c^2 - 2*a*c^3)*d^2*e^4 - 6*(b^3*c - a*b*c^2)*d*e^
5 + (b^4 - 2*a*b^2*c + a^2*c^2)*e^6)/(b^2*c^6 - 4*a*c^7)))/(b^2*c^3 - 4*a*c
^4)) - (2*(b^2*c^4 - 4*a*c^5)*d^3 - 2*(b^3*c^3 - 4*a*b*c^4)*d^2*e + 2*(a*b^
2*c^3 - 4*a^2*c^4)*d*e^2 + ((b^2*c^4 - 4*a*c^5)*d^2*e - (b^3*c^3 - 4*a*b*c^
4)*d*e^2 + (a*b^2*c^3 - 4*a^2*c^4)*e^3)*x^2)*sqrt((9*c^4*d^4*e^2 - 18*b*c^3
*d^3*e^3 + 3*(5*b^2*c^2 - 2*a*c^3)*d^2*e^4 - 6*(b^3*c - a*b*c^2)*d*e^5 + (b
^4 - 2*a*b^2*c + a^2*c^2)*e^6)/(b^2*c^6 - 4*a*c^7)))/x^2) - sqrt(1/2)*c*sqr
t((2*c^3*d^3 - 3*b*c^2*d^2*e + 3*(b^2*c - 2*a*c^2)*d*e^2 - (b^3 - 3*a*b*c)*
e^3 - (b^2*c^3 - 4*a*c^4)*sqrt((9*c^4*d^4*e^2 - 18*b*c^3*d^3*e^3 + 3*(5*b^2
*c^2 - 2*a*c^3)*d^2*e^4 - 6*(b^3*c - a*b*c^2)*d*e^5 + (b^4 - 2*a*b^2*c + a^
2*c^2)*e^6)/(b^2*c^6 - 4*a*c^7)))/(b^2*c^3 - 4*a*c^4))*log(-(6*b*c^3*d^5*e
- 6*(2*b^2*c^2 + a*c^3)*d^4*e^2 + 8*(b^3*c + 2*a*b*c^2)*d^3*e^3 - 2*(b^4 +
6*a*b^2*c + 2*a^2*c^2)*d^2*e^4 + 2*(2*a*b^3 + a^2*b*c)*d*e^5 - 2*(a^2*b^2 -
a^3*c)*e^6 + (3*b*c^3*d^4*e^2 - 6*b^2*c^2*d^3*e^3 + 2*(2*b^3*c + a*b*c^2)*
d^2*e^4 - (b^4 + 2*a*b^2*c)*d*e^5 + (a*b^3 - a^2*b*c)*e^6)*x^2 - 2*sqrt(1/2
)*(3*(b^2*c^3 - 4*a*c^4)*d^3*e - 6*(b^3*c^2 - 4*a*b*c^3)*d^2*e^2 + (4*b^4*c
- 17*a*b^2*c^2 + 4*a^2*c^3)*d*e^3 - (b^5 - 5*a*b^3*c + 4*a^2*b*c^2)*e^4 -
((b^3*c^4 - 4*a*b*c^5)*d - (b^4*c^3 - 6*a*b^2*c^4 + 8*a^2*c^5)*e)*sqrt((9*c
^4*d^4*e^2 - 18*b*c^3*d^3*e^3 + 3*(5*b^2*c^2 - 2*a*c^3)*d^2*e^4 - 6*(b^3*c
- a*b*c^2)*d*e^5 + (b^4 - 2*a*b^2*c + a^2*c^2)*e^6)/(b^2*c^6 - 4*a*c^7)))*s
qrt(e*x^2 + d)*sqrt((2*c^3*d^3 - 3*b*c^2*d^2*e + 3*(b^2*c - 2*a*c^2)*d*e^2
- (b^3 - 3*a*b*c)*e^3 - (b^2*c^3 - 4*a*c^4)*sqrt((9*c^4*d^4*e^2 - 18*b*c^3*
d^3*e^3 + 3*(5*b^2*c^2 - 2*a*c^3)*d^2*e^4 - 6*(b^3*c - a*b*c^2)*d*e^5 + (b^
4 - 2*a*b^2*c + a^2*c^2)*e^6)/(b^2*c^6 - 4*a*c^7)))/(b^2*c^3 - 4*a*c^4)) -
(2*(b^2*c^4 - 4*a*c^5)*d^3 - 2*(b^3*c^3 - 4*a*b*c^4)*d^2*e + 2*(a*b^2*c^3 -
4*a^2*c^4)*d*e^2 + ((b^2*c^4 - 4*a*c^5)*d^2*e - (b^3*c^3 - 4*a*b*c^4)*d*e^
2 + (a*b^2*c^3 - 4*a^2*c^4)*e^3)*x^2)*sqrt((9*c^4*d^4*e^2 - 18*b*c^3*d^3*e^
3 + 3*(5*b^2*c^2 - 2*a*c^3)*d^2*e^4 - 6*(b^3*c - a*b*c^2)*d*e^5 + (b^4 - 2*
a*b^2*c + a^2*c^2)*e^6)/(b^2*c^6 - 4*a*c^7)))/x^2) - 4*sqrt(e*x^2 + d)*e)/c

Sympy [F]

$$\int \frac{x(d+ex^2)^{3/2}}{a+bx^2+cx^4}\,dx = \int \frac{x(d+ex^2)^{\frac{3}{2}}}{a+bx^2+cx^4}\,dx$$

```
[In] integrate(x*(e*x**2+d)**(3/2)/(c*x**4+b*x**2+a),x)
[Out] Integral(x*(d + e*x**2)**(3/2)/(a + b*x**2 + c*x**4), x)
```

Maxima [F]

$$\int \frac{x(d+ex^2)^{3/2}}{a+bx^2+cx^4}\,dx = \int \frac{(ex^2+d)^{\frac{3}{2}}x}{cx^4+bx^2+a}\,dx$$

```
[In] integrate(x*(e*x^2+d)^(3/2)/(c*x^4+b*x^2+a),x, algorithm="maxima")
[Out] integrate((e*x^2 + d)^(3/2)*x/(c*x^4 + b*x^2 + a), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 802 vs. 2(278) = 556.

Time = 0.34 (sec) , antiderivative size = 802, normalized size of antiderivative = 2.45

$$\int \frac{x(d+ex^2)^{3/2}}{a+bx^2+cx^4}\,dx = \frac{\sqrt{ex^2+d}e}{c}$$

$$+ \frac{\left(\sqrt{-4\,c^2d+2\,(bc-\sqrt{b^2-4\,ac}c)e}(2\,(b^2c-4\,ac^2)de-(b^3-4\,abc)e^2)c^2e^2-2\,(\sqrt{b^2-4\,ac}c^3d^2e-\sqrt{b^2-4}\right.}{}$$

$$- \frac{\left(\sqrt{-4\,c^2d+2\,(bc+\sqrt{b^2-4\,ac}c)e}(2\,(b^2c-4\,ac^2)de-(b^3-4\,abc)e^2)c^2e^2+2\,(\sqrt{b^2-4\,ac}c^3d^2e-\sqrt{b^2-4}\right.}{}$$

```
[In] integrate(x*(e*x^2+d)^(3/2)/(c*x^4+b*x^2+a),x, algorithm="giac")
[Out] sqrt(e*x^2 + d)*e/c + 1/8*(sqrt(-4*c^2*d + 2*(b*c - sqrt(b^2 - 4*a*c)*c)*e)
*(2*(b^2*c - 4*a*c^2)*d*e - (b^3 - 4*a*b*c)*e^2)*c^2*e^2 - 2*(sqrt(b^2 - 4*
a*c)*c^3*d^2*e - sqrt(b^2 - 4*a*c)*b*c^2*d*e^2 + sqrt(b^2 - 4*a*c)*a*c^2*e^
3)*sqrt(-4*c^2*d + 2*(b*c - sqrt(b^2 - 4*a*c)*c)*e)*abs(c)*abs(e) - (4*c^5*
```

```
d^3*e - 6*b*c^4*d^2*e^2 + 4*(b^2*c^3 - a*c^4)*d*e^3 - (b^3*c^2 - 2*a*b*c^3)
*e^4)*sqrt(-4*c^2*d + 2*(b*c - sqrt(b^2 - 4*a*c)*c)*e))*arctan(2*sqrt(1/2)*
sqrt(e*x^2 + d)/sqrt(-(2*c^2*d - b*c*e + sqrt(-4*(c^2*d^2 - b*c*d*e + a*c*e
^2)*c^2 + (2*c^2*d - b*c*e)^2))/c^2))/((sqrt(b^2 - 4*a*c)*c^4*d^2 - sqrt(b^
2 - 4*a*c)*b*c^3*d*e + sqrt(b^2 - 4*a*c)*a*c^3*e^2)*c^2*abs(e)) - 1/8*(sqrt
(-4*c^2*d + 2*(b*c + sqrt(b^2 - 4*a*c)*c)*e)*(2*(b^2*c - 4*a*c^2)*d*e - (b^
3 - 4*a*b*c)*e^2)*c^2*e^2 + 2*(sqrt(b^2 - 4*a*c)*c^3*d^2*e - sqrt(b^2 - 4*a
*c)*b*c^2*d*e^2 + sqrt(b^2 - 4*a*c)*a*c^2*e^3)*sqrt(-4*c^2*d + 2*(b*c + sqr
t(b^2 - 4*a*c)*c)*e)*abs(c)*abs(e) - (4*c^5*d^3*e - 6*b*c^4*d^2*e^2 + 4*(b^
2*c^3 - a*c^4)*d*e^3 - (b^3*c^2 - 2*a*b*c^3)*e^4)*sqrt(-4*c^2*d + 2*(b*c +
sqrt(b^2 - 4*a*c)*c)*e))*arctan(2*sqrt(1/2)*sqrt(e*x^2 + d)/sqrt(-(2*c^2*d
- b*c*e - sqrt(-4*(c^2*d^2 - b*c*d*e + a*c*e^2)*c^2 + (2*c^2*d - b*c*e)^2))
/c^2))/((sqrt(b^2 - 4*a*c)*c^4*d^2 - sqrt(b^2 - 4*a*c)*b*c^3*d*e + sqrt(b^2
 - 4*a*c)*a*c^3*e^2)*c^2*abs(e))
```

Mupad [B] (verification not implemented)

Time = 9.76 (sec) , antiderivative size = 12392, normalized size of antiderivative = 37.90

$$\int \frac{x(d+ex^2)^{3/2}}{a+bx^2+cx^4}\,dx = \text{Too large to display}$$

```
[In] int((x*(d + e*x^2)^(3/2))/(a + b*x^2 + c*x^4),x)
```

```
[Out] (e*(d + e*x^2)^(1/2))/c - atan(((((16*a^2*c^3*e^5 - 4*a*b^2*c^2*e^5 + 16*a*
c^4*d^2*e^3 + 4*b^3*c^2*d*e^4 - 4*b^2*c^3*d^2*e^3 - 16*a*b*c^3*d*e^4)/c - (
2*(d + e*x^2)^(1/2)*(-(((4*b^5*e^3 + 32*a*c^4*d^3 - 8*b^2*c^3*d^3 + 48*a^2*
b*c^2*e^3 - 96*a^2*c^3*d*e^2 + 12*b^3*c^2*d^2*e - 28*a*b^3*c*e^3 - 12*b^4*c
*d*e^2 - 48*a*b*c^3*d^2*e + 72*a*b^2*c^2*d*e^2)^2/4 - (256*a^2*c^5 + 16*b^4
*c^3 - 128*a*b^2*c^4)*(a^3*e^6 + c^3*d^6 - b^3*d^3*e^3 + 3*a*b^2*d^2*e^4 +
3*a*c^2*d^4*e^2 + 3*a^2*c*d^2*e^4 + 3*b^2*c*d^4*e^2 - 3*a^2*b*d*e^5 - 3*b*c
^2*d^5*e - 6*a*b*c*d^3*e^3))^(1/2) + 2*b^5*e^3 + 16*a*c^4*d^3 - 4*b^2*c^3*d
^3 + 24*a^2*b*c^2*e^3 - 48*a^2*c^3*d*e^2 + 6*b^3*c^2*d^2*e - 14*a*b^3*c*e^3
 - 6*b^4*c*d*e^2 - 24*a*b*c^3*d^2*e + 36*a*b^2*c^2*d*e^2)/(16*(16*a^2*c^5 +
 b^4*c^3 - 8*a*b^2*c^4)))^(1/2)*(4*b^3*c^3*e^3 - 8*b^2*c^4*d*e^2 - 16*a*b*c
^4*e^3 + 32*a*c^5*d*e^2))/c)*(-(((4*b^5*e^3 + 32*a*c^4*d^3 - 8*b^2*c^3*d^3
+ 48*a^2*b*c^2*e^3 - 96*a^2*c^3*d*e^2 + 12*b^3*c^2*d^2*e - 28*a*b^3*c*e^3 -
 12*b^4*c*d*e^2 - 48*a*b*c^3*d^2*e + 72*a*b^2*c^2*d*e^2)^2/4 - (256*a^2*c^5
 + 16*b^4*c^3 - 128*a*b^2*c^4)*(a^3*e^6 + c^3*d^6 - b^3*d^3*e^3 + 3*a*b^2*d
^2*e^4 + 3*a*c^2*d^4*e^2 + 3*a^2*c*d^2*e^4 + 3*b^2*c*d^4*e^2 - 3*a^2*b*d*e^
5 - 3*b*c^2*d^5*e - 6*a*b*c*d^3*e^3))^(1/2) + 2*b^5*e^3 + 16*a*c^4*d^3 - 4*
b^2*c^3*d^3 + 24*a^2*b*c^2*e^3 - 48*a^2*c^3*d*e^2 + 6*b^3*c^2*d^2*e - 14*a*
b^3*c*e^3 - 6*b^4*c*d*e^2 - 24*a*b*c^3*d^2*e + 36*a*b^2*c^2*d*e^2)/(16*(16*
a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2) - (2*(d + e*x^2)^(1/2)*(b^4*e^6 +
2*a^2*c^2*e^6 + 2*c^4*d^4*e^2 - 12*a*c^3*d^2*e^4 - 4*b*c^3*d^3*e^3 + 6*b^2*
```

c^2*d^2*e^4 - 4*a*b^2*c*e^6 - 4*b^3*c*d*e^5 + 12*a*b*c^2*d*e^5))/c)*(-(((4*
b^5*e^3 + 32*a*c^4*d^3 - 8*b^2*c^3*d^3 + 48*a^2*b*c^2*e^3 - 96*a^2*c^3*d*e^
2 + 12*b^3*c^2*d^2*e - 28*a*b^3*c*e^3 - 12*b^4*c*d*e^2 - 48*a*b*c^3*d^2*e +
 72*a*b^2*c^2*d*e^2)^2/4 - (256*a^2*c^5 + 16*b^4*c^3 - 128*a*b^2*c^4)*(a^3*
e^6 + c^3*d^6 - b^3*d^3*e^3 + 3*a*b^2*d^2*e^4 + 3*a*c^2*d^4*e^2 + 3*a^2*c*d
^2*e^4 + 3*b^2*c*d^4*e^2 - 3*a^2*b*d*e^5 - 3*b*c^2*d^5*e - 6*a*b*c*d^3*e^3)
)^(1/2) + 2*b^5*e^3 + 16*a*c^4*d^3 - 4*b^2*c^3*d^3 + 24*a^2*b*c^2*e^3 - 48*
a^2*c^3*d*e^2 + 6*b^3*c^2*d^2*e - 14*a*b^3*c*e^3 - 6*b^4*c*d*e^2 - 24*a*b*c
^3*d^2*e + 36*a*b^2*c^2*d*e^2)/(16*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(
1/2)*1i - (((16*a^2*c^3*e^5 - 4*a*b^2*c^2*e^5 + 16*a*c^4*d^2*e^3 + 4*b^3*c^
2*d*e^4 - 4*b^2*c^3*d^2*e^3 - 16*a*b*c^3*d*e^4)/c + (2*(d + e*x^2)^(1/2)*(-
(((4*b^5*e^3 + 32*a*c^4*d^3 - 8*b^2*c^3*d^3 + 48*a^2*b*c^2*e^3 - 96*a^2*c^3
*d*e^2 + 12*b^3*c^2*d^2*e - 28*a*b^3*c*e^3 - 12*b^4*c*d*e^2 - 48*a*b*c^3*d^
2*e + 72*a*b^2*c^2*d*e^2)^2/4 - (256*a^2*c^5 + 16*b^4*c^3 - 128*a*b^2*c^4)*
(a^3*e^6 + c^3*d^6 - b^3*d^3*e^3 + 3*a*b^2*d^2*e^4 + 3*a*c^2*d^4*e^2 + 3*a^
2*c*d^2*e^4 + 3*b^2*c*d^4*e^2 - 3*a^2*b*d*e^5 - 3*b*c^2*d^5*e - 6*a*b*c*d^3
*e^3))^(1/2) + 2*b^5*e^3 + 16*a*c^4*d^3 - 4*b^2*c^3*d^3 + 24*a^2*b*c^2*e^3
 - 48*a^2*c^3*d*e^2 + 6*b^3*c^2*d^2*e - 14*a*b^3*c*e^3 - 6*b^4*c*d*e^2 - 24*
a*b*c^3*d^2*e + 36*a*b^2*c^2*d*e^2)/(16*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4
)))^(1/2)*(4*b^3*c^3*e^3 - 8*b^2*c^4*d*e^2 - 16*a*b*c^4*e^3 + 32*a*c^5*d*e^
2))/c)*(-(((4*b^5*e^3 + 32*a*c^4*d^3 - 8*b^2*c^3*d^3 + 48*a^2*b*c^2*e^3 - 9
6*a^2*c^3*d*e^2 + 12*b^3*c^2*d^2*e - 28*a*b^3*c*e^3 - 12*b^4*c*d*e^2 - 48*a
*b*c^3*d^2*e + 72*a*b^2*c^2*d*e^2)^2/4 - (256*a^2*c^5 + 16*b^4*c^3 - 128*a*
b^2*c^4)*(a^3*e^6 + c^3*d^6 - b^3*d^3*e^3 + 3*a*b^2*d^2*e^4 + 3*a*c^2*d^4*e
^2 + 3*a^2*c*d^2*e^4 + 3*b^2*c*d^4*e^2 - 3*a^2*b*d*e^5 - 3*b*c^2*d^5*e - 6*
a*b*c*d^3*e^3))^(1/2) + 2*b^5*e^3 + 16*a*c^4*d^3 - 4*b^2*c^3*d^3 + 24*a^2*b
*c^2*e^3 - 48*a^2*c^3*d*e^2 + 6*b^3*c^2*d^2*e - 14*a*b^3*c*e^3 - 6*b^4*c*d*
e^2 - 24*a*b*c^3*d^2*e + 36*a*b^2*c^2*d*e^2)/(16*(16*a^2*c^5 + b^4*c^3 - 8*
a*b^2*c^4)))^(1/2) + (2*(d + e*x^2)^(1/2)*(b^4*e^6 + 2*a^2*c^2*e^6 + 2*c^4*
d^4*e^2 - 12*a*c^3*d^2*e^4 - 4*b*c^3*d^3*e^3 + 6*b^2*c^2*d^2*e^4 - 4*a*b^2*
c*e^6 - 4*b^3*c*d*e^5 + 12*a*b*c^2*d*e^5))/c)*(-(((4*b^5*e^3 + 32*a*c^4*d^3
 - 8*b^2*c^3*d^3 + 48*a^2*b*c^2*e^3 - 96*a^2*c^3*d*e^2 + 12*b^3*c^2*d^2*e -
 28*a*b^3*c*e^3 - 12*b^4*c*d*e^2 - 48*a*b*c^3*d^2*e + 72*a*b^2*c^2*d*e^2)^2
/4 - (256*a^2*c^5 + 16*b^4*c^3 - 128*a*b^2*c^4)*(a^3*e^6 + c^3*d^6 - b^3*d^
3*e^3 + 3*a*b^2*d^2*e^4 + 3*a*c^2*d^4*e^2 + 3*a^2*c*d^2*e^4 + 3*b^2*c*d^4*e
^2 - 3*a^2*b*d*e^5 - 3*b*c^2*d^5*e - 6*a*b*c*d^3*e^3))^(1/2) + 2*b^5*e^3 +
16*a*c^4*d^3 - 4*b^2*c^3*d^3 + 24*a^2*b*c^2*e^3 - 48*a^2*c^3*d*e^2 + 6*b^3*
c^2*d^2*e - 14*a*b^3*c*e^3 - 6*b^4*c*d*e^2 - 24*a*b*c^3*d^2*e + 36*a*b^2*c^
2*d*e^2)/(16*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2)*1i)/((((16*a^2*c^
3*e^5 - 4*a*b^2*c^2*e^5 + 16*a*c^4*d^2*e^3 + 4*b^3*c^2*d*e^4 - 4*b^2*c^3*d^
2*e^3 - 16*a*b*c^3*d*e^4)/c - (2*(d + e*x^2)^(1/2)*(-(((4*b^5*e^3 + 32*a*c^
4*d^3 - 8*b^2*c^3*d^3 + 48*a^2*b*c^2*e^3 - 96*a^2*c^3*d*e^2 + 12*b^3*c^2*d^
2*e - 28*a*b^3*c*e^3 - 12*b^4*c*d*e^2 - 48*a*b*c^3*d^2*e + 72*a*b^2*c^2*d*e
^2)^2/4 - (256*a^2*c^5 + 16*b^4*c^3 - 128*a*b^2*c^4)*(a^3*e^6 + c^3*d^6 - b
^3*d^3*e^3 + 3*a*b^2*d^2*e^4 + 3*a*c^2*d^4*e^2 + 3*a^2*c*d^2*e^4 + 3*b^2*c*

d^4*e^2 - 3*a^2*b*d*e^5 - 3*b*c^2*d^5*e - 6*a*b*c*d^3*e^3))^(1/2) + 2*b^5*e^3 + 16*a*c^4*d^3 - 4*b^2*c^3*d^3 + 24*a^2*b*c^2*e^3 - 48*a^2*c^3*d*e^2 + 6*b^3*c^2*d^2*e - 14*a*b^3*c*e^3 - 6*b^4*c*d*e^2 - 24*a*b*c^3*d^2*e + 36*a*b^2*c^2*d*e^2)/(16*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2)*(4*b^3*c^3*e^3 - 8*b^2*c^4*d*e^2 - 16*a*b*c^4*e^3 + 32*a*c^5*d*e^2))/c)*(-(((4*b^5*e^3 + 32*a*c^4*d^3 - 8*b^2*c^3*d^3 + 48*a^2*b*c^2*e^3 - 96*a^2*c^3*d*e^2 + 12*b^3*c^2*d^2*e - 28*a*b^3*c*e^3 - 12*b^4*c*d*e^2 - 48*a*b*c^3*d^2*e + 72*a*b^2*c^2*d*e^2)^2/4 - (256*a^2*c^5 + 16*b^4*c^3 - 128*a*b^2*c^4)*(a^3*e^6 + c^3*d^6 - b^3*d^3*e^3 + 3*a*b^2*d^2*e^4 + 3*a*c^2*d^4*e^2 + 3*a^2*c*d^2*e^4 + 3*b^2*c*d^4*e^2 - 3*a^2*b*d*e^5 - 3*b*c^2*d^5*e - 6*a*b*c*d^3*e^3))^(1/2) + 2*b^5*e^3 + 16*a*c^4*d^3 - 4*b^2*c^3*d^3 + 24*a^2*b*c^2*e^3 - 48*a^2*c^3*d*e^2 + 6*b^3*c^2*d^2*e - 14*a*b^3*c*e^3 - 6*b^4*c*d*e^2 - 24*a*b*c^3*d^2*e + 36*a*b^2*c^2*d*e^2)/(16*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2) - (2*(d + e*x^2)^(1/2)*(b^4*e^6 + 2*a^2*c^2*e^6 + 2*c^4*d^4*e^2 - 12*a*c^3*d^2*e^4 - 4*b*c^3*d^3*e^3 + 6*b^2*c^2*d^2*e^4 - 4*a*b^2*c*e^6 - 4*b^3*c*d*e^5 + 12*a*b*c^2*d*e^5))/c)*(-(((4*b^5*e^3 + 32*a*c^4*d^3 - 8*b^2*c^3*d^3 + 48*a^2*b*c^2*e^3 - 96*a^2*c^3*d*e^2 + 12*b^3*c^2*d^2*e - 28*a*b^3*c*e^3 - 12*b^4*c*d*e^2 - 48*a*b*c^3*d^2*e + 72*a*b^2*c^2*d*e^2)^2/4 - (256*a^2*c^5 + 16*b^4*c^3 - 128*a*b^2*c^4)*(a^3*e^6 + c^3*d^6 - b^3*d^3*e^3 + 3*a*b^2*d^2*e^4 + 3*a*c^2*d^4*e^2 + 3*a^2*c*d^2*e^4 + 3*b^2*c*d^4*e^2 - 3*a^2*b*d*e^5 - 3*b*c^2*d^5*e - 6*a*b*c*d^3*e^3))^(1/2) + 2*b^5*e^3 + 16*a*c^4*d^3 - 4*b^2*c^3*d^3 + 24*a^2*b*c^2*e^3 - 48*a^2*c^3*d*e^2 + 6*b^3*c^2*d^2*e - 14*a*b^3*c*e^3 - 6*b^4*c*d*e^2 - 24*a*b*c^3*d^2*e + 36*a*b^2*c^2*d*e^2)/(16*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2) + (((16*a^2*c^3*e^5 - 4*a*b^2*c^2*e^5 + 16*a*c^4*d^2*e^3 + 4*b^3*c^2*d*e^4 - 4*b^2*c^3*d^2*e^3 - 16*a*b*c^3*d*e^4)/c + (2*(d + e*x^2)^(1/2)*(-(((4*b^5*e^3 + 32*a*c^4*d^3 - 8*b^2*c^3*d^3 + 48*a^2*b*c^2*e^3 - 96*a^2*c^3*d*e^2 + 12*b^3*c^2*d^2*e - 28*a*b^3*c*e^3 - 12*b^4*c*d*e^2 - 48*a*b*c^3*d^2*e + 72*a*b^2*c^2*d*e^2)^2/4 - (256*a^2*c^5 + 16*b^4*c^3 - 128*a*b^2*c^4)*(a^3*e^6 + c^3*d^6 - b^3*d^3*e^3 + 3*a*b^2*d^2*e^4 + 3*a*c^2*d^4*e^2 + 3*a^2*c*d^2*e^4 + 3*b^2*c*d^4*e^2 - 3*a^2*b*d*e^5 - 3*b*c^2*d^5*e - 6*a*b*c*d^3*e^3))^(1/2) + 2*b^5*e^3 + 16*a*c^4*d^3 - 4*b^2*c^3*d^3 + 24*a^2*b*c^2*e^3 - 48*a^2*c^3*d*e^2 + 6*b^3*c^2*d^2*e - 14*a*b^3*c*e^3 - 6*b^4*c*d*e^2 - 24*a*b*c^3*d^2*e + 36*a*b^2*c^2*d*e^2)/(16*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2)*(4*b^3*c^3*e^3 - 8*b^2*c^4*d*e^2 - 16*a*b*c^4*e^3 + 32*a*c^5*d*e^2))/c)*(-(((4*b^5*e^3 + 32*a*c^4*d^3 - 8*b^2*c^3*d^3 + 48*a^2*b*c^2*e^3 - 96*a^2*c^3*d*e^2 + 12*b^3*c^2*d^2*e - 28*a*b^3*c*e^3 - 12*b^4*c*d*e^2 - 48*a*b*c^3*d^2*e + 72*a*b^2*c^2*d*e^2)^2/4 - (256*a^2*c^5 + 16*b^4*c^3 - 128*a*b^2*c^4)*(a^3*e^6 + c^3*d^6 - b^3*d^3*e^3 + 3*a*b^2*d^2*e^4 + 3*a*c^2*d^4*e^2 + 3*a^2*c*d^2*e^4 + 3*b^2*c*d^4*e^2 - 3*a^2*b*d*e^5 - 3*b*c^2*d^5*e - 6*a*b*c*d^3*e^3))^(1/2) + 2*b^5*e^3 + 16*a*c^4*d^3 - 4*b^2*c^3*d^3 + 24*a^2*b*c^2*e^3 - 48*a^2*c^3*d*e^2 + 6*b^3*c^2*d^2*e - 14*a*b^3*c*e^3 - 6*b^4*c*d*e^2 - 24*a*b*c^3*d^2*e + 36*a*b^2*c^2*d*e^2)/(16*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2) + (2*(d + e*x^2)^(1/2)*(b^4*e^6 + 2*a^2*c^2*e^6 + 2*c^4*d^4*e^2 - 12*a*c^3*d^2*e^4 - 4*b*c^3*d^3*e^3 + 6*b^2*c^2*d^2*e^4 - 4*a*b^2*c*e^6 - 4*b^3*c*d*e^5 + 12*a*b*c^2*d*e^5))/c)*(

-(((4*b^5*e^3 + 32*a*c^4*d^3 - 8*b^2*c^3*d^3 + 48*a^2*b*c^2*e^3 - 96*a^2*c^
3*d*e^2 + 12*b^3*c^2*d^2*e - 28*a*b^3*c*e^3 - 12*b^4*c*d*e^2 - 48*a*b*c^3*d
^2*e + 72*a*b^2*c^2*d*e^2)^2/4 - (256*a^2*c^5 + 16*b^4*c^3 - 128*a*b^2*c^4)
*(a^3*e^6 + c^3*d^6 - b^3*d^3*e^3 + 3*a*b^2*d^2*e^4 + 3*a*c^2*d^4*e^2 + 3*a
^2*c*d^2*e^4 + 3*b^2*c*d^4*e^2 - 3*a^2*b*d*e^5 - 3*b*c^2*d^5*e - 6*a*b*c*d^
3*e^3))^(1/2) + 2*b^5*e^3 + 16*a*c^4*d^3 - 4*b^2*c^3*d^3 + 24*a^2*b*c^2*e^3
 - 48*a^2*c^3*d*e^2 + 6*b^3*c^2*d^2*e - 14*a*b^3*c*e^3 - 6*b^4*c*d*e^2 - 24
*a*b*c^3*d^2*e + 36*a*b^2*c^2*d*e^2)/(16*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^
4)))^(1/2) - (2*(2*c^3*d^5*e^3 - b^3*d^2*e^6 - a^2*b*e^8 + 4*a*c^2*d^3*e^5
- 5*b*c^2*d^4*e^4 + 4*b^2*c*d^3*e^5 + 2*a*b^2*d*e^7 + 2*a^2*c*d*e^7 - 6*a*b
*c*d^2*e^6))/c))*(-(((4*b^5*e^3 + 32*a*c^4*d^3 - 8*b^2*c^3*d^3 + 48*a^2*b*c
^2*e^3 - 96*a^2*c^3*d*e^2 + 12*b^3*c^2*d^2*e - 28*a*b^3*c*e^3 - 12*b^4*c*d*
e^2 - 48*a*b*c^3*d^2*e + 72*a*b^2*c^2*d*e^2)^2/4 - (256*a^2*c^5 + 16*b^4*c^
3 - 128*a*b^2*c^4)*(a^3*e^6 + c^3*d^6 - b^3*d^3*e^3 + 3*a*b^2*d^2*e^4 + 3*a
*c^2*d^4*e^2 + 3*a^2*c*d^2*e^4 + 3*b^2*c*d^4*e^2 - 3*a^2*b*d*e^5 - 3*b*c^2*
d^5*e - 6*a*b*c*d^3*e^3))^(1/2) + 2*b^5*e^3 + 16*a*c^4*d^3 - 4*b^2*c^3*d^3
+ 24*a^2*b*c^2*e^3 - 48*a^2*c^3*d*e^2 + 6*b^3*c^2*d^2*e - 14*a*b^3*c*e^3 -
6*b^4*c*d*e^2 - 24*a*b*c^3*d^2*e + 36*a*b^2*c^2*d*e^2)/(16*(16*a^2*c^5 + b^
4*c^3 - 8*a*b^2*c^4)))^(1/2)*2i - atan(((((16*a^2*c^3*e^5 - 4*a*b^2*c^2*e^5
 + 16*a*c^4*d^2*e^3 + 4*b^3*c^2*d*e^4 - 4*b^2*c^3*d^2*e^3 - 16*a*b*c^3*d*e^
4)/c - (2*(d + e*x^2)^(1/2)*((((4*b^5*e^3 + 32*a*c^4*d^3 - 8*b^2*c^3*d^3 +
48*a^2*b*c^2*e^3 - 96*a^2*c^3*d*e^2 + 12*b^3*c^2*d^2*e - 28*a*b^3*c*e^3 - 1
2*b^4*c*d*e^2 - 48*a*b*c^3*d^2*e + 72*a*b^2*c^2*d*e^2)^2/4 - (256*a^2*c^5 +
 16*b^4*c^3 - 128*a*b^2*c^4)*(a^3*e^6 + c^3*d^6 - b^3*d^3*e^3 + 3*a*b^2*d^2
*e^4 + 3*a*c^2*d^4*e^2 + 3*a^2*c*d^2*e^4 + 3*b^2*c*d^4*e^2 - 3*a^2*b*d*e^5
- 3*b*c^2*d^5*e - 6*a*b*c*d^3*e^3))^(1/2) - 2*b^5*e^3 - 16*a*c^4*d^3 + 4*b^
2*c^3*d^3 - 24*a^2*b*c^2*e^3 + 48*a^2*c^3*d*e^2 - 6*b^3*c^2*d^2*e + 14*a*b^
3*c*e^3 + 6*b^4*c*d*e^2 + 24*a*b*c^3*d^2*e - 36*a*b^2*c^2*d*e^2)/(16*(16*a^
2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2)*(4*b^3*c^3*e^3 - 8*b^2*c^4*d*e^2 - 1
6*a*b*c^4*e^3 + 32*a*c^5*d*e^2))/c)*((((4*b^5*e^3 + 32*a*c^4*d^3 - 8*b^2*c^
3*d^3 + 48*a^2*b*c^2*e^3 - 96*a^2*c^3*d*e^2 + 12*b^3*c^2*d^2*e - 28*a*b^3*c
*e^3 - 12*b^4*c*d*e^2 - 48*a*b*c^3*d^2*e + 72*a*b^2*c^2*d*e^2)^2/4 - (256*a
^2*c^5 + 16*b^4*c^3 - 128*a*b^2*c^4)*(a^3*e^6 + c^3*d^6 - b^3*d^3*e^3 + 3*a
*b^2*d^2*e^4 + 3*a*c^2*d^4*e^2 + 3*a^2*c*d^2*e^4 + 3*b^2*c*d^4*e^2 - 3*a^2*
b*d*e^5 - 3*b*c^2*d^5*e - 6*a*b*c*d^3*e^3))^(1/2) - 2*b^5*e^3 - 16*a*c^4*d^
3 + 4*b^2*c^3*d^3 - 24*a^2*b*c^2*e^3 + 48*a^2*c^3*d*e^2 - 6*b^3*c^2*d^2*e +
 14*a*b^3*c*e^3 + 6*b^4*c*d*e^2 + 24*a*b*c^3*d^2*e - 36*a*b^2*c^2*d*e^2)/(1
6*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2) - (2*(d + e*x^2)^(1/2)*(b^4*
e^6 + 2*a^2*c^2*e^6 + 2*c^4*d^4*e^2 - 12*a*c^3*d^2*e^4 - 4*b*c^3*d^3*e^3 +
6*b^2*c^2*d^2*e^4 - 4*a*b^2*c*e^6 - 4*b^3*c*d*e^5 + 12*a*b*c^2*d*e^5))/c)*(
(((4*b^5*e^3 + 32*a*c^4*d^3 - 8*b^2*c^3*d^3 + 48*a^2*b*c^2*e^3 - 96*a^2*c^3
*d*e^2 + 12*b^3*c^2*d^2*e - 28*a*b^3*c*e^3 - 12*b^4*c*d*e^2 - 48*a*b*c^3*d^
2*e + 72*a*b^2*c^2*d*e^2)^2/4 - (256*a^2*c^5 + 16*b^4*c^3 - 128*a*b^2*c^4)*
(a^3*e^6 + c^3*d^6 - b^3*d^3*e^3 + 3*a*b^2*d^2*e^4 + 3*a*c^2*d^4*e^2 + 3*a^
2*c*d^2*e^4 + 3*b^2*c*d^4*e^2 - 3*a^2*b*d*e^5 - 3*b*c^2*d^5*e - 6*a*b*c*d^3

*e^3))^(1/2) - 2*b^5*e^3 - 16*a*c^4*d^3 + 4*b^2*c^3*d^3 - 24*a^2*b*c^2*e^3 + 48*a^2*c^3*d*e^2 - 6*b^3*c^2*d^2*e + 14*a*b^3*c*e^3 + 6*b^4*c*d*e^2 + 24*a*b*c^3*d^2*e - 36*a*b^2*c^2*d*e^2)/(16*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2)*1i - (((16*a^2*c^3*e^5 - 4*a*b^2*c^2*e^5 + 16*a*c^4*d^2*e^3 + 4*b^3*c^2*d*e^4 - 4*b^2*c^3*d^2*e^3 - 16*a*b*c^3*d*e^4)/c + (2*(d + e*x^2)^(1/2)*((((4*b^5*e^3 + 32*a*c^4*d^3 - 8*b^2*c^3*d^3 + 48*a^2*b*c^2*e^3 - 96*a^2*c^3*d*e^2 + 12*b^3*c^2*d^2*e - 28*a*b^3*c*e^3 - 12*b^4*c*d*e^2 - 48*a*b*c^3*d^2*e + 72*a*b^2*c^2*d*e^2)^2/4 - (256*a^2*c^5 + 16*b^4*c^3 - 128*a*b^2*c^4)*(a^3*e^6 + c^3*d^6 - b^3*d^3*e^3 + 3*a*b^2*d^2*e^4 + 3*a*c^2*d^4*e^2 + 3*a^2*c*d^2*e^4 + 3*b^2*c*d^4*e^2 - 3*a^2*b*d*e^5 - 3*b*c^2*d^5*e - 6*a*b*c*d^3*e^3))^(1/2) - 2*b^5*e^3 - 16*a*c^4*d^3 + 4*b^2*c^3*d^3 - 24*a^2*b*c^2*e^3 + 48*a^2*c^3*d*e^2 - 6*b^3*c^2*d^2*e + 14*a*b^3*c*e^3 + 6*b^4*c*d*e^2 + 24*a*b*c^3*d^2*e - 36*a*b^2*c^2*d*e^2)/(16*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2)*(4*b^3*c^3*e^3 - 8*b^2*c^4*d*e^2 - 16*a*b*c^4*e^3 + 32*a*c^5*d*e^2))/c)*((((4*b^5*e^3 + 32*a*c^4*d^3 - 8*b^2*c^3*d^3 + 48*a^2*b*c^2*e^3 - 96*a^2*c^3*d*e^2 + 12*b^3*c^2*d^2*e - 28*a*b^3*c*e^3 - 12*b^4*c*d*e^2 - 48*a*b*c^3*d^2*e + 72*a*b^2*c^2*d*e^2)^2/4 - (256*a^2*c^5 + 16*b^4*c^3 - 128*a*b^2*c^4)*(a^3*e^6 + c^3*d^6 - b^3*d^3*e^3 + 3*a*b^2*d^2*e^4 + 3*a*c^2*d^4*e^2 + 3*a^2*c*d^2*e^4 + 3*b^2*c*d^4*e^2 - 3*a^2*b*d*e^5 - 3*b*c^2*d^5*e - 6*a*b*c*d^3*e^3))^(1/2) - 2*b^5*e^3 - 16*a*c^4*d^3 + 4*b^2*c^3*d^3 - 24*a^2*b*c^2*e^3 + 48*a^2*c^3*d*e^2 - 6*b^3*c^2*d^2*e + 14*a*b^3*c*e^3 + 6*b^4*c*d*e^2 + 24*a*b*c^3*d^2*e - 36*a*b^2*c^2*d*e^2)/(16*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2) + (2*(d + e*x^2)^(1/2)*(b^4*e^6 + 2*a^2*c^2*e^6 + 2*c^4*d^4*e^2 - 12*a*c^3*d^2*e^4 - 4*b*c^3*d^3*e^3 + 6*b^2*c^2*d^2*e^4 - 4*a*b^2*c*e^6 - 4*b^3*c*d*e^5 + 12*a*b*c^2*d*e^5))/c)*((((4*b^5*e^3 + 32*a*c^4*d^3 - 8*b^2*c^3*d^3 + 48*a^2*b*c^2*e^3 - 96*a^2*c^3*d*e^2 + 12*b^3*c^2*d^2*e - 28*a*b^3*c*e^3 - 12*b^4*c*d*e^2 - 48*a*b*c^3*d^2*e + 72*a*b^2*c^2*d*e^2)^2/4 - (256*a^2*c^5 + 16*b^4*c^3 - 128*a*b^2*c^4)*(a^3*e^6 + c^3*d^6 - b^3*d^3*e^3 + 3*a*b^2*d^2*e^4 + 3*a*c^2*d^4*e^2 + 3*a^2*c*d^2*e^4 + 3*b^2*c*d^4*e^2 - 3*a^2*b*d*e^5 - 3*b*c^2*d^5*e - 6*a*b*c*d^3*e^3))^(1/2) - 2*b^5*e^3 - 16*a*c^4*d^3 + 4*b^2*c^3*d^3 - 24*a^2*b*c^2*e^3 + 48*a^2*c^3*d*e^2 - 6*b^3*c^2*d^2*e + 14*a*b^3*c*e^3 + 6*b^4*c*d*e^2 + 24*a*b*c^3*d^2*e - 36*a*b^2*c^2*d*e^2)/(16*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2)*1i)/((((16*a^2*c^3*e^5 - 4*a*b^2*c^2*e^5 + 16*a*c^4*d^2*e^3 + 4*b^3*c^2*d*e^4 - 4*b^2*c^3*d^2*e^3 - 16*a*b*c^3*d*e^4)/c - (2*(d + e*x^2)^(1/2)*((((4*b^5*e^3 + 32*a*c^4*d^3 - 8*b^2*c^3*d^3 + 48*a^2*b*c^2*e^3 - 96*a^2*c^3*d*e^2 + 12*b^3*c^2*d^2*e - 28*a*b^3*c*e^3 - 12*b^4*c*d*e^2 - 48*a*b*c^3*d^2*e + 72*a*b^2*c^2*d*e^2)^2/4 - (256*a^2*c^5 + 16*b^4*c^3 - 128*a*b^2*c^4)*(a^3*e^6 + c^3*d^6 - b^3*d^3*e^3 + 3*a*b^2*d^2*e^4 + 3*a*c^2*d^4*e^2 + 3*a^2*c*d^2*e^4 + 3*b^2*c*d^4*e^2 - 3*a^2*b*d*e^5 - 3*b*c^2*d^5*e - 6*a*b*c*d^3*e^3))^(1/2) - 2*b^5*e^3 - 16*a*c^4*d^3 + 4*b^2*c^3*d^3 - 24*a^2*b*c^2*e^3 + 48*a^2*c^3*d*e^2 - 6*b^3*c^2*d^2*e + 14*a*b^3*c*e^3 + 6*b^4*c*d*e^2 + 24*a*b*c^3*d^2*e - 36*a*b^2*c^2*d*e^2)/(16*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2)*(4*b^3*c^3*e^3 - 8*b^2*c^4*d*e^2 - 16*a*b*c^4*e^3 + 32*a*c^5*d*e^2))/c)*((((4*b^5*e^3 + 32*a*c^4*d^3 - 8*b^2*c^3*d^3 + 48*a^2*b*c^2*e^3 - 96*a^2*c^3*d*e^2 + 12*b

^3*c^2*d^2*e - 28*a*b^3*c*e^3 - 12*b^4*c*d*e^2 - 48*a*b*c^3*d^2*e + 72*a*b^
2*c^2*d*e^2)^2/4 - (256*a^2*c^5 + 16*b^4*c^3 - 128*a*b^2*c^4)*(a^3*e^6 + c^
3*d^6 - b^3*d^3*e^3 + 3*a*b^2*d^2*e^4 + 3*a*c^2*d^4*e^2 + 3*a^2*c*d^2*e^4 +
 3*b^2*c*d^4*e^2 - 3*a^2*b*d*e^5 - 3*b*c^2*d^5*e - 6*a*b*c*d^3*e^3))^(1/2)
- 2*b^5*e^3 - 16*a*c^4*d^3 + 4*b^2*c^3*d^3 - 24*a^2*b*c^2*e^3 + 48*a^2*c^3*
d*e^2 - 6*b^3*c^2*d^2*e + 14*a*b^3*c*e^3 + 6*b^4*c*d*e^2 + 24*a*b*c^3*d^2*e
 - 36*a*b^2*c^2*d*e^2)/(16*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2) - (
2*(d + e*x^2)^(1/2)*(b^4*e^6 + 2*a^2*c^2*e^6 + 2*c^4*d^4*e^2 - 12*a*c^3*d^2
*e^4 - 4*b*c^3*d^3*e^3 + 6*b^2*c^2*d^2*e^4 - 4*a*b^2*c*e^6 - 4*b^3*c*d*e^5
+ 12*a*b*c^2*d*e^5))/c)*((((4*b^5*e^3 + 32*a*c^4*d^3 - 8*b^2*c^3*d^3 + 48*a
^2*b*c^2*e^3 - 96*a^2*c^3*d*e^2 + 12*b^3*c^2*d^2*e - 28*a*b^3*c*e^3 - 12*b^
4*c*d*e^2 - 48*a*b*c^3*d^2*e + 72*a*b^2*c^2*d*e^2)^2/4 - (256*a^2*c^5 + 16*
b^4*c^3 - 128*a*b^2*c^4)*(a^3*e^6 + c^3*d^6 - b^3*d^3*e^3 + 3*a*b^2*d^2*e^4
 + 3*a*c^2*d^4*e^2 + 3*a^2*c*d^2*e^4 + 3*b^2*c*d^4*e^2 - 3*a^2*b*d*e^5 - 3*
b*c^2*d^5*e - 6*a*b*c*d^3*e^3))^(1/2) - 2*b^5*e^3 - 16*a*c^4*d^3 + 4*b^2*c^
3*d^3 - 24*a^2*b*c^2*e^3 + 48*a^2*c^3*d*e^2 - 6*b^3*c^2*d^2*e + 14*a*b^3*c*
e^3 + 6*b^4*c*d*e^2 + 24*a*b*c^3*d^2*e - 36*a*b^2*c^2*d*e^2)/(16*(16*a^2*c^
5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2) + (((16*a^2*c^3*e^5 - 4*a*b^2*c^2*e^5 +
16*a*c^4*d^2*e^3 + 4*b^3*c^2*d*e^4 - 4*b^2*c^3*d^2*e^3 - 16*a*b*c^3*d*e^4)/
c + (2*(d + e*x^2)^(1/2)*((((4*b^5*e^3 + 32*a*c^4*d^3 - 8*b^2*c^3*d^3 + 48*
a^2*b*c^2*e^3 - 96*a^2*c^3*d*e^2 + 12*b^3*c^2*d^2*e - 28*a*b^3*c*e^3 - 12*b
^4*c*d*e^2 - 48*a*b*c^3*d^2*e + 72*a*b^2*c^2*d*e^2)^2/4 - (256*a^2*c^5 + 16
*b^4*c^3 - 128*a*b^2*c^4)*(a^3*e^6 + c^3*d^6 - b^3*d^3*e^3 + 3*a*b^2*d^2*e^
4 + 3*a*c^2*d^4*e^2 + 3*a^2*c*d^2*e^4 + 3*b^2*c*d^4*e^2 - 3*a^2*b*d*e^5 - 3
*b*c^2*d^5*e - 6*a*b*c*d^3*e^3))^(1/2) - 2*b^5*e^3 - 16*a*c^4*d^3 + 4*b^2*c
^3*d^3 - 24*a^2*b*c^2*e^3 + 48*a^2*c^3*d*e^2 - 6*b^3*c^2*d^2*e + 14*a*b^3*c
*e^3 + 6*b^4*c*d*e^2 + 24*a*b*c^3*d^2*e - 36*a*b^2*c^2*d*e^2)/(16*(16*a^2*c
^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2)*(4*b^3*c^3*e^3 - 8*b^2*c^4*d*e^2 - 16*a
*b*c^4*e^3 + 32*a*c^5*d*e^2))/c)*((((4*b^5*e^3 + 32*a*c^4*d^3 - 8*b^2*c^3*d
^3 + 48*a^2*b*c^2*e^3 - 96*a^2*c^3*d*e^2 + 12*b^3*c^2*d^2*e - 28*a*b^3*c*e^
3 - 12*b^4*c*d*e^2 - 48*a*b*c^3*d^2*e + 72*a*b^2*c^2*d*e^2)^2/4 - (256*a^2*
c^5 + 16*b^4*c^3 - 128*a*b^2*c^4)*(a^3*e^6 + c^3*d^6 - b^3*d^3*e^3 + 3*a*b^
2*d^2*e^4 + 3*a*c^2*d^4*e^2 + 3*a^2*c*d^2*e^4 + 3*b^2*c*d^4*e^2 - 3*a^2*b*d
*e^5 - 3*b*c^2*d^5*e - 6*a*b*c*d^3*e^3))^(1/2) - 2*b^5*e^3 - 16*a*c^4*d^3 +
 4*b^2*c^3*d^3 - 24*a^2*b*c^2*e^3 + 48*a^2*c^3*d*e^2 - 6*b^3*c^2*d^2*e + 14
*a*b^3*c*e^3 + 6*b^4*c*d*e^2 + 24*a*b*c^3*d^2*e - 36*a*b^2*c^2*d*e^2)/(16*(
16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2) + (2*(d + e*x^2)^(1/2)*(b^4*e^6
 + 2*a^2*c^2*e^6 + 2*c^4*d^4*e^2 - 12*a*c^3*d^2*e^4 - 4*b*c^3*d^3*e^3 + 6*b
^2*c^2*d^2*e^4 - 4*a*b^2*c*e^6 - 4*b^3*c*d*e^5 + 12*a*b*c^2*d*e^5))/c)*((((
4*b^5*e^3 + 32*a*c^4*d^3 - 8*b^2*c^3*d^3 + 48*a^2*b*c^2*e^3 - 96*a^2*c^3*d*
e^2 + 12*b^3*c^2*d^2*e - 28*a*b^3*c*e^3 - 12*b^4*c*d*e^2 - 48*a*b*c^3*d^2*e
 + 72*a*b^2*c^2*d*e^2)^2/4 - (256*a^2*c^5 + 16*b^4*c^3 - 128*a*b^2*c^4)*(a^
3*e^6 + c^3*d^6 - b^3*d^3*e^3 + 3*a*b^2*d^2*e^4 + 3*a*c^2*d^4*e^2 + 3*a^2*c
*d^2*e^4 + 3*b^2*c*d^4*e^2 - 3*a^2*b*d*e^5 - 3*b*c^2*d^5*e - 6*a*b*c*d^3*e^
3))^(1/2) - 2*b^5*e^3 - 16*a*c^4*d^3 + 4*b^2*c^3*d^3 - 24*a^2*b*c^2*e^3 + 4

8*a^2*c^3*d*e^2 - 6*b^3*c^2*d^2*e + 14*a*b^3*c*e^3 + 6*b^4*c*d*e^2 + 24*a*b
*c^3*d^2*e - 36*a*b^2*c^2*d*e^2)/(16*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))
^(1/2) - (2*(2*c^3*d^5*e^3 - b^3*d^2*e^6 - a^2*b*e^8 + 4*a*c^2*d^3*e^5 - 5*
b*c^2*d^4*e^4 + 4*b^2*c*d^3*e^5 + 2*a*b^2*d*e^7 + 2*a^2*c*d*e^7 - 6*a*b*c*d
^2*e^6))/c))*((((4*b^5*e^3 + 32*a*c^4*d^3 - 8*b^2*c^3*d^3 + 48*a^2*b*c^2*e^
3 - 96*a^2*c^3*d*e^2 + 12*b^3*c^2*d^2*e - 28*a*b^3*c*e^3 - 12*b^4*c*d*e^2 -
 48*a*b*c^3*d^2*e + 72*a*b^2*c^2*d*e^2)^2/4 - (256*a^2*c^5 + 16*b^4*c^3 - 1
28*a*b^2*c^4)*(a^3*e^6 + c^3*d^6 - b^3*d^3*e^3 + 3*a*b^2*d^2*e^4 + 3*a*c^2*
d^4*e^2 + 3*a^2*c*d^2*e^4 + 3*b^2*c*d^4*e^2 - 3*a^2*b*d*e^5 - 3*b*c^2*d^5*e
 - 6*a*b*c*d^3*e^3))^(1/2) - 2*b^5*e^3 - 16*a*c^4*d^3 + 4*b^2*c^3*d^3 - 24*
a^2*b*c^2*e^3 + 48*a^2*c^3*d*e^2 - 6*b^3*c^2*d^2*e + 14*a*b^3*c*e^3 + 6*b^4
*c*d*e^2 + 24*a*b*c^3*d^2*e - 36*a*b^2*c^2*d*e^2)/(16*(16*a^2*c^5 + b^4*c^3
 - 8*a*b^2*c^4)))^(1/2)*2i

3.369 $\int \frac{(d+ex^2)^{3/2}}{x(a+bx^2+cx^4)}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 346

$$\int \frac{(d+ex^2)^{3/2}}{x(a+bx^2+cx^4)}\,dx = -\frac{d^{3/2}\text{arctanh}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)}{a} - \frac{\left(a\sqrt{b^2-4ac}e^2-cd\left(\sqrt{b^2-4ac}d-4ae\right)-b(cd^2+ae^2)\right)\text{arctanh}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex^2}}{\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}}\right)}{\sqrt{2}a\sqrt{c}\sqrt{b^2-4ac}\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}} - \frac{\left(a\sqrt{b^2-4ac}e^2-cd\left(\sqrt{b^2-4ac}d+4ae\right)+b(cd^2+ae^2)\right)\text{arctanh}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex^2}}{\sqrt{2cd-\left(b+\sqrt{b^2-4ac}\right)e}}\right)}{\sqrt{2}a\sqrt{c}\sqrt{b^2-4ac}\sqrt{2cd-\left(b+\sqrt{b^2-4ac}\right)e}}$$

```
[Out] -d^(3/2)*arctanh((e*x^2+d)^(1/2)/d^(1/2))/a-1/2*arctanh(2^(1/2)*c^(1/2)*(e*
x^2+d)^(1/2)/(2*c*d-e*(b-(-4*a*c+b^2)^(1/2)))^(1/2))*(-b*(a*e^2+c*d^2)+a*e^
2*(-4*a*c+b^2)^(1/2)-c*d*(-4*a*e+d*(-4*a*c+b^2)^(1/2)))/a*2^(1/2)/c^(1/2)/(
-4*a*c+b^2)^(1/2)/(2*c*d-e*(b-(-4*a*c+b^2)^(1/2)))^(1/2)-1/2*arctanh(2^(1/2
)*c^(1/2)*(e*x^2+d)^(1/2)/(2*c*d-e*(b+(-4*a*c+b^2)^(1/2)))^(1/2))*(b*(a*e^2
+c*d^2)+a*e^2*(-4*a*c+b^2)^(1/2)-c*d*(4*a*e+d*(-4*a*c+b^2)^(1/2)))/a*2^(1/2
)/c^(1/2)/(-4*a*c+b^2)^(1/2)/(2*c*d-e*(b+(-4*a*c+b^2)^(1/2)))^(1/2)
```

Rubi [A] (verified)

Time = 1.23 (sec) , antiderivative size = 346, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.207, Rules used = {1265, 911, 1301, 212, 1180, 214}

$$\int \frac{(d+ex^2)^{3/2}}{x(a+bx^2+cx^4)}\,dx =$$

$$-\frac{\left(-cd\left(d\sqrt{b^2-4ac}-4ae\right)+ae^2\sqrt{b^2-4ac}-b(ae^2+cd^2)\right)\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex^2}}{\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}\right)}{\sqrt{2}a\sqrt{c}\sqrt{b^2-4ac}\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}$$
$$-\frac{\left(-cd\left(d\sqrt{b^2-4ac}+4ae\right)+ae^2\sqrt{b^2-4ac}+b(ae^2+cd^2)\right)\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex^2}}{\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}\right)}{\sqrt{2}a\sqrt{c}\sqrt{b^2-4ac}\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}$$
$$-\frac{d^{3/2}\operatorname{arctanh}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)}{a}$$

```
[In] Int[(d + e*x^2)^(3/2)/(x*(a + b*x^2 + c*x^4)),x]
```

```
[Out] -((d^(3/2)*ArcTanh[Sqrt[d + e*x^2]/Sqrt[d]])/a) - ((a*Sqrt[b^2 - 4*a*c]*e^2
 - c*d*(Sqrt[b^2 - 4*a*c]*d - 4*a*e) - b*(c*d^2 + a*e^2))*ArcTanh[(Sqrt[2]*
Sqrt[c]*Sqrt[d + e*x^2])/Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]])/(Sqrt[2]
*a*Sqrt[c]*Sqrt[b^2 - 4*a*c]*Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]) - ((a
*Sqrt[b^2 - 4*a*c]*e^2 - c*d*(Sqrt[b^2 - 4*a*c]*d + 4*a*e) + b*(c*d^2 + a*e
^2))*ArcTanh[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x^2])/Sqrt[2*c*d - (b + Sqrt[b^2 -
 4*a*c])*e]])/(Sqrt[2]*a*Sqrt[c]*Sqrt[b^2 - 4*a*c]*Sqrt[2*c*d - (b + Sqrt[b
^2 - 4*a*c])*e])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 911

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> With[{q = Denominator[m]}, Dist[q/e, S
```

```
ubst[Int[x^(q*(m + 1) - 1)*((e*f - d*g)/e + g*(x^q/e))^n*((c*d^2 - b*d*e +
a*e^2)/e^2 - (2*c*d - b*e)*(x^q/e^2) + c*(x^(2*q)/e^2))^p, x], x, (d + e*x)
^(1/q)], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegersQ[n, p] && Fra
ctionQ[m]
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1265

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rule 1301

```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.))/((a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*((d + e*x^2)^q/(a
+ b*x^2 + c*x^4)), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b^2 - 4
*a*c, 0] && IntegerQ[q] && IntegerQ[m]
```

Rubi steps

$$\text{integral} = \frac{1}{2}\text{Subst}\left(\int \frac{(d+ex)^{3/2}}{x\,(a+bx+cx^2)}\,dx, x, x^2\right)$$

$$= \frac{\text{Subst}\left(\int \frac{x^4}{\left(-\frac{d}{e}+\frac{x^2}{e}\right)\left(\frac{cd^2-bde+ae^2}{e^2}-\frac{(2cd-be)x^2}{e^2}+\frac{cx^4}{e^2}\right)}\,dx, x, \sqrt{d+ex^2}\right)}{e}$$

$$= \frac{\text{Subst}\left(\int \left(-\frac{d^2e}{a(d-x^2)}+\frac{e\left(d(cd^2-bde+ae^2)-(cd^2-ae^2)x^2\right)}{a(cd^2-bde+ae^2-(2cd-be)x^2+cx^4)}\right)\,dx, x, \sqrt{d+ex^2}\right)}{e}$$

$$= \frac{\text{Subst}\left(\int \frac{d(cd^2-bde+ae^2)+(-cd^2+ae^2)x^2}{cd^2-bde+ae^2+(-2cd+be)x^2+cx^4}\,dx, x, \sqrt{d+ex^2}\right)}{a} - \frac{d^2\text{Subst}\left(\int \frac{1}{d-x^2}\,dx, x, \sqrt{d+ex^2}\right)}{a}$$

$$= -\frac{d^{3/2}\tanh^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)}{a} + \frac{\left(a\sqrt{b^2-4ac}e^2 - cd\left(\sqrt{b^2-4ac}d-4ae\right) - b(cd^2+ae^2)\right)\text{Subst}\left(\int \frac{1}{-\frac{1}{2}\sqrt{b^2-4ac}e+\frac{1}{2}(-2cd+be)+cx^2}\,dx,\right.}{2a\sqrt{b^2-4ac}} + \frac{\left(a\sqrt{b^2-4ac}e^2 - cd\left(\sqrt{b^2-4ac}d+4ae\right) + b(cd^2+ae^2)\right)\text{Subst}\left(\int \frac{1}{\frac{1}{2}\sqrt{b^2-4ac}e+\frac{1}{2}(-2cd+be)+cx^2}\,dx, x\right.}{2a\sqrt{b^2-4ac}}$$

$$= -\frac{d^{3/2}\tanh^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)}{a} - \frac{\left(a\sqrt{b^2-4ac}e^2 - cd\left(\sqrt{b^2-4ac}d-4ae\right) - b(cd^2+ae^2)\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex^2}}{\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}}\right)}{\sqrt{2}a\sqrt{c}\sqrt{b^2-4ac}\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}} - \frac{\left(a\sqrt{b^2-4ac}e^2 - cd\left(\sqrt{b^2-4ac}d+4ae\right) + b(cd^2+ae^2)\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex^2}}{\sqrt{2cd-\left(b+\sqrt{b^2-4ac}\right)e}}\right)}{\sqrt{2}a\sqrt{c}\sqrt{b^2-4ac}\sqrt{2cd-\left(b+\sqrt{b^2-4ac}\right)e}}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 1.39 (sec) , antiderivative size = 379, normalized size of antiderivative = 1.10

$$\int \frac{(d+ex^2)^{3/2}}{x(a+bx^2+cx^4)}\,dx = -\frac{\frac{\sqrt{2}\left(-a\sqrt{-b^2+4ac}e^2+cd\left(\sqrt{-b^2+4ac}d+4iae\right)-ib(cd^2+ae^2)\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex^2}}{\sqrt{-2cd+be-i\sqrt{-b^2+4ac}e}}\right)}{\sqrt{c}\sqrt{-b^2+4ac}\sqrt{-2cd+\left(b-i\sqrt{-b^2+4ac}\right)e}} + \frac{\sqrt{2}\left(-a\sqrt{-b^2+4ac}e^2+cd\left(\sqrt{-b^2+4ac}d-4i\right.\right.}{\sqrt{c}\sqrt{-b^2+4ac}\sqrt{}}}{2a}$$

```
[In] Integrate[(d + e*x^2)^(3/2)/(x*(a + b*x^2 + c*x^4)),x]
```

```
[Out] -1/2*((Sqrt[2]*(-(a*Sqrt[-b^2 + 4*a*c]*e^2) + c*d*(Sqrt[-b^2 + 4*a*c]*d + (
4*I)*a*e) - I*b*(c*d^2 + a*e^2))*ArcTan[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x^2])/S
qrt[-2*c*d + b*e - I*Sqrt[-b^2 + 4*a*c]*e]])/(Sqrt[c]*Sqrt[-b^2 + 4*a*c]*Sq
rt[-2*c*d + (b - I*Sqrt[-b^2 + 4*a*c])*e]) + (Sqrt[2]*(-(a*Sqrt[-b^2 + 4*a*
c]*e^2) + c*d*(Sqrt[-b^2 + 4*a*c]*d - (4*I)*a*e) + I*b*(c*d^2 + a*e^2))*Arc
Tan[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x^2])/Sqrt[-2*c*d + b*e + I*Sqrt[-b^2 + 4*a
*c]*e]])/(Sqrt[c]*Sqrt[-b^2 + 4*a*c]*Sqrt[-2*c*d + (b + I*Sqrt[-b^2 + 4*a*c
])*e]) + 2*d^(3/2)*ArcTanh[Sqrt[d + e*x^2]/Sqrt[d]])/a
```

Maple [A] (verified)

Time = 0.52 (sec) , antiderivative size = 402, normalized size of antiderivative = 1.16

method	result
pseudoelliptic	$\frac{-\sqrt{2}\sqrt{\left(be-2cd+\sqrt{-4e^2\left(ac-\frac{b^2}{4}\right)}\right)c}\left(\left(a e^2-c d^2\right)\sqrt{-4e^2\left(ac-\frac{b^2}{4}\right)}-e^3ab+4acd e^2-d^2ebc\right)\operatorname{arctanh}\left(\frac{c\sqrt{e x^2+d}\sqrt{\cdots}}{\sqrt{\left(-be+2cd+\sqrt{-4e^2(\cdots}\right.}}\cdots\right.}{\cdots}$
default	$\frac{\frac{\left(e x^2+d\right)^{\frac{3}{2}}}{3}+d\left(\sqrt{e x^2+d}-\sqrt{d}\,\ln\left(\frac{2d+2\sqrt{d}\,\sqrt{e x^2+d}}{x}\right)\right)}{a}+\frac{-\sqrt{2}\sqrt{\left(be-2cd+\sqrt{-4e^2\left(ac-\frac{b^2}{4}\right)}\right)c}\left(\left(a e^2-c d^2\right)\sqrt{-4e^2\left(ac-\cdots\right.}\right.}{\cdots}$

```
[In] int((e*x^2+d)^(3/2)/x/(c*x^4+b*x^2+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2/((-b*e+2*c*d+(-4*e^2*(a*c-1/4*b^2))^(1/2))*c)^(1/2)*(-2^(1/2)*((b*e-2*c
*d+(-4*e^2*(a*c-1/4*b^2))^(1/2))*c)^(1/2)*((a*e^2-c*d^2)*(-4*e^2*(a*c-1/4*b
^2))^(1/2)-e^3*a*b+4*a*c*d*e^2-d^2*e*b*c)*arctanh(c*(e*x^2+d)^(1/2)*2^(1/2)
/((-b*e+2*c*d+(-4*e^2*(a*c-1/4*b^2))^(1/2))*c)^(1/2))+((-b*e+2*c*d+(-4*e^2*
(a*c-1/4*b^2))^(1/2))*c)^(1/2)*(2^(1/2)*((a*e^2-c*d^2)*(-4*e^2*(a*c-1/4*b^2
))^(1/2)+e*(a*b*e^2-4*a*c*d*e+b*c*d^2))*arctan(c*(e*x^2+d)^(1/2)*2^(1/2)/((
b*e-2*c*d+(-4*e^2*(a*c-1/4*b^2))^(1/2))*c)^(1/2))-2*d^(3/2)*arctanh((e*x^2+
d)^(1/2)/d^(1/2))*(-4*e^2*(a*c-1/4*b^2))^(1/2)*((b*e-2*c*d+(-4*e^2*(a*c-1/4
*b^2))^(1/2))*c)^(1/2)))/((b*e-2*c*d+(-4*e^2*(a*c-1/4*b^2))^(1/2))*c)^(1/2)
/(-4*e^2*(a*c-1/4*b^2))^(1/2)/a
```

Fricas [F(-1)]

Timed out.

$$\int \frac{\left(d+ex^2\right)^{3/2}}{x\left(a+bx^2+cx^4\right)}\,dx=\text{Timed out}$$

```
[In] integrate((e*x^2+d)^(3/2)/x/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]

$$\int \frac{(d+ex^2)^{3/2}}{x(a+bx^2+cx^4)}\,dx = \int \frac{(d+ex^2)^{\frac{3}{2}}}{x(a+bx^2+cx^4)}\,dx$$

[In] integrate((e*x**2+d)**(3/2)/x/(c*x**4+b*x**2+a),x)

[Out] Integral((d + e*x**2)**(3/2)/(x*(a + b*x**2 + c*x**4)), x)

Maxima [F]

$$\int \frac{(d+ex^2)^{3/2}}{x(a+bx^2+cx^4)}\,dx = \int \frac{(ex^2+d)^{\frac{3}{2}}}{(cx^4+bx^2+a)x}\,dx$$

[In] integrate((e*x^2+d)^(3/2)/x/(c*x^4+b*x^2+a),x, algorithm="maxima")

[Out] integrate((e*x^2 + d)^(3/2)/((c*x^4 + b*x^2 + a)*x), x)

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 838 vs. 2(292) = 584.

Time = 0.34 (sec) , antiderivative size = 838, normalized size of antiderivative = 2.42

$$\int \frac{(d+ex^2)^{3/2}}{x(a+bx^2+cx^4)}\,dx = \frac{d^2\arctan\left(\frac{\sqrt{ex^2+d}}{\sqrt{-d}}\right)}{a\sqrt{-d}}$$

$$-\frac{\left(\sqrt{-4\,c^2d+2\,(bc-\sqrt{b^2-4\,ac}c)e}((b^2c-4\,ac^2)d^2-(ab^2-4\,a^2c)e^2)a^2e^2-2\,(\sqrt{b^2-4\,ac}ac^2d^3-\sqrt{b^2-\cdots}\right.}{\cdots}$$

$$+\frac{\left(\sqrt{-4\,c^2d+2\,(bc+\sqrt{b^2-4\,ac}c)e}((b^2c-4\,ac^2)d^2-(ab^2-4\,a^2c)e^2)a^2e^2+2\,(\sqrt{b^2-4\,ac}ac^2d^3-\sqrt{b^2-\cdots}\right.}{\cdots}$$

[In] integrate((e*x^2+d)^(3/2)/x/(c*x^4+b*x^2+a),x, algorithm="giac")

[Out] d^2*arctan(sqrt(e*x^2 + d)/sqrt(-d))/(a*sqrt(-d)) - 1/8*(sqrt(-4*c^2*d + 2*(b*c - sqrt(b^2 - 4*a*c)*c)*e)*((b^2*c - 4*a*c^2)*d^2 - (a*b^2 - 4*a^2*c)*e^2)*a^2*e^2 - 2*(sqrt(b^2 - 4*a*c)*a*c^2*d^3 - sqrt(b^2 - 4*a*c)*a*b*c*d^2*

```
e + sqrt(b^2 - 4*a*c)*a^2*c*d*e^2)*sqrt(-4*c^2*d + 2*(b*c - sqrt(b^2 - 4*a*
c)*c)*e)*abs(a)*abs(e) - (2*a^2*b*c^2*d^3*e + 6*a^3*b*c*d*e^3 - a^3*b^2*e^4
 - (a^2*b^2*c + 8*a^3*c^2)*d^2*e^2)*sqrt(-4*c^2*d + 2*(b*c - sqrt(b^2 - 4*a
*c)*c)*e))*arctan(2*sqrt(1/2)*sqrt(e*x^2 + d)/sqrt(-(2*a*c*d - a*b*e + sqrt
(-4*(a*c*d^2 - a*b*d*e + a^2*e^2)*a*c + (2*a*c*d - a*b*e)^2))/(a*c)))/((sqr
t(b^2 - 4*a*c)*a^2*c^2*d^2 - sqrt(b^2 - 4*a*c)*a^2*b*c*d*e + sqrt(b^2 - 4*a
*c)*a^3*c*e^2)*abs(a)*abs(c)*abs(e)) + 1/8*(sqrt(-4*c^2*d + 2*(b*c + sqrt(b
^2 - 4*a*c)*c)*e)*((b^2*c - 4*a*c^2)*d^2 - (a*b^2 - 4*a^2*c)*e^2)*a^2*e^2 +
 2*(sqrt(b^2 - 4*a*c)*a*c^2*d^3 - sqrt(b^2 - 4*a*c)*a*b*c*d^2*e + sqrt(b^2
- 4*a*c)*a^2*c*d*e^2)*sqrt(-4*c^2*d + 2*(b*c + sqrt(b^2 - 4*a*c)*c)*e)*abs(
a)*abs(e) - (2*a^2*b*c^2*d^3*e + 6*a^3*b*c*d*e^3 - a^3*b^2*e^4 - (a^2*b^2*c
 + 8*a^3*c^2)*d^2*e^2)*sqrt(-4*c^2*d + 2*(b*c + sqrt(b^2 - 4*a*c)*c)*e))*ar
ctan(2*sqrt(1/2)*sqrt(e*x^2 + d)/sqrt(-(2*a*c*d - a*b*e - sqrt(-4*(a*c*d^2
- a*b*d*e + a^2*e^2)*a*c + (2*a*c*d - a*b*e)^2))/(a*c)))/((sqrt(b^2 - 4*a*c
)*a^2*c^2*d^2 - sqrt(b^2 - 4*a*c)*a^2*b*c*d*e + sqrt(b^2 - 4*a*c)*a^3*c*e^2
)*abs(a)*abs(c)*abs(e))
```

Mupad [B] (verification not implemented)

Time = 11.99 (sec) , antiderivative size = 28434, normalized size of antiderivative = 82.18

$$\int \frac{(d+ex^2)^{3/2}}{x(a+bx^2+cx^4)}\,dx = \text{Too large to display}$$

```
[In] int((d + e*x^2)^(3/2)/(x*(a + b*x^2 + c*x^4)),x)
```

```
[Out] atan((((d + e*x^2)^(1/2)*(2*a^4*c*e^16 + 6*c^5*d^8*e^8 - 16*a*c^4*d^6*e^10
- 16*b*c^4*d^7*e^9 + 4*b^4*c*d^4*e^12 + 16*a^2*c^3*d^4*e^12 + 8*a^3*c^2*d^2
*e^14 + 24*b^2*c^3*d^6*e^10 - 16*b^3*c^2*d^5*e^11 - 8*a^3*b*c*d*e^15 - 8*a*
b^3*c*d^3*e^13 + 16*a*b^2*c^2*d^4*e^12 - 24*a^2*b*c^2*d^3*e^13 + 12*a^2*b^2
*c*d^2*e^14) + (-(((4*b^4*c*d^3 - 4*a^2*b^3*e^3 + 32*a^2*c^3*d^3 - 24*a*b^2
*c^2*d^3 - 96*a^3*c^2*d*e^2 + 16*a^3*b*c*e^3 - 12*a*b^3*c*d^2*e + 48*a^2*b*
c^2*d^2*e + 24*a^2*b^2*c*d*e^2)^2/4 - (256*a^4*c^3 + 16*a^2*b^4*c - 128*a^3
*b^2*c^2)*(a^3*e^6 + c^3*d^6 - b^3*d^3*e^3 + 3*a*b^2*d^2*e^4 + 3*a*c^2*d^4*
e^2 + 3*a^2*c*d^2*e^4 + 3*b^2*c*d^4*e^2 - 3*a^2*b*d*e^5 - 3*b*c^2*d^5*e - 6
*a*b*c*d^3*e^3))^(1/2) - 2*b^4*c*d^3 + 2*a^2*b^3*e^3 - 16*a^2*c^3*d^3 + 12*
a*b^2*c^2*d^3 + 48*a^3*c^2*d*e^2 - 8*a^3*b*c*e^3 + 6*a*b^3*c*d^2*e - 24*a^2
*b*c^2*d^2*e - 12*a^2*b^2*c*d*e^2)/(16*(16*a^4*c^3 + a^2*b^4*c - 8*a^3*b^2*
c^2)))^(1/2)*(((-(((4*b^4*c*d^3 - 4*a^2*b^3*e^3 + 32*a^2*c^3*d^3 - 24*a*b^2
*c^2*d^3 - 96*a^3*c^2*d*e^2 + 16*a^3*b*c*e^3 - 12*a*b^3*c*d^2*e + 48*a^2*b*
c^2*d^2*e + 24*a^2*b^2*c*d*e^2)^2/4 - (256*a^4*c^3 + 16*a^2*b^4*c - 128*a^3
*b^2*c^2)*(a^3*e^6 + c^3*d^6 - b^3*d^3*e^3 + 3*a*b^2*d^2*e^4 + 3*a*c^2*d^4*
e^2 + 3*a^2*c*d^2*e^4 + 3*b^2*c*d^4*e^2 - 3*a^2*b*d*e^5 - 3*b*c^2*d^5*e - 6
*a*b*c*d^3*e^3))^(1/2) - 2*b^4*c*d^3 + 2*a^2*b^3*e^3 - 16*a^2*c^3*d^3 + 12*
a*b^2*c^2*d^3 + 48*a^3*c^2*d*e^2 - 8*a^3*b*c*e^3 + 6*a*b^3*c*d^2*e - 24*a^2
```

*b*c^2*d^2*e - 12*a^2*b^2*c*d*e^2)/(16*(16*a^4*c^3 + a^2*b^4*c - 8*a^3*b^2*
c^2)))^(1/2)*((d + e*x^2)^(1/2)*(-(((4*b^4*c*d^3 - 4*a^2*b^3*e^3 + 32*a^2*c
^3*d^3 - 24*a*b^2*c^2*d^3 - 96*a^3*c^2*d*e^2 + 16*a^3*b*c*e^3 - 12*a*b^3*c*
d^2*e + 48*a^2*b*c^2*d^2*e + 24*a^2*b^2*c*d*e^2)^2/4 - (256*a^4*c^3 + 16*a^
2*b^4*c - 128*a^3*b^2*c^2)*(a^3*e^6 + c^3*d^6 - b^3*d^3*e^3 + 3*a*b^2*d^2*e
^4 + 3*a*c^2*d^4*e^2 + 3*a^2*c*d^2*e^4 + 3*b^2*c*d^4*e^2 - 3*a^2*b*d*e^5 -
3*b*c^2*d^5*e - 6*a*b*c*d^3*e^3))^(1/2) - 2*b^4*c*d^3 + 2*a^2*b^3*e^3 - 16*
a^2*c^3*d^3 + 12*a*b^2*c^2*d^3 + 48*a^3*c^2*d*e^2 - 8*a^3*b*c*e^3 + 6*a*b^3
*c*d^2*e - 24*a^2*b*c^2*d^2*e - 12*a^2*b^2*c*d*e^2)/(16*(16*a^4*c^3 + a^2*b
^4*c - 8*a^3*b^2*c^2)))^(1/2)*(512*a^5*c^4*e^10 + 32*a^3*b^4*c^2*e^10 - 256
*a^4*b^2*c^3*e^10 + 768*a^4*c^5*d^2*e^8 + 64*a^2*b^4*c^3*d^2*e^8 - 448*a^3*
b^2*c^4*d^2*e^8 - 896*a^4*b*c^4*d*e^9 - 64*a^2*b^5*c^2*d*e^9 + 480*a^3*b^3*
c^3*d*e^9) - 192*a^3*c^5*d^4*e^8 - 192*a^4*c^4*d^2*e^10 + 48*a^2*b^2*c^4*d^
4*e^8 - 64*a^2*b^3*c^3*d^3*e^9 + 16*a^2*b^4*c^2*d^2*e^10 - 16*a^3*b^2*c^3*d
^2*e^10 + 64*a^4*b*c^3*d*e^11 + 256*a^3*b*c^4*d^3*e^9 - 16*a^3*b^3*c^2*d*e^
11) + (d + e*x^2)^(1/2)*(8*a^3*b^3*c*e^13 - 32*a^4*b*c^2*e^13 + 176*a^4*c^3
*d*e^12 - 144*a^2*c^5*d^5*e^8 + 224*a^3*c^4*d^3*e^10 - 16*b^4*c^3*d^5*e^8 +
16*b^5*c^2*d^4*e^9 + 48*a^2*b^2*c^3*d^3*e^10 + 112*a^2*b^3*c^2*d^2*e^11 -
16*a^2*b^4*c*d*e^12 + 96*a*b^2*c^4*d^5*e^8 - 80*a*b^3*c^3*d^4*e^9 - 32*a*b^
4*c^2*d^3*e^10 + 96*a^2*b*c^4*d^4*e^9 - 416*a^3*b*c^3*d^2*e^11 + 16*a^3*b^2
*c^2*d*e^12))*(-(((4*b^4*c*d^3 - 4*a^2*b^3*e^3 + 32*a^2*c^3*d^3 - 24*a*b^2*
c^2*d^3 - 96*a^3*c^2*d*e^2 + 16*a^3*b*c*e^3 - 12*a*b^3*c*d^2*e + 48*a^2*b*c
^2*d^2*e + 24*a^2*b^2*c*d*e^2)^2/4 - (256*a^4*c^3 + 16*a^2*b^4*c - 128*a^3*
b^2*c^2)*(a^3*e^6 + c^3*d^6 - b^3*d^3*e^3 + 3*a*b^2*d^2*e^4 + 3*a*c^2*d^4*e
^2 + 3*a^2*c*d^2*e^4 + 3*b^2*c*d^4*e^2 - 3*a^2*b*d*e^5 - 3*b*c^2*d^5*e - 6*
a*b*c*d^3*e^3))^(1/2) - 2*b^4*c*d^3 + 2*a^2*b^3*e^3 - 16*a^2*c^3*d^3 + 12*a
*b^2*c^2*d^3 + 48*a^3*c^2*d*e^2 - 8*a^3*b*c*e^3 + 6*a*b^3*c*d^2*e - 24*a^2*
b*c^2*d^2*e - 12*a^2*b^2*c*d*e^2)/(16*(16*a^4*c^3 + a^2*b^4*c - 8*a^3*b^2*c
^2)))^(1/2) + 12*a*c^5*d^7*e^8 + 4*a^4*c^2*d*e^14 - 84*a^2*c^4*d^5*e^10 - 9
2*a^3*c^3*d^3*e^12 - 4*b^2*c^4*d^7*e^8 - 4*b^3*c^3*d^6*e^9 + 8*b^4*c^2*d^5*
e^10 - 12*a^2*b^2*c^2*d^3*e^12 + 32*a*b*c^4*d^6*e^9 - 4*a^3*b^2*c*d*e^14 -
36*a*b^2*c^3*d^5*e^10 - 20*a*b^3*c^2*d^4*e^11 + 160*a^2*b*c^3*d^4*e^11 + 4*
a^2*b^3*c*d^2*e^13 + 16*a^3*b*c^2*d^2*e^13))*(-(((4*b^4*c*d^3 - 4*a^2*b^3*e
^3 + 32*a^2*c^3*d^3 - 24*a*b^2*c^2*d^3 - 96*a^3*c^2*d*e^2 + 16*a^3*b*c*e^3
- 12*a*b^3*c*d^2*e + 48*a^2*b*c^2*d^2*e + 24*a^2*b^2*c*d*e^2)^2/4 - (256*a^
4*c^3 + 16*a^2*b^4*c - 128*a^3*b^2*c^2)*(a^3*e^6 + c^3*d^6 - b^3*d^3*e^3 +
3*a*b^2*d^2*e^4 + 3*a*c^2*d^4*e^2 + 3*a^2*c*d^2*e^4 + 3*b^2*c*d^4*e^2 - 3*a
^2*b*d*e^5 - 3*b*c^2*d^5*e - 6*a*b*c*d^3*e^3))^(1/2) - 2*b^4*c*d^3 + 2*a^2*
b^3*e^3 - 16*a^2*c^3*d^3 + 12*a*b^2*c^2*d^3 + 48*a^3*c^2*d*e^2 - 8*a^3*b*c*
e^3 + 6*a*b^3*c*d^2*e - 24*a^2*b*c^2*d^2*e - 12*a^2*b^2*c*d*e^2)/(16*(16*a^
4*c^3 + a^2*b^4*c - 8*a^3*b^2*c^2)))^(1/2)*1i + ((d + e*x^2)^(1/2)*(2*a^4*c
*e^16 + 6*c^5*d^8*e^8 - 16*a*c^4*d^6*e^10 - 16*b*c^4*d^7*e^9 + 4*b^4*c*d^4*
e^12 + 16*a^2*c^3*d^4*e^12 + 8*a^3*c^2*d^2*e^14 + 24*b^2*c^3*d^6*e^10 - 16*
b^3*c^2*d^5*e^11 - 8*a^3*b*c*d*e^15 - 8*a*b^3*c*d^3*e^13 + 16*a*b^2*c^2*d^4
*e^12 - 24*a^2*b*c^2*d^3*e^13 + 12*a^2*b^2*c*d^2*e^14) + (-(((4*b^4*c*d^3 -

4*a^2*b^3*e^3 + 32*a^2*c^3*d^3 - 24*a*b^2*c^2*d^3 - 96*a^3*c^2*d*e^2 + 16*
a^3*b*c*e^3 - 12*a*b^3*c*d^2*e + 48*a^2*b*c^2*d^2*e + 24*a^2*b^2*c*d*e^2)^2
/4 - (256*a^4*c^3 + 16*a^2*b^4*c - 128*a^3*b^2*c^2)*(a^3*e^6 + c^3*d^6 - b^
3*d^3*e^3 + 3*a*b^2*d^2*e^4 + 3*a*c^2*d^4*e^2 + 3*a^2*c*d^2*e^4 + 3*b^2*c*d
^4*e^2 - 3*a^2*b*d*e^5 - 3*b*c^2*d^5*e - 6*a*b*c*d^3*e^3))^(1/2) - 2*b^4*c*
d^3 + 2*a^2*b^3*e^3 - 16*a^2*c^3*d^3 + 12*a*b^2*c^2*d^3 + 48*a^3*c^2*d*e^2
- 8*a^3*b*c*e^3 + 6*a*b^3*c*d^2*e - 24*a^2*b*c^2*d^2*e - 12*a^2*b^2*c*d*e^2
)/(16*(16*a^4*c^3 + a^2*b^4*c - 8*a^3*b^2*c^2)))^(1/2)*(((-(((4*b^4*c*d^3 -
4*a^2*b^3*e^3 + 32*a^2*c^3*d^3 - 24*a*b^2*c^2*d^3 - 96*a^3*c^2*d*e^2 + 16*
a^3*b*c*e^3 - 12*a*b^3*c*d^2*e + 48*a^2*b*c^2*d^2*e + 24*a^2*b^2*c*d*e^2)^2
/4 - (256*a^4*c^3 + 16*a^2*b^4*c - 128*a^3*b^2*c^2)*(a^3*e^6 + c^3*d^6 - b^
3*d^3*e^3 + 3*a*b^2*d^2*e^4 + 3*a*c^2*d^4*e^2 + 3*a^2*c*d^2*e^4 + 3*b^2*c*d
^4*e^2 - 3*a^2*b*d*e^5 - 3*b*c^2*d^5*e - 6*a*b*c*d^3*e^3))^(1/2) - 2*b^4*c*
d^3 + 2*a^2*b^3*e^3 - 16*a^2*c^3*d^3 + 12*a*b^2*c^2*d^3 + 48*a^3*c^2*d*e^2
- 8*a^3*b*c*e^3 + 6*a*b^3*c*d^2*e - 24*a^2*b*c^2*d^2*e - 12*a^2*b^2*c*d*e^2
)/(16*(16*a^4*c^3 + a^2*b^4*c - 8*a^3*b^2*c^2)))^(1/2)*((d + e*x^2)^(1/2)*(
-(((4*b^4*c*d^3 - 4*a^2*b^3*e^3 + 32*a^2*c^3*d^3 - 24*a*b^2*c^2*d^3 - 96*a^
3*c^2*d*e^2 + 16*a^3*b*c*e^3 - 12*a*b^3*c*d^2*e + 48*a^2*b*c^2*d^2*e + 24*a
^2*b^2*c*d*e^2)^2/4 - (256*a^4*c^3 + 16*a^2*b^4*c - 128*a^3*b^2*c^2)*(a^3*e
^6 + c^3*d^6 - b^3*d^3*e^3 + 3*a*b^2*d^2*e^4 + 3*a*c^2*d^4*e^2 + 3*a^2*c*d^
2*e^4 + 3*b^2*c*d^4*e^2 - 3*a^2*b*d*e^5 - 3*b*c^2*d^5*e - 6*a*b*c*d^3*e^3))
^(1/2) - 2*b^4*c*d^3 + 2*a^2*b^3*e^3 - 16*a^2*c^3*d^3 + 12*a*b^2*c^2*d^3 +
48*a^3*c^2*d*e^2 - 8*a^3*b*c*e^3 + 6*a*b^3*c*d^2*e - 24*a^2*b*c^2*d^2*e - 1
2*a^2*b^2*c*d*e^2)/(16*(16*a^4*c^3 + a^2*b^4*c - 8*a^3*b^2*c^2)))^(1/2)*(51
2*a^5*c^4*e^10 + 32*a^3*b^4*c^2*e^10 - 256*a^4*b^2*c^3*e^10 + 768*a^4*c^5*d
^2*e^8 + 64*a^2*b^4*c^3*d^2*e^8 - 448*a^3*b^2*c^4*d^2*e^8 - 896*a^4*b*c^4*d
*e^9 - 64*a^2*b^5*c^2*d*e^9 + 480*a^3*b^3*c^3*d*e^9) + 192*a^3*c^5*d^4*e^8
+ 192*a^4*c^4*d^2*e^10 - 48*a^2*b^2*c^4*d^4*e^8 + 64*a^2*b^3*c^3*d^3*e^9 -
16*a^2*b^4*c^2*d^2*e^10 + 16*a^3*b^2*c^3*d^2*e^10 - 64*a^4*b*c^3*d*e^11 - 2
56*a^3*b*c^4*d^3*e^9 + 16*a^3*b^3*c^2*d*e^11) + (d + e*x^2)^(1/2)*(8*a^3*b^
3*c*e^13 - 32*a^4*b*c^2*e^13 + 176*a^4*c^3*d*e^12 - 144*a^2*c^5*d^5*e^8 + 2
24*a^3*c^4*d^3*e^10 - 16*b^4*c^3*d^5*e^8 + 16*b^5*c^2*d^4*e^9 + 48*a^2*b^2*
c^3*d^3*e^10 + 112*a^2*b^3*c^2*d^2*e^11 - 16*a^2*b^4*c*d*e^12 + 96*a*b^2*c^
4*d^5*e^8 - 80*a*b^3*c^3*d^4*e^9 - 32*a*b^4*c^2*d^3*e^10 + 96*a^2*b*c^4*d^4
*e^9 - 416*a^3*b*c^3*d^2*e^11 + 16*a^3*b^2*c^2*d*e^12))*(-(((4*b^4*c*d^3 -
4*a^2*b^3*e^3 + 32*a^2*c^3*d^3 - 24*a*b^2*c^2*d^3 - 96*a^3*c^2*d*e^2 + 16*a
^3*b*c*e^3 - 12*a*b^3*c*d^2*e + 48*a^2*b*c^2*d^2*e + 24*a^2*b^2*c*d*e^2)^2/
4 - (256*a^4*c^3 + 16*a^2*b^4*c - 128*a^3*b^2*c^2)*(a^3*e^6 + c^3*d^6 - b^3
*d^3*e^3 + 3*a*b^2*d^2*e^4 + 3*a*c^2*d^4*e^2 + 3*a^2*c*d^2*e^4 + 3*b^2*c*d^
4*e^2 - 3*a^2*b*d*e^5 - 3*b*c^2*d^5*e - 6*a*b*c*d^3*e^3))^(1/2) - 2*b^4*c*d
^3 + 2*a^2*b^3*e^3 - 16*a^2*c^3*d^3 + 12*a*b^2*c^2*d^3 + 48*a^3*c^2*d*e^2 -
8*a^3*b*c*e^3 + 6*a*b^3*c*d^2*e - 24*a^2*b*c^2*d^2*e - 12*a^2*b^2*c*d*e^2)
/(16*(16*a^4*c^3 + a^2*b^4*c - 8*a^3*b^2*c^2)))^(1/2) - 12*a*c^5*d^7*e^8 -
4*a^4*c^2*d*e^14 + 84*a^2*c^4*d^5*e^10 + 92*a^3*c^3*d^3*e^12 + 4*b^2*c^4*d^
7*e^8 + 4*b^3*c^3*d^6*e^9 - 8*b^4*c^2*d^5*e^10 + 12*a^2*b^2*c^2*d^3*e^12 -

32*a*b*c^4*d^6*e^9 + 4*a^3*b^2*c*d*e^14 + 36*a*b^2*c^3*d^5*e^10 + 20*a*b^3*
c^2*d^4*e^11 - 160*a^2*b*c^3*d^4*e^11 - 4*a^2*b^3*c*d^2*e^13 - 16*a^3*b*c^2
*d^2*e^13))*(-(((4*b^4*c*d^3 - 4*a^2*b^3*e^3 + 32*a^2*c^3*d^3 - 24*a*b^2*c^
2*d^3 - 96*a^3*c^2*d*e^2 + 16*a^3*b*c*e^3 - 12*a*b^3*c*d^2*e + 48*a^2*b*c^2
*d^2*e + 24*a^2*b^2*c*d*e^2)^2/4 - (256*a^4*c^3 + 16*a^2*b^4*c - 128*a^3*b^
2*c^2)*(a^3*e^6 + c^3*d^6 - b^3*d^3*e^3 + 3*a*b^2*d^2*e^4 + 3*a*c^2*d^4*e^2
 + 3*a^2*c*d^2*e^4 + 3*b^2*c*d^4*e^2 - 3*a^2*b*d*e^5 - 3*b*c^2*d^5*e - 6*a*
b*c*d^3*e^3))^(1/2) - 2*b^4*c*d^3 + 2*a^2*b^3*e^3 - 16*a^2*c^3*d^3 + 12*a*b
^2*c^2*d^3 + 48*a^3*c^2*d*e^2 - 8*a^3*b*c*e^3 + 6*a*b^3*c*d^2*e - 24*a^2*b*
c^2*d^2*e - 12*a^2*b^2*c*d*e^2)/(16*(16*a^4*c^3 + a^2*b^4*c - 8*a^3*b^2*c^2
)))^(1/2)*1i)/(((d + e*x^2)^(1/2)*(2*a^4*c*e^16 + 6*c^5*d^8*e^8 - 16*a*c^4*
d^6*e^10 - 16*b*c^4*d^7*e^9 + 4*b^4*c*d^4*e^12 + 16*a^2*c^3*d^4*e^12 + 8*a^
3*c^2*d^2*e^14 + 24*b^2*c^3*d^6*e^10 - 16*b^3*c^2*d^5*e^11 - 8*a^3*b*c*d*e^
15 - 8*a*b^3*c*d^3*e^13 + 16*a*b^2*c^2*d^4*e^12 - 24*a^2*b*c^2*d^3*e^13 + 1
2*a^2*b^2*c*d^2*e^14) + (-(((4*b^4*c*d^3 - 4*a^2*b^3*e^3 + 32*a^2*c^3*d^3 -
 24*a*b^2*c^2*d^3 - 96*a^3*c^2*d*e^2 + 16*a^3*b*c*e^3 - 12*a*b^3*c*d^2*e +
48*a^2*b*c^2*d^2*e + 24*a^2*b^2*c*d*e^2)^2/4 - (256*a^4*c^3 + 16*a^2*b^4*c
- 128*a^3*b^2*c^2)*(a^3*e^6 + c^3*d^6 - b^3*d^3*e^3 + 3*a*b^2*d^2*e^4 + 3*a
*c^2*d^4*e^2 + 3*a^2*c*d^2*e^4 + 3*b^2*c*d^4*e^2 - 3*a^2*b*d*e^5 - 3*b*c^2*
d^5*e - 6*a*b*c*d^3*e^3))^(1/2) - 2*b^4*c*d^3 + 2*a^2*b^3*e^3 - 16*a^2*c^3*
d^3 + 12*a*b^2*c^2*d^3 + 48*a^3*c^2*d*e^2 - 8*a^3*b*c*e^3 + 6*a*b^3*c*d^2*e
 - 24*a^2*b*c^2*d^2*e - 12*a^2*b^2*c*d*e^2)/(16*(16*a^4*c^3 + a^2*b^4*c - 8
*a^3*b^2*c^2)))^(1/2)*(((-(((4*b^4*c*d^3 - 4*a^2*b^3*e^3 + 32*a^2*c^3*d^3 -
 24*a*b^2*c^2*d^3 - 96*a^3*c^2*d*e^2 + 16*a^3*b*c*e^3 - 12*a*b^3*c*d^2*e +
48*a^2*b*c^2*d^2*e + 24*a^2*b^2*c*d*e^2)^2/4 - (256*a^4*c^3 + 16*a^2*b^4*c
- 128*a^3*b^2*c^2)*(a^3*e^6 + c^3*d^6 - b^3*d^3*e^3 + 3*a*b^2*d^2*e^4 + 3*a
*c^2*d^4*e^2 + 3*a^2*c*d^2*e^4 + 3*b^2*c*d^4*e^2 - 3*a^2*b*d*e^5 - 3*b*c^2*
d^5*e - 6*a*b*c*d^3*e^3))^(1/2) - 2*b^4*c*d^3 + 2*a^2*b^3*e^3 - 16*a^2*c^3*
d^3 + 12*a*b^2*c^2*d^3 + 48*a^3*c^2*d*e^2 - 8*a^3*b*c*e^3 + 6*a*b^3*c*d^2*e
 - 24*a^2*b*c^2*d^2*e - 12*a^2*b^2*c*d*e^2)/(16*(16*a^4*c^3 + a^2*b^4*c - 8
*a^3*b^2*c^2)))^(1/2)*((d + e*x^2)^(1/2)*(-(((4*b^4*c*d^3 - 4*a^2*b^3*e^3 +
 32*a^2*c^3*d^3 - 24*a*b^2*c^2*d^3 - 96*a^3*c^2*d*e^2 + 16*a^3*b*c*e^3 - 12
*a*b^3*c*d^2*e + 48*a^2*b*c^2*d^2*e + 24*a^2*b^2*c*d*e^2)^2/4 - (256*a^4*c^
3 + 16*a^2*b^4*c - 128*a^3*b^2*c^2)*(a^3*e^6 + c^3*d^6 - b^3*d^3*e^3 + 3*a*
b^2*d^2*e^4 + 3*a*c^2*d^4*e^2 + 3*a^2*c*d^2*e^4 + 3*b^2*c*d^4*e^2 - 3*a^2*b
*d*e^5 - 3*b*c^2*d^5*e - 6*a*b*c*d^3*e^3))^(1/2) - 2*b^4*c*d^3 + 2*a^2*b^3*
e^3 - 16*a^2*c^3*d^3 + 12*a*b^2*c^2*d^3 + 48*a^3*c^2*d*e^2 - 8*a^3*b*c*e^3
+ 6*a*b^3*c*d^2*e - 24*a^2*b*c^2*d^2*e - 12*a^2*b^2*c*d*e^2)/(16*(16*a^4*c^
3 + a^2*b^4*c - 8*a^3*b^2*c^2)))^(1/2)*(512*a^5*c^4*e^10 + 32*a^3*b^4*c^2*e
^10 - 256*a^4*b^2*c^3*e^10 + 768*a^4*c^5*d^2*e^8 + 64*a^2*b^4*c^3*d^2*e^8 -
 448*a^3*b^2*c^4*d^2*e^8 - 896*a^4*b*c^4*d*e^9 - 64*a^2*b^5*c^2*d*e^9 + 480
*a^3*b^3*c^3*d*e^9) + 192*a^3*c^5*d^4*e^8 + 192*a^4*c^4*d^2*e^10 - 48*a^2*b
^2*c^4*d^4*e^8 + 64*a^2*b^3*c^3*d^3*e^9 - 16*a^2*b^4*c^2*d^2*e^10 + 16*a^3*
b^2*c^3*d^2*e^10 - 64*a^4*b*c^3*d*e^11 - 256*a^3*b*c^4*d^3*e^9 + 16*a^3*b^3
*c^2*d*e^11) + (d + e*x^2)^(1/2)*(8*a^3*b^3*c*e^13 - 32*a^4*b*c^2*e^13 + 17

6*a^4*c^3*d*e^12 - 144*a^2*c^5*d^5*e^8 + 224*a^3*c^4*d^3*e^10 - 16*b^4*c^3*
d^5*e^8 + 16*b^5*c^2*d^4*e^9 + 48*a^2*b^2*c^3*d^3*e^10 + 112*a^2*b^3*c^2*d^
2*e^11 - 16*a^2*b^4*c*d*e^12 + 96*a*b^2*c^4*d^5*e^8 - 80*a*b^3*c^3*d^4*e^9
- 32*a*b^4*c^2*d^3*e^10 + 96*a^2*b*c^4*d^4*e^9 - 416*a^3*b*c^3*d^2*e^11 + 1
6*a^3*b^2*c^2*d*e^12))*(-(((4*b^4*c*d^3 - 4*a^2*b^3*e^3 + 32*a^2*c^3*d^3 -
24*a*b^2*c^2*d^3 - 96*a^3*c^2*d*e^2 + 16*a^3*b*c*e^3 - 12*a*b^3*c*d^2*e + 4
8*a^2*b*c^2*d^2*e + 24*a^2*b^2*c*d*e^2)^2/4 - (256*a^4*c^3 + 16*a^2*b^4*c -
128*a^3*b^2*c^2)*(a^3*e^6 + c^3*d^6 - b^3*d^3*e^3 + 3*a*b^2*d^2*e^4 + 3*a*
c^2*d^4*e^2 + 3*a^2*c*d^2*e^4 + 3*b^2*c*d^4*e^2 - 3*a^2*b*d*e^5 - 3*b*c^2*d
^5*e - 6*a*b*c*d^3*e^3))^(1/2) - 2*b^4*c*d^3 + 2*a^2*b^3*e^3 - 16*a^2*c^3*d
^3 + 12*a*b^2*c^2*d^3 + 48*a^3*c^2*d*e^2 - 8*a^3*b*c*e^3 + 6*a*b^3*c*d^2*e
- 24*a^2*b*c^2*d^2*e - 12*a^2*b^2*c*d*e^2)/(16*(16*a^4*c^3 + a^2*b^4*c - 8*
a^3*b^2*c^2)))^(1/2) - 12*a*c^5*d^7*e^8 - 4*a^4*c^2*d*e^14 + 84*a^2*c^4*d^5
*e^10 + 92*a^3*c^3*d^3*e^12 + 4*b^2*c^4*d^7*e^8 + 4*b^3*c^3*d^6*e^9 - 8*b^4
*c^2*d^5*e^10 + 12*a^2*b^2*c^2*d^3*e^12 - 32*a*b*c^4*d^6*e^9 + 4*a^3*b^2*c*
d*e^14 + 36*a*b^2*c^3*d^5*e^10 + 20*a*b^3*c^2*d^4*e^11 - 160*a^2*b*c^3*d^4*
e^11 - 4*a^2*b^3*c*d^2*e^13 - 16*a^3*b*c^2*d^2*e^13))*(-(((4*b^4*c*d^3 - 4*
a^2*b^3*e^3 + 32*a^2*c^3*d^3 - 24*a*b^2*c^2*d^3 - 96*a^3*c^2*d*e^2 + 16*a^3
*b*c*e^3 - 12*a*b^3*c*d^2*e + 48*a^2*b*c^2*d^2*e + 24*a^2*b^2*c*d*e^2)^2/4
- (256*a^4*c^3 + 16*a^2*b^4*c - 128*a^3*b^2*c^2)*(a^3*e^6 + c^3*d^6 - b^3*d
^3*e^3 + 3*a*b^2*d^2*e^4 + 3*a*c^2*d^4*e^2 + 3*a^2*c*d^2*e^4 + 3*b^2*c*d^4*
e^2 - 3*a^2*b*d*e^5 - 3*b*c^2*d^5*e - 6*a*b*c*d^3*e^3))^(1/2) - 2*b^4*c*d^3
+ 2*a^2*b^3*e^3 - 16*a^2*c^3*d^3 + 12*a*b^2*c^2*d^3 + 48*a^3*c^2*d*e^2 - 8
*a^3*b*c*e^3 + 6*a*b^3*c*d^2*e - 24*a^2*b*c^2*d^2*e - 12*a^2*b^2*c*d*e^2)/(
16*(16*a^4*c^3 + a^2*b^4*c - 8*a^3*b^2*c^2)))^(1/2) - ((d + e*x^2)^(1/2)*(2
*a^4*c*e^16 + 6*c^5*d^8*e^8 - 16*a*c^4*d^6*e^10 - 16*b*c^4*d^7*e^9 + 4*b^4*
c*d^4*e^12 + 16*a^2*c^3*d^4*e^12 + 8*a^3*c^2*d^2*e^14 + 24*b^2*c^3*d^6*e^10
- 16*b^3*c^2*d^5*e^11 - 8*a^3*b*c*d*e^15 - 8*a*b^3*c*d^3*e^13 + 16*a*b^2*c
^2*d^4*e^12 - 24*a^2*b*c^2*d^3*e^13 + 12*a^2*b^2*c*d^2*e^14) + (-(((4*b^4*c
*d^3 - 4*a^2*b^3*e^3 + 32*a^2*c^3*d^3 - 24*a*b^2*c^2*d^3 - 96*a^3*c^2*d*e^2
+ 16*a^3*b*c*e^3 - 12*a*b^3*c*d^2*e + 48*a^2*b*c^2*d^2*e + 24*a^2*b^2*c*d*
e^2)^2/4 - (256*a^4*c^3 + 16*a^2*b^4*c - 128*a^3*b^2*c^2)*(a^3*e^6 + c^3*d^
6 - b^3*d^3*e^3 + 3*a*b^2*d^2*e^4 + 3*a*c^2*d^4*e^2 + 3*a^2*c*d^2*e^4 + 3*b
^2*c*d^4*e^2 - 3*a^2*b*d*e^5 - 3*b*c^2*d^5*e - 6*a*b*c*d^3*e^3))^(1/2) - 2*
b^4*c*d^3 + 2*a^2*b^3*e^3 - 16*a^2*c^3*d^3 + 12*a*b^2*c^2*d^3 + 48*a^3*c^2*
d*e^2 - 8*a^3*b*c*e^3 + 6*a*b^3*c*d^2*e - 24*a^2*b*c^2*d^2*e - 12*a^2*b^2*c
*d*e^2)/(16*(16*a^4*c^3 + a^2*b^4*c - 8*a^3*b^2*c^2)))^(1/2)*(((-(((4*b^4*c
*d^3 - 4*a^2*b^3*e^3 + 32*a^2*c^3*d^3 - 24*a*b^2*c^2*d^3 - 96*a^3*c^2*d*e^2
+ 16*a^3*b*c*e^3 - 12*a*b^3*c*d^2*e + 48*a^2*b*c^2*d^2*e + 24*a^2*b^2*c*d*
e^2)^2/4 - (256*a^4*c^3 + 16*a^2*b^4*c - 128*a^3*b^2*c^2)*(a^3*e^6 + c^3*d^
6 - b^3*d^3*e^3 + 3*a*b^2*d^2*e^4 + 3*a*c^2*d^4*e^2 + 3*a^2*c*d^2*e^4 + 3*b
^2*c*d^4*e^2 - 3*a^2*b*d*e^5 - 3*b*c^2*d^5*e - 6*a*b*c*d^3*e^3))^(1/2) - 2*
b^4*c*d^3 + 2*a^2*b^3*e^3 - 16*a^2*c^3*d^3 + 12*a*b^2*c^2*d^3 + 48*a^3*c^2*
d*e^2 - 8*a^3*b*c*e^3 + 6*a*b^3*c*d^2*e - 24*a^2*b*c^2*d^2*e - 12*a^2*b^2*c
*d*e^2)/(16*(16*a^4*c^3 + a^2*b^4*c - 8*a^3*b^2*c^2)))^(1/2)*((d + e*x^2)^(

1/2)*(-(((4*b^4*c*d^3 - 4*a^2*b^3*e^3 + 32*a^2*c^3*d^3 - 24*a*b^2*c^2*d^3 - 96*a^3*c^2*d*e^2 + 16*a^3*b*c*e^3 - 12*a*b^3*c*d^2*e + 48*a^2*b*c^2*d^2*e + 24*a^2*b^2*c*d*e^2)^2/4 - (256*a^4*c^3 + 16*a^2*b^4*c - 128*a^3*b^2*c^2)*(a^3*e^6 + c^3*d^6 - b^3*d^3*e^3 + 3*a*b^2*d^2*e^4 + 3*a*c^2*d^4*e^2 + 3*a^2*c*d^2*e^4 + 3*b^2*c*d^4*e^2 - 3*a^2*b*d*e^5 - 3*b*c^2*d^5*e - 6*a*b*c*d^3*e^3))^(1/2) - 2*b^4*c*d^3 + 2*a^2*b^3*e^3 - 16*a^2*c^3*d^3 + 12*a*b^2*c^2*d^3 + 48*a^3*c^2*d*e^2 - 8*a^3*b*c*e^3 + 6*a*b^3*c*d^2*e - 24*a^2*b*c^2*d^2*e - 12*a^2*b^2*c*d*e^2)/(16*(16*a^4*c^3 + a^2*b^4*c - 8*a^3*b^2*c^2)))^(1/2)*(512*a^5*c^4*e^10 + 32*a^3*b^4*c^2*e^10 - 256*a^4*b^2*c^3*e^10 + 768*a^4*c^5*d^2*e^8 + 64*a^2*b^4*c^3*d^2*e^8 - 448*a^3*b^2*c^4*d^2*e^8 - 896*a^4*b*c^4*d*e^9 - 64*a^2*b^5*c^2*d*e^9 + 480*a^3*b^3*c^3*d*e^9) - 192*a^3*c^5*d^4*e^8 - 192*a^4*c^4*d^2*e^10 + 48*a^2*b^2*c^4*d^4*e^8 - 64*a^2*b^3*c^3*d^3*e^9 + 16*a^2*b^4*c^2*d^2*e^10 - 16*a^3*b^2*c^3*d^2*e^10 + 64*a^4*b*c^3*d*e^11 + 256*a^3*b*c^4*d^3*e^9 - 16*a^3*b^3*c^2*d*e^11) + (d + e*x^2)^(1/2)*(8*a^3*b^3*c*e^13 - 32*a^4*b*c^2*e^13 + 176*a^4*c^3*d*e^12 - 144*a^2*c^5*d^5*e^8 + 224*a^3*c^4*d^3*e^10 - 16*b^4*c^3*d^5*e^8 + 16*b^5*c^2*d^4*e^9 + 48*a^2*b^2*c^3*d^3*e^10 + 112*a^2*b^3*c^2*d^2*e^11 - 16*a^2*b^4*c*d*e^12 + 96*a*b^2*c^4*d^5*e^8 - 80*a*b^3*c^3*d^4*e^9 - 32*a*b^4*c^2*d^3*e^10 + 96*a^2*b*c^4*d^4*e^9 - 416*a^3*b*c^3*d^2*e^11 + 16*a^3*b^2*c^2*d*e^12))*(-(((4*b^4*c*d^3 - 4*a^2*b^3*e^3 + 32*a^2*c^3*d^3 - 24*a*b^2*c^2*d^3 - 96*a^3*c^2*d*e^2 + 16*a^3*b*c*e^3 - 12*a*b^3*c*d^2*e + 48*a^2*b*c^2*d^2*e + 24*a^2*b^2*c*d*e^2)^2/4 - (256*a^4*c^3 + 16*a^2*b^4*c - 128*a^3*b^2*c^2)*(a^3*e^6 + c^3*d^6 - b^3*d^3*e^3 + 3*a*b^2*d^2*e^4 + 3*a*c^2*d^4*e^2 + 3*a^2*c*d^2*e^4 + 3*b^2*c*d^4*e^2 - 3*a^2*b*d*e^5 - 3*b*c^2*d^5*e - 6*a*b*c*d^3*e^3))^(1/2) - 2*b^4*c*d^3 + 2*a^2*b^3*e^3 - 16*a^2*c^3*d^3 + 12*a*b^2*c^2*d^3 + 48*a^3*c^2*d*e^2 - 8*a^3*b*c*e^3 + 6*a*b^3*c*d^2*e - 24*a^2*b*c^2*d^2*e - 12*a^2*b^2*c*d*e^2)/(16*(16*a^4*c^3 + a^2*b^4*c - 8*a^3*b^2*c^2)))^(1/2) + 12*a*c^5*d^7*e^8 + 4*a^4*c^2*d*e^14 - 84*a^2*c^4*d^5*e^10 - 92*a^3*c^3*d^3*e^12 - 4*b^2*c^4*d^7*e^8 - 4*b^3*c^3*d^6*e^9 + 8*b^4*c^2*d^5*e^10 - 12*a^2*b^2*c^2*d^3*e^12 + 32*a*b*c^4*d^6*e^9 - 4*a^3*b^2*c*d*e^14 - 36*a*b^2*c^3*d^5*e^10 - 20*a*b^3*c^2*d^4*e^11 + 160*a^2*b*c^3*d^4*e^11 + 4*a^2*b^3*c*d^2*e^13 + 16*a^3*b*c^2*d^2*e^13))*(-(((4*b^4*c*d^3 - 4*a^2*b^3*e^3 + 32*a^2*c^3*d^3 - 24*a*b^2*c^2*d^3 - 96*a^3*c^2*d*e^2 + 16*a^3*b*c*e^3 - 12*a*b^3*c*d^2*e + 48*a^2*b*c^2*d^2*e + 24*a^2*b^2*c*d*e^2)^2/4 - (256*a^4*c^3 + 16*a^2*b^4*c - 128*a^3*b^2*c^2)*(a^3*e^6 + c^3*d^6 - b^3*d^3*e^3 + 3*a*b^2*d^2*e^4 + 3*a*c^2*d^4*e^2 + 3*a^2*c*d^2*e^4 + 3*b^2*c*d^4*e^2 - 3*a^2*b*d*e^5 - 3*b*c^2*d^5*e - 6*a*b*c*d^3*e^3))^(1/2) - 2*b^4*c*d^3 + 2*a^2*b^3*e^3 - 16*a^2*c^3*d^3 + 12*a*b^2*c^2*d^3 + 48*a^3*c^2*d*e^2 - 8*a^3*b*c*e^3 + 6*a*b^3*c*d^2*e - 24*a^2*b*c^2*d^2*e - 12*a^2*b^2*c*d*e^2)/(16*(16*a^4*c^3 + a^2*b^4*c - 8*a^3*b^2*c^2)))^(1/2) + 6*c^4*d^8*e^10 + 14*a*c^3*d^6*e^12 + 2*a^3*c*d^2*e^16 - 16*b*c^3*d^7*e^11 - 4*b^3*c*d^5*e^13 + 10*a^2*c^2*d^4*e^14 + 14*b^2*c^2*d^6*e^12 - 24*a*b*c^2*d^5*e^13 + 10*a*b^2*c*d^4*e^14 - 8*a^2*b*c*d^3*e^15))*(-(((4*b^4*c*d^3 - 4*a^2*b^3*e^3 + 32*a^2*c^3*d^3 - 24*a*b^2*c^2*d^3 - 96*a^3*c^2*d*e^2 + 16*a^3*b*c*e^3 - 12*a*b^3*c*d^2*e + 48*a^2*b*c^2*d^2*e + 24*a^2*b^2*c*d*e^2)^2/4 - (256*a^4*c^3 + 16*a^2*b^4*c - 128*a^3*b^2*c^2)*(a^3*e^6

+ c^3*d^6 - b^3*d^3*e^3 + 3*a*b^2*d^2*e^4 + 3*a*c^2*d^4*e^2 + 3*a^2*c*d^2*
e^4 + 3*b^2*c*d^4*e^2 - 3*a^2*b*d*e^5 - 3*b*c^2*d^5*e - 6*a*b*c*d^3*e^3))^(
1/2) - 2*b^4*c*d^3 + 2*a^2*b^3*e^3 - 16*a^2*c^3*d^3 + 12*a*b^2*c^2*d^3 + 48
*a^3*c^2*d*e^2 - 8*a^3*b*c*e^3 + 6*a*b^3*c*d^2*e - 24*a^2*b*c^2*d^2*e - 12*
a^2*b^2*c*d*e^2)/(16*(16*a^4*c^3 + a^2*b^4*c - 8*a^3*b^2*c^2)))^(1/2)*2i +
atan((((d + e*x^2)^(1/2)*(2*a^4*c*e^16 + 6*c^5*d^8*e^8 - 16*a*c^4*d^6*e^10
- 16*b*c^4*d^7*e^9 + 4*b^4*c*d^4*e^12 + 16*a^2*c^3*d^4*e^12 + 8*a^3*c^2*d^2
*e^14 + 24*b^2*c^3*d^6*e^10 - 16*b^3*c^2*d^5*e^11 - 8*a^3*b*c*d*e^15 - 8*a*
b^3*c*d^3*e^13 + 16*a*b^2*c^2*d^4*e^12 - 24*a^2*b*c^2*d^3*e^13 + 12*a^2*b^2
*c*d^2*e^14) + ((((4*b^4*c*d^3 - 4*a^2*b^3*e^3 + 32*a^2*c^3*d^3 - 24*a*b^2*
c^2*d^3 - 96*a^3*c^2*d*e^2 + 16*a^3*b*c*e^3 - 12*a*b^3*c*d^2*e + 48*a^2*b*c
^2*d^2*e + 24*a^2*b^2*c*d*e^2)^2/4 - (256*a^4*c^3 + 16*a^2*b^4*c - 128*a^3*
b^2*c^2)*(a^3*e^6 + c^3*d^6 - b^3*d^3*e^3 + 3*a*b^2*d^2*e^4 + 3*a*c^2*d^4*e
^2 + 3*a^2*c*d^2*e^4 + 3*b^2*c*d^4*e^2 - 3*a^2*b*d*e^5 - 3*b*c^2*d^5*e - 6*
a*b*c*d^3*e^3))^(1/2) + 2*b^4*c*d^3 - 2*a^2*b^3*e^3 + 16*a^2*c^3*d^3 - 12*a
*b^2*c^2*d^3 - 48*a^3*c^2*d*e^2 + 8*a^3*b*c*e^3 - 6*a*b^3*c*d^2*e + 24*a^2*
b*c^2*d^2*e + 12*a^2*b^2*c*d*e^2)/(16*(16*a^4*c^3 + a^2*b^4*c - 8*a^3*b^2*c
^2)))^(1/2)*((((((4*b^4*c*d^3 - 4*a^2*b^3*e^3 + 32*a^2*c^3*d^3 - 24*a*b^2*c
^2*d^3 - 96*a^3*c^2*d*e^2 + 16*a^3*b*c*e^3 - 12*a*b^3*c*d^2*e + 48*a^2*b*c^
2*d^2*e + 24*a^2*b^2*c*d*e^2)^2/4 - (256*a^4*c^3 + 16*a^2*b^4*c - 128*a^3*b
^2*c^2)*(a^3*e^6 + c^3*d^6 - b^3*d^3*e^3 + 3*a*b^2*d^2*e^4 + 3*a*c^2*d^4*e^
2 + 3*a^2*c*d^2*e^4 + 3*b^2*c*d^4*e^2 - 3*a^2*b*d*e^5 - 3*b*c^2*d^5*e - 6*a
*b*c*d^3*e^3))^(1/2) + 2*b^4*c*d^3 - 2*a^2*b^3*e^3 + 16*a^2*c^3*d^3 - 12*a*
b^2*c^2*d^3 - 48*a^3*c^2*d*e^2 + 8*a^3*b*c*e^3 - 6*a*b^3*c*d^2*e + 24*a^2*b
*c^2*d^2*e + 12*a^2*b^2*c*d*e^2)/(16*(16*a^4*c^3 + a^2*b^4*c - 8*a^3*b^2*c^
2)))^(1/2)*((d + e*x^2)^(1/2)*((((4*b^4*c*d^3 - 4*a^2*b^3*e^3 + 32*a^2*c^3*
d^3 - 24*a*b^2*c^2*d^3 - 96*a^3*c^2*d*e^2 + 16*a^3*b*c*e^3 - 12*a*b^3*c*d^2
*e + 48*a^2*b*c^2*d^2*e + 24*a^2*b^2*c*d*e^2)^2/4 - (256*a^4*c^3 + 16*a^2*b
^4*c - 128*a^3*b^2*c^2)*(a^3*e^6 + c^3*d^6 - b^3*d^3*e^3 + 3*a*b^2*d^2*e^4
+ 3*a*c^2*d^4*e^2 + 3*a^2*c*d^2*e^4 + 3*b^2*c*d^4*e^2 - 3*a^2*b*d*e^5 - 3*b
*c^2*d^5*e - 6*a*b*c*d^3*e^3))^(1/2) + 2*b^4*c*d^3 - 2*a^2*b^3*e^3 + 16*a^2
*c^3*d^3 - 12*a*b^2*c^2*d^3 - 48*a^3*c^2*d*e^2 + 8*a^3*b*c*e^3 - 6*a*b^3*c*
d^2*e + 24*a^2*b*c^2*d^2*e + 12*a^2*b^2*c*d*e^2)/(16*(16*a^4*c^3 + a^2*b^4*
c - 8*a^3*b^2*c^2)))^(1/2)*(512*a^5*c^4*e^10 + 32*a^3*b^4*c^2*e^10 - 256*a^
4*b^2*c^3*e^10 + 768*a^4*c^5*d^2*e^8 + 64*a^2*b^4*c^3*d^2*e^8 - 448*a^3*b^2
*c^4*d^2*e^8 - 896*a^4*b*c^4*d*e^9 - 64*a^2*b^5*c^2*d*e^9 + 480*a^3*b^3*c^3
*d*e^9) - 192*a^3*c^5*d^4*e^8 - 192*a^4*c^4*d^2*e^10 + 48*a^2*b^2*c^4*d^4*e
^8 - 64*a^2*b^3*c^3*d^3*e^9 + 16*a^2*b^4*c^2*d^2*e^10 - 16*a^3*b^2*c^3*d^2*
e^10 + 64*a^4*b*c^3*d*e^11 + 256*a^3*b*c^4*d^3*e^9 - 16*a^3*b^3*c^2*d*e^11)
+ (d + e*x^2)^(1/2)*(8*a^3*b^3*c*e^13 - 32*a^4*b*c^2*e^13 + 176*a^4*c^3*d*
e^12 - 144*a^2*c^5*d^5*e^8 + 224*a^3*c^4*d^3*e^10 - 16*b^4*c^3*d^5*e^8 + 16
*b^5*c^2*d^4*e^9 + 48*a^2*b^2*c^3*d^3*e^10 + 112*a^2*b^3*c^2*d^2*e^11 - 16*
a^2*b^4*c*d*e^12 + 96*a*b^2*c^4*d^5*e^8 - 80*a*b^3*c^3*d^4*e^9 - 32*a*b^4*c
^2*d^3*e^10 + 96*a^2*b*c^4*d^4*e^9 - 416*a^3*b*c^3*d^2*e^11 + 16*a^3*b^2*c^
2*d*e^12))*((((4*b^4*c*d^3 - 4*a^2*b^3*e^3 + 32*a^2*c^3*d^3 - 24*a*b^2*c^2*

d^3 - 96*a^3*c^2*d*e^2 + 16*a^3*b*c*e^3 - 12*a*b^3*c*d^2*e + 48*a^2*b*c^2*d
^2*e + 24*a^2*b^2*c*d*e^2)^2/4 - (256*a^4*c^3 + 16*a^2*b^4*c - 128*a^3*b^2*
c^2)*(a^3*e^6 + c^3*d^6 - b^3*d^3*e^3 + 3*a*b^2*d^2*e^4 + 3*a*c^2*d^4*e^2 +
3*a^2*c*d^2*e^4 + 3*b^2*c*d^4*e^2 - 3*a^2*b*d*e^5 - 3*b*c^2*d^5*e - 6*a*b*
c*d^3*e^3))^(1/2) + 2*b^4*c*d^3 - 2*a^2*b^3*e^3 + 16*a^2*c^3*d^3 - 12*a*b^2
*c^2*d^3 - 48*a^3*c^2*d*e^2 + 8*a^3*b*c*e^3 - 6*a*b^3*c*d^2*e + 24*a^2*b*c^
2*d^2*e + 12*a^2*b^2*c*d*e^2)/(16*(16*a^4*c^3 + a^2*b^4*c - 8*a^3*b^2*c^2))
)^(1/2) + 12*a*c^5*d^7*e^8 + 4*a^4*c^2*d*e^14 - 84*a^2*c^4*d^5*e^10 - 92*a^
3*c^3*d^3*e^12 - 4*b^2*c^4*d^7*e^8 - 4*b^3*c^3*d^6*e^9 + 8*b^4*c^2*d^5*e^10
- 12*a^2*b^2*c^2*d^3*e^12 + 32*a*b*c^4*d^6*e^9 - 4*a^3*b^2*c*d*e^14 - 36*a
*b^2*c^3*d^5*e^10 - 20*a*b^3*c^2*d^4*e^11 + 160*a^2*b*c^3*d^4*e^11 + 4*a^2*
b^3*c*d^2*e^13 + 16*a^3*b*c^2*d^2*e^13))*((((4*b^4*c*d^3 - 4*a^2*b^3*e^3 +
32*a^2*c^3*d^3 - 24*a*b^2*c^2*d^3 - 96*a^3*c^2*d*e^2 + 16*a^3*b*c*e^3 - 12*
a*b^3*c*d^2*e + 48*a^2*b*c^2*d^2*e + 24*a^2*b^2*c*d*e^2)^2/4 - (256*a^4*c^3
+ 16*a^2*b^4*c - 128*a^3*b^2*c^2)*(a^3*e^6 + c^3*d^6 - b^3*d^3*e^3 + 3*a*b
^2*d^2*e^4 + 3*a*c^2*d^4*e^2 + 3*a^2*c*d^2*e^4 + 3*b^2*c*d^4*e^2 - 3*a^2*b*
d*e^5 - 3*b*c^2*d^5*e - 6*a*b*c*d^3*e^3))^(1/2) + 2*b^4*c*d^3 - 2*a^2*b^3*e
^3 + 16*a^2*c^3*d^3 - 12*a*b^2*c^2*d^3 - 48*a^3*c^2*d*e^2 + 8*a^3*b*c*e^3 -
6*a*b^3*c*d^2*e + 24*a^2*b*c^2*d^2*e + 12*a^2*b^2*c*d*e^2)/(16*(16*a^4*c^3
+ a^2*b^4*c - 8*a^3*b^2*c^2)))^(1/2)*1i + ((d + e*x^2)^(1/2)*(2*a^4*c*e^16
+ 6*c^5*d^8*e^8 - 16*a*c^4*d^6*e^10 - 16*b*c^4*d^7*e^9 + 4*b^4*c*d^4*e^12
+ 16*a^2*c^3*d^4*e^12 + 8*a^3*c^2*d^2*e^14 + 24*b^2*c^3*d^6*e^10 - 16*b^3*c
^2*d^5*e^11 - 8*a^3*b*c*d*e^15 - 8*a*b^3*c*d^3*e^13 + 16*a*b^2*c^2*d^4*e^12
- 24*a^2*b*c^2*d^3*e^13 + 12*a^2*b^2*c*d^2*e^14) + ((((4*b^4*c*d^3 - 4*a^2
*b^3*e^3 + 32*a^2*c^3*d^3 - 24*a*b^2*c^2*d^3 - 96*a^3*c^2*d*e^2 + 16*a^3*b*
c*e^3 - 12*a*b^3*c*d^2*e + 48*a^2*b*c^2*d^2*e + 24*a^2*b^2*c*d*e^2)^2/4 - (
256*a^4*c^3 + 16*a^2*b^4*c - 128*a^3*b^2*c^2)*(a^3*e^6 + c^3*d^6 - b^3*d^3*
e^3 + 3*a*b^2*d^2*e^4 + 3*a*c^2*d^4*e^2 + 3*a^2*c*d^2*e^4 + 3*b^2*c*d^4*e^2
- 3*a^2*b*d*e^5 - 3*b*c^2*d^5*e - 6*a*b*c*d^3*e^3))^(1/2) + 2*b^4*c*d^3 -
2*a^2*b^3*e^3 + 16*a^2*c^3*d^3 - 12*a*b^2*c^2*d^3 - 48*a^3*c^2*d*e^2 + 8*a^
3*b*c*e^3 - 6*a*b^3*c*d^2*e + 24*a^2*b*c^2*d^2*e + 12*a^2*b^2*c*d*e^2)/(16*
(16*a^4*c^3 + a^2*b^4*c - 8*a^3*b^2*c^2)))^(1/2)*((((((4*b^4*c*d^3 - 4*a^2*
b^3*e^3 + 32*a^2*c^3*d^3 - 24*a*b^2*c^2*d^3 - 96*a^3*c^2*d*e^2 + 16*a^3*b*c
*e^3 - 12*a*b^3*c*d^2*e + 48*a^2*b*c^2*d^2*e + 24*a^2*b^2*c*d*e^2)^2/4 - (2
56*a^4*c^3 + 16*a^2*b^4*c - 128*a^3*b^2*c^2)*(a^3*e^6 + c^3*d^6 - b^3*d^3*e
^3 + 3*a*b^2*d^2*e^4 + 3*a*c^2*d^4*e^2 + 3*a^2*c*d^2*e^4 + 3*b^2*c*d^4*e^2
- 3*a^2*b*d*e^5 - 3*b*c^2*d^5*e - 6*a*b*c*d^3*e^3))^(1/2) + 2*b^4*c*d^3 - 2
*a^2*b^3*e^3 + 16*a^2*c^3*d^3 - 12*a*b^2*c^2*d^3 - 48*a^3*c^2*d*e^2 + 8*a^3
*b*c*e^3 - 6*a*b^3*c*d^2*e + 24*a^2*b*c^2*d^2*e + 12*a^2*b^2*c*d*e^2)/(16*(
16*a^4*c^3 + a^2*b^4*c - 8*a^3*b^2*c^2)))^(1/2)*((d + e*x^2)^(1/2)*((((4*b^
4*c*d^3 - 4*a^2*b^3*e^3 + 32*a^2*c^3*d^3 - 24*a*b^2*c^2*d^3 - 96*a^3*c^2*d*
e^2 + 16*a^3*b*c*e^3 - 12*a*b^3*c*d^2*e + 48*a^2*b*c^2*d^2*e + 24*a^2*b^2*c
*d*e^2)^2/4 - (256*a^4*c^3 + 16*a^2*b^4*c - 128*a^3*b^2*c^2)*(a^3*e^6 + c^3
*d^6 - b^3*d^3*e^3 + 3*a*b^2*d^2*e^4 + 3*a*c^2*d^4*e^2 + 3*a^2*c*d^2*e^4 +
3*b^2*c*d^4*e^2 - 3*a^2*b*d*e^5 - 3*b*c^2*d^5*e - 6*a*b*c*d^3*e^3))^(1/2) +

2*b^4*c*d^3 - 2*a^2*b^3*e^3 + 16*a^2*c^3*d^3 - 12*a*b^2*c^2*d^3 - 48*a^3*c
^2*d*e^2 + 8*a^3*b*c*e^3 - 6*a*b^3*c*d^2*e + 24*a^2*b*c^2*d^2*e + 12*a^2*b^
2*c*d*e^2)/(16*(16*a^4*c^3 + a^2*b^4*c - 8*a^3*b^2*c^2)))^(1/2)*(512*a^5*c^
4*e^10 + 32*a^3*b^4*c^2*e^10 - 256*a^4*b^2*c^3*e^10 + 768*a^4*c^5*d^2*e^8 +
64*a^2*b^4*c^3*d^2*e^8 - 448*a^3*b^2*c^4*d^2*e^8 - 896*a^4*b*c^4*d*e^9 - 6
4*a^2*b^5*c^2*d*e^9 + 480*a^3*b^3*c^3*d*e^9) + 192*a^3*c^5*d^4*e^8 + 192*a^
4*c^4*d^2*e^10 - 48*a^2*b^2*c^4*d^4*e^8 + 64*a^2*b^3*c^3*d^3*e^9 - 16*a^2*b
^4*c^2*d^2*e^10 + 16*a^3*b^2*c^3*d^2*e^10 - 64*a^4*b*c^3*d*e^11 - 256*a^3*b
*c^4*d^3*e^9 + 16*a^3*b^3*c^2*d*e^11) + (d + e*x^2)^(1/2)*(8*a^3*b^3*c*e^13
- 32*a^4*b*c^2*e^13 + 176*a^4*c^3*d*e^12 - 144*a^2*c^5*d^5*e^8 + 224*a^3*c
^4*d^3*e^10 - 16*b^4*c^3*d^5*e^8 + 16*b^5*c^2*d^4*e^9 + 48*a^2*b^2*c^3*d^3*
e^10 + 112*a^2*b^3*c^2*d^2*e^11 - 16*a^2*b^4*c*d*e^12 + 96*a*b^2*c^4*d^5*e^
8 - 80*a*b^3*c^3*d^4*e^9 - 32*a*b^4*c^2*d^3*e^10 + 96*a^2*b*c^4*d^4*e^9 - 4
16*a^3*b*c^3*d^2*e^11 + 16*a^3*b^2*c^2*d*e^12))*((((4*b^4*c*d^3 - 4*a^2*b^3
*e^3 + 32*a^2*c^3*d^3 - 24*a*b^2*c^2*d^3 - 96*a^3*c^2*d*e^2 + 16*a^3*b*c*e^
3 - 12*a*b^3*c*d^2*e + 48*a^2*b*c^2*d^2*e + 24*a^2*b^2*c*d*e^2)^2/4 - (256*
a^4*c^3 + 16*a^2*b^4*c - 128*a^3*b^2*c^2)*(a^3*e^6 + c^3*d^6 - b^3*d^3*e^3
+ 3*a*b^2*d^2*e^4 + 3*a*c^2*d^4*e^2 + 3*a^2*c*d^2*e^4 + 3*b^2*c*d^4*e^2 - 3
*a^2*b*d*e^5 - 3*b*c^2*d^5*e - 6*a*b*c*d^3*e^3))^(1/2) + 2*b^4*c*d^3 - 2*a^
2*b^3*e^3 + 16*a^2*c^3*d^3 - 12*a*b^2*c^2*d^3 - 48*a^3*c^2*d*e^2 + 8*a^3*b*
c*e^3 - 6*a*b^3*c*d^2*e + 24*a^2*b*c^2*d^2*e + 12*a^2*b^2*c*d*e^2)/(16*(16*
a^4*c^3 + a^2*b^4*c - 8*a^3*b^2*c^2)))^(1/2) - 12*a*c^5*d^7*e^8 - 4*a^4*c^2
*d*e^14 + 84*a^2*c^4*d^5*e^10 + 92*a^3*c^3*d^3*e^12 + 4*b^2*c^4*d^7*e^8 + 4
*b^3*c^3*d^6*e^9 - 8*b^4*c^2*d^5*e^10 + 12*a^2*b^2*c^2*d^3*e^12 - 32*a*b*c^
4*d^6*e^9 + 4*a^3*b^2*c*d*e^14 + 36*a*b^2*c^3*d^5*e^10 + 20*a*b^3*c^2*d^4*e
^11 - 160*a^2*b*c^3*d^4*e^11 - 4*a^2*b^3*c*d^2*e^13 - 16*a^3*b*c^2*d^2*e^13
))*((((4*b^4*c*d^3 - 4*a^2*b^3*e^3 + 32*a^2*c^3*d^3 - 24*a*b^2*c^2*d^3 - 96
*a^3*c^2*d*e^2 + 16*a^3*b*c*e^3 - 12*a*b^3*c*d^2*e + 48*a^2*b*c^2*d^2*e + 2
4*a^2*b^2*c*d*e^2)^2/4 - (256*a^4*c^3 + 16*a^2*b^4*c - 128*a^3*b^2*c^2)*(a^
3*e^6 + c^3*d^6 - b^3*d^3*e^3 + 3*a*b^2*d^2*e^4 + 3*a*c^2*d^4*e^2 + 3*a^2*c
*d^2*e^4 + 3*b^2*c*d^4*e^2 - 3*a^2*b*d*e^5 - 3*b*c^2*d^5*e - 6*a*b*c*d^3*e^
3))^(1/2) + 2*b^4*c*d^3 - 2*a^2*b^3*e^3 + 16*a^2*c^3*d^3 - 12*a*b^2*c^2*d^3
- 48*a^3*c^2*d*e^2 + 8*a^3*b*c*e^3 - 6*a*b^3*c*d^2*e + 24*a^2*b*c^2*d^2*e
+ 12*a^2*b^2*c*d*e^2)/(16*(16*a^4*c^3 + a^2*b^4*c - 8*a^3*b^2*c^2)))^(1/2)*
1i)/(((d + e*x^2)^(1/2)*(2*a^4*c*e^16 + 6*c^5*d^8*e^8 - 16*a*c^4*d^6*e^10 -
16*b*c^4*d^7*e^9 + 4*b^4*c*d^4*e^12 + 16*a^2*c^3*d^4*e^12 + 8*a^3*c^2*d^2*
e^14 + 24*b^2*c^3*d^6*e^10 - 16*b^3*c^2*d^5*e^11 - 8*a^3*b*c*d*e^15 - 8*a*b
^3*c*d^3*e^13 + 16*a*b^2*c^2*d^4*e^12 - 24*a^2*b*c^2*d^3*e^13 + 12*a^2*b^2*
c*d^2*e^14) + ((((4*b^4*c*d^3 - 4*a^2*b^3*e^3 + 32*a^2*c^3*d^3 - 24*a*b^2*c
^2*d^3 - 96*a^3*c^2*d*e^2 + 16*a^3*b*c*e^3 - 12*a*b^3*c*d^2*e + 48*a^2*b*c^
2*d^2*e + 24*a^2*b^2*c*d*e^2)^2/4 - (256*a^4*c^3 + 16*a^2*b^4*c - 128*a^3*b
^2*c^2)*(a^3*e^6 + c^3*d^6 - b^3*d^3*e^3 + 3*a*b^2*d^2*e^4 + 3*a*c^2*d^4*e^
2 + 3*a^2*c*d^2*e^4 + 3*b^2*c*d^4*e^2 - 3*a^2*b*d*e^5 - 3*b*c^2*d^5*e - 6*a
*b*c*d^3*e^3))^(1/2) + 2*b^4*c*d^3 - 2*a^2*b^3*e^3 + 16*a^2*c^3*d^3 - 12*a*
b^2*c^2*d^3 - 48*a^3*c^2*d*e^2 + 8*a^3*b*c*e^3 - 6*a*b^3*c*d^2*e + 24*a^2*b

*c^2*d^2*e + 12*a^2*b^2*c*d*e^2)/(16*(16*a^4*c^3 + a^2*b^4*c - 8*a^3*b^2*c^
2)))^(1/2)*((((((4*b^4*c*d^3 - 4*a^2*b^3*e^3 + 32*a^2*c^3*d^3 - 24*a*b^2*c^
2*d^3 - 96*a^3*c^2*d*e^2 + 16*a^3*b*c*e^3 - 12*a*b^3*c*d^2*e + 48*a^2*b*c^2
*d^2*e + 24*a^2*b^2*c*d*e^2)^2/4 - (256*a^4*c^3 + 16*a^2*b^4*c - 128*a^3*b^
2*c^2)*(a^3*e^6 + c^3*d^6 - b^3*d^3*e^3 + 3*a*b^2*d^2*e^4 + 3*a*c^2*d^4*e^2
 + 3*a^2*c*d^2*e^4 + 3*b^2*c*d^4*e^2 - 3*a^2*b*d*e^5 - 3*b*c^2*d^5*e - 6*a*
b*c*d^3*e^3))^(1/2) + 2*b^4*c*d^3 - 2*a^2*b^3*e^3 + 16*a^2*c^3*d^3 - 12*a*b
^2*c^2*d^3 - 48*a^3*c^2*d*e^2 + 8*a^3*b*c*e^3 - 6*a*b^3*c*d^2*e + 24*a^2*b*
c^2*d^2*e + 12*a^2*b^2*c*d*e^2)/(16*(16*a^4*c^3 + a^2*b^4*c - 8*a^3*b^2*c^2
)))^(1/2)*((d + e*x^2)^(1/2)*((((4*b^4*c*d^3 - 4*a^2*b^3*e^3 + 32*a^2*c^3*d
^3 - 24*a*b^2*c^2*d^3 - 96*a^3*c^2*d*e^2 + 16*a^3*b*c*e^3 - 12*a*b^3*c*d^2*
e + 48*a^2*b*c^2*d^2*e + 24*a^2*b^2*c*d*e^2)^2/4 - (256*a^4*c^3 + 16*a^2*b^
4*c - 128*a^3*b^2*c^2)*(a^3*e^6 + c^3*d^6 - b^3*d^3*e^3 + 3*a*b^2*d^2*e^4 +
 3*a*c^2*d^4*e^2 + 3*a^2*c*d^2*e^4 + 3*b^2*c*d^4*e^2 - 3*a^2*b*d*e^5 - 3*b*
c^2*d^5*e - 6*a*b*c*d^3*e^3))^(1/2) + 2*b^4*c*d^3 - 2*a^2*b^3*e^3 + 16*a^2*
c^3*d^3 - 12*a*b^2*c^2*d^3 - 48*a^3*c^2*d*e^2 + 8*a^3*b*c*e^3 - 6*a*b^3*c*d
^2*e + 24*a^2*b*c^2*d^2*e + 12*a^2*b^2*c*d*e^2)/(16*(16*a^4*c^3 + a^2*b^4*c
 - 8*a^3*b^2*c^2)))^(1/2)*(512*a^5*c^4*e^10 + 32*a^3*b^4*c^2*e^10 - 256*a^4
*b^2*c^3*e^10 + 768*a^4*c^5*d^2*e^8 + 64*a^2*b^4*c^3*d^2*e^8 - 448*a^3*b^2*
c^4*d^2*e^8 - 896*a^4*b*c^4*d*e^9 - 64*a^2*b^5*c^2*d*e^9 + 480*a^3*b^3*c^3*
d*e^9) + 192*a^3*c^5*d^4*e^8 + 192*a^4*c^4*d^2*e^10 - 48*a^2*b^2*c^4*d^4*e^
8 + 64*a^2*b^3*c^3*d^3*e^9 - 16*a^2*b^4*c^2*d^2*e^10 + 16*a^3*b^2*c^3*d^2*e
^10 - 64*a^4*b*c^3*d*e^11 - 256*a^3*b*c^4*d^3*e^9 + 16*a^3*b^3*c^2*d*e^11)
+ (d + e*x^2)^(1/2)*(8*a^3*b^3*c*e^13 - 32*a^4*b*c^2*e^13 + 176*a^4*c^3*d*e
^12 - 144*a^2*c^5*d^5*e^8 + 224*a^3*c^4*d^3*e^10 - 16*b^4*c^3*d^5*e^8 + 16*
b^5*c^2*d^4*e^9 + 48*a^2*b^2*c^3*d^3*e^10 + 112*a^2*b^3*c^2*d^2*e^11 - 16*a
^2*b^4*c*d*e^12 + 96*a*b^2*c^4*d^5*e^8 - 80*a*b^3*c^3*d^4*e^9 - 32*a*b^4*c^
2*d^3*e^10 + 96*a^2*b*c^4*d^4*e^9 - 416*a^3*b*c^3*d^2*e^11 + 16*a^3*b^2*c^2
*d*e^12))*((((4*b^4*c*d^3 - 4*a^2*b^3*e^3 + 32*a^2*c^3*d^3 - 24*a*b^2*c^2*d
^3 - 96*a^3*c^2*d*e^2 + 16*a^3*b*c*e^3 - 12*a*b^3*c*d^2*e + 48*a^2*b*c^2*d^
2*e + 24*a^2*b^2*c*d*e^2)^2/4 - (256*a^4*c^3 + 16*a^2*b^4*c - 128*a^3*b^2*c
^2)*(a^3*e^6 + c^3*d^6 - b^3*d^3*e^3 + 3*a*b^2*d^2*e^4 + 3*a*c^2*d^4*e^2 +
3*a^2*c*d^2*e^4 + 3*b^2*c*d^4*e^2 - 3*a^2*b*d*e^5 - 3*b*c^2*d^5*e - 6*a*b*c
*d^3*e^3))^(1/2) + 2*b^4*c*d^3 - 2*a^2*b^3*e^3 + 16*a^2*c^3*d^3 - 12*a*b^2*
c^2*d^3 - 48*a^3*c^2*d*e^2 + 8*a^3*b*c*e^3 - 6*a*b^3*c*d^2*e + 24*a^2*b*c^2
*d^2*e + 12*a^2*b^2*c*d*e^2)/(16*(16*a^4*c^3 + a^2*b^4*c - 8*a^3*b^2*c^2)))
^(1/2) - 12*a*c^5*d^7*e^8 - 4*a^4*c^2*d*e^14 + 84*a^2*c^4*d^5*e^10 + 92*a^3
*c^3*d^3*e^12 + 4*b^2*c^4*d^7*e^8 + 4*b^3*c^3*d^6*e^9 - 8*b^4*c^2*d^5*e^10
+ 12*a^2*b^2*c^2*d^3*e^12 - 32*a*b*c^4*d^6*e^9 + 4*a^3*b^2*c*d*e^14 + 36*a*
b^2*c^3*d^5*e^10 + 20*a*b^3*c^2*d^4*e^11 - 160*a^2*b*c^3*d^4*e^11 - 4*a^2*b
^3*c*d^2*e^13 - 16*a^3*b*c^2*d^2*e^13))*((((4*b^4*c*d^3 - 4*a^2*b^3*e^3 + 3
2*a^2*c^3*d^3 - 24*a*b^2*c^2*d^3 - 96*a^3*c^2*d*e^2 + 16*a^3*b*c*e^3 - 12*a
*b^3*c*d^2*e + 48*a^2*b*c^2*d^2*e + 24*a^2*b^2*c*d*e^2)^2/4 - (256*a^4*c^3
+ 16*a^2*b^4*c - 128*a^3*b^2*c^2)*(a^3*e^6 + c^3*d^6 - b^3*d^3*e^3 + 3*a*b^
2*d^2*e^4 + 3*a*c^2*d^4*e^2 + 3*a^2*c*d^2*e^4 + 3*b^2*c*d^4*e^2 - 3*a^2*b*d

*e^5 - 3*b*c^2*d^5*e - 6*a*b*c*d^3*e^3))^(1/2) + 2*b^4*c*d^3 - 2*a^2*b^3*e^
3 + 16*a^2*c^3*d^3 - 12*a*b^2*c^2*d^3 - 48*a^3*c^2*d*e^2 + 8*a^3*b*c*e^3 -
6*a*b^3*c*d^2*e + 24*a^2*b*c^2*d^2*e + 12*a^2*b^2*c*d*e^2)/(16*(16*a^4*c^3
+ a^2*b^4*c - 8*a^3*b^2*c^2)))^(1/2) - ((d + e*x^2)^(1/2)*(2*a^4*c*e^16 + 6
*c^5*d^8*e^8 - 16*a*c^4*d^6*e^10 - 16*b*c^4*d^7*e^9 + 4*b^4*c*d^4*e^12 + 16
*a^2*c^3*d^4*e^12 + 8*a^3*c^2*d^2*e^14 + 24*b^2*c^3*d^6*e^10 - 16*b^3*c^2*d
^5*e^11 - 8*a^3*b*c*d*e^15 - 8*a*b^3*c*d^3*e^13 + 16*a*b^2*c^2*d^4*e^12 - 2
4*a^2*b*c^2*d^3*e^13 + 12*a^2*b^2*c*d^2*e^14) + ((((4*b^4*c*d^3 - 4*a^2*b^3
*e^3 + 32*a^2*c^3*d^3 - 24*a*b^2*c^2*d^3 - 96*a^3*c^2*d*e^2 + 16*a^3*b*c*e^
3 - 12*a*b^3*c*d^2*e + 48*a^2*b*c^2*d^2*e + 24*a^2*b^2*c*d*e^2)^2/4 - (256*
a^4*c^3 + 16*a^2*b^4*c - 128*a^3*b^2*c^2)*(a^3*e^6 + c^3*d^6 - b^3*d^3*e^3
+ 3*a*b^2*d^2*e^4 + 3*a*c^2*d^4*e^2 + 3*a^2*c*d^2*e^4 + 3*b^2*c*d^4*e^2 - 3
*a^2*b*d*e^5 - 3*b*c^2*d^5*e - 6*a*b*c*d^3*e^3))^(1/2) + 2*b^4*c*d^3 - 2*a^
2*b^3*e^3 + 16*a^2*c^3*d^3 - 12*a*b^2*c^2*d^3 - 48*a^3*c^2*d*e^2 + 8*a^3*b*
c*e^3 - 6*a*b^3*c*d^2*e + 24*a^2*b*c^2*d^2*e + 12*a^2*b^2*c*d*e^2)/(16*(16*
a^4*c^3 + a^2*b^4*c - 8*a^3*b^2*c^2)))^(1/2)*((((((4*b^4*c*d^3 - 4*a^2*b^3*
e^3 + 32*a^2*c^3*d^3 - 24*a*b^2*c^2*d^3 - 96*a^3*c^2*d*e^2 + 16*a^3*b*c*e^3
 - 12*a*b^3*c*d^2*e + 48*a^2*b*c^2*d^2*e + 24*a^2*b^2*c*d*e^2)^2/4 - (256*a
^4*c^3 + 16*a^2*b^4*c - 128*a^3*b^2*c^2)*(a^3*e^6 + c^3*d^6 - b^3*d^3*e^3 +
 3*a*b^2*d^2*e^4 + 3*a*c^2*d^4*e^2 + 3*a^2*c*d^2*e^4 + 3*b^2*c*d^4*e^2 - 3*
a^2*b*d*e^5 - 3*b*c^2*d^5*e - 6*a*b*c*d^3*e^3))^(1/2) + 2*b^4*c*d^3 - 2*a^2
*b^3*e^3 + 16*a^2*c^3*d^3 - 12*a*b^2*c^2*d^3 - 48*a^3*c^2*d*e^2 + 8*a^3*b*c
*e^3 - 6*a*b^3*c*d^2*e + 24*a^2*b*c^2*d^2*e + 12*a^2*b^2*c*d*e^2)/(16*(16*a
^4*c^3 + a^2*b^4*c - 8*a^3*b^2*c^2)))^(1/2)*((d + e*x^2)^(1/2)*((((4*b^4*c*
d^3 - 4*a^2*b^3*e^3 + 32*a^2*c^3*d^3 - 24*a*b^2*c^2*d^3 - 96*a^3*c^2*d*e^2
+ 16*a^3*b*c*e^3 - 12*a*b^3*c*d^2*e + 48*a^2*b*c^2*d^2*e + 24*a^2*b^2*c*d*e
^2)^2/4 - (256*a^4*c^3 + 16*a^2*b^4*c - 128*a^3*b^2*c^2)*(a^3*e^6 + c^3*d^6
 - b^3*d^3*e^3 + 3*a*b^2*d^2*e^4 + 3*a*c^2*d^4*e^2 + 3*a^2*c*d^2*e^4 + 3*b^
2*c*d^4*e^2 - 3*a^2*b*d*e^5 - 3*b*c^2*d^5*e - 6*a*b*c*d^3*e^3))^(1/2) + 2*b
^4*c*d^3 - 2*a^2*b^3*e^3 + 16*a^2*c^3*d^3 - 12*a*b^2*c^2*d^3 - 48*a^3*c^2*d
*e^2 + 8*a^3*b*c*e^3 - 6*a*b^3*c*d^2*e + 24*a^2*b*c^2*d^2*e + 12*a^2*b^2*c*
d*e^2)/(16*(16*a^4*c^3 + a^2*b^4*c - 8*a^3*b^2*c^2)))^(1/2)*(512*a^5*c^4*e^
10 + 32*a^3*b^4*c^2*e^10 - 256*a^4*b^2*c^3*e^10 + 768*a^4*c^5*d^2*e^8 + 64*
a^2*b^4*c^3*d^2*e^8 - 448*a^3*b^2*c^4*d^2*e^8 - 896*a^4*b*c^4*d*e^9 - 64*a^
2*b^5*c^2*d*e^9 + 480*a^3*b^3*c^3*d*e^9) - 192*a^3*c^5*d^4*e^8 - 192*a^4*c^
4*d^2*e^10 + 48*a^2*b^2*c^4*d^4*e^8 - 64*a^2*b^3*c^3*d^3*e^9 + 16*a^2*b^4*c
^2*d^2*e^10 - 16*a^3*b^2*c^3*d^2*e^10 + 64*a^4*b*c^3*d*e^11 + 256*a^3*b*c^4
*d^3*e^9 - 16*a^3*b^3*c^2*d*e^11) + (d + e*x^2)^(1/2)*(8*a^3*b^3*c*e^13 - 3
2*a^4*b*c^2*e^13 + 176*a^4*c^3*d*e^12 - 144*a^2*c^5*d^5*e^8 + 224*a^3*c^4*d
^3*e^10 - 16*b^4*c^3*d^5*e^8 + 16*b^5*c^2*d^4*e^9 + 48*a^2*b^2*c^3*d^3*e^10
 + 112*a^2*b^3*c^2*d^2*e^11 - 16*a^2*b^4*c*d*e^12 + 96*a*b^2*c^4*d^5*e^8 -
80*a*b^3*c^3*d^4*e^9 - 32*a*b^4*c^2*d^3*e^10 + 96*a^2*b*c^4*d^4*e^9 - 416*a
^3*b*c^3*d^2*e^11 + 16*a^3*b^2*c^2*d*e^12))*((((4*b^4*c*d^3 - 4*a^2*b^3*e^3
 + 32*a^2*c^3*d^3 - 24*a*b^2*c^2*d^3 - 96*a^3*c^2*d*e^2 + 16*a^3*b*c*e^3 -
12*a*b^3*c*d^2*e + 48*a^2*b*c^2*d^2*e + 24*a^2*b^2*c*d*e^2)^2/4 - (256*a^4*

c^3 + 16*a^2*b^4*c - 128*a^3*b^2*c^2)*(a^3*e^6 + c^3*d^6 - b^3*d^3*e^3 + 3*a*b^2*d^2*e^4 + 3*a*c^2*d^4*e^2 + 3*a^2*c*d^2*e^4 + 3*b^2*c*d^4*e^2 - 3*a^2*b*d*e^5 - 3*b*c^2*d^5*e - 6*a*b*c*d^3*e^3))^(1/2) + 2*b^4*c*d^3 - 2*a^2*b^3*e^3 + 16*a^2*c^3*d^3 - 12*a*b^2*c^2*d^3 - 48*a^3*c^2*d*e^2 + 8*a^3*b*c*e^3 - 6*a*b^3*c*d^2*e + 24*a^2*b*c^2*d^2*e + 12*a^2*b^2*c*d*e^2)/(16*(16*a^4*c^3 + a^2*b^4*c - 8*a^3*b^2*c^2)))^(1/2) + 12*a*c^5*d^7*e^8 + 4*a^4*c^2*d*e^14 - 84*a^2*c^4*d^5*e^10 - 92*a^3*c^3*d^3*e^12 - 4*b^2*c^4*d^7*e^8 - 4*b^3*c^3*d^6*e^9 + 8*b^4*c^2*d^5*e^10 - 12*a^2*b^2*c^2*d^3*e^12 + 32*a*b*c^4*d^6*e^9 - 4*a^3*b^2*c*d*e^14 - 36*a*b^2*c^3*d^5*e^10 - 20*a*b^3*c^2*d^4*e^11 + 160*a^2*b*c^3*d^4*e^11 + 4*a^2*b^3*c*d^2*e^13 + 16*a^3*b*c^2*d^2*e^13))*((((4*b^4*c*d^3 - 4*a^2*b^3*e^3 + 32*a^2*c^3*d^3 - 24*a*b^2*c^2*d^3 - 96*a^3*c^2*d*e^2 + 16*a^3*b*c*e^3 - 12*a*b^3*c*d^2*e + 48*a^2*b*c^2*d^2*e + 24*a^2*b^2*c*d*e^2)^2/4 - (256*a^4*c^3 + 16*a^2*b^4*c - 128*a^3*b^2*c^2)*(a^3*e^6 + c^3*d^6 - b^3*d^3*e^3 + 3*a*b^2*d^2*e^4 + 3*a*c^2*d^4*e^2 + 3*a^2*c*d^2*e^4 + 3*b^2*c*d^4*e^2 - 3*a^2*b*d*e^5 - 3*b*c^2*d^5*e - 6*a*b*c*d^3*e^3))^(1/2) + 2*b^4*c*d^3 - 2*a^2*b^3*e^3 + 16*a^2*c^3*d^3 - 12*a*b^2*c^2*d^3 - 48*a^3*c^2*d*e^2 + 8*a^3*b*c*e^3 - 6*a*b^3*c*d^2*e + 24*a^2*b*c^2*d^2*e + 12*a^2*b^2*c*d*e^2)/(16*(16*a^4*c^3 + a^2*b^4*c - 8*a^3*b^2*c^2)))^(1/2) + 6*c^4*d^8*e^10 + 14*a*c^3*d^6*e^12 + 2*a^3*c*d^2*e^16 - 16*b*c^3*d^7*e^11 - 4*b^3*c*d^5*e^13 + 10*a^2*c^2*d^4*e^14 + 14*b^2*c^2*d^6*e^12 - 24*a*b*c^2*d^5*e^13 + 10*a*b^2*c*d^4*e^14 - 8*a^2*b*c*d^3*e^15))*((((4*b^4*c*d^3 - 4*a^2*b^3*e^3 + 32*a^2*c^3*d^3 - 24*a*b^2*c^2*d^3 - 96*a^3*c^2*d*e^2 + 16*a^3*b*c*e^3 - 12*a*b^3*c*d^2*e + 48*a^2*b*c^2*d^2*e + 24*a^2*b^2*c*d*e^2)^2/4 - (256*a^4*c^3 + 16*a^2*b^4*c - 128*a^3*b^2*c^2)*(a^3*e^6 + c^3*d^6 - b^3*d^3*e^3 + 3*a*b^2*d^2*e^4 + 3*a*c^2*d^4*e^2 + 3*a^2*c*d^2*e^4 + 3*b^2*c*d^4*e^2 - 3*a^2*b*d*e^5 - 3*b*c^2*d^5*e - 6*a*b*c*d^3*e^3))^(1/2) + 2*b^4*c*d^3 - 2*a^2*b^3*e^3 + 16*a^2*c^3*d^3 - 12*a*b^2*c^2*d^3 - 48*a^3*c^2*d*e^2 + 8*a^3*b*c*e^3 - 6*a*b^3*c*d^2*e + 24*a^2*b*c^2*d^2*e + 12*a^2*b^2*c*d*e^2)/(16*(16*a^4*c^3 + a^2*b^4*c - 8*a^3*b^2*c^2)))^(1/2)*2i - (atanh((72*c^4*d^6*e^10*(d + e*x^2)^(1/2)*(d^3)^(1/2))/(72*c^4*d^8*e^10 + 60*a*c^3*d^6*e^12 + 2*a^3*c*d^2*e^16 - 104*b*c^3*d^7*e^11 - 6*b^3*c*d^5*e^13 + 8*a^2*c^2*d^4*e^14 + (18*c^5*d^10*e^8)/a + 20*b^2*c^2*d^6*e^12 + (20*b^2*c^3*d^8*e^10)/a + (12*b^3*c^2*d^7*e^11)/a - (4*b^2*c^4*d^10*e^8)/a^2 + (10*b^3*c^3*d^9*e^9)/a^2 - (6*b^4*c^2*d^8*e^10)/a^2 - 32*a*b*c^2*d^5*e^13 + 12*a*b^2*c*d^4*e^14 - 8*a^2*b*c*d^3*e^15 - (48*b*c^4*d^9*e^9)/a) + (2*a^3*c*e^16*(d + e*x^2)^(1/2)*(d^3)^(1/2))/(72*c^4*d^8*e^10 + 60*a*c^3*d^6*e^12 + 2*a^3*c*d^2*e^16 - 104*b*c^3*d^7*e^11 - 6*b^3*c*d^5*e^13 + 8*a^2*c^2*d^4*e^14 + (18*c^5*d^10*e^8)/a + 20*b^2*c^2*d^6*e^12 + (20*b^2*c^3*d^8*e^10)/a + (12*b^3*c^2*d^7*e^11)/a - (4*b^2*c^4*d^10*e^8)/a^2 + (10*b^3*c^3*d^9*e^9)/a^2 - (6*b^4*c^2*d^8*e^10)/a^2 - 32*a*b*c^2*d^5*e^13 + 12*a*b^2*c*d^4*e^14 - 8*a^2*b*c*d^3*e^15 - (48*b*c^4*d^9*e^9)/a) + (18*c^5*d^8*e^8*(d + e*x^2)^(1/2)*(d^3)^(1/2))/(18*c^5*d^10*e^8 + 72*a*c^4*d^8*e^10 + 2*a^4*c*d^2*e^16 - 48*b*c^4*d^9*e^9 + 60*a^2*c^3*d^6*e^12 + 8*a^3*c^2*d^4*e^14 + 20*b^2*c^3*d^8*e^10 + 12*b^3*c^2*d^7*e^11 - (4*b^2*c^4*d^10*e^8)/a + (10*b^3*c^3*d^9*e^9)/a - (6*b^4*c^2*d^8*e^10)/a - 104*a*b*c^3*d^7*e^11 - 6*a*b^3*c*d^5*e^13 - 8*a^3*b*c*d^3*e^15 +

20*a*b^2*c^2*d^6*e^12 - 32*a^2*b*c^2*d^5*e^13 + 12*a^2*b^2*c*d^4*e^14) + (8
*a^2*c^2*d^2*e^14*(d + e*x^2)^(1/2)*(d^3)^(1/2))/(72*c^4*d^8*e^10 + 60*a*c^
3*d^6*e^12 + 2*a^3*c*d^2*e^16 - 104*b*c^3*d^7*e^11 - 6*b^3*c*d^5*e^13 + 8*a
^2*c^2*d^4*e^14 + (18*c^5*d^10*e^8)/a + 20*b^2*c^2*d^6*e^12 + (20*b^2*c^3*d
^8*e^10)/a + (12*b^3*c^2*d^7*e^11)/a - (4*b^2*c^4*d^10*e^8)/a^2 + (10*b^3*c
^3*d^9*e^9)/a^2 - (6*b^4*c^2*d^8*e^10)/a^2 - 32*a*b*c^2*d^5*e^13 + 12*a*b^2
*c*d^4*e^14 - 8*a^2*b*c*d^3*e^15 - (48*b*c^4*d^9*e^9)/a) + (20*b^2*c^2*d^4*
e^12*(d + e*x^2)^(1/2)*(d^3)^(1/2))/(72*c^4*d^8*e^10 + 60*a*c^3*d^6*e^12 +
2*a^3*c*d^2*e^16 - 104*b*c^3*d^7*e^11 - 6*b^3*c*d^5*e^13 + 8*a^2*c^2*d^4*e^
14 + (18*c^5*d^10*e^8)/a + 20*b^2*c^2*d^6*e^12 + (20*b^2*c^3*d^8*e^10)/a +
(12*b^3*c^2*d^7*e^11)/a - (4*b^2*c^4*d^10*e^8)/a^2 + (10*b^3*c^3*d^9*e^9)/a
^2 - (6*b^4*c^2*d^8*e^10)/a^2 - 32*a*b*c^2*d^5*e^13 + 12*a*b^2*c*d^4*e^14 -
8*a^2*b*c*d^3*e^15 - (48*b*c^4*d^9*e^9)/a) - (48*b*c^4*d^7*e^9*(d + e*x^2)
^(1/2)*(d^3)^(1/2))/(18*c^5*d^10*e^8 + 72*a*c^4*d^8*e^10 + 2*a^4*c*d^2*e^16
- 48*b*c^4*d^9*e^9 + 60*a^2*c^3*d^6*e^12 + 8*a^3*c^2*d^4*e^14 + 20*b^2*c^3
*d^8*e^10 + 12*b^3*c^2*d^7*e^11 - (4*b^2*c^4*d^10*e^8)/a + (10*b^3*c^3*d^9*
e^9)/a - (6*b^4*c^2*d^8*e^10)/a - 104*a*b*c^3*d^7*e^11 - 6*a*b^3*c*d^5*e^13
- 8*a^3*b*c*d^3*e^15 + 20*a*b^2*c^2*d^6*e^12 - 32*a^2*b*c^2*d^5*e^13 + 12*
a^2*b^2*c*d^4*e^14) - (4*b^2*c^4*d^8*e^8*(d + e*x^2)^(1/2)*(d^3)^(1/2))/(18
*a*c^5*d^10*e^8 + 2*a^5*c*d^2*e^16 + 72*a^2*c^4*d^8*e^10 + 60*a^3*c^3*d^6*e
^12 + 8*a^4*c^2*d^4*e^14 - 4*b^2*c^4*d^10*e^8 + 10*b^3*c^3*d^9*e^9 - 6*b^4*
c^2*d^8*e^10 + 20*a^2*b^2*c^2*d^6*e^12 - 48*a*b*c^4*d^9*e^9 - 8*a^4*b*c*d^3
*e^15 + 20*a*b^2*c^3*d^8*e^10 + 12*a*b^3*c^2*d^7*e^11 - 104*a^2*b*c^3*d^7*e
^11 - 6*a^2*b^3*c*d^5*e^13 - 32*a^3*b*c^2*d^5*e^13 + 12*a^3*b^2*c*d^4*e^14)
+ (10*b^3*c^3*d^7*e^9*(d + e*x^2)^(1/2)*(d^3)^(1/2))/(18*a*c^5*d^10*e^8 +
2*a^5*c*d^2*e^16 + 72*a^2*c^4*d^8*e^10 + 60*a^3*c^3*d^6*e^12 + 8*a^4*c^2*d^
4*e^14 - 4*b^2*c^4*d^10*e^8 + 10*b^3*c^3*d^9*e^9 - 6*b^4*c^2*d^8*e^10 + 20*
a^2*b^2*c^2*d^6*e^12 - 48*a*b*c^4*d^9*e^9 - 8*a^4*b*c*d^3*e^15 + 20*a*b^2*c
^3*d^8*e^10 + 12*a*b^3*c^2*d^7*e^11 - 104*a^2*b*c^3*d^7*e^11 - 6*a^2*b^3*c*
d^5*e^13 - 32*a^3*b*c^2*d^5*e^13 + 12*a^3*b^2*c*d^4*e^14) - (6*b^4*c^2*d^6*
e^10*(d + e*x^2)^(1/2)*(d^3)^(1/2))/(18*a*c^5*d^10*e^8 + 2*a^5*c*d^2*e^16 +
72*a^2*c^4*d^8*e^10 + 60*a^3*c^3*d^6*e^12 + 8*a^4*c^2*d^4*e^14 - 4*b^2*c^4
*d^10*e^8 + 10*b^3*c^3*d^9*e^9 - 6*b^4*c^2*d^8*e^10 + 20*a^2*b^2*c^2*d^6*e^
12 - 48*a*b*c^4*d^9*e^9 - 8*a^4*b*c*d^3*e^15 + 20*a*b^2*c^3*d^8*e^10 + 12*a
*b^3*c^2*d^7*e^11 - 104*a^2*b*c^3*d^7*e^11 - 6*a^2*b^3*c*d^5*e^13 - 32*a^3*
b*c^2*d^5*e^13 + 12*a^3*b^2*c*d^4*e^14) + (60*a*c^3*d^4*e^12*(d + e*x^2)^(1
/2)*(d^3)^(1/2))/(72*c^4*d^8*e^10 + 60*a*c^3*d^6*e^12 + 2*a^3*c*d^2*e^16 -
104*b*c^3*d^7*e^11 - 6*b^3*c*d^5*e^13 + 8*a^2*c^2*d^4*e^14 + (18*c^5*d^10*e
^8)/a + 20*b^2*c^2*d^6*e^12 + (20*b^2*c^3*d^8*e^10)/a + (12*b^3*c^2*d^7*e^1
1)/a - (4*b^2*c^4*d^10*e^8)/a^2 + (10*b^3*c^3*d^9*e^9)/a^2 - (6*b^4*c^2*d^8
*e^10)/a^2 - 32*a*b*c^2*d^5*e^13 + 12*a*b^2*c*d^4*e^14 - 8*a^2*b*c*d^3*e^15
- (48*b*c^4*d^9*e^9)/a) - (104*b*c^3*d^5*e^11*(d + e*x^2)^(1/2)*(d^3)^(1/2
))/(72*c^4*d^8*e^10 + 60*a*c^3*d^6*e^12 + 2*a^3*c*d^2*e^16 - 104*b*c^3*d^7*
e^11 - 6*b^3*c*d^5*e^13 + 8*a^2*c^2*d^4*e^14 + (18*c^5*d^10*e^8)/a + 20*b^2
*c^2*d^6*e^12 + (20*b^2*c^3*d^8*e^10)/a + (12*b^3*c^2*d^7*e^11)/a - (4*b^2*

c^4*d^10*e^8)/a^2 + (10*b^3*c^3*d^9*e^9)/a^2 - (6*b^4*c^2*d^8*e^10)/a^2 - 3
2*a*b*c^2*d^5*e^13 + 12*a*b^2*c*d^4*e^14 - 8*a^2*b*c*d^3*e^15 - (48*b*c^4*d
^9*e^9)/a) - (6*b^3*c*d^3*e^13*(d + e*x^2)^(1/2)*(d^3)^(1/2))/(72*c^4*d^8*e
^10 + 60*a*c^3*d^6*e^12 + 2*a^3*c*d^2*e^16 - 104*b*c^3*d^7*e^11 - 6*b^3*c*d
^5*e^13 + 8*a^2*c^2*d^4*e^14 + (18*c^5*d^10*e^8)/a + 20*b^2*c^2*d^6*e^12 +
(20*b^2*c^3*d^8*e^10)/a + (12*b^3*c^2*d^7*e^11)/a - (4*b^2*c^4*d^10*e^8)/a^
2 + (10*b^3*c^3*d^9*e^9)/a^2 - (6*b^4*c^2*d^8*e^10)/a^2 - 32*a*b*c^2*d^5*e^
13 + 12*a*b^2*c*d^4*e^14 - 8*a^2*b*c*d^3*e^15 - (48*b*c^4*d^9*e^9)/a) + (20
*b^2*c^3*d^6*e^10*(d + e*x^2)^(1/2)*(d^3)^(1/2))/(18*c^5*d^10*e^8 + 72*a*c^
4*d^8*e^10 + 2*a^4*c*d^2*e^16 - 48*b*c^4*d^9*e^9 + 60*a^2*c^3*d^6*e^12 + 8*
a^3*c^2*d^4*e^14 + 20*b^2*c^3*d^8*e^10 + 12*b^3*c^2*d^7*e^11 - (4*b^2*c^4*d
^10*e^8)/a + (10*b^3*c^3*d^9*e^9)/a - (6*b^4*c^2*d^8*e^10)/a - 104*a*b*c^3*
d^7*e^11 - 6*a*b^3*c*d^5*e^13 - 8*a^3*b*c*d^3*e^15 + 20*a*b^2*c^2*d^6*e^12
- 32*a^2*b*c^2*d^5*e^13 + 12*a^2*b^2*c*d^4*e^14) + (12*b^3*c^2*d^5*e^11*(d
+ e*x^2)^(1/2)*(d^3)^(1/2))/(18*c^5*d^10*e^8 + 72*a*c^4*d^8*e^10 + 2*a^4*c*
d^2*e^16 - 48*b*c^4*d^9*e^9 + 60*a^2*c^3*d^6*e^12 + 8*a^3*c^2*d^4*e^14 + 20
*b^2*c^3*d^8*e^10 + 12*b^3*c^2*d^7*e^11 - (4*b^2*c^4*d^10*e^8)/a + (10*b^3*
c^3*d^9*e^9)/a - (6*b^4*c^2*d^8*e^10)/a - 104*a*b*c^3*d^7*e^11 - 6*a*b^3*c*
d^5*e^13 - 8*a^3*b*c*d^3*e^15 + 20*a*b^2*c^2*d^6*e^12 - 32*a^2*b*c^2*d^5*e^
13 + 12*a^2*b^2*c*d^4*e^14) - (32*a*b*c^2*d^3*e^13*(d + e*x^2)^(1/2)*(d^3)^
(1/2))/(72*c^4*d^8*e^10 + 60*a*c^3*d^6*e^12 + 2*a^3*c*d^2*e^16 - 104*b*c^3*
d^7*e^11 - 6*b^3*c*d^5*e^13 + 8*a^2*c^2*d^4*e^14 + (18*c^5*d^10*e^8)/a + 20
*b^2*c^2*d^6*e^12 + (20*b^2*c^3*d^8*e^10)/a + (12*b^3*c^2*d^7*e^11)/a - (4*
b^2*c^4*d^10*e^8)/a^2 + (10*b^3*c^3*d^9*e^9)/a^2 - (6*b^4*c^2*d^8*e^10)/a^2
 - 32*a*b*c^2*d^5*e^13 + 12*a*b^2*c*d^4*e^14 - 8*a^2*b*c*d^3*e^15 - (48*b*c
^4*d^9*e^9)/a) + (12*a*b^2*c*d^2*e^14*(d + e*x^2)^(1/2)*(d^3)^(1/2))/(72*c^
4*d^8*e^10 + 60*a*c^3*d^6*e^12 + 2*a^3*c*d^2*e^16 - 104*b*c^3*d^7*e^11 - 6*
b^3*c*d^5*e^13 + 8*a^2*c^2*d^4*e^14 + (18*c^5*d^10*e^8)/a + 20*b^2*c^2*d^6*
e^12 + (20*b^2*c^3*d^8*e^10)/a + (12*b^3*c^2*d^7*e^11)/a - (4*b^2*c^4*d^10*
e^8)/a^2 + (10*b^3*c^3*d^9*e^9)/a^2 - (6*b^4*c^2*d^8*e^10)/a^2 - 32*a*b*c^2
*d^5*e^13 + 12*a*b^2*c*d^4*e^14 - 8*a^2*b*c*d^3*e^15 - (48*b*c^4*d^9*e^9)/a
) - (8*a^2*b*c*d*e^15*(d + e*x^2)^(1/2)*(d^3)^(1/2))/(72*c^4*d^8*e^10 + 60*
a*c^3*d^6*e^12 + 2*a^3*c*d^2*e^16 - 104*b*c^3*d^7*e^11 - 6*b^3*c*d^5*e^13 +
 8*a^2*c^2*d^4*e^14 + (18*c^5*d^10*e^8)/a + 20*b^2*c^2*d^6*e^12 + (20*b^2*c
^3*d^8*e^10)/a + (12*b^3*c^2*d^7*e^11)/a - (4*b^2*c^4*d^10*e^8)/a^2 + (10*b
^3*c^3*d^9*e^9)/a^2 - (6*b^4*c^2*d^8*e^10)/a^2 - 32*a*b*c^2*d^5*e^13 + 12*a
*b^2*c*d^4*e^14 - 8*a^2*b*c*d^3*e^15 - (48*b*c^4*d^9*e^9)/a))*(d^3)^(1/2))/
a

3.370 $\int \frac{(d+ex^2)^{3/2}}{x^3(a+bx^2+cx^4)}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 417

$$\int \frac{(d+ex^2)^{3/2}}{x^3(a+bx^2+cx^4)}\,dx = -\frac{d\sqrt{d+ex^2}}{2ax^2} + \frac{\sqrt{d}e\,\text{arctanh}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)}{2a} + \frac{\sqrt{d}(bd-2ae)\text{arctanh}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)}{a^2} - \frac{\sqrt{c}\left(b^2d^2+bd\left(\sqrt{b^2-4ac}d-2ae\right)-2a\left(cd^2+e\left(\sqrt{b^2-4ac}d-ae\right)\right)\right)\text{arctanh}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex^2}}{\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}}\right)}{\sqrt{2}a^2\sqrt{b^2-4ac}\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}} + \frac{\sqrt{c}\left(b^2d^2-bd\left(\sqrt{b^2-4ac}d+2ae\right)-2a\left(cd^2-e\left(\sqrt{b^2-4ac}d+ae\right)\right)\right)\text{arctanh}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex^2}}{\sqrt{2cd-\left(b+\sqrt{b^2-4ac}\right)e}}\right)}{\sqrt{2}a^2\sqrt{b^2-4ac}\sqrt{2cd-\left(b+\sqrt{b^2-4ac}\right)e}}$$

```
[Out] 1/2*e*arctanh((e*x^2+d)^(1/2)/d^(1/2))*d^(1/2)/a+(-2*a*e+b*d)*arctanh((e*x^
2+d)^(1/2)/d^(1/2))*d^(1/2)/a^2-1/2*d*(e*x^2+d)^(1/2)/a/x^2-1/2*arctanh(2^(
1/2)*c^(1/2)*(e*x^2+d)^(1/2)/(2*c*d-e*(b-(-4*a*c+b^2)^(1/2)))^(1/2))*c^(1/2
)*(b^2*d^2+b*d*(-2*a*e+d*(-4*a*c+b^2)^(1/2))-2*a*(c*d^2+e*(-a*e+d*(-4*a*c+b
^2)^(1/2))))/a^2*2^(1/2)/(-4*a*c+b^2)^(1/2)/(2*c*d-e*(b-(-4*a*c+b^2)^(1/2))
)^(1/2)+1/2*arctanh(2^(1/2)*c^(1/2)*(e*x^2+d)^(1/2)/(2*c*d-e*(b+(-4*a*c+b^2
)^(1/2)))^(1/2))*c^(1/2)*(b^2*d^2-b*d*(2*a*e+d*(-4*a*c+b^2)^(1/2))-2*a*(c*d
^2-e*(a*e+d*(-4*a*c+b^2)^(1/2))))/a^2*2^(1/2)/(-4*a*c+b^2)^(1/2)/(2*c*d-e*(
b+(-4*a*c+b^2)^(1/2)))^(1/2)
```

Rubi [A] (verified)

Time = 2.47 (sec) , antiderivative size = 416, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.241, Rules used = {1265, 911, 1301, 205, 212, 1180, 214}

$$\int \frac{(d+ex^2)^{3/2}}{x^3(a+bx^2+cx^4)}\,dx =$$

$$-\frac{\sqrt{c}\left(bd\left(d\sqrt{b^2-4ac}-2ae\right)-2ae\left(d\sqrt{b^2-4ac}-ae\right)-2acd^2+b^2d^2\right)\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex^2}}{\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}\right)}{\sqrt{2}a^2\sqrt{b^2-4ac}\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}$$
$$+\frac{\sqrt{c}\left(-bd\left(d\sqrt{b^2-4ac}+2ae\right)+2ae\left(d\sqrt{b^2-4ac}+ae\right)-2acd^2+b^2d^2\right)\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex^2}}{\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}\right)}{\sqrt{2}a^2\sqrt{b^2-4ac}\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}$$
$$+\frac{\sqrt{d}(bd-2ae)\operatorname{arctanh}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)}{a^2}+\frac{\sqrt{d}e\operatorname{arctanh}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)}{2a}-\frac{d\sqrt{d+ex^2}}{2ax^2}$$

```
[In] Int[(d + e*x^2)^(3/2)/(x^3*(a + b*x^2 + c*x^4)),x]
```

```
[Out] -1/2*(d*Sqrt[d + e*x^2])/(a*x^2) + (Sqrt[d]*e*ArcTanh[Sqrt[d + e*x^2]/Sqrt[
d]])/(2*a) + (Sqrt[d]*(b*d - 2*a*e)*ArcTanh[Sqrt[d + e*x^2]/Sqrt[d]])/a^2 -
 (Sqrt[c]*(b^2*d^2 - 2*a*c*d^2 + b*d*(Sqrt[b^2 - 4*a*c]*d - 2*a*e) - 2*a*e*
(Sqrt[b^2 - 4*a*c]*d - a*e))*ArcTanh[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x^2])/Sqrt
[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]])/(Sqrt[2]*a^2*Sqrt[b^2 - 4*a*c]*Sqrt[2
*c*d - (b - Sqrt[b^2 - 4*a*c])*e]) + (Sqrt[c]*(b^2*d^2 - 2*a*c*d^2 + 2*a*e*
(Sqrt[b^2 - 4*a*c]*d + a*e) - b*d*(Sqrt[b^2 - 4*a*c]*d + 2*a*e))*ArcTanh[(S
qrt[2]*Sqrt[c]*Sqrt[d + e*x^2])/Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]])/(
Sqrt[2]*a^2*Sqrt[b^2 - 4*a*c]*Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integ
erQ[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denom
inator[p + 1/n] < Denominator[p])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 911

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> With[{q = Denominator[m]}, Dist[q/e, S
ubst[Int[x^(q*(m + 1) - 1)*((e*f - d*g)/e + g*(x^q/e))^n*((c*d^2 - b*d*e +
a*e^2)/e^2 - (2*c*d - b*e)*(x^q/e^2) + c*(x^(2*q)/e^2))^p, x], x, (d + e*x)
^(1/q)], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegersQ[n, p] && Fra
ctionQ[m]
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1265

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rule 1301

```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.))/((a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*((d + e*x^2)^q/(a
+ b*x^2 + c*x^4)), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b^2 - 4
*a*c, 0] && IntegerQ[q] && IntegerQ[m]
```

Rubi steps

$$\text{integral} = \frac{1}{2}\text{Subst}\left(\int \frac{(d+ex)^{3/2}}{x^2(a+bx+cx^2)}\,dx, x, x^2\right)$$

$$= \frac{\text{Subst}\left(\int \frac{x^4}{\left(-\frac{d}{e}+\frac{x^2}{e}\right)^2\left(\frac{cd^2-bde+ae^2}{e^2}-\frac{(2cd-be)x^2}{e^2}+\frac{cx^4}{e^2}\right)}\,dx, x, \sqrt{d+ex^2}\right)}{e}$$

$$= \frac{\text{Subst}\left(\int \left(\frac{d^2e^2}{a(d-x^2)^2} - \frac{de(-bd+2ae)}{a^2(d-x^2)} + \frac{e(-((bd-ae)(cd^2-bde+ae^2))+cd(bd-2ae)x^2)}{a^2(cd^2-bde+ae^2-(2cd-be)x^2+cx^4)}\right) dx, x, \sqrt{d+ex^2}\right)}{e}$$

$$= \frac{\text{Subst}\left(\int \frac{-((bd-ae)(cd^2-bde+ae^2))+cd(bd-2ae)x^2}{cd^2-bde+ae^2+(-2cd+be)x^2+cx^4} dx, x, \sqrt{d+ex^2}\right)}{a^2} + \frac{(d^2e)\,\text{Subst}\left(\int \frac{1}{(d-x^2)^2} dx, x, \sqrt{d+ex^2}\right)}{a} + \frac{(d(bd-2ae))\text{Subst}\left(\int \frac{1}{d-x^2} dx, x, \sqrt{d+ex^2}\right)}{a^2}$$

$$= -\frac{d\sqrt{d+ex^2}}{2ax^2} + \frac{\sqrt{d}(bd-2ae)\tanh^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)}{a^2} + \frac{(de)\text{Subst}\left(\int \frac{1}{d-x^2} dx, x, \sqrt{d+ex^2}\right)}{2a} + \frac{\left(c\left(b^2d^2-2acd^2+bd\left(\sqrt{b^2-4ac}d-2ae\right)-2ae\left(\sqrt{b^2-4ac}d-ae\right)\right)\right)\text{Subst}\left(\int \frac{1}{-\frac{1}{2}\sqrt{b^2-4ac}e+\frac{1}{2}(-2c}\right.}{2a^2\sqrt{b^2-4ac}} - \frac{\left(c\left(b^2d^2-2acd^2+2ae\left(\sqrt{b^2-4ac}d+ae\right)-bd\left(\sqrt{b^2-4ac}d+2ae\right)\right)\right)\text{Subst}\left(\int \frac{1}{\frac{1}{2}\sqrt{b^2-4ac}e+\frac{1}{2}(-2cd}\right.}{2a^2\sqrt{b^2-4ac}}$$

$$= -\frac{d\sqrt{d+ex^2}}{2ax^2} + \frac{\sqrt{d}e\tanh^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)}{2a} + \frac{\sqrt{d}(bd-2ae)\tanh^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)}{a^2} - \frac{\sqrt{c}\left(b^2d^2-2acd^2+bd\left(\sqrt{b^2-4ac}d-2ae\right)-2ae\left(\sqrt{b^2-4ac}d-ae\right)\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex^2}}{\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)}}\right.}{\sqrt{2}a^2\sqrt{b^2-4ac}\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}} + \frac{\sqrt{c}\left(b^2d^2-2acd^2+2ae\left(\sqrt{b^2-4ac}d+ae\right)-bd\left(\sqrt{b^2-4ac}d+2ae\right)\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex^2}}{\sqrt{2cd-\left(b+\sqrt{b^2-4ac}\right)}}\right.}{\sqrt{2}a^2\sqrt{b^2-4ac}\sqrt{2cd-\left(b+\sqrt{b^2-4ac}\right)e}}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 1.74 (sec) , antiderivative size = 427, normalized size of antiderivative = 1.02

$$\int \frac{(d+ex^2)^{3/2}}{x^3(a+bx^2+cx^4)} dx = \frac{-\frac{ad\sqrt{d+ex^2}}{x^2} + \frac{\sqrt{2}\sqrt{c}\left(-ib^2d^2+bd\left(\sqrt{-b^2+4ac}d+2iae\right)-2ia\left(-cd^2+e\left(-i\sqrt{-b^2+4ac}d+ae\right)\right)\right)\arctan\left(\frac{}{\sqrt{-}}\right.}{\sqrt{-b^2+4ac}\sqrt{-2cd+\left(b-i\sqrt{-b^2+4ac}\right)e}}}{}$$

```
[In] Integrate[(d + e*x^2)^(3/2)/(x^3*(a + b*x^2 + c*x^4)),x]
```

```
[Out] (-((a*d*Sqrt[d + e*x^2])/x^2) + (Sqrt[2]*Sqrt[c]*((-I)*b^2*d^2 + b*d*(Sqrt[
-b^2 + 4*a*c]*d + (2*I)*a*e) - (2*I)*a*(-(c*d^2) + e*((-I)*Sqrt[-b^2 + 4*a*
c]*d + a*e)))*ArcTan[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x^2])/Sqrt[-2*c*d + b*e -
I*Sqrt[-b^2 + 4*a*c]*e]])/(Sqrt[-b^2 + 4*a*c]*Sqrt[-2*c*d + (b - I*Sqrt[-b^
2 + 4*a*c])*e]) + (Sqrt[2]*Sqrt[c]*(I*b^2*d^2 + b*d*(Sqrt[-b^2 + 4*a*c]*d -
 (2*I)*a*e) + (2*I)*a*(-(c*d^2) + e*(I*Sqrt[-b^2 + 4*a*c]*d + a*e)))*ArcTan
[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x^2])/Sqrt[-2*c*d + b*e + I*Sqrt[-b^2 + 4*a*c]
*e]])/(Sqrt[-b^2 + 4*a*c]*Sqrt[-2*c*d + (b + I*Sqrt[-b^2 + 4*a*c])*e]) + Sq
rt[d]*(2*b*d - 3*a*e)*ArcTanh[Sqrt[d + e*x^2]/Sqrt[d]])/(2*a^2)
```

Maple [A] (verified)

Time = 0.85 (sec) , antiderivative size = 399, normalized size of antiderivative = 0.96

method	result
risch	$-\frac{d\sqrt{e x^2+d}}{2 a x^2}-\frac{\frac{\sqrt{d}\,(3ae-2bd)\ln\left(\frac{2d+2\sqrt{d}\,\sqrt{e x^2+d}}{x}\right)}{a}+\frac{c\sqrt{2}\left(-\frac{\left(-2e^3a^2+2abd\,e^2+2ac\,d^2e-b^2d^2e+2\sqrt{-e^2\left(4ac-b^2\right)}\,ade-\sqrt{-e^2\left(4ac\cdots\right.}\right.}{\sqrt{\left(-be+2cd+\sqrt{-e^2\left(4\cdots\right.}\right.}}\cdots\right)}{\cdots}}{\cdots}$
pseudoelliptic	$-\frac{\sqrt{\left(be-2cd+\sqrt{-4e^2\left(ac-\frac{b^2}{4}\right)}\right)c}\,\sqrt{2}\,c\left(\left(\left(-d^{\frac{3}{2}}ae+\frac{b\,d^{\frac{5}{2}}}{2}\right)\sqrt{-4e^2\left(ac-\frac{b^2}{4}\right)}+e\left(\left(-ac+\frac{b^2}{2}\right)d^{\frac{5}{2}}+ae\left(ea\sqrt{d}-d^{\frac{3}{2}}b\right)\right)\right)x^2\,\mathrm{arcta}\cdots\right.}{\cdots}$
default	$\frac{-\frac{\left(e x^2+d\right)^{\frac{5}{2}}}{2d\,x^2}+\frac{3e\left(\frac{\left(e x^2+d\right)^{\frac{3}{2}}}{3}+d\left(\sqrt{e x^2+d}-\sqrt{d}\,\ln\left(\frac{2d+2\sqrt{d}\,\sqrt{e x^2+d}}{x}\right)\right)\right)}{2d}}{a}-\frac{b\left(\frac{\left(e x^2+d\right)^{\frac{3}{2}}}{3}+d\left(\sqrt{e x^2+d}-\sqrt{d}\,\ln\left(\frac{2d+2\sqrt{d}\,\sqrt{e\cdots}}{x}\right.\right.\right.\cdots}{a^2}\cdots$

```
[In] int((e*x^2+d)^(3/2)/x^3/(c*x^4+b*x^2+a),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*d*(e*x^2+d)^(1/2)/a/x^2-1/2/a*(d^(1/2)*(3*a*e-2*b*d)/a*ln((2*d+2*d^(1/
2)*(e*x^2+d)^(1/2))/x)+1/a*c*2^(1/2)/(-e^2*(4*a*c-b^2))^(1/2)*(-(-2*e^3*a^2
+2*a*b*d*e^2+2*a*c*d^2*e-b^2*d^2*e+2*(-e^2*(4*a*c-b^2))^(1/2)*a*d*e-(-e^2*(
4*a*c-b^2))^(1/2)*b*d^2)/((-b*e+2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2)*ar
ctanh(c*(e*x^2+d)^(1/2)*2^(1/2)/((-b*e+2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(
1/2))+(2*e^3*a^2-2*a*b*d*e^2-2*a*c*d^2*e+b^2*d^2*e+2*(-e^2*(4*a*c-b^2))^(1/
2)*a*d*e-(-e^2*(4*a*c-b^2))^(1/2)*b*d^2)/((b*e-2*c*d+(-e^2*(4*a*c-b^2))^(1/
2))*c)^(1/2)*arctan(c*(e*x^2+d)^(1/2)*2^(1/2)/((b*e-2*c*d+(-e^2*(4*a*c-b^2)
)^(1/2))*c)^(1/2))))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{(d+ex^2)^{3/2}}{x^3(a+bx^2+cx^4)}\,dx = \text{Timed out}$$

```
[In] integrate((e*x^2+d)^(3/2)/x^3/(c*x^4+b*x^2+a),x, algorithm="fricas")
[Out] Timed out
```

Sympy [F]

$$\int \frac{(d+ex^2)^{3/2}}{x^3(a+bx^2+cx^4)}\,dx = \int \frac{(d+ex^2)^{\frac{3}{2}}}{x^3(a+bx^2+cx^4)}\,dx$$

```
[In] integrate((e*x**2+d)**(3/2)/x**3/(c*x**4+b*x**2+a),x)
[Out] Integral((d + e*x**2)**(3/2)/(x**3*(a + b*x**2 + c*x**4)), x)
```

Maxima [F]

$$\int \frac{(d+ex^2)^{3/2}}{x^3(a+bx^2+cx^4)}\,dx = \int \frac{(ex^2+d)^{\frac{3}{2}}}{(cx^4+bx^2+a)x^3}\,dx$$

```
[In] integrate((e*x^2+d)^(3/2)/x^3/(c*x^4+b*x^2+a),x, algorithm="maxima")
[Out] integrate((e*x^2 + d)^(3/2)/((c*x^4 + b*x^2 + a)*x^3), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 898 vs. $2(355) = 710$.

Time = 0.33 (sec) , antiderivative size = 898, normalized size of antiderivative = 2.15

$$\int \frac{(d+ex^2)^{3/2}}{x^3(a+bx^2+cx^4)}\,dx = -\frac{(2\,bd^2-3\,ade)\arctan\left(\frac{\sqrt{ex^2+d}}{\sqrt{-d}}\right)}{2\,a^2\sqrt{-d}}$$

$$+\frac{\left(\sqrt{-4\,c^2d+2\,(bc-\sqrt{b^2-4\,ac}c)e}((b^3-4\,abc)d^2-2\,(ab^2-4\,a^2c)de)e^2-2\,(\sqrt{b^2-4\,ac}bcd^3+2\,\sqrt{b^2-4\,a\right.}{}$$

$$-\frac{\left(\sqrt{-4\,c^2d+2\,(bc+\sqrt{b^2-4\,ac}c)e}((b^3-4\,abc)d^2-2\,(ab^2-4\,a^2c)de)e^2+2\,(\sqrt{b^2-4\,ac}bcd^3+2\,\sqrt{b^2-4\,a\right.}{}$$

$$-\frac{\sqrt{ex^2+d}d}{2\,ax^2}$$

[In] integrate((e*x^2+d)^(3/2)/x^3/(c*x^4+b*x^2+a),x, algorithm="giac")

[Out] -1/2*(2*b*d^2 - 3*a*d*e)*arctan(sqrt(e*x^2 + d)/sqrt(-d))/(a^2*sqrt(-d)) + 1/8*(sqrt(-4*c^2*d + 2*(b*c - sqrt(b^2 - 4*a*c)*c)*e)*((b^3 - 4*a*b*c)*d^2 - 2*(a*b^2 - 4*a^2*c)*d*e)*e^2 - 2*(sqrt(b^2 - 4*a*c)*b*c*d^3 + 2*sqrt(b^2 - 4*a*c)*a*b*d*e^2 - sqrt(b^2 - 4*a*c)*a^2*e^3 - (b^2 + a*c)*sqrt(b^2 - 4*a*c)*d^2*e)*sqrt(-4*c^2*d + 2*(b*c - sqrt(b^2 - 4*a*c)*c)*e)*abs(e) + (2*a^2*b*e^4 - 2*(b^2*c - 2*a*c^2)*d^3*e + (b^3 + 2*a*b*c)*d^2*e^2 - 2*(a*b^2 + 2*a^2*c)*d*e^3)*sqrt(-4*c^2*d + 2*(b*c - sqrt(b^2 - 4*a*c)*c)*e))*arctan(2*sqrt(1/2)*sqrt(e*x^2 + d)/sqrt(-(2*a^2*c*d - a^2*b*e + sqrt(-4*(a^2*c*d^2 - a^2*b*d*e + a^3*e^2)*a^2*c + (2*a^2*c*d - a^2*b*e)^2))/(a^2*c)))/((sqrt(b^2 - 4*a*c)*a^2*c*d^2 - sqrt(b^2 - 4*a*c)*a^2*b*d*e + sqrt(b^2 - 4*a*c)*a^3*e^2)*abs(c)*abs(e)) - 1/8*(sqrt(-4*c^2*d + 2*(b*c + sqrt(b^2 - 4*a*c)*c)*e)*((b^3 - 4*a*b*c)*d^2 - 2*(a*b^2 - 4*a^2*c)*d*e)*e^2 + 2*(sqrt(b^2 - 4*a*c)*b*c*d^3 + 2*sqrt(b^2 - 4*a*c)*a*b*d*e^2 - sqrt(b^2 - 4*a*c)*a^2*e^3 - (b^2 + a*c)*sqrt(b^2 - 4*a*c)*d^2*e)*sqrt(-4*c^2*d + 2*(b*c + sqrt(b^2 - 4*a*c)*c)*e)*abs(e) + (2*a^2*b*e^4 - 2*(b^2*c - 2*a*c^2)*d^3*e + (b^3 + 2*a*b*c)*d^2*e^2 - 2*(a*b^2 + 2*a^2*c)*d*e^3)*sqrt(-4*c^2*d + 2*(b*c + sqrt(b^2 - 4*a*c)*c)*e))*arctan(2*sqrt(1/2)*sqrt(e*x^2 + d)/sqrt(-(2*a^2*c*d - a^2*b*e - sqrt(-4*(a^2*c*d^2 - a^2*b*d*e + a^3*e^2)*a^2*c + (2*a^2*c*d - a^2*b*e)^2))/(a^2*c)))/((sqrt(b^2 - 4*a*c)*a^2*c*d^2 - sqrt(b^2 - 4*a*c)*a^2*b*d*e + sqrt(b^2 - 4*a*c)*a^3*e^2)*abs(c)*abs(e)) - 1/2*sqrt(e*x^2 + d)*d/(a*x^2)

Mupad [B] (verification not implemented)

Time = 11.06 (sec) , antiderivative size = 35855, normalized size of antiderivative = 85.98

$$\int \frac{(d+ex^2)^{3/2}}{x^3(a+bx^2+cx^4)}\,dx = \text{Too large to display}$$

```
[In] int((d + e*x^2)^(3/2)/(x^3*(a + b*x^2 + c*x^4)),x)
```

```
[Out] (d^(1/2)*atan(((d^(1/2)*(3*a*e - 2*b*d)*(((d + e*x^2)^(1/2)*(4*a^6*c^3*e^16
 + 4*a^2*c^7*d^8*e^8 - 2*a^3*c^6*d^6*e^10 + 132*a^4*c^5*d^4*e^12 - 2*a^5*c^
4*d^2*e^14 + 4*b^4*c^5*d^8*e^8 + 129*a^2*b^2*c^5*d^6*e^10 - 32*a^2*b^3*c^4*
d^5*e^11 + 8*a^2*b^4*c^3*d^4*e^12 + 88*a^3*b^2*c^4*d^4*e^12 - 28*a^3*b^3*c^
3*d^3*e^13 + 33*a^4*b^2*c^3*d^2*e^14 - 16*a^5*b*c^3*d*e^15 - 8*a*b^2*c^6*d^
8*e^8 - 28*a*b^3*c^5*d^7*e^9 + 8*a^2*b*c^6*d^7*e^9 - 228*a^3*b*c^5*d^5*e^11
 - 60*a^4*b*c^4*d^3*e^13))/(2*a^4) - (d^(1/2)*((56*a^4*c^6*d^6*e^9 - 44*a^5
*c^5*d^4*e^11 - 100*a^6*c^4*d^2*e^13 + 40*a^2*b^3*c^5*d^7*e^8 - 39*a^2*b^5*
c^3*d^5*e^10 - 11*a^2*b^6*c^2*d^4*e^11 - 108*a^3*b^2*c^5*d^6*e^9 + 96*a^3*b
^3*c^4*d^5*e^10 + 111*a^3*b^4*c^3*d^4*e^11 + 22*a^3*b^5*c^2*d^3*e^12 - 237*
a^4*b^2*c^4*d^4*e^11 - 161*a^4*b^3*c^3*d^3*e^12 - 19*a^4*b^4*c^2*d^2*e^13 +
 111*a^5*b^2*c^3*d^2*e^13 - 28*a^6*b*c^3*d*e^14 - 8*a*b^5*c^4*d^7*e^8 + 6*a
*b^6*c^3*d^6*e^9 + 2*a*b^7*c^2*d^5*e^10 - 32*a^3*b*c^6*d^7*e^8 + 92*a^4*b*c
^5*d^5*e^10 + 252*a^5*b*c^4*d^3*e^12 + 6*a^5*b^3*c^2*d*e^14)/a^4 + (d^(1/2)
*(3*a*e - 2*b*d)*(((d + e*x^2)^(1/2)*(64*a^7*b*c^3*e^13 + 352*a^7*c^4*d*e^1
2 - 16*a^6*b^3*c^2*e^13 - 160*a^5*c^6*d^5*e^8 + 736*a^6*c^5*d^3*e^10 + 32*a
^2*b^6*c^3*d^5*e^8 - 32*a^2*b^7*c^2*d^4*e^9 - 224*a^3*b^4*c^4*d^5*e^8 + 144
*a^3*b^5*c^3*d^4*e^9 + 112*a^3*b^6*c^2*d^3*e^10 + 432*a^4*b^2*c^5*d^5*e^8 +
 144*a^4*b^3*c^4*d^4*e^9 - 716*a^4*b^4*c^3*d^3*e^10 - 132*a^4*b^5*c^2*d^2*e
^11 + 936*a^5*b^2*c^4*d^3*e^10 + 860*a^5*b^3*c^3*d^2*e^11 - 896*a^5*b*c^5*d
^4*e^9 + 64*a^5*b^4*c^2*d*e^12 - 1392*a^6*b*c^4*d^2*e^11 - 336*a^6*b^2*c^3*
d*e^12))/(2*a^4) + (d^(1/2)*((320*a^8*c^4*d*e^11 + 320*a^7*c^5*d^3*e^9 + 32
*a^5*b^3*c^4*d^4*e^8 - 24*a^5*b^4*c^3*d^3*e^9 - 8*a^5*b^5*c^2*d^2*e^10 + 16
*a^6*b^2*c^4*d^3*e^9 + 144*a^6*b^3*c^3*d^2*e^10 - 128*a^6*b*c^5*d^4*e^8 + 8
*a^6*b^4*c^2*d*e^11 - 448*a^7*b*c^4*d^2*e^10 - 112*a^7*b^2*c^3*d*e^11)/a^4
- (d^(1/2)*(d + e*x^2)^(1/2)*(3*a*e - 2*b*d)*(1024*a^9*c^4*e^10 + 64*a^7*b^
4*c^2*e^10 - 512*a^8*b^2*c^3*e^10 + 1536*a^8*c^5*d^2*e^8 + 128*a^6*b^4*c^3*
d^2*e^8 - 896*a^7*b^2*c^4*d^2*e^8 - 1792*a^8*b*c^4*d*e^9 - 128*a^6*b^5*c^2*
d*e^9 + 960*a^7*b^3*c^3*d*e^9))/(8*a^6))*(3*a*e - 2*b*d))/(4*a^2)))/(4*a^2)
)*(3*a*e - 2*b*d))/(4*a^2))*1i)/(4*a^2) + (d^(1/2)*(3*a*e - 2*b*d)*(((d + e
*x^2)^(1/2)*(4*a^6*c^3*e^16 + 4*a^2*c^7*d^8*e^8 - 2*a^3*c^6*d^6*e^10 + 132*
a^4*c^5*d^4*e^12 - 2*a^5*c^4*d^2*e^14 + 4*b^4*c^5*d^8*e^8 + 129*a^2*b^2*c^5
*d^6*e^10 - 32*a^2*b^3*c^4*d^5*e^11 + 8*a^2*b^4*c^3*d^4*e^12 + 88*a^3*b^2*c
^4*d^4*e^12 - 28*a^3*b^3*c^3*d^3*e^13 + 33*a^4*b^2*c^3*d^2*e^14 - 16*a^5*b*
c^3*d*e^15 - 8*a*b^2*c^6*d^8*e^8 - 28*a*b^3*c^5*d^7*e^9 + 8*a^2*b*c^6*d^7*e
^9 - 228*a^3*b*c^5*d^5*e^11 - 60*a^4*b*c^4*d^3*e^13))/(2*a^4) + (d^(1/2)*((
```

56*a^4*c^6*d^6*e^9 - 44*a^5*c^5*d^4*e^11 - 100*a^6*c^4*d^2*e^13 + 40*a^2*b^3*c^5*d^7*e^8 - 39*a^2*b^5*c^3*d^5*e^10 - 11*a^2*b^6*c^2*d^4*e^11 - 108*a^3*b^2*c^5*d^6*e^9 + 96*a^3*b^3*c^4*d^5*e^10 + 111*a^3*b^4*c^3*d^4*e^11 + 22*a^3*b^5*c^2*d^3*e^12 - 237*a^4*b^2*c^4*d^4*e^11 - 161*a^4*b^3*c^3*d^3*e^12 - 19*a^4*b^4*c^2*d^2*e^13 + 111*a^5*b^2*c^3*d^2*e^13 - 28*a^6*b*c^3*d*e^14 - 8*a*b^5*c^4*d^7*e^8 + 6*a*b^6*c^3*d^6*e^9 + 2*a*b^7*c^2*d^5*e^10 - 32*a^3*b*c^6*d^7*e^8 + 92*a^4*b*c^5*d^5*e^10 + 252*a^5*b*c^4*d^3*e^12 + 6*a^5*b^3*c^2*d*e^14)/a^4 - (d^(1/2)*(3*a*e - 2*b*d)*(((d + e*x^2)^(1/2)*(64*a^7*b*c^3*e^13 + 352*a^7*c^4*d*e^12 - 16*a^6*b^3*c^2*e^13 - 160*a^5*c^6*d^5*e^8 + 736*a^6*c^5*d^3*e^10 + 32*a^2*b^6*c^3*d^5*e^8 - 32*a^2*b^7*c^2*d^4*e^9 - 224*a^3*b^4*c^4*d^5*e^8 + 144*a^3*b^5*c^3*d^4*e^9 + 112*a^3*b^6*c^2*d^3*e^10 + 432*a^4*b^2*c^5*d^5*e^8 + 144*a^4*b^3*c^4*d^4*e^9 - 716*a^4*b^4*c^3*d^3*e^10 - 132*a^4*b^5*c^2*d^2*e^11 + 936*a^5*b^2*c^4*d^3*e^10 + 860*a^5*b^3*c^3*d^2*e^11 - 896*a^5*b*c^5*d^4*e^9 + 64*a^5*b^4*c^2*d*e^12 - 1392*a^6*b*c^4*d^2*e^11 - 336*a^6*b^2*c^3*d*e^12))/(2*a^4) - (d^(1/2)*((320*a^8*c^4*d*e^11 + 320*a^7*c^5*d^3*e^9 + 32*a^5*b^3*c^4*d^4*e^8 - 24*a^5*b^4*c^3*d^3*e^9 - 8*a^5*b^5*c^2*d^2*e^10 + 16*a^6*b^2*c^4*d^3*e^9 + 144*a^6*b^3*c^3*d^2*e^10 - 128*a^6*b*c^5*d^4*e^8 + 8*a^6*b^4*c^2*d*e^11 - 448*a^7*b*c^4*d^2*e^10 - 112*a^7*b^2*c^3*d*e^11)/a^4 + (d^(1/2)*(d + e*x^2)^(1/2)*(3*a*e - 2*b*d)*(1024*a^9*c^4*e^10 + 64*a^7*b^4*c^2*e^10 - 512*a^8*b^2*c^3*e^10 + 1536*a^8*c^5*d^2*e^8 + 128*a^6*b^4*c^3*d^2*e^8 - 896*a^7*b^2*c^4*d^2*e^8 - 1792*a^8*b*c^4*d*e^9 - 128*a^6*b^5*c^2*d*e^9 + 960*a^7*b^3*c^3*d*e^9))/(8*a^6))*(3*a*e - 2*b*d))/(4*a^2)))/(4*a^2))*(3*a*e - 2*b*d))/(4*a^2))*1i)/(4*a^2))/((3*a*c^7*d^9*e^9 + 3*a^5*c^3*d*e^17 - 2*b*c^7*d^10*e^8 + 3*a^2*c^6*d^7*e^11 + 3*a^4*c^4*d^3*e^15 + 4*b^2*c^6*d^9*e^9 - 2*b^3*c^5*d^8*e^10 + 2*a^2*b^2*c^4*d^5*e^13 - (11*a^2*b^3*c^3*d^4*e^14)/2 + 11*a^3*b^2*c^3*d^3*e^15 - 8*a*b*c^6*d^8*e^10 + 4*a*b^2*c^5*d^7*e^11 + a*b^4*c^3*d^5*e^13 - (3*a^2*b*c^5*d^6*e^12)/2 - 5*a^3*b*c^4*d^4*e^14 - (19*a^4*b*c^3*d^2*e^16)/2)/a^4 - (d^(1/2)*(3*a*e - 2*b*d)*(((d + e*x^2)^(1/2)*(4*a^6*c^3*e^16 + 4*a^2*c^7*d^8*e^8 - 2*a^3*c^6*d^6*e^10 + 132*a^4*c^5*d^4*e^12 - 2*a^5*c^4*d^2*e^14 + 4*b^4*c^5*d^8*e^8 + 129*a^2*b^2*c^5*d^6*e^10 - 32*a^2*b^3*c^4*d^5*e^11 + 8*a^2*b^4*c^3*d^4*e^12 + 88*a^3*b^2*c^4*d^4*e^12 - 28*a^3*b^3*c^3*d^3*e^13 + 33*a^4*b^2*c^3*d^2*e^14 - 16*a^5*b*c^3*d*e^15 - 8*a*b^2*c^6*d^8*e^8 - 28*a*b^3*c^5*d^7*e^9 + 8*a^2*b*c^6*d^7*e^9 - 228*a^3*b*c^5*d^5*e^11 - 60*a^4*b*c^4*d^3*e^13))/(2*a^4) - (d^(1/2)*((56*a^4*c^6*d^6*e^9 - 44*a^5*c^5*d^4*e^11 - 100*a^6*c^4*d^2*e^13 + 40*a^2*b^3*c^5*d^7*e^8 - 39*a^2*b^5*c^3*d^5*e^10 - 11*a^2*b^6*c^2*d^4*e^11 - 108*a^3*b^2*c^5*d^6*e^9 + 96*a^3*b^3*c^4*d^5*e^10 + 111*a^3*b^4*c^3*d^4*e^11 + 22*a^3*b^5*c^2*d^3*e^12 - 237*a^4*b^2*c^4*d^4*e^11 - 161*a^4*b^3*c^3*d^3*e^12 - 19*a^4*b^4*c^2*d^2*e^13 + 111*a^5*b^2*c^3*d^2*e^13 - 28*a^6*b*c^3*d*e^14 - 8*a*b^5*c^4*d^7*e^8 + 6*a*b^6*c^3*d^6*e^9 + 2*a*b^7*c^2*d^5*e^10 - 32*a^3*b*c^6*d^7*e^8 + 92*a^4*b*c^5*d^5*e^10 + 252*a^5*b*c^4*d^3*e^12 + 6*a^5*b^3*c^2*d*e^14)/a^4 + (d^(1/2)*(3*a*e - 2*b*d)*(((d + e*x^2)^(1/2)*(64*a^7*b*c^3*e^13 + 352*a^7*c^4*d*e^12 - 16*a^6*b^3*c^2*e^13 - 160*a^5*c^6*d^5*e^8 + 736*a^6*c^5*d^3*e^10 + 32*a^2*b^6*c^3*d^5*e^8 - 32*a^2*b^7*c^2*d^4*e^9 - 224*a^3*b^4*c^4*d^5*e^8 + 144*a^3*b^5*c^3*d^4*e^9 + 112* a^3*b^6*c^2*d^3*e^10 + 432*a^4*b^2*c^5*d^5*e^8 + 144*a^4*b^3*c^4*d^4*e^9 - 716*a^4*b^4*c^3*d^3*e^10 - 132*a^4*b^5*c^2*d^2*e^11 + 936*a^5*b^2*c^4*d^3*e^10 + 860*a^5*b^3*c^3*d^2*e^11 - 896*a^5*b*c^5*d^4*e^9 + 64*a^5*b^4*c^2*d*e^12 - 1392*a^6*b*c^4*d^2*e^11 - 336*a^6*b^2*c^3*d*e^12))/(2*a^4) + (d^(1/2)*((320*a^8*c^4*d*e^11 + 320*a^7*c^5*d^3*e^9 + 32*a^5*b^3*c^4*d^4*e^8 - 24*a^5*b^4*c^3*d^3*e^9 - 8*a^5*b^5*c^2*d^2*e^10 + 16*a^6*b^2*c^4*d^3*e^9 + 144*a^6*b^3*c^3*d^2*e^10 - 128*a^6*b*c^5*d^4*e^8 + 8*a^6*b^4*c^2*d*e^11 - 448*a^7*b*c^4*d^2*e^10 - 112*a^7*b^2*c^3*d*e^11)/a^4 - (d^(1/2)*(d + e*x^2)^(1/2)*(3*a*e - 2*b*d)*(1024*a^9*c^4*e^10 + 64*a^7*b^4*c^2*e^10 - 512*a^8*b^2*c^3*e^10 + 1536*a^8*c^5*d^2*e^8 + 128*a^6*b^4*c^3*d^2*e^8 - 896*a^7*b^2*c^4*d^2*e^8 - 1792*a^8*b*c^4*d*e^9 - 128*a^6*b^5*c^2*d*e^9 + 960*a^7*b^3*c^3*d*e^9))/(8*a^6))*(3*a*e - 2*b*d))/(4*a^2)))/(4*a^2))*(3*a*e - 2*b*d))/(4*a^2)))/(4*a^2) + (d^(1/2)*(3*a*e - 2*b*d)*(((d + e*x^2)^(1/2)*(4*a^6*c^3*e^16 + 4*a^2*c^7*d^8*e^8 - 2*a^3*c^6*d^6*e^10 + 132*a^4*c^5*d^4*e^12 - 2*a^5*c^4*d^2*e^14 + 4*b^4*c^5*d^8*e^8 + 129*a^2*b^2*c^5*d^6*e^10 - 32*a^2*b^3*c^4*d^5*e^11 + 8*a^2*b^4*c^3*d^4*e^12 + 88*a^3*b^2*c^4*d^4*e^12 - 28*a^3*b^3*c^3*d^3*e^13 + 33*a^4*b^2*c^3*d^2*e^14 - 16*a^5*b*c^3*d*e^15 - 8*a*b^2*c^6*d^8*e^8 - 28*a*b^3*c^5*d^7*e^9 + 8*a^2*b*c^6*d^7*e^9 - 228*a^3*b*c^5*d^5*e^11 - 60*a^4*b*c^4*d^3*e^13))/(2*a^4) + (d^(1/2)*((56*a^4*c^6*d^6*e^9 - 44*a^5*c^5*d^4*e^11 - 100*a^6*c^4*d^2*e^13 + 40*a^2*b^3*c^5*d^7*e^8 - 39*a^2*b^5*c^3*d^5*e^10 - 11*a^2*b^6*c^2*d^4*e^11 - 108*a^3*b^2*c^5*d^6*e^9 + 96*a^3*b^3*c^4*d^5*e^10 + 111*a^3*b^4*c^3*d^4*e^11 + 22*a^3*b^5*c^2*d^3*e^12 - 237*a^4*b^2*c^4*d^4*e^11 - 161*a^4*b^3*c^3*d^3*e^12 - 19*a^4*b^4*c^2*d^2*e^13 + 111*a^5*b^2*c^3*d^2*e^13 - 28*a^6*b*c^3*d*e^14 - 8*a*b^5*c^4*d^7*e^8 + 6*a*b^6*c^3*d^6*e^9 + 2*a*b^7*c^2*d^5*e^10 - 32*a^3*b*c^6*d^7*e^8 + 92*a^4*b*c^5*d^5*e^10 + 252*a^5*b*c^4*d^3*e^12 + 6*a^5*b^3*c^2*d*e^14)/a^4 - (d^(1/2)*(3*a*e - 2*b*d)*(((d + e*x^2)^(1/2)*(64*a^7*b*c^3*e^13 + 352*a^7*c^4*d*e^12 - 16*a^6*b^3*c^2*e^13 - 160*a^5*c^6*d^5*e^8 + 736*a^6*c^5*d^3*e^10 + 32*a^2*b^6*c^3*d^5*e^8 - 32*a^2*b^7*c^2*d^4*e^9 - 224*a^3*b^4*c^4*d^5*e^8 + 144*a^3*b^5*c^3*d^4*e^9 + 112*a^3*b^6*c^2*d^3*e^10 + 432*a^4*b^2*c^5*d^5*e^8 + 144*a^4*b^3*c^4*d^4*e^9 - 716*a^4*b^4*c^3*d^3*e^10 - 132*a^4*b^5*c^2*d^2*e^11 + 936*a^5*b^2*c^4*d^3*e^10 + 860*a^5*b^3*c^3*d^2*e^11 - 896*a^5*b*c^5*d^4*e^9 + 64*a^5*b^4*c^2*d*e^12 - 1392*a^6*b*c^4*d^2*e^11 - 336*a^6*b^2*c^3*d*e^12))/(2*a^4) - (d^(1/2)*((320*a^8*c^4*d*e^11 + 320*a^7*c^5*d^3*e^9 + 32*a^5*b^3*c^4*d^4*e^8 - 24*a^5*b^4*c^3*d^3*e^9 - 8*a^5*b^5*c^2*d^2*e^10 + 16*a^6*b^2*c^4*d^3*e^9 + 144*a^6*b^3*c^3*d^2*e^10 - 128*a^6*b*c^5*d^4*e^8 + 8*a^6*b^4*c^2*d*e^11 - 448*a^7*b*c^4*d^2*e^10 - 112*a^7*b^2*c^3*d*e^11)/a^4 + (d^(1/2)*(d + e*x^2)^(1/2)*(3*a*e - 2*b*d)*(1024*a^9*c^4*e^10 + 64*a^7*b^4*c^2*e^10 - 512*a^8*b^2*c^3*e^10 + 1536*a^8*c^5*d^2*e^8 + 128*a^6*b^4*c^3*d^2*e^8 - 896*a^7*b^2*c^4*d^2*e^8 - 1792*a^8*b*c^4*d*e^9 - 128*a^6*b^5*c^2*d*e^9 + 960*a^7*b^3*c^3*d*e^9))/(8*a^6))*(3*a*e - 2*b*d))/(4*a^2)))/(4*a^2))*(3*a*e - 2*b*d))/(4*a^2)))/(4*a^2)))*(3*a*e - 2*b*d)*1i)/(2*a^2) - atan(((((224*a^4*c^6*d^6*e^9 - 176*a^5*c^5*d^4*e^11 - 400*a^6*c^4*d^2*e^13 + 160*a^2*b^3*c^5*d^7*e^8 - 156*a^2*b^5*c^3*d^5*e^10 - 44*a^2*b^6*c^2*d^4*e^11 - 432*a^3*b^2*c^5*d^6*e^9 + 384*a^3*b^3*c^4*d^5*e^10 + 444*a^3*b^4*c^3*d^4*e^11

+ 88*a^3*b^5*c^2*d^3*e^12 - 948*a^4*b^2*c^4*d^4*e^11 - 644*a^4*b^3*c^3*d^3*
e^12 - 76*a^4*b^4*c^2*d^2*e^13 + 444*a^5*b^2*c^3*d^2*e^13 - 112*a^6*b*c^3*d
*e^14 - 32*a*b^5*c^4*d^7*e^8 + 24*a*b^6*c^3*d^6*e^9 + 8*a*b^7*c^2*d^5*e^10
- 128*a^3*b*c^6*d^7*e^8 + 368*a^4*b*c^5*d^5*e^10 + 1008*a^5*b*c^4*d^3*e^12
+ 24*a^5*b^3*c^2*d*e^14)/(4*a^4) + (((1280*a^8*c^4*d*e^11 + 1280*a^7*c^5*d^
3*e^9 + 128*a^5*b^3*c^4*d^4*e^8 - 96*a^5*b^4*c^3*d^3*e^9 - 32*a^5*b^5*c^2*d
^2*e^10 + 64*a^6*b^2*c^4*d^3*e^9 + 576*a^6*b^3*c^3*d^2*e^10 - 512*a^6*b*c^5
*d^4*e^8 + 32*a^6*b^4*c^2*d*e^11 - 1792*a^7*b*c^4*d^2*e^10 - 448*a^7*b^2*c^
3*d*e^11)/(4*a^4) - ((d + e*x^2)^(1/2)*((((4*b^6*d^3 - 4*a^3*b^3*e^3 - 32*a
^3*c^3*d^3 + 12*a^2*b^4*d*e^2 + 96*a^4*c^2*d*e^2 + 72*a^2*b^2*c^2*d^3 - 32*
a*b^4*c*d^3 + 16*a^4*b*c*e^3 - 12*a*b^5*d^2*e + 84*a^2*b^3*c*d^2*e - 144*a^
3*b*c^2*d^2*e - 72*a^3*b^2*c*d*e^2)^2/4 - (16*a^4*b^4 + 256*a^6*c^2 - 128*a
^5*b^2*c)*(c^4*d^6 + a^3*c*e^6 + 3*a*c^3*d^4*e^2 - b^3*c*d^3*e^3 + 3*a^2*c^
2*d^2*e^4 + 3*b^2*c^2*d^4*e^2 - 3*b*c^3*d^5*e - 3*a^2*b*c*d*e^5 - 6*a*b*c^2
*d^3*e^3 + 3*a*b^2*c*d^2*e^4))^(1/2) + 2*b^6*d^3 - 2*a^3*b^3*e^3 - 16*a^3*c
^3*d^3 + 6*a^2*b^4*d*e^2 + 48*a^4*c^2*d*e^2 + 36*a^2*b^2*c^2*d^3 - 16*a*b^4
*c*d^3 + 8*a^4*b*c*e^3 - 6*a*b^5*d^2*e + 42*a^2*b^3*c*d^2*e - 72*a^3*b*c^2*
d^2*e - 36*a^3*b^2*c*d*e^2)/(16*(a^4*b^4 + 16*a^6*c^2 - 8*a^5*b^2*c)))^(1/2
)*(1024*a^9*c^4*e^10 + 64*a^7*b^4*c^2*e^10 - 512*a^8*b^2*c^3*e^10 + 1536*a^
8*c^5*d^2*e^8 + 128*a^6*b^4*c^3*d^2*e^8 - 896*a^7*b^2*c^4*d^2*e^8 - 1792*a^
8*b*c^4*d*e^9 - 128*a^6*b^5*c^2*d*e^9 + 960*a^7*b^3*c^3*d*e^9))/(2*a^4))*((
((4*b^6*d^3 - 4*a^3*b^3*e^3 - 32*a^3*c^3*d^3 + 12*a^2*b^4*d*e^2 + 96*a^4*c^
2*d*e^2 + 72*a^2*b^2*c^2*d^3 - 32*a*b^4*c*d^3 + 16*a^4*b*c*e^3 - 12*a*b^5*d
^2*e + 84*a^2*b^3*c*d^2*e - 144*a^3*b*c^2*d^2*e - 72*a^3*b^2*c*d*e^2)^2/4 -
(16*a^4*b^4 + 256*a^6*c^2 - 128*a^5*b^2*c)*(c^4*d^6 + a^3*c*e^6 + 3*a*c^3*
d^4*e^2 - b^3*c*d^3*e^3 + 3*a^2*c^2*d^2*e^4 + 3*b^2*c^2*d^4*e^2 - 3*b*c^3*d
^5*e - 3*a^2*b*c*d*e^5 - 6*a*b*c^2*d^3*e^3 + 3*a*b^2*c*d^2*e^4))^(1/2) + 2*
b^6*d^3 - 2*a^3*b^3*e^3 - 16*a^3*c^3*d^3 + 6*a^2*b^4*d*e^2 + 48*a^4*c^2*d*e
^2 + 36*a^2*b^2*c^2*d^3 - 16*a*b^4*c*d^3 + 8*a^4*b*c*e^3 - 6*a*b^5*d^2*e +
42*a^2*b^3*c*d^2*e - 72*a^3*b*c^2*d^2*e - 36*a^3*b^2*c*d*e^2)/(16*(a^4*b^4
+ 16*a^6*c^2 - 8*a^5*b^2*c)))^(1/2) + ((d + e*x^2)^(1/2)*(64*a^7*b*c^3*e^13
+ 352*a^7*c^4*d*e^12 - 16*a^6*b^3*c^2*e^13 - 160*a^5*c^6*d^5*e^8 + 736*a^6
*c^5*d^3*e^10 + 32*a^2*b^6*c^3*d^5*e^8 - 32*a^2*b^7*c^2*d^4*e^9 - 224*a^3*b
^4*c^4*d^5*e^8 + 144*a^3*b^5*c^3*d^4*e^9 + 112*a^3*b^6*c^2*d^3*e^10 + 432*a
^4*b^2*c^5*d^5*e^8 + 144*a^4*b^3*c^4*d^4*e^9 - 716*a^4*b^4*c^3*d^3*e^10 - 1
32*a^4*b^5*c^2*d^2*e^11 + 936*a^5*b^2*c^4*d^3*e^10 + 860*a^5*b^3*c^3*d^2*e^
11 - 896*a^5*b*c^5*d^4*e^9 + 64*a^5*b^4*c^2*d*e^12 - 1392*a^6*b*c^4*d^2*e^1
1 - 336*a^6*b^2*c^3*d*e^12))/(2*a^4))*((((4*b^6*d^3 - 4*a^3*b^3*e^3 - 32*a^
3*c^3*d^3 + 12*a^2*b^4*d*e^2 + 96*a^4*c^2*d*e^2 + 72*a^2*b^2*c^2*d^3 - 32*a
*b^4*c*d^3 + 16*a^4*b*c*e^3 - 12*a*b^5*d^2*e + 84*a^2*b^3*c*d^2*e - 144*a^3
*b*c^2*d^2*e - 72*a^3*b^2*c*d*e^2)^2/4 - (16*a^4*b^4 + 256*a^6*c^2 - 128*a^
5*b^2*c)*(c^4*d^6 + a^3*c*e^6 + 3*a*c^3*d^4*e^2 - b^3*c*d^3*e^3 + 3*a^2*c^2
*d^2*e^4 + 3*b^2*c^2*d^4*e^2 - 3*b*c^3*d^5*e - 3*a^2*b*c*d*e^5 - 6*a*b*c^2*
d^3*e^3 + 3*a*b^2*c*d^2*e^4))^(1/2) + 2*b^6*d^3 - 2*a^3*b^3*e^3 - 16*a^3*c^
3*d^3 + 6*a^2*b^4*d*e^2 + 48*a^4*c^2*d*e^2 + 36*a^2*b^2*c^2*d^3 - 16*a*b^4*

c*d^3 + 8*a^4*b*c*e^3 - 6*a*b^5*d^2*e + 42*a^2*b^3*c*d^2*e - 72*a^3*b*c^2*d
^2*e - 36*a^3*b^2*c*d*e^2)/(16*(a^4*b^4 + 16*a^6*c^2 - 8*a^5*b^2*c)))^(1/2)
)*((((4*b^6*d^3 - 4*a^3*b^3*e^3 - 32*a^3*c^3*d^3 + 12*a^2*b^4*d*e^2 + 96*a^
4*c^2*d*e^2 + 72*a^2*b^2*c^2*d^3 - 32*a*b^4*c*d^3 + 16*a^4*b*c*e^3 - 12*a*b
^5*d^2*e + 84*a^2*b^3*c*d^2*e - 144*a^3*b*c^2*d^2*e - 72*a^3*b^2*c*d*e^2)^2
/4 - (16*a^4*b^4 + 256*a^6*c^2 - 128*a^5*b^2*c)*(c^4*d^6 + a^3*c*e^6 + 3*a*
c^3*d^4*e^2 - b^3*c*d^3*e^3 + 3*a^2*c^2*d^2*e^4 + 3*b^2*c^2*d^4*e^2 - 3*b*c
^3*d^5*e - 3*a^2*b*c*d*e^5 - 6*a*b*c^2*d^3*e^3 + 3*a*b^2*c*d^2*e^4))^(1/2)
+ 2*b^6*d^3 - 2*a^3*b^3*e^3 - 16*a^3*c^3*d^3 + 6*a^2*b^4*d*e^2 + 48*a^4*c^2
*d*e^2 + 36*a^2*b^2*c^2*d^3 - 16*a*b^4*c*d^3 + 8*a^4*b*c*e^3 - 6*a*b^5*d^2*
e + 42*a^2*b^3*c*d^2*e - 72*a^3*b*c^2*d^2*e - 36*a^3*b^2*c*d*e^2)/(16*(a^4*
b^4 + 16*a^6*c^2 - 8*a^5*b^2*c)))^(1/2) - ((d + e*x^2)^(1/2)*(4*a^6*c^3*e^1
6 + 4*a^2*c^7*d^8*e^8 - 2*a^3*c^6*d^6*e^10 + 132*a^4*c^5*d^4*e^12 - 2*a^5*c
^4*d^2*e^14 + 4*b^4*c^5*d^8*e^8 + 129*a^2*b^2*c^5*d^6*e^10 - 32*a^2*b^3*c^4
*d^5*e^11 + 8*a^2*b^4*c^3*d^4*e^12 + 88*a^3*b^2*c^4*d^4*e^12 - 28*a^3*b^3*c
^3*d^3*e^13 + 33*a^4*b^2*c^3*d^2*e^14 - 16*a^5*b*c^3*d*e^15 - 8*a*b^2*c^6*d
^8*e^8 - 28*a*b^3*c^5*d^7*e^9 + 8*a^2*b*c^6*d^7*e^9 - 228*a^3*b*c^5*d^5*e^1
1 - 60*a^4*b*c^4*d^3*e^13))/(2*a^4))*((((4*b^6*d^3 - 4*a^3*b^3*e^3 - 32*a^3
*c^3*d^3 + 12*a^2*b^4*d*e^2 + 96*a^4*c^2*d*e^2 + 72*a^2*b^2*c^2*d^3 - 32*a*
b^4*c*d^3 + 16*a^4*b*c*e^3 - 12*a*b^5*d^2*e + 84*a^2*b^3*c*d^2*e - 144*a^3*
b*c^2*d^2*e - 72*a^3*b^2*c*d*e^2)^2/4 - (16*a^4*b^4 + 256*a^6*c^2 - 128*a^5
*b^2*c)*(c^4*d^6 + a^3*c*e^6 + 3*a*c^3*d^4*e^2 - b^3*c*d^3*e^3 + 3*a^2*c^2*
d^2*e^4 + 3*b^2*c^2*d^4*e^2 - 3*b*c^3*d^5*e - 3*a^2*b*c*d*e^5 - 6*a*b*c^2*d
^3*e^3 + 3*a*b^2*c*d^2*e^4))^(1/2) + 2*b^6*d^3 - 2*a^3*b^3*e^3 - 16*a^3*c^3
*d^3 + 6*a^2*b^4*d*e^2 + 48*a^4*c^2*d*e^2 + 36*a^2*b^2*c^2*d^3 - 16*a*b^4*c
*d^3 + 8*a^4*b*c*e^3 - 6*a*b^5*d^2*e + 42*a^2*b^3*c*d^2*e - 72*a^3*b*c^2*d^
2*e - 36*a^3*b^2*c*d*e^2)/(16*(a^4*b^4 + 16*a^6*c^2 - 8*a^5*b^2*c)))^(1/2)*
1i - (((224*a^4*c^6*d^6*e^9 - 176*a^5*c^5*d^4*e^11 - 400*a^6*c^4*d^2*e^13 +
160*a^2*b^3*c^5*d^7*e^8 - 156*a^2*b^5*c^3*d^5*e^10 - 44*a^2*b^6*c^2*d^4*e^
11 - 432*a^3*b^2*c^5*d^6*e^9 + 384*a^3*b^3*c^4*d^5*e^10 + 444*a^3*b^4*c^3*d
^4*e^11 + 88*a^3*b^5*c^2*d^3*e^12 - 948*a^4*b^2*c^4*d^4*e^11 - 644*a^4*b^3*
c^3*d^3*e^12 - 76*a^4*b^4*c^2*d^2*e^13 + 444*a^5*b^2*c^3*d^2*e^13 - 112*a^6
*b*c^3*d*e^14 - 32*a*b^5*c^4*d^7*e^8 + 24*a*b^6*c^3*d^6*e^9 + 8*a*b^7*c^2*d
^5*e^10 - 128*a^3*b*c^6*d^7*e^8 + 368*a^4*b*c^5*d^5*e^10 + 1008*a^5*b*c^4*d
^3*e^12 + 24*a^5*b^3*c^2*d*e^14)/(4*a^4) + (((1280*a^8*c^4*d*e^11 + 1280*a^
7*c^5*d^3*e^9 + 128*a^5*b^3*c^4*d^4*e^8 - 96*a^5*b^4*c^3*d^3*e^9 - 32*a^5*b
^5*c^2*d^2*e^10 + 64*a^6*b^2*c^4*d^3*e^9 + 576*a^6*b^3*c^3*d^2*e^10 - 512*a
^6*b*c^5*d^4*e^8 + 32*a^6*b^4*c^2*d*e^11 - 1792*a^7*b*c^4*d^2*e^10 - 448*a^
7*b^2*c^3*d*e^11)/(4*a^4) + ((d + e*x^2)^(1/2)*((((4*b^6*d^3 - 4*a^3*b^3*e^
3 - 32*a^3*c^3*d^3 + 12*a^2*b^4*d*e^2 + 96*a^4*c^2*d*e^2 + 72*a^2*b^2*c^2*d
^3 - 32*a*b^4*c*d^3 + 16*a^4*b*c*e^3 - 12*a*b^5*d^2*e + 84*a^2*b^3*c*d^2*e
- 144*a^3*b*c^2*d^2*e - 72*a^3*b^2*c*d*e^2)^2/4 - (16*a^4*b^4 + 256*a^6*c^2
- 128*a^5*b^2*c)*(c^4*d^6 + a^3*c*e^6 + 3*a*c^3*d^4*e^2 - b^3*c*d^3*e^3 +
3*a^2*c^2*d^2*e^4 + 3*b^2*c^2*d^4*e^2 - 3*b*c^3*d^5*e - 3*a^2*b*c*d*e^5 - 6
*a*b*c^2*d^3*e^3 + 3*a*b^2*c*d^2*e^4))^(1/2) + 2*b^6*d^3 - 2*a^3*b^3*e^3 -

16*a^3*c^3*d^3 + 6*a^2*b^4*d*e^2 + 48*a^4*c^2*d*e^2 + 36*a^2*b^2*c^2*d^3 - 16*a*b^4*c*d^3 + 8*a^4*b*c*e^3 - 6*a*b^5*d^2*e + 42*a^2*b^3*c*d^2*e - 72*a^3*b*c^2*d^2*e - 36*a^3*b^2*c*d*e^2)/(16*(a^4*b^4 + 16*a^6*c^2 - 8*a^5*b^2*c)))^(1/2)*(1024*a^9*c^4*e^10 + 64*a^7*b^4*c^2*e^10 - 512*a^8*b^2*c^3*e^10 + 1536*a^8*c^5*d^2*e^8 + 128*a^6*b^4*c^3*d^2*e^8 - 896*a^7*b^2*c^4*d^2*e^8 - 1792*a^8*b*c^4*d*e^9 - 128*a^6*b^5*c^2*d*e^9 + 960*a^7*b^3*c^3*d*e^9))/(2*a^4))*((((4*b^6*d^3 - 4*a^3*b^3*e^3 - 32*a^3*c^3*d^3 + 12*a^2*b^4*d*e^2 + 96*a^4*c^2*d*e^2 + 72*a^2*b^2*c^2*d^3 - 32*a*b^4*c*d^3 + 16*a^4*b*c*e^3 - 12*a*b^5*d^2*e + 84*a^2*b^3*c*d^2*e - 144*a^3*b*c^2*d^2*e - 72*a^3*b^2*c*d*e^2)^2/4 - (16*a^4*b^4 + 256*a^6*c^2 - 128*a^5*b^2*c)*(c^4*d^6 + a^3*c*e^6 + 3*a*c^3*d^4*e^2 - b^3*c*d^3*e^3 + 3*a^2*c^2*d^2*e^4 + 3*b^2*c^2*d^4*e^2 - 3*b*c^3*d^5*e - 3*a^2*b*c*d*e^5 - 6*a*b*c^2*d^3*e^3 + 3*a*b^2*c*d^2*e^4))^(1/2) + 2*b^6*d^3 - 2*a^3*b^3*e^3 - 16*a^3*c^3*d^3 + 6*a^2*b^4*d*e^2 + 48*a^4*c^2*d*e^2 + 36*a^2*b^2*c^2*d^3 - 16*a*b^4*c*d^3 + 8*a^4*b*c*e^3 - 6*a*b^5*d^2*e + 42*a^2*b^3*c*d^2*e - 72*a^3*b*c^2*d^2*e - 36*a^3*b^2*c*d*e^2)/(16*(a^4*b^4 + 16*a^6*c^2 - 8*a^5*b^2*c)))^(1/2) - ((d + e*x^2)^(1/2)*(64*a^7*b*c^3*e^13 + 352*a^7*c^4*d*e^12 - 16*a^6*b^3*c^2*e^13 - 160*a^5*c^6*d^5*e^8 + 736*a^6*c^5*d^3*e^10 + 32*a^2*b^6*c^3*d^5*e^8 - 32*a^2*b^7*c^2*d^4*e^9 - 224*a^3*b^4*c^4*d^5*e^8 + 144*a^3*b^5*c^3*d^4*e^9 + 112*a^3*b^6*c^2*d^3*e^10 + 432*a^4*b^2*c^5*d^5*e^8 + 144*a^4*b^3*c^4*d^4*e^9 - 716*a^4*b^4*c^3*d^3*e^10 - 132*a^4*b^5*c^2*d^2*e^11 + 936*a^5*b^2*c^4*d^3*e^10 + 860*a^5*b^3*c^3*d^2*e^11 - 896*a^5*b*c^5*d^4*e^9 + 64*a^5*b^4*c^2*d*e^12 - 1392*a^6*b*c^4*d^2*e^11 - 336*a^6*b^2*c^3*d*e^12))/(2*a^4))*((((4*b^6*d^3 - 4*a^3*b^3*e^3 - 32*a^3*c^3*d^3 + 12*a^2*b^4*d*e^2 + 96*a^4*c^2*d*e^2 + 72*a^2*b^2*c^2*d^3 - 32*a*b^4*c*d^3 + 16*a^4*b*c*e^3 - 12*a*b^5*d^2*e + 84*a^2*b^3*c*d^2*e - 144*a^3*b*c^2*d^2*e - 72*a^3*b^2*c*d*e^2)^2/4 - (16*a^4*b^4 + 256*a^6*c^2 - 128*a^5*b^2*c)*(c^4*d^6 + a^3*c*e^6 + 3*a*c^3*d^4*e^2 - b^3*c*d^3*e^3 + 3*a^2*c^2*d^2*e^4 + 3*b^2*c^2*d^4*e^2 - 3*b*c^3*d^5*e - 3*a^2*b*c*d*e^5 - 6*a*b*c^2*d^3*e^3 + 3*a*b^2*c*d^2*e^4))^(1/2) + 2*b^6*d^3 - 2*a^3*b^3*e^3 - 16*a^3*c^3*d^3 + 6*a^2*b^4*d*e^2 + 48*a^4*c^2*d*e^2 + 36*a^2*b^2*c^2*d^3 - 16*a*b^4*c*d^3 + 8*a^4*b*c*e^3 - 6*a*b^5*d^2*e + 42*a^2*b^3*c*d^2*e - 72*a^3*b*c^2*d^2*e - 36*a^3*b^2*c*d*e^2)/(16*(a^4*b^4 + 16*a^6*c^2 - 8*a^5*b^2*c)))^(1/2))*((((4*b^6*d^3 - 4*a^3*b^3*e^3 - 32*a^3*c^3*d^3 + 12*a^2*b^4*d*e^2 + 96*a^4*c^2*d*e^2 + 72*a^2*b^2*c^2*d^3 - 32*a*b^4*c*d^3 + 16*a^4*b*c*e^3 - 12*a*b^5*d^2*e + 84*a^2*b^3*c*d^2*e - 144*a^3*b*c^2*d^2*e - 72*a^3*b^2*c*d*e^2)^2/4 - (16*a^4*b^4 + 256*a^6*c^2 - 128*a^5*b^2*c)*(c^4*d^6 + a^3*c*e^6 + 3*a*c^3*d^4*e^2 - b^3*c*d^3*e^3 + 3*a^2*c^2*d^2*e^4 + 3*b^2*c^2*d^4*e^2 - 3*b*c^3*d^5*e - 3*a^2*b*c*d*e^5 - 6*a*b*c^2*d^3*e^3 + 3*a*b^2*c*d^2*e^4))^(1/2) + 2*b^6*d^3 - 2*a^3*b^3*e^3 - 16*a^3*c^3*d^3 + 6*a^2*b^4*d*e^2 + 48*a^4*c^2*d*e^2 + 36*a^2*b^2*c^2*d^3 - 16*a*b^4*c*d^3 + 8*a^4*b*c*e^3 - 6*a*b^5*d^2*e + 42*a^2*b^3*c*d^2*e - 72*a^3*b*c^2*d^2*e - 36*a^3*b^2*c*d*e^2)/(16*(a^4*b^4 + 16*a^6*c^2 - 8*a^5*b^2*c)))^(1/2) + ((d + e*x^2)^(1/2)*(4*a^6*c^3*e^16 + 4*a^2*c^7*d^8*e^8 - 2*a^3*c^6*d^6*e^10 + 132*a^4*c^5*d^4*e^12 - 2*a^5*c^4*d^2*e^14 + 4*b^4*c^5*d^8*e^8 + 129*a^2*b^2*c^5*d^6*e^10 - 32*a^2*b^3*c^4*d^5*e^11 + 8*a^2*b^4*c^3*d^4*e^12 + 88*a^3*b^2*c^4*d^4*e^12 - 28*a

^3*b^3*c^3*d^3*e^13 + 33*a^4*b^2*c^3*d^2*e^14 - 16*a^5*b*c^3*d*e^15 - 8*a*b
^2*c^6*d^8*e^8 - 28*a*b^3*c^5*d^7*e^9 + 8*a^2*b*c^6*d^7*e^9 - 228*a^3*b*c^5
*d^5*e^11 - 60*a^4*b*c^4*d^3*e^13))/(2*a^4))*((((4*b^6*d^3 - 4*a^3*b^3*e^3
- 32*a^3*c^3*d^3 + 12*a^2*b^4*d*e^2 + 96*a^4*c^2*d*e^2 + 72*a^2*b^2*c^2*d^3
- 32*a*b^4*c*d^3 + 16*a^4*b*c*e^3 - 12*a*b^5*d^2*e + 84*a^2*b^3*c*d^2*e -
144*a^3*b*c^2*d^2*e - 72*a^3*b^2*c*d*e^2)^2/4 - (16*a^4*b^4 + 256*a^6*c^2 -
128*a^5*b^2*c)*(c^4*d^6 + a^3*c*e^6 + 3*a*c^3*d^4*e^2 - b^3*c*d^3*e^3 + 3*
a^2*c^2*d^2*e^4 + 3*b^2*c^2*d^4*e^2 - 3*b*c^3*d^5*e - 3*a^2*b*c*d*e^5 - 6*a
*b*c^2*d^3*e^3 + 3*a*b^2*c*d^2*e^4))^(1/2) + 2*b^6*d^3 - 2*a^3*b^3*e^3 - 16
*a^3*c^3*d^3 + 6*a^2*b^4*d*e^2 + 48*a^4*c^2*d*e^2 + 36*a^2*b^2*c^2*d^3 - 16
*a*b^4*c*d^3 + 8*a^4*b*c*e^3 - 6*a*b^5*d^2*e + 42*a^2*b^3*c*d^2*e - 72*a^3*
b*c^2*d^2*e - 36*a^3*b^2*c*d*e^2)/(16*(a^4*b^4 + 16*a^6*c^2 - 8*a^5*b^2*c))
)^(1/2)*1i)/((6*a*c^7*d^9*e^9 + 6*a^5*c^3*d*e^17 - 4*b*c^7*d^10*e^8 + 6*a^2
*c^6*d^7*e^11 + 6*a^4*c^4*d^3*e^15 + 8*b^2*c^6*d^9*e^9 - 4*b^3*c^5*d^8*e^10
+ 4*a^2*b^2*c^4*d^5*e^13 - 11*a^2*b^3*c^3*d^4*e^14 + 22*a^3*b^2*c^3*d^3*e^
15 - 16*a*b*c^6*d^8*e^10 + 8*a*b^2*c^5*d^7*e^11 + 2*a*b^4*c^3*d^5*e^13 - 3*
a^2*b*c^5*d^6*e^12 - 10*a^3*b*c^4*d^4*e^14 - 19*a^4*b*c^3*d^2*e^16)/(2*a^4)
+ (((224*a^4*c^6*d^6*e^9 - 176*a^5*c^5*d^4*e^11 - 400*a^6*c^4*d^2*e^13 + 1
60*a^2*b^3*c^5*d^7*e^8 - 156*a^2*b^5*c^3*d^5*e^10 - 44*a^2*b^6*c^2*d^4*e^11
- 432*a^3*b^2*c^5*d^6*e^9 + 384*a^3*b^3*c^4*d^5*e^10 + 444*a^3*b^4*c^3*d^4
*e^11 + 88*a^3*b^5*c^2*d^3*e^12 - 948*a^4*b^2*c^4*d^4*e^11 - 644*a^4*b^3*c^
3*d^3*e^12 - 76*a^4*b^4*c^2*d^2*e^13 + 444*a^5*b^2*c^3*d^2*e^13 - 112*a^6*b
*c^3*d*e^14 - 32*a*b^5*c^4*d^7*e^8 + 24*a*b^6*c^3*d^6*e^9 + 8*a*b^7*c^2*d^5
*e^10 - 128*a^3*b*c^6*d^7*e^8 + 368*a^4*b*c^5*d^5*e^10 + 1008*a^5*b*c^4*d^3
*e^12 + 24*a^5*b^3*c^2*d*e^14)/(4*a^4) + (((1280*a^8*c^4*d*e^11 + 1280*a^7*
c^5*d^3*e^9 + 128*a^5*b^3*c^4*d^4*e^8 - 96*a^5*b^4*c^3*d^3*e^9 - 32*a^5*b^5
*c^2*d^2*e^10 + 64*a^6*b^2*c^4*d^3*e^9 + 576*a^6*b^3*c^3*d^2*e^10 - 512*a^6
*b*c^5*d^4*e^8 + 32*a^6*b^4*c^2*d*e^11 - 1792*a^7*b*c^4*d^2*e^10 - 448*a^7*
b^2*c^3*d*e^11)/(4*a^4) - ((d + e*x^2)^(1/2)*((((4*b^6*d^3 - 4*a^3*b^3*e^3
- 32*a^3*c^3*d^3 + 12*a^2*b^4*d*e^2 + 96*a^4*c^2*d*e^2 + 72*a^2*b^2*c^2*d^3
- 32*a*b^4*c*d^3 + 16*a^4*b*c*e^3 - 12*a*b^5*d^2*e + 84*a^2*b^3*c*d^2*e -
144*a^3*b*c^2*d^2*e - 72*a^3*b^2*c*d*e^2)^2/4 - (16*a^4*b^4 + 256*a^6*c^2 -
128*a^5*b^2*c)*(c^4*d^6 + a^3*c*e^6 + 3*a*c^3*d^4*e^2 - b^3*c*d^3*e^3 + 3*
a^2*c^2*d^2*e^4 + 3*b^2*c^2*d^4*e^2 - 3*b*c^3*d^5*e - 3*a^2*b*c*d*e^5 - 6*a
*b*c^2*d^3*e^3 + 3*a*b^2*c*d^2*e^4))^(1/2) + 2*b^6*d^3 - 2*a^3*b^3*e^3 - 16
*a^3*c^3*d^3 + 6*a^2*b^4*d*e^2 + 48*a^4*c^2*d*e^2 + 36*a^2*b^2*c^2*d^3 - 16
*a*b^4*c*d^3 + 8*a^4*b*c*e^3 - 6*a*b^5*d^2*e + 42*a^2*b^3*c*d^2*e - 72*a^3*
b*c^2*d^2*e - 36*a^3*b^2*c*d*e^2)/(16*(a^4*b^4 + 16*a^6*c^2 - 8*a^5*b^2*c))
)^(1/2)*(1024*a^9*c^4*e^10 + 64*a^7*b^4*c^2*e^10 - 512*a^8*b^2*c^3*e^10 + 1
536*a^8*c^5*d^2*e^8 + 128*a^6*b^4*c^3*d^2*e^8 - 896*a^7*b^2*c^4*d^2*e^8 - 1
792*a^8*b*c^4*d*e^9 - 128*a^6*b^5*c^2*d*e^9 + 960*a^7*b^3*c^3*d*e^9))/(2*a^
4))*((((4*b^6*d^3 - 4*a^3*b^3*e^3 - 32*a^3*c^3*d^3 + 12*a^2*b^4*d*e^2 + 96*
a^4*c^2*d*e^2 + 72*a^2*b^2*c^2*d^3 - 32*a*b^4*c*d^3 + 16*a^4*b*c*e^3 - 12*a
*b^5*d^2*e + 84*a^2*b^3*c*d^2*e - 144*a^3*b*c^2*d^2*e - 72*a^3*b^2*c*d*e^2)
^2/4 - (16*a^4*b^4 + 256*a^6*c^2 - 128*a^5*b^2*c)*(c^4*d^6 + a^3*c*e^6 + 3*

a*c^3*d^4*e^2 - b^3*c*d^3*e^3 + 3*a^2*c^2*d^2*e^4 + 3*b^2*c^2*d^4*e^2 - 3*b*c^3*d^5*e - 3*a^2*b*c*d*e^5 - 6*a*b*c^2*d^3*e^3 + 3*a*b^2*c*d^2*e^4))^(1/2) + 2*b^6*d^3 - 2*a^3*b^3*e^3 - 16*a^3*c^3*d^3 + 6*a^2*b^4*d*e^2 + 48*a^4*c^2*d*e^2 + 36*a^2*b^2*c^2*d^3 - 16*a*b^4*c*d^3 + 8*a^4*b*c*e^3 - 6*a*b^5*d^2*e + 42*a^2*b^3*c*d^2*e - 72*a^3*b*c^2*d^2*e - 36*a^3*b^2*c*d*e^2)/(16*(a^4*b^4 + 16*a^6*c^2 - 8*a^5*b^2*c)))^(1/2) + ((d + e*x^2)^(1/2)*(64*a^7*b*c^3*e^13 + 352*a^7*c^4*d*e^12 - 16*a^6*b^3*c^2*e^13 - 160*a^5*c^6*d^5*e^8 + 736*a^6*c^5*d^3*e^10 + 32*a^2*b^6*c^3*d^5*e^8 - 32*a^2*b^7*c^2*d^4*e^9 - 224*a^3*b^4*c^4*d^5*e^8 + 144*a^3*b^5*c^3*d^4*e^9 + 112*a^3*b^6*c^2*d^3*e^10 + 432*a^4*b^2*c^5*d^5*e^8 + 144*a^4*b^3*c^4*d^4*e^9 - 716*a^4*b^4*c^3*d^3*e^10 - 132*a^4*b^5*c^2*d^2*e^11 + 936*a^5*b^2*c^4*d^3*e^10 + 860*a^5*b^3*c^3*d^2*e^11 - 896*a^5*b*c^5*d^4*e^9 + 64*a^5*b^4*c^2*d*e^12 - 1392*a^6*b*c^4*d^2*e^11 - 336*a^6*b^2*c^3*d*e^12))/(2*a^4))*((((4*b^6*d^3 - 4*a^3*b^3*e^3 - 32*a^3*c^3*d^3 + 12*a^2*b^4*d*e^2 + 96*a^4*c^2*d*e^2 + 72*a^2*b^2*c^2*d^3 - 32*a*b^4*c*d^3 + 16*a^4*b*c*e^3 - 12*a*b^5*d^2*e + 84*a^2*b^3*c*d^2*e - 144*a^3*b*c^2*d^2*e - 72*a^3*b^2*c*d*e^2)^2/4 - (16*a^4*b^4 + 256*a^6*c^2 - 128*a^5*b^2*c)*(c^4*d^6 + a^3*c*e^6 + 3*a*c^3*d^4*e^2 - b^3*c*d^3*e^3 + 3*a^2*c^2*d^2*e^4 + 3*b^2*c^2*d^4*e^2 - 3*b*c^3*d^5*e - 3*a^2*b*c*d*e^5 - 6*a*b*c^2*d^3*e^3 + 3*a*b^2*c*d^2*e^4))^(1/2) + 2*b^6*d^3 - 2*a^3*b^3*e^3 - 16*a^3*c^3*d^3 + 6*a^2*b^4*d*e^2 + 48*a^4*c^2*d*e^2 + 36*a^2*b^2*c^2*d^3 - 16*a*b^4*c*d^3 + 8*a^4*b*c*e^3 - 6*a*b^5*d^2*e + 42*a^2*b^3*c*d^2*e - 72*a^3*b*c^2*d^2*e - 36*a^3*b^2*c*d*e^2)/(16*(a^4*b^4 + 16*a^6*c^2 - 8*a^5*b^2*c)))^(1/2))*((((4*b^6*d^3 - 4*a^3*b^3*e^3 - 32*a^3*c^3*d^3 + 12*a^2*b^4*d*e^2 + 96*a^4*c^2*d*e^2 + 72*a^2*b^2*c^2*d^3 - 32*a*b^4*c*d^3 + 16*a^4*b*c*e^3 - 12*a*b^5*d^2*e + 84*a^2*b^3*c*d^2*e - 144*a^3*b*c^2*d^2*e - 72*a^3*b^2*c*d*e^2)^2/4 - (16*a^4*b^4 + 256*a^6*c^2 - 128*a^5*b^2*c)*(c^4*d^6 + a^3*c*e^6 + 3*a*c^3*d^4*e^2 - b^3*c*d^3*e^3 + 3*a^2*c^2*d^2*e^4 + 3*b^2*c^2*d^4*e^2 - 3*b*c^3*d^5*e - 3*a^2*b*c*d*e^5 - 6*a*b*c^2*d^3*e^3 + 3*a*b^2*c*d^2*e^4))^(1/2) + 2*b^6*d^3 - 2*a^3*b^3*e^3 - 16*a^3*c^3*d^3 + 6*a^2*b^4*d*e^2 + 48*a^4*c^2*d*e^2 + 36*a^2*b^2*c^2*d^3 - 16*a*b^4*c*d^3 + 8*a^4*b*c*e^3 - 6*a*b^5*d^2*e + 42*a^2*b^3*c*d^2*e - 72*a^3*b*c^2*d^2*e - 36*a^3*b^2*c*d*e^2)/(16*(a^4*b^4 + 16*a^6*c^2 - 8*a^5*b^2*c)))^(1/2) - ((d + e*x^2)^(1/2)*(4*a^6*c^3*e^16 + 4*a^2*c^7*d^8*e^8 - 2*a^3*c^6*d^6*e^10 + 132*a^4*c^5*d^4*e^12 - 2*a^5*c^4*d^2*e^14 + 4*b^4*c^5*d^8*e^8 + 129*a^2*b^2*c^5*d^6*e^10 - 32*a^2*b^3*c^4*d^5*e^11 + 8*a^2*b^4*c^3*d^4*e^12 + 88*a^3*b^2*c^4*d^4*e^12 - 28*a^3*b^3*c^3*d^3*e^13 + 33*a^4*b^2*c^3*d^2*e^14 - 16*a^5*b*c^3*d*e^15 - 8*a*b^2*c^6*d^8*e^8 - 28*a*b^3*c^5*d^7*e^9 + 8*a^2*b*c^6*d^7*e^9 - 228*a^3*b*c^5*d^5*e^11 - 60*a^4*b*c^4*d^3*e^13))/(2*a^4))*((((4*b^6*d^3 - 4*a^3*b^3*e^3 - 32*a^3*c^3*d^3 + 12*a^2*b^4*d*e^2 + 96*a^4*c^2*d*e^2 + 72*a^2*b^2*c^2*d^3 - 32*a*b^4*c*d^3 + 16*a^4*b*c*e^3 - 12*a*b^5*d^2*e + 84*a^2*b^3*c*d^2*e - 144*a^3*b*c^2*d^2*e - 72*a^3*b^2*c*d*e^2)^2/4 - (16*a^4*b^4 + 256*a^6*c^2 - 128*a^5*b^2*c)*(c^4*d^6 + a^3*c*e^6 + 3*a*c^3*d^4*e^2 - b^3*c*d^3*e^3 + 3*a^2*c^2*d^2*e^4 + 3*b^2*c^2*d^4*e^2 - 3*b*c^3*d^5*e - 3*a^2*b*c*d*e^5 - 6*a*b*c^2*d^3*e^3 + 3*a*b^2*c*d^2*e^4))^(1/2) + 2*b^6*d^3 - 2*a^3*b^3*e^3 - 16*a^3*c^3*d^3 + 6*a^2*b^4*d*e^2 + 48*a^4*c^2*d*e^2 + 36*a^2*b^2*c^2*d^3 - 16*a *b^4*c*d^3 + 8*a^4*b*c*e^3 - 6*a*b^5*d^2*e + 42*a^2*b^3*c*d^2*e - 72*a^3*b*c^2*d^2*e - 36*a^3*b^2*c*d*e^2)/(16*(a^4*b^4 + 16*a^6*c^2 - 8*a^5*b^2*c)))^(1/2) + (((224*a^4*c^6*d^6*e^9 - 176*a^5*c^5*d^4*e^11 - 400*a^6*c^4*d^2*e^13 + 160*a^2*b^3*c^5*d^7*e^8 - 156*a^2*b^5*c^3*d^5*e^10 - 44*a^2*b^6*c^2*d^4*e^11 - 432*a^3*b^2*c^5*d^6*e^9 + 384*a^3*b^3*c^4*d^5*e^10 + 444*a^3*b^4*c^3*d^4*e^11 + 88*a^3*b^5*c^2*d^3*e^12 - 948*a^4*b^2*c^4*d^4*e^11 - 644*a^4*b^3*c^3*d^3*e^12 - 76*a^4*b^4*c^2*d^2*e^13 + 444*a^5*b^2*c^3*d^2*e^13 - 112*a^6*b*c^3*d*e^14 - 32*a*b^5*c^4*d^7*e^8 + 24*a*b^6*c^3*d^6*e^9 + 8*a*b^7*c^2*d^5*e^10 - 128*a^3*b*c^6*d^7*e^8 + 368*a^4*b*c^5*d^5*e^10 + 1008*a^5*b*c^4*d^3*e^12 + 24*a^5*b^3*c^2*d*e^14)/(4*a^4) + (((1280*a^8*c^4*d*e^11 + 1280*a^7*c^5*d^3*e^9 + 128*a^5*b^3*c^4*d^4*e^8 - 96*a^5*b^4*c^3*d^3*e^9 - 32*a^5*b^5*c^2*d^2*e^10 + 64*a^6*b^2*c^4*d^3*e^9 + 576*a^6*b^3*c^3*d^2*e^10 - 512*a^6*b*c^5*d^4*e^8 + 32*a^6*b^4*c^2*d*e^11 - 1792*a^7*b*c^4*d^2*e^10 - 448*a^7*b^2*c^3*d*e^11)/(4*a^4) + ((d + e*x^2)^(1/2)*((((4*b^6*d^3 - 4*a^3*b^3*e^3 - 32*a^3*c^3*d^3 + 12*a^2*b^4*d*e^2 + 96*a^4*c^2*d*e^2 + 72*a^2*b^2*c^2*d^3 - 32*a*b^4*c*d^3 + 16*a^4*b*c*e^3 - 12*a*b^5*d^2*e + 84*a^2*b^3*c*d^2*e - 144*a^3*b*c^2*d^2*e - 72*a^3*b^2*c*d*e^2)^2/4 - (16*a^4*b^4 + 256*a^6*c^2 - 128*a^5*b^2*c)*(c^4*d^6 + a^3*c*e^6 + 3*a*c^3*d^4*e^2 - b^3*c*d^3*e^3 + 3*a^2*c^2*d^2*e^4 + 3*b^2*c^2*d^4*e^2 - 3*b*c^3*d^5*e - 3*a^2*b*c*d*e^5 - 6*a*b*c^2*d^3*e^3 + 3*a*b^2*c*d^2*e^4))^(1/2) + 2*b^6*d^3 - 2*a^3*b^3*e^3 - 16*a^3*c^3*d^3 + 6*a^2*b^4*d*e^2 + 48*a^4*c^2*d*e^2 + 36*a^2*b^2*c^2*d^3 - 16*a*b^4*c*d^3 + 8*a^4*b*c*e^3 - 6*a*b^5*d^2*e + 42*a^2*b^3*c*d^2*e - 72*a^3*b*c^2*d^2*e - 36*a^3*b^2*c*d*e^2)/(16*(a^4*b^4 + 16*a^6*c^2 - 8*a^5*b^2*c)))^(1/2)*(1024*a^9*c^4*e^10 + 64*a^7*b^4*c^2*e^10 - 512*a^8*b^2*c^3*e^10 + 1536*a^8*c^5*d^2*e^8 + 128*a^6*b^4*c^3*d^2*e^8 - 896*a^7*b^2*c^4*d^2*e^8 - 1792*a^8*b*c^4*d*e^9 - 128*a^6*b^5*c^2*d*e^9 + 960*a^7*b^3*c^3*d*e^9))/(2*a^4))*((((4*b^6*d^3 - 4*a^3*b^3*e^3 - 32*a^3*c^3*d^3 + 12*a^2*b^4*d*e^2 + 96*a^4*c^2*d*e^2 + 72*a^2*b^2*c^2*d^3 - 32*a*b^4*c*d^3 + 16*a^4*b*c*e^3 - 12*a*b^5*d^2*e + 84*a^2*b^3*c*d^2*e - 144*a^3*b*c^2*d^2*e - 72*a^3*b^2*c*d*e^2)^2/4 - (16*a^4*b^4 + 256*a^6*c^2 - 128*a^5*b^2*c)*(c^4*d^6 + a^3*c*e^6 + 3*a*c^3*d^4*e^2 - b^3*c*d^3*e^3 + 3*a^2*c^2*d^2*e^4 + 3*b^2*c^2*d^4*e^2 - 3*b*c^3*d^5*e - 3*a^2*b*c*d*e^5 - 6*a*b*c^2*d^3*e^3 + 3*a*b^2*c*d^2*e^4))^(1/2) + 2*b^6*d^3 - 2*a^3*b^3*e^3 - 16*a^3*c^3*d^3 + 6*a^2*b^4*d*e^2 + 48*a^4*c^2*d*e^2 + 36*a^2*b^2*c^2*d^3 - 16*a*b^4*c*d^3 + 8*a^4*b*c*e^3 - 6*a*b^5*d^2*e + 42*a^2*b^3*c*d^2*e - 72*a^3*b*c^2*d^2*e - 36*a^3*b^2*c*d*e^2)/(16*(a^4*b^4 + 16*a^6*c^2 - 8*a^5*b^2*c)))^(1/2) - ((d + e*x^2)^(1/2)*(64*a^7*b*c^3*e^13 + 352*a^7*c^4*d*e^12 - 16*a^6*b^3*c^2*e^13 - 160*a^5*c^6*d^5*e^8 + 736*a^6*c^5*d^3*e^10 + 32*a^2*b^6*c^3*d^5*e^8 - 32*a^2*b^7*c^2*d^4*e^9 - 224*a^3*b^4*c^4*d^5*e^8 + 144*a^3*b^5*c^3*d^4*e^9 + 112*a^3*b^6*c^2*d^3*e^10 + 432*a^4*b^2*c^5*d^5*e^8 + 144*a^4*b^3*c^4*d^4*e^9 - 716*a^4*b^4*c^3*d^3*e^10 - 132*a^4*b^5*c^2*d^2*e^11 + 936*a^5*b^2*c^4*d^3*e^10 + 860*a^5*b^3*c^3*d^2*e^11 - 896*a^5*b*c^5*d^4*e^9 + 64*a^5*b^4*c^2*d*e^12 - 1392*a^6*b*c^4*d^2*e^11 - 336*a^6*b^2*c^3*d*e^12))/(2*a^4))*((((4*b^6*d^3 - 4*a^3*b^3*e^3 - 32*a^3*c^3*d^3 + 12*a^2*b^4*d*e^2 + 96*a^4*c^2*d*e^2 + 72*a^2*b^2*c^2*d^3 - 32*a*b^4*c*d^3 + 16*a^4*b*c*e^3 - 12*a*b^5*d^2*e + 84*a^2*b^3*c*d^2* e - 144*a^3*b*c^2*d^2*e - 72*a^3*b^2*c*d*e^2)^2/4 - (16*a^4*b^4 + 256*a^6*c^2 - 128*a^5*b^2*c)*(c^4*d^6 + a^3*c*e^6 + 3*a*c^3*d^4*e^2 - b^3*c*d^3*e^3 + 3*a^2*c^2*d^2*e^4 + 3*b^2*c^2*d^4*e^2 - 3*b*c^3*d^5*e - 3*a^2*b*c*d*e^5 - 6*a*b*c^2*d^3*e^3 + 3*a*b^2*c*d^2*e^4))^(1/2) + 2*b^6*d^3 - 2*a^3*b^3*e^3 - 16*a^3*c^3*d^3 + 6*a^2*b^4*d*e^2 + 48*a^4*c^2*d*e^2 + 36*a^2*b^2*c^2*d^3 - 16*a*b^4*c*d^3 + 8*a^4*b*c*e^3 - 6*a*b^5*d^2*e + 42*a^2*b^3*c*d^2*e - 72*a^3*b*c^2*d^2*e - 36*a^3*b^2*c*d*e^2)/(16*(a^4*b^4 + 16*a^6*c^2 - 8*a^5*b^2*c)))^(1/2))*((((4*b^6*d^3 - 4*a^3*b^3*e^3 - 32*a^3*c^3*d^3 + 12*a^2*b^4*d*e^2 + 96*a^4*c^2*d*e^2 + 72*a^2*b^2*c^2*d^3 - 32*a*b^4*c*d^3 + 16*a^4*b*c*e^3 - 12*a*b^5*d^2*e + 84*a^2*b^3*c*d^2*e - 144*a^3*b*c^2*d^2*e - 72*a^3*b^2*c*d*e^2)^2/4 - (16*a^4*b^4 + 256*a^6*c^2 - 128*a^5*b^2*c)*(c^4*d^6 + a^3*c*e^6 + 3*a*c^3*d^4*e^2 - b^3*c*d^3*e^3 + 3*a^2*c^2*d^2*e^4 + 3*b^2*c^2*d^4*e^2 - 3*b*c^3*d^5*e - 3*a^2*b*c*d*e^5 - 6*a*b*c^2*d^3*e^3 + 3*a*b^2*c*d^2*e^4))^(1/2) + 2*b^6*d^3 - 2*a^3*b^3*e^3 - 16*a^3*c^3*d^3 + 6*a^2*b^4*d*e^2 + 48*a^4*c^2*d*e^2 + 36*a^2*b^2*c^2*d^3 - 16*a*b^4*c*d^3 + 8*a^4*b*c*e^3 - 6*a*b^5*d^2*e + 42*a^2*b^3*c*d^2*e - 72*a^3*b*c^2*d^2*e - 36*a^3*b^2*c*d*e^2)/(16*(a^4*b^4 + 16*a^6*c^2 - 8*a^5*b^2*c)))^(1/2) + ((d + e*x^2)^(1/2)*(4*a^6*c^3*e^16 + 4*a^2*c^7*d^8*e^8 - 2*a^3*c^6*d^6*e^10 + 132*a^4*c^5*d^4*e^12 - 2*a^5*c^4*d^2*e^14 + 4*b^4*c^5*d^8*e^8 + 129*a^2*b^2*c^5*d^6*e^10 - 32*a^2*b^3*c^4*d^5*e^11 + 8*a^2*b^4*c^3*d^4*e^12 + 88*a^3*b^2*c^4*d^4*e^12 - 28*a^3*b^3*c^3*d^3*e^13 + 33*a^4*b^2*c^3*d^2*e^14 - 16*a^5*b*c^3*d*e^15 - 8*a*b^2*c^6*d^8*e^8 - 28*a*b^3*c^5*d^7*e^9 + 8*a^2*b*c^6*d^7*e^9 - 228*a^3*b*c^5*d^5*e^11 - 60*a^4*b*c^4*d^3*e^13))/(2*a^4))*((((4*b^6*d^3 - 4*a^3*b^3*e^3 - 32*a^3*c^3*d^3 + 12*a^2*b^4*d*e^2 + 96*a^4*c^2*d*e^2 + 72*a^2*b^2*c^2*d^3 - 32*a*b^4*c*d^3 + 16*a^4*b*c*e^3 - 12*a*b^5*d^2*e + 84*a^2*b^3*c*d^2*e - 144*a^3*b*c^2*d^2*e - 72*a^3*b^2*c*d*e^2)^2/4 - (16*a^4*b^4 + 256*a^6*c^2 - 128*a^5*b^2*c)*(c^4*d^6 + a^3*c*e^6 + 3*a*c^3*d^4*e^2 - b^3*c*d^3*e^3 + 3*a^2*c^2*d^2*e^4 + 3*b^2*c^2*d^4*e^2 - 3*b*c^3*d^5*e - 3*a^2*b*c*d*e^5 - 6*a*b*c^2*d^3*e^3 + 3*a*b^2*c*d^2*e^4))^(1/2) + 2*b^6*d^3 - 2*a^3*b^3*e^3 - 16*a^3*c^3*d^3 + 6*a^2*b^4*d*e^2 + 48*a^4*c^2*d*e^2 + 36*a^2*b^2*c^2*d^3 - 16*a*b^4*c*d^3 + 8*a^4*b*c*e^3 - 6*a*b^5*d^2*e + 42*a^2*b^3*c*d^2*e - 72*a^3*b*c^2*d^2*e - 36*a^3*b^2*c*d*e^2)/(16*(a^4*b^4 + 16*a^6*c^2 - 8*a^5*b^2*c)))^(1/2)))*((((4*b^6*d^3 - 4*a^3*b^3*e^3 - 32*a^3*c^3*d^3 + 12*a^2*b^4*d*e^2 + 96*a^4*c^2*d*e^2 + 72*a^2*b^2*c^2*d^3 - 32*a*b^4*c*d^3 + 16*a^4*b*c*e^3 - 12*a*b^5*d^2*e + 84*a^2*b^3*c*d^2*e - 144*a^3*b*c^2*d^2*e - 72*a^3*b^2*c*d*e^2)^2/4 - (16*a^4*b^4 + 256*a^6*c^2 - 128*a^5*b^2*c)*(c^4*d^6 + a^3*c*e^6 + 3*a*c^3*d^4*e^2 - b^3*c*d^3*e^3 + 3*a^2*c^2*d^2*e^4 + 3*b^2*c^2*d^4*e^2 - 3*b*c^3*d^5*e - 3*a^2*b*c*d*e^5 - 6*a*b*c^2*d^3*e^3 + 3*a*b^2*c*d^2*e^4))^(1/2) + 2*b^6*d^3 - 2*a^3*b^3*e^3 - 16*a^3*c^3*d^3 + 6*a^2*b^4*d*e^2 + 48*a^4*c^2*d*e^2 + 36*a^2*b^2*c^2*d^3 - 16*a*b^4*c*d^3 + 8*a^4*b*c*e^3 - 6*a*b^5*d^2*e + 42*a^2*b^3*c*d^2*e - 72*a^3*b*c^2*d^2*e - 36*a^3*b^2*c*d*e^2)/(16*(a^4*b^4 + 16*a^6*c^2 - 8*a^5*b^2*c)))^(1/2)*2i - (d*(d + e*x^2)^(1/2))/(2*a*x^2) - atan(((((224*a^4*c^6*d^6*e^9 - 176*a^5*c^5*d^4*e^11 - 400*a^6*c^4*d^2*e^13 + 160*a^2*b^3*c^5*d^7*e^8 - 156*a^2*b^5*c^3*d^5*e^10 - 44*a^2*b^6*c^2*d^4*e^11 - 432*a^3*b^2*c^5*d^6*e^9 + 384*a^3*b^3*c^4*d^5*e^10 + 4

44*a^3*b^4*c^3*d^4*e^11 + 88*a^3*b^5*c^2*d^3*e^12 - 948*a^4*b^2*c^4*d^4*e^1
1 - 644*a^4*b^3*c^3*d^3*e^12 - 76*a^4*b^4*c^2*d^2*e^13 + 444*a^5*b^2*c^3*d^
2*e^13 - 112*a^6*b*c^3*d*e^14 - 32*a*b^5*c^4*d^7*e^8 + 24*a*b^6*c^3*d^6*e^9
 + 8*a*b^7*c^2*d^5*e^10 - 128*a^3*b*c^6*d^7*e^8 + 368*a^4*b*c^5*d^5*e^10 +
1008*a^5*b*c^4*d^3*e^12 + 24*a^5*b^3*c^2*d*e^14)/(4*a^4) + (((1280*a^8*c^4*
d*e^11 + 1280*a^7*c^5*d^3*e^9 + 128*a^5*b^3*c^4*d^4*e^8 - 96*a^5*b^4*c^3*d^
3*e^9 - 32*a^5*b^5*c^2*d^2*e^10 + 64*a^6*b^2*c^4*d^3*e^9 + 576*a^6*b^3*c^3*
d^2*e^10 - 512*a^6*b*c^5*d^4*e^8 + 32*a^6*b^4*c^2*d*e^11 - 1792*a^7*b*c^4*d
^2*e^10 - 448*a^7*b^2*c^3*d*e^11)/(4*a^4) - ((d + e*x^2)^(1/2)*(-(((4*b^6*d
^3 - 4*a^3*b^3*e^3 - 32*a^3*c^3*d^3 + 12*a^2*b^4*d*e^2 + 96*a^4*c^2*d*e^2 +
 72*a^2*b^2*c^2*d^3 - 32*a*b^4*c*d^3 + 16*a^4*b*c*e^3 - 12*a*b^5*d^2*e + 84
*a^2*b^3*c*d^2*e - 144*a^3*b*c^2*d^2*e - 72*a^3*b^2*c*d*e^2)^2/4 - (16*a^4*
b^4 + 256*a^6*c^2 - 128*a^5*b^2*c)*(c^4*d^6 + a^3*c*e^6 + 3*a*c^3*d^4*e^2 -
 b^3*c*d^3*e^3 + 3*a^2*c^2*d^2*e^4 + 3*b^2*c^2*d^4*e^2 - 3*b*c^3*d^5*e - 3*
a^2*b*c*d*e^5 - 6*a*b*c^2*d^3*e^3 + 3*a*b^2*c*d^2*e^4))^(1/2) - 2*b^6*d^3 +
 2*a^3*b^3*e^3 + 16*a^3*c^3*d^3 - 6*a^2*b^4*d*e^2 - 48*a^4*c^2*d*e^2 - 36*a
^2*b^2*c^2*d^3 + 16*a*b^4*c*d^3 - 8*a^4*b*c*e^3 + 6*a*b^5*d^2*e - 42*a^2*b^
3*c*d^2*e + 72*a^3*b*c^2*d^2*e + 36*a^3*b^2*c*d*e^2)/(16*(a^4*b^4 + 16*a^6*
c^2 - 8*a^5*b^2*c)))^(1/2)*(1024*a^9*c^4*e^10 + 64*a^7*b^4*c^2*e^10 - 512*a
^8*b^2*c^3*e^10 + 1536*a^8*c^5*d^2*e^8 + 128*a^6*b^4*c^3*d^2*e^8 - 896*a^7*
b^2*c^4*d^2*e^8 - 1792*a^8*b*c^4*d*e^9 - 128*a^6*b^5*c^2*d*e^9 + 960*a^7*b^
3*c^3*d*e^9))/(2*a^4))*(-(((4*b^6*d^3 - 4*a^3*b^3*e^3 - 32*a^3*c^3*d^3 + 12
*a^2*b^4*d*e^2 + 96*a^4*c^2*d*e^2 + 72*a^2*b^2*c^2*d^3 - 32*a*b^4*c*d^3 + 1
6*a^4*b*c*e^3 - 12*a*b^5*d^2*e + 84*a^2*b^3*c*d^2*e - 144*a^3*b*c^2*d^2*e -
 72*a^3*b^2*c*d*e^2)^2/4 - (16*a^4*b^4 + 256*a^6*c^2 - 128*a^5*b^2*c)*(c^4*
d^6 + a^3*c*e^6 + 3*a*c^3*d^4*e^2 - b^3*c*d^3*e^3 + 3*a^2*c^2*d^2*e^4 + 3*b
^2*c^2*d^4*e^2 - 3*b*c^3*d^5*e - 3*a^2*b*c*d*e^5 - 6*a*b*c^2*d^3*e^3 + 3*a*
b^2*c*d^2*e^4))^(1/2) - 2*b^6*d^3 + 2*a^3*b^3*e^3 + 16*a^3*c^3*d^3 - 6*a^2*
b^4*d*e^2 - 48*a^4*c^2*d*e^2 - 36*a^2*b^2*c^2*d^3 + 16*a*b^4*c*d^3 - 8*a^4*
b*c*e^3 + 6*a*b^5*d^2*e - 42*a^2*b^3*c*d^2*e + 72*a^3*b*c^2*d^2*e + 36*a^3*
b^2*c*d*e^2)/(16*(a^4*b^4 + 16*a^6*c^2 - 8*a^5*b^2*c)))^(1/2) + ((d + e*x^2
)^(1/2)*(64*a^7*b*c^3*e^13 + 352*a^7*c^4*d*e^12 - 16*a^6*b^3*c^2*e^13 - 160
*a^5*c^6*d^5*e^8 + 736*a^6*c^5*d^3*e^10 + 32*a^2*b^6*c^3*d^5*e^8 - 32*a^2*b
^7*c^2*d^4*e^9 - 224*a^3*b^4*c^4*d^5*e^8 + 144*a^3*b^5*c^3*d^4*e^9 + 112*a^
3*b^6*c^2*d^3*e^10 + 432*a^4*b^2*c^5*d^5*e^8 + 144*a^4*b^3*c^4*d^4*e^9 - 71
6*a^4*b^4*c^3*d^3*e^10 - 132*a^4*b^5*c^2*d^2*e^11 + 936*a^5*b^2*c^4*d^3*e^1
0 + 860*a^5*b^3*c^3*d^2*e^11 - 896*a^5*b*c^5*d^4*e^9 + 64*a^5*b^4*c^2*d*e^1
2 - 1392*a^6*b*c^4*d^2*e^11 - 336*a^6*b^2*c^3*d*e^12))/(2*a^4))*(-(((4*b^6*
d^3 - 4*a^3*b^3*e^3 - 32*a^3*c^3*d^3 + 12*a^2*b^4*d*e^2 + 96*a^4*c^2*d*e^2
+ 72*a^2*b^2*c^2*d^3 - 32*a*b^4*c*d^3 + 16*a^4*b*c*e^3 - 12*a*b^5*d^2*e + 8
4*a^2*b^3*c*d^2*e - 144*a^3*b*c^2*d^2*e - 72*a^3*b^2*c*d*e^2)^2/4 - (16*a^4
*b^4 + 256*a^6*c^2 - 128*a^5*b^2*c)*(c^4*d^6 + a^3*c*e^6 + 3*a*c^3*d^4*e^2
- b^3*c*d^3*e^3 + 3*a^2*c^2*d^2*e^4 + 3*b^2*c^2*d^4*e^2 - 3*b*c^3*d^5*e - 3
*a^2*b*c*d*e^5 - 6*a*b*c^2*d^3*e^3 + 3*a*b^2*c*d^2*e^4))^(1/2) - 2*b^6*d^3
+ 2*a^3*b^3*e^3 + 16*a^3*c^3*d^3 - 6*a^2*b^4*d*e^2 - 48*a^4*c^2*d*e^2 - 36*

a^2*b^2*c^2*d^3 + 16*a*b^4*c*d^3 - 8*a^4*b*c*e^3 + 6*a*b^5*d^2*e - 42*a^2*b ^3*c*d^2*e + 72*a^3*b*c^2*d^2*e + 36*a^3*b^2*c*d*e^2)/(16*(a^4*b^4 + 16*a^6 *c^2 - 8*a^5*b^2*c)))^(1/2))*(-(((4*b^6*d^3 - 4*a^3*b^3*e^3 - 32*a^3*c^3*d^ 3 + 12*a^2*b^4*d*e^2 + 96*a^4*c^2*d*e^2 + 72*a^2*b^2*c^2*d^3 - 32*a*b^4*c*d ^3 + 16*a^4*b*c*e^3 - 12*a*b^5*d^2*e + 84*a^2*b^3*c*d^2*e - 144*a^3*b*c^2*d ^2*e - 72*a^3*b^2*c*d*e^2)^2/4 - (16*a^4*b^4 + 256*a^6*c^2 - 128*a^5*b^2*c) *(c^4*d^6 + a^3*c*e^6 + 3*a*c^3*d^4*e^2 - b^3*c*d^3*e^3 + 3*a^2*c^2*d^2*e^4 + 3*b^2*c^2*d^4*e^2 - 3*b*c^3*d^5*e - 3*a^2*b*c*d*e^5 - 6*a*b*c^2*d^3*e^3 + 3*a*b^2*c*d^2*e^4))^(1/2) - 2*b^6*d^3 + 2*a^3*b^3*e^3 + 16*a^3*c^3*d^3 - 6*a^2*b^4*d*e^2 - 48*a^4*c^2*d*e^2 - 36*a^2*b^2*c^2*d^3 + 16*a*b^4*c*d^3 - 8*a^4*b*c*e^3 + 6*a*b^5*d^2*e - 42*a^2*b^3*c*d^2*e + 72*a^3*b*c^2*d^2*e + 3 6*a^3*b^2*c*d*e^2)/(16*(a^4*b^4 + 16*a^6*c^2 - 8*a^5*b^2*c)))^(1/2) - ((d + e*x^2)^(1/2)*(4*a^6*c^3*e^16 + 4*a^2*c^7*d^8*e^8 - 2*a^3*c^6*d^6*e^10 + 13 2*a^4*c^5*d^4*e^12 - 2*a^5*c^4*d^2*e^14 + 4*b^4*c^5*d^8*e^8 + 129*a^2*b^2*c ^5*d^6*e^10 - 32*a^2*b^3*c^4*d^5*e^11 + 8*a^2*b^4*c^3*d^4*e^12 + 88*a^3*b^2 *c^4*d^4*e^12 - 28*a^3*b^3*c^3*d^3*e^13 + 33*a^4*b^2*c^3*d^2*e^14 - 16*a^5* b*c^3*d*e^15 - 8*a*b^2*c^6*d^8*e^8 - 28*a*b^3*c^5*d^7*e^9 + 8*a^2*b*c^6*d^7 *e^9 - 228*a^3*b*c^5*d^5*e^11 - 60*a^4*b*c^4*d^3*e^13))/(2*a^4))*(-(((4*b^6 *d^3 - 4*a^3*b^3*e^3 - 32*a^3*c^3*d^3 + 12*a^2*b^4*d*e^2 + 96*a^4*c^2*d*e^2 + 72*a^2*b^2*c^2*d^3 - 32*a*b^4*c*d^3 + 16*a^4*b*c*e^3 - 12*a*b^5*d^2*e + 84*a^2*b^3*c*d^2*e - 144*a^3*b*c^2*d^2*e - 72*a^3*b^2*c*d*e^2)^2/4 - (16*a^ 4*b^4 + 256*a^6*c^2 - 128*a^5*b^2*c)*(c^4*d^6 + a^3*c*e^6 + 3*a*c^3*d^4*e^2 - b^3*c*d^3*e^3 + 3*a^2*c^2*d^2*e^4 + 3*b^2*c^2*d^4*e^2 - 3*b*c^3*d^5*e - 3*a^2*b*c*d*e^5 - 6*a*b*c^2*d^3*e^3 + 3*a*b^2*c*d^2*e^4))^(1/2) - 2*b^6*d^3 + 2*a^3*b^3*e^3 + 16*a^3*c^3*d^3 - 6*a^2*b^4*d*e^2 - 48*a^4*c^2*d*e^2 - 36 *a^2*b^2*c^2*d^3 + 16*a*b^4*c*d^3 - 8*a^4*b*c*e^3 + 6*a*b^5*d^2*e - 42*a^2* b^3*c*d^2*e + 72*a^3*b*c^2*d^2*e + 36*a^3*b^2*c*d*e^2)/(16*(a^4*b^4 + 16*a^ 6*c^2 - 8*a^5*b^2*c)))^(1/2)*1i - (((224*a^4*c^6*d^6*e^9 - 176*a^5*c^5*d^4* e^11 - 400*a^6*c^4*d^2*e^13 + 160*a^2*b^3*c^5*d^7*e^8 - 156*a^2*b^5*c^3*d^5 *e^10 - 44*a^2*b^6*c^2*d^4*e^11 - 432*a^3*b^2*c^5*d^6*e^9 + 384*a^3*b^3*c^4 *d^5*e^10 + 444*a^3*b^4*c^3*d^4*e^11 + 88*a^3*b^5*c^2*d^3*e^12 - 948*a^4*b^ 2*c^4*d^4*e^11 - 644*a^4*b^3*c^3*d^3*e^12 - 76*a^4*b^4*c^2*d^2*e^13 + 444*a ^5*b^2*c^3*d^2*e^13 - 112*a^6*b*c^3*d*e^14 - 32*a*b^5*c^4*d^7*e^8 + 24*a*b^ 6*c^3*d^6*e^9 + 8*a*b^7*c^2*d^5*e^10 - 128*a^3*b*c^6*d^7*e^8 + 368*a^4*b*c^ 5*d^5*e^10 + 1008*a^5*b*c^4*d^3*e^12 + 24*a^5*b^3*c^2*d*e^14)/(4*a^4) + (((1280*a^8*c^4*d*e^11 + 1280*a^7*c^5*d^3*e^9 + 128*a^5*b^3*c^4*d^4*e^8 - 96*a ^5*b^4*c^3*d^3*e^9 - 32*a^5*b^5*c^2*d^2*e^10 + 64*a^6*b^2*c^4*d^3*e^9 + 576 *a^6*b^3*c^3*d^2*e^10 - 512*a^6*b*c^5*d^4*e^8 + 32*a^6*b^4*c^2*d*e^11 - 179 2*a^7*b*c^4*d^2*e^10 - 448*a^7*b^2*c^3*d*e^11)/(4*a^4) + ((d + e*x^2)^(1/2) *(-(((4*b^6*d^3 - 4*a^3*b^3*e^3 - 32*a^3*c^3*d^3 + 12*a^2*b^4*d*e^2 + 96*a^ 4*c^2*d*e^2 + 72*a^2*b^2*c^2*d^3 - 32*a*b^4*c*d^3 + 16*a^4*b*c*e^3 - 12*a*b ^5*d^2*e + 84*a^2*b^3*c*d^2*e - 144*a^3*b*c^2*d^2*e - 72*a^3*b^2*c*d*e^2)^2 /4 - (16*a^4*b^4 + 256*a^6*c^2 - 128*a^5*b^2*c)*(c^4*d^6 + a^3*c*e^6 + 3*a* c^3*d^4*e^2 - b^3*c*d^3*e^3 + 3*a^2*c^2*d^2*e^4 + 3*b^2*c^2*d^4*e^2 - 3*b*c ^3*d^5*e - 3*a^2*b*c*d*e^5 - 6*a*b*c^2*d^3*e^3 + 3*a*b^2*c*d^2*e^4))^(1/2)

- 2*b^6*d^3 + 2*a^3*b^3*e^3 + 16*a^3*c^3*d^3 - 6*a^2*b^4*d*e^2 - 48*a^4*c^2
*d*e^2 - 36*a^2*b^2*c^2*d^3 + 16*a*b^4*c*d^3 - 8*a^4*b*c*e^3 + 6*a*b^5*d^2*
e - 42*a^2*b^3*c*d^2*e + 72*a^3*b*c^2*d^2*e + 36*a^3*b^2*c*d*e^2)/(16*(a^4*
b^4 + 16*a^6*c^2 - 8*a^5*b^2*c)))^(1/2)*(1024*a^9*c^4*e^10 + 64*a^7*b^4*c^2
*e^10 - 512*a^8*b^2*c^3*e^10 + 1536*a^8*c^5*d^2*e^8 + 128*a^6*b^4*c^3*d^2*e
^8 - 896*a^7*b^2*c^4*d^2*e^8 - 1792*a^8*b*c^4*d*e^9 - 128*a^6*b^5*c^2*d*e^9
 + 960*a^7*b^3*c^3*d*e^9))/(2*a^4))*(-(((4*b^6*d^3 - 4*a^3*b^3*e^3 - 32*a^3
*c^3*d^3 + 12*a^2*b^4*d*e^2 + 96*a^4*c^2*d*e^2 + 72*a^2*b^2*c^2*d^3 - 32*a*
b^4*c*d^3 + 16*a^4*b*c*e^3 - 12*a*b^5*d^2*e + 84*a^2*b^3*c*d^2*e - 144*a^3*
b*c^2*d^2*e - 72*a^3*b^2*c*d*e^2)^2/4 - (16*a^4*b^4 + 256*a^6*c^2 - 128*a^5
*b^2*c)*(c^4*d^6 + a^3*c*e^6 + 3*a*c^3*d^4*e^2 - b^3*c*d^3*e^3 + 3*a^2*c^2*
d^2*e^4 + 3*b^2*c^2*d^4*e^2 - 3*b*c^3*d^5*e - 3*a^2*b*c*d*e^5 - 6*a*b*c^2*d
^3*e^3 + 3*a*b^2*c*d^2*e^4))^(1/2) - 2*b^6*d^3 + 2*a^3*b^3*e^3 + 16*a^3*c^3
*d^3 - 6*a^2*b^4*d*e^2 - 48*a^4*c^2*d*e^2 - 36*a^2*b^2*c^2*d^3 + 16*a*b^4*c
*d^3 - 8*a^4*b*c*e^3 + 6*a*b^5*d^2*e - 42*a^2*b^3*c*d^2*e + 72*a^3*b*c^2*d^
2*e + 36*a^3*b^2*c*d*e^2)/(16*(a^4*b^4 + 16*a^6*c^2 - 8*a^5*b^2*c)))^(1/2)
- ((d + e*x^2)^(1/2)*(64*a^7*b*c^3*e^13 + 352*a^7*c^4*d*e^12 - 16*a^6*b^3*c
^2*e^13 - 160*a^5*c^6*d^5*e^8 + 736*a^6*c^5*d^3*e^10 + 32*a^2*b^6*c^3*d^5*e
^8 - 32*a^2*b^7*c^2*d^4*e^9 - 224*a^3*b^4*c^4*d^5*e^8 + 144*a^3*b^5*c^3*d^4
*e^9 + 112*a^3*b^6*c^2*d^3*e^10 + 432*a^4*b^2*c^5*d^5*e^8 + 144*a^4*b^3*c^4
*d^4*e^9 - 716*a^4*b^4*c^3*d^3*e^10 - 132*a^4*b^5*c^2*d^2*e^11 + 936*a^5*b^
2*c^4*d^3*e^10 + 860*a^5*b^3*c^3*d^2*e^11 - 896*a^5*b*c^5*d^4*e^9 + 64*a^5*
b^4*c^2*d*e^12 - 1392*a^6*b*c^4*d^2*e^11 - 336*a^6*b^2*c^3*d*e^12))/(2*a^4)
)*(-(((4*b^6*d^3 - 4*a^3*b^3*e^3 - 32*a^3*c^3*d^3 + 12*a^2*b^4*d*e^2 + 96*a
^4*c^2*d*e^2 + 72*a^2*b^2*c^2*d^3 - 32*a*b^4*c*d^3 + 16*a^4*b*c*e^3 - 12*a*
b^5*d^2*e + 84*a^2*b^3*c*d^2*e - 144*a^3*b*c^2*d^2*e - 72*a^3*b^2*c*d*e^2)^
2/4 - (16*a^4*b^4 + 256*a^6*c^2 - 128*a^5*b^2*c)*(c^4*d^6 + a^3*c*e^6 + 3*a
*c^3*d^4*e^2 - b^3*c*d^3*e^3 + 3*a^2*c^2*d^2*e^4 + 3*b^2*c^2*d^4*e^2 - 3*b*
c^3*d^5*e - 3*a^2*b*c*d*e^5 - 6*a*b*c^2*d^3*e^3 + 3*a*b^2*c*d^2*e^4))^(1/2)
 - 2*b^6*d^3 + 2*a^3*b^3*e^3 + 16*a^3*c^3*d^3 - 6*a^2*b^4*d*e^2 - 48*a^4*c^
2*d*e^2 - 36*a^2*b^2*c^2*d^3 + 16*a*b^4*c*d^3 - 8*a^4*b*c*e^3 + 6*a*b^5*d^2
*e - 42*a^2*b^3*c*d^2*e + 72*a^3*b*c^2*d^2*e + 36*a^3*b^2*c*d*e^2)/(16*(a^4
*b^4 + 16*a^6*c^2 - 8*a^5*b^2*c)))^(1/2))*(-(((4*b^6*d^3 - 4*a^3*b^3*e^3 -
32*a^3*c^3*d^3 + 12*a^2*b^4*d*e^2 + 96*a^4*c^2*d*e^2 + 72*a^2*b^2*c^2*d^3 -
 32*a*b^4*c*d^3 + 16*a^4*b*c*e^3 - 12*a*b^5*d^2*e + 84*a^2*b^3*c*d^2*e - 14
4*a^3*b*c^2*d^2*e - 72*a^3*b^2*c*d*e^2)^2/4 - (16*a^4*b^4 + 256*a^6*c^2 - 1
28*a^5*b^2*c)*(c^4*d^6 + a^3*c*e^6 + 3*a*c^3*d^4*e^2 - b^3*c*d^3*e^3 + 3*a^
2*c^2*d^2*e^4 + 3*b^2*c^2*d^4*e^2 - 3*b*c^3*d^5*e - 3*a^2*b*c*d*e^5 - 6*a*b
*c^2*d^3*e^3 + 3*a*b^2*c*d^2*e^4))^(1/2) - 2*b^6*d^3 + 2*a^3*b^3*e^3 + 16*a
^3*c^3*d^3 - 6*a^2*b^4*d*e^2 - 48*a^4*c^2*d*e^2 - 36*a^2*b^2*c^2*d^3 + 16*a
*b^4*c*d^3 - 8*a^4*b*c*e^3 + 6*a*b^5*d^2*e - 42*a^2*b^3*c*d^2*e + 72*a^3*b*
c^2*d^2*e + 36*a^3*b^2*c*d*e^2)/(16*(a^4*b^4 + 16*a^6*c^2 - 8*a^5*b^2*c)))^
(1/2) + ((d + e*x^2)^(1/2)*(4*a^6*c^3*e^16 + 4*a^2*c^7*d^8*e^8 - 2*a^3*c^6*
d^6*e^10 + 132*a^4*c^5*d^4*e^12 - 2*a^5*c^4*d^2*e^14 + 4*b^4*c^5*d^8*e^8 +
129*a^2*b^2*c^5*d^6*e^10 - 32*a^2*b^3*c^4*d^5*e^11 + 8*a^2*b^4*c^3*d^4*e^12

+ 88*a^3*b^2*c^4*d^4*e^12 - 28*a^3*b^3*c^3*d^3*e^13 + 33*a^4*b^2*c^3*d^2*e
^14 - 16*a^5*b*c^3*d*e^15 - 8*a*b^2*c^6*d^8*e^8 - 28*a*b^3*c^5*d^7*e^9 + 8*
a^2*b*c^6*d^7*e^9 - 228*a^3*b*c^5*d^5*e^11 - 60*a^4*b*c^4*d^3*e^13))/(2*a^4
))*(-(((4*b^6*d^3 - 4*a^3*b^3*e^3 - 32*a^3*c^3*d^3 + 12*a^2*b^4*d*e^2 + 96*
a^4*c^2*d*e^2 + 72*a^2*b^2*c^2*d^3 - 32*a*b^4*c*d^3 + 16*a^4*b*c*e^3 - 12*a
*b^5*d^2*e + 84*a^2*b^3*c*d^2*e - 144*a^3*b*c^2*d^2*e - 72*a^3*b^2*c*d*e^2)
^2/4 - (16*a^4*b^4 + 256*a^6*c^2 - 128*a^5*b^2*c)*(c^4*d^6 + a^3*c*e^6 + 3*
a*c^3*d^4*e^2 - b^3*c*d^3*e^3 + 3*a^2*c^2*d^2*e^4 + 3*b^2*c^2*d^4*e^2 - 3*b
*c^3*d^5*e - 3*a^2*b*c*d*e^5 - 6*a*b*c^2*d^3*e^3 + 3*a*b^2*c*d^2*e^4))^(1/2
) - 2*b^6*d^3 + 2*a^3*b^3*e^3 + 16*a^3*c^3*d^3 - 6*a^2*b^4*d*e^2 - 48*a^4*c
^2*d*e^2 - 36*a^2*b^2*c^2*d^3 + 16*a*b^4*c*d^3 - 8*a^4*b*c*e^3 + 6*a*b^5*d^
2*e - 42*a^2*b^3*c*d^2*e + 72*a^3*b*c^2*d^2*e + 36*a^3*b^2*c*d*e^2)/(16*(a^
4*b^4 + 16*a^6*c^2 - 8*a^5*b^2*c)))^(1/2)*1i)/((6*a*c^7*d^9*e^9 + 6*a^5*c^3
*d*e^17 - 4*b*c^7*d^10*e^8 + 6*a^2*c^6*d^7*e^11 + 6*a^4*c^4*d^3*e^15 + 8*b^
2*c^6*d^9*e^9 - 4*b^3*c^5*d^8*e^10 + 4*a^2*b^2*c^4*d^5*e^13 - 11*a^2*b^3*c^
3*d^4*e^14 + 22*a^3*b^2*c^3*d^3*e^15 - 16*a*b*c^6*d^8*e^10 + 8*a*b^2*c^5*d^
7*e^11 + 2*a*b^4*c^3*d^5*e^13 - 3*a^2*b*c^5*d^6*e^12 - 10*a^3*b*c^4*d^4*e^1
4 - 19*a^4*b*c^3*d^2*e^16)/(2*a^4) + (((224*a^4*c^6*d^6*e^9 - 176*a^5*c^5*d
^4*e^11 - 400*a^6*c^4*d^2*e^13 + 160*a^2*b^3*c^5*d^7*e^8 - 156*a^2*b^5*c^3*
d^5*e^10 - 44*a^2*b^6*c^2*d^4*e^11 - 432*a^3*b^2*c^5*d^6*e^9 + 384*a^3*b^3*
c^4*d^5*e^10 + 444*a^3*b^4*c^3*d^4*e^11 + 88*a^3*b^5*c^2*d^3*e^12 - 948*a^4
*b^2*c^4*d^4*e^11 - 644*a^4*b^3*c^3*d^3*e^12 - 76*a^4*b^4*c^2*d^2*e^13 + 44
4*a^5*b^2*c^3*d^2*e^13 - 112*a^6*b*c^3*d*e^14 - 32*a*b^5*c^4*d^7*e^8 + 24*a
*b^6*c^3*d^6*e^9 + 8*a*b^7*c^2*d^5*e^10 - 128*a^3*b*c^6*d^7*e^8 + 368*a^4*b
*c^5*d^5*e^10 + 1008*a^5*b*c^4*d^3*e^12 + 24*a^5*b^3*c^2*d*e^14)/(4*a^4) +
(((1280*a^8*c^4*d*e^11 + 1280*a^7*c^5*d^3*e^9 + 128*a^5*b^3*c^4*d^4*e^8 - 9
6*a^5*b^4*c^3*d^3*e^9 - 32*a^5*b^5*c^2*d^2*e^10 + 64*a^6*b^2*c^4*d^3*e^9 +
576*a^6*b^3*c^3*d^2*e^10 - 512*a^6*b*c^5*d^4*e^8 + 32*a^6*b^4*c^2*d*e^11 -
1792*a^7*b*c^4*d^2*e^10 - 448*a^7*b^2*c^3*d*e^11)/(4*a^4) - ((d + e*x^2)^(1
/2)*(-(((4*b^6*d^3 - 4*a^3*b^3*e^3 - 32*a^3*c^3*d^3 + 12*a^2*b^4*d*e^2 + 96
*a^4*c^2*d*e^2 + 72*a^2*b^2*c^2*d^3 - 32*a*b^4*c*d^3 + 16*a^4*b*c*e^3 - 12*
a*b^5*d^2*e + 84*a^2*b^3*c*d^2*e - 144*a^3*b*c^2*d^2*e - 72*a^3*b^2*c*d*e^2
)^2/4 - (16*a^4*b^4 + 256*a^6*c^2 - 128*a^5*b^2*c)*(c^4*d^6 + a^3*c*e^6 + 3
*a*c^3*d^4*e^2 - b^3*c*d^3*e^3 + 3*a^2*c^2*d^2*e^4 + 3*b^2*c^2*d^4*e^2 - 3*
b*c^3*d^5*e - 3*a^2*b*c*d*e^5 - 6*a*b*c^2*d^3*e^3 + 3*a*b^2*c*d^2*e^4))^(1/
2) - 2*b^6*d^3 + 2*a^3*b^3*e^3 + 16*a^3*c^3*d^3 - 6*a^2*b^4*d*e^2 - 48*a^4*
c^2*d*e^2 - 36*a^2*b^2*c^2*d^3 + 16*a*b^4*c*d^3 - 8*a^4*b*c*e^3 + 6*a*b^5*d
^2*e - 42*a^2*b^3*c*d^2*e + 72*a^3*b*c^2*d^2*e + 36*a^3*b^2*c*d*e^2)/(16*(a
^4*b^4 + 16*a^6*c^2 - 8*a^5*b^2*c)))^(1/2)*(1024*a^9*c^4*e^10 + 64*a^7*b^4*
c^2*e^10 - 512*a^8*b^2*c^3*e^10 + 1536*a^8*c^5*d^2*e^8 + 128*a^6*b^4*c^3*d^
2*e^8 - 896*a^7*b^2*c^4*d^2*e^8 - 1792*a^8*b*c^4*d*e^9 - 128*a^6*b^5*c^2*d*
e^9 + 960*a^7*b^3*c^3*d*e^9))/(2*a^4))*(-(((4*b^6*d^3 - 4*a^3*b^3*e^3 - 32*
a^3*c^3*d^3 + 12*a^2*b^4*d*e^2 + 96*a^4*c^2*d*e^2 + 72*a^2*b^2*c^2*d^3 - 32
*a*b^4*c*d^3 + 16*a^4*b*c*e^3 - 12*a*b^5*d^2*e + 84*a^2*b^3*c*d^2*e - 144*a
^3*b*c^2*d^2*e - 72*a^3*b^2*c*d*e^2)^2/4 - (16*a^4*b^4 + 256*a^6*c^2 - 128*

a^5*b^2*c)*(c^4*d^6 + a^3*c*e^6 + 3*a*c^3*d^4*e^2 - b^3*c*d^3*e^3 + 3*a^2*c
^2*d^2*e^4 + 3*b^2*c^2*d^4*e^2 - 3*b*c^3*d^5*e - 3*a^2*b*c*d*e^5 - 6*a*b*c^
2*d^3*e^3 + 3*a*b^2*c*d^2*e^4))^(1/2) - 2*b^6*d^3 + 2*a^3*b^3*e^3 + 16*a^3*
c^3*d^3 - 6*a^2*b^4*d*e^2 - 48*a^4*c^2*d*e^2 - 36*a^2*b^2*c^2*d^3 + 16*a*b^
4*c*d^3 - 8*a^4*b*c*e^3 + 6*a*b^5*d^2*e - 42*a^2*b^3*c*d^2*e + 72*a^3*b*c^2
*d^2*e + 36*a^3*b^2*c*d*e^2)/(16*(a^4*b^4 + 16*a^6*c^2 - 8*a^5*b^2*c)))^(1/
2) + ((d + e*x^2)^(1/2)*(64*a^7*b*c^3*e^13 + 352*a^7*c^4*d*e^12 - 16*a^6*b^
3*c^2*e^13 - 160*a^5*c^6*d^5*e^8 + 736*a^6*c^5*d^3*e^10 + 32*a^2*b^6*c^3*d^
5*e^8 - 32*a^2*b^7*c^2*d^4*e^9 - 224*a^3*b^4*c^4*d^5*e^8 + 144*a^3*b^5*c^3*
d^4*e^9 + 112*a^3*b^6*c^2*d^3*e^10 + 432*a^4*b^2*c^5*d^5*e^8 + 144*a^4*b^3*
c^4*d^4*e^9 - 716*a^4*b^4*c^3*d^3*e^10 - 132*a^4*b^5*c^2*d^2*e^11 + 936*a^5
*b^2*c^4*d^3*e^10 + 860*a^5*b^3*c^3*d^2*e^11 - 896*a^5*b*c^5*d^4*e^9 + 64*a
^5*b^4*c^2*d*e^12 - 1392*a^6*b*c^4*d^2*e^11 - 336*a^6*b^2*c^3*d*e^12))/(2*a
^4))*(-(((4*b^6*d^3 - 4*a^3*b^3*e^3 - 32*a^3*c^3*d^3 + 12*a^2*b^4*d*e^2 + 9
6*a^4*c^2*d*e^2 + 72*a^2*b^2*c^2*d^3 - 32*a*b^4*c*d^3 + 16*a^4*b*c*e^3 - 12
*a*b^5*d^2*e + 84*a^2*b^3*c*d^2*e - 144*a^3*b*c^2*d^2*e - 72*a^3*b^2*c*d*e^
2)^2/4 - (16*a^4*b^4 + 256*a^6*c^2 - 128*a^5*b^2*c)*(c^4*d^6 + a^3*c*e^6 +
3*a*c^3*d^4*e^2 - b^3*c*d^3*e^3 + 3*a^2*c^2*d^2*e^4 + 3*b^2*c^2*d^4*e^2 - 3
*b*c^3*d^5*e - 3*a^2*b*c*d*e^5 - 6*a*b*c^2*d^3*e^3 + 3*a*b^2*c*d^2*e^4))^(1
/2) - 2*b^6*d^3 + 2*a^3*b^3*e^3 + 16*a^3*c^3*d^3 - 6*a^2*b^4*d*e^2 - 48*a^4
*c^2*d*e^2 - 36*a^2*b^2*c^2*d^3 + 16*a*b^4*c*d^3 - 8*a^4*b*c*e^3 + 6*a*b^5*
d^2*e - 42*a^2*b^3*c*d^2*e + 72*a^3*b*c^2*d^2*e + 36*a^3*b^2*c*d*e^2)/(16*(
a^4*b^4 + 16*a^6*c^2 - 8*a^5*b^2*c)))^(1/2))*(-(((4*b^6*d^3 - 4*a^3*b^3*e^3
 - 32*a^3*c^3*d^3 + 12*a^2*b^4*d*e^2 + 96*a^4*c^2*d*e^2 + 72*a^2*b^2*c^2*d^
3 - 32*a*b^4*c*d^3 + 16*a^4*b*c*e^3 - 12*a*b^5*d^2*e + 84*a^2*b^3*c*d^2*e -
 144*a^3*b*c^2*d^2*e - 72*a^3*b^2*c*d*e^2)^2/4 - (16*a^4*b^4 + 256*a^6*c^2
- 128*a^5*b^2*c)*(c^4*d^6 + a^3*c*e^6 + 3*a*c^3*d^4*e^2 - b^3*c*d^3*e^3 + 3
*a^2*c^2*d^2*e^4 + 3*b^2*c^2*d^4*e^2 - 3*b*c^3*d^5*e - 3*a^2*b*c*d*e^5 - 6*
a*b*c^2*d^3*e^3 + 3*a*b^2*c*d^2*e^4))^(1/2) - 2*b^6*d^3 + 2*a^3*b^3*e^3 + 1
6*a^3*c^3*d^3 - 6*a^2*b^4*d*e^2 - 48*a^4*c^2*d*e^2 - 36*a^2*b^2*c^2*d^3 + 1
6*a*b^4*c*d^3 - 8*a^4*b*c*e^3 + 6*a*b^5*d^2*e - 42*a^2*b^3*c*d^2*e + 72*a^3
*b*c^2*d^2*e + 36*a^3*b^2*c*d*e^2)/(16*(a^4*b^4 + 16*a^6*c^2 - 8*a^5*b^2*c)
))^(1/2) - ((d + e*x^2)^(1/2)*(4*a^6*c^3*e^16 + 4*a^2*c^7*d^8*e^8 - 2*a^3*c
^6*d^6*e^10 + 132*a^4*c^5*d^4*e^12 - 2*a^5*c^4*d^2*e^14 + 4*b^4*c^5*d^8*e^8
 + 129*a^2*b^2*c^5*d^6*e^10 - 32*a^2*b^3*c^4*d^5*e^11 + 8*a^2*b^4*c^3*d^4*e
^12 + 88*a^3*b^2*c^4*d^4*e^12 - 28*a^3*b^3*c^3*d^3*e^13 + 33*a^4*b^2*c^3*d^
2*e^14 - 16*a^5*b*c^3*d*e^15 - 8*a*b^2*c^6*d^8*e^8 - 28*a*b^3*c^5*d^7*e^9 +
 8*a^2*b*c^6*d^7*e^9 - 228*a^3*b*c^5*d^5*e^11 - 60*a^4*b*c^4*d^3*e^13))/(2*
a^4))*(-(((4*b^6*d^3 - 4*a^3*b^3*e^3 - 32*a^3*c^3*d^3 + 12*a^2*b^4*d*e^2 +
96*a^4*c^2*d*e^2 + 72*a^2*b^2*c^2*d^3 - 32*a*b^4*c*d^3 + 16*a^4*b*c*e^3 - 1
2*a*b^5*d^2*e + 84*a^2*b^3*c*d^2*e - 144*a^3*b*c^2*d^2*e - 72*a^3*b^2*c*d*e
^2)^2/4 - (16*a^4*b^4 + 256*a^6*c^2 - 128*a^5*b^2*c)*(c^4*d^6 + a^3*c*e^6 +
 3*a*c^3*d^4*e^2 - b^3*c*d^3*e^3 + 3*a^2*c^2*d^2*e^4 + 3*b^2*c^2*d^4*e^2 -
3*b*c^3*d^5*e - 3*a^2*b*c*d*e^5 - 6*a*b*c^2*d^3*e^3 + 3*a*b^2*c*d^2*e^4))^(
1/2) - 2*b^6*d^3 + 2*a^3*b^3*e^3 + 16*a^3*c^3*d^3 - 6*a^2*b^4*d*e^2 - 48*a^

4*c^2*d*e^2 - 36*a^2*b^2*c^2*d^3 + 16*a*b^4*c*d^3 - 8*a^4*b*c*e^3 + 6*a*b^5
*d^2*e - 42*a^2*b^3*c*d^2*e + 72*a^3*b*c^2*d^2*e + 36*a^3*b^2*c*d*e^2)/(16*
(a^4*b^4 + 16*a^6*c^2 - 8*a^5*b^2*c)))^(1/2) + (((224*a^4*c^6*d^6*e^9 - 176
*a^5*c^5*d^4*e^11 - 400*a^6*c^4*d^2*e^13 + 160*a^2*b^3*c^5*d^7*e^8 - 156*a^
2*b^5*c^3*d^5*e^10 - 44*a^2*b^6*c^2*d^4*e^11 - 432*a^3*b^2*c^5*d^6*e^9 + 38
4*a^3*b^3*c^4*d^5*e^10 + 444*a^3*b^4*c^3*d^4*e^11 + 88*a^3*b^5*c^2*d^3*e^12
 - 948*a^4*b^2*c^4*d^4*e^11 - 644*a^4*b^3*c^3*d^3*e^12 - 76*a^4*b^4*c^2*d^2
*e^13 + 444*a^5*b^2*c^3*d^2*e^13 - 112*a^6*b*c^3*d*e^14 - 32*a*b^5*c^4*d^7*
e^8 + 24*a*b^6*c^3*d^6*e^9 + 8*a*b^7*c^2*d^5*e^10 - 128*a^3*b*c^6*d^7*e^8 +
 368*a^4*b*c^5*d^5*e^10 + 1008*a^5*b*c^4*d^3*e^12 + 24*a^5*b^3*c^2*d*e^14)/
(4*a^4) + (((1280*a^8*c^4*d*e^11 + 1280*a^7*c^5*d^3*e^9 + 128*a^5*b^3*c^4*d
^4*e^8 - 96*a^5*b^4*c^3*d^3*e^9 - 32*a^5*b^5*c^2*d^2*e^10 + 64*a^6*b^2*c^4*
d^3*e^9 + 576*a^6*b^3*c^3*d^2*e^10 - 512*a^6*b*c^5*d^4*e^8 + 32*a^6*b^4*c^2
*d*e^11 - 1792*a^7*b*c^4*d^2*e^10 - 448*a^7*b^2*c^3*d*e^11)/(4*a^4) + ((d +
 e*x^2)^(1/2)*(-(((4*b^6*d^3 - 4*a^3*b^3*e^3 - 32*a^3*c^3*d^3 + 12*a^2*b^4*
d*e^2 + 96*a^4*c^2*d*e^2 + 72*a^2*b^2*c^2*d^3 - 32*a*b^4*c*d^3 + 16*a^4*b*c
*e^3 - 12*a*b^5*d^2*e + 84*a^2*b^3*c*d^2*e - 144*a^3*b*c^2*d^2*e - 72*a^3*b
^2*c*d*e^2)^2/4 - (16*a^4*b^4 + 256*a^6*c^2 - 128*a^5*b^2*c)*(c^4*d^6 + a^3
*c*e^6 + 3*a*c^3*d^4*e^2 - b^3*c*d^3*e^3 + 3*a^2*c^2*d^2*e^4 + 3*b^2*c^2*d^
4*e^2 - 3*b*c^3*d^5*e - 3*a^2*b*c*d*e^5 - 6*a*b*c^2*d^3*e^3 + 3*a*b^2*c*d^2
*e^4))^(1/2) - 2*b^6*d^3 + 2*a^3*b^3*e^3 + 16*a^3*c^3*d^3 - 6*a^2*b^4*d*e^2
 - 48*a^4*c^2*d*e^2 - 36*a^2*b^2*c^2*d^3 + 16*a*b^4*c*d^3 - 8*a^4*b*c*e^3 +
 6*a*b^5*d^2*e - 42*a^2*b^3*c*d^2*e + 72*a^3*b*c^2*d^2*e + 36*a^3*b^2*c*d*e
^2)/(16*(a^4*b^4 + 16*a^6*c^2 - 8*a^5*b^2*c)))^(1/2)*(1024*a^9*c^4*e^10 + 6
4*a^7*b^4*c^2*e^10 - 512*a^8*b^2*c^3*e^10 + 1536*a^8*c^5*d^2*e^8 + 128*a^6*
b^4*c^3*d^2*e^8 - 896*a^7*b^2*c^4*d^2*e^8 - 1792*a^8*b*c^4*d*e^9 - 128*a^6*
b^5*c^2*d*e^9 + 960*a^7*b^3*c^3*d*e^9))/(2*a^4))*(-(((4*b^6*d^3 - 4*a^3*b^3
*e^3 - 32*a^3*c^3*d^3 + 12*a^2*b^4*d*e^2 + 96*a^4*c^2*d*e^2 + 72*a^2*b^2*c^
2*d^3 - 32*a*b^4*c*d^3 + 16*a^4*b*c*e^3 - 12*a*b^5*d^2*e + 84*a^2*b^3*c*d^2
*e - 144*a^3*b*c^2*d^2*e - 72*a^3*b^2*c*d*e^2)^2/4 - (16*a^4*b^4 + 256*a^6*
c^2 - 128*a^5*b^2*c)*(c^4*d^6 + a^3*c*e^6 + 3*a*c^3*d^4*e^2 - b^3*c*d^3*e^3
 + 3*a^2*c^2*d^2*e^4 + 3*b^2*c^2*d^4*e^2 - 3*b*c^3*d^5*e - 3*a^2*b*c*d*e^5
- 6*a*b*c^2*d^3*e^3 + 3*a*b^2*c*d^2*e^4))^(1/2) - 2*b^6*d^3 + 2*a^3*b^3*e^3
 + 16*a^3*c^3*d^3 - 6*a^2*b^4*d*e^2 - 48*a^4*c^2*d*e^2 - 36*a^2*b^2*c^2*d^3
 + 16*a*b^4*c*d^3 - 8*a^4*b*c*e^3 + 6*a*b^5*d^2*e - 42*a^2*b^3*c*d^2*e + 72
*a^3*b*c^2*d^2*e + 36*a^3*b^2*c*d*e^2)/(16*(a^4*b^4 + 16*a^6*c^2 - 8*a^5*b^
2*c)))^(1/2) - ((d + e*x^2)^(1/2)*(64*a^7*b*c^3*e^13 + 352*a^7*c^4*d*e^12 -
 16*a^6*b^3*c^2*e^13 - 160*a^5*c^6*d^5*e^8 + 736*a^6*c^5*d^3*e^10 + 32*a^2*
b^6*c^3*d^5*e^8 - 32*a^2*b^7*c^2*d^4*e^9 - 224*a^3*b^4*c^4*d^5*e^8 + 144*a^
3*b^5*c^3*d^4*e^9 + 112*a^3*b^6*c^2*d^3*e^10 + 432*a^4*b^2*c^5*d^5*e^8 + 14
4*a^4*b^3*c^4*d^4*e^9 - 716*a^4*b^4*c^3*d^3*e^10 - 132*a^4*b^5*c^2*d^2*e^11
 + 936*a^5*b^2*c^4*d^3*e^10 + 860*a^5*b^3*c^3*d^2*e^11 - 896*a^5*b*c^5*d^4*
e^9 + 64*a^5*b^4*c^2*d*e^12 - 1392*a^6*b*c^4*d^2*e^11 - 336*a^6*b^2*c^3*d*e
^12))/(2*a^4))*(-(((4*b^6*d^3 - 4*a^3*b^3*e^3 - 32*a^3*c^3*d^3 + 12*a^2*b^4
*d*e^2 + 96*a^4*c^2*d*e^2 + 72*a^2*b^2*c^2*d^3 - 32*a*b^4*c*d^3 + 16*a^4*b*

c*e^3 - 12*a*b^5*d^2*e + 84*a^2*b^3*c*d^2*e - 144*a^3*b*c^2*d^2*e - 72*a^3*
b^2*c*d*e^2)^2/4 - (16*a^4*b^4 + 256*a^6*c^2 - 128*a^5*b^2*c)*(c^4*d^6 + a^
3*c*e^6 + 3*a*c^3*d^4*e^2 - b^3*c*d^3*e^3 + 3*a^2*c^2*d^2*e^4 + 3*b^2*c^2*d
^4*e^2 - 3*b*c^3*d^5*e - 3*a^2*b*c*d*e^5 - 6*a*b*c^2*d^3*e^3 + 3*a*b^2*c*d^
2*e^4))^(1/2) - 2*b^6*d^3 + 2*a^3*b^3*e^3 + 16*a^3*c^3*d^3 - 6*a^2*b^4*d*e^
2 - 48*a^4*c^2*d*e^2 - 36*a^2*b^2*c^2*d^3 + 16*a*b^4*c*d^3 - 8*a^4*b*c*e^3
+ 6*a*b^5*d^2*e - 42*a^2*b^3*c*d^2*e + 72*a^3*b*c^2*d^2*e + 36*a^3*b^2*c*d*
e^2)/(16*(a^4*b^4 + 16*a^6*c^2 - 8*a^5*b^2*c)))^(1/2))*(-(((4*b^6*d^3 - 4*a
^3*b^3*e^3 - 32*a^3*c^3*d^3 + 12*a^2*b^4*d*e^2 + 96*a^4*c^2*d*e^2 + 72*a^2*
b^2*c^2*d^3 - 32*a*b^4*c*d^3 + 16*a^4*b*c*e^3 - 12*a*b^5*d^2*e + 84*a^2*b^3
*c*d^2*e - 144*a^3*b*c^2*d^2*e - 72*a^3*b^2*c*d*e^2)^2/4 - (16*a^4*b^4 + 25
6*a^6*c^2 - 128*a^5*b^2*c)*(c^4*d^6 + a^3*c*e^6 + 3*a*c^3*d^4*e^2 - b^3*c*d
^3*e^3 + 3*a^2*c^2*d^2*e^4 + 3*b^2*c^2*d^4*e^2 - 3*b*c^3*d^5*e - 3*a^2*b*c*
d*e^5 - 6*a*b*c^2*d^3*e^3 + 3*a*b^2*c*d^2*e^4))^(1/2) - 2*b^6*d^3 + 2*a^3*b
^3*e^3 + 16*a^3*c^3*d^3 - 6*a^2*b^4*d*e^2 - 48*a^4*c^2*d*e^2 - 36*a^2*b^2*c
^2*d^3 + 16*a*b^4*c*d^3 - 8*a^4*b*c*e^3 + 6*a*b^5*d^2*e - 42*a^2*b^3*c*d^2*
e + 72*a^3*b*c^2*d^2*e + 36*a^3*b^2*c*d*e^2)/(16*(a^4*b^4 + 16*a^6*c^2 - 8*
a^5*b^2*c)))^(1/2) + ((d + e*x^2)^(1/2)*(4*a^6*c^3*e^16 + 4*a^2*c^7*d^8*e^8
 - 2*a^3*c^6*d^6*e^10 + 132*a^4*c^5*d^4*e^12 - 2*a^5*c^4*d^2*e^14 + 4*b^4*c
^5*d^8*e^8 + 129*a^2*b^2*c^5*d^6*e^10 - 32*a^2*b^3*c^4*d^5*e^11 + 8*a^2*b^4
*c^3*d^4*e^12 + 88*a^3*b^2*c^4*d^4*e^12 - 28*a^3*b^3*c^3*d^3*e^13 + 33*a^4*
b^2*c^3*d^2*e^14 - 16*a^5*b*c^3*d*e^15 - 8*a*b^2*c^6*d^8*e^8 - 28*a*b^3*c^5
*d^7*e^9 + 8*a^2*b*c^6*d^7*e^9 - 228*a^3*b*c^5*d^5*e^11 - 60*a^4*b*c^4*d^3*
e^13))/(2*a^4))*(-(((4*b^6*d^3 - 4*a^3*b^3*e^3 - 32*a^3*c^3*d^3 + 12*a^2*b^
4*d*e^2 + 96*a^4*c^2*d*e^2 + 72*a^2*b^2*c^2*d^3 - 32*a*b^4*c*d^3 + 16*a^4*b
*c*e^3 - 12*a*b^5*d^2*e + 84*a^2*b^3*c*d^2*e - 144*a^3*b*c^2*d^2*e - 72*a^3
*b^2*c*d*e^2)^2/4 - (16*a^4*b^4 + 256*a^6*c^2 - 128*a^5*b^2*c)*(c^4*d^6 + a
^3*c*e^6 + 3*a*c^3*d^4*e^2 - b^3*c*d^3*e^3 + 3*a^2*c^2*d^2*e^4 + 3*b^2*c^2*
d^4*e^2 - 3*b*c^3*d^5*e - 3*a^2*b*c*d*e^5 - 6*a*b*c^2*d^3*e^3 + 3*a*b^2*c*d
^2*e^4))^(1/2) - 2*b^6*d^3 + 2*a^3*b^3*e^3 + 16*a^3*c^3*d^3 - 6*a^2*b^4*d*e
^2 - 48*a^4*c^2*d*e^2 - 36*a^2*b^2*c^2*d^3 + 16*a*b^4*c*d^3 - 8*a^4*b*c*e^3
 + 6*a*b^5*d^2*e - 42*a^2*b^3*c*d^2*e + 72*a^3*b*c^2*d^2*e + 36*a^3*b^2*c*d
e^2)/(16(a^4*b^4 + 16*a^6*c^2 - 8*a^5*b^2*c)))^(1/2)))*(-(((4*b^6*d^3 - 4
*a^3*b^3*e^3 - 32*a^3*c^3*d^3 + 12*a^2*b^4*d*e^2 + 96*a^4*c^2*d*e^2 + 72*a^
2*b^2*c^2*d^3 - 32*a*b^4*c*d^3 + 16*a^4*b*c*e^3 - 12*a*b^5*d^2*e + 84*a^2*b
^3*c*d^2*e - 144*a^3*b*c^2*d^2*e - 72*a^3*b^2*c*d*e^2)^2/4 - (16*a^4*b^4 +
256*a^6*c^2 - 128*a^5*b^2*c)*(c^4*d^6 + a^3*c*e^6 + 3*a*c^3*d^4*e^2 - b^3*c
*d^3*e^3 + 3*a^2*c^2*d^2*e^4 + 3*b^2*c^2*d^4*e^2 - 3*b*c^3*d^5*e - 3*a^2*b*
c*d*e^5 - 6*a*b*c^2*d^3*e^3 + 3*a*b^2*c*d^2*e^4))^(1/2) - 2*b^6*d^3 + 2*a^3
*b^3*e^3 + 16*a^3*c^3*d^3 - 6*a^2*b^4*d*e^2 - 48*a^4*c^2*d*e^2 - 36*a^2*b^2
*c^2*d^3 + 16*a*b^4*c*d^3 - 8*a^4*b*c*e^3 + 6*a*b^5*d^2*e - 42*a^2*b^3*c*d^
2*e + 72*a^3*b*c^2*d^2*e + 36*a^3*b^2*c*d*e^2)/(16*(a^4*b^4 + 16*a^6*c^2 -
8*a^5*b^2*c)))^(1/2)*2i

3.371 $\int \frac{x^4(d+ex^2)^{3/2}}{a+bx^2+cx^4}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 595

$$\int \frac{x^4(d+ex^2)^{3/2}}{a+bx^2+cx^4}\,dx = \frac{(3cd-4be)x\sqrt{d+ex^2}}{8c^2} + \frac{x(d+ex^2)^{3/2}}{4c}$$
$$-\frac{\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}\left(bcd-b^2e+ace-\frac{b^2cd-2ac^2d-b^3e+3abce}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}\,x}{\sqrt{b-\sqrt{b^2-4ac}}\sqrt{d+ex^2}}\right)}{2c^3\sqrt{b-\sqrt{b^2-4ac}}}$$
$$-\frac{\sqrt{2cd-\left(b+\sqrt{b^2-4ac}\right)e}\left(bcd-b^2e+ace+\frac{b^2cd-2ac^2d-b^3e+3abce}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2cd-\left(b+\sqrt{b^2-4ac}\right)e}\,x}{\sqrt{b+\sqrt{b^2-4ac}}\sqrt{d+ex^2}}\right)}{2c^3\sqrt{b+\sqrt{b^2-4ac}}}$$
$$+\frac{d(3cd-4be)\operatorname{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{8c^2\sqrt{e}}$$
$$-\frac{\sqrt{e}\left(bcd-b^2e+ace-\frac{b^2cd-2ac^2d-b^3e+3abce}{\sqrt{b^2-4ac}}\right)\operatorname{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{2c^3}$$
$$-\frac{\sqrt{e}\left(bcd-b^2e+ace+\frac{b^2cd-2ac^2d-b^3e+3abce}{\sqrt{b^2-4ac}}\right)\operatorname{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{2c^3}$$

```
[Out] 1/4*x*(e*x^2+d)^(3/2)/c+1/8*d*(-4*b*e+3*c*d)*arctanh(x*e^(1/2)/(e*x^2+d)^(1
/2))/c^2/e^(1/2)-1/2*arctanh(x*e^(1/2)/(e*x^2+d)^(1/2))*(b*c*d-b^2*e+a*c*e+
(-3*a*b*c*e+2*a*c^2*d+b^3*e-b^2*c*d)/(-4*a*c+b^2)^(1/2))*e^(1/2)/c^3-1/2*ar
ctanh(x*e^(1/2)/(e*x^2+d)^(1/2))*(b*c*d-b^2*e+a*c*e+(3*a*b*c*e-2*a*c^2*d-b^
3*e+b^2*c*d)/(-4*a*c+b^2)^(1/2))*e^(1/2)/c^3+1/8*(-4*b*e+3*c*d)*x*(e*x^2+d)
^(1/2)/c^2-1/2*arctan(x*(2*c*d-e*(b-(-4*a*c+b^2)^(1/2)))^(1/2)/(e*x^2+d)^(1
/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2))*(b*c*d-b^2*e+a*c*e+(-3*a*b*c*e+2*a*c^2*d+
```

```
b^3*e-b^2*c*d)/(-4*a*c+b^2)^(1/2))*(2*c*d-e*(b-(-4*a*c+b^2)^(1/2)))^(1/2)/c
^3/(b-(-4*a*c+b^2)^(1/2))^(1/2)-1/2*arctan(x*(2*c*d-e*(b+(-4*a*c+b^2)^(1/2)
))^(1/2)/(e*x^2+d)^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2))*(b*c*d-b^2*e+a*c*e+(
3*a*b*c*e-2*a*c^2*d-b^3*e+b^2*c*d)/(-4*a*c+b^2)^(1/2))*(2*c*d-e*(b+(-4*a*c+
b^2)^(1/2)))^(1/2)/c^3/(b+(-4*a*c+b^2)^(1/2))^(1/2)
```

Rubi [A] (verified)

Time = 2.25 (sec) , antiderivative size = 595, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.310$, Rules used = {1305, 396, 201, 223, 212, 1706, 399, 385, 211}

$$\int \frac{x^4(d+ex^2)^{3/2}}{a+bx^2+cx^4}\,dx =$$

$$-\frac{\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}\left(-\frac{3abce-2ac^2d+b^3(-e)+b^2cd}{\sqrt{b^2-4ac}}+ace+b^2(-e)+bcd\right)\arctan\left(\frac{x\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}{\sqrt{b-\sqrt{b^2-4ac}}\sqrt{d+ex^2}}\right)}{2c^3\sqrt{b-\sqrt{b^2-4ac}}}$$
$$-\frac{\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}\left(\frac{3abce-2ac^2d+b^3(-e)+b^2cd}{\sqrt{b^2-4ac}}+ace+b^2(-e)+bcd\right)\arctan\left(\frac{x\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}{\sqrt{\sqrt{b^2-4ac}+b}\sqrt{d+ex^2}}\right)}{2c^3\sqrt{\sqrt{b^2-4ac}+b}}$$
$$-\frac{\sqrt{e}\operatorname{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)\left(-\frac{3abce-2ac^2d+b^3(-e)+b^2cd}{\sqrt{b^2-4ac}}+ace+b^2(-e)+bcd\right)}{2c^3}$$
$$-\frac{\sqrt{e}\operatorname{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)\left(\frac{3abce-2ac^2d+b^3(-e)+b^2cd}{\sqrt{b^2-4ac}}+ace+b^2(-e)+bcd\right)}{2c^3}$$
$$+\frac{d\operatorname{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)(3cd-4be)}{8c^2\sqrt{e}}+\frac{x\sqrt{d+ex^2}(3cd-4be)}{8c^2}+\frac{x(d+ex^2)^{3/2}}{4c}$$

```
[In] Int[(x^4*(d + e*x^2)^(3/2))/(a + b*x^2 + c*x^4),x]
```

```
[Out] ((3*c*d - 4*b*e)*x*Sqrt[d + e*x^2])/(8*c^2) + (x*(d + e*x^2)^(3/2))/(4*c) -
 (Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]*(b*c*d - b^2*e + a*c*e - (b^2*c*d
 - 2*a*c^2*d - b^3*e + 3*a*b*c*e)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2*c*d - (
b - Sqrt[b^2 - 4*a*c])*e]*x)/(Sqrt[b - Sqrt[b^2 - 4*a*c]]*Sqrt[d + e*x^2])]
)/(2*c^3*Sqrt[b - Sqrt[b^2 - 4*a*c]]) - (Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c
])*e]*(b*c*d - b^2*e + a*c*e + (b^2*c*d - 2*a*c^2*d - b^3*e + 3*a*b*c*e)/Sq
rt[b^2 - 4*a*c])*ArcTan[(Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]*x)/(Sqrt[b
 + Sqrt[b^2 - 4*a*c]]*Sqrt[d + e*x^2])])/(2*c^3*Sqrt[b + Sqrt[b^2 - 4*a*c]]
) + (d*(3*c*d - 4*b*e)*ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]])/(8*c^2*Sqrt[e]
) - (Sqrt[e]*(b*c*d - b^2*e + a*c*e - (b^2*c*d - 2*a*c^2*d - b^3*e + 3*a*b*
c*e)/Sqrt[b^2 - 4*a*c])*ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]])/(2*c^3) - (Sq
```

rt[e]*(b*c*d - b^2*e + a*c*e + (b^2*c*d - 2*a*c^2*d - b^3*e + 3*a*b*c*e)/Sqrt[b^2 - 4*a*c])*ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]])/(2*c^3)

Rule 201

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Dist[a*n*(p/(n*p + 1)), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 396

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 399

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Di
st[b/d, Int[(a + b*x^n)^(p - 1), x], x] - Dist[(b*c - a*d)/d, Int[(a + b*x^
n)^(p - 1)/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, p}, x] && NeQ[b*c - a*
d, 0] && EqQ[n*(p - 1) + 1, 0] && IntegerQ[n]
```

Rule 1305

```
Int[(((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x_)^2)^(q_))/((a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4), x_Symbol] :> Dist[f^4/c^2, Int[(f*x)^(m - 4)*(c*d - b*e + c
*e*x^2)*(d + e*x^2)^(q - 1), x], x] - Dist[f^4/c^2, Int[(f*x)^(m - 4)*(d +
e*x^2)^(q - 1)*(Simp[a*(c*d - b*e) + (b*c*d - b^2*e + a*c*e)*x^2, x]/(a + b
*x^2 + c*x^4)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b^2 - 4*a*c,
0] &&  !IntegerQ[q] && GtQ[q, 0] && GtQ[m, 3]
```

Rule 1706

```
Int[(Px_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^
(p_.), x_Symbol] :> Int[ExpandIntegrand[Px*(d + e*x^2)^q*(a + b*x^2 + c*x^4
)^p, x], x] /; FreeQ[{a, b, c, d, e, q}, x] && PolyQ[Px, x^2] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[p]
```

Rubi steps

$$\text{integral} = \frac{\int \sqrt{d+ex^2}(cd-be+cex^2)\,dx}{c^2} - \frac{\int \frac{\sqrt{d+ex^2}(a(cd-be)+(bcd-b^2e+ace)x^2)}{a+bx^2+cx^4}\,dx}{c^2}$$

$$= \frac{x(d+ex^2)^{3/2}}{4c} - \frac{\int \left(\frac{\left(bcd-b^2e+ace+\frac{-b^2cd+2ac^2d+b^3e-3abce}{\sqrt{b^2-4ac}}\right)\sqrt{d+ex^2}}{b-\sqrt{b^2-4ac}+2cx^2} + \frac{\left(bcd-b^2e+ace-\frac{-b^2cd+2ac^2d+b^3e-3abce}{\sqrt{b^2-4ac}}\right)\sqrt{d+ex^2}}{b+\sqrt{b^2-4ac}+2cx^2}\right)dx}{c^2} + \frac{(3cd-4be)\int \sqrt{d+ex^2}\,dx}{4c^2}$$

$$= \frac{(3cd-4be)x\sqrt{d+ex^2}}{8c^2} + \frac{x(d+ex^2)^{3/2}}{4c} + \frac{(d(3cd-4be))\int \frac{1}{\sqrt{d+ex^2}}\,dx}{8c^2} - \frac{\left(bcd-b^2e+ace-\frac{b^2cd-2ac^2d-b^3e+3abce}{\sqrt{b^2-4ac}}\right)\int \frac{\sqrt{d+ex^2}}{b-\sqrt{b^2-4ac}+2cx^2}\,dx}{c^2} - \frac{\left(bcd-b^2e+ace+\frac{b^2cd-2ac^2d-b^3e+3abce}{\sqrt{b^2-4ac}}\right)\int \frac{\sqrt{d+ex^2}}{b+\sqrt{b^2-4ac}+2cx^2}\,dx}{c^2}$$

$$= \frac{(3cd-4be)x\sqrt{d+ex^2}}{8c^2} + \frac{x(d+ex^2)^{3/2}}{4c} + \frac{(d(3cd-4be))\text{Subst}\left(\int \frac{1}{1-ex^2}\,dx, x, \frac{x}{\sqrt{d+ex^2}}\right)}{8c^2} - \frac{\left(e\left(bcd-b^2e+ace-\frac{b^2cd-2ac^2d-b^3e+3abce}{\sqrt{b^2-4ac}}\right)\right)\int \frac{1}{\sqrt{d+ex^2}}\,dx}{2c^3} - \frac{\left(\left(2cd-\left(b-\sqrt{b^2-4ac}\right)e\right)\left(bcd-b^2e+ace-\frac{b^2cd-2ac^2d-b^3e+3abce}{\sqrt{b^2-4ac}}\right)\right)\int \frac{1}{\left(b-\sqrt{b^2-4ac}+2cx^2\right)\sqrt{d+ex^2}}\,dx}{2c^3} - \frac{\left(e\left(bcd-b^2e+ace+\frac{b^2cd-2ac^2d-b^3e+3abce}{\sqrt{b^2-4ac}}\right)\right)\int \frac{1}{\sqrt{d+ex^2}}\,dx}{2c^3} - \frac{\left(\left(2cd-\left(b+\sqrt{b^2-4ac}\right)e\right)\left(bcd-b^2e+ace+\frac{b^2cd-2ac^2d-b^3e+3abce}{\sqrt{b^2-4ac}}\right)\right)\int \frac{1}{\left(b+\sqrt{b^2-4ac}+2cx^2\right)\sqrt{d+ex^2}}\,dx}{2c^3}$$

$$= \frac{(3cd-4be)x\sqrt{d+ex^2}}{8c^2} + \frac{x(d+ex^2)^{3/2}}{4c} + \frac{d(3cd-4be)\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{8c^2\sqrt{e}} - \frac{\left(e\left(bcd-b^2e+ace-\frac{b^2cd-2ac^2d-b^3e+3abce}{\sqrt{b^2-4ac}}\right)\right)\text{Subst}\left(\int \frac{1}{1-ex^2}\,dx, x, \frac{x}{\sqrt{d+ex^2}}\right)}{2c^3} - \frac{\left(\left(2cd-\left(b-\sqrt{b^2-4ac}\right)e\right)\left(bcd-b^2e+ace-\frac{b^2cd-2ac^2d-b^3e+3abce}{\sqrt{b^2-4ac}}\right)\right)\text{Subst}\left(\int \frac{1}{b-\sqrt{b^2-4ac}-\left(-2cd+\right.}\right.}{2c^3} - \frac{\left(e\left(bcd-b^2e+ace+\frac{b^2cd-2ac^2d-b^3e+3abce}{\sqrt{b^2-4ac}}\right)\right)\text{Subst}\left(\int \frac{1}{1-ex^2}\,dx, x, \frac{x}{\sqrt{d+ex^2}}\right)}{2c^3} - \frac{\left(\left(2cd-\left(b+\sqrt{b^2-4ac}\right)e\right)\left(bcd-b^2e+ace+\frac{b^2cd-2ac^2d-b^3e+3abce}{\sqrt{b^2-4ac}}\right)\right)\text{Subst}\left(\int \frac{1}{b+\sqrt{b^2-4ac}-\left(-2cd+\right.}\right.}{2c^3}$$

$$= \frac{(3cd-4be)x\sqrt{d+ex^2}}{8c^2} + \frac{x(d+ex^2)^{3/2}}{4c}$$

$$- \frac{\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}\left(bcd-b^2e+ace-\frac{b^2cd-2ac^2d-b^3e+3abce}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}x}{\sqrt{b-\sqrt{b^2-4ac}}\sqrt{d+ex^2}}\right)}{2c^3\sqrt{b-\sqrt{b^2-4ac}}}$$

$$- \frac{\sqrt{2cd-\left(b+\sqrt{b^2-4ac}\right)e}\left(bcd-b^2e+ace+\frac{b^2cd-2ac^2d-b^3e+3abce}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2cd-\left(b+\sqrt{b^2-4ac}\right)e}x}{\sqrt{b+\sqrt{b^2-4ac}}\sqrt{d+ex^2}}\right)}{2c^3\sqrt{b+\sqrt{b^2-4ac}}}$$

$$+ \frac{d(3cd-4be)\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{8c^2\sqrt{e}}$$

$$- \frac{\sqrt{e}\left(bcd-b^2e+ace-\frac{b^2cd-2ac^2d-b^3e+3abce}{\sqrt{b^2-4ac}}\right)\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{2c^3}$$

$$- \frac{\sqrt{e}\left(bcd-b^2e+ace+\frac{b^2cd-2ac^2d-b^3e+3abce}{\sqrt{b^2-4ac}}\right)\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{2c^3}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 2.30 (sec) , antiderivative size = 1407, normalized size of antiderivative = 2.36

$$\int \frac{x^4(d+ex^2)^{3/2}}{a+bx^2+cx^4}\,dx = \frac{cx\sqrt{d+ex^2}(5cd-4be+2cex^2) + \frac{2(3c^2d^2+8b^2e^2-4ce(3bd+2ae))\text{arctanh}\left(\frac{\sqrt{e}x}{-\sqrt{d}+\sqrt{d+ex^2}}\right)}{\sqrt{e}} - 2\text{I}}{}$$

```
[In] Integrate[(x^4*(d + e*x^2)^(3/2))/(a + b*x^2 + c*x^4),x]
```

```
[Out] (c*x*Sqrt[d + e*x^2]*(5*c*d - 4*b*e + 2*c*e*x^2) + (2*(3*c^2*d^2 + 8*b^2*e^
2 - 4*c*e*(3*b*d + 2*a*e))*ArcTanh[(Sqrt[e]*x)/(-Sqrt[d] + Sqrt[d + e*x^2])
])/Sqrt[e] - 2*RootSum[a*e^4 + 4*b*d*e^2*#1^2 - 4*a*e^3*#1^2 + 16*c*d^2*#1^
4 - 8*b*d*e*#1^4 + 6*a*e^2*#1^4 + 4*b*d*#1^6 - 4*a*e*#1^6 + a*#1^8 & , (-(a
*c^2*d^2*e^3*Log[x]) + 2*a*b*c*d*e^4*Log[x] - a*b^2*e^5*Log[x] + a^2*c*e^5*
Log[x] + a*c^2*d^2*e^3*Log[-Sqrt[d] + Sqrt[d + e*x^2] - x*#1] - 2*a*b*c*d*e
^4*Log[-Sqrt[d] + Sqrt[d + e*x^2] - x*#1] + a*b^2*e^5*Log[-Sqrt[d] + Sqrt[d
 + e*x^2] - x*#1] - a^2*c*e^5*Log[-Sqrt[d] + Sqrt[d + e*x^2] - x*#1] - 4*b*
c^2*d^3*e*Log[x]*#1^2 + 8*b^2*c*d^2*e^2*Log[x]*#1^2 - 5*a*c^2*d^2*e^2*Log[x
]*#1^2 - 4*b^3*d*e^3*Log[x]*#1^2 + 2*a*b*c*d*e^3*Log[x]*#1^2 + 3*a*b^2*e^4*
Log[x]*#1^2 - 3*a^2*c*e^4*Log[x]*#1^2 + 4*b*c^2*d^3*e*Log[-Sqrt[d] + Sqrt[d
 + e*x^2] - x*#1]*#1^2 - 8*b^2*c*d^2*e^2*Log[-Sqrt[d] + Sqrt[d + e*x^2] - x
*#1]*#1^2 + 5*a*c^2*d^2*e^2*Log[-Sqrt[d] + Sqrt[d + e*x^2] - x*#1]*#1^2 + 4
```

```
*b^3*d*e^3*Log[-Sqrt[d] + Sqrt[d + e*x^2] - x*#1]*#1^2 - 2*a*b*c*d*e^3*Log[
-Sqrt[d] + Sqrt[d + e*x^2] - x*#1]*#1^2 - 3*a*b^2*e^4*Log[-Sqrt[d] + Sqrt[d
 + e*x^2] - x*#1]*#1^2 + 3*a^2*c*e^4*Log[-Sqrt[d] + Sqrt[d + e*x^2] - x*#1]
*#1^2 + 4*b*c^2*d^3*Log[x]*#1^4 - 8*b^2*c*d^2*e*Log[x]*#1^4 + 5*a*c^2*d^2*e
*Log[x]*#1^4 + 4*b^3*d*e^2*Log[x]*#1^4 - 2*a*b*c*d*e^2*Log[x]*#1^4 - 3*a*b^
2*e^3*Log[x]*#1^4 + 3*a^2*c*e^3*Log[x]*#1^4 - 4*b*c^2*d^3*Log[-Sqrt[d] + Sq
rt[d + e*x^2] - x*#1]*#1^4 + 8*b^2*c*d^2*e*Log[-Sqrt[d] + Sqrt[d + e*x^2] -
 x*#1]*#1^4 - 5*a*c^2*d^2*e*Log[-Sqrt[d] + Sqrt[d + e*x^2] - x*#1]*#1^4 - 4
*b^3*d*e^2*Log[-Sqrt[d] + Sqrt[d + e*x^2] - x*#1]*#1^4 + 2*a*b*c*d*e^2*Log[
-Sqrt[d] + Sqrt[d + e*x^2] - x*#1]*#1^4 + 3*a*b^2*e^3*Log[-Sqrt[d] + Sqrt[d
 + e*x^2] - x*#1]*#1^4 - 3*a^2*c*e^3*Log[-Sqrt[d] + Sqrt[d + e*x^2] - x*#1]
*#1^4 + a*c^2*d^2*Log[x]*#1^6 - 2*a*b*c*d*e*Log[x]*#1^6 + a*b^2*e^2*Log[x]*
#1^6 - a^2*c*e^2*Log[x]*#1^6 - a*c^2*d^2*Log[-Sqrt[d] + Sqrt[d + e*x^2] - x
*#1]*#1^6 + 2*a*b*c*d*e*Log[-Sqrt[d] + Sqrt[d + e*x^2] - x*#1]*#1^6 - a*b^2
*e^2*Log[-Sqrt[d] + Sqrt[d + e*x^2] - x*#1]*#1^6 + a^2*c*e^2*Log[-Sqrt[d] +
 Sqrt[d + e*x^2] - x*#1]*#1^6)/(b*d*e^2*#1 - a*e^3*#1 + 8*c*d^2*#1^3 - 4*b*
d*e*#1^3 + 3*a*e^2*#1^3 + 3*b*d*#1^5 - 3*a*e*#1^5 + a*#1^7) & ])/(8*c^3)
```

Maple [A] (verified)

Time = 1.37 (sec) , antiderivative size = 460, normalized size of antiderivative = 0.77

method	result
risch	$-\frac{x\left(-2c\,x^{2}e+4be-5cd\right)\sqrt{e\,x^{2}+d}}{8c^{2}}-\frac{\frac{\left(8e^{2}ac-8b^{2}e^{2}+12bcde-3c^{2}d^{2}\right)\ln\left(x\sqrt{e}+\sqrt{e\,x^{2}+d}\right)}{c\sqrt{e}}-\frac{4a\sqrt{2}\left(\left(\left(e^{2}ac-(be-cd)^{2}\right)\sqrt{-4d^{2}\left(ac-\frac{b^{2}}{4}\right.}\right.\right.}{}}{}$
pseudoelliptic	$-\frac{2\left(a\sqrt{2}\sqrt{\left(-2ae+bd+\sqrt{-4d^{2}\left(ac-\frac{b^{2}}{4}\right)}\right)a}\left(\frac{\left(\frac{\left(-ac+b^{2}\right)e^{\frac{5}{2}}}{2}+\frac{c^{2}d^{2}\sqrt{e}}{2}-b\,e^{\frac{3}{2}}cd\right)\sqrt{-4d^{2}\left(ac-\frac{b^{2}}{4}\right)}}{2}+\left(dc\left(ac-\frac{b^{2}}{2}\right)e^{\frac{3}{2}}-\frac{3b\left(\left(a\right.\right.}{}\right.\right.\right.}{}$
default	$\frac{\frac{x\left(e\,x^{2}+d\right)^{\frac{3}{2}}}{4}+\frac{3d\left(\frac{x\sqrt{e\,x^{2}+d}}{2}+\frac{d\ln\left(x\sqrt{e}+\sqrt{e\,x^{2}+d}\right)}{2\sqrt{e}}\right)}{4}}{c}-\frac{2\left(a\sqrt{2}\sqrt{\left(-2ae+bd+\sqrt{-4d^{2}\left(ac-\frac{b^{2}}{4}\right)}\right)a}\left(\frac{\left(\frac{\left(-ac+b^{2}\right)e^{\frac{5}{2}}}{2}+\frac{c^{2}d^{2}\sqrt{e}}{2}\right.}{}\right.\right.}{}$

```
[In] int(x^4*(e*x^2+d)^(3/2)/(c*x^4+b*x^2+a),x,method=_RETURNVERBOSE)
```

```
[Out] -1/8*x*(-2*c*e*x^2+4*b*e-5*c*d)*(e*x^2+d)^(1/2)/c^2-1/8/c^2*((8*a*c*e^2-8*b
^2*e^2+12*b*c*d*e-3*c^2*d^2)/c*ln(x*e^(1/2)+(e*x^2+d)^(1/2))/e^(1/2)-4/c/((
2*a*e-b*d+(-4*d^2*(a*c-1/4*b^2))^(1/2))*a)^(1/2)*a*2^(1/2)/((-2*a*e+b*d+(-4
*d^2*(a*c-1/4*b^2))^(1/2))*a)^(1/2)*(((e^2*a*c-(b*e-c*d)^2)*(-4*d^2*(a*c-1/
4*b^2))^(1/2)+3*(e*c*(b*e-4/3*c*d)*a-1/3*b*(b*e-c*d)^2)*d)*((-2*a*e+b*d+(-4
*d^2*(a*c-1/4*b^2))^(1/2))*a)^(1/2)*arctanh(a/x*(e*x^2+d)^(1/2)*2^(1/2)/((2
*a*e-b*d+(-4*d^2*(a*c-1/4*b^2))^(1/2))*a)^(1/2))-((2*a*e-b*d+(-4*d^2*(a*c-1
/4*b^2))^(1/2))*a)^(1/2)*arctan(a/x*(e*x^2+d)^(1/2)*2^(1/2)/((-2*a*e+b*d+(-
4*d^2*(a*c-1/4*b^2))^(1/2))*a)^(1/2))*((e^2*a*c-(b*e-c*d)^2)*(-4*d^2*(a*c-1
/4*b^2))^(1/2)-3*(e*c*(b*e-4/3*c*d)*a-1/3*b*(b*e-c*d)^2)*d))/(-4*d^2*(a*c-1
/4*b^2))^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 9908 vs. $2(515) = 1030$.

Time = 290.82 (sec) , antiderivative size = 19825, normalized size of antiderivative = 33.32

$$\int \frac{x^4(d+ex^2)^{3/2}}{a+bx^2+cx^4}\,dx = \text{Too large to display}$$

```
[In] integrate(x^4*(e*x^2+d)^(3/2)/(c*x^4+b*x^2+a),x, algorithm="fricas")
[Out] Too large to include
```

Sympy [F]

$$\int \frac{x^4(d+ex^2)^{3/2}}{a+bx^2+cx^4}\,dx = \int \frac{x^4(d+ex^2)^{\frac{3}{2}}}{a+bx^2+cx^4}\,dx$$

```
[In] integrate(x**4*(e*x**2+d)**(3/2)/(c*x**4+b*x**2+a),x)
[Out] Integral(x**4*(d + e*x**2)**(3/2)/(a + b*x**2 + c*x**4), x)
```

Maxima [F]

$$\int \frac{x^4(d+ex^2)^{3/2}}{a+bx^2+cx^4}\,dx = \int \frac{(ex^2+d)^{\frac{3}{2}}x^4}{cx^4+bx^2+a}\,dx$$

```
[In] integrate(x^4*(e*x^2+d)^(3/2)/(c*x^4+b*x^2+a),x, algorithm="maxima")
[Out] integrate((e*x^2 + d)^(3/2)*x^4/(c*x^4 + b*x^2 + a), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{x^4(d+ex^2)^{3/2}}{a+bx^2+cx^4}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(x^4*(e*x^2+d)^(3/2)/(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:index.cc index_m i_lex_is_greater Err
or: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^4(d+ex^2)^{3/2}}{a+bx^2+cx^4}\,dx = \int \frac{x^4\,(e\,x^2+d)^{3/2}}{c\,x^4+b\,x^2+a}\,dx$$

```
[In] int((x^4*(d + e*x^2)^(3/2))/(a + b*x^2 + c*x^4),x)
```

```
[Out] int((x^4*(d + e*x^2)^(3/2))/(a + b*x^2 + c*x^4), x)
```

3.372 $\int \frac{x^2(d+ex^2)^{3/2}}{a+bx^2+cx^4}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 491

$$\begin{aligned}\int \frac{x^2(d+ex^2)^{3/2}}{a+bx^2+cx^4}\,dx &= \frac{ex\sqrt{d+ex^2}}{2c} \\ &+ \frac{\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}\left(cd-be-\frac{bcd-b^2e+2ace}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}\,x}{\sqrt{b-\sqrt{b^2-4ac}}\sqrt{d+ex^2}}\right)}{2c^2\sqrt{b-\sqrt{b^2-4ac}}} \\ &+ \frac{\sqrt{2cd-\left(b+\sqrt{b^2-4ac}\right)e}\left(cd-be+\frac{bcd-b^2e+2ace}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2cd-\left(b+\sqrt{b^2-4ac}\right)e}\,x}{\sqrt{b+\sqrt{b^2-4ac}}\sqrt{d+ex^2}}\right)}{2c^2\sqrt{b+\sqrt{b^2-4ac}}} \\ &+ \frac{d\sqrt{e}\,\text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{2c} + \frac{\sqrt{e}\left(cd-be-\frac{bcd-b^2e+2ace}{\sqrt{b^2-4ac}}\right)\text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{2c^2} \\ &+ \frac{\sqrt{e}\left(cd-be+\frac{bcd-b^2e+2ace}{\sqrt{b^2-4ac}}\right)\text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{2c^2}\end{aligned}$$

```
[Out] 1/2*d*arctanh(x*e^(1/2)/(e*x^2+d)^(1/2))*e^(1/2)/c+1/2*arctanh(x*e^(1/2)/(e
*x^2+d)^(1/2))*(c*d-b*e+(-2*a*c*e+b^2*e-b*c*d)/(-4*a*c+b^2)^(1/2))*e^(1/2)/
c^2+1/2*arctanh(x*e^(1/2)/(e*x^2+d)^(1/2))*(c*d-b*e+(2*a*c*e-b^2*e+b*c*d)/(
-4*a*c+b^2)^(1/2))*e^(1/2)/c^2+1/2*e*x*(e*x^2+d)^(1/2)/c+1/2*arctan(x*(2*c*
d-e*(b-(-4*a*c+b^2)^(1/2)))^(1/2)/(e*x^2+d)^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1
/2))*(c*d-b*e+(-2*a*c*e+b^2*e-b*c*d)/(-4*a*c+b^2)^(1/2))*(2*c*d-e*(b-(-4*a*
c+b^2)^(1/2)))^(1/2)/c^2/(b-(-4*a*c+b^2)^(1/2))^(1/2)+1/2*arctan(x*(2*c*d-e
*(b+(-4*a*c+b^2)^(1/2)))^(1/2)/(e*x^2+d)^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2)
)*(c*d-b*e+(2*a*c*e-b^2*e+b*c*d)/(-4*a*c+b^2)^(1/2))*(2*c*d-e*(b+(-4*a*c+b^
2)^(1/2)))^(1/2)/c^2/(b+(-4*a*c+b^2)^(1/2))^(1/2)
```

Rubi [A] (verified)

Time = 1.25 (sec) , antiderivative size = 491, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.276$, Rules used = {1307, 201, 223, 212, 1706, 399, 385, 211}

$$\int \frac{x^2(d+ex^2)^{3/2}}{a+bx^2+cx^4}\,dx = \frac{\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}\left(-\frac{2ace+b^2(-e)+bcd}{\sqrt{b^2-4ac}}-be+cd\right)\arctan\left(\frac{x\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}{\sqrt{b-\sqrt{b^2-4ac}}\sqrt{d+ex^2}}\right)}{2c^2\sqrt{b-\sqrt{b^2-4ac}}}$$
$$+\frac{\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}\left(\frac{2ace+b^2(-e)+bcd}{\sqrt{b^2-4ac}}-be+cd\right)\arctan\left(\frac{x\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}{\sqrt{\sqrt{b^2-4ac}+b}\sqrt{d+ex^2}}\right)}{2c^2\sqrt{\sqrt{b^2-4ac}+b}}$$
$$+\frac{\sqrt{e}\operatorname{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)\left(-\frac{2ace+b^2(-e)+bcd}{\sqrt{b^2-4ac}}-be+cd\right)}{2c^2}$$
$$+\frac{\sqrt{e}\operatorname{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)\left(\frac{2ace+b^2(-e)+bcd}{\sqrt{b^2-4ac}}-be+cd\right)}{2c^2}+\frac{d\sqrt{e}\operatorname{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{2c}+\frac{ex\sqrt{d+ex^2}}{2c}$$

```
[In] Int[(x^2*(d + e*x^2)^(3/2))/(a + b*x^2 + c*x^4),x]
```

```
[Out] (e*x*Sqrt[d + e*x^2])/(2*c) + (Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]*(c*d
 - b*e - (b*c*d - b^2*e + 2*a*c*e)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2*c*d -
(b - Sqrt[b^2 - 4*a*c])*e]*x)/(Sqrt[b - Sqrt[b^2 - 4*a*c]]*Sqrt[d + e*x^2])
])/(2*c^2*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*
c])*e]*(c*d - b*e + (b*c*d - b^2*e + 2*a*c*e)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sq
rt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]*x)/(Sqrt[b + Sqrt[b^2 - 4*a*c]]*Sqrt[
d + e*x^2])])/(2*c^2*Sqrt[b + Sqrt[b^2 - 4*a*c]]) + (d*Sqrt[e]*ArcTanh[(Sqr
t[e]*x)/Sqrt[d + e*x^2]])/(2*c) + (Sqrt[e]*(c*d - b*e - (b*c*d - b^2*e + 2*
a*c*e)/Sqrt[b^2 - 4*a*c])*ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]])/(2*c^2) + (
Sqrt[e]*(c*d - b*e + (b*c*d - b^2*e + 2*a*c*e)/Sqrt[b^2 - 4*a*c])*ArcTanh[(
Sqrt[e]*x)/Sqrt[d + e*x^2]])/(2*c^2)
```

Rule 201

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Dist[a*n*(p/(n*p + 1)), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 399

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Di
st[b/d, Int[(a + b*x^n)^(p - 1), x], x] - Dist[(b*c - a*d)/d, Int[(a + b*x^
n)^(p - 1)/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, p}, x] && NeQ[b*c - a*
d, 0] && EqQ[n*(p - 1) + 1, 0] && IntegerQ[n]
```

Rule 1307

```
Int[(((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x_)^2)^(q_))/((a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4), x_Symbol] :> Dist[e*(f^2/c), Int[(f*x)^(m - 2)*(d + e*x^2)^
(q - 1), x], x] - Dist[f^2/c, Int[(f*x)^(m - 2)*(d + e*x^2)^(q - 1)*(Simp[a
*e - (c*d - b*e)*x^2, x]/(a + b*x^2 + c*x^4)), x], x] /; FreeQ[{a, b, c, d,
 e, f}, x] && NeQ[b^2 - 4*a*c, 0] &&  !IntegerQ[q] && GtQ[q, 0] && GtQ[m, 1
] && LeQ[m, 3]
```

Rule 1706

```
Int[(Px_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^
(p_.), x_Symbol] :> Int[ExpandIntegrand[Px*(d + e*x^2)^q*(a + b*x^2 + c*x^4
)^p, x], x] /; FreeQ[{a, b, c, d, e, q}, x] && PolyQ[Px, x^2] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[p]
```

Rubi steps

$$\text{integral} = -\frac{\int \frac{\sqrt{d+ex^2}(ae-(cd-be)x^2)}{a+bx^2+cx^4}\,dx}{c} + \frac{e\int \sqrt{d+ex^2}\,dx}{c}$$

$$= \frac{ex\sqrt{d+ex^2}}{2c} - \frac{\int \left(\frac{\left(-cd+be+\frac{bcd-b^2e+2ace}{\sqrt{b^2-4ac}}\right)\sqrt{d+ex^2}}{b-\sqrt{b^2-4ac}+2cx^2} + \frac{\left(-cd+be-\frac{bcd-b^2e+2ace}{\sqrt{b^2-4ac}}\right)\sqrt{d+ex^2}}{b+\sqrt{b^2-4ac}+2cx^2}\right) dx}{c} + \frac{(de)\int \frac{1}{\sqrt{d+ex^2}}\, dx}{2c}$$

$$= \frac{ex\sqrt{d+ex^2}}{2c} + \frac{(de)\text{Subst}\left(\int \frac{1}{1-ex^2}\, dx, x, \frac{x}{\sqrt{d+ex^2}}\right)}{2c} + \frac{\left(cd-be+\frac{bcd-b^2e+2ace}{\sqrt{b^2-4ac}}\right)\int \frac{\sqrt{d+ex^2}}{b+\sqrt{b^2-4ac}+2cx^2}\, dx}{c} - \frac{\left(-cd+be+\frac{bcd-b^2e+2ace}{\sqrt{b^2-4ac}}\right)\int \frac{\sqrt{d+ex^2}}{b-\sqrt{b^2-4ac}+2cx^2}\, dx}{c}$$

$$= \frac{ex\sqrt{d+ex^2}}{2c} + \frac{d\sqrt{e}\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{2c} + \frac{\left(e\left(cd-be-\frac{bcd-b^2e+2ace}{\sqrt{b^2-4ac}}\right)\right)\int \frac{1}{\sqrt{d+ex^2}}\, dx}{2c^2} + \frac{\left(\left(2cd-\left(b-\sqrt{b^2-4ac}\right)e\right)\left(cd-be-\frac{bcd-b^2e+2ace}{\sqrt{b^2-4ac}}\right)\right)\int \frac{1}{\left(b-\sqrt{b^2-4ac}+2cx^2\right)\sqrt{d+ex^2}}\, dx}{2c^2} + \frac{\left(e\left(cd-be+\frac{bcd-b^2e+2ace}{\sqrt{b^2-4ac}}\right)\right)\int \frac{1}{\sqrt{d+ex^2}}\, dx}{2c^2} + \frac{\left(\left(2cd-\left(b+\sqrt{b^2-4ac}\right)e\right)\left(cd-be+\frac{bcd-b^2e+2ace}{\sqrt{b^2-4ac}}\right)\right)\int \frac{1}{\left(b+\sqrt{b^2-4ac}+2cx^2\right)\sqrt{d+ex^2}}\, dx}{2c^2}$$

$$= \frac{ex\sqrt{d+ex^2}}{2c} + \frac{d\sqrt{e}\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{2c} + \frac{\left(e\left(cd-be-\frac{bcd-b^2e+2ace}{\sqrt{b^2-4ac}}\right)\right)\text{Subst}\left(\int \frac{1}{1-ex^2}\, dx, x, \frac{x}{\sqrt{d+ex^2}}\right)}{2c^2} + \frac{\left(\left(2cd-\left(b-\sqrt{b^2-4ac}\right)e\right)\left(cd-be-\frac{bcd-b^2e+2ace}{\sqrt{b^2-4ac}}\right)\right)\text{Subst}\left(\int \frac{1}{b-\sqrt{b^2-4ac}-\left(-2cd+\left(b-\sqrt{b^2-4ac}\right)e\right)x^2}\, d\right.}{2c^2} + \frac{\left(e\left(cd-be+\frac{bcd-b^2e+2ace}{\sqrt{b^2-4ac}}\right)\right)\text{Subst}\left(\int \frac{1}{1-ex^2}\, dx, x, \frac{x}{\sqrt{d+ex^2}}\right)}{2c^2} + \frac{\left(\left(2cd-\left(b+\sqrt{b^2-4ac}\right)e\right)\left(cd-be+\frac{bcd-b^2e+2ace}{\sqrt{b^2-4ac}}\right)\right)\text{Subst}\left(\int \frac{1}{b+\sqrt{b^2-4ac}-\left(-2cd+\left(b+\sqrt{b^2-4ac}\right)e\right)x^2}\, d\right.}{2c^2}$$

$$= \frac{ex\sqrt{d+ex^2}}{2c} + \frac{\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}\left(cd-be-\frac{bcd-b^2e+2ace}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}x}{\sqrt{b-\sqrt{b^2-4ac}}\sqrt{d+ex^2}}\right)}{2c^2\sqrt{b-\sqrt{b^2-4ac}}} + \frac{\sqrt{2cd-\left(b+\sqrt{b^2-4ac}\right)e}\left(cd-be+\frac{bcd-b^2e+2ace}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2cd-\left(b+\sqrt{b^2-4ac}\right)e}x}{\sqrt{b+\sqrt{b^2-4ac}}\sqrt{d+ex^2}}\right)}{2c^2\sqrt{b+\sqrt{b^2-4ac}}} + \frac{d\sqrt{e}\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{2c} + \frac{\sqrt{e}\left(cd-be-\frac{bcd-b^2e+2ace}{\sqrt{b^2-4ac}}\right)\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{2c^2} + \frac{\sqrt{e}\left(cd-be+\frac{bcd-b^2e+2ace}{\sqrt{b^2-4ac}}\right)\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{2c^2}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 1.38 (sec) , antiderivative size = 916, normalized size of antiderivative = 1.87

$$\int \frac{x^2(d+ex^2)^{3/2}}{a+bx^2+cx^4}\,dx = \frac{2cex\sqrt{d+ex^2}+4\sqrt{e}(3cd-2be)\text{arctanh}\left(\frac{\sqrt{e}x}{-\sqrt{d}+\sqrt{d+ex^2}}\right)+\text{RootSum}\left[ae^4+4bde^2\#1\right.}{}$$

```
[In] Integrate[(x^2*(d + e*x^2)^(3/2))/(a + b*x^2 + c*x^4),x]
```

```
[Out] (2*c*e*x*Sqrt[d + e*x^2] + 4*Sqrt[e]*(3*c*d - 2*b*e)*ArcTanh[(Sqrt[e]*x)/(-
Sqrt[d] + Sqrt[d + e*x^2])] + RootSum[a*e^4 + 4*b*d*e^2*#1^2 - 4*a*e^3*#1^2
 + 16*c*d^2*#1^4 - 8*b*d*e*#1^4 + 6*a*e^2*#1^4 + 4*b*d*#1^6 - 4*a*e*#1^6 +
a*#1^8 & , (2*a*c*d*e^4*Log[x] - a*b*e^5*Log[x] - 2*a*c*d*e^4*Log[-Sqrt[d]
+ Sqrt[d + e*x^2] - x*#1] + a*b*e^5*Log[-Sqrt[d] + Sqrt[d + e*x^2] - x*#1]
- 4*c^2*d^3*e*Log[x]*#1^2 + 8*b*c*d^2*e^2*Log[x]*#1^2 - 4*b^2*d*e^3*Log[x]*
#1^2 - 2*a*c*d*e^3*Log[x]*#1^2 + 3*a*b*e^4*Log[x]*#1^2 + 4*c^2*d^3*e*Log[-S
qrt[d] + Sqrt[d + e*x^2] - x*#1]*#1^2 - 8*b*c*d^2*e^2*Log[-Sqrt[d] + Sqrt[d
 + e*x^2] - x*#1]*#1^2 + 4*b^2*d*e^3*Log[-Sqrt[d] + Sqrt[d + e*x^2] - x*#1]
*#1^2 + 2*a*c*d*e^3*Log[-Sqrt[d] + Sqrt[d + e*x^2] - x*#1]*#1^2 - 3*a*b*e^4
*Log[-Sqrt[d] + Sqrt[d + e*x^2] - x*#1]*#1^2 + 4*c^2*d^3*Log[x]*#1^4 - 8*b*
c*d^2*e*Log[x]*#1^4 + 4*b^2*d*e^2*Log[x]*#1^4 + 2*a*c*d*e^2*Log[x]*#1^4 - 3
*a*b*e^3*Log[x]*#1^4 - 4*c^2*d^3*Log[-Sqrt[d] + Sqrt[d + e*x^2] - x*#1]*#1^
4 + 8*b*c*d^2*e*Log[-Sqrt[d] + Sqrt[d + e*x^2] - x*#1]*#1^4 - 4*b^2*d*e^2*L
og[-Sqrt[d] + Sqrt[d + e*x^2] - x*#1]*#1^4 - 2*a*c*d*e^2*Log[-Sqrt[d] + Sqr
t[d + e*x^2] - x*#1]*#1^4 + 3*a*b*e^3*Log[-Sqrt[d] + Sqrt[d + e*x^2] - x*#1
]*#1^4 - 2*a*c*d*e*Log[x]*#1^6 + a*b*e^2*Log[x]*#1^6 + 2*a*c*d*e*Log[-Sqrt[
```

```
d] + Sqrt[d + e*x^2] - x*#1]*#1^6 - a*b*e^2*Log[-Sqrt[d] + Sqrt[d + e*x^2]
- x*#1]*#1^6)/(b*d*e^2*#1 - a*e^3*#1 + 8*c*d^2*#1^3 - 4*b*d*e*#1^3 + 3*a*e^
2*#1^3 + 3*b*d*#1^5 - 3*a*e*#1^5 + a*#1^7) & ])/(4*c^2)
```

Maple [A] (verified)

Time = 0.75 (sec) , antiderivative size = 394, normalized size of antiderivative = 0.80

method	result
risch	$\frac{ex\sqrt{e\,x^2+d}}{2c}-\frac{\frac{\sqrt{e}\,(2be-3cd)\ln\left(x\sqrt{e}+\sqrt{e\,x^2+d}\right)}{c}+\frac{a\sqrt{2}\left(\frac{\left(2acd\,e^2-b^2d\,e^2+2d^2ebc-2c^2d^3+\sqrt{-d^2\left(4ac-b^2\right)}\,b\,e^2-2\sqrt{-d^2\left(4ac-b^2\right)}\,cde\right.}{\sqrt{\left(-2ae+bd+\sqrt{-d^2\left(4ac-b^2\right)}\right)}}\right.}{}}{}$
default	$-\frac{a\left(\left(e^{\frac{3}{2}}cd-\frac{b\,e^{\frac{5}{2}}}{2}\right)\sqrt{-4d^2\left(ac-\frac{b^2}{4}\right)}+d\left(\left(ac-\frac{b^2}{2}\right)e^{\frac{5}{2}}+dc\left(-cd\sqrt{e}+e^{\frac{3}{2}}b\right)\right)\right)\sqrt{2}\sqrt{\left(-2ae+bd+\sqrt{-4d^2\left(ac-\frac{b^2}{4}\right)}\right)a}\ \arctan}{}$
pseudoelliptic	$-\frac{a\left(\left(e^{\frac{3}{2}}cd-\frac{b\,e^{\frac{5}{2}}}{2}\right)\sqrt{-4d^2\left(ac-\frac{b^2}{4}\right)}+d\left(\left(ac-\frac{b^2}{2}\right)e^{\frac{5}{2}}+dc\left(-cd\sqrt{e}+e^{\frac{3}{2}}b\right)\right)\right)\sqrt{2}\sqrt{\left(-2ae+bd+\sqrt{-4d^2\left(ac-\frac{b^2}{4}\right)}\right)a}\ \arctan}{}$

```
[In] int(x^2*(e*x^2+d)^(3/2)/(c*x^4+b*x^2+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*e*x*(e*x^2+d)^(1/2)/c-1/2/c*(e^(1/2)*(2*b*e-3*c*d)/c*ln(x*e^(1/2)+(e*x^
2+d)^(1/2))+1/c*a*2^(1/2)/(-d^2*(4*a*c-b^2))^(1/2)*((2*a*c*d*e^2-b^2*d*e^2+
2*d^2*e*b*c-2*c^2*d^3+(-d^2*(4*a*c-b^2))^(1/2)*b*e^2-2*(-d^2*(4*a*c-b^2))^(
1/2)*c*d*e)/((-2*a*e+b*d+(-d^2*(4*a*c-b^2))^(1/2))*a)^(1/2)*arctan(a/x*(e*x
^2+d)^(1/2)*2^(1/2)/((-2*a*e+b*d+(-d^2*(4*a*c-b^2))^(1/2))*a)^(1/2))-(-2*a*
c*d*e^2+b^2*d*e^2-2*d^2*e*b*c+2*c^2*d^3+(-d^2*(4*a*c-b^2))^(1/2)*b*e^2-2*(-
d^2*(4*a*c-b^2))^(1/2)*c*d*e)/((2*a*e-b*d+(-d^2*(4*a*c-b^2))^(1/2))*a)^(1/2
)*arctanh(a/x*(e*x^2+d)^(1/2)*2^(1/2)/((2*a*e-b*d+(-d^2*(4*a*c-b^2))^(1/2))
*a)^(1/2))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 5955 vs. $2(415) = 830$.

Time = 54.07 (sec) , antiderivative size = 11917, normalized size of antiderivative = 24.27

$$\int \frac{x^2(d+ex^2)^{3/2}}{a+bx^2+cx^4}\,dx = \text{Too large to display}$$

```
[In] integrate(x^2*(e*x^2+d)^(3/2)/(c*x^4+b*x^2+a),x, algorithm="fricas")
[Out] Too large to include
```

Sympy [F]

$$\int \frac{x^2(d+ex^2)^{3/2}}{a+bx^2+cx^4}\,dx = \int \frac{x^2(d+ex^2)^{\frac{3}{2}}}{a+bx^2+cx^4}\,dx$$

```
[In] integrate(x**2*(e*x**2+d)**(3/2)/(c*x**4+b*x**2+a),x)
[Out] Integral(x**2*(d + e*x**2)**(3/2)/(a + b*x**2 + c*x**4), x)
```

Maxima [F]

$$\int \frac{x^2(d+ex^2)^{3/2}}{a+bx^2+cx^4}\,dx = \int \frac{(ex^2+d)^{\frac{3}{2}}x^2}{cx^4+bx^2+a}\,dx$$

```
[In] integrate(x^2*(e*x^2+d)^(3/2)/(c*x^4+b*x^2+a),x, algorithm="maxima")
[Out] integrate((e*x^2 + d)^(3/2)*x^2/(c*x^4 + b*x^2 + a), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{x^2(d+ex^2)^{3/2}}{a+bx^2+cx^4}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(x^2*(e*x^2+d)^(3/2)/(c*x^4+b*x^2+a),x, algorithm="giac")
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:index.cc index_m i_lex_is_greater Err
or: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2(d+ex^2)^{3/2}}{a+bx^2+cx^4}\,dx=\int \frac{x^2\,\left(e\,x^2+d\right)^{3/2}}{c\,x^4+b\,x^2+a}\,dx$$

```
[In] int((x^2*(d + e*x^2)^(3/2))/(a + b*x^2 + c*x^4),x)
[Out] int((x^2*(d + e*x^2)^(3/2))/(a + b*x^2 + c*x^4), x)
```

3.373 $\int \frac{(d+ex^2)^{3/2}}{a+bx^2+cx^4}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 487

$$\int \frac{(d+ex^2)^{3/2}}{a+bx^2+cx^4}\,dx = \frac{\left(2c^2d^2+b\left(b-\sqrt{b^2-4ac}\right)e^2-2ce\left(bd-\sqrt{b^2-4ac}\,d+ae\right)\right)\arctan\left(\frac{\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}\,x}{\sqrt{b-\sqrt{b^2-4ac}}\sqrt{d+ex^2}}\right)}{c\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}}$$
$$-\frac{\left(2c^2d^2+b\left(b+\sqrt{b^2-4ac}\right)e^2-2ce\left(bd+\sqrt{b^2-4ac}\,d+ae\right)\right)\arctan\left(\frac{\sqrt{2cd-\left(b+\sqrt{b^2-4ac}\right)e}\,x}{\sqrt{b+\sqrt{b^2-4ac}}\sqrt{d+ex^2}}\right)}{c\sqrt{b^2-4ac}\sqrt{b+\sqrt{b^2-4ac}}\sqrt{2cd-\left(b+\sqrt{b^2-4ac}\right)e}}$$
$$+\frac{\sqrt{e}\left(3cd-\left(b-\sqrt{b^2-4ac}\right)e\right)\operatorname{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{2c\sqrt{b^2-4ac}}$$
$$-\frac{\sqrt{e}\left(3cd-\left(b+\sqrt{b^2-4ac}\right)e\right)\operatorname{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{2c\sqrt{b^2-4ac}}$$

```
[Out] 1/2*arctanh(x*e^(1/2)/(e*x^2+d)^(1/2))*(3*c*d-e*(b-(-4*a*c+b^2)^(1/2)))*e^(
1/2)/c/(-4*a*c+b^2)^(1/2)-1/2*arctanh(x*e^(1/2)/(e*x^2+d)^(1/2))*(3*c*d-e*(
b+(-4*a*c+b^2)^(1/2)))*e^(1/2)/c/(-4*a*c+b^2)^(1/2)+arctan(x*(2*c*d-e*(b-(-
4*a*c+b^2)^(1/2)))^(1/2)/(e*x^2+d)^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2))*(2*c
^2*d^2+b*e^2*(b-(-4*a*c+b^2)^(1/2))-2*c*e*(b*d+a*e-d*(-4*a*c+b^2)^(1/2)))/c
/(-4*a*c+b^2)^(1/2)/(2*c*d-e*(b-(-4*a*c+b^2)^(1/2)))^(1/2)/(b-(-4*a*c+b^2)^
(1/2))^(1/2)-arctan(x*(2*c*d-e*(b+(-4*a*c+b^2)^(1/2)))^(1/2)/(e*x^2+d)^(1/2
)/(b+(-4*a*c+b^2)^(1/2))^(1/2))*(2*c^2*d^2+b*e^2*(b+(-4*a*c+b^2)^(1/2))-2*c
*e*(b*d+a*e+d*(-4*a*c+b^2)^(1/2)))/c/(-4*a*c+b^2)^(1/2)/(b+(-4*a*c+b^2)^(1/
2))^(1/2)/(2*c*d-e*(b+(-4*a*c+b^2)^(1/2)))^(1/2)
```

Rubi [A] (verified)

Time = 1.14 (sec) , antiderivative size = 487, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.269, Rules used = {1188, 427, 537, 223, 212, 385, 211}

$$\int \frac{(d+ex^2)^{3/2}}{a+bx^2+cx^4}\,dx = \frac{\left(-2ce\left(-d\sqrt{b^2-4ac}+ae+bd\right)+be^2\left(b-\sqrt{b^2-4ac}\right)+2c^2d^2\right)\arctan\left(\frac{x\sqrt{2cd-e\left(b-\sqrt{b\right.}}}{\sqrt{b-\sqrt{b^2-4ac}}\sqrt{\vphantom{b}}}\right.}{c\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}$$
$$-\frac{\left(-2ce\left(d\sqrt{b^2-4ac}+ae+bd\right)+be^2\left(\sqrt{b^2-4ac}+b\right)+2c^2d^2\right)\arctan\left(\frac{x\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}{\sqrt{\sqrt{b^2-4ac}+b}\sqrt{d+ex^2}}\right)}{c\sqrt{b^2-4ac}\sqrt{\sqrt{b^2-4ac}+b}\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}$$
$$+\frac{\sqrt{e}\operatorname{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)\left(3cd-e\left(b-\sqrt{b^2-4ac}\right)\right)}{2c\sqrt{b^2-4ac}}$$
$$-\frac{\sqrt{e}\operatorname{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)\left(3cd-e\left(\sqrt{b^2-4ac}+b\right)\right)}{2c\sqrt{b^2-4ac}}$$

```
[In] Int[(d + e*x^2)^(3/2)/(a + b*x^2 + c*x^4),x]
```

```
[Out] ((2*c^2*d^2 + b*(b - Sqrt[b^2 - 4*a*c])*e^2 - 2*c*e*(b*d - Sqrt[b^2 - 4*a*c
]*d + a*e))*ArcTan[(Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]*x)/(Sqrt[b - Sq
rt[b^2 - 4*a*c]]*Sqrt[d + e*x^2])]])/(c*Sqrt[b^2 - 4*a*c]*Sqrt[b - Sqrt[b^2
- 4*a*c]]*Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]) - ((2*c^2*d^2 + b*(b + S
qrt[b^2 - 4*a*c])*e^2 - 2*c*e*(b*d + Sqrt[b^2 - 4*a*c]*d + a*e))*ArcTan[(Sq
rt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]*x)/(Sqrt[b + Sqrt[b^2 - 4*a*c]]*Sqrt[
d + e*x^2])])/(c*Sqrt[b^2 - 4*a*c]*Sqrt[b + Sqrt[b^2 - 4*a*c]]*Sqrt[2*c*d -
 (b + Sqrt[b^2 - 4*a*c])*e]) + (Sqrt[e]*(3*c*d - (b - Sqrt[b^2 - 4*a*c])*e)
*ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]])/(2*c*Sqrt[b^2 - 4*a*c]) - (Sqrt[e]*(
3*c*d - (b + Sqrt[b^2 - 4*a*c])*e)*ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]])/(2
*c*Sqrt[b^2 - 4*a*c])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 427

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[d*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(b*(n*(p + q) + 1))),
x] + Dist[1/(b*(n*(p + q) + 1)), Int[(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Simp
[c*(b*c*(n*(p + q) + 1) - a*d) + d*(b*c*(n*(p + 2*q - 1) + 1) - a*d*(n*(q -
 1) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d,
 0] && GtQ[q, 1] && NeQ[n*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[a
, b, c, d, n, p, q, x]
```

Rule 537

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 1188

```
Int[((d_) + (e_.)*(x_)^2)^(q_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symb
ol] :> With[{r = Rt[b^2 - 4*a*c, 2]}, Dist[2*(c/r), Int[(d + e*x^2)^q/(b -
r + 2*c*x^2), x], x] - Dist[2*(c/r), Int[(d + e*x^2)^q/(b + r + 2*c*x^2), x
], x]] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
- b*d*e + a*e^2, 0] &&  !IntegerQ[q]
```

Rubi steps

$$\text{integral} = \frac{(2c)\int \frac{(d+ex^2)^{3/2}}{b-\sqrt{b^2-4ac}+2cx^2}\,dx}{\sqrt{b^2-4ac}} - \frac{(2c)\int \frac{(d+ex^2)^{3/2}}{b+\sqrt{b^2-4ac}+2cx^2}\,dx}{\sqrt{b^2-4ac}}$$

$$= \frac{\int \frac{d\left(4cd-\left(b-\sqrt{b^2-4ac}\right)e\right)+2e\left(3cd-\left(b-\sqrt{b^2-4ac}\right)e\right)x^2}{\left(b-\sqrt{b^2-4ac}+2cx^2\right)\sqrt{d+ex^2}}\,dx}{2\sqrt{b^2-4ac}} - \frac{\int \frac{d\left(4cd-\left(b+\sqrt{b^2-4ac}\right)e\right)+2e\left(3cd-\left(b+\sqrt{b^2-4ac}\right)e\right)x^2}{\left(b+\sqrt{b^2-4ac}+2cx^2\right)\sqrt{d+ex^2}}\,dx}{2\sqrt{b^2-4ac}}$$

$$=\frac{\left(e\left(3cd-\left(b-\sqrt{b^2-4ac}\right)e\right)\right)\int\frac{1}{\sqrt{d+ex^2}}\,dx}{2c\sqrt{b^2-4ac}}-\frac{\left(e\left(3cd-\left(b+\sqrt{b^2-4ac}\right)e\right)\right)\int\frac{1}{\sqrt{d+ex^2}}\,dx}{2c\sqrt{b^2-4ac}}$$
$$+\frac{\left(2c^2d^2+b\left(b-\sqrt{b^2-4ac}\right)e^2-2ce\left(bd-\sqrt{b^2-4ac}d+ae\right)\right)\int\frac{1}{\left(b-\sqrt{b^2-4ac}+2cx^2\right)\sqrt{d+ex^2}}\,dx}{c\sqrt{b^2-4ac}}$$
$$-\frac{\left(2c^2d^2+b\left(b+\sqrt{b^2-4ac}\right)e^2-2ce\left(bd+\sqrt{b^2-4ac}d+ae\right)\right)\int\frac{1}{\left(b+\sqrt{b^2-4ac}+2cx^2\right)\sqrt{d+ex^2}}\,dx}{c\sqrt{b^2-4ac}}$$

$$=\frac{\left(e\left(3cd-\left(b-\sqrt{b^2-4ac}\right)e\right)\right)\text{Subst}\left(\int\frac{1}{1-ex^2}\,dx,x,\frac{x}{\sqrt{d+ex^2}}\right)}{2c\sqrt{b^2-4ac}}$$
$$-\frac{\left(e\left(3cd-\left(b+\sqrt{b^2-4ac}\right)e\right)\right)\text{Subst}\left(\int\frac{1}{1-ex^2}\,dx,x,\frac{x}{\sqrt{d+ex^2}}\right)}{2c\sqrt{b^2-4ac}}$$
$$+\frac{\left(2c^2d^2+b\left(b-\sqrt{b^2-4ac}\right)e^2-2ce\left(bd-\sqrt{b^2-4ac}d+ae\right)\right)\text{Subst}\left(\int\frac{1}{b-\sqrt{b^2-4ac}-\left(-2cd+\left(b-\sqrt{b^2-4a\right.\right.}}\right.}{c\sqrt{b^2-4ac}}$$
$$-\frac{\left(2c^2d^2+b\left(b+\sqrt{b^2-4ac}\right)e^2-2ce\left(bd+\sqrt{b^2-4ac}d+ae\right)\right)\text{Subst}\left(\int\frac{1}{b+\sqrt{b^2-4ac}-\left(-2cd+\left(b+\sqrt{b^2-4a\right.\right.}}\right.}{c\sqrt{b^2-4ac}}$$

$$=\frac{\left(2c^2d^2+b\left(b-\sqrt{b^2-4ac}\right)e^2-2ce\left(bd-\sqrt{b^2-4ac}d+ae\right)\right)\tan^{-1}\left(\frac{\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}x}{\sqrt{b-\sqrt{b^2-4ac}}\sqrt{d+ex^2}}\right)}{c\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}}$$
$$-\frac{\left(2c^2d^2+b\left(b+\sqrt{b^2-4ac}\right)e^2-2ce\left(bd+\sqrt{b^2-4ac}d+ae\right)\right)\tan^{-1}\left(\frac{\sqrt{2cd-\left(b+\sqrt{b^2-4ac}\right)e}x}{\sqrt{b+\sqrt{b^2-4ac}}\sqrt{d+ex^2}}\right)}{c\sqrt{b^2-4ac}\sqrt{b+\sqrt{b^2-4ac}}\sqrt{2cd-\left(b+\sqrt{b^2-4ac}\right)e}}$$
$$+\frac{\sqrt{e}\left(3cd-\left(b-\sqrt{b^2-4ac}\right)e\right)\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{2c\sqrt{b^2-4ac}}$$
$$-\frac{\sqrt{e}\left(3cd-\left(b+\sqrt{b^2-4ac}\right)e\right)\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{2c\sqrt{b^2-4ac}}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 8958 vs. 2(487) = 974.

Time = 16.20 (sec) , antiderivative size = 8958, normalized size of antiderivative = 18.39

$$\int \frac{(d+ex^2)^{3/2}}{a+bx^2+cx^4}\,dx = \text{Result too large to show}$$

```
[In] Integrate[(d + e*x^2)^(3/2)/(a + b*x^2 + c*x^4),x]
[Out] Result too large to show
```

Maple [A] (verified)

Time = 0.41 (sec) , antiderivative size = 411, normalized size of antiderivative = 0.84

method	result
default	$-\frac{\sqrt{\left(-2ae+bd+\sqrt{-4d^2\left(ac-\frac{b^2}{4}\right)}\right)a}\,\sqrt{2}\left(\left(a\,e^2-c\,d^2\right)\sqrt{-4d^2\left(ac-\frac{b^2}{4}\right)}+d\left(ab\,e^2-4acde+bc\,d^2\right)\right)\operatorname{arctanh}\left(\frac{a\sqrt{e}}{x\sqrt{\left(2ae-bd+\right.}}\right.}{}$
pseudoelliptic	$-\frac{\sqrt{\left(-2ae+bd+\sqrt{-4d^2\left(ac-\frac{b^2}{4}\right)}\right)a}\,\sqrt{2}\left(\left(a\,e^2-c\,d^2\right)\sqrt{-4d^2\left(ac-\frac{b^2}{4}\right)}+d\left(ab\,e^2-4acde+bc\,d^2\right)\right)\operatorname{arctanh}\left(\frac{a\sqrt{e}}{x\sqrt{\left(2ae-bd+\right.}}\right.}{}$

```
[In] int((e*x^2+d)^(3/2)/(c*x^4+b*x^2+a),x,method=_RETURNVERBOSE)
[Out] -1/2/((-2*a*e+b*d+(-4*d^2*(a*c-1/4*b^2))^(1/2))*a)^(1/2)*(((-2*a*e+b*d+(-4*
d^2*(a*c-1/4*b^2))^(1/2))*a)^(1/2)*2^(1/2)*((a*e^2-c*d^2)*(-4*d^2*(a*c-1/4*
b^2))^(1/2)+d*(a*b*e^2-4*a*c*d*e+b*c*d^2))*arctanh(a/x*(e*x^2+d)^(1/2)*2^(1
/2)/((2*a*e-b*d+(-4*d^2*(a*c-1/4*b^2))^(1/2))*a)^(1/2))-(2^(1/2)*((a*e^2-c*
d^2)*(-4*d^2*(a*c-1/4*b^2))^(1/2)-a*b*d*e^2+4*a*c*d^2*e-b*c*d^3)*arctan(a/x
*(e*x^2+d)^(1/2)*2^(1/2)/((-2*a*e+b*d+(-4*d^2*(a*c-1/4*b^2))^(1/2))*a)^(1/2
))+2*(-4*d^2*(a*c-1/4*b^2))^(1/2)*arctanh((e*x^2+d)^(1/2)/x/e^(1/2))*((-2*a
*e+b*d+(-4*d^2*(a*c-1/4*b^2))^(1/2))*a)^(1/2)*e^(3/2))*((2*a*e-b*d+(-4*d^2*
(a*c-1/4*b^2))^(1/2))*a)^(1/2))/((2*a*e-b*d+(-4*d^2*(a*c-1/4*b^2))^(1/2))*a
)^(1/2)/(-4*d^2*(a*c-1/4*b^2))^(1/2)/c
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3857 vs. 2(415) = 830.

Time = 15.60 (sec) , antiderivative size = 7721, normalized size of antiderivative = 15.85

$$\int \frac{(d+ex^2)^{3/2}}{a+bx^2+cx^4}\,dx = \text{Too large to display}$$

```
[In] integrate((e*x^2+d)^(3/2)/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] Too large to include
```

Sympy [F]

$$\int \frac{(d+ex^2)^{3/2}}{a+bx^2+cx^4}\,dx = \int \frac{(d+ex^2)^{\frac{3}{2}}}{a+bx^2+cx^4}\,dx$$

```
[In] integrate((e*x**2+d)**(3/2)/(c*x**4+b*x**2+a),x)
```

```
[Out] Integral((d + e*x**2)**(3/2)/(a + b*x**2 + c*x**4), x)
```

Maxima [F]

$$\int \frac{(d+ex^2)^{3/2}}{a+bx^2+cx^4}\,dx = \int \frac{(ex^2+d)^{\frac{3}{2}}}{cx^4+bx^2+a}\,dx$$

```
[In] integrate((e*x^2+d)^(3/2)/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

```
[Out] integrate((e*x^2 + d)^(3/2)/(c*x^4 + b*x^2 + a), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(d+ex^2)^{3/2}}{a+bx^2+cx^4}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate((e*x^2+d)^(3/2)/(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:index.cc index_m i_lex_is_greater Err
or: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex^2)^{3/2}}{a+bx^2+cx^4}\,dx=\int \frac{{\left(e\,x^2+d\right)}^{3/2}}{c\,x^4+b\,x^2+a} \,d x$$

```
[In] int((d + e*x^2)^(3/2)/(a + b*x^2 + c*x^4),x)
[Out] int((d + e*x^2)^(3/2)/(a + b*x^2 + c*x^4), x)
```

3.374 $\int \frac{(d+ex^2)^{3/2}}{x^2(a+bx^2+cx^4)}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 260

$$\int \frac{(d+ex^2)^{3/2}}{x^2(a+bx^2+cx^4)}\,dx = -\frac{d\sqrt{d+ex^2}}{ax} - \frac{\left(2cd-\left(b-\sqrt{b^2-4ac}\right)e\right)^{3/2}\arctan\left(\frac{\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}\,x}{\sqrt{b-\sqrt{b^2-4ac}}\sqrt{d+ex^2}}\right)}{\sqrt{b^2-4ac}\left(b-\sqrt{b^2-4ac}\right)^{3/2}} + \frac{\left(2cd-\left(b+\sqrt{b^2-4ac}\right)e\right)^{3/2}\arctan\left(\frac{\sqrt{2cd-\left(b+\sqrt{b^2-4ac}\right)e}\,x}{\sqrt{b+\sqrt{b^2-4ac}}\sqrt{d+ex^2}}\right)}{\sqrt{b^2-4ac}\left(b+\sqrt{b^2-4ac}\right)^{3/2}}$$

```
[Out] -arctan(x*(2*c*d-e*(b-(-4*a*c+b^2)^(1/2)))^(1/2)/(e*x^2+d)^(1/2)/(b-(-4*a*c
+b^2)^(1/2))^(1/2))*(2*c*d-e*(b-(-4*a*c+b^2)^(1/2)))^(3/2)/(b-(-4*a*c+b^2)^
(1/2))^(3/2)/(-4*a*c+b^2)^(1/2)+arctan(x*(2*c*d-e*(b+(-4*a*c+b^2)^(1/2)))^(
1/2)/(e*x^2+d)^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2))*(2*c*d-e*(b+(-4*a*c+b^2)
^(1/2)))^(3/2)/(-4*a*c+b^2)^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(3/2)-d*(e*x^2+d)^
(1/2)/a/x
```

Rubi [A] (verified)

Time = 0.60 (sec) , antiderivative size = 432, normalized size of antiderivative = 1.66, number of steps used = 16, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.276, Rules used = {1309, 283, 223, 212, 1706, 399, 385, 211}

$$\int \frac{(d+ex^2)^{3/2}}{x^2(a+bx^2+cx^4)}\,dx = -\frac{\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}\left(\frac{bd-2ae}{\sqrt{b^2-4ac}}+d\right)\arctan\left(\frac{x\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}{\sqrt{b-\sqrt{b^2-4ac}}\sqrt{d+ex^2}}\right)}{2a\sqrt{b-\sqrt{b^2-4ac}}} - \frac{\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}\left(d-\frac{bd-2ae}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{x\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}{\sqrt{\sqrt{b^2-4ac}+b}\sqrt{d+ex^2}}\right)}{2a\sqrt{\sqrt{b^2-4ac}+b}} - \frac{\sqrt{e}\text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)\left(d-\frac{bd-2ae}{\sqrt{b^2-4ac}}\right)}{2a} - \frac{\sqrt{e}\text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)\left(\frac{bd-2ae}{\sqrt{b^2-4ac}}+d\right)}{2a} + \frac{d\sqrt{e}\text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{a} - \frac{d\sqrt{d+ex^2}}{ax}$$

```
[In] Int[(d + e*x^2)^(3/2)/(x^2*(a + b*x^2 + c*x^4)),x]
```

```
[Out] -((d*Sqrt[d + e*x^2])/(a*x)) - (Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]*(d
+ (b*d - 2*a*e)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a
*c])*e]*x)/(Sqrt[b - Sqrt[b^2 - 4*a*c]]*Sqrt[d + e*x^2])])/(2*a*Sqrt[b - Sq
rt[b^2 - 4*a*c]]) - (Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]*(d - (b*d - 2*
a*e)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]*x)/
(Sqrt[b + Sqrt[b^2 - 4*a*c]]*Sqrt[d + e*x^2])])/(2*a*Sqrt[b + Sqrt[b^2 - 4*
a*c]]) + (d*Sqrt[e]*ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]])/a - (Sqrt[e]*(d -
 (b*d - 2*a*e)/Sqrt[b^2 - 4*a*c])*ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]])/(2*
a) - (Sqrt[e]*(d + (b*d - 2*a*e)/Sqrt[b^2 - 4*a*c])*ArcTanh[(Sqrt[e]*x)/Sqr
t[d + e*x^2]])/(2*a)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 283

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + 1))), x] - Dist[b*n*(p/(c^n*(m + 1))), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 399

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Di
st[b/d, Int[(a + b*x^n)^(p - 1), x], x] - Dist[(b*c - a*d)/d, Int[(a + b*x^
n)^(p - 1)/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, p}, x] && NeQ[b*c - a*
d, 0] && EqQ[n*(p - 1) + 1, 0] && IntegerQ[n]
```

Rule 1309

```
Int[(((f_.)*(x_))^(m_)*((d_.) + (e_.)*(x_)^2)^(q_))/((a_) + (b_.)*(x_)^2 +
(c_.)*(x_)^4), x_Symbol] :> Dist[d/a, Int[(f*x)^m*(d + e*x^2)^(q - 1), x],
x] - Dist[1/(a*f^2), Int[(f*x)^(m + 2)*(d + e*x^2)^(q - 1)*(Simp[b*d - a*e
+ c*d*x^2, x]/(a + b*x^2 + c*x^4)), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
&& NeQ[b^2 - 4*a*c, 0] &&  !IntegerQ[q] && GtQ[q, 0] && LtQ[m, 0]
```

Rule 1706

```
Int[(Px_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^
(p_.), x_Symbol] :> Int[ExpandIntegrand[Px*(d + e*x^2)^q*(a + b*x^2 + c*x^4
)^p, x], x] /; FreeQ[{a, b, c, d, e, q}, x] && PolyQ[Px, x^2] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[p]
```

Rubi steps

$$\text{integral} = -\frac{\int \frac{(bd-ae+cdx^2)\sqrt{d+ex^2}}{a+bx^2+cx^4}\,dx}{a} + \frac{d\int \frac{\sqrt{d+ex^2}}{x^2}\,dx}{a}$$

$$=-\frac{d\sqrt{d+ex^2}}{ax}-\frac{\int \left(\frac{\left(cd+\frac{c(bd-2ae)}{\sqrt{b^2-4ac}}\right)\sqrt{d+ex^2}}{b-\sqrt{b^2-4ac}+2cx^2}+\frac{\left(cd-\frac{c(bd-2ae)}{\sqrt{b^2-4ac}}\right)\sqrt{d+ex^2}}{b+\sqrt{b^2-4ac}+2cx^2}\right)dx}{a}+\frac{(de)\int \frac{1}{\sqrt{d+ex^2}}\,dx}{a}$$

$$=-\frac{d\sqrt{d+ex^2}}{ax}+\frac{(de)\text{Subst}\left(\int \frac{1}{1-ex^2}\,dx,x,\frac{x}{\sqrt{d+ex^2}}\right)}{a}-\frac{\left(c\left(d-\frac{bd-2ae}{\sqrt{b^2-4ac}}\right)\right)\int \frac{\sqrt{d+ex^2}}{b+\sqrt{b^2-4ac}+2cx^2}\,dx}{a}-\frac{\left(c\left(d+\frac{bd-2ae}{\sqrt{b^2-4ac}}\right)\right)\int \frac{\sqrt{d+ex^2}}{b-\sqrt{b^2-4ac}+2cx^2}\,dx}{a}$$

$$=-\frac{d\sqrt{d+ex^2}}{ax}+\frac{d\sqrt{e}\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{a}-\frac{\left(e\left(d-\frac{bd-2ae}{\sqrt{b^2-4ac}}\right)\right)\int \frac{1}{\sqrt{d+ex^2}}\,dx}{2a}-\frac{\left(\left(2cd-\left(b+\sqrt{b^2-4ac}\right)e\right)\left(d-\frac{bd-2ae}{\sqrt{b^2-4ac}}\right)\right)\int \frac{1}{\left(b+\sqrt{b^2-4ac}+2cx^2\right)\sqrt{d+ex^2}}\,dx}{2a}-\frac{\left(e\left(d+\frac{bd-2ae}{\sqrt{b^2-4ac}}\right)\right)\int \frac{1}{\sqrt{d+ex^2}}\,dx}{2a}-\frac{\left(\left(2cd-\left(b-\sqrt{b^2-4ac}\right)e\right)\left(d+\frac{bd-2ae}{\sqrt{b^2-4ac}}\right)\right)\int \frac{1}{\left(b-\sqrt{b^2-4ac}+2cx^2\right)\sqrt{d+ex^2}}\,dx}{2a}$$

$$=-\frac{d\sqrt{d+ex^2}}{ax}+\frac{d\sqrt{e}\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{a}-\frac{\left(e\left(d-\frac{bd-2ae}{\sqrt{b^2-4ac}}\right)\right)\text{Subst}\left(\int \frac{1}{1-ex^2}\,dx,x,\frac{x}{\sqrt{d+ex^2}}\right)}{2a}-\frac{\left(\left(2cd-\left(b+\sqrt{b^2-4ac}\right)e\right)\left(d-\frac{bd-2ae}{\sqrt{b^2-4ac}}\right)\right)\text{Subst}\left(\int \frac{1}{b+\sqrt{b^2-4ac}-\left(-2cd+\left(b+\sqrt{b^2-4ac}\right)e\right)x^2}\,dx,x,\frac{x}{\sqrt{d+ex^2}}\right)}{2a}-\frac{\left(e\left(d+\frac{bd-2ae}{\sqrt{b^2-4ac}}\right)\right)\text{Subst}\left(\int \frac{1}{1-ex^2}\,dx,x,\frac{x}{\sqrt{d+ex^2}}\right)}{2a}-\frac{\left(\left(2cd-\left(b-\sqrt{b^2-4ac}\right)e\right)\left(d+\frac{bd-2ae}{\sqrt{b^2-4ac}}\right)\right)\text{Subst}\left(\int \frac{1}{b-\sqrt{b^2-4ac}-\left(-2cd+\left(b-\sqrt{b^2-4ac}\right)e\right)x^2}\,dx,x,\frac{x}{\sqrt{d+ex^2}}\right)}{2a}$$

$$
\begin{aligned}
&= -\frac{d\sqrt{d+ex^2}}{ax} \\
&- \frac{\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}\left(d+\frac{bd-2ae}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}x}{\sqrt{b-\sqrt{b^2-4ac}}\sqrt{d+ex^2}}\right)}{2a\sqrt{b-\sqrt{b^2-4ac}}} \\
&- \frac{\sqrt{2cd-\left(b+\sqrt{b^2-4ac}\right)e}\left(d-\frac{bd-2ae}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2cd-\left(b+\sqrt{b^2-4ac}\right)e}x}{\sqrt{b+\sqrt{b^2-4ac}}\sqrt{d+ex^2}}\right)}{2a\sqrt{b+\sqrt{b^2-4ac}}} \\
&+ \frac{d\sqrt{e}\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{a} - \frac{\sqrt{e}\left(d-\frac{bd-2ae}{\sqrt{b^2-4ac}}\right)\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{2a} \\
&- \frac{\sqrt{e}\left(d+\frac{bd-2ae}{\sqrt{b^2-4ac}}\right)\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{2a}
\end{aligned}
$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 7491 vs. 2(260) = 520.

Time = 16.33 (sec) , antiderivative size = 7491, normalized size of antiderivative = 28.81

$$
\int \frac{(d+ex^2)^{3/2}}{x^2(a+bx^2+cx^4)}\,dx = \text{Result too large to show}
$$

```
[In] Integrate[(d + e*x^2)^(3/2)/(x^2*(a + b*x^2 + c*x^4)),x]
[Out] Result too large to show
```

Maple [A] (verified)

Time = 0.94 (sec) , antiderivative size = 347, normalized size of antiderivative = 1.33

method	result
risch	$-\frac{d\sqrt{e x^2+d}}{a x}-\frac{d\sqrt{2}\left(\frac{\left(-2e^2a^2+2abde+2d^2ac-b^2d^2+2\sqrt{-d^2\left(4ac-b^2\right)}\,ae-\sqrt{-d^2\left(4ac-b^2\right)}\,bd\right)\arctan\left(\frac{a\sqrt{e x^2+d}\,\sqrt{2}}{x\sqrt{\left(-2ae+bd+\sqrt{-d^2\left(4a\right.}\right.}}\right.}{\sqrt{\left(-2ae+bd+\sqrt{-d^2\left(4ac-b^2\right)}\right)a}}\right.}{2}$
pseudoelliptic	$-\frac{d\left(\frac{2\sqrt{e x^2+d}}{x}+\frac{\left(-2e^2a^2+2abde+2d^2ac-b^2d^2+2\sqrt{-d^2\left(4ac-b^2\right)}\,ae-\sqrt{-d^2\left(4ac-b^2\right)}\,bd\right)\sqrt{2}\arctan\left(\frac{a\sqrt{e x^2+d}\,\sqrt{2}}{x\sqrt{\left(-2ae+bd+\sqrt{-d^2\left(4ac-b\right.}\right.}}\right.}{\sqrt{-d^2\left(4ac-b^2\right)}\sqrt{\left(-2ae+bd+\sqrt{-d^2\left(4ac-b^2\right)}\right)a}}\right.}{2}$
default	$\frac{-\frac{\left(e x^2+d\right)^{\frac{5}{2}}}{dx}+\frac{4e\left(\frac{x\left(e x^2+d\right)^{\frac{3}{2}}}{4}+\frac{3d\left(\frac{x\sqrt{e x^2+d}}{2}+\frac{d\ln\left(x\sqrt{e}+\sqrt{e x^2+d}\right)}{2\sqrt{e}}\right)}{4}\right)}{d}}{a}+\frac{\left(\left(-\frac{b\sqrt{e}\,d}{2}+e^{\frac{3}{2}}a\right)\sqrt{-4d^2\left(ac-\frac{b^2}{4}\right)}-b\,e^{\frac{3}{2}}ad+a\right.}{}$

```
[In] int((e*x^2+d)^(3/2)/x^2/(c*x^4+b*x^2+a),x,method=_RETURNVERBOSE)
```

```
[Out] -d*(e*x^2+d)^(1/2)/a/x-1/2/a*d*2^(1/2)/(-d^2*(4*a*c-b^2))^(1/2)*((-2*e^2*a^
2+2*a*b*d*e+2*d^2*a*c-b^2*d^2+2*(-d^2*(4*a*c-b^2))^(1/2)*a*e-(-d^2*(4*a*c-b
^2))^(1/2)*b*d)/((-2*a*e+b*d+(-d^2*(4*a*c-b^2))^(1/2))*a)^(1/2)*arctan(a/x*
(e*x^2+d)^(1/2)*2^(1/2)/((-2*a*e+b*d+(-d^2*(4*a*c-b^2))^(1/2))*a)^(1/2))-(2
*e^2*a^2-2*a*b*d*e-2*d^2*a*c+b^2*d^2+2*(-d^2*(4*a*c-b^2))^(1/2)*a*e-(-d^2*(
4*a*c-b^2))^(1/2)*b*d)/((2*a*e-b*d+(-d^2*(4*a*c-b^2))^(1/2))*a)^(1/2)*arcta
nh(a/x*(e*x^2+d)^(1/2)*2^(1/2)/((2*a*e-b*d+(-d^2*(4*a*c-b^2))^(1/2))*a)^(1/
2)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 4059 vs. 2(218) = 436.

Time = 7.92 (sec) , antiderivative size = 4059, normalized size of antiderivative = 15.61

$$\int \frac{\left(d+ex^2\right)^{3/2}}{x^2\left(a+bx^2+cx^4\right)}\,dx=\text{Too large to display}$$

```
[In] integrate((e*x^2+d)^(3/2)/x^2/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] -1/4*(sqrt(1/2)*a*x*sqrt(-(3*a^2*b*d*e^2 - 2*a^3*e^3 + (b^3 - 3*a*b*c)*d^3
- 3*(a*b^2 - 2*a^2*c)*d^2*e + (a^3*b^2 - 4*a^4*c)*sqrt(-(18*a^3*b*d^3*e^3 -
 9*a^4*d^2*e^4 - (b^4 - 2*a*b^2*c + a^2*c^2)*d^6 + 6*(a*b^3 - a^2*b*c)*d^5*
e - 3*(5*a^2*b^2 - 2*a^3*c)*d^4*e^2)/(a^6*b^2 - 4*a^7*c)))/(a^3*b^2 - 4*a^4
*c))*log(-(12*a^3*b*d^3*e^3 - 6*a^4*d^2*e^4 - 2*(a*b^2*c - a^2*c^2)*d^6 + 2
*(a*b^3 + 2*a^2*b*c)*d^5*e - 4*(2*a^2*b^2 + a^3*c)*d^4*e^2 + ((a^3*b^2*c -
4*a^4*c^2)*d^3 - (a^3*b^3 - 4*a^4*b*c)*d^2*e + (a^4*b^2 - 4*a^5*c)*d*e^2)*x
^2*sqrt(-(18*a^3*b*d^3*e^3 - 9*a^4*d^2*e^4 - (b^4 - 2*a*b^2*c + a^2*c^2)*d^
6 + 6*(a*b^3 - a^2*b*c)*d^5*e - 3*(5*a^2*b^2 - 2*a^3*c)*d^4*e^2)/(a^6*b^2 -
 4*a^7*c)) + (27*a^3*b*d^2*e^4 - 12*a^4*d*e^5 + (b^3*c - a*b*c^2)*d^6 - (b^
4 + 6*a*b^2*c - 4*a^2*c^2)*d^5*e + 2*(4*a*b^3 + 5*a^2*b*c)*d^4*e^2 - 2*(11*
a^2*b^2 + 4*a^3*c)*d^3*e^3)*x^2 + 2*sqrt(1/2)*sqrt(e*x^2 + d)*(((a^4*b^3 -
4*a^5*b*c)*d - 2*(a^5*b^2 - 4*a^6*c)*e)*x*sqrt(-(18*a^3*b*d^3*e^3 - 9*a^4*d
^2*e^4 - (b^4 - 2*a*b^2*c + a^2*c^2)*d^6 + 6*(a*b^3 - a^2*b*c)*d^5*e - 3*(5
*a^2*b^2 - 2*a^3*c)*d^4*e^2)/(a^6*b^2 - 4*a^7*c)) - ((a*b^4 - 5*a^2*b^2*c +
 4*a^3*c^2)*d^4 - 3*(a^2*b^3 - 4*a^3*b*c)*d^3*e + 3*(a^3*b^2 - 4*a^4*c)*d^2
*e^2)*x)*sqrt(-(3*a^2*b*d*e^2 - 2*a^3*e^3 + (b^3 - 3*a*b*c)*d^3 - 3*(a*b^2
- 2*a^2*c)*d^2*e + (a^3*b^2 - 4*a^4*c)*sqrt(-(18*a^3*b*d^3*e^3 - 9*a^4*d^2*
e^4 - (b^4 - 2*a*b^2*c + a^2*c^2)*d^6 + 6*(a*b^3 - a^2*b*c)*d^5*e - 3*(5*a^
2*b^2 - 2*a^3*c)*d^4*e^2)/(a^6*b^2 - 4*a^7*c)))/(a^3*b^2 - 4*a^4*c)))/x^2)
- sqrt(1/2)*a*x*sqrt(-(3*a^2*b*d*e^2 - 2*a^3*e^3 + (b^3 - 3*a*b*c)*d^3 - 3*
(a*b^2 - 2*a^2*c)*d^2*e + (a^3*b^2 - 4*a^4*c)*sqrt(-(18*a^3*b*d^3*e^3 - 9*a
^4*d^2*e^4 - (b^4 - 2*a*b^2*c + a^2*c^2)*d^6 + 6*(a*b^3 - a^2*b*c)*d^5*e -
3*(5*a^2*b^2 - 2*a^3*c)*d^4*e^2)/(a^6*b^2 - 4*a^7*c)))/(a^3*b^2 - 4*a^4*c))
*log(-(12*a^3*b*d^3*e^3 - 6*a^4*d^2*e^4 - 2*(a*b^2*c - a^2*c^2)*d^6 + 2*(a*
b^3 + 2*a^2*b*c)*d^5*e - 4*(2*a^2*b^2 + a^3*c)*d^4*e^2 + ((a^3*b^2*c - 4*a^
4*c^2)*d^3 - (a^3*b^3 - 4*a^4*b*c)*d^2*e + (a^4*b^2 - 4*a^5*c)*d*e^2)*x^2*s
qrt(-(18*a^3*b*d^3*e^3 - 9*a^4*d^2*e^4 - (b^4 - 2*a*b^2*c + a^2*c^2)*d^6 +
6*(a*b^3 - a^2*b*c)*d^5*e - 3*(5*a^2*b^2 - 2*a^3*c)*d^4*e^2)/(a^6*b^2 - 4*a
^7*c)) + (27*a^3*b*d^2*e^4 - 12*a^4*d*e^5 + (b^3*c - a*b*c^2)*d^6 - (b^4 +
6*a*b^2*c - 4*a^2*c^2)*d^5*e + 2*(4*a*b^3 + 5*a^2*b*c)*d^4*e^2 - 2*(11*a^2*
b^2 + 4*a^3*c)*d^3*e^3)*x^2 - 2*sqrt(1/2)*sqrt(e*x^2 + d)*(((a^4*b^3 - 4*a^
5*b*c)*d - 2*(a^5*b^2 - 4*a^6*c)*e)*x*sqrt(-(18*a^3*b*d^3*e^3 - 9*a^4*d^2*e
^4 - (b^4 - 2*a*b^2*c + a^2*c^2)*d^6 + 6*(a*b^3 - a^2*b*c)*d^5*e - 3*(5*a^2
*b^2 - 2*a^3*c)*d^4*e^2)/(a^6*b^2 - 4*a^7*c)) - ((a*b^4 - 5*a^2*b^2*c + 4*a
^3*c^2)*d^4 - 3*(a^2*b^3 - 4*a^3*b*c)*d^3*e + 3*(a^3*b^2 - 4*a^4*c)*d^2*e^2
)*x)*sqrt(-(3*a^2*b*d*e^2 - 2*a^3*e^3 + (b^3 - 3*a*b*c)*d^3 - 3*(a*b^2 - 2*
a^2*c)*d^2*e + (a^3*b^2 - 4*a^4*c)*sqrt(-(18*a^3*b*d^3*e^3 - 9*a^4*d^2*e^4
- (b^4 - 2*a*b^2*c + a^2*c^2)*d^6 + 6*(a*b^3 - a^2*b*c)*d^5*e - 3*(5*a^2*b^
2 - 2*a^3*c)*d^4*e^2)/(a^6*b^2 - 4*a^7*c)))/(a^3*b^2 - 4*a^4*c)))/x^2) - sq
rt(1/2)*a*x*sqrt(-(3*a^2*b*d*e^2 - 2*a^3*e^3 + (b^3 - 3*a*b*c)*d^3 - 3*(a*b
^2 - 2*a^2*c)*d^2*e - (a^3*b^2 - 4*a^4*c)*sqrt(-(18*a^3*b*d^3*e^3 - 9*a^4*d
^2*e^4 - (b^4 - 2*a*b^2*c + a^2*c^2)*d^6 + 6*(a*b^3 - a^2*b*c)*d^5*e - 3*(5
*a^2*b^2 - 2*a^3*c)*d^4*e^2)/(a^6*b^2 - 4*a^7*c)))/(a^3*b^2 - 4*a^4*c))*log
(-(12*a^3*b*d^3*e^3 - 6*a^4*d^2*e^4 - 2*(a*b^2*c - a^2*c^2)*d^6 + 2*(a*b^3
```

```
+ 2*a^2*b*c)*d^5*e - 4*(2*a^2*b^2 + a^3*c)*d^4*e^2 - ((a^3*b^2*c - 4*a^4*c^
2)*d^3 - (a^3*b^3 - 4*a^4*b*c)*d^2*e + (a^4*b^2 - 4*a^5*c)*d*e^2)*x^2*sqrt(
-(18*a^3*b*d^3*e^3 - 9*a^4*d^2*e^4 - (b^4 - 2*a*b^2*c + a^2*c^2)*d^6 + 6*(a
*b^3 - a^2*b*c)*d^5*e - 3*(5*a^2*b^2 - 2*a^3*c)*d^4*e^2)/(a^6*b^2 - 4*a^7*c
)) + (27*a^3*b*d^2*e^4 - 12*a^4*d*e^5 + (b^3*c - a*b*c^2)*d^6 - (b^4 + 6*a*
b^2*c - 4*a^2*c^2)*d^5*e + 2*(4*a*b^3 + 5*a^2*b*c)*d^4*e^2 - 2*(11*a^2*b^2
+ 4*a^3*c)*d^3*e^3)*x^2 + 2*sqrt(1/2)*sqrt(e*x^2 + d)*(((a^4*b^3 - 4*a^5*b*
c)*d - 2*(a^5*b^2 - 4*a^6*c)*e)*x*sqrt(-(18*a^3*b*d^3*e^3 - 9*a^4*d^2*e^4 -
 (b^4 - 2*a*b^2*c + a^2*c^2)*d^6 + 6*(a*b^3 - a^2*b*c)*d^5*e - 3*(5*a^2*b^2
 - 2*a^3*c)*d^4*e^2)/(a^6*b^2 - 4*a^7*c)) + ((a*b^4 - 5*a^2*b^2*c + 4*a^3*c
^2)*d^4 - 3*(a^2*b^3 - 4*a^3*b*c)*d^3*e + 3*(a^3*b^2 - 4*a^4*c)*d^2*e^2)*x)
*sqrt(-(3*a^2*b*d*e^2 - 2*a^3*e^3 + (b^3 - 3*a*b*c)*d^3 - 3*(a*b^2 - 2*a^2*
c)*d^2*e - (a^3*b^2 - 4*a^4*c)*sqrt(-(18*a^3*b*d^3*e^3 - 9*a^4*d^2*e^4 - (b
^4 - 2*a*b^2*c + a^2*c^2)*d^6 + 6*(a*b^3 - a^2*b*c)*d^5*e - 3*(5*a^2*b^2 -
2*a^3*c)*d^4*e^2)/(a^6*b^2 - 4*a^7*c)))/(a^3*b^2 - 4*a^4*c)))/x^2) + sqrt(1
/2)*a*x*sqrt(-(3*a^2*b*d*e^2 - 2*a^3*e^3 + (b^3 - 3*a*b*c)*d^3 - 3*(a*b^2 -
 2*a^2*c)*d^2*e - (a^3*b^2 - 4*a^4*c)*sqrt(-(18*a^3*b*d^3*e^3 - 9*a^4*d^2*e
^4 - (b^4 - 2*a*b^2*c + a^2*c^2)*d^6 + 6*(a*b^3 - a^2*b*c)*d^5*e - 3*(5*a^2
*b^2 - 2*a^3*c)*d^4*e^2)/(a^6*b^2 - 4*a^7*c)))/(a^3*b^2 - 4*a^4*c))*log(-(1
2*a^3*b*d^3*e^3 - 6*a^4*d^2*e^4 - 2*(a*b^2*c - a^2*c^2)*d^6 + 2*(a*b^3 + 2*
a^2*b*c)*d^5*e - 4*(2*a^2*b^2 + a^3*c)*d^4*e^2 - ((a^3*b^2*c - 4*a^4*c^2)*d
^3 - (a^3*b^3 - 4*a^4*b*c)*d^2*e + (a^4*b^2 - 4*a^5*c)*d*e^2)*x^2*sqrt(-(18
*a^3*b*d^3*e^3 - 9*a^4*d^2*e^4 - (b^4 - 2*a*b^2*c + a^2*c^2)*d^6 + 6*(a*b^3
 - a^2*b*c)*d^5*e - 3*(5*a^2*b^2 - 2*a^3*c)*d^4*e^2)/(a^6*b^2 - 4*a^7*c)) +
 (27*a^3*b*d^2*e^4 - 12*a^4*d*e^5 + (b^3*c - a*b*c^2)*d^6 - (b^4 + 6*a*b^2*
c - 4*a^2*c^2)*d^5*e + 2*(4*a*b^3 + 5*a^2*b*c)*d^4*e^2 - 2*(11*a^2*b^2 + 4*
a^3*c)*d^3*e^3)*x^2 - 2*sqrt(1/2)*sqrt(e*x^2 + d)*(((a^4*b^3 - 4*a^5*b*c)*d
 - 2*(a^5*b^2 - 4*a^6*c)*e)*x*sqrt(-(18*a^3*b*d^3*e^3 - 9*a^4*d^2*e^4 - (b^
4 - 2*a*b^2*c + a^2*c^2)*d^6 + 6*(a*b^3 - a^2*b*c)*d^5*e - 3*(5*a^2*b^2 - 2
*a^3*c)*d^4*e^2)/(a^6*b^2 - 4*a^7*c)) + ((a*b^4 - 5*a^2*b^2*c + 4*a^3*c^2)*
d^4 - 3*(a^2*b^3 - 4*a^3*b*c)*d^3*e + 3*(a^3*b^2 - 4*a^4*c)*d^2*e^2)*x)*sqr
t(-(3*a^2*b*d*e^2 - 2*a^3*e^3 + (b^3 - 3*a*b*c)*d^3 - 3*(a*b^2 - 2*a^2*c)*d
^2*e - (a^3*b^2 - 4*a^4*c)*sqrt(-(18*a^3*b*d^3*e^3 - 9*a^4*d^2*e^4 - (b^4 -
 2*a*b^2*c + a^2*c^2)*d^6 + 6*(a*b^3 - a^2*b*c)*d^5*e - 3*(5*a^2*b^2 - 2*a^
3*c)*d^4*e^2)/(a^6*b^2 - 4*a^7*c)))/(a^3*b^2 - 4*a^4*c)))/x^2) + 4*sqrt(e*x
^2 + d)*d)/(a*x)
```

Sympy [F]

$$\int \frac{(d+ex^2)^{3/2}}{x^2\left(a+bx^2+cx^4\right)}\,dx=\int \frac{(d+ex^2)^{\frac{3}{2}}}{x^2\left(a+bx^2+cx^4\right)}\,dx$$

```
[In] integrate((e*x**2+d)**(3/2)/x**2/(c*x**4+b*x**2+a),x)
[Out] Integral((d + e*x**2)**(3/2)/(x**2*(a + b*x**2 + c*x**4)), x)
```

Maxima [F]

$$\int \frac{(d+ex^2)^{3/2}}{x^2\left(a+bx^2+cx^4\right)}\,dx=\int \frac{(ex^2+d)^{\frac{3}{2}}}{(cx^4+bx^2+a)x^2}\,dx$$

```
[In] integrate((e*x^2+d)^(3/2)/x^2/(c*x^4+b*x^2+a),x, algorithm="maxima")
[Out] integrate((e*x^2 + d)^(3/2)/((c*x^4 + b*x^2 + a)*x^2), x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{(d+ex^2)^{3/2}}{x^2\left(a+bx^2+cx^4\right)}\,dx=\text{Timed out}$$

```
[In] integrate((e*x^2+d)^(3/2)/x^2/(c*x^4+b*x^2+a),x, algorithm="giac")
[Out] Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex^2)^{3/2}}{x^2\left(a+bx^2+cx^4\right)}\,dx=\int \frac{{\left(e\,x^2+d\right)}^{3/2}}{x^2\,\left(c\,x^4+b\,x^2+a\right)}\,dx$$

```
[In] int((d + e*x^2)^(3/2)/(x^2*(a + b*x^2 + c*x^4)),x)
[Out] int((d + e*x^2)^(3/2)/(x^2*(a + b*x^2 + c*x^4)), x)
```

3.375 $\int \frac{(d+ex^2)^{3/2}}{x^4(a+bx^2+cx^4)}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 523

$$\int \frac{(d+ex^2)^{3/2}}{x^4(a+bx^2+cx^4)}\,dx = \frac{(bd-ae)\sqrt{d+ex^2}}{a^2x} - \frac{(d+ex^2)^{3/2}}{3ax^3} + \frac{\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}\left(bd-ae+\frac{b^2d-2acd-abe}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}\,x}{\sqrt{b-\sqrt{b^2-4ac}}\sqrt{d+ex^2}}\right)}{2a^2\sqrt{b-\sqrt{b^2-4ac}}} + \frac{\sqrt{2cd-\left(b+\sqrt{b^2-4ac}\right)e}\left(bd-ae-\frac{b^2d-2acd-abe}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2cd-\left(b+\sqrt{b^2-4ac}\right)e}\,x}{\sqrt{b+\sqrt{b^2-4ac}}\sqrt{d+ex^2}}\right)}{2a^2\sqrt{b+\sqrt{b^2-4ac}}} - \frac{\sqrt{e}(bd-ae)\operatorname{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{a^2} + \frac{\sqrt{e}\left(bd-ae-\frac{b^2d-2acd-abe}{\sqrt{b^2-4ac}}\right)\operatorname{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{2a^2} + \frac{\sqrt{e}\left(bd-ae+\frac{b^2d-2acd-abe}{\sqrt{b^2-4ac}}\right)\operatorname{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{2a^2}$$

```
[Out] -1/3*(e*x^2+d)^(3/2)/a/x^3-(-a*e+b*d)*arctanh(x*e^(1/2)/(e*x^2+d)^(1/2))*e^
(1/2)/a^2+1/2*arctanh(x*e^(1/2)/(e*x^2+d)^(1/2))*(b*d-a*e+(a*b*e+2*a*c*d-b^
2*d)/(-4*a*c+b^2)^(1/2))*e^(1/2)/a^2+1/2*arctanh(x*e^(1/2)/(e*x^2+d)^(1/2))
*(b*d-a*e+(-a*b*e-2*a*c*d+b^2*d)/(-4*a*c+b^2)^(1/2))*e^(1/2)/a^2+(-a*e+b*d)
*(e*x^2+d)^(1/2)/a^2/x+1/2*arctan(x*(2*c*d-e*(b-(-4*a*c+b^2)^(1/2)))^(1/2)/
(e*x^2+d)^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2))*(b*d-a*e+(-a*b*e-2*a*c*d+b^2*
d)/(-4*a*c+b^2)^(1/2))*(2*c*d-e*(b-(-4*a*c+b^2)^(1/2)))^(1/2)/a^2/(b-(-4*a*
c+b^2)^(1/2))^(1/2)+1/2*arctan(x*(2*c*d-e*(b+(-4*a*c+b^2)^(1/2)))^(1/2)/(e*
x^2+d)^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2))*(b*d-a*e+(a*b*e+2*a*c*d-b^2*d)/(
```

```
-4*a*c+b^2)^(1/2))*(2*c*d-e*(b+(-4*a*c+b^2)^(1/2)))^(1/2)/a^2/(b+(-4*a*c+b^
2)^(1/2))^(1/2)
```

Rubi [A] (verified)

Time = 1.87 (sec) , antiderivative size = 523, normalized size of antiderivative = 1.00, number of steps used = 19, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.345$, Rules used = {1309, 270, 6860, 283, 223, 212, 1706, 399, 385, 211}

$$\int \frac{(d+ex^2)^{3/2}}{x^4(a+bx^2+cx^4)}\,dx = \frac{\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}\left(\frac{-abe-2acd+b^2d}{\sqrt{b^2-4ac}}-ae+bd\right)\arctan\left(\frac{x\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}{\sqrt{b-\sqrt{b^2-4ac}}\sqrt{d+ex^2}}\right)}{2a^2\sqrt{b-\sqrt{b^2-4ac}}}$$
$$+\frac{\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}\left(-\frac{-abe-2acd+b^2d}{\sqrt{b^2-4ac}}-ae+bd\right)\arctan\left(\frac{x\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}{\sqrt{\sqrt{b^2-4ac}+b}\sqrt{d+ex^2}}\right)}{2a^2\sqrt{\sqrt{b^2-4ac}+b}}$$
$$+\frac{\sqrt{e}\text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)\left(-\frac{-abe-2acd+b^2d}{\sqrt{b^2-4ac}}-ae+bd\right)}{2a^2}$$
$$+\frac{\sqrt{e}\text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)\left(\frac{-abe-2acd+b^2d}{\sqrt{b^2-4ac}}-ae+bd\right)}{2a^2}$$
$$-\frac{\sqrt{e}(bd-ae)\text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{a^2}+\frac{\sqrt{d+ex^2}(bd-ae)}{a^2x}-\frac{(d+ex^2)^{3/2}}{3ax^3}$$

```
[In] Int[(d + e*x^2)^(3/2)/(x^4*(a + b*x^2 + c*x^4)),x]
```

```
[Out] ((b*d - a*e)*Sqrt[d + e*x^2])/(a^2*x) - (d + e*x^2)^(3/2)/(3*a*x^3) + (Sqrt
[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]*(b*d - a*e + (b^2*d - 2*a*c*d - a*b*e)/
Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]*x)/(Sqrt
[b - Sqrt[b^2 - 4*a*c]]*Sqrt[d + e*x^2])])/(2*a^2*Sqrt[b - Sqrt[b^2 - 4*a*c
]]) + (Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]*(b*d - a*e - (b^2*d - 2*a*c*
d - a*b*e)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*
e]*x)/(Sqrt[b + Sqrt[b^2 - 4*a*c]]*Sqrt[d + e*x^2])])/(2*a^2*Sqrt[b + Sqrt[
b^2 - 4*a*c]]) - (Sqrt[e]*(b*d - a*e)*ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]])
/a^2 + (Sqrt[e]*(b*d - a*e - (b^2*d - 2*a*c*d - a*b*e)/Sqrt[b^2 - 4*a*c])*A
rcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]])/(2*a^2) + (Sqrt[e]*(b*d - a*e + (b^2*d
 - 2*a*c*d - a*b*e)/Sqrt[b^2 - 4*a*c])*ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]]
)/(2*a^2)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 283

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + 1))), x] - Dist[b*n*(p/(c^n*(m + 1))), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 399

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Di
st[b/d, Int[(a + b*x^n)^(p - 1), x], x] - Dist[(b*c - a*d)/d, Int[(a + b*x^
n)^(p - 1)/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, p}, x] && NeQ[b*c - a*
d, 0] && EqQ[n*(p - 1) + 1, 0] && IntegerQ[n]
```

Rule 1309

```
Int[(((f_.)*(x_))^(m_)*((d_.) + (e_.)*(x_)^2)^(q_))/((a_) + (b_.)*(x_)^2 +
(c_.)*(x_)^4), x_Symbol] :> Dist[d/a, Int[(f*x)^m*(d + e*x^2)^(q - 1), x],
x] - Dist[1/(a*f^2), Int[(f*x)^(m + 2)*(d + e*x^2)^(q - 1)*(Simp[b*d - a*e
+ c*d*x^2, x]/(a + b*x^2 + c*x^4)), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
&& NeQ[b^2 - 4*a*c, 0] &&  !IntegerQ[q] && GtQ[q, 0] && LtQ[m, 0]
```

Rule 1706

```
Int[(Px_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^
(p_.), x_Symbol] :> Int[ExpandIntegrand[Px*(d + e*x^2)^q*(a + b*x^2 + c*x^4
)^p, x], x] /; FreeQ[{a, b, c, d, e, q}, x] && PolyQ[Px, x^2] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[p]
```

Rule 6860

```
Int[(u_)/((a_.) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.)), x_Symbol] :> With[
{v = RationalFunctionExpand[u/(a + b*x^n + c*x^(2*n)), x]}, Int[v, x] /; Su
mQ[v]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && IGtQ[n, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{\int \frac{(bd-ae+cdx^2)\sqrt{d+ex^2}}{x^2(a+bx^2+cx^4)}\,dx}{a}+\frac{d\int \frac{\sqrt{d+ex^2}}{x^4}\,dx}{a}\\
&= -\frac{(d+ex^2)^{3/2}}{3ax^3}-\frac{\int \left(\frac{(bd-ae)\sqrt{d+ex^2}}{ax^2}+\frac{\sqrt{d+ex^2}\,(-b^2d+acd+abe-c(bd-ae)x^2)}{a(a+bx^2+cx^4)}\right)dx}{a}\\
&= -\frac{(d+ex^2)^{3/2}}{3ax^3}-\frac{\int \frac{\sqrt{d+ex^2}\,(-b^2d+acd+abe-c(bd-ae)x^2)}{a+bx^2+cx^4}\,dx}{a^2}-\frac{(bd-ae)\int \frac{\sqrt{d+ex^2}}{x^2}\,dx}{a^2}\\
&= \frac{(bd-ae)\sqrt{d+ex^2}}{a^2x}-\frac{(d+ex^2)^{3/2}}{3ax^3}\\
&\quad -\frac{\int \left(\frac{\left(-c(bd-ae)-\frac{c\left(b^2d-2acd-abe\right)}{\sqrt{b^2-4ac}}\right)\sqrt{d+ex^2}}{b-\sqrt{b^2-4ac}+2cx^2}+\frac{\left(-c(bd-ae)+\frac{c\left(b^2d-2acd-abe\right)}{\sqrt{b^2-4ac}}\right)\sqrt{d+ex^2}}{b+\sqrt{b^2-4ac}+2cx^2}\right)dx}{a^2}\\
&\quad -\frac{(e(bd-ae))\int \frac{1}{\sqrt{d+ex^2}}\,dx}{a^2}\\
&= \frac{(bd-ae)\sqrt{d+ex^2}}{a^2x}-\frac{(d+ex^2)^{3/2}}{3ax^3}-\frac{(e(bd-ae))\text{Subst}\left(\int \frac{1}{1-ex^2}\,dx,x,\frac{x}{\sqrt{d+ex^2}}\right)}{a^2}\\
&\quad +\frac{\left(c\left(bd-ae-\frac{b^2d-2acd-abe}{\sqrt{b^2-4ac}}\right)\right)\int \frac{\sqrt{d+ex^2}}{b+\sqrt{b^2-4ac}+2cx^2}\,dx}{a^2}\\
&\quad +\frac{\left(c\left(bd-ae+\frac{b^2d-2acd-abe}{\sqrt{b^2-4ac}}\right)\right)\int \frac{\sqrt{d+ex^2}}{b-\sqrt{b^2-4ac}+2cx^2}\,dx}{a^2}
\end{aligned}
$$

$$= \frac{(bd-ae)\sqrt{d+ex^2}}{a^2x} - \frac{(d+ex^2)^{3/2}}{3ax^3} - \frac{\sqrt{e}(bd-ae)\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{a^2} + \frac{\left(e\left(bd-ae-\frac{b^2d-2acd-abe}{\sqrt{b^2-4ac}}\right)\right)\int\frac{1}{\sqrt{d+ex^2}}\,dx}{2a^2} + \frac{\left(\left(2cd-\left(b+\sqrt{b^2-4ac}\right)e\right)\left(bd-ae-\frac{b^2d-2acd-abe}{\sqrt{b^2-4ac}}\right)\right)\int\frac{1}{\left(b+\sqrt{b^2-4ac}+2cx^2\right)\sqrt{d+ex^2}}\,dx}{2a^2} + \frac{\left(e\left(bd-ae+\frac{b^2d-2acd-abe}{\sqrt{b^2-4ac}}\right)\right)\int\frac{1}{\sqrt{d+ex^2}}\,dx}{2a^2} + \frac{\left(\left(2cd-\left(b-\sqrt{b^2-4ac}\right)e\right)\left(bd-ae+\frac{b^2d-2acd-abe}{\sqrt{b^2-4ac}}\right)\right)\int\frac{1}{\left(b-\sqrt{b^2-4ac}+2cx^2\right)\sqrt{d+ex^2}}\,dx}{2a^2}$$

$$= \frac{(bd-ae)\sqrt{d+ex^2}}{a^2x} - \frac{(d+ex^2)^{3/2}}{3ax^3} - \frac{\sqrt{e}(bd-ae)\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{a^2} + \frac{\left(e\left(bd-ae-\frac{b^2d-2acd-abe}{\sqrt{b^2-4ac}}\right)\right)\text{Subst}\left(\int\frac{1}{1-ex^2}\,dx,x,\frac{x}{\sqrt{d+ex^2}}\right)}{2a^2} + \frac{\left(\left(2cd-\left(b+\sqrt{b^2-4ac}\right)e\right)\left(bd-ae-\frac{b^2d-2acd-abe}{\sqrt{b^2-4ac}}\right)\right)\text{Subst}\left(\int\frac{1}{b+\sqrt{b^2-4ac}-\left(-2cd+\left(b+\sqrt{b^2-4ac}\right)e\right)x}\right.}{2a^2} + \frac{\left(e\left(bd-ae+\frac{b^2d-2acd-abe}{\sqrt{b^2-4ac}}\right)\right)\text{Subst}\left(\int\frac{1}{1-ex^2}\,dx,x,\frac{x}{\sqrt{d+ex^2}}\right)}{2a^2} + \frac{\left(\left(2cd-\left(b-\sqrt{b^2-4ac}\right)e\right)\left(bd-ae+\frac{b^2d-2acd-abe}{\sqrt{b^2-4ac}}\right)\right)\text{Subst}\left(\int\frac{1}{b-\sqrt{b^2-4ac}-\left(-2cd+\left(b-\sqrt{b^2-4ac}\right)e\right)x}\right.}{2a^2}$$

$$= \frac{(bd-ae)\sqrt{d+ex^2}}{a^2x} - \frac{(d+ex^2)^{3/2}}{3ax^3} + \frac{\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}\left(bd-ae+\frac{b^2d-2acd-abe}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}x}{\sqrt{b-\sqrt{b^2-4ac}}\sqrt{d+ex^2}}\right)}{2a^2\sqrt{b-\sqrt{b^2-4ac}}} + \frac{\sqrt{2cd-\left(b+\sqrt{b^2-4ac}\right)e}\left(bd-ae-\frac{b^2d-2acd-abe}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2cd-\left(b+\sqrt{b^2-4ac}\right)e}x}{\sqrt{b+\sqrt{b^2-4ac}}\sqrt{d+ex^2}}\right)}{2a^2\sqrt{b+\sqrt{b^2-4ac}}} - \frac{\sqrt{e}(bd-ae)\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{a^2} + \frac{\sqrt{e}\left(bd-ae-\frac{b^2d-2acd-abe}{\sqrt{b^2-4ac}}\right)\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{2a^2} + \frac{\sqrt{e}\left(bd-ae+\frac{b^2d-2acd-abe}{\sqrt{b^2-4ac}}\right)\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{2a^2}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 1.49 (sec) , antiderivative size = 1095, normalized size of antiderivative = 2.09

$$\int \frac{(d+ex^2)^{3/2}}{x^4(a+bx^2+cx^4)}\,dx = \frac{\sqrt{d+ex^2}(-ad+3bdx^2-4aex^2)}{3a^2x^3}$$
$$-\frac{\text{RootSum}\left[ae^4+4bde^2\#1^2-4ae^3\#1^2+16cd^2\#1^4-8bde\#1^4+6ae^2\#1^4+4bd\#1^6-4ae\#1^6+a\#1^8\&,\ldots\right.}{\ldots}$$

```
[In] Integrate[(d + e*x^2)^(3/2)/(x^4*(a + b*x^2 + c*x^4)),x]
```

```
[Out] (Sqrt[d + e*x^2]*(-(a*d) + 3*b*d*x^2 - 4*a*e*x^2))/(3*a^2*x^3) - RootSum[a*
e^4 + 4*b*d*e^2*#1^2 - 4*a*e^3*#1^2 + 16*c*d^2*#1^4 - 8*b*d*e*#1^4 + 6*a*e^
2*#1^4 + 4*b*d*#1^6 - 4*a*e*#1^6 + a*#1^8 & , (b^2*d^2*e^3*Log[x] - a*c*d^2
*e^3*Log[x] - 2*a*b*d*e^4*Log[x] + a^2*e^5*Log[x] - b^2*d^2*e^3*Log[-Sqrt[d
] + Sqrt[d + e*x^2] - x*#1] + a*c*d^2*e^3*Log[-Sqrt[d] + Sqrt[d + e*x^2] -
x*#1] + 2*a*b*d*e^4*Log[-Sqrt[d] + Sqrt[d + e*x^2] - x*#1] - a^2*e^5*Log[-S
qrt[d] + Sqrt[d + e*x^2] - x*#1] + 4*b*c*d^3*e*Log[x]*#1^2 - 3*b^2*d^2*e^2*
Log[x]*#1^2 - 5*a*c*d^2*e^2*Log[x]*#1^2 + 6*a*b*d*e^3*Log[x]*#1^2 - 3*a^2*e
^4*Log[x]*#1^2 - 4*b*c*d^3*e*Log[-Sqrt[d] + Sqrt[d + e*x^2] - x*#1]*#1^2 +
3*b^2*d^2*e^2*Log[-Sqrt[d] + Sqrt[d + e*x^2] - x*#1]*#1^2 + 5*a*c*d^2*e^2*L
og[-Sqrt[d] + Sqrt[d + e*x^2] - x*#1]*#1^2 - 6*a*b*d*e^3*Log[-Sqrt[d] + Sqr
t[d + e*x^2] - x*#1]*#1^2 + 3*a^2*e^4*Log[-Sqrt[d] + Sqrt[d + e*x^2] - x*#1
]*#1^2 - 4*b*c*d^3*Log[x]*#1^4 + 3*b^2*d^2*e*Log[x]*#1^4 + 5*a*c*d^2*e*Log[
x]*#1^4 - 6*a*b*d*e^2*Log[x]*#1^4 + 3*a^2*e^3*Log[x]*#1^4 + 4*b*c*d^3*Log[-
Sqrt[d] + Sqrt[d + e*x^2] - x*#1]*#1^4 - 3*b^2*d^2*e*Log[-Sqrt[d] + Sqrt[d
+ e*x^2] - x*#1]*#1^4 - 5*a*c*d^2*e*Log[-Sqrt[d] + Sqrt[d + e*x^2] - x*#1]*
#1^4 + 6*a*b*d*e^2*Log[-Sqrt[d] + Sqrt[d + e*x^2] - x*#1]*#1^4 - 3*a^2*e^3*
Log[-Sqrt[d] + Sqrt[d + e*x^2] - x*#1]*#1^4 - b^2*d^2*Log[x]*#1^6 + a*c*d^2
*Log[x]*#1^6 + 2*a*b*d*e*Log[x]*#1^6 - a^2*e^2*Log[x]*#1^6 + b^2*d^2*Log[-S
qrt[d] + Sqrt[d + e*x^2] - x*#1]*#1^6 - a*c*d^2*Log[-Sqrt[d] + Sqrt[d + e*x
^2] - x*#1]*#1^6 - 2*a*b*d*e*Log[-Sqrt[d] + Sqrt[d + e*x^2] - x*#1]*#1^6 +
a^2*e^2*Log[-Sqrt[d] + Sqrt[d + e*x^2] - x*#1]*#1^6)/(b*d*e^2*#1 - a*e^3*#1
 + 8*c*d^2*#1^3 - 4*b*d*e*#1^3 + 3*a*e^2*#1^3 + 3*b*d*#1^5 - 3*a*e*#1^5 + a
*#1^7) & ]/(4*a^2)
```

Maple [A] (verified)

Time = 1.09 (sec) , antiderivative size = 451, normalized size of antiderivative = 0.86

method	result
risch	$-\frac{\sqrt{e\,x^2+d}\,(4ae\,x^2-3bd\,x^2+da)}{3a^2x^3}-\frac{\sqrt{2}\left(-\left(\left(e^2a^2+(-2bde-c\,d^2)a+b^2d^2\right)\sqrt{-4d^2\left(ac-\frac{b^2}{4}\right)}-\left((b\,e^2+4dce)a^2+(-2b^2de-\right.\right.\right.}{}$
pseudoelliptic	$\frac{\sqrt{\left(-2ae+bd+\sqrt{-4d^2\left(ac-\frac{b^2}{4}\right)}\right)a}\left(\left(\left((-ac+b^2)d^2-2abde+e^2a^2\right)\sqrt{-4d^2\left(ac-\frac{b^2}{4}\right)}-\left((-3abc+b^3)d^2+2e(2c\,a^2-b^2a)d+a^2\right.\right.\right.}{}$
default	$\frac{-\frac{(e\,x^2+d)^{\frac{5}{2}}}{3d\,x^3}+\frac{2e\left(-\frac{(e\,x^2+d)^{\frac{5}{2}}}{dx}+\frac{4e\left(\frac{x(e\,x^2+d)^{\frac{3}{2}}}{4}+\frac{3d\left(\frac{x\sqrt{e\,x^2+d}}{2}+\frac{d\ln\left(x\sqrt{e}+\sqrt{e\,x^2+d}\right)}{2\sqrt{e}}\right)}{4}\right)}{d}\right)}{3d}}{a}-\frac{b\left(-\frac{(e\,x^2+d)^{\frac{5}{2}}}{dx}+\frac{4e\left(\frac{x(e\,x\right.}{}\right.}{}$

[In] int((e*x^2+d)^(3/2)/x^4/(c*x^4+b*x^2+a),x,method=_RETURNVERBOSE)

[Out] -1/3*(e*x^2+d)^(1/2)*(4*a*e*x^2-3*b*d*x^2+a*d)/a^2/x^3-1/2/a^2/((2*a*e-b*d+(-4*d^2*(a*c-1/4*b^2))^(1/2))*a)^(1/2)*2^(1/2)*(-((e^2*a^2+(-2*b*d*e-c*d^2)*a+b^2*d^2)*(-4*d^2*(a*c-1/4*b^2))^(1/2)-((b*e^2+4*c*d*e)*a^2+(-2*b^2*d*e-3*b*c*d^2)*a+b^3*d^2)*d)*((-2*a*e+b*d+(-4*d^2*(a*c-1/4*b^2))^(1/2))*a)^(1/2)*arctanh(a/x*(e*x^2+d)^(1/2)*2^(1/2)/((2*a*e-b*d+(-4*d^2*(a*c-1/4*b^2))^(1/2))*a)^(1/2))+((e^2*a^2+(-2*b*d*e-c*d^2)*a+b^2*d^2)*(-4*d^2*(a*c-1/4*b^2))^(1/2)+((b*e^2+4*c*d*e)*a^2+(-2*b^2*d*e-3*b*c*d^2)*a+b^3*d^2)*d)*((2*a*e-b*d+(-4*d^2*(a*c-1/4*b^2))^(1/2))*a)^(1/2)*arctan(a/x*(e*x^2+d)^(1/2)*2^(1/2)/((-2*a*e+b*d+(-4*d^2*(a*c-1/4*b^2))^(1/2))*a)^(1/2)))/((-2*a*e+b*d+(-4*d^2*(a*c-1/4*b^2))^(1/2))*a)^(1/2)/(-4*d^2*(a*c-1/4*b^2))^(1/2)

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 7830 vs. $2(447) = 894$.

Time = 32.46 (sec) , antiderivative size = 7830, normalized size of antiderivative = 14.97

$$\int \frac{(d+ex^2)^{3/2}}{x^4 (a+bx^2+cx^4)} \, dx = \text{Too large to display}$$

```
[In] integrate((e*x^2+d)^(3/2)/x^4/(c*x^4+b*x^2+a),x, algorithm="fricas")
[Out] Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(d+ex^2)^{3/2}}{x^4 (a+bx^2+cx^4)} \, dx = \text{Timed out}$$

```
[In] integrate((e*x**2+d)**(3/2)/x**4/(c*x**4+b*x**2+a),x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{(d+ex^2)^{3/2}}{x^4 (a+bx^2+cx^4)} \, dx = \int \frac{(ex^2+d)^{\frac{3}{2}}}{(cx^4+bx^2+a)x^4} \, dx$$

```
[In] integrate((e*x^2+d)^(3/2)/x^4/(c*x^4+b*x^2+a),x, algorithm="maxima")
[Out] integrate((e*x^2 + d)^(3/2)/((c*x^4 + b*x^2 + a)*x^4), x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{(d+ex^2)^{3/2}}{x^4 (a+bx^2+cx^4)} \, dx = \text{Timed out}$$

```
[In] integrate((e*x^2+d)^(3/2)/x^4/(c*x^4+b*x^2+a),x, algorithm="giac")
[Out] Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex^2)^{3/2}}{x^4\,(a+bx^2+cx^4)}\,dx=\int \frac{{\left(e\,x^2+d\right)}^{3/2}}{x^4\,\left(c\,x^4+b\,x^2+a\right)}\,dx$$

```
[In] int((d + e*x^2)^(3/2)/(x^4*(a + b*x^2 + c*x^4)),x)
[Out] int((d + e*x^2)^(3/2)/(x^4*(a + b*x^2 + c*x^4)), x)
```

3.376 $\int \frac{x^5\sqrt{1-x^2}}{a+bx^2+cx^4}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 281

$$\int \frac{x^5\sqrt{1-x^2}}{a+bx^2+cx^4}\,dx = -\frac{b\sqrt{1-x^2}}{c^2}-\frac{(1-x^2)^{3/2}}{3c} + \frac{\left(b^2-ac+bc-\frac{b^3-3abc+b^2c-2ac^2}{\sqrt{b^2-4ac}}\right)\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{1-x^2}}{\sqrt{b+2c-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}c^{5/2}\sqrt{b+2c-\sqrt{b^2-4ac}}} + \frac{\left(b^2-ac+bc+\frac{b^3-3abc+b^2c-2ac^2}{\sqrt{b^2-4ac}}\right)\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{1-x^2}}{\sqrt{b+2c+\sqrt{b^2-4ac}}}\right)}{\sqrt{2}c^{5/2}\sqrt{b+2c+\sqrt{b^2-4ac}}}$$

```
[Out] -1/3*(-x^2+1)^(3/2)/c-b*(-x^2+1)^(1/2)/c^2+1/2*arctanh(2^(1/2)*c^(1/2)*(-x^
2+1)^(1/2)/(b+2*c-(-4*a*c+b^2)^(1/2))^(1/2))*(b^2-a*c+b*c+(3*a*b*c+2*a*c^2-
b^3-b^2*c)/(-4*a*c+b^2)^(1/2))/c^(5/2)*2^(1/2)/(b+2*c-(-4*a*c+b^2)^(1/2))^(
1/2)+1/2*arctanh(2^(1/2)*c^(1/2)*(-x^2+1)^(1/2)/(b+2*c+(-4*a*c+b^2)^(1/2))^
(1/2))*(b^2-a*c+b*c+(-3*a*b*c-2*a*c^2+b^3+b^2*c)/(-4*a*c+b^2)^(1/2))/c^(5/2
)*2^(1/2)/(b+2*c+(-4*a*c+b^2)^(1/2))^(1/2)
```

Rubi [A] (verified)

Time = 4.70 (sec) , antiderivative size = 281, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.172$, Rules used

$= \{1265, 911, 1301, 1180, 214\}$

$$\int \frac{x^5\sqrt{1-x^2}}{a+bx^2+cx^4}\,dx = \frac{\left(-\frac{-3abc-2ac^2+b^3+b^2c}{\sqrt{b^2-4ac}}-ac+b^2+bc\right)\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{1-x^2}}{\sqrt{-\sqrt{b^2-4ac}+b+2c}}\right)}{\sqrt{2}c^{5/2}\sqrt{-\sqrt{b^2-4ac}+b+2c}} + \frac{\left(\frac{-3abc-2ac^2+b^3+b^2c}{\sqrt{b^2-4ac}}-ac+b^2+bc\right)\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{1-x^2}}{\sqrt{\sqrt{b^2-4ac}+b+2c}}\right)}{\sqrt{2}c^{5/2}\sqrt{\sqrt{b^2-4ac}+b+2c}} - \frac{b\sqrt{1-x^2}}{c^2} - \frac{(1-x^2)^{3/2}}{3c}$$

```
[In] Int[(x^5*Sqrt[1 - x^2])/(a + b*x^2 + c*x^4),x]
```

```
[Out] -((b*Sqrt[1 - x^2])/c^2) - (1 - x^2)^(3/2)/(3*c) + ((b^2 - a*c + b*c - (b^3
 - 3*a*b*c + b^2*c - 2*a*c^2)/Sqrt[b^2 - 4*a*c])*ArcTanh[(Sqrt[2]*Sqrt[c]*S
qrt[1 - x^2])/Sqrt[b + 2*c - Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*c^(5/2)*Sqrt[b +
 2*c - Sqrt[b^2 - 4*a*c]]) + ((b^2 - a*c + b*c + (b^3 - 3*a*b*c + b^2*c - 2
*a*c^2)/Sqrt[b^2 - 4*a*c])*ArcTanh[(Sqrt[2]*Sqrt[c]*Sqrt[1 - x^2])/Sqrt[b +
 2*c + Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*c^(5/2)*Sqrt[b + 2*c + Sqrt[b^2 - 4*a*
c]])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 911

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> With[{q = Denominator[m]}, Dist[q/e, S
ubst[Int[x^(q*(m + 1) - 1)*((e*f - d*g)/e + g*(x^q/e))^n*((c*d^2 - b*d*e +
a*e^2)/e^2 - (2*c*d - b*e)*(x^q/e^2) + c*(x^(2*q)/e^2))^p, x], x, (d + e*x)
^(1/q)], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegersQ[n, p] && Fra
ctionQ[m]
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1265

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
```

```
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rule 1301

```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.))/((a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*((d + e*x^2)^q/(a
+ b*x^2 + c*x^4)), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b^2 - 4
*a*c, 0] && IntegerQ[q] && IntegerQ[m]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{1}{2}\text{Subst}\left(\int \frac{\sqrt{1-x}x^2}{a+bx+cx^2}\,dx, x, x^2\right)\\
&= -\text{Subst}\left(\int \frac{x^2(1-x^2)^2}{a+b+c+(-b-2c)x^2+cx^4}\,dx, x, \sqrt{1-x^2}\right)\\
&= -\text{Subst}\left(\int \left(\frac{b}{c^2}+\frac{x^2}{c}-\frac{b(a+b+c)-(b^2-ac+bc)\,x^2}{c^2\,(a+b+c+(-b-2c)x^2+cx^4)}\right)\,dx, x, \sqrt{1-x^2}\right)\\
&= -\frac{b\sqrt{1-x^2}}{c^2}-\frac{(1-x^2)^{3/2}}{3c}+\frac{\text{Subst}\left(\int \frac{b(a+b+c)+(-b^2+ac-bc)x^2}{a+b+c+(-b-2c)x^2+cx^4}\,dx, x, \sqrt{1-x^2}\right)}{c^2}\\
&= -\frac{b\sqrt{1-x^2}}{c^2}-\frac{(1-x^2)^{3/2}}{3c}\\
&\quad-\frac{\left(b^2-ac+bc-\frac{b^3-3abc+b^2c-2ac^2}{\sqrt{b^2-4ac}}\right)\text{Subst}\left(\int \frac{1}{\frac{1}{2}(-b-2c)+\frac{1}{2}\sqrt{b^2-4ac}+cx^2}\,dx, x, \sqrt{1-x^2}\right)}{2c^2}\\
&\quad-\frac{\left(b^2-ac+bc+\frac{b^3-3abc+b^2c-2ac^2}{\sqrt{b^2-4ac}}\right)\text{Subst}\left(\int \frac{1}{\frac{1}{2}(-b-2c)-\frac{1}{2}\sqrt{b^2-4ac}+cx^2}\,dx, x, \sqrt{1-x^2}\right)}{2c^2}\\
&= -\frac{b\sqrt{1-x^2}}{c^2}-\frac{(1-x^2)^{3/2}}{3c}\\
&\quad+\frac{\left(b^2-ac+bc-\frac{b^3-3abc+b^2c-2ac^2}{\sqrt{b^2-4ac}}\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{1-x^2}}{\sqrt{b+2c-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}c^{5/2}\sqrt{b+2c-\sqrt{b^2-4ac}}}\\
&\quad+\frac{\left(b^2-ac+bc+\frac{b^3-3abc+b^2c-2ac^2}{\sqrt{b^2-4ac}}\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{1-x^2}}{\sqrt{b+2c+\sqrt{b^2-4ac}}}\right)}{\sqrt{2}c^{5/2}\sqrt{b+2c+\sqrt{b^2-4ac}}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.79 (sec) , antiderivative size = 346, normalized size of antiderivative = 1.23

$$\int \frac{x^5\sqrt{1-x^2}}{a+bx^2+cx^4}\,dx$$
$$= \frac{2\sqrt{c}\sqrt{1-x^2}(-3b+c(-1+x^2)) - \frac{3\sqrt{2}\left(b^3+bc\left(-3a+\sqrt{b^2-4ac}\right)+b^2\left(c+\sqrt{b^2-4ac}\right)-ac\left(2c+\sqrt{b^2-4ac}\right)\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}\sqrt{1-x^2}}{\sqrt{-b-2c-\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}\sqrt{-b-2c-\sqrt{b^2-4ac}}}}{6c^{5/2}}$$

```
[In] Integrate[(x^5*Sqrt[1 - x^2])/(a + b*x^2 + c*x^4),x]
[Out] (2*Sqrt[c]*Sqrt[1 - x^2]*(-3*b + c*(-1 + x^2)) - (3*Sqrt[2]*(b^3 + b*c*(-3*
a + Sqrt[b^2 - 4*a*c]) + b^2*(c + Sqrt[b^2 - 4*a*c]) - a*c*(2*c + Sqrt[b^2
- 4*a*c]))*ArcTan[(Sqrt[2]*Sqrt[c]*Sqrt[1 - x^2])/Sqrt[-b - 2*c - Sqrt[b^2
- 4*a*c]]])/(Sqrt[b^2 - 4*a*c]*Sqrt[-b - 2*c - Sqrt[b^2 - 4*a*c]]) - (3*Sqr
t[2]*(-b^3 + a*c*(2*c - Sqrt[b^2 - 4*a*c]) + b*c*(3*a + Sqrt[b^2 - 4*a*c])
+ b^2*(-c + Sqrt[b^2 - 4*a*c]))*ArcTan[(Sqrt[2]*Sqrt[c]*Sqrt[1 - x^2])/Sqrt
[-b - 2*c + Sqrt[b^2 - 4*a*c]]])/(Sqrt[b^2 - 4*a*c]*Sqrt[-b - 2*c + Sqrt[b^
2 - 4*a*c]]))/(6*c^(5/2))
```

Maple [A] (verified)

Time = 1.06 (sec) , antiderivative size = 316, normalized size of antiderivative = 1.12

method	result
pseudoelliptic	$-\frac{3\left(\left(\left(\frac{((-a+b)c+b^2)\sqrt{-4ac+b^2}}{3}+\frac{2ac^2}{3}+b\left(a-\frac{b}{3}\right)c-\frac{b^3}{3}\right)\sqrt{2}\sqrt{\left(b+2c+\sqrt{-4ac+b^2}\right)c}\,\arctan\left(\frac{c\sqrt{-x^2+1}\,\sqrt{2}}{\sqrt{\left(\sqrt{-4ac+b^2}-b-2c\right)c}}\right)+\sqrt{(\ldots}\right.\right.}{2\sqrt{-4\ldots}}$
risch	$\frac{(-cx^2+3b+c)(x^2-1)}{3c^2\sqrt{-x^2+1}} - \frac{2a\left(\frac{\left(-2ac\sqrt{-4ac+b^2}+b^2\sqrt{-4ac+b^2}+\sqrt{-4ac+b^2}\,bc+4abc+4a\,c^2-b^3-b^2c\right)\arctan\left(\frac{\frac{2a\left(\sqrt{-x^2+1}-1\right)\ldots}{x^2}}{2\sqrt{4ac-2b^2-2\sqrt{-4a\ldots}}}\right)}{\left(8ac-2b^2\right)\sqrt{4ac-2b^2-2\sqrt{-4ac+b^2}\,a-2b\sqrt{-4ac+b^2}-2ab}}\ldots\right.}{\ldots} \ldots \frac{2a\left(\frac{\left(-2ac\sqrt{-4ac+b^2}+b^2\sqrt{-4ac+b^2}+\sqrt{-4ac+b^2}\,bc+4abc+4a\,c^2-b^3-b^2c\right)\arctan\left(\frac{\frac{2a\left(\sqrt{-x^2+1}-1\right)^2}{x^2}+2\sqrt{-4\ldots}}{2\sqrt{4ac-2b^2-2\sqrt{-4ac+b^2}\,a-2\ldots}}\right)}{\left(8ac-2b^2\right)\sqrt{4ac-2b^2-2\sqrt{-4ac+b^2}\,a-2b\sqrt{-4ac+b^2}-2ab}}\ldots\right.}{\ldots}$
default	$-\frac{(-x^2+1)^{\frac{3}{2}}}{3c} - \ldots$

```
[In] int(x^5*(-x^2+1)^(1/2)/(c*x^4+b*x^2+a),x,method=_RETURNVERBOSE)
```

```
[Out] -3/2/(-4*a*c+b^2)^(1/2)/(((-4*a*c+b^2)^(1/2)-b-2*c)*c)^(1/2)/((b+2*c+(-4*a*
c+b^2)^(1/2))*c)^(1/2)*((1/3*((-a+b)*c+b^2)*(-4*a*c+b^2)^(1/2)+2/3*a*c^2+b*
(a-1/3*b)*c-1/3*b^3)*2^(1/2)*((b+2*c+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*
(-x^2+1)^(1/2)*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b-2*c)*c)^(1/2))+(((-4*a*c+b^2)
^(1/2)-b-2*c)*c)^(1/2)*(2^(1/2)*(1/3*((a-b)*c-b^2)*(-4*a*c+b^2)^(1/2)+2/3*a
*c^2+b*(a-1/3*b)*c-1/3*b^3)*arctanh(c*(-x^2+1)^(1/2)*2^(1/2)/((b+2*c+(-4*a*
c+b^2)^(1/2))*c)^(1/2))+2/3*(b+1/3*c-1/3*c*x^2)*(-4*a*c+b^2)^(1/2)*((b+2*c+
(-4*a*c+b^2)^(1/2))*c)^(1/2)*(-x^2+1)^(1/2)))/c^2
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3615 vs. $2(237) = 474$.

Time = 3.73 (sec) , antiderivative size = 3615, normalized size of antiderivative = 12.86

$$\int \frac{x^5\sqrt{1-x^2}}{a+bx^2+cx^4}\,dx = \text{Too large to display}$$

```
[In] integrate(x^5*(-x^2+1)^(1/2)/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] -1/6*(3*sqrt(1/2)*c^2*sqrt((b^5 + 2*a^2*c^3 + (5*a^2*b - 4*a*b^2)*c^2 - (5*
a*b^3 - b^4)*c - (b^2*c^5 - 4*a*c^6)*sqrt((b^8 + (a^4 - 4*a^3*b + 4*a^2*b^2
)*c^4 - 2*(3*a^3*b^2 - 7*a^2*b^3 + 2*a*b^4)*c^3 + (11*a^2*b^4 - 10*a*b^5 +
b^6)*c^2 - 2*(3*a*b^6 - b^7)*c)/(b^2*c^10 - 4*a*c^11)))/(b^2*c^5 - 4*a*c^6)
)*log(-(2*a^3*b^4 + (a^2*b^2*c^5 - 4*a^3*c^6)*x^2*sqrt((b^8 + (a^4 - 4*a^3*
b + 4*a^2*b^2)*c^4 - 2*(3*a^3*b^2 - 7*a^2*b^3 + 2*a*b^4)*c^3 + (11*a^2*b^4
- 10*a*b^5 + b^6)*c^2 - 2*(3*a*b^6 - b^7)*c)/(b^2*c^10 - 4*a*c^11)) + 2*(a^
5 - 2*a^4*b)*c^2 + (a^2*b^5 + (a^4*b - 2*a^3*b^2)*c^2 - (3*a^3*b^3 - a^2*b^
4)*c)*x^2 - 2*(3*a^4*b^2 - a^3*b^3)*c + sqrt(1/2)*((b^5*c^5 - 7*a*b^3*c^6 +
 12*a^2*b*c^7)*x^2*sqrt((b^8 + (a^4 - 4*a^3*b + 4*a^2*b^2)*c^4 - 2*(3*a^3*b
^2 - 7*a^2*b^3 + 2*a*b^4)*c^3 + (11*a^2*b^4 - 10*a*b^5 + b^6)*c^2 - 2*(3*a*
b^6 - b^7)*c)/(b^2*c^10 - 4*a*c^11)) + (b^8 + 4*(a^4 - 2*a^3*b)*c^4 - (17*a
^3*b^2 - 14*a^2*b^3)*c^3 + (20*a^2*b^4 - 7*a*b^5)*c^2 - (8*a*b^6 - b^7)*c)*
x^2)*sqrt((b^5 + 2*a^2*c^3 + (5*a^2*b - 4*a*b^2)*c^2 - (5*a*b^3 - b^4)*c -
(b^2*c^5 - 4*a*c^6)*sqrt((b^8 + (a^4 - 4*a^3*b + 4*a^2*b^2)*c^4 - 2*(3*a^3*
b^2 - 7*a^2*b^3 + 2*a*b^4)*c^3 + (11*a^2*b^4 - 10*a*b^5 + b^6)*c^2 - 2*(3*a
*b^6 - b^7)*c)/(b^2*c^10 - 4*a*c^11)))/(b^2*c^5 - 4*a*c^6)) - 2*(a^3*b^4 +
(a^5 - 2*a^4*b)*c^2 - (3*a^4*b^2 - a^3*b^3)*c)*sqrt(-x^2 + 1))/x^2) - 3*sqr
t(1/2)*c^2*sqrt((b^5 + 2*a^2*c^3 + (5*a^2*b - 4*a*b^2)*c^2 - (5*a*b^3 - b^4
)*c - (b^2*c^5 - 4*a*c^6)*sqrt((b^8 + (a^4 - 4*a^3*b + 4*a^2*b^2)*c^4 - 2*(
3*a^3*b^2 - 7*a^2*b^3 + 2*a*b^4)*c^3 + (11*a^2*b^4 - 10*a*b^5 + b^6)*c^2 -
2*(3*a*b^6 - b^7)*c)/(b^2*c^10 - 4*a*c^11)))/(b^2*c^5 - 4*a*c^6))*log(-(2*a
^3*b^4 + (a^2*b^2*c^5 - 4*a^3*c^6)*x^2*sqrt((b^8 + (a^4 - 4*a^3*b + 4*a^2*b
^2)*c^4 - 2*(3*a^3*b^2 - 7*a^2*b^3 + 2*a*b^4)*c^3 + (11*a^2*b^4 - 10*a*b^5
```

+ b^6)*c^2 - 2*(3*a*b^6 - b^7)*c)/(b^2*c^10 - 4*a*c^11)) + 2*(a^5 - 2*a^4*b
)*c^2 + (a^2*b^5 + (a^4*b - 2*a^3*b^2)*c^2 - (3*a^3*b^3 - a^2*b^4)*c)*x^2 -
2*(3*a^4*b^2 - a^3*b^3)*c - sqrt(1/2)*((b^5*c^5 - 7*a*b^3*c^6 + 12*a^2*b*c
^7)*x^2*sqrt((b^8 + (a^4 - 4*a^3*b + 4*a^2*b^2)*c^4 - 2*(3*a^3*b^2 - 7*a^2*
b^3 + 2*a*b^4)*c^3 + (11*a^2*b^4 - 10*a*b^5 + b^6)*c^2 - 2*(3*a*b^6 - b^7)*
c)/(b^2*c^10 - 4*a*c^11)) + (b^8 + 4*(a^4 - 2*a^3*b)*c^4 - (17*a^3*b^2 - 14
*a^2*b^3)*c^3 + (20*a^2*b^4 - 7*a*b^5)*c^2 - (8*a*b^6 - b^7)*c)*x^2)*sqrt((
b^5 + 2*a^2*c^3 + (5*a^2*b - 4*a*b^2)*c^2 - (5*a*b^3 - b^4)*c - (b^2*c^5 -
4*a*c^6)*sqrt((b^8 + (a^4 - 4*a^3*b + 4*a^2*b^2)*c^4 - 2*(3*a^3*b^2 - 7*a^2
*b^3 + 2*a*b^4)*c^3 + (11*a^2*b^4 - 10*a*b^5 + b^6)*c^2 - 2*(3*a*b^6 - b^7)
*c)/(b^2*c^10 - 4*a*c^11)))/(b^2*c^5 - 4*a*c^6)) - 2*(a^3*b^4 + (a^5 - 2*a^
4*b)*c^2 - (3*a^4*b^2 - a^3*b^3)*c)*sqrt(-x^2 + 1))/x^2) - 3*sqrt(1/2)*c^2*
sqrt((b^5 + 2*a^2*c^3 + (5*a^2*b - 4*a*b^2)*c^2 - (5*a*b^3 - b^4)*c + (b^2*
c^5 - 4*a*c^6)*sqrt((b^8 + (a^4 - 4*a^3*b + 4*a^2*b^2)*c^4 - 2*(3*a^3*b^2 -
7*a^2*b^3 + 2*a*b^4)*c^3 + (11*a^2*b^4 - 10*a*b^5 + b^6)*c^2 - 2*(3*a*b^6
- b^7)*c)/(b^2*c^10 - 4*a*c^11)))/(b^2*c^5 - 4*a*c^6))*log(-(2*a^3*b^4 - (a
^2*b^2*c^5 - 4*a^3*c^6)*x^2*sqrt((b^8 + (a^4 - 4*a^3*b + 4*a^2*b^2)*c^4 - 2
*(3*a^3*b^2 - 7*a^2*b^3 + 2*a*b^4)*c^3 + (11*a^2*b^4 - 10*a*b^5 + b^6)*c^2
- 2*(3*a*b^6 - b^7)*c)/(b^2*c^10 - 4*a*c^11)) + 2*(a^5 - 2*a^4*b)*c^2 + (a^
2*b^5 + (a^4*b - 2*a^3*b^2)*c^2 - (3*a^3*b^3 - a^2*b^4)*c)*x^2 - 2*(3*a^4*b
^2 - a^3*b^3)*c + sqrt(1/2)*((b^5*c^5 - 7*a*b^3*c^6 + 12*a^2*b*c^7)*x^2*sqr
t((b^8 + (a^4 - 4*a^3*b + 4*a^2*b^2)*c^4 - 2*(3*a^3*b^2 - 7*a^2*b^3 + 2*a*b
^4)*c^3 + (11*a^2*b^4 - 10*a*b^5 + b^6)*c^2 - 2*(3*a*b^6 - b^7)*c)/(b^2*c^1
0 - 4*a*c^11)) - (b^8 + 4*(a^4 - 2*a^3*b)*c^4 - (17*a^3*b^2 - 14*a^2*b^3)*c
^3 + (20*a^2*b^4 - 7*a*b^5)*c^2 - (8*a*b^6 - b^7)*c)*x^2)*sqrt((b^5 + 2*a^2
*c^3 + (5*a^2*b - 4*a*b^2)*c^2 - (5*a*b^3 - b^4)*c + (b^2*c^5 - 4*a*c^6)*sq
rt((b^8 + (a^4 - 4*a^3*b + 4*a^2*b^2)*c^4 - 2*(3*a^3*b^2 - 7*a^2*b^3 + 2*a*
b^4)*c^3 + (11*a^2*b^4 - 10*a*b^5 + b^6)*c^2 - 2*(3*a*b^6 - b^7)*c)/(b^2*c^
10 - 4*a*c^11)))/(b^2*c^5 - 4*a*c^6)) - 2*(a^3*b^4 + (a^5 - 2*a^4*b)*c^2 -
(3*a^4*b^2 - a^3*b^3)*c)*sqrt(-x^2 + 1))/x^2) + 3*sqrt(1/2)*c^2*sqrt((b^5 +
2*a^2*c^3 + (5*a^2*b - 4*a*b^2)*c^2 - (5*a*b^3 - b^4)*c + (b^2*c^5 - 4*a*c
^6)*sqrt((b^8 + (a^4 - 4*a^3*b + 4*a^2*b^2)*c^4 - 2*(3*a^3*b^2 - 7*a^2*b^3
+ 2*a*b^4)*c^3 + (11*a^2*b^4 - 10*a*b^5 + b^6)*c^2 - 2*(3*a*b^6 - b^7)*c)/(
b^2*c^10 - 4*a*c^11)))/(b^2*c^5 - 4*a*c^6))*log(-(2*a^3*b^4 - (a^2*b^2*c^5
- 4*a^3*c^6)*x^2*sqrt((b^8 + (a^4 - 4*a^3*b + 4*a^2*b^2)*c^4 - 2*(3*a^3*b^2
- 7*a^2*b^3 + 2*a*b^4)*c^3 + (11*a^2*b^4 - 10*a*b^5 + b^6)*c^2 - 2*(3*a*b^
6 - b^7)*c)/(b^2*c^10 - 4*a*c^11)) + 2*(a^5 - 2*a^4*b)*c^2 + (a^2*b^5 + (a^
4*b - 2*a^3*b^2)*c^2 - (3*a^3*b^3 - a^2*b^4)*c)*x^2 - 2*(3*a^4*b^2 - a^3*b^
3)*c - sqrt(1/2)*((b^5*c^5 - 7*a*b^3*c^6 + 12*a^2*b*c^7)*x^2*sqrt((b^8 + (a
^4 - 4*a^3*b + 4*a^2*b^2)*c^4 - 2*(3*a^3*b^2 - 7*a^2*b^3 + 2*a*b^4)*c^3 + (
11*a^2*b^4 - 10*a*b^5 + b^6)*c^2 - 2*(3*a*b^6 - b^7)*c)/(b^2*c^10 - 4*a*c^1
1)) - (b^8 + 4*(a^4 - 2*a^3*b)*c^4 - (17*a^3*b^2 - 14*a^2*b^3)*c^3 + (20*a^
2*b^4 - 7*a*b^5)*c^2 - (8*a*b^6 - b^7)*c)*x^2)*sqrt((b^5 + 2*a^2*c^3 + (5*a
^2*b - 4*a*b^2)*c^2 - (5*a*b^3 - b^4)*c + (b^2*c^5 - 4*a*c^6)*sqrt((b^8 + (
a^4 - 4*a^3*b + 4*a^2*b^2)*c^4 - 2*(3*a^3*b^2 - 7*a^2*b^3 + 2*a*b^4)*c^3 +

```
(11*a^2*b^4 - 10*a*b^5 + b^6)*c^2 - 2*(3*a*b^6 - b^7)*c)/(b^2*c^10 - 4*a*c^
11)))/(b^2*c^5 - 4*a*c^6)) - 2*(a^3*b^4 + (a^5 - 2*a^4*b)*c^2 - (3*a^4*b^2
- a^3*b^3)*c)*sqrt(-x^2 + 1))/x^2) - 2*(c*x^2 - 3*b - c)*sqrt(-x^2 + 1))/c^
2
```

Sympy [F]

$$\int \frac{x^5\sqrt{1-x^2}}{a+bx^2+cx^4}\,dx = \int \frac{x^5\sqrt{-(x-1)\,(x+1)}}{a+bx^2+cx^4}\,dx$$

```
[In] integrate(x**5*(-x**2+1)**(1/2)/(c*x**4+b*x**2+a),x)
[Out] Integral(x**5*sqrt(-(x - 1)*(x + 1))/(a + b*x**2 + c*x**4), x)
```

Maxima [F]

$$\int \frac{x^5\sqrt{1-x^2}}{a+bx^2+cx^4}\,dx = \int \frac{\sqrt{-x^2+1}x^5}{cx^4+bx^2+a}\,dx$$

```
[In] integrate(x^5*(-x^2+1)^(1/2)/(c*x^4+b*x^2+a),x, algorithm="maxima")
[Out] integrate(sqrt(-x^2 + 1)*x^5/(c*x^4 + b*x^2 + a), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 4637 vs. $2(237) = 474$.

Time = 1.15 (sec) , antiderivative size = 4637, normalized size of antiderivative = 16.50

$$\int \frac{x^5\sqrt{1-x^2}}{a+bx^2+cx^4}\,dx = \text{Too large to display}$$

```
[In] integrate(x^5*(-x^2+1)^(1/2)/(c*x^4+b*x^2+a),x, algorithm="giac")
[Out] 1/8*(2*b^6*c^4 - 14*a*b^4*c^5 + 6*b^5*c^5 + 24*a^2*b^2*c^6 - 40*a*b^3*c^6 +
 4*b^4*c^6 + 64*a^2*b*c^7 - 24*a*b^2*c^7 + 32*a^2*c^8 - sqrt(2)*sqrt(b^2 -
4*a*c)*sqrt(-b*c - 2*c^2 - sqrt(b^2 - 4*a*c)*c)*b^6*c^2 + 7*sqrt(2)*sqrt(b^
2 - 4*a*c)*sqrt(-b*c - 2*c^2 - sqrt(b^2 - 4*a*c)*c)*a*b^4*c^3 - 5*sqrt(2)*s
qrt(b^2 - 4*a*c)*sqrt(-b*c - 2*c^2 - sqrt(b^2 - 4*a*c)*c)*b^5*c^3 - 12*sqrt
(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - 2*c^2 - sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^4
+ 26*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - 2*c^2 - sqrt(b^2 - 4*a*c)*c)*a*b
^3*c^4 - 13*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - 2*c^2 - sqrt(b^2 - 4*a*c)
*c)*b^4*c^4 - 32*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - 2*c^2 - sqrt(b^2 - 4
```

*a*c)*c)*a^2*b*c^5 + 43*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - 2*c^2 - sqrt(
b^2 - 4*a*c)*c)*a*b^2*c^5 - 19*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - 2*c^2
- sqrt(b^2 - 4*a*c)*c)*b^3*c^5 - 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - 2
*c^2 - sqrt(b^2 - 4*a*c)*c)*a^2*c^6 + 48*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*
c - 2*c^2 - sqrt(b^2 - 4*a*c)*c)*a*b*c^6 - 10*sqrt(2)*sqrt(b^2 - 4*a*c)*sqr
t(-b*c - 2*c^2 - sqrt(b^2 - 4*a*c)*c)*b^2*c^6 + 20*sqrt(2)*sqrt(b^2 - 4*a*c
)*sqrt(-b*c - 2*c^2 - sqrt(b^2 - 4*a*c)*c)*a*c^7 - 2*(b^2 - 4*a*c)*b^4*c^4
+ 6*(b^2 - 4*a*c)*a*b^2*c^5 - 6*(b^2 - 4*a*c)*b^3*c^5 + 16*(b^2 - 4*a*c)*a*
b*c^6 - 4*(b^2 - 4*a*c)*b^2*c^6 + 8*(b^2 - 4*a*c)*a*c^7 - (2*b^6*c^2 - 18*a
*b^4*c^3 + 2*b^5*c^3 + 48*a^2*b^2*c^4 - 16*a*b^3*c^4 - 32*a^3*c^5 + 32*a^2*
b*c^5 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - 2*c^2 - sqrt(b^2 - 4*a*c)*c)*
b^6 + 9*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - 2*c^2 - sqrt(b^2 - 4*a*c)*c)*
a*b^4*c - 3*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - 2*c^2 - sqrt(b^2 - 4*a*c)
*c)*b^5*c - 24*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - 2*c^2 - sqrt(b^2 - 4*a
*c)*c)*a^2*b^2*c^2 + 18*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - 2*c^2 - sqrt(
b^2 - 4*a*c)*c)*a*b^3*c^2 - 7*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - 2*c^2 -
sqrt(b^2 - 4*a*c)*c)*b^4*c^2 + 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - 2*
c^2 - sqrt(b^2 - 4*a*c)*c)*a^3*c^3 - 24*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c
- 2*c^2 - sqrt(b^2 - 4*a*c)*c)*a^2*b*c^3 + 33*sqrt(2)*sqrt(b^2 - 4*a*c)*sq
rt(-b*c - 2*c^2 - sqrt(b^2 - 4*a*c)*c)*a*b^2*c^3 - 5*sqrt(2)*sqrt(b^2 - 4*a
*c)*sqrt(-b*c - 2*c^2 - sqrt(b^2 - 4*a*c)*c)*b^3*c^3 - 20*sqrt(2)*sqrt(b^2
- 4*a*c)*sqrt(-b*c - 2*c^2 - sqrt(b^2 - 4*a*c)*c)*a^2*c^4 + 20*sqrt(2)*sqrt
(b^2 - 4*a*c)*sqrt(-b*c - 2*c^2 - sqrt(b^2 - 4*a*c)*c)*a*b*c^4 - 2*(b^2 - 4
*a*c)*b^4*c^2 + 10*(b^2 - 4*a*c)*a*b^2*c^3 - 2*(b^2 - 4*a*c)*b^3*c^3 - 8*(b
^2 - 4*a*c)*a^2*c^4 + 8*(b^2 - 4*a*c)*a*b*c^4)*c^2 + 2*(sqrt(2)*sqrt(-b*c -
2*c^2 - sqrt(b^2 - 4*a*c)*c)*a*b^5*c^2 + sqrt(2)*sqrt(-b*c - 2*c^2 - sqrt(
b^2 - 4*a*c)*c)*b^6*c^2 - 8*sqrt(2)*sqrt(-b*c - 2*c^2 - sqrt(b^2 - 4*a*c)*c
)*a^2*b^3*c^3 - 6*sqrt(2)*sqrt(-b*c - 2*c^2 - sqrt(b^2 - 4*a*c)*c)*a*b^4*c^
3 + 3*sqrt(2)*sqrt(-b*c - 2*c^2 - sqrt(b^2 - 4*a*c)*c)*b^5*c^3 + 2*a*b^5*c^
3 + 2*b^6*c^3 + 16*sqrt(2)*sqrt(-b*c - 2*c^2 - sqrt(b^2 - 4*a*c)*c)*a^3*b*c
^4 + 8*sqrt(2)*sqrt(-b*c - 2*c^2 - sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^4 - 11*sq
rt(2)*sqrt(-b*c - 2*c^2 - sqrt(b^2 - 4*a*c)*c)*a*b^3*c^4 - 16*a^2*b^3*c^4 +
7*sqrt(2)*sqrt(-b*c - 2*c^2 - sqrt(b^2 - 4*a*c)*c)*b^4*c^4 - 16*a*b^4*c^4
+ 2*b^5*c^4 - 4*sqrt(2)*sqrt(-b*c - 2*c^2 - sqrt(b^2 - 4*a*c)*c)*a^2*b*c^5
+ 32*a^3*b*c^5 - 28*sqrt(2)*sqrt(-b*c - 2*c^2 - sqrt(b^2 - 4*a*c)*c)*a*b^2*
c^5 + 32*a^2*b^2*c^5 + 5*sqrt(2)*sqrt(-b*c - 2*c^2 - sqrt(b^2 - 4*a*c)*c)*b
^3*c^5 - 16*a*b^3*c^5 - 20*sqrt(2)*sqrt(-b*c - 2*c^2 - sqrt(b^2 - 4*a*c)*c)
*a*b*c^6 + 32*a^2*b*c^6 - 2*(b^2 - 4*a*c)*a*b^3*c^3 - 2*(b^2 - 4*a*c)*b^4*c
^3 + 8*(b^2 - 4*a*c)*a^2*b*c^4 + 8*(b^2 - 4*a*c)*a*b^2*c^4 - 2*(b^2 - 4*a*c
)*b^3*c^4 + 8*(b^2 - 4*a*c)*a*b*c^5)*abs(c))*arctan(2*sqrt(1/2)*sqrt(-x^2 +
1)/sqrt(-(b*c^3 + 2*c^4 + sqrt(-4*(a*c^3 + b*c^3 + c^4)*c^4 + (b*c^3 + 2*c
^4)^2))/c^4))/((a*b^4*c^4 + b^5*c^4 - 8*a^2*b^2*c^5 - 6*a*b^3*c^5 + 3*b^4*c
^5 + 16*a^3*c^6 + 8*a^2*b*c^6 - 11*a*b^2*c^6 + 7*b^3*c^6 - 4*a^2*c^7 - 28*a
*b*c^7 + 5*b^2*c^7 - 20*a*c^8)*c^2) + 1/8*(2*b^6*c^4 - 14*a*b^4*c^5 + 6*b^5
*c^5 + 24*a^2*b^2*c^6 - 40*a*b^3*c^6 + 4*b^4*c^6 + 64*a^2*b*c^7 - 24*a*b^2*

c^7 + 32*a^2*c^8 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - 2*c^2 + sqrt(b^2 - 4*a*c)*c)*b^6*c^2 + 7*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - 2*c^2 + sqrt(b^2 - 4*a*c)*c)*a*b^4*c^3 - 5*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - 2*c^2 + sqrt(b^2 - 4*a*c)*c)*b^5*c^3 - 12*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - 2*c^2 + sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^4 + 26*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - 2*c^2 + sqrt(b^2 - 4*a*c)*c)*a*b^3*c^4 - 13*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - 2*c^2 + sqrt(b^2 - 4*a*c)*c)*b^4*c^4 - 32*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - 2*c^2 + sqrt(b^2 - 4*a*c)*c)*a^2*b*c^5 + 43*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - 2*c^2 + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^5 - 19*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - 2*c^2 + sqrt(b^2 - 4*a*c)*c)*b^3*c^5 - 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - 2*c^2 + sqrt(b^2 - 4*a*c)*c)*a^2*c^6 + 48*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - 2*c^2 + sqrt(b^2 - 4*a*c)*c)*a*b*c^6 - 10*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - 2*c^2 + sqrt(b^2 - 4*a*c)*c)*b^2*c^6 + 20*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - 2*c^2 + sqrt(b^2 - 4*a*c)*c)*a*c^7 - 2*(b^2 - 4*a*c)*b^4*c^4 + 6*(b^2 - 4*a*c)*a*b^2*c^5 - 6*(b^2 - 4*a*c)*b^3*c^5 + 16*(b^2 - 4*a*c)*a*b*c^6 - 4*(b^2 - 4*a*c)*b^2*c^6 + 8*(b^2 - 4*a*c)*a*c^7 - (2*b^6*c^2 - 18*a*b^4*c^3 + 2*b^5*c^3 + 48*a^2*b^2*c^4 - 16*a*b^3*c^4 - 32*a^3*c^5 + 32*a^2*b*c^5 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - 2*c^2 + sqrt(b^2 - 4*a*c)*c)*b^6 + 9*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - 2*c^2 + sqrt(b^2 - 4*a*c)*c)*a*b^4*c - 3*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - 2*c^2 + sqrt(b^2 - 4*a*c)*c)*b^5*c - 24*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - 2*c^2 + sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^2 + 18*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - 2*c^2 + sqrt(b^2 - 4*a*c)*c)*a*b^3*c^2 - 7*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - 2*c^2 + sqrt(b^2 - 4*a*c)*c)*b^4*c^2 + 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - 2*c^2 + sqrt(b^2 - 4*a*c)*c)*a^3*c^3 - 24*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - 2*c^2 + sqrt(b^2 - 4*a*c)*c)*a^2*b*c^3 + 33*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - 2*c^2 + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^3 - 5*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - 2*c^2 + sqrt(b^2 - 4*a*c)*c)*b^3*c^3 - 20*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - 2*c^2 + sqrt(b^2 - 4*a*c)*c)*a^2*c^4 + 20*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - 2*c^2 + sqrt(b^2 - 4*a*c)*c)*a*b*c^4 - 2*(b^2 - 4*a*c)*b^4*c^2 + 10*(b^2 - 4*a*c)*a*b^2*c^3 - 2*(b^2 - 4*a*c)*b^3*c^3 - 8*(b^2 - 4*a*c)*a^2*c^4 + 8*(b^2 - 4*a*c)*a*b*c^4)*c^2 + 2*(sqrt(2)*sqrt(-b*c - 2*c^2 + sqrt(b^2 - 4*a*c)*c)*a*b^5*c^2 + sqrt(2)*sqrt(-b*c - 2*c^2 + sqrt(b^2 - 4*a*c)*c)*b^6*c^2 - 8*sqrt(2)*sqrt(-b*c - 2*c^2 + sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c^3 - 6*sqrt(2)*sqrt(-b*c - 2*c^2 + sqrt(b^2 - 4*a*c)*c)*a*b^4*c^3 + 3*sqrt(2)*sqrt(-b*c - 2*c^2 + sqrt(b^2 - 4*a*c)*c)*b^5*c^3 - 2*a*b^5*c^3 - 2*b^6*c^3 + 16*sqrt(2)*sqrt(-b*c - 2*c^2 + sqrt(b^2 - 4*a*c)*c)*a^3*b*c^4 + 8*sqrt(2)*sqrt(-b*c - 2*c^2 + sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^4 - 11*sqrt(2)*sqrt(-b*c - 2*c^2 + sqrt(b^2 - 4*a*c)*c)*a*b^3*c^4 + 16*a^2*b^3*c^4 + 7*sqrt(2)*sqrt(-b*c - 2*c^2 + sqrt(b^2 - 4*a*c)*c)*b^4*c^4 + 16*a*b^4*c^4 - 2*b^5*c^4 - 4*sqrt(2)*sqrt(-b*c - 2*c^2 + sqrt(b^2 - 4*a*c)*c)*a^2*b*c^5 - 32*a^3*b*c^5 - 28*sqrt(2)*sqrt(-b*c - 2*c^2 + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^5 - 32*a^2*b^2*c^5 + 5*sqrt(2)*sqrt(-b*c - 2*c^2 + sqrt(b^2 - 4*a*c)*c)*b^3*c^5 + 16*a*b^3*c^5 - 20*sqrt(2)*sqrt(-b*c - 2*c^2 + sqrt(b^2 - 4*a*c)*c)*a*b*c^6 - 32*a^2*b*c^6 + 2*(b^2 - 4*

```
a*c)*a*b^3*c^3 + 2*(b^2 - 4*a*c)*b^4*c^3 - 8*(b^2 - 4*a*c)*a^2*b*c^4 - 8*(b
^2 - 4*a*c)*a*b^2*c^4 + 2*(b^2 - 4*a*c)*b^3*c^4 - 8*(b^2 - 4*a*c)*a*b*c^5)*
abs(c))*arctan(2*sqrt(1/2)*sqrt(-x^2 + 1)/sqrt(-(b*c^3 + 2*c^4 - sqrt(-4*(a
*c^3 + b*c^3 + c^4)*c^4 + (b*c^3 + 2*c^4)^2))/c^4))/((a*b^4*c^4 + b^5*c^4 -
 8*a^2*b^2*c^5 - 6*a*b^3*c^5 + 3*b^4*c^5 + 16*a^3*c^6 + 8*a^2*b*c^6 - 11*a*
b^2*c^6 + 7*b^3*c^6 - 4*a^2*c^7 - 28*a*b*c^7 + 5*b^2*c^7 - 20*a*c^8)*c^2) -
 1/3*((-x^2 + 1)^(3/2)*c^2 + 3*sqrt(-x^2 + 1)*b*c)/c^3
```

Mupad [B] (verification not implemented)

Time = 8.34 (sec) , antiderivative size = 917, normalized size of antiderivative = 3.26

$$\int \frac{x^5\sqrt{1-x^2}}{a+bx^2+cx^4}\,dx = \sqrt{1-x^2}\left(\frac{2}{3\,c}-\frac{\frac{b}{c}+1}{c}+\frac{x^2}{3\,c}\right)$$

$$-\frac{\ln\left(\frac{\frac{\left(x\,\sqrt{-\frac{b-\sqrt{b^2-4\,a\,c}}{2\,c}}-1\right)\,1i}{\sqrt{\frac{b-\sqrt{b^2-4\,a\,c}}{2\,c}+1}}-\sqrt{1-x^2}\,1i}{x-\sqrt{-\frac{b-\sqrt{b^2-4\,a\,c}}{2\,c}}}\right)\,\left(b^3\,c+b^4-b^3\,\sqrt{b^2-4\,a\,c}+4\,a^2\,c^2+2\,a\,c^2\,\sqrt{b^2-4\,a\,c}-b^2\,c\,\sqrt{b^2-\right.}{4\,c^3\,\sqrt{\frac{b-\sqrt{b^2-4\,a\,c}}{2\,c}+1}\,\left(4\,a\,c-b^2\right)}$$

$$-\frac{\ln\left(\frac{\frac{\left(x\,\sqrt{-\frac{b+\sqrt{b^2-4\,a\,c}}{2\,c}}+1\right)\,1i}{\sqrt{\frac{b+\sqrt{b^2-4\,a\,c}}{2\,c}+1}}+\sqrt{1-x^2}\,1i}{x+\sqrt{-\frac{b+\sqrt{b^2-4\,a\,c}}{2\,c}}}\right)\,\left(b^3\,c+b^4+b^3\,\sqrt{b^2-4\,a\,c}+4\,a^2\,c^2-2\,a\,c^2\,\sqrt{b^2-4\,a\,c}+b^2\,c\,\sqrt{b^2-\right.}{4\,c^3\,\left(4\,a\,c-b^2\right)\,\sqrt{\frac{b+\sqrt{b^2-4\,a\,c}}{2\,c}+1}}$$

$$-\frac{\ln\left(\frac{\frac{\left(x\,\sqrt{-\frac{b+\sqrt{b^2-4\,a\,c}}{2\,c}}-1\right)\,1i}{\sqrt{\frac{b+\sqrt{b^2-4\,a\,c}}{2\,c}+1}}-\sqrt{1-x^2}\,1i}{x-\sqrt{-\frac{b+\sqrt{b^2-4\,a\,c}}{2\,c}}}\right)\,\left(b^3\,c+b^4+b^3\,\sqrt{b^2-4\,a\,c}+4\,a^2\,c^2-2\,a\,c^2\,\sqrt{b^2-4\,a\,c}+b^2\,c\,\sqrt{b^2-\right.}{4\,c^3\,\left(4\,a\,c-b^2\right)\,\sqrt{\frac{b+\sqrt{b^2-4\,a\,c}}{2\,c}+1}}$$

$$-\frac{\ln\left(\frac{\frac{\left(x\,\sqrt{-\frac{b-\sqrt{b^2-4\,a\,c}}{2\,c}}+1\right)\,1i}{\sqrt{\frac{b-\sqrt{b^2-4\,a\,c}}{2\,c}+1}}+\sqrt{1-x^2}\,1i}{x+\sqrt{-\frac{b-\sqrt{b^2-4\,a\,c}}{2\,c}}}\right)\,\left(b^3\,c+b^4-b^3\,\sqrt{b^2-4\,a\,c}+4\,a^2\,c^2+2\,a\,c^2\,\sqrt{b^2-4\,a\,c}-b^2\,c\,\sqrt{b^2-\right.}{4\,c^3\,\sqrt{\frac{b-\sqrt{b^2-4\,a\,c}}{2\,c}+1}\,\left(4\,a\,c-b^2\right)}$$

```
[In] int((x^5*(1 - x^2)^(1/2))/(a + b*x^2 + c*x^4),x)
```

```
[Out] (1 - x^2)^(1/2)*(2/(3*c) - (b/c + 1)/c + x^2/(3*c)) - (log((((x*(-(b - (b^2
 - 4*a*c)^(1/2))/(2*c))^(1/2) - 1)*1i)/((b - (b^2 - 4*a*c)^(1/2))/(2*c) + 1
)^(1/2) - (1 - x^2)^(1/2)*1i)/(x - (-(b - (b^2 - 4*a*c)^(1/2))/(2*c))^(1/2)
))*(b^3*c + b^4 - b^3*(b^2 - 4*a*c)^(1/2) + 4*a^2*c^2 + 2*a*c^2*(b^2 - 4*a*
c)^(1/2) - b^2*c*(b^2 - 4*a*c)^(1/2) - 4*a*b*c^2 - 5*a*b^2*c + 3*a*b*c*(b^2
 - 4*a*c)^(1/2)))/(4*c^3*((b - (b^2 - 4*a*c)^(1/2))/(2*c) + 1)^(1/2)*(4*a*c
 - b^2)) - (log((((x*(-(b + (b^2 - 4*a*c)^(1/2))/(2*c))^(1/2) + 1)*1i)/((b
+ (b^2 - 4*a*c)^(1/2))/(2*c) + 1)^(1/2) + (1 - x^2)^(1/2)*1i)/(x + (-(b + (
b^2 - 4*a*c)^(1/2))/(2*c))^(1/2)))*(b^3*c + b^4 + b^3*(b^2 - 4*a*c)^(1/2) +
 4*a^2*c^2 - 2*a*c^2*(b^2 - 4*a*c)^(1/2) + b^2*c*(b^2 - 4*a*c)^(1/2) - 4*a*
b*c^2 - 5*a*b^2*c - 3*a*b*c*(b^2 - 4*a*c)^(1/2)))/(4*c^3*(4*a*c - b^2)*((b
+ (b^2 - 4*a*c)^(1/2))/(2*c) + 1)^(1/2)) - (log((((x*(-(b + (b^2 - 4*a*c)^(
1/2))/(2*c))^(1/2) - 1)*1i)/((b + (b^2 - 4*a*c)^(1/2))/(2*c) + 1)^(1/2) - (
1 - x^2)^(1/2)*1i)/(x - (-(b + (b^2 - 4*a*c)^(1/2))/(2*c))^(1/2)))*(b^3*c +
 b^4 + b^3*(b^2 - 4*a*c)^(1/2) + 4*a^2*c^2 - 2*a*c^2*(b^2 - 4*a*c)^(1/2) +
b^2*c*(b^2 - 4*a*c)^(1/2) - 4*a*b*c^2 - 5*a*b^2*c - 3*a*b*c*(b^2 - 4*a*c)^(
1/2)))/(4*c^3*(4*a*c - b^2)*((b + (b^2 - 4*a*c)^(1/2))/(2*c) + 1)^(1/2)) -
(log((((x*(-(b - (b^2 - 4*a*c)^(1/2))/(2*c))^(1/2) + 1)*1i)/((b - (b^2 - 4*
a*c)^(1/2))/(2*c) + 1)^(1/2) + (1 - x^2)^(1/2)*1i)/(x + (-(b - (b^2 - 4*a*c
)^(1/2))/(2*c))^(1/2)))*(b^3*c + b^4 - b^3*(b^2 - 4*a*c)^(1/2) + 4*a^2*c^2
+ 2*a*c^2*(b^2 - 4*a*c)^(1/2) - b^2*c*(b^2 - 4*a*c)^(1/2) - 4*a*b*c^2 - 5*a
*b^2*c + 3*a*b*c*(b^2 - 4*a*c)^(1/2)))/(4*c^3*((b - (b^2 - 4*a*c)^(1/2))/(2
*c) + 1)^(1/2)*(4*a*c - b^2))
```

3.377 $\int \frac{x^3\sqrt{1-x^2}}{a+bx^2+cx^4}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 229

$$\int \frac{x^3\sqrt{1-x^2}}{a+bx^2+cx^4}\,dx = \frac{\sqrt{1-x^2}}{c} - \frac{\left(b+c-\frac{b^2-2ac+bc}{\sqrt{b^2-4ac}}\right)\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{1-x^2}}{\sqrt{b+2c-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}c^{3/2}\sqrt{b+2c-\sqrt{b^2-4ac}}} - \frac{\left(b+c+\frac{b^2-2ac+bc}{\sqrt{b^2-4ac}}\right)\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{1-x^2}}{\sqrt{b+2c+\sqrt{b^2-4ac}}}\right)}{\sqrt{2}c^{3/2}\sqrt{b+2c+\sqrt{b^2-4ac}}}$$

```
[Out] (-x^2+1)^(1/2)/c-1/2*arctanh(2^(1/2)*c^(1/2)*(-x^2+1)^(1/2)/(b+2*c-(-4*a*c+
b^2)^(1/2))^(1/2))*(b+c+(2*a*c-b^2-b*c)/(-4*a*c+b^2)^(1/2))/c^(3/2)*2^(1/2)
/(b+2*c-(-4*a*c+b^2)^(1/2))^(1/2)-1/2*arctanh(2^(1/2)*c^(1/2)*(-x^2+1)^(1/2
)/(b+2*c+(-4*a*c+b^2)^(1/2))^(1/2))*(b+c+(-2*a*c+b^2+b*c)/(-4*a*c+b^2)^(1/2
))/c^(3/2)*2^(1/2)/(b+2*c+(-4*a*c+b^2)^(1/2))^(1/2)
```

Rubi [A] (verified)

Time = 1.05 (sec) , antiderivative size = 229, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.172, Rules used = {1265, 838, 840, 1180, 214}

$$\int \frac{x^3\sqrt{1-x^2}}{a+bx^2+cx^4}\,dx = -\frac{\left(-\frac{-2ac+b^2+bc}{\sqrt{b^2-4ac}}+b+c\right)\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{1-x^2}}{\sqrt{-\sqrt{b^2-4ac}+b+2c}}\right)}{\sqrt{2}c^{3/2}\sqrt{-\sqrt{b^2-4ac}+b+2c}} - \frac{\left(\frac{-2ac+b^2+bc}{\sqrt{b^2-4ac}}+b+c\right)\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{1-x^2}}{\sqrt{\sqrt{b^2-4ac}+b+2c}}\right)}{\sqrt{2}c^{3/2}\sqrt{\sqrt{b^2-4ac}+b+2c}} + \frac{\sqrt{1-x^2}}{c}$$

```
[In] Int[(x^3*Sqrt[1 - x^2])/(a + b*x^2 + c*x^4),x]
```

```
[Out] Sqrt[1 - x^2]/c - ((b + c - (b^2 - 2*a*c + b*c)/Sqrt[b^2 - 4*a*c])*ArcTanh[
(Sqrt[2]*Sqrt[c]*Sqrt[1 - x^2])/Sqrt[b + 2*c - Sqrt[b^2 - 4*a*c]]])/(Sqrt[2
]*c^(3/2)*Sqrt[b + 2*c - Sqrt[b^2 - 4*a*c]]) - ((b + c + (b^2 - 2*a*c + b*c
)/Sqrt[b^2 - 4*a*c])*ArcTanh[(Sqrt[2]*Sqrt[c]*Sqrt[1 - x^2])/Sqrt[b + 2*c +
 Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*c^(3/2)*Sqrt[b + 2*c + Sqrt[b^2 - 4*a*c]])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 838

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Simp[g*((d + e*x)^m/(c*m)), x] + Dist[1/c, Int[
(d + e*x)^(m - 1)*(Simp[c*d*f - a*e*g + (g*c*d - b*e*g + c*e*f)*x, x]/(a +
b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*c
, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && FractionQ[m] && GtQ[m, 0]
```

Rule 840

```
Int[((f_.) + (g_.)*(x_))/(Sqrt[(d_.) + (e_.)*(x_)]*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)), x_Symbol] :> Dist[2, Subst[Int[(e*f - d*g + g*x^2)/(c*d^2 - b
*d*e + a*e^2 - (2*c*d - b*e)*x^2 + c*x^4), x], x, Sqrt[d + e*x]], x] /; Fre
eQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e +
a*e^2, 0]
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1265

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rubi steps

$$\text{integral} = \frac{1}{2}\text{Subst}\left(\int \frac{\sqrt{1-x}x}{a+bx+cx^2}\,dx,x,x^2\right)$$

$$= \frac{\sqrt{1-x^2}}{c} + \frac{\text{Subst}\left(\int \frac{a+(b+c)x}{\sqrt{1-x}(a+bx+cx^2)}\,dx, x, x^2\right)}{2c}$$

$$= \frac{\sqrt{1-x^2}}{c} + \frac{\text{Subst}\left(\int \frac{-a-b-c+(b+c)x^2}{a+b+c+(-b-2c)x^2+cx^4}\,dx, x, \sqrt{1-x^2}\right)}{c}$$

$$= \frac{\sqrt{1-x^2}}{c} + \frac{\left(b+c-\frac{b^2-2ac+bc}{\sqrt{b^2-4ac}}\right)\text{Subst}\left(\int \frac{1}{\frac{1}{2}(-b-2c)+\frac{1}{2}\sqrt{b^2-4ac}+cx^2}\,dx, x, \sqrt{1-x^2}\right)}{2c} + \frac{\left(b+c+\frac{b^2-2ac+bc}{\sqrt{b^2-4ac}}\right)\text{Subst}\left(\int \frac{1}{\frac{1}{2}(-b-2c)-\frac{1}{2}\sqrt{b^2-4ac}+cx^2}\,dx, x, \sqrt{1-x^2}\right)}{2c}$$

$$= \frac{\sqrt{1-x^2}}{c} - \frac{\left(b+c-\frac{b^2-2ac+bc}{\sqrt{b^2-4ac}}\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{1-x^2}}{\sqrt{b+2c-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}c^{3/2}\sqrt{b+2c-\sqrt{b^2-4ac}}} - \frac{\left(b+c+\frac{b^2-2ac+bc}{\sqrt{b^2-4ac}}\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{1-x^2}}{\sqrt{b+2c+\sqrt{b^2-4ac}}}\right)}{\sqrt{2}c^{3/2}\sqrt{b+2c+\sqrt{b^2-4ac}}}$$

Mathematica [A] (verified)

Time = 0.54 (sec) , antiderivative size = 285, normalized size of antiderivative = 1.24

$$\int \frac{x^3\sqrt{1-x^2}}{a+bx^2+cx^4}\,dx$$
$$= \frac{\sqrt{1-x^2}}{c} + \frac{\left(b^2-2ac+bc+b\sqrt{b^2-4ac}+c\sqrt{b^2-4ac}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}\sqrt{1-x^2}}{\sqrt{-b-2c-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}c^{3/2}\sqrt{b^2-4ac}\sqrt{-b-2c-\sqrt{b^2-4ac}}} + \frac{\left(-b^2+2ac-bc+b\sqrt{b^2-4ac}+c\sqrt{b^2-4ac}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}\sqrt{1-x^2}}{\sqrt{-b-2c+\sqrt{b^2-4ac}}}\right)}{\sqrt{2}c^{3/2}\sqrt{b^2-4ac}\sqrt{-b-2c+\sqrt{b^2-4ac}}}$$

```
[In] Integrate[(x^3*Sqrt[1 - x^2])/(a + b*x^2 + c*x^4),x]
```

```
[Out] Sqrt[1 - x^2]/c + ((b^2 - 2*a*c + b*c + b*Sqrt[b^2 - 4*a*c] + c*Sqrt[b^2 -
4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*Sqrt[1 - x^2])/Sqrt[-b - 2*c - Sqrt[b^2 - 4
*a*c]]])/(Sqrt[2]*c^(3/2)*Sqrt[b^2 - 4*a*c]*Sqrt[-b - 2*c - Sqrt[b^2 - 4*a*
c]]) + ((-b^2 + 2*a*c - b*c + b*Sqrt[b^2 - 4*a*c] + c*Sqrt[b^2 - 4*a*c])*Ar
cTan[(Sqrt[2]*Sqrt[c]*Sqrt[1 - x^2])/Sqrt[-b - 2*c + Sqrt[b^2 - 4*a*c]]])/(
Sqrt[2]*c^(3/2)*Sqrt[b^2 - 4*a*c]*Sqrt[-b - 2*c + Sqrt[b^2 - 4*a*c]])
```

Maple [A] (verified)

Time = 0.54 (sec) , antiderivative size = 228, normalized size of antiderivative = 1.00

method	result
pseudoelliptic	$\frac{\sqrt{-x^2+1}+\frac{\left(b\sqrt{-4ac+b^2}+\sqrt{-4ac+b^2}\,c+2ac-b^2-bc\right)\sqrt{2}\,\arctan\left(\frac{c\sqrt{-x^2+1}\,\sqrt{2}}{\sqrt{\left(\sqrt{-4ac+b^2}-b-2c\right)c}}\right)}{2\sqrt{-4ac+b^2}\,\sqrt{\left(\sqrt{-4ac+b^2}-b-2c\right)c}}-\frac{\left(b\sqrt{-4ac+b^2}+\sqrt{-4ac+b^2}\,c-2ac+b^2+bc\right)\sqrt{\cdots}}{2\sqrt{-4ac+b^2}\,\sqrt{\left(b+2c\cdots\right.}}}{c}$
risch	$-\frac{x^2-1}{c\sqrt{-x^2+1}}+\frac{2a\left(\frac{\left(b\sqrt{-4ac+b^2}+2\sqrt{-4ac+b^2}\,c+4ac-b^2\right)\arctan\left(\frac{\frac{2a\left(\sqrt{-x^2+1}-1\right)^2}{x^2}+2\sqrt{-4ac+b^2}+2a+2b}{2\sqrt{4ac-2b^2-2\sqrt{-4ac+b^2}\,a-2b\sqrt{-4ac+b^2}-2ab}}\right)}{\left(8ac-2b^2\right)\sqrt{4ac-2b^2-2\sqrt{-4ac+b^2}\,a-2b\sqrt{-4ac+b^2}-2ab}}-\frac{\left(-b\sqrt{-4ac+\cdots\right.}{\cdots}\right.}{c}$
default	$\frac{2a\left(\frac{\left(b\sqrt{-4ac+b^2}+2\sqrt{-4ac+b^2}\,c+4ac-b^2\right)\arctan\left(\frac{\frac{2a\left(\sqrt{-x^2+1}-1\right)^2}{x^2}+2\sqrt{-4ac+b^2}+2a+2b}{2\sqrt{4ac-2b^2-2\sqrt{-4ac+b^2}\,a-2b\sqrt{-4ac+b^2}-2ab}}\right)}{\left(8ac-2b^2\right)\sqrt{4ac-2b^2-2\sqrt{-4ac+b^2}\,a-2b\sqrt{-4ac+b^2}-2ab}}-\frac{\left(-b\sqrt{-4ac+b^2}-2\sqrt{-4ac+b^2}\,c+\cdots\right.}{\left(8ac-2b^2\right)\cdots}\right.}{c}$

[In] int(x^3*(-x^2+1)^(1/2)/(c*x^4+b*x^2+a),x,method=_RETURNVERBOSE)

[Out] 1/c*((-x^2+1)^(1/2)+1/2*(b*(-4*a*c+b^2)^(1/2)+(-4*a*c+b^2)^(1/2)*c+2*a*c-b^2-b*c)/(-4*a*c+b^2)^(1/2)*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b-2*c)*c)^(1/2)*arctan(c*(-x^2+1)^(1/2)*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b-2*c)*c)^(1/2))-1/2*(b*(-4*a*c+b^2)^(1/2)+(-4*a*c+b^2)^(1/2)*c-2*a*c+b^2+b*c)/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+2*c+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(c*(-x^2+1)^(1/2)*2^(1/2)/((b+2*c+(-4*a*c+b^2)^(1/2))*c)^(1/2)))

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2053 vs. $2(187) = 374$.

Time = 1.47 (sec) , antiderivative size = 2053, normalized size of antiderivative = 8.97

$$\int \frac{x^3\sqrt{1-x^2}}{a+bx^2+cx^4}\,dx = \text{Too large to display}$$

[In] integrate(x^3*(-x^2+1)^(1/2)/(c*x^4+b*x^2+a),x, algorithm="fricas")

[Out] 1/2*(sqrt(1/2)*c*sqrt((b^3 - 2*a*c^2 - (3*a*b - b^2)*c - (b^2*c^3 - 4*a*c^4)*sqrt((b^4 + (a^2 - 2*a*b + b^2)*c^2 - 2*(a*b^2 - b^3)*c)/(b^2*c^6 - 4*a*c^7)))/(b^2*c^3 - 4*a*c^4))*log((2*a^2*b^2 + (a*b^2*c^3 - 4*a^2*c^4)*x^2*sqr

```
t((b^4 + (a^2 - 2*a*b + b^2)*c^2 - 2*(a*b^2 - b^3)*c)/(b^2*c^6 - 4*a*c^7))
+ (a*b^3 - (a^2*b - a*b^2)*c)*x^2 - 2*(a^3 - a^2*b)*c + sqrt(1/2)*((b^4*c^3
 - 6*a*b^2*c^4 + 8*a^2*c^5)*x^2*sqrt((b^4 + (a^2 - 2*a*b + b^2)*c^2 - 2*(a*
b^2 - b^3)*c)/(b^2*c^6 - 4*a*c^7)) + (b^5 + 4*(a^2*b - a*b^2)*c^2 - (5*a*b^
3 - b^4)*c)*x^2)*sqrt((b^3 - 2*a*c^2 - (3*a*b - b^2)*c - (b^2*c^3 - 4*a*c^4
)*sqrt((b^4 + (a^2 - 2*a*b + b^2)*c^2 - 2*(a*b^2 - b^3)*c)/(b^2*c^6 - 4*a*c
^7)))/(b^2*c^3 - 4*a*c^4)) - 2*(a^2*b^2 - (a^3 - a^2*b)*c)*sqrt(-x^2 + 1))/
x^2) - sqrt(1/2)*c*sqrt((b^3 - 2*a*c^2 - (3*a*b - b^2)*c - (b^2*c^3 - 4*a*c
^4)*sqrt((b^4 + (a^2 - 2*a*b + b^2)*c^2 - 2*(a*b^2 - b^3)*c)/(b^2*c^6 - 4*a
*c^7)))/(b^2*c^3 - 4*a*c^4))*log((2*a^2*b^2 + (a*b^2*c^3 - 4*a^2*c^4)*x^2*s
qrt((b^4 + (a^2 - 2*a*b + b^2)*c^2 - 2*(a*b^2 - b^3)*c)/(b^2*c^6 - 4*a*c^7)
) + (a*b^3 - (a^2*b - a*b^2)*c)*x^2 - 2*(a^3 - a^2*b)*c - sqrt(1/2)*((b^4*c
^3 - 6*a*b^2*c^4 + 8*a^2*c^5)*x^2*sqrt((b^4 + (a^2 - 2*a*b + b^2)*c^2 - 2*(
a*b^2 - b^3)*c)/(b^2*c^6 - 4*a*c^7)) + (b^5 + 4*(a^2*b - a*b^2)*c^2 - (5*a*
b^3 - b^4)*c)*x^2)*sqrt((b^3 - 2*a*c^2 - (3*a*b - b^2)*c - (b^2*c^3 - 4*a*c
^4)*sqrt((b^4 + (a^2 - 2*a*b + b^2)*c^2 - 2*(a*b^2 - b^3)*c)/(b^2*c^6 - 4*a
*c^7)))/(b^2*c^3 - 4*a*c^4)) - 2*(a^2*b^2 - (a^3 - a^2*b)*c)*sqrt(-x^2 + 1)
)/x^2) - sqrt(1/2)*c*sqrt((b^3 - 2*a*c^2 - (3*a*b - b^2)*c + (b^2*c^3 - 4*a
*c^4)*sqrt((b^4 + (a^2 - 2*a*b + b^2)*c^2 - 2*(a*b^2 - b^3)*c)/(b^2*c^6 - 4
*a*c^7)))/(b^2*c^3 - 4*a*c^4))*log((2*a^2*b^2 - (a*b^2*c^3 - 4*a^2*c^4)*x^2
*sqrt((b^4 + (a^2 - 2*a*b + b^2)*c^2 - 2*(a*b^2 - b^3)*c)/(b^2*c^6 - 4*a*c^
7)) + (a*b^3 - (a^2*b - a*b^2)*c)*x^2 - 2*(a^3 - a^2*b)*c + sqrt(1/2)*((b^4
*c^3 - 6*a*b^2*c^4 + 8*a^2*c^5)*x^2*sqrt((b^4 + (a^2 - 2*a*b + b^2)*c^2 - 2
*(a*b^2 - b^3)*c)/(b^2*c^6 - 4*a*c^7)) - (b^5 + 4*(a^2*b - a*b^2)*c^2 - (5*
a*b^3 - b^4)*c)*x^2)*sqrt((b^3 - 2*a*c^2 - (3*a*b - b^2)*c + (b^2*c^3 - 4*a
*c^4)*sqrt((b^4 + (a^2 - 2*a*b + b^2)*c^2 - 2*(a*b^2 - b^3)*c)/(b^2*c^6 - 4
*a*c^7)))/(b^2*c^3 - 4*a*c^4)) - 2*(a^2*b^2 - (a^3 - a^2*b)*c)*sqrt(-x^2 +
1))/x^2) + sqrt(1/2)*c*sqrt((b^3 - 2*a*c^2 - (3*a*b - b^2)*c + (b^2*c^3 - 4
*a*c^4)*sqrt((b^4 + (a^2 - 2*a*b + b^2)*c^2 - 2*(a*b^2 - b^3)*c)/(b^2*c^6 -
 4*a*c^7)))/(b^2*c^3 - 4*a*c^4))*log((2*a^2*b^2 - (a*b^2*c^3 - 4*a^2*c^4)*x
^2*sqrt((b^4 + (a^2 - 2*a*b + b^2)*c^2 - 2*(a*b^2 - b^3)*c)/(b^2*c^6 - 4*a*
c^7)) + (a*b^3 - (a^2*b - a*b^2)*c)*x^2 - 2*(a^3 - a^2*b)*c - sqrt(1/2)*((b
^4*c^3 - 6*a*b^2*c^4 + 8*a^2*c^5)*x^2*sqrt((b^4 + (a^2 - 2*a*b + b^2)*c^2 -
 2*(a*b^2 - b^3)*c)/(b^2*c^6 - 4*a*c^7)) - (b^5 + 4*(a^2*b - a*b^2)*c^2 - (
5*a*b^3 - b^4)*c)*x^2)*sqrt((b^3 - 2*a*c^2 - (3*a*b - b^2)*c + (b^2*c^3 - 4
*a*c^4)*sqrt((b^4 + (a^2 - 2*a*b + b^2)*c^2 - 2*(a*b^2 - b^3)*c)/(b^2*c^6 -
 4*a*c^7)))/(b^2*c^3 - 4*a*c^4)) - 2*(a^2*b^2 - (a^3 - a^2*b)*c)*sqrt(-x^2
+ 1))/x^2) + 2*sqrt(-x^2 + 1))/c
```

Sympy [F]

$$\int \frac{x^3\sqrt{1-x^2}}{a+bx^2+cx^4}\,dx = \int \frac{x^3\sqrt{-(x-1)(x+1)}}{a+bx^2+cx^4}\,dx$$

```
[In] integrate(x**3*(-x**2+1)**(1/2)/(c*x**4+b*x**2+a),x)
[Out] Integral(x**3*sqrt(-(x - 1)*(x + 1))/(a + b*x**2 + c*x**4), x)
```

Maxima [F]

$$\int \frac{x^3\sqrt{1-x^2}}{a+bx^2+cx^4}\,dx = \int \frac{\sqrt{-x^2+1}x^3}{cx^4+bx^2+a}\,dx$$

```
[In] integrate(x^3*(-x^2+1)^(1/2)/(c*x^4+b*x^2+a),x, algorithm="maxima")
[Out] integrate(sqrt(-x^2 + 1)*x^3/(c*x^4 + b*x^2 + a), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 4060 vs. $2(187) = 374$.

Time = 1.07 (sec) , antiderivative size = 4060, normalized size of antiderivative = 17.73

$$\int \frac{x^3\sqrt{1-x^2}}{a+bx^2+cx^4}\,dx = \text{Too large to display}$$

```
[In] integrate(x^3*(-x^2+1)^(1/2)/(c*x^4+b*x^2+a),x, algorithm="giac")
[Out] sqrt(-x^2 + 1)/c - 1/8*(2*b^5*c^4 - 12*a*b^3*c^5 + 6*b^4*c^5 + 16*a^2*b*c^6
 - 32*a*b^2*c^6 + 4*b^3*c^6 + 32*a^2*c^7 - 16*a*b*c^7 - sqrt(2)*sqrt(b^2 -
4*a*c)*sqrt(-b*c - 2*c^2 - sqrt(b^2 - 4*a*c)*c)*b^5*c^2 + 6*sqrt(2)*sqrt(b^
2 - 4*a*c)*sqrt(-b*c - 2*c^2 - sqrt(b^2 - 4*a*c)*c)*a*b^3*c^3 - 5*sqrt(2)*s
qrt(b^2 - 4*a*c)*sqrt(-b*c - 2*c^2 - sqrt(b^2 - 4*a*c)*c)*b^4*c^3 - 8*sqrt(
2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - 2*c^2 - sqrt(b^2 - 4*a*c)*c)*a^2*b*c^4 + 2
0*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - 2*c^2 - sqrt(b^2 - 4*a*c)*c)*a*b^2*
c^4 - 13*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - 2*c^2 - sqrt(b^2 - 4*a*c)*c)
*b^3*c^4 - 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - 2*c^2 - sqrt(b^2 - 4*a*
c)*c)*a^2*c^5 + 26*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - 2*c^2 - sqrt(b^2 -
 4*a*c)*c)*a*b*c^5 - 19*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - 2*c^2 - sqrt(
b^2 - 4*a*c)*c)*b^2*c^5 + 20*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - 2*c^2 -
sqrt(b^2 - 4*a*c)*c)*a*c^6 - 10*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - 2*c^2
 - sqrt(b^2 - 4*a*c)*c)*b*c^6 - 2*(b^2 - 4*a*c)*b^3*c^4 + 4*(b^2 - 4*a*c)*a
```

*b*c^5 - 6*(b^2 - 4*a*c)*b^2*c^5 + 8*(b^2 - 4*a*c)*a*c^6 - 4*(b^2 - 4*a*c)*b*c^6 - (2*b^5*c^2 - 16*a*b^3*c^3 + 2*b^4*c^3 + 32*a^2*b*c^4 - 16*a*b^2*c^4 + 32*a^2*c^5 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - 2*c^2 - sqrt(b^2 - 4*a*c)*c)*b^5 + 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - 2*c^2 - sqrt(b^2 - 4*a*c)*c)*a*b^3*c - 3*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - 2*c^2 - sqrt(b^2 - 4*a*c)*c)*b^4*c - 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - 2*c^2 - sqrt(b^2 - 4*a*c)*c)*a^2*b*c^2 + 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - 2*c^2 - sqrt(b^2 - 4*a*c)*c)*a*b^2*c^2 - 7*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - 2*c^2 - sqrt(b^2 - 4*a*c)*c)*b^3*c^2 - 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - 2*c^2 - sqrt(b^2 - 4*a*c)*c)*a^2*c^3 + 28*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - 2*c^2 - sqrt(b^2 - 4*a*c)*c)*a*b*c^3 - 5*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - 2*c^2 - sqrt(b^2 - 4*a*c)*c)*b^2*c^3 + 20*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - 2*c^2 - sqrt(b^2 - 4*a*c)*c)*a*c^4 - 2*(b^2 - 4*a*c)*b^3*c^2 + 8*(b^2 - 4*a*c)*a*b*c^3 - 2*(b^2 - 4*a*c)*b^2*c^3 + 8*(b^2 - 4*a*c)*a*c^4)*c^2 + 2*(sqrt(2)*sqrt(-b*c - 2*c^2 - sqrt(b^2 - 4*a*c)*c)*a*b^4*c^2 + sqrt(2)*sqrt(-b*c - 2*c^2 - sqrt(b^2 - 4*a*c)*c)*b^5*c^2 - 8*sqrt(2)*sqrt(-b*c - 2*c^2 - sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^3 - 6*sqrt(2)*sqrt(-b*c - 2*c^2 - sqrt(b^2 - 4*a*c)*c)*a*b^3*c^3 + 3*sqrt(2)*sqrt(-b*c - 2*c^2 - sqrt(b^2 - 4*a*c)*c)*b^4*c^3 + 2*a*b^4*c^3 + 2*b^5*c^3 + 16*sqrt(2)*sqrt(-b*c - 2*c^2 - sqrt(b^2 - 4*a*c)*c)*a^3*c^4 + 8*sqrt(2)*sqrt(-b*c - 2*c^2 - sqrt(b^2 - 4*a*c)*c)*a^2*b*c^4 - 11*sqrt(2)*sqrt(-b*c - 2*c^2 - sqrt(b^2 - 4*a*c)*c)*a*b^2*c^4 - 16*a^2*b^2*c^4 + 7*sqrt(2)*sqrt(-b*c - 2*c^2 - sqrt(b^2 - 4*a*c)*c)*b^3*c^4 - 16*a*b^3*c^4 + 2*b^4*c^4 - 4*sqrt(2)*sqrt(-b*c - 2*c^2 - sqrt(b^2 - 4*a*c)*c)*a^2*c^5 + 32*a^3*c^5 - 28*sqrt(2)*sqrt(-b*c - 2*c^2 - sqrt(b^2 - 4*a*c)*c)*a*b*c^5 + 32*a^2*b*c^5 + 5*sqrt(2)*sqrt(-b*c - 2*c^2 - sqrt(b^2 - 4*a*c)*c)*b^2*c^5 - 16*a*b^2*c^5 - 20*sqrt(2)*sqrt(-b*c - 2*c^2 - sqrt(b^2 - 4*a*c)*c)*a*c^6 + 32*a^2*c^6 - 2*(b^2 - 4*a*c)*a*b^2*c^3 - 2*(b^2 - 4*a*c)*b^3*c^3 + 8*(b^2 - 4*a*c)*a^2*c^4 + 8*(b^2 - 4*a*c)*a*b*c^4 - 2*(b^2 - 4*a*c)*b^2*c^4 + 8*(b^2 - 4*a*c)*a*c^5)*abs(c))*arctan(2*sqrt(1/2)*sqrt(-x^2 + 1)/sqrt(-(b*c + 2*c^2 + sqrt(-4*(a*c + b*c + c^2)*c^2 + (b*c + 2*c^2)^2))/c^2))/((a*b^4*c^3 + b^5*c^3 - 8*a^2*b^2*c^4 - 6*a*b^3*c^4 + 3*b^4*c^4 + 16*a^3*c^5 + 8*a^2*b*c^5 - 11*a*b^2*c^5 + 7*b^3*c^5 - 4*a^2*c^6 - 28*a*b*c^6 + 5*b^2*c^6 - 20*a*c^7)*c^2) - 1/8*(2*b^5*c^4 - 12*a*b^3*c^5 + 6*b^4*c^5 + 16*a^2*b*c^6 - 32*a*b^2*c^6 + 4*b^3*c^6 + 32*a^2*c^7 - 16*a*b*c^7 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - 2*c^2 + sqrt(b^2 - 4*a*c)*c)*b^5*c^2 + 6*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - 2*c^2 + sqrt(b^2 - 4*a*c)*c)*a*b^3*c^3 - 5*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - 2*c^2 + sqrt(b^2 - 4*a*c)*c)*b^4*c^3 - 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - 2*c^2 + sqrt(b^2 - 4*a*c)*c)*a^2*b*c^4 + 20*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - 2*c^2 + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^4 - 13*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - 2*c^2 + sqrt(b^2 - 4*a*c)*c)*b^3*c^4 - 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - 2*c^2 + sqrt(b^2 - 4*a*c)*c)*a^2*c^5 + 26*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - 2*c^2 + sqrt(b^2 - 4*a*c)*c)*a*b*c^5 - 19*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - 2*c^2 + sqrt(b^2 - 4*a*c)*c)*b^2*c^5 + 20*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - 2*c^2 + sqrt(b^2 - 4*a*c)*c)*a*c^6 - 10*sqrt(2)*sqrt(b^2 - 4

```
*a*c)*sqrt(-b*c - 2*c^2 + sqrt(b^2 - 4*a*c)*c)*b*c^6 - 2*(b^2 - 4*a*c)*b^3*
c^4 + 4*(b^2 - 4*a*c)*a*b*c^5 - 6*(b^2 - 4*a*c)*b^2*c^5 + 8*(b^2 - 4*a*c)*a
*c^6 - 4*(b^2 - 4*a*c)*b*c^6 - (2*b^5*c^2 - 16*a*b^3*c^3 + 2*b^4*c^3 + 32*a
^2*b*c^4 - 16*a*b^2*c^4 + 32*a^2*c^5 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c
- 2*c^2 + sqrt(b^2 - 4*a*c)*c)*b^5 + 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c
- 2*c^2 + sqrt(b^2 - 4*a*c)*c)*a*b^3*c - 3*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-
b*c - 2*c^2 + sqrt(b^2 - 4*a*c)*c)*b^4*c - 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqr
t(-b*c - 2*c^2 + sqrt(b^2 - 4*a*c)*c)*a^2*b*c^2 + 16*sqrt(2)*sqrt(b^2 - 4*a
*c)*sqrt(-b*c - 2*c^2 + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^2 - 7*sqrt(2)*sqrt(b^2
 - 4*a*c)*sqrt(-b*c - 2*c^2 + sqrt(b^2 - 4*a*c)*c)*b^3*c^2 - 16*sqrt(2)*sqr
t(b^2 - 4*a*c)*sqrt(-b*c - 2*c^2 + sqrt(b^2 - 4*a*c)*c)*a^2*c^3 + 28*sqrt(2
)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - 2*c^2 + sqrt(b^2 - 4*a*c)*c)*a*b*c^3 - 5*sq
rt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - 2*c^2 + sqrt(b^2 - 4*a*c)*c)*b^2*c^3 +
20*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - 2*c^2 + sqrt(b^2 - 4*a*c)*c)*a*c^4
 - 2*(b^2 - 4*a*c)*b^3*c^2 + 8*(b^2 - 4*a*c)*a*b*c^3 - 2*(b^2 - 4*a*c)*b^2*
c^3 + 8*(b^2 - 4*a*c)*a*c^4)*c^2 + 2*(sqrt(2)*sqrt(-b*c - 2*c^2 + sqrt(b^2
- 4*a*c)*c)*a*b^4*c^2 + sqrt(2)*sqrt(-b*c - 2*c^2 + sqrt(b^2 - 4*a*c)*c)*b^
5*c^2 - 8*sqrt(2)*sqrt(-b*c - 2*c^2 + sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^3 - 6*
sqrt(2)*sqrt(-b*c - 2*c^2 + sqrt(b^2 - 4*a*c)*c)*a*b^3*c^3 + 3*sqrt(2)*sqrt
(-b*c - 2*c^2 + sqrt(b^2 - 4*a*c)*c)*b^4*c^3 - 2*a*b^4*c^3 - 2*b^5*c^3 + 16
*sqrt(2)*sqrt(-b*c - 2*c^2 + sqrt(b^2 - 4*a*c)*c)*a^3*c^4 + 8*sqrt(2)*sqrt(
-b*c - 2*c^2 + sqrt(b^2 - 4*a*c)*c)*a^2*b*c^4 - 11*sqrt(2)*sqrt(-b*c - 2*c^
2 + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^4 + 16*a^2*b^2*c^4 + 7*sqrt(2)*sqrt(-b*c -
 2*c^2 + sqrt(b^2 - 4*a*c)*c)*b^3*c^4 + 16*a*b^3*c^4 - 2*b^4*c^4 - 4*sqrt(2
)*sqrt(-b*c - 2*c^2 + sqrt(b^2 - 4*a*c)*c)*a^2*c^5 - 32*a^3*c^5 - 28*sqrt(2
)*sqrt(-b*c - 2*c^2 + sqrt(b^2 - 4*a*c)*c)*a*b*c^5 - 32*a^2*b*c^5 + 5*sqrt(
2)*sqrt(-b*c - 2*c^2 + sqrt(b^2 - 4*a*c)*c)*b^2*c^5 + 16*a*b^2*c^5 - 20*sqr
t(2)*sqrt(-b*c - 2*c^2 + sqrt(b^2 - 4*a*c)*c)*a*c^6 - 32*a^2*c^6 + 2*(b^2 -
 4*a*c)*a*b^2*c^3 + 2*(b^2 - 4*a*c)*b^3*c^3 - 8*(b^2 - 4*a*c)*a^2*c^4 - 8*(
b^2 - 4*a*c)*a*b*c^4 + 2*(b^2 - 4*a*c)*b^2*c^4 - 8*(b^2 - 4*a*c)*a*c^5)*abs
(c))*arctan(2*sqrt(1/2)*sqrt(-x^2 + 1)/sqrt(-(b*c + 2*c^2 - sqrt(-4*(a*c +
b*c + c^2)*c^2 + (b*c + 2*c^2)^2))/c^2))/((a*b^4*c^3 + b^5*c^3 - 8*a^2*b^2*
c^4 - 6*a*b^3*c^4 + 3*b^4*c^4 + 16*a^3*c^5 + 8*a^2*b*c^5 - 11*a*b^2*c^5 + 7
*b^3*c^5 - 4*a^2*c^6 - 28*a*b*c^6 + 5*b^2*c^6 - 20*a*c^7)*c^2)
```

Mupad [B] (verification not implemented)

Time = 8.35 (sec) , antiderivative size = 776, normalized size of antiderivative = 3.39

$$\int \frac{x^3\sqrt{1-x^2}}{a+bx^2+cx^4}\,dx = \frac{\sqrt{1-x^2}}{c} - \frac{\ln\left(\frac{\frac{\left(x\sqrt{-\frac{b-\sqrt{b^2-4ac}}{2c}}-1\right)1i}{\sqrt{\frac{b-\sqrt{b^2-4ac}}{2c}+1}}-\sqrt{1-x^2}\,1i}{x-\sqrt{-\frac{b-\sqrt{b^2-4ac}}{2c}}}\right)\left(4ac^2-b^2c-b^3+b^2\sqrt{b^2-4ac}+4abc-2ac\sqrt{b^2-4ac}+bc\sqrt{\right.}{4c^2\sqrt{\frac{b-\sqrt{b^2-4ac}}{2c}+1}\,(4ac-b^2)} + \frac{\ln\left(\frac{\frac{\left(x\sqrt{-\frac{b+\sqrt{b^2-4ac}}{2c}}-1\right)1i}{\sqrt{\frac{b+\sqrt{b^2-4ac}}{2c}+1}}-\sqrt{1-x^2}\,1i}{x-\sqrt{-\frac{b+\sqrt{b^2-4ac}}{2c}}}\right)\left(b^2c-4ac^2+b^3+b^2\sqrt{b^2-4ac}-4abc-2ac\sqrt{b^2-4ac}+bc\sqrt{\right.}{4c^2\,(4ac-b^2)\sqrt{\frac{b+\sqrt{b^2-4ac}}{2c}+1}} - \frac{\ln\left(\frac{\frac{\left(x\sqrt{-\frac{b-\sqrt{b^2-4ac}}{2c}}+1\right)1i}{\sqrt{\frac{b-\sqrt{b^2-4ac}}{2c}+1}}+\sqrt{1-x^2}\,1i}{x+\sqrt{-\frac{b-\sqrt{b^2-4ac}}{2c}}}\right)\left(4ac^2-b^2c-b^3+b^2\sqrt{b^2-4ac}+4abc-2ac\sqrt{b^2-4ac}+bc\sqrt{\right.}{4c^2\sqrt{\frac{b-\sqrt{b^2-4ac}}{2c}+1}\,(4ac-b^2)} + \frac{\ln\left(\frac{\frac{\left(x\sqrt{-\frac{b+\sqrt{b^2-4ac}}{2c}}+1\right)1i}{\sqrt{\frac{b+\sqrt{b^2-4ac}}{2c}+1}}+\sqrt{1-x^2}\,1i}{x+\sqrt{-\frac{b+\sqrt{b^2-4ac}}{2c}}}\right)\left(b^2c-4ac^2+b^3+b^2\sqrt{b^2-4ac}-4abc-2ac\sqrt{b^2-4ac}+bc\sqrt{\right.}{4c^2\,(4ac-b^2)\sqrt{\frac{b+\sqrt{b^2-4ac}}{2c}+1}}$$

```
[In] int((x^3*(1 - x^2)^(1/2))/(a + b*x^2 + c*x^4),x)
```

```
[Out] (1 - x^2)^(1/2)/c - (log((((x*(-(b - (b^2 - 4*a*c)^(1/2))/(2*c))^(1/2) - 1)
*1i)/((b - (b^2 - 4*a*c)^(1/2))/(2*c) + 1)^(1/2) - (1 - x^2)^(1/2)*1i)/(x -
 (-(b - (b^2 - 4*a*c)^(1/2))/(2*c))^(1/2)))*(4*a*c^2 - b^2*c - b^3 + b^2*(b
^2 - 4*a*c)^(1/2) + 4*a*b*c - 2*a*c*(b^2 - 4*a*c)^(1/2) + b*c*(b^2 - 4*a*c)
^(1/2)))/(4*c^2*((b - (b^2 - 4*a*c)^(1/2))/(2*c) + 1)^(1/2)*(4*a*c - b^2))
+ (log((((x*(-(b + (b^2 - 4*a*c)^(1/2))/(2*c))^(1/2) - 1)*1i)/((b + (b^2 -
4*a*c)^(1/2))/(2*c) + 1)^(1/2) - (1 - x^2)^(1/2)*1i)/(x - (-(b + (b^2 - 4*a
*c)^(1/2))/(2*c))^(1/2)))*(b^2*c - 4*a*c^2 + b^3 + b^2*(b^2 - 4*a*c)^(1/2)
```

```
- 4*a*b*c - 2*a*c*(b^2 - 4*a*c)^(1/2) + b*c*(b^2 - 4*a*c)^(1/2)))/(4*c^2*(4
*a*c - b^2)*((b + (b^2 - 4*a*c)^(1/2))/(2*c) + 1)^(1/2)) - (log((((x*(-(b -
 (b^2 - 4*a*c)^(1/2))/(2*c))^(1/2) + 1)*1i)/((b - (b^2 - 4*a*c)^(1/2))/(2*c
) + 1)^(1/2) + (1 - x^2)^(1/2)*1i)/(x + (-(b - (b^2 - 4*a*c)^(1/2))/(2*c))^
(1/2)))*(4*a*c^2 - b^2*c - b^3 + b^2*(b^2 - 4*a*c)^(1/2) + 4*a*b*c - 2*a*c*
(b^2 - 4*a*c)^(1/2) + b*c*(b^2 - 4*a*c)^(1/2)))/(4*c^2*((b - (b^2 - 4*a*c)^
(1/2))/(2*c) + 1)^(1/2)*(4*a*c - b^2)) + (log((((x*(-(b + (b^2 - 4*a*c)^(1/
2))/(2*c))^(1/2) + 1)*1i)/((b + (b^2 - 4*a*c)^(1/2))/(2*c) + 1)^(1/2) + (1
- x^2)^(1/2)*1i)/(x + (-(b + (b^2 - 4*a*c)^(1/2))/(2*c))^(1/2)))*(b^2*c - 4
*a*c^2 + b^3 + b^2*(b^2 - 4*a*c)^(1/2) - 4*a*b*c - 2*a*c*(b^2 - 4*a*c)^(1/2
) + b*c*(b^2 - 4*a*c)^(1/2)))/(4*c^2*(4*a*c - b^2)*((b + (b^2 - 4*a*c)^(1/2
))/(2*c) + 1)^(1/2))
```

3.378 $\int \frac{x\sqrt{1-x^2}}{a+bx^2+cx^4}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 182

$$\int \frac{x\sqrt{1-x^2}}{a+bx^2+cx^4}\,dx = -\frac{\sqrt{b+2c-\sqrt{b^2-4ac}}\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{1-x^2}}{\sqrt{b+2c-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b^2-4ac}} + \frac{\sqrt{b+2c+\sqrt{b^2-4ac}}\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{1-x^2}}{\sqrt{b+2c+\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b^2-4ac}}$$

```
[Out] -1/2*arctanh(2^(1/2)*c^(1/2)*(-x^2+1)^(1/2)/(b+2*c-(-4*a*c+b^2)^(1/2))^(1/2
))*(b+2*c-(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2)/c^(1/2)/(-4*a*c+b^2)^(1/2)+1/2*
arctanh(2^(1/2)*c^(1/2)*(-x^2+1)^(1/2)/(b+2*c+(-4*a*c+b^2)^(1/2))^(1/2))*(b
+2*c+(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2)/c^(1/2)/(-4*a*c+b^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 182, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {1261, 713, 1144, 214}

$$\int \frac{x\sqrt{1-x^2}}{a+bx^2+cx^4}\,dx = \frac{\sqrt{\sqrt{b^2-4ac}+b+2c}\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{1-x^2}}{\sqrt{\sqrt{b^2-4ac}+b+2c}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b^2-4ac}} - \frac{\sqrt{-\sqrt{b^2-4ac}+b+2c}\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{1-x^2}}{\sqrt{-\sqrt{b^2-4ac}+b+2c}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b^2-4ac}}$$

```
[In] Int[(x*Sqrt[1 - x^2])/(a + b*x^2 + c*x^4),x]
```

```
[Out] -((Sqrt[b + 2*c - Sqrt[b^2 - 4*a*c]]*ArcTanh[(Sqrt[2]*Sqrt[c]*Sqrt[1 - x^2]
)/Sqrt[b + 2*c - Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*Sqrt[c]*Sqrt[b^2 - 4*a*c]))
```

```
+ (Sqrt[b + 2*c + Sqrt[b^2 - 4*a*c]]*ArcTanh[(Sqrt[2]*Sqrt[c]*Sqrt[1 - x^2]
)/Sqrt[b + 2*c + Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*Sqrt[c]*Sqrt[b^2 - 4*a*c])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 713

```
Int[Sqrt[(d_.) + (e_.)*(x_)]/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol]
 :> Dist[2*e, Subst[Int[x^2/(c*d^2 - b*d*e + a*e^2 - (2*c*d - b*e)*x^2 + c*
x^4), x], x, Sqrt[d + e*x]], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4
*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 1144

```
Int[((d_.)*(x_))^(m_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> Wi
th[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(d^2/2)*(b/q + 1), Int[(d*x)^(m - 2)/(b/2
 + q/2 + c*x^2), x], x] - Dist[(d^2/2)*(b/q - 1), Int[(d*x)^(m - 2)/(b/2 -
q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - 4*a*c, 0] && G
eQ[m, 2]
```

Rule 1261

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(
p_.), x_Symbol] :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x],
 x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{1}{2}\text{Subst}\left(\int \frac{\sqrt{1-x}}{a+bx+cx^2}\,dx, x, x^2\right) \\
&= -\text{Subst}\left(\int \frac{x^2}{a+b+c+(-b-2c)x^2+cx^4}\,dx, x, \sqrt{1-x^2}\right) \\
&= \frac{1}{2}\left(-1-\frac{b+2c}{\sqrt{b^2-4ac}}\right)\text{Subst}\left(\int \frac{1}{\frac{1}{2}(-b-2c)-\frac{1}{2}\sqrt{b^2-4ac}+cx^2}\,dx, x, \sqrt{1-x^2}\right) \\
&\quad -\frac{1}{2}\left(1-\frac{b+2c}{\sqrt{b^2-4ac}}\right)\text{Subst}\left(\int \frac{1}{\frac{1}{2}(-b-2c)+\frac{1}{2}\sqrt{b^2-4ac}+cx^2}\,dx, x, \sqrt{1-x^2}\right) \\
&= -\frac{\sqrt{b+2c-\sqrt{b^2-4ac}}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{1-x^2}}{\sqrt{b+2c-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b^2-4ac}} \\
&\quad +\frac{\sqrt{b+2c+\sqrt{b^2-4ac}}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{1-x^2}}{\sqrt{b+2c+\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b^2-4ac}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 169, normalized size of antiderivative = 0.93

$$\int \frac{x\sqrt{1-x^2}}{a+bx^2+cx^4}\,dx = \frac{\sqrt{-b-2c-\sqrt{b^2-4ac}}\arctan\left(\frac{\sqrt{2}\sqrt{c}\sqrt{1-x^2}}{\sqrt{-b-2c-\sqrt{b^2-4ac}}}\right)-\sqrt{-b-2c+\sqrt{b^2-4ac}}\arctan\left(\frac{\sqrt{2}\sqrt{c}\sqrt{1-x^2}}{\sqrt{-b-2c+\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b^2-4ac}}$$

```
[In] Integrate[(x*Sqrt[1 - x^2])/(a + b*x^2 + c*x^4),x]
```

```
[Out] (Sqrt[-b - 2*c - Sqrt[b^2 - 4*a*c]]*ArcTan[(Sqrt[2]*Sqrt[c]*Sqrt[1 - x^2])/
Sqrt[-b - 2*c - Sqrt[b^2 - 4*a*c]]] - Sqrt[-b - 2*c + Sqrt[b^2 - 4*a*c]]*Ar
cTan[(Sqrt[2]*Sqrt[c]*Sqrt[1 - x^2])/Sqrt[-b - 2*c + Sqrt[b^2 - 4*a*c]]])/(
Sqrt[2]*Sqrt[c]*Sqrt[b^2 - 4*a*c])
```

Maple [A] (verified)

Time = 0.45 (sec) , antiderivative size = 162, normalized size of antiderivative = 0.89

method	result
pseudoelliptic	$-\frac{\sqrt{2}\left(\frac{\left(\sqrt{-4ac+b^2}-b-2c\right)\arctan\left(\frac{c\sqrt{-x^2+1}\,\sqrt{2}}{\sqrt{\left(\sqrt{-4ac+b^2}-b-2c\right)c}}\right)}{\sqrt{\left(\sqrt{-4ac+b^2}-b-2c\right)c}}-\frac{\left(b+2c+\sqrt{-4ac+b^2}\right)\operatorname{arctanh}\left(\frac{c\sqrt{-x^2+1}\,\sqrt{2}}{\sqrt{\left(b+2c+\sqrt{-4ac+b^2}\right)c}}\right)}{\sqrt{\left(b+2c+\sqrt{-4ac+b^2}\right)c}}\right)}{2\sqrt{-4ac+b^2}}$
default	$2a\left(\frac{\left(-2\sqrt{-4ac+b^2}\,a-b\sqrt{-4ac+b^2}+4ac-b^2\right)\arctan\left(\frac{\frac{2a\left(\sqrt{-x^2+1}-1\right)^2}{x^2}+2\sqrt{-4ac+b^2}+2a+2b}{2\sqrt{4ac-2b^2-2\sqrt{-4ac+b^2}\,a-2b\sqrt{-4ac+b^2}-2ab}}\right)}{2a(4ac-b^2)\sqrt{4ac-2b^2-2\sqrt{-4ac+b^2}\,a-2b\sqrt{-4ac+b^2}-2ab}}-\frac{\left(2\sqrt{-4ac+b^2}\,a\right.}{}\right.$

```
[In] int(x*(-x^2+1)^(1/2)/(c*x^4+b*x^2+a),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*2^(1/2)/(-4*a*c+b^2)^(1/2)*(((-4*a*c+b^2)^(1/2)-b-2*c)/(((-4*a*c+b^2)^
(1/2)-b-2*c)*c)^(1/2)*arctan(c*(-x^2+1)^(1/2)*2^(1/2)/(((-4*a*c+b^2)^(1/2)-
b-2*c)*c)^(1/2))-(b+2*c+(-4*a*c+b^2)^(1/2))/((b+2*c+(-4*a*c+b^2)^(1/2))*c)^
(1/2)*arctanh(c*(-x^2+1)^(1/2)*2^(1/2)/((b+2*c+(-4*a*c+b^2)^(1/2))*c)^(1/2)
))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 871 vs. $2(143) = 286$.

Time = 0.75 (sec) , antiderivative size = 871, normalized size of antiderivative = 4.79

$$\int \frac{x\sqrt{1-x^2}}{a+bx^2+cx^4}\,dx =$$

$$-\frac{1}{2}\sqrt{\frac{1}{2}}\sqrt{\frac{b+2\,c-\frac{b^2c-4\,ac^2}{\sqrt{b^2c^2-4\,ac^3}}}{b^2c-4\,ac^2}}\log\left(\frac{bx^2+\frac{(b^2c-4\,ac^2)x^2}{\sqrt{b^2c^2-4\,ac^3}}+\sqrt{\frac{1}{2}}\left((b^2-4\,ac)x^2+\frac{(b^3c-4\,abc^2)x^2}{\sqrt{b^2c^2-4\,ac^3}}\right)\sqrt{\frac{b+2\,c-\frac{b^2c-4\,ac^2}{\sqrt{b^2c^2-4\,ac^3}}}{b^2c-4\,ac^2}}\cdots}{x^2}\right.$$

$$+\frac{1}{2}\sqrt{\frac{1}{2}}\sqrt{\frac{b+2\,c-\frac{b^2c-4\,ac^2}{\sqrt{b^2c^2-4\,ac^3}}}{b^2c-4\,ac^2}}\log\left(\frac{bx^2+\frac{(b^2c-4\,ac^2)x^2}{\sqrt{b^2c^2-4\,ac^3}}-\sqrt{\frac{1}{2}}\left((b^2-4\,ac)x^2+\frac{(b^3c-4\,abc^2)x^2}{\sqrt{b^2c^2-4\,ac^3}}\right)\sqrt{\frac{b+2\,c-\frac{b^2c-4\,ac^2}{\sqrt{b^2c^2-4\,ac^3}}}{b^2c-4\,ac^2}}\cdots}{x^2}\right.$$

$$-\frac{1}{2}\sqrt{\frac{1}{2}}\sqrt{\frac{b+2\,c+\frac{b^2c-4\,ac^2}{\sqrt{b^2c^2-4\,ac^3}}}{b^2c-4\,ac^2}}\log\left(\frac{bx^2-\frac{(b^2c-4\,ac^2)x^2}{\sqrt{b^2c^2-4\,ac^3}}+\sqrt{\frac{1}{2}}\left((b^2-4\,ac)x^2-\frac{(b^3c-4\,abc^2)x^2}{\sqrt{b^2c^2-4\,ac^3}}\right)\sqrt{\frac{b+2\,c+\frac{b^2c-4\,ac^2}{\sqrt{b^2c^2-4\,ac^3}}}{b^2c-4\,ac^2}}\cdots}{x^2}\right.$$

$$+\frac{1}{2}\sqrt{\frac{1}{2}}\sqrt{\frac{b+2\,c+\frac{b^2c-4\,ac^2}{\sqrt{b^2c^2-4\,ac^3}}}{b^2c-4\,ac^2}}\log\left(\frac{bx^2-\frac{(b^2c-4\,ac^2)x^2}{\sqrt{b^2c^2-4\,ac^3}}-\sqrt{\frac{1}{2}}\left((b^2-4\,ac)x^2-\frac{(b^3c-4\,abc^2)x^2}{\sqrt{b^2c^2-4\,ac^3}}\right)\sqrt{\frac{b+2\,c+\frac{b^2c-4\,ac^2}{\sqrt{b^2c^2-4\,ac^3}}}{b^2c-4\,ac^2}}\cdots}{x^2}\right.$$

```
[In] integrate(x*(-x^2+1)^(1/2)/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] -1/2*sqrt(1/2)*sqrt((b + 2*c - (b^2*c - 4*a*c^2)/sqrt(b^2*c^2 - 4*a*c^3))/(
b^2*c - 4*a*c^2))*log((b*x^2 + (b^2*c - 4*a*c^2)*x^2/sqrt(b^2*c^2 - 4*a*c^3
) + sqrt(1/2)*((b^2 - 4*a*c)*x^2 + (b^3*c - 4*a*b*c^2)*x^2/sqrt(b^2*c^2 - 4
*a*c^3))*sqrt((b + 2*c - (b^2*c - 4*a*c^2)/sqrt(b^2*c^2 - 4*a*c^3))/(b^2*c
- 4*a*c^2)) - 2*sqrt(-x^2 + 1)*a + 2*a)/x^2) + 1/2*sqrt(1/2)*sqrt((b + 2*c
- (b^2*c - 4*a*c^2)/sqrt(b^2*c^2 - 4*a*c^3))/(b^2*c - 4*a*c^2))*log((b*x^2
+ (b^2*c - 4*a*c^2)*x^2/sqrt(b^2*c^2 - 4*a*c^3) - sqrt(1/2)*((b^2 - 4*a*c)*
x^2 + (b^3*c - 4*a*b*c^2)*x^2/sqrt(b^2*c^2 - 4*a*c^3))*sqrt((b + 2*c - (b^2
*c - 4*a*c^2)/sqrt(b^2*c^2 - 4*a*c^3))/(b^2*c - 4*a*c^2)) - 2*sqrt(-x^2 + 1
)*a + 2*a)/x^2) - 1/2*sqrt(1/2)*sqrt((b + 2*c + (b^2*c - 4*a*c^2)/sqrt(b^2*
c^2 - 4*a*c^3))/(b^2*c - 4*a*c^2))*log((b*x^2 - (b^2*c - 4*a*c^2)*x^2/sqrt(
b^2*c^2 - 4*a*c^3) + sqrt(1/2)*((b^2 - 4*a*c)*x^2 - (b^3*c - 4*a*b*c^2)*x^2
/sqrt(b^2*c^2 - 4*a*c^3))*sqrt((b + 2*c + (b^2*c - 4*a*c^2)/sqrt(b^2*c^2 -
4*a*c^3))/(b^2*c - 4*a*c^2)) - 2*sqrt(-x^2 + 1)*a + 2*a)/x^2) + 1/2*sqrt(1/
2)*sqrt((b + 2*c + (b^2*c - 4*a*c^2)/sqrt(b^2*c^2 - 4*a*c^3))/(b^2*c - 4*a*
c^2))*log((b*x^2 - (b^2*c - 4*a*c^2)*x^2/sqrt(b^2*c^2 - 4*a*c^3) - sqrt(1/2
```

```
)*((b^2 - 4*a*c)*x^2 - (b^3*c - 4*a*b*c^2)*x^2/sqrt(b^2*c^2 - 4*a*c^3))*sqr
t((b + 2*c + (b^2*c - 4*a*c^2)/sqrt(b^2*c^2 - 4*a*c^3))/(b^2*c - 4*a*c^2))
- 2*sqrt(-x^2 + 1)*a + 2*a)/x^2)
```

Sympy [F]

$$\int \frac{x\sqrt{1-x^2}}{a+bx^2+cx^4}\,dx = \int \frac{x\sqrt{-(x-1)(x+1)}}{a+bx^2+cx^4}\,dx$$

```
[In] integrate(x*(-x**2+1)**(1/2)/(c*x**4+b*x**2+a),x)
[Out] Integral(x*sqrt(-(x - 1)*(x + 1))/(a + b*x**2 + c*x**4), x)
```

Maxima [F]

$$\int \frac{x\sqrt{1-x^2}}{a+bx^2+cx^4}\,dx = \int \frac{\sqrt{-x^2+1}x}{cx^4+bx^2+a}\,dx$$

```
[In] integrate(x*(-x^2+1)^(1/2)/(c*x^4+b*x^2+a),x, algorithm="maxima")
[Out] integrate(sqrt(-x^2 + 1)*x/(c*x^4 + b*x^2 + a), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 591 vs. $2(143) = 286$.

Time = 1.57 (sec) , antiderivative size = 591, normalized size of antiderivative = 3.25

$$\int \frac{x\sqrt{1-x^2}}{a+bx^2+cx^4}\,dx$$

$$= \frac{\left(2\,b^2c^2 - 8\,ac^3 - \sqrt{2}\sqrt{b^2-4\,ac}\sqrt{-bc-2\,c^2-\sqrt{b^2-4\,ac}c}b^2 + 4\,\sqrt{2}\sqrt{b^2-4\,ac}\sqrt{-bc-2\,c^2-\sqrt{b^2-4\,a}}\right.}{2\,(b^4}$$

$$+ \frac{\left(2\,b^2c^2 - 8\,ac^3 - \sqrt{2}\sqrt{b^2-4\,ac}\sqrt{-bc-2\,c^2+\sqrt{b^2-4\,ac}c}b^2 + 4\,\sqrt{2}\sqrt{b^2-4\,ac}\sqrt{-bc-2\,c^2+\sqrt{b^2-}}\right.}{2\,(b}$$

```
[In] integrate(x*(-x^2+1)^(1/2)/(c*x^4+b*x^2+a),x, algorithm="giac")
[Out] 1/2*(2*b^2*c^2 - 8*a*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - 2*c^2 - sq
rt(b^2 - 4*a*c)*c)*b^2 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - 2*c^2 - sq
```

```
rt(b^2 - 4*a*c)*c)*a*c - 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - 2*c^2 - sq
rt(b^2 - 4*a*c)*c)*b*c - 5*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - 2*c^2 - sq
rt(b^2 - 4*a*c)*c)*c^2 - 2*(b^2 - 4*a*c)*c^2)*arctan(2*sqrt(1/2)*sqrt(-x^2
+ 1)/sqrt(-(b + 2*c + sqrt((b + 2*c)^2 - 4*(a + b + c)*c))/c))/((b^4 - 8*a*
b^2*c + 2*b^3*c + 16*a^2*c^2 - 8*a*b*c^2 + 5*b^2*c^2 - 20*a*c^3)*abs(c)) +
1/2*(2*b^2*c^2 - 8*a*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - 2*c^2 + sq
rt(b^2 - 4*a*c)*c)*b^2 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - 2*c^2 + sq
rt(b^2 - 4*a*c)*c)*a*c - 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - 2*c^2 + sq
rt(b^2 - 4*a*c)*c)*b*c - 5*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - 2*c^2 + sq
rt(b^2 - 4*a*c)*c)*c^2 - 2*(b^2 - 4*a*c)*c^2)*arctan(2*sqrt(1/2)*sqrt(-x^2
+ 1)/sqrt(-(b + 2*c - sqrt((b + 2*c)^2 - 4*(a + b + c)*c))/c))/((b^4 - 8*a*
b^2*c + 2*b^3*c + 16*a^2*c^2 - 8*a*b*c^2 + 5*b^2*c^2 - 20*a*c^3)*abs(c))
```

Mupad [B] (verification not implemented)

Time = 8.30 (sec) , antiderivative size = 649, normalized size of antiderivative = 3.57

$$\int \frac{x\sqrt{1-x^2}}{a+bx^2+cx^4}\,dx = \frac{\ln\left(\frac{\frac{\left(x\sqrt{-\frac{b-\sqrt{b^2-4\,a\,c}}{2\,c}}-1\right)\,1\mathrm{i}}{\sqrt{\frac{b-\sqrt{b^2-4\,a\,c}}{2\,c}+1}}-\sqrt{1-x^2}\,1\mathrm{i}}{x-\sqrt{-\frac{b-\sqrt{b^2-4\,a\,c}}{2\,c}}}\right)\left(4\,a\,c+b\,\sqrt{b^2-4\,a\,c}+2\,c\,\sqrt{b^2-4\,a\,c}-b^2\right)}{4\,c\,\sqrt{\frac{b-\sqrt{b^2-4\,a\,c}}{2\,c}+1}\,\left(4\,a\,c-b^2\right)} - \frac{\ln\left(\frac{\frac{\left(x\sqrt{-\frac{b+\sqrt{b^2-4\,a\,c}}{2\,c}}-1\right)\,1\mathrm{i}}{\sqrt{\frac{b+\sqrt{b^2-4\,a\,c}}{2\,c}+1}}-\sqrt{1-x^2}\,1\mathrm{i}}{x-\sqrt{-\frac{b+\sqrt{b^2-4\,a\,c}}{2\,c}}}\right)\left(b\,\sqrt{b^2-4\,a\,c}-4\,a\,c+2\,c\,\sqrt{b^2-4\,a\,c}+b^2\right)}{4\,c\,\left(4\,a\,c-b^2\right)\,\sqrt{\frac{b+\sqrt{b^2-4\,a\,c}}{2\,c}+1}} + \frac{\ln\left(\frac{\frac{\left(x\sqrt{-\frac{b-\sqrt{b^2-4\,a\,c}}{2\,c}}+1\right)\,1\mathrm{i}}{\sqrt{\frac{b-\sqrt{b^2-4\,a\,c}}{2\,c}+1}}+\sqrt{1-x^2}\,1\mathrm{i}}{x+\sqrt{-\frac{b-\sqrt{b^2-4\,a\,c}}{2\,c}}}\right)\left(4\,a\,c+b\,\sqrt{b^2-4\,a\,c}+2\,c\,\sqrt{b^2-4\,a\,c}-b^2\right)}{4\,c\,\sqrt{\frac{b-\sqrt{b^2-4\,a\,c}}{2\,c}+1}\,\left(4\,a\,c-b^2\right)} - \frac{\ln\left(\frac{\frac{\left(x\sqrt{-\frac{b+\sqrt{b^2-4\,a\,c}}{2\,c}}+1\right)\,1\mathrm{i}}{\sqrt{\frac{b+\sqrt{b^2-4\,a\,c}}{2\,c}+1}}+\sqrt{1-x^2}\,1\mathrm{i}}{x+\sqrt{-\frac{b+\sqrt{b^2-4\,a\,c}}{2\,c}}}\right)\left(b\,\sqrt{b^2-4\,a\,c}-4\,a\,c+2\,c\,\sqrt{b^2-4\,a\,c}+b^2\right)}{4\,c\,\left(4\,a\,c-b^2\right)\,\sqrt{\frac{b+\sqrt{b^2-4\,a\,c}}{2\,c}+1}}$$

```
[In] int((x*(1 - x^2)^(1/2))/(a + b*x^2 + c*x^4),x)
```

```
[Out] (log((((x*(-(b - (b^2 - 4*a*c)^(1/2))/(2*c))^(1/2) - 1)*1i)/((b - (b^2 - 4*
a*c)^(1/2))/(2*c) + 1)^(1/2) - (1 - x^2)^(1/2)*1i)/(x - (-(b - (b^2 - 4*a*c
)^(1/2))/(2*c))^(1/2)))*(4*a*c + b*(b^2 - 4*a*c)^(1/2) + 2*c*(b^2 - 4*a*c)^
(1/2) - b^2))/(4*c*((b - (b^2 - 4*a*c)^(1/2))/(2*c) + 1)^(1/2)*(4*a*c - b^2
)) - (log((((x*(-(b + (b^2 - 4*a*c)^(1/2))/(2*c))^(1/2) - 1)*1i)/((b + (b^2
 - 4*a*c)^(1/2))/(2*c) + 1)^(1/2) - (1 - x^2)^(1/2)*1i)/(x - (-(b + (b^2 -
4*a*c)^(1/2))/(2*c))^(1/2)))*(b*(b^2 - 4*a*c)^(1/2) - 4*a*c + 2*c*(b^2 - 4*
a*c)^(1/2) + b^2))/(4*c*(4*a*c - b^2)*((b + (b^2 - 4*a*c)^(1/2))/(2*c) + 1)
```

```
^(1/2)) + (log((((x*(-(b - (b^2 - 4*a*c)^(1/2))/(2*c))^(1/2) + 1)*1i)/((b -
 (b^2 - 4*a*c)^(1/2))/(2*c) + 1)^(1/2) + (1 - x^2)^(1/2)*1i)/(x + (-(b - (b
^2 - 4*a*c)^(1/2))/(2*c))^(1/2)))*(4*a*c + b*(b^2 - 4*a*c)^(1/2) + 2*c*(b^2
 - 4*a*c)^(1/2) - b^2))/(4*c*((b - (b^2 - 4*a*c)^(1/2))/(2*c) + 1)^(1/2)*(4
*a*c - b^2)) - (log((((x*(-(b + (b^2 - 4*a*c)^(1/2))/(2*c))^(1/2) + 1)*1i)/
((b + (b^2 - 4*a*c)^(1/2))/(2*c) + 1)^(1/2) + (1 - x^2)^(1/2)*1i)/(x + (-(b
 + (b^2 - 4*a*c)^(1/2))/(2*c))^(1/2)))*(b*(b^2 - 4*a*c)^(1/2) - 4*a*c + 2*c
*(b^2 - 4*a*c)^(1/2) + b^2))/(4*c*(4*a*c - b^2)*((b + (b^2 - 4*a*c)^(1/2))/
(2*c) + 1)^(1/2))
```

3.379 $\int \frac{\sqrt{1-x^2}}{x(a+bx^2+cx^4)}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 241

$$\int \frac{\sqrt{1-x^2}}{x\left(a+bx^2+cx^4\right)}\,dx = -\frac{\operatorname{arctanh}\left(\sqrt{1-x^2}\right)}{a} + \frac{\sqrt{c}\left(2a+b+\sqrt{b^2-4ac}\right)\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{1-x^2}}{\sqrt{b+2c-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}a\sqrt{b^2-4ac}\sqrt{b+2c-\sqrt{b^2-4ac}}} - \frac{\sqrt{c}\left(2a+b-\sqrt{b^2-4ac}\right)\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{1-x^2}}{\sqrt{b+2c+\sqrt{b^2-4ac}}}\right)}{\sqrt{2}a\sqrt{b^2-4ac}\sqrt{b+2c+\sqrt{b^2-4ac}}}$$

```
[Out] -arctanh((-x^2+1)^(1/2))/a+1/2*arctanh(2^(1/2)*c^(1/2)*(-x^2+1)^(1/2)/(b+2*
c-(-4*a*c+b^2)^(1/2))^(1/2))*c^(1/2)*(2*a+b+(-4*a*c+b^2)^(1/2))/a*2^(1/2)/(
-4*a*c+b^2)^(1/2)/(b+2*c-(-4*a*c+b^2)^(1/2))^(1/2)-1/2*arctanh(2^(1/2)*c^(1
/2)*(-x^2+1)^(1/2)/(b+2*c+(-4*a*c+b^2)^(1/2))^(1/2))*c^(1/2)*(2*a+b-(-4*a*c
+b^2)^(1/2))/a*2^(1/2)/(-4*a*c+b^2)^(1/2)/(b+2*c+(-4*a*c+b^2)^(1/2))^(1/2)
```

Rubi [A] (verified)

Time = 0.98 (sec) , antiderivative size = 241, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.207, Rules used

$= \{1265, 911, 1301, 213, 1180, 214\}$

$$\int \frac{\sqrt{1-x^2}}{x\left(a+bx^2+cx^4\right)}\,dx = \frac{\sqrt{c}\left(\sqrt{b^2-4ac}+2a+b\right)\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{1-x^2}}{\sqrt{-\sqrt{b^2-4ac}+b+2c}}\right)}{\sqrt{2}a\sqrt{b^2-4ac}\sqrt{-\sqrt{b^2-4ac}+b+2c}} - \frac{\sqrt{c}\left(-\sqrt{b^2-4ac}+2a+b\right)\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{1-x^2}}{\sqrt{\sqrt{b^2-4ac}+b+2c}}\right)}{\sqrt{2}a\sqrt{b^2-4ac}\sqrt{\sqrt{b^2-4ac}+b+2c}} - \frac{\operatorname{arctanh}\left(\sqrt{1-x^2}\right)}{a}$$

```
[In] Int[Sqrt[1 - x^2]/(x*(a + b*x^2 + c*x^4)),x]
```

```
[Out] -(ArcTanh[Sqrt[1 - x^2]]/a) + (Sqrt[c]*(2*a + b + Sqrt[b^2 - 4*a*c])*ArcTan
h[(Sqrt[2]*Sqrt[c]*Sqrt[1 - x^2])/Sqrt[b + 2*c - Sqrt[b^2 - 4*a*c]]])/(Sqrt
[2]*a*Sqrt[b^2 - 4*a*c]*Sqrt[b + 2*c - Sqrt[b^2 - 4*a*c]]) - (Sqrt[c]*(2*a
+ b - Sqrt[b^2 - 4*a*c])*ArcTanh[(Sqrt[2]*Sqrt[c]*Sqrt[1 - x^2])/Sqrt[b + 2
*c + Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*a*Sqrt[b^2 - 4*a*c]*Sqrt[b + 2*c + Sqrt[
b^2 - 4*a*c]])
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 911

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> With[{q = Denominator[m]}, Dist[q/e, S
ubst[Int[x^(q*(m + 1) - 1)*((e*f - d*g)/e + g*(x^q/e))^n*((c*d^2 - b*d*e +
a*e^2)/e^2 - (2*c*d - b*e)*(x^q/e^2) + c*(x^(2*q)/e^2))^p, x], x, (d + e*x)
^(1/q)], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegersQ[n, p] && Fra
ctionQ[m]
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
```

```
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1265

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rule 1301

```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.))/((a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*((d + e*x^2)^q/(a
+ b*x^2 + c*x^4)), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b^2 - 4
*a*c, 0] && IntegerQ[q] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{2}\text{Subst}\left(\int \frac{\sqrt{1-x}}{x\left(a+bx+cx^2\right)}\,dx,x,x^2\right)\\
&= -\text{Subst}\left(\int \frac{x^2}{\left(1-x^2\right)\left(a+b+c+(-b-2c)x^2+cx^4\right)}\,dx,x,\sqrt{1-x^2}\right)\\
&= -\text{Subst}\left(\int \left(-\frac{1}{a\left(-1+x^2\right)}+\frac{-a-b-c+cx^2}{a\left(a+b+c-(b+2c)x^2+cx^4\right)}\right)dx,x,\sqrt{1-x^2}\right)\\
&= \frac{\text{Subst}\left(\int \frac{1}{-1+x^2}\,dx,x,\sqrt{1-x^2}\right)}{a}-\frac{\text{Subst}\left(\int \frac{-a-b-c+cx^2}{a+b+c+(-b-2c)x^2+cx^4}\,dx,x,\sqrt{1-x^2}\right)}{a}\\
&= -\frac{\tanh^{-1}\left(\sqrt{1-x^2}\right)}{a}\\
&\quad+\frac{\left(c\left(2a+b-\sqrt{b^2-4ac}\right)\right)\text{Subst}\left(\int \frac{1}{\frac{1}{2}(-b-2c)-\frac{1}{2}\sqrt{b^2-4ac}+cx^2}\,dx,x,\sqrt{1-x^2}\right)}{2a\sqrt{b^2-4ac}}\\
&\quad-\frac{\left(c\left(2a+b+\sqrt{b^2-4ac}\right)\right)\text{Subst}\left(\int \frac{1}{\frac{1}{2}(-b-2c)+\frac{1}{2}\sqrt{b^2-4ac}+cx^2}\,dx,x,\sqrt{1-x^2}\right)}{2a\sqrt{b^2-4ac}}\\
&= -\frac{\tanh^{-1}\left(\sqrt{1-x^2}\right)}{a}+\frac{\sqrt{c}\left(2a+b+\sqrt{b^2-4ac}\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{1-x^2}}{\sqrt{b+2c-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}a\sqrt{b^2-4ac}\sqrt{b+2c-\sqrt{b^2-4ac}}}\\
&\quad-\frac{\sqrt{c}\left(2a+b-\sqrt{b^2-4ac}\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{1-x^2}}{\sqrt{b+2c+\sqrt{b^2-4ac}}}\right)}{\sqrt{2}a\sqrt{b^2-4ac}\sqrt{b+2c+\sqrt{b^2-4ac}}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.52 (sec) , antiderivative size = 264, normalized size of antiderivative = 1.10

$$\int \frac{\sqrt{1-x^2}}{x\left(a+bx^2+cx^4\right)}\,dx = -\frac{\frac{\sqrt{2}\sqrt{c}\left(-2a-b+\sqrt{b^2-4ac}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}\sqrt{1-x^2}}{\sqrt{-b-2c-\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}\sqrt{-b-2c-\sqrt{b^2-4ac}}}+\frac{\sqrt{2}\sqrt{c}\left(2a+b+\sqrt{b^2-4ac}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}\sqrt{1-x^2}}{\sqrt{-b-2c+\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}\sqrt{-b-2c+\sqrt{b^2-4ac}}}-\log\left(-1+\sqrt{1-\right.}{2a}$$

```
[In] Integrate[Sqrt[1 - x^2]/(x*(a + b*x^2 + c*x^4)),x]
```

```
[Out] -1/2*((Sqrt[2]*Sqrt[c]*(-2*a - b + Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[
c]*Sqrt[1 - x^2])/Sqrt[-b - 2*c - Sqrt[b^2 - 4*a*c]]])/(Sqrt[b^2 - 4*a*c]*S
qrt[-b - 2*c - Sqrt[b^2 - 4*a*c]]) + (Sqrt[2]*Sqrt[c]*(2*a + b + Sqrt[b^2 -
 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*Sqrt[1 - x^2])/Sqrt[-b - 2*c + Sqrt[b^2 -
4*a*c]]])/(Sqrt[b^2 - 4*a*c]*Sqrt[-b - 2*c + Sqrt[b^2 - 4*a*c]]) - Log[-1 +
 Sqrt[1 - x^2]] + Log[a*(1 + Sqrt[1 - x^2])])/a
```

Maple [A] (verified)

Time = 0.46 (sec) , antiderivative size = 271, normalized size of antiderivative = 1.12

method	result
pseudoelliptic	$-\frac{\sqrt{2}\sqrt{\left(b+2c+\sqrt{-4ac+b^2}\right)c}\,c\left(a+\frac{b}{2}+\frac{\sqrt{-4ac+b^2}}{2}\right)\arctan\left(\frac{c\sqrt{-x^2+1}\,\sqrt{2}}{\sqrt{\left(\sqrt{-4ac+b^2}-b-2c\right)c}}\right)+\sqrt{\left(\sqrt{-4ac+b^2}-b-2c\right)c}\left(c\sqrt{2}\left(a+\frac{b}{2}\right.\right.}{\sqrt{-4ac+b^2}\sqrt{\left(\sqrt{-4ac+b^2}-b-2\right.}}$
default	$\frac{\sqrt{-x^2+1}-\operatorname{arctanh}\left(\frac{1}{\sqrt{-x^2+1}}\right)}{a}-\frac{2a\left(\frac{\left(-\sqrt{-4ac+b^2}\,ab+2ac\sqrt{-4ac+b^2}-b^2\sqrt{-4ac+b^2}+4c\,a^2-b^2a+4abc-b^3\right)\arctan\left(\frac{2a\left(\sqrt{-\right.}{2\sqrt{4ac-2b^2}}\right)}{2a\left(4ac-b^2\right)\sqrt{4ac-2b^2-2\sqrt{-4ac+b^2}\,a-2b\sqrt{-4ac+b^2}-2}}\right.}{}$

```
[In] int((-x^2+1)^(1/2)/x/(c*x^4+b*x^2+a),x,method=_RETURNVERBOSE)
```

```
[Out] -1/(-4*a*c+b^2)^(1/2)*(2^(1/2)*((b+2*c+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*(a+1/
2*b+1/2*(-4*a*c+b^2)^(1/2))*arctan(c*(-x^2+1)^(1/2)*2^(1/2)/(((-4*a*c+b^2)^
(1/2)-b-2*c)*c)^(1/2))+(((-4*a*c+b^2)^(1/2)-b-2*c)*c)^(1/2)*(c*2^(1/2)*(a+1
/2*b-1/2*(-4*a*c+b^2)^(1/2))*arctanh(c*(-x^2+1)^(1/2)*2^(1/2)/((b+2*c+(-4*a
*c+b^2)^(1/2))*c)^(1/2))-1/2*(-4*a*c+b^2)^(1/2)*((b+2*c+(-4*a*c+b^2)^(1/2))
*c)^(1/2)*(ln((-x^2+1)^(1/2)-1)-ln(1+(-x^2+1)^(1/2)))))/(((-4*a*c+b^2)^(1/2
)-b-2*c)*c)^(1/2)/((b+2*c+(-4*a*c+b^2)^(1/2))*c)^(1/2)/a
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1232 vs. 2(196) = 392.

Time = 2.72 (sec) , antiderivative size = 1232, normalized size of antiderivative = 5.11

$$\int \frac{\sqrt{1-x^2}}{x\left(a+bx^2+cx^4\right)}\,dx = \text{Too large to display}$$

```
[In] integrate((-x^2+1)^(1/2)/x/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] 1/2*(sqrt(1/2)*a*sqrt((a*b + b^2 - 2*a*c + (a^2*b^2 - 4*a^3*c)*sqrt((a^2 +
2*a*b + b^2)/(a^4*b^2 - 4*a^5*c)))/(a^2*b^2 - 4*a^3*c))*log((2*sqrt(1/2)*(a
^3*b^2 - 4*a^4*c)*x^2*sqrt((a^2 + 2*a*b + b^2)/(a^4*b^2 - 4*a^5*c))*sqrt((a
*b + b^2 - 2*a*c + (a^2*b^2 - 4*a^3*c)*sqrt((a^2 + 2*a*b + b^2)/(a^4*b^2 -
4*a^5*c)))/(a^2*b^2 - 4*a^3*c)) + (a^2*b^2 - 4*a^3*c)*x^2*sqrt((a^2 + 2*a*b
 + b^2)/(a^4*b^2 - 4*a^5*c)) + (a*b + b^2)*x^2 + 2*a^2 + 2*a*b - 2*(a^2 + a
*b)*sqrt(-x^2 + 1))/x^2) - sqrt(1/2)*a*sqrt((a*b + b^2 - 2*a*c + (a^2*b^2 -
 4*a^3*c)*sqrt((a^2 + 2*a*b + b^2)/(a^4*b^2 - 4*a^5*c)))/(a^2*b^2 - 4*a^3*c
))*log(-(2*sqrt(1/2)*(a^3*b^2 - 4*a^4*c)*x^2*sqrt((a^2 + 2*a*b + b^2)/(a^4*
b^2 - 4*a^5*c))*sqrt((a*b + b^2 - 2*a*c + (a^2*b^2 - 4*a^3*c)*sqrt((a^2 + 2
*a*b + b^2)/(a^4*b^2 - 4*a^5*c)))/(a^2*b^2 - 4*a^3*c)) - (a^2*b^2 - 4*a^3*c
)*x^2*sqrt((a^2 + 2*a*b + b^2)/(a^4*b^2 - 4*a^5*c)) - (a*b + b^2)*x^2 - 2*a
^2 - 2*a*b + 2*(a^2 + a*b)*sqrt(-x^2 + 1))/x^2) + sqrt(1/2)*a*sqrt((a*b + b
^2 - 2*a*c - (a^2*b^2 - 4*a^3*c)*sqrt((a^2 + 2*a*b + b^2)/(a^4*b^2 - 4*a^5*
c)))/(a^2*b^2 - 4*a^3*c))*log(-(2*sqrt(1/2)*(a^3*b^2 - 4*a^4*c)*x^2*sqrt((a
^2 + 2*a*b + b^2)/(a^4*b^2 - 4*a^5*c))*sqrt((a*b + b^2 - 2*a*c - (a^2*b^2 -
 4*a^3*c)*sqrt((a^2 + 2*a*b + b^2)/(a^4*b^2 - 4*a^5*c)))/(a^2*b^2 - 4*a^3*c
)) + (a^2*b^2 - 4*a^3*c)*x^2*sqrt((a^2 + 2*a*b + b^2)/(a^4*b^2 - 4*a^5*c))
- (a*b + b^2)*x^2 - 2*a^2 - 2*a*b + 2*(a^2 + a*b)*sqrt(-x^2 + 1))/x^2) - sq
rt(1/2)*a*sqrt((a*b + b^2 - 2*a*c - (a^2*b^2 - 4*a^3*c)*sqrt((a^2 + 2*a*b +
 b^2)/(a^4*b^2 - 4*a^5*c)))/(a^2*b^2 - 4*a^3*c))*log((2*sqrt(1/2)*(a^3*b^2
- 4*a^4*c)*x^2*sqrt((a^2 + 2*a*b + b^2)/(a^4*b^2 - 4*a^5*c))*sqrt((a*b + b^
2 - 2*a*c - (a^2*b^2 - 4*a^3*c)*sqrt((a^2 + 2*a*b + b^2)/(a^4*b^2 - 4*a^5*c
)))/(a^2*b^2 - 4*a^3*c)) - (a^2*b^2 - 4*a^3*c)*x^2*sqrt((a^2 + 2*a*b + b^2)
/(a^4*b^2 - 4*a^5*c)) + (a*b + b^2)*x^2 + 2*a^2 + 2*a*b - 2*(a^2 + a*b)*sqr
t(-x^2 + 1))/x^2) + 2*log((sqrt(-x^2 + 1) - 1)/x))/a
```

Sympy [F]

$$\int \frac{\sqrt{1-x^2}}{x\left(a+bx^2+cx^4\right)}\,dx=\int \frac{\sqrt{-\left(x-1\right)\left(x+1\right)}}{x\left(a+bx^2+cx^4\right)}\,dx$$

```
[In] integrate((-x**2+1)**(1/2)/x/(c*x**4+b*x**2+a),x)
[Out] Integral(sqrt(-(x - 1)*(x + 1))/(x*(a + b*x**2 + c*x**4)), x)
```

Maxima [F]

$$\int \frac{\sqrt{1-x^2}}{x\left(a+bx^2+cx^4\right)}\,dx=\int \frac{\sqrt{-x^2+1}}{(cx^4+bx^2+a)x}\,dx$$

```
[In] integrate((-x^2+1)^(1/2)/x/(c*x^4+b*x^2+a),x, algorithm="maxima")
[Out] integrate(sqrt(-x^2 + 1)/((c*x^4 + b*x^2 + a)*x), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3639 vs. $2(196) = 392$.

Time = 0.97 (sec) , antiderivative size = 3639, normalized size of antiderivative = 15.10

$$\int \frac{\sqrt{1-x^2}}{x\left(a+bx^2+cx^4\right)}\,dx=\text{Too large to display}$$

```
[In] integrate((-x^2+1)^(1/2)/x/(c*x^4+b*x^2+a),x, algorithm="giac")
[Out] -1/2*log(sqrt(-x^2 + 1) + 1)/a + 1/2*log(-sqrt(-x^2 + 1) + 1)/a + 1/8*(4*a^
3*b^3*c^2 + 2*a^2*b^4*c^2 - 16*a^4*b*c^3 + 4*a^2*b^3*c^3 - 32*a^4*c^4 - 16*
a^3*b*c^4 - 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - 2*c^2 - sqrt(b^2 - 4*a*
c)*c)*a^3*b^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - 2*c^2 - sqrt(b^2 - 4*
a*c)*c)*a^2*b^4 + 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - 2*c^2 - sqrt(b^2
- 4*a*c)*c)*a^4*b*c - 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - 2*c^2 - sqrt(
b^2 - 4*a*c)*c)*a^3*b^2*c - 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - 2*c^2 -
 sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c + 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c -
2*c^2 - sqrt(b^2 - 4*a*c)*c)*a^4*c^2 - 10*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b
*c - 2*c^2 - sqrt(b^2 - 4*a*c)*c)*a^3*b*c^2 - 9*sqrt(2)*sqrt(b^2 - 4*a*c)*s
qrt(-b*c - 2*c^2 - sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^2 - 20*sqrt(2)*sqrt(b^2 -
 4*a*c)*sqrt(-b*c - 2*c^2 - sqrt(b^2 - 4*a*c)*c)*a^3*c^3 - 10*sqrt(2)*sqrt(
b^2 - 4*a*c)*sqrt(-b*c - 2*c^2 - sqrt(b^2 - 4*a*c)*c)*a^2*b*c^3 - 4*(b^2 -
4*a*c)*a^3*b*c^2 - 2*(b^2 - 4*a*c)*a^2*b^2*c^2 - 8*(b^2 - 4*a*c)*a^3*c^3 -
```

```
4*(b^2 - 4*a*c)*a^2*b*c^3 + (2*b^4*c^2 - 16*a*b^2*c^3 + 32*a^2*c^4 - sqrt(2
)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - 2*c^2 - sqrt(b^2 - 4*a*c)*c)*b^4 + 8*sqrt(2
)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - 2*c^2 - sqrt(b^2 - 4*a*c)*c)*a*b^2*c - 2*sq
rt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - 2*c^2 - sqrt(b^2 - 4*a*c)*c)*b^3*c - 16
*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - 2*c^2 - sqrt(b^2 - 4*a*c)*c)*a^2*c^2
 + 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - 2*c^2 - sqrt(b^2 - 4*a*c)*c)*a*b
*c^2 - 5*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - 2*c^2 - sqrt(b^2 - 4*a*c)*c)
*b^2*c^2 + 20*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - 2*c^2 - sqrt(b^2 - 4*a*
c)*c)*a*c^3 - 2*(b^2 - 4*a*c)*b^2*c^2 + 8*(b^2 - 4*a*c)*a*c^3)*a^2 + 2*(sqr
t(2)*sqrt(-b*c - 2*c^2 - sqrt(b^2 - 4*a*c)*c)*a^2*b^4 + sqrt(2)*sqrt(-b*c -
 2*c^2 - sqrt(b^2 - 4*a*c)*c)*a*b^5 - 8*sqrt(2)*sqrt(-b*c - 2*c^2 - sqrt(b^
2 - 4*a*c)*c)*a^3*b^2*c - 6*sqrt(2)*sqrt(-b*c - 2*c^2 - sqrt(b^2 - 4*a*c)*c
)*a^2*b^3*c + 3*sqrt(2)*sqrt(-b*c - 2*c^2 - sqrt(b^2 - 4*a*c)*c)*a*b^4*c +
2*a^2*b^4*c + 2*a*b^5*c + 16*sqrt(2)*sqrt(-b*c - 2*c^2 - sqrt(b^2 - 4*a*c)*
c)*a^4*c^2 + 8*sqrt(2)*sqrt(-b*c - 2*c^2 - sqrt(b^2 - 4*a*c)*c)*a^3*b*c^2 -
 11*sqrt(2)*sqrt(-b*c - 2*c^2 - sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^2 - 16*a^3*b
^2*c^2 + 7*sqrt(2)*sqrt(-b*c - 2*c^2 - sqrt(b^2 - 4*a*c)*c)*a*b^3*c^2 - 16*
a^2*b^3*c^2 + 2*a*b^4*c^2 - 4*sqrt(2)*sqrt(-b*c - 2*c^2 - sqrt(b^2 - 4*a*c)
*c)*a^3*c^3 + 32*a^4*c^3 - 28*sqrt(2)*sqrt(-b*c - 2*c^2 - sqrt(b^2 - 4*a*c)
*c)*a^2*b*c^3 + 32*a^3*b*c^3 + 5*sqrt(2)*sqrt(-b*c - 2*c^2 - sqrt(b^2 - 4*a
*c)*c)*a*b^2*c^3 - 16*a^2*b^2*c^3 - 20*sqrt(2)*sqrt(-b*c - 2*c^2 - sqrt(b^2
 - 4*a*c)*c)*a^2*c^4 + 32*a^3*c^4 - 2*(b^2 - 4*a*c)*a^2*b^2*c - 2*(b^2 - 4*
a*c)*a*b^3*c + 8*(b^2 - 4*a*c)*a^3*c^2 + 8*(b^2 - 4*a*c)*a^2*b*c^2 - 2*(b^2
 - 4*a*c)*a*b^2*c^2 + 8*(b^2 - 4*a*c)*a^2*c^3)*abs(a))*arctan(2*sqrt(1/2)*s
qrt(-x^2 + 1)/sqrt(-(a*b + 2*a*c + sqrt(-4*(a^2 + a*b + a*c)*a*c + (a*b + 2
*a*c)^2))/(a*c)))/((a^3*b^4 + a^2*b^5 - 8*a^4*b^2*c - 6*a^3*b^3*c + 3*a^2*b
^4*c + 16*a^5*c^2 + 8*a^4*b*c^2 - 11*a^3*b^2*c^2 + 7*a^2*b^3*c^2 - 4*a^4*c^
3 - 28*a^3*b*c^3 + 5*a^2*b^2*c^3 - 20*a^3*c^4)*abs(a)*abs(c)) - 1/8*(4*a^3*
b^3*c^2 + 2*a^2*b^4*c^2 - 16*a^4*b*c^3 + 4*a^2*b^3*c^3 - 32*a^4*c^4 - 16*a^
3*b*c^4 - 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - 2*c^2 + sqrt(b^2 - 4*a*c)
*c)*a^3*b^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - 2*c^2 + sqrt(b^2 - 4*a*
c)*c)*a^2*b^4 + 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - 2*c^2 + sqrt(b^2 -
4*a*c)*c)*a^4*b*c - 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - 2*c^2 + sqrt(b^
2 - 4*a*c)*c)*a^3*b^2*c - 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - 2*c^2 + s
qrt(b^2 - 4*a*c)*c)*a^2*b^3*c + 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - 2*
c^2 + sqrt(b^2 - 4*a*c)*c)*a^4*c^2 - 10*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c
 - 2*c^2 + sqrt(b^2 - 4*a*c)*c)*a^3*b*c^2 - 9*sqrt(2)*sqrt(b^2 - 4*a*c)*sqr
t(-b*c - 2*c^2 + sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^2 - 20*sqrt(2)*sqrt(b^2 - 4
*a*c)*sqrt(-b*c - 2*c^2 + sqrt(b^2 - 4*a*c)*c)*a^3*c^3 - 10*sqrt(2)*sqrt(b^
2 - 4*a*c)*sqrt(-b*c - 2*c^2 + sqrt(b^2 - 4*a*c)*c)*a^2*b*c^3 - 4*(b^2 - 4*
a*c)*a^3*b*c^2 - 2*(b^2 - 4*a*c)*a^2*b^2*c^2 - 8*(b^2 - 4*a*c)*a^3*c^3 - 4*
(b^2 - 4*a*c)*a^2*b*c^3 + (2*b^4*c^2 - 16*a*b^2*c^3 + 32*a^2*c^4 - sqrt(2)*
sqrt(b^2 - 4*a*c)*sqrt(-b*c - 2*c^2 + sqrt(b^2 - 4*a*c)*c)*b^4 + 8*sqrt(2)*
sqrt(b^2 - 4*a*c)*sqrt(-b*c - 2*c^2 + sqrt(b^2 - 4*a*c)*c)*a*b^2*c - 2*sqrt
(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - 2*c^2 + sqrt(b^2 - 4*a*c)*c)*b^3*c - 16*s
```

```
qrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - 2*c^2 + sqrt(b^2 - 4*a*c)*c)*a^2*c^2 +
 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - 2*c^2 + sqrt(b^2 - 4*a*c)*c)*a*b*c
^2 - 5*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - 2*c^2 + sqrt(b^2 - 4*a*c)*c)*b
^2*c^2 + 20*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - 2*c^2 + sqrt(b^2 - 4*a*c)
*c)*a*c^3 - 2*(b^2 - 4*a*c)*b^2*c^2 + 8*(b^2 - 4*a*c)*a*c^3)*a^2 - 2*(sqrt(
2)*sqrt(-b*c - 2*c^2 + sqrt(b^2 - 4*a*c)*c)*a^2*b^4 + sqrt(2)*sqrt(-b*c - 2
*c^2 + sqrt(b^2 - 4*a*c)*c)*a*b^5 - 8*sqrt(2)*sqrt(-b*c - 2*c^2 + sqrt(b^2
- 4*a*c)*c)*a^3*b^2*c - 6*sqrt(2)*sqrt(-b*c - 2*c^2 + sqrt(b^2 - 4*a*c)*c)*
a^2*b^3*c + 3*sqrt(2)*sqrt(-b*c - 2*c^2 + sqrt(b^2 - 4*a*c)*c)*a*b^4*c - 2*
a^2*b^4*c - 2*a*b^5*c + 16*sqrt(2)*sqrt(-b*c - 2*c^2 + sqrt(b^2 - 4*a*c)*c)
*a^4*c^2 + 8*sqrt(2)*sqrt(-b*c - 2*c^2 + sqrt(b^2 - 4*a*c)*c)*a^3*b*c^2 - 1
1*sqrt(2)*sqrt(-b*c - 2*c^2 + sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^2 + 16*a^3*b^2
*c^2 + 7*sqrt(2)*sqrt(-b*c - 2*c^2 + sqrt(b^2 - 4*a*c)*c)*a*b^3*c^2 + 16*a^
2*b^3*c^2 - 2*a*b^4*c^2 - 4*sqrt(2)*sqrt(-b*c - 2*c^2 + sqrt(b^2 - 4*a*c)*c
)*a^3*c^3 - 32*a^4*c^3 - 28*sqrt(2)*sqrt(-b*c - 2*c^2 + sqrt(b^2 - 4*a*c)*c
)*a^2*b*c^3 - 32*a^3*b*c^3 + 5*sqrt(2)*sqrt(-b*c - 2*c^2 + sqrt(b^2 - 4*a*c
)*c)*a*b^2*c^3 + 16*a^2*b^2*c^3 - 20*sqrt(2)*sqrt(-b*c - 2*c^2 + sqrt(b^2 -
 4*a*c)*c)*a^2*c^4 - 32*a^3*c^4 + 2*(b^2 - 4*a*c)*a^2*b^2*c + 2*(b^2 - 4*a*
c)*a*b^3*c - 8*(b^2 - 4*a*c)*a^3*c^2 - 8*(b^2 - 4*a*c)*a^2*b*c^2 + 2*(b^2 -
 4*a*c)*a*b^2*c^2 - 8*(b^2 - 4*a*c)*a^2*c^3)*abs(a))*arctan(2*sqrt(1/2)*sqr
t(-x^2 + 1)/sqrt(-(a*b + 2*a*c - sqrt(-4*(a^2 + a*b + a*c)*a*c + (a*b + 2*a
*c)^2))/(a*c)))/((a^3*b^4 + a^2*b^5 - 8*a^4*b^2*c - 6*a^3*b^3*c + 3*a^2*b^4
*c + 16*a^5*c^2 + 8*a^4*b*c^2 - 11*a^3*b^2*c^2 + 7*a^2*b^3*c^2 - 4*a^4*c^3
- 28*a^3*b*c^3 + 5*a^2*b^2*c^3 - 20*a^3*c^4)*abs(a)*abs(c))
```

Mupad [B] (verification not implemented)

Time = 8.40 (sec) , antiderivative size = 669, normalized size of antiderivative = 2.78

$$\int \frac{\sqrt{1-x^2}}{x\,(a+bx^2+cx^4)}\,dx$$
$$= \frac{\ln\left(\sqrt{\frac{1}{x^2}-1}-\sqrt{\frac{1}{x^2}}\right)}{a} + \frac{\ln\left(\frac{\frac{\left(x\sqrt{-\frac{b+\sqrt{b^2-4\,a\,c}}{2\,c}}+1\right)\,1\mathrm{i}}{\sqrt{\frac{b+\sqrt{b^2-4\,a\,c}}{2\,c}+1}}+\sqrt{1-x^2}\,1\mathrm{i}}{x+\sqrt{-\frac{b+\sqrt{b^2-4\,a\,c}}{2\,c}}}\right)\left(4\,a\,c+2\,a\,\sqrt{b^2-4\,a\,c}+b\,\sqrt{b^2-4\,a\,c}-b^2\right)}{4\,a\,\left(4\,a\,c-b^2\right)\,\sqrt{\frac{b+\sqrt{b^2-4\,a\,c}}{2\,c}+1}}$$
$$- \frac{\ln\left(\frac{\frac{\left(x\sqrt{-\frac{b-\sqrt{b^2-4\,a\,c}}{2\,c}}+1\right)\,1\mathrm{i}}{\sqrt{\frac{b-\sqrt{b^2-4\,a\,c}}{2\,c}+1}}+\sqrt{1-x^2}\,1\mathrm{i}}{x+\sqrt{-\frac{b-\sqrt{b^2-4\,a\,c}}{2\,c}}}\right)\left(2\,a\,\sqrt{b^2-4\,a\,c}-4\,a\,c+b\,\sqrt{b^2-4\,a\,c}+b^2\right)}{4\,a\,\sqrt{\frac{b-\sqrt{b^2-4\,a\,c}}{2\,c}+1}\,\left(4\,a\,c-b^2\right)}$$
$$+ \frac{\ln\left(\frac{\frac{\left(x\sqrt{-\frac{b+\sqrt{b^2-4\,a\,c}}{2\,c}}-1\right)\,1\mathrm{i}}{\sqrt{\frac{b+\sqrt{b^2-4\,a\,c}}{2\,c}+1}}-\sqrt{1-x^2}\,1\mathrm{i}}{x-\sqrt{-\frac{b+\sqrt{b^2-4\,a\,c}}{2\,c}}}\right)\left(4\,a\,c+2\,a\,\sqrt{b^2-4\,a\,c}+b\,\sqrt{b^2-4\,a\,c}-b^2\right)}{4\,a\,\left(4\,a\,c-b^2\right)\,\sqrt{\frac{b+\sqrt{b^2-4\,a\,c}}{2\,c}+1}}$$
$$- \frac{\ln\left(\frac{\frac{\left(x\sqrt{-\frac{b-\sqrt{b^2-4\,a\,c}}{2\,c}}-1\right)\,1\mathrm{i}}{\sqrt{\frac{b-\sqrt{b^2-4\,a\,c}}{2\,c}+1}}-\sqrt{1-x^2}\,1\mathrm{i}}{x-\sqrt{-\frac{b-\sqrt{b^2-4\,a\,c}}{2\,c}}}\right)\left(2\,a\,\sqrt{b^2-4\,a\,c}-4\,a\,c+b\,\sqrt{b^2-4\,a\,c}+b^2\right)}{4\,a\,\sqrt{\frac{b-\sqrt{b^2-4\,a\,c}}{2\,c}+1}\,\left(4\,a\,c-b^2\right)}$$

```
[In] int((1 - x^2)^(1/2)/(x*(a + b*x^2 + c*x^4)),x)
```

```
[Out] log((1/x^2 - 1)^(1/2) - (1/x^2)^(1/2))/a + (log((((x*(-(b + (b^2 - 4*a*c)^(
1/2))/(2*c))^(1/2) + 1)*1i)/((b + (b^2 - 4*a*c)^(1/2))/(2*c) + 1)^(1/2) + (
1 - x^2)^(1/2)*1i)/(x + (-(b + (b^2 - 4*a*c)^(1/2))/(2*c))^(1/2)))*(4*a*c +
 2*a*(b^2 - 4*a*c)^(1/2) + b*(b^2 - 4*a*c)^(1/2) - b^2))/(4*a*(4*a*c - b^2)
*((b + (b^2 - 4*a*c)^(1/2))/(2*c) + 1)^(1/2)) - (log((((x*(-(b - (b^2 - 4*a
*c)^(1/2))/(2*c))^(1/2) + 1)*1i)/((b - (b^2 - 4*a*c)^(1/2))/(2*c) + 1)^(1/2
```

```
) + (1 - x^2)^(1/2)*1i)/(x + (-(b - (b^2 - 4*a*c)^(1/2))/(2*c))^(1/2)))*(2*
a*(b^2 - 4*a*c)^(1/2) - 4*a*c + b*(b^2 - 4*a*c)^(1/2) + b^2))/(4*a*((b - (b
^2 - 4*a*c)^(1/2))/(2*c) + 1)^(1/2)*(4*a*c - b^2)) + (log((((x*(-(b + (b^2
- 4*a*c)^(1/2))/(2*c))^(1/2) - 1)*1i)/((b + (b^2 - 4*a*c)^(1/2))/(2*c) + 1)
^(1/2) - (1 - x^2)^(1/2)*1i)/(x - (-(b + (b^2 - 4*a*c)^(1/2))/(2*c))^(1/2))
)*(4*a*c + 2*a*(b^2 - 4*a*c)^(1/2) + b*(b^2 - 4*a*c)^(1/2) - b^2))/(4*a*(4*
a*c - b^2)*((b + (b^2 - 4*a*c)^(1/2))/(2*c) + 1)^(1/2)) - (log((((x*(-(b -
(b^2 - 4*a*c)^(1/2))/(2*c))^(1/2) - 1)*1i)/((b - (b^2 - 4*a*c)^(1/2))/(2*c)
 + 1)^(1/2) - (1 - x^2)^(1/2)*1i)/(x - (-(b - (b^2 - 4*a*c)^(1/2))/(2*c))^(
1/2)))*(2*a*(b^2 - 4*a*c)^(1/2) - 4*a*c + b*(b^2 - 4*a*c)^(1/2) + b^2))/(4*
a*((b - (b^2 - 4*a*c)^(1/2))/(2*c) + 1)^(1/2)*(4*a*c - b^2))
```

3.380 $\int \frac{\sqrt{1-x^2}}{x^3(a+bx^2+cx^4)} \, dx$

Optimal result

Integrand size = 29, antiderivative size = 290

$$\begin{aligned}\int \frac{\sqrt{1-x^2}}{x^3 (a+bx^2+cx^4)} \, dx = & -\frac{1}{4a\left(1-\sqrt{1-x^2}\right)}+\frac{1}{4a\left(1+\sqrt{1-x^2}\right)} \\ & +\frac{(a+2b)\text{arctanh}\left(\sqrt{1-x^2}\right)}{2a^2} \\ & -\frac{\sqrt{c}\left(a+b+\frac{b^2+a(b-2c)}{\sqrt{b^2-4ac}}\right)\text{arctanh}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{1-x^2}}{\sqrt{b+2c-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}a^2\sqrt{b+2c-\sqrt{b^2-4ac}}} \\ & -\frac{\sqrt{c}\left(a+b-\frac{b^2+a(b-2c)}{\sqrt{b^2-4ac}}\right)\text{arctanh}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{1-x^2}}{\sqrt{b+2c+\sqrt{b^2-4ac}}}\right)}{\sqrt{2}a^2\sqrt{b+2c+\sqrt{b^2-4ac}}}\end{aligned}$$

```
[Out] 1/2*(a+2*b)*arctanh((-x^2+1)^(1/2))/a^2-1/4/a/(1-(-x^2+1)^(1/2))+1/4/a/(1+(
-x^2+1)^(1/2))-1/2*arctanh(2^(1/2)*c^(1/2)*(-x^2+1)^(1/2)/(b+2*c-(-4*a*c+b^
2)^(1/2))^(1/2))*c^(1/2)*(a+b+(b^2+a*(b-2*c))/(-4*a*c+b^2)^(1/2))/a^2*2^(1/
2)/(b+2*c-(-4*a*c+b^2)^(1/2))^(1/2)-1/2*arctanh(2^(1/2)*c^(1/2)*(-x^2+1)^(1
/2)/(b+2*c+(-4*a*c+b^2)^(1/2))^(1/2))*c^(1/2)*(a+b+(-b^2-a*(b-2*c))/(-4*a*c
+b^2)^(1/2))/a^2*2^(1/2)/(b+2*c+(-4*a*c+b^2)^(1/2))^(1/2)
```

Rubi [A] (verified)

Time = 1.50 (sec) , antiderivative size = 290, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.207, Rules used = {1265, 911, 1301, 213, 1180, 214}

$$\int \frac{\sqrt{1-x^2}}{x^3(a+bx^2+cx^4)}\,dx = -\frac{\sqrt{c}\left(\frac{a(b-2c)+b^2}{\sqrt{b^2-4ac}}+a+b\right)\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{1-x^2}}{\sqrt{-\sqrt{b^2-4ac}+b+2c}}\right)}{\sqrt{2}a^2\sqrt{-\sqrt{b^2-4ac}+b+2c}} - \frac{\sqrt{c}\left(-\frac{a(b-2c)+b^2}{\sqrt{b^2-4ac}}+a+b\right)\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{1-x^2}}{\sqrt{\sqrt{b^2-4ac}+b+2c}}\right)}{\sqrt{2}a^2\sqrt{\sqrt{b^2-4ac}+b+2c}} + \frac{(a+2b)\operatorname{arctanh}\left(\sqrt{1-x^2}\right)}{2a^2} - \frac{1}{4a\left(1-\sqrt{1-x^2}\right)} + \frac{1}{4a\left(\sqrt{1-x^2}+1\right)}$$

```
[In] Int[Sqrt[1 - x^2]/(x^3*(a + b*x^2 + c*x^4)),x]
```

```
[Out] -1/4*1/(a*(1 - Sqrt[1 - x^2])) + 1/(4*a*(1 + Sqrt[1 - x^2])) + ((a + 2*b)*A
rcTanh[Sqrt[1 - x^2]])/(2*a^2) - (Sqrt[c]*(a + b + (b^2 + a*(b - 2*c))/Sqrt
[b^2 - 4*a*c])*ArcTanh[(Sqrt[2]*Sqrt[c]*Sqrt[1 - x^2])/Sqrt[b + 2*c - Sqrt[
b^2 - 4*a*c]]])/(Sqrt[2]*a^2*Sqrt[b + 2*c - Sqrt[b^2 - 4*a*c]]) - (Sqrt[c]*
(a + b - (b^2 + a*(b - 2*c))/Sqrt[b^2 - 4*a*c])*ArcTanh[(Sqrt[2]*Sqrt[c]*Sq
rt[1 - x^2])/Sqrt[b + 2*c + Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*a^2*Sqrt[b + 2*c
+ Sqrt[b^2 - 4*a*c]])
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 911

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> With[{q = Denominator[m]}, Dist[q/e, S
ubst[Int[x^(q*(m + 1) - 1)*((e*f - d*g)/e + g*(x^q/e))^n*((c*d^2 - b*d*e +
a*e^2)/e^2 - (2*c*d - b*e)*(x^q/e^2) + c*(x^(2*q)/e^2))^p, x], x, (d + e*x)
^(1/q)], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegersQ[n, p] && Fra
```

```
ctionQ[m]
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1265

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rule 1301

```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.))/((a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*((d + e*x^2)^q/(a
+ b*x^2 + c*x^4)), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b^2 - 4
*a*c, 0] && IntegerQ[q] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{2}\text{Subst}\left(\int \frac{\sqrt{1-x}}{x^2\left(a+bx+cx^2\right)}\,dx, x, x^2\right) \\
&= -\text{Subst}\left(\int \frac{x^2}{\left(1-x^2\right)^2\left(a+b+c+(-b-2c)x^2+cx^4\right)}\,dx, x, \sqrt{1-x^2}\right) \\
&= -\text{Subst}\left(\int \left(\frac{1}{4a(-1+x)^2}+\frac{1}{4a(1+x)^2}+\frac{a+2b}{2a^2\left(-1+x^2\right)}\right.\right. \\
&\qquad\left.\left.+\frac{b(a+b+c)-(a+b)cx^2}{a^2\left(a+b+c-(b+2c)x^2+cx^4\right)}\right)dx, x, \sqrt{1-x^2}\right) \\
&= -\frac{1}{4a\left(1-\sqrt{1-x^2}\right)}+\frac{1}{4a\left(1+\sqrt{1-x^2}\right)} \\
&\quad-\frac{\text{Subst}\left(\int \frac{b(a+b+c)-(a+b)cx^2}{a+b+c+(-b-2c)x^2+cx^4}\,dx, x, \sqrt{1-x^2}\right)}{a^2} \\
&\quad-\frac{(a+2b)\text{Subst}\left(\int \frac{1}{-1+x^2}\,dx, x, \sqrt{1-x^2}\right)}{2a^2}
\end{aligned}$$

$$= -\frac{1}{4a\left(1-\sqrt{1-x^2}\right)} + \frac{1}{4a\left(1+\sqrt{1-x^2}\right)} + \frac{(a+2b)\tanh^{-1}\left(\sqrt{1-x^2}\right)}{2a^2}$$
$$+ \frac{\left(c\left(a+b-\frac{b^2+a(b-2c)}{\sqrt{b^2-4ac}}\right)\right)\text{Subst}\left(\int \frac{1}{\frac{1}{2}(-b-2c)-\frac{1}{2}\sqrt{b^2-4ac}+cx^2}\,dx, x, \sqrt{1-x^2}\right)}{2a^2}$$
$$+ \frac{\left(c\left(a+b+\frac{b^2+a(b-2c)}{\sqrt{b^2-4ac}}\right)\right)\text{Subst}\left(\int \frac{1}{\frac{1}{2}(-b-2c)+\frac{1}{2}\sqrt{b^2-4ac}+cx^2}\,dx, x, \sqrt{1-x^2}\right)}{2a^2}$$

$$= -\frac{1}{4a\left(1-\sqrt{1-x^2}\right)} + \frac{1}{4a\left(1+\sqrt{1-x^2}\right)} + \frac{(a+2b)\tanh^{-1}\left(\sqrt{1-x^2}\right)}{2a^2}$$
$$- \frac{\sqrt{c}\left(a+b+\frac{b^2+a(b-2c)}{\sqrt{b^2-4ac}}\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{1-x^2}}{\sqrt{b+2c-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}a^2\sqrt{b+2c-\sqrt{b^2-4ac}}}$$
$$- \frac{\sqrt{c}\left(a+b-\frac{b^2+a(b-2c)}{\sqrt{b^2-4ac}}\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{1-x^2}}{\sqrt{b+2c+\sqrt{b^2-4ac}}}\right)}{\sqrt{2}a^2\sqrt{b+2c+\sqrt{b^2-4ac}}}$$

Mathematica [A] (verified)

Time = 1.45 (sec) , antiderivative size = 307, normalized size of antiderivative = 1.06

$$\int \frac{\sqrt{1-x^2}}{x^3\left(a+bx^2+cx^4\right)}\,dx$$
$$= \frac{-\frac{a\sqrt{1-x^2}}{x^2} + \frac{\sqrt{2}\sqrt{c}\left(b\left(-b+\sqrt{b^2-4ac}\right)+a\left(-b+2c+\sqrt{b^2-4ac}\right)\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}\sqrt{1-x^2}}{\sqrt{-b-2c-\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}\sqrt{-b-2c-\sqrt{b^2-4ac}}} + \frac{\sqrt{2}\sqrt{c}\left(b\left(b+\sqrt{b^2-4ac}\right)+a\left(b-2c+\sqrt{b^2-4ac}\right)\cdots\right.}{\sqrt{b^2-4ac}\sqrt{-b-2c+\sqrt{\cdots}}}\cdots}{2a^2}$$

```
[In] Integrate[Sqrt[1 - x^2]/(x^3*(a + b*x^2 + c*x^4)),x]
```

```
[Out] (-((a*Sqrt[1 - x^2])/x^2) + (Sqrt[2]*Sqrt[c]*(b*(-b + Sqrt[b^2 - 4*a*c]) +
a*(-b + 2*c + Sqrt[b^2 - 4*a*c]))*ArcTan[(Sqrt[2]*Sqrt[c]*Sqrt[1 - x^2])/Sq
rt[-b - 2*c - Sqrt[b^2 - 4*a*c]]])/(Sqrt[b^2 - 4*a*c]*Sqrt[-b - 2*c - Sqrt[
b^2 - 4*a*c]]) + (Sqrt[2]*Sqrt[c]*(b*(b + Sqrt[b^2 - 4*a*c]) + a*(b - 2*c +
 Sqrt[b^2 - 4*a*c]))*ArcTan[(Sqrt[2]*Sqrt[c]*Sqrt[1 - x^2])/Sqrt[-b - 2*c +
 Sqrt[b^2 - 4*a*c]]])/(Sqrt[b^2 - 4*a*c]*Sqrt[-b - 2*c + Sqrt[b^2 - 4*a*c]]
) + (a + 2*b)*ArcTanh[Sqrt[1 - x^2]])/(2*a^2)
```

Maple [A] (verified)

Time = 0.67 (sec) , antiderivative size = 330, normalized size of antiderivative = 1.14

method	result
pseudoelliptic	$-\frac{-2\sqrt{\left(b+2c+\sqrt{-4ac+b^2}\right)c}\,\sqrt{2}\,c\,x^2\left((a+b)\sqrt{-4ac+b^2}+a(b-2c)+b^2\right)\arctan\left(\frac{c\sqrt{-x^2+1}\,\sqrt{2}}{\sqrt{\left(\sqrt{-4ac+b^2}-b-2c\right)c}}\right)+\left(-2\left((-a-b)\sqrt{-\right.\right.}{}$
risch	$\frac{x^2-1}{2a\,x^2\sqrt{-x^2+1}}+\frac{-\frac{(-a-2b)\operatorname{arctanh}\left(\frac{1}{\sqrt{-x^2+1}}\right)}{a}+\frac{2\left(2\sqrt{-4ac+b^2}\,a^2c-\sqrt{-4ac+b^2}\,a\,b^2+3\sqrt{-4ac+b^2}\,abc-\sqrt{-4ac+b^2}\,b^3+4cb\,a^2-4\right.}{a\left(4ac-b^2\right)\sqrt{4ac-2b^2-2\sqrt{-}}}}{}$
default	$\frac{-\frac{\left(-x^2+1\right)^{\frac{3}{2}}}{2x^2}-\frac{\sqrt{-x^2+1}}{2}+\frac{\operatorname{arctanh}\left(\frac{1}{\sqrt{-x^2+1}}\right)}{2}}{a}-\frac{b\left(\sqrt{-x^2+1}-\operatorname{arctanh}\left(\frac{1}{\sqrt{-x^2+1}}\right)\right)}{a^2}-\frac{-2a\left(\frac{\left(2\sqrt{-4ac+b^2}\,a^2c-\sqrt{-4ac+b\right.}{}\right.}{}}{}$

```
[In] int((-x^2+1)^(1/2)/x^3/(c*x^4+b*x^2+a),x,method=_RETURNVERBOSE)
```

```
[Out] -1/4/(-4*a*c+b^2)^(1/2)*(-2*((b+2*c+(-4*a*c+b^2)^(1/2))*c)^(1/2)*2^(1/2)*c*
x^2*((a+b)*(-4*a*c+b^2)^(1/2)+a*(b-2*c)+b^2)*arctan(c*(-x^2+1)^(1/2)*2^(1/2
)/(((-4*a*c+b^2)^(1/2)-b-2*c)*c)^(1/2))+(-2*((-a-b)*(-4*a*c+b^2)^(1/2)+a*(b
-2*c)+b^2)*2^(1/2)*c*x^2*arctanh(c*(-x^2+1)^(1/2)*2^(1/2)/((b+2*c+(-4*a*c+b
^2)^(1/2))*c)^(1/2))+((b+2*c+(-4*a*c+b^2)^(1/2))*c)^(1/2)*(-4*a*c+b^2)^(1/2
)*(-x^2*(a+2*b)*ln(1+(-x^2+1)^(1/2))+x^2*(a+2*b)*ln((-x^2+1)^(1/2)-1)+2*a*(
-x^2+1)^(1/2)))*(((-4*a*c+b^2)^(1/2)-b-2*c)*c)^(1/2))/(((-4*a*c+b^2)^(1/2)-
b-2*c)*c)^(1/2)/((b+2*c+(-4*a*c+b^2)^(1/2))*c)^(1/2)/a^2/x^2
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2799 vs. $2(236) = 472$.

Time = 7.50 (sec) , antiderivative size = 2799, normalized size of antiderivative = 9.65

$$\int \frac{\sqrt{1-x^2}}{x^3\left(a+bx^2+cx^4\right)}\,dx = \text{Too large to display}$$

```
[In] integrate((-x^2+1)^(1/2)/x^3/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] -1/2*(sqrt(1/2)*a^2*x^2*sqrt((a*b^3 + b^4 + 2*a^2*c^2 - (3*a^2*b + 4*a*b^2)
*c - (a^4*b^2 - 4*a^5*c)*sqrt((a^2*b^4 + 2*a*b^5 + b^6 + (a^4 + 4*a^3*b + 4
*a^2*b^2)*c^2 - 2*(a^3*b^2 + 3*a^2*b^3 + 2*a*b^4)*c)/(a^8*b^2 - 4*a^9*c)))/
(a^4*b^2 - 4*a^5*c))*log(((a^4*b^2*c - 4*a^5*c^2)*x^2*sqrt((a^2*b^4 + 2*a*b
```

```
^5 + b^6 + (a^4 + 4*a^3*b + 4*a^2*b^2)*c^2 - 2*(a^3*b^2 + 3*a^2*b^3 + 2*a*b
^4)*c)/(a^8*b^2 - 4*a^9*c)) + 2*(a^3 + 2*a^2*b)*c^2 + ((a^2*b + 2*a*b^2)*c^
2 - (a*b^3 + b^4)*c)*x^2 - 2*(a^2*b^2 + a*b^3)*c + sqrt(1/2)*((a^5*b^3 - 4*
a^6*b*c)*x^2*sqrt((a^2*b^4 + 2*a*b^5 + b^6 + (a^4 + 4*a^3*b + 4*a^2*b^2)*c^
2 - 2*(a^3*b^2 + 3*a^2*b^3 + 2*a*b^4)*c)/(a^8*b^2 - 4*a^9*c)) + (a^2*b^4 +
a*b^5 + 4*(a^4 + 2*a^3*b)*c^2 - (5*a^3*b^2 + 6*a^2*b^3)*c)*x^2)*sqrt((a*b^3
 + b^4 + 2*a^2*c^2 - (3*a^2*b + 4*a*b^2)*c - (a^4*b^2 - 4*a^5*c)*sqrt((a^2*
b^4 + 2*a*b^5 + b^6 + (a^4 + 4*a^3*b + 4*a^2*b^2)*c^2 - 2*(a^3*b^2 + 3*a^2*
b^3 + 2*a*b^4)*c)/(a^8*b^2 - 4*a^9*c)))/(a^4*b^2 - 4*a^5*c)) - 2*((a^3 + 2*
a^2*b)*c^2 - (a^2*b^2 + a*b^3)*c)*sqrt(-x^2 + 1))/x^2) - sqrt(1/2)*a^2*x^2*
sqrt((a*b^3 + b^4 + 2*a^2*c^2 - (3*a^2*b + 4*a*b^2)*c - (a^4*b^2 - 4*a^5*c)
*sqrt((a^2*b^4 + 2*a*b^5 + b^6 + (a^4 + 4*a^3*b + 4*a^2*b^2)*c^2 - 2*(a^3*b
^2 + 3*a^2*b^3 + 2*a*b^4)*c)/(a^8*b^2 - 4*a^9*c)))/(a^4*b^2 - 4*a^5*c))*log
(((a^4*b^2*c - 4*a^5*c^2)*x^2*sqrt((a^2*b^4 + 2*a*b^5 + b^6 + (a^4 + 4*a^3*
b + 4*a^2*b^2)*c^2 - 2*(a^3*b^2 + 3*a^2*b^3 + 2*a*b^4)*c)/(a^8*b^2 - 4*a^9*
c)) + 2*(a^3 + 2*a^2*b)*c^2 + ((a^2*b + 2*a*b^2)*c^2 - (a*b^3 + b^4)*c)*x^2
 - 2*(a^2*b^2 + a*b^3)*c - sqrt(1/2)*((a^5*b^3 - 4*a^6*b*c)*x^2*sqrt((a^2*b
^4 + 2*a*b^5 + b^6 + (a^4 + 4*a^3*b + 4*a^2*b^2)*c^2 - 2*(a^3*b^2 + 3*a^2*b
^3 + 2*a*b^4)*c)/(a^8*b^2 - 4*a^9*c)) + (a^2*b^4 + a*b^5 + 4*(a^4 + 2*a^3*b
)*c^2 - (5*a^3*b^2 + 6*a^2*b^3)*c)*x^2)*sqrt((a*b^3 + b^4 + 2*a^2*c^2 - (3*
a^2*b + 4*a*b^2)*c - (a^4*b^2 - 4*a^5*c)*sqrt((a^2*b^4 + 2*a*b^5 + b^6 + (a
^4 + 4*a^3*b + 4*a^2*b^2)*c^2 - 2*(a^3*b^2 + 3*a^2*b^3 + 2*a*b^4)*c)/(a^8*b
^2 - 4*a^9*c)))/(a^4*b^2 - 4*a^5*c)) - 2*((a^3 + 2*a^2*b)*c^2 - (a^2*b^2 +
a*b^3)*c)*sqrt(-x^2 + 1))/x^2) + sqrt(1/2)*a^2*x^2*sqrt((a*b^3 + b^4 + 2*a^
2*c^2 - (3*a^2*b + 4*a*b^2)*c + (a^4*b^2 - 4*a^5*c)*sqrt((a^2*b^4 + 2*a*b^5
 + b^6 + (a^4 + 4*a^3*b + 4*a^2*b^2)*c^2 - 2*(a^3*b^2 + 3*a^2*b^3 + 2*a*b^4
)*c)/(a^8*b^2 - 4*a^9*c)))/(a^4*b^2 - 4*a^5*c))*log(-((a^4*b^2*c - 4*a^5*c^
2)*x^2*sqrt((a^2*b^4 + 2*a*b^5 + b^6 + (a^4 + 4*a^3*b + 4*a^2*b^2)*c^2 - 2*
(a^3*b^2 + 3*a^2*b^3 + 2*a*b^4)*c)/(a^8*b^2 - 4*a^9*c)) - 2*(a^3 + 2*a^2*b)
*c^2 - ((a^2*b + 2*a*b^2)*c^2 - (a*b^3 + b^4)*c)*x^2 + 2*(a^2*b^2 + a*b^3)*
c + sqrt(1/2)*((a^5*b^3 - 4*a^6*b*c)*x^2*sqrt((a^2*b^4 + 2*a*b^5 + b^6 + (a
^4 + 4*a^3*b + 4*a^2*b^2)*c^2 - 2*(a^3*b^2 + 3*a^2*b^3 + 2*a*b^4)*c)/(a^8*b
^2 - 4*a^9*c)) - (a^2*b^4 + a*b^5 + 4*(a^4 + 2*a^3*b)*c^2 - (5*a^3*b^2 + 6*
a^2*b^3)*c)*x^2)*sqrt((a*b^3 + b^4 + 2*a^2*c^2 - (3*a^2*b + 4*a*b^2)*c + (a
^4*b^2 - 4*a^5*c)*sqrt((a^2*b^4 + 2*a*b^5 + b^6 + (a^4 + 4*a^3*b + 4*a^2*b^
2)*c^2 - 2*(a^3*b^2 + 3*a^2*b^3 + 2*a*b^4)*c)/(a^8*b^2 - 4*a^9*c)))/(a^4*b^
2 - 4*a^5*c)) + 2*((a^3 + 2*a^2*b)*c^2 - (a^2*b^2 + a*b^3)*c)*sqrt(-x^2 + 1
))/x^2) - sqrt(1/2)*a^2*x^2*sqrt((a*b^3 + b^4 + 2*a^2*c^2 - (3*a^2*b + 4*a*
b^2)*c + (a^4*b^2 - 4*a^5*c)*sqrt((a^2*b^4 + 2*a*b^5 + b^6 + (a^4 + 4*a^3*b
 + 4*a^2*b^2)*c^2 - 2*(a^3*b^2 + 3*a^2*b^3 + 2*a*b^4)*c)/(a^8*b^2 - 4*a^9*c
)))/(a^4*b^2 - 4*a^5*c))*log(-((a^4*b^2*c - 4*a^5*c^2)*x^2*sqrt((a^2*b^4 +
2*a*b^5 + b^6 + (a^4 + 4*a^3*b + 4*a^2*b^2)*c^2 - 2*(a^3*b^2 + 3*a^2*b^3 +
2*a*b^4)*c)/(a^8*b^2 - 4*a^9*c)) - 2*(a^3 + 2*a^2*b)*c^2 - ((a^2*b + 2*a*b^
2)*c^2 - (a*b^3 + b^4)*c)*x^2 + 2*(a^2*b^2 + a*b^3)*c - sqrt(1/2)*((a^5*b^3
 - 4*a^6*b*c)*x^2*sqrt((a^2*b^4 + 2*a*b^5 + b^6 + (a^4 + 4*a^3*b + 4*a^2*b^
```

```
2)*c^2 - 2*(a^3*b^2 + 3*a^2*b^3 + 2*a*b^4)*c)/(a^8*b^2 - 4*a^9*c)) - (a^2*b
^4 + a*b^5 + 4*(a^4 + 2*a^3*b)*c^2 - (5*a^3*b^2 + 6*a^2*b^3)*c)*x^2)*sqrt((
a*b^3 + b^4 + 2*a^2*c^2 - (3*a^2*b + 4*a*b^2)*c + (a^4*b^2 - 4*a^5*c)*sqrt(
(a^2*b^4 + 2*a*b^5 + b^6 + (a^4 + 4*a^3*b + 4*a^2*b^2)*c^2 - 2*(a^3*b^2 + 3
*a^2*b^3 + 2*a*b^4)*c)/(a^8*b^2 - 4*a^9*c)))/(a^4*b^2 - 4*a^5*c)) + 2*((a^3
 + 2*a^2*b)*c^2 - (a^2*b^2 + a*b^3)*c)*sqrt(-x^2 + 1))/x^2) + (a + 2*b)*x^2
*log((sqrt(-x^2 + 1) - 1)/x) + sqrt(-x^2 + 1)*a)/(a^2*x^2)
```

Sympy [F]

$$\int \frac{\sqrt{1-x^2}}{x^3\left(a+bx^2+cx^4\right)}\,dx=\int \frac{\sqrt{-\left(x-1\right)\left(x+1\right)}}{x^3\left(a+bx^2+cx^4\right)}\,dx$$

```
[In] integrate((-x**2+1)**(1/2)/x**3/(c*x**4+b*x**2+a),x)
[Out] Integral(sqrt(-(x - 1)*(x + 1))/(x**3*(a + b*x**2 + c*x**4)), x)
```

Maxima [F]

$$\int \frac{\sqrt{1-x^2}}{x^3\left(a+bx^2+cx^4\right)}\,dx=\int \frac{\sqrt{-x^2+1}}{(cx^4+bx^2+a)x^3}\,dx$$

```
[In] integrate((-x^2+1)^(1/2)/x^3/(c*x^4+b*x^2+a),x, algorithm="maxima")
[Out] integrate(sqrt(-x^2 + 1)/((c*x^4 + b*x^2 + a)*x^3), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1675 vs. 2(236) = 472.

Time = 1.57 (sec) , antiderivative size = 1675, normalized size of antiderivative = 5.78

$$\int \frac{\sqrt{1-x^2}}{x^3\left(a+bx^2+cx^4\right)}\,dx=\text{Too large to display}$$

```
[In] integrate((-x^2+1)^(1/2)/x^3/(c*x^4+b*x^2+a),x, algorithm="giac")
[Out] -1/4*(sqrt(2)*sqrt(-b*c - 2*c^2 - sqrt(b^2 - 4*a*c)*c)*b^5 - 8*sqrt(2)*sqrt
(-b*c - 2*c^2 - sqrt(b^2 - 4*a*c)*c)*a*b^3*c + 2*sqrt(2)*sqrt(-b*c - 2*c^2
- sqrt(b^2 - 4*a*c)*c)*b^4*c + 2*b^5*c + 16*sqrt(2)*sqrt(-b*c - 2*c^2 - sqr
t(b^2 - 4*a*c)*c)*a^2*b*c^2 - 8*sqrt(2)*sqrt(-b*c - 2*c^2 - sqrt(b^2 - 4*a*
c)*c)*a*b^2*c^2 + 5*sqrt(2)*sqrt(-b*c - 2*c^2 - sqrt(b^2 - 4*a*c)*c)*b^3*c^
2 - 16*a*b^3*c^2 + 2*b^4*c^2 - 20*sqrt(2)*sqrt(-b*c - 2*c^2 - sqrt(b^2 - 4*
```

```
a*c)*c)*a*b*c^3 + 32*a^2*b*c^3 - 12*a*b^2*c^3 + 16*a^2*c^4 - sqrt(2)*sqrt(b
^2 - 4*a*c)*sqrt(-b*c - 2*c^2 - sqrt(b^2 - 4*a*c)*c)*b^4 + 6*sqrt(2)*sqrt(b
^2 - 4*a*c)*sqrt(-b*c - 2*c^2 - sqrt(b^2 - 4*a*c)*c)*a*b^2*c - 2*sqrt(2)*sq
rt(b^2 - 4*a*c)*sqrt(-b*c - 2*c^2 - sqrt(b^2 - 4*a*c)*c)*b^3*c - 8*sqrt(2)*
sqrt(b^2 - 4*a*c)*sqrt(-b*c - 2*c^2 - sqrt(b^2 - 4*a*c)*c)*a^2*c^2 + 4*sqrt
(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - 2*c^2 - sqrt(b^2 - 4*a*c)*c)*a*b*c^2 - 5*
sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - 2*c^2 - sqrt(b^2 - 4*a*c)*c)*b^2*c^2
+ 10*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - 2*c^2 - sqrt(b^2 - 4*a*c)*c)*a*c
^3 - 2*(b^2 - 4*a*c)*b^3*c + 8*(b^2 - 4*a*c)*a*b*c^2 - 2*(b^2 - 4*a*c)*b^2*
c^2 + 4*(b^2 - 4*a*c)*a*c^3)*arctan(2*sqrt(1/2)*sqrt(-x^2 + 1)/sqrt(-(a^2*b
 + 2*a^2*c + sqrt(-4*(a^3 + a^2*b + a^2*c)*a^2*c + (a^2*b + 2*a^2*c)^2))/(a
^2*c)))/((a^2*b^4 - 8*a^3*b^2*c + 2*a^2*b^3*c + 16*a^4*c^2 - 8*a^3*b*c^2 +
5*a^2*b^2*c^2 - 20*a^3*c^3)*abs(c)) - 1/4*(sqrt(2)*sqrt(-b*c - 2*c^2 + sqrt
(b^2 - 4*a*c)*c)*b^5 - 8*sqrt(2)*sqrt(-b*c - 2*c^2 + sqrt(b^2 - 4*a*c)*c)*a
*b^3*c + 2*sqrt(2)*sqrt(-b*c - 2*c^2 + sqrt(b^2 - 4*a*c)*c)*b^4*c - 2*b^5*c
 + 16*sqrt(2)*sqrt(-b*c - 2*c^2 + sqrt(b^2 - 4*a*c)*c)*a^2*b*c^2 - 8*sqrt(2
)*sqrt(-b*c - 2*c^2 + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^2 + 5*sqrt(2)*sqrt(-b*c
- 2*c^2 + sqrt(b^2 - 4*a*c)*c)*b^3*c^2 + 16*a*b^3*c^2 - 2*b^4*c^2 - 20*sqrt
(2)*sqrt(-b*c - 2*c^2 + sqrt(b^2 - 4*a*c)*c)*a*b*c^3 - 32*a^2*b*c^3 + 12*a*
b^2*c^3 - 16*a^2*c^4 + sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - 2*c^2 + sqrt(b
^2 - 4*a*c)*c)*b^4 - 6*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - 2*c^2 + sqrt(b
^2 - 4*a*c)*c)*a*b^2*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - 2*c^2 + sq
rt(b^2 - 4*a*c)*c)*b^3*c + 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - 2*c^2 +
sqrt(b^2 - 4*a*c)*c)*a^2*c^2 - 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - 2*c^
2 + sqrt(b^2 - 4*a*c)*c)*a*b*c^2 + 5*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c -
2*c^2 + sqrt(b^2 - 4*a*c)*c)*b^2*c^2 - 10*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b
*c - 2*c^2 + sqrt(b^2 - 4*a*c)*c)*a*c^3 + 2*(b^2 - 4*a*c)*b^3*c - 8*(b^2 -
4*a*c)*a*b*c^2 + 2*(b^2 - 4*a*c)*b^2*c^2 - 4*(b^2 - 4*a*c)*a*c^3)*arctan(2*
sqrt(1/2)*sqrt(-x^2 + 1)/sqrt(-(a^2*b + 2*a^2*c - sqrt(-4*(a^3 + a^2*b + a^
2*c)*a^2*c + (a^2*b + 2*a^2*c)^2))/(a^2*c)))/((a^2*b^4 - 8*a^3*b^2*c + 2*a^
2*b^3*c + 16*a^4*c^2 - 8*a^3*b*c^2 + 5*a^2*b^2*c^2 - 20*a^3*c^3)*abs(c)) +
1/4*(a + 2*b)*log(sqrt(-x^2 + 1) + 1)/a^2 - 1/4*(a + 2*b)*log(-sqrt(-x^2 +
1) + 1)/a^2 - 1/2*sqrt(-x^2 + 1)/(a*x^2)
```

Mupad [B] (verification not implemented)

Time = 8.24 (sec) , antiderivative size = 825, normalized size of antiderivative = 2.84

$$\int \frac{\sqrt{1-x^2}}{x^3\,(a+bx^2+cx^4)}\,dx = \frac{\ln\left(\sqrt{\frac{1}{x^2}-1}-\sqrt{\frac{1}{x^2}}\right)}{2\,a} - \frac{\ln\left(\sqrt{\frac{1}{x^2}-1}-\sqrt{\frac{1}{x^2}}\right)\,(a+b)}{a^2} - \frac{\sqrt{1-x^2}}{2\,a\,x^2}$$

$$-\frac{\ln\left(\frac{\frac{\left(x\,\sqrt{-\frac{b+\sqrt{b^2-4\,a\,c}}{2\,c}}+1\right)\,1\mathrm{i}}{\sqrt{\frac{b+\sqrt{b^2-4\,a\,c}}{2\,c}+1}}+\sqrt{1-x^2}\,1\mathrm{i}}{x+\sqrt{-\frac{b+\sqrt{b^2-4\,a\,c}}{2\,c}}}\right)\,\left(4\,a^2\,c-a\,b^2-b^3+b^2\,\sqrt{b^2-4\,a\,c}+4\,a\,b\,c+a\,b\,\sqrt{b^2-4\,a\,c}-2\,a\,c\cdots}{4\,a^2\,\left(4\,a\,c-b^2\right)\,\sqrt{\frac{b+\sqrt{b^2-4\,a\,c}}{2\,c}+1}}$$

$$+\frac{\ln\left(\frac{\frac{\left(x\,\sqrt{-\frac{b-\sqrt{b^2-4\,a\,c}}{2\,c}}+1\right)\,1\mathrm{i}}{\sqrt{\frac{b-\sqrt{b^2-4\,a\,c}}{2\,c}+1}}+\sqrt{1-x^2}\,1\mathrm{i}}{x+\sqrt{-\frac{b-\sqrt{b^2-4\,a\,c}}{2\,c}}}\right)\,\left(a\,b^2-4\,a^2\,c+b^3+b^2\,\sqrt{b^2-4\,a\,c}-4\,a\,b\,c+a\,b\,\sqrt{b^2-4\,a\,c}-2\,a\,c\cdots}{4\,a^2\,\sqrt{\frac{b-\sqrt{b^2-4\,a\,c}}{2\,c}+1}\,\left(4\,a\,c-b^2\right)}$$

$$-\frac{\ln\left(\frac{\frac{\left(x\,\sqrt{-\frac{b+\sqrt{b^2-4\,a\,c}}{2\,c}}-1\right)\,1\mathrm{i}}{\sqrt{\frac{b+\sqrt{b^2-4\,a\,c}}{2\,c}+1}}-\sqrt{1-x^2}\,1\mathrm{i}}{x-\sqrt{-\frac{b+\sqrt{b^2-4\,a\,c}}{2\,c}}}\right)\,\left(4\,a^2\,c-a\,b^2-b^3+b^2\,\sqrt{b^2-4\,a\,c}+4\,a\,b\,c+a\,b\,\sqrt{b^2-4\,a\,c}-2\,a\,c\cdots}{4\,a^2\,\left(4\,a\,c-b^2\right)\,\sqrt{\frac{b+\sqrt{b^2-4\,a\,c}}{2\,c}+1}}$$

$$+\frac{\ln\left(\frac{\frac{\left(x\,\sqrt{-\frac{b-\sqrt{b^2-4\,a\,c}}{2\,c}}-1\right)\,1\mathrm{i}}{\sqrt{\frac{b-\sqrt{b^2-4\,a\,c}}{2\,c}+1}}-\sqrt{1-x^2}\,1\mathrm{i}}{x-\sqrt{-\frac{b-\sqrt{b^2-4\,a\,c}}{2\,c}}}\right)\,\left(a\,b^2-4\,a^2\,c+b^3+b^2\,\sqrt{b^2-4\,a\,c}-4\,a\,b\,c+a\,b\,\sqrt{b^2-4\,a\,c}-2\,a\,c\cdots}{4\,a^2\,\sqrt{\frac{b-\sqrt{b^2-4\,a\,c}}{2\,c}+1}\,\left(4\,a\,c-b^2\right)}$$

```
[In] int((1 - x^2)^(1/2)/(x^3*(a + b*x^2 + c*x^4)),x)
```

```
[Out] log((1/x^2 - 1)^(1/2) - (1/x^2)^(1/2))/(2*a) - (log((1/x^2 - 1)^(1/2) - (1/
x^2)^(1/2))*(a + b))/a^2 - (1 - x^2)^(1/2)/(2*a*x^2) - (log((((x*(-(b + (b^
2 - 4*a*c)^(1/2))/(2*c))^(1/2) + 1)*1i)/((b + (b^2 - 4*a*c)^(1/2))/(2*c) +
1)^(1/2) + (1 - x^2)^(1/2)*1i)/(x + (-(b + (b^2 - 4*a*c)^(1/2))/(2*c))^(1/2
)))*(4*a^2*c - a*b^2 - b^3 + b^2*(b^2 - 4*a*c)^(1/2) + 4*a*b*c + a*b*(b^2 -
 4*a*c)^(1/2) - 2*a*c*(b^2 - 4*a*c)^(1/2)))/(4*a^2*(4*a*c - b^2)*((b + (b^2
 - 4*a*c)^(1/2))/(2*c) + 1)^(1/2)) + (log((((x*(-(b - (b^2 - 4*a*c)^(1/2))/
(2*c))^(1/2) + 1)*1i)/((b - (b^2 - 4*a*c)^(1/2))/(2*c) + 1)^(1/2) + (1 - x^
```

```
2)^(1/2)*1i)/(x + (-(b - (b^2 - 4*a*c)^(1/2))/(2*c))^(1/2)))*(a*b^2 - 4*a^2
*c + b^3 + b^2*(b^2 - 4*a*c)^(1/2) - 4*a*b*c + a*b*(b^2 - 4*a*c)^(1/2) - 2*
a*c*(b^2 - 4*a*c)^(1/2)))/(4*a^2*((b - (b^2 - 4*a*c)^(1/2))/(2*c) + 1)^(1/2
)*(4*a*c - b^2)) - (log((((x*(-(b + (b^2 - 4*a*c)^(1/2))/(2*c))^(1/2) - 1)*
1i)/((b + (b^2 - 4*a*c)^(1/2))/(2*c) + 1)^(1/2) - (1 - x^2)^(1/2)*1i)/(x -
(-(b + (b^2 - 4*a*c)^(1/2))/(2*c))^(1/2)))*(4*a^2*c - a*b^2 - b^3 + b^2*(b^
2 - 4*a*c)^(1/2) + 4*a*b*c + a*b*(b^2 - 4*a*c)^(1/2) - 2*a*c*(b^2 - 4*a*c)^
(1/2)))/(4*a^2*(4*a*c - b^2)*((b + (b^2 - 4*a*c)^(1/2))/(2*c) + 1)^(1/2)) +
 (log((((x*(-(b - (b^2 - 4*a*c)^(1/2))/(2*c))^(1/2) - 1)*1i)/((b - (b^2 - 4
*a*c)^(1/2))/(2*c) + 1)^(1/2) - (1 - x^2)^(1/2)*1i)/(x - (-(b - (b^2 - 4*a*
c)^(1/2))/(2*c))^(1/2)))*(a*b^2 - 4*a^2*c + b^3 + b^2*(b^2 - 4*a*c)^(1/2) -
 4*a*b*c + a*b*(b^2 - 4*a*c)^(1/2) - 2*a*c*(b^2 - 4*a*c)^(1/2)))/(4*a^2*((b
 - (b^2 - 4*a*c)^(1/2))/(2*c) + 1)^(1/2)*(4*a*c - b^2))
```

3.381 $\int \frac{x^4\sqrt{1-x^2}}{a+bx^2+cx^4}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 325

$$\int \frac{x^4\sqrt{1-x^2}}{a+bx^2+cx^4}\,dx = \frac{x\sqrt{1-x^2}}{2c}+\frac{(2b+c)\arcsin(x)}{2c^2} - \frac{\left(b^2-ac+bc-\frac{b^3-3abc+b^2c-2ac^2}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{b+2c-\sqrt{b^2-4ac}}x}{\sqrt{b-\sqrt{b^2-4ac}}\sqrt{1-x^2}}\right)}{c^2\sqrt{b-\sqrt{b^2-4ac}}\sqrt{b+2c-\sqrt{b^2-4ac}}} - \frac{\left(b^2-ac+bc+\frac{b^3-3abc+b^2c-2ac^2}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{b+2c+\sqrt{b^2-4ac}}x}{\sqrt{b+\sqrt{b^2-4ac}}\sqrt{1-x^2}}\right)}{c^2\sqrt{b+\sqrt{b^2-4ac}}\sqrt{b+2c+\sqrt{b^2-4ac}}}$$

```
[Out] 1/2*(2*b+c)*arcsin(x)/c^2+1/2*x*(-x^2+1)^(1/2)/c-arctan(x*(b+2*c-(-4*a*c+b^
2)^(1/2))^(1/2)/(-x^2+1)^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2))*(b^2-a*c+b*c+(
3*a*b*c+2*a*c^2-b^3-b^2*c)/(-4*a*c+b^2)^(1/2))/c^2/(b-(-4*a*c+b^2)^(1/2))^(
1/2)/(b+2*c-(-4*a*c+b^2)^(1/2))^(1/2)-arctan(x*(b+2*c+(-4*a*c+b^2)^(1/2))^(
1/2)/(-x^2+1)^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2))*(b^2-a*c+b*c+(-3*a*b*c-2*
a*c^2+b^3+b^2*c)/(-4*a*c+b^2)^(1/2))/c^2/(b+(-4*a*c+b^2)^(1/2))^(1/2)/(b+2*
c+(-4*a*c+b^2)^(1/2))^(1/2)
```

Rubi [A] (verified)

Time = 3.55 (sec) , antiderivative size = 325, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.207$, Rules used

= {1305, 396, 222, 1706, 385, 211}

$$\int \frac{x^4\sqrt{1-x^2}}{a+bx^2+cx^4}\,dx = -\frac{\left(-\frac{-3abc-2ac^2+b^3+b^2c}{\sqrt{b^2-4ac}}-ac+b^2+bc\right)\arctan\left(\frac{x\sqrt{-\sqrt{b^2-4ac}+b+2c}}{\sqrt{1-x^2}\sqrt{b-\sqrt{b^2-4ac}}}\right)}{c^2\sqrt{b-\sqrt{b^2-4ac}}\sqrt{-\sqrt{b^2-4ac}+b+2c}} - \frac{\left(\frac{-3abc-2ac^2+b^3+b^2c}{\sqrt{b^2-4ac}}-ac+b^2+bc\right)\arctan\left(\frac{x\sqrt{\sqrt{b^2-4ac}+b+2c}}{\sqrt{1-x^2}\sqrt{\sqrt{b^2-4ac}+b}}\right)}{c^2\sqrt{\sqrt{b^2-4ac}+b}\sqrt{\sqrt{b^2-4ac}+b+2c}} + \frac{\arcsin(x)(2b+c)}{2c^2} + \frac{\sqrt{1-x^2}x}{2c}$$

```
[In] Int[(x^4*Sqrt[1 - x^2])/(a + b*x^2 + c*x^4),x]
```

```
[Out] (x*Sqrt[1 - x^2])/(2*c) + ((2*b + c)*ArcSin[x])/(2*c^2) - ((b^2 - a*c + b*c
 - (b^3 - 3*a*b*c + b^2*c - 2*a*c^2)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[b + 2*
c - Sqrt[b^2 - 4*a*c]]*x)/(Sqrt[b - Sqrt[b^2 - 4*a*c]]*Sqrt[1 - x^2])])/(c^
2*Sqrt[b - Sqrt[b^2 - 4*a*c]]*Sqrt[b + 2*c - Sqrt[b^2 - 4*a*c]]) - ((b^2 -
a*c + b*c + (b^3 - 3*a*b*c + b^2*c - 2*a*c^2)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sq
rt[b + 2*c + Sqrt[b^2 - 4*a*c]]*x)/(Sqrt[b + Sqrt[b^2 - 4*a*c]]*Sqrt[1 - x^
2])])/(c^2*Sqrt[b + Sqrt[b^2 - 4*a*c]]*Sqrt[b + 2*c + Sqrt[b^2 - 4*a*c]])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 396

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 1305

```
Int[(((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x_)^2)^(q_))/((a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4), x_Symbol] :> Dist[f^4/c^2, Int[(f*x)^(m - 4)*(c*d - b*e + c
```

```
*e*x^2)*(d + e*x^2)^(q - 1), x], x] - Dist[f^4/c^2, Int[(f*x)^(m - 4)*(d +
e*x^2)^(q - 1)*(Simp[a*(c*d - b*e) + (b*c*d - b^2*e + a*c*e)*x^2, x]/(a + b
*x^2 + c*x^4)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b^2 - 4*a*c,
0] &&  !IntegerQ[q] && GtQ[q, 0] && GtQ[m, 3]
```

Rule 1706

```
Int[(Px_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^
(p_.), x_Symbol] :> Int[ExpandIntegrand[Px*(d + e*x^2)^q*(a + b*x^2 + c*x^4
)^p, x], x] /; FreeQ[{a, b, c, d, e, q}, x] && PolyQ[Px, x^2] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[p]
```

Rubi steps

$$\text{integral} = \frac{\int \frac{b+c-cx^2}{\sqrt{1-x^2}}\,dx}{c^2} - \frac{\int \frac{a(b+c)+(b^2-ac+bc)x^2}{\sqrt{1-x^2}(a+bx^2+cx^4)}\,dx}{c^2}$$

$$= \frac{x\sqrt{1-x^2}}{2c} - \frac{\int \left(\frac{b^2-ac+bc+\frac{-b^3+3abc-b^2c+2ac^2}{\sqrt{b^2-4ac}}}{\sqrt{1-x^2}\left(b-\sqrt{b^2-4ac}+2cx^2\right)} + \frac{b^2-ac+bc-\frac{-b^3+3abc-b^2c+2ac^2}{\sqrt{b^2-4ac}}}{\sqrt{1-x^2}\left(b+\sqrt{b^2-4ac}+2cx^2\right)}\right)dx}{c^2} + \frac{(2b+c)\int \frac{1}{\sqrt{1-x^2}}\,dx}{2c^2}$$

$$= \frac{x\sqrt{1-x^2}}{2c} + \frac{(2b+c)\sin^{-1}(x)}{2c^2} - \frac{\left(b^2-ac+bc-\frac{b^3-3abc+b^2c-2ac^2}{\sqrt{b^2-4ac}}\right)\int \frac{1}{\sqrt{1-x^2}\left(b-\sqrt{b^2-4ac}+2cx^2\right)}\,dx}{c^2} - \frac{\left(b^2-ac+bc+\frac{b^3-3abc+b^2c-2ac^2}{\sqrt{b^2-4ac}}\right)\int \frac{1}{\sqrt{1-x^2}\left(b+\sqrt{b^2-4ac}+2cx^2\right)}\,dx}{c^2}$$

$$= \frac{x\sqrt{1-x^2}}{2c} + \frac{(2b+c)\sin^{-1}(x)}{2c^2} - \frac{\left(b^2-ac+bc-\frac{b^3-3abc+b^2c-2ac^2}{\sqrt{b^2-4ac}}\right)\text{Subst}\left(\int \frac{1}{b-\sqrt{b^2-4ac}-\left(-b-2c+\sqrt{b^2-4ac}\right)x^2}\,dx, x, \frac{x}{\sqrt{1-x^2}}\right)}{c^2} - \frac{\left(b^2-ac+bc+\frac{b^3-3abc+b^2c-2ac^2}{\sqrt{b^2-4ac}}\right)\text{Subst}\left(\int \frac{1}{b+\sqrt{b^2-4ac}-\left(-b-2c-\sqrt{b^2-4ac}\right)x^2}\,dx, x, \frac{x}{\sqrt{1-x^2}}\right)}{c^2}$$

$$= \frac{x\sqrt{1-x^2}}{2c} + \frac{(2b+c)\sin^{-1}(x)}{2c^2} - \frac{\left(b^2-ac+bc-\frac{b^3-3abc+b^2c-2ac^2}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{b+2c-\sqrt{b^2-4ac}}x}{\sqrt{b-\sqrt{b^2-4ac}}\sqrt{1-x^2}}\right)}{c^2\sqrt{b-\sqrt{b^2-4ac}}\sqrt{b+2c-\sqrt{b^2-4ac}}} - \frac{\left(b^2-ac+bc+\frac{b^3-3abc+b^2c-2ac^2}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{b+2c+\sqrt{b^2-4ac}}x}{\sqrt{b+\sqrt{b^2-4ac}}\sqrt{1-x^2}}\right)}{c^2\sqrt{b+\sqrt{b^2-4ac}}\sqrt{b+2c+\sqrt{b^2-4ac}}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.64 (sec) , antiderivative size = 588, normalized size of antiderivative = 1.81

$$\int \frac{x^4\sqrt{1-x^2}}{a+bx^2+cx^4}\,dx$$

$$= \frac{2cx\sqrt{1-x^2}+4(2b+c)\arctan\left(\frac{x}{-1+\sqrt{1-x^2}}\right)+\text{RootSum}\left[a+4a\#1^2+4b\#1^2+6a\#1^4+8b\#1^4+16c\#1^4\right.}{}$$

```
[In] Integrate[(x^4*Sqrt[1 - x^2])/(a + b*x^2 + c*x^4),x]
```

```
[Out] (2*c*x*Sqrt[1 - x^2] + 4*(2*b + c)*ArcTan[x/(-1 + Sqrt[1 - x^2])] + RootSum
[a + 4*a*#1^2 + 4*b*#1^2 + 6*a*#1^4 + 8*b*#1^4 + 16*c*#1^4 + 4*a*#1^6 + 4*b
*#1^6 + a*#1^8 & , (-(a*b*Log[x]) - a*c*Log[x] + a*b*Log[-1 + Sqrt[1 - x^2]
 - x*#1] + a*c*Log[-1 + Sqrt[1 - x^2] - x*#1] - 3*a*b*Log[x]*#1^2 - 4*b^2*L
og[x]*#1^2 + a*c*Log[x]*#1^2 - 4*b*c*Log[x]*#1^2 + 3*a*b*Log[-1 + Sqrt[1 -
x^2] - x*#1]*#1^2 + 4*b^2*Log[-1 + Sqrt[1 - x^2] - x*#1]*#1^2 - a*c*Log[-1
+ Sqrt[1 - x^2] - x*#1]*#1^2 + 4*b*c*Log[-1 + Sqrt[1 - x^2] - x*#1]*#1^2 -
3*a*b*Log[x]*#1^4 - 4*b^2*Log[x]*#1^4 + a*c*Log[x]*#1^4 - 4*b*c*Log[x]*#1^4
 + 3*a*b*Log[-1 + Sqrt[1 - x^2] - x*#1]*#1^4 + 4*b^2*Log[-1 + Sqrt[1 - x^2]
 - x*#1]*#1^4 - a*c*Log[-1 + Sqrt[1 - x^2] - x*#1]*#1^4 + 4*b*c*Log[-1 + Sq
rt[1 - x^2] - x*#1]*#1^4 - a*b*Log[x]*#1^6 - a*c*Log[x]*#1^6 + a*b*Log[-1 +
 Sqrt[1 - x^2] - x*#1]*#1^6 + a*c*Log[-1 + Sqrt[1 - x^2] - x*#1]*#1^6)/(a*#
1 + b*#1 + 3*a*#1^3 + 4*b*#1^3 + 8*c*#1^3 + 3*a*#1^5 + 3*b*#1^5 + a*#1^7) &
 ])/(4*c^2)
```

Maple [A] (verified)

Time = 2.12 (sec) , antiderivative size = 250, normalized size of antiderivative = 0.77

method	result
risch	$-\frac{x(x^2-1)}{2c\sqrt{-x^2+1}}+\frac{\frac{(2b+c)\arcsin(x)}{c}+\frac{a\sqrt{2}\left(\frac{\left(b\sqrt{-4ac+b^2}+\sqrt{-4ac+b^2}\,c+2ac-b^2-bc\right)\arctan\left(\frac{a\sqrt{-x^2+1}\,\sqrt{2}}{x\sqrt{\left(2a+b+\sqrt{-4ac+b^2}\right)a}}\right)}{\sqrt{\left(2a+b+\sqrt{-4ac+b^2}\right)a}}-\frac{\left(b\sqrt{-4a\cdots\right.}{\cdots}\right.}{c\sqrt{-4ac+b^2}}}{2c}$
pseudoelliptic	$\frac{a\sqrt{2}\sqrt{\left(2a+b+\sqrt{-4ac+b^2}\right)a}\left(\frac{(-b-c)\sqrt{-4ac+b^2}}{2}+ac-\frac{b(b+c)}{2}\right)\operatorname{arctanh}\left(\frac{a\sqrt{-x^2+1}\,\sqrt{2}}{x\sqrt{\left(-b+\sqrt{-4ac+b^2}-2a\right)a}}\right)+\left(a\sqrt{2}\left(\frac{(b+c)\sqrt{-4ac\cdots}}{2}\cdots\right.\right.}{\sqrt{\left(-b+\sqrt{-4\cdots\right.}}}$
default	$\frac{\frac{x\sqrt{-x^2+1}}{2}+\frac{\arcsin(x)}{2}}{c}+\frac{a\sqrt{2}\sqrt{\left(2a+b+\sqrt{-4ac+b^2}\right)a}\left(\frac{(-b-c)\sqrt{-4ac+b^2}}{2}+ac-\frac{b(b+c)}{2}\right)\operatorname{arctanh}\left(\frac{a\sqrt{-x^2+1}\,\sqrt{2}}{x\sqrt{\left(-b+\sqrt{-4ac+b^2}-2a\right)a\cdots}}\right.}{\cdots}$

```
[In] int(x^4*(-x^2+1)^(1/2)/(c*x^4+b*x^2+a),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*x/c*(x^2-1)/(-x^2+1)^(1/2)+1/2/c*((2*b+c)/c*arcsin(x)+1/c*a*2^(1/2)/(-
4*a*c+b^2)^(1/2)*((b*(-4*a*c+b^2)^(1/2)+(-4*a*c+b^2)^(1/2)*c+2*a*c-b^2-b*c)
/((2*a+b+(-4*a*c+b^2)^(1/2))*a)^(1/2)*arctan(a/x*(-x^2+1)^(1/2)*2^(1/2)/((2
*a+b+(-4*a*c+b^2)^(1/2))*a)^(1/2))-(b*(-4*a*c+b^2)^(1/2)+(-4*a*c+b^2)^(1/2)
*c-2*a*c+b^2+b*c)/((-b+(-4*a*c+b^2)^(1/2)-2*a)*a)^(1/2)*arctanh(a/x*(-x^2+1
)^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2)-2*a)*a)^(1/2))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2860 vs. 2(279) = 558.

Time = 1.30 (sec) , antiderivative size = 2860, normalized size of antiderivative = 8.80

$$\int \frac{x^4\sqrt{1-x^2}}{a+bx^2+cx^4}\,dx = \text{Too large to display}$$

```
[In] integrate(x^4*(-x^2+1)^(1/2)/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] -1/2*(sqrt(1/2)*c^2*sqrt(-(b^4 + (2*a^2 - 3*a*b)*c^2 - (4*a*b^2 - b^3)*c +
(b^2*c^4 - 4*a*c^5)*sqrt((b^6 + a^2*c^4 + 2*(2*a^2*b - a*b^2)*c^3 + (4*a^2*
b^2 - 6*a*b^3 + b^4)*c^2 - 2*(2*a*b^4 - b^5)*c)/(b^2*c^8 - 4*a*c^9)))/(b^2*
c^4 - 4*a*c^5))*log(-(2*a^2*b^3 - 2*a^3*c^2 - 2*(a^2*b^3 - a^3*c^2 - (2*a^3
*b - a^2*b^2)*c)*x^2 - 2*(2*a^3*b - a^2*b^2)*c + sqrt(1/2)*((b^6 + 4*a^2*b*
c^3 + (8*a^2*b^2 - 5*a*b^3)*c^2 - (6*a*b^4 - b^5)*c)*sqrt(-x^2 + 1)*x - (b^
6 + 4*a^2*b*c^3 + (8*a^2*b^2 - 5*a*b^3)*c^2 - (6*a*b^4 - b^5)*c)*x - ((b^4*
```

c^4 - 6*a*b^2*c^5 + 8*a^2*c^6)*sqrt(-x^2 + 1)*x - (b^4*c^4 - 6*a*b^2*c^5 + 8*a^2*c^6)*x)*sqrt((b^6 + a^2*c^4 + 2*(2*a^2*b - a*b^2)*c^3 + (4*a^2*b^2 - 6*a*b^3 + b^4)*c^2 - 2*(2*a*b^4 - b^5)*c)/(b^2*c^8 - 4*a*c^9)))*sqrt(-(b^4 + (2*a^2 - 3*a*b)*c^2 - (4*a*b^2 - b^3)*c + (b^2*c^4 - 4*a*c^5)*sqrt((b^6 + a^2*c^4 + 2*(2*a^2*b - a*b^2)*c^3 + (4*a^2*b^2 - 6*a*b^3 + b^4)*c^2 - 2*(2*a*b^4 - b^5)*c)/(b^2*c^8 - 4*a*c^9)))/(b^2*c^4 - 4*a*c^5)) - 2*(a^2*b^3 - a^3*c^2 - (2*a^3*b - a^2*b^2)*c)*sqrt(-x^2 + 1))/x^2) - sqrt(1/2)*c^2*sqrt(-(b^4 + (2*a^2 - 3*a*b)*c^2 - (4*a*b^2 - b^3)*c + (b^2*c^4 - 4*a*c^5)*sqrt((b^6 + a^2*c^4 + 2*(2*a^2*b - a*b^2)*c^3 + (4*a^2*b^2 - 6*a*b^3 + b^4)*c^2 - 2*(2*a*b^4 - b^5)*c)/(b^2*c^8 - 4*a*c^9)))/(b^2*c^4 - 4*a*c^5))*log(-(2*a^2*b^3 - 2*a^3*c^2 - 2*(a^2*b^3 - a^3*c^2 - (2*a^3*b - a^2*b^2)*c)*x^2 - 2*(2*a^3*b - a^2*b^2)*c - sqrt(1/2)*((b^6 + 4*a^2*b*c^3 + (8*a^2*b^2 - 5*a*b^3)*c^2 - (6*a*b^4 - b^5)*c)*sqrt(-x^2 + 1)*x - (b^6 + 4*a^2*b*c^3 + (8*a^2*b^2 - 5*a*b^3)*c^2 - (6*a*b^4 - b^5)*c)*x - ((b^4*c^4 - 6*a*b^2*c^5 + 8*a^2*c^6)*sqrt(-x^2 + 1)*x - (b^4*c^4 - 6*a*b^2*c^5 + 8*a^2*c^6)*x)*sqrt((b^6 + a^2*c^4 + 2*(2*a^2*b - a*b^2)*c^3 + (4*a^2*b^2 - 6*a*b^3 + b^4)*c^2 - 2*(2*a*b^4 - b^5)*c)/(b^2*c^8 - 4*a*c^9)))*sqrt(-(b^4 + (2*a^2 - 3*a*b)*c^2 - (4*a*b^2 - b^3)*c + (b^2*c^4 - 4*a*c^5)*sqrt((b^6 + a^2*c^4 + 2*(2*a^2*b - a*b^2)*c^3 + (4*a^2*b^2 - 6*a*b^3 + b^4)*c^2 - 2*(2*a*b^4 - b^5)*c)/(b^2*c^8 - 4*a*c^9)))/(b^2*c^4 - 4*a*c^5)) - 2*(a^2*b^3 - a^3*c^2 - (2*a^3*b - a^2*b^2)*c)*sqrt(-x^2 + 1))/x^2) + sqrt(1/2)*c^2*sqrt(-(b^4 + (2*a^2 - 3*a*b)*c^2 - (4*a*b^2 - b^3)*c - (b^2*c^4 - 4*a*c^5)*sqrt((b^6 + a^2*c^4 + 2*(2*a^2*b - a*b^2)*c^3 + (4*a^2*b^2 - 6*a*b^3 + b^4)*c^2 - 2*(2*a*b^4 - b^5)*c)/(b^2*c^8 - 4*a*c^9)))/(b^2*c^4 - 4*a*c^5))*log(-(2*a^2*b^3 - 2*a^3*c^2 - 2*(a^2*b^3 - a^3*c^2 - (2*a^3*b - a^2*b^2)*c)*x^2 - 2*(2*a^3*b - a^2*b^2)*c + sqrt(1/2)*((b^6 + 4*a^2*b*c^3 + (8*a^2*b^2 - 5*a*b^3)*c^2 - (6*a*b^4 - b^5)*c)*sqrt(-x^2 + 1)*x - (b^6 + 4*a^2*b*c^3 + (8*a^2*b^2 - 5*a*b^3)*c^2 - (6*a*b^4 - b^5)*c)*x + ((b^4*c^4 - 6*a*b^2*c^5 + 8*a^2*c^6)*sqrt(-x^2 + 1)*x - (b^4*c^4 - 6*a*b^2*c^5 + 8*a^2*c^6)*x)*sqrt((b^6 + a^2*c^4 + 2*(2*a^2*b - a*b^2)*c^3 + (4*a^2*b^2 - 6*a*b^3 + b^4)*c^2 - 2*(2*a*b^4 - b^5)*c)/(b^2*c^8 - 4*a*c^9)))*sqrt(-(b^4 + (2*a^2 - 3*a*b)*c^2 - (4*a*b^2 - b^3)*c - (b^2*c^4 - 4*a*c^5)*sqrt((b^6 + a^2*c^4 + 2*(2*a^2*b - a*b^2)*c^3 + (4*a^2*b^2 - 6*a*b^3 + b^4)*c^2 - 2*(2*a*b^4 - b^5)*c)/(b^2*c^8 - 4*a*c^9)))/(b^2*c^4 - 4*a*c^5)) - 2*(a^2*b^3 - a^3*c^2 - (2*a^3*b - a^2*b^2)*c)*sqrt(-x^2 + 1))/x^2) - sqrt(1/2)*c^2*sqrt(-(b^4 + (2*a^2 - 3*a*b)*c^2 - (4*a*b^2 - b^3)*c - (b^2*c^4 - 4*a*c^5)*sqrt((b^6 + a^2*c^4 + 2*(2*a^2*b - a*b^2)*c^3 + (4*a^2*b^2 - 6*a*b^3 + b^4)*c^2 - 2*(2*a*b^4 - b^5)*c)/(b^2*c^8 - 4*a*c^9)))/(b^2*c^4 - 4*a*c^5))*log(-(2*a^2*b^3 - 2*a^3*c^2 - 2*(a^2*b^3 - a^3*c^2 - (2*a^3*b - a^2*b^2)*c)*x^2 - 2*(2*a^3*b - a^2*b^2)*c - sqrt(1/2)*((b^6 + 4*a^2*b*c^3 + (8*a^2*b^2 - 5*a*b^3)*c^2 - (6*a*b^4 - b^5)*c)*sqrt(-x^2 + 1)*x - (b^6 + 4*a^2*b*c^3 + (8*a^2*b^2 - 5*a*b^3)*c^2 - (6*a*b^4 - b^5)*c)*x + ((b^4*c^4 - 6*a*b^2*c^5 + 8*a^2*c^6)*sqrt(-x^2 + 1)*x - (b^4*c^4 - 6*a*b^2*c^5 + 8*a^2*c^6)*x)*sqrt((b^6 + a^2*c^4 + 2*(2*a^2*b - a*b^2)*c^3 + (4*a^2*b^2 - 6*a*b^3 + b^4)*c^2 - 2*(2*a*b^4 - b^5)*c)/(b^2*c^8 - 4*a*c^9)))*sqrt(-(b^4 + (2*a^2 - 3*a*b)*c^2 - (4*a*b^2 - b^3)*c - (b^2*c^4 - 4*a*c^5)*sqrt((b^6 +

```
a^2*c^4 + 2*(2*a^2*b - a*b^2)*c^3 + (4*a^2*b^2 - 6*a*b^3 + b^4)*c^2 - 2*(2
*a*b^4 - b^5)*c)/(b^2*c^8 - 4*a*c^9)))/(b^2*c^4 - 4*a*c^5)) - 2*(a^2*b^3 -
a^3*c^2 - (2*a^3*b - a^2*b^2)*c)*sqrt(-x^2 + 1))/x^2) - sqrt(-x^2 + 1)*c*x
+ 2*(2*b + c)*arctan((sqrt(-x^2 + 1) - 1)/x))/c^2
```

Sympy [F]

$$\int \frac{x^4\sqrt{1-x^2}}{a+bx^2+cx^4}\,dx = \int \frac{x^4\sqrt{-(x-1)(x+1)}}{a+bx^2+cx^4}\,dx$$

```
[In] integrate(x**4*(-x**2+1)**(1/2)/(c*x**4+b*x**2+a),x)
[Out] Integral(x**4*sqrt(-(x - 1)*(x + 1))/(a + b*x**2 + c*x**4), x)
```

Maxima [F]

$$\int \frac{x^4\sqrt{1-x^2}}{a+bx^2+cx^4}\,dx = \int \frac{\sqrt{-x^2+1}x^4}{cx^4+bx^2+a}\,dx$$

```
[In] integrate(x^4*(-x^2+1)^(1/2)/(c*x^4+b*x^2+a),x, algorithm="maxima")
[Out] integrate(sqrt(-x^2 + 1)*x^4/(c*x^4 + b*x^2 + a), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1710 vs. $2(279) = 558$.

Time = 1.42 (sec) , antiderivative size = 1710, normalized size of antiderivative = 5.26

$$\int \frac{x^4\sqrt{1-x^2}}{a+bx^2+cx^4}\,dx = \text{Too large to display}$$

```
[In] integrate(x^4*(-x^2+1)^(1/2)/(c*x^4+b*x^2+a),x, algorithm="giac")
[Out] 1/4*(3*sqrt(2)*sqrt(2*a^2 + a*b + sqrt(b^2 - 4*a*c)*a)*a^2*b^3 + 2*sqrt(2)*
sqrt(2*a^2 + a*b + sqrt(b^2 - 4*a*c)*a)*a*b^4 - 2*a^2*b^4 - sqrt(2)*sqrt(2*
a^2 + a*b + sqrt(b^2 - 4*a*c)*a)*b^5 + 2*a*b^5 - 12*sqrt(2)*sqrt(2*a^2 + a*
b + sqrt(b^2 - 4*a*c)*a)*a^3*b*c - 8*sqrt(2)*sqrt(2*a^2 + a*b + sqrt(b^2 -
4*a*c)*a)*a^2*b^2*c + 12*a^3*b^2*c + 8*sqrt(2)*sqrt(2*a^2 + a*b + sqrt(b^2
- 4*a*c)*a)*a*b^3*c - 16*a^2*b^3*c - 16*a^4*c^2 - 16*sqrt(2)*sqrt(2*a^2 + a
*b + sqrt(b^2 - 4*a*c)*a)*a^2*b*c^2 + 32*a^3*b*c^2 - 3*sqrt(2)*sqrt(2*a^2 +
a*b + sqrt(b^2 - 4*a*c)*a)*sqrt(b^2 - 4*a*c)*a^2*b^2 - 2*sqrt(2)*sqrt(2*a^
2 + a*b + sqrt(b^2 - 4*a*c)*a)*sqrt(b^2 - 4*a*c)*a*b^3 + sqrt(2)*sqrt(2*a^2
```

+ a*b + sqrt(b^2 - 4*a*c)*a)*sqrt(b^2 - 4*a*c)*b^4 + 6*sqrt(2)*sqrt(2*a^2 + a*b + sqrt(b^2 - 4*a*c)*a)*sqrt(b^2 - 4*a*c)*a^3*c + 4*sqrt(2)*sqrt(2*a^2 + a*b + sqrt(b^2 - 4*a*c)*a)*sqrt(b^2 - 4*a*c)*a^2*b*c - 6*sqrt(2)*sqrt(2*a^2 + a*b + sqrt(b^2 - 4*a*c)*a)*sqrt(b^2 - 4*a*c)*a*b^2*c + 8*sqrt(2)*sqrt(2*a^2 + a*b + sqrt(b^2 - 4*a*c)*a)*sqrt(b^2 - 4*a*c)*a^2*c^2 + 2*(b^2 - 4*a*c)*a^2*b^2 - 2*(b^2 - 4*a*c)*a*b^3 - 4*(b^2 - 4*a*c)*a^3*c + 8*(b^2 - 4*a*c)*a^2*b*c)*abs(a)*arctan(-1/2*sqrt(2)*(x/(sqrt(-x^2 + 1) - 1) - (sqrt(-x^2 + 1) - 1)/x)/sqrt((2*a*c^2 + b*c^2 + sqrt(-4*(a*c^2 + b*c^2 + c^3)*a*c^2 + (2*a*c^2 + b*c^2)^2))/(a*c^2)))/(3*a^4*b^2*c^2 + 2*a^3*b^3*c^2 - a^2*b^4*c^2 - 12*a^5*c^3 - 8*a^4*b*c^3 + 8*a^3*b^2*c^3 - 16*a^4*c^4) + 1/4*(3*sqrt(2)*sqrt(2*a^2 + a*b - sqrt(b^2 - 4*a*c)*a)*a^2*b^3 + 2*sqrt(2)*sqrt(2*a^2 + a*b - sqrt(b^2 - 4*a*c)*a)*a*b^4 + 2*a^2*b^4 - sqrt(2)*sqrt(2*a^2 + a*b - sqrt(b^2 - 4*a*c)*a)*b^5 - 2*a*b^5 - 12*sqrt(2)*sqrt(2*a^2 + a*b - sqrt(b^2 - 4*a*c)*a)*a^3*b*c - 8*sqrt(2)*sqrt(2*a^2 + a*b - sqrt(b^2 - 4*a*c)*a)*a^2*b^2*c - 12*a^3*b^2*c + 8*sqrt(2)*sqrt(2*a^2 + a*b - sqrt(b^2 - 4*a*c)*a)*a*b^3*c + 16*a^2*b^3*c + 16*a^4*c^2 - 16*sqrt(2)*sqrt(2*a^2 + a*b - sqrt(b^2 - 4*a*c)*a)*a^2*b*c^2 - 32*a^3*b*c^2 + 3*sqrt(2)*sqrt(2*a^2 + a*b - sqrt(b^2 - 4*a*c)*a)*sqrt(b^2 - 4*a*c)*a^2*b^2 + 2*sqrt(2)*sqrt(2*a^2 + a*b - sqrt(b^2 - 4*a*c)*a)*sqrt(b^2 - 4*a*c)*a*b^3 - sqrt(2)*sqrt(2*a^2 + a*b - sqrt(b^2 - 4*a*c)*a)*sqrt(b^2 - 4*a*c)*b^4 - 6*sqrt(2)*sqrt(2*a^2 + a*b - sqrt(b^2 - 4*a*c)*a)*sqrt(b^2 - 4*a*c)*a^3*c - 4*sqrt(2)*sqrt(2*a^2 + a*b - sqrt(b^2 - 4*a*c)*a)*sqrt(b^2 - 4*a*c)*a^2*b*c + 6*sqrt(2)*sqrt(2*a^2 + a*b - sqrt(b^2 - 4*a*c)*a)*sqrt(b^2 - 4*a*c)*a*b^2*c - 8*sqrt(2)*sqrt(2*a^2 + a*b - sqrt(b^2 - 4*a*c)*a)*sqrt(b^2 - 4*a*c)*a^2*c^2 - 2*(b^2 - 4*a*c)*a^2*b^2 + 2*(b^2 - 4*a*c)*a*b^3 + 4*(b^2 - 4*a*c)*a^3*c - 8*(b^2 - 4*a*c)*a^2*b*c)*abs(a)*arctan(-1/2*sqrt(2)*(x/(sqrt(-x^2 + 1) - 1) - (sqrt(-x^2 + 1) - 1)/x)/sqrt((2*a*c^2 + b*c^2 - sqrt(-4*(a*c^2 + b*c^2 + c^3)*a*c^2 + (2*a*c^2 + b*c^2)^2))/(a*c^2)))/(3*a^4*b^2*c^2 + 2*a^3*b^3*c^2 - a^2*b^4*c^2 - 12*a^5*c^3 - 8*a^4*b*c^3 + 8*a^3*b^2*c^3 - 16*a^4*c^4) + 1/2*sqrt(-x^2 + 1)*x/c + 1/4*(pi*sgn(x) + 2*arctan(-1/2*x*((sqrt(-x^2 + 1) - 1)^2/x^2 - 1)/(sqrt(-x^2 + 1) - 1)))*(2*b + c)/c^2

Mupad [B] (verification not implemented)

Time = 8.24 (sec) , antiderivative size = 1024, normalized size of antiderivative = 3.15

$$\int \frac{x^4\sqrt{1-x^2}}{a+bx^2+cx^4}\,dx = \operatorname{asin}(x)\left(\frac{\frac{b}{c}+1}{c}-\frac{1}{2\,c}\right)+\frac{x\,\sqrt{1-x^2}}{2\,c}$$

$$-\frac{\ln\left(\frac{\frac{\left(x\,\sqrt{-\frac{b-\sqrt{b^2-4\,a\,c}}{2\,c}}-1\right)\,1\mathrm{i}}{\sqrt{\frac{b-\sqrt{b^2-4\,a\,c}}{2\,c}+1}}-\sqrt{1-x^2}\,1\mathrm{i}}{x-\sqrt{-\frac{b-\sqrt{b^2-4\,a\,c}}{2\,c}}}\right)\left(b^2\left(-\frac{b-\sqrt{b^2-4\,a\,c}}{2\,c}\right)^{3/2}+a\,b\,\sqrt{-\frac{b-\sqrt{b^2-4\,a\,c}}{2\,c}}+2\,a\,c\,\sqrt{-\frac{b-\sqrt{b^2-4\,a\,c}}{2\,c}}-2}{2\,c\,\sqrt{\frac{b-\sqrt{b^2-4\,a\,c}}{2\,c}+1}\,\left(4\,a\,c-b^2\right)}$$

$$+\frac{\ln\left(\frac{\frac{\left(x\,\sqrt{-\frac{b+\sqrt{b^2-4\,a\,c}}{2\,c}}+1\right)\,1\mathrm{i}}{\sqrt{\frac{b+\sqrt{b^2-4\,a\,c}}{2\,c}+1}}+\sqrt{1-x^2}\,1\mathrm{i}}{x+\sqrt{-\frac{b+\sqrt{b^2-4\,a\,c}}{2\,c}}}\right)\left(b^2\left(-\frac{b+\sqrt{b^2-4\,a\,c}}{2\,c}\right)^{3/2}+a\,b\,\sqrt{-\frac{b+\sqrt{b^2-4\,a\,c}}{2\,c}}+2\,a\,c\,\sqrt{-\frac{b+\sqrt{b^2-4\,a\,c}}{2\,c}}-2}{2\,c\,\left(4\,a\,c-b^2\right)\,\sqrt{\frac{b+\sqrt{b^2-4\,a\,c}}{2\,c}+1}}$$

$$+\frac{\ln\left(\frac{\frac{\left(x\,\sqrt{-\frac{b-\sqrt{b^2-4\,a\,c}}{2\,c}}+1\right)\,1\mathrm{i}}{\sqrt{\frac{b-\sqrt{b^2-4\,a\,c}}{2\,c}+1}}+\sqrt{1-x^2}\,1\mathrm{i}}{x+\sqrt{-\frac{b-\sqrt{b^2-4\,a\,c}}{2\,c}}}\right)\left(b^2\left(-\frac{b-\sqrt{b^2-4\,a\,c}}{2\,c}\right)^{3/2}+a\,b\,\sqrt{-\frac{b-\sqrt{b^2-4\,a\,c}}{2\,c}}+2\,a\,c\,\sqrt{-\frac{b-\sqrt{b^2-4\,a\,c}}{2\,c}}-2}{2\,c\,\sqrt{\frac{b-\sqrt{b^2-4\,a\,c}}{2\,c}+1}\,\left(4\,a\,c-b^2\right)}$$

$$-\frac{\ln\left(\frac{\frac{\left(x\,\sqrt{-\frac{b+\sqrt{b^2-4\,a\,c}}{2\,c}}-1\right)\,1\mathrm{i}}{\sqrt{\frac{b+\sqrt{b^2-4\,a\,c}}{2\,c}+1}}-\sqrt{1-x^2}\,1\mathrm{i}}{x-\sqrt{-\frac{b+\sqrt{b^2-4\,a\,c}}{2\,c}}}\right)\left(b^2\left(-\frac{b+\sqrt{b^2-4\,a\,c}}{2\,c}\right)^{3/2}+a\,b\,\sqrt{-\frac{b+\sqrt{b^2-4\,a\,c}}{2\,c}}+2\,a\,c\,\sqrt{-\frac{b+\sqrt{b^2-4\,a\,c}}{2\,c}}-2}{2\,c\,\left(4\,a\,c-b^2\right)\,\sqrt{\frac{b+\sqrt{b^2-4\,a\,c}}{2\,c}+1}}$$

```
[In] int((x^4*(1 - x^2)^(1/2))/(a + b*x^2 + c*x^4),x)
```

```
[Out] asin(x)*((b/c + 1)/c - 1/(2*c)) + (x*(1 - x^2)^(1/2))/(2*c) - (log((((x*(-(
b - (b^2 - 4*a*c)^(1/2))/(2*c))^(1/2) - 1)*1i)/((b - (b^2 - 4*a*c)^(1/2))/(
2*c) + 1)^(1/2) - (1 - x^2)^(1/2)*1i)/(x - (-(b - (b^2 - 4*a*c)^(1/2))/(2*c
))^(1/2)))*(b^2*(-(b - (b^2 - 4*a*c)^(1/2))/(2*c))^(3/2) + a*b*(-(b - (b^2
- 4*a*c)^(1/2))/(2*c))^(1/2) + 2*a*c*(-(b - (b^2 - 4*a*c)^(1/2))/(2*c))^(1/
2) - 2*a*c*(-(b - (b^2 - 4*a*c)^(1/2))/(2*c))^(3/2) + b*c*(-(b - (b^2 - 4*a
*c)^(1/2))/(2*c))^(3/2)))/(2*c*((b - (b^2 - 4*a*c)^(1/2))/(2*c) + 1)^(1/2)*
(4*a*c - b^2)) + (log((((x*(-(b + (b^2 - 4*a*c)^(1/2))/(2*c))^(1/2) + 1)*1i
```

```
)/((b + (b^2 - 4*a*c)^(1/2))/(2*c) + 1)^(1/2) + (1 - x^2)^(1/2)*1i)/(x + (-
(b + (b^2 - 4*a*c)^(1/2))/(2*c))^(1/2)))*(b^2*(-(b + (b^2 - 4*a*c)^(1/2))/(
2*c))^(3/2) + a*b*(-(b + (b^2 - 4*a*c)^(1/2))/(2*c))^(1/2) + 2*a*c*(-(b + (
b^2 - 4*a*c)^(1/2))/(2*c))^(1/2) - 2*a*c*(-(b + (b^2 - 4*a*c)^(1/2))/(2*c))
^(3/2) + b*c*(-(b + (b^2 - 4*a*c)^(1/2))/(2*c))^(3/2)))/(2*c*(4*a*c - b^2)*
((b + (b^2 - 4*a*c)^(1/2))/(2*c) + 1)^(1/2)) + (log((((x*(-(b - (b^2 - 4*a*
c)^(1/2))/(2*c))^(1/2) + 1)*1i)/((b - (b^2 - 4*a*c)^(1/2))/(2*c) + 1)^(1/2)
 + (1 - x^2)^(1/2)*1i)/(x + (-(b - (b^2 - 4*a*c)^(1/2))/(2*c))^(1/2)))*(b^2
*(-(b - (b^2 - 4*a*c)^(1/2))/(2*c))^(3/2) + a*b*(-(b - (b^2 - 4*a*c)^(1/2))
/(2*c))^(1/2) + 2*a*c*(-(b - (b^2 - 4*a*c)^(1/2))/(2*c))^(1/2) - 2*a*c*(-(b
 - (b^2 - 4*a*c)^(1/2))/(2*c))^(3/2) + b*c*(-(b - (b^2 - 4*a*c)^(1/2))/(2*c
))^(3/2)))/(2*c*((b - (b^2 - 4*a*c)^(1/2))/(2*c) + 1)^(1/2)*(4*a*c - b^2))
- (log((((x*(-(b + (b^2 - 4*a*c)^(1/2))/(2*c))^(1/2) - 1)*1i)/((b + (b^2 -
4*a*c)^(1/2))/(2*c) + 1)^(1/2) - (1 - x^2)^(1/2)*1i)/(x - (-(b + (b^2 - 4*a
*c)^(1/2))/(2*c))^(1/2)))*(b^2*(-(b + (b^2 - 4*a*c)^(1/2))/(2*c))^(3/2) + a
*b*(-(b + (b^2 - 4*a*c)^(1/2))/(2*c))^(1/2) + 2*a*c*(-(b + (b^2 - 4*a*c)^(1
/2))/(2*c))^(1/2) - 2*a*c*(-(b + (b^2 - 4*a*c)^(1/2))/(2*c))^(3/2) + b*c*(-
(b + (b^2 - 4*a*c)^(1/2))/(2*c))^(3/2)))/(2*c*(4*a*c - b^2)*((b + (b^2 - 4*
a*c)^(1/2))/(2*c) + 1)^(1/2))
```

3.382 $\int \frac{x^2\sqrt{1-x^2}}{a+bx^2+cx^4}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 263

$$\int \frac{x^2\sqrt{1-x^2}}{a+bx^2+cx^4}\,dx=-\frac{\arcsin(x)}{c}+\frac{\left(b+c-\frac{b^2-2ac+bc}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{b+2c-\sqrt{b^2-4ac}}\,x}{\sqrt{b-\sqrt{b^2-4ac}}\sqrt{1-x^2}}\right)}{c\sqrt{b-\sqrt{b^2-4ac}}\sqrt{b+2c-\sqrt{b^2-4ac}}}+\frac{\left(b+c+\frac{b^2-2ac+bc}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{b+2c+\sqrt{b^2-4ac}}\,x}{\sqrt{b+\sqrt{b^2-4ac}}\sqrt{1-x^2}}\right)}{c\sqrt{b+\sqrt{b^2-4ac}}\sqrt{b+2c+\sqrt{b^2-4ac}}}$$

```
[Out] -arcsin(x)/c+arctan(x*(b+2*c-(-4*a*c+b^2)^(1/2))^(1/2)/(-x^2+1)^(1/2)/(b-(-
4*a*c+b^2)^(1/2))^(1/2))*(b+c+(2*a*c-b^2-b*c)/(-4*a*c+b^2)^(1/2))/c/(b-(-4*
a*c+b^2)^(1/2))^(1/2)/(b+2*c-(-4*a*c+b^2)^(1/2))^(1/2)+arctan(x*(b+2*c+(-4*
a*c+b^2)^(1/2))^(1/2)/(-x^2+1)^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2))*(b+c+(-2
*a*c+b^2+b*c)/(-4*a*c+b^2)^(1/2))/c/(b+(-4*a*c+b^2)^(1/2))^(1/2)/(b+2*c+(-4
*a*c+b^2)^(1/2))^(1/2)
```

Rubi [A] (verified)

Time = 1.36 (sec) , antiderivative size = 263, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.172, Rules used = {1307, 222, 1706, 385, 211}

$$\int \frac{x^2\sqrt{1-x^2}}{a+bx^2+cx^4}\,dx=\frac{\left(-\frac{-2ac+b^2+bc}{\sqrt{b^2-4ac}}+b+c\right)\arctan\left(\frac{x\sqrt{-\sqrt{b^2-4ac}+b+2c}}{\sqrt{1-x^2}\sqrt{b-\sqrt{b^2-4ac}}}\right)}{c\sqrt{b-\sqrt{b^2-4ac}}\sqrt{-\sqrt{b^2-4ac}+b+2c}}+\frac{\left(\frac{-2ac+b^2+bc}{\sqrt{b^2-4ac}}+b+c\right)\arctan\left(\frac{x\sqrt{\sqrt{b^2-4ac}+b+2c}}{\sqrt{1-x^2}\sqrt{\sqrt{b^2-4ac}+b}}\right)}{c\sqrt{\sqrt{b^2-4ac}+b}\sqrt{\sqrt{b^2-4ac}+b+2c}}-\frac{\arcsin(x)}{c}$$

```
[In] Int[(x^2*Sqrt[1 - x^2])/(a + b*x^2 + c*x^4),x]
```

```
[Out] -(ArcSin[x]/c) + ((b + c - (b^2 - 2*a*c + b*c)/Sqrt[b^2 - 4*a*c])*ArcTan[(S
qrt[b + 2*c - Sqrt[b^2 - 4*a*c]]*x)/(Sqrt[b - Sqrt[b^2 - 4*a*c]]*Sqrt[1 - x
^2])])/(c*Sqrt[b - Sqrt[b^2 - 4*a*c]]*Sqrt[b + 2*c - Sqrt[b^2 - 4*a*c]]) +
((b + c + (b^2 - 2*a*c + b*c)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[b + 2*c + Sqr
t[b^2 - 4*a*c]]*x)/(Sqrt[b + Sqrt[b^2 - 4*a*c]]*Sqrt[1 - x^2])])/(c*Sqrt[b
+ Sqrt[b^2 - 4*a*c]]*Sqrt[b + 2*c + Sqrt[b^2 - 4*a*c]])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 1307

```
Int[(((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x_)^2)^(q_))/((a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4), x_Symbol] :> Dist[e*(f^2/c), Int[(f*x)^(m - 2)*(d + e*x^2)^
(q - 1), x], x] - Dist[f^2/c, Int[(f*x)^(m - 2)*(d + e*x^2)^(q - 1)*(Simp[a
*e - (c*d - b*e)*x^2, x]/(a + b*x^2 + c*x^4)), x], x] /; FreeQ[{a, b, c, d,
 e, f}, x] && NeQ[b^2 - 4*a*c, 0] &&  !IntegerQ[q] && GtQ[q, 0] && GtQ[m, 1
] && LeQ[m, 3]
```

Rule 1706

```
Int[(Px_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^
(p_.), x_Symbol] :> Int[ExpandIntegrand[Px*(d + e*x^2)^q*(a + b*x^2 + c*x^4
)^p, x], x] /; FreeQ[{a, b, c, d, e, q}, x] && PolyQ[Px, x^2] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[p]
```

Rubi steps

$$\text{integral} = -\frac{\int \frac{1}{\sqrt{1-x^2}}\,dx}{c} - \frac{\int \frac{-a-(b+c)x^2}{\sqrt{1-x^2}(a+bx^2+cx^4)}\,dx}{c}$$

$$= -\frac{\sin^{-1}(x)}{c} - \frac{\int \left(\frac{-b-c+\frac{b^2-2ac+bc}{\sqrt{b^2-4ac}}}{\sqrt{1-x^2}\left(b-\sqrt{b^2-4ac}+2cx^2\right)} + \frac{-b-c-\frac{b^2-2ac+bc}{\sqrt{b^2-4ac}}}{\sqrt{1-x^2}\left(b+\sqrt{b^2-4ac}+2cx^2\right)}\right)dx}{c}$$

$$= -\frac{\sin^{-1}(x)}{c} + \frac{\left(b+c-\frac{b^2-2ac+bc}{\sqrt{b^2-4ac}}\right)\int \frac{1}{\sqrt{1-x^2}\left(b-\sqrt{b^2-4ac}+2cx^2\right)}\,dx}{c} + \frac{\left(b+c+\frac{b^2-2ac+bc}{\sqrt{b^2-4ac}}\right)\int \frac{1}{\sqrt{1-x^2}\left(b+\sqrt{b^2-4ac}+2cx^2\right)}\,dx}{c}$$

$$= -\frac{\sin^{-1}(x)}{c} + \frac{\left(b+c-\frac{b^2-2ac+bc}{\sqrt{b^2-4ac}}\right)\text{Subst}\left(\int \frac{1}{b-\sqrt{b^2-4ac}-\left(-b-2c+\sqrt{b^2-4ac}\right)x^2}\,dx, x, \frac{x}{\sqrt{1-x^2}}\right)}{c} + \frac{\left(b+c+\frac{b^2-2ac+bc}{\sqrt{b^2-4ac}}\right)\text{Subst}\left(\int \frac{1}{b+\sqrt{b^2-4ac}-\left(-b-2c-\sqrt{b^2-4ac}\right)x^2}\,dx, x, \frac{x}{\sqrt{1-x^2}}\right)}{c}$$

$$= -\frac{\sin^{-1}(x)}{c} + \frac{\left(b+c-\frac{b^2-2ac+bc}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{b+2c-\sqrt{b^2-4ac}}\,x}{\sqrt{b-\sqrt{b^2-4ac}}\sqrt{1-x^2}}\right)}{c\sqrt{b-\sqrt{b^2-4ac}}\sqrt{b+2c-\sqrt{b^2-4ac}}} + \frac{\left(b+c+\frac{b^2-2ac+bc}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{b+2c+\sqrt{b^2-4ac}}\,x}{\sqrt{b+\sqrt{b^2-4ac}}\sqrt{1-x^2}}\right)}{c\sqrt{b+\sqrt{b^2-4ac}}\sqrt{b+2c+\sqrt{b^2-4ac}}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.41 (sec) , antiderivative size = 412, normalized size of antiderivative = 1.57

$$\int \frac{x^2\sqrt{1-x^2}}{a+bx^2+cx^4}\,dx =$$

$$-\frac{8\arctan\left(\frac{x}{-1+\sqrt{1-x^2}}\right) + \text{RootSum}\left[a+4a\#1^2+4b\#1^2+6a\#1^4+8b\#1^4+16c\#1^4+4a\#1^6+4b\#1^6\right.}{}$$

```
[In] Integrate[(x^2*Sqrt[1 - x^2])/(a + b*x^2 + c*x^4),x]
```

```
[Out] -1/4*(8*ArcTan[x/(-1 + Sqrt[1 - x^2])] + RootSum[a + 4*a*#1^2 + 4*b*#1^2 +
6*a*#1^4 + 8*b*#1^4 + 16*c*#1^4 + 4*a*#1^6 + 4*b*#1^6 + a*#1^8 & , (-(a*Log
[x]) + a*Log[-1 + Sqrt[1 - x^2] - x*#1] - 3*a*Log[x]*#1^2 - 4*b*Log[x]*#1^2
 - 4*c*Log[x]*#1^2 + 3*a*Log[-1 + Sqrt[1 - x^2] - x*#1]*#1^2 + 4*b*Log[-1 +
 Sqrt[1 - x^2] - x*#1]*#1^2 + 4*c*Log[-1 + Sqrt[1 - x^2] - x*#1]*#1^2 - 3*a
*Log[x]*#1^4 - 4*b*Log[x]*#1^4 - 4*c*Log[x]*#1^4 + 3*a*Log[-1 + Sqrt[1 - x^
2] - x*#1]*#1^4 + 4*b*Log[-1 + Sqrt[1 - x^2] - x*#1]*#1^4 + 4*c*Log[-1 + Sq
rt[1 - x^2] - x*#1]*#1^4 - a*Log[x]*#1^6 + a*Log[-1 + Sqrt[1 - x^2] - x*#1]
*#1^6)/(a*#1 + b*#1 + 3*a*#1^3 + 4*b*#1^3 + 8*c*#1^3 + 3*a*#1^5 + 3*b*#1^5
+ a*#1^7) & ])/c
```

Maple [A] (verified)

Time = 0.32 (sec) , antiderivative size = 262, normalized size of antiderivative = 1.00

method	result
default	$\frac{a\sqrt{\left(2a+b+\sqrt{-4ac+b^2}\right)a}\,\sqrt{2}\left(b+2c+\sqrt{-4ac+b^2}\right)\operatorname{arctanh}\left(\frac{a\sqrt{-x^2+1}\,\sqrt{2}}{x\sqrt{\left(-b+\sqrt{-4ac+b^2}-2a\right)a}}\right)+\sqrt{\left(-b+\sqrt{-4ac+b^2}-2a\right)a}\left(a\sqrt{2}\right(}{2\sqrt{\left(-b+\sqrt{-4ac+b^2}-2a\right)a}\,\sqrt{-4ac+b}}$
pseudoelliptic	$\frac{a\sqrt{\left(2a+b+\sqrt{-4ac+b^2}\right)a}\,\sqrt{2}\left(b+2c+\sqrt{-4ac+b^2}\right)\operatorname{arctanh}\left(\frac{a\sqrt{-x^2+1}\,\sqrt{2}}{x\sqrt{\left(-b+\sqrt{-4ac+b^2}-2a\right)a}}\right)+\sqrt{\left(-b+\sqrt{-4ac+b^2}-2a\right)a}\left(a\sqrt{2}\right(}{2\sqrt{\left(-b+\sqrt{-4ac+b^2}-2a\right)a}\,\sqrt{-4ac+b}}$

```
[In] int(x^2*(-x^2+1)^(1/2)/(c*x^4+b*x^2+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2/((-b+(-4*a*c+b^2)^(1/2)-2*a)*a)^(1/2)/(-4*a*c+b^2)^(1/2)/((2*a+b+(-4*a*
c+b^2)^(1/2))*a)^(1/2)*(a*((2*a+b+(-4*a*c+b^2)^(1/2))*a)^(1/2)*2^(1/2)*(b+2
*c+(-4*a*c+b^2)^(1/2))*arctanh(a/x*(-x^2+1)^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2)
^(1/2)-2*a)*a)^(1/2))+((-b+(-4*a*c+b^2)^(1/2)-2*a)*a)^(1/2)*(a*2^(1/2)*(b+2
*c-(-4*a*c+b^2)^(1/2))*arctan(a/x*(-x^2+1)^(1/2)*2^(1/2)/((2*a+b+(-4*a*c+b^
2)^(1/2))*a)^(1/2))+2*arctan(1/x*(-x^2+1)^(1/2))*(-4*a*c+b^2)^(1/2)*((2*a+b
+(-4*a*c+b^2)^(1/2))*a)^(1/2)))/c
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1430 vs. 2(223) = 446.

Time = 0.67 (sec) , antiderivative size = 1430, normalized size of antiderivative = 5.44

$$\int \frac{x^2\sqrt{1-x^2}}{a+bx^2+cx^4}\,dx = \text{Too large to display}$$

```
[In] integrate(x^2*(-x^2+1)^(1/2)/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] -1/2*(sqrt(1/2)*c*sqrt(-(b^2 - (2*a - b)*c + (b^2*c^2 - 4*a*c^3)*sqrt((b^2
+ 2*b*c + c^2)/(b^2*c^4 - 4*a*c^5)))/(b^2*c^2 - 4*a*c^3))*log(-(2*(a*b + a*
c)*x^2 - 2*a*b - 2*a*c + sqrt(1/2)*((b^3 - 4*a*c^2 - (4*a*b - b^2)*c)*sqrt(
-x^2 + 1)*x - (b^3 - 4*a*c^2 - (4*a*b - b^2)*c)*x - ((b^3*c^2 - 4*a*b*c^3)*
sqrt(-x^2 + 1)*x - (b^3*c^2 - 4*a*b*c^3)*x)*sqrt((b^2 + 2*b*c + c^2)/(b^2*c
^4 - 4*a*c^5)))*sqrt(-(b^2 - (2*a - b)*c + (b^2*c^2 - 4*a*c^3)*sqrt((b^2 +
2*b*c + c^2)/(b^2*c^4 - 4*a*c^5)))/(b^2*c^2 - 4*a*c^3)) + 2*(a*b + a*c)*sqr
t(-x^2 + 1))/x^2) - sqrt(1/2)*c*sqrt(-(b^2 - (2*a - b)*c + (b^2*c^2 - 4*a*c
^3)*sqrt((b^2 + 2*b*c + c^2)/(b^2*c^4 - 4*a*c^5)))/(b^2*c^2 - 4*a*c^3))*log
(-(2*(a*b + a*c)*x^2 - 2*a*b - 2*a*c - sqrt(1/2)*((b^3 - 4*a*c^2 - (4*a*b -
 b^2)*c)*sqrt(-x^2 + 1)*x - (b^3 - 4*a*c^2 - (4*a*b - b^2)*c)*x - ((b^3*c^2
```

```
 - 4*a*b*c^3)*sqrt(-x^2 + 1)*x - (b^3*c^2 - 4*a*b*c^3)*x)*sqrt((b^2 + 2*b*c
 + c^2)/(b^2*c^4 - 4*a*c^5)))*sqrt(-(b^2 - (2*a - b)*c + (b^2*c^2 - 4*a*c^3
)*sqrt((b^2 + 2*b*c + c^2)/(b^2*c^4 - 4*a*c^5)))/(b^2*c^2 - 4*a*c^3)) + 2*(
a*b + a*c)*sqrt(-x^2 + 1))/x^2) + sqrt(1/2)*c*sqrt(-(b^2 - (2*a - b)*c - (b
^2*c^2 - 4*a*c^3)*sqrt((b^2 + 2*b*c + c^2)/(b^2*c^4 - 4*a*c^5)))/(b^2*c^2 -
 4*a*c^3))*log(-(2*(a*b + a*c)*x^2 - 2*a*b - 2*a*c + sqrt(1/2)*((b^3 - 4*a*
c^2 - (4*a*b - b^2)*c)*sqrt(-x^2 + 1)*x - (b^3 - 4*a*c^2 - (4*a*b - b^2)*c)
*x + ((b^3*c^2 - 4*a*b*c^3)*sqrt(-x^2 + 1)*x - (b^3*c^2 - 4*a*b*c^3)*x)*sqr
t((b^2 + 2*b*c + c^2)/(b^2*c^4 - 4*a*c^5)))*sqrt(-(b^2 - (2*a - b)*c - (b^2
*c^2 - 4*a*c^3)*sqrt((b^2 + 2*b*c + c^2)/(b^2*c^4 - 4*a*c^5)))/(b^2*c^2 - 4
*a*c^3)) + 2*(a*b + a*c)*sqrt(-x^2 + 1))/x^2) - sqrt(1/2)*c*sqrt(-(b^2 - (2
*a - b)*c - (b^2*c^2 - 4*a*c^3)*sqrt((b^2 + 2*b*c + c^2)/(b^2*c^4 - 4*a*c^5
)))/(b^2*c^2 - 4*a*c^3))*log(-(2*(a*b + a*c)*x^2 - 2*a*b - 2*a*c - sqrt(1/2
)*((b^3 - 4*a*c^2 - (4*a*b - b^2)*c)*sqrt(-x^2 + 1)*x - (b^3 - 4*a*c^2 - (4
*a*b - b^2)*c)*x + ((b^3*c^2 - 4*a*b*c^3)*sqrt(-x^2 + 1)*x - (b^3*c^2 - 4*a
*b*c^3)*x)*sqrt((b^2 + 2*b*c + c^2)/(b^2*c^4 - 4*a*c^5)))*sqrt(-(b^2 - (2*a
 - b)*c - (b^2*c^2 - 4*a*c^3)*sqrt((b^2 + 2*b*c + c^2)/(b^2*c^4 - 4*a*c^5))
)/(b^2*c^2 - 4*a*c^3)) + 2*(a*b + a*c)*sqrt(-x^2 + 1))/x^2) - 4*arctan((sqr
t(-x^2 + 1) - 1)/x))/c
```

Sympy [F]

$$\int \frac{x^2\sqrt{1-x^2}}{a+bx^2+cx^4}\,dx = \int \frac{x^2\sqrt{-(x-1)(x+1)}}{a+bx^2+cx^4}\,dx$$

```
[In] integrate(x**2*(-x**2+1)**(1/2)/(c*x**4+b*x**2+a),x)
[Out] Integral(x**2*sqrt(-(x - 1)*(x + 1))/(a + b*x**2 + c*x**4), x)
```

Maxima [F]

$$\int \frac{x^2\sqrt{1-x^2}}{a+bx^2+cx^4}\,dx = \int \frac{\sqrt{-x^2+1}x^2}{cx^4+bx^2+a}\,dx$$

```
[In] integrate(x^2*(-x^2+1)^(1/2)/(c*x^4+b*x^2+a),x, algorithm="maxima")
[Out] integrate(sqrt(-x^2 + 1)*x^2/(c*x^4 + b*x^2 + a), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3580 vs. 2(223) = 446.

Time = 1.02 (sec) , antiderivative size = 3580, normalized size of antiderivative = 13.61

$$\int \frac{x^2\sqrt{1-x^2}}{a+bx^2+cx^4}\,dx = \text{Too large to display}$$

```
[In] integrate(x^2*(-x^2+1)^(1/2)/(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] -1/2*(pi*sgn(x) + 2*arctan(-1/2*x*((sqrt(-x^2 + 1) - 1)^2/x^2 - 1)/(sqrt(-x
^2 + 1) - 1)))/c + 1/8*((2*a^2*b^4 - 16*a^3*b^2*c + 32*a^4*c^2 + 3*sqrt(2)*
sqrt(2*a^2 + a*b + sqrt(b^2 - 4*a*c)*a)*sqrt(b^2 - 4*a*c)*a^2*b^2 + 2*sqrt(
2)*sqrt(2*a^2 + a*b + sqrt(b^2 - 4*a*c)*a)*sqrt(b^2 - 4*a*c)*a*b^3 - sqrt(2
)*sqrt(2*a^2 + a*b + sqrt(b^2 - 4*a*c)*a)*sqrt(b^2 - 4*a*c)*b^4 - 12*sqrt(2
)*sqrt(2*a^2 + a*b + sqrt(b^2 - 4*a*c)*a)*sqrt(b^2 - 4*a*c)*a^3*c - 8*sqrt(
2)*sqrt(2*a^2 + a*b + sqrt(b^2 - 4*a*c)*a)*sqrt(b^2 - 4*a*c)*a^2*b*c + 8*sq
rt(2)*sqrt(2*a^2 + a*b + sqrt(b^2 - 4*a*c)*a)*sqrt(b^2 - 4*a*c)*a*b^2*c - 1
6*sqrt(2)*sqrt(2*a^2 + a*b + sqrt(b^2 - 4*a*c)*a)*sqrt(b^2 - 4*a*c)*a^2*c^2
 - 2*(b^2 - 4*a*c)*a^2*b^2 + 8*(b^2 - 4*a*c)*a^3*c)*c^2*abs(a) - 2*(3*sqrt(
2)*sqrt(2*a^2 + a*b + sqrt(b^2 - 4*a*c)*a)*a^3*b^2*c + 5*sqrt(2)*sqrt(2*a^2
 + a*b + sqrt(b^2 - 4*a*c)*a)*a^2*b^3*c + sqrt(2)*sqrt(2*a^2 + a*b + sqrt(b
^2 - 4*a*c)*a)*a*b^4*c + 2*a^2*b^4*c - sqrt(2)*sqrt(2*a^2 + a*b + sqrt(b^2
- 4*a*c)*a)*b^5*c + 2*a*b^5*c - 12*sqrt(2)*sqrt(2*a^2 + a*b + sqrt(b^2 - 4*
a*c)*a)*a^4*c^2 - 20*sqrt(2)*sqrt(2*a^2 + a*b + sqrt(b^2 - 4*a*c)*a)*a^3*b*
c^2 + 3*sqrt(2)*sqrt(2*a^2 + a*b + sqrt(b^2 - 4*a*c)*a)*a^2*b^2*c^2 - 16*a^
3*b^2*c^2 + 10*sqrt(2)*sqrt(2*a^2 + a*b + sqrt(b^2 - 4*a*c)*a)*a*b^3*c^2 -
16*a^2*b^3*c^2 - sqrt(2)*sqrt(2*a^2 + a*b + sqrt(b^2 - 4*a*c)*a)*b^4*c^2 +
2*a*b^4*c^2 - 28*sqrt(2)*sqrt(2*a^2 + a*b + sqrt(b^2 - 4*a*c)*a)*a^3*c^3 +
32*a^4*c^3 - 24*sqrt(2)*sqrt(2*a^2 + a*b + sqrt(b^2 - 4*a*c)*a)*a^2*b*c^3 +
 32*a^3*b*c^3 + 8*sqrt(2)*sqrt(2*a^2 + a*b + sqrt(b^2 - 4*a*c)*a)*a*b^2*c^3
 - 16*a^2*b^2*c^3 - 16*sqrt(2)*sqrt(2*a^2 + a*b + sqrt(b^2 - 4*a*c)*a)*a^2*
c^4 + 32*a^3*c^4 - 2*(b^2 - 4*a*c)*a^2*b^2*c - 2*(b^2 - 4*a*c)*a*b^3*c + 8*
(b^2 - 4*a*c)*a^3*c^2 + 8*(b^2 - 4*a*c)*a^2*b*c^2 - 2*(b^2 - 4*a*c)*a*b^2*c
^2 + 8*(b^2 - 4*a*c)*a^2*c^3)*abs(a)*abs(c) + (4*a^3*b^3*c^2 + 2*a^2*b^4*c^
2 - 16*a^4*b*c^3 + 4*a^2*b^3*c^3 - 32*a^4*c^4 - 16*a^3*b*c^4 + 6*sqrt(2)*sq
rt(2*a^2 + a*b + sqrt(b^2 - 4*a*c)*a)*sqrt(b^2 - 4*a*c)*a^3*b*c^2 + 7*sqrt(
2)*sqrt(2*a^2 + a*b + sqrt(b^2 - 4*a*c)*a)*sqrt(b^2 - 4*a*c)*a^2*b^2*c^2 -
sqrt(2)*sqrt(2*a^2 + a*b + sqrt(b^2 - 4*a*c)*a)*sqrt(b^2 - 4*a*c)*b^4*c^2 +
 12*sqrt(2)*sqrt(2*a^2 + a*b + sqrt(b^2 - 4*a*c)*a)*sqrt(b^2 - 4*a*c)*a^3*c
^3 + 22*sqrt(2)*sqrt(2*a^2 + a*b + sqrt(b^2 - 4*a*c)*a)*sqrt(b^2 - 4*a*c)*a
^2*b*c^3 + 4*sqrt(2)*sqrt(2*a^2 + a*b + sqrt(b^2 - 4*a*c)*a)*sqrt(b^2 - 4*a
*c)*a*b^2*c^3 - 2*sqrt(2)*sqrt(2*a^2 + a*b + sqrt(b^2 - 4*a*c)*a)*sqrt(b^2
- 4*a*c)*b^3*c^3 + 16*sqrt(2)*sqrt(2*a^2 + a*b + sqrt(b^2 - 4*a*c)*a)*sqrt(
b^2 - 4*a*c)*a^2*c^4 + 8*sqrt(2)*sqrt(2*a^2 + a*b + sqrt(b^2 - 4*a*c)*a)*sq
```

rt(b^2 - 4*a*c)*a*b*c^4 - 4*(b^2 - 4*a*c)*a^3*b*c^2 - 2*(b^2 - 4*a*c)*a^2*b ^2*c^2 - 8*(b^2 - 4*a*c)*a^3*c^3 - 4*(b^2 - 4*a*c)*a^2*b*c^3)*abs(a))*arcta n(-1/2*sqrt(2)*(x/(sqrt(-x^2 + 1) - 1) - (sqrt(-x^2 + 1) - 1)/x)/sqrt((2*a* c + b*c + sqrt(-4*(a*c + b*c + c^2)*a*c + (2*a*c + b*c)^2))/(a*c)))/((3*a^5 *b^2*c^2 + 5*a^4*b^3*c^2 + a^3*b^4*c^2 - a^2*b^5*c^2 - 12*a^6*c^3 - 20*a^5* b*c^3 + 3*a^4*b^2*c^3 + 10*a^3*b^3*c^3 - a^2*b^4*c^3 - 28*a^5*c^4 - 24*a^4* b*c^4 + 8*a^3*b^2*c^4 - 16*a^4*c^5)*abs(c)) - 1/8*((2*a^2*b^4 - 16*a^3*b^2* c + 32*a^4*c^2 + 3*sqrt(2)*sqrt(2*a^2 + a*b - sqrt(b^2 - 4*a*c)*a)*sqrt(b^2 - 4*a*c)*a^2*b^2 + 2*sqrt(2)*sqrt(2*a^2 + a*b - sqrt(b^2 - 4*a*c)*a)*sqrt(b^2 - 4*a*c)*a*b^3 - sqrt(2)*sqrt(2*a^2 + a*b - sqrt(b^2 - 4*a*c)*a)*sqrt(b ^2 - 4*a*c)*b^4 - 12*sqrt(2)*sqrt(2*a^2 + a*b - sqrt(b^2 - 4*a*c)*a)*sqrt(b ^2 - 4*a*c)*a^3*c - 8*sqrt(2)*sqrt(2*a^2 + a*b - sqrt(b^2 - 4*a*c)*a)*sqrt(b^2 - 4*a*c)*a^2*b*c + 8*sqrt(2)*sqrt(2*a^2 + a*b - sqrt(b^2 - 4*a*c)*a)*sq rt(b^2 - 4*a*c)*a*b^2*c - 16*sqrt(2)*sqrt(2*a^2 + a*b - sqrt(b^2 - 4*a*c)*a)*sqrt(b^2 - 4*a*c)*a^2*c^2 - 2*(b^2 - 4*a*c)*a^2*b^2 + 8*(b^2 - 4*a*c)*a^3 *c)*c^2*abs(a) + 2*(3*sqrt(2)*sqrt(2*a^2 + a*b - sqrt(b^2 - 4*a*c)*a)*a^3*b ^2*c + 5*sqrt(2)*sqrt(2*a^2 + a*b - sqrt(b^2 - 4*a*c)*a)*a^2*b^3*c + sqrt(2)*sqrt(2*a^2 + a*b - sqrt(b^2 - 4*a*c)*a)*a*b^4*c - 2*a^2*b^4*c - sqrt(2)*s qrt(2*a^2 + a*b - sqrt(b^2 - 4*a*c)*a)*b^5*c - 2*a*b^5*c - 12*sqrt(2)*sqrt(2*a^2 + a*b - sqrt(b^2 - 4*a*c)*a)*a^4*c^2 - 20*sqrt(2)*sqrt(2*a^2 + a*b - sqrt(b^2 - 4*a*c)*a)*a^3*b*c^2 + 3*sqrt(2)*sqrt(2*a^2 + a*b - sqrt(b^2 - 4* a*c)*a)*a^2*b^2*c^2 + 16*a^3*b^2*c^2 + 10*sqrt(2)*sqrt(2*a^2 + a*b - sqrt(b ^2 - 4*a*c)*a)*a*b^3*c^2 + 16*a^2*b^3*c^2 - sqrt(2)*sqrt(2*a^2 + a*b - sqrt (b^2 - 4*a*c)*a)*b^4*c^2 - 2*a*b^4*c^2 - 28*sqrt(2)*sqrt(2*a^2 + a*b - sqrt (b^2 - 4*a*c)*a)*a^3*c^3 - 32*a^4*c^3 - 24*sqrt(2)*sqrt(2*a^2 + a*b - sqrt(b^2 - 4*a*c)*a)*a^2*b*c^3 - 32*a^3*b*c^3 + 8*sqrt(2)*sqrt(2*a^2 + a*b - sqr t(b^2 - 4*a*c)*a)*a*b^2*c^3 + 16*a^2*b^2*c^3 - 16*sqrt(2)*sqrt(2*a^2 + a*b - sqrt(b^2 - 4*a*c)*a)*a^2*c^4 - 32*a^3*c^4 + 2*(b^2 - 4*a*c)*a^2*b^2*c + 2 *(b^2 - 4*a*c)*a*b^3*c - 8*(b^2 - 4*a*c)*a^3*c^2 - 8*(b^2 - 4*a*c)*a^2*b*c^ 2 + 2*(b^2 - 4*a*c)*a*b^2*c^2 - 8*(b^2 - 4*a*c)*a^2*c^3)*abs(a)*abs(c) + (4 *a^3*b^3*c^2 + 2*a^2*b^4*c^2 - 16*a^4*b*c^3 + 4*a^2*b^3*c^3 - 32*a^4*c^4 - 16*a^3*b*c^4 + 6*sqrt(2)*sqrt(2*a^2 + a*b - sqrt(b^2 - 4*a*c)*a)*sqrt(b^2 - 4*a*c)*a^3*b*c^2 + 7*sqrt(2)*sqrt(2*a^2 + a*b - sqrt(b^2 - 4*a*c)*a)*sqrt(b^2 - 4*a*c)*a^2*b^2*c^2 - sqrt(2)*sqrt(2*a^2 + a*b - sqrt(b^2 - 4*a*c)*a)* sqrt(b^2 - 4*a*c)*b^4*c^2 + 12*sqrt(2)*sqrt(2*a^2 + a*b - sqrt(b^2 - 4*a*c) *a)*sqrt(b^2 - 4*a*c)*a^3*c^3 + 22*sqrt(2)*sqrt(2*a^2 + a*b - sqrt(b^2 - 4* a*c)*a)*sqrt(b^2 - 4*a*c)*a^2*b*c^3 + 4*sqrt(2)*sqrt(2*a^2 + a*b - sqrt(b^2 - 4*a*c)*a)*sqrt(b^2 - 4*a*c)*a*b^2*c^3 - 2*sqrt(2)*sqrt(2*a^2 + a*b - sqr t(b^2 - 4*a*c)*a)*sqrt(b^2 - 4*a*c)*b^3*c^3 + 16*sqrt(2)*sqrt(2*a^2 + a*b - sqrt(b^2 - 4*a*c)*a)*sqrt(b^2 - 4*a*c)*a^2*c^4 + 8*sqrt(2)*sqrt(2*a^2 + a* b - sqrt(b^2 - 4*a*c)*a)*sqrt(b^2 - 4*a*c)*a*b*c^4 - 4*(b^2 - 4*a*c)*a^3*b* c^2 - 2*(b^2 - 4*a*c)*a^2*b^2*c^2 - 8*(b^2 - 4*a*c)*a^3*c^3 - 4*(b^2 - 4*a* c)*a^2*b*c^3)*abs(a))*arctan(-1/2*sqrt(2)*(x/(sqrt(-x^2 + 1) - 1) - (sqrt(- x^2 + 1) - 1)/x)/sqrt((2*a*c + b*c - sqrt(-4*(a*c + b*c + c^2)*a*c + (2*a*c + b*c)^2))/(a*c)))/((3*a^5*b^2*c^2 + 5*a^4*b^3*c^2 + a^3*b^4*c^2 - a^2*b^5

```
*c^2 - 12*a^6*c^3 - 20*a^5*b*c^3 + 3*a^4*b^2*c^3 + 10*a^3*b^3*c^3 - a^2*b^4
*c^3 - 28*a^5*c^4 - 24*a^4*b*c^4 + 8*a^3*b^2*c^4 - 16*a^4*c^5)*abs(c))
```

Mupad [B] (verification not implemented)

Time = 8.34 (sec) , antiderivative size = 870, normalized size of antiderivative = 3.31

$$\int \frac{x^2\sqrt{1-x^2}}{a+bx^2+cx^4}\,dx = -\frac{\operatorname{asin}(x)}{c}$$

$$-\frac{\ln\left(\frac{\frac{\left(x\sqrt{-\frac{b-\sqrt{b^2-4ac}}{2c}}+1\right)1i}{\sqrt{\frac{b-\sqrt{b^2-4ac}}{2c}+1}}+\sqrt{1-x^2}\,1i}{x+\sqrt{-\frac{b-\sqrt{b^2-4ac}}{2c}}}\right)\left(2a\sqrt{-\frac{b-\sqrt{b^2-4ac}}{2c}}+b\sqrt{-\frac{b-\sqrt{b^2-4ac}}{2c}}+b\left(-\frac{b-\sqrt{b^2-4ac}}{2c}\right)^{3/2}+2c\left(-\right.\right.}{\sqrt{\frac{b-\sqrt{b^2-4ac}}{2c}+1}\,(8ac-2b^2)}$$

$$-\frac{\ln\left(\frac{\frac{\left(x\sqrt{-\frac{b+\sqrt{b^2-4ac}}{2c}}+1\right)1i}{\sqrt{\frac{b+\sqrt{b^2-4ac}}{2c}+1}}+\sqrt{1-x^2}\,1i}{x+\sqrt{-\frac{b+\sqrt{b^2-4ac}}{2c}}}\right)\left(2a\sqrt{-\frac{b+\sqrt{b^2-4ac}}{2c}}+b\sqrt{-\frac{b+\sqrt{b^2-4ac}}{2c}}+b\left(-\frac{b+\sqrt{b^2-4ac}}{2c}\right)^{3/2}+2c\left(-\right.\right.}{(8ac-2b^2)\,\sqrt{\frac{b+\sqrt{b^2-4ac}}{2c}+1}}$$

$$+\frac{\ln\left(\frac{\frac{\left(x\sqrt{-\frac{b-\sqrt{b^2-4ac}}{2c}}-1\right)1i}{\sqrt{\frac{b-\sqrt{b^2-4ac}}{2c}+1}}-\sqrt{1-x^2}\,1i}{x-\sqrt{-\frac{b-\sqrt{b^2-4ac}}{2c}}}\right)\left(2a\sqrt{-\frac{b-\sqrt{b^2-4ac}}{2c}}+b\sqrt{-\frac{b-\sqrt{b^2-4ac}}{2c}}+b\left(-\frac{b-\sqrt{b^2-4ac}}{2c}\right)^{3/2}+2c\left(-\right.\right.}{\sqrt{\frac{b-\sqrt{b^2-4ac}}{2c}+1}\,(8ac-2b^2)}$$

$$+\frac{\ln\left(\frac{\frac{\left(x\sqrt{-\frac{b+\sqrt{b^2-4ac}}{2c}}-1\right)1i}{\sqrt{\frac{b+\sqrt{b^2-4ac}}{2c}+1}}-\sqrt{1-x^2}\,1i}{x-\sqrt{-\frac{b+\sqrt{b^2-4ac}}{2c}}}\right)\left(2a\sqrt{-\frac{b+\sqrt{b^2-4ac}}{2c}}+b\sqrt{-\frac{b+\sqrt{b^2-4ac}}{2c}}+b\left(-\frac{b+\sqrt{b^2-4ac}}{2c}\right)^{3/2}+2c\left(-\right.\right.}{(8ac-2b^2)\,\sqrt{\frac{b+\sqrt{b^2-4ac}}{2c}+1}}$$

```
[In] int((x^2*(1 - x^2)^(1/2))/(a + b*x^2 + c*x^4),x)
```

```
[Out] (log((((x*(-(b - (b^2 - 4*a*c)^(1/2))/(2*c))^(1/2) - 1)*1i)/((b - (b^2 - 4*
a*c)^(1/2))/(2*c) + 1)^(1/2) - (1 - x^2)^(1/2)*1i)/(x - (-(b - (b^2 - 4*a*c
)^(1/2))/(2*c))^(1/2)))*(2*a*(-(b - (b^2 - 4*a*c)^(1/2))/(2*c))^(1/2) + b*(
-(b - (b^2 - 4*a*c)^(1/2))/(2*c))^(1/2) + b*(-(b - (b^2 - 4*a*c)^(1/2))/(2*
c))^(3/2) + 2*c*(-(b - (b^2 - 4*a*c)^(1/2))/(2*c))^(3/2)))/(((b - (b^2 - 4*
```

```
a*c)^(1/2))/(2*c) + 1)^(1/2)*(8*a*c - 2*b^2)) - (log((((x*(-(b - (b^2 - 4*a
*c)^(1/2))/(2*c))^(1/2) + 1)*1i)/((b - (b^2 - 4*a*c)^(1/2))/(2*c) + 1)^(1/2
) + (1 - x^2)^(1/2)*1i)/(x + (-(b - (b^2 - 4*a*c)^(1/2))/(2*c))^(1/2)))*(2*
a*(-(b - (b^2 - 4*a*c)^(1/2))/(2*c))^(1/2) + b*(-(b - (b^2 - 4*a*c)^(1/2))/
(2*c))^(1/2) + b*(-(b - (b^2 - 4*a*c)^(1/2))/(2*c))^(3/2) + 2*c*(-(b - (b^2
 - 4*a*c)^(1/2))/(2*c))^(3/2)))/(((b - (b^2 - 4*a*c)^(1/2))/(2*c) + 1)^(1/2
)*(8*a*c - 2*b^2)) - (log((((x*(-(b + (b^2 - 4*a*c)^(1/2))/(2*c))^(1/2) + 1
)*1i)/((b + (b^2 - 4*a*c)^(1/2))/(2*c) + 1)^(1/2) + (1 - x^2)^(1/2)*1i)/(x
+ (-(b + (b^2 - 4*a*c)^(1/2))/(2*c))^(1/2)))*(2*a*(-(b + (b^2 - 4*a*c)^(1/2
))/(2*c))^(1/2) + b*(-(b + (b^2 - 4*a*c)^(1/2))/(2*c))^(1/2) + b*(-(b + (b^
2 - 4*a*c)^(1/2))/(2*c))^(3/2) + 2*c*(-(b + (b^2 - 4*a*c)^(1/2))/(2*c))^(3/
2)))/((8*a*c - 2*b^2)*((b + (b^2 - 4*a*c)^(1/2))/(2*c) + 1)^(1/2)) - asin(x
)/c + (log((((x*(-(b + (b^2 - 4*a*c)^(1/2))/(2*c))^(1/2) - 1)*1i)/((b + (b^
2 - 4*a*c)^(1/2))/(2*c) + 1)^(1/2) - (1 - x^2)^(1/2)*1i)/(x - (-(b + (b^2 -
 4*a*c)^(1/2))/(2*c))^(1/2)))*(2*a*(-(b + (b^2 - 4*a*c)^(1/2))/(2*c))^(1/2)
 + b*(-(b + (b^2 - 4*a*c)^(1/2))/(2*c))^(1/2) + b*(-(b + (b^2 - 4*a*c)^(1/2
))/(2*c))^(3/2) + 2*c*(-(b + (b^2 - 4*a*c)^(1/2))/(2*c))^(3/2)))/((8*a*c -
2*b^2)*((b + (b^2 - 4*a*c)^(1/2))/(2*c) + 1)^(1/2))
```

3.383 $\int \frac{\sqrt{1-x^2}}{a+bx^2+cx^4}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 220

$$\int \frac{\sqrt{1-x^2}}{a+bx^2+cx^4}\,dx = \frac{\sqrt{b+2c-\sqrt{b^2-4ac}}\arctan\left(\frac{\sqrt{b+2c-\sqrt{b^2-4ac}}x}{\sqrt{b-\sqrt{b^2-4ac}}\sqrt{1-x^2}}\right)}{\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}} - \frac{\sqrt{b+2c+\sqrt{b^2-4ac}}\arctan\left(\frac{\sqrt{b+2c+\sqrt{b^2-4ac}}x}{\sqrt{b+\sqrt{b^2-4ac}}\sqrt{1-x^2}}\right)}{\sqrt{b^2-4ac}\sqrt{b+\sqrt{b^2-4ac}}}$$

```
[Out] arctan(x*(b+2*c-(-4*a*c+b^2)^(1/2))^(1/2)/(-x^2+1)^(1/2)/(b-(-4*a*c+b^2)^(1
/2))^(1/2))*(b+2*c-(-4*a*c+b^2)^(1/2))^(1/2)/(-4*a*c+b^2)^(1/2)/(b-(-4*a*c+
b^2)^(1/2))^(1/2)-arctan(x*(b+2*c+(-4*a*c+b^2)^(1/2))^(1/2)/(-x^2+1)^(1/2)/
(b+(-4*a*c+b^2)^(1/2))^(1/2))*(b+2*c+(-4*a*c+b^2)^(1/2))^(1/2)/(-4*a*c+b^2)
^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2)
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 220, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {1188, 399, 222, 385, 211}

$$\int \frac{\sqrt{1-x^2}}{a+bx^2+cx^4}\,dx = \frac{\sqrt{-\sqrt{b^2-4ac}+b+2c}\arctan\left(\frac{x\sqrt{-\sqrt{b^2-4ac}+b+2c}}{\sqrt{1-x^2}\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}} - \frac{\sqrt{\sqrt{b^2-4ac}+b+2c}\arctan\left(\frac{x\sqrt{\sqrt{b^2-4ac}+b+2c}}{\sqrt{1-x^2}\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{b^2-4ac}\sqrt{\sqrt{b^2-4ac}+b}}$$

```
[In] Int[Sqrt[1 - x^2]/(a + b*x^2 + c*x^4),x]
```

```
[Out] (Sqrt[b + 2*c - Sqrt[b^2 - 4*a*c]]*ArcTan[(Sqrt[b + 2*c - Sqrt[b^2 - 4*a*c]
]*x)/(Sqrt[b - Sqrt[b^2 - 4*a*c]]*Sqrt[1 - x^2])])/(Sqrt[b^2 - 4*a*c]*Sqrt[
b - Sqrt[b^2 - 4*a*c]]) - (Sqrt[b + 2*c + Sqrt[b^2 - 4*a*c]]*ArcTan[(Sqrt[b
 + 2*c + Sqrt[b^2 - 4*a*c]]*x)/(Sqrt[b + Sqrt[b^2 - 4*a*c]]*Sqrt[1 - x^2])]
)/(Sqrt[b^2 - 4*a*c]*Sqrt[b + Sqrt[b^2 - 4*a*c]])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 399

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Di
st[b/d, Int[(a + b*x^n)^(p - 1), x], x] - Dist[(b*c - a*d)/d, Int[(a + b*x^
n)^(p - 1)/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, p}, x] && NeQ[b*c - a*
d, 0] && EqQ[n*(p - 1) + 1, 0] && IntegerQ[n]
```

Rule 1188

```
Int[((d_) + (e_.)*(x_)^2)^(q_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symb
ol] :> With[{r = Rt[b^2 - 4*a*c, 2]}, Dist[2*(c/r), Int[(d + e*x^2)^q/(b -
r + 2*c*x^2), x], x] - Dist[2*(c/r), Int[(d + e*x^2)^q/(b + r + 2*c*x^2), x
], x]] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
- b*d*e + a*e^2, 0] &&  !IntegerQ[q]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{(2c)\int \frac{\sqrt{1-x^2}}{b-\sqrt{b^2-4ac}+2cx^2}\,dx}{\sqrt{b^2-4ac}} - \frac{(2c)\int \frac{\sqrt{1-x^2}}{b+\sqrt{b^2-4ac}+2cx^2}\,dx}{\sqrt{b^2-4ac}} \\
&= \frac{\left(b+2c-\sqrt{b^2-4ac}\right)\int \frac{1}{\sqrt{1-x^2}\left(b-\sqrt{b^2-4ac}+2cx^2\right)}\,dx}{\sqrt{b^2-4ac}} \\
&\quad - \frac{\left(b+2c+\sqrt{b^2-4ac}\right)\int \frac{1}{\sqrt{1-x^2}\left(b+\sqrt{b^2-4ac}+2cx^2\right)}\,dx}{\sqrt{b^2-4ac}}
\end{aligned}$$

$$= \frac{\left(b+2c-\sqrt{b^2-4ac}\right)\text{Subst}\left(\int \frac{1}{b-\sqrt{b^2-4ac}-\left(-b-2c+\sqrt{b^2-4ac}\right)x^2}\,dx,x,\frac{x}{\sqrt{1-x^2}}\right)}{\sqrt{b^2-4ac}} - \frac{\left(b+2c+\sqrt{b^2-4ac}\right)\text{Subst}\left(\int \frac{1}{b+\sqrt{b^2-4ac}-\left(-b-2c-\sqrt{b^2-4ac}\right)x^2}\,dx,x,\frac{x}{\sqrt{1-x^2}}\right)}{\sqrt{b^2-4ac}}$$

$$= \frac{\sqrt{b+2c-\sqrt{b^2-4ac}}\tan^{-1}\left(\frac{\sqrt{b+2c-\sqrt{b^2-4ac}}\,x}{\sqrt{b-\sqrt{b^2-4ac}}\sqrt{1-x^2}}\right)}{\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}} - \frac{\sqrt{b+2c+\sqrt{b^2-4ac}}\tan^{-1}\left(\frac{\sqrt{b+2c+\sqrt{b^2-4ac}}\,x}{\sqrt{b+\sqrt{b^2-4ac}}\sqrt{1-x^2}}\right)}{\sqrt{b^2-4ac}\sqrt{b+\sqrt{b^2-4ac}}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.30 (sec) , antiderivative size = 237, normalized size of antiderivative = 1.08

$$\int \frac{\sqrt{1-x^2}}{a+bx^2+cx^4}\,dx$$
$$= -\frac{1}{4}\text{RootSum}\Big[a+4a\#1^2+4b\#1^2+6a\#1^4+8b\#1^4+16c\#1^4+4a\#1^6+4b\#1^6$$
$$+a\#1^8\&,\frac{-\log(x)+\log\left(-1+\sqrt{1-x^2}-x\#1\right)+\log(x)\#1^2-\log\left(-1+\sqrt{1-x^2}-x\#1\right)\#1^2+\log(x)}{a\#1+b\#1+3a\#1^3+4b\#1^3+8c\#}$$

```
[In] Integrate[Sqrt[1 - x^2]/(a + b*x^2 + c*x^4),x]
```

```
[Out] -1/4*RootSum[a + 4*a*#1^2 + 4*b*#1^2 + 6*a*#1^4 + 8*b*#1^4 + 16*c*#1^4 + 4*
a*#1^6 + 4*b*#1^6 + a*#1^8 & , (-Log[x] + Log[-1 + Sqrt[1 - x^2] - x*#1] +
Log[x]*#1^2 - Log[-1 + Sqrt[1 - x^2] - x*#1]*#1^2 + Log[x]*#1^4 - Log[-1 +
Sqrt[1 - x^2] - x*#1]*#1^4 - Log[x]*#1^6 + Log[-1 + Sqrt[1 - x^2] - x*#1]*#
1^6)/(a*#1 + b*#1 + 3*a*#1^3 + 4*b*#1^3 + 8*c*#1^3 + 3*a*#1^5 + 3*b*#1^5 +
a*#1^7) & ]
```

Maple [A] (verified)

Time = 0.39 (sec) , antiderivative size = 168, normalized size of antiderivative = 0.76

method	result
default	$-\frac{\sqrt{2}\left(-\frac{\left(-b+\sqrt{-4ac+b^{2}}-2a\right)\operatorname{arctanh}\left(\frac{a\sqrt{-x^{2}+1}\,\sqrt{2}}{x\sqrt{\left(-b+\sqrt{-4ac+b^{2}}-2a\right)a}}\right)}{\sqrt{\left(-b+\sqrt{-4ac+b^{2}}-2a\right)a}}+\frac{\left(2a+b+\sqrt{-4ac+b^{2}}\right)\arctan\left(\frac{a\sqrt{-x^{2}+1}\,\sqrt{2}}{x\sqrt{\left(2a+b+\sqrt{-4ac+b^{2}}\right)a}}\right)}{\sqrt{\left(2a+b+\sqrt{-4ac+b^{2}}\right)a}}\right)}{2\sqrt{-4ac+b^{2}}}$
pseudoelliptic	$-\frac{\sqrt{2}\left(-\frac{\left(-b+\sqrt{-4ac+b^{2}}-2a\right)\operatorname{arctanh}\left(\frac{a\sqrt{-x^{2}+1}\,\sqrt{2}}{x\sqrt{\left(-b+\sqrt{-4ac+b^{2}}-2a\right)a}}\right)}{\sqrt{\left(-b+\sqrt{-4ac+b^{2}}-2a\right)a}}+\frac{\left(2a+b+\sqrt{-4ac+b^{2}}\right)\arctan\left(\frac{a\sqrt{-x^{2}+1}\,\sqrt{2}}{x\sqrt{\left(2a+b+\sqrt{-4ac+b^{2}}\right)a}}\right)}{\sqrt{\left(2a+b+\sqrt{-4ac+b^{2}}\right)a}}\right)}{2\sqrt{-4ac+b^{2}}}$

```
[In] int((-x^2+1)^(1/2)/(c*x^4+b*x^2+a),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*2^(1/2)/(-4*a*c+b^2)^(1/2)*(-(-b+(-4*a*c+b^2)^(1/2)-2*a)/((-b+(-4*a*c+
b^2)^(1/2)-2*a)*a)^(1/2)*arctanh(a/x*(-x^2+1)^(1/2)*2^(1/2)/((-b+(-4*a*c+b^
2)^(1/2)-2*a)*a)^(1/2))+(2*a+b+(-4*a*c+b^2)^(1/2))/((2*a+b+(-4*a*c+b^2)^(1/
2))*a)^(1/2)*arctan(a/x*(-x^2+1)^(1/2)*2^(1/2)/((2*a+b+(-4*a*c+b^2)^(1/2))*
a)^(1/2)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 759 vs. $2(180) = 360$.

Time = 0.35 (sec) , antiderivative size = 759, normalized size of antiderivative = 3.45

$$\int \frac{\sqrt{1-x^2}}{a+bx^2+cx^4}\,dx$$

$$= \frac{1}{2}\sqrt{\frac{1}{2}}\sqrt{-\frac{2\,a+b+\frac{ab^2-4\,a^2c}{\sqrt{a^2b^2-4\,a^3c}}}{ab^2-4\,a^2c}}\log\left(-\frac{x^2+\frac{\sqrt{\frac{1}{2}}\left((ab^2-4\,a^2c)\sqrt{-x^2+1}x-(ab^2-4\,a^2c)x\right)\sqrt{-\frac{2\,a+b+\frac{ab^2-4\,a^2c}{\sqrt{a^2b^2-4\,a^3c}}}{ab^2-4\,a^2c}}}{\sqrt{a^2b^2-4\,a^3c}}+\sqrt{-x^2+\right.\right.$$

$$-\frac{1}{2}\sqrt{\frac{1}{2}}\sqrt{-\frac{2\,a+b+\frac{ab^2-4\,a^2c}{\sqrt{a^2b^2-4\,a^3c}}}{ab^2-4\,a^2c}}\log\left(-\frac{x^2-\frac{\sqrt{\frac{1}{2}}\left((ab^2-4\,a^2c)\sqrt{-x^2+1}x-(ab^2-4\,a^2c)x\right)\sqrt{-\frac{2\,a+b+\frac{ab^2-4\,a^2c}{\sqrt{a^2b^2-4\,a^3c}}}{ab^2-4\,a^2c}}}{\sqrt{a^2b^2-4\,a^3c}}+\sqrt{-x^2\right.\right.$$

$$-\frac{1}{2}\sqrt{\frac{1}{2}}\sqrt{-\frac{2\,a+b-\frac{ab^2-4\,a^2c}{\sqrt{a^2b^2-4\,a^3c}}}{ab^2-4\,a^2c}}\log\left(-\frac{x^2+\frac{\sqrt{\frac{1}{2}}\left((ab^2-4\,a^2c)\sqrt{-x^2+1}x-(ab^2-4\,a^2c)x\right)\sqrt{-\frac{2\,a+b-\frac{ab^2-4\,a^2c}{\sqrt{a^2b^2-4\,a^3c}}}{ab^2-4\,a^2c}}}{\sqrt{a^2b^2-4\,a^3c}}+\sqrt{-x^2\right.\right.$$

$$+\frac{1}{2}\sqrt{\frac{1}{2}}\sqrt{-\frac{2\,a+b-\frac{ab^2-4\,a^2c}{\sqrt{a^2b^2-4\,a^3c}}}{ab^2-4\,a^2c}}\log\left(-\frac{x^2-\frac{\sqrt{\frac{1}{2}}\left((ab^2-4\,a^2c)\sqrt{-x^2+1}x-(ab^2-4\,a^2c)x\right)\sqrt{-\frac{2\,a+b-\frac{ab^2-4\,a^2c}{\sqrt{a^2b^2-4\,a^3c}}}{ab^2-4\,a^2c}}}{\sqrt{a^2b^2-4\,a^3c}}+\sqrt{-x^2\right.\right.$$

```
[In] integrate((-x^2+1)^(1/2)/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] 1/2*sqrt(1/2)*sqrt(-(2*a + b + (a*b^2 - 4*a^2*c)/sqrt(a^2*b^2 - 4*a^3*c))/(
a*b^2 - 4*a^2*c))*log(-(x^2 + sqrt(1/2)*((a*b^2 - 4*a^2*c)*sqrt(-x^2 + 1)*x
 - (a*b^2 - 4*a^2*c)*x)*sqrt(-(2*a + b + (a*b^2 - 4*a^2*c)/sqrt(a^2*b^2 - 4
*a^3*c))/(a*b^2 - 4*a^2*c))/sqrt(a^2*b^2 - 4*a^3*c) + sqrt(-x^2 + 1) - 1)/x
^2) - 1/2*sqrt(1/2)*sqrt(-(2*a + b + (a*b^2 - 4*a^2*c)/sqrt(a^2*b^2 - 4*a^3
*c))/(a*b^2 - 4*a^2*c))*log(-(x^2 - sqrt(1/2)*((a*b^2 - 4*a^2*c)*sqrt(-x^2
+ 1)*x - (a*b^2 - 4*a^2*c)*x)*sqrt(-(2*a + b + (a*b^2 - 4*a^2*c)/sqrt(a^2*b
^2 - 4*a^3*c))/(a*b^2 - 4*a^2*c))/sqrt(a^2*b^2 - 4*a^3*c) + sqrt(-x^2 + 1)
- 1)/x^2) - 1/2*sqrt(1/2)*sqrt(-(2*a + b - (a*b^2 - 4*a^2*c)/sqrt(a^2*b^2 -
 4*a^3*c))/(a*b^2 - 4*a^2*c))*log(-(x^2 + sqrt(1/2)*((a*b^2 - 4*a^2*c)*sqrt
(-x^2 + 1)*x - (a*b^2 - 4*a^2*c)*x)*sqrt(-(2*a + b - (a*b^2 - 4*a^2*c)/sqrt
(a^2*b^2 - 4*a^3*c))/(a*b^2 - 4*a^2*c))/sqrt(a^2*b^2 - 4*a^3*c) + sqrt(-x^2
 + 1) - 1)/x^2) + 1/2*sqrt(1/2)*sqrt(-(2*a + b - (a*b^2 - 4*a^2*c)/sqrt(a^2
*b^2 - 4*a^3*c))/(a*b^2 - 4*a^2*c))*log(-(x^2 - sqrt(1/2)*((a*b^2 - 4*a^2*c
)*sqrt(-x^2 + 1)*x - (a*b^2 - 4*a^2*c)*x)*sqrt(-(2*a + b - (a*b^2 - 4*a^2*c
)/sqrt(a^2*b^2 - 4*a^3*c))/(a*b^2 - 4*a^2*c))/sqrt(a^2*b^2 - 4*a^3*c) + sqr
t(-x^2 + 1) - 1)/x^2)
```

Sympy [F]

$$\int \frac{\sqrt{1-x^2}}{a+bx^2+cx^4}\,dx = \int \frac{\sqrt{-\left(x-1\right)\left(x+1\right)}}{a+bx^2+cx^4}\,dx$$

```
[In] integrate((-x**2+1)**(1/2)/(c*x**4+b*x**2+a),x)
[Out] Integral(sqrt(-(x - 1)*(x + 1))/(a + b*x**2 + c*x**4), x)
```

Maxima [F]

$$\int \frac{\sqrt{1-x^2}}{a+bx^2+cx^4}\,dx = \int \frac{\sqrt{-x^2+1}}{cx^4+bx^2+a}\,dx$$

```
[In] integrate((-x^2+1)^(1/2)/(c*x^4+b*x^2+a),x, algorithm="maxima")
[Out] integrate(sqrt(-x^2 + 1)/(c*x^4 + b*x^2 + a), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 641 vs. $2(180) = 360$.

Time = 1.08 (sec) , antiderivative size = 641, normalized size of antiderivative = 2.91

$$\int \frac{\sqrt{1-x^2}}{a+bx^2+cx^4}\,dx =$$

$$-\frac{\left(2\,a^2b^2-8\,a^3c+3\,\sqrt{2}\sqrt{2\,a^2+ab+\sqrt{b^2-4\,ac}a}\sqrt{b^2-4\,ac}a^2+2\,\sqrt{2}\sqrt{2\,a^2+ab+\sqrt{b^2-4\,ac}a}\sqrt{b^2-\right.}{2\,(3}$$

$$+\frac{\left(2\,a^2b^2-8\,a^3c+3\,\sqrt{2}\sqrt{2\,a^2+ab-\sqrt{b^2-4\,ac}a}\sqrt{b^2-4\,ac}a^2+2\,\sqrt{2}\sqrt{2\,a^2+ab-\sqrt{b^2-4\,ac}a}\sqrt{b^2-\right.}{2\,(3}$$

```
[In] integrate((-x^2+1)^(1/2)/(c*x^4+b*x^2+a),x, algorithm="giac")
[Out] -1/2*(2*a^2*b^2 - 8*a^3*c + 3*sqrt(2)*sqrt(2*a^2 + a*b + sqrt(b^2 - 4*a*c)*
a)*sqrt(b^2 - 4*a*c)*a^2 + 2*sqrt(2)*sqrt(2*a^2 + a*b + sqrt(b^2 - 4*a*c)*a
)*sqrt(b^2 - 4*a*c)*a*b - sqrt(2)*sqrt(2*a^2 + a*b + sqrt(b^2 - 4*a*c)*a)*s
qrt(b^2 - 4*a*c)*b^2 + 4*sqrt(2)*sqrt(2*a^2 + a*b + sqrt(b^2 - 4*a*c)*a)*sq
rt(b^2 - 4*a*c)*a*c - 2*(b^2 - 4*a*c)*a^2)*abs(a)*arctan(-1/2*sqrt(2)*(x/(s
qrt(-x^2 + 1) - 1) - (sqrt(-x^2 + 1) - 1)/x)/sqrt((2*a + b + sqrt((2*a + b)
```

```
^2 - 4*(a + b + c)*a))/a))/(3*a^4*b^2 + 2*a^3*b^3 - a^2*b^4 - 12*a^5*c - 8*
a^4*b*c + 8*a^3*b^2*c - 16*a^4*c^2) + 1/2*(2*a^2*b^2 - 8*a^3*c + 3*sqrt(2)*
sqrt(2*a^2 + a*b - sqrt(b^2 - 4*a*c)*a)*sqrt(b^2 - 4*a*c)*a^2 + 2*sqrt(2)*s
qrt(2*a^2 + a*b - sqrt(b^2 - 4*a*c)*a)*sqrt(b^2 - 4*a*c)*a*b - sqrt(2)*sqrt
(2*a^2 + a*b - sqrt(b^2 - 4*a*c)*a)*sqrt(b^2 - 4*a*c)*b^2 + 4*sqrt(2)*sqrt(
2*a^2 + a*b - sqrt(b^2 - 4*a*c)*a)*sqrt(b^2 - 4*a*c)*a*c - 2*(b^2 - 4*a*c)*
a^2)*abs(a)*arctan(-1/2*sqrt(2)*(x/(sqrt(-x^2 + 1) - 1) - (sqrt(-x^2 + 1) -
 1)/x)/sqrt((2*a + b - sqrt((2*a + b)^2 - 4*(a + b + c)*a))/a))/(3*a^4*b^2
+ 2*a^3*b^3 - a^2*b^4 - 12*a^5*c - 8*a^4*b*c + 8*a^3*b^2*c - 16*a^4*c^2)
```

Mupad [B] (verification not implemented)

Time = 8.27 (sec) , antiderivative size = 989, normalized size of antiderivative = 4.50

$$\int \frac{\sqrt{1-x^2}}{a+bx^2+cx^4}\,dx =$$

$$-\frac{\ln\left(\frac{\frac{\left(x\sqrt{-\frac{b-\sqrt{b^2-4ac}}{2c}}-1\right)1i}{\sqrt{\frac{b-\sqrt{b^2-4ac}}{2c}+1}}-\sqrt{1-x^2}\,1i}{x-\sqrt{-\frac{b-\sqrt{b^2-4ac}}{2c}}}\right)\Bigg(b^2\sqrt{-\frac{b-\sqrt{b^2-4ac}}{2c}}+ab\sqrt{-\frac{b-\sqrt{b^2-4ac}}{2c}}-2ac\sqrt{-\frac{b-\sqrt{b^2-4ac}}{2c}}+2ac\Bigg(}{2a\sqrt{\frac{b-\sqrt{b^2-4ac}}{2c}+1}\,(4ac-b^2)}$$

$$+\frac{\ln\left(\frac{\frac{\left(x\sqrt{-\frac{b+\sqrt{b^2-4ac}}{2c}}+1\right)1i}{\sqrt{\frac{b+\sqrt{b^2-4ac}}{2c}+1}}+\sqrt{1-x^2}\,1i}{x+\sqrt{-\frac{b+\sqrt{b^2-4ac}}{2c}}}\right)\Bigg(b^2\sqrt{-\frac{b+\sqrt{b^2-4ac}}{2c}}+ab\sqrt{-\frac{b+\sqrt{b^2-4ac}}{2c}}-2ac\sqrt{-\frac{b+\sqrt{b^2-4ac}}{2c}}+2ac\Bigg(}{2a\,(4ac-b^2)\,\sqrt{\frac{b+\sqrt{b^2-4ac}}{2c}+1}}$$

$$+\frac{\ln\left(\frac{\frac{\left(x\sqrt{-\frac{b-\sqrt{b^2-4ac}}{2c}}+1\right)1i}{\sqrt{\frac{b-\sqrt{b^2-4ac}}{2c}+1}}+\sqrt{1-x^2}\,1i}{x+\sqrt{-\frac{b-\sqrt{b^2-4ac}}{2c}}}\right)\Bigg(b^2\sqrt{-\frac{b-\sqrt{b^2-4ac}}{2c}}+ab\sqrt{-\frac{b-\sqrt{b^2-4ac}}{2c}}-2ac\sqrt{-\frac{b-\sqrt{b^2-4ac}}{2c}}+2ac\Bigg(}{2a\sqrt{\frac{b-\sqrt{b^2-4ac}}{2c}+1}\,(4ac-b^2)}$$

$$-\frac{\ln\left(\frac{\frac{\left(x\sqrt{-\frac{b+\sqrt{b^2-4ac}}{2c}}-1\right)1i}{\sqrt{\frac{b+\sqrt{b^2-4ac}}{2c}+1}}-\sqrt{1-x^2}\,1i}{x-\sqrt{-\frac{b+\sqrt{b^2-4ac}}{2c}}}\right)\Bigg(b^2\sqrt{-\frac{b+\sqrt{b^2-4ac}}{2c}}+ab\sqrt{-\frac{b+\sqrt{b^2-4ac}}{2c}}-2ac\sqrt{-\frac{b+\sqrt{b^2-4ac}}{2c}}+2ac\Bigg(}{2a\,(4ac-b^2)\,\sqrt{\frac{b+\sqrt{b^2-4ac}}{2c}+1}}$$

```
[In] int((1 - x^2)^(1/2)/(a + b*x^2 + c*x^4),x)

[Out] (log((((x*(-(b + (b^2 - 4*a*c)^(1/2))/(2*c))^(1/2) + 1)*1i)/((b + (b^2 - 4*
a*c)^(1/2))/(2*c) + 1)^(1/2) + (1 - x^2)^(1/2)*1i)/(x + (-(b + (b^2 - 4*a*c
)^(1/2))/(2*c))^(1/2)))*(b^2*(-(b + (b^2 - 4*a*c)^(1/2))/(2*c))^(1/2) + a*b
*(-(b + (b^2 - 4*a*c)^(1/2))/(2*c))^(1/2) - 2*a*c*(-(b + (b^2 - 4*a*c)^(1/2
))/(2*c))^(1/2) + 2*a*c*(-(b + (b^2 - 4*a*c)^(1/2))/(2*c))^(3/2) + b*c*(-(b
 + (b^2 - 4*a*c)^(1/2))/(2*c))^(3/2)))/(2*a*(4*a*c - b^2)*((b + (b^2 - 4*a*
c)^(1/2))/(2*c) + 1)^(1/2)) - (log((((x*(-(b - (b^2 - 4*a*c)^(1/2))/(2*c))^
(1/2) - 1)*1i)/((b - (b^2 - 4*a*c)^(1/2))/(2*c) + 1)^(1/2) - (1 - x^2)^(1/2
)*1i)/(x - (-(b - (b^2 - 4*a*c)^(1/2))/(2*c))^(1/2)))*(b^2*(-(b - (b^2 - 4*
a*c)^(1/2))/(2*c))^(1/2) + a*b*(-(b - (b^2 - 4*a*c)^(1/2))/(2*c))^(1/2) - 2
*a*c*(-(b - (b^2 - 4*a*c)^(1/2))/(2*c))^(1/2) + 2*a*c*(-(b - (b^2 - 4*a*c)^
(1/2))/(2*c))^(3/2) + b*c*(-(b - (b^2 - 4*a*c)^(1/2))/(2*c))^(3/2)))/(2*a*(
(b - (b^2 - 4*a*c)^(1/2))/(2*c) + 1)^(1/2)*(4*a*c - b^2)) + (log((((x*(-(b
- (b^2 - 4*a*c)^(1/2))/(2*c))^(1/2) + 1)*1i)/((b - (b^2 - 4*a*c)^(1/2))/(2*
c) + 1)^(1/2) + (1 - x^2)^(1/2)*1i)/(x + (-(b - (b^2 - 4*a*c)^(1/2))/(2*c))
^(1/2)))*(b^2*(-(b - (b^2 - 4*a*c)^(1/2))/(2*c))^(1/2) + a*b*(-(b - (b^2 -
4*a*c)^(1/2))/(2*c))^(1/2) - 2*a*c*(-(b - (b^2 - 4*a*c)^(1/2))/(2*c))^(1/2)
 + 2*a*c*(-(b - (b^2 - 4*a*c)^(1/2))/(2*c))^(3/2) + b*c*(-(b - (b^2 - 4*a*c
)^(1/2))/(2*c))^(3/2)))/(2*a*((b - (b^2 - 4*a*c)^(1/2))/(2*c) + 1)^(1/2)*(4
*a*c - b^2)) - (log((((x*(-(b + (b^2 - 4*a*c)^(1/2))/(2*c))^(1/2) - 1)*1i)/
((b + (b^2 - 4*a*c)^(1/2))/(2*c) + 1)^(1/2) - (1 - x^2)^(1/2)*1i)/(x - (-(b
 + (b^2 - 4*a*c)^(1/2))/(2*c))^(1/2)))*(b^2*(-(b + (b^2 - 4*a*c)^(1/2))/(2*
c))^(1/2) + a*b*(-(b + (b^2 - 4*a*c)^(1/2))/(2*c))^(1/2) - 2*a*c*(-(b + (b^
2 - 4*a*c)^(1/2))/(2*c))^(1/2) + 2*a*c*(-(b + (b^2 - 4*a*c)^(1/2))/(2*c))^(
3/2) + b*c*(-(b + (b^2 - 4*a*c)^(1/2))/(2*c))^(3/2)))/(2*a*(4*a*c - b^2)*((
b + (b^2 - 4*a*c)^(1/2))/(2*c) + 1)^(1/2))
```

3.384 $\int \frac{\sqrt{1-x^2}}{x^2(a+bx^2+cx^4)}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 265

$$\int \frac{\sqrt{1-x^2}}{x^2\left(a+bx^2+cx^4\right)}\,dx = -\frac{\sqrt{1-x^2}}{ax} - \frac{c\left(1+\frac{2a+b}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{b+2c-\sqrt{b^2-4ac}}\,x}{\sqrt{b-\sqrt{b^2-4ac}}\sqrt{1-x^2}}\right)}{a\sqrt{b-\sqrt{b^2-4ac}}\sqrt{b+2c-\sqrt{b^2-4ac}}} - \frac{c\left(1-\frac{2a+b}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{b+2c+\sqrt{b^2-4ac}}\,x}{\sqrt{b+\sqrt{b^2-4ac}}\sqrt{1-x^2}}\right)}{a\sqrt{b+\sqrt{b^2-4ac}}\sqrt{b+2c+\sqrt{b^2-4ac}}}$$

```
[Out] -(-x^2+1)^(1/2)/a/x-c*arctan(x*(b+2*c-(-4*a*c+b^2)^(1/2))^(1/2)/(-x^2+1)^(1
/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2))*(1+(2*a+b)/(-4*a*c+b^2)^(1/2))/a/(b-(-4*a
*c+b^2)^(1/2))^(1/2)/(b+2*c-(-4*a*c+b^2)^(1/2))^(1/2)-c*arctan(x*(b+2*c+(-4
*a*c+b^2)^(1/2))^(1/2)/(-x^2+1)^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2))*(1+(-2*
a-b)/(-4*a*c+b^2)^(1/2))/a/(b+(-4*a*c+b^2)^(1/2))^(1/2)/(b+2*c+(-4*a*c+b^2)
^(1/2))^(1/2)
```

Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 265, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.172, Rules used = {1309, 270, 1706, 385, 211}

$$\int \frac{\sqrt{1-x^2}}{x^2\left(a+bx^2+cx^4\right)}\,dx = -\frac{c\left(\frac{2a+b}{\sqrt{b^2-4ac}}+1\right)\arctan\left(\frac{x\sqrt{-\sqrt{b^2-4ac}+b+2c}}{\sqrt{1-x^2}\sqrt{b-\sqrt{b^2-4ac}}}\right)}{a\sqrt{b-\sqrt{b^2-4ac}}\sqrt{-\sqrt{b^2-4ac}+b+2c}} - \frac{c\left(1-\frac{2a+b}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{x\sqrt{\sqrt{b^2-4ac}+b+2c}}{\sqrt{1-x^2}\sqrt{\sqrt{b^2-4ac}+b}}\right)}{a\sqrt{\sqrt{b^2-4ac}+b}\sqrt{\sqrt{b^2-4ac}+b+2c}} - \frac{\sqrt{1-x^2}}{ax}$$

```
[In] Int[Sqrt[1 - x^2]/(x^2*(a + b*x^2 + c*x^4)),x]
```

```
[Out] -(Sqrt[1 - x^2]/(a*x)) - (c*(1 + (2*a + b)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[
b + 2*c - Sqrt[b^2 - 4*a*c]]*x)/(Sqrt[b - Sqrt[b^2 - 4*a*c]]*Sqrt[1 - x^2])
])/(a*Sqrt[b - Sqrt[b^2 - 4*a*c]]*Sqrt[b + 2*c - Sqrt[b^2 - 4*a*c]]) - (c*(
1 - (2*a + b)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[b + 2*c + Sqrt[b^2 - 4*a*c]]*
x)/(Sqrt[b + Sqrt[b^2 - 4*a*c]]*Sqrt[1 - x^2])])/(a*Sqrt[b + Sqrt[b^2 - 4*a
*c]]*Sqrt[b + 2*c + Sqrt[b^2 - 4*a*c]])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 1309

```
Int[(((f_.)*(x_))^(m_)*((d_.) + (e_.)*(x_)^2)^(q_))/((a_) + (b_.)*(x_)^2 +
(c_.)*(x_)^4), x_Symbol] :> Dist[d/a, Int[(f*x)^m*(d + e*x^2)^(q - 1), x],
x] - Dist[1/(a*f^2), Int[(f*x)^(m + 2)*(d + e*x^2)^(q - 1)*(Simp[b*d - a*e
+ c*d*x^2, x]/(a + b*x^2 + c*x^4)), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
&& NeQ[b^2 - 4*a*c, 0] &&  !IntegerQ[q] && GtQ[q, 0] && LtQ[m, 0]
```

Rule 1706

```
Int[(Px_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^
(p_.), x_Symbol] :> Int[ExpandIntegrand[Px*(d + e*x^2)^q*(a + b*x^2 + c*x^4
)^p, x], x] /; FreeQ[{a, b, c, d, e, q}, x] && PolyQ[Px, x^2] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[p]
```

Rubi steps

$$\text{integral} = \frac{\int \frac{1}{x^2\sqrt{1-x^2}}\,dx}{a} - \frac{\int \frac{a+b+cx^2}{\sqrt{1-x^2}(a+bx^2+cx^4)}\,dx}{a}$$

$$= -\frac{\sqrt{1-x^2}}{ax} - \frac{\int \left(\frac{c+\frac{(2a+b)c}{\sqrt{b^2-4ac}}}{\sqrt{1-x^2}\left(b-\sqrt{b^2-4ac}+2cx^2\right)} + \frac{c-\frac{(2a+b)c}{\sqrt{b^2-4ac}}}{\sqrt{1-x^2}\left(b+\sqrt{b^2-4ac}+2cx^2\right)}\right)dx}{a}$$

$$= -\frac{\sqrt{1-x^2}}{ax} - \frac{\left(c\left(1-\frac{2a+b}{\sqrt{b^2-4ac}}\right)\right)\int \frac{1}{\sqrt{1-x^2}\left(b+\sqrt{b^2-4ac}+2cx^2\right)}\,dx}{a} - \frac{\left(c\left(1+\frac{2a+b}{\sqrt{b^2-4ac}}\right)\right)\int \frac{1}{\sqrt{1-x^2}\left(b-\sqrt{b^2-4ac}+2cx^2\right)}\,dx}{a}$$

$$= -\frac{\sqrt{1-x^2}}{ax} - \frac{\left(c\left(1-\frac{2a+b}{\sqrt{b^2-4ac}}\right)\right)\text{Subst}\left(\int \frac{1}{b+\sqrt{b^2-4ac}-\left(-b-2c-\sqrt{b^2-4ac}\right)x^2}\,dx, x, \frac{x}{\sqrt{1-x^2}}\right)}{a} - \frac{\left(c\left(1+\frac{2a+b}{\sqrt{b^2-4ac}}\right)\right)\text{Subst}\left(\int \frac{1}{b-\sqrt{b^2-4ac}-\left(-b-2c+\sqrt{b^2-4ac}\right)x^2}\,dx, x, \frac{x}{\sqrt{1-x^2}}\right)}{a}$$

$$= -\frac{\sqrt{1-x^2}}{ax} - \frac{c\left(1+\frac{2a+b}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{b+2c-\sqrt{b^2-4ac}}\,x}{\sqrt{b-\sqrt{b^2-4ac}}\sqrt{1-x^2}}\right)}{a\sqrt{b-\sqrt{b^2-4ac}}\sqrt{b+2c-\sqrt{b^2-4ac}}} - \frac{c\left(1-\frac{2a+b}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{b+2c+\sqrt{b^2-4ac}}\,x}{\sqrt{b+\sqrt{b^2-4ac}}\sqrt{1-x^2}}\right)}{a\sqrt{b+\sqrt{b^2-4ac}}\sqrt{b+2c+\sqrt{b^2-4ac}}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.45 (sec) , antiderivative size = 471, normalized size of antiderivative = 1.78

$$\int \frac{\sqrt{1-x^2}}{x^2\left(a+bx^2+cx^4\right)}\,dx = -\frac{\sqrt{1-x^2}}{ax} + \frac{\text{RootSum}\left[a+4a\#1^2+4b\#1^2+6a\#1^4+8b\#1^4+16c\#1^4+4a\#1^6+4b\#1^6+a\#1^8\&, \frac{-a\log(x)-b\log(x)-}{}\right.}{}$$

```
[In] Integrate[Sqrt[1 - x^2]/(x^2*(a + b*x^2 + c*x^4)),x]
```

```
[Out] -(Sqrt[1 - x^2]/(a*x)) + RootSum[a + 4*a*#1^2 + 4*b*#1^2 + 6*a*#1^4 + 8*b*#
1^4 + 16*c*#1^4 + 4*a*#1^6 + 4*b*#1^6 + a*#1^8 & , (-(a*Log[x]) - b*Log[x]
+ a*Log[-1 + Sqrt[1 - x^2] - x*#1] + b*Log[-1 + Sqrt[1 - x^2] - x*#1] - 3*a
*Log[x]*#1^2 - 3*b*Log[x]*#1^2 - 4*c*Log[x]*#1^2 + 3*a*Log[-1 + Sqrt[1 - x^
2] - x*#1]*#1^2 + 3*b*Log[-1 + Sqrt[1 - x^2] - x*#1]*#1^2 + 4*c*Log[-1 + Sq
rt[1 - x^2] - x*#1]*#1^2 - 3*a*Log[x]*#1^4 - 3*b*Log[x]*#1^4 - 4*c*Log[x]*#
1^4 + 3*a*Log[-1 + Sqrt[1 - x^2] - x*#1]*#1^4 + 3*b*Log[-1 + Sqrt[1 - x^2]
- x*#1]*#1^4 + 4*c*Log[-1 + Sqrt[1 - x^2] - x*#1]*#1^4 - a*Log[x]*#1^6 - b*
Log[x]*#1^6 + a*Log[-1 + Sqrt[1 - x^2] - x*#1]*#1^6 + b*Log[-1 + Sqrt[1 - x
^2] - x*#1]*#1^6)/(a*#1 + b*#1 + 3*a*#1^3 + 4*b*#1^3 + 8*c*#1^3 + 3*a*#1^5
+ 3*b*#1^5 + a*#1^7) & ]/(4*a)
```

Maple [A] (verified)

Time = 0.74 (sec) , antiderivative size = 239, normalized size of antiderivative = 0.90

method	result
pseudoelliptic	$\frac{-\frac{\sqrt{-x^2+1}}{x}+\frac{\left(-2ac+b^2+\sqrt{-4ac+b^2}\,a+b\sqrt{-4ac+b^2}+ab\right)\sqrt{2}\,\arctan\left(\frac{a\sqrt{-x^2+1}\,\sqrt{2}}{x\sqrt{\left(2a+b+\sqrt{-4ac+b^2}\right)a}}\right)}{2\sqrt{-4ac+b^2}\,\sqrt{\left(2a+b+\sqrt{-4ac+b^2}\right)a}}-\frac{\left(2ac-b^2+\sqrt{-4ac+b^2}\,a+b\sqrt{-4ac+\cdots}\right.}{2\sqrt{-4ac+b^2}\cdots}}{a}$
risch	$\frac{x^2-1}{ax\sqrt{-x^2+1}}+\frac{\sqrt{2}\left(\left(\left(-a-b\right)\sqrt{-4ac+b^2}+a(b-2c)+b^2\right)\sqrt{\left(2a+b+\sqrt{-4ac+b^2}\right)a}\,\operatorname{arctanh}\left(\frac{a\sqrt{-x^2+1}\,\sqrt{2}}{x\sqrt{\left(-b+\sqrt{-4ac+b^2}-2a\right)a}}\right)+\cdots\right.}{2a\sqrt{-4ac+b^2}\,\sqrt{\left(-b+\sqrt{-4ac+b^2}-2a\cdots\right.}}$
default	$\frac{-\frac{\left(-x^2+1\right)^{\frac{3}{2}}}{x}-x\sqrt{-x^2+1}-\arcsin(x)}{a}+\frac{\left(\left(-a-b\right)\sqrt{-4ac+b^2}+a(b-2c)+b^2\right)\sqrt{2}\,\sqrt{\left(2a+b+\sqrt{-4ac+b^2}\right)a}\,\operatorname{arctanh}\left(\frac{a\cdots}{x\sqrt{\left(-b\cdots\right.}}\right.}{\cdots}$

```
[In] int((-x^2+1)^(1/2)/x^2/(c*x^4+b*x^2+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/a*(-1/x*(-x^2+1)^(1/2)+1/2*(-2*a*c+b^2+(-4*a*c+b^2)^(1/2)*a+b*(-4*a*c+b^2
)^(1/2)+a*b)/(-4*a*c+b^2)^(1/2)*2^(1/2)/((2*a+b+(-4*a*c+b^2)^(1/2))*a)^(1/2
)*arctan(a/x*(-x^2+1)^(1/2)*2^(1/2)/((2*a+b+(-4*a*c+b^2)^(1/2))*a)^(1/2))-1
/2*(2*a*c-b^2+(-4*a*c+b^2)^(1/2)*a+b*(-4*a*c+b^2)^(1/2)-a*b)/(-4*a*c+b^2)^(
1/2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2)-2*a)*a)^(1/2)*arctanh(a/x*(-x^2+1)^(1/
2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2)-2*a)*a)^(1/2)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1998 vs. 2(221) = 442.

Time = 0.49 (sec) , antiderivative size = 1998, normalized size of antiderivative = 7.54

$$\int \frac{\sqrt{1-x^2}}{x^2\left(a+bx^2+cx^4\right)}\,dx = \text{Too large to display}$$

```
[In] integrate((-x^2+1)^(1/2)/x^2/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] 1/2*(sqrt(1/2)*a*x*sqrt(-(a*b^2 + b^3 - (2*a^2 + 3*a*b)*c + (a^3*b^2 - 4*a^
4*c)*sqrt((a^2*b^2 + 2*a*b^3 + b^4 + a^2*c^2 - 2*(a^2*b + a*b^2)*c)/(a^6*b^
2 - 4*a^7*c)))/(a^3*b^2 - 4*a^4*c))*log((2*a*c^2 - 2*(a*c^2 - (a*b + b^2)*c
)*x^2 - 2*(a*b + b^2)*c + sqrt(1/2)*((a*b^3 + b^4 + 4*a^2*c^2 - (4*a^2*b +
5*a*b^2)*c)*sqrt(-x^2 + 1)*x - (a*b^3 + b^4 + 4*a^2*c^2 - (4*a^2*b + 5*a*b^
2)*c)*x - ((a^3*b^3 - 4*a^4*b*c)*sqrt(-x^2 + 1)*x - (a^3*b^3 - 4*a^4*b*c)*x
)*sqrt((a^2*b^2 + 2*a*b^3 + b^4 + a^2*c^2 - 2*(a^2*b + a*b^2)*c)/(a^6*b^2 -
 4*a^7*c))*sqrt(-(a*b^2 + b^3 - (2*a^2 + 3*a*b)*c + (a^3*b^2 - 4*a^4*c)*sq
```

```
rt((a^2*b^2 + 2*a*b^3 + b^4 + a^2*c^2 - 2*(a^2*b + a*b^2)*c)/(a^6*b^2 - 4*a
^7*c)))/(a^3*b^2 - 4*a^4*c)) - 2*(a*c^2 - (a*b + b^2)*c)*sqrt(-x^2 + 1))/x^
2) - sqrt(1/2)*a*x*sqrt(-(a*b^2 + b^3 - (2*a^2 + 3*a*b)*c + (a^3*b^2 - 4*a^
4*c)*sqrt((a^2*b^2 + 2*a*b^3 + b^4 + a^2*c^2 - 2*(a^2*b + a*b^2)*c)/(a^6*b^
2 - 4*a^7*c)))/(a^3*b^2 - 4*a^4*c))*log((2*a*c^2 - 2*(a*c^2 - (a*b + b^2)*c
)*x^2 - 2*(a*b + b^2)*c - sqrt(1/2)*((a*b^3 + b^4 + 4*a^2*c^2 - (4*a^2*b +
5*a*b^2)*c)*sqrt(-x^2 + 1)*x - (a*b^3 + b^4 + 4*a^2*c^2 - (4*a^2*b + 5*a*b^
2)*c)*x - ((a^3*b^3 - 4*a^4*b*c)*sqrt(-x^2 + 1)*x - (a^3*b^3 - 4*a^4*b*c)*x
)*sqrt((a^2*b^2 + 2*a*b^3 + b^4 + a^2*c^2 - 2*(a^2*b + a*b^2)*c)/(a^6*b^2 -
 4*a^7*c)))*sqrt(-(a*b^2 + b^3 - (2*a^2 + 3*a*b)*c + (a^3*b^2 - 4*a^4*c)*sq
rt((a^2*b^2 + 2*a*b^3 + b^4 + a^2*c^2 - 2*(a^2*b + a*b^2)*c)/(a^6*b^2 - 4*a
^7*c)))/(a^3*b^2 - 4*a^4*c)) - 2*(a*c^2 - (a*b + b^2)*c)*sqrt(-x^2 + 1))/x^
2) + sqrt(1/2)*a*x*sqrt(-(a*b^2 + b^3 - (2*a^2 + 3*a*b)*c - (a^3*b^2 - 4*a^
4*c)*sqrt((a^2*b^2 + 2*a*b^3 + b^4 + a^2*c^2 - 2*(a^2*b + a*b^2)*c)/(a^6*b^
2 - 4*a^7*c)))/(a^3*b^2 - 4*a^4*c))*log((2*a*c^2 - 2*(a*c^2 - (a*b + b^2)*c
)*x^2 - 2*(a*b + b^2)*c + sqrt(1/2)*((a*b^3 + b^4 + 4*a^2*c^2 - (4*a^2*b +
5*a*b^2)*c)*sqrt(-x^2 + 1)*x - (a*b^3 + b^4 + 4*a^2*c^2 - (4*a^2*b + 5*a*b^
2)*c)*x + ((a^3*b^3 - 4*a^4*b*c)*sqrt(-x^2 + 1)*x - (a^3*b^3 - 4*a^4*b*c)*x
)*sqrt((a^2*b^2 + 2*a*b^3 + b^4 + a^2*c^2 - 2*(a^2*b + a*b^2)*c)/(a^6*b^2 -
 4*a^7*c)))*sqrt(-(a*b^2 + b^3 - (2*a^2 + 3*a*b)*c - (a^3*b^2 - 4*a^4*c)*sq
rt((a^2*b^2 + 2*a*b^3 + b^4 + a^2*c^2 - 2*(a^2*b + a*b^2)*c)/(a^6*b^2 - 4*a
^7*c)))/(a^3*b^2 - 4*a^4*c)) - 2*(a*c^2 - (a*b + b^2)*c)*sqrt(-x^2 + 1))/x^
2) - sqrt(1/2)*a*x*sqrt(-(a*b^2 + b^3 - (2*a^2 + 3*a*b)*c - (a^3*b^2 - 4*a^
4*c)*sqrt((a^2*b^2 + 2*a*b^3 + b^4 + a^2*c^2 - 2*(a^2*b + a*b^2)*c)/(a^6*b^
2 - 4*a^7*c)))/(a^3*b^2 - 4*a^4*c))*log((2*a*c^2 - 2*(a*c^2 - (a*b + b^2)*c
)*x^2 - 2*(a*b + b^2)*c - sqrt(1/2)*((a*b^3 + b^4 + 4*a^2*c^2 - (4*a^2*b +
5*a*b^2)*c)*sqrt(-x^2 + 1)*x - (a*b^3 + b^4 + 4*a^2*c^2 - (4*a^2*b + 5*a*b^
2)*c)*x + ((a^3*b^3 - 4*a^4*b*c)*sqrt(-x^2 + 1)*x - (a^3*b^3 - 4*a^4*b*c)*x
)*sqrt((a^2*b^2 + 2*a*b^3 + b^4 + a^2*c^2 - 2*(a^2*b + a*b^2)*c)/(a^6*b^2 -
 4*a^7*c)))*sqrt(-(a*b^2 + b^3 - (2*a^2 + 3*a*b)*c - (a^3*b^2 - 4*a^4*c)*sq
rt((a^2*b^2 + 2*a*b^3 + b^4 + a^2*c^2 - 2*(a^2*b + a*b^2)*c)/(a^6*b^2 - 4*a
^7*c)))/(a^3*b^2 - 4*a^4*c)) - 2*(a*c^2 - (a*b + b^2)*c)*sqrt(-x^2 + 1))/x^
2) - 2*sqrt(-x^2 + 1))/(a*x)
```

Sympy [F]

$$\int \frac{\sqrt{1-x^2}}{x^2\left(a+bx^2+cx^4\right)}\,dx=\int \frac{\sqrt{-\left(x-1\right)\left(x+1\right)}}{x^2\left(a+bx^2+cx^4\right)}\,dx$$

```
[In] integrate((-x**2+1)**(1/2)/x**2/(c*x**4+b*x**2+a),x)
```

```
[Out] Integral(sqrt(-(x - 1)*(x + 1))/(x**2*(a + b*x**2 + c*x**4)), x)
```

Maxima [F]

$$\int \frac{\sqrt{1-x^2}}{x^2\,(a+bx^2+cx^4)}\,dx = \int \frac{\sqrt{-x^2+1}}{(cx^4+bx^2+a)x^2}\,dx$$

```
[In] integrate((-x^2+1)^(1/2)/x^2/(c*x^4+b*x^2+a),x, algorithm="maxima")
[Out] integrate(sqrt(-x^2 + 1)/((c*x^4 + b*x^2 + a)*x^2), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3965 vs. $2(221) = 442$.

Time = 0.85 (sec) , antiderivative size = 3965, normalized size of antiderivative = 14.96

$$\int \frac{\sqrt{1-x^2}}{x^2\,(a+bx^2+cx^4)}\,dx = \text{Too large to display}$$

```
[In] integrate((-x^2+1)^(1/2)/x^2/(c*x^4+b*x^2+a),x, algorithm="giac")
[Out] 1/8*(4*a^6*b^3 + 6*a^5*b^4 + 2*a^4*b^5 - 16*a^7*b*c - 32*a^6*b^2*c - 12*a^5
*b^3*c + 32*a^7*c^2 + 16*a^6*b*c^2 + 6*sqrt(2)*sqrt(2*a^2 + a*b + sqrt(b^2
- 4*a*c)*a)*sqrt(b^2 - 4*a*c)*a^6*b + 13*sqrt(2)*sqrt(2*a^2 + a*b + sqrt(b^
2 - 4*a*c)*a)*sqrt(b^2 - 4*a*c)*a^5*b^2 + 7*sqrt(2)*sqrt(2*a^2 + a*b + sqrt
(b^2 - 4*a*c)*a)*sqrt(b^2 - 4*a*c)*a^4*b^3 - sqrt(2)*sqrt(2*a^2 + a*b + sqr
t(b^2 - 4*a*c)*a)*sqrt(b^2 - 4*a*c)*a^3*b^4 - sqrt(2)*sqrt(2*a^2 + a*b + sq
rt(b^2 - 4*a*c)*a)*sqrt(b^2 - 4*a*c)*a^2*b^5 - 12*sqrt(2)*sqrt(2*a^2 + a*b
+ sqrt(b^2 - 4*a*c)*a)*sqrt(b^2 - 4*a*c)*a^6*c - 6*sqrt(2)*sqrt(2*a^2 + a*b
 + sqrt(b^2 - 4*a*c)*a)*sqrt(b^2 - 4*a*c)*a^5*b*c + 12*sqrt(2)*sqrt(2*a^2 +
 a*b + sqrt(b^2 - 4*a*c)*a)*sqrt(b^2 - 4*a*c)*a^4*b^2*c + 6*sqrt(2)*sqrt(2*
a^2 + a*b + sqrt(b^2 - 4*a*c)*a)*sqrt(b^2 - 4*a*c)*a^3*b^3*c - 16*sqrt(2)*s
qrt(2*a^2 + a*b + sqrt(b^2 - 4*a*c)*a)*sqrt(b^2 - 4*a*c)*a^5*c^2 - 8*sqrt(2
)*sqrt(2*a^2 + a*b + sqrt(b^2 - 4*a*c)*a)*sqrt(b^2 - 4*a*c)*a^4*b*c^2 - 4*(
b^2 - 4*a*c)*a^6*b - 6*(b^2 - 4*a*c)*a^5*b^2 - 2*(b^2 - 4*a*c)*a^4*b^3 + 8*
(b^2 - 4*a*c)*a^6*c + 4*(b^2 - 4*a*c)*a^5*b*c - (2*a^3*b^4 + 2*a^2*b^5 - 16
*a^4*b^2*c - 16*a^3*b^3*c + 32*a^5*c^2 + 32*a^4*b*c^2 + 3*sqrt(2)*sqrt(2*a^
2 + a*b + sqrt(b^2 - 4*a*c)*a)*sqrt(b^2 - 4*a*c)*a^3*b^2 + 5*sqrt(2)*sqrt(2
*a^2 + a*b + sqrt(b^2 - 4*a*c)*a)*sqrt(b^2 - 4*a*c)*a^2*b^3 + sqrt(2)*sqrt(
2*a^2 + a*b + sqrt(b^2 - 4*a*c)*a)*sqrt(b^2 - 4*a*c)*a*b^4 - sqrt(2)*sqrt(2
*a^2 + a*b + sqrt(b^2 - 4*a*c)*a)*sqrt(b^2 - 4*a*c)*b^5 - 12*sqrt(2)*sqrt(2
*a^2 + a*b + sqrt(b^2 - 4*a*c)*a)*sqrt(b^2 - 4*a*c)*a^4*c - 20*sqrt(2)*sqrt
(2*a^2 + a*b + sqrt(b^2 - 4*a*c)*a)*sqrt(b^2 - 4*a*c)*a^3*b*c + 8*sqrt(2)*s
qrt(2*a^2 + a*b + sqrt(b^2 - 4*a*c)*a)*sqrt(b^2 - 4*a*c)*a*b^3*c - 16*sqrt(
2)*sqrt(2*a^2 + a*b + sqrt(b^2 - 4*a*c)*a)*sqrt(b^2 - 4*a*c)*a^3*c^2 - 16*s
```

qrt(2)*sqrt(2*a^2 + a*b + sqrt(b^2 - 4*a*c)*a)*sqrt(b^2 - 4*a*c)*a^2*b*c^2
- 2*(b^2 - 4*a*c)*a^3*b^2 - 2*(b^2 - 4*a*c)*a^2*b^3 + 8*(b^2 - 4*a*c)*a^4*c
 + 8*(b^2 - 4*a*c)*a^3*b*c)*a^2 + 2*(3*sqrt(2)*sqrt(2*a^2 + a*b + sqrt(b^2
- 4*a*c)*a)*a^5*b^2 + 5*sqrt(2)*sqrt(2*a^2 + a*b + sqrt(b^2 - 4*a*c)*a)*a^4
*b^3 + sqrt(2)*sqrt(2*a^2 + a*b + sqrt(b^2 - 4*a*c)*a)*a^3*b^4 + 2*a^4*b^4
- sqrt(2)*sqrt(2*a^2 + a*b + sqrt(b^2 - 4*a*c)*a)*a^2*b^5 + 2*a^3*b^5 - 12*
sqrt(2)*sqrt(2*a^2 + a*b + sqrt(b^2 - 4*a*c)*a)*a^6*c - 20*sqrt(2)*sqrt(2*a
^2 + a*b + sqrt(b^2 - 4*a*c)*a)*a^5*b*c + 3*sqrt(2)*sqrt(2*a^2 + a*b + sqrt
(b^2 - 4*a*c)*a)*a^4*b^2*c - 16*a^5*b^2*c + 10*sqrt(2)*sqrt(2*a^2 + a*b + s
qrt(b^2 - 4*a*c)*a)*a^3*b^3*c - 16*a^4*b^3*c - sqrt(2)*sqrt(2*a^2 + a*b + s
qrt(b^2 - 4*a*c)*a)*a^2*b^4*c + 2*a^3*b^4*c - 28*sqrt(2)*sqrt(2*a^2 + a*b +
 sqrt(b^2 - 4*a*c)*a)*a^5*c^2 + 32*a^6*c^2 - 24*sqrt(2)*sqrt(2*a^2 + a*b +
sqrt(b^2 - 4*a*c)*a)*a^4*b*c^2 + 32*a^5*b*c^2 + 8*sqrt(2)*sqrt(2*a^2 + a*b
+ sqrt(b^2 - 4*a*c)*a)*a^3*b^2*c^2 - 16*a^4*b^2*c^2 - 16*sqrt(2)*sqrt(2*a^2
 + a*b + sqrt(b^2 - 4*a*c)*a)*a^4*c^3 + 32*a^5*c^3 - 2*(b^2 - 4*a*c)*a^4*b^
2 - 2*(b^2 - 4*a*c)*a^3*b^3 + 8*(b^2 - 4*a*c)*a^5*c + 8*(b^2 - 4*a*c)*a^4*b
c - 2(b^2 - 4*a*c)*a^3*b^2*c + 8*(b^2 - 4*a*c)*a^4*c^2)*abs(a))*arctan(-1
/2*sqrt(2)*(x/(sqrt(-x^2 + 1) - 1) - (sqrt(-x^2 + 1) - 1)/x)/sqrt((2*a^2 +
a*b + sqrt(-4*(a^2 + a*b + a*c)*a^2 + (2*a^2 + a*b)^2))/a^2))/(3*a^8*b^2 +
5*a^7*b^3 + a^6*b^4 - a^5*b^5 - 12*a^9*c - 20*a^8*b*c + 3*a^7*b^2*c + 10*a^
6*b^3*c - a^5*b^4*c - 28*a^8*c^2 - 24*a^7*b*c^2 + 8*a^6*b^2*c^2 - 16*a^7*c^
3) + 1/8*(4*a^6*b^3 + 6*a^5*b^4 + 2*a^4*b^5 - 16*a^7*b*c - 32*a^6*b^2*c - 1
2*a^5*b^3*c + 32*a^7*c^2 + 16*a^6*b*c^2 + 6*sqrt(2)*sqrt(2*a^2 + a*b - sqrt
(b^2 - 4*a*c)*a)*sqrt(b^2 - 4*a*c)*a^6*b + 13*sqrt(2)*sqrt(2*a^2 + a*b - sq
rt(b^2 - 4*a*c)*a)*sqrt(b^2 - 4*a*c)*a^5*b^2 + 7*sqrt(2)*sqrt(2*a^2 + a*b -
 sqrt(b^2 - 4*a*c)*a)*sqrt(b^2 - 4*a*c)*a^4*b^3 - sqrt(2)*sqrt(2*a^2 + a*b
- sqrt(b^2 - 4*a*c)*a)*sqrt(b^2 - 4*a*c)*a^3*b^4 - sqrt(2)*sqrt(2*a^2 + a*b
 - sqrt(b^2 - 4*a*c)*a)*sqrt(b^2 - 4*a*c)*a^2*b^5 - 12*sqrt(2)*sqrt(2*a^2 +
 a*b - sqrt(b^2 - 4*a*c)*a)*sqrt(b^2 - 4*a*c)*a^6*c - 6*sqrt(2)*sqrt(2*a^2
+ a*b - sqrt(b^2 - 4*a*c)*a)*sqrt(b^2 - 4*a*c)*a^5*b*c + 12*sqrt(2)*sqrt(2*
a^2 + a*b - sqrt(b^2 - 4*a*c)*a)*sqrt(b^2 - 4*a*c)*a^4*b^2*c + 6*sqrt(2)*sq
rt(2*a^2 + a*b - sqrt(b^2 - 4*a*c)*a)*sqrt(b^2 - 4*a*c)*a^3*b^3*c - 16*sqrt
(2)*sqrt(2*a^2 + a*b - sqrt(b^2 - 4*a*c)*a)*sqrt(b^2 - 4*a*c)*a^5*c^2 - 8*s
qrt(2)*sqrt(2*a^2 + a*b - sqrt(b^2 - 4*a*c)*a)*sqrt(b^2 - 4*a*c)*a^4*b*c^2
- 4*(b^2 - 4*a*c)*a^6*b - 6*(b^2 - 4*a*c)*a^5*b^2 - 2*(b^2 - 4*a*c)*a^4*b^3
 + 8*(b^2 - 4*a*c)*a^6*c + 4*(b^2 - 4*a*c)*a^5*b*c - (2*a^3*b^4 + 2*a^2*b^5
 - 16*a^4*b^2*c - 16*a^3*b^3*c + 32*a^5*c^2 + 32*a^4*b*c^2 + 3*sqrt(2)*sqrt
(2*a^2 + a*b - sqrt(b^2 - 4*a*c)*a)*sqrt(b^2 - 4*a*c)*a^3*b^2 + 5*sqrt(2)*s
qrt(2*a^2 + a*b - sqrt(b^2 - 4*a*c)*a)*sqrt(b^2 - 4*a*c)*a^2*b^3 + sqrt(2)*
sqrt(2*a^2 + a*b - sqrt(b^2 - 4*a*c)*a)*sqrt(b^2 - 4*a*c)*a*b^4 - sqrt(2)*s
qrt(2*a^2 + a*b - sqrt(b^2 - 4*a*c)*a)*sqrt(b^2 - 4*a*c)*b^5 - 12*sqrt(2)*s
qrt(2*a^2 + a*b - sqrt(b^2 - 4*a*c)*a)*sqrt(b^2 - 4*a*c)*a^4*c - 20*sqrt(2)
*sqrt(2*a^2 + a*b - sqrt(b^2 - 4*a*c)*a)*sqrt(b^2 - 4*a*c)*a^3*b*c + 8*sqrt
(2)*sqrt(2*a^2 + a*b - sqrt(b^2 - 4*a*c)*a)*sqrt(b^2 - 4*a*c)*a*b^3*c - 16*
sqrt(2)*sqrt(2*a^2 + a*b - sqrt(b^2 - 4*a*c)*a)*sqrt(b^2 - 4*a*c)*a^3*c^2 -

```
16*sqrt(2)*sqrt(2*a^2 + a*b - sqrt(b^2 - 4*a*c)*a)*sqrt(b^2 - 4*a*c)*a^2*b
*c^2 - 2*(b^2 - 4*a*c)*a^3*b^2 - 2*(b^2 - 4*a*c)*a^2*b^3 + 8*(b^2 - 4*a*c)*
a^4*c + 8*(b^2 - 4*a*c)*a^3*b*c)*a^2 + 2*(3*sqrt(2)*sqrt(2*a^2 + a*b - sqrt
(b^2 - 4*a*c)*a)*a^5*b^2 + 5*sqrt(2)*sqrt(2*a^2 + a*b - sqrt(b^2 - 4*a*c)*a
)*a^4*b^3 + sqrt(2)*sqrt(2*a^2 + a*b - sqrt(b^2 - 4*a*c)*a)*a^3*b^4 - 2*a^4
*b^4 - sqrt(2)*sqrt(2*a^2 + a*b - sqrt(b^2 - 4*a*c)*a)*a^2*b^5 - 2*a^3*b^5
- 12*sqrt(2)*sqrt(2*a^2 + a*b - sqrt(b^2 - 4*a*c)*a)*a^6*c - 20*sqrt(2)*sqr
t(2*a^2 + a*b - sqrt(b^2 - 4*a*c)*a)*a^5*b*c + 3*sqrt(2)*sqrt(2*a^2 + a*b -
 sqrt(b^2 - 4*a*c)*a)*a^4*b^2*c + 16*a^5*b^2*c + 10*sqrt(2)*sqrt(2*a^2 + a*
b - sqrt(b^2 - 4*a*c)*a)*a^3*b^3*c + 16*a^4*b^3*c - sqrt(2)*sqrt(2*a^2 + a*
b - sqrt(b^2 - 4*a*c)*a)*a^2*b^4*c - 2*a^3*b^4*c - 28*sqrt(2)*sqrt(2*a^2 +
a*b - sqrt(b^2 - 4*a*c)*a)*a^5*c^2 - 32*a^6*c^2 - 24*sqrt(2)*sqrt(2*a^2 + a
*b - sqrt(b^2 - 4*a*c)*a)*a^4*b*c^2 - 32*a^5*b*c^2 + 8*sqrt(2)*sqrt(2*a^2 +
 a*b - sqrt(b^2 - 4*a*c)*a)*a^3*b^2*c^2 + 16*a^4*b^2*c^2 - 16*sqrt(2)*sqrt(
2*a^2 + a*b - sqrt(b^2 - 4*a*c)*a)*a^4*c^3 - 32*a^5*c^3 + 2*(b^2 - 4*a*c)*a
^4*b^2 + 2*(b^2 - 4*a*c)*a^3*b^3 - 8*(b^2 - 4*a*c)*a^5*c - 8*(b^2 - 4*a*c)*
a^4*b*c + 2*(b^2 - 4*a*c)*a^3*b^2*c - 8*(b^2 - 4*a*c)*a^4*c^2)*abs(a))*arct
an(-1/2*sqrt(2)*(x/(sqrt(-x^2 + 1) - 1) - (sqrt(-x^2 + 1) - 1)/x)/sqrt((2*a
^2 + a*b - sqrt(-4*(a^2 + a*b + a*c)*a^2 + (2*a^2 + a*b)^2))/a^2))/(3*a^8*b
^2 + 5*a^7*b^3 + a^6*b^4 - a^5*b^5 - 12*a^9*c - 20*a^8*b*c + 3*a^7*b^2*c +
10*a^6*b^3*c - a^5*b^4*c - 28*a^8*c^2 - 24*a^7*b*c^2 + 8*a^6*b^2*c^2 - 16*a
^7*c^3) + 1/2*(x/(sqrt(-x^2 + 1) - 1) - (sqrt(-x^2 + 1) - 1)/x)/a
```

Mupad [B] (verification not implemented)

Time = 8.17 (sec) , antiderivative size = 1234, normalized size of antiderivative = 4.66

$$\int \frac{\sqrt{1-x^2}}{x^2 (a+bx^2+cx^4)} dx = -\frac{\sqrt{1-x^2}}{a\,x}$$

$$+ \frac{\ln\left(\frac{\frac{\left(x\sqrt{-\frac{b+\sqrt{b^2-4ac}}{2c}}-1\right)1i}{\sqrt{\frac{b+\sqrt{b^2-4ac}}{2c}+1}}-\sqrt{1-x^2}\,1i}{x-\sqrt{-\frac{b+\sqrt{b^2-4ac}}{2c}}}\right)\left(b^3\sqrt{-\frac{b+\sqrt{b^2-4ac}}{2c}}+a\,b^2\sqrt{-\frac{b+\sqrt{b^2-4ac}}{2c}}-2\,a^2\,c\sqrt{-\frac{b+\sqrt{b^2-4ac}}{2c}}-2\,a\,c\right.}{2\,a^2\,(4\,a\,c-b^2)\,\sqrt{\frac{b+\sqrt{}}{}}}$$

$$+ \frac{\ln\left(\frac{\frac{\left(x\sqrt{-\frac{b-\sqrt{b^2-4ac}}{2c}}-1\right)1i}{\sqrt{\frac{b-\sqrt{b^2-4ac}}{2c}+1}}-\sqrt{1-x^2}\,1i}{x-\sqrt{-\frac{b-\sqrt{b^2-4ac}}{2c}}}\right)\left(b^3\sqrt{-\frac{b-\sqrt{b^2-4ac}}{2c}}+a\,b^2\sqrt{-\frac{b-\sqrt{b^2-4ac}}{2c}}-2\,a^2\,c\sqrt{-\frac{b-\sqrt{b^2-4ac}}{2c}}-2\,a\,c\right.}{2\,a^2\sqrt{\frac{b-\sqrt{b^2-4ac}}{2c}+1}\,(}$$

$$- \frac{\ln\left(\frac{\frac{\left(x\sqrt{-\frac{b+\sqrt{b^2-4ac}}{2c}}+1\right)1i}{\sqrt{\frac{b+\sqrt{b^2-4ac}}{2c}+1}}+\sqrt{1-x^2}\,1i}{x+\sqrt{-\frac{b+\sqrt{b^2-4ac}}{2c}}}\right)\left(b^3\sqrt{-\frac{b+\sqrt{b^2-4ac}}{2c}}+a\,b^2\sqrt{-\frac{b+\sqrt{b^2-4ac}}{2c}}-2\,a^2\,c\sqrt{-\frac{b+\sqrt{b^2-4ac}}{2c}}-2\,a\,c\right.}{2\,a^2\,(4\,a\,c-b^2)\,\sqrt{\frac{b+\sqrt{}}{}}}$$

$$- \frac{\ln\left(\frac{\frac{\left(x\sqrt{-\frac{b-\sqrt{b^2-4ac}}{2c}}+1\right)1i}{\sqrt{\frac{b-\sqrt{b^2-4ac}}{2c}+1}}+\sqrt{1-x^2}\,1i}{x+\sqrt{-\frac{b-\sqrt{b^2-4ac}}{2c}}}\right)\left(b^3\sqrt{-\frac{b-\sqrt{b^2-4ac}}{2c}}+a\,b^2\sqrt{-\frac{b-\sqrt{b^2-4ac}}{2c}}-2\,a^2\,c\sqrt{-\frac{b-\sqrt{b^2-4ac}}{2c}}-2\,a\,c\right.}{2\,a^2\sqrt{\frac{b-\sqrt{b^2-4ac}}{2c}+1}\,(}$$

```
[In] int((1 - x^2)^(1/2)/(x^2*(a + b*x^2 + c*x^4)),x)
```

```
[Out] (log((((x*(-(b + (b^2 - 4*a*c)^(1/2))/(2*c))^(1/2) - 1)*1i)/((b + (b^2 - 4*
a*c)^(1/2))/(2*c) + 1)^(1/2) - (1 - x^2)^(1/2)*1i)/(x - (-(b + (b^2 - 4*a*c
)^(1/2))/(2*c))^(1/2)))*(b^3*(-(b + (b^2 - 4*a*c)^(1/2))/(2*c))^(1/2) + a*b
^2*(-(b + (b^2 - 4*a*c)^(1/2))/(2*c))^(1/2) - 2*a^2*c*(-(b + (b^2 - 4*a*c)^
(1/2))/(2*c))^(1/2) - 2*a*c^2*(-(b + (b^2 - 4*a*c)^(1/2))/(2*c))^(3/2) + b^
2*c*(-(b + (b^2 - 4*a*c)^(1/2))/(2*c))^(3/2) - 3*a*b*c*(-(b + (b^2 - 4*a*c)
^(1/2))/(2*c))^(1/2) + a*b*c*(-(b + (b^2 - 4*a*c)^(1/2))/(2*c))^(3/2)))/(2*
a^2*(4*a*c - b^2)*((b + (b^2 - 4*a*c)^(1/2))/(2*c) + 1)^(1/2)) - (1 - x^2)^
```

```
(1/2)/(a*x) + (log((((x*(-(b - (b^2 - 4*a*c)^(1/2))/(2*c))^(1/2) - 1)*1i)/(
(b - (b^2 - 4*a*c)^(1/2))/(2*c) + 1)^(1/2) - (1 - x^2)^(1/2)*1i)/(x - (-(b
- (b^2 - 4*a*c)^(1/2))/(2*c))^(1/2)))*(b^3*(-(b - (b^2 - 4*a*c)^(1/2))/(2*c
))^(1/2) + a*b^2*(-(b - (b^2 - 4*a*c)^(1/2))/(2*c))^(1/2) - 2*a^2*c*(-(b -
(b^2 - 4*a*c)^(1/2))/(2*c))^(1/2) - 2*a*c^2*(-(b - (b^2 - 4*a*c)^(1/2))/(2*
c))^(3/2) + b^2*c*(-(b - (b^2 - 4*a*c)^(1/2))/(2*c))^(3/2) - 3*a*b*c*(-(b -
 (b^2 - 4*a*c)^(1/2))/(2*c))^(1/2) + a*b*c*(-(b - (b^2 - 4*a*c)^(1/2))/(2*c
))^(3/2)))/(2*a^2*((b - (b^2 - 4*a*c)^(1/2))/(2*c) + 1)^(1/2)*(4*a*c - b^2)
) - (log((((x*(-(b + (b^2 - 4*a*c)^(1/2))/(2*c))^(1/2) + 1)*1i)/((b + (b^2
- 4*a*c)^(1/2))/(2*c) + 1)^(1/2) + (1 - x^2)^(1/2)*1i)/(x + (-(b + (b^2 - 4
*a*c)^(1/2))/(2*c))^(1/2)))*(b^3*(-(b + (b^2 - 4*a*c)^(1/2))/(2*c))^(1/2) +
 a*b^2*(-(b + (b^2 - 4*a*c)^(1/2))/(2*c))^(1/2) - 2*a^2*c*(-(b + (b^2 - 4*a
*c)^(1/2))/(2*c))^(1/2) - 2*a*c^2*(-(b + (b^2 - 4*a*c)^(1/2))/(2*c))^(3/2)
+ b^2*c*(-(b + (b^2 - 4*a*c)^(1/2))/(2*c))^(3/2) - 3*a*b*c*(-(b + (b^2 - 4*
a*c)^(1/2))/(2*c))^(1/2) + a*b*c*(-(b + (b^2 - 4*a*c)^(1/2))/(2*c))^(3/2)))
/(2*a^2*(4*a*c - b^2)*((b + (b^2 - 4*a*c)^(1/2))/(2*c) + 1)^(1/2)) - (log((
((x*(-(b - (b^2 - 4*a*c)^(1/2))/(2*c))^(1/2) + 1)*1i)/((b - (b^2 - 4*a*c)^(
1/2))/(2*c) + 1)^(1/2) + (1 - x^2)^(1/2)*1i)/(x + (-(b - (b^2 - 4*a*c)^(1/2
))/(2*c))^(1/2)))*(b^3*(-(b - (b^2 - 4*a*c)^(1/2))/(2*c))^(1/2) + a*b^2*(-(
b - (b^2 - 4*a*c)^(1/2))/(2*c))^(1/2) - 2*a^2*c*(-(b - (b^2 - 4*a*c)^(1/2))
/(2*c))^(1/2) - 2*a*c^2*(-(b - (b^2 - 4*a*c)^(1/2))/(2*c))^(3/2) + b^2*c*(-
(b - (b^2 - 4*a*c)^(1/2))/(2*c))^(3/2) - 3*a*b*c*(-(b - (b^2 - 4*a*c)^(1/2)
)/(2*c))^(1/2) + a*b*c*(-(b - (b^2 - 4*a*c)^(1/2))/(2*c))^(3/2)))/(2*a^2*((
b - (b^2 - 4*a*c)^(1/2))/(2*c) + 1)^(1/2)*(4*a*c - b^2))
```

3.385 $\int \frac{x^2\sqrt{1-x^2}}{-1+x^2+x^4}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 96

$$\int \frac{x^2\sqrt{1-x^2}}{-1+x^2+x^4}\,dx = -\arcsin(x)+\sqrt{\frac{1}{5}\left(2+\sqrt{5}\right)}\arctan\left(\frac{\sqrt{\frac{1}{2}\left(1+\sqrt{5}\right)}x}{\sqrt{1-x^2}}\right) - \sqrt{\frac{1}{5}\left(-2+\sqrt{5}\right)}\operatorname{arctanh}\left(\frac{\sqrt{\frac{1}{2}\left(-1+\sqrt{5}\right)}x}{\sqrt{1-x^2}}\right)$$

```
[Out] -arcsin(x)-1/5*arctanh(1/2*x*(-2+2*5^(1/2))^(1/2)/(-x^2+1)^(1/2))*(-10+5*5^
(1/2))^(1/2)+1/5*arctan(1/2*x*(2+2*5^(1/2))^(1/2)/(-x^2+1)^(1/2))*(10+5*5^(
1/2))^(1/2)
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {1307, 222, 1706, 385, 213, 209}

$$\int \frac{x^2\sqrt{1-x^2}}{-1+x^2+x^4}\,dx = -\arcsin(x)+\sqrt{\frac{1}{5}\left(2+\sqrt{5}\right)}\arctan\left(\frac{\sqrt{\frac{1}{2}\left(1+\sqrt{5}\right)}x}{\sqrt{1-x^2}}\right) - \sqrt{\frac{1}{5}\left(\sqrt{5}-2\right)}\operatorname{arctanh}\left(\frac{\sqrt{\frac{1}{2}\left(\sqrt{5}-1\right)}x}{\sqrt{1-x^2}}\right)$$

```
[In] Int[(x^2*Sqrt[1 - x^2])/(-1 + x^2 + x^4),x]
```

```
[Out] -ArcSin[x] + Sqrt[(2 + Sqrt[5])/5]*ArcTan[(Sqrt[(1 + Sqrt[5])/2]*x)/Sqrt[1
- x^2]] - Sqrt[(-2 + Sqrt[5])/5]*ArcTanh[(Sqrt[(-1 + Sqrt[5])/2]*x)/Sqrt[1
- x^2]]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 1307

```
Int[(((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x_)^2)^(q_))/((a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4), x_Symbol] :> Dist[e*(f^2/c), Int[(f*x)^(m - 2)*(d + e*x^2)^
(q - 1), x], x] - Dist[f^2/c, Int[(f*x)^(m - 2)*(d + e*x^2)^(q - 1)*(Simp[a
*e - (c*d - b*e)*x^2, x]/(a + b*x^2 + c*x^4)), x], x] /; FreeQ[{a, b, c, d,
 e, f}, x] && NeQ[b^2 - 4*a*c, 0] &&  !IntegerQ[q] && GtQ[q, 0] && GtQ[m, 1
] && LeQ[m, 3]
```

Rule 1706

```
Int[(Px_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^
(p_.), x_Symbol] :> Int[ExpandIntegrand[Px*(d + e*x^2)^q*(a + b*x^2 + c*x^4
)^p, x], x] /; FreeQ[{a, b, c, d, e, q}, x] && PolyQ[Px, x^2] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[p]
```

Rubi steps

$$\text{integral} = -\int \frac{1}{\sqrt{1-x^2}}\,dx - \int \frac{1-2x^2}{\sqrt{1-x^2}\,(-1+x^2+x^4)}\,dx$$

$$= -\sin^{-1}(x) - \int \left(\frac{-2+\frac{4}{\sqrt{5}}}{\sqrt{1-x^2}\,(1-\sqrt{5}+2x^2)} + \frac{-2-\frac{4}{\sqrt{5}}}{\sqrt{1-x^2}\,(1+\sqrt{5}+2x^2)} \right) dx$$

$$= -\sin^{-1}(x) + \frac{1}{5}\left(2\left(5-2\sqrt{5}\right)\right) \int \frac{1}{\sqrt{1-x^2}\,(1-\sqrt{5}+2x^2)}\,dx + \frac{1}{5}\left(2\left(5+2\sqrt{5}\right)\right) \int \frac{1}{\sqrt{1-x^2}\,(1+\sqrt{5}+2x^2)}\,dx$$

$$= -\sin^{-1}(x) + \frac{1}{5}\left(2\left(5-2\sqrt{5}\right)\right) \text{Subst}\left(\int \frac{1}{1-\sqrt{5}-(-3+\sqrt{5})\,x^2}\,dx, x, \frac{x}{\sqrt{1-x^2}}\right) + \frac{1}{5}\left(2\left(5+2\sqrt{5}\right)\right) \text{Subst}\left(\int \frac{1}{1+\sqrt{5}-(-3-\sqrt{5})\,x^2}\,dx, x, \frac{x}{\sqrt{1-x^2}}\right)$$

$$= -\sin^{-1}(x) + \sqrt{\frac{1}{5}\left(2+\sqrt{5}\right)} \tan^{-1}\left(\frac{\sqrt{\frac{1}{2}\left(1+\sqrt{5}\right)}x}{\sqrt{1-x^2}}\right) - \sqrt{\frac{1}{5}\left(-2+\sqrt{5}\right)} \tanh^{-1}\left(\frac{\sqrt{\frac{1}{2}\left(-1+\sqrt{5}\right)}x}{\sqrt{1-x^2}}\right)$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 199 vs. $2(96) = 192$.

Time = 0.44 (sec) , antiderivative size = 199, normalized size of antiderivative = 2.07

$$\int \frac{x^2\sqrt{1-x^2}}{-1+x^2+x^4}\,dx = \frac{1}{5}\left(-10\arctan\left(\frac{x}{-1+\sqrt{1-x^2}}\right) + \sqrt{5\left(2+\sqrt{5}\right)}\arctan\left(\frac{\sqrt{-2+\sqrt{5}}x}{-1+\sqrt{1-x^2}}\right) + \sqrt{5\left(2+\sqrt{5}\right)}\arctan\left(\frac{\sqrt{2+\sqrt{5}}x}{-1+\sqrt{1-x^2}}\right) + \sqrt{5\left(-2+\sqrt{5}\right)}\text{arctanh}\left(\frac{\sqrt{2+\sqrt{5}}x}{1-\sqrt{1-x^2}}\right) + \sqrt{5\left(-2+\sqrt{5}\right)}\text{arctanh}\left(\frac{\sqrt{-2+\sqrt{5}}x}{-1+\sqrt{1-x^2}}\right)\right)$$

```
[In] Integrate[(x^2*Sqrt[1 - x^2])/(-1 + x^2 + x^4),x]
[Out] (-10*ArcTan[x/(-1 + Sqrt[1 - x^2])] + Sqrt[5*(2 + Sqrt[5])]*ArcTan[(Sqrt[-2
 + Sqrt[5]]*x)/(-1 + Sqrt[1 - x^2])] + Sqrt[5*(2 + Sqrt[5])]*ArcTan[(Sqrt[2
```

```
 + Sqrt[5]]*x)/(-1 + Sqrt[1 - x^2])] + Sqrt[5*(-2 + Sqrt[5])]*ArcTanh[(Sqrt
[2 + Sqrt[5]]*x)/(1 - Sqrt[1 - x^2])] + Sqrt[5*(-2 + Sqrt[5])]*ArcTanh[(Sqr
t[-2 + Sqrt[5]]*x)/(-1 + Sqrt[1 - x^2])])/5
```

Maple [A] (verified)

Time = 2.02 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.04

method	result
pseudoelliptic	$\frac{\left(5-3\sqrt{5}\right)\sqrt{2\sqrt{5}+2}\,\operatorname{arctanh}\left(\frac{2\sqrt{-x^{2}+1}}{x\sqrt{2\sqrt{5}-2}}\right)}{20}+\frac{\left(-3\sqrt{5}-5\right)\sqrt{2\sqrt{5}-2}\,\arctan\left(\frac{2\sqrt{-x^{2}+1}}{x\sqrt{2\sqrt{5}+2}}\right)}{20}+\arctan\left(\frac{\sqrt{-x^{2}+1}}{x}\right)$
default	$-\frac{\sqrt{2+\sqrt{5}}\,\sqrt{5}\,\arctan\left(\frac{\sqrt{-x^{2}+1}-1}{x\sqrt{2+\sqrt{5}}}\right)}{5}+\frac{\sqrt{\sqrt{5}-2}\,\sqrt{5}\,\operatorname{arctanh}\left(\frac{\sqrt{-x^{2}+1}-1}{x\sqrt{\sqrt{5}-2}}\right)}{5}+2\arctan\left(\frac{\sqrt{-x^{2}+1}-1}{x}\right)-\frac{\sqrt{5}\,\text{arcta}}{}$
trager	$\text{RootOf}\left(\_Z^{2}+1\right)\ln\left(-\text{RootOf}\left(\_Z^{2}+1\right)\sqrt{-x^{2}+1}+x\right)-\text{RootOf}\left(400\_Z^{4}+80\_Z^{2}\right.$

```
[In] int(x^2*(-x^2+1)^(1/2)/(x^4+x^2-1),x,method=_RETURNVERBOSE)
```

```
[Out] 1/20*(5-3*5^(1/2))*(2*5^(1/2)+2)^(1/2)*arctanh(2/x*(-x^2+1)^(1/2)/(2*5^(1/2
)-2)^(1/2))+1/20*(-3*5^(1/2)-5)*(2*5^(1/2)-2)^(1/2)*arctan(2/x*(-x^2+1)^(1/
2)/(2*5^(1/2)+2)^(1/2))+arctan(1/x*(-x^2+1)^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 318 vs. $2(71) = 142$.

Time = 0.29 (sec) , antiderivative size = 318, normalized size of antiderivative = 3.31

$$\int \frac{x^2\sqrt{1-x^2}}{-1+x^2+x^4}\,dx$$
$$= \frac{1}{10}\sqrt{5}\sqrt{-\sqrt{5}-2}\log\left(-\frac{2x^2+\sqrt{-x^2+1}\left((\sqrt{5}x-x)\sqrt{-\sqrt{5}-2}+2\right)-(\sqrt{5}x-x)\sqrt{-\sqrt{5}-2}-2}{x^2}\right)$$
$$-\frac{1}{10}\sqrt{5}\sqrt{-\sqrt{5}-2}\log\left(-\frac{2x^2-\sqrt{-x^2+1}\left((\sqrt{5}x-x)\sqrt{-\sqrt{5}-2}-2\right)+(\sqrt{5}x-x)\sqrt{-\sqrt{5}-2}-2}{x^2}\right)$$
$$+\frac{1}{10}\sqrt{5}\sqrt{\sqrt{5}-2}\log\left(-\frac{2x^2+\left(\sqrt{-x^2+1}(\sqrt{5}x+x)-\sqrt{5}x-x\right)\sqrt{\sqrt{5}-2}+2\sqrt{-x^2+1}-2}{x^2}\right)$$
$$-\frac{1}{10}\sqrt{5}\sqrt{\sqrt{5}-2}\log\left(-\frac{2x^2-\left(\sqrt{-x^2+1}(\sqrt{5}x+x)-\sqrt{5}x-x\right)\sqrt{\sqrt{5}-2}+2\sqrt{-x^2+1}-2}{x^2}\right)$$
$$+2\arctan\left(\frac{\sqrt{-x^2+1}-1}{x}\right)$$

```
[In] integrate(x^2*(-x^2+1)^(1/2)/(x^4+x^2-1),x, algorithm="fricas")
```

```
[Out] 1/10*sqrt(5)*sqrt(-sqrt(5) - 2)*log(-(2*x^2 + sqrt(-x^2 + 1)*((sqrt(5)*x -
x)*sqrt(-sqrt(5) - 2) + 2) - (sqrt(5)*x - x)*sqrt(-sqrt(5) - 2) - 2)/x^2) -
 1/10*sqrt(5)*sqrt(-sqrt(5) - 2)*log(-(2*x^2 - sqrt(-x^2 + 1)*((sqrt(5)*x -
 x)*sqrt(-sqrt(5) - 2) - 2) + (sqrt(5)*x - x)*sqrt(-sqrt(5) - 2) - 2)/x^2)
+ 1/10*sqrt(5)*sqrt(sqrt(5) - 2)*log(-(2*x^2 + (sqrt(-x^2 + 1)*(sqrt(5)*x +
 x) - sqrt(5)*x - x)*sqrt(sqrt(5) - 2) + 2*sqrt(-x^2 + 1) - 2)/x^2) - 1/10*
sqrt(5)*sqrt(sqrt(5) - 2)*log(-(2*x^2 - (sqrt(-x^2 + 1)*(sqrt(5)*x + x) - s
qrt(5)*x - x)*sqrt(sqrt(5) - 2) + 2*sqrt(-x^2 + 1) - 2)/x^2) + 2*arctan((sq
rt(-x^2 + 1) - 1)/x)
```

Sympy [F]

$$\int \frac{x^2\sqrt{1-x^2}}{-1+x^2+x^4}\,dx = \int \frac{x^2\sqrt{-(x-1)(x+1)}}{x^4+x^2-1}\,dx$$

```
[In] integrate(x**2*(-x**2+1)**(1/2)/(x**4+x**2-1),x)
```

```
[Out] Integral(x**2*sqrt(-(x - 1)*(x + 1))/(x**4 + x**2 - 1), x)
```

Maxima [F]

$$\int \frac{x^2\sqrt{1-x^2}}{-1+x^2+x^4}\,dx = \int \frac{\sqrt{-x^2+1}x^2}{x^4+x^2-1}\,dx$$

```
[In] integrate(x^2*(-x^2+1)^(1/2)/(x^4+x^2-1),x, algorithm="maxima")
[Out] integrate(sqrt(-x^2 + 1)*x^2/(x^4 + x^2 - 1), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 209 vs. $2(71) = 142$.

Time = 0.33 (sec) , antiderivative size = 209, normalized size of antiderivative = 2.18

$$\begin{aligned}
&\int \frac{x^2\sqrt{1-x^2}}{-1+x^2+x^4}\,dx \\
&= -\frac{1}{2}\pi\mathrm{sgn}(x) - \frac{1}{5}\sqrt{5\sqrt{5}+10}\arctan\left(-\frac{\frac{x}{\sqrt{-x^2+1}-1} - \frac{\sqrt{-x^2+1}-1}{x}}{\sqrt{2\sqrt{5}+2}}\right) \\
&\quad - \frac{1}{10}\sqrt{5\sqrt{5}-10}\log\left(\left|\sqrt{2\sqrt{5}-2} - \frac{x}{\sqrt{-x^2+1}-1} + \frac{\sqrt{-x^2+1}-1}{x}\right|\right) \\
&\quad + \frac{1}{10}\sqrt{5\sqrt{5}-10}\log\left(\left|-\sqrt{2\sqrt{5}-2} - \frac{x}{\sqrt{-x^2+1}-1} + \frac{\sqrt{-x^2+1}-1}{x}\right|\right) \\
&\quad - \arctan\left(-\frac{x\left(\frac{\left(\sqrt{-x^2+1}-1\right)^2}{x^2} - 1\right)}{2\left(\sqrt{-x^2+1}-1\right)}\right)
\end{aligned}$$

```
[In] integrate(x^2*(-x^2+1)^(1/2)/(x^4+x^2-1),x, algorithm="giac")
[Out] -1/2*pi*sgn(x) - 1/5*sqrt(5*sqrt(5) + 10)*arctan(-(x/(sqrt(-x^2 + 1) - 1) -
 (sqrt(-x^2 + 1) - 1)/x)/sqrt(2*sqrt(5) + 2)) - 1/10*sqrt(5*sqrt(5) - 10)*l
og(abs(sqrt(2*sqrt(5) - 2) - x/(sqrt(-x^2 + 1) - 1) + (sqrt(-x^2 + 1) - 1)/
x)) + 1/10*sqrt(5*sqrt(5) - 10)*log(abs(-sqrt(2*sqrt(5) - 2) - x/(sqrt(-x^2
 + 1) - 1) + (sqrt(-x^2 + 1) - 1)/x)) - arctan(-1/2*x*((sqrt(-x^2 + 1) - 1)
^2/x^2 - 1)/(sqrt(-x^2 + 1) - 1))
```

Mupad [B] (verification not implemented)

Time = 8.39 (sec) , antiderivative size = 383, normalized size of antiderivative = 3.99

$$\int \frac{x^2\sqrt{1-x^2}}{-1+x^2+x^4}\,dx = -\mathrm{asin}(x) - \frac{\ln\left(\frac{\frac{\left(x\sqrt{\frac{\sqrt{5}}{2}-\frac{1}{2}}-1\right)1\mathrm{i}}{\sqrt{\frac{3}{2}-\frac{\sqrt{5}}{2}}}-\sqrt{1-x^2}\,1\mathrm{i}}{x-\sqrt{\frac{\sqrt{5}}{2}-\frac{1}{2}}}\right)\left(\sqrt{5}-2\right)}{\left(2\sqrt{\frac{\sqrt{5}}{2}-\frac{1}{2}}+4\left(\frac{\sqrt{5}}{2}-\frac{1}{2}\right)^{3/2}\right)\sqrt{\frac{3}{2}-\frac{\sqrt{5}}{2}}} + \frac{\ln\left(\frac{\frac{\left(x\sqrt{-\frac{\sqrt{5}}{2}-\frac{1}{2}}-1\right)1\mathrm{i}}{\sqrt{\frac{\sqrt{5}}{2}+\frac{3}{2}}}-\sqrt{1-x^2}\,1\mathrm{i}}{x-\sqrt{-\frac{\sqrt{5}}{2}-\frac{1}{2}}}\right)\left(\sqrt{5}+2\right)}{\left(2\sqrt{-\frac{\sqrt{5}}{2}-\frac{1}{2}}+4\left(-\frac{\sqrt{5}}{2}-\frac{1}{2}\right)^{3/2}\right)\sqrt{\frac{\sqrt{5}}{2}+\frac{3}{2}}} + \frac{\ln\left(\frac{\frac{\left(x\sqrt{\frac{\sqrt{5}}{2}-\frac{1}{2}}+1\right)1\mathrm{i}}{\sqrt{\frac{3}{2}-\frac{\sqrt{5}}{2}}}+\sqrt{1-x^2}\,1\mathrm{i}}{x+\sqrt{\frac{\sqrt{5}}{2}-\frac{1}{2}}}\right)\left(\sqrt{5}-2\right)}{\left(2\sqrt{\frac{\sqrt{5}}{2}-\frac{1}{2}}+4\left(\frac{\sqrt{5}}{2}-\frac{1}{2}\right)^{3/2}\right)\sqrt{\frac{3}{2}-\frac{\sqrt{5}}{2}}} - \frac{\ln\left(\frac{\frac{\left(x\sqrt{-\frac{\sqrt{5}}{2}-\frac{1}{2}}+1\right)1\mathrm{i}}{\sqrt{\frac{\sqrt{5}}{2}+\frac{3}{2}}}+\sqrt{1-x^2}\,1\mathrm{i}}{x+\sqrt{-\frac{\sqrt{5}}{2}-\frac{1}{2}}}\right)\left(\sqrt{5}+2\right)}{\left(2\sqrt{-\frac{\sqrt{5}}{2}-\frac{1}{2}}+4\left(-\frac{\sqrt{5}}{2}-\frac{1}{2}\right)^{3/2}\right)\sqrt{\frac{\sqrt{5}}{2}+\frac{3}{2}}}$$

```
[In] int((x^2*(1 - x^2)^(1/2))/(x^2 + x^4 - 1),x)
```

```
[Out] (log((((x*(- 5^(1/2)/2 - 1/2)^(1/2) - 1)*1i)/(5^(1/2)/2 + 3/2)^(1/2) - (1 -
 x^2)^(1/2)*1i)/(x - (- 5^(1/2)/2 - 1/2)^(1/2)))*(5^(1/2) + 2))/((2*(- 5^(1
/2)/2 - 1/2)^(1/2) + 4*(- 5^(1/2)/2 - 1/2)^(3/2))*(5^(1/2)/2 + 3/2)^(1/2))
- (log((((x*(5^(1/2)/2 - 1/2)^(1/2) - 1)*1i)/(3/2 - 5^(1/2)/2)^(1/2) - (1 -
 x^2)^(1/2)*1i)/(x - (5^(1/2)/2 - 1/2)^(1/2)))*(5^(1/2) - 2))/((2*(5^(1/2)/
2 - 1/2)^(1/2) + 4*(5^(1/2)/2 - 1/2)^(3/2))*(3/2 - 5^(1/2)/2)^(1/2)) - asin
(x) + (log((((x*(5^(1/2)/2 - 1/2)^(1/2) + 1)*1i)/(3/2 - 5^(1/2)/2)^(1/2) +
(1 - x^2)^(1/2)*1i)/(x + (5^(1/2)/2 - 1/2)^(1/2)))*(5^(1/2) - 2))/((2*(5^(1
/2)/2 - 1/2)^(1/2) + 4*(5^(1/2)/2 - 1/2)^(3/2))*(3/2 - 5^(1/2)/2)^(1/2)) -
(log((((x*(- 5^(1/2)/2 - 1/2)^(1/2) + 1)*1i)/(5^(1/2)/2 + 3/2)^(1/2) + (1 -
```

```
x^2)^(1/2)*1i)/(x + (- 5^(1/2)/2 - 1/2)^(1/2)))*(5^(1/2) + 2))/((2*(- 5^(1
/2)/2 - 1/2)^(1/2) + 4*(- 5^(1/2)/2 - 1/2)^(3/2))*(5^(1/2)/2 + 3/2)^(1/2))
```

3.386 $\int \frac{x^8}{\sqrt{d+ex^2}(a+bx^2+cx^4)}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 479

$$\begin{aligned}\int \frac{x^8}{\sqrt{d+ex^2}\,(a+bx^2+cx^4)}\,dx &= -\frac{3dx\sqrt{d+ex^2}}{8ce^2}-\frac{bx\sqrt{d+ex^2}}{2c^2e}+\frac{x^3\sqrt{d+ex^2}}{4ce} \\ &\quad -\frac{\left(b^3-2abc-\frac{b^4-4ab^2c+2a^2c^2}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}x}{\sqrt{b-\sqrt{b^2-4ac}}\sqrt{d+ex^2}}\right)}{c^3\sqrt{b-\sqrt{b^2-4ac}}\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}} \\ &\quad -\frac{\left(b^3-2abc+\frac{b^4-4ab^2c+2a^2c^2}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2cd-\left(b+\sqrt{b^2-4ac}\right)e}x}{\sqrt{b+\sqrt{b^2-4ac}}\sqrt{d+ex^2}}\right)}{c^3\sqrt{b+\sqrt{b^2-4ac}}\sqrt{2cd-\left(b+\sqrt{b^2-4ac}\right)e}} \\ &\quad +\frac{3d^2\text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{8ce^{5/2}}+\frac{bd\text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{2c^2e^{3/2}}+\frac{(b^2-ac)\,\text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{c^3\sqrt{e}}\end{aligned}$$

```
[Out] 3/8*d^2*arctanh(x*e^(1/2)/(e*x^2+d)^(1/2))/c/e^(5/2)+1/2*b*d*arctanh(x*e^(1
/2)/(e*x^2+d)^(1/2))/c^2/e^(3/2)+(-a*c+b^2)*arctanh(x*e^(1/2)/(e*x^2+d)^(1/
2))/c^3/e^(1/2)-3/8*d*x*(e*x^2+d)^(1/2)/c/e^2-1/2*b*x*(e*x^2+d)^(1/2)/c^2/e
+1/4*x^3*(e*x^2+d)^(1/2)/c/e-arctan(x*(2*c*d-e*(b-(-4*a*c+b^2)^(1/2)))^(1/2
)/(e*x^2+d)^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2))*(b^3-2*a*b*c+(-2*a^2*c^2+4*
a*b^2*c-b^4)/(-4*a*c+b^2)^(1/2))/c^3/(2*c*d-e*(b-(-4*a*c+b^2)^(1/2)))^(1/2)
/(b-(-4*a*c+b^2)^(1/2))^(1/2)-arctan(x*(2*c*d-e*(b+(-4*a*c+b^2)^(1/2)))^(1/
2)/(e*x^2+d)^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2))*(b^3-2*a*b*c+(2*a^2*c^2-4*
a*b^2*c+b^4)/(-4*a*c+b^2)^(1/2))/c^3/(b+(-4*a*c+b^2)^(1/2))^(1/2)/(2*c*d-e*
(b+(-4*a*c+b^2)^(1/2)))^(1/2)
```

Rubi [A] (verified)

Time = 1.22 (sec) , antiderivative size = 479, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.241$, Rules used = {1317, 223, 212, 327, 1706, 385, 211}

$$\int \frac{x^8}{\sqrt{d+ex^2}\,(a+bx^2+cx^4)}\,dx$$
$$= -\frac{\left(-\frac{2a^2c^2-4ab^2c+b^4}{\sqrt{b^2-4ac}}-2abc+b^3\right)\arctan\left(\frac{x\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}{\sqrt{b-\sqrt{b^2-4ac}}\sqrt{d+ex^2}}\right)}{c^3\sqrt{b-\sqrt{b^2-4ac}}\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}$$
$$-\frac{\left(\frac{2a^2c^2-4ab^2c+b^4}{\sqrt{b^2-4ac}}-2abc+b^3\right)\arctan\left(\frac{x\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}{\sqrt{\sqrt{b^2-4ac}+b}\sqrt{d+ex^2}}\right)}{c^3\sqrt{\sqrt{b^2-4ac}+b}\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}$$
$$+\frac{(b^2-ac)\,\text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{c^3\sqrt{e}}+\frac{bd\text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{2c^2e^{3/2}}$$
$$+\frac{3d^2\text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{8ce^{5/2}}-\frac{bx\sqrt{d+ex^2}}{2c^2e}-\frac{3dx\sqrt{d+ex^2}}{8ce^2}+\frac{x^3\sqrt{d+ex^2}}{4ce}$$

```
[In] Int[x^8/(Sqrt[d + e*x^2]*(a + b*x^2 + c*x^4)),x]
```

```
[Out] (-3*d*x*Sqrt[d + e*x^2])/(8*c*e^2) - (b*x*Sqrt[d + e*x^2])/(2*c^2*e) + (x^3
*Sqrt[d + e*x^2])/(4*c*e) - ((b^3 - 2*a*b*c - (b^4 - 4*a*b^2*c + 2*a^2*c^2)
/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]*x)/(Sqr
t[b - Sqrt[b^2 - 4*a*c]]*Sqrt[d + e*x^2])])/(c^3*Sqrt[b - Sqrt[b^2 - 4*a*c]
]*Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]) - ((b^3 - 2*a*b*c + (b^4 - 4*a*b
^2*c + 2*a^2*c^2)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2*c*d - (b + Sqrt[b^2 - 4
*a*c])*e]*x)/(Sqrt[b + Sqrt[b^2 - 4*a*c]]*Sqrt[d + e*x^2])])/(c^3*Sqrt[b +
Sqrt[b^2 - 4*a*c]]*Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]) + (3*d^2*ArcTan
h[(Sqrt[e]*x)/Sqrt[d + e*x^2]])/(8*c*e^(5/2)) + (b*d*ArcTanh[(Sqrt[e]*x)/Sq
rt[d + e*x^2]])/(2*c^2*e^(3/2)) + ((b^2 - a*c)*ArcTanh[(Sqrt[e]*x)/Sqrt[d +
 e*x^2]])/(c^3*Sqrt[e])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 1317

```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_))/((a_) + (b_.)*(x_)^2 +
(c_.)*(x_)^4), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^q, (f*x)^m/(a +
 b*x^2 + c*x^4), x], x] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ[b^2 - 4*a
*c, 0] &&  !IntegerQ[q] && IntegerQ[m]
```

Rule 1706

```
Int[(Px_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^
(p_.), x_Symbol] :> Int[ExpandIntegrand[Px*(d + e*x^2)^q*(a + b*x^2 + c*x^4
)^p, x], x] /; FreeQ[{a, b, c, d, e, q}, x] && PolyQ[Px, x^2] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[p]
```

Rubi steps

$$\text{integral} = \int \left(\frac{b^2-ac}{c^3\sqrt{d+ex^2}} - \frac{bx^2}{c^2\sqrt{d+ex^2}} + \frac{x^4}{c\sqrt{d+ex^2}} - \frac{a(b^2-ac)+b(b^2-2ac)\,x^2}{c^3\sqrt{d+ex^2}\,(a+bx^2+cx^4)} \right) dx$$

$$= -\frac{\int \frac{a(b^2-ac)+b(b^2-2ac)x^2}{\sqrt{d+ex^2}(a+bx^2+cx^4)}\,dx}{c^3} - \frac{b\int \frac{x^2}{\sqrt{d+ex^2}}\,dx}{c^2} + \frac{\int \frac{x^4}{\sqrt{d+ex^2}}\,dx}{c} + \frac{(b^2-ac)\int \frac{1}{\sqrt{d+ex^2}}\,dx}{c^3}$$

$$=-\frac{bx\sqrt{d+ex^2}}{2c^2e}+\frac{x^3\sqrt{d+ex^2}}{4ce}-\frac{\int\left(\frac{b(b^2-2ac)+\frac{-b^4+4ab^2c-2a^2c^2}{\sqrt{b^2-4ac}}}{\left(b-\sqrt{b^2-4ac}+2cx^2\right)\sqrt{d+ex^2}}+\frac{b(b^2-2ac)-\frac{-b^4+4ab^2c-2a^2c^2}{\sqrt{b^2-4ac}}}{\left(b+\sqrt{b^2-4ac}+2cx^2\right)\sqrt{d+ex^2}}\right)dx}{c^3}+\frac{(b^2-ac)\,\text{Subst}\left(\int\frac{1}{1-ex^2}\,dx,x,\frac{x}{\sqrt{d+ex^2}}\right)}{c^3}+\frac{(bd)\int\frac{1}{\sqrt{d+ex^2}}\,dx}{2c^2e}-\frac{(3d)\int\frac{x^2}{\sqrt{d+ex^2}}\,dx}{4ce}$$

$$=-\frac{3dx\sqrt{d+ex^2}}{8ce^2}-\frac{bx\sqrt{d+ex^2}}{2c^2e}+\frac{x^3\sqrt{d+ex^2}}{4ce}+\frac{(b^2-ac)\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{c^3\sqrt{e}}-\frac{\left(b^3-2abc-\frac{b^4-4ab^2c+2a^2c^2}{\sqrt{b^2-4ac}}\right)\int\frac{1}{\left(b-\sqrt{b^2-4ac}+2cx^2\right)\sqrt{d+ex^2}}\,dx}{c^3}-\frac{\left(b^3-2abc+\frac{b^4-4ab^2c+2a^2c^2}{\sqrt{b^2-4ac}}\right)\int\frac{1}{\left(b+\sqrt{b^2-4ac}+2cx^2\right)\sqrt{d+ex^2}}\,dx}{c^3}+\frac{(3d^2)\int\frac{1}{\sqrt{d+ex^2}}\,dx}{8ce^2}+\frac{(bd)\text{Subst}\left(\int\frac{1}{1-ex^2}\,dx,x,\frac{x}{\sqrt{d+ex^2}}\right)}{2c^2e}$$

$$=-\frac{3dx\sqrt{d+ex^2}}{8ce^2}-\frac{bx\sqrt{d+ex^2}}{2c^2e}+\frac{x^3\sqrt{d+ex^2}}{4ce}+\frac{bd\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{2c^2e^{3/2}}+\frac{(b^2-ac)\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{c^3\sqrt{e}}-\frac{\left(b^3-2abc-\frac{b^4-4ab^2c+2a^2c^2}{\sqrt{b^2-4ac}}\right)\text{Subst}\left(\int\frac{1}{b-\sqrt{b^2-4ac}-\left(-2cd+\left(b-\sqrt{b^2-4ac}\right)e\right)x^2}\,dx,x,\frac{x}{\sqrt{d+ex^2}}\right)}{c^3}-\frac{\left(b^3-2abc+\frac{b^4-4ab^2c+2a^2c^2}{\sqrt{b^2-4ac}}\right)\text{Subst}\left(\int\frac{1}{b+\sqrt{b^2-4ac}-\left(-2cd+\left(b+\sqrt{b^2-4ac}\right)e\right)x^2}\,dx,x,\frac{x}{\sqrt{d+ex^2}}\right)}{c^3}+\frac{(3d^2)\,\text{Subst}\left(\int\frac{1}{1-ex^2}\,dx,x,\frac{x}{\sqrt{d+ex^2}}\right)}{8ce^2}$$

$$= -\frac{3dx\sqrt{d+ex^2}}{8ce^2} - \frac{bx\sqrt{d+ex^2}}{2c^2e} + \frac{x^3\sqrt{d+ex^2}}{4ce}$$
$$- \frac{\left(b^3 - 2abc - \frac{b^4-4ab^2c+2a^2c^2}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}x}{\sqrt{b-\sqrt{b^2-4ac}}\sqrt{d+ex^2}}\right)}{c^3\sqrt{b-\sqrt{b^2-4ac}}\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}}$$
$$- \frac{\left(b^3 - 2abc + \frac{b^4-4ab^2c+2a^2c^2}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2cd-\left(b+\sqrt{b^2-4ac}\right)e}x}{\sqrt{b+\sqrt{b^2-4ac}}\sqrt{d+ex^2}}\right)}{c^3\sqrt{b+\sqrt{b^2-4ac}}\sqrt{2cd-\left(b+\sqrt{b^2-4ac}\right)e}}$$
$$+ \frac{3d^2\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{8ce^{5/2}} + \frac{bd\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{2c^2e^{3/2}} + \frac{(b^2-ac)\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{c^3\sqrt{e}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 1.37 (sec) , antiderivative size = 838, normalized size of antiderivative = 1.75

$$\int \frac{x^8}{\sqrt{d+ex^2}\,(a+bx^2+cx^4)}\,dx$$
$$= \frac{\frac{c\sqrt{d+ex^2}(-3cdx-4bex+2cex^3)}{e^2} + \frac{2(3c^2d^2+8b^2e^2+4ce(bd-2ae))\operatorname{arctanh}\left(\frac{\sqrt{e}x}{-\sqrt{d}+\sqrt{d+ex^2}}\right)}{e^{5/2}} - 2\mathrm{RootSum}\left[ae^4+4bde^2\#1^2-4a\right.}{}$$

```
[In] Integrate[x^8/(Sqrt[d + e*x^2]*(a + b*x^2 + c*x^4)),x]
```

```
[Out] ((c*Sqrt[d + e*x^2]*(-3*c*d*x - 4*b*e*x + 2*c*e*x^3))/e^2 + (2*(3*c^2*d^2 +
 8*b^2*e^2 + 4*c*e*(b*d - 2*a*e))*ArcTanh[(Sqrt[e]*x)/(-Sqrt[d] + Sqrt[d +
e*x^2])])/e^(5/2) - 2*RootSum[a*e^4 + 4*b*d*e^2*#1^2 - 4*a*e^3*#1^2 + 16*c*
d^2*#1^4 - 8*b*d*e*#1^4 + 6*a*e^2*#1^4 + 4*b*d*#1^6 - 4*a*e*#1^6 + a*#1^8 &
 , (-(a*b^2*e^3*Log[x]) + a^2*c*e^3*Log[x] + a*b^2*e^3*Log[-Sqrt[d] + Sqrt[
d + e*x^2] - x*#1] - a^2*c*e^3*Log[-Sqrt[d] + Sqrt[d + e*x^2] - x*#1] - 4*b
^3*d*e*Log[x]*#1^2 + 8*a*b*c*d*e*Log[x]*#1^2 + 3*a*b^2*e^2*Log[x]*#1^2 - 3*
a^2*c*e^2*Log[x]*#1^2 + 4*b^3*d*e*Log[-Sqrt[d] + Sqrt[d + e*x^2] - x*#1]*#1
^2 - 8*a*b*c*d*e*Log[-Sqrt[d] + Sqrt[d + e*x^2] - x*#1]*#1^2 - 3*a*b^2*e^2*
Log[-Sqrt[d] + Sqrt[d + e*x^2] - x*#1]*#1^2 + 3*a^2*c*e^2*Log[-Sqrt[d] + Sq
rt[d + e*x^2] - x*#1]*#1^2 + 4*b^3*d*Log[x]*#1^4 - 8*a*b*c*d*Log[x]*#1^4 -
3*a*b^2*e*Log[x]*#1^4 + 3*a^2*c*e*Log[x]*#1^4 - 4*b^3*d*Log[-Sqrt[d] + Sqrt
[d + e*x^2] - x*#1]*#1^4 + 8*a*b*c*d*Log[-Sqrt[d] + Sqrt[d + e*x^2] - x*#1]
*#1^4 + 3*a*b^2*e*Log[-Sqrt[d] + Sqrt[d + e*x^2] - x*#1]*#1^4 - 3*a^2*c*e*L
og[-Sqrt[d] + Sqrt[d + e*x^2] - x*#1]*#1^4 + a*b^2*Log[x]*#1^6 - a^2*c*Log[
```

```
x]*#1^6 - a*b^2*Log[-Sqrt[d] + Sqrt[d + e*x^2] - x*#1]*#1^6 + a^2*c*Log[-Sq
rt[d] + Sqrt[d + e*x^2] - x*#1]*#1^6)/(b*d*e^2*#1 - a*e^3*#1 + 8*c*d^2*#1^3
 - 4*b*d*e*#1^3 + 3*a*e^2*#1^3 + 3*b*d*#1^5 - 3*a*e*#1^5 + a*#1^7) & ])/(8*
c^3)
```

Maple [A] (verified)

Time = 1.06 (sec) , antiderivative size = 394, normalized size of antiderivative = 0.82

method	result
risch	$-\frac{x(-2c\,x^2e+4be+3cd)\sqrt{e\,x^2+d}}{8e^2c^2}-\frac{\frac{(8e^2ac-8b^2e^2-4bcde-3c^2d^2)\ln\left(x\sqrt{e}+\sqrt{e\,x^2+d}\right)}{c\sqrt{e}}+\frac{4e^2a\sqrt{2}\left(\frac{\left(-3abcd+b^3d+\sqrt{-d^2\left(4ac-b^2\cdots\right.\right.}{\cdots}\right.}{\cdots}}{\cdots}$
default	$-\frac{\frac{ac\ln\left(x\sqrt{e}+\sqrt{e\,x^2+d}\right)}{\sqrt{e}}-\frac{b^2\ln\left(x\sqrt{e}+\sqrt{e\,x^2+d}\right)}{\sqrt{e}}-c^2\left(\frac{x^3\sqrt{e\,x^2+d}}{4e}-\frac{3d\left(\frac{x\sqrt{e\,x^2+d}}{2e}-\frac{d\ln\left(x\sqrt{e}+\sqrt{e\,x^2+d}\right)}{2e^{\frac{3}{2}}}\right)}{4e}\right)+bc\left(\frac{x\sqrt{e\,x^2+d}}{2e}-\frac{d}{\cdots}\right.}{c^3}$
pseudoelliptic	$-\frac{-a\sqrt{2}\,e^{\frac{9}{2}}\sqrt{\left(-2ae+bd+\sqrt{-4d^2\left(ac-\frac{b^2}{4}\right)}\right)a}\left(\left(ac-b^2\right)\sqrt{-4d^2\left(ac-\frac{b^2}{4}\right)}+3abcd-b^3d\right)\operatorname{arctanh}\left(\frac{a\sqrt{e\,x^2+d}\,\sqrt{2}}{x\sqrt{\left(2ae-bd+\sqrt{-4d^2\left(a\cdots\right.\right.}}\right.}{\cdots}$

```
[In] int(x^8/(c*x^4+b*x^2+a)/(e*x^2+d)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/8*x*(-2*c*e*x^2+4*b*e+3*c*d)*(e*x^2+d)^(1/2)/e^2/c^2-1/8/e^2/c^2*((8*a*c
*e^2-8*b^2*e^2-4*b*c*d*e-3*c^2*d^2)/c*ln(x*e^(1/2)+(e*x^2+d)^(1/2))/e^(1/2)
+4*e^2/c*a*2^(1/2)/(-d^2*(4*a*c-b^2))^(1/2)*((-3*a*b*c*d+b^3*d+(-d^2*(4*a*c
-b^2))^(1/2)*a*c-(-d^2*(4*a*c-b^2))^(1/2)*b^2)/((-2*a*e+b*d+(-d^2*(4*a*c-b^
2))^(1/2))*a)^(1/2)*arctan(a/x*(e*x^2+d)^(1/2)*2^(1/2)/((-2*a*e+b*d+(-d^2*(
4*a*c-b^2))^(1/2))*a)^(1/2))-(3*a*b*c*d-b^3*d+(-d^2*(4*a*c-b^2))^(1/2)*a*c-
(-d^2*(4*a*c-b^2))^(1/2)*b^2)/((2*a*e-b*d+(-d^2*(4*a*c-b^2))^(1/2))*a)^(1/2
)*arctanh(a/x*(e*x^2+d)^(1/2)*2^(1/2)/((2*a*e-b*d+(-d^2*(4*a*c-b^2))^(1/2))
*a)^(1/2))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 9131 vs. $2(405) = 810$.

Time = 142.83 (sec) , antiderivative size = 18271, normalized size of antiderivative = 38.14

$$\int \frac{x^8}{\sqrt{d+ex^2}\,(a+bx^2+cx^4)}\,dx = \text{Too large to display}$$

```
[In] integrate(x^8/(c*x^4+b*x^2+a)/(e*x^2+d)^(1/2),x, algorithm="fricas")
[Out] Too large to include
```

Sympy [F]

$$\int \frac{x^8}{\sqrt{d+ex^2}\,(a+bx^2+cx^4)}\,dx = \int \frac{x^8}{\sqrt{d+ex^2}\,(a+bx^2+cx^4)}\,dx$$

```
[In] integrate(x**8/(c*x**4+b*x**2+a)/(e*x**2+d)**(1/2),x)
[Out] Integral(x**8/(sqrt(d + e*x**2)*(a + b*x**2 + c*x**4)), x)
```

Maxima [F]

$$\int \frac{x^8}{\sqrt{d+ex^2}\,(a+bx^2+cx^4)}\,dx = \int \frac{x^8}{(cx^4+bx^2+a)\sqrt{ex^2+d}}\,dx$$

```
[In] integrate(x^8/(c*x^4+b*x^2+a)/(e*x^2+d)^(1/2),x, algorithm="maxima")
[Out] integrate(x^8/((c*x^4 + b*x^2 + a)*sqrt(e*x^2 + d)), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{x^8}{\sqrt{d+ex^2}\,(a+bx^2+cx^4)}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(x^8/(c*x^4+b*x^2+a)/(e*x^2+d)^(1/2),x, algorithm="giac")
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:index.cc index_m i_lex_is_greater Err
or: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^8}{\sqrt{d+ex^2}\,(a+bx^2+cx^4)}\,dx = \int \frac{x^8}{\sqrt{e\,x^2+d}\,\left(c\,x^4+b\,x^2+a\right)}\,dx$$

```
[In] int(x^8/((d + e*x^2)^(1/2)*(a + b*x^2 + c*x^4)),x)
[Out] int(x^8/((d + e*x^2)^(1/2)*(a + b*x^2 + c*x^4)), x)
```

3.387 $\int \frac{x^6}{\sqrt{d+ex^2}(a+bx^2+cx^4)}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 366

$$\int \frac{x^6}{\sqrt{d+ex^2}\,(a+bx^2+cx^4)}\,dx = \frac{x\sqrt{d+ex^2}}{2ce} + \frac{\left(b^2-ac-\frac{b(b^2-3ac)}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}\,x}{\sqrt{b-\sqrt{b^2-4ac}}\sqrt{d+ex^2}}\right)}{c^2\sqrt{b-\sqrt{b^2-4ac}}\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}} + \frac{\left(b^2-ac+\frac{b(b^2-3ac)}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2cd-\left(b+\sqrt{b^2-4ac}\right)e}\,x}{\sqrt{b+\sqrt{b^2-4ac}}\sqrt{d+ex^2}}\right)}{c^2\sqrt{b+\sqrt{b^2-4ac}}\sqrt{2cd-\left(b+\sqrt{b^2-4ac}\right)e}} - \frac{d\,\text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{2ce^{3/2}} - \frac{b\,\text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{c^2\sqrt{e}}$$

```
[Out] -1/2*d*arctanh(x*e^(1/2)/(e*x^2+d)^(1/2))/c/e^(3/2)-b*arctanh(x*e^(1/2)/(e*
x^2+d)^(1/2))/c^2/e^(1/2)+1/2*x*(e*x^2+d)^(1/2)/c/e+arctan(x*(2*c*d-e*(b-(-
4*a*c+b^2)^(1/2)))^(1/2)/(e*x^2+d)^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2))*(b^2
-a*c-b*(-3*a*c+b^2)/(-4*a*c+b^2)^(1/2))/c^2/(2*c*d-e*(b-(-4*a*c+b^2)^(1/2))
)^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2)+arctan(x*(2*c*d-e*(b+(-4*a*c+b^2)^(1/2
)))^(1/2)/(e*x^2+d)^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2))*(b^2-a*c+b*(-3*a*c+
b^2)/(-4*a*c+b^2)^(1/2))/c^2/(b+(-4*a*c+b^2)^(1/2))^(1/2)/(2*c*d-e*(b+(-4*a
*c+b^2)^(1/2)))^(1/2)
```

Rubi [A] (verified)

Time = 0.86 (sec) , antiderivative size = 366, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.241, Rules used = {1317, 223, 212, 327, 1706, 385, 211}

$$\int \frac{x^6}{\sqrt{d+ex^2}\,(a+bx^2+cx^4)}\,dx = \frac{\left(-\frac{b(b^2-3ac)}{\sqrt{b^2-4ac}}-ac+b^2\right)\arctan\left(\frac{x\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}{\sqrt{b-\sqrt{b^2-4ac}}\sqrt{d+ex^2}}\right)}{c^2\sqrt{b-\sqrt{b^2-4ac}}\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}} + \frac{\left(\frac{b(b^2-3ac)}{\sqrt{b^2-4ac}}-ac+b^2\right)\arctan\left(\frac{x\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}{\sqrt{\sqrt{b^2-4ac}+b}\sqrt{d+ex^2}}\right)}{c^2\sqrt{\sqrt{b^2-4ac}+b}\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}} - \frac{b\text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{c^2\sqrt{e}} - \frac{d\text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{2ce^{3/2}} + \frac{x\sqrt{d+ex^2}}{2ce}$$

```
[In] Int[x^6/(Sqrt[d + e*x^2]*(a + b*x^2 + c*x^4)),x]
```

```
[Out] (x*Sqrt[d + e*x^2])/(2*c*e) + ((b^2 - a*c - (b*(b^2 - 3*a*c))/Sqrt[b^2 - 4*
a*c])*ArcTan[(Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]*x)/(Sqrt[b - Sqrt[b^2
 - 4*a*c]]*Sqrt[d + e*x^2])])/(c^2*Sqrt[b - Sqrt[b^2 - 4*a*c]]*Sqrt[2*c*d -
 (b - Sqrt[b^2 - 4*a*c])*e]) + ((b^2 - a*c + (b*(b^2 - 3*a*c))/Sqrt[b^2 - 4
*a*c])*ArcTan[(Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]*x)/(Sqrt[b + Sqrt[b^
2 - 4*a*c]]*Sqrt[d + e*x^2])])/(c^2*Sqrt[b + Sqrt[b^2 - 4*a*c]]*Sqrt[2*c*d
- (b + Sqrt[b^2 - 4*a*c])*e]) - (d*ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]])/(2
*c*e^(3/2)) - (b*ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]])/(c^2*Sqrt[e])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 1317

```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_))/((a_) + (b_.)*(x_)^2 +
(c_.)*(x_)^4), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^q, (f*x)^m/(a +
 b*x^2 + c*x^4), x], x] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ[b^2 - 4*a
*c, 0] &&  !IntegerQ[q] && IntegerQ[m]
```

Rule 1706

```
Int[(Px_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^
(p_.), x_Symbol] :> Int[ExpandIntegrand[Px*(d + e*x^2)^q*(a + b*x^2 + c*x^4
)^p, x], x] /; FreeQ[{a, b, c, d, e, q}, x] && PolyQ[Px, x^2] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(-\frac{b}{c^2\sqrt{d+ex^2}}+\frac{x^2}{c\sqrt{d+ex^2}}+\frac{ab+(b^2-ac)\,x^2}{c^2\sqrt{d+ex^2}\,(a+bx^2+cx^4)}\right)dx \\
&= \frac{\int \frac{ab+(b^2-ac)x^2}{\sqrt{d+ex^2}(a+bx^2+cx^4)}\,dx}{c^2}-\frac{b\int \frac{1}{\sqrt{d+ex^2}}\,dx}{c^2}+\frac{\int \frac{x^2}{\sqrt{d+ex^2}}\,dx}{c} \\
&= \frac{x\sqrt{d+ex^2}}{2ce}+\frac{\int \left(\frac{b^2-ac+\frac{b\left(-b^2+3ac\right)}{\sqrt{b^2-4ac}}}{\left(b-\sqrt{b^2-4ac}+2cx^2\right)\sqrt{d+ex^2}}+\frac{b^2-ac-\frac{b\left(-b^2+3ac\right)}{\sqrt{b^2-4ac}}}{\left(b+\sqrt{b^2-4ac}+2cx^2\right)\sqrt{d+ex^2}}\right)dx}{c^2} \\
&\quad -\frac{b\text{Subst}\left(\int \frac{1}{1-ex^2}\,dx,x,\frac{x}{\sqrt{d+ex^2}}\right)}{c^2}-\frac{d\int \frac{1}{\sqrt{d+ex^2}}\,dx}{2ce}
\end{aligned}$$

$$=\frac{x\sqrt{d+ex^2}}{2ce}-\frac{b\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{c^2\sqrt{e}}+\frac{\left(b^2-ac-\frac{b(b^2-3ac)}{\sqrt{b^2-4ac}}\right)\int\frac{1}{\left(b-\sqrt{b^2-4ac}+2cx^2\right)\sqrt{d+ex^2}}dx}{c^2}+\frac{\left(b^2-ac+\frac{b(b^2-3ac)}{\sqrt{b^2-4ac}}\right)\int\frac{1}{\left(b+\sqrt{b^2-4ac}+2cx^2\right)\sqrt{d+ex^2}}dx}{c^2}-\frac{d\text{Subst}\left(\int\frac{1}{1-ex^2}dx,x,\frac{x}{\sqrt{d+ex^2}}\right)}{2ce}$$

$$=\frac{x\sqrt{d+ex^2}}{2ce}-\frac{d\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{2ce^{3/2}}-\frac{b\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{c^2\sqrt{e}}+\frac{\left(b^2-ac-\frac{b(b^2-3ac)}{\sqrt{b^2-4ac}}\right)\text{Subst}\left(\int\frac{1}{b-\sqrt{b^2-4ac}-\left(-2cd+\left(b-\sqrt{b^2-4ac}\right)e\right)x^2}dx,x,\frac{x}{\sqrt{d+ex^2}}\right)}{c^2}+\frac{\left(b^2-ac+\frac{b(b^2-3ac)}{\sqrt{b^2-4ac}}\right)\text{Subst}\left(\int\frac{1}{b+\sqrt{b^2-4ac}-\left(-2cd+\left(b+\sqrt{b^2-4ac}\right)e\right)x^2}dx,x,\frac{x}{\sqrt{d+ex^2}}\right)}{c^2}$$

$$=\frac{x\sqrt{d+ex^2}}{2ce}+\frac{\left(b^2-ac-\frac{b(b^2-3ac)}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}x}{\sqrt{b-\sqrt{b^2-4ac}}\sqrt{d+ex^2}}\right)}{c^2\sqrt{b-\sqrt{b^2-4ac}}\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}}+\frac{\left(b^2-ac+\frac{b(b^2-3ac)}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2cd-\left(b+\sqrt{b^2-4ac}\right)e}x}{\sqrt{b+\sqrt{b^2-4ac}}\sqrt{d+ex^2}}\right)}{c^2\sqrt{b+\sqrt{b^2-4ac}}\sqrt{2cd-\left(b+\sqrt{b^2-4ac}\right)e}}-\frac{d\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{2ce^{3/2}}-\frac{b\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{c^2\sqrt{e}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.78 (sec) , antiderivative size = 594, normalized size of antiderivative = 1.62

$$\int\frac{x^6}{\sqrt{d+ex^2}\left(a+bx^2+cx^4\right)}dx=\frac{x\sqrt{d+ex^2}}{2ce}+\frac{(-cd-2be)\text{arctanh}\left(\frac{\sqrt{e}x}{-\sqrt{d}+\sqrt{d+ex^2}}\right)}{c^2e^{3/2}}+\frac{\text{RootSum}\left[ae^4+4bde^2\#1^2-4ae^3\#1^2+16cd^2\#1^4-8bde\#1^4+6ae^2\#1^4+4bd\#1^6-4ae\#1^6+a\#1^8\right.}{}$$

```
[In] Integrate[x^6/(Sqrt[d + e*x^2]*(a + b*x^2 + c*x^4)),x]
```

```
[Out] (x*Sqrt[d + e*x^2])/(2*c*e) + ((-(c*d) - 2*b*e)*ArcTanh[(Sqrt[e]*x)/(-Sqrt[
d] + Sqrt[d + e*x^2])])/(c^2*e^(3/2)) + RootSum[a*e^4 + 4*b*d*e^2*#1^2 - 4*
```

```
a*e^3*#1^2 + 16*c*d^2*#1^4 - 8*b*d*e*#1^4 + 6*a*e^2*#1^4 + 4*b*d*#1^6 - 4*a
*e*#1^6 + a*#1^8 & , (-(a*b*e^3*Log[x]) + a*b*e^3*Log[-Sqrt[d] + Sqrt[d + e
*x^2] - x*#1] - 4*b^2*d*e*Log[x]*#1^2 + 4*a*c*d*e*Log[x]*#1^2 + 3*a*b*e^2*L
og[x]*#1^2 + 4*b^2*d*e*Log[-Sqrt[d] + Sqrt[d + e*x^2] - x*#1]*#1^2 - 4*a*c*
d*e*Log[-Sqrt[d] + Sqrt[d + e*x^2] - x*#1]*#1^2 - 3*a*b*e^2*Log[-Sqrt[d] +
Sqrt[d + e*x^2] - x*#1]*#1^2 + 4*b^2*d*Log[x]*#1^4 - 4*a*c*d*Log[x]*#1^4 -
3*a*b*e*Log[x]*#1^4 - 4*b^2*d*Log[-Sqrt[d] + Sqrt[d + e*x^2] - x*#1]*#1^4 +
 4*a*c*d*Log[-Sqrt[d] + Sqrt[d + e*x^2] - x*#1]*#1^4 + 3*a*b*e*Log[-Sqrt[d]
 + Sqrt[d + e*x^2] - x*#1]*#1^4 + a*b*Log[x]*#1^6 - a*b*Log[-Sqrt[d] + Sqrt
[d + e*x^2] - x*#1]*#1^6)/(b*d*e^2*#1 - a*e^3*#1 + 8*c*d^2*#1^3 - 4*b*d*e*#
1^3 + 3*a*e^2*#1^3 + 3*b*d*#1^5 - 3*a*e*#1^5 + a*#1^7) & ]/(4*c^2)
```

Maple [A] (verified)

Time = 0.80 (sec) , antiderivative size = 305, normalized size of antiderivative = 0.83

method	result
risch	$\frac{x\sqrt{e\,x^2+d}}{2ce}-\frac{\frac{(2be+cd)\ln\left(x\sqrt{e}+\sqrt{e\,x^2+d}\right)}{c\sqrt{e}}+\frac{ea\sqrt{2}\left(\frac{\left(2acd-b^2d+\sqrt{-d^2\left(4ac-b^2\right)}\,b\right)\arctan\left(\frac{a\sqrt{e\,x^2+d}\,\sqrt{2}}{x\sqrt{\left(-2ae+bd+\sqrt{-d^2\left(4ac-b^2\right)}\right)a}}\right)}{\sqrt{\left(-2ae+bd+\sqrt{-d^2\left(4ac-b^2\right)}\right)a}}-\cdots\right)}{c\sqrt{-d^2\left(4ac-\cdots\right.}}}{2ec}$
default	$\frac{\frac{x\sqrt{e\,x^2+d}}{2e}-\frac{d\ln\left(x\sqrt{e}+\sqrt{e\,x^2+d}\right)}{2e^{\frac{3}{2}}}}{c}-\frac{b\ln\left(x\sqrt{e}+\sqrt{e\,x^2+d}\right)}{c^2\sqrt{e}}-\frac{a\sqrt{2}\left(\frac{\left(2acd-b^2d+\sqrt{-d^2\left(4ac-b^2\right)}\,b\right)\arctan\left(\frac{a\sqrt{e\,x^2+d}\cdots}{x\sqrt{\left(-2ae+bd+\sqrt{-d\cdots}\right.}}\right.}{\sqrt{\left(-2ae+bd+\sqrt{-d^2\left(4ac-b^2\right)}\right)a}}\cdots\right.}{\cdots}$
pseudoelliptic	$-\frac{a\sqrt{2}\sqrt{\left(-2ae+bd+\sqrt{-4d^2\left(ac-\frac{b^2}{4}\right)}\right)a}\,e^{\frac{3}{2}}\left(-\frac{\sqrt{-4d^2\left(ac-\frac{b^2}{4}\right)}\,b}{2}+d\left(ac-\frac{b^2}{2}\right)\right)\operatorname{arctanh}\left(\frac{a\sqrt{e\,x^2+d}\,\sqrt{2}}{x\sqrt{\left(2ae-bd+\sqrt{-4d^2\left(ac-\frac{b^2}{4}\right)}\right)a}}\cdots\right.}{\cdots}$

```
[In] int(x^6/(c*x^4+b*x^2+a)/(e*x^2+d)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*x*(e*x^2+d)^(1/2)/c/e-1/2/e/c*((2*b*e+c*d)/c*ln(x*e^(1/2)+(e*x^2+d)^(1/
2))/e^(1/2)+e/c*a*2^(1/2)/(-d^2*(4*a*c-b^2))^(1/2)*((2*a*c*d-b^2*d+(-d^2*(4
*a*c-b^2))^(1/2)*b)/((-2*a*e+b*d+(-d^2*(4*a*c-b^2))^(1/2))*a)^(1/2)*arctan(
a/x*(e*x^2+d)^(1/2)*2^(1/2)/((-2*a*e+b*d+(-d^2*(4*a*c-b^2))^(1/2))*a)^(1/2)
)-(-2*a*c*d+b^2*d+(-d^2*(4*a*c-b^2))^(1/2)*b)/((2*a*e-b*d+(-d^2*(4*a*c-b^2)
)^(1/2))*a)^(1/2)*arctanh(a/x*(e*x^2+d)^(1/2)*2^(1/2)/((2*a*e-b*d+(-d^2*(4*
a*c-b^2))^(1/2))*a)^(1/2))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 7323 vs. $2(308) = 616$.

Time = 52.68 (sec) , antiderivative size = 14654, normalized size of antiderivative = 40.04

$$\int \frac{x^6}{\sqrt{d+ex^2}\,(a+bx^2+cx^4)}\,dx = \text{Too large to display}$$

```
[In] integrate(x^6/(c*x^4+b*x^2+a)/(e*x^2+d)^(1/2),x, algorithm="fricas")
[Out] Too large to include
```

Sympy [F]

$$\int \frac{x^6}{\sqrt{d+ex^2}\,(a+bx^2+cx^4)}\,dx = \int \frac{x^6}{\sqrt{d+ex^2}\,(a+bx^2+cx^4)}\,dx$$

```
[In] integrate(x**6/(c*x**4+b*x**2+a)/(e*x**2+d)**(1/2),x)
[Out] Integral(x**6/(sqrt(d + e*x**2)*(a + b*x**2 + c*x**4)), x)
```

Maxima [F]

$$\int \frac{x^6}{\sqrt{d+ex^2}\,(a+bx^2+cx^4)}\,dx = \int \frac{x^6}{(cx^4+bx^2+a)\sqrt{ex^2+d}}\,dx$$

```
[In] integrate(x^6/(c*x^4+b*x^2+a)/(e*x^2+d)^(1/2),x, algorithm="maxima")
[Out] integrate(x^6/((c*x^4 + b*x^2 + a)*sqrt(e*x^2 + d)), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{x^6}{\sqrt{d+ex^2}\,(a+bx^2+cx^4)}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(x^6/(c*x^4+b*x^2+a)/(e*x^2+d)^(1/2),x, algorithm="giac")
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:index.cc index_m i_lex_is_greater Err
or: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^6}{\sqrt{d+ex^2}\,(a+bx^2+cx^4)}\,dx=\int \frac{x^6}{\sqrt{e\,x^2+d}\,\left(c\,x^4+b\,x^2+a\right)}\,dx$$

```
[In] int(x^6/((d + e*x^2)^(1/2)*(a + b*x^2 + c*x^4)),x)
[Out] int(x^6/((d + e*x^2)^(1/2)*(a + b*x^2 + c*x^4)), x)
```

3.388 $\int \frac{x^4}{\sqrt{d+ex^2}(a+bx^2+cx^4)}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 298

$$\int \frac{x^4}{\sqrt{d+ex^2}\,(a+bx^2+cx^4)}\,dx = -\frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}\,x}{\sqrt{b-\sqrt{b^2-4ac}}\sqrt{d+ex^2}}\right)}{c\sqrt{b-\sqrt{b^2-4ac}}\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}} - \frac{\left(b+\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2cd-\left(b+\sqrt{b^2-4ac}\right)e}\,x}{\sqrt{b+\sqrt{b^2-4ac}}\sqrt{d+ex^2}}\right)}{c\sqrt{b+\sqrt{b^2-4ac}}\sqrt{2cd-\left(b+\sqrt{b^2-4ac}\right)e}} + \frac{\operatorname{arctanh}\left(\frac{\sqrt{e}\,x}{\sqrt{d+ex^2}}\right)}{c\sqrt{e}}$$

```
[Out] arctanh(x*e^(1/2)/(e*x^2+d)^(1/2))/c/e^(1/2)-arctan(x*(2*c*d-e*(b-(-4*a*c+b
^2)^(1/2)))^(1/2)/(e*x^2+d)^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2))*(b+(2*a*c-b
^2)/(-4*a*c+b^2)^(1/2))/c/(2*c*d-e*(b-(-4*a*c+b^2)^(1/2)))^(1/2)/(b-(-4*a*c
+b^2)^(1/2))^(1/2)-arctan(x*(2*c*d-e*(b+(-4*a*c+b^2)^(1/2)))^(1/2)/(e*x^2+d
)^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2))*(b+(-2*a*c+b^2)/(-4*a*c+b^2)^(1/2))/c
/(b+(-4*a*c+b^2)^(1/2))^(1/2)/(2*c*d-e*(b+(-4*a*c+b^2)^(1/2)))^(1/2)
```

Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 298, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.207, Rules used = {1317, 223, 212, 1706, 385, 211}

$$\int \frac{x^4}{\sqrt{d+ex^2}\,(a+bx^2+cx^4)}\,dx = -\frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{x\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}{\sqrt{b-\sqrt{b^2-4ac}}\sqrt{d+ex^2}}\right)}{c\sqrt{b-\sqrt{b^2-4ac}}\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}} - \frac{\left(\frac{b^2-2ac}{\sqrt{b^2-4ac}}+b\right)\arctan\left(\frac{x\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}{\sqrt{\sqrt{b^2-4ac}+b}\sqrt{d+ex^2}}\right)}{c\sqrt{\sqrt{b^2-4ac}+b}\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}} + \frac{\text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{c\sqrt{e}}$$

```
[In] Int[x^4/(Sqrt[d + e*x^2]*(a + b*x^2 + c*x^4)),x]
```

```
[Out] -(((b - (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2*c*d - (b - Sqrt[b^2
 - 4*a*c])*e]*x)/(Sqrt[b - Sqrt[b^2 - 4*a*c]]*Sqrt[d + e*x^2])]])/(c*Sqrt[b
- Sqrt[b^2 - 4*a*c]]*Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e])) - ((b + (b^2
 - 2*a*c)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e
]*x)/(Sqrt[b + Sqrt[b^2 - 4*a*c]]*Sqrt[d + e*x^2])]])/(c*Sqrt[b + Sqrt[b^2 -
 4*a*c]]*Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]) + ArcTanh[(Sqrt[e]*x)/Sqr
t[d + e*x^2]]/(c*Sqrt[e])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 1317

```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_))/((a_) + (b_.)*(x_)^2 +
(c_.)*(x_)^4), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^q, (f*x)^m/(a +
 b*x^2 + c*x^4), x], x] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ[b^2 - 4*a
*c, 0] &&  !IntegerQ[q] && IntegerQ[m]
```

Rule 1706

```
Int[(Px_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^
(p_.), x_Symbol] :> Int[ExpandIntegrand[Px*(d + e*x^2)^q*(a + b*x^2 + c*x^4
)^p, x], x] /; FreeQ[{a, b, c, d, e, q}, x] && PolyQ[Px, x^2] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[p]
```

Rubi steps

$$\text{integral} = \int \left(\frac{1}{c\sqrt{d+ex^2}} - \frac{a+bx^2}{c\sqrt{d+ex^2}\,(a+bx^2+cx^4)}\right) dx$$

$$= \frac{\int \frac{1}{\sqrt{d+ex^2}}\,dx}{c} - \frac{\int \frac{a+bx^2}{\sqrt{d+ex^2}(a+bx^2+cx^4)}\,dx}{c}$$

$$= -\frac{\int \left(\frac{b+\frac{-b^2+2ac}{\sqrt{b^2-4ac}}}{\left(b-\sqrt{b^2-4ac}+2cx^2\right)\sqrt{d+ex^2}} + \frac{b-\frac{-b^2+2ac}{\sqrt{b^2-4ac}}}{\left(b+\sqrt{b^2-4ac}+2cx^2\right)\sqrt{d+ex^2}}\right) dx}{c} + \frac{\text{Subst}\left(\int \frac{1}{1-ex^2}\,dx, x, \frac{x}{\sqrt{d+ex^2}}\right)}{c}$$

$$= \frac{\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{c\sqrt{e}} - \frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\int \frac{1}{\left(b-\sqrt{b^2-4ac}+2cx^2\right)\sqrt{d+ex^2}}\,dx}{c} - \frac{\left(b+\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\int \frac{1}{\left(b+\sqrt{b^2-4ac}+2cx^2\right)\sqrt{d+ex^2}}\,dx}{c}$$

$$= \frac{\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{c\sqrt{e}} - \frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\text{Subst}\left(\int \frac{1}{b-\sqrt{b^2-4ac}-\left(-2cd+\left(b-\sqrt{b^2-4ac}\right)e\right)x^2}\,dx, x, \frac{x}{\sqrt{d+ex^2}}\right)}{c} - \frac{\left(b+\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\text{Subst}\left(\int \frac{1}{b+\sqrt{b^2-4ac}-\left(-2cd+\left(b+\sqrt{b^2-4ac}\right)e\right)x^2}\,dx, x, \frac{x}{\sqrt{d+ex^2}}\right)}{c}$$

$$= -\frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}x}{\sqrt{b-\sqrt{b^2-4ac}}\sqrt{d+ex^2}}\right)}{c\sqrt{b-\sqrt{b^2-4ac}}\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}} - \frac{\left(b+\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2cd-\left(b+\sqrt{b^2-4ac}\right)e}x}{\sqrt{b+\sqrt{b^2-4ac}}\sqrt{d+ex^2}}\right)}{c\sqrt{b+\sqrt{b^2-4ac}}\sqrt{2cd-\left(b+\sqrt{b^2-4ac}\right)e}} + \frac{\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{c\sqrt{e}}$$

Mathematica [A] (verified)

Time = 10.67 (sec) , antiderivative size = 292, normalized size of antiderivative = 0.98

$$\int \frac{x^4}{\sqrt{d+ex^2}\,(a+bx^2+cx^4)}\,dx$$

$$= \frac{-\frac{\left(b+\frac{-b^2+2ac}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2cd-be+\sqrt{b^2-4ac}e}x}{\sqrt{b-\sqrt{b^2-4ac}}\sqrt{d+ex^2}}\right)}{\sqrt{b-\sqrt{b^2-4ac}}\sqrt{2cd+\left(-b+\sqrt{b^2-4ac}\right)e}} - \frac{\left(b+\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2cd-\left(b+\sqrt{b^2-4ac}\right)e}x}{\sqrt{b+\sqrt{b^2-4ac}}\sqrt{d+ex^2}}\right)}{\sqrt{b+\sqrt{b^2-4ac}}\sqrt{2cd-\left(b+\sqrt{b^2-4ac}\right)e}} + \frac{\operatorname{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{\sqrt{e}}}{c}$$

```
[In] Integrate[x^4/(Sqrt[d + e*x^2]*(a + b*x^2 + c*x^4)),x]
```

```
[Out] (-(((b + (-b^2 + 2*a*c)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2*c*d - b*e + Sqrt[
b^2 - 4*a*c]*e]*x)/(Sqrt[b - Sqrt[b^2 - 4*a*c]]*Sqrt[d + e*x^2])])/(Sqrt[b
- Sqrt[b^2 - 4*a*c]]*Sqrt[2*c*d + (-b + Sqrt[b^2 - 4*a*c])*e])) - ((b + (b^
2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*
e]*x)/(Sqrt[b + Sqrt[b^2 - 4*a*c]]*Sqrt[d + e*x^2])])/(Sqrt[b + Sqrt[b^2 -
4*a*c]]*Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]) + ArcTanh[(Sqrt[e]*x)/Sqrt
[d + e*x^2]]/Sqrt[e])/c
```

Maple [A] (verified)

Time = 0.46 (sec) , antiderivative size = 252, normalized size of antiderivative = 0.85

method	result
default	$\frac{\ln\left(x\sqrt{e}+\sqrt{e\,x^2+d}\right)}{c\sqrt{e}}+\frac{a\sqrt{2}\left(\frac{\left(-bd+\sqrt{-d^2\left(4ac-b^2\right)}\right)\arctan\left(\frac{a\sqrt{e\,x^2+d}\,\sqrt{2}}{x\sqrt{\left(-2ae+bd+\sqrt{-d^2\left(4ac-b^2\right)}\right)a}}\right)}{\sqrt{\left(-2ae+bd+\sqrt{-d^2\left(4ac-b^2\right)}\right)a}}-\frac{\left(bd+\sqrt{-d^2\left(4ac-b^2\right)}\right)\text{arct}}{\sqrt{\left(2ae-\right.}}\right.}{2c\sqrt{-d^2(4ac-b^2)}}$
pseudoelliptic	$-\frac{a\sqrt{\left(-2ae+bd+\sqrt{-4d^2\left(ac-\frac{b^2}{4}\right)}\right)a}\,\sqrt{2}\,\sqrt{e}\left(bd+\sqrt{-4d^2\left(ac-\frac{b^2}{4}\right)}\right)\text{arctanh}\left(\frac{a\sqrt{e\,x^2+d}\,\sqrt{2}}{x\sqrt{\left(2ae-bd+\sqrt{-4d^2\left(ac-\frac{b^2}{4}\right)}\right)a}}\right)-\sqrt{\left(2\right.}}{2\sqrt{e}\sqrt{\left(-2a\right.}}$

```
[In] int(x^4/(c*x^4+b*x^2+a)/(e*x^2+d)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/c*ln(x*e^(1/2)+(e*x^2+d)^(1/2))/e^(1/2)+1/2/c*a*2^(1/2)/(-d^2*(4*a*c-b^2)
)^(1/2)*((-b*d+(-d^2*(4*a*c-b^2))^(1/2))/((-2*a*e+b*d+(-d^2*(4*a*c-b^2))^(1
/2))*a)^(1/2)*arctan(a/x*(e*x^2+d)^(1/2)*2^(1/2)/((-2*a*e+b*d+(-d^2*(4*a*c-
b^2))^(1/2))*a)^(1/2))-(b*d+(-d^2*(4*a*c-b^2))^(1/2))/((2*a*e-b*d+(-d^2*(4*
a*c-b^2))^(1/2))*a)^(1/2)*arctanh(a/x*(e*x^2+d)^(1/2)*2^(1/2)/((2*a*e-b*d+(
-d^2*(4*a*c-b^2))^(1/2))*a)^(1/2)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 5543 vs. $2(252) = 504$.

Time = 9.05 (sec) , antiderivative size = 11094, normalized size of antiderivative = 37.23

$$\int \frac{x^4}{\sqrt{d+ex^2}\,(a+bx^2+cx^4)}\,dx = \text{Too large to display}$$

```
[In] integrate(x^4/(c*x^4+b*x^2+a)/(e*x^2+d)^(1/2),x, algorithm="fricas")
```

```
[Out] Too large to include
```

Sympy [F]

$$\int \frac{x^4}{\sqrt{d+ex^2}\,(a+bx^2+cx^4)}\,dx = \int \frac{x^4}{\sqrt{d+ex^2}\,(a+bx^2+cx^4)}\,dx$$

```
[In] integrate(x**4/(c*x**4+b*x**2+a)/(e*x**2+d)**(1/2),x)
[Out] Integral(x**4/(sqrt(d + e*x**2)*(a + b*x**2 + c*x**4)), x)
```

Maxima [F]

$$\int \frac{x^4}{\sqrt{d+ex^2}\,(a+bx^2+cx^4)}\,dx = \int \frac{x^4}{(cx^4+bx^2+a)\sqrt{ex^2+d}}\,dx$$

```
[In] integrate(x^4/(c*x^4+b*x^2+a)/(e*x^2+d)^(1/2),x, algorithm="maxima")
[Out] integrate(x^4/((c*x^4 + b*x^2 + a)*sqrt(e*x^2 + d)), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{x^4}{\sqrt{d+ex^2}\,(a+bx^2+cx^4)}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(x^4/(c*x^4+b*x^2+a)/(e*x^2+d)^(1/2),x, algorithm="giac")
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:index.cc index_m i_lex_is_greater Err
or: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^4}{\sqrt{d+ex^2}\,(a+bx^2+cx^4)}\,dx = \int \frac{x^4}{\sqrt{e\,x^2+d}\,\left(c\,x^4+b\,x^2+a\right)}\,dx$$

```
[In] int(x^4/((d + e*x^2)^(1/2)*(a + b*x^2 + c*x^4)),x)
[Out] int(x^4/((d + e*x^2)^(1/2)*(a + b*x^2 + c*x^4)), x)
```

3.389 $\int \frac{x^2}{\sqrt{d+ex^2}(a+bx^2+cx^4)}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 240

$$\int \frac{x^2}{\sqrt{d+ex^2}\,(a+bx^2+cx^4)}\,dx = -\frac{\sqrt{b-\sqrt{b^2-4ac}}\arctan\left(\frac{\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}\,x}{\sqrt{b-\sqrt{b^2-4ac}}\sqrt{d+ex^2}}\right)}{\sqrt{b^2-4ac}\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}} + \frac{\sqrt{b+\sqrt{b^2-4ac}}\arctan\left(\frac{\sqrt{2cd-\left(b+\sqrt{b^2-4ac}\right)e}\,x}{\sqrt{b+\sqrt{b^2-4ac}}\sqrt{d+ex^2}}\right)}{\sqrt{b^2-4ac}\sqrt{2cd-\left(b+\sqrt{b^2-4ac}\right)e}}$$

```
[Out] -arctan(x*(2*c*d-e*(b-(-4*a*c+b^2)^(1/2)))^(1/2)/(e*x^2+d)^(1/2)/(b-(-4*a*c
+b^2)^(1/2))^(1/2))*(b-(-4*a*c+b^2)^(1/2))^(1/2)/(-4*a*c+b^2)^(1/2)/(2*c*d-
e*(b-(-4*a*c+b^2)^(1/2)))^(1/2)+arctan(x*(2*c*d-e*(b+(-4*a*c+b^2)^(1/2)))^(
1/2)/(e*x^2+d)^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2))*(b+(-4*a*c+b^2)^(1/2))^(
1/2)/(-4*a*c+b^2)^(1/2)/(2*c*d-e*(b+(-4*a*c+b^2)^(1/2)))^(1/2)
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 240, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used

$= \{1317, 385, 211\}$

$$\int \frac{x^2}{\sqrt{d+ex^2}\,(a+bx^2+cx^4)}\,dx = \frac{\sqrt{\sqrt{b^2-4ac}+b}\arctan\left(\frac{x\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}{\sqrt{\sqrt{b^2-4ac}+b}\sqrt{d+ex^2}}\right)}{\sqrt{b^2-4ac}\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}} - \frac{\sqrt{b-\sqrt{b^2-4ac}}\arctan\left(\frac{x\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}{\sqrt{b-\sqrt{b^2-4ac}}\sqrt{d+ex^2}}\right)}{\sqrt{b^2-4ac}\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}$$

```
[In] Int[x^2/(Sqrt[d + e*x^2]*(a + b*x^2 + c*x^4)),x]
```

```
[Out] -((Sqrt[b - Sqrt[b^2 - 4*a*c]]*ArcTan[(Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])
*e]*x)/(Sqrt[b - Sqrt[b^2 - 4*a*c]]*Sqrt[d + e*x^2])])/(Sqrt[b^2 - 4*a*c]*S
qrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e])) + (Sqrt[b + Sqrt[b^2 - 4*a*c]]*Arc
Tan[(Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]*x)/(Sqrt[b + Sqrt[b^2 - 4*a*c]
]*Sqrt[d + e*x^2])])/(Sqrt[b^2 - 4*a*c]*Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c]
)*e])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 1317

```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_))/((a_) + (b_.)*(x_)^2 +
(c_.)*(x_)^4), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^q, (f*x)^m/(a +
 b*x^2 + c*x^4), x], x] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ[b^2 - 4*a
*c, 0] &&  !IntegerQ[q] && IntegerQ[m]
```

Rubi steps

$$\text{integral} = \int \left(\frac{1-\frac{b}{\sqrt{b^2-4ac}}}{\left(b-\sqrt{b^2-4ac}+2cx^2\right)\sqrt{d+ex^2}} + \frac{1+\frac{b}{\sqrt{b^2-4ac}}}{\left(b+\sqrt{b^2-4ac}+2cx^2\right)\sqrt{d+ex^2}}\right) dx$$

$$= \left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\int \frac{1}{\left(b-\sqrt{b^2-4ac}+2cx^2\right)\sqrt{d+ex^2}}\,dx + \left(1+\frac{b}{\sqrt{b^2-4ac}}\right)\int \frac{1}{\left(b+\sqrt{b^2-4ac}+2cx^2\right)\sqrt{d+ex^2}}\,dx$$

$$= \left(1 - \frac{b}{\sqrt{b^2-4ac}}\right)\text{Subst}\left(\int \frac{1}{b-\sqrt{b^2-4ac}-\left(-2cd+\left(b-\sqrt{b^2-4ac}\right)e\right)x^2}\,dx, x, \frac{x}{\sqrt{d+ex^2}}\right) + \left(1 + \frac{b}{\sqrt{b^2-4ac}}\right)\text{Subst}\left(\int \frac{1}{b+\sqrt{b^2-4ac}-\left(-2cd+\left(b+\sqrt{b^2-4ac}\right)e\right)x^2}\,dx, x, \frac{x}{\sqrt{d+ex^2}}\right)$$

$$= -\frac{\sqrt{b-\sqrt{b^2-4ac}}\tan^{-1}\left(\frac{\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}x}{\sqrt{b-\sqrt{b^2-4ac}}\sqrt{d+ex^2}}\right)}{\sqrt{b^2-4ac}\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}} + \frac{\sqrt{b+\sqrt{b^2-4ac}}\tan^{-1}\left(\frac{\sqrt{2cd-\left(b+\sqrt{b^2-4ac}\right)e}x}{\sqrt{b+\sqrt{b^2-4ac}}\sqrt{d+ex^2}}\right)}{\sqrt{b^2-4ac}\sqrt{2cd-\left(b+\sqrt{b^2-4ac}\right)e}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 8.66 (sec) , antiderivative size = 227, normalized size of antiderivative = 0.95

$$\int \frac{x^2}{\sqrt{d+ex^2}\,(a+bx^2+cx^4)}\,dx = -d\text{RootSum}\Big[ae^4+4bde^2\#1^2-4ae^3\#1^2+16cd^2\#1^4 - 8bde\#1^4+6ae^2\#1^4+4bd\#1^6-4ae\#1^6 + a\#1^8\&, \frac{-e\log(x)\#1+e\log\left(-\sqrt{d}+\sqrt{d+ex^2}-x\#1\right)\#1+\log(x)\#1^3-\log\left(-\sqrt{d}+\sqrt{d+ex^2}-x\right.}{-bde^2+ae^3-8cd^2\#1^2+4bde\#1^2-3ae^2\#1^2-3bd\#1^4+3ae\#1^4-a\#1^6}$$

```
[In] Integrate[x^2/(Sqrt[d + e*x^2]*(a + b*x^2 + c*x^4)),x]
```

```
[Out] -(d*RootSum[a*e^4 + 4*b*d*e^2*#1^2 - 4*a*e^3*#1^2 + 16*c*d^2*#1^4 - 8*b*d*e
*#1^4 + 6*a*e^2*#1^4 + 4*b*d*#1^6 - 4*a*e*#1^6 + a*#1^8 & , (-(e*Log[x]*#1)
 + e*Log[-Sqrt[d] + Sqrt[d + e*x^2] - x*#1]*#1 + Log[x]*#1^3 - Log[-Sqrt[d]
 + Sqrt[d + e*x^2] - x*#1]*#1^3)/(-(b*d*e^2) + a*e^3 - 8*c*d^2*#1^2 + 4*b*d
*e*#1^2 - 3*a*e^2*#1^2 - 3*b*d*#1^4 + 3*a*e*#1^4 - a*#1^6) & ])
```

Maple [A] (verified)

Time = 0.40 (sec) , antiderivative size = 233, normalized size of antiderivative = 0.97

method	result
default	$\frac{\sqrt{2}\,ad\left(\operatorname{arctanh}\left(\frac{a\sqrt{e\,x^2+d}\,\sqrt{2}}{x\sqrt{\left(2ae-bd+\sqrt{-4d^2\left(ac-\frac{b^2}{4}\right)}\right)a}}\right)\sqrt{\left(-2ae+bd+\sqrt{-4d^2\left(ac-\frac{b^2}{4}\right)}\right)a}+\arctan\left(\frac{a\sqrt{e\,x^2+d}\,\sqrt{2}}{x\sqrt{\left(-2ae+bd+\sqrt{-4d^2\left(ac-\cdots\right.}}}\right.\right.}{\sqrt{-4d^2\left(ac-\frac{b^2}{4}\right)}\,\sqrt{\left(-2ae+bd+\sqrt{-4d^2\left(ac-\frac{b^2}{4}\right)}\right)a}\,\sqrt{\left(2ae-bd+\sqrt{-4d^2\left(ac-\frac{b^2}{4}\right)}\right.\cdots}}$
pseudoelliptic	$\frac{\sqrt{2}\,ad\left(\operatorname{arctanh}\left(\frac{a\sqrt{e\,x^2+d}\,\sqrt{2}}{x\sqrt{\left(2ae-bd+\sqrt{-4d^2\left(ac-\frac{b^2}{4}\right)}\right)a}}\right)\sqrt{\left(-2ae+bd+\sqrt{-4d^2\left(ac-\frac{b^2}{4}\right)}\right)a}+\arctan\left(\frac{a\sqrt{e\,x^2+d}\,\sqrt{2}}{x\sqrt{\left(-2ae+bd+\sqrt{-4d^2\left(ac-\cdots\right.}}}\right.\right.}{\sqrt{-4d^2\left(ac-\frac{b^2}{4}\right)}\,\sqrt{\left(-2ae+bd+\sqrt{-4d^2\left(ac-\frac{b^2}{4}\right)}\right)a}\,\sqrt{\left(2ae-bd+\sqrt{-4d^2\left(ac-\frac{b^2}{4}\right)}\right.\cdots}}$

```
[In] int(x^2/(c*x^4+b*x^2+a)/(e*x^2+d)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2^(1/2)*a*d*(arctanh(a/x*(e*x^2+d)^(1/2)*2^(1/2)/((2*a*e-b*d+(-4*d^2*(a*c-1
/4*b^2))^(1/2))*a)^(1/2))*((-2*a*e+b*d+(-4*d^2*(a*c-1/4*b^2))^(1/2))*a)^(1/
2)+arctan(a/x*(e*x^2+d)^(1/2)*2^(1/2)/((-2*a*e+b*d+(-4*d^2*(a*c-1/4*b^2))^(
1/2))*a)^(1/2))*((2*a*e-b*d+(-4*d^2*(a*c-1/4*b^2))^(1/2))*a)^(1/2))/(-4*d^2
*(a*c-1/4*b^2))^(1/2)/((-2*a*e+b*d+(-4*d^2*(a*c-1/4*b^2))^(1/2))*a)^(1/2)/(
(2*a*e-b*d+(-4*d^2*(a*c-1/4*b^2))^(1/2))*a)^(1/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3395 vs. 2(200) = 400.

Time = 2.46 (sec) , antiderivative size = 3395, normalized size of antiderivative = 14.15

$$\int \frac{x^2}{\sqrt{d+ex^2}\,(a+bx^2+cx^4)}\,dx = \text{Too large to display}$$

```
[In] integrate(x^2/(c*x^4+b*x^2+a)/(e*x^2+d)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/4*sqrt(1/2)*sqrt(-(b*d - 2*a*e + ((b^2*c - 4*a*c^2)*d^2 - (b^3 - 4*a*b*c)
*d*e + (a*b^2 - 4*a^2*c)*e^2)*sqrt(d^2/((b^2*c^2 - 4*a*c^3)*d^4 - 2*(b^3*c
- 4*a*b*c^2)*d^3*e + (b^4 - 2*a*b^2*c - 8*a^2*c^2)*d^2*e^2 - 2*(a*b^3 - 4*a
^2*b*c)*d*e^3 + (a^2*b^2 - 4*a^3*c)*e^4)))/((b^2*c - 4*a*c^2)*d^2 - (b^3 -
4*a*b*c)*d*e + (a*b^2 - 4*a^2*c)*e^2))*log((((b^2*c - 4*a*c^2)*d^3 - (b^3 -
 4*a*b*c)*d^2*e + (a*b^2 - 4*a^2*c)*d*e^2)*sqrt(d^2/((b^2*c^2 - 4*a*c^3)*d^
4 - 2*(b^3*c - 4*a*b*c^2)*d^3*e + (b^4 - 2*a*b^2*c - 8*a^2*c^2)*d^2*e^2 - 2
*(a*b^3 - 4*a^2*b*c)*d*e^3 + (a^2*b^2 - 4*a^3*c)*e^4))*x^2 + 2*a*d^2 - (b*d
^2 - 4*a*d*e)*x^2 + 2*sqrt(1/2)*((b^2 - 4*a*c)*d^2*x - ((b^3*c - 4*a*b*c^2)
*d^3 - (b^4 - 2*a*b^2*c - 8*a^2*c^2)*d^2*e + 3*(a*b^3 - 4*a^2*b*c)*d*e^2 -
```

2*(a^2*b^2 - 4*a^3*c)*e^3)*sqrt(d^2/((b^2*c^2 - 4*a*c^3)*d^4 - 2*(b^3*c - 4*a*b*c^2)*d^3*e + (b^4 - 2*a*b^2*c - 8*a^2*c^2)*d^2*e^2 - 2*(a*b^3 - 4*a^2*b*c)*d*e^3 + (a^2*b^2 - 4*a^3*c)*e^4))*x)*sqrt(e*x^2 + d)*sqrt(-(b*d - 2*a*e + ((b^2*c - 4*a*c^2)*d^2 - (b^3 - 4*a*b*c)*d*e + (a*b^2 - 4*a^2*c)*e^2)*sqrt(d^2/((b^2*c^2 - 4*a*c^3)*d^4 - 2*(b^3*c - 4*a*b*c^2)*d^3*e + (b^4 - 2*a*b^2*c - 8*a^2*c^2)*d^2*e^2 - 2*(a*b^3 - 4*a^2*b*c)*d*e^3 + (a^2*b^2 - 4*a^3*c)*e^4)))/((b^2*c - 4*a*c^2)*d^2 - (b^3 - 4*a*b*c)*d*e + (a*b^2 - 4*a^2*c)*e^2)))/x^2) - 1/4*sqrt(1/2)*sqrt(-(b*d - 2*a*e + ((b^2*c - 4*a*c^2)*d^2 - (b^3 - 4*a*b*c)*d*e + (a*b^2 - 4*a^2*c)*e^2)*sqrt(d^2/((b^2*c^2 - 4*a*c^3)*d^4 - 2*(b^3*c - 4*a*b*c^2)*d^3*e + (b^4 - 2*a*b^2*c - 8*a^2*c^2)*d^2*e^2 - 2*(a*b^3 - 4*a^2*b*c)*d*e^3 + (a^2*b^2 - 4*a^3*c)*e^4)))/((b^2*c - 4*a*c^2)*d^2 - (b^3 - 4*a*b*c)*d*e + (a*b^2 - 4*a^2*c)*e^2))*log((((b^2*c - 4*a*c^2)*d^3 - (b^3 - 4*a*b*c)*d^2*e + (a*b^2 - 4*a^2*c)*d*e^2)*sqrt(d^2/((b^2*c^2 - 4*a*c^3)*d^4 - 2*(b^3*c - 4*a*b*c^2)*d^3*e + (b^4 - 2*a*b^2*c - 8*a^2*c^2)*d^2*e^2 - 2*(a*b^3 - 4*a^2*b*c)*d*e^3 + (a^2*b^2 - 4*a^3*c)*e^4))*x^2 + 2*a*d^2 - (b*d^2 - 4*a*d*e)*x^2 - 2*sqrt(1/2)*((b^2 - 4*a*c)*d^2*x - ((b^3*c - 4*a*b*c^2)*d^3 - (b^4 - 2*a*b^2*c - 8*a^2*c^2)*d^2*e + 3*(a*b^3 - 4*a^2*b*c)*d*e^2 - 2*(a^2*b^2 - 4*a^3*c)*e^3)*sqrt(d^2/((b^2*c^2 - 4*a*c^3)*d^4 - 2*(b^3*c - 4*a*b*c^2)*d^3*e + (b^4 - 2*a*b^2*c - 8*a^2*c^2)*d^2*e^2 - 2*(a*b^3 - 4*a^2*b*c)*d*e^3 + (a^2*b^2 - 4*a^3*c)*e^4))*x)*sqrt(e*x^2 + d)*sqrt(-(b*d - 2*a*e + ((b^2*c - 4*a*c^2)*d^2 - (b^3 - 4*a*b*c)*d*e + (a*b^2 - 4*a^2*c)*e^2)*sqrt(d^2/((b^2*c^2 - 4*a*c^3)*d^4 - 2*(b^3*c - 4*a*b*c^2)*d^3*e + (b^4 - 2*a*b^2*c - 8*a^2*c^2)*d^2*e^2 - 2*(a*b^3 - 4*a^2*b*c)*d*e^3 + (a^2*b^2 - 4*a^3*c)*e^4)))/((b^2*c - 4*a*c^2)*d^2 - (b^3 - 4*a*b*c)*d*e + (a*b^2 - 4*a^2*c)*e^2)))/x^2) - 1/4*sqrt(1/2)*sqrt(-(b*d - 2*a*e - ((b^2*c - 4*a*c^2)*d^2 - (b^3 - 4*a*b*c)*d*e + (a*b^2 - 4*a^2*c)*e^2)*sqrt(d^2/((b^2*c^2 - 4*a*c^3)*d^4 - 2*(b^3*c - 4*a*b*c^2)*d^3*e + (b^4 - 2*a*b^2*c - 8*a^2*c^2)*d^2*e^2 - 2*(a*b^3 - 4*a^2*b*c)*d*e^3 + (a^2*b^2 - 4*a^3*c)*e^4)))/((b^2*c - 4*a*c^2)*d^2 - (b^3 - 4*a*b*c)*d*e + (a*b^2 - 4*a^2*c)*e^2))*log(-(((b^2*c - 4*a*c^2)*d^3 - (b^3 - 4*a*b*c)*d^2*e + (a*b^2 - 4*a^2*c)*d*e^2)*sqrt(d^2/((b^2*c^2 - 4*a*c^3)*d^4 - 2*(b^3*c - 4*a*b*c^2)*d^3*e + (b^4 - 2*a*b^2*c - 8*a^2*c^2)*d^2*e^2 - 2*(a*b^3 - 4*a^2*b*c)*d*e^3 + (a^2*b^2 - 4*a^3*c)*e^4))*x^2 - 2*a*d^2 + (b*d^2 - 4*a*d*e)*x^2 + 2*sqrt(1/2)*((b^2 - 4*a*c)*d^2*x + ((b^3*c - 4*a*b*c^2)*d^3 - (b^4 - 2*a*b^2*c - 8*a^2*c^2)*d^2*e + 3*(a*b^3 - 4*a^2*b*c)*d*e^2 - 2*(a^2*b^2 - 4*a^3*c)*e^3)*sqrt(d^2/((b^2*c^2 - 4*a*c^3)*d^4 - 2*(b^3*c - 4*a*b*c^2)*d^3*e + (b^4 - 2*a*b^2*c - 8*a^2*c^2)*d^2*e^2 - 2*(a*b^3 - 4*a^2*b*c)*d*e^3 + (a^2*b^2 - 4*a^3*c)*e^4))*x)*sqrt(e*x^2 + d)*sqrt(-(b*d - 2*a*e - ((b^2*c - 4*a*c^2)*d^2 - (b^3 - 4*a*b*c)*d*e + (a*b^2 - 4*a^2*c)*e^2)*sqrt(d^2/((b^2*c^2 - 4*a*c^3)*d^4 - 2*(b^3*c - 4*a*b*c^2)*d^3*e + (b^4 - 2*a*b^2*c - 8*a^2*c^2)*d^2*e^2 - 2*(a*b^3 - 4*a^2*b*c)*d*e^3 + (a^2*b^2 - 4*a^3*c)*e^4)))/((b^2*c - 4*a*c^2)*d^2 - (b^3 - 4*a*b*c)*d*e + (a*b^2 - 4*a^2*c)*e^2)))/x^2) + 1/4*sqrt(1/2)*sqrt(-(b*d - 2*a*e - ((b^2*c - 4*a*c^2)*d^2 - (b^3 - 4*a*b*c)*d*e + (a*b^2 - 4*a^2*c)*e^2)*sqrt(d^2/((b^2*c^2 - 4*a*c^3)*d^4 - 2*(b^3*c - 4*a*b*c^2)*d^3*e + (b^4 - 2*a*b^2*c - 8*a^2*c^2)*d^2*e^2 - 2*(a*b^3 - 4*a^2*b*c)*d*e^3 + (a^2*b^2 -

```
4*a^3*c)*e^4)))/((b^2*c - 4*a*c^2)*d^2 - (b^3 - 4*a*b*c)*d*e + (a*b^2 - 4*
a^2*c)*e^2))*log(-(((b^2*c - 4*a*c^2)*d^3 - (b^3 - 4*a*b*c)*d^2*e + (a*b^2
- 4*a^2*c)*d*e^2)*sqrt(d^2/((b^2*c^2 - 4*a*c^3)*d^4 - 2*(b^3*c - 4*a*b*c^2)
*d^3*e + (b^4 - 2*a*b^2*c - 8*a^2*c^2)*d^2*e^2 - 2*(a*b^3 - 4*a^2*b*c)*d*e^
3 + (a^2*b^2 - 4*a^3*c)*e^4))*x^2 - 2*a*d^2 + (b*d^2 - 4*a*d*e)*x^2 - 2*sqr
t(1/2)*((b^2 - 4*a*c)*d^2*x + ((b^3*c - 4*a*b*c^2)*d^3 - (b^4 - 2*a*b^2*c -
 8*a^2*c^2)*d^2*e + 3*(a*b^3 - 4*a^2*b*c)*d*e^2 - 2*(a^2*b^2 - 4*a^3*c)*e^3
)*sqrt(d^2/((b^2*c^2 - 4*a*c^3)*d^4 - 2*(b^3*c - 4*a*b*c^2)*d^3*e + (b^4 -
2*a*b^2*c - 8*a^2*c^2)*d^2*e^2 - 2*(a*b^3 - 4*a^2*b*c)*d*e^3 + (a^2*b^2 - 4
*a^3*c)*e^4))*x)*sqrt(e*x^2 + d)*sqrt(-(b*d - 2*a*e - ((b^2*c - 4*a*c^2)*d^
2 - (b^3 - 4*a*b*c)*d*e + (a*b^2 - 4*a^2*c)*e^2)*sqrt(d^2/((b^2*c^2 - 4*a*c
^3)*d^4 - 2*(b^3*c - 4*a*b*c^2)*d^3*e + (b^4 - 2*a*b^2*c - 8*a^2*c^2)*d^2*e
^2 - 2*(a*b^3 - 4*a^2*b*c)*d*e^3 + (a^2*b^2 - 4*a^3*c)*e^4)))/((b^2*c - 4*a
*c^2)*d^2 - (b^3 - 4*a*b*c)*d*e + (a*b^2 - 4*a^2*c)*e^2)))/x^2)
```

Sympy [F]

$$\int \frac{x^2}{\sqrt{d+ex^2}\,(a+bx^2+cx^4)}\,dx = \int \frac{x^2}{\sqrt{d+ex^2}\,(a+bx^2+cx^4)}\,dx$$

```
[In] integrate(x**2/(c*x**4+b*x**2+a)/(e*x**2+d)**(1/2),x)
[Out] Integral(x**2/(sqrt(d + e*x**2)*(a + b*x**2 + c*x**4)), x)
```

Maxima [F]

$$\int \frac{x^2}{\sqrt{d+ex^2}\,(a+bx^2+cx^4)}\,dx = \int \frac{x^2}{(cx^4+bx^2+a)\sqrt{ex^2+d}}\,dx$$

```
[In] integrate(x^2/(c*x^4+b*x^2+a)/(e*x^2+d)^(1/2),x, algorithm="maxima")
[Out] integrate(x^2/((c*x^4 + b*x^2 + a)*sqrt(e*x^2 + d)), x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{x^2}{\sqrt{d+ex^2}\,(a+bx^2+cx^4)}\,dx = \text{Timed out}$$

```
[In] integrate(x^2/(c*x^4+b*x^2+a)/(e*x^2+d)^(1/2),x, algorithm="giac")
[Out] Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{\sqrt{d+ex^2}\,(a+bx^2+cx^4)}\,dx = \int \frac{x^2}{\sqrt{e\,x^2+d}\,\left(c\,x^4+b\,x^2+a\right)}\,dx$$

```
[In] int(x^2/((d + e*x^2)^(1/2)*(a + b*x^2 + c*x^4)),x)
[Out] int(x^2/((d + e*x^2)^(1/2)*(a + b*x^2 + c*x^4)), x)
```

3.390 $\int \frac{1}{\sqrt{d+ex^2}(a+bx^2+cx^4)}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 243

$$\int \frac{1}{\sqrt{d+ex^2}\,(a+bx^2+cx^4)}\,dx=\frac{2c\arctan\left(\frac{\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}\,x}{\sqrt{b-\sqrt{b^2-4ac}}\sqrt{d+ex^2}}\right)}{\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}}-\frac{2c\arctan\left(\frac{\sqrt{2cd-\left(b+\sqrt{b^2-4ac}\right)e}\,x}{\sqrt{b+\sqrt{b^2-4ac}}\sqrt{d+ex^2}}\right)}{\sqrt{b^2-4ac}\sqrt{b+\sqrt{b^2-4ac}}\sqrt{2cd-\left(b+\sqrt{b^2-4ac}\right)e}}$$

```
[Out] 2*c*arctan(x*(2*c*d-e*(b-(-4*a*c+b^2)^(1/2)))^(1/2)/(e*x^2+d)^(1/2)/(b-(-4*
a*c+b^2)^(1/2))^(1/2))/(-4*a*c+b^2)^(1/2)/(2*c*d-e*(b-(-4*a*c+b^2)^(1/2)))^
(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2)-2*c*arctan(x*(2*c*d-e*(b+(-4*a*c+b^2)^(1
/2)))^(1/2)/(e*x^2+d)^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2))/(-4*a*c+b^2)^(1/2
)/(b+(-4*a*c+b^2)^(1/2))^(1/2)/(2*c*d-e*(b+(-4*a*c+b^2)^(1/2)))^(1/2)
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 243, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used

$= \{1188, 385, 211\}$

$$\int \frac{1}{\sqrt{d+ex^2}\,(a+bx^2+cx^4)}\,dx = \frac{2c\arctan\left(\frac{x\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}{\sqrt{b-\sqrt{b^2-4ac}}\sqrt{d+ex^2}}\right)}{\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}} - \frac{2c\arctan\left(\frac{x\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}{\sqrt{\sqrt{b^2-4ac}+b}\sqrt{d+ex^2}}\right)}{\sqrt{b^2-4ac}\sqrt{\sqrt{b^2-4ac}+b}\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}$$

```
[In] Int[1/(Sqrt[d + e*x^2]*(a + b*x^2 + c*x^4)),x]
```

```
[Out] (2*c*ArcTan[(Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]*x)/(Sqrt[b - Sqrt[b^2
- 4*a*c]]*Sqrt[d + e*x^2])])/(Sqrt[b^2 - 4*a*c]*Sqrt[b - Sqrt[b^2 - 4*a*c]]
*Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]) - (2*c*ArcTan[(Sqrt[2*c*d - (b +
Sqrt[b^2 - 4*a*c])*e]*x)/(Sqrt[b + Sqrt[b^2 - 4*a*c]]*Sqrt[d + e*x^2])])/(S
qrt[b^2 - 4*a*c]*Sqrt[b + Sqrt[b^2 - 4*a*c]]*Sqrt[2*c*d - (b + Sqrt[b^2 - 4
*a*c])*e])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 1188

```
Int[((d_) + (e_.)*(x_)^2)^(q_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symb
ol] :> With[{r = Rt[b^2 - 4*a*c, 2]}, Dist[2*(c/r), Int[(d + e*x^2)^q/(b -
r + 2*c*x^2), x], x] - Dist[2*(c/r), Int[(d + e*x^2)^q/(b + r + 2*c*x^2), x
], x]] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
- b*d*e + a*e^2, 0] &&  !IntegerQ[q]
```

Rubi steps

$$\text{integral} = \frac{(2c)\int \frac{1}{\left(b-\sqrt{b^2-4ac}+2cx^2\right)\sqrt{d+ex^2}}\,dx}{\sqrt{b^2-4ac}} - \frac{(2c)\int \frac{1}{\left(b+\sqrt{b^2-4ac}+2cx^2\right)\sqrt{d+ex^2}}\,dx}{\sqrt{b^2-4ac}}$$

$$= \frac{(2c)\mathrm{Subst}\left(\int \frac{1}{b-\sqrt{b^2-4ac}-\left(-2cd+\left(b-\sqrt{b^2-4ac}\right)e\right)x^2}\,dx,x,\frac{x}{\sqrt{d+ex^2}}\right)}{\sqrt{b^2-4ac}} - \frac{(2c)\mathrm{Subst}\left(\int \frac{1}{b+\sqrt{b^2-4ac}-\left(-2cd+\left(b+\sqrt{b^2-4ac}\right)e\right)x^2}\,dx,x,\frac{x}{\sqrt{d+ex^2}}\right)}{\sqrt{b^2-4ac}}$$

$$= \frac{2c\tan^{-1}\left(\frac{\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}x}{\sqrt{b-\sqrt{b^2-4ac}}\sqrt{d+ex^2}}\right)}{\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}} - \frac{2c\tan^{-1}\left(\frac{\sqrt{2cd-\left(b+\sqrt{b^2-4ac}\right)e}x}{\sqrt{b+\sqrt{b^2-4ac}}\sqrt{d+ex^2}}\right)}{\sqrt{b^2-4ac}\sqrt{b+\sqrt{b^2-4ac}}\sqrt{2cd-\left(b+\sqrt{b^2-4ac}\right)e}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.14 (sec) , antiderivative size = 179, normalized size of antiderivative = 0.74

$$\int \frac{1}{\sqrt{d+ex^2}\,(a+bx^2+cx^4)}\,dx$$
$$= -2e^{3/2}\mathrm{RootSum}\left[cd^4-4cd^3\#1+4bd^2e\#1+6cd^2\#1^2-8bde\#1^2+16ae^2\#1^2 - 4cd\#1^3+4be\#1^3 + c\#1^4\&, \frac{\log\left(d+2ex^2-2\sqrt{e}x\sqrt{d+ex^2}-\#1\right)\#1}{-cd^3+bd^2e+3cd^2\#1-4bde\#1+8ae^2\#1-3cd\#1^2+3be\#1^2+c\#1^3}\&\right]$$

```
[In] Integrate[1/(Sqrt[d + e*x^2]*(a + b*x^2 + c*x^4)),x]
```

```
[Out] -2*e^(3/2)*RootSum[c*d^4 - 4*c*d^3*#1 + 4*b*d^2*e*#1 + 6*c*d^2*#1^2 - 8*b*d
*e*#1^2 + 16*a*e^2*#1^2 - 4*c*d*#1^3 + 4*b*e*#1^3 + c*#1^4 & , (Log[d + 2*e
*x^2 - 2*Sqrt[e]*x*Sqrt[d + e*x^2] - #1]*#1)/(-(c*d^3) + b*d^2*e + 3*c*d^2*
#1 - 4*b*d*e*#1 + 8*a*e^2*#1 - 3*c*d*#1^2 + 3*b*e*#1^2 + c*#1^3) & ]
```

Maple [A] (verified)

Time = 0.38 (sec) , antiderivative size = 224, normalized size of antiderivative = 0.92

method	result
default	$-\frac{\sqrt{2}\left(\frac{\left(bd+\sqrt{-d^2\left(4ac-b^2\right)}\right)\arctan\left(\frac{a\sqrt{e\,x^2+d}\,\sqrt{2}}{x\sqrt{\left(-2ae+bd+\sqrt{-d^2\left(4ac-b^2\right)}\right)a}}\right)}{\sqrt{\left(-2ae+bd+\sqrt{-d^2\left(4ac-b^2\right)}\right)a}}-\frac{\left(-bd+\sqrt{-d^2\left(4ac-b^2\right)}\right)\operatorname{arctanh}\left(\frac{a\sqrt{e\,x^2+d}}{x\sqrt{\left(2ae-bd+\sqrt{-d^2}\right.}}\right.}{\sqrt{\left(2ae-bd+\sqrt{-d^2\left(4ac-b^2\right)}\right)a}}\right.}{2\sqrt{-d^2(4ac-b^2)}}$
pseudoelliptic	$-\frac{\sqrt{2}\left(\frac{\left(bd+\sqrt{-d^2\left(4ac-b^2\right)}\right)\arctan\left(\frac{a\sqrt{e\,x^2+d}\,\sqrt{2}}{x\sqrt{\left(-2ae+bd+\sqrt{-d^2\left(4ac-b^2\right)}\right)a}}\right)}{\sqrt{\left(-2ae+bd+\sqrt{-d^2\left(4ac-b^2\right)}\right)a}}-\frac{\left(-bd+\sqrt{-d^2\left(4ac-b^2\right)}\right)\operatorname{arctanh}\left(\frac{a\sqrt{e\,x^2+d}}{x\sqrt{\left(2ae-bd+\sqrt{-d^2}\right.}}\right.}{\sqrt{\left(2ae-bd+\sqrt{-d^2\left(4ac-b^2\right)}\right)a}}\right.}{2\sqrt{-d^2(4ac-b^2)}}$

```
[In] int(1/(c*x^4+b*x^2+a)/(e*x^2+d)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*2^(1/2)/(-d^2*(4*a*c-b^2))^(1/2)*((b*d+(-d^2*(4*a*c-b^2))^(1/2))/((-2*
a*e+b*d+(-d^2*(4*a*c-b^2))^(1/2))*a)^(1/2)*arctan(a/x*(e*x^2+d)^(1/2)*2^(1/
2)/((-2*a*e+b*d+(-d^2*(4*a*c-b^2))^(1/2))*a)^(1/2))-(-b*d+(-d^2*(4*a*c-b^2)
)^(1/2))/((2*a*e-b*d+(-d^2*(4*a*c-b^2))^(1/2))*a)^(1/2)*arctanh(a/x*(e*x^2+
d)^(1/2)*2^(1/2)/((2*a*e-b*d+(-d^2*(4*a*c-b^2))^(1/2))*a)^(1/2)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 4557 vs. $2(203) = 406$.

Time = 6.17 (sec) , antiderivative size = 4557, normalized size of antiderivative = 18.75

$$\int \frac{1}{\sqrt{d+ex^2}\,(a+bx^2+cx^4)}\,dx = \text{Too large to display}$$

```
[In] integrate(1/(c*x^4+b*x^2+a)/(e*x^2+d)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/4*sqrt(1/2)*sqrt(-(b*c*d - (b^2 - 2*a*c)*e - ((a*b^2*c - 4*a^2*c^2)*d^2 -
 (a*b^3 - 4*a^2*b*c)*d*e + (a^2*b^2 - 4*a^3*c)*e^2)*sqrt((c^2*d^2 - 2*b*c*d
*e + b^2*e^2)/((a^2*b^2*c^2 - 4*a^3*c^3)*d^4 - 2*(a^2*b^3*c - 4*a^3*b*c^2)*
d^3*e + (a^2*b^4 - 2*a^3*b^2*c - 8*a^4*c^2)*d^2*e^2 - 2*(a^3*b^3 - 4*a^4*b*
c)*d*e^3 + (a^4*b^2 - 4*a^5*c)*e^4)))/((a*b^2*c - 4*a^2*c^2)*d^2 - (a*b^3 -
 4*a^2*b*c)*d*e + (a^2*b^2 - 4*a^3*c)*e^2))*log(-(2*a*c^2*d^2 - 2*a*b*c*d*e
 + ((a*b^2*c^2 - 4*a^2*c^3)*d^3 - (a*b^3*c - 4*a^2*b*c^2)*d^2*e + (a^2*b^2*
c - 4*a^3*c^2)*d*e^2)*x^2*sqrt((c^2*d^2 - 2*b*c*d*e + b^2*e^2)/((a^2*b^2*c^
2 - 4*a^3*c^3)*d^4 - 2*(a^2*b^3*c - 4*a^3*b*c^2)*d^3*e + (a^2*b^4 - 2*a^3*b
```

^2*c - 8*a^4*c^2)*d^2*e^2 - 2*(a^3*b^3 - 4*a^4*b*c)*d*e^3 + (a^4*b^2 - 4*a^5*c)*e^4)) - (b*c^2*d^2 + 4*a*b*c*e^2 - (b^2*c + 4*a*c^2)*d*e)*x^2 + 2*sqrt(1/2)*sqrt(e*x^2 + d)*((2*(a^2*b^2*c^2 - 4*a^3*c^3)*d^3 - 3*(a^2*b^3*c - 4*a^3*b*c^2)*d^2*e + (a^2*b^4 - 2*a^3*b^2*c - 8*a^4*c^2)*d*e^2 - (a^3*b^3 - 4*a^4*b*c)*e^3)*x*sqrt((c^2*d^2 - 2*b*c*d*e + b^2*e^2)/((a^2*b^2*c^2 - 4*a^3*c^3)*d^4 - 2*(a^2*b^3*c - 4*a^3*b*c^2)*d^3*e + (a^2*b^4 - 2*a^3*b^2*c - 8*a^4*c^2)*d^2*e^2 - 2*(a^3*b^3 - 4*a^4*b*c)*d*e^3 + (a^4*b^2 - 4*a^5*c)*e^4)) - ((a*b^2*c - 4*a^2*c^2)*d*e - (a*b^3 - 4*a^2*b*c)*e^2)*x)*sqrt(-(b*c*d - (b^2 - 2*a*c)*e - ((a*b^2*c - 4*a^2*c^2)*d^2 - (a*b^3 - 4*a^2*b*c)*d*e + (a^2*b^2 - 4*a^3*c)*e^2)*sqrt((c^2*d^2 - 2*b*c*d*e + b^2*e^2)/((a^2*b^2*c^2 - 4*a^3*c^3)*d^4 - 2*(a^2*b^3*c - 4*a^3*b*c^2)*d^3*e + (a^2*b^4 - 2*a^3*b^2*c - 8*a^4*c^2)*d^2*e^2 - 2*(a^3*b^3 - 4*a^4*b*c)*d*e^3 + (a^4*b^2 - 4*a^5*c)*e^4)))/((a*b^2*c - 4*a^2*c^2)*d^2 - (a*b^3 - 4*a^2*b*c)*d*e + (a^2*b^2 - 4*a^3*c)*e^2)))/x^2) - 1/4*sqrt(1/2)*sqrt(-(b*c*d - (b^2 - 2*a*c)*e - ((a*b^2*c - 4*a^2*c^2)*d^2 - (a*b^3 - 4*a^2*b*c)*d*e + (a^2*b^2 - 4*a^3*c)*e^2)*sqrt((c^2*d^2 - 2*b*c*d*e + b^2*e^2)/((a^2*b^2*c^2 - 4*a^3*c^3)*d^4 - 2*(a^2*b^3*c - 4*a^3*b*c^2)*d^3*e + (a^2*b^4 - 2*a^3*b^2*c - 8*a^4*c^2)*d^2*e^2 - 2*(a^3*b^3 - 4*a^4*b*c)*d*e^3 + (a^4*b^2 - 4*a^5*c)*e^4)))/((a*b^2*c - 4*a^2*c^2)*d^2 - (a*b^3 - 4*a^2*b*c)*d*e + (a^2*b^2 - 4*a^3*c)*e^2))*log(-(2*a*c^2*d^2 - 2*a*b*c*d*e + ((a*b^2*c^2 - 4*a^2*c^3)*d^3 - (a*b^3*c - 4*a^2*b*c^2)*d^2*e + (a^2*b^2*c - 4*a^3*c^2)*d*e^2)*x^2*sqrt((c^2*d^2 - 2*b*c*d*e + b^2*e^2)/((a^2*b^2*c^2 - 4*a^3*c^3)*d^4 - 2*(a^2*b^3*c - 4*a^3*b*c^2)*d^3*e + (a^2*b^4 - 2*a^3*b^2*c - 8*a^4*c^2)*d^2*e^2 - 2*(a^3*b^3 - 4*a^4*b*c)*d*e^3 + (a^4*b^2 - 4*a^5*c)*e^4)) - (b*c^2*d^2 + 4*a*b*c*e^2 - (b^2*c + 4*a*c^2)*d*e)*x^2 - 2*sqrt(1/2)*sqrt(e*x^2 + d)*((2*(a^2*b^2*c^2 - 4*a^3*c^3)*d^3 - 3*(a^2*b^3*c - 4*a^3*b*c^2)*d^2*e + (a^2*b^4 - 2*a^3*b^2*c - 8*a^4*c^2)*d*e^2 - (a^3*b^3 - 4*a^4*b*c)*e^3)*x*sqrt((c^2*d^2 - 2*b*c*d*e + b^2*e^2)/((a^2*b^2*c^2 - 4*a^3*c^3)*d^4 - 2*(a^2*b^3*c - 4*a^3*b*c^2)*d^3*e + (a^2*b^4 - 2*a^3*b^2*c - 8*a^4*c^2)*d^2*e^2 - 2*(a^3*b^3 - 4*a^4*b*c)*d*e^3 + (a^4*b^2 - 4*a^5*c)*e^4)) - ((a*b^2*c - 4*a^2*c^2)*d*e - (a*b^3 - 4*a^2*b*c)*e^2)*x)*sqrt(-(b*c*d - (b^2 - 2*a*c)*e - ((a*b^2*c - 4*a^2*c^2)*d^2 - (a*b^3 - 4*a^2*b*c)*d*e + (a^2*b^2 - 4*a^3*c)*e^2)*sqrt((c^2*d^2 - 2*b*c*d*e + b^2*e^2)/((a^2*b^2*c^2 - 4*a^3*c^3)*d^4 - 2*(a^2*b^3*c - 4*a^3*b*c^2)*d^3*e + (a^2*b^4 - 2*a^3*b^2*c - 8*a^4*c^2)*d^2*e^2 - 2*(a^3*b^3 - 4*a^4*b*c)*d*e^3 + (a^4*b^2 - 4*a^5*c)*e^4)))/((a*b^2*c - 4*a^2*c^2)*d^2 - (a*b^3 - 4*a^2*b*c)*d*e + (a^2*b^2 - 4*a^3*c)*e^2)))/x^2) - 1/4*sqrt(1/2)*sqrt(-(b*c*d - (b^2 - 2*a*c)*e + ((a*b^2*c - 4*a^2*c^2)*d^2 - (a*b^3 - 4*a^2*b*c)*d*e + (a^2*b^2 - 4*a^3*c)*e^2)*sqrt((c^2*d^2 - 2*b*c*d*e + b^2*e^2)/((a^2*b^2*c^2 - 4*a^3*c^3)*d^4 - 2*(a^2*b^3*c - 4*a^3*b*c^2)*d^3*e + (a^2*b^4 - 2*a^3*b^2*c - 8*a^4*c^2)*d^2*e^2 - 2*(a^3*b^3 - 4*a^4*b*c)*d*e^3 + (a^4*b^2 - 4*a^5*c)*e^4)))/((a*b^2*c - 4*a^2*c^2)*d^2 - (a*b^3 - 4*a^2*b*c)*d*e + (a^2*b^2 - 4*a^3*c)*e^2))*log(-(2*a*c^2*d^2 - 2*a*b*c*d*e - ((a*b^2*c^2 - 4*a^2*c^3)*d^3 - (a*b^3*c - 4*a^2*b*c^2)*d^2*e + (a^2*b^2*c - 4*a^3*c^2)*d*e^2)*x^2*sqrt((c^2*d^2 - 2*b*c*d*e + b^2*e^2)/((a^2*b^2*c^2 - 4*a^3*c^3)*d^4 - 2*(a^2*b^3*c - 4*a^3*b*c^2)*d^3*e + (a^2*b^4 - 2*a^3*b^2*c - 8*a^4*c^2)*d^2*e^2 -

```
2*(a^3*b^3 - 4*a^4*b*c)*d*e^3 + (a^4*b^2 - 4*a^5*c)*e^4)) - (b*c^2*d^2 + 4
*a*b*c*e^2 - (b^2*c + 4*a*c^2)*d*e)*x^2 + 2*sqrt(1/2)*sqrt(e*x^2 + d)*((2*(
a^2*b^2*c^2 - 4*a^3*c^3)*d^3 - 3*(a^2*b^3*c - 4*a^3*b*c^2)*d^2*e + (a^2*b^4
 - 2*a^3*b^2*c - 8*a^4*c^2)*d*e^2 - (a^3*b^3 - 4*a^4*b*c)*e^3)*x*sqrt((c^2*
d^2 - 2*b*c*d*e + b^2*e^2)/((a^2*b^2*c^2 - 4*a^3*c^3)*d^4 - 2*(a^2*b^3*c -
4*a^3*b*c^2)*d^3*e + (a^2*b^4 - 2*a^3*b^2*c - 8*a^4*c^2)*d^2*e^2 - 2*(a^3*b
^3 - 4*a^4*b*c)*d*e^3 + (a^4*b^2 - 4*a^5*c)*e^4)) + ((a*b^2*c - 4*a^2*c^2)*
d*e - (a*b^3 - 4*a^2*b*c)*e^2)*x)*sqrt(-(b*c*d - (b^2 - 2*a*c)*e + ((a*b^2*
c - 4*a^2*c^2)*d^2 - (a*b^3 - 4*a^2*b*c)*d*e + (a^2*b^2 - 4*a^3*c)*e^2)*sqr
t((c^2*d^2 - 2*b*c*d*e + b^2*e^2)/((a^2*b^2*c^2 - 4*a^3*c^3)*d^4 - 2*(a^2*b
^3*c - 4*a^3*b*c^2)*d^3*e + (a^2*b^4 - 2*a^3*b^2*c - 8*a^4*c^2)*d^2*e^2 - 2
*(a^3*b^3 - 4*a^4*b*c)*d*e^3 + (a^4*b^2 - 4*a^5*c)*e^4)))/((a*b^2*c - 4*a^2
*c^2)*d^2 - (a*b^3 - 4*a^2*b*c)*d*e + (a^2*b^2 - 4*a^3*c)*e^2)))/x^2) + 1/4
*sqrt(1/2)*sqrt(-(b*c*d - (b^2 - 2*a*c)*e + ((a*b^2*c - 4*a^2*c^2)*d^2 - (a
*b^3 - 4*a^2*b*c)*d*e + (a^2*b^2 - 4*a^3*c)*e^2)*sqrt((c^2*d^2 - 2*b*c*d*e
+ b^2*e^2)/((a^2*b^2*c^2 - 4*a^3*c^3)*d^4 - 2*(a^2*b^3*c - 4*a^3*b*c^2)*d^3
*e + (a^2*b^4 - 2*a^3*b^2*c - 8*a^4*c^2)*d^2*e^2 - 2*(a^3*b^3 - 4*a^4*b*c)*
d*e^3 + (a^4*b^2 - 4*a^5*c)*e^4)))/((a*b^2*c - 4*a^2*c^2)*d^2 - (a*b^3 - 4*
a^2*b*c)*d*e + (a^2*b^2 - 4*a^3*c)*e^2))*log(-(2*a*c^2*d^2 - 2*a*b*c*d*e -
((a*b^2*c^2 - 4*a^2*c^3)*d^3 - (a*b^3*c - 4*a^2*b*c^2)*d^2*e + (a^2*b^2*c -
 4*a^3*c^2)*d*e^2)*x^2*sqrt((c^2*d^2 - 2*b*c*d*e + b^2*e^2)/((a^2*b^2*c^2 -
 4*a^3*c^3)*d^4 - 2*(a^2*b^3*c - 4*a^3*b*c^2)*d^3*e + (a^2*b^4 - 2*a^3*b^2*
c - 8*a^4*c^2)*d^2*e^2 - 2*(a^3*b^3 - 4*a^4*b*c)*d*e^3 + (a^4*b^2 - 4*a^5*c
)*e^4)) - (b*c^2*d^2 + 4*a*b*c*e^2 - (b^2*c + 4*a*c^2)*d*e)*x^2 - 2*sqrt(1/
2)*sqrt(e*x^2 + d)*((2*(a^2*b^2*c^2 - 4*a^3*c^3)*d^3 - 3*(a^2*b^3*c - 4*a^3
*b*c^2)*d^2*e + (a^2*b^4 - 2*a^3*b^2*c - 8*a^4*c^2)*d*e^2 - (a^3*b^3 - 4*a^
4*b*c)*e^3)*x*sqrt((c^2*d^2 - 2*b*c*d*e + b^2*e^2)/((a^2*b^2*c^2 - 4*a^3*c^
3)*d^4 - 2*(a^2*b^3*c - 4*a^3*b*c^2)*d^3*e + (a^2*b^4 - 2*a^3*b^2*c - 8*a^4
*c^2)*d^2*e^2 - 2*(a^3*b^3 - 4*a^4*b*c)*d*e^3 + (a^4*b^2 - 4*a^5*c)*e^4)) +
 ((a*b^2*c - 4*a^2*c^2)*d*e - (a*b^3 - 4*a^2*b*c)*e^2)*x)*sqrt(-(b*c*d - (b
^2 - 2*a*c)*e + ((a*b^2*c - 4*a^2*c^2)*d^2 - (a*b^3 - 4*a^2*b*c)*d*e + (a^2
*b^2 - 4*a^3*c)*e^2)*sqrt((c^2*d^2 - 2*b*c*d*e + b^2*e^2)/((a^2*b^2*c^2 - 4
*a^3*c^3)*d^4 - 2*(a^2*b^3*c - 4*a^3*b*c^2)*d^3*e + (a^2*b^4 - 2*a^3*b^2*c
- 8*a^4*c^2)*d^2*e^2 - 2*(a^3*b^3 - 4*a^4*b*c)*d*e^3 + (a^4*b^2 - 4*a^5*c)*
e^4)))/((a*b^2*c - 4*a^2*c^2)*d^2 - (a*b^3 - 4*a^2*b*c)*d*e + (a^2*b^2 - 4*
a^3*c)*e^2)))/x^2)
```

Sympy [F]

$$\int \frac{1}{\sqrt{d+ex^2}\,(a+bx^2+cx^4)}\,dx = \int \frac{1}{\sqrt{d+ex^2}\,(a+bx^2+cx^4)}\,dx$$

```
[In] integrate(1/(c*x**4+b*x**2+a)/(e*x**2+d)**(1/2),x)
[Out] Integral(1/(sqrt(d + e*x**2)*(a + b*x**2 + c*x**4)), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{d+ex^2}\,(a+bx^2+cx^4)}\,dx = \int \frac{1}{(cx^4+bx^2+a)\sqrt{ex^2+d}}\,dx$$

```
[In] integrate(1/(c*x^4+b*x^2+a)/(e*x^2+d)^(1/2),x, algorithm="maxima")
[Out] integrate(1/((c*x^4 + b*x^2 + a)*sqrt(e*x^2 + d)), x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{d+ex^2}\,(a+bx^2+cx^4)}\,dx = \text{Timed out}$$

```
[In] integrate(1/(c*x^4+b*x^2+a)/(e*x^2+d)^(1/2),x, algorithm="giac")
[Out] Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{d+ex^2}\,(a+bx^2+cx^4)}\,dx = \int \frac{1}{\sqrt{e\,x^2+d}\,\left(c\,x^4+b\,x^2+a\right)}\,dx$$

```
[In] int(1/((d + e*x^2)^(1/2)*(a + b*x^2 + c*x^4)),x)
[Out] int(1/((d + e*x^2)^(1/2)*(a + b*x^2 + c*x^4)), x)
```

3.391 $\int \frac{1}{x^2\sqrt{d+ex^2}(a+bx^2+cx^4)}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 280

$$\int \frac{1}{x^2\sqrt{d+ex^2}\,(a+bx^2+cx^4)}\,dx = -\frac{\sqrt{d+ex^2}}{adx} - \frac{c\left(1+\frac{b}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)ex}}{\sqrt{b-\sqrt{b^2-4ac}}\sqrt{d+ex^2}}\right)}{a\sqrt{b-\sqrt{b^2-4ac}}\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}} - \frac{c\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2cd-\left(b+\sqrt{b^2-4ac}\right)ex}}{\sqrt{b+\sqrt{b^2-4ac}}\sqrt{d+ex^2}}\right)}{a\sqrt{b+\sqrt{b^2-4ac}}\sqrt{2cd-\left(b+\sqrt{b^2-4ac}\right)e}}$$

```
[Out] -(e*x^2+d)^(1/2)/a/d/x-c*arctan(x*(2*c*d-e*(b-(-4*a*c+b^2)^(1/2)))^(1/2)/(e
*x^2+d)^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2))*(1+b/(-4*a*c+b^2)^(1/2))/a/(2*c
*d-e*(b-(-4*a*c+b^2)^(1/2)))^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2)-c*arctan(x*
(2*c*d-e*(b+(-4*a*c+b^2)^(1/2)))^(1/2)/(e*x^2+d)^(1/2)/(b+(-4*a*c+b^2)^(1/2
))^(1/2))*(1-b/(-4*a*c+b^2)^(1/2))/a/(b+(-4*a*c+b^2)^(1/2))^(1/2)/(2*c*d-e*
(b+(-4*a*c+b^2)^(1/2)))^(1/2)
```

Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 280, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.172, Rules used = {1317, 270, 1706, 385, 211}

$$\int \frac{1}{x^2\sqrt{d+ex^2}\,(a+bx^2+cx^4)}\,dx = -\frac{c\left(\frac{b}{\sqrt{b^2-4ac}}+1\right)\arctan\left(\frac{x\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}{\sqrt{b-\sqrt{b^2-4ac}}\sqrt{d+ex^2}}\right)}{a\sqrt{b-\sqrt{b^2-4ac}}\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}} - \frac{c\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{x\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}{\sqrt{\sqrt{b^2-4ac}+b}\sqrt{d+ex^2}}\right)}{a\sqrt{\sqrt{b^2-4ac}+b}\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}} - \frac{\sqrt{d+ex^2}}{adx}$$

```
[In] Int[1/(x^2*Sqrt[d + e*x^2]*(a + b*x^2 + c*x^4)),x]
```

```
[Out] -(Sqrt[d + e*x^2]/(a*d*x)) - (c*(1 + b/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2*c*
d - (b - Sqrt[b^2 - 4*a*c])*e]*x)/(Sqrt[b - Sqrt[b^2 - 4*a*c]]*Sqrt[d + e*x
^2])])/(a*Sqrt[b - Sqrt[b^2 - 4*a*c]]*Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*
e]) - (c*(1 - b/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a
*c])*e]*x)/(Sqrt[b + Sqrt[b^2 - 4*a*c]]*Sqrt[d + e*x^2])])/(a*Sqrt[b + Sqrt
[b^2 - 4*a*c]]*Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 1317

```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_))/((a_) + (b_.)*(x_)^2 +
(c_.)*(x_)^4), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^q, (f*x)^m/(a +
 b*x^2 + c*x^4), x], x] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ[b^2 - 4*a
*c, 0] &&  !IntegerQ[q] && IntegerQ[m]
```

Rule 1706

```
Int[(Px_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^
(p_.), x_Symbol] :> Int[ExpandIntegrand[Px*(d + e*x^2)^q*(a + b*x^2 + c*x^4
)^p, x], x] /; FreeQ[{a, b, c, d, e, q}, x] && PolyQ[Px, x^2] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[p]
```

Rubi steps

$$\text{integral} = \int \left(\frac{1}{ax^2\sqrt{d+ex^2}} + \frac{-b-cx^2}{a\sqrt{d+ex^2}\,(a+bx^2+cx^4)}\right) dx$$

$$= \frac{\int \frac{1}{x^2\sqrt{d+ex^2}}\,dx}{a} + \frac{\int \frac{-b-cx^2}{\sqrt{d+ex^2}(a+bx^2+cx^4)}\,dx}{a}$$

$$= -\frac{\sqrt{d+ex^2}}{adx} + \frac{\int \left(\frac{-c-\frac{bc}{\sqrt{b^2-4ac}}}{\left(b-\sqrt{b^2-4ac}+2cx^2\right)\sqrt{d+ex^2}} + \frac{-c+\frac{bc}{\sqrt{b^2-4ac}}}{\left(b+\sqrt{b^2-4ac}+2cx^2\right)\sqrt{d+ex^2}}\right) dx}{a}$$

$$= -\frac{\sqrt{d+ex^2}}{adx} - \frac{\left(c\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\right)\int \frac{1}{\left(b+\sqrt{b^2-4ac}+2cx^2\right)\sqrt{d+ex^2}}\,dx}{a} - \frac{\left(c\left(1+\frac{b}{\sqrt{b^2-4ac}}\right)\right)\int \frac{1}{\left(b-\sqrt{b^2-4ac}+2cx^2\right)\sqrt{d+ex^2}}\,dx}{a}$$

$$= -\frac{\sqrt{d+ex^2}}{adx} - \frac{\left(c\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\right)\text{Subst}\left(\int \frac{1}{b+\sqrt{b^2-4ac}-\left(-2cd+\left(b+\sqrt{b^2-4ac}\right)e\right)x^2}\,dx, x, \frac{x}{\sqrt{d+ex^2}}\right)}{a} - \frac{\left(c\left(1+\frac{b}{\sqrt{b^2-4ac}}\right)\right)\text{Subst}\left(\int \frac{1}{b-\sqrt{b^2-4ac}-\left(-2cd+\left(b-\sqrt{b^2-4ac}\right)e\right)x^2}\,dx, x, \frac{x}{\sqrt{d+ex^2}}\right)}{a}$$

$$= -\frac{\sqrt{d+ex^2}}{adx} - \frac{c\left(1+\frac{b}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}\,x}{\sqrt{b-\sqrt{b^2-4ac}}\sqrt{d+ex^2}}\right)}{a\sqrt{b-\sqrt{b^2-4ac}}\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}} - \frac{c\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2cd-\left(b+\sqrt{b^2-4ac}\right)e}\,x}{\sqrt{b+\sqrt{b^2-4ac}}\sqrt{d+ex^2}}\right)}{a\sqrt{b+\sqrt{b^2-4ac}}\sqrt{2cd-\left(b+\sqrt{b^2-4ac}\right)e}}$$

Mathematica [A] (verified)

Time = 10.80 (sec) , antiderivative size = 271, normalized size of antiderivative = 0.97

$$\int \frac{1}{x^2\sqrt{d+ex^2}\,(a+bx^2+cx^4)}\,dx$$
$$= -\frac{\frac{\sqrt{d+ex^2}}{dx}+\frac{c\left(1+\frac{b}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2cd-be+\sqrt{b^2-4ac}ex}}{\sqrt{b-\sqrt{b^2-4ac}}\sqrt{d+ex^2}}\right)}{\sqrt{b-\sqrt{b^2-4ac}}\sqrt{2cd+\left(-b+\sqrt{b^2-4ac}\right)e}}+\frac{c\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2cd-\left(b+\sqrt{b^2-4ac}\right)ex}}{\sqrt{b+\sqrt{b^2-4ac}}\sqrt{d+ex^2}}\right)}{\sqrt{b+\sqrt{b^2-4ac}}\sqrt{2cd-\left(b+\sqrt{b^2-4ac}\right)e}}}{a}$$

```
[In] Integrate[1/(x^2*Sqrt[d + e*x^2]*(a + b*x^2 + c*x^4)),x]
```

```
[Out] -((Sqrt[d + e*x^2]/(d*x) + (c*(1 + b/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2*c*d
- b*e + Sqrt[b^2 - 4*a*c]*e]*x)/(Sqrt[b - Sqrt[b^2 - 4*a*c]]*Sqrt[d + e*x^2
]])])/(Sqrt[b - Sqrt[b^2 - 4*a*c]]*Sqrt[2*c*d + (-b + Sqrt[b^2 - 4*a*c])*e])
 + (c*(1 - b/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c]
)*e]*x)/(Sqrt[b + Sqrt[b^2 - 4*a*c]]*Sqrt[d + e*x^2])])/(Sqrt[b + Sqrt[b^2
- 4*a*c]]*Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]))/a)
```

Maple [A] (verified)

Time = 0.62 (sec) , antiderivative size = 266, normalized size of antiderivative = 0.95

method	result
default	$-\frac{\sqrt{e x^2+d}}{a d x}+\frac{\sqrt{2}\left(\frac{\left(-2acd+b^2d+\sqrt{-d^2\left(4ac-b^2\right)}\,b\right)\arctan\left(\frac{a\sqrt{e x^2+d}\,\sqrt{2}}{x\sqrt{\left(-2ae+bd+\sqrt{-d^2\left(4ac-b^2\right)}\right)a}}\right)}{\sqrt{\left(-2ae+bd+\sqrt{-d^2\left(4ac-b^2\right)}\right)a}}-\frac{\left(2acd-b^2d+\sqrt{-d^2\left(4ac-b^2\right)}\,b\right. \cdots}{\sqrt{\left(2ae- \cdots\right.}}\right.}{2a\sqrt{-d^2(4ac-b^2)}}$
risch	$-\frac{\sqrt{e x^2+d}}{a d x}+\frac{\sqrt{2}\left(\frac{\left(-2acd+b^2d+\sqrt{-d^2\left(4ac-b^2\right)}\,b\right)\arctan\left(\frac{a\sqrt{e x^2+d}\,\sqrt{2}}{x\sqrt{\left(-2ae+bd+\sqrt{-d^2\left(4ac-b^2\right)}\right)a}}\right)}{\sqrt{\left(-2ae+bd+\sqrt{-d^2\left(4ac-b^2\right)}\right)a}}-\frac{\left(2acd-b^2d+\sqrt{-d^2\left(4ac-b^2\right)}\,b\right. \cdots}{\sqrt{\left(2ae- \cdots\right.}}\right.}{2a\sqrt{-d^2(4ac-b^2)}}$
pseudoelliptic	$-\frac{\sqrt{\left(-2ae+bd+\sqrt{-4d^2\left(ac-\frac{b^2}{4}\right)}\right)a}\,\sqrt{2}\,dx\left(\left(\frac{\sqrt{-4d^2\left(ac-\frac{b^2}{4}\right)}\,b}{2}+d\left(ac-\frac{b^2}{2}\right)\right)\operatorname{arctanh}\left(\frac{a\sqrt{e x^2+d}\,\sqrt{2}}{x\sqrt{\left(2ae-bd+\sqrt{-4d^2\left(ac-\frac{b^2}{4}\right)}\right)a}}\right)+\cdots\right.}{\sqrt{\left(2ae-bd \cdots\right.}}$

```
[In] int(1/x^2/(c*x^4+b*x^2+a)/(e*x^2+d)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -(e*x^2+d)^(1/2)/a/d/x+1/2*2^(1/2)/a/(-d^2*(4*a*c-b^2))^(1/2)*((-2*a*c*d+b^
2*d+(-d^2*(4*a*c-b^2))^(1/2)*b)/((-2*a*e+b*d+(-d^2*(4*a*c-b^2))^(1/2))*a)^(
1/2)*arctan(a/x*(e*x^2+d)^(1/2)*2^(1/2)/((-2*a*e+b*d+(-d^2*(4*a*c-b^2))^(1/
2))*a)^(1/2))-(2*a*c*d-b^2*d+(-d^2*(4*a*c-b^2))^(1/2)*b)/((2*a*e-b*d+(-d^2*
(4*a*c-b^2))^(1/2))*a)^(1/2)*arctanh(a/x*(e*x^2+d)^(1/2)*2^(1/2)/((2*a*e-b*
d+(-d^2*(4*a*c-b^2))^(1/2))*a)^(1/2)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 6431 vs. $2(236) = 472$.

Time = 12.66 (sec) , antiderivative size = 6431, normalized size of antiderivative = 22.97

$$\int \frac{1}{x^2\sqrt{d+ex^2}\,(a+bx^2+cx^4)}\,dx = \text{Too large to display}$$

```
[In] integrate(1/x^2/(c*x^4+b*x^2+a)/(e*x^2+d)^(1/2),x, algorithm="fricas")
```

```
[Out] Too large to include
```

Sympy [F]

$$\int \frac{1}{x^2\sqrt{d+ex^2}\,(a+bx^2+cx^4)}\,dx = \int \frac{1}{x^2\sqrt{d+ex^2}\,(a+bx^2+cx^4)}\,dx$$

```
[In] integrate(1/x**2/(c*x**4+b*x**2+a)/(e*x**2+d)**(1/2),x)
```

```
[Out] Integral(1/(x**2*sqrt(d + e*x**2)*(a + b*x**2 + c*x**4)), x)
```

Maxima [F]

$$\int \frac{1}{x^2\sqrt{d+ex^2}\,(a+bx^2+cx^4)}\,dx = \int \frac{1}{(cx^4+bx^2+a)\sqrt{ex^2+d}\,x^2}\,dx$$

```
[In] integrate(1/x^2/(c*x^4+b*x^2+a)/(e*x^2+d)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((c*x^4 + b*x^2 + a)*sqrt(e*x^2 + d)*x^2), x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{1}{x^2\sqrt{d+ex^2}\,(a+bx^2+cx^4)}\,dx = \text{Timed out}$$

```
[In] integrate(1/x^2/(c*x^4+b*x^2+a)/(e*x^2+d)^(1/2),x, algorithm="giac")
[Out] Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^2\sqrt{d+ex^2}\,(a+bx^2+cx^4)}\,dx = \int \frac{1}{x^2\,\sqrt{e\,x^2+d}\,\left(c\,x^4+b\,x^2+a\right)}\,dx$$

```
[In] int(1/(x^2*(d + e*x^2)^(1/2)*(a + b*x^2 + c*x^4)),x)
[Out] int(1/(x^2*(d + e*x^2)^(1/2)*(a + b*x^2 + c*x^4)), x)
```

3.392 $\int \frac{1}{x^4\sqrt{d+ex^2}(a+bx^2+cx^4)}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 341

$$\int \frac{1}{x^4\sqrt{d+ex^2}\,(a+bx^2+cx^4)}\,dx = -\frac{\sqrt{d+ex^2}}{3adx^3}+\frac{b\sqrt{d+ex^2}}{a^2dx}+\frac{2e\sqrt{d+ex^2}}{3ad^2x} + \frac{c\left(b+\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}\,x}{\sqrt{b-\sqrt{b^2-4ac}}\sqrt{d+ex^2}}\right)}{a^2\sqrt{b-\sqrt{b^2-4ac}}\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}} + \frac{c\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2cd-\left(b+\sqrt{b^2-4ac}\right)e}\,x}{\sqrt{b+\sqrt{b^2-4ac}}\sqrt{d+ex^2}}\right)}{a^2\sqrt{b+\sqrt{b^2-4ac}}\sqrt{2cd-\left(b+\sqrt{b^2-4ac}\right)e}}$$

```
[Out] -1/3*(e*x^2+d)^(1/2)/a/d/x^3+b*(e*x^2+d)^(1/2)/a^2/d/x+2/3*e*(e*x^2+d)^(1/2
)/a/d^2/x+c*arctan(x*(2*c*d-e*(b-(-4*a*c+b^2)^(1/2)))^(1/2)/(e*x^2+d)^(1/2)
/(b-(-4*a*c+b^2)^(1/2))^(1/2))*(b+(-2*a*c+b^2)/(-4*a*c+b^2)^(1/2))/a^2/(2*c
*d-e*(b-(-4*a*c+b^2)^(1/2)))^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2)+c*arctan(x*
(2*c*d-e*(b+(-4*a*c+b^2)^(1/2)))^(1/2)/(e*x^2+d)^(1/2)/(b+(-4*a*c+b^2)^(1/2
))^(1/2))*(b+(2*a*c-b^2)/(-4*a*c+b^2)^(1/2))/a^2/(b+(-4*a*c+b^2)^(1/2))^(1/
2)/(2*c*d-e*(b+(-4*a*c+b^2)^(1/2)))^(1/2)
```

Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 341, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.207, Rules used = {1317, 277, 270, 1706, 385, 211}

$$\int \frac{1}{x^4\sqrt{d+ex^2}\,(a+bx^2+cx^4)}\,dx = \frac{c\left(\frac{b^2-2ac}{\sqrt{b^2-4ac}}+b\right)\arctan\left(\frac{x\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}{\sqrt{b-\sqrt{b^2-4ac}}\sqrt{d+ex^2}}\right)}{a^2\sqrt{b-\sqrt{b^2-4ac}}\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}} + \frac{c\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{x\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}{\sqrt{\sqrt{b^2-4ac}+b}\sqrt{d+ex^2}}\right)}{a^2\sqrt{\sqrt{b^2-4ac}+b}\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}} + \frac{b\sqrt{d+ex^2}}{a^2dx} + \frac{2e\sqrt{d+ex^2}}{3ad^2x} - \frac{\sqrt{d+ex^2}}{3adx^3}$$

```
[In] Int[1/(x^4*Sqrt[d + e*x^2]*(a + b*x^2 + c*x^4)),x]
```

```
[Out] -1/3*Sqrt[d + e*x^2]/(a*d*x^3) + (b*Sqrt[d + e*x^2])/(a^2*d*x) + (2*e*Sqrt[
d + e*x^2])/(3*a*d^2*x) + (c*(b + (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*ArcTan[(
Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]*x)/(Sqrt[b - Sqrt[b^2 - 4*a*c]]*Sqr
t[d + e*x^2])])/(a^2*Sqrt[b - Sqrt[b^2 - 4*a*c]]*Sqrt[2*c*d - (b - Sqrt[b^2
 - 4*a*c])*e]) + (c*(b - (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2*c*
d - (b + Sqrt[b^2 - 4*a*c])*e]*x)/(Sqrt[b + Sqrt[b^2 - 4*a*c]]*Sqrt[d + e*x
^2])])/(a^2*Sqrt[b + Sqrt[b^2 - 4*a*c]]*Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c]
)*e])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 277

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x^(m + 1)*((
a + b*x^n)^(p + 1)/(a*(m + 1))), x] - Dist[b*((m + n*(p + 1) + 1)/(a*(m + 1
))), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 1317

```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_))/((a_) + (b_.)*(x_)^2 +
(c_.)*(x_)^4), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^q, (f*x)^m/(a +
 b*x^2 + c*x^4), x], x] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ[b^2 - 4*a
*c, 0] &&  !IntegerQ[q] && IntegerQ[m]
```

Rule 1706

```
Int[(Px_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^
(p_.), x_Symbol] :> Int[ExpandIntegrand[Px*(d + e*x^2)^q*(a + b*x^2 + c*x^4
)^p, x], x] /; FreeQ[{a, b, c, d, e, q}, x] && PolyQ[Px, x^2] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(\frac{1}{ax^4\sqrt{d+ex^2}}-\frac{b}{a^2x^2\sqrt{d+ex^2}}+\frac{b^2-ac+bcx^2}{a^2\sqrt{d+ex^2}\,(a+bx^2+cx^4)}\right)dx\\
&= \frac{\int \frac{b^2-ac+bcx^2}{\sqrt{d+ex^2}(a+bx^2+cx^4)}\,dx}{a^2}+\frac{\int \frac{1}{x^4\sqrt{d+ex^2}}\,dx}{a}-\frac{b\int \frac{1}{x^2\sqrt{d+ex^2}}\,dx}{a^2}\\
&= -\frac{\sqrt{d+ex^2}}{3adx^3}+\frac{b\sqrt{d+ex^2}}{a^2dx}\\
&\quad+\frac{\int \left(\frac{bc+\frac{c\left(b^2-2ac\right)}{\sqrt{b^2-4ac}}}{\left(b-\sqrt{b^2-4ac}+2cx^2\right)\sqrt{d+ex^2}}+\frac{bc-\frac{c\left(b^2-2ac\right)}{\sqrt{b^2-4ac}}}{\left(b+\sqrt{b^2-4ac}+2cx^2\right)\sqrt{d+ex^2}}\right)dx}{a^2}-\frac{(2e)\int \frac{1}{x^2\sqrt{d+ex^2}}\,dx}{3ad}\\
&= -\frac{\sqrt{d+ex^2}}{3adx^3}+\frac{b\sqrt{d+ex^2}}{a^2dx}+\frac{2e\sqrt{d+ex^2}}{3ad^2x}\\
&\quad+\frac{\left(c\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\right)\int \frac{1}{\left(b+\sqrt{b^2-4ac}+2cx^2\right)\sqrt{d+ex^2}}\,dx}{a^2}\\
&\quad+\frac{\left(c\left(b+\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\right)\int \frac{1}{\left(b-\sqrt{b^2-4ac}+2cx^2\right)\sqrt{d+ex^2}}\,dx}{a^2}
\end{aligned}$$

$$= -\frac{\sqrt{d+ex^2}}{3adx^3} + \frac{b\sqrt{d+ex^2}}{a^2dx} + \frac{2e\sqrt{d+ex^2}}{3ad^2x} + \frac{\left(c\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\right)\text{Subst}\left(\int \frac{1}{b+\sqrt{b^2-4ac}-\left(-2cd+\left(b+\sqrt{b^2-4ac}\right)e\right)x^2}\,dx, x, \frac{x}{\sqrt{d+ex^2}}\right)}{a^2} + \frac{\left(c\left(b+\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\right)\text{Subst}\left(\int \frac{1}{b-\sqrt{b^2-4ac}-\left(-2cd+\left(b-\sqrt{b^2-4ac}\right)e\right)x^2}\,dx, x, \frac{x}{\sqrt{d+ex^2}}\right)}{a^2}$$

$$= -\frac{\sqrt{d+ex^2}}{3adx^3} + \frac{b\sqrt{d+ex^2}}{a^2dx} + \frac{2e\sqrt{d+ex^2}}{3ad^2x} + \frac{c\left(b+\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}x}{\sqrt{b-\sqrt{b^2-4ac}}\sqrt{d+ex^2}}\right)}{a^2\sqrt{b-\sqrt{b^2-4ac}}\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}} + \frac{c\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2cd-\left(b+\sqrt{b^2-4ac}\right)e}x}{\sqrt{b+\sqrt{b^2-4ac}}\sqrt{d+ex^2}}\right)}{a^2\sqrt{b+\sqrt{b^2-4ac}}\sqrt{2cd-\left(b+\sqrt{b^2-4ac}\right)e}}$$

Mathematica [A] (verified)

Time = 10.59 (sec) , antiderivative size = 320, normalized size of antiderivative = 0.94

$$\int \frac{1}{x^4\sqrt{d+ex^2}\,(a+bx^2+cx^4)}\,dx$$

$$= \frac{\frac{3b\sqrt{d+ex^2}}{dx} - \frac{a(d-2ex^2)\sqrt{d+ex^2}}{d^2x^3} + \frac{3c\left(b+\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2cd-be+\sqrt{b^2-4ac}ex}}{\sqrt{b-\sqrt{b^2-4ac}}\sqrt{d+ex^2}}\right)}{\sqrt{b-\sqrt{b^2-4ac}}\sqrt{2cd+\left(-b+\sqrt{b^2-4ac}\right)e}} + \frac{3c\left(b+\frac{-b^2+2ac}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2cd-\left(b+\sqrt{b^2-4ac}\right)e}x}{\sqrt{b+\sqrt{b^2-4ac}}\sqrt{d+ex^2}}\right)}{\sqrt{b+\sqrt{b^2-4ac}}\sqrt{2cd-\left(b+\sqrt{b^2-4ac}\right)e}}}{3a^2}$$

```
[In] Integrate[1/(x^4*Sqrt[d + e*x^2]*(a + b*x^2 + c*x^4)),x]
```

```
[Out] ((3*b*Sqrt[d + e*x^2])/(d*x) - (a*(d - 2*e*x^2)*Sqrt[d + e*x^2])/(d^2*x^3)
+ (3*c*(b + (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2*c*d - b*e + Sqr
t[b^2 - 4*a*c]*e]*x)/(Sqrt[b - Sqrt[b^2 - 4*a*c]]*Sqrt[d + e*x^2])])/(Sqrt[
b - Sqrt[b^2 - 4*a*c]]*Sqrt[2*c*d + (-b + Sqrt[b^2 - 4*a*c])*e]) + (3*c*(b
+ (-b^2 + 2*a*c)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2*c*d - (b + Sqrt[b^2 - 4*
a*c])*e]*x)/(Sqrt[b + Sqrt[b^2 - 4*a*c]]*Sqrt[d + e*x^2])])/(Sqrt[b + Sqrt[
b^2 - 4*a*c]]*Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]))/(3*a^2)
```

Maple [A] (verified)

Time = 0.71 (sec) , antiderivative size = 332, normalized size of antiderivative = 0.97

method	result
risch	$-\frac{\sqrt{e\,x^2+d}\,(-2ae\,x^2-3bd\,x^2+da)}{3d^2a^2x^3}+\frac{\sqrt{2}\left(\frac{\left(3abcd-b^3d+\sqrt{-d^2\left(4ac-b^2\right)}\,ac-\sqrt{-d^2\left(4ac-b^2\right)}\,b^2\right)\arctan\left(\frac{a\sqrt{e\,x^2+d}\,\sqrt{}}{x\sqrt{\left(-2ae+bd+\sqrt{-d^2}\right.}}\right.}{\sqrt{\left(-2ae+bd+\sqrt{-d^2\left(4ac-b^2\right)}\right)a}}\right.}{}$
default	$\frac{-\frac{\sqrt{e\,x^2+d}}{3d\,x^3}+\frac{2e\sqrt{e\,x^2+d}}{3d^2x}}{a}+\frac{b\sqrt{e\,x^2+d}}{a^2dx}+\frac{\sqrt{2}\left(\frac{\left(3abcd-b^3d+\sqrt{-d^2\left(4ac-b^2\right)}\,ac-\sqrt{-d^2\left(4ac-b^2\right)}\,b^2\right)\arctan\left(\frac{a\sqrt{e\,x^2}}{x\sqrt{\left(-2ae+bd+\sqrt{}\right.}}\right.}{\sqrt{\left(-2ae+bd+\sqrt{-d^2\left(4ac-b^2\right)}\right)a}}\right.}{}$
pseudoelliptic	$-\frac{3\sqrt{\left(-2ae+bd+\sqrt{-4d^2\left(ac-\frac{b^2}{4}\right)}\right)a}\,\sqrt{2}\,d^2\left(\left(ac-b^2\right)\sqrt{-4d^2\left(ac-\frac{b^2}{4}\right)}+\left(-3abc+b^3\right)d\right)x^3\operatorname{arctanh}\left(\frac{a\sqrt{e\,x^2+d}\,\sqrt{2}}{x\sqrt{\left(2ae-bd+\sqrt{-4d^2\left(ac-\frac{b^2}{4}\right)}\right.}}\right.}{2}$

```
[In] int(1/x^4/(c*x^4+b*x^2+a)/(e*x^2+d)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/3*(e*x^2+d)^(1/2)*(-2*a*e*x^2-3*b*d*x^2+a*d)/d^2/a^2/x^3+1/2/a^2*2^(1/2)
/(-d^2*(4*a*c-b^2))^(1/2)*((3*a*b*c*d-b^3*d+(-d^2*(4*a*c-b^2))^(1/2)*a*c-(-
d^2*(4*a*c-b^2))^(1/2)*b^2)/((-2*a*e+b*d+(-d^2*(4*a*c-b^2))^(1/2))*a)^(1/2)
*arctan(a/x*(e*x^2+d)^(1/2)*2^(1/2)/((-2*a*e+b*d+(-d^2*(4*a*c-b^2))^(1/2))*
a)^(1/2))-(-3*a*b*c*d+b^3*d+(-d^2*(4*a*c-b^2))^(1/2)*a*c-(-d^2*(4*a*c-b^2))
^(1/2)*b^2)/((2*a*e-b*d+(-d^2*(4*a*c-b^2))^(1/2))*a)^(1/2)*arctanh(a/x*(e*x
^2+d)^(1/2)*2^(1/2)/((2*a*e-b*d+(-d^2*(4*a*c-b^2))^(1/2))*a)^(1/2)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 8187 vs. 2(291) = 582.

Time = 62.57 (sec) , antiderivative size = 8187, normalized size of antiderivative = 24.01

$$\int \frac{1}{x^4\sqrt{d+ex^2}\,(a+bx^2+cx^4)}\,dx = \text{Too large to display}$$

```
[In] integrate(1/x^4/(c*x^4+b*x^2+a)/(e*x^2+d)^(1/2),x, algorithm="fricas")
```

```
[Out] Too large to include
```

Sympy [F]

$$\int \frac{1}{x^4\sqrt{d+ex^2}\,(a+bx^2+cx^4)}\,dx = \int \frac{1}{x^4\sqrt{d+ex^2}\,(a+bx^2+cx^4)}\,dx$$

```
[In] integrate(1/x**4/(c*x**4+b*x**2+a)/(e*x**2+d)**(1/2),x)
[Out] Integral(1/(x**4*sqrt(d + e*x**2)*(a + b*x**2 + c*x**4)), x)
```

Maxima [F]

$$\int \frac{1}{x^4\sqrt{d+ex^2}\,(a+bx^2+cx^4)}\,dx = \int \frac{1}{(cx^4+bx^2+a)\sqrt{ex^2+dx^4}}\,dx$$

```
[In] integrate(1/x^4/(c*x^4+b*x^2+a)/(e*x^2+d)^(1/2),x, algorithm="maxima")
[Out] integrate(1/((c*x^4 + b*x^2 + a)*sqrt(e*x^2 + d)*x^4), x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{1}{x^4\sqrt{d+ex^2}\,(a+bx^2+cx^4)}\,dx = \text{Timed out}$$

```
[In] integrate(1/x^4/(c*x^4+b*x^2+a)/(e*x^2+d)^(1/2),x, algorithm="giac")
[Out] Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^4\sqrt{d+ex^2}\,(a+bx^2+cx^4)}\,dx = \int \frac{1}{x^4\,\sqrt{e\,x^2+d}\,\left(c\,x^4+b\,x^2+a\right)}\,dx$$

```
[In] int(1/(x^4*(d + e*x^2)^(1/2)*(a + b*x^2 + c*x^4)),x)
[Out] int(1/(x^4*(d + e*x^2)^(1/2)*(a + b*x^2 + c*x^4)), x)
```

3.393 $\int \frac{1}{x^6\sqrt{d+ex^2}(a+bx^2+cx^4)}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 443

$$\begin{aligned}\int \frac{1}{x^6\sqrt{d+ex^2}\,(a+bx^2+cx^4)}\,dx =& -\frac{\sqrt{d+ex^2}}{5adx^5}+\frac{b\sqrt{d+ex^2}}{3a^2dx^3}+\frac{4e\sqrt{d+ex^2}}{15ad^2x^3}\\ &-\frac{(b^2-ac)\sqrt{d+ex^2}}{a^3dx}-\frac{2be\sqrt{d+ex^2}}{3a^2d^2x}-\frac{8e^2\sqrt{d+ex^2}}{15ad^3x}\\ &-\frac{c\left(b^2-ac+\frac{b(b^2-3ac)}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}\,x}{\sqrt{b-\sqrt{b^2-4ac}}\sqrt{d+ex^2}}\right)}{a^3\sqrt{b-\sqrt{b^2-4ac}}\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}}\\ &-\frac{c\left(b^2-ac-\frac{b(b^2-3ac)}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2cd-\left(b+\sqrt{b^2-4ac}\right)e}\,x}{\sqrt{b+\sqrt{b^2-4ac}}\sqrt{d+ex^2}}\right)}{a^3\sqrt{b+\sqrt{b^2-4ac}}\sqrt{2cd-\left(b+\sqrt{b^2-4ac}\right)e}}\end{aligned}$$

```
[Out] -1/5*(e*x^2+d)^(1/2)/a/d/x^5+1/3*b*(e*x^2+d)^(1/2)/a^2/d/x^3+4/15*e*(e*x^2+
d)^(1/2)/a/d^2/x^3-(-a*c+b^2)*(e*x^2+d)^(1/2)/a^3/d/x-2/3*b*e*(e*x^2+d)^(1/
2)/a^2/d^2/x-8/15*e^2*(e*x^2+d)^(1/2)/a/d^3/x-c*arctan(x*(2*c*d-e*(b-(-4*a*
c+b^2)^(1/2)))^(1/2)/(e*x^2+d)^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2))*(b^2-a*c
+b*(-3*a*c+b^2)/(-4*a*c+b^2)^(1/2))/a^3/(2*c*d-e*(b-(-4*a*c+b^2)^(1/2)))^(1
/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2)-c*arctan(x*(2*c*d-e*(b+(-4*a*c+b^2)^(1/2))
)^(1/2)/(e*x^2+d)^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2))*(b^2-a*c-b*(-3*a*c+b^
2)/(-4*a*c+b^2)^(1/2))/a^3/(b+(-4*a*c+b^2)^(1/2))^(1/2)/(2*c*d-e*(b+(-4*a*c
+b^2)^(1/2)))^(1/2)
```

Rubi [A] (verified)

Time = 1.07 (sec) , antiderivative size = 443, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.207, Rules used = {1317, 277, 270, 1706, 385, 211}

$$\int \frac{1}{x^6\sqrt{d+ex^2}\,(a+bx^2+cx^4)}\,dx$$
$$= -\frac{c\left(\frac{b(b^2-3ac)}{\sqrt{b^2-4ac}}-ac+b^2\right)\arctan\left(\frac{x\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}{\sqrt{b-\sqrt{b^2-4ac}}\sqrt{d+ex^2}}\right)}{a^3\sqrt{b-\sqrt{b^2-4ac}}\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}$$
$$-\frac{c\left(-\frac{b(b^2-3ac)}{\sqrt{b^2-4ac}}-ac+b^2\right)\arctan\left(\frac{x\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}{\sqrt{\sqrt{b^2-4ac}+b}\sqrt{d+ex^2}}\right)}{a^3\sqrt{\sqrt{b^2-4ac}+b}\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}-\frac{(b^2-ac)\sqrt{d+ex^2}}{a^3dx}$$
$$-\frac{2be\sqrt{d+ex^2}}{3a^2d^2x}+\frac{b\sqrt{d+ex^2}}{3a^2dx^3}-\frac{8e^2\sqrt{d+ex^2}}{15ad^3x}+\frac{4e\sqrt{d+ex^2}}{15ad^2x^3}-\frac{\sqrt{d+ex^2}}{5adx^5}$$

```
[In] Int[1/(x^6*Sqrt[d + e*x^2]*(a + b*x^2 + c*x^4)),x]
```

```
[Out] -1/5*Sqrt[d + e*x^2]/(a*d*x^5) + (b*Sqrt[d + e*x^2])/(3*a^2*d*x^3) + (4*e*S
qrt[d + e*x^2])/(15*a*d^2*x^3) - ((b^2 - a*c)*Sqrt[d + e*x^2])/(a^3*d*x) -
(2*b*e*Sqrt[d + e*x^2])/(3*a^2*d^2*x) - (8*e^2*Sqrt[d + e*x^2])/(15*a*d^3*x
) - (c*(b^2 - a*c + (b*(b^2 - 3*a*c))/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2*c*d
 - (b - Sqrt[b^2 - 4*a*c])*e]*x)/(Sqrt[b - Sqrt[b^2 - 4*a*c]]*Sqrt[d + e*x^
2])])/(a^3*Sqrt[b - Sqrt[b^2 - 4*a*c]]*Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])
*e]) - (c*(b^2 - a*c - (b*(b^2 - 3*a*c))/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2*
c*d - (b + Sqrt[b^2 - 4*a*c])*e]*x)/(Sqrt[b + Sqrt[b^2 - 4*a*c]]*Sqrt[d + e
*x^2])])/(a^3*Sqrt[b + Sqrt[b^2 - 4*a*c]]*Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*
c])*e])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 277

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x^(m + 1)*((
a + b*x^n)^(p + 1)/(a*(m + 1))), x] - Dist[b*((m + n*(p + 1) + 1)/(a*(m + 1
))), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 1317

```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_))/((a_) + (b_.)*(x_)^2 +
(c_.)*(x_)^4), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^q, (f*x)^m/(a +
 b*x^2 + c*x^4), x], x] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ[b^2 - 4*a
*c, 0] &&  !IntegerQ[q] && IntegerQ[m]
```

Rule 1706

```
Int[(Px_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^
(p_.), x_Symbol] :> Int[ExpandIntegrand[Px*(d + e*x^2)^q*(a + b*x^2 + c*x^4
)^p, x], x] /; FreeQ[{a, b, c, d, e, q}, x] && PolyQ[Px, x^2] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[p]
```

Rubi steps

$$\text{integral} = \int \left(\frac{1}{ax^6\sqrt{d+ex^2}} - \frac{b}{a^2x^4\sqrt{d+ex^2}} + \frac{b^2-ac}{a^3x^2\sqrt{d+ex^2}} + \frac{-b(b^2-2ac)-c(b^2-ac)\,x^2}{a^3\sqrt{d+ex^2}\,(a+bx^2+cx^4)}\right) dx$$

$$= \frac{\int \frac{-b(b^2-2ac)-c(b^2-ac)x^2}{\sqrt{d+ex^2}(a+bx^2+cx^4)}\,dx}{a^3} + \frac{\int \frac{1}{x^6\sqrt{d+ex^2}}\,dx}{a} - \frac{b\int \frac{1}{x^4\sqrt{d+ex^2}}\,dx}{a^2} + \frac{(b^2-ac)\int \frac{1}{x^2\sqrt{d+ex^2}}\,dx}{a^3}$$

$$= -\frac{\sqrt{d+ex^2}}{5adx^5} + \frac{b\sqrt{d+ex^2}}{3a^2dx^3} - \frac{(b^2-ac)\sqrt{d+ex^2}}{a^3dx} + \frac{\int \left(\frac{-\frac{bc(b^2-3ac)}{\sqrt{b^2-4ac}}-c(b^2-ac)}{\left(b-\sqrt{b^2-4ac}+2cx^2\right)\sqrt{d+ex^2}} + \frac{\frac{bc(b^2-3ac)}{\sqrt{b^2-4ac}}-c(b^2-ac)}{\left(b+\sqrt{b^2-4ac}+2cx^2\right)\sqrt{d+ex^2}}\right) dx}{a^3} - \frac{(4e)\int \frac{1}{x^4\sqrt{d+ex^2}}\,dx}{5ad} + \frac{(2be)\int \frac{1}{x^2\sqrt{d+ex^2}}\,dx}{3a^2d}$$

$$= -\frac{\sqrt{d+ex^2}}{5adx^5} + \frac{b\sqrt{d+ex^2}}{3a^2dx^3} + \frac{4e\sqrt{d+ex^2}}{15ad^2x^3} - \frac{(b^2-ac)\sqrt{d+ex^2}}{a^3dx} - \frac{2be\sqrt{d+ex^2}}{3a^2d^2x} - \frac{\left(c\left(b^2-ac-\frac{b(b^2-3ac)}{\sqrt{b^2-4ac}}\right)\right)\int \frac{1}{\left(b+\sqrt{b^2-4ac}+2cx^2\right)\sqrt{d+ex^2}}\,dx}{a^3} - \frac{\left(c\left(b^2-ac+\frac{b(b^2-3ac)}{\sqrt{b^2-4ac}}\right)\right)\int \frac{1}{\left(b-\sqrt{b^2-4ac}+2cx^2\right)\sqrt{d+ex^2}}\,dx}{a^3} + \frac{(8e^2)\int \frac{1}{x^2\sqrt{d+ex^2}}\,dx}{15ad^2}$$

$$= -\frac{\sqrt{d+ex^2}}{5adx^5} + \frac{b\sqrt{d+ex^2}}{3a^2dx^3} + \frac{4e\sqrt{d+ex^2}}{15ad^2x^3} - \frac{(b^2-ac)\sqrt{d+ex^2}}{a^3dx} - \frac{2be\sqrt{d+ex^2}}{3a^2d^2x} - \frac{8e^2\sqrt{d+ex^2}}{15ad^3x} - \frac{\left(c\left(b^2-ac-\frac{b(b^2-3ac)}{\sqrt{b^2-4ac}}\right)\right)\text{Subst}\left(\int \frac{1}{b+\sqrt{b^2-4ac}-\left(-2cd+\left(b+\sqrt{b^2-4ac}\right)e\right)x^2}\,dx, x, \frac{x}{\sqrt{d+ex^2}}\right)}{a^3} - \frac{\left(c\left(b^2-ac+\frac{b(b^2-3ac)}{\sqrt{b^2-4ac}}\right)\right)\text{Subst}\left(\int \frac{1}{b-\sqrt{b^2-4ac}-\left(-2cd+\left(b-\sqrt{b^2-4ac}\right)e\right)x^2}\,dx, x, \frac{x}{\sqrt{d+ex^2}}\right)}{a^3}$$

$$= -\frac{\sqrt{d+ex^2}}{5adx^5} + \frac{b\sqrt{d+ex^2}}{3a^2dx^3} + \frac{4e\sqrt{d+ex^2}}{15ad^2x^3} - \frac{(b^2-ac)\sqrt{d+ex^2}}{a^3dx} - \frac{2be\sqrt{d+ex^2}}{3a^2d^2x} - \frac{8e^2\sqrt{d+ex^2}}{15ad^3x} - \frac{c\left(b^2-ac+\frac{b(b^2-3ac)}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}\,x}{\sqrt{b-\sqrt{b^2-4ac}}\sqrt{d+ex^2}}\right)}{a^3\sqrt{b-\sqrt{b^2-4ac}}\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}} - \frac{c\left(b^2-ac-\frac{b(b^2-3ac)}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2cd-\left(b+\sqrt{b^2-4ac}\right)e}\,x}{\sqrt{b+\sqrt{b^2-4ac}}\sqrt{d+ex^2}}\right)}{a^3\sqrt{b+\sqrt{b^2-4ac}}\sqrt{2cd-\left(b+\sqrt{b^2-4ac}\right)e}}$$

Mathematica [A] (verified)

Time = 11.23 (sec) , antiderivative size = 383, normalized size of antiderivative = 0.86

$$\int \frac{1}{x^6\sqrt{d+ex^2}\,(a+bx^2+cx^4)}\,dx = -\frac{\frac{15(b^2-ac)\sqrt{d+ex^2}}{dx} - \frac{5ab(d-2ex^2)\sqrt{d+ex^2}}{d^2x^3} + \frac{a^2\sqrt{d+ex^2}\,(3d^2-4dex^2+8e^2x^4)}{d^3x^5} + \frac{15c\left(b^2-ac+\frac{b(b^2-3ac)}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2cd-be+\sqrt{b^2-4ac}ex}}{\sqrt{b-\sqrt{b^2-4ac}}\sqrt{d+ex^2}}\right)}{\sqrt{b-\sqrt{b^2-4ac}}\sqrt{2cd+\left(-b+\sqrt{b^2-4ac}\right)e}}}{15a^3}$$

```
[In] Integrate[1/(x^6*Sqrt[d + e*x^2]*(a + b*x^2 + c*x^4)),x]
```

```
[Out] -1/15*((15*(b^2 - a*c)*Sqrt[d + e*x^2])/(d*x) - (5*a*b*(d - 2*e*x^2)*Sqrt[d
 + e*x^2])/(d^2*x^3) + (a^2*Sqrt[d + e*x^2]*(3*d^2 - 4*d*e*x^2 + 8*e^2*x^4)
)/(d^3*x^5) + (15*c*(b^2 - a*c + (b*(b^2 - 3*a*c))/Sqrt[b^2 - 4*a*c])*ArcTa
n[(Sqrt[2*c*d - b*e + Sqrt[b^2 - 4*a*c]*e]*x)/(Sqrt[b - Sqrt[b^2 - 4*a*c]]*
Sqrt[d + e*x^2])])/(Sqrt[b - Sqrt[b^2 - 4*a*c]]*Sqrt[2*c*d + (-b + Sqrt[b^2
 - 4*a*c])*e]) + (15*c*(b^2 - a*c - (b*(b^2 - 3*a*c))/Sqrt[b^2 - 4*a*c])*Ar
cTan[(Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]*x)/(Sqrt[b + Sqrt[b^2 - 4*a*c
]]*Sqrt[d + e*x^2])])/(Sqrt[b + Sqrt[b^2 - 4*a*c]]*Sqrt[2*c*d - (b + Sqrt[b
^2 - 4*a*c])*e]))/a^3
```

Maple [A] (verified)

Time = 0.98 (sec) , antiderivative size = 410, normalized size of antiderivative = 0.93

method	result
risch	$-\frac{\sqrt{e\,x^2+d}\,\left(8a^2e^2x^4+10abde\,x^4-15ac\,d^2x^4+15b^2d^2x^4-4a^2de\,x^2-5ab\,d^2x^2+3a^2d^2\right)}{15d^3a^3x^5}-\frac{\sqrt{2}\left(\frac{\left(-2a^2c^2d+4a\,b^2cd-d\,b^4+2\sqrt{\cdots}\right.}{\cdots}\right.}{\cdots}$
default	$\frac{-\frac{\sqrt{e\,x^2+d}}{5d\,x^5}-\frac{4e\left(-\frac{\sqrt{e\,x^2+d}}{3d\,x^3}+\frac{2e\sqrt{e\,x^2+d}}{3d^2x}\right)}{5d}}{a}-\frac{b\left(-\frac{\sqrt{e\,x^2+d}}{3d\,x^3}+\frac{2e\sqrt{e\,x^2+d}}{3d^2x}\right)}{a^2}-\frac{\left(-ac+b^2\right)\sqrt{e\,x^2+d}}{a^3dx}-\frac{\sqrt{2}\left(\frac{\left(-2a^2c^2d+4a\,b^2\cdots\right.}{\cdots}\right.}{\cdots}$
pseudoelliptic	$-\frac{-5\sqrt{\left(-2ae+bd+\sqrt{-4d^2\left(ac-\frac{b^2}{4}\right)}\right)a}\,\left(\left(abc-\frac{1}{2}b^3\right)\sqrt{-4d^2\left(ac-\frac{b^2}{4}\right)}+d\left(-2a\,b^2c+\frac{1}{2}b^4+a^2c^2\right)\right)\sqrt{2}\,d^3x^5\,\mathrm{arctanh}\left(\frac{\cdots}{x\sqrt{\left(2\cdots\right.}}\right.}{\cdots}$

```
[In] int(1/x^6/(c*x^4+b*x^2+a)/(e*x^2+d)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/15*(e*x^2+d)^(1/2)*(8*a^2*e^2*x^4+10*a*b*d*e*x^4-15*a*c*d^2*x^4+15*b^2*d
^2*x^4-4*a^2*d*e*x^2-5*a*b*d^2*x^2+3*a^2*d^2)/d^3/a^3/x^5-1/2/a^3*2^(1/2)/(
-d^2*(4*a*c-b^2))^(1/2)*((-2*a^2*c^2*d+4*a*b^2*c*d-d*b^4+2*(-d^2*(4*a*c-b^2
))^(1/2)*a*b*c-(-d^2*(4*a*c-b^2))^(1/2)*b^3)/((-2*a*e+b*d+(-d^2*(4*a*c-b^2)
)^(1/2))*a)^(1/2)*arctan(a/x*(e*x^2+d)^(1/2)*2^(1/2)/((-2*a*e+b*d+(-d^2*(4*
a*c-b^2))^(1/2))*a)^(1/2))-(2*a^2*c^2*d-4*a*b^2*c*d+d*b^4+2*(-d^2*(4*a*c-b^
2))^(1/2)*a*b*c-(-d^2*(4*a*c-b^2))^(1/2)*b^3)/((2*a*e-b*d+(-d^2*(4*a*c-b^2)
)^(1/2))*a)^(1/2)*arctanh(a/x*(e*x^2+d)^(1/2)*2^(1/2)/((2*a*e-b*d+(-d^2*(4*
a*c-b^2))^(1/2))*a)^(1/2)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 9998 vs. $2(381) = 762$.

Time = 165.00 (sec) , antiderivative size = 9998, normalized size of antiderivative = 22.57

$$\int \frac{1}{x^6\sqrt{d+ex^2}\,(a+bx^2+cx^4)}\,dx = \text{Too large to display}$$

```
[In] integrate(1/x^6/(c*x^4+b*x^2+a)/(e*x^2+d)^(1/2),x, algorithm="fricas")
[Out] Too large to include
```

Sympy [F]

$$\int \frac{1}{x^6\sqrt{d+ex^2}\,(a+bx^2+cx^4)}\,dx = \int \frac{1}{x^6\sqrt{d+ex^2}\,(a+bx^2+cx^4)}\,dx$$

```
[In] integrate(1/x**6/(c*x**4+b*x**2+a)/(e*x**2+d)**(1/2),x)
[Out] Integral(1/(x**6*sqrt(d + e*x**2)*(a + b*x**2 + c*x**4)), x)
```

Maxima [F]

$$\int \frac{1}{x^6\sqrt{d+ex^2}\,(a+bx^2+cx^4)}\,dx = \int \frac{1}{(cx^4+bx^2+a)\sqrt{ex^2+d}x^6}\,dx$$

```
[In] integrate(1/x^6/(c*x^4+b*x^2+a)/(e*x^2+d)^(1/2),x, algorithm="maxima")
[Out] integrate(1/((c*x^4 + b*x^2 + a)*sqrt(e*x^2 + d)*x^6), x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{1}{x^6\sqrt{d+ex^2}\,(a+bx^2+cx^4)}\,dx = \text{Timed out}$$

```
[In] integrate(1/x^6/(c*x^4+b*x^2+a)/(e*x^2+d)^(1/2),x, algorithm="giac")
[Out] Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^6\sqrt{d+ex^2}\,(a+bx^2+cx^4)}\,dx = \int \frac{1}{x^6\,\sqrt{e\,x^2+d}\,\left(c\,x^4+b\,x^2+a\right)}\,dx$$

```
[In] int(1/(x^6*(d + e*x^2)^(1/2)*(a + b*x^2 + c*x^4)),x)
[Out] int(1/(x^6*(d + e*x^2)^(1/2)*(a + b*x^2 + c*x^4)), x)
```

3.394 $\int \frac{x^6}{(d+ex^2)^{3/2}(a+bx^2+cx^4)}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 350

$$\int \frac{x^6}{(d+ex^2)^{3/2}(a+bx^2+cx^4)}\,dx = -\frac{d^2x}{e(cd^2-bde+ae^2)\sqrt{d+ex^2}} + \frac{2\left(b^2-ac-\frac{b(b^2-3ac)}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}\,x}{\sqrt{b-\sqrt{b^2-4ac}}\sqrt{d+ex^2}}\right)}{c\sqrt{b-\sqrt{b^2-4ac}}\left(2cd-\left(b-\sqrt{b^2-4ac}\right)e\right)^{3/2}} + \frac{2\left(b^2-ac+\frac{b(b^2-3ac)}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2cd-\left(b+\sqrt{b^2-4ac}\right)e}\,x}{\sqrt{b+\sqrt{b^2-4ac}}\sqrt{d+ex^2}}\right)}{c\sqrt{b+\sqrt{b^2-4ac}}\left(2cd-\left(b+\sqrt{b^2-4ac}\right)e\right)^{3/2}} + \frac{\operatorname{arctanh}\left(\frac{\sqrt{e}\,x}{\sqrt{d+ex^2}}\right)}{ce^{3/2}}$$

```
[Out] arctanh(x*e^(1/2)/(e*x^2+d)^(1/2))/c/e^(3/2)-d^2*x/e/(a*e^2-b*d*e+c*d^2)/(e
*x^2+d)^(1/2)+2*arctan(x*(2*c*d-e*(b-(-4*a*c+b^2)^(1/2)))^(1/2)/(e*x^2+d)^(
1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2))*(b^2-a*c-b*(-3*a*c+b^2)/(-4*a*c+b^2)^(1/
2))/c/(2*c*d-e*(b-(-4*a*c+b^2)^(1/2)))^(3/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2)+2
*arctan(x*(2*c*d-e*(b+(-4*a*c+b^2)^(1/2)))^(1/2)/(e*x^2+d)^(1/2)/(b+(-4*a*c
+b^2)^(1/2))^(1/2))*(b^2-a*c+b*(-3*a*c+b^2)/(-4*a*c+b^2)^(1/2))/c/(2*c*d-e*
(b+(-4*a*c+b^2)^(1/2)))^(3/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2)
```

Rubi [A] (verified)

Time = 2.97 (sec) , antiderivative size = 507, normalized size of antiderivative = 1.45, number of steps used = 14, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.241$, Rules used = {1311, 294, 223, 212, 1706, 385, 211}

$$\int \frac{x^6}{(d+ex^2)^{3/2}(a+bx^2+cx^4)}dx = \frac{\left(-\frac{2a^2ce-ab^2e-3abcd+b^3d}{\sqrt{b^2-4ac}}-abe-acd+b^2d\right)\arctan\left(\frac{x\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}{\sqrt{b-\sqrt{b^2-4ac}}\sqrt{d+ex^2}}\right)}{c\sqrt{b-\sqrt{b^2-4ac}}\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}(ae^2-bde+cd^2)} + \frac{\left(\frac{2a^2ce-ab^2e-3abcd+b^3d}{\sqrt{b^2-4ac}}-abe-acd+b^2d\right)\arctan\left(\frac{x\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}{\sqrt{\sqrt{b^2-4ac}+b}\sqrt{d+ex^2}}\right)}{c\sqrt{\sqrt{b^2-4ac}+b}\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}(ae^2-bde+cd^2)} - \frac{(bd-ae)\text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{c\sqrt{e}(ae^2-bde+cd^2)} + \frac{d^2\text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{e^{3/2}(ae^2-bde+cd^2)} - \frac{d^2x}{e\sqrt{d+ex^2}(ae^2-bde+cd^2)}$$

```
[In] Int[x^6/((d + e*x^2)^(3/2)*(a + b*x^2 + c*x^4)),x]
```

```
[Out] -((d^2*x)/(e*(c*d^2 - b*d*e + a*e^2)*Sqrt[d + e*x^2])) + ((b^2*d - a*c*d -
a*b*e - (b^3*d - 3*a*b*c*d - a*b^2*e + 2*a^2*c*e)/Sqrt[b^2 - 4*a*c])*ArcTan
[(Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]*x)/(Sqrt[b - Sqrt[b^2 - 4*a*c]]*S
qrt[d + e*x^2])])/(c*Sqrt[b - Sqrt[b^2 - 4*a*c]]*Sqrt[2*c*d - (b - Sqrt[b^2
 - 4*a*c])*e]*(c*d^2 - b*d*e + a*e^2)) + ((b^2*d - a*c*d - a*b*e + (b^3*d -
 3*a*b*c*d - a*b^2*e + 2*a^2*c*e)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2*c*d - (
b + Sqrt[b^2 - 4*a*c])*e]*x)/(Sqrt[b + Sqrt[b^2 - 4*a*c]]*Sqrt[d + e*x^2])]
)/(c*Sqrt[b + Sqrt[b^2 - 4*a*c]]*Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]*(c
*d^2 - b*d*e + a*e^2)) + (d^2*ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]])/(e^(3/2
)*(c*d^2 - b*d*e + a*e^2)) - ((b*d - a*e)*ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^
2]])/(c*Sqrt[e]*(c*d^2 - b*d*e + a*e^2))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 1311

```
Int[(((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x_)^2)^(q_))/((a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4), x_Symbol] :> Dist[d^2*(f^4/(c*d^2 - b*d*e + a*e^2)), Int[(f
*x)^(m - 4)*(d + e*x^2)^q, x], x] - Dist[f^4/(c*d^2 - b*d*e + a*e^2), Int[(
f*x)^(m - 4)*(d + e*x^2)^(q + 1)*(Simp[a*d + (b*d - a*e)*x^2, x]/(a + b*x^2
 + c*x^4)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b^2 - 4*a*c, 0] &
&  !IntegerQ[q] && LtQ[q, -1] && GtQ[m, 3]
```

Rule 1706

```
Int[(Px_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^
(p_.), x_Symbol] :> Int[ExpandIntegrand[Px*(d + e*x^2)^q*(a + b*x^2 + c*x^4
)^p, x], x] /; FreeQ[{a, b, c, d, e, q}, x] && PolyQ[Px, x^2] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{\int \frac{x^2(ad+(bd-ae)x^2)}{\sqrt{d+ex^2}(a+bx^2+cx^4)}\,dx}{cd^2-bde+ae^2}+\frac{d^2\int \frac{x^2}{(d+ex^2)^{3/2}}\,dx}{cd^2-bde+ae^2}\\
&= -\frac{d^2x}{e\left(cd^2-bde+ae^2\right)\sqrt{d+ex^2}}\\
&\quad -\frac{\int \left(\frac{bd-ae}{c\sqrt{d+ex^2}}-\frac{a(bd-ae)+(b^2d-acd-abe)x^2}{c\sqrt{d+ex^2}(a+bx^2+cx^4)}\right)dx}{cd^2-bde+ae^2}+\frac{d^2\int \frac{1}{\sqrt{d+ex^2}}\,dx}{e\left(cd^2-bde+ae^2\right)}
\end{aligned}$$

$$= -\frac{d^2x}{e\left(cd^2-bde+ae^2\right)\sqrt{d+ex^2}} + \frac{\int \frac{a(bd-ae)+\left(b^2d-acd-abe\right)x^2}{\sqrt{d+ex^2}\left(a+bx^2+cx^4\right)}\,dx}{c\left(cd^2-bde+ae^2\right)} + \frac{d^2\text{Subst}\left(\int \frac{1}{1-ex^2}\,dx, x, \frac{x}{\sqrt{d+ex^2}}\right)}{e\left(cd^2-bde+ae^2\right)} - \frac{(bd-ae)\int \frac{1}{\sqrt{d+ex^2}}\,dx}{c\left(cd^2-bde+ae^2\right)}$$

$$= -\frac{d^2x}{e\left(cd^2-bde+ae^2\right)\sqrt{d+ex^2}} + \frac{d^2\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{e^{3/2}\left(cd^2-bde+ae^2\right)} + \frac{\int \left(\frac{b^2d-acd-abe+\frac{-b^3d+3abcd+ab^2e-2a^2ce}{\sqrt{b^2-4ac}}}{\left(b-\sqrt{b^2-4ac}+2cx^2\right)\sqrt{d+ex^2}} + \frac{b^2d-acd-abe-\frac{-b^3d+3abcd+ab^2e-2a^2ce}{\sqrt{b^2-4ac}}}{\left(b+\sqrt{b^2-4ac}+2cx^2\right)\sqrt{d+ex^2}}\right)dx}{c\left(cd^2-bde+ae^2\right)} - \frac{(bd-ae)\text{Subst}\left(\int \frac{1}{1-ex^2}\,dx, x, \frac{x}{\sqrt{d+ex^2}}\right)}{c\left(cd^2-bde+ae^2\right)}$$

$$= -\frac{d^2x}{e\left(cd^2-bde+ae^2\right)\sqrt{d+ex^2}} + \frac{d^2\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{e^{3/2}\left(cd^2-bde+ae^2\right)} - \frac{(bd-ae)\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{c\sqrt{e}\left(cd^2-bde+ae^2\right)} + \frac{\left(b^2d-acd-abe-\frac{b^3d-3abcd-ab^2e+2a^2ce}{\sqrt{b^2-4ac}}\right)\int \frac{1}{\left(b-\sqrt{b^2-4ac}+2cx^2\right)\sqrt{d+ex^2}}}{c\left(cd^2-bde+ae^2\right)} + \frac{\left(b^2d-acd-abe+\frac{b^3d-3abcd-ab^2e+2a^2ce}{\sqrt{b^2-4ac}}\right)\int \frac{1}{\left(b+\sqrt{b^2-4ac}+2cx^2\right)\sqrt{d+ex^2}}\,dx}{c\left(cd^2-bde+ae^2\right)}$$

$$= -\frac{d^2x}{e\left(cd^2-bde+ae^2\right)\sqrt{d+ex^2}} + \frac{d^2\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{e^{3/2}\left(cd^2-bde+ae^2\right)} - \frac{(bd-ae)\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{c\sqrt{e}\left(cd^2-bde+ae^2\right)} + \frac{\left(b^2d-acd-abe-\frac{b^3d-3abcd-ab^2e+2a^2ce}{\sqrt{b^2-4ac}}\right)\text{Subst}\left(\int \frac{1}{b-\sqrt{b^2-4ac}-\left(-2cd+\left(b-\sqrt{b^2-4ac}\right)e\right)x^2}\,dx, x, \frac{x}{\sqrt{d+ex^2}}\right.}{c\left(cd^2-bde+ae^2\right)} + \frac{\left(b^2d-acd-abe+\frac{b^3d-3abcd-ab^2e+2a^2ce}{\sqrt{b^2-4ac}}\right)\text{Subst}\left(\int \frac{1}{b+\sqrt{b^2-4ac}-\left(-2cd+\left(b+\sqrt{b^2-4ac}\right)e\right)x^2}\,dx, x, \frac{x}{\sqrt{d+ex^2}}\right.}{c\left(cd^2-bde+ae^2\right)}$$

$$
\begin{aligned}
&= -\frac{d^2x}{e\left(cd^2-bde+ae^2\right)\sqrt{d+ex^2}} \\
&+ \frac{\left(b^2d-acd-abe-\frac{b^3d-3abcd-ab^2e+2a^2ce}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}x}{\sqrt{b-\sqrt{b^2-4ac}}\sqrt{d+ex^2}}\right)}{c\sqrt{b-\sqrt{b^2-4ac}}\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}\left(cd^2-bde+ae^2\right)} \\
&+ \frac{\left(b^2d-acd-abe+\frac{b^3d-3abcd-ab^2e+2a^2ce}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2cd-\left(b+\sqrt{b^2-4ac}\right)e}x}{\sqrt{b+\sqrt{b^2-4ac}}\sqrt{d+ex^2}}\right)}{c\sqrt{b+\sqrt{b^2-4ac}}\sqrt{2cd-\left(b+\sqrt{b^2-4ac}\right)e}\left(cd^2-bde+ae^2\right)} \\
&+ \frac{d^2\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{e^{3/2}\left(cd^2-bde+ae^2\right)} - \frac{(bd-ae)\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{c\sqrt{e}\left(cd^2-bde+ae^2\right)}
\end{aligned}
$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 10546 vs. $2(350) = 700$.

Time = 21.27 (sec) , antiderivative size = 10546, normalized size of antiderivative = 30.13

$$
\int \frac{x^6}{\left(d+ex^2\right)^{3/2}\left(a+bx^2+cx^4\right)}\,dx = \text{Result too large to show}
$$

```
[In] Integrate[x^6/((d + e*x^2)^(3/2)*(a + b*x^2 + c*x^4)),x]
[Out] Result too large to show
```

Maple [A] (verified)

Time = 0.61 (sec) , antiderivative size = 502, normalized size of antiderivative = 1.43

method	result
pseudoelliptic	$-\frac{\sqrt{\left(-2ae+bd+\sqrt{-4d^2\left(ac-\frac{b^2}{4}\right)}\right)a}\,a\sqrt{2}\,\sqrt{e\,x^2+d}\left(\frac{\left(-e^{\frac{3}{2}}bd+a\,e^{\frac{5}{2}}\right)\sqrt{-4d^2\left(ac-\frac{b^2}{4}\right)}}{2}+d\left(d\left(ac-\frac{b^2}{2}\right)e^{\frac{3}{2}}+\frac{ab\,e^{\frac{5}{2}}}{2}\right)\right)\text{arctanh}}{}$
default	$\frac{-\frac{x}{e\sqrt{e\,x^2+d}}+\frac{\ln\left(x\sqrt{e}+\sqrt{e\,x^2+d}\right)}{e^{\frac{3}{2}}}}{c}-\frac{bx}{c^2d\sqrt{e\,x^2+d}}-\frac{\left((ae-bd)\sqrt{-4d^2\left(ac-\frac{b^2}{4}\right)}+\left(\left(2ac-b^2\right)d+abe\right)d\right)a\sqrt{2}\,cd\sqrt{\left(-2ae+bd+\right.}}{}$

```
[In] int(x^6/(e*x^2+d)^(3/2)/(c*x^4+b*x^2+a),x,method=_RETURNVERBOSE)
```

```
[Out] -1/(-4*d^2*(a*c-1/4*b^2))^(1/2)/e^(3/2)/((-2*a*e+b*d+(-4*d^2*(a*c-1/4*b^2))
^(1/2))*a)^(1/2)/((2*a*e-b*d+(-4*d^2*(a*c-1/4*b^2))^(1/2))*a)^(1/2)*(((-2*a
*e+b*d+(-4*d^2*(a*c-1/4*b^2))^(1/2))*a)^(1/2)*a*2^(1/2)*(e*x^2+d)^(1/2)*(1/
2*(-e^(3/2)*b*d+a*e^(5/2))*(-4*d^2*(a*c-1/4*b^2))^(1/2)+d*(d*(a*c-1/2*b^2)*
e^(3/2)+1/2*a*b*e^(5/2)))*arctanh(a/x*(e*x^2+d)^(1/2)*2^(1/2)/((2*a*e-b*d+(
-4*d^2*(a*c-1/4*b^2))^(1/2))*a)^(1/2))+((2*a*e-b*d+(-4*d^2*(a*c-1/4*b^2))^(
1/2))*a)^(1/2)*(a*2^(1/2)*(1/2*(e^(3/2)*b*d-a*e^(5/2))*(-4*d^2*(a*c-1/4*b^2
))^(1/2)+d*(d*(a*c-1/2*b^2)*e^(3/2)+1/2*a*b*e^(5/2)))*(e*x^2+d)^(1/2)*arcta
n(a/x*(e*x^2+d)^(1/2)*2^(1/2)/((-2*a*e+b*d+(-4*d^2*(a*c-1/4*b^2))^(1/2))*a)
^(1/2))-((e*x^2+d)^(1/2)*(a*e^2-b*d*e+c*d^2)*arctanh((e*x^2+d)^(1/2)/x/e^(1
/2))-e^(1/2)*c*d^2*x)*((-2*a*e+b*d+(-4*d^2*(a*c-1/4*b^2))^(1/2))*a)^(1/2)*(
-4*d^2*(a*c-1/4*b^2))^(1/2)))/(e*x^2+d)^(1/2)/(a*e^2-b*d*e+c*d^2)/c
```

Fricas [F(-1)]

Timed out.

$$\int \frac{x^6}{(d+ex^2)^{3/2}(a+bx^2+cx^4)}\,dx = \text{Timed out}$$

```
[In] integrate(x^6/(e*x^2+d)^(3/2)/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]

$$\int \frac{x^6}{(d+ex^2)^{3/2}(a+bx^2+cx^4)}\,dx = \int \frac{x^6}{(d+ex^2)^{\frac{3}{2}}(a+bx^2+cx^4)}\,dx$$

```
[In] integrate(x**6/(e*x**2+d)**(3/2)/(c*x**4+b*x**2+a),x)
```

```
[Out] Integral(x**6/((d + e*x**2)**(3/2)*(a + b*x**2 + c*x**4)), x)
```

Maxima [F]

$$\int \frac{x^6}{(d+ex^2)^{3/2}(a+bx^2+cx^4)}\,dx = \int \frac{x^6}{(cx^4+bx^2+a)(ex^2+d)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(x^6/(e*x^2+d)^(3/2)/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

```
[Out] integrate(x^6/((c*x^4 + b*x^2 + a)*(e*x^2 + d)^(3/2)), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{x^6}{(d+ex^2)^{3/2}(a+bx^2+cx^4)}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(x^6/(e*x^2+d)^(3/2)/(c*x^4+b*x^2+a),x, algorithm="giac")
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:index.cc index_m i_lex_is_greater Err
or: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^6}{(d+ex^2)^{3/2}(a+bx^2+cx^4)}\,dx = \int \frac{x^6}{(e\,x^2+d)^{3/2}\,(c\,x^4+b\,x^2+a)}\,dx$$

```
[In] int(x^6/((d + e*x^2)^(3/2)*(a + b*x^2 + c*x^4)),x)
[Out] int(x^6/((d + e*x^2)^(3/2)*(a + b*x^2 + c*x^4)), x)
```

3.395 $\int \frac{x^4}{(d+ex^2)^{3/2}(a+bx^2+cx^4)}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 360

$$\int \frac{x^4}{(d+ex^2)^{3/2}(a+bx^2+cx^4)}\,dx = \frac{dx}{(cd^2-bde+ae^2)\sqrt{d+ex^2}} - \frac{\left(bd-ae-\frac{b^2d-2acd-abe}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}\,x}{\sqrt{b-\sqrt{b^2-4ac}}\sqrt{d+ex^2}}\right)}{\sqrt{b-\sqrt{b^2-4ac}}\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}\,(cd^2-bde+ae^2)} - \frac{\left(bd-ae+\frac{b^2d-2acd-abe}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2cd-\left(b+\sqrt{b^2-4ac}\right)e}\,x}{\sqrt{b+\sqrt{b^2-4ac}}\sqrt{d+ex^2}}\right)}{\sqrt{b+\sqrt{b^2-4ac}}\sqrt{2cd-\left(b+\sqrt{b^2-4ac}\right)e}\,(cd^2-bde+ae^2)}$$

```
[Out] d*x/(a*e^2-b*d*e+c*d^2)/(e*x^2+d)^(1/2)-arctan(x*(2*c*d-e*(b-(-4*a*c+b^2)^(
1/2)))^(1/2)/(e*x^2+d)^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2))*(b*d-a*e+(a*b*e+
2*a*c*d-b^2*d)/(-4*a*c+b^2)^(1/2))/(a*e^2-b*d*e+c*d^2)/(2*c*d-e*(b-(-4*a*c+
b^2)^(1/2)))^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2)-arctan(x*(2*c*d-e*(b+(-4*a*
c+b^2)^(1/2)))^(1/2)/(e*x^2+d)^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2))*(b*d-a*e
+(-a*b*e-2*a*c*d+b^2*d)/(-4*a*c+b^2)^(1/2))/(a*e^2-b*d*e+c*d^2)/(b+(-4*a*c+
b^2)^(1/2))^(1/2)/(2*c*d-e*(b+(-4*a*c+b^2)^(1/2)))^(1/2)
```

Rubi [A] (verified)

Time = 0.93 (sec) , antiderivative size = 360, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.172$, Rules used = {1311, 197, 1706, 385, 211}

$$\int \frac{x^4}{(d+ex^2)^{3/2}(a+bx^2+cx^4)}\,dx = -\frac{\left(-\frac{-abe-2acd+b^2d}{\sqrt{b^2-4ac}}-ae+bd\right)\arctan\left(\frac{x\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}{\sqrt{b-\sqrt{b^2-4ac}}\sqrt{d+ex^2}}\right)}{\sqrt{b-\sqrt{b^2-4ac}}\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}\,(ae^2-bde+cd^2)} - \frac{\left(\frac{-abe-2acd+b^2d}{\sqrt{b^2-4ac}}-ae+bd\right)\arctan\left(\frac{x\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}{\sqrt{\sqrt{b^2-4ac}+b}\sqrt{d+ex^2}}\right)}{\sqrt{\sqrt{b^2-4ac}+b}\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}\,(ae^2-bde+cd^2)} + \frac{dx}{\sqrt{d+ex^2}\,(ae^2-bde+cd^2)}$$

```
[In] Int[x^4/((d + e*x^2)^(3/2)*(a + b*x^2 + c*x^4)),x]
```

```
[Out] (d*x)/((c*d^2 - b*d*e + a*e^2)*Sqrt[d + e*x^2]) - ((b*d - a*e - (b^2*d - 2*
a*c*d - a*b*e)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*
c])*e]*x)/(Sqrt[b - Sqrt[b^2 - 4*a*c]]*Sqrt[d + e*x^2])]])/(Sqrt[b - Sqrt[b^
2 - 4*a*c]]*Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]*(c*d^2 - b*d*e + a*e^2)
) - ((b*d - a*e + (b^2*d - 2*a*c*d - a*b*e)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt
[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]*x)/(Sqrt[b + Sqrt[b^2 - 4*a*c]]*Sqrt[d
+ e*x^2])]])/(Sqrt[b + Sqrt[b^2 - 4*a*c]]*Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c
])*e]*(c*d^2 - b*d*e + a*e^2))
```

Rule 197

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
```

```
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 1311

```
Int[(((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x_)^2)^(q_))/((a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4), x_Symbol] :> Dist[d^2*(f^4/(c*d^2 - b*d*e + a*e^2)), Int[(f
*x)^(m - 4)*(d + e*x^2)^q, x], x] - Dist[f^4/(c*d^2 - b*d*e + a*e^2), Int[(
f*x)^(m - 4)*(d + e*x^2)^(q + 1)*(Simp[a*d + (b*d - a*e)*x^2, x]/(a + b*x^2
 + c*x^4)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b^2 - 4*a*c, 0] &
&  !IntegerQ[q] && LtQ[q, -1] && GtQ[m, 3]
```

Rule 1706

```
Int[(Px_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^
(p_.), x_Symbol] :> Int[ExpandIntegrand[Px*(d + e*x^2)^q*(a + b*x^2 + c*x^4
)^p, x], x] /; FreeQ[{a, b, c, d, e, q}, x] && PolyQ[Px, x^2] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{\int \frac{ad+(bd-ae)x^2}{\sqrt{d+ex^2}(a+bx^2+cx^4)}\,dx}{cd^2-bde+ae^2}+\frac{d^2\int \frac{1}{(d+ex^2)^{3/2}}\,dx}{cd^2-bde+ae^2} \\
&= \frac{dx}{(cd^2-bde+ae^2)\sqrt{d+ex^2}}-\frac{\int \left(\frac{bd-ae+\frac{-b^2d+2acd+abe}{\sqrt{b^2-4ac}}}{\left(b-\sqrt{b^2-4ac}+2cx^2\right)\sqrt{d+ex^2}}+\frac{bd-ae-\frac{-b^2d+2acd+abe}{\sqrt{b^2-4ac}}}{\left(b+\sqrt{b^2-4ac}+2cx^2\right)\sqrt{d+ex^2}}\right)dx}{cd^2-bde+ae^2} \\
&= \frac{dx}{(cd^2-bde+ae^2)\sqrt{d+ex^2}}-\frac{\left(bd-ae-\frac{b^2d-2acd-abe}{\sqrt{b^2-4ac}}\right)\int \frac{1}{\left(b-\sqrt{b^2-4ac}+2cx^2\right)\sqrt{d+ex^2}}\,dx}{cd^2-bde+ae^2} \\
&\quad -\frac{\left(bd-ae+\frac{b^2d-2acd-abe}{\sqrt{b^2-4ac}}\right)\int \frac{1}{\left(b+\sqrt{b^2-4ac}+2cx^2\right)\sqrt{d+ex^2}}\,dx}{cd^2-bde+ae^2} \\
&= \frac{dx}{(cd^2-bde+ae^2)\sqrt{d+ex^2}} \\
&\quad -\frac{\left(bd-ae-\frac{b^2d-2acd-abe}{\sqrt{b^2-4ac}}\right)\text{Subst}\left(\int \frac{1}{b-\sqrt{b^2-4ac}-\left(-2cd+\left(b-\sqrt{b^2-4ac}\right)e\right)x^2}\,dx,x,\frac{x}{\sqrt{d+ex^2}}\right)}{cd^2-bde+ae^2} \\
&\quad -\frac{\left(bd-ae+\frac{b^2d-2acd-abe}{\sqrt{b^2-4ac}}\right)\text{Subst}\left(\int \frac{1}{b+\sqrt{b^2-4ac}-\left(-2cd+\left(b+\sqrt{b^2-4ac}\right)e\right)x^2}\,dx,x,\frac{x}{\sqrt{d+ex^2}}\right)}{cd^2-bde+ae^2}
\end{aligned}$$

$$= \frac{dx}{(cd^2 - bde + ae^2)\sqrt{d + ex^2}} - \frac{\left(bd - ae - \frac{b^2d - 2acd - abe}{\sqrt{b^2 - 4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2cd - \left(b - \sqrt{b^2 - 4ac}\right)e}x}{\sqrt{b - \sqrt{b^2 - 4ac}}\sqrt{d + ex^2}}\right)}{\sqrt{b - \sqrt{b^2 - 4ac}}\sqrt{2cd - \left(b - \sqrt{b^2 - 4ac}\right)e}\,(cd^2 - bde + ae^2)} - \frac{\left(bd - ae + \frac{b^2d - 2acd - abe}{\sqrt{b^2 - 4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2cd - \left(b + \sqrt{b^2 - 4ac}\right)e}x}{\sqrt{b + \sqrt{b^2 - 4ac}}\sqrt{d + ex^2}}\right)}{\sqrt{b + \sqrt{b^2 - 4ac}}\sqrt{2cd - \left(b + \sqrt{b^2 - 4ac}\right)e}\,(cd^2 - bde + ae^2)}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.69 (sec) , antiderivative size = 449, normalized size of antiderivative = 1.25

$$\int \frac{x^4}{(d + ex^2)^{3/2}(a + bx^2 + cx^4)}\,dx = \frac{\frac{4dx}{\sqrt{d+ex^2}} - d\text{RootSum}\Big[ae^4 + 4bde^2\#1^2 - 4ae^3\#1^2 + 16cd^2\#1^4 - 8bde\#1}{}$$

```
[In] Integrate[x^4/((d + e*x^2)^(3/2)*(a + b*x^2 + c*x^4)),x]
```

```
[Out] ((4*d*x)/Sqrt[d + e*x^2] - d*RootSum[a*e^4 + 4*b*d*e^2*#1^2 - 4*a*e^3*#1^2
+ 16*c*d^2*#1^4 - 8*b*d*e*#1^4 + 6*a*e^2*#1^4 + 4*b*d*#1^6 - 4*a*e*#1^6 + a
*#1^8 & , (-(a*e^3*Log[x]) + a*e^3*Log[-Sqrt[d] + Sqrt[d + e*x^2] - x*#1] -
 4*b*d*e*Log[x]*#1^2 + 7*a*e^2*Log[x]*#1^2 + 4*b*d*e*Log[-Sqrt[d] + Sqrt[d
+ e*x^2] - x*#1]*#1^2 - 7*a*e^2*Log[-Sqrt[d] + Sqrt[d + e*x^2] - x*#1]*#1^2
 + 4*b*d*Log[x]*#1^4 - 7*a*e*Log[x]*#1^4 - 4*b*d*Log[-Sqrt[d] + Sqrt[d + e*
x^2] - x*#1]*#1^4 + 7*a*e*Log[-Sqrt[d] + Sqrt[d + e*x^2] - x*#1]*#1^4 + a*L
og[x]*#1^6 - a*Log[-Sqrt[d] + Sqrt[d + e*x^2] - x*#1]*#1^6)/(b*d*e^2*#1 - a
*e^3*#1 + 8*c*d^2*#1^3 - 4*b*d*e*#1^3 + 3*a*e^2*#1^3 + 3*b*d*#1^5 - 3*a*e*#
1^5 + a*#1^7) & ])/(4*c*d^2 - 4*b*d*e + 4*a*e^2)
```

Maple [A] (verified)

Time = 0.54 (sec) , antiderivative size = 382, normalized size of antiderivative = 1.06

method	result
pseudoelliptic	$\frac{d\left(\sqrt{2}\,a\sqrt{\left(-2ae+bd+\sqrt{-4d^2\left(ac-\frac{b^2}{4}\right)}\right)a}\,\sqrt{e\,x^2+d}\left(ae-\frac{bd}{2}-\frac{\sqrt{-4d^2\left(ac-\frac{b^2}{4}\right)}}{2}\right)\operatorname{arctanh}\left(\frac{a\sqrt{e\,x^2+d}\,\sqrt{2}}{x\sqrt{\left(2ae-bd+\sqrt{-4d^2\left(ac-\frac{b^2}{4}\right)}\right)}}\right.}{\sqrt{\left(-2ae+bd+\sqrt{-4d^2}\right.}}$
default	$\frac{x}{cd\sqrt{e\,x^2+d}}+\frac{\sqrt{2}\,ac\,d^2\sqrt{\left(-2ae+bd+\sqrt{-4d^2\left(ac-\frac{b^2}{4}\right)}\right)a}\,\sqrt{e\,x^2+d}\left(ae-\frac{bd}{2}-\frac{\sqrt{-4d^2\left(ac-\frac{b^2}{4}\right)}}{2}\right)\operatorname{arctanh}\left(\frac{a\sqrt{e}}{x\sqrt{\left(2ae-bd+\right.}}\right.}{c\sqrt{}}$

[In] int(x^4/(e*x^2+d)^(3/2)/(c*x^4+b*x^2+a),x,method=_RETURNVERBOSE)

[Out] 1/((-2*a*e+b*d+(-4*d^2*(a*c-1/4*b^2))^(1/2))*a)^(1/2)/(e*x^2+d)^(1/2)/((2*a*e-b*d+(-4*d^2*(a*c-1/4*b^2))^(1/2))*a)^(1/2)*d/(-4*d^2*(a*c-1/4*b^2))^(1/2)*(2^(1/2)*a*((-2*a*e+b*d+(-4*d^2*(a*c-1/4*b^2))^(1/2))*a)^(1/2)*(e*x^2+d)^(1/2)*(a*e-1/2*b*d-1/2*(-4*d^2*(a*c-1/4*b^2))^(1/2))*arctanh(a/x*(e*x^2+d)^(1/2)*2^(1/2)/((2*a*e-b*d+(-4*d^2*(a*c-1/4*b^2))^(1/2))*a)^(1/2))+((2*a*e-b*d+(-4*d^2*(a*c-1/4*b^2))^(1/2))*a)^(1/2)*(a*2^(1/2)*(e*x^2+d)^(1/2)*(a*e-1/2*b*d+1/2*(-4*d^2*(a*c-1/4*b^2))^(1/2))*arctan(a/x*(e*x^2+d)^(1/2)*2^(1/2)/((-2*a*e+b*d+(-4*d^2*(a*c-1/4*b^2))^(1/2))*a)^(1/2))+x*(-4*d^2*(a*c-1/4*b^2))^(1/2)*((-2*a*e+b*d+(-4*d^2*(a*c-1/4*b^2))^(1/2))*a)^(1/2)))/(a*e^2-b*d*e+c*d^2)

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 14462 vs. $2(318) = 636$.

Time = 156.12 (sec) , antiderivative size = 14462, normalized size of antiderivative = 40.17

$$\int \frac{x^4}{(d+ex^2)^{3/2}\,(a+bx^2+cx^4)}\,dx = \text{Too large to display}$$

[In] integrate(x^4/(e*x^2+d)^(3/2)/(c*x^4+b*x^2+a),x, algorithm="fricas")

[Out] Too large to include

Sympy [F]

$$\int \frac{x^4}{(d+ex^2)^{3/2}(a+bx^2+cx^4)}\,dx = \int \frac{x^4}{(d+ex^2)^{\frac{3}{2}}(a+bx^2+cx^4)}\,dx$$

```
[In] integrate(x**4/(e*x**2+d)**(3/2)/(c*x**4+b*x**2+a),x)
[Out] Integral(x**4/((d + e*x**2)**(3/2)*(a + b*x**2 + c*x**4)), x)
```

Maxima [F]

$$\int \frac{x^4}{(d+ex^2)^{3/2}(a+bx^2+cx^4)}\,dx = \int \frac{x^4}{(cx^4+bx^2+a)(ex^2+d)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(x^4/(e*x^2+d)^(3/2)/(c*x^4+b*x^2+a),x, algorithm="maxima")
[Out] integrate(x^4/((c*x^4 + b*x^2 + a)*(e*x^2 + d)^(3/2)), x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{x^4}{(d+ex^2)^{3/2}(a+bx^2+cx^4)}\,dx = \text{Timed out}$$

```
[In] integrate(x^4/(e*x^2+d)^(3/2)/(c*x^4+b*x^2+a),x, algorithm="giac")
[Out] Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^4}{(d+ex^2)^{3/2}(a+bx^2+cx^4)}\,dx = \int \frac{x^4}{(e\,x^2+d)^{3/2}\,(c\,x^4+b\,x^2+a)}\,dx$$

```
[In] int(x^4/((d + e*x^2)^(3/2)*(a + b*x^2 + c*x^4)),x)
[Out] int(x^4/((d + e*x^2)^(3/2)*(a + b*x^2 + c*x^4)), x)
```

3.396 $\int \frac{x^2}{(d+ex^2)^{3/2}(a+bx^2+cx^4)}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 333

$$\int \frac{x^2}{(d+ex^2)^{3/2}(a+bx^2+cx^4)}\,dx = -\frac{ex}{(cd^2-bde+ae^2)\sqrt{d+ex^2}} + \frac{c\left(d-\frac{bd-2ae}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}\,x}{\sqrt{b-\sqrt{b^2-4ac}}\sqrt{d+ex^2}}\right)}{\sqrt{b-\sqrt{b^2-4ac}}\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}\,(cd^2-bde+ae^2)} + \frac{c\left(d+\frac{bd-2ae}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2cd-\left(b+\sqrt{b^2-4ac}\right)e}\,x}{\sqrt{b+\sqrt{b^2-4ac}}\sqrt{d+ex^2}}\right)}{\sqrt{b+\sqrt{b^2-4ac}}\sqrt{2cd-\left(b+\sqrt{b^2-4ac}\right)e}\,(cd^2-bde+ae^2)}$$

```
[Out] -e*x/(a*e^2-b*d*e+c*d^2)/(e*x^2+d)^(1/2)+c*arctan(x*(2*c*d-e*(b-(-4*a*c+b^2
)^(1/2)))^(1/2)/(e*x^2+d)^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2))*(d+(2*a*e-b*d
)/(-4*a*c+b^2)^(1/2))/(a*e^2-b*d*e+c*d^2)/(2*c*d-e*(b-(-4*a*c+b^2)^(1/2)))^
(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2)+c*arctan(x*(2*c*d-e*(b+(-4*a*c+b^2)^(1/2
)))^(1/2)/(e*x^2+d)^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2))*(d+(-2*a*e+b*d)/(-4
*a*c+b^2)^(1/2))/(a*e^2-b*d*e+c*d^2)/(b+(-4*a*c+b^2)^(1/2))^(1/2)/(2*c*d-e*
(b+(-4*a*c+b^2)^(1/2)))^(1/2)
```

Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 333, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.172$, Rules used = {1313, 197, 1706, 385, 211}

$$\int \frac{x^2}{(d+ex^2)^{3/2}(a+bx^2+cx^4)}\,dx = \frac{c\left(d-\frac{bd-2ae}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{x\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}{\sqrt{b-\sqrt{b^2-4ac}}\sqrt{d+ex^2}}\right)}{\sqrt{b-\sqrt{b^2-4ac}}\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}\,(ae^2-bde+cd^2)} + \frac{c\left(\frac{bd-2ae}{\sqrt{b^2-4ac}}+d\right)\arctan\left(\frac{x\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}{\sqrt{\sqrt{b^2-4ac}+b}\sqrt{d+ex^2}}\right)}{\sqrt{\sqrt{b^2-4ac}+b}\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}\,(ae^2-bde+cd^2)} - \frac{ex}{\sqrt{d+ex^2}\,(ae^2-bde+cd^2)}$$

```
[In] Int[x^2/((d + e*x^2)^(3/2)*(a + b*x^2 + c*x^4)),x]
```

```
[Out] -((e*x)/((c*d^2 - b*d*e + a*e^2)*Sqrt[d + e*x^2])) + (c*(d - (b*d - 2*a*e)/
Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]*x)/(Sqrt
[b - Sqrt[b^2 - 4*a*c]]*Sqrt[d + e*x^2])])/(Sqrt[b - Sqrt[b^2 - 4*a*c]]*Sqr
t[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]*(c*d^2 - b*d*e + a*e^2)) + (c*(d + (b*
d - 2*a*e)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*
e]*x)/(Sqrt[b + Sqrt[b^2 - 4*a*c]]*Sqrt[d + e*x^2])])/(Sqrt[b + Sqrt[b^2 -
4*a*c]]*Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]*(c*d^2 - b*d*e + a*e^2))
```

Rule 197

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 1313

```
Int[(((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x_)^2)^(q_))/((a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4), x_Symbol] :> Dist[(-d)*e*(f^2/(c*d^2 - b*d*e + a*e^2)), Int
```

```
[(f*x)^(m - 2)*(d + e*x^2)^q, x], x] + Dist[f^2/(c*d^2 - b*d*e + a*e^2), In
t[(f*x)^(m - 2)*(d + e*x^2)^(q + 1)*(Simp[a*e + c*d*x^2, x]/(a + b*x^2 + c*
x^4)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b^2 - 4*a*c, 0] &&  !I
ntegerQ[q] && LtQ[q, -1] && GtQ[m, 1] && LeQ[m, 3]
```

Rule 1706

```
Int[(Px_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^
(p_.), x_Symbol] :> Int[ExpandIntegrand[Px*(d + e*x^2)^q*(a + b*x^2 + c*x^4
)^p, x], x] /; FreeQ[{a, b, c, d, e, q}, x] && PolyQ[Px, x^2] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[p]
```

Rubi steps

$$\text{integral} = \frac{\int \frac{ae+cdx^2}{\sqrt{d+ex^2}(a+bx^2+cx^4)}\,dx}{cd^2-bde+ae^2} - \frac{(de)\int \frac{1}{(d+ex^2)^{3/2}}\,dx}{cd^2-bde+ae^2}$$

$$= -\frac{ex}{(cd^2-bde+ae^2)\sqrt{d+ex^2}} + \frac{\int \left(\frac{cd+\frac{c(-bd+2ae)}{\sqrt{b^2-4ac}}}{\left(b-\sqrt{b^2-4ac}+2cx^2\right)\sqrt{d+ex^2}} + \frac{cd-\frac{c(-bd+2ae)}{\sqrt{b^2-4ac}}}{\left(b+\sqrt{b^2-4ac}+2cx^2\right)\sqrt{d+ex^2}}\right)dx}{cd^2-bde+ae^2}$$

$$= -\frac{ex}{(cd^2-bde+ae^2)\sqrt{d+ex^2}} + \frac{\left(c\left(d-\frac{bd-2ae}{\sqrt{b^2-4ac}}\right)\right)\int \frac{1}{\left(b-\sqrt{b^2-4ac}+2cx^2\right)\sqrt{d+ex^2}}\,dx}{cd^2-bde+ae^2} + \frac{\left(c\left(d+\frac{bd-2ae}{\sqrt{b^2-4ac}}\right)\right)\int \frac{1}{\left(b+\sqrt{b^2-4ac}+2cx^2\right)\sqrt{d+ex^2}}\,dx}{cd^2-bde+ae^2}$$

$$= -\frac{ex}{(cd^2-bde+ae^2)\sqrt{d+ex^2}} + \frac{\left(c\left(d-\frac{bd-2ae}{\sqrt{b^2-4ac}}\right)\right)\text{Subst}\left(\int \frac{1}{b-\sqrt{b^2-4ac}-\left(-2cd+\left(b-\sqrt{b^2-4ac}\right)e\right)x^2}\,dx, x, \frac{x}{\sqrt{d+ex^2}}\right)}{cd^2-bde+ae^2} + \frac{\left(c\left(d+\frac{bd-2ae}{\sqrt{b^2-4ac}}\right)\right)\text{Subst}\left(\int \frac{1}{b+\sqrt{b^2-4ac}-\left(-2cd+\left(b+\sqrt{b^2-4ac}\right)e\right)x^2}\,dx, x, \frac{x}{\sqrt{d+ex^2}}\right)}{cd^2-bde+ae^2}$$

$$= -\frac{ex}{(cd^2 - bde + ae^2)\sqrt{d + ex^2}} + \frac{c\left(d - \frac{bd-2ae}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}x}{\sqrt{b-\sqrt{b^2-4ac}}\sqrt{d+ex^2}}\right)}{\sqrt{b-\sqrt{b^2-4ac}}\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}\,(cd^2-bde+ae^2)} + \frac{c\left(d + \frac{bd-2ae}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2cd-\left(b+\sqrt{b^2-4ac}\right)e}x}{\sqrt{b+\sqrt{b^2-4ac}}\sqrt{d+ex^2}}\right)}{\sqrt{b+\sqrt{b^2-4ac}}\sqrt{2cd-\left(b+\sqrt{b^2-4ac}\right)e}\,(cd^2-bde+ae^2)}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.64 (sec) , antiderivative size = 463, normalized size of antiderivative = 1.39

$$\int \frac{x^2}{(d+ex^2)^{3/2}(a+bx^2+cx^4)}\,dx =$$

$$-\frac{\frac{4ex}{\sqrt{d+ex^2}} - \text{RootSum}\Big[ae^4 + 4bde^2\#1^2 - 4ae^3\#1^2 + 16cd^2\#1^4 - 8bde\#1^4 + 6ae^2\#1^4 + 4bd\#1^6 - 4ae\#1^6 +}{}$$

```
[In] Integrate[x^2/((d + e*x^2)^(3/2)*(a + b*x^2 + c*x^4)),x]
```

```
[Out] -(((4*e*x)/Sqrt[d + e*x^2] - RootSum[a*e^4 + 4*b*d*e^2*#1^2 - 4*a*e^3*#1^2
+ 16*c*d^2*#1^4 - 8*b*d*e*#1^4 + 6*a*e^2*#1^4 + 4*b*d*#1^6 - 4*a*e*#1^6 + a
*#1^8 & , (-(a*e^4*Log[x]) + a*e^4*Log[-Sqrt[d] + Sqrt[d + e*x^2] - x*#1] -
 4*c*d^2*e*Log[x]*#1^2 + 3*a*e^3*Log[x]*#1^2 + 4*c*d^2*e*Log[-Sqrt[d] + Sqr
t[d + e*x^2] - x*#1]*#1^2 - 3*a*e^3*Log[-Sqrt[d] + Sqrt[d + e*x^2] - x*#1]*
#1^2 + 4*c*d^2*Log[x]*#1^4 - 3*a*e^2*Log[x]*#1^4 - 4*c*d^2*Log[-Sqrt[d] + S
qrt[d + e*x^2] - x*#1]*#1^4 + 3*a*e^2*Log[-Sqrt[d] + Sqrt[d + e*x^2] - x*#1
]*#1^4 + a*e*Log[x]*#1^6 - a*e*Log[-Sqrt[d] + Sqrt[d + e*x^2] - x*#1]*#1^6)
/(b*d*e^2*#1 - a*e^3*#1 + 8*c*d^2*#1^3 - 4*b*d*e*#1^3 + 3*a*e^2*#1^3 + 3*b*
d*#1^5 - 3*a*e*#1^5 + a*#1^7) & ])/(4*c*d^2 - 4*b*d*e + 4*a*e^2))
```

Maple [A] (verified)

Time = 0.40 (sec) , antiderivative size = 393, normalized size of antiderivative = 1.18

method	result
default	$\frac{a\sqrt{\left(-2ae+bd+\sqrt{-4d^2\left(ac-\frac{b^2}{4}\right)}\right)a}\,\sqrt{2}\,\sqrt{e\,x^2+d}\left(-bde+2c\,d^2+\sqrt{-4d^2\left(ac-\frac{b^2}{4}\right)}\,e\right)\operatorname{arctanh}\left(\frac{a\sqrt{e\,x^2+d}\,\sqrt{2}}{x\sqrt{\left(2ae-bd+\sqrt{-4d^2\left(ac-\frac{b^2}{4}\right.}\right.}}\right.}{\sqrt{\left(-2ae+bd+\sqrt{-4\right.}}}$
pseudoelliptic	$\frac{a\sqrt{\left(-2ae+bd+\sqrt{-4d^2\left(ac-\frac{b^2}{4}\right)}\right)a}\,\sqrt{2}\,\sqrt{e\,x^2+d}\left(-bde+2c\,d^2+\sqrt{-4d^2\left(ac-\frac{b^2}{4}\right)}\,e\right)\operatorname{arctanh}\left(\frac{a\sqrt{e\,x^2+d}\,\sqrt{2}}{x\sqrt{\left(2ae-bd+\sqrt{-4d^2\left(ac-\frac{b^2}{4}\right.}\right.}}\right.}{\sqrt{\left(-2ae+bd+\sqrt{-4\right.}}}$

```
[In] int(x^2/(e*x^2+d)^(3/2)/(c*x^4+b*x^2+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/((-2*a*e+b*d+(-4*d^2*(a*c-1/4*b^2))^(1/2))*a)^(1/2)*(a*((-2*a*e+b*d+(-4*d
^2*(a*c-1/4*b^2))^(1/2))*a)^(1/2)*2^(1/2)*(e*x^2+d)^(1/2)*(-b*d*e+2*c*d^2+(
-4*d^2*(a*c-1/4*b^2))^(1/2)*e)*arctanh(a/x*(e*x^2+d)^(1/2)*2^(1/2)/((2*a*e-
b*d+(-4*d^2*(a*c-1/4*b^2))^(1/2))*a)^(1/2))-((2*a*e-b*d+(-4*d^2*(a*c-1/4*b^
2))^(1/2))*a)^(1/2)*(a*2^(1/2)*(e*x^2+d)^(1/2)*(b*d*e-2*c*d^2+(-4*d^2*(a*c-
1/4*b^2))^(1/2)*e)*arctan(a/x*(e*x^2+d)^(1/2)*2^(1/2)/((-2*a*e+b*d+(-4*d^2*
(a*c-1/4*b^2))^(1/2))*a)^(1/2))+2*(-4*d^2*(a*c-1/4*b^2))^(1/2)*((-2*a*e+b*d
+(-4*d^2*(a*c-1/4*b^2))^(1/2))*a)^(1/2)*e*x))/(e*x^2+d)^(1/2)/((2*a*e-b*d+(
-4*d^2*(a*c-1/4*b^2))^(1/2))*a)^(1/2)/(-4*d^2*(a*c-1/4*b^2))^(1/2)/(2*a*e^2
-2*b*d*e+2*c*d^2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 14146 vs. 2(291) = 582.

Time = 111.97 (sec) , antiderivative size = 14146, normalized size of antiderivative = 42.48

$$\int \frac{x^2}{(d+ex^2)^{3/2}\,(a+bx^2+cx^4)}\,dx = \text{Too large to display}$$

```
[In] integrate(x^2/(e*x^2+d)^(3/2)/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] Too large to include
```

Sympy [F]

$$\int \frac{x^2}{(d+ex^2)^{3/2}\,(a+bx^2+cx^4)}\,dx = \int \frac{x^2}{(d+ex^2)^{\frac{3}{2}}\,(a+bx^2+cx^4)}\,dx$$

```
[In] integrate(x**2/(e*x**2+d)**(3/2)/(c*x**4+b*x**2+a),x)
[Out] Integral(x**2/((d + e*x**2)**(3/2)*(a + b*x**2 + c*x**4)), x)
```

Maxima [F]

$$\int \frac{x^2}{(d+ex^2)^{3/2}\,(a+bx^2+cx^4)}\,dx = \int \frac{x^2}{(cx^4+bx^2+a)(ex^2+d)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(x^2/(e*x^2+d)^(3/2)/(c*x^4+b*x^2+a),x, algorithm="maxima")
[Out] integrate(x^2/((c*x^4 + b*x^2 + a)*(e*x^2 + d)^(3/2)), x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{x^2}{(d+ex^2)^{3/2}\,(a+bx^2+cx^4)}\,dx = \text{Timed out}$$

```
[In] integrate(x^2/(e*x^2+d)^(3/2)/(c*x^4+b*x^2+a),x, algorithm="giac")
[Out] Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{(d+ex^2)^{3/2}\,(a+bx^2+cx^4)}\,dx = \int \frac{x^2}{(e\,x^2+d)^{3/2}\,(c\,x^4+b\,x^2+a)}\,dx$$

```
[In] int(x^2/((d + e*x^2)^(3/2)*(a + b*x^2 + c*x^4)),x)
[Out] int(x^2/((d + e*x^2)^(3/2)*(a + b*x^2 + c*x^4)), x)
```

3.397 $\int \frac{1}{(d+ex^2)^{3/2}(a+bx^2+cx^4)}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 341

$$\int \frac{1}{(d+ex^2)^{3/2}(a+bx^2+cx^4)}\,dx = \frac{e^2x}{d(cd^2-bde+ae^2)\sqrt{d+ex^2}} - \frac{c\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}\,x}{\sqrt{b-\sqrt{b^2-4ac}}\sqrt{d+ex^2}}\right)}{\sqrt{b-\sqrt{b^2-4ac}}\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}\,(cd^2-bde+ae^2)} - \frac{c\left(e+\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2cd-\left(b+\sqrt{b^2-4ac}\right)e}\,x}{\sqrt{b+\sqrt{b^2-4ac}}\sqrt{d+ex^2}}\right)}{\sqrt{b+\sqrt{b^2-4ac}}\sqrt{2cd-\left(b+\sqrt{b^2-4ac}\right)e}\,(cd^2-bde+ae^2)}$$

```
[Out] e^2*x/d/(a*e^2-b*d*e+c*d^2)/(e*x^2+d)^(1/2)-c*arctan(x*(2*c*d-e*(b-(-4*a*c+
b^2)^(1/2)))^(1/2)/(e*x^2+d)^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2))*(e+(b*e-2*
c*d)/(-4*a*c+b^2)^(1/2))/(a*e^2-b*d*e+c*d^2)/(2*c*d-e*(b-(-4*a*c+b^2)^(1/2)
))^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2)-c*arctan(x*(2*c*d-e*(b+(-4*a*c+b^2)^(
1/2)))^(1/2)/(e*x^2+d)^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2))*(e+(-b*e+2*c*d)/
(-4*a*c+b^2)^(1/2))/(a*e^2-b*d*e+c*d^2)/(b+(-4*a*c+b^2)^(1/2))^(1/2)/(2*c*d
-e*(b+(-4*a*c+b^2)^(1/2)))^(1/2)
```

Rubi [A] (verified)

Time = 0.60 (sec) , antiderivative size = 341, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {1186, 197, 1706, 385, 211}

$$\int \frac{1}{(d+ex^2)^{3/2}(a+bx^2+cx^4)}\,dx = -\frac{c\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{x\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}{\sqrt{b-\sqrt{b^2-4ac}}\sqrt{d+ex^2}}\right)}{\sqrt{b-\sqrt{b^2-4ac}}\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}\,(ae^2-bde+cd^2)} - \frac{c\left(\frac{2cd-be}{\sqrt{b^2-4ac}}+e\right)\arctan\left(\frac{x\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}{\sqrt{\sqrt{b^2-4ac}+b}\sqrt{d+ex^2}}\right)}{\sqrt{\sqrt{b^2-4ac}+b}\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}\,(ae^2-bde+cd^2)} + \frac{e^2x}{d\sqrt{d+ex^2}\,(ae^2-bde+cd^2)}$$

```
[In] Int[1/((d + e*x^2)^(3/2)*(a + b*x^2 + c*x^4)),x]
```

```
[Out] (e^2*x)/(d*(c*d^2 - b*d*e + a*e^2)*Sqrt[d + e*x^2]) - (c*(e - (2*c*d - b*e)
/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]*x)/(Sqr
t[b - Sqrt[b^2 - 4*a*c]]*Sqrt[d + e*x^2])])/(Sqrt[b - Sqrt[b^2 - 4*a*c]]*Sq
rt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]*(c*d^2 - b*d*e + a*e^2)) - (c*(e + (2
*c*d - b*e)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])
*e]*x)/(Sqrt[b + Sqrt[b^2 - 4*a*c]]*Sqrt[d + e*x^2])])/(Sqrt[b + Sqrt[b^2 -
 4*a*c]]*Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]*(c*d^2 - b*d*e + a*e^2))
```

Rule 197

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 1186

```
Int[((d_) + (e_.)*(x_)^2)^(q_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symb
ol] :> Dist[e^2/(c*d^2 - b*d*e + a*e^2), Int[(d + e*x^2)^q, x], x] + Dist[1
/(c*d^2 - b*d*e + a*e^2), Int[(d + e*x^2)^(q + 1)*((c*d - b*e - c*e*x^2)/(a
 + b*x^2 + c*x^4)), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c,
 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[q] && LtQ[q, -1]
```

Rule 1706

```
Int[(Px_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^
(p_.), x_Symbol] :> Int[ExpandIntegrand[Px*(d + e*x^2)^q*(a + b*x^2 + c*x^4
)^p, x], x] /; FreeQ[{a, b, c, d, e, q}, x] && PolyQ[Px, x^2] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\int \frac{cd-be-cex^2}{\sqrt{d+ex^2}(a+bx^2+cx^4)}\,dx}{cd^2-bde+ae^2}+\frac{e^2\int \frac{1}{(d+ex^2)^{3/2}}\,dx}{cd^2-bde+ae^2}\\
&= \frac{e^2x}{d\,(cd^2-bde+ae^2)\sqrt{d+ex^2}}+\frac{\int \left(\frac{-ce-\frac{c(-2cd+be)}{\sqrt{b^2-4ac}}}{\left(b-\sqrt{b^2-4ac}+2cx^2\right)\sqrt{d+ex^2}}+\frac{-ce+\frac{c(-2cd+be)}{\sqrt{b^2-4ac}}}{\left(b+\sqrt{b^2-4ac}+2cx^2\right)\sqrt{d+ex^2}}\right)dx}{cd^2-bde+ae^2}\\
&= \frac{e^2x}{d\,(cd^2-bde+ae^2)\sqrt{d+ex^2}}-\frac{\left(c\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\right)\int \frac{1}{\left(b-\sqrt{b^2-4ac}+2cx^2\right)\sqrt{d+ex^2}}\,dx}{cd^2-bde+ae^2}\\
&\quad -\frac{\left(c\left(e+\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\right)\int \frac{1}{\left(b+\sqrt{b^2-4ac}+2cx^2\right)\sqrt{d+ex^2}}\,dx}{cd^2-bde+ae^2}\\
&= \frac{e^2x}{d\,(cd^2-bde+ae^2)\sqrt{d+ex^2}}\\
&\quad -\frac{\left(c\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\right)\text{Subst}\left(\int \frac{1}{b-\sqrt{b^2-4ac}-\left(-2cd+\left(b-\sqrt{b^2-4ac}\right)e\right)x^2}\,dx,x,\frac{x}{\sqrt{d+ex^2}}\right)}{cd^2-bde+ae^2}\\
&\quad -\frac{\left(c\left(e+\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\right)\text{Subst}\left(\int \frac{1}{b+\sqrt{b^2-4ac}-\left(-2cd+\left(b+\sqrt{b^2-4ac}\right)e\right)x^2}\,dx,x,\frac{x}{\sqrt{d+ex^2}}\right)}{cd^2-bde+ae^2}
\end{aligned}$$

$$= \frac{e^2x}{d(cd^2-bde+ae^2)\sqrt{d+ex^2}} - \frac{c\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}x}{\sqrt{b-\sqrt{b^2-4ac}}\sqrt{d+ex^2}}\right)}{\sqrt{b-\sqrt{b^2-4ac}}\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}\,(cd^2-bde+ae^2)} - \frac{c\left(e+\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2cd-\left(b+\sqrt{b^2-4ac}\right)e}x}{\sqrt{b+\sqrt{b^2-4ac}}\sqrt{d+ex^2}}\right)}{\sqrt{b+\sqrt{b^2-4ac}}\sqrt{2cd-\left(b+\sqrt{b^2-4ac}\right)e}\,(cd^2-bde+ae^2)}$$

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 18.25 (sec) , antiderivative size = 2061, normalized size of antiderivative = 6.04

$$\int \frac{1}{(d+ex^2)^{3/2}(a+bx^2+cx^4)}\,dx = \text{Result too large to show}$$

```
[In] Integrate[1/((d + e*x^2)^(3/2)*(a + b*x^2 + c*x^4)),x]
[Out] (2*c*x*(45*Sqrt[-(((-b + Sqrt[b^2 - 4*a*c])*(2*c*d + (-b + Sqrt[b^2 - 4*a*c
])*e)*x^2*(d + e*x^2))/(d^2*(b - Sqrt[b^2 - 4*a*c] + 2*c*x^2)^2))] + (30*e*
x^2*Sqrt[-(((-b + Sqrt[b^2 - 4*a*c])*(2*c*d + (-b + Sqrt[b^2 - 4*a*c])*e)*x
^2*(d + e*x^2))/(d^2*(b - Sqrt[b^2 - 4*a*c] + 2*c*x^2)^2))])/d - 45*ArcSin[
Sqrt[-(((2*c*d + (-b + Sqrt[b^2 - 4*a*c])*e)*x^2)/(d*(-b + Sqrt[b^2 - 4*a*c
] - 2*c*x^2)))]] - (30*e*x^2*ArcSin[Sqrt[-(((2*c*d + (-b + Sqrt[b^2 - 4*a*c
])*e)*x^2)/(d*(-b + Sqrt[b^2 - 4*a*c] - 2*c*x^2)))]])/d - (45*(2*c*d + (-b
+ Sqrt[b^2 - 4*a*c])*e)*x^2*ArcSin[Sqrt[-(((2*c*d + (-b + Sqrt[b^2 - 4*a*c]
)*e)*x^2)/(d*(-b + Sqrt[b^2 - 4*a*c] - 2*c*x^2)))]])/(d*(-b + Sqrt[b^2 - 4*
a*c] - 2*c*x^2)) - (30*e*(2*c*d + (-b + Sqrt[b^2 - 4*a*c])*e)*x^4*ArcSin[Sq
rt[-(((2*c*d + (-b + Sqrt[b^2 - 4*a*c])*e)*x^2)/(d*(-b + Sqrt[b^2 - 4*a*c]
- 2*c*x^2)))]])/(d^2*(-b + Sqrt[b^2 - 4*a*c] - 2*c*x^2)) + 4*(-(((2*c*d + (
-b + Sqrt[b^2 - 4*a*c])*e)*x^2)/(d*(-b + Sqrt[b^2 - 4*a*c] - 2*c*x^2))))^(5
/2)*Sqrt[((-b + Sqrt[b^2 - 4*a*c])*(d + e*x^2))/(d*(-b + Sqrt[b^2 - 4*a*c]
- 2*c*x^2))]*Hypergeometric2F1[2, 2, 7/2, -(((2*c*d + (-b + Sqrt[b^2 - 4*a*
c])*e)*x^2)/(d*(-b + Sqrt[b^2 - 4*a*c] - 2*c*x^2)))] + (4*e*x^2*(-(((2*c*d
+ (-b + Sqrt[b^2 - 4*a*c])*e)*x^2)/(d*(-b + Sqrt[b^2 - 4*a*c] - 2*c*x^2))))
^(5/2)*Sqrt[((-b + Sqrt[b^2 - 4*a*c])*(d + e*x^2))/(d*(-b + Sqrt[b^2 - 4*a*
c] - 2*c*x^2))]*Hypergeometric2F1[2, 2, 7/2, -(((2*c*d + (-b + Sqrt[b^2 - 4
*a*c])*e)*x^2)/(d*(-b + Sqrt[b^2 - 4*a*c] - 2*c*x^2)))])/d))/(15*Sqrt[b^2 -
 4*a*c]*d*(-(((2*c*d + (-b + Sqrt[b^2 - 4*a*c])*e)*x^2)/(d*(-b + Sqrt[b^2 -
 4*a*c] - 2*c*x^2))))^(3/2)*(b - Sqrt[b^2 - 4*a*c] + 2*c*x^2)*Sqrt[d + e*x^
```

```
2]*Sqrt[((-b + Sqrt[b^2 - 4*a*c])*(d + e*x^2))/(d*(-b + Sqrt[b^2 - 4*a*c] -
 2*c*x^2))]]) - (2*c*x*Sqrt[((b + Sqrt[b^2 - 4*a*c])*(d + e*x^2))/(d*(b + Sq
rt[b^2 - 4*a*c] + 2*c*x^2))]*(45*Sqrt[-(((b + Sqrt[b^2 - 4*a*c])*(-2*c*d +
(b + Sqrt[b^2 - 4*a*c])*e)*x^2*(d + e*x^2))/(d^2*(b + Sqrt[b^2 - 4*a*c] + 2
*c*x^2)^2))] + (30*e*x^2*Sqrt[-(((b + Sqrt[b^2 - 4*a*c])*(-2*c*d + (b + Sqr
t[b^2 - 4*a*c])*e)*x^2*(d + e*x^2))/(d^2*(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)^
2))])/d - 45*ArcSin[Sqrt[((2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)*x^2)/(d*(b +
Sqrt[b^2 - 4*a*c] + 2*c*x^2))]] - (30*e*x^2*ArcSin[Sqrt[((2*c*d - (b + Sqrt
[b^2 - 4*a*c])*e)*x^2)/(d*(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2))]])/d + (45*(2*
c*d - (b + Sqrt[b^2 - 4*a*c])*e)*x^2*ArcSin[Sqrt[((2*c*d - (b + Sqrt[b^2 -
4*a*c])*e)*x^2)/(d*(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2))]])/(d*(b + Sqrt[b^2 -
 4*a*c] + 2*c*x^2)) - (30*e*(-2*c*d + (b + Sqrt[b^2 - 4*a*c])*e)*x^4*ArcSin
[Sqrt[((2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)*x^2)/(d*(b + Sqrt[b^2 - 4*a*c] +
 2*c*x^2))]])/(d^2*(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)) + 4*(((2*c*d - (b + S
qrt[b^2 - 4*a*c])*e)*x^2)/(d*(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)))^(5/2)*Sqrt
[((b + Sqrt[b^2 - 4*a*c])*(d + e*x^2))/(d*(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)
)]*Hypergeometric2F1[2, 2, 7/2, ((2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)*x^2)/(
d*(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2))] + (4*e*x^2*(((2*c*d - (b + Sqrt[b^2 -
 4*a*c])*e)*x^2)/(d*(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)))^(5/2)*Sqrt[((b + Sq
rt[b^2 - 4*a*c])*(d + e*x^2))/(d*(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2))]*Hyperg
eometric2F1[2, 2, 7/2, ((2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)*x^2)/(d*(b + Sq
rt[b^2 - 4*a*c] + 2*c*x^2))])/d))/(15*Sqrt[b^2 - 4*a*c]*(b + Sqrt[b^2 - 4*a
*c])*(((2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)*x^2)/(d*(b + Sqrt[b^2 - 4*a*c] +
 2*c*x^2)))^(3/2)*(d + e*x^2)^(3/2))
```

Maple [A] (verified)

Time = 0.41 (sec) , antiderivative size = 429, normalized size of antiderivative = 1.26

method	result
default	$-\frac{\left(\frac{(be-cd)\sqrt{-4d^2\left(ac-\frac{b^2}{4}\right)}}{2}+\left(\frac{bcd}{2}+e\left(ac-\frac{b^2}{2}\right)\right)d\right)\sqrt{2}\,d\sqrt{\left(-2ae+bd+\sqrt{-4d^2\left(ac-\frac{b^2}{4}\right)}\right)a}\,\sqrt{e\,x^2+d}\,\operatorname{arctanh}\left(\frac{\cdots}{x\sqrt{\left(2ae-b\cdots\right.}}\right.}{\sqrt{\cdots}}$
pseudoelliptic	$-\frac{\left(\frac{(be-cd)\sqrt{-4d^2\left(ac-\frac{b^2}{4}\right)}}{2}+\left(\frac{bcd}{2}+e\left(ac-\frac{b^2}{2}\right)\right)d\right)\sqrt{2}\,d\sqrt{\left(-2ae+bd+\sqrt{-4d^2\left(ac-\frac{b^2}{4}\right)}\right)a}\,\sqrt{e\,x^2+d}\,\operatorname{arctanh}\left(\frac{\cdots}{x\sqrt{\left(2ae-b\cdots\right.}}\right.}{\sqrt{\cdots}}$

```
[In] int(1/(e*x^2+d)^(3/2)/(c*x^4+b*x^2+a),x,method=_RETURNVERBOSE)

[Out] -1/((-2*a*e+b*d+(-4*d^2*(a*c-1/4*b^2))^(1/2))*a)^(1/2)/(e*x^2+d)^(1/2)/((2*
a*e-b*d+(-4*d^2*(a*c-1/4*b^2))^(1/2))*a)^(1/2)/(-4*d^2*(a*c-1/4*b^2))^(1/2)
```

```
*((1/2*(b*e-c*d)*(-4*d^2*(a*c-1/4*b^2))^(1/2)+(1/2*b*c*d+e*(a*c-1/2*b^2))*d
)*2^(1/2)*d*((-2*a*e+b*d+(-4*d^2*(a*c-1/4*b^2))^(1/2))*a)^(1/2)*(e*x^2+d)^(
1/2)*arctanh(a/x*(e*x^2+d)^(1/2)*2^(1/2)/((2*a*e-b*d+(-4*d^2*(a*c-1/4*b^2))
^(1/2))*a)^(1/2))+(2^(1/2)*(1/2*(-b*e+c*d)*(-4*d^2*(a*c-1/4*b^2))^(1/2)+(1/
2*b*c*d+e*(a*c-1/2*b^2))*d)*d*(e*x^2+d)^(1/2)*arctan(a/x*(e*x^2+d)^(1/2)*2^
(1/2)/((-2*a*e+b*d+(-4*d^2*(a*c-1/4*b^2))^(1/2))*a)^(1/2))-e^2*x*(-4*d^2*(a
*c-1/4*b^2))^(1/2)*((-2*a*e+b*d+(-4*d^2*(a*c-1/4*b^2))^(1/2))*a)^(1/2))*((2
*a*e-b*d+(-4*d^2*(a*c-1/4*b^2))^(1/2))*a)^(1/2))/(a*e^2-b*d*e+c*d^2)/d
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 17249 vs. 2(299) = 598.

Time = 217.47 (sec) , antiderivative size = 17249, normalized size of antiderivative = 50.58

$$\int \frac{1}{(d+ex^2)^{3/2}(a+bx^2+cx^4)}\,dx = \text{Too large to display}$$

```
[In] integrate(1/(e*x^2+d)^(3/2)/(c*x^4+b*x^2+a),x, algorithm="fricas")
[Out] Too large to include
```

Sympy [F]

$$\int \frac{1}{(d+ex^2)^{3/2}(a+bx^2+cx^4)}\,dx = \int \frac{1}{(d+ex^2)^{\frac{3}{2}}(a+bx^2+cx^4)}\,dx$$

```
[In] integrate(1/(e*x**2+d)**(3/2)/(c*x**4+b*x**2+a),x)
[Out] Integral(1/((d + e*x**2)**(3/2)*(a + b*x**2 + c*x**4)), x)
```

Maxima [F]

$$\int \frac{1}{(d+ex^2)^{3/2}(a+bx^2+cx^4)}\,dx = \int \frac{1}{(cx^4+bx^2+a)(ex^2+d)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/(e*x^2+d)^(3/2)/(c*x^4+b*x^2+a),x, algorithm="maxima")
[Out] integrate(1/((c*x^4 + b*x^2 + a)*(e*x^2 + d)^(3/2)), x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{1}{(d+ex^2)^{3/2}(a+bx^2+cx^4)}\,dx = \text{Timed out}$$

```
[In] integrate(1/(e*x^2+d)^(3/2)/(c*x^4+b*x^2+a),x, algorithm="giac")
[Out] Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(d+ex^2)^{3/2}(a+bx^2+cx^4)}\,dx = \int \frac{1}{(e\,x^2+d)^{3/2}\,(c\,x^4+b\,x^2+a)}\,dx$$

```
[In] int(1/((d + e*x^2)^(3/2)*(a + b*x^2 + c*x^4)),x)
[Out] int(1/((d + e*x^2)^(3/2)*(a + b*x^2 + c*x^4)), x)
```

3.398 $\int \frac{1}{x^2(d+ex^2)^{3/2}(a+bx^2+cx^4)}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 339

$$\int \frac{1}{x^2\left(d+ex^2\right)^{3/2}\left(a+bx^2+cx^4\right)}\,dx = \frac{e(cd-be)x}{ad\left(cd^2+e(-bd+ae)\right)\sqrt{d+ex^2}} + \frac{-d-2ex^2}{ad^2x\sqrt{d+ex^2}} - \frac{2c^2\left(1+\frac{b}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}\,x}{\sqrt{b-\sqrt{b^2-4ac}}\sqrt{d+ex^2}}\right)}{a\sqrt{b-\sqrt{b^2-4ac}}\left(2cd-\left(b-\sqrt{b^2-4ac}\right)e\right)^{3/2}} - \frac{2c^2\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2cd-\left(b+\sqrt{b^2-4ac}\right)e}\,x}{\sqrt{b+\sqrt{b^2-4ac}}\sqrt{d+ex^2}}\right)}{a\sqrt{b+\sqrt{b^2-4ac}}\left(2cd-\left(b+\sqrt{b^2-4ac}\right)e\right)^{3/2}}$$

```
[Out] e*(-b*e+c*d)*x/a/d/(c*d^2+e*(a*e-b*d))/(e*x^2+d)^(1/2)+(-2*e*x^2-d)/a/d^2/x
/(e*x^2+d)^(1/2)-2*c^2*arctan(x*(2*c*d-e*(b-(-4*a*c+b^2)^(1/2)))^(1/2)/(e*x
^2+d)^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2))*(1+b/(-4*a*c+b^2)^(1/2))/a/(2*c*d
-e*(b-(-4*a*c+b^2)^(1/2)))^(3/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2)-2*c^2*arctan(
x*(2*c*d-e*(b+(-4*a*c+b^2)^(1/2)))^(1/2)/(e*x^2+d)^(1/2)/(b+(-4*a*c+b^2)^(1
/2))^(1/2))*(1-b/(-4*a*c+b^2)^(1/2))/a/(2*c*d-e*(b+(-4*a*c+b^2)^(1/2)))^(3/
2)/(b+(-4*a*c+b^2)^(1/2))^(1/2)
```

Rubi [A] (verified)

Time = 2.00 (sec) , antiderivative size = 462, normalized size of antiderivative = 1.36, number of steps used = 12, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.276, Rules used = {1315, 277, 197, 6860, 270, 1706, 385, 211}

$$\int \frac{1}{x^2 (d+ex^2)^{3/2} (a+bx^2+cx^4)} dx = -\frac{c\left(\frac{2ace+b^2(-e)+bcd}{\sqrt{b^2-4ac}}-be+cd\right)\arctan\left(\frac{x\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}{\sqrt{b-\sqrt{b^2-4ac}}\sqrt{d+ex^2}}\right)}{a\sqrt{b-\sqrt{b^2-4ac}}\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}\left(ae^2-bde+cd^2\right)} - \frac{c\left(-\frac{2ace+b^2(-e)+bcd}{\sqrt{b^2-4ac}}-be+cd\right)\arctan\left(\frac{x\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}{\sqrt{\sqrt{b^2-4ac}+b}\sqrt{d+ex^2}}\right)}{a\sqrt{\sqrt{b^2-4ac}+b}\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}\left(ae^2-bde+cd^2\right)} - \frac{e^2}{dx\sqrt{d+ex^2}\left(ae^2-bde+cd^2\right)} - \frac{\sqrt{d+ex^2}(cd-be)}{adx\left(ae^2-bde+cd^2\right)} - \frac{2e^3x}{d^2\sqrt{d+ex^2}\left(ae^2-bde+cd^2\right)}$$

```
[In] Int[1/(x^2*(d + e*x^2)^(3/2)*(a + b*x^2 + c*x^4)),x]
```

```
[Out] -(e^2/(d*(c*d^2 - b*d*e + a*e^2)*x*Sqrt[d + e*x^2])) - (2*e^3*x)/(d^2*(c*d^
2 - b*d*e + a*e^2)*Sqrt[d + e*x^2]) - ((c*d - b*e)*Sqrt[d + e*x^2])/(a*d*(c
*d^2 - b*d*e + a*e^2)*x) - (c*(c*d - b*e + (b*c*d - b^2*e + 2*a*c*e)/Sqrt[b
^2 - 4*a*c])*ArcTan[(Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]*x)/(Sqrt[b - S
qrt[b^2 - 4*a*c]]*Sqrt[d + e*x^2])]])/(a*Sqrt[b - Sqrt[b^2 - 4*a*c]]*Sqrt[2*
c*d - (b - Sqrt[b^2 - 4*a*c])*e]*(c*d^2 - b*d*e + a*e^2)) - (c*(c*d - b*e -
 (b*c*d - b^2*e + 2*a*c*e)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2*c*d - (b + Sqr
t[b^2 - 4*a*c])*e]*x)/(Sqrt[b + Sqrt[b^2 - 4*a*c]]*Sqrt[d + e*x^2])]])/(a*Sq
rt[b + Sqrt[b^2 - 4*a*c]]*Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]*(c*d^2 -
b*d*e + a*e^2))
```

Rule 197

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 277

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x^(m + 1)*((
a + b*x^n)^(p + 1)/(a*(m + 1))), x] - Dist[b*((m + n*(p + 1) + 1)/(a*(m + 1
))), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 1315

```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_))/((a_) + (b_.)*(x_)^2 +
(c_.)*(x_)^4), x_Symbol] :> Dist[e^2/(c*d^2 - b*d*e + a*e^2), Int[(f*x)^m*(
d + e*x^2)^q, x], x] + Dist[1/(c*d^2 - b*d*e + a*e^2), Int[(f*x)^m*(d + e*x
^2)^(q + 1)*(Simp[c*d - b*e - c*e*x^2, x]/(a + b*x^2 + c*x^4)), x], x] /; F
reeQ[{a, b, c, d, e, f, m}, x] && NeQ[b^2 - 4*a*c, 0] &&  !IntegerQ[q] && L
tQ[q, -1]
```

Rule 1706

```
Int[(Px_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^
(p_.), x_Symbol] :> Int[ExpandIntegrand[Px*(d + e*x^2)^q*(a + b*x^2 + c*x^4
)^p, x], x] /; FreeQ[{a, b, c, d, e, q}, x] && PolyQ[Px, x^2] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[p]
```

Rule 6860

```
Int[(u_)/((a_.) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.)), x_Symbol] :> With[
{v = RationalFunctionExpand[u/(a + b*x^n + c*x^(2*n)), x]}, Int[v, x] /; Su
mQ[v]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && IGtQ[n, 0]
```

Rubi steps

$$\text{integral} = \frac{\int \frac{cd-be-cex^2}{x^2\sqrt{d+ex^2}(a+bx^2+cx^4)}\,dx}{cd^2-bde+ae^2} + \frac{e^2\int \frac{1}{x^2(d+ex^2)^{3/2}}\,dx}{cd^2-bde+ae^2}$$

$$= -\frac{e^2}{d\left(cd^2-bde+ae^2\right)x\sqrt{d+ex^2}} + \frac{\int \left(\frac{cd-be}{ax^2\sqrt{d+ex^2}} + \frac{-bcd+b^2e-ace-c(cd-be)x^2}{a\sqrt{d+ex^2}(a+bx^2+cx^4)}\right)dx}{cd^2-bde+ae^2} - \frac{\left(2e^3\right)\int \frac{1}{(d+ex^2)^{3/2}}\,dx}{d\left(cd^2-bde+ae^2\right)}$$

$$= -\frac{e^2}{d\left(cd^2-bde+ae^2\right)x\sqrt{d+ex^2}} - \frac{2e^3x}{d^2\left(cd^2-bde+ae^2\right)\sqrt{d+ex^2}} + \frac{\int \frac{-bcd+b^2e-ace-c(cd-be)x^2}{\sqrt{d+ex^2}(a+bx^2+cx^4)}\,dx}{a\left(cd^2-bde+ae^2\right)} + \frac{(cd-be)\int \frac{1}{x^2\sqrt{d+ex^2}}\,dx}{a\left(cd^2-bde+ae^2\right)}$$

$$= -\frac{e^2}{d\left(cd^2-bde+ae^2\right)x\sqrt{d+ex^2}} - \frac{2e^3x}{d^2\left(cd^2-bde+ae^2\right)\sqrt{d+ex^2}} - \frac{(cd-be)\sqrt{d+ex^2}}{ad\left(cd^2-bde+ae^2\right)x} + \frac{\int \left(\frac{-c(cd-be)+\frac{c\left(-bcd+b^2e-2ace\right)}{\sqrt{b^2-4ac}}}{\left(b-\sqrt{b^2-4ac}+2cx^2\right)\sqrt{d+ex^2}} + \frac{-c(cd-be)-\frac{c\left(-bcd+b^2e-2ace\right)}{\sqrt{b^2-4ac}}}{\left(b+\sqrt{b^2-4ac}+2cx^2\right)\sqrt{d+ex^2}}\right)dx}{a\left(cd^2-bde+ae^2\right)}$$

$$= -\frac{e^2}{d\left(cd^2-bde+ae^2\right)x\sqrt{d+ex^2}} - \frac{2e^3x}{d^2\left(cd^2-bde+ae^2\right)\sqrt{d+ex^2}} - \frac{(cd-be)\sqrt{d+ex^2}}{ad\left(cd^2-bde+ae^2\right)x} - \frac{\left(c\left(cd-be-\frac{bcd-b^2e+2ace}{\sqrt{b^2-4ac}}\right)\right)\int \frac{1}{\left(b+\sqrt{b^2-4ac}+2cx^2\right)\sqrt{d+ex^2}}\,dx}{a\left(cd^2-bde+ae^2\right)} - \frac{\left(c\left(cd-be+\frac{bcd-b^2e+2ace}{\sqrt{b^2-4ac}}\right)\right)\int \frac{1}{\left(b-\sqrt{b^2-4ac}+2cx^2\right)\sqrt{d+ex^2}}\,dx}{a\left(cd^2-bde+ae^2\right)}$$

$$= -\frac{e^2}{d\left(cd^2-bde+ae^2\right)x\sqrt{d+ex^2}} - \frac{2e^3x}{d^2\left(cd^2-bde+ae^2\right)\sqrt{d+ex^2}} - \frac{(cd-be)\sqrt{d+ex^2}}{ad\left(cd^2-bde+ae^2\right)x} - \frac{\left(c\left(cd-be-\frac{bcd-b^2e+2ace}{\sqrt{b^2-4ac}}\right)\right)\text{Subst}\left(\int \frac{1}{b+\sqrt{b^2-4ac}-\left(-2cd+\left(b+\sqrt{b^2-4ac}\right)e\right)x^2}\,dx, x, \frac{x}{\sqrt{d+ex^2}}\right)}{a\left(cd^2-bde+ae^2\right)} - \frac{\left(c\left(cd-be+\frac{bcd-b^2e+2ace}{\sqrt{b^2-4ac}}\right)\right)\text{Subst}\left(\int \frac{1}{b-\sqrt{b^2-4ac}-\left(-2cd+\left(b-\sqrt{b^2-4ac}\right)e\right)x^2}\,dx, x, \frac{x}{\sqrt{d+ex^2}}\right)}{a\left(cd^2-bde+ae^2\right)}$$

$$= -\frac{e^2}{d\left(cd^2 - bde + ae^2\right)x\sqrt{d+ex^2}} - \frac{2e^3x}{d^2\left(cd^2-bde+ae^2\right)\sqrt{d+ex^2}} - \frac{(cd-be)\sqrt{d+ex^2}}{ad\left(cd^2-bde+ae^2\right)x} - \frac{c\left(cd-be+\frac{bcd-b^2e+2ace}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}x}{\sqrt{b-\sqrt{b^2-4ac}}\sqrt{d+ex^2}}\right)}{a\sqrt{b-\sqrt{b^2-4ac}}\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}\left(cd^2-bde+ae^2\right)} - \frac{c\left(cd-be-\frac{bcd-b^2e+2ace}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2cd-\left(b+\sqrt{b^2-4ac}\right)e}x}{\sqrt{b+\sqrt{b^2-4ac}}\sqrt{d+ex^2}}\right)}{a\sqrt{b+\sqrt{b^2-4ac}}\sqrt{2cd-\left(b+\sqrt{b^2-4ac}\right)e}\left(cd^2-bde+ae^2\right)}$$

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 17.54 (sec) , antiderivative size = 2158, normalized size of antiderivative = 6.37

$$\int \frac{1}{x^2\left(d+ex^2\right)^{3/2}\left(a+bx^2+cx^4\right)}\,dx = \text{Result too large to show}$$

```
[In] Integrate[1/(x^2*(d + e*x^2)^(3/2)*(a + b*x^2 + c*x^4)),x]
```

```
[Out] -((d + 2*e*x^2)/(a*d^2*x*Sqrt[d + e*x^2])) - ((c + (b*c)/Sqrt[b^2 - 4*a*c])
*x*(45*Sqrt[-(((-b + Sqrt[b^2 - 4*a*c])*(2*c*d + (-b + Sqrt[b^2 - 4*a*c])*e
)*x^2*(d + e*x^2))/(d^2*(b - Sqrt[b^2 - 4*a*c] + 2*c*x^2)^2))] + (30*e*x^2*
Sqrt[-(((-b + Sqrt[b^2 - 4*a*c])*(2*c*d + (-b + Sqrt[b^2 - 4*a*c])*e)*x^2*(
d + e*x^2))/(d^2*(b - Sqrt[b^2 - 4*a*c] + 2*c*x^2)^2))])/d - 45*ArcSin[Sqrt
[-(((2*c*d + (-b + Sqrt[b^2 - 4*a*c])*e)*x^2)/(d*(-b + Sqrt[b^2 - 4*a*c] -
2*c*x^2)))]] - (30*e*x^2*ArcSin[Sqrt[-(((2*c*d + (-b + Sqrt[b^2 - 4*a*c])*e
)*x^2)/(d*(-b + Sqrt[b^2 - 4*a*c] - 2*c*x^2)))]])/d - (45*(2*c*d + (-b + Sq
rt[b^2 - 4*a*c])*e)*x^2*ArcSin[Sqrt[-(((2*c*d + (-b + Sqrt[b^2 - 4*a*c])*e)
*x^2)/(d*(-b + Sqrt[b^2 - 4*a*c] - 2*c*x^2)))]])/(d*(-b + Sqrt[b^2 - 4*a*c]
 - 2*c*x^2)) - (30*e*(2*c*d + (-b + Sqrt[b^2 - 4*a*c])*e)*x^4*ArcSin[Sqrt[-
(((2*c*d + (-b + Sqrt[b^2 - 4*a*c])*e)*x^2)/(d*(-b + Sqrt[b^2 - 4*a*c] - 2*
c*x^2)))]])/(d^2*(-b + Sqrt[b^2 - 4*a*c] - 2*c*x^2)) + 4*(-(((2*c*d + (-b +
 Sqrt[b^2 - 4*a*c])*e)*x^2)/(d*(-b + Sqrt[b^2 - 4*a*c] - 2*c*x^2))))^(5/2)*
Sqrt[((-b + Sqrt[b^2 - 4*a*c])*(d + e*x^2))/(d*(-b + Sqrt[b^2 - 4*a*c] - 2*
c*x^2))]*Hypergeometric2F1[2, 2, 7/2, -(((2*c*d + (-b + Sqrt[b^2 - 4*a*c])*
e)*x^2)/(d*(-b + Sqrt[b^2 - 4*a*c] - 2*c*x^2)))] + (4*e*x^2*(-(((2*c*d + (-
b + Sqrt[b^2 - 4*a*c])*e)*x^2)/(d*(-b + Sqrt[b^2 - 4*a*c] - 2*c*x^2))))^(5/
2)*Sqrt[((-b + Sqrt[b^2 - 4*a*c])*(d + e*x^2))/(d*(-b + Sqrt[b^2 - 4*a*c] -
```

```
2*c*x^2))]*Hypergeometric2F1[2, 2, 7/2, -(((2*c*d + (-b + Sqrt[b^2 - 4*a*c
])*e)*x^2)/(d*(-b + Sqrt[b^2 - 4*a*c] - 2*c*x^2)))])/d))/(15*a*(b - Sqrt[b^
2 - 4*a*c])*d*(-(((2*c*d + (-b + Sqrt[b^2 - 4*a*c])*e)*x^2)/(d*(-b + Sqrt[b
^2 - 4*a*c] - 2*c*x^2))))^(3/2)*(1 - (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c]))*Sq
rt[d + e*x^2]*Sqrt[((-b + Sqrt[b^2 - 4*a*c])*(d + e*x^2))/(d*(-b + Sqrt[b^2
 - 4*a*c] - 2*c*x^2))]) + ((-c + (b*c)/Sqrt[b^2 - 4*a*c])*x*(45*Sqrt[-(((b
+ Sqrt[b^2 - 4*a*c])*(-2*c*d + (b + Sqrt[b^2 - 4*a*c])*e)*x^2*(d + e*x^2))/
(d^2*(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)^2))] + (30*e*x^2*Sqrt[-(((b + Sqrt[b
^2 - 4*a*c])*(-2*c*d + (b + Sqrt[b^2 - 4*a*c])*e)*x^2*(d + e*x^2))/(d^2*(b
+ Sqrt[b^2 - 4*a*c] + 2*c*x^2)^2))])/d - 45*ArcSin[Sqrt[((2*c*d - (b + Sqrt
[b^2 - 4*a*c])*e)*x^2)/(d*(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2))]] - (30*e*x^2*
ArcSin[Sqrt[((2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)*x^2)/(d*(b + Sqrt[b^2 - 4*
a*c] + 2*c*x^2))]])/d + (45*(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)*x^2*ArcSin[
Sqrt[((2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)*x^2)/(d*(b + Sqrt[b^2 - 4*a*c] +
2*c*x^2))]])/(d*(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)) - (30*e*(-2*c*d + (b + S
qrt[b^2 - 4*a*c])*e)*x^4*ArcSin[Sqrt[((2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)*x
^2)/(d*(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2))]])/(d^2*(b + Sqrt[b^2 - 4*a*c] +
2*c*x^2)) + 4*(((2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)*x^2)/(d*(b + Sqrt[b^2 -
 4*a*c] + 2*c*x^2)))^(5/2)*Sqrt[((b + Sqrt[b^2 - 4*a*c])*(d + e*x^2))/(d*(b
 + Sqrt[b^2 - 4*a*c] + 2*c*x^2))]*Hypergeometric2F1[2, 2, 7/2, ((2*c*d - (b
 + Sqrt[b^2 - 4*a*c])*e)*x^2)/(d*(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2))] + (4*e
*x^2*(((2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)*x^2)/(d*(b + Sqrt[b^2 - 4*a*c] +
 2*c*x^2)))^(5/2)*Sqrt[((b + Sqrt[b^2 - 4*a*c])*(d + e*x^2))/(d*(b + Sqrt[b
^2 - 4*a*c] + 2*c*x^2))]*Hypergeometric2F1[2, 2, 7/2, ((2*c*d - (b + Sqrt[b
^2 - 4*a*c])*e)*x^2)/(d*(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2))])/d))/(15*a*(b +
 Sqrt[b^2 - 4*a*c])*d*(((2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)*x^2)/(d*(b + Sq
rt[b^2 - 4*a*c] + 2*c*x^2)))^(3/2)*(1 + (2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]))*
Sqrt[d + e*x^2]*Sqrt[((b + Sqrt[b^2 - 4*a*c])*(d + e*x^2))/(d*(b + Sqrt[b^2
 - 4*a*c] + 2*c*x^2))])
```

Maple [A] (verified)

Time = 1.14 (sec) , antiderivative size = 513, normalized size of antiderivative = 1.51

method	result
default	$\frac{-\frac{1}{dx\sqrt{e\,x^2+d}}-\frac{2ex}{d^2\sqrt{e\,x^2+d}}}{a}-\frac{\sqrt{2}\,d\left(\left((ac-b^2)e+bcd\right)\sqrt{-4d^2\left(ac-\frac{b^2}{4}\right)}+(2a\,c^2-b^2c)d^2+e(-3abc+b^3)d\right)\sqrt{\left(-2ae+bd+\sqrt{-\ldots}\right.}}{\ldots}$
pseudoelliptic	$-\frac{\frac{\sqrt{2}\,d^2\left(\left(\left(ac-b^2\right)e+bcd\right)\sqrt{-4d^2\left(ac-\frac{b^2}{4}\right)}+\left(2a\,c^2-b^2c\right)d^2+e\left(-3abc+b^3\right)d\right)\sqrt{\left(-2ae+bd+\sqrt{-4d^2\left(ac-\frac{b^2}{4}\right)}\right)a}\,x\sqrt{e\,x^2+d}\,\operatorname{arctanh}\left(\ldots\right)}{2}}{\ldots}$
risch	$-\frac{\sqrt{e\,x^2+d}}{d^2ax}-\frac{\frac{e^2a\sqrt{\left(x+\frac{\sqrt{-ed}}{e}\right)^2e-2\sqrt{-ed}\left(x+\frac{\sqrt{-ed}}{e}\right)}}{2d\left(a\,e^2-bde+c\,d^2\right)\left(x+\frac{\sqrt{-ed}}{e}\right)}+\frac{e^2a\sqrt{\left(x-\frac{\sqrt{-ed}}{e}\right)^2e+2\sqrt{-ed}\left(x-\frac{\sqrt{-ed}}{e}\right)}}{2d\left(a\,e^2-bde+c\,d^2\right)\left(x-\frac{\sqrt{-ed}}{e}\right)}-\frac{d\sqrt{2}\left(-\left(\left(ace-b^2e+bcd\right)\sqrt{-\ldots}\right.\right.}{\ldots}}{\ldots}$

```
[In] int(1/x^2/(e*x^2+d)^(3/2)/(c*x^4+b*x^2+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/a*(-1/d/x/(e*x^2+d)^(1/2)-2*e/d^2*x/(e*x^2+d)^(1/2))-1/2/a/((-2*a*e+b*d+(
-4*d^2*(a*c-1/4*b^2))^(1/2))*a)^(1/2)/(e*x^2+d)^(1/2)/((2*a*e-b*d+(-4*d^2*(
a*c-1/4*b^2))^(1/2))*a)^(1/2)*(2^(1/2)*d*(((a*c-b^2)*e+b*c*d)*(-4*d^2*(a*c-
1/4*b^2))^(1/2)+(2*a*c^2-b^2*c)*d^2+e*(-3*a*b*c+b^3)*d)*((-2*a*e+b*d+(-4*d^
2*(a*c-1/4*b^2))^(1/2))*a)^(1/2)*(e*x^2+d)^(1/2)*arctanh(a/x*(e*x^2+d)^(1/2
)*2^(1/2)/((2*a*e-b*d+(-4*d^2*(a*c-1/4*b^2))^(1/2))*a)^(1/2))-((2*a*e-b*d+(
-4*d^2*(a*c-1/4*b^2))^(1/2))*a)^(1/2)*(2^(1/2)*d*(e*x^2+d)^(1/2)*(((a*c-b^2
)*e+b*c*d)*(-4*d^2*(a*c-1/4*b^2))^(1/2)+(-2*a*c^2+b^2*c)*d^2+e*(3*a*b*c-b^3
)*d)*arctan(a/x*(e*x^2+d)^(1/2)*2^(1/2)/((-2*a*e+b*d+(-4*d^2*(a*c-1/4*b^2))
^(1/2))*a)^(1/2))-2*(-4*d^2*(a*c-1/4*b^2))^(1/2)*((-2*a*e+b*d+(-4*d^2*(a*c-
1/4*b^2))^(1/2))*a)^(1/2)*e*x*(b*e-c*d)))/(-4*d^2*(a*c-1/4*b^2))^(1/2)/(a*e
^2-b*d*e+c*d^2)/d
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{x^2\left(d+ex^2\right)^{3/2}\left(a+bx^2+cx^4\right)}\,dx = \text{Timed out}$$

```
[In] integrate(1/x^2/(e*x^2+d)^(3/2)/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]

$$\int \frac{1}{x^2 (d+ex^2)^{3/2} (a+bx^2+cx^4)}\,dx = \int \frac{1}{x^2 (d+ex^2)^{\frac{3}{2}} (a+bx^2+cx^4)}\,dx$$

```
[In] integrate(1/x**2/(e*x**2+d)**(3/2)/(c*x**4+b*x**2+a),x)
[Out] Integral(1/(x**2*(d + e*x**2)**(3/2)*(a + b*x**2 + c*x**4)), x)
```

Maxima [F]

$$\int \frac{1}{x^2 (d+ex^2)^{3/2} (a+bx^2+cx^4)}\,dx = \int \frac{1}{(cx^4+bx^2+a)(ex^2+d)^{\frac{3}{2}}x^2}\,dx$$

```
[In] integrate(1/x^2/(e*x^2+d)^(3/2)/(c*x^4+b*x^2+a),x, algorithm="maxima")
[Out] integrate(1/((c*x^4 + b*x^2 + a)*(e*x^2 + d)^(3/2)*x^2), x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{1}{x^2 (d+ex^2)^{3/2} (a+bx^2+cx^4)}\,dx = \text{Timed out}$$

```
[In] integrate(1/x^2/(e*x^2+d)^(3/2)/(c*x^4+b*x^2+a),x, algorithm="giac")
[Out] Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^2 (d+ex^2)^{3/2} (a+bx^2+cx^4)}\,dx = \int \frac{1}{x^2\,(e\,x^2+d)^{3/2}\,(c\,x^4+b\,x^2+a)}\,dx$$

```
[In] int(1/(x^2*(d + e*x^2)^(3/2)*(a + b*x^2 + c*x^4)),x)
[Out] int(1/(x^2*(d + e*x^2)^(3/2)*(a + b*x^2 + c*x^4)), x)
```

3.399 $\int \frac{1}{x^4(d+ex^2)^{3/2}(a+bx^2+cx^4)}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 419

$$\int \frac{1}{x^4 (d+ex^2)^{3/2} (a+bx^2+cx^4)}\,dx = -\frac{1}{3adx^3\sqrt{d+ex^2}} + \frac{3bd+4ae}{3a^2d^2x\sqrt{d+ex^2}} + \frac{2e(3bd+4ae)x}{3a^2d^3\sqrt{d+ex^2}} - \frac{e(bcd-b^2e+ace)\,x}{a^2d\,(cd^2+e(-bd+ae))\sqrt{d+ex^2}} + \frac{2c^2\left(b+\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}\,x}{\sqrt{b-\sqrt{b^2-4ac}}\sqrt{d+ex^2}}\right)}{a^2\sqrt{b-\sqrt{b^2-4ac}}\left(2cd-\left(b-\sqrt{b^2-4ac}\right)e\right)^{3/2}} + \frac{2c^2\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2cd-\left(b+\sqrt{b^2-4ac}\right)e}\,x}{\sqrt{b+\sqrt{b^2-4ac}}\sqrt{d+ex^2}}\right)}{a^2\sqrt{b+\sqrt{b^2-4ac}}\left(2cd-\left(b+\sqrt{b^2-4ac}\right)e\right)^{3/2}}$$

```
[Out] -1/3/a/d/x^3/(e*x^2+d)^(1/2)+1/3*(4*a*e+3*b*d)/a^2/d^2/x/(e*x^2+d)^(1/2)+2/
3*e*(4*a*e+3*b*d)*x/a^2/d^3/(e*x^2+d)^(1/2)-e*(a*c*e-b^2*e+b*c*d)*x/a^2/d/(
c*d^2+e*(a*e-b*d))/(e*x^2+d)^(1/2)+2*c^2*arctan(x*(2*c*d-e*(b-(-4*a*c+b^2)^
(1/2)))^(1/2)/(e*x^2+d)^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2))*(b+(-2*a*c+b^2)
/(-4*a*c+b^2)^(1/2))/a^2/(2*c*d-e*(b-(-4*a*c+b^2)^(1/2)))^(3/2)/(b-(-4*a*c+
b^2)^(1/2))^(1/2)+2*c^2*arctan(x*(2*c*d-e*(b+(-4*a*c+b^2)^(1/2)))^(1/2)/(e*
x^2+d)^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2))*(b+(2*a*c-b^2)/(-4*a*c+b^2)^(1/2
))/a^2/(2*c*d-e*(b+(-4*a*c+b^2)^(1/2)))^(3/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2)
```

Rubi [A] (verified)

Time = 3.37 (sec) , antiderivative size = 647, normalized size of antiderivative = 1.54, number of steps used = 15, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.276$, Rules used = {1315, 277, 197, 6860, 270, 1706, 385, 211}

$$\int \frac{1}{x^4 (d+ex^2)^{3/2} (a+bx^2+cx^4)} dx = \frac{c\left(\frac{3abce-2ac^2d+b^3(-e)+b^2cd}{\sqrt{b^2-4ac}}+ace+b^2(-e)+bcd\right)\arctan\left(\frac{x\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}{\sqrt{b-\sqrt{b^2-4ac}}\sqrt{d+ex^2}}\right)}{a^2\sqrt{b-\sqrt{b^2-4ac}}\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}\left(ae^2-bde+cd^2\right)} + \frac{c\left(-\frac{3abce-2ac^2d+b^3(-e)+b^2cd}{\sqrt{b^2-4ac}}+ace+b^2(-e)+bcd\right)\arctan\left(\frac{x\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}{\sqrt{\sqrt{b^2-4ac}+b}\sqrt{d+ex^2}}\right)}{a^2\sqrt{\sqrt{b^2-4ac}+b}\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}\left(ae^2-bde+cd^2\right)} + \frac{\sqrt{d+ex^2}(ace+b^2(-e)+bcd)}{a^2dx\left(ae^2-bde+cd^2\right)} + \frac{2e\sqrt{d+ex^2}(cd-be)}{3ad^2x\left(ae^2-bde+cd^2\right)} - \frac{e^2}{3dx^3\sqrt{d+ex^2}\left(ae^2-bde+cd^2\right)} - \frac{\sqrt{d+ex^2}(cd-be)}{3adx^3\left(ae^2-bde+cd^2\right)} + \frac{4e^3}{3d^2x\sqrt{d+ex^2}\left(ae^2-bde+cd^2\right)} + \frac{8e^4x}{3d^3\sqrt{d+ex^2}\left(ae^2-bde+cd^2\right)}$$

```
[In] Int[1/(x^4*(d + e*x^2)^(3/2)*(a + b*x^2 + c*x^4)),x]
```

```
[Out] -1/3*e^2/(d*(c*d^2 - b*d*e + a*e^2)*x^3*Sqrt[d + e*x^2]) + (4*e^3)/(3*d^2*(
c*d^2 - b*d*e + a*e^2)*x*Sqrt[d + e*x^2]) + (8*e^4*x)/(3*d^3*(c*d^2 - b*d*e
 + a*e^2)*Sqrt[d + e*x^2]) - ((c*d - b*e)*Sqrt[d + e*x^2])/(3*a*d*(c*d^2 -
b*d*e + a*e^2)*x^3) + (2*e*(c*d - b*e)*Sqrt[d + e*x^2])/(3*a*d^2*(c*d^2 - b
*d*e + a*e^2)*x) + ((b*c*d - b^2*e + a*c*e)*Sqrt[d + e*x^2])/(a^2*d*(c*d^2
- b*d*e + a*e^2)*x) + (c*(b*c*d - b^2*e + a*c*e + (b^2*c*d - 2*a*c^2*d - b^
3*e + 3*a*b*c*e)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2*c*d - (b - Sqrt[b^2 - 4*
a*c])*e]*x)/(Sqrt[b - Sqrt[b^2 - 4*a*c]]*Sqrt[d + e*x^2])]])/(a^2*Sqrt[b - S
qrt[b^2 - 4*a*c]]*Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]*(c*d^2 - b*d*e +
a*e^2)) + (c*(b*c*d - b^2*e + a*c*e - (b^2*c*d - 2*a*c^2*d - b^3*e + 3*a*b*
c*e)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]*x)/
(Sqrt[b + Sqrt[b^2 - 4*a*c]]*Sqrt[d + e*x^2])]])/(a^2*Sqrt[b + Sqrt[b^2 - 4*
a*c]]*Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]*(c*d^2 - b*d*e + a*e^2))
```

Rule 197

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 277

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x^(m + 1)*((
a + b*x^n)^(p + 1)/(a*(m + 1))), x] - Dist[b*((m + n*(p + 1) + 1)/(a*(m + 1
))), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 1315

```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_))/((a_) + (b_.)*(x_)^2 +
(c_.)*(x_)^4), x_Symbol] :> Dist[e^2/(c*d^2 - b*d*e + a*e^2), Int[(f*x)^m*(
d + e*x^2)^q, x], x] + Dist[1/(c*d^2 - b*d*e + a*e^2), Int[(f*x)^m*(d + e*x
^2)^(q + 1)*(Simp[c*d - b*e - c*e*x^2, x]/(a + b*x^2 + c*x^4)), x], x] /; F
reeQ[{a, b, c, d, e, f, m}, x] && NeQ[b^2 - 4*a*c, 0] &&  !IntegerQ[q] && L
tQ[q, -1]
```

Rule 1706

```
Int[(Px_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^
(p_.), x_Symbol] :> Int[ExpandIntegrand[Px*(d + e*x^2)^q*(a + b*x^2 + c*x^4
)^p, x], x] /; FreeQ[{a, b, c, d, e, q}, x] && PolyQ[Px, x^2] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[p]
```

Rule 6860

```
Int[(u_)/((a_.) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.)), x_Symbol] :> With[
{v = RationalFunctionExpand[u/(a + b*x^n + c*x^(2*n)), x]}, Int[v, x] /; Su
mQ[v]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && IGtQ[n, 0]
```

Rubi steps

$$\text{integral} = \frac{\int \frac{cd-be-cex^2}{x^4\sqrt{d+ex^2}(a+bx^2+cx^4)}\,dx}{cd^2-bde+ae^2} + \frac{e^2\int \frac{1}{x^4(d+ex^2)^{3/2}}\,dx}{cd^2-bde+ae^2}$$

$$= -\frac{e^2}{3d\,(cd^2-bde+ae^2)\,x^3\sqrt{d+ex^2}} + \frac{\int \left(\frac{cd-be}{ax^4\sqrt{d+ex^2}} + \frac{-bcd+b^2e-ace}{a^2x^2\sqrt{d+ex^2}} + \frac{b^2cd-ac^2d-b^3e+2abce+c(bcd-b^2e+ace)x^2}{a^2\sqrt{d+ex^2}(a+bx^2+cx^4)}\right)dx}{cd^2-bde+ae^2} - \frac{(4e^3)\int \frac{1}{x^2(d+ex^2)^{3/2}}\,dx}{3d\,(cd^2-bde+ae^2)}$$

$$= -\frac{e^2}{3d\,(cd^2-bde+ae^2)\,x^3\sqrt{d+ex^2}} + \frac{4e^3}{3d^2\,(cd^2-bde+ae^2)\,x\sqrt{d+ex^2}} + \frac{\int \frac{b^2cd-ac^2d-b^3e+2abce+c(bcd-b^2e+ace)x^2}{\sqrt{d+ex^2}(a+bx^2+cx^4)}\,dx}{a^2\,(cd^2-bde+ae^2)} + \frac{(8e^4)\int \frac{1}{(d+ex^2)^{3/2}}\,dx}{3d^2\,(cd^2-bde+ae^2)} + \frac{(cd-be)\int \frac{1}{x^4\sqrt{d+ex^2}}\,dx}{a\,(cd^2-bde+ae^2)} - \frac{(bcd-b^2e+ace)\int \frac{1}{x^2\sqrt{d+ex^2}}\,dx}{a^2\,(cd^2-bde+ae^2)}$$

$$= -\frac{e^2}{3d\,(cd^2-bde+ae^2)\,x^3\sqrt{d+ex^2}} + \frac{4e^3}{3d^2\,(cd^2-bde+ae^2)\,x\sqrt{d+ex^2}} + \frac{8e^4x}{3d^3\,(cd^2-bde+ae^2)\,\sqrt{d+ex^2}} - \frac{(cd-be)\sqrt{d+ex^2}}{3ad\,(cd^2-bde+ae^2)\,x^3} + \frac{(bcd-b^2e+ace)\,\sqrt{d+ex^2}}{a^2d\,(cd^2-bde+ae^2)\,x} + \frac{\int \left(\frac{c(bcd-b^2e+ace)-\frac{c\left(-b^2cd+2ac^2d+b^3e-3abce\right)}{\sqrt{b^2-4ac}}}{\left(b-\sqrt{b^2-4ac}+2cx^2\right)\sqrt{d+ex^2}} + \frac{c(bcd-b^2e+ace)+\frac{c\left(-b^2cd+2ac^2d+b^3e-3abce\right)}{\sqrt{b^2-4ac}}}{\left(b+\sqrt{b^2-4ac}+2cx^2\right)\sqrt{d+ex^2}}\right)dx}{a^2\,(cd^2-bde+ae^2)} - \frac{(2e(cd-be))\int \frac{1}{x^2\sqrt{d+ex^2}}\,dx}{3ad\,(cd^2-bde+ae^2)}$$

$$= -\frac{e^2}{3d\left(cd^2-bde+ae^2\right)x^3\sqrt{d+ex^2}}+\frac{4e^3}{3d^2\left(cd^2-bde+ae^2\right)x\sqrt{d+ex^2}} + \frac{8e^4x}{3d^3\left(cd^2-bde+ae^2\right)\sqrt{d+ex^2}}-\frac{(cd-be)\sqrt{d+ex^2}}{3ad\left(cd^2-bde+ae^2\right)x^3} + \frac{2e(cd-be)\sqrt{d+ex^2}}{3ad^2\left(cd^2-bde+ae^2\right)x}+\frac{\left(bcd-b^2e+ace\right)\sqrt{d+ex^2}}{a^2d\left(cd^2-bde+ae^2\right)x} + \frac{\left(c\left(bcd-b^2e+ace-\frac{b^2cd-2ac^2d-b^3e+3abce}{\sqrt{b^2-4ac}}\right)\right)\int\frac{1}{\left(b+\sqrt{b^2-4ac}+2cx^2\right)\sqrt{d+ex^2}}\,dx}{a^2\left(cd^2-bde+ae^2\right)} + \frac{\left(c\left(bcd-b^2e+ace+\frac{b^2cd-2ac^2d-b^3e+3abce}{\sqrt{b^2-4ac}}\right)\right)\int\frac{1}{\left(b-\sqrt{b^2-4ac}+2cx^2\right)\sqrt{d+ex^2}}\,dx}{a^2\left(cd^2-bde+ae^2\right)}$$

$$= -\frac{e^2}{3d\left(cd^2-bde+ae^2\right)x^3\sqrt{d+ex^2}}+\frac{4e^3}{3d^2\left(cd^2-bde+ae^2\right)x\sqrt{d+ex^2}} + \frac{8e^4x}{3d^3\left(cd^2-bde+ae^2\right)\sqrt{d+ex^2}}-\frac{(cd-be)\sqrt{d+ex^2}}{3ad\left(cd^2-bde+ae^2\right)x^3} + \frac{2e(cd-be)\sqrt{d+ex^2}}{3ad^2\left(cd^2-bde+ae^2\right)x}+\frac{\left(bcd-b^2e+ace\right)\sqrt{d+ex^2}}{a^2d\left(cd^2-bde+ae^2\right)x} + \frac{\left(c\left(bcd-b^2e+ace-\frac{b^2cd-2ac^2d-b^3e+3abce}{\sqrt{b^2-4ac}}\right)\right)\text{Subst}\left(\int\frac{1}{b+\sqrt{b^2-4ac}-\left(-2cd+\left(b+\sqrt{b^2-4ac}\right)e\right)x^2}\,dx,x,\frac{x}{\sqrt{d+ex}}\right.}{a^2\left(cd^2-bde+ae^2\right)} + \frac{\left(c\left(bcd-b^2e+ace+\frac{b^2cd-2ac^2d-b^3e+3abce}{\sqrt{b^2-4ac}}\right)\right)\text{Subst}\left(\int\frac{1}{b-\sqrt{b^2-4ac}-\left(-2cd+\left(b-\sqrt{b^2-4ac}\right)e\right)x^2}\,dx,x,\frac{x}{\sqrt{d+ex}}\right.}{a^2\left(cd^2-bde+ae^2\right)}$$

$$= -\frac{e^2}{3d\left(cd^2-bde+ae^2\right)x^3\sqrt{d+ex^2}}+\frac{4e^3}{3d^2\left(cd^2-bde+ae^2\right)x\sqrt{d+ex^2}} + \frac{8e^4x}{3d^3\left(cd^2-bde+ae^2\right)\sqrt{d+ex^2}}-\frac{(cd-be)\sqrt{d+ex^2}}{3ad\left(cd^2-bde+ae^2\right)x^3} + \frac{2e(cd-be)\sqrt{d+ex^2}}{3ad^2\left(cd^2-bde+ae^2\right)x}+\frac{\left(bcd-b^2e+ace\right)\sqrt{d+ex^2}}{a^2d\left(cd^2-bde+ae^2\right)x} + \frac{c\left(bcd-b^2e+ace+\frac{b^2cd-2ac^2d-b^3e+3abce}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}x}{\sqrt{b-\sqrt{b^2-4ac}}\sqrt{d+ex^2}}\right)}{a^2\sqrt{b-\sqrt{b^2-4ac}}\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}\left(cd^2-bde+ae^2\right)} + \frac{c\left(bcd-b^2e+ace-\frac{b^2cd-2ac^2d-b^3e+3abce}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2cd-\left(b+\sqrt{b^2-4ac}\right)e}x}{\sqrt{b+\sqrt{b^2-4ac}}\sqrt{d+ex^2}}\right)}{a^2\sqrt{b+\sqrt{b^2-4ac}}\sqrt{2cd-\left(b+\sqrt{b^2-4ac}\right)e}\left(cd^2-bde+ae^2\right)}$$

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 17.82 (sec) , antiderivative size = 2218, normalized size of antiderivative = 5.29

$$\int \frac{1}{x^4 (d+ex^2)^{3/2} (a+bx^2+cx^4)} \, dx = \text{Result too large to show}$$

```
[In] Integrate[1/(x^4*(d + e*x^2)^(3/2)*(a + b*x^2 + c*x^4)),x]
```

```
[Out] (b*(d + 2*e*x^2))/(a^2*d^2*x*Sqrt[d + e*x^2]) - (d^2 - 4*d*e*x^2 - 8*e^2*x^
4)/(3*a*d^3*x^3*Sqrt[d + e*x^2]) + ((b*c + (c*(b^2 - 2*a*c))/Sqrt[b^2 - 4*a
*c])*x*(45*Sqrt[-(((-b + Sqrt[b^2 - 4*a*c])*(2*c*d + (-b + Sqrt[b^2 - 4*a*c
])*e)*x^2*(d + e*x^2))/(d^2*(b - Sqrt[b^2 - 4*a*c] + 2*c*x^2)^2))] + (30*e*
x^2*Sqrt[-(((-b + Sqrt[b^2 - 4*a*c])*(2*c*d + (-b + Sqrt[b^2 - 4*a*c])*e)*x
^2*(d + e*x^2))/(d^2*(b - Sqrt[b^2 - 4*a*c] + 2*c*x^2)^2))])/d - 45*ArcSin[
Sqrt[-(((2*c*d + (-b + Sqrt[b^2 - 4*a*c])*e)*x^2)/(d*(-b + Sqrt[b^2 - 4*a*c
] - 2*c*x^2)))]] - (30*e*x^2*ArcSin[Sqrt[-(((2*c*d + (-b + Sqrt[b^2 - 4*a*c
])*e)*x^2)/(d*(-b + Sqrt[b^2 - 4*a*c] - 2*c*x^2)))]])/d - (45*(2*c*d + (-b
+ Sqrt[b^2 - 4*a*c])*e)*x^2*ArcSin[Sqrt[-(((2*c*d + (-b + Sqrt[b^2 - 4*a*c]
)*e)*x^2)/(d*(-b + Sqrt[b^2 - 4*a*c] - 2*c*x^2)))]])/(d*(-b + Sqrt[b^2 - 4*
a*c] - 2*c*x^2)) - (30*e*(2*c*d + (-b + Sqrt[b^2 - 4*a*c])*e)*x^4*ArcSin[Sq
rt[-(((2*c*d + (-b + Sqrt[b^2 - 4*a*c])*e)*x^2)/(d*(-b + Sqrt[b^2 - 4*a*c]
- 2*c*x^2)))]])/(d^2*(-b + Sqrt[b^2 - 4*a*c] - 2*c*x^2)) + 4*(-(((2*c*d + (
-b + Sqrt[b^2 - 4*a*c])*e)*x^2)/(d*(-b + Sqrt[b^2 - 4*a*c] - 2*c*x^2))))^(5
/2)*Sqrt[((-b + Sqrt[b^2 - 4*a*c])*(d + e*x^2))/(d*(-b + Sqrt[b^2 - 4*a*c]
- 2*c*x^2))]*Hypergeometric2F1[2, 2, 7/2, -(((2*c*d + (-b + Sqrt[b^2 - 4*a*
c])*e)*x^2)/(d*(-b + Sqrt[b^2 - 4*a*c] - 2*c*x^2)))] + (4*e*x^2*(-(((2*c*d
+ (-b + Sqrt[b^2 - 4*a*c])*e)*x^2)/(d*(-b + Sqrt[b^2 - 4*a*c] - 2*c*x^2))))
^(5/2)*Sqrt[((-b + Sqrt[b^2 - 4*a*c])*(d + e*x^2))/(d*(-b + Sqrt[b^2 - 4*a*
c] - 2*c*x^2))]*Hypergeometric2F1[2, 2, 7/2, -(((2*c*d + (-b + Sqrt[b^2 - 4
*a*c])*e)*x^2)/(d*(-b + Sqrt[b^2 - 4*a*c] - 2*c*x^2)))])/d))/(15*a^2*(b - S
qrt[b^2 - 4*a*c])*d*(-(((2*c*d + (-b + Sqrt[b^2 - 4*a*c])*e)*x^2)/(d*(-b +
Sqrt[b^2 - 4*a*c] - 2*c*x^2))))^(3/2)*(1 - (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c
]))*Sqrt[d + e*x^2]*Sqrt[((-b + Sqrt[b^2 - 4*a*c])*(d + e*x^2))/(d*(-b + Sq
rt[b^2 - 4*a*c] - 2*c*x^2))]) + ((b*c - (c*(b^2 - 2*a*c))/Sqrt[b^2 - 4*a*c]
)*x*(45*Sqrt[-(((b + Sqrt[b^2 - 4*a*c])*(-2*c*d + (b + Sqrt[b^2 - 4*a*c])*e
)*x^2*(d + e*x^2))/(d^2*(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)^2))] + (30*e*x^2*
Sqrt[-(((b + Sqrt[b^2 - 4*a*c])*(-2*c*d + (b + Sqrt[b^2 - 4*a*c])*e)*x^2*(d
 + e*x^2))/(d^2*(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)^2))])/d - 45*ArcSin[Sqrt[
((2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)*x^2)/(d*(b + Sqrt[b^2 - 4*a*c] + 2*c*x
^2))]] - (30*e*x^2*ArcSin[Sqrt[((2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)*x^2)/(d
*(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2))]])/d + (45*(2*c*d - (b + Sqrt[b^2 - 4*a
*c])*e)*x^2*ArcSin[Sqrt[((2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)*x^2)/(d*(b + S
qrt[b^2 - 4*a*c] + 2*c*x^2))]])/(d*(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)) - (30
```

```
*e*(-2*c*d + (b + Sqrt[b^2 - 4*a*c])*e)*x^4*ArcSin[Sqrt[((2*c*d - (b + Sqrt
[b^2 - 4*a*c])*e)*x^2)/(d*(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2))]]])/(d^2*(b + S
qrt[b^2 - 4*a*c] + 2*c*x^2)) + 4*(((2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)*x^2)
/(d*(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)))^(5/2)*Sqrt[((b + Sqrt[b^2 - 4*a*c])
*(d + e*x^2))/(d*(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2))]*Hypergeometric2F1[2, 2
, 7/2, ((2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)*x^2)/(d*(b + Sqrt[b^2 - 4*a*c]
+ 2*c*x^2))] + (4*e*x^2*(((2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)*x^2)/(d*(b +
Sqrt[b^2 - 4*a*c] + 2*c*x^2)))^(5/2)*Sqrt[((b + Sqrt[b^2 - 4*a*c])*(d + e*x
^2))/(d*(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2))]*Hypergeometric2F1[2, 2, 7/2, ((
2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)*x^2)/(d*(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2
))])/d))/(15*a^2*(b + Sqrt[b^2 - 4*a*c])*d*(((2*c*d - (b + Sqrt[b^2 - 4*a*c
])*e)*x^2)/(d*(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)))^(3/2)*(1 + (2*c*x^2)/(b +
 Sqrt[b^2 - 4*a*c]))*Sqrt[d + e*x^2]*Sqrt[((b + Sqrt[b^2 - 4*a*c])*(d + e*x
^2))/(d*(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2))])
```

Maple [A] (verified)

Time = 1.32 (sec) , antiderivative size = 610, normalized size of antiderivative = 1.46

method	result
pseudoelliptic	$-\frac{-3\sqrt{\left(-2ae+bd+\sqrt{-4d^2\left(ac-\frac{b^2}{4}\right)}\right)a}\,\sqrt{2}\,d^3\left(\left(c\left(be-\frac{cd}{2}\right)a-\frac{b^2(be-cd)}{2}\right)\sqrt{-4d^2\left(ac-\frac{b^2}{4}\right)}+d\left(a^2c^2e+\left(-2b^2ce+\frac{3}{2}bc^2d\right)a\cdots\right.\right.}{\cdots}$
default	$\frac{-\frac{1}{3d\,x^3\sqrt{e\,x^2+d}}-\frac{4e\left(-\frac{1}{dx\sqrt{e\,x^2+d}}-\frac{2ex}{d^2\sqrt{e\,x^2+d}}\right)}{3d}}{a}-\frac{b\left(-\frac{1}{dx\sqrt{e\,x^2+d}}-\frac{2ex}{d^2\sqrt{e\,x^2+d}}\right)}{a^2}+\frac{\sqrt{2}\,d\left(\left(-\frac{dc\left(ac-b^2\right)}{2}+be\left(ac-\frac{b^2}{2}\right)\right)\cdots\right.}{\cdots}$
risch	$-\frac{\sqrt{e\,x^2+d}\,\left(-5ae\,x^2-3bd\,x^2+da\right)}{3d^3a^2x^3}+\frac{\frac{e^3a^2\sqrt{\left(x-\frac{\sqrt{-ed}}{e}\right)^2e+2\sqrt{-ed}\,\left(x-\frac{\sqrt{-ed}}{e}\right)}}{2d\left(a\,e^2-bde+c\,d^2\right)\left(x-\frac{\sqrt{-ed}}{e}\right)}+\frac{e^3a^2\sqrt{\left(x+\frac{\sqrt{-ed}}{e}\right)^2e-2\sqrt{-ed}\,\left(x+\frac{\sqrt{-ed}}{e}\right)}}{2d\left(a\,e^2-bde+c\,d^2\right)\left(x+\frac{\sqrt{-ed}}{e}\right)}+\cdots}{\cdots}$

```
[In] int(1/x^4/(e*x^2+d)^(3/2)/(c*x^4+b*x^2+a),x,method=_RETURNVERBOSE)
```

```
[Out] -1/3*(-3*((-2*a*e+b*d+(-4*d^2*(a*c-1/4*b^2))^(1/2))*a)^(1/2)*2^(1/2)*d^3*((
c*(b*e-1/2*c*d)*a-1/2*b^2*(b*e-c*d))*(-4*d^2*(a*c-1/4*b^2))^(1/2)+d*(a^2*c^
2*e+(-2*b^2*c*e+3/2*b*c^2*d)*a+1/2*b^3*(b*e-c*d)))*x^3*(e*x^2+d)^(1/2)*arct
anh(a/x*(e*x^2+d)^(1/2)*2^(1/2)/((2*a*e-b*d+(-4*d^2*(a*c-1/4*b^2))^(1/2))*a
)^(1/2))+((2*a*e-b*d+(-4*d^2*(a*c-1/4*b^2))^(1/2))*a)^(1/2)*(-3*2^(1/2)*d^3
*(((-e*b*c+1/2*c^2*d)*a+1/2*b^2*(b*e-c*d))*(-4*d^2*(a*c-1/4*b^2))^(1/2)+d*(
```

```
a^2*c^2*e+(-2*b^2*c*e+3/2*b*c^2*d)*a+1/2*b^3*(b*e-c*d)))*x^3*(e*x^2+d)^(1/2
)*arctan(a/x*(e*x^2+d)^(1/2)*2^(1/2)/((-2*a*e+b*d+(-4*d^2*(a*c-1/4*b^2))^(1
/2))*a)^(1/2))+((-2*a*e+b*d+(-4*d^2*(a*c-1/4*b^2))^(1/2))*a)^(1/2)*(e^2*(-8
*e^2*x^4-4*d*e*x^2+d^2)*a^2-(-c*d^2+e*(5*c*x^2+b)*d-2*b*e^2*x^2)*(e*x^2+d)*
d*a+3*b*d^2*x^2*(e*x^2+d)*(b*e-c*d))*(-4*d^2*(a*c-1/4*b^2))^(1/2)))/((-2*a*
e+b*d+(-4*d^2*(a*c-1/4*b^2))^(1/2))*a)^(1/2)/(e*x^2+d)^(1/2)/(-4*d^2*(a*c-1
/4*b^2))^(1/2)/((2*a*e-b*d+(-4*d^2*(a*c-1/4*b^2))^(1/2))*a)^(1/2)/a^2/x^3/(
a*e^2-b*d*e+c*d^2)/d^3
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{x^4 (d+ex^2)^{3/2} (a+bx^2+cx^4)} dx = \text{Timed out}$$

```
[In] integrate(1/x^4/(e*x^2+d)^(3/2)/(c*x^4+b*x^2+a),x, algorithm="fricas")
[Out] Timed out
```

Sympy [F]

$$\int \frac{1}{x^4 (d+ex^2)^{3/2} (a+bx^2+cx^4)} dx = \int \frac{1}{x^4 (d+ex^2)^{\frac{3}{2}} (a+bx^2+cx^4)} dx$$

```
[In] integrate(1/x**4/(e*x**2+d)**(3/2)/(c*x**4+b*x**2+a),x)
[Out] Integral(1/(x**4*(d + e*x**2)**(3/2)*(a + b*x**2 + c*x**4)), x)
```

Maxima [F]

$$\int \frac{1}{x^4 (d+ex^2)^{3/2} (a+bx^2+cx^4)} dx = \int \frac{1}{(cx^4+bx^2+a)(ex^2+d)^{\frac{3}{2}} x^4} dx$$

```
[In] integrate(1/x^4/(e*x^2+d)^(3/2)/(c*x^4+b*x^2+a),x, algorithm="maxima")
[Out] integrate(1/((c*x^4 + b*x^2 + a)*(e*x^2 + d)^(3/2)*x^4), x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{1}{x^4 (d+ex^2)^{3/2} (a+bx^2+cx^4)} dx = \text{Timed out}$$

```
[In] integrate(1/x^4/(e*x^2+d)^(3/2)/(c*x^4+b*x^2+a),x, algorithm="giac")
[Out] Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^4 (d+ex^2)^{3/2} (a+bx^2+cx^4)} dx = \int \frac{1}{x^4 (e\,x^2+d)^{3/2} (c\,x^4+b\,x^2+a)} dx$$

```
[In] int(1/(x^4*(d + e*x^2)^(3/2)*(a + b*x^2 + c*x^4)),x)
[Out] int(1/(x^4*(d + e*x^2)^(3/2)*(a + b*x^2 + c*x^4)), x)
```

3.400 $\int \frac{(fx)^m(d+ex^2)^q}{a+bx^2+cx^4}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 243

$$\int \frac{(fx)^m (d+ex^2)^q}{a+bx^2+cx^4}\,dx = \frac{2c(fx)^{1+m}(d+ex^2)^q\left(1+\frac{ex^2}{d}\right)^{-q}\operatorname{AppellF1}\left(\frac{1+m}{2},1,-q,\frac{3+m}{2},-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{ex^2}{d}\right)}{\sqrt{b^2-4ac}\left(b-\sqrt{b^2-4ac}\right)f(1+m)} - \frac{2c(fx)^{1+m}(d+ex^2)^q\left(1+\frac{ex^2}{d}\right)^{-q}\operatorname{AppellF1}\left(\frac{1+m}{2},1,-q,\frac{3+m}{2},-\frac{2cx^2}{b+\sqrt{b^2-4ac}},-\frac{ex^2}{d}\right)}{\sqrt{b^2-4ac}\left(b+\sqrt{b^2-4ac}\right)f(1+m)}$$

```
[Out] 2*c*(f*x)^(1+m)*(e*x^2+d)^q*AppellF1(1/2+1/2*m,1,-q,3/2+1/2*m,-2*c*x^2/(b-(
-4*a*c+b^2)^(1/2)),-e*x^2/d)/f/(1+m)/((1+e*x^2/d)^q)/(b-(-4*a*c+b^2)^(1/2))
/(-4*a*c+b^2)^(1/2)-2*c*(f*x)^(1+m)*(e*x^2+d)^q*AppellF1(1/2+1/2*m,1,-q,3/2
+1/2*m,-2*c*x^2/(b+(-4*a*c+b^2)^(1/2)),-e*x^2/d)/f/(1+m)/((1+e*x^2/d)^q)/(-
4*a*c+b^2)^(1/2)/(b+(-4*a*c+b^2)^(1/2))
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 243, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used

$= \{1319, 525, 524\}$

$$\int \frac{(fx)^m (d+ex^2)^q}{a+bx^2+cx^4}\,dx$$
$$= \frac{2c(fx)^{m+1}(d+ex^2)^q\left(\frac{ex^2}{d}+1\right)^{-q}\text{AppellF1}\left(\frac{m+1}{2},1,-q,\frac{m+3}{2},-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{ex^2}{d}\right)}{f(m+1)\sqrt{b^2-4ac}\left(b-\sqrt{b^2-4ac}\right)} - \frac{2c(fx)^{m+1}(d+ex^2)^q\left(\frac{ex^2}{d}+1\right)^{-q}\text{AppellF1}\left(\frac{m+1}{2},1,-q,\frac{m+3}{2},-\frac{2cx^2}{b+\sqrt{b^2-4ac}},-\frac{ex^2}{d}\right)}{f(m+1)\sqrt{b^2-4ac}\left(\sqrt{b^2-4ac}+b\right)}$$

```
[In] Int[((f*x)^m*(d + e*x^2)^q)/(a + b*x^2 + c*x^4),x]
```

```
[Out] (2*c*(f*x)^(1 + m)*(d + e*x^2)^q*AppellF1[(1 + m)/2, 1, -q, (3 + m)/2, (-2*
c*x^2)/(b - Sqrt[b^2 - 4*a*c]), -((e*x^2)/d)])/(Sqrt[b^2 - 4*a*c]*(b - Sqrt
[b^2 - 4*a*c])*f*(1 + m)*(1 + (e*x^2)/d)^q) - (2*c*(f*x)^(1 + m)*(d + e*x^2
)^q*AppellF1[(1 + m)/2, 1, -q, (3 + m)/2, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]
), -((e*x^2)/d)])/(Sqrt[b^2 - 4*a*c]*(b + Sqrt[b^2 - 4*a*c])*f*(1 + m)*(1 +
 (e*x^2)/d)^q)
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 525

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p]), Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 1319

```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_))/((a_) + (b_.)*(x_)^2 +
(c_.)*(x_)^4), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q, 1/(a
 + b*x^2 + c*x^4), x], x] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && NeQ[b^2
- 4*a*c, 0] &&  !IntegerQ[q] &&  !IntegerQ[m]
```

Rubi steps

$$\text{integral} = \int \left(\frac{2c(fx)^m (d+ex^2)^q}{\sqrt{b^2-4ac}\left(b-\sqrt{b^2-4ac}+2cx^2\right)} - \frac{2c(fx)^m (d+ex^2)^q}{\sqrt{b^2-4ac}\left(b+\sqrt{b^2-4ac}+2cx^2\right)} \right) dx$$

$$= \frac{(2c)\int \frac{(fx)^m (d+ex^2)^q}{b-\sqrt{b^2-4ac}+2cx^2}\,dx}{\sqrt{b^2-4ac}} - \frac{(2c)\int \frac{(fx)^m (d+ex^2)^q}{b+\sqrt{b^2-4ac}+2cx^2}\,dx}{\sqrt{b^2-4ac}}$$

$$= \frac{\left(2c(d+ex^2)^q \left(1+\frac{ex^2}{d}\right)^{-q}\right)\int \frac{(fx)^m \left(1+\frac{ex^2}{d}\right)^q}{b-\sqrt{b^2-4ac}+2cx^2}\,dx}{\sqrt{b^2-4ac}} - \frac{\left(2c(d+ex^2)^q \left(1+\frac{ex^2}{d}\right)^{-q}\right)\int \frac{(fx)^m \left(1+\frac{ex^2}{d}\right)^q}{b+\sqrt{b^2-4ac}+2cx^2}\,dx}{\sqrt{b^2-4ac}}$$

$$= \frac{2c(fx)^{1+m} (d+ex^2)^q \left(1+\frac{ex^2}{d}\right)^{-q} F_1\left(\frac{1+m}{2};1,-q;\frac{3+m}{2};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{ex^2}{d}\right)}{\sqrt{b^2-4ac}\left(b-\sqrt{b^2-4ac}\right) f(1+m)} - \frac{2c(fx)^{1+m} (d+ex^2)^q \left(1+\frac{ex^2}{d}\right)^{-q} F_1\left(\frac{1+m}{2};1,-q;\frac{3+m}{2};-\frac{2cx^2}{b+\sqrt{b^2-4ac}},-\frac{ex^2}{d}\right)}{\sqrt{b^2-4ac}\left(b+\sqrt{b^2-4ac}\right) f(1+m)}$$

Mathematica [F]

$$\int \frac{(fx)^m (d+ex^2)^q}{a+bx^2+cx^4}\,dx = \int \frac{(fx)^m (d+ex^2)^q}{a+bx^2+cx^4}\,dx$$

```
[In] Integrate[((f*x)^m*(d + e*x^2)^q)/(a + b*x^2 + c*x^4),x]
[Out] Integrate[((f*x)^m*(d + e*x^2)^q)/(a + b*x^2 + c*x^4), x]
```

Maple [F]

$$\int \frac{(fx)^m (e\,x^2+d)^q}{c\,x^4+b\,x^2+a}dx$$

```
[In] int((f*x)^m*(e*x^2+d)^q/(c*x^4+b*x^2+a),x)
[Out] int((f*x)^m*(e*x^2+d)^q/(c*x^4+b*x^2+a),x)
```

Fricas [F]

$$\int \frac{(fx)^m (d+ex^2)^q}{a+bx^2+cx^4} \, dx = \int \frac{(ex^2+d)^q (fx)^m}{cx^4+bx^2+a} \, dx$$

```
[In] integrate((f*x)^m*(e*x^2+d)^q/(c*x^4+b*x^2+a),x, algorithm="fricas")
[Out] integral((e*x^2 + d)^q*(f*x)^m/(c*x^4 + b*x^2 + a), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(fx)^m (d+ex^2)^q}{a+bx^2+cx^4} \, dx = \text{Timed out}$$

```
[In] integrate((f*x)**m*(e*x**2+d)**q/(c*x**4+b*x**2+a),x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{(fx)^m (d+ex^2)^q}{a+bx^2+cx^4} \, dx = \int \frac{(ex^2+d)^q (fx)^m}{cx^4+bx^2+a} \, dx$$

```
[In] integrate((f*x)^m*(e*x^2+d)^q/(c*x^4+b*x^2+a),x, algorithm="maxima")
[Out] integrate((e*x^2 + d)^q*(f*x)^m/(c*x^4 + b*x^2 + a), x)
```

Giac [F]

$$\int \frac{(fx)^m (d+ex^2)^q}{a+bx^2+cx^4} \, dx = \int \frac{(ex^2+d)^q (fx)^m}{cx^4+bx^2+a} \, dx$$

```
[In] integrate((f*x)^m*(e*x^2+d)^q/(c*x^4+b*x^2+a),x, algorithm="giac")
[Out] integrate((e*x^2 + d)^q*(f*x)^m/(c*x^4 + b*x^2 + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(fx)^m (d+ex^2)^q}{a+bx^2+cx^4}\,dx = \int \frac{(f\,x)^m\,(e\,x^2+d)^q}{c\,x^4+b\,x^2+a}\,dx$$

```
[In] int(((f*x)^m*(d + e*x^2)^q)/(a + b*x^2 + c*x^4),x)
[Out] int(((f*x)^m*(d + e*x^2)^q)/(a + b*x^2 + c*x^4), x)
```

3.401 $\int \frac{x^7(d+ex^2)^q}{a+bx^2+cx^4}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 313

$$\int \frac{x^7(d+ex^2)^q}{a+bx^2+cx^4}\,dx = -\frac{(cd+be)\left(d+ex^2\right)^{1+q}}{2c^2e^2(1+q)}+\frac{\left(d+ex^2\right)^{2+q}}{2ce^2(2+q)} + \frac{\left(a-\frac{b^2}{c}+\frac{b(b^2-3ac)}{c\sqrt{b^2-4ac}}\right)\left(d+ex^2\right)^{1+q}\,\text{Hypergeometric2F1}\left(1,1+q,2+q,\frac{2c(d+ex^2)}{2cd-\left(b-\sqrt{b^2-4ac}\right)e}\right)}{2c\left(2cd-\left(b-\sqrt{b^2-4ac}\right)e\right)(1+q)} + \frac{\left(a-\frac{b^2}{c}-\frac{b(b^2-3ac)}{c\sqrt{b^2-4ac}}\right)\left(d+ex^2\right)^{1+q}\,\text{Hypergeometric2F1}\left(1,1+q,2+q,\frac{2c(d+ex^2)}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}\right)}{2c\left(2cd-\left(b+\sqrt{b^2-4ac}\right)e\right)(1+q)}$$

```
[Out] -1/2*(b*e+c*d)*(e*x^2+d)^(1+q)/c^2/e^2/(1+q)+1/2*(e*x^2+d)^(2+q)/c/e^2/(2+q
)+1/2*(e*x^2+d)^(1+q)*hypergeom([1, 1+q],[2+q],2*c*(e*x^2+d)/(2*c*d-e*(b-(-
4*a*c+b^2)^(1/2))))*(a-b^2/c+b*(-3*a*c+b^2)/c/(-4*a*c+b^2)^(1/2))/c/(1+q)/(
2*c*d-e*(b-(-4*a*c+b^2)^(1/2)))+1/2*(e*x^2+d)^(1+q)*hypergeom([1, 1+q],[2+q
],2*c*(e*x^2+d)/(2*c*d-e*(b+(-4*a*c+b^2)^(1/2))))*(a-b^2/c-b*(-3*a*c+b^2)/c
/(-4*a*c+b^2)^(1/2))/c/(1+q)/(2*c*d-e*(b+(-4*a*c+b^2)^(1/2)))
```

Rubi [A] (verified)

Time = 0.63 (sec) , antiderivative size = 313, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used

$= \{1265, 1642, 70\}$

$$\int \frac{x^7(d+ex^2)^q}{a+bx^2+cx^4}\,dx$$
$$= \frac{\left(\frac{b(b^2-3ac)}{c\sqrt{b^2-4ac}}+a-\frac{b^2}{c}\right)(d+ex^2)^{q+1}\,\text{Hypergeometric2F1}\left(1,q+1,q+2,\frac{2c(ex^2+d)}{2cd-\left(b-\sqrt{b^2-4ac}\right)e}\right)}{2c(q+1)\left(2cd-e\left(b-\sqrt{b^2-4ac}\right)\right)}$$
$$+\frac{\left(-\frac{b(b^2-3ac)}{c\sqrt{b^2-4ac}}+a-\frac{b^2}{c}\right)(d+ex^2)^{q+1}\,\text{Hypergeometric2F1}\left(1,q+1,q+2,\frac{2c(ex^2+d)}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}\right)}{2c(q+1)\left(2cd-e\left(\sqrt{b^2-4ac}+b\right)\right)}$$
$$-\frac{(be+cd)(d+ex^2)^{q+1}}{2c^2e^2(q+1)}+\frac{(d+ex^2)^{q+2}}{2ce^2(q+2)}$$

```
[In] Int[(x^7*(d + e*x^2)^q)/(a + b*x^2 + c*x^4),x]
```

```
[Out] -1/2*((c*d + b*e)*(d + e*x^2)^(1 + q))/(c^2*e^2*(1 + q)) + (d + e*x^2)^(2 +
 q)/(2*c*e^2*(2 + q)) + ((a - b^2/c + (b*(b^2 - 3*a*c))/(c*Sqrt[b^2 - 4*a*c
]))*(d + e*x^2)^(1 + q)*Hypergeometric2F1[1, 1 + q, 2 + q, (2*c*(d + e*x^2)
)/(2*c*d - (b - Sqrt[b^2 - 4*a*c])*e)])/(2*c*(2*c*d - (b - Sqrt[b^2 - 4*a*c
])*e)*(1 + q)) + ((a - b^2/c - (b*(b^2 - 3*a*c))/(c*Sqrt[b^2 - 4*a*c]))*(d
+ e*x^2)^(1 + q)*Hypergeometric2F1[1, 1 + q, 2 + q, (2*c*(d + e*x^2))/(2*c*
d - (b + Sqrt[b^2 - 4*a*c])*e)])/(2*c*(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)*(
1 + q))
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 1265

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rule 1642

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{1}{2}\text{Subst}\left(\int \frac{x^3(d+ex)^q}{a+bx+cx^2}\,dx,x,x^2\right)\\
&= \frac{1}{2}\text{Subst}\left(\int \left(\frac{(-cd-be)(d+ex)^q}{c^2e}+\frac{\left(\frac{b^2}{c^2}-\frac{a}{c}-\frac{b(b^2-3ac)}{c^2\sqrt{b^2-4ac}}\right)(d+ex)^q}{b-\sqrt{b^2-4ac}+2cx}\right.\right.\\
&\qquad\left.\left.+\frac{\left(\frac{b^2}{c^2}-\frac{a}{c}+\frac{b(b^2-3ac)}{c^2\sqrt{b^2-4ac}}\right)(d+ex)^q}{b+\sqrt{b^2-4ac}+2cx}+\frac{(d+ex)^{1+q}}{ce}\right)\,dx,x,x^2\right)\\
&= -\frac{(cd+be)\left(d+ex^2\right)^{1+q}}{2c^2e^2(1+q)}+\frac{\left(d+ex^2\right)^{2+q}}{2ce^2(2+q)}\\
&\quad-\frac{\left(a-\frac{b^2}{c}-\frac{b(b^2-3ac)}{c\sqrt{b^2-4ac}}\right)\text{Subst}\left(\int \frac{(d+ex)^q}{b+\sqrt{b^2-4ac}+2cx}\,dx,x,x^2\right)}{2c}\\
&\quad-\frac{\left(a-\frac{b^2}{c}+\frac{b(b^2-3ac)}{c\sqrt{b^2-4ac}}\right)\text{Subst}\left(\int \frac{(d+ex)^q}{b-\sqrt{b^2-4ac}+2cx}\,dx,x,x^2\right)}{2c}\\
&= -\frac{(cd+be)\left(d+ex^2\right)^{1+q}}{2c^2e^2(1+q)}+\frac{\left(d+ex^2\right)^{2+q}}{2ce^2(2+q)}\\
&\quad+\frac{\left(a-\frac{b^2}{c}+\frac{b(b^2-3ac)}{c\sqrt{b^2-4ac}}\right)\left(d+ex^2\right)^{1+q}\,{}_2F_1\left(1,1+q;2+q;\frac{2c(d+ex^2)}{2cd-\left(b-\sqrt{b^2-4ac}\right)e}\right)}{2c\left(2cd-\left(b-\sqrt{b^2-4ac}\right)e\right)(1+q)}\\
&\quad+\frac{\left(a-\frac{b^2}{c}-\frac{b(b^2-3ac)}{c\sqrt{b^2-4ac}}\right)\left(d+ex^2\right)^{1+q}\,{}_2F_1\left(1,1+q;2+q;\frac{2c(d+ex^2)}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}\right)}{2c\left(2cd-\left(b+\sqrt{b^2-4ac}\right)e\right)(1+q)}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 1.10 (sec) , antiderivative size = 272, normalized size of antiderivative = 0.87

$$
\int \frac{x^7(d+ex^2)^q}{a+bx^2+cx^4}\,dx = \frac{\left(d+ex^2\right)^{1+q}\left(-\frac{cd+be}{e^2(1+q)}+\frac{c(d+ex^2)}{e^2(2+q)}+\frac{c\left(a-\frac{b^2}{c}+\frac{b(b^2-3ac)}{c\sqrt{b^2-4ac}}\right)\text{Hypergeometric2F1}\left(1,1+q,2+q,\frac{2c(d+ex^2)}{2cd+\left(-b+\sqrt{b^2-4ac}\right)e}\right)}{\left(2cd+\left(-b+\sqrt{b^2-4ac}\right)e\right)(1+q)}+\frac{c\left(a-\frac{b^2}{c}-\cdots\right.}{\cdots}\right.}{2c^2}
$$

```
[In] Integrate[(x^7*(d + e*x^2)^q)/(a + b*x^2 + c*x^4),x]
```

```
[Out] ((d + e*x^2)^(1 + q)*(-((c*d + b*e)/(e^2*(1 + q))) + (c*(d + e*x^2))/(e^2*(
2 + q)) + (c*(a - b^2/c + (b*(b^2 - 3*a*c))/(c*Sqrt[b^2 - 4*a*c]))*Hypergeo
```

```
metric2F1[1, 1 + q, 2 + q, (2*c*(d + e*x^2))/(2*c*d + (-b + Sqrt[b^2 - 4*a*
c])*e)])/((2*c*d + (-b + Sqrt[b^2 - 4*a*c])*e)*(1 + q)) + (c*(a - b^2/c - (
b*(b^2 - 3*a*c))/(c*Sqrt[b^2 - 4*a*c]))*Hypergeometric2F1[1, 1 + q, 2 + q,
(2*c*(d + e*x^2))/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)])/((2*c*d - (b + Sqrt
[b^2 - 4*a*c])*e)*(1 + q))))/(2*c^2)
```

Maple [F]

$$\int \frac{x^{7} \left(e\,x^{2}+d\right)^{q}}{c\,x^{4}+b\,x^{2}+a}dx$$

```
[In] int(x^7*(e*x^2+d)^q/(c*x^4+b*x^2+a),x)
[Out] int(x^7*(e*x^2+d)^q/(c*x^4+b*x^2+a),x)
```

Fricas [F]

$$\int \frac{x^7(d+ex^2)^q}{a+bx^2+cx^4}\,dx=\int \frac{(ex^2+d)^q x^7}{cx^4+bx^2+a}\,dx$$

```
[In] integrate(x^7*(e*x^2+d)^q/(c*x^4+b*x^2+a),x, algorithm="fricas")
[Out] integral((e*x^2 + d)^q*x^7/(c*x^4 + b*x^2 + a), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^7(d+ex^2)^q}{a+bx^2+cx^4}\,dx=\text{Timed out}$$

```
[In] integrate(x**7*(e*x**2+d)**q/(c*x**4+b*x**2+a),x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{x^7(d+ex^2)^q}{a+bx^2+cx^4}\,dx=\int \frac{(ex^2+d)^q x^7}{cx^4+bx^2+a}\,dx$$

```
[In] integrate(x^7*(e*x^2+d)^q/(c*x^4+b*x^2+a),x, algorithm="maxima")
[Out] integrate((e*x^2 + d)^q*x^7/(c*x^4 + b*x^2 + a), x)
```

Giac [F]

$$\int \frac{x^7(d+ex^2)^q}{a+bx^2+cx^4}\,dx = \int \frac{(ex^2+d)^q x^7}{cx^4+bx^2+a}\,dx$$

```
[In] integrate(x^7*(e*x^2+d)^q/(c*x^4+b*x^2+a),x, algorithm="giac")
[Out] integrate((e*x^2 + d)^q*x^7/(c*x^4 + b*x^2 + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^7(d+ex^2)^q}{a+bx^2+cx^4}\,dx = \int \frac{x^7\,(e\,x^2+d)^q}{c\,x^4+b\,x^2+a}\,dx$$

```
[In] int((x^7*(d + e*x^2)^q)/(a + b*x^2 + c*x^4),x)
[Out] int((x^7*(d + e*x^2)^q)/(a + b*x^2 + c*x^4), x)
```

3.402 $\int \frac{x^5(d+ex^2)^q}{a+bx^2+cx^4}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 256

$$\int \frac{x^5(d+ex^2)^q}{a+bx^2+cx^4}\,dx = \frac{(d+ex^2)^{1+q}}{2ce(1+q)} + \frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)(d+ex^2)^{1+q}\,\text{Hypergeometric2F1}\left(1,1+q,2+q,\frac{2c(d+ex^2)}{2cd-\left(b-\sqrt{b^2-4ac}\right)e}\right)}{2c\left(2cd-\left(b-\sqrt{b^2-4ac}\right)e\right)(1+q)} + \frac{\left(b+\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)(d+ex^2)^{1+q}\,\text{Hypergeometric2F1}\left(1,1+q,2+q,\frac{2c(d+ex^2)}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}\right)}{2c\left(2cd-\left(b+\sqrt{b^2-4ac}\right)e\right)(1+q)}$$

```
[Out] 1/2*(e*x^2+d)^(1+q)/c/e/(1+q)+1/2*(e*x^2+d)^(1+q)*hypergeom([1, 1+q],[2+q],
2*c*(e*x^2+d)/(2*c*d-e*(b-(-4*a*c+b^2)^(1/2))))*(b+(2*a*c-b^2)/(-4*a*c+b^2)
^(1/2))/c/(1+q)/(2*c*d-e*(b-(-4*a*c+b^2)^(1/2)))+1/2*(e*x^2+d)^(1+q)*hyperg
eom([1, 1+q],[2+q],2*c*(e*x^2+d)/(2*c*d-e*(b+(-4*a*c+b^2)^(1/2))))*(b+(-2*a
*c+b^2)/(-4*a*c+b^2)^(1/2))/c/(1+q)/(2*c*d-e*(b+(-4*a*c+b^2)^(1/2)))
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 256, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {1265, 1642, 70}

$$\int \frac{x^5(d+ex^2)^q}{a+bx^2+cx^4}\,dx$$
$$= \frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)(d+ex^2)^{q+1}\text{Hypergeometric2F1}\left(1,q+1,q+2,\frac{2c(ex^2+d)}{2cd-\left(b-\sqrt{b^2-4ac}\right)e}\right)}{2c(q+1)\left(2cd-e\left(b-\sqrt{b^2-4ac}\right)\right)}$$
$$+\frac{\left(\frac{b^2-2ac}{\sqrt{b^2-4ac}}+b\right)(d+ex^2)^{q+1}\text{Hypergeometric2F1}\left(1,q+1,q+2,\frac{2c(ex^2+d)}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}\right)}{2c(q+1)\left(2cd-e\left(\sqrt{b^2-4ac}+b\right)\right)}$$
$$+\frac{(d+ex^2)^{q+1}}{2ce(q+1)}$$

```
[In] Int[(x^5*(d + e*x^2)^q)/(a + b*x^2 + c*x^4),x]
```

```
[Out] (d + e*x^2)^(1 + q)/(2*c*e*(1 + q)) + ((b - (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c]
)*(d + e*x^2)^(1 + q)*Hypergeometric2F1[1, 1 + q, 2 + q, (2*c*(d + e*x^2))/
(2*c*d - (b - Sqrt[b^2 - 4*a*c])*e)])/(2*c*(2*c*d - (b - Sqrt[b^2 - 4*a*c])
*e)*(1 + q)) + ((b + (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*(d + e*x^2)^(1 + q)*H
ypergeometric2F1[1, 1 + q, 2 + q, (2*c*(d + e*x^2))/(2*c*d - (b + Sqrt[b^2
- 4*a*c])*e)])/(2*c*(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)*(1 + q))
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 1265

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rule 1642

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{2}\text{Subst}\left(\int \frac{x^2(d+ex)^q}{a+bx+cx^2}\,dx, x, x^2\right) \\
&= \frac{1}{2}\text{Subst}\left(\int \left(\frac{(d+ex)^q}{c} + \frac{\left(-\frac{b}{c}+\frac{b^2-2ac}{c\sqrt{b^2-4ac}}\right)(d+ex)^q}{b-\sqrt{b^2-4ac}+2cx} + \frac{\left(-\frac{b}{c}-\frac{b^2-2ac}{c\sqrt{b^2-4ac}}\right)(d+ex)^q}{b+\sqrt{b^2-4ac}+2cx}\right) dx, x, x^2\right) \\
&= \frac{(d+ex^2)^{1+q}}{2ce(1+q)} - \frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\text{Subst}\left(\int \frac{(d+ex)^q}{b-\sqrt{b^2-4ac}+2cx}\,dx, x, x^2\right)}{2c} \\
&\quad - \frac{\left(b+\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\text{Subst}\left(\int \frac{(d+ex)^q}{b+\sqrt{b^2-4ac}+2cx}\,dx, x, x^2\right)}{2c} \\
&= \frac{(d+ex^2)^{1+q}}{2ce(1+q)} + \frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)(d+ex^2)^{1+q}\,{}_2F_1\left(1,1+q;2+q;\frac{2c(d+ex^2)}{2cd-\left(b-\sqrt{b^2-4ac}\right)e}\right)}{2c\left(2cd-\left(b-\sqrt{b^2-4ac}\right)e\right)(1+q)} \\
&\quad + \frac{\left(b+\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)(d+ex^2)^{1+q}\,{}_2F_1\left(1,1+q;2+q;\frac{2c(d+ex^2)}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}\right)}{2c\left(2cd-\left(b+\sqrt{b^2-4ac}\right)e\right)(1+q)}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.53 (sec) , antiderivative size = 211, normalized size of antiderivative = 0.82

$$\int \frac{x^5(d+ex^2)^q}{a+bx^2+cx^4}\,dx$$

$$= \frac{(d+ex^2)^{1+q}\left(\frac{1}{e} + \frac{\left(b+\frac{-b^2+2ac}{\sqrt{b^2-4ac}}\right)\text{Hypergeometric2F1}\left(1,1+q,2+q,\frac{2c(d+ex^2)}{2cd+\left(-b+\sqrt{b^2-4ac}\right)e}\right)}{2cd+\left(-b+\sqrt{b^2-4ac}\right)e} + \frac{\left(b+\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\text{Hypergeometric2F1}\left(1,1 \ldots\right.}{2cd-\left(b+\sqrt{b^2-4 \ldots\right.}\right.}{2c(1+q)}$$

```
[In] Integrate[(x^5*(d + e*x^2)^q)/(a + b*x^2 + c*x^4),x]
```

```
[Out] ((d + e*x^2)^(1 + q)*(e^(-1) + ((b + (-b^2 + 2*a*c)/Sqrt[b^2 - 4*a*c])*Hype
rgeometric2F1[1, 1 + q, 2 + q, (2*c*(d + e*x^2))/(2*c*d + (-b + Sqrt[b^2 -
4*a*c])*e)])/(2*c*d + (-b + Sqrt[b^2 - 4*a*c])*e) + ((b + (b^2 - 2*a*c)/Sqr
t[b^2 - 4*a*c])*Hypergeometric2F1[1, 1 + q, 2 + q, (2*c*(d + e*x^2))/(2*c*d
 - (b + Sqrt[b^2 - 4*a*c])*e)])/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)))/(2*c*
(1 + q))
```

Maple [F]

$$\int \frac{x^5(e\,x^2+d)^q}{c\,x^4+b\,x^2+a}dx$$

```
[In] int(x^5*(e*x^2+d)^q/(c*x^4+b*x^2+a),x)
[Out] int(x^5*(e*x^2+d)^q/(c*x^4+b*x^2+a),x)
```

Fricas [F]

$$\int \frac{x^5(d+ex^2)^q}{a+bx^2+cx^4}\,dx = \int \frac{(ex^2+d)^q x^5}{cx^4+bx^2+a}\,dx$$

```
[In] integrate(x^5*(e*x^2+d)^q/(c*x^4+b*x^2+a),x, algorithm="fricas")
[Out] integral((e*x^2 + d)^q*x^5/(c*x^4 + b*x^2 + a), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^5(d+ex^2)^q}{a+bx^2+cx^4}\,dx = \text{Timed out}$$

```
[In] integrate(x**5*(e*x**2+d)**q/(c*x**4+b*x**2+a),x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{x^5(d+ex^2)^q}{a+bx^2+cx^4}\,dx = \int \frac{(ex^2+d)^q x^5}{cx^4+bx^2+a}\,dx$$

```
[In] integrate(x^5*(e*x^2+d)^q/(c*x^4+b*x^2+a),x, algorithm="maxima")
[Out] integrate((e*x^2 + d)^q*x^5/(c*x^4 + b*x^2 + a), x)
```

Giac [F]

$$\int \frac{x^5(d+ex^2)^q}{a+bx^2+cx^4}\,dx = \int \frac{(ex^2+d)^q x^5}{cx^4+bx^2+a}\,dx$$

```
[In] integrate(x^5*(e*x^2+d)^q/(c*x^4+b*x^2+a),x, algorithm="giac")
[Out] integrate((e*x^2 + d)^q*x^5/(c*x^4 + b*x^2 + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^5(d+ex^2)^q}{a+bx^2+cx^4}\,dx = \int \frac{x^5\,(e\,x^2+d)^q}{c\,x^4+b\,x^2+a}\,dx$$

```
[In] int((x^5*(d + e*x^2)^q)/(a + b*x^2 + c*x^4),x)
[Out] int((x^5*(d + e*x^2)^q)/(a + b*x^2 + c*x^4), x)
```

3.403 $\int \frac{x^3(d+ex^2)^q}{a+bx^2+cx^4}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 210

$$\int \frac{x^3(d+ex^2)^q}{a+bx^2+cx^4}\,dx = -\frac{\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)(d+ex^2)^{1+q}\,\text{Hypergeometric2F1}\left(1,1+q,2+q,\frac{2c(d+ex^2)}{2cd-\left(b-\sqrt{b^2-4ac}\right)e}\right)}{2\left(2cd-\left(b-\sqrt{b^2-4ac}\right)e\right)(1+q)} - \frac{\left(1+\frac{b}{\sqrt{b^2-4ac}}\right)(d+ex^2)^{1+q}\,\text{Hypergeometric2F1}\left(1,1+q,2+q,\frac{2c(d+ex^2)}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}\right)}{2\left(2cd-\left(b+\sqrt{b^2-4ac}\right)e\right)(1+q)}$$

```
[Out] -1/2*(e*x^2+d)^(1+q)*hypergeom([1, 1+q],[2+q],2*c*(e*x^2+d)/(2*c*d-e*(b-(-4
*a*c+b^2)^(1/2))))*(1-b/(-4*a*c+b^2)^(1/2))/(1+q)/(2*c*d-e*(b-(-4*a*c+b^2)^
(1/2)))-1/2*(e*x^2+d)^(1+q)*hypergeom([1, 1+q],[2+q],2*c*(e*x^2+d)/(2*c*d-e
*(b+(-4*a*c+b^2)^(1/2))))*(1+b/(-4*a*c+b^2)^(1/2))/(1+q)/(2*c*d-e*(b+(-4*a*
c+b^2)^(1/2)))
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 210, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used

$= \{1265, 844, 70\}$

$$\int \frac{x^3(d+ex^2)^q}{a+bx^2+cx^4}\,dx$$
$$= -\frac{\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)(d+ex^2)^{q+1}\,\text{Hypergeometric2F1}\left(1,q+1,q+2,\frac{2c(ex^2+d)}{2cd-\left(b-\sqrt{b^2-4ac}\right)e}\right)}{2(q+1)\left(2cd-e\left(b-\sqrt{b^2-4ac}\right)\right)} - \frac{\left(\frac{b}{\sqrt{b^2-4ac}}+1\right)(d+ex^2)^{q+1}\,\text{Hypergeometric2F1}\left(1,q+1,q+2,\frac{2c(ex^2+d)}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}\right)}{2(q+1)\left(2cd-e\left(\sqrt{b^2-4ac}+b\right)\right)}$$

```
[In] Int[(x^3*(d + e*x^2)^q)/(a + b*x^2 + c*x^4),x]
```

```
[Out] -1/2*((1 - b/Sqrt[b^2 - 4*a*c])*(d + e*x^2)^(1 + q)*Hypergeometric2F1[1, 1
+ q, 2 + q, (2*c*(d + e*x^2))/(2*c*d - (b - Sqrt[b^2 - 4*a*c])*e)])/((2*c*d
 - (b - Sqrt[b^2 - 4*a*c])*e)*(1 + q)) - ((1 + b/Sqrt[b^2 - 4*a*c])*(d + e*
x^2)^(1 + q)*Hypergeometric2F1[1, 1 + q, 2 + q, (2*c*(d + e*x^2))/(2*c*d -
(b + Sqrt[b^2 - 4*a*c])*e)])/(2*(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)*(1 + q)
)
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 844

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m, (f + g*x)/(a +
 b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*c
, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !RationalQ[m]
```

Rule 1265

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rubi steps

$$\text{integral} = \frac{1}{2}\text{Subst}\left(\int \frac{x(d+ex)^q}{a+bx+cx^2}\,dx, x, x^2\right)$$

$$= \frac{1}{2}\text{Subst}\left(\int \left(\frac{\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)(d+ex)^q}{b-\sqrt{b^2-4ac}+2cx}+\frac{\left(1+\frac{b}{\sqrt{b^2-4ac}}\right)(d+ex)^q}{b+\sqrt{b^2-4ac}+2cx}\right)dx,x,x^2\right)$$

$$= \frac{1}{2}\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\text{Subst}\left(\int \frac{(d+ex)^q}{b-\sqrt{b^2-4ac}+2cx}dx,x,x^2\right) + \frac{1}{2}\left(1+\frac{b}{\sqrt{b^2-4ac}}\right)\text{Subst}\left(\int \frac{(d+ex)^q}{b+\sqrt{b^2-4ac}+2cx}dx,x,x^2\right)$$

$$= -\frac{\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)(d+ex^2)^{1+q}\ {}_2F_1\left(1,1+q;2+q;\frac{2c(d+ex^2)}{2cd-\left(b-\sqrt{b^2-4ac}\right)e}\right)}{2\left(2cd-\left(b-\sqrt{b^2-4ac}\right)e\right)(1+q)} - \frac{\left(1+\frac{b}{\sqrt{b^2-4ac}}\right)(d+ex^2)^{1+q}\ {}_2F_1\left(1,1+q;2+q;\frac{2c(d+ex^2)}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}\right)}{2\left(2cd-\left(b+\sqrt{b^2-4ac}\right)e\right)(1+q)}$$

Mathematica [A] (verified)

Time = 0.35 (sec) , antiderivative size = 183, normalized size of antiderivative = 0.87

$$\int \frac{x^3(d+ex^2)^q}{a+bx^2+cx^4}dx = -\frac{(d+ex^2)^{1+q}\left(\left(-bd+\sqrt{b^2-4ac}d+2ae\right)\text{Hypergeometric2F1}\left(1,1+q,2+q,\frac{2c(d+ex^2)}{2cd+\left(-b+\sqrt{b^2-4ac}\right)e}\right)+(bd\right.}{4\sqrt{b^2-4ac}\,(cd^2+e(-bd+ae))}$$

```
[In] Integrate[(x^3*(d + e*x^2)^q)/(a + b*x^2 + c*x^4),x]
```

```
[Out] -1/4*((d + e*x^2)^(1 + q)*((-(b*d) + Sqrt[b^2 - 4*a*c]*d + 2*a*e)*Hypergeom
etric2F1[1, 1 + q, 2 + q, (2*c*(d + e*x^2))/(2*c*d + (-b + Sqrt[b^2 - 4*a*c
])*e)] + (b*d + Sqrt[b^2 - 4*a*c]*d - 2*a*e)*Hypergeometric2F1[1, 1 + q, 2
+ q, (2*c*(d + e*x^2))/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)]))/(Sqrt[b^2 - 4
*a*c]*(c*d^2 + e*(-(b*d) + a*e))*(1 + q))
```

Maple [F]

$$\int \frac{x^3(e\,x^2+d)^q}{c\,x^4+b\,x^2+a}dx$$

```
[In] int(x^3*(e*x^2+d)^q/(c*x^4+b*x^2+a),x)
```

```
[Out] int(x^3*(e*x^2+d)^q/(c*x^4+b*x^2+a),x)
```

Fricas [F]

$$\int \frac{x^3(d+ex^2)^q}{a+bx^2+cx^4}\,dx = \int \frac{(ex^2+d)^q x^3}{cx^4+bx^2+a}\,dx$$

```
[In] integrate(x^3*(e*x^2+d)^q/(c*x^4+b*x^2+a),x, algorithm="fricas")
[Out] integral((e*x^2 + d)^q*x^3/(c*x^4 + b*x^2 + a), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^3(d+ex^2)^q}{a+bx^2+cx^4}\,dx = \text{Timed out}$$

```
[In] integrate(x**3*(e*x**2+d)**q/(c*x**4+b*x**2+a),x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{x^3(d+ex^2)^q}{a+bx^2+cx^4}\,dx = \int \frac{(ex^2+d)^q x^3}{cx^4+bx^2+a}\,dx$$

```
[In] integrate(x^3*(e*x^2+d)^q/(c*x^4+b*x^2+a),x, algorithm="maxima")
[Out] integrate((e*x^2 + d)^q*x^3/(c*x^4 + b*x^2 + a), x)
```

Giac [F]

$$\int \frac{x^3(d+ex^2)^q}{a+bx^2+cx^4}\,dx = \int \frac{(ex^2+d)^q x^3}{cx^4+bx^2+a}\,dx$$

```
[In] integrate(x^3*(e*x^2+d)^q/(c*x^4+b*x^2+a),x, algorithm="giac")
[Out] integrate((e*x^2 + d)^q*x^3/(c*x^4 + b*x^2 + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3(d+ex^2)^q}{a+bx^2+cx^4}\,dx=\int \frac{x^3\,(e\,x^2+d)^q}{c\,x^4+b\,x^2+a}\,dx$$

```
[In] int((x^3*(d + e*x^2)^q)/(a + b*x^2 + c*x^4),x)
[Out] int((x^3*(d + e*x^2)^q)/(a + b*x^2 + c*x^4), x)
```

3.404 $\int \frac{x(d+ex^2)^q}{a+bx^2+cx^4} dx$

Optimal result

Integrand size = 25, antiderivative size = 198

$$\int \frac{x(d+ex^2)^q}{a+bx^2+cx^4} dx = -\frac{c(d+ex^2)^{1+q} \text{Hypergeometric2F1}\left(1,1+q,2+q,\frac{2c(d+ex^2)}{2cd-\left(b-\sqrt{b^2-4ac}\right)e}\right)}{\sqrt{b^2-4ac}\left(2cd-\left(b-\sqrt{b^2-4ac}\right)e\right)(1+q)} + \frac{c(d+ex^2)^{1+q} \text{Hypergeometric2F1}\left(1,1+q,2+q,\frac{2c(d+ex^2)}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}\right)}{\sqrt{b^2-4ac}\left(2cd-\left(b+\sqrt{b^2-4ac}\right)e\right)(1+q)}$$

```
[Out] -c*(e*x^2+d)^(1+q)*hypergeom([1, 1+q],[2+q],2*c*(e*x^2+d)/(2*c*d-e*(b-(-4*a
*c+b^2)^(1/2))))/(1+q)/(2*c*d-e*(b-(-4*a*c+b^2)^(1/2)))/(-4*a*c+b^2)^(1/2)+
c*(e*x^2+d)^(1+q)*hypergeom([1, 1+q],[2+q],2*c*(e*x^2+d)/(2*c*d-e*(b+(-4*a*
c+b^2)^(1/2))))/(1+q)/(-4*a*c+b^2)^(1/2)/(2*c*d-e*(b+(-4*a*c+b^2)^(1/2)))
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 198, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used

$= \{1261, 725, 70\}$

$$\int \frac{x(d+ex^2)^q}{a+bx^2+cx^4}\,dx$$
$$= \frac{c(d+ex^2)^{q+1}\,\text{Hypergeometric2F1}\left(1,q+1,q+2,\frac{2c(ex^2+d)}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}\right)}{(q+1)\sqrt{b^2-4ac}\left(2cd-e\left(\sqrt{b^2-4ac}+b\right)\right)}$$
$$- \frac{c(d+ex^2)^{q+1}\,\text{Hypergeometric2F1}\left(1,q+1,q+2,\frac{2c(ex^2+d)}{2cd-\left(b-\sqrt{b^2-4ac}\right)e}\right)}{(q+1)\sqrt{b^2-4ac}\left(2cd-e\left(b-\sqrt{b^2-4ac}\right)\right)}$$

```
[In] Int[(x*(d + e*x^2)^q)/(a + b*x^2 + c*x^4),x]
```

```
[Out] -((c*(d + e*x^2)^(1 + q)*Hypergeometric2F1[1, 1 + q, 2 + q, (2*c*(d + e*x^2
))/(2*c*d - (b - Sqrt[b^2 - 4*a*c])*e)])/(Sqrt[b^2 - 4*a*c]*(2*c*d - (b - S
qrt[b^2 - 4*a*c])*e)*(1 + q))) + (c*(d + e*x^2)^(1 + q)*Hypergeometric2F1[1
, 1 + q, 2 + q, (2*c*(d + e*x^2))/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)])/(Sq
rt[b^2 - 4*a*c]*(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)*(1 + q))
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 725

```
Int[((d_.) + (e_.)*(x_))^(m_)/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol
] :> Int[ExpandIntegrand[(d + e*x)^m, 1/(a + b*x + c*x^2), x], x] /; FreeQ[
{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2,
 0] && NeQ[2*c*d - b*e, 0] &&  !IntegerQ[m]
```

Rule 1261

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(
p_.), x_Symbol] :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x],
 x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]
```

Rubi steps

$$\text{integral} = \frac{1}{2}\text{Subst}\left(\int \frac{(d+ex)^q}{a+bx+cx^2}\,dx, x, x^2\right)$$

$$= \frac{1}{2}\text{Subst}\left(\int \left(\frac{2c(d+ex)^q}{\sqrt{b^2-4ac}\left(b-\sqrt{b^2-4ac}+2cx\right)} - \frac{2c(d+ex)^q}{\sqrt{b^2-4ac}\left(b+\sqrt{b^2-4ac}+2cx\right)}\right)dx, x, x^2\right)$$

$$= \frac{c\text{Subst}\left(\int \frac{(d+ex)^q}{b-\sqrt{b^2-4ac}+2cx}dx, x, x^2\right)}{\sqrt{b^2-4ac}} - \frac{c\text{Subst}\left(\int \frac{(d+ex)^q}{b+\sqrt{b^2-4ac}+2cx}dx, x, x^2\right)}{\sqrt{b^2-4ac}}$$

$$= -\frac{c(d+ex^2)^{1+q}\ {}_2F_1\left(1,1+q;2+q;\frac{2c(d+ex^2)}{2cd-\left(b-\sqrt{b^2-4ac}\right)e}\right)}{\sqrt{b^2-4ac}\left(2cd-\left(b-\sqrt{b^2-4ac}\right)e\right)(1+q)} + \frac{c(d+ex^2)^{1+q}\ {}_2F_1\left(1,1+q;2+q;\frac{2c(d+ex^2)}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}\right)}{\sqrt{b^2-4ac}\left(2cd-\left(b+\sqrt{b^2-4ac}\right)e\right)(1+q)}$$

Mathematica [A] (verified)

Time = 0.32 (sec) , antiderivative size = 168, normalized size of antiderivative = 0.85

$$\int \frac{x(d+ex^2)^q}{a+bx^2+cx^4}\,dx = \frac{c(d+ex^2)^{1+q}\left(-\frac{\text{Hypergeometric2F1}\left(1,1+q,2+q,\frac{2c(d+ex^2)}{2cd+\left(-b+\sqrt{b^2-4ac}\right)e}\right)}{2cd+\left(-b+\sqrt{b^2-4ac}\right)e} + \frac{\text{Hypergeometric2F1}\left(1,1+q,2+q,\frac{2c(d+ex^2)}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}\right)}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}\right)}{\sqrt{b^2-4ac}(1+q)}$$

```
[In] Integrate[(x*(d + e*x^2)^q)/(a + b*x^2 + c*x^4),x]
```

```
[Out] (c*(d + e*x^2)^(1 + q)*(-(Hypergeometric2F1[1, 1 + q, 2 + q, (2*c*(d + e*x^
2))/(2*c*d + (-b + Sqrt[b^2 - 4*a*c])*e)]/(2*c*d + (-b + Sqrt[b^2 - 4*a*c])
*e)) + Hypergeometric2F1[1, 1 + q, 2 + q, (2*c*(d + e*x^2))/(2*c*d - (b + S
qrt[b^2 - 4*a*c])*e)]/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)))/(Sqrt[b^2 - 4*a
*c]*(1 + q))
```

Maple [F]

$$\int \frac{x(e\,x^2+d)^q}{c\,x^4+b\,x^2+a}dx$$

```
[In] int(x*(e*x^2+d)^q/(c*x^4+b*x^2+a),x)
[Out] int(x*(e*x^2+d)^q/(c*x^4+b*x^2+a),x)
```

Fricas [F]

$$\int \frac{x(d+ex^2)^q}{a+bx^2+cx^4}\,dx = \int \frac{(ex^2+d)^q x}{cx^4+bx^2+a}\,dx$$

```
[In] integrate(x*(e*x^2+d)^q/(c*x^4+b*x^2+a),x, algorithm="fricas")
[Out] integral((e*x^2 + d)^q*x/(c*x^4 + b*x^2 + a), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x(d+ex^2)^q}{a+bx^2+cx^4}\,dx = \text{Timed out}$$

```
[In] integrate(x*(e*x**2+d)**q/(c*x**4+b*x**2+a),x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{x(d+ex^2)^q}{a+bx^2+cx^4}\,dx = \int \frac{(ex^2+d)^q x}{cx^4+bx^2+a}\,dx$$

```
[In] integrate(x*(e*x^2+d)^q/(c*x^4+b*x^2+a),x, algorithm="maxima")
[Out] integrate((e*x^2 + d)^q*x/(c*x^4 + b*x^2 + a), x)
```

Giac [F]

$$\int \frac{x(d+ex^2)^q}{a+bx^2+cx^4}\,dx = \int \frac{(ex^2+d)^q x}{cx^4+bx^2+a}\,dx$$

```
[In] integrate(x*(e*x^2+d)^q/(c*x^4+b*x^2+a),x, algorithm="giac")
[Out] integrate((e*x^2 + d)^q*x/(c*x^4 + b*x^2 + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x(d+ex^2)^q}{a+bx^2+cx^4}\,dx = \int \frac{x\,(e\,x^2+d)^q}{c\,x^4+b\,x^2+a}\,dx$$

```
[In] int((x*(d + e*x^2)^q)/(a + b*x^2 + c*x^4),x)
[Out] int((x*(d + e*x^2)^q)/(a + b*x^2 + c*x^4), x)
```

3.405 $\int \frac{(d+ex^2)^q}{x(a+bx^2+cx^4)} dx$

Optimal result

Integrand size = 27, antiderivative size = 262

$$\int \frac{(d+ex^2)^q}{x(a+bx^2+cx^4)} dx = \frac{c\left(1+\frac{b}{\sqrt{b^2-4ac}}\right)(d+ex^2)^{1+q}\text{Hypergeometric2F1}\left(1,1+q,2+q,\frac{2c(d+ex^2)}{2cd-\left(b-\sqrt{b^2-4ac}\right)e}\right)}{2a\left(2cd-\left(b-\sqrt{b^2-4ac}\right)e\right)(1+q)} + \frac{c\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)(d+ex^2)^{1+q}\text{Hypergeometric2F1}\left(1,1+q,2+q,\frac{2c(d+ex^2)}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}\right)}{2a\left(2cd-\left(b+\sqrt{b^2-4ac}\right)e\right)(1+q)} - \frac{(d+ex^2)^{1+q}\text{Hypergeometric2F1}\left(1,1+q,2+q,1+\frac{ex^2}{d}\right)}{2ad(1+q)}$$

```
[Out] -1/2*(e*x^2+d)^(1+q)*hypergeom([1, 1+q],[2+q],1+e*x^2/d)/a/d/(1+q)+1/2*c*(e
*x^2+d)^(1+q)*hypergeom([1, 1+q],[2+q],2*c*(e*x^2+d)/(2*c*d-e*(b-(-4*a*c+b^
2)^(1/2))))*(1+b/(-4*a*c+b^2)^(1/2))/a/(1+q)/(2*c*d-e*(b-(-4*a*c+b^2)^(1/2)
))+1/2*c*(e*x^2+d)^(1+q)*hypergeom([1, 1+q],[2+q],2*c*(e*x^2+d)/(2*c*d-e*(b
+(-4*a*c+b^2)^(1/2))))*(1-b/(-4*a*c+b^2)^(1/2))/a/(1+q)/(2*c*d-e*(b+(-4*a*c
+b^2)^(1/2)))
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 262, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {1265, 974, 67, 844, 70}

$$\int \frac{(d+ex^2)^q}{x(a+bx^2+cx^4)}\,dx$$
$$= \frac{c\left(\frac{b}{\sqrt{b^2-4ac}}+1\right)(d+ex^2)^{q+1}\text{Hypergeometric2F1}\left(1,q+1,q+2,\frac{2c(ex^2+d)}{2cd-\left(b-\sqrt{b^2-4ac}\right)e}\right)}{2a(q+1)\left(2cd-e\left(b-\sqrt{b^2-4ac}\right)\right)}$$
$$+\frac{c\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)(d+ex^2)^{q+1}\text{Hypergeometric2F1}\left(1,q+1,q+2,\frac{2c(ex^2+d)}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}\right)}{2a(q+1)\left(2cd-e\left(\sqrt{b^2-4ac}+b\right)\right)}$$
$$-\frac{(d+ex^2)^{q+1}\text{Hypergeometric2F1}\left(1,q+1,q+2,\frac{ex^2}{d}+1\right)}{2ad(q+1)}$$

```
[In] Int[(d + e*x^2)^q/(x*(a + b*x^2 + c*x^4)),x]
```

```
[Out] (c*(1 + b/Sqrt[b^2 - 4*a*c])*(d + e*x^2)^(1 + q)*Hypergeometric2F1[1, 1 + q
, 2 + q, (2*c*(d + e*x^2))/(2*c*d - (b - Sqrt[b^2 - 4*a*c])*e)])/(2*a*(2*c*
d - (b - Sqrt[b^2 - 4*a*c])*e)*(1 + q)) + (c*(1 - b/Sqrt[b^2 - 4*a*c])*(d +
 e*x^2)^(1 + q)*Hypergeometric2F1[1, 1 + q, 2 + q, (2*c*(d + e*x^2))/(2*c*d
 - (b + Sqrt[b^2 - 4*a*c])*e)])/(2*a*(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)*(1
 + q)) - ((d + e*x^2)^(1 + q)*Hypergeometric2F1[1, 1 + q, 2 + q, 1 + (e*x^2
)/d])/(2*a*d*(1 + q))
```

Rule 67

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 844

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m, (f + g*x)/(a +
```

```
 b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*c
, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !RationalQ[m]
```

Rule 974

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g
*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ
[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && (
IntegerQ[p] || (ILtQ[m, 0] && ILtQ[n, 0])) &&  !(IGtQ[m, 0] || IGtQ[n, 0])
```

Rule 1265

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{1}{2}\text{Subst}\left(\int \frac{(d+ex)^q}{x\,(a+bx+cx^2)}\,dx,x,x^2\right)\\
&= \frac{1}{2}\text{Subst}\left(\int \left(\frac{(d+ex)^q}{ax}+\frac{(-b-cx)(d+ex)^q}{a\,(a+bx+cx^2)}\right)\,dx,x,x^2\right)\\
&= \frac{\text{Subst}\left(\int \frac{(d+ex)^q}{x}\,dx,x,x^2\right)}{2a}+\frac{\text{Subst}\left(\int \frac{(-b-cx)(d+ex)^q}{a+bx+cx^2}\,dx,x,x^2\right)}{2a}\\
&= -\frac{(d+ex^2)^{1+q}\;{}_2F_1\left(1,1+q;2+q;1+\frac{ex^2}{d}\right)}{2ad(1+q)}\\
&\quad+\frac{\text{Subst}\left(\int \left(\frac{\left(-c-\frac{bc}{\sqrt{b^2-4ac}}\right)(d+ex)^q}{b-\sqrt{b^2-4ac}+2cx}+\frac{\left(-c+\frac{bc}{\sqrt{b^2-4ac}}\right)(d+ex)^q}{b+\sqrt{b^2-4ac}+2cx}\right)\,dx,x,x^2\right)}{2a}\\
&= -\frac{(d+ex^2)^{1+q}\;{}_2F_1\left(1,1+q;2+q;1+\frac{ex^2}{d}\right)}{2ad(1+q)}\\
&\quad-\frac{\left(c\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\right)\text{Subst}\left(\int \frac{(d+ex)^q}{b+\sqrt{b^2-4ac}+2cx}\,dx,x,x^2\right)}{2a}\\
&\quad-\frac{\left(c\left(1+\frac{b}{\sqrt{b^2-4ac}}\right)\right)\text{Subst}\left(\int \frac{(d+ex)^q}{b-\sqrt{b^2-4ac}+2cx}\,dx,x,x^2\right)}{2a}
\end{aligned}
$$

$$= \frac{c\left(1+\frac{b}{\sqrt{b^2-4ac}}\right)\left(d+ex^2\right)^{1+q}\ {}_2F_1\left(1,1+q;2+q;\frac{2c(d+ex^2)}{2cd-\left(b-\sqrt{b^2-4ac}\right)e}\right)}{2a\left(2cd-\left(b-\sqrt{b^2-4ac}\right)e\right)(1+q)} + \frac{c\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\left(d+ex^2\right)^{1+q}\ {}_2F_1\left(1,1+q;2+q;\frac{2c(d+ex^2)}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}\right)}{2a\left(2cd-\left(b+\sqrt{b^2-4ac}\right)e\right)(1+q)} - \frac{\left(d+ex^2\right)^{1+q}\ {}_2F_1\left(1,1+q;2+q;1+\frac{ex^2}{d}\right)}{2ad(1+q)}$$

Mathematica [A] (verified)

Time = 0.42 (sec) , antiderivative size = 218, normalized size of antiderivative = 0.83

$$\int \frac{(d+ex^2)^q}{x\left(a+bx^2+cx^4\right)}\,dx$$

$$= \frac{\left(d+ex^2\right)^{1+q}\left(\frac{c\left(1+\frac{b}{\sqrt{b^2-4ac}}\right)\text{Hypergeometric2F1}\left(1,1+q,2+q,\frac{2c\left(d+ex^2\right)}{2cd+\left(-b+\sqrt{b^2-4ac}\right)e}\right)}{2cd+\left(-b+\sqrt{b^2-4ac}\right)e} + \frac{c\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\text{Hypergeometric2F1}\left(1,1+q,\ldots\right)}{2cd-\left(b+\sqrt{b^2-4ac}\right)\ldots}\ldots\right.}{2a(1+q)}$$

```
[In] Integrate[(d + e*x^2)^q/(x*(a + b*x^2 + c*x^4)),x]
```

```
[Out] ((d + e*x^2)^(1 + q)*((c*(1 + b/Sqrt[b^2 - 4*a*c])*Hypergeometric2F1[1, 1 +
 q, 2 + q, (2*c*(d + e*x^2))/(2*c*d + (-b + Sqrt[b^2 - 4*a*c])*e)])/(2*c*d
+ (-b + Sqrt[b^2 - 4*a*c])*e) + (c*(1 - b/Sqrt[b^2 - 4*a*c])*Hypergeometric
2F1[1, 1 + q, 2 + q, (2*c*(d + e*x^2))/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)]
)/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e) - Hypergeometric2F1[1, 1 + q, 2 + q,
1 + (e*x^2)/d]/d))/(2*a*(1 + q))
```

Maple [F]

$$\int \frac{(e\,x^2+d)^q}{x\,(c\,x^4+b\,x^2+a)}dx$$

```
[In] int((e*x^2+d)^q/x/(c*x^4+b*x^2+a),x)
```

```
[Out] int((e*x^2+d)^q/x/(c*x^4+b*x^2+a),x)
```

Fricas [F]

$$\int \frac{(d+ex^2)^q}{x(a+bx^2+cx^4)}\,dx = \int \frac{(ex^2+d)^q}{(cx^4+bx^2+a)x}\,dx$$

```
[In] integrate((e*x^2+d)^q/x/(c*x^4+b*x^2+a),x, algorithm="fricas")
[Out] integral((e*x^2 + d)^q/(c*x^5 + b*x^3 + a*x), x)
```

Sympy [F]

$$\int \frac{(d+ex^2)^q}{x(a+bx^2+cx^4)}\,dx = \int \frac{(d+ex^2)^q}{x(a+bx^2+cx^4)}\,dx$$

```
[In] integrate((e*x**2+d)**q/x/(c*x**4+b*x**2+a),x)
[Out] Integral((d + e*x**2)**q/(x*(a + b*x**2 + c*x**4)), x)
```

Maxima [F]

$$\int \frac{(d+ex^2)^q}{x(a+bx^2+cx^4)}\,dx = \int \frac{(ex^2+d)^q}{(cx^4+bx^2+a)x}\,dx$$

```
[In] integrate((e*x^2+d)^q/x/(c*x^4+b*x^2+a),x, algorithm="maxima")
[Out] integrate((e*x^2 + d)^q/((c*x^4 + b*x^2 + a)*x), x)
```

Giac [F]

$$\int \frac{(d+ex^2)^q}{x(a+bx^2+cx^4)}\,dx = \int \frac{(ex^2+d)^q}{(cx^4+bx^2+a)x}\,dx$$

```
[In] integrate((e*x^2+d)^q/x/(c*x^4+b*x^2+a),x, algorithm="giac")
[Out] integrate((e*x^2 + d)^q/((c*x^4 + b*x^2 + a)*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex^2)^q}{x\,(a+bx^2+cx^4)}\,dx = \int \frac{(e\,x^2+d)^q}{x\,\left(c\,x^4+b\,x^2+a\right)}\,dx$$

```
[In] int((d + e*x^2)^q/(x*(a + b*x^2 + c*x^4)),x)
[Out] int((d + e*x^2)^q/(x*(a + b*x^2 + c*x^4)), x)
```

3.406 $\int \frac{(d+ex^2)^q}{x^3(a+bx^2+cx^4)}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 322

$$\int \frac{(d+ex^2)^q}{x^3(a+bx^2+cx^4)}\,dx = -\frac{c\left(b+\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)(d+ex^2)^{1+q}\,\text{Hypergeometric2F1}\left(1,1+q,2+q,\frac{2c(d+ex^2)}{2cd-\left(b-\sqrt{b^2-4ac}\right)e}\right)}{2a^2\left(2cd-\left(b-\sqrt{b^2-4ac}\right)e\right)(1+q)} - \frac{c\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)(d+ex^2)^{1+q}\,\text{Hypergeometric2F1}\left(1,1+q,2+q,\frac{2c(d+ex^2)}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}\right)}{2a^2\left(2cd-\left(b+\sqrt{b^2-4ac}\right)e\right)(1+q)} + \frac{b(d+ex^2)^{1+q}\,\text{Hypergeometric2F1}\left(1,1+q,2+q,1+\frac{ex^2}{d}\right)}{2a^2d(1+q)} + \frac{e(d+ex^2)^{1+q}\,\text{Hypergeometric2F1}\left(2,1+q,2+q,1+\frac{ex^2}{d}\right)}{2ad^2(1+q)}$$

```
[Out] 1/2*b*(e*x^2+d)^(1+q)*hypergeom([1, 1+q],[2+q],1+e*x^2/d)/a^2/d/(1+q)+1/2*e
*(e*x^2+d)^(1+q)*hypergeom([2, 1+q],[2+q],1+e*x^2/d)/a/d^2/(1+q)-1/2*c*(e*x
^2+d)^(1+q)*hypergeom([1, 1+q],[2+q],2*c*(e*x^2+d)/(2*c*d-e*(b-(-4*a*c+b^2)
^(1/2))))*(b+(-2*a*c+b^2)/(-4*a*c+b^2)^(1/2))/a^2/(1+q)/(2*c*d-e*(b-(-4*a*c
+b^2)^(1/2)))-1/2*c*(e*x^2+d)^(1+q)*hypergeom([1, 1+q],[2+q],2*c*(e*x^2+d)/
(2*c*d-e*(b+(-4*a*c+b^2)^(1/2))))*(b+(2*a*c-b^2)/(-4*a*c+b^2)^(1/2))/a^2/(1
+q)/(2*c*d-e*(b+(-4*a*c+b^2)^(1/2)))
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 322, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {1265, 974, 67, 844, 70}

$$\int \frac{(d+ex^2)^q}{x^3(a+bx^2+cx^4)}\,dx$$
$$= -\frac{c\left(\frac{b^2-2ac}{\sqrt{b^2-4ac}}+b\right)(d+ex^2)^{q+1}\,\text{Hypergeometric2F1}\left(1,q+1,q+2,\frac{2c(ex^2+d)}{2cd-\left(b-\sqrt{b^2-4ac}\right)e}\right)}{2a^2(q+1)\left(2cd-e\left(b-\sqrt{b^2-4ac}\right)\right)}$$
$$-\frac{c\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)(d+ex^2)^{q+1}\,\text{Hypergeometric2F1}\left(1,q+1,q+2,\frac{2c(ex^2+d)}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}\right)}{2a^2(q+1)\left(2cd-e\left(\sqrt{b^2-4ac}+b\right)\right)}$$
$$+\frac{b(d+ex^2)^{q+1}\,\text{Hypergeometric2F1}\left(1,q+1,q+2,\frac{ex^2}{d}+1\right)}{2a^2d(q+1)}$$
$$+\frac{e(d+ex^2)^{q+1}\,\text{Hypergeometric2F1}\left(2,q+1,q+2,\frac{ex^2}{d}+1\right)}{2ad^2(q+1)}$$

```
[In] Int[(d + e*x^2)^q/(x^3*(a + b*x^2 + c*x^4)),x]
```

```
[Out] -1/2*(c*(b + (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*(d + e*x^2)^(1 + q)*Hypergeom
etric2F1[1, 1 + q, 2 + q, (2*c*(d + e*x^2))/(2*c*d - (b - Sqrt[b^2 - 4*a*c]
)*e)])/(a^2*(2*c*d - (b - Sqrt[b^2 - 4*a*c])*e)*(1 + q)) - (c*(b - (b^2 - 2
*a*c)/Sqrt[b^2 - 4*a*c])*(d + e*x^2)^(1 + q)*Hypergeometric2F1[1, 1 + q, 2
+ q, (2*c*(d + e*x^2))/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)])/(2*a^2*(2*c*d
- (b + Sqrt[b^2 - 4*a*c])*e)*(1 + q)) + (b*(d + e*x^2)^(1 + q)*Hypergeometr
ic2F1[1, 1 + q, 2 + q, 1 + (e*x^2)/d])/(2*a^2*d*(1 + q)) + (e*(d + e*x^2)^(
1 + q)*Hypergeometric2F1[2, 1 + q, 2 + q, 1 + (e*x^2)/d])/(2*a*d^2*(1 + q))
```

Rule 67

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] && !IntegerQ[m] && IntegerQ[n]
```

Rule 844

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m, (f + g*x)/(a +
 b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*c
, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !RationalQ[m]
```

Rule 974

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g
*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ
[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && (
IntegerQ[p] || (ILtQ[m, 0] && ILtQ[n, 0])) &&  !(IGtQ[m, 0] || IGtQ[n, 0])
```

Rule 1265

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{2}\text{Subst}\left(\int \frac{(d+ex)^q}{x^2\left(a+bx+cx^2\right)}\,dx,x,x^2\right)\\
&= \frac{1}{2}\text{Subst}\left(\int \left(\frac{(d+ex)^q}{ax^2}-\frac{b(d+ex)^q}{a^2x}+\frac{\left(b^2-ac+bcx\right)(d+ex)^q}{a^2\left(a+bx+cx^2\right)}\right)dx,x,x^2\right)\\
&= \frac{\text{Subst}\left(\int \frac{\left(b^2-ac+bcx\right)(d+ex)^q}{a+bx+cx^2}\,dx,x,x^2\right)}{2a^2}\\
&\quad+\frac{\text{Subst}\left(\int \frac{(d+ex)^q}{x^2}\,dx,x,x^2\right)}{2a}-\frac{b\text{Subst}\left(\int \frac{(d+ex)^q}{x}\,dx,x,x^2\right)}{2a^2}\\
&= \frac{b\left(d+ex^2\right)^{1+q}\,{}_2F_1\left(1,1+q;2+q;1+\frac{ex^2}{d}\right)}{2a^2d(1+q)}\\
&\quad+\frac{e\left(d+ex^2\right)^{1+q}\,{}_2F_1\left(2,1+q;2+q;1+\frac{ex^2}{d}\right)}{2ad^2(1+q)}\\
&\quad+\frac{\text{Subst}\left(\int \left(\frac{\left(bc+\frac{c\left(b^2-2ac\right)}{\sqrt{b^2-4ac}}\right)(d+ex)^q}{b-\sqrt{b^2-4ac}+2cx}+\frac{\left(bc-\frac{c\left(b^2-2ac\right)}{\sqrt{b^2-4ac}}\right)(d+ex)^q}{b+\sqrt{b^2-4ac}+2cx}\right)dx,x,x^2\right)}{2a^2}
\end{aligned}$$

$$= \frac{b(d+ex^2)^{1+q} \,{}_2F_1\left(1,1+q;2+q;1+\frac{ex^2}{d}\right)}{2a^2d(1+q)} + \frac{e(d+ex^2)^{1+q} \,{}_2F_1\left(2,1+q;2+q;1+\frac{ex^2}{d}\right)}{2ad^2(1+q)} + \frac{\left(c\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\right) \text{Subst}\left(\int \frac{(d+ex)^q}{b+\sqrt{b^2-4ac}+2cx}\,dx,x,x^2\right)}{2a^2} + \frac{\left(c\left(b+\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\right) \text{Subst}\left(\int \frac{(d+ex)^q}{b-\sqrt{b^2-4ac}+2cx}\,dx,x,x^2\right)}{2a^2}$$

$$= -\frac{c\left(b+\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)(d+ex^2)^{1+q} \,{}_2F_1\left(1,1+q;2+q;\frac{2c(d+ex^2)}{2cd-\left(b-\sqrt{b^2-4ac}\right)e}\right)}{2a^2\left(2cd-\left(b-\sqrt{b^2-4ac}\right)e\right)(1+q)} - \frac{c\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)(d+ex^2)^{1+q} \,{}_2F_1\left(1,1+q;2+q;\frac{2c(d+ex^2)}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}\right)}{2a^2\left(2cd-\left(b+\sqrt{b^2-4ac}\right)e\right)(1+q)} + \frac{b(d+ex^2)^{1+q} \,{}_2F_1\left(1,1+q;2+q;1+\frac{ex^2}{d}\right)}{2a^2d(1+q)} + \frac{e(d+ex^2)^{1+q} \,{}_2F_1\left(2,1+q;2+q;1+\frac{ex^2}{d}\right)}{2ad^2(1+q)}$$

Mathematica [A] (verified)

Time = 0.45 (sec) , antiderivative size = 259, normalized size of antiderivative = 0.80

$$\int \frac{(d+ex^2)^q}{x^3(a+bx^2+cx^4)}\,dx = \frac{(d+ex^2)^{1+q}\left(-\frac{c\left(b+\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\text{Hypergeometric2F1}\left(1,1+q,2+q,\frac{2c(d+ex^2)}{2cd+\left(-b+\sqrt{b^2-4ac}\right)e}\right)}{2cd+\left(-b+\sqrt{b^2-4ac}\right)e} - \frac{c\left(b+\frac{-b^2+2ac}{\sqrt{b^2-4ac}}\right)\text{Hypergeometric2F1}\left(1,1+\ldots\right.}{2cd-\left(b+\sqrt{b^2-4a\ldots\right.}\right.}{2a^2(1+q)}$$

```
[In] Integrate[(d + e*x^2)^q/(x^3*(a + b*x^2 + c*x^4)),x]
```

```
[Out] ((d + e*x^2)^(1 + q)*(-((c*(b + (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*Hypergeome
tric2F1[1, 1 + q, 2 + q, (2*c*(d + e*x^2))/(2*c*d + (-b + Sqrt[b^2 - 4*a*c]
)*e)])/(2*c*d + (-b + Sqrt[b^2 - 4*a*c])*e)) - (c*(b + (-b^2 + 2*a*c)/Sqrt[
b^2 - 4*a*c])*Hypergeometric2F1[1, 1 + q, 2 + q, (2*c*(d + e*x^2))/(2*c*d -
 (b + Sqrt[b^2 - 4*a*c])*e)])/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e) + (b*Hype
rgeometric2F1[1, 1 + q, 2 + q, 1 + (e*x^2)/d])/d + (a*e*Hypergeometric2F1[2
, 1 + q, 2 + q, 1 + (e*x^2)/d])/d^2))/(2*a^2*(1 + q))
```

Maple [F]

$$\int \frac{(e\,x^2+d)^q}{x^3\,(c\,x^4+b\,x^2+a)}dx$$

```
[In] int((e*x^2+d)^q/x^3/(c*x^4+b*x^2+a),x)
[Out] int((e*x^2+d)^q/x^3/(c*x^4+b*x^2+a),x)
```

Fricas [F]

$$\int \frac{(d+ex^2)^q}{x^3\,(a+bx^2+cx^4)}\,dx = \int \frac{(ex^2+d)^q}{(cx^4+bx^2+a)x^3}\,dx$$

```
[In] integrate((e*x^2+d)^q/x^3/(c*x^4+b*x^2+a),x, algorithm="fricas")
[Out] integral((e*x^2 + d)^q/(c*x^7 + b*x^5 + a*x^3), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(d+ex^2)^q}{x^3\,(a+bx^2+cx^4)}\,dx = \text{Timed out}$$

```
[In] integrate((e*x**2+d)**q/x**3/(c*x**4+b*x**2+a),x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{(d+ex^2)^q}{x^3\,(a+bx^2+cx^4)}\,dx = \int \frac{(ex^2+d)^q}{(cx^4+bx^2+a)x^3}\,dx$$

```
[In] integrate((e*x^2+d)^q/x^3/(c*x^4+b*x^2+a),x, algorithm="maxima")
[Out] integrate((e*x^2 + d)^q/((c*x^4 + b*x^2 + a)*x^3), x)
```

Giac [F]

$$\int \frac{(d+ex^2)^q}{x^3(a+bx^2+cx^4)}\,dx = \int \frac{(ex^2+d)^q}{(cx^4+bx^2+a)x^3}\,dx$$

```
[In] integrate((e*x^2+d)^q/x^3/(c*x^4+b*x^2+a),x, algorithm="giac")
[Out] integrate((e*x^2 + d)^q/((c*x^4 + b*x^2 + a)*x^3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex^2)^q}{x^3(a+bx^2+cx^4)}\,dx = \int \frac{(e\,x^2+d)^q}{x^3\,(c\,x^4+b\,x^2+a)}\,dx$$

```
[In] int((d + e*x^2)^q/(x^3*(a + b*x^2 + c*x^4)),x)
[Out] int((d + e*x^2)^q/(x^3*(a + b*x^2 + c*x^4)), x)
```

3.407 $\int \frac{x^6(d+ex^2)^q}{a+bx^2+cx^4}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 339

$$\begin{aligned}
&\int \frac{x^6(d+ex^2)^q}{a+bx^2+cx^4}\,dx \\
&= \frac{\left(b^2-ac-\frac{b(b^2-3ac)}{\sqrt{b^2-4ac}}\right)x(d+ex^2)^q\left(1+\frac{ex^2}{d}\right)^{-q}\text{AppellF1}\left(\frac{1}{2},1,-q,\frac{3}{2},-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{ex^2}{d}\right)}{c^2\left(b-\sqrt{b^2-4ac}\right)} \\
&\quad+\frac{\left(b^2-ac+\frac{b(b^2-3ac)}{\sqrt{b^2-4ac}}\right)x(d+ex^2)^q\left(1+\frac{ex^2}{d}\right)^{-q}\text{AppellF1}\left(\frac{1}{2},1,-q,\frac{3}{2},-\frac{2cx^2}{b+\sqrt{b^2-4ac}},-\frac{ex^2}{d}\right)}{c^2\left(b+\sqrt{b^2-4ac}\right)} \\
&\quad-\frac{bx(d+ex^2)^q\left(1+\frac{ex^2}{d}\right)^{-q}\text{Hypergeometric2F1}\left(\frac{1}{2},-q,\frac{3}{2},-\frac{ex^2}{d}\right)}{c^2} \\
&\quad+\frac{x^3(d+ex^2)^q\left(1+\frac{ex^2}{d}\right)^{-q}\text{Hypergeometric2F1}\left(\frac{3}{2},-q,\frac{5}{2},-\frac{ex^2}{d}\right)}{3c}
\end{aligned}$$

```
[Out] -b*x*(e*x^2+d)^q*hypergeom([1/2, -q],[3/2],-e*x^2/d)/c^2/((1+e*x^2/d)^q)+1/
3*x^3*(e*x^2+d)^q*hypergeom([3/2, -q],[5/2],-e*x^2/d)/c/((1+e*x^2/d)^q)+x*(
e*x^2+d)^q*AppellF1(1/2,1,-q,3/2,-2*c*x^2/(b-(-4*a*c+b^2)^(1/2)),-e*x^2/d)*
(b^2-a*c-b*(-3*a*c+b^2)/(-4*a*c+b^2)^(1/2))/c^2/((1+e*x^2/d)^q)/(b-(-4*a*c+
b^2)^(1/2))+x*(e*x^2+d)^q*AppellF1(1/2,1,-q,3/2,-2*c*x^2/(b+(-4*a*c+b^2)^(1
/2)),-e*x^2/d)*(b^2-a*c+b*(-3*a*c+b^2)/(-4*a*c+b^2)^(1/2))/c^2/((1+e*x^2/d)
^q)/(b+(-4*a*c+b^2)^(1/2))
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 339, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.296$, Rules used = {1317, 252, 251, 372, 371, 1706, 441, 440}

$$\int \frac{x^6(d+ex^2)^q}{a+bx^2+cx^4}\,dx$$
$$= \frac{x\left(-\frac{b(b^2-3ac)}{\sqrt{b^2-4ac}}-ac+b^2\right)(d+ex^2)^q\left(\frac{ex^2}{d}+1\right)^{-q}\text{AppellF1}\left(\frac{1}{2},1,-q,\frac{3}{2},-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{ex^2}{d}\right)}{c^2\left(b-\sqrt{b^2-4ac}\right)}$$
$$+\frac{x\left(\frac{b(b^2-3ac)}{\sqrt{b^2-4ac}}-ac+b^2\right)(d+ex^2)^q\left(\frac{ex^2}{d}+1\right)^{-q}\text{AppellF1}\left(\frac{1}{2},1,-q,\frac{3}{2},-\frac{2cx^2}{b+\sqrt{b^2-4ac}},-\frac{ex^2}{d}\right)}{c^2\left(\sqrt{b^2-4ac}+b\right)}$$
$$-\frac{bx(d+ex^2)^q\left(\frac{ex^2}{d}+1\right)^{-q}\text{Hypergeometric2F1}\left(\frac{1}{2},-q,\frac{3}{2},-\frac{ex^2}{d}\right)}{c^2}$$
$$+\frac{x^3(d+ex^2)^q\left(\frac{ex^2}{d}+1\right)^{-q}\text{Hypergeometric2F1}\left(\frac{3}{2},-q,\frac{5}{2},-\frac{ex^2}{d}\right)}{3c}$$

```
[In] Int[(x^6*(d + e*x^2)^q)/(a + b*x^2 + c*x^4),x]
```

```
[Out] ((b^2 - a*c - (b*(b^2 - 3*a*c))/Sqrt[b^2 - 4*a*c])*x*(d + e*x^2)^q*AppellF1
[1/2, 1, -q, 3/2, (-2*c*x^2)/(b - Sqrt[b^2 - 4*a*c]), -((e*x^2)/d)])/(c^2*(
b - Sqrt[b^2 - 4*a*c])*(1 + (e*x^2)/d)^q) + ((b^2 - a*c + (b*(b^2 - 3*a*c))
/Sqrt[b^2 - 4*a*c])*x*(d + e*x^2)^q*AppellF1[1/2, 1, -q, 3/2, (-2*c*x^2)/(b
 + Sqrt[b^2 - 4*a*c]), -((e*x^2)/d)])/(c^2*(b + Sqrt[b^2 - 4*a*c])*(1 + (e*
x^2)/d)^q) - (b*x*(d + e*x^2)^q*Hypergeometric2F1[1/2, -q, 3/2, -((e*x^2)/d
)])/(c^2*(1 + (e*x^2)/d)^q) + (x^3*(d + e*x^2)^q*Hypergeometric2F1[3/2, -q,
 5/2, -((e*x^2)/d)])/(3*c*(1 + (e*x^2)/d)^q)
```

Rule 251

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 252

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x
^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(1 + b*(x^n/a))^p, x], x]
 /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Sim
plify[1/n + p], 0] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 372

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(c*x)^
m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0]
 && !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 440

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 441

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]),
 Int[(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] && !(IntegerQ[p] || GtQ[a, 0])
```

Rule 1317

```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_))/((a_) + (b_.)*(x_)^2 +
(c_.)*(x_)^4), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^q, (f*x)^m/(a +
 b*x^2 + c*x^4), x], x] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ[b^2 - 4*a
*c, 0] && !IntegerQ[q] && IntegerQ[m]
```

Rule 1706

```
Int[(Px_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^
(p_.), x_Symbol] :> Int[ExpandIntegrand[Px*(d + e*x^2)^q*(a + b*x^2 + c*x^4
)^p, x], x] /; FreeQ[{a, b, c, d, e, q}, x] && PolyQ[Px, x^2] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \left(-\frac{b(d+ex^2)^q}{c^2}+\frac{x^2(d+ex^2)^q}{c}+\frac{(ab+(b^2-ac)\,x^2)\,(d+ex^2)^q}{c^2\,(a+bx^2+cx^4)}\right)dx\\ &= \frac{\int \frac{(ab+(b^2-ac)x^2)\,(d+ex^2)^q}{a+bx^2+cx^4}\,dx}{c^2}-\frac{b\int (d+ex^2)^q\,dx}{c^2}+\frac{\int x^2(d+ex^2)^q\,dx}{c}\end{aligned}$$

$$= \frac{\int \left(\frac{\left(b^2-ac+\frac{b(-b^2+3ac)}{\sqrt{b^2-4ac}}\right)(d+ex^2)^q}{b-\sqrt{b^2-4ac}+2cx^2} + \frac{\left(b^2-ac-\frac{b(-b^2+3ac)}{\sqrt{b^2-4ac}}\right)(d+ex^2)^q}{b+\sqrt{b^2-4ac}+2cx^2} \right) dx}{c^2} - \frac{\left(b(d+ex^2)^q \left(1+\frac{ex^2}{d}\right)^{-q}\right) \int \left(1+\frac{ex^2}{d}\right)^q dx}{c^2} + \frac{\left((d+ex^2)^q \left(1+\frac{ex^2}{d}\right)^{-q}\right) \int x^2 \left(1+\frac{ex^2}{d}\right)^q dx}{c}$$

$$= -\frac{bx(d+ex^2)^q \left(1+\frac{ex^2}{d}\right)^{-q} {}_2F_1\left(\frac{1}{2},-q;\frac{3}{2};-\frac{ex^2}{d}\right)}{c^2} + \frac{x^3(d+ex^2)^q \left(1+\frac{ex^2}{d}\right)^{-q} {}_2F_1\left(\frac{3}{2},-q;\frac{5}{2};-\frac{ex^2}{d}\right)}{3c} + \frac{\left(b^2-ac-\frac{b(b^2-3ac)}{\sqrt{b^2-4ac}}\right) \int \frac{(d+ex^2)^q}{b-\sqrt{b^2-4ac}+2cx^2} dx}{c^2} + \frac{\left(b^2-ac+\frac{b(b^2-3ac)}{\sqrt{b^2-4ac}}\right) \int \frac{(d+ex^2)^q}{b+\sqrt{b^2-4ac}+2cx^2} dx}{c^2}$$

$$= -\frac{bx(d+ex^2)^q \left(1+\frac{ex^2}{d}\right)^{-q} {}_2F_1\left(\frac{1}{2},-q;\frac{3}{2};-\frac{ex^2}{d}\right)}{c^2} + \frac{x^3(d+ex^2)^q \left(1+\frac{ex^2}{d}\right)^{-q} {}_2F_1\left(\frac{3}{2},-q;\frac{5}{2};-\frac{ex^2}{d}\right)}{3c} + \frac{\left(\left(b^2-ac-\frac{b(b^2-3ac)}{\sqrt{b^2-4ac}}\right)(d+ex^2)^q \left(1+\frac{ex^2}{d}\right)^{-q}\right) \int \frac{\left(1+\frac{ex^2}{d}\right)^q}{b-\sqrt{b^2-4ac}+2cx^2} dx}{c^2} + \frac{\left(\left(b^2-ac+\frac{b(b^2-3ac)}{\sqrt{b^2-4ac}}\right)(d+ex^2)^q \left(1+\frac{ex^2}{d}\right)^{-q}\right) \int \frac{\left(1+\frac{ex^2}{d}\right)^q}{b+\sqrt{b^2-4ac}+2cx^2} dx}{c^2}$$

$$= \frac{\left(b^2-ac-\frac{b(b^2-3ac)}{\sqrt{b^2-4ac}}\right) x(d+ex^2)^q \left(1+\frac{ex^2}{d}\right)^{-q} F_1\left(\frac{1}{2};1,-q;\frac{3}{2};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{ex^2}{d}\right)}{c^2\left(b-\sqrt{b^2-4ac}\right)} + \frac{\left(b^2-ac+\frac{b(b^2-3ac)}{\sqrt{b^2-4ac}}\right) x(d+ex^2)^q \left(1+\frac{ex^2}{d}\right)^{-q} F_1\left(\frac{1}{2};1,-q;\frac{3}{2};-\frac{2cx^2}{b+\sqrt{b^2-4ac}},-\frac{ex^2}{d}\right)}{c^2\left(b+\sqrt{b^2-4ac}\right)} - \frac{bx(d+ex^2)^q \left(1+\frac{ex^2}{d}\right)^{-q} {}_2F_1\left(\frac{1}{2},-q;\frac{3}{2};-\frac{ex^2}{d}\right)}{c^2} + \frac{x^3(d+ex^2)^q \left(1+\frac{ex^2}{d}\right)^{-q} {}_2F_1\left(\frac{3}{2},-q;\frac{5}{2};-\frac{ex^2}{d}\right)}{3c}$$

Mathematica [F]

$$\int \frac{x^6(d+ex^2)^q}{a+bx^2+cx^4}\,dx=\int \frac{x^6(d+ex^2)^q}{a+bx^2+cx^4}\,dx$$

```
[In] Integrate[(x^6*(d + e*x^2)^q)/(a + b*x^2 + c*x^4),x]
[Out] Integrate[(x^6*(d + e*x^2)^q)/(a + b*x^2 + c*x^4), x]
```

Maple [F]

$$\int \frac{x^6(e\,x^2+d)^q}{c\,x^4+b\,x^2+a}dx$$

```
[In] int(x^6*(e*x^2+d)^q/(c*x^4+b*x^2+a),x)
[Out] int(x^6*(e*x^2+d)^q/(c*x^4+b*x^2+a),x)
```

Fricas [F]

$$\int \frac{x^6(d+ex^2)^q}{a+bx^2+cx^4}\,dx=\int \frac{(ex^2+d)^q x^6}{cx^4+bx^2+a}\,dx$$

```
[In] integrate(x^6*(e*x^2+d)^q/(c*x^4+b*x^2+a),x, algorithm="fricas")
[Out] integral((e*x^2 + d)^q*x^6/(c*x^4 + b*x^2 + a), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^6(d+ex^2)^q}{a+bx^2+cx^4}\,dx=\text{Timed out}$$

```
[In] integrate(x**6*(e*x**2+d)**q/(c*x**4+b*x**2+a),x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{x^6(d+ex^2)^q}{a+bx^2+cx^4}\,dx = \int \frac{(ex^2+d)^q x^6}{cx^4+bx^2+a}\,dx$$

```
[In] integrate(x^6*(e*x^2+d)^q/(c*x^4+b*x^2+a),x, algorithm="maxima")
[Out] integrate((e*x^2 + d)^q*x^6/(c*x^4 + b*x^2 + a), x)
```

Giac [F]

$$\int \frac{x^6(d+ex^2)^q}{a+bx^2+cx^4}\,dx = \int \frac{(ex^2+d)^q x^6}{cx^4+bx^2+a}\,dx$$

```
[In] integrate(x^6*(e*x^2+d)^q/(c*x^4+b*x^2+a),x, algorithm="giac")
[Out] integrate((e*x^2 + d)^q*x^6/(c*x^4 + b*x^2 + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^6(d+ex^2)^q}{a+bx^2+cx^4}\,dx = \int \frac{x^6\,(e\,x^2+d)^q}{c\,x^4+b\,x^2+a}\,dx$$

```
[In] int((x^6*(d + e*x^2)^q)/(a + b*x^2 + c*x^4),x)
[Out] int((x^6*(d + e*x^2)^q)/(a + b*x^2 + c*x^4), x)
```

3.408 $\int \frac{x^4(d+ex^2)^q}{a+bx^2+cx^4} dx$

Optimal result

Integrand size = 27, antiderivative size = 273

$$\int \frac{x^4(d+ex^2)^q}{a+bx^2+cx^4} dx = -\frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)x(d+ex^2)^q\left(1+\frac{ex^2}{d}\right)^{-q}\text{AppellF1}\left(\frac{1}{2},1,-q,\frac{3}{2},-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{ex^2}{d}\right)}{c\left(b-\sqrt{b^2-4ac}\right)} - \frac{\left(b+\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)x(d+ex^2)^q\left(1+\frac{ex^2}{d}\right)^{-q}\text{AppellF1}\left(\frac{1}{2},1,-q,\frac{3}{2},-\frac{2cx^2}{b+\sqrt{b^2-4ac}},-\frac{ex^2}{d}\right)}{c\left(b+\sqrt{b^2-4ac}\right)} + \frac{x(d+ex^2)^q\left(1+\frac{ex^2}{d}\right)^{-q}\text{Hypergeometric2F1}\left(\frac{1}{2},-q,\frac{3}{2},-\frac{ex^2}{d}\right)}{c}$$

```
[Out] x*(e*x^2+d)^q*hypergeom([1/2, -q],[3/2],-e*x^2/d)/c/((1+e*x^2/d)^q)-x*(e*x^
2+d)^q*AppellF1(1/2,1,-q,3/2,-2*c*x^2/(b-(-4*a*c+b^2)^(1/2)),-e*x^2/d)*(b+(
2*a*c-b^2)/(-4*a*c+b^2)^(1/2))/c/((1+e*x^2/d)^q)/(b-(-4*a*c+b^2)^(1/2))-x*(
e*x^2+d)^q*AppellF1(1/2,1,-q,3/2,-2*c*x^2/(b+(-4*a*c+b^2)^(1/2)),-e*x^2/d)*
(b+(-2*a*c+b^2)/(-4*a*c+b^2)^(1/2))/c/((1+e*x^2/d)^q)/(b+(-4*a*c+b^2)^(1/2)
)
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 273, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {1317, 252, 251, 1706, 441, 440}

$$\int \frac{x^4(d+ex^2)^q}{a+bx^2+cx^4}\,dx$$
$$= -\frac{x\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)(d+ex^2)^q\left(\frac{ex^2}{d}+1\right)^{-q}\text{AppellF1}\left(\frac{1}{2},1,-q,\frac{3}{2},-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{ex^2}{d}\right)}{c\left(b-\sqrt{b^2-4ac}\right)}$$
$$-\frac{x\left(\frac{b^2-2ac}{\sqrt{b^2-4ac}}+b\right)(d+ex^2)^q\left(\frac{ex^2}{d}+1\right)^{-q}\text{AppellF1}\left(\frac{1}{2},1,-q,\frac{3}{2},-\frac{2cx^2}{b+\sqrt{b^2-4ac}},-\frac{ex^2}{d}\right)}{c\left(\sqrt{b^2-4ac}+b\right)}$$
$$+\frac{x(d+ex^2)^q\left(\frac{ex^2}{d}+1\right)^{-q}\text{Hypergeometric2F1}\left(\frac{1}{2},-q,\frac{3}{2},-\frac{ex^2}{d}\right)}{c}$$

```
[In] Int[(x^4*(d + e*x^2)^q)/(a + b*x^2 + c*x^4),x]
```

```
[Out] -(((b - (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*x*(d + e*x^2)^q*AppellF1[1/2, 1, -
q, 3/2, (-2*c*x^2)/(b - Sqrt[b^2 - 4*a*c]), -((e*x^2)/d)])/(c*(b - Sqrt[b^2
 - 4*a*c])*(1 + (e*x^2)/d)^q)) - ((b + (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*x*(
d + e*x^2)^q*AppellF1[1/2, 1, -q, 3/2, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]),
-((e*x^2)/d)])/(c*(b + Sqrt[b^2 - 4*a*c])*(1 + (e*x^2)/d)^q) + (x*(d + e*x^
2)^q*Hypergeometric2F1[1/2, -q, 3/2, -((e*x^2)/d)])/(c*(1 + (e*x^2)/d)^q)
```

Rule 251

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 252

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x
^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(1 + b*(x^n/a))^p, x], x]
 /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Sim
plify[1/n + p], 0] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 440

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
```

```
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 441

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]),
 Int[(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 1317

```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_))/((a_) + (b_.)*(x_)^2 +
(c_.)*(x_)^4), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^q, (f*x)^m/(a +
 b*x^2 + c*x^4), x], x] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ[b^2 - 4*a
*c, 0] &&  !IntegerQ[q] && IntegerQ[m]
```

Rule 1706

```
Int[(Px_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^
(p_.), x_Symbol] :> Int[ExpandIntegrand[Px*(d + e*x^2)^q*(a + b*x^2 + c*x^4
)^p, x], x] /; FreeQ[{a, b, c, d, e, q}, x] && PolyQ[Px, x^2] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[p]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \left(\frac{(d+ex^2)^q}{c}-\frac{(a+bx^2)\,(d+ex^2)^q}{c\,(a+bx^2+cx^4)}\right)\,dx\\
&= \frac{\int (d+ex^2)^q\,dx}{c}-\frac{\int \frac{(a+bx^2)(d+ex^2)^q}{a+bx^2+cx^4}\,dx}{c}\\
&= -\frac{\int \left(\frac{\left(b+\frac{-b^2+2ac}{\sqrt{b^2-4ac}}\right)(d+ex^2)^q}{b-\sqrt{b^2-4ac}+2cx^2}+\frac{\left(b-\frac{-b^2+2ac}{\sqrt{b^2-4ac}}\right)(d+ex^2)^q}{b+\sqrt{b^2-4ac}+2cx^2}\right)\,dx}{c}\\
&\quad+\frac{\left((d+ex^2)^q\left(1+\frac{ex^2}{d}\right)^{-q}\right)\int\left(1+\frac{ex^2}{d}\right)^q\,dx}{c}\\
&= \frac{x(d+ex^2)^q\left(1+\frac{ex^2}{d}\right)^{-q}\,{}_2F_1\left(\frac{1}{2},-q;\frac{3}{2};-\frac{ex^2}{d}\right)}{c}\\
&\quad-\frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\int\frac{(d+ex^2)^q}{b-\sqrt{b^2-4ac}+2cx^2}\,dx}{c}-\frac{\left(b+\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\int\frac{(d+ex^2)^q}{b+\sqrt{b^2-4ac}+2cx^2}\,dx}{c}
\end{aligned}
$$

$$= \frac{x(d+ex^2)^q \left(1+\frac{ex^2}{d}\right)^{-q} {}_2F_1\left(\frac{1}{2},-q;\frac{3}{2};-\frac{ex^2}{d}\right)}{c} - \frac{\left(\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)(d+ex^2)^q \left(1+\frac{ex^2}{d}\right)^{-q}\right)\int \frac{\left(1+\frac{ex^2}{d}\right)^q}{b-\sqrt{b^2-4ac}+2cx^2}\,dx}{c} - \frac{\left(\left(b+\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)(d+ex^2)^q \left(1+\frac{ex^2}{d}\right)^{-q}\right)\int \frac{\left(1+\frac{ex^2}{d}\right)^q}{b+\sqrt{b^2-4ac}+2cx^2}\,dx}{c}$$

$$= -\frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)x(d+ex^2)^q \left(1+\frac{ex^2}{d}\right)^{-q} F_1\left(\frac{1}{2};1,-q;\frac{3}{2};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{ex^2}{d}\right)}{c\left(b-\sqrt{b^2-4ac}\right)} - \frac{\left(b+\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)x(d+ex^2)^q \left(1+\frac{ex^2}{d}\right)^{-q} F_1\left(\frac{1}{2};1,-q;\frac{3}{2};-\frac{2cx^2}{b+\sqrt{b^2-4ac}},-\frac{ex^2}{d}\right)}{c\left(b+\sqrt{b^2-4ac}\right)} + \frac{x(d+ex^2)^q \left(1+\frac{ex^2}{d}\right)^{-q} {}_2F_1\left(\frac{1}{2},-q;\frac{3}{2};-\frac{ex^2}{d}\right)}{c}$$

Mathematica [F]

$$\int \frac{x^4(d+ex^2)^q}{a+bx^2+cx^4}\,dx = \int \frac{x^4(d+ex^2)^q}{a+bx^2+cx^4}\,dx$$

```
[In] Integrate[(x^4*(d + e*x^2)^q)/(a + b*x^2 + c*x^4),x]
[Out] Integrate[(x^4*(d + e*x^2)^q)/(a + b*x^2 + c*x^4), x]
```

Maple [F]

$$\int \frac{x^4(e\,x^2+d)^q}{c\,x^4+b\,x^2+a}dx$$

```
[In] int(x^4*(e*x^2+d)^q/(c*x^4+b*x^2+a),x)
[Out] int(x^4*(e*x^2+d)^q/(c*x^4+b*x^2+a),x)
```

Fricas [F]

$$\int \frac{x^4(d+ex^2)^q}{a+bx^2+cx^4}\,dx = \int \frac{(ex^2+d)^q x^4}{cx^4+bx^2+a}\,dx$$

```
[In] integrate(x^4*(e*x^2+d)^q/(c*x^4+b*x^2+a),x, algorithm="fricas")
[Out] integral((e*x^2 + d)^q*x^4/(c*x^4 + b*x^2 + a), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^4(d+ex^2)^q}{a+bx^2+cx^4}\,dx = \text{Timed out}$$

```
[In] integrate(x**4*(e*x**2+d)**q/(c*x**4+b*x**2+a),x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{x^4(d+ex^2)^q}{a+bx^2+cx^4}\,dx = \int \frac{(ex^2+d)^q x^4}{cx^4+bx^2+a}\,dx$$

```
[In] integrate(x^4*(e*x^2+d)^q/(c*x^4+b*x^2+a),x, algorithm="maxima")
[Out] integrate((e*x^2 + d)^q*x^4/(c*x^4 + b*x^2 + a), x)
```

Giac [F]

$$\int \frac{x^4(d+ex^2)^q}{a+bx^2+cx^4}\,dx = \int \frac{(ex^2+d)^q x^4}{cx^4+bx^2+a}\,dx$$

```
[In] integrate(x^4*(e*x^2+d)^q/(c*x^4+b*x^2+a),x, algorithm="giac")
[Out] integrate((e*x^2 + d)^q*x^4/(c*x^4 + b*x^2 + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^4(d+ex^2)^q}{a+bx^2+cx^4}\,dx = \int \frac{x^4\,(e\,x^2+d)^q}{c\,x^4+b\,x^2+a}\,dx$$

```
[In] int((x^4*(d + e*x^2)^q)/(a + b*x^2 + c*x^4),x)
[Out] int((x^4*(d + e*x^2)^q)/(a + b*x^2 + c*x^4), x)
```

3.409 $\int \frac{x^2(d+ex^2)^q}{a+bx^2+cx^4}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 162

$$\int \frac{x^2(d+ex^2)^q}{a+bx^2+cx^4}\,dx = -\frac{x(d+ex^2)^q\left(1+\frac{ex^2}{d}\right)^{-q}\operatorname{AppellF1}\left(\frac{1}{2},1,-q,\frac{3}{2},-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{ex^2}{d}\right)}{\sqrt{b^2-4ac}} + \frac{x(d+ex^2)^q\left(1+\frac{ex^2}{d}\right)^{-q}\operatorname{AppellF1}\left(\frac{1}{2},1,-q,\frac{3}{2},-\frac{2cx^2}{b+\sqrt{b^2-4ac}},-\frac{ex^2}{d}\right)}{\sqrt{b^2-4ac}}$$

```
[Out] -x*(e*x^2+d)^q*AppellF1(1/2,1,-q,3/2,-2*c*x^2/(b-(-4*a*c+b^2)^(1/2)),-e*x^2
/d)/((1+e*x^2/d)^q)/(-4*a*c+b^2)^(1/2)+x*(e*x^2+d)^q*AppellF1(1/2,1,-q,3/2,
-2*c*x^2/(b+(-4*a*c+b^2)^(1/2)),-e*x^2/d)/((1+e*x^2/d)^q)/(-4*a*c+b^2)^(1/2
)
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 162, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {1317, 441, 440}

$$\int \frac{x^2(d+ex^2)^q}{a+bx^2+cx^4}\,dx = \frac{x(d+ex^2)^q\left(\frac{ex^2}{d}+1\right)^{-q}\operatorname{AppellF1}\left(\frac{1}{2},1,-q,\frac{3}{2},-\frac{2cx^2}{b+\sqrt{b^2-4ac}},-\frac{ex^2}{d}\right)}{\sqrt{b^2-4ac}} - \frac{x(d+ex^2)^q\left(\frac{ex^2}{d}+1\right)^{-q}\operatorname{AppellF1}\left(\frac{1}{2},1,-q,\frac{3}{2},-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{ex^2}{d}\right)}{\sqrt{b^2-4ac}}$$

```
[In] Int[(x^2*(d + e*x^2)^q)/(a + b*x^2 + c*x^4),x]
```

```
[Out] -((x*(d + e*x^2)^q*AppellF1[1/2, 1, -q, 3/2, (-2*c*x^2)/(b - Sqrt[b^2 - 4*a
*c]), -((e*x^2)/d)])/(Sqrt[b^2 - 4*a*c]*(1 + (e*x^2)/d)^q)) + (x*(d + e*x^2
)^q*AppellF1[1/2, 1, -q, 3/2, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]), -((e*x^2)
/d)])/(Sqrt[b^2 - 4*a*c]*(1 + (e*x^2)/d)^q)
```

Rule 440

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 441

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]),
 Int[(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 1317

```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_))/((a_) + (b_.)*(x_)^2 +
(c_.)*(x_)^4), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^q, (f*x)^m/(a +
 b*x^2 + c*x^4), x], x] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ[b^2 - 4*a
*c, 0] &&  !IntegerQ[q] && IntegerQ[m]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \left(\frac{\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)(d+ex^2)^q}{b-\sqrt{b^2-4ac}+2cx^2}+\frac{\left(1+\frac{b}{\sqrt{b^2-4ac}}\right)(d+ex^2)^q}{b+\sqrt{b^2-4ac}+2cx^2}\right) dx \\
&= \left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\int \frac{(d+ex^2)^q}{b-\sqrt{b^2-4ac}+2cx^2}\, dx \\
&\quad +\left(1+\frac{b}{\sqrt{b^2-4ac}}\right)\int \frac{(d+ex^2)^q}{b+\sqrt{b^2-4ac}+2cx^2}\, dx \\
&= \left(\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)(d+ex^2)^q\left(1+\frac{ex^2}{d}\right)^{-q}\right)\int \frac{\left(1+\frac{ex^2}{d}\right)^q}{b-\sqrt{b^2-4ac}+2cx^2}\, dx \\
&\quad +\left(\left(1+\frac{b}{\sqrt{b^2-4ac}}\right)(d+ex^2)^q\left(1+\frac{ex^2}{d}\right)^{-q}\right)\int \frac{\left(1+\frac{ex^2}{d}\right)^q}{b+\sqrt{b^2-4ac}+2cx^2}\, dx
\end{aligned}
$$

$$= -\frac{x(d+ex^2)^q \left(1+\frac{ex^2}{d}\right)^{-q} F_1\left(\frac{1}{2};1,-q;\frac{3}{2};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{ex^2}{d}\right)}{\sqrt{b^2-4ac}} + \frac{x(d+ex^2)^q \left(1+\frac{ex^2}{d}\right)^{-q} F_1\left(\frac{1}{2};1,-q;\frac{3}{2};-\frac{2cx^2}{b+\sqrt{b^2-4ac}},-\frac{ex^2}{d}\right)}{\sqrt{b^2-4ac}}$$

Mathematica [F]

$$\int \frac{x^2(d+ex^2)^q}{a+bx^2+cx^4}\,dx = \int \frac{x^2(d+ex^2)^q}{a+bx^2+cx^4}\,dx$$

```
[In] Integrate[(x^2*(d + e*x^2)^q)/(a + b*x^2 + c*x^4),x]
[Out] Integrate[(x^2*(d + e*x^2)^q)/(a + b*x^2 + c*x^4), x]
```

Maple [F]

$$\int \frac{x^2(e\,x^2+d)^q}{c\,x^4+b\,x^2+a}dx$$

```
[In] int(x^2*(e*x^2+d)^q/(c*x^4+b*x^2+a),x)
[Out] int(x^2*(e*x^2+d)^q/(c*x^4+b*x^2+a),x)
```

Fricas [F]

$$\int \frac{x^2(d+ex^2)^q}{a+bx^2+cx^4}\,dx = \int \frac{(ex^2+d)^q x^2}{cx^4+bx^2+a}\,dx$$

```
[In] integrate(x^2*(e*x^2+d)^q/(c*x^4+b*x^2+a),x, algorithm="fricas")
[Out] integral((e*x^2 + d)^q*x^2/(c*x^4 + b*x^2 + a), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^2(d+ex^2)^q}{a+bx^2+cx^4}\,dx = \text{Timed out}$$

```
[In] integrate(x**2*(e*x**2+d)**q/(c*x**4+b*x**2+a),x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{x^2(d+ex^2)^q}{a+bx^2+cx^4}\,dx = \int \frac{(ex^2+d)^q x^2}{cx^4+bx^2+a}\,dx$$

```
[In] integrate(x^2*(e*x^2+d)^q/(c*x^4+b*x^2+a),x, algorithm="maxima")
[Out] integrate((e*x^2 + d)^q*x^2/(c*x^4 + b*x^2 + a), x)
```

Giac [F]

$$\int \frac{x^2(d+ex^2)^q}{a+bx^2+cx^4}\,dx = \int \frac{(ex^2+d)^q x^2}{cx^4+bx^2+a}\,dx$$

```
[In] integrate(x^2*(e*x^2+d)^q/(c*x^4+b*x^2+a),x, algorithm="giac")
[Out] integrate((e*x^2 + d)^q*x^2/(c*x^4 + b*x^2 + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2(d+ex^2)^q}{a+bx^2+cx^4}\,dx = \int \frac{x^2\,(e\,x^2+d)^q}{c\,x^4+b\,x^2+a}\,dx$$

```
[In] int((x^2*(d + e*x^2)^q)/(a + b*x^2 + c*x^4),x)
[Out] int((x^2*(d + e*x^2)^q)/(a + b*x^2 + c*x^4), x)
```

3.410 $\int \frac{(d+ex^2)^q}{a+bx^2+cx^4}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 190

$$\int \frac{(d+ex^2)^q}{a+bx^2+cx^4}\,dx = -\frac{2cx(d+ex^2)^q\left(1+\frac{ex^2}{d}\right)^{-q}\text{AppellF1}\left(\frac{1}{2},1,-q,\frac{3}{2},-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{ex^2}{d}\right)}{b^2-4ac-b\sqrt{b^2-4ac}} - \frac{2cx(d+ex^2)^q\left(1+\frac{ex^2}{d}\right)^{-q}\text{AppellF1}\left(\frac{1}{2},1,-q,\frac{3}{2},-\frac{2cx^2}{b+\sqrt{b^2-4ac}},-\frac{ex^2}{d}\right)}{b^2-4ac+b\sqrt{b^2-4ac}}$$

```
[Out] -2*c*x*(e*x^2+d)^q*AppellF1(1/2,1,-q,3/2,-2*c*x^2/(b-(-4*a*c+b^2)^(1/2)),-e
*x^2/d)/((1+e*x^2/d)^q)/(b^2-4*a*c-b*(-4*a*c+b^2)^(1/2))-2*c*x*(e*x^2+d)^q*
AppellF1(1/2,1,-q,3/2,-2*c*x^2/(b+(-4*a*c+b^2)^(1/2)),-e*x^2/d)/((1+e*x^2/d
)^q)/(b^2-4*a*c+b*(-4*a*c+b^2)^(1/2))
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 190, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1188, 441, 440}

$$\int \frac{(d+ex^2)^q}{a+bx^2+cx^4}\,dx = -\frac{2cx(d+ex^2)^q\left(\frac{ex^2}{d}+1\right)^{-q}\text{AppellF1}\left(\frac{1}{2},1,-q,\frac{3}{2},-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{ex^2}{d}\right)}{-b\sqrt{b^2-4ac}-4ac+b^2} - \frac{2cx(d+ex^2)^q\left(\frac{ex^2}{d}+1\right)^{-q}\text{AppellF1}\left(\frac{1}{2},1,-q,\frac{3}{2},-\frac{2cx^2}{b+\sqrt{b^2-4ac}},-\frac{ex^2}{d}\right)}{b\sqrt{b^2-4ac}-4ac+b^2}$$

```
[In] Int[(d + e*x^2)^q/(a + b*x^2 + c*x^4),x]
```

```
[Out] (-2*c*x*(d + e*x^2)^q*AppellF1[1/2, 1, -q, 3/2, (-2*c*x^2)/(b - Sqrt[b^2 -
4*a*c]), -((e*x^2)/d)])/((b^2 - 4*a*c - b*Sqrt[b^2 - 4*a*c])*(1 + (e*x^2)/d
)^q) - (2*c*x*(d + e*x^2)^q*AppellF1[1/2, 1, -q, 3/2, (-2*c*x^2)/(b + Sqrt[
b^2 - 4*a*c]), -((e*x^2)/d)])/((b^2 - 4*a*c + b*Sqrt[b^2 - 4*a*c])*(1 + (e*
x^2)/d)^q)
```

Rule 440

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 441

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]),
 Int[(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 1188

```
Int[((d_) + (e_.)*(x_)^2)^(q_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symb
ol] :> With[{r = Rt[b^2 - 4*a*c, 2]}, Dist[2*(c/r), Int[(d + e*x^2)^q/(b -
r + 2*c*x^2), x], x] - Dist[2*(c/r), Int[(d + e*x^2)^q/(b + r + 2*c*x^2), x
], x]] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
- b*d*e + a*e^2, 0] &&  !IntegerQ[q]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{(2c)\int \frac{(d+ex^2)^q}{b-\sqrt{b^2-4ac}+2cx^2}\,dx}{\sqrt{b^2-4ac}} - \frac{(2c)\int \frac{(d+ex^2)^q}{b+\sqrt{b^2-4ac}+2cx^2}\,dx}{\sqrt{b^2-4ac}}\\
&= \frac{\left(2c(d+ex^2)^q\left(1+\frac{ex^2}{d}\right)^{-q}\right)\int \frac{\left(1+\frac{ex^2}{d}\right)^q}{b-\sqrt{b^2-4ac}+2cx^2}\,dx}{\sqrt{b^2-4ac}}\\
&\quad - \frac{\left(2c(d+ex^2)^q\left(1+\frac{ex^2}{d}\right)^{-q}\right)\int \frac{\left(1+\frac{ex^2}{d}\right)^q}{b+\sqrt{b^2-4ac}+2cx^2}\,dx}{\sqrt{b^2-4ac}}\\
&= -\frac{2cx(d+ex^2)^q\left(1+\frac{ex^2}{d}\right)^{-q}F_1\left(\frac{1}{2};1,-q;\frac{3}{2};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{ex^2}{d}\right)}{b^2-4ac-b\sqrt{b^2-4ac}}\\
&\quad - \frac{2cx(d+ex^2)^q\left(1+\frac{ex^2}{d}\right)^{-q}F_1\left(\frac{1}{2};1,-q;\frac{3}{2};-\frac{2cx^2}{b+\sqrt{b^2-4ac}},-\frac{ex^2}{d}\right)}{b^2-4ac+b\sqrt{b^2-4ac}}
\end{aligned}$$

Mathematica [F]

$$\int \frac{(d+ex^2)^q}{a+bx^2+cx^4}\,dx = \int \frac{(d+ex^2)^q}{a+bx^2+cx^4}\,dx$$

```
[In] Integrate[(d + e*x^2)^q/(a + b*x^2 + c*x^4),x]
[Out] Integrate[(d + e*x^2)^q/(a + b*x^2 + c*x^4), x]
```

Maple [F]

$$\int \frac{(e\,x^2+d)^q}{c\,x^4+b\,x^2+a}dx$$

```
[In] int((e*x^2+d)^q/(c*x^4+b*x^2+a),x)
[Out] int((e*x^2+d)^q/(c*x^4+b*x^2+a),x)
```

Fricas [F]

$$\int \frac{(d+ex^2)^q}{a+bx^2+cx^4}\,dx = \int \frac{(ex^2+d)^q}{cx^4+bx^2+a}\,dx$$

```
[In] integrate((e*x^2+d)^q/(c*x^4+b*x^2+a),x, algorithm="fricas")
[Out] integral((e*x^2 + d)^q/(c*x^4 + b*x^2 + a), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(d+ex^2)^q}{a+bx^2+cx^4}\,dx = \text{Timed out}$$

```
[In] integrate((e*x**2+d)**q/(c*x**4+b*x**2+a),x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{(d+ex^2)^q}{a+bx^2+cx^4}\,dx = \int \frac{(ex^2+d)^q}{cx^4+bx^2+a}\,dx$$

```
[In] integrate((e*x^2+d)^q/(c*x^4+b*x^2+a),x, algorithm="maxima")
[Out] integrate((e*x^2 + d)^q/(c*x^4 + b*x^2 + a), x)
```

Giac [F]

$$\int \frac{(d+ex^2)^q}{a+bx^2+cx^4}\,dx = \int \frac{(ex^2+d)^q}{cx^4+bx^2+a}\,dx$$

```
[In] integrate((e*x^2+d)^q/(c*x^4+b*x^2+a),x, algorithm="giac")
[Out] integrate((e*x^2 + d)^q/(c*x^4 + b*x^2 + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex^2)^q}{a+bx^2+cx^4}\,dx = \int \frac{(e\,x^2+d)^q}{c\,x^4+b\,x^2+a}\,dx$$

```
[In] int((d + e*x^2)^q/(a + b*x^2 + c*x^4),x)
[Out] int((d + e*x^2)^q/(a + b*x^2 + c*x^4), x)
```

3.411 $\int \frac{(d+ex^2)^q}{x^2(a+bx^2+cx^4)}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 264

$$\begin{aligned}
&\int \frac{(d+ex^2)^q}{x^2(a+bx^2+cx^4)}\,dx\\
&=-\frac{c\left(1+\frac{b}{\sqrt{b^2-4ac}}\right)x(d+ex^2)^q\left(1+\frac{ex^2}{d}\right)^{-q}\text{AppellF1}\left(\frac{1}{2},1,-q,\frac{3}{2},-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{ex^2}{d}\right)}{a\left(b-\sqrt{b^2-4ac}\right)}\\
&\quad-\frac{c\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)x(d+ex^2)^q\left(1+\frac{ex^2}{d}\right)^{-q}\text{AppellF1}\left(\frac{1}{2},1,-q,\frac{3}{2},-\frac{2cx^2}{b+\sqrt{b^2-4ac}},-\frac{ex^2}{d}\right)}{a\left(b+\sqrt{b^2-4ac}\right)}\\
&\quad-\frac{(d+ex^2)^q\left(1+\frac{ex^2}{d}\right)^{-q}\text{Hypergeometric2F1}\left(-\frac{1}{2},-q,\frac{1}{2},-\frac{ex^2}{d}\right)}{ax}
\end{aligned}$$

```
[Out] -(e*x^2+d)^q*hypergeom([-1/2, -q],[1/2],-e*x^2/d)/a/x/((1+e*x^2/d)^q)-c*x*(
e*x^2+d)^q*AppellF1(1/2,1,-q,3/2,-2*c*x^2/(b-(-4*a*c+b^2)^(1/2)),-e*x^2/d)*
(1+b/(-4*a*c+b^2)^(1/2))/a/((1+e*x^2/d)^q)/(b-(-4*a*c+b^2)^(1/2))-c*x*(e*x^
2+d)^q*AppellF1(1/2,1,-q,3/2,-2*c*x^2/(b+(-4*a*c+b^2)^(1/2)),-e*x^2/d)*(1-b
/(-4*a*c+b^2)^(1/2))/a/((1+e*x^2/d)^q)/(b+(-4*a*c+b^2)^(1/2))
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 264, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {1317, 372, 371, 1706, 441, 440}

$$\begin{aligned}
&\int \frac{(d+ex^2)^q}{x^2\left(a+bx^2+cx^4\right)}\,dx \\
&= -\frac{cx\left(\frac{b}{\sqrt{b^2-4ac}}+1\right)(d+ex^2)^q\left(\frac{ex^2}{d}+1\right)^{-q}\text{AppellF1}\left(\frac{1}{2},1,-q,\frac{3}{2},-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{ex^2}{d}\right)}{a\left(b-\sqrt{b^2-4ac}\right)} \\
&\quad -\frac{cx\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)(d+ex^2)^q\left(\frac{ex^2}{d}+1\right)^{-q}\text{AppellF1}\left(\frac{1}{2},1,-q,\frac{3}{2},-\frac{2cx^2}{b+\sqrt{b^2-4ac}},-\frac{ex^2}{d}\right)}{a\left(\sqrt{b^2-4ac}+b\right)} \\
&\quad -\frac{(d+ex^2)^q\left(\frac{ex^2}{d}+1\right)^{-q}\text{Hypergeometric2F1}\left(-\frac{1}{2},-q,\frac{1}{2},-\frac{ex^2}{d}\right)}{ax}
\end{aligned}$$

```
[In] Int[(d + e*x^2)^q/(x^2*(a + b*x^2 + c*x^4)),x]
```

```
[Out] -((c*(1 + b/Sqrt[b^2 - 4*a*c])*x*(d + e*x^2)^q*AppellF1[1/2, 1, -q, 3/2, (-
2*c*x^2)/(b - Sqrt[b^2 - 4*a*c]), -((e*x^2)/d)])/(a*(b - Sqrt[b^2 - 4*a*c])
*(1 + (e*x^2)/d)^q)) - (c*(1 - b/Sqrt[b^2 - 4*a*c])*x*(d + e*x^2)^q*AppellF
1[1/2, 1, -q, 3/2, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]), -((e*x^2)/d)])/(a*(b
 + Sqrt[b^2 - 4*a*c])*(1 + (e*x^2)/d)^q) - ((d + e*x^2)^q*Hypergeometric2F1
[-1/2, -q, 1/2, -((e*x^2)/d)])/(a*x*(1 + (e*x^2)/d)^q)
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 372

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(c*x)^
m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 440

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
```

```
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 441

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]),
 Int[(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 1317

```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_))/((a_) + (b_.)*(x_)^2 +
(c_.)*(x_)^4), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^q, (f*x)^m/(a +
 b*x^2 + c*x^4), x], x] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ[b^2 - 4*a
*c, 0] &&  !IntegerQ[q] && IntegerQ[m]
```

Rule 1706

```
Int[(Px_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^
(p_.), x_Symbol] :> Int[ExpandIntegrand[Px*(d + e*x^2)^q*(a + b*x^2 + c*x^4
)^p, x], x] /; FreeQ[{a, b, c, d, e, q}, x] && PolyQ[Px, x^2] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[p]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \left(\frac{(d+ex^2)^q}{ax^2}+\frac{(-b-cx^2)\,(d+ex^2)^q}{a\,(a+bx^2+cx^4)}\right)dx\\
&= \frac{\int \frac{(d+ex^2)^q}{x^2}\,dx}{a}+\frac{\int \frac{(-b-cx^2)(d+ex^2)^q}{a+bx^2+cx^4}\,dx}{a}\\
&= \frac{\int \left(\frac{\left(-c-\frac{bc}{\sqrt{b^2-4ac}}\right)(d+ex^2)^q}{b-\sqrt{b^2-4ac}+2cx^2}+\frac{\left(-c+\frac{bc}{\sqrt{b^2-4ac}}\right)(d+ex^2)^q}{b+\sqrt{b^2-4ac}+2cx^2}\right)dx}{a}\\
&\quad+\frac{\left((d+ex^2)^q\left(1+\frac{ex^2}{d}\right)^{-q}\right)\int \frac{\left(1+\frac{ex^2}{d}\right)^q}{x^2}\,dx}{a}\\
&= -\frac{(d+ex^2)^q\left(1+\frac{ex^2}{d}\right)^{-q}\,{}_2F_1\left(-\frac{1}{2},-q;\frac{1}{2};-\frac{ex^2}{d}\right)}{ax}\\
&\quad-\frac{\left(c\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\right)\int \frac{(d+ex^2)^q}{b+\sqrt{b^2-4ac}+2cx^2}\,dx}{a}-\frac{\left(c\left(1+\frac{b}{\sqrt{b^2-4ac}}\right)\right)\int \frac{(d+ex^2)^q}{b-\sqrt{b^2-4ac}+2cx^2}\,dx}{a}
\end{aligned}
$$

$$= -\frac{(d+ex^2)^q \left(1+\frac{ex^2}{d}\right)^{-q} {}_2F_1\left(-\frac{1}{2},-q;\frac{1}{2};-\frac{ex^2}{d}\right)}{ax} - \frac{\left(c\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)(d+ex^2)^q \left(1+\frac{ex^2}{d}\right)^{-q}\right)\int \frac{\left(1+\frac{ex^2}{d}\right)^q}{b+\sqrt{b^2-4ac}+2cx^2}\,dx}{a} - \frac{\left(c\left(1+\frac{b}{\sqrt{b^2-4ac}}\right)(d+ex^2)^q \left(1+\frac{ex^2}{d}\right)^{-q}\right)\int \frac{\left(1+\frac{ex^2}{d}\right)^q}{b-\sqrt{b^2-4ac}+2cx^2}\,dx}{a}$$

$$= -\frac{c\left(1+\frac{b}{\sqrt{b^2-4ac}}\right)x(d+ex^2)^q\left(1+\frac{ex^2}{d}\right)^{-q} F_1\left(\frac{1}{2};1,-q;\frac{3}{2};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{ex^2}{d}\right)}{a\left(b-\sqrt{b^2-4ac}\right)} - \frac{c\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)x(d+ex^2)^q\left(1+\frac{ex^2}{d}\right)^{-q} F_1\left(\frac{1}{2};1,-q;\frac{3}{2};-\frac{2cx^2}{b+\sqrt{b^2-4ac}},-\frac{ex^2}{d}\right)}{a\left(b+\sqrt{b^2-4ac}\right)} - \frac{(d+ex^2)^q \left(1+\frac{ex^2}{d}\right)^{-q} {}_2F_1\left(-\frac{1}{2},-q;\frac{1}{2};-\frac{ex^2}{d}\right)}{ax}$$

Mathematica [F]

$$\int \frac{(d+ex^2)^q}{x^2(a+bx^2+cx^4)}\,dx = \int \frac{(d+ex^2)^q}{x^2(a+bx^2+cx^4)}\,dx$$

```
[In] Integrate[(d + e*x^2)^q/(x^2*(a + b*x^2 + c*x^4)),x]
[Out] Integrate[(d + e*x^2)^q/(x^2*(a + b*x^2 + c*x^4)), x]
```

Maple [F]

$$\int \frac{(e\,x^2+d)^q}{x^2\,(c\,x^4+b\,x^2+a)}dx$$

```
[In] int((e*x^2+d)^q/x^2/(c*x^4+b*x^2+a),x)
[Out] int((e*x^2+d)^q/x^2/(c*x^4+b*x^2+a),x)
```

Fricas [F]

$$\int \frac{(d+ex^2)^q}{x^2\left(a+bx^2+cx^4\right)}\,dx=\int \frac{(ex^2+d)^q}{(cx^4+bx^2+a)x^2}\,dx$$

```
[In] integrate((e*x^2+d)^q/x^2/(c*x^4+b*x^2+a),x, algorithm="fricas")
[Out] integral((e*x^2 + d)^q/(c*x^6 + b*x^4 + a*x^2), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(d+ex^2)^q}{x^2\left(a+bx^2+cx^4\right)}\,dx=\text{Timed out}$$

```
[In] integrate((e*x**2+d)**q/x**2/(c*x**4+b*x**2+a),x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{(d+ex^2)^q}{x^2\left(a+bx^2+cx^4\right)}\,dx=\int \frac{(ex^2+d)^q}{(cx^4+bx^2+a)x^2}\,dx$$

```
[In] integrate((e*x^2+d)^q/x^2/(c*x^4+b*x^2+a),x, algorithm="maxima")
[Out] integrate((e*x^2 + d)^q/((c*x^4 + b*x^2 + a)*x^2), x)
```

Giac [F]

$$\int \frac{(d+ex^2)^q}{x^2\left(a+bx^2+cx^4\right)}\,dx=\int \frac{(ex^2+d)^q}{(cx^4+bx^2+a)x^2}\,dx$$

```
[In] integrate((e*x^2+d)^q/x^2/(c*x^4+b*x^2+a),x, algorithm="giac")
[Out] integrate((e*x^2 + d)^q/((c*x^4 + b*x^2 + a)*x^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex^2)^q}{x^2(a+bx^2+cx^4)}\,dx=\int \frac{(e\,x^2+d)^q}{x^2\,\left(c\,x^4+b\,x^2+a\right)}\,dx$$

```
[In] int((d + e*x^2)^q/(x^2*(a + b*x^2 + c*x^4)),x)
[Out] int((d + e*x^2)^q/(x^2*(a + b*x^2 + c*x^4)), x)
```

3.412 $\int \frac{(d+ex^2)^q}{x^4(a+bx^2+cx^4)}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 328

$$\begin{aligned}
&\int \frac{(d+ex^2)^q}{x^4(a+bx^2+cx^4)}\,dx \\
&= \frac{c\left(b+\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)x(d+ex^2)^q\left(1+\frac{ex^2}{d}\right)^{-q}\operatorname{AppellF1}\left(\frac{1}{2},1,-q,\frac{3}{2},-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{ex^2}{d}\right)}{a^2\left(b-\sqrt{b^2-4ac}\right)} \\
&\quad+\frac{c\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)x(d+ex^2)^q\left(1+\frac{ex^2}{d}\right)^{-q}\operatorname{AppellF1}\left(\frac{1}{2},1,-q,\frac{3}{2},-\frac{2cx^2}{b+\sqrt{b^2-4ac}},-\frac{ex^2}{d}\right)}{a^2\left(b+\sqrt{b^2-4ac}\right)} \\
&\quad-\frac{(d+ex^2)^q\left(1+\frac{ex^2}{d}\right)^{-q}\operatorname{Hypergeometric2F1}\left(-\frac{3}{2},-q,-\frac{1}{2},-\frac{ex^2}{d}\right)}{3ax^3} \\
&\quad+\frac{b(d+ex^2)^q\left(1+\frac{ex^2}{d}\right)^{-q}\operatorname{Hypergeometric2F1}\left(-\frac{1}{2},-q,\frac{1}{2},-\frac{ex^2}{d}\right)}{a^2x}
\end{aligned}$$

```
[Out] -1/3*(e*x^2+d)^q*hypergeom([-3/2, -q],[-1/2],-e*x^2/d)/a/x^3/((1+e*x^2/d)^q
)+b*(e*x^2+d)^q*hypergeom([-1/2, -q],[1/2],-e*x^2/d)/a^2/x/((1+e*x^2/d)^q)+
c*x*(e*x^2+d)^q*AppellF1(1/2,1,-q,3/2,-2*c*x^2/(b-(-4*a*c+b^2)^(1/2)),-e*x^
2/d)*(b+(-2*a*c+b^2)/(-4*a*c+b^2)^(1/2))/a^2/((1+e*x^2/d)^q)/(b-(-4*a*c+b^2
)^(1/2))+c*x*(e*x^2+d)^q*AppellF1(1/2,1,-q,3/2,-2*c*x^2/(b+(-4*a*c+b^2)^(1/
2)),-e*x^2/d)*(b+(2*a*c-b^2)/(-4*a*c+b^2)^(1/2))/a^2/((1+e*x^2/d)^q)/(b+(-4
*a*c+b^2)^(1/2))
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 328, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {1317, 372, 371, 1706, 441, 440}

$$\int \frac{(d+ex^2)^q}{x^4(a+bx^2+cx^4)}\,dx = \frac{cx\left(\frac{b^2-2ac}{\sqrt{b^2-4ac}}+b\right)(d+ex^2)^q\left(\frac{ex^2}{d}+1\right)^{-q}\text{AppellF1}\left(\frac{1}{2},1,-q,\frac{3}{2},-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{ex^2}{d}\right)}{a^2\left(b-\sqrt{b^2-4ac}\right)} + \frac{cx\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)(d+ex^2)^q\left(\frac{ex^2}{d}+1\right)^{-q}\text{AppellF1}\left(\frac{1}{2},1,-q,\frac{3}{2},-\frac{2cx^2}{b+\sqrt{b^2-4ac}},-\frac{ex^2}{d}\right)}{a^2\left(\sqrt{b^2-4ac}+b\right)} + \frac{b(d+ex^2)^q\left(\frac{ex^2}{d}+1\right)^{-q}\text{Hypergeometric2F1}\left(-\frac{1}{2},-q,\frac{1}{2},-\frac{ex^2}{d}\right)}{a^2x} - \frac{(d+ex^2)^q\left(\frac{ex^2}{d}+1\right)^{-q}\text{Hypergeometric2F1}\left(-\frac{3}{2},-q,-\frac{1}{2},-\frac{ex^2}{d}\right)}{3ax^3}$$

```
[In] Int[(d + e*x^2)^q/(x^4*(a + b*x^2 + c*x^4)),x]
```

```
[Out] (c*(b + (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*x*(d + e*x^2)^q*AppellF1[1/2, 1, -
q, 3/2, (-2*c*x^2)/(b - Sqrt[b^2 - 4*a*c]), -((e*x^2)/d)])/(a^2*(b - Sqrt[b
^2 - 4*a*c])*(1 + (e*x^2)/d)^q) + (c*(b - (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*
x*(d + e*x^2)^q*AppellF1[1/2, 1, -q, 3/2, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]
), -((e*x^2)/d)])/(a^2*(b + Sqrt[b^2 - 4*a*c])*(1 + (e*x^2)/d)^q) - ((d + e
*x^2)^q*Hypergeometric2F1[-3/2, -q, -1/2, -((e*x^2)/d)])/(3*a*x^3*(1 + (e*x
^2)/d)^q) + (b*(d + e*x^2)^q*Hypergeometric2F1[-1/2, -q, 1/2, -((e*x^2)/d)]
)/(a^2*x*(1 + (e*x^2)/d)^q)
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 372

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(c*x)^
m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0]
 && !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 440

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 441

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]),
 Int[(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 1317

```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_))/((a_) + (b_.)*(x_)^2 +
(c_.)*(x_)^4), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^q, (f*x)^m/(a +
 b*x^2 + c*x^4), x], x] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ[b^2 - 4*a
*c, 0] &&  !IntegerQ[q] && IntegerQ[m]
```

Rule 1706

```
Int[(Px_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^
(p_.), x_Symbol] :> Int[ExpandIntegrand[Px*(d + e*x^2)^q*(a + b*x^2 + c*x^4
)^p, x], x] /; FreeQ[{a, b, c, d, e, q}, x] && PolyQ[Px, x^2] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(\frac{(d+ex^2)^q}{ax^4} - \frac{b(d+ex^2)^q}{a^2x^2} + \frac{(b^2-ac+bcx^2)\,(d+ex^2)^q}{a^2\,(a+bx^2+cx^4)}\right) dx \\
&= \frac{\int \frac{(b^2-ac+bcx^2)(d+ex^2)^q}{a+bx^2+cx^4}\,dx}{a^2} + \frac{\int \frac{(d+ex^2)^q}{x^4}\,dx}{a} - \frac{b\int \frac{(d+ex^2)^q}{x^2}\,dx}{a^2} \\
&= \frac{\int \left(\frac{\left(bc+\frac{c(b^2-2ac)}{\sqrt{b^2-4ac}}\right)(d+ex^2)^q}{b-\sqrt{b^2-4ac}+2cx^2} + \frac{\left(bc-\frac{c(b^2-2ac)}{\sqrt{b^2-4ac}}\right)(d+ex^2)^q}{b+\sqrt{b^2-4ac}+2cx^2}\right) dx}{a^2} \\
&\quad + \frac{\left((d+ex^2)^q\left(1+\frac{ex^2}{d}\right)^{-q}\right)\int \frac{\left(1+\frac{ex^2}{d}\right)^q}{x^4}\,dx}{a} \\
&\quad - \frac{\left(b(d+ex^2)^q\left(1+\frac{ex^2}{d}\right)^{-q}\right)\int \frac{\left(1+\frac{ex^2}{d}\right)^q}{x^2}\,dx}{a^2}
\end{aligned}$$

$$= -\frac{(d+ex^2)^q \left(1+\frac{ex^2}{d}\right)^{-q} {}_2F_1\left(-\frac{3}{2},-q;-\frac{1}{2};-\frac{ex^2}{d}\right)}{3ax^3} + \frac{b(d+ex^2)^q \left(1+\frac{ex^2}{d}\right)^{-q} {}_2F_1\left(-\frac{1}{2},-q;\frac{1}{2};-\frac{ex^2}{d}\right)}{a^2x} + \frac{\left(c\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\right)\int \frac{(d+ex^2)^q}{b+\sqrt{b^2-4ac}+2cx^2}\,dx}{a^2} + \frac{\left(c\left(b+\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\right)\int \frac{(d+ex^2)^q}{b-\sqrt{b^2-4ac}+2cx^2}\,dx}{a^2}$$

$$= -\frac{(d+ex^2)^q \left(1+\frac{ex^2}{d}\right)^{-q} {}_2F_1\left(-\frac{3}{2},-q;-\frac{1}{2};-\frac{ex^2}{d}\right)}{3ax^3} + \frac{b(d+ex^2)^q \left(1+\frac{ex^2}{d}\right)^{-q} {}_2F_1\left(-\frac{1}{2},-q;\frac{1}{2};-\frac{ex^2}{d}\right)}{a^2x} + \frac{\left(c\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)(d+ex^2)^q\left(1+\frac{ex^2}{d}\right)^{-q}\right)\int \frac{\left(1+\frac{ex^2}{d}\right)^q}{b+\sqrt{b^2-4ac}+2cx^2}\,dx}{a^2} + \frac{\left(c\left(b+\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)(d+ex^2)^q\left(1+\frac{ex^2}{d}\right)^{-q}\right)\int \frac{\left(1+\frac{ex^2}{d}\right)^q}{b-\sqrt{b^2-4ac}+2cx^2}\,dx}{a^2}$$

$$= \frac{c\left(b+\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)x(d+ex^2)^q\left(1+\frac{ex^2}{d}\right)^{-q} F_1\left(\frac{1}{2};1,-q;\frac{3}{2};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{ex^2}{d}\right)}{a^2\left(b-\sqrt{b^2-4ac}\right)} + \frac{c\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)x(d+ex^2)^q\left(1+\frac{ex^2}{d}\right)^{-q} F_1\left(\frac{1}{2};1,-q;\frac{3}{2};-\frac{2cx^2}{b+\sqrt{b^2-4ac}},-\frac{ex^2}{d}\right)}{a^2\left(b+\sqrt{b^2-4ac}\right)} - \frac{(d+ex^2)^q \left(1+\frac{ex^2}{d}\right)^{-q} {}_2F_1\left(-\frac{3}{2},-q;-\frac{1}{2};-\frac{ex^2}{d}\right)}{3ax^3} + \frac{b(d+ex^2)^q \left(1+\frac{ex^2}{d}\right)^{-q} {}_2F_1\left(-\frac{1}{2},-q;\frac{1}{2};-\frac{ex^2}{d}\right)}{a^2x}$$

Mathematica [F]

$$\int \frac{(d+ex^2)^q}{x^4(a+bx^2+cx^4)}\,dx = \int \frac{(d+ex^2)^q}{x^4(a+bx^2+cx^4)}\,dx$$

```
[In] Integrate[(d + e*x^2)^q/(x^4*(a + b*x^2 + c*x^4)),x]
[Out] Integrate[(d + e*x^2)^q/(x^4*(a + b*x^2 + c*x^4)), x]
```

Maple [F]

$$\int \frac{(e\,x^2+d)^q}{x^4\,(c\,x^4+b\,x^2+a)}dx$$

```
[In] int((e*x^2+d)^q/x^4/(c*x^4+b*x^2+a),x)
[Out] int((e*x^2+d)^q/x^4/(c*x^4+b*x^2+a),x)
```

Fricas [F]

$$\int \frac{(d+ex^2)^q}{x^4\,(a+bx^2+cx^4)}\,dx = \int \frac{(ex^2+d)^q}{(cx^4+bx^2+a)x^4}\,dx$$

```
[In] integrate((e*x^2+d)^q/x^4/(c*x^4+b*x^2+a),x, algorithm="fricas")
[Out] integral((e*x^2 + d)^q/(c*x^8 + b*x^6 + a*x^4), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(d+ex^2)^q}{x^4\,(a+bx^2+cx^4)}\,dx = \text{Timed out}$$

```
[In] integrate((e*x**2+d)**q/x**4/(c*x**4+b*x**2+a),x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{(d+ex^2)^q}{x^4\,(a+bx^2+cx^4)}\,dx = \int \frac{(ex^2+d)^q}{(cx^4+bx^2+a)x^4}\,dx$$

```
[In] integrate((e*x^2+d)^q/x^4/(c*x^4+b*x^2+a),x, algorithm="maxima")
[Out] integrate((e*x^2 + d)^q/((c*x^4 + b*x^2 + a)*x^4), x)
```

Giac [F]

$$\int \frac{(d + ex^2)^q}{x^4 (a + bx^2 + cx^4)} \, dx = \int \frac{(ex^2 + d)^q}{(cx^4 + bx^2 + a)x^4} \, dx$$

```
[In] integrate((e*x^2+d)^q/x^4/(c*x^4+b*x^2+a),x, algorithm="giac")
[Out] integrate((e*x^2 + d)^q/((c*x^4 + b*x^2 + a)*x^4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d + ex^2)^q}{x^4 (a + bx^2 + cx^4)} \, dx = \int \frac{(e\,x^2 + d)^q}{x^4\,(c\,x^4 + b\,x^2 + a)} \, dx$$

```
[In] int((d + e*x^2)^q/(x^4*(a + b*x^2 + c*x^4)),x)
[Out] int((d + e*x^2)^q/(x^4*(a + b*x^2 + c*x^4)), x)
```

3.413 $\int \frac{\sqrt{1+\frac{1}{c^2x^2}}}{\sqrt{1-c^4x^4}}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 40

$$\int \frac{\sqrt{1+\frac{1}{c^2x^2}}}{\sqrt{1-c^4x^4}}\,dx = -\frac{\operatorname{arctanh}\left(\frac{\sqrt{1-c^4x^4}}{c\sqrt{1+\frac{1}{c^2x^2}}\,x}\right)}{c}$$

```
[Out] -arctanh((-c^4*x^4+1)^(1/2)/c/x/(1+1/c^2/x^2)^(1/2))/c
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.10, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.179, Rules used = {1462, 1266, 862, 65, 214}

$$\int \frac{\sqrt{1+\frac{1}{c^2x^2}}}{\sqrt{1-c^4x^4}}\,dx = -\frac{x\sqrt{\frac{1}{c^2x^2}+1}\operatorname{arctanh}\left(\sqrt{1-c^2x^2}\right)}{\sqrt{c^2x^2+1}}$$

```
[In] Int[Sqrt[1 + 1/(c^2*x^2)]/Sqrt[1 - c^4*x^4],x]
```

```
[Out] -((Sqrt[1 + 1/(c^2*x^2)]*x*ArcTanh[Sqrt[1 - c^2*x^2]])/Sqrt[1 + c^2*x^2])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 862

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_.), x_Symbol] :> Int[(d + e*x)^(m + p)*(f + g*x)^n*(a/d + (c/e)*x)^p,
x] /; FreeQ[{a, c, d, e, f, g, m, n}, x] && NeQ[e*f - d*g, 0] && EqQ[c*d^2
+ a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && EqQ[m + p, 0]))
```

Rule 1266

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_S
ymbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + c*x^2)^p, x],
x, x^2], x] /; FreeQ[{a, c, d, e, p, q}, x] && IntegerQ[(m + 1)/2]
```

Rule 1462

```
Int[((d_) + (e_.)*(x_)^(mn_.))^(q_)*((a_) + (c_.)*(x_)^(n2_.))^(p_.), x_Sym
bol] :> Dist[(e^IntPart[q]*((d + e*x^mn)^FracPart[q]/(1 + d*(1/(x^mn*e)))^F
racPart[q]))/x^(mn*FracPart[q]), Int[x^(mn*q)*(1 + d*(1/(x^mn*e)))^q*(a + c
*x^n2)^p, x], x] /; FreeQ[{a, c, d, e, mn, p, q}, x] && EqQ[n2, -2*mn] &&
!IntegerQ[p] &&  !IntegerQ[q] && PosQ[n2]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\left(\sqrt{1+\frac{1}{c^2x^2}}x\right)\int \frac{\sqrt{1+c^2x^2}}{x\sqrt{1-c^4x^4}}\,dx}{\sqrt{1+c^2x^2}}\\
&= \frac{\left(\sqrt{1+\frac{1}{c^2x^2}}x\right)\text{Subst}\left(\int \frac{\sqrt{1+c^2x}}{x\sqrt{1-c^4x^2}}\,dx,x,x^2\right)}{2\sqrt{1+c^2x^2}}\\
&= \frac{\left(\sqrt{1+\frac{1}{c^2x^2}}x\right)\text{Subst}\left(\int \frac{1}{x\sqrt{1-c^2x}}\,dx,x,x^2\right)}{2\sqrt{1+c^2x^2}}\\
&= -\frac{\left(\sqrt{1+\frac{1}{c^2x^2}}x\right)\text{Subst}\left(\int \frac{1}{\frac{1}{c^2}-\frac{x^2}{c^2}}\,dx,x,\sqrt{1-c^2x^2}\right)}{c^2\sqrt{1+c^2x^2}}\\
&= -\frac{\sqrt{1+\frac{1}{c^2x^2}}x\tanh^{-1}\left(\sqrt{1-c^2x^2}\right)}{\sqrt{1+c^2x^2}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.87 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.45

$$\int \frac{\sqrt{1+\frac{1}{c^2x^2}}}{\sqrt{1-c^4x^4}}\,dx = -\frac{\sqrt{1+\frac{1}{c^2x^2}}\,x\operatorname{arctanh}\left(\frac{\sqrt{1-c^4x^4}}{\sqrt{1+c^2x^2}}\right)}{\sqrt{1+c^2x^2}}$$

```
[In] Integrate[Sqrt[1 + 1/(c^2*x^2)]/Sqrt[1 - c^4*x^4],x]
```

```
[Out] -((Sqrt[1 + 1/(c^2*x^2)]*x*ArcTanh[Sqrt[1 - c^4*x^4]/Sqrt[1 + c^2*x^2]])/Sq
rt[1 + c^2*x^2])
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.38 (sec) , antiderivative size = 101, normalized size of antiderivative = 2.52

method	result	size
default	$-\frac{\sqrt{\frac{c^2x^2+1}{c^2x^2}}\,x\sqrt{-c^4x^4+1}\,\operatorname{csgn}\left(\frac{1}{c}\right)\ln\left(\frac{2\operatorname{csgn}\left(\frac{1}{c}\right)c\sqrt{-\frac{c^2x^2-1}{c^2}}+2}{c^2x}\right)}{\left(c^2x^2+1\right)\sqrt{-\frac{c^2x^2-1}{c^2}}\,c}$	101

```
[In] int((1+1/c^2/x^2)^(1/2)/(-c^4*x^4+1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -((c^2*x^2+1)/c^2/x^2)^(1/2)*x*(-c^4*x^4+1)^(1/2)*csgn(1/c)*ln(2*(csgn(1/c)
*c*(-1/c^2*(c^2*x^2-1))^(1/2)+1)/c^2/x)/(c^2*x^2+1)/(-1/c^2*(c^2*x^2-1))^(1
/2)/c
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 120 vs. 2(36) = 72.

Time = 0.25 (sec) , antiderivative size = 120, normalized size of antiderivative = 3.00

$$\int \frac{\sqrt{1+\frac{1}{c^2x^2}}}{\sqrt{1-c^4x^4}}\,dx = -\frac{\log\left(\frac{c^2x^2+\sqrt{-c^4x^4+1}cx\sqrt{\frac{c^2x^2+1}{c^2x^2}}+1}{c^2x^2+1}\right)-\log\left(-\frac{c^2x^2-\sqrt{-c^4x^4+1}cx\sqrt{\frac{c^2x^2+1}{c^2x^2}}+1}{c^2x^2+1}\right)}{2\,c}$$

```
[In] integrate((1+1/c^2/x^2)^(1/2)/(-c^4*x^4+1)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/2*(log((c^2*x^2 + sqrt(-c^4*x^4 + 1)*c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) +
 1)/(c^2*x^2 + 1)) - log(-(c^2*x^2 - sqrt(-c^4*x^4 + 1)*c*x*sqrt((c^2*x^2 +
 1)/(c^2*x^2)) + 1)/(c^2*x^2 + 1)))/c
```

Sympy [F]

$$\int \frac{\sqrt{1+\frac{1}{c^2x^2}}}{\sqrt{1-c^4x^4}}\,dx = \int \frac{\sqrt{1+\frac{1}{c^2x^2}}}{\sqrt{-\left(cx-1\right)\left(cx+1\right)\left(c^2x^2+1\right)}}\,dx$$

```
[In] integrate((1+1/c**2/x**2)**(1/2)/(-c**4*x**4+1)**(1/2),x)
[Out] Integral(sqrt(1 + 1/(c**2*x**2))/sqrt(-(c*x - 1)*(c*x + 1)*(c**2*x**2 + 1))
, x)
```

Maxima [F]

$$\int \frac{\sqrt{1+\frac{1}{c^2x^2}}}{\sqrt{1-c^4x^4}}\,dx = \int \frac{\sqrt{\frac{1}{c^2x^2}+1}}{\sqrt{-c^4x^4+1}}\,dx$$

```
[In] integrate((1+1/c^2/x^2)^(1/2)/(-c^4*x^4+1)^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(1/(c^2*x^2) + 1)/sqrt(-c^4*x^4 + 1), x)
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.05

$$\int \frac{\sqrt{1+\frac{1}{c^2x^2}}}{\sqrt{1-c^4x^4}}\,dx = -\frac{\left(\log\left(\sqrt{-c^2x^2+1}+1\right)-\log\left(-\sqrt{-c^2x^2+1}+1\right)\right)|c|}{2\,c^2}$$

```
[In] integrate((1+1/c^2/x^2)^(1/2)/(-c^4*x^4+1)^(1/2),x, algorithm="giac")
[Out] -1/2*(log(sqrt(-c^2*x^2 + 1) + 1) - log(-sqrt(-c^2*x^2 + 1) + 1))*abs(c)/c^
2
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{1+\frac{1}{c^2x^2}}}{\sqrt{1-c^4x^4}}\,dx = \int \frac{\sqrt{\frac{1}{c^2\,x^2}+1}}{\sqrt{1-c^4\,x^4}}\,dx$$

```
[In] int((1/(c^2*x^2) + 1)^(1/2)/(1 - c^4*x^4)^(1/2),x)
[Out] int((1/(c^2*x^2) + 1)^(1/2)/(1 - c^4*x^4)^(1/2), x)
```

CHAPTER 4

APPENDIX

4.1 Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* Nasser: April 7,2022. add second output which gives reason for the grade *)
(*                       Small rewrite of logic in main function to make it*)
(*                       match Maple's logic. No change in functionality otherwise*)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)


(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
```

```
(*          GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)


GradeAntiderivative[result_,optimal_] := Module[{expnResult,expnOptimal,leafCountResult,leafCou
    expnResult  = ExpnType[result];
    expnOptimal = ExpnType[optimal];
    leafCountResult = LeafCount[result];
    leafCountOptimal = LeafCount[optimal];

    (*Print["expnResult=",expnResult," expnOptimal=",expnOptimal];*)
    If[expnResult<=expnOptimal,
        If[Not[FreeQ[result,Complex]], (*result contains complex*)
            If[Not[FreeQ[optimal,Complex]], (*optimal contains complex*)
                If[leafCountResult<=2*leafCountOptimal,
                    finalresult={"A",""}
                    ,(*ELSE*)
                    finalresult={"B","Both result and optimal contain complex but leaf count is
                  ]
            ,(*ELSE*)
                finalresult={"C","Result contains complex when optimal does not."}
            ]
        ,(*ELSE*)(*result does not contains complex*)
            If[leafCountResult<=2*leafCountOptimal,
               finalresult={"A",""}
            ,(*ELSE*)
               finalresult={"B","Leaf count is larger than twice the leaf count of optimal. $"<
              ]
        ]
    ,(*ELSE*) (*expnResult>expnOptimal*)
        If[FreeQ[result,Integrate] && FreeQ[result,Int],
            finalresult={"C","Result contains higher order function than in optimal. Order "<>T
            ,
            finalresult={"F","Contains unresolved integral."}
        ]
    ];

    finalresult
]

(* ::Text:: *)
(*The following summarizes the type number assigned an *)
```

```
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
  Exp,Log,
  Sin,Cos,Tan,Cot,Sec,Csc,
  ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
```

```
  Sinh,Cosh,Tanh,Coth,Sech,Csch,
  ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
  Erf, Erfc, Erfi,
  FresnelS, FresnelC,
  ExpIntegralE, ExpIntegralEi, LogIntegral,
  SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
  Gamma, LogGamma, PolyGamma,
  Zeta, PolyLog, ProductLog,
  EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]


AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems
#Nasser 4/07/2022   add second output which gives reason for the grade

GradeAntiderivative := proc(result,optimal)
local leaf_count_result,
        leaf_count_optimal,
        ExpnType_result,
        ExpnType_optimal,
        debug:=false;

        leaf_count_result:=leafcount(result);
```

```
        #do NOT call ExpnType() if leaf size is too large. Recursion problem
        if leaf_count_result > 500000 then
            return "B","result has leaf size over 500,000. Avoiding possible recursion issues
        fi;

        leaf_count_optimal := leafcount(optimal);
        ExpnType_result  := ExpnType(result);
        ExpnType_optimal := ExpnType(optimal);

        if debug then
            print("ExpnType_result",ExpnType_result," ExpnType_optimal=",ExpnType_optimal);
        fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
#       antiderivative
#   "A" if result can be considered optimal

    #This check below actually is not needed, since I only
    #call this grading only for passed integrals. i.e. I check
    #for "F" before calling this. But no harm of keeping it here.
    #just in case.


    if not type(result,freeof('int')) then
        return "F","Result contains unresolved integral";
    fi;


    if ExpnType_result<=ExpnType_optimal then
        if debug then
             print("ExpnType_result<=ExpnType_optimal");
        fi;
        if is_contains_complex(result) then
            if is_contains_complex(optimal) then
                if debug then
                        print("both result and optimal complex");
                fi;
                if leaf_count_result<=2*leaf_count_optimal then
                    return "A"," ";
                else
                   return "B",cat("Both result and optimal contain complex but leaf count of
                                convert(leaf_count_result,string)," vs. $2 (",
```

```
                                    convert(leaf_count_optimal,string)," ) = ",convert(2*leaf_co
                end if
            else #result contains complex but optimal is not
                if debug then
                        print("result contains complex but optimal is not");
                fi;
                return "C","Result contains complex when optimal does not.";
            fi;
        else # result do not contain complex
             # this assumes optimal do not as well. No check is needed here.
            if debug then
                   print("result do not contain complex, this assumes optimal do not as well")
            fi;
            if leaf_count_result<=2*leaf_count_optimal then
                if debug then
                    print("leaf_count_result<=2*leaf_count_optimal");
                fi;
                return "A"," ";
            else
                if debug then
                    print("leaf_count_result>2*leaf_count_optimal");
                fi;
                return "B",cat("Leaf count of result is larger than twice the leaf count of opt
                                    convert(leaf_count_result,string),"$ vs. $2(",
                                    convert(leaf_count_optimal,string),")=",convert(2*leaf_count
            fi;
        fi;
    else #ExpnType(result) > ExpnType(optimal)
        if debug then
            print("ExpnType(result) > ExpnType(optimal)");
        fi;
        return "C",cat("Result contains higher order function than in optimal. Order ",
                        convert(ExpnType_result,string)," vs. order ",
                        convert(ExpnType_optimal,string),".");
    fi;

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:
```

```
# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
```

```
  9
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#              Port of original Maple grading function by
#              Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#              added 'exp_polar'
from sympy import *

def leaf_count(expr):
    #sympy do not have leaf count function. This is approximation
    return round(1.7*count_ops(expr))

def is_sqrt(expr):
    if isinstance(expr,Pow):
        if expr.args[1] == Rational(1,2):
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
            asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
            asinh,acosh,atanh,acoth,asech,acsch
        ]

def is_special_function(func):
    return func in [ erf,erfc,erfi,
             fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
             gamma,loggamma,digamma,zeta,polylog,LambertW,
             elliptic_f,elliptic_e,elliptic_pi,exp_polar
         ]

def is_hypergeometric_function(func):
    return func in [hyper]

def is_appell_function(func):
    return func in [appellf1]

def is_atom(expn):
    try:
        if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
          return True
        else:
          return False
```

```
    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
       return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow):   #type(expn,'`^`')
        if isinstance(expn.args[1],Integer):  #type(op(2,expn),'integer')
            return expnType(expn.args[0])   #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational):  #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
        else:
            return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,ExpnType(op(1,expn)),ExpnTy
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type(expn,'`*`')
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):  #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))  #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
```

```
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    #print ("Enter grade_antiderivative for sagemath")
    #print("Enter grade_antiderivative, result=",result," optimal=",optimal)

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    #print("leaf_count_result=",leaf_count_result)
    #print("leaf_count_optimal=",leaf_count_optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        grade = "F"
        grade_annotation =""
    else:
        if expnType_result <= expnType_optimal:
            if result.has(I):
                if optimal.has(I): #both result and optimal complex
                    if leaf_count_result <= 2*leaf_count_optimal:
                        grade = "A"
                        grade_annotation =""
                    else:
                        grade = "B"
                        grade_annotation ="Both result and optimal contain complex but leaf count of result is large
                else: #result contains complex but optimal is not
                    grade = "C"
                    grade_annotation ="Result contains complex when optimal does not."
            else: # result do not contain complex, this assumes optimal do not as well
                if leaf_count_result <= 2*leaf_count_optimal:
                    grade = "A"
                    grade_annotation =""
                else:
                    grade = "B"
                    grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(le
        else:
            grade = "C"
            grade_annotation ="Result contains higher order function than in optimal. Order "+str(ExpnType_
```

```
    #print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

    return grade, grade_annotation
```

SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#               Albert Rich to use with Sagemath. This is used to
#               grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#               'arctan2','floor','abs','log_integral'
#June 4, 2022  Made default grade_annotation "none" instead of "" due
#               issue later when reading the file.
#July 14, 2022. Added ellipticF. This is until they fix sagemath, then remove it.

from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    #print("Enter tree_size, expr is ",expr)

    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
            return False
    else:
        return False
```

```
def is_elementary_function(func):
    #debug=False
    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    #debug=False
    if debug:
        print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
          'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','sinh_integral'
          'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
          'polylog','lambert_w','elliptic_f','elliptic_e','ellipticF',
          'elliptic_pi','exp_integral_e','log_integral']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m


def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U']

def is_appell_function(func):
    return func.name() in ['hypergeometric']   #[appellf1] can't find this in sagemath
```

```
def is_atom(expn):

   #debug=False
   if debug:
       print ("Enter is_atom, expn=",expn)

   if not hasattr(expn, 'parent'):
      return False


   #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic-t
   try:
      if expn.parent() is SR:
         return expn.operator() is None
      if expn.parent() in (ZZ, QQ, AA, QQbar):
         return expn in expn.parent() # Should always return True
      if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
         return expn in expn.parent().base_ring() or expn in expn.parent().gens()

      return False

   except AttributeError as error:
      print("Exception,AttributeError in  is_atom")
      print ("cought exception" , type(error).__name__ )
      return False


def expnType(expn):

   if debug:
      print (">>>>>Enter expnType, expn=", expn)
      print (">>>>>is_atom(expn)=", is_atom(expn))

   if is_atom(expn):
      return 1
   elif type(expn)==list:   #isinstance(expn,list):
      return max(map(expnType, expn))    #apply(max,map(ExpnType,expn))
   elif  is_sqrt(expn):
      if  type(expn.operands()[0])==Rational:  #type(isinstance(expn.args[0],Rational):
         return 1
      else:
         return max(2,expnType(expn.operands()[0]))   #max(2,expnType(expn.args[0]))
   elif expn.operator() == operator.pow:    #isinstance(expn,Pow)
      if type(expn.operands()[1])==Integer:   #isinstance(expn.args[1],Integer)
         return expnType(expn.operands()[0])    #expnType(expn.args[0])
      elif type(expn.operands()[1])==Rational:   #isinstance(expn.args[1],Rational)
         if type(expn.operands()[0])==Rational:  #isinstance(expn.args[0],Rational)
```

```
                return 1
            else:
                return max(2,expnType(expn.operands()[0]))   #max(2,expnType(expn.args[0]))
        else:
            return max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1]))  #max(3,expnType(expn.
    elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #isinstance(expn,Add) or isinsta
        m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
        m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.operator()):  #is_elementary_function(expn.func)
        return max(3,expnType(expn.operands()[0]))
    elif is_special_function(expn.operator()): #is_special_function(expn.func)
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,m1)
    elif is_hypergeometric_function(expn.operator()): #is_hypergeometric_function(expn.func)
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,m1)
    elif is_appell_function(expn.operator()):
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(6,m1)
    elif str(expn).find("Integral") != -1: #this will never happen, since it
                #is checked before calling the grading function that is passed.
                #but kept it here.
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):



    if debug:
        print ("Enter grade_antiderivative for sagemath")
        print("Enter grade_antiderivative, result=",result)
        print("Enter grade_antiderivative, optimal=",optimal)
        print("type(anti)=",type(result))
        print("type(optimal)=",type(optimal))

    leaf_count_result  = tree_size(result) #leaf_count(result)
    leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

    #if debug: print ("leaf_count_result=", leaf_count_result, "leaf_count_optimal=",leaf_count_optimal)



    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)
```

```
if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",expnType_optimal)

if expnType_result <= expnType_optimal:
    if result.has(I):
        if optimal.has(I): #both result and optimal complex
            if leaf_count_result <= 2*leaf_count_optimal:
                grade = "A"
                grade_annotation ="none"
            else:
                grade = "B"
                grade_annotation ="Both result and optimal contain complex but leaf count of result is larger th
        else: #result contains complex but optimal is not
            grade = "C"
            grade_annotation ="Result contains complex when optimal does not."
    else: # result do not contain complex, this assumes optimal do not as well
        if leaf_count_result <= 2*leaf_count_optimal:
            grade = "A"
            grade_annotation ="none"
        else:
            grade = "B"
            grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(leaf_c
else:
    grade = "C"
    grade_annotation ="Result contains higher order function than in optimal. Order "+str(expnType_resul


print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

return grade, grade_annotation
```